Title: Dynamics of Linear Perturbations in Modified Gravity Theories

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Abstract: A modification of Gravity in the low-curvature regime may account for the late time acceleration of our universe, and is therefore an interesting alternative to Dark Energy.In such models, the modified Einstein equations admit self-accelerated solutions in the presence of negligible matter. At the level of perturbation theory,the modified equations give rise to new dynamics for the perturbations of the metric and matter. I will consider scalar perturbations, presenting in some details the dynamics of linear perturbations for two specific models, f(R) Gravity and Modified Source Gravity. I will conclude by demonstrating how some characteristic features of these models are likely to be common to general models of Modified Gravity and how Large Scale Structure formation and the Integrated Sachs Wolfe effect might be useful probes to distinguish between Dark Energy and Modified Gravity.

Outline

- Different approaches to the phenomenon of cosmic acceleration...Modified Gravity
- f(R) and Modified Source Gravity (MSG)
- Dynamics of linear perturbations in Modified Gravity
- Characteristic signatures of Modified Gravity: Large Scale Structures, ISW and Weak Lensing as test for modified theory ?

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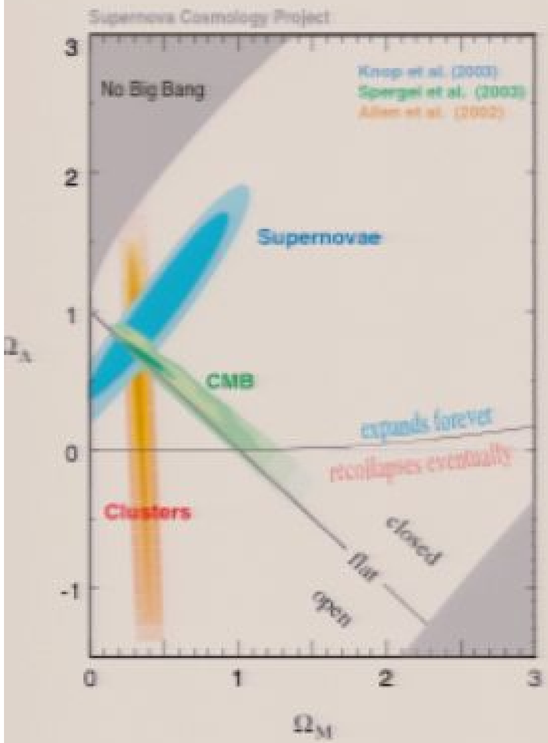
Cosmic acceleration



A very good fit to all these data is a Universe in which 70% of the energy budget is in the COSMOLOGICAL CONSTANT.

Anyhow it is important to explore the whole space of explanations that fit these data and could have testable features...

Dark Energy

$$G_{\mu\nu} = \frac{1}{M_P^2}\tilde{T}_{\mu\nu}$$

X matter fields with dynamics such as to cause the late universe to accelerate (quintessence, k-essence, ...)

Modified Gravity

$$\tilde{G}_{\mu\nu} = \frac{1}{M_P^2}T_{\mu\nu}$$

modification of GR on large scales, admitting self-accelerating solutions

Modified Gravity

There are different ways in which one can modify gravity

I will focus on direct covariant modifications
of the 4-dim Einstein-Hilbert action

f(R) theories
(Capozziello et al. astro-ph/0303041,
Carroll et al. PRD'04)

Modified Source Gravity
(Carroll et al. astro-ph/0611321, PRD'07)

modified Einstein equations

extra d.o.f.

~~extra d.o.f.~~

new equations → new dynamics

- LCDM (and std quintessence) models modify "directly" the background dynamics, (giving late-time acceleration), but modify in a more "indirect" way the dynamics of perturbations
- Modified Gravity changes the background dynamics similarly to DE models, and modifies "directly" the dynamics for perturbations

(...generalized Dark Energy...)

There is typically enough freedom in a modified gravity model to reproduce any desired expansion history. However the behavior of perturbations might manifest characteristic signatures of modifications

Structure formation and ISW are important tests for modified gravity

f(R) Gravity

$$S = \frac{M_P^2}{2}\int dx^4\sqrt{-g}\,[R+f(R)] + \int d^4x\sqrt{-g}\mathcal{L}_m[\chi_i, g_{\mu\nu}]$$

$$\begin{cases} (1+f_R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R+f) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R = \frac{T_{\mu\nu}}{M_P^2} \\ \nabla_\mu T^{\mu\nu} = 0 \end{cases}$$

the Einstein equations are fourth order. The trace-equation becomes:

$$(1-f_R)R + 2f - 3\Box f_R = \frac{T}{M_P^2}$$

NOT an algebraic equation!

Friedmann eq.:

$$(1-f_R)\mathcal{H}^2 + \frac{a^2}{6}f - \frac{a''}{a}f_R + \mathcal{H}f'_R = \frac{1}{3M_P^2}a^2\rho$$

Issues

- Background evolution: we need $f_{RR} > 0$ to reproduce the correct matter era

(Amendola et al. astro-ph/0603703-0612180)

- Curvature instabilities: curvature scalar is rapidly driven to small values in presence of matter; to avoid this we need $f_{RR} > 0$

(Dolgov & Kawasaki, Phys.Lett.B 573 (2003), V.Faraoni astro-ph/061073; I.Navarro & K.Van Acoyelen gr-qc/0611127, Amendola et al. astro-ph/0603703 Y.S.Song,W.Hu and I.Sawicki Phys.Rev.D75:044004,(2007),I.Sawicki and W.Hu astro-ph/0702278)

- Local tests of Gravity: rich literature, Luca Amendola and Wayne Hu's talks, tight costraints making the models cloes to LCDM, but still leaving space for departures at linear perturbations level. We need $f_{RR} > 0$ and small f_R gradients

(Navarro & Van Acoyelen, gr-qc/0611127, Chiba Phys.Lett.B 575 (2003), Chiba, Smith, Erickcek astro-ph/0611867 Hu, Sawicki astro-ph/0705.1158, Amendola, Tsujiakawa astro-ph/0705.0396)

We will focus on linear perturbations and investigate which evolution/constraints arise. We can learn how modifications of gravity could manifest themselves in CMB, LSS and ISW

Dynamics of Linear Perturbations in f(R) Gravity

R.Bean, D.Bernat, L.Pogosian, A.S., M.Trodden astro-ph/0611321, PRD'07

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Rachel's talk

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Scalar perturbations in Conformal Newtonian gauge

$$ds^2 = -a^2(\tau)\,(1+2\Psi)\,d\tau^2 + a^2(\tau)\,(1-2\Phi)\,d\vec{x}^2$$

$$\begin{cases} T^0_0 = -\rho(1+\delta) \\ T^0_j = (\rho+P)v_j \\ T^i_j = (P+\delta P)\delta^i_j \end{cases}$$

Dynamics of Linear Perturbations in f(R) Gravity

CDM equation:

$$\delta'' + \mathcal{H}\delta' + k^2\Psi - 3\Phi'' - 3\mathcal{H}\Phi' = 0$$

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Einstein equations:

dynamical eq.

anisotropy eq. $(\Psi - \Phi) = -\frac{f_{RR}}{1+f_R}\delta R$

dynamical eq.

Poisson eq. $k^2(1+f_R)\frac{(\Phi+\Psi)}{2} = -\frac{a^2}{2M_P^2}\rho\Delta + \frac{3}{2}(\mathcal{H}^2 - \mathcal{H}')f_{RR}\delta R - \frac{3}{2}f_R'(\mathcal{H}\Psi + \Phi')$

time dependent G

extra terms

$$\left(\delta R = \frac{2}{a^2}\left[-6\frac{a''}{a}\Psi - 3\mathcal{H}\Psi' + k^2\Psi - 9\mathcal{H}\Phi' - 3\Phi'' - 2k^2\Phi\right] \ , \ \Delta \equiv \delta + 3\mathcal{H}v/k \right)$$

Extra Dynamics

new variables:

$$\begin{cases} \Phi_+ \equiv \dfrac{\Phi + \Psi}{2} & \longrightarrow \quad \text{ISW } (\Phi'_+) \text{ \& WL} \\ \chi \equiv f_{RR}\delta R & \longrightarrow \quad \text{potentials slip} \end{cases}$$

anisotropy eq. :

$$\Psi - \Phi = -\frac{f_{RR}}{1+f_R}\delta R \quad \xrightarrow[F \equiv 1 + f_R]{} \quad \boxed{\Psi = \Phi_+ - \frac{\chi}{2F}}$$

CONSTRAINT eq. !

and two coupled first order differential equations

momentum & Poisson eqs.:

$$\begin{cases} \Phi'_+ = \dfrac{3}{2}\dfrac{a\Omega v}{HkF} - \left(1 + \dfrac{1}{2}\dfrac{F'}{F}\right)\Phi_+ + \dfrac{3}{4}\dfrac{F'}{F}\dfrac{\chi}{F} \\ \chi' = -\dfrac{2\Omega\Delta}{H^2}\dfrac{F}{F'} + \left(1 + \dfrac{F'}{F} - 2\dfrac{H'}{H}\dfrac{F}{F'}\right)\chi - 2F\Phi'_+ - 2F\left(1 + \dfrac{2}{3}\dfrac{k^2}{a^2H^2}\dfrac{F}{F'}\right)\Phi_+ \end{cases}$$

Modified Evolution

CDM equation:

$$\delta'' + \mathcal{H}\delta' + k^2\Phi_+ - k^2\frac{\chi}{2F} = 0$$

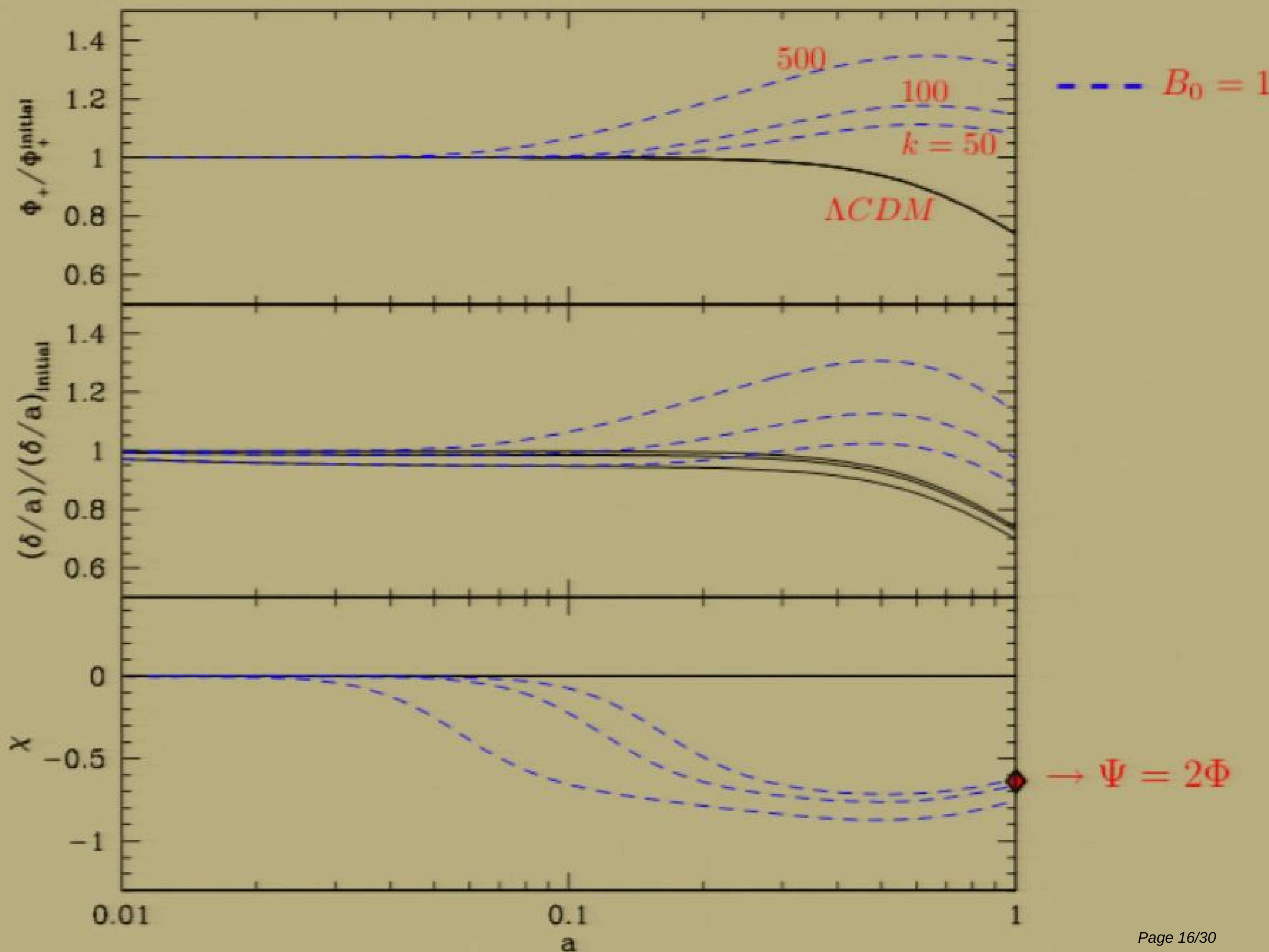
Poisson eq.

$$k^2\Phi_+ = -\frac{a^2}{2M_P^2 F}\rho\Delta + \frac{3}{2}(\mathcal{H}^2 - \mathcal{H}')\frac{\chi}{F} - \frac{3}{2}\frac{F'}{F}\left(\mathcal{H}\left(\Phi_+ - \frac{\chi}{2F}\right) + \Phi'_+ + \frac{\chi'}{2F} - \frac{F'}{2F}\frac{\chi}{F}\right)$$

temporary modification of growth

$$\delta'' + \mathcal{H}\delta' - \frac{a^2}{2M_P^2 F}\rho\delta + \frac{3}{2}(\mathcal{H}^2 - \mathcal{H}')\frac{\chi}{F} - \frac{3}{2}\frac{F'}{F}\left(\mathcal{H}\left(\Phi_+ - \frac{\chi}{2F}\right) + \Phi'_+ + \frac{\chi'}{2F} - \frac{F'}{2F}\frac{\chi}{F}\right) - k^2\frac{\chi}{2F} = 0$$

$w = -1$



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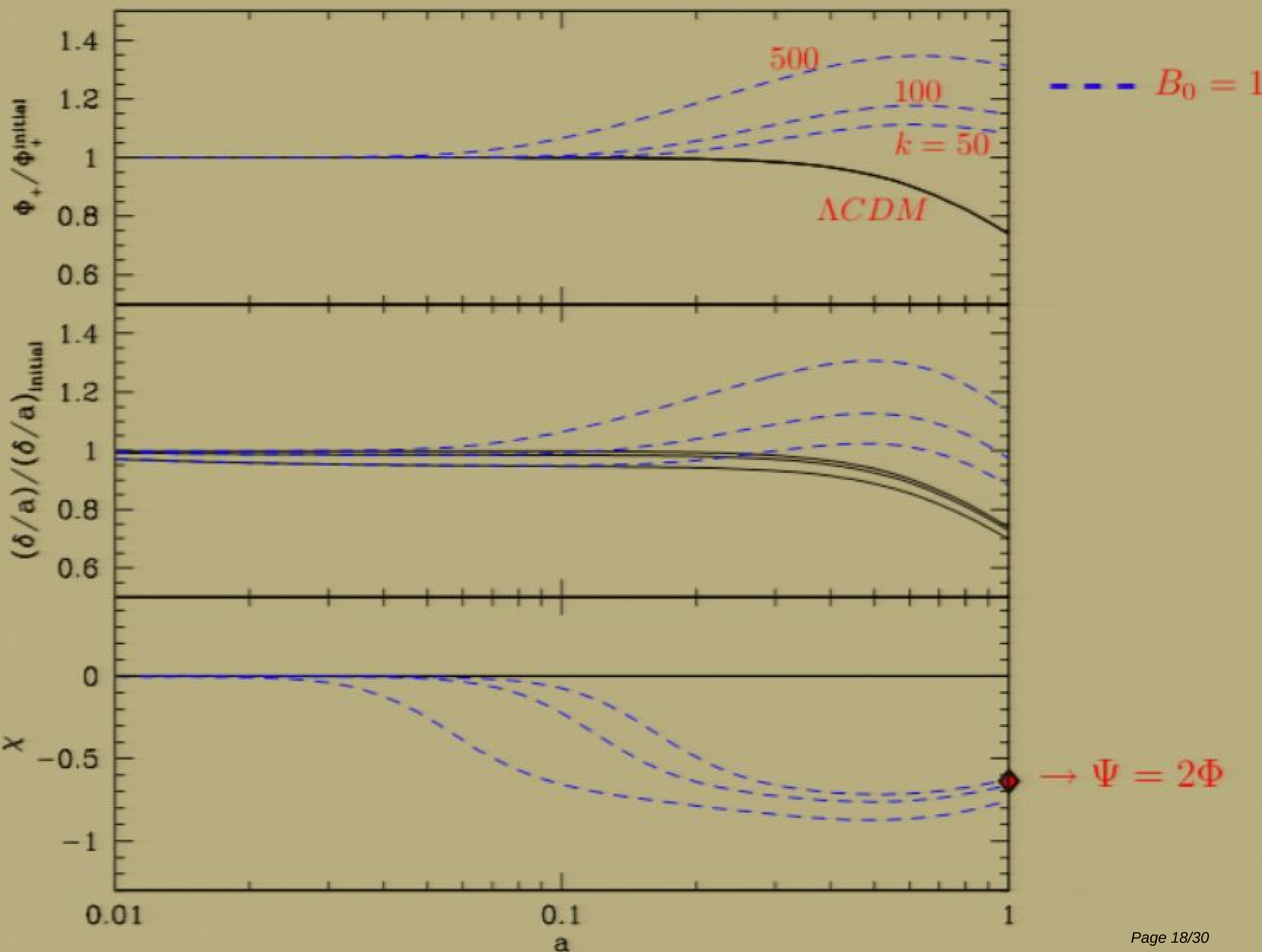
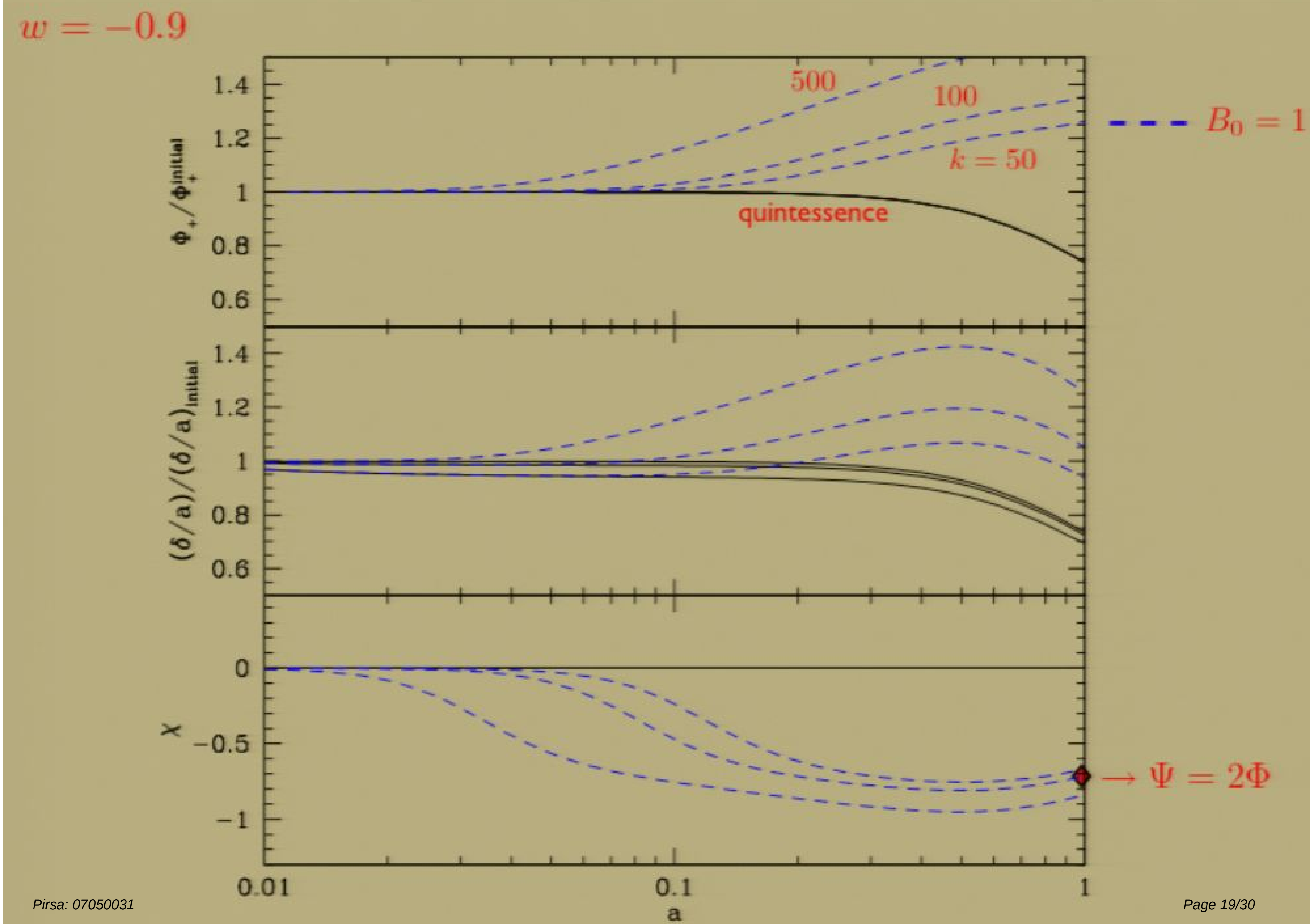
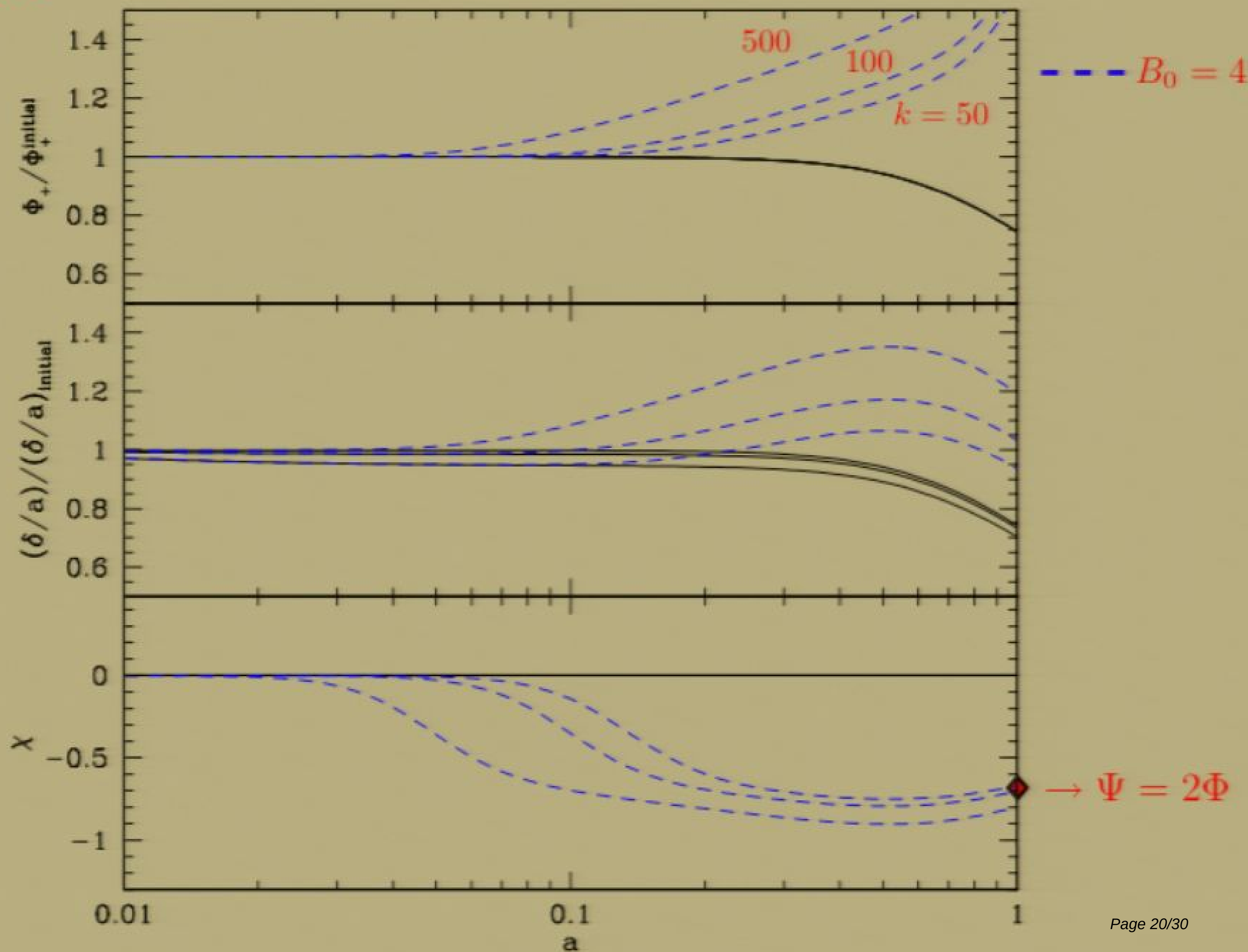


1.4
1.2
1
0.8
0.6
$\Phi_+/\Phi_+^{\rm initial}$
500
100
$k = 50$
ΛCDM
$B_0 = 1$
1.4
1.2
1
0.8
0.6
$(\delta/a)/(\delta/a)_{\rm initial}$
0
−0.5
−1
χ
$\rightarrow \Psi = 2\Phi$
0.01
0.1
1
a



$w = -0.9$
500
100
$B_0 = 1$
$k = 50$
quintessence
$\Phi_+/\Phi_+^{initial}$
$(\delta/a)/(\delta/a)_{initial}$
χ
$\rightarrow \Psi = 2\Phi$
1.4
1.2
1
0.8
0.6
0
−0.5
−1
0.01
0.1
1
a

$w = -1.02$



500
100
$k = 50$
$B_0 = 4$
$\Phi_+/\Phi_+^{initial}$
1.4
1.2
1
0.8
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$(\delta/a)/(\delta/a)_{initial}$
1.4
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a
$\rightarrow \Psi = 2\Phi$

Modifying Gravity without new d.o.f.

The Einstein-Hilbert action for gravity has the peculiarity of giving dynamical equations only for the massless spin-2 graviton. The other equations are constraints.

When we modify the action we typically free up some d.o.f., for f(R) a scalar.

It would be interesting to modify the equations without introducing new d.o.f.

An example is Modified Source Gravity

See also Cuscuton cosmology
Afshordi et al. PRD'07 (hep-th/0609150)

Dynamics of Linear Perturbations in Modified Source Gravity

$$S = \int dx^4 \sqrt{-g} \left[\frac{M_P^2}{2} e^{2\psi} R + 3e^{2\psi} (\nabla\psi)^2 - U(\psi) \right] + s_m[g, \chi_i]$$

Carroll et al. astro-ph/0607458, NJP'06

$$e^{2\psi} G_{\mu\nu} = \kappa^2 \left(T_{\mu\nu} + T_{\mu\nu}^{(\psi)} \right)$$

EOMs

$$\Box\psi + (\nabla\psi)^2 + \frac{1}{6M_P^2} e^{-2\psi} \frac{dU}{d\psi} - \frac{1}{6} R = 0$$

trace of Einstein eq.

$$\frac{dU}{d\psi} - 4U(\psi) = -T$$

↓

$$\psi = \psi(T) = \psi(\rho_m)$$

$$G_{\mu\nu} = \tilde{T}_{\mu\nu}(\rho)$$

Scalar perturbations in Conformal Newtonian gauge

Matter equation

$$\delta'' + \mathcal{H}\delta' + k^2\Psi - 3\Phi'' - 3\mathcal{H}\Phi' = 0$$

$$\delta\psi = -\frac{1}{3}\frac{d\psi}{dlna}\delta$$

Scalar perturbations in Conformal Newtonian gauge

Matter equation

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$$\delta\psi = -\frac{1}{3}\frac{d\psi}{dlna}\delta$$

Einstein equations

$$\Psi - \Phi = -2\delta\psi$$

$$-k^2\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}) = \frac{e^{-2\psi}}{2M_P^2}(\rho\delta + (U_{,\psi} - 2U - 2\rho)\delta\psi) - k^2\delta\psi - 3\psi'(\delta\psi' - \Phi' + \phi'\Psi + 6\mathcal{H}\Psi) - 3\mathcal{H}\delta\psi'$$

Scalar perturbations in Conformal Newtonian gauge

Matter equation

$$\delta'' + \mathcal{H}\delta' + k^2\Psi - 3\Phi'' - 3\mathcal{H}\Phi' = 0 \qquad \delta\psi = -\frac{1}{3}\frac{d\psi}{dlna}\delta$$

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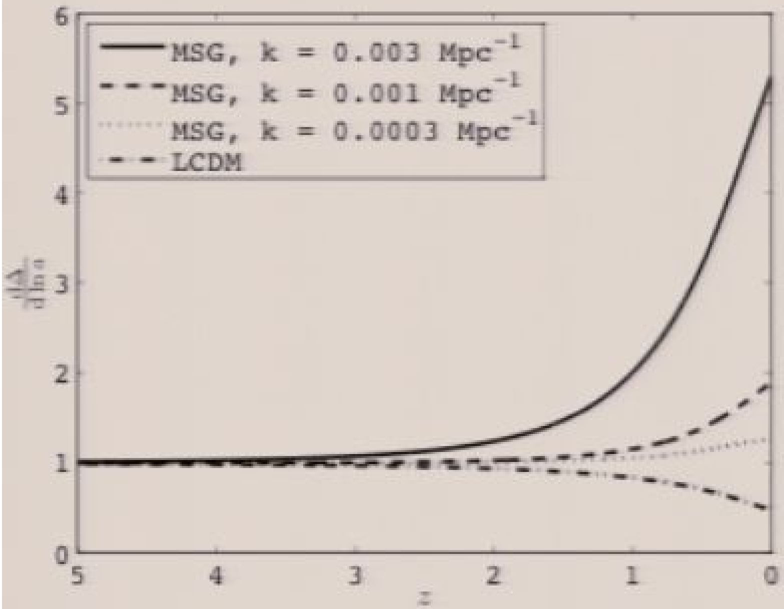
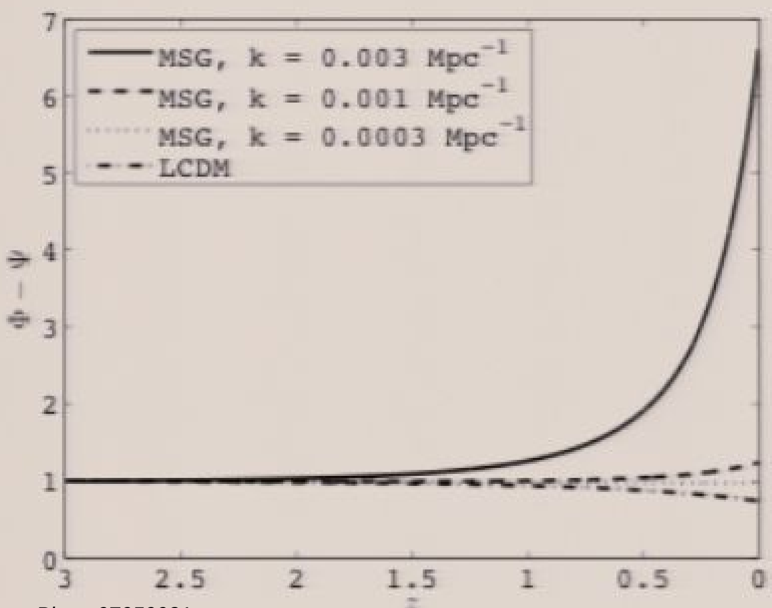
late-time

negative sound speed

$$\delta'' + \mathcal{H}\delta' - \left[\frac{e^{-2\psi}}{2M_P^2}\left(1 + \frac{d\psi}{dlna}\right)\rho - \frac{k^2}{3a^2}\frac{d\psi}{dlna}\right]\delta = 0$$

- scale-dependence
- runaway growth for small scales





- scale-dependent runaway growth
- rapid structure formation drives the growth of gravitational potentials
- the ISW effect is enhanced at the lowest multipoles
- negative LSS-ISW correlation
- important to study the non-linear regime

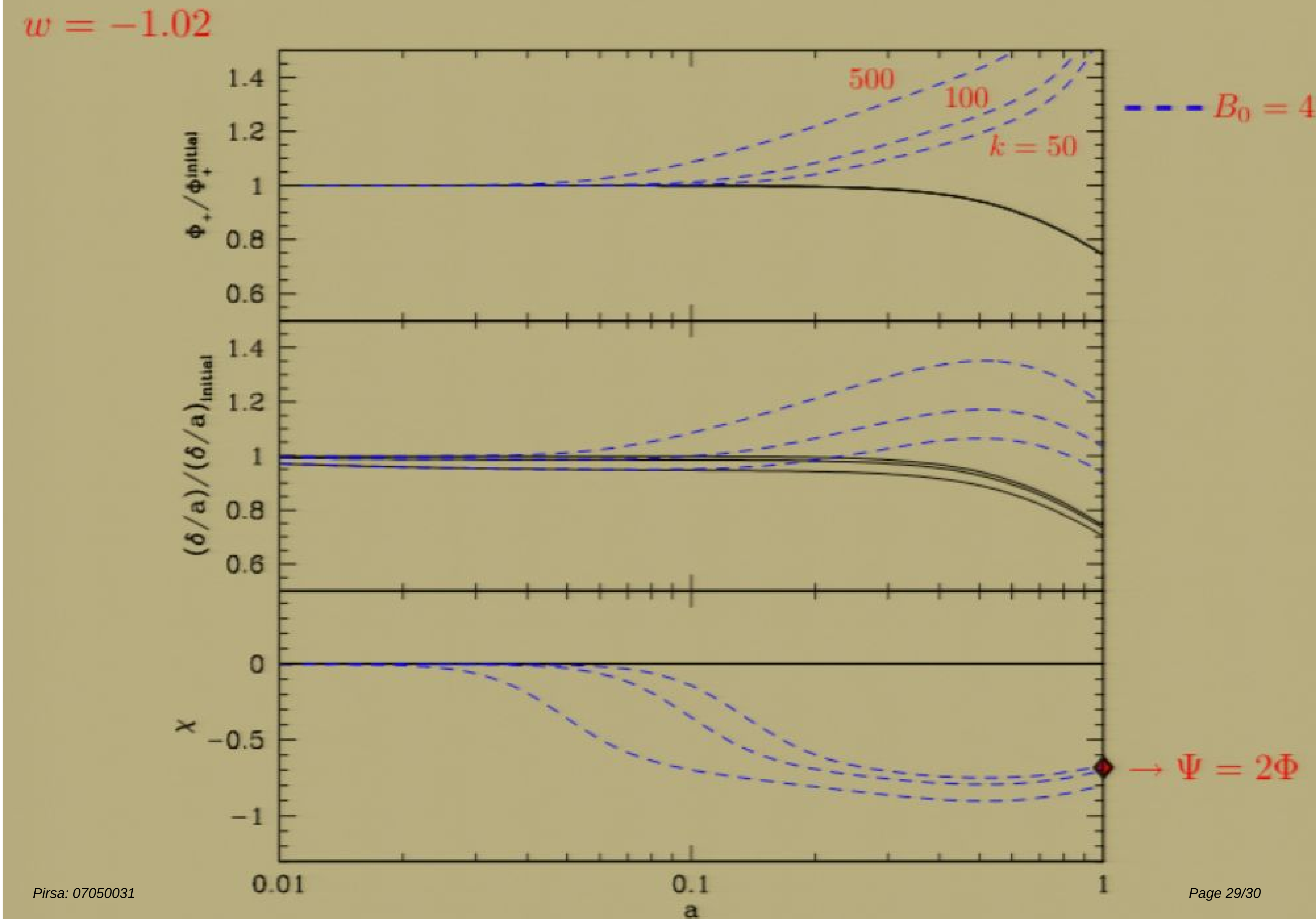
CONCLUSIONS

For modified gravity models that reproduce the desired background evolution, specifically f(R) & MSG, we investigated the dynamics of linear perturbations, finding:

- effective shear ⟹ slip between metric potentials Ψ and Φ
- non trivial 'clustering of modifications'
- modified, scale-dependent evolution of the metric potentials ⟹ modified ISW signal
- modified, scale-dependent evolution of matter perturbations

The ISW, its correlation with LSS and Weak Lensing might be very useful probes of modifications of gravity

THANK YOU!



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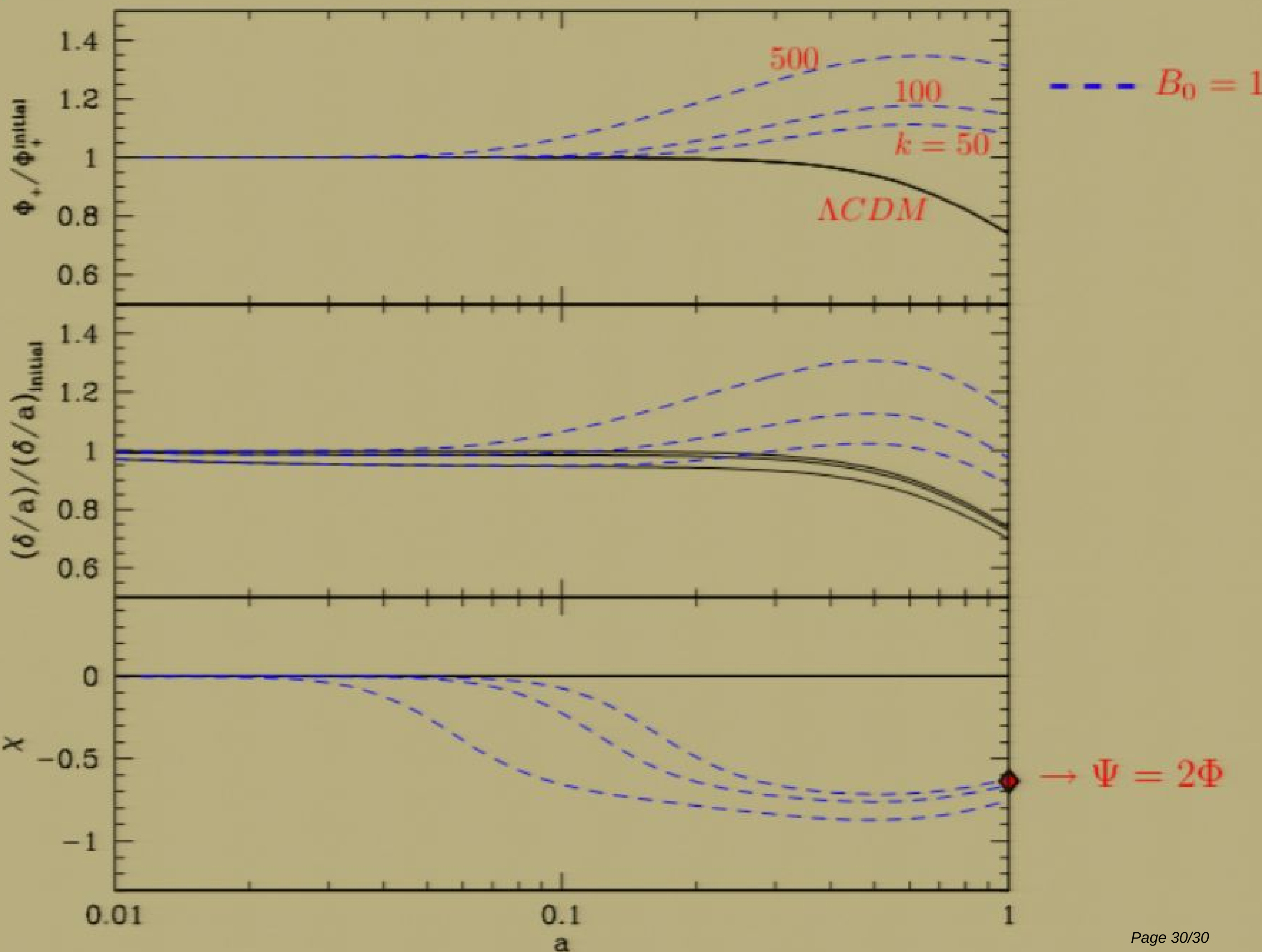

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