Title: Gravity Probes and Graviton Hair

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Abstract:

GRAVITON (QUANTUM) HAIR AND GRAVITY PROBES

Gia Dvali

NYU

Based on: hep-th/0605295

hep-th/0607144

&

hep-th/0612016

Gregory Gabadadze,

Oriol Pujolas, Rakib Rahman.

+ SOME NEW IDEAS.

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OUTLINE

* Black hole no hair for massive fields;
* Loophole: black holes with massive quantum integer spin hair;
* Detecting quantum hair in the Lab (Planck scale physics?)
* Implications for Planck scale physics, large distance modified gravity, QCD ....
* Domain walls as probes of gravity

COMPLEMENTARY (VERY INTERESTING) POSSIBILITY:

BLACK HOLE HAIR UNDER **DISCRETE** GUAGE SYMMETRIES

Krauss & Wilczek
Preskill & Krauss

WHEN,

$$U(1) \longrightarrow Z_N$$

BLACK HOLES CAN HAVE Z_N CHARGE.

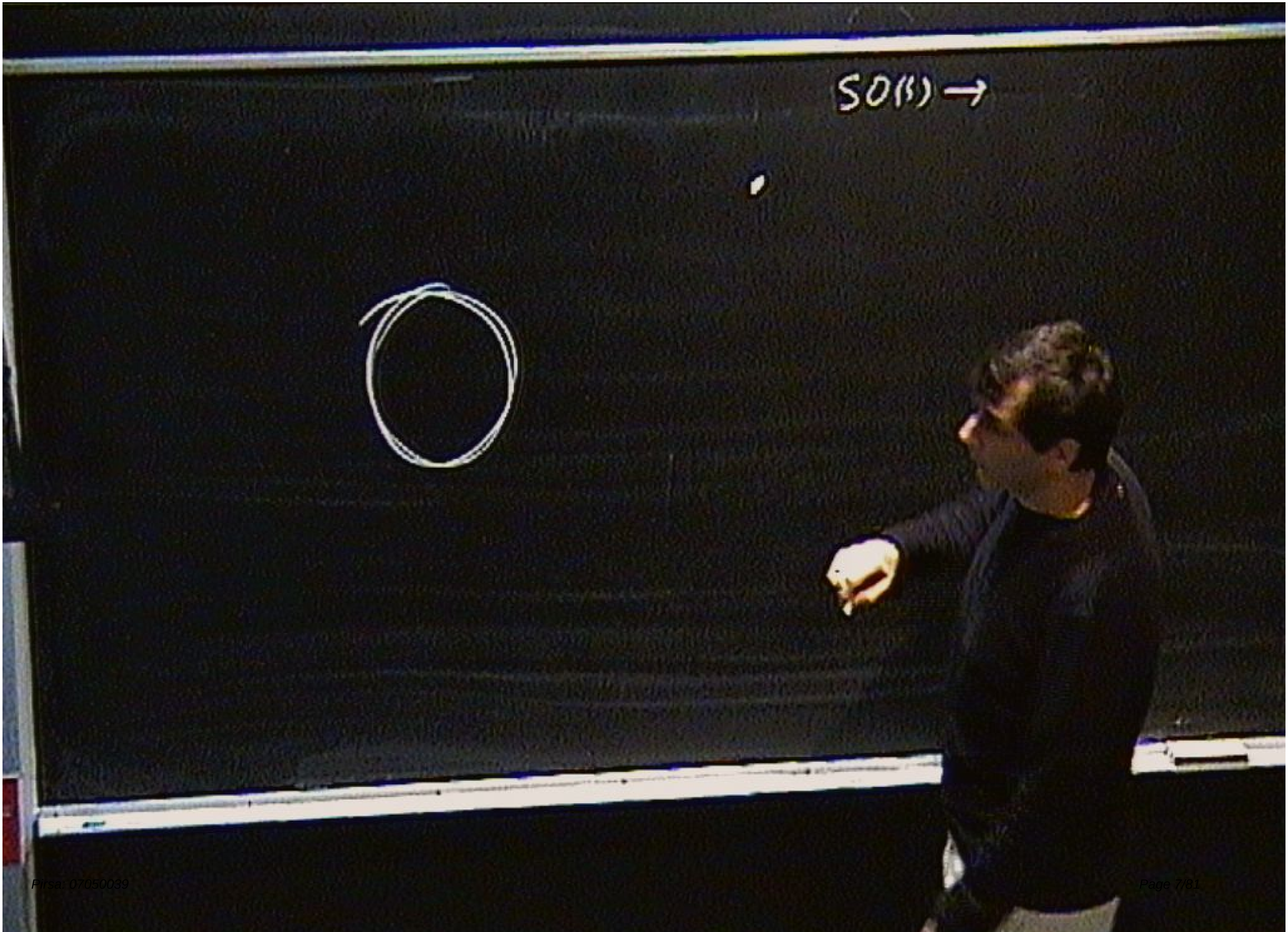
GOOD QUANTUM NUMBERS FOR THE BLACK HOLES

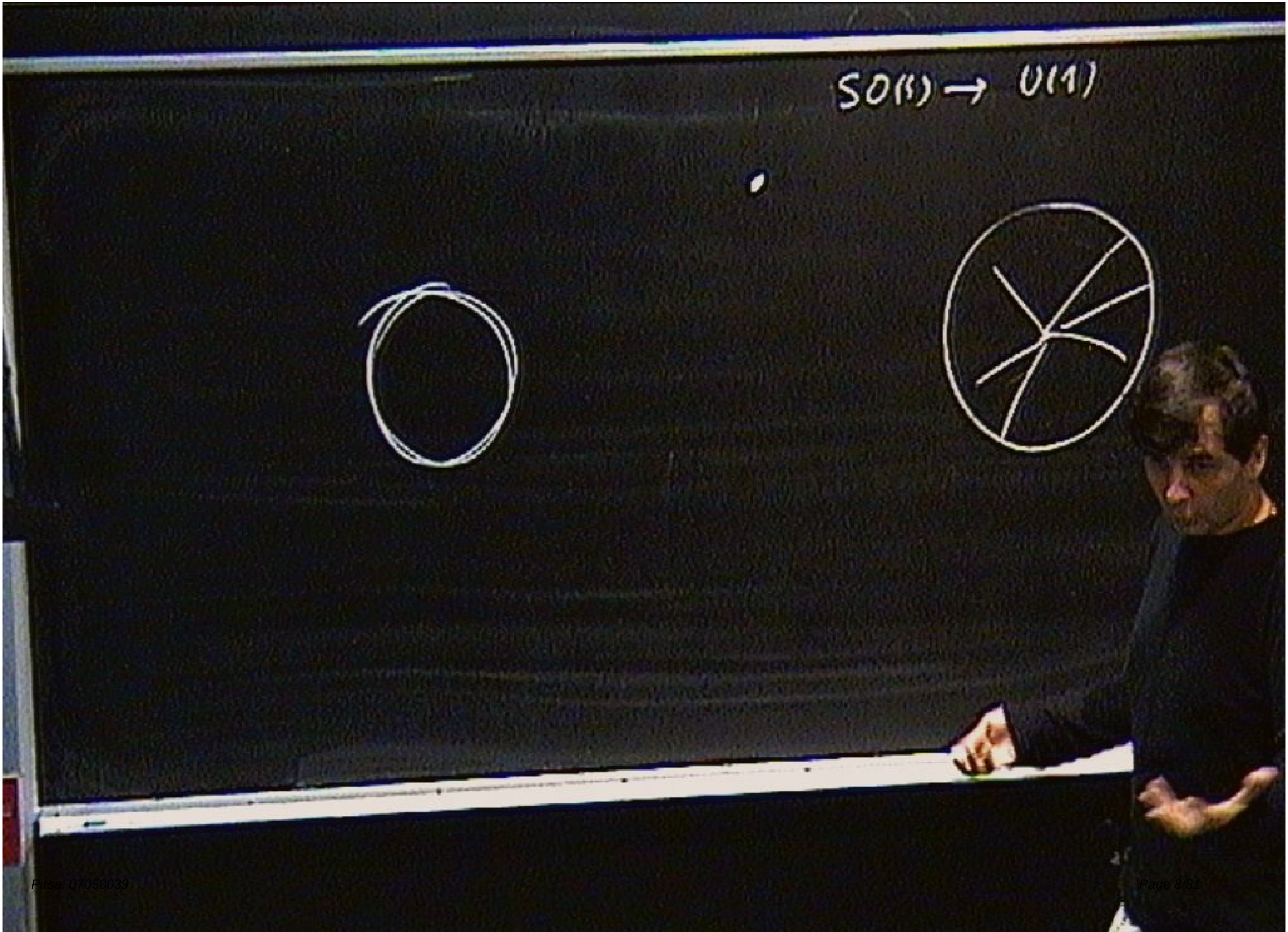
MASS M

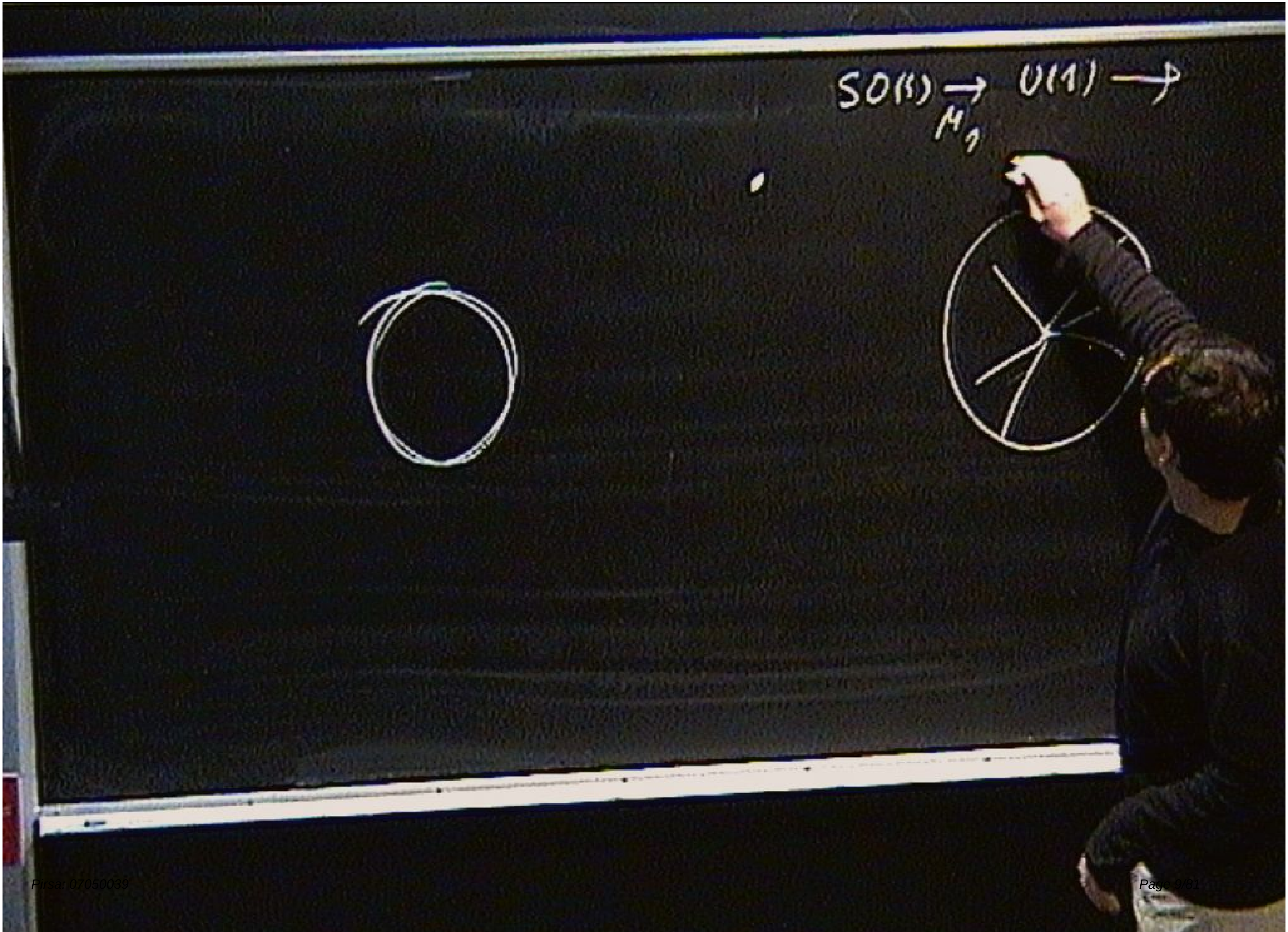
SPIN J

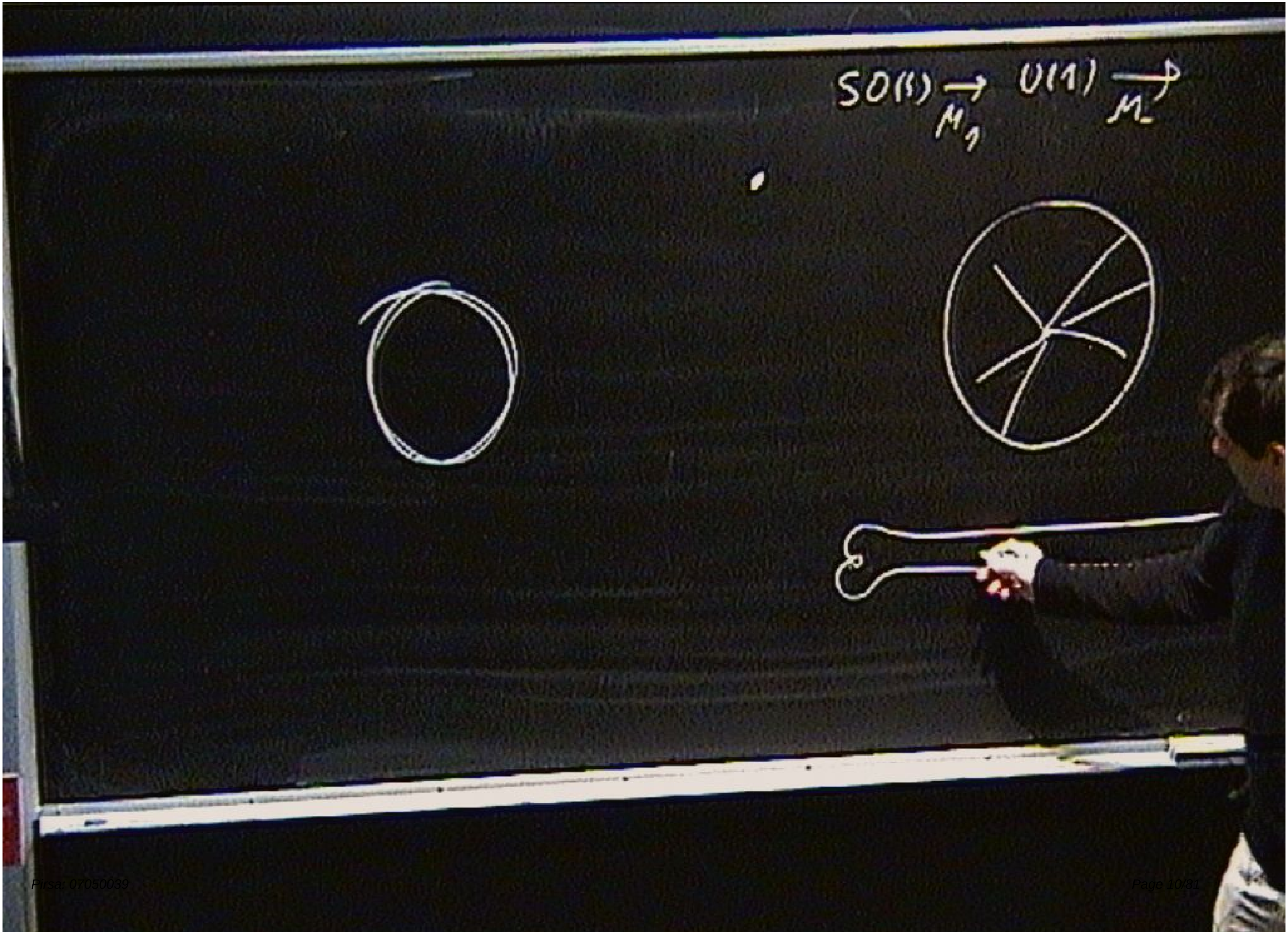
AND CHARGES UNDER THE MASSLESS GAUGE FIELDS

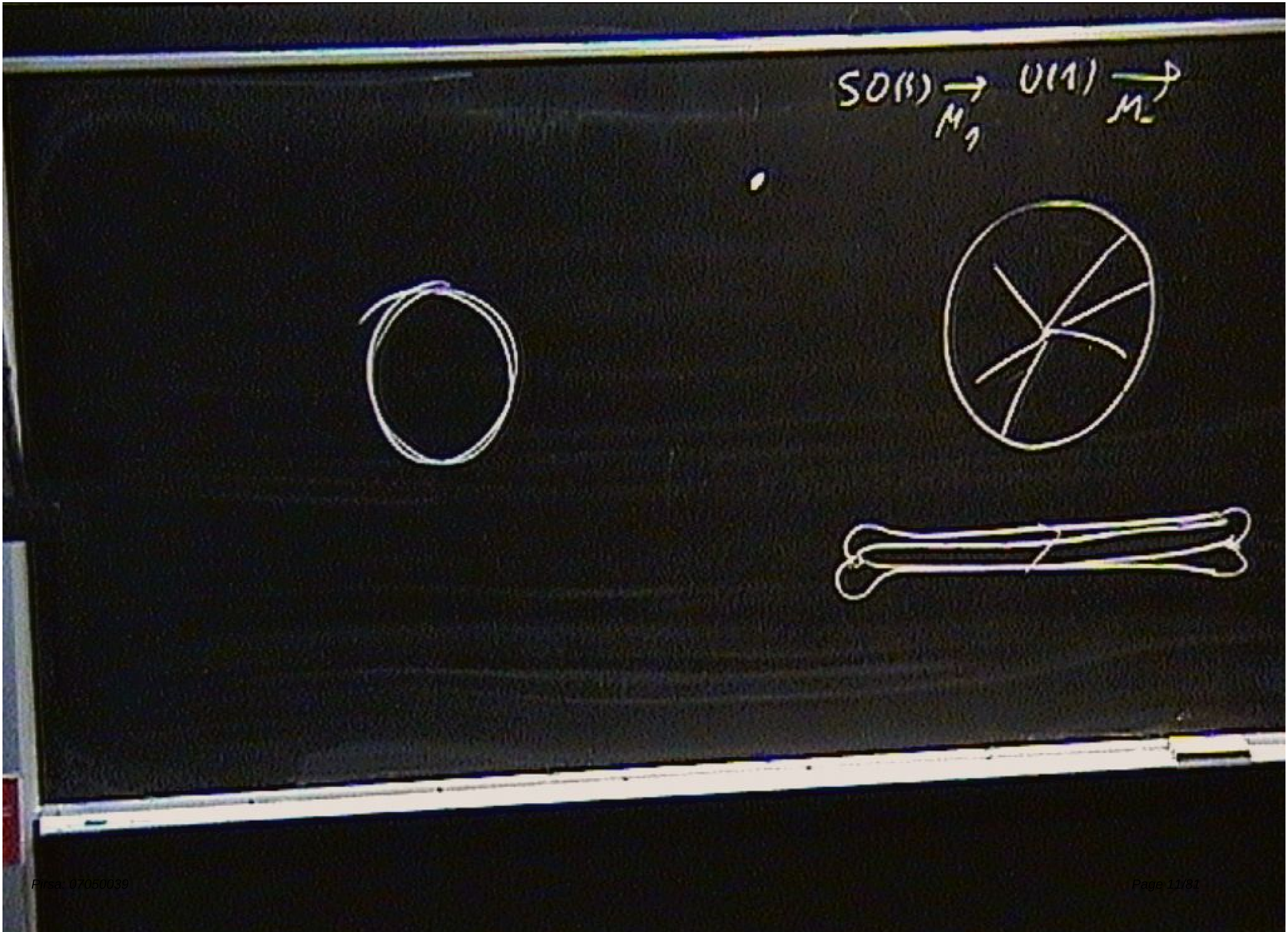
(E.G. ELECTRIC CHARGE Q)

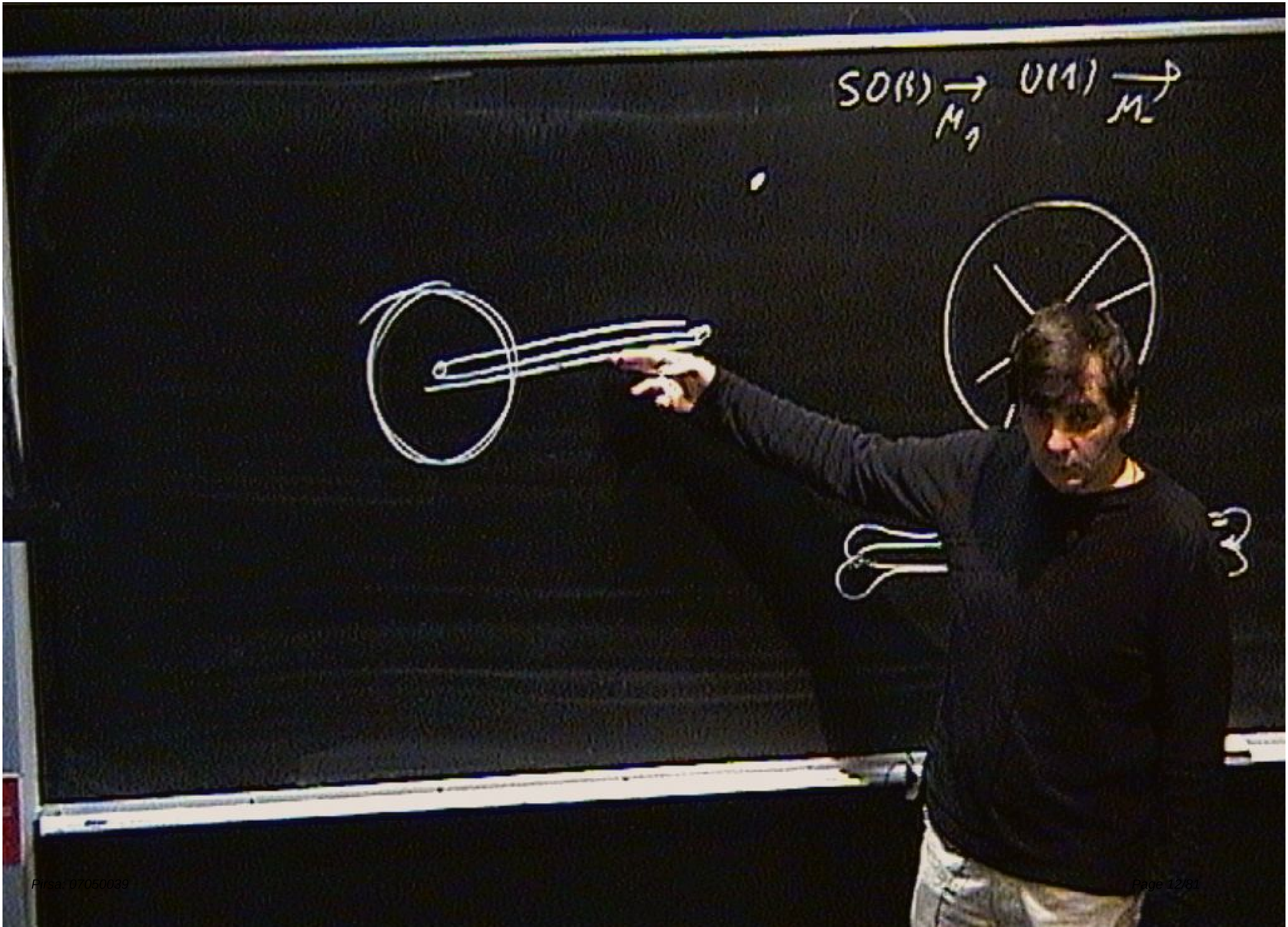

SO(3) →


SO(3) → U(1)

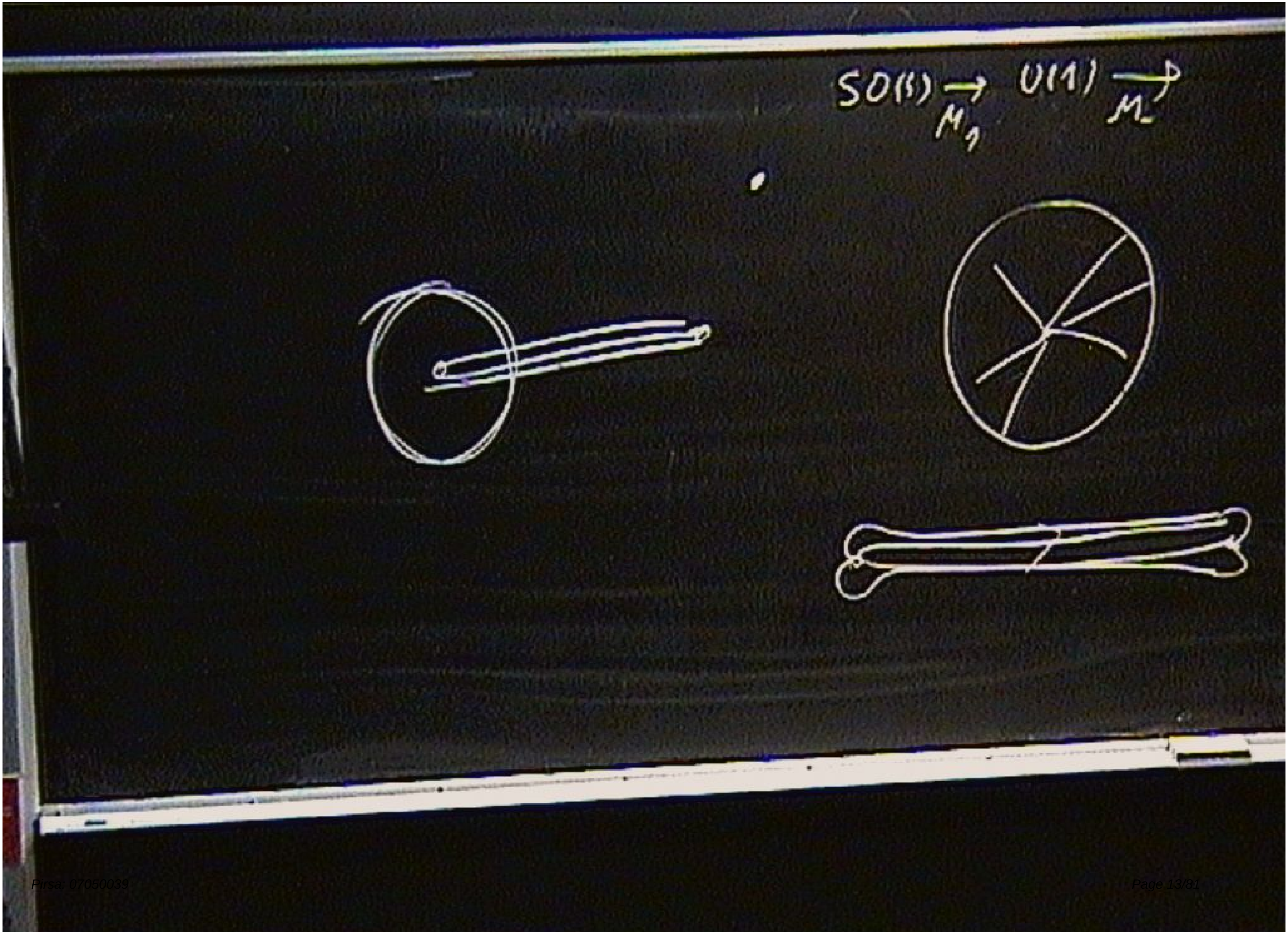

SO(3) → U(1) →
M1


SO(3) → U(1) →
M1

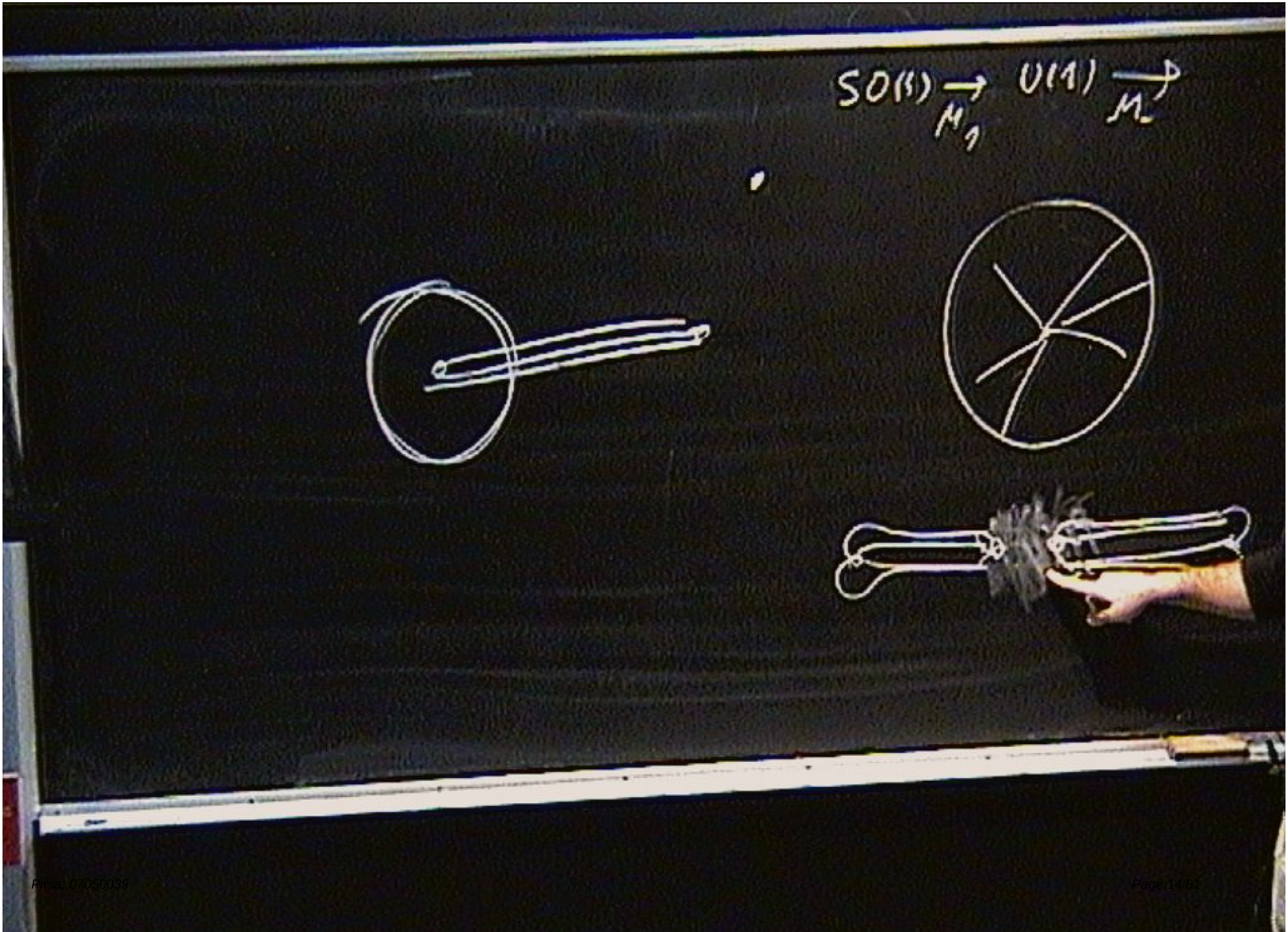

$SO(3) \xrightarrow[M_1]{} U(1) \xrightarrow[M_2]{}$



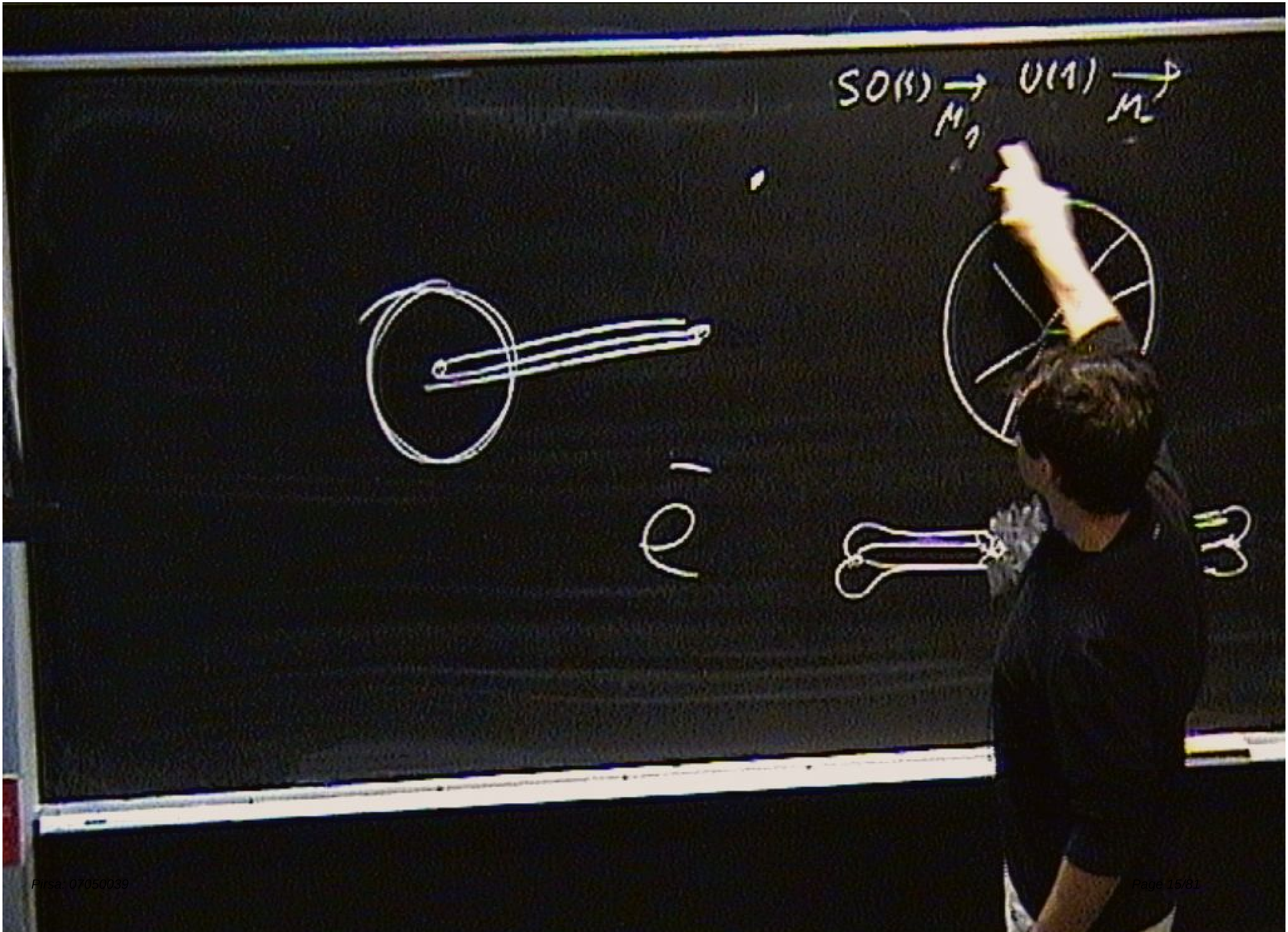
SO(3) $\xrightarrow[M_1]{}$ U(1) $\xrightarrow[M_-]{}$

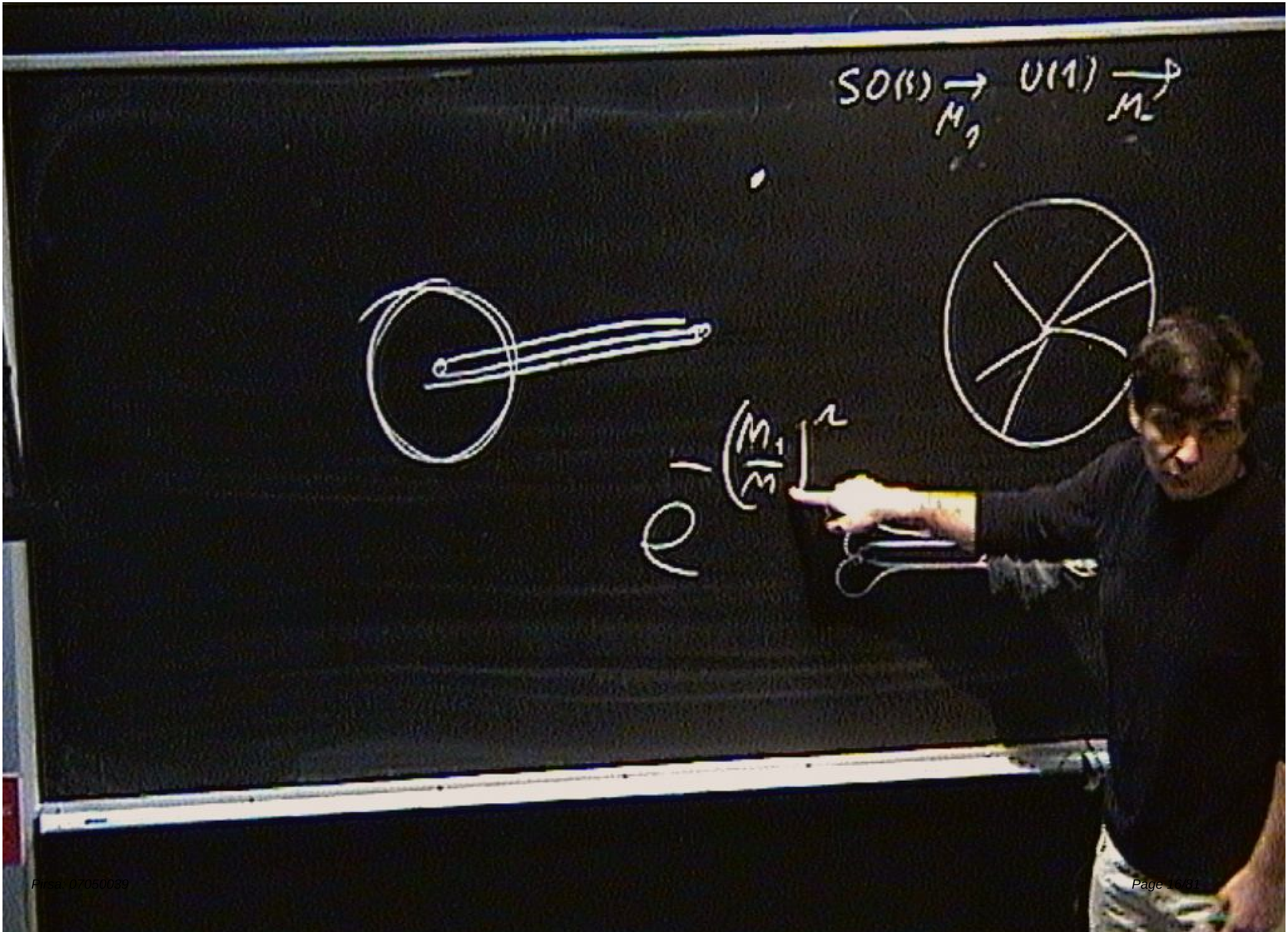


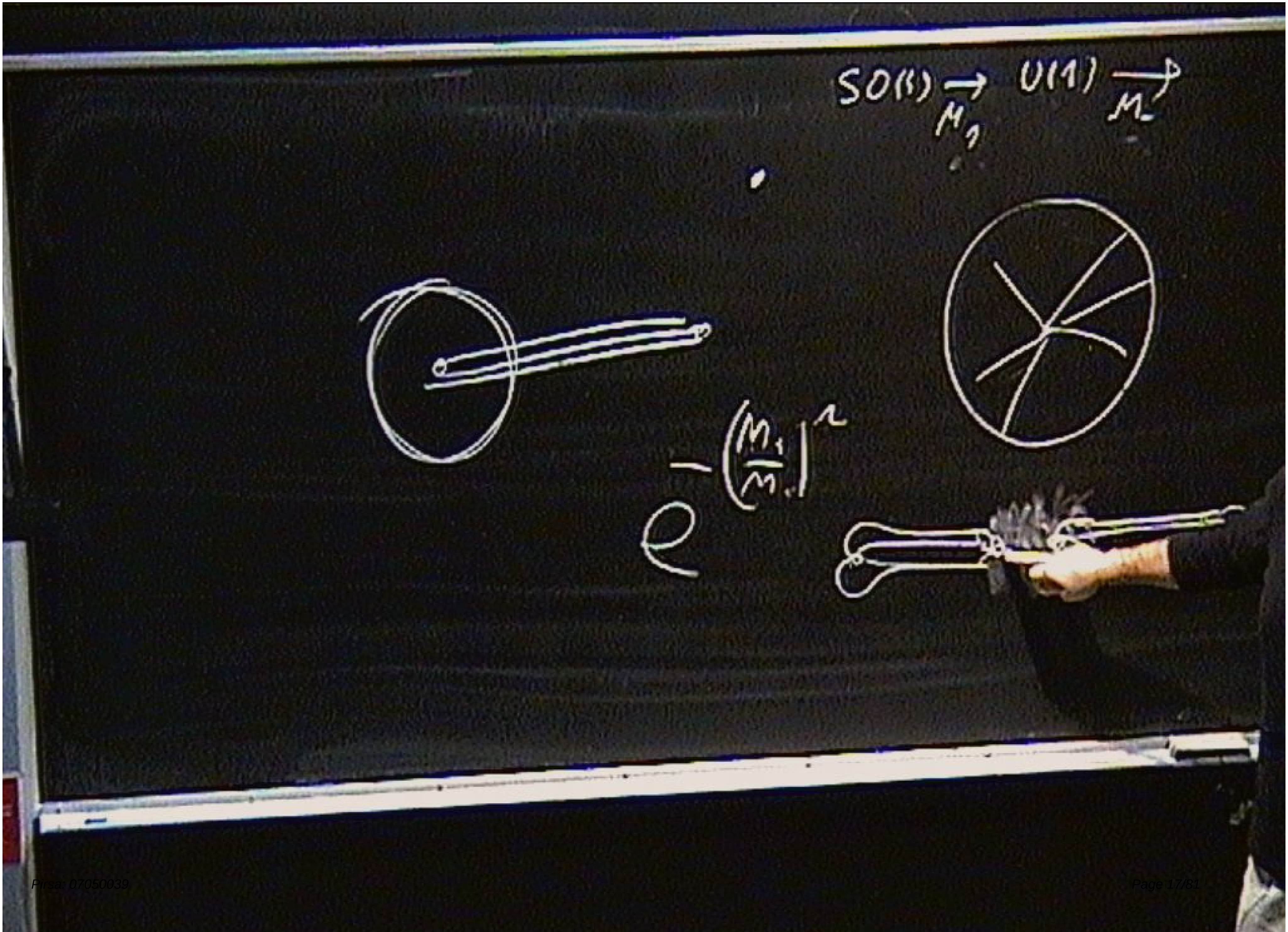
$SO(3) \xrightarrow{M_1} U(1) \xrightarrow{M_2}$

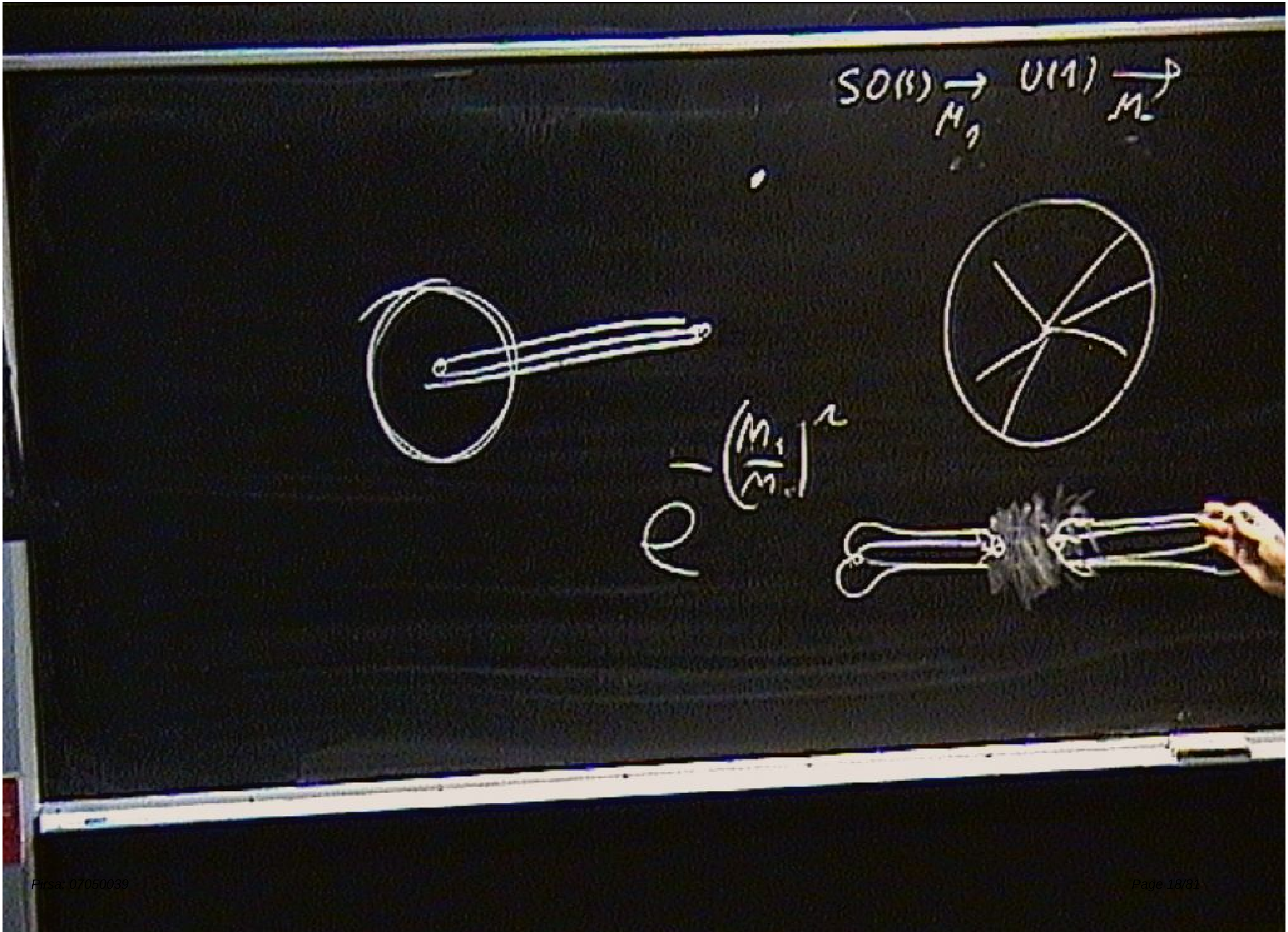


$SO(3) \xrightarrow[M_1]{} U(1) \xrightarrow[M_2]{}$


SO(3) → U(1) →
M1


SO(3) $\xrightarrow[M_1]{}$ U(1) $\xrightarrow[M_2]{}$
$e^{-\left(\frac{M_1}{M}\right)^2}$


SO(3) → U(1) →
M1
M2
e^-(M1/M2)^2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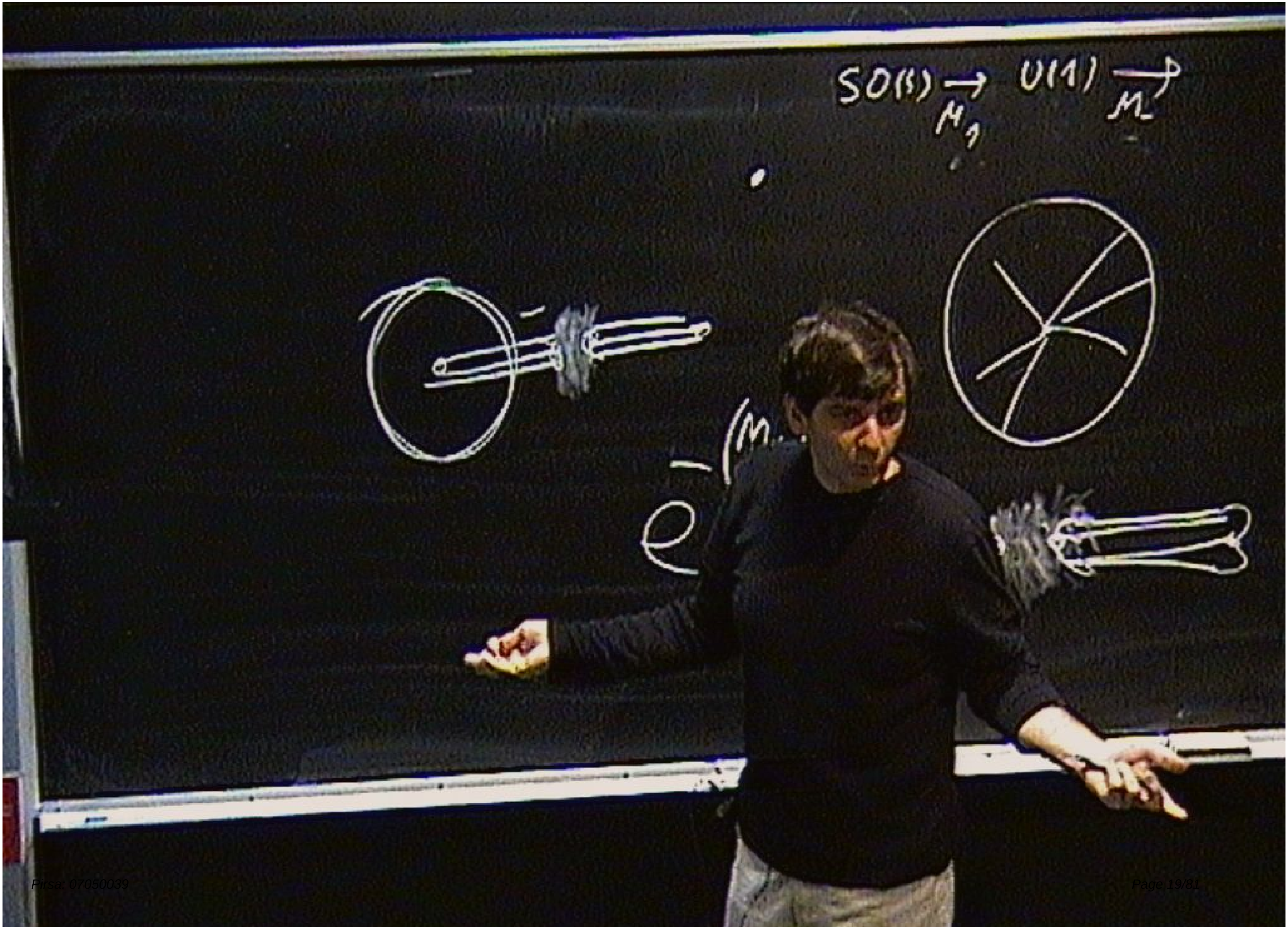
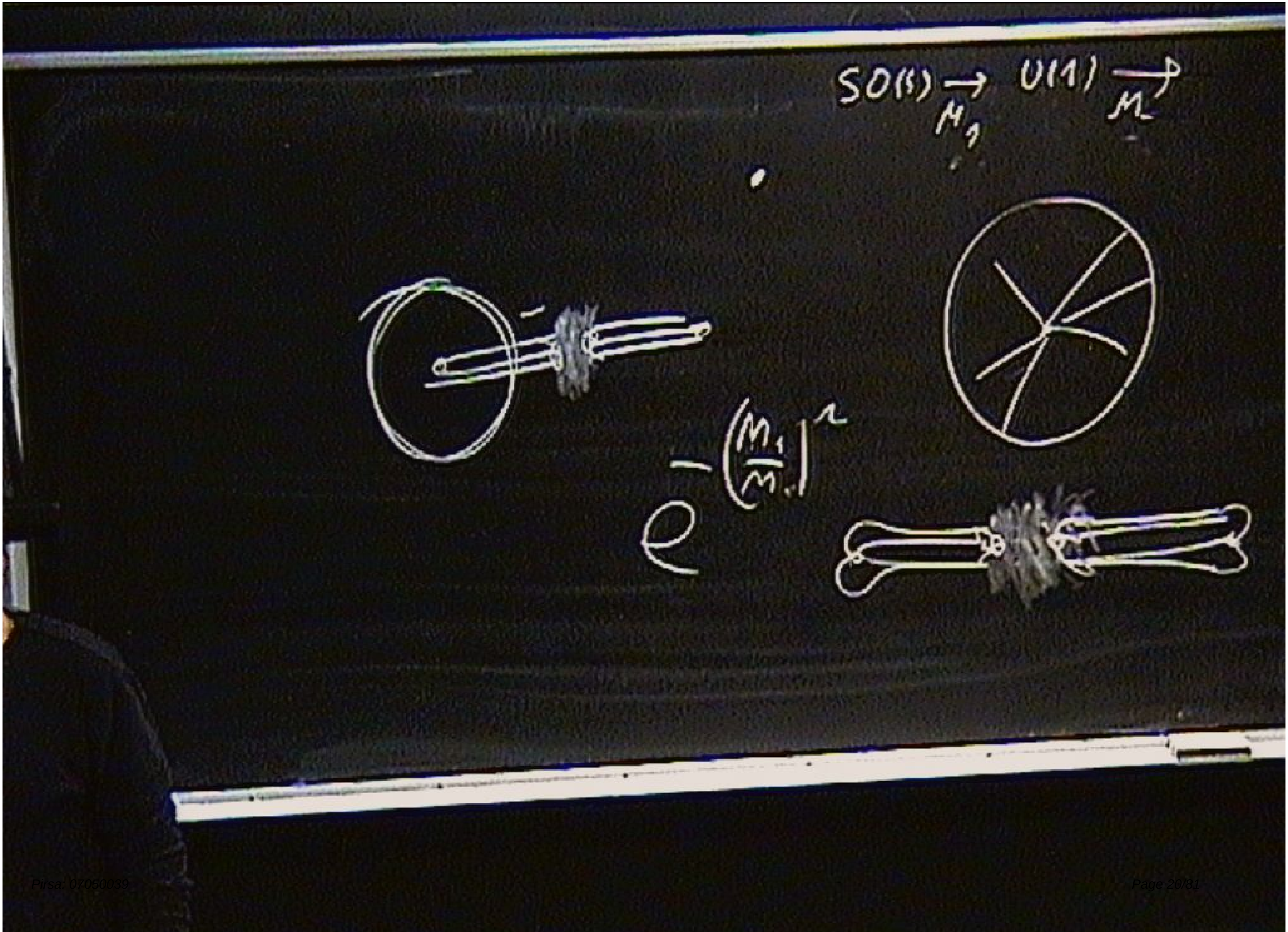
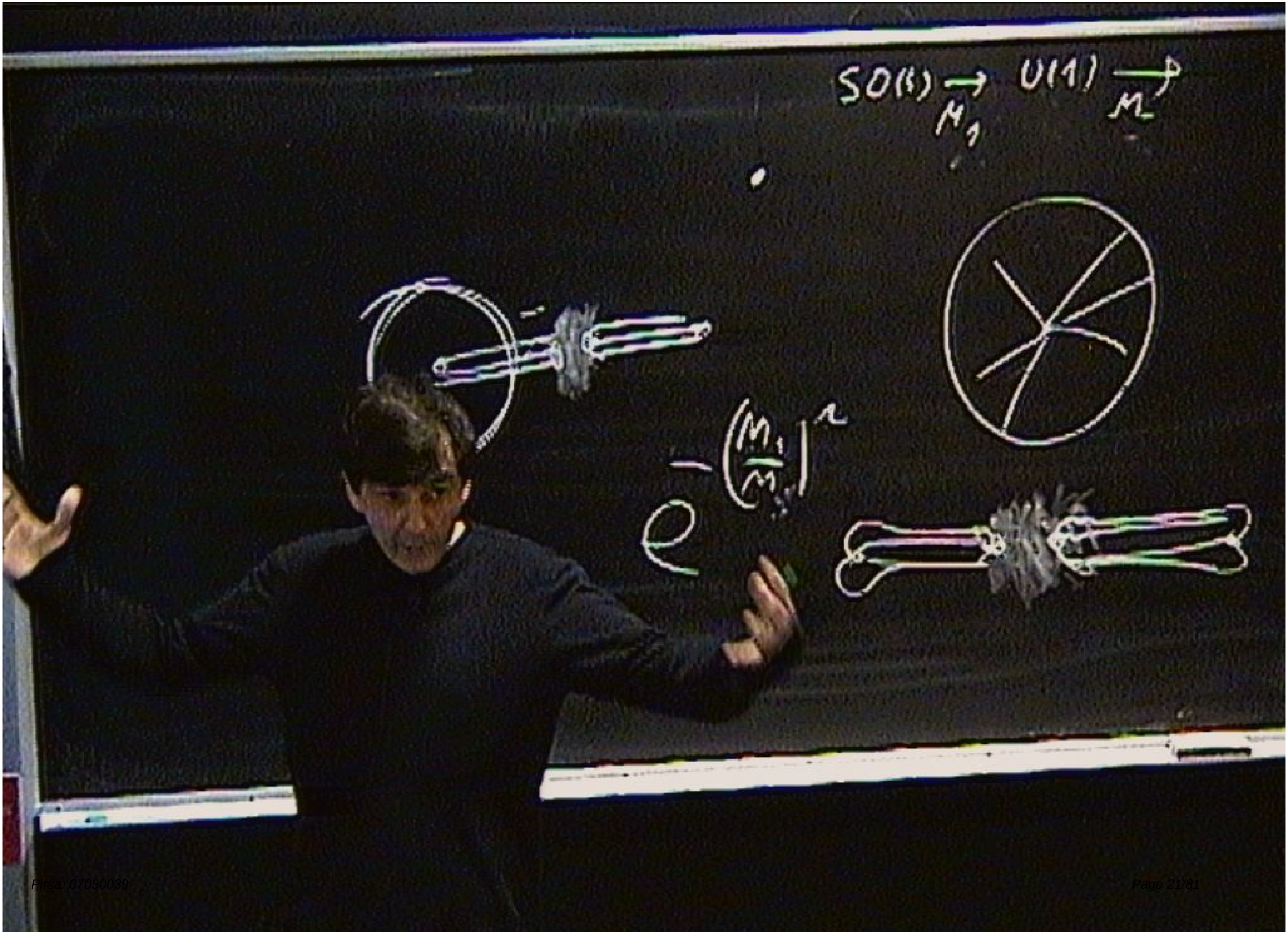
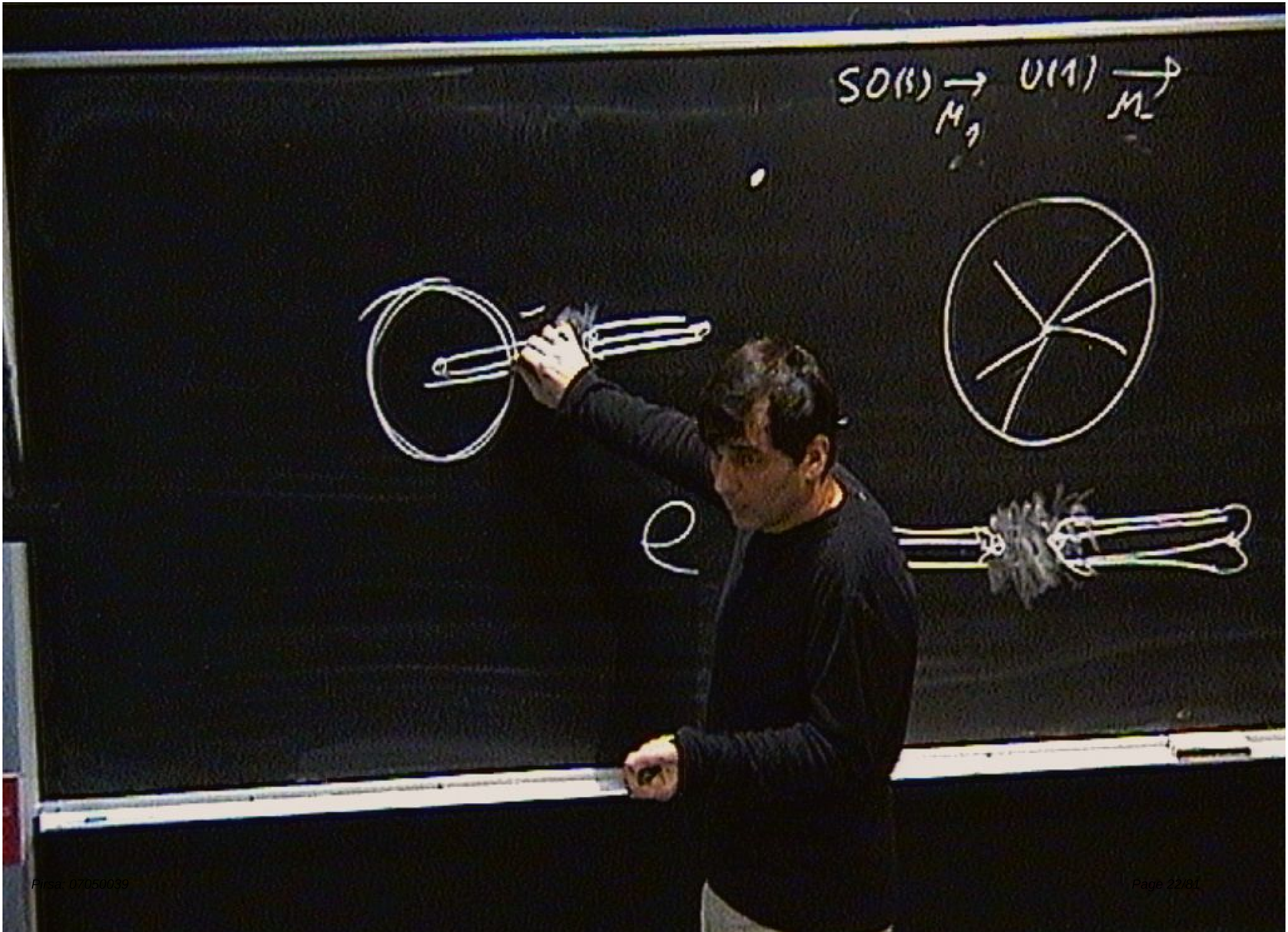

$SO(3) \xrightarrow[M_1]{} U(1) \xrightarrow[M]{}$


$SO(5) \xrightarrow[M_1]{} U(1) \xrightarrow[M_2]{}$
$e^{-\left(\frac{M_1}{M_2}\right)^2}$


SO(1) → U(1) →


SO(3) → U(1) →
M_1
M
e

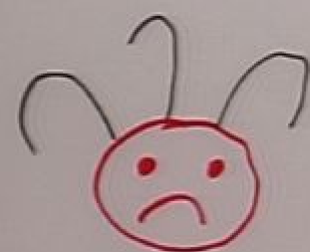
GOOD QUANTUM NUMBERS
FOR THE BLACK HOLES

MASS M
SPIN J

AND CHARGES UNDER THE
MASSLESS GAUGE FIELDS
(E.G. ELECTRIC CHARGE Q)

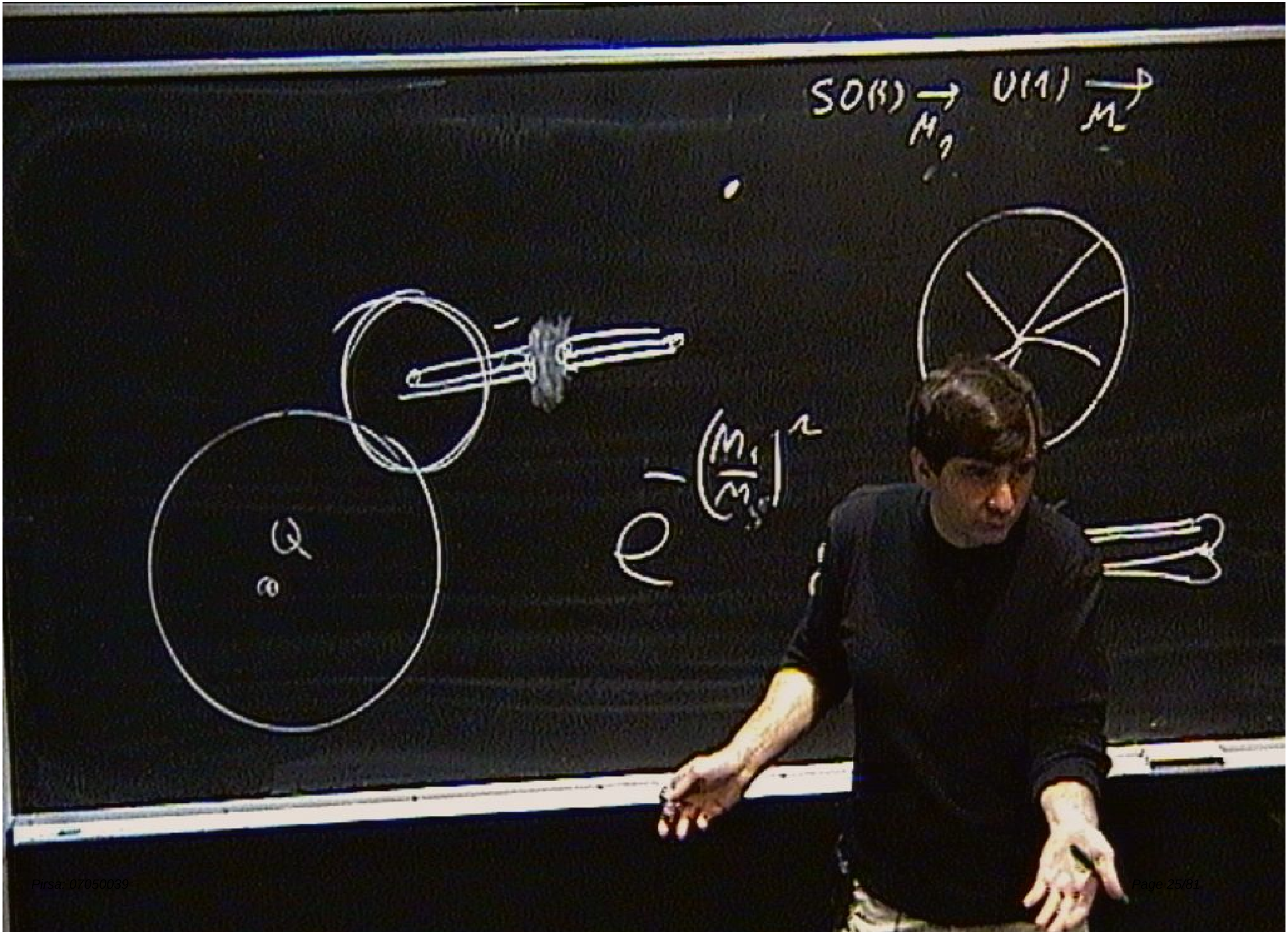
BLACK HOLES HAVE NO MASSIVE HAIR

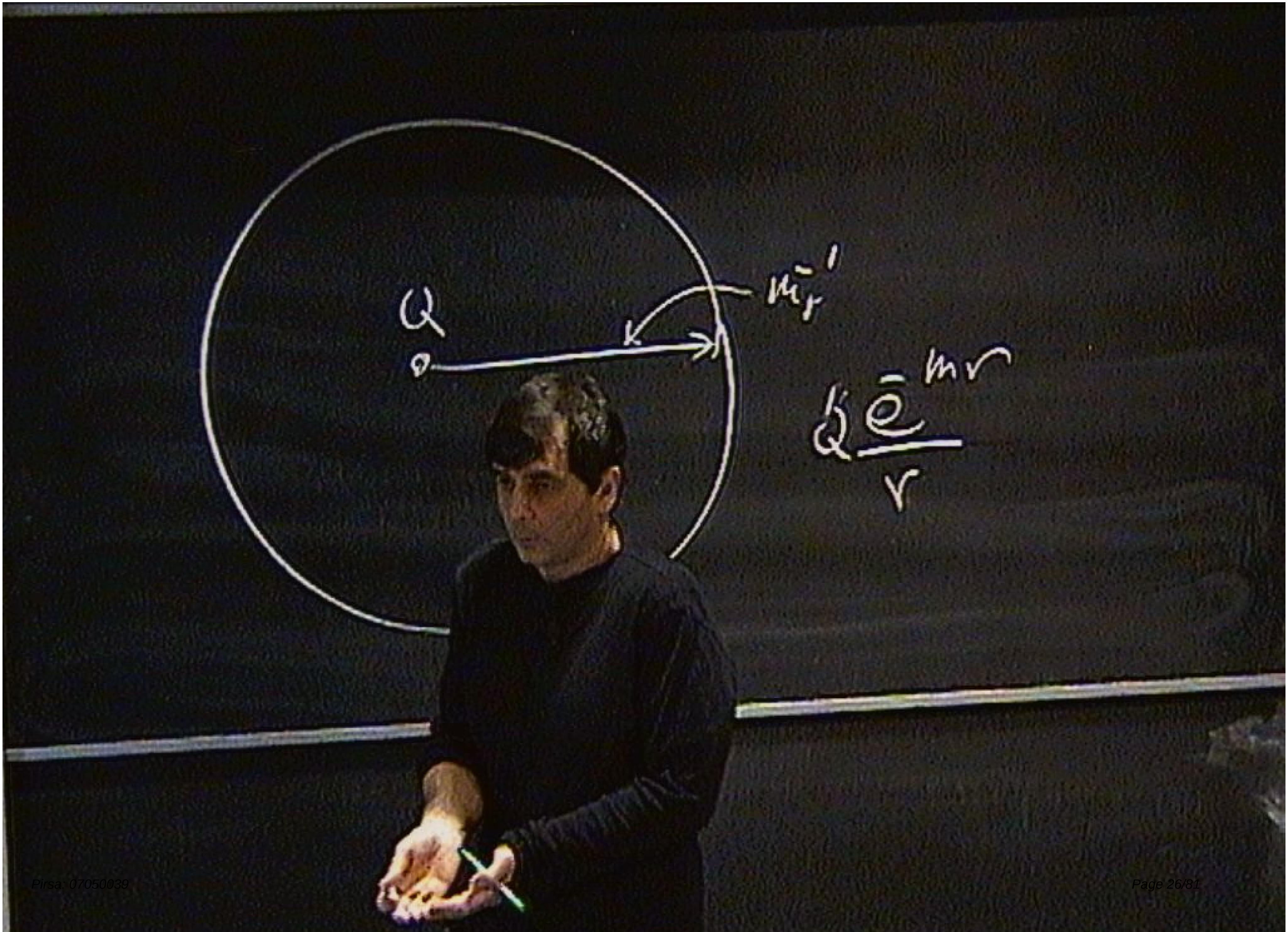
Bekenstein
Teitelboim



- IN PARTICULAR, NO HAIR FOR MASSIVE SPIN-2 FIELD

$h_{\mu\nu}$


SO(3) → U(1) →
Q


Q
m_γ^{-1}
$Q\frac{e^{-mr}}{r}$

PAULI-FIERZ ACTION

$$h^{\mu\nu}\mathcal{E}h_{\mu\nu} - m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

LINEARIZED EINSTEIN

$$\mathcal{E}h_{\mu\nu} - m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = 0$$

FOR $m^2 = 0$, THERE IS A GAUGE INVARIANCE

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

BUT, FOR $m^2 \neq 0$,

$h_{\mu\nu}$ IS OBSERVABLE, AND MUST VANISH AT THE HORIZON, AND EVERYWHERE.

BUT, THERE IS A LOOPHOLE:

QUANTUM HAIR!

IN FACT, $m^2 \neq 0$ THEORY IS ALSO HIDDENLY GAUGE INVARIANT.

$m^2 \to 0$ LIMIT IS DISCONTINUOUS:

van Dam, Veltman; Zakharov.

For $m^2 \neq 0$,

$h_{\mu\nu} \leftarrow$ 5 DEGREES OF FREEDOM

FOR $m^2 = 0$,

$h_{\mu\nu} \leftarrow$ 2 DEGREES OF FREEDOM

BUT, THERE IS A LOOPHOLE:

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For $m^2 \neq 0$,

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FOR $m^2 = 0$,

$h_{\mu\nu} \leftarrow$ 2 DEGREES OF FREEDOM

WE CAN'T TALK ABOUT GAUGE SYMMETRY, UNLESS WE KNOW HOW DEGREES OF FREEDOM TRANSFORM.

$$h_{\mu\nu} \equiv \hat{h}_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$$

NOW PAULI-FIERZ BECOMES:

$$\hat{h}_{\mu\nu} \mathcal{E} \hat{h}^{\mu\nu} - m^2 \left[\left(\hat{h}_{\mu\nu} + \underline{\partial_{\{\mu} A_{\nu\}}} \right)^2 - \left(\hat{h} + 2 \underline{\partial_\mu A^\mu} \right)^2 \right]$$

WHICH IS GAUGE INVARIANT UNDER:

$$\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$A_\mu \to A_\mu - \xi_\mu$$

THE EQUATIONS:

$$\mathcal{E}\hat{h}_{\mu\nu} - m^2\left[(\hat{h}_{\mu\nu} + \partial_{\{\mu}A_{\nu\}}) - \eta_{\mu\nu}(\hat{h} + 2\partial_\alpha A^\alpha)\right] = 0$$

$$\partial^\mu F_{\mu\nu} = -\partial^\mu(\hat{h}_{\mu\nu} - \eta_{\mu\nu}\hat{h})$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu .$$

THERE IS A LOCALLY-PURE-GAUGE SOLUTION:

$$h_{\mu\nu} = 0 \longleftrightarrow \hat{h}_{\mu\nu} = -\partial_{\{\mu}A_{\nu\}}$$

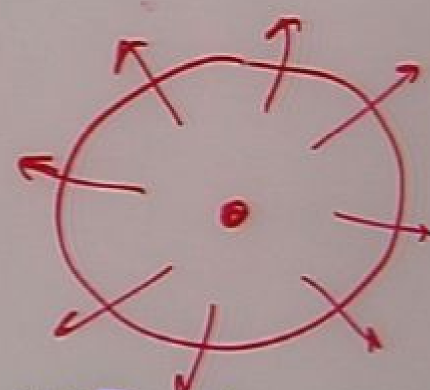
$$F_{\mu\nu} = \mu\,\frac{\epsilon_{\mu\nu}}{r^2}$$

MAGNETIC MONOPOLE TYPE CONFIGURATION FOR A_μ:

$$F_{\mu\nu} = \mu \frac{\epsilon_{\mu\nu}}{r^2}$$

$$A_\phi = \mu \frac{1-\cos\Theta}{\sin\Theta}, \quad A_\Theta = A_r = 0$$

$$\vec{M} = \mu \frac{\vec{r}}{r^3}$$

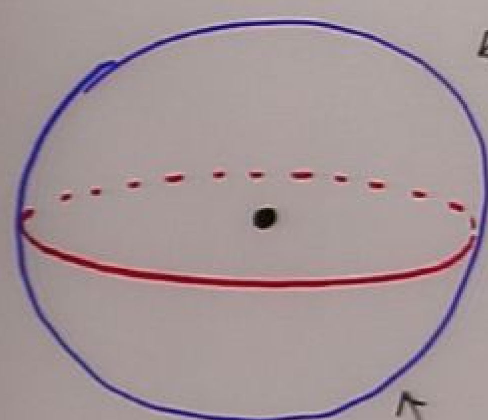


BUT, THIS IS NOT A DIRAC MAGNETIC MONOPOLE, BECAUSE THERE IS NO DETECTABLE MAGNETIC FIELD.

REMEMBER $h_{\mu\nu} \equiv 0$!!!

NO REFERENCE TO DIRAC STRING

Wu & Yang



$$A^{U}_{\phi} = \mu \frac{(1-\cos\Theta)}{\sin\Theta}$$

$$A^{L}_{\phi} = -\mu \frac{(1+\cos\Theta)}{\sin\Theta}$$

$$A^{U}_{\mu} - A^{L}_{\mu} \Big|_{\theta=\frac{\pi}{2}} = \mu\, \partial_\mu \phi \Big|_{\theta=\frac{\pi}{2}}$$

$$h^{U}_{\mu\nu} - h^{L}_{\mu\nu} \Big|_{\theta=\frac{\pi}{2}} = 2\mu\, \partial_\mu \partial_\nu \phi \Big|_{\theta=\frac{\pi}{2}}$$

$h^{U}_{\mu\nu}$ and $h^{L}_{\mu\nu}$ describe the same physics.

NO REFERENCE TO DIRAC STRING

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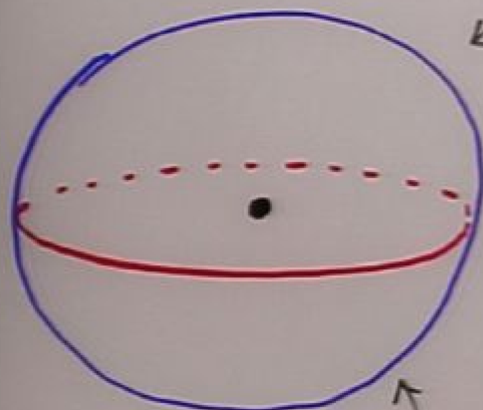

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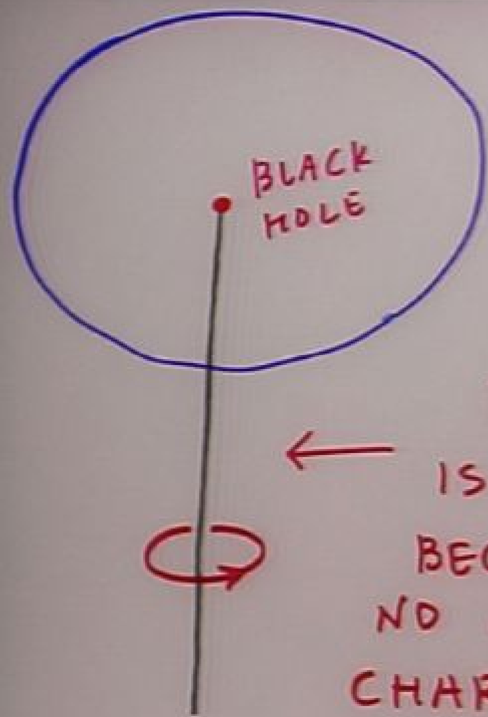
$$h^U_{\mu\nu} - h^L_{\mu\nu} \Big|_{\theta=\frac{\pi}{2}} = 2\mu\, \partial_\mu \partial_\nu \phi \Big|_{\theta=\frac{\pi}{2}}$$

$h^U_{\mu\nu}$ and $h^L_{\mu\nu}$ describe the same physics.



DIRAC STRING

IS UNOBSERVABLE,

BECAUSE THERE ARE

NO ELECTRICALLY

CHARGED PARTICLES

UNDER A_μ

$$\int dX^\mu A_\mu$$

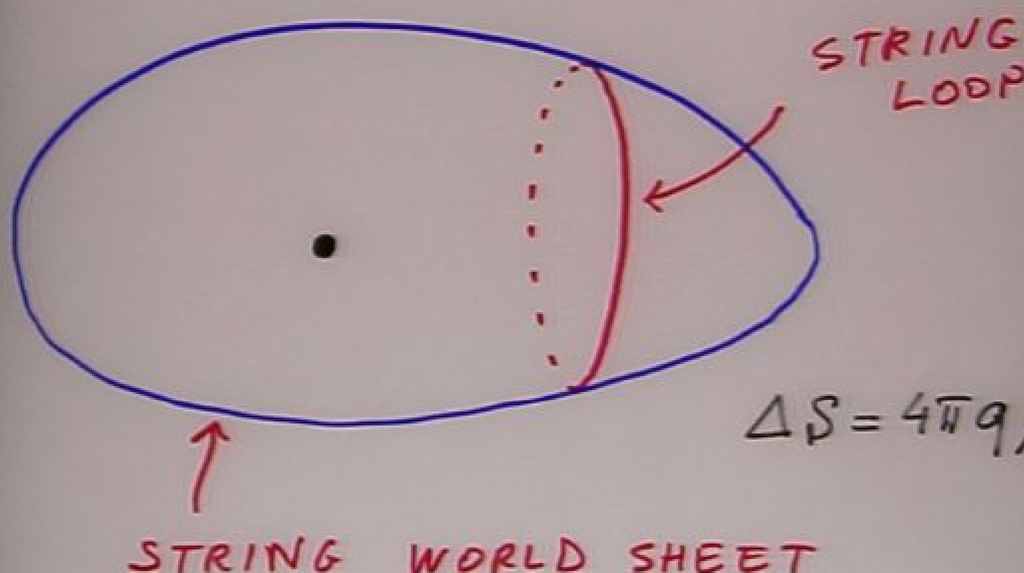
IS NOT GAUGE INVARIAT

UNDER:

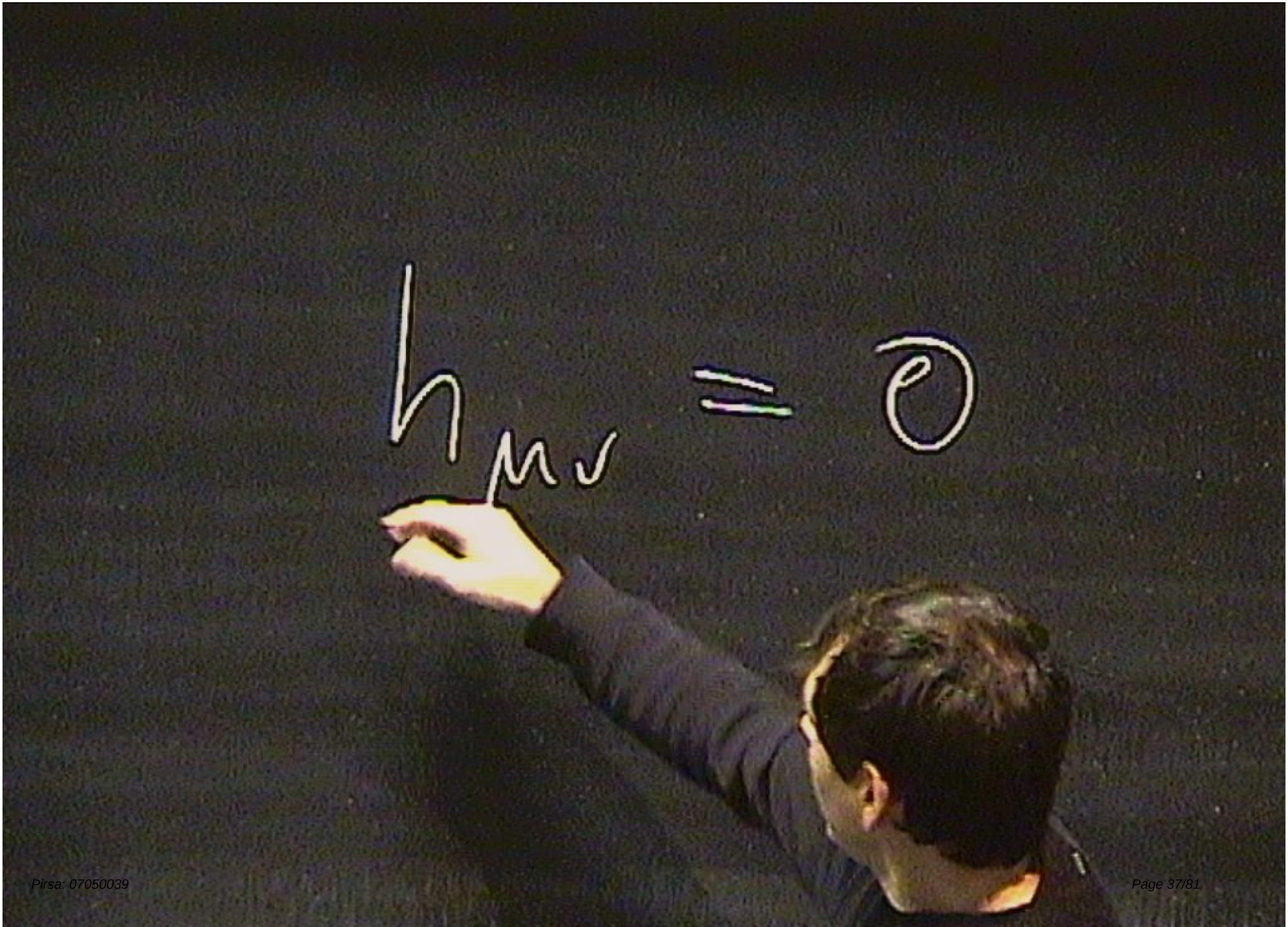
$$A_\mu \longrightarrow A_\mu + \xi_\mu$$

THE MASSIVE SPIN-2 HAIR IS CLASSICALLY UNDETECTABLE, BUT CAN BE DETECTED QUANTUM-MECHANICALLY BY STRINGY AHARONOV-BOHM EXPERIMENT

$$q \int dx^{\mu} \wedge dx^{\nu} F_{\mu\nu}$$

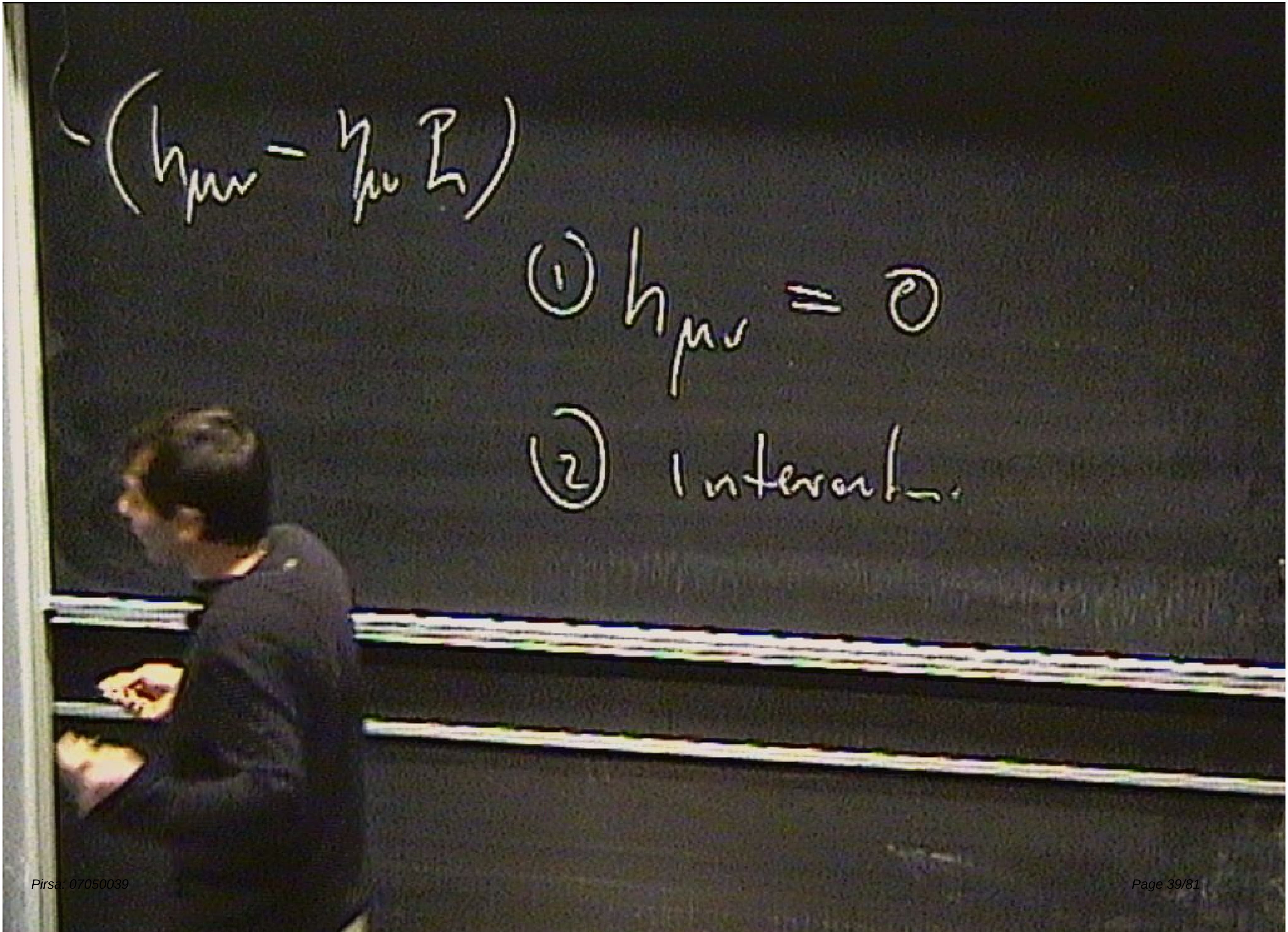


$$\Delta S = 4\pi q \mu$$


$h_{\mu\nu} = 0$

(1) $h_{\mu\nu} = 0$

(2) Interaction.



$(h_{\mu\nu} - \eta_{\mu\nu} h)$
① $h_{\mu\nu} = 0$
② interaction

$$\partial^\mu \partial^\nu \left(h_{\mu\nu} - \eta_{\mu\nu} h \right)$$

$$h_{\mu\nu} = \partial_\mu \partial_\nu \phi$$

① $h_{\mu\nu} = 0$

② Interaction.

$$\partial^\mu \partial^\nu (h_{\mu\nu} - \eta_{\mu\nu} h) = 0$$

$$h_{\mu\nu} = \partial_\mu \partial_\nu \phi$$

① $h_{\mu\nu} = \mathcal{O}$

② Interact.

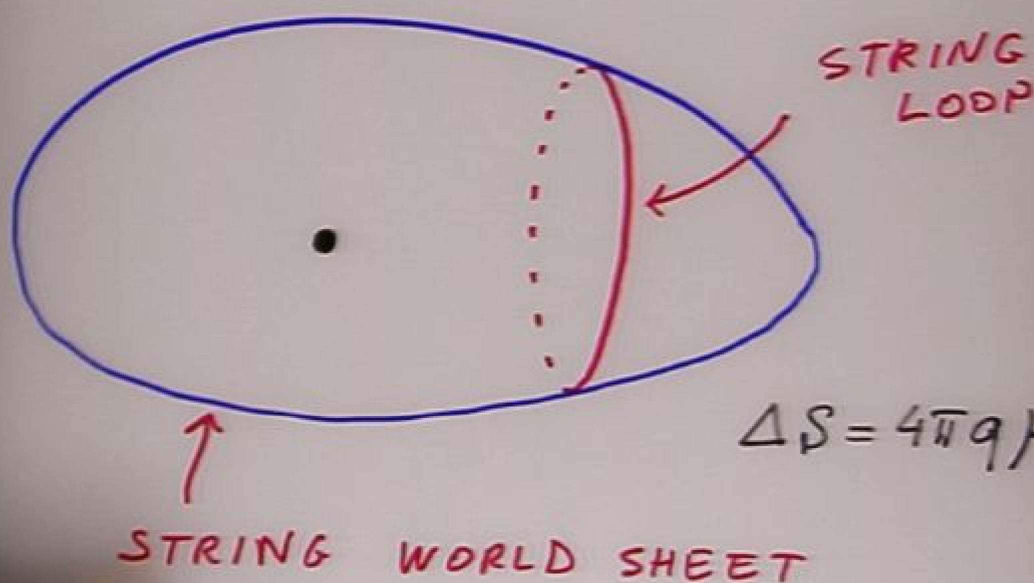
$$\partial^\mu \partial^\nu \left(h_{\mu\nu} - \eta_{\mu\nu} h \right) = 0$$

$$h_{\mu\nu} = \partial_\mu \partial_\nu \phi$$

1. $h_{\mu\nu} = 0$
2. 1 interact.

THE MASSIVE SPIN-2 HAIR IS CLASSICALLY UNDETECTABLE, BUT CAN BE DETECTED QUANTUM-MECHANICALLY BY STRINGY AHARONOV-BOHM EXPERIMENT

$$q \int dx^\mu \wedge dx^\nu F_{\mu\nu}$$



$$\Delta S = 4\pi q \mu$$

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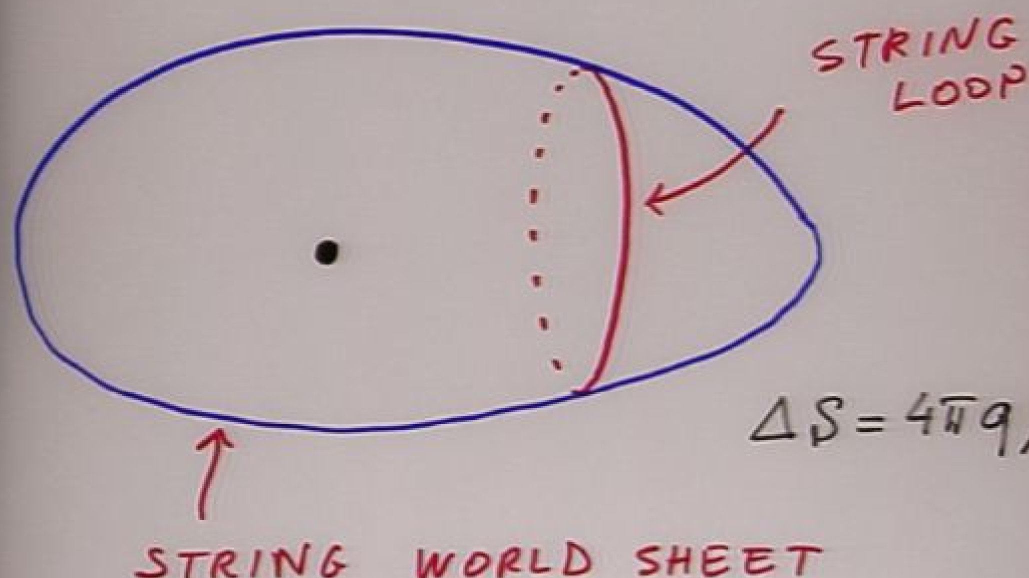

$$\Delta S = 4\pi q\mu$$

TO SUMMARIZE, IN THE PRESENCE OF STRINGS, PAULI-FIERZ ACTION CAN BE REPRESENTED IN THE FORM:

$$\hat{h}^{\mu\nu}\mathcal{E}\hat{h}_{\mu\nu} - m^2\left(\left(\hat{h}_{\mu\nu} + \partial_{[\mu} A_{\nu]}\right)^2 - \left(\hat{h} + 2\partial_\alpha A^\alpha\right)^2\right)$$

$$+\; g\int dx^\mu \wedge dx^\nu F_{\mu\nu}$$

BLACK HOLES ARE LABELED BY THE QUANTUM SPIN-2 CHARGE μ.

IT IS DETECTABLE IF:

$$g\mu \neq \frac{n}{2}$$

$G_{\mu\nu}$

$$\mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} + \theta G \tilde{G}$$

$G_{\mu\nu}$

$$\mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} + \theta\, G\tilde{G}$$

$$q \int dx^\mu \wedge dx^\nu F_{\mu\nu}$$

$G_{\mu\nu}$

$$\mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} + \theta\, G \tilde{G}$$

$$PF + q \int dx^\mu \wedge dx^\nu F_{\mu\nu}$$

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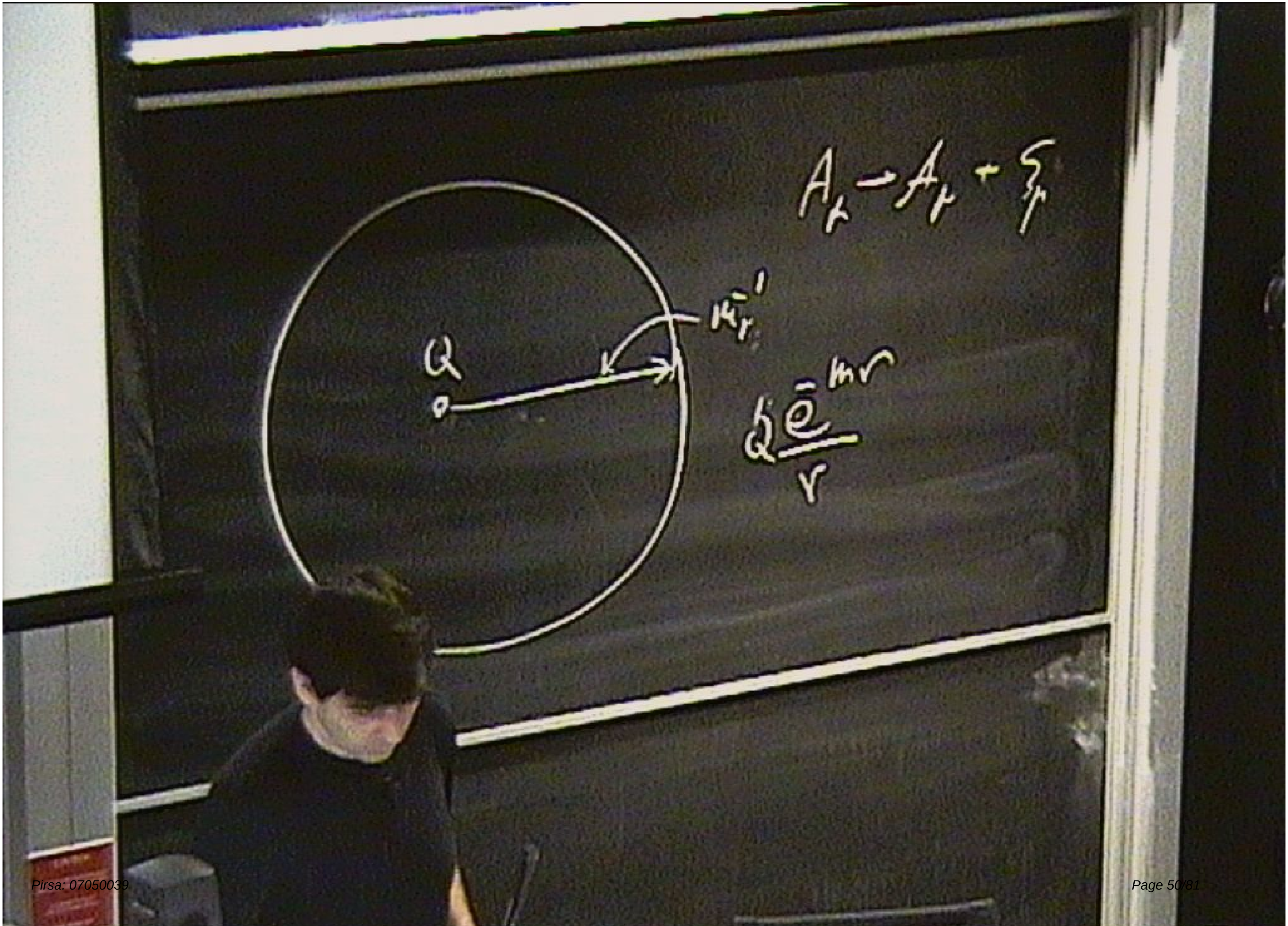
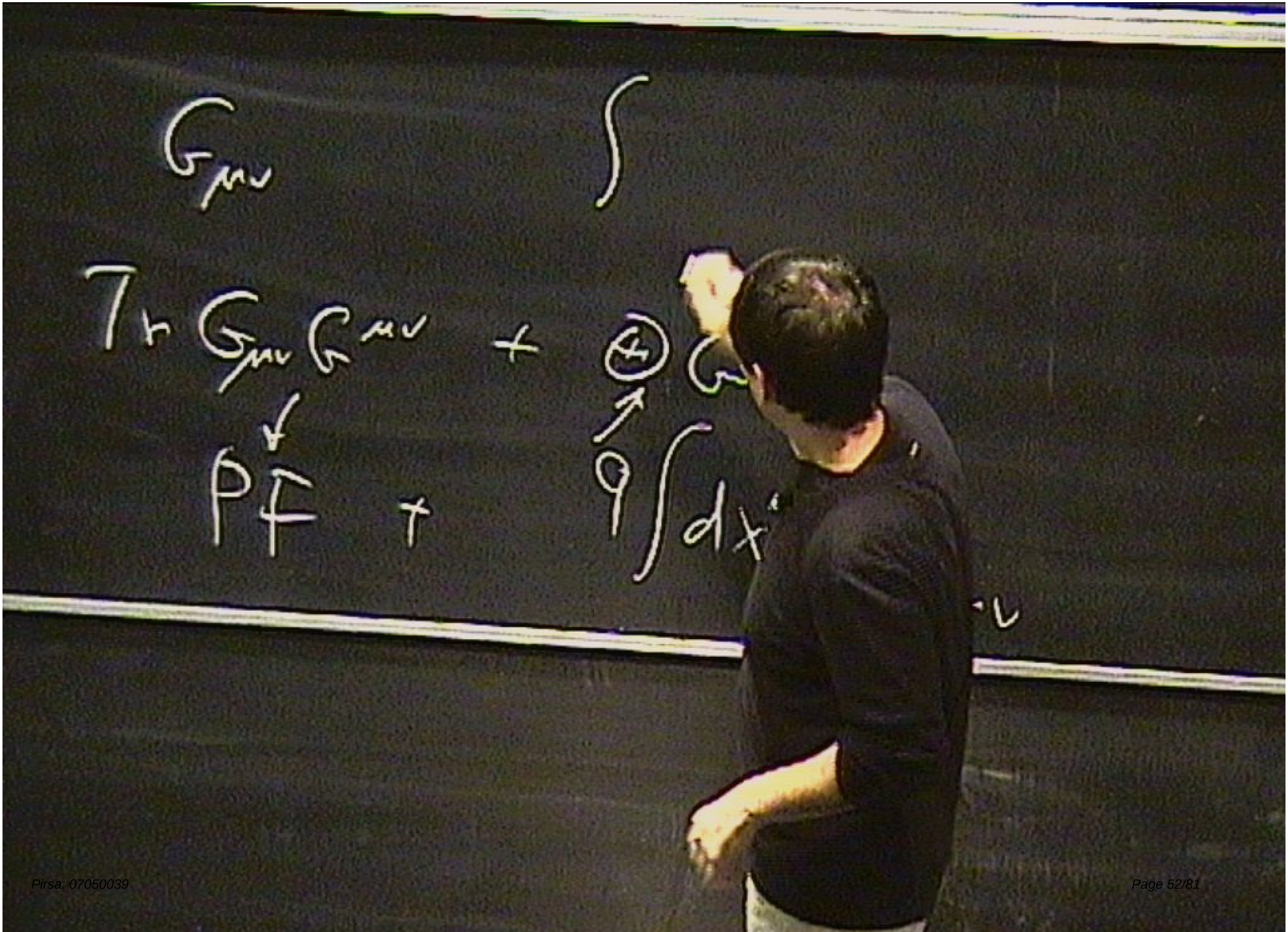

$A_\mu \to A_\mu + \xi_\mu$
m_γ^{-1}
Q
$Q\frac{e^{-mr}}{r}$

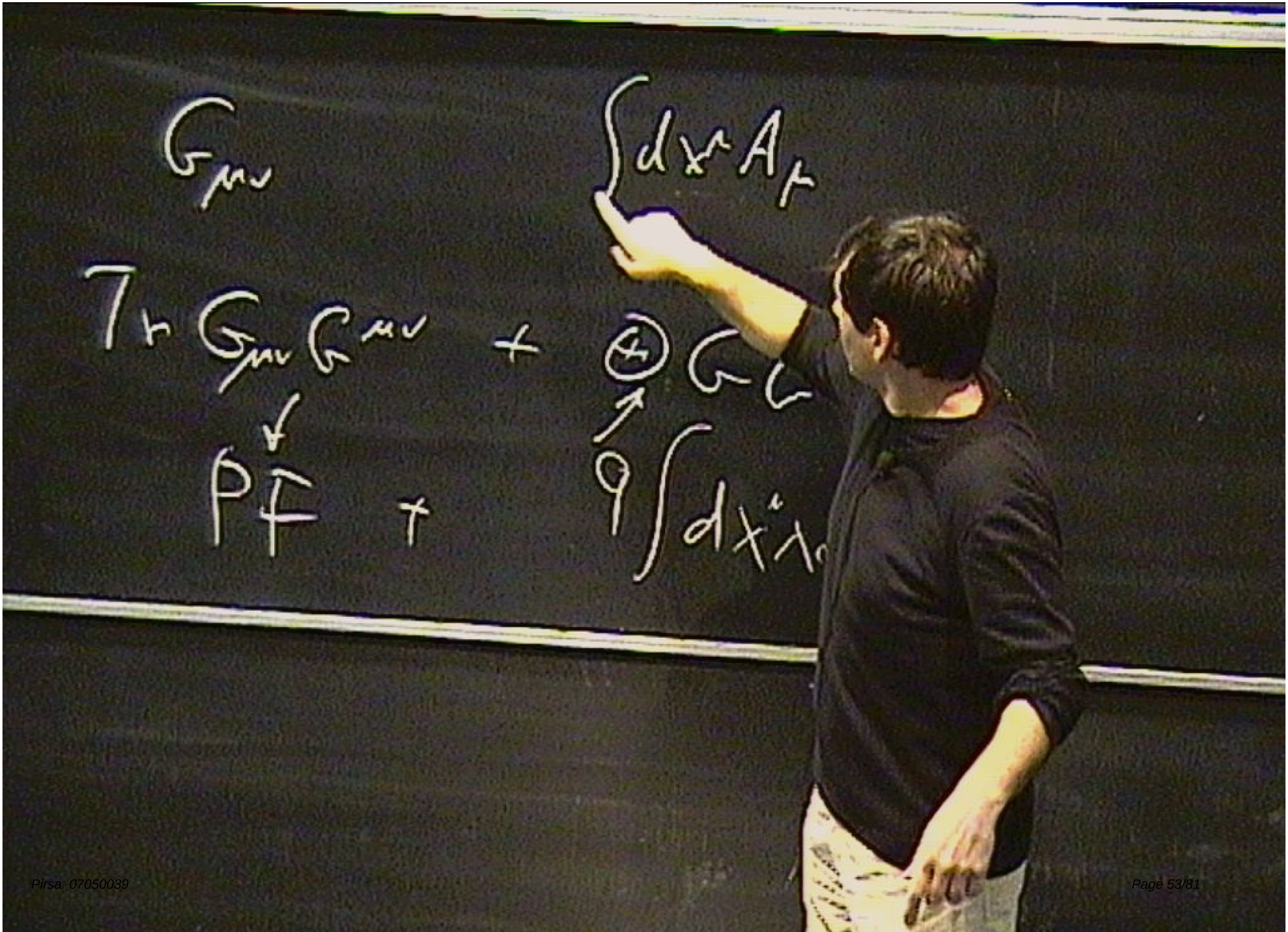
INTERESTINGLY, THE SUPERMASSIVE QUANTUM HAIR OF A BLACK HOLE CAN BE DETECTED BY AN ORDINARY ELECTROMAGNETIC AHARONOV-BOHM EXPERIMENT!

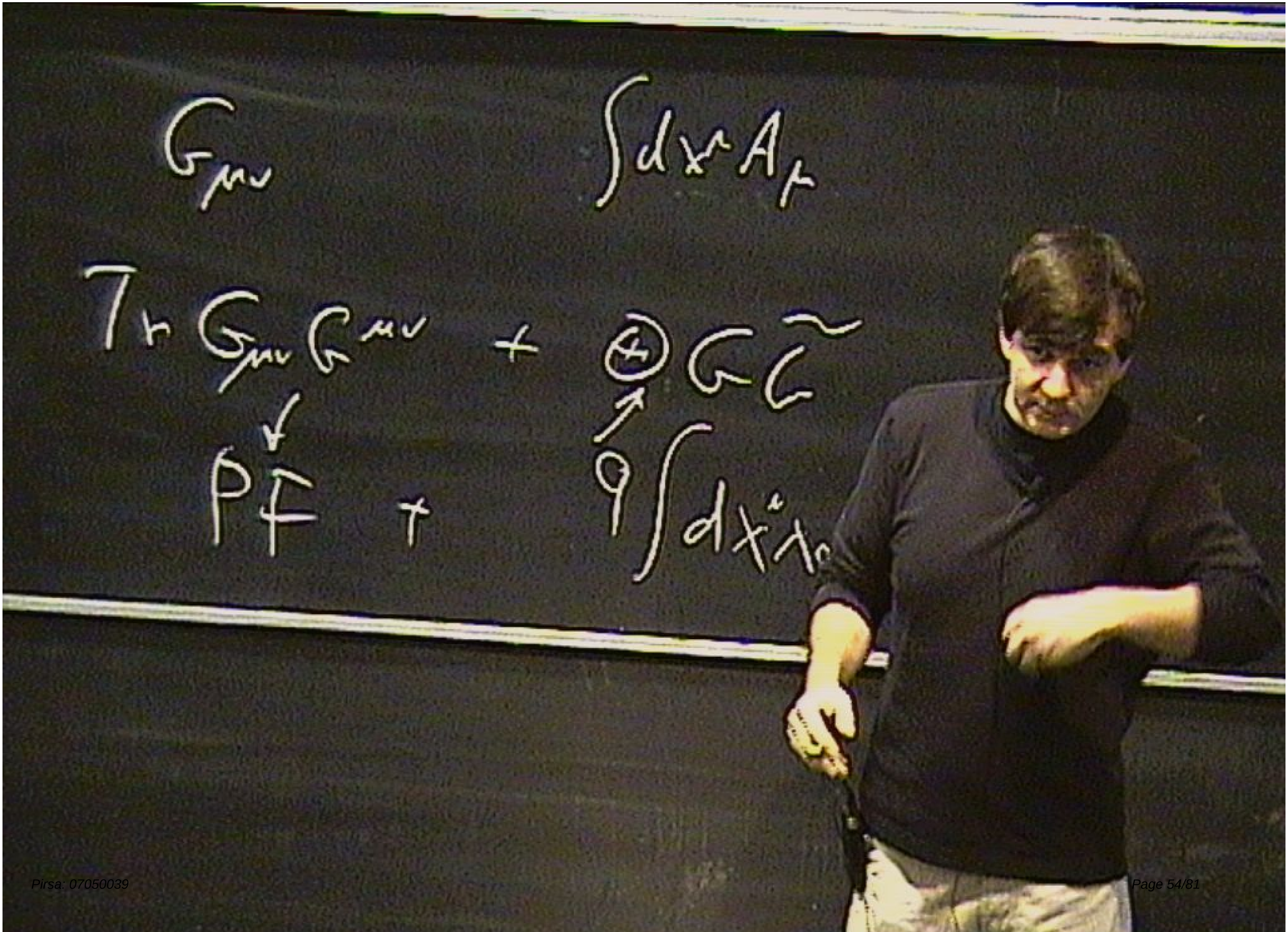
PROVIDED THERE IS A BOUNDARY TERM

$$F_{\mu\nu} F^{(EM)}_{\alpha\beta} \varepsilon^{\mu\nu\alpha\beta}$$

USUAL ELECTROMAGNETIC FIELD STRENGTH


Gμν
∫
Tr Gμν Gμν + θ G
PF + q∫dx


$G_{\mu\nu}$
$\int dx^\mu A_\mu$
$\mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} + \theta\, G G$
$PF + q \int dx^i A$


$G_{\mu\nu}$
$\int dx^\mu A_\mu$
$\mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} + \theta\, G \tilde{G}$
$PF + q \int dx^i A_i$

$G_{\mu\nu}$ $\int dx^\mu A_\mu$

$\mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} + \theta\, G\tilde{G}$

$PF + q \int dx^\mu \wedge dx^\nu F_{\mu\nu}$

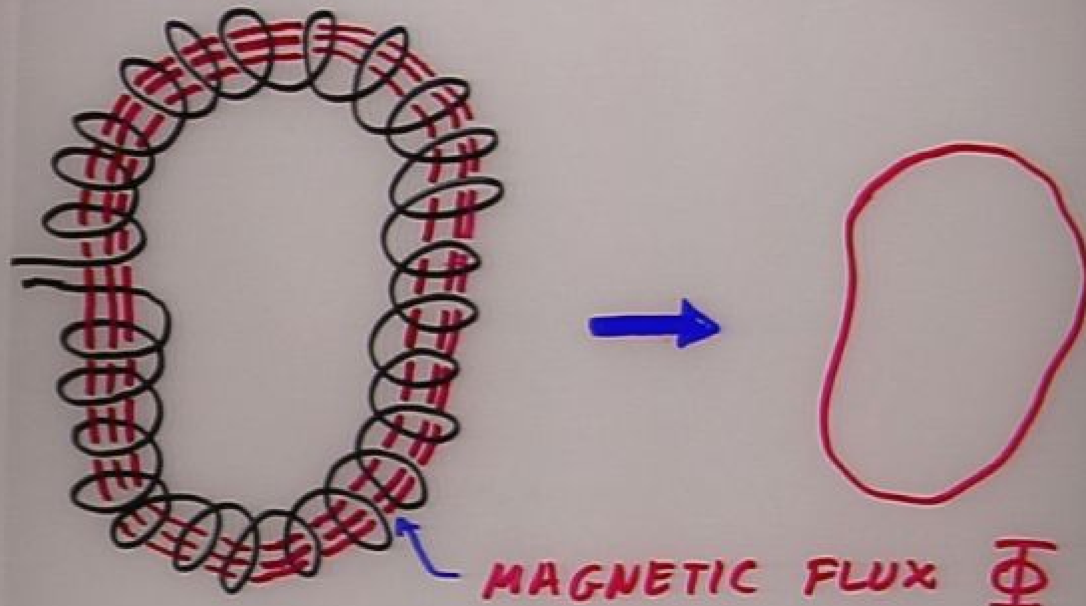
INTERESTINGLY, THE SUPERMASSIVE QUANTUM HAIR OF A BLACK HOLE CAN BE DETECTED BY AN ORDINARY ELECTROMAGNETIC AHARONOV-BOHM EXPERIMENT !

PROVIDED THERE IS A BOUNDARY TERM

$$F_{\mu\nu} F^{(EM)}_{\alpha\beta} \varepsilon^{\mu\nu\alpha\beta}$$

USUAL ELECTROMAGNETIC FIELD STRENGTH

NOW, TAKE A LOOP OF AN ORDINARY MAGNETIC SOLENOID

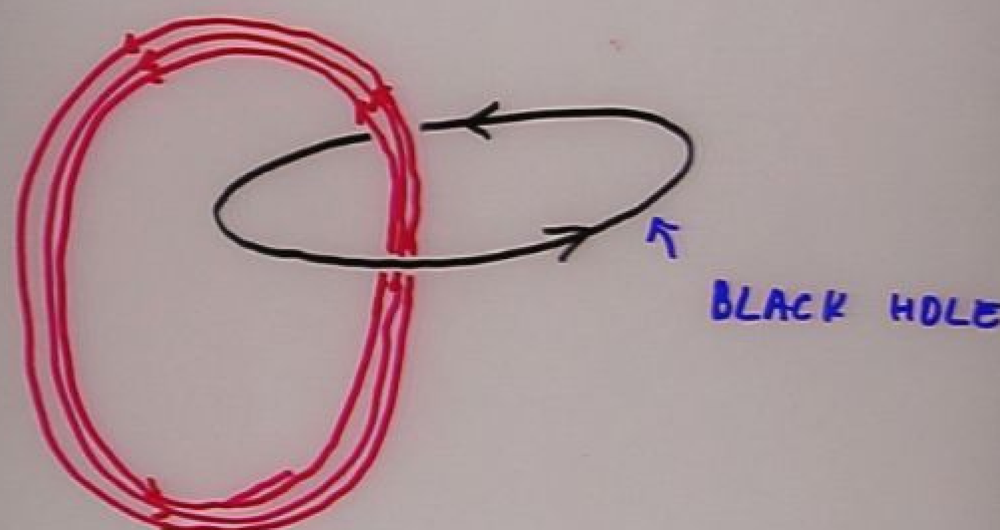


FROM LARGE DISTANCE SOLENOID BECOMES A STRING

$$F^{(EM)}_{\alpha\beta}\,\varepsilon^{\alpha\beta\mu\nu} \longrightarrow \underset{\text{FLUX}}{\Phi} \int dx^{\mu} \wedge dx^{\nu}$$

COUPLING TO A SOLENOID BECOMES EFFECTIVELY A COUPLING TO THE STRING

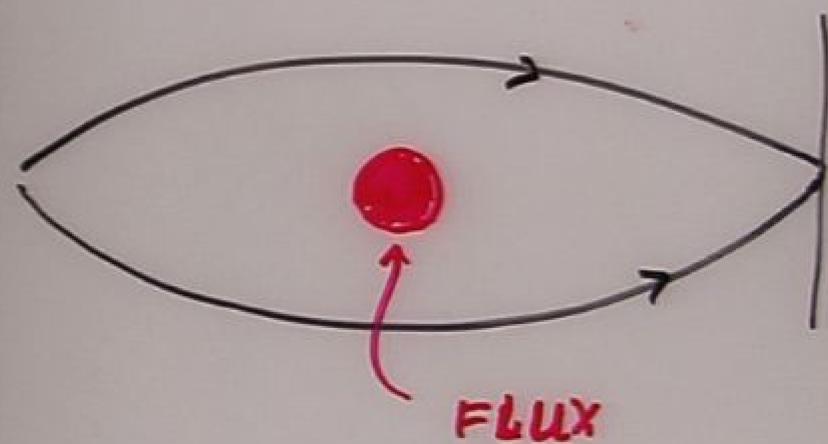
$$F_{\mu\nu} F^{(EM)}_{\alpha\beta} \varepsilon^{\mu\nu\alpha\beta} \longrightarrow \Phi \int dx^{\mu} \wedge dx^{\nu} F_{\mu\nu}$$



ENCIRCLING SUCH A SOLENOID WE GET A PHASE SHIFT:

$$\Delta S = 4\pi \Phi \mu \;!$$

SUCH A BLACK HOLE OR A REMNANT PARTICLE CAN BE DETECTED BY AN ORDINARY AHARONOV-BOHM EXPERIMENT.



FOR EXAMPLE, SHOUD ONE DISCOVER A NON ZERO EFFECT FOR ELECTRICALLY-NEUTRAL PARTICLES, THIS WOULD BE A SIGNAL FOR THE QUANTUM HAIR!

THIS DISCUSSION CAN BE GENERALIZED TO THE MASSIVE FIELDS OF HIGHER SPIN.

ALL WE NEED IS THE BOUNDARY COUPLING

$$F \wedge F^{(EM)}$$

WHERE $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ AND A_μ IS THE STÜCKELBERG FIELD THAT GIVES MASS TO ANY HIGH SPIN FIELD IN A GAUGE INVARIANT WAY:

$$A_\mu \longrightarrow A_\mu + \xi_\mu$$

STRING AND KALUZA-KLEIN THEORIES CONTAIN INFINITE TOWER OF MASSIVE TENSORIAL FIELDS.

SO, ARE THERE INFINITELY MANY QUANTUM CHARGES THAT BLACK HOLES CAN HAVE?

BLACK HOLES WITH QUANTUM HAIR IN LARGE DISTANCE MODIFICATION OF GRAVITY

GRAVITON IS NO LONGER MASSLESS.

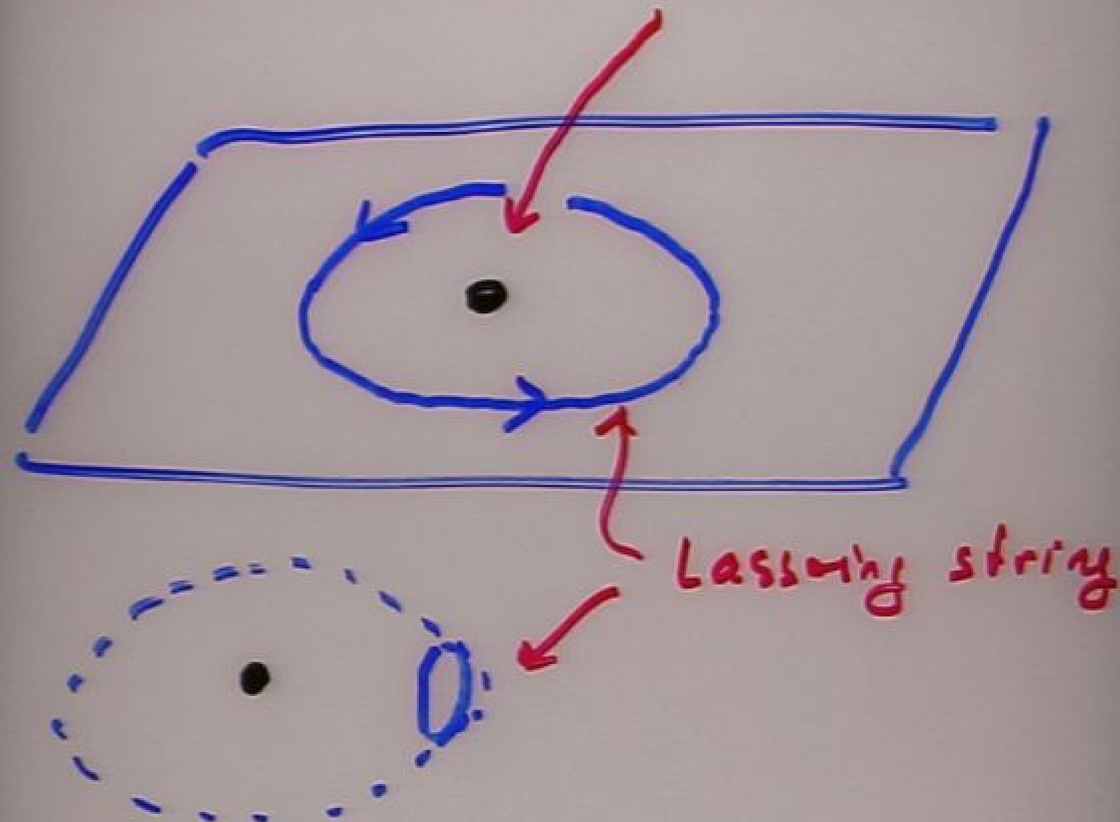
IN DGP:

$$h_{\mu\nu} = \int_0^\infty \frac{dm}{(mr_c)^2 + 1} \, h_{\mu\nu}^{(m)}$$

massive spin-2

$$h_{\mu\nu}^{(m)} = h_{\mu\nu}^{(m)} + \partial_\mu A_\nu^{(m)} + \partial_\nu A_\mu^{(m)}$$

BLACK HOLE WITH QUANTUM HAIR ON DGP BRANE



we measure

$$\int_{S_2} dx^A \wedge dx^B \; \partial_{[A}\, g^{CD}\, \partial_C\, g_{DB]}$$

Because gravity for $r \to \infty$ is 5-dimensional, we can think in terms of 5D topological invariants

$$X_{AB} \equiv \partial_{[A} \partial^{c} h_{cB]}$$

$$\downarrow$$

$$\int dx^A \wedge dx^B \, X_{AB} = 4\pi f''(y)$$

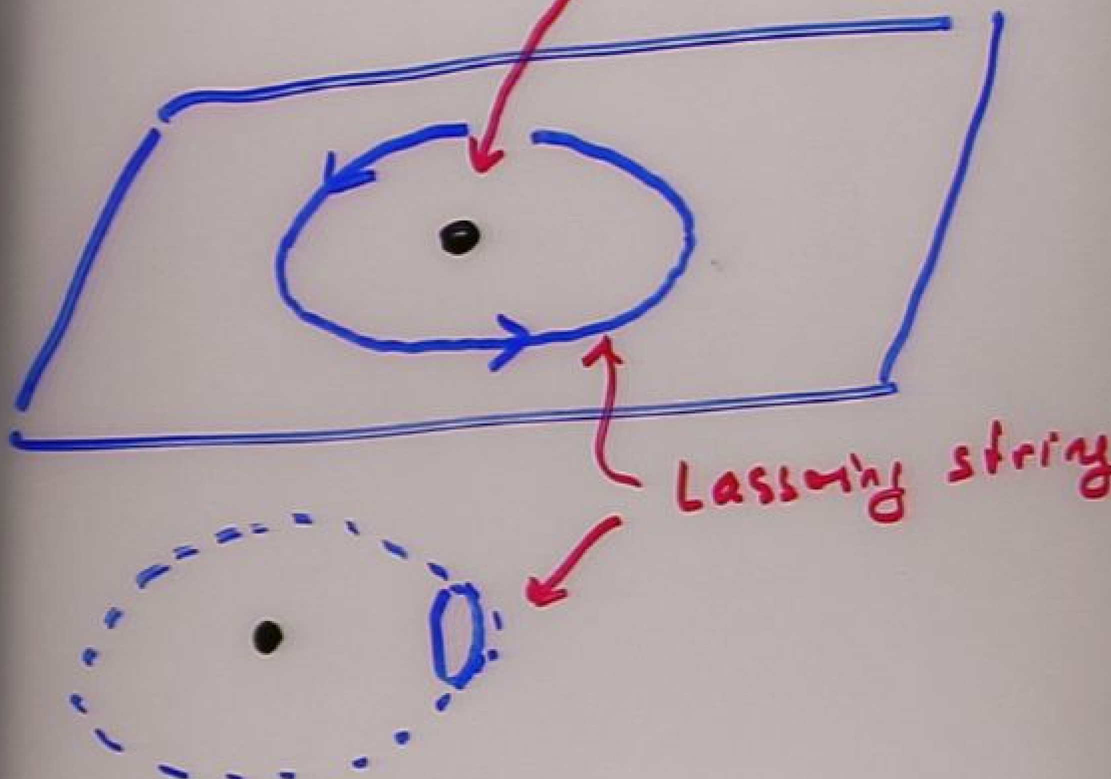
sphere at $r \to \infty$

$$h_{AB} = \partial_A \xi_B + \partial_B \xi_A$$

$$\xi_5 = 0, \quad \xi_\mu = f(y) A_\mu$$

monopole

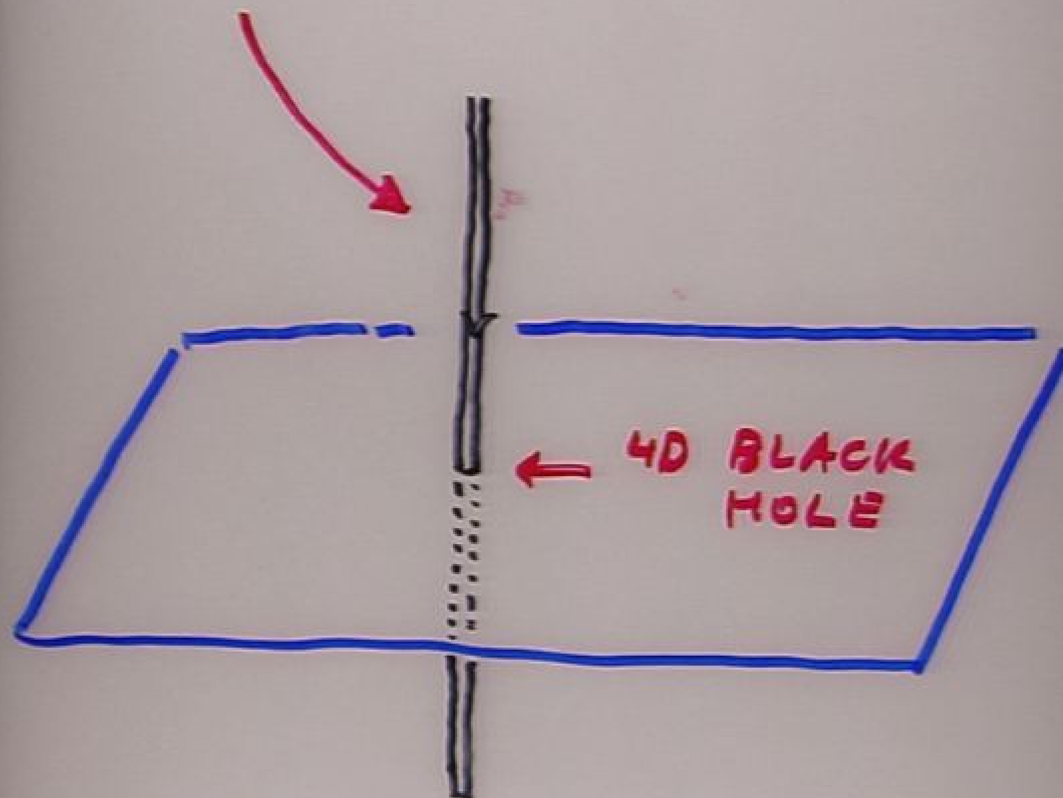
BLACK HOLE WITH QUANTUM HAIR ON DδP BRANE



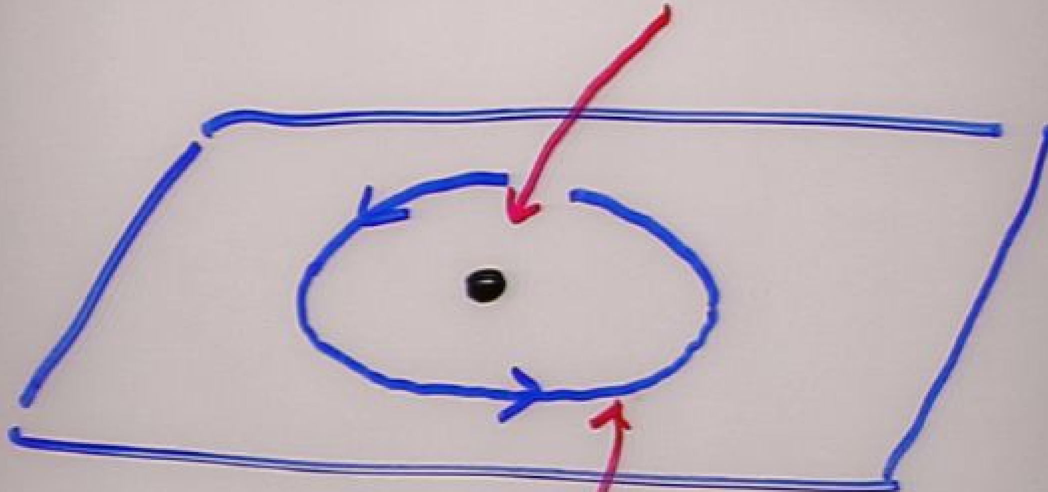
we measure

$$\int_{S_2} dx^A \wedge dx^B \; \partial_{[A}\, g^{CD}\, \partial_C\, g_{DB]}$$

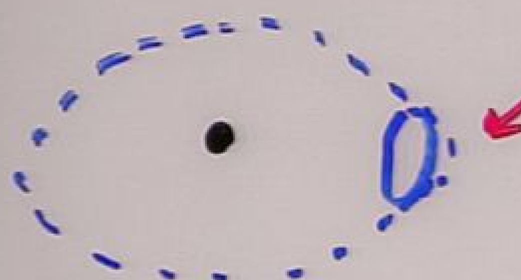
FLUX OF "COMBED" GRAVITATIONAL MONOPOLES



BLACK HOLE WITH QUANTUM HAIR ON DσP BRANE

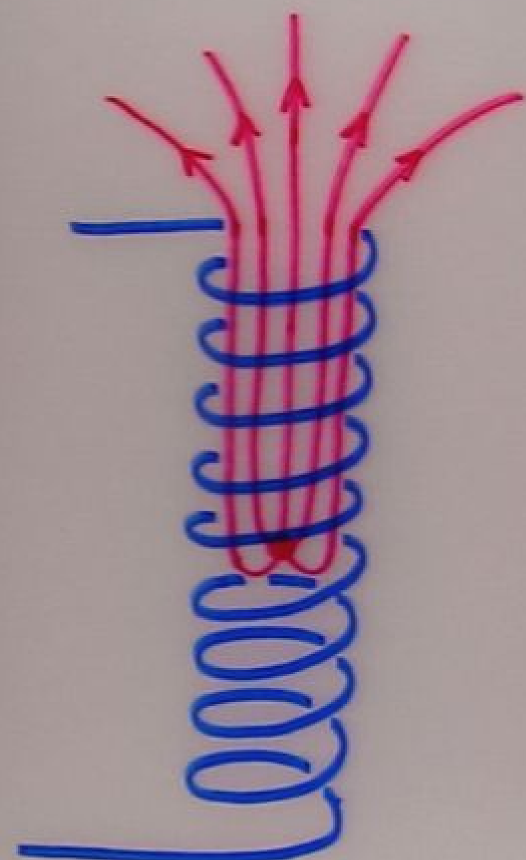


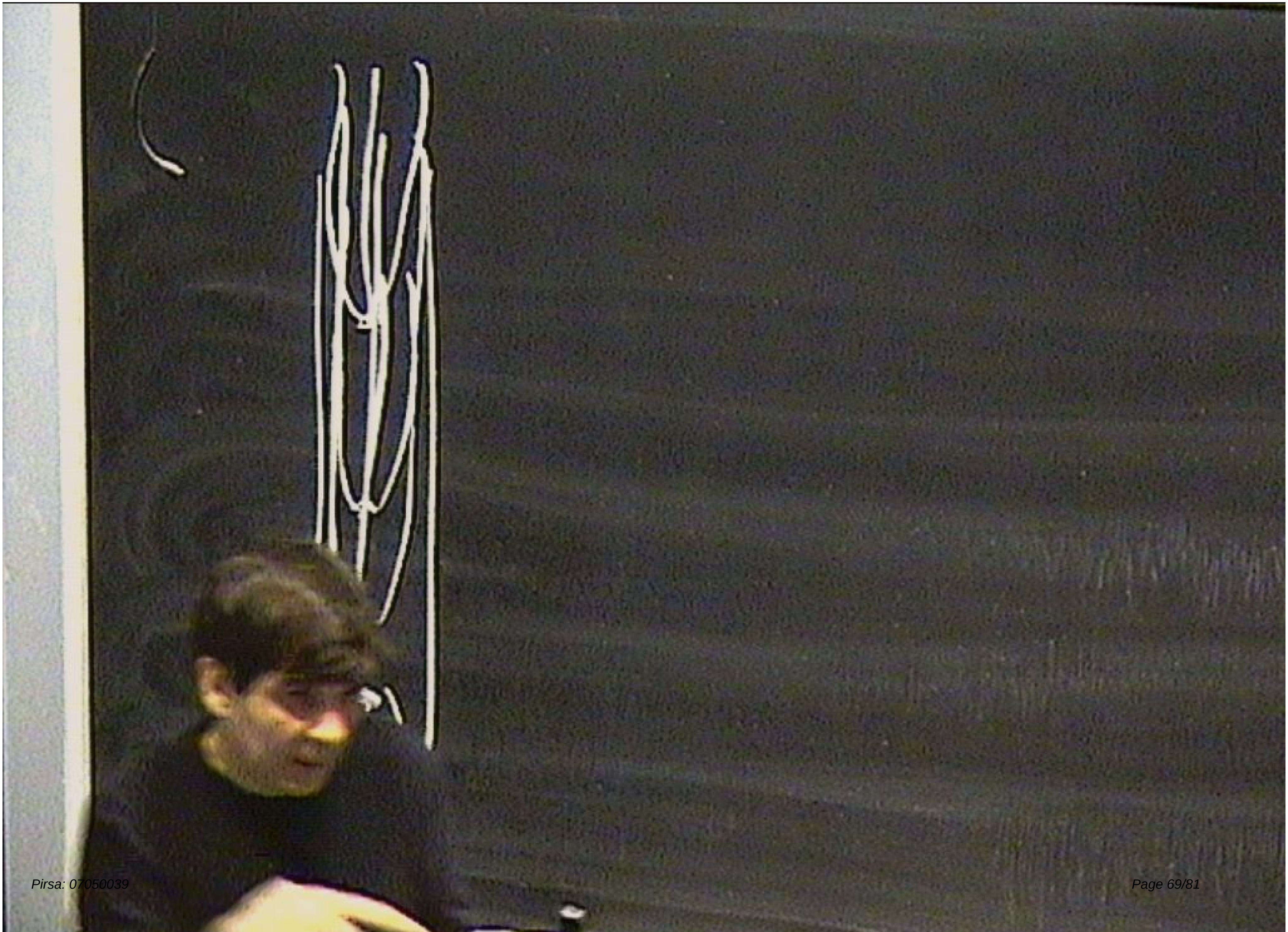
Lassoing string

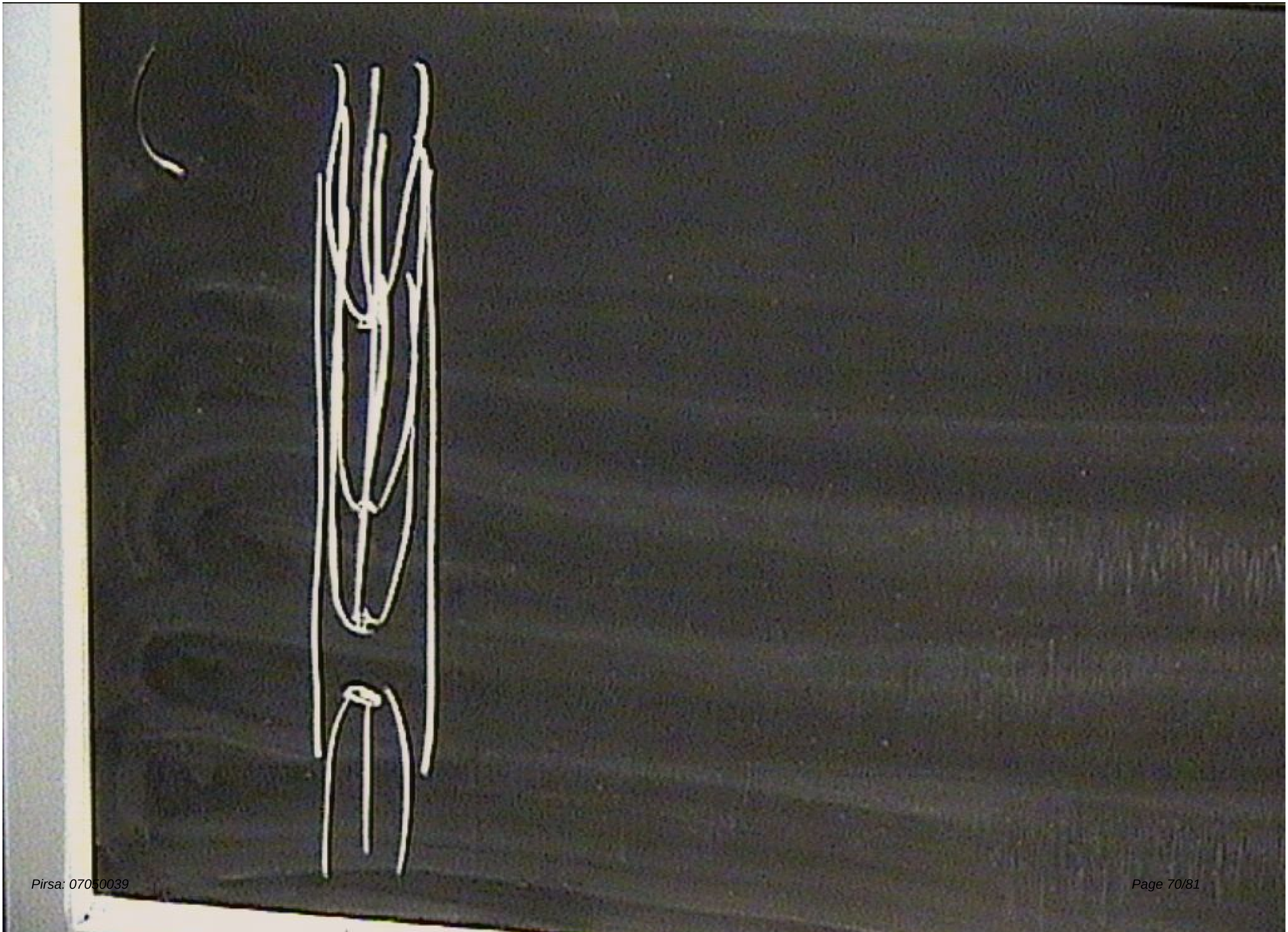


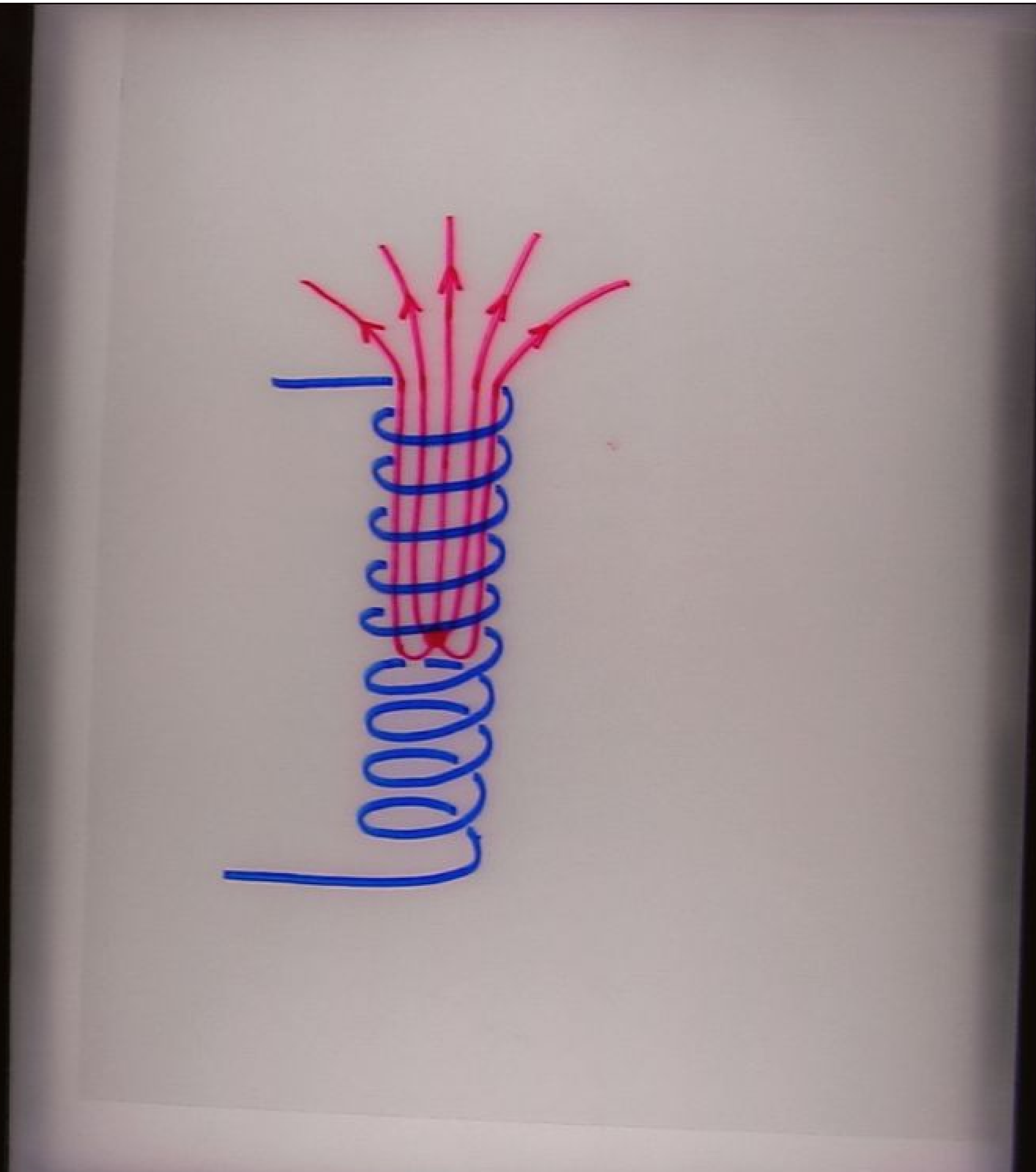
we measure

$$\int_{S_2} dx^A \wedge dx^B \; \partial_{[A} g^{CD} \partial_C g_{DB]}$$



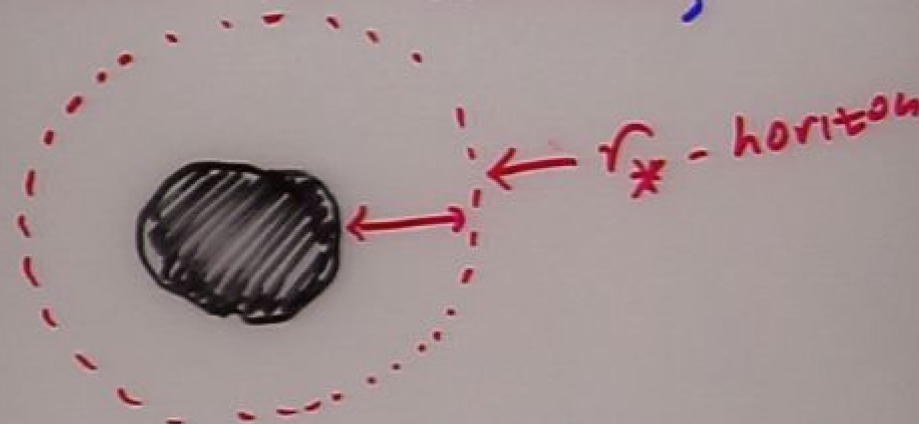






The rule:

For a source that is localized within its own r_*, gravity is almost Einsteinian,



with small corrections given by:

$$\delta \simeq \left(\frac{r}{10^{28}\,\mathrm{cm}}\right)^{2-2\alpha}\sqrt{\frac{r}{r_g}}$$

Domain walls as probes of gravity.

G.D., Gabadadze, Pujolas, Rahman

In GR domain walls

$$T_{\mu\nu} = \sigma\,\delta(z)\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 0 \end{pmatrix}$$

produce Rindler-type space, with 3D deSitter

Vilenkin; Ipser & Sikivie

$$ds^2 = (1 - H|z|)^2\left(-dt^2 + e^{2Ht}(dx^2 + dy^2)\right) + dz^2$$

$$H \equiv 2\pi G_N \sigma$$

In large-distance-modified gravity, sub-M_* walls <u>do not</u> gravitate.

E.g. in linear approximation

$$h_{\mu\nu} = \delta_\mu^3 \delta_\nu^3 \frac{1}{\Box + m^2(\Box)} \sigma G_N \delta(z)$$

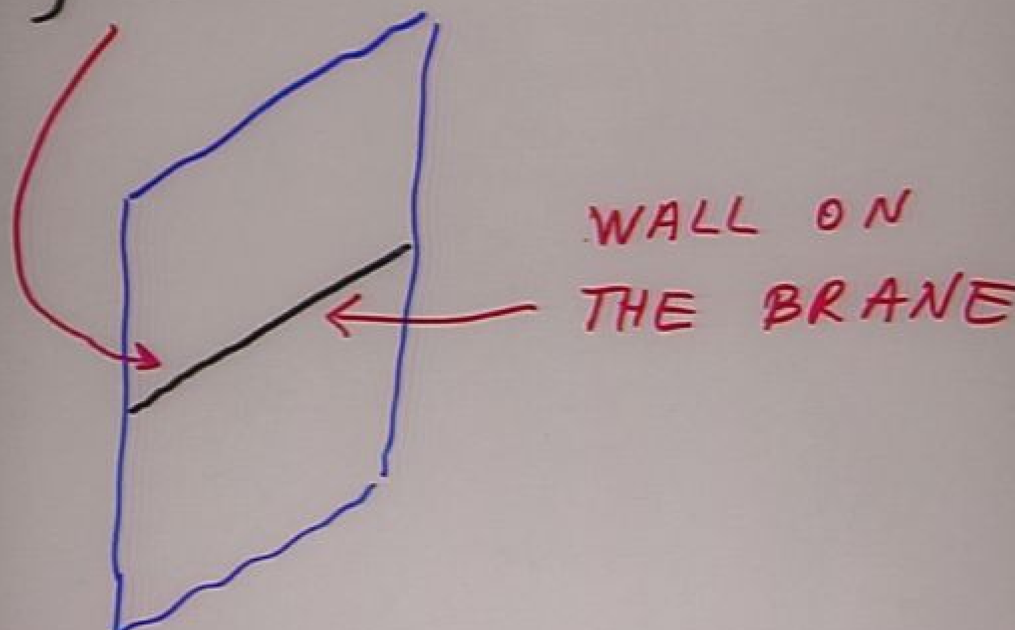
pure gauge

Exact solutions

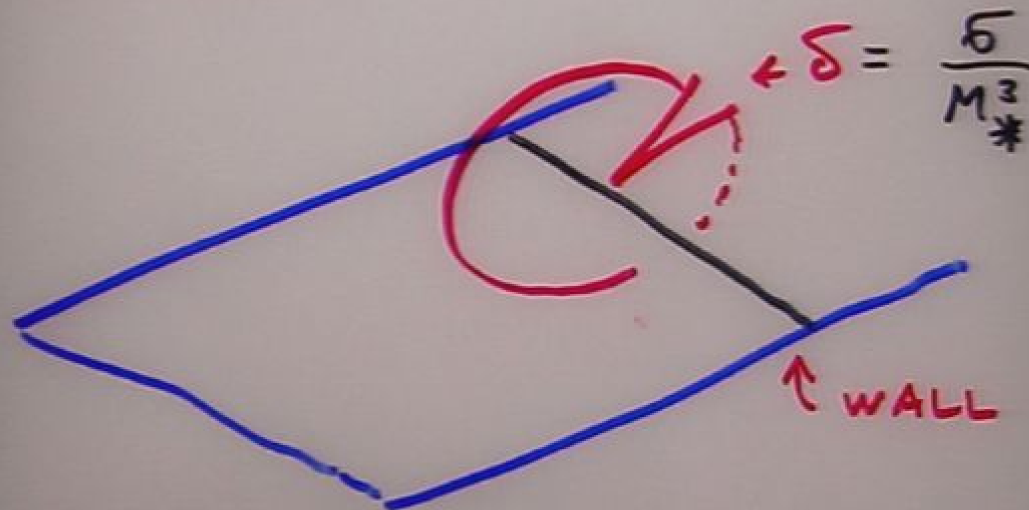
$$S = \int d^5x \sqrt{-g_{(5)}}\, \frac{M_*^3}{2} R_{(5)} +$$

$$+ \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R_{(4)} - \tau \right)$$

$$- \int d^3x \sqrt{-\gamma}\, \sigma$$

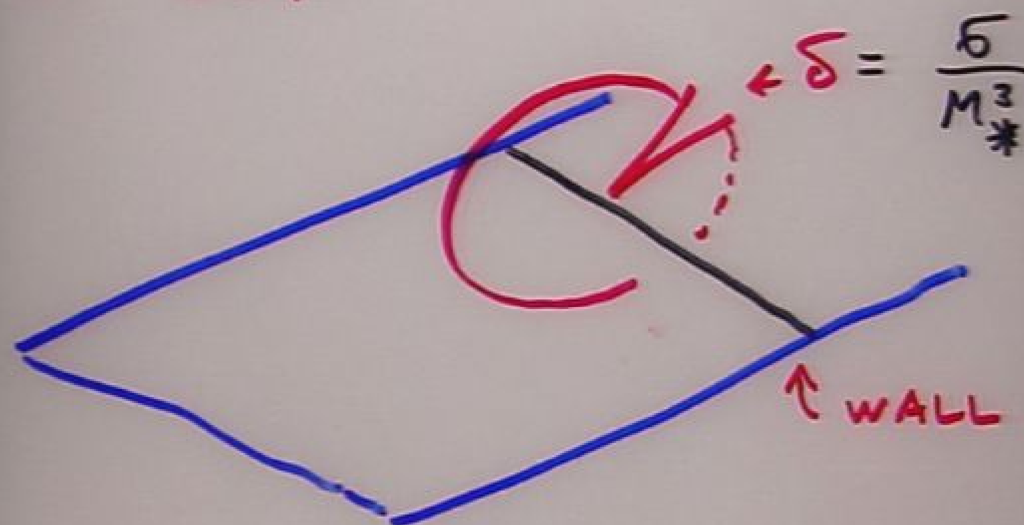


Full non-linear story in DGP



$$\underbrace{G^{(4)}_{\mu\nu} + \frac{1}{r_c}\left(K_{\mu\nu} - g_{\mu\nu} K\right) = 8\pi G_N T_{\mu\nu}}_{T^{eff}_{\mu\nu} = 0\,!}$$

Full non-linear story in DGP



$$G^{(4)}_{\mu\nu} + \underbrace{\frac{1}{r_c}\left(K_{\mu\nu} - g_{\mu\nu} K\right) = 8\pi G_N T_{\mu\nu}}_{T^{eff}_{\mu\nu} = 0\,!}$$

Bulk metric:

$$ds^2 = \{ dz^2 + dR^2 + R^{2K} ds_K^2$$

$K = 0, 1$

Brane location: $Z(\xi), R(\xi)$

Induced metric:

$$ds_4^2 = d\xi^2 + R^{2K}(\xi) ds_K^2$$

Equation:

$$\epsilon K \frac{\sqrt{1-R'^2}}{r_c R} = -K \frac{1-R'^2}{R^2} + \frac{\tau}{3M_p^2}$$

$\epsilon = \pm 1$ $\qquad r_c = \frac{M_p^2}{2M_*^3}$

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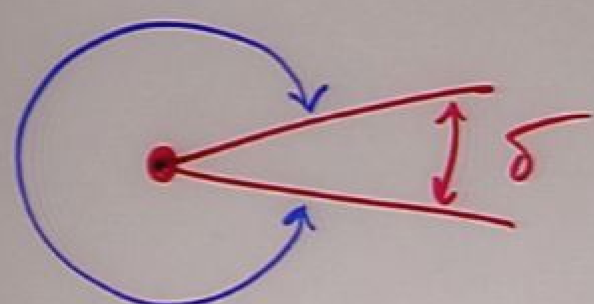
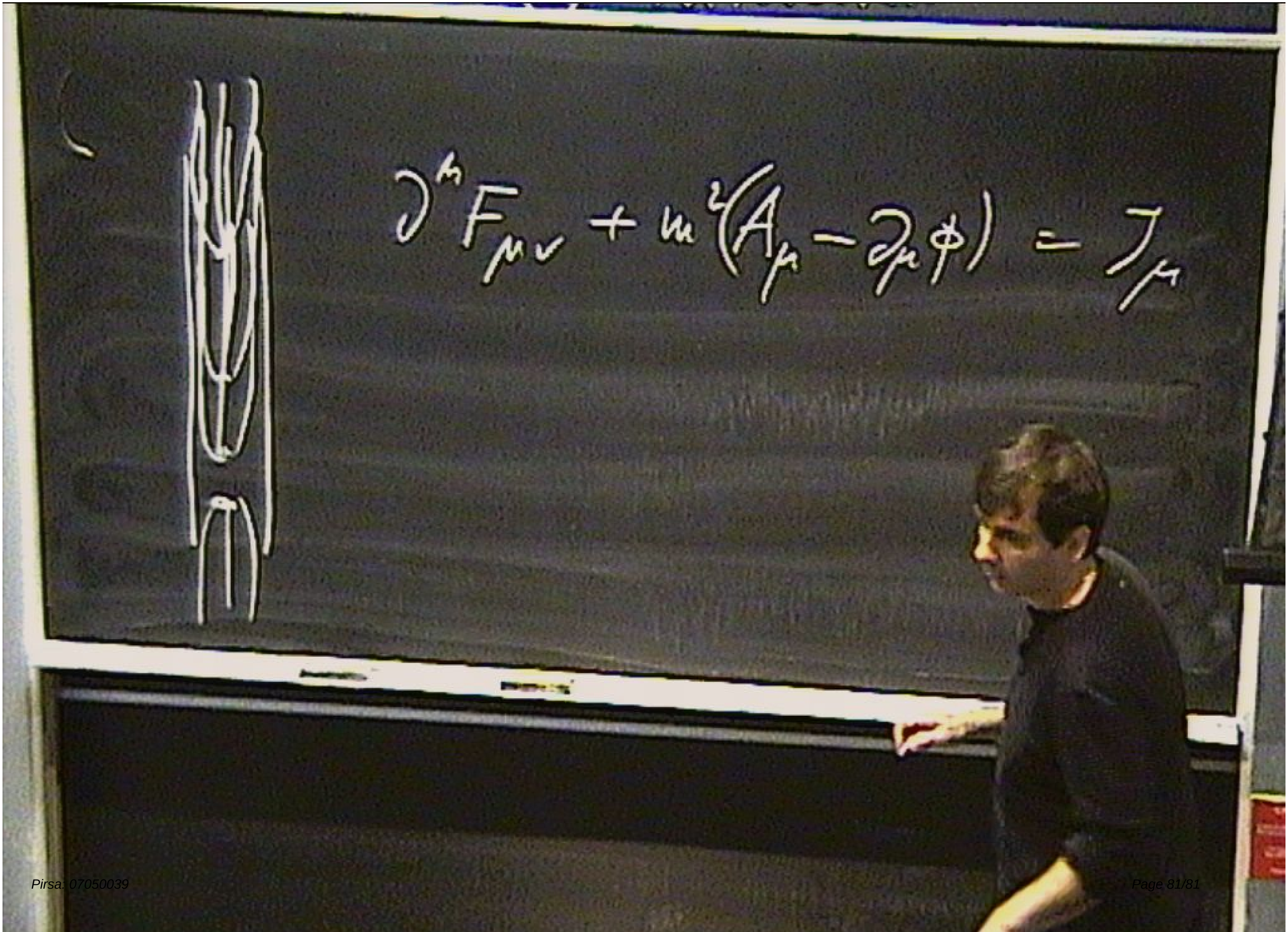
The conventional branch:
wall creates a deficit angle:

$$\delta \simeq \frac{\sigma}{M_*^3}$$

On the self-accelerating branch
the wall create an excess angle:

$$\delta \simeq - \frac{\sigma}{M_*^3}$$




$\partial^{\mu} F_{\mu\nu} + m^2(A_\mu - \partial_\mu \phi) = J_\mu$