Title: Weak Coupling Approach to Thermal QCD

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Abstract:

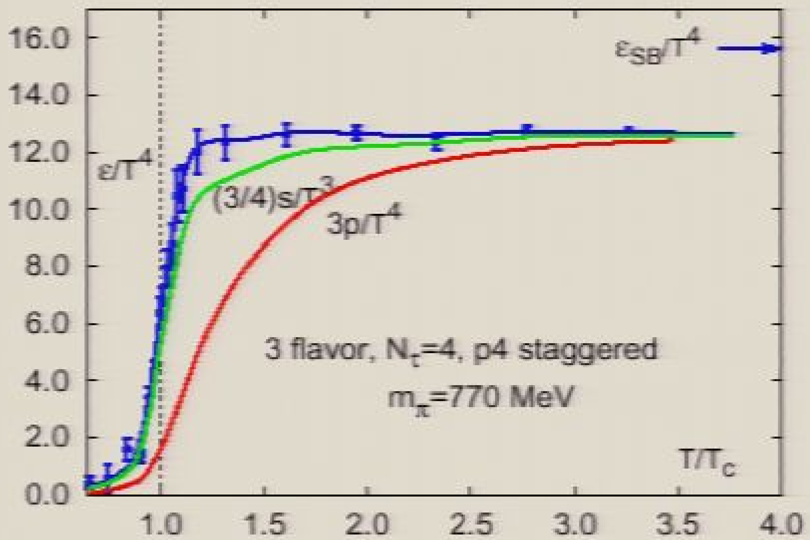
Weak coupling approach to thermal QCD

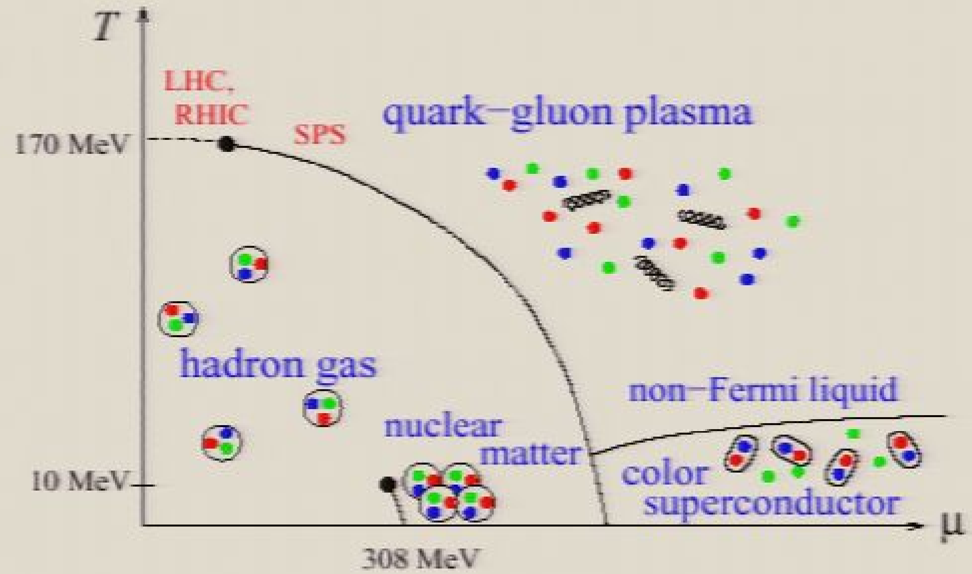
Anton Rebhan

rebhana@hep.itp.tuwien.ac.at

Institute for Theoretical Physics, Technical University Vienna, Austria

High-temperature/density QCD





Asymptotic freedom: $g \to 0$ for $T/\Lambda_{\rm QCD}$ or $\mu/\Lambda_{\rm QCD} \to \infty$

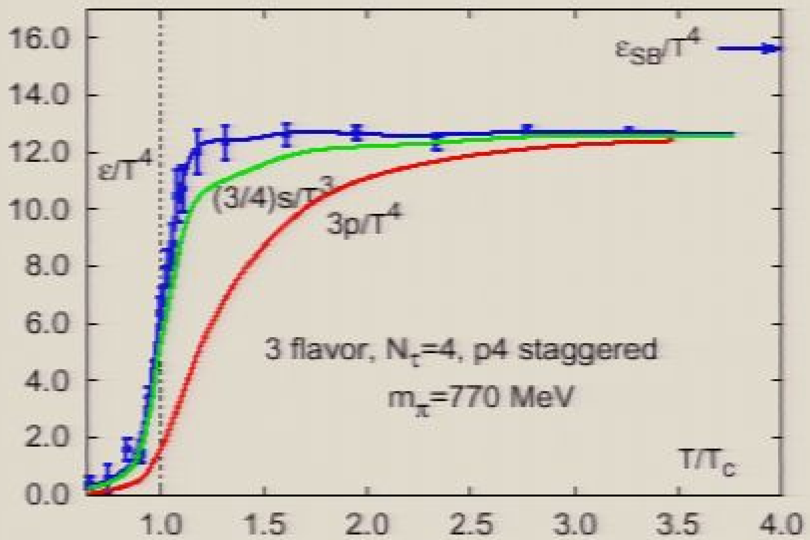
Q: Can one use weak-coupling techniques at T a few times T_c (μ a few times μ_c)?

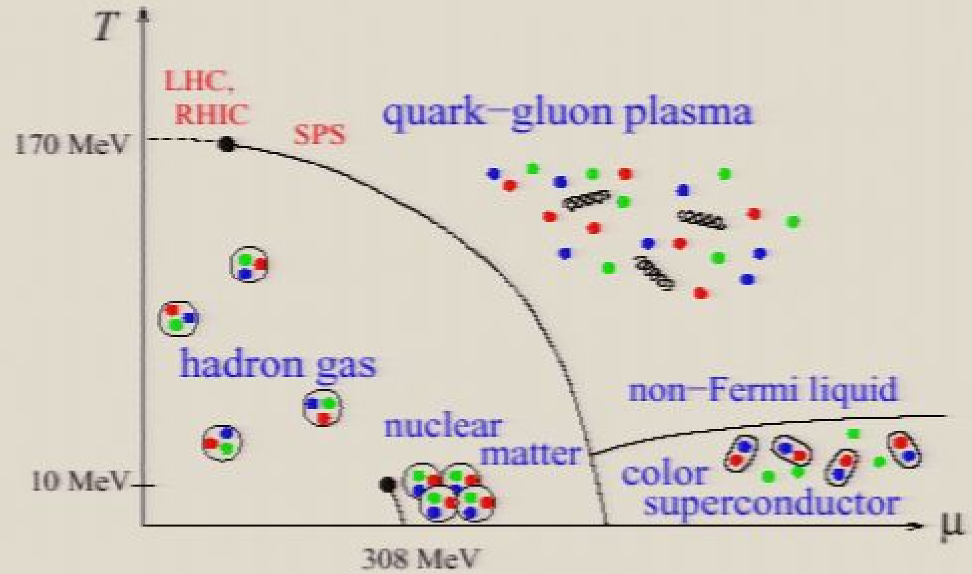
Matsubara frequencies $2\pi T > 2\pi T_c > 1$ GeV

RHIC: sQGP

LHC: sQGP or wQGP/pQCD?

High-temperature/density QCD





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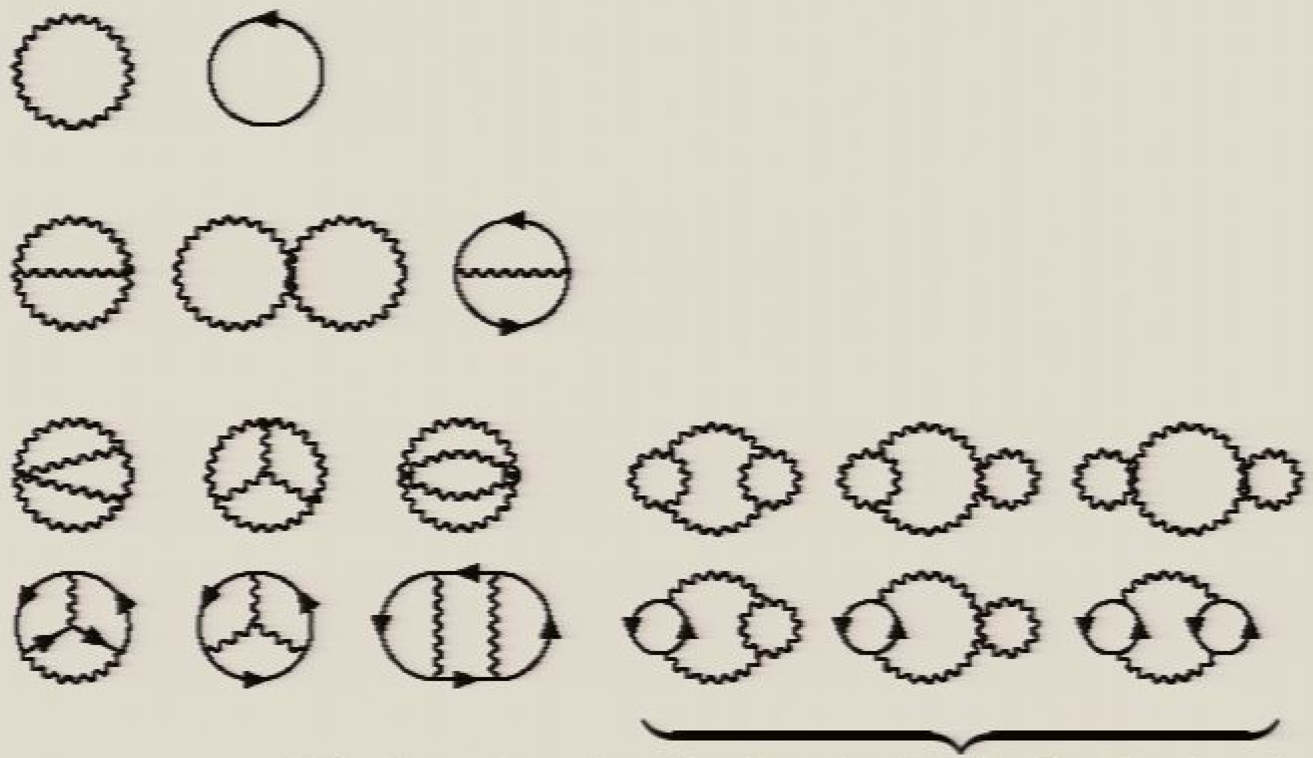
Perturbative calculation of QCD thermodynamics

$P = (\beta V)^{-1} \ln Z =$



IR singular due to (perturbative) electrostatic screening

4-loop: IR singular due to nonperturbative magnetostatic "screening"

"Linde(-Polyakov) problem" at momentum scales $g^2 T$

Other quantities already earlier: Debye mass at one-loop (resummed) order

$m_D = (\frac{N_c + N_f/2}{3})^{1/2} gT + \frac{N_c}{4\pi} g^2 T (\ln \frac{gT}{g^2 T} + c) + \ldots$ AR (1993), Arnold & Yaffe (1995), Laine & Philipsen (199

Perturbative results for the QCD pressure @ $\mu = 0, T > T_c$

$$P = \frac{8\pi^2}{45}T^4\Big\{(1+\tfrac{21}{32}N_f) - \tfrac{15}{4}(1+\tfrac{5}{12}N_f)\frac{\alpha_s}{\pi} + 30[(1+\tfrac{1}{6}N_f)(\frac{\alpha_s}{\pi})]^{3/2} + \mathcal{O}(\alpha_s^2)\Big\}.$$

Shuryak 1978

Kapusta 1979

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&+\Big\{\tfrac{135}{2}(1+\tfrac{1}{6}N_f)\ln[\frac{\alpha_s}{\pi}(1+\tfrac{1}{6}N_f)] \\
&-\tfrac{165}{8}(1+\tfrac{5}{12}N_f)(1-\tfrac{2}{33}N_f)\ln\frac{\bar{\mu}}{2\pi T}\Big\}(\frac{\alpha_s}{\pi})^2 \quad + \mathcal{O}(\alpha_s^2)\Big\}.
\end{aligned}
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$$+\Big\{\tfrac{135}{2}(1+\tfrac{1}{6}N_f)\ln[\frac{\alpha_s}{\pi}(1+\tfrac{1}{6}N_f)] + 237.2 + 15.97N_f - 0.413N_f^2$$
$$-\tfrac{165}{8}(1+\tfrac{5}{12}N_f)(1-\tfrac{2}{33}N_f)\ln\frac{\bar{\mu}}{2\pi T}\Big\}(\frac{\alpha_s}{\pi})^2\Big\}.$$

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$$+(1+\tfrac{1}{6}N_f)^{1/2}\Big[-799.2 - 21.96N_f - 1.926N_f^2$$

$$+\tfrac{495}{2}(1+\tfrac{1}{6}N_f)(1-\tfrac{2}{33}N_f)\ln\frac{\bar{\mu}}{2\pi T}\Big](\frac{\alpha_s}{\pi})^{5/2} + \mathcal{O}(\alpha_s^3\ln\alpha_s)\Big\}.$$

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Shuryak 1978
Kapusta 1979
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Zhai & Kastening 1995
Braaten & Nieto 1996

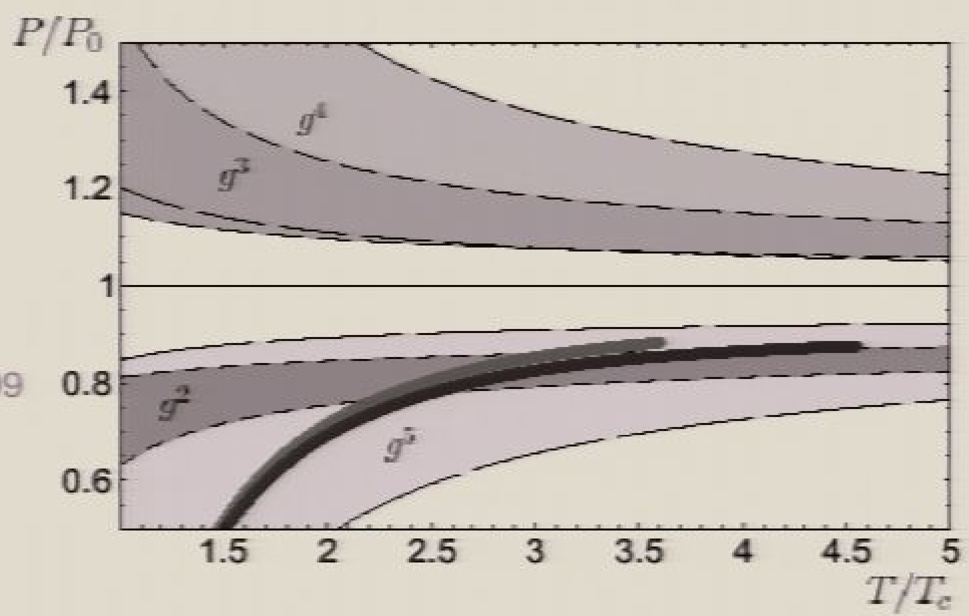
No apparent convergence; steadily increasing renormalization scale ($\bar{\mu}$) dependence:

$N_f = 0$

Lattice data:

Boyd et al. (BI) 1996

Okamoto et al. (CP-PACS) 1999



$\bar{\mu} = \pi T \ldots 4\pi T$

Effective Field Theory when $T \gg g\mu$: 3d Yang-Mills+Scalar

Scales when $g \ll 1$:

T: *hard* — scale of Matsubara frequencies $\omega_n = \pi i n T$ (n even/odd for bosons/fermions

gT: *soft*, g^2T: *ultrasoft*, $\ldots$ contributions only from zero-modes $n = 0$ (no fermions)

Braaten & Nieto 1996: Pressure of QCD

$$P = P^{\text{hard}} + P^{\text{soft}}, \qquad P^{\text{hard}} = T^4(c_1 + c_2 g^2 + c_3 g^4 + c_4 g^6 + \ldots)$$

with P^{soft} from effective 3-d theory EQCD (electrostatic QCD)

$$\mathcal{L}_E = \frac{1}{2}\,\mathrm{tr}\,F_{ij}^2 + \mathrm{tr}\,[D_i, A_0]^2 + m_E^2\,\mathrm{tr}\,A_0^2 + \frac{1}{2}\lambda_E(\mathrm{tr}\,A_0^2)^2 + \ldots,$$

perturbative matching:

$$g_E^2 = g^2 T + \ldots, \quad m_E^2 = (1 + N_f/6) g^2 T^2 + \ldots, \quad \lambda_E = \frac{9 - N_f}{12\pi^2} g^4 T + \ldots,$$

$$P_{\text{soft}}/T \;=\; \frac{2}{3\pi} m_E^3 - \frac{3}{8\pi^2}\left(4 \ln \frac{\Lambda_E}{2 m_E} + 3\right) g_E^2 m_E^2$$

$$- \frac{9}{8\pi^3}\left(\frac{89}{24} - \frac{11}{6}\ln 2 + \frac{1}{6}\pi^2\right) g_E^4\, m_E + \ldots$$

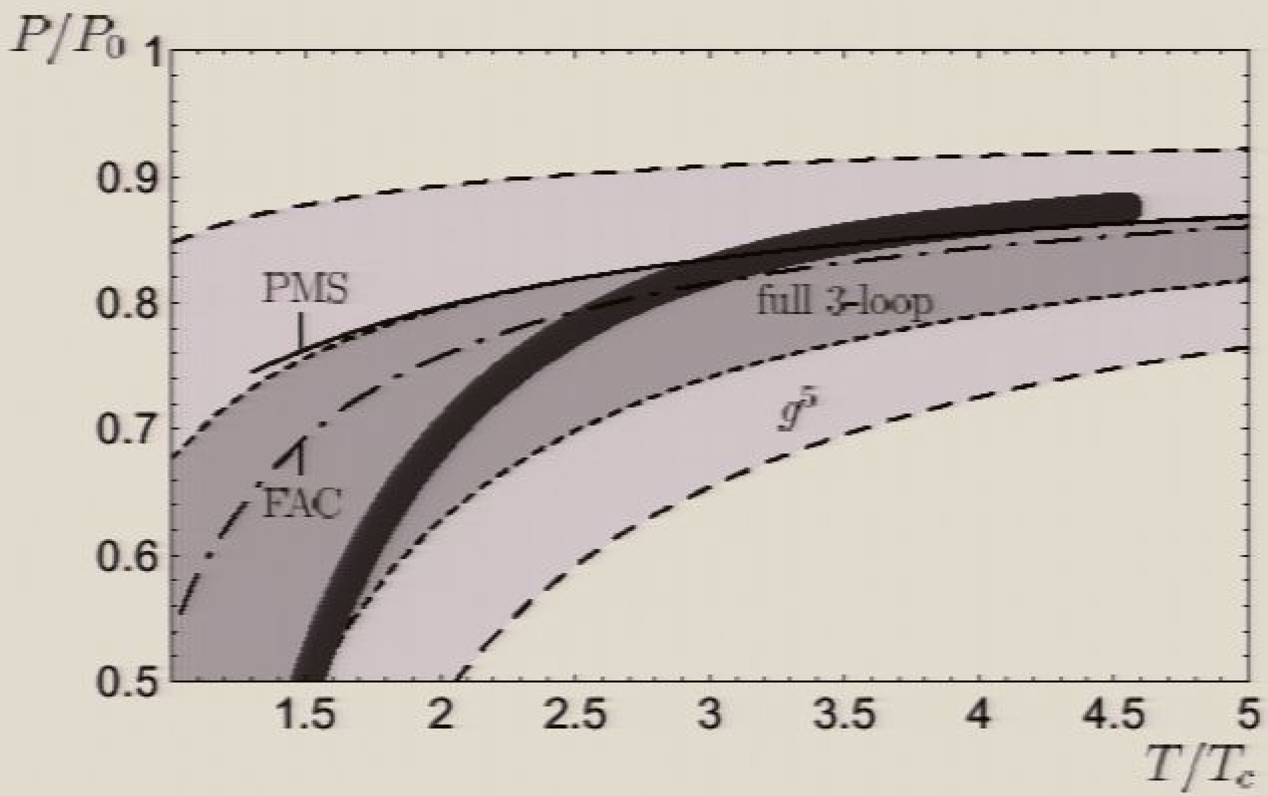
Improving apparent convergence in dimensional reduction

Expanding $P = P^{\rm hard} + P^{\rm soft}$ in powers (and log's) of g

$\rightarrow$ perturbative series with bad convergence

Not expanding $m_E^2(g), g_E^2(g), \ldots \rightarrow$ improved convergence for $T \gtrsim 3T_c$

J.-P. Blaizot, E. Iancu, AR, PRD68 (2003) 025011:



$\bar\mu_{\rm MS} = \Lambda_E = \pi T \ldots 4\pi T$

$O(g^6 \ln(g))$-contribution

Last perturbatively calculable coefficient done by

Kajantie, Laine, Rummukainen & Schröder (2003):

$$P \ni N_g \frac{(N g_E^2)^3}{(4\pi)^4} \left[\left(\tfrac{43}{12} - \tfrac{157\pi^2}{768} \right) \ln \frac{\Lambda_E}{g_E^2} + \left(\tfrac{43}{4} - \tfrac{491\pi^2}{768} \right) \ln \frac{\Lambda_E}{m_E} + \tilde{\delta} \right]$$

$\tilde{\delta}$ determined by 3-d effective field theory MQCD (magnetostatic QCD)

$\mathcal{L}_M = \frac{1}{2}\,\mathrm{tr}\, F_{ij}^2 + \ldots,$ adjoint scalar A_0 integrated out, too

- nonperturbative mass gap $\sim g^2 T$, requires lattice calculation

  (and matching using 4-loop lattice perturbation theory) $\longrightarrow$ contribution #$(g^2T)^3 T$

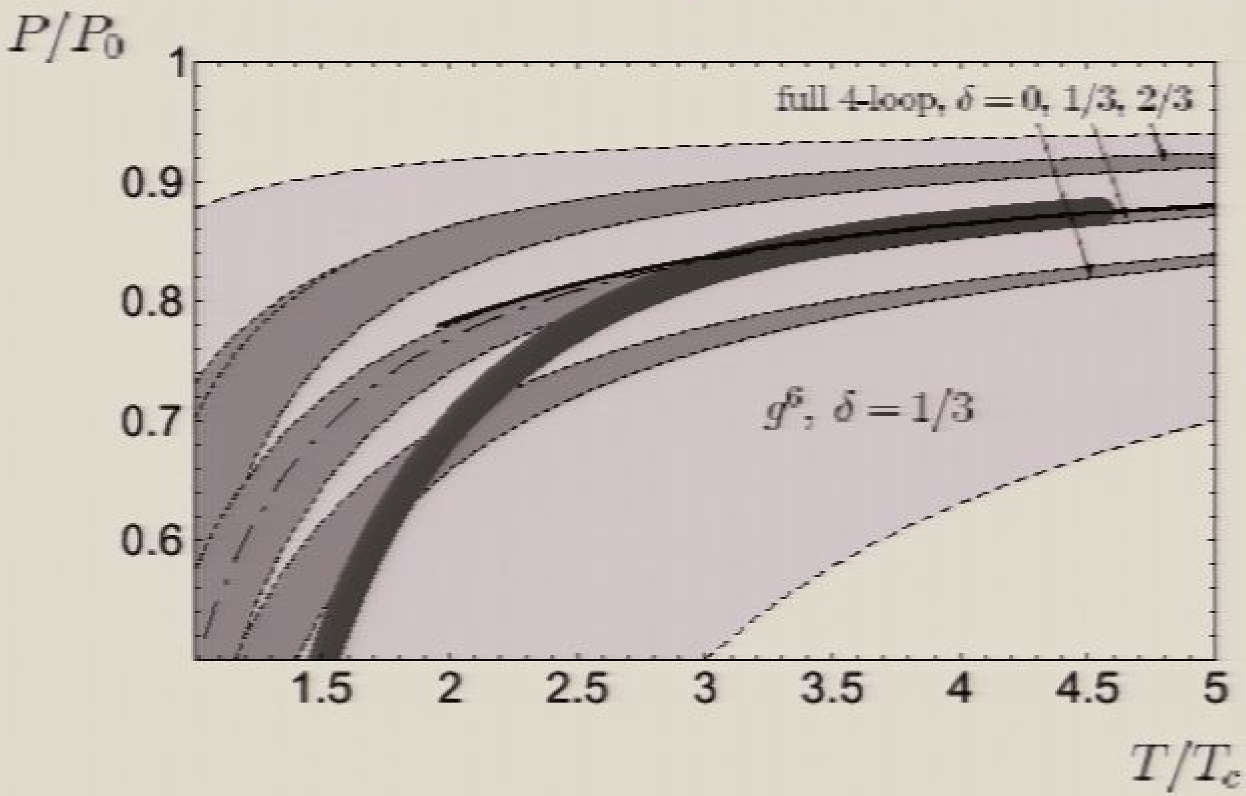
  steady progress: Di Renzo, Laine, Miccio, Schröder & Torrero, JHEP 07 (2006)

Q: How does it look for some particular value $\tilde{\delta} \sim O(1)$?

Improving apparent convergence in dimensional reduction (cont'd)

$P^{\rm hard} + P^{\rm soft}$ to order $g^6[\log(g) + \delta]$ with some $\delta \sim O(1)$: even stronger renormalization scale dependence

even greater improvement by not expanding out $m_E^2(g)$ and $g_E^2(g)$ and truncating



Meanwhile, other improvement schemes
(applied separately to $P^{\rm hard}$, $P^{\rm soft}$; $\Lambda_E \neq \bar{\mu}_{\rm MS}$):

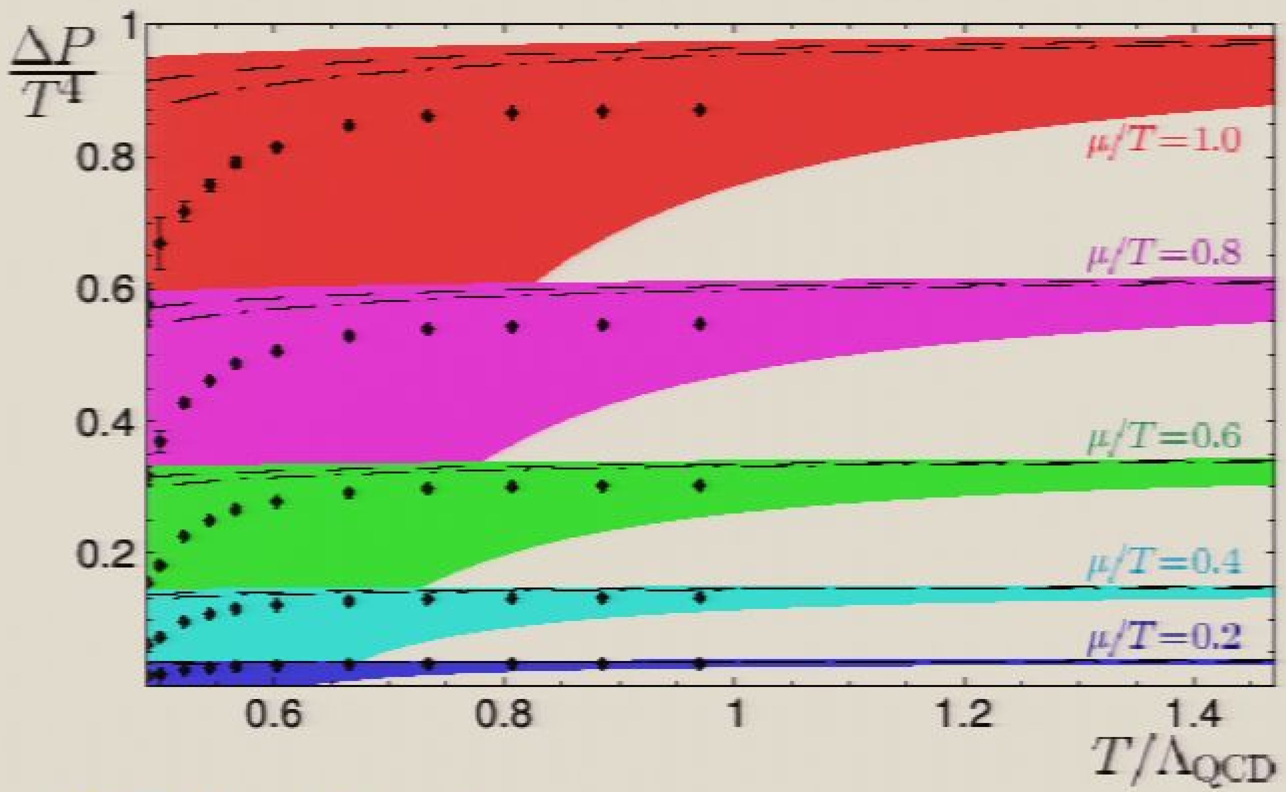
Padé : Cvetič & Kögerler, PRD70 (2004)

PMS: Inui, Niégawa & Ozaki, hep-ph/0501277

Improving apparent convergence in dimensional reduction

Works also at finite chemical potential $\mu \lesssim T$:

→Vuorinen, PRD68 (2003) 054017; Ipp, AR & Vuorinen, PRD69 (2004) 077901



$\Delta P = P(T,\mu) - P(T,0)$ for $N_f = 2$,

unexpanded 3-loop results with $\bar{\mu}_{\rm MS}$ varied by a factor of 4 and two FAC schemes (dashed)

vs. lattice data from Allton et al, PRD68 (2003) 014507 (not yet continuum extrapolated!)

(consistent with Fodor, Katz & Szabó, PLB568 (2003))

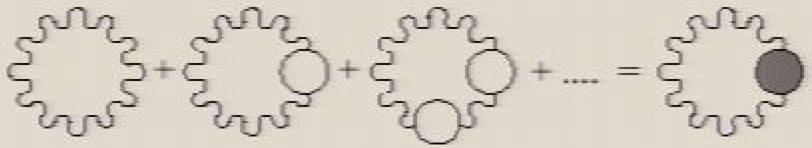
Large N_f limit of QCD and QED

G. D. Moore, JHEP 10 (2002) 055: $N_f \to \infty, N_c \sim 1, g^2 N_f \sim 1$
as testing ground for weak-coupling techniques at high T

Much simpler than large-N_c:

order N_f^1:

order N_f^0:



dressed gluon propagator contains typical gauge-theory phenomena such as

- Debye screening for electrostatic modes
- unscreened magnetostatic modes
- complicated dispersion laws, Landau damping, plasmon damping

and can be solved exactly (nonperturbative w.r.t. $g_{\text{eff}}^2 \propto g^2 N_f$)

Large N_f limit of QCD and QED

Effective coupling constant $g^2_{\rm eff} = g^2 T_F = \begin{cases} \dfrac{g^2 N_f}{2}, & \text{QCD}, \\ e^2 N_f, & \text{QED}. \end{cases}$

One-loop beta function exact: $\dfrac{1}{g^2_{\rm eff}(\mu)} = \dfrac{1}{g^2_{\rm eff}(\mu')} + \dfrac{1}{6\pi^2} \ln(\mu'/\mu)$.

No asymptotic freedom — instead: Landau singularity at exponentially large

$$\Lambda_{\rm L} = \bar\mu_{\rm MS} e^{5/6} e^{6\pi^2/g^2_{\rm eff}(\bar\mu_{\rm MS})}.$$

Theory only exists as cutoff-theory with $\Lambda_{\rm Cutoff} < \Lambda_L$

But thermodynamic potential insensitive to cutoff as long as $T, \mu \ll \Lambda_{\rm L}$

Technicality: cutoff needs to be imposed in Euclidean invariant manner, otherwise spurious singularities

Thermodynamic potential of large-N_f QCD and QED

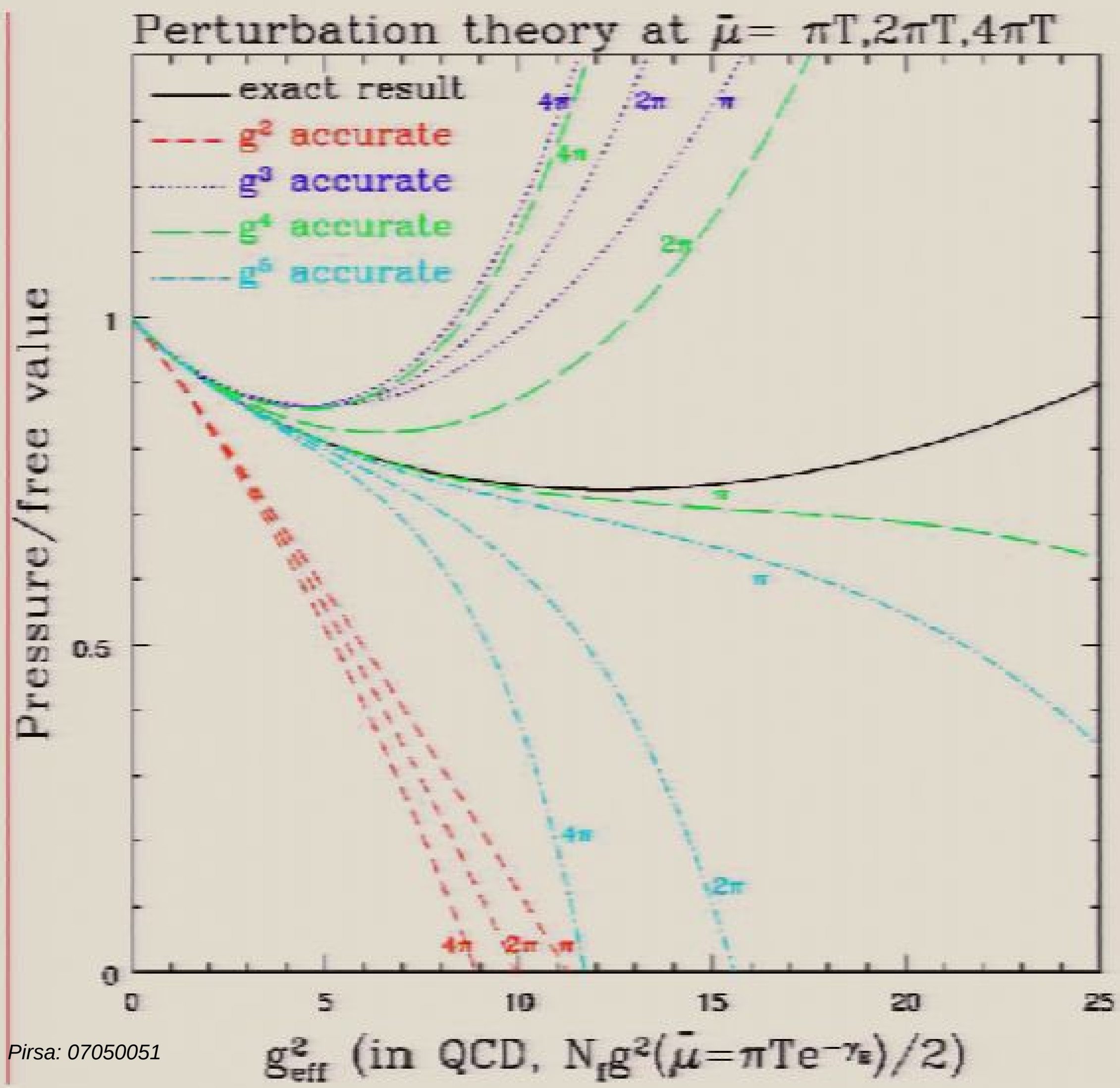
$$
\begin{aligned}
P &= N N_f \left(\frac{7\pi^2 T^4}{180} + \frac{\mu^2 T^2}{6} + \frac{\mu^4}{12\pi^2} \right) \\
&+ N_g \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} \Bigg[2 \Big\{ \left[n_b + \tfrac{1}{2}\right] \mathrm{Im}\ln\left(\mathrm{q}^2 - \omega^2 + \Pi_{\mathrm{T}} + \Pi_{\mathrm{vac}}\right) \\
&\qquad - \tfrac{1}{2}\,\mathrm{Im}\ln\left(\mathrm{q}^2 - \omega^2 + \Pi_{\mathrm{vac}}\right) \Big\} \\
&\quad + \Big\{ \left[n_b + \tfrac{1}{2}\right] \mathrm{Im}\ln \frac{\mathrm{q}^2 - \omega^2 + \Pi_{\mathrm{L}} + \Pi_{\mathrm{vac}}}{\mathrm{q}^2 - \omega^2} - \tfrac{1}{2}\,\mathrm{Im}\ln \frac{\mathrm{q}^2 - \omega^2 + \Pi_{\mathrm{vac}}}{\mathrm{q}^2 - \omega^2} \Big\} \Bigg] \\
&\quad + O(N_f^{-1})
\end{aligned}
$$

with $\Pi^{\mu\nu} = \Pi^{\mu\nu}_{\mathrm{vac}} + \Pi^{\mu\nu}_{mat}$, $\Pi^{\mu\nu}_{mat} \ni \Pi_T, \Pi_L$, 2 distinct structure functions

Interaction pressure $P - \underbrace{P_0}_{SB}$ finite as $N_f \to \infty$

Pressure of large-N_f QCD and QED @ $\mu = 0$

G. D. Moore, JHEP 10 (2002) 055, E: hep-ph/0209190, A. Ipp. G. D. Moore, AR, JHEP 01 (2003) 037



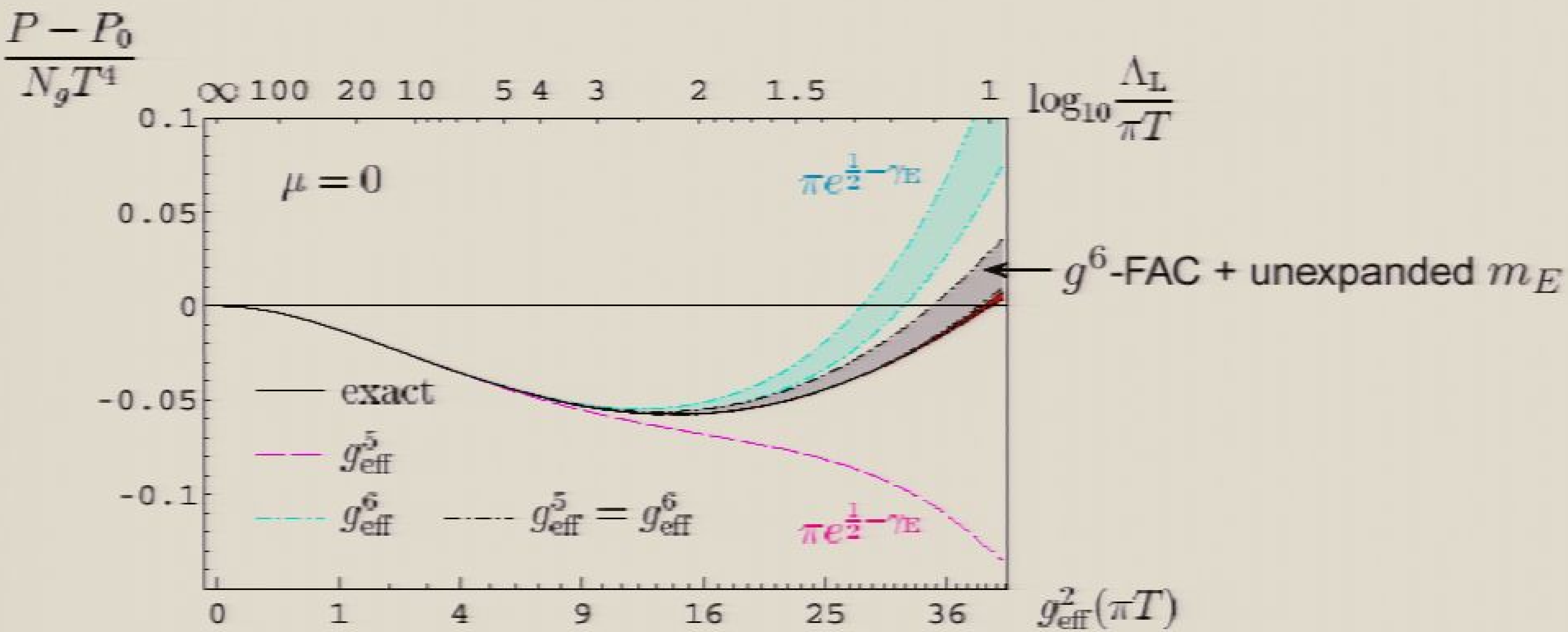
Comparison with strict perturbation theory

Pressure of large-N_f QCD and QED @ $\mu = 0$

Numerical result sufficiently accurate to verify perturbative results through order $g_{\rm eff}^5$ and to extract $g_{\rm eff}^6$ term (no log here!)

$$P\Big|_{g_{\rm eff}^6,\mu=0,\bar\mu_{\rm MS}=\pi T} = +20(2) N_g \left(\frac{g_{\rm eff}}{4\pi}\right)^6 T^4$$

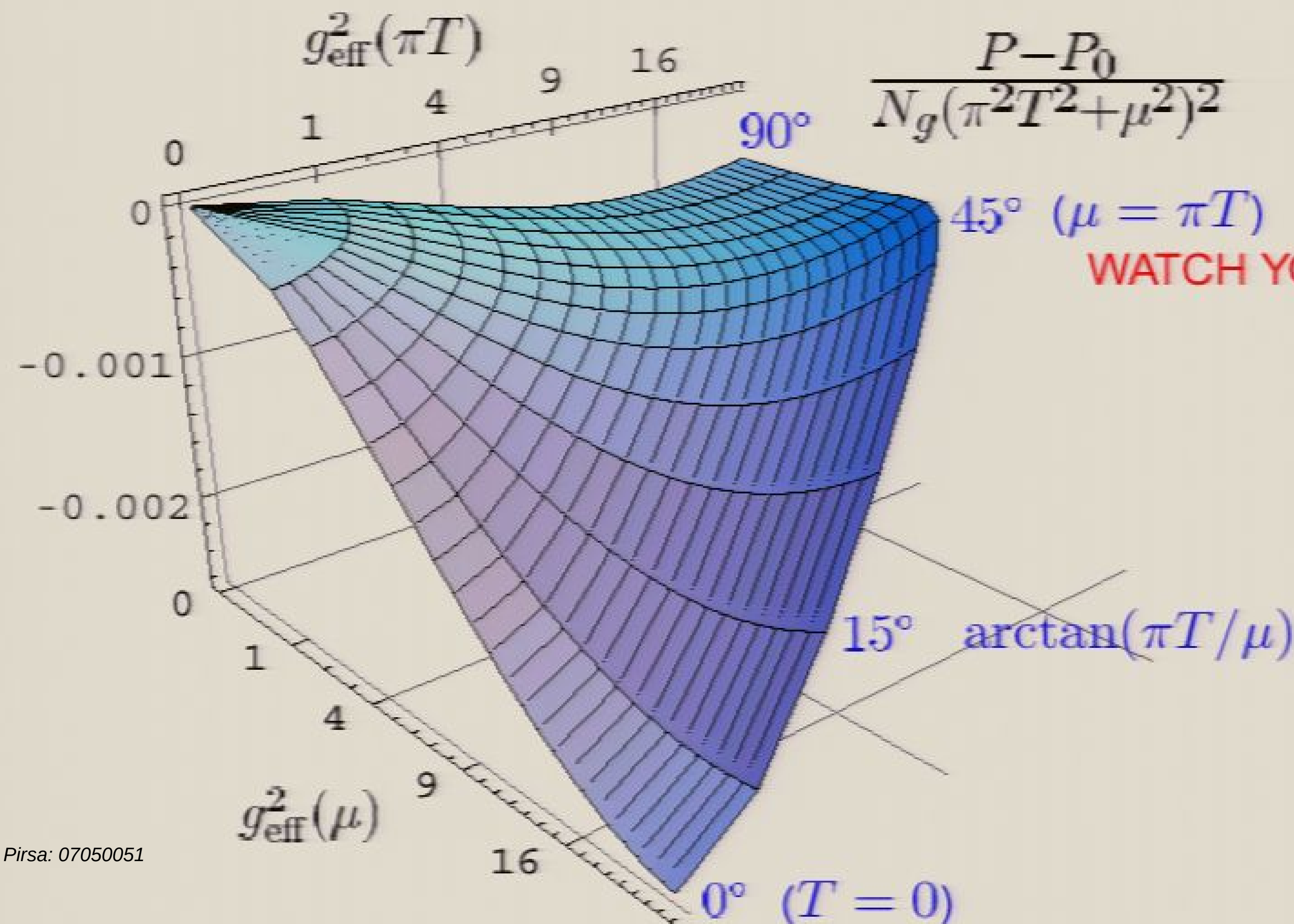
Strict perturbative $O(g^6)$-result vs. unexpanded m_E:



Large-N_f pressure at finite chemical potential μ

Straightforward generalization to finite chemical potential by evaluating $\Pi_{T,L}$ with

$$n_f(k,T,\mu) = \frac{1}{2}\left(\frac{1}{e^{(k-\mu)/T}+1} + \frac{1}{e^{(k+\mu)/T}+1}\right)$$



WATCH YOUR STEP!

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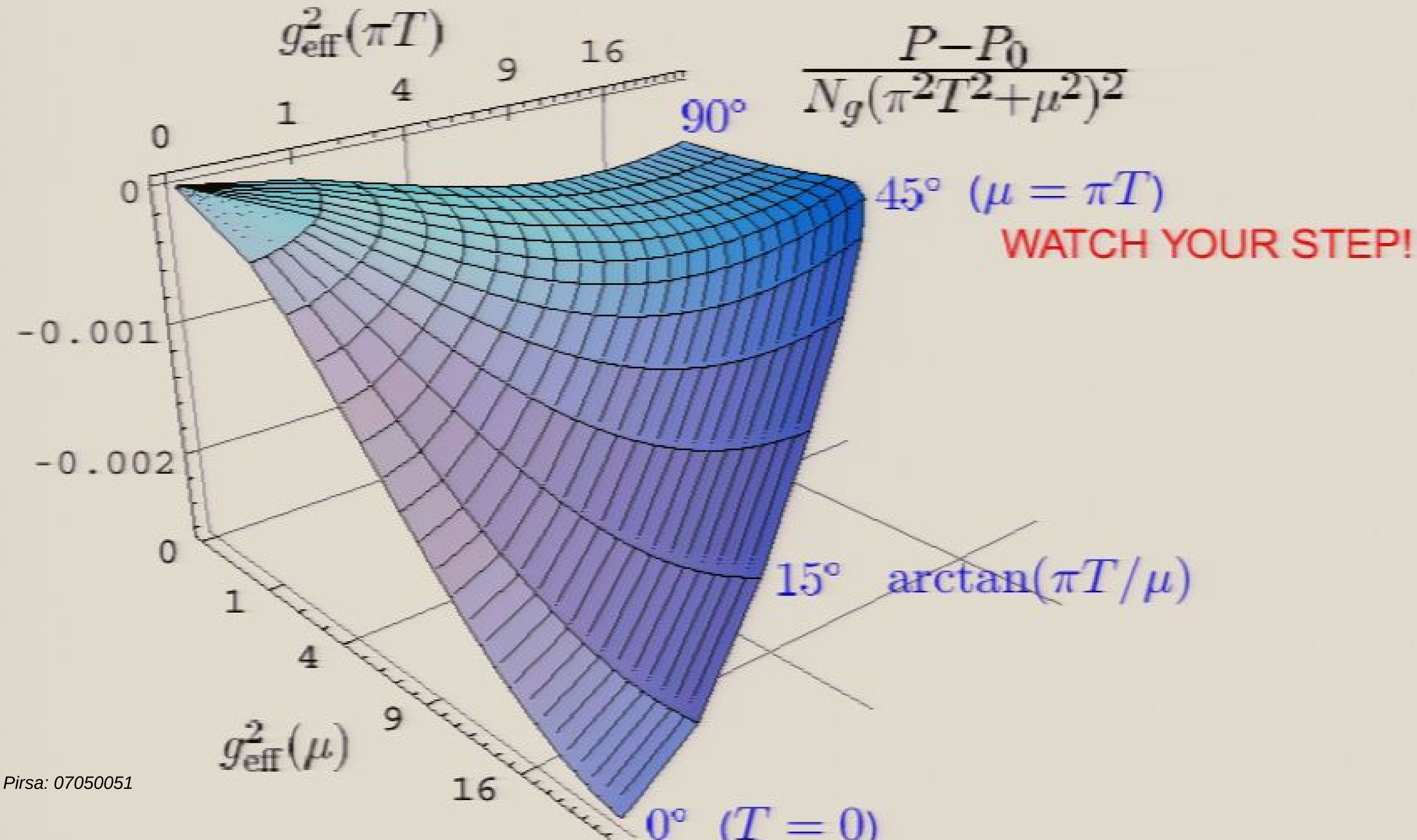

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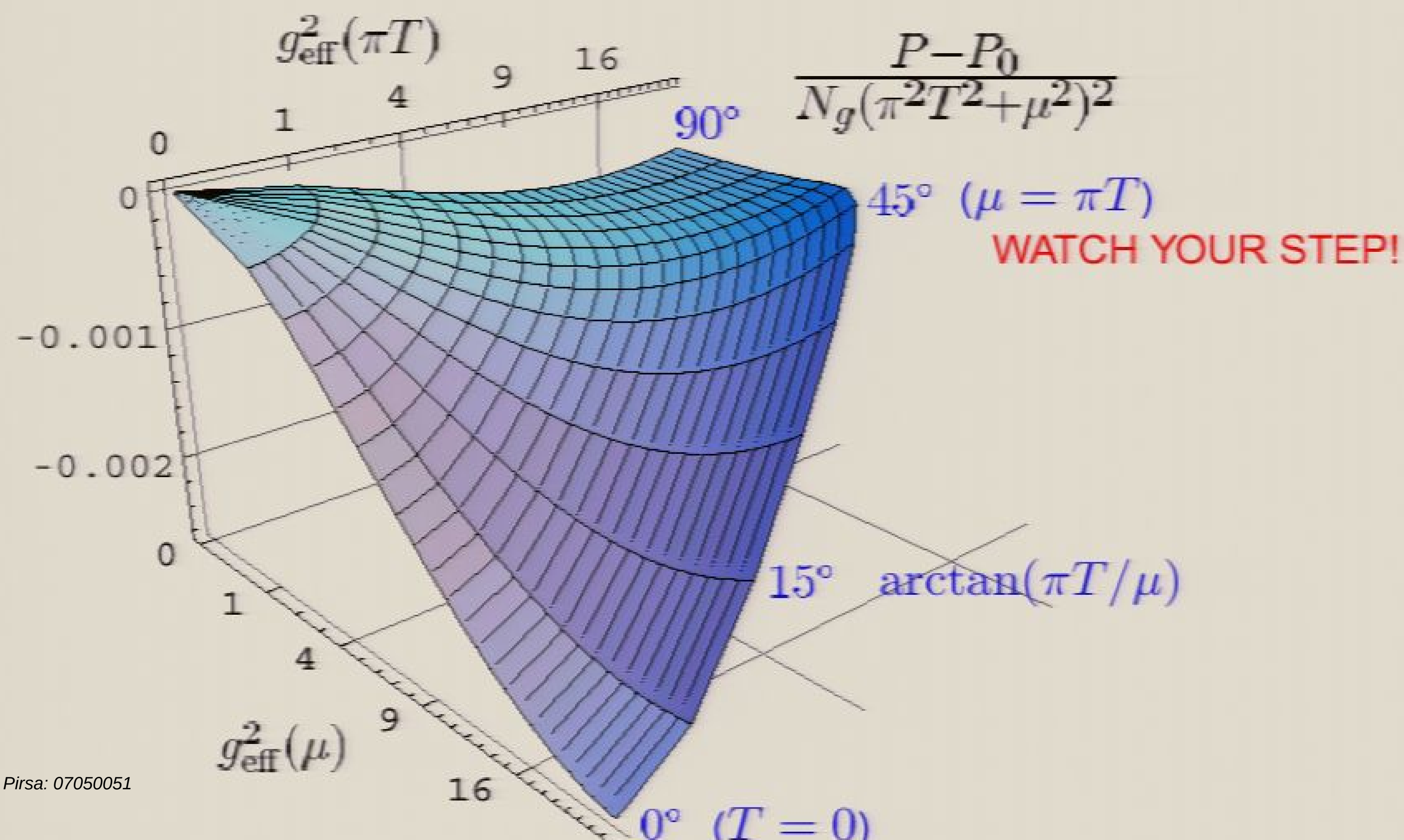
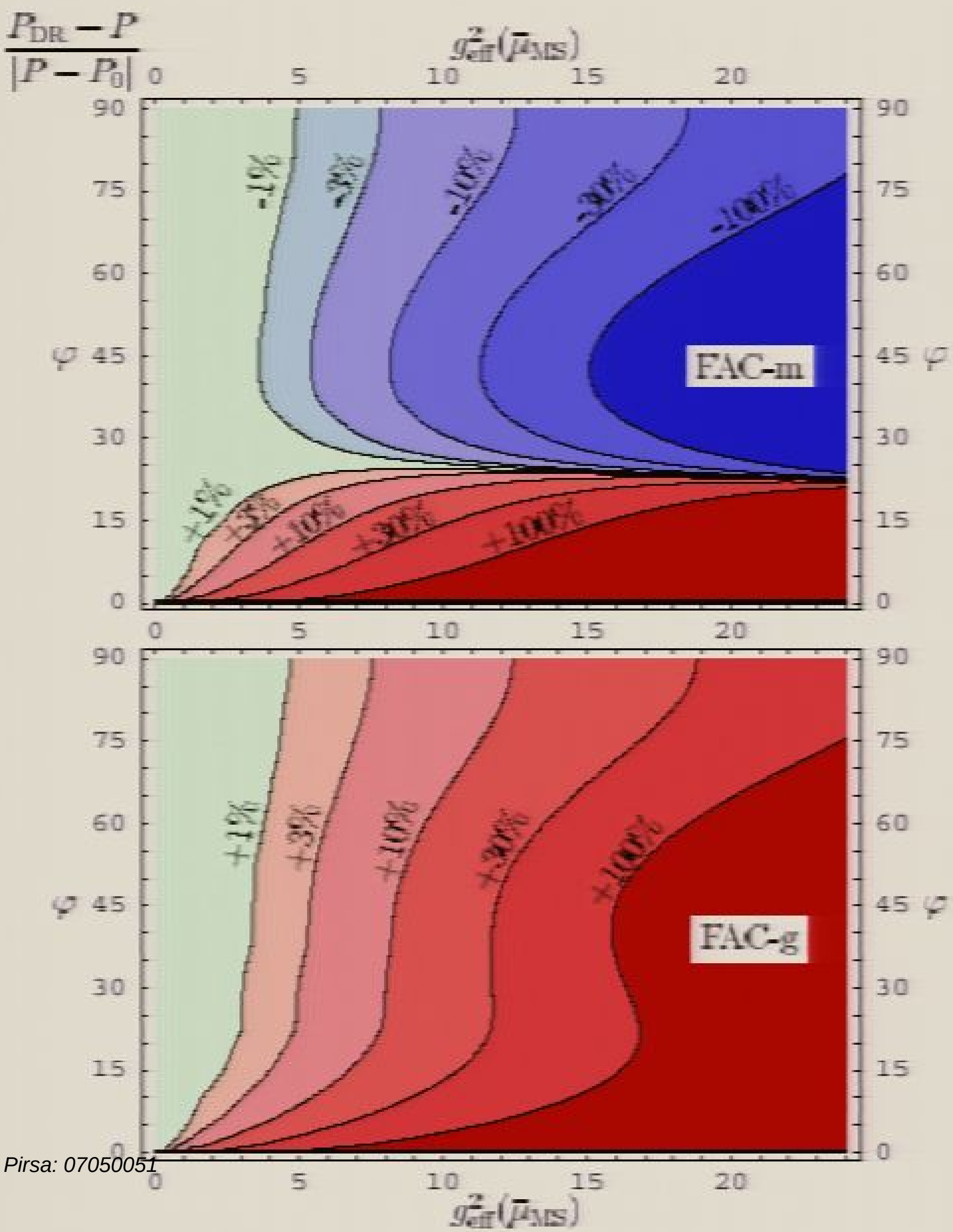


Comparison with complete dimensional reduction results for all μ, T



Ipp, AR & Vuorinen, PRD69 (2004) 077901

Comparison for two Fastest Apparent Convergence scales (for m_E^2 and g_E^2, resp.)

Breakdown of Dim.Red.

for $T \lesssim g_{\text{eff}}\mu/\pi$

$$\varphi \equiv \arctan(\pi T/\mu)$$

Behavior at small $T \neq 0$

LO (2-loop) term in interaction pressure gives

$$P - P_0 = -N_g \left[\tfrac{5}{9}T^4 + \tfrac{2}{\pi^2}\mu^2 T^2 + \tfrac{1}{\pi^4}\mu^4\right] \frac{g_{\text{eff}}^2}{32} + \dots$$

Entropy $\mathcal{S} = \left(\frac{\partial P}{\partial T}\right)_\mu$ at small T should start as

$$\mathcal{S} - \mathcal{S}_0 = - N_g \frac{g_{\text{eff}}^2}{8\pi^2} \mu^2 T + \dots$$

Non-Fermi-Liquid Behavior at small $T \neq 0$

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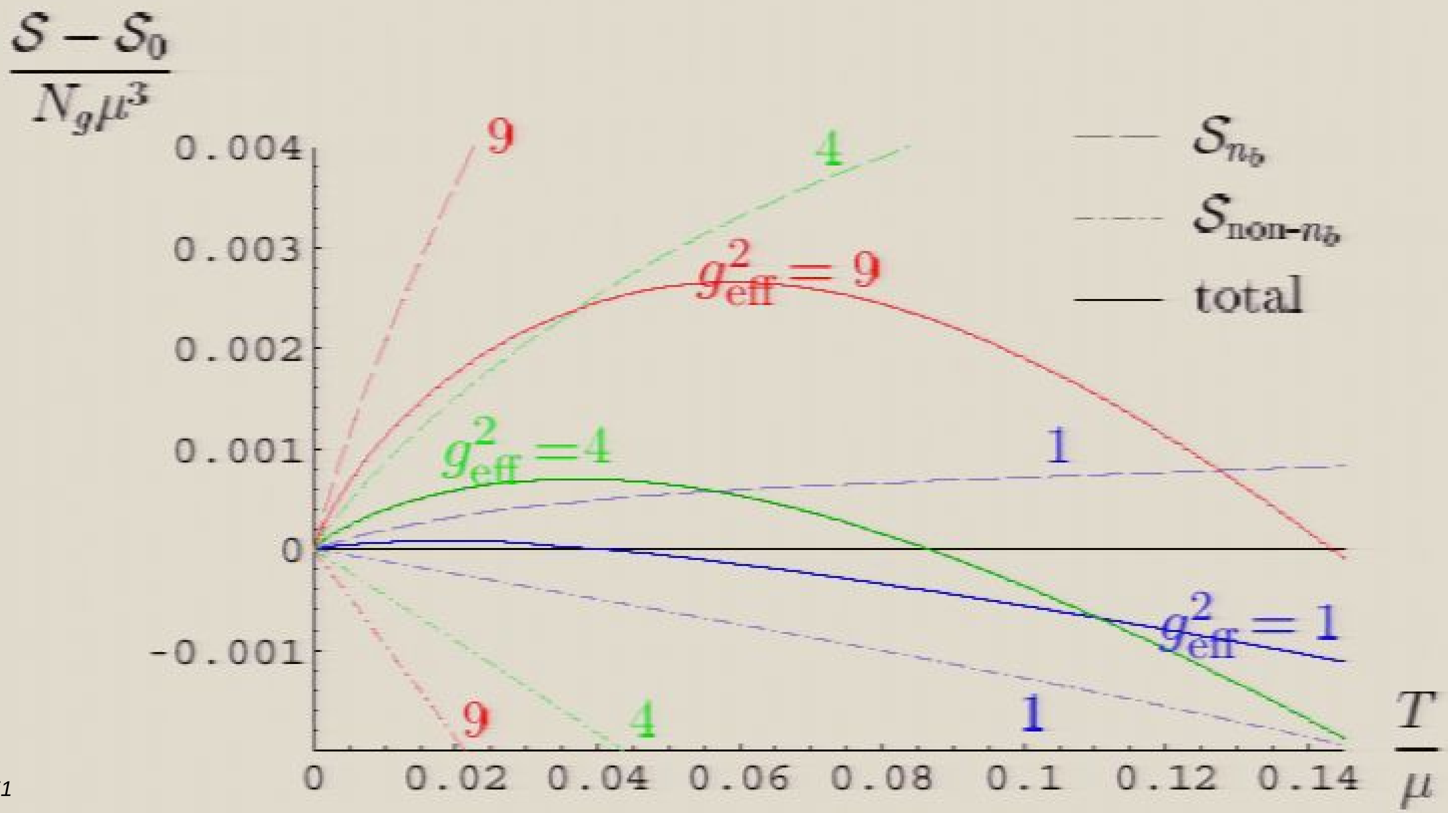
BUT:



Hard (Thermal/Dense) Loop Effective Theory

Actual effective theory at soft scales when dimensional reduction not applicable:
HTL/HDL EFT (Braaten & Pisarski, Frenkel & Taylor & Wong 1990):

$$\mathcal{L}^{\rm HTL} = \mathcal{L}_f^{\rm HTL} + \mathcal{L}_g^{\rm HTL}$$

$$= \hat{M}^2 \int \frac{d\Omega_{\mathbf{v}}}{4\pi} \bar{\psi} \gamma^\mu \frac{v_\mu}{iv \cdot D(A)} \psi + \frac{\hat{m}_D^2}{2} \mathrm{tr} \int \frac{d\Omega_{\mathbf{v}}}{4\pi} F^{\mu\alpha} \frac{v_\alpha v^\beta}{(v \cdot D_{adj.}(A))^2} F_{\mu\beta}$$

$v = (1, \mathbf{v})$ with $\mathbf{v}^2 = 1$ is direction of hard particles' momenta $p^\mu \sim T v^\mu$

- gauge invariant also in the non-static case
- nonlocal (because modes integrated out are real rather than virtual)

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HL EFT: generalization to nonthermal anisotropic distribution functions

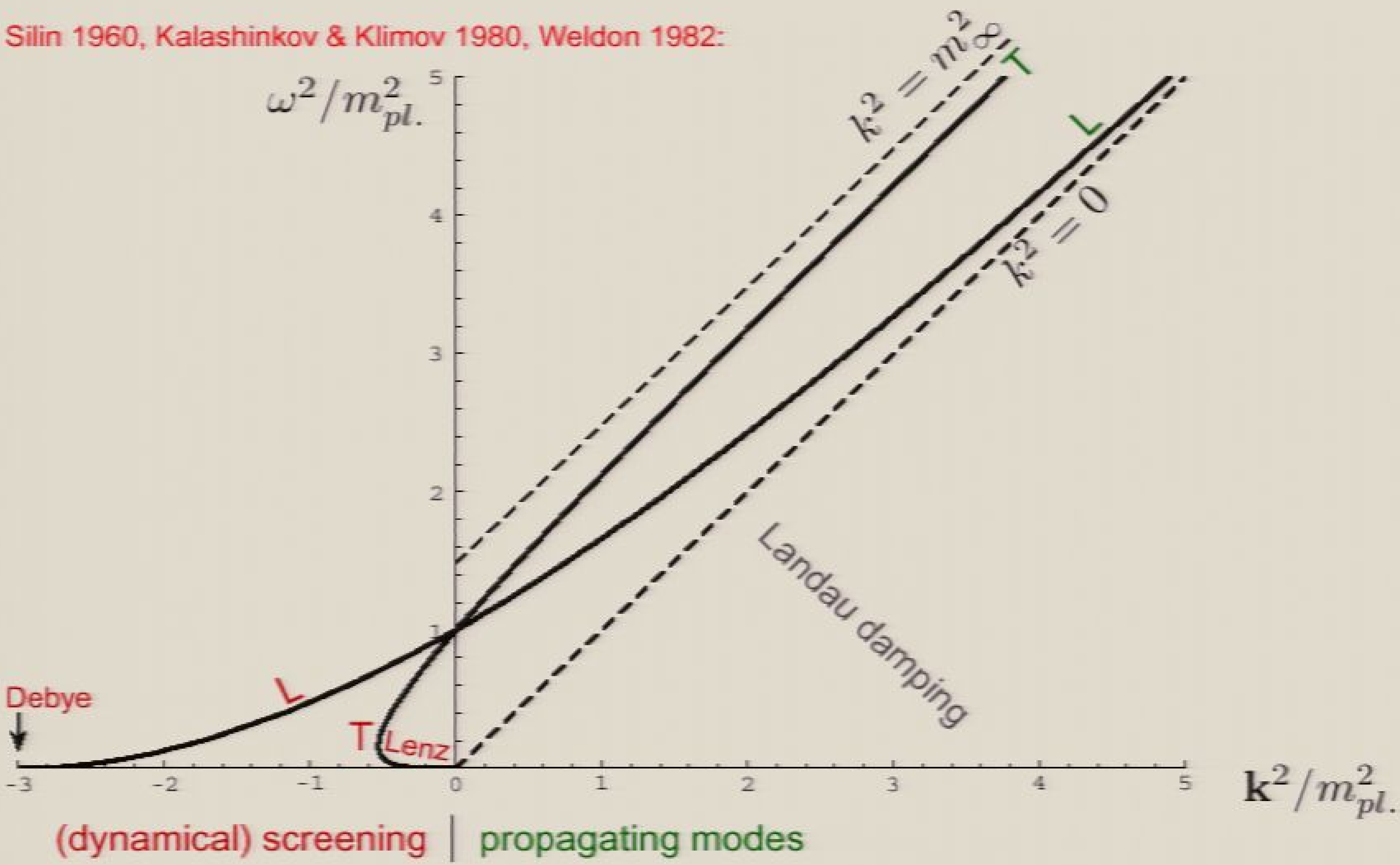
(Pisarski 1997; Mrówczyński, AR & Strickland 2004

- effective theory for QGP instabilities!

(Arnold & Lenaghan & Moore & Yaffe; AR & Romatschke & Strickland; Bödeker & Rummukainen)

Dispersion laws of HTL/HDL gauge bosons

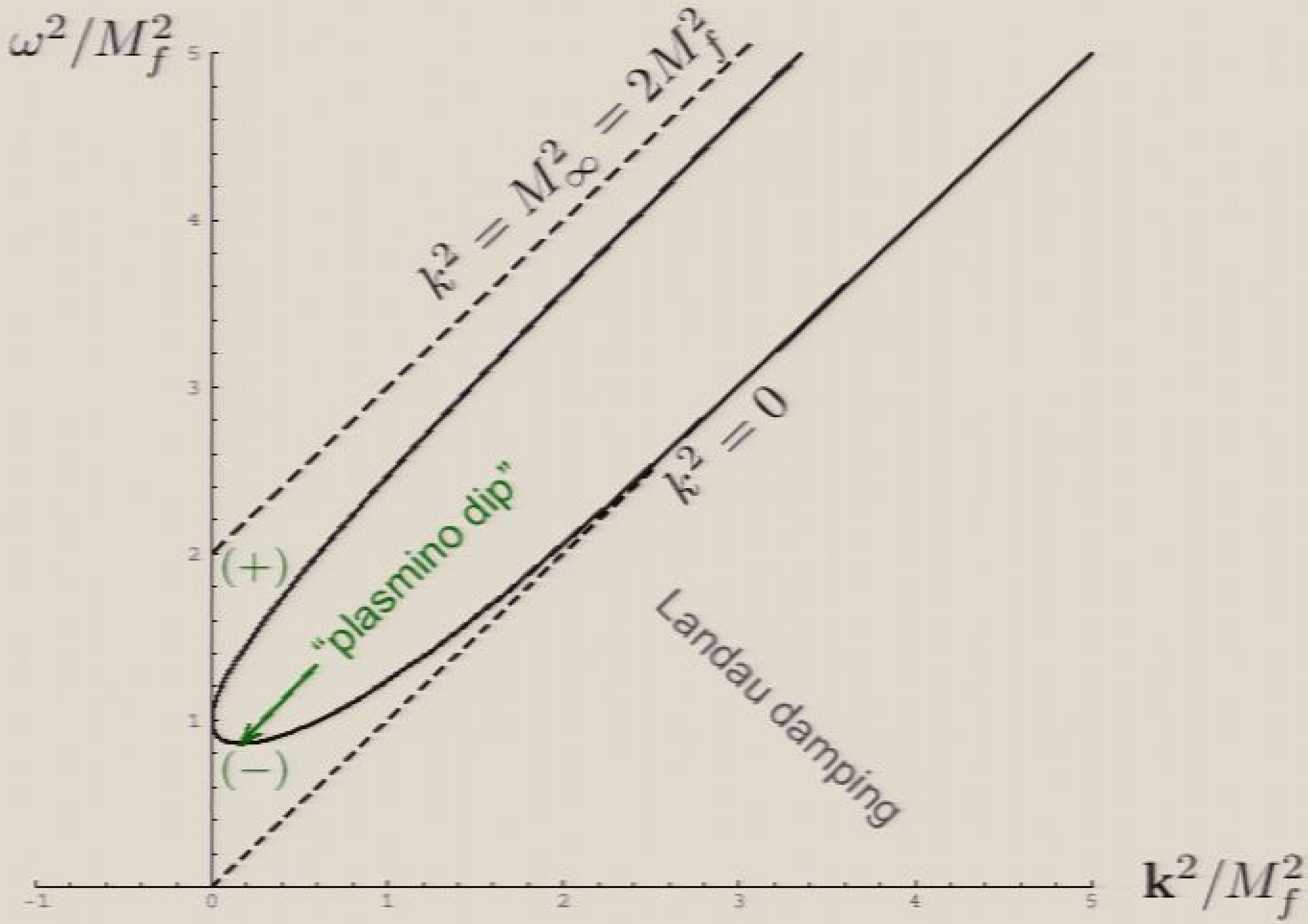
Silin 1960, Kalashinkov & Klimov 1980, Weldon 1982:



- Debye screening of electrostatic modes with $m_D^2 = 3m^2_{pl.} = 2m^2_\infty$
- Weak screening of quasistatic magnetic modes: $\kappa = \sqrt{-\mathbf{k}^2} = [\pi m_\mathrm{D}\omega/4]^{1/3}$

Dispersion laws of HTL/HDL fermionic excitations

Klimov 1981, Weldon 1982, 1989:



(no screening) | propagating modes

- Extra collective mode $(-)$ with negative helicity over chirality ratio

Low-temperature expansion of entropy

Dynamical magnetic screening scale $\kappa = [\pi m_{\rm D}\omega/4]^{1/3}$
$\rightarrow$ low-T entropy with log's and fractional powers in T:

"Anomalous specific heat"

T. Holstein, R.E. Norton & P. Pincus, PRB8 (1973) 2649; Chakravarty, Norton & Syljuasen, PRL 74 (1995) 1423

A. Ipp, A. Gerhold & AR, PRD69 (2004) 011901R; PRD70 (2004) 105015

$$\frac{S-S_0}{N_g} = \frac{g_{\rm eff}^2\mu^2 T}{36\pi^2}\left(\ln\frac{4g_{\rm eff}\mu}{\pi^2 T} - 2 + \gamma_E - \frac{6}{\pi^2}\zeta'(2)\right)$$
$$-\frac{8\,2^{2/3}\Gamma(\frac{8}{3})\zeta(\frac{8}{3})}{9\sqrt{3}\pi^{11/3}}(g_{\rm eff}\mu)^{4/3}T^{5/3} + \frac{80\,2^{1/3}\Gamma(\frac{10}{3})\zeta(\frac{10}{3})}{27\sqrt{3}\pi^{13/3}}(g_{\rm eff}\mu)^{2/3}T^{7/3}$$
$$+\frac{2048-256\pi^2-36\pi^4+3\pi^6}{540\pi^2}T^3\left[\ln\frac{g_{\rm eff}\mu}{T} - 4.3493485\ldots\right] + O(T^{11/3})$$

Low-temperature expansion of entropy

Dynamical magnetic screening scale $\kappa = [\pi m_D \omega/4]^{1/3}$
$\rightarrow$ low-T entropy with log's and fractional powers in T:

"Anomalous specific heat"

T. Holstein, R.E. Norton & P. Pincus, PRB8 (1973) 2649; Chakravarty, Norton & Syljuasen, PRL 74 (1995) 1423

A. Ipp, A. Gerhold & AR, PRD69 (2004) 011901R; PRD70 (2004) 105015

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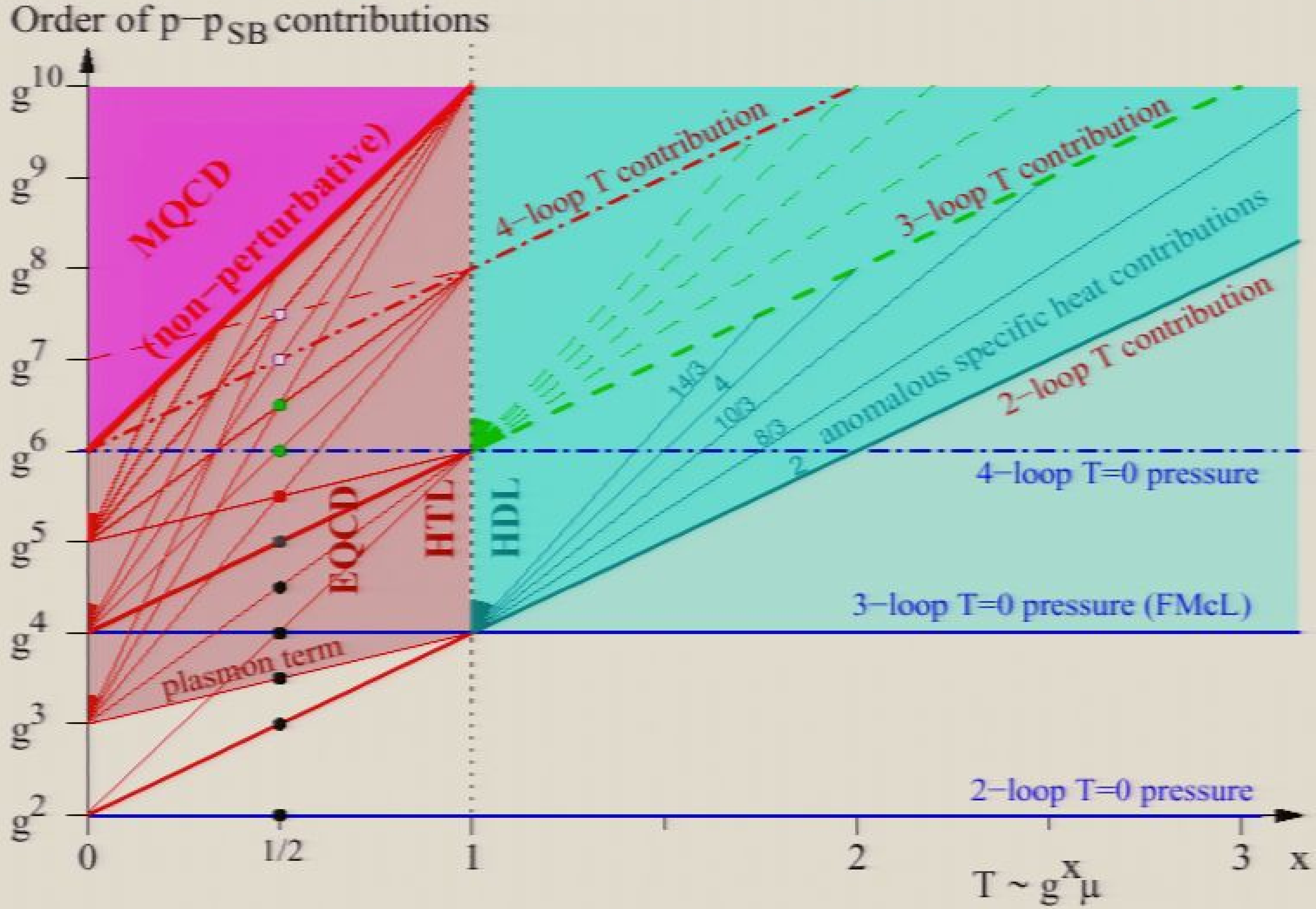
- Systematic expansion for $T/\mu \sim g_{\rm eff}^{1+\delta}$ with $\delta > 0$:

$$\frac{S-S_0}{N_g \mu^3} \sim g_{\rm eff}^{3+\delta}\ln\frac{c}{g_{\rm eff}} + g_{\rm eff}^{3+(5/3)\delta} + g_{\rm eff}^{3+(7/3)\delta} + g_{\rm eff}^{3+3\delta}\ln\frac{c}{g_{\rm eff}} + g_{\rm eff}^{3+(11/3)\delta} + \ldots$$

Structure of perturbation theory at parametrically small $T/$

Individual terms in coupling expansion for given $x \geq 0$ in $T \sim g^x \mu$



Leading term of interaction entropy for $T \sim g\mu$

Anomalous low-temperature series is applicable only for $T \ll g_{\rm eff}\mu$

complete infinite low-temperature series is contained in
HDL-resummed expression Gerhold, Ipp & AR, PRD70 (2004)

$$\frac{1}{N_g}(\mathcal{S}-\mathcal{S}^0) = -\frac{g_{\rm eff}^2\mu^2 T}{24\pi^2} - \frac{1}{2\pi^3}\int_0^\infty dq_0\,\frac{\partial n_b(q_0)}{\partial T}\int_0^\infty dq\,q^2\Bigg[2\,{\rm Im}\,\ln\left(\frac{q^2-q_0^2+\Pi_T^{\rm HDL}}{q^2-q_0^2}\right)+{\rm Im}\,\ln\left(\frac{q^2-q_0^2+\Pi_L^{\rm HDL}}{q^2-q_0^2}\right)\Bigg] +O(g_{\rm eff}^4\mu^2 T)$$

- full leading-order result $\forall\, T \ll \mu$

$g_{\rm eff}\mu \ll T \ll \mu$:

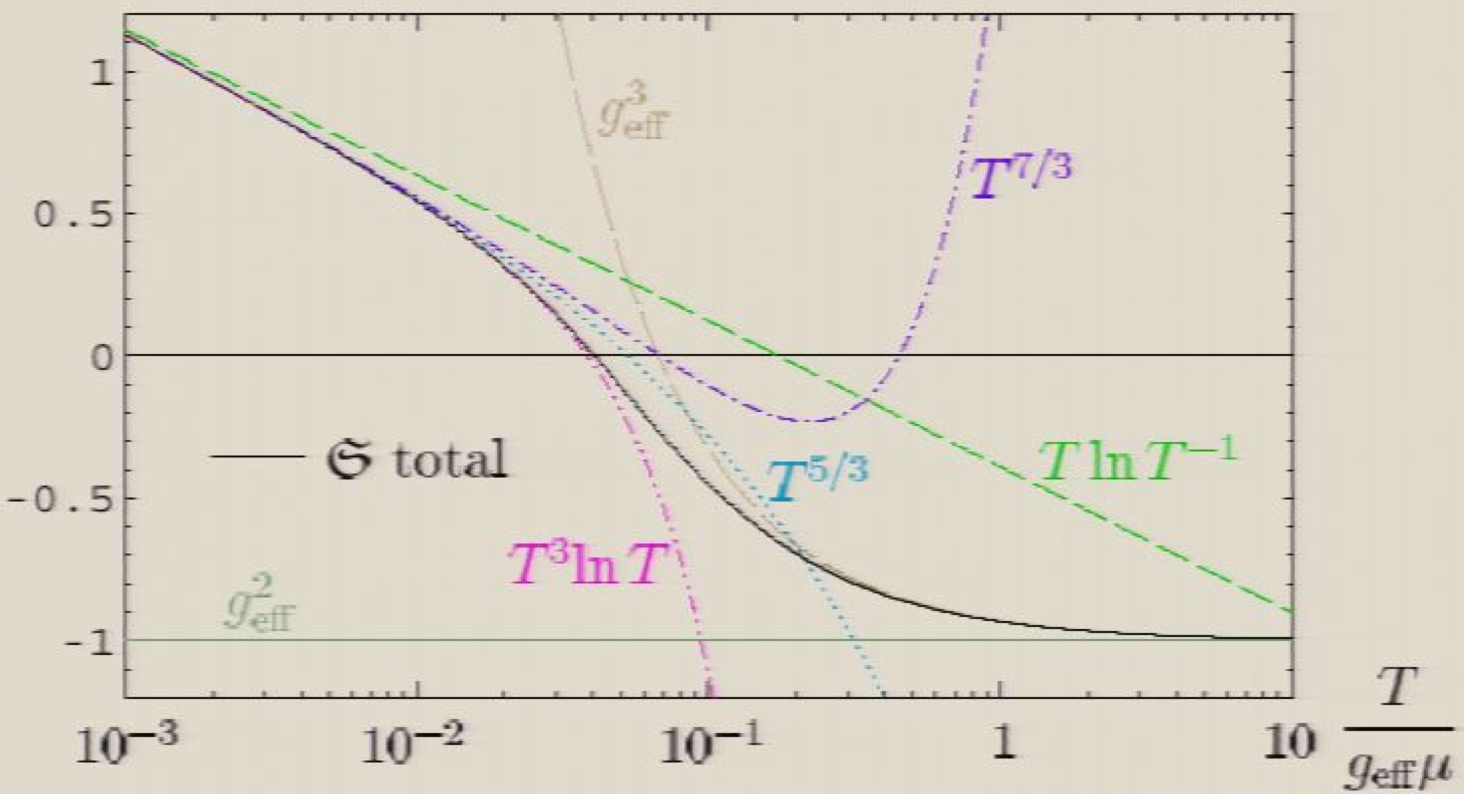
dominant resummation effect now *longitudinal plasmon effect* (Debye screening)

$$\frac{1}{N_g}(\mathcal{S}-\mathcal{S}_0) \simeq -\frac{g_{\rm eff}^2\mu^2 T}{8\pi^2}+\frac{g_{\rm eff}^3\mu^3}{12\pi^4}$$

$\longleftarrow$ also from dimensional reduction

HDL-resummed low-T entropy

$$\mathfrak{S} = \frac{\mathcal{S} - \mathcal{S}_0}{N_g (g_{\mathrm{eff}}\mu)^2 T/(8\pi^2)}$$



low-temperature expansion to order $T \ln T$, $T^{5/3}$, $T^{7/3}$, $T^3 \ln T$, resp.

g_{eff}^2, g_{eff}^3: perturbative result for $g_{\mathrm{eff}}\mu \ll T \ll \mu$

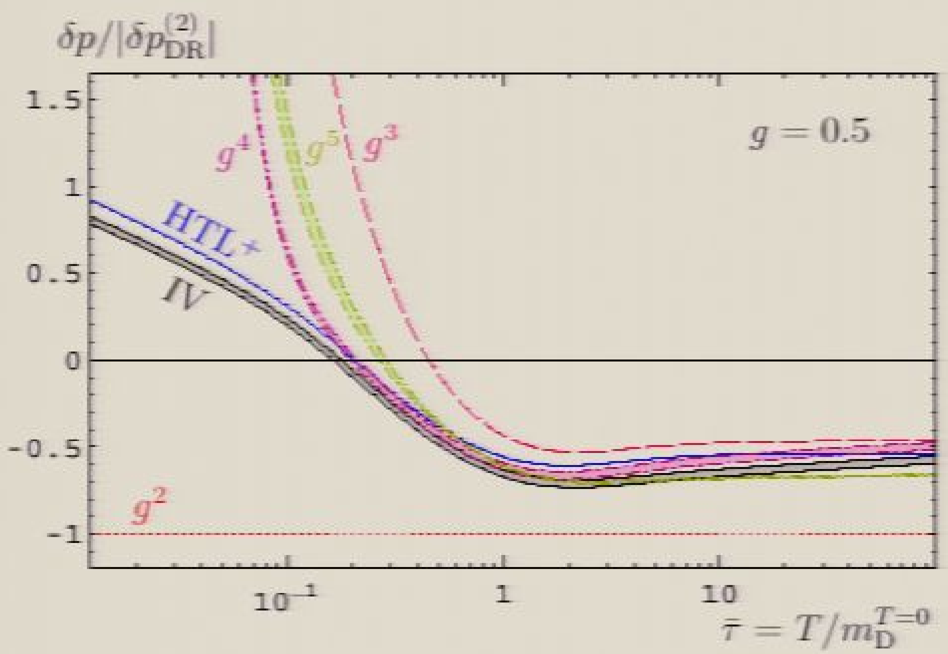
Unified calculation of QCD pressure

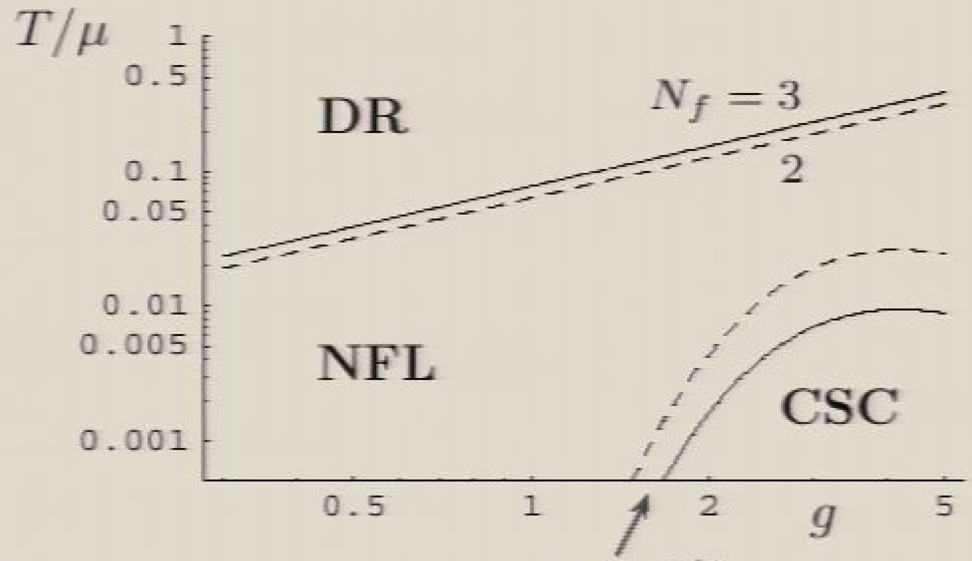
Ipp, Kajantie, AR & Vuorinen, PRD74 (2006

Full fourth-order calculation (IV): Perturbative calculation of IR-safe diagrams
+ full one-loop resummation of 2GR (2-gluon-reducible) diagrams

- Confirmation of both HDL/HTL resummation and dimensional reduction results and their range of applicability:

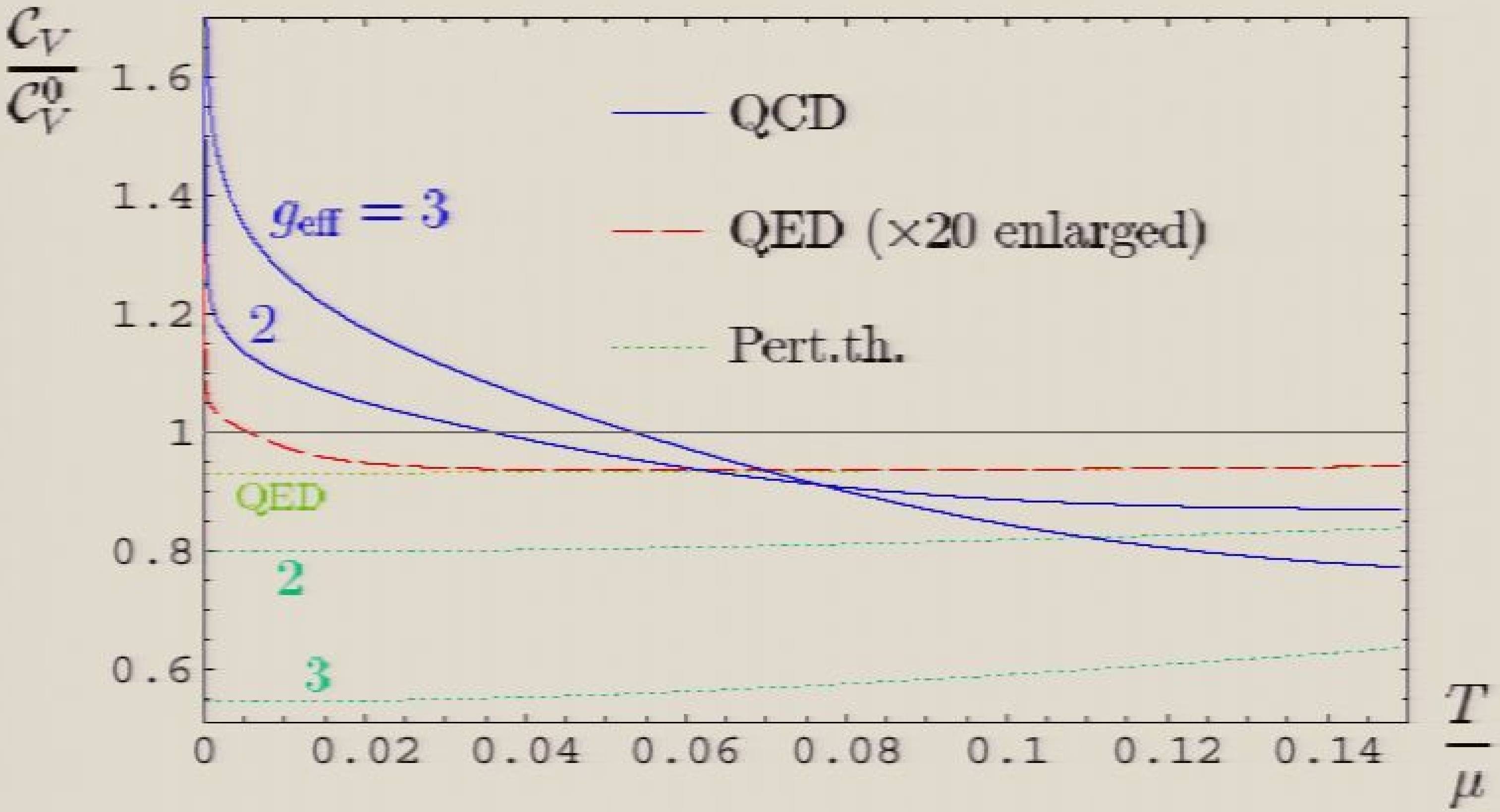
$$\delta p \equiv p - p_{\rm SB} - (p - p_{\rm SB})|_{T=0}$$





$$\frac{T_c^{\rm 2SC}}{\mu} \simeq \frac{T_c^{\rm CFL}}{\mu} \simeq 2\frac{e^\gamma}{\pi} e^{-(\pi^2+4)/8}(4\pi)^4\left(\frac{2}{N_f}\right)^{5/2} g^{-5} e^{-3\pi^2/\sqrt{2}g}$$

HDL-resummed result for the specific heat



$N_f = 2$ QCD: $g_{\rm eff} = 2$ and 3 correspond to $\alpha_s \approx 0.32$ and 0.72

significant deviations from naive perturbative result for low-T specific heat in QCD for $T/\mu \lesssim 0.05$

$\rightarrow$ cooling of (proto-)neutron stars with normal quark matter component

Non-Fermi-liquid effects also in neutrino emissitivity (T. Schäfer & K. Schwenzer, PRD70 (2004) 114037)

HDL-resummed result for NFL quark dispersion laws

Gerhold & AR, PRD 71 (2005) 085010

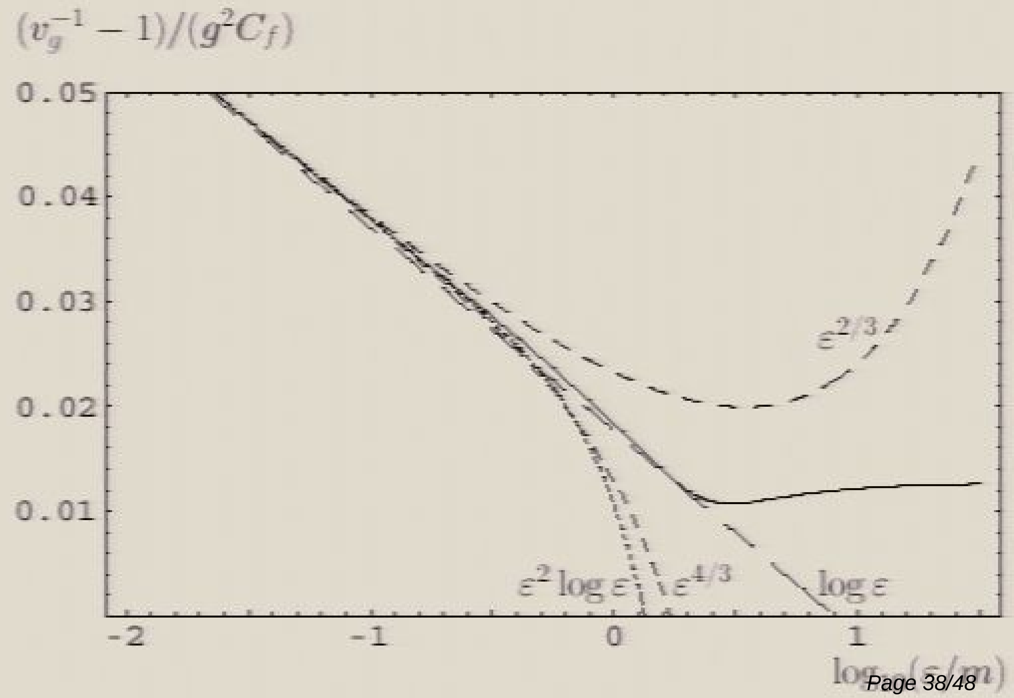
Logarithms and fractional powers of $\varepsilon = E - \mu$ in
group velocity of quark quasiparticles near would-be Fermi surface:

$$T = 0: \quad v_g^{-1}(\varepsilon) = 1 + \frac{g^2 C_f}{12\pi^2} \ln \frac{8.07m}{|\varepsilon|} + O((\varepsilon/m)^{2/3}) \qquad m^2 = N_f (g\mu/2\pi)^2$$

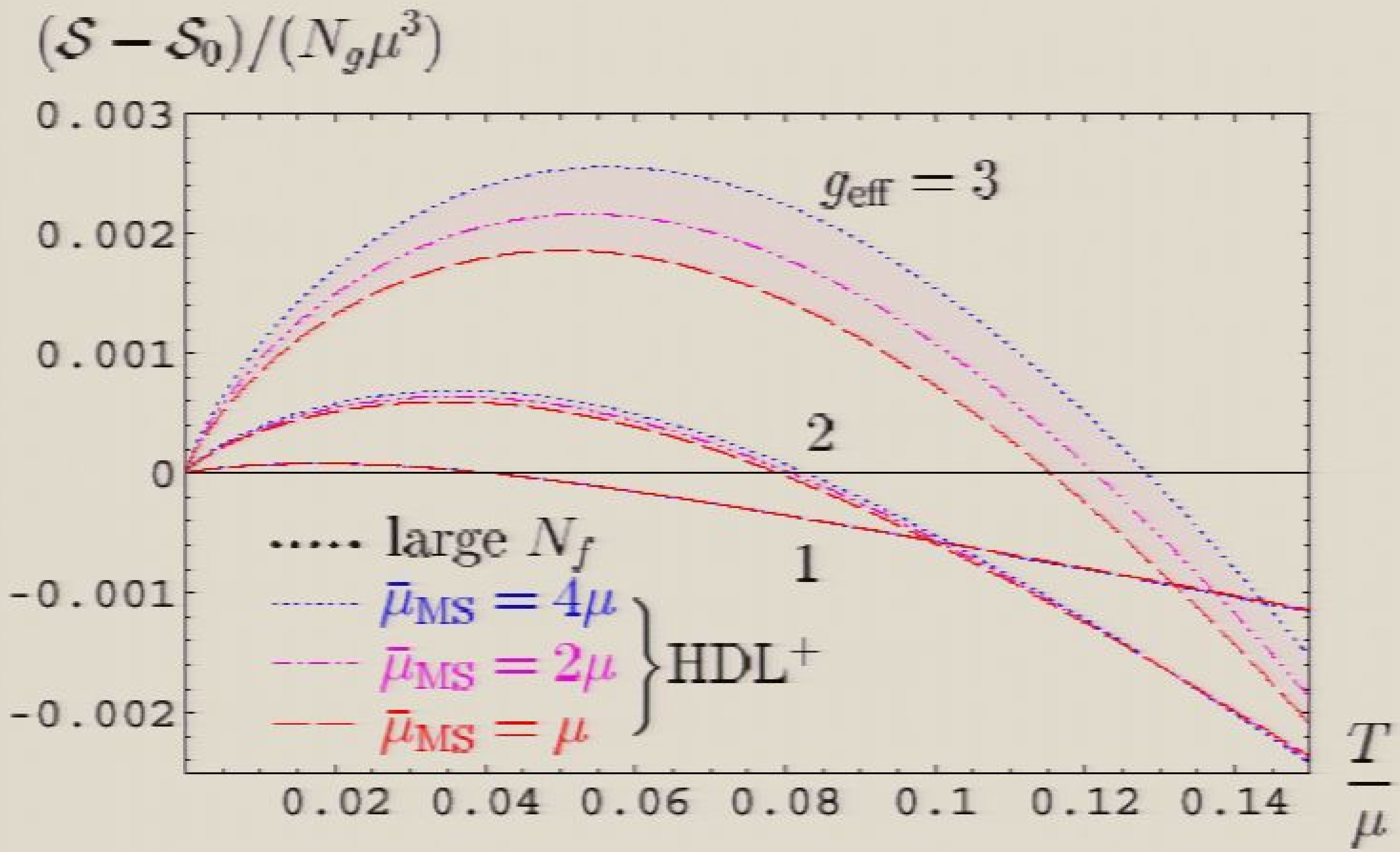
$$g\mu \gg T \neq 0: \quad v_g^{-1}(0) = 1 + \frac{g^2 C_f}{12\pi^2} \ln \frac{9.15m}{T} + O((T/m)^3)$$

compare with scale of log in $\mathcal{C}_V$

$$\mathcal{C}_V - \mathcal{C}_V^0 = \frac{N_g m^2 T}{9} \ln \frac{0.282m}{T} + \ldots$$



HDL-resummed entropy vs. nonperturbative large-N_f resul



... $\bar{\mu}_{\text{MS}}$-dependence displays uncertainties due to contributions suppressed by powers of $\frac{g_{\text{eff}}^2}{4\pi}$

Very good agreement for small T; less good at higher T, g_{eff}

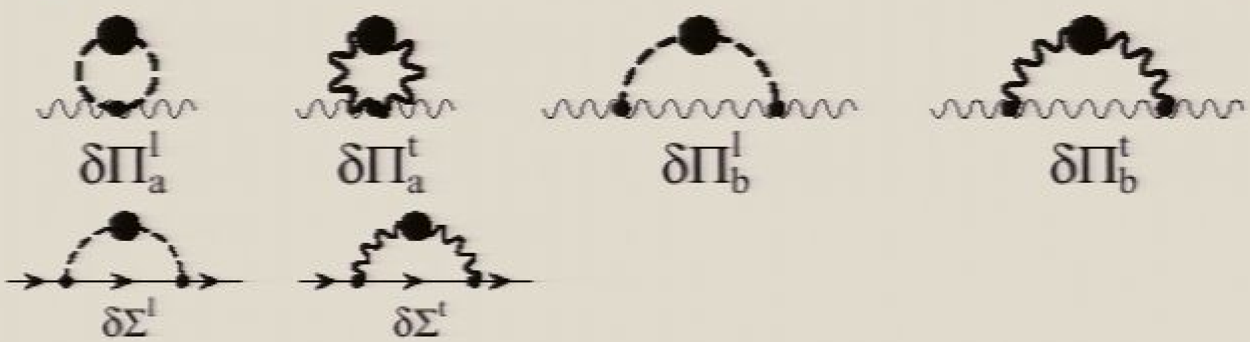
HTL-resummed entropy at high T, $\mu = 0$

Blaizot, Iancu & AR (1999):

HTL resummation through 2-loop Φ-derivable (2PI) entropy expression:

$$\mathcal{S} \;=\; -\,\mathrm{tr}\int_K \frac{\partial n(k_0)}{\partial T}\left[\Im m \log G^{-1} - \Im m\Pi\, \Re e G\right]$$
$$-2\,\mathrm{tr}\int_K \frac{\partial f(k_0)}{\partial T}\left[\Im m \log S^{-1} - \Im m\Sigma\, \Re e S\right] + \mathcal{S}_{3-\mathrm{loop}},$$

- nontrivial reorganization of perturbation theory:
  convergence-spoiling g^3 contribution kept in nonpolynomial form;
  3/4 of g^3 contribution contained in NLO correction to m_∞^2, M_∞^2 at $k \sim T$:



(momentum-dependent even for $k \gg gT$)

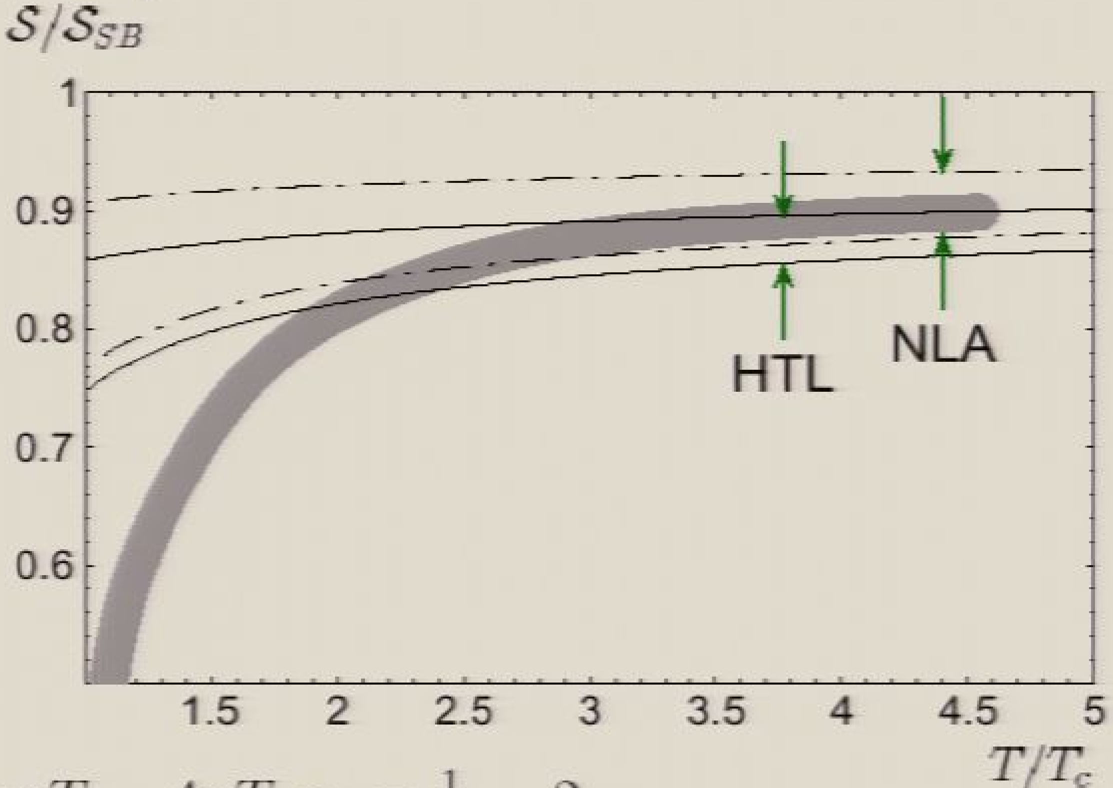
Application to QCD

Approximately self-consistent evaluations:

1) HTL self energies and propagators (full lines)

2) NLA (dash-dotted): momentum averaged NLO corrections to m_∞^2, M_∞^2, included through quadratic gap equation for hard momenta $p > \sqrt{c_\Lambda 2\pi T m_D}$

pure glue, $N_f = 0$:



$\bar{\mu}_{MS} = \pi T \ldots 4\pi T, c_\Lambda = \frac{1}{2} \ldots 2$

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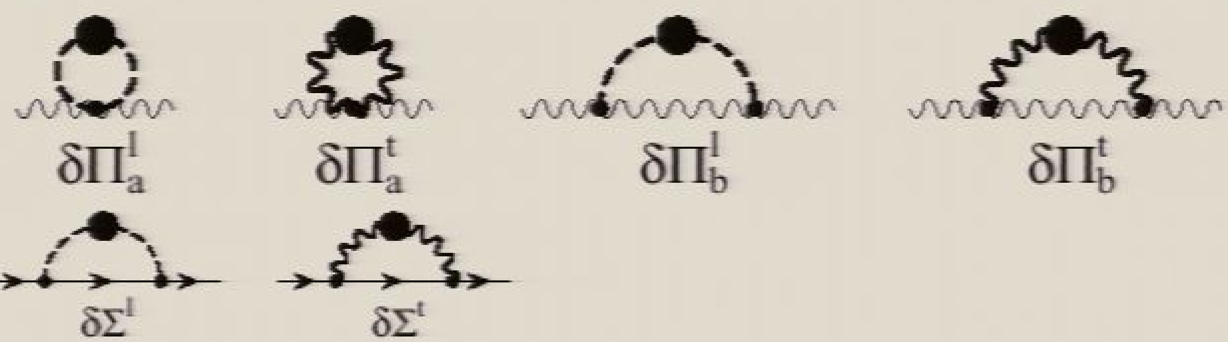

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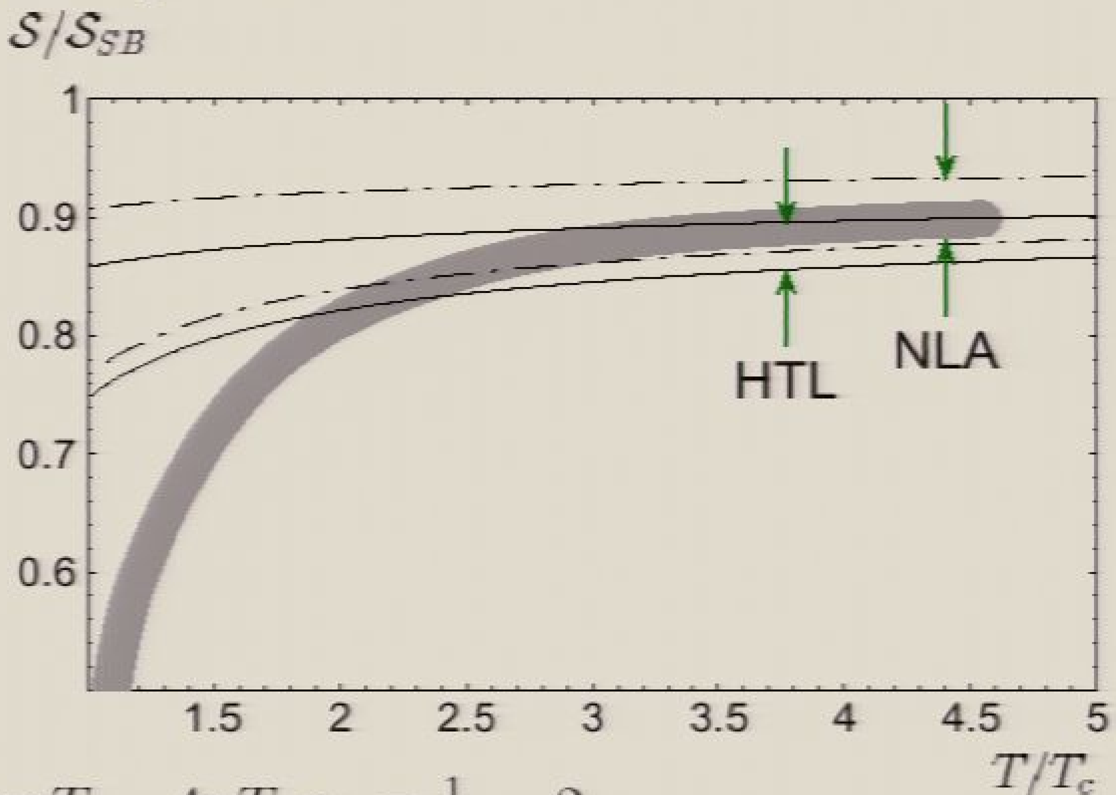

$\bar{\mu}_{MS} = \pi T \ldots 4\pi T, c_\Lambda = \frac{1}{2} \ldots 2$

Application to $\mathcal{N} = 4$ super-Yang-Mills

Weak-coupling: $\mathcal{S}/\mathcal{S}_0 = 1 - \frac{3}{2\pi^2}\lambda + \frac{\sqrt{2}+3}{\pi^3}\lambda^{3/2} + \ldots$

even more poorly convergent: $\mathcal{S}/\mathcal{S}_0 \geq 1$ for $\lambda \equiv g^2 N \geq 1.14$

$(1.85$ for pure glue QCD

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Strong coupling (AdS/CFT): $S/\mathcal{S}_0 = \frac{3}{4}\left(1 + \frac{15\zeta(3)}{8}\lambda^{-3/2} + \ldots\right)$

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Possible interpolation: Padé

Just enough information to fix uniquely all coefficients of [4,4] Padé approximant:

$$R_{[4,4]} = \frac{1+\alpha\lambda^{1/2}+\beta\lambda+\gamma\lambda^{3/2}+\delta\lambda^2}{1+\bar{\alpha}\lambda^{1/2}+\bar{\beta}\lambda+\bar{\gamma}\lambda^{3/2}+\bar{\delta}\lambda^2}$$

$$\bar{\alpha} = \alpha, \quad \bar{\beta} = \frac{4}{3}\beta, \quad \bar{\gamma} = \frac{4}{3}\gamma, \quad \bar{\delta} = \frac{4}{3}\delta,$$

$$\alpha = \frac{2(9+3\sqrt{2}+\gamma\pi^3)}{9\pi}, \quad \beta = \frac{9}{2\pi^2}, \quad \gamma = \frac{2}{15\zeta(3)}, \quad \delta = \frac{2}{15\zeta(3)}\alpha$$

All coefficients positive: no poles anywhere, smooth monotonic interpolation

Blaizot, Iancu, Kraemmer & AR, hep-ph/061139

Application to $\mathcal{N} = 4$ super-Yang-Mills

(Blaizot, Iancu, Kraemmer & AR, hep-ph/0611393

Compare to HTL/NLA resummation of weak-coupling result:

HTL energies

scalar: $\Pi_s \equiv m^2_{\infty(s)}$,

gluons: $\Pi_T = m^2_{\infty(g)} + \frac{\omega^2-k^2}{2k^2}\Pi_L$, $\Pi_L = 2m^2_{\infty(g)}\left(1 - \frac{\omega}{2k}\log\frac{\omega+k}{\omega-k}\right)$,

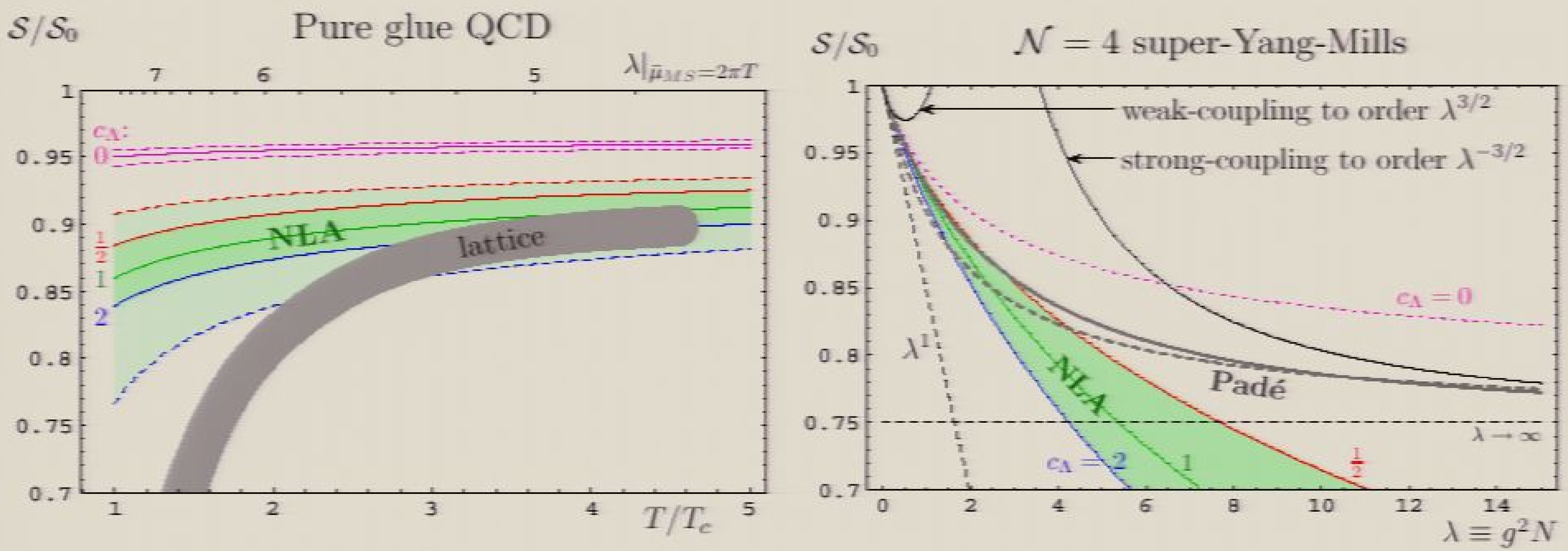
gluinos: $\Sigma_\pm = \frac{m^2_{\infty(f)}}{2k}\left(1 - \frac{\omega\mp k}{2k}\log\frac{\omega+k}{\omega-k}\right)$

with $$m^2_{\infty(s)} = m^2_{\infty(g)} = \frac{2+n_s+n_f/2}{12}\lambda T^2 = \lambda T^2,$$
$$m^2_{\infty(f)} = \frac{2+n_s}{8}\lambda T^2 = \lambda T^2$$

weighted NLO correction of (hard) thermal masses for all excitations

$$\bar{\delta} m^2_\infty = \frac{\int dk\, k\, n'(k) \Re e \delta\Pi(\omega{=}k)}{\int dk\, k\, n'(k)} = -\lambda T m_\infty \frac{2\sqrt{2}+n_s}{4\pi} = -\lambda T m_\infty \frac{\sqrt{2}+3}{2\pi}$$

Comparison pure-glue QCD and $\mathcal{N} = 4$ super-Yang-Mills



- roughly $\lambda_{\rm SYM} \leftrightarrow \frac{1}{2}\lambda_{\rm QCD}$
- QCD at $T \gtrsim 3T_c$ corresponds to $\lambda_{\rm SYM} \lesssim 2.5$
  where unresummed perturbative result fails,
  but simple HTL/NLA resummation agrees well with Padé extrapolation
- (numerically) important additional nonpert. physics in QCD for $T \lesssim 2.5T_c$
  [in SYM for $\lambda_{\rm SYM} \gtrsim 4$]
  (but there behavior of entropy no longer comparable)