

Title: Heavy Quark Diffusion from AdS/CFT

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Abstract:

# Heavy Quark Transport in AdS/CFT

Derek Teaney

Arkansas State University & SUNY Stonybrook

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  - Jorge Casalderrey-Solana, DT; hep-th/0701123
  - Jorge Casalderrey-Solana, DT; hep-ph/0605199

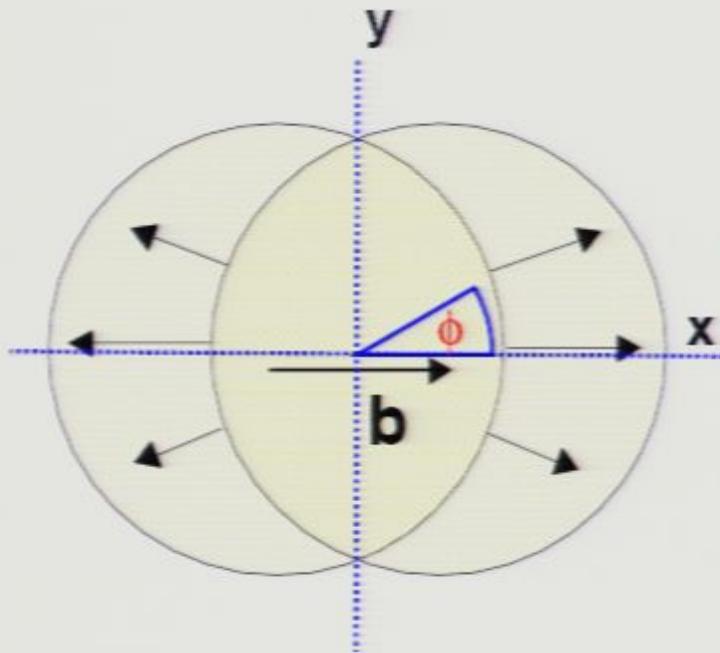
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Observation:



There is a large momentum anisotropy:

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \approx 20\%$$

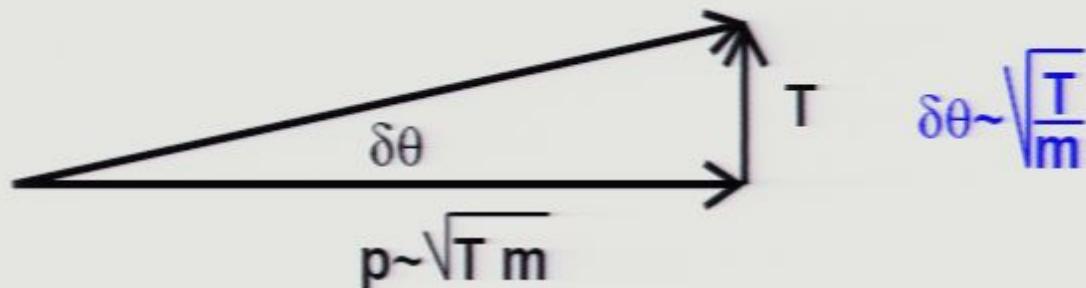
Interpretation

- The medium responds as a fluid to differences in  $X$  and  $Y$  pressure gradients

Hydro models “work”

## Estimate of transport times with Heavy Quarks

- Put a heavy quark in this medium



$$\delta\theta \sim \sqrt{\frac{T}{m}}$$

- The charm quark undergoes a random walk suffering many collisions
- The relaxation time of the heavy quark is:

$$\tau_R^{\text{charm}} \sim \frac{M}{T} \tau_R^{\text{light}}$$

If you think you know the relaxation time you should be able to compute the charm spectrum.

## Langevin description of heavy quark thermalization:

- Write down an equation of motion for the heavy quarks.

$$\begin{aligned}\frac{dx}{dt} &= -\frac{p}{M} \\ \frac{dp}{dt} &= -\underbrace{\eta_D p}_{\text{Drag}} + \underbrace{\xi(t)}_{\text{Random Force}}\end{aligned}$$

- The drag and the random force are related

$$\langle \xi_i(t) \xi_j(t') \rangle = \frac{\kappa}{3} \delta_{ij} \delta(t - t') \quad \eta_D = \frac{\kappa}{2MT}$$

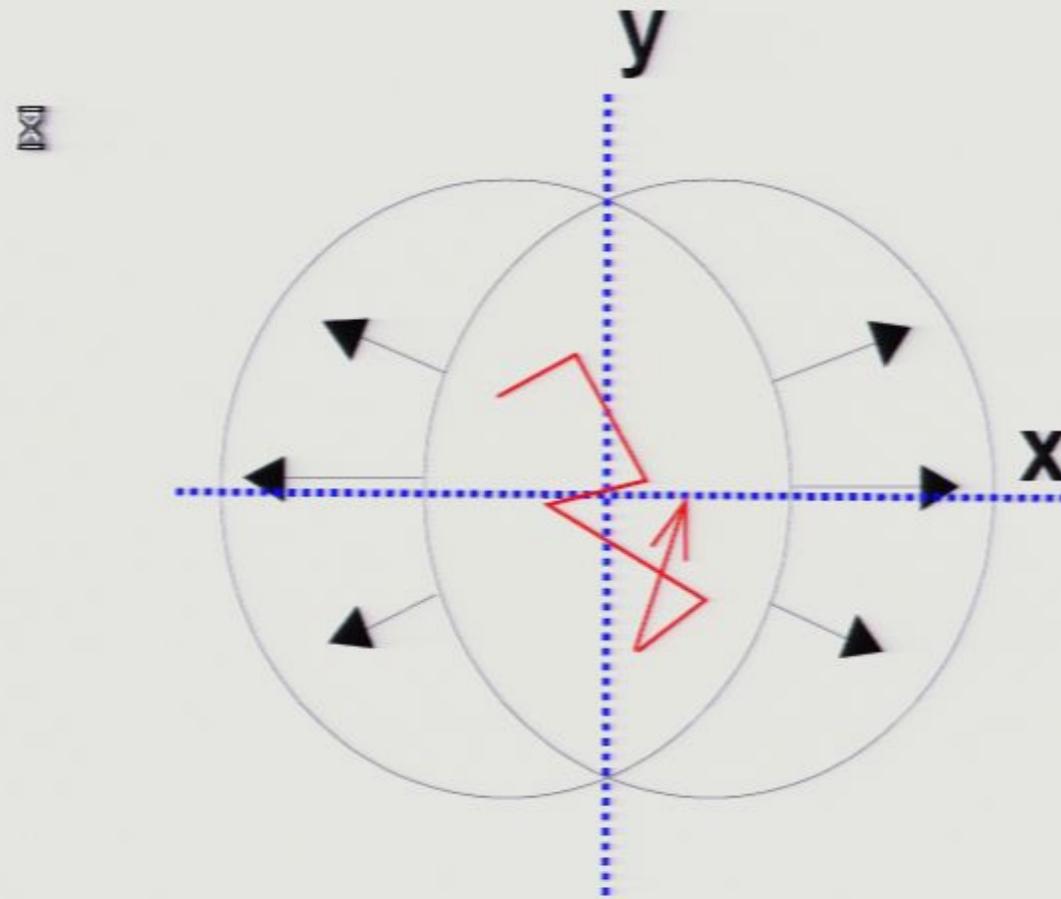
$\kappa$  = Mean Squared Momentum Transfer per Time

- Einstein related the diffusion coefficient to the mean squared momentum transfer

$$D = 2T^2/\kappa$$

All parameters are related to the heavy quark diffusion coefficient or  $\kappa$

## Hydro + Brownian Heavy Quarks



The heavy quarks will either relax to the thermal spectrum and show the same  $v_2$  as all thermal particles or not depending on the Drag/Diffusion coefficients and  $p_T$ .

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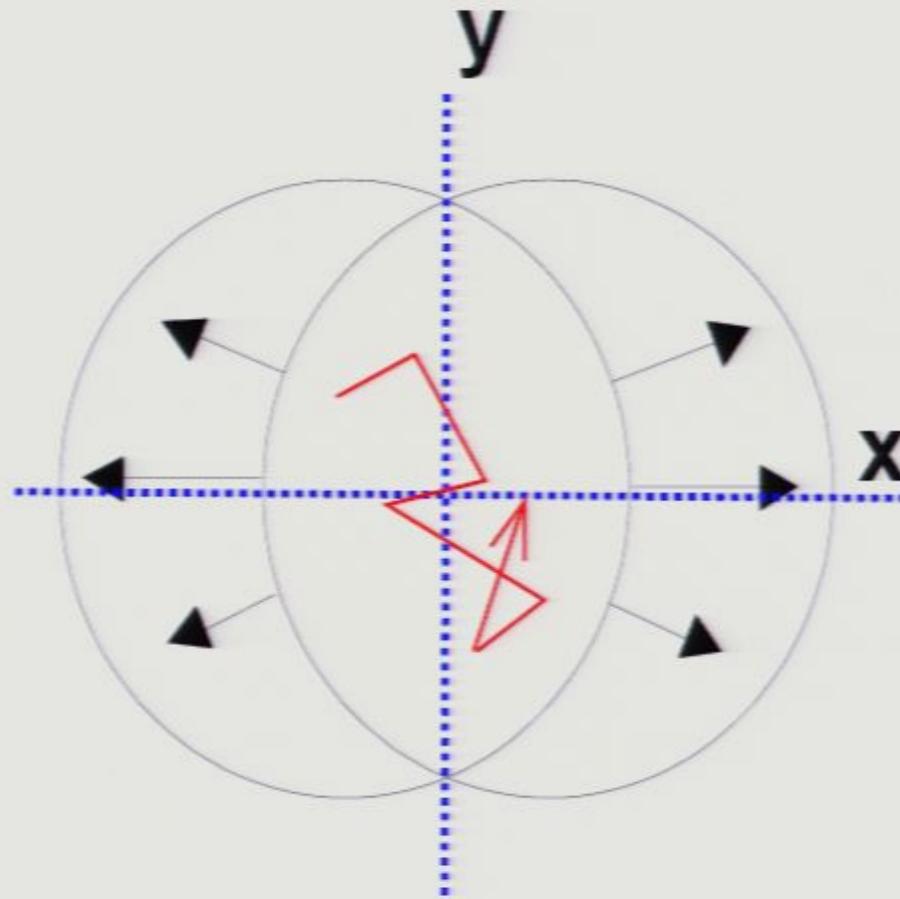
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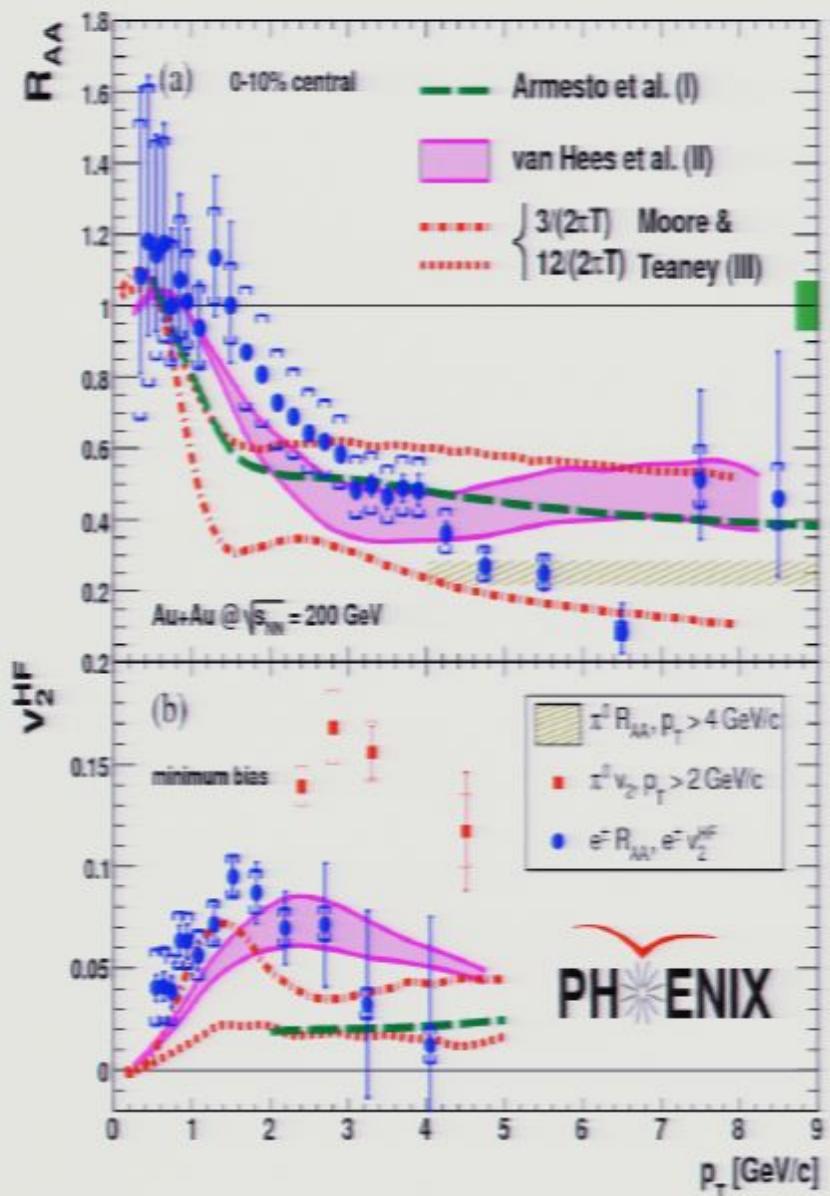
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## Summary

1. Suppression and Elliptic Flow are intimately related.
2. From the suppression pattern, we estimate that

$$D \lesssim \frac{12}{2\pi T}$$

With this diffusion coefficient, I can't produce enough elliptic flow.

I want to know the heavy quark diffusion coefficient

- Compute at weak coupling → Kinetic Theory
- Lattice → Hard
- Compute at strong coupling → AdS/CFT

Extrapolate to reality.

## Matching Langevin to a Microscopic Theory

- Heavy Quarks are Quasi Classical

$$\lambda_{\text{de Broglie}} \sim \frac{\hbar}{\sqrt{MT}} \ll \frac{\hbar}{T}$$

- Compare the Langevin process to the microscopic theory

Langevin

$$\frac{dp}{dt} = -\eta_D p + \xi(t)$$

Microscopic Theory

$$\frac{dp}{dt} = \mathcal{F}(t, \mathbf{x}) = qE(t, \mathbf{x})$$

- Match the Langevin to the Microscopic Theory

Langevin

$$\kappa = \int dt \langle \xi(t) \xi(0) \rangle$$

Microscopic Theory

$$\kappa = \int dt \langle \mathcal{F}(t, \mathbf{x}) \mathcal{F}(0, \mathbf{x}) \rangle_{HQ}$$

Diffusion Coefficient  $\leftrightarrow$  Electric Field Correlator

**Formula:**

$$\kappa = \int dt \langle \mathcal{F}(t, \mathbf{x}) \mathcal{F}(0, \mathbf{x}) \rangle_{HQ}$$

**Issues:**

1. What is  $\mathcal{F}(t, x)$  ?

$$\mathcal{F} \equiv \int d^3\mathbf{x} \underbrace{Q^\dagger(t, \mathbf{x}) T^a Q(t, \mathbf{x})}_{\text{charge density}} \times \underbrace{E_a(t, \mathbf{x})}_{\text{E-Field}}$$

2. How precisely is  $\langle \dots \rangle_{HQ}$  defined ?

**Formula:**

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How to define  $\langle \dots \rangle_{HQ}$ ?

$$Z_{HQ} = \sum_{s'} \left\langle s' \left| e^{-\beta H} Q(\mathbf{x}, -i\beta) Q^\dagger(\mathbf{x}) \right| s' \right\rangle$$

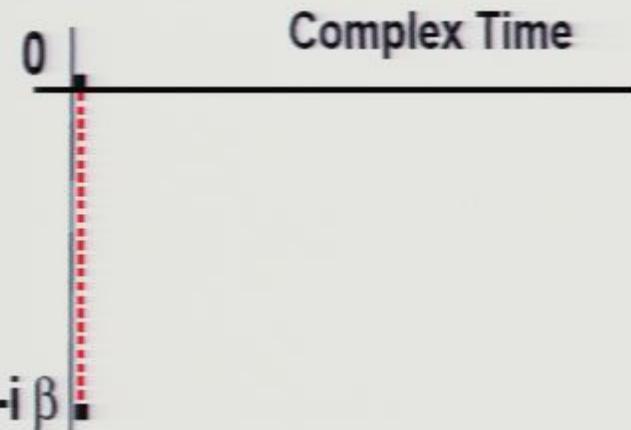
- The heavy quark Lagrangian is

$$\mathcal{L} = Q^\dagger(\mathbf{x}, t) (i\partial_t - M - A_0) Q(\mathbf{x}, t)$$

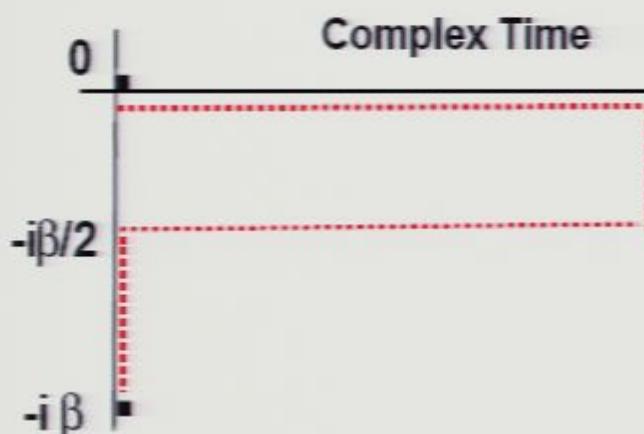
- Going through the path integral construction we have

$$\begin{aligned} Z_{HQ} &= \left\langle \int [DQ] [DQ^\dagger] e^{i \int_0^\beta Q^\dagger(i\partial_\tau - M - A_0 - i\epsilon) Q} Q(\mathbf{x}, -i\beta) Q^\dagger(\mathbf{x}) \right\rangle_{YM} \\ &= \text{Tr} \exp \left( i \int_0^\beta d\tau A_0(\tau, \mathbf{x}) \right) \end{aligned}$$

How to define  $\langle \dots \rangle_{HQ}$ ?



$$Z_{HQ} = \sum_s \left\langle s \left| e^{-\beta H} \right| s \right\rangle$$



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The path in the time plane is arbitrary

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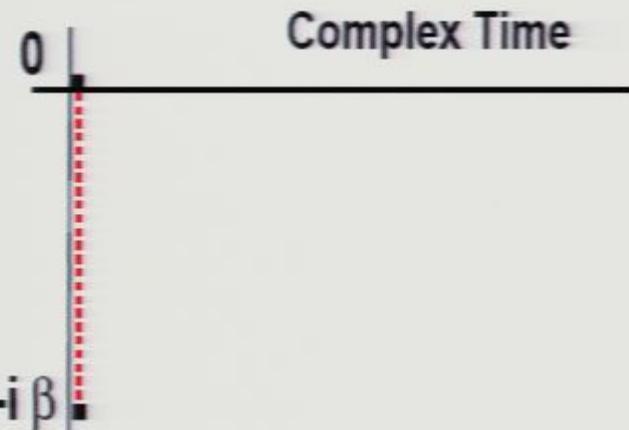
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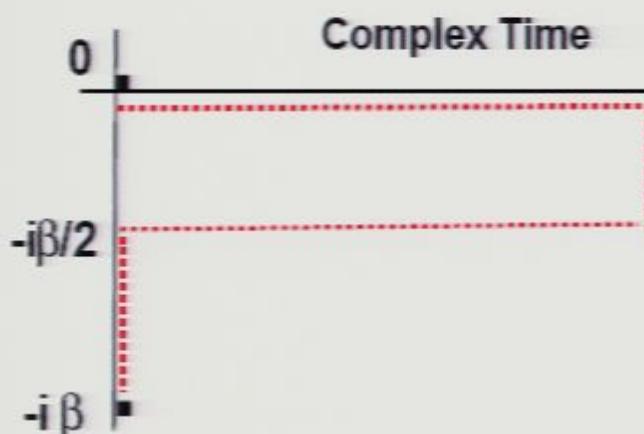
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The path in the time plane is arbitrary

Formula:

$$\langle \mathcal{F}(t)\mathcal{F}(0) \rangle_{HQ}$$

## Contour Ordered Products of Forces:

$$\langle T_C[\mathcal{F}(t_C) \mathcal{F}(0)] \rangle_{HQ}$$



- Usually divide the operators into "1" and "2" type operators

$$\langle TO_1(t)O_1(t') \rangle \quad \langle O_1(t)O_2(t') \rangle \quad \langle O_2(t)O_1(t') \rangle \quad \langle \tilde{T}O_2(t)O_2(t') \rangle$$

- Example:

$$G_{12}(0, t) = \langle \mathcal{F}_2(t) \mathcal{F}_1(0) \rangle_{HQ} \quad \mathcal{F}_2(t) \equiv \mathcal{F}(t - i\beta/2)$$

## Thermal Field Theory Symbol Soup

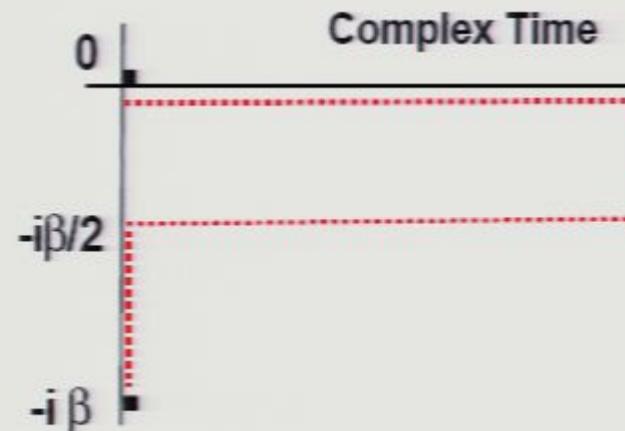
- Everything Related to the Retarded Greens Function

$$iG_R(t) = \theta(t) \langle [\mathcal{F}(t), \mathcal{F}(0)] \rangle_{HQ},$$

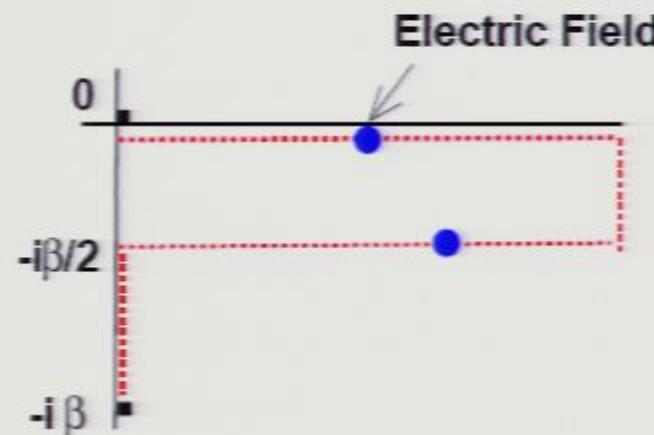
- Thus

$$\begin{aligned}\kappa &= \int dt \langle \mathcal{F}(t) \mathcal{F}(0) \rangle_{HQ} \\ &= \lim_{\omega \rightarrow 0} G_{12}(\omega) \\ &= \lim_{\omega \rightarrow 0} -\frac{2T}{\omega} \text{Im}G_R(\omega)\end{aligned}$$

## Relation to Wilson Lines



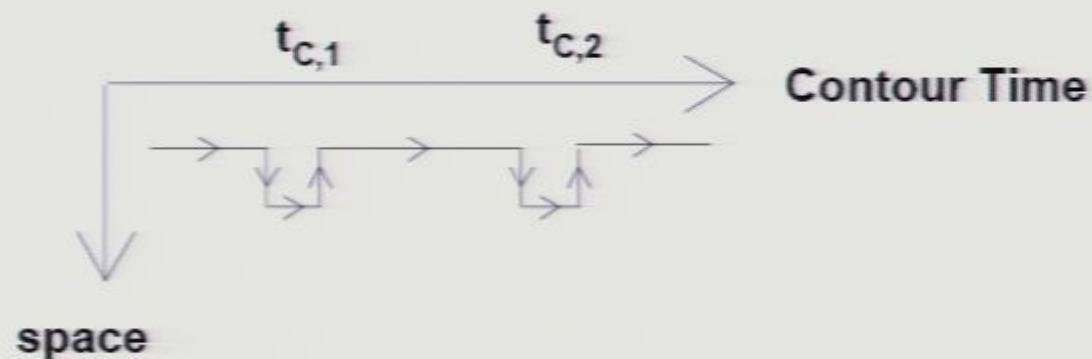
$$Z_{HQ} = \left\langle \text{Tr} \exp \left( i \int_C dt_C A_0(t_C) \right) \right\rangle$$
$$\equiv \langle W_C[0] \rangle$$



$$\langle T_C[\mathcal{F}(t_C) \mathcal{F}(0)] \rangle_{HQ} = \frac{1}{\langle W_C[0] \rangle}$$
$$\times \langle T_C[E(t_C) E(0)] \rangle_{YM}$$

## Wilson Lines and the Diffusion Coefficient:

- Variations of Wilson Lines give electric field correlators



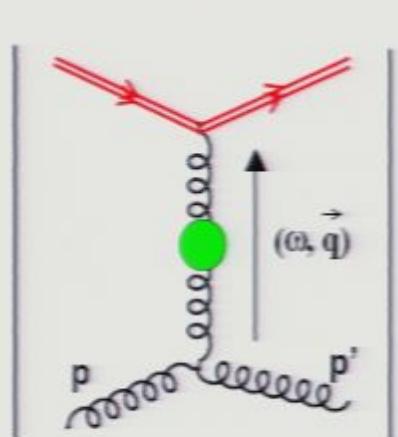
$$\frac{1}{\langle W_C[0] \rangle} \left\langle \frac{\delta^2 W[\delta y]}{\delta y(t_1) \delta y(t_2)} \right\rangle_{YM} = \frac{1}{\langle W_C[0] \rangle} \langle T_C [E(t_2) E(t_1)] \rangle$$

- The varied Wilson line = Force generating functional.

$$\left\langle e^{i \int_C dt \delta y(t) \mathcal{F}(t)} \right\rangle_{HQ} = \frac{1}{\langle W_C[0] \rangle} \langle W_C[\delta y] \rangle$$

## Computing $\kappa$ – Kinetic Theory vs. Correlators

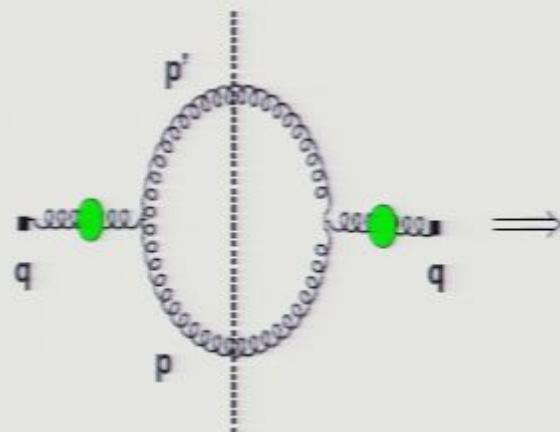
- $\kappa$  is the mean squared momentum transfer per unit time:



Feynman diagram illustrating the calculation of  $\kappa$  via kinetic theory. It shows a green gluon loop with momentum  $p$  and frequency  $\omega$ . The loop is coupled to two external gluons with momenta  $p'$  and  $p''$ , represented by red lines. The loop is labeled with  $2$  above it, indicating a factor of  $2$  from the loop's self-energy.

$$\kappa = \int_{\mathbf{p}, \mathbf{q}} \mathbf{q}^2 \ n(p)(1+n(p')) \ |M_{\text{glue}}|^2$$

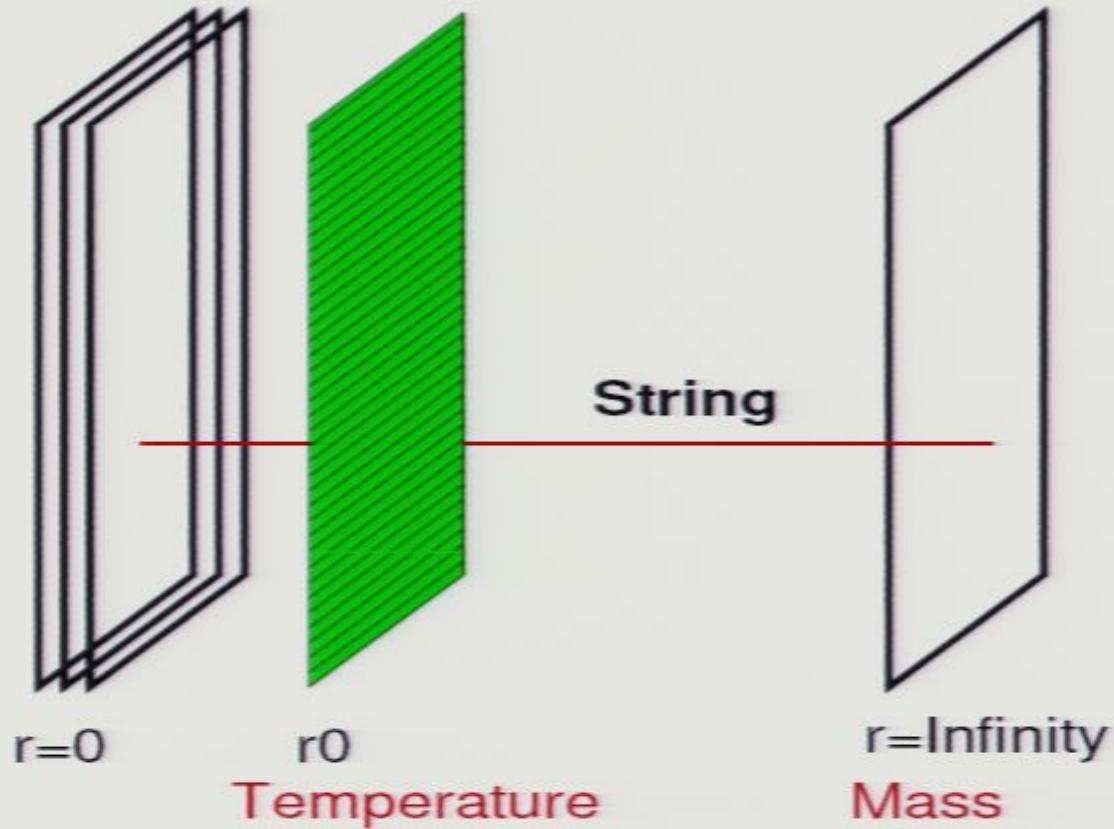
- $\kappa$  is the Electric Field Correlator:



The Same Thing

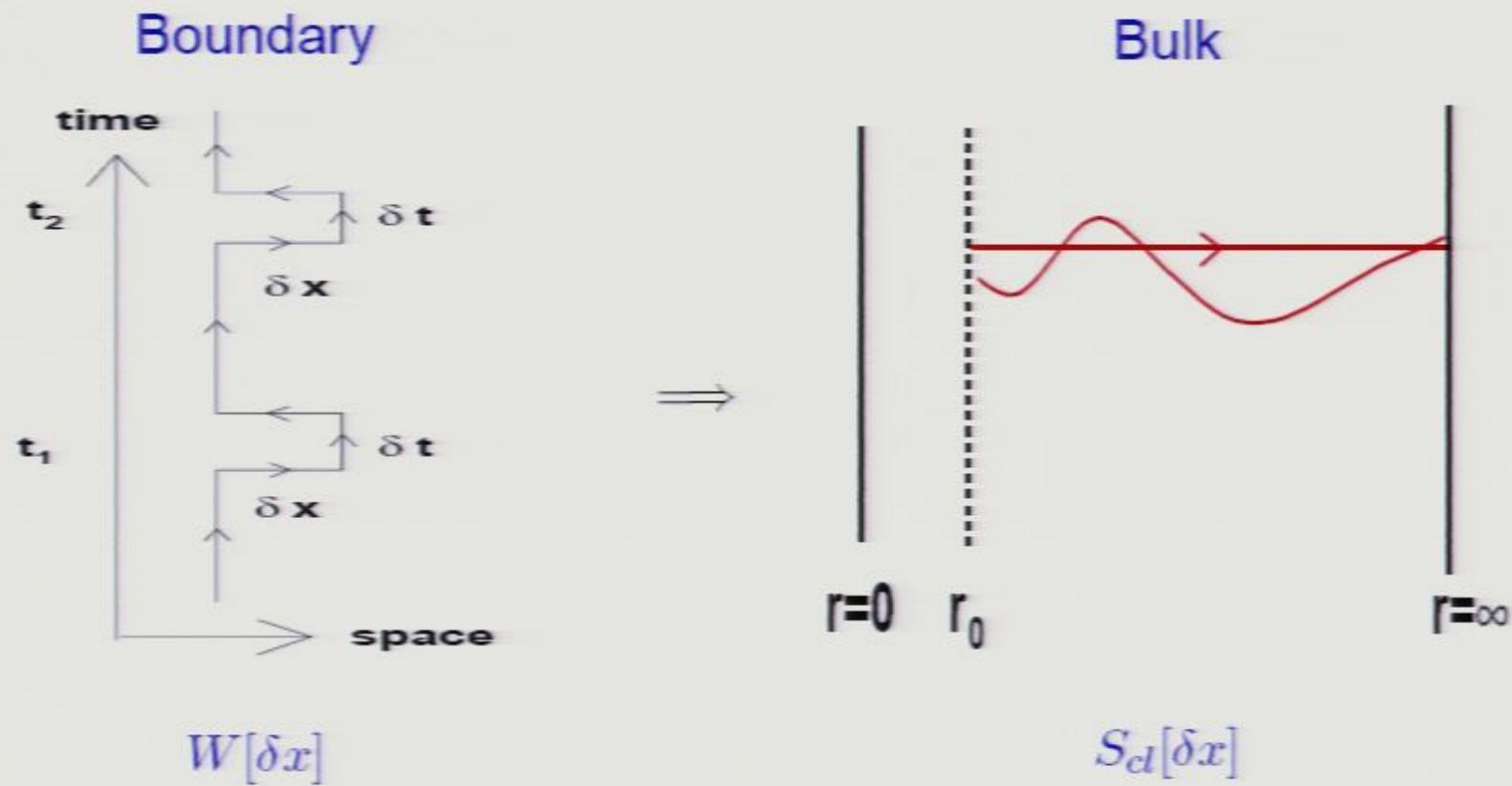
N=4 at Finite Temperature

**N-1 D3 Branes      Event Horizon      Test Brane**



Temperature of gauge theory is the hawking temperature of the Black hole

Testing the string dynamics:



$$\frac{1}{\langle W_C[0] \rangle} \langle W_C[\delta y] \rangle = \frac{1}{e^{iS_{cl}[0]}} e^{iS_{cl}[\delta y]}$$

## Summary of Technical Steps

- Fluctuate the end point of a string at  $r = \infty$  at very small frequencies:  $u = 1/r^2$

$$S_{NG} = \frac{R^2}{2\pi\ell_s^2} \underbrace{\int dt dr}_{\text{Area}} \left[ 1 - \underbrace{\frac{1}{2} \left( \frac{\dot{y}_\parallel^2}{f} - 4fu(y'_\parallel)^2 \right)}_{\text{Quadratic Fluctuations}} \right]$$

- Solve for the waves along the string in the  $AdS_5 \times S_5$  metric
  - Impose incoming boundary conditions for retarded propagator .
- Differentiating twice with respect to  $\delta y$  yields the retarded force force correlator

$$\langle \mathcal{F}(t)\mathcal{F}(0) \rangle \sim \frac{\delta^2}{\delta y(t)\delta y(0)} \langle W[\delta y] \rangle \sim \frac{\delta^2}{\delta y(t)\delta y(0)} e^{iS_{NG}[\delta y]}$$

$\kappa$  is the imaginary part of the retarded correlator

## The Real Time Thermal Fields and the Correspondence – Son& Herzog (2002)

- The AdS Black Hole Metric in canonical coordinates,  $f \equiv 1 - (r_0/r)^4$

$$ds^2 = \frac{r^2}{R^2} \left[ -f(r)dt^2 + d\mathbf{x}_{\parallel}^2 \right] + \frac{R^2}{f(r)r^2} dr^2 + R^2 d\Omega_5^2$$

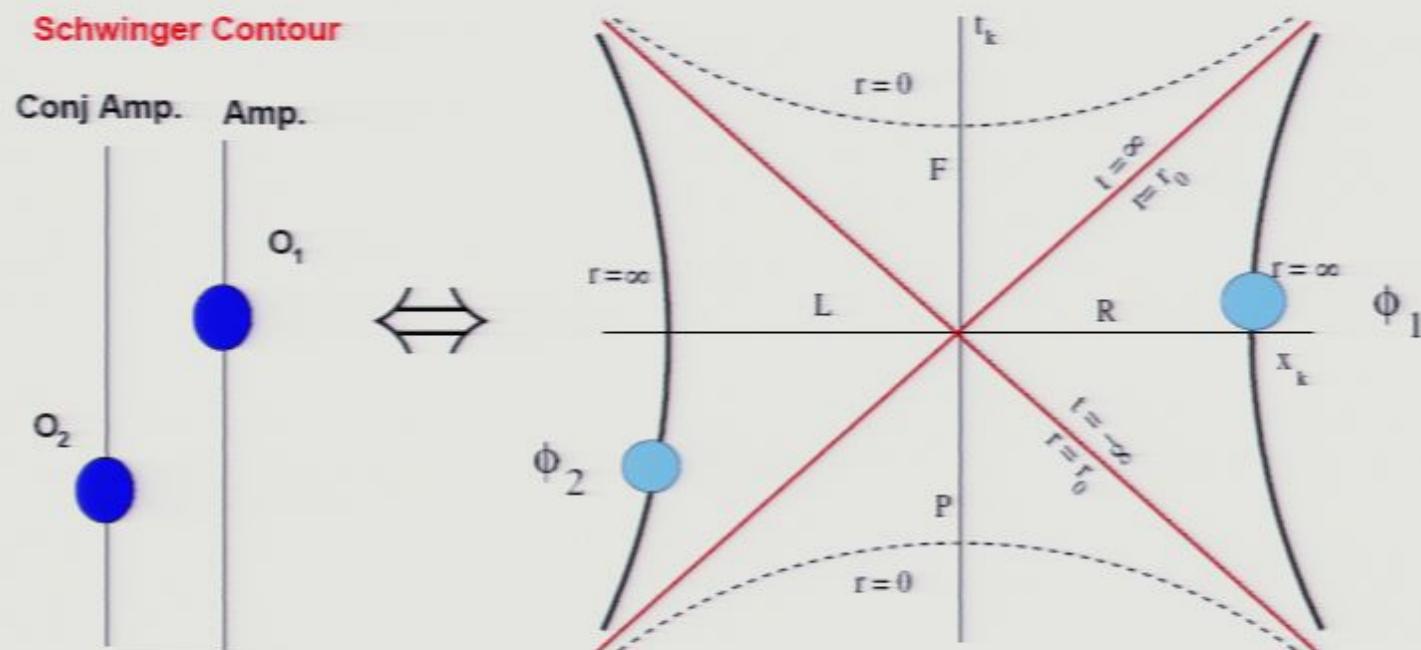
- The canonical observer sees only one patch of space time
- Change to Kruskal Coordinates
- Consider the scalar wave equation

$$\frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \phi = 0$$

## Kruskal Coordinates and the Correspondence (Son-Herzog/Unruh/Israel):

- Source fields for "1" and "2" operators live on the right and left quadrants

$$\mathcal{O}_1, \mathcal{O}_2 \Leftrightarrow \phi_1, \phi_2$$



$$\left\langle e^{i \int dt_1 \phi_1 \mathcal{O}_1} e^{-i \int dt_2 \phi_2 \mathcal{O}_2} \right\rangle_{SYM} = e^{i S[\phi_1, \phi_2]}$$

## Boundary conditions Herzog-Son/Unruh/Israel

- Solve the scalar wave eq. in  $(t, r)$  coordinates

$$e^{-i\omega t}(r-1)^{+i\frac{\omega}{4}} \quad \text{out-falling}$$

$$e^{-i\omega t}(r-1)^{-i\frac{\omega}{4}} \quad \text{in-falling}$$

- In Kruskal coordinates these are

$$e^{+i\frac{\omega}{2}\log(-U)} \quad \text{out-falling at future infinity}$$

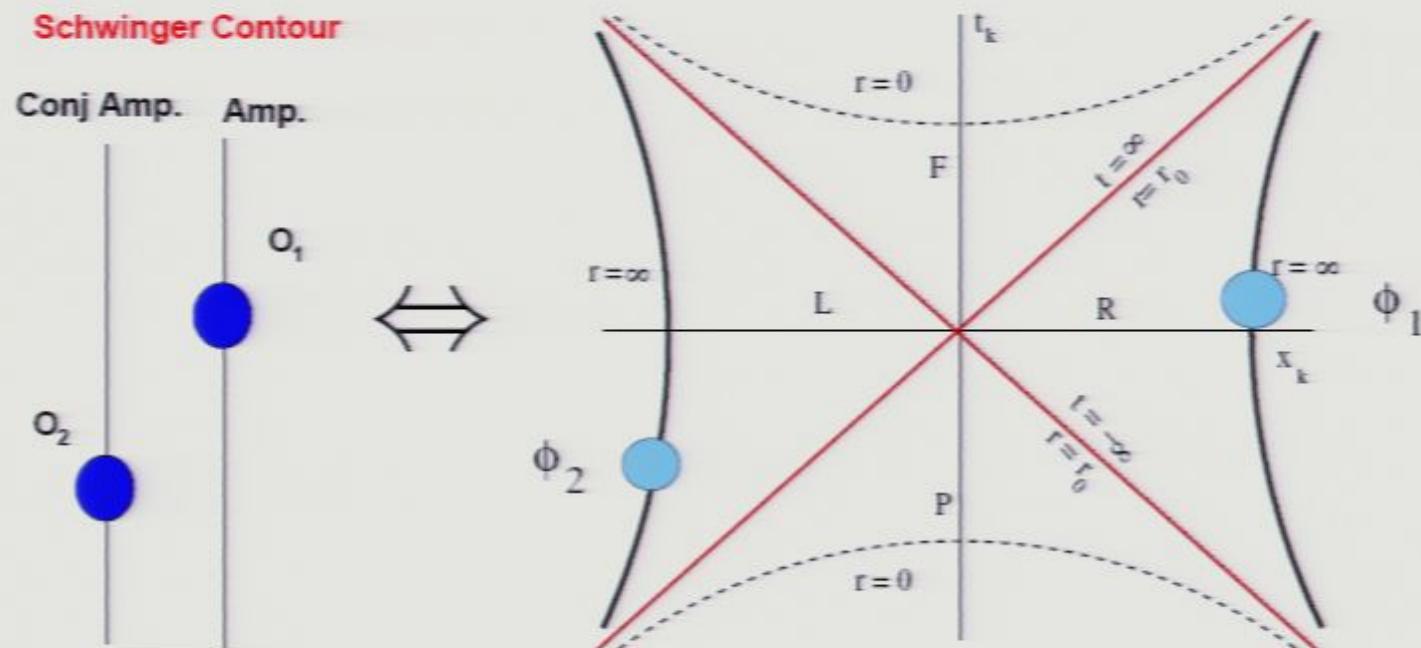
$$e^{-i\frac{\omega}{2}\log(V)} \quad \text{in-falling at past infinity}$$

- Decide how to extend these to the full kruskal plane
  - Negative energy modes are outfalling at the future event horizon
    - \* Anal. in upper U plane
  - Positive energy modes are infalling at the past event horizon
    - \* Anal. in lower V plane

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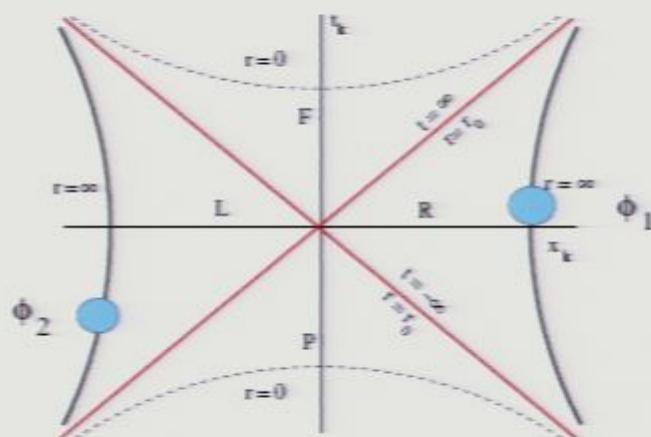
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Solution in the Kruskal Plane:



- Can now find the solution

$$\phi_1 \underbrace{d\phi_1/dr}_{\text{fixed by b.c.}} \leftrightarrow \phi_2 \underbrace{d\phi_2/dr}_{\text{fixed by b.c.}}$$

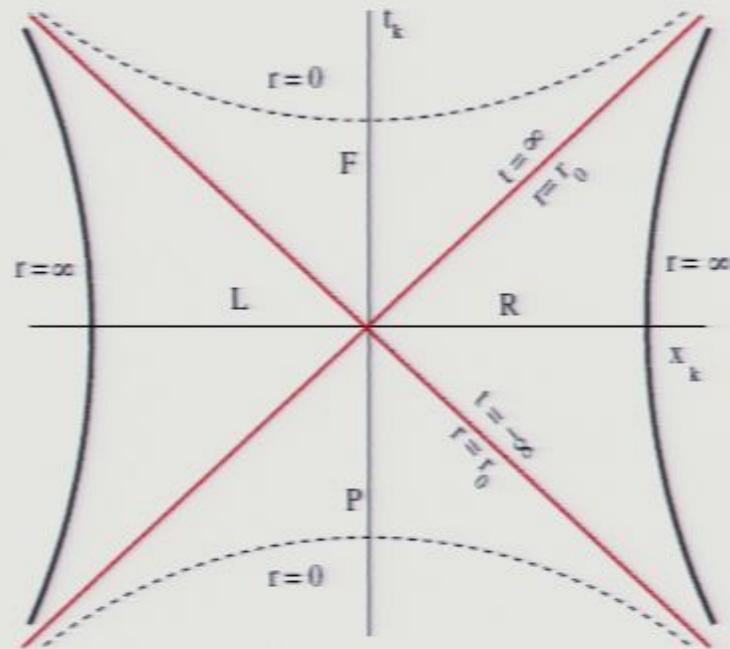
- Substituting in the action

$$\left\langle e^{i \int dt_1 \phi_1 \mathcal{O}_1} e^{-i \int dt_2 \phi_2 \mathcal{O}_2} \right\rangle_{SYM} = e^{iS[\phi_1, \phi_2]}$$

- Differentiating the action

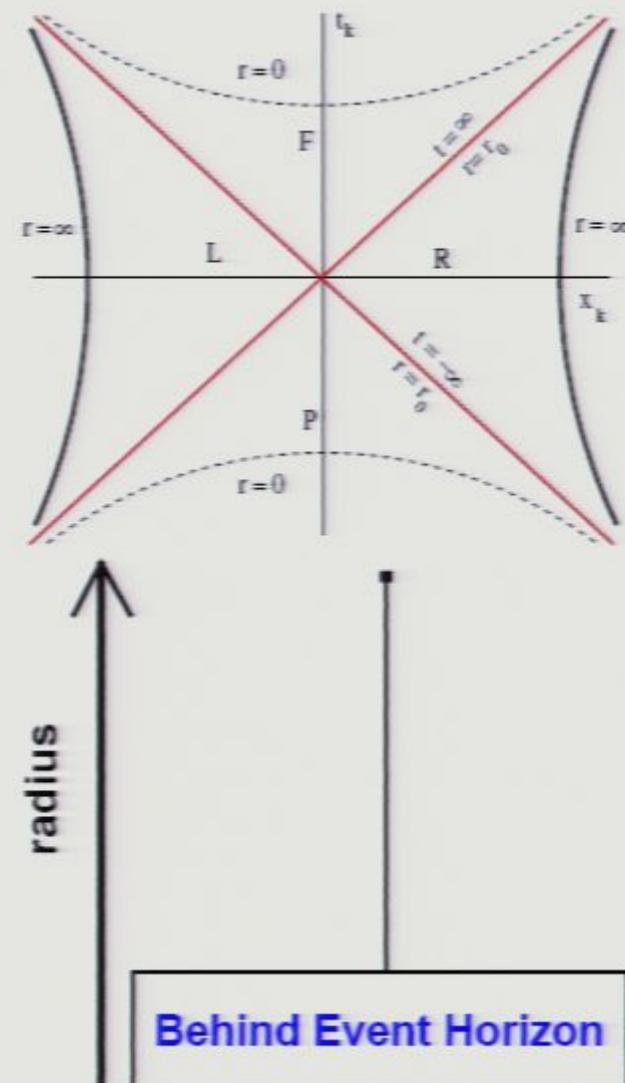
$$\langle \mathcal{O}_2(t) \mathcal{O}_1(0) \rangle \sim \frac{\delta}{\delta \phi_2(t)} \frac{\delta}{\delta \phi_1(0)} e^{iS[\phi_1, \phi_2]}$$

## Wilson Lines in the "1" and "2" Formalism



1. Break the  $U(N)$  gauge to  $U(N - 1) \times U(1)$  giving a mass to a heavy gauge boson through the Higgs Mechanism.
2. At finite temperature there are "1" type w-bosons and "2" type w-bosons
3. Our Wilson line runs along the "1" axis and back along the "2" axis
4. We are looking for classical string solution which connects the two boundaries.

## Kruskal Observer and $(t, r)$ Observer



## Summary of Technical Steps

- Fluctuate the end point of a string at  $r = \infty$  at very small frequencies:  $u = 1/r^2$

$$S_{NG} = \frac{R^2}{2\pi\ell_s^2} \int \underbrace{dt dr}_{\text{Area}} \underbrace{\left[ 1 - \frac{1}{2} \left( \frac{\dot{y}_\parallel^2}{f} - 4fu(y'_\parallel)^2 \right) \right]}_{\text{Quadratic Fluctuations}}$$

- Solve for the waves along the string in the full Kruskal plane – Boundary Conditions!
- Compute the classical string action as a quadratic functional of  $\delta y_1$  and  $\delta y_2$ .

$\delta y_1$  = Right boundary string endpoint

$\delta y_2$  = Left boundary string endpoint

- Differentiating with respect to  $\delta y_1$  and  $\delta y_2$  yields

$$\langle \mathcal{F}_2(t) F_1(0) \rangle = \frac{\delta^2}{\delta y_2(t) \delta y_1(0)} e^{iS[\delta y_2, \delta y_1]}$$

## Results:

- Mean Squared momentum Transfer

$$\kappa = \pi \underbrace{\sqrt{\lambda}}_{\frac{R^2}{\ell_s^2} \text{ from string action}} \times \underbrace{T^3}_{\text{dimension}}$$

- Answer:

$$\begin{aligned} D &= \frac{2T^2}{\kappa} \\ &= \frac{2}{\sqrt{\lambda}\pi T} \end{aligned}$$

## QCD Guesses: Strong Coupling

- Strong Coupling:  $\mathcal{N} = 4$  SUSY.  $\lambda \approx 5 \leftrightarrow 20$

$$D = \frac{1}{\sqrt{g_{YM}^2 N_c}} \frac{4}{2\pi T} \quad \longrightarrow \quad D \simeq \frac{1.0 \leftrightarrow 2.0}{2\pi T}$$

- Weak coupling (Aleski Vuorinen)

2 gluons + 6 scalars + 8 fermions  $\neq$  2 gluons

$$\frac{D_{QCD}}{D_{SYM}} = \frac{6}{1 + \frac{N_f}{2N_c}} \approx 4$$

- Best guess for QCD from strong coupling

$$D \approx \frac{4.0 \leftrightarrow 8.0}{2\pi T}$$

Compare to weak coupling best guess  $D \approx 6/(2\pi T)$

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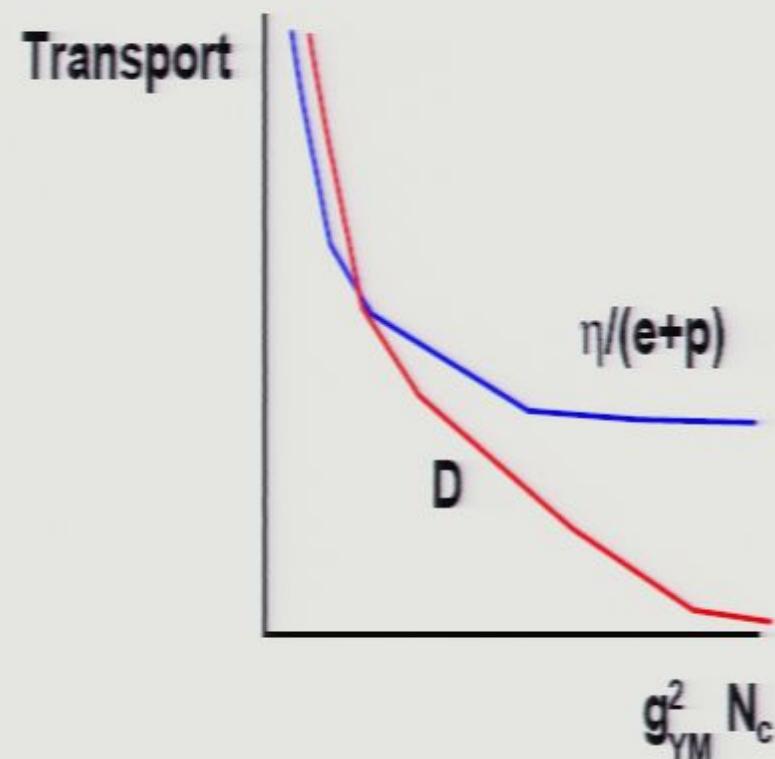
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Heavy Quark Diffusion is Parametrically Small

$$D = \frac{1}{\sqrt{g_{YM}^2 N_c}} \frac{4}{2\pi T} \quad \frac{\eta}{e+p} = \frac{1}{4\pi T}$$



## Constraint On The Heavy Quark Mass

- To treat the heavy quark as a quasi-classical quasi-particle we need

$$\tau_R \gg \frac{\hbar}{T}$$

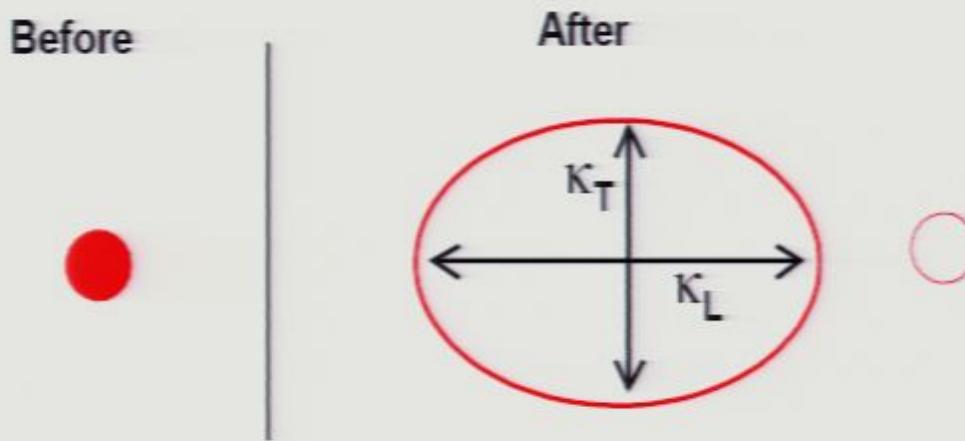
- Then we have

$$\tau_R \sim \frac{M}{T} D \quad D = \frac{2}{\sqrt{\lambda} \pi T}$$

- This leads to a constraint on Mass/String Length

$$M \gg \frac{\pi T}{2} \sqrt{\lambda} \quad L \gg r_o$$

## Generalize to Relativistic Heavy Quarks



- Transverse Momentum Broadening of a heavy quark (analogous to  $\hat{q}$ )

$\kappa_T(v)$  = Mean squared transverse momentum transfer per unit time

$\kappa_L(v)$  = Mean squared longitudinal momentum transfer per unit time

## Field Theory Formulation

- Non-Relativistic

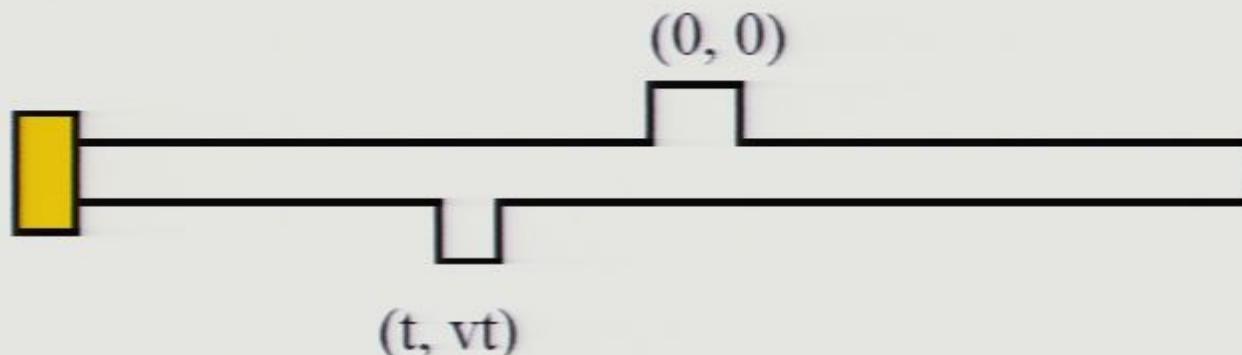
$$\kappa = \int dt \langle E^y(t) E^y(0) \rangle_{HQ}$$

- Relativistic

$$\kappa_T = \int dt \langle F^{y\mu}(t, vt) v_\mu F^{y\nu}(0, 0) v_\nu \rangle_{HQ}$$

- The field strengths are variations on the contour

$$\langle F^{y\mu}(t, vt) v_\mu F^{y\nu}(0, 0) v_\nu \rangle_{HQ} \sim \left\langle \frac{\delta^2}{\delta y_2(t) \delta y_1(0)} W[y_2, y_1] \right\rangle$$



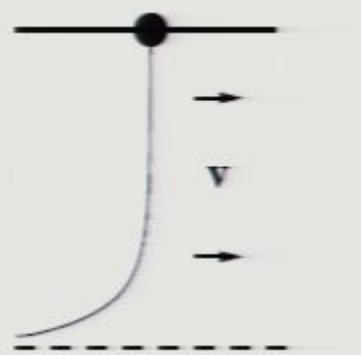
## General AdS/CFT Procedure

1. Find a semi-classical string dual to a moving heavy quark
  - understand the Kruskal Structure
2. Fluctuate the end points solve for waves
  - Apply boundary conditions with the same content as before
3. Determine  $\kappa_T$

$$\kappa_T \sim \left\langle \frac{\delta^2}{\delta y_2 \delta y_1} W[y_2, y_1] \right\rangle \sim \frac{\delta^2}{\delta y_2 \delta y_1} e^{+iS_{NG}[y_2, y_1]}$$

## Step 1: Finding the semi-classical string (HKKKY and S. Gubser)

- Turn on an electric field to accelerate the quark

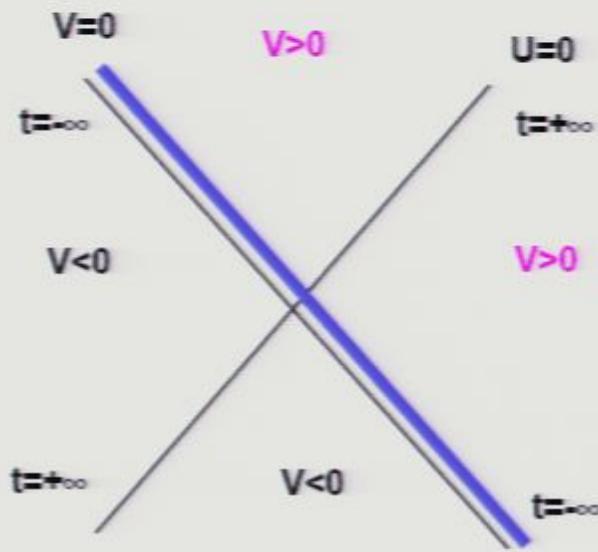


- A semiclassical string trails behind the quark

$$x_3 = vt + \frac{v}{2} [\arctan(z) - \operatorname{arctanh}(z)]$$

- Rewrite in terms of Kruskal coordinates

$$x_3 = \underbrace{\frac{v}{2} \log(V)}_{\text{Why the singularity?}} + v \arctan(z) \quad \text{for } V > 0$$



- Applying the same boundary conditions as before (the lower  $V$ ) plane

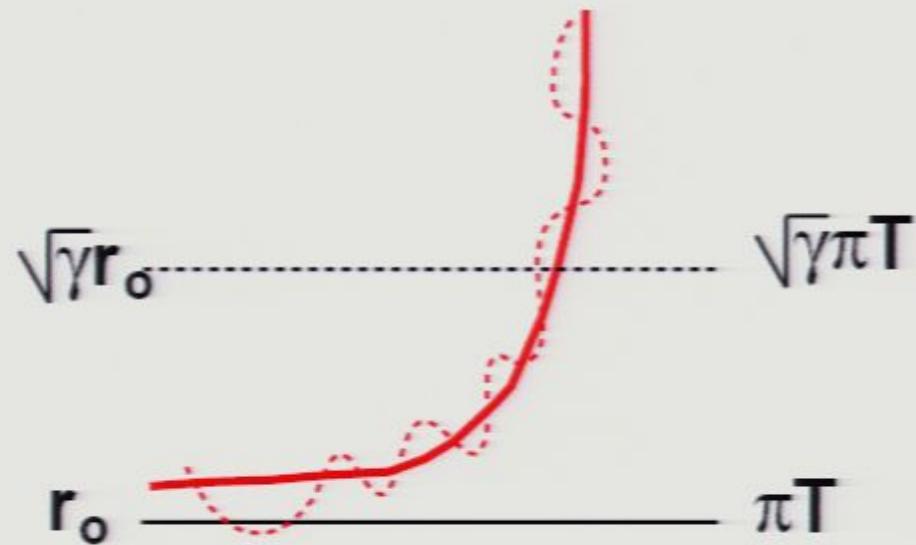
$$x_3 = \frac{v}{2} \log(V) + \dots \quad \text{for } V > 0$$

$$x_3 = \frac{v}{2} \log(|V|) - iv \underbrace{\pi/2}_{\beta/2} + \dots \quad \text{for } V < 0$$

- Good Thing. We are evaluating operators of the form

$$\langle \mathcal{O}(t - i\sigma, vt - iv\sigma) \mathcal{O}(0, 0) \rangle \quad \text{for } \sigma = \beta/2$$

## Fluctuating the String



- At  $\sqrt{\gamma}r_o$  have: One infalling and one outfalling mode.
- At  $r_o$  have: Two infalling modes

No fluctuation dissipation theorem – What to do?

## Change Coordinates/Gauge

- Find a change of coordinates where the induced metric is diagonal

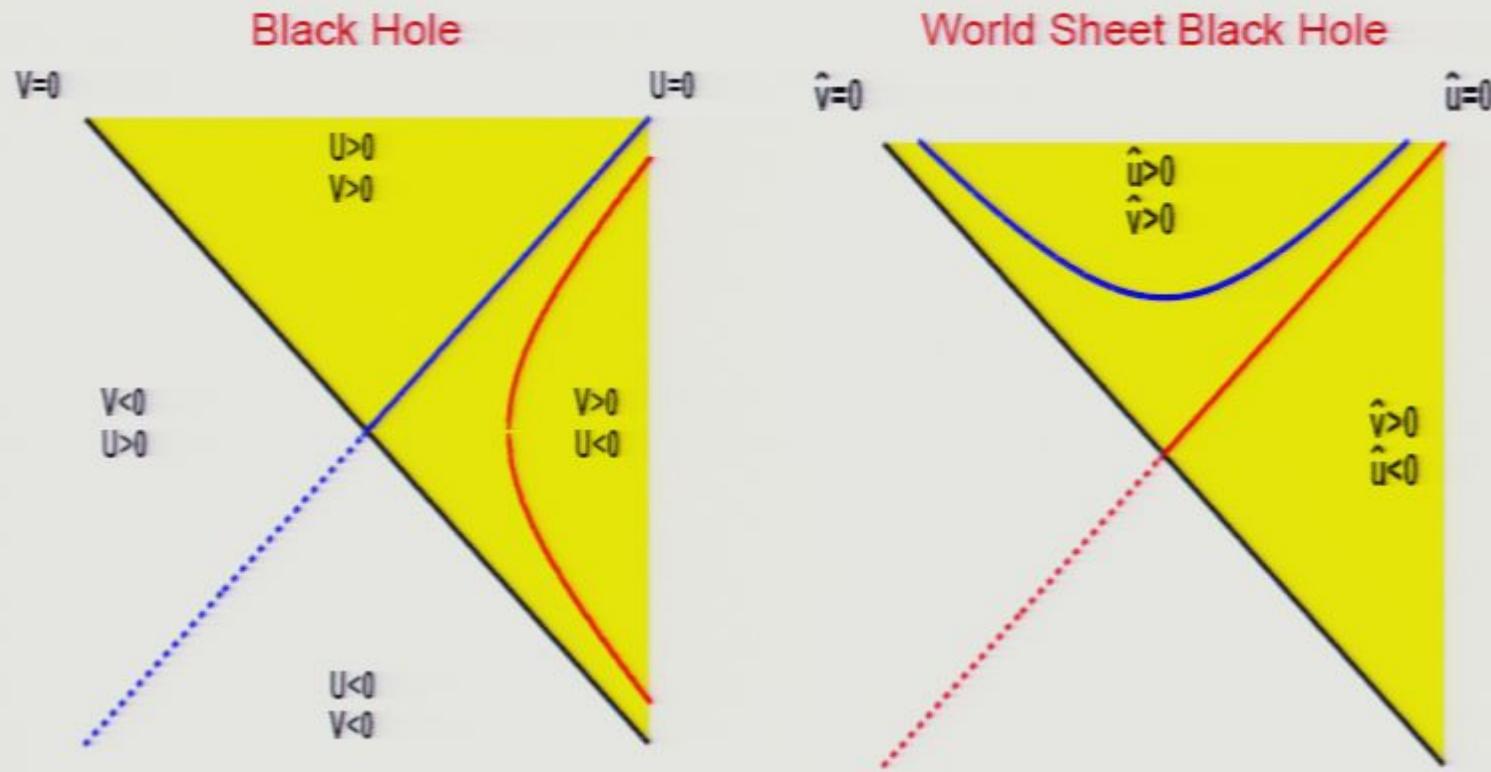
$$h_{ab} = G_{MN} \partial_a X^M \partial_b X^N$$

$$\begin{aligned}\tau = \hat{t} &= \frac{1}{\sqrt{\gamma}} \left( t - \frac{1}{2} \operatorname{arctanh}(z) + \frac{1}{2} \sqrt{\gamma} \operatorname{artanh}(\sqrt{\gamma} z) + \text{reg. funcs} \right) \\ \sigma = \hat{z} &= \sqrt{\gamma} z\end{aligned}$$

- The induced metric becomes

$$h_{ab} d\sigma^a d\sigma^b = \frac{R^2}{\hat{z}^2} \left[ -f(\hat{z}) d\tau^2 + \frac{1}{f(\hat{z})} d\sigma^2 \right] \quad f(\hat{z}) = 1 - \hat{z}^4$$

The radius  $r = r_o \sqrt{\gamma}$  is playing the role of the future event horizon



The radius  $r = \sqrt{\gamma}r_o$  is a future event horizon for the world sheet metric

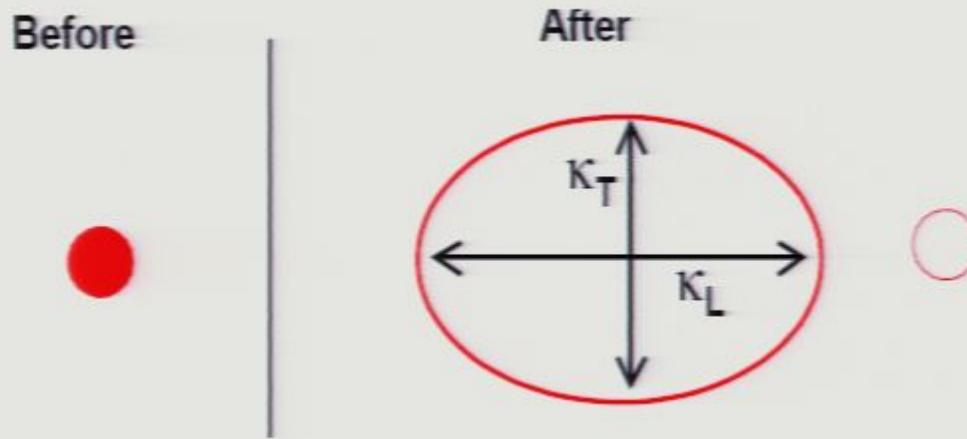
Apply the same boundary conditions as before.

## Final Step

- Solve for waves on the moving string
  - Negative energy modes are outfalling at the future event horizon
    - \* Anal. in upper  $\hat{U}$  plane
  - Positive energy modes are infalling at the past event horizon
    - \* Anal. in lower  $V$  plane
- Then the transverse momentum broadening is

$$\begin{aligned}\kappa_T &\sim \int dt \left\langle \frac{\delta^2}{\delta y_2 \delta y_1} W[y_2, y_1] \right\rangle \\ &\sim \int dt \frac{\delta^2}{\delta y_2(t) \delta y_1(0)} e^{iS_{NG}[y_2, y_1]}\end{aligned}$$

## Results Relativistic Heavy Quarks



- Transverse Momentum Broadening of a heavy quark (analogous to  $\hat{q}$ )

$$\kappa_T(v) = \underbrace{\sqrt{\lambda} \pi T^3}_{\text{Non-relativistic result}} \times \sqrt{\gamma}$$

Non-relativistic result

$$\kappa_L(v) = \underbrace{\sqrt{\lambda} \pi T^3}_{\text{Non-relativistic result}} \times \gamma^{3/2} \Leftarrow (\text{Gubser})$$

Non-relativistic result

The strong dependence on the energy could have important phenomenological consequences and theoretical implications.