Title: QCD vs. N=4 SYM: Shear Viscosity

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Abstract:

QCD versus $\mathcal{N}=4$ SYM: Shear viscosity

Guy D. Moore, with Simon Caron-Huot, Sangyong Jeon

Outline:

- What does a Heavy Ion Collision look like?
- What is viscosity and is the QGP an ideal fluid?
- Claimed bound on η/s: $\mathcal{N}=4$ Super-Yang-Mills
- What is $\mathcal{N}=4$ Super-Yang-Mills anyway
- Really comparing QCD and $\mathcal{N}=4$ SYM

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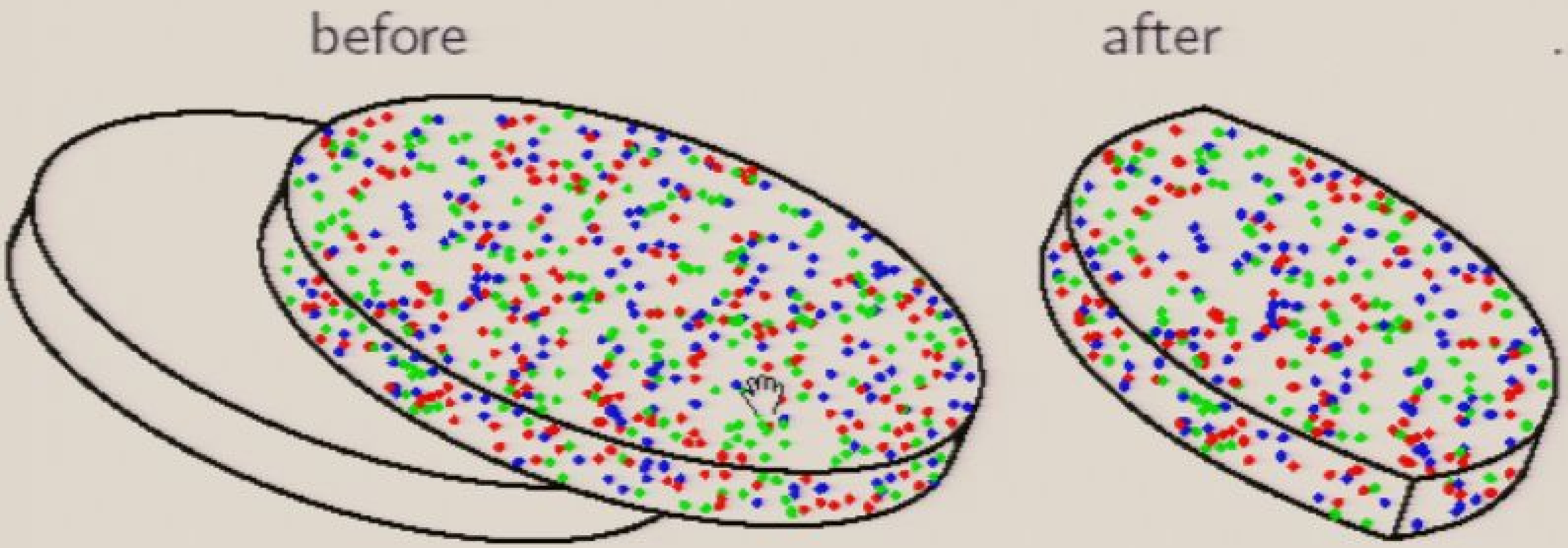
What a heavy ion collision looks like

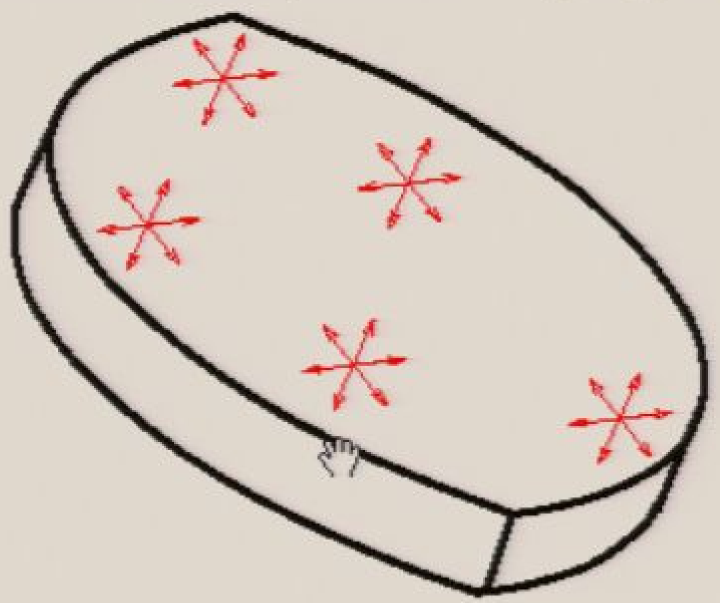


Lorentz flattened nuclei collide, form "flat almond" shaped region of plasma

Weak coupling picture

Almost free quarks+gluons $\Rightarrow$ fly in straight lines

Each chunk of almond has particles flying in each direction

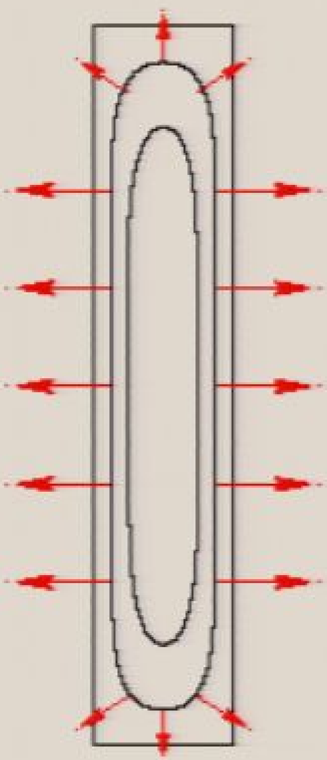


If they NEVER re-scatter: this is hadron distribution too

No preferred direction in detector: azimuthal symmetry

Strong coupling picture

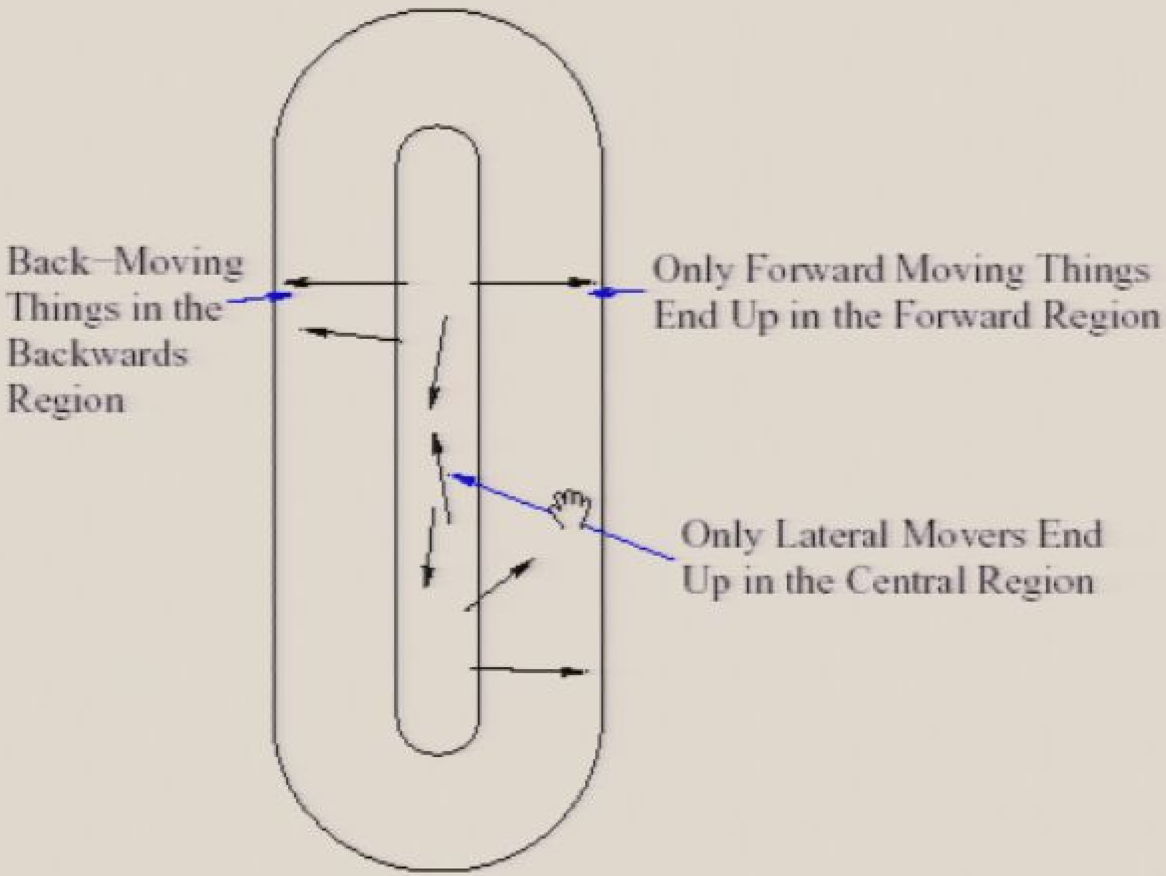
Plasma acts like a fluid, with pressure and press. gradients



Large pressure gradients forward and backward. Vertical gradient small and in small area. Most of fluid starts flowing forward or backward. Particle momenta will be thermal **PLUS** fluid CM component. More forward & backward, less in-plane.

Momentum Selection

Another way of thinking of the same thing



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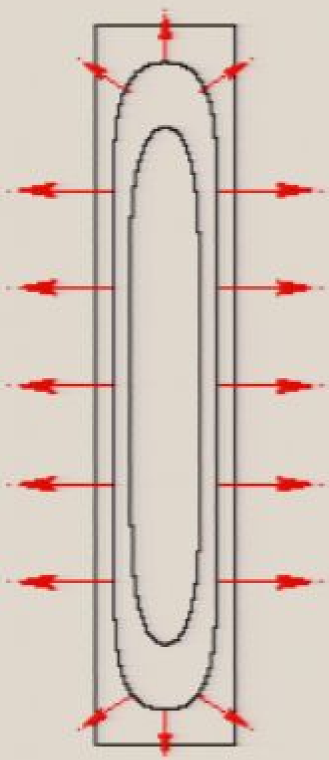

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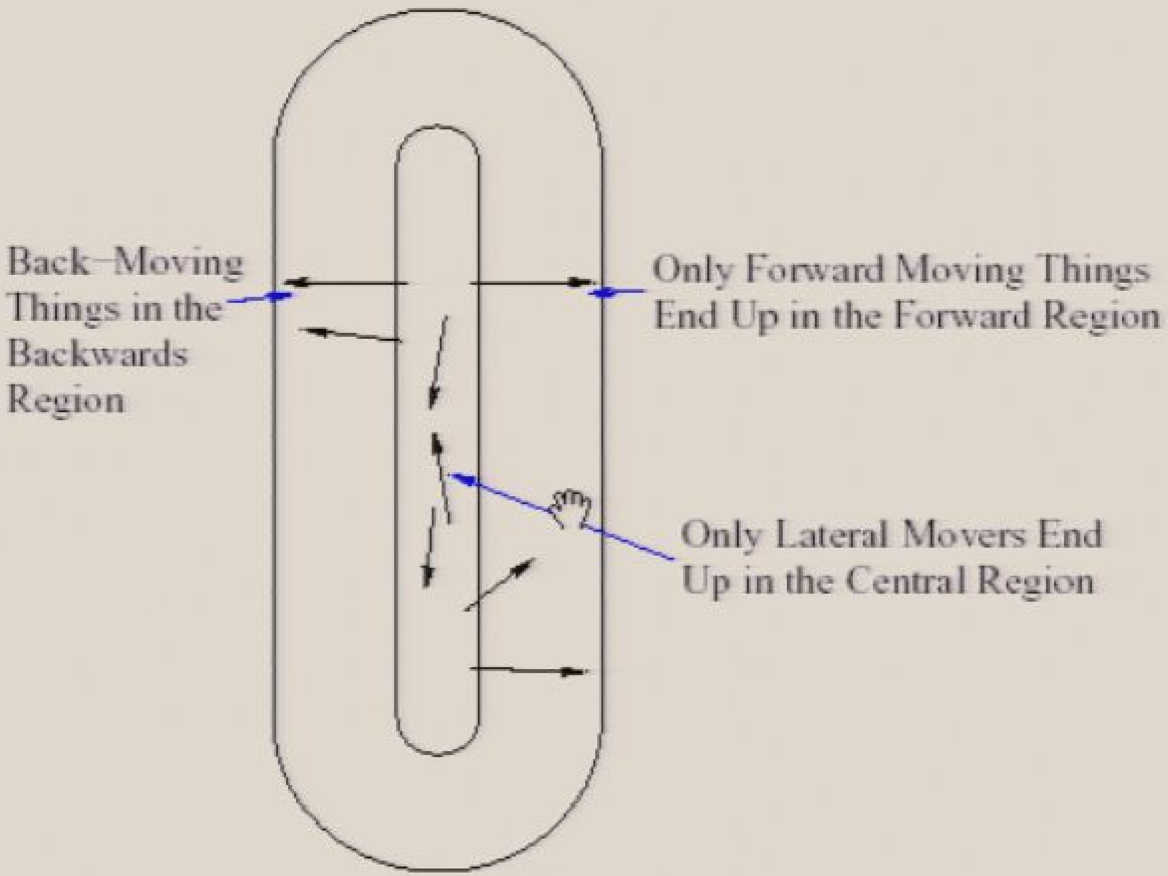

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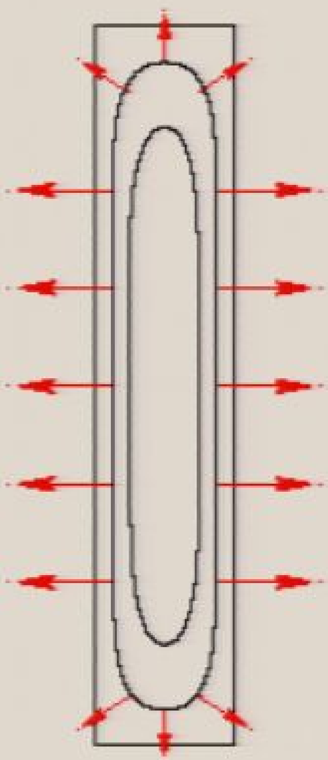

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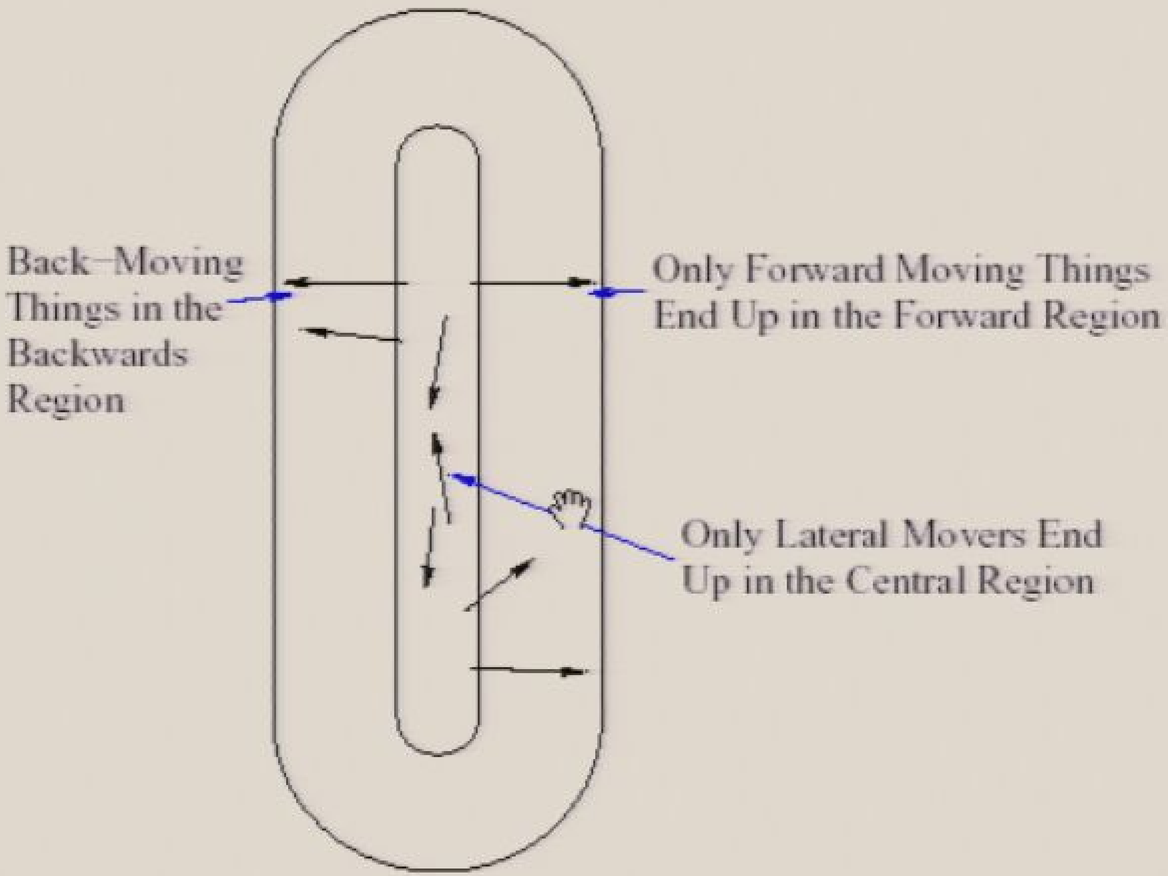
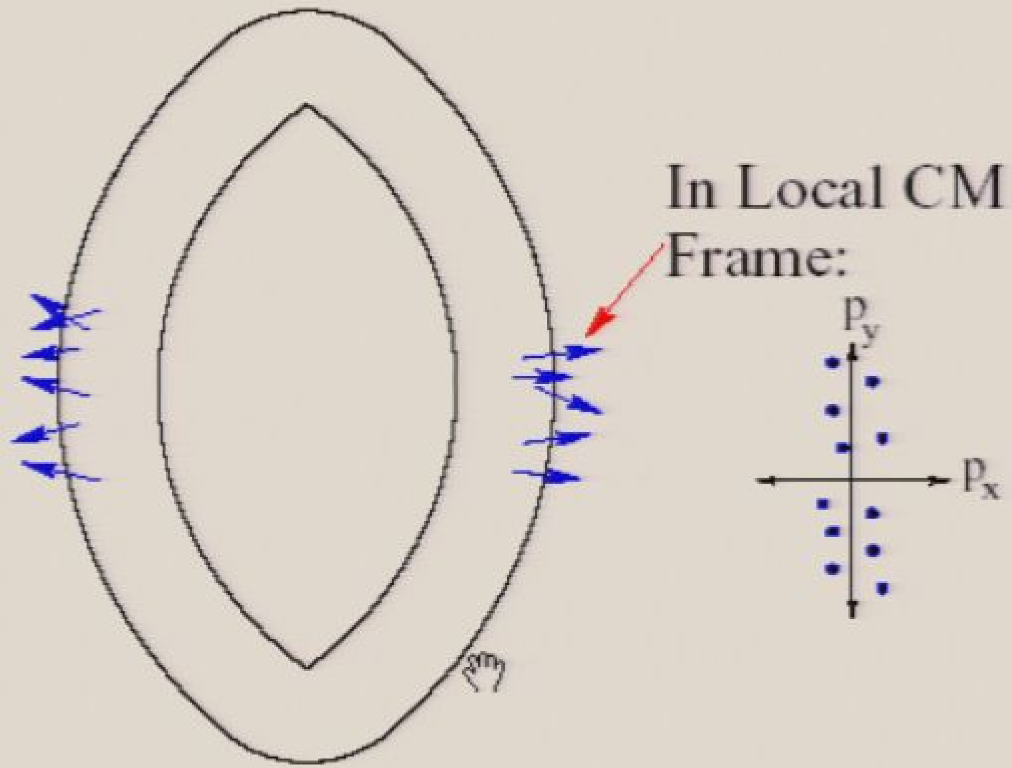


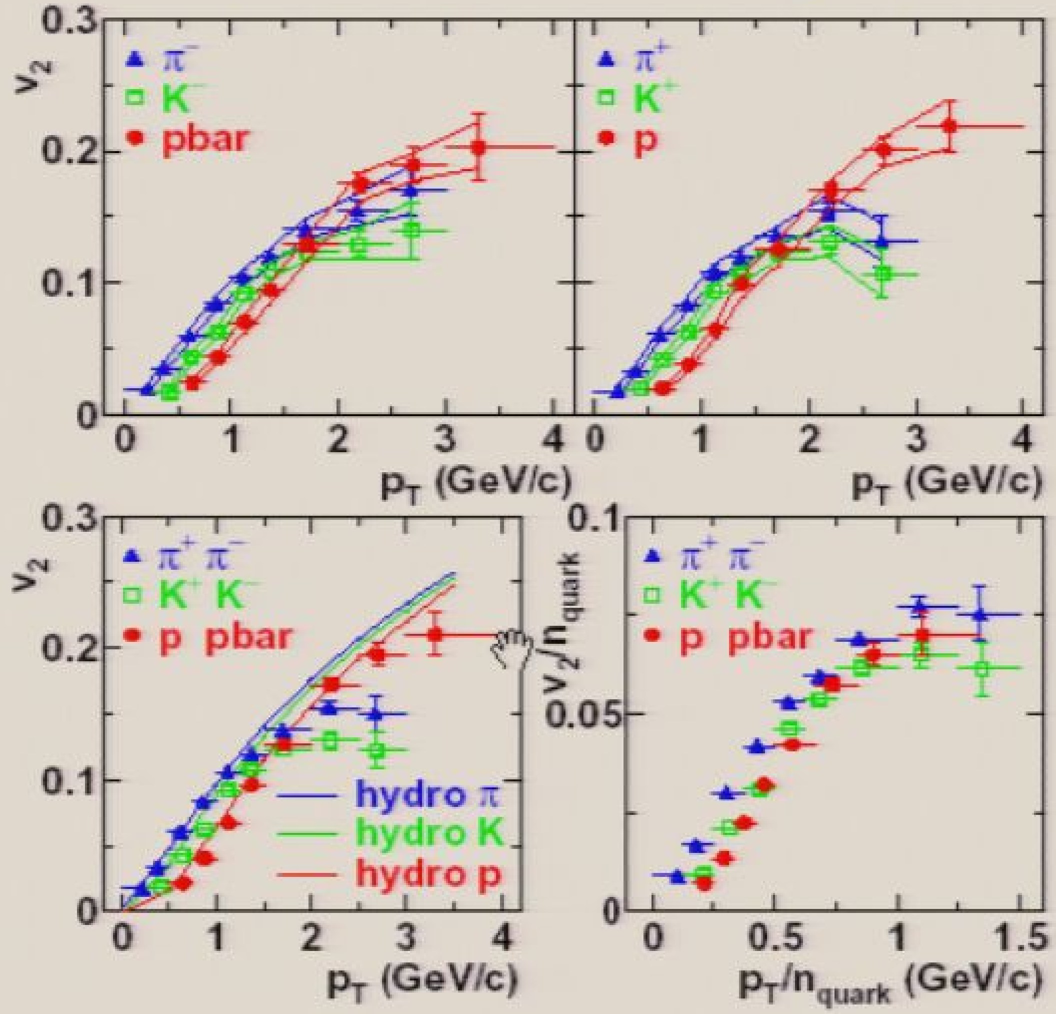
Same is true in transverse plane



Scattering converts p_y^2 into p_x^2.

A measure: $v_2 \equiv \langle \cos 2\theta \rangle \simeq \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2}$

And the data says



Observed v_2 as large as can be.

Namely, v_2 maximum if rescattering perfect–ideal hydrodynamics. Ideal hydro calculations get v_2 right.

Ideal hydro: stress conservation and an equation of state

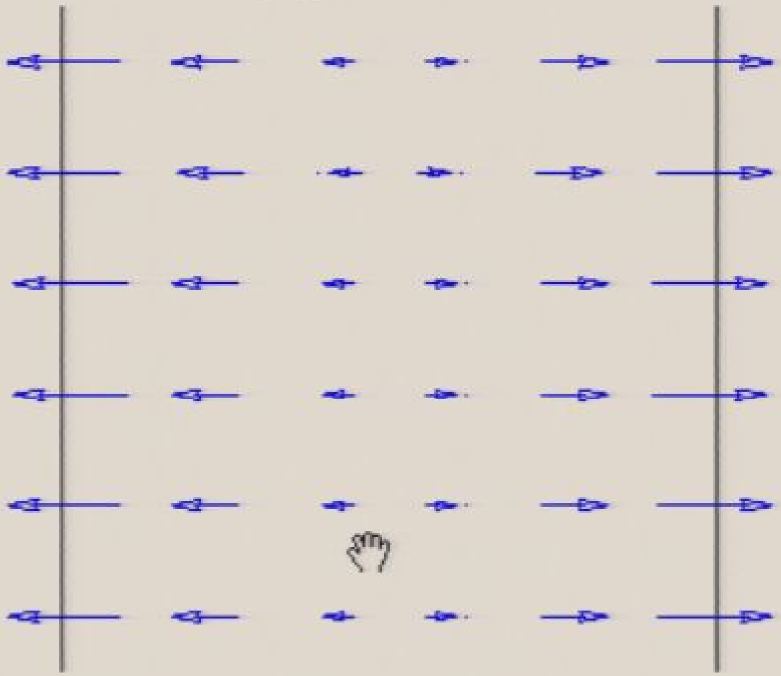
$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = P\eta^{\mu\nu} + (P+\epsilon)u^\mu u^\nu \qquad P = P(\epsilon)$$

Leading corrections: viscosity. In local rest frame $\mathbf{u} = 0$,

$$T_{ij} = P\delta_{ij} - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2\delta_{ij}}{3}\partial_k u_k\right) - \zeta\delta_{ij}\partial_k u_k$$

Longitudinal expansion again

Expansion has nonzero $\partial_1 v_1$



Reduces force in 1 direction, reduces system expansion.

Viscosity similarly reduces elliptic flow, v_2.

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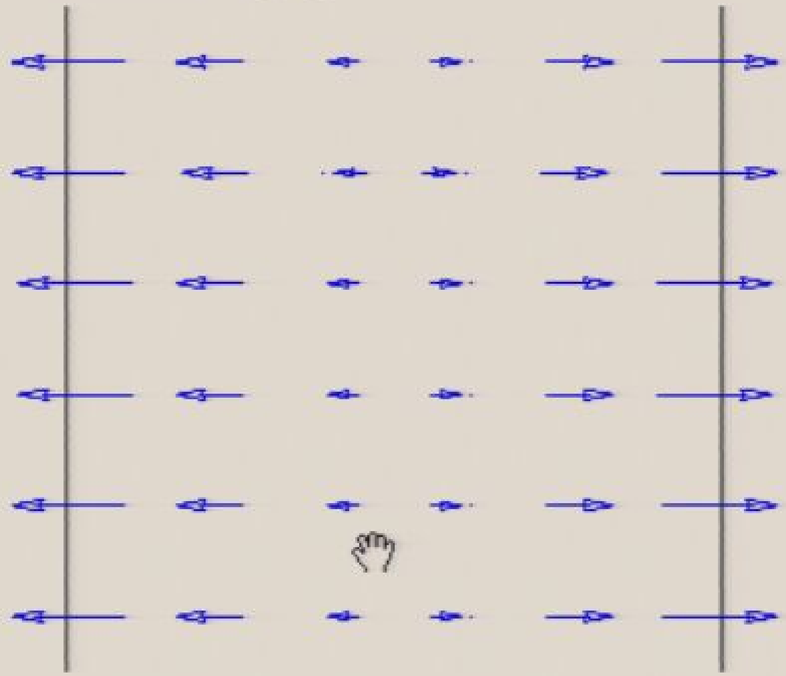

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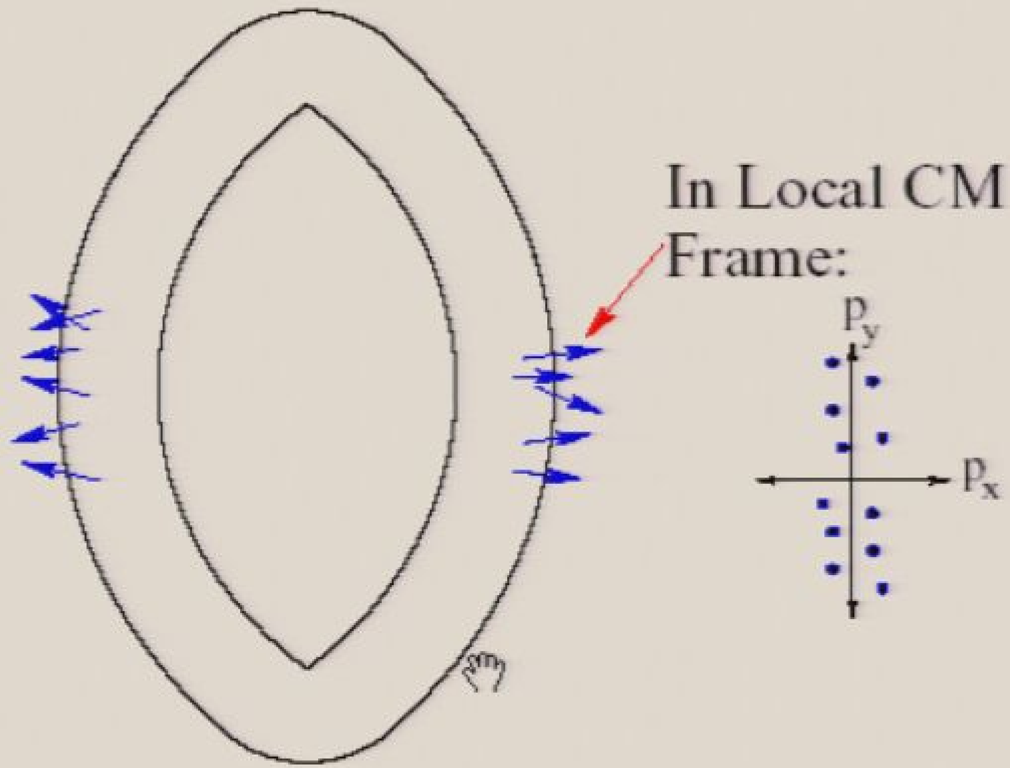

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Computing η in QCD

We can only do it reliably at weak coupling!

Quasiparticle picture: long lived quarks and gluons.

Approach to equilibrium determined by collisions $\mathcal{C}$

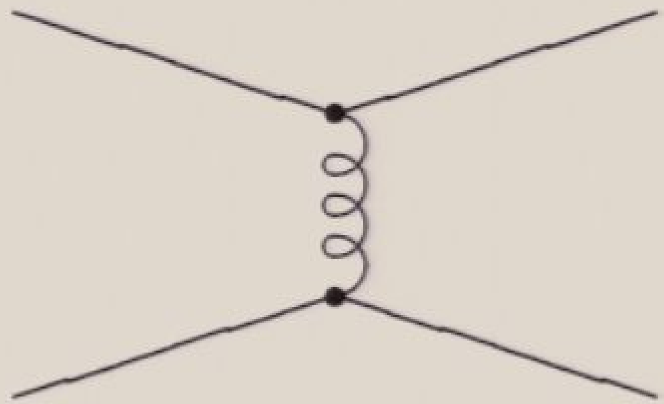
η: failure of equilibrium. Involves *inverse* of collision rate. Roughly

$$\begin{aligned} \delta T_{ij} &= \int \frac{d^3p}{(2\pi)^3 2E} p_i p_j \delta f \\ \delta f &= \mathcal{C}^{-1} \frac{p_i p_j}{E} \partial_i v_j f(1 \pm f) \end{aligned}$$

More collisions–closer to equilibrium, smaller η

Dominant collisions in QCD

The most important collisions are Coulomb scattering

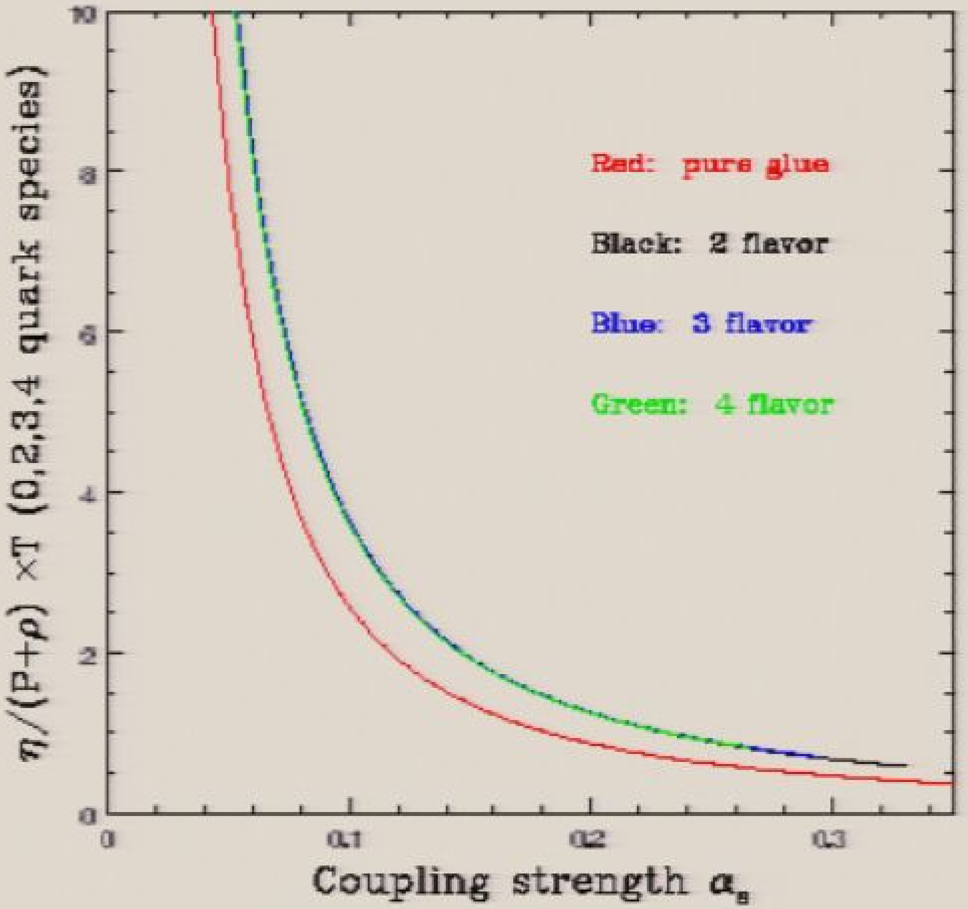


Vacuum cross-section divergent as $g^4 \int d^2Q/Q^4$

Small angle scatterings' importance $\propto Q^2$.

Thermal medium effects: importance
$\propto \int d^2Q Q^2/Q^2(Q^2 + g^2T^2)$. Finite but IR dominant.

Results in QCD



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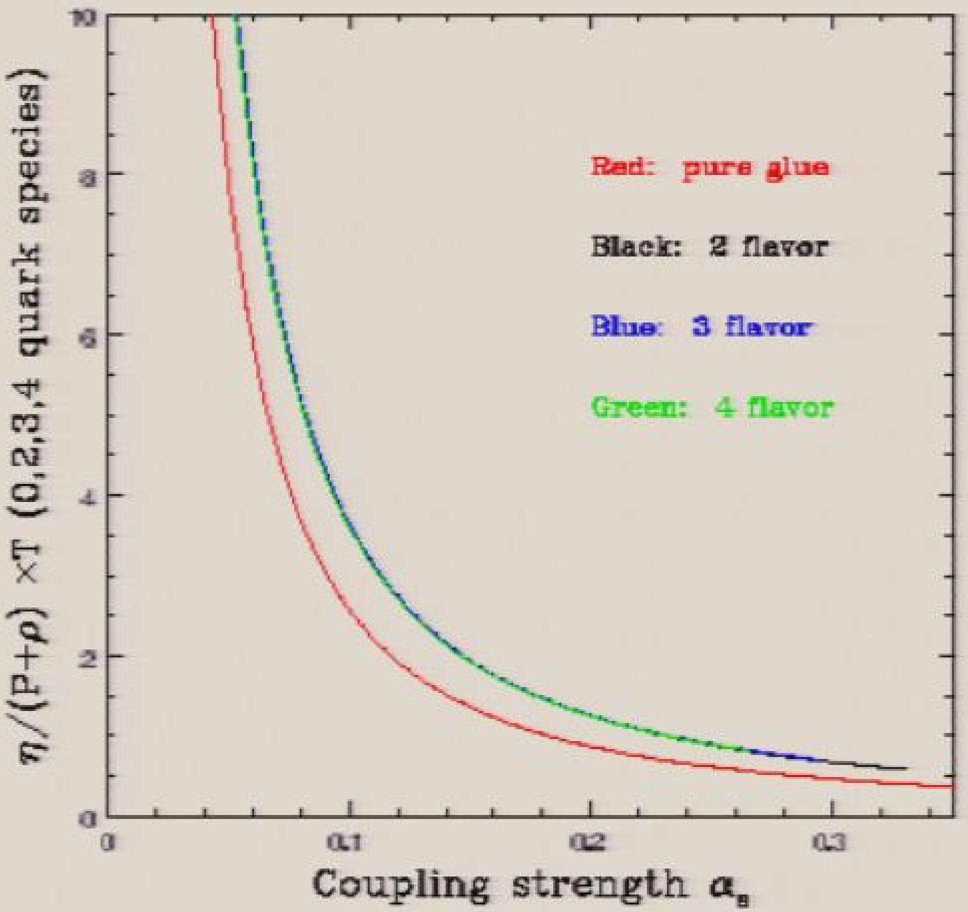

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How else can you compute η in QCD?

- Lattice? Only does statics. Dynamics by analytic continuation, fraught with error
- Chiral perturbation theory? Only works at $T \ll 200$ MeV. Breaks down where it's interesting
- Instantons? Quark models? No quantitative, reliable techniques.
- Similar, solvable theories? Let's explore

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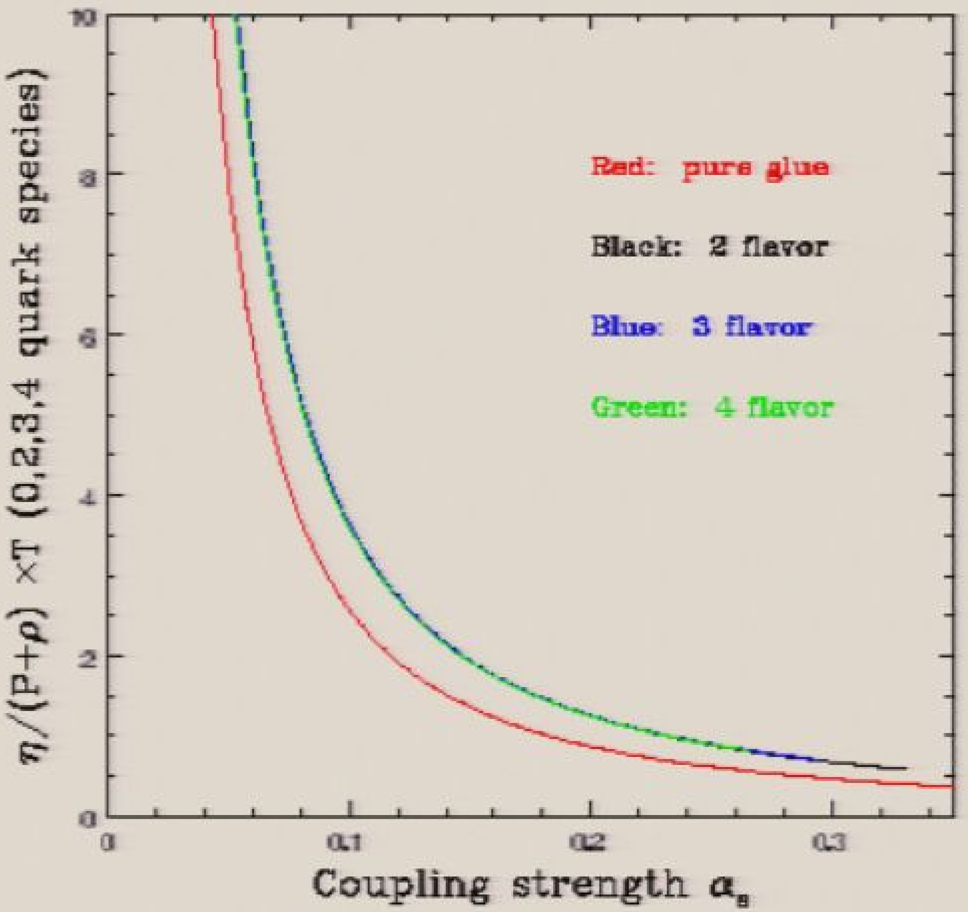

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$\mathcal{N}=4$ Super-Yang-Mills

A theory you can solve!

- Yang-Mills theory with gauge group SU(N_c)
- 4 adjoint Weyl fermion + 6 real adjoint scalar fields
- Yukawa, scalar interactions fixed by (high) supersymmetry
- Exactly conformal: no masses, scale invariant coupling
- Large N_c and $g^2 N_c$ limit solvable by string theory methods

$\mathcal{N}=4$ Super-Yang-Mills

Consider minimally SUSY theory in 10 dimensions:

- Gauge fields for SU(N_c): 8 polarizations
- single 16-component Majorana-Weyl fermion field

Now make 6 dimensions small and compact:

- Gauge fields $G_{A=4...9}$ are scalars ϕ_A in 4-dimensions
- $F^2_{\mu A}$ field strengths give $(D_\mu \phi_A)^2$ kinetic terms
- 16-component fermion is 4 Majoranas in 4-D
- $\bar{\psi}\gamma_A D_A \psi$ become Yukawa interactions
- F^2_{AB} become $[\phi_A, \phi_B]^2$ scalar quartics

16 real supercharges is $\mathcal{N}=4$ in 4-D

Policastro, Son, Starinets

Solve large N_c, $g^2 N_c \equiv \lambda$ theory at T using string methods

Viscosity η computed and has simple form:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

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Determine η/s in several theories with gravity duals. Find

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in all of them. Ratio is dimensionless and all known substances have $\eta/s > 1/4\pi$. May be universal bound! but see Cohen, hep-th/0702136

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η/s as a dimensionless ratio

Definition of η: stress divided by dv/dx.

Stress: $\frac{\text{Force}}{\text{area}} = \frac{m}{lt^2}$

dv/dx: $\frac{l/t}{l} = \frac{1}{t}$.

$\eta : \frac{m}{lt}$.

Entropy density: $s = \frac{1}{l^3}$ in natural units

$\eta/s = \frac{ml^2}{t} =$ energy$\times$time $=$ action.

$\eta/s \propto \hbar$.

(Roughly, η/s is time between scatterings $\times$ particle energy.)

QCD and $\mathcal{N}$=4 SYM

QCD and SYM are not that different. Could QCD at strong coupling saturate the bound?

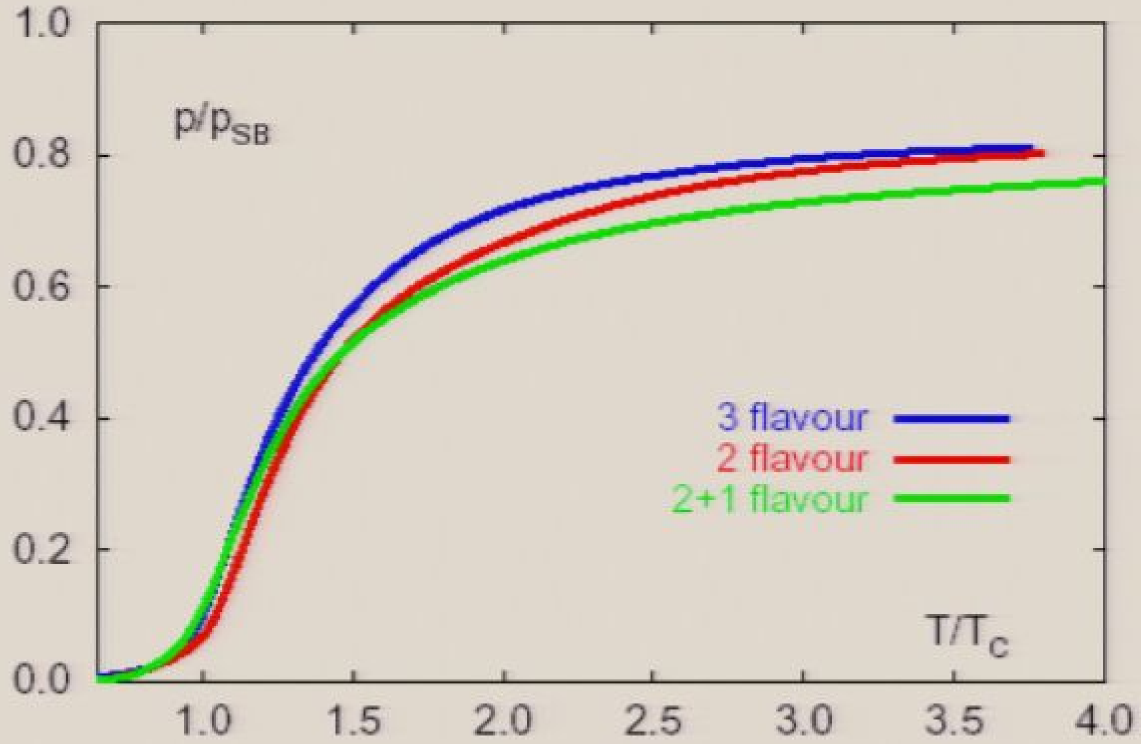
SYM at strong coupling: $\epsilon(T)/\epsilon_{\lambda=0}(T) = 3/4$

QCD at 1–3 T_c: $\epsilon/\epsilon_{g^2=0}$ very close to 3/4

SYM has a few more fields and they are adjoint, but it's not that different, is it?

QCD seems to demand small η/s. Speculation: it's near bound.

Comparing pressure with free theory value in QCD:



Near $P/P_{\mathrm{ideal}} = 3/4$ in a range, 2–4T_c.

Below: conformal breaking important. Above: not strongly coupled.

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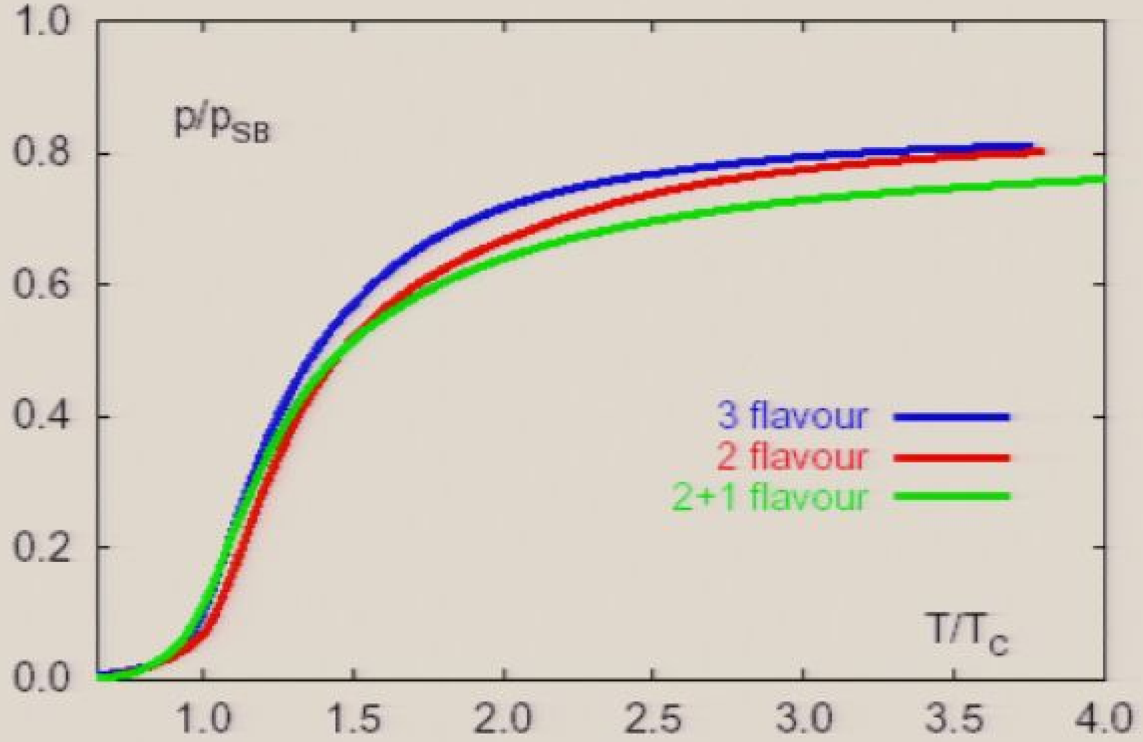

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Rash of papers using SYM to study heavy ion collisions

- Viscosity Papers mentioned. Shuryak and Zahed, hep-th/0308073 hep-ph/0405066 hep-ph/0307267
- Heavy quark diffusion 3 groups: UW group hep-th/0605158, Teaney hep-ph/0605199, Gubser hep-th/0605182 hep-th/0605292 all heavily cited
- Hard quark energy loss hep-ph/0605178 and its 57 citations
- Photon and dilepton production hep-th/0607237
- Full heavy-ion dynamics Shuryak and Zahed, hep-th/0511199

Actually testing QCD-SYM comparison

Before believing any of this, we should see if QCD and SYM give close to the same answers for anything.

Weak coupling: calculations possible in both theories

Compare weak coupling–see if they're at all the same.

Goal: look at η/s in Weak-Coupled $\mathcal{N}=4$ SYM and compare to QCD.

Quasiparticle picture

State described up to small corrections by 2-point function
$G^{>}(x_1, x_2) = \langle \phi(x_1)\phi(x_2) = G^{>}(p, x)$

$$G^{>}(p,x) = \frac{-i\pi}{\omega}\Big(f\delta(p-\omega) + (1 \pm f)\delta(p+\omega)\Big)$$

Equilibrium:

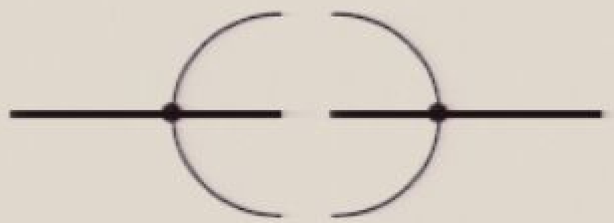
$$f(p,x) = \left[\exp\left(\beta P^{\mu} u_{\mu}(x)\right) \pm 1\right]^{-1}$$

Propagation: $\Box G^{>} = \Sigma G$ becomes

$$p^{\mu}\partial_{\mu} f = E_p \mathcal{C}[f]$$

with $\mathcal{C}[f]$ a collision term arising from self-energies which resembles a momentum-integrated matrix element squared with external population functions.

Naive collision term



Sum over all lowest-order $(2 \leftrightarrow 2)$ processes

$$\mathcal{C}_a[f] \;=\; \frac{1}{4}\sum_{bcd}\frac{1}{2p^0}\int_{kp'k'}|\mathcal{M}_{ab\to cd}(p,p';k,k')|^2(2\pi)^4\delta^4(p+p'-k-k')$$
$$\times\Big(f_p f_{p'}(1\pm f_k)(1\pm f_{k'}) - f_k f_{k'}(1\pm f_p)(1\pm f_{p'})\Big)$$

Halves of self-energy: $\mathcal{M}$ and $\mathcal{M}^*$

"cut" lines: on-shell external states

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Extra process: collinear splitting

Massless/light external states have $O(g^2)$ or $O(\lambda)$ chance to "split" into 2 particles DGLAP equations

Coulomb scattering has $\sigma \propto g^2$ or λ rather than expected $\propto g^4$ or λ^2.

Splitting is *as common* as hard scattering up to logs

We must include splitting.

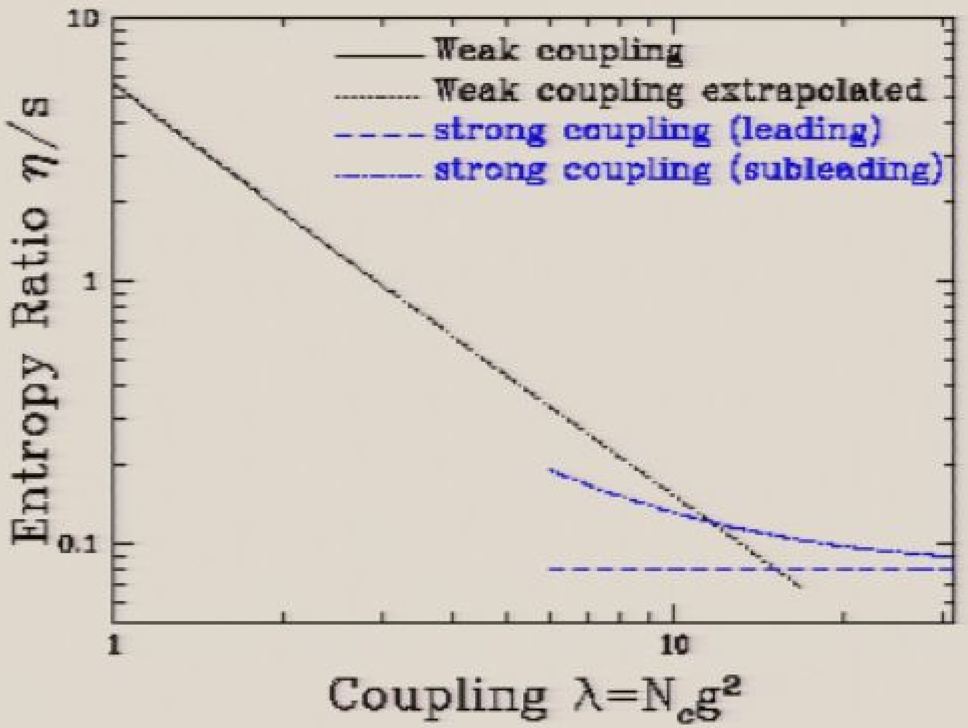
Sensitive to small (thermal-induced) masses

Sensitive to exact nature of thermal-corrected Coulomb

Sensitive to multiple-scattering interference (LPM effect)

Must be careful

Results in SYM theory



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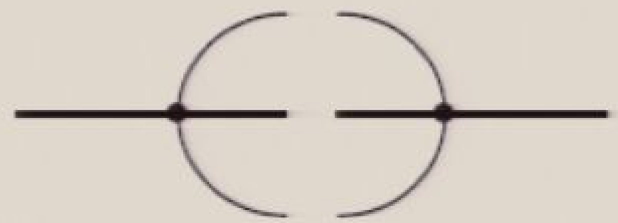

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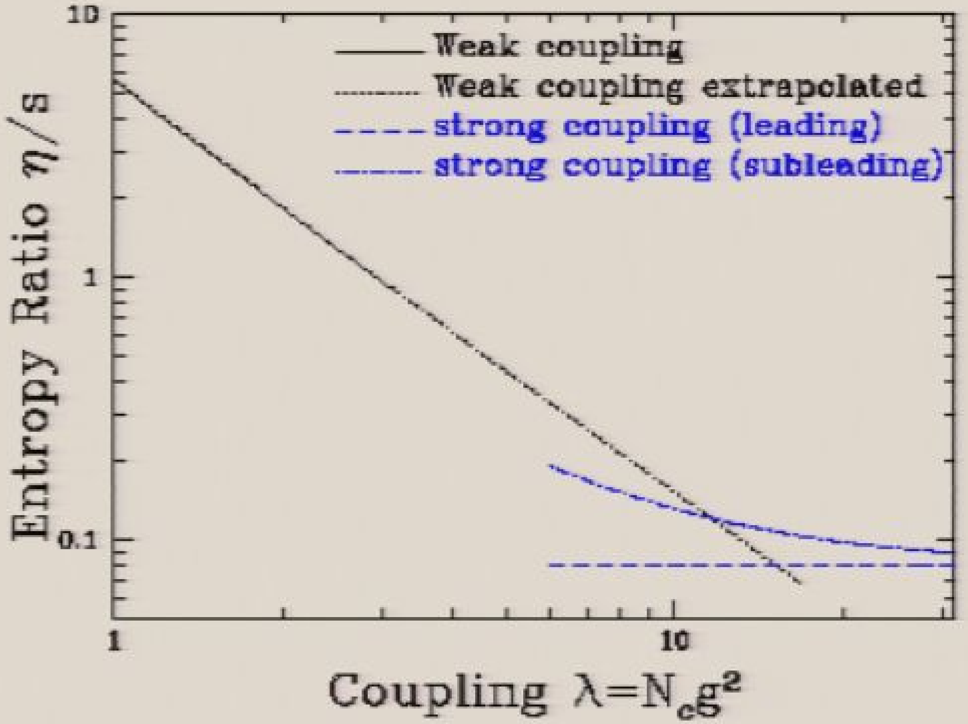

Approaches strong-coupling around $\lambda = 15$ ($\alpha_s = .4$ for $N_c = 3$)

Big differences between QCD and SYM

- More scattering targets in SYM than in QCD

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  SYM: $m_{\mathrm{D}}^2 = \frac{g^2T^2}{3}(N_{\mathrm{c}\,\mathrm{gluons}} + 2N_{\mathrm{c}\,\mathrm{fermions}} + 3N_{\mathrm{c}\,\mathrm{scalars}}) = 2N_{\mathrm{c}}g^2T^2$

- Larger Casimir to couple to gauge bosons N_{c} rather than $(N_{\mathrm{c}}^2-1)/2N_{\mathrm{c}}$

- Extra scattering processes due to Yukawa, scalar interactions

- Extra collinear processes due to scalar-gauge and Yukawa couplings QCD: GGG and FFG. SYM: GGG, FFG, SSG, FFS

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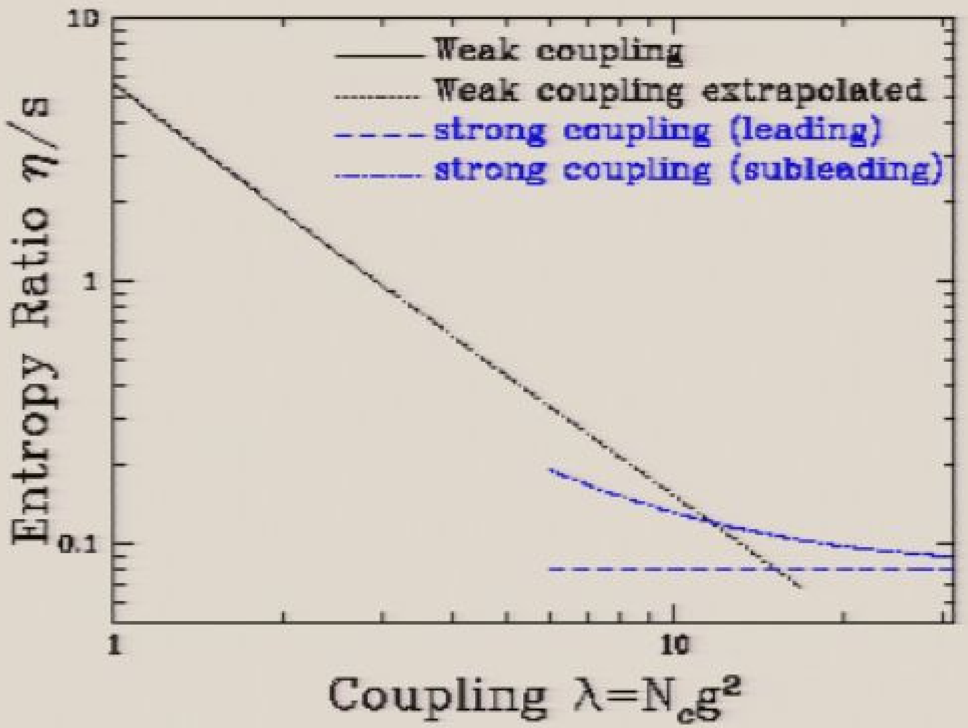

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$$\lambda = g^2 N_c$$

$$=$$

$$\tilde{\lambda} = g^2 N_c N_{DOF}$$

Big differences between QCD and SYM

- More scattering targets in SYM than in QCD

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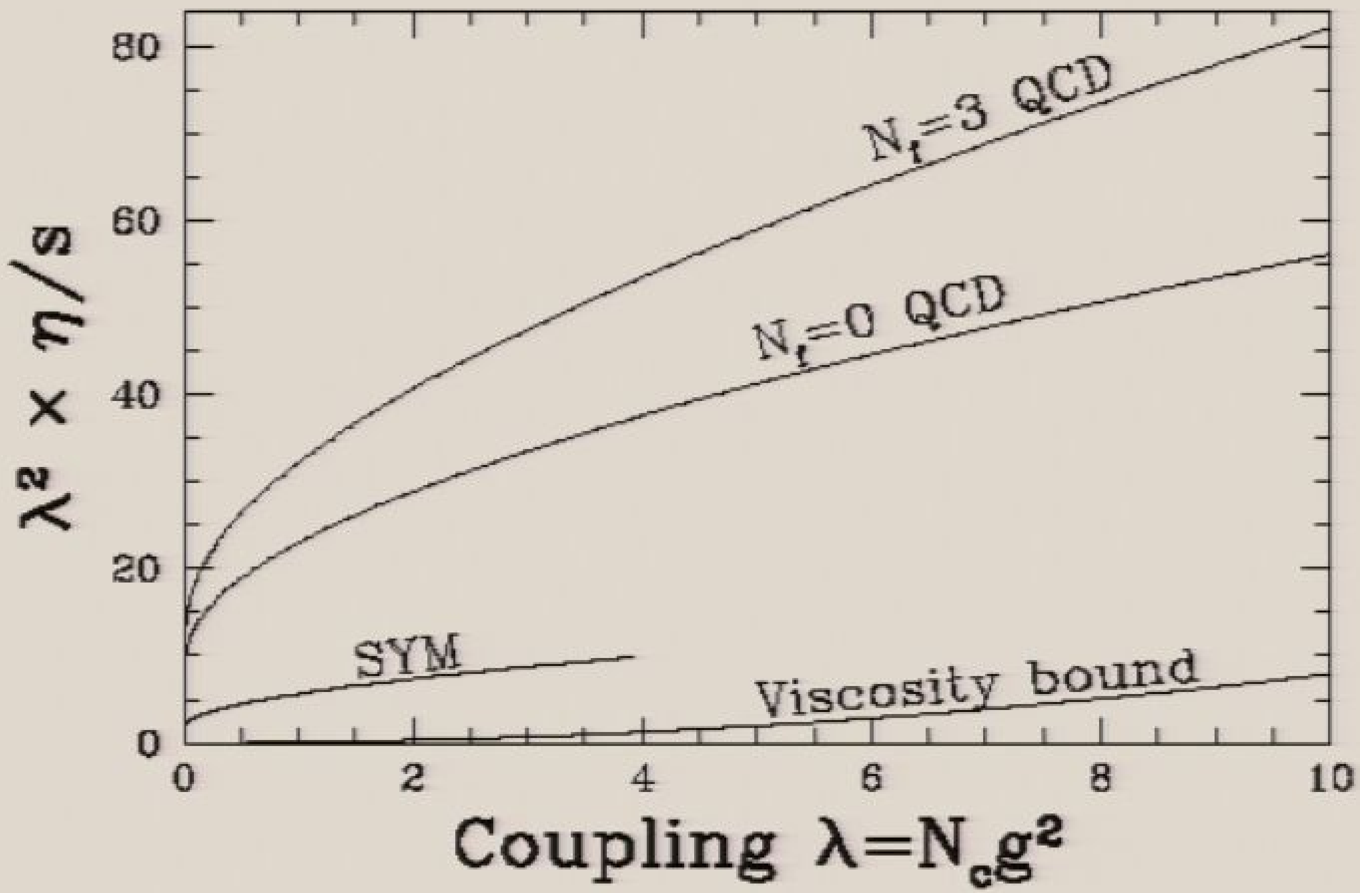
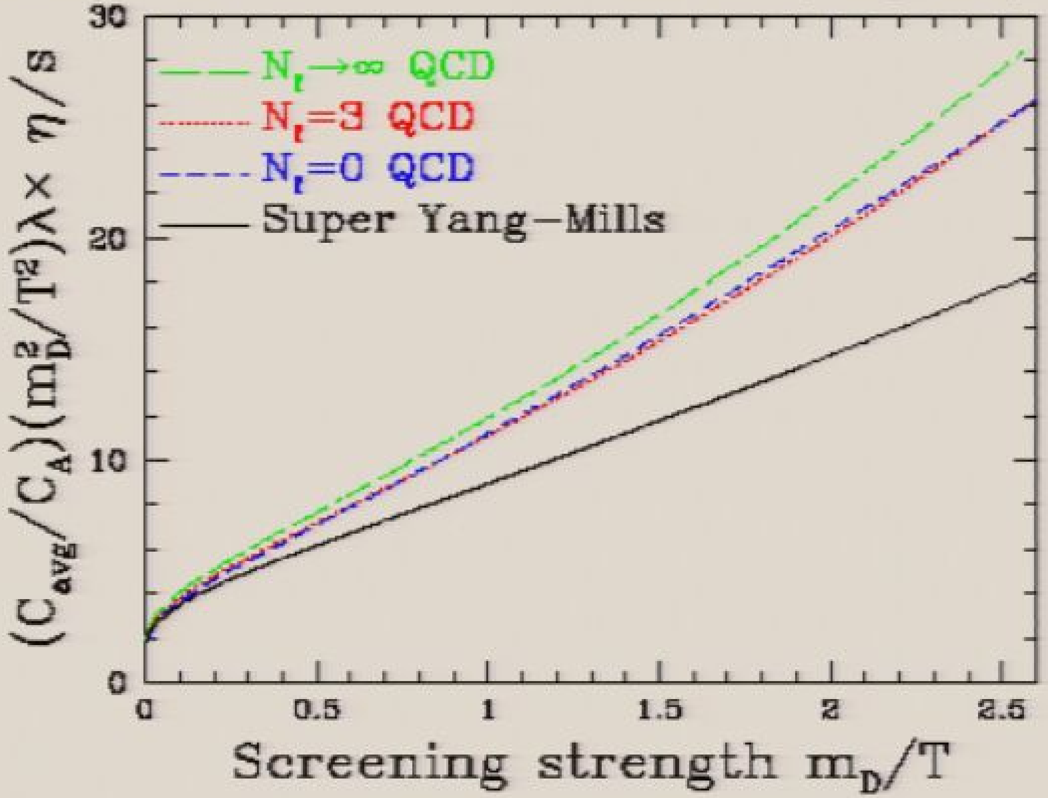
QCD and SYM compared



η/s in SYM is pretty drastically lower!

Why so much lower?

Number of scatterers: $4\times$ Casimir coupling to quarks: $> 2\times$



Same result rescaling by these differences

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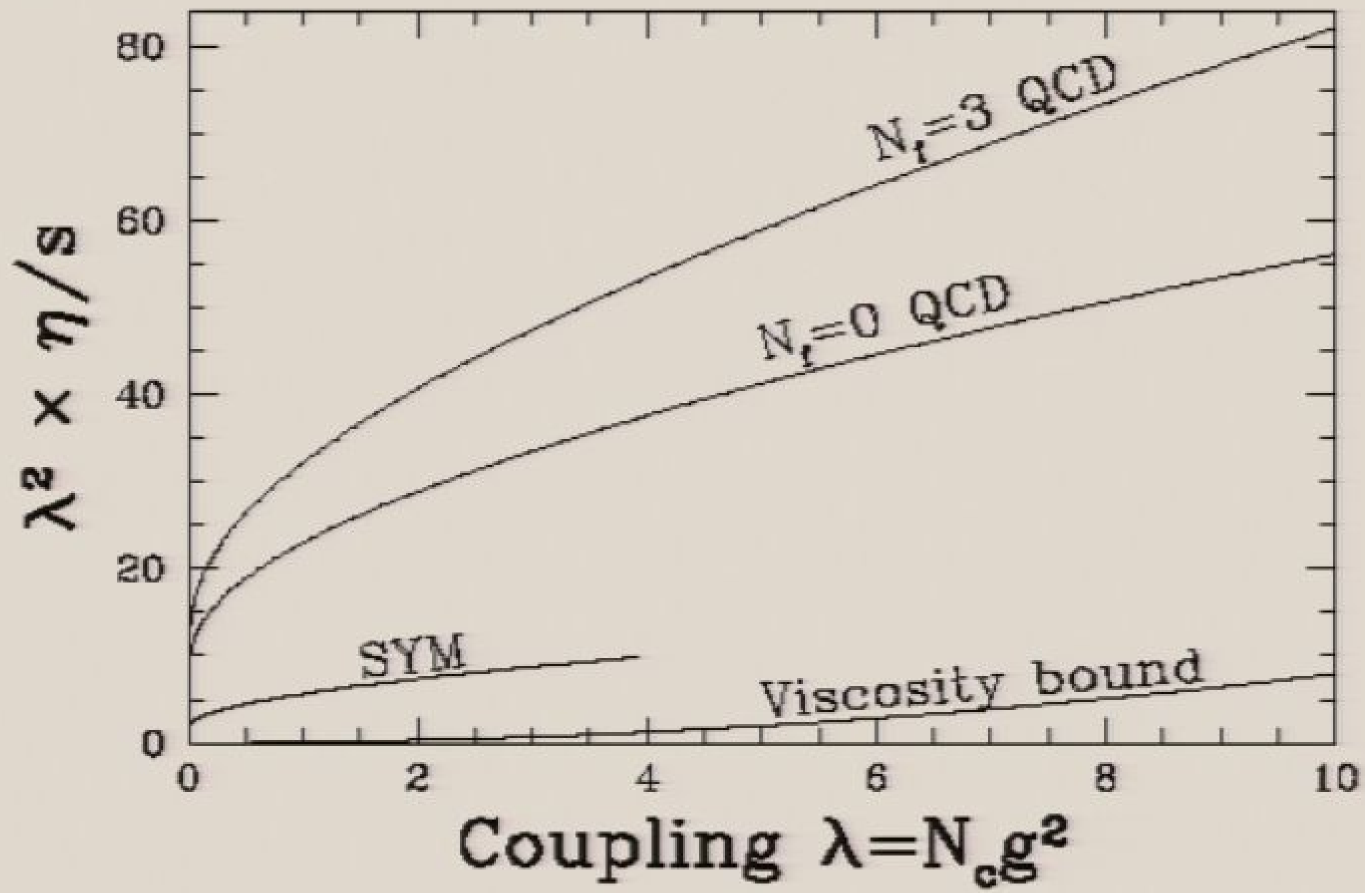

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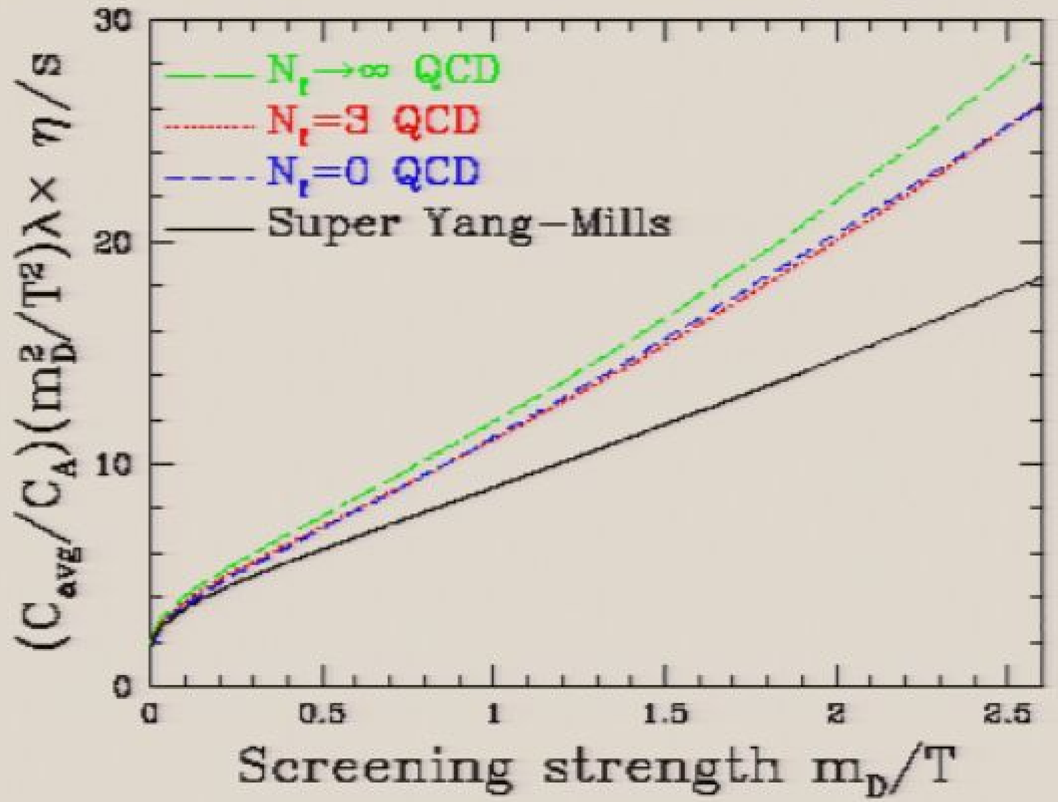

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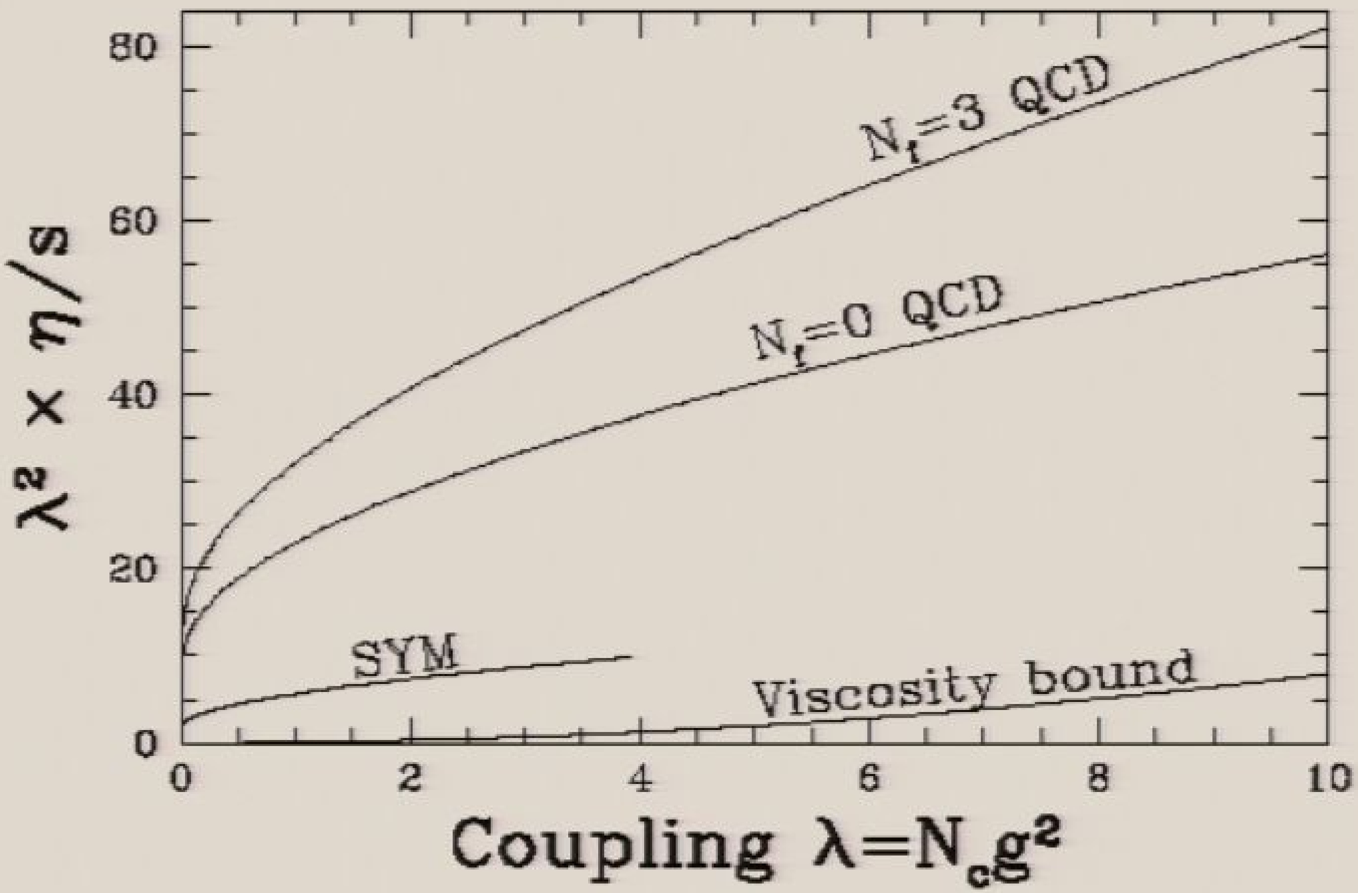

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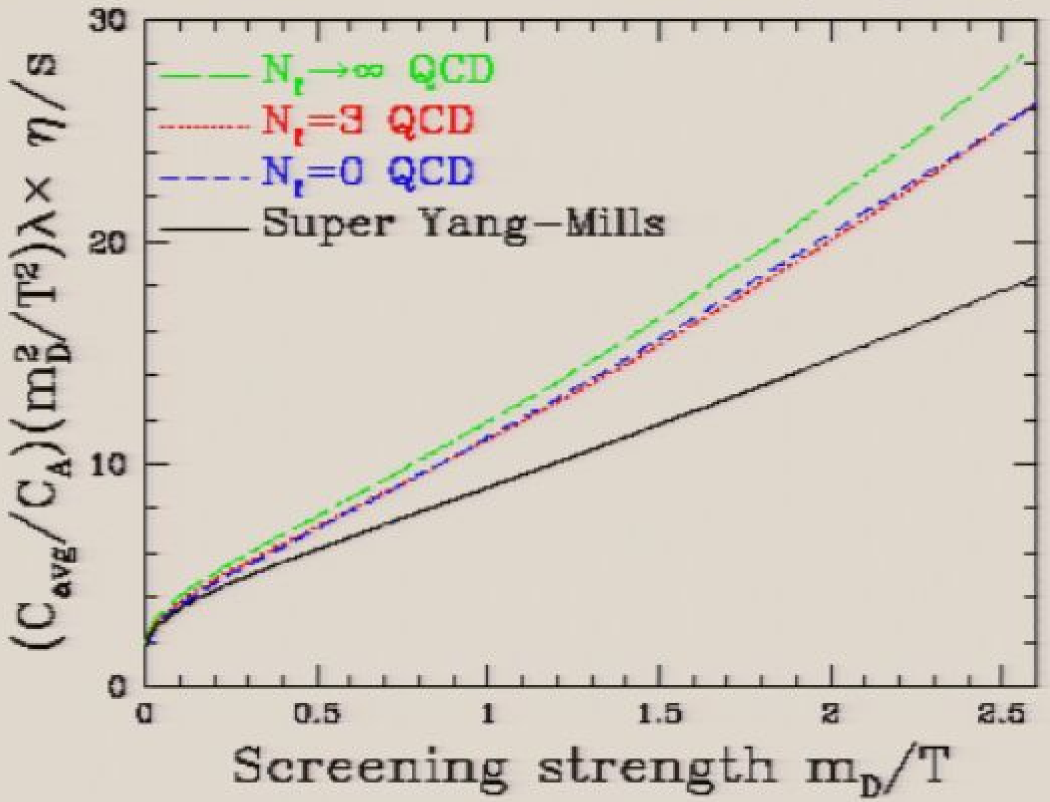

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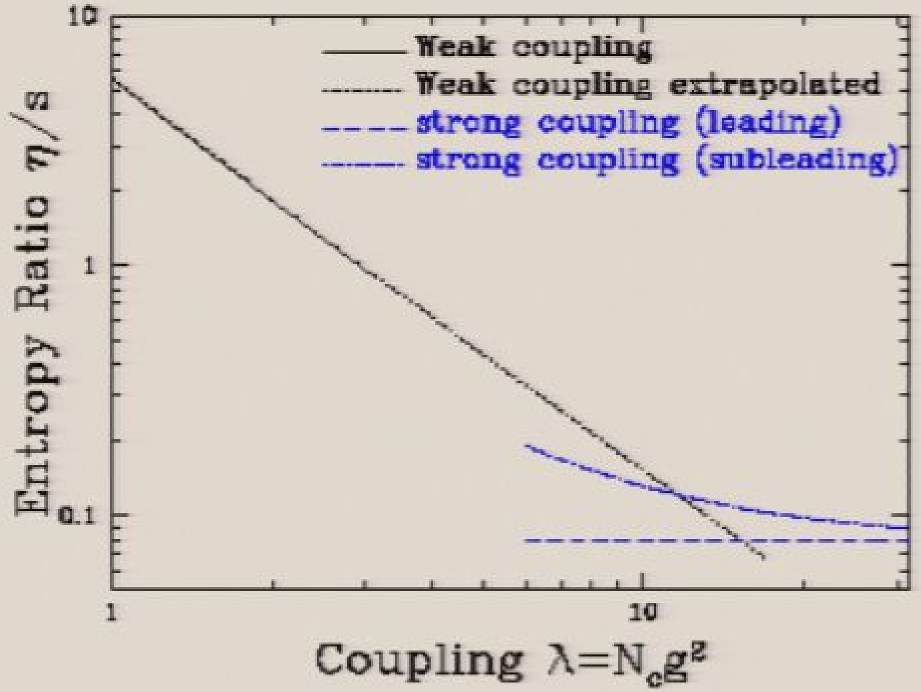
4

Lesson: compare m_D^2 not $g^2 N_c$

But that means $\alpha_s = 0.5$ in QCD equates with $\lambda = 4.5$ rather than $\lambda = 18$ in SYM.



Far from $1/4\pi$ limit

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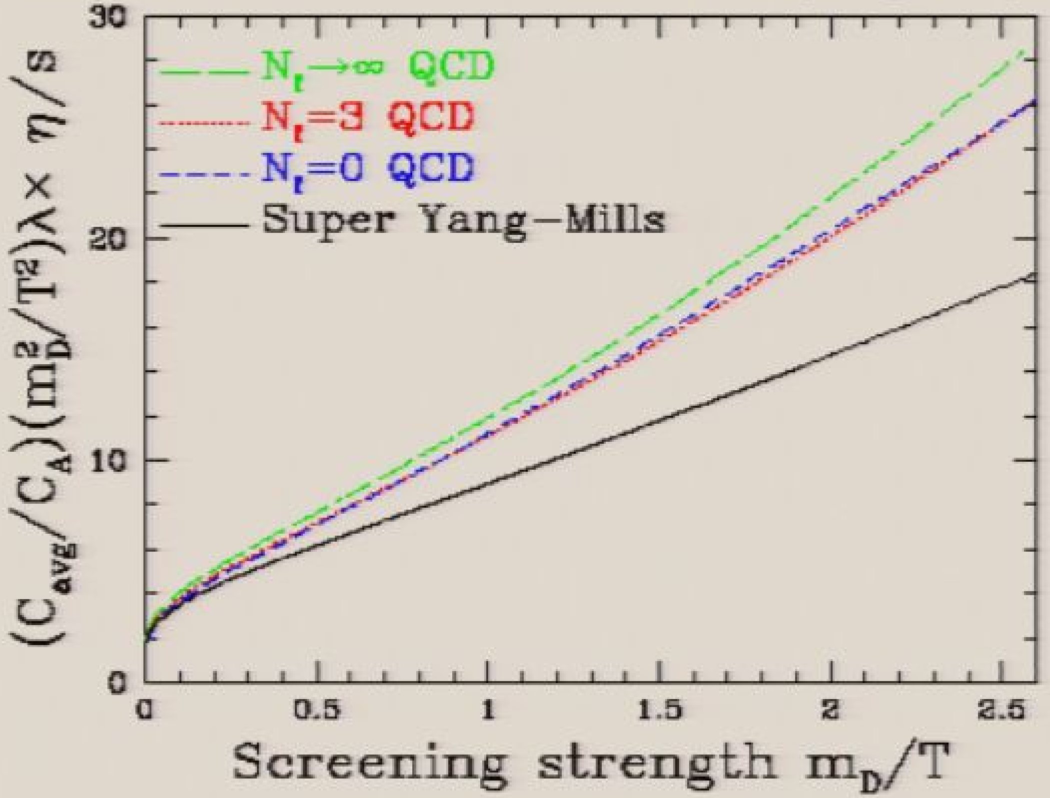

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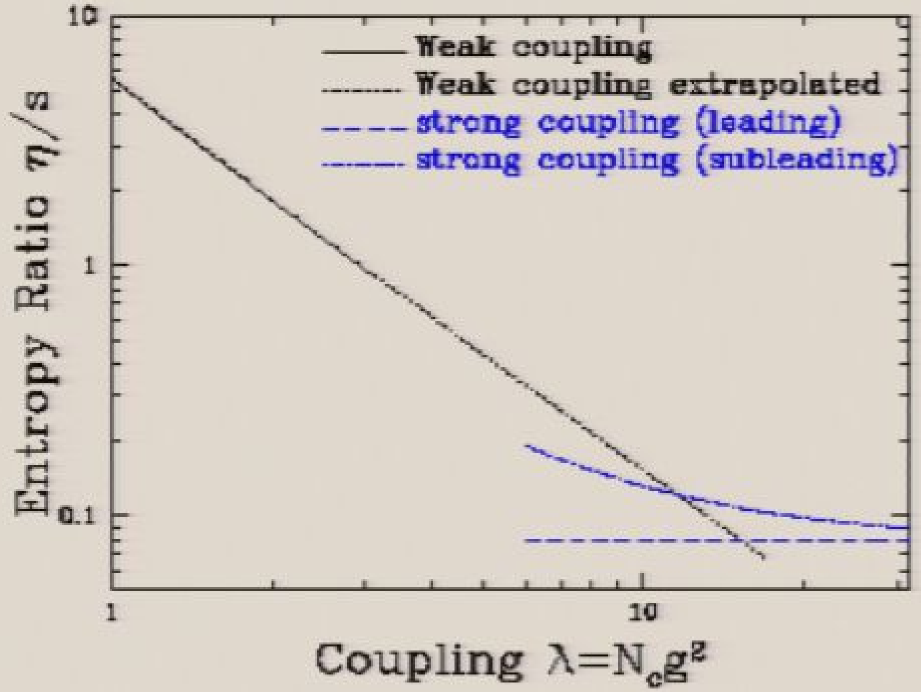

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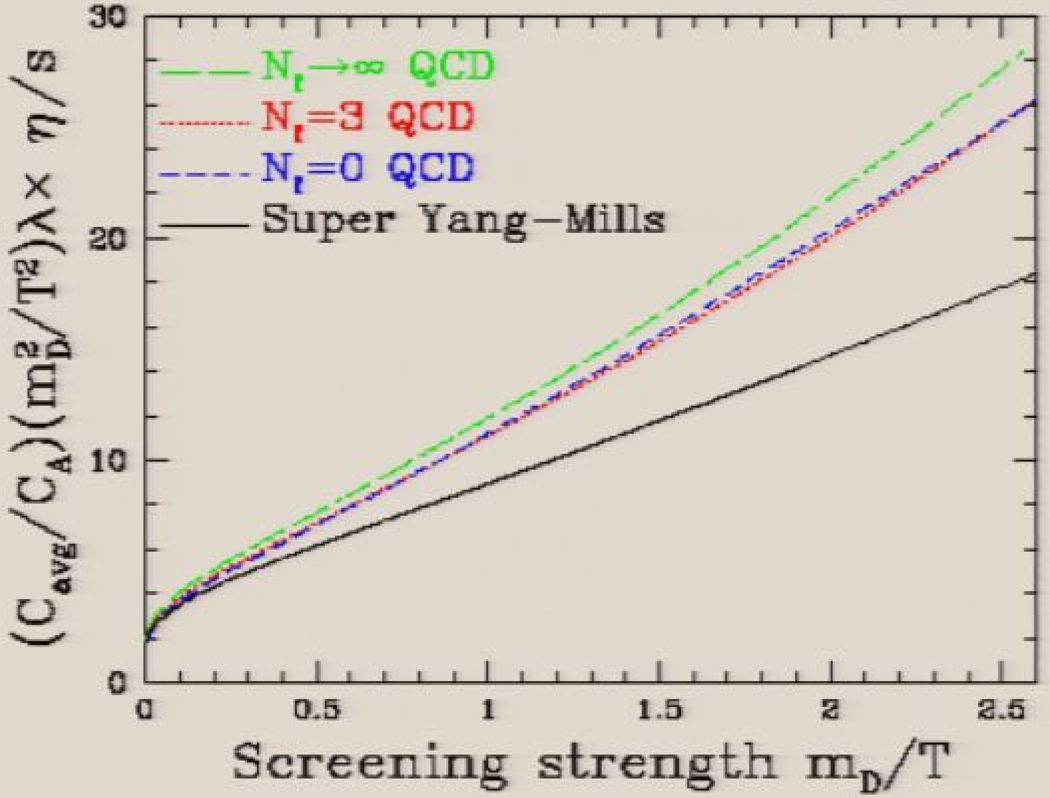

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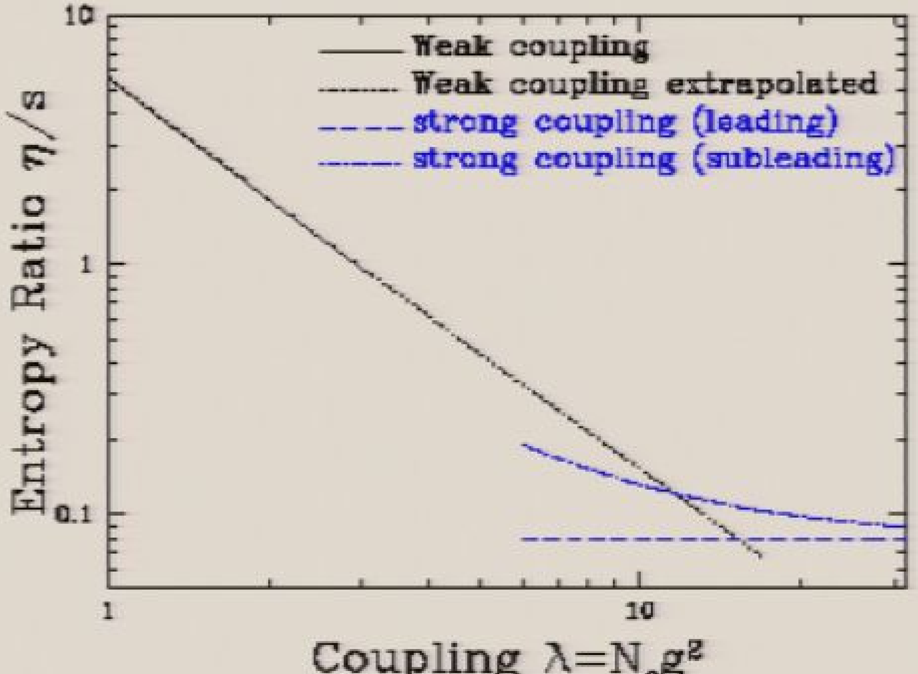

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Also: SYM thermo approach strong coupling behavior around $\lambda = 2 - 4$. We see η/s approaches strong coupling nearer $\lambda = 10$. Takes much more coupling to get $\eta \sim 1/4\pi$ than $\epsilon/\epsilon_{\lambda=0} = 3/4$.

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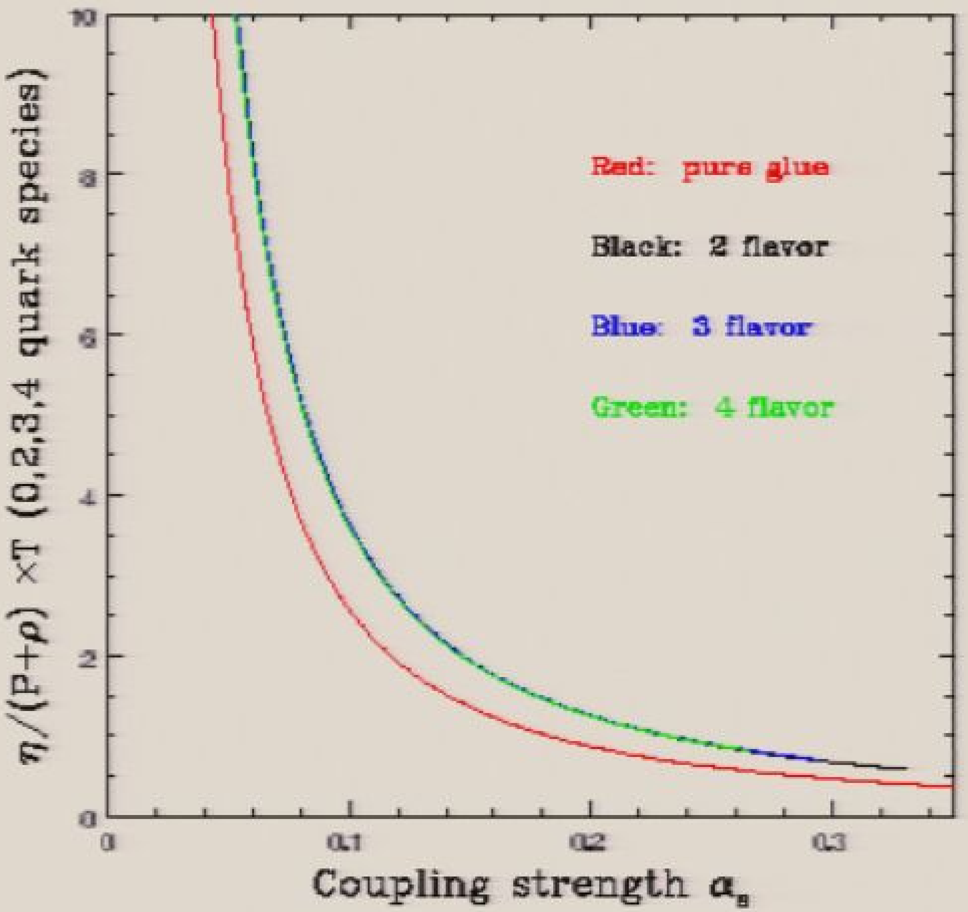
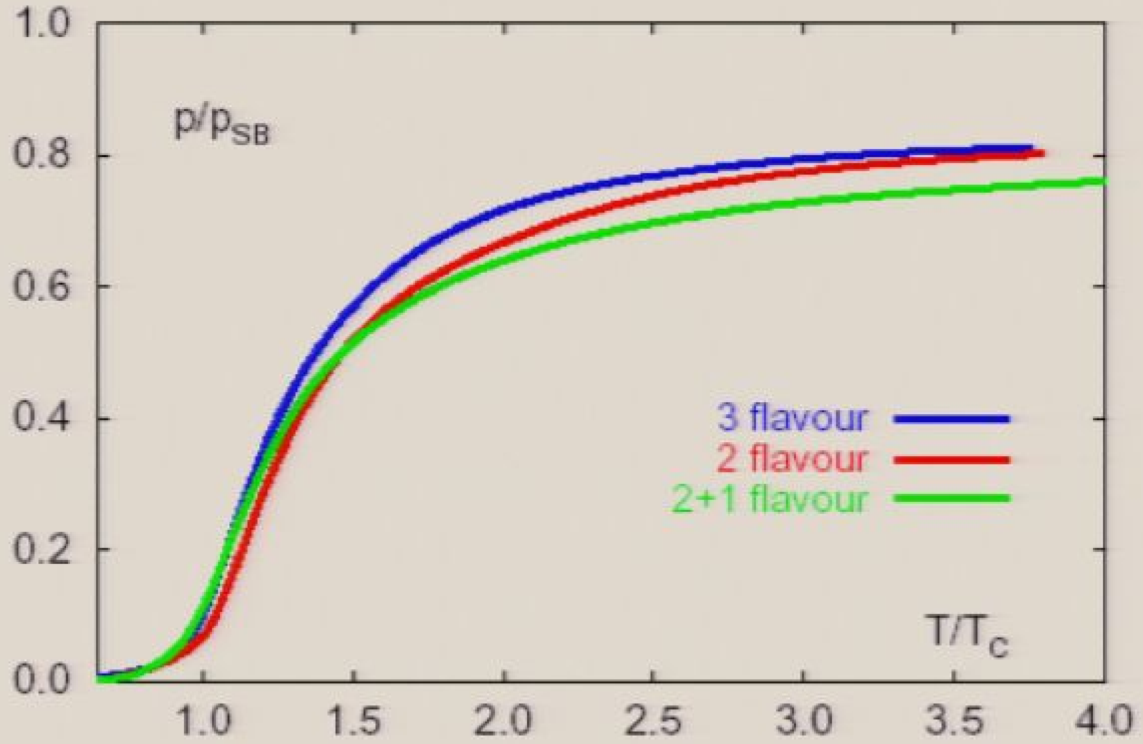
Results in QCD



η/s is large except where you can't believe it

Comparing pressure with free theory value in QCD:



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Below: conformal breaking important. Above: not strongly coupled.

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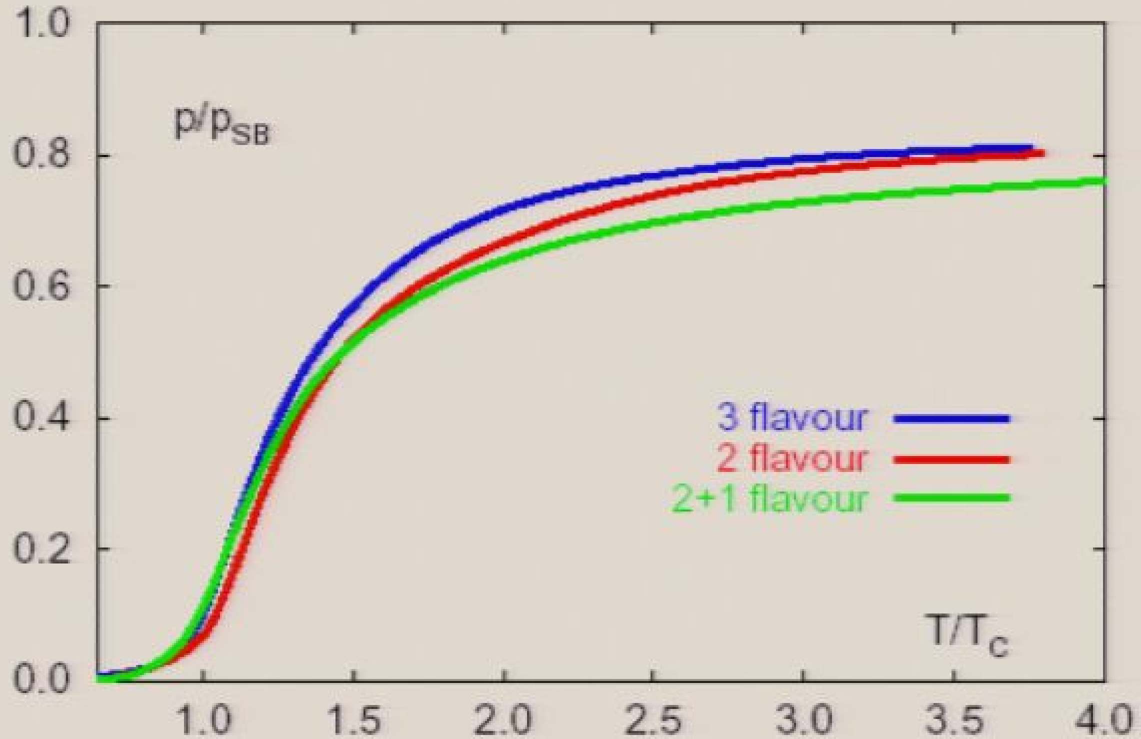

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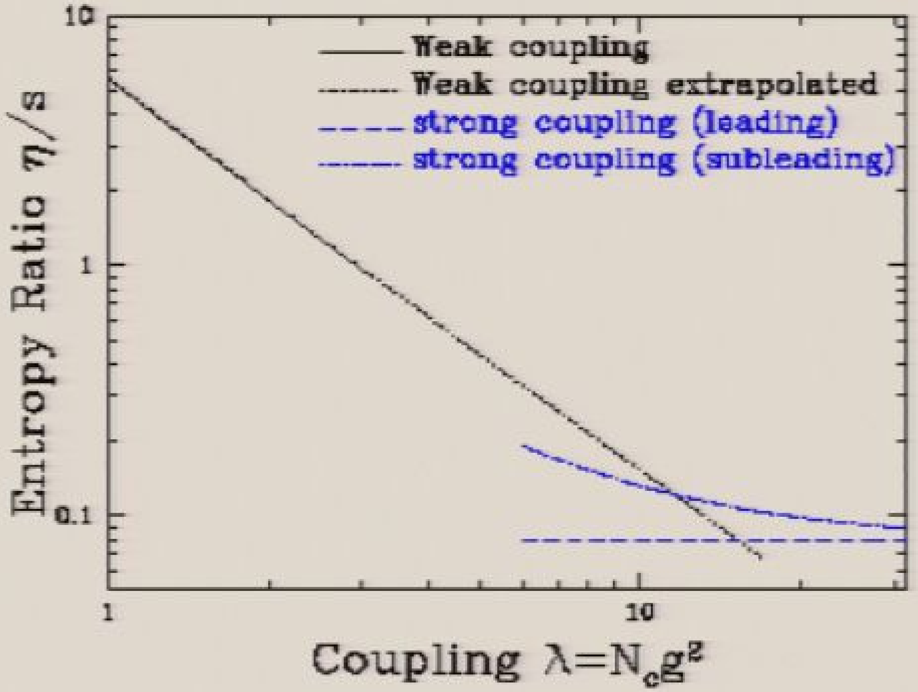

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Conclusions

- $\mathcal{N}=4$ SYM is a beautiful theory
- To test analogy to QCD, look at weak coupling
- Quantitatively quite different from QCD:
  - More fields, especially when you count by size of representation
  - Larger Casimirs, hence larger couplings
  - Extra (scalar and Yukawa) couplings
- Weak-coupling analysis suggests η/s in QCD actually quite far from $1/4\pi$

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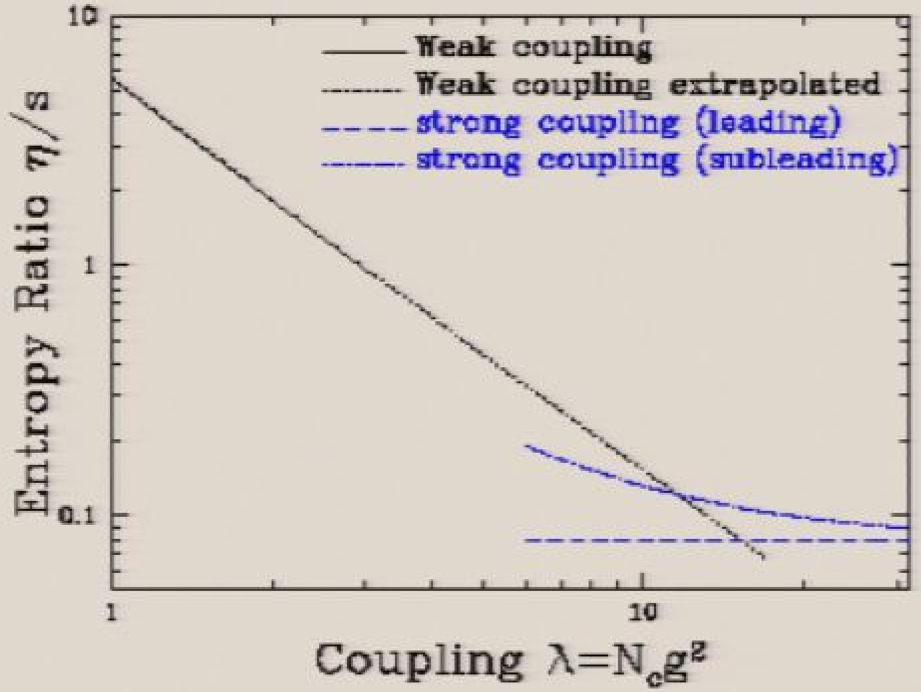

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