Title: Consistent Lorentz Violation in Flat and Curved Space

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Abstract: Motivated by the severity of the bounds on Lorentz violation in the presence of ordinary gravity, we study frameworks in which Lorentz violation does not affect the spacetime geometry. We show that there are at least two inequivalent classes of spontaneous Lorentz breaking that even in the presence of gravity result in Minkowski space. The first one generically corresponds to the condensation of tensor fields with tachyonic mass, which in turn is related to ghost-condensation. In the second class, realized in the DGP model or in theories of massive gravity, spontaneous Lorentz breaking is induced by the expectation value of sources. The generalization to de-Sitter space is also discussed.

Consistent Lorentz Violation In flat and curved Space

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hep-th/0702117

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Excursions in the Dark

Perimeter Institute

May 29th 2007

Bounds on Lorentz Violation

At phenomenological level, one introduces explicit Lorentz breaking. For example, in the kinetic terms:

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From cosmic rays,

$$1 - c_\gamma^2 = \epsilon < 10^{-23}$$

(Coleman & Glashow)

Including Gravity…

In GR, Lorentz group is gauged. => Explicit breaking is not an option, unless we add degrees of freedom (depart from GR).

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Is it possible to *spontaneously* break Lorentz invariance *without affecting the background* ?

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$$(\Lambda_1 \sim TeV)$$

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$$\partial_t \phi < (10^{-3} eV)^2 \Rightarrow \epsilon \sim \frac{1}{\Lambda_1^4} \partial_\mu \phi \partial_\nu \phi \sim 10^{-60}$$

The effect would become observable if $\Lambda_1 \leq 10 eV$

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Two ways:

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Observation of Lorentz Violation in the near future would strongly hint to modifications of gravity in the IR

OUTLINE

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- Motivations and bounds on LV

- (In)Consistent Lorentz Violation:

  A) High spin tachyon (ghost-)condensation
  B) Massive gravity (DGP)

  - Minkowski Lorentz Violating Solutions

  - LV in (Anti-) de Sitter

- Conclusions

A) High Spin Tachyon Condensation

A natural way to break Lorentz is by following the analogy with the breaking of internal symmetries. This requires an integer spin field that acquires a VEV,

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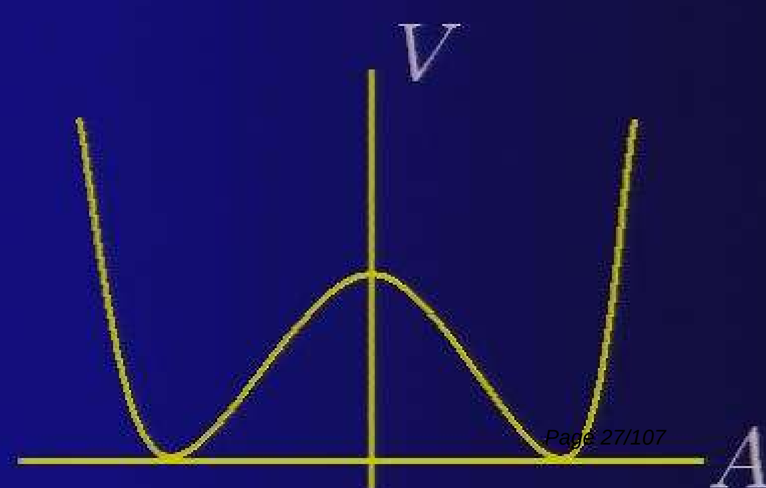
Schematically

$$V = -\mu^2 A^2 + \lambda A^4$$



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$$T_{\mu\nu} = V' A_\mu A_\nu - V g_{\mu\nu}$$

Ghost Condensation

(Arkani-Hamed, Cheng, Luty & Mukohyama)

This is related to ghost condensation, where what condenses is the derivative of a scalar field

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A_μ tachyonic $\longleftrightarrow$ ϕ ghost

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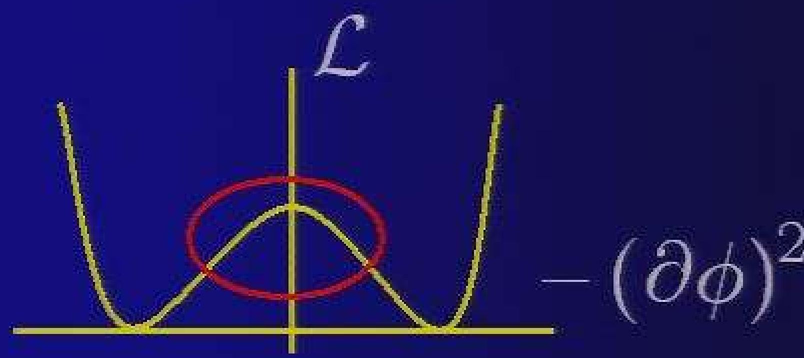
Around the origin ϕ is a ghost.



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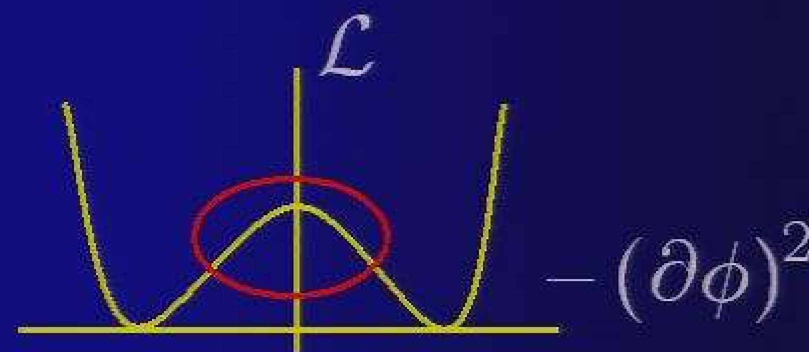

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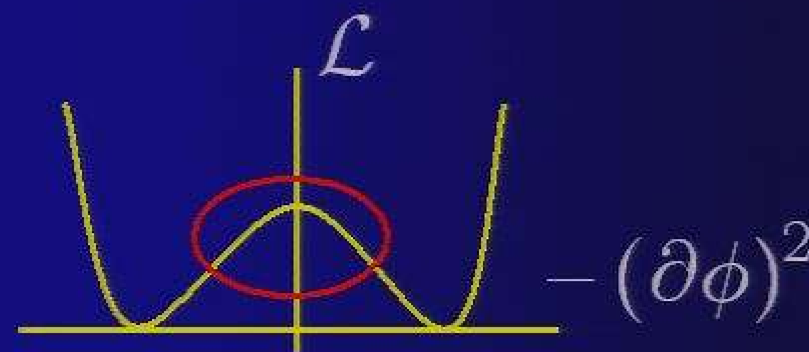

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However there is a consistent effective theory on a Lorentz Violating background, $\phi = \alpha t$

B) Massive Gravity (DGP)

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Another way to break Lorentz is to make gravity ‘massive’.

The breaking of Lorentz is accomplished by the sources, but this does not necessarily curve spacetime.

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The same happens in massive gravity,

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \frac{m^2}{2}(h_{\mu\nu} - \eta_{\mu\nu} h) = \frac{T_{\mu\nu}}{M_4^2}$$

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With $T_{\mu\nu}$ constant, this is pure gauge → flat space

Dvali Gabadadze Porrati (DGP)

$$S = \int d^5x \sqrt{-g}\, \frac{M_*^3}{2}\, R_5 + \int d^4x \sqrt{-h}\, \frac{m_P^2}{2}\, R_4$$

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$$M_*^3 G^{(5)}_{\mu\nu} = \left[T_{\mu\nu} - m_P^2 G^{(4)}_{\mu\nu}\right]\delta(y)$$

$$M_*^3 \left(K_{\mu\nu} - K h_{\mu\nu}\right) = T_{\mu\nu} - m_P^2\, G^{(4)}_{\mu\nu}$$

Dvali Gabadadze Porrati (DGP)

At linear level, DGP is a theory of massive gravitons

$$h_{\mu\nu}(x,y) = \int_0^\infty dm\, h_{\mu\nu}^{(m)}(x)\, \psi^{(m)}(y)$$

$$\left(\partial_y^2 + m^2 + m^2 r_c \delta(y)\right) \psi^{(m)}(y) = 0$$

$$(\mathcal{E}h^{(m)})_{\mu\nu} + \frac{m^2}{2}(h_{\mu\nu}^{(m)} - \eta_{\mu\nu} h^{(m)}) = \frac{1}{M_5^3} \int dy\, \psi^{(m)}(y) T_{\mu\nu}(y)$$

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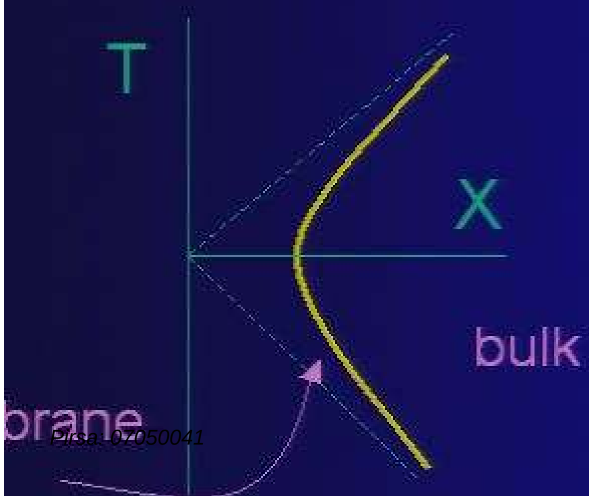

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To linear order this works for any constant $T_{\mu\nu}$

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ii) Gauss-Codacci equations:

$$G_{\mu\nu} = -\frac{\Lambda_5}{2M_5^3} g_{\mu\nu} + KK_{\mu\nu} - K_\mu^\rho K_{\rho\nu} - \frac{1}{2} g_{\mu\nu}(K^2 - K_{\rho\sigma}K^{\rho\sigma}) + E_{\mu\nu}$$

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bulk cosmological constant

'dark radiation'

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$$E^{\mu}_{\nu} = -(w+1)\frac{\rho_0^2}{8M_5^6}\,\mathrm{diag}\left(-1,\ \frac{1}{3},\ \frac{1}{3},\ \frac{1}{3}\right)$$

$$\Lambda_5 = (1+3w)\frac{\rho_0^2}{12M_5^3}$$

Full Solution

(see also Csaki, Erlich, Grojean)

From the symmetries, the bulk is Schwarzchild-Anti de Sitter

$$ds_5^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Omega_\kappa^2 \qquad f(R) = \kappa - \frac{R^2}{\ell^2} - \frac{C}{R^2}$$

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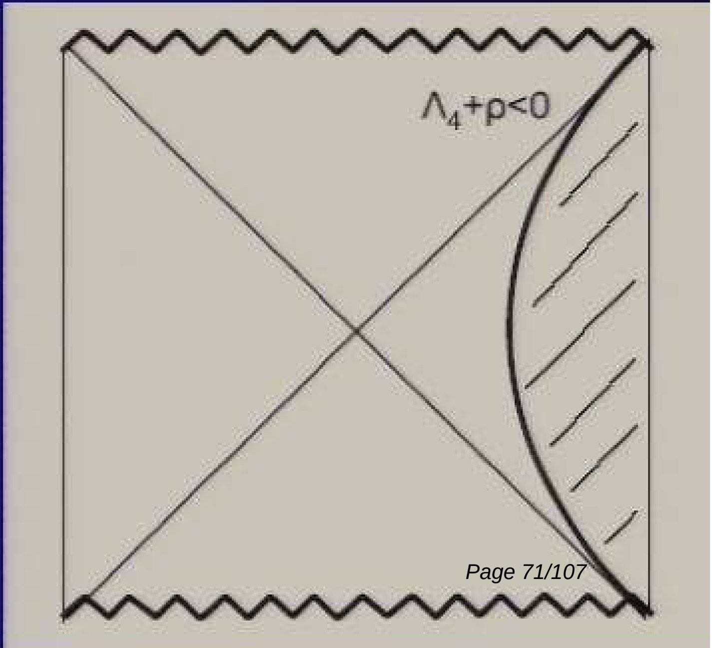
The brane is a trajectory (T(t), R(t))

Upon gauge choice

the induced metric is,

$$ds_4^2 = -dt^2 + R^2(t)d\Omega_\kappa^2$$

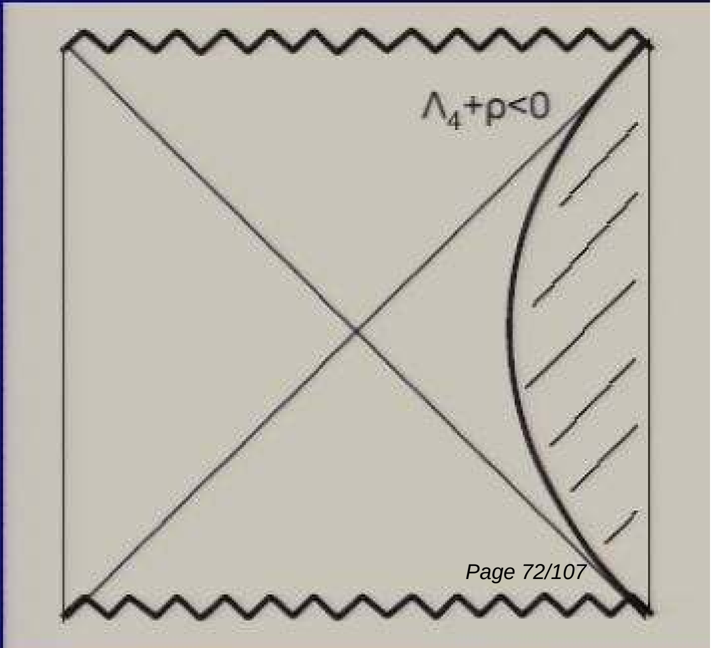


The Israel junction conditions determine the trajectory

$$6\epsilon\, M_5^3\,\frac{\sqrt{f(R)+\dot{R}^2}}{R} = -3M_4^2\,\frac{\dot{R}^2+\kappa}{R^2}+\rho(t)+\Lambda_4$$

For $\rho_{\text{Total}} < 0$ the bulk is the exterior → no singularities/sources.

This can be achieved with negative c.c. and $\rho > 0$



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$$ds_5^2 = dy^2 - \frac{\sqrt{C}}{\ell}\, \frac{\sinh^2\left(2(|y|+y_0)/\ell\right)}{\cosh\left(2(|y|+y_0)/\ell\right)}\, dt^2 + \sqrt{C}\,\ell \cosh\left(2(|y|+y_0)/\ell\right) d\vec{x}^2$$

$$\cosh^2(2y_0/\ell) = -\frac{24}{1+w}\left(\frac{\ell\rho_0}{6M_5^3}\right)^2$$

$$\ell^2 = \frac{6M_5^3}{\Lambda_5}$$

Stability

Solutions may be affected by

- ghosts
- Jeans instability
- tachyons

For the simplest LV solution K_{tt} is non-zero

$$\pi = \frac{\rho_0}{12M_4}t^2$$

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Consider fluctuations, $\pi = \pi_0 + \delta$

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$$\rho_0 < \frac{M_4^2}{r_c^2} \approx (10^{-3} eV)^4$$

For large ρ_0 there is Jeans instability but no ghosts.

‘Tachyonic’ instability

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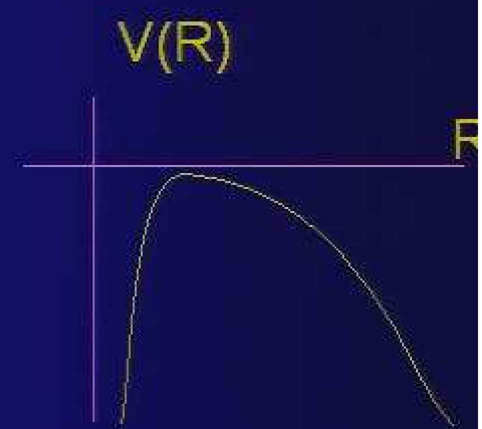
(Without DGP term)   Friedman equation:

$$\dot{R}^2 - \frac{(\Lambda_4 + \rho)^2}{36 M_5^6} R^2 = -\kappa$$



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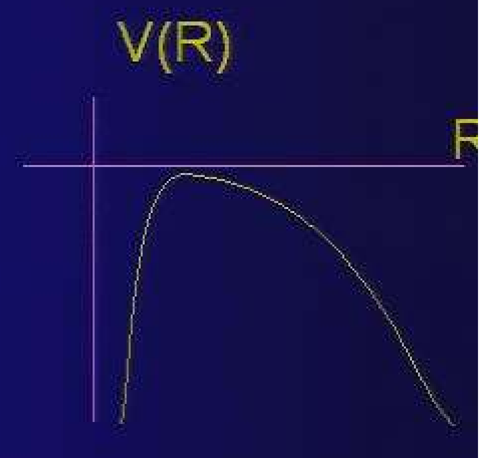

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$$\Lambda_5 < 0 \qquad C < 0$$

Including the DGP term,

$$V'' = -\frac{(w'+1)\rho_0' \left[(3w'-1)\Lambda_4 + 2(3w'+1)\rho_0'\right]}{6M_5^6 + [\Lambda_4 + \rho_0']M_4^2}$$

De Sitter Breaking

Can we break the isometries of de-Sitter?

Take a massive graviton on dS,

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \frac{m^2}{2}(h_{\mu\nu} - \hat{g}_{\mu\nu} h) = \frac{T_{\mu\nu}}{M_4^2}$$

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→ is $h_{\mu\nu} - \hat{g}_{\mu\nu} h = -\frac{2\,T_{\mu\nu}}{m^2\, M_4^2}$ pure gauge?

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However, $\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} \propto T_{\mu\nu}$

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Hence, the solution of the massive gravity equations is

$$h^{\mu}_{\nu} = \frac{2}{(m^2 - 2H^2)M_4^2} \frac{\rho_0}{a^{3(1+w)}} \mathrm{diag}(w + \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$$

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Massless and massive gravitons respond to a fluid with opposite sign !

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For a de-Sitter brane in 5D, the perturbation induced by a fluid is

$$h^{\mu}_{\nu}(x,y) = \left(\int \frac{\psi^{(m)}(y)\psi^{(m)}(0)}{2H^2 - m^2}\, dm\right) \frac{2\,\rho_0}{M_4^2\, a^{3(1+w)}}\, \mathrm{diag}(w + \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$$

The metric on the brane stays de Sitter if the zero mode cancels the KK tower:

$$\frac{1}{2H^2}\psi_{z.m.}(0)^2 + \int_{KK} dm\, \frac{\psi^m(0)^2}{2H^2 - m^2} = 0$$

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Can we realize this ?? Yes, if $\Lambda_4 = r_c \Lambda_5 \quad (H_{bulk} = H_{brane}$

De Sitter Breaking

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i) Israel junction conditions

$$M_4^2 G_{\mu\nu} - 2M_5^3 (K_{\mu\nu} - g_{\mu\nu} K) = T_{\mu\nu} - \Lambda_4 g_{\mu\nu}$$

ii) Gauss-Codacci equations:

$$G_{\mu\nu} = -\frac{\Lambda_5}{2M_5^3} g_{\mu\nu} + K K_{\mu\nu} - K_\mu^\rho K_{\rho\nu} - \frac{1}{2} g_{\mu\nu} (K^2 - K_{\rho\sigma} K^{\rho\sigma}) + E_{\mu\nu}$$

De Sitter Breaking

$$w = -\frac{1}{3}$$

$$\Lambda_4 = r_c \Lambda_5$$

$$E_0 = -\frac{1}{12}\left(\frac{\rho_0}{M^3}\right)^2$$

(Stealth brane)

The bulk is 5D Sch-dS. The Friedman equation is

$$6\epsilon M_5^3 \frac{\sqrt{-C}}{R^2} = -3M_4^2 H^2 + \Lambda_4 + \rho(t)$$

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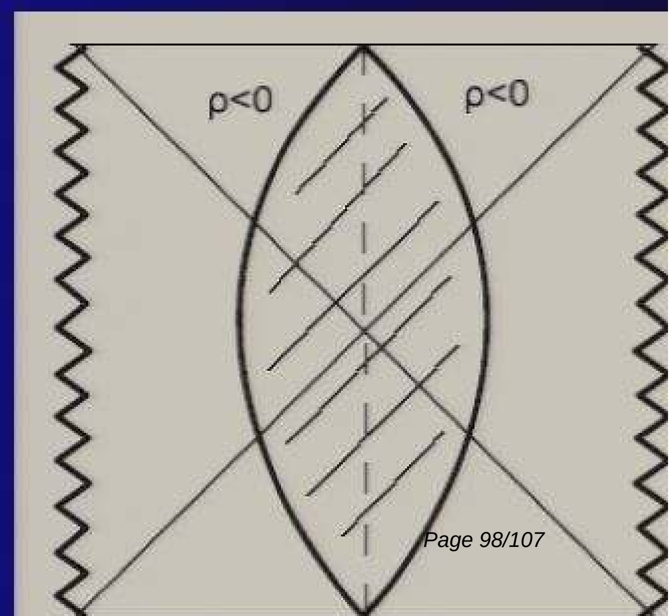
C must be negative. The naked singularity can be avoided by taking the 'exterior'.

However, this requires $\rho < 0$!!



Anti De Sitter Breaking

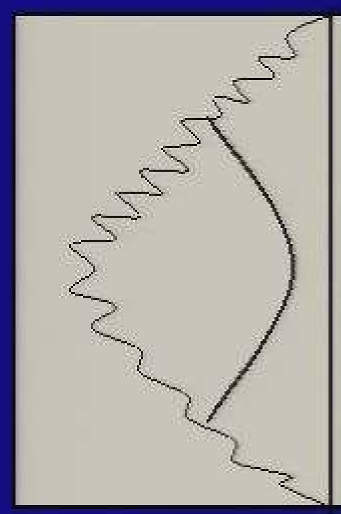
$$w = -\frac{1}{3}$$

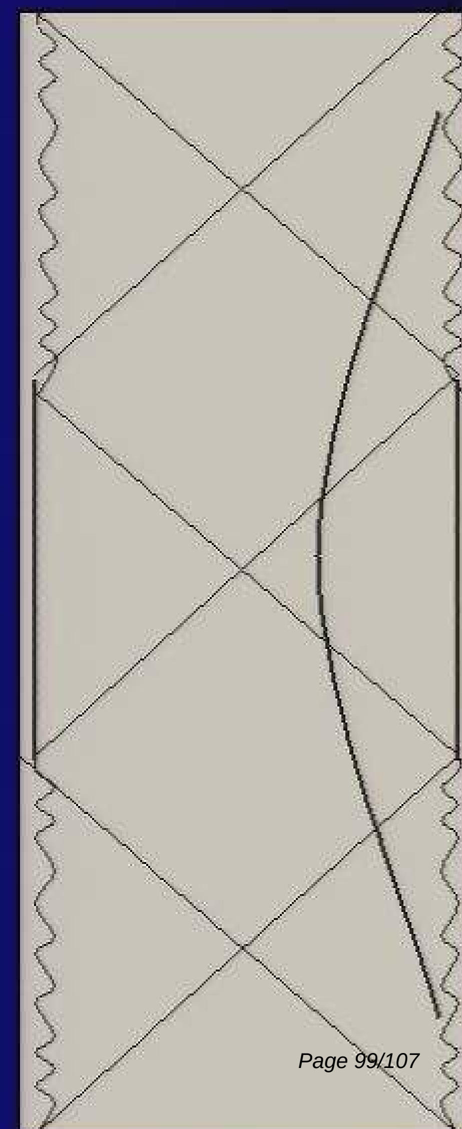
$$\begin{aligned} \Lambda_4 &= r_c \Lambda_5 \\ E_0 &= -\frac{1}{12}\left(\frac{\rho_0}{M^3}\right)^2 \end{aligned}$$

Impossible.

Massless and massive gravitons contribute with the same sign.

Nonlinear solution is singular.





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Bounds on LV operators:

$$\frac{1}{\Lambda_1^4}\partial_\mu\phi\partial_\nu\phi\, F^\mu_\alpha F^{\nu\alpha} \rightarrow \frac{1}{\Lambda}K_{\mu\nu}\, F^\mu_\alpha F^{\alpha\nu}$$

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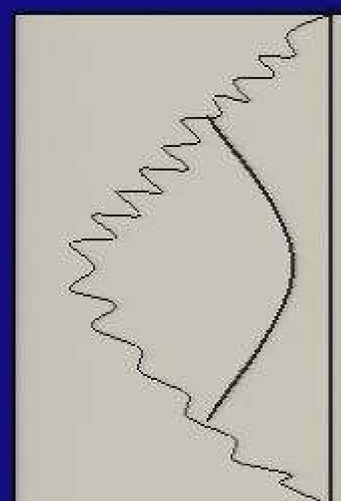

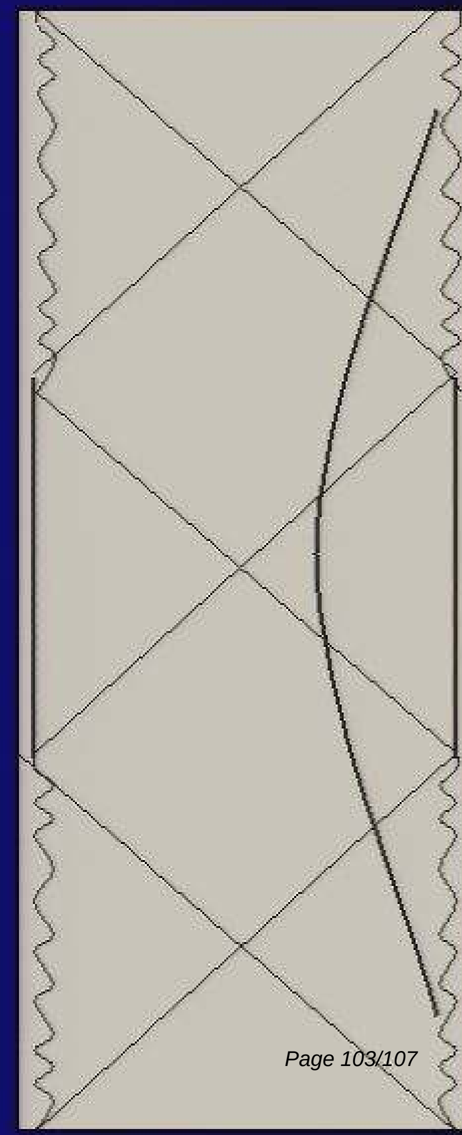

Summary

• Flat space: Lorentz can be broken only if there is no massless graviton.

• dS: Breaking the isometries requires cancellation between massless and massive gravitons.

In DGP this requires negative energy density.

LV source redshifts away.

•AdS: Impossible. Massless and massive gravitons contribute with the same sign.

Nonlinear solution is singular.

Is this generic??

Conclusions

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- Most studies consider explicit Lorentz breaking. However, in the presence of gravity it should be *spontaneous*.
- Bounds on LV relaxed if source of LV does not gravitate.
- Breaking Lorentz without curving space requires modifications of gravity:

A) High Spin Tachyon Condensation: it only makes sense in the Lorentz Violating phase.

B) Massive Gravitons (Extra-dimensions): Lorentz violating sources don't curve spacetime! (screening)

- New solutions: arbitrary fluid on the brane (flat)
- Size of LV limited and small
- in de Sitter -> negative energy density

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Observation of Lorentz Violation in the future would strongly hint to modifications of gravity in the IR !!