Title: Quantum Black Holes and High Energy

Date: May 22, 2007 04:00 PM

URL: http://pirsa.org/07050053

Abstract:

The role of (broken) scale invariance of QCD at RHIC

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Outline

Running QCD coupling and the initial conditions in hydrodynamical calculations

Based on work with E.Levin, M.Nardi;
T.Hirano, U.Heinz, Y.Nara, R.Lacey

Broken scale invariance and bulk viscosity close to T_c

Recent work with K.Tuchin

QCD and quantum anomalies

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \bar{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f;$$

Classical scale invariance is broken by quantum effects:

scale anomaly

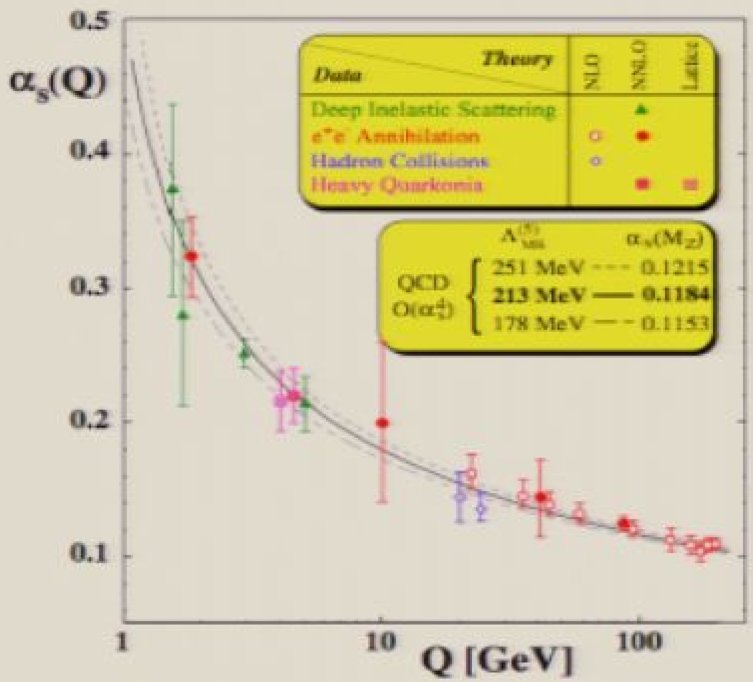
$$\theta^\mu_\mu = \frac{\beta(g)}{2g} G^{\alpha\beta a} G^a_{\alpha\beta} + \sum_q m_q \bar{q} q$$

trace of the energy-momentum tensor

"beta-function"; describes the dependence of coupling on momentum $\mu \frac{dg(\mu)}{d\mu} = \beta($

Hadrons get masses ⟺ coupling runs with the distance

Asymptotic Freedom



At short distances,
the strong force becomes weak
(anti-screening) -
one can access the "asymptotically free" regime in hard processes

and in super-dense matter
(inter-particle distances ~ 1/T)

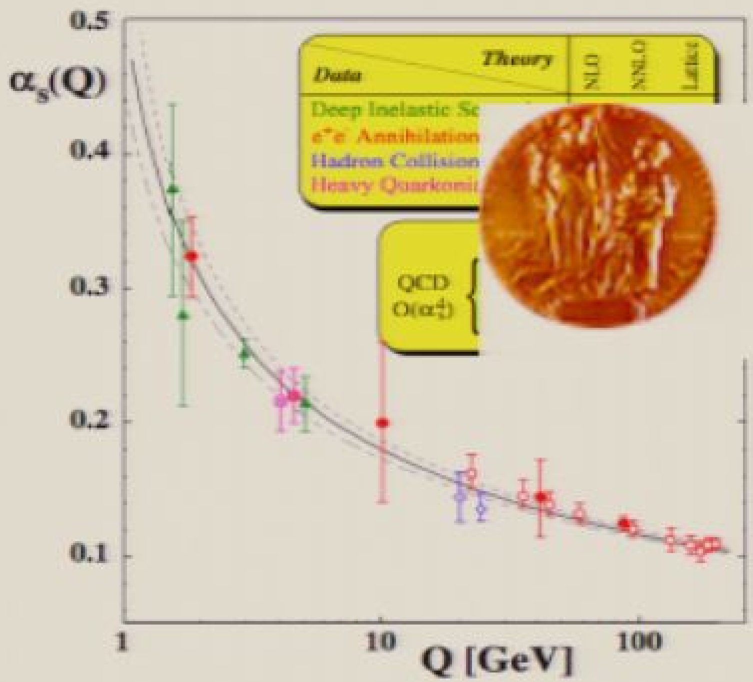
$$\alpha_s(Q) \simeq \frac{4\pi}{b\ln(Q^2/\Lambda^2)}$$

number of colors

number of flavors

$$b = (11N_c - 2N_f)/3$$

Asymptotic Freedom



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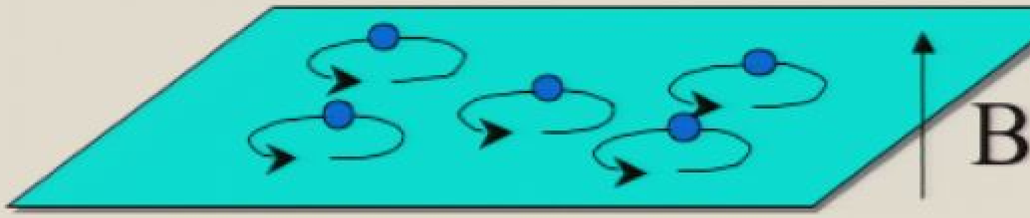
Asymptotic freedom and Landau levels of 2D parton gas

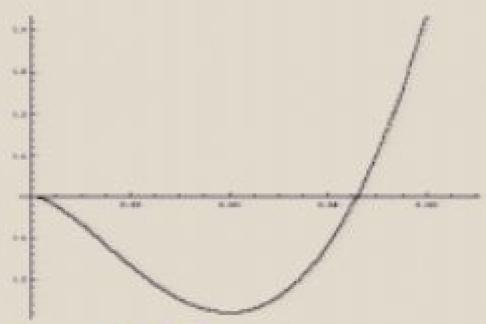


The effective potential: sum over 2D Landau levels

$$V_{\text{pert}}(H) = \frac{g\,H}{4\,\pi^2} \int dp_z \sum_{n=0}^{\infty} \sum_{s_z=\pm 1} \sqrt{2\,g\,H\,(n+1/2-s_z)+p_z^2}.$$

Paramagnetic response of the vacuum:

$$\mathrm{Re}\,V_{\text{pert}}(H) = \frac{1}{2}H^2 + (g\,H)^2\,\frac{b}{32\,\pi^2}\left(\ln\frac{g\,H}{\mu^2} - \frac{1}{2}\right)$$



1. The lowest level n=0 of radius $\sim (gH)^{-1/2}$ is unstable!

2. Strong fields ⟷ Short distances

QCD and the classical limit

Classical dynamics applies when the action $S = \int d^4x \ \mathcal{L}(x)$ is large in units of the Planck constant (Bohr-Sommerfeld quantization)

(gA -> A

$$\frac{S_{QCD}}{\hbar} \sim \frac{1}{g^2\hbar} \int d^4x \ \mathrm{tr}\, G^{\mu\nu}(x) G_{\mu\nu}(x) \gg 1$$

(equivalent to setting $\hbar \to 0$)

=> Need weak coupling and strong fields

$$A^2 \ll \frac{p^2}{g^2}$$

weak field

$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

strong field

$$A^2 \sim \frac{p^2}{g^2}$$

Renormalization group and the effective action

RG constraints the form of the effective action:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4\bar{g}^2(t)}\, G^2, \quad t \equiv \ln\left(\frac{G^2}{\Lambda^4}\right)$$

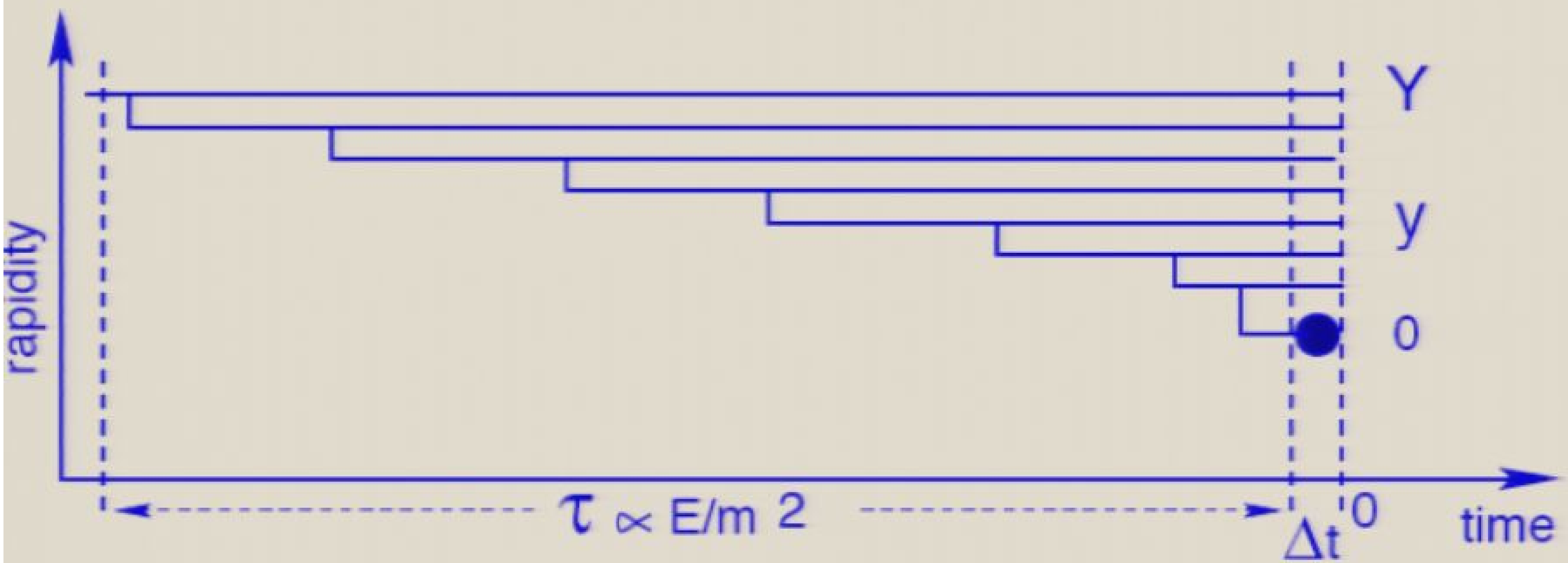
the coupling is defined through

$$t = \int_g^{\bar{g}(t)} \frac{dg}{\beta(g)}.$$

At large t (strong color field),

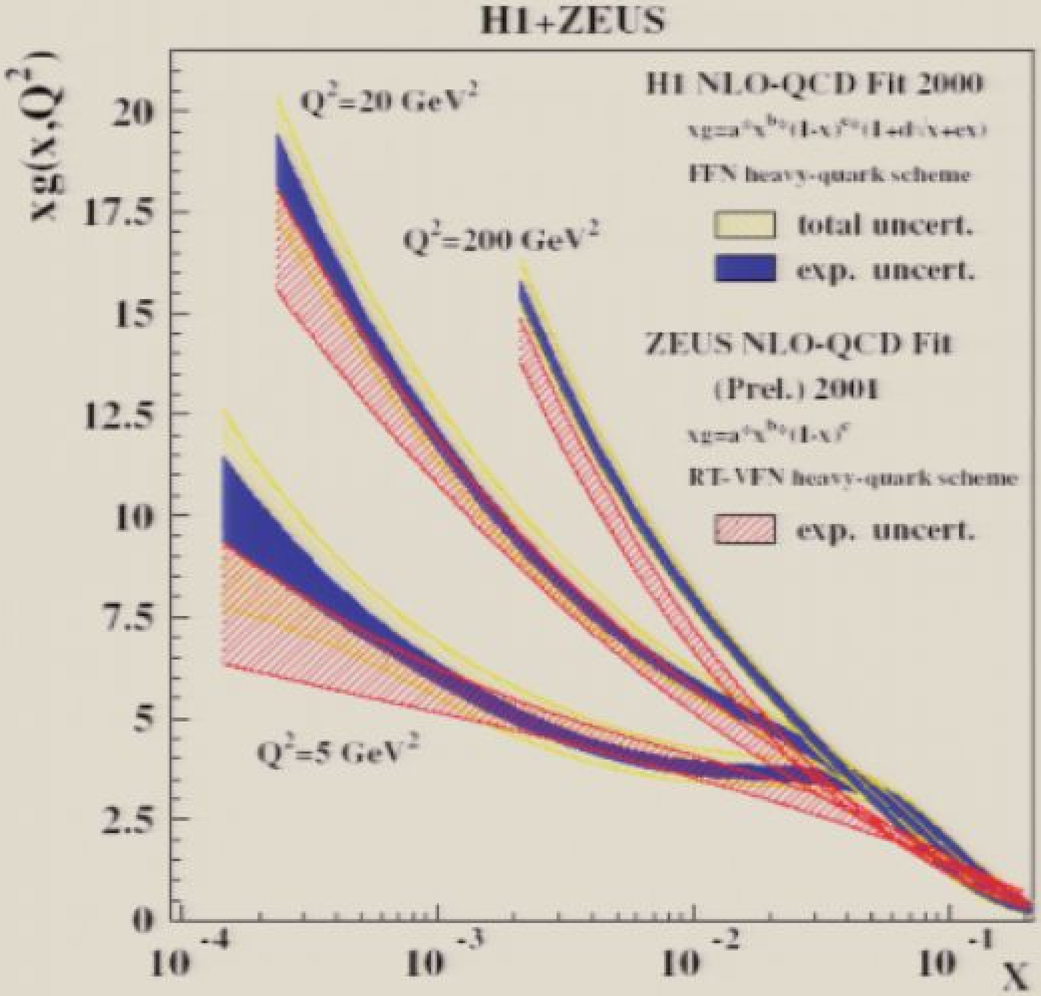
$$\frac{1}{\bar{g}^2(t)} \sim t + \ldots \quad \text{and} \quad \mathcal{L}_{\text{eff}} \sim G^2 \,\ln\left(\frac{G^2}{\Lambda^4}\right)$$

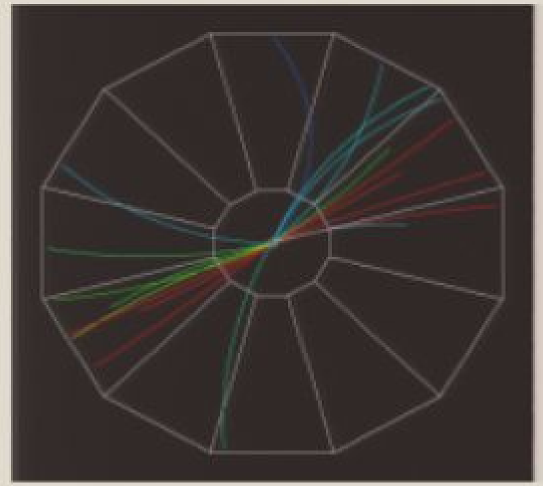
The space-time picture of high-energy interactions in QCD

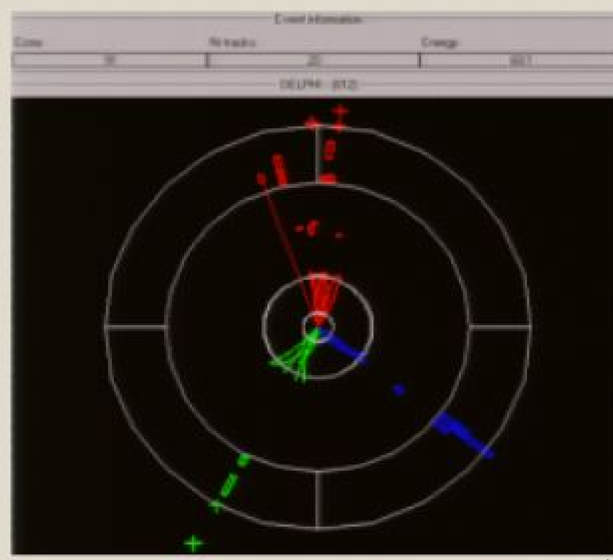


1. Fast (large y) partons live for a long time;
2. Parton splitting probability is $\sim \alpha_s y$ - not small!

Resolving the gluon cloud at small x and short distances ~ $1/Q^2$







number of gluons

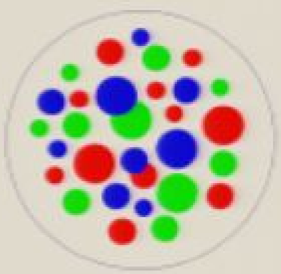
"jets": high momentum partons

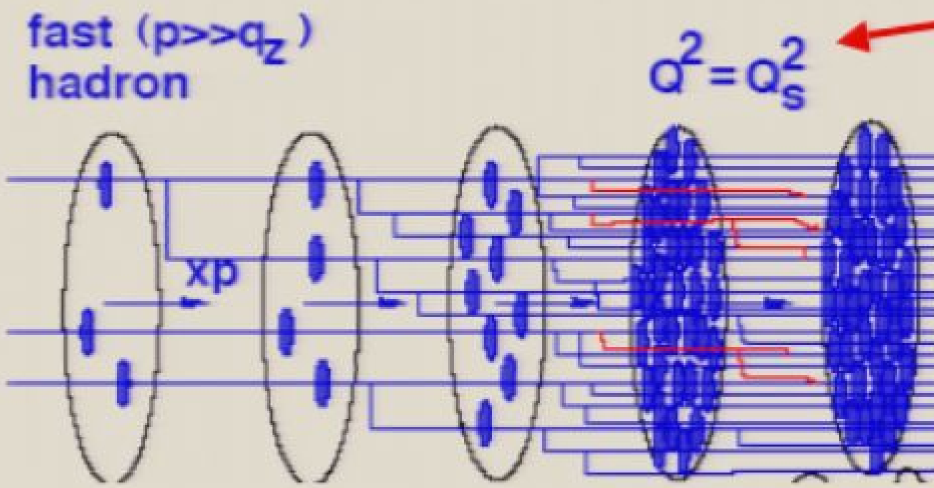
Building up strong color fields:
small x (high energy) and large A (heavy nuclei)

Bjorken x : the fraction of hadron's momentum carried by a parton; high energies s open access to small $x = Q^2/s$

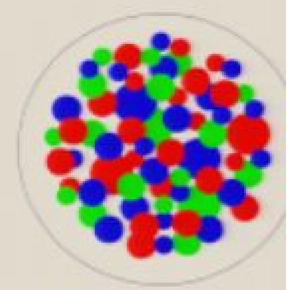
the boundary of non-linear regime: partons of size $1/Q > 1/Q$ overlap

Large x





small x

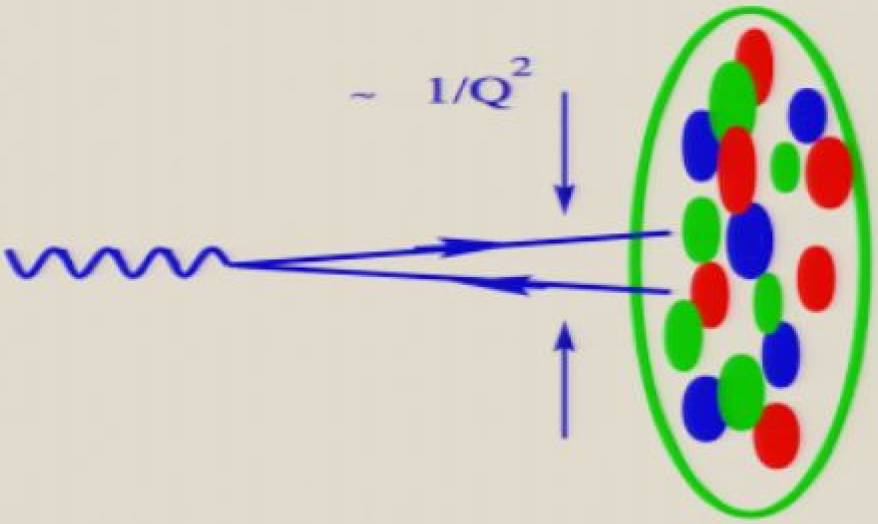


Because the probability to emit an extra gluon is $\sim \alpha_s \ln(1/x) \sim 1$, the number of gluons at small x grows; the transverse area is limited

→ transverse density becomes large

Strong color fields in heavy nuclei

At small Bjorken x, hard processes develop over large longitudinal distances $l_c \sim \frac{2\nu}{Q^2} = \frac{1}{mx}$



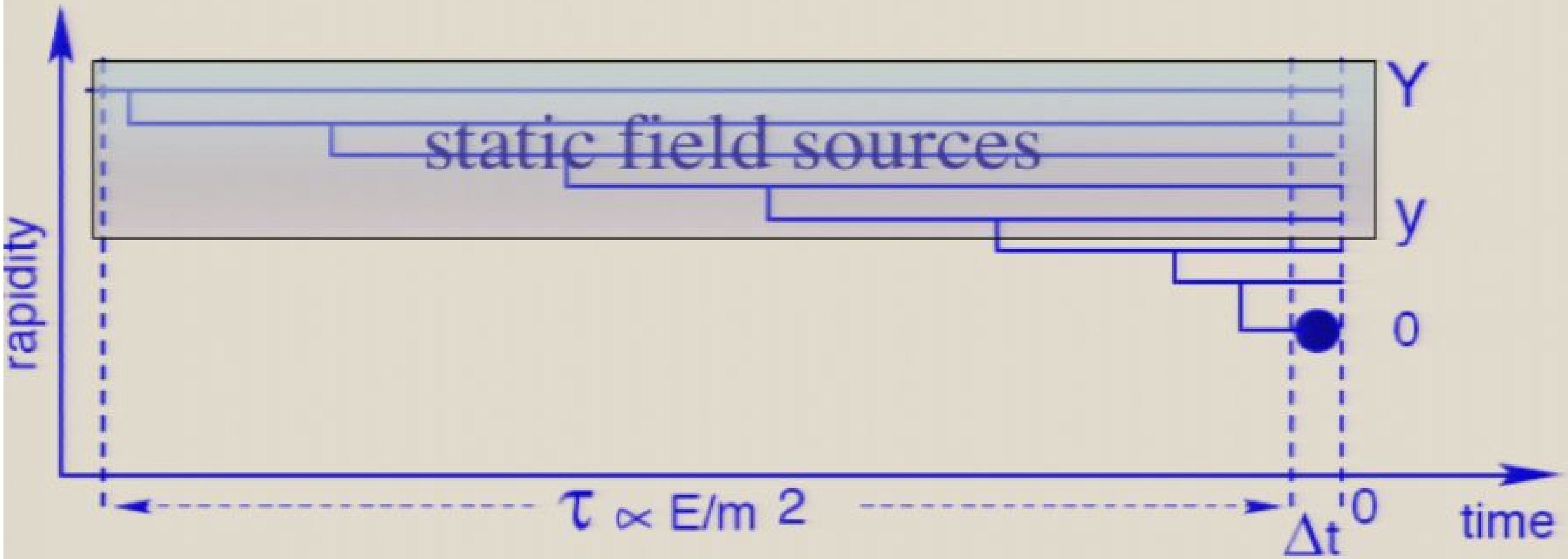
Density of partons in the transverse plane as a new dimensionful parameter Q_s ("saturation scale")

Gribov, Levin, Ryskin

All partons contribute coherently => at sufficiently small x and/or large A strong fields, weak coupling!

McLerran, Venugopalan

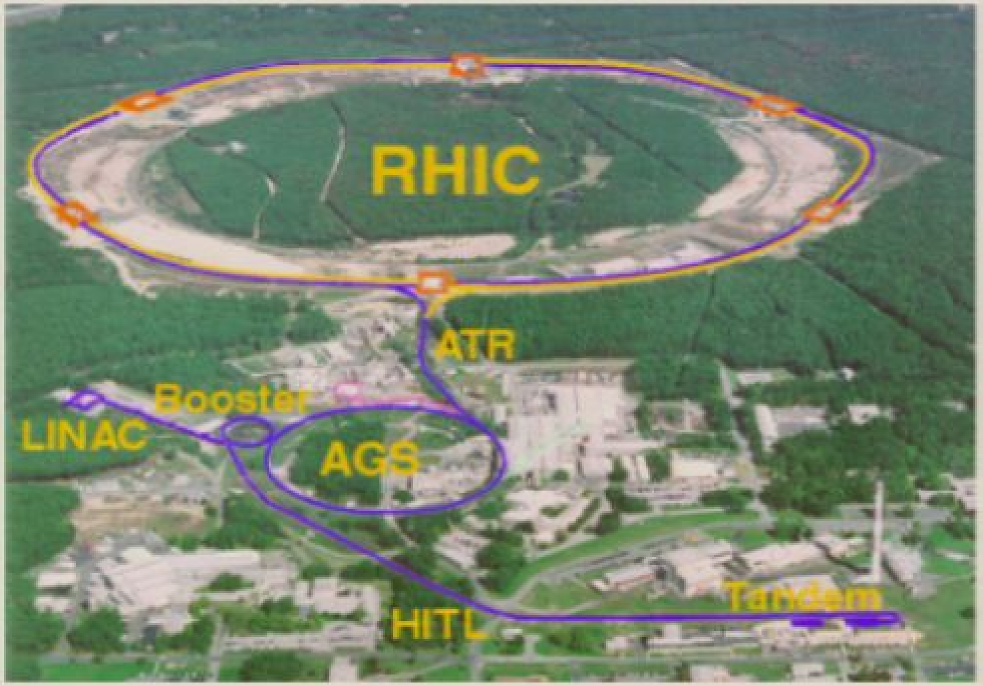
The origin of classical background field



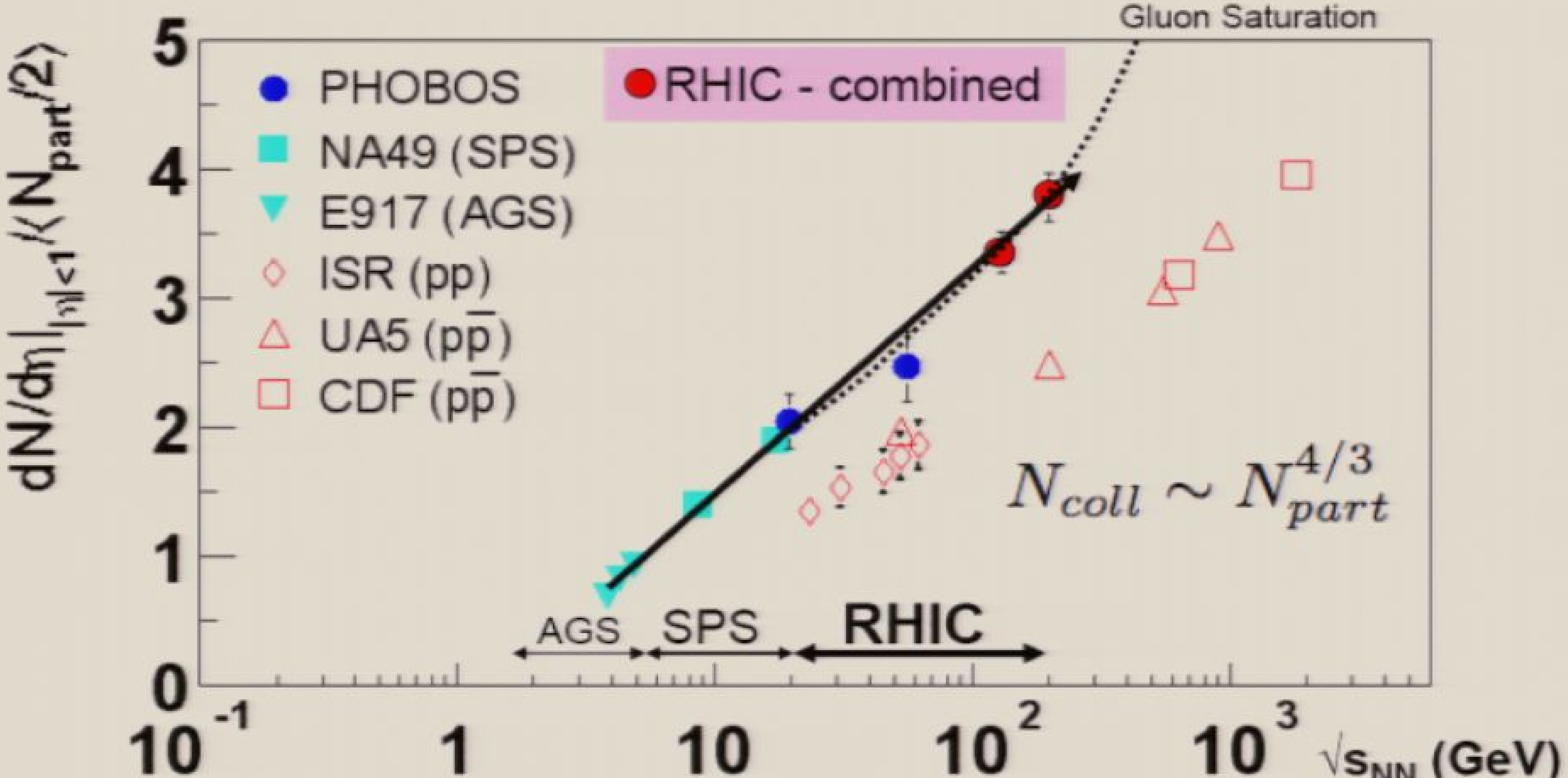
Gluons with large rapidity and large occupation number act as a background field for the production of slower gluons

"Color Glass Condensate"

Semi-classical QCD: experimental tests



Hadron multiplicities: the effect of parton coherence

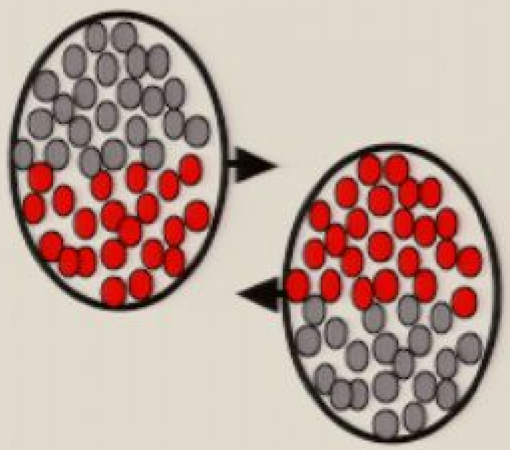


Semi-classical QCD and total multiplicities in heavy ion collisions

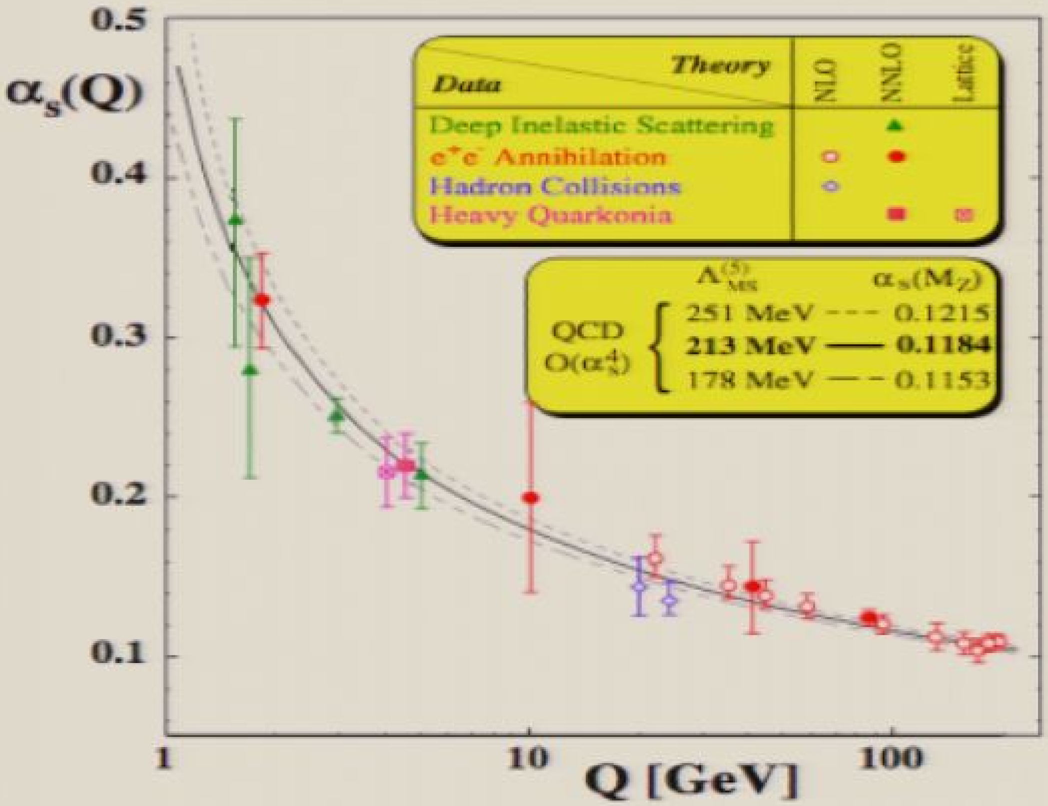
Expect very simple dependence of multiplicity on atomic number A / N_{part}:

$$n \sim \frac{S_A Q_s^2}{\alpha_s(Q_s^2)} \sim N_{part}\ \ln N_{part}$$

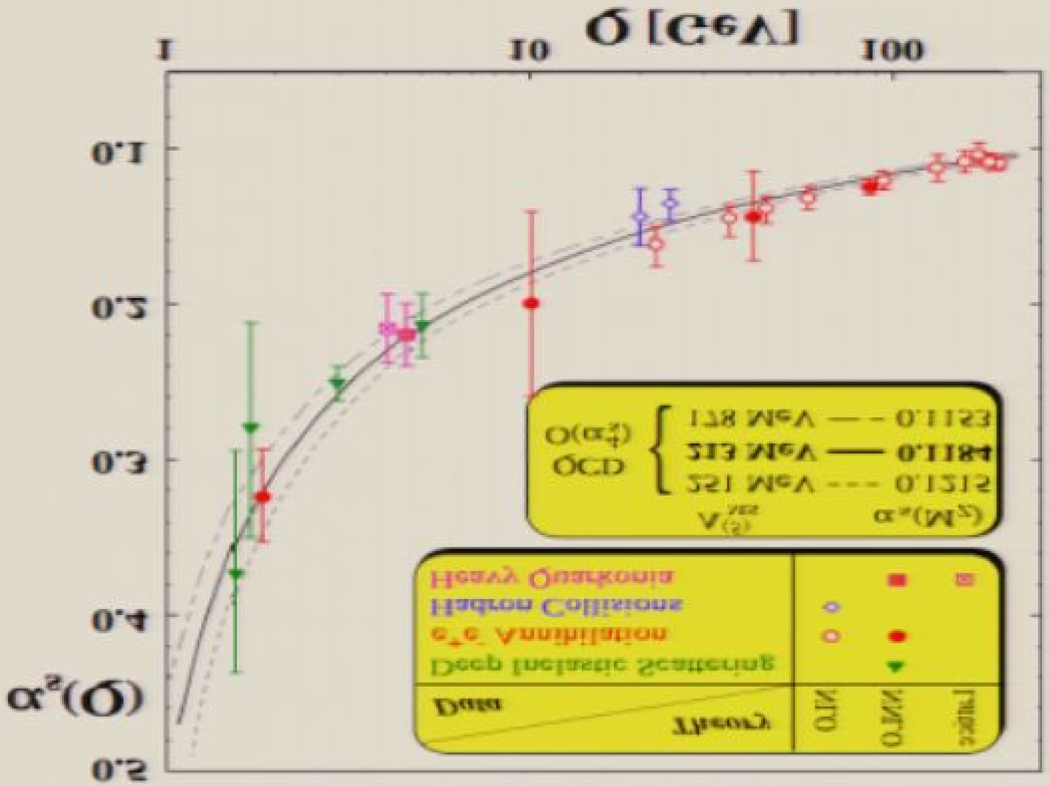
N_{part}:



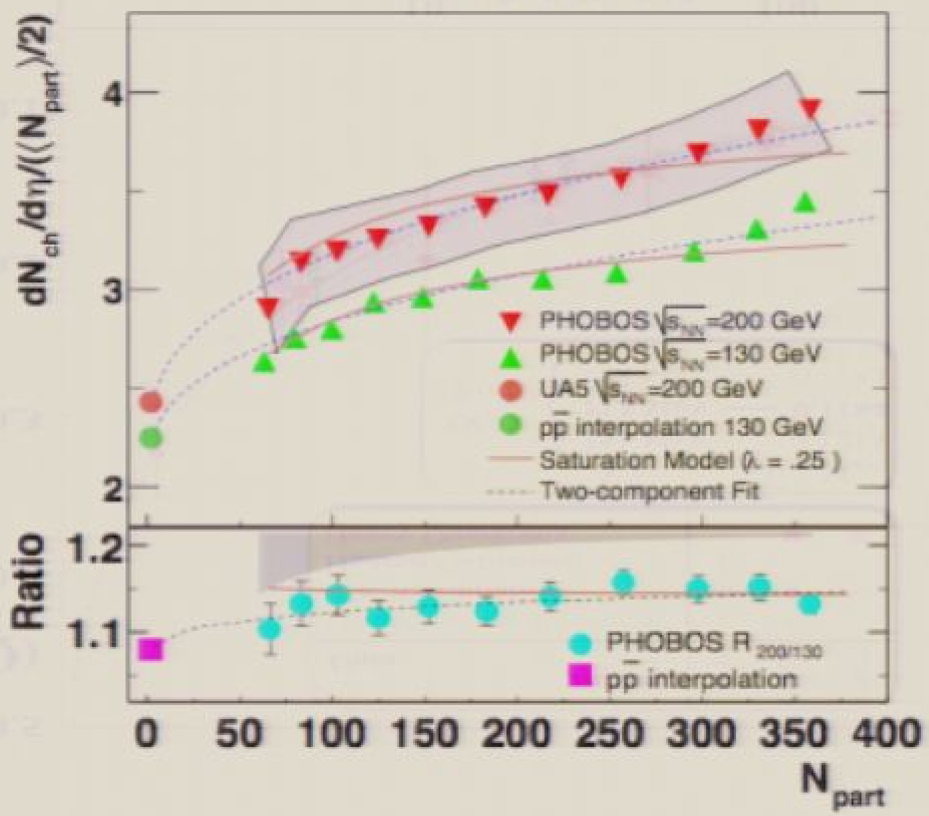
Classical QCD in action



$$\frac{1}{N_{part}} \frac{dN}{d\eta} \sim \frac{1}{\alpha_s(Q$$



$$\frac{1}{N_{part}} \frac{dN}{d\eta} \sim \frac{1}{\alpha_s(Q^2)}$$



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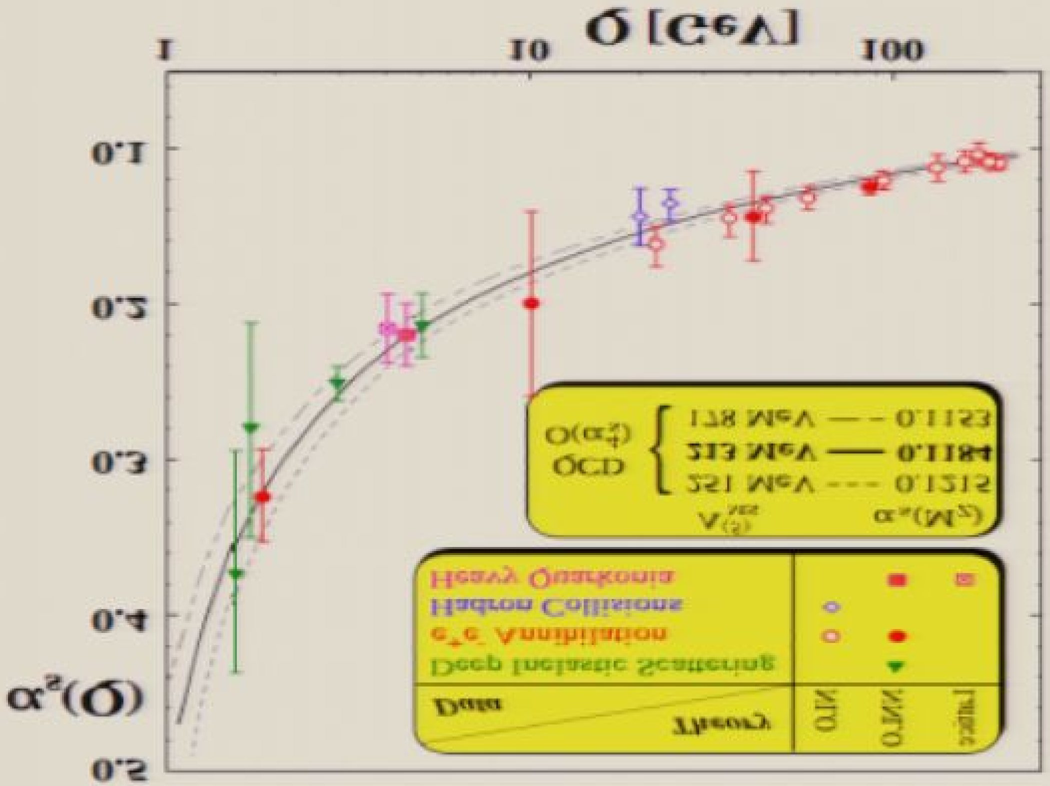
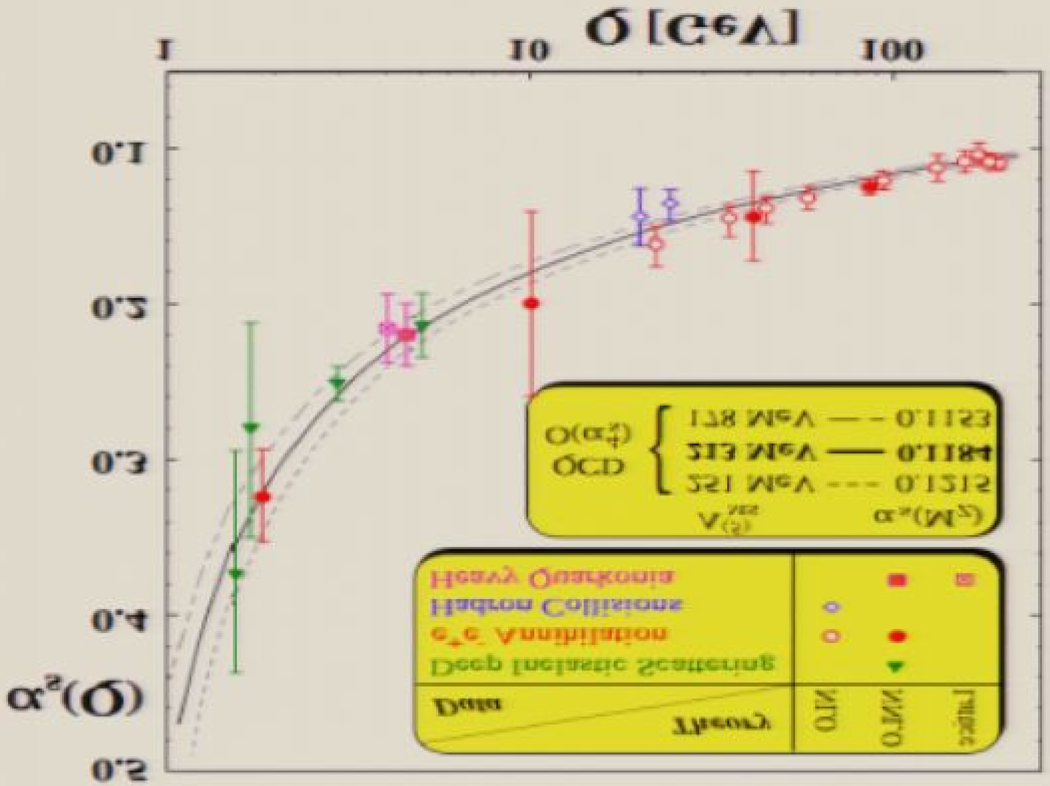
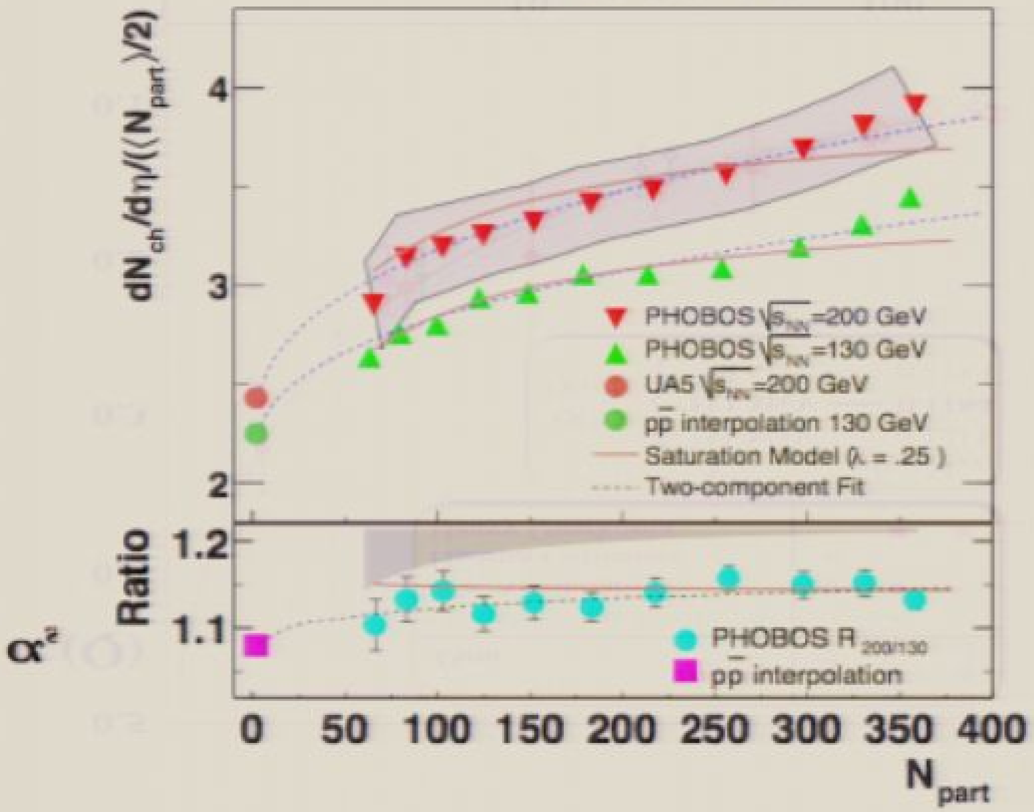


$$\frac{1}{N_{part}} \frac{dN}{d\eta} \sim \frac{1}{\alpha_s(Q}$$



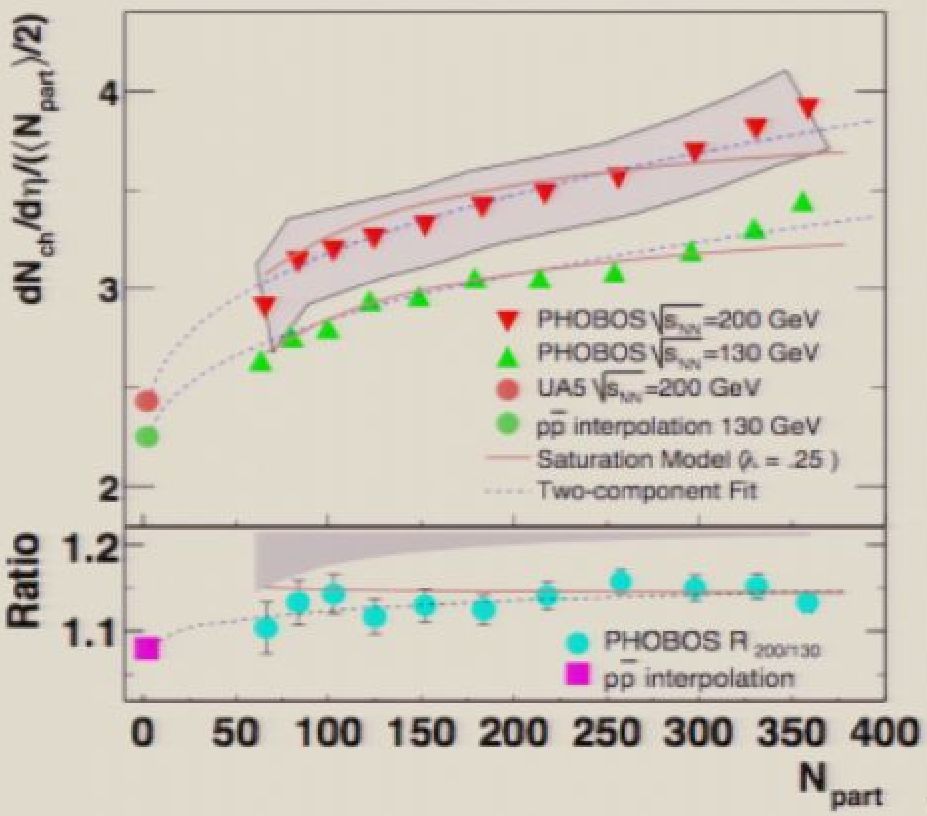
$$\frac{1}{N_{part}} \frac{dN}{d\eta} \sim \frac{1}{\alpha_s(Q\ldots}$$

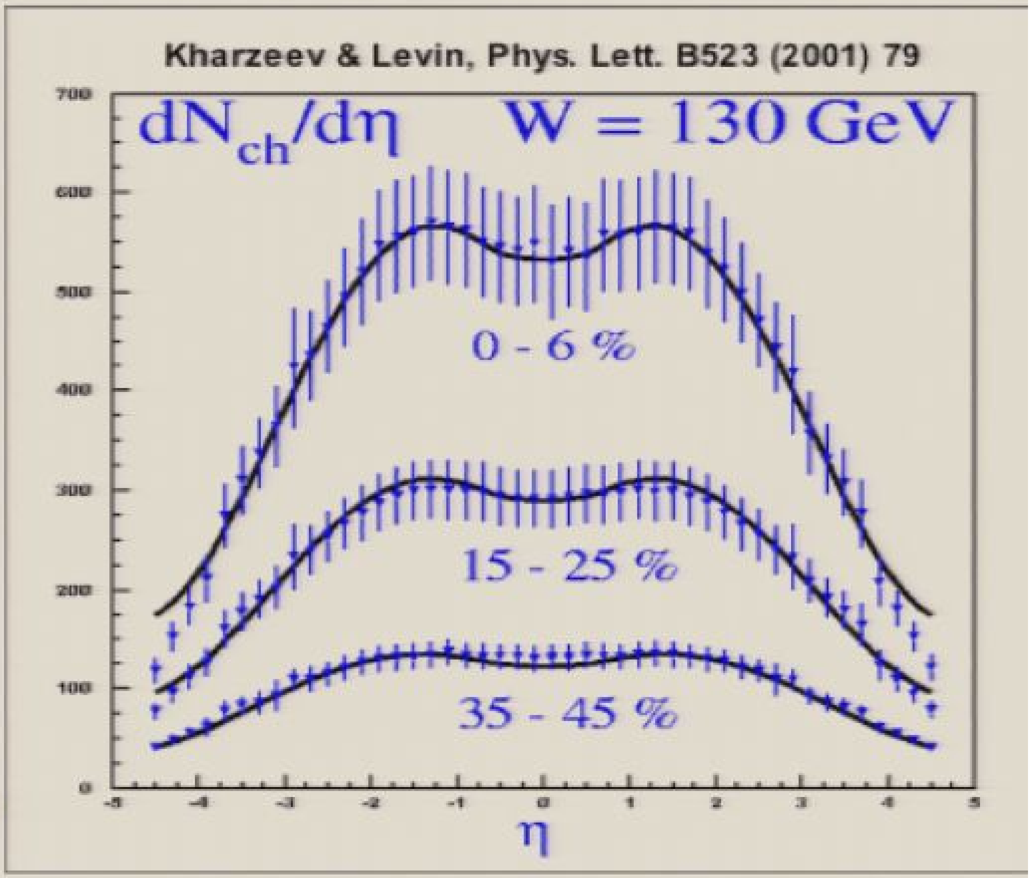
$$\frac{1}{N_{part}} \; \frac{dN}{d\eta} \sim \frac{1}{\alpha_s(Q^2_s)}$$



Classical QCD dynamics in action

The data on hadron multiplicities in Au-Au and d-Au collisions support the quasi-classical picture





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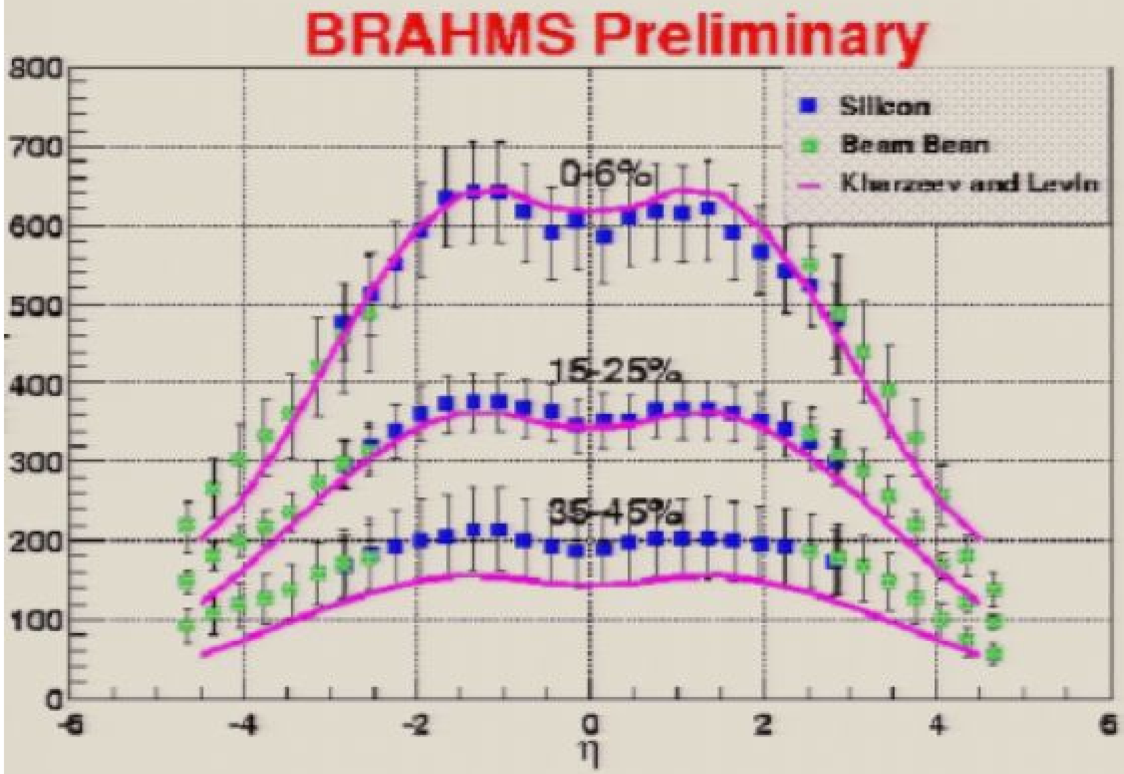
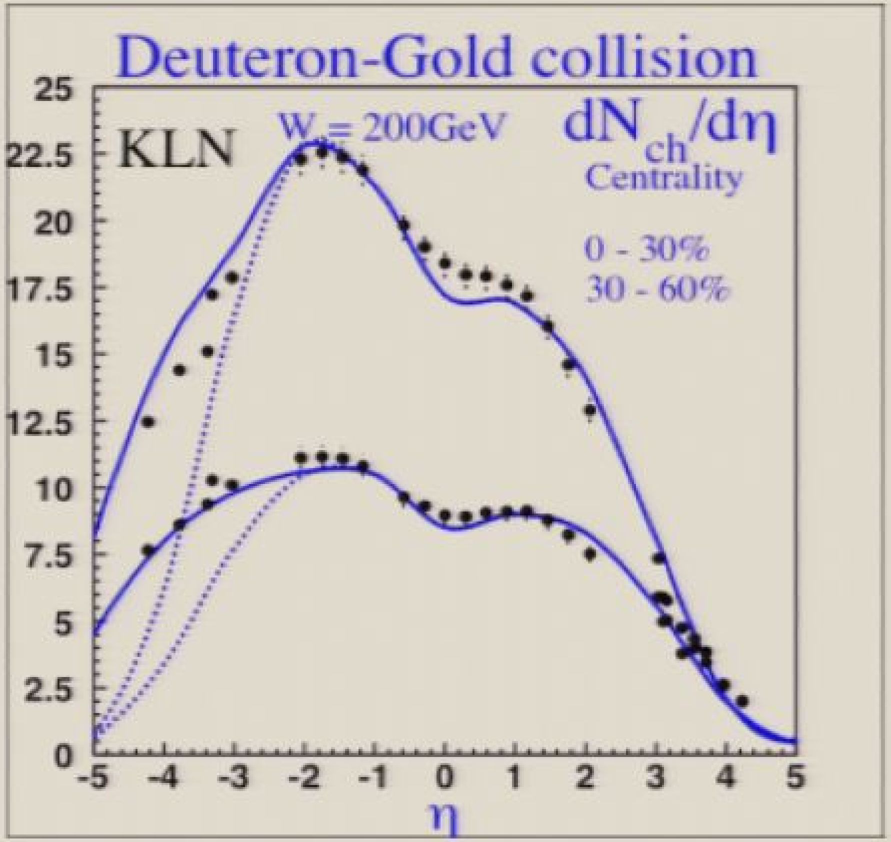
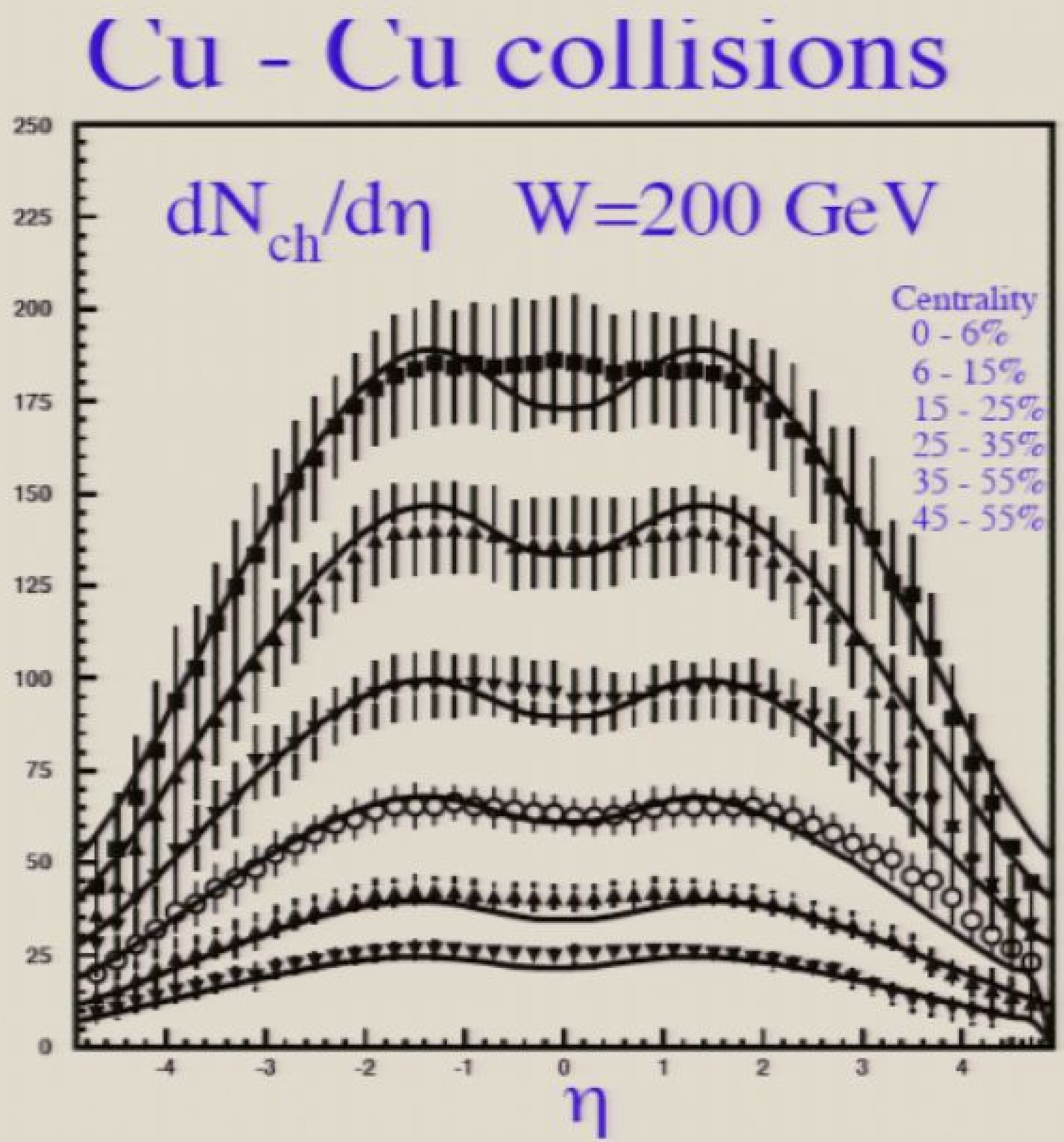


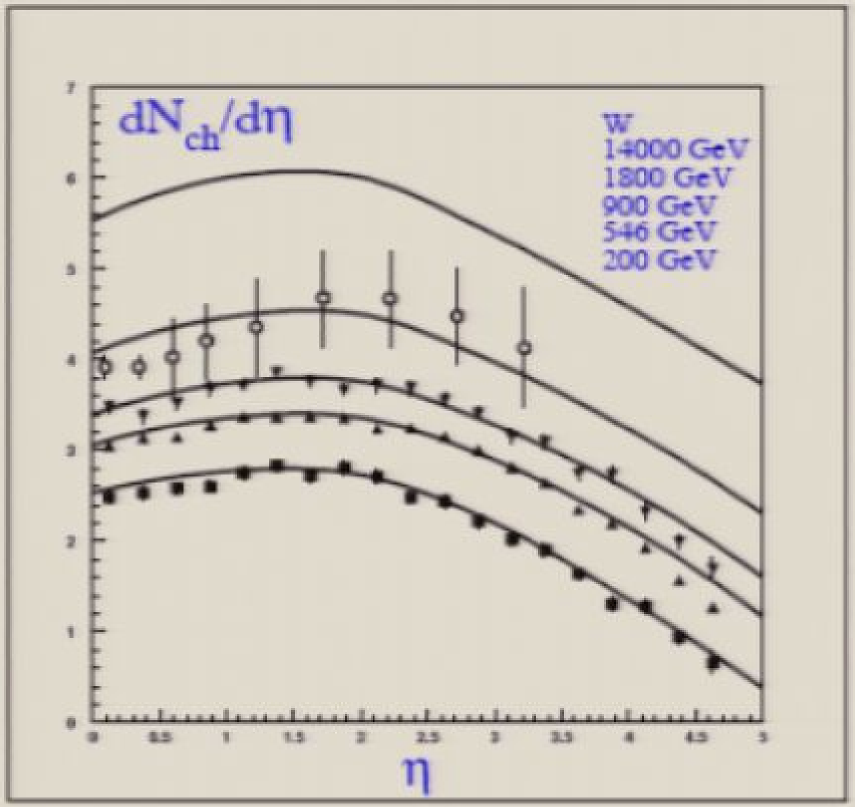


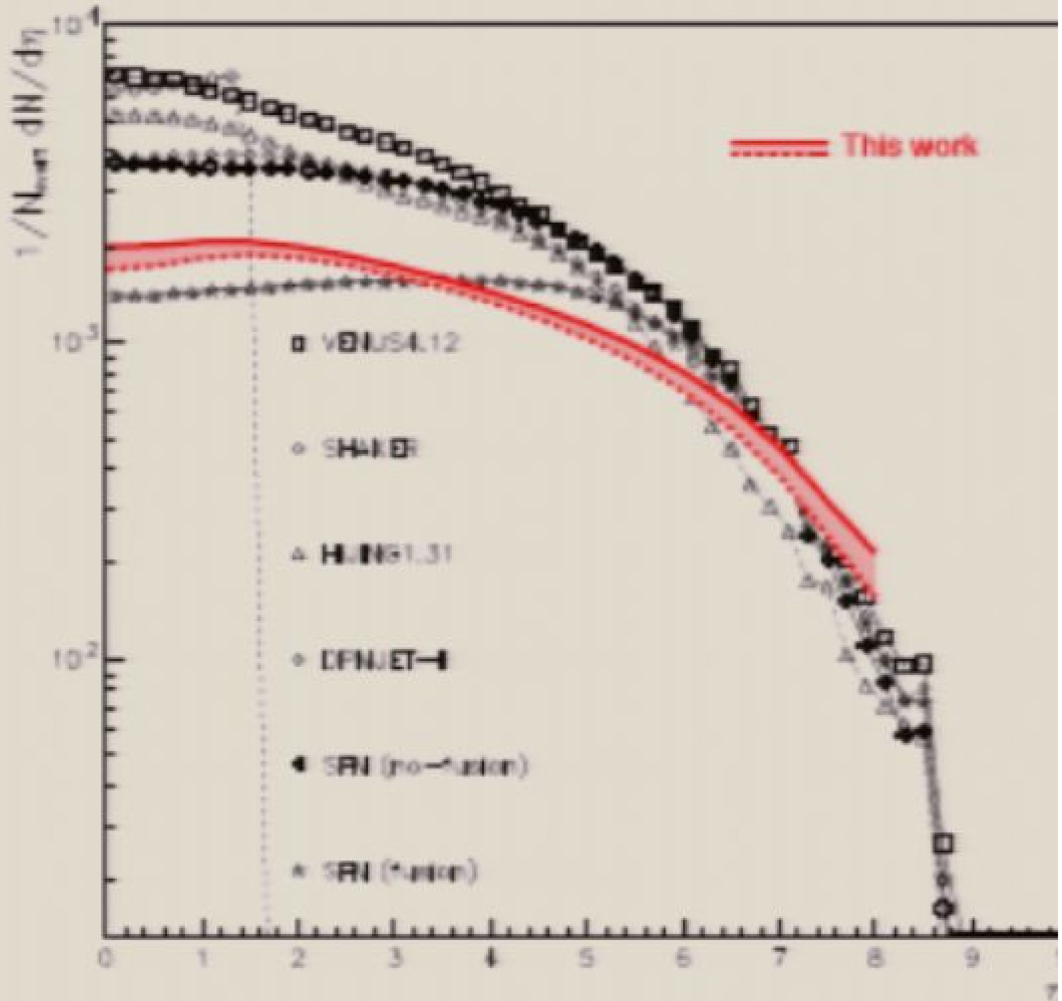
First look at the CuCu data



KLN

Predictions for the LHC





KLN, hep-ph/0408050

How dense is the produced matter?

The initial energy density achieved:

$$\epsilon_{initial} \simeq \frac{< k_t >}{\tau_0} \frac{d^2 N}{d^2 b\ d\eta} \simeq Q_s^2 \frac{d^2 N}{d^2 b\ d\eta} \simeq 18\ \text{GeV}/\text{fm}^3$$

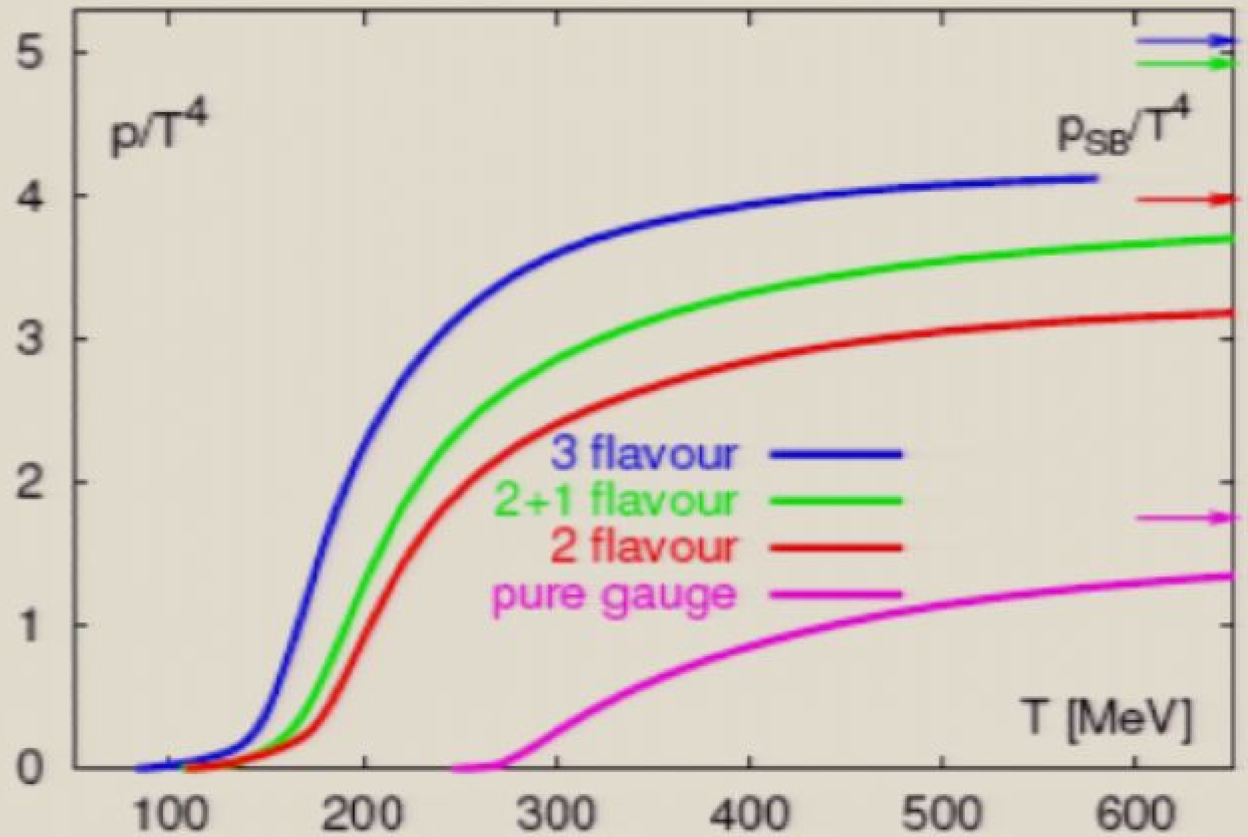
mean transverse momentum of produced gluons

gluon formation time

the density of the gluons in the transverse plane and in rapidity

about 100 times nuclear density !

What happens at such energy densities?



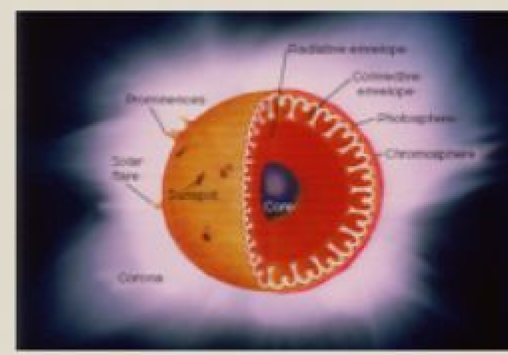
Data from lattice QCD simulations F. Karsch et al

Phase transitions:

deconfinement

Chiral symmetry restoration

$U_A(1)$ restoration



critical temperature ~ 10^{12} K; cf temperature inside the Sun ~ 10^7 K

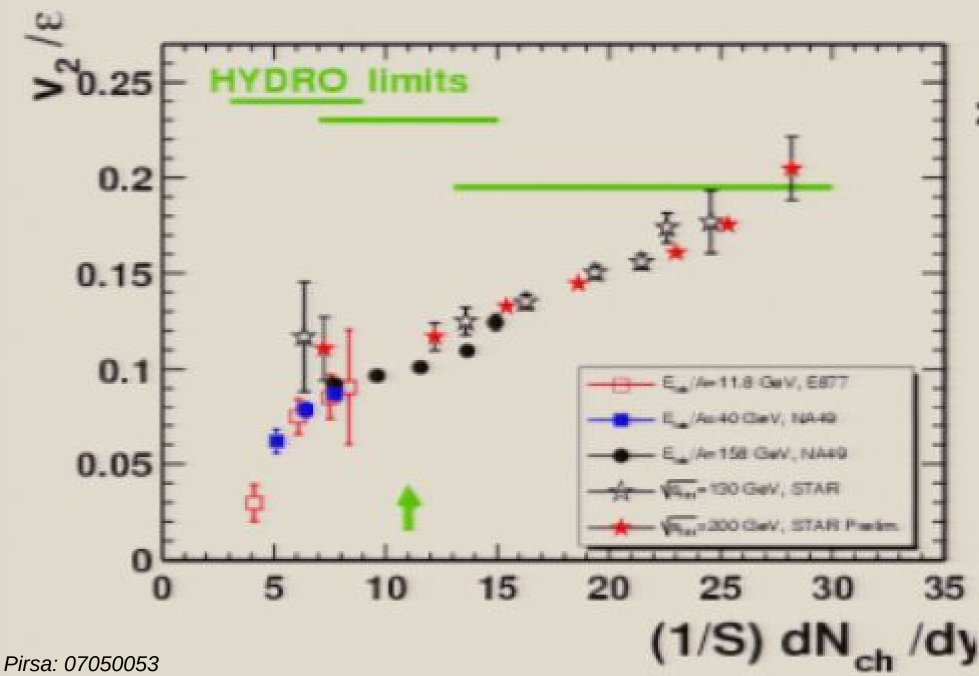
Do we see the QCD matter at RHIC ?

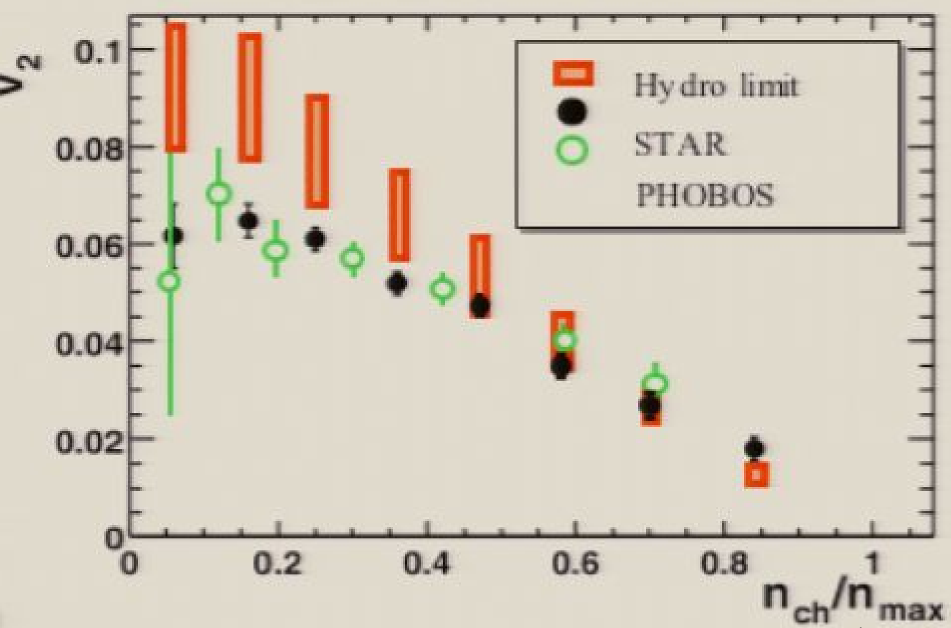
Collective flow =>

Au-Au collisions at RHIC produce strongly interacting matte

shear viscosity - to - entropy rati

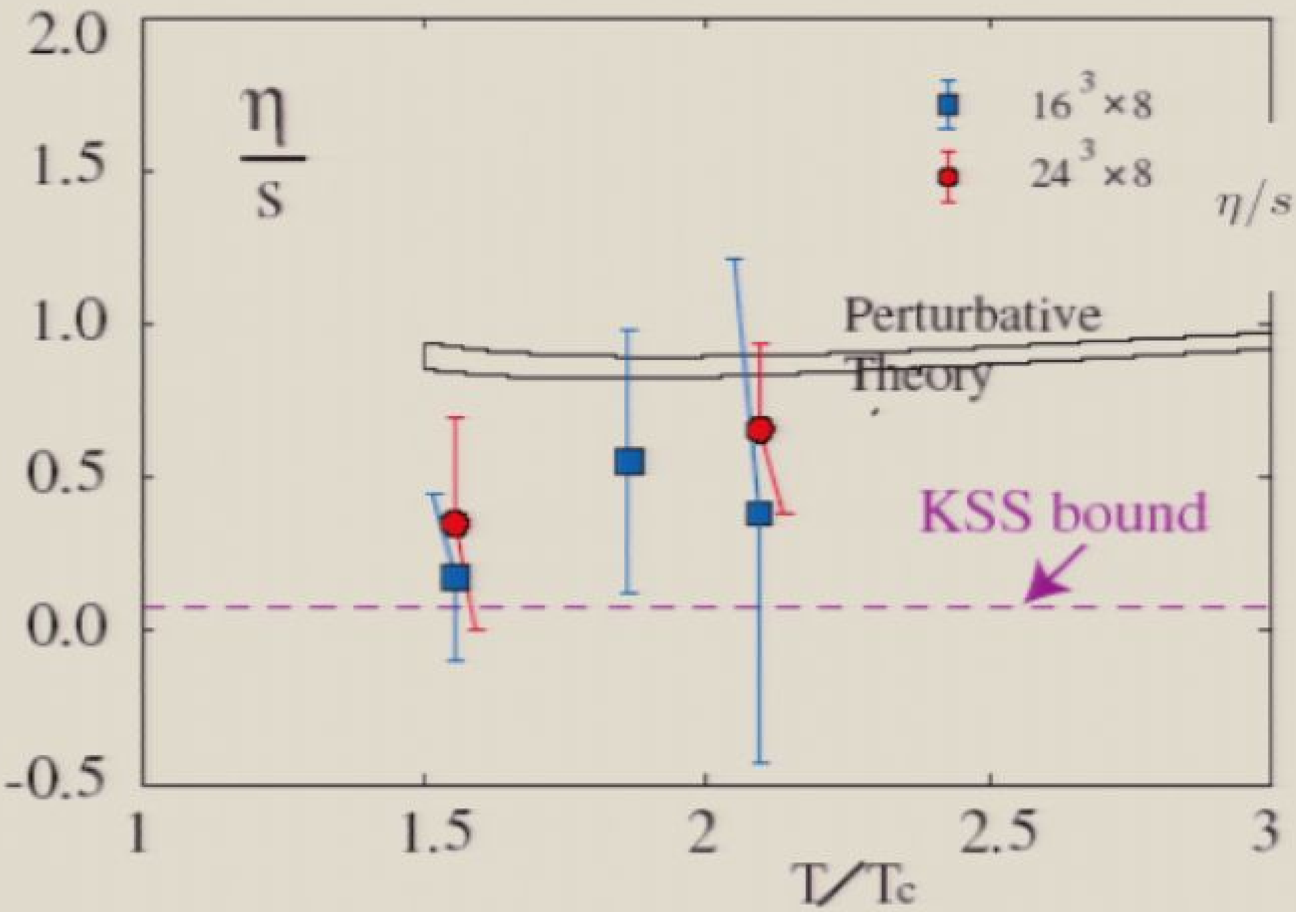
hydrodynamics: QCD liquid is more fluid than water





Shear viscosity of sQGP

A.Nakamura and S.Sakai, hep-lat/0406009;
Recent work:
H.Meyer, 0704.1801



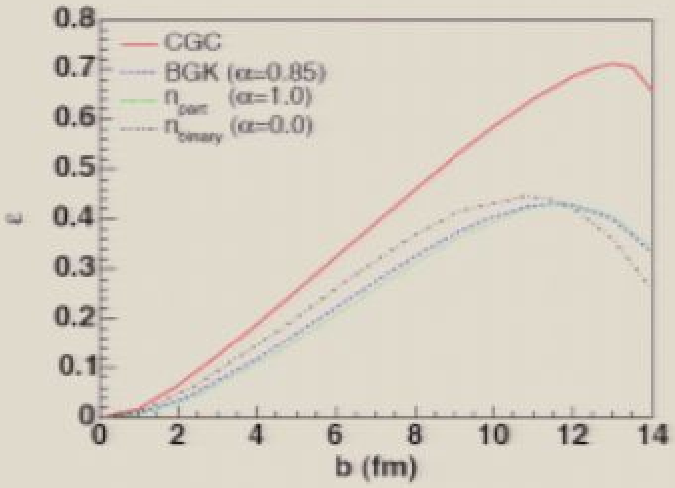
$$\eta/s = \begin{cases} 0.134(33) & (T = 1.65T \\ 0.102(56) & (T = 1.24T \end{cases}$$

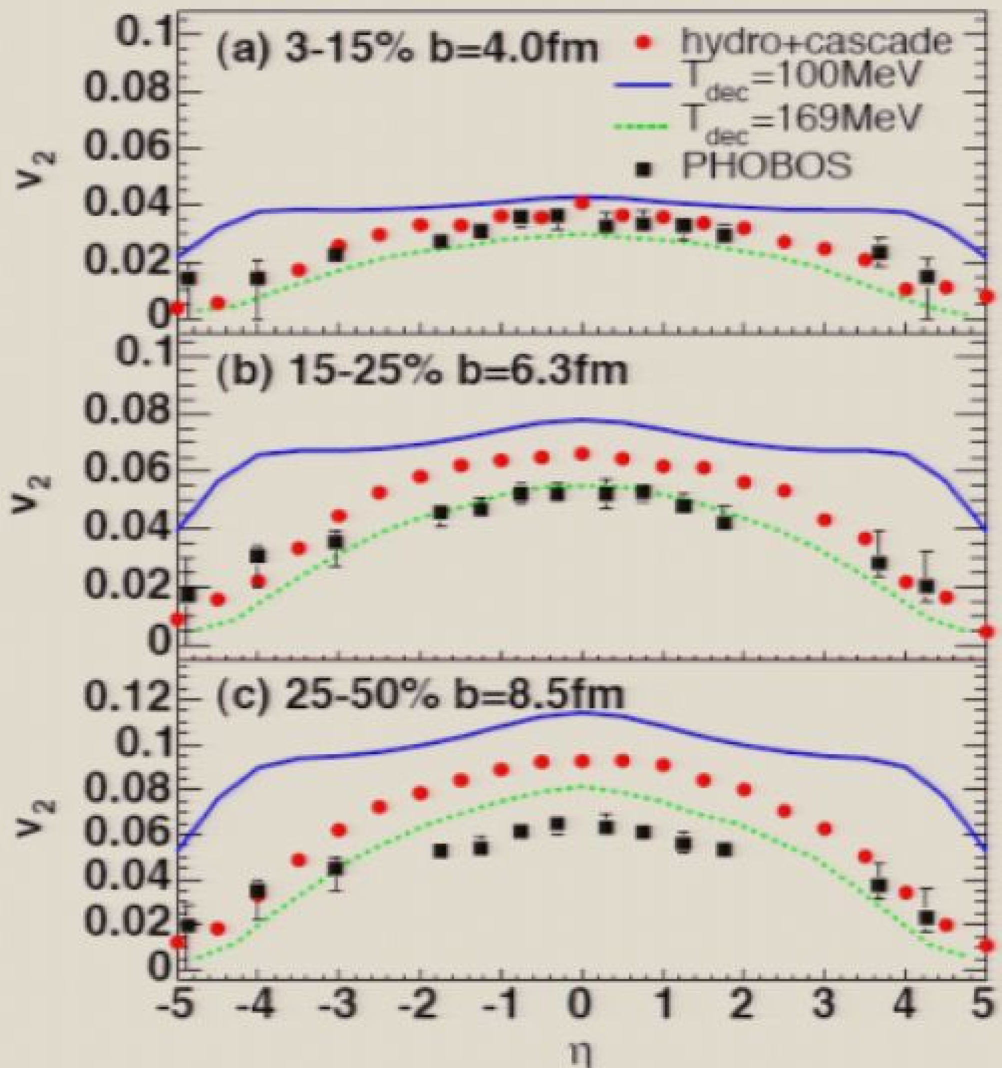
Perfect liquid

Kovtun - Son - Starinets bound:
strongly coupled SUSY QCD = classical supergravity

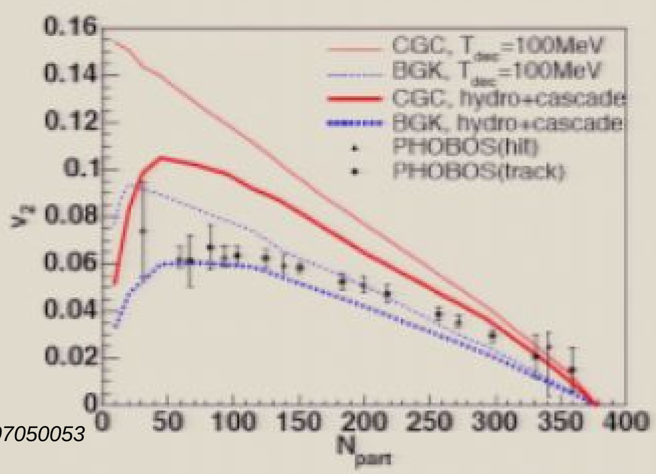
How small really is the viscosity?

CGC initial conditions lead to larger ellipticity,





require some viscous effects:



T.Hirano, U.Heinz, DK, R.Lacey, Y. Nara, hep-ph/051104

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T.Hirano, U.Heinz, DK, R.Lacey, Y. Nara, hep-ph/0511046

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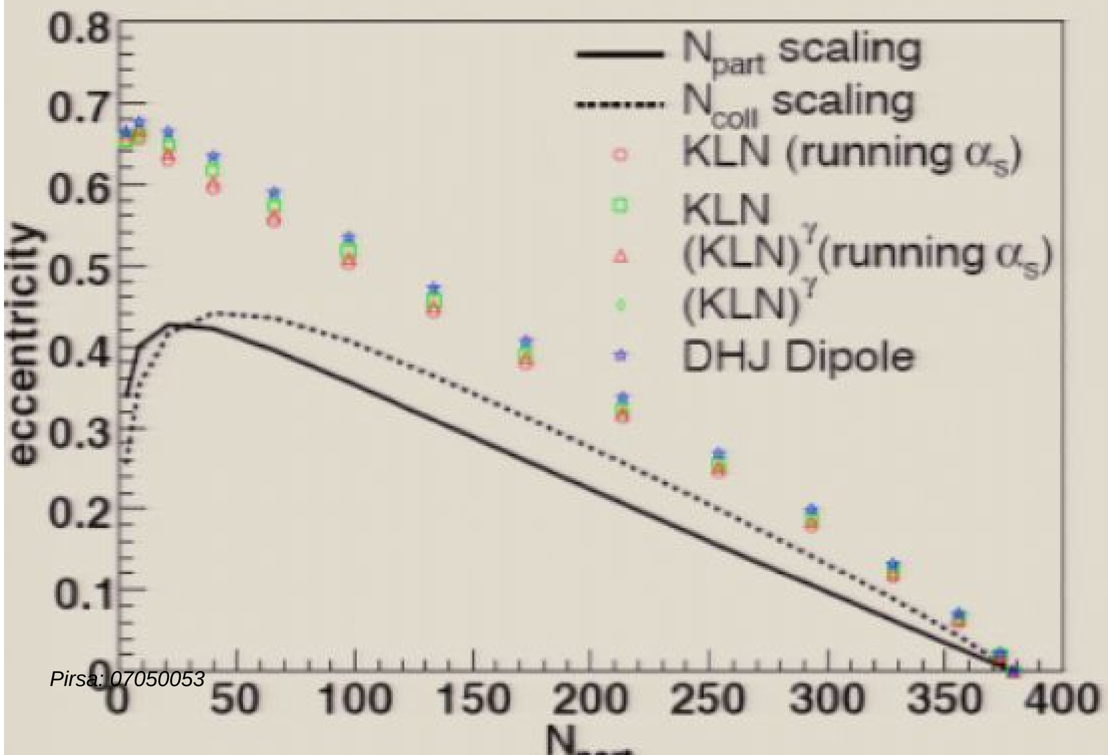
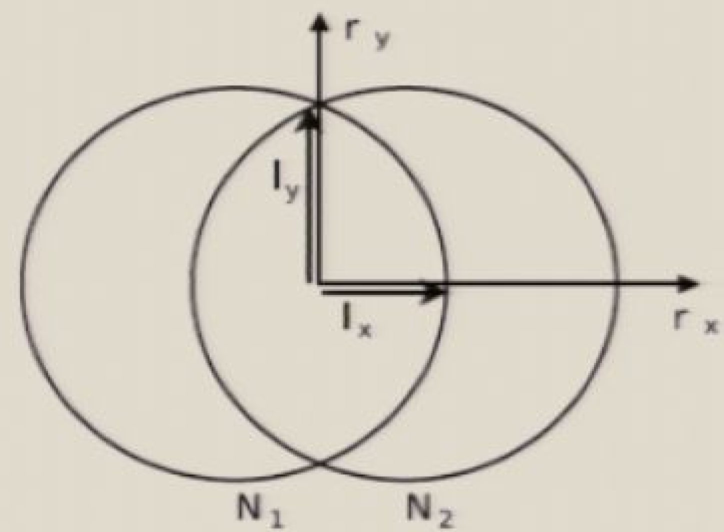
H.Drescher, A.Dumitru,A.Hayashigaki,Y.Nara, nucl-th/0605012





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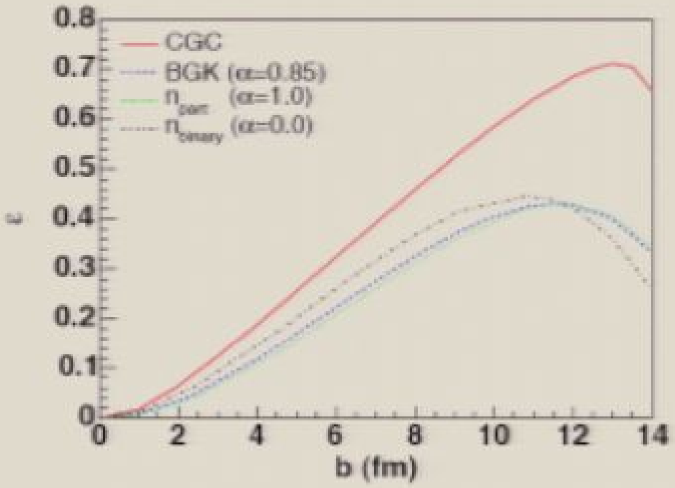

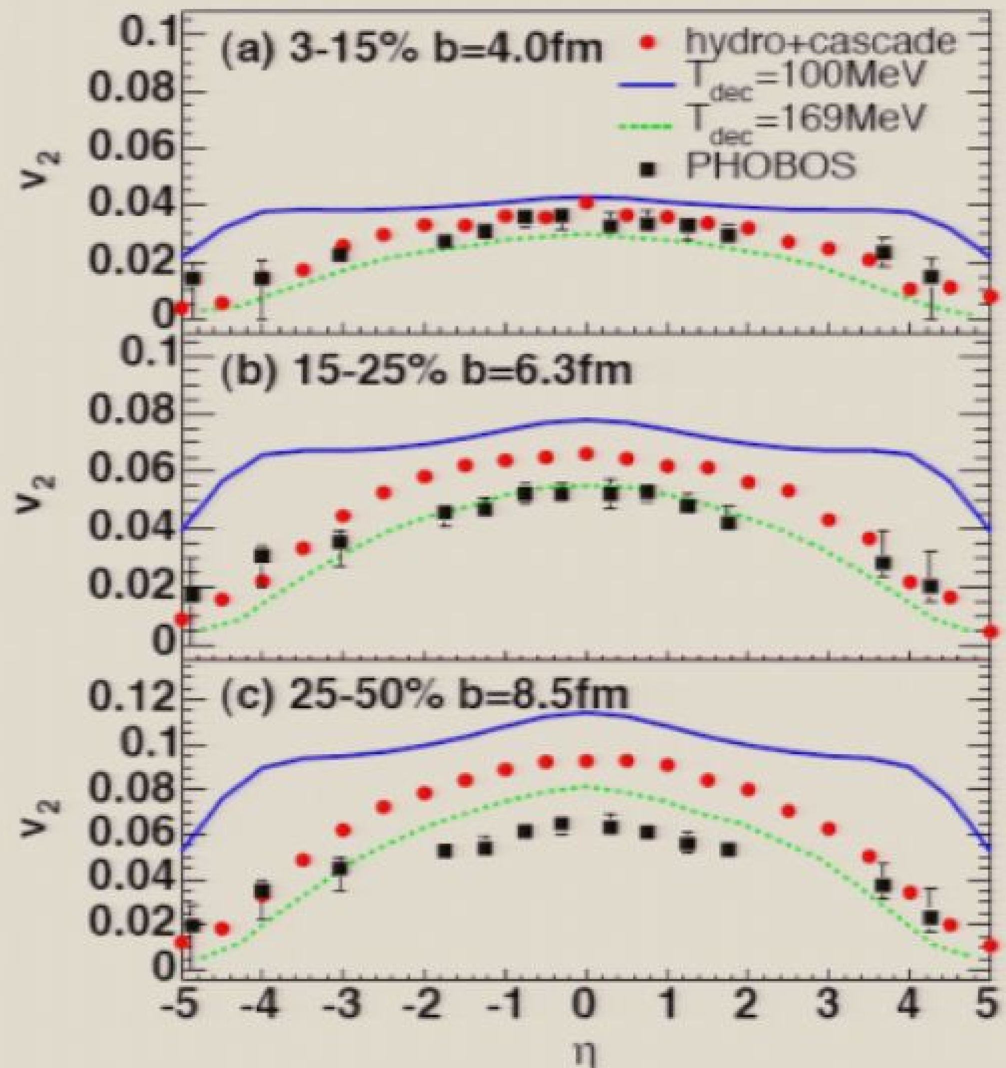

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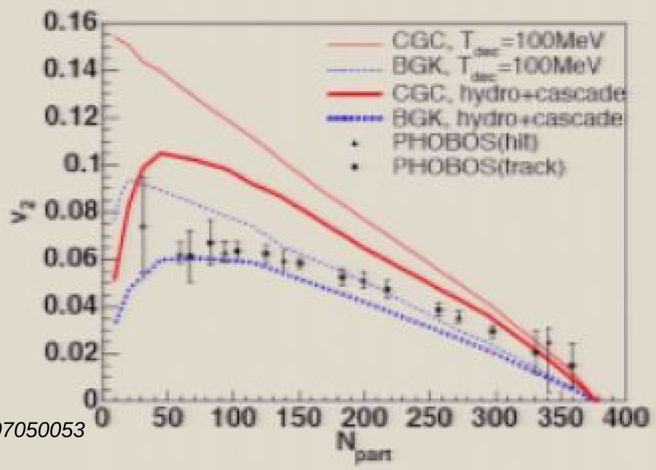

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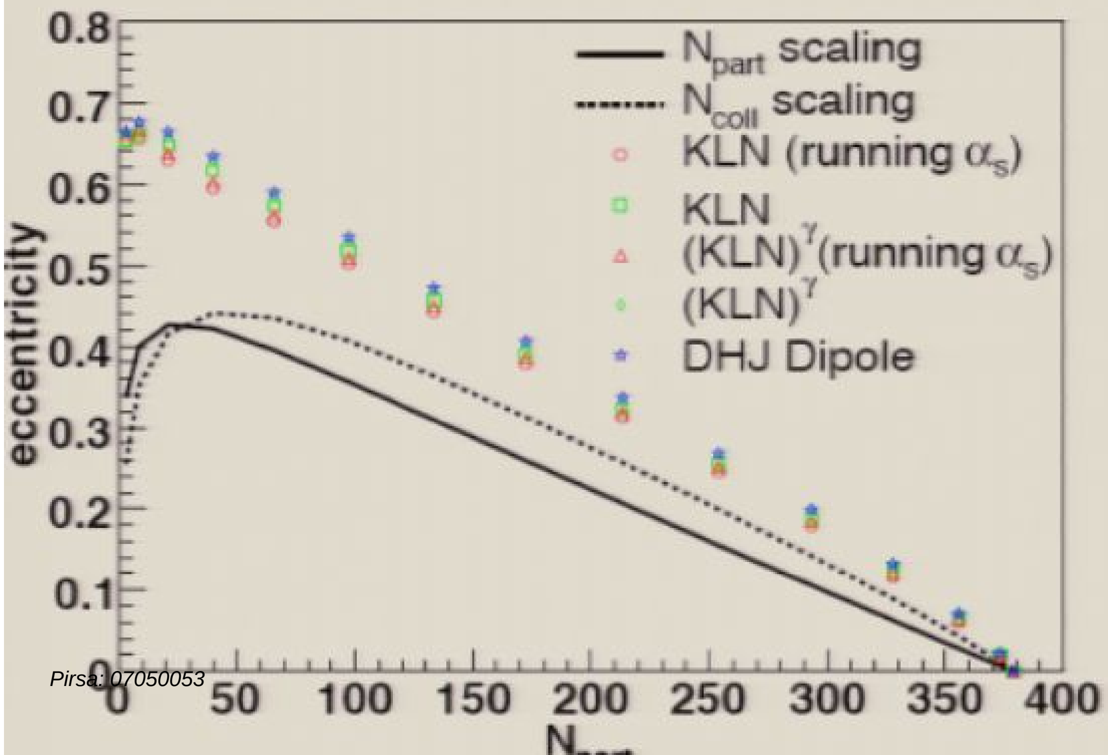

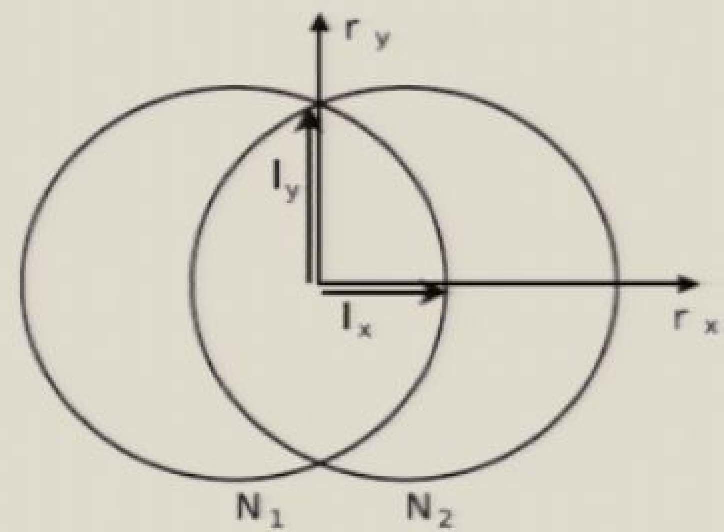

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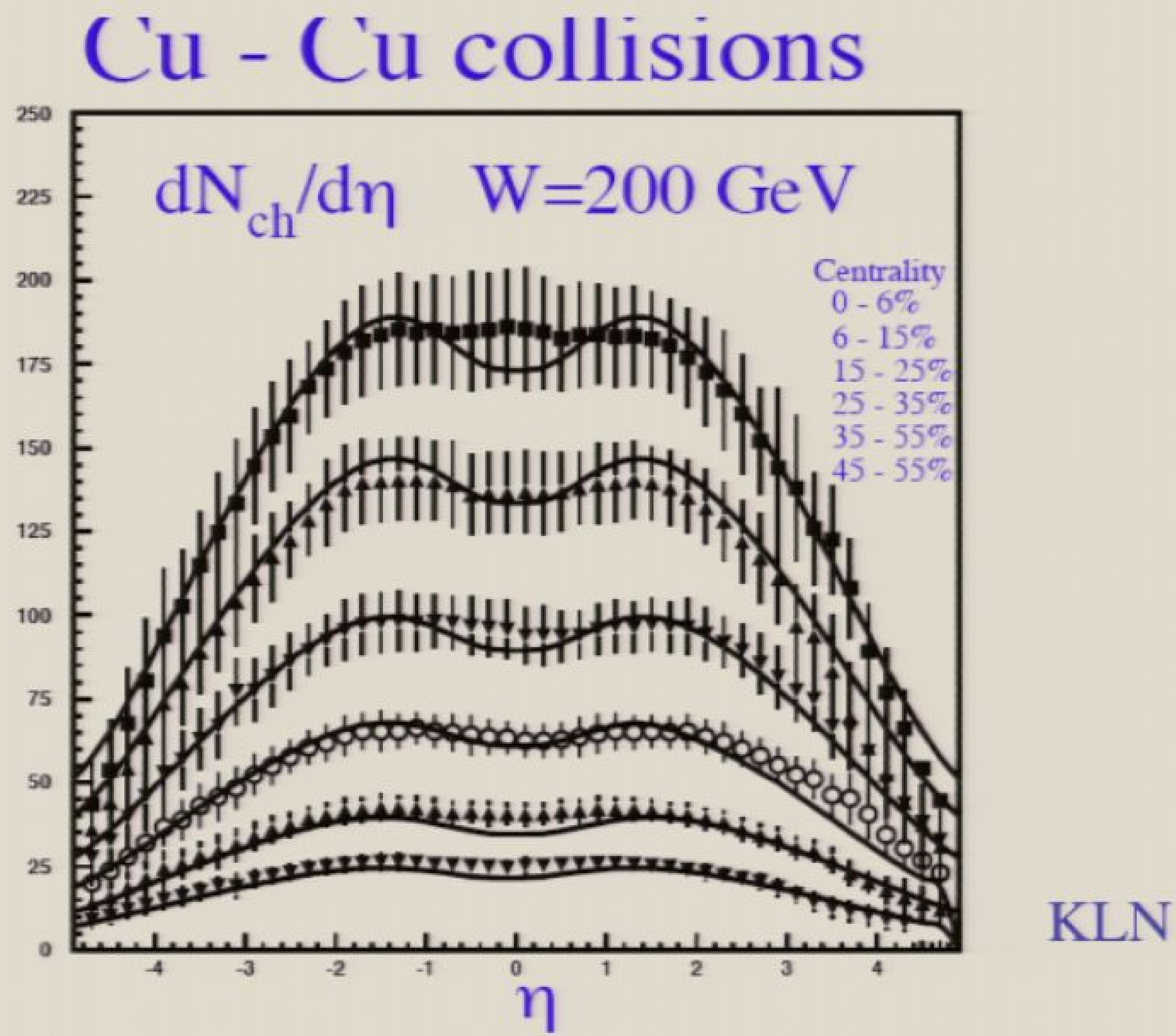

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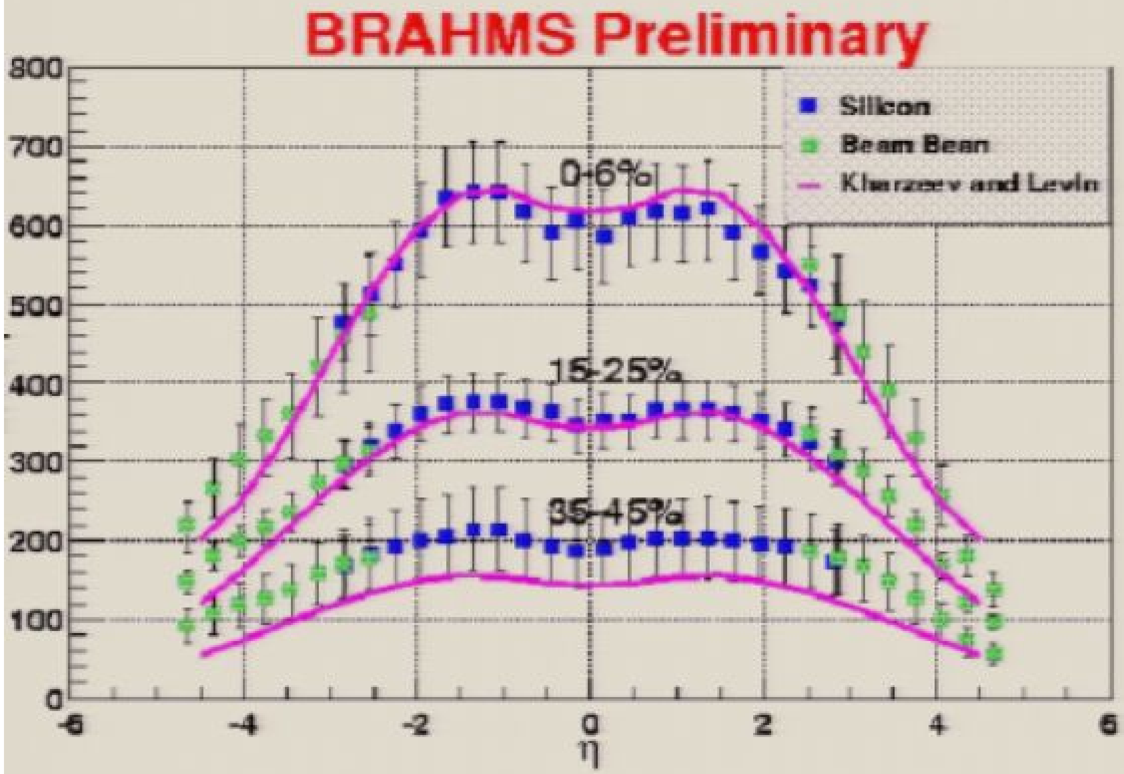

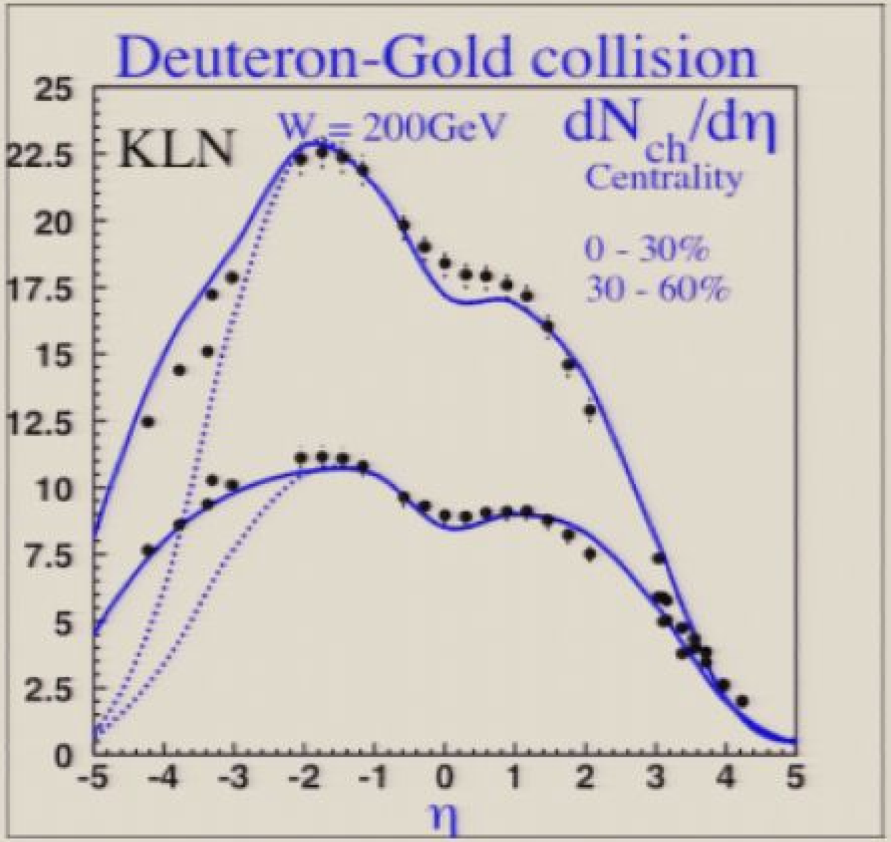

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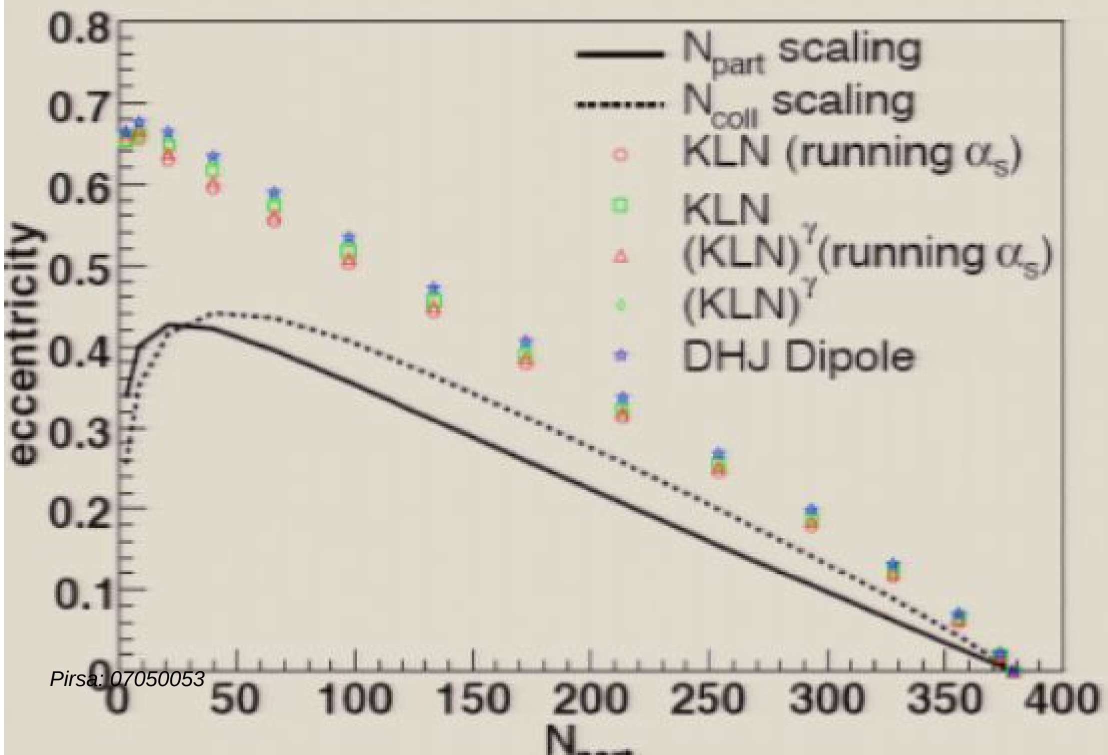

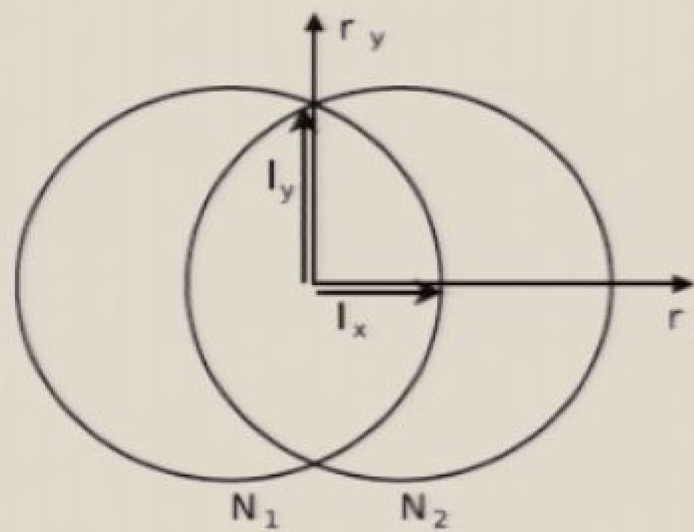

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Low energy theorems and bulk viscosity

Sum rule for the spectral density:

$$2\int_0^\infty \frac{\rho(u,\vec{0})}{u}\,du = -\left(4 - T\frac{\partial}{\partial T}\right)\langle\theta\rangle_T = T^5\frac{\partial}{\partial T}\frac{(\mathcal{E}-3P)_{\mathrm{LAT}}}{T^4} + 16|\epsilon_v|$$

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$$\frac{\rho(\omega,\vec{0})}{\omega} = \frac{9\,\zeta}{\pi}\frac{\omega_0^2}{\omega_0^2+\omega^2}$$

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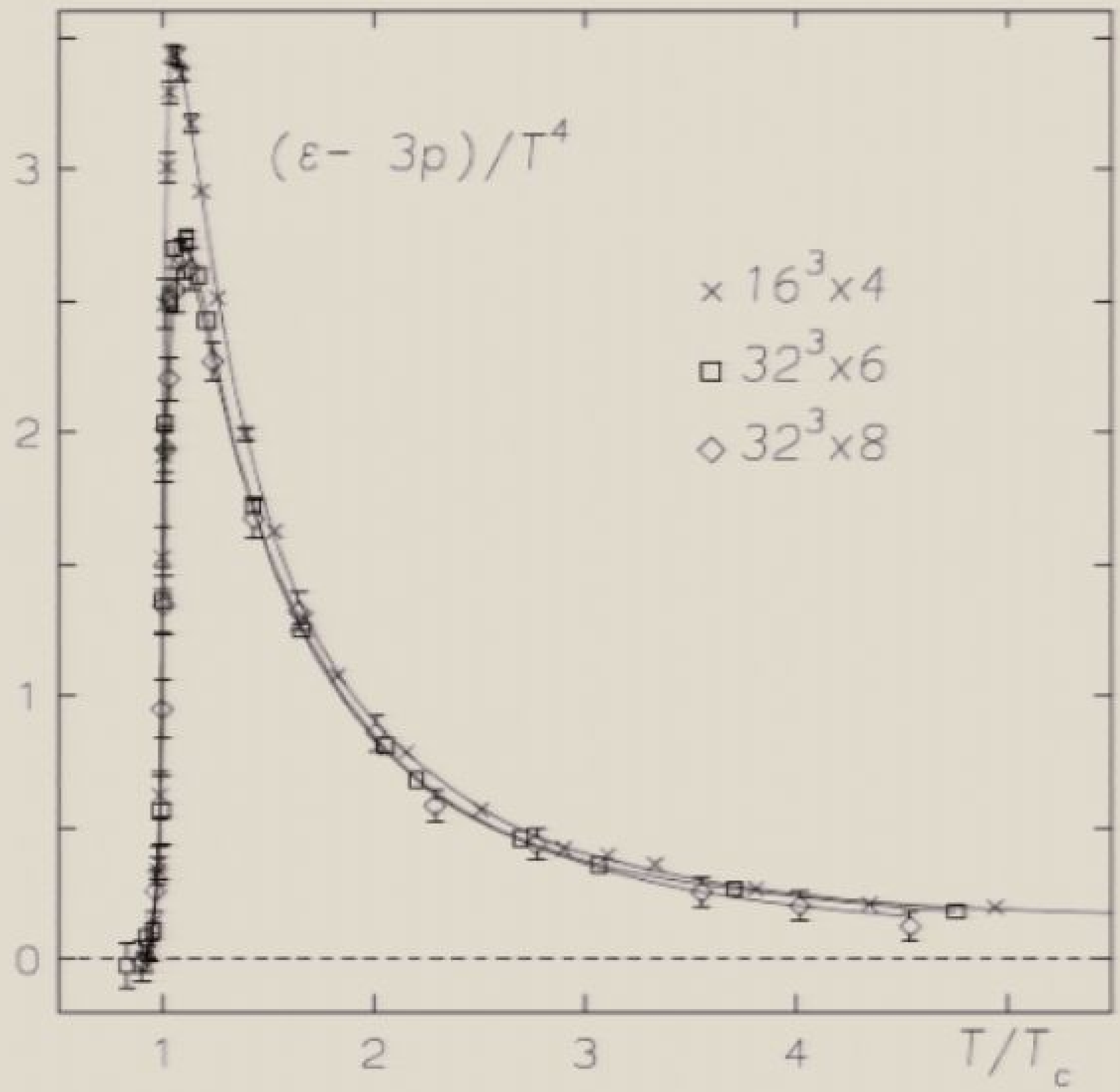
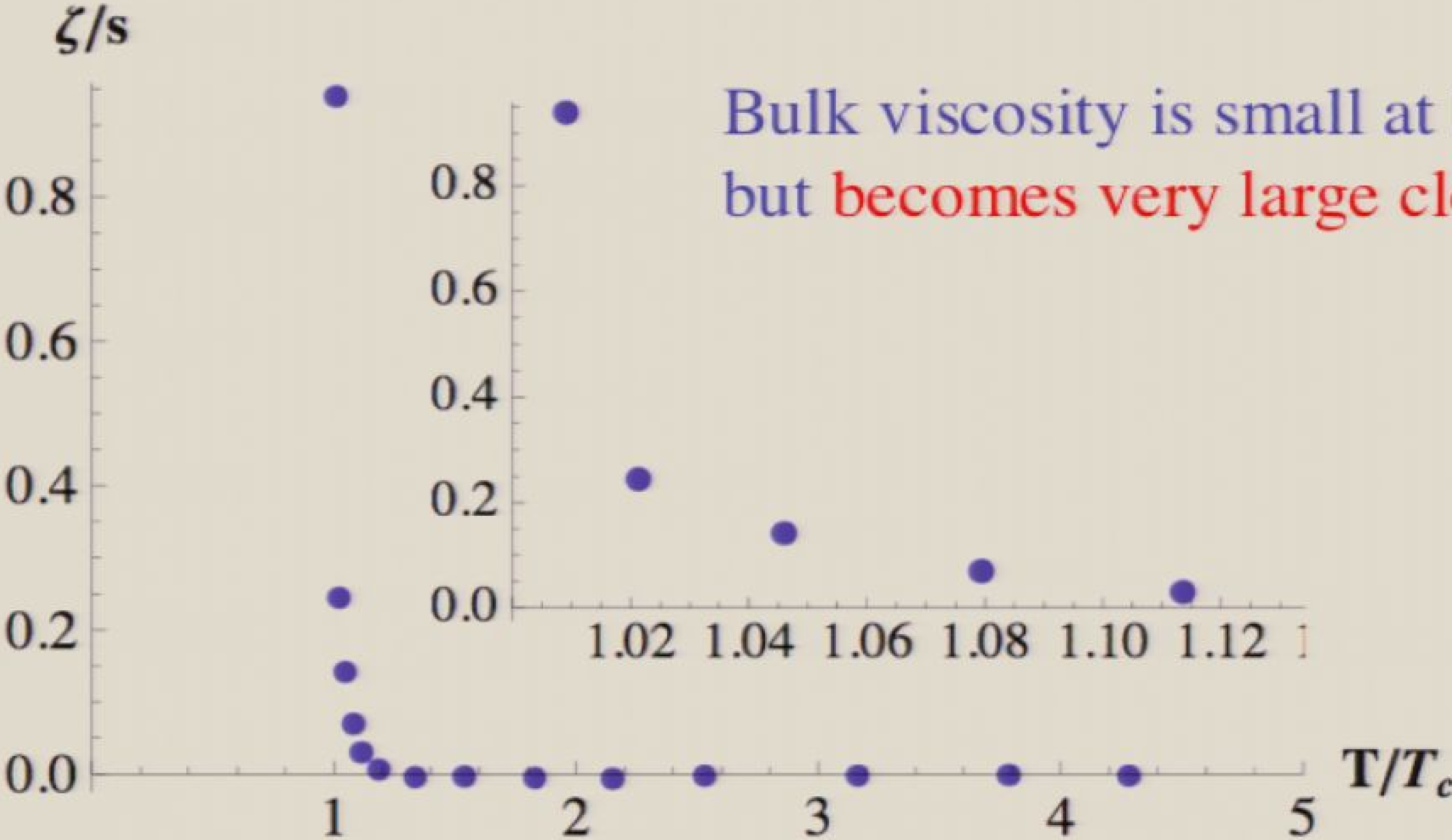
pure gauge



Use the lattice data from G.Boyd, J.Engels, F.Karsch, E.Laermann, C.Legeland, M.Lutgeimer, B.Petersson, hep-lat/9602007

The result



Bulk viscosity is small at high T, but becomes very large close to T

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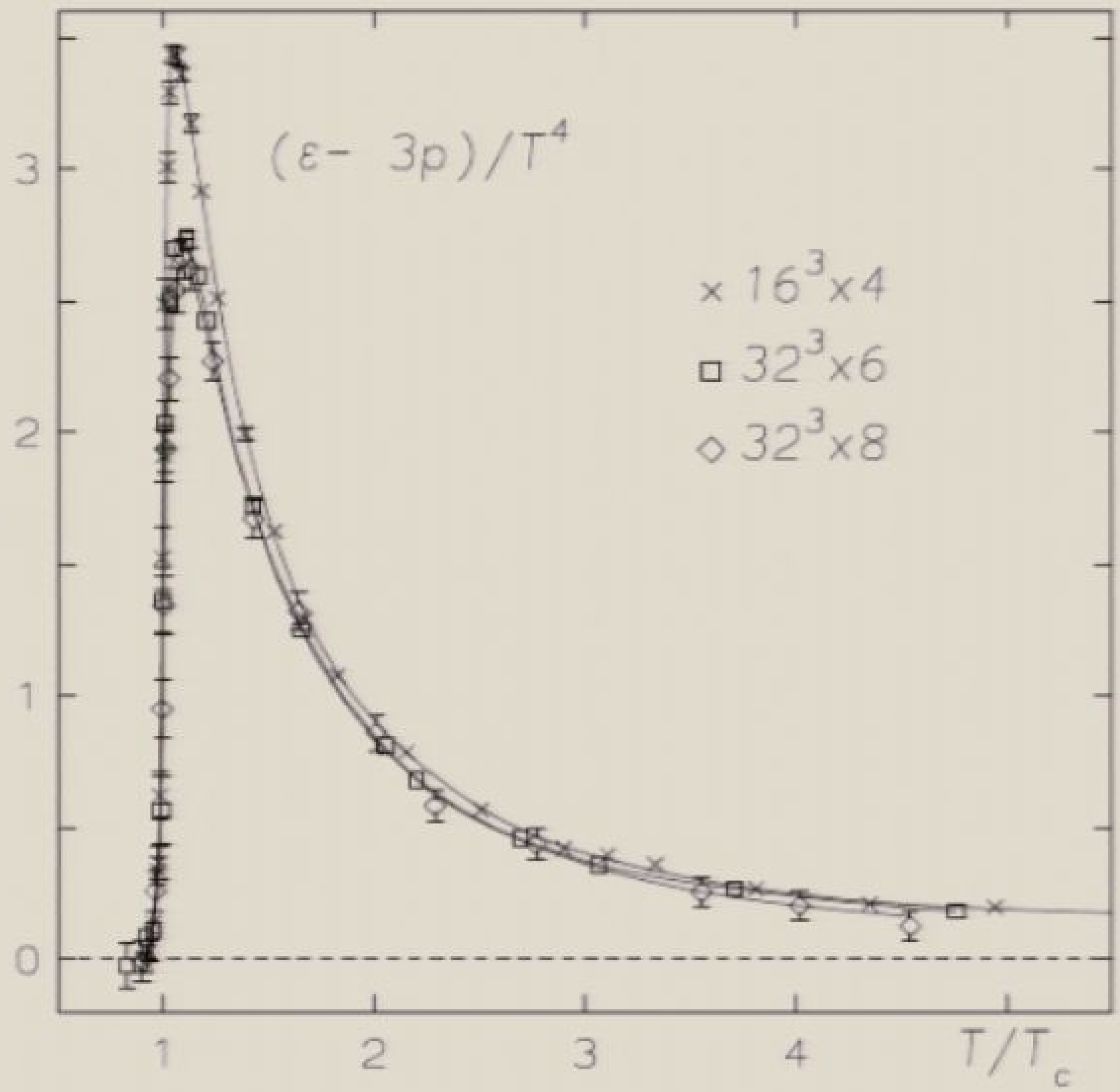

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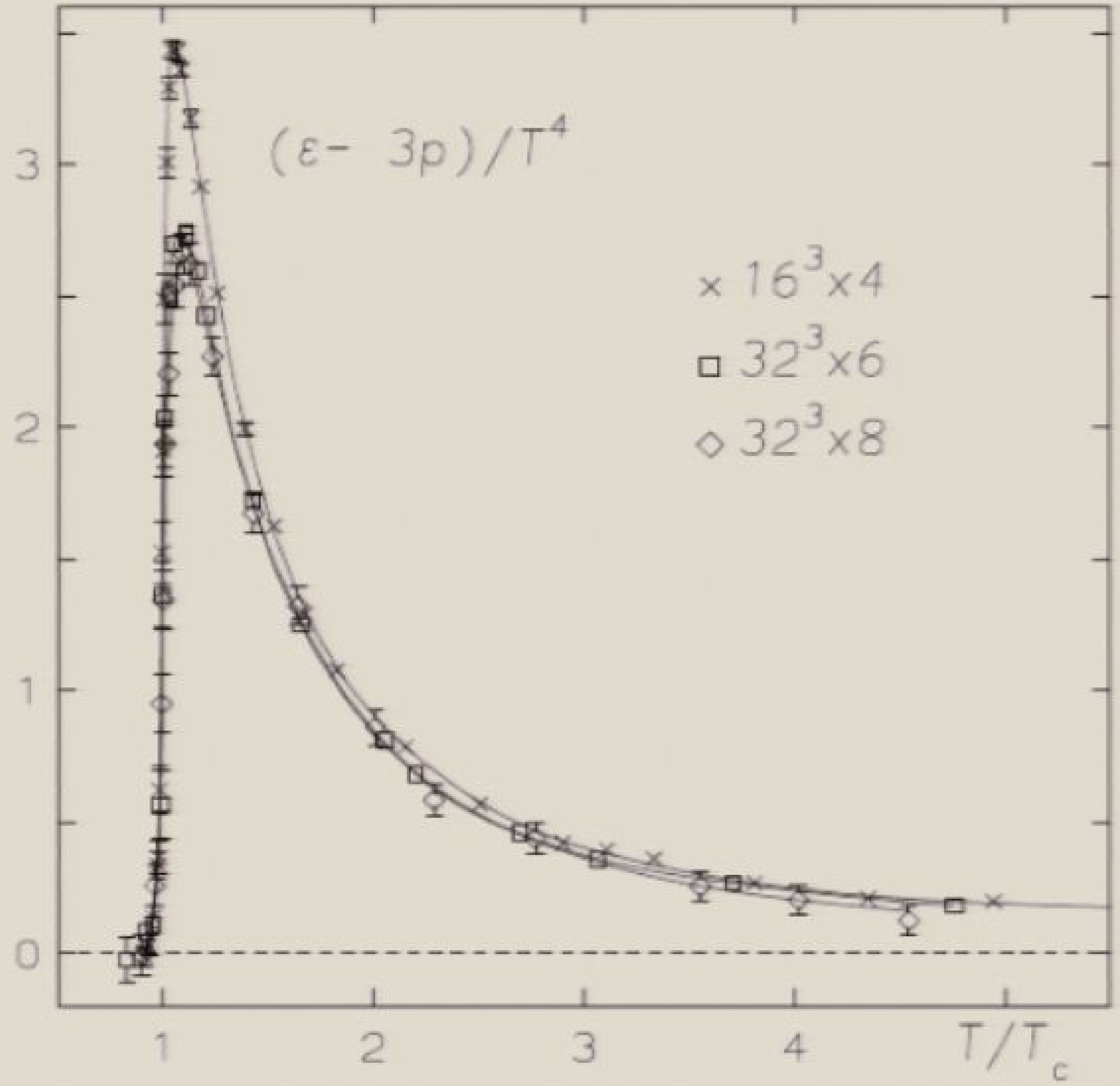

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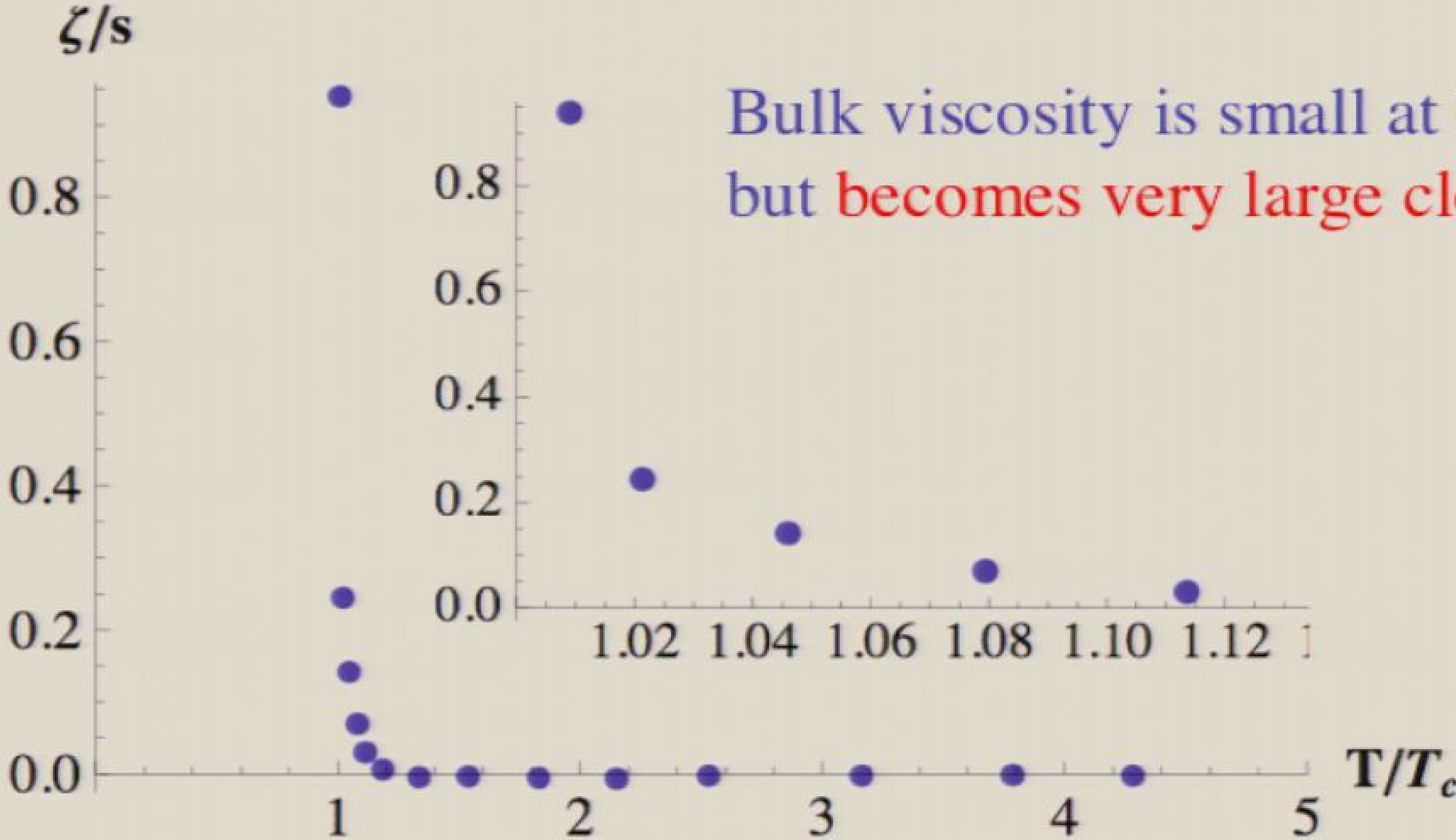

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Can we say anything about non-perturbative effects?

At zero temperature, broken scale invariance leads to a chain of low-energy theorems for the correlation functions of $\partial^\mu s_\mu = \theta^\mu_\mu$

Novikov, Shifman, Vainshtein, Zakharov '81

(elegant geometrical interpretation - classical theory in a curved background)

Migdal, Shifman '82; DK, Levin, Tuchin '04

These theorems have been generalized to finite T:

$$G^E(0,\vec{0}) = \int d^4x \, \langle T\theta(x), \theta(0)\rangle = \left(T\frac{\partial}{\partial T} - 4 \right) \langle \theta \rangle_T$$

Ellis, Kapusta, Tang '98

What is the role of broken scale invariance in the transport properties of QCD plasma?

Consider the bulk viscosity:

$$T_{ij} = P_{\rm eq}(\epsilon)\,\delta_{ij} - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k\right) - \zeta\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{u}$$

it can be computed through

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3r\, e^{i\omega t}\,\langle[\theta^\mu_\mu(x),\theta^\mu_\mu(0)]\rangle$$

Since $\partial^\mu s_\mu = \theta^\mu_\mu$ is a measure of deviation from conformal invariance, bulk viscosity characterizes the importance of scale anomaly

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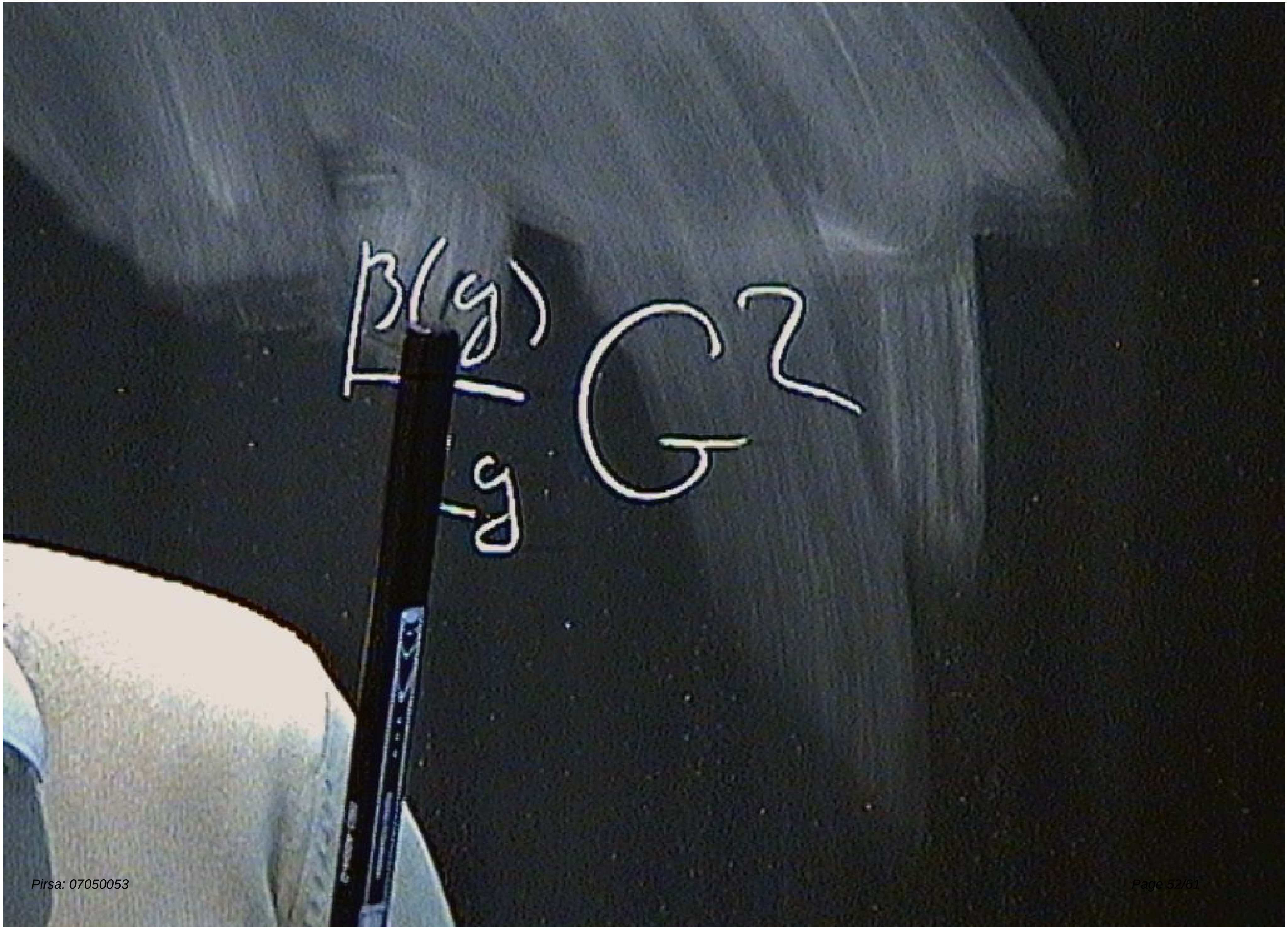
$\frac{\beta(g)}{g} G^2$

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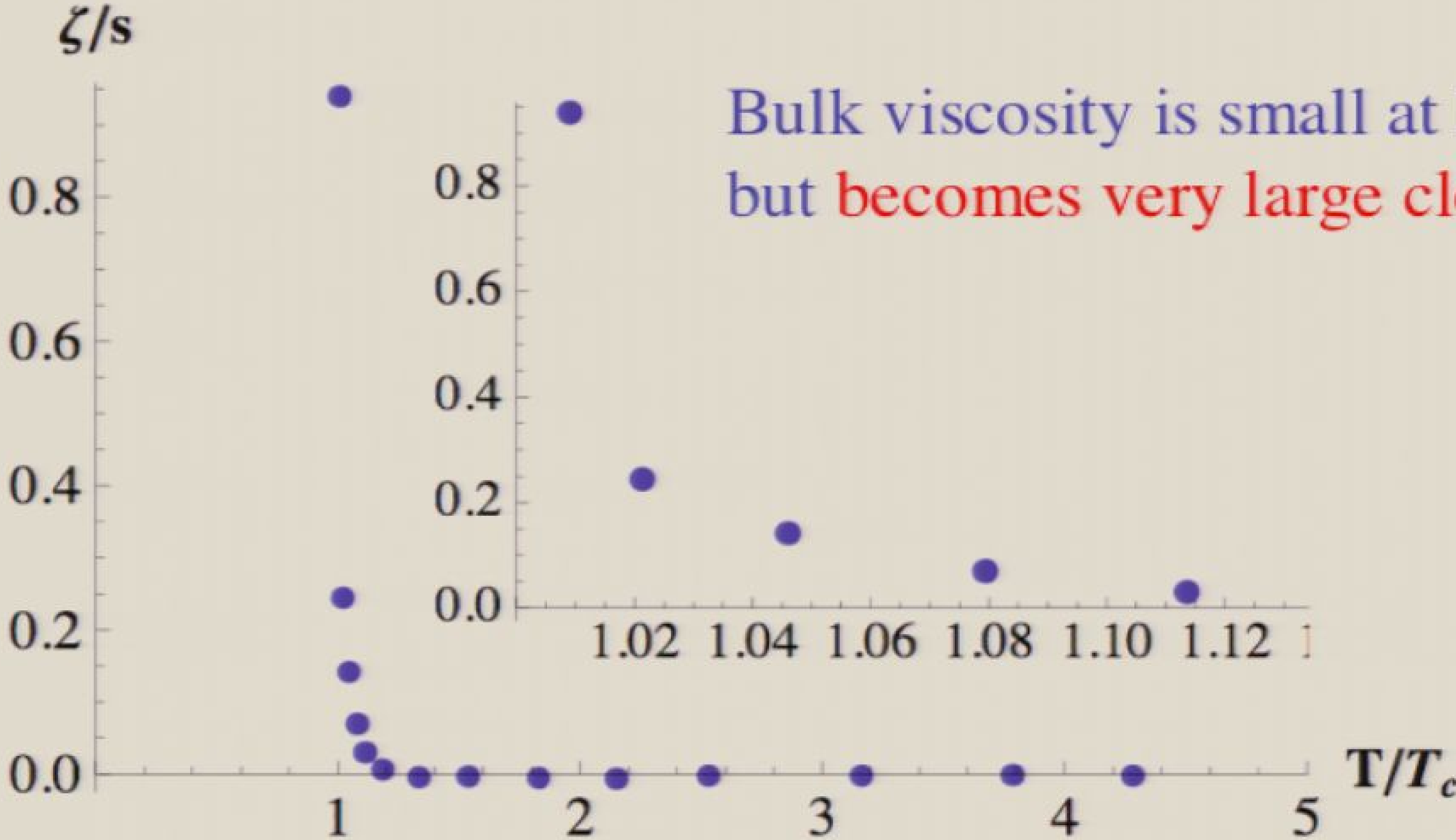

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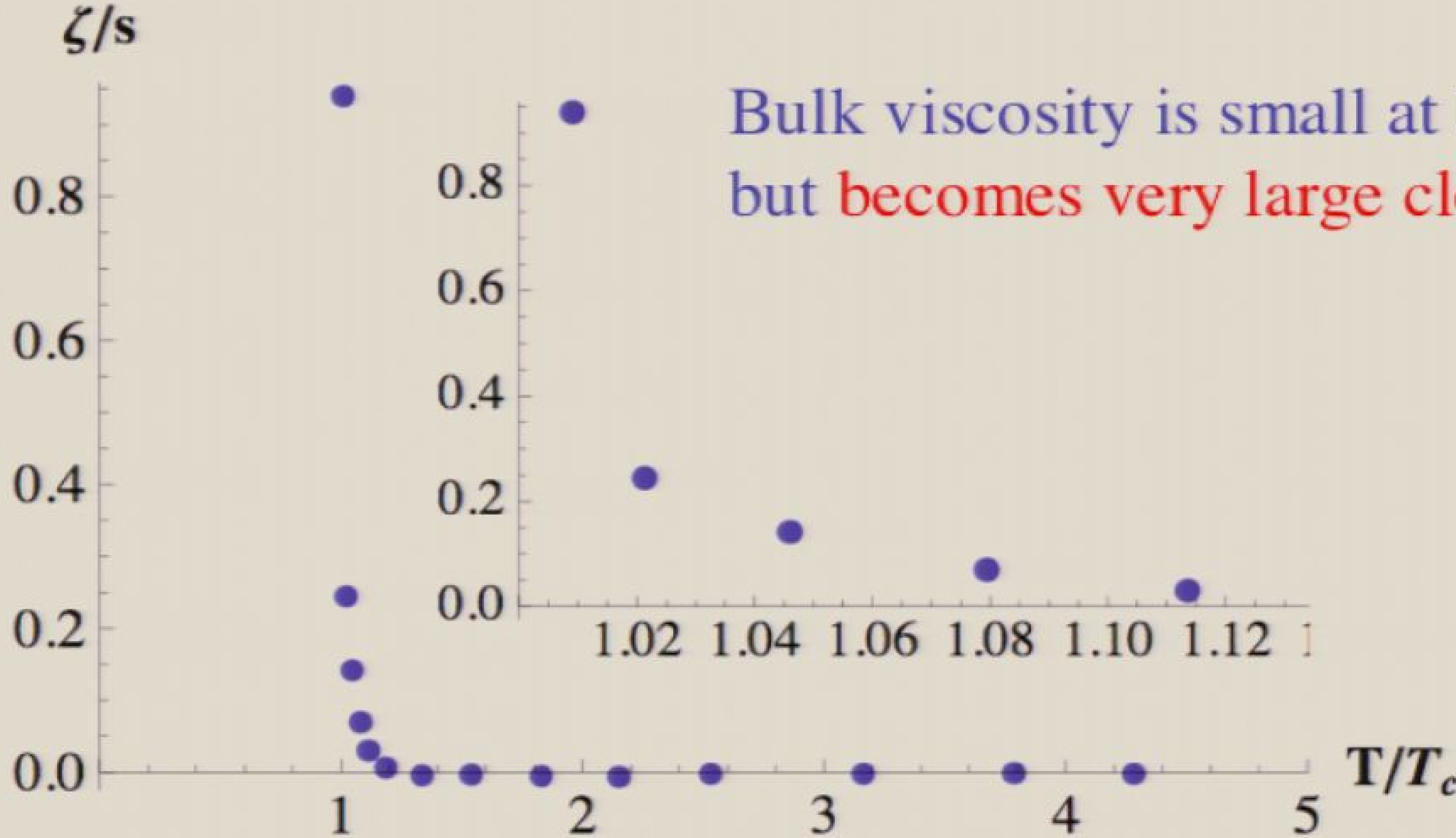

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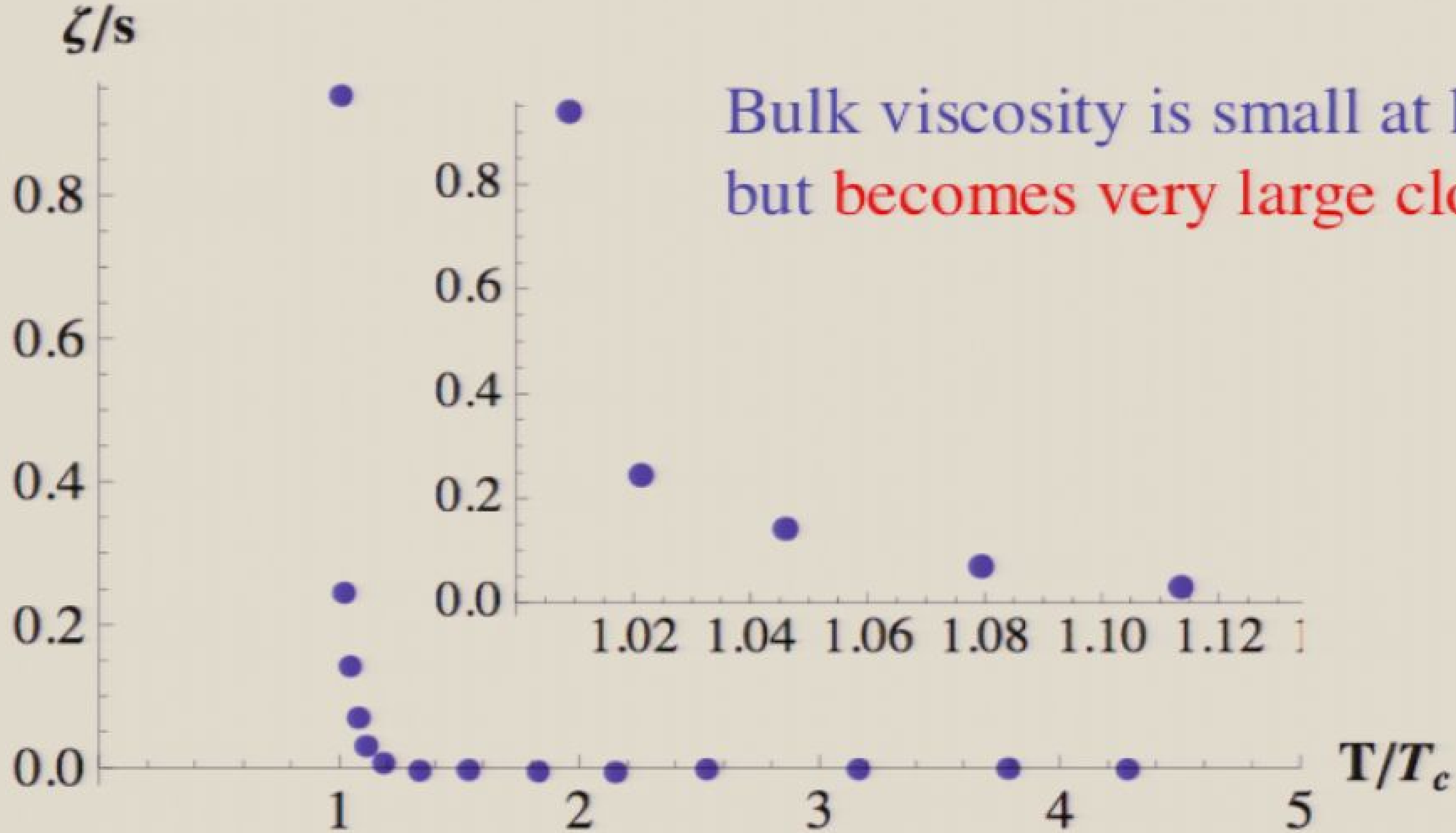

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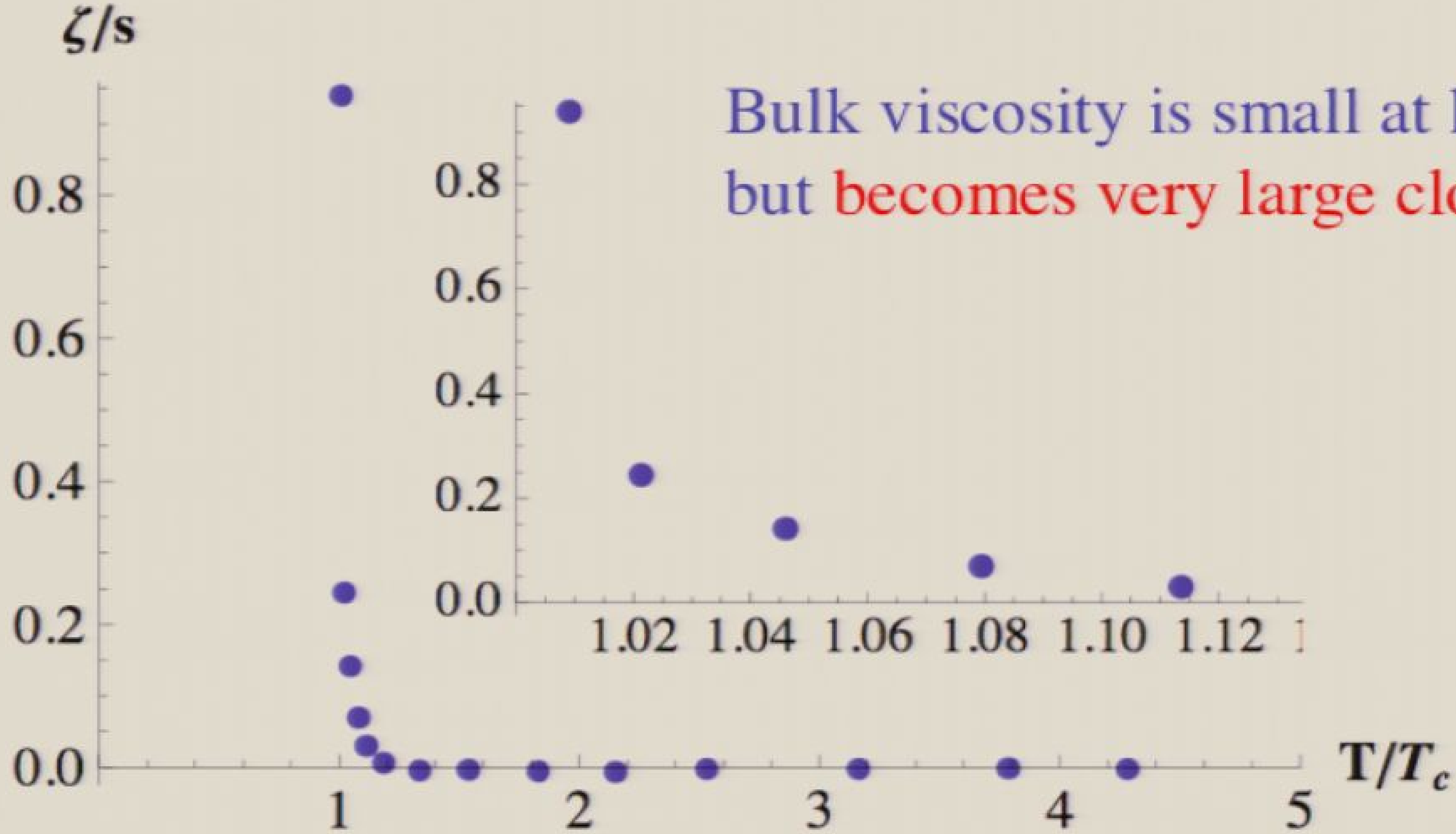

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Summary

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Incorporate scale invariance breaking in the dual description?

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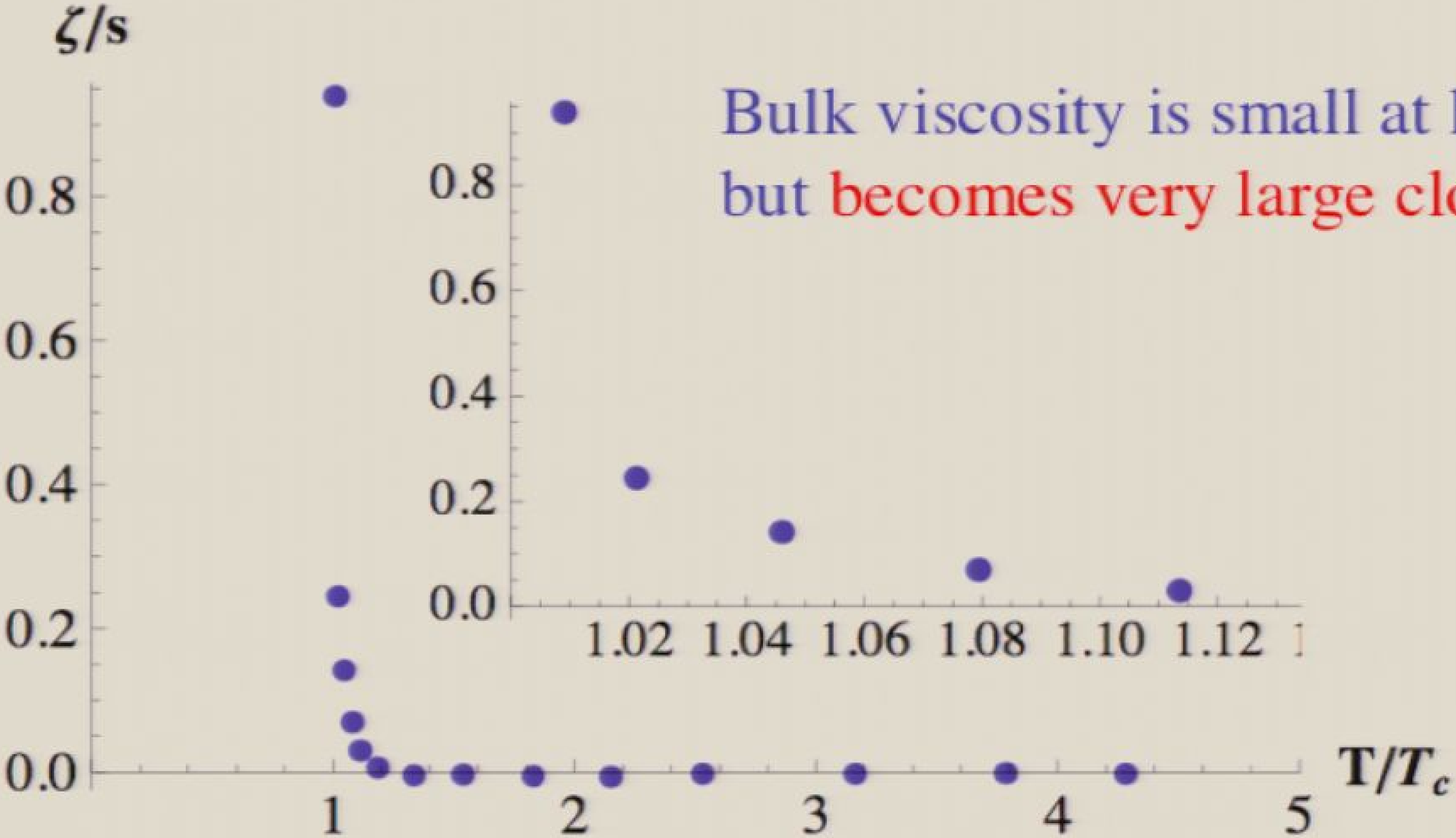

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