Title: Dynamics with Vector Condensates at Finite Density in QCD and Beyond

Date: May 22, 2007 04:40 PM

URL: http://pirsa.org/07050054

Abstract:

Dynamics with Vector Condensates at Finite Density in QCD and Beyond

Gluonic phase in neutral two-flavor dense QCD

E. Gorbar, M. Hashimoto, and V. M., Phys. Lett. B 632 (2006) 305; Phys. Rev. Lett. 96 (2006) 022005; Phys. Rev. D 75 (2007) 085012

E. Gorbar, M. Hashimoto, V. M., and I. Shovkovy, Phys. Rev. D 73 (2006) 111502(R)

M. Hashimoto and V. M., hep-ph/0705.2399

Dense quark matter:

$n = \kappa n_0$, $\kappa \simeq 2-5$, $n_0 \simeq 0.17\ \mathrm{fm}^{-3}$

nucleon density

Beyond dense QCD

V. Gusynin, V. M., and I. Shovkovy,
Phys. Lett. B581 (2004) 82;
E. Gorbar, J. Jia, and V. M.,
Phys. Rev. D73 (2006) 045001;
A. Buchel, J. Jia, and V. M.,
Phys. Lett. B 647 (2007) 305;
Nucl. Phys. B 772 (2007) 323

Dynamics in an essentially soluble model with vector condensates of gauge fields

Phases with vector condensates of gauge fields yield an unconventional type of dynamics: in this case, the Higgs mechanism is provided by condensates of gauge (or gauge + scalar) fields.

These dynamics admit a dual, gauge invariant, description as dynamics with a condensation of colorless vector composites (vector hadrons).

The dynamics are very rich

Gluonic Phases in Neutral Two-Flavor Dense QCD

E. Gorbar, M. Hashimoto, V.M,
Phys. Lett. B632 (2006) 305;
Phys. Rev. D75 (2007) 085012
M. Hashimoto and V.M., hep-ph/0705.2329

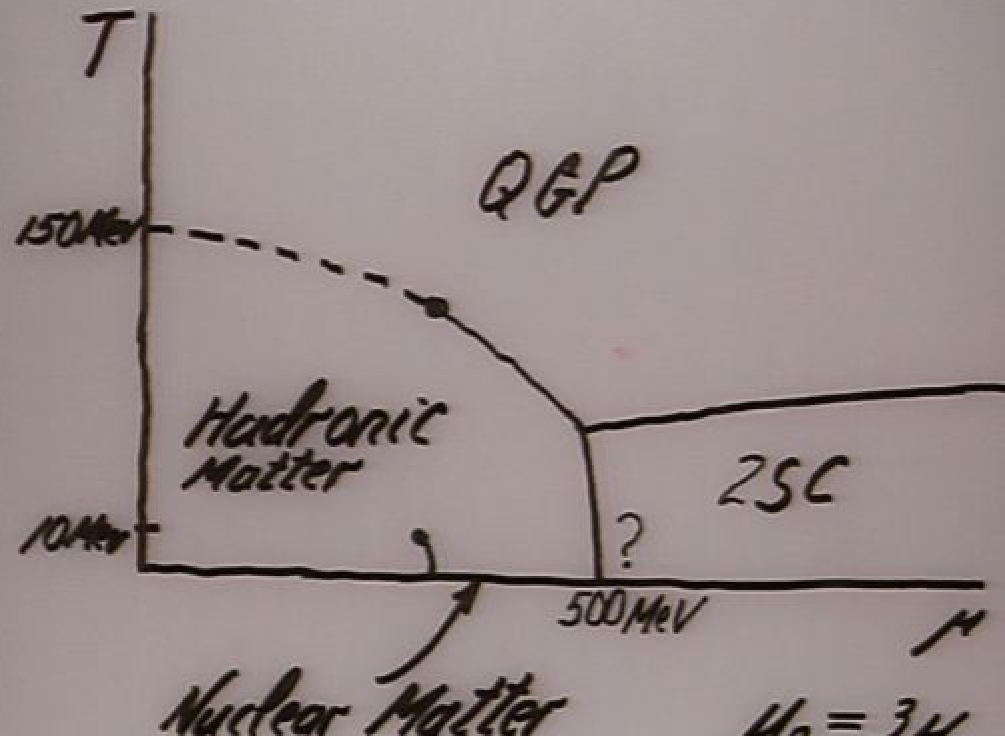
a) Gluonic degrees of freedom play crucial role in dynamics. Vector condensates of gluons.

b) Higgs picture: color $SU(3)_c$ is completely broken.

c) Rotational $SO(3)$ is spontaneously broken.

d) Landscape of vacua.

e) Rich dynamics. Exotic hadron states. Electric superconductivity in some of the vacua.



$\mu_B = 3\mu$

$T_c^{(SC)} \simeq 0.6\, |\Delta_0^-|$

11
3

The attractive bifermion channels:

$$\{3\}\times\{3^*\}\to\{1\}:\ C^{\{1\}}_{\{3\}\{3^*\}}=\frac{4}{3},$$

$$\{3\}\times\{3\}\to\{3^*\}:\ C^{\{3^*\}}_{\{3\}\{3\}}=\frac{2}{3}.$$

The first channel leads to the chiral condensate.

The second channel may lead to the color (diquark) condensate.

Diquark Condensate

Diquark composite field:

$$\Delta^{(a)} \sim i\varepsilon^{ij}\varepsilon^{acd}\,\bar{q}^{C}_{ic}\gamma_5 q_{jd}\,,$$

a, c, d = blue (b), red (r), green (g)

i, j = up (u), down (d)

Diquark gap:

$$\bar{\Delta} \equiv \langle\Delta^a\rangle\big|_{a=b\,(\text{blue})}$$

$$SU(3)_c \longrightarrow SU(2)_c$$

$Q(\Delta^a) = \frac{1}{3}$, $B(\Delta^a) = \frac{2}{3}$,

$I_3(\Delta^a) = 0$

2SC phase

Neutrality Conditions in β-Equilibrium

μ_e – chemical potential of electrons

Diagonal matrix of quark chemical potential:

$$\hat{\mu}_{ij,ab} = (\mu - \mu_e Q_i)\delta_{ij}\delta_{ab} + \frac{2}{\sqrt{3}}\mu_8 \delta_{ij} (T^8)_{ab}$$

$\mu_8 \equiv \frac{\sqrt{3}}{2} g \langle A_0^{(8)} \rangle$ (follows from Gauss's law)

$$\mu_{ur} = \mu_{ug} = \mu - \frac{2}{3}\mu_e + \frac{1}{3}\mu_8$$

$$\mu_{dr} = \mu_{dg} = \mu + \frac{1}{3}\mu_e + \frac{1}{3}\mu_8$$

$$\mu_{ub} = \mu - \frac{2}{3}\mu_e - \frac{2}{3}\mu_8, \quad \mu_{db} = \mu + \frac{1}{3}\mu_e - \frac{2}{3}\mu_8$$

$$n_Q = -\frac{\partial \Omega}{\partial \mu_e} = 0, \quad n_8 = -\frac{\partial \Omega}{\partial \mu_8} = 0$$

$\Omega \equiv V_{eff}$ – free energy density (effective potential)

Chromomagnetic Instability

M. Huang and I. Shovkovy, Phys. Rev. D70 (2004) 051501(R); ibid D70 (2004) 094030.

For $A^{(4)}-A^{(7)}$ gluons, the Meissner mass is

$$m_{M,4}^2 = \frac{g^2\tilde{\mu}^2}{6\pi^2}\left(1-\frac{2\delta\mu^2}{\Delta^2}\right), \quad \delta\mu = \frac{\mu_e}{2},$$

$\tilde{\mu} = \mu - \frac{1}{3}(\delta\mu - \mu_8)$.

For $\delta\mu > \frac{\bar{\Delta}}{\sqrt{2}}$, $m_{M,4}^2$ is negative (chromomagnetic instability).

For $\delta\mu > \bar{\Delta}$, $m_{M,8}^2$ becomes negative ($\delta\mu > \bar{\Delta} \longrightarrow$ g2SC phase).

Collective Excitations, Instabilities, and the Ground State in Dense Quark Matter

E. Gorbar, M. Hashimoto, V.M., and I. Shovkovy,
Phys. Rev. D 73 (2006) 111502 (R)

The basic idea: while for calculating screening Meissner and Debye masses, it is sufficient to study the gluon polarization operator only at one point, $(p_0, \vec{p}) = (0, \vec{p} \to 0)$ (static regime), we studied the spectrum of light ($|M^2| \ll \mu^2$) plasmons.

$$\Pi^{\mu\nu}(p_0, \vec{p}) = \left(g^{\mu\nu} - u^\mu u^\nu + \frac{\vec{p}^\mu \vec{p}^\nu}{p^2}\right) H +$$
$$+ u^\mu u^\nu K - \frac{\vec{p}^\mu \vec{p}^\nu}{p^2} L + \left(u^\mu \frac{\vec{p}^\nu}{p} + u^\nu \frac{\vec{p}^\mu}{p}\right) N,$$

$u^\mu = (1, 0, 0, 0)$, $p = |\vec{p}|$, $\vec{p}^\mu \equiv (0, \vec{p})$.

Dispersion relations for plasmons:

Magnetic modes: $p_0^2 - p^2 + H = 0$

Electric modes:

$$p_0^2 K - p^2 L - 2p_0 p N + KL + N^2 = 0$$

Results

4–7th gluonic channels:

(magnetic) $p_0^2 \simeq M^2 + v^2 p^2$, $v^2 \simeq 0.03$;

$M^2 > 0$ for $\delta\mu < \frac{\bar{\Delta}}{\sqrt{2}}$,

$M^2 < 0$ for $\delta\mu > \frac{\bar{\Delta}}{\sqrt{2}}$

Bose-Einstein (BE) condensation phenomenon

(electric) $p_0^2 \to M^2$ as $p \to 0$, with the same M^2, i.e., the same BE phenomenon.

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It is the origin of the instability in 2SC phase.

Gauged NJL Model

$$\mathcal{L} = \bar{q}(i\not{D} + \hat{\mu}\gamma^0)q + G_\Delta[(\bar{q}i\varepsilon\epsilon^a\gamma_5 q^c)\cdot(\bar{q}^c i\varepsilon\epsilon^a\gamma_5 q)] + \mathcal{L}_g + \bar{e}(i\not{\partial} + \mu_e\gamma^0)e,$$

$\mathcal{L}_g = -\frac{1}{4}F^\alpha_{\mu\nu}F^{\alpha\mu\nu}$, $D_\mu = \partial_\mu - igA^\alpha_\mu T^\alpha$; $\alpha = 1,2,\ldots 8$

$\varepsilon \equiv \varepsilon^{ij}$, $i,j = u, d$ (up, down)

$\epsilon^a \equiv \epsilon^{acd}$, $a,c,d = r,g,b$ (red, green, blue)

$\hat{\mu}$ is a diagonal chemical potential matrix:

$\mu_{ur} = \mu_{ug} = \tilde{\mu} - \delta\mu$, $\mu_{dr} = \mu_{dg} = \tilde{\mu} + \delta\mu$

$\mu_{ub} = \tilde{\mu} - \delta\mu - \mu_8$, $\mu_{db} = \tilde{\mu} + \delta\mu - \mu_8$

$$\tilde{\mu} = \mu - \frac{\delta\mu}{3} + \frac{\mu_8}{3}, \quad \delta\mu \equiv \frac{\mu_e}{2}.$$

$\mu_8 = \frac{g\sqrt{3}}{2}\langle A_0^8\rangle$ is determined from Gauss's law.

Introduce auxiliary diquark field

$$\Delta^{(a)} \sim i\bar{q}^C \varepsilon \epsilon^a \gamma_5 q$$

$$\mathcal{L} = \bar{q}(i\not{D} + \hat{\mu}\gamma^0)q - \frac{1}{2}\Delta^{(a)}[i\bar{q}\varepsilon\epsilon^a\gamma_5 q^C] - $$
$$- \frac{1}{2}[i\bar{q}^C\varepsilon\epsilon^a\gamma_5 q]\Delta^{*(a)} - \frac{|\Delta^{(a)}|^2}{4G_\Delta} + \mathcal{L}_g$$

The gauge: $[\Delta^{(a)}]^T = (0, 0, \Delta^b \equiv \Delta)$.

Nambu-Gorkov spinor: $\begin{pmatrix} q \\ q^C \end{pmatrix} \equiv \psi$.

Inverse propagator S_g^{-1} of ψ:

$$S_g^{-1} = \begin{pmatrix} [G_{0,g}^+]^{-1} & \Delta \\ \Delta^+ & [G_{0,g}^-]^{-1} \end{pmatrix},$$

$$[G_{0,g}^+]^{-1} = (p^0 + \tilde{\mu} - \delta\mu\tau^3 - \mu_8 I_b)\gamma^0 - \vec{\gamma}\vec{p} + g\not{A}^\alpha T^\alpha$$
$$[G_{0,g}^-]^{-1} = (p^0 - \tilde{\mu} + \delta\mu\tau^3 + \mu_8 I_b)\gamma^0 - \vec{\gamma}\vec{p} - g\not{A}^\alpha T^\alpha$$
$$\tau^3 = \mathrm{diag}(1, -1), \quad I_b = \mathrm{diag}(0, 0, 1)$$

Constant fields A_μ^α represent potential gluonic condensates

$$S_g^{-1} = \begin{pmatrix} [G_{0,g}^{+}]^{-1} & \Delta^{-} \\ \Delta^{+} & [G_{0,g}^{-}]^{-1} \end{pmatrix}$$

$\Delta^{-} \equiv -i\epsilon^{b}\varepsilon\gamma_5\Delta$, $\Delta^{+} \equiv -i\epsilon^{b}\varepsilon\gamma_5\Delta^{*}$,

Δ yields the quark energy gap:

$$\bar{\Delta} = \langle\Delta\rangle$$

The effective potential for scalar field Δ and gluon fields A_μ^α:

$$V_{eff} = \frac{|\Delta|^2}{4G_\Delta} + \frac{g^2}{4} f^{\alpha\beta\gamma} f^{\alpha\delta\sigma} A_\mu^\beta A_\nu^\gamma A^{\delta\mu} A^{\sigma\nu} - \frac{1}{2i}\int \frac{d^4p}{(2\pi)^4} \ln\det S_g^{-1}$$

Hard dense loop approximation

Major observation

The gap $\bar{\Delta}$ breaks $SU(3)_c \to SU(2)_c$.

Then, adjoint $\{8\} = 3 \oplus 2 \oplus \bar{2} \oplus 1$,

i.e.,

$$\{A_\mu^{(a)}\} = (A_\mu^{(1)}, A_\mu^{(2)}, A_\mu^{(3)}) \oplus \varphi_\mu \oplus \varphi_\mu^+ \oplus A_\mu^{(8)},$$

$$\varphi_\mu \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} A_\mu^{(4)} - i A_\mu^{(5)} \\ A_\mu^{(6)} - i A_\mu^{(7)} \end{pmatrix} \text{ is}$$

vector Higgs like matter (with respect to $SU(2)_c$ field).

Because of the chromomagnetic instability, a spatial φ_{i_0} should condense. Due to $SO(3)_{rot}$, one can take $\langle \varphi_z \rangle \neq 0$. And because of the $SU(2)_c$, one can take $\langle A_z^{(6)} \rangle \neq 0$ (Higgs like condensate). Then, $SO(3)_{rot} \to SO(2)_{rot}$, $SU(2)_c \to$ nothing

But it is not the end of the story. An analysis, based on the Ginzburg-Landau (GL) approach, shows that at $\delta\mu \gtrsim \bar{\Delta}/\sqrt{2}$ there is a solution with

$$\underline{\mu_8 = \frac{\sqrt{3}}{2} g \langle A_0^{(8)} \rangle} \neq 0, \quad \underline{B \equiv g \langle A_z^{(6)} \rangle} \neq 0,$$

$$\underline{C = g \langle A_z^{(1)} \rangle} \neq 0, \quad \mu_3 \equiv \underline{D = g \langle A_0^{(3)} \rangle} \neq 0$$

Together, $\langle A_z^{(6)} \rangle$, $\langle A_0^{(3)} \rangle$, and $\langle A_z^{(1)} \rangle$ imply that

$SO(3)_{rot} \longrightarrow SO(2)_{rot}$

$SU(2)_c \longrightarrow$ nothing

$\tilde{U}(1)_{em} \longrightarrow$ nothing

<u>Anisotropic superconductor</u> (<u>gluonic cylindrical</u> <u>phase I</u>)

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Anisotropic superconductor (gluonic cylindrical phase I)

The initial symmetry

$$[SU(3)_c]_{local} \times [U(1)_{em} \times U(1)^{3}_{J_L} \times U(1)^{3}_{J_R}]_{glob} \times SO(3)_{rot}$$

$\bar{\Delta} \neq 0 \Rightarrow SU(3)_c \rightarrow SU(2)_c$, $U(1)_{em}$ is broken but

$$\tilde{Q} = Q - \frac{1}{\sqrt{3}} T^8$$

$$\tilde{B} = 2(\tilde{Q} - I_3)$$

are unbroken: $U(1)_{em} \Rightarrow \tilde{U}_{em}(1)$

The VEV $\langle A^{(6)}_3 \rangle$ breaks $SU(2)_c$ but

$$\tilde{\tilde{Q}} = \tilde{Q} - T^3 = Q - \frac{1}{\sqrt{3}} T^8 - T^3$$

$$\tilde{\tilde{B}} = 2(\tilde{\tilde{Q}} - I_3)$$

are unbroken: $\tilde{U}(1)_{em} \Rightarrow \tilde{\tilde{U}}(1)_{em}$.

At last, because T^3 and T^1 does not commute, the VEV $\langle A^{(1)}_3 \rangle$ breaks $\tilde{\tilde{U}}(1)_{em}$ with $\tilde{\tilde{Q}}$ as its generator. $\tilde{\tilde{B}}$ is also broken.

Near-critical solution ($\delta\mu_{cr} = \Delta/\sqrt{2}$):

$$B_{sol} \equiv g\langle A_3^{(6)}\rangle = \frac{\delta\mu^2-\delta\mu_{cr}^2}{\delta\mu_{cr}^2}\,\frac{16\tilde{\mu}\sqrt{10\left(1+\frac{\pi}{\alpha_s}\right)}}{27\left[1+\frac{\mu_8}{\tilde{\mu}}\left(3+\frac{2\pi}{\alpha_s}\right)\right]},$$

$$C_{sol} \equiv g\langle A_3^{(1)}\rangle = \frac{\sqrt{\delta\mu^2-\delta\mu_{cr}^2}}{\delta\mu_{cr}}\,\frac{4\sqrt{5}\,\tilde{\mu}}{9},$$

$$D_{sol} \equiv g\langle A_0^{(3)}\rangle = \frac{\delta\mu^2-\delta\mu_{cr}^2}{\delta\mu_{cr}^2}\,\frac{8\tilde{\mu}}{9\left[1+\frac{\mu_8}{\tilde{\mu}}\left(3+\frac{2\pi}{\alpha_s}\right)\right]},$$

$$\tilde{\mu} = \mu - \frac{\delta\mu}{3} + \frac{\mu_8}{3}, \quad \delta\mu \equiv \frac{\mu_e}{2}$$

This solution describes non-abelian constant electric fields in the ground state:

$$E_3^{(2)} \equiv F_{03}^{(2)} = g f^{2\alpha\beta} A_0^{\alpha} A_3^{\beta} = \frac{1}{g} C_{sol} D_{sol},$$

$$E_3^{(7)} \equiv F_{03}^{(7)} = g f^{7\alpha\beta} A_0^{\alpha} A_3^{\beta} = \frac{1}{2g} B_{sol}\left(2\mu_8 - D_{sol}\right)$$

Unitary gauge in gluonic phase

$$\begin{cases} \Delta^r = 0\,, \quad \Delta^g = 0, \quad \Delta^b = \Delta \\ \varphi_Z^T = \frac{1}{\sqrt{2}}\left(0, A_Z^{(6)}\right), \end{cases}$$

where $\Delta = \langle \Delta \rangle + \delta \equiv \bar{\Delta} + \delta$ and $A_Z^{(6)} = \langle A_Z^{(6)} \rangle + a_Z^{(6)} \equiv B + a_Z^{(6)}$ are real.

Unitary gauge in gluonic phase

$$
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\end{cases}
$$

where $\Delta = \langle\Delta\rangle + \delta \equiv \bar{\Delta} + \delta$ and $A_z^{(6)} = \langle A_z^{(6)}\rangle + a_z^{(6)} \equiv B + a_z^{(6)}$ are real.

Complementarity Gauge Invariant Description

Complementarity Principle: There is no phase transition between the confined and Higgs phases, if Higgs field is in the fundamental representation of the gauge group.

(E. Fradkin and S. Shenker, Phys. Rev. D19 (1979) 3682;
S. Dimopoulos, S. Raby and L. Susskind, Nucl. Phys. B173 (1980) 208.

$\Phi^T \equiv (\Phi^r, \Phi^g, \Phi^b)$ is the diquark field.

In the unitary gauge,

$\Phi^T = (0, 0, \Delta)$

	Q_{em}	B	$\tilde{Q}_{em}$	$\tilde{B}$	$\tilde{\tilde{Q}}_{em}$	$\tilde{\tilde{B}}$	I_3
ψ_{ur}	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0	0	-1	$\frac{1}{2}$
ψ_{ug}	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0	1	1	$\frac{1}{2}$
ψ_{ub}	$\frac{2}{3}$	$\frac{1}{3}$	1	1	1	1	$\frac{1}{2}$
ψ_{dr}	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	0	-1	-1	$-\frac{1}{2}$
ψ_{dg}	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	0	0	1	$-\frac{1}{2}$
ψ_{db}	$-\frac{1}{3}$	$\frac{1}{3}$	0	1	0	1	$-\frac{1}{2}$
Φ^r	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	1	1	2	0
Φ^g	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	1	0	0	0
Φ^b	$\frac{1}{3}$	$\frac{2}{3}$	0	0	0	0	0

	Q_{em}	B	$\bar{Q}_{em}$	$\bar{B}$	$\bar{\bar{Q}}_{em}$	$\bar{\bar{B}}$	I_3
$A_\mu^+ \equiv \frac{1}{\sqrt{2}}(A_\mu^{(1)} + iA_\mu^{(2)})$	0	0	0	0	1	2	0
$A_\mu^{(3)}$	0	0	0	0	0	0	0
$A_\mu^- \equiv \frac{1}{\sqrt{2}}(A_\mu^{(1)} - iA_\mu^{(2)})$	0	0	0	0	-1	-2	0
$\phi_\mu^{*r} \equiv \frac{1}{\sqrt{2}}(A_\mu^{(4)} + iA_\mu^{(5)})$	0	0	$\frac{1}{2}$	1	1	2	0
$\phi_\mu^{*g} \equiv \frac{1}{\sqrt{2}}(A_\mu^{(6)} + iA_\mu^{(7)})$	0	0	$\frac{1}{2}$	1	0	0	0
$\phi_\mu^{r} \equiv \frac{1}{\sqrt{2}}(A_\mu^{(4)} - iA_\mu^{(5)})$	0	0	$-\frac{1}{2}$	-1	-1	-2	0
$\phi_\mu^{g} \equiv \frac{1}{\sqrt{2}}(A_\mu^{(6)} - iA_\mu^{(7)})$	0	0	$-\frac{1}{2}$	-1	0	0	0
$A_\mu^{(8)}$	0	0	0	0	0	0	0

TABLE II: The quantum numbers of gluons.

	Q_{em}	$\mathcal{B}$	I_3
$\epsilon^{\alpha\beta\gamma}(F^*_{z\mu}\Phi)_\alpha(D^*_z\Phi)_\beta(\Phi)_\gamma \sim A^+_\mu$	1	2	0
$(D_z\Phi^*)^\dagger(F_{z\mu}\Phi^*) + (\text{h.c.}) \sim A^3_\mu, A^8_\mu$	0	0	0
$\epsilon^{\alpha\beta\gamma}(F_{z\mu}\Phi^*)_\alpha(D_z\Phi^*)_\beta(\Phi^*)_\gamma \sim A^-_\mu$	−1	−2	0
$\epsilon^{\alpha\beta\gamma}(F^*_{0j}D^*_z\Phi)_\alpha(D^*_z\Phi)_\beta(\Phi)_\gamma \sim A^+_j$	1	2	0
$i(D_z\Phi^*)^\dagger(D_\mu D_z\Phi^*) + (\text{h.c.}) \sim A^3_\mu, A^8_\mu$	0	0	0
$\epsilon^{\alpha\beta\gamma}(F_{0j}D_z\Phi^*)_\alpha(D_z\Phi^*)_\beta(\Phi^*)_\gamma \sim A^-_j$	−1	−2	0
$\epsilon^{\alpha\beta\gamma}(D^*_\mu\Phi)_\alpha(D^*_z\Phi)_\beta\Phi_\gamma \sim \phi^{*r}_\mu$	1	2	0
$(D^*_z\Phi)^\dagger(D^*_\mu\Phi) \sim \phi^{*g}_\mu$	0	0	0
$\epsilon^{\alpha\beta\gamma}(F^*_{0j}\Phi)_\alpha(D^*_z\Phi)_\beta(\Phi)_\gamma \sim \phi^{*r}_j$	1	2	0
$(D^*_z\Phi)^\dagger(F^*_{0j}\Phi) \sim \phi^{*g}_j$	0	0	0
$\epsilon^{\alpha\beta\gamma}(D_\mu\Phi^*)_\alpha(D_z\Phi^*)_\beta(\Phi^*)_\gamma \sim \phi^r_\mu$	−1	−2	0
$(D_z\Phi^*)^\dagger(D_\mu\Phi^*) \sim \phi^g_\mu$	0	0	0
$\epsilon^{\alpha\beta\gamma}(F_{0j}\Phi^*)_\alpha(D_z\Phi^*)_\beta(\Phi^*)_\gamma \sim \phi^r_j$	−1	−2	0
$(D_z\Phi^*)^\dagger(F_{0j}\Phi^*) \sim \phi^g_j$	0	0	0
$\Phi^T(iD_\mu\Phi^*) + (\text{h.c.}) \sim A^8_\mu$	0	0	0

TABLE IV: Composite vectors in the confinement picture.

$$A^+_\mu = \frac{1}{\sqrt{2}}(A^1_\mu + iA^2_\mu) \qquad \phi^r_\mu = \frac{1}{\sqrt{2}}(A^4_\mu - iA^5_\mu)$$

$$A^-_\mu = \frac{1}{\sqrt{2}}(A^1_\mu - iA^2_\mu) \qquad \phi^g_\mu = \frac{1}{\sqrt{2}}(A^6_\mu - iA^7_\mu)$$

Two other gluonic phases connected with a first order phase transition

a) Gluonic cylindrical phase II.

Take $C \equiv g\langle A_z^{(1)}\rangle = 0$ in gluonic cylindrical phase I:

$\mu_8 = \frac{\sqrt{3}}{2} g\langle A_0^{(8)}\rangle \neq 0$, $B \equiv g\langle A_z^{(6)}\rangle \neq 0$,

and $\mu_3 \equiv D = g\langle A_0^{(3)}\rangle \neq 0$

$SO(3)_{rot} \rightarrow SO(2)_{rot}$

electromagnetic $\tilde{U}(1)$ is preserved

b) Gluonic color-spin locked (GCSL) phase

$\mu_8 = \frac{\sqrt{3}}{2} g\langle A_0^{(8)}\rangle \neq 0$, $K \equiv g\langle A_y^{(4)}\rangle = g\langle A_z^{(6)}\rangle \neq 0$

$SU(2)_c \times \tilde{U}(1)_{em} \times SO(3)_{rot} \rightarrow SO(2)_{diag}$

$T_{diag} = \tilde{T}_2 + T_{yz}$; $\tilde{T}_2 \in SU(2)_c$, $T_{yz} \in SO(3)_{rot}$

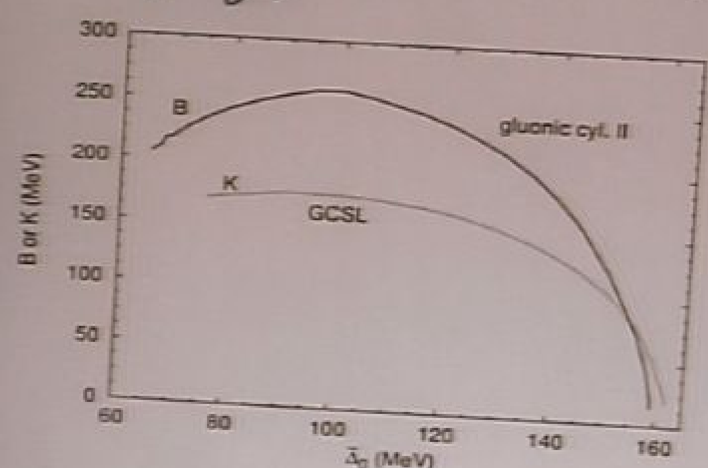
Anisotropic superconductor.

Numerical analysis: these phases exist

M. Hashimoto and V. M., hep-ph/0705.2399

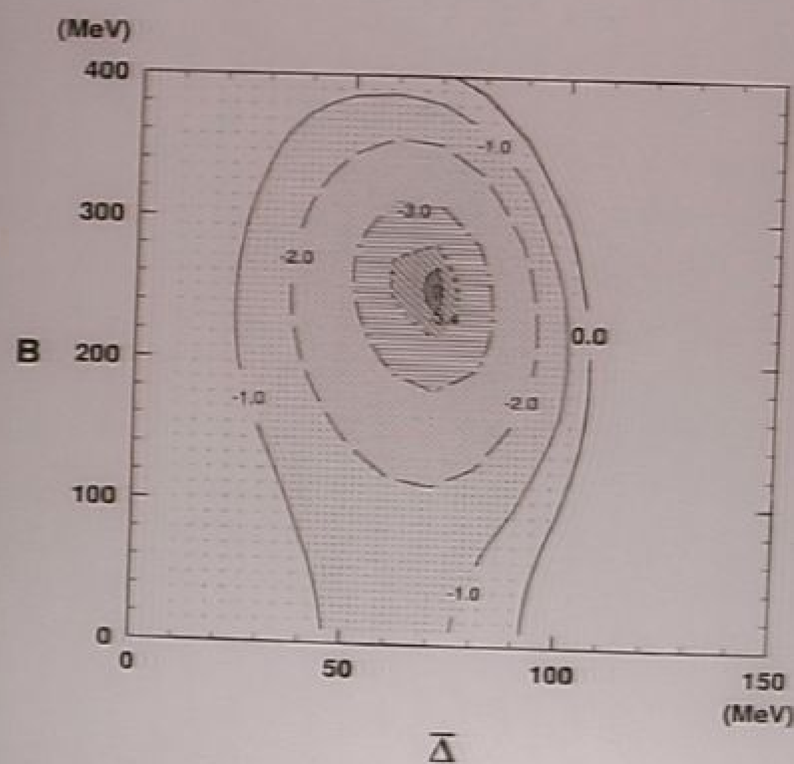
$\mu = 400$ MeV

$\bar{\Delta}_0$ is the 2SC gap parameter defined at $\delta\mu = 0$ (it essentially represents the strength of the NJL coupling)



$\mu = 400$ MeV

$\bar{\Delta}_0 = 110$ MeV



Gluonic phases are most stable in the wide region $6.7\times10^1\,\mathrm{MeV} < \bar{\Delta}_0 < 1.6\times10^2\,\mathrm{MeV}$

Single plane LOFF phase is favorable in the window $6.5\times10^1\,\mathrm{MeV} < \bar{\Delta}_0 < 6.7\times10^1\,\mathrm{MeV}$

The normal phase is the ground state at $\bar{\Delta}_0 < 6.5\times10^1\,\mathrm{MeV}$ (weak coupling)

The 2SC phase is realized at $\bar{\Delta}_0 > 1.6\times10^2\,\mathrm{MeV}$ (strong coupling)