Title: Dark Energy from variation of the fundamental scale

Date: May 18, 2007 09:00 AM

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Abstract: We explore the hypothesis that a dynamical dark energy is related to a time-variation of the fundamental mass scale. A dilatation anomaly induced by quantum fluctuations could explain the small value of the present dark energy. Reformulated as a scalar field theory this would predict a quintessence potential which asymptotically relaxes to zero rather than to a nonvanishing constant. An observable consequence of such a scenario results in "early dark energy " contributing a few percent to the energy density of the Universe even at high redshift. Quintessence would be related to a new "fundamental" macroscopic force and induce a small time variation of fundamental constants.

What is our universe made of ?

What is our universe made of ?

Dark Energy dominates the Universe

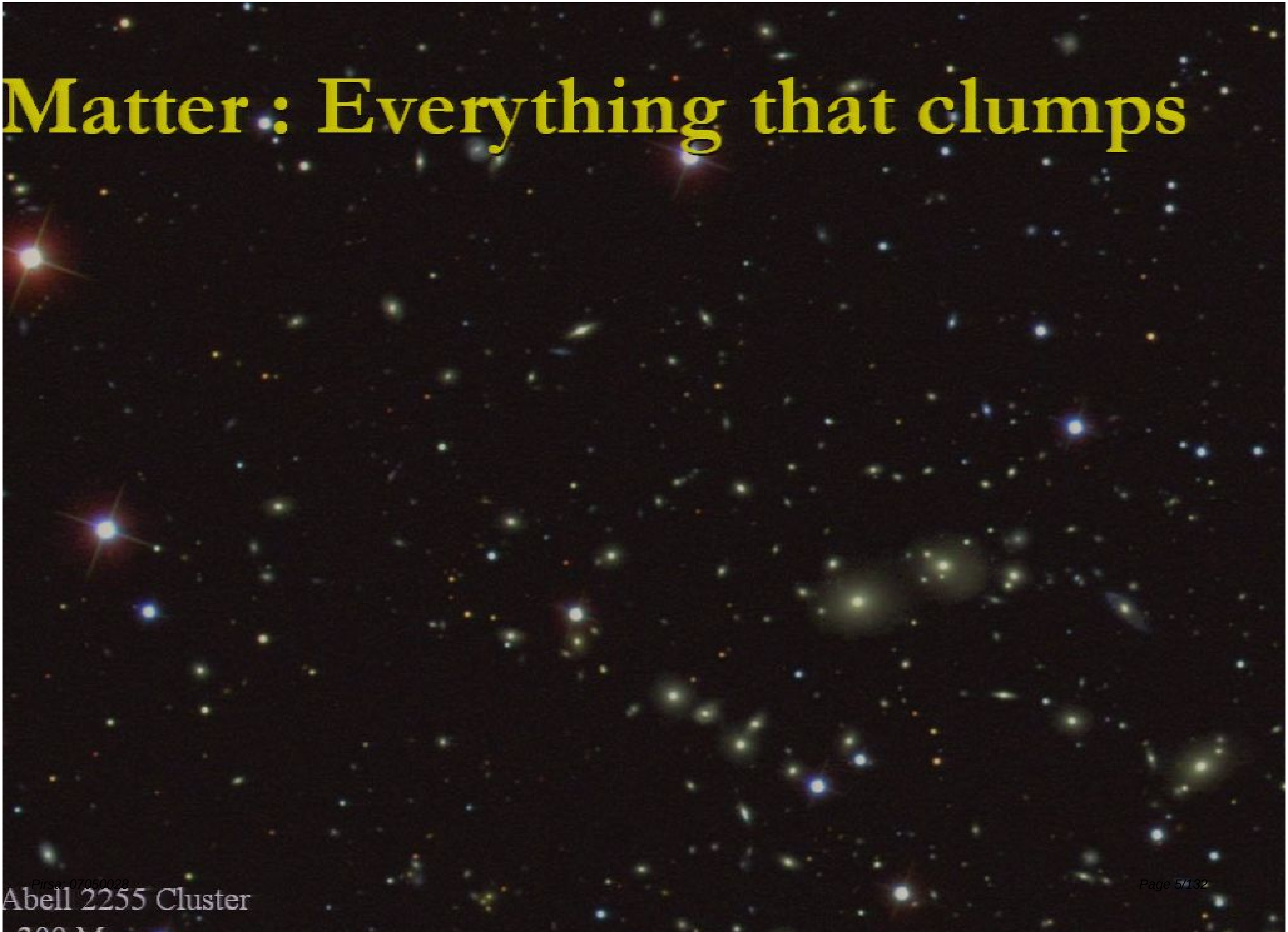
Energy - density in the Universe

=

Matter + Dark Energy

25 % + 75 %

Matter : Everything that clumps



Abell 2255 Cluster

Dark Energy density is
the same at every point of space

" homogeneous "

No local force –
" In what direction should it draw ? "

What is Dark Energy ?

Cosmological Constant

or

Quintessence ?

Quintessence and solution of cosmological constant problem should be related !

Cosmological Constant
- Einstein -

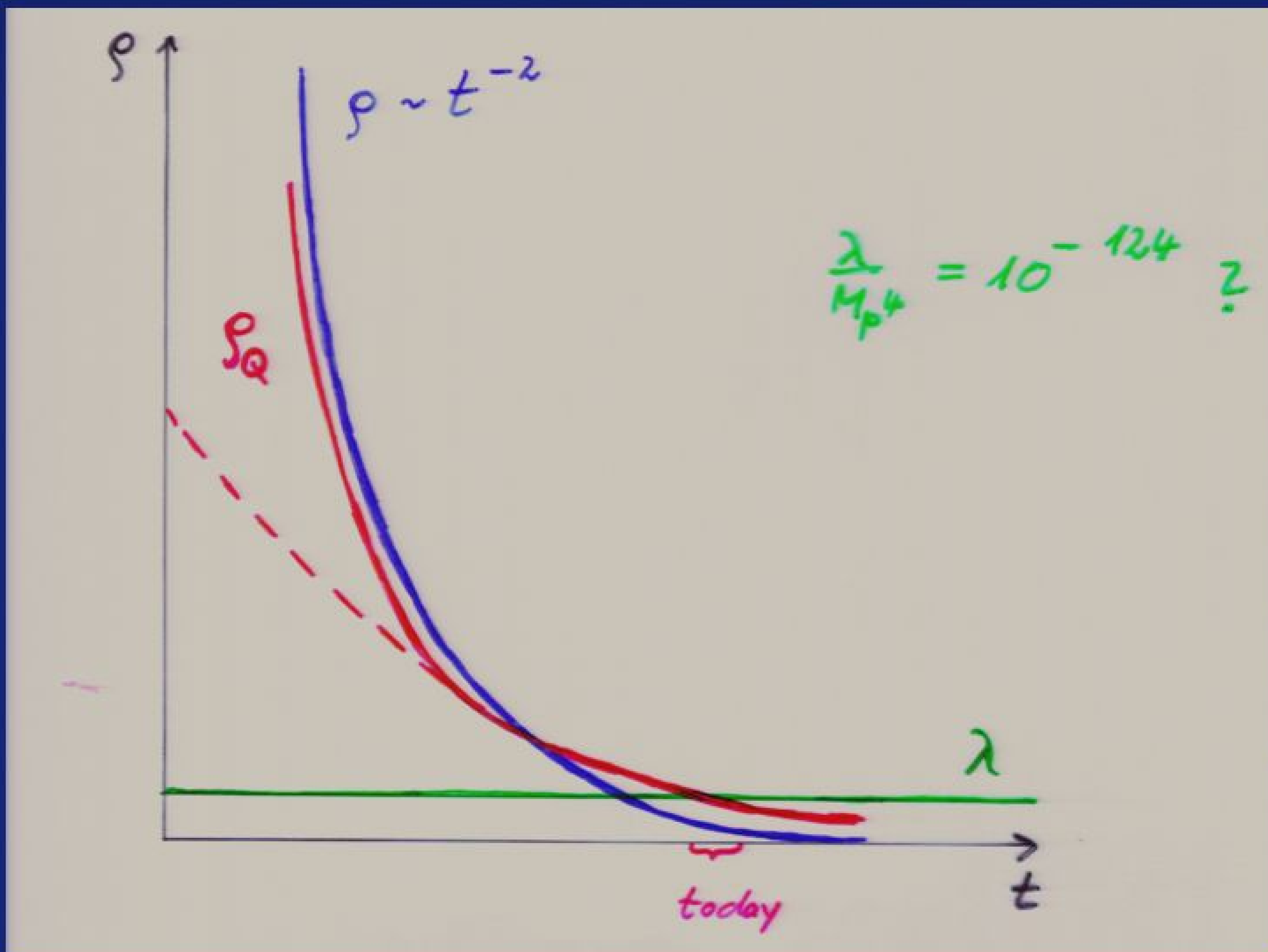
- Constant λ compatible with all symmetries
- No time variation in contribution to energy density

- Why so small ? $\lambda/M^4 = 10^{-120}$

- Why important just today ?

Cosm. Const. | Quintessence

static | dynamical



challenge

- explain why Dark Energy goes to zero asymptotically ,
- not to a constant !

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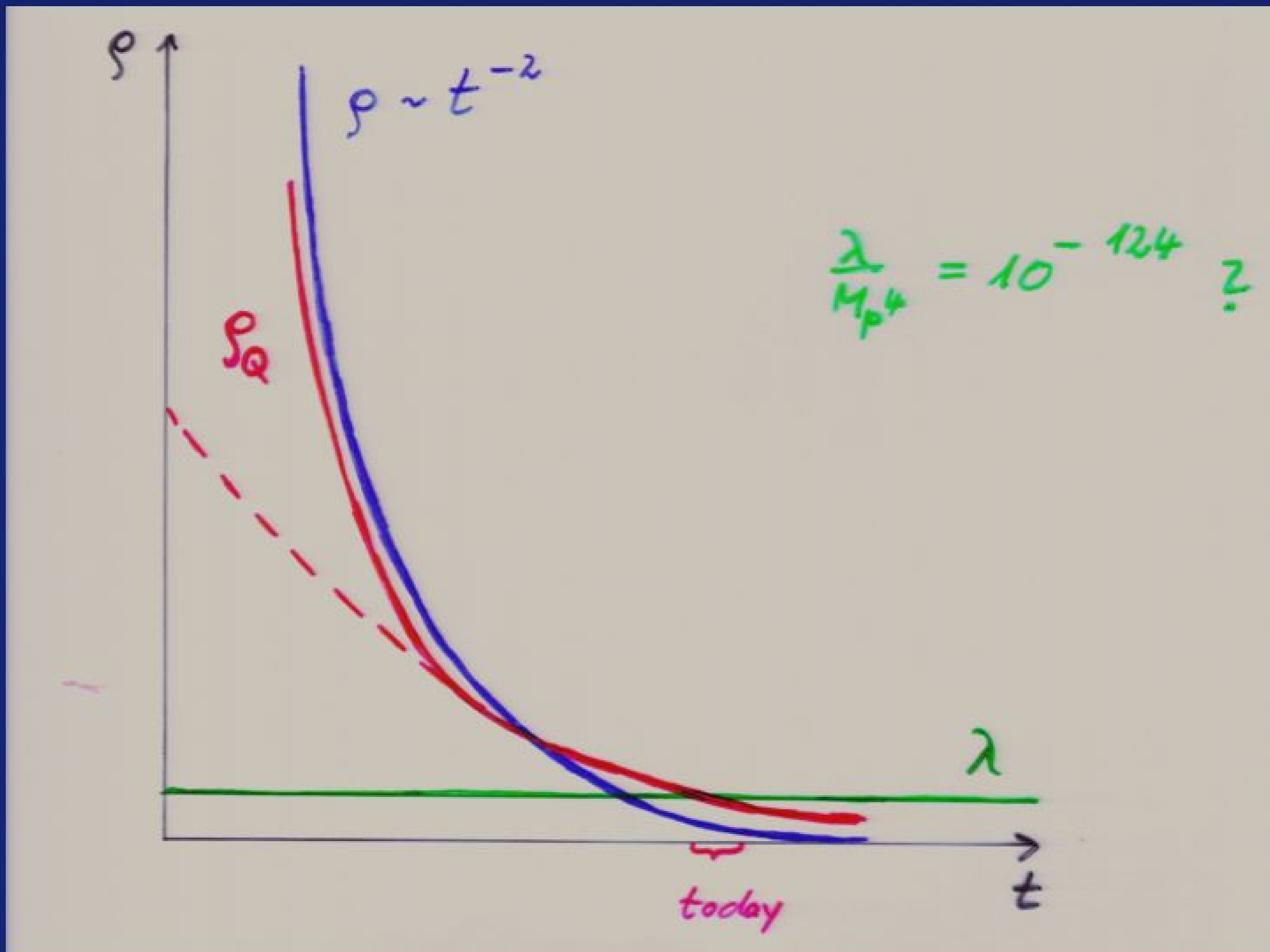

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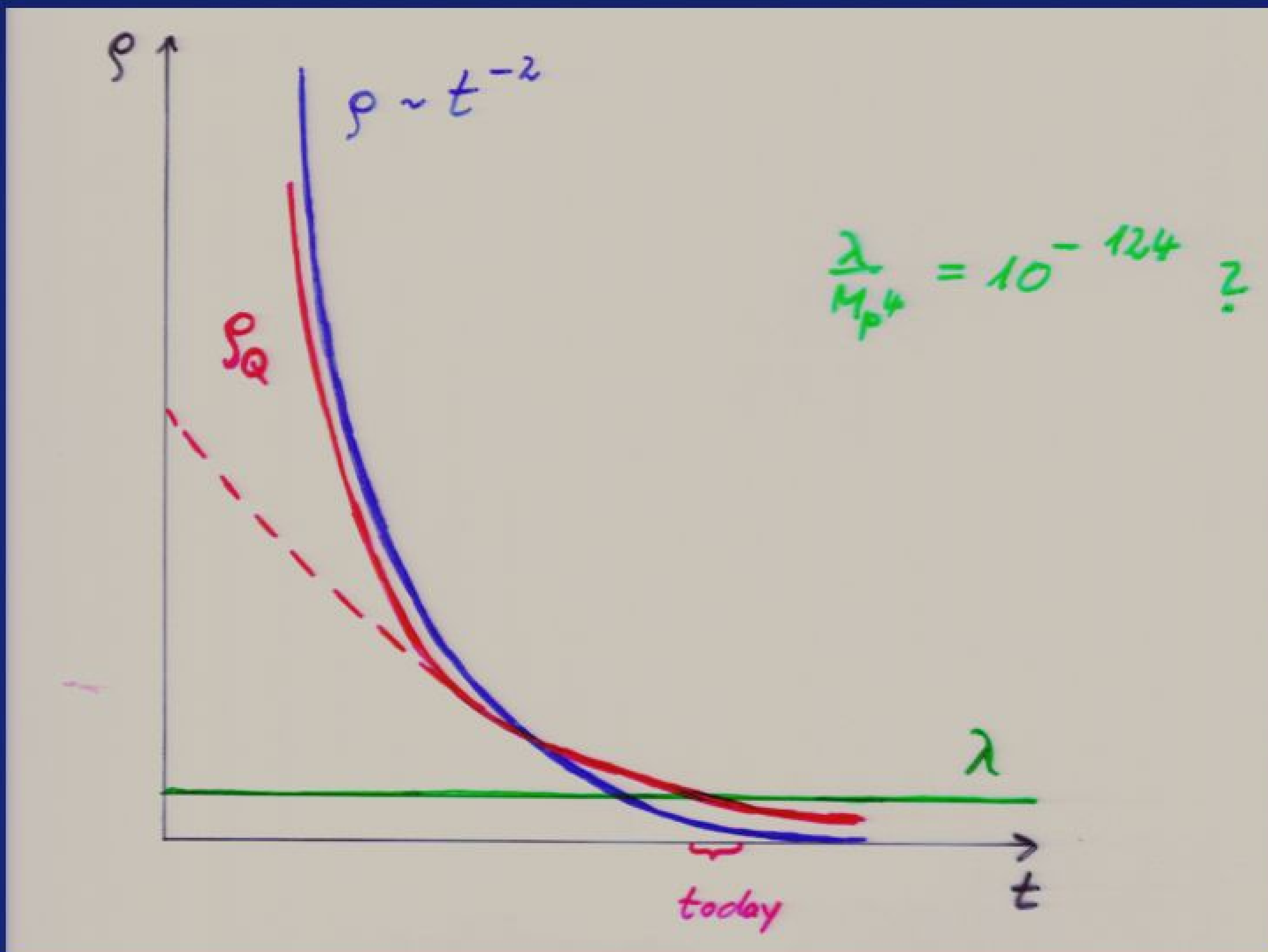

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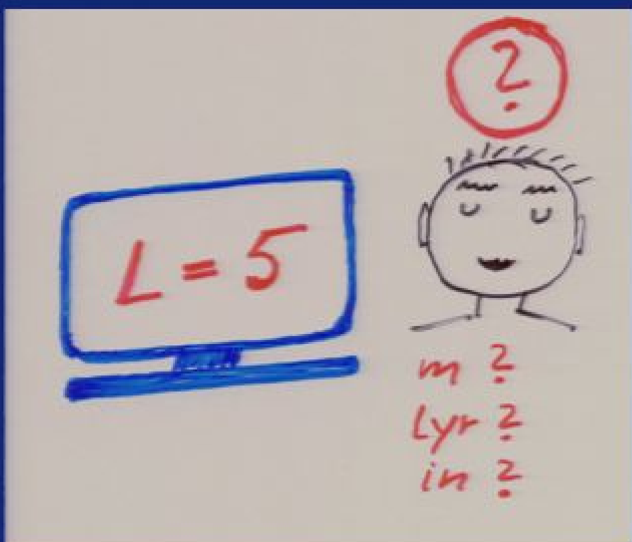
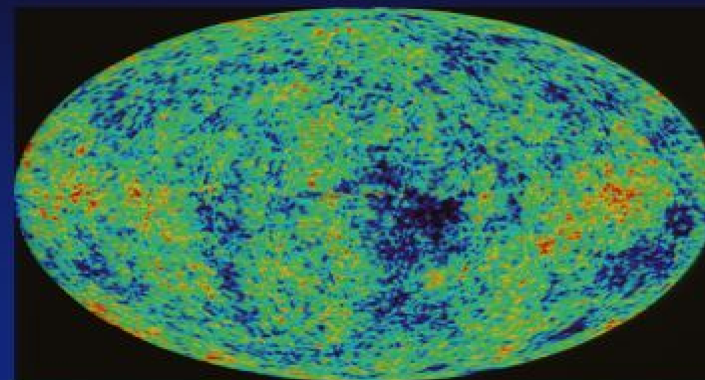
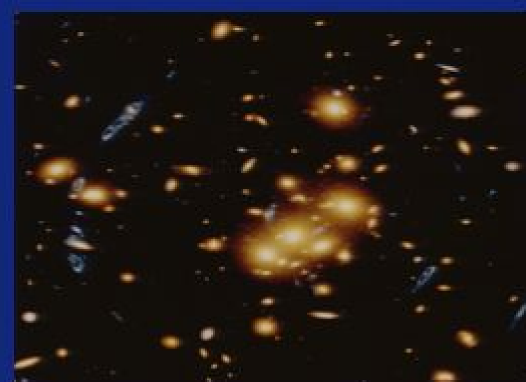


$$\Omega_m + X = 1$$



$$\Omega_m \; : \; 25\%$$



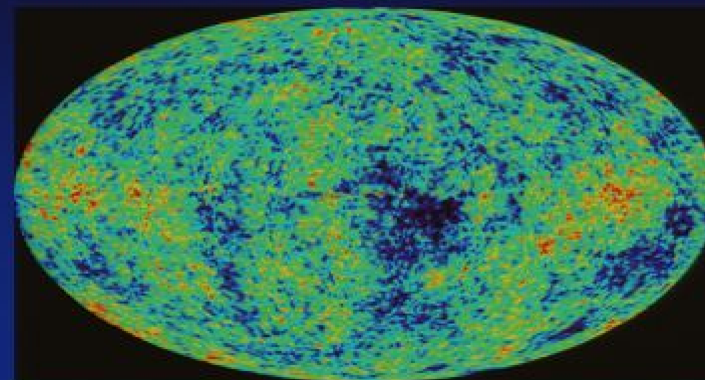
$$\Omega_h \; : \; 75\%$$

Dark Energy

Time dependent Dark Energy : Quintessence

- What changes in time ?
- **Only dimensionless ratios of mass scales are observable !**
- V : potential energy of scalar field or cosmological constant
- V/M^4 is observable
- **Imagine the Planck mass M increases …**

$$\Omega_m + X = 1$$



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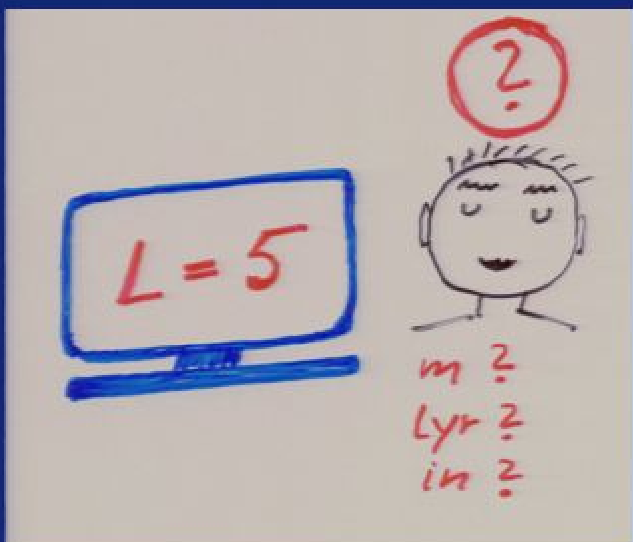
Dark Energy



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Fundamental mass scale

- Unification fixes parameters with dimensions

- Special relativity : c
- Quantum theory : h
- Unification with gravity :

fundamental mass scale

(Planck mass , string tension , …)

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Fundamental mass scale

- Fixed parameter or dynamical scale ?
- Dynamical scale ⟷ Field
- Dynamical scale compared to what ?

momentum versus mass

(or other parameter with dimension)

Cosmon and fundamental mass scale

- Assume all mass parameters are proportional to scalar field χ (GUTs, superstrings,…)
- $M_p \sim \chi$, $m_{proton} \sim \chi$, $\Lambda_{QCD} \sim \chi$, $M_W \sim \chi$,…
- χ may evolve with time : **cosmon**
- m_n/M : (almost) constant - ***observation*** !

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Example :

Field χ is connected to scale of transition
from higher dimensional physics
to effective four dimensional description
in theory without fundamental mass parameter

(except for running of dimensionless couplings…)

theory without explicit mass scale

- Lagrange density:

$$L = \sqrt{g}\left(-\frac{1}{2}\chi^2 R + \frac{1}{2}(\delta - 6)\partial^\mu \chi \partial_\mu \chi + V(\chi) + h\chi\overline{\psi}\psi\right)$$

realistic theory

- χ has no gauge interactions
- χ is effective scalar field after "integrating out" all other scalar fields

Dilatation symmetry

- Lagrange density:

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- Dilatation symmetry for

$$V = \lambda \chi^4, \lambda = const., \delta = const., h = const.$$

- Conformal symmetry for $\delta=0$

Dilatation anomaly

- Quantum fluctuations responsible for dilatation anomaly
- Running couplings: **hypothesis**

$$\partial\lambda/\partial \ln\chi = -A\lambda\,,\ \partial\delta/\partial \ln\chi = E\delta^2$$

- Renormalization scale μ : (momentum scale)
- $\lambda \sim (\chi/\mu)^{-A}$
- $E > 0$: crossover Quintessence

Dilatation symmetry

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Asymptotic behavior of effective potential

- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A}\, \chi^4$

$$V \sim \chi^{4-A}$$

crucial : behavior for large χ !

Dilatation anomaly and quantum fluctuations

- Computation of running couplings (beta functions) needs unified theory !
- Dominant contribution from modes with momenta $\sim\chi$!
- No prejudice on "natural value " of anomalous dimension should be inferred from tiny contributions at QCD- momentum scale !

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e.g. $V \sim \chi^{2}$ or $V \sim \text{const.}$

crucial : behavior for large χ !

Cosmology

Cosmology : χ increases with time !

(due to coupling of χ to curvature scalar)

for large χ the ratio V/M^4 decreases to zero

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Effective cosmological constant vanishes asymptotically for large t !

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Asymptotically vanishing effective "cosmological constant"

- Effective cosmological constant ~ V/M^4
- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A}\, \chi^4$
- $M = \chi$

$$V/M^4 \sim (\chi/\mu)^{-A}$$

Weyl scaling

Weyl scaling : $g_{\mu\nu} \rightarrow (M/\chi)^2 g_{\mu\nu}$,

$$\varphi/M = \ln(\chi^4/V(\chi))$$

$$L = \sqrt{g}\left(-\frac{1}{2}M^2 R + \frac{1}{2}k^2(\phi)\partial^\mu\phi\partial_\mu\phi + V(\phi) + m(\phi)\overline{\psi}\psi\right)$$

Exponential potential : $V = M^4 \exp(-\varphi/M)$

No additional constant !

Without dilatation – anomaly :

V= const.

Massless Goldstone boson = dilaton

Dilatation – anomaly :

V (φ)

Scalar with tiny time dependent mass :

cosmon

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quantum fluctuations and naturalness

- Jordan- and Einstein frame completely equivalent on level of effective action and field equations (**after** computation of quantum fluctuations !)
- Treatment of quantum fluctuations depends on frame : Jacobian for variable transformation in functional integral
- What is natural in one frame may look unnatural in another frame

quantum fluctuations and frames

- Einstein frame : quantum fluctuations make zero cosmological constant look unnatural
- Jordan frame : quantum fluctuations are at the origin of dilatation anomaly;
- key ingredient for solution of cosmological constant problem !

fixed points and fluctuation contributions of individual components

If running couplings influenced by fixed points:
individual fluctuation contribution can be huge overestimate !

here : fixed point at vanishing quartic coupling and anomalous dimension $\Rightarrow$ $V \sim \chi^{4-A}$

it makes no sense to use naïve scaling argument to infer individual contribution $V \sim h\,\chi^{4}$

Exponential cosmon potential

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Cosmic Attractors

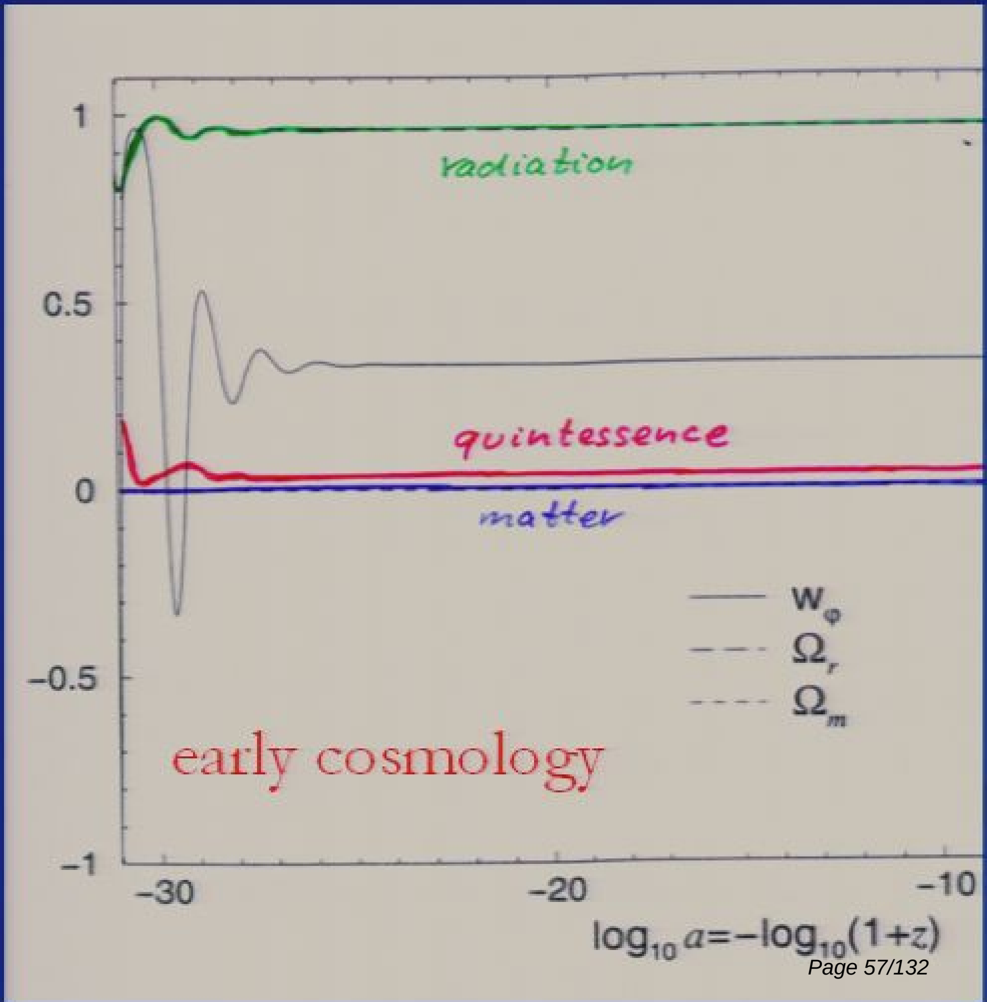
Solutions independent of initial conditions

typically $V \sim t^{-2}$

$\varphi \sim \ln(t)$

$\Omega_h \sim$ const.

details depend on $V(\varphi)$ or kinetic term



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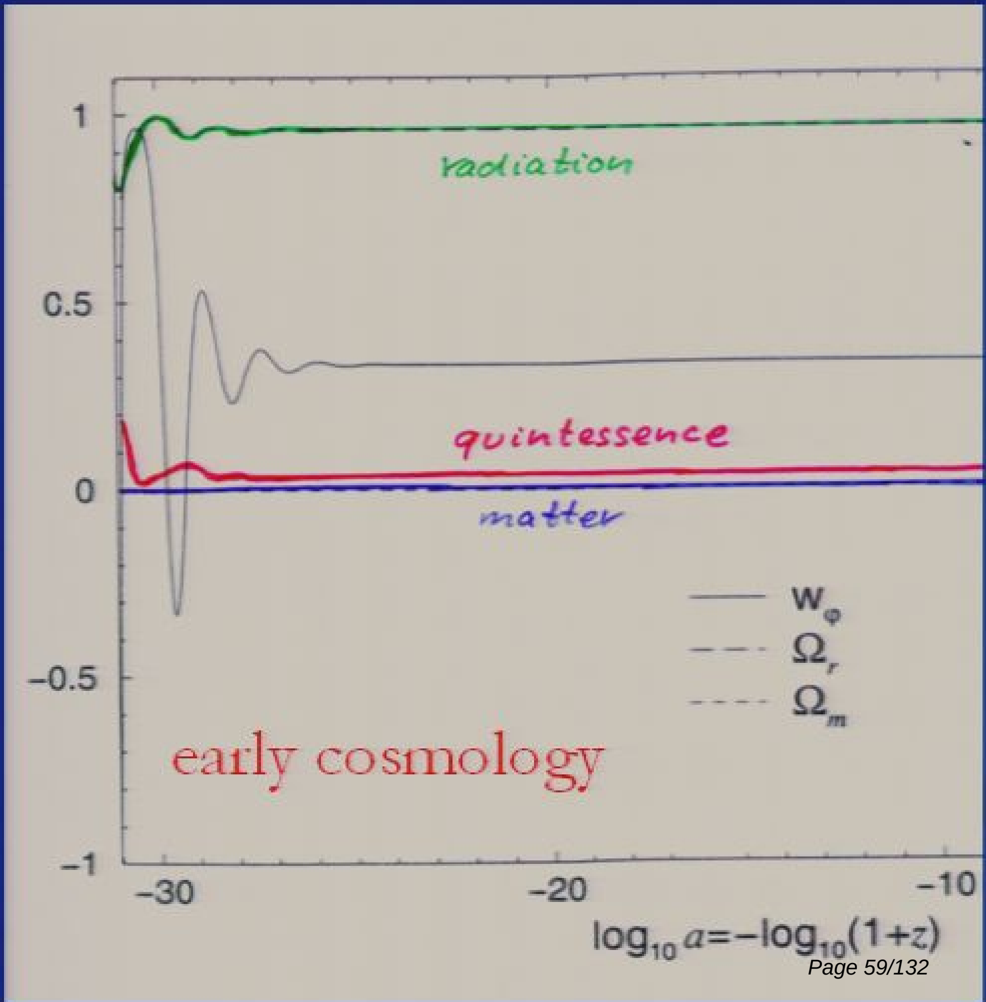

partial solution of cosmological constant problem

$$\Omega_h \sim \text{const.}$$

Dark Energy and Matter of similar size !

Cosmological mass scales

- Energy density

$\rho \sim (2.4\times10^{-3}\ \mathrm{eV})^{-4}$

- Reduced Planck mass

$M=2.44\times10^{18}\,\mathrm{GeV}$

- Newton's constant

$G_N=(8\pi M^2)$

Only ratios of mass scales are observable !

homogeneous dark energy: $\rho_h/M^4 = 6.5\ 10^{-121}$

matter: $\rho_m/M^4 = 3.5\ 10^{-121}$

Time evolution

- $\rho_m / M^4 \sim a^{-3} \sim$
  - t^{-2} matter dominated universe
  - $t^{-3/2}$ radiation dominated universe

- $\rho_r / M^4 \sim a^{-4} \sim t^{-2}$ radiation dominated universe

Huge age ➡ small ratio

Same explanation for small dark energy?

Quintessence

Dynamical dark energy ,
generated by scalar **field**

(cosmon)

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations
…. modifications

realistic quintessence

fraction in dark energy has to increase in "recent time" !

Crossover Quintessence

$$\partial\delta/\partial \ln \chi = E\delta^2$$

(like QCD gauge coupling)

critical χ where δ grows large

critical φ where k grows large

$k^2(\varphi) = \delta(\chi)/4$

$k^2(\varphi) =$ "$1/(2E(\varphi_c - \varphi)/M)$"

if $\varphi_c \approx 276/M$ (tuning !) :

this will be responsible for relative increase of dark energy in **present** cosmological epoch

Realistic cosmology

Hypothesis on running couplings
yields realistic cosmology
for suitable values of A , E , φ_c

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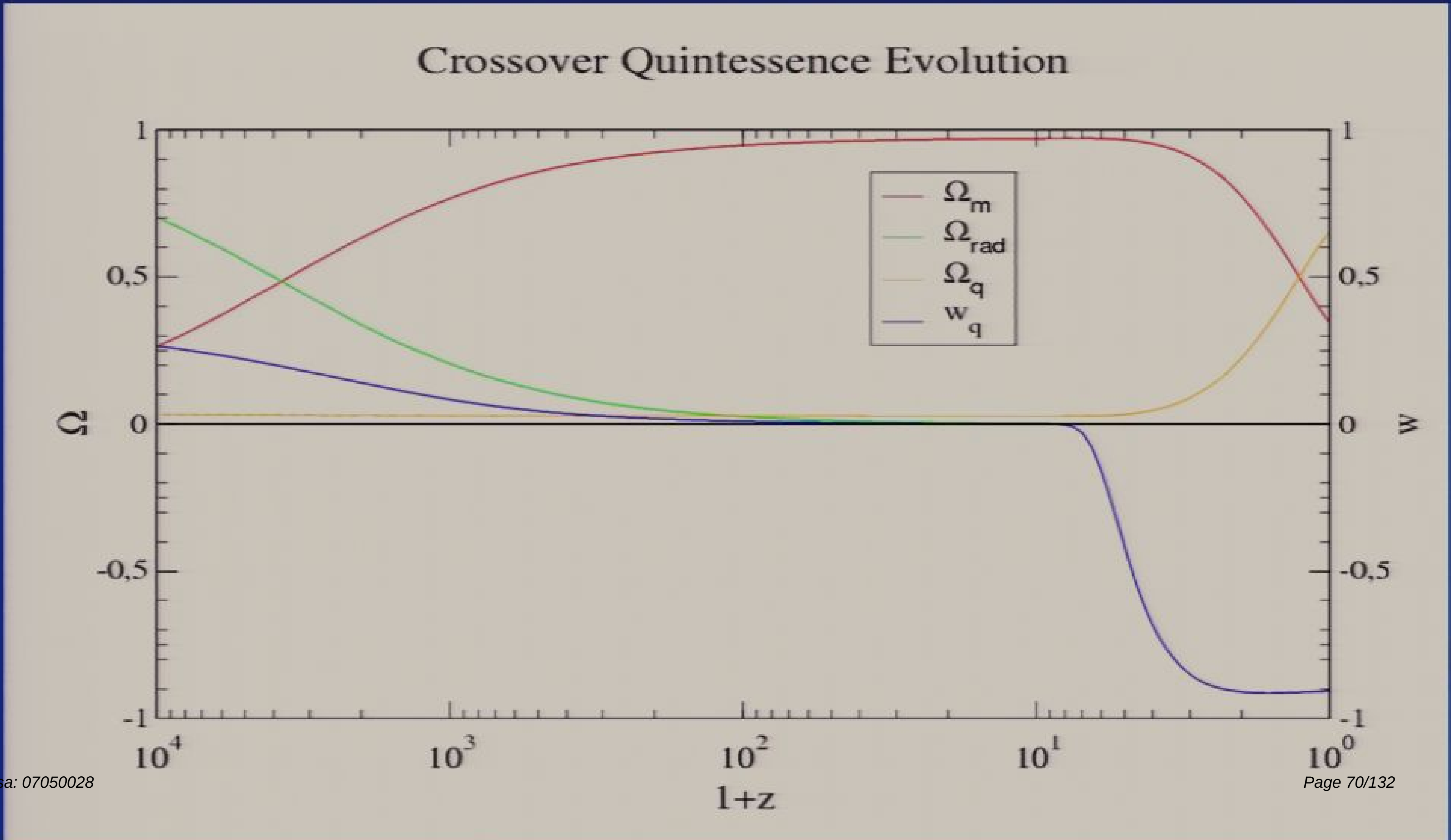
Quintessence becomes important "today"



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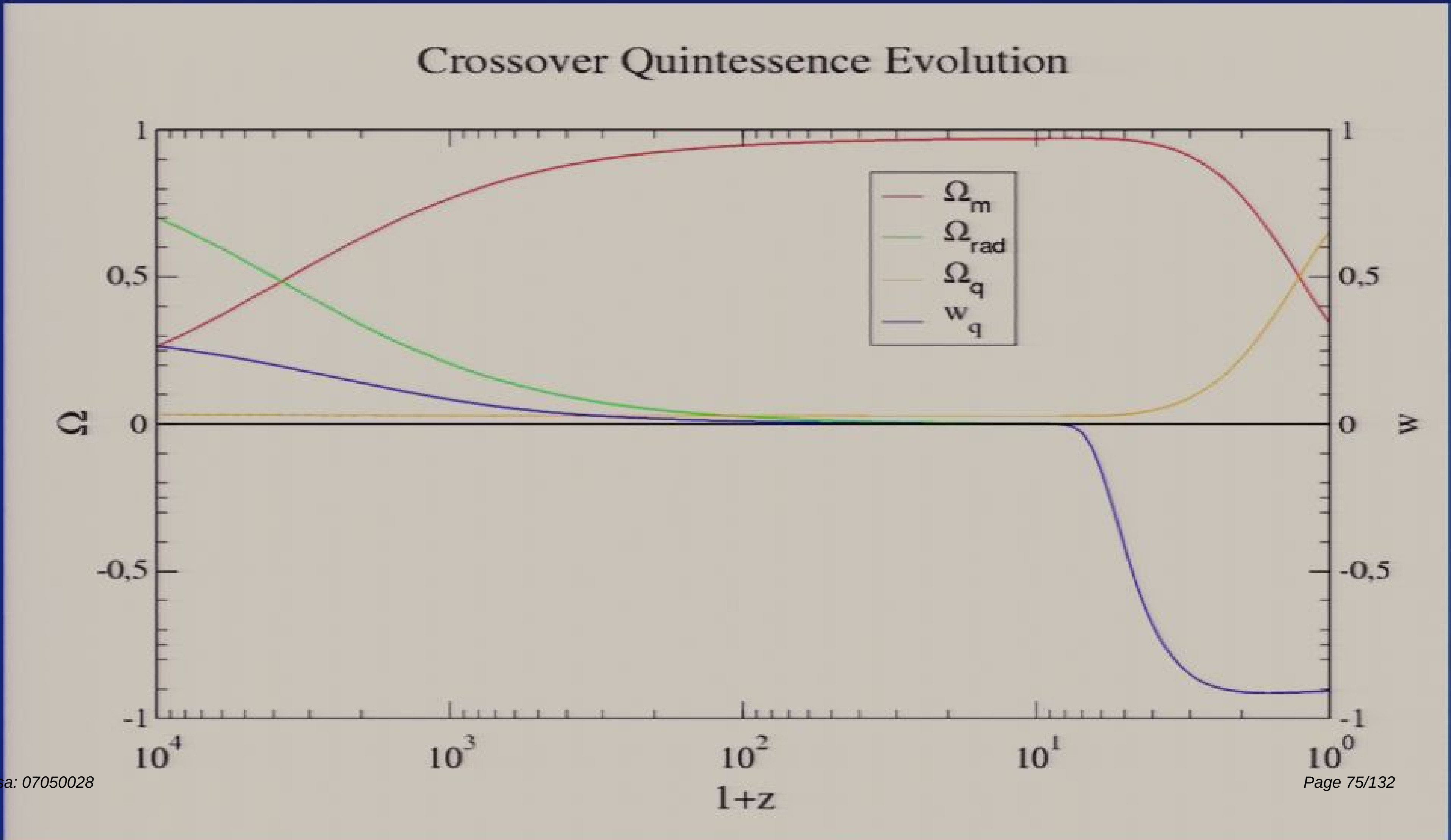

many models…

the quintessence of Quintessence

Cosmon – Field $\varphi(x,y,z,t)$

similar to electric field , but no direction (scalar field)

may be fundamental or composite (effective) field

Homogeneous und isotropic Universe : $\varphi(x,y,z,t)=\varphi(t)$

Potential und kinetic energy of the cosmon -field contribute to a dynamical energy density of the Universe !

Cosmon

- *Scalar field changes its value even in the* ***present*** *cosmological epoch*
- *Potential und kinetic energy of cosmon contribute to the energy density of the Universe*
- *Time - variable dark energy :*

  $\rho_h(t)$ *decreases with time !*

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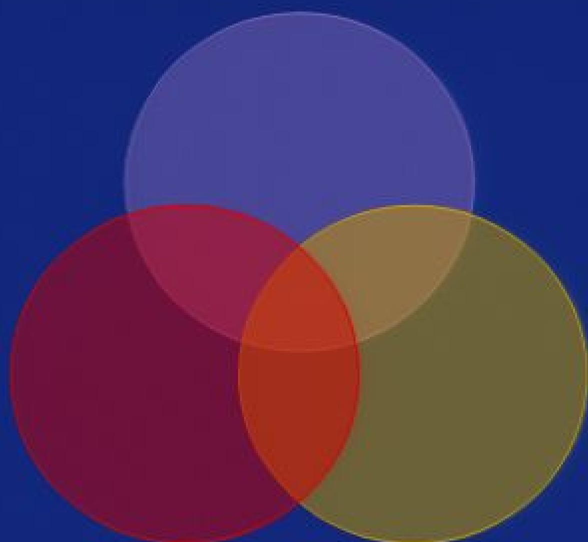
Cosmon

- *Tiny (time varying) mass*
- $m_c \sim H$
- *New long - range interaction*

"Fundamental" Interactions

Strong, electromagnetic, weak interactions



gravitation cosmodynamics

On astronomical length scales:

graviton

+

cosmon

Dynamics of quintessence

- Cosmon φ : scalar singlet field
- Lagrange density $L = V + \frac{1}{2}\, k(\varphi)\, \partial\varphi\, \partial\varphi$

  (units: reduced Planck mass M=1)
- Potential : $V = \exp[-\varphi]$
- "Natural initial value" in Planck era $\varphi=0$
- today: $\varphi=276$

kinetial

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\varphi)^2\, k^2(\varphi) + \exp[-\varphi]$$

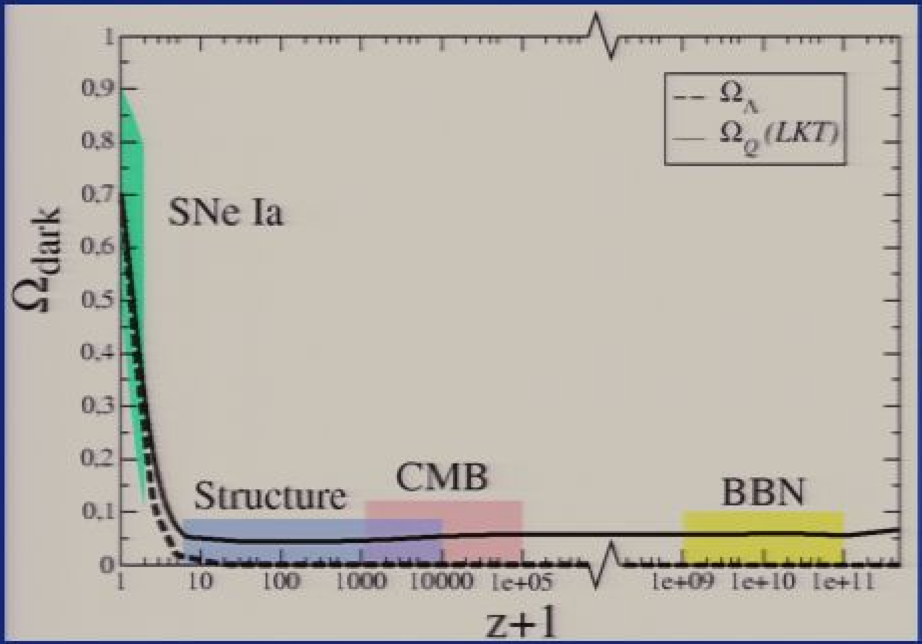
Small almost constant k :

- Small almost constant Ω_h

Large k :

- Cosmon dominated universe (like inflation)

Why has quintessence become important "now" ?



Doran,…

$$w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)}$$

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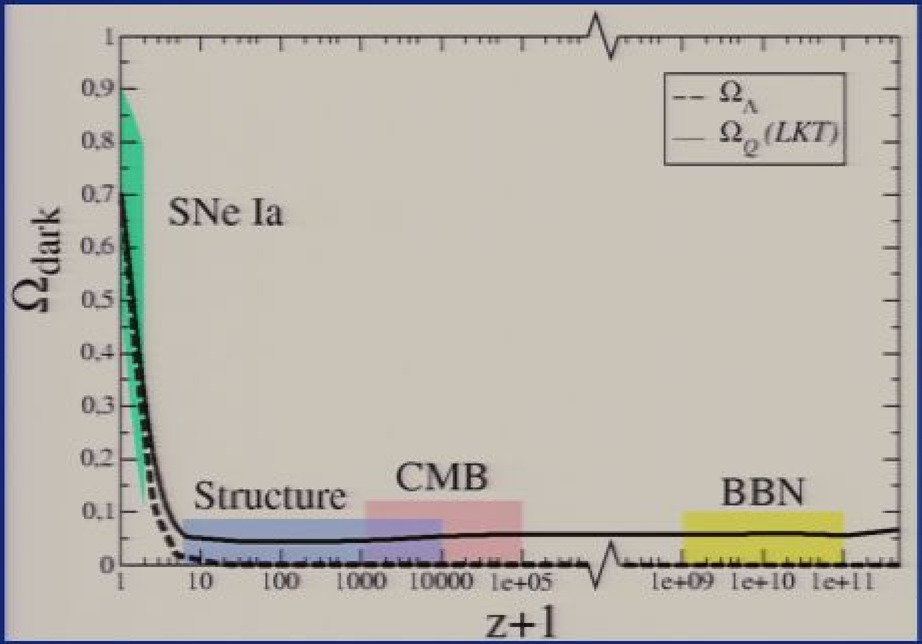

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coincidence problem

What is responsible for increase of Ω_h for $z < 10$?

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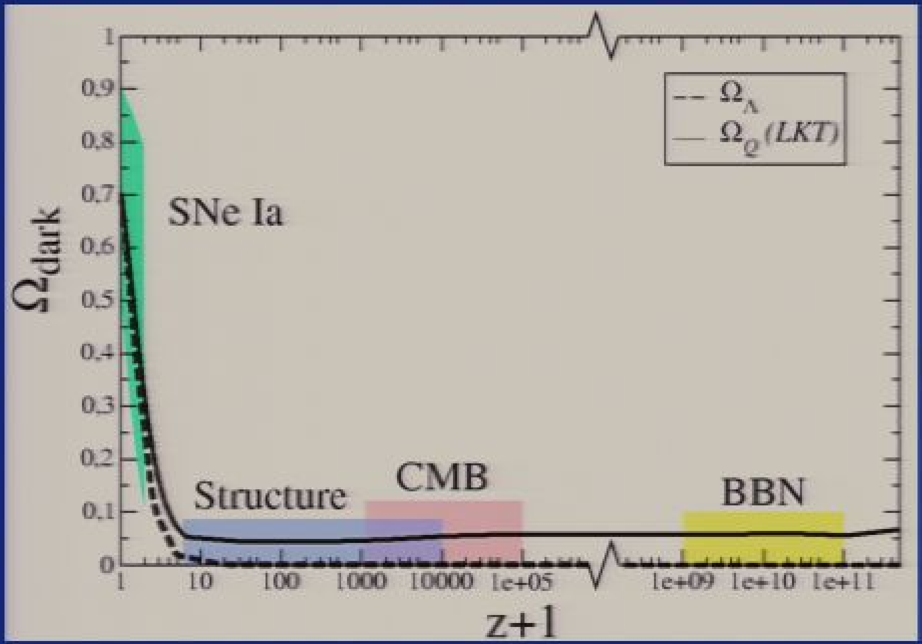

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coincidence problem

What is responsible for increase of Ω_h for $z < 10$?

a) Properties of cosmon potential or kinetic term

Late quintessence

- w close to -1
- Ω_h negligible in early cosmology
- needs tiny parameter, similar to cosmological constant

Early quintessence

- Ω_h changes only modestly
- w changes in time

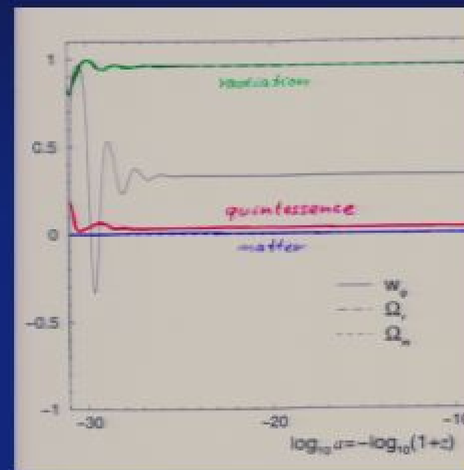
transition

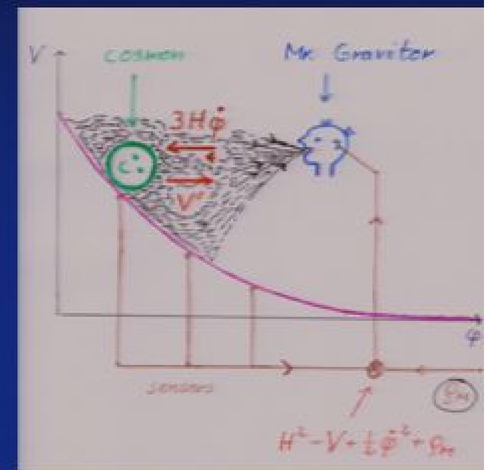
- special feature in cosmon potential or kinetic term becomes important "now"
- tuning at ‰ level

attractor solutions

Small almost constant k :

- Small almost constant Ω_h





⇒ This can explain tiny value of Dark Energy !

Large k :

- Cosmon dominated universe (like inflation)

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\varphi)^2 k^2(\varphi) + \exp[-\varphi]$$

Transition to cosmon dominated universe

- Large value k >> 1 : universe is dominated by scalar field
- k increases rapidly : evolution of scalar fied essentially stops
- Realistic and natural quintessence:

k changes from small to large values after structure formation

b) Quintessence reacts to some special event in cosmology

- Onset of
  matter dominance

K- essence

Amendariz-Picon, Mukhanov, Steinhardt

needs higher derivative kinetic term

- Appearance of
  non-linear structure

Back-reaction effect

needs coupling between Dark Matter and Dark Energy

Back-reaction effect

scalar evolution equation

$$\langle \ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) \rangle = 0$$

$$0 = \ddot{\varphi}_0 + 3H\dot{\varphi}_0 + V'(\varphi_0) + V'''(\varphi_0)\langle\chi\rangle + \frac{1}{2}V''(\varphi_0)\langle\chi^2\rangle$$

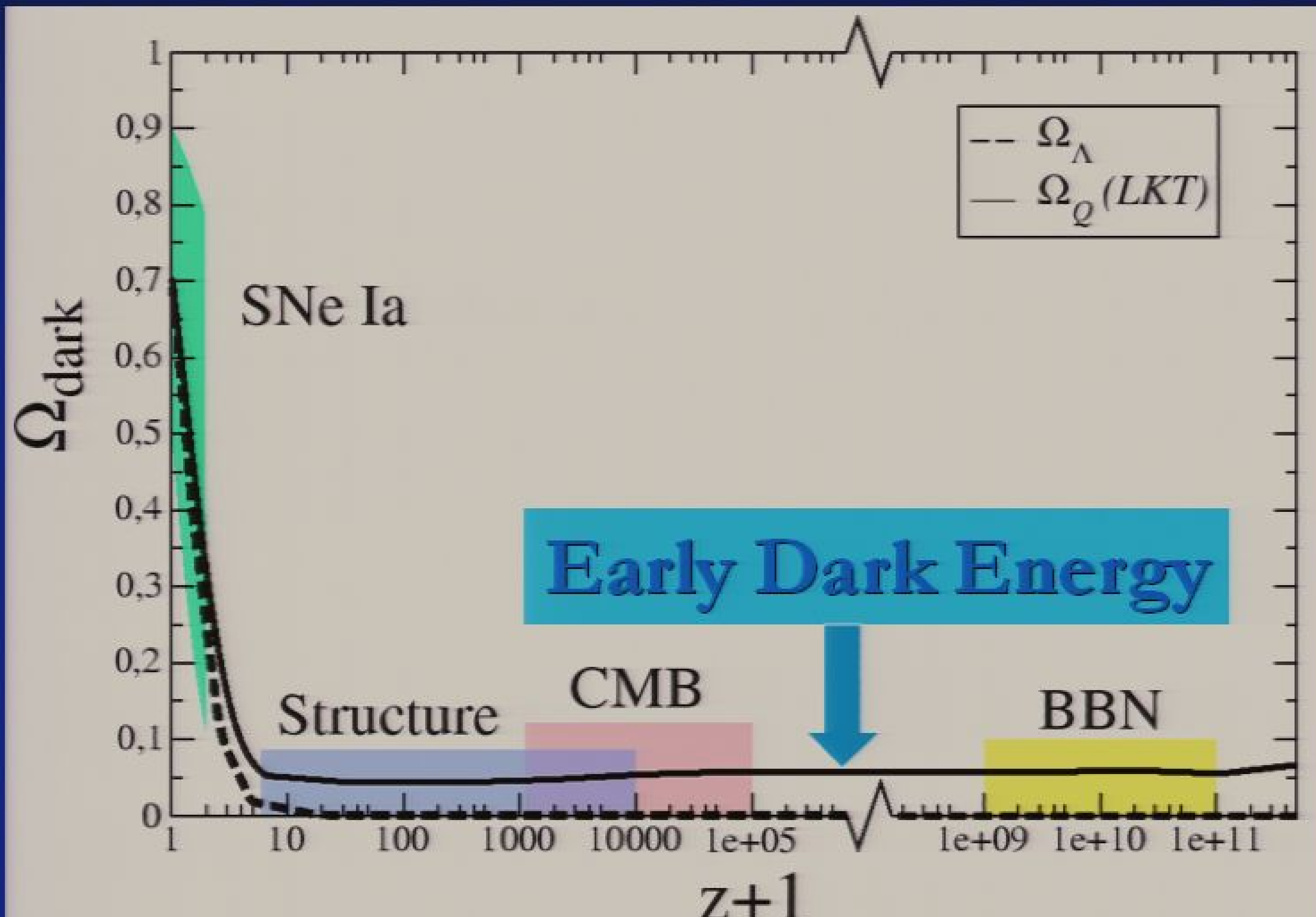
fluctuation effect

backreaction

(In principle, same for metric, but small effect)

- Needs large inhomogeneities after structure has been formed
- Local cosmon field participates in structure

Time dependence of dark energy



cosmological constant : $\Omega_h \sim t^2 \sim (1+z)^{-3}$

M.Doran,…

early dark energy

expected in models

which explain same order of

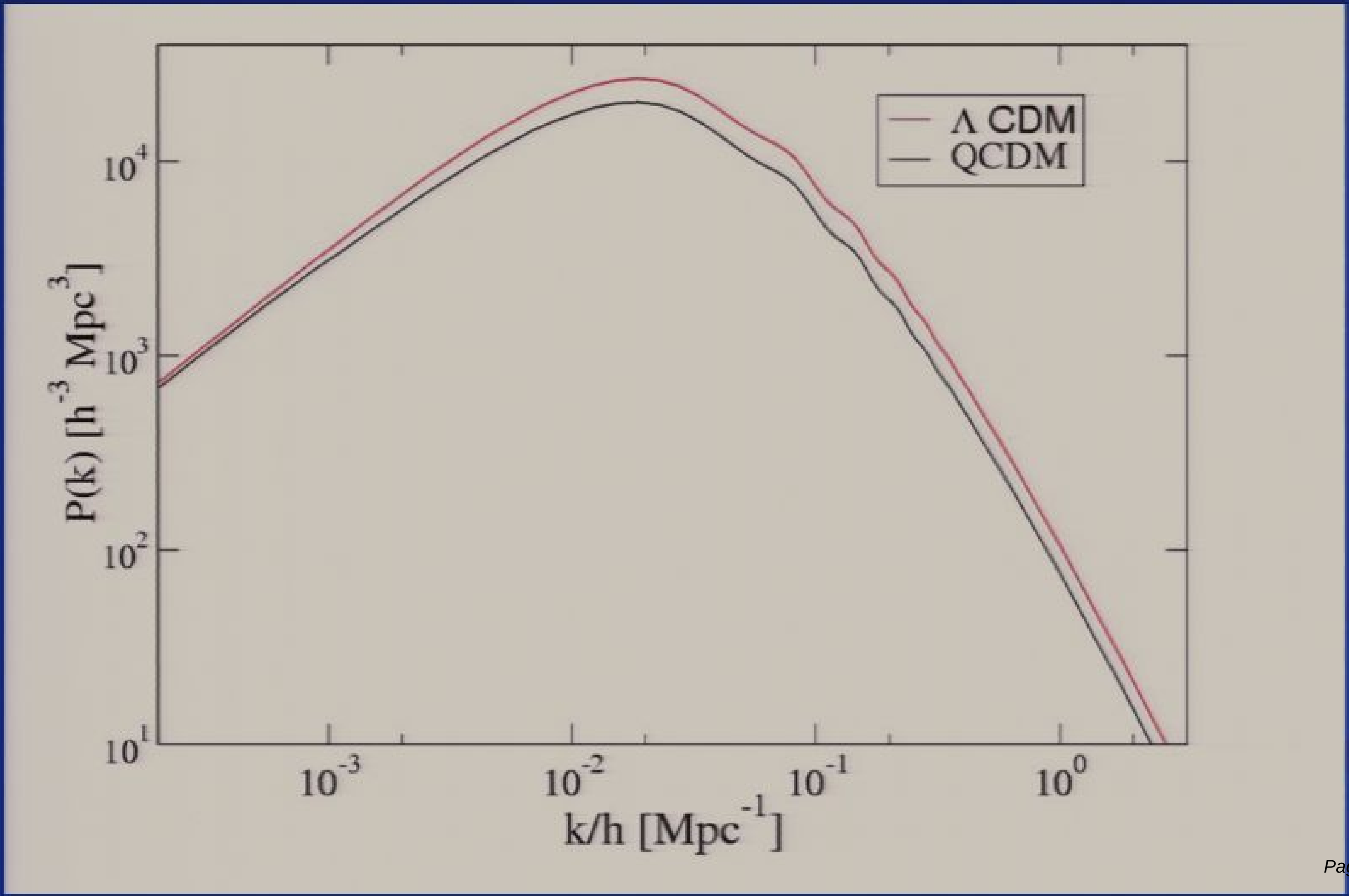
magnitude of

dark energy and matter naturally

effects of early dark energy

- modifies cosmological evolution (CMB)
- slows down the growth of structure

Early quintessence slows down the growth of structure



Growth of density fluctuations

- Matter dominated universe with constant Ω_h :

$$\Delta\rho \sim a^{1-\frac{\epsilon}{2}}\ ,\ \epsilon = \frac{5}{2}\left(1 - \sqrt{1 - \frac{24}{25}\Omega_h}\right)$$

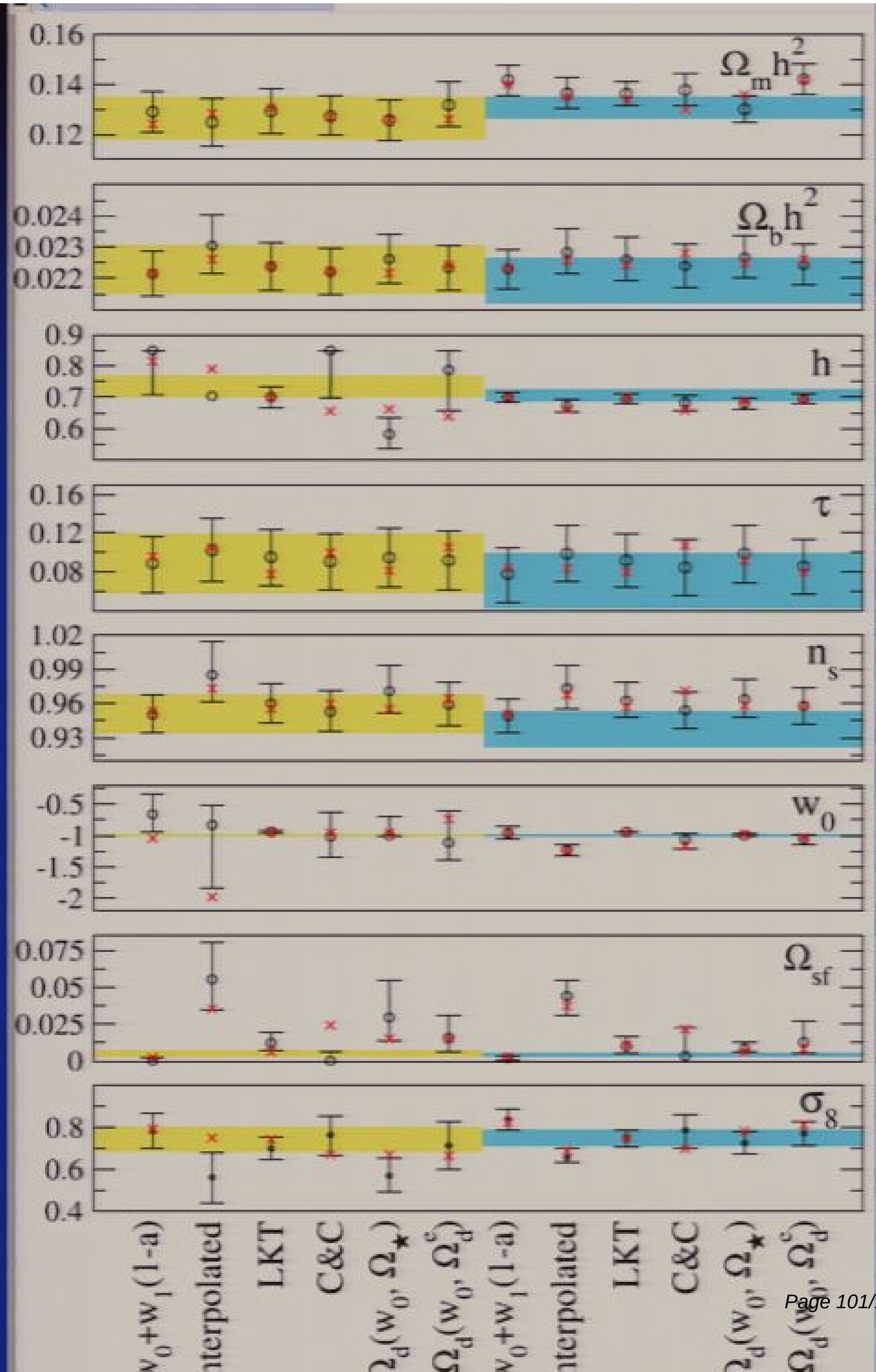
P.Ferreira,M.Joyce

- Dark energy slows down structure formation

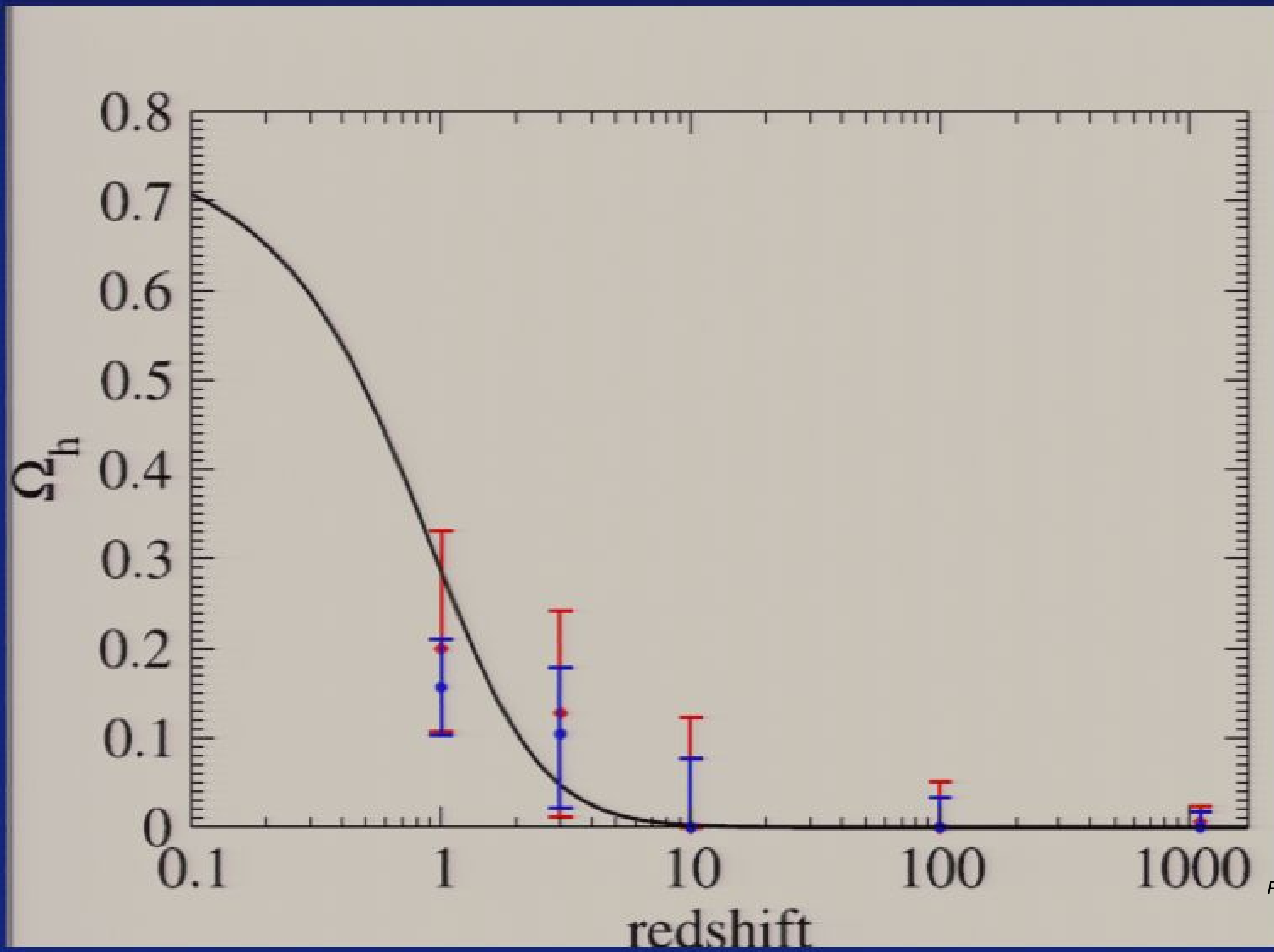
  ⟹ $\Omega_h < 10\%$ during structure formation

bounds on
Early Dark Energy
after WMAP'06

G.Robbers,M.Doran,…

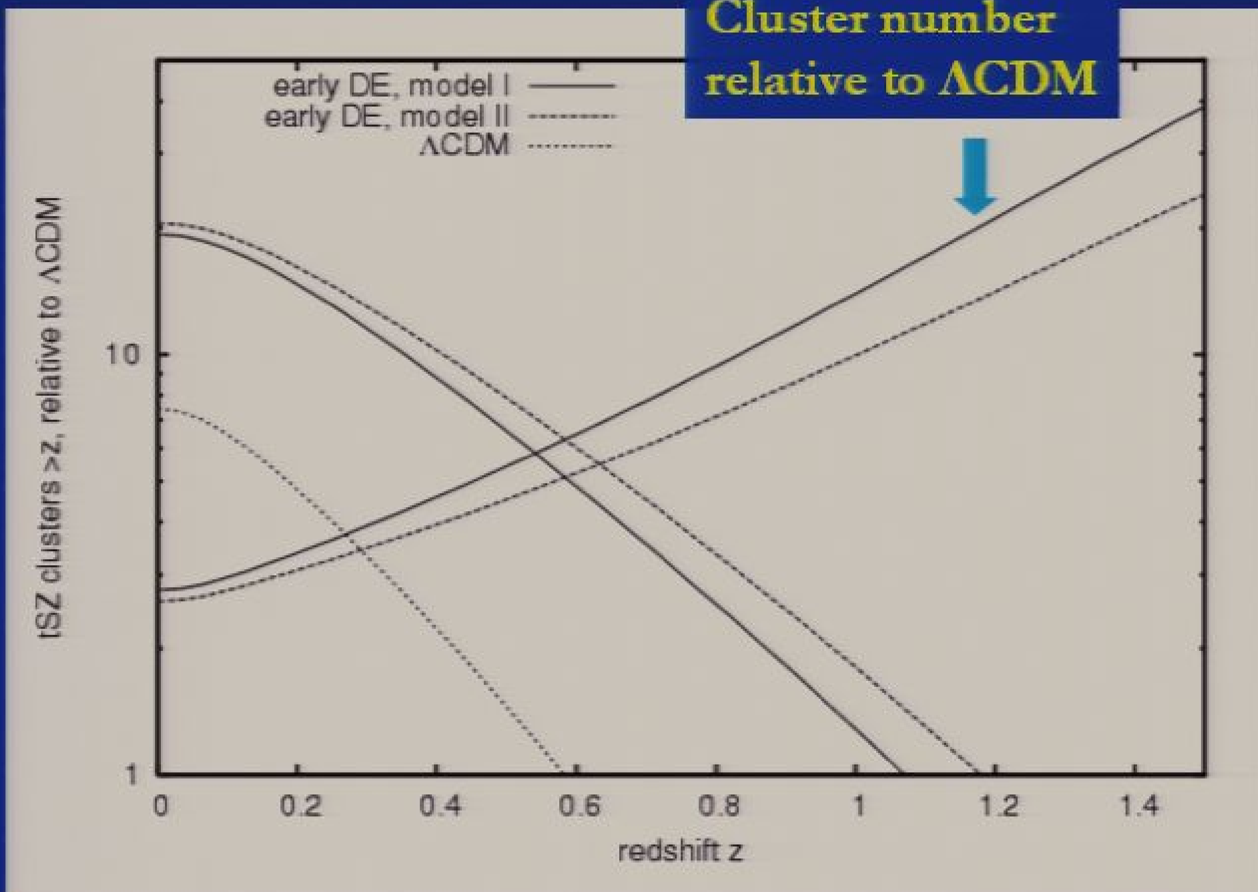


interpolation of Ω_h



Little Early Dark Energy can make large effect !
Non – linear enhancement

Cluster number relative to ΛCDM



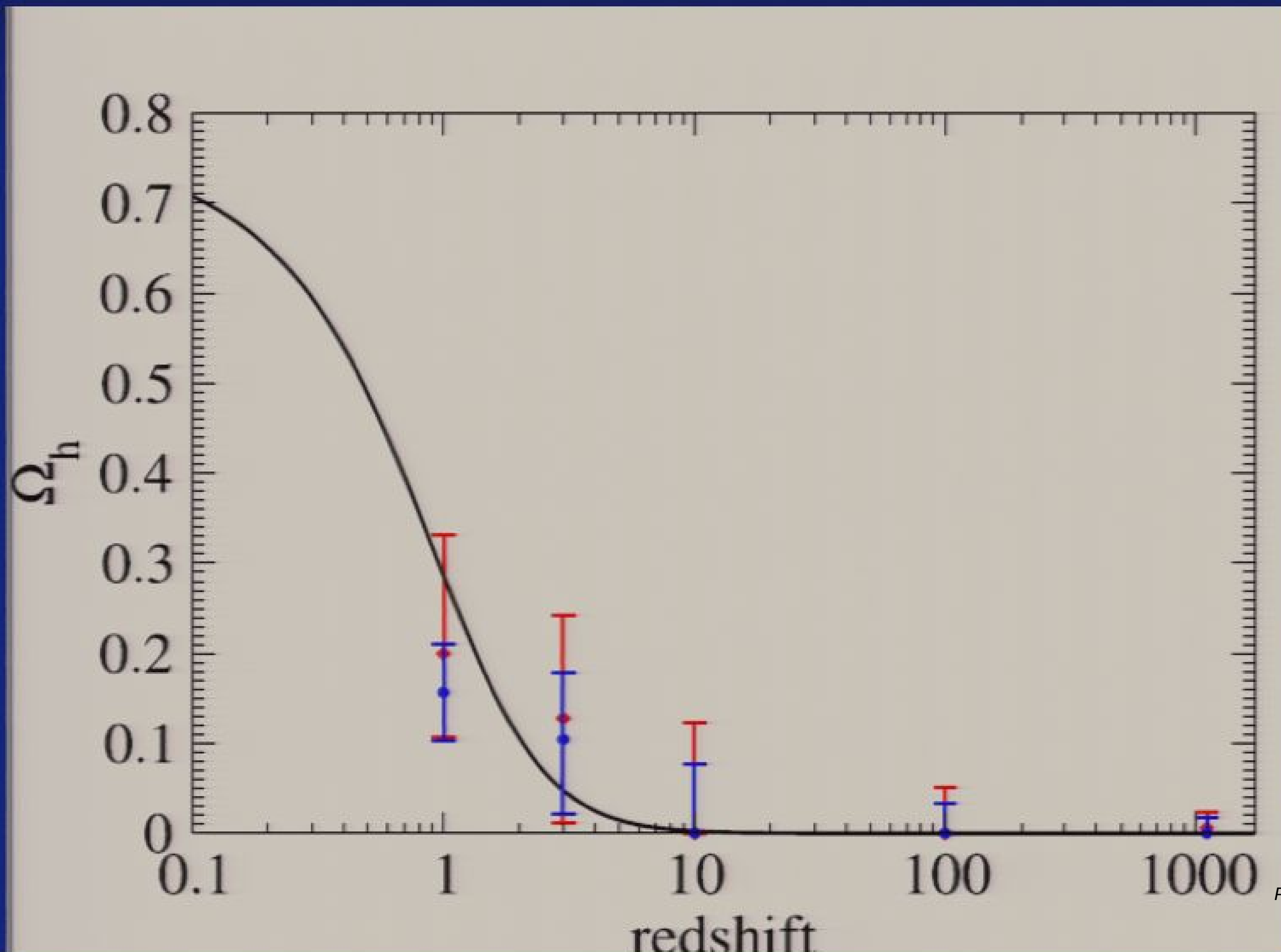
Two models with 4% Dark Energy during structure formation

Fixed σ_8
(normalization dependence !)

More clusters at high redshift !

Bartelmann,Doran,…

interpolation of Ω_h



Quintessence from higher dimensions - an instructive example -

work with J. Schwindt

hep-th/0501049

Time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional or string theories
- Exponential form rather generic

  (after Weyl scaling)
- But most models show too strong time dependence of constants !

Quintessence from higher dimensions

An instructive example:

Einstein – Maxwell theory in six dimensions

$$S = \int d^6x \sqrt{-g} \left\{ -\frac{M_6^4}{2} R + \lambda_6 + \frac{1}{4} F^{AB} F_{AB} \right\}$$

Warning : not scale - free !

Dilatation anomaly replaced by explicit mass scales.

Energy momentum tensor

$$T^{(F)}_{AB} = F_{AC}F_B{}^C - \frac{1}{4}F_{CD}F^{CD}g_{AB}$$

$$R_{AB} - \frac{1}{2}Rg_{AB} = M_6^{-4}(T^{(F)}_{AB} + T^{(M)}_{AB} - \lambda_6 g_{AB}),$$

$$\partial_A(\sqrt{-g}F^{AB}) = 0.$$

Metric

Ansatz with particular metric (not most general !)

which is consistent with

d=4 homogeneous and isotropic Universe

and internal U(1) x Z_2 isometry

$$ds^2 = \exp\left(-\frac{\phi(t)}{\bar{M}}\right)\{-dt^2 + a^2(t)d\vec{x}d\vec{x}\}$$

$$+\exp\left(\frac{\phi(t)}{\bar{M}}\right) r_0^2\{d\rho^2 + B^2 \sin^2\rho \, d\theta^2\}$$

$$r_0^2 = \frac{\bar{M}^2}{4\pi B M_6^4}$$

B ≠ 1 : football shaped internal geometry

Exact solution

$$A_\theta = \frac{m}{2e_6}(1-\cos\rho)$$

m : monopole number (integer)

$$H^2 = \frac{1}{3\bar{M}^2}(\frac{1}{2}\dot{\phi}^2 + V(\phi))$$

cosmology with scalar

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

and potential V :

$$V(\phi) = \bar{M}^4 \left\{ \frac{\lambda_6}{M_6^4 \bar{M}^2}\, e^{-\frac{\phi}{\bar{M}}} - 4\pi B \frac{M_6^4}{\bar{M}^4}\, e^{-\frac{2\phi}{\bar{M}}} + 2\pi^2 m^2 \frac{M_6^4}{e_6^2 \bar{M}^6}\, e^{-\frac{3\phi}{\bar{M}}} \right\}$$

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Free integration constants

M , B , $\Phi(t=0)$, $(d\Phi/dt)(t=0)$: continuous

m : discrete

Conical singularities

deficit angle

$$\Delta = 2\pi(1 - B)$$

singularities can be included with
energy momentum tensor on brane

$$(T^{(B)})^{\nu}_{\mu} = \frac{B-1}{B r_0^2 e^{\phi/\bar{M}}}\, M_6^4 \left(\frac{\delta(\rho)}{\rho} + \frac{\delta(\rho-\pi)}{\pi-\rho} \right) \delta^{\nu}_{\mu}$$

bulk point of view :
describe everything in terms of bulk geometry
(not possible for modes on brane without tail in bulk)

Warped branes

- model is similar to first co-dimension two brane model : C.W. Nucl.Phys.B255,480(1985);

  see also B253,366(1985)
- first realistic warped model
- see Rubakov and Shaposhnikov for earlier work (no stable solutions, infinitely many chiral fermions)
- see Randjbar-Daemi, C.W. for arbitrary dimensions

Asymptotic solution for large t

$$H = 2t^{-1}, \quad \phi = 2\bar{M} \ln \frac{t}{\sqrt{10} M_6^2 \lambda_6^{-1/2}}$$

$$\Omega_h = \frac{V + \frac{1}{2}\dot{\phi}^2}{3\bar{M}^2 H^2} \to 1$$

$$V + \frac{1}{2}\dot{\phi}^2 \propto t^{-2}$$

Naturalness

- No tuning of parameters or integration constants
- Radiation and matter can be implemented
- Asymptotic solution depends on details of model, e.g. solutions with constant $\Omega_h \neq 1$

problem :

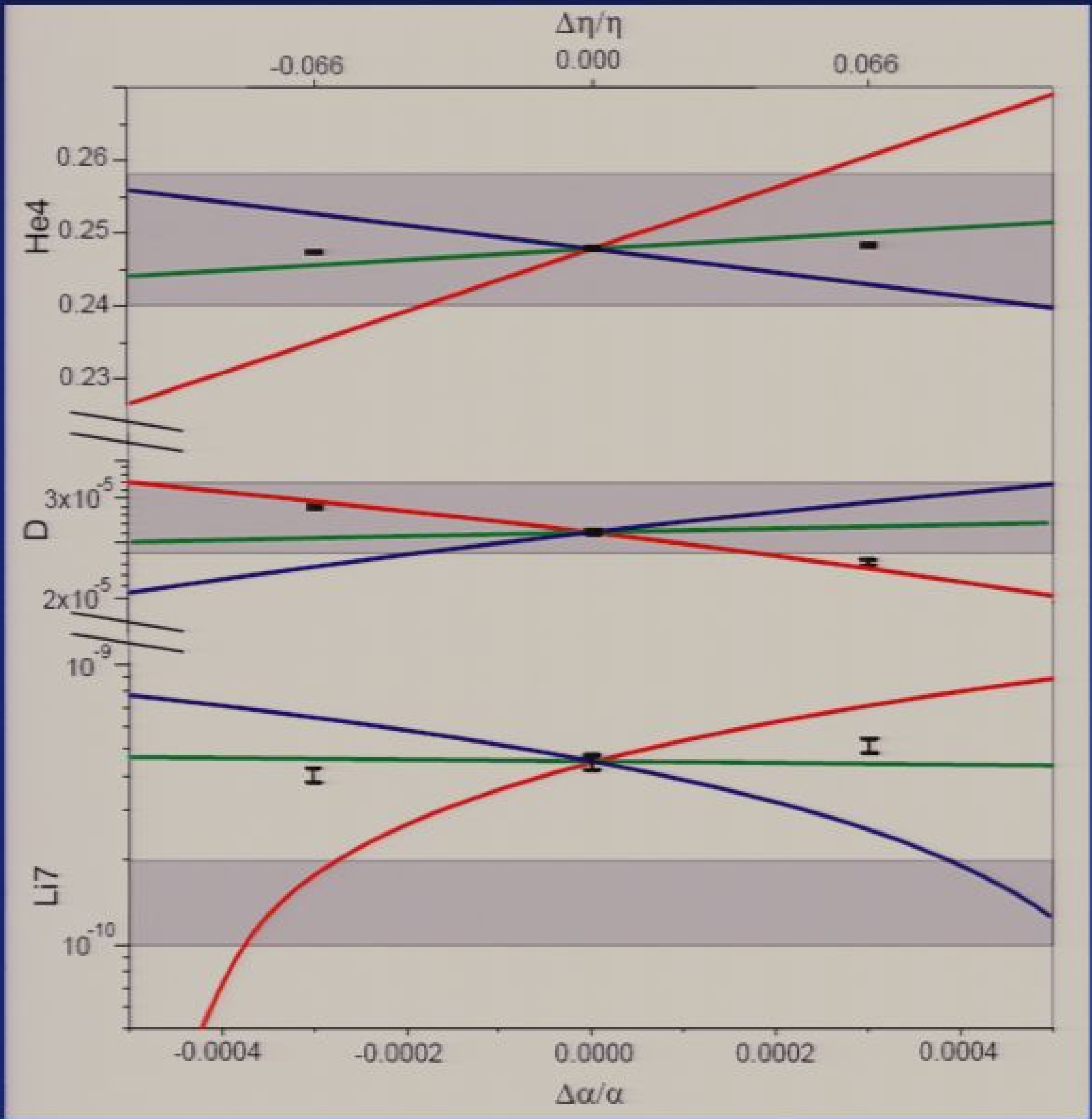
time variation of fundamental constants

primordial abundances for three GUT models

He

D

Li



present observations : 1σ

T.Dent, S.Stern, . . .

three GUT models

- unification scale ~ Planck scale
- 1) All particle physics scales $\sim \Lambda_{QCD}$
- 2) Fermi scale and fermion masses ~ unification scale
- 3) Fermi scale varies more rapidly than Λ_{QCD}

$\Delta\alpha/\alpha \approx 4\ 10^{-4}$ allowed for GUT 1 and 3 , larger for GUT 2

$\Delta \ln(M_n/M_P) \approx 40\ \Delta\alpha/\alpha \approx 0.015$ allowed

Dimensional reduction

$$L^{(4)} = -\frac{\bar{M}^2}{2} R + \frac{Z_1(\phi)}{4} F^{(1)}_{\mu\nu} F^{\mu\nu(1)}$$

$$+\frac{Z_2(\phi)}{4} F^{(2)}_{\mu\nu} F^{\mu\nu(2)}$$

$$+i \sum_j \bar{\psi}_j \gamma^\mu (\partial_\mu - i Q_j^{(1)} \bar{e}_1 A^{(1)}_\mu - i Q_j^{(2)} \bar{e}_2 A^{(2)}_\mu) \psi_j$$

$$+\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi)$$

Time dependent gauge coupling

$$e_{1(2)} = \frac{\bar{e}_{1(2)}}{\sqrt{Z_{1(2)}}}$$

$$Z_1 = e^{\phi/\bar{M}}, \quad Z_2 = e^{2\phi/\bar{M}}$$

stabilizing the couplings…

gauge couplings go to zero as volume of internal space increases

two ways to solve this problem:

- irrelevant for modes on branes
- possible stabilization by fixed points in scale free models

????????????????????????

Why becomes Quintessence dominant in the present cosmological epoch ?

Are dark energy and dark matter related ?

Can Quintessence be explained in a fundamental unified theory ?

Transition to cosmon dominated universe

- Large value k >> 1 : universe is dominated by scalar field
- k increases rapidly : evolution of scalar fied essentially stops
- Realistic and natural quintessence:

k changes from small to large values after structure formation

coincidence problem

What is responsible for increase of Ω_h for $z < 10$?

Cosmon

- *Tiny mass*

- $m_c \sim H$

- *New long - range interaction*

cosmon mass changes with time !

for standard kinetic term

- $m_c^2 = V''$

for standard exponential potential , $k \approx$ const.

- $m_c^2 = V''/k^2 = V/(k^2 M^2)$

$$= 3\,\Omega_h\,(1 - w_h)\,H^2/(2\,k^2)$$

quantum fluctuations and naturalness

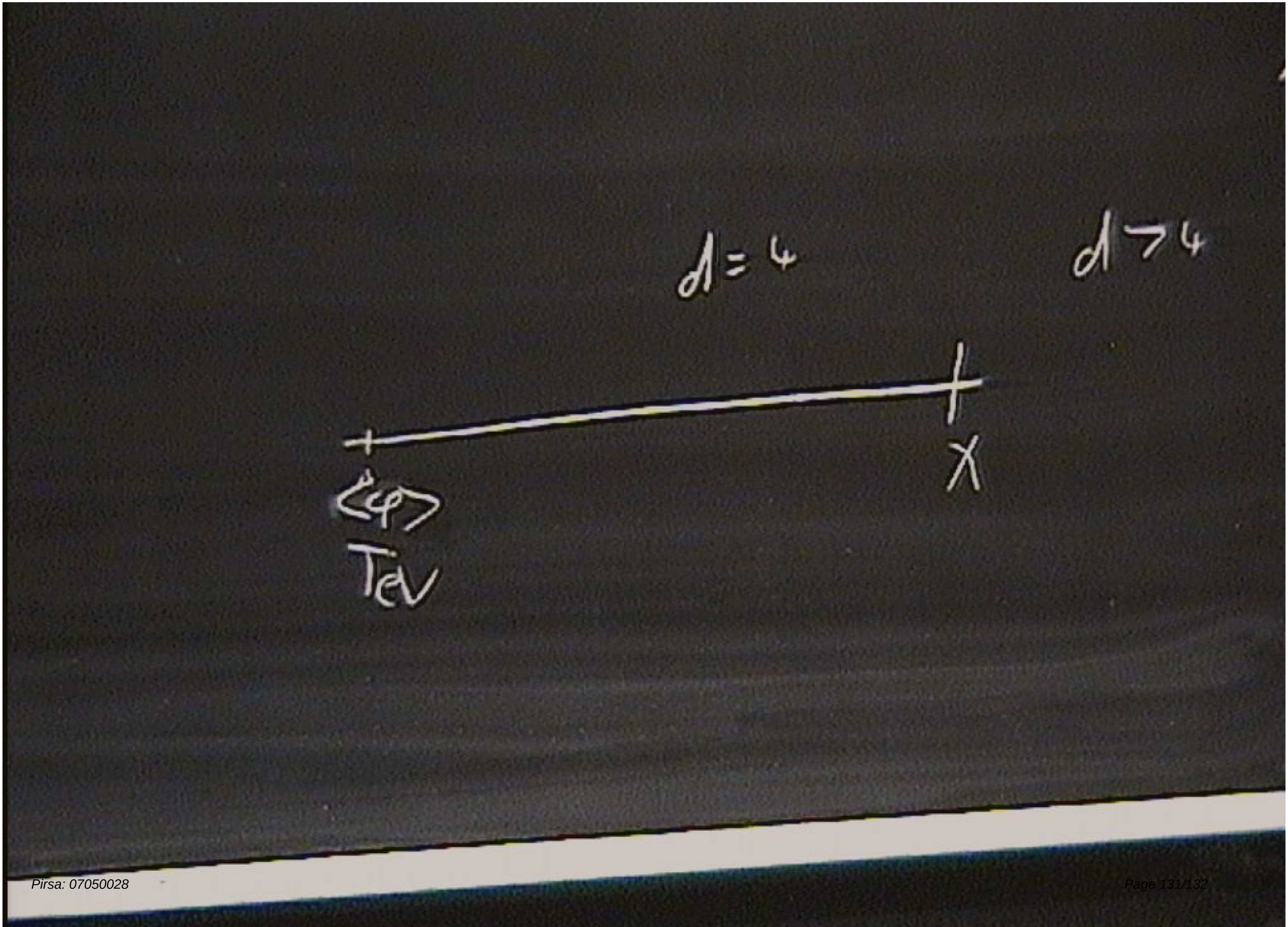
- Jordan- and Einstein frame completely equivalent on level of effective action and field equations (**after** computation of quantum fluctuations !)
- Treatment of quantum fluctuations depends on frame : Jacobian for variable transformation in functional integral
- What is natural in one frame may look unnatural in another frame

Time evolution

- $\rho_m/M^4 \sim a^{-3} \sim$
  - t^{-2} matter dominated universe
  - $t^{-3/2}$ radiation dominated universe
- $\rho_r/M^4 \sim a^{-4} \sim t^{-2}$ radiation dominated universe

Huge age ➡ small ratio

Same explanation for small dark energy?


d = 4
d > 4
X
<φ>
TeV

$m = 10^{-20} X$ $d = 4$ $d > 4$