

Title: Why there is something rather than nothing

Date: May 20, 2007 04:00 PM

URL: <http://pirsa.org/07050044>

Abstract: We show that the recently suggested Euclidean quantum gravity density matrix of the Universe which generates the set of inflationary universes is, in fact, the density matrix of the microcanonical ensemble in {\em Lorentzian} quantum cosmology. This ensemble corresponds to the uniform (weight one) distribution over an entire phase space of {\em true physical variables}, but in terms of the observable spacetime geometry it is peaked about complex saddle-points of the Lorentzian path integral, given by the recently obtained Euclidean gravitational instantons belonging to the bounded range of the cosmological constant. It is shown that the conformal rotation, designed to solve the problem of unboundedness of the Euclidean gravitational action, is, in fact, the manifestation of correct integration range over the lapse and shift functions in Lorentzian quantum gravity, enforcing the Hamiltonian and momentum constraints. Inflationary cosmologies generated by the gravitational instantons at late stages of expansion incorporate cosmological acceleration phenomenon whose low-energy scale can be attained by invoking the ideas of dynamically evolving compactified extra dimensions. Thus, together with the bounded range of the early cosmological constant, this initial state of the Universe simultaneously suggests the dynamical restriction mechanism for the landscape of string vacua and the solution of the dark energy problem.

Why there is Something rather than Nothing (from Everything):

origin of the cosmological constant and dark energy

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Introduction



Problems:

- indefiniteness of the Euclidean gravitational action;

- infrared catastrophe of small cosmological constant Λ ,

$$\Psi_{\text{HH}} \sim \exp\left(\frac{3\pi}{2G\Lambda}\right) \rightarrow \infty, \quad \Lambda \rightarrow 0;$$

- cosmology debate of the no-boundary vs tunneling proposals

EQG density matrix of the Universe

elimination of the infrared catastrophe of small Λ

ensemble of universes in a limited range of Λ

selection mechanism for string landscape

A.B. & A.Yu.Kamenshchik,
JCAP, 09, 014 (2006)
[hep-th/0605132];
Phys. Rev. D74, 121502 (2006)
[hep-th/0611206]

Justification of these results from Lorentzian quantum gravity (LQG) and suggestion of a new mechanism of dark energy (A.B., hep-th/0704.0083)

Plan

- **EQG density matrix:**
effects of conformal anomaly and radiation --- limiting the cosmological constant ensemble
- **LQG density matrix --- microcanonical ensemble**
- **Sum over Everything**
- **Conformal rotation in EQG --- selection of thermal instantons**
- **Dark energy from the microcanonical state of the Universe:**
hierarchy problem, strings and evolving extra dimensions
- **Conclusions: Something rather than Nothing comes from Everything**

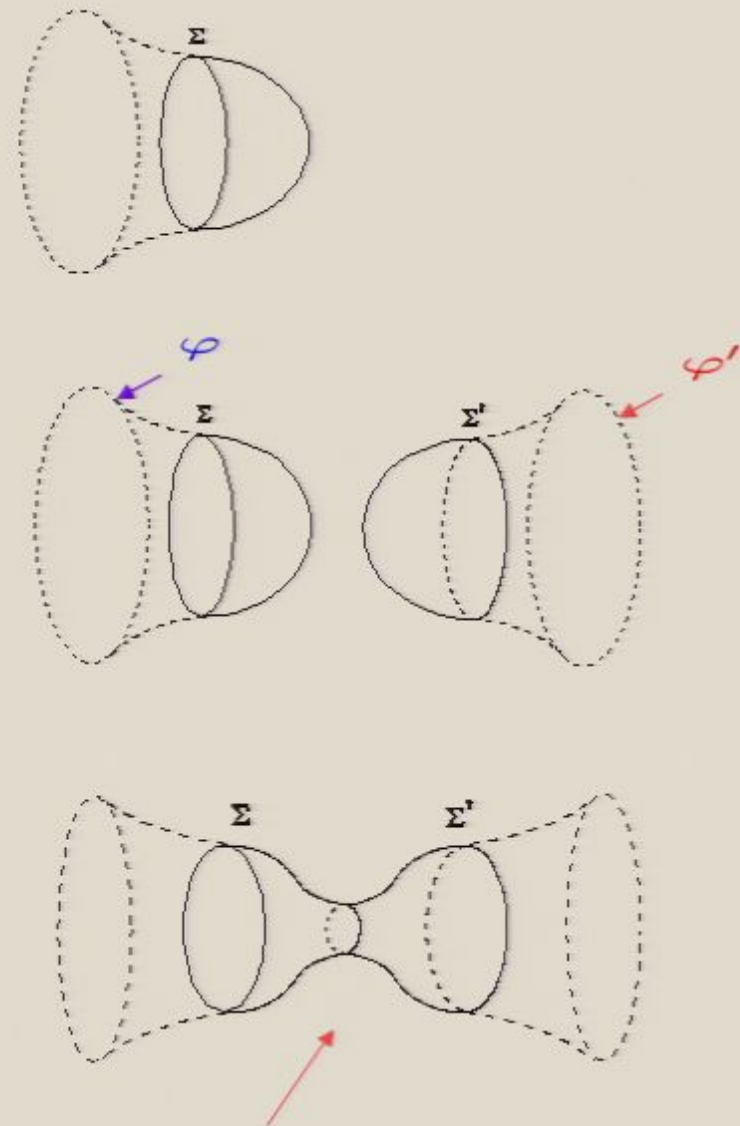
From the pure Hartle-Hawking state to a statistical ensemble – the density matrix:

$$|\Psi_{HH}\rangle = \Psi_{HH}[\varphi]$$

$$|\Psi_{HH}\rangle\langle\Psi_{HH}| = \rho_{HH}[\varphi, \varphi']$$



$$\hat{\rho}_{\text{mixed}} = \rho_{\text{mixed}}[\varphi, \varphi']$$



instanton bridge mediates density matrix correlations

EQG density matrix

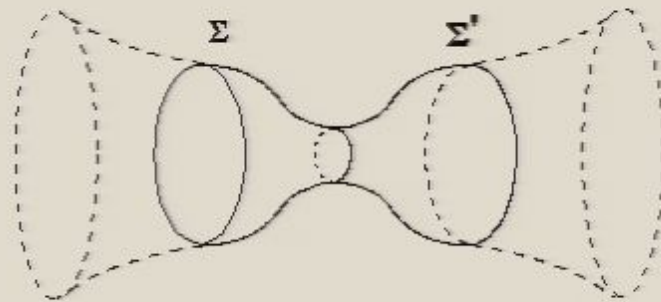
$$\rho[\varphi, \varphi'] = e^{\Gamma} \int_{g, \phi |_{\Sigma, \Sigma'} = (\varphi, \varphi')} D[g, \phi] \exp(-S_E[g, \phi])$$

D. Page
(1986)

$$\text{tr } \hat{\rho} = 1$$

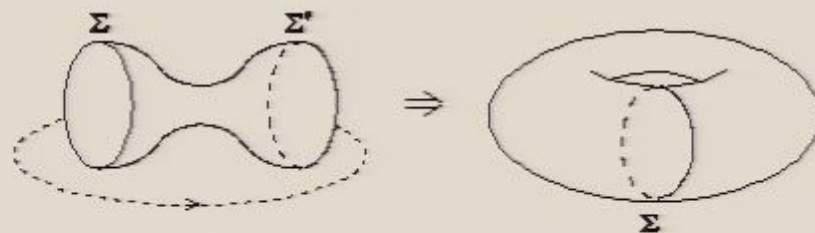


**Effective action:
statistical sum**



$$e^{-\Gamma} = \int_{g, \phi |_{\Sigma} = g, \phi |_{\Sigma'}} D[g, \phi] \exp(-S_E[g, \phi])$$

integration over periodic fields on a torus:



Euclidean FRW metric

$$ds^2 = N^2 d\tau^2 + a^2 d^2\Omega^{(3)},$$

↖ lapse
↖ scale factor

↙ 3-sphere of a unit size

$$[g, \phi] = [a(\tau), N(\tau); \Phi(x)]$$

↗ minisuperspace background

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \dots)$$

quantum "matter" – cosmological perturbations

gravitons



$$e^{-\Gamma} = \int_{\text{periodic}} D[a, N] e^{-\Gamma_E[a, N]}$$

$$e^{-\Gamma_E[a, N]} = \int_{\text{periodic}} D\Phi(x) e^{-S_E[a, N; \Phi(x)]}$$

quantum effective action
of Φ on minisuperspace
background

Effective action for **conformally** coupled fields

$$\Gamma_E[a, N] = \int d\tau N \mathcal{L}(a, a') + F(\eta)$$

local part
nonlocal (thermal) part

$$\mathcal{L}(a, a') = -aa'^2 - a + H^2 a^3 + B \left(\frac{a'^2}{a} - \frac{a'^4}{6a} + \frac{1}{2a} \right)$$

classical part
conformal anomaly part
vacuum (Casimir) energy

$a' \equiv \frac{1}{N} \frac{da}{d\tau}, \quad \Lambda = 3H^2$
Hubble constant

B -- coefficient of the **Gauss-Bonnet** term in the total conformal anomaly

Nonlocal part -- free energy

$$F(\eta) = \pm \sum_{\omega} \ln(1 \mp e^{-\omega\eta}), \quad \eta = \int d\tau \frac{N}{a}$$

energies of field oscillators on a 3-sphere
instanton period in units of conformal time --- inverse temperature

Saddle points of the path integral — solutions of **Euclidean** effective equation of motion

$$\frac{\delta \Gamma_E[a, N]}{\delta N(\tau)} = 0$$

quantum anomaly term

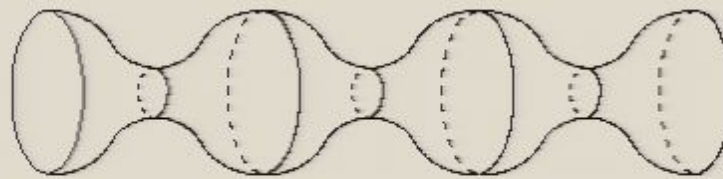
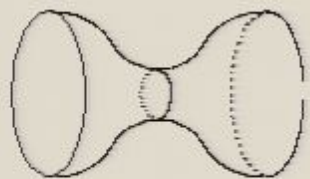
$$\frac{a'^2}{a^2} + B \left(\frac{1}{2} \frac{a'^4}{a^4} - \frac{a'^2}{a^4} \right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4}$$

amount of radiation constant

$$C = \frac{B}{2} + \frac{dF(\eta)}{d\eta}, \quad \eta = \int d\tau \frac{N}{a}$$

Firouzjahi, Sarangi & Tye (2004);
Sarangi & Tye (2005);
Brustein & de Alwis (2006)

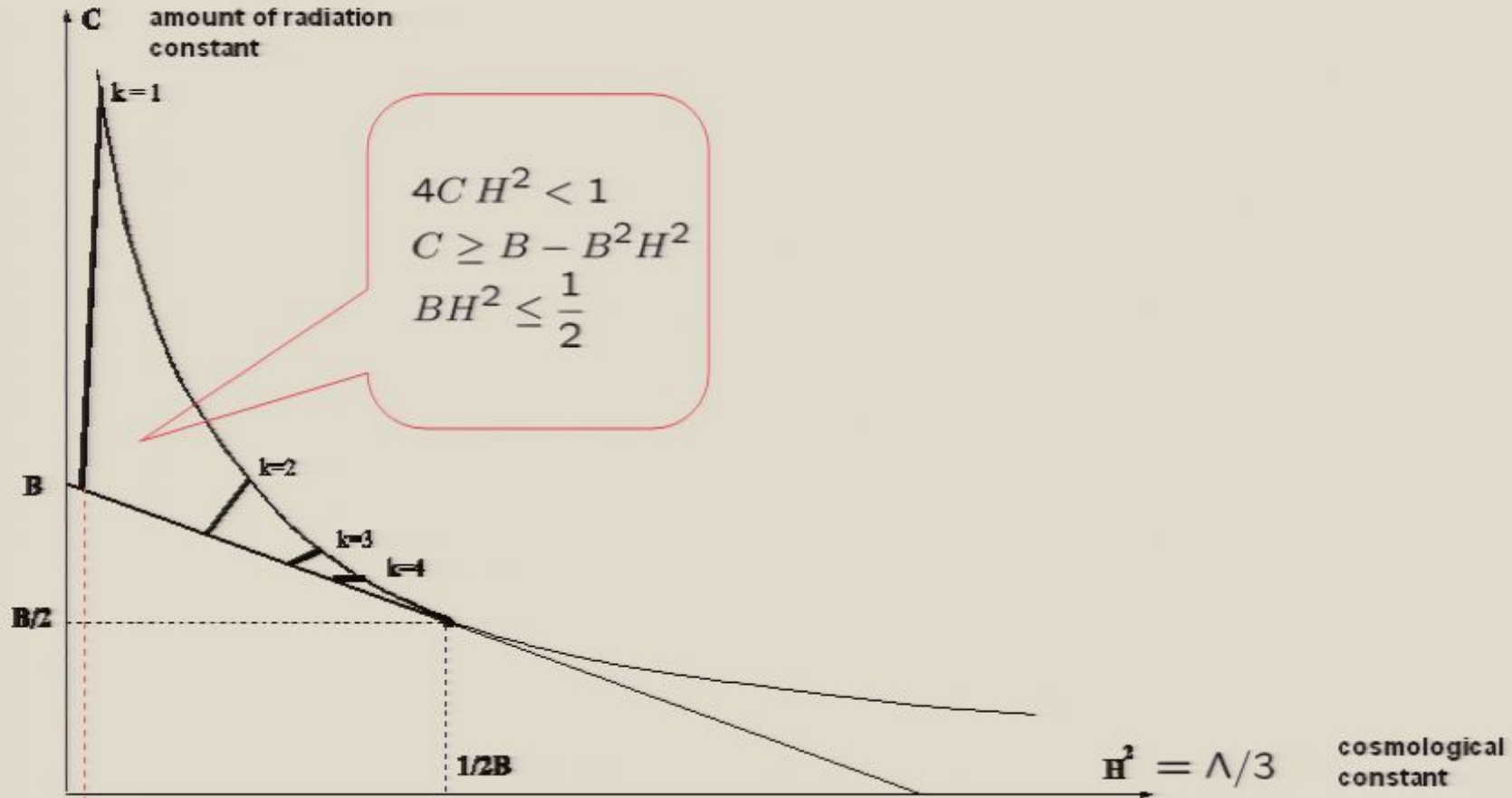
Solutions --- set of tubular periodic garland-type instantons with oscillating scale factor



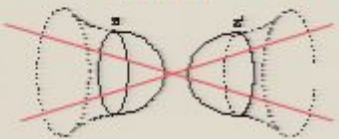
Halliwell & Myers (1989);
Fischler, Morgan & Polchinski (1990)

1- fold, $k=1$

k - folded garland, $k=1,2,3,\dots$



- $\Lambda_{\min} < \Lambda < \Lambda_{\max}$ bounded range of the cosmological constant \rightarrow selection rule for string landscape



- for any $\Lambda > 0$ elimination of the infrared catastrophe (anomaly effect)

- $\Lambda_{\max} = 3m_P^2/2B$ new quantum gravity scale (anomaly effect)

- $\Lambda_{\min}, \Lambda_{\max} \rightarrow \frac{\Lambda_{\min}}{N}, \frac{\Lambda_{\max}}{N}; N = \# \text{ of quantum fields: } 1/N\text{-approximation}$

LQG density matrix

Representation of three-metric and matter fields $q = (g_{ij}(\mathbf{x}), \phi(\mathbf{x}))$; p – conjugated momenta

$$\rho(q_+, q_-) = e^\Gamma \int_{q(t_\pm) = q_\pm} D[q, p, N] \exp i \int_{t_-}^{t_+} dt (p \dot{q} - N^\mu H_\mu)$$

\leftarrow includes gauge fixing factor
 \nearrow lapse and shift functions
 \nwarrow constraints $H_\mu = H_\mu(q, p)$

Range of integration over N^μ : $-\infty < N^\mu < \infty$



Wheeler-DeWitt equations

$$\hat{H}_\mu(q, \partial/i\partial q) \rho(q, q_-) = 0$$



Microcanonical density matrix

$$\hat{\rho} \sim \left(\prod_\mu \delta(\hat{H}_\mu) \right)$$

A simplest analogy — an unconstrained system with a conserved Hamiltonian \hat{H} in the microcanonical state with a fixed energy E ,

$$\hat{\rho} \sim \delta(\hat{H} - E)$$

Spatially closed cosmology does not have *freely specifiable* constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ , all having a particular value — *zero*.



The microcanonical ensemble with

$$\hat{\rho} \sim \left(\prod_{\mu} \delta(\hat{H}_{\mu}) \right)$$

is as a most general and natural candidate for the quantum state of the *closed* Universe.

The microcanonical statistical sum of the Universe is just a uniformly distributed (with a unit weight) integral over entire phase space of true physical degrees of freedom — *Sum over Everything*.

Sum over Everything

Statistical sum

$$e^{-\Gamma} = e^{-\Gamma} \text{Tr}_{\text{phys}} \hat{\rho} = \int_{\text{periodic}} D[q, p, N] e^{i \int dt (p \dot{q} - N^\mu H_\mu)}$$

Physical reduction in the unitary gauge, $\chi^\mu(q, p) = 0$, $(q, p) \rightarrow (q_{\text{phys}}, p_{\text{phys}})$.

Canonical Faddeev-Popov integral in terms of physical variables:

$$\begin{aligned} \int D[q, p, N] e^{i \int dt (p \dot{q} - N^\mu H_\mu)} &= \int Dq_{\text{phys}} Dp_{\text{phys}} e^{i \int dt (p_{\text{phys}} \dot{q}_{\text{phys}} - H_{\text{phys}}(t))} \\ &= \text{Tr}_{\text{phys}} \left(\mathbf{T} e^{-i \int dt \hat{H}_{\text{phys}}(t)} \right) \end{aligned}$$

physical Hamiltonian
↙
chronological ordering

On-shell Faddeev-Popov path integral is gauge-independent.

In static gauges, $\partial_t \chi^\mu(q, p, t) = 0$, $\hat{H}_{\text{phys}}(t) = 0$ (closed cosmology)



$$e^{-\Gamma} = \text{Tr}_{\text{phys}} \mathbf{I}_{\text{phys}} = \int dq_{\text{phys}} dp_{\text{phys}} = \text{sum over Everything.}$$

Gaussian integration over momenta --
Lagrangian path integral:

$$e^{-\Gamma} = \int D[q, N] e^{iS_L[q, N]}$$

↙ Lorentzian variables (L)

Field decomposition: $[q, N] \rightarrow [a_L(t), N_L(t); \Phi_L(x)]$, $D[q, N] = D[a_L, N_L] \times D\Phi_L(x)$

⏟
minisuperspace



LQG path integrals with
real integration variables

$$\left\{ \begin{array}{l} e^{-\Gamma} = \int D[a_L, N_L] e^{i\Gamma_L[a_L, N_L]} \\ e^{i\Gamma_L[a_L, N_L]} = \int D\Phi_L(x) e^{iS_L[a_L, N_L; \Phi_L(x)]} \end{array} \right.$$

No periodic solutions of Lorentzian effective equation in real time and real geometry!
Saddle points of the Lorentzian path integral exist in Wick-rotated (Euclidean) geometry:

$$t = \tau, \quad N_L = -iN, \quad iS_L[a_L, N_L; \Phi_L] = -S_E[a, N; \Phi],$$

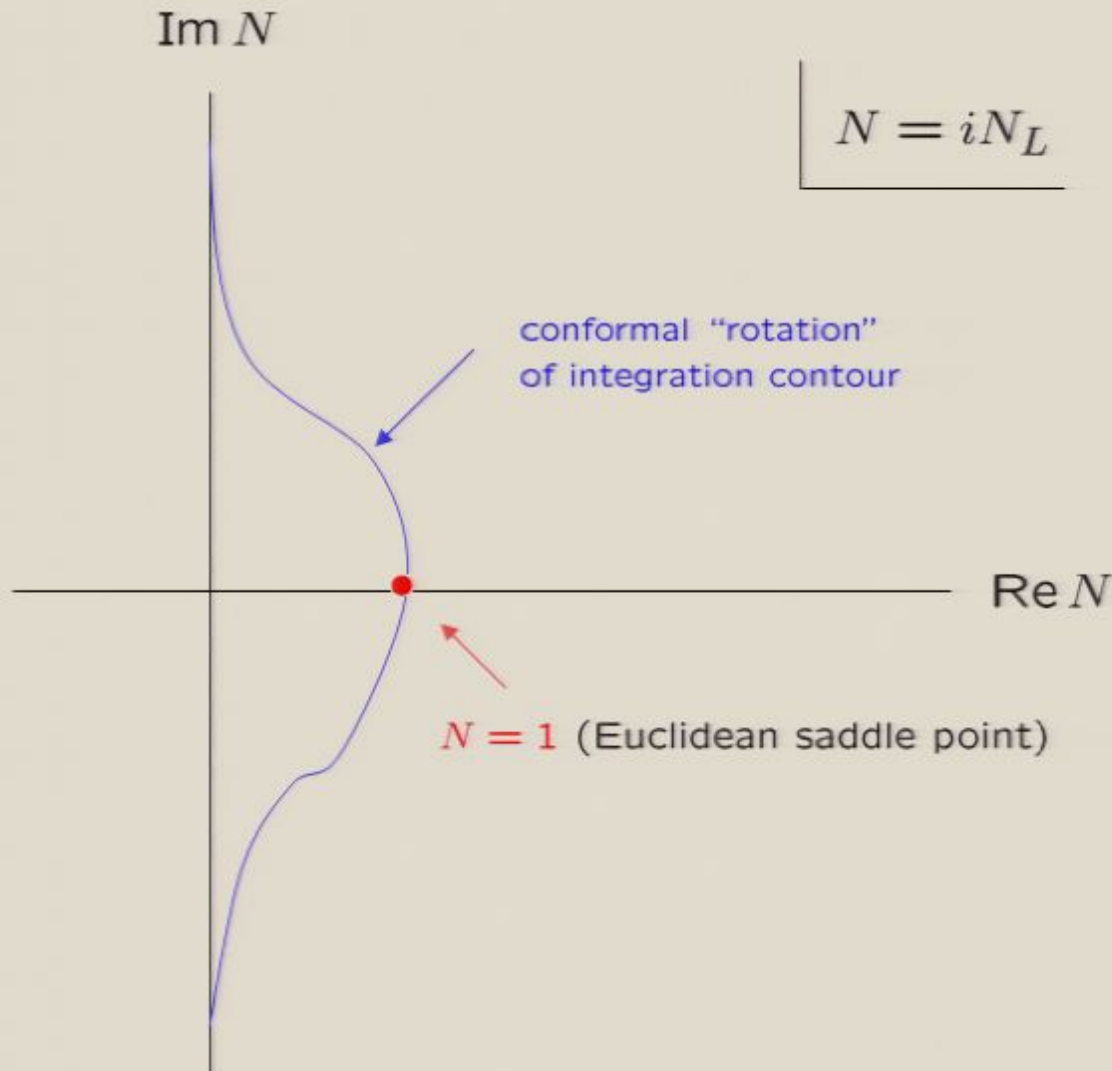


$$\left. \begin{array}{l} e^{i\Gamma_L[a_L, N_L]} = e^{-\Gamma_E[a, N]} \\ e^{i\Gamma} = e^{-\Gamma_E} \end{array} \right\}$$

EQG path integral with the imaginary
lapse integration contour:

$$N \in [-i\infty, i\infty]$$

Steepest decent integration contour for the Euclidean lapse -- conformal rotation in EQG



Deformation of the original contour of integration

$$-\infty < N_L < \infty$$

into the complex plane to pass through the saddle point

Conformal rotation in the one-loop approximation -- selection of thermal instantons with a fixed Euclidean time period.

Integration measure in the Faddeev-Popov path integral: $D[a, N] = Da DN \mu[a, N] \delta[\chi] \text{Det } Q$

$$\mu_{1\text{-loop}} = \prod_{\tau} \left(\frac{\partial^2(N\mathcal{L})}{\partial \dot{a} \partial \dot{a}} \right)^{1/2} = \prod_{\tau} \left(\frac{D}{N a^2 a'^2} \right)^{1/2}$$

local Lagrangian in the action
local measure
gauge-fixing factor

$$D = a a'^2 (a^2 - B + B a'^2) > 0$$

↪ on background instantons

The gauge disentangling conformal mode perturbations: $\chi(a, N) = \delta N - (N/a) \delta a$

The Faddeev-Popov operator: $Q = a(d/d\tau)a^{-1}$

↪ perturbations on background

Quadratic part of the action in terms of the conformal mode σ : $\delta a = \sigma a, \delta N = \sigma N$

$$\delta_{\sigma}^2 \Gamma_E = -\frac{3\pi m_P^2}{2} \int d\tau N D \left[\left(\frac{\sigma}{a'} \right)' \right]^2 < 0$$

But the integration range over σ is imaginary!

Density Matrix Reloaded:

Minimum set of assumptions -- an ultimate equipartition in physical phase space in the form of the microcanonical state of closed quantum cosmology



- Constraining the ensemble of Λ (and possibly landscape of string vacua)
- A new dark energy mechanism transcending the inflationary and matter-domination stages as a quasi-equilibrium **decay** of the initial microcanonical state

“Nothing comes from Nothing”

Sidney Coleman: “There is Nothing rather than Something”

Something (rather than Nothing) comes from Everything

Конец показа слайдов. Щелкните для выхода.

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Mechanism of variable B from string/Kaluza-Klein theory

$B \Rightarrow B(t)$ indefinitely growing with evolving size of X-tra dimension(s) $R(t)$

B --- coefficient of the Gauss-Bonnet term in the total conformal anomaly;

$B \sim \mathcal{N}$ number of conformally invariant massless modes --- KK and winding modes

Masses of KK and winding modes:

$$m_{n,w}^2 = \frac{n^2}{R^2} + \frac{w^2}{\alpha'^2} R^2 \ll H_+^2 \sim \frac{m_P^2}{\mathcal{N}}$$

approximate masslessness and conformal invariance

- Growing tower of superhorizon KK modes ($w = 0, n \leq \mathcal{N}$):

$$\mathcal{N} \sim (m_P R)^{2/3} \Rightarrow H_+ \sim \frac{m_P}{(m_P R)^{1/3}} \quad \text{indefinitely decreases with } R \rightarrow \infty$$

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$\Lambda \sim m_P^2 \Rightarrow \Lambda_{\text{present}}$ (fine-tuning the expansion/contraction of X-tra space size???)

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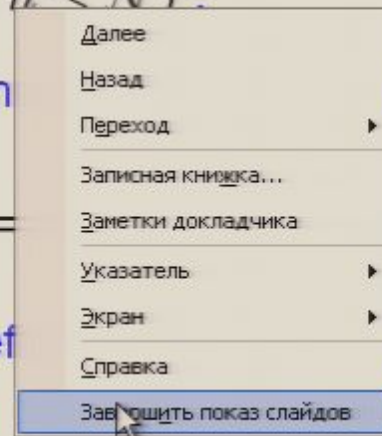
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9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9

periodic boundary conditions

no quantum corrections from α -integration
 --- selection of saddle-point instantons

||
1

Заметки к слайду



$$e^{-\Gamma_{1-loop}} = e^{-\Gamma_0} \text{Det } Q_0 \int_{-i\infty}^{i\infty} D\sigma \left(\prod_{\tau} D/a^2 \right)^{1/2} e^{-\frac{1}{2}\delta\sigma^2 \Gamma_E}$$

ghost factor

conformal mode contribution factor

$$= e^{-\Gamma_0} \times \text{Det} \left(\frac{d}{d\tau} \right) \left[\text{Det} \left(-\frac{1}{\sqrt{D}} \frac{d}{d\tau} D \frac{d}{d\tau} \frac{1}{\sqrt{D}} \right) \right]^{-1/2} = e^{-\Gamma_0}$$

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periodic boundary conditions

→ $\begin{matrix} \parallel \\ 1 \end{matrix}$

no quantum corrections from σ -integration
 --- selection of saddle-point instantons

Dark energy from the microcanonical state of the Universe

Lorentzian Universe with initial conditions set by the saddle-point instanton

Analytic continuation of the instanton solutions:

$$\tau = it, \quad a_L(t) = a(it)$$

Two quasi-exponential branches of the evolution (analogue of DGP model):

$$a_L(t) \sim e^{H_{\pm} t}, \quad t \rightarrow \infty, \quad H_{\pm}^2 = \frac{m_P^2}{B} \left(1 \pm \sqrt{1 - 2BH^2/m_P^2} \right)$$

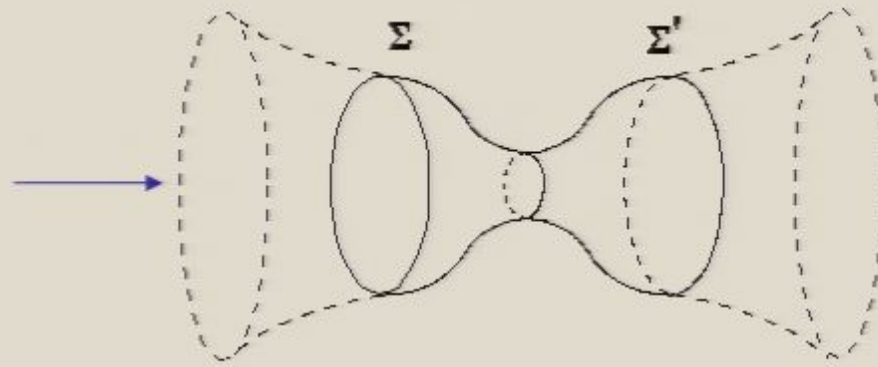
Decay of a composite H^2 in the end of inflation:

$$H^2 \rightarrow 0, \quad H_+^2 \rightarrow \frac{2m_P^2}{B}$$

Original (fundamental or composite --- inflaton field) cosmological constant $H^2 = \Lambda/3$



Cosmological acceleration with $\Lambda_+ = 4\Lambda_{\max} = \frac{6m_P^2}{B}$ --- new QG scale (upper bound on Λ)



Hierarchy problem, strings and extra dimensions

- Early Universe: constraints from large-scale structure formation

$$\Lambda_{\text{GUT}} \simeq \Lambda_{\text{early}} \ll \Lambda_{\text{Planck}}$$

$$\left(10^{16} \text{ GeV}\right)^2 \qquad \left(10^{19} \text{ GeV}\right)^2$$



B_{early}

- Present Universe: cosmological acceleration or dark energy

$$\Lambda_{\text{present}} \simeq 0.7 \epsilon_{\text{crit}}$$

$$\left(10^{-33} \text{ eV}\right)^2$$



B_{present}



$B = B(t)$

should be a function of time

String theory vs EQG density matrix:

Limited instanton ensemble is generated due to the **nonlocal infrared** effect of the conformal anomaly
 — should fit into string theory at its **low energy field-theoretic** level



Constraining the landscape of string vacua:

Mechanism of variable B from string/Kaluza-Klein theory

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