Title: New Phases of N = SYM

Date: May 16, 2007 11:00 AM

URL: http://pirsa.org/07050026

Abstract: TBA

Motivation

- The AdS/CFT correspondence provides a unique method to approach the mysteries of black holes: the singularity, the horizon, their evaporation.
- From a traditional gravitational perspective, we have next to no handle on quantum gravity effects.
- AdS/CFT conjectures that black hole physics should be completely describable by a well defined dual finite temperature field theory.
- However, the black hole is described by the field theory at strong coupling, whereas what is computationally accessible is the weakly coupled theory. It is therefore critical to understand the behaviour of the theory as a function of coupling.

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Outline

- The AdS/CFT correspondence in a nutshell
- Phase structure of $\mathcal{N} = 4$ SYM theory
  - The Polyakov loop as an order parameter.
  - Including the (six) scalar fields.
- Joint eigenvalue distributions
  - Low temperature: $S^1 \times S^5$.
  - Intermediate temperatures: S^6 ellipsoid.
  - A new second order transition: $S^6 \to S^5$.
- Geometrical speculations
- Conclusions and implications

AdS/CFT in a nutshell

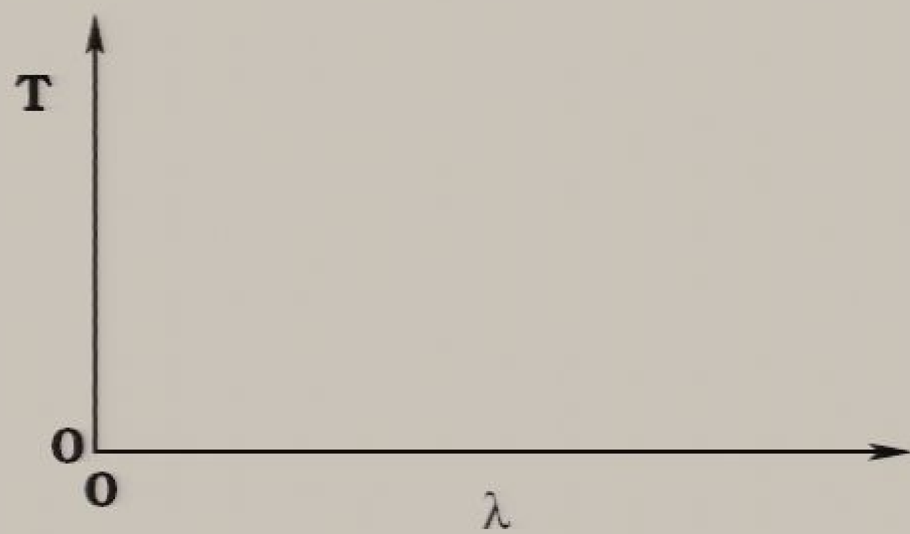
Maldacena (1997) + 4000 or so papers

- The superselection sector of IIB string theory on geometries that asymptote to $AdS_5 \times S^5$ is equivalent to $\mathcal{N} = 4$ Super Yang-Mills theory on the 'boundary' $\mathbb{R} \times S^3$.
- A particularly tractable limit of the correspondence is the 't Hooft $N \to \infty$ limit. In field theory N is the rank of the $SU(N)$ gauge group. In string theory $N \sim 1/g_s$, so we can neglect string interactions.
- The 't Hooft coupling of the field theory, $\lambda = g_{YM}^2 N$, is a free dimensionless parameter. In string theory $\lambda \sim (R_{AdS}/L_s)^4$. Large λ implies that the AdS curvature is small and we can use classical supergravity (Einstein gravity + some extra fields).

Phase structure of $\mathcal{N} = 4$ SYM theory on $S^3 \times S^1$

Hawking-Page (1983), Witten (1998), Sundborg (1999), Aharony *et al.* (2003) ...

- Parameters: 't Hooft coupling λ and temperature T.



- Interpolation from strong to weak coupling?
- e.g. how similar are black holes and weakly coupled plasmas?

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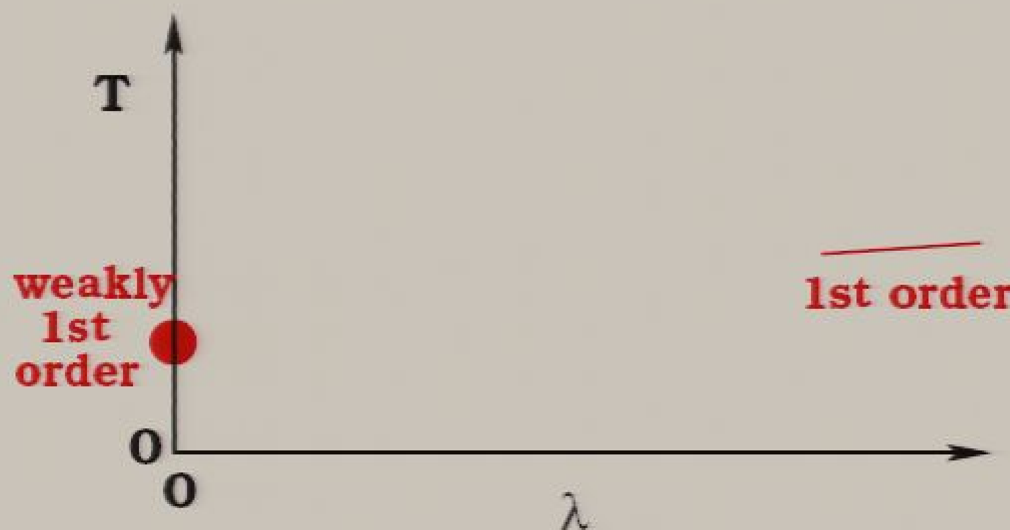

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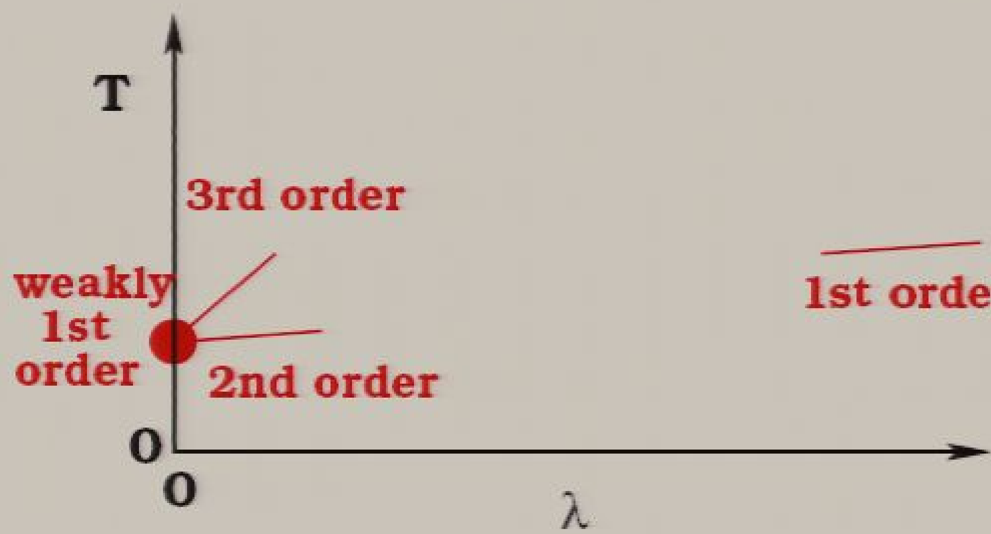

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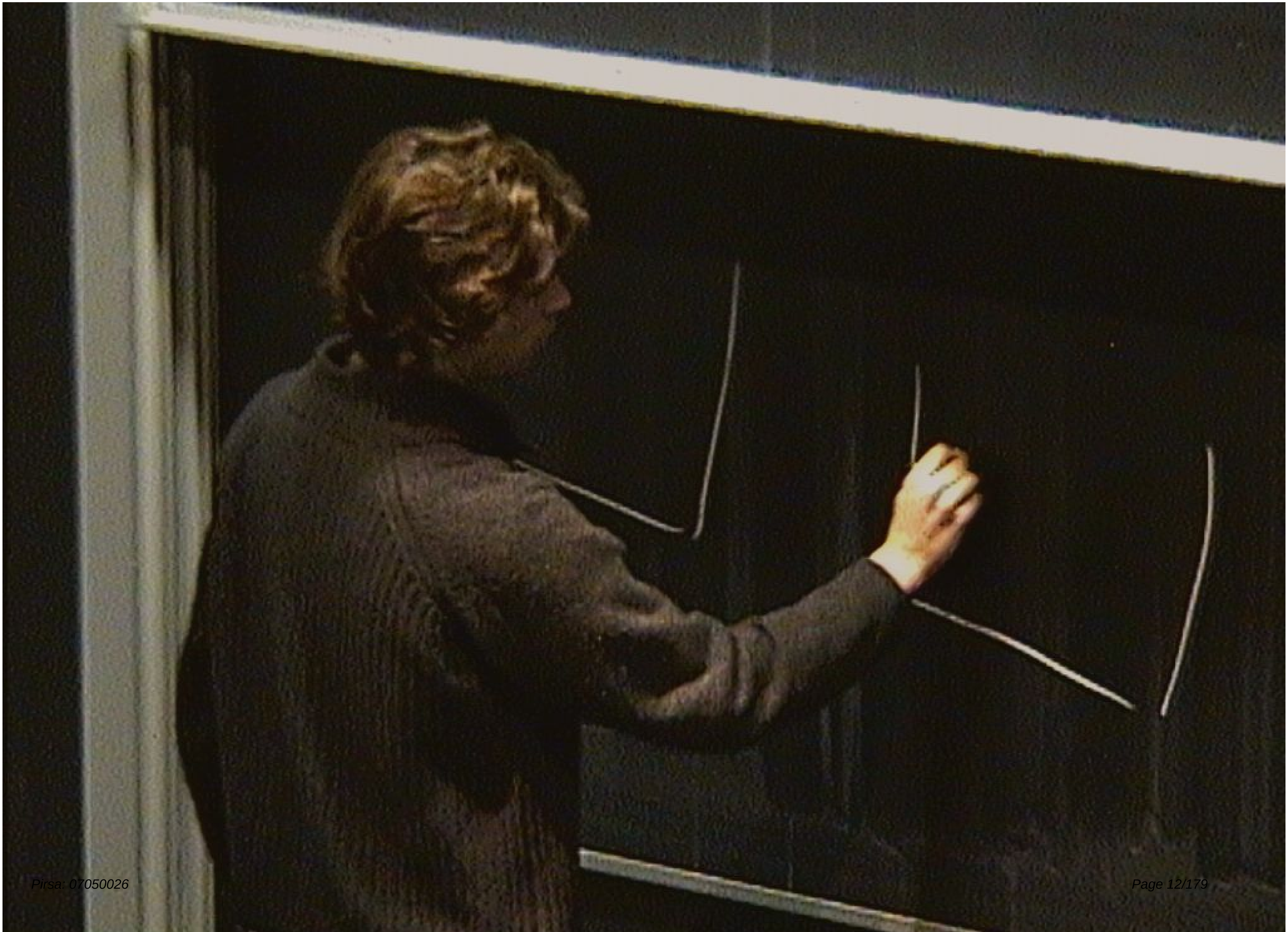

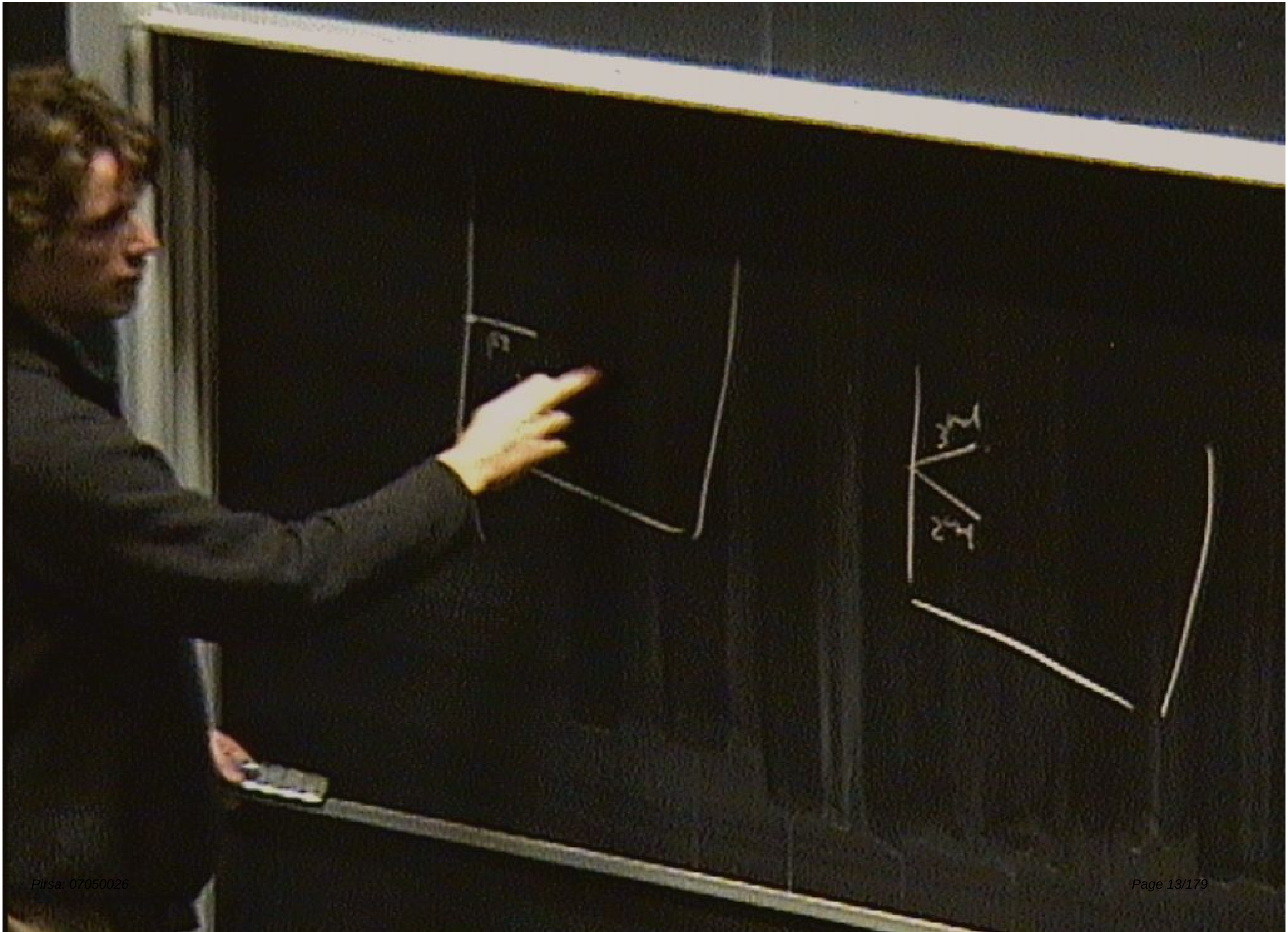

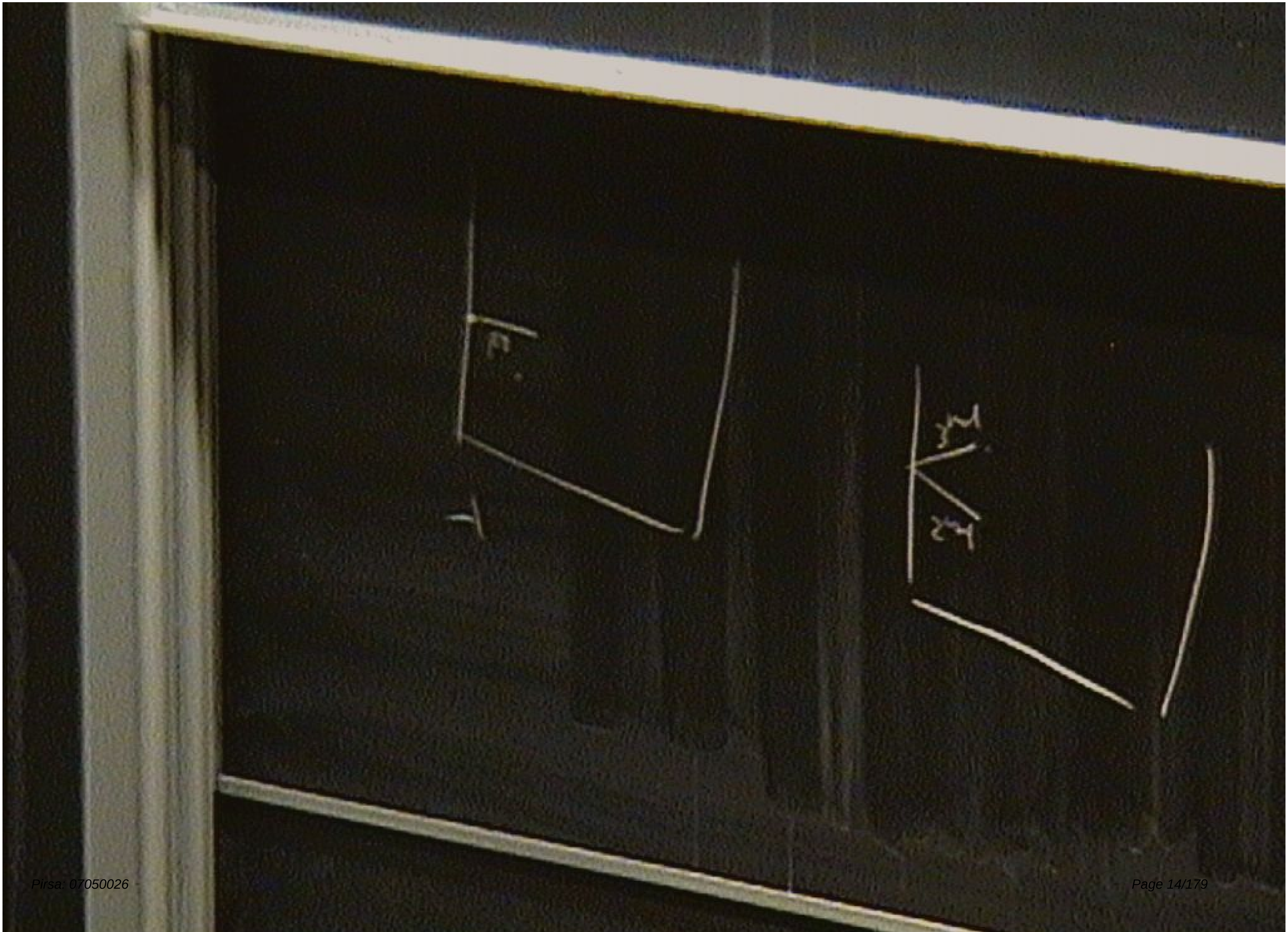

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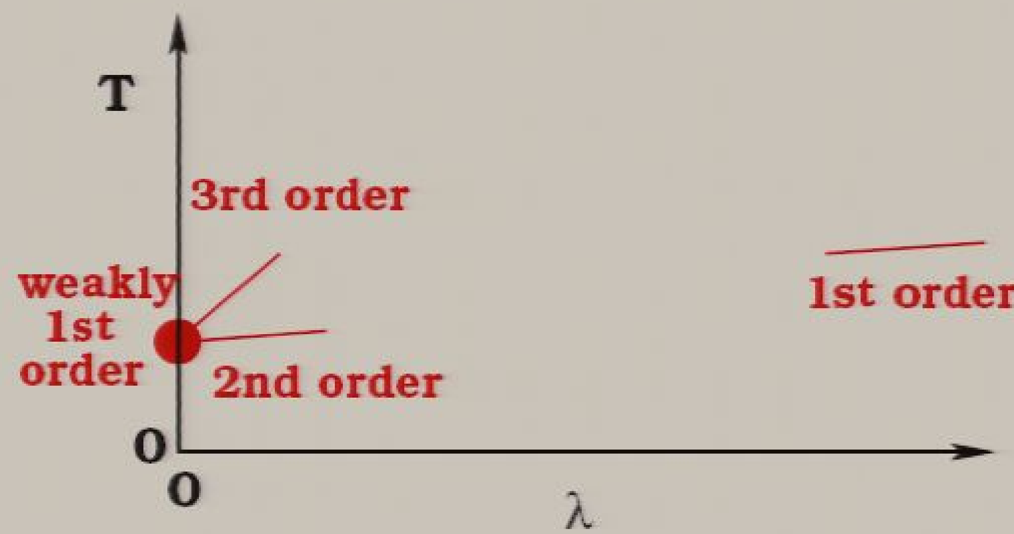

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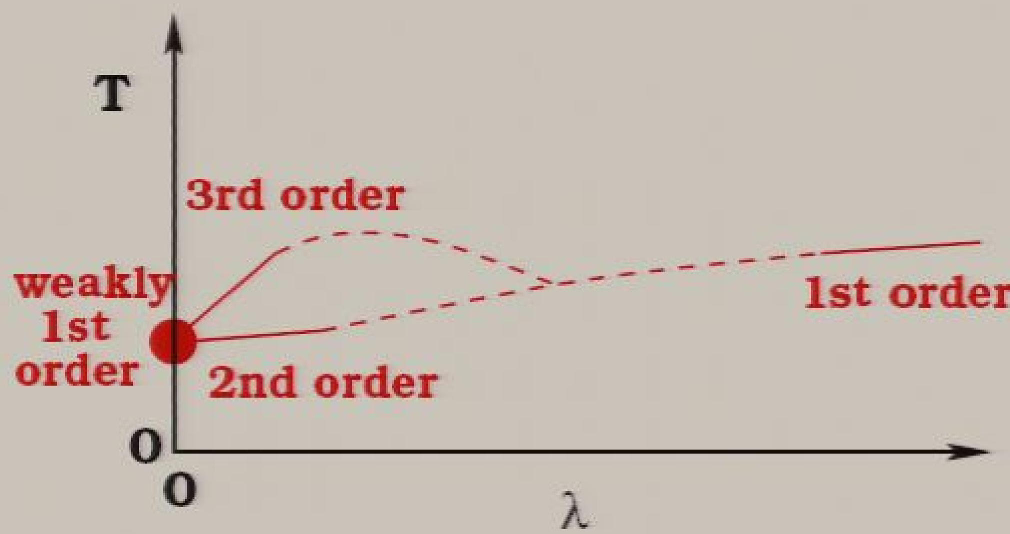

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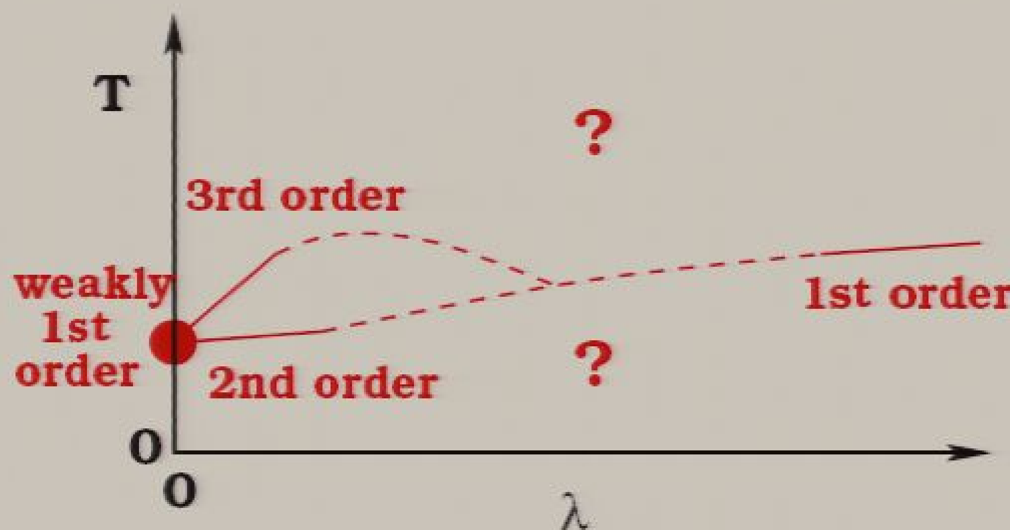

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Current framework: Polyakov loops

Aharony *et al.* (2003), Alvarez-Gaume *et al.* (2005) ...

- The order parameter for these transitions is the large N eigenvalue distribution of the Polyakov loop

$$U = P\, e^{i \oint A_0\, d\tau} \,.$$

- In practice, people study the eigenvalue distribution of the spatially homogeneous and time independent mode of βA_0. The eigenvalues θ_p take values on a circle of radius 2π, and become the distribution $\rho(\theta)$ in the large N limit.

- 'Deconfinement': separates uniform vs. non uniform.
- 'Gross-Witten': separates gapped vs. non gapped.

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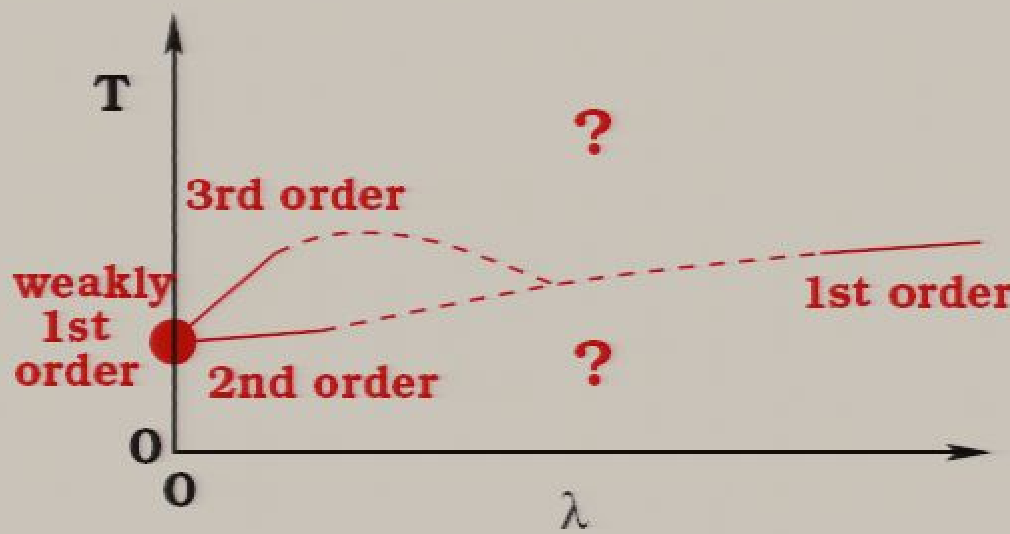

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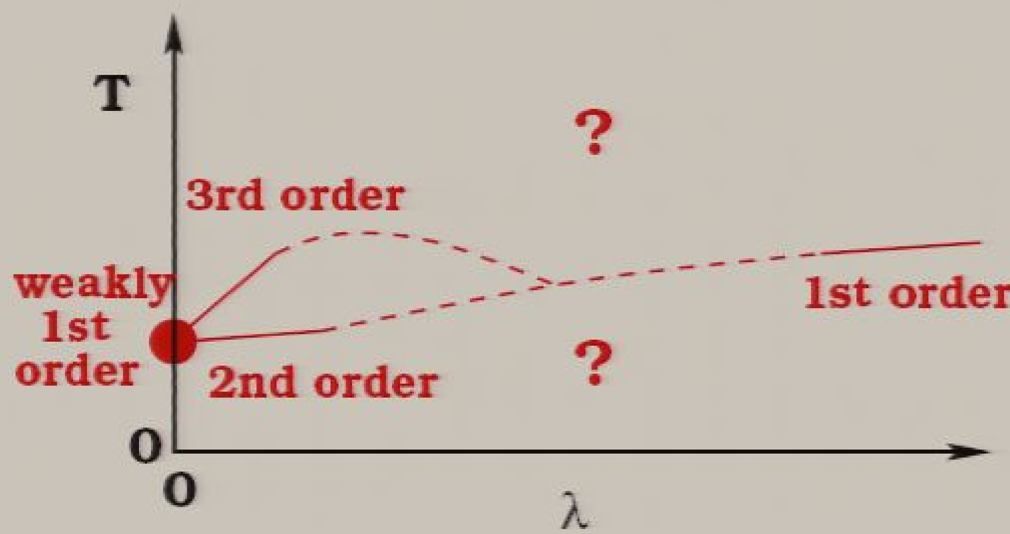

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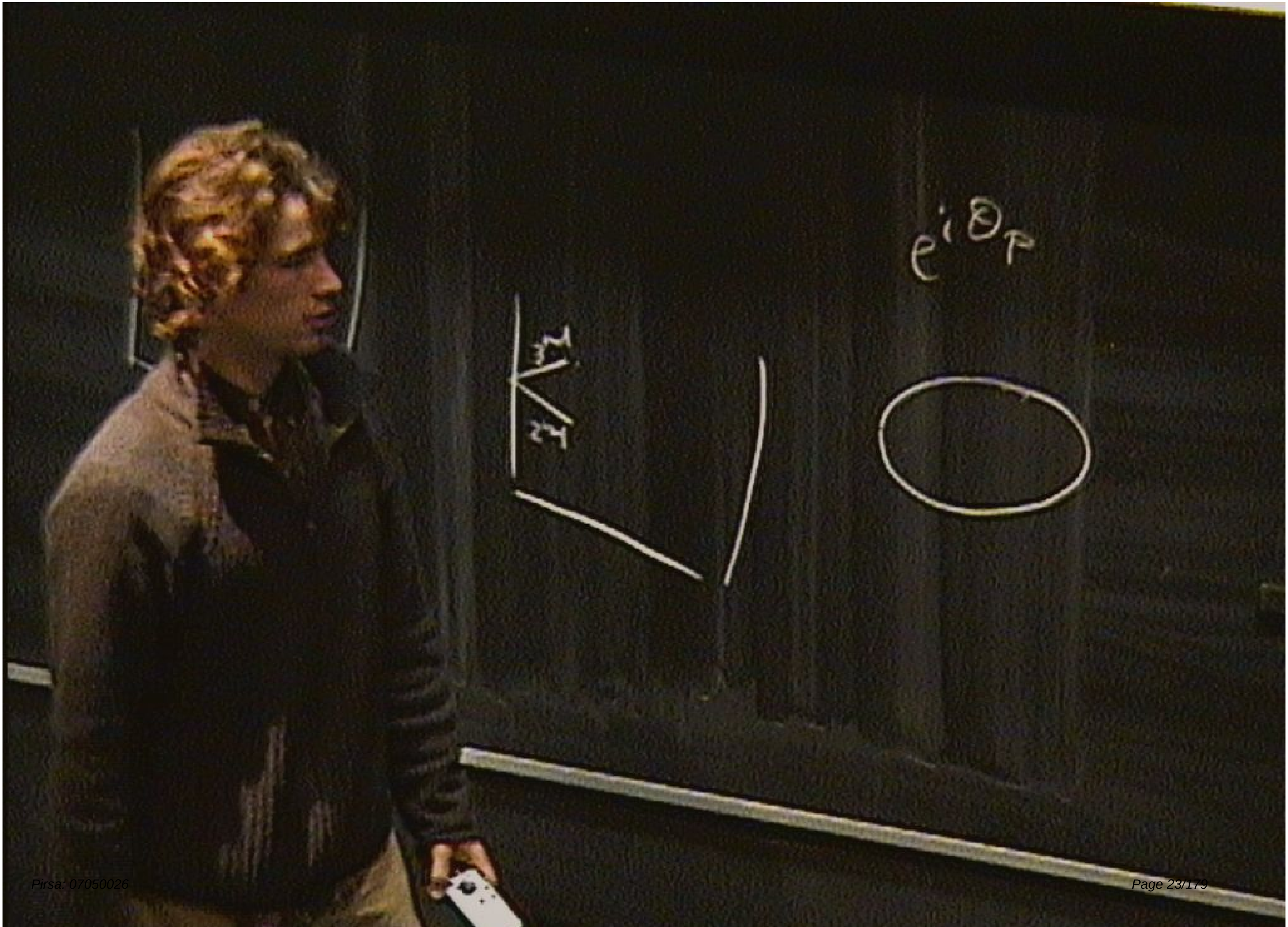
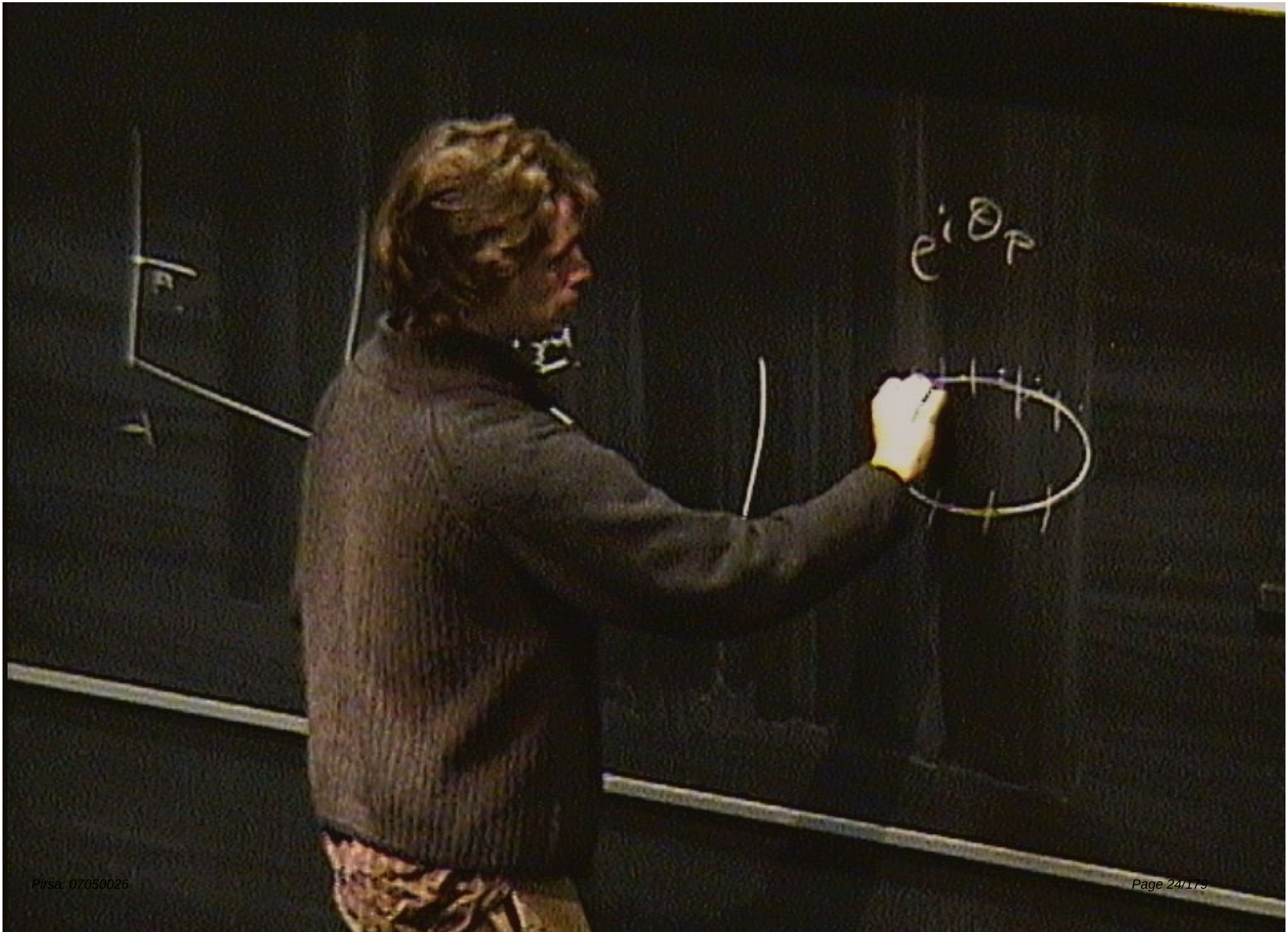

$e^{i\theta_P}$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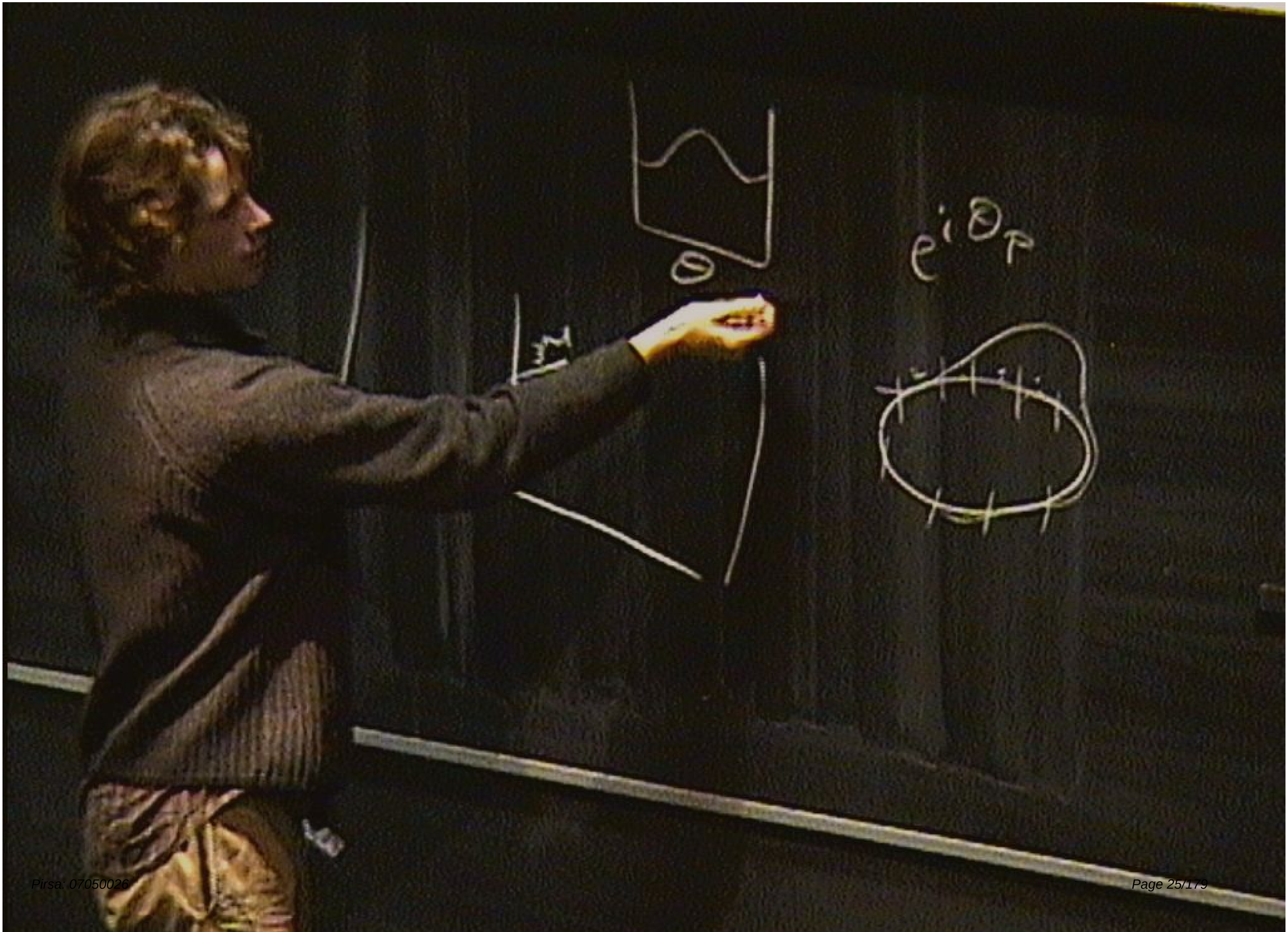
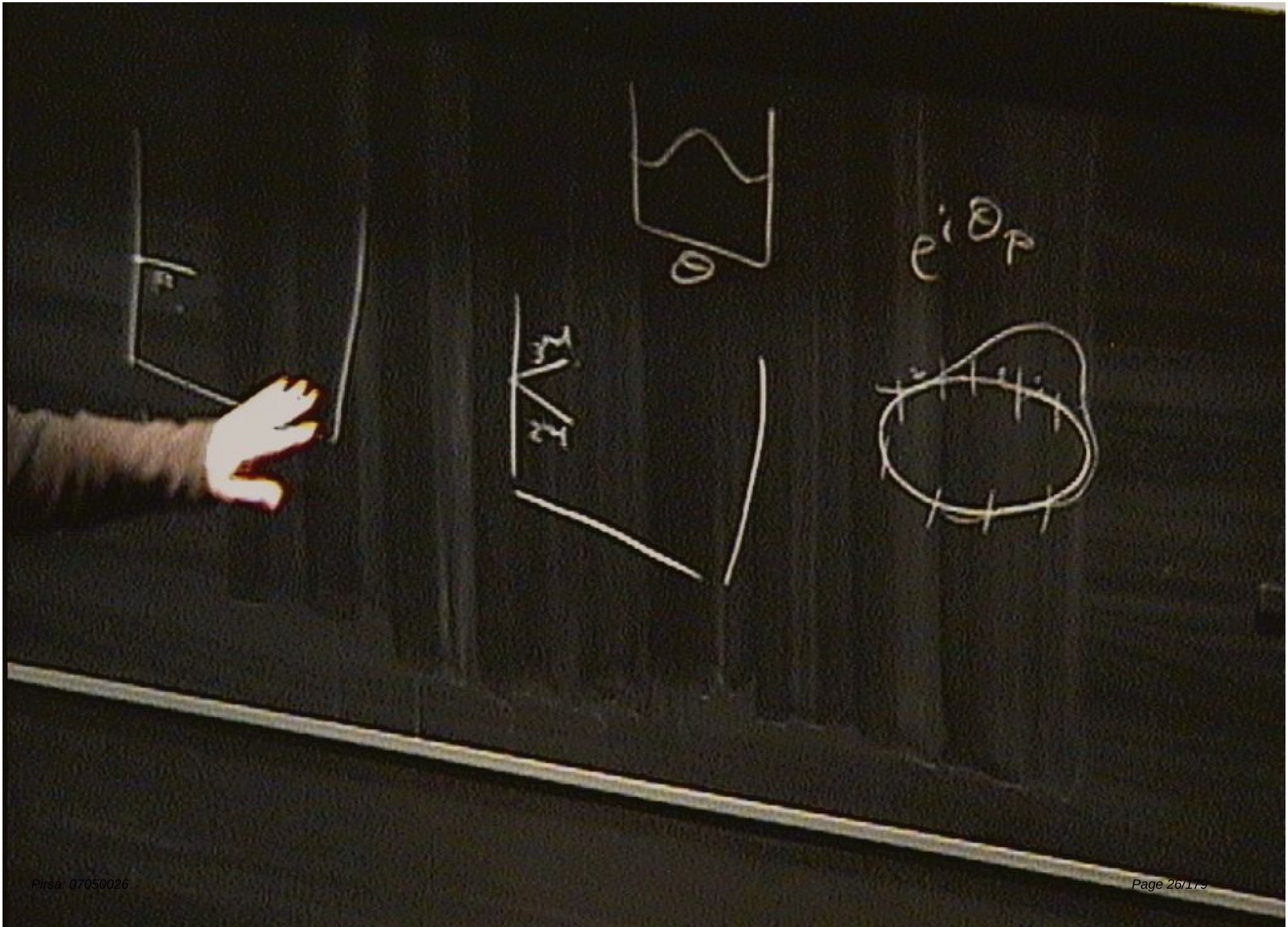

$e^{i\theta_P}$
θ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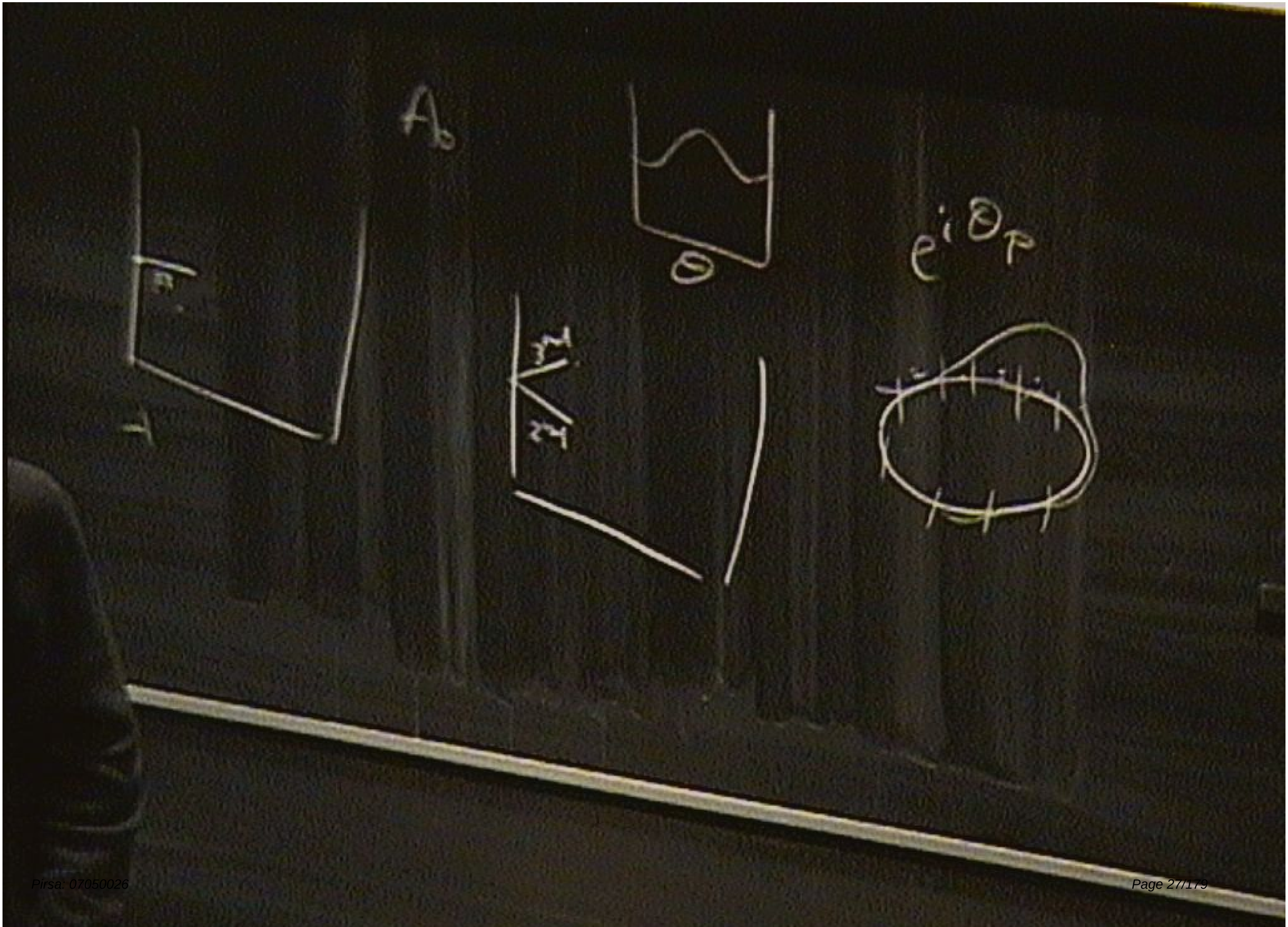
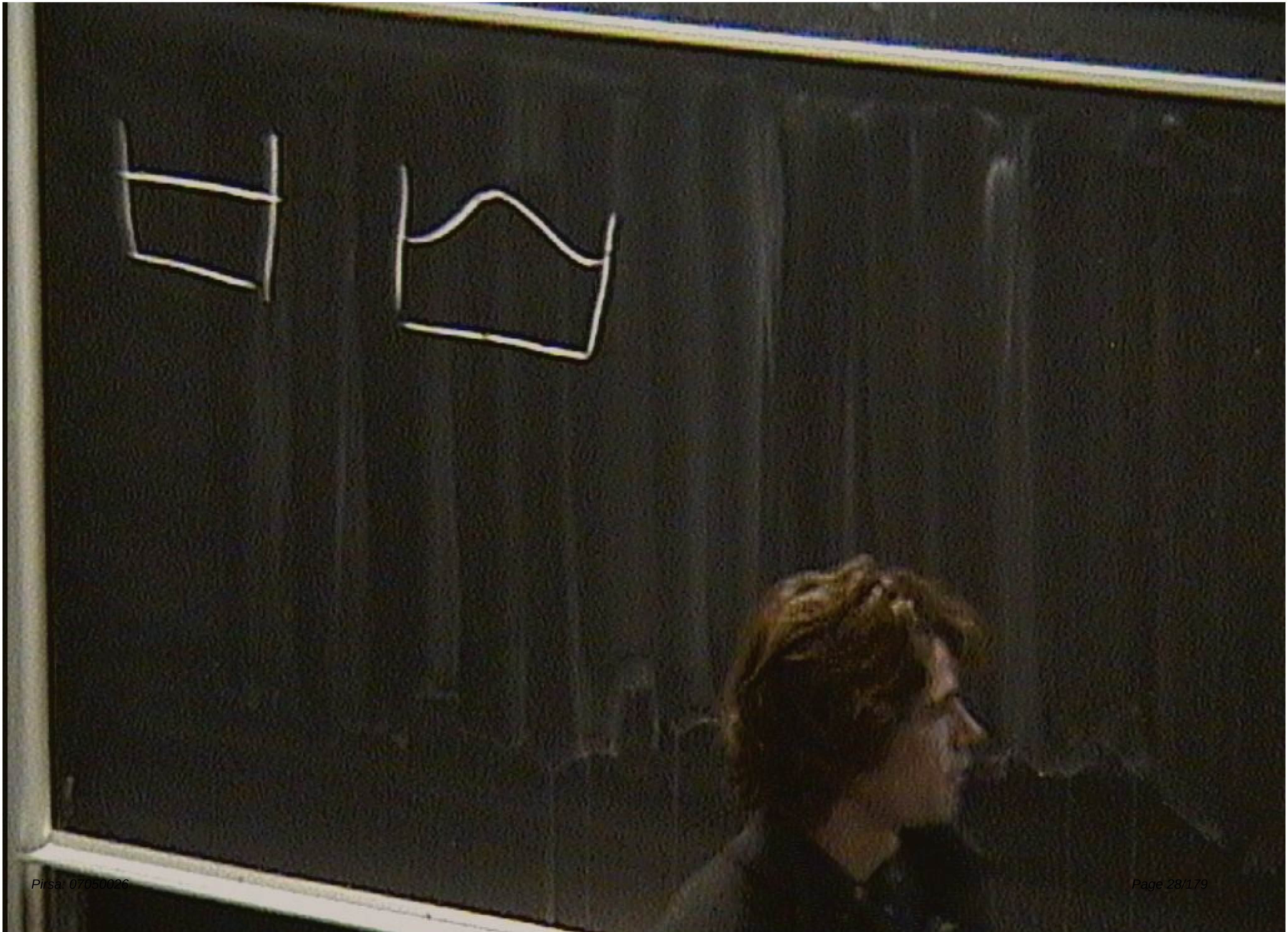
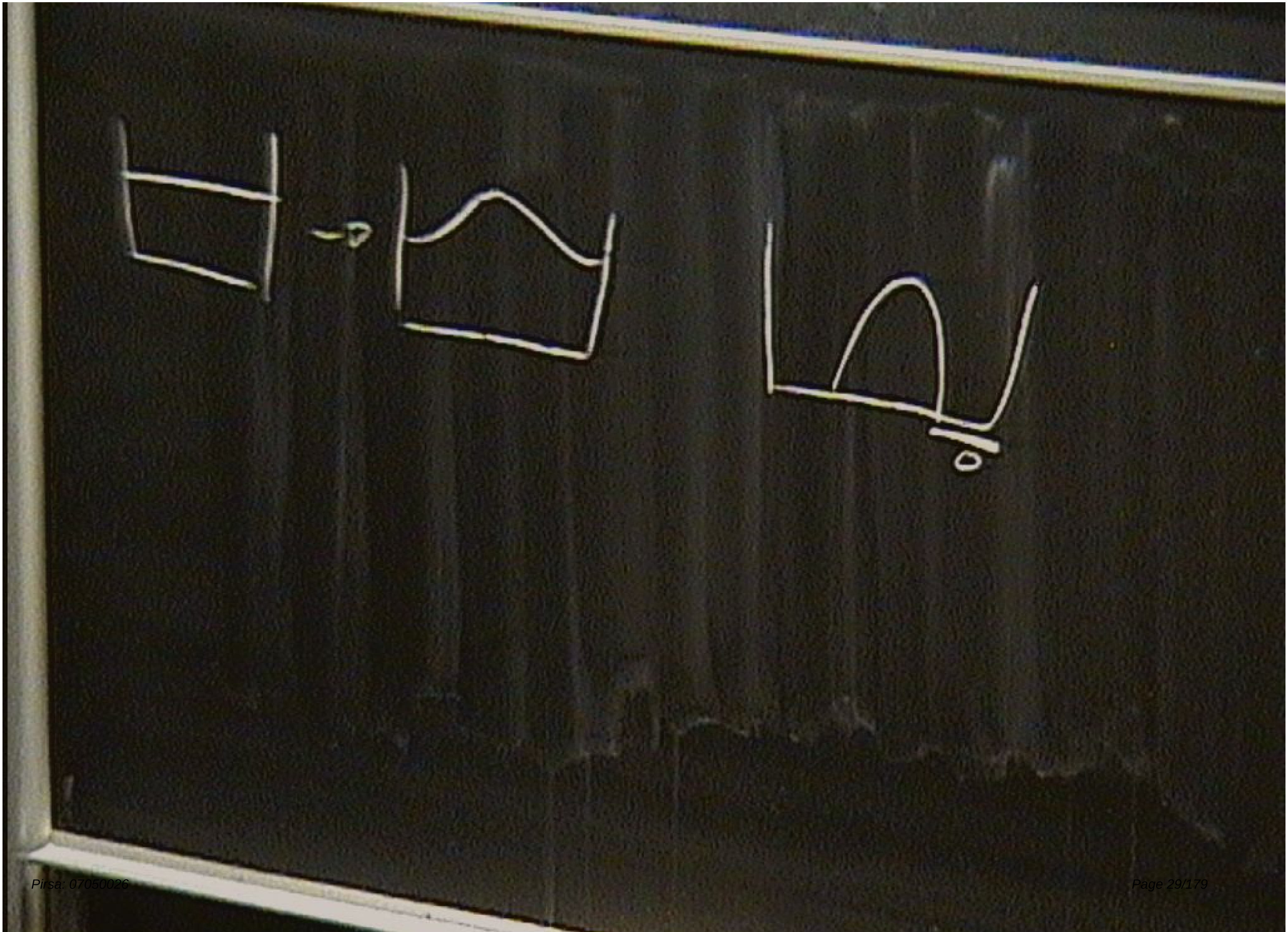
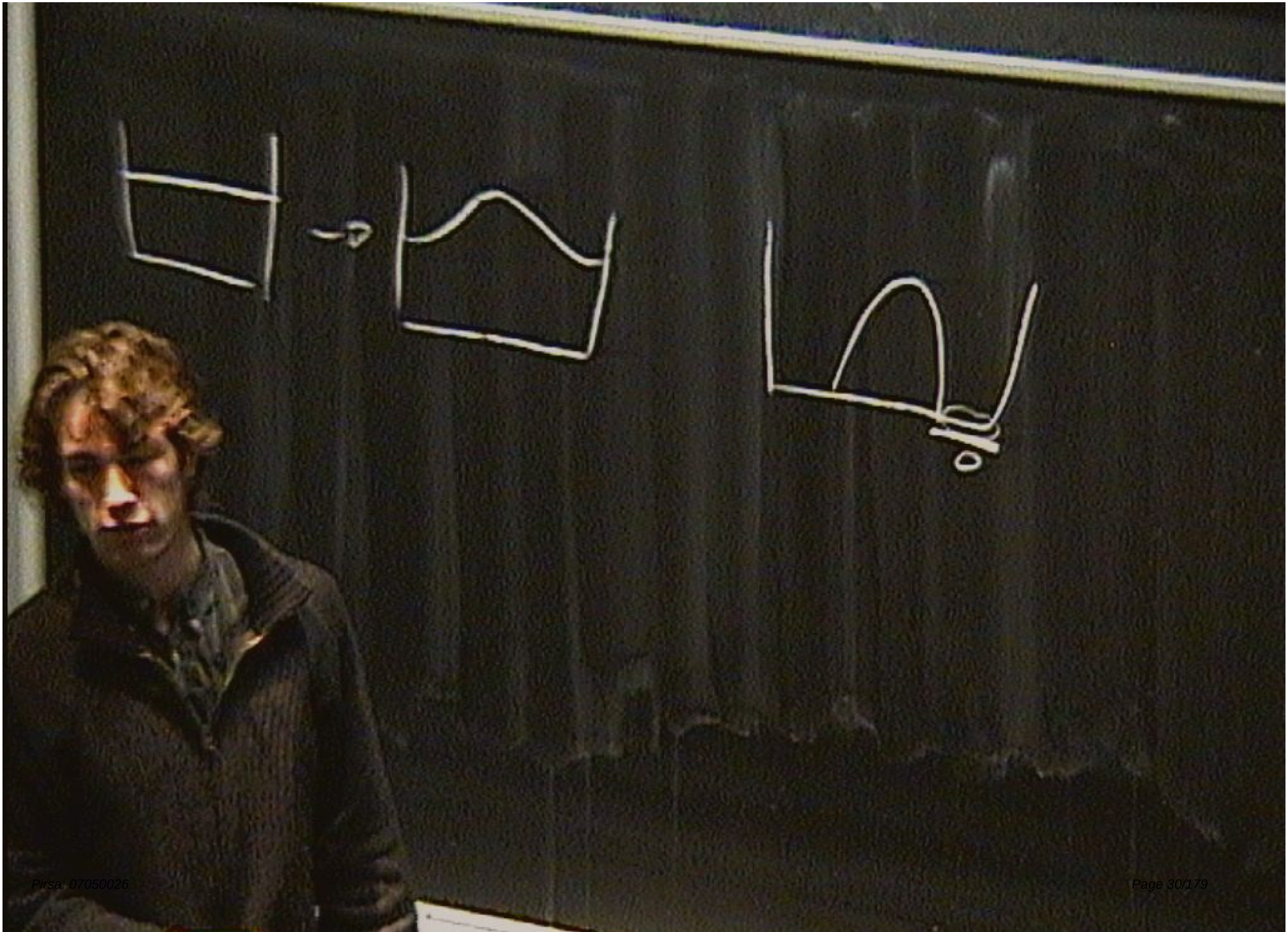
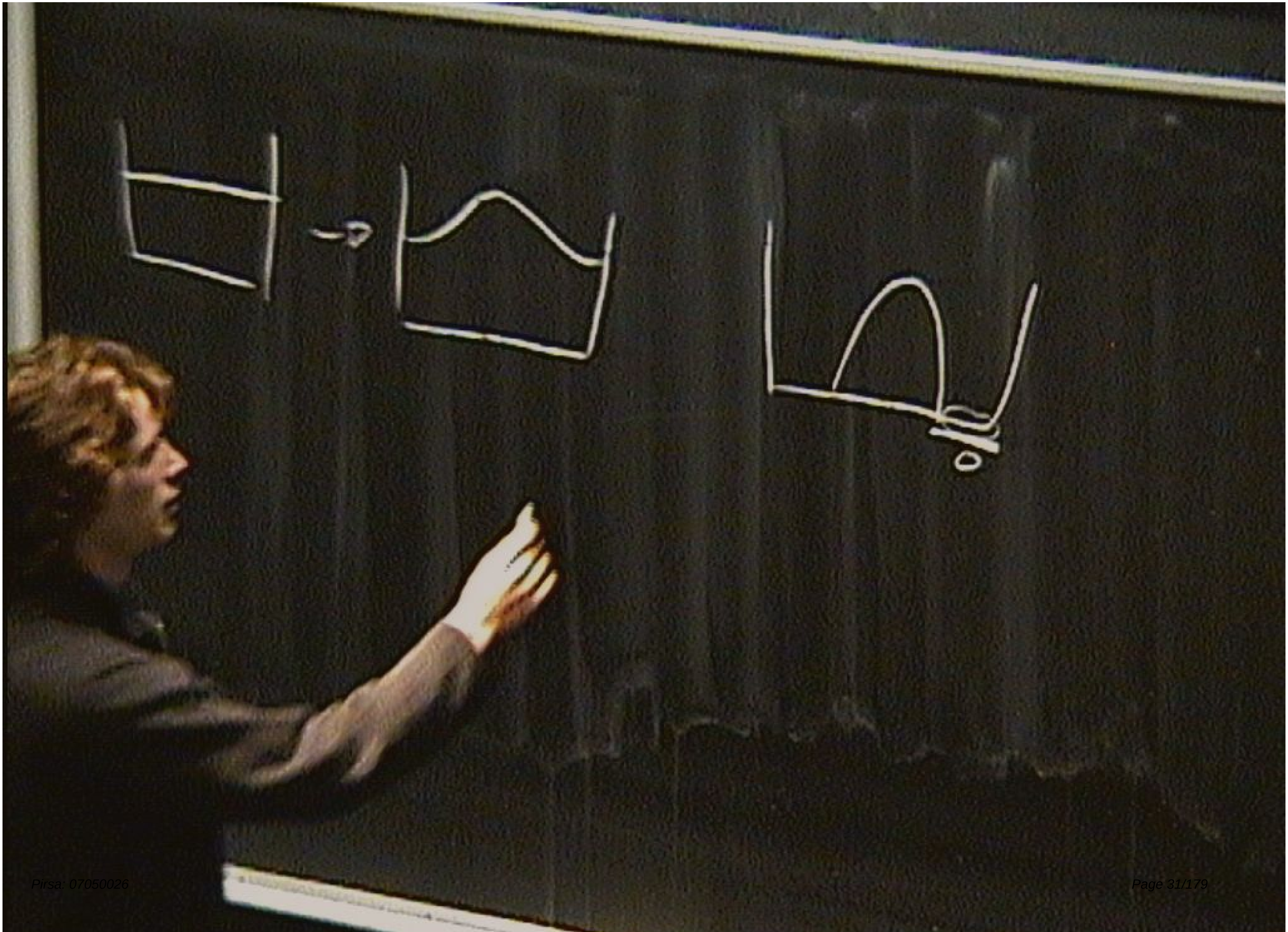
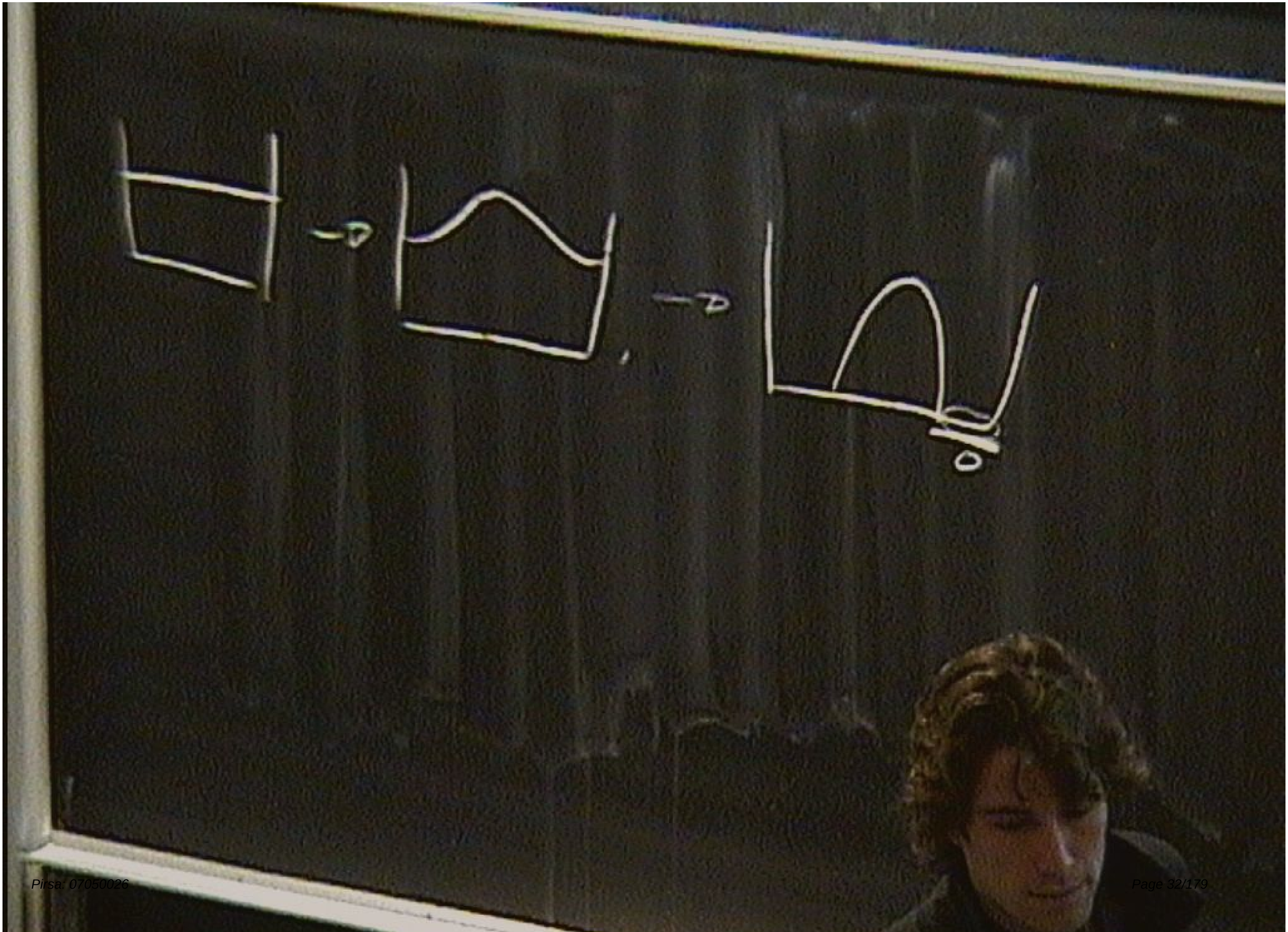

A_b
θ
$e^{i\theta_P}$











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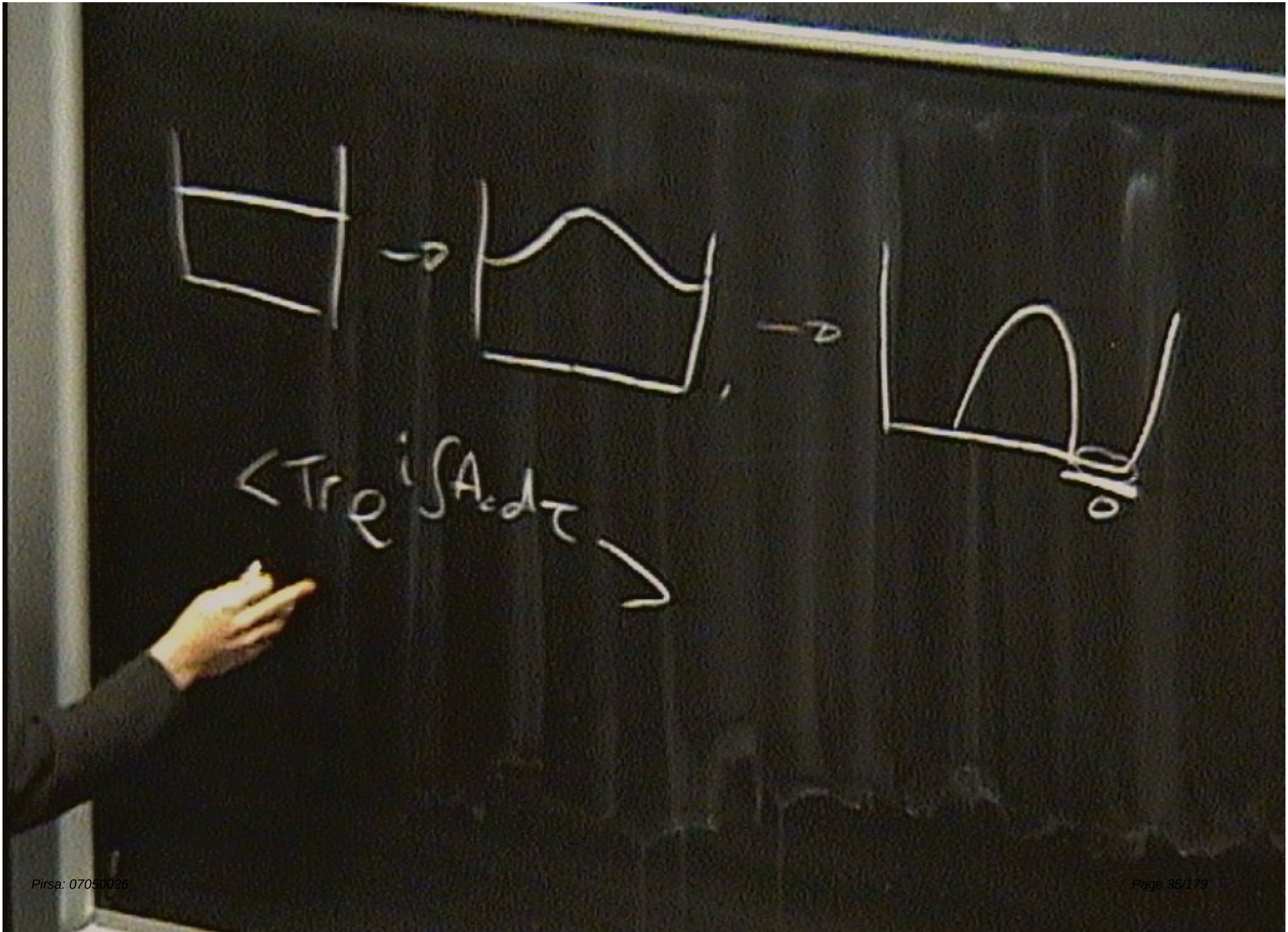
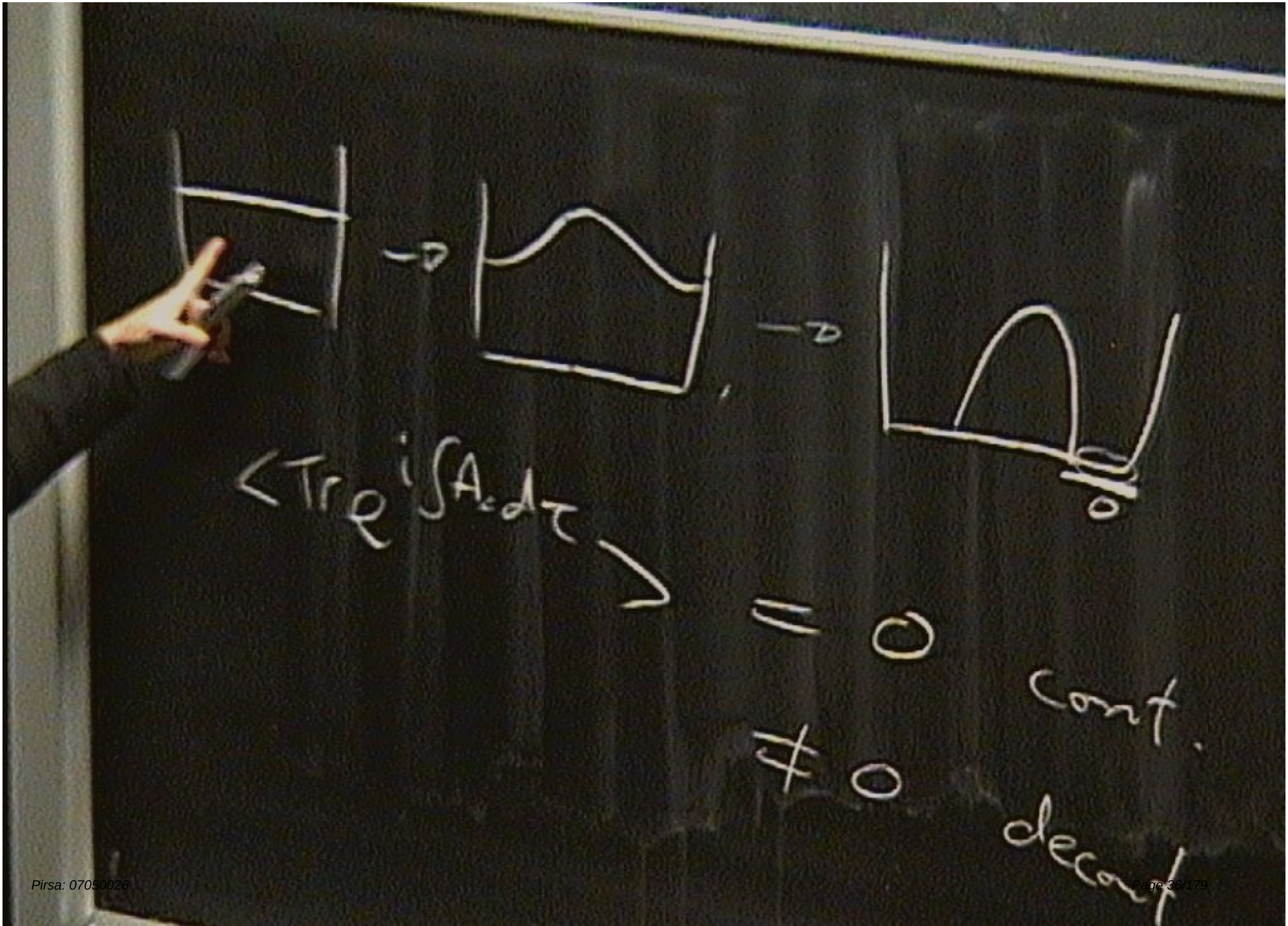
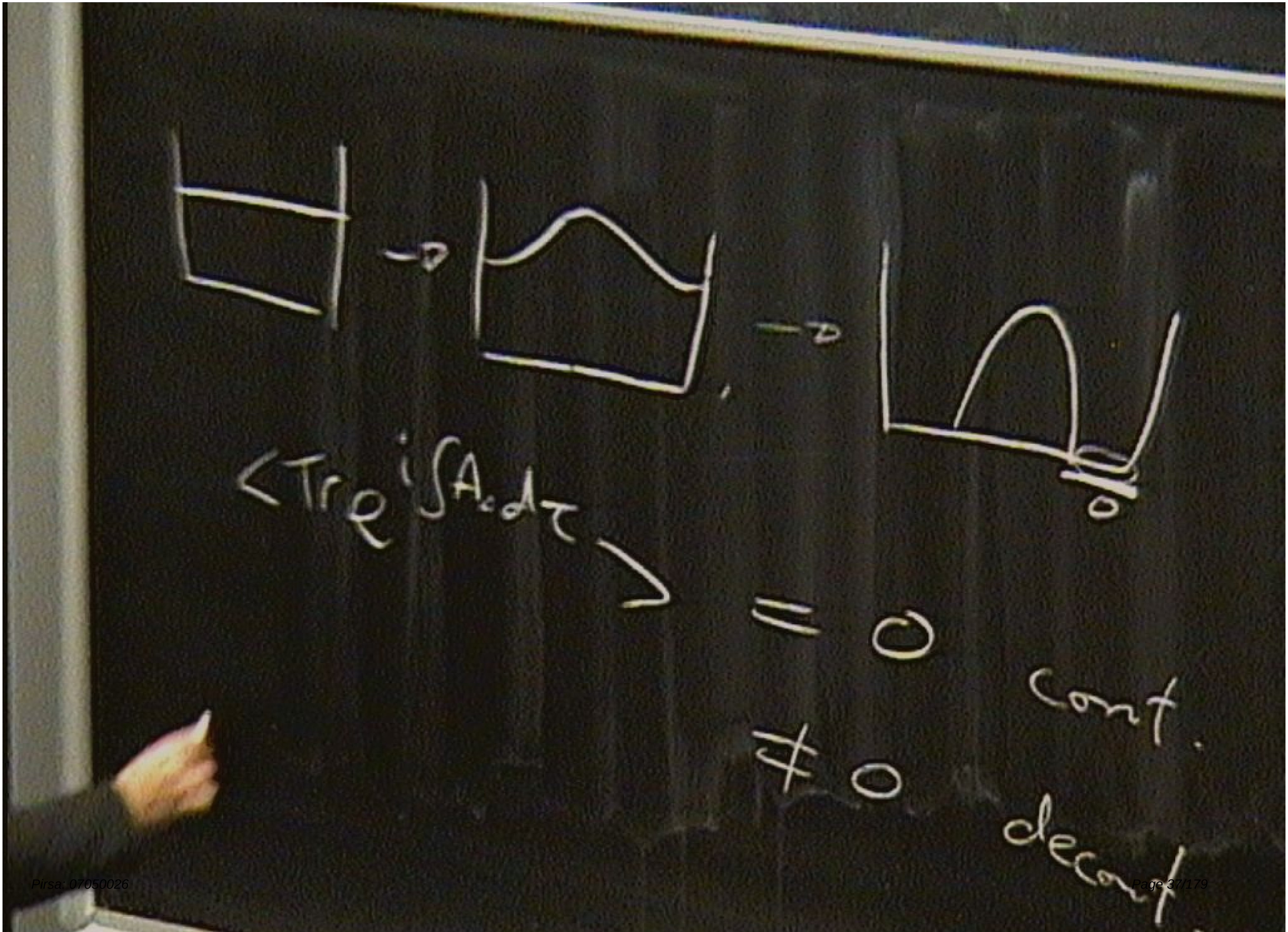

<Tr e^{i∫A_t dτ} >


<Tr e^{i∫A_t dτ}>
= 0
cont.
≠ 0
decont.


<Tr e^{i∫A₀dτ}> = 0
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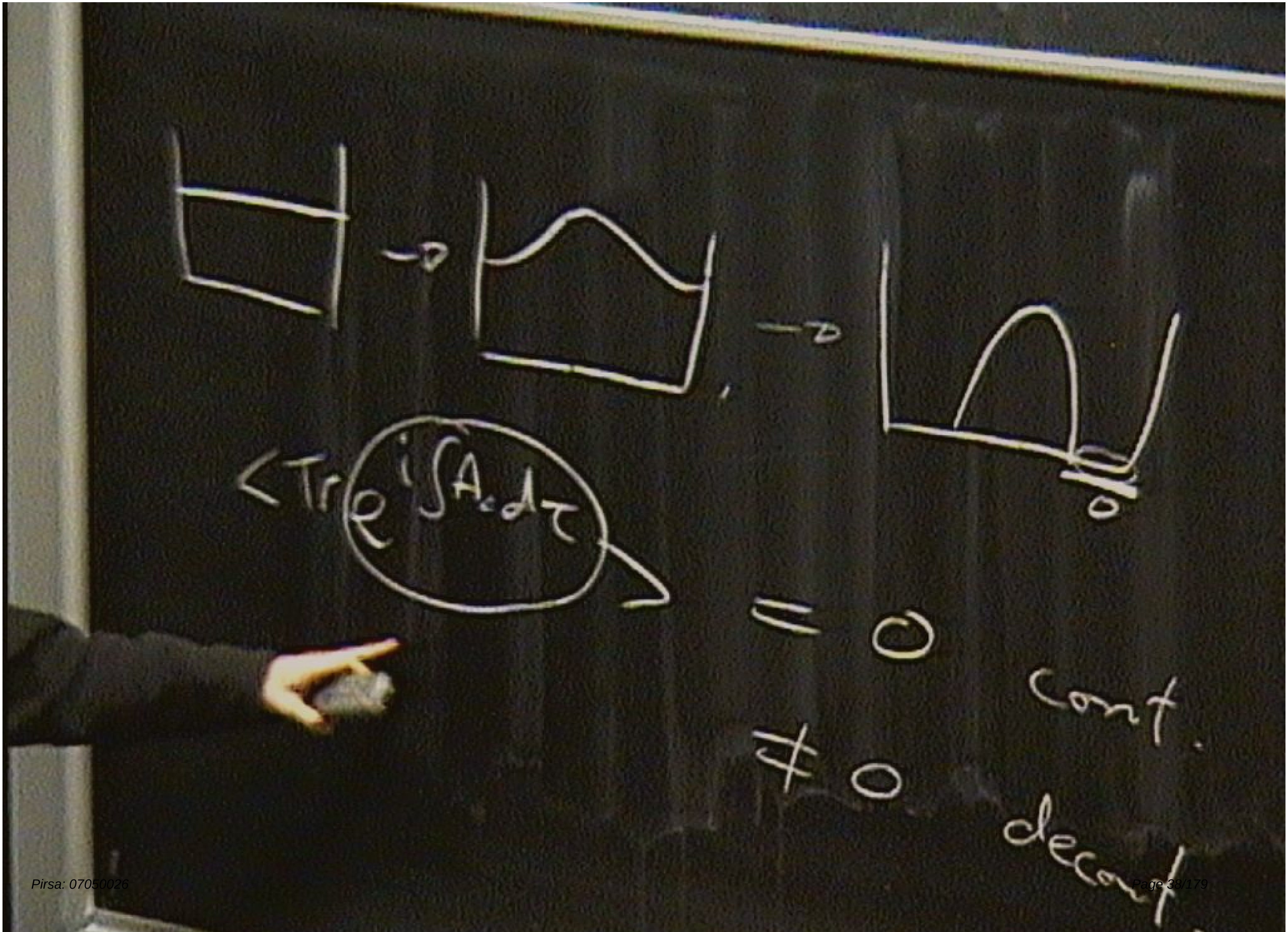
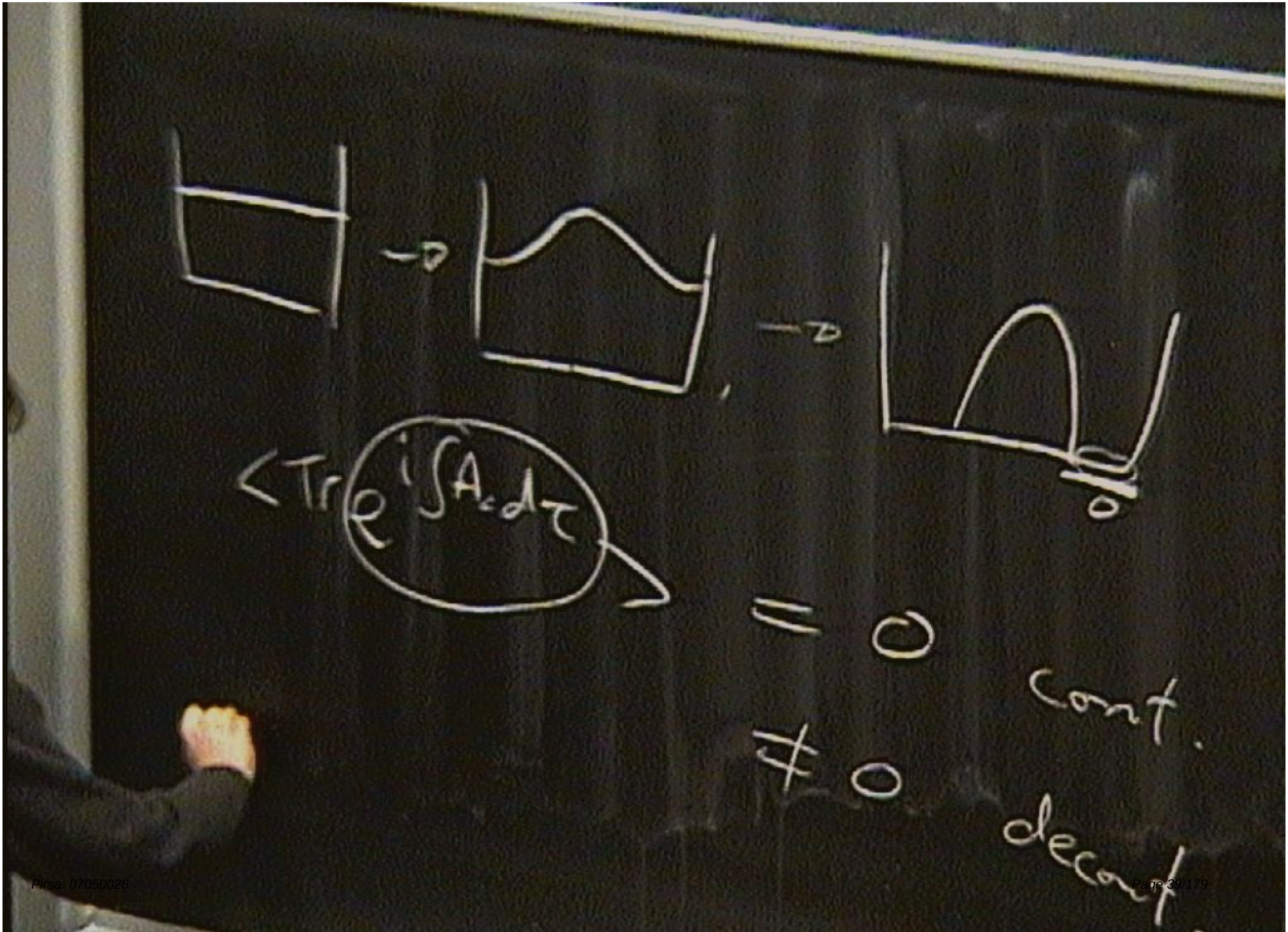

= 0 cont.
≠ 0 decont.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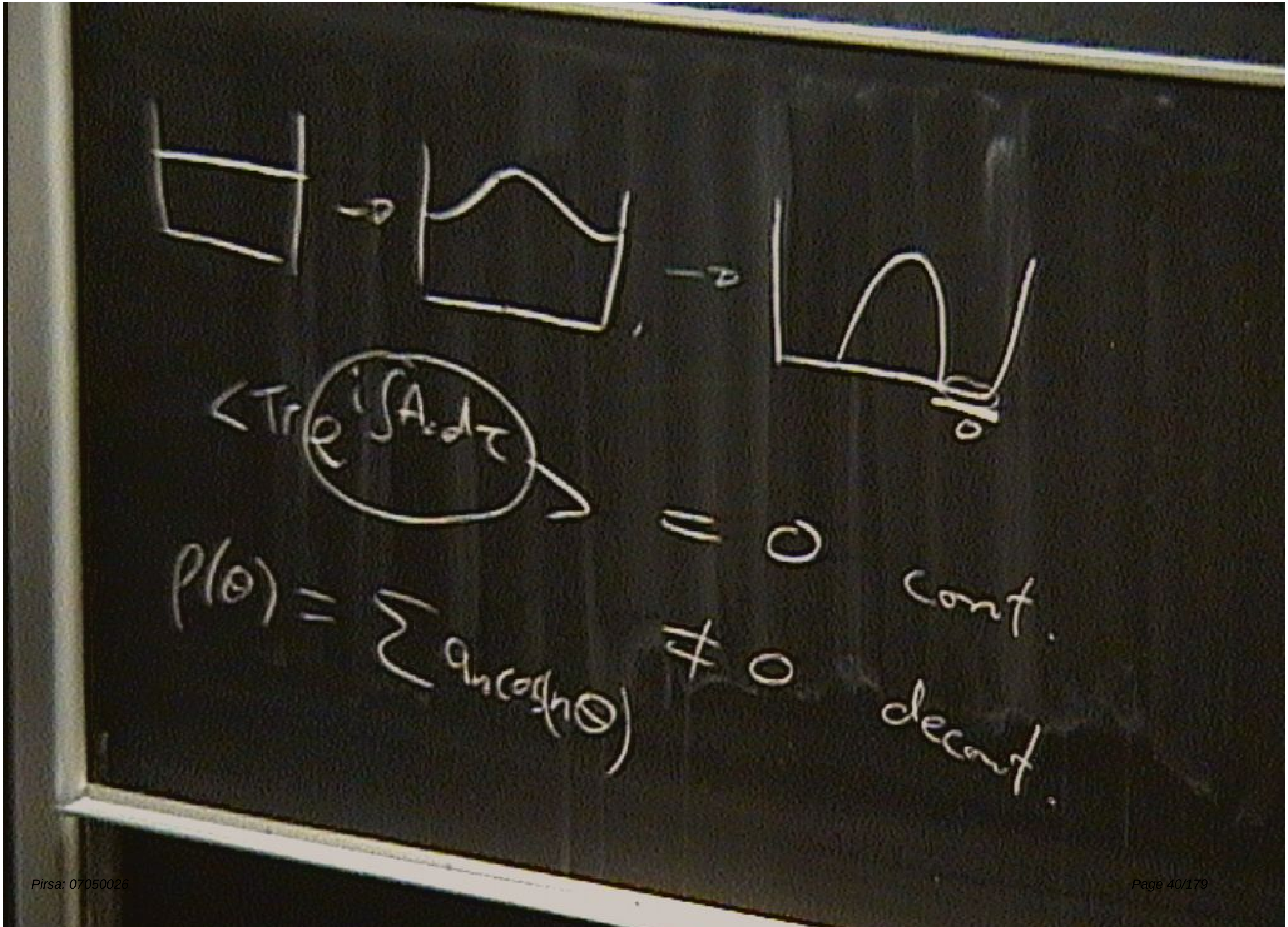
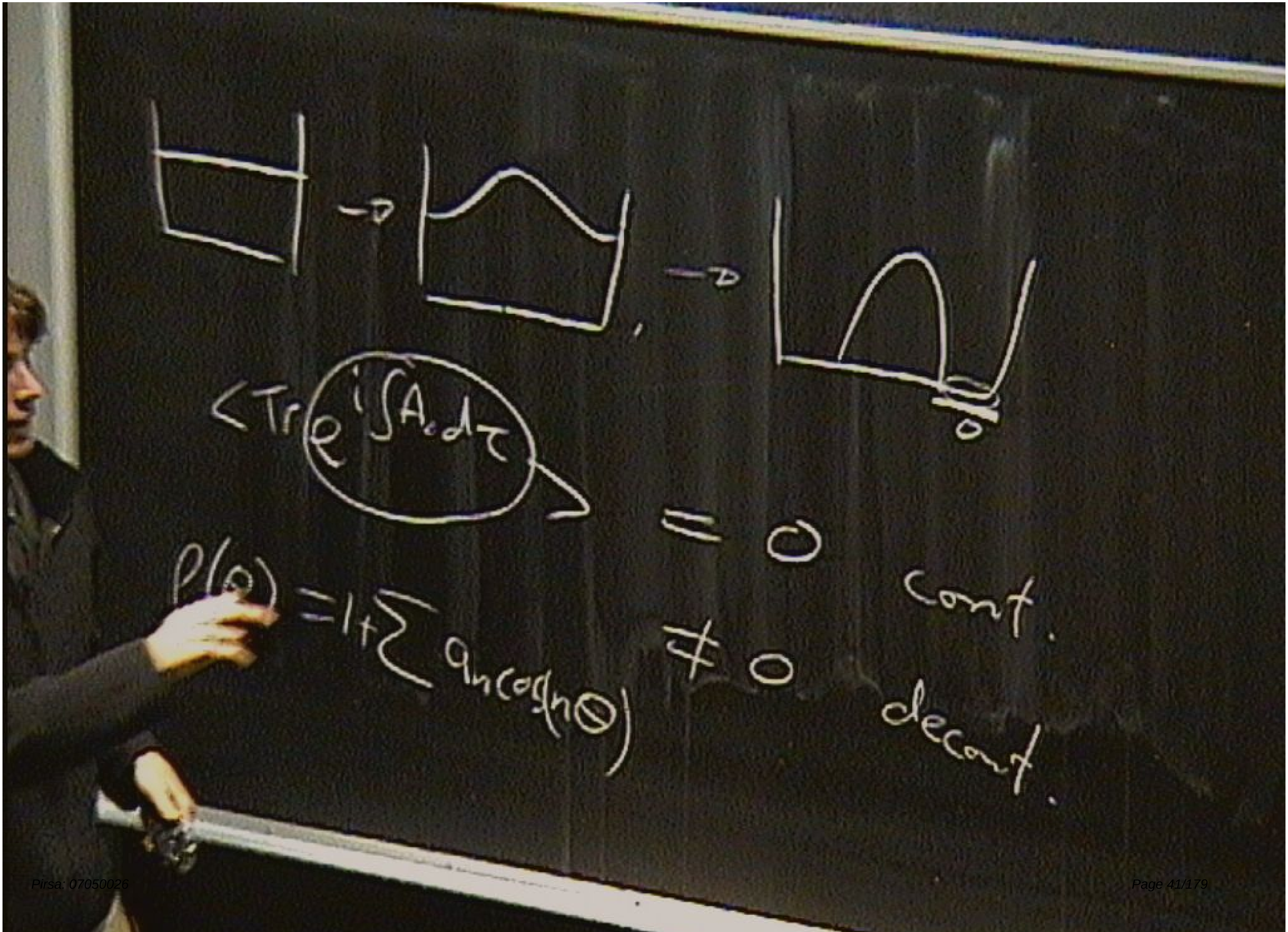
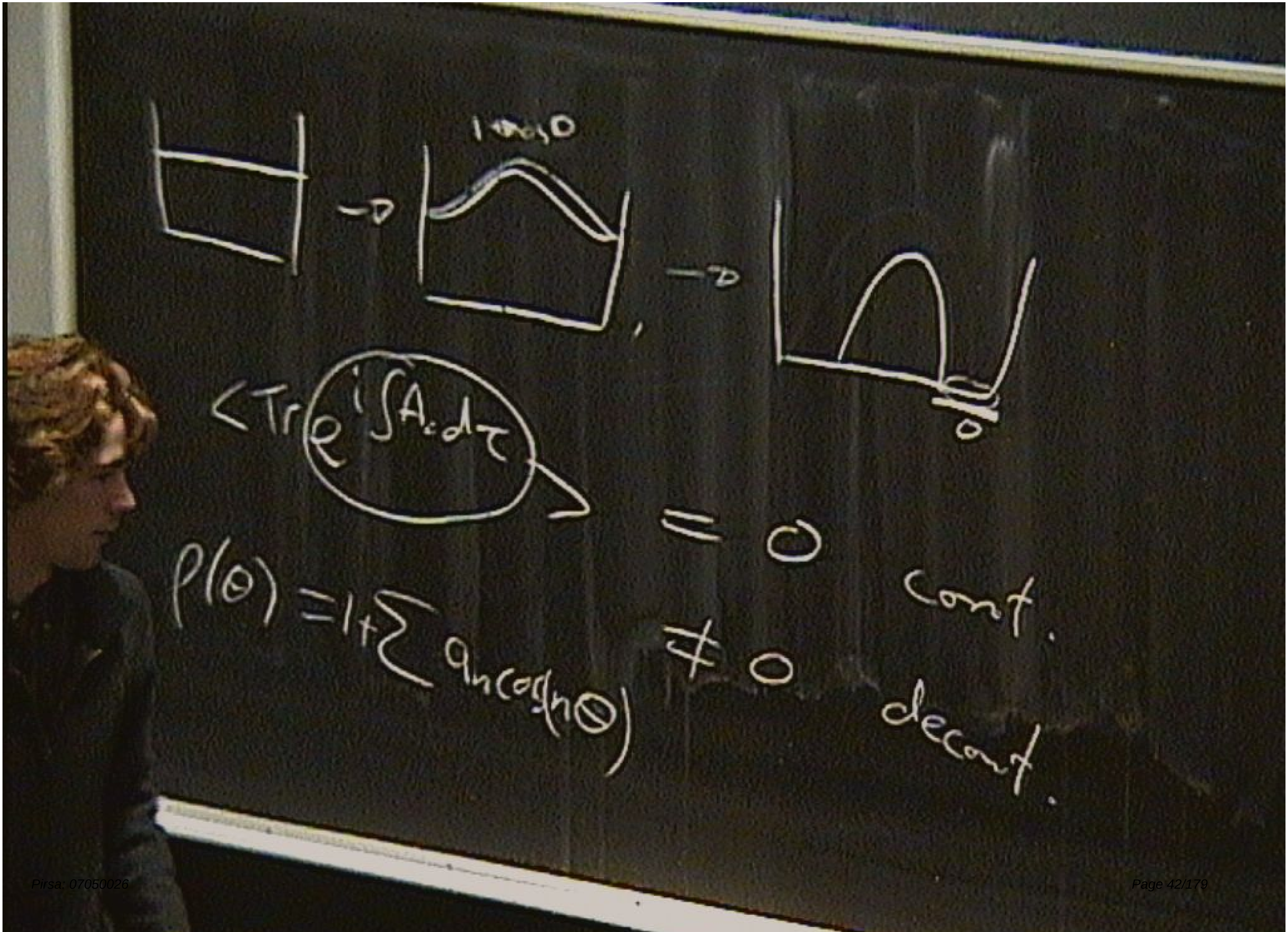

<Tr e^{i∫A·dτ}> = 0 cont.
ρ(θ) = Σ a_n cos(nθ) ≠ 0 decont.


<Tr e^{i∫A₀dτ}> = 0
ρ(θ) = 1 + Σ aₙcos(nθ) ≠ 0
cont.
decont.


<Tr e^{i∫A_0 dτ}> = 0
cont.
ρ(θ) = 1 + Σ a_n cos(nθ) ≠ 0
deconf.

Including scalar fields

Hollowood-Kumar-Naqvi (2006), Hartnoll-Kumar (2006)

- At low and intermediate temperature, the six scalar fields Φ^J have a conformal mass term whereas A_0 is classically massless. Sensible to integrate out the scalars.
- At higher temperatures, $RT \sim \lambda^{-1/2}$, the one loop mass of A_0 is comparable to that of the scalars. We will allow for condensates of A_0 and Φ^J.
- Integrate out the off diagonal modes to obtain an effective potential for the respective eigenvalues $\{\theta_p\}_{p=1}^N$ and $\{\phi_p^J\}_{p=1}^N$. This truncation to commuting VEVs is consistent.
- Strategy:
  - Compute one loop effective potential $S_{\text{eff.}}[\theta, \phi^J]$.
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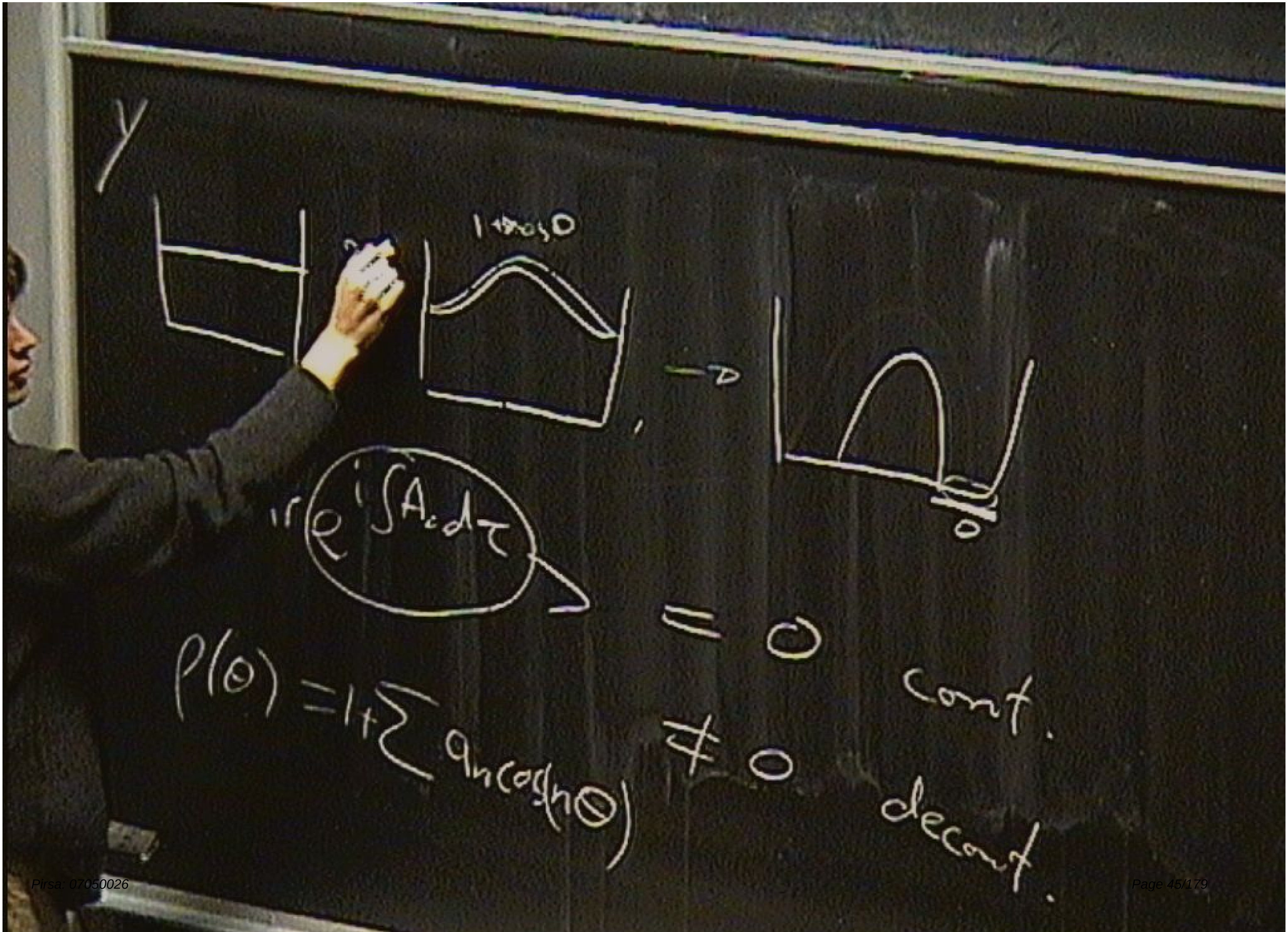
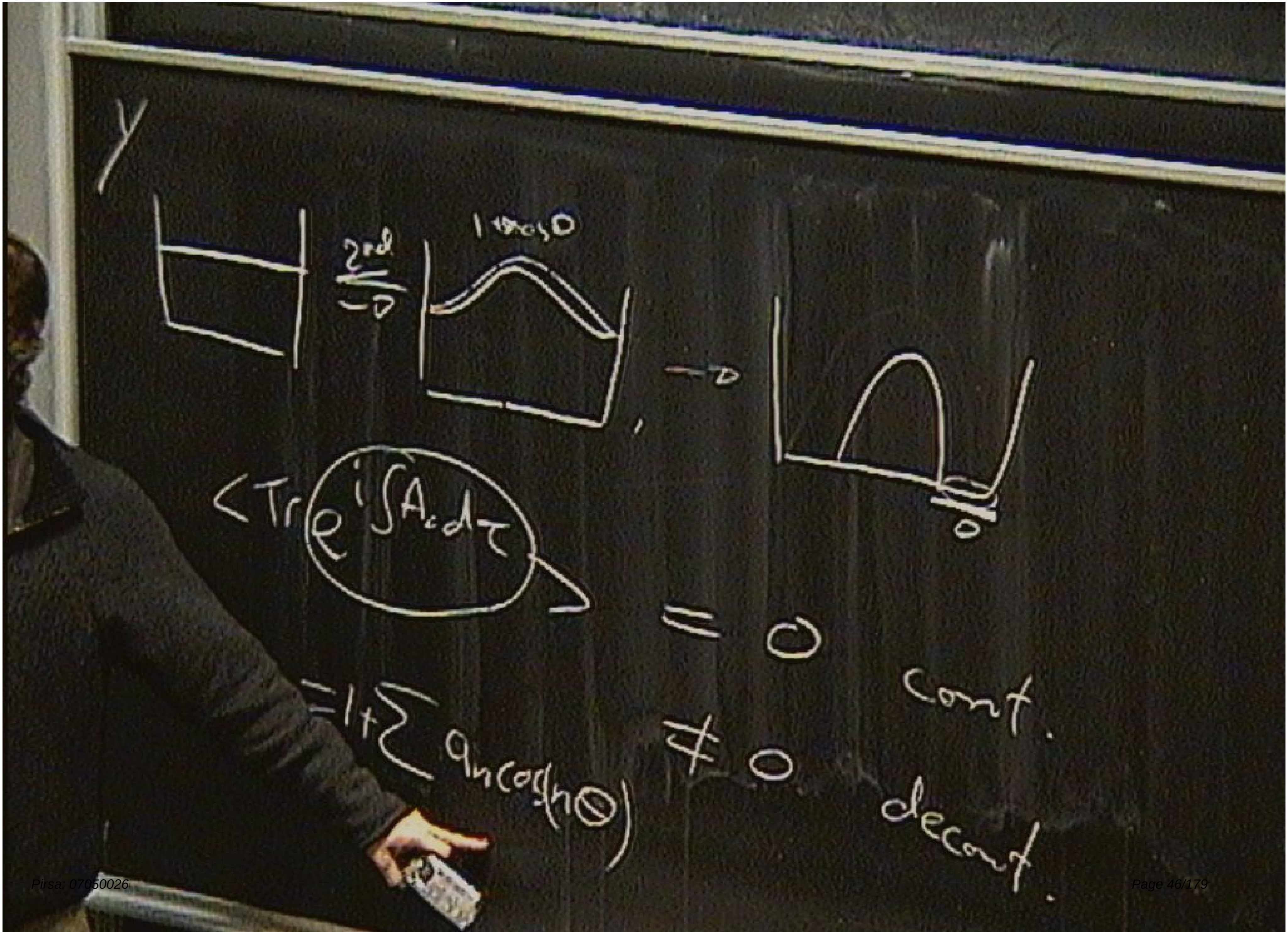
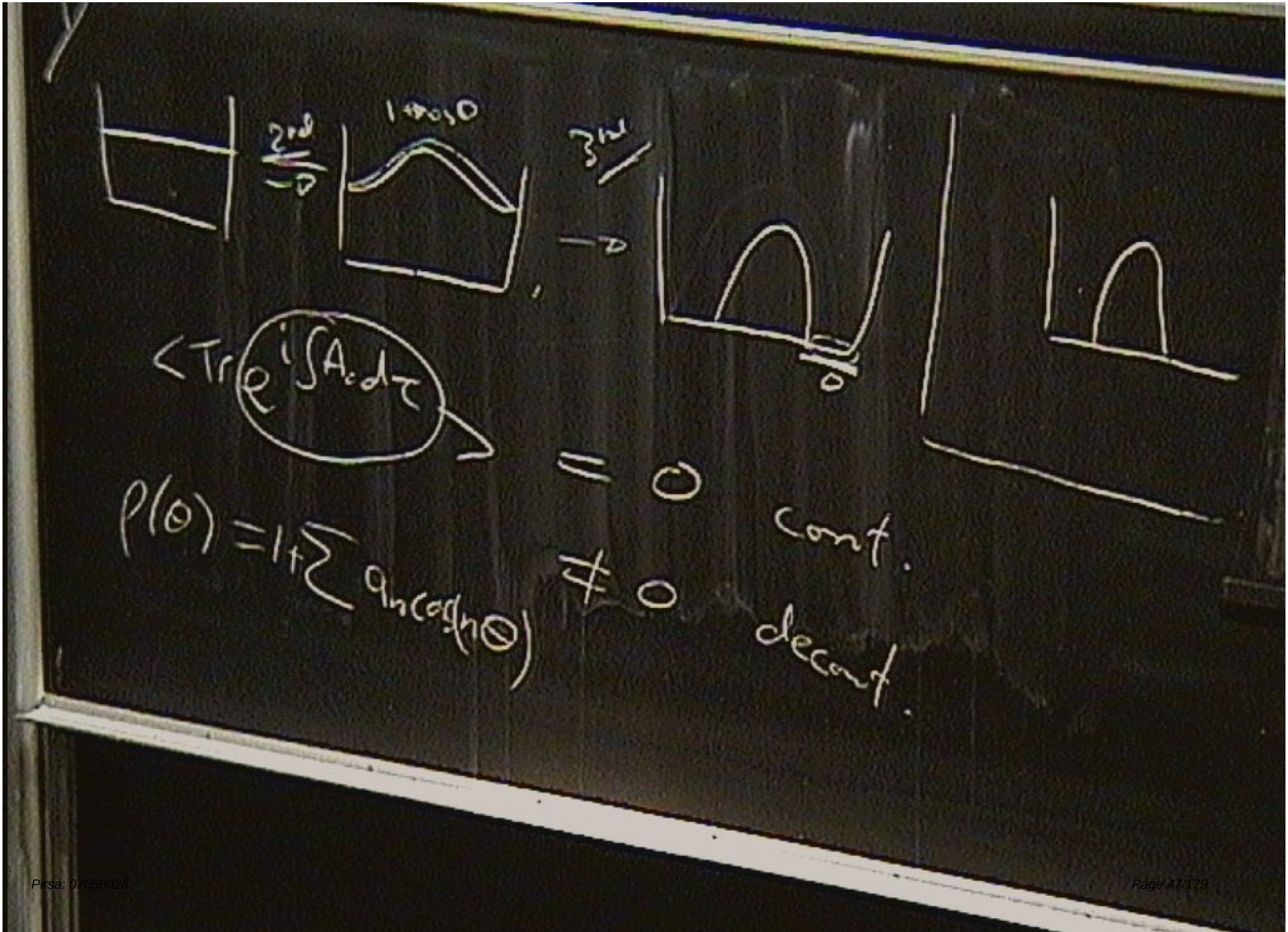
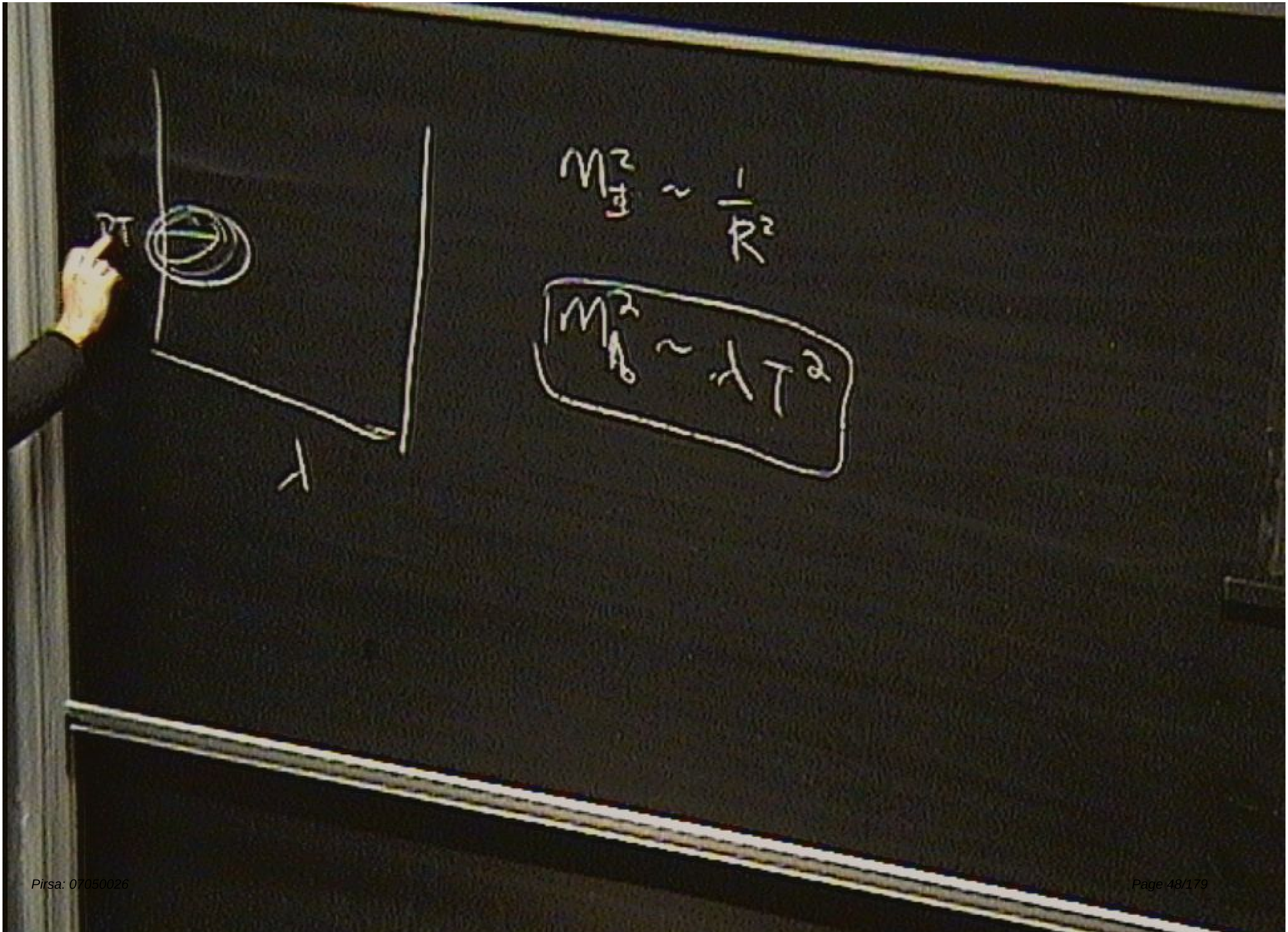
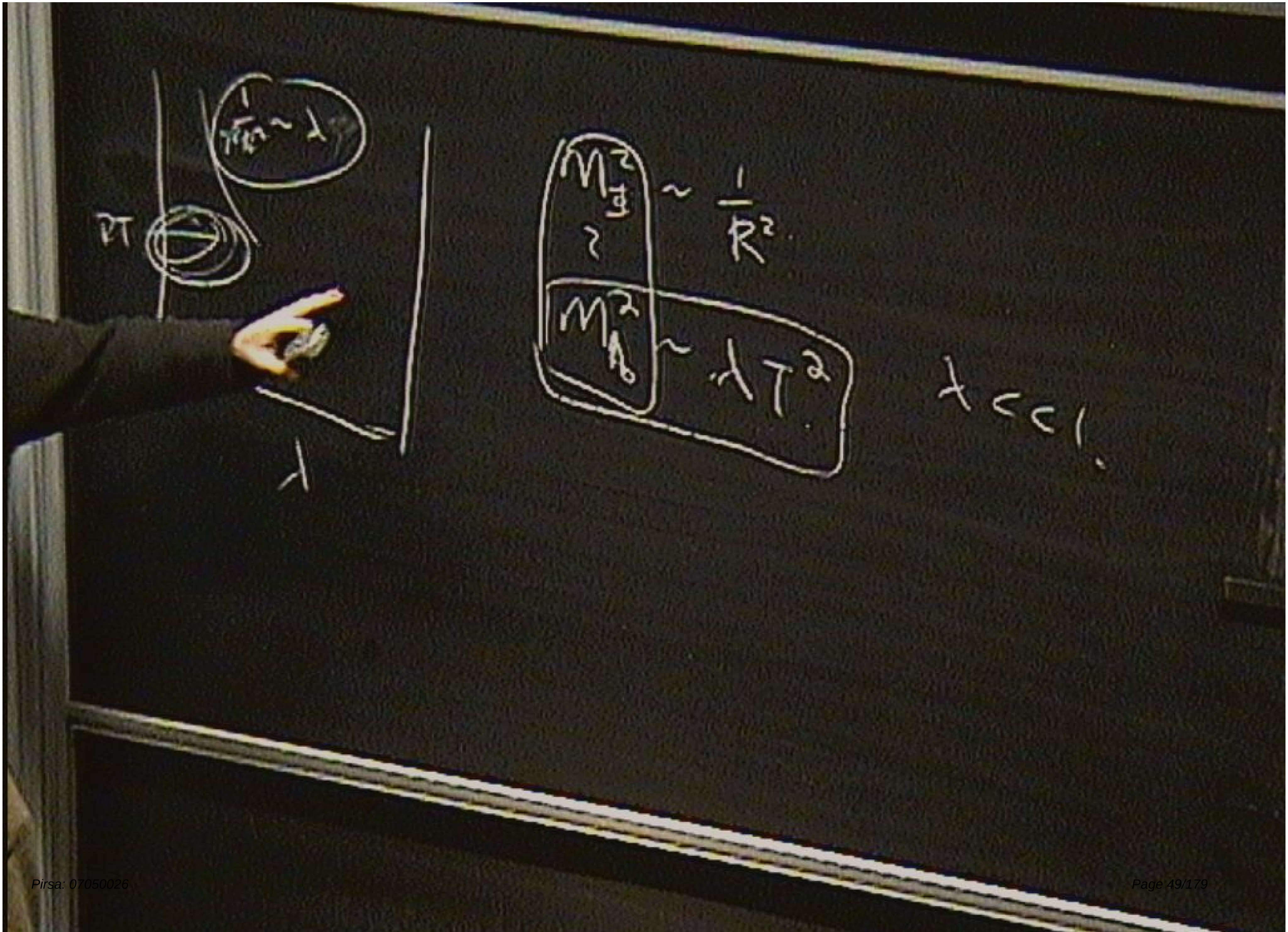
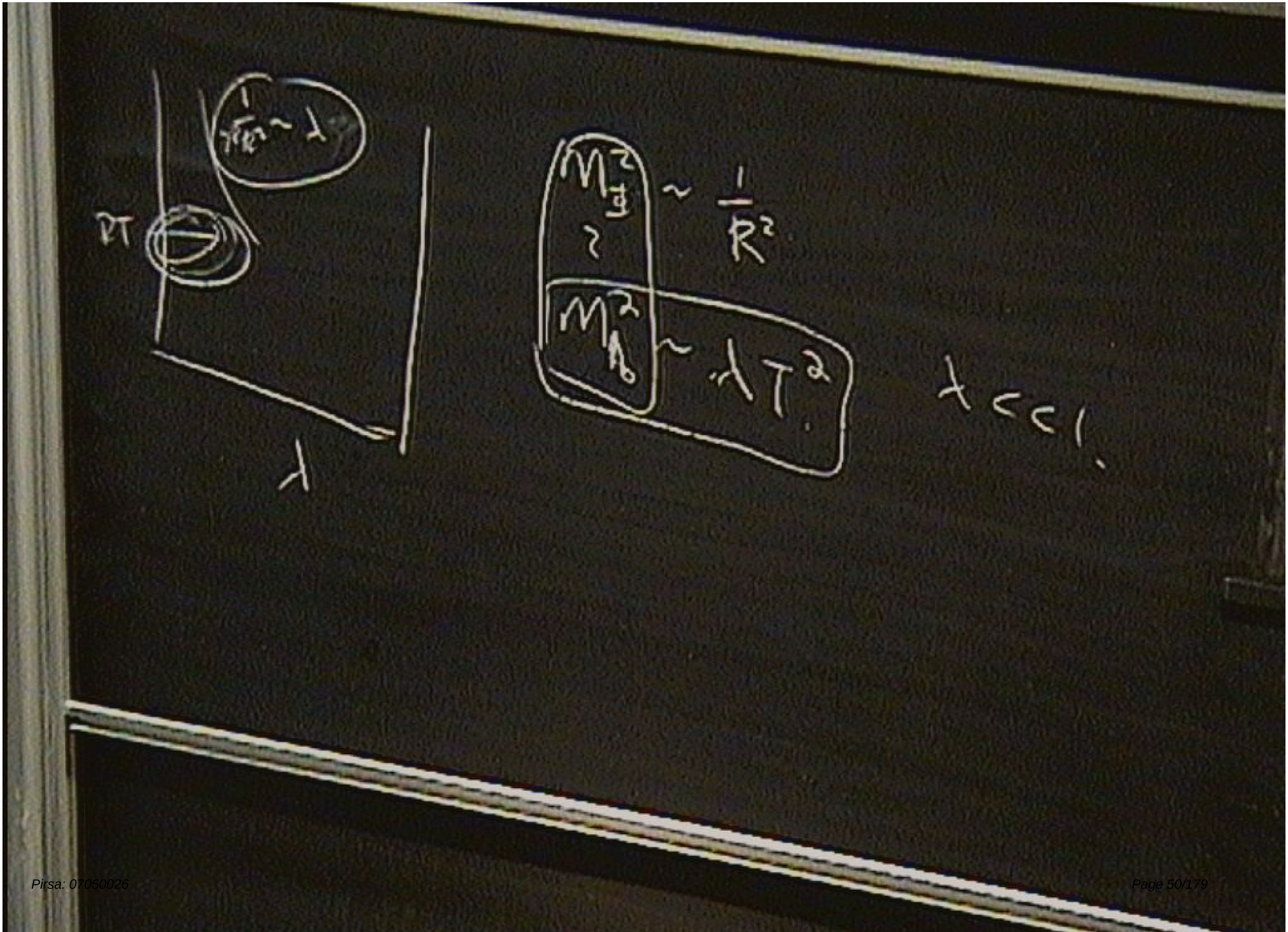

= 0 cont.
ρ(θ) = 1+Σ ancos(nθ) ≠ 0 decont.


2nd
cont.
decont.


2nd
1+cosθ
3rd
<Tr e^{i∫A₀dτ}
= 0 cont.
ρ(θ) = 1 + Σ aₙ cos(nθ)
≠ 0 deconf.


T
λ
m² ~ 1/R²
m² ~ λT²


λ << 1
λ


RT
λ
M² ~ 1/R²
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λ << 1.

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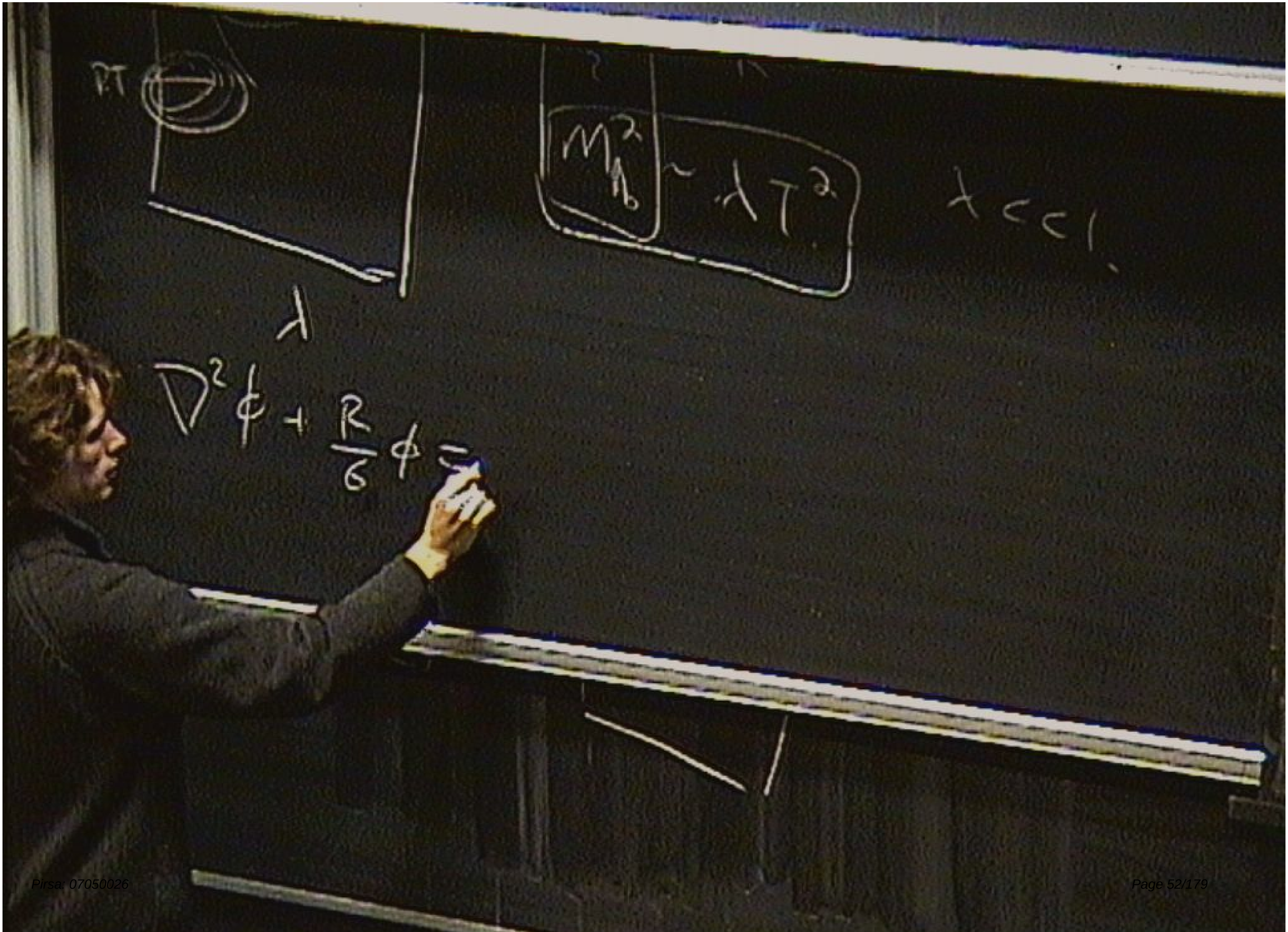
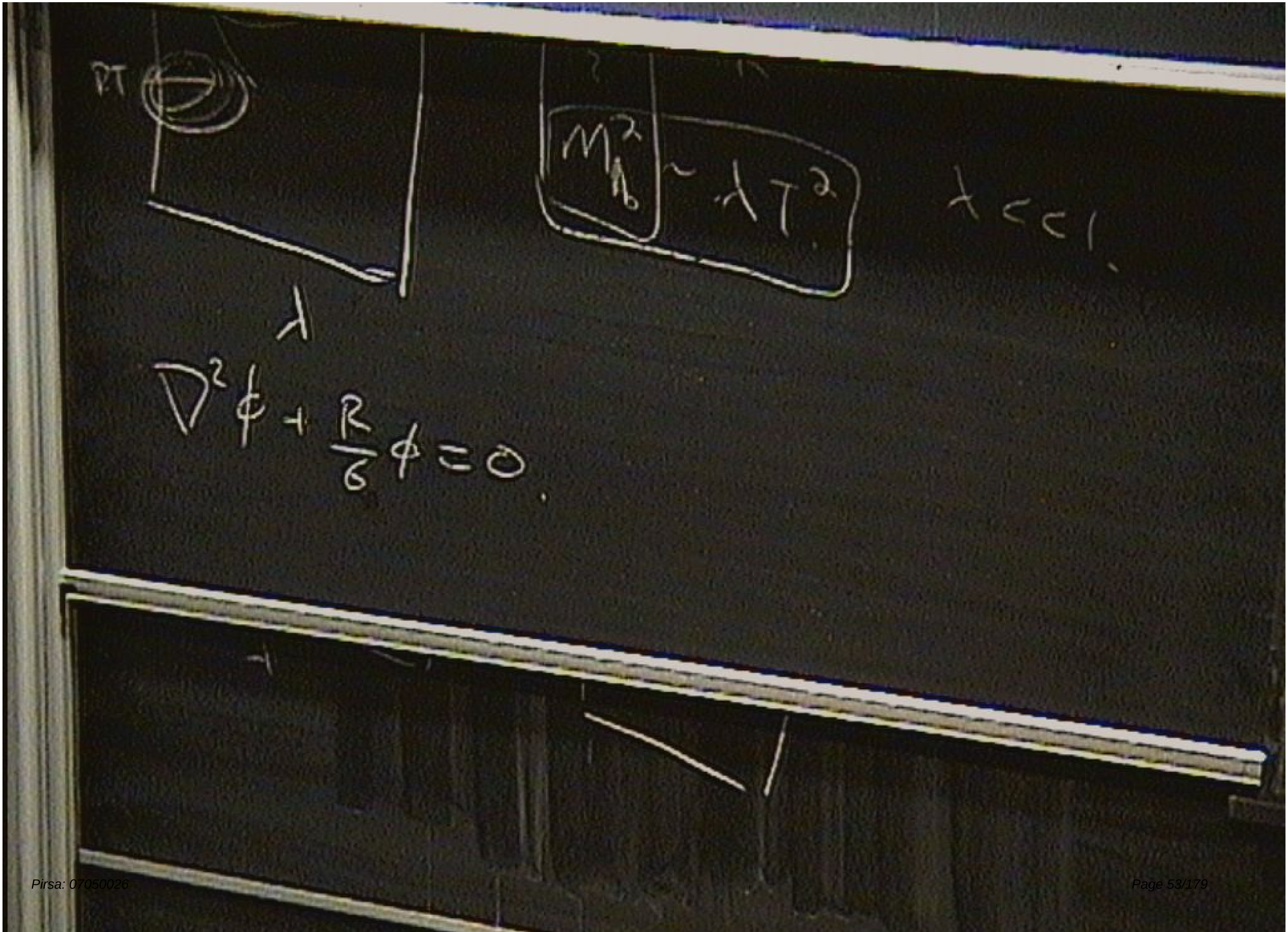

m_h^2 ~ λT^2
λ << 1
λ
∇^2 φ + R/6 φ =


$m^2 \sim \lambda T^2$
$\lambda \ll 1$
λ
$\nabla^2\phi + \frac{R}{6}\phi = 0$

Aside: scalars and emergent geometry

Berenstein (2005)

- A very similar effective potential has been conjectured recently to describe the $1/8^{\text{th}}$ BPS sector of $\mathcal{N} = 4$ SYM theory at zero temperature and strong coupling.
- Berenstein considers a matrix model for the six scalar fields of the theory

$$S = \sum_p \phi_p^2 - \frac{1}{2} \sum_{p \neq q} \log |\phi_p - \phi_q|^2$$

- In the ground state of this model, the eigenvalues form an S^5. This is to be identified with the S^5 in the dual geometry.
- The geometry dual to an operator $\mathrm{tr}\mathcal{O}$ is to be obtained by solving the model

$$\mathrm{tr}\mathcal{O} e^{-S} .$$

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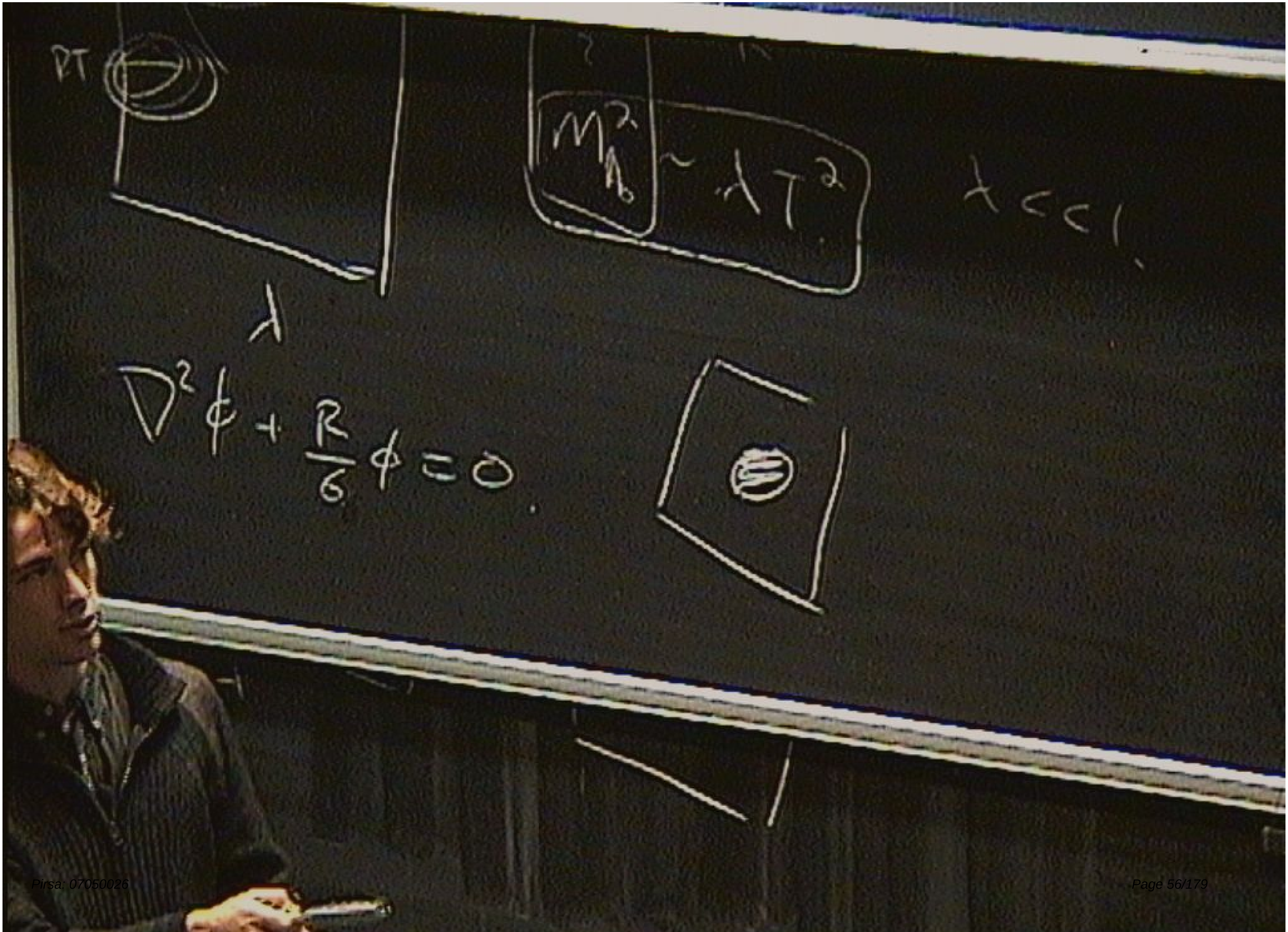
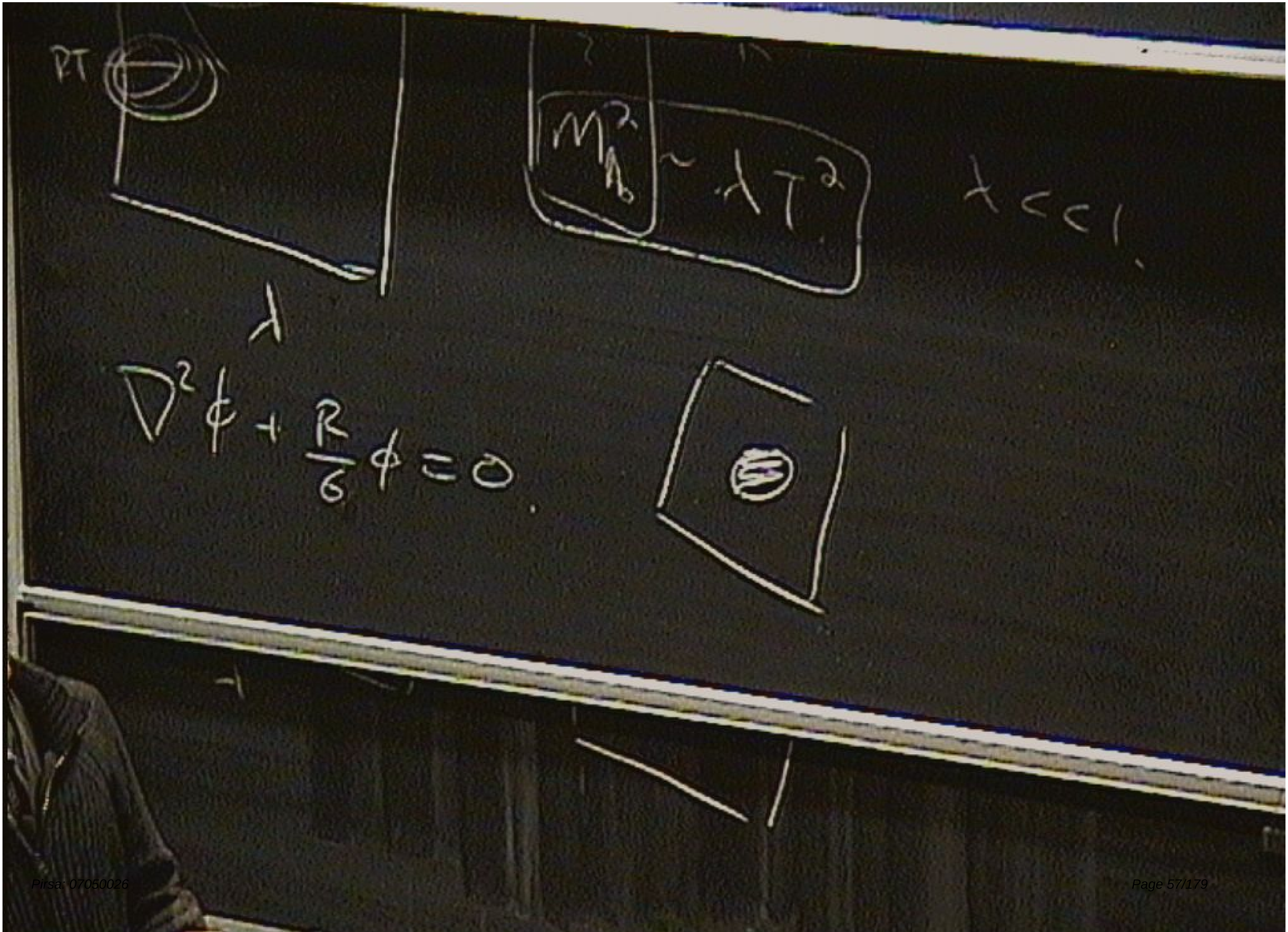

PT
$m_h^2 \sim \lambda T^2$
$\lambda \ll 1$
λ
$D^2\phi + \frac{R}{6}\phi = 0$


PT
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Aside: scalars and emergent geometry

Berenstein (2005)

- A very similar effective potential has been conjectured recently to describe the $1/8^{\text{th}}$ BPS sector of $\mathcal{N}=4$ SYM theory at zero temperature and strong coupling.
- Berenstein considers a matrix model for the six scalar fields of the theory

$$S = \sum_{p} \phi_p^2 - \frac{1}{2} \sum_{p \neq q} \log |\phi_p - \phi_q|^2$$

- In the ground state of this model, the eigenvalues form an S^5. This is to be identified with the S^5 in the dual geometry.
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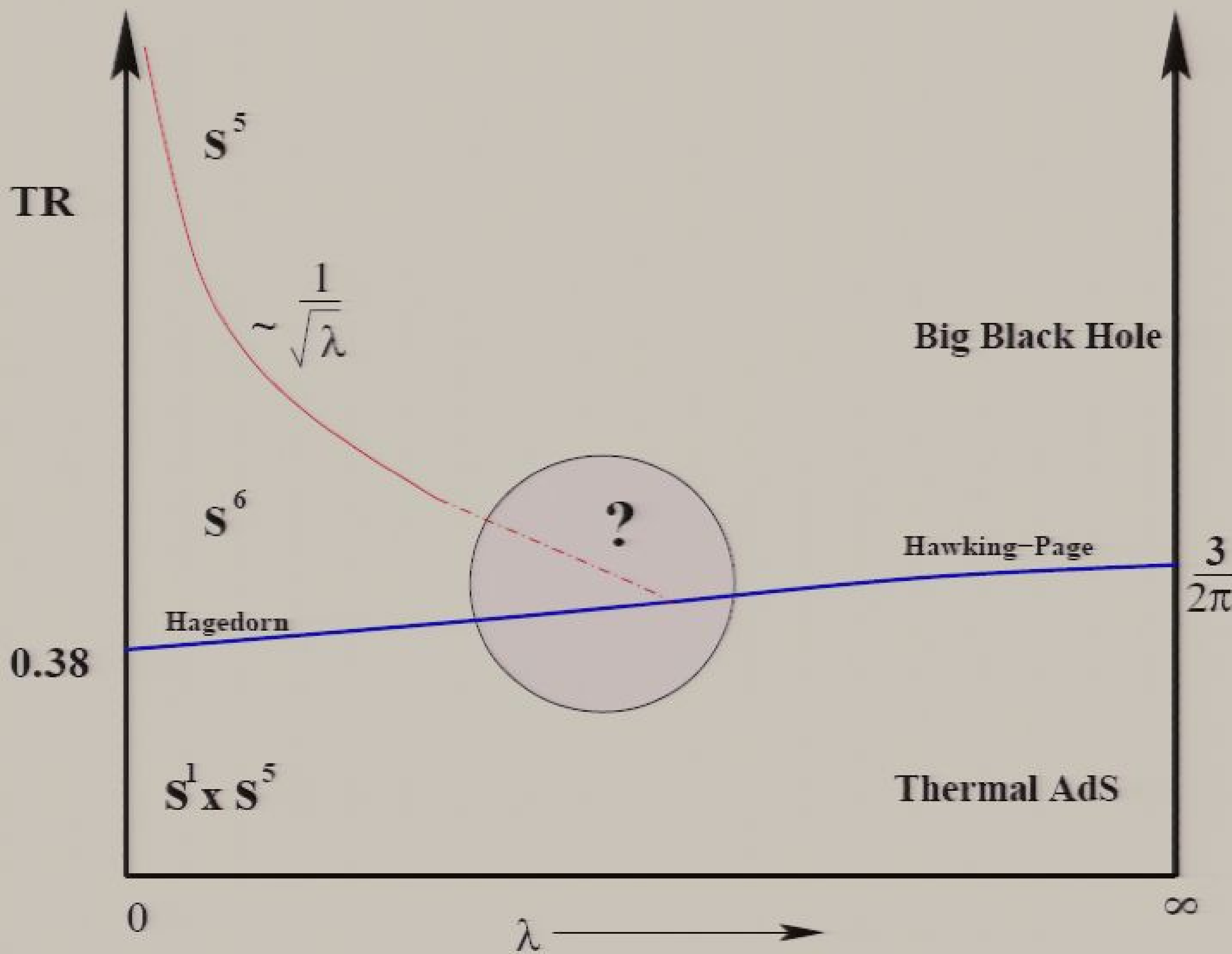
Summary of results

Gürsoy-Hartnoll-Hollowood-Kumar (2007)



The one loop effective potential

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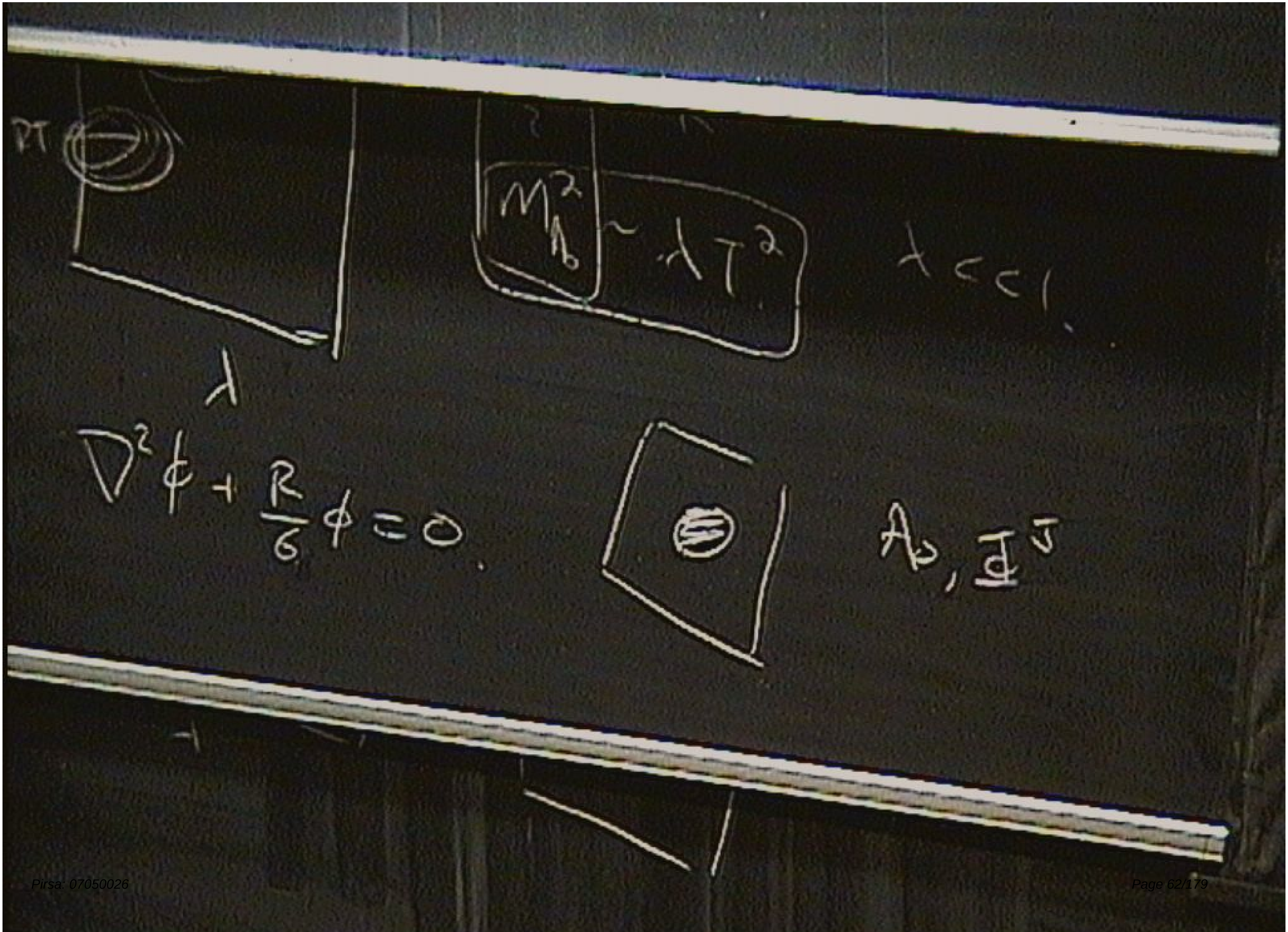
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λ
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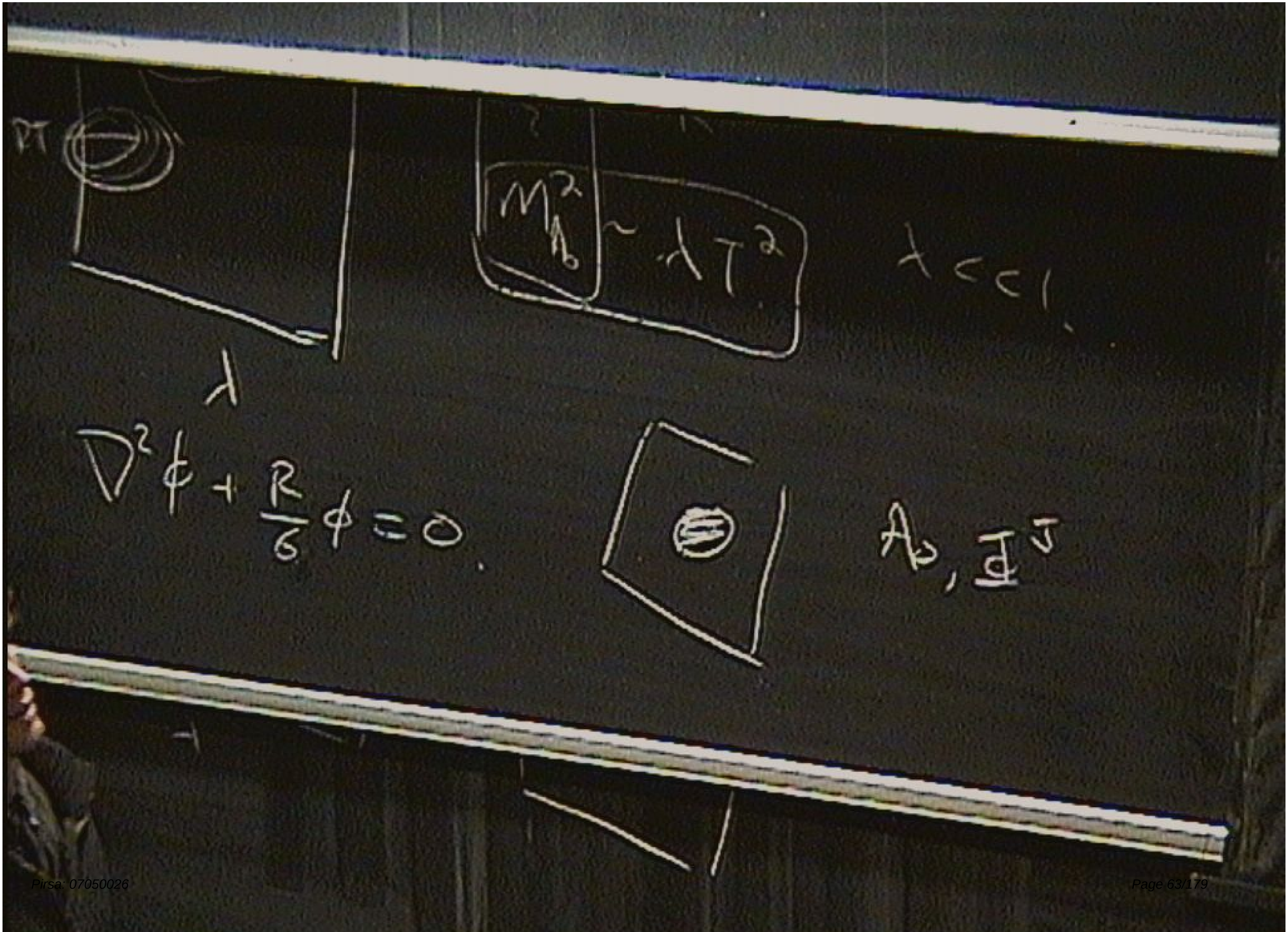
A_b, Φ^J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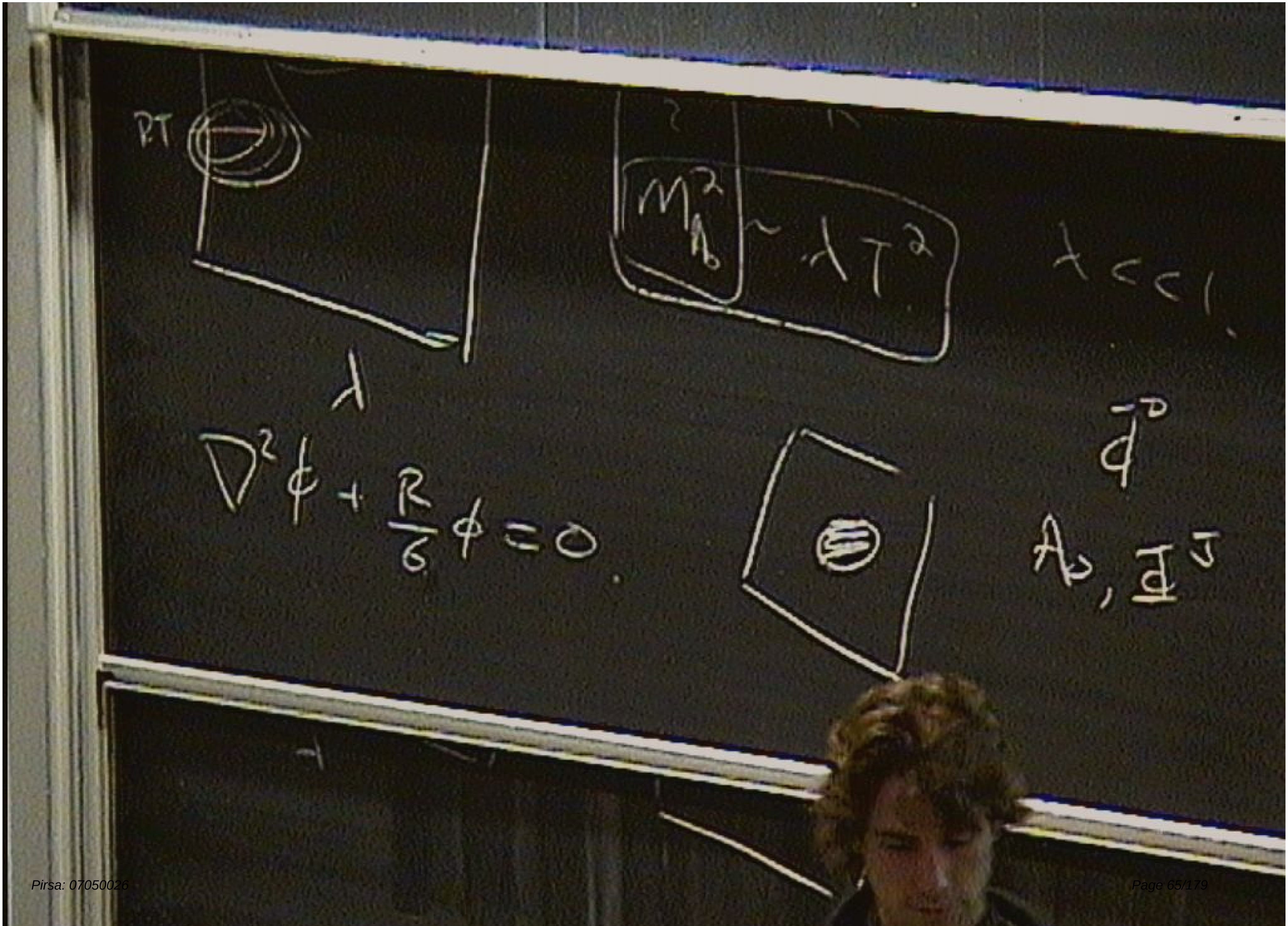
P.T
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λ
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φ⃗
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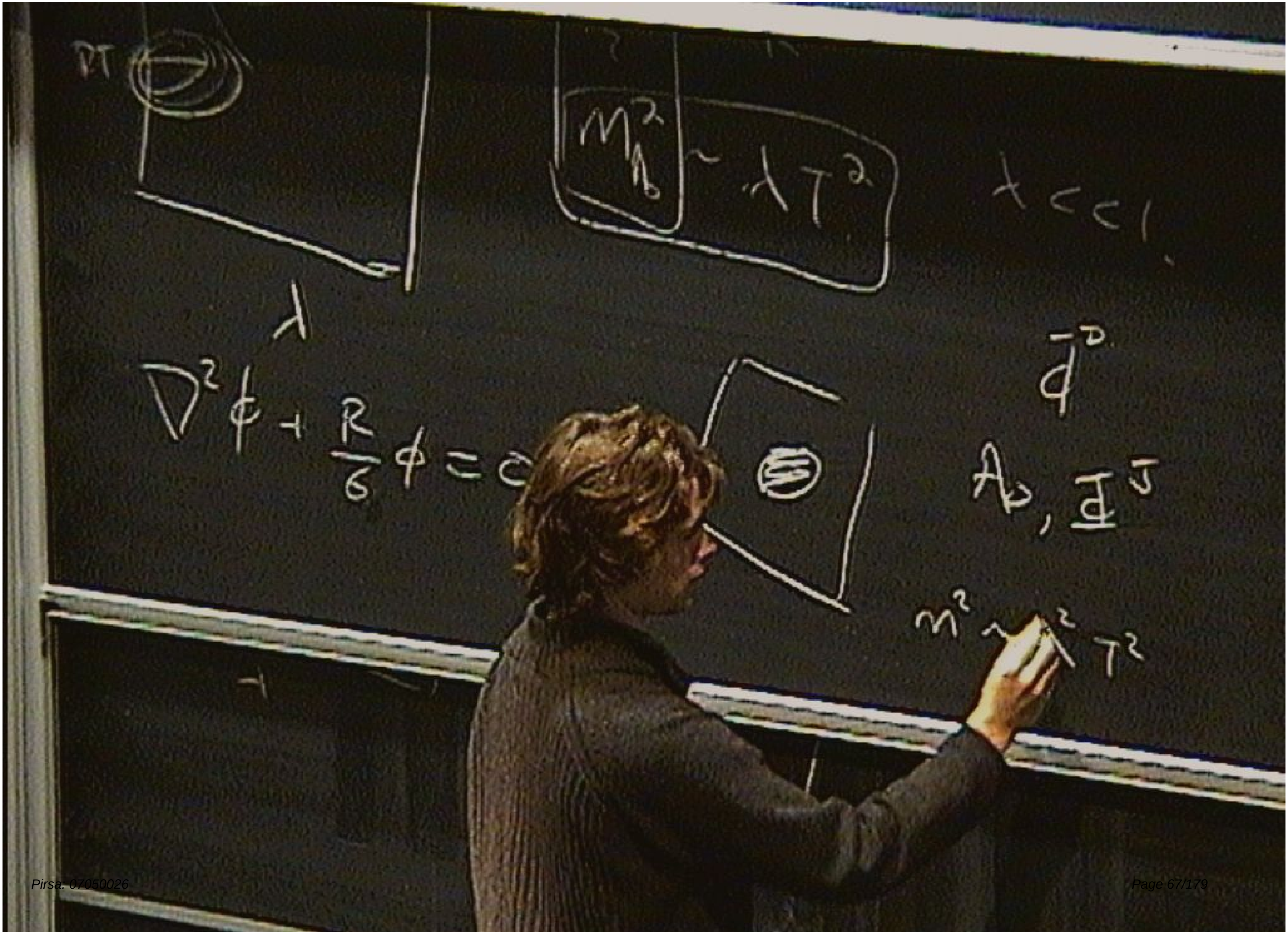
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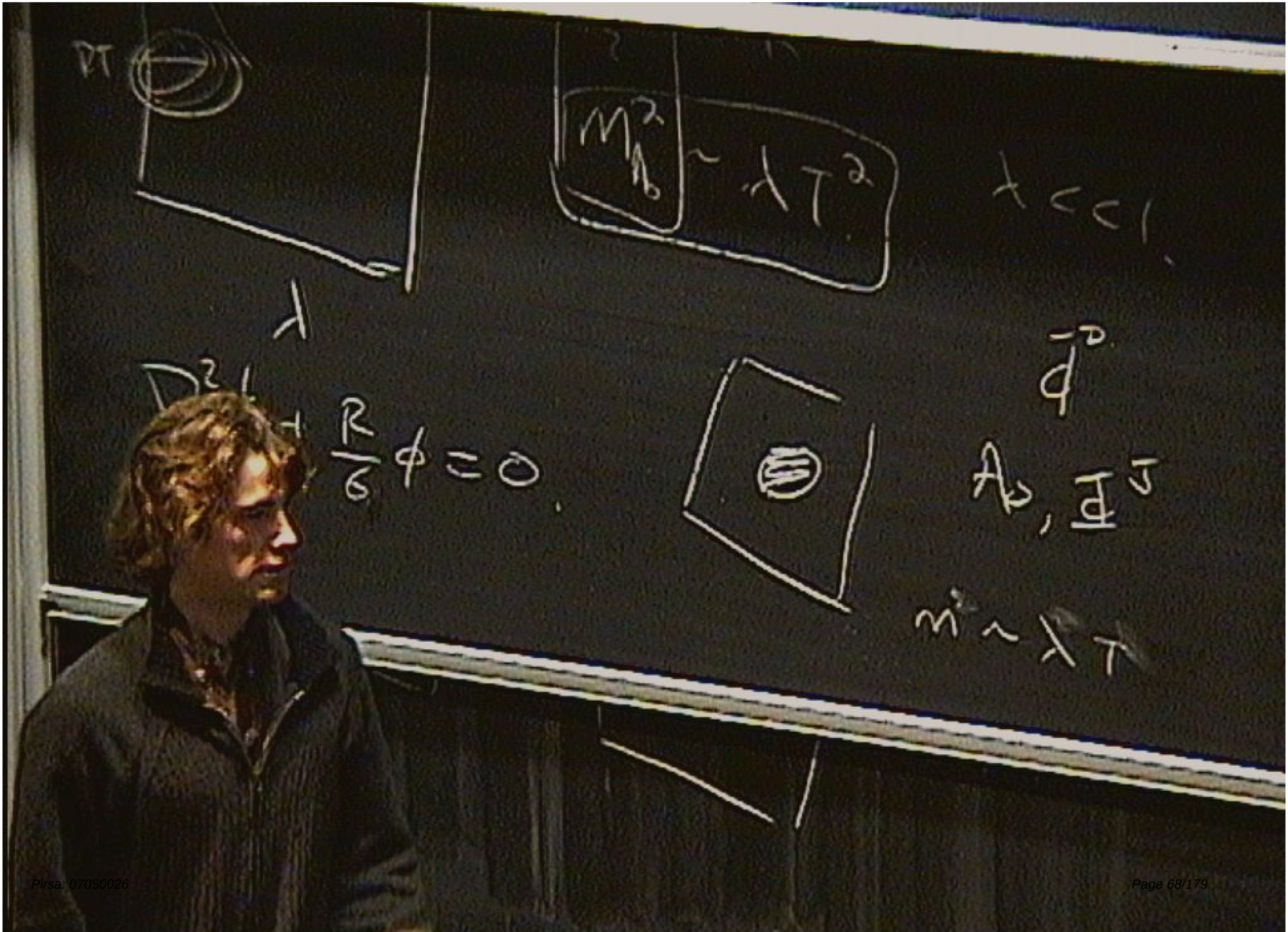
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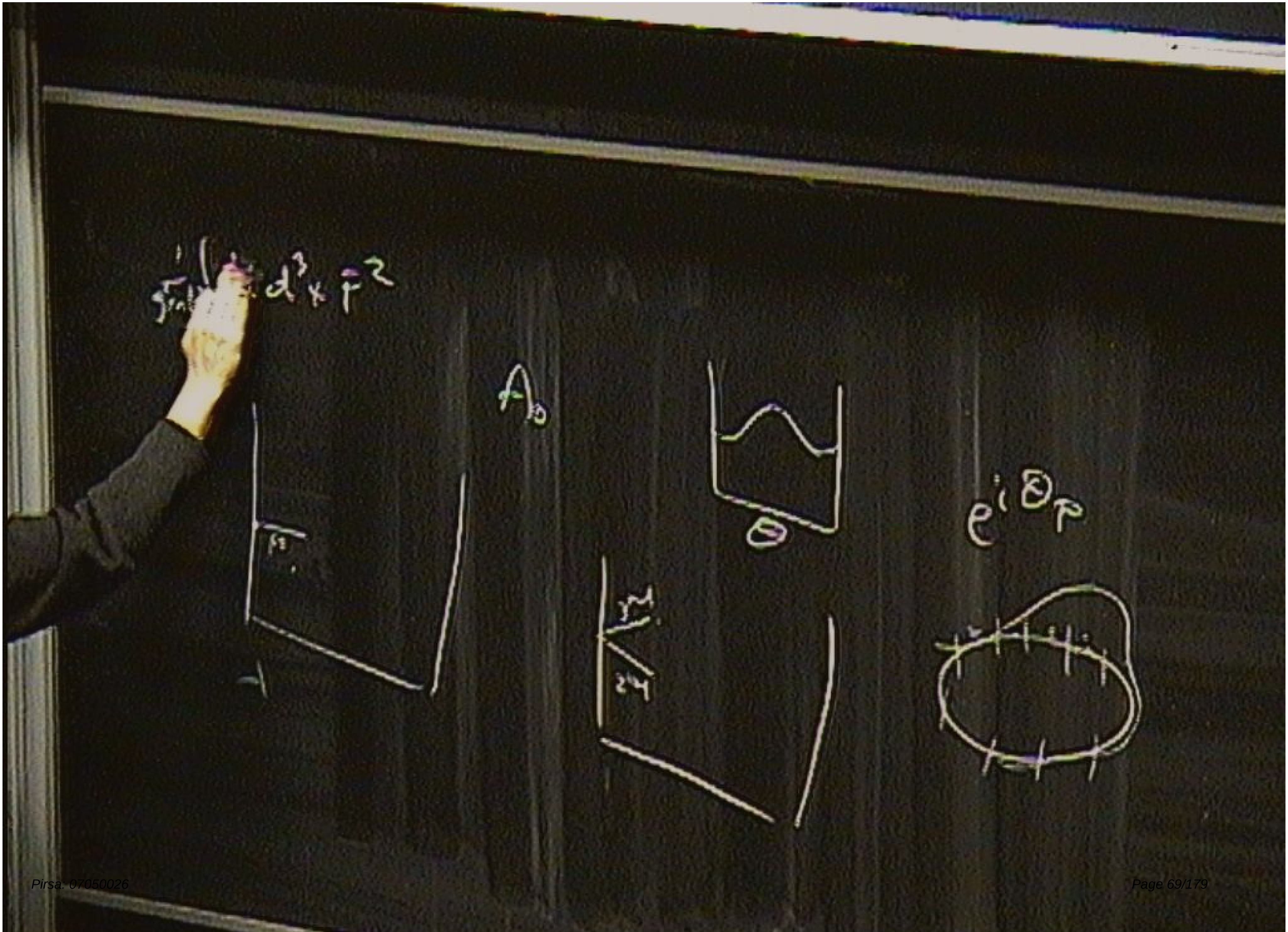
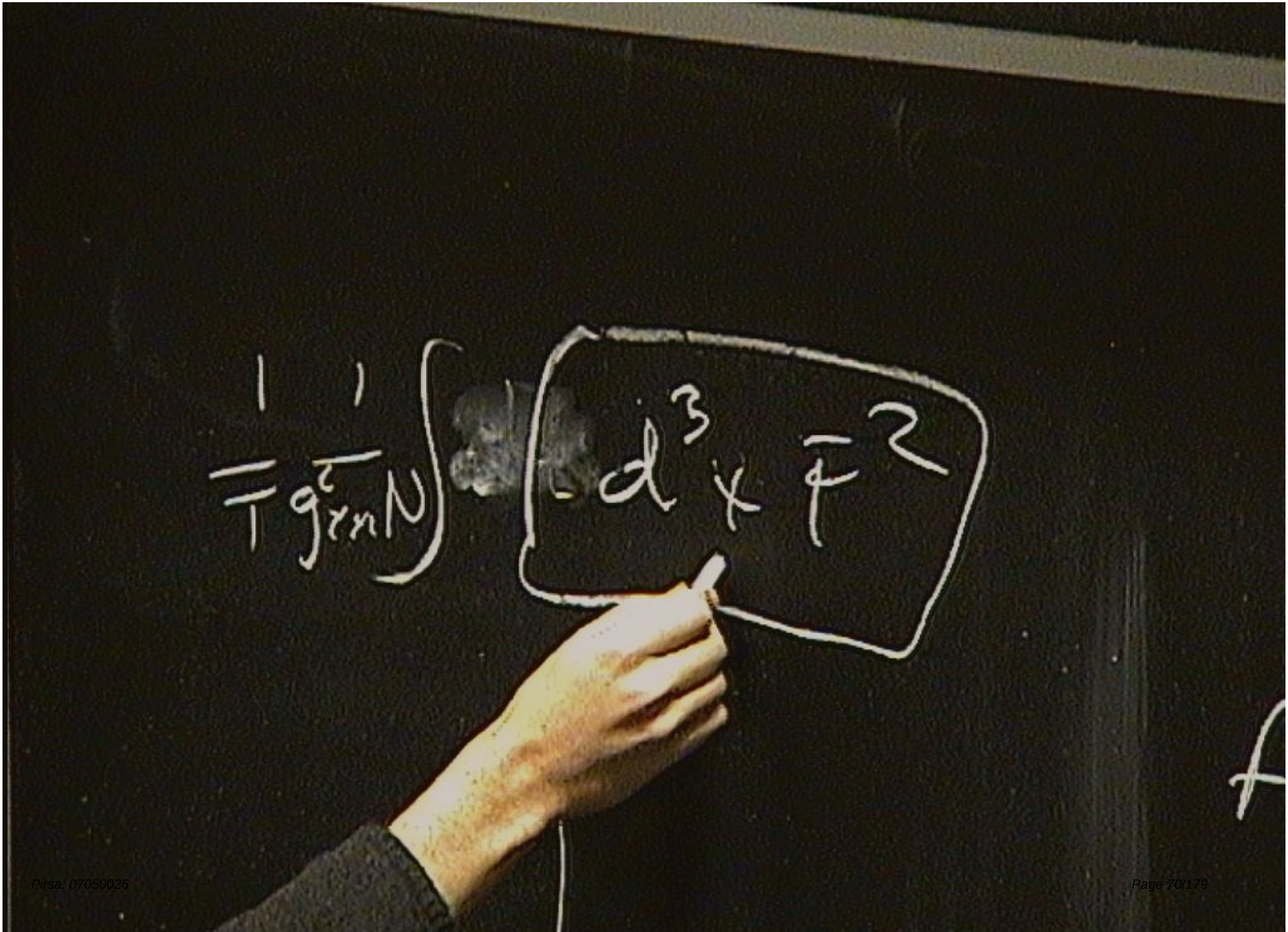
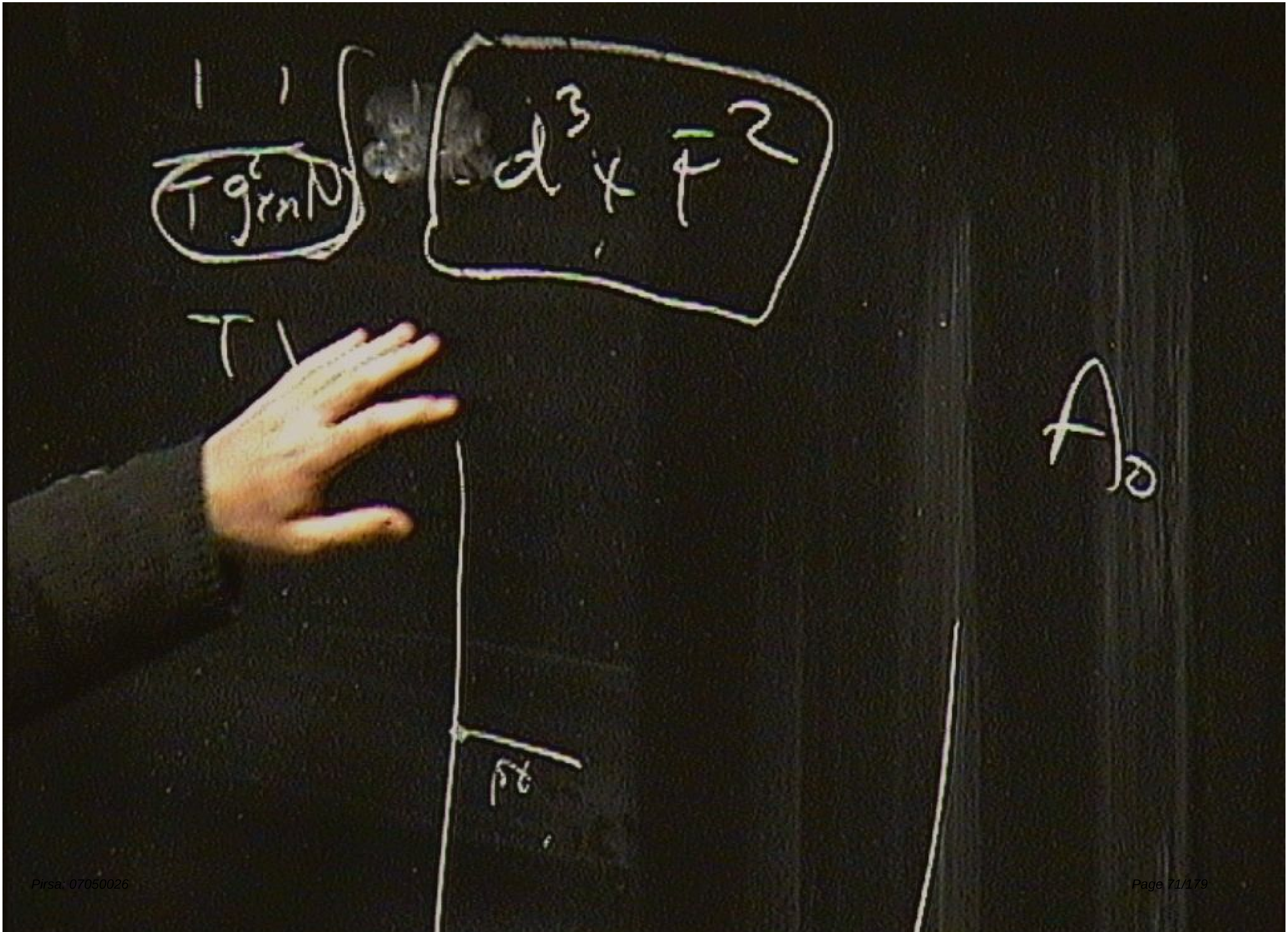
R/6 φ = 0
A_b, Φ^J
m^2 ~ λT


A_0
Θ
$e^{i\Theta_P}$

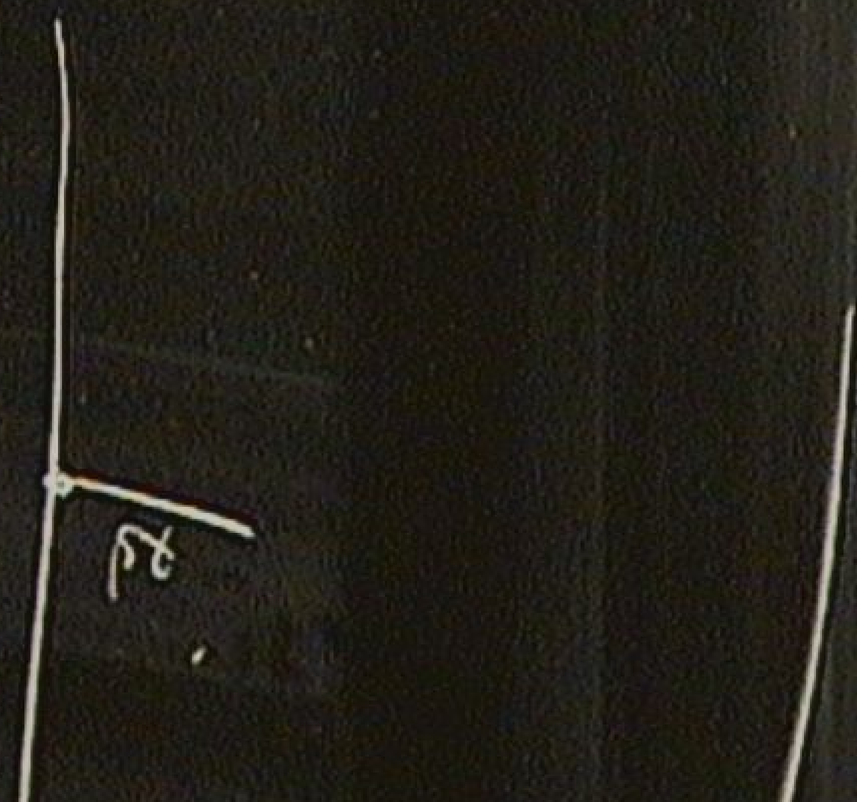

$\int d^3x\, F^2$


d^3x F^2
A_0

$$\frac{1}{T g_{YM}^2 N} \int d^3x\, F^2$$

$T\lambda$

A_0



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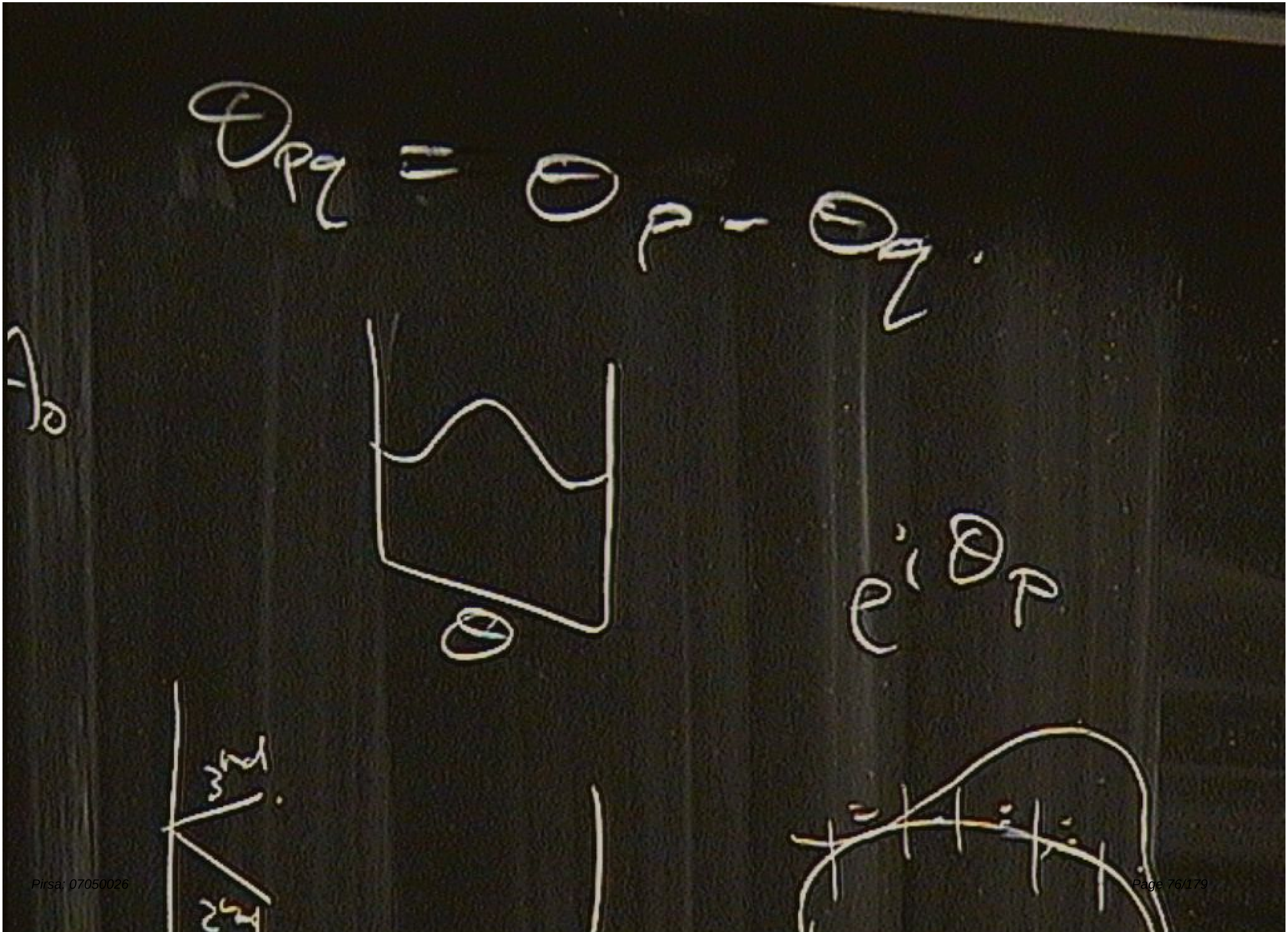
Θpq = Θp − Θq
Θ
e^{iΘp}
3rd
2nd

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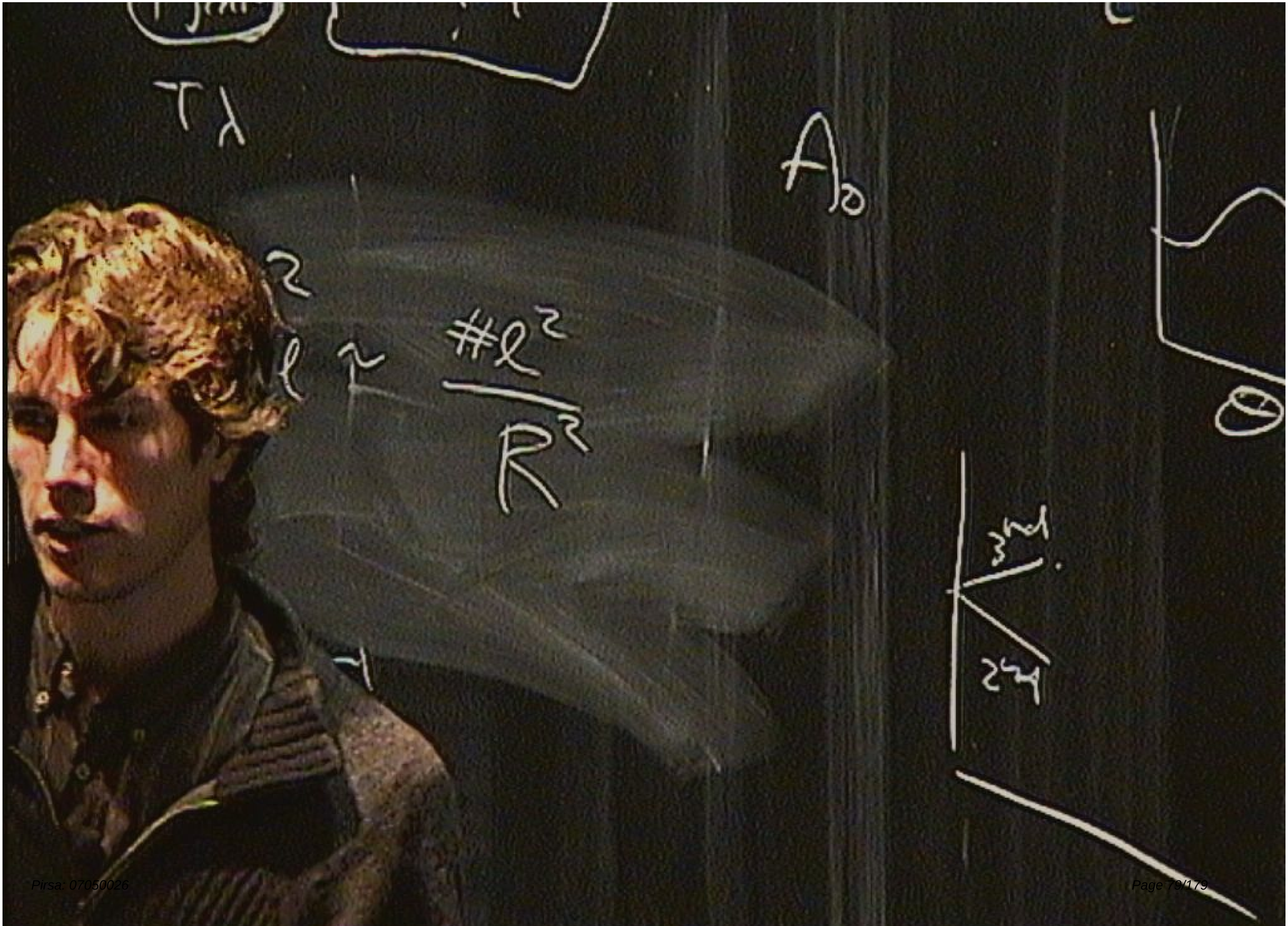
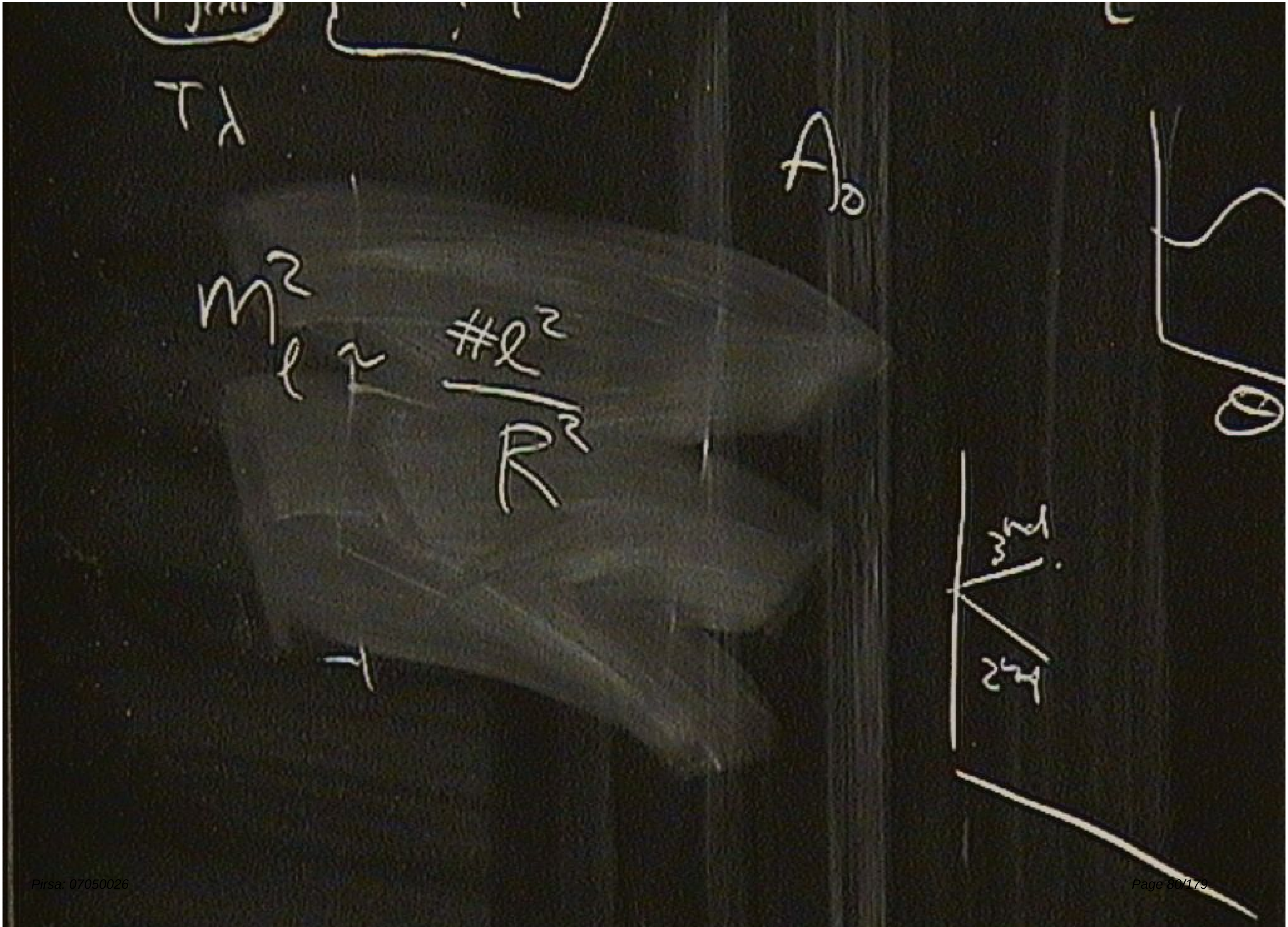

Tλ
A0
#ℓ²
R²
3rd
2nd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3rd
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- For $RT \ll \lambda^{-1/2}$, expect condensate of scalars will be small compared to the θ_p condensate.

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$$S = \lambda^0 S^{(0)}(\theta) + \lambda \left(S^{(1)}(\theta, \tilde{\mathbf{x}}) + S_{\text{2-loop}}(\theta) \right) + \mathcal{O}(\lambda^2) \ ,$$

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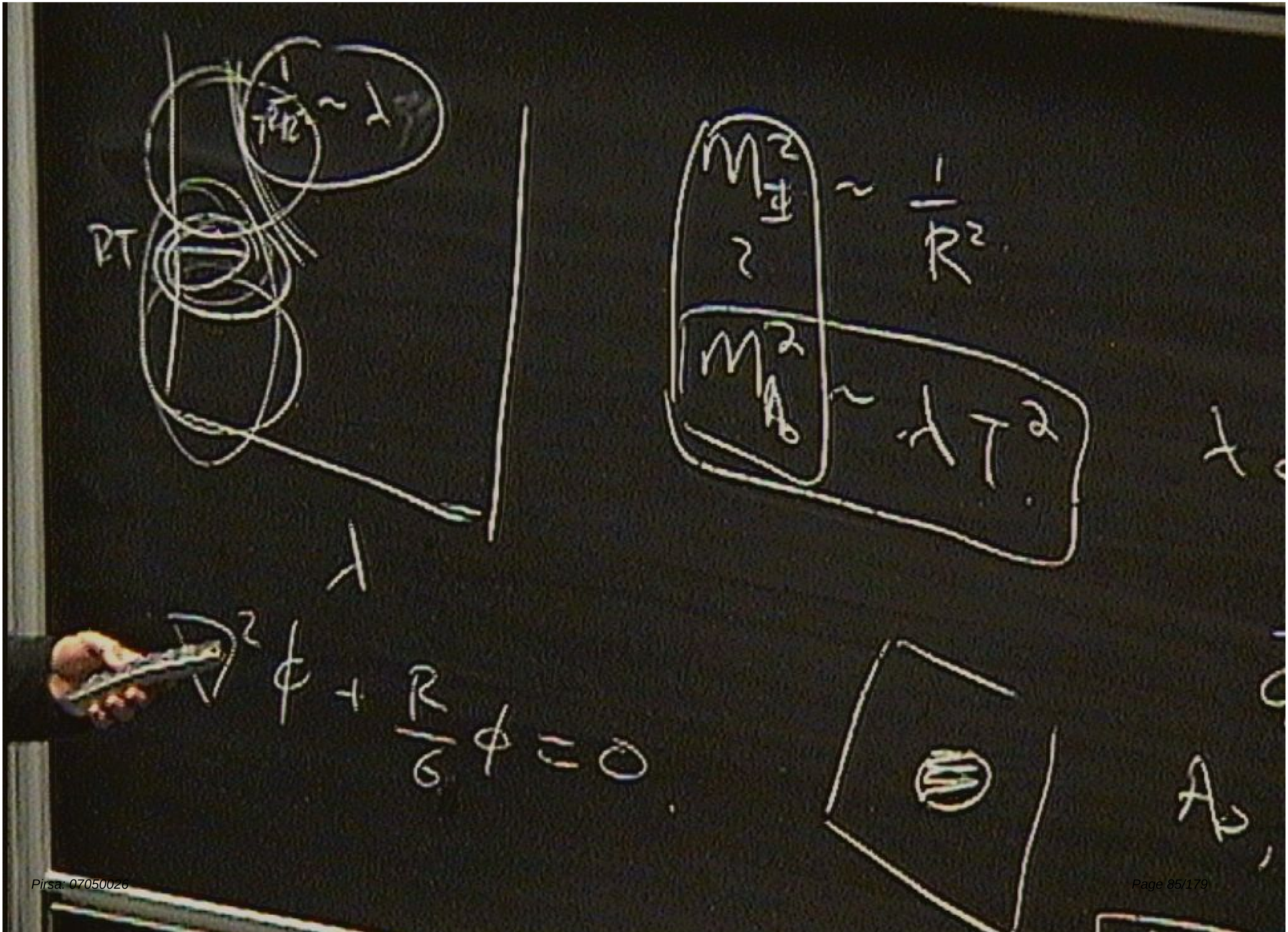
RT
λ
M² ~ 1/R²
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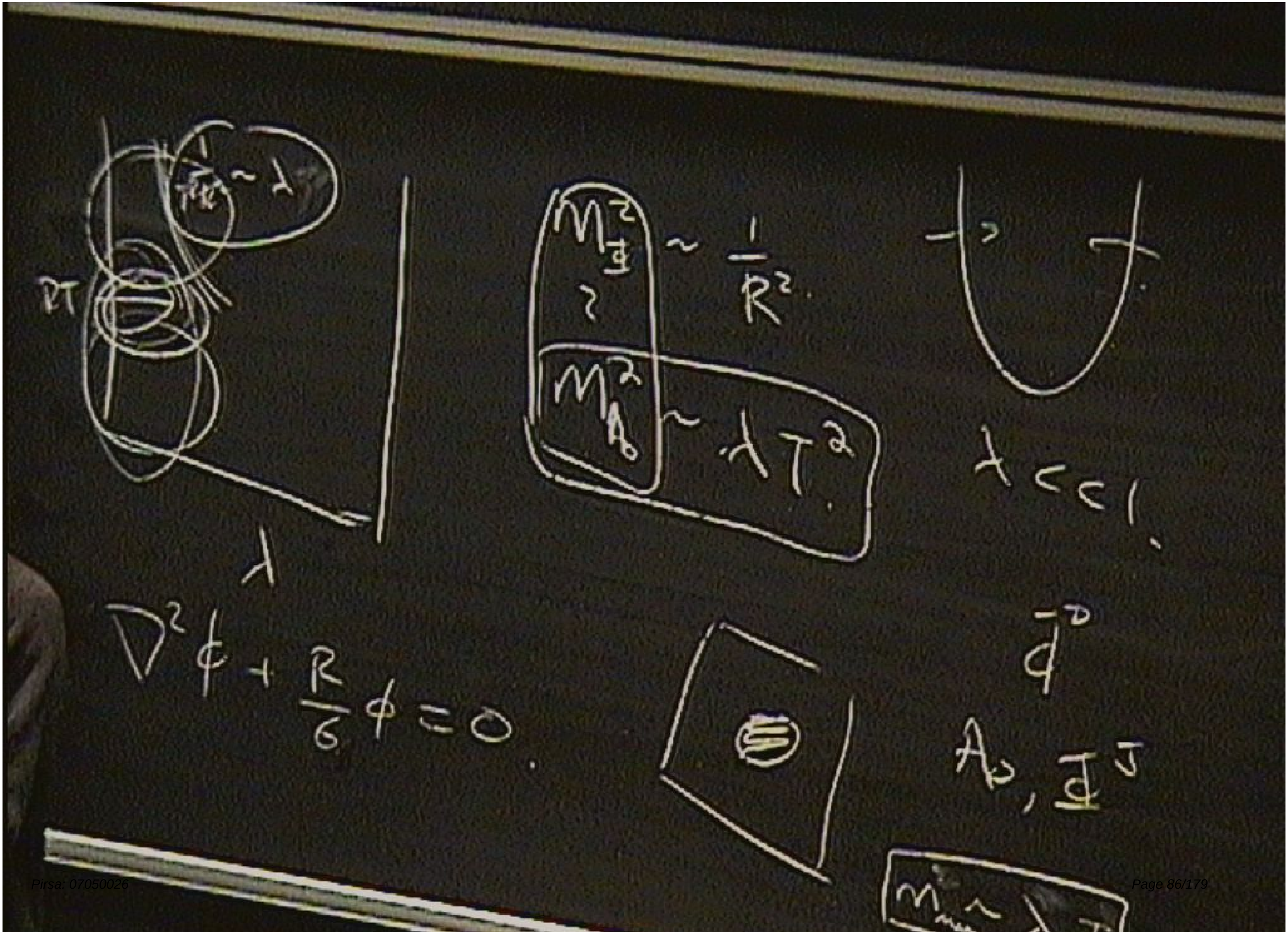
M_Φ^2 ~ 1/R^2
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λ
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A_0, Φ^J

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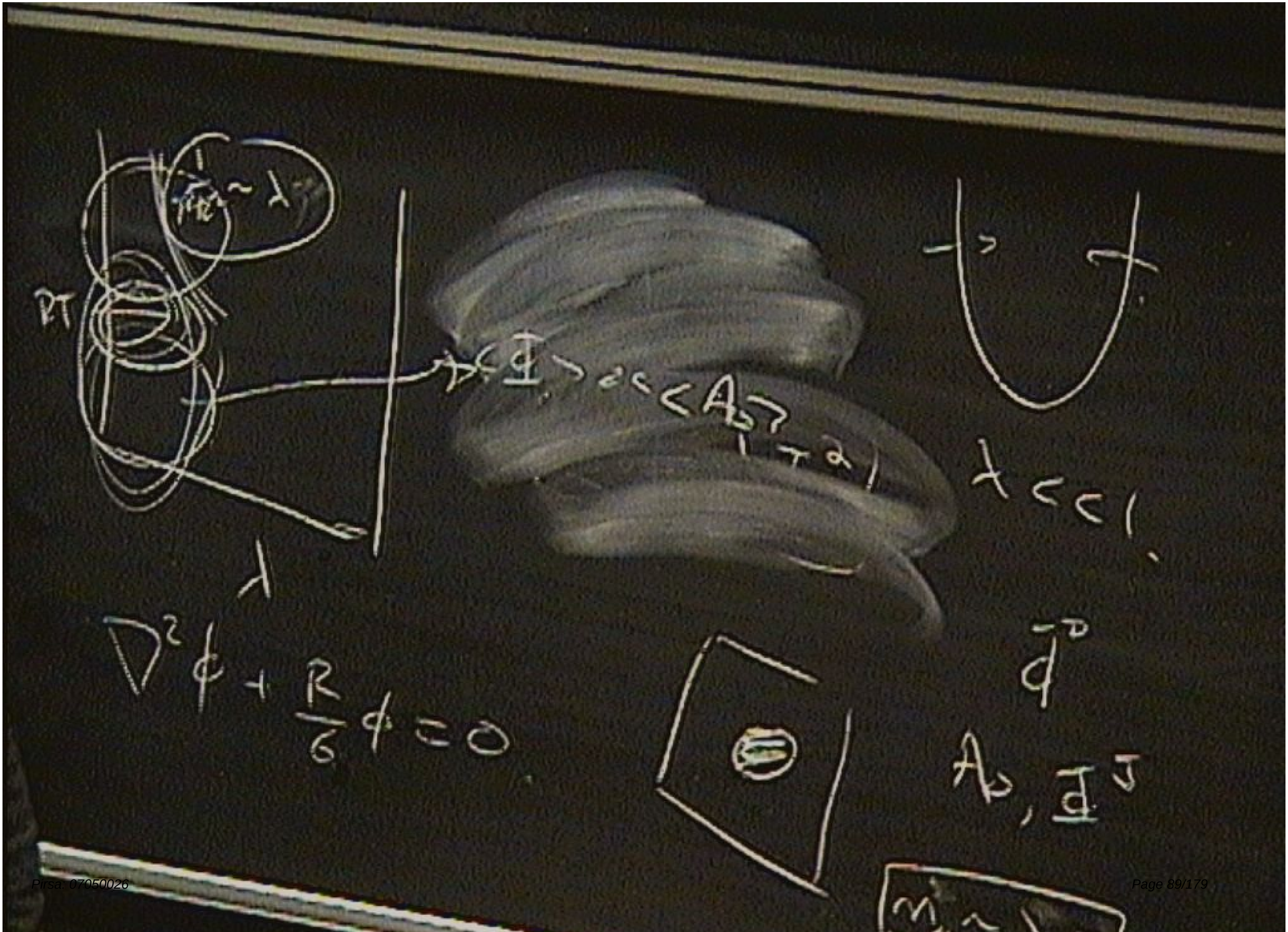
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- Continuum limit as $N \to \infty$

$$\frac{1}{N}\sum_{p=1}^{N} \to \int d^6x d\theta\, \rho(\mathbf{x},\theta) = 1\ .$$

- Action becomes

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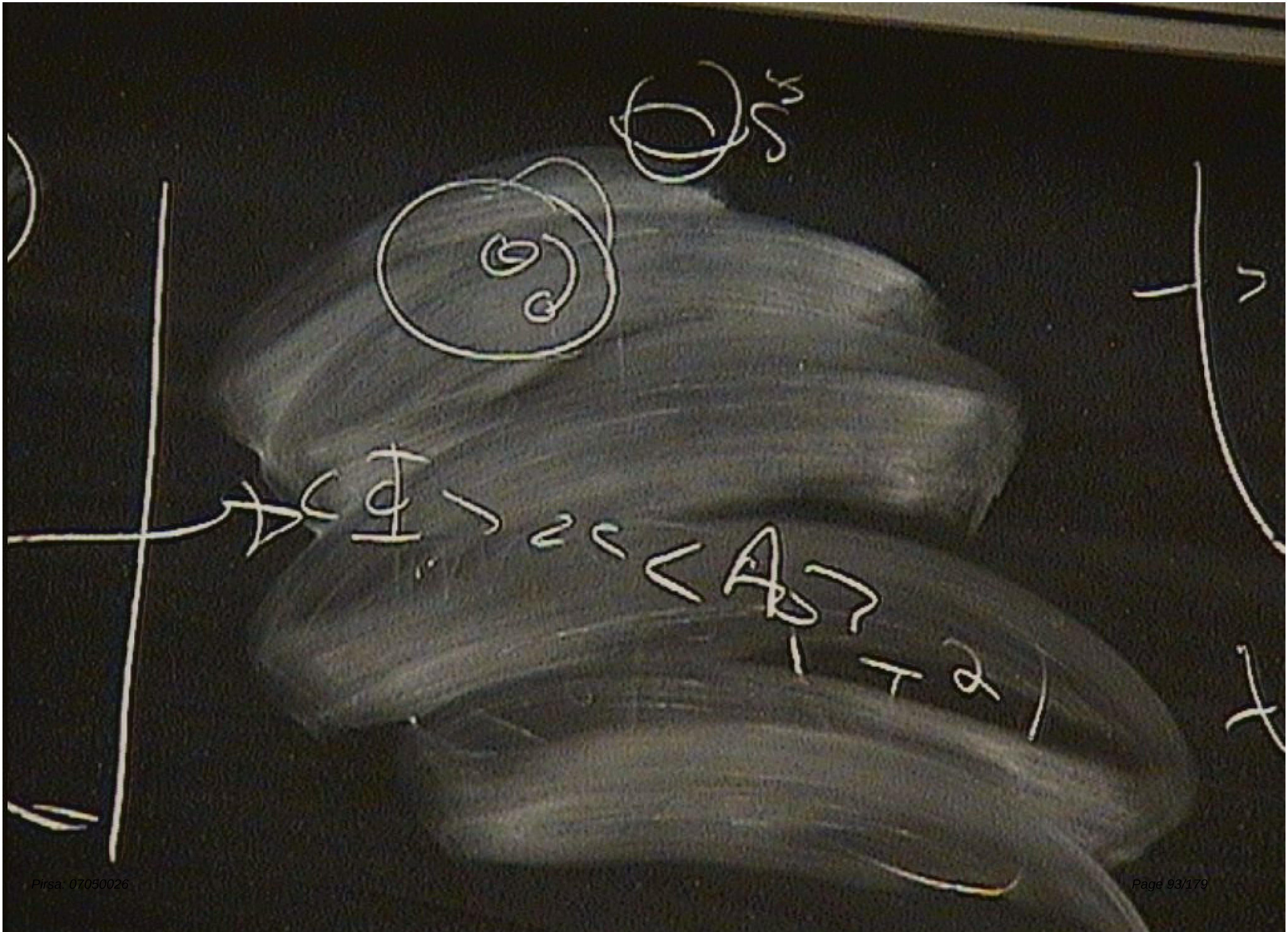

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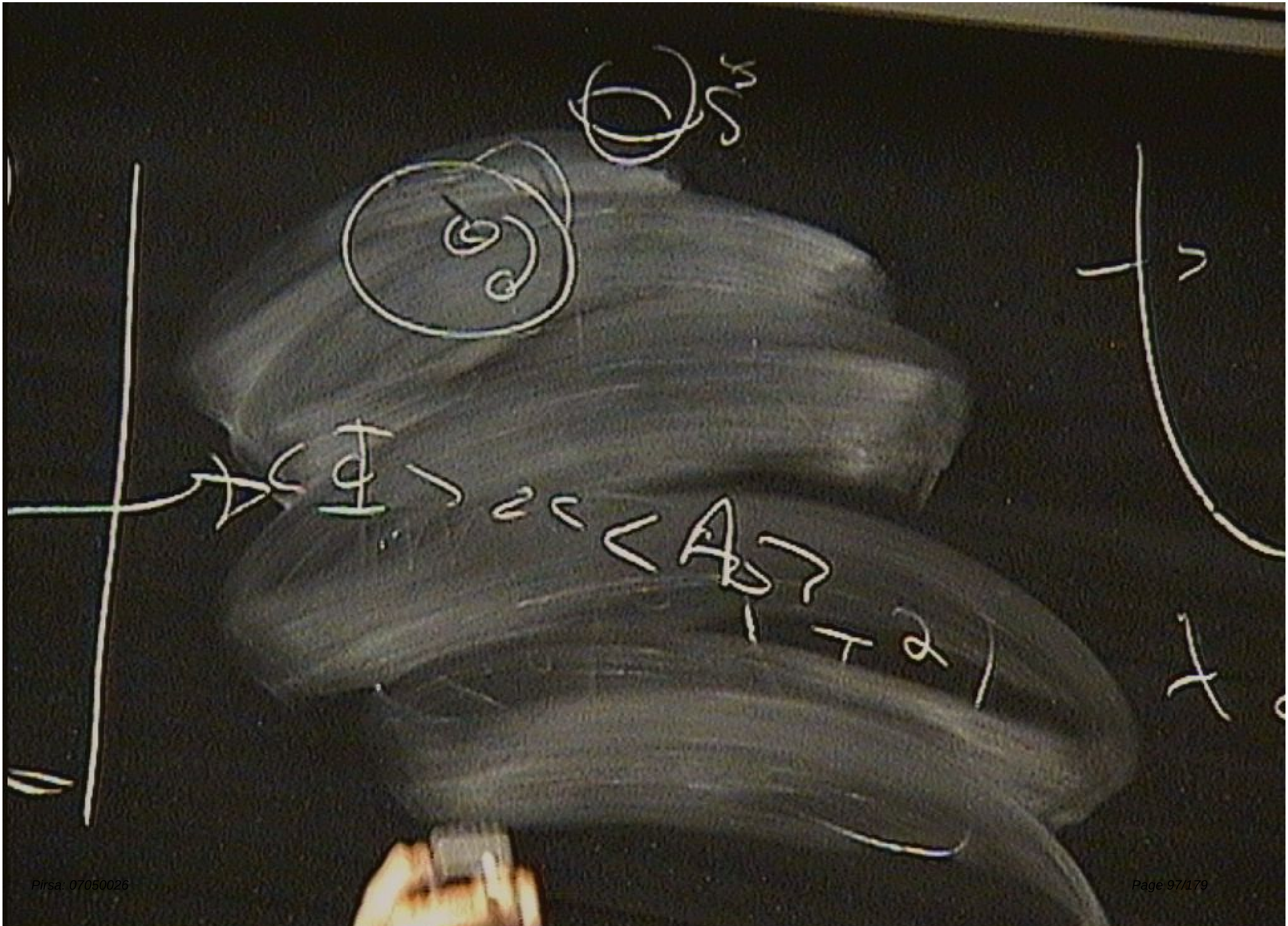
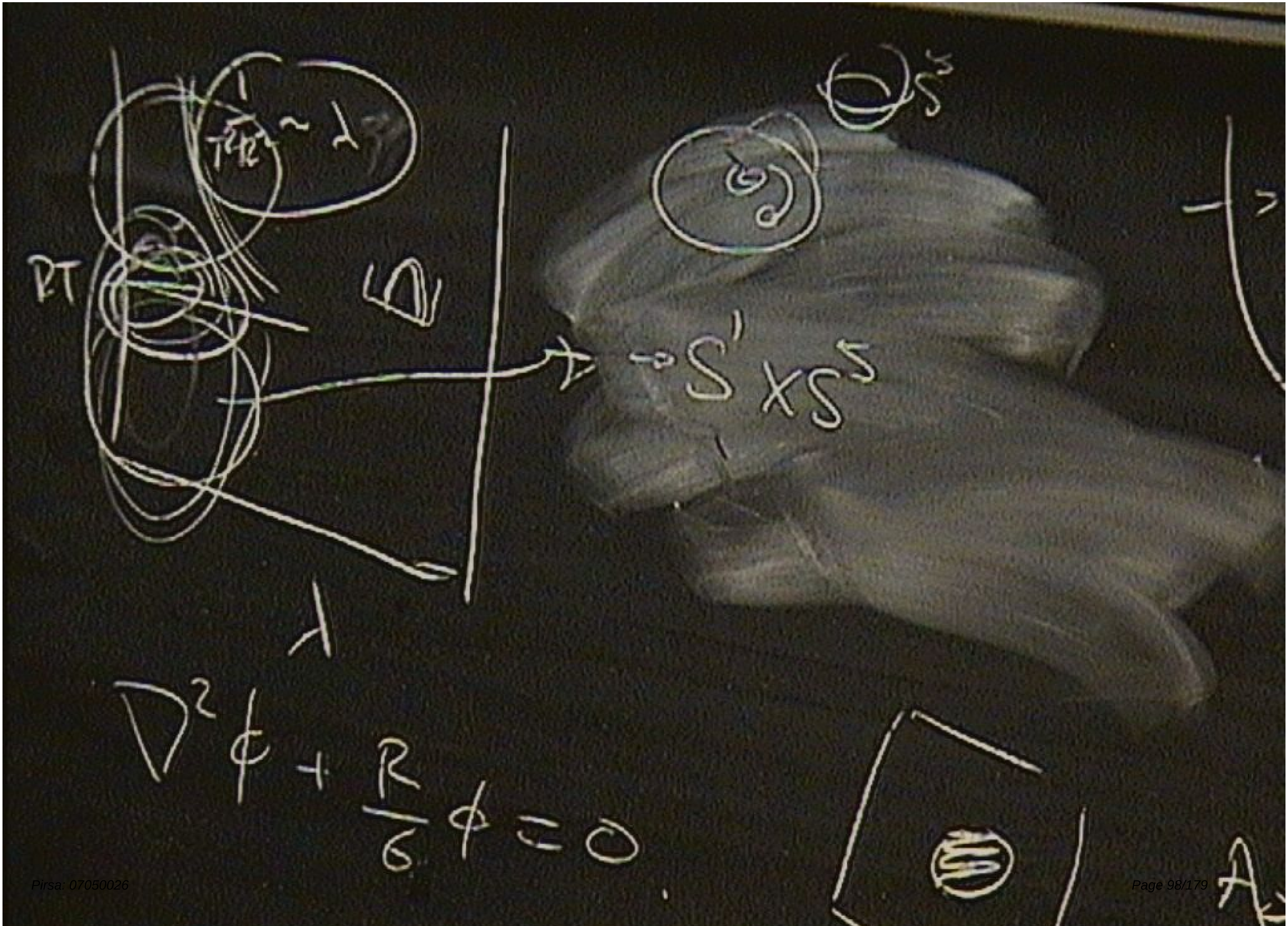
- Find the solution

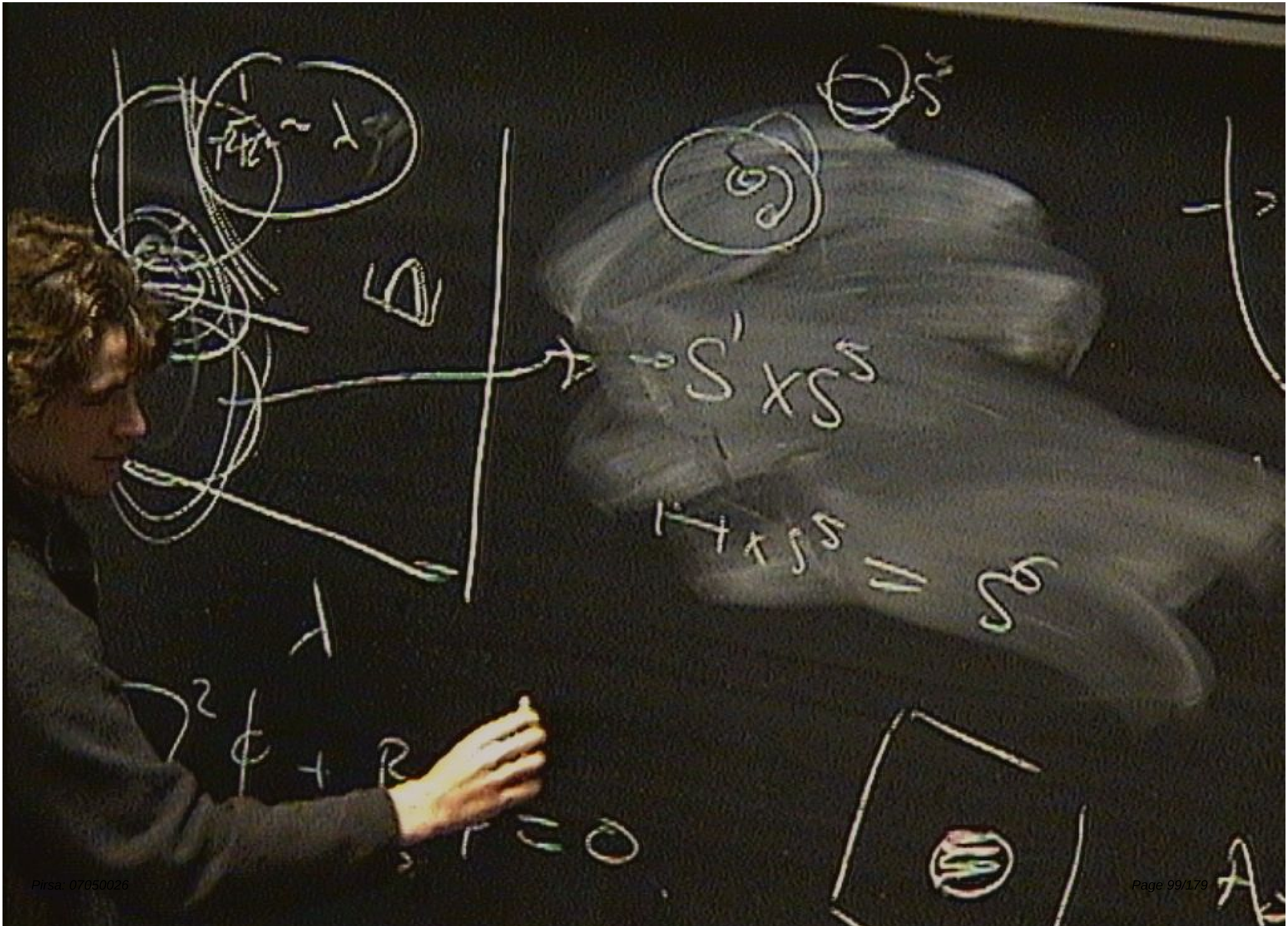
$$r(\theta) = \frac{2048}{945\pi^2}\frac{1}{RT}\rho(\theta)$$

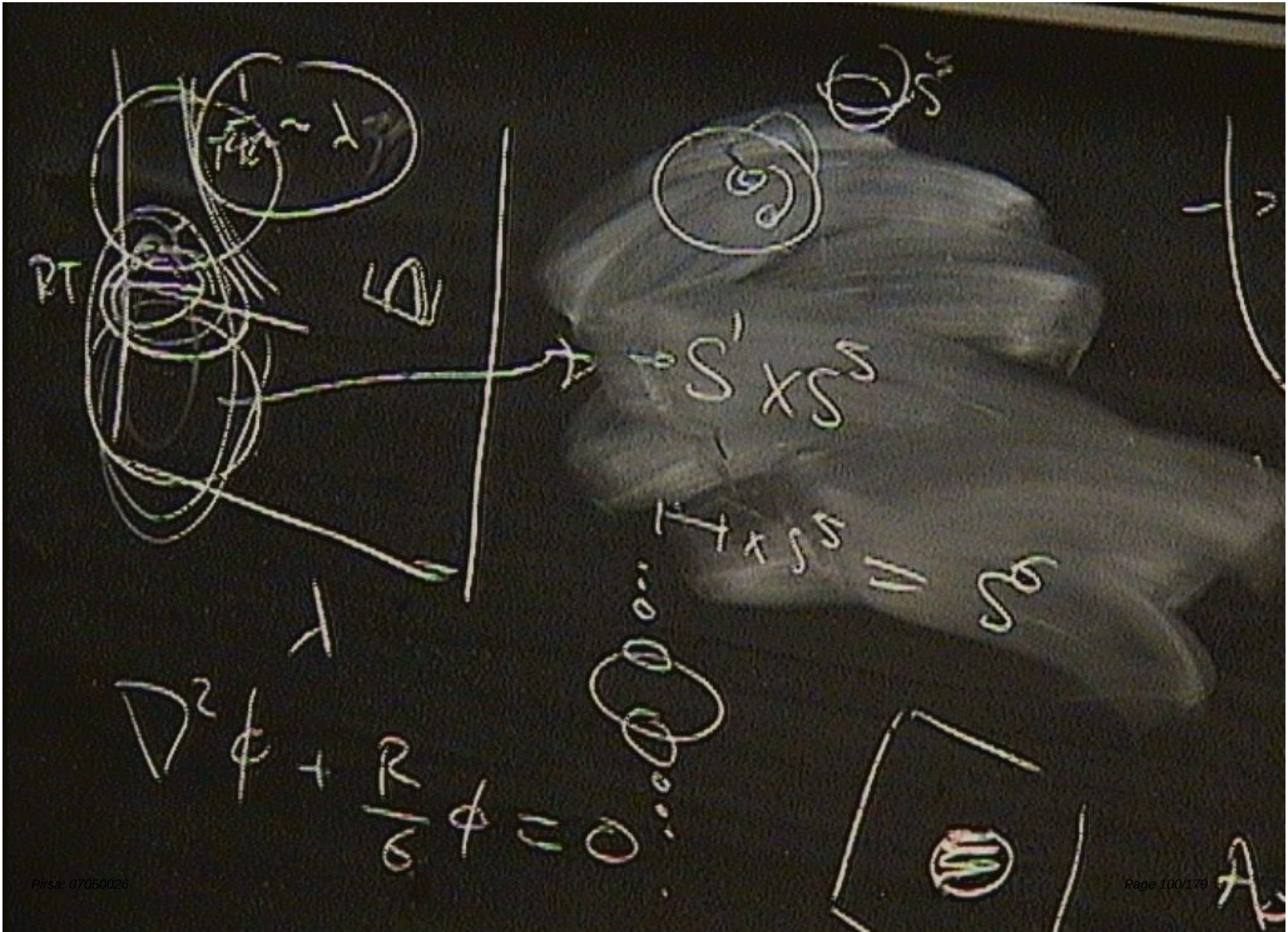
- Below the transition, $\rho(\theta)$ uniform (i.e. constant) and hence full solution: $S^1 \times S^5$.
- Above the transition, $\rho(\theta)$ is gapped, and hence full solution is topologically: S^6.
- At temperatures well above the transition, $1 \ll RT \ll \lambda^{-1/2}$, can show that $\rho(\theta) = 4\pi(TR)^3\sqrt{\theta_0^2 - \theta^2}$, which implies the full solution is an ellipsoid

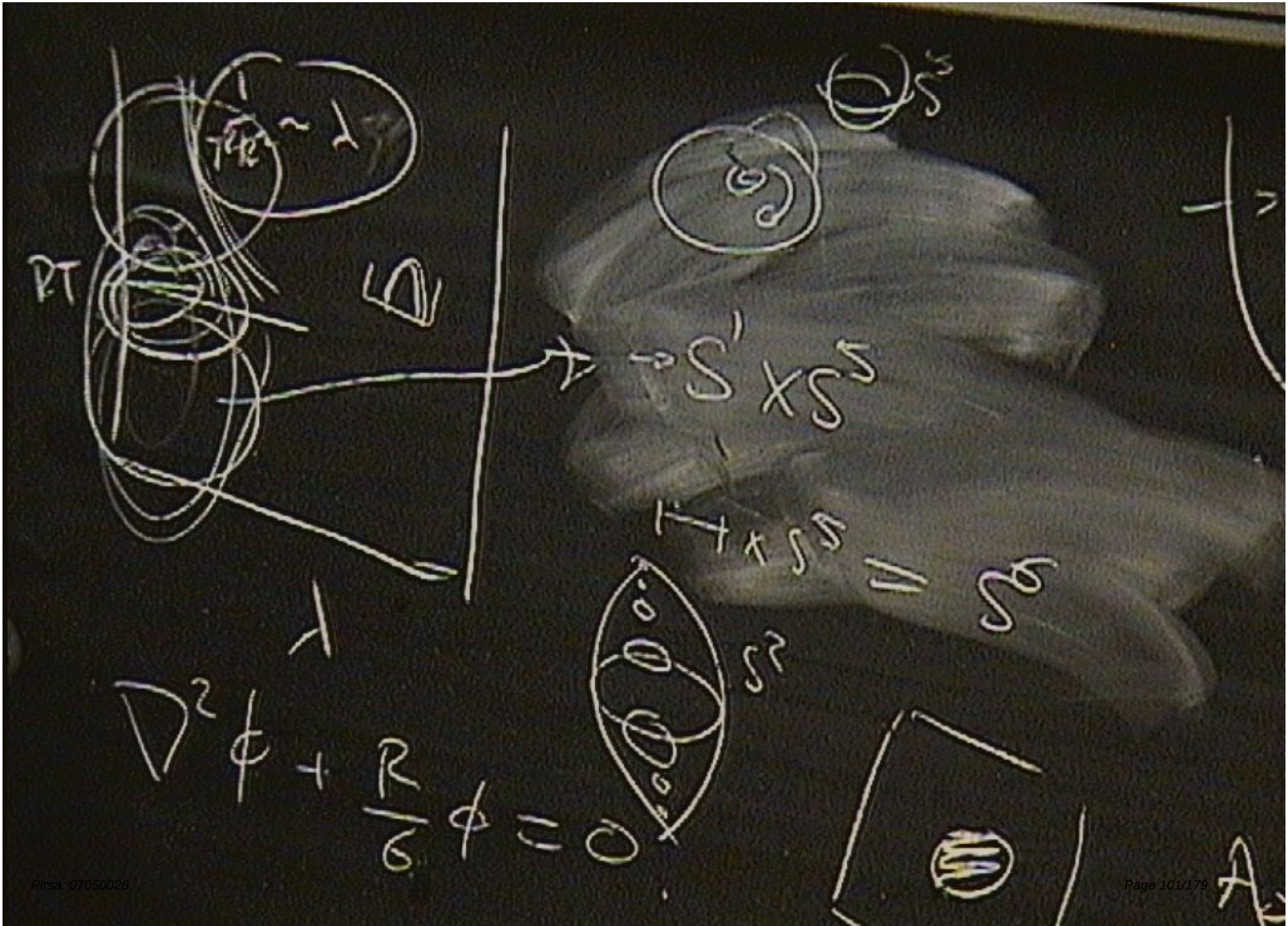
$$\frac{\pi^4 945^2}{8 \times 2048^2 \lambda^2 RT}\mathbf{x}^2 + 2\pi^2(TR)^3\theta^2 = 1\ .$$




RT
S'xS⁵
λ
D²φ + R/6 φ = 0




RT
S' x S5
D2φ + R/6 φ = 0


PT
S^5
$\to S^1 \times S^5$
S^2
$D^2\phi + \frac{R}{6}\phi = 0$

- Find the solution

$$r(\theta) = \frac{2048}{945\pi^2}\frac{1}{RT}\rho(\theta)$$

- Below the transition, $\rho(\theta)$ uniform (i.e. constant) and hence full solution: $S^1 \times S^5$.
- Above the transition, $\rho(\theta)$ is gapped, and hence full solution is topologically: S^6.
- At temperatures well above the transition, $1 \ll RT \ll \lambda^{-1/2}$, can show that $\rho(\theta) = 4\pi(TR)^3\sqrt{\theta_0^2 - \theta^2}$, which implies the full solution is an ellipsoid

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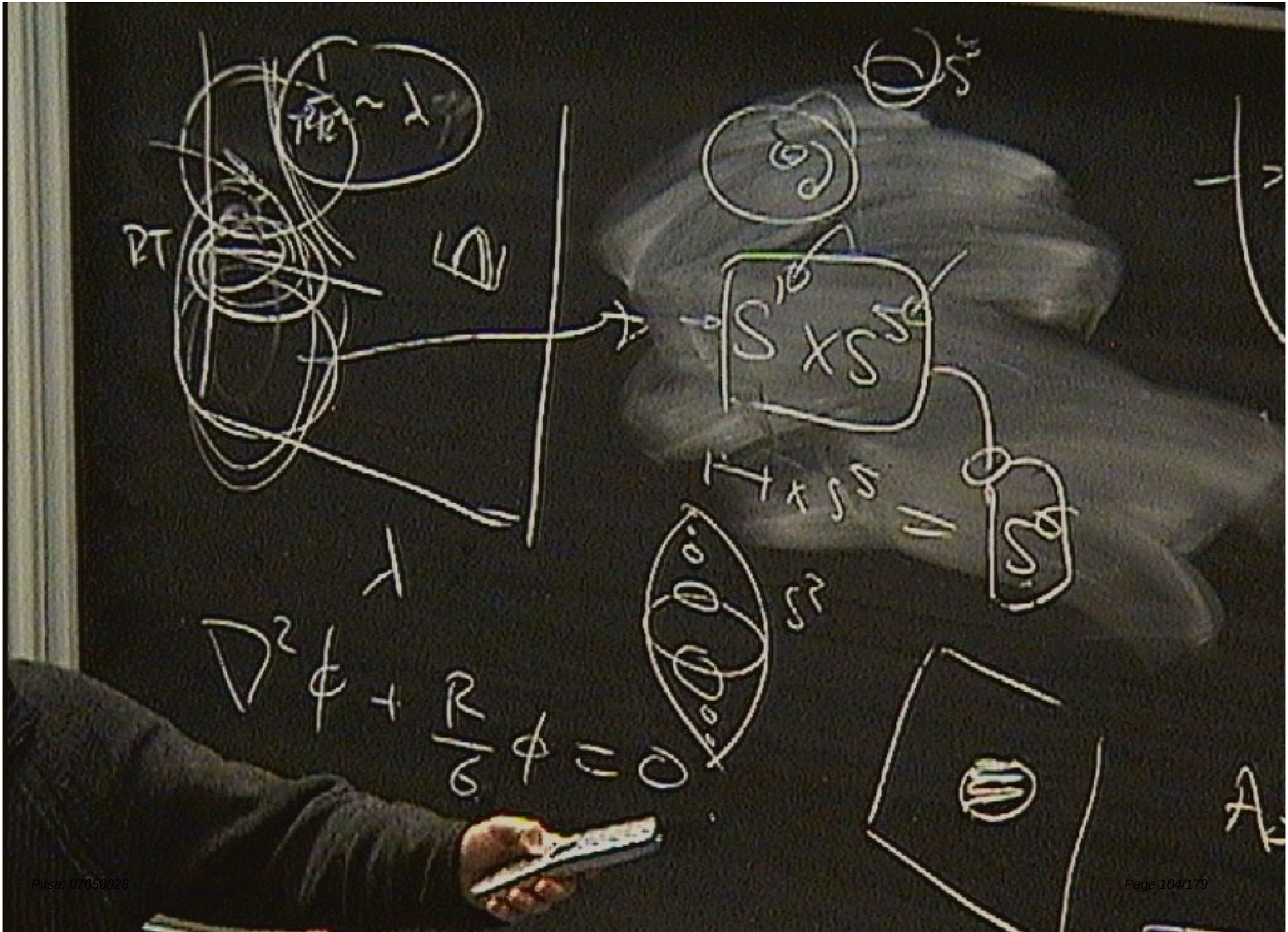
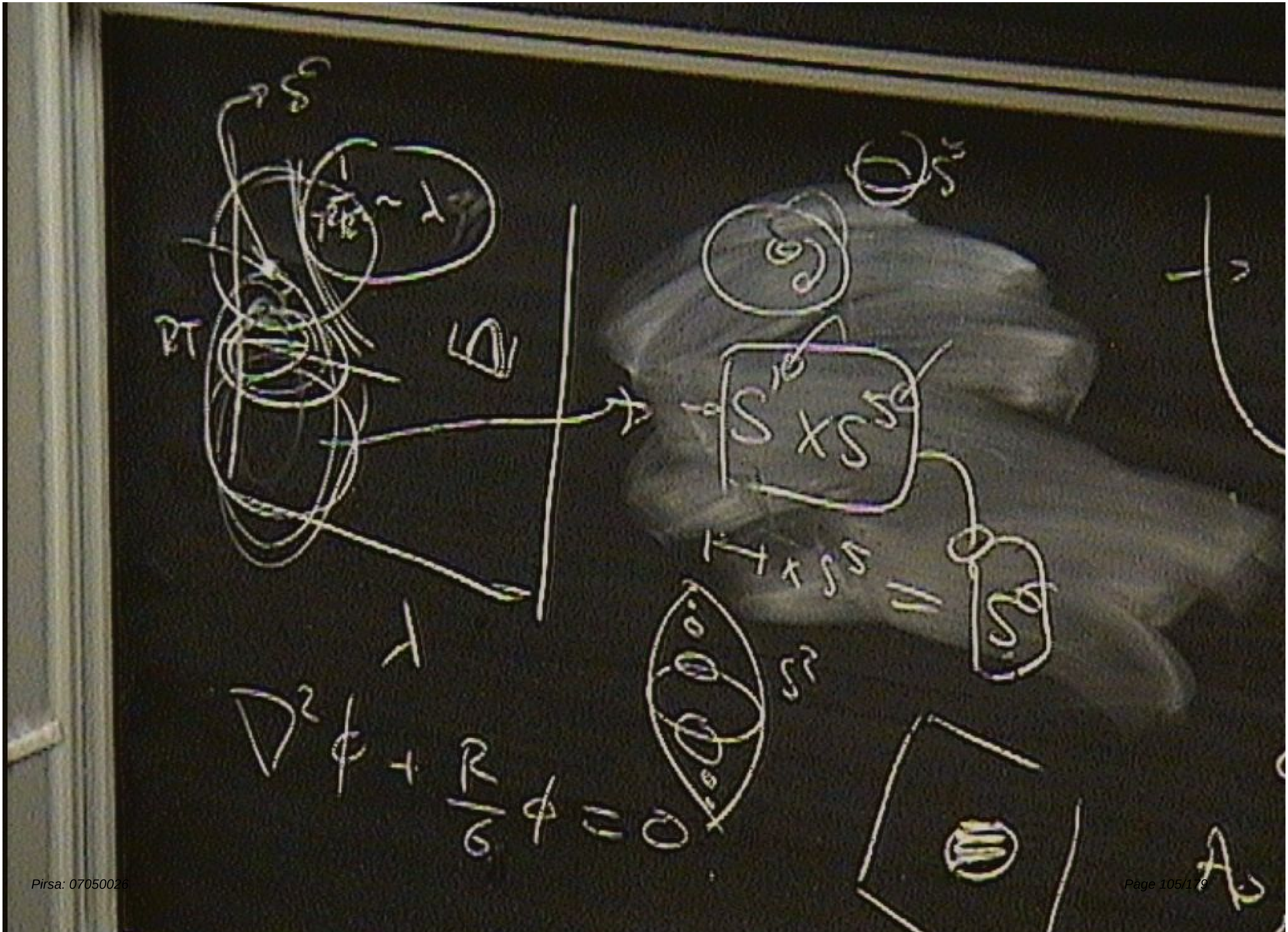
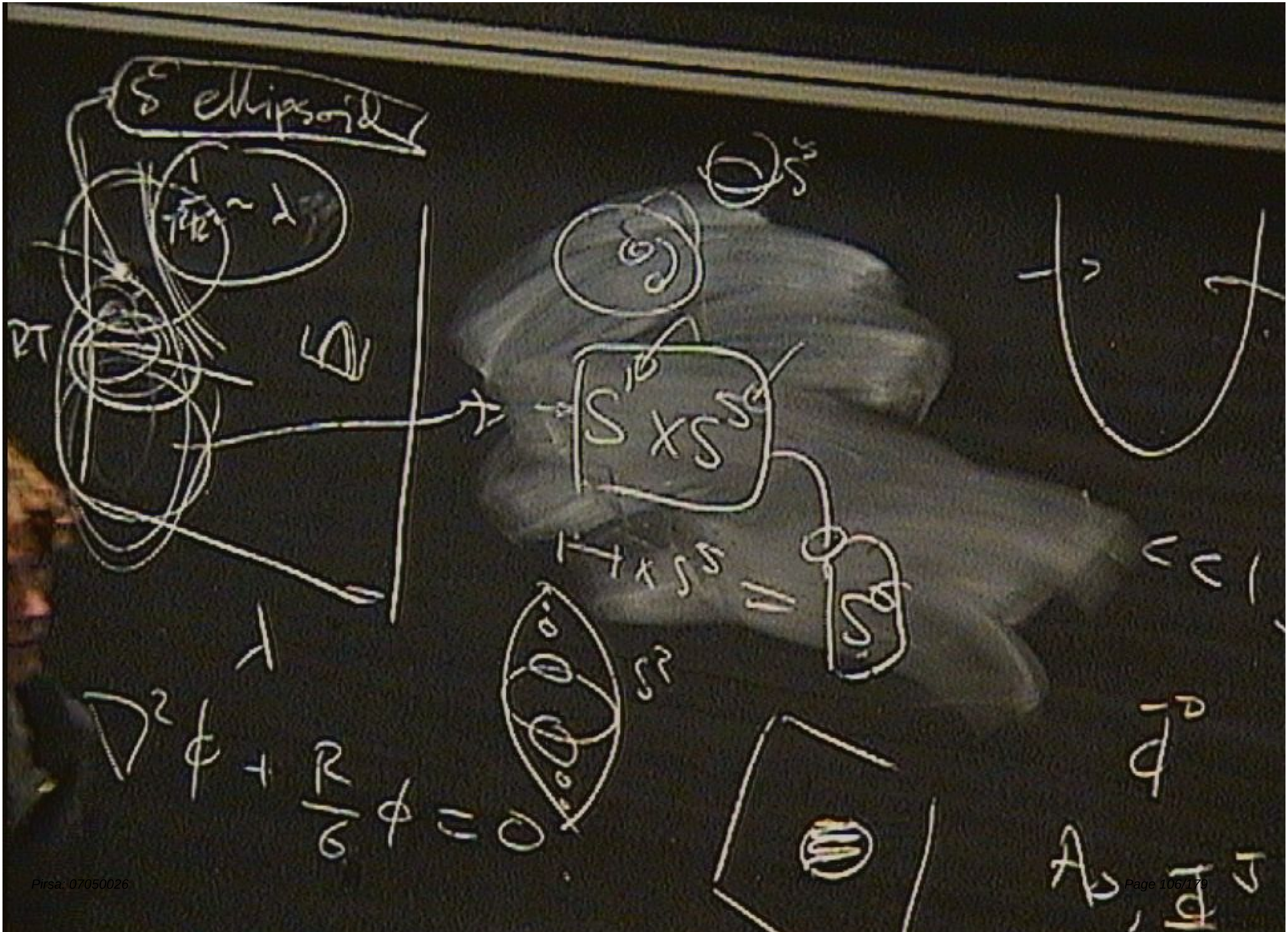

RT
λ
S¹ × S³
S³
D²φ + R/6 φ = 0


S⁵
λ
S¹ × S⁵
S⁵
S⁵
λ
D²φ + R/6 φ = 0


ellipsoid

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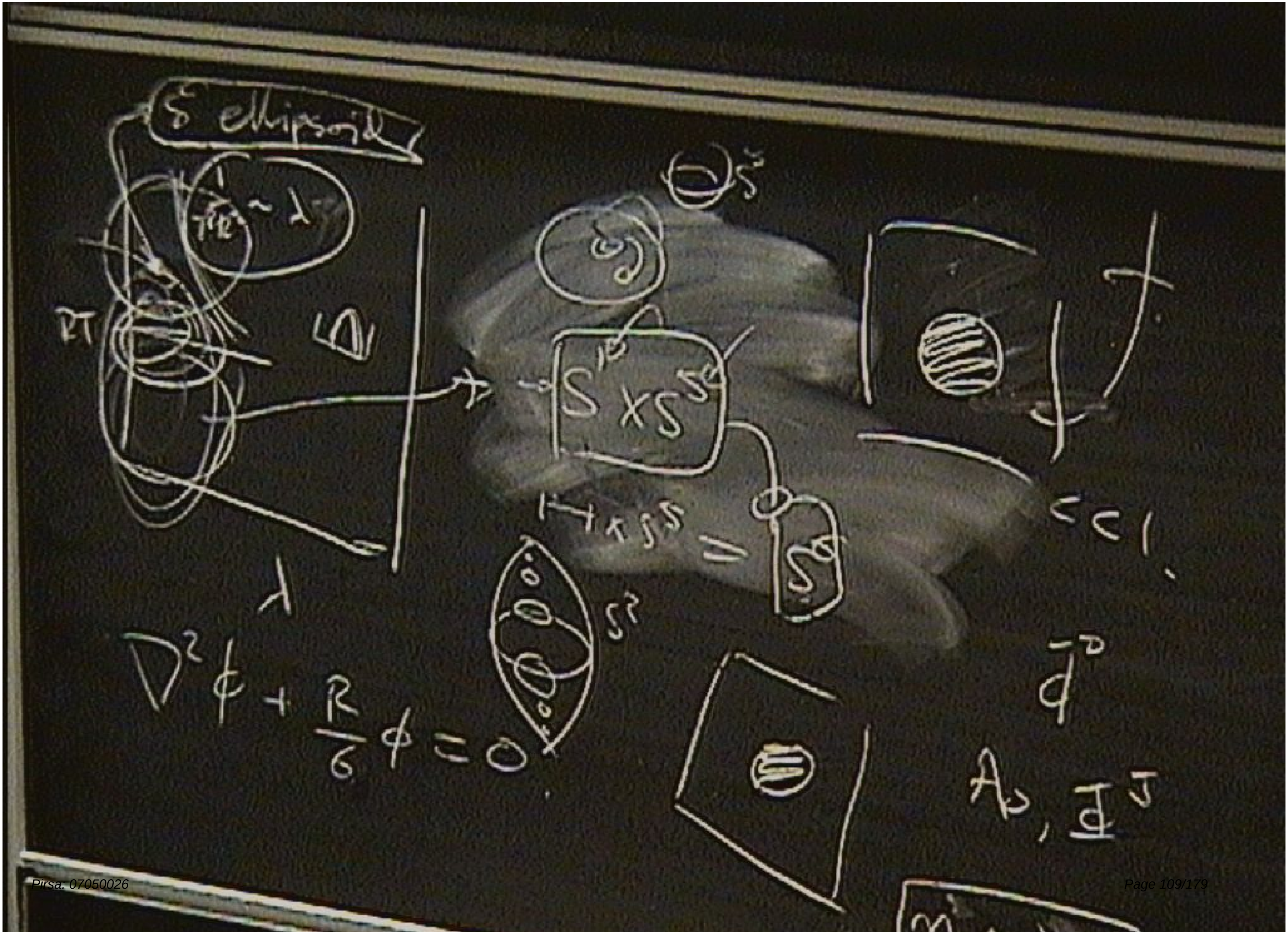
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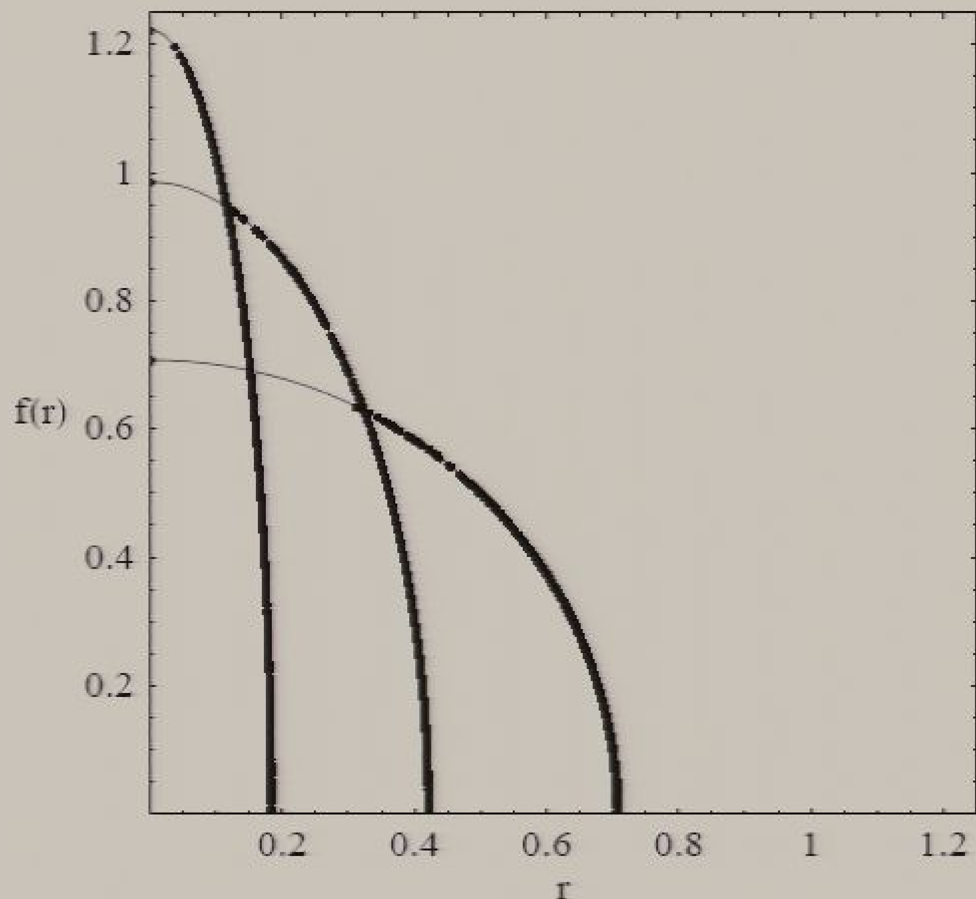
High temperature solutions



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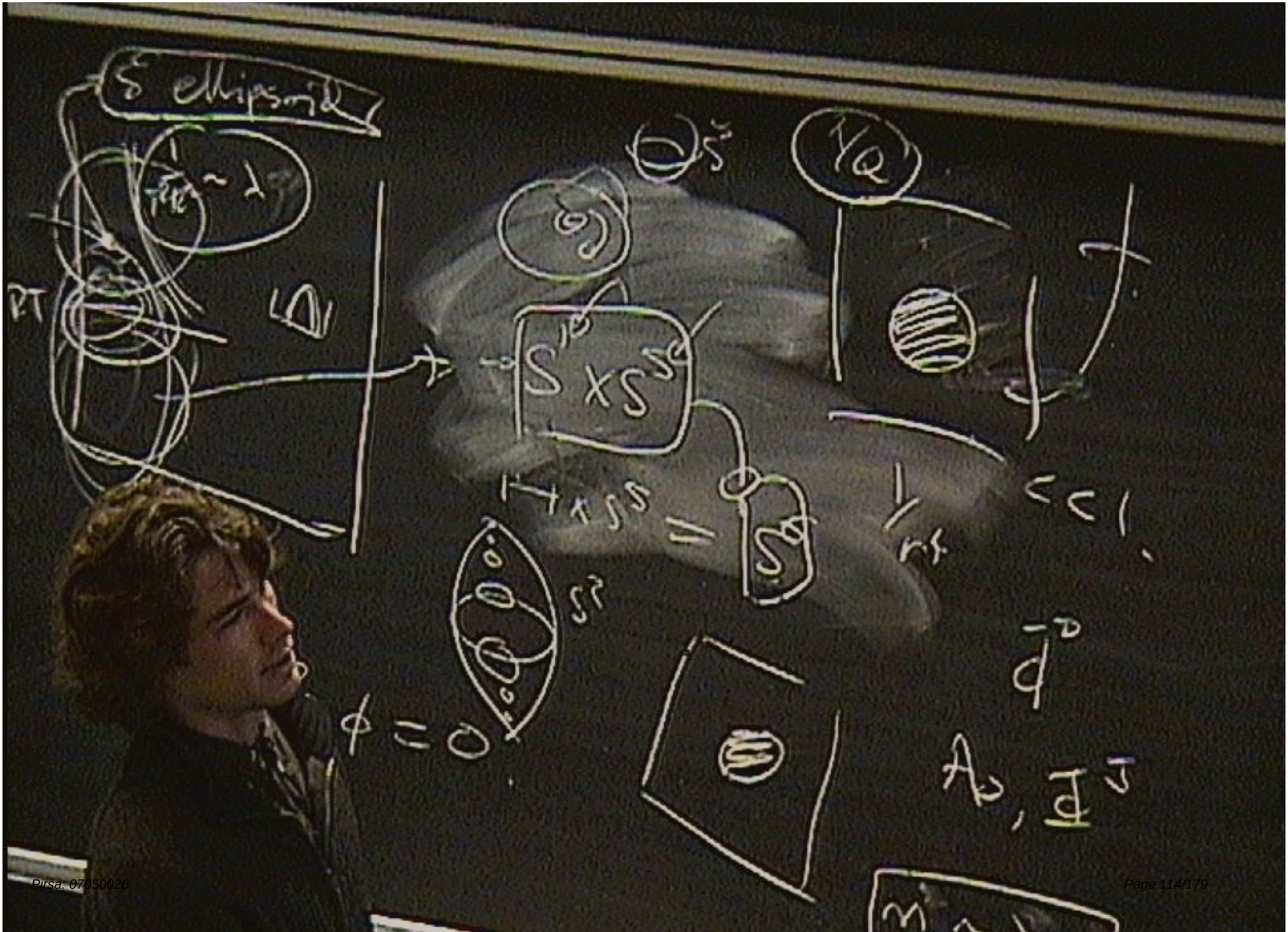
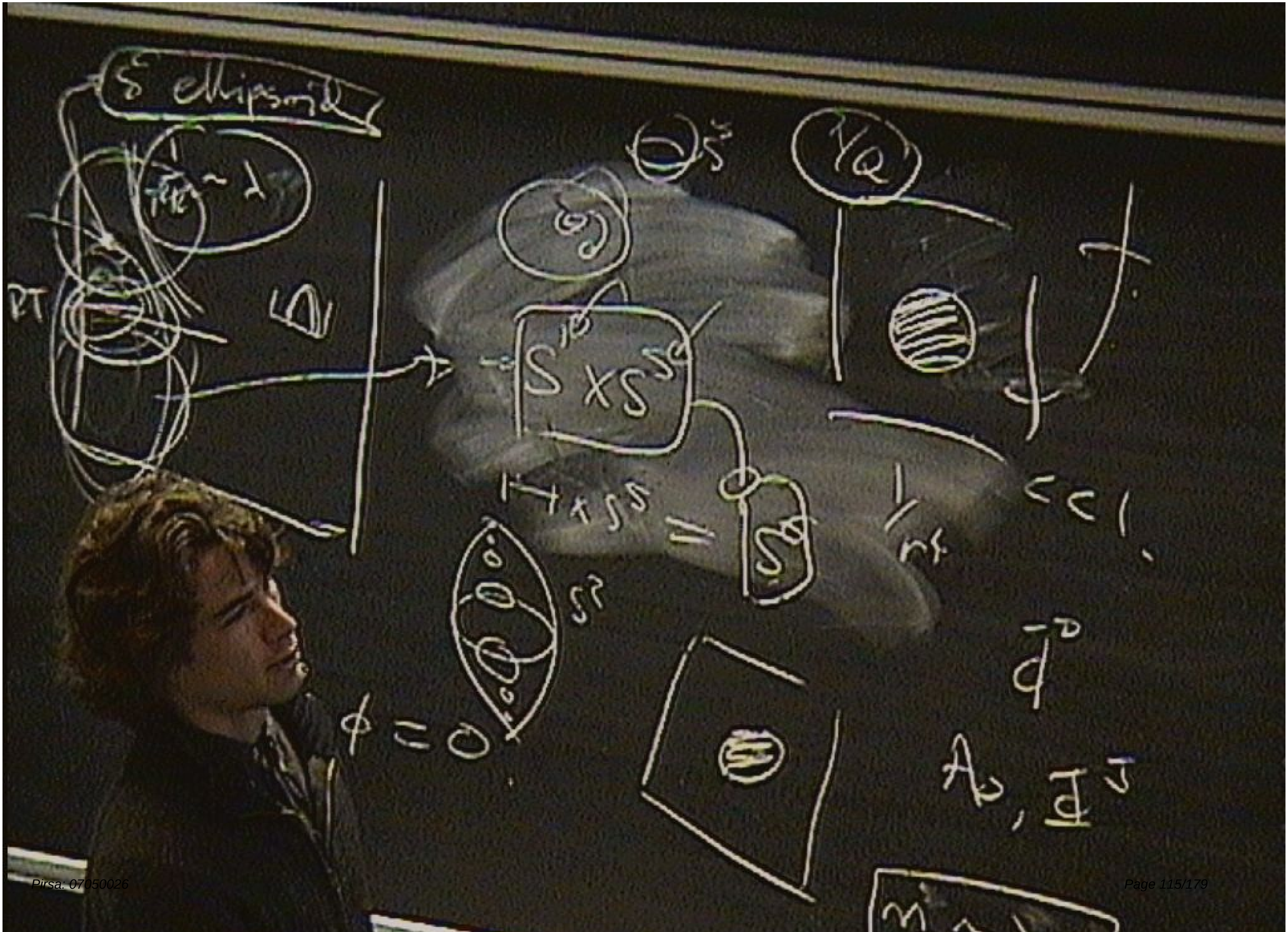

φ = 0
<< 1



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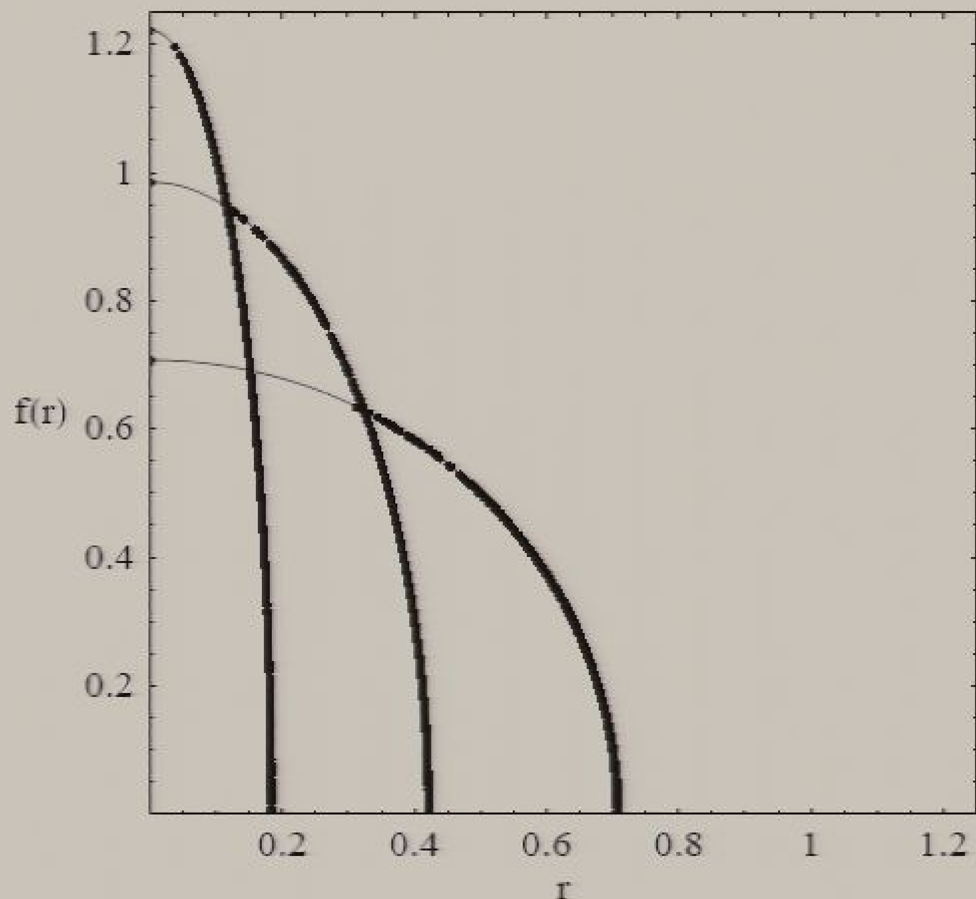

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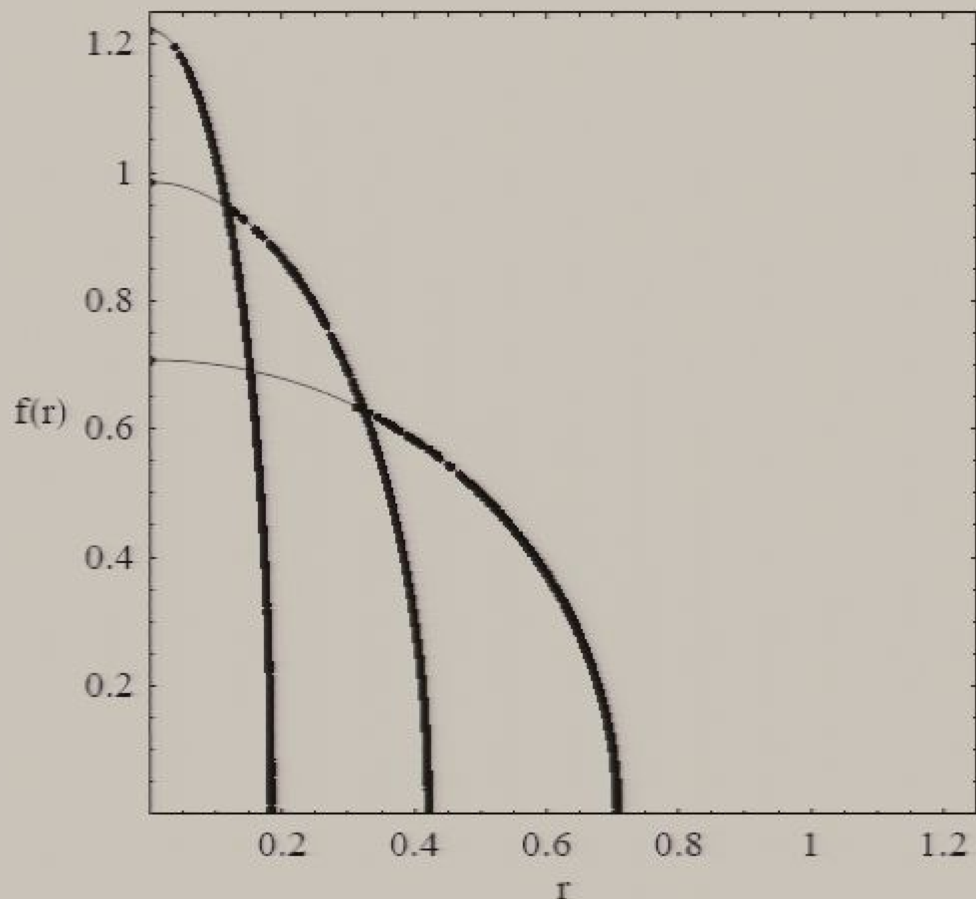

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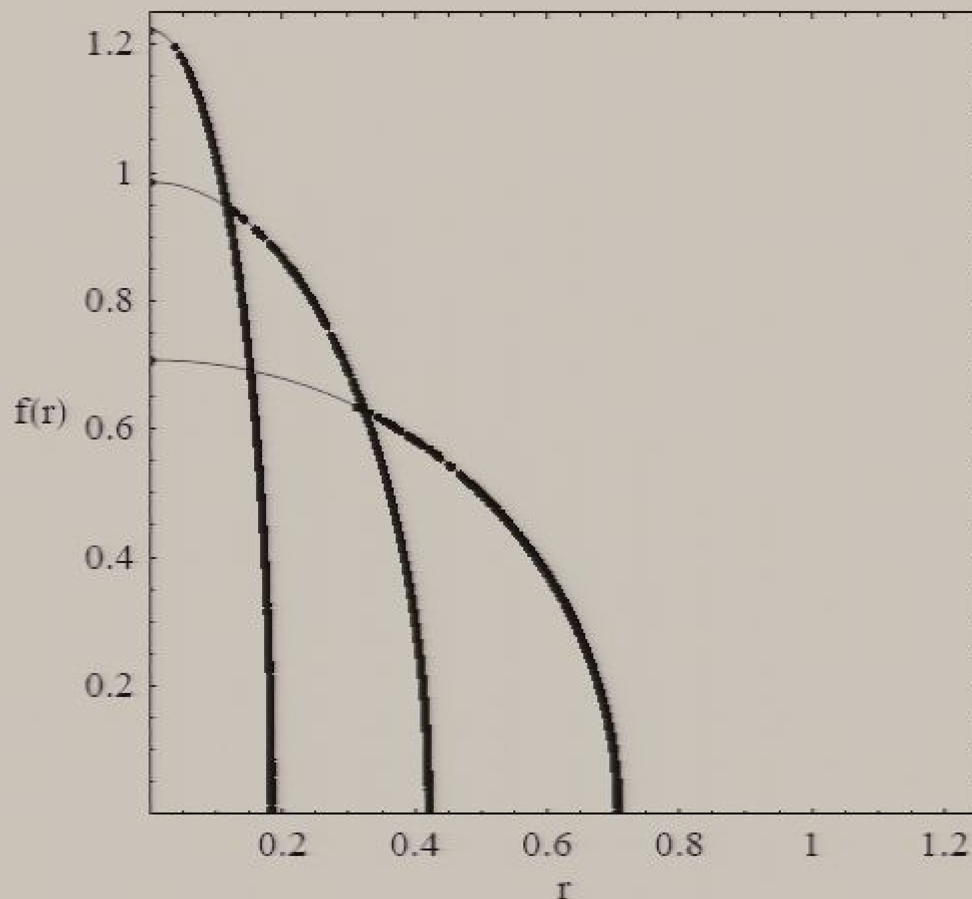

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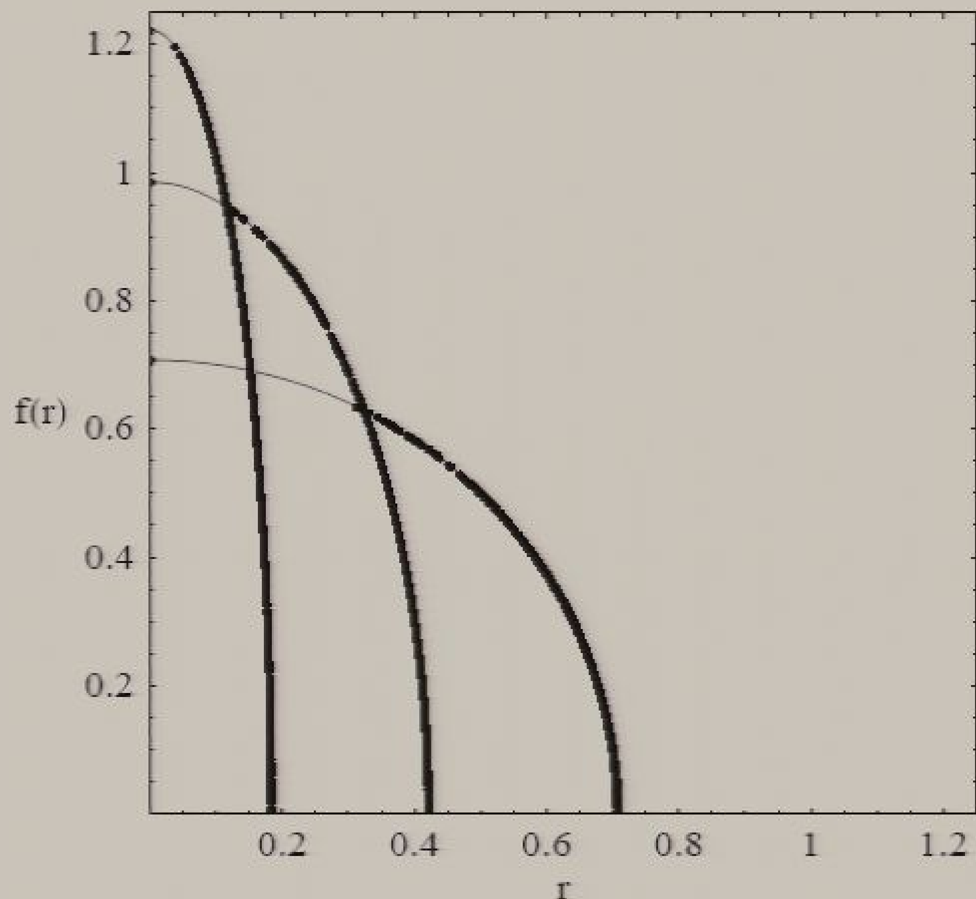

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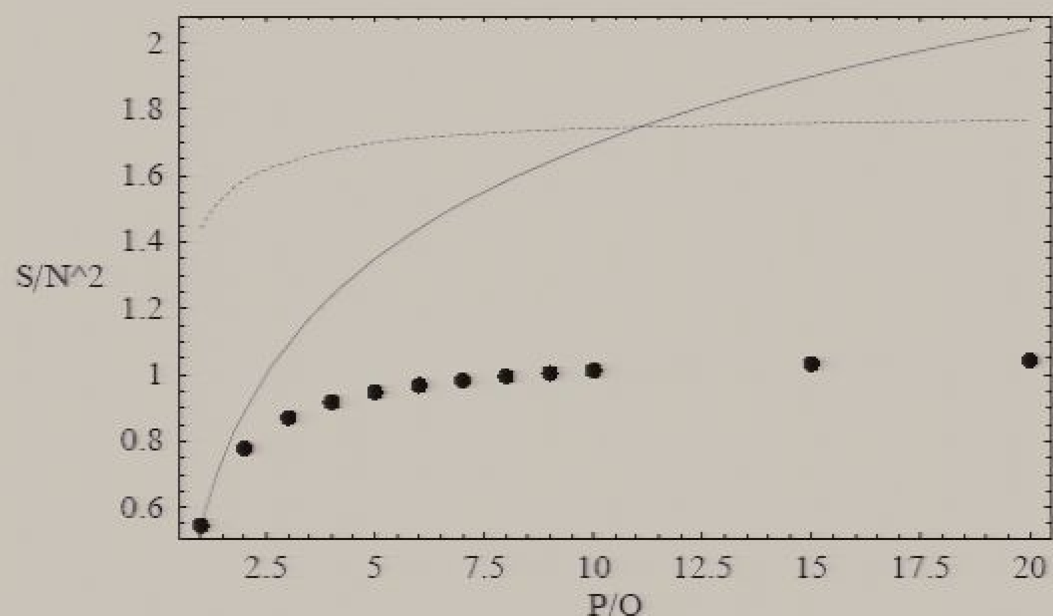

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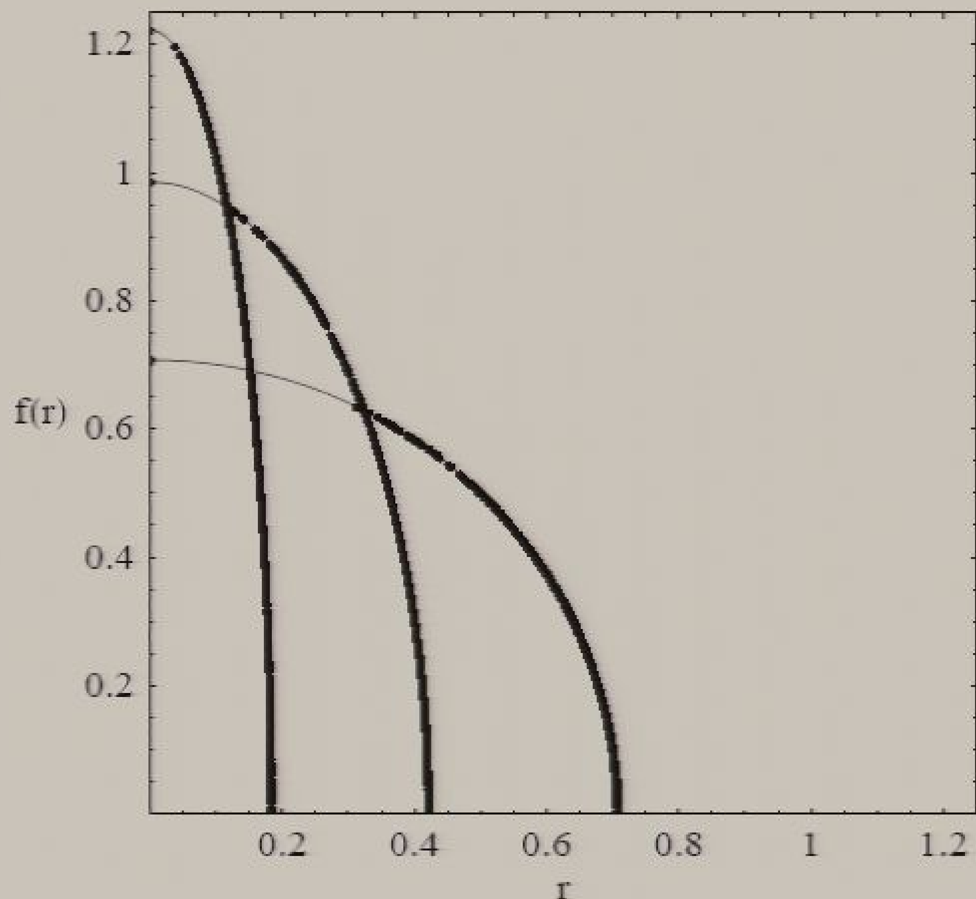

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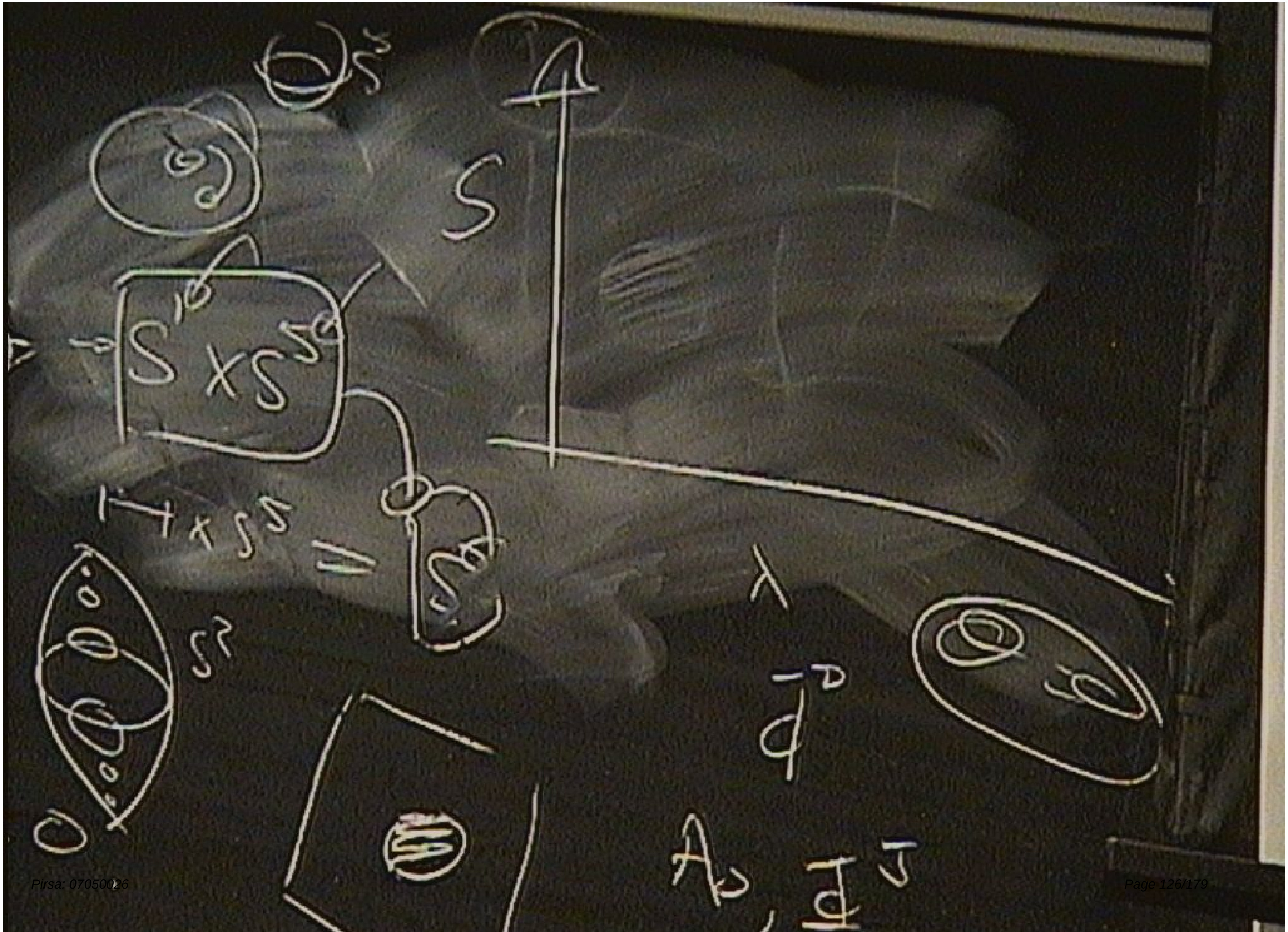

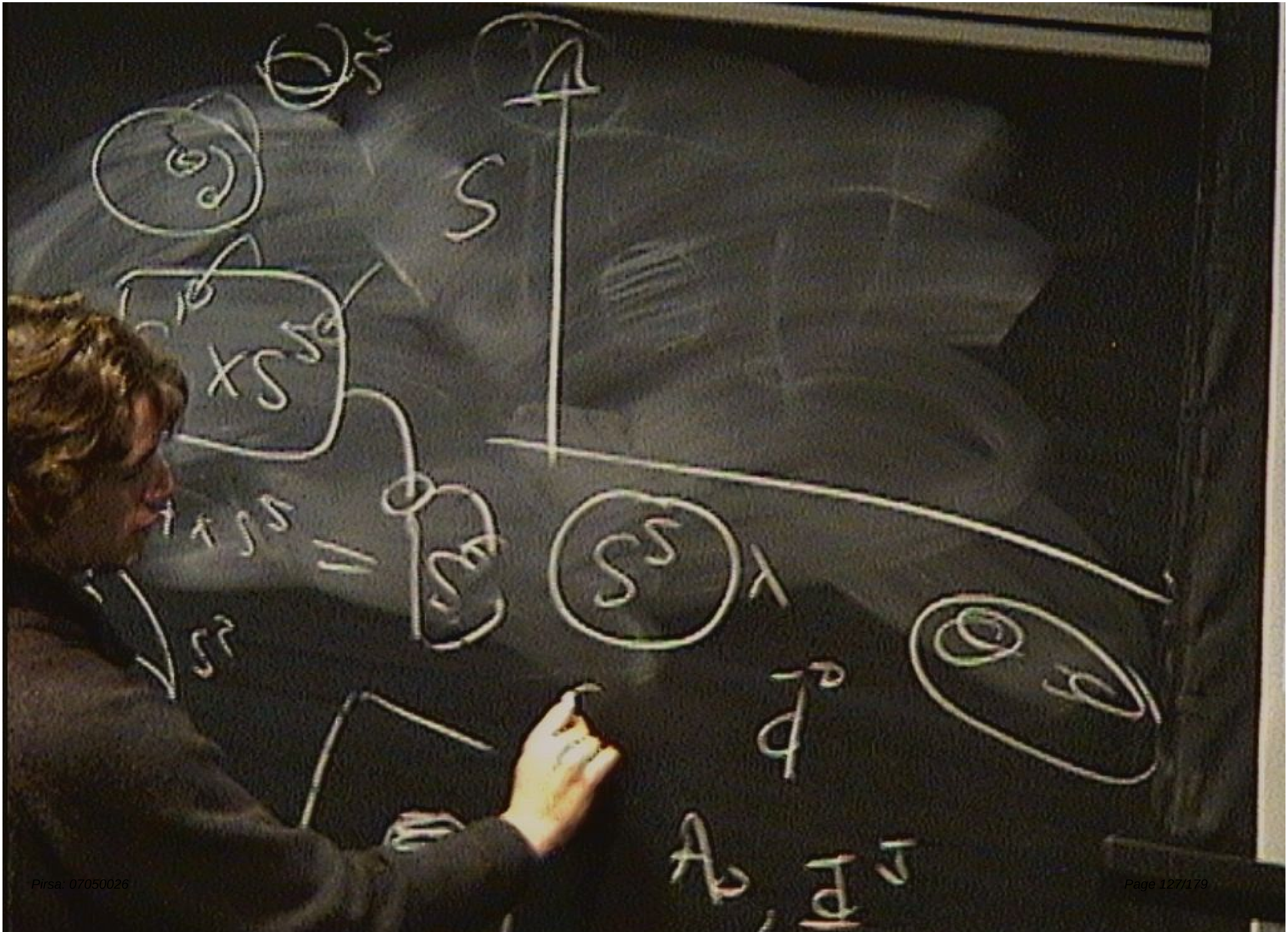

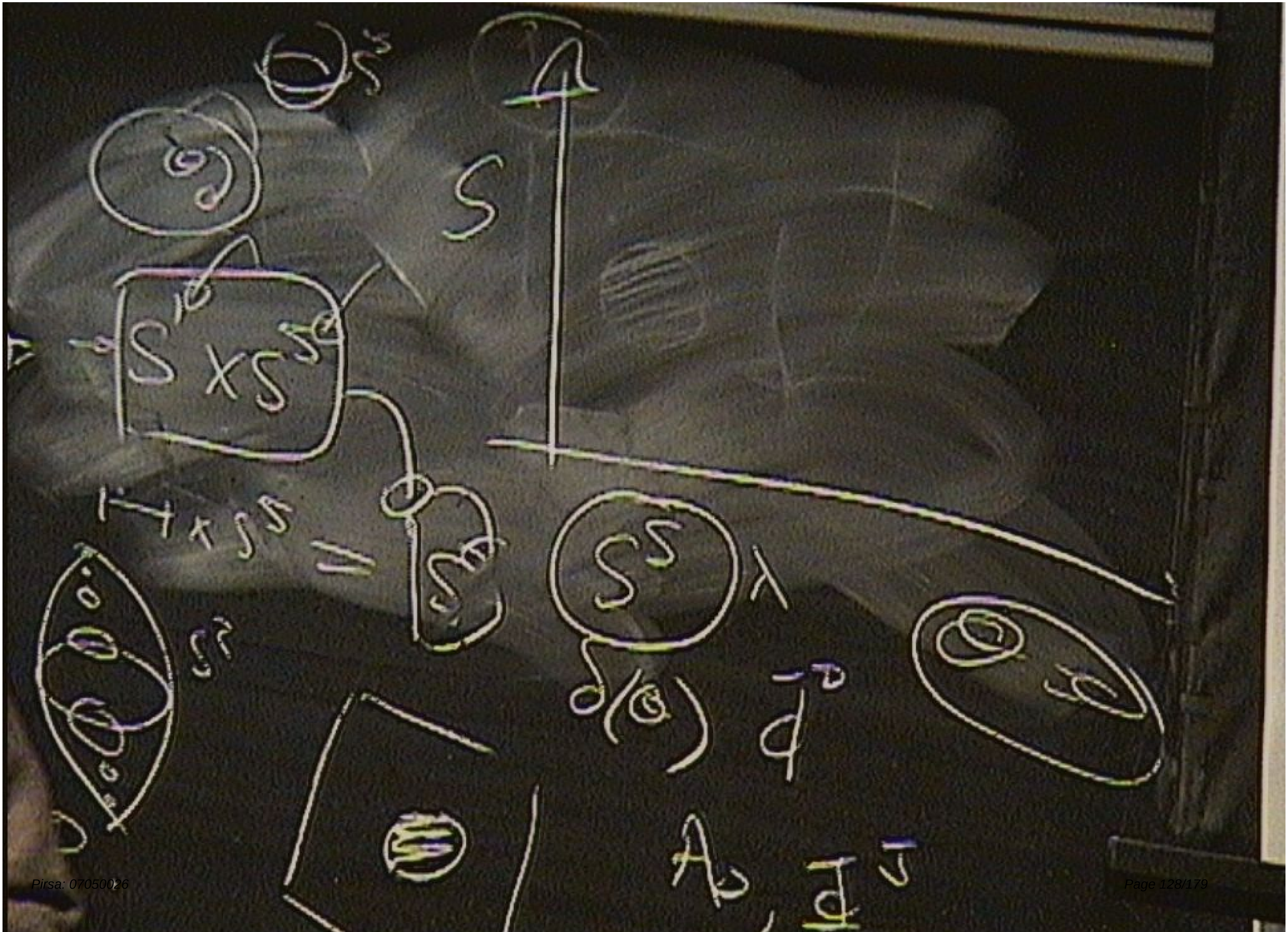

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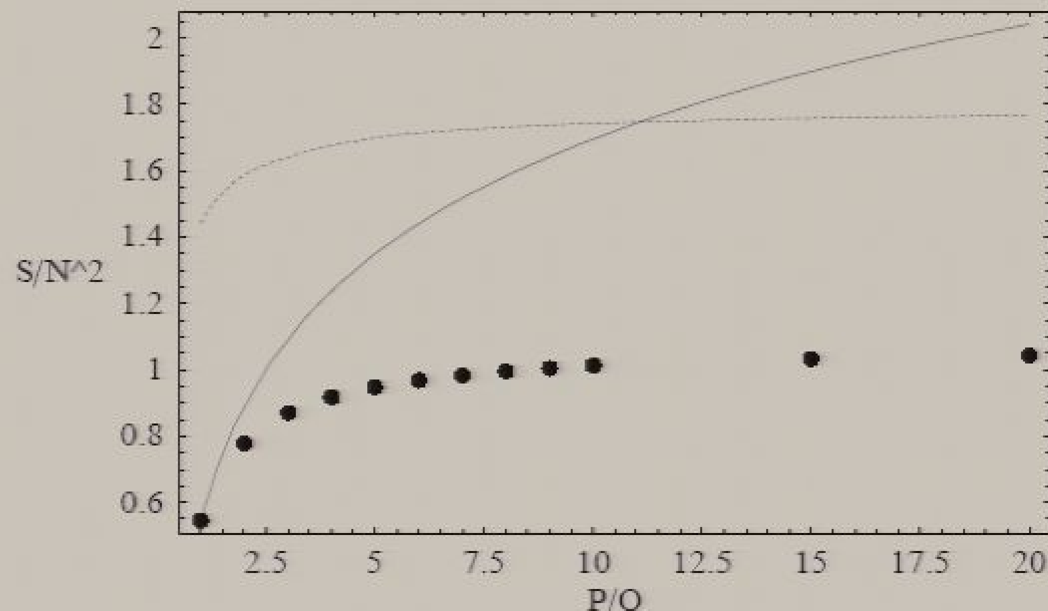

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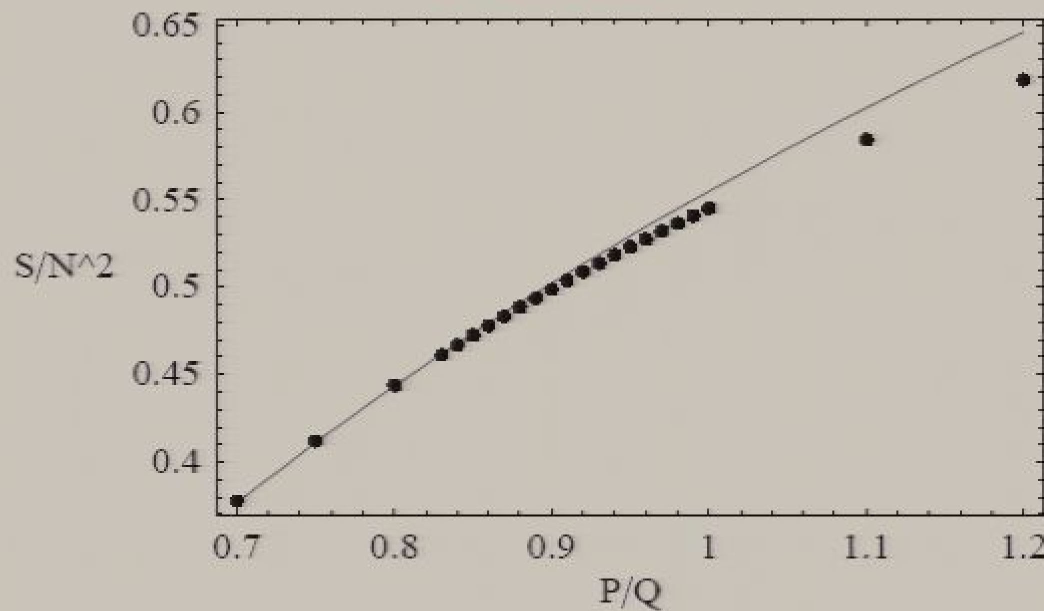

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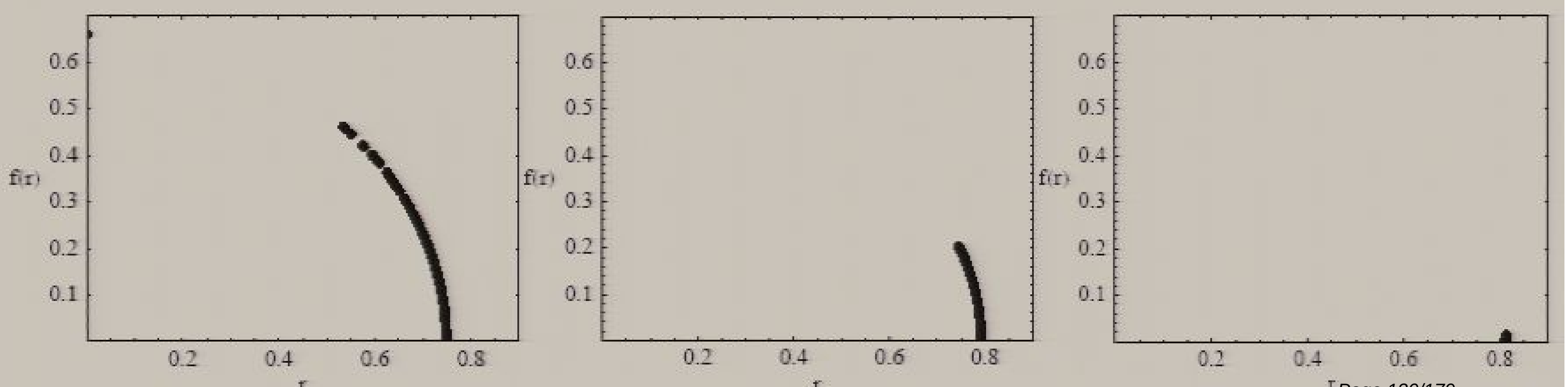
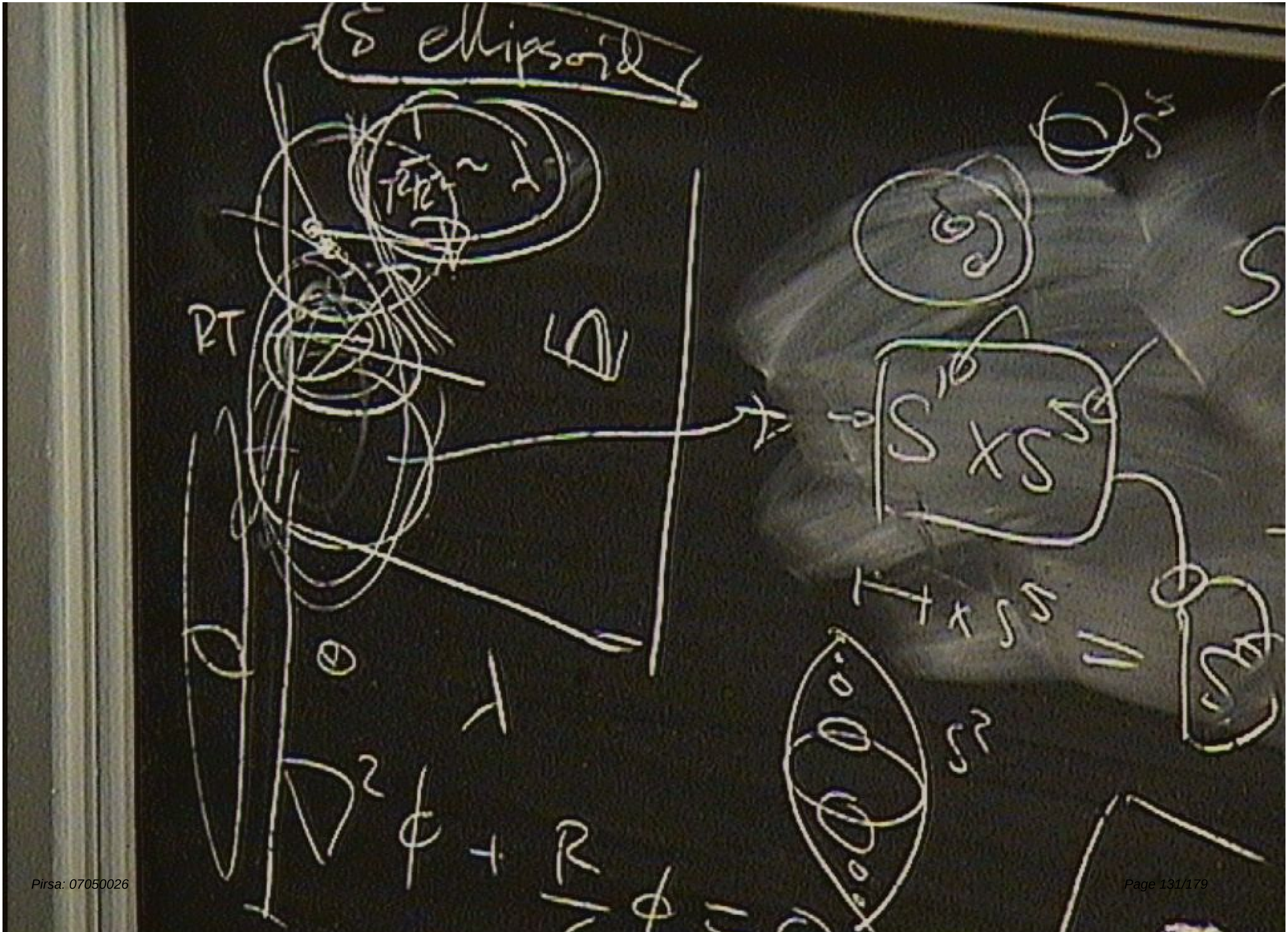
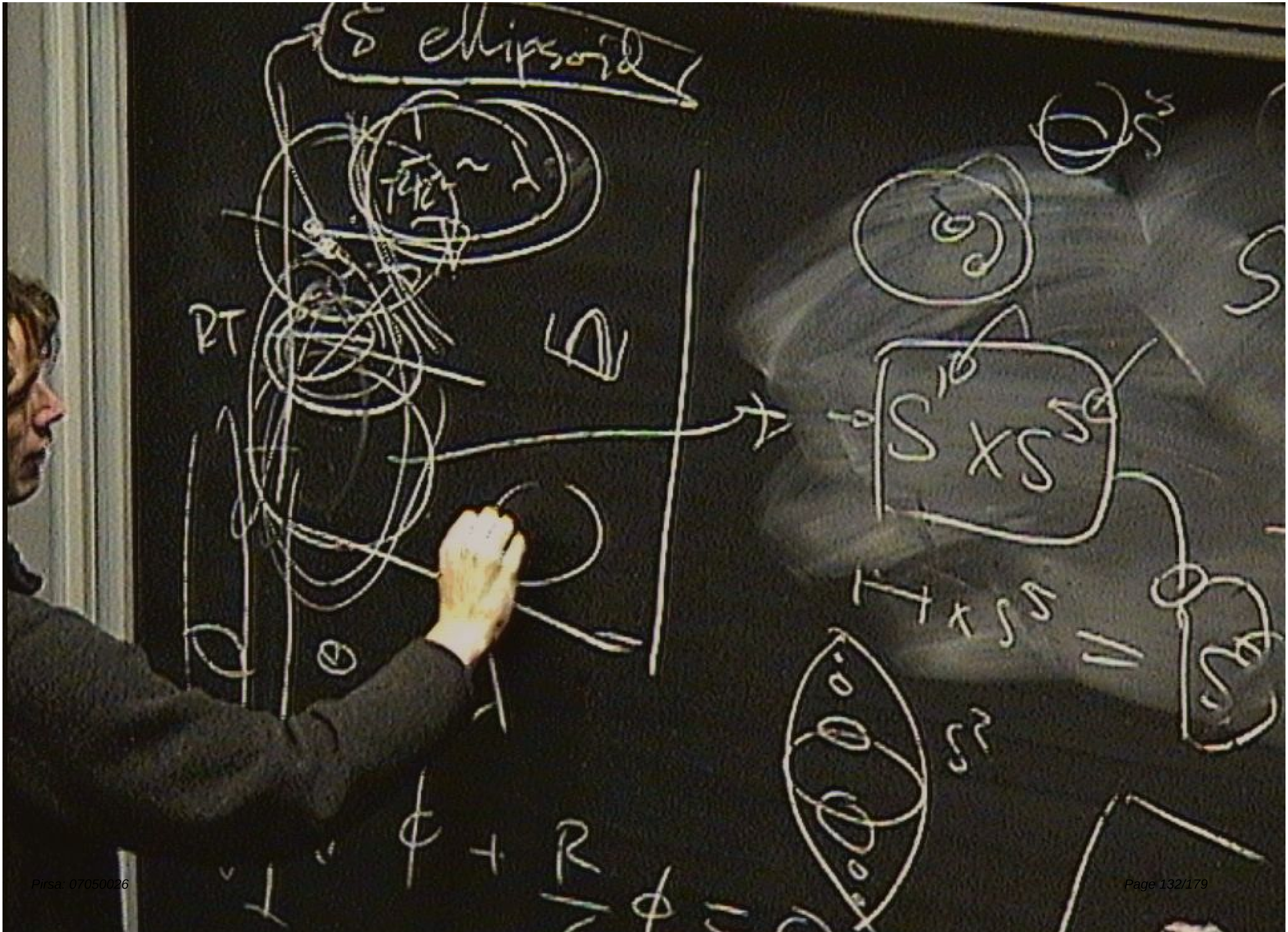
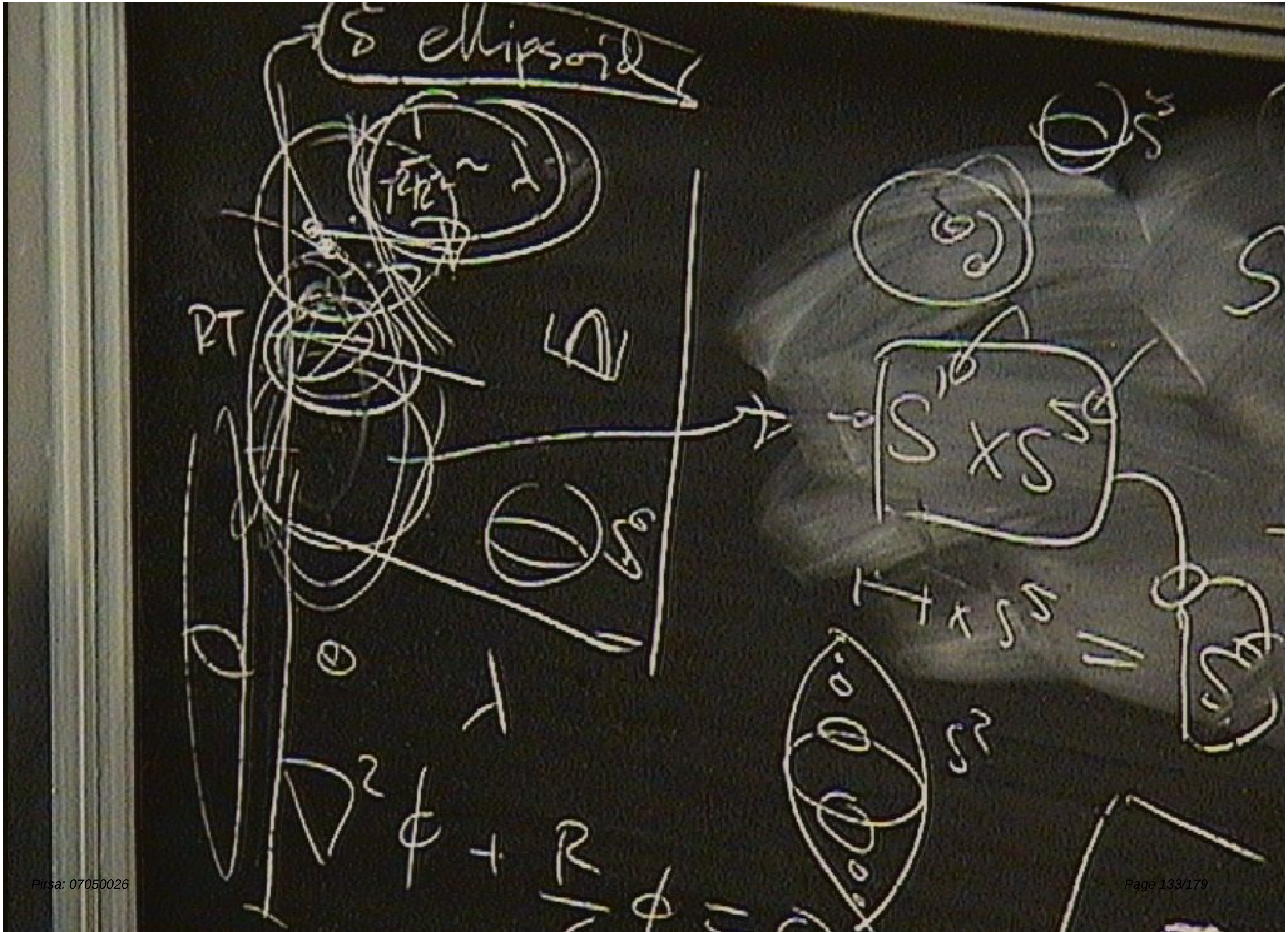
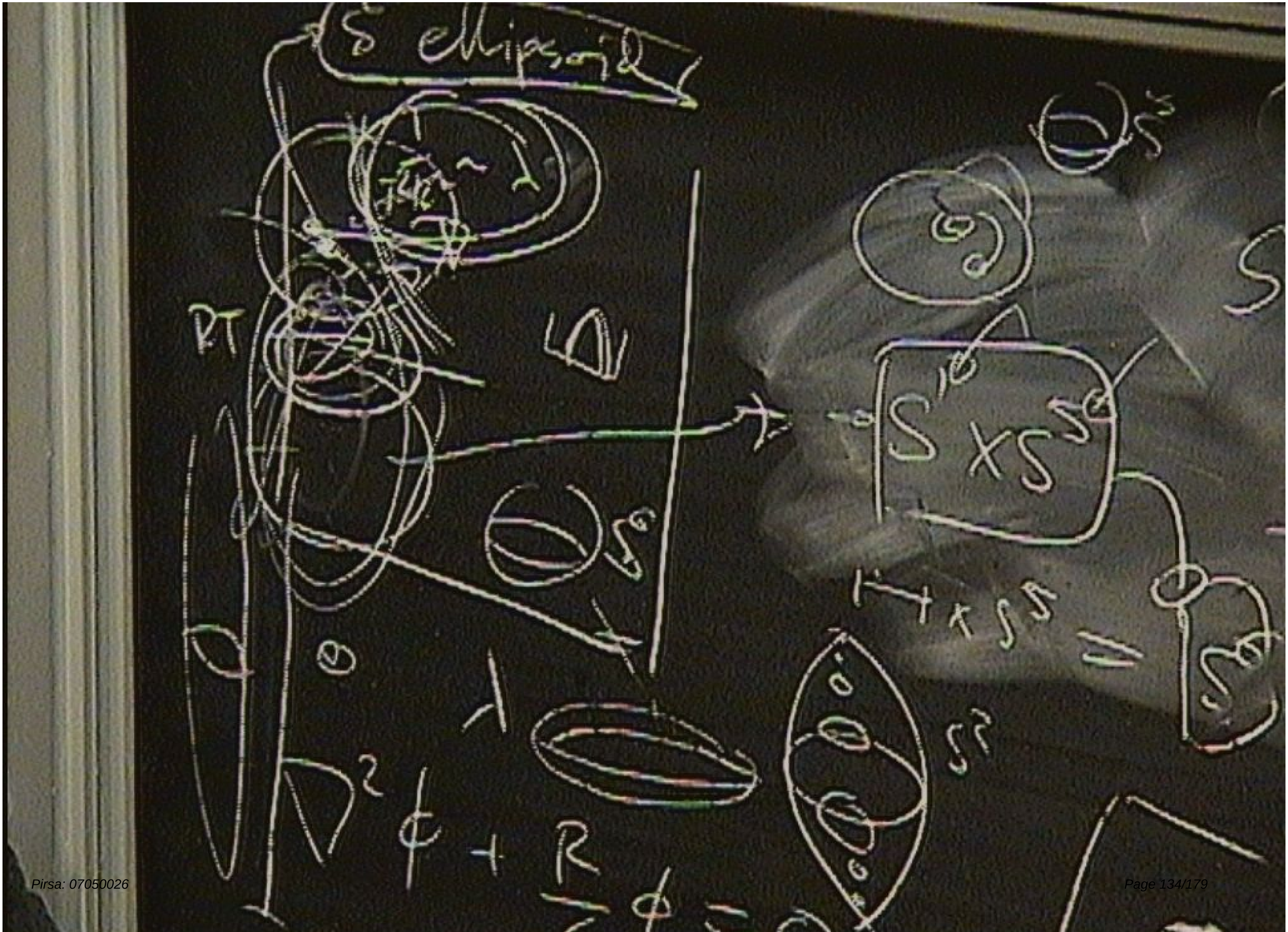



ellipsoid
RT



ellipsoid
RT
$S^1 \times S^3$
S^2


ellipsoid
RT


ellipsoid
RT

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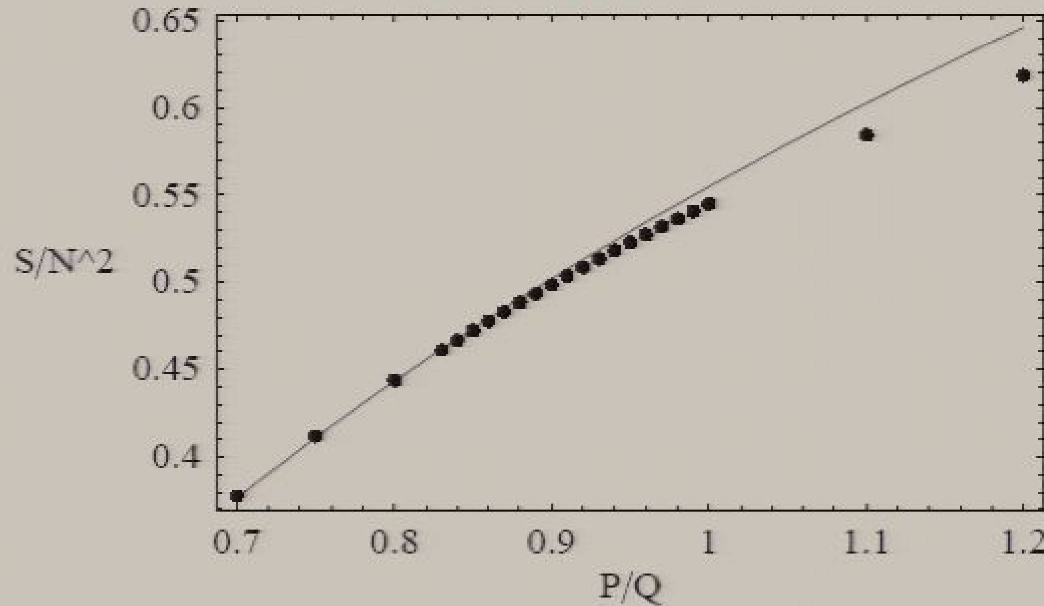

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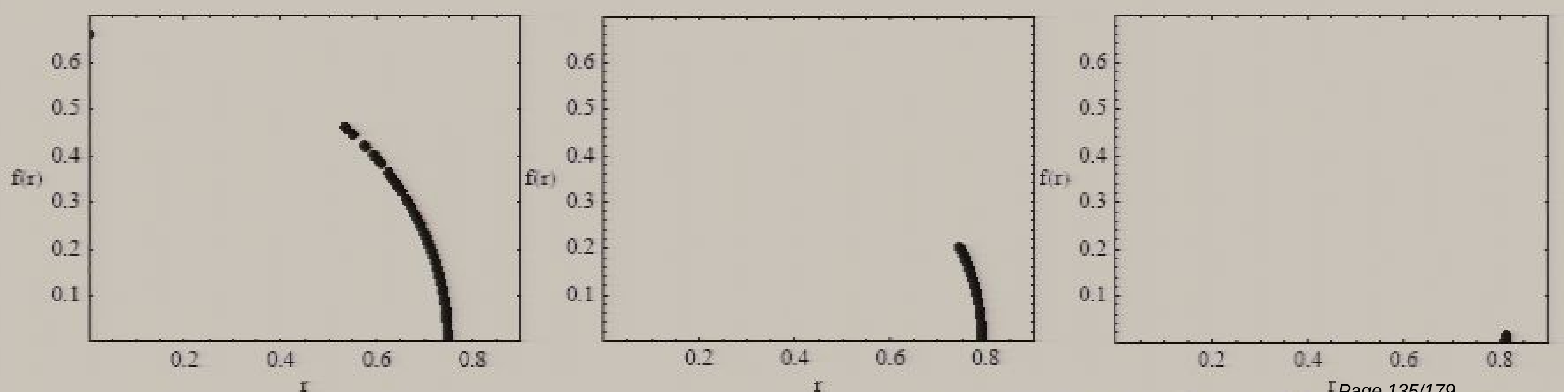

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- Consider the stability of the S^5 solution. The action for quadratic fluctuations in $\delta\theta_p$ is

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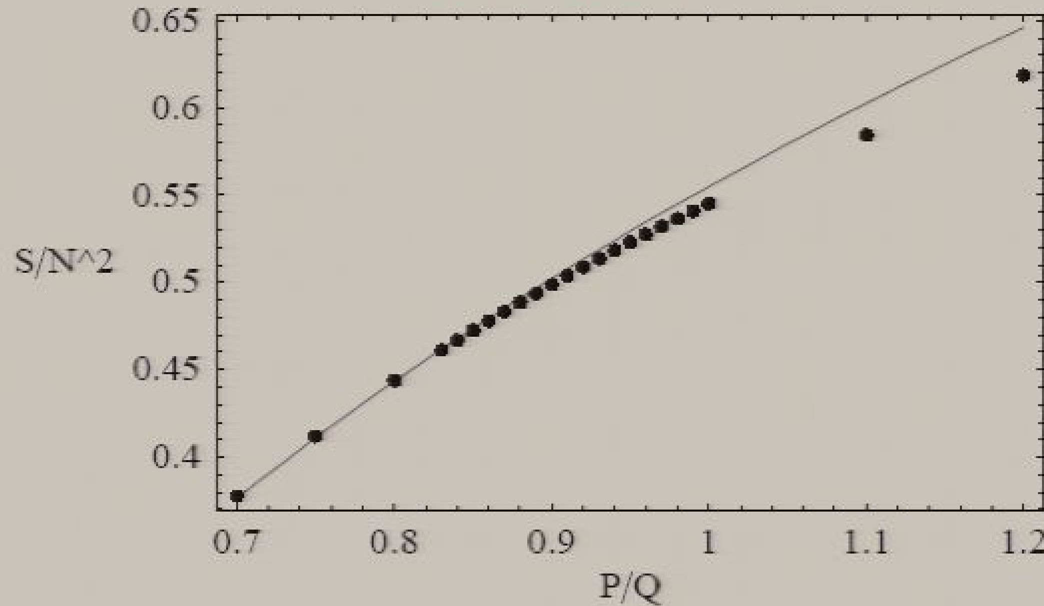

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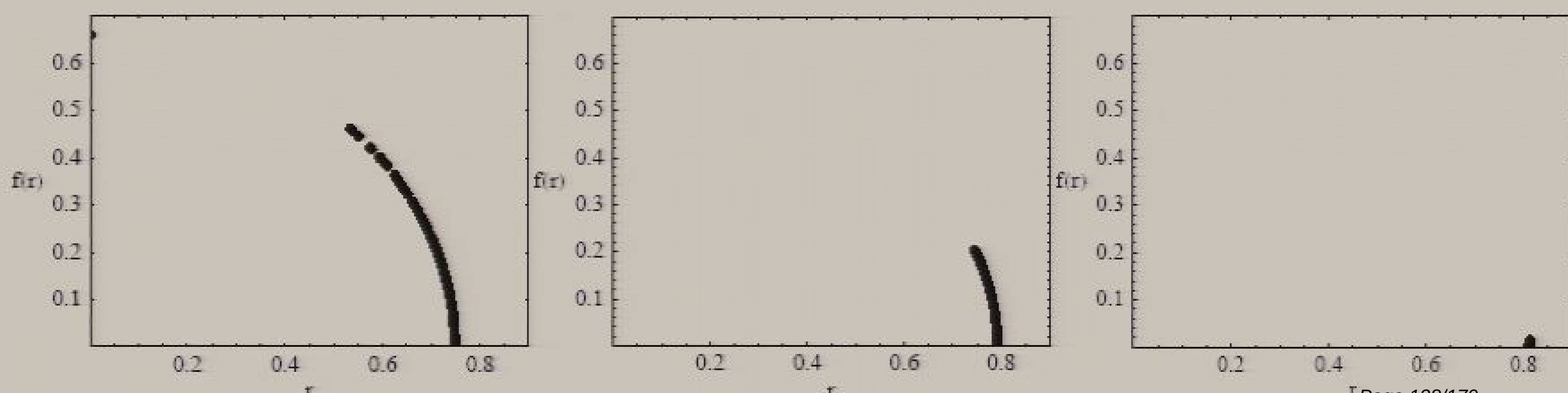

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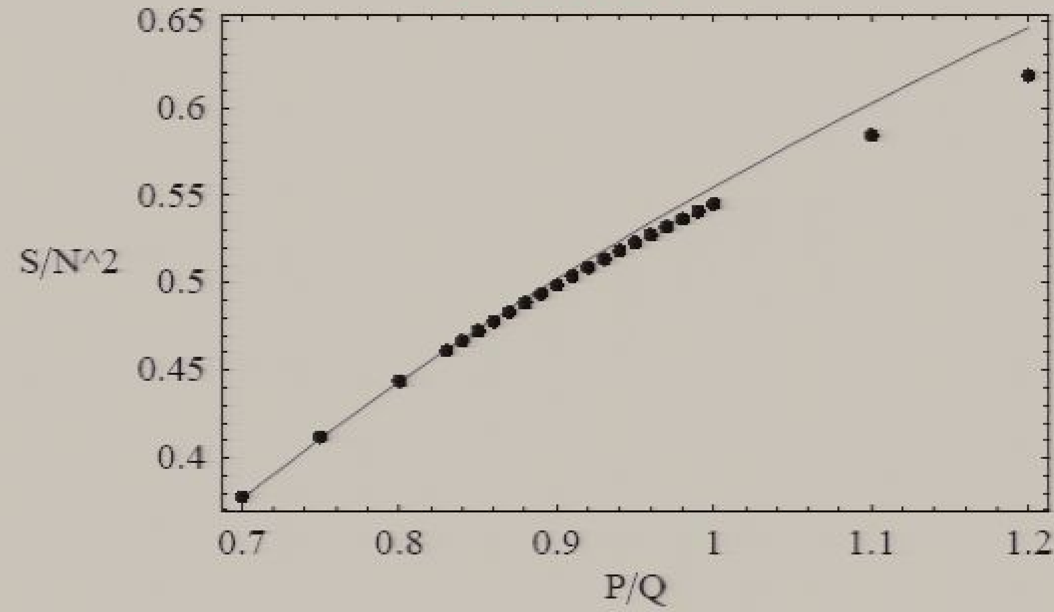

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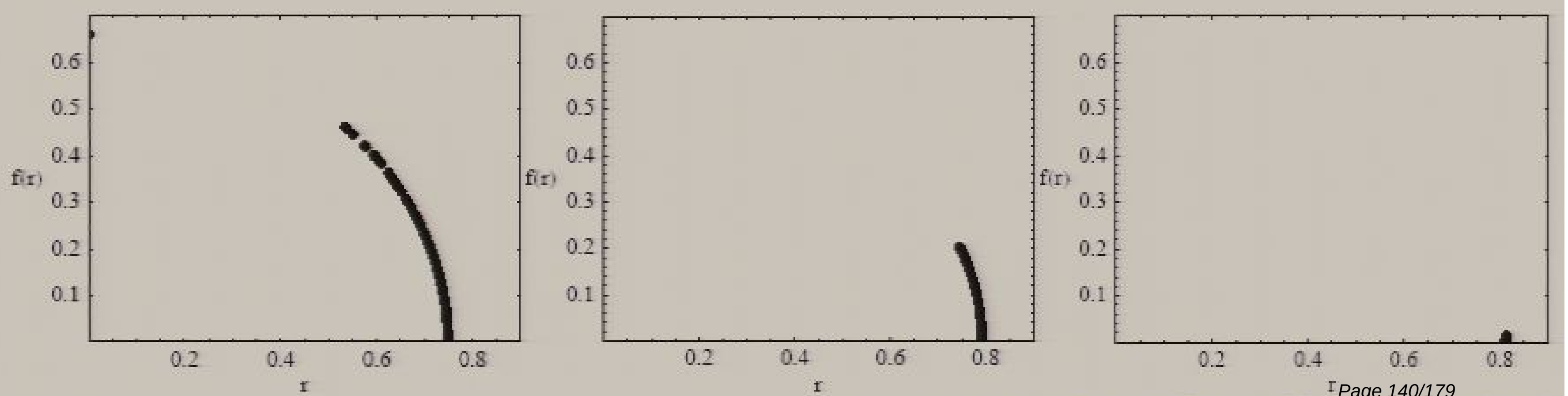

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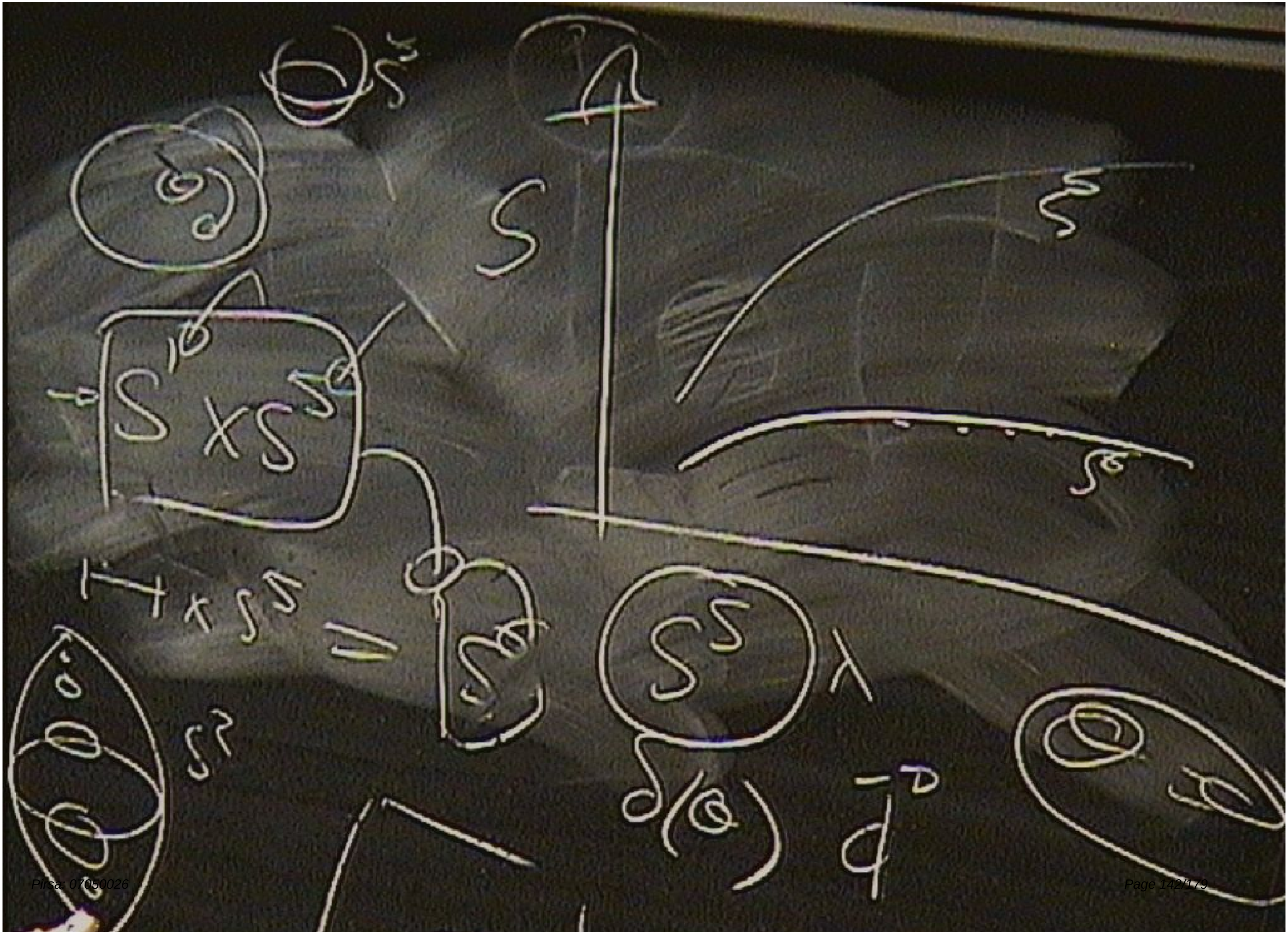

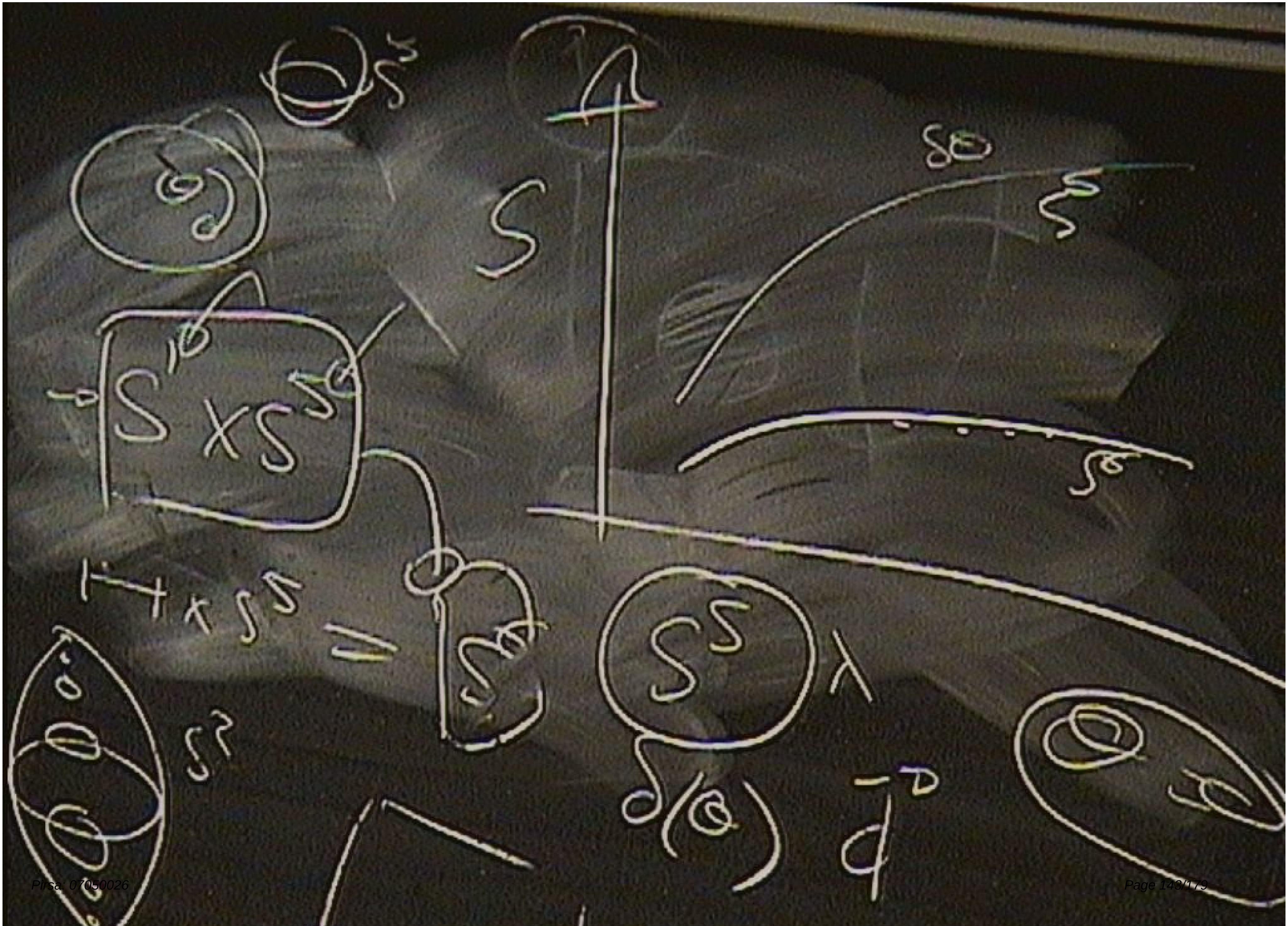

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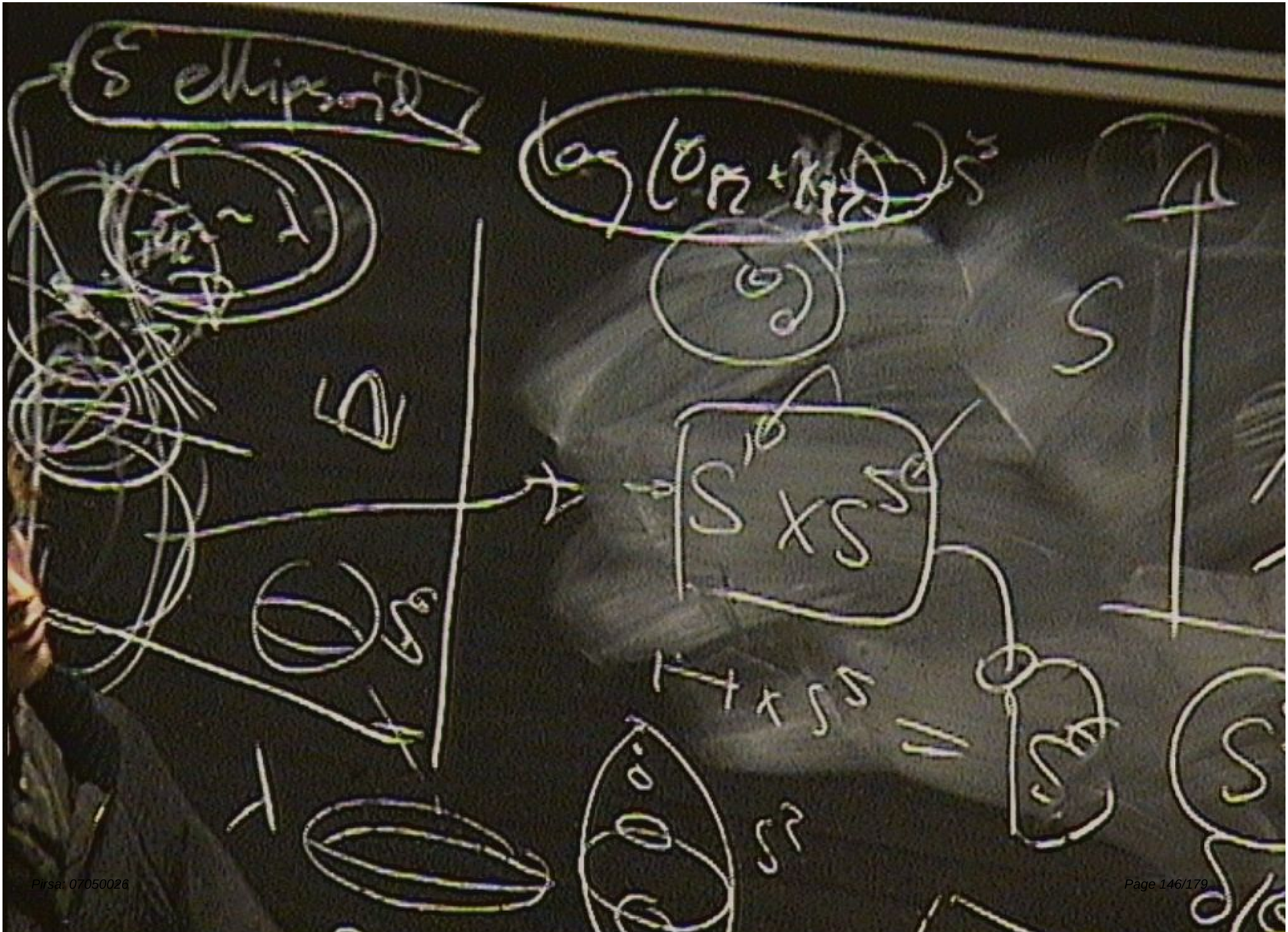
ellipsoid

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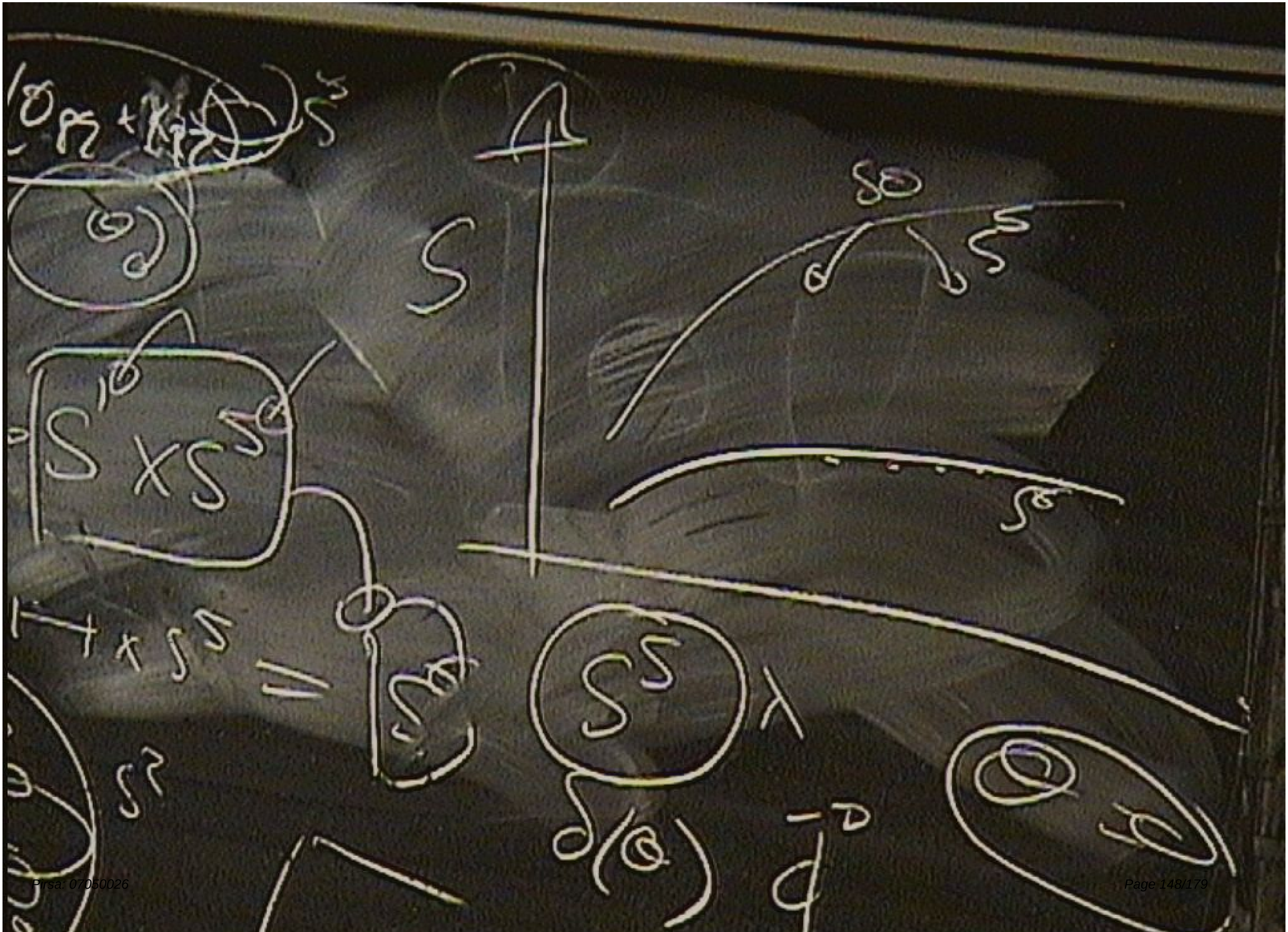
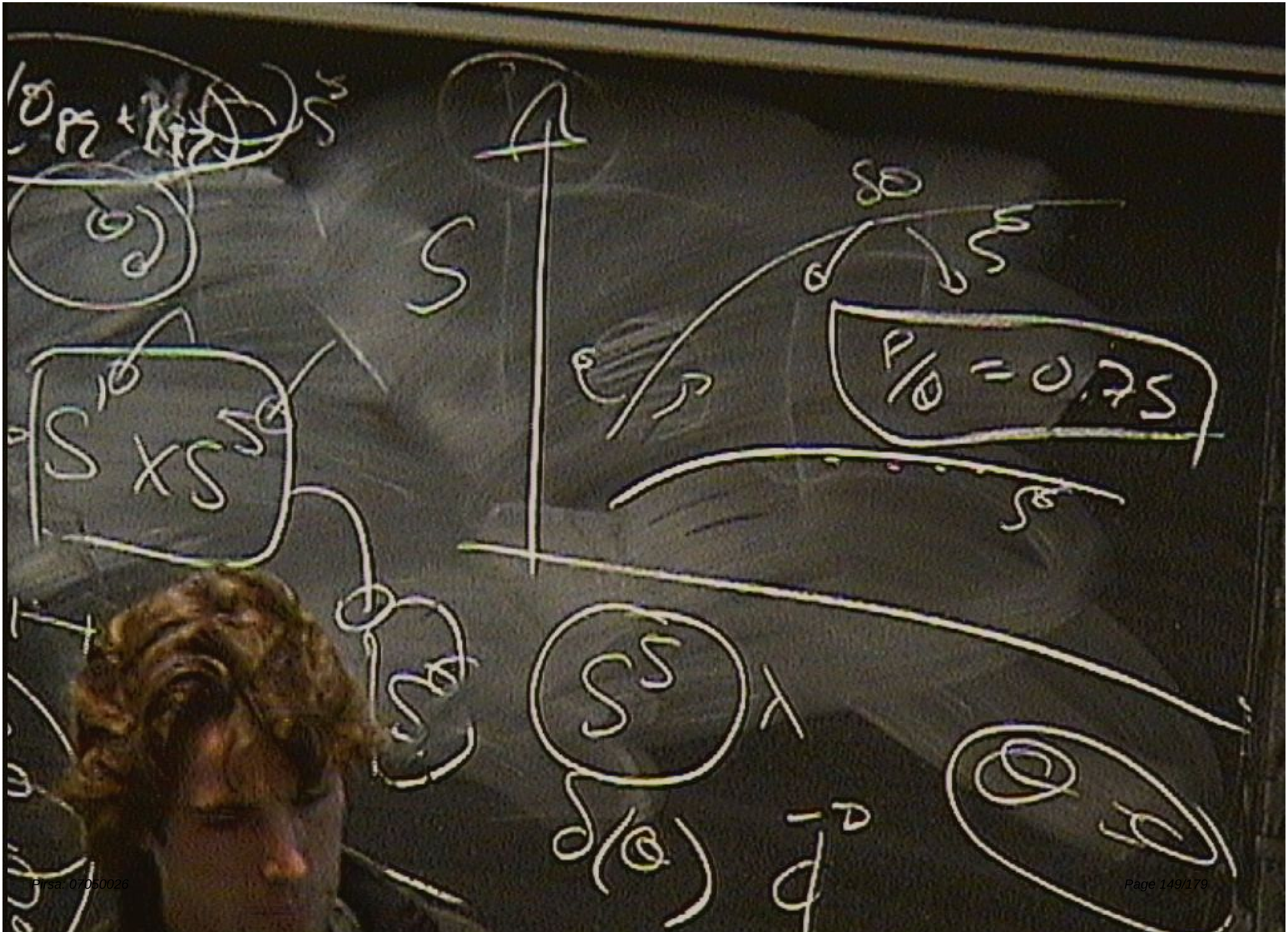
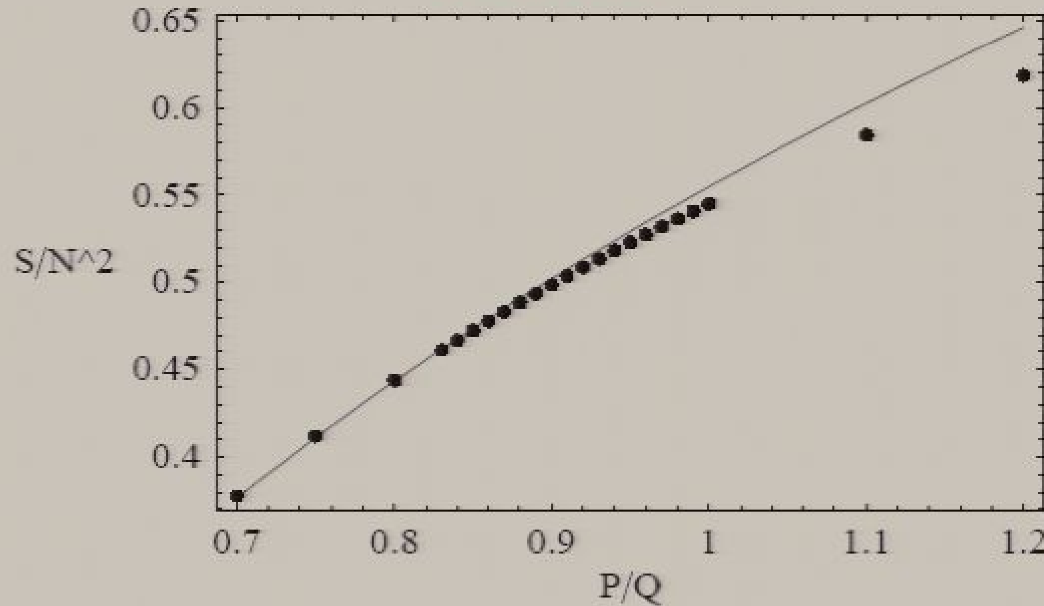




- We can zoom in to see the crossover



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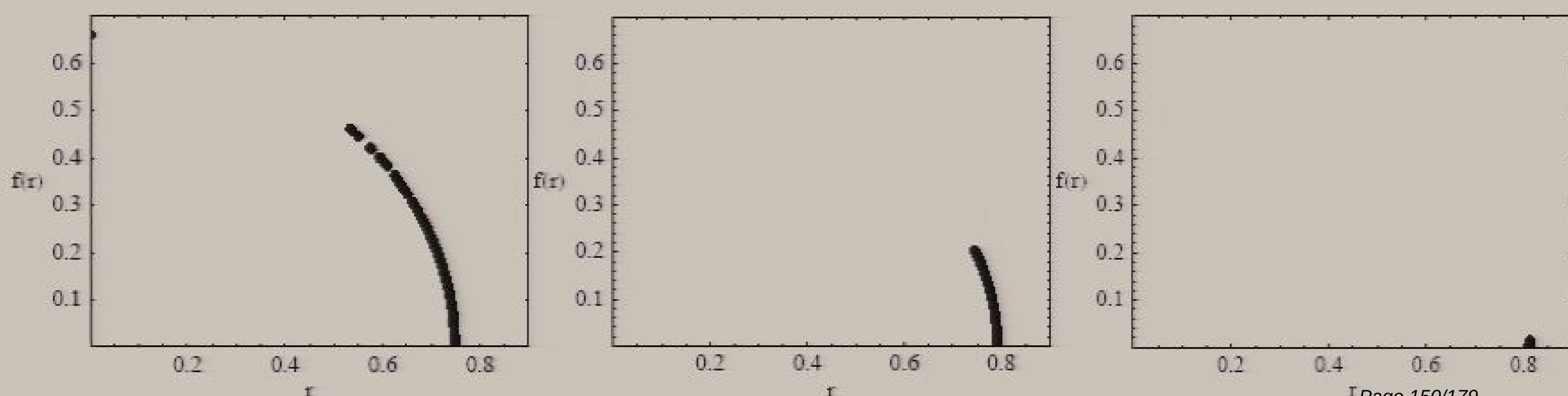

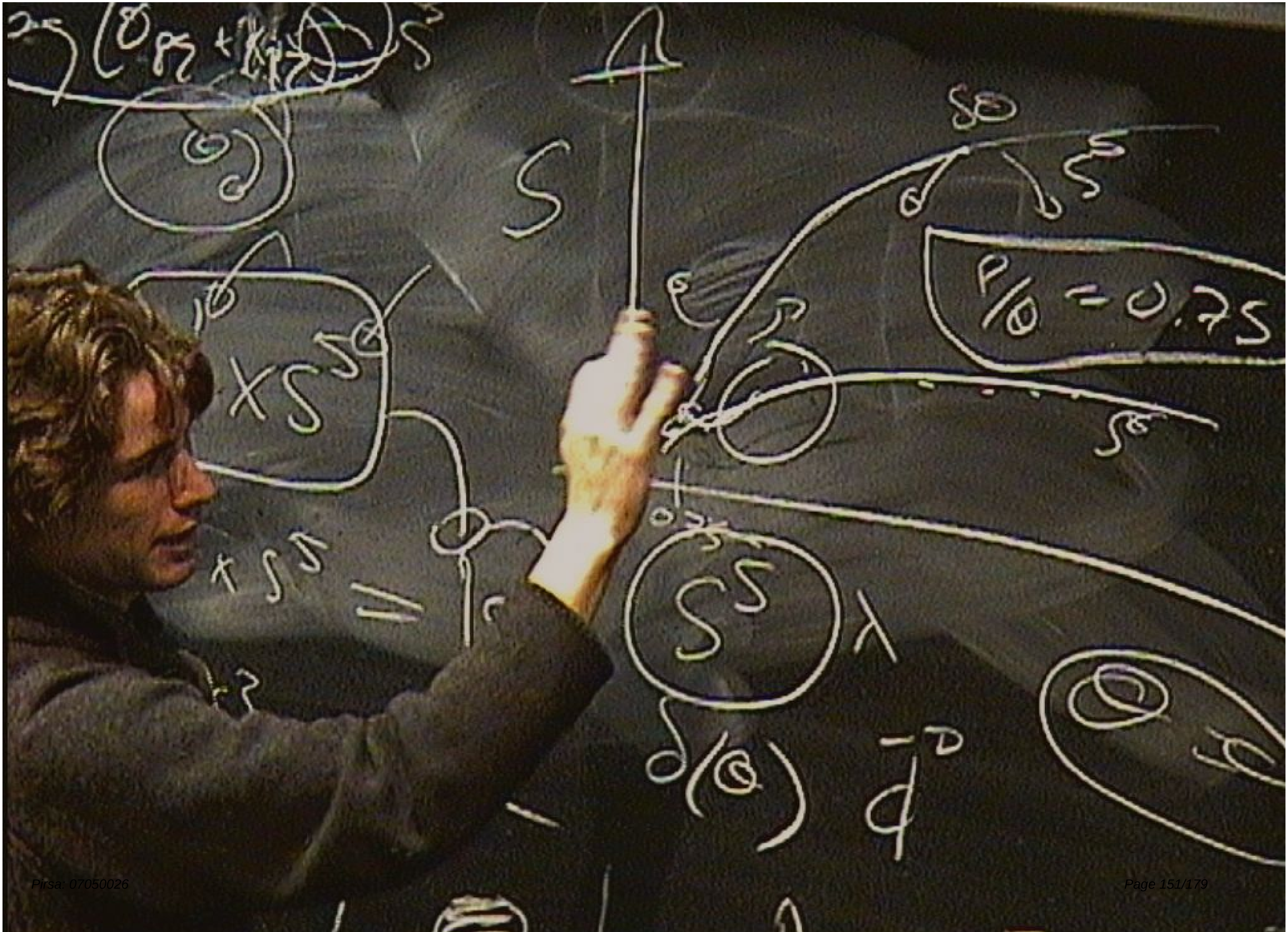

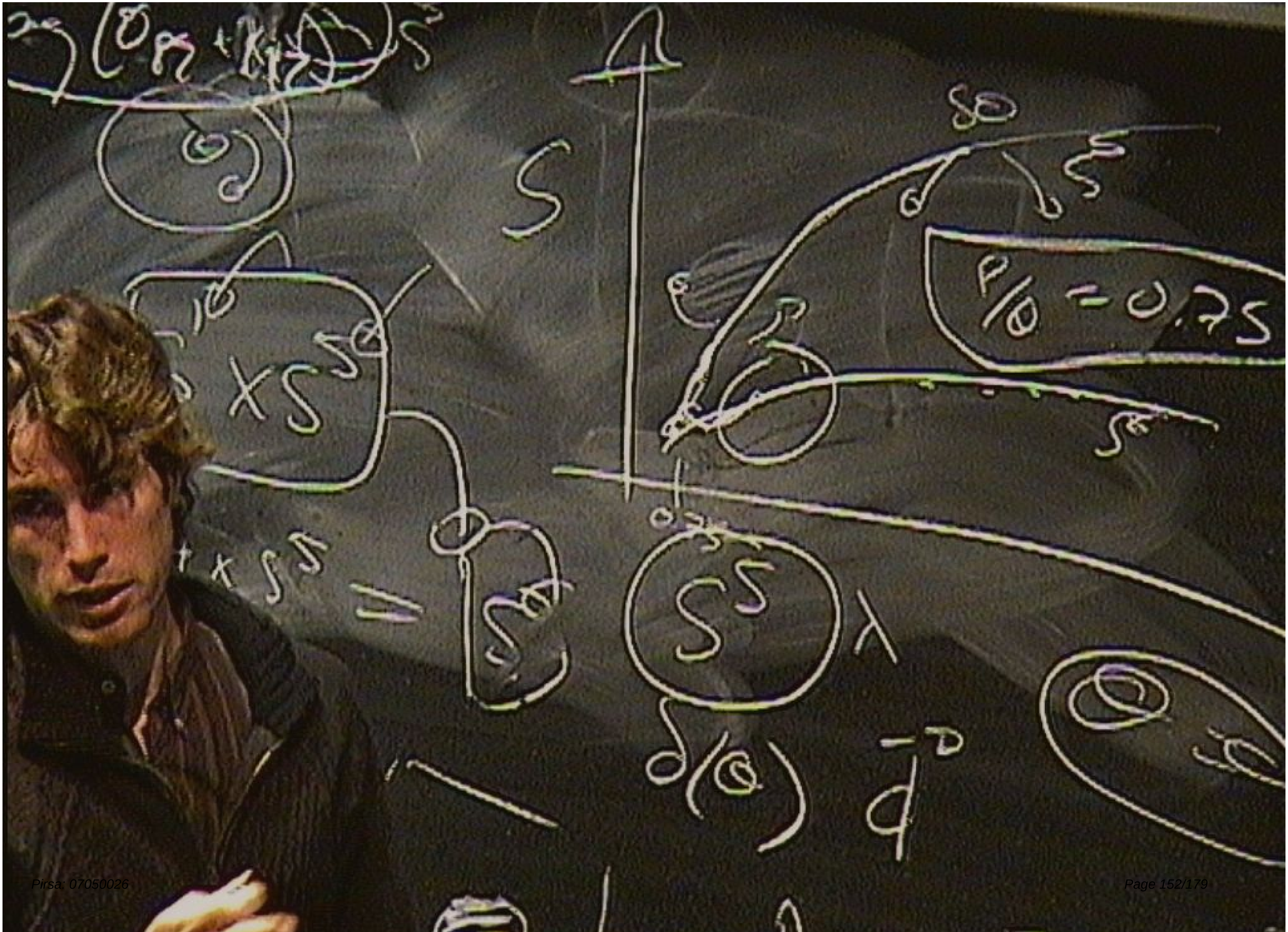
P/Θ = 0.75

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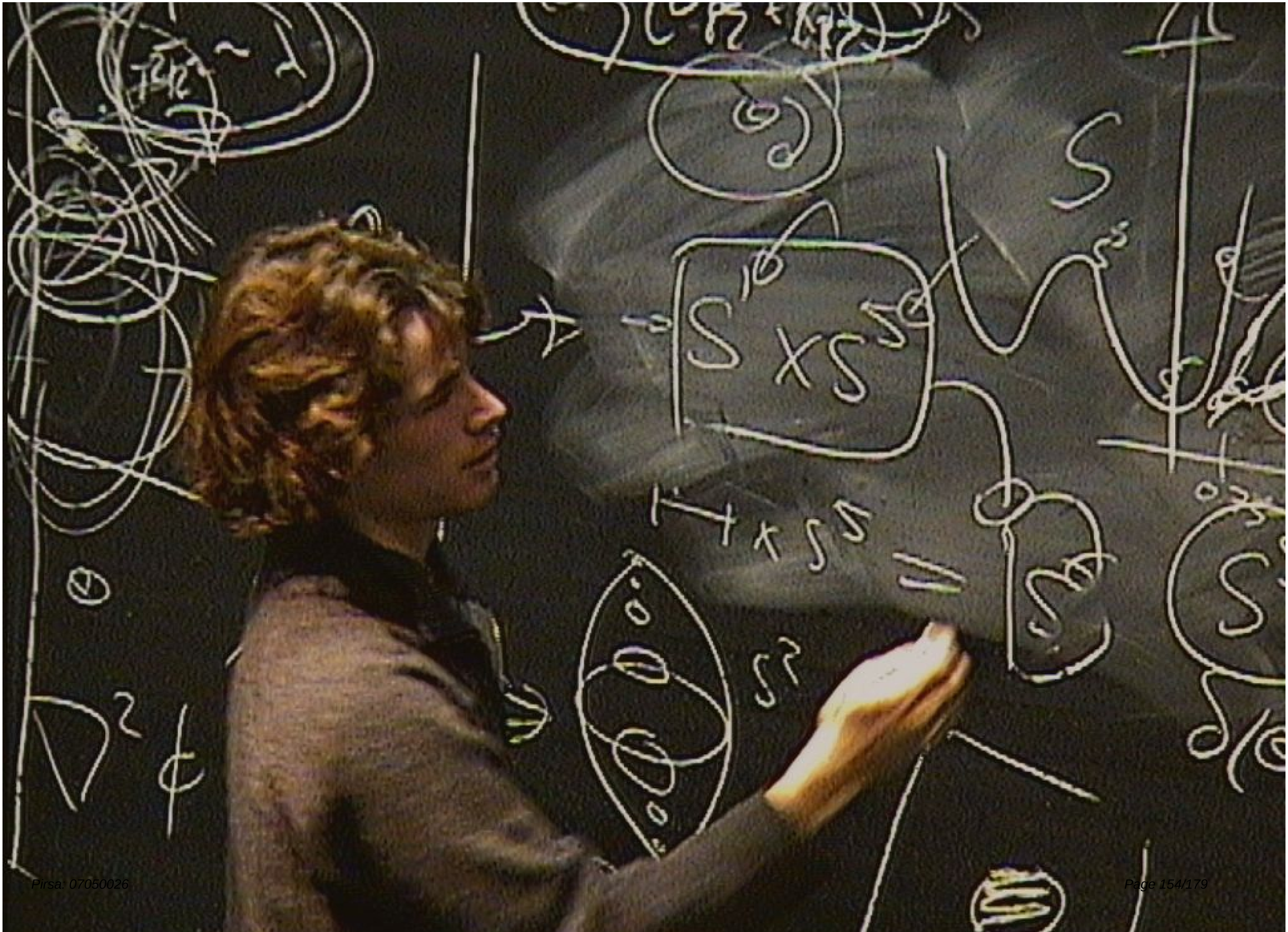

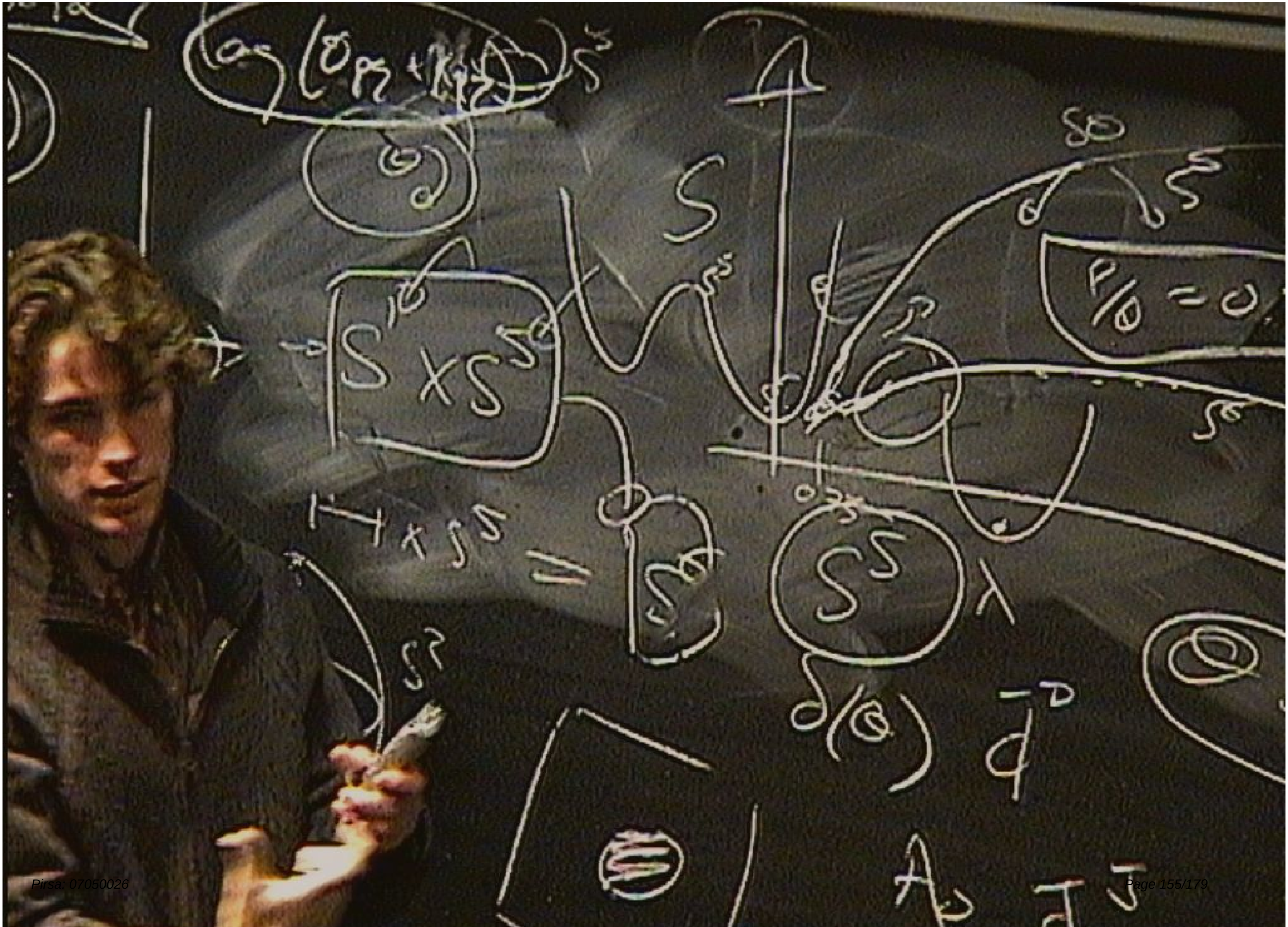

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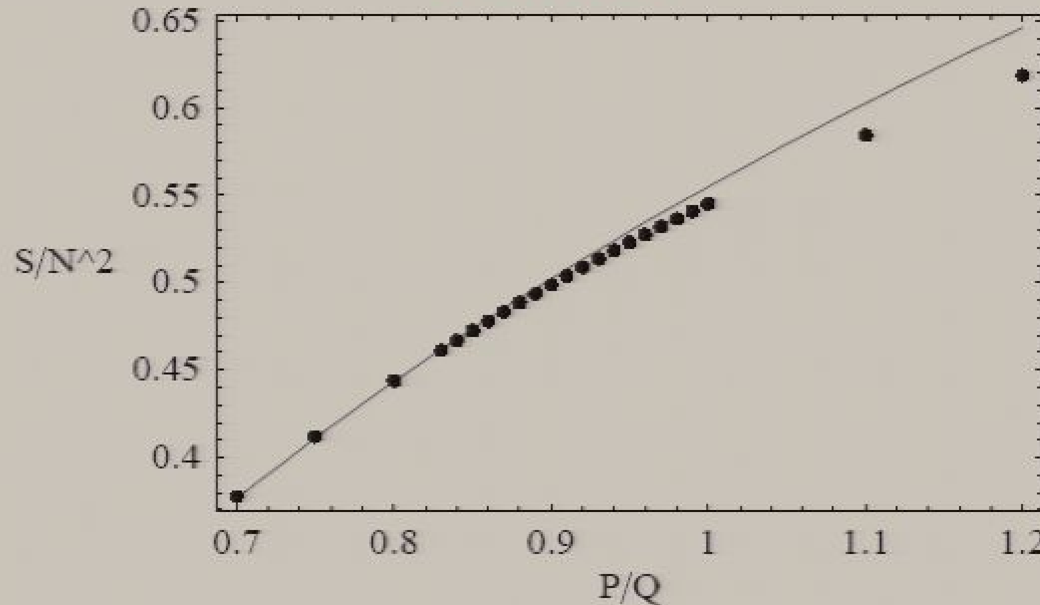

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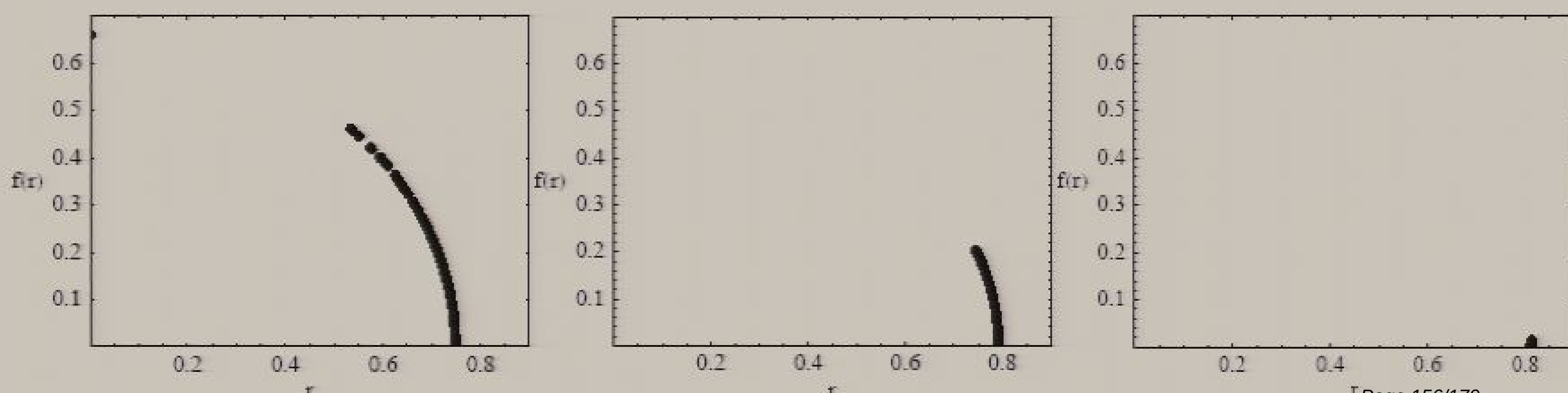

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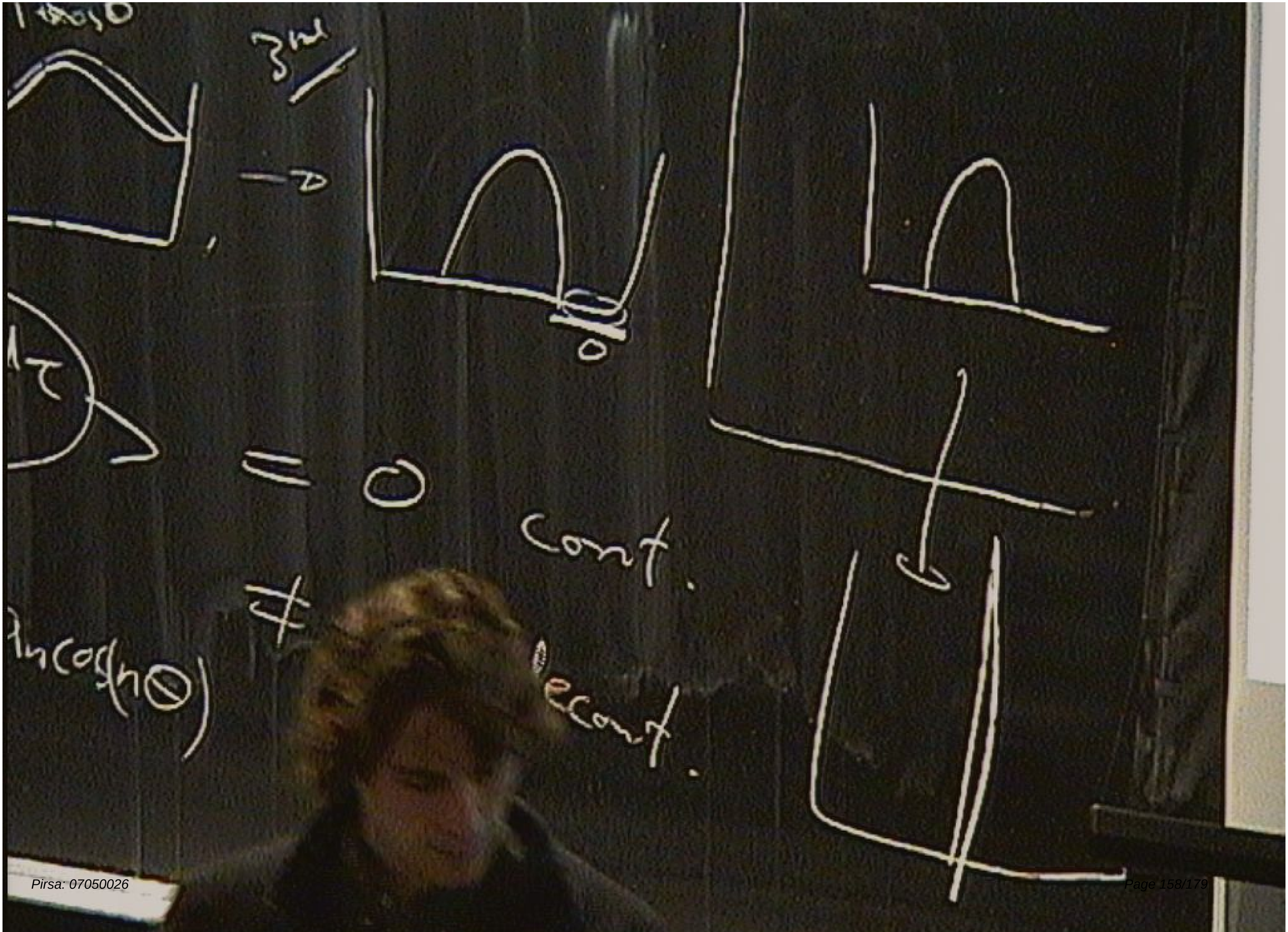
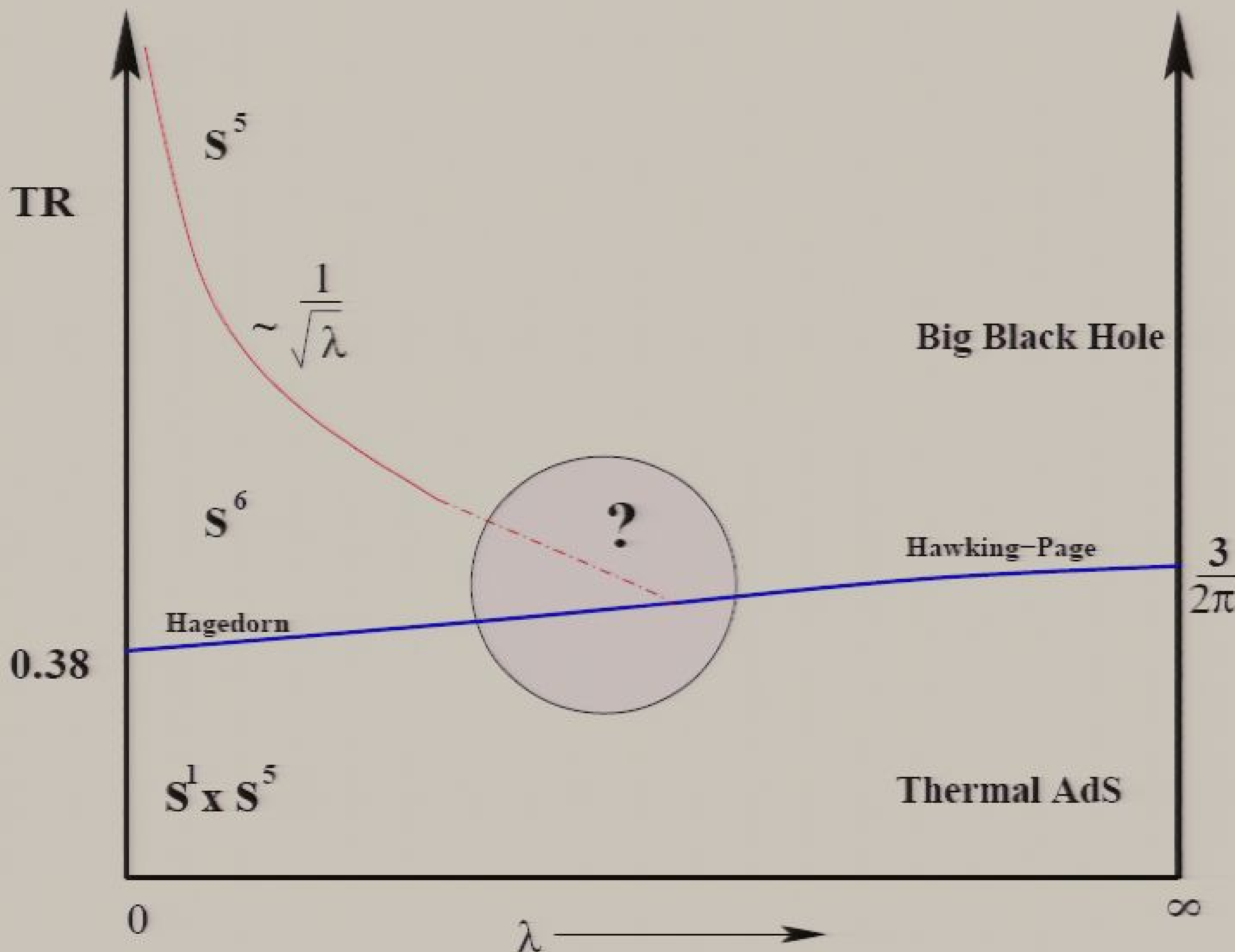
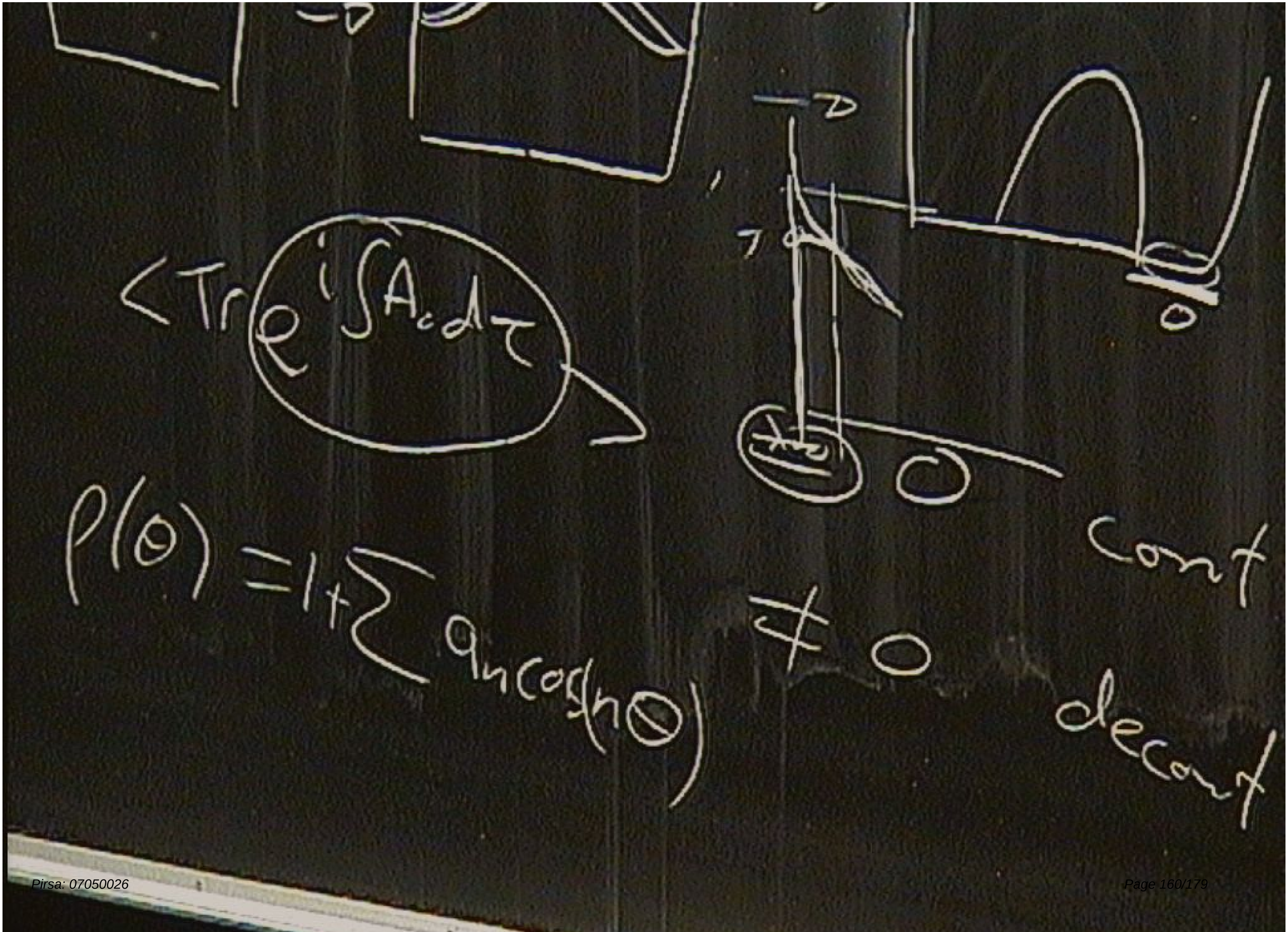
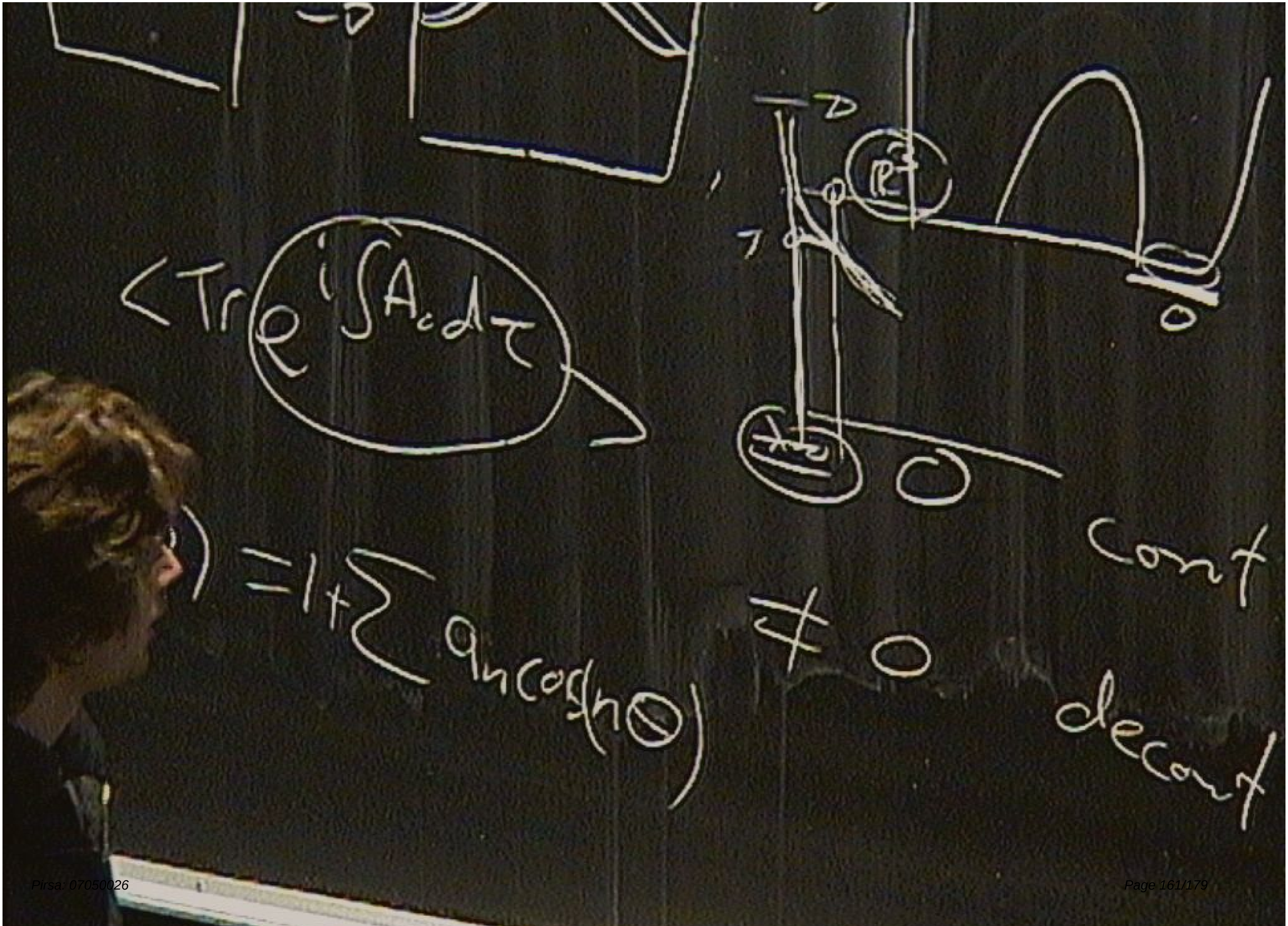

= 0
cont.
cont.

Summary of results

Gürsoy-Hartnoll-Hollowood-Kumar (2007)




<Tr e^{i∫A_0 dτ}
ρ(θ) = 1 + Σ a_n cos(nθ)
≠ 0
0
cont
decont



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≠ 0
cont
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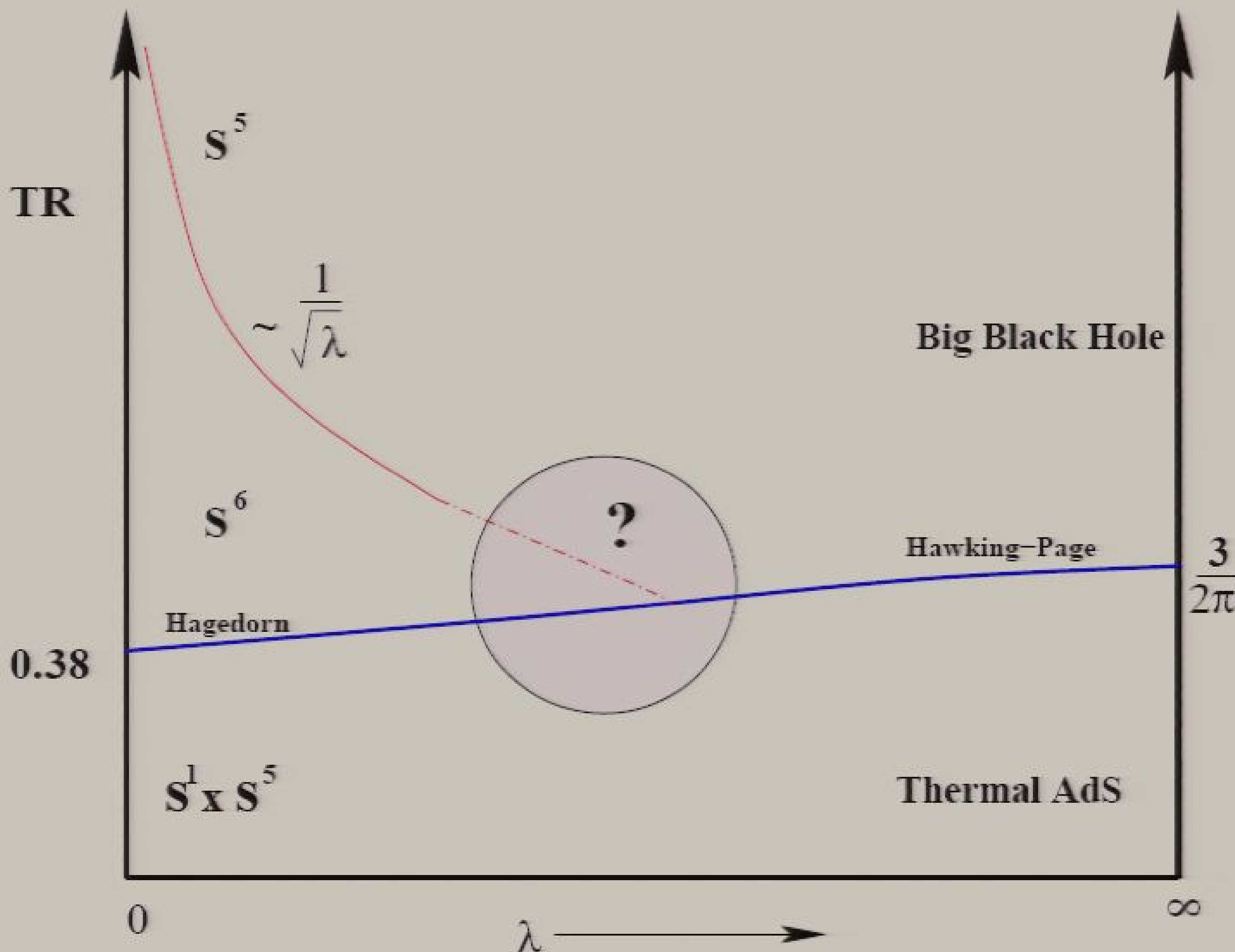

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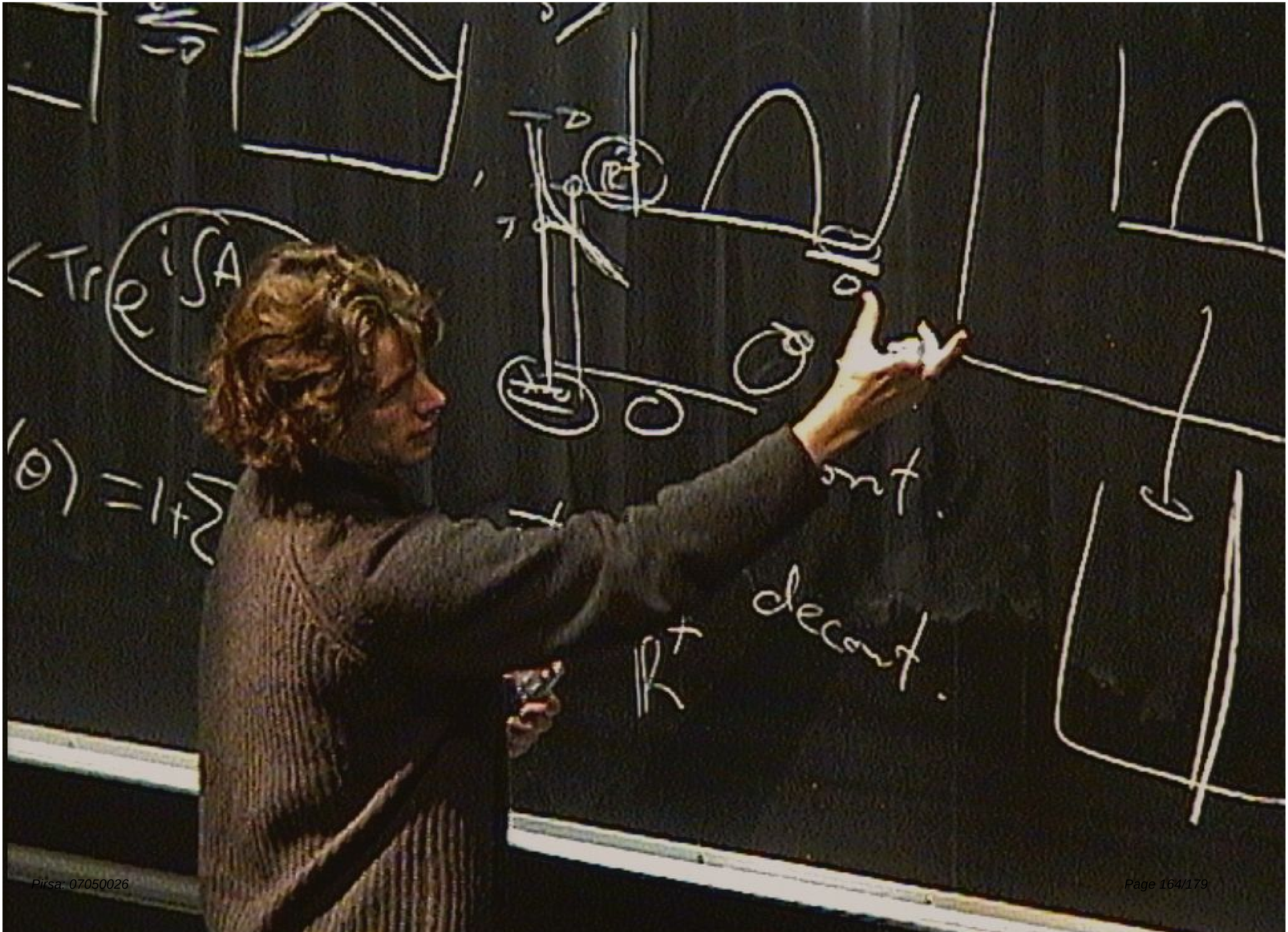
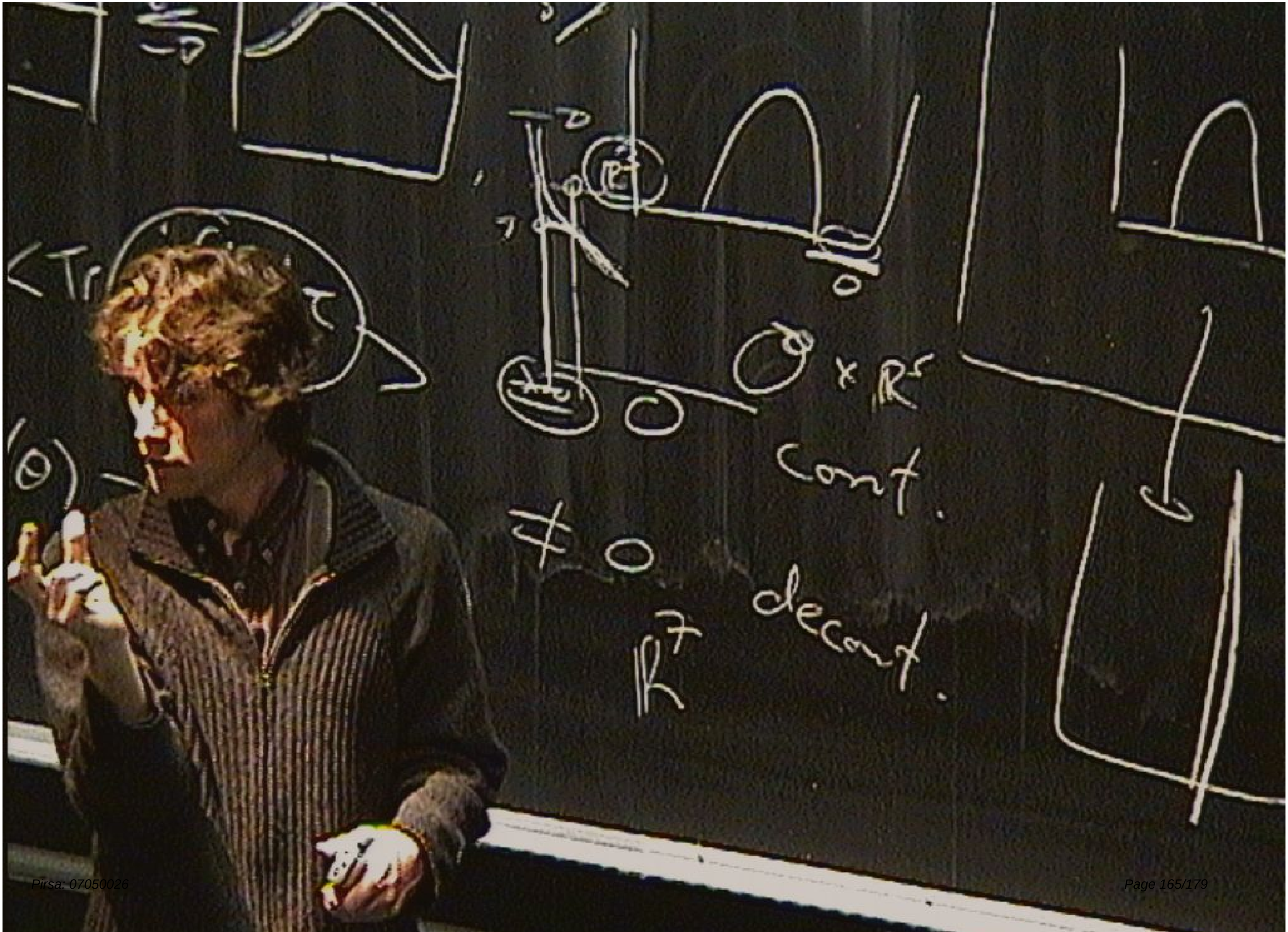
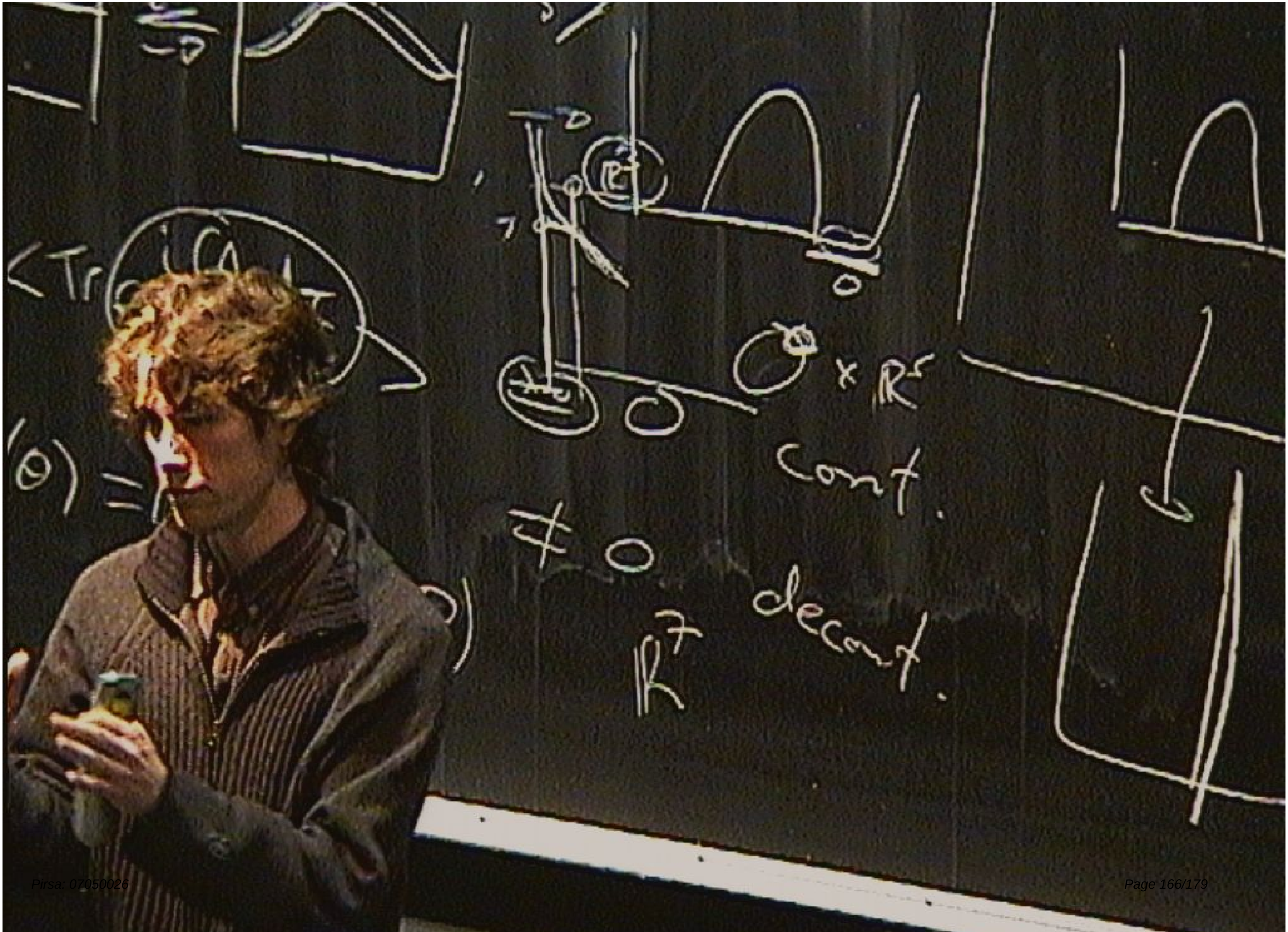
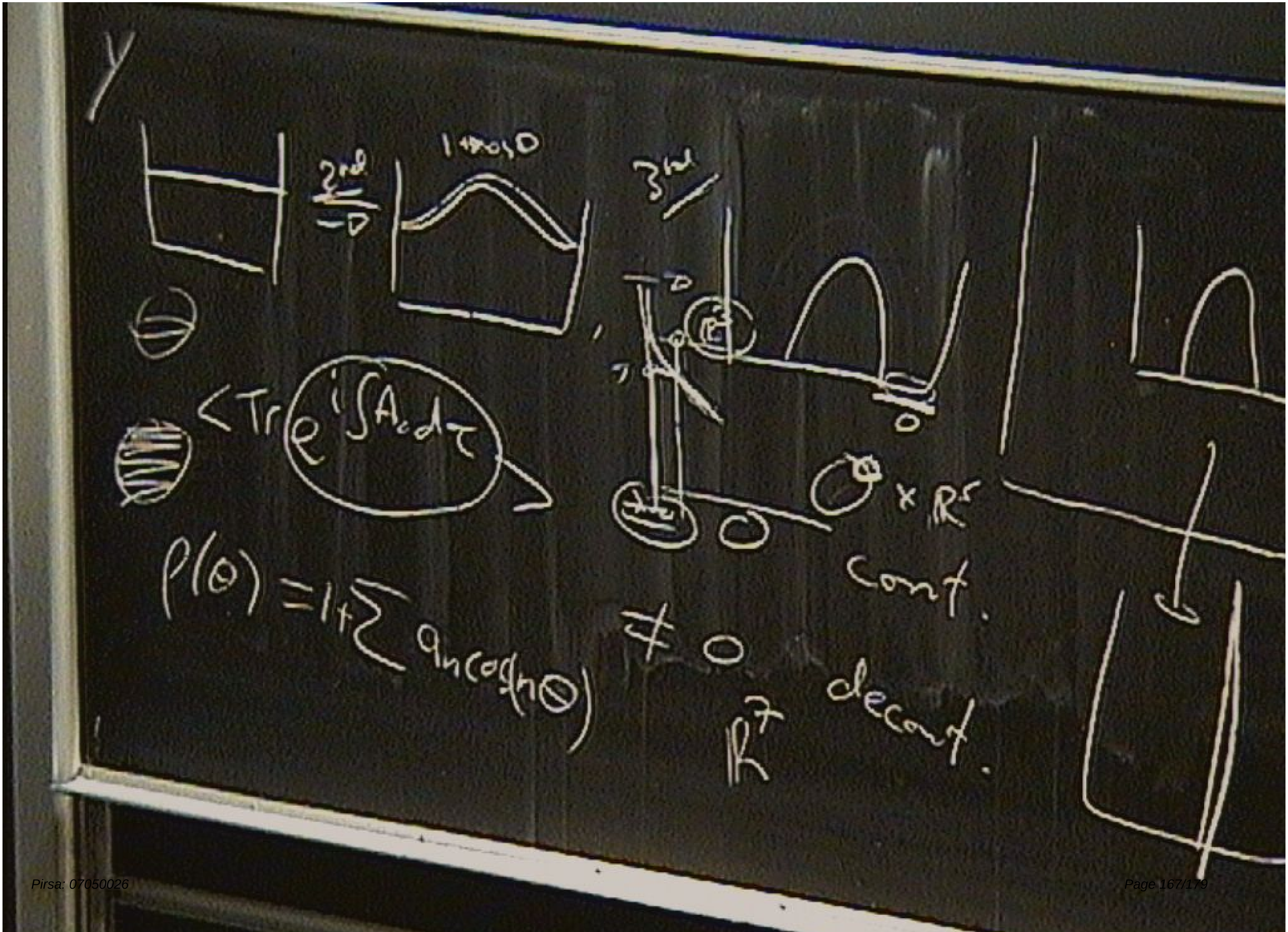
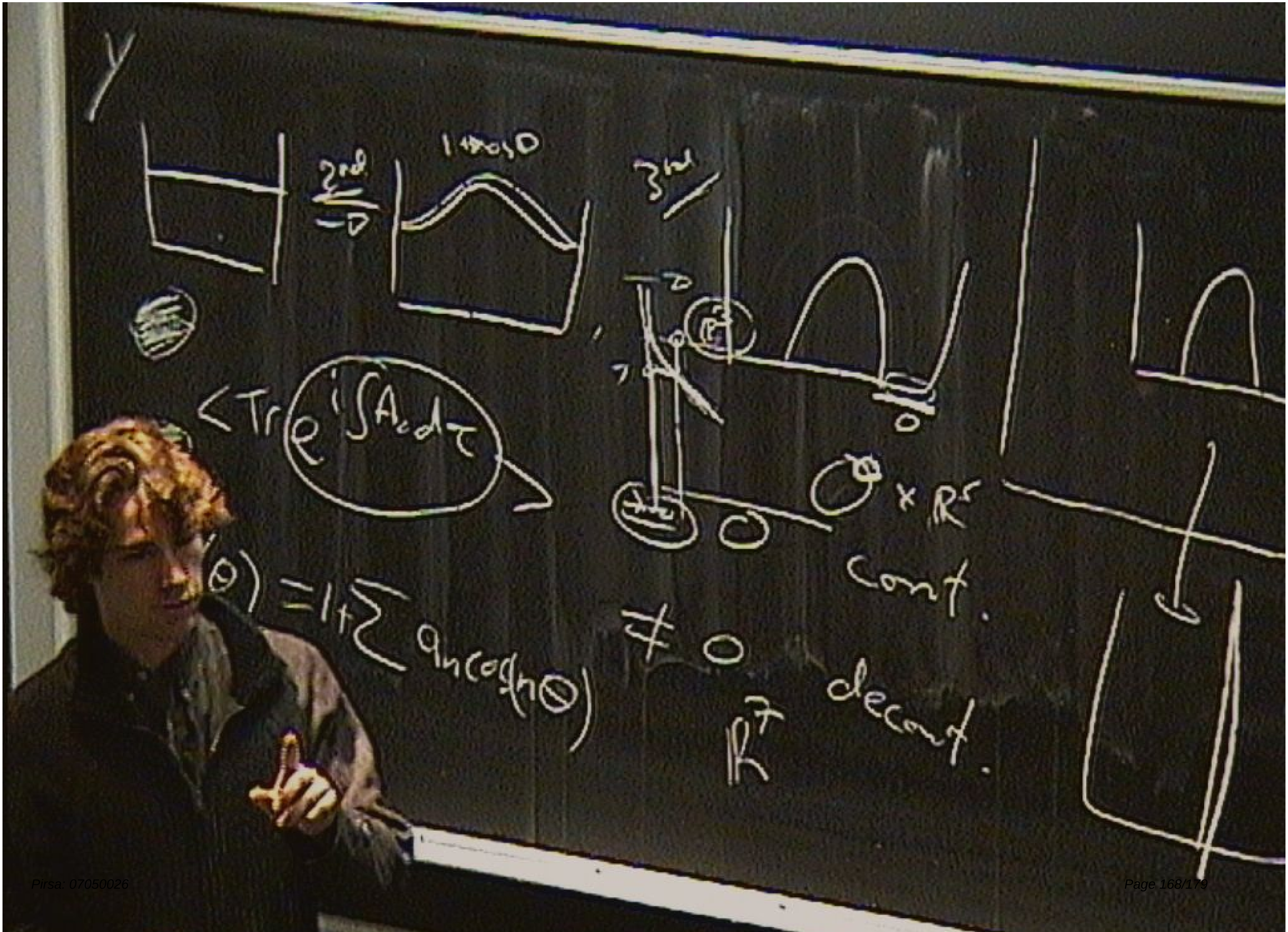
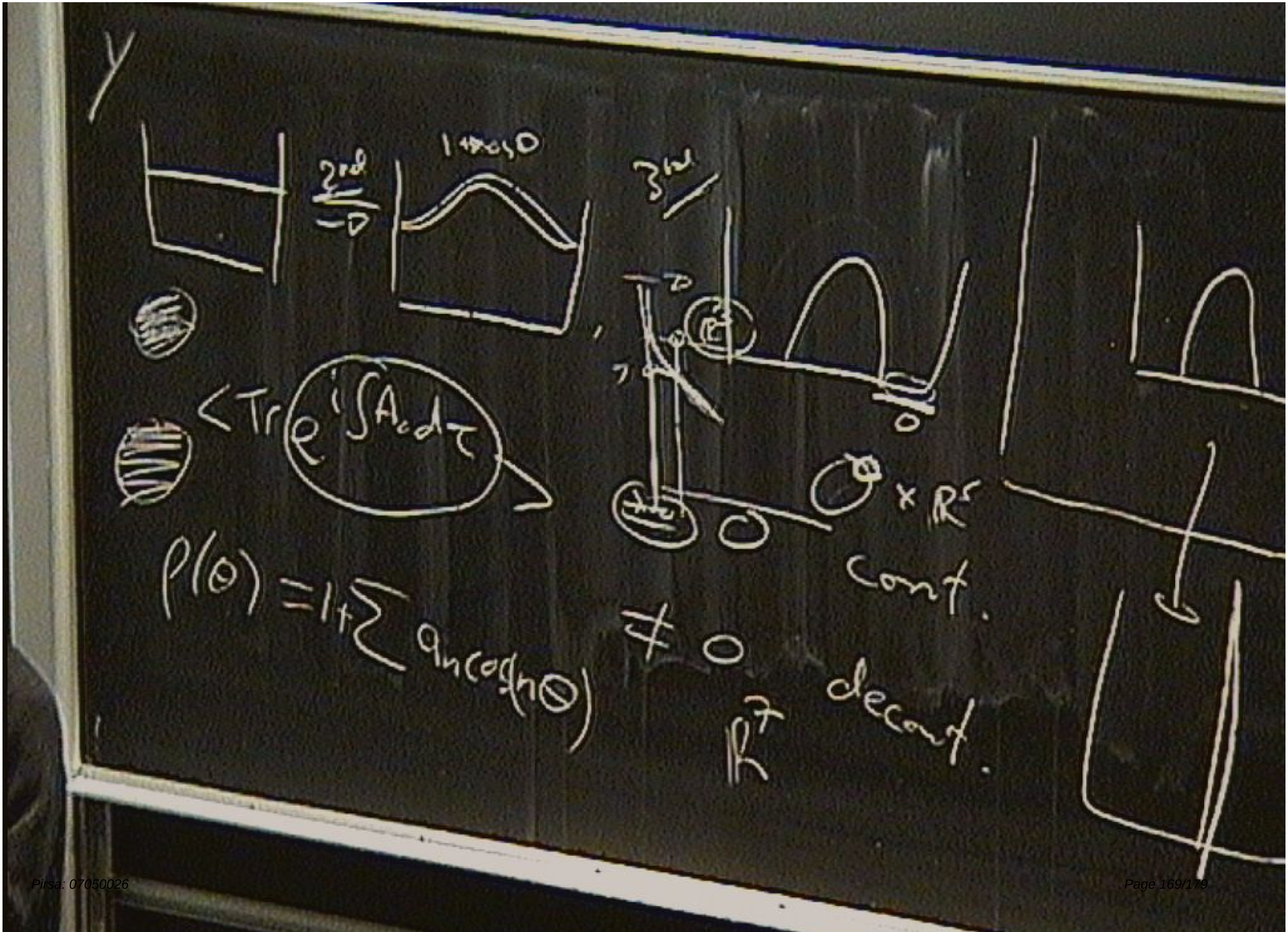

decont.


× R5
cont.
≠ 0
decont.
R7


cont.
decont.


2nd
1+cosθ
3rd
<Tr e^{i∫A_0 dτ}
ρ(θ) = 1+Σ a_n cos(nθ)
≠ 0
× R^5
cont.
decont.
R^7


2nd
1+cosθ
3rd
<Tr e^{i∫A_0 dτ}
= 1 + Σ a_n cos(nθ)
≠ 0
× R^5
cont.
decont.
R^7


2nd
3rd
< Tr e^{i∫A₀dτ}
ρ(Θ) = 1 + Σ aₙ cos(nΘ)
≠ 0
× R⁵
conf.
deconf.
R⁷

Geometrical speculations

- At zero temperature, the S^5 in the eigenvalue distribution is interpreted as the S^5 in the dual spacetime: $AdS_5 \times S^5$.
- The S^1 in our $S^1 \times S^5$ is naturally interpreted as the thermal circle in Euclidean thermal AdS_5 (in fact it's the 'T-dual').
- What about the S^6 and S^5 phases? It would be nice to associate the disappearance of the product S^1 factor with the formation of a horizon (Euclidean cigar).
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Conclusions and implications

- Finite temperature $\mathcal{N} = 4$ SYM theory on S^3 may be described by a joint eigenvalue distribution for A_0 and Φ^J.
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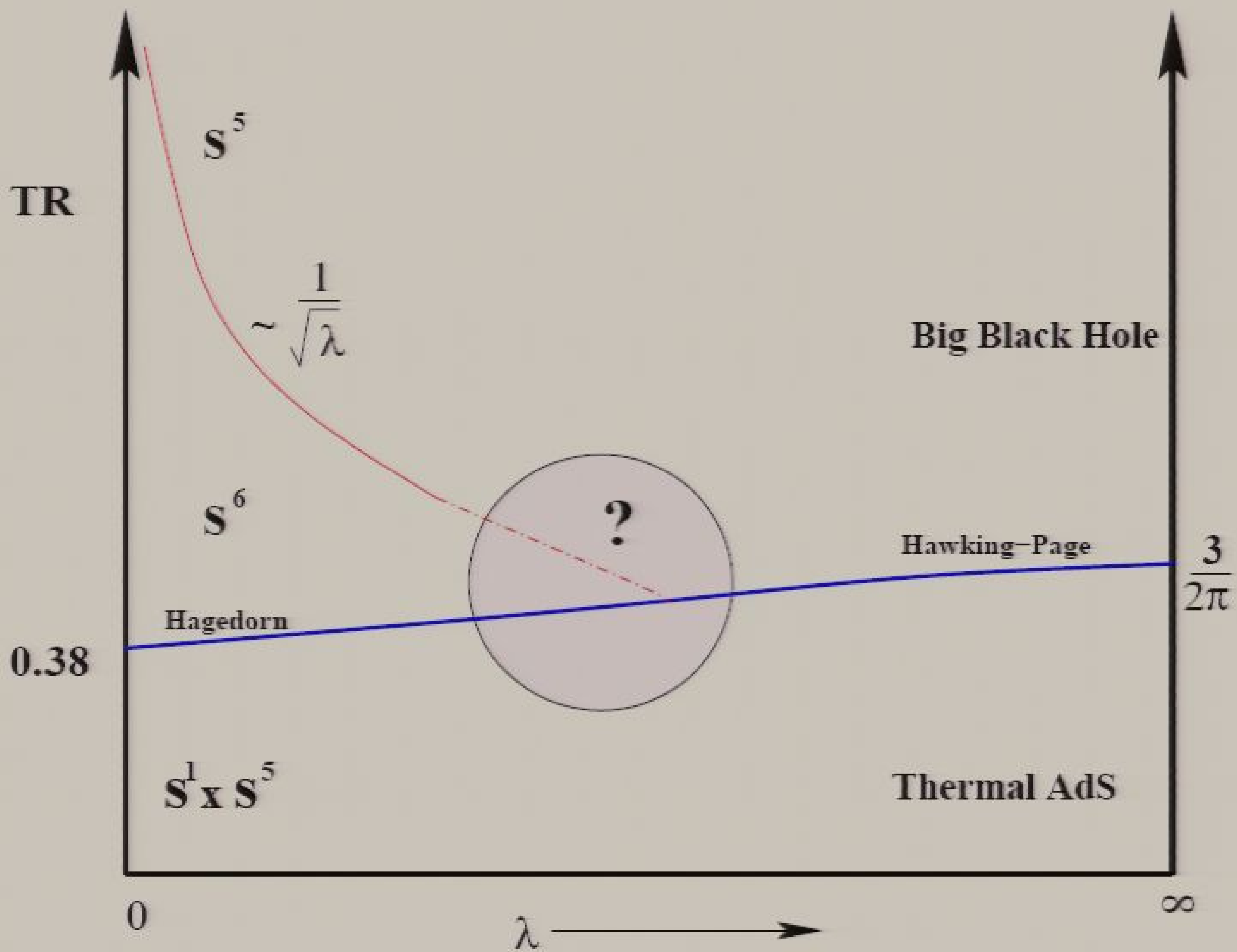
- One implication: suppose we want to trace the AdS big black hole saddle (the S^5?) to weak coupling (c.f. Fidkowski--Hubeny-Kleban-Shenker '03). This is possible, but the saddle is a local maximum at very weak coupling.

Summary of results

Gürsoy-Hartnoll-Hollowood-Kumar (2007)



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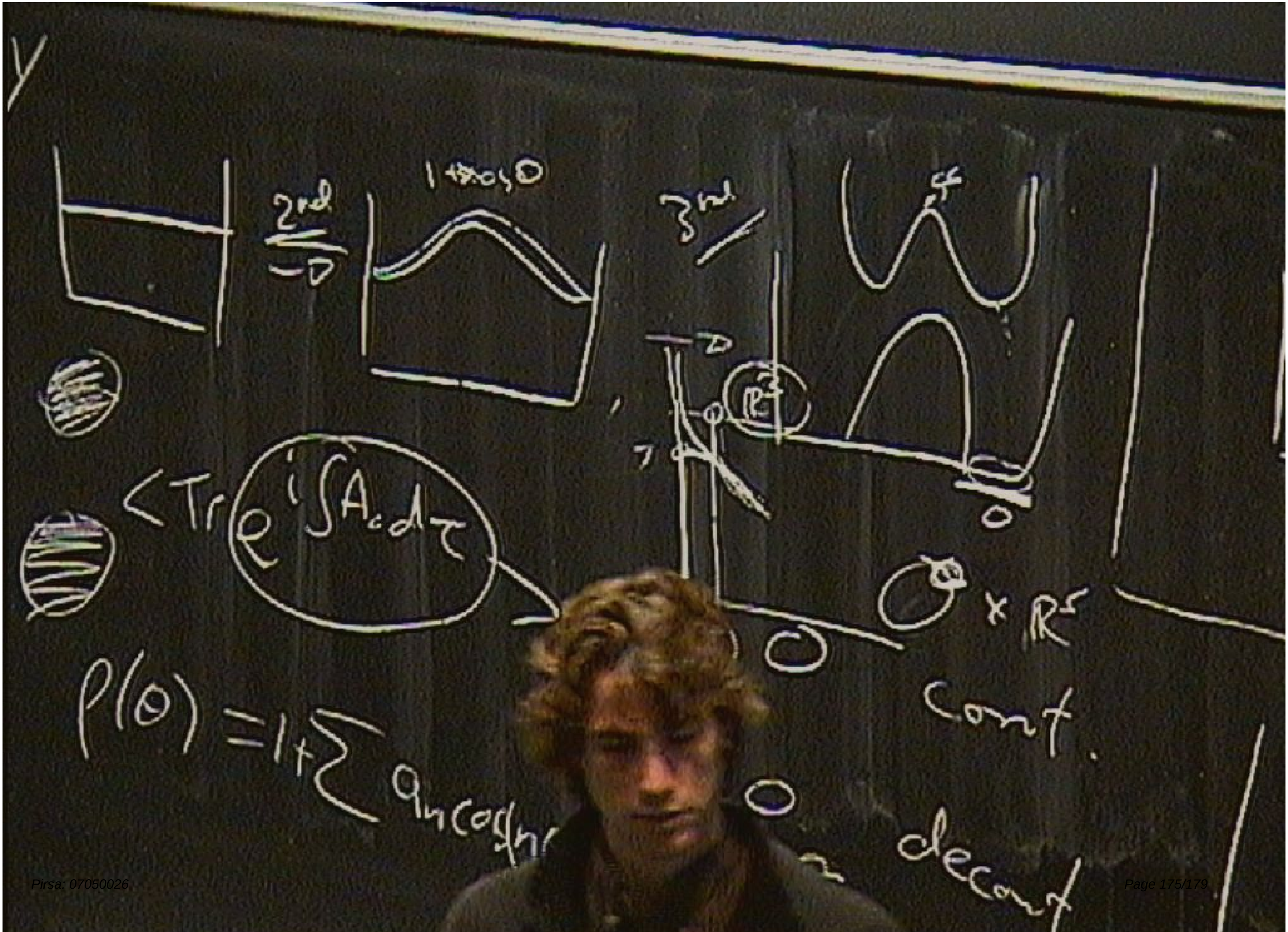
2nd
3rd
cont.
decont

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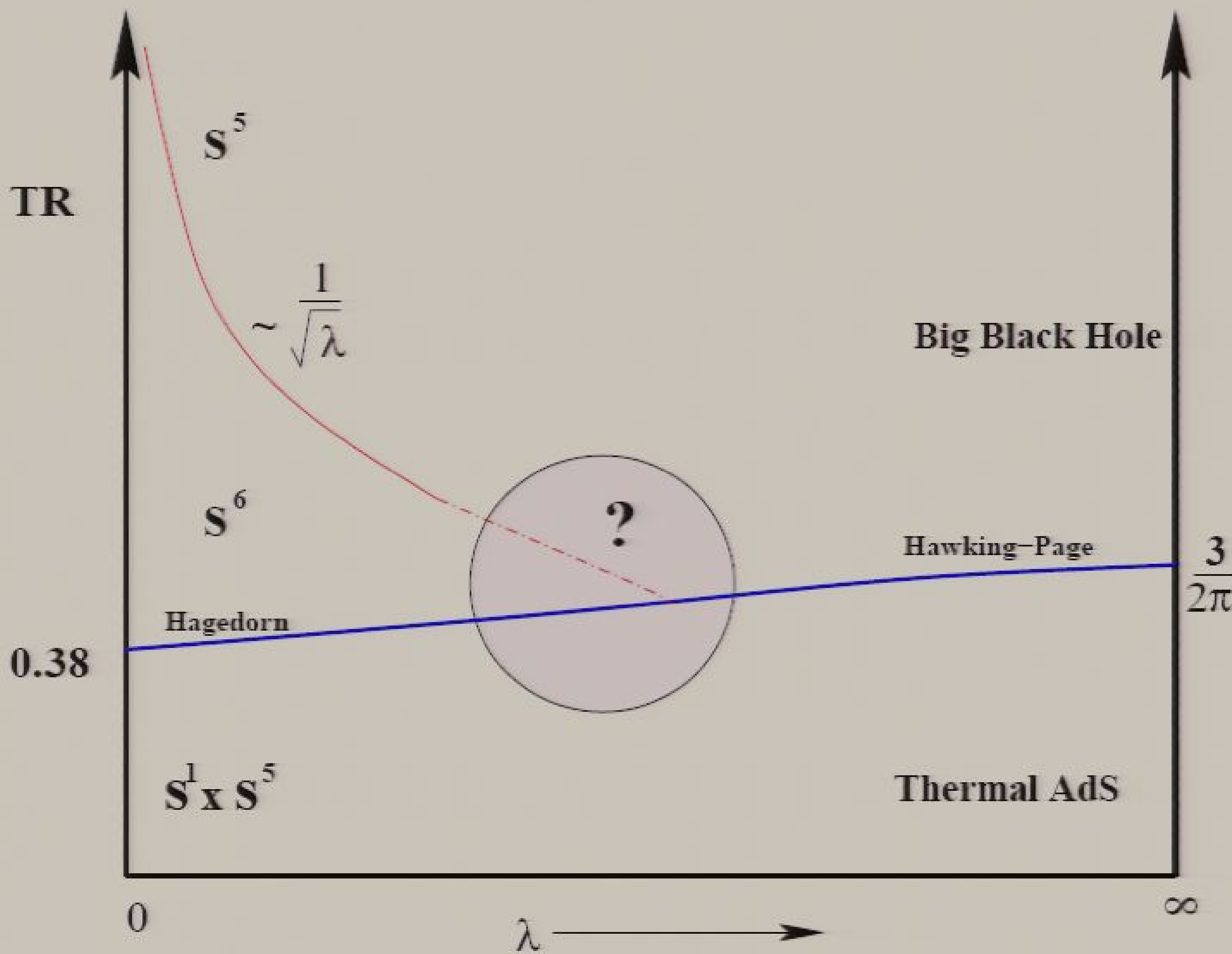

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