

Title: RHIC Serves the Perfect Liquid? - Fact or Fiction? Success and Puzzles From 5 Years of RHIC Experiments

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Abstract:

**'RHIC Serves the Perfect Fluid' – fact or fiction?
Successes and puzzles from 5 years of RHIC Experiments***



Ulrich Heinz

Department of Physics
The Ohio State University
191 West Woodruff Avenue
Columbus, OH 43210

presented at
Exotic States of Hot and Dense Matter and their Dual Description
Perimeter Institute, May 22-25, 2007

Collaborators:

Asis Chaudhuri, Magdalena Djordjevic, Evan Frodermann, Amy Hummel,
Pasi Huovinen, Peter Kolb, Anthony Kuhlman, Zi-Wei Lin, Mike Lisa,
Dénes Molnár, Huichao Song, Sergey Voloshin

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Successes and puzzles from 5 years of RHIC Experiments***



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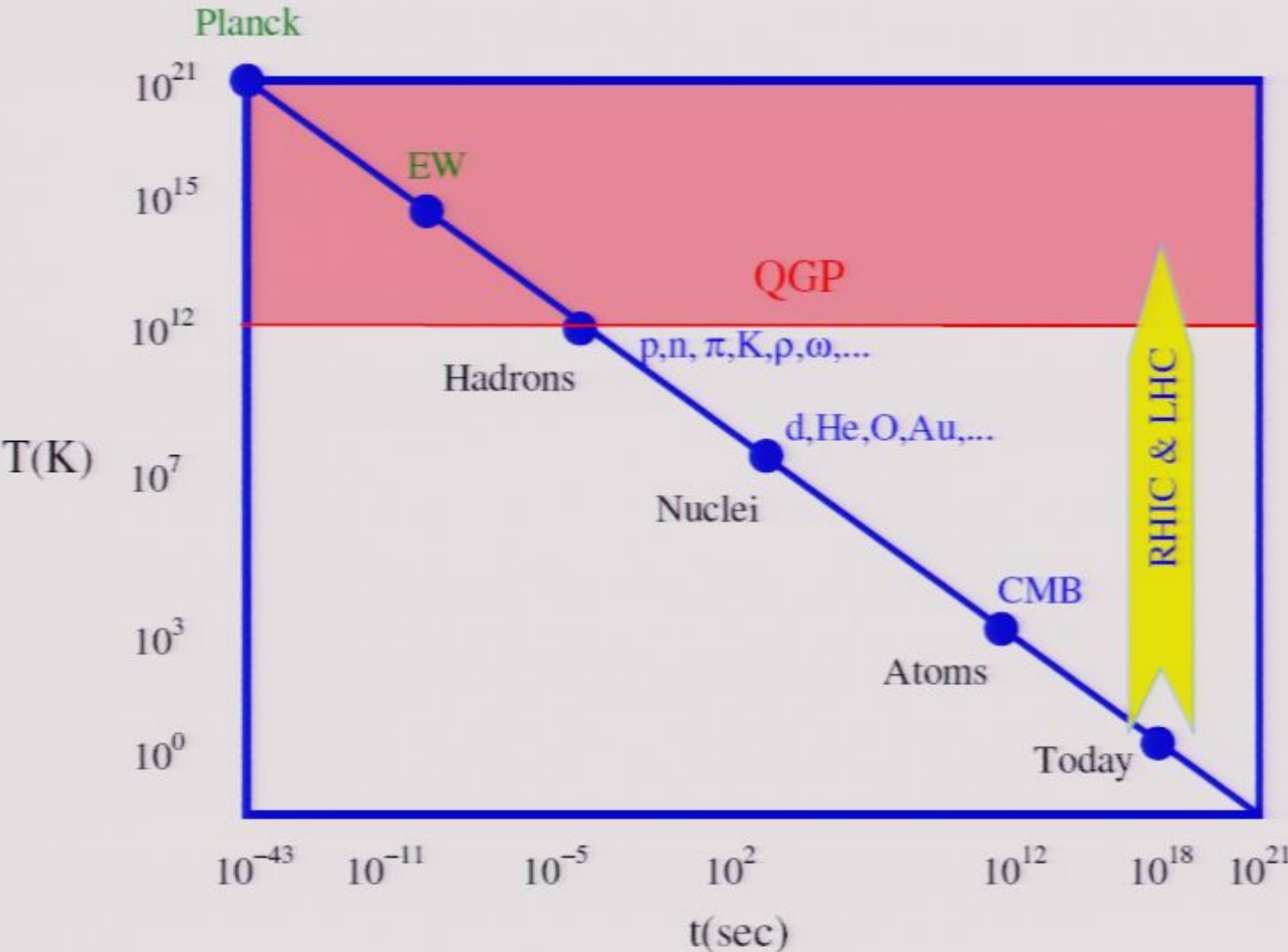
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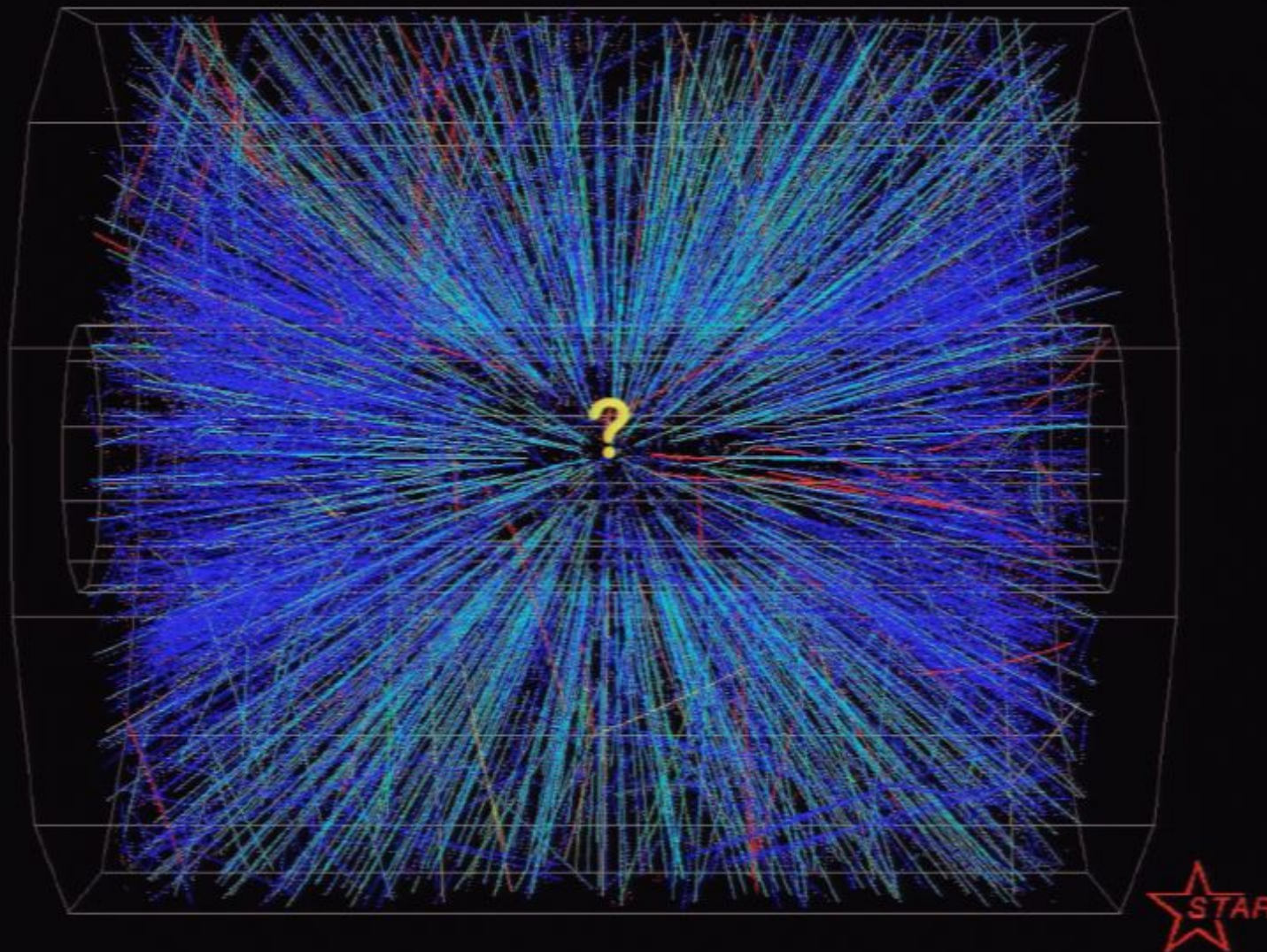
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Recreating the Early Universe in the laboratory . . .

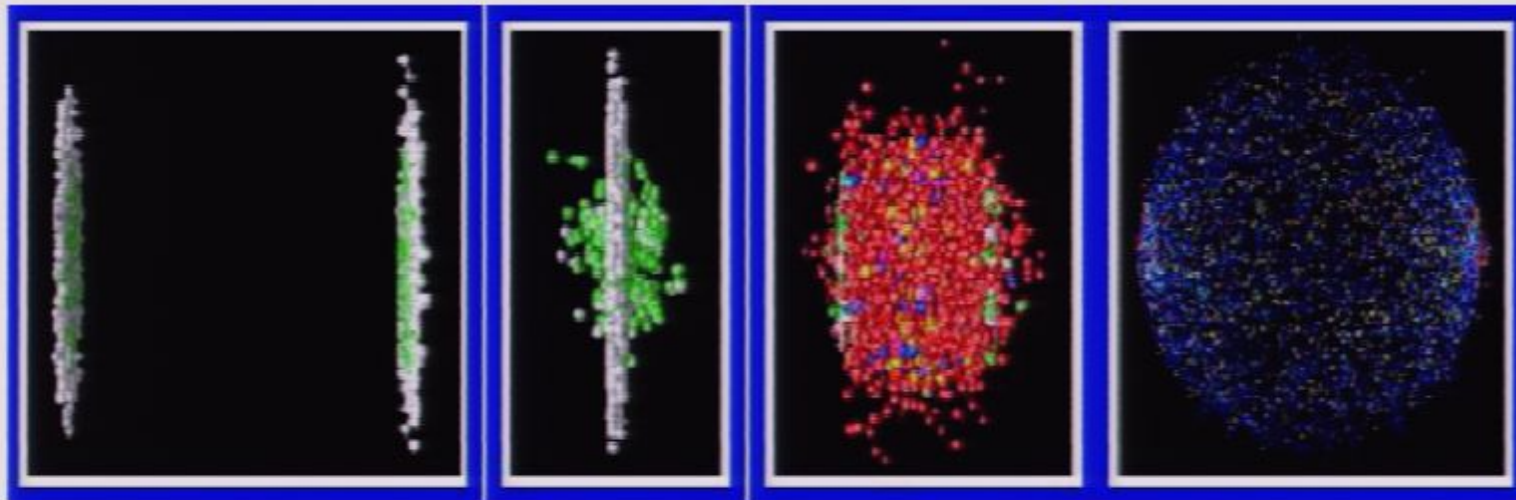


... with Little Bangs:

(100 AGeV) Au \longrightarrow \longleftarrow (100 AGeV) Au

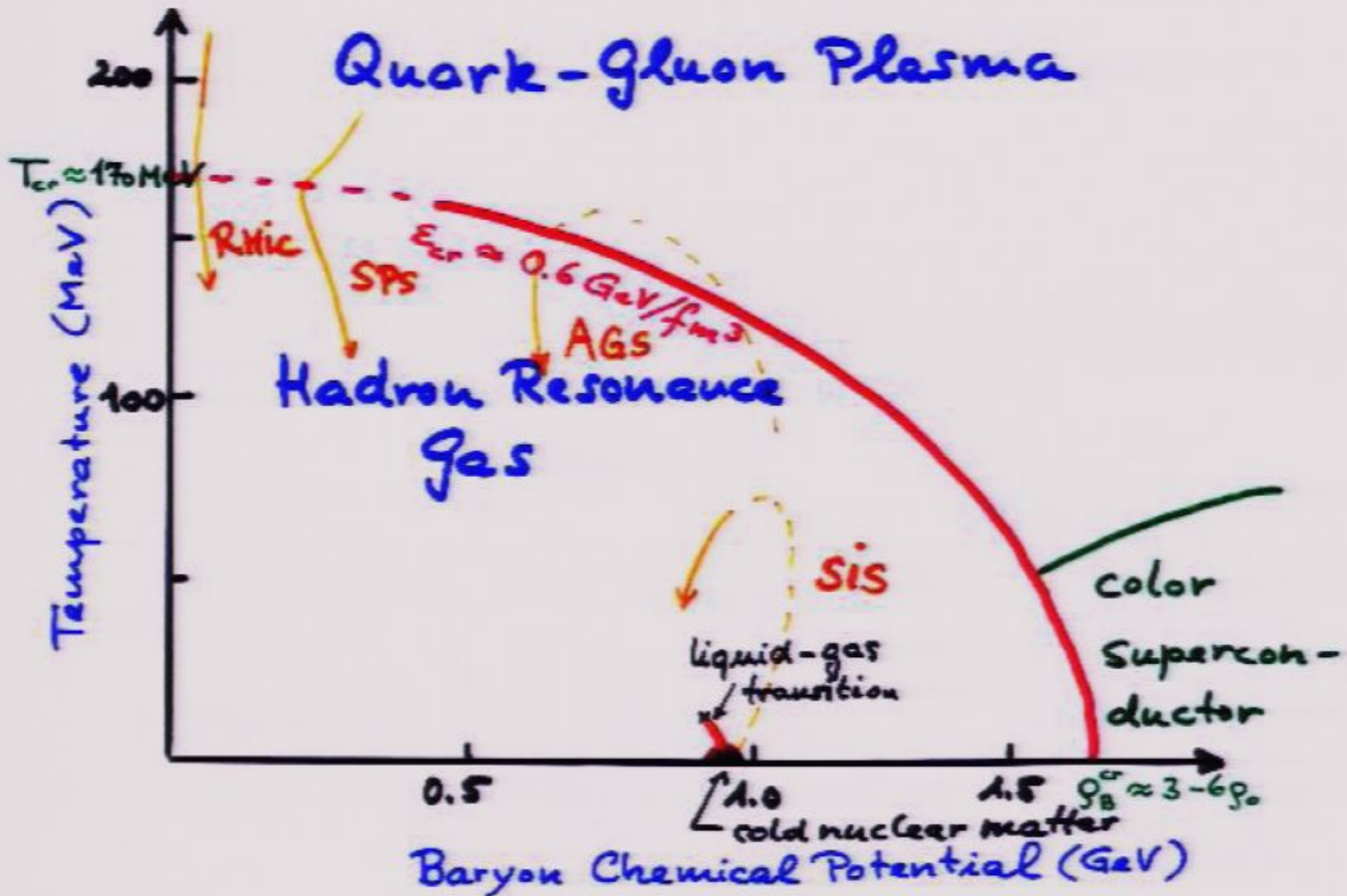


Stages of the “Little Bang”:



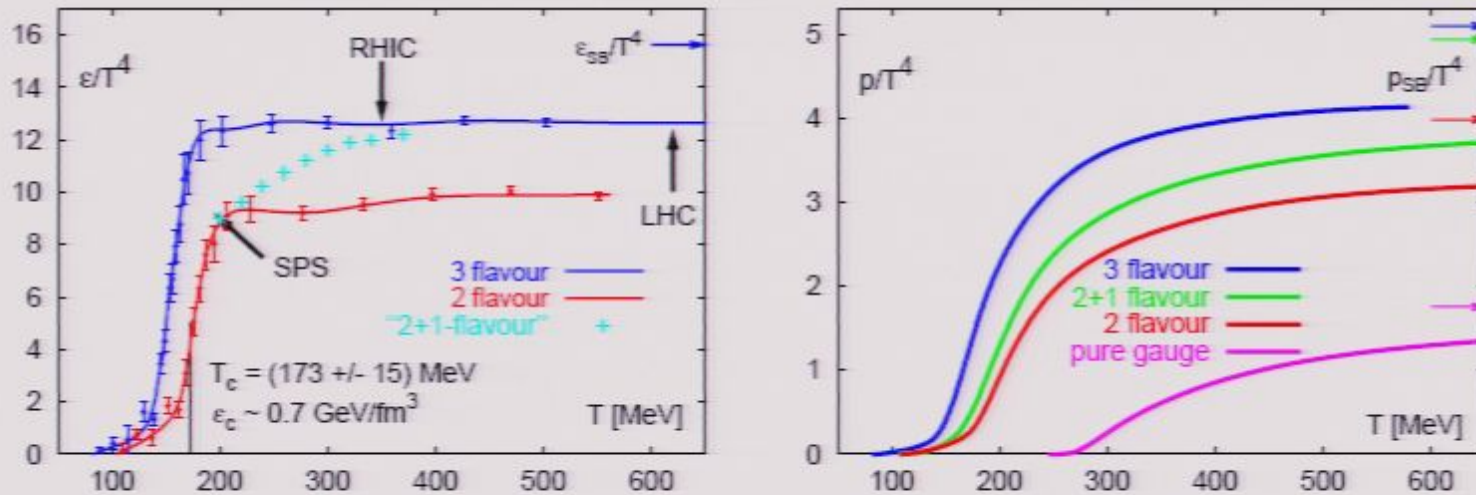
1. Two Lorentz-contracted disks (Au or Pb nuclei) approach each other
2. Hard collisions produce high- p_T particles
3. Soft collisions thermalize the quark-gluon plasma; the fluid expands hydrodynamically
4. The QGP hadronizes; the hadron gas continues to expand until freeze-out; hadrons reach the detector.

The QCD Phase Diagram and Heavy-Ion Collisions



The QCD equation of state (EOS) at zero baryon density

F. Karsch and E. Laermann, hep-lat/0305025, in "Quark-Gluon Plasma 3"



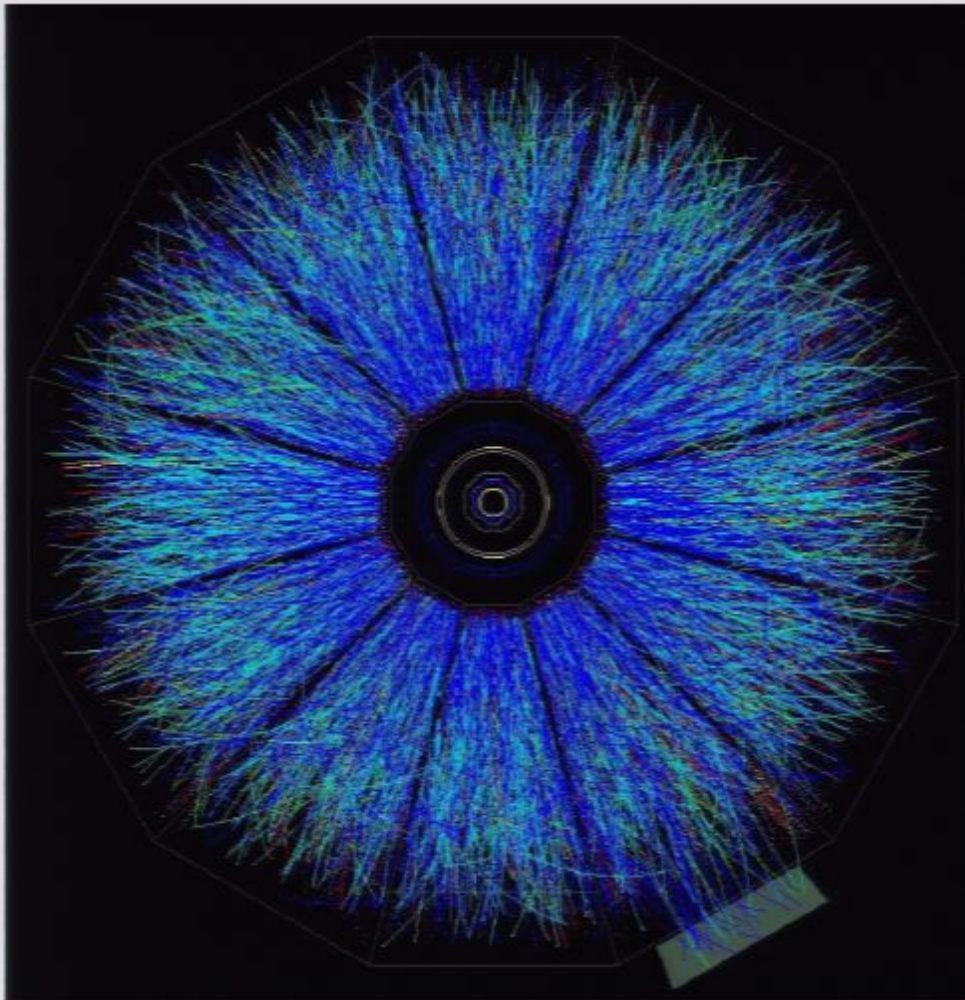
- Critical temperature $T_{\text{cr}} = 173 \pm 15 \text{ MeV}$ ($\approx 100\,000 \times T_{\text{center of sun}}$)
- Critical energy density $\epsilon_{\text{cr}} \approx 0.7 \text{ GeV/fm}^3$
- $\epsilon \approx 0.8 \epsilon_{\text{SB}}$ for $T \gtrsim 1.3 T_{\text{cr}}$, $\epsilon \approx 3p$ for $T \gtrsim 2 T_{\text{cr}}$

\Rightarrow Weakly coupled QGP? NO!

Questions driving the Heavy-Ion Program:

- What is the **equation of state** of QCD matter?
- How did hadrons first form, and what is the **origin of the mass** of visible matter in the universe?
- What are the **properties of matter at the highest energy densities**?
 - **degrees of freedom?**
 - **viscosity?**
 - **color and heat conductivity?**
 - **transport of conserved charges (susceptibilities)?**
- Which microscopic QCD mechanisms control **non-equilibrium dynamics** and **thermalization** of dense QCD matter?
 - **parton energy loss?**
 - **plasma instabilities?**
 - **color turbulence and dynamical chaos?**
- How does the medium affect **hadronization** of hard partons, and how does it react to their energy loss?

How to extract physics from Little Bangs?



- hadron yield ratios (chemistry)
- hadron spectra (thermal radiation)
- collective flow (radial flow, flow anisotropies)
- hard direct probes (jets, charm, direct electromagnetic radiation)
- 2-particle correlations (HBT interferometry with femtometer spatial and yoctosecond temporal resolution)

Overview:

0. Goals of RHIC Physics (already done)

The four key observations (so far) at RHIC:

1. Collective flow and (im)perfect fluidity: The Little Bang
2. Primordial Hadrosynthesis at $T_c = 170$ MeV:
statistical hadronization of the QGP
3. Quark Coalescence: first signs of color deconfinement
4. JET: thermalization of fast partons
and Jet Emission Tomography of the QGP
5. Hydrodynamics of viscous relativistic fluids

6. Summary

Collective Flow – the “Bang”

Collective flow tests the Equation of State:

Hydrodynamic equations, ideal fluid limit:

(\dot{f} = time derivative in local rest frame, $\partial \cdot u$ = local expansion rate)

$$\dot{n}_B = -n_B (\partial \cdot u)$$

$$\dot{\epsilon} = -(\epsilon + p) (\partial \cdot u)$$

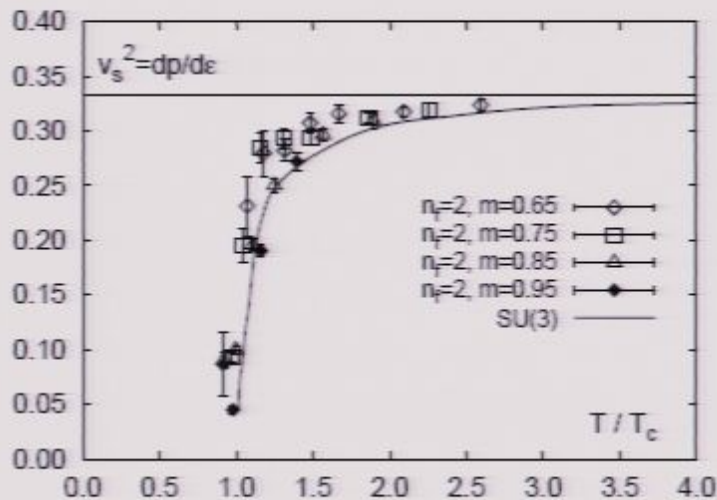
$$\dot{u}^\mu = \frac{\nabla^\mu p}{\epsilon + p}$$

- flow driven by pressure gradients $\nabla^\mu p$

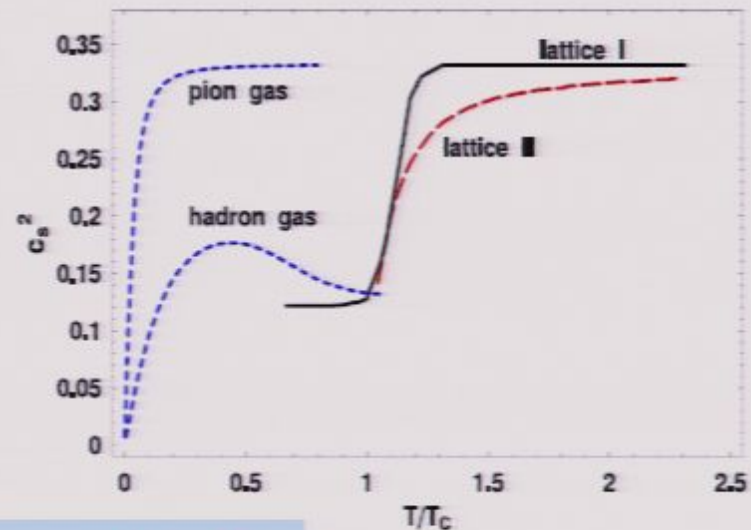
- acceleration $\frac{\nabla^\mu p}{\epsilon + p}$ closely related to

$$\text{speed of sound } c_s^2 = \frac{\partial p}{\partial \epsilon}$$

Karsch+Laermann, hep-lat/0305025



Chojnacki et al., nucl-th/0410036



“Softest point” near $T = T_{cr}$.

Hydrodynamics – the natural tool to study flow:

Relativistic Hydrodynamics:

Conservation of energy, momentum and baryon number

$$\partial_\mu T^{\mu\nu} = 0$$

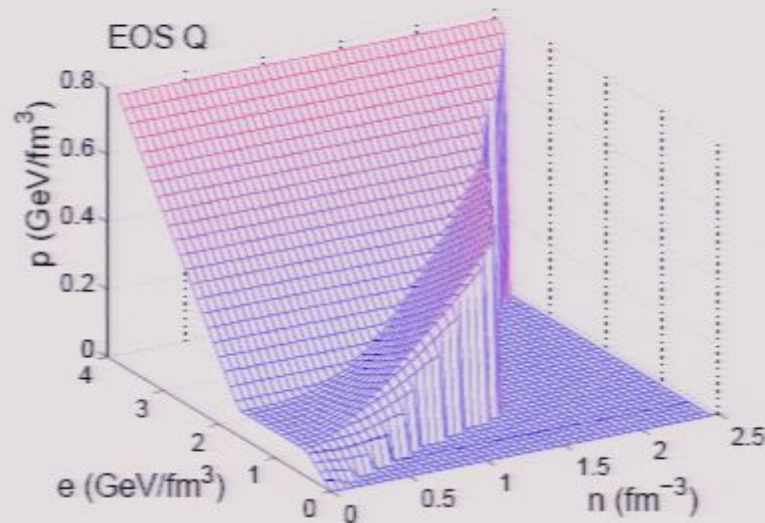
$$\partial_\mu j^\mu = 0$$

with energy momentum tensor $T^{\mu\nu}(x) = (e(x) + p(x)) u^\mu(x) u^\nu(x) - g^{\mu\nu} p(x)$
and baryon current $j^\mu(x) = n(x) u^\mu(x)$

Equation of State:

- EOS I: ultrarelativistic ideal gas, $p = \frac{1}{3} e$
- EOS H: hadron resonance gas, $p \sim 0.15 e$
- EOS Q: Maxwell construction between EOS I and EOS H

critical temperature $T_{crit} = 0.164$ GeV
 \Rightarrow bag constant $B^{1/4} = 0.23$ GeV
 latent heat $\Delta e = 1.15$ GeV/fm³



Implement exact longitudinal boost invariance for simplicity ($Y \approx 0$ only)

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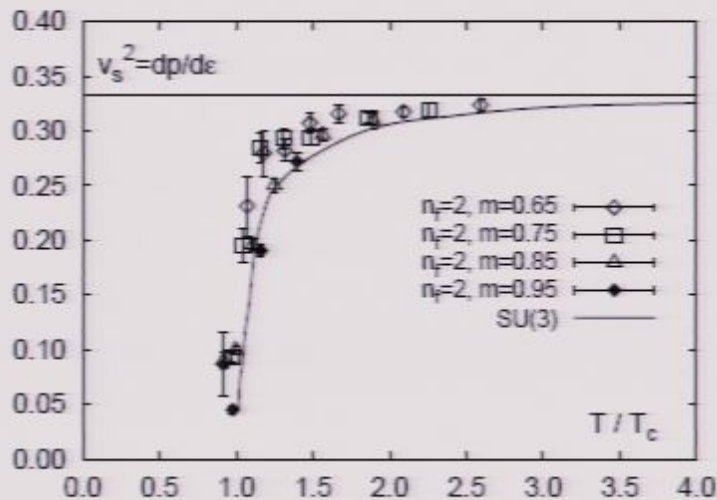
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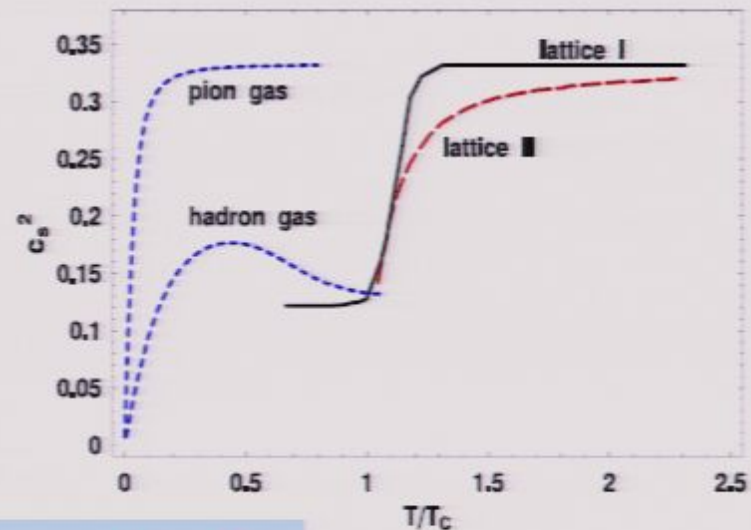
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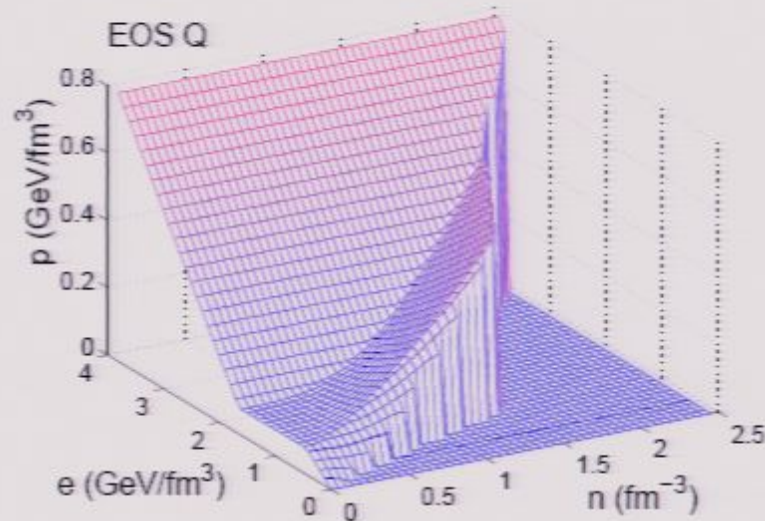
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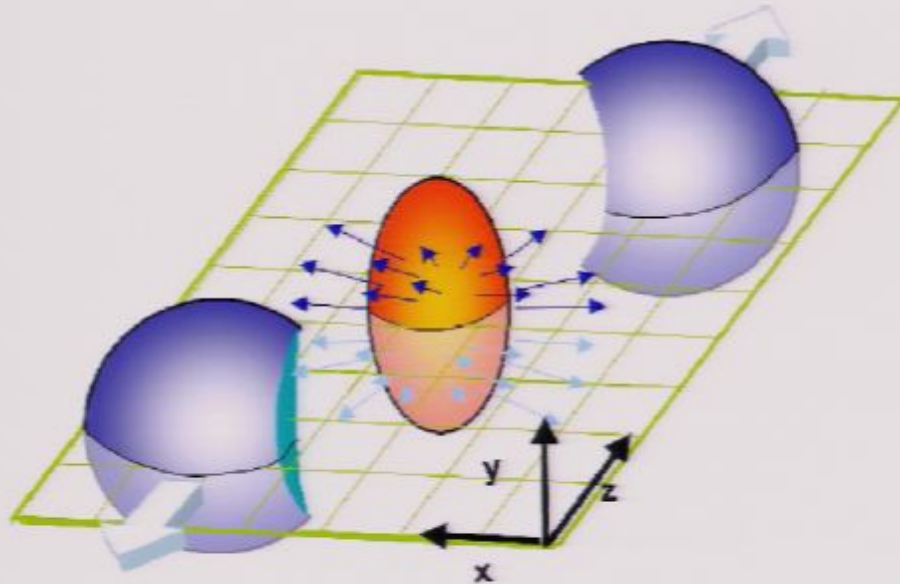
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Implement exact longitudinal boost invariance for simplicity ($Y \approx 0$ only)

Elliptic Flow



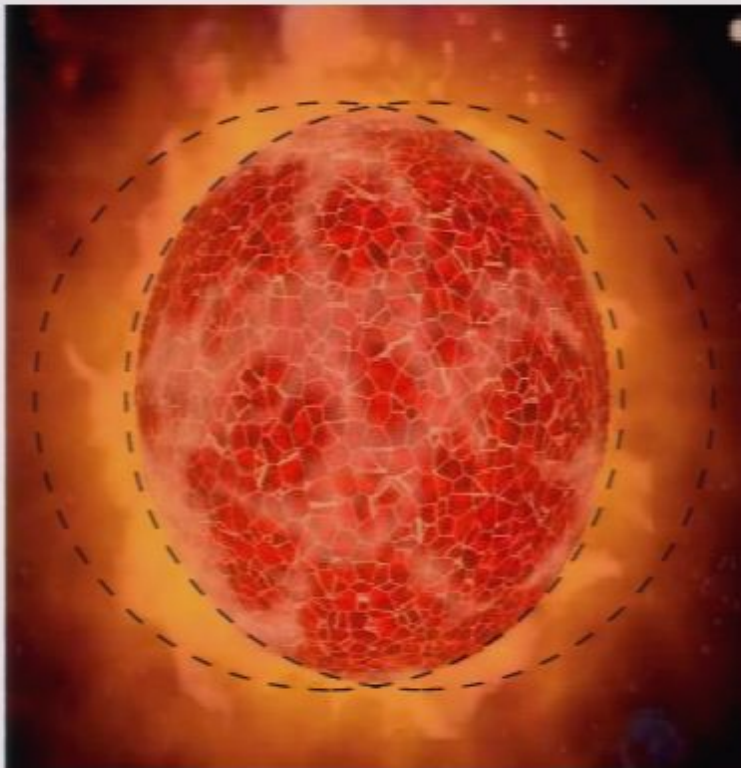
In non-central collisions the overlap region is elliptically deformed
 \Rightarrow anisotropic pressure gradients
 \Rightarrow anisotropic (“elliptic”) collective flow.

Elliptic flow

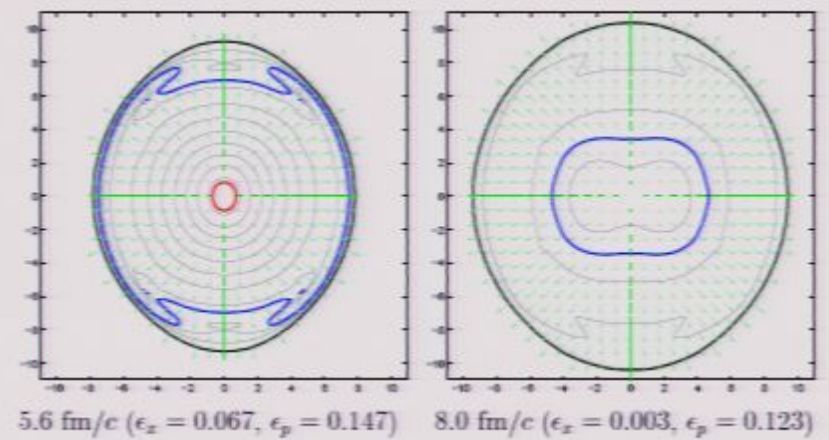
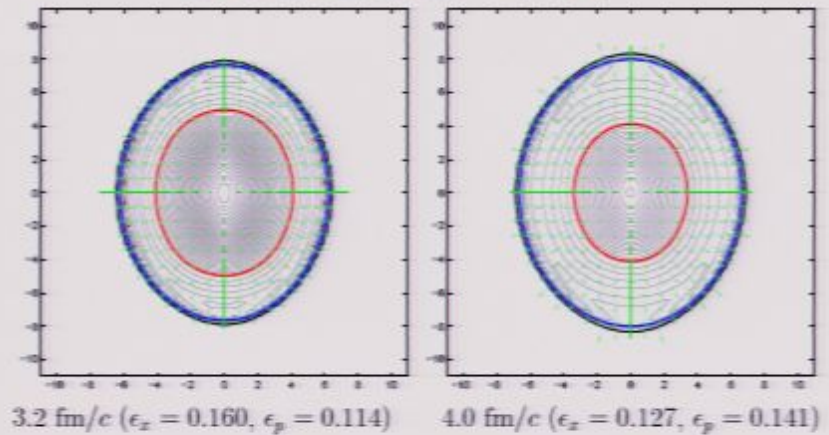
- peaks at midrapidity
- driven by spatial deformation of reaction zone at thermalization
- magnitude of signal probes degree and time of thermalization
- “self-quenching”: it shuts itself off as dynamics reduces deformation (H. Sorge)
- sensitive to Equation of State during first ~ 5 fm/c

$$v_2(y, p_T, b) = \langle \cos(2\phi) \rangle_{y, p_T, b} = \frac{\int d\phi \cos(2\phi) \frac{dN}{dy p_T dp_T d\phi}(b)}{\int d\phi \frac{dN}{dy p_T dp_T d\phi}(b)}$$

Radial and elliptic flow from hydrodynamics:

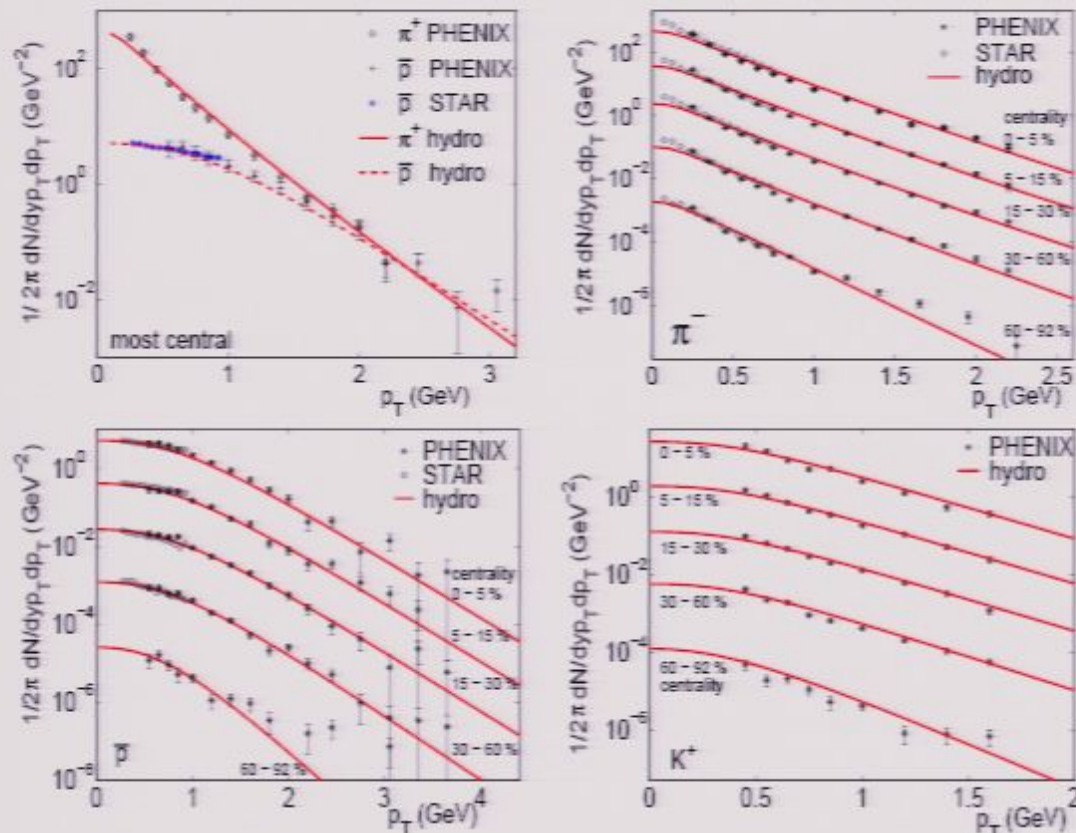


Au+Au at $b = 7$ fm



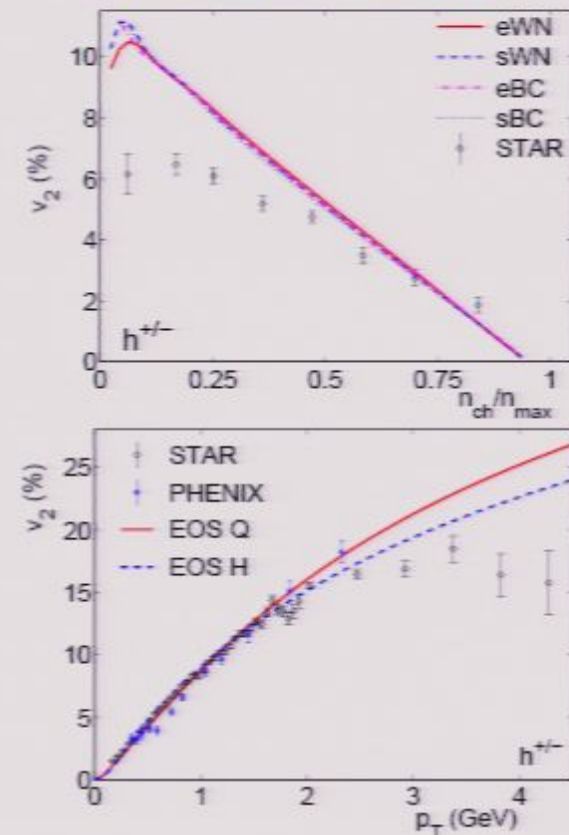
Successes of hydrodynamics at RHIC:

Single particle spectra from central and peripheral Au+Au @ 130 A GeV (STAR, PHENIX):



Model parameters fixed with π , \bar{p} spectra at $b = 0$;
all other spectra predicted (UH&P.Kolb, hep-ph/0204061).

Centrality and momentum dependence of elliptic flow v_2 (STAR, PHENIX, PHOBOS):



$$v_2 = \langle \cos(2\phi) \rangle$$

What is fitted, what is predicted?

Au+Au @ 130 A GeV:

$$\tau_{\text{eq}} = 0.6 \text{ fm}/c, \quad e_{\text{max}}(b=0) = 24.6 \text{ GeV}/\text{fm}^3, \quad \langle e \rangle(\tau=1 \text{ fm}/c) = 5.4 \text{ GeV}/\text{fm}^3$$
$$T_{\text{max}}(b=0) = 340 \text{ MeV}, \quad T_{\text{chem}} = T_{\text{had}} = 165 \text{ MeV}, \quad T_{\text{dec}} = 130 \text{ MeV}$$

All fit parameters are fixed in central ($b=0$) collisions:

- Glauber model \Rightarrow shape of initial transverse entropy and baryon density profiles $s(\mathbf{r}, \tau_{\text{eq}}), n_B(\mathbf{r}, \tau_{\text{eq}})$
 \Rightarrow free parameters $s_0(\tau_{\text{eq}}), n_0(\tau_{\text{eq}})$, soft/hard fraction
- Measured p/π ratio \Rightarrow fixes n_0/s_0
- Total charged multiplicity $dN_{\text{ch}}/dy \Rightarrow$ fixes product $\tau_{\text{eq}} \cdot s_0(\tau_{\text{eq}})$
- soft/hard fraction \Rightarrow fixed through centrality dependence of dN_{ch}/dy
- Shape of π, p spectra \Rightarrow fixes decoupling temperature T_{dec} and radial flow $\langle v_{\perp} \rangle$
- Final radial flow $\langle v_{\perp} \rangle \Rightarrow$ "fixes" τ_{eq} [upper limit] (flow needs time and pressure to develop)
- Equation of State \Rightarrow compute $e_0 = e_{\text{max}}(b=0), T_{\text{max}}(b=0)$ from s_0, n_0

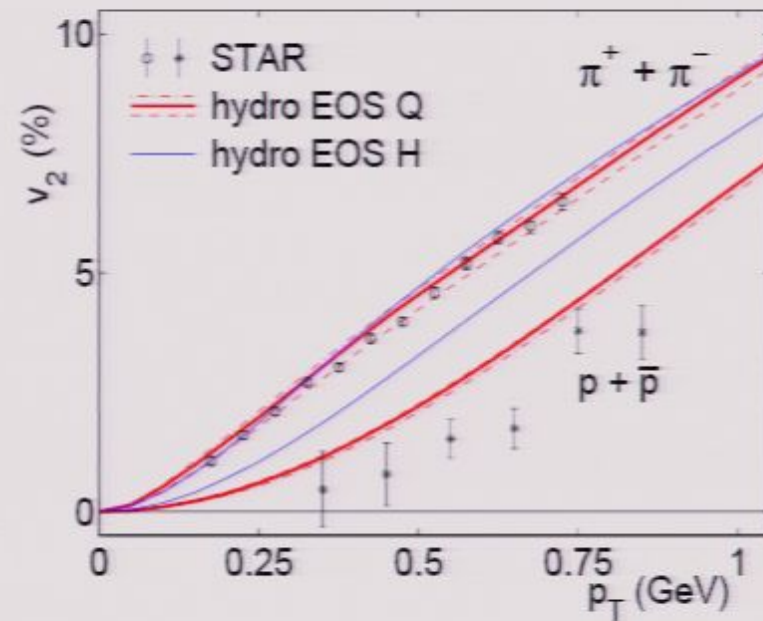
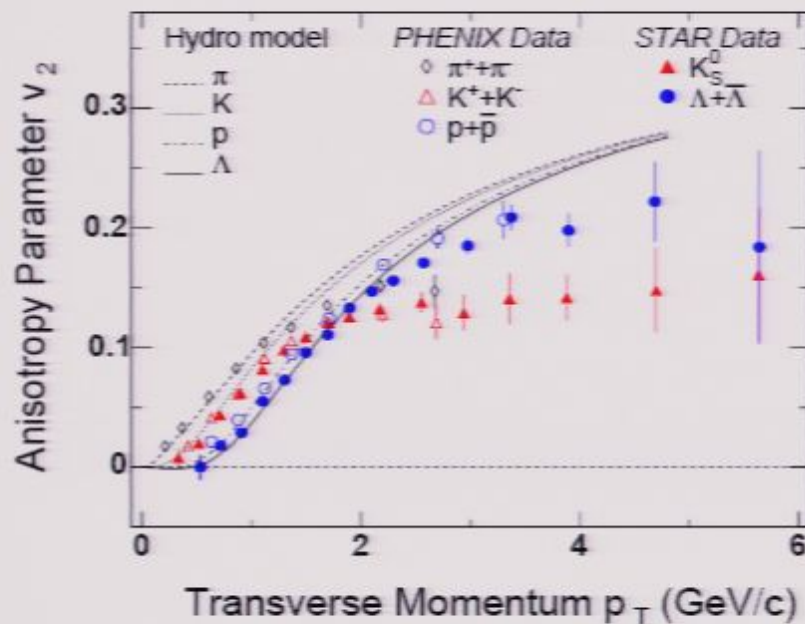
Predictions (no additional parameters!):

- All hadron spectra other than p, π in $b=0$ collisions
- All hadron spectra and elliptic flow coefficients for non-central collisions at any impact parameter

Final radial flow $\langle v_{\perp} \rangle > 0.5 c \Rightarrow$ bang!

Rest mass dependence of differential elliptic flow (the “fine structure”)

STAR Coll., PRL 87, 182301 (2001) and PRL 92, 052302 (2004); PHENIX Coll., PRL 91, 182301 (2003)



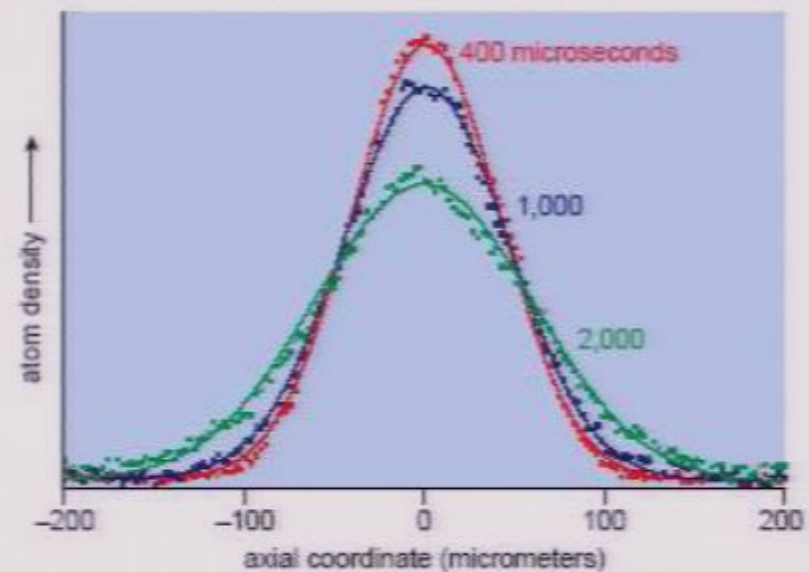
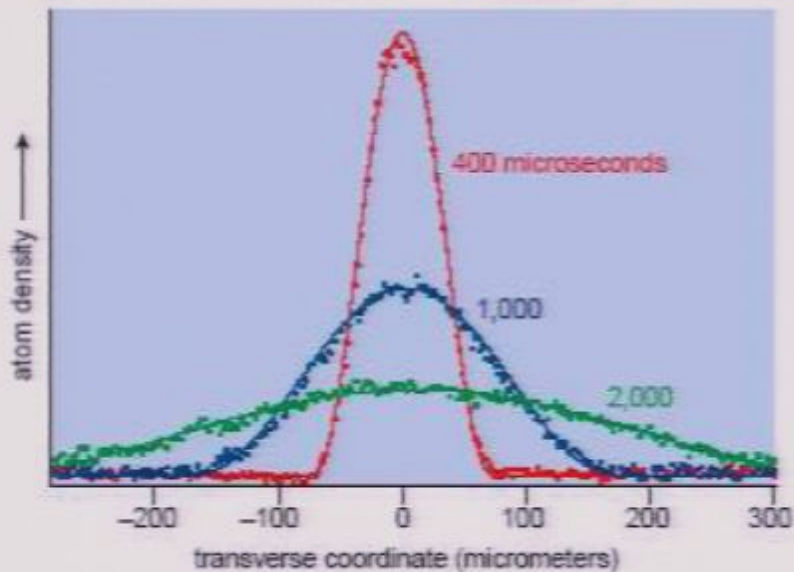
Data follow the hydrodynamically predicted rest mass dependence of $v_2(p_\perp)$ out to $p_\perp \sim 1.5$ GeV for mesons and out to $p_\perp \sim 2.3$ GeV for baryons
 \Rightarrow bulk of matter (> 99% of all particles) behaves hydrodynamically!

Note: mass-splitting of v_2 (“fine structure”) sensitive to EOS!

so hydro works –
why? what does this mean?

Elliptic collective flow of strongly coupled atoms at $T = 10^{-6}$ K:

J.E.Thomas et al., Am. Scientist 92 (2004) 238



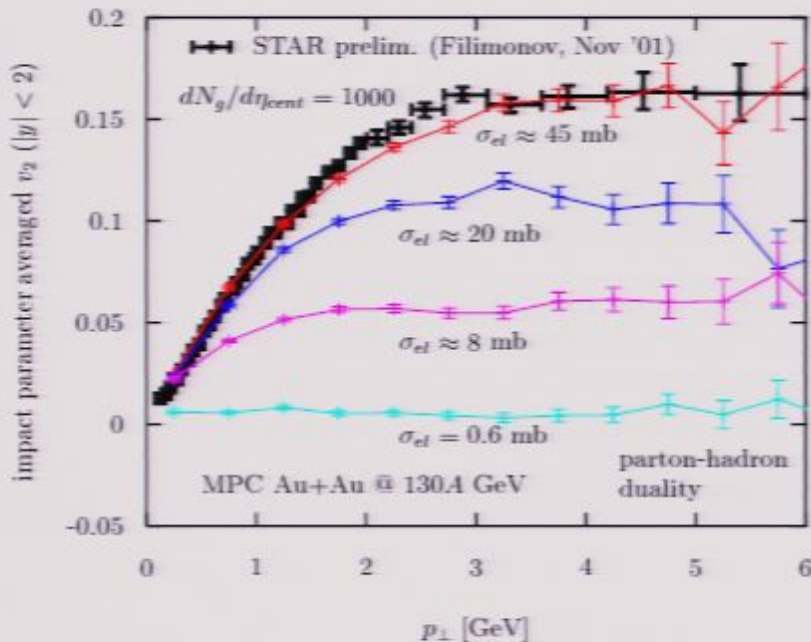
Interaction strength can be tuned (Feshbach resonance):

Strong interaction: elliptic collective flow

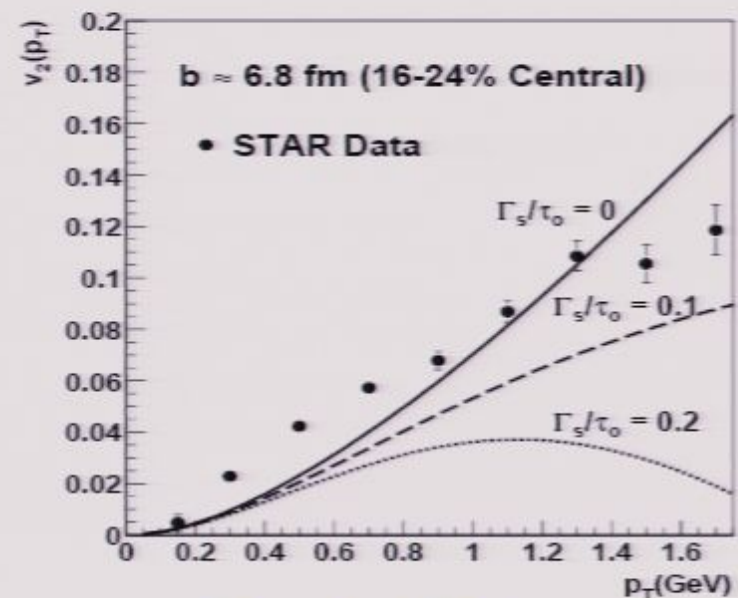
Weak interaction: ballistic expansion with aspect ratio $\rightarrow 1$

Breakdown of hydrodynamics at high p_{\perp} : upper limits for the QGP viscosity

D. Molnár and M. Gyulassy, NPA 697 (2002) 495



D. Teaney, PRC 68 (2003) 034913



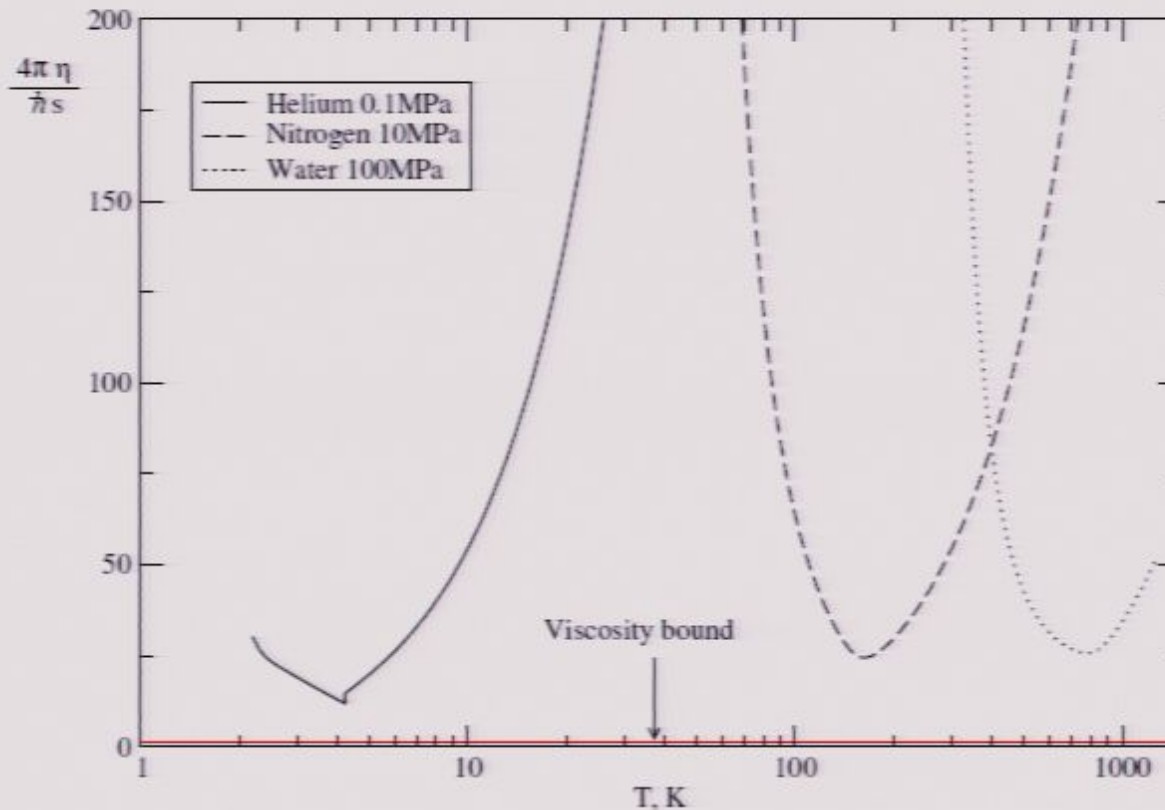
$$\Gamma_s = \frac{4}{3}\eta/(T \cdot s)$$

- For sufficiently (very) large σ_{el} , $v_2(p_{\perp})$ from covariant parton transport model MPC follows hydrodynamic curve at low p_{\perp} and reproduces observed saturation at high p_{\perp}
- Similar pattern is seen in viscous hydrodynamics: viscous corrections increase $\sim p_{\perp}^2$
- v_2 data suggest $\frac{\Gamma_s}{\tau} < 0.1$, close to **minimum viscosity** $\frac{\eta}{s} = \frac{\hbar}{4\pi}$ (Son et al. 2002)

QGP – the most ideal fluid ever observed!

AdS/CFT universal lower viscosity bound conjecture: $\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$

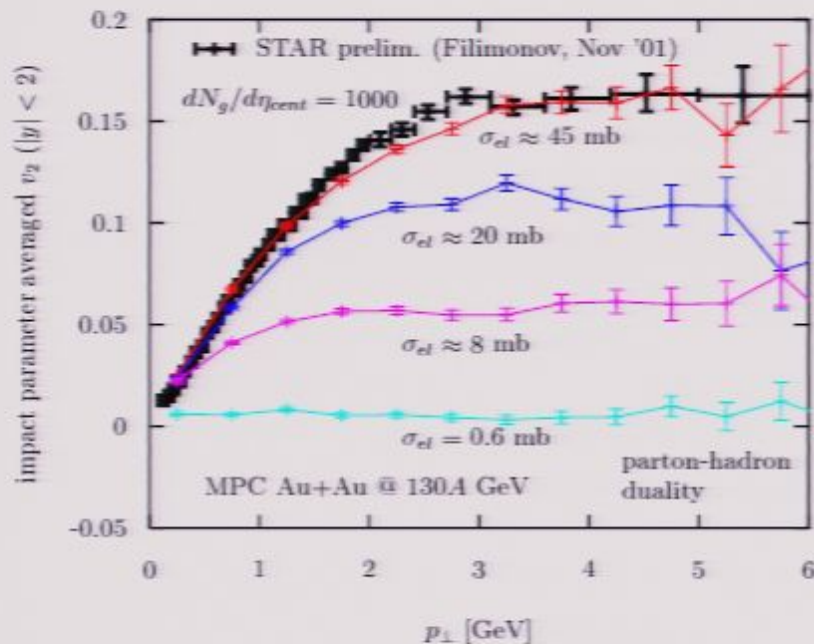
Kovtun, Son, Starinets, PRL 94 (2005) 111601



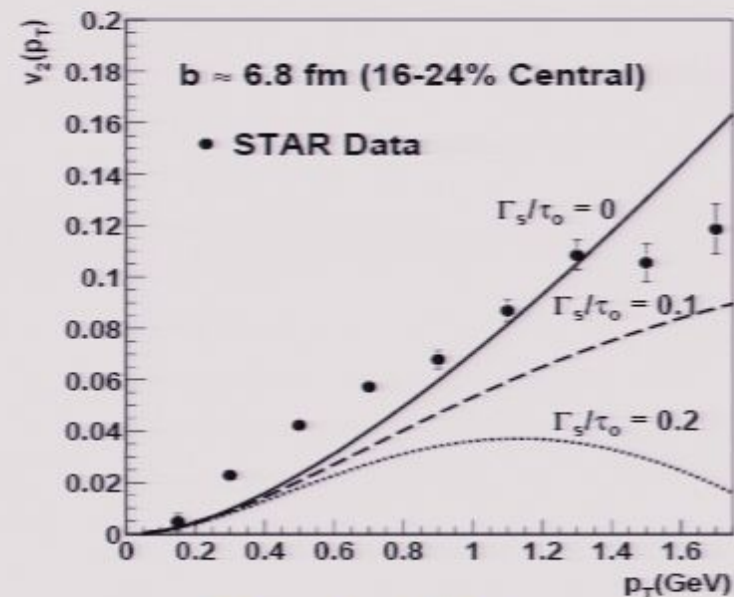
Upper limit for QGP viscosity from Teaney's estimate is close to this bound!

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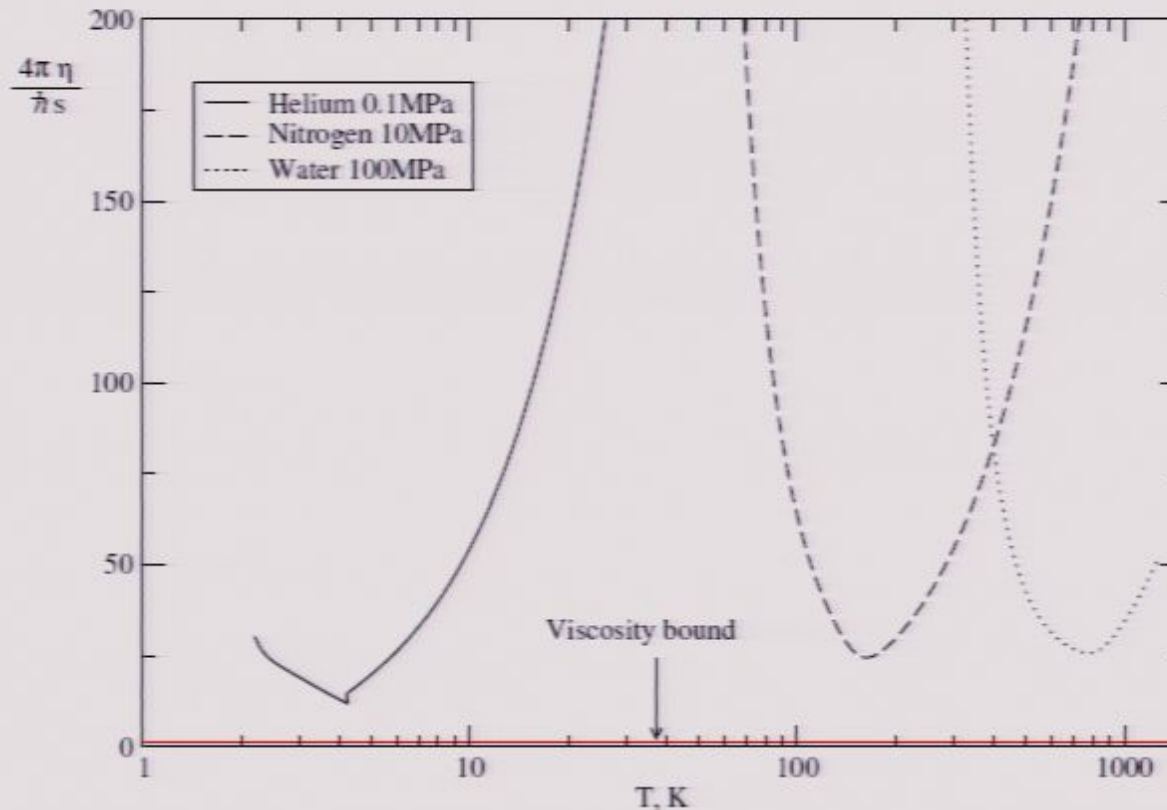
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Kovtun, Son, Starinets, PRL 94 (2005) 111601

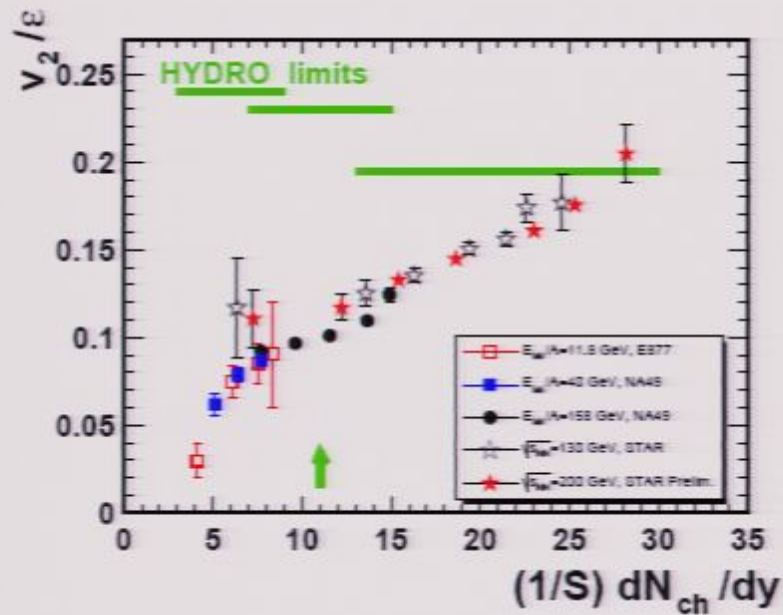


Upper limit for QGP viscosity from Teaney's estimate is close to this bound!

Limits of ideal fluid dynamics: hadronic viscosity

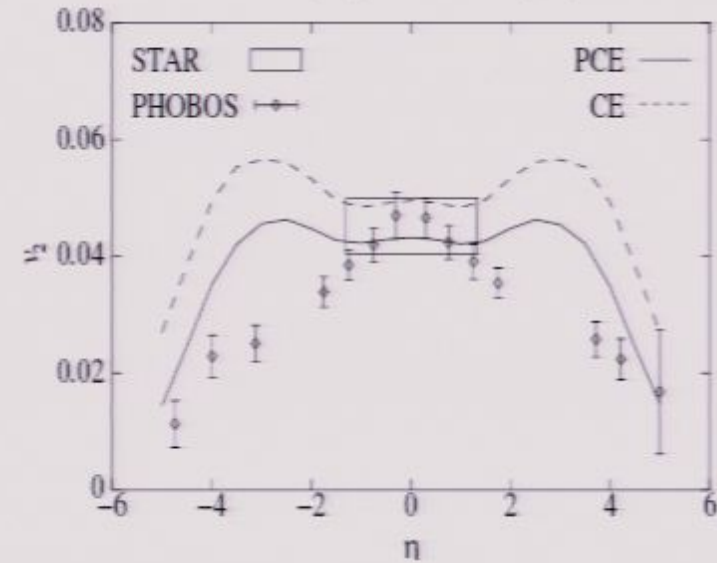
Limits of ideal fluid dynamics: smaller, less dense systems

STAR, PRC 66 ('02) 034904; NA49, PRC 68 ('03) 034903



(3+1)-d hydrodynamics:

T. Hirano, PRC 65 ('02) 011901; 66 ('02) 054905



- $\frac{v_2^{\text{measured}}}{v_2^{\text{hydro}}}$ scales with $\frac{1}{S} \frac{dN_{\text{ch}}}{dy} \propto s_{\text{init}}$
- $e_{\text{init}} > 10 \text{ GeV}/\text{fm}^3$ needed for v_2 to saturate before hadronization and exhaust ideal hydro limit!
- hydrodynamics predicts non-monotonic v_2/ϵ : between AGS and RHIC it **decreases**, due to softening of EOS by quark-hadron transition (Kolb, Sollfrank, UH, PRC 62 (2000) 054909)
- data show instead monotonous **increase** of v_2/ϵ with \sqrt{s} !?

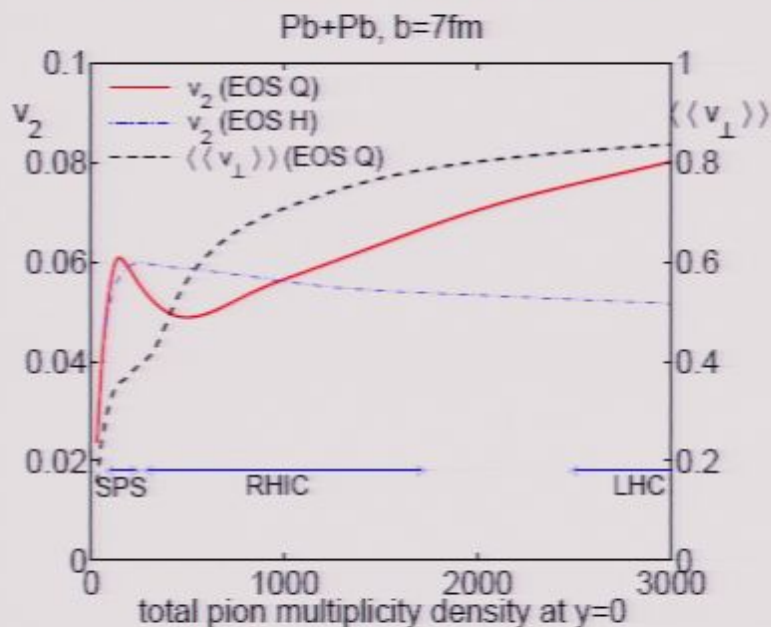
What's going on??

Breakdown of ideal hydro: the viscous hadron fluid

Excitation function of elliptic flow:

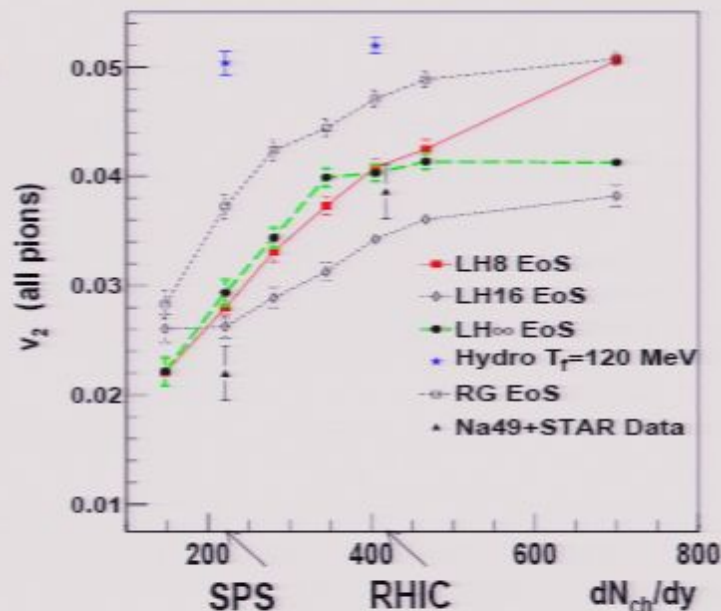
Ideal hydro

P. Kolb, J. Sollfrank, U.H., PRC 62 ('00) 054909



Hydro + RQMD

D. Teaney, J. Lauret, E. Shuryak, nucl-th/0110037

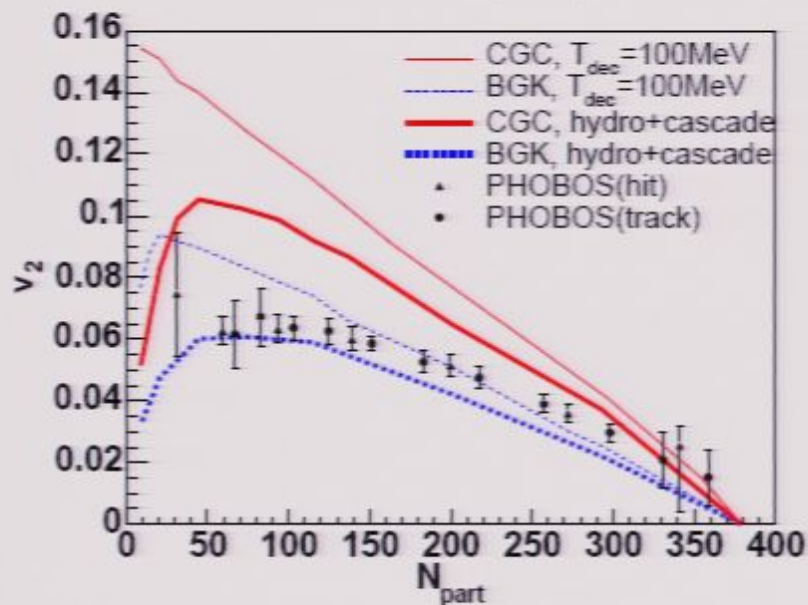


Hadron resonance gas is very viscous and does not respond strongly to spatial eccentricity \Rightarrow non-monotonic behaviour of v_2 resulting from dip in c_s^2 near phase-transition is erased!

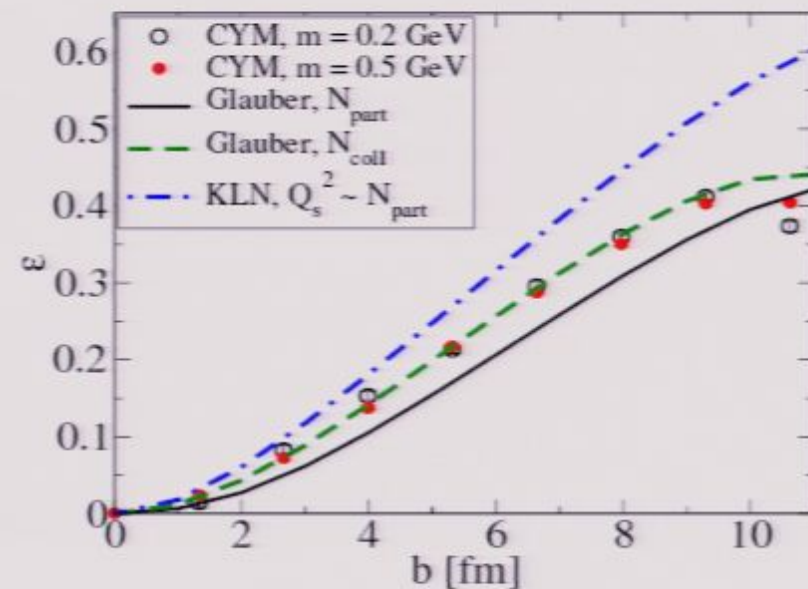
Breakdown of ideal hydro: the viscous hadronic fluid (II)

3D Hydro+Cascade Model: Ideal fluid dynamics for QGP above T_c , hadronic cascade with realistic cross sections (JAM) below T_c

Hirano et al., PLB 636 (2006) 299



Lappi & Venugopalan, PRC 74 (2006) 054905

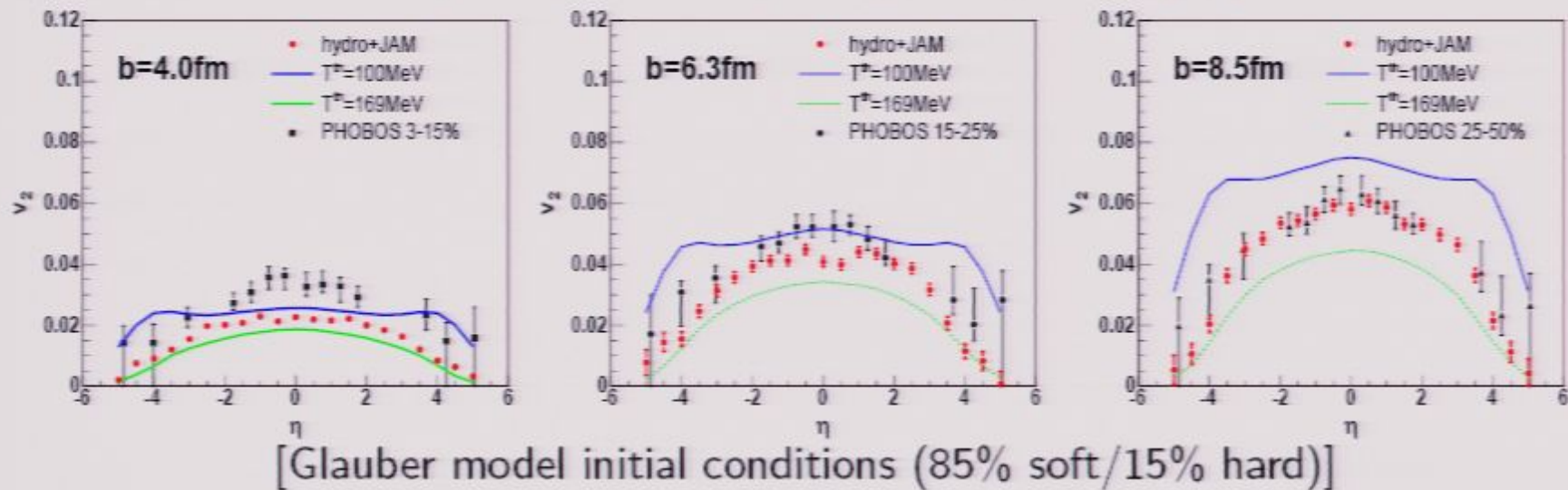


- Hadronic dissipation reduces elliptic flow buildup in peripheral collisions
- **Color Glass Condensate** (CGC-KLN) model (McLerran & Venugopalan 1994; Kharzeev, Levin, Nardi 2001) produces steeper edge of initial distribution, resulting in larger eccentricities ϵ than in Glauber model
- Ideal hydrodynamics turns larger spatial eccentricity ϵ into larger elliptic flow v_2
- For Glauber model initial conditions, hadronic dissipation fully explains the data; for CGC/KLN initial conditions hadronic dissipation not enough – need additional **QGP viscosity!**

⇒ Need better control over initial conditions!

Hadronic dissipation explains reduced elliptic flow at forward rapidity:

T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, PLB 636 (2006) 299



- Not enough elliptic flow from perfect QGP fluid – some hadronic contribution to v_2 is required
- Treating the hadronic stage as ideal fluid overpredicts v_2 in peripheral collisions and at forward rapidities
- Dissipation in hadronic cascade brings theory in line with data (except for small b – excess in data due to event-by-event geometry fluctuations (Miller & Snellings, PHOBOS))

Early evolution of elliptic flow via photons and dileptons

Photon and dilepton elliptic flow:

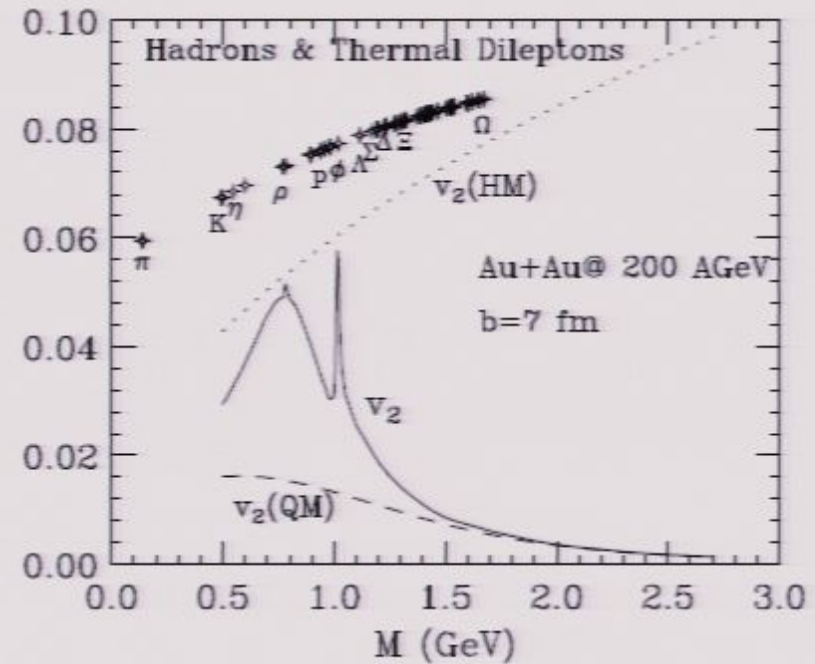
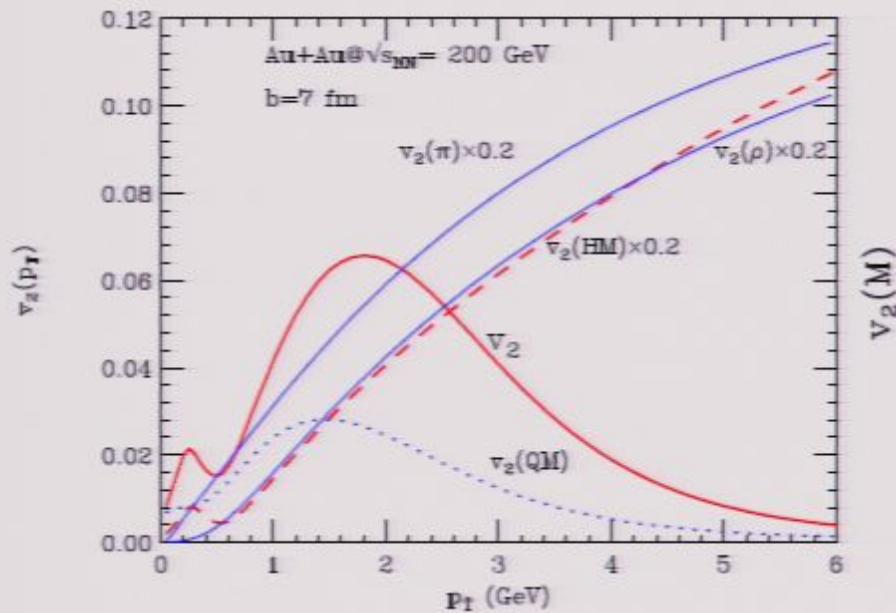
High- p_T photons and high-mass dileptons probe early fireball evolution:

R. Chatterjee, E. Frodermann, UH, D.K. Srivastava,
PRL 96 (2006) 202302

Photon elliptic flow:

R. Chatterjee, D.K. Srivastava, UH, C. Gale,
nucl-th/0702039 (PRC, in press)

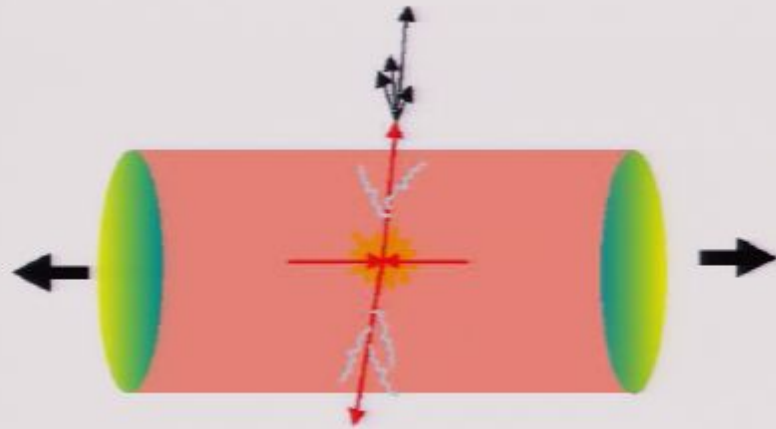
Dilepton elliptic flow:



What causes the fast
thermalization at RHIC?

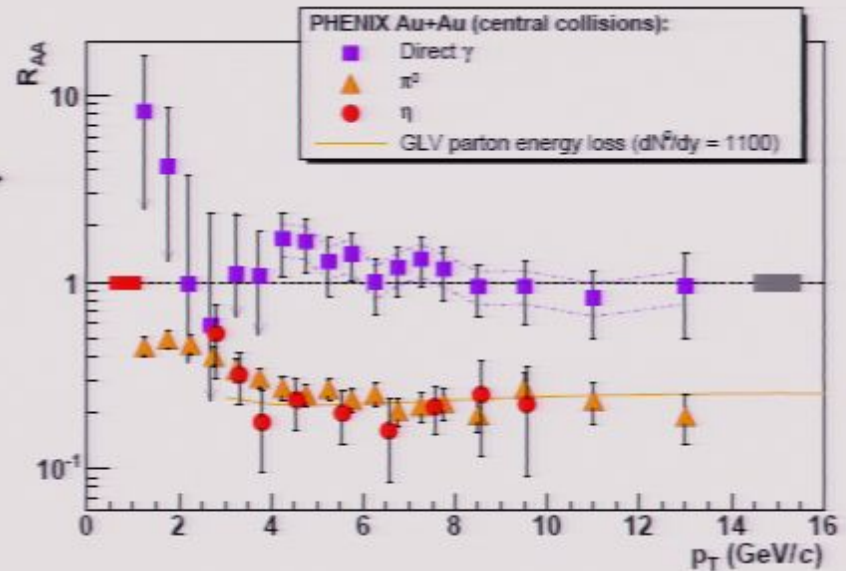
Study behaviour of non-thermal
partons!

Suppression of high p_T hadron production in Au+Au:



PHENIX Coll., PRL 96 (2006) 202301

$$R_{AA}(p_T; b) = \frac{\frac{dN_{AA}}{dp_T}(b)}{N_{\text{coll}}(b) \frac{dN_{pp}}{dp_T}}$$



$$\begin{aligned} &\Rightarrow \frac{dN_g}{dy} = 1000 \pm 200 \\ &\Rightarrow \langle e \rangle (\tau_0 = 0.2 \text{ fm}) \approx 20 \text{ GeV/fm}^3 ! \end{aligned}$$

High- p_T suppression absent in d+Au \Rightarrow suppression in Au+Au not due to nuclear wavefunction (e.g. CGC) but a **final state effect**

JET –

Jet Emission Tomography of the QGP

The “opacity problem”

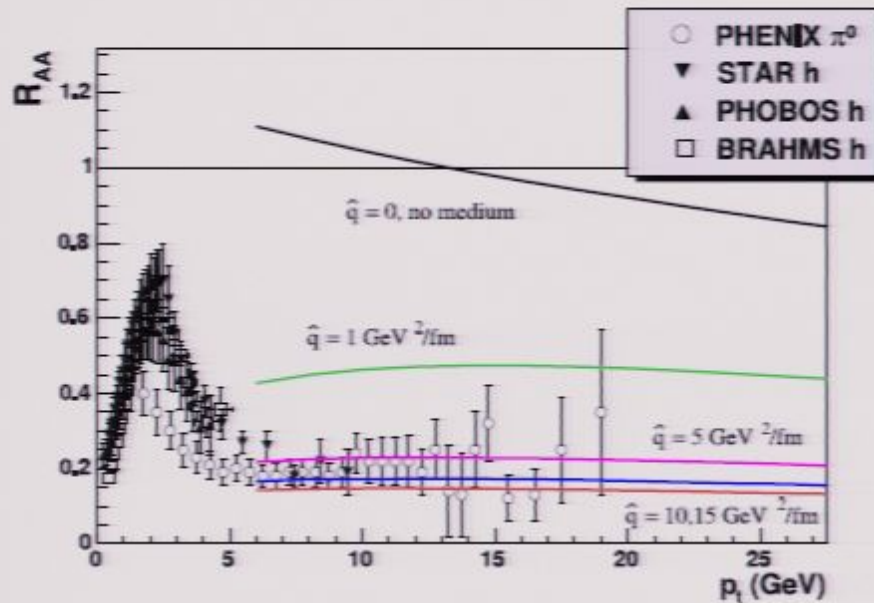
Bjorken 1982: collisional energy loss \implies “monojet production”

Gyulassy, Wang, et al. (80's): radiative energy loss dominates

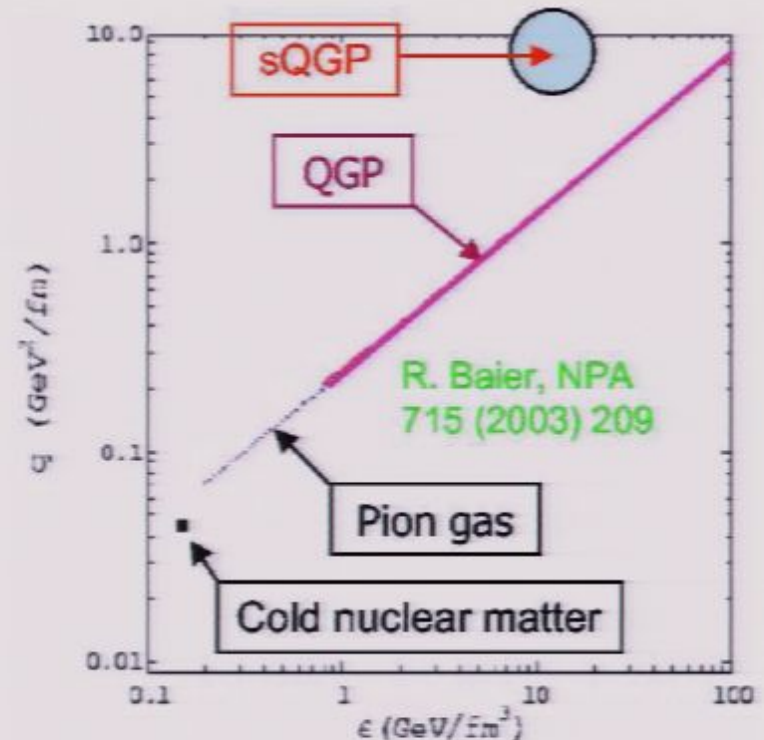
Baier, Dokshitzer, Mueller, Peigne, Schiff ('96), Zakharov ('97), Wiedemann ('00), Gyulassy, Levai, Vitev ('00), ... :

Non-abelian radiative energy loss controlled by “transport coefficient” $\hat{q} = \frac{2\mu_D}{\lambda}$; perturbatively $\hat{q}(\tau) \approx 2e^{3/4}(\tau)$

Eskola et al., NPA 747 (2005) 511



RHIC data



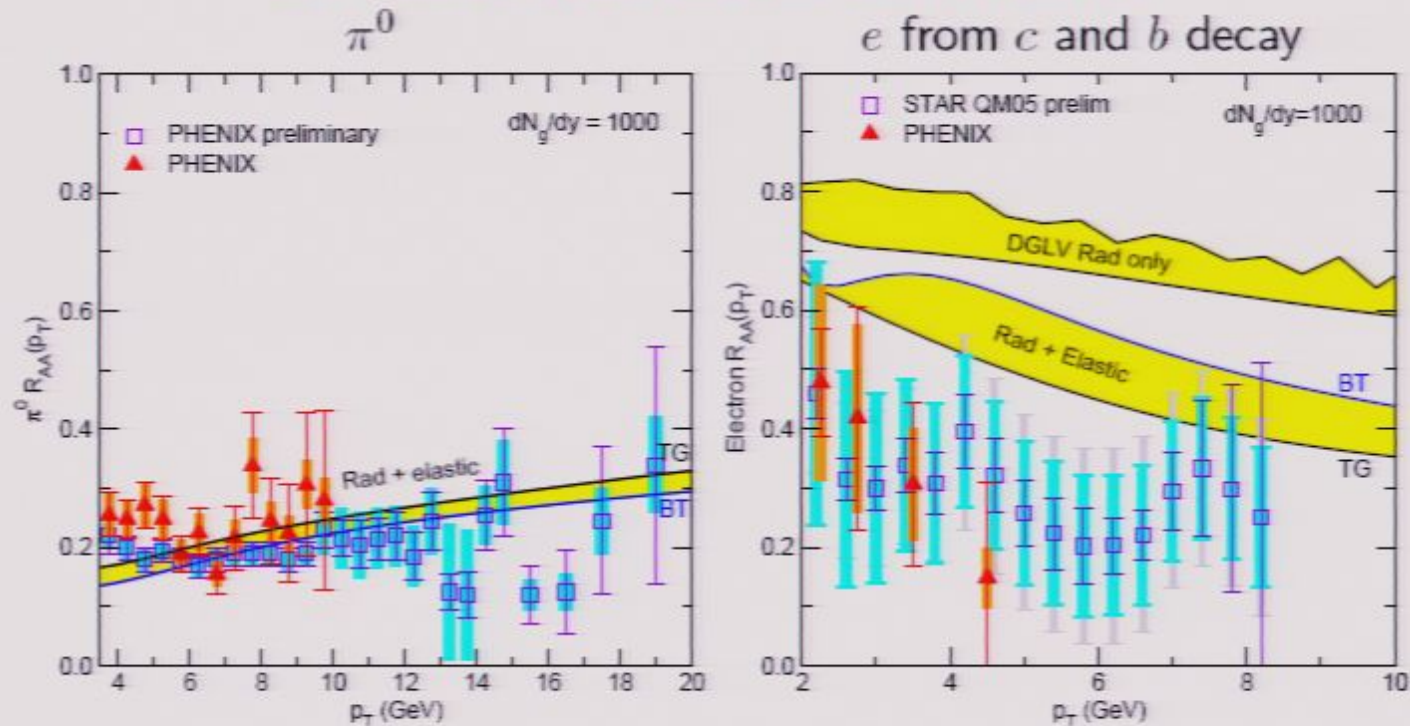
The return of elastic collisional energy loss:

Mustafa (2005), Mustafa & Thoma (2005):

At RHIC, elastic energy loss can compete with radiative loss.

But both together still not sufficient to explain RHIC data:

Wicks, Horowitz, Gyulassy, Djordjevic, NPA 784 (2007) 426



How to tell elastic from radiative energy loss

M. Djordjevic, nucl-th/0603066

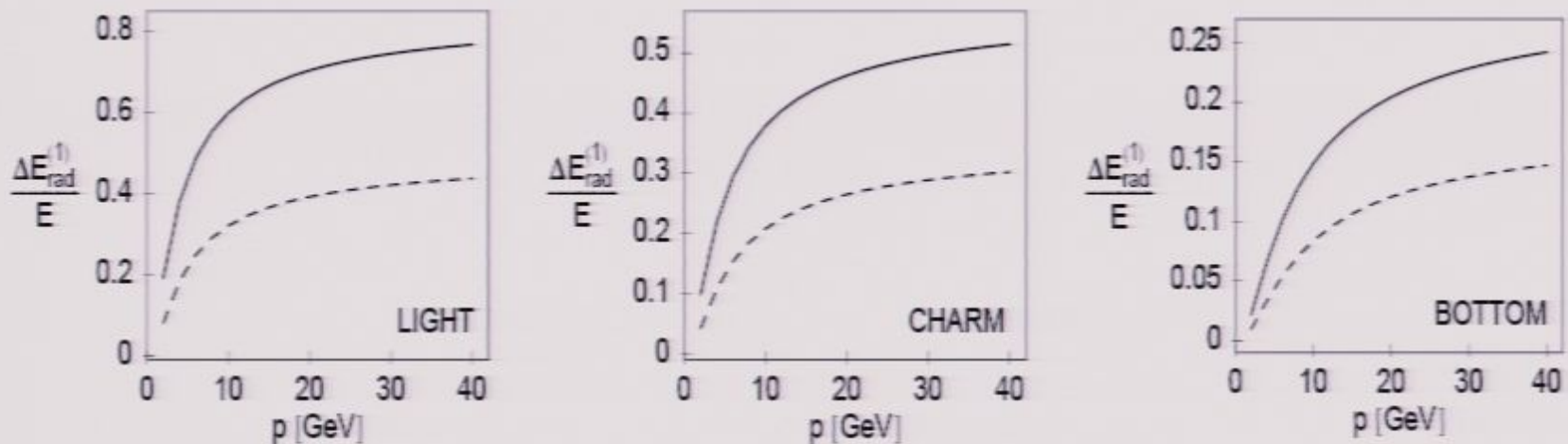


- Elastic collisional energy loss always depends linearly on path length L
- Radiative energy loss shows non-abelian quadratic L dependence for small mass and/or high E_T of leading parton
- Study azimuthal angle dependence of energy loss in non-central Au+Au/Pb+Pb or central U+U collisions (Kuhlman and UH, PRL 94 (2005) 132301)

Radiative energy loss may be larger than previously thought, too!

M. Djordjevic and U. Heinz (2007)

first-order radiative energy loss after $L = 5$ fm,
in an infinite QCD medium made of static (dashed) or dynamical scatterers (solid)



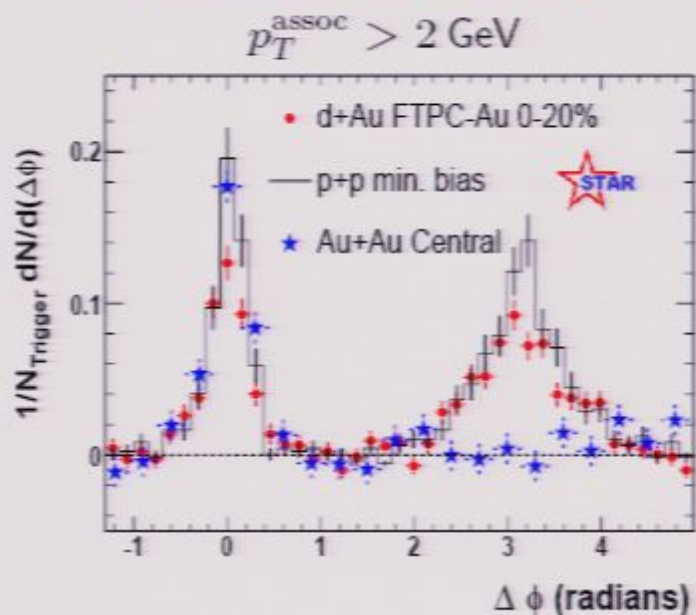
First-order radiative energy loss is ≈ 1.7 times bigger for a dynamical medium,
due to recoil of scatterers \implies new hope for perturbative QCD?

Higher order effects, LPM suppression, finite size medium corrections still to be calculated . . .

Mach cones from quenching jets?

Jet quenching in central Au+Au collisions:

STAR Coll., PRL 91 (2003) 072304

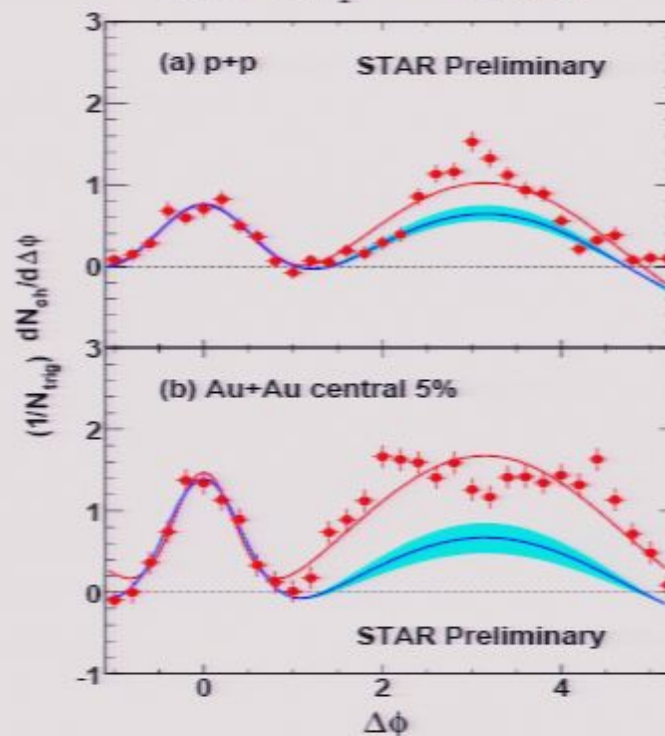


- trigger particle for near-side jet has $4 < p_T < 6 \text{ GeV}$

- away-side jet ($p_T > 2 \text{ GeV}$) visible in p+p and d+Au, but fully quenched in central Au+Au
- energy of quenched jet appears as additional multiplicity of low- p_T particles opposite to trigger particle
- \Rightarrow "thermalization" of intermediate- p_T jets!

STAR Coll., F. Wang, Quark Matter 2004

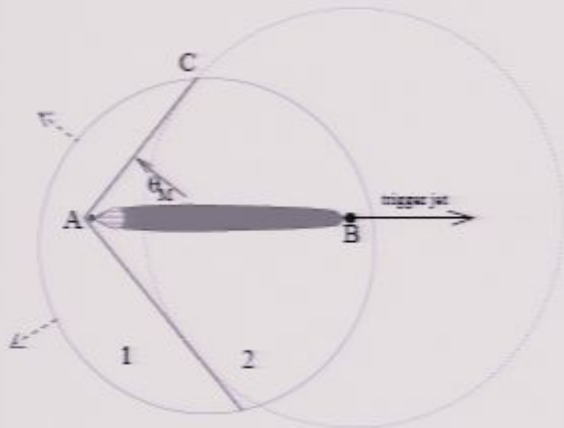
$0.15 < p_T^{\text{assoc}} < 4 \text{ GeV}$



Evidence for a “sonic boom”?!

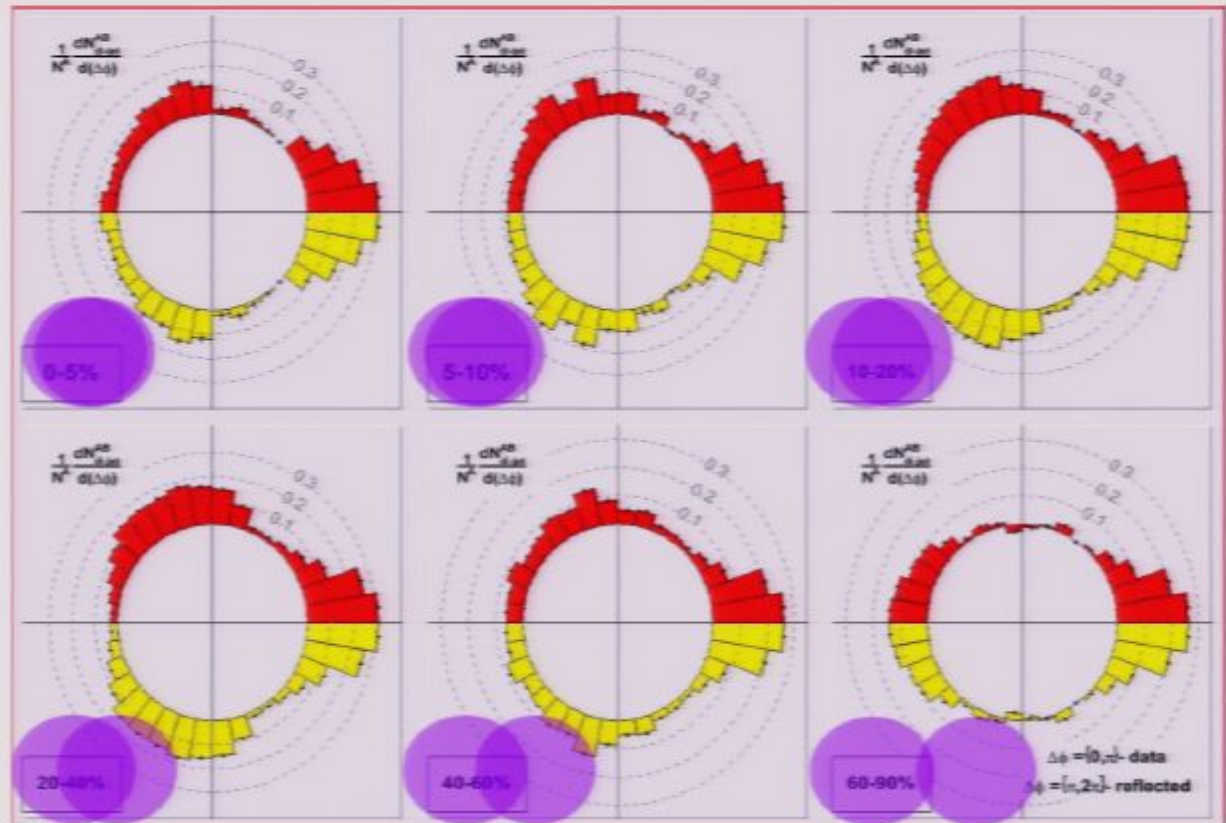
Shuryak et al., hep-ph/0411315

PHENIX Coll., PRL 97 (2006) 052301 (fig. courtesy W.A.Zajc)



Away-side jet creates
Mach cone at $\cos \theta_M = \frac{c_s}{c}$

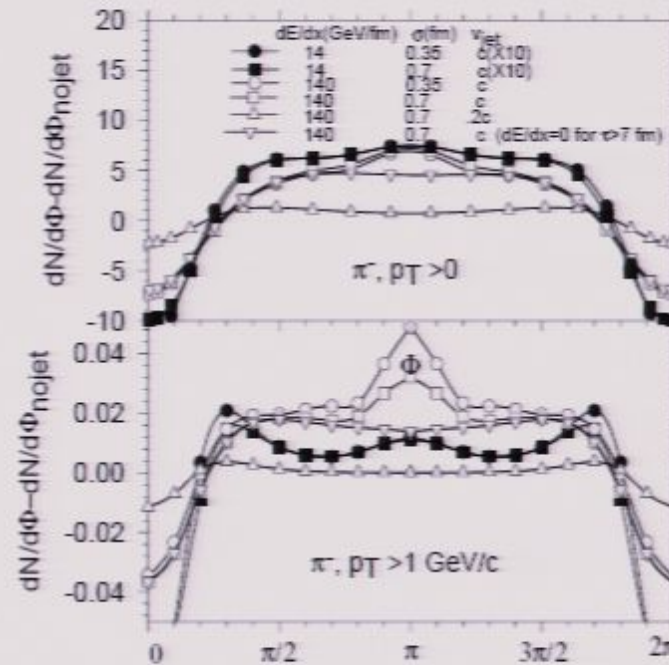
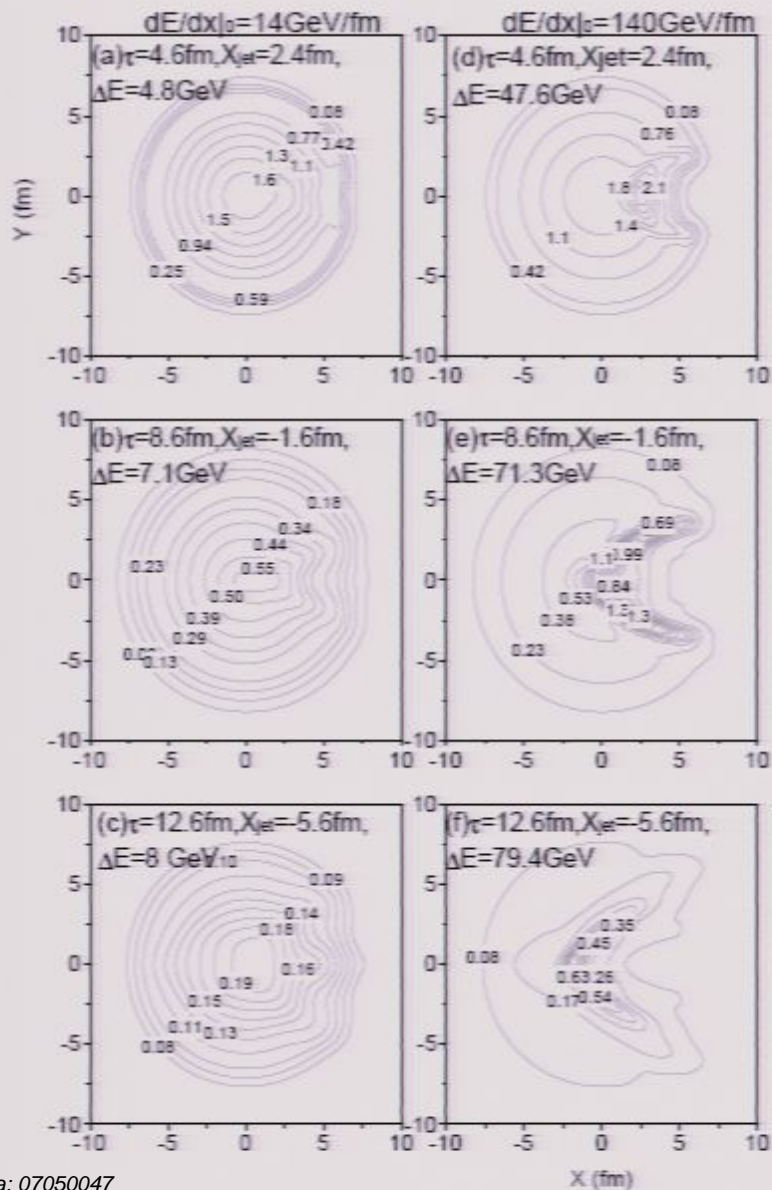
$$\Rightarrow \theta_M \approx 63^\circ \approx 1.1 \text{ rad}$$



Jet induced Mach cones from hydro?

A. Chaudhuri, UH, PRL 97 (2006) 062301

Pions, $T_{dec} = 100$ MeV



- No peaks at the predicted Mach angle, no dip at $\phi = \pi$!
- **Peak** at $\phi = \pi$ reflects momentum imparted by fast parton on medium (absent if parton gets "lost")
- Broad shoulder in $dN/d\phi$ extends into right hemisphere; exists also for subsonic parton speed
- For $p_T > 1$ GeV/c, shoulder edges turn into small peaks (sharper for smaller σ)

⇒ Backsplash!

Relativistic hydrodynamics for *viscous* fluids

Viscous relativistic hydrodynamics (Israel & Stewart 1979)

Include shear viscosity η , neglect bulk viscosity (massless partons) and heat conduction ($\mu_B \approx 0$); solve

$$\partial_\mu T^{\mu\nu} = 0$$

with modified energy momentum tensor

$$T^{\mu\nu}(x) = (e(x)+p(x))u^\mu(x)u^\nu(x) - g^{\mu\nu}p(x) + \pi^{\mu\nu}.$$

$\pi^{\mu\nu}$ = traceless viscous pressure tensor which relaxes locally to 2η times the shear tensor $\nabla^{\langle\mu} u^{\nu\rangle}$ on a microscopic kinetic time scale τ_π :

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi} (\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu} u^{\nu\rangle}) - (u^\mu\pi^{\nu\lambda} + u^\nu\pi^{\mu\lambda}) Du_\lambda$$

where $D \equiv u^\mu\partial_\mu$ is the time derivative in the local rest frame.

Kinetic theory relates η and τ_π , but for a strongly coupled QGP neither η nor this relation are known \implies treat η and τ_π as independent phenomenological parameters.

For consistency: $\tau_\pi\theta \ll 1$ ($\theta = \partial^\mu u_\mu =$ local expansion rate).

(1+1)-d viscous hydrodynamic equations

(Muronga & Rischke 2004, Chaudhuri & Heinz 2005)

[For (2+1)-d viscous hydrodynamic equations see Heinz, Song & Chaudhuri, PRC 73 (2006) 034904]

Azimuthally symmetric transverse dynamics with long. boost invariance:
Use (τ, r, ϕ, η) coordinates and solve

- hydrodynamic equations for $T^{\tau\tau} = (e + \mathcal{P})\gamma_r^2 - \mathcal{P}$, $T^{\tau r} = (e + \mathcal{P})\gamma_r^2 v_r$
(with "effective pressure" $\mathcal{P} = p - r^2 \pi^{\phi\phi} - \tau^2 \pi^{\eta\eta}$) together with
- kinetic relaxation equations for $\pi^{\phi\phi}$, $\pi^{\eta\eta}$:

$$\begin{aligned} \frac{1}{\tau} \partial_\tau (\tau T^{\tau\tau}) + \frac{1}{r} \partial_r (r (T^{\tau\tau} + \mathcal{P}) v_r) &= - \frac{p + \tau^2 \pi^{\eta\eta}}{\tau}, \\ \frac{1}{\tau} \partial_\tau (\tau T^{\tau r}) + \frac{1}{r} \partial_r (r (T^{\tau r} v_r + \mathcal{P})) &= + \frac{p + r^2 \pi^{\phi\phi}}{r}, \\ (\partial_\tau + v_r \partial_r) \pi^{\eta\eta} &= - \frac{1}{\gamma_r \tau \pi} \left[\pi^{\eta\eta} - \frac{2\eta}{\tau^2} \left(\frac{\theta}{3} - \frac{\gamma_r}{\tau} \right) \right], \\ (\partial_\tau + v_r \partial_r) \pi^{\phi\phi} &= - \frac{1}{\gamma_r \tau \pi} \left[\pi^{\phi\phi} - \frac{2\eta}{r^2} \left(\frac{\theta}{3} - \frac{\gamma_r v_r}{r} \right) \right]. \end{aligned}$$

Close equations with EOS $p(e)$ where $e = T^{\tau\tau} - v_r T^{\tau r}$ and $v_r = T^{\tau r} / (T^{\tau\tau} + \mathcal{P})$.

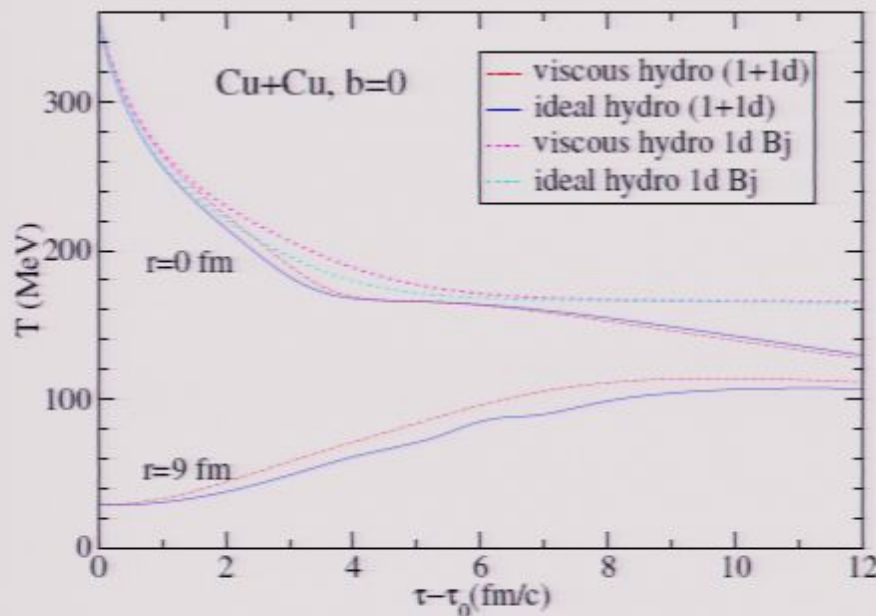
(2+1)-d viscous hydro: less longitudinal work, more radial flow

Huichao Song, hot off the press

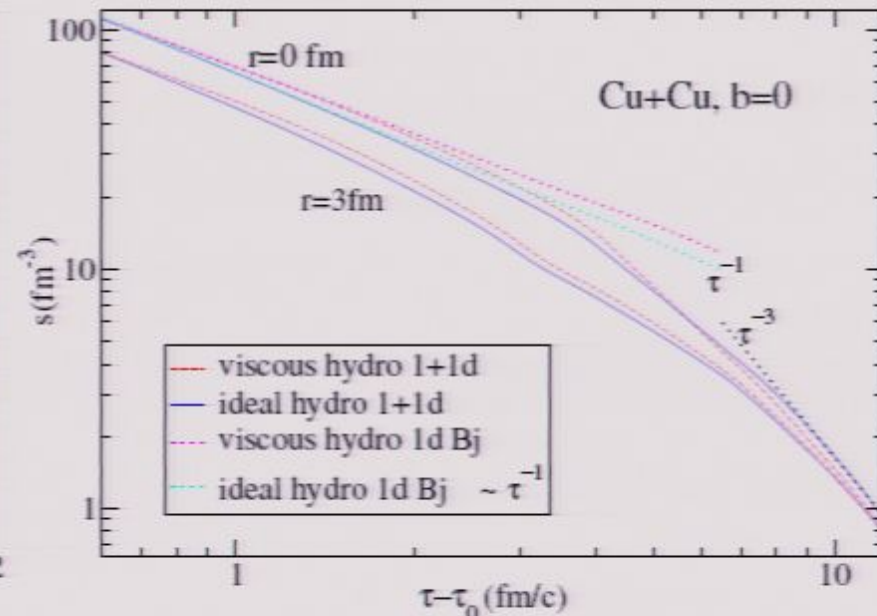
Cu+Cu @ $b = 0$, EOS Q

$$\tau_0 = 0.6 \frac{\text{fm}}{c}, \epsilon_0 = 30 \frac{\text{GeV}}{\text{fm}^3}, \frac{\eta}{s} = \frac{1}{4\pi}, \tau_\pi = 0.24 \left(\frac{200 \text{ MeV}}{T} \right) \frac{\text{fm}}{c}, T_{\text{dec}} = 130 \text{ MeV}$$

Temperature vs. time



Entropy density vs. time



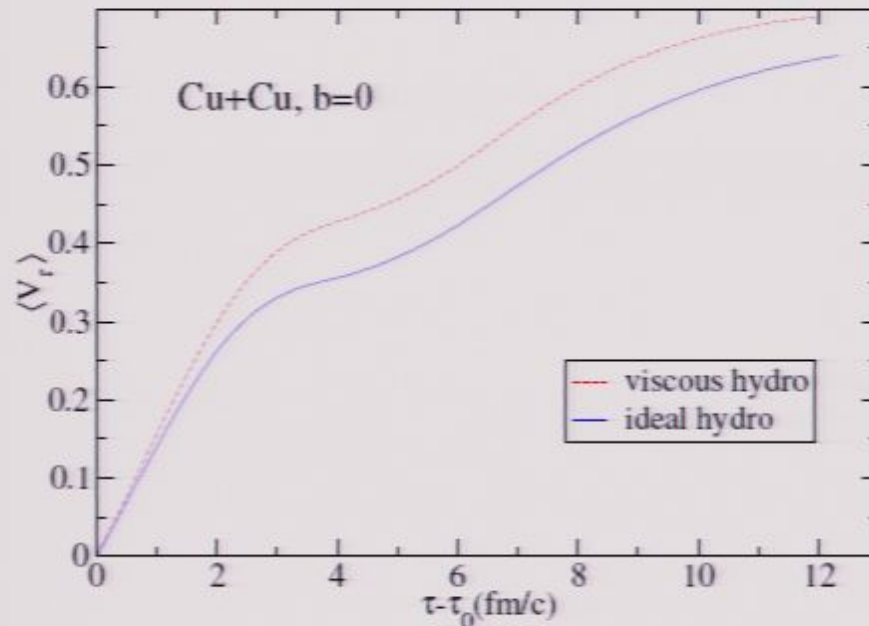
- Radial flow develops much faster, expansion becomes earlier 3-dimensional
- Shear viscosity initially reduces the cooling due to longitudinal work, but then leads to faster cooling than for ideal fluid later, due to stronger radial flow (seen also by Chaudhuri 2006,2007; Romatschke et al. 2006,2007)

(2+1)-d viscous hydro: more radial flow, flatter spectra

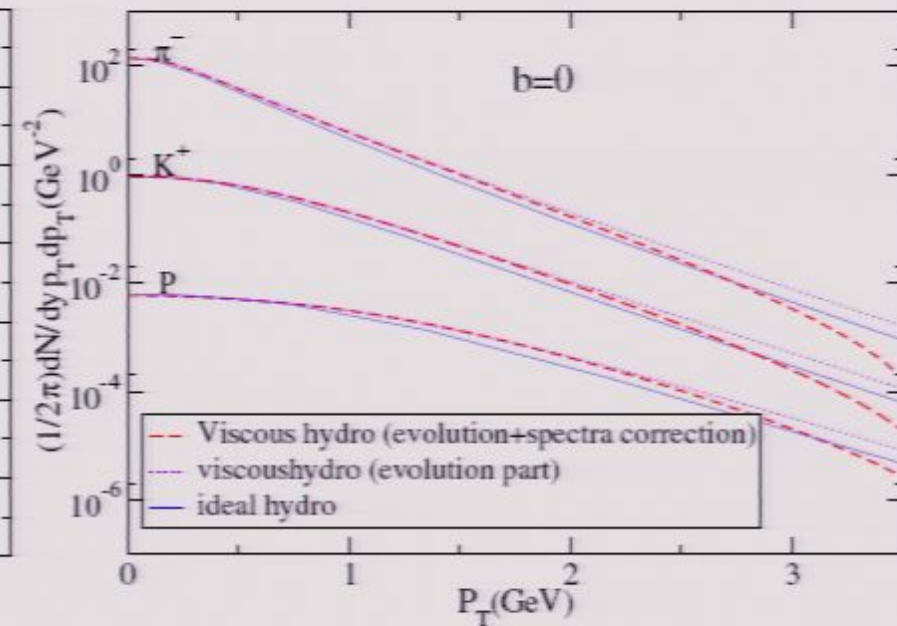
Cu+Cu @ $b = 0$, EOS Q

$$\tau_0 = 0.6 \frac{\text{fm}}{c}, \quad e_0 = 30 \frac{\text{GeV}}{\text{fm}^3}, \quad \frac{\eta}{s} = \frac{1}{4\pi}, \quad \tau_\pi = 0.24 \left(\frac{200 \text{ MeV}}{T} \right) \frac{\text{fm}}{c}, \quad T_{\text{dec}} = 130 \text{ MeV}$$

radial velocity vs. time



hadron p_T -spectra

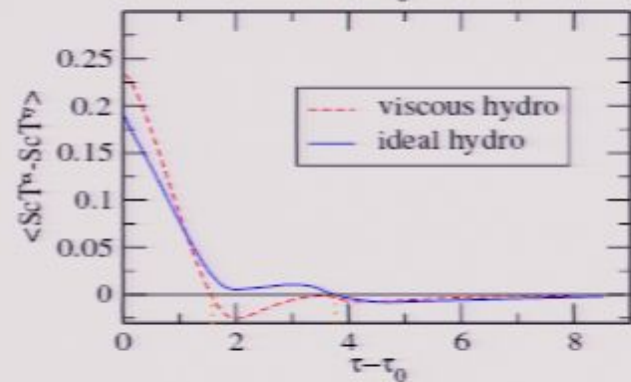
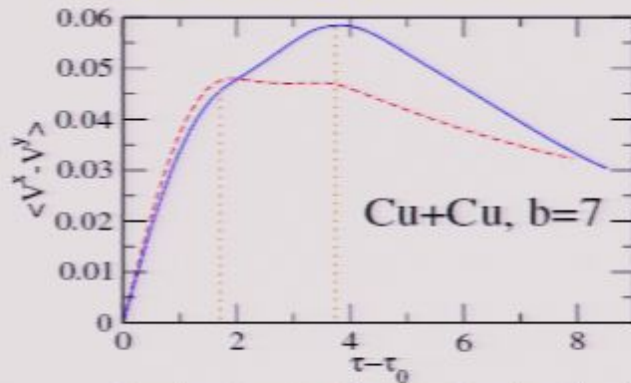


- For identical initial and freeze-out conditions, viscous evolution yields more radial flow and flatter spectra (as previously seen by Chaudhuri 2006,2007; Romatschke 2007)
- Effect on $b = 0$ spectra can be largely absorbed by starting viscous hydro later with lower initial density (Romatschke et al., 2006,2007)

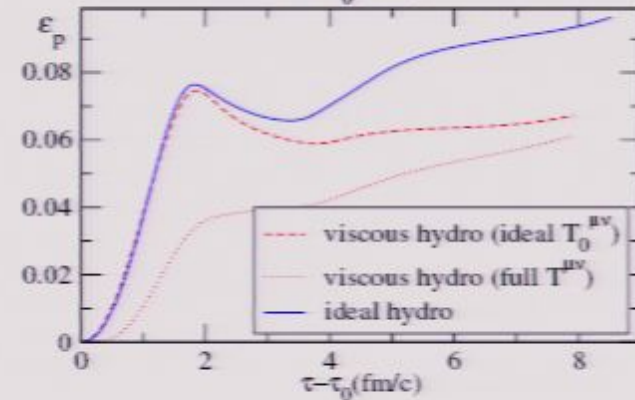
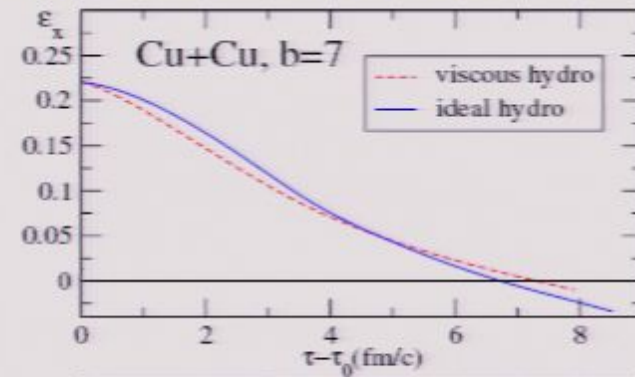
(2+1)-d viscous hydro: less momentum anisotropy

Cu+Cu @ $b = 7$ fm, EOS Q, same initial and final conditions

flow anisotropy vs. time



spatial eccentricity and momentum anisotropy



- Flow anisotropy develops faster initially, but stalls earlier than for ideal fluids;
- Source eccentricity decays initially faster, but more slowly later;
- Total momentum anisotropy is reduced by almost 50% relative to ideal fluid;

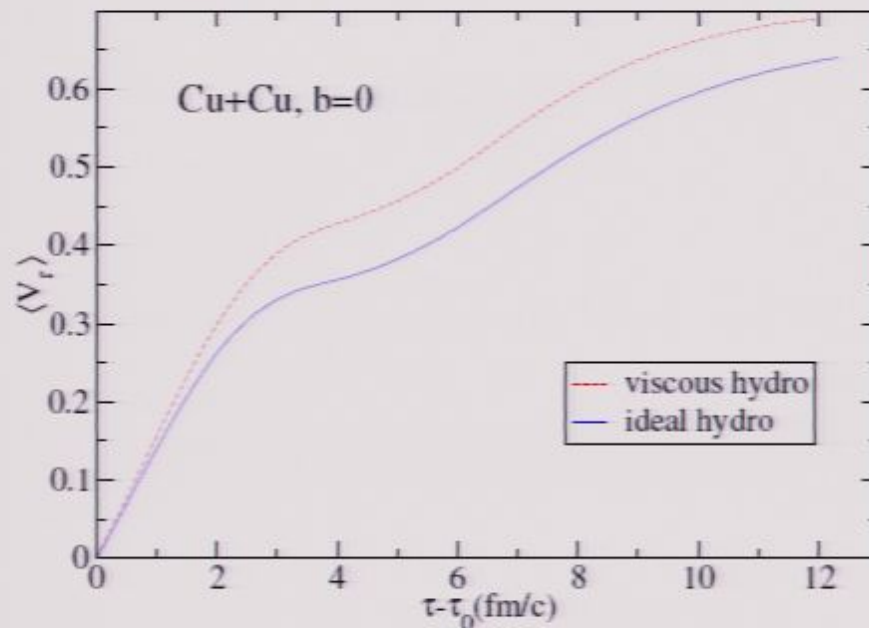
viscous pressure components contribute very strongly to this reduction during the first 3-4 fm/c

(2+1)-d viscous hydro: more radial flow, flatter spectra

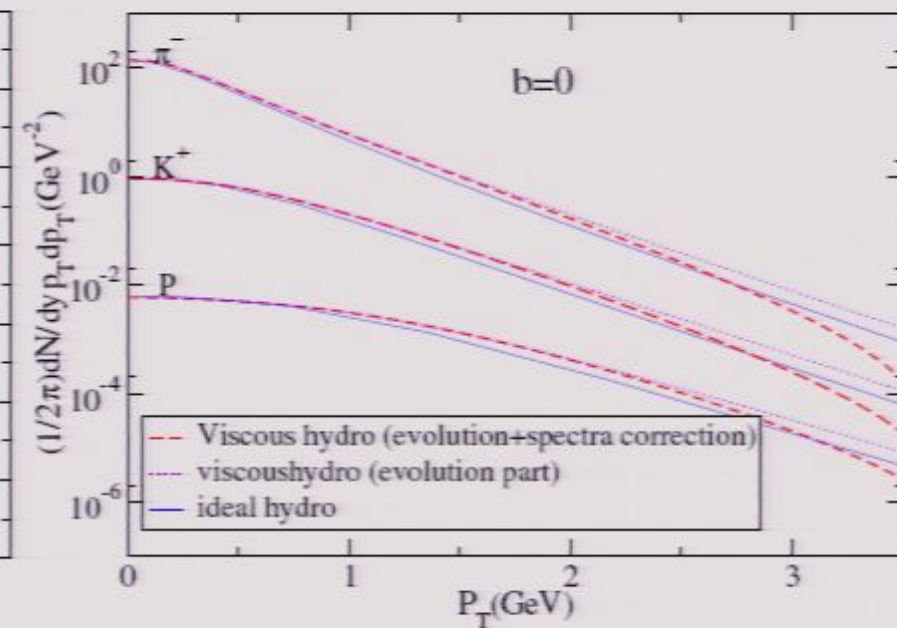
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radial velocity vs. time



hadron p_T -spectra

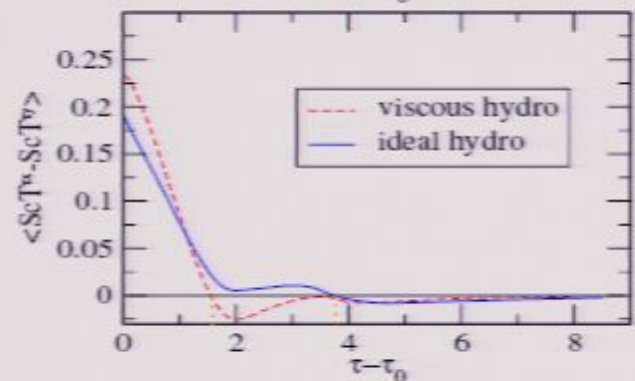
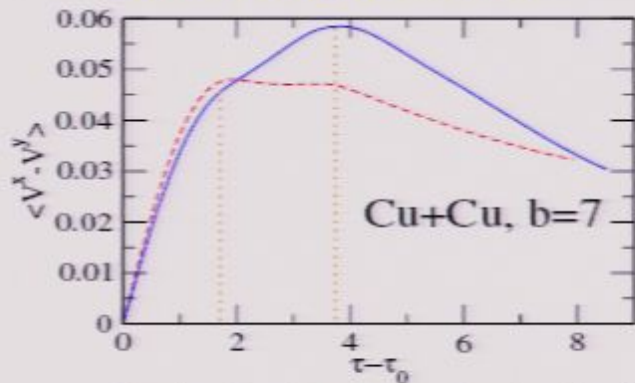


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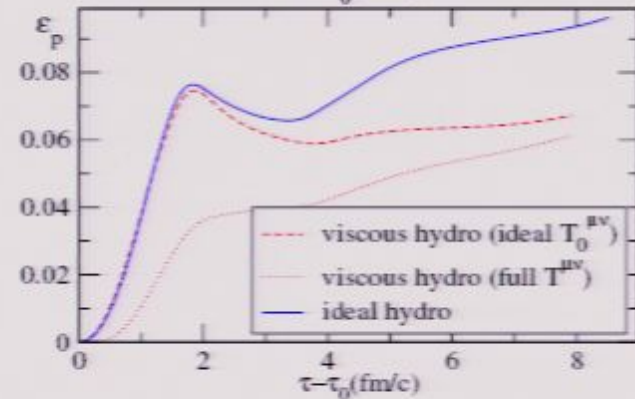
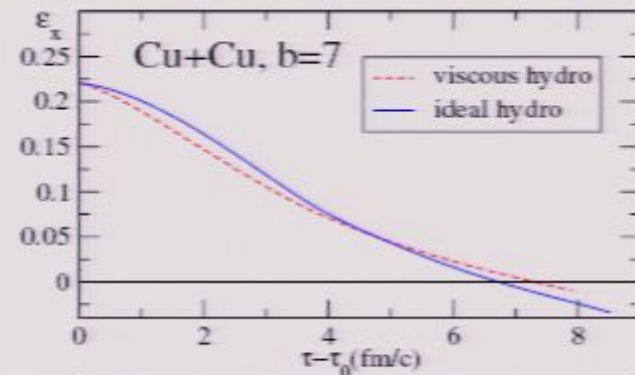
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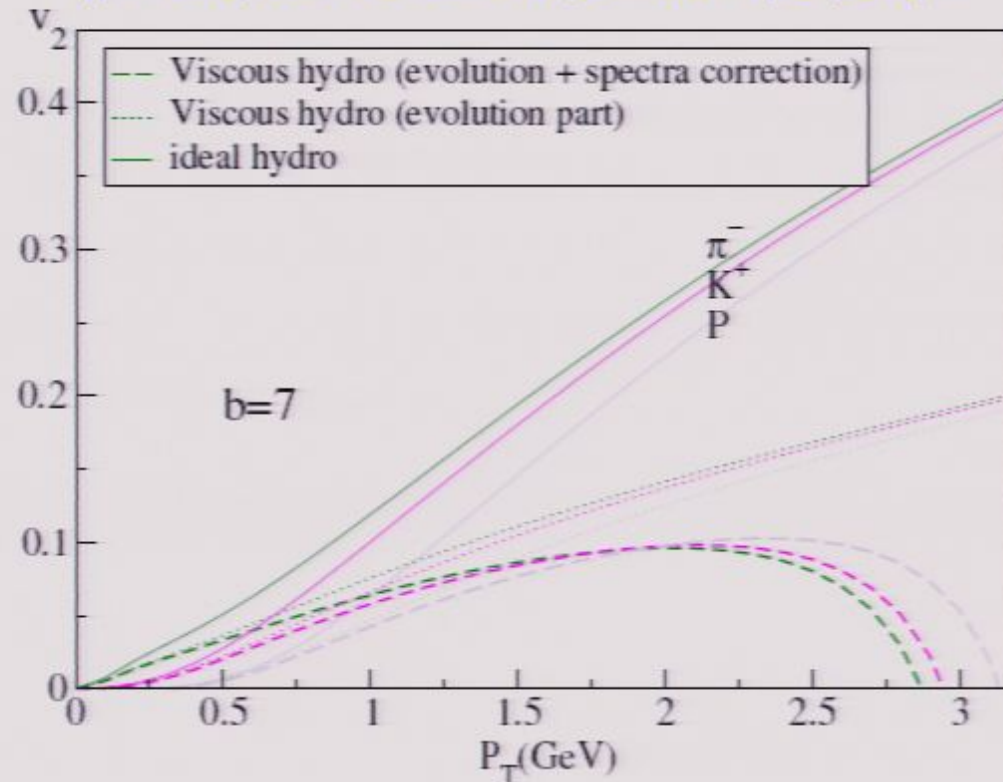
viscous pressure components contribute very strongly to this reduction during the first 3-4 fm/c

Elliptic flow from (2+1)-dim. viscous hydrodynamics

Cu+Cu @ $b=7$ fm

(Initial conditions adjusted such that same single-particle spectra for π , \bar{p} at $b=0$)

$$\eta/s = 1/4\pi, \tau_\pi = 0.24(200 \text{ MeV}/T) \text{ fm}/c$$



- **Elliptic flow very sensitive to even minimal shear viscosity!**
- Viscous corrections to equilibrium distribution fct. have significant effect on v_2 (Teaney 2003), but at low p_T the effects from the reduced hydrodynamic flow anisotropy are larger

Is the perturbative QCD
approach to the QGP dead?

Anomalous viscosity and energy loss from color chaos

Asakawa, Bass, Müller, PRL 96 (2006) 252301 and Prog. Theor. Phys. 116 (2007) 725

- Viscosity in an anisotropically expanding fluid
⇒ local momentum anisotropies $\sim \eta$ (Heiselberg, Wang 1996; Teaney 2003)
- Anisotropic momentum distributions ⇒ plasma instabilities
(Weibel '59, Mrówczyński '88-'03, Arnold, Lenaghan & Moore '02-'06, Rebhan, Romatschke & Strickland '03-'05)
- Plasma instabilities ⇒ turbulent (color) magnetic fields
⇒ anomalous viscosity and energy loss of (color) charged particles, $\eta_B \sim 1/\langle B^2 \rangle \sim 1/\eta$
(Malone et al. 1975, Niu et al. 1978-1980)

⇒ large viscosity leads to small anomalous viscosity ⇒ self-regulating!

Collisional and turbulent mean-field scattering rates add

$$\Rightarrow \text{total viscosity } \eta^{-1} = \eta_{\text{coll}}^{-1} + \eta_B^{-1}$$

For $g \ll 1$ Asakawa et al. find $\eta_B \sim \frac{s}{g^{3/2}}$ while $\eta_{\text{coll}} \sim \frac{s}{g^4 \ln g}$.

So at early times (high T , small g) anomalous viscosity η_B may dominate and render η anomalously small! ⇒ hope for perturbative QCD?!

(NB: ⇒ Early Universe was not a perfect fluid!)

Summary and Outlook

The QGP at RHIC:

- $v_2 \implies$ short thermalization time $\tau_{\text{therm}} < 1 \text{ fm}/c \implies$ matter initially in QGP state, $e_0 > 10 e_{\text{cr}}$, $T_0 \sim 2T_{\text{cr}}$, lives $\sim 5 - 7 \text{ fm}/c$ before hadronizing
- Bulk of matter created at RHIC = almost ideal fluid with very low (anomalous?) viscosity \implies QGP = strongly coupled plasma (non-perturbative??)
NB: hadron gas is *not* an ideal fluid, but very viscous!

Other RHIC data support this picture:

- $T_{\text{cr}} \approx 170 \text{ MeV}$ measured (statistical hadronization), consistent/w LQCD
- strangeness enhancement, quark number scaling of v_2 , R_{CP} of identified hadrons at intermediate p_T indicate quark deconfinement
- jet emission tomography (JET), heavy quark energy loss \Rightarrow QGP is color opaque
- JET and hydro yield independent, consistent estimates for initial energy density $e_{\text{init}} \approx 15 e_{\text{cr}} \approx 100 e_{\text{nucl.mat.}}$

So how perfect is this QGP liquid?

\implies Relativistic hydrodynamics for viscous fluids (ongoing)

\implies QGP viscosity will soon be extracted from RHIC and LHC data