

Title: Cosmological Black Hole Formation and the QCD Phase Transition

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Abstract:

# Why Study Primordial Black Holes?

- **Evaporation has implications for**
  - Baryogenesis [Dokuchaev et. al, 2004]
  - Nucleosynthesis [Kohri and Yokoyama, 1999]
  - Cosmic Rays [Kim, Lee, MacGibbon, 1999]
  - Quantum Gravity [Coleman et. al, 1991]
- **Dark Matter**
  - MACHOs in galactic halo [Alcock et. al, 2001]
  - Remnants from evaporated PBHs?  
[MacGibbon, 1986]

## Formation – Qualitative Description

- Over-dense region crosses the horizon
- Over-dense region expands
- Over-dense region stops expanding
- Depending on conditions, over-dense region will either collapse or disperse

# Classifying Over-dense Regions

Regions are specified by:

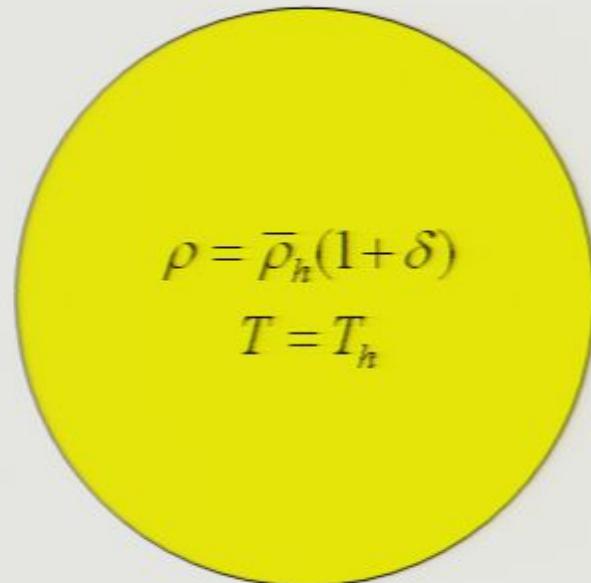
- When they enter the horizon

- Their over-density       $\frac{\delta\rho}{\rho} \equiv \delta$

At horizon crossing:

$$\rho = \bar{\rho}_h$$

$$T = \bar{T}_h$$


$$\rho = \bar{\rho}_h(1+\delta)$$

# General Approach

- Adapted from an approach put forth by Cardall and Fuller [astro-ph/9801103]
- Specify a region with given  $\bar{\rho}_h, \delta$
- Determine the turnaround point and size at turnaround
- Apply collapse condition
- Calculate the resulting mass spectrum

# Evolution of Overdense Region

- Outside the metric is a flat FRW universe:

$$ds^2 = -dt^2 + R^2(t) \left[ dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

- Inside the metric is a closed FRW universe:

$$ds^2 = -d\tau^2 + S^2(\tau) \left[ \frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

- The size  $S$  of region evolves as:

$$\left( \frac{dS}{d\tau} \right)^2 = \frac{8\pi G}{3} \rho S^2 - \kappa$$

$\rho$  = energy density of region

$\tau$  = time coordinate for region

$\kappa$  = constant (from matching regions)

# Evolution of Overdense Region

- Outside of the overdense region the scale factor  $R$  evolves as:

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G}{3} \bar{\rho} R^2$$

- Match the inner and outer regions at horizon crossing:

$$S_h = R_h$$

$$\left(\frac{dS}{d\tau}\right)_h = \left(\frac{dR}{dt}\right)_h$$

# Turn-around Condition

- Evolution of over-dense region is given by:

$$\left( \frac{dS(\tau)}{d\tau} \right)^2 = \frac{8\pi G}{3} [\rho(\tau)S^2(\tau) - \bar{\rho}_h R_h^2 \delta]$$

- Expansion of over-dense region stops when:

$$\rho_* S_*^2 = \bar{\rho}_h R_h^2 \delta$$

- Solve for  $T$  at turn-around

# Size of Over-dense Region

The over-dense region's proper size at the time of collapse is

$$d_* = \frac{S_*}{S_h} r_h = \frac{S_*}{S_h} \int_0^{t_h} \frac{dt}{R(t)}$$

$r_h$  = particle horizon at crossing

$S_h$  = scale factor at crossing

$S_*$  = scale factor at collapse

# Collapse Condition

A fluctuation in energy density  $\rho'$  satisfies the equations of General Relativity.

$$\frac{\partial^2 \rho'}{\partial t^2} = v_s^2 \nabla^2 \rho' + 4\pi G w(1+3v_s^2) \rho'$$

$$w = P + \rho = \text{enthalpy}$$

$$k_J^2 = \frac{4\pi G w(1+3v_s^2)}{v_s^2} = (\text{Jeans wavenumber})^2$$

# Collapse Condition

Size of region must exceed relativistic Jeans length at turnaround.

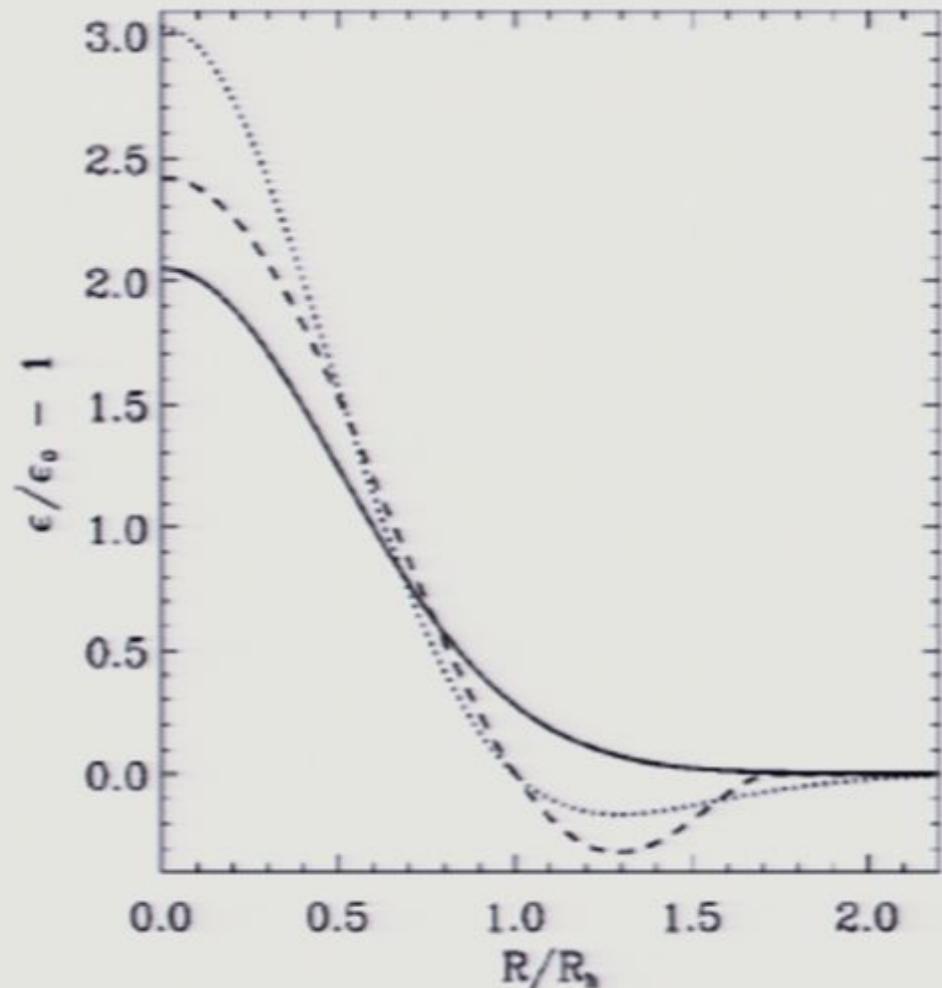
$$d_* > \frac{\pi}{k_J} = \sqrt{\frac{\pi v_s^2}{4G(1+3v_s^2)w}}$$

For a constant sound speed:  $\delta > \delta_c(v_s^2) = \frac{8\pi^2}{3} \frac{v_s^2}{(1+v_s^2)(1+3v_s^2)^3}$

$$\delta_c(v_s^2 = 1/3) = \frac{\pi^2}{12} \approx 0.822$$

# Collapse Condition

This is roughly consistent with Niemeyer and Jedamzik (1999) who got  $\delta_c \approx 0.70 \pm 0.02$  in a numerical approach.



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This is roughly consistent with Niemeyer and Jedamzik (1999) who got  $\delta_c \approx 0.70 \pm 0.02$  in a numerical approach.

However, Green et al. (2004) and Musco et al. (2005) got  $\delta_c \approx 0.45 \pm 0.02$  because they only included the growing mode, not the decaying mode. This halves the critical value of over-density.

$$\delta_+ \sim t \text{ (growing mode)} \quad \delta_- \sim t^{-1} \text{ (decaying mode)}$$

$$\delta(t) = A\left(\frac{t}{t_0}\right) + B\left(\frac{t_0}{t}\right)$$

$$\frac{d\delta}{dt}(t_0) = 0 \Rightarrow A = B \text{ (50% mixture)}$$

- Machinery in place to:
  - Find temperate at turn-around
  - Find size of collapsing region
  - Find Jeans Length
  - Evaluate collapse criterion
- Study various equations of state.

# Models of Degrees of Freedom

## Bag Model

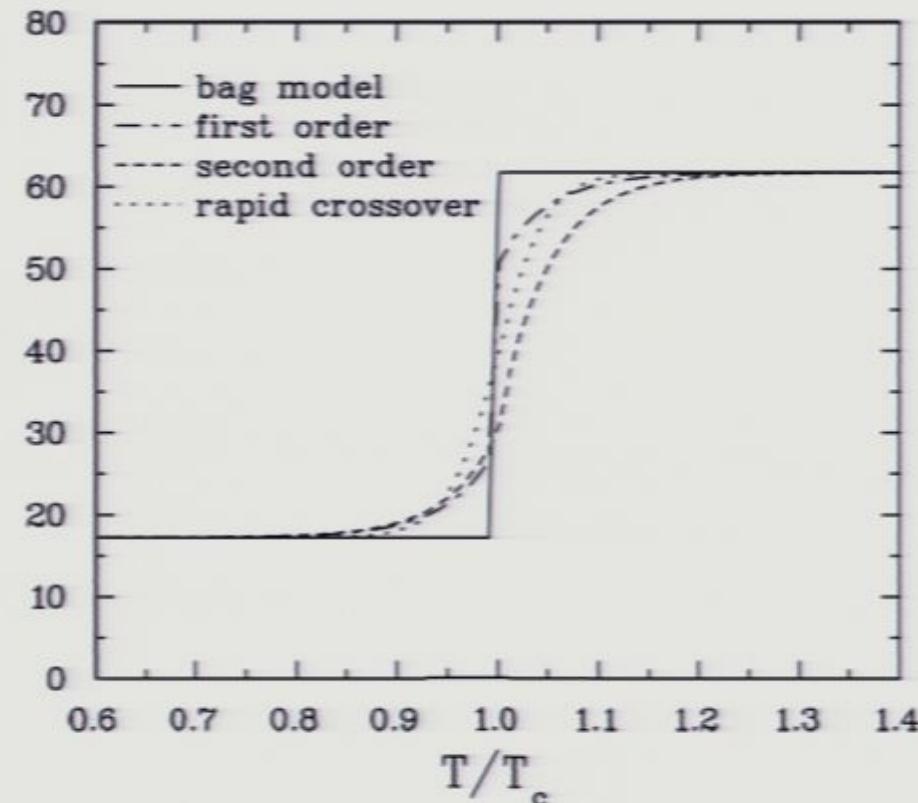
$$N(T) = \begin{cases} N_q & (T > T_c) \\ N_h & (T < T_c) \end{cases}$$

## Softened 1st or 2nd Order

$$N(T) = \begin{cases} N_q - \beta \exp\left(\frac{T_c - T}{\Delta}\right) & (T > T_c) \\ N_h + \alpha \exp\left(\frac{T - T_c}{\Delta}\right) & (T < T_c) \end{cases}$$

## Rapid Crossover

$$N(T) = \frac{1}{2} \left[ N_q + N_h + (N_q - N_h) \tanh\left(\frac{T - T_c}{\Delta}\right) \right]$$



$N_q = 61.75$  (u, d, s quarks, gluons)

$N_h = 17.25$  (leptons, pions)

$\Delta = 0.05 T_c$

$\alpha = 11.125$  (1st)  $13$  (2nd)

$\beta = 11.125$  (1st)  $33.375$  (2nd)

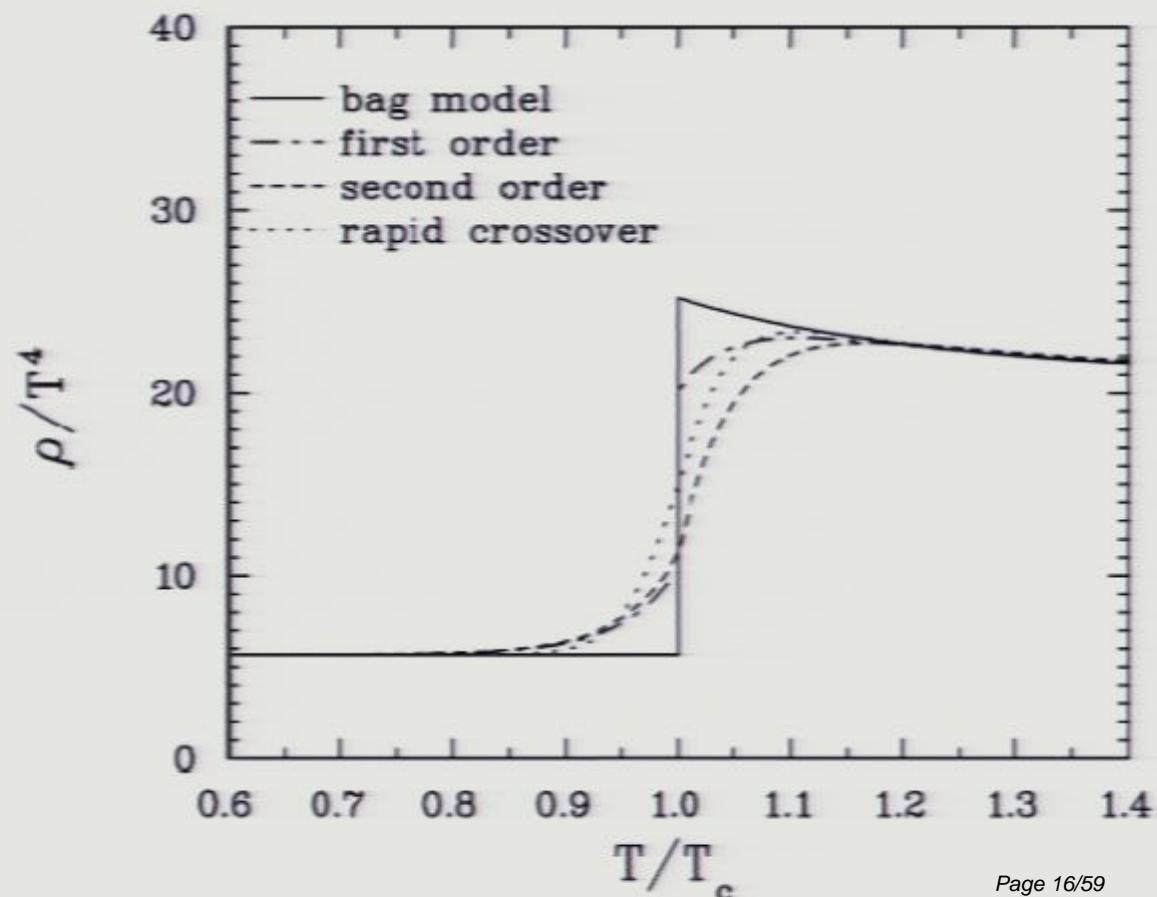
# Energy Density

Obtained from  $N(T)$  and thermodynamic relations

$$s(T) = \frac{4\pi^2}{90} T^3 N(T)$$

$$P(T) = \int_0^T s(T') dT'$$

$$\rho(T) = -P(T) + Ts(T)$$



# Pressure

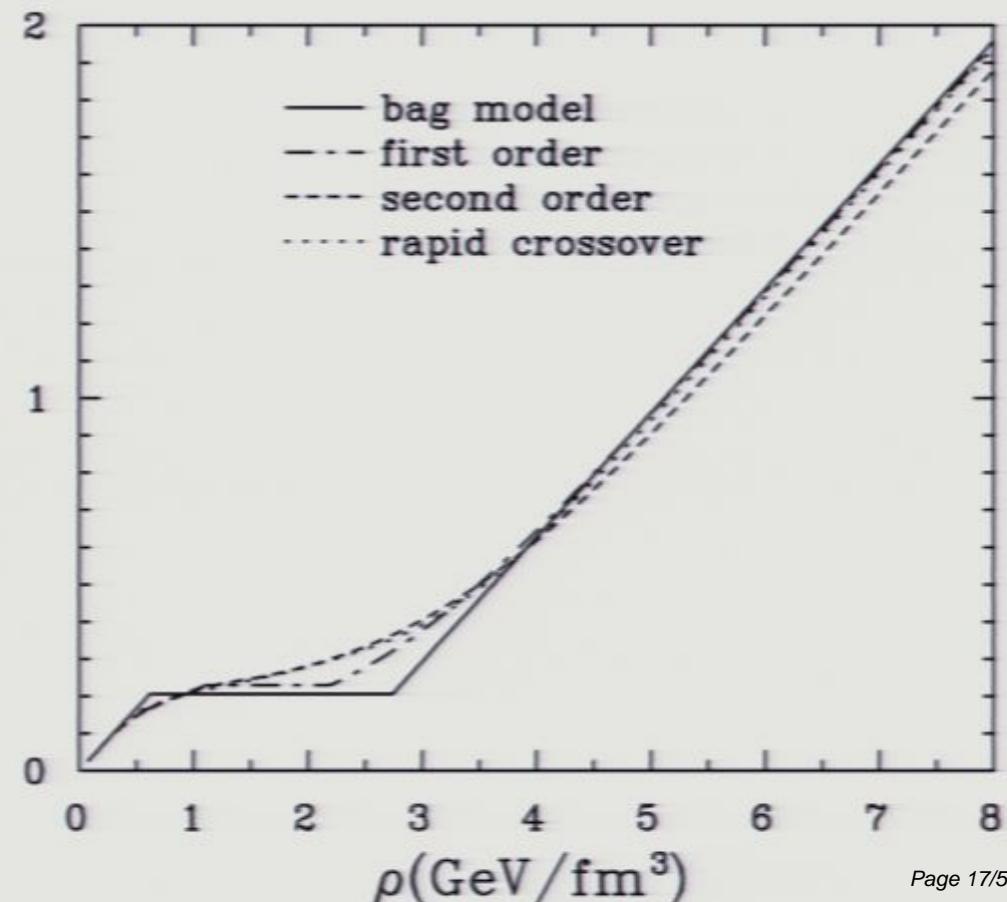
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$P(\text{GeV/fm}^3)$



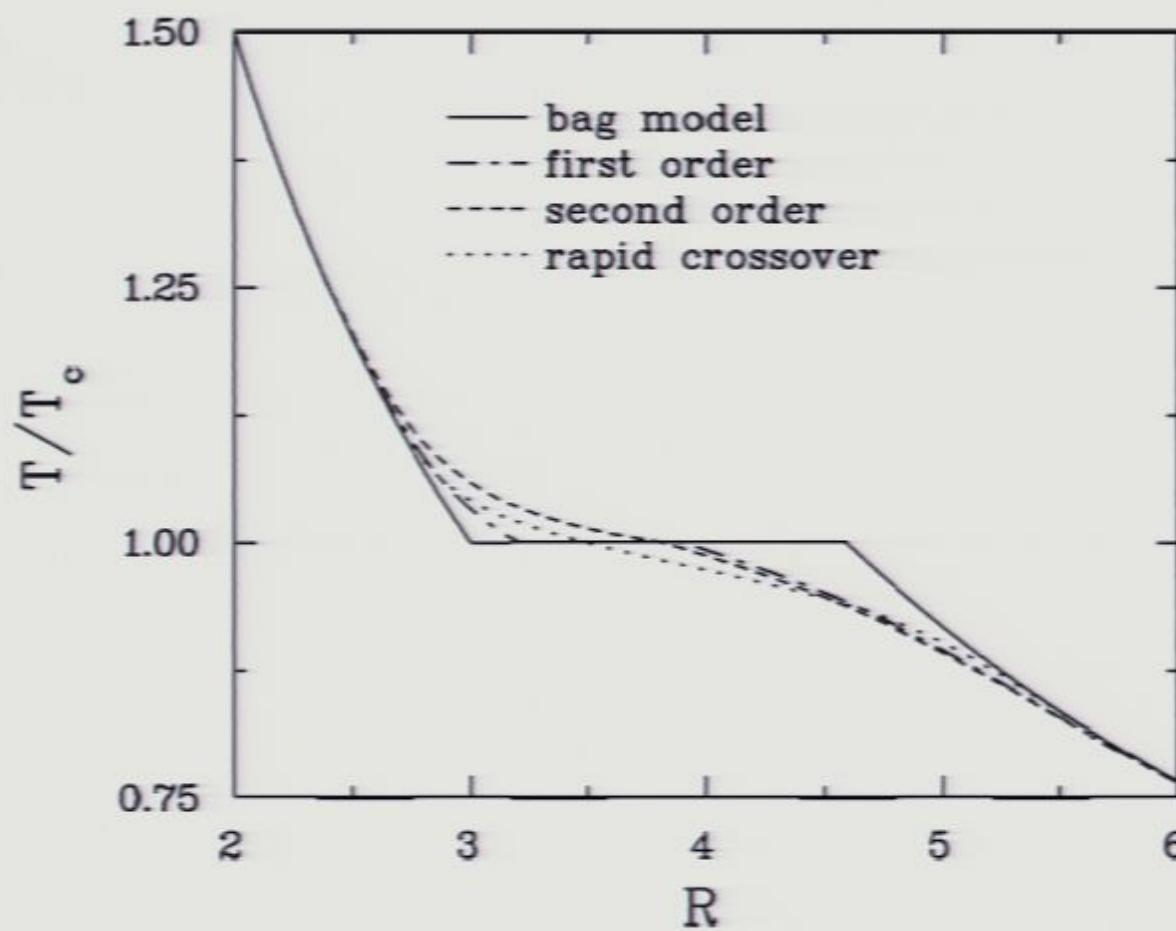
# Scale Factor

Conservation of entropy relates scale factor and temperature

$$s(T)R^3(T) = s_0 R_0^3$$

with

$$R(3T_c) = 1$$

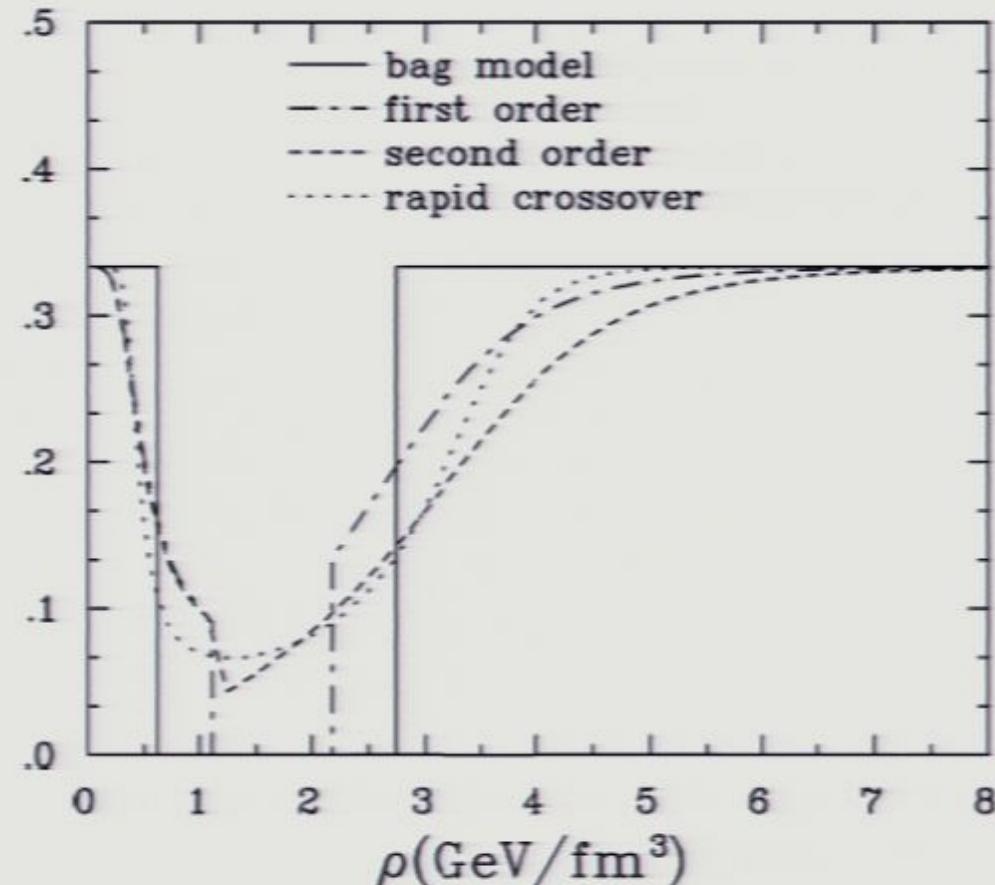


# Softening of the Equation of State

Reduction in the speed of sound

$$v_s^2 = \frac{dP}{d\rho} = \frac{dP/dT}{d\rho/dT}$$

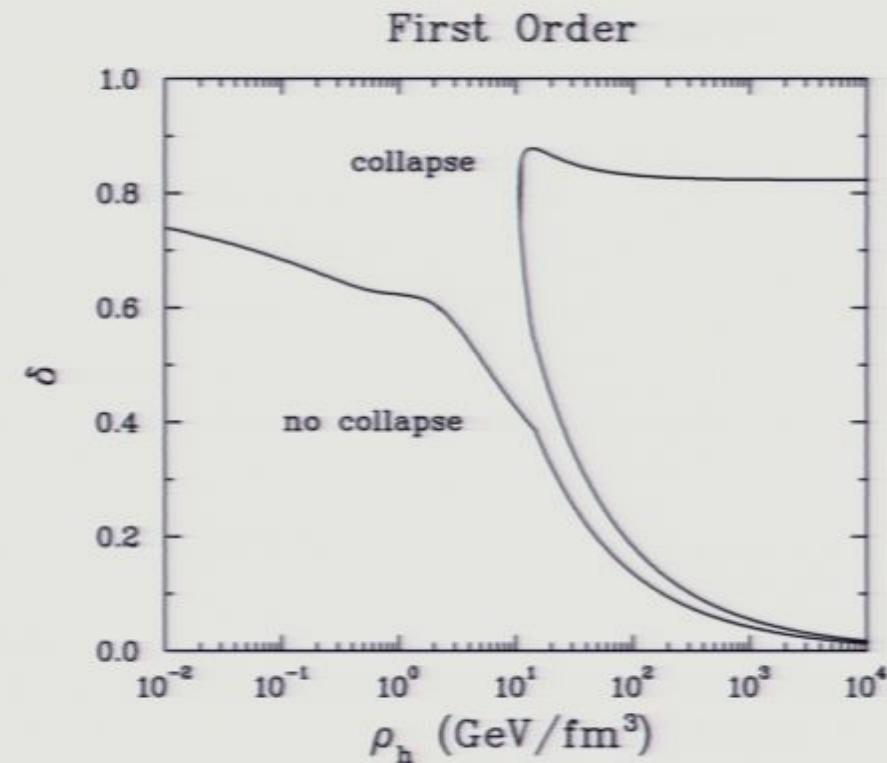
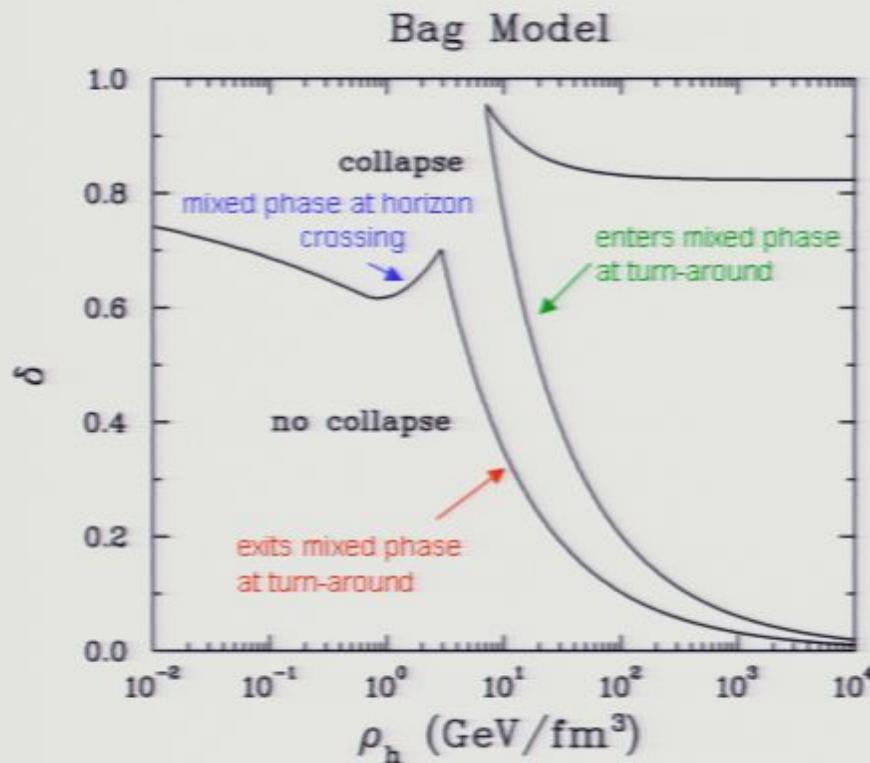
$$= \frac{s}{d\rho/dT}$$



# Determination of Critical Deltas

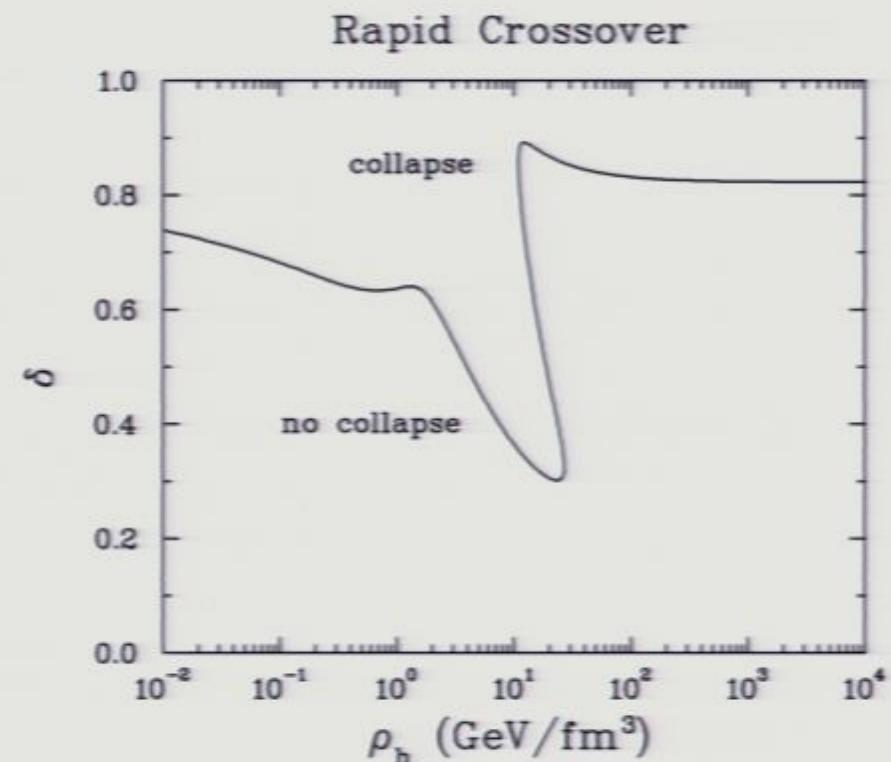
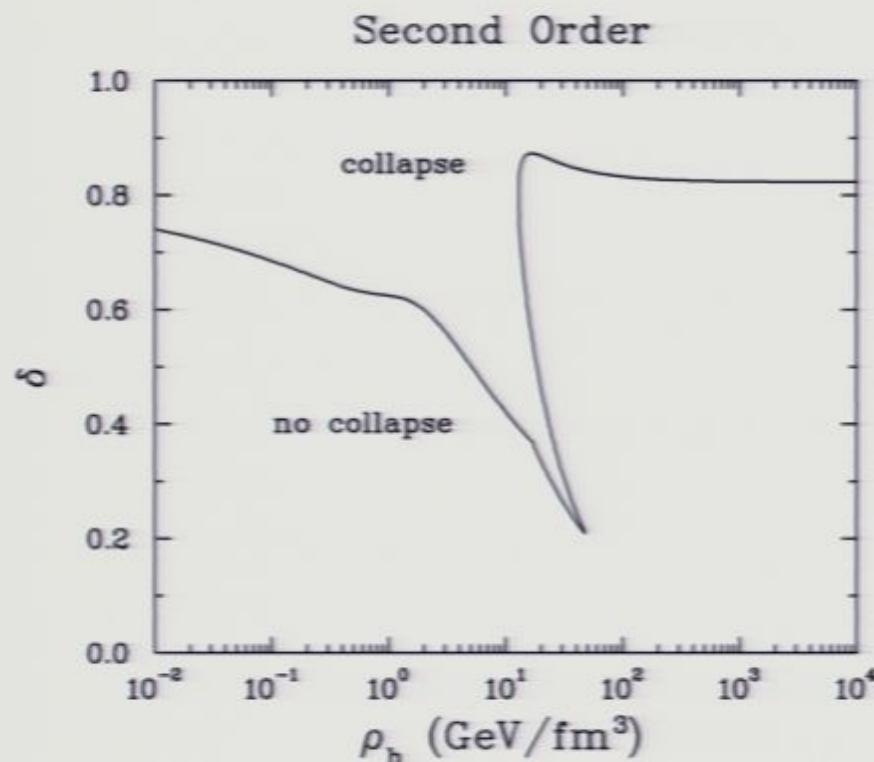
- Specify
  - When over-dense region enters horizon
  - Over-density  $\delta$
  - Model of phase transition
- Determine
  - Point of turn-around
  - Jeans length and size of region at this point
- Find critical deltas using collapse condition

# Critical $\delta$ – First Order Transitions



$\rho_h$  is the density at horizon crossing  
green region denotes the mixed phase

# Critical $\delta$ – Higher Order Transitions



$\rho_h$  is the density at horizon crossing

## Notable Features – Critical Deltas

- Greatest reduction in  $\delta$  for regions crossing horizon **before** softening and entering the mixed phase **at** turn-around.
- Slight reduction in  $\delta$  for regions which cross horizon near softest points.
- The tail goes to infinity for 1<sup>st</sup> order phase transitions.

# Spectrum of Perturbations

Assume Gaussian distribution

$$P(\delta, M_h) = \frac{1}{\sqrt{2\pi}\sigma(M_h)} \exp\left(-\frac{\delta^2}{2\sigma^2(M_h)}\right) \quad \sigma = 9.5 \times 10^{-5} \left(\frac{M_h}{10^{22} M_\odot}\right)^{\frac{1-n}{4}}$$

Width  $\sigma$  is COBE normalized variance which depends on the spectral index  $n$ . [Green and Liddle, 1997]

Power spectrum of fluctuations  $P(k) \sim k^n$

Limits from inflation etc. give  $n \leq 1.3$  but they come from larger length scales.

# Integrate Formation Rate

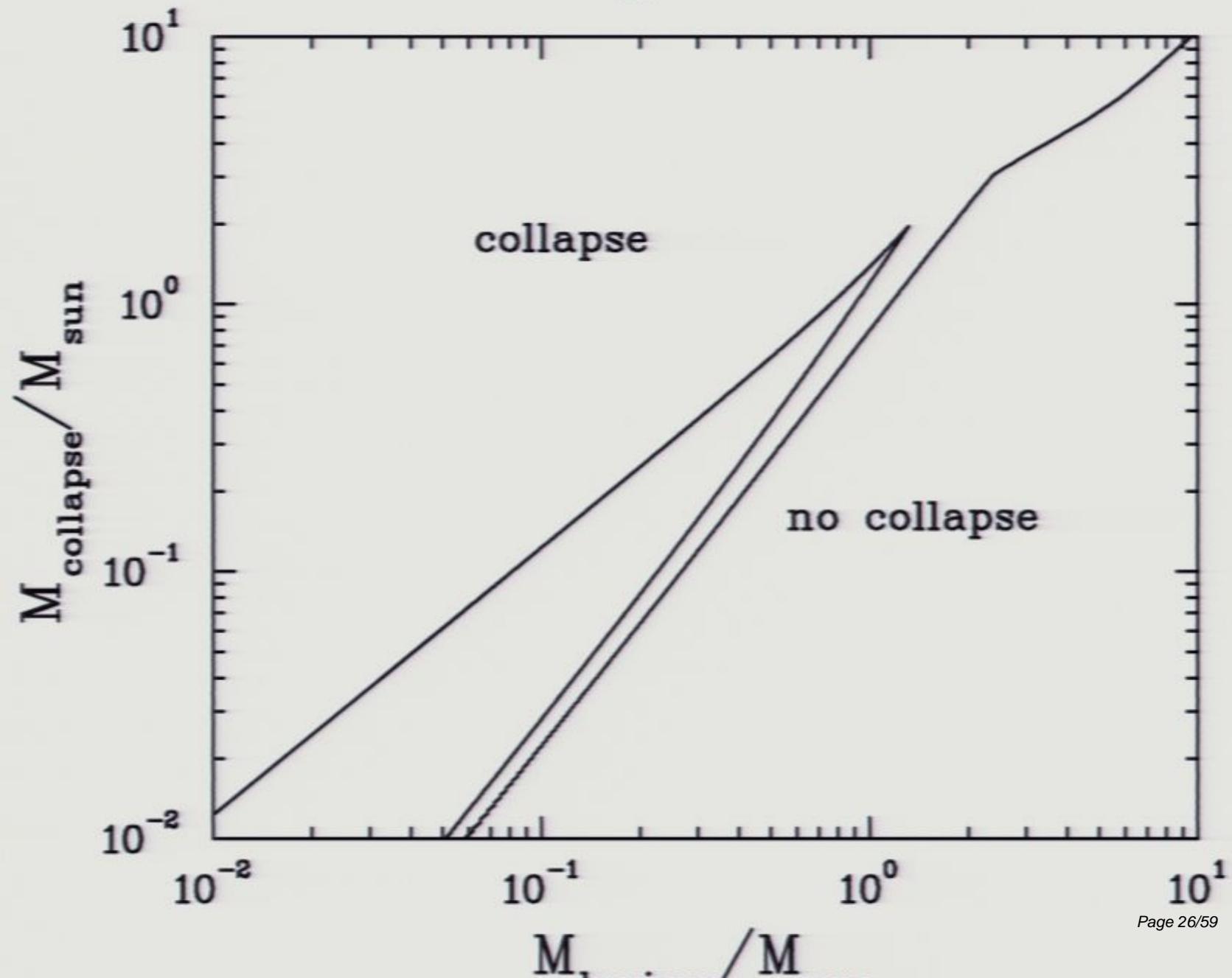
$$n(t) = \int_0^t dt' \left[ \frac{R(t')}{R(t)} \right]^3 \left| \frac{1}{V_h(t')} \frac{dV_h(t')}{dt'} \right| \frac{1}{V_h(t')} \int d\delta P(\delta, M_h(t'))$$

↑                   ↑                   ↑                   ↑                   ↑  
Present density   Dilution from formation to present time   Horizon crossing rate   Density factor (1 per horizon)   Integration range in delta  
Probability that overdensity at horizon crossing leads to collapse

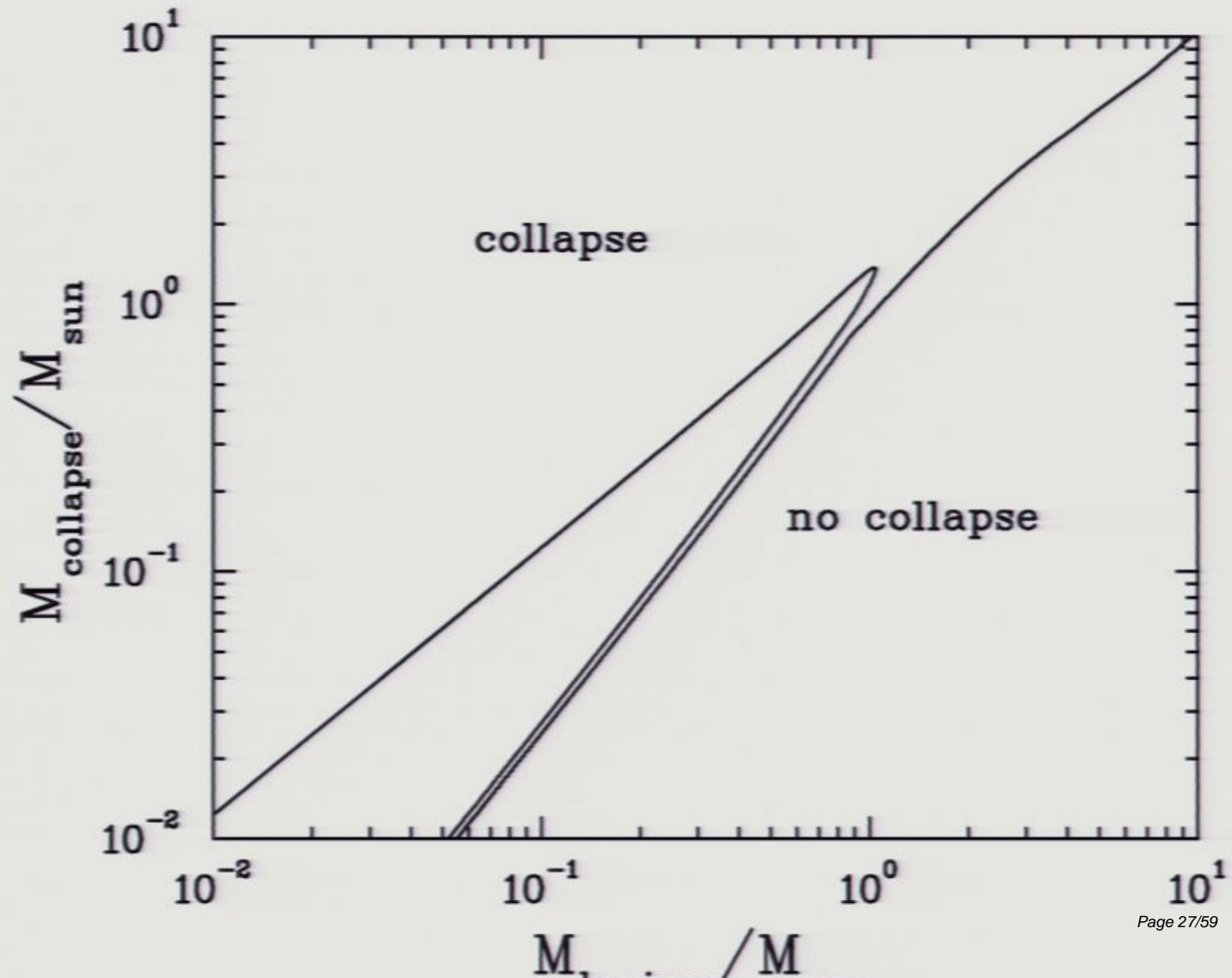
To calculate the mass distribution  $dn/dM$  insert the factor

$$\delta(M_{\text{collapse}}(\delta, M_h) - M)$$

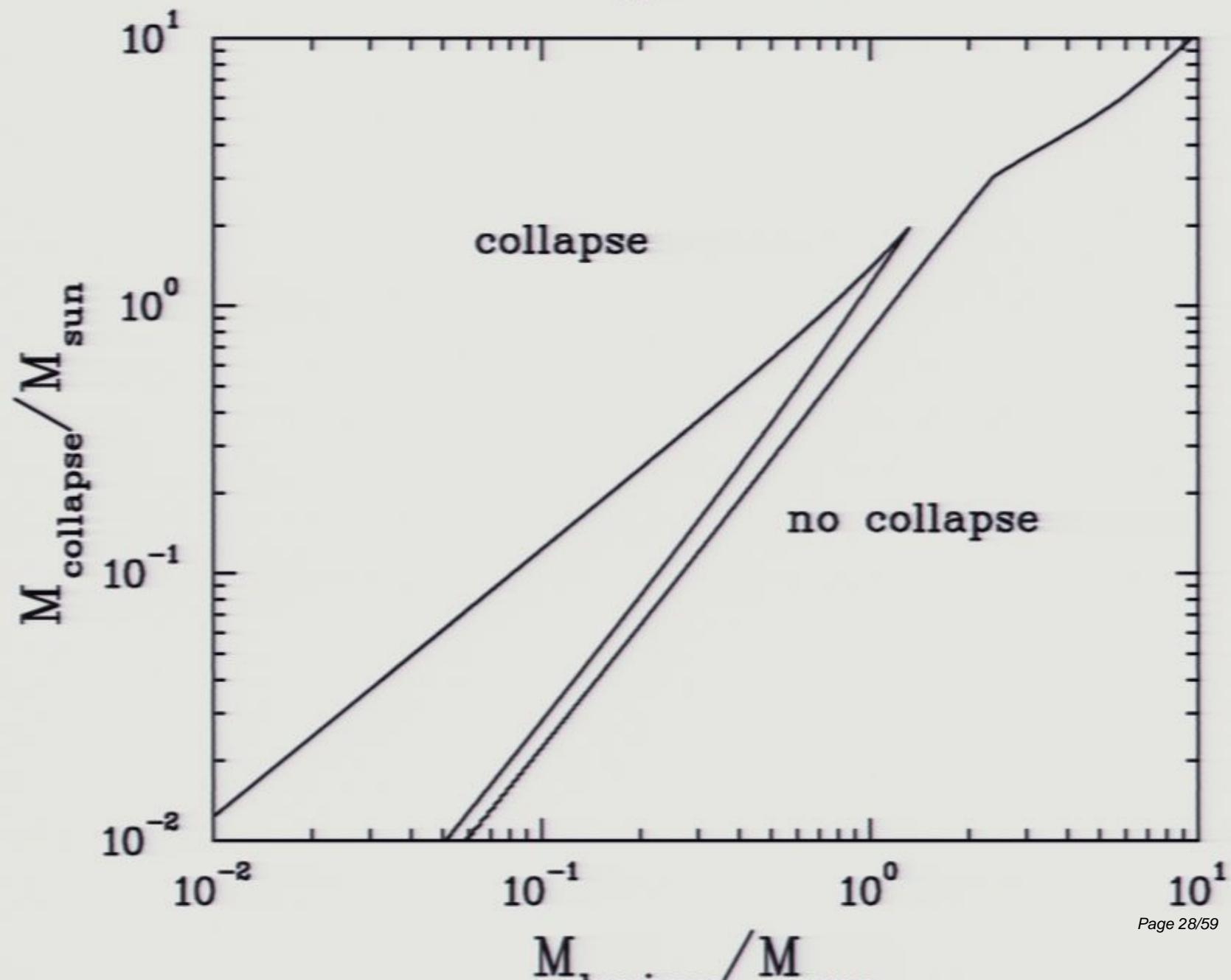
## Bag Model



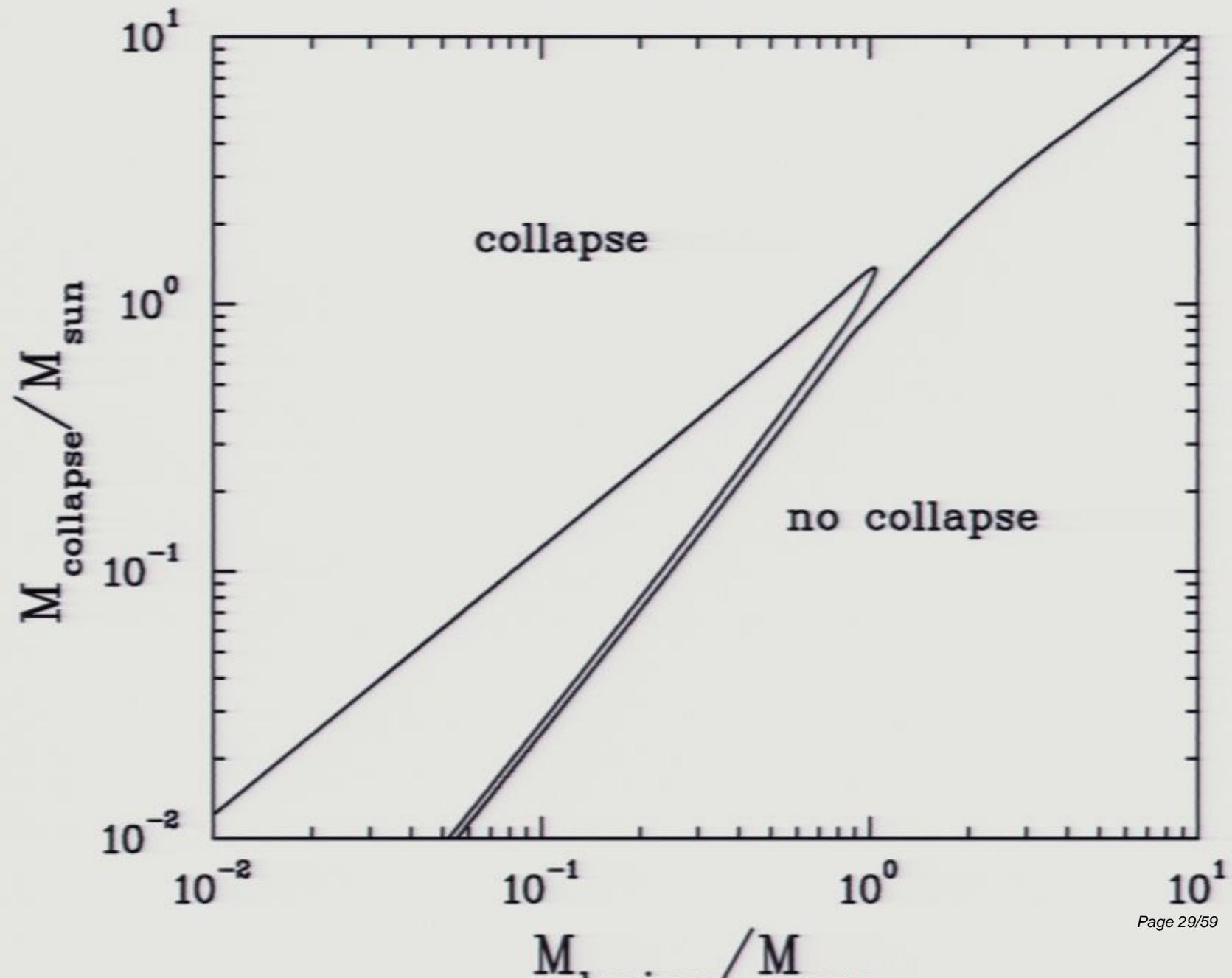
## First Order



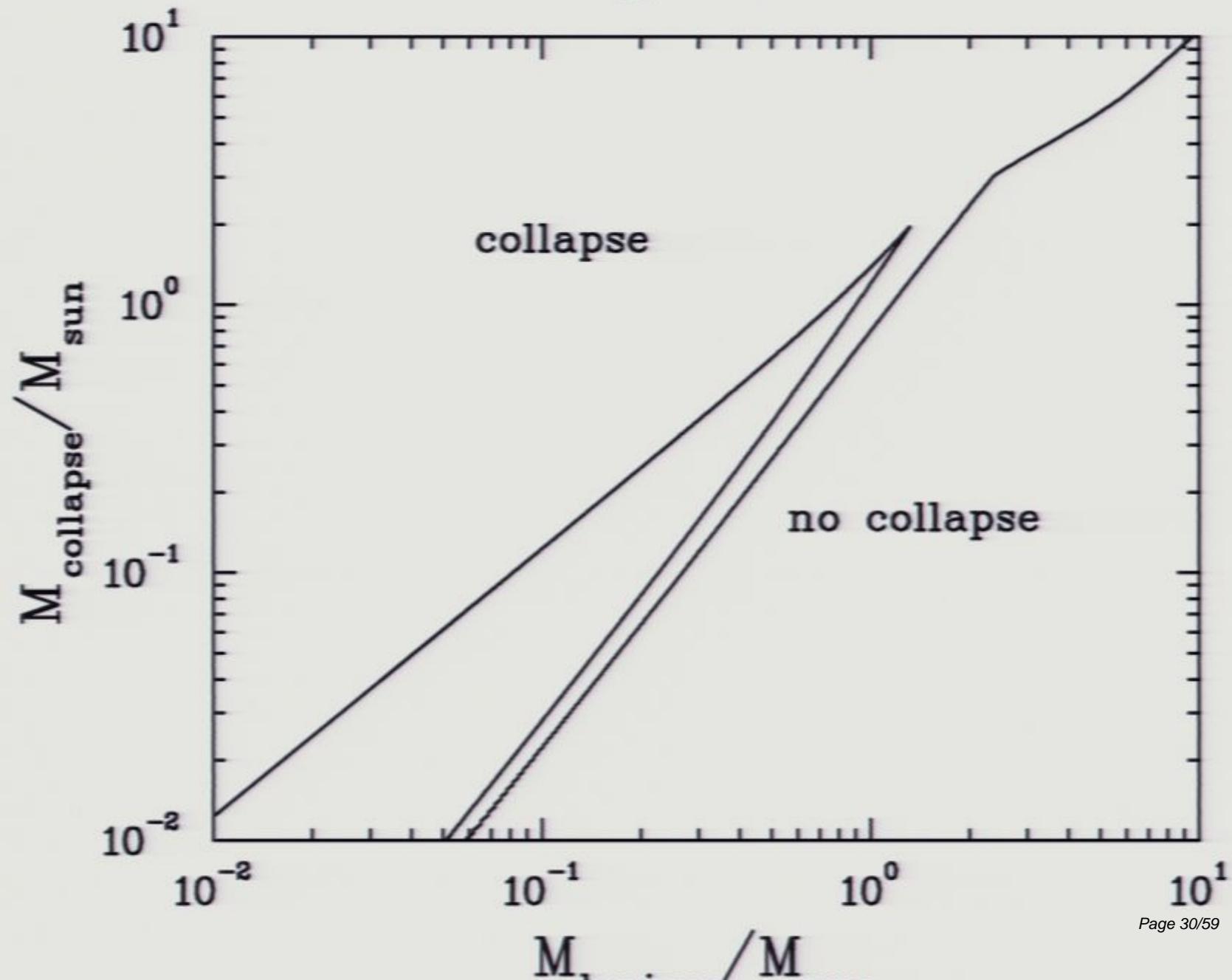
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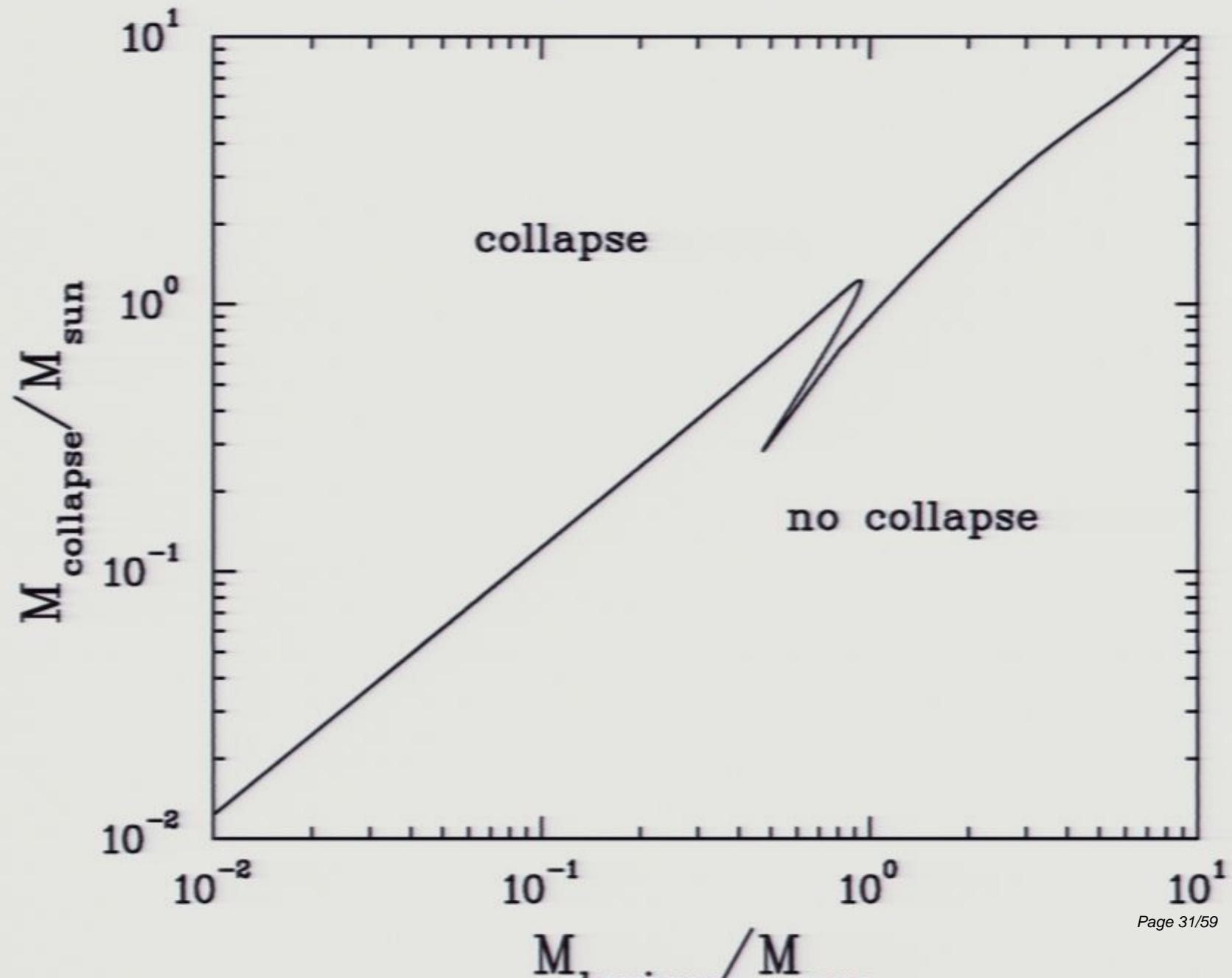
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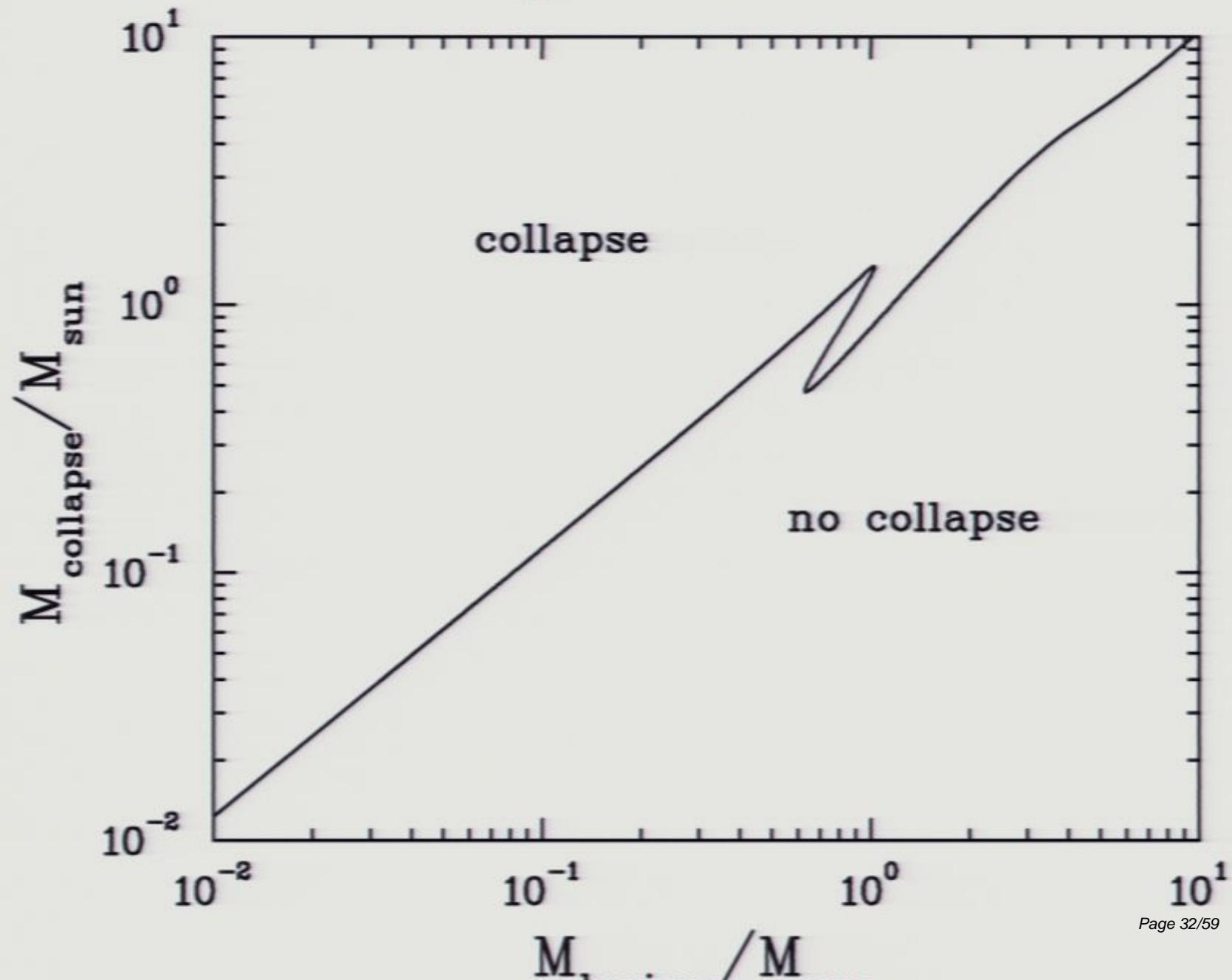
## Bag Model



## Second Order



## Rapid Crossover



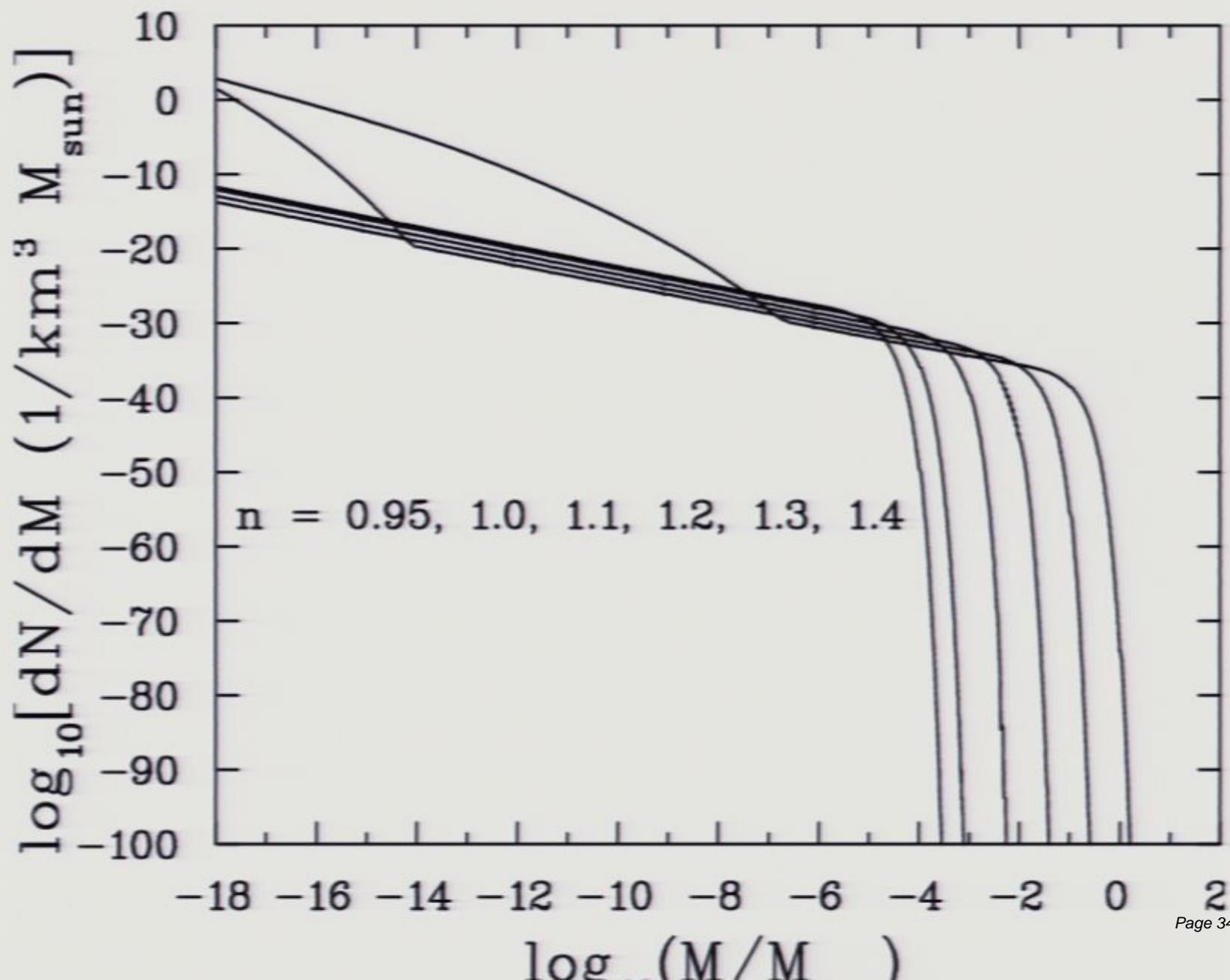
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$$\frac{dn^{\text{net}}}{dM}(M,t) = \frac{dn}{dM}(M,t) \exp(-S(M,t))$$

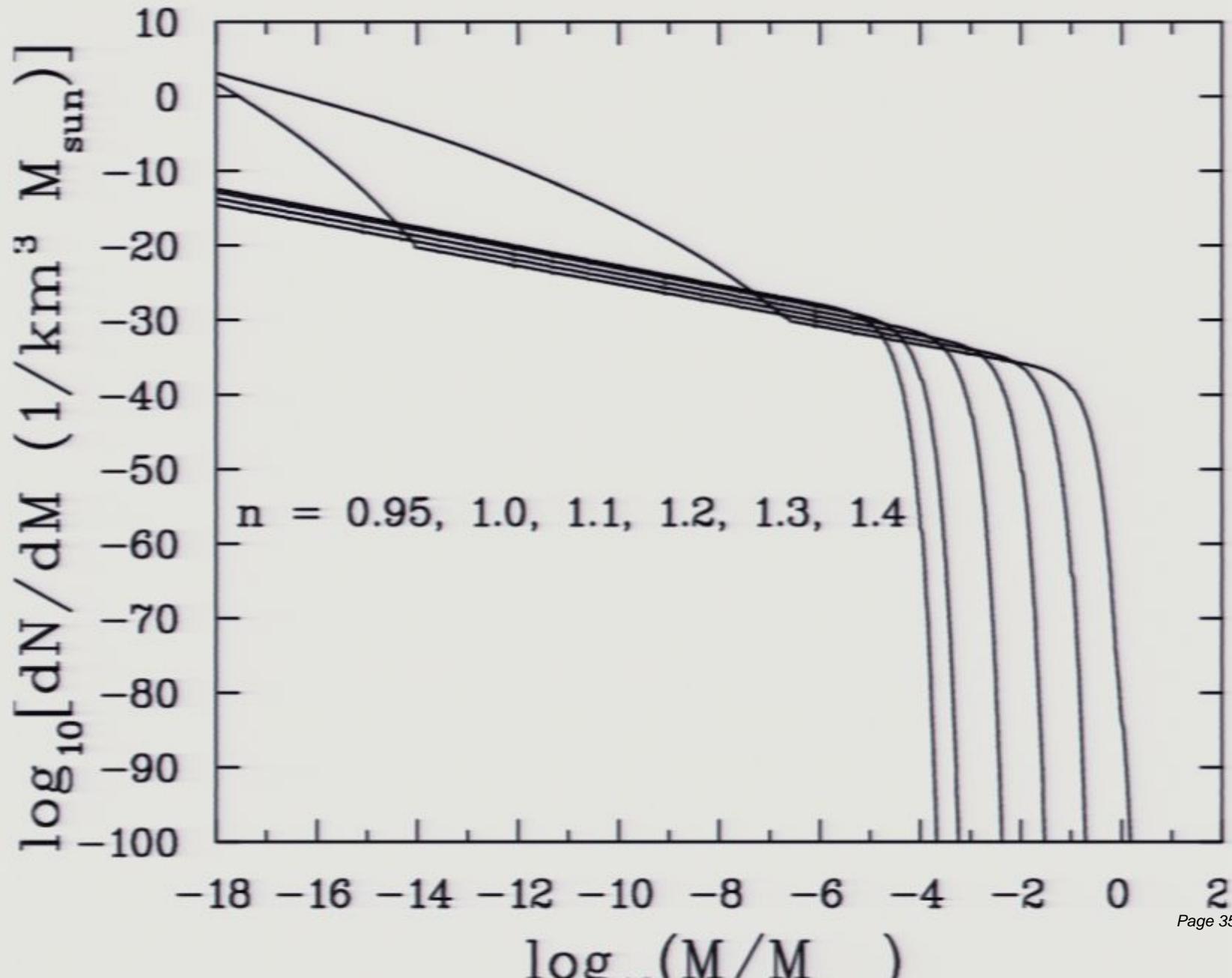

↑  
Suppression factor

$$S(M,t) = \int_0^t dt' \left| \frac{1}{V_h(t')} \frac{dV_h(t')}{dt'} \right|_{\Delta(t')} \int d\delta P(\delta, M_h(t')) \times \theta(M_{\text{collapse}}(\delta, M_h(t')) - M)$$

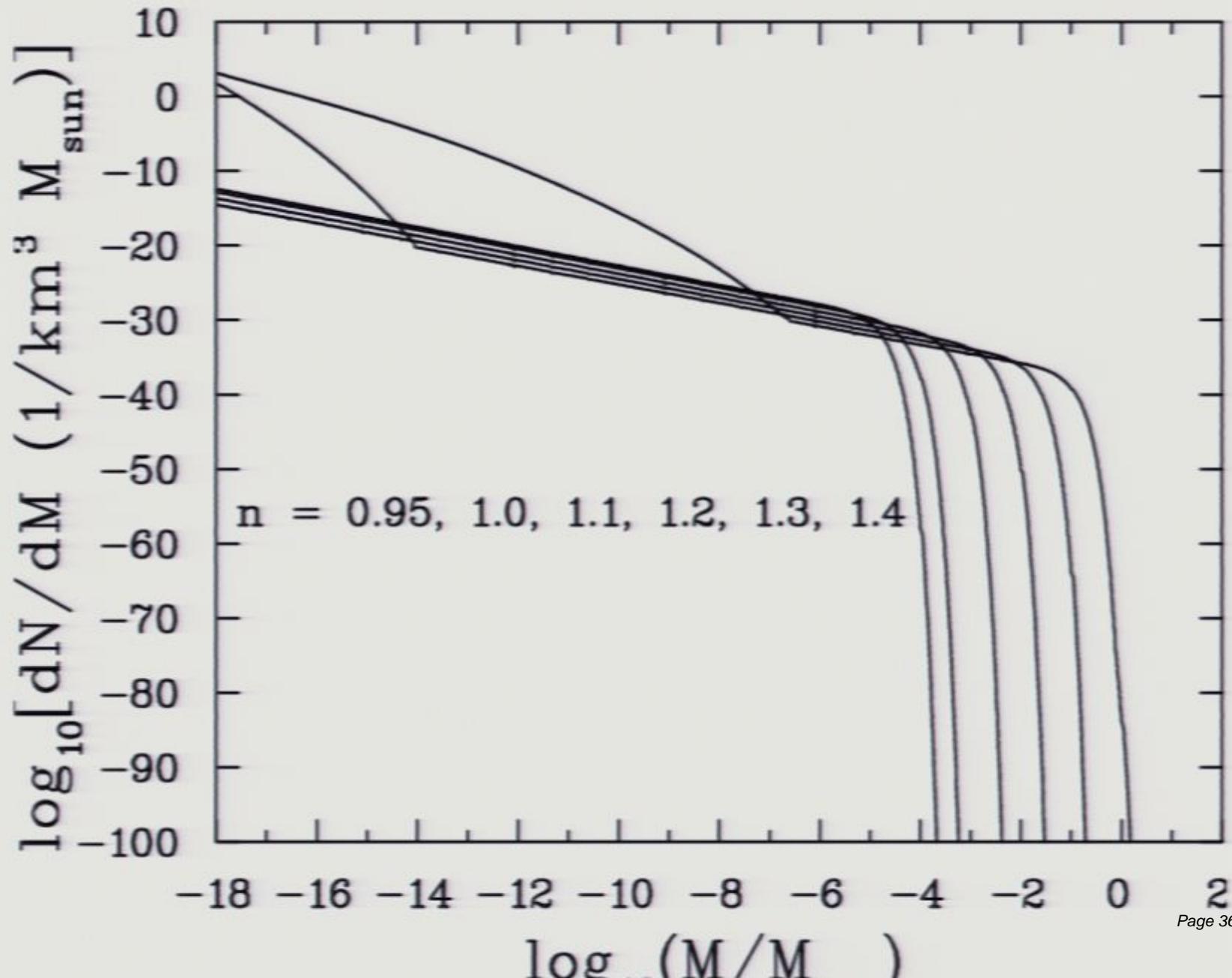
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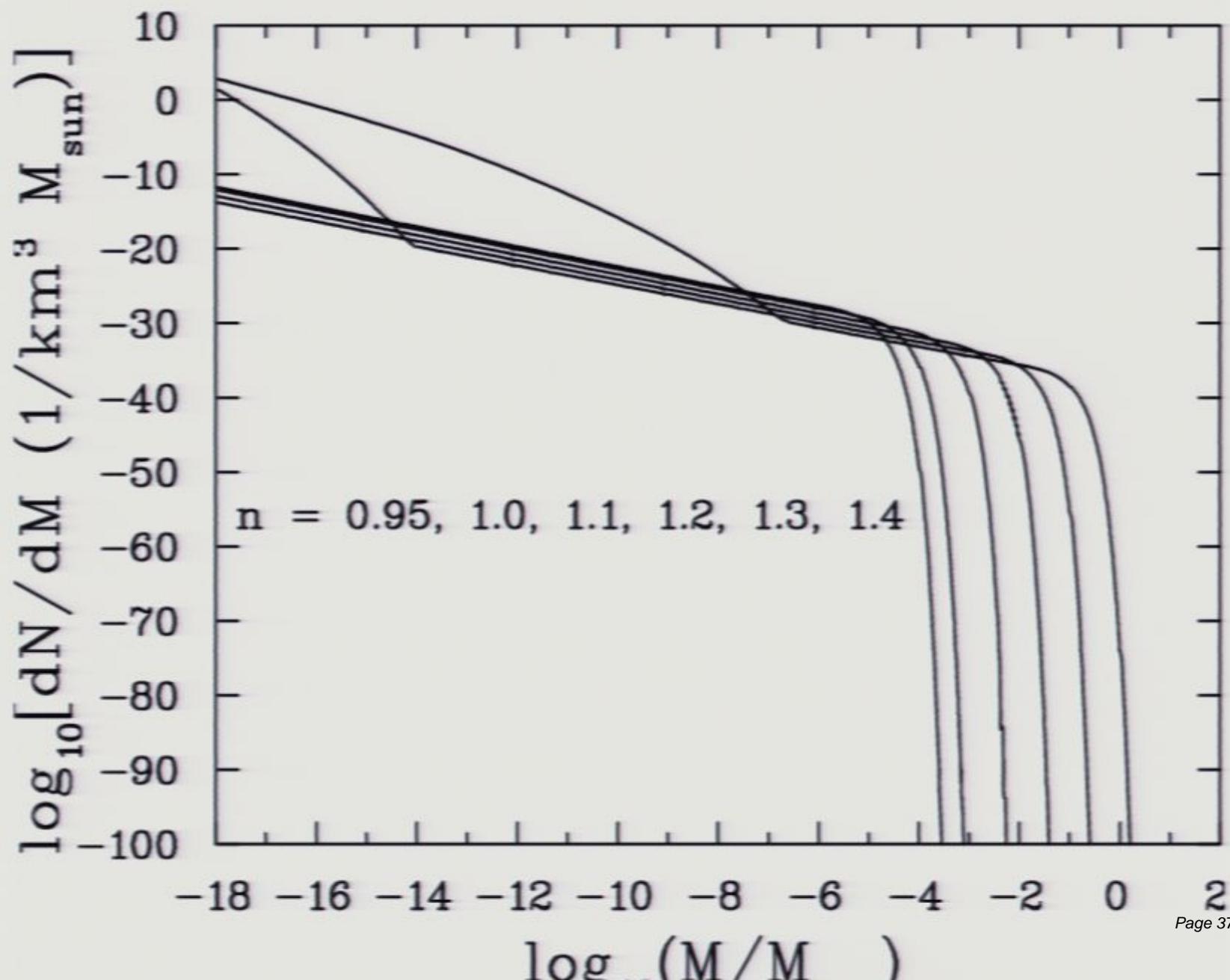
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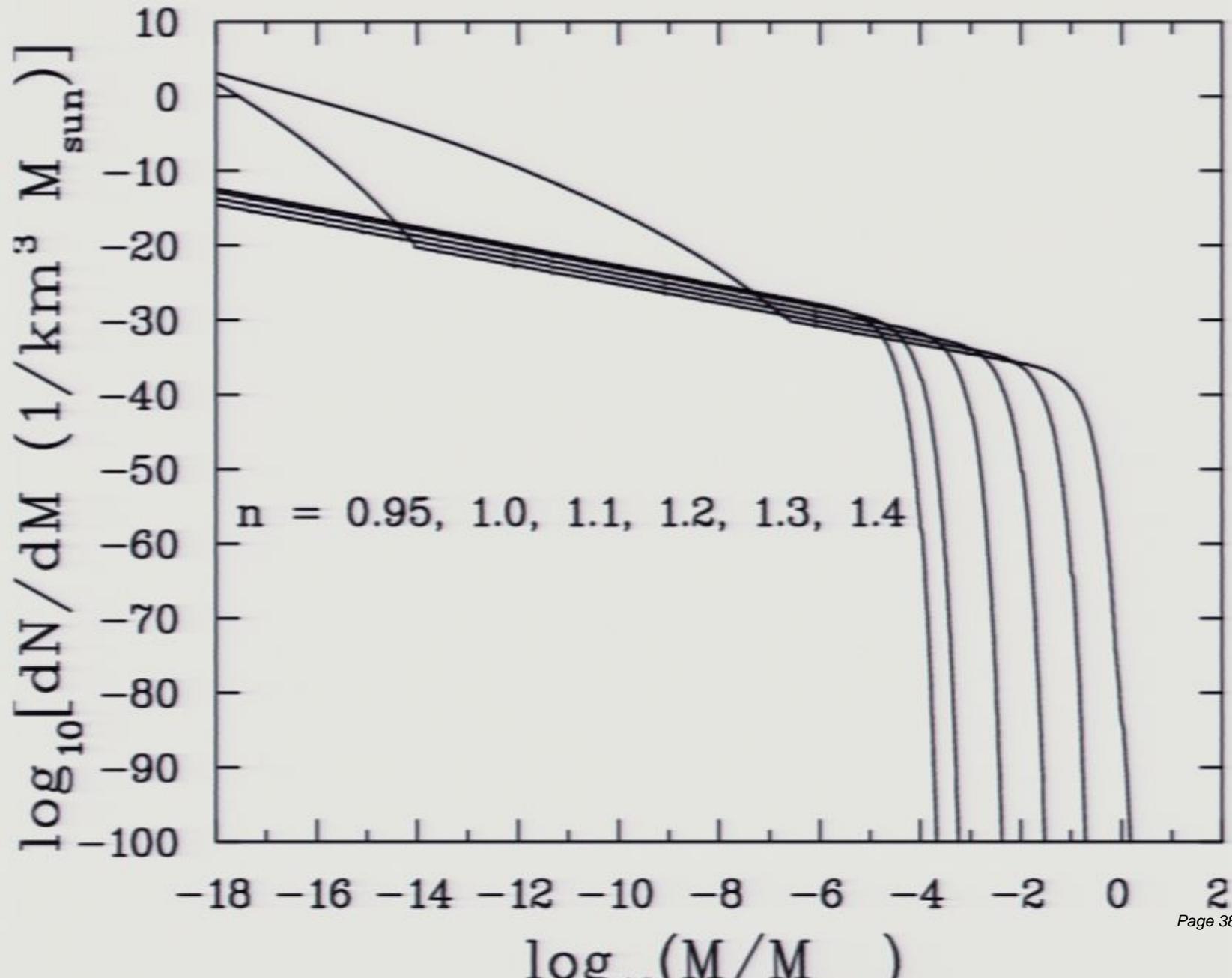
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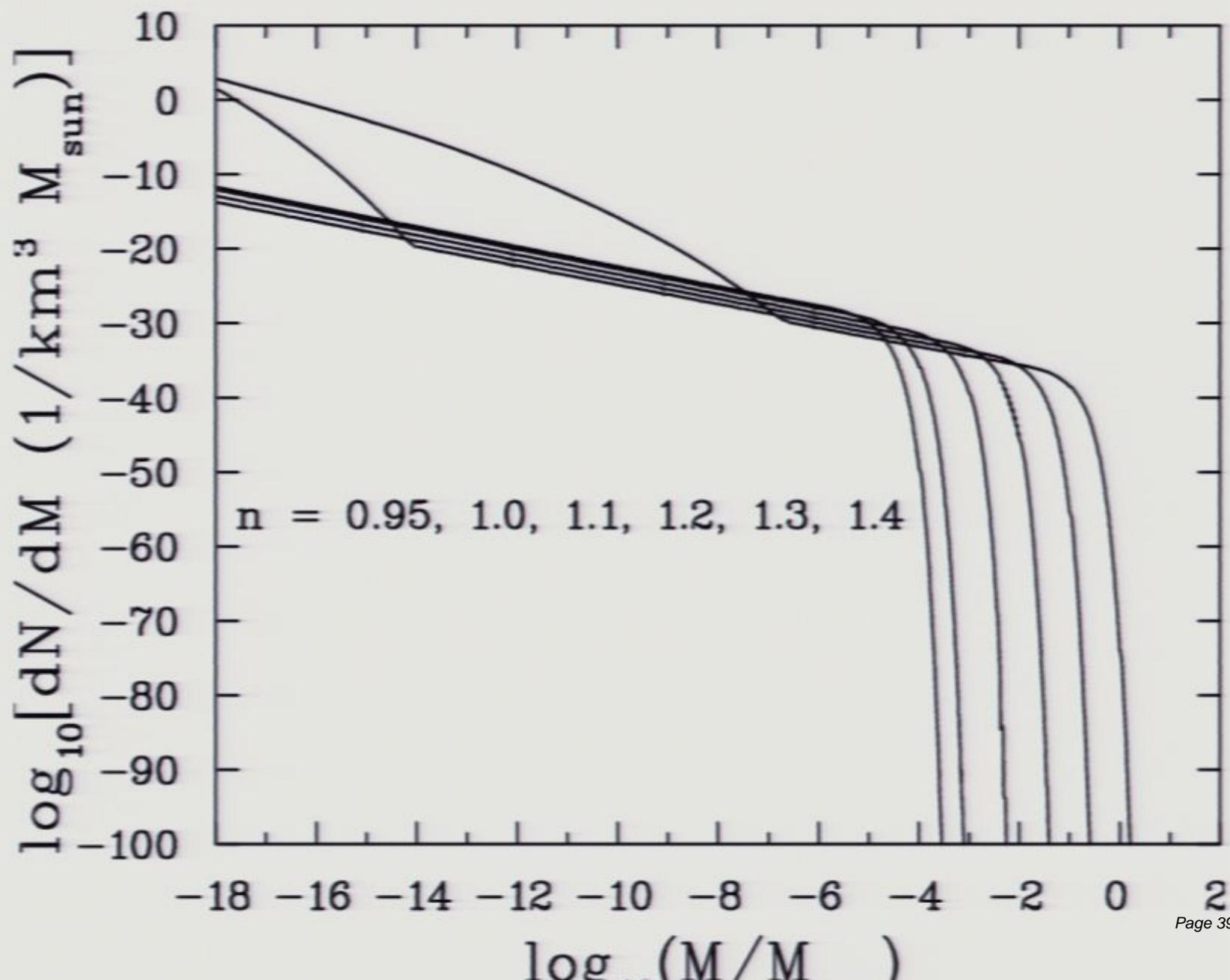
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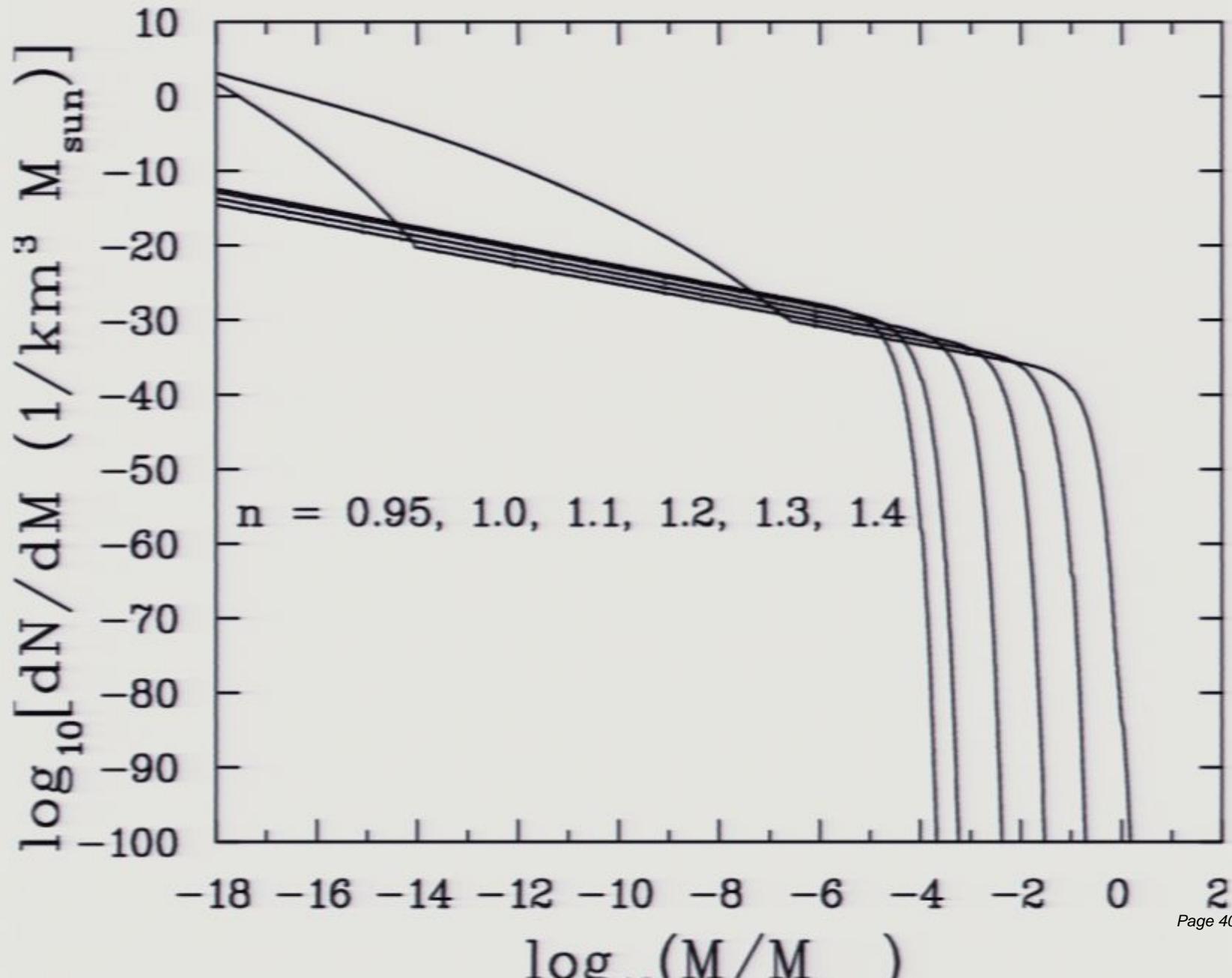
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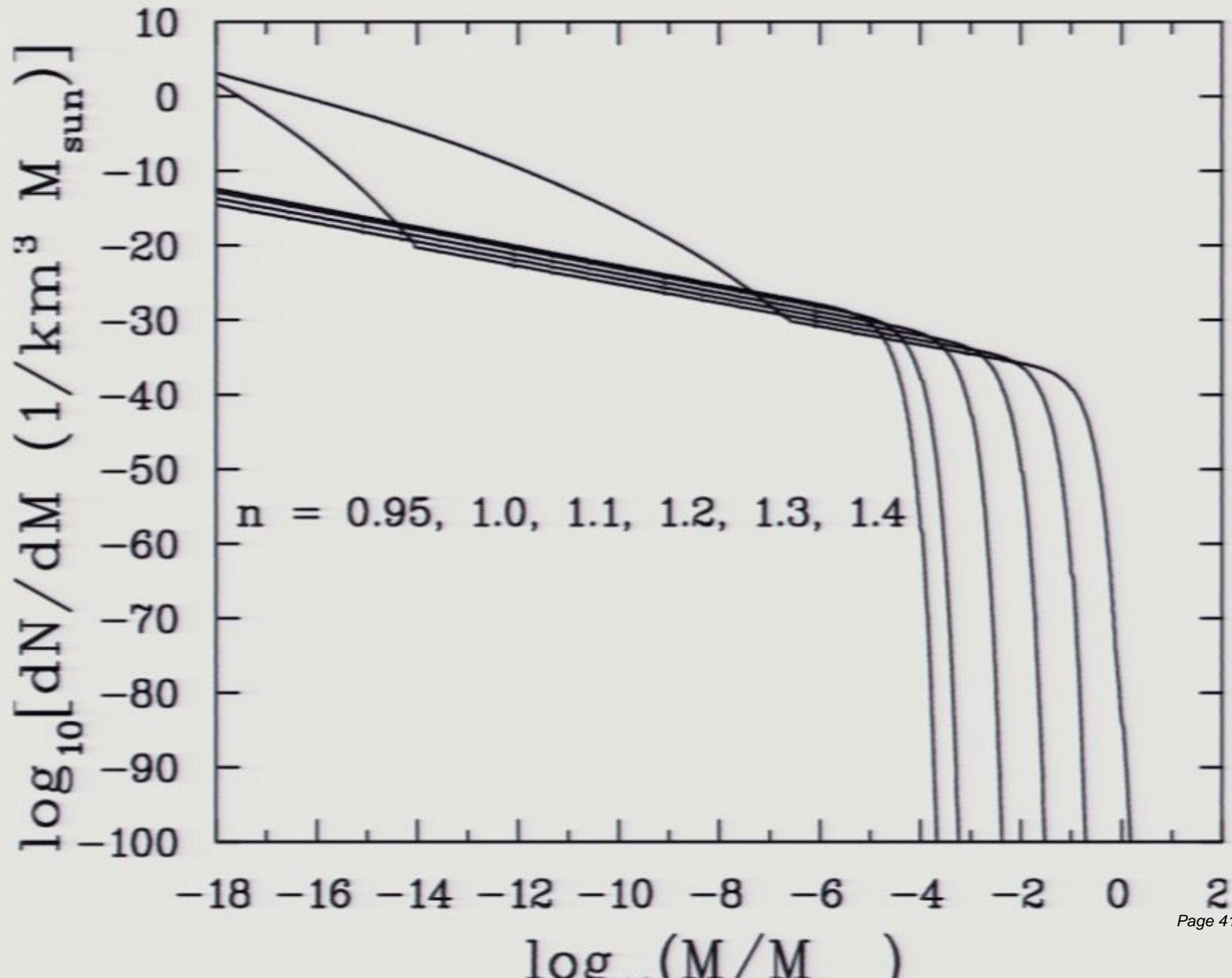
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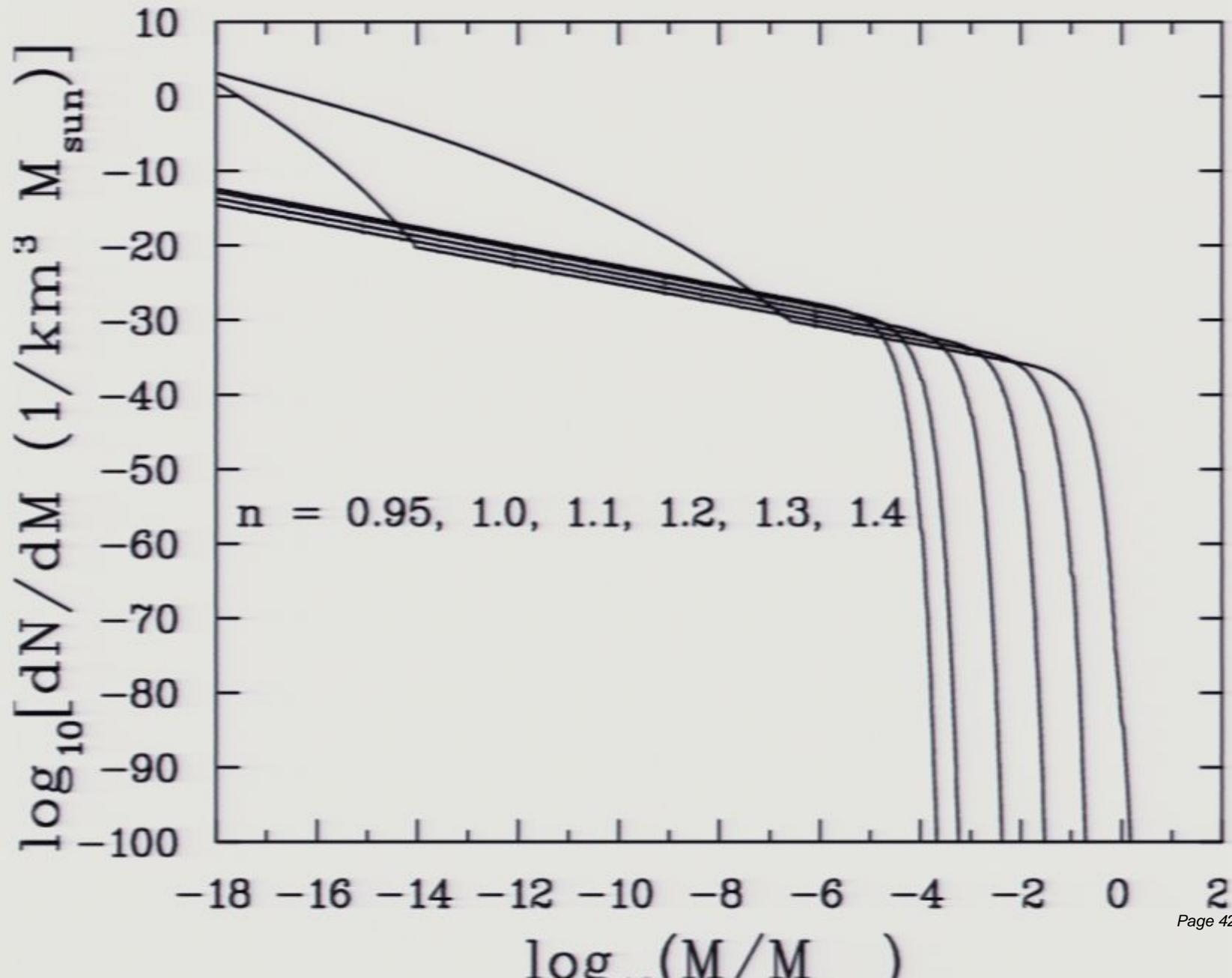
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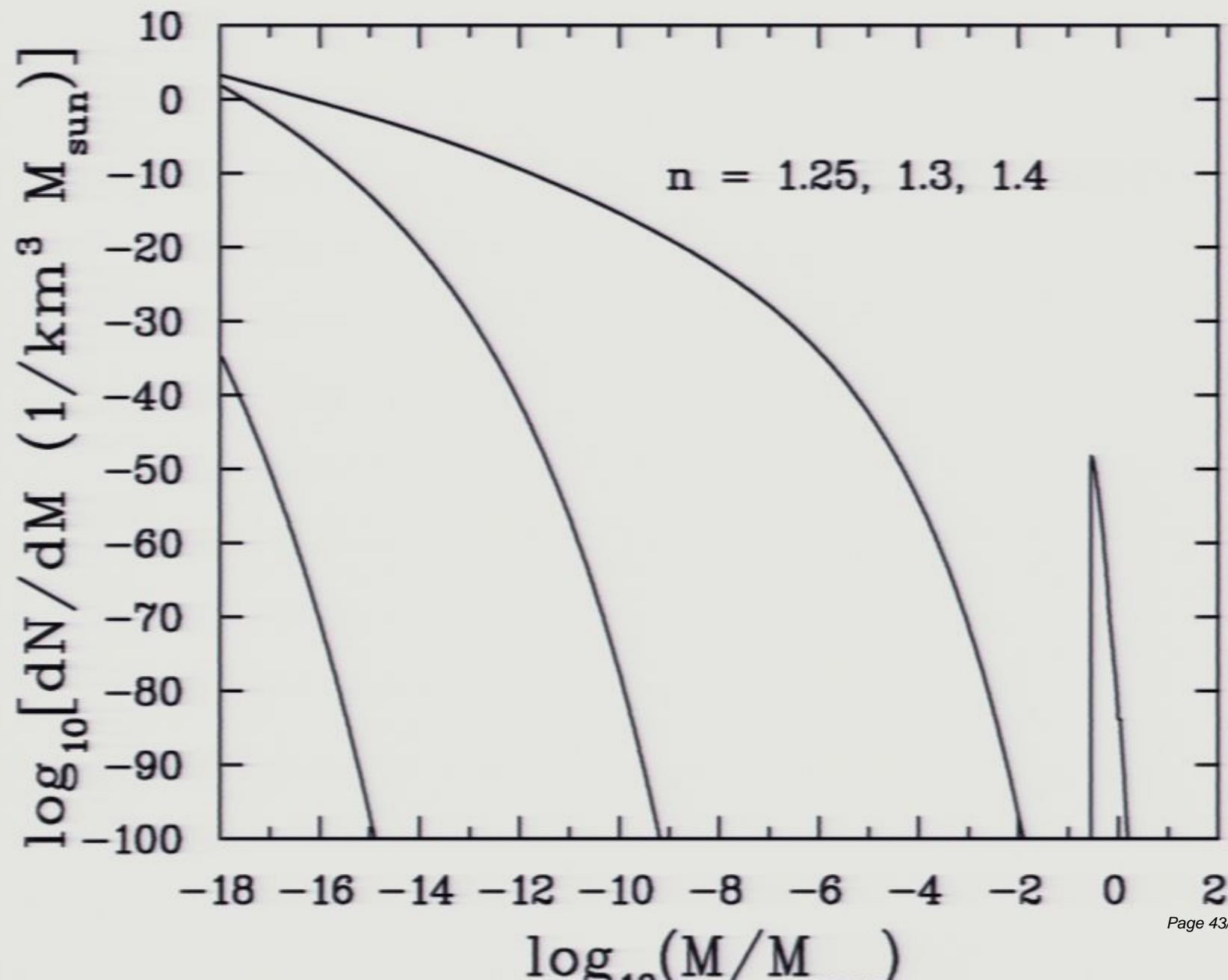
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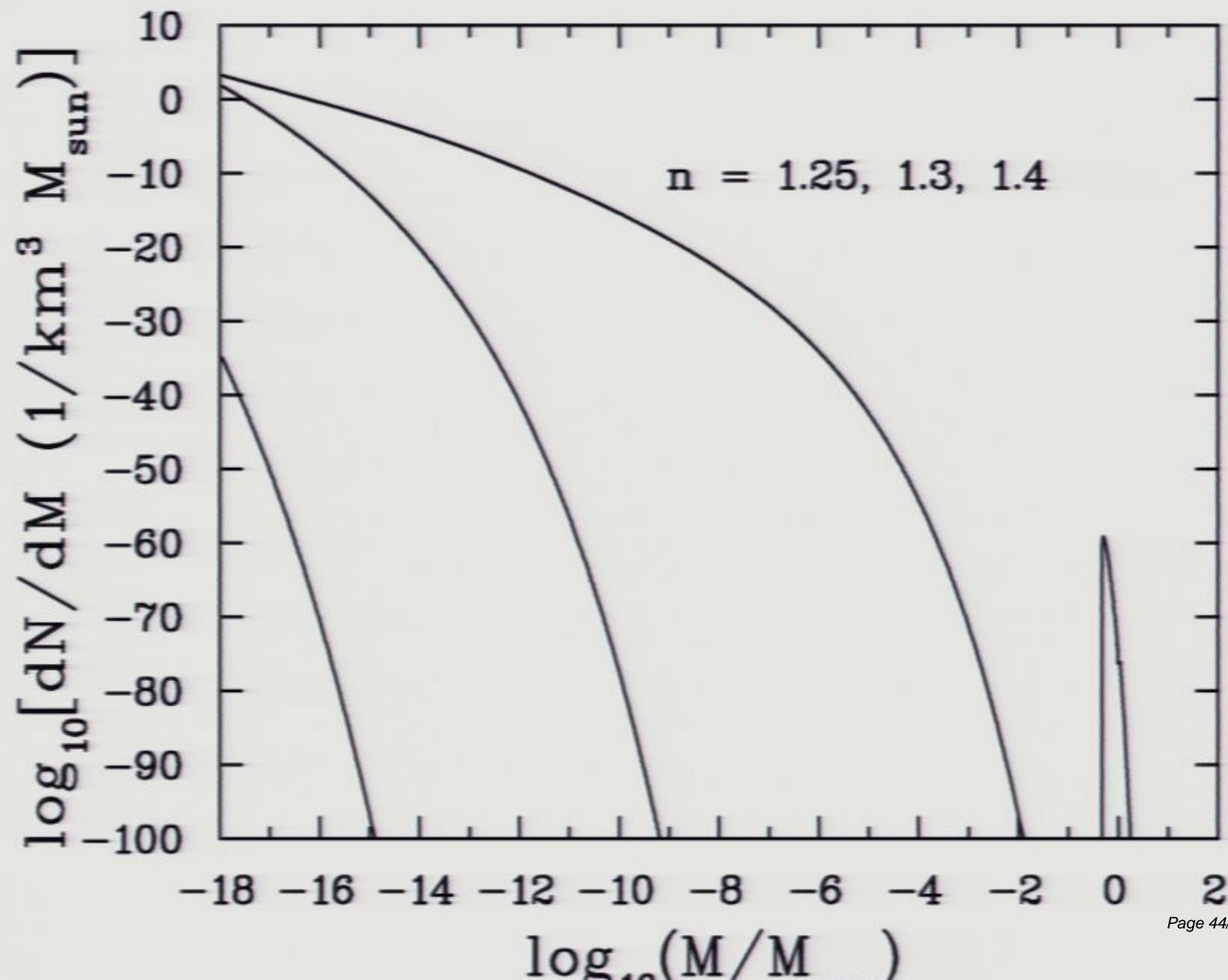
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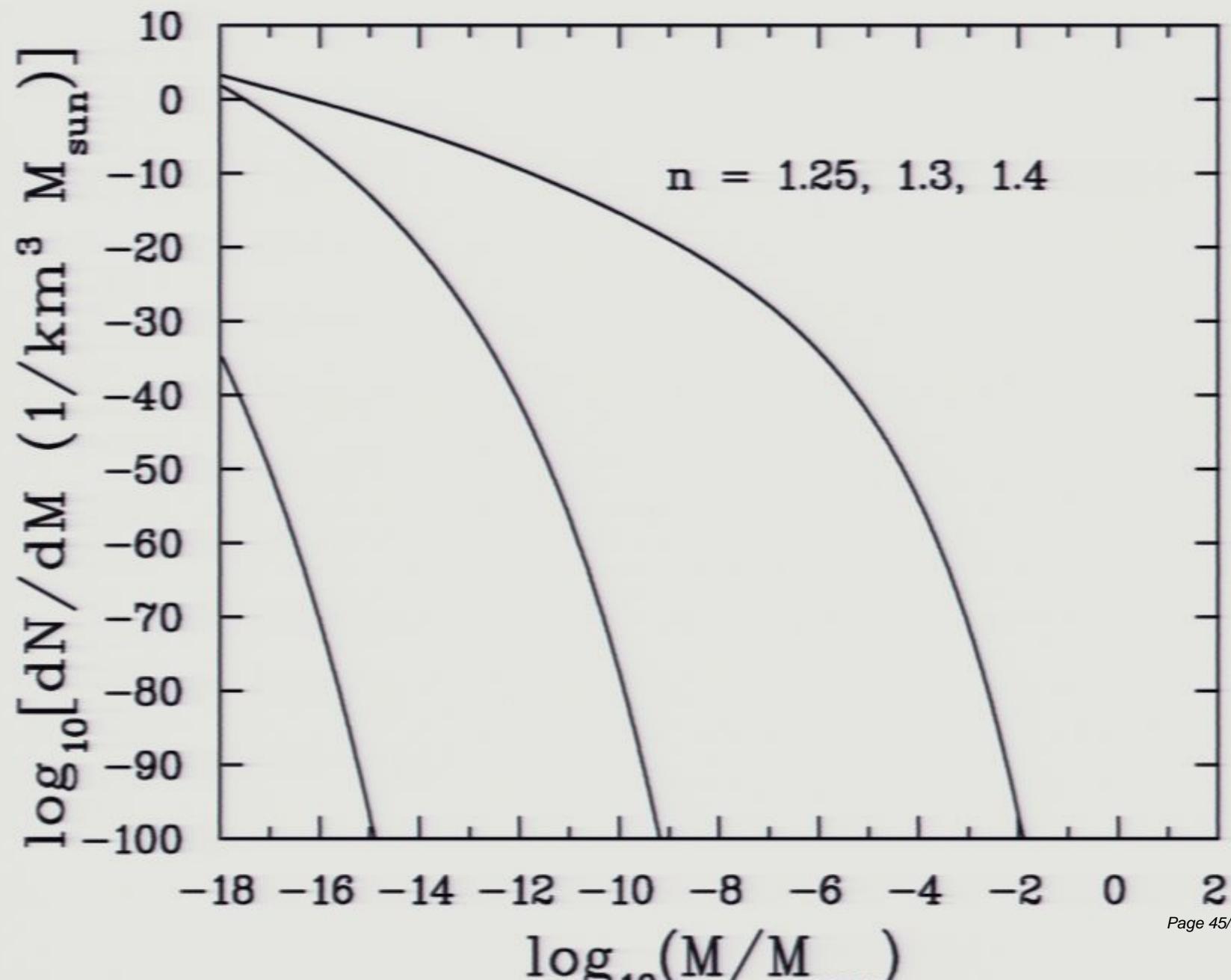
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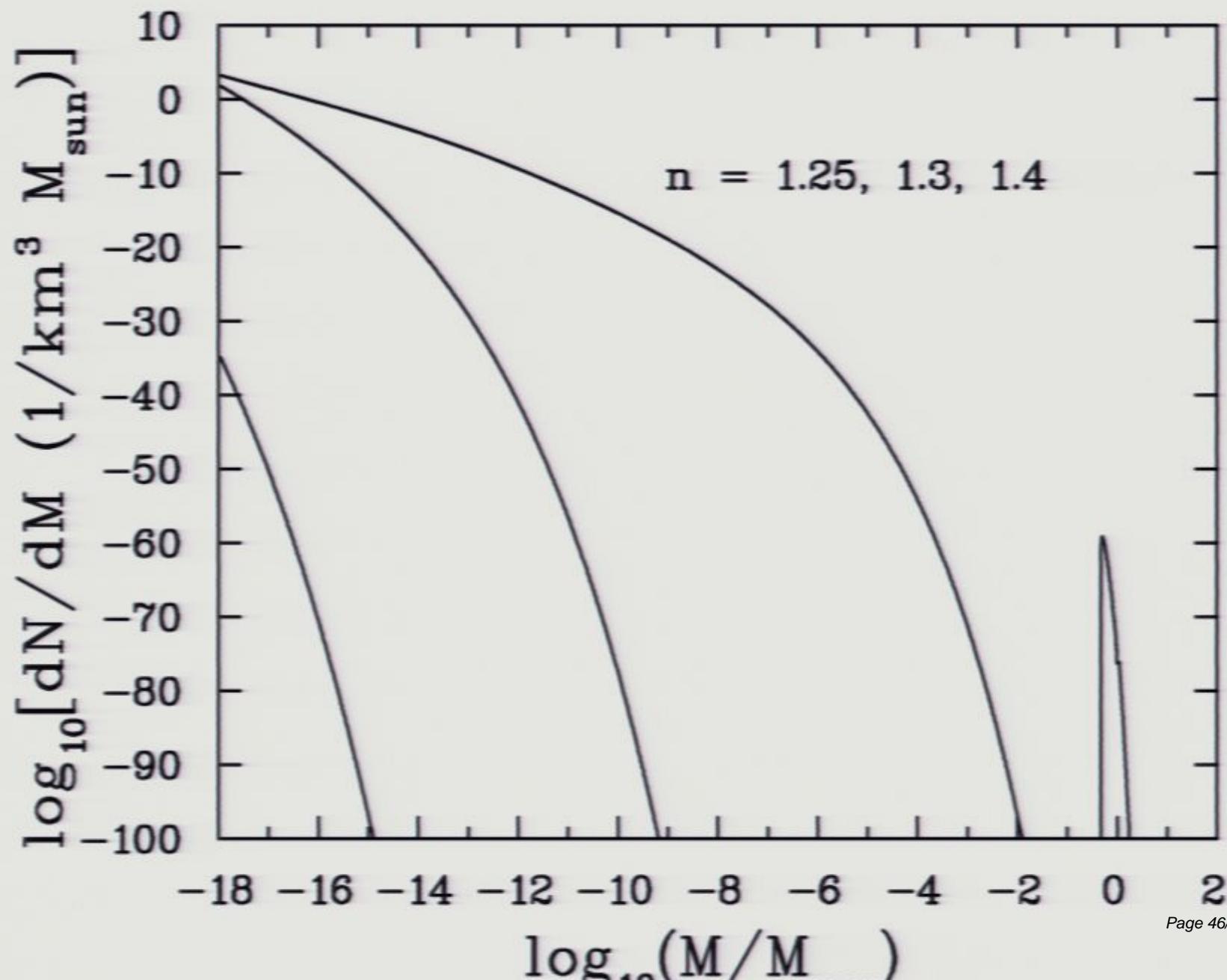
## Rapid Crossover



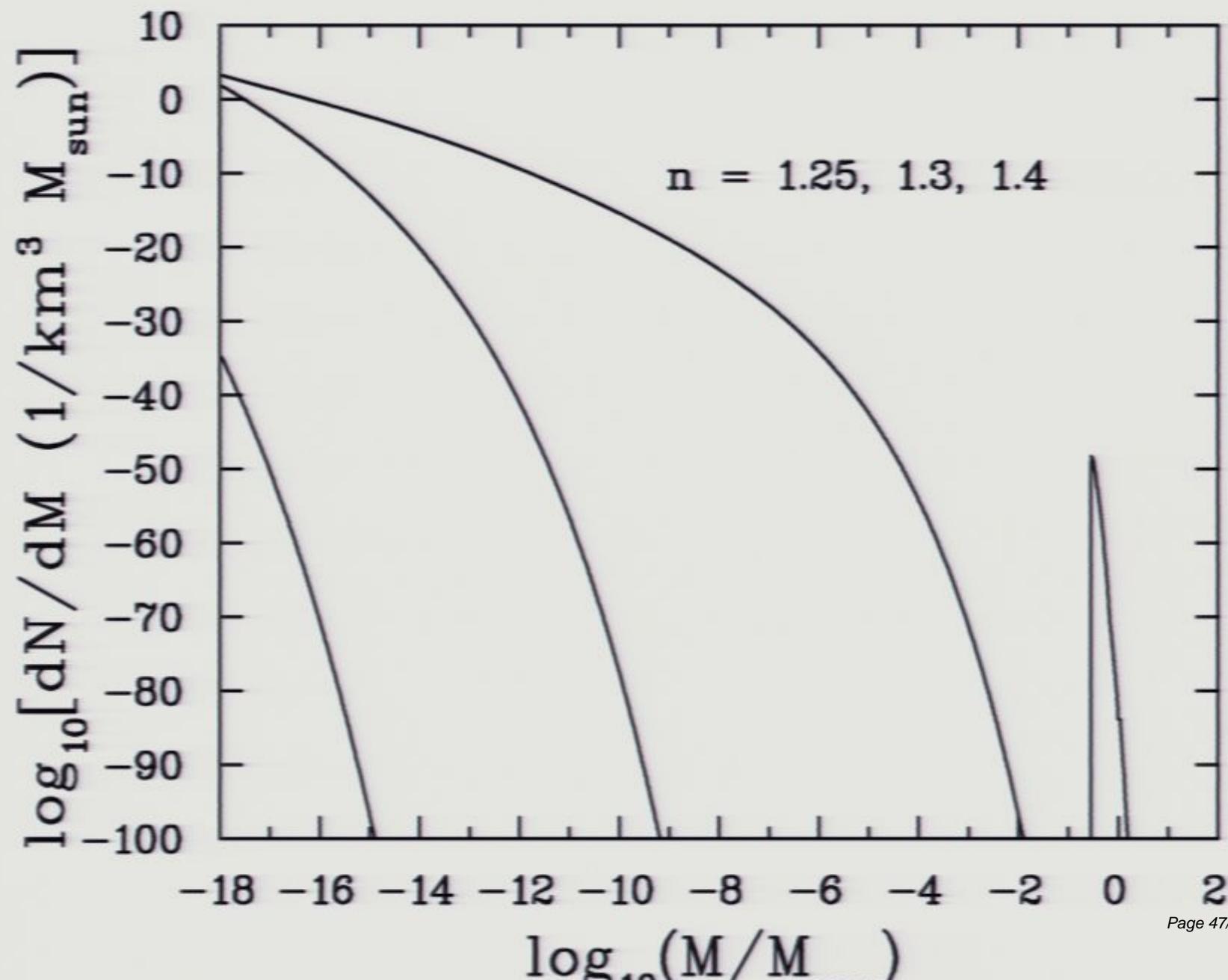
## Fixed Speed



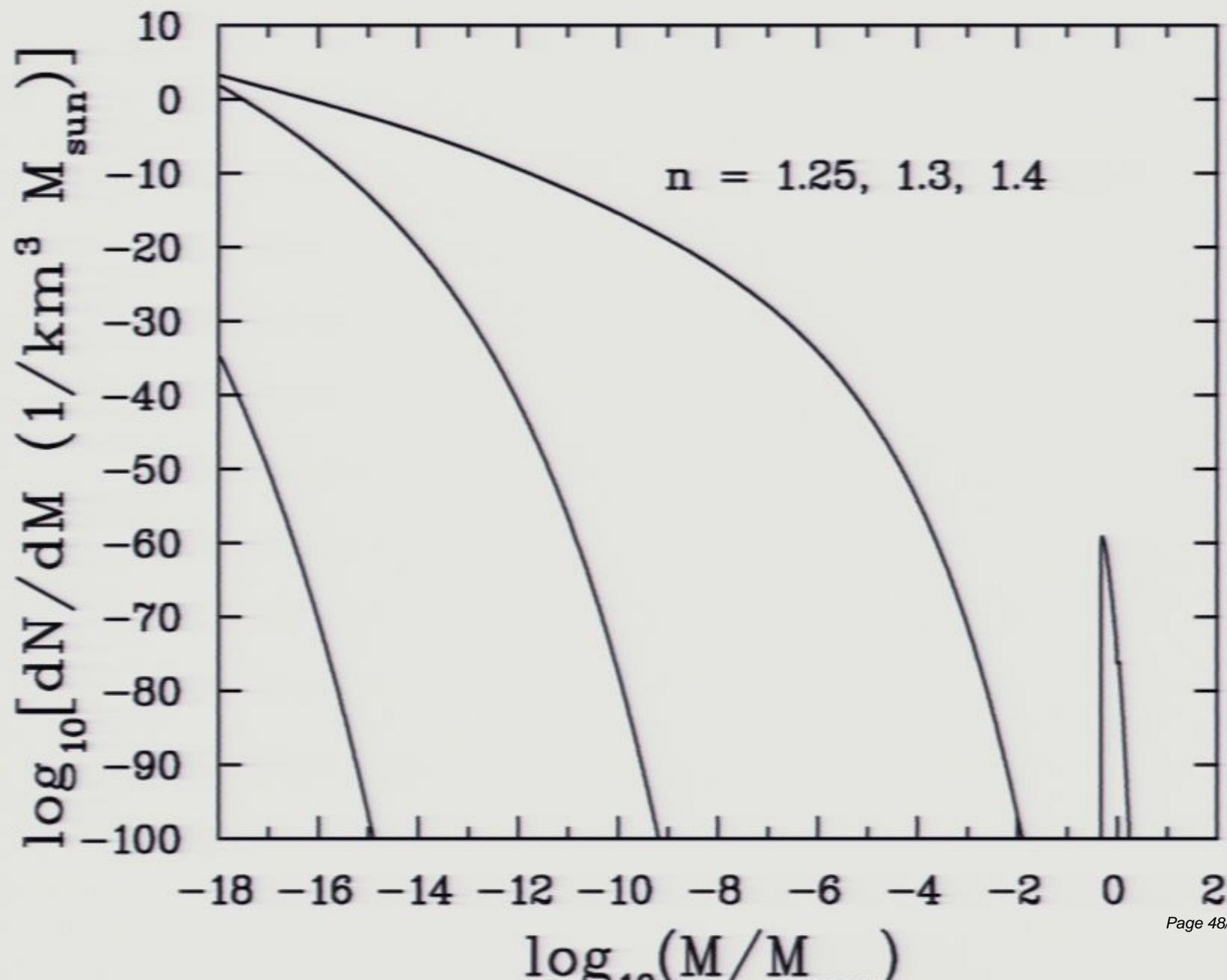
## Rapid Crossover



## Second Order



## Rapid Crossover



# PBH Abundance

| $\Omega_{\text{PBH}}$  | n=1.4                | n=1.3                | n=1.25                | n=1.2             | n=1.1             | n=1.0                | n=0.95               |
|------------------------|----------------------|----------------------|-----------------------|-------------------|-------------------|----------------------|----------------------|
| <b>Bag Model</b>       | $7.8 \times 10^{14}$ | $7.8 \times 10^{11}$ | $4.4 \times 10^7$     | $4.5 \times 10^7$ | $4.6 \times 10^7$ | $4.7 \times 10^7$    | $4.7 \times 10^7$    |
| <b>First Order</b>     | $1.3 \times 10^{15}$ | $1.3 \times 10^{12}$ | $2.3 \times 10^7$     | $2.3 \times 10^7$ | $2.3 \times 10^7$ | $2.3 \times 10^7$    | $2.2 \times 10^7$    |
| <b>2nd Order</b>       | $1.8 \times 10^{15}$ | $1.9 \times 10^{12}$ | $9.7 \times 10^{-26}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| <b>Crossover</b>       | $1.8 \times 10^{15}$ | $1.9 \times 10^{12}$ | $9.7 \times 10^{-26}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| <b>Fixed Speed</b>     | $1.8 \times 10^{15}$ | $1.9 \times 10^{12}$ | $9.7 \times 10^{-26}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| Number/pc <sup>3</sup> | n=1.4                | n=1.3                | n=1.25                | n=1.2             | n=1.1             | n=1.0                | n=0.95               |
| <b>Bag Model</b>       | $6.4 \times 10^{24}$ | $7.5 \times 10^{22}$ | $1.5 \times 10^8$     | $4.0 \times 10^8$ | $2.6 \times 10^9$ | $1.7 \times 10^{10}$ | $4.1 \times 10^{10}$ |
| <b>First Order</b>     | $1.0 \times 10^{25}$ | $1.2 \times 10^{23}$ | $3.1 \times 10^7$     | $8.1 \times 10^7$ | $5.4 \times 10^8$ | $3.6 \times 10^9$    | $9.3 \times 10^9$    |
| <b>2nd Order</b>       | $1.5 \times 10^{25}$ | $1.8 \times 10^{23}$ | $1.3 \times 10^{-14}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
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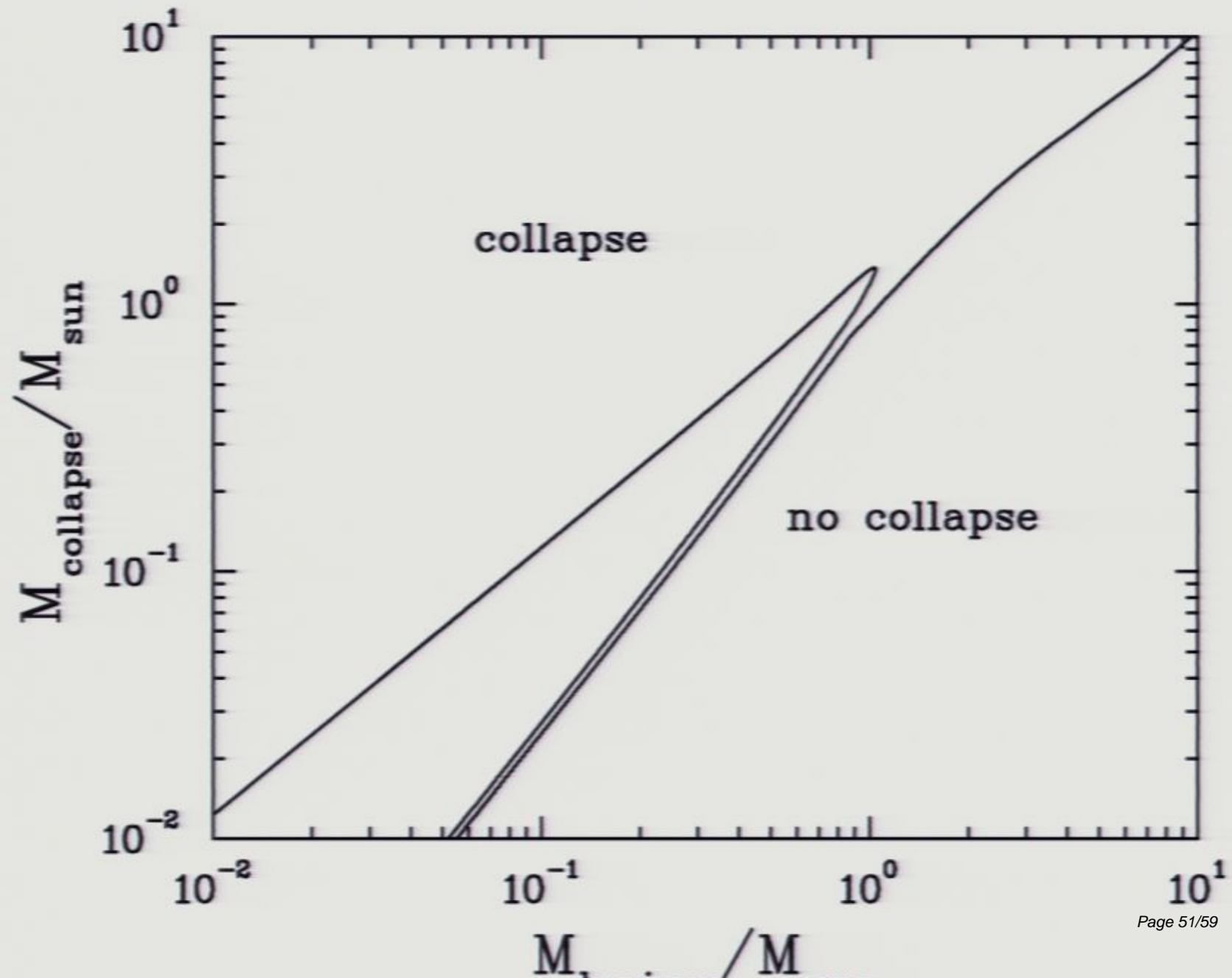
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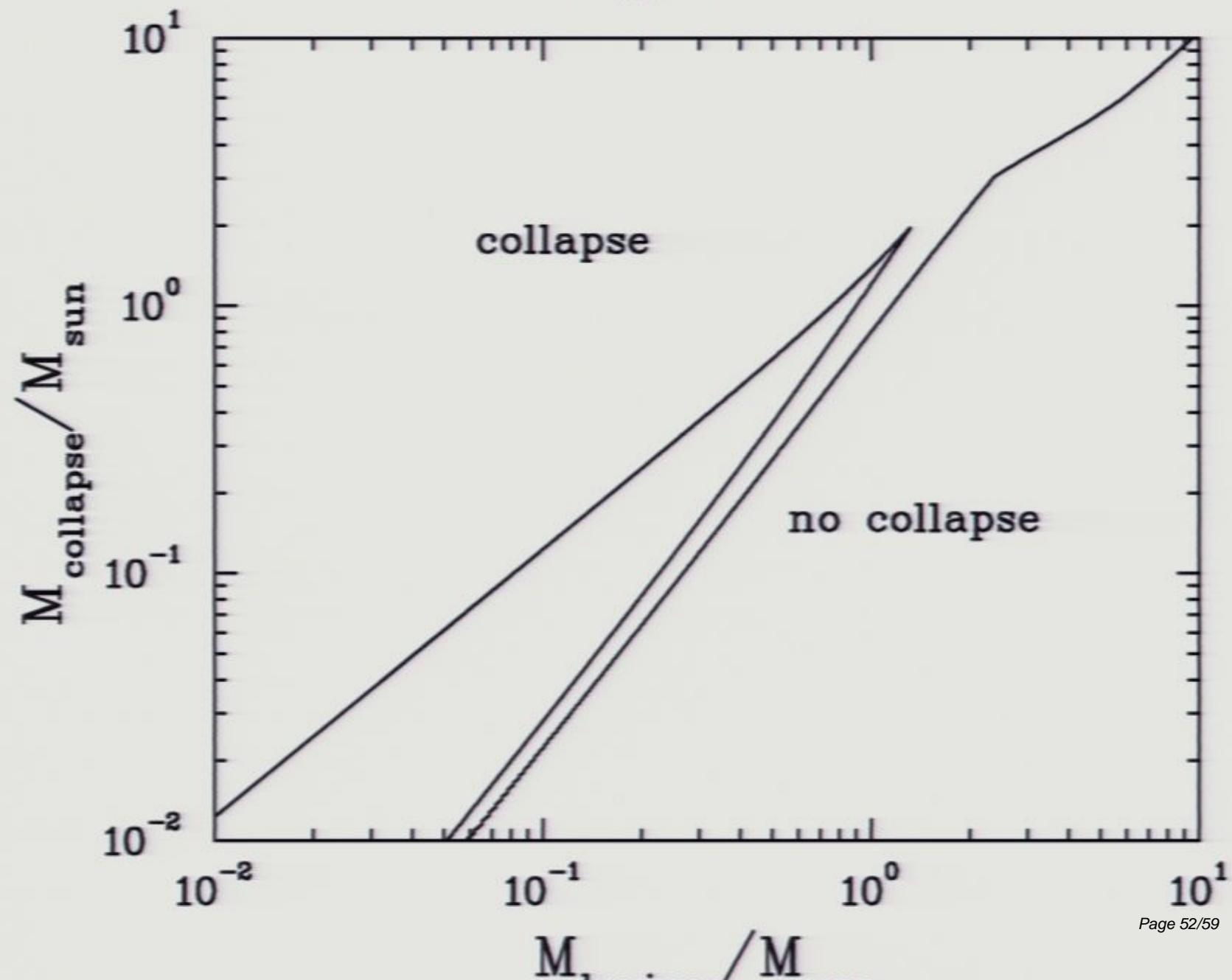

↑  
Suppression factor

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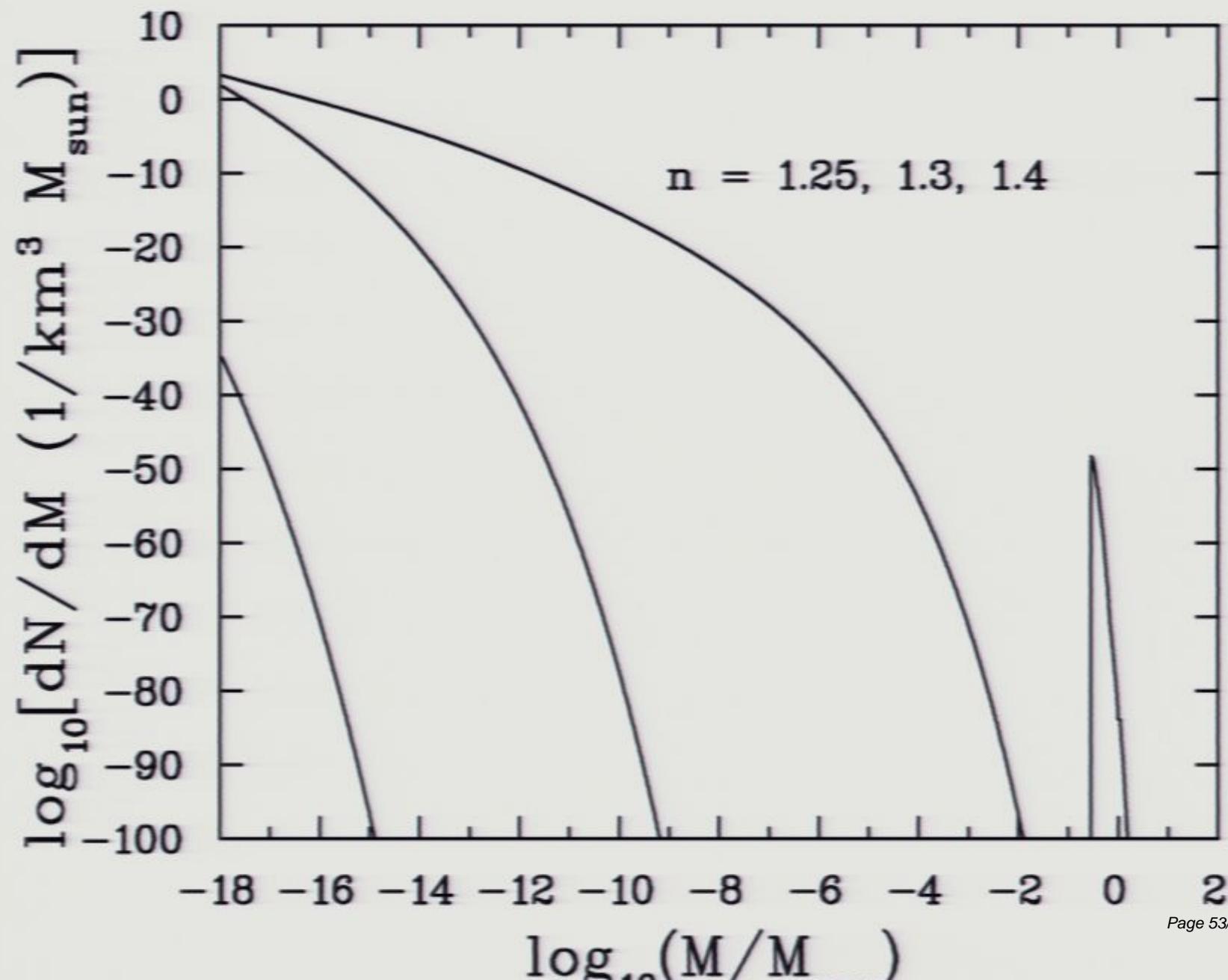
## First Order



## Bag Model



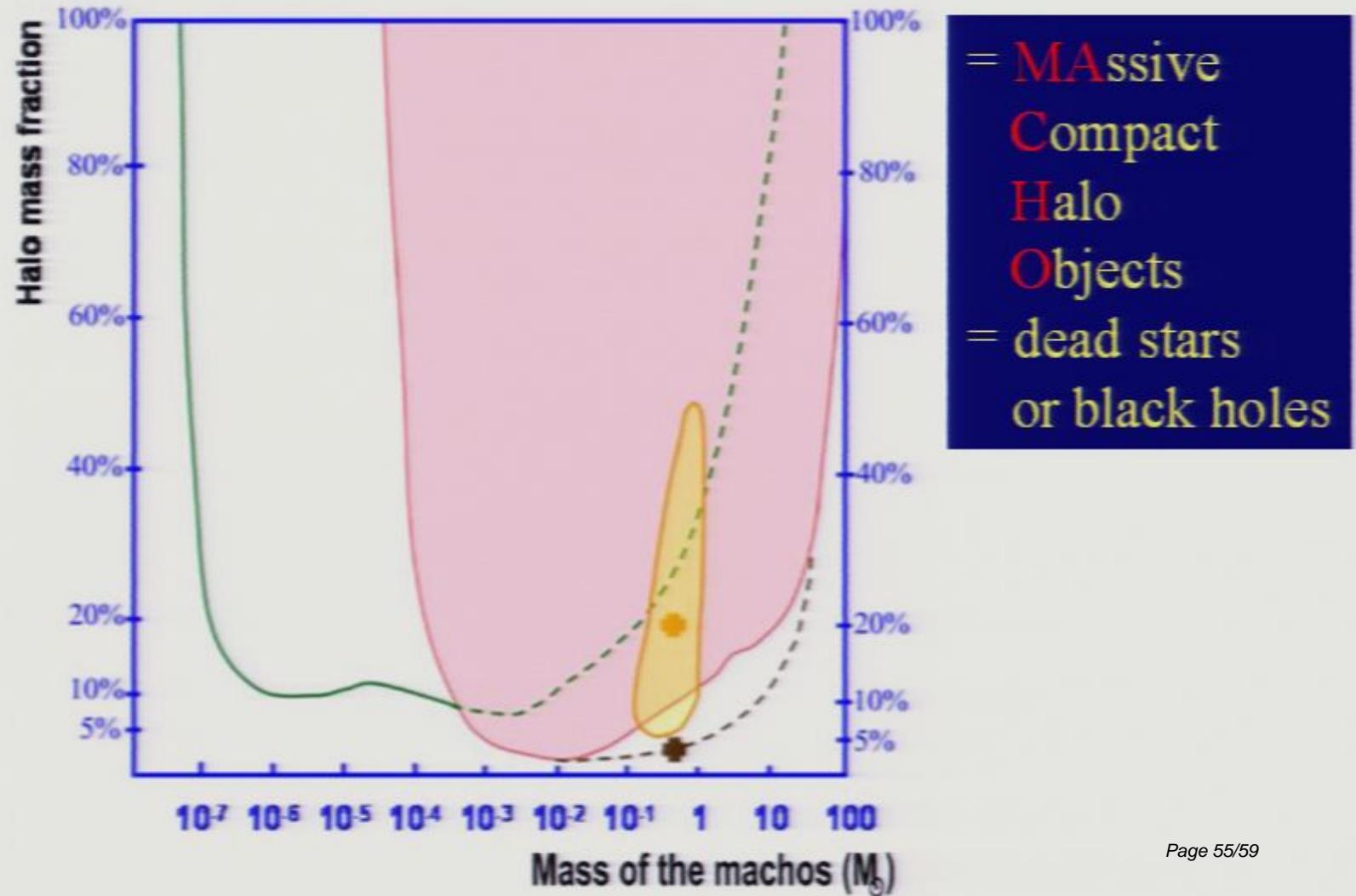
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| Number/pc <sup>3</sup> | n=1.4                | n=1.3                | n=1.25                | n=1.2             | n=1.1             | n=1.0                | n=0.95               |
| <b>Bag Model</b>       | $6.4 \times 10^{24}$ | $7.5 \times 10^{22}$ | $1.5 \times 10^8$     | $4.0 \times 10^8$ | $2.6 \times 10^9$ | $1.7 \times 10^{10}$ | $4.1 \times 10^{10}$ |
| <b>First Order</b>     | $1.0 \times 10^{25}$ | $1.2 \times 10^{23}$ | $3.1 \times 10^7$     | $8.1 \times 10^7$ | $5.4 \times 10^8$ | $3.6 \times 10^9$    | $9.3 \times 10^9$    |
| <b>2nd Order</b>       | $1.5 \times 10^{25}$ | $1.8 \times 10^{23}$ | $1.3 \times 10^{-14}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| <b>Crossover</b>       | $1.5 \times 10^{25}$ | $1.8 \times 10^{23}$ | $1.3 \times 10^{-14}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| <b>Fixed Speed</b>     | $1.5 \times 10^{25}$ | $1.8 \times 10^{23}$ | $1.3 \times 10^{-14}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |

# Our Halo is not made of Machos?



# Conclusion

- Black holes would have been produced over a wide mass range when the universe passed through the QCD phase transition/rapid crossover due to softening of the equation of state.
- Observation of the mass distribution and abundance would provide information on the QCD equation of state if the spectrum of primordial fluctuations is known.
- Observation of the mass distribution and abundance would provide information on the spectrum of primordial fluctuations if the QCD equation of state is known.
- A first order QCD phase transition would seem to have overclosed the universe; is it therefore ruled out?

**Ask me about the  
Electroweak transition!**

# PBH Abundance

| $\Omega_{\text{PBH}}$  | n=1.4                | n=1.3                | n=1.25                | n=1.2             | n=1.1             | n=1.0                | n=0.95               |
|------------------------|----------------------|----------------------|-----------------------|-------------------|-------------------|----------------------|----------------------|
| <b>Bag Model</b>       | $7.8 \times 10^{14}$ | $7.8 \times 10^{11}$ | $4.4 \times 10^7$     | $4.5 \times 10^7$ | $4.6 \times 10^7$ | $4.7 \times 10^7$    | $4.7 \times 10^7$    |
| <b>First Order</b>     | $1.3 \times 10^{15}$ | $1.3 \times 10^{12}$ | $2.3 \times 10^7$     | $2.3 \times 10^7$ | $2.3 \times 10^7$ | $2.3 \times 10^7$    | $2.2 \times 10^7$    |
| <b>2nd Order</b>       | $1.8 \times 10^{15}$ | $1.9 \times 10^{12}$ | $9.7 \times 10^{-26}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| <b>Crossover</b>       | $1.8 \times 10^{15}$ | $1.9 \times 10^{12}$ | $9.7 \times 10^{-26}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| <b>Fixed Speed</b>     | $1.8 \times 10^{15}$ | $1.9 \times 10^{12}$ | $9.7 \times 10^{-26}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| Number/pc <sup>3</sup> | n=1.4                | n=1.3                | n=1.25                | n=1.2             | n=1.1             | n=1.0                | n=0.95               |
| <b>Bag Model</b>       | $6.4 \times 10^{24}$ | $7.5 \times 10^{22}$ | $1.5 \times 10^8$     | $4.0 \times 10^8$ | $2.6 \times 10^9$ | $1.7 \times 10^{10}$ | $4.1 \times 10^{10}$ |
| <b>First Order</b>     | $1.0 \times 10^{25}$ | $1.2 \times 10^{23}$ | $3.1 \times 10^7$     | $8.1 \times 10^7$ | $5.4 \times 10^8$ | $3.6 \times 10^9$    | $9.3 \times 10^9$    |
| <b>2nd Order</b>       | $1.5 \times 10^{25}$ | $1.8 \times 10^{23}$ | $1.3 \times 10^{-14}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| <b>Crossover</b>       | $1.5 \times 10^{25}$ | $1.8 \times 10^{23}$ | $1.3 \times 10^{-14}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |
| <b>Fixed Speed</b>     | $1.5 \times 10^{25}$ | $1.8 \times 10^{23}$ | $1.3 \times 10^{-14}$ | ~ 0               | ~ 0               | ~ 0                  | ~ 0                  |

# Softening of the Equation of State

Reduction in the speed of sound

$$v_s^2 = \frac{dP}{d\rho} = \frac{dP/dT}{d\rho/dT}$$

$$= \frac{s}{d\rho/dT}$$

