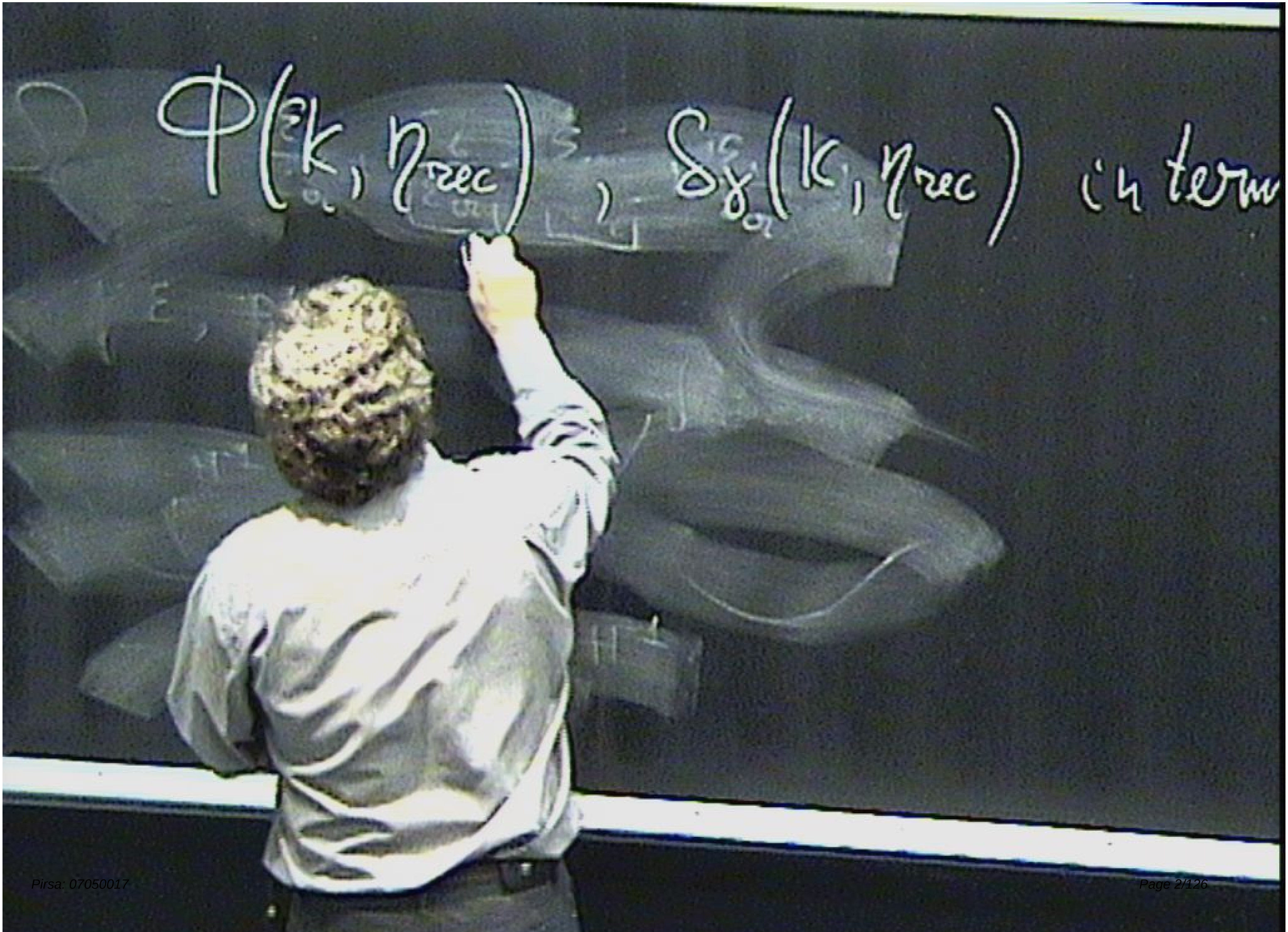
Title: Advanced Topics in Cosmology

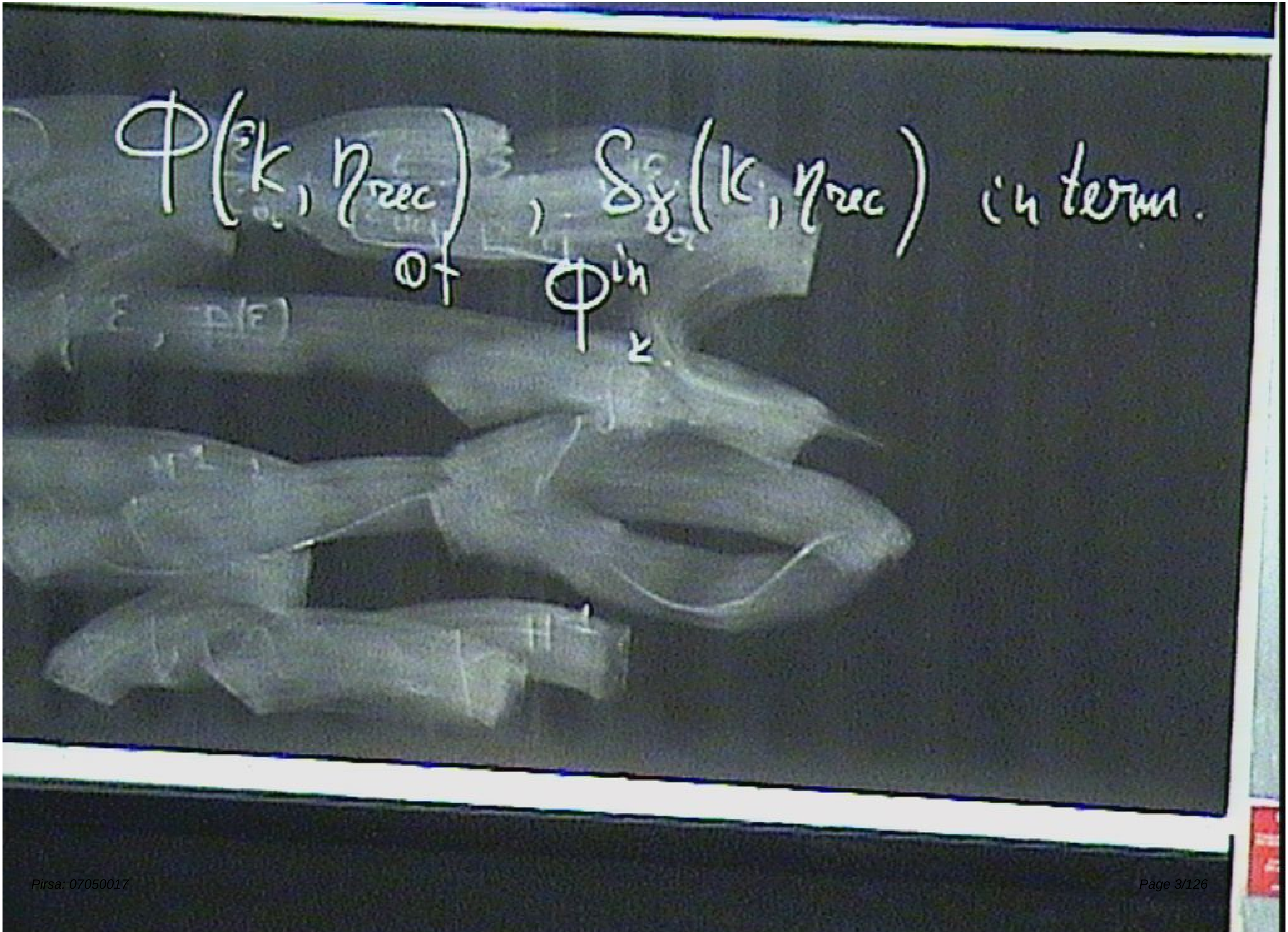
Date: May 24, 2007 12:00 PM

URL: http://pirsa.org/07050017

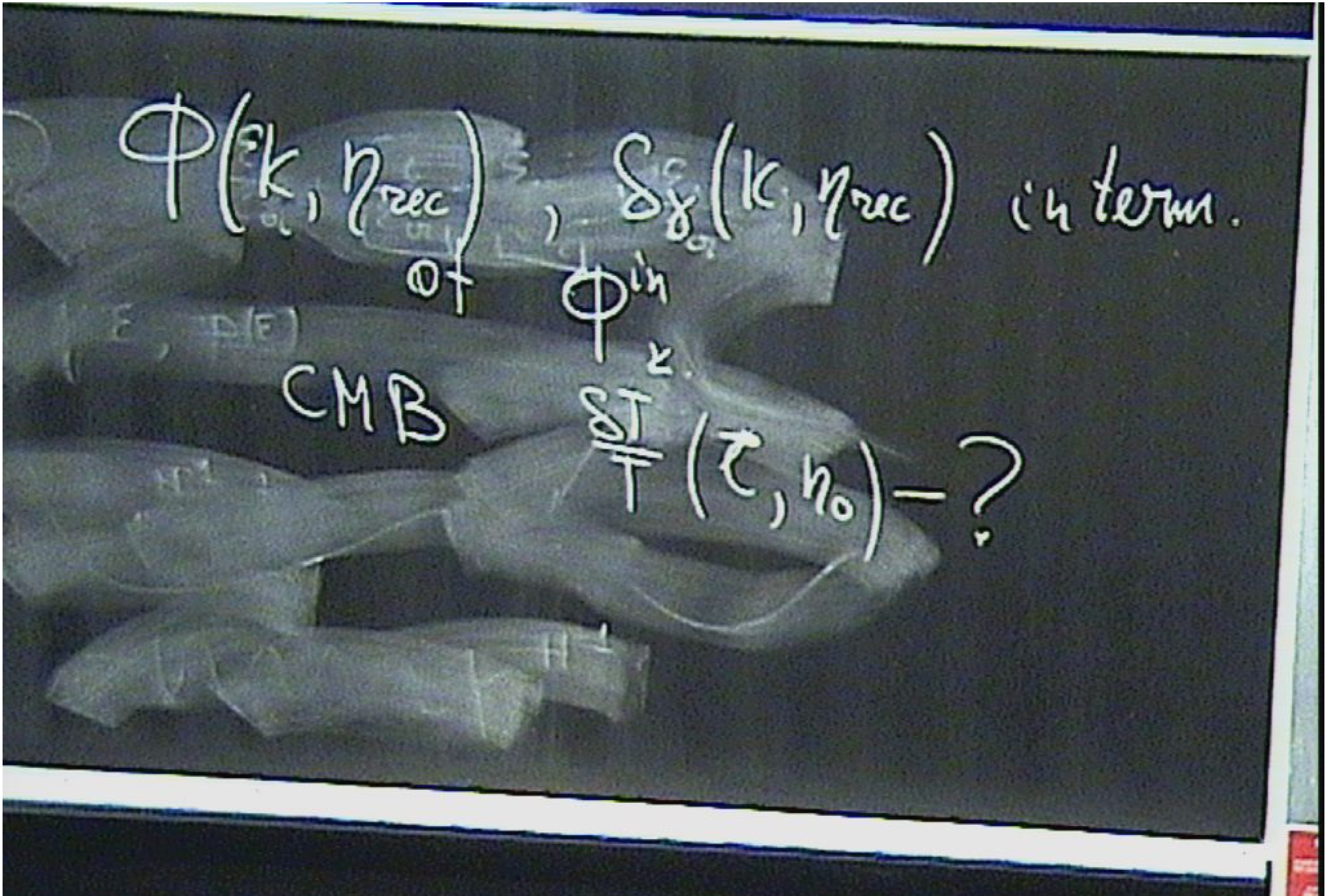
Abstract: Class 6 part 2



$\Phi(k, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term

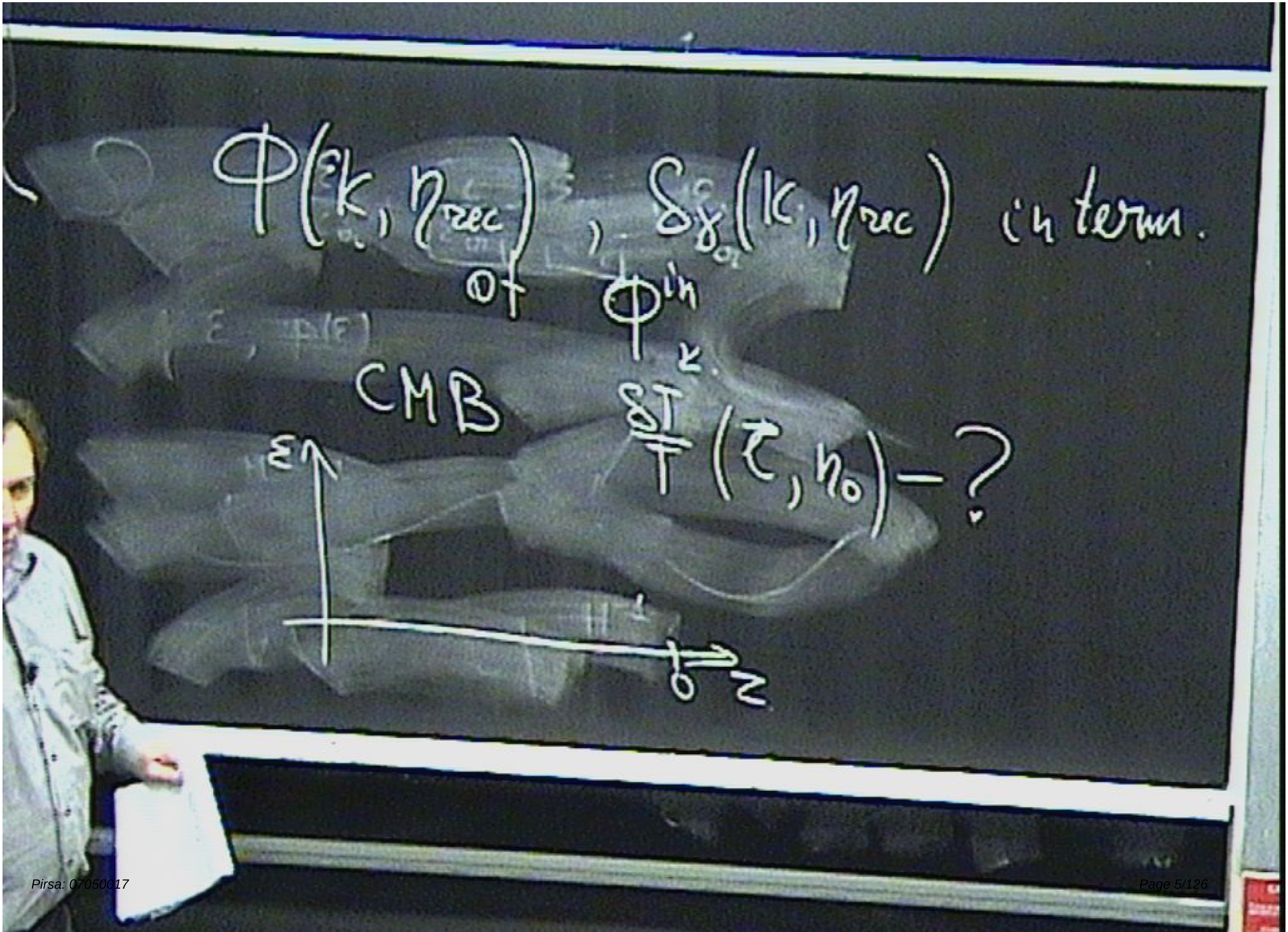


$\Phi(k, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of Φ^{in}_k.



$\Phi(k, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of Φ^{in}_k

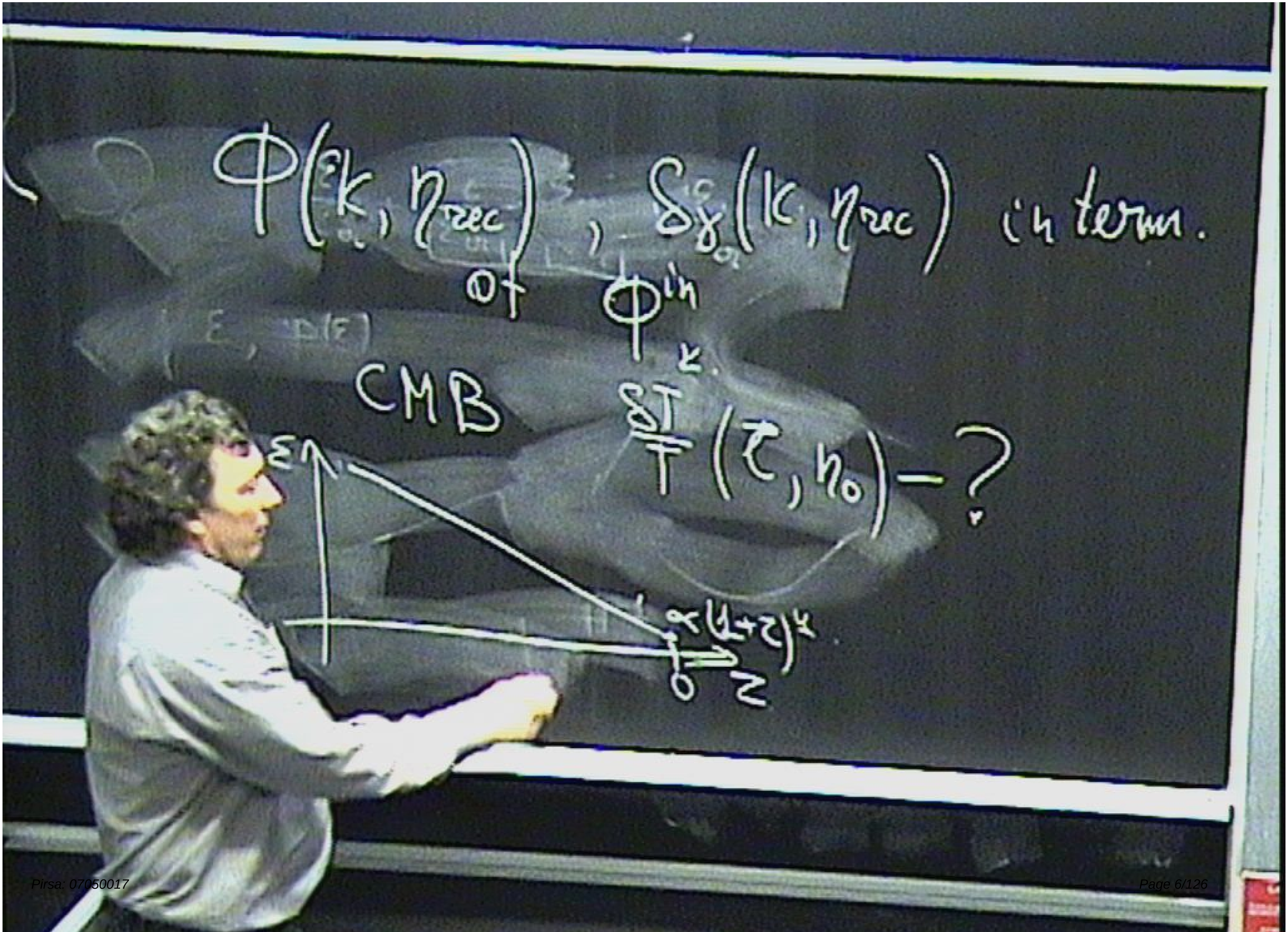
CMB $\frac{\delta T}{T}(\vec{n}, \eta_0)$ — ?



$\Phi(\vec{k}, \eta_{rec})$, $\delta_{\gamma}(k, \eta_{rec})$ in term. of Φ^{in}

$\downarrow$

CMB $\frac{\delta T}{T}(\vec{e}, \eta_0)$ − ?

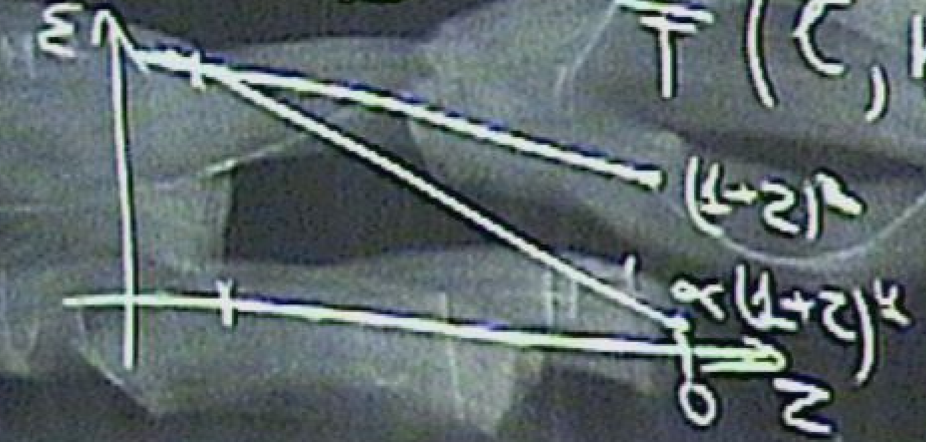


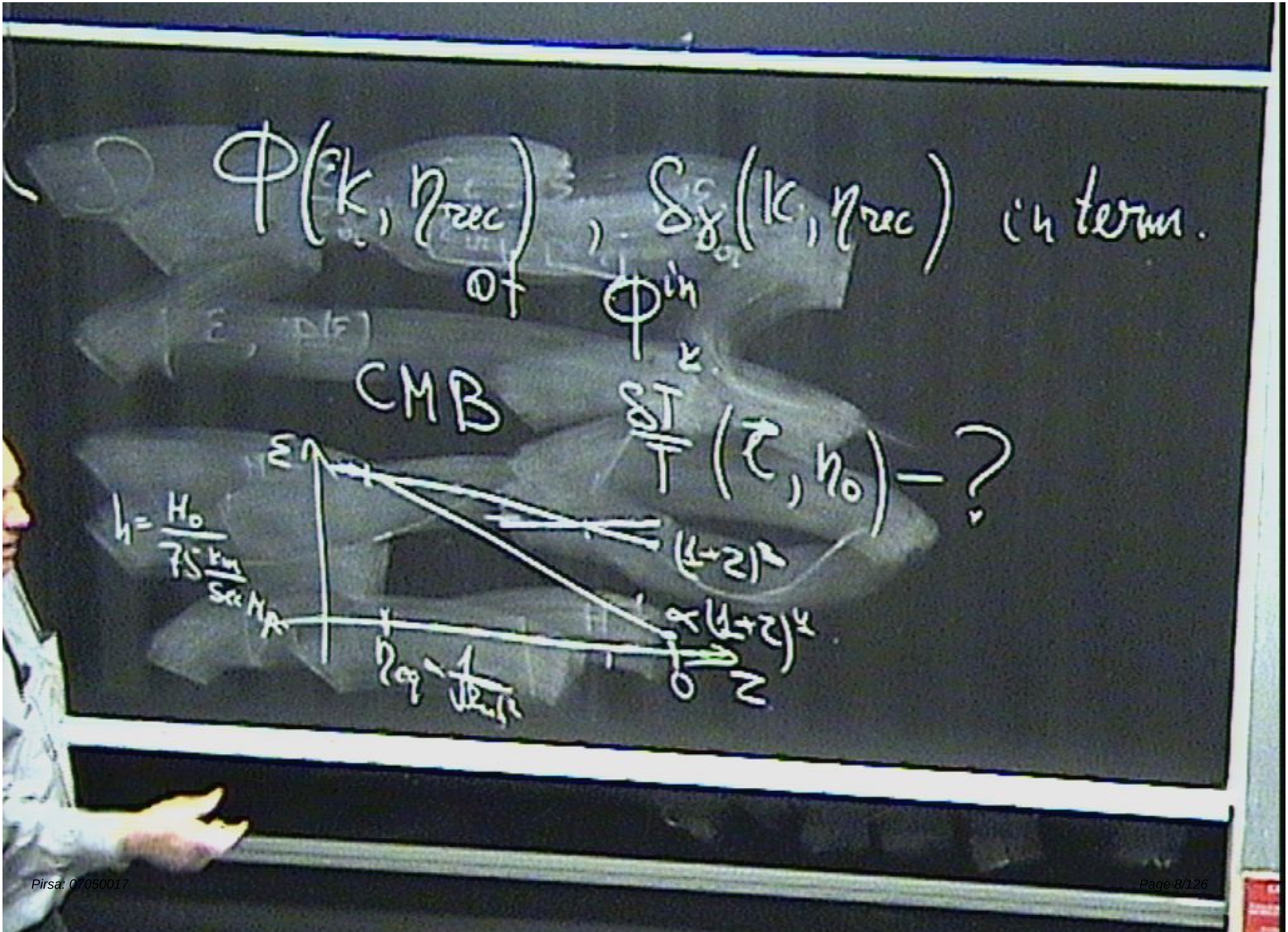
$\Phi(\vec{k}, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of Φ^{in}_{k}

CMB $\frac{\delta T}{T}(\vec{n}, \eta_0)$ – ?

$\Phi(k, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of Φ^{in}_k

CMB $\frac{\delta T}{T}(\vec{n}, \eta_0)$ – ?





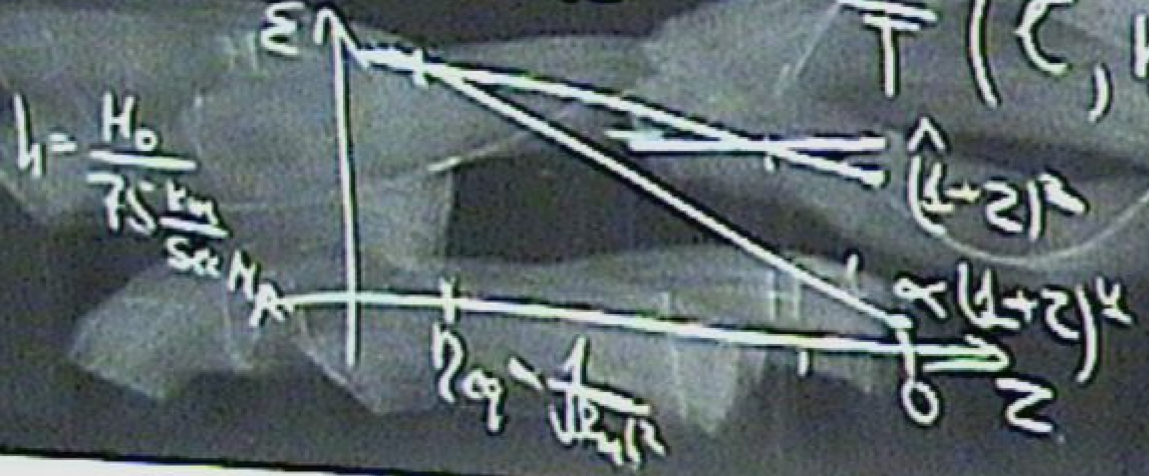
$\Phi(\vec{k}, \eta_{rec})$, $\delta_\gamma(\vec{k}, \eta_{rec})$ in term. of Φ^{in}_{k}

CMB $\frac{\delta T}{T}(\vec{n}, \eta_0)$ – ?

$h = \frac{H_0}{75 \frac{km}{sec\, Mpc}}$

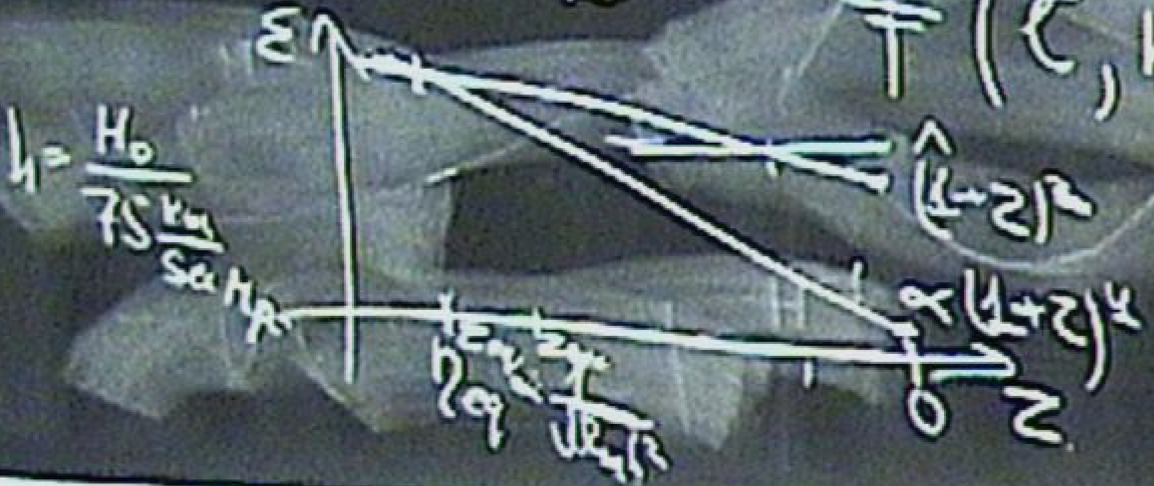
$\Phi(\vec{k}, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of Φ^{in}_{k}.

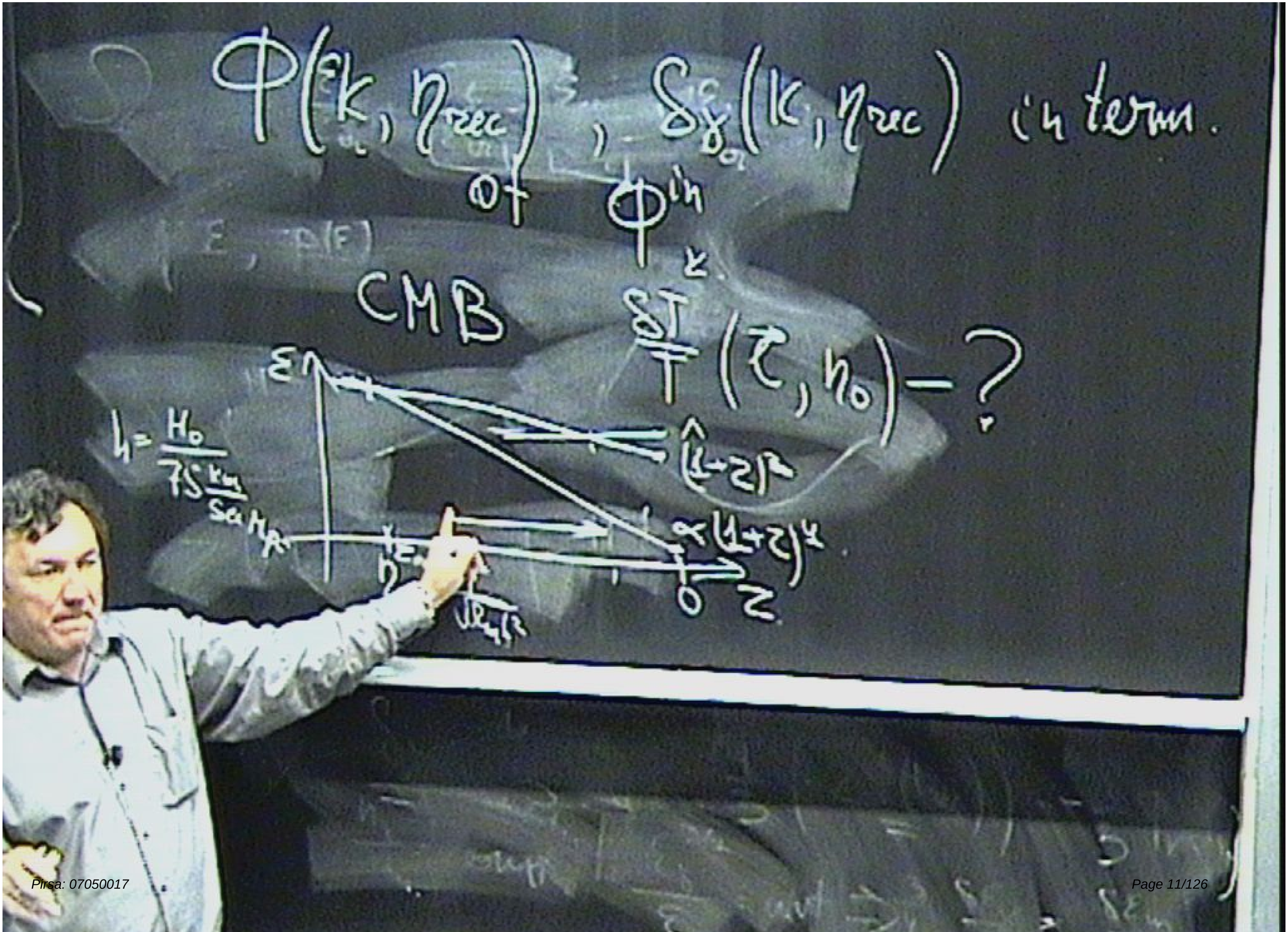
CMB $\frac{\delta T}{T}(\vec{n}, \eta_0) - ?$



$\Phi(k, \eta_{rec})$, $\delta_{\gamma}(k, \eta_{rec})$ in term. of Φ^{in}_{k}

CMB $\frac{\delta T}{T}(\vec{n}, \eta_0)$ — ?



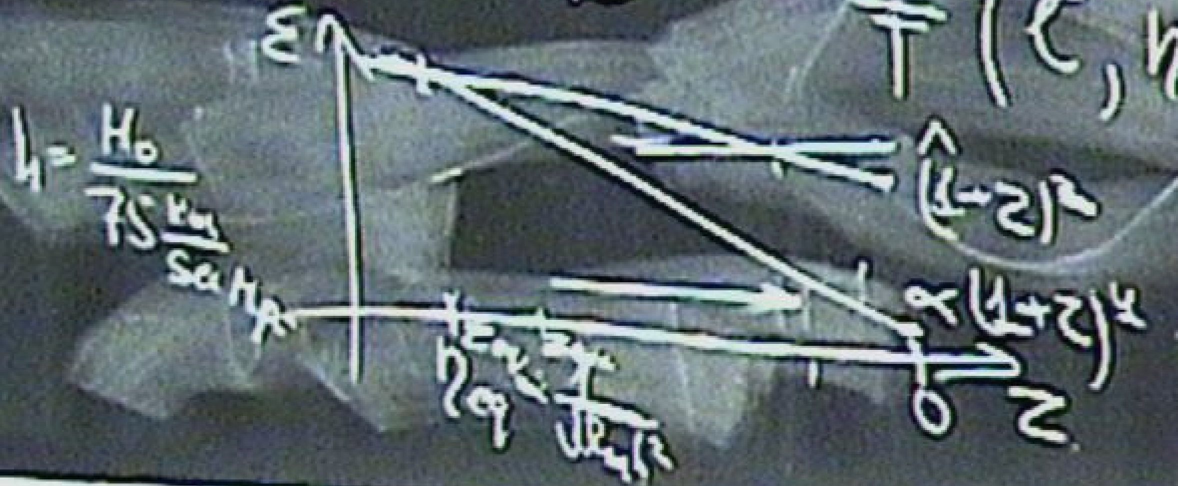


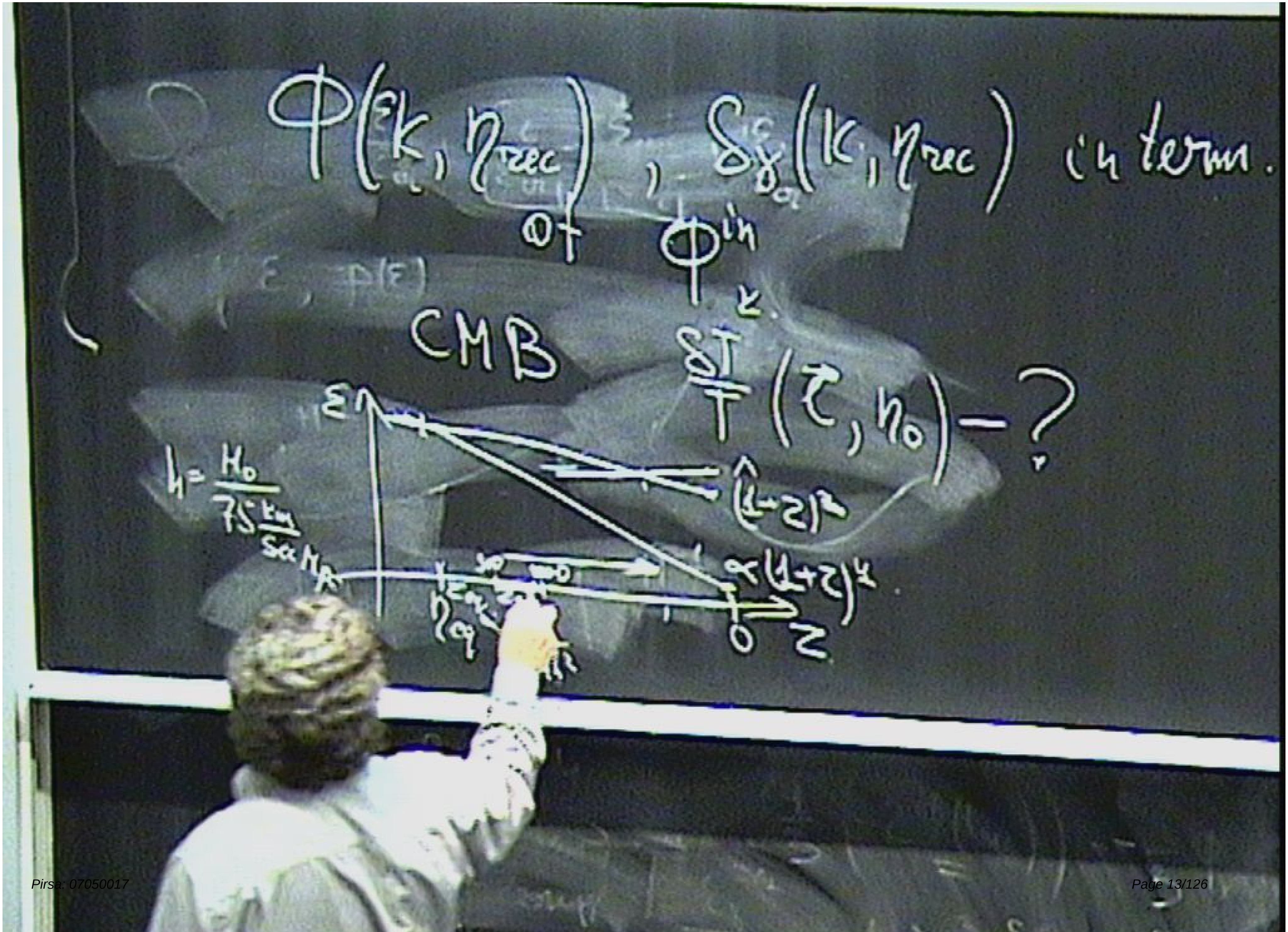
$\Phi(\vec{k}, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of Φ^{in}_k

CMB $\frac{\delta T}{T}(\vec{\tau}, \eta_0) - ?$

$\Phi(\vec{k}, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of Φ^{in}_{k}

CMB $\frac{\delta T}{T}(\vec{n}, \eta_0)$ — ?





$\Phi(k, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of Φ^{in}

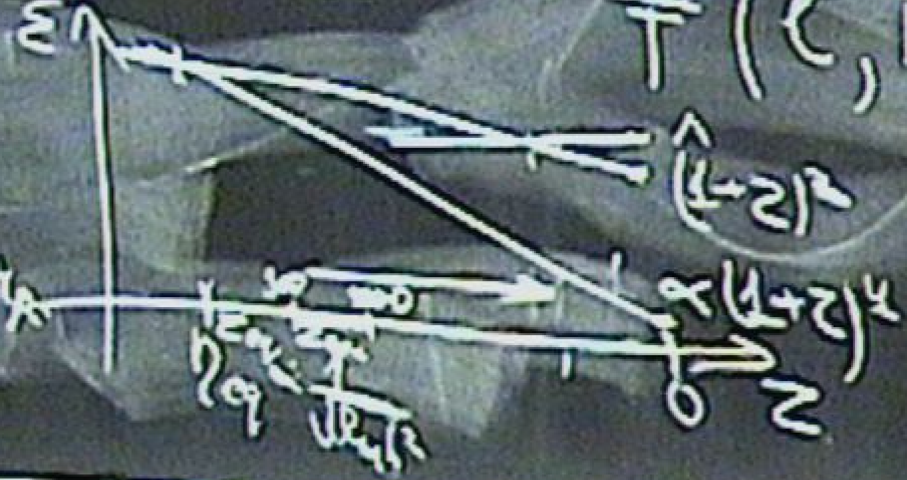
CMB $\frac{\delta T}{T}(\vec{n}, \eta_0)$ — ?

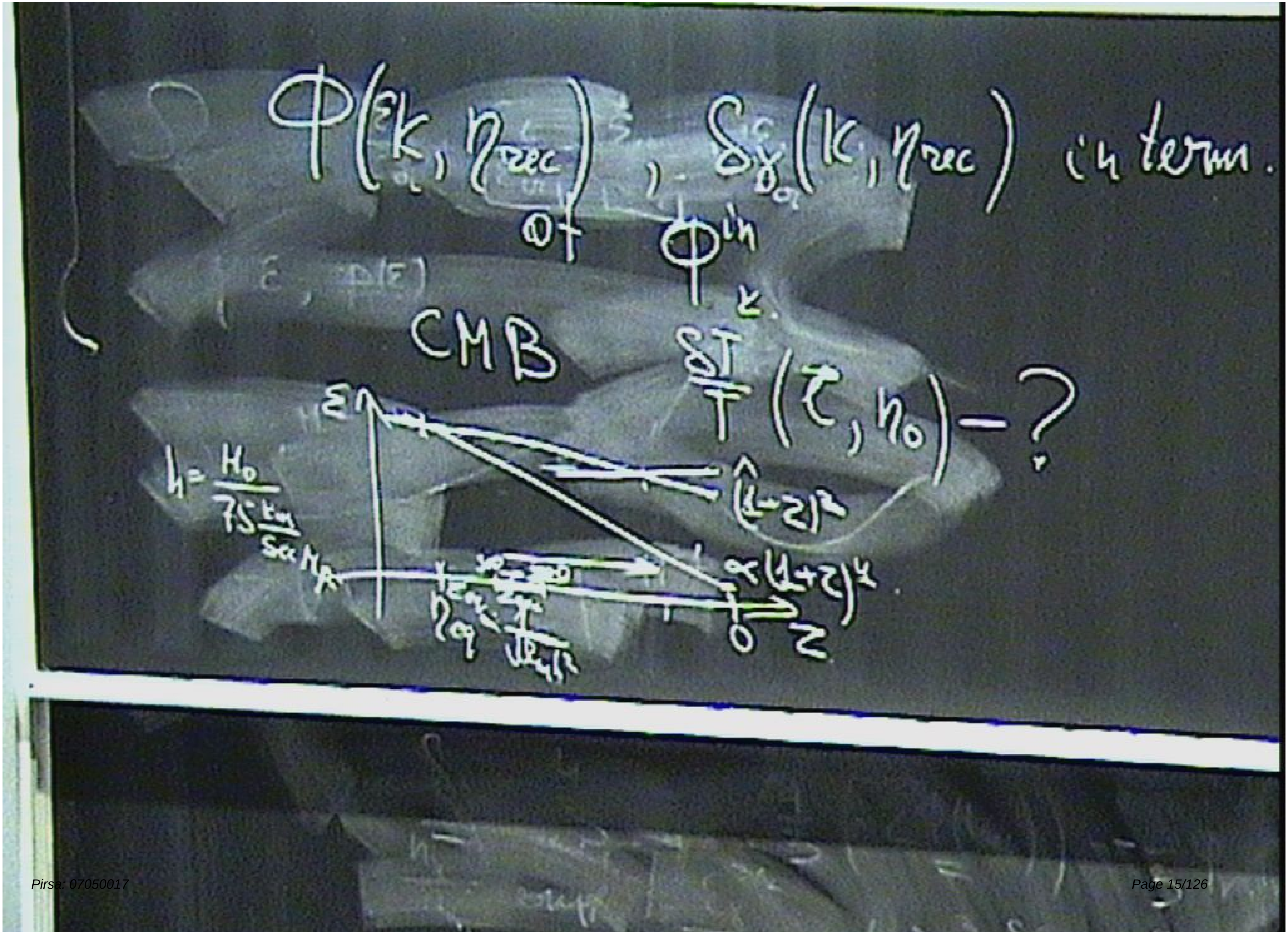
$h = \frac{H_0}{75 \frac{km}{sec\, Mpc}}$

$\Phi(\vec{k}, \eta_{rec})$, $\delta_\gamma(\vec{k}, \eta_{rec})$ in term. of Φ^{in}_{k}

CMB $\frac{\delta T}{T}(\vec{n}, \eta_0)$ — ?

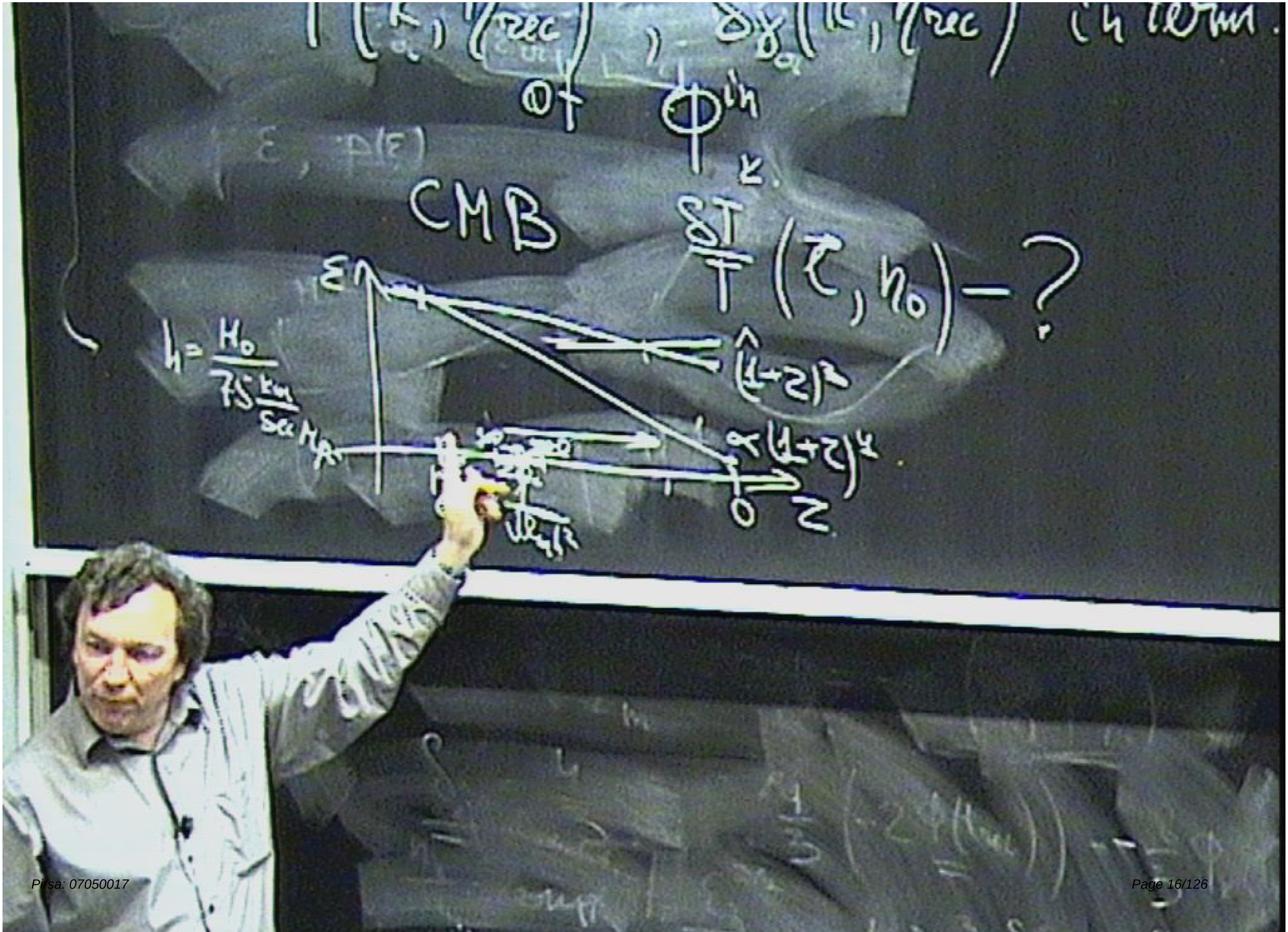
$h = \frac{H_0}{75 \frac{km}{sec\, Mpc}}$



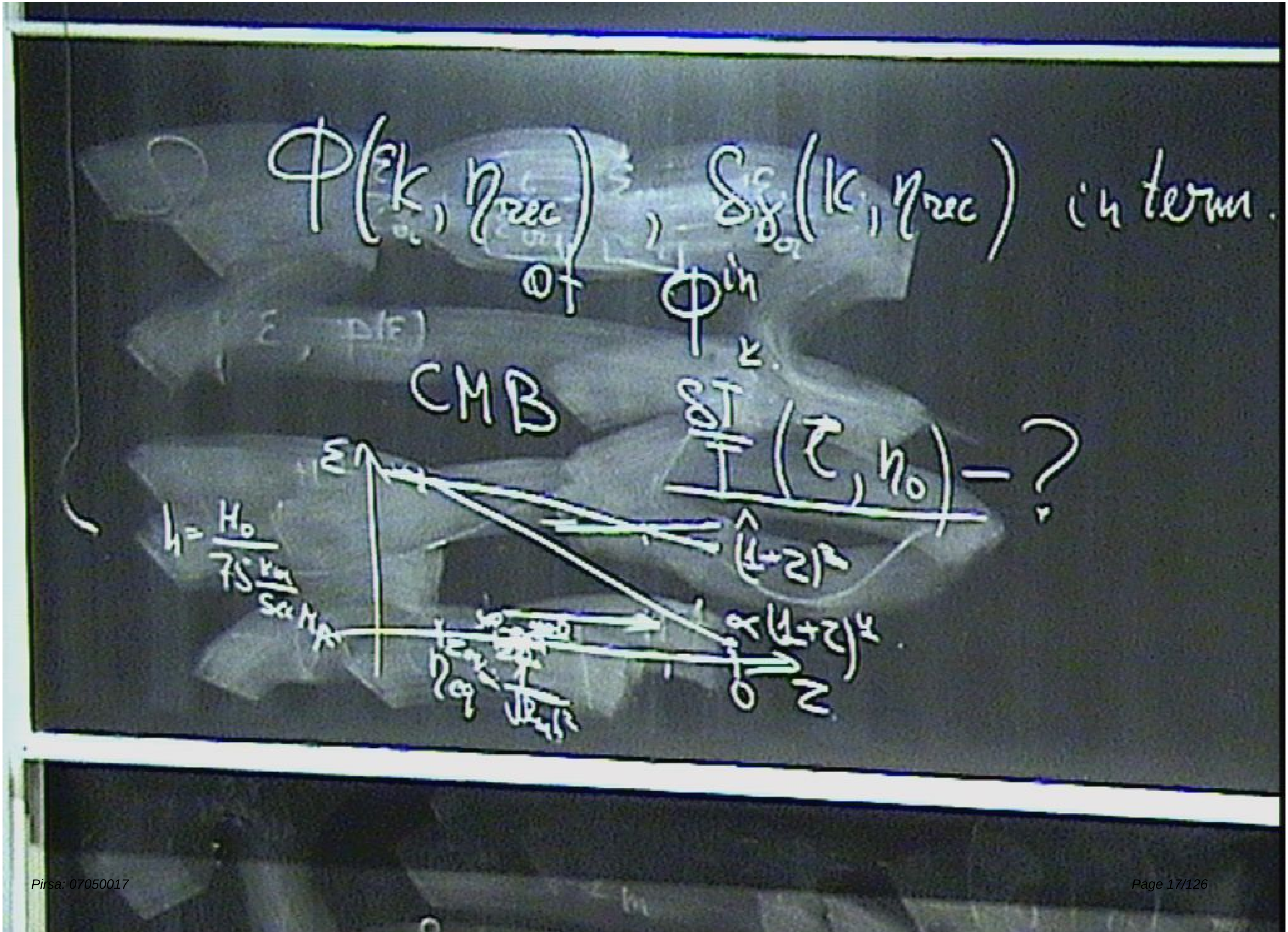


$\Phi(\vec{k}, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term. of $\Phi^{in}_{\vec{k}}$

CMB $\frac{\delta T}{T}(\vec{\tau}, \eta_0) - ?$



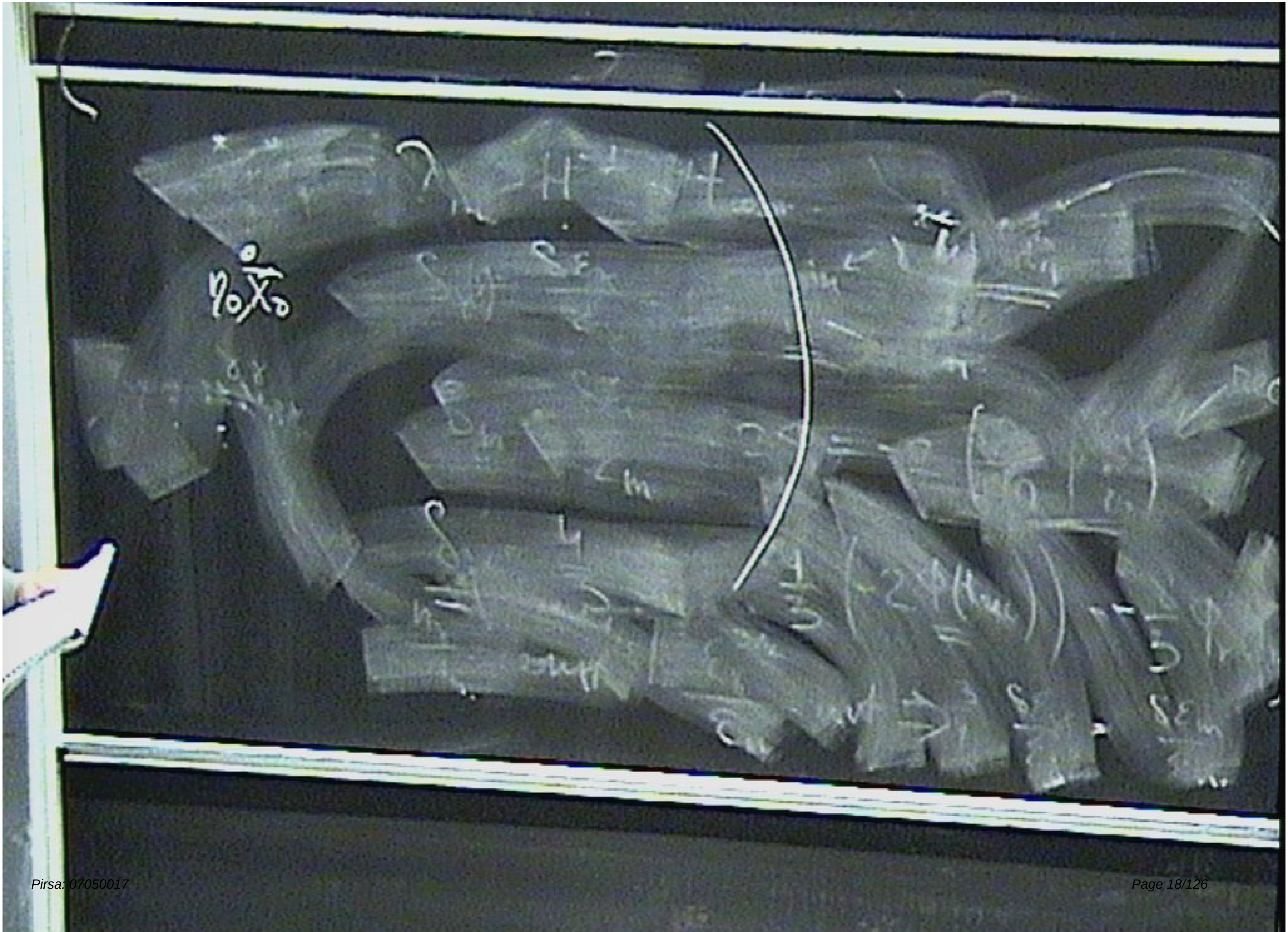
T(k, η_rec), δ_γ(k, η_rec) in terms
of Φ^in_k
CMB
δT/T (n, η_0) − ?
ε
h = H_0 / 75 km/sec Mpc
∝(1+z)^3
∝(1+z)^4
0
z

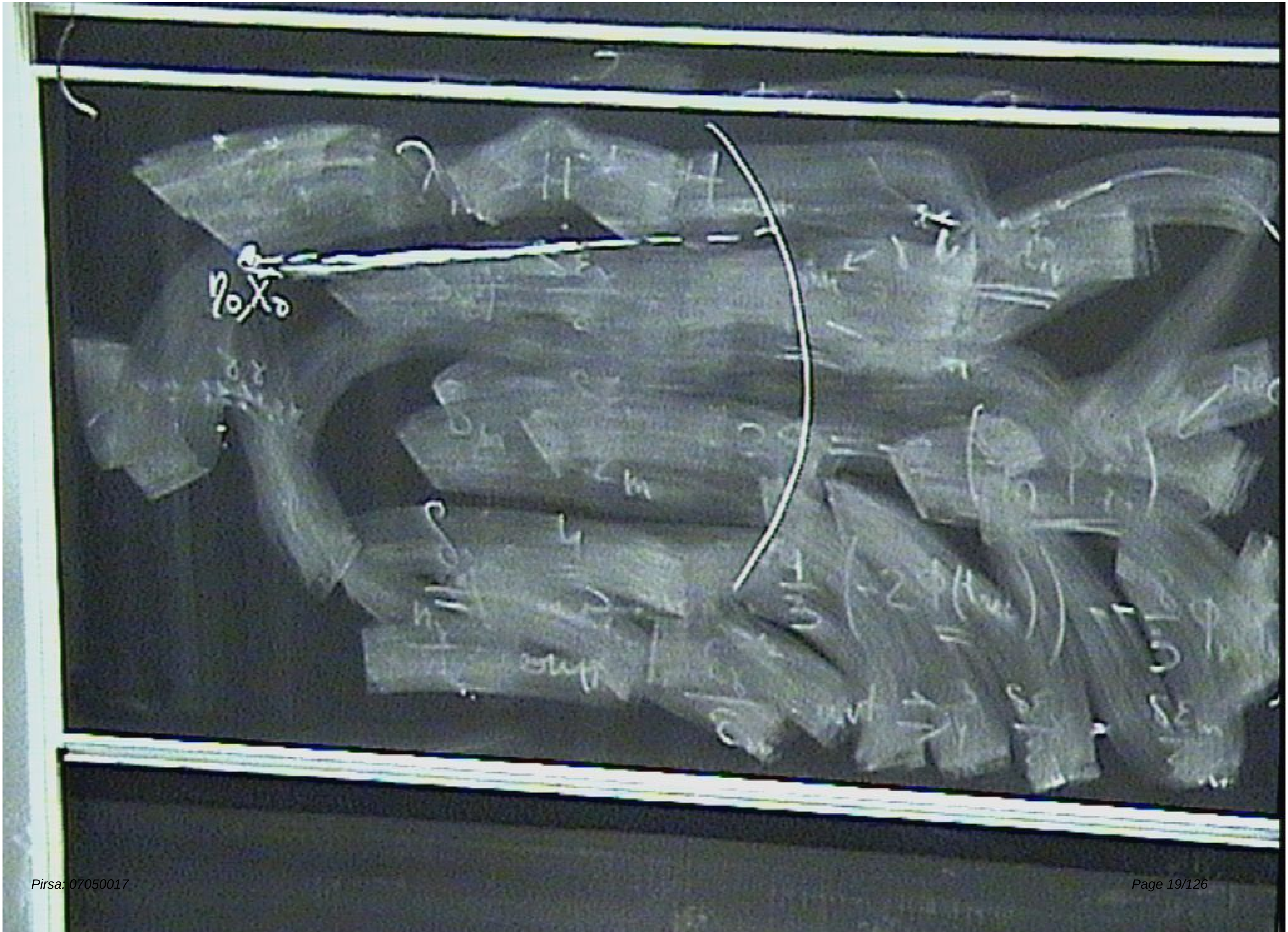


$\Phi(k, \eta_{rec})$, $\delta_\gamma(k, \eta_{rec})$ in term of Φ^{in}_k.

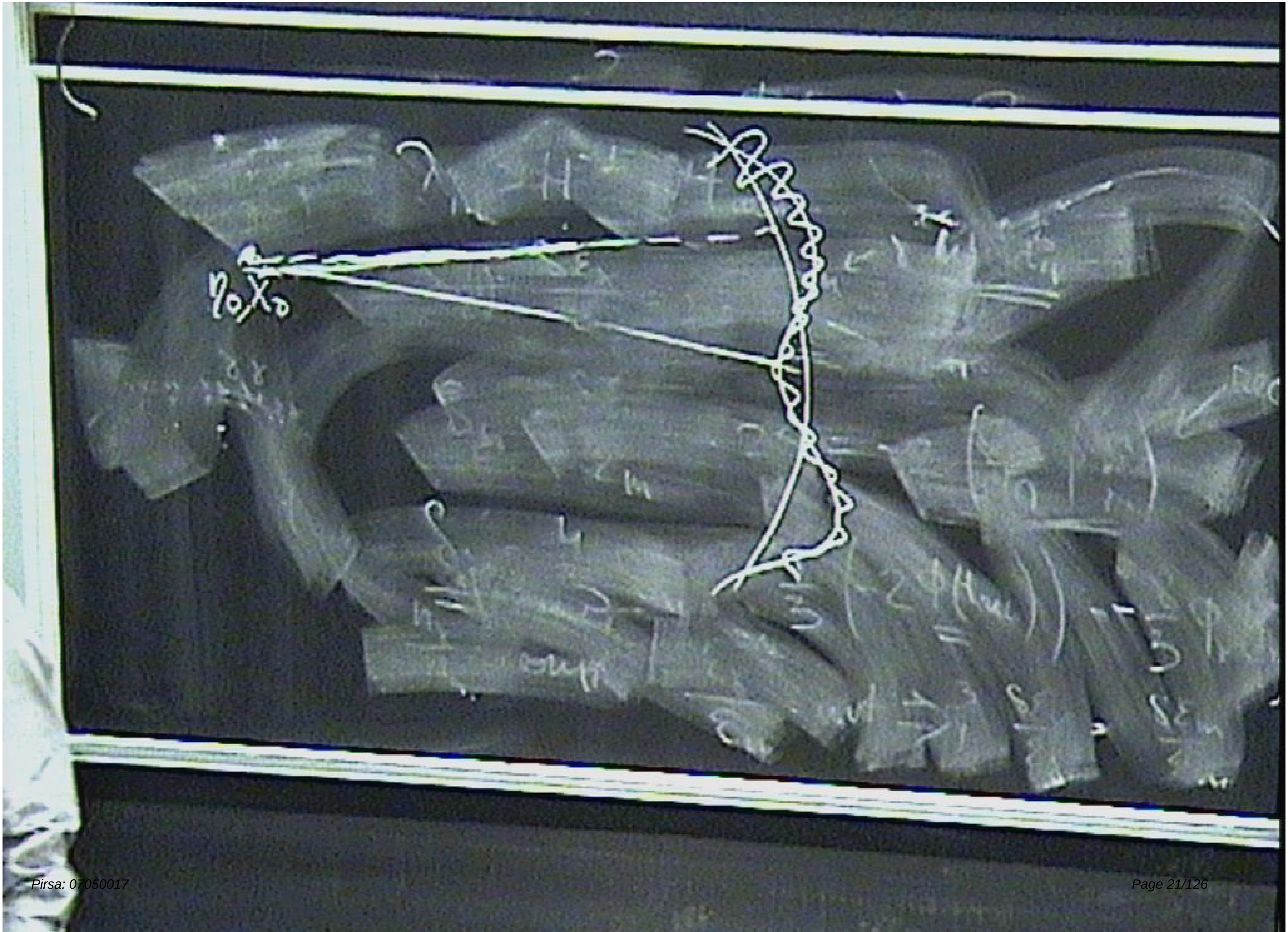
CMB $\frac{\delta T}{T}(\hat{n}, \eta_0)$ — ?

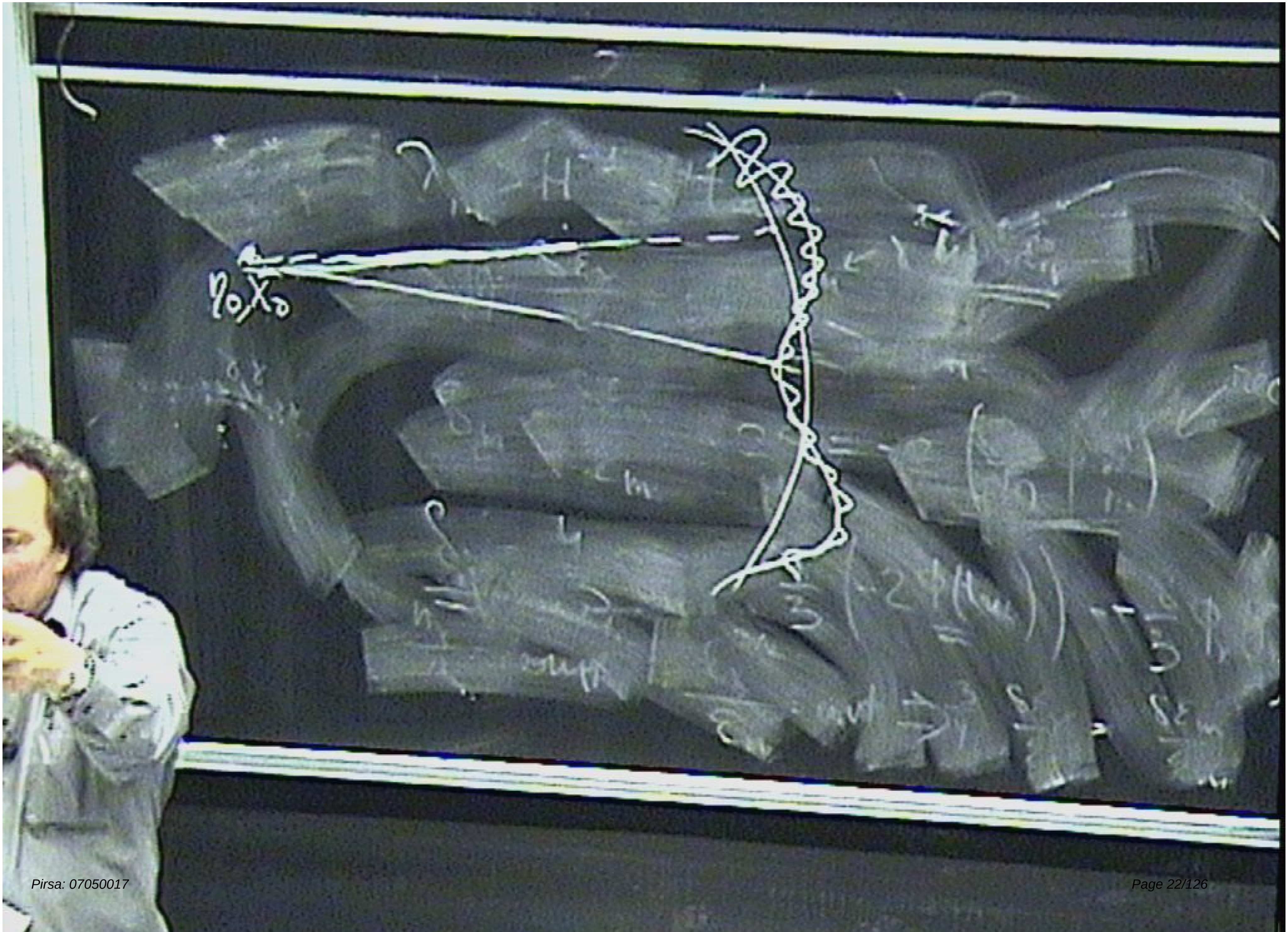
$h = \frac{H_0}{75 \frac{km}{sec\,Mpc}}$

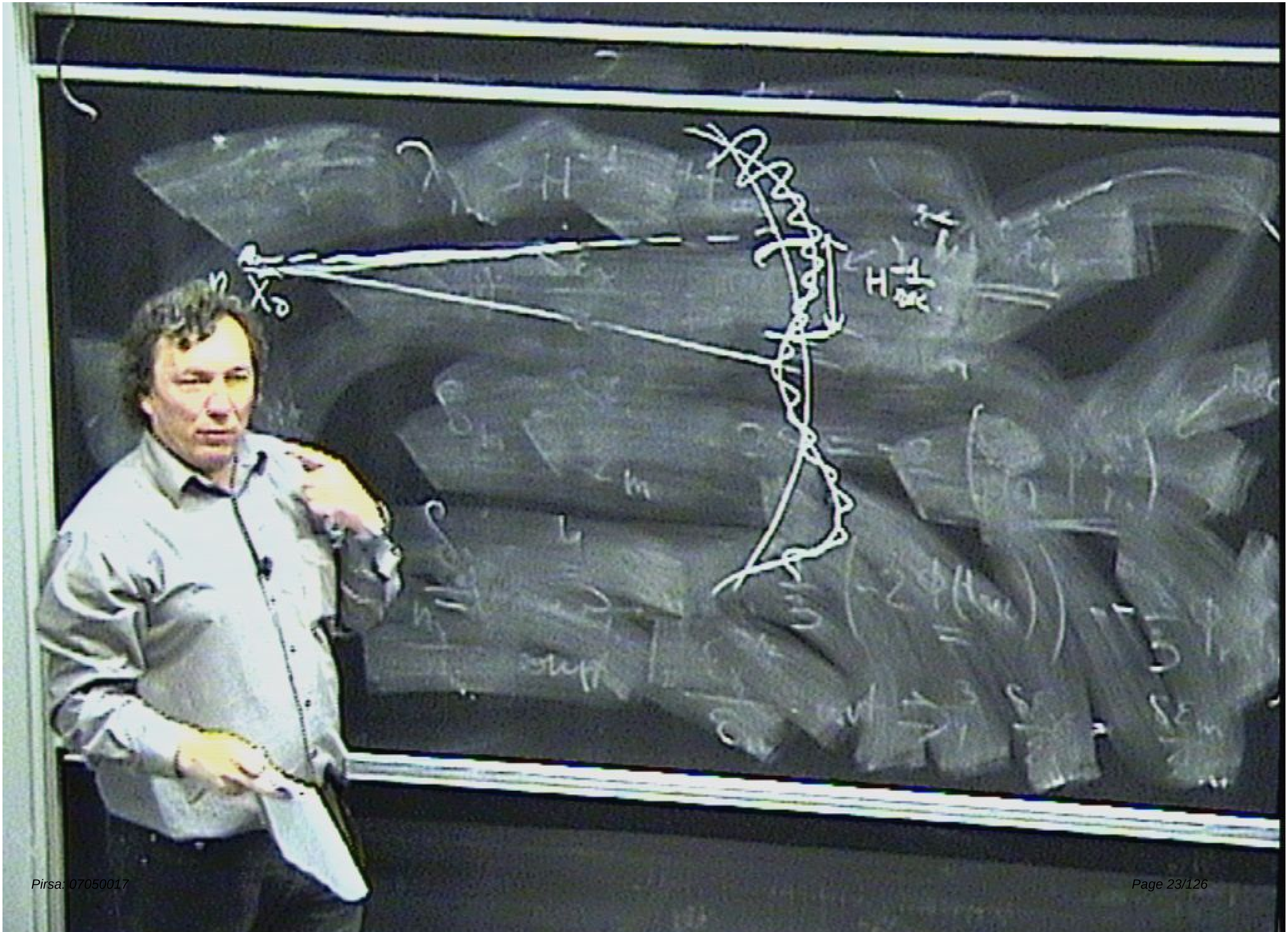

η₀X̊₀

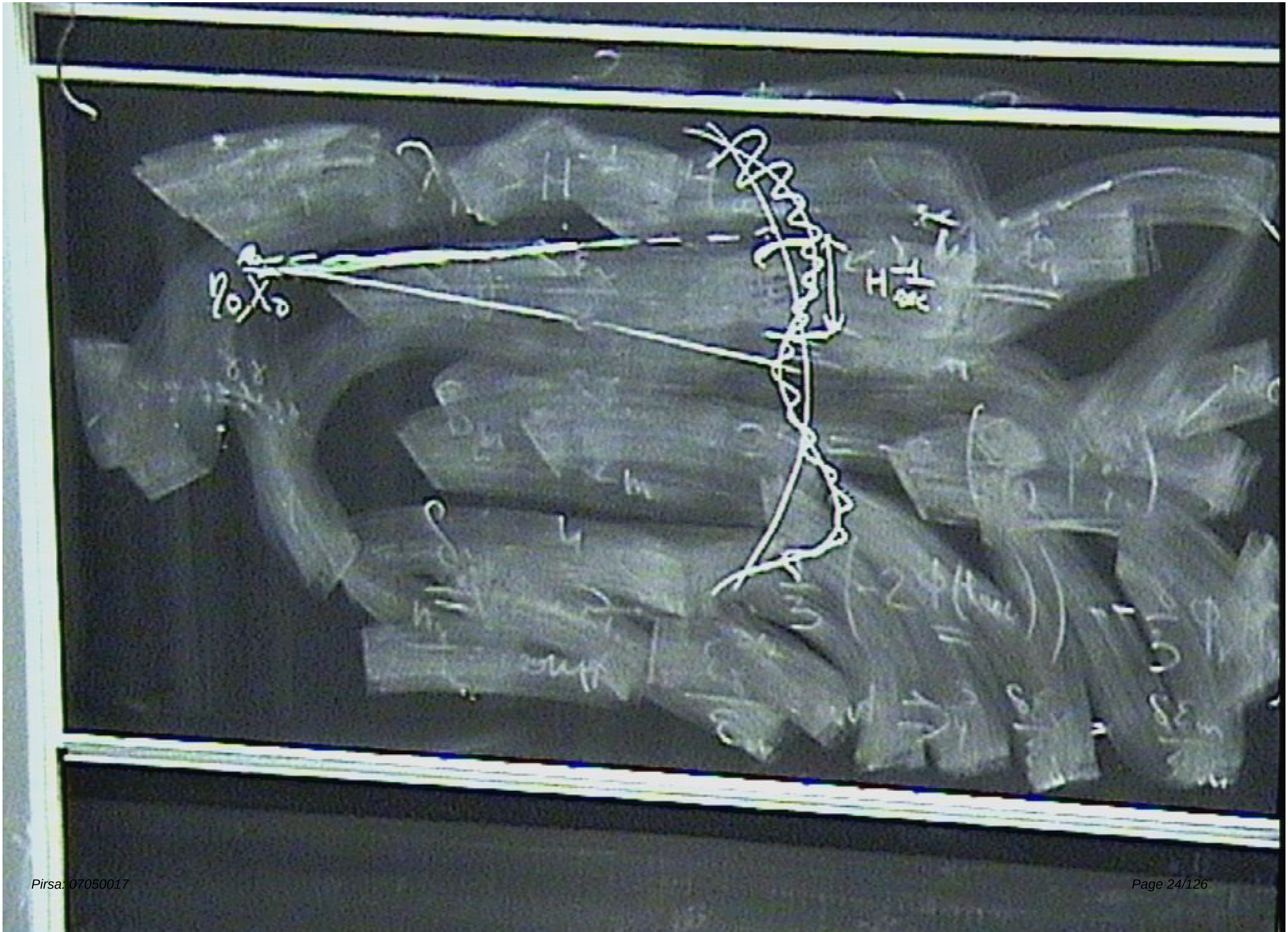


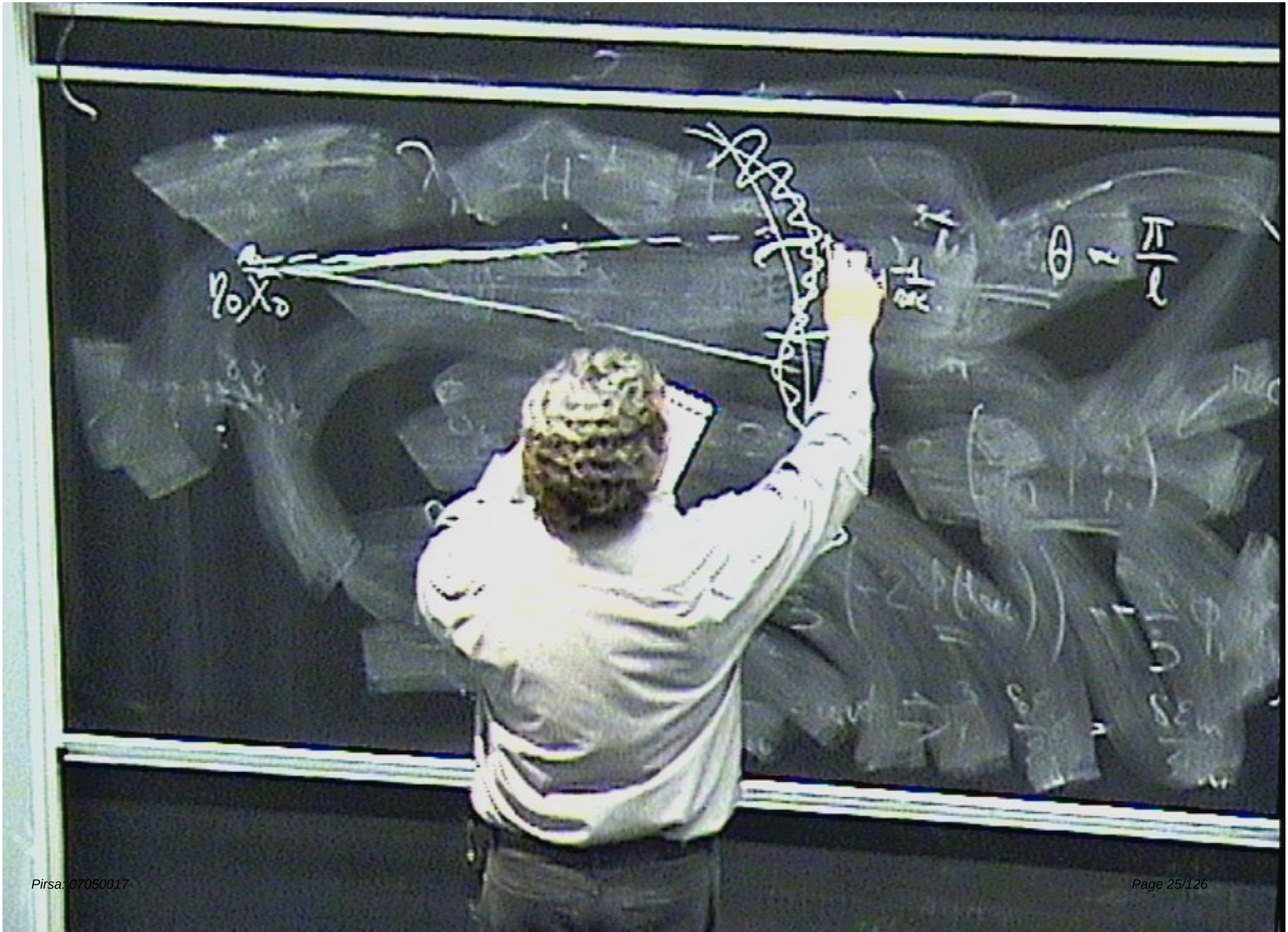
$\eta_0 X_0$

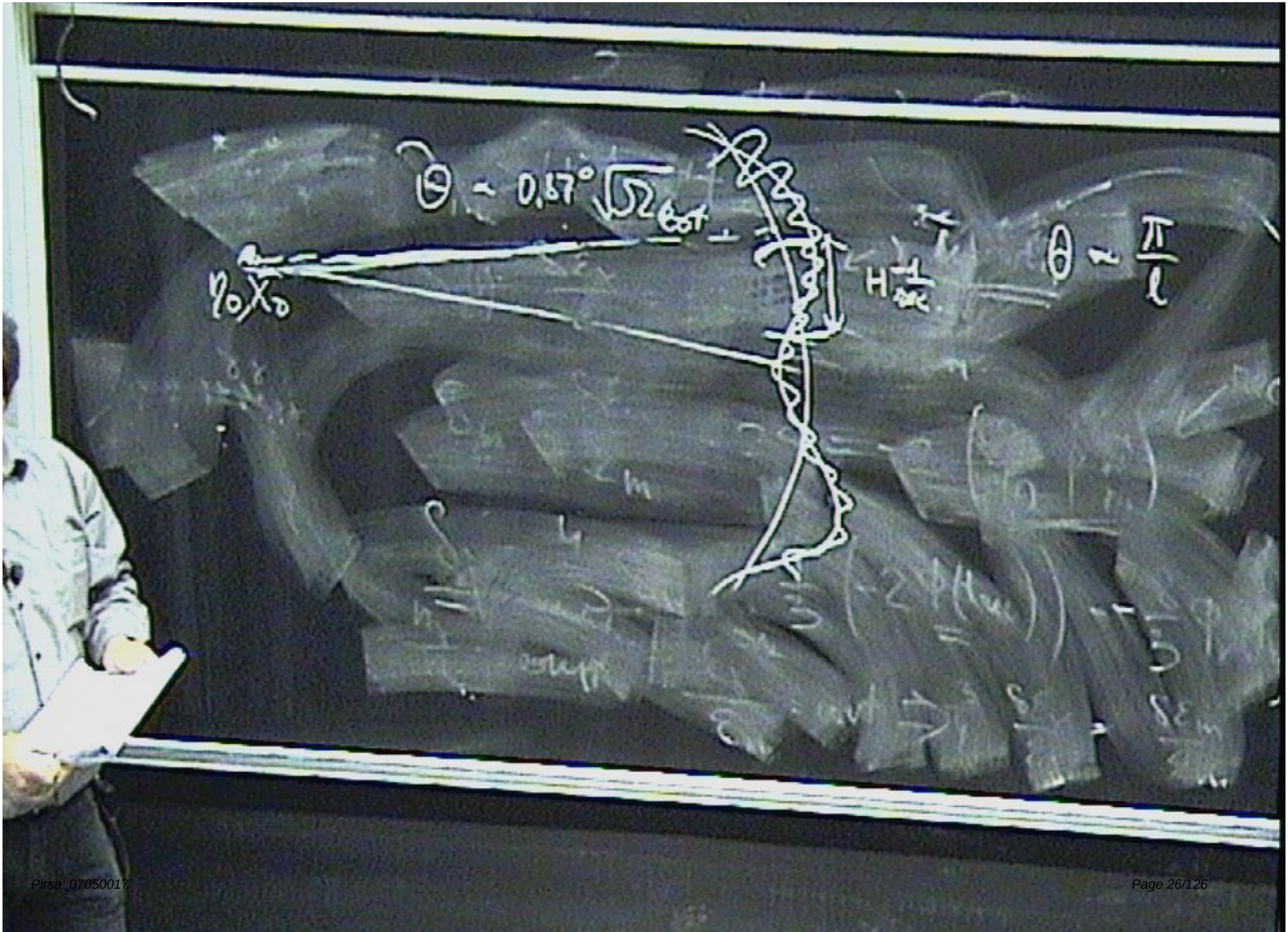

η_0,χ_0


η0,χ0


λ > H^{-1}
X_0
H^{-1}_{osc}

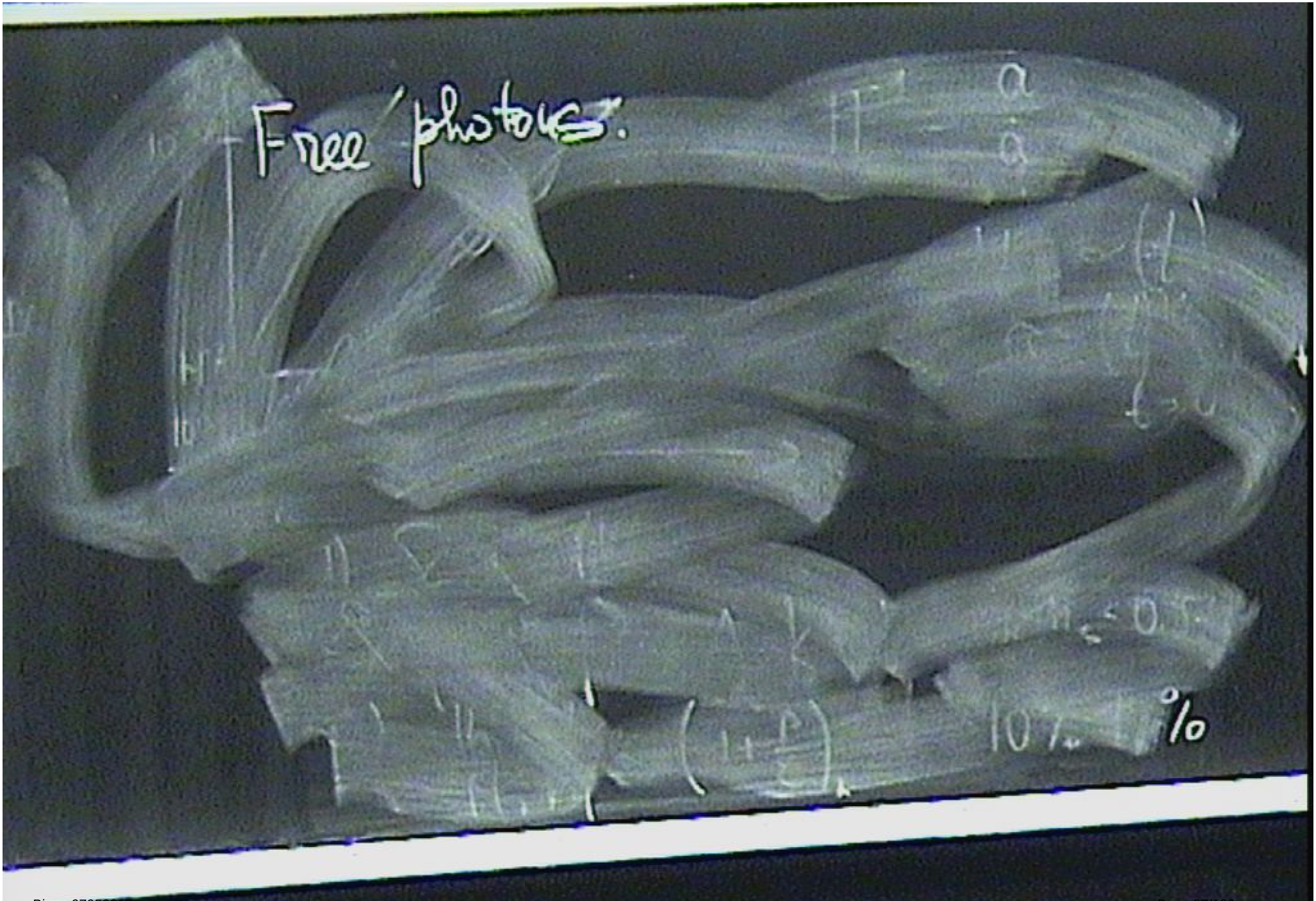

η0,χ0
H⁻¹osc


$\eta_0 X_0$
$\theta \sim \frac{\pi}{\ell}$



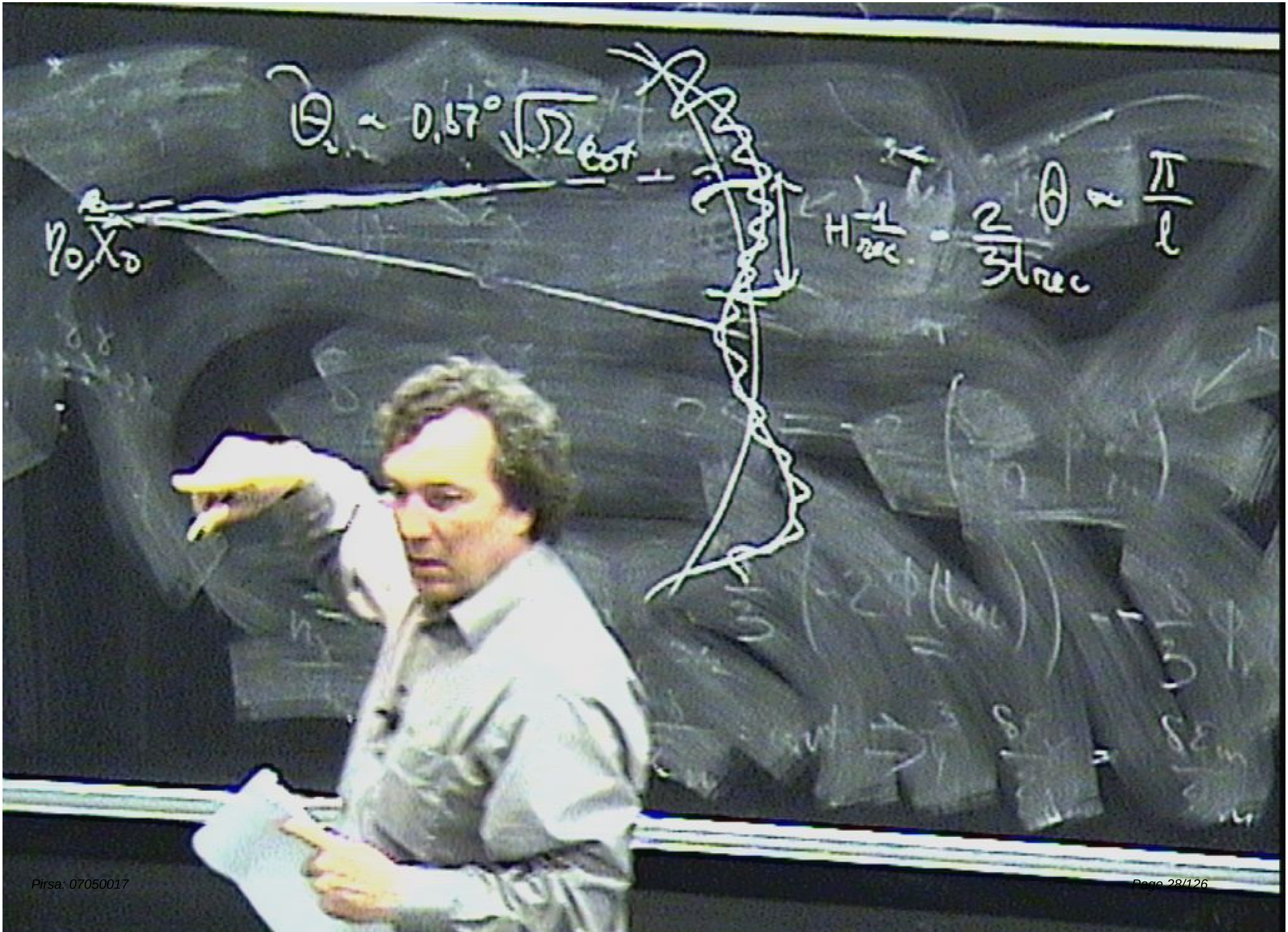
$$\Theta \sim 0.87^\circ \sqrt{\Omega_{tot}}$$

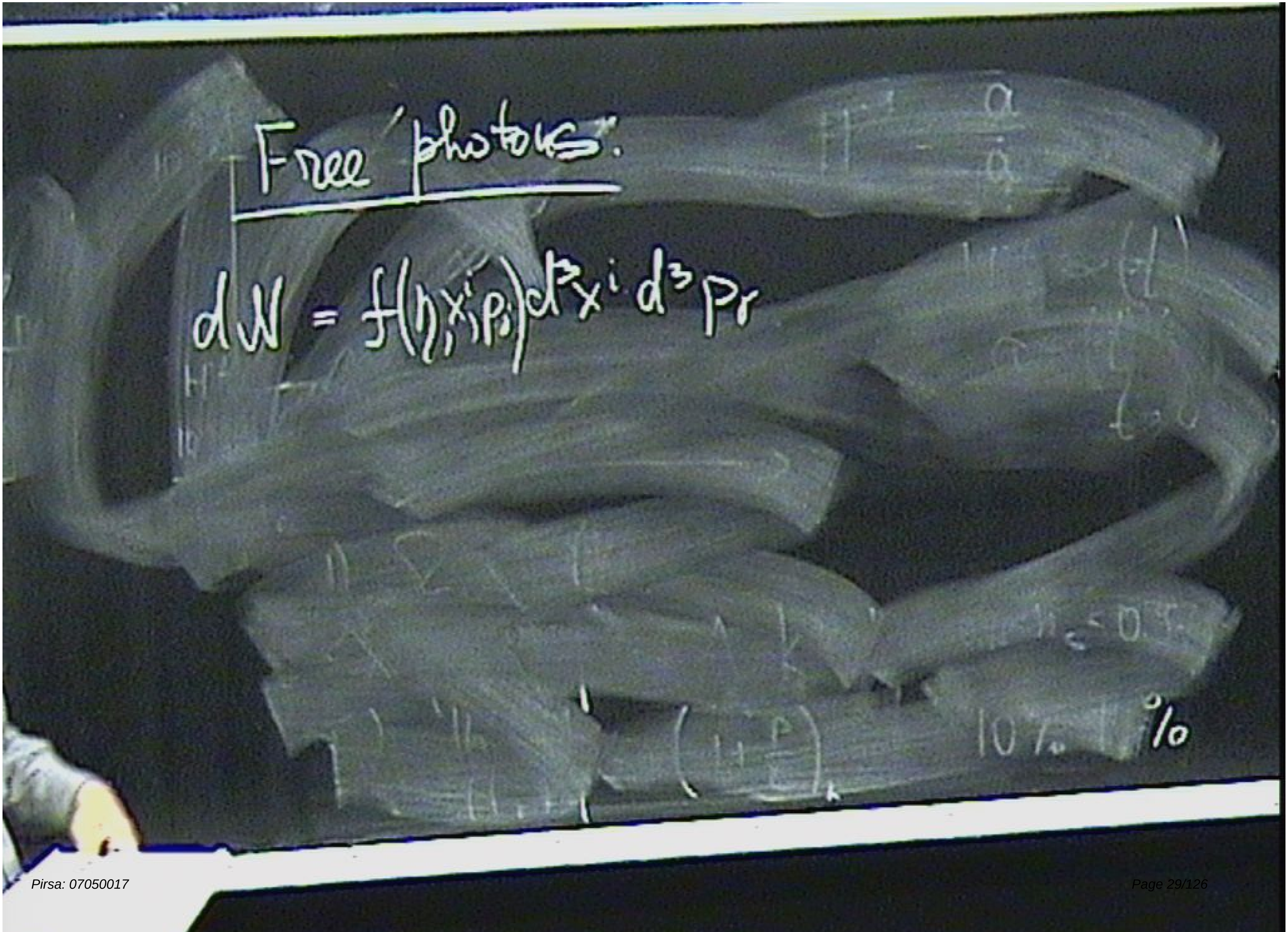
$$\Theta \sim \frac{\pi}{l}$$



Free photons:

%

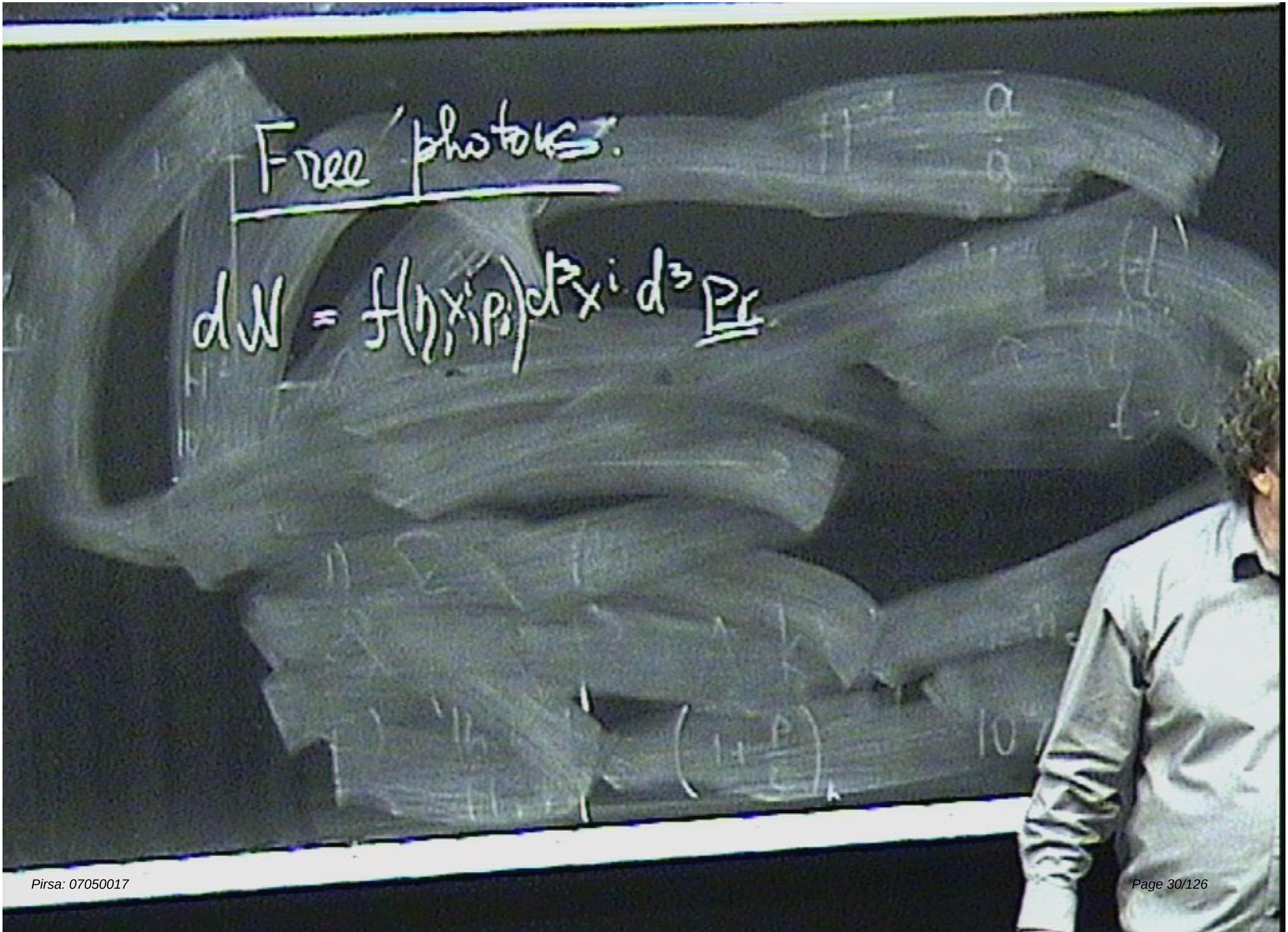

Θ1 ~ 0.87° √Ωtot
η0, X0
H^-1 rec = 2/(3 t rec)
θ ~ π/ℓ



Free photons:

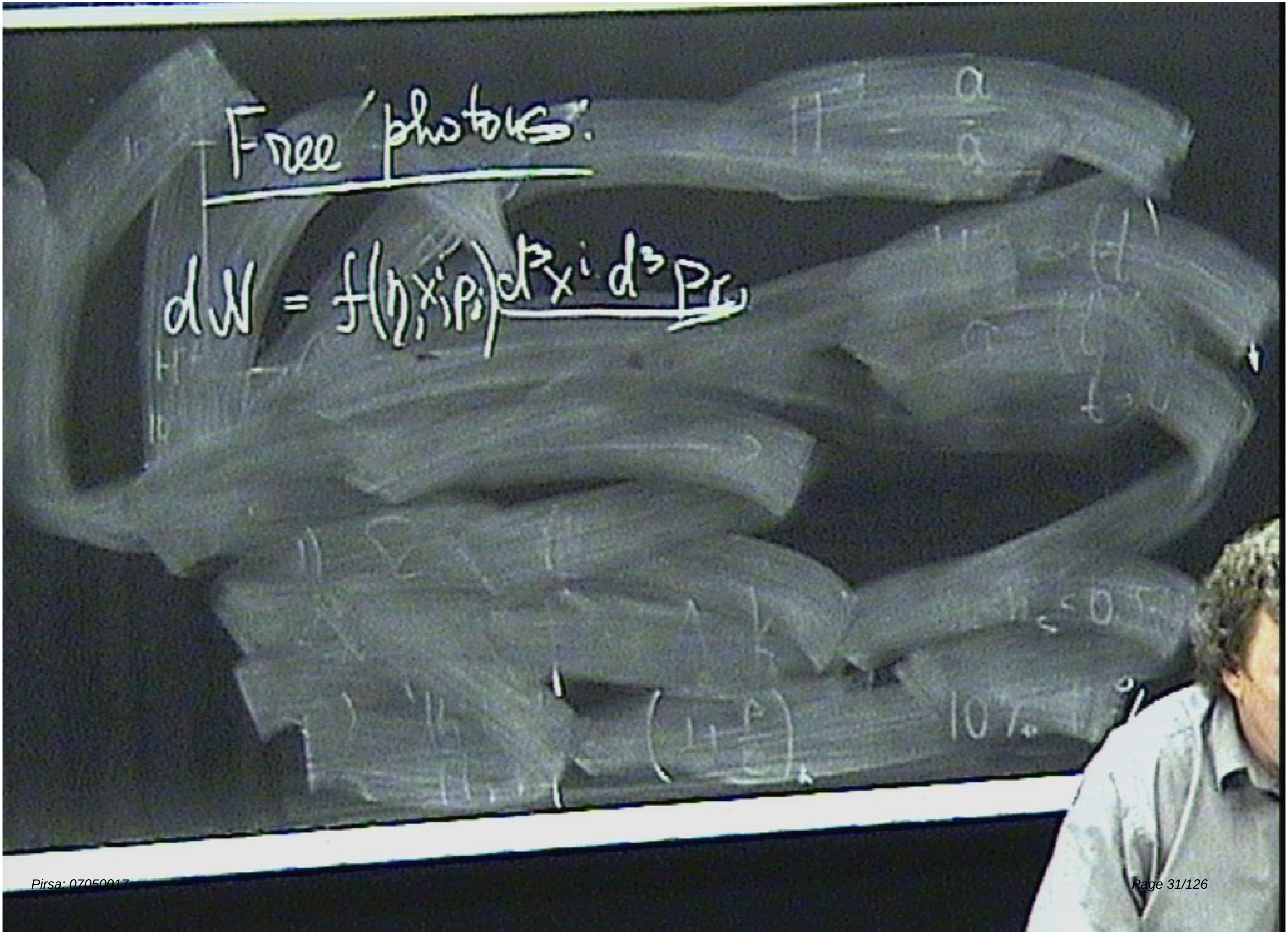
$$dN = f(\eta, x^i, p_i)\, d^3x^i\, d^3p_r$$

%



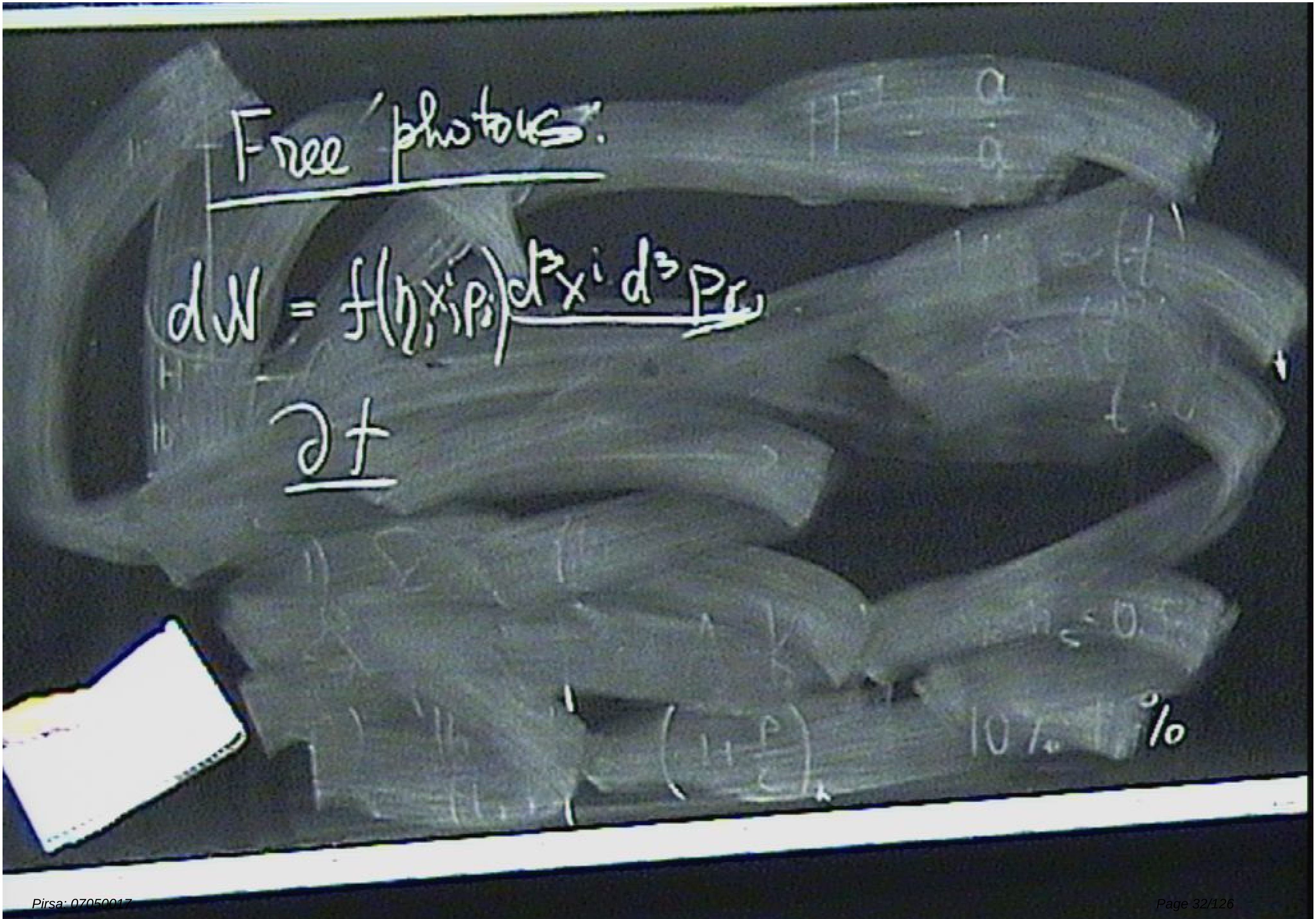
Free photons:

$$dN = f(\eta, x^i, p_i)\, d^3x^i\, d^3 \underline{P_r}$$



Free photons:

$$dN = f(\eta, x^i, p_i)\,\underline{d^3x^i\, d^3p_i}$$

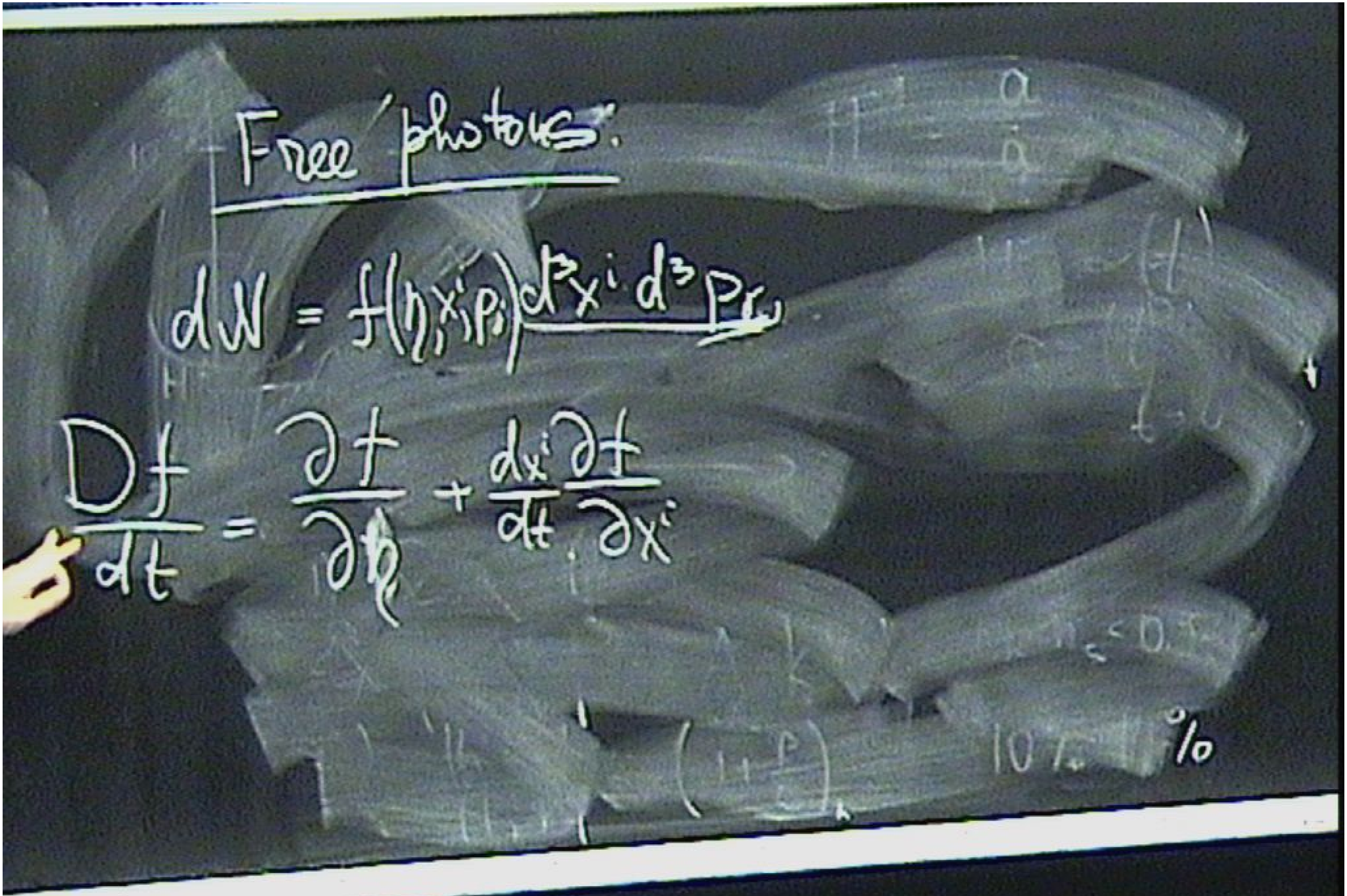


Free photons:

$$dN = f(\eta, x^i, p_i)\, d^3x^i\, d^3p_i$$

$$\partial t$$

%

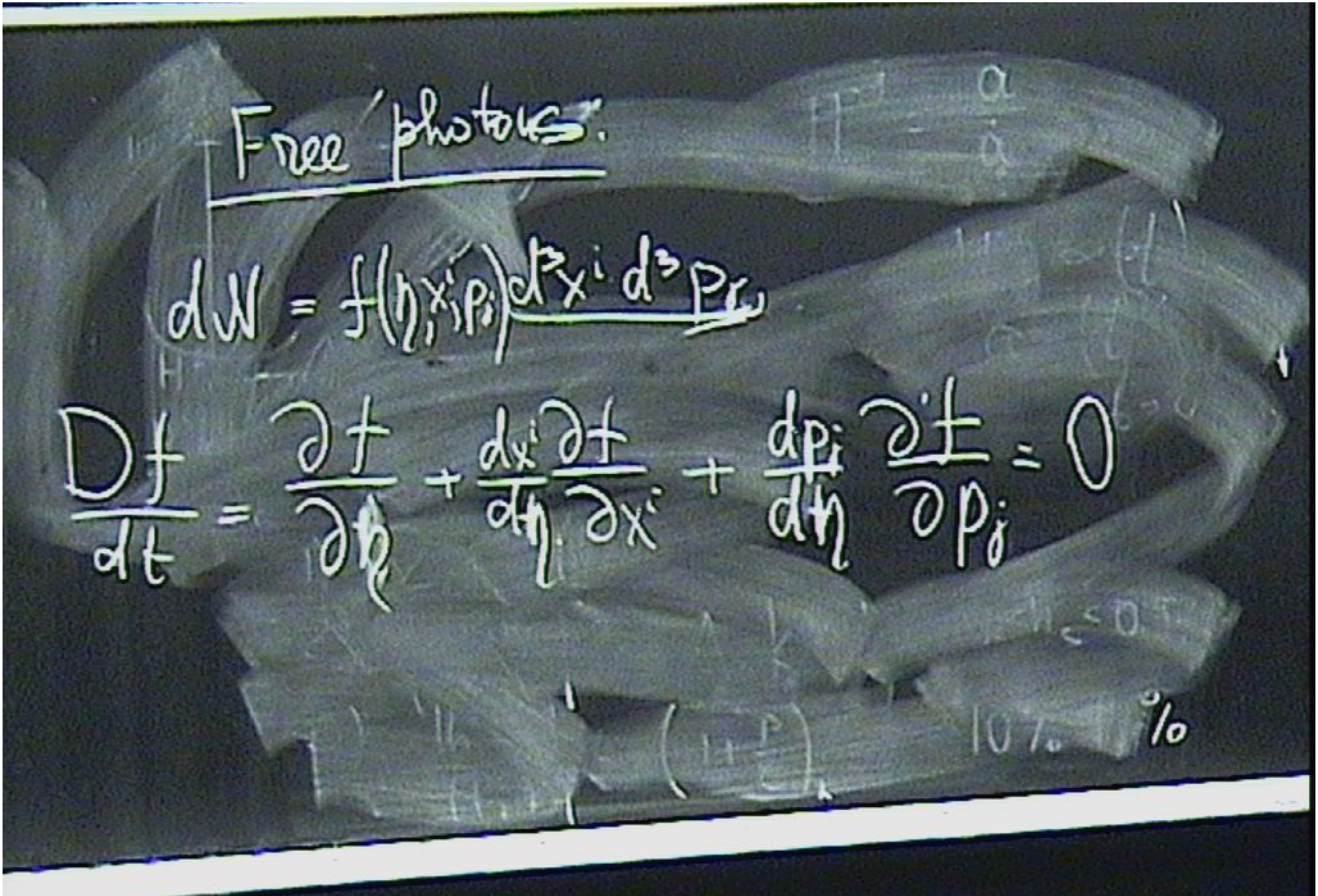


Free photons:

$$dN = f(\eta, x^i, p_i)\, \underline{d^3x^i\, d^3p_i}$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{dt}\frac{\partial f}{\partial x^i}$$

%

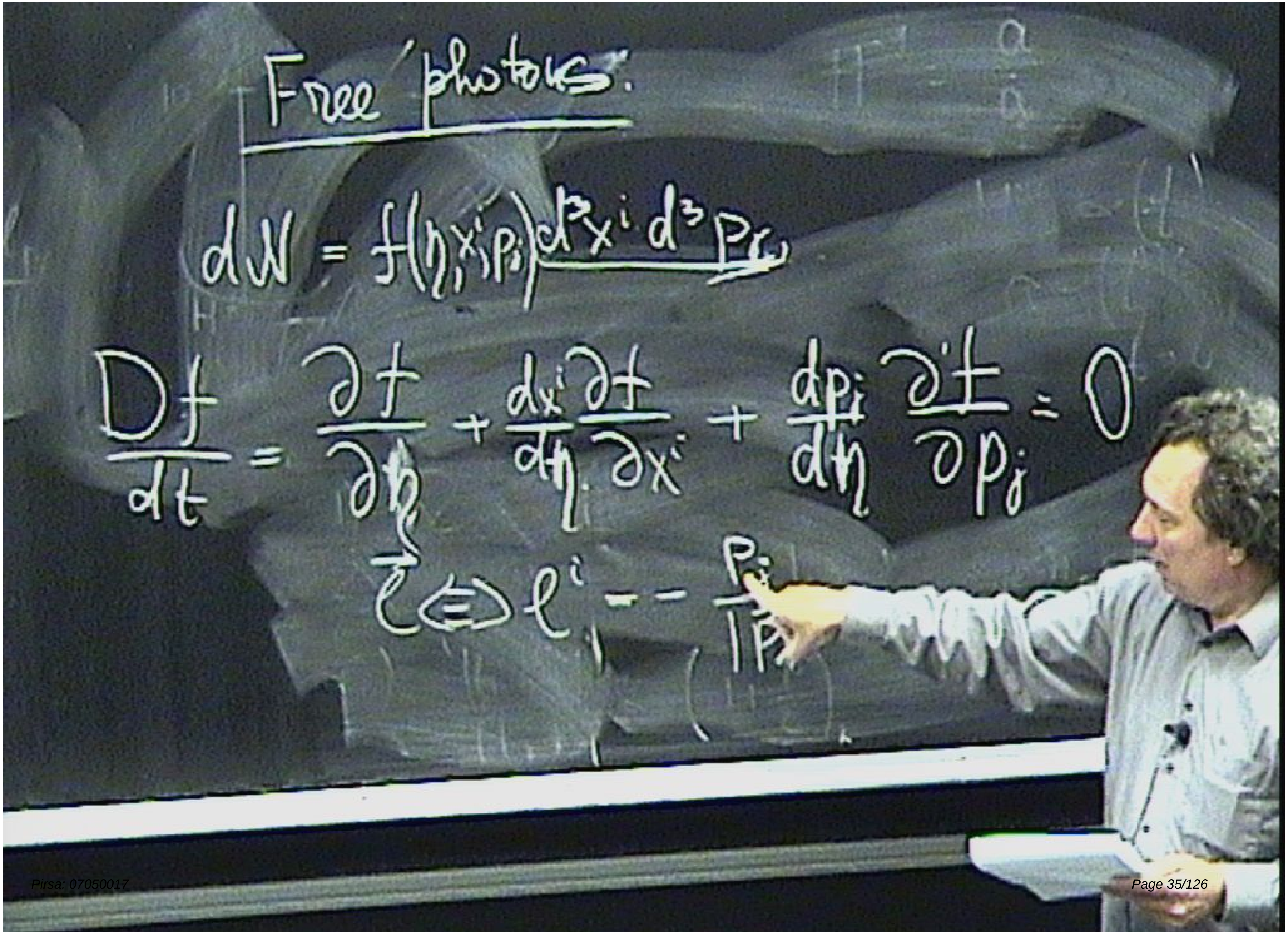


Free photons:

$$dN = f(\eta, x^i, p_i)\, d^3x^i\, d^3p_i$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta}\frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta}\frac{\partial f}{\partial p_i} = 0$$

%

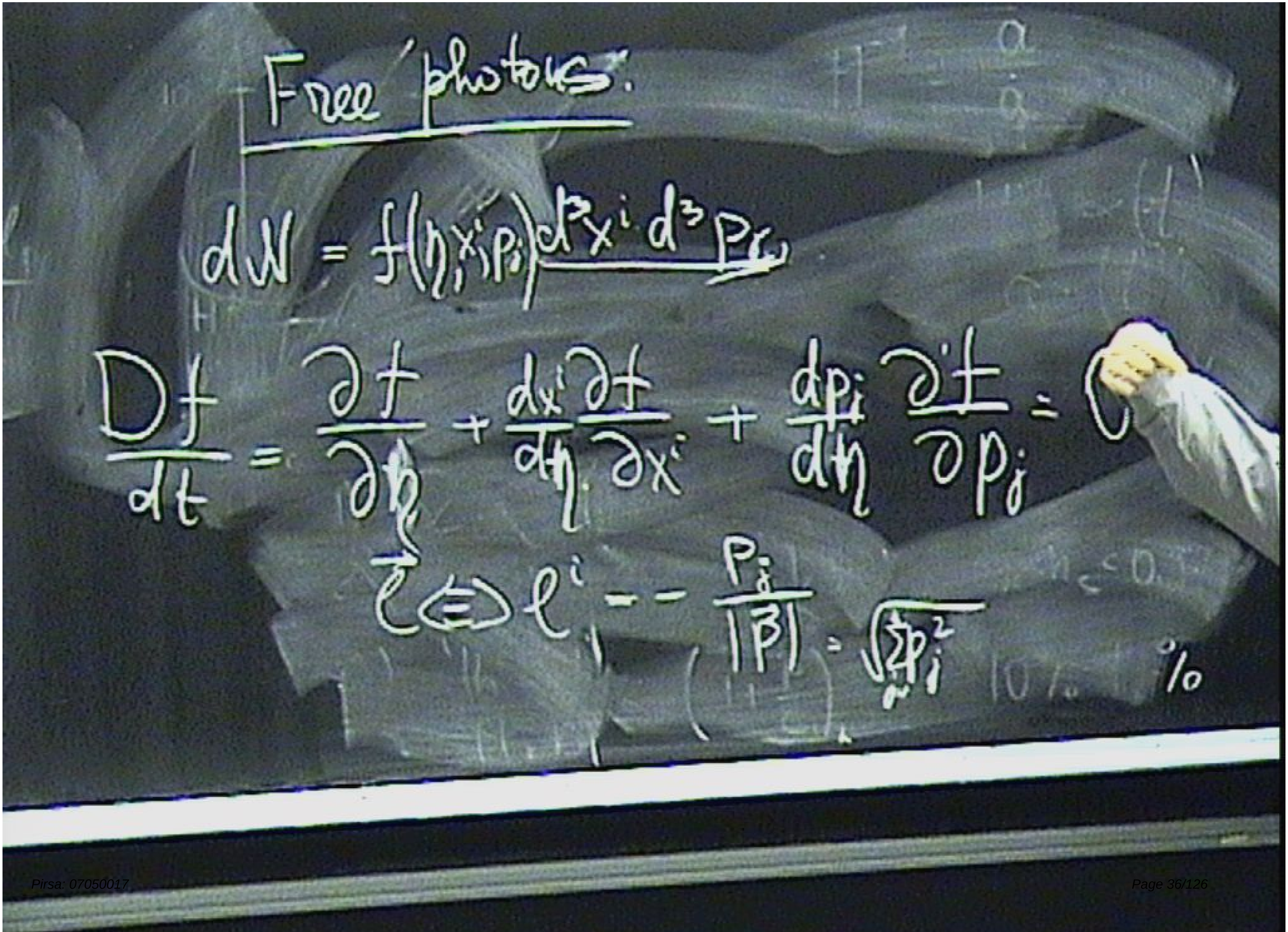


Free photons:

$$dN = f(\eta, x^i, p_i)\, d^3x^i\, d^3p_i$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta}\frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta}\frac{\partial f}{\partial p_j} = 0$$

$$\vec{l} \Leftrightarrow l^i = -\frac{p_i}{|p|}$$



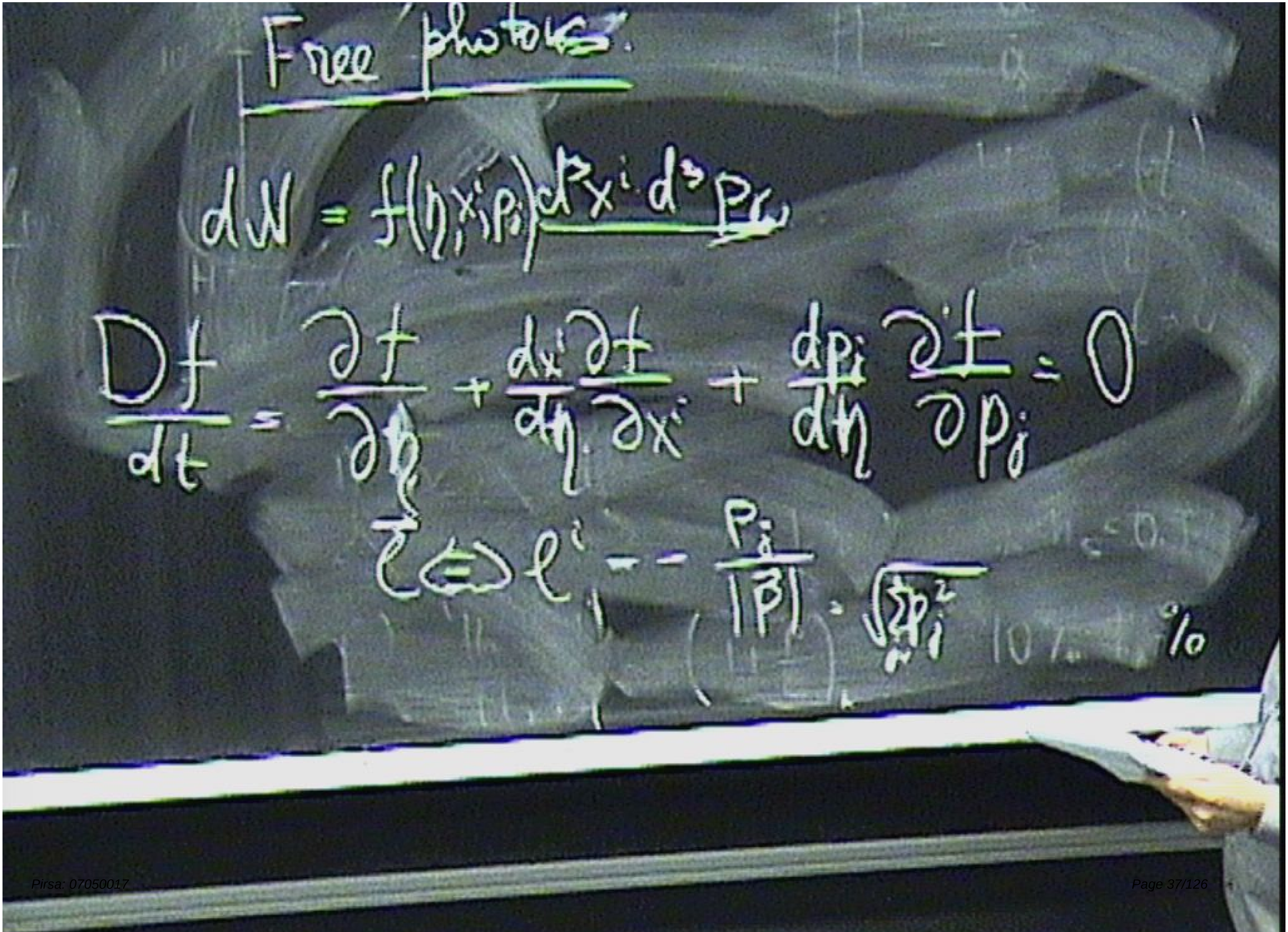
Free photons:

$$dN = f(\eta, x^i, p_i)\, \underline{d^3x^i\, d^3p_i}$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta}\frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta}\frac{\partial f}{\partial p_i} = 0$$

$$\hat{l} \Leftrightarrow l^i = -\frac{p_i}{|\vec{p}|} \qquad |\vec{p}| = \sqrt{\sum_i p_i^2}$$

%



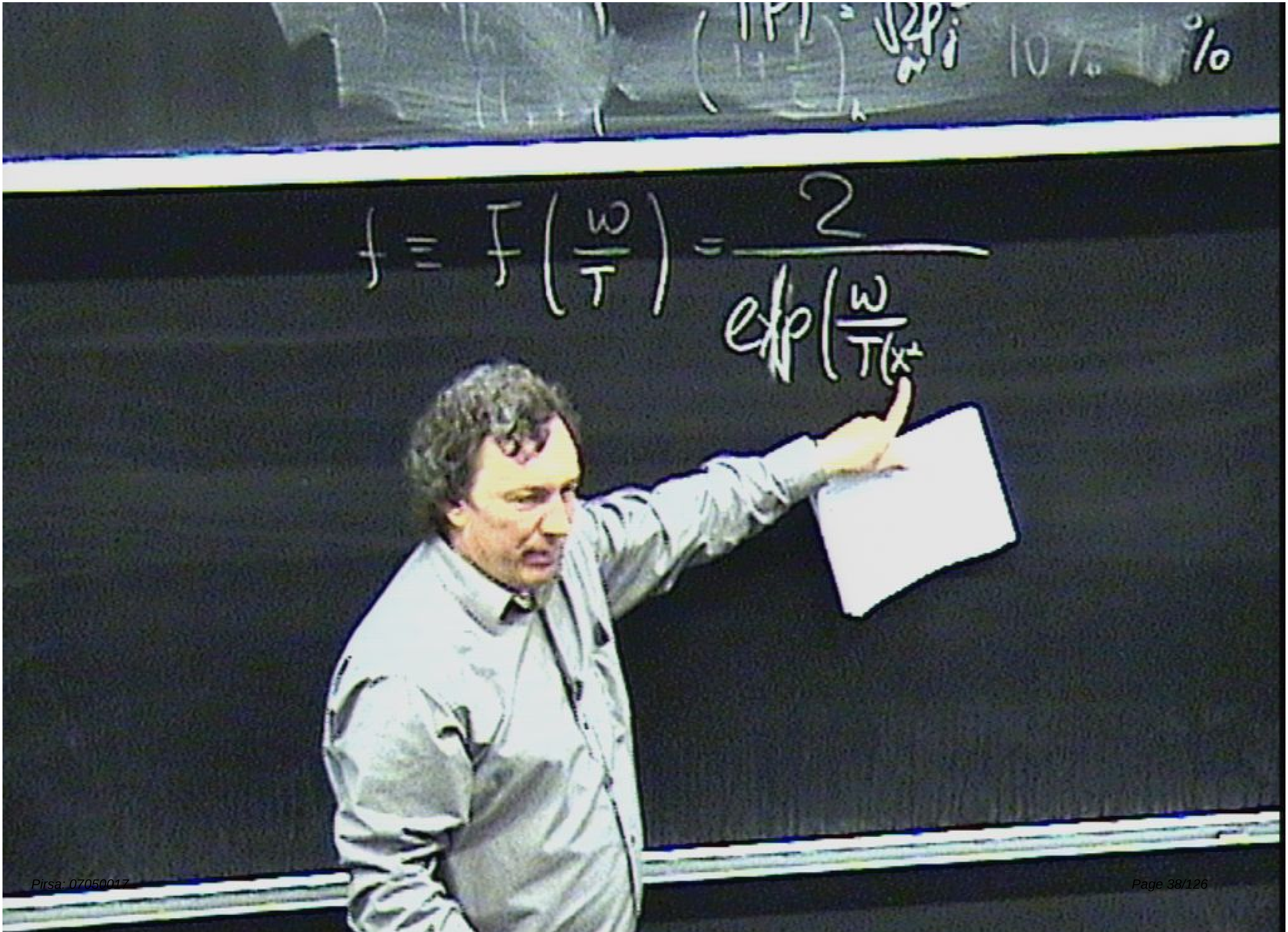
Free photons:

$$dN = f(\eta, x^i, p_i)\, d^3x^i\, d^3p_i$$

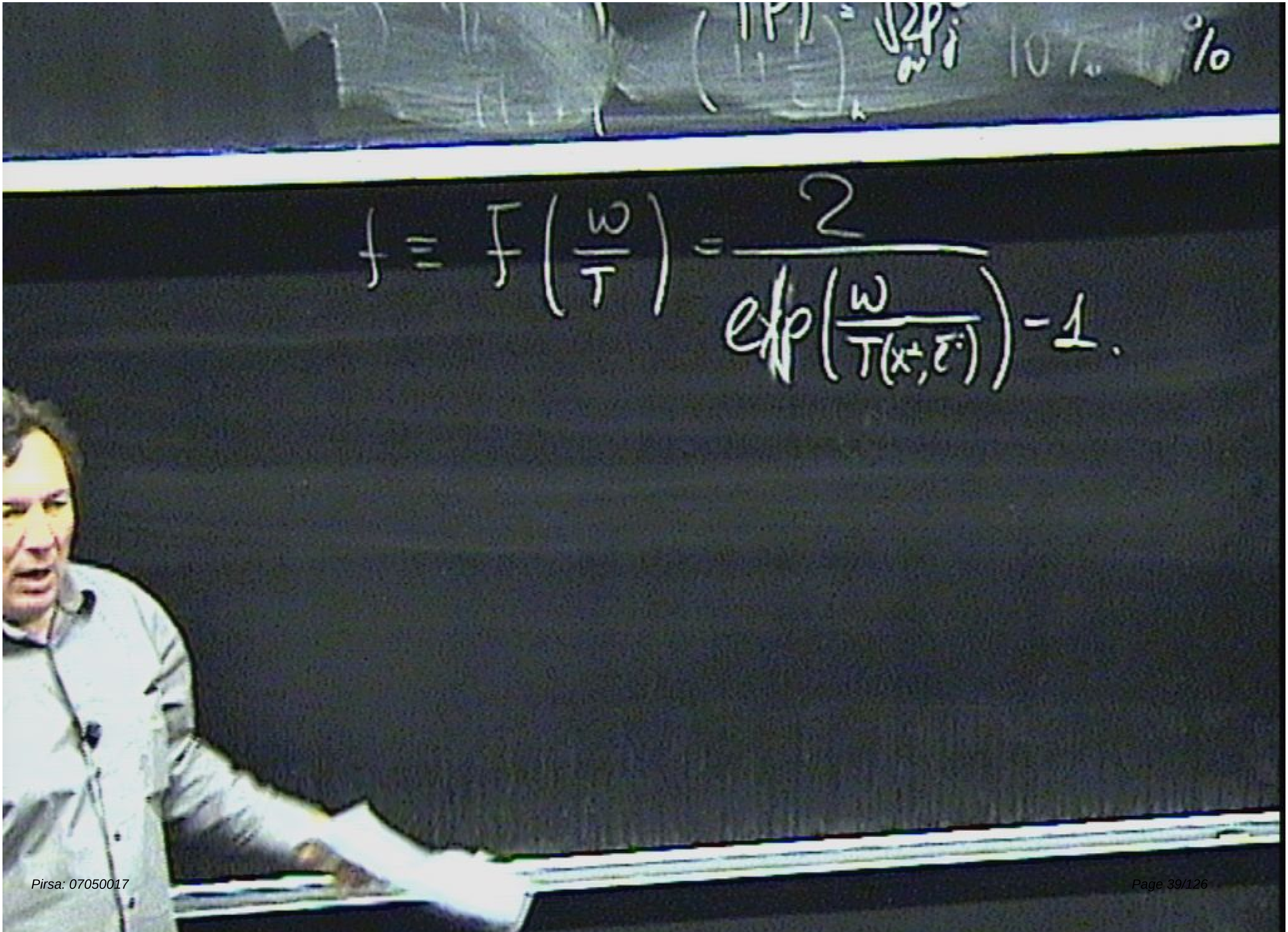
$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta}\frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta}\frac{\partial f}{\partial p_i} = 0$$

$$\vec{\ell} \Leftrightarrow \ell^i = -\frac{p_i}{|\vec{p}|} \cdot \sqrt{\sum p_i^2}$$

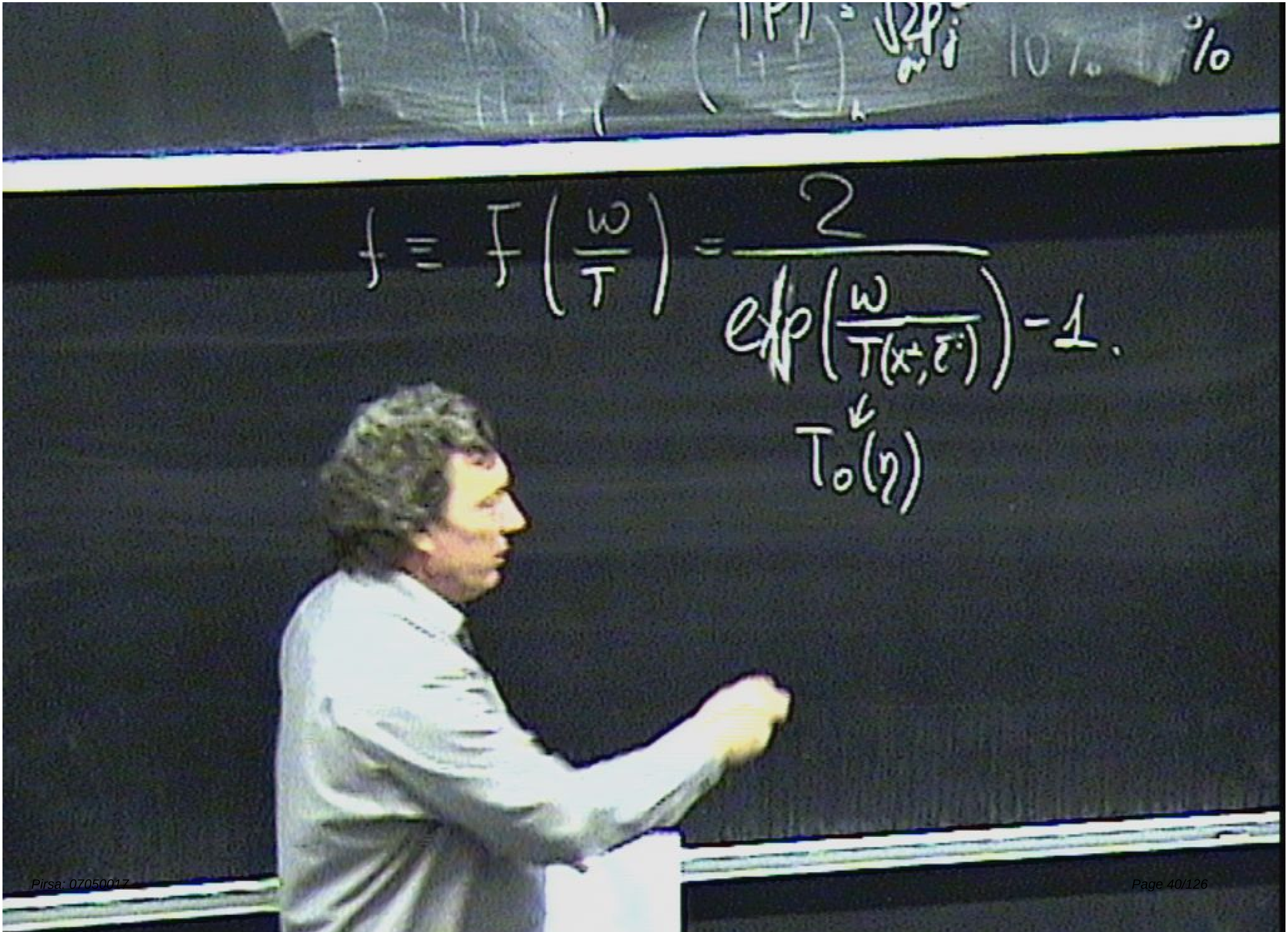
%



$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^+}\right.}$$



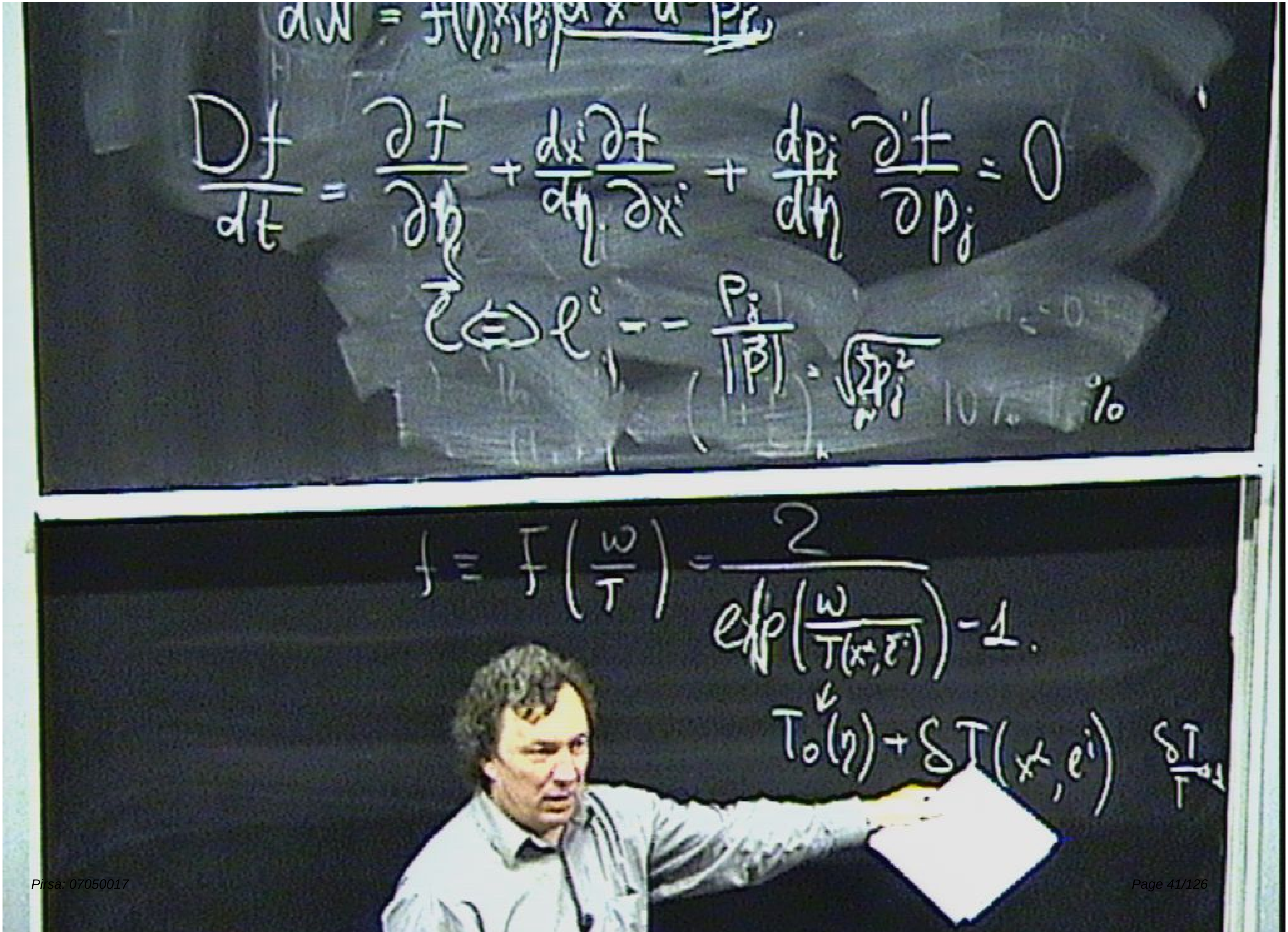
$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^+,\tau^\cdot)}\right)-1}.$$



$|p| = \sqrt{2p_i}$ 10% %

$$f \equiv F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{\underset{\swarrow}{T(x^+,\vec{x}^\perp)}}\right) - 1}.$$

$$T_0(\eta)$$



$$dN = f(\eta, x^i, p_i)\, d^3x\, d^3p_i$$

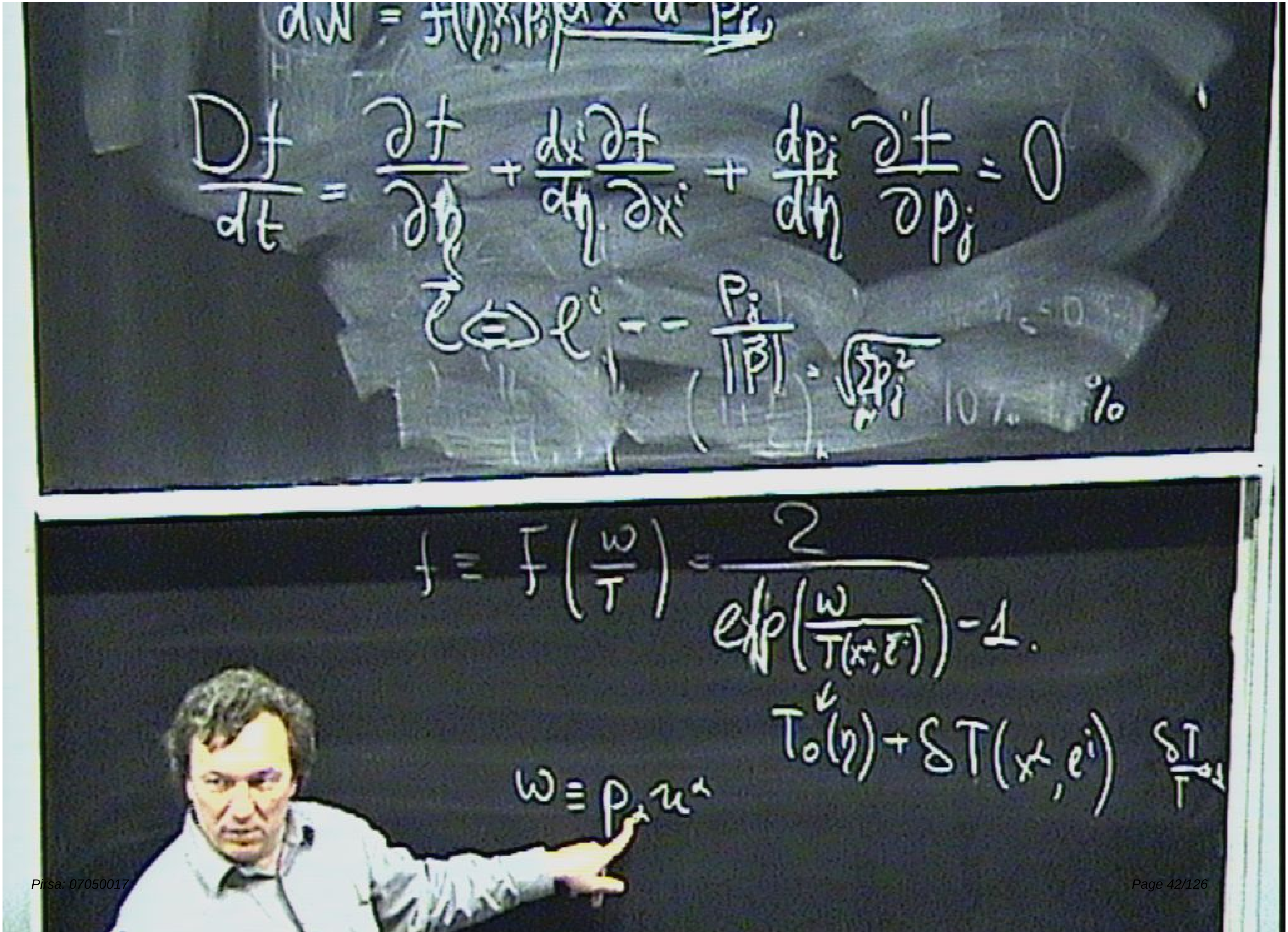
$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta}\frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta}\frac{\partial f}{\partial p_i} = 0$$

$$\vec{e} \Leftrightarrow e^i = -\frac{p_i}{|p|} \quad |p| = \sqrt{\sum p_i^2}$$

%

$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^i, e^i)}\right) - 1}.$$

$$T_0(\eta) + \delta T(x^i, e^i) \qquad \frac{\delta T}{T}$$



$dN = f(\eta, x^i, p_i)\, d^3x\, d^3p_i$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta}\frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta}\frac{\partial f}{\partial p_i} = 0$$

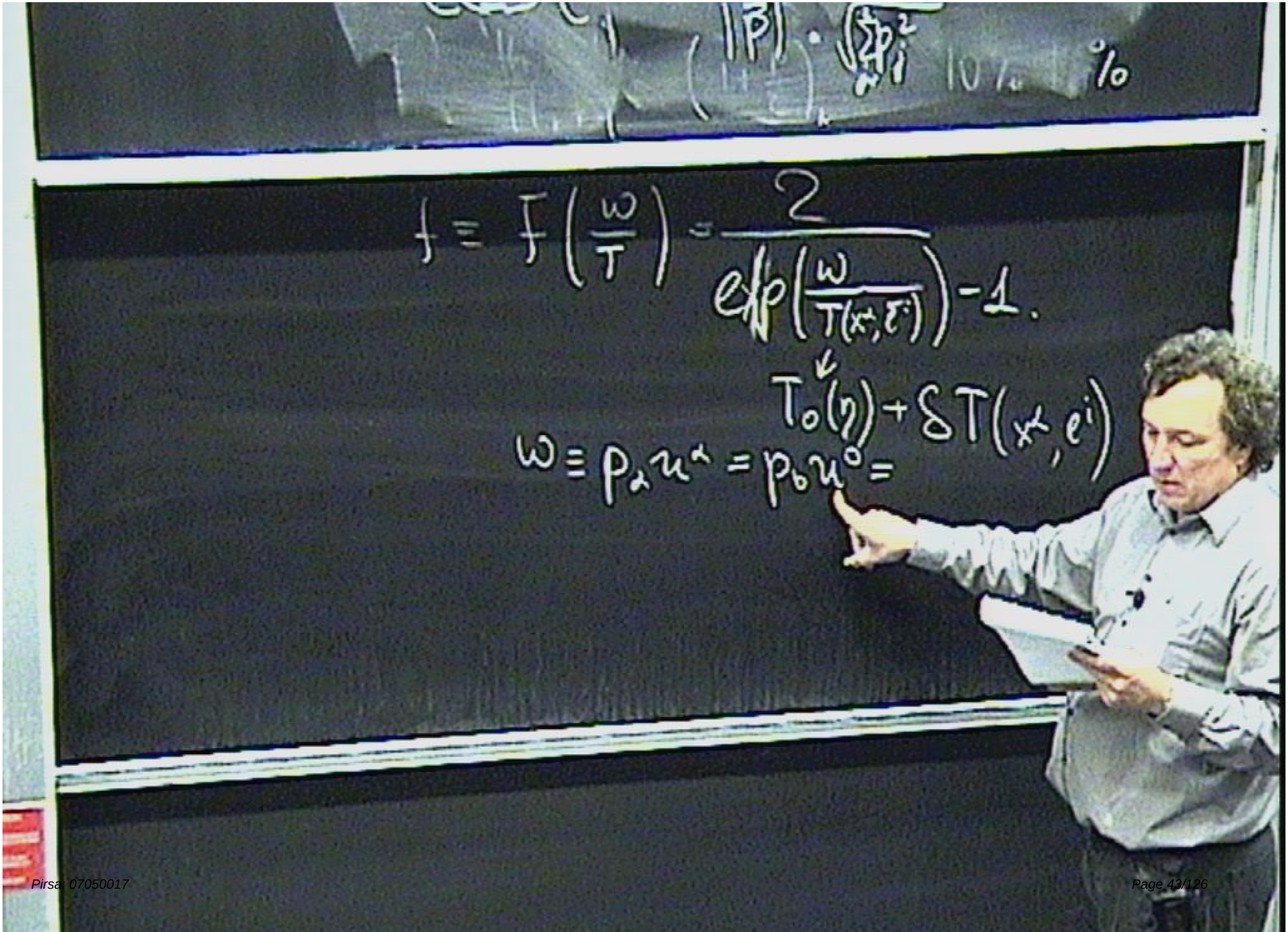
$$\vec{e} \Leftrightarrow e^i = -\frac{p_i}{|\vec{p}|} \quad |\vec{p}| = \sqrt{\sum p_i^2}$$

10% %

$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^\alpha, e^i)}\right) - 1}.$$

$T_0(\eta) + \delta T(x^\alpha, e^i)$ $\frac{\delta T}{T}$

$\omega \equiv p_\alpha u^\alpha$



$|p|$ $\sqrt{\sum p_i^2}$ 10 % 1 %

$$f \equiv F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^\alpha, e^i)}\right) - 1}.$$

$$T_0(\eta) + \delta T(x^\alpha, e^i)$$

$$\omega \equiv p_\alpha u^\alpha = p_0 u^0 =$$

10 %   %

$$f \equiv F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^\mu, \epsilon^i)}\right) - 1}.$$

$$\swarrow$$

$$T_0(\eta) + \delta T(x^\mu, e^i) \qquad \frac{\delta T}{T}$$

$$\omega \equiv p_\alpha u^\alpha = p_0 u^0 =$$

$$\parallel$$

$$u_0 u^0 = g_{00} u^{02} =$$

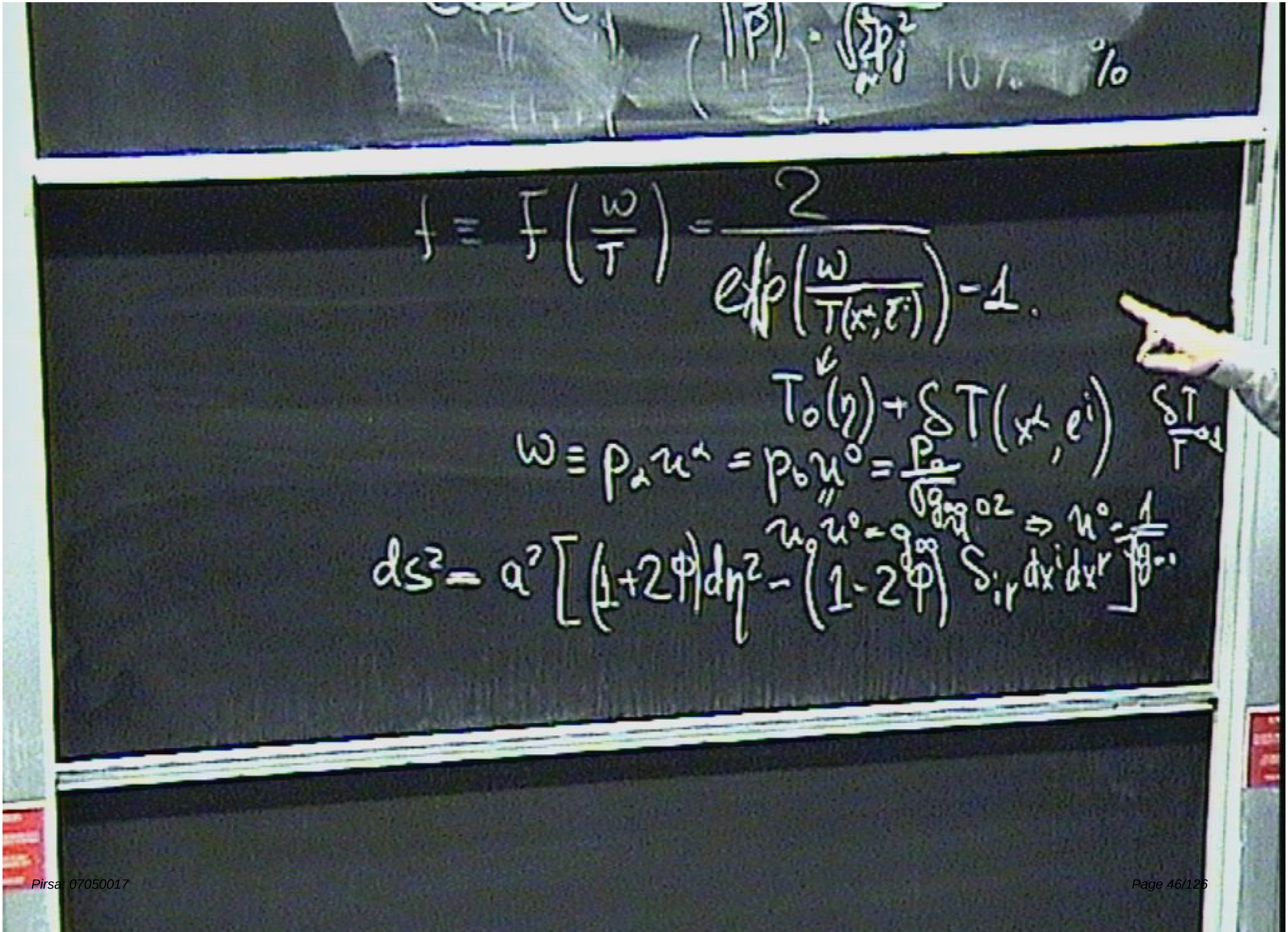
$|\vec{p}|$ $\sqrt{\sum p_i^2}$ 10% 1%

$$f \equiv F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^\alpha, e^i)}\right) - 1}.$$

$$T(x^\alpha, e^i) \swarrow T_0(\eta) + \delta T(x^\alpha, e^i) \qquad \frac{\delta T}{T} \ll 1$$

$$\omega \equiv p_\alpha u^\alpha = p_0 u^0 =$$

$$u_0 u^0 = g_{00} u^{02} \Rightarrow u^0 = \frac{1}{\sqrt{g_{00}}}$$



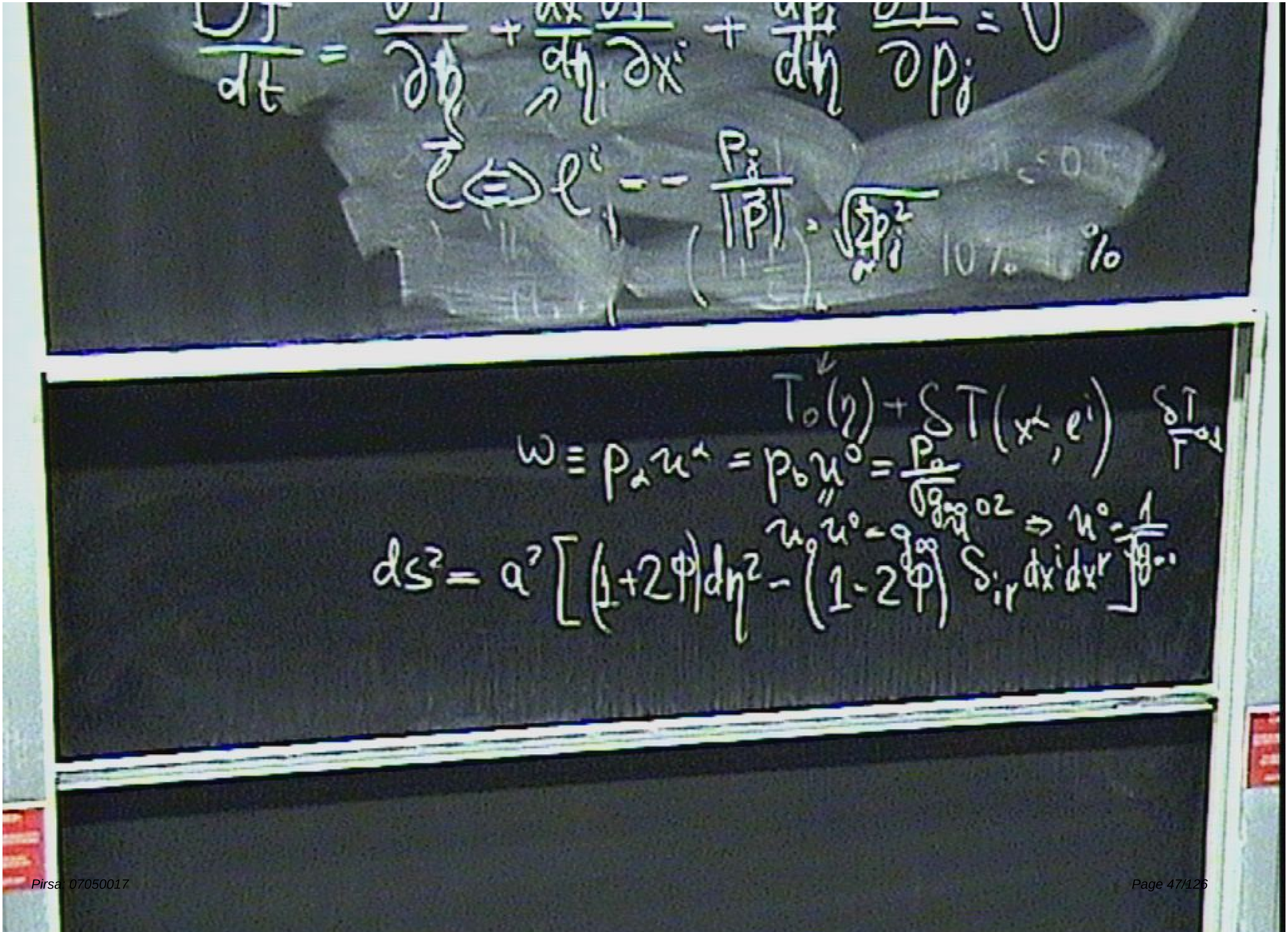
$$f \equiv F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^\alpha, e^i)}\right) - 1}.$$

$$T_0(\eta) + \delta T(x^\alpha, e^i) \qquad \frac{\delta T}{T}$$

$$\omega \equiv p_\alpha u^\alpha = p_0 u^0 = \frac{p_0}{\sqrt{g_{00}}}$$

$$u_0 u^0 = g_{00} u^0 u^0 = 1 \Rightarrow u^0 = \frac{1}{\sqrt{g_{00}}}$$

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - (1-2\Phi)\delta_{ir}dx^i dx^r\right] g_{00}$$



$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta}\frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta}\frac{\partial f}{\partial p_i} = 0$$

$$\vec{\ell} \Leftrightarrow \ell^i = -\frac{p_i}{|p|} \cdot \sqrt{\sum p_i^2}$$

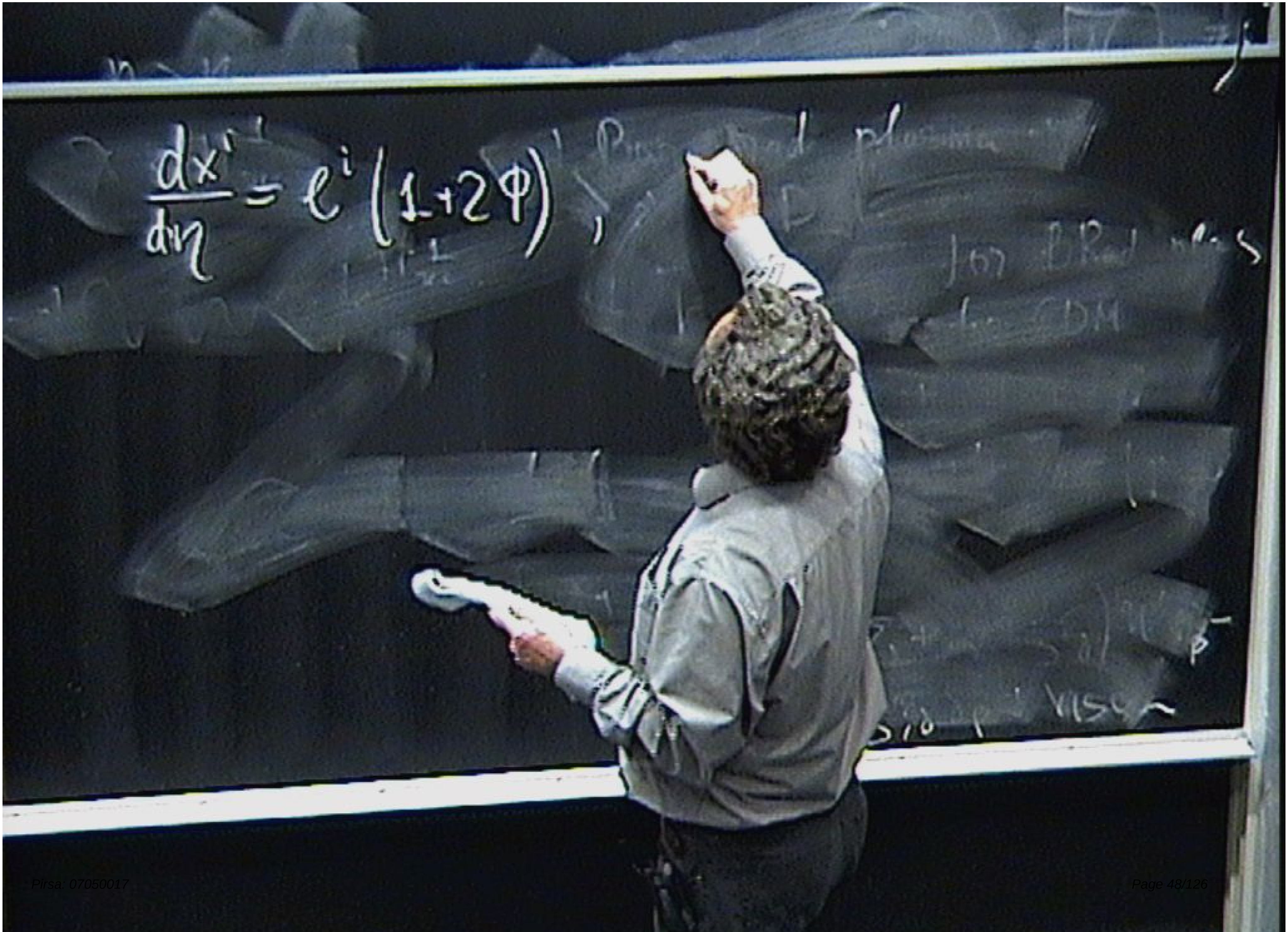
10% %

$$T_0(\eta) + \delta T(x^\alpha, \ell^i) \qquad \frac{\delta T}{T}$$

$$\omega \equiv p_\alpha u^\alpha = p_0 u^0 = \frac{p_0}{\sqrt{g_{00}}}$$

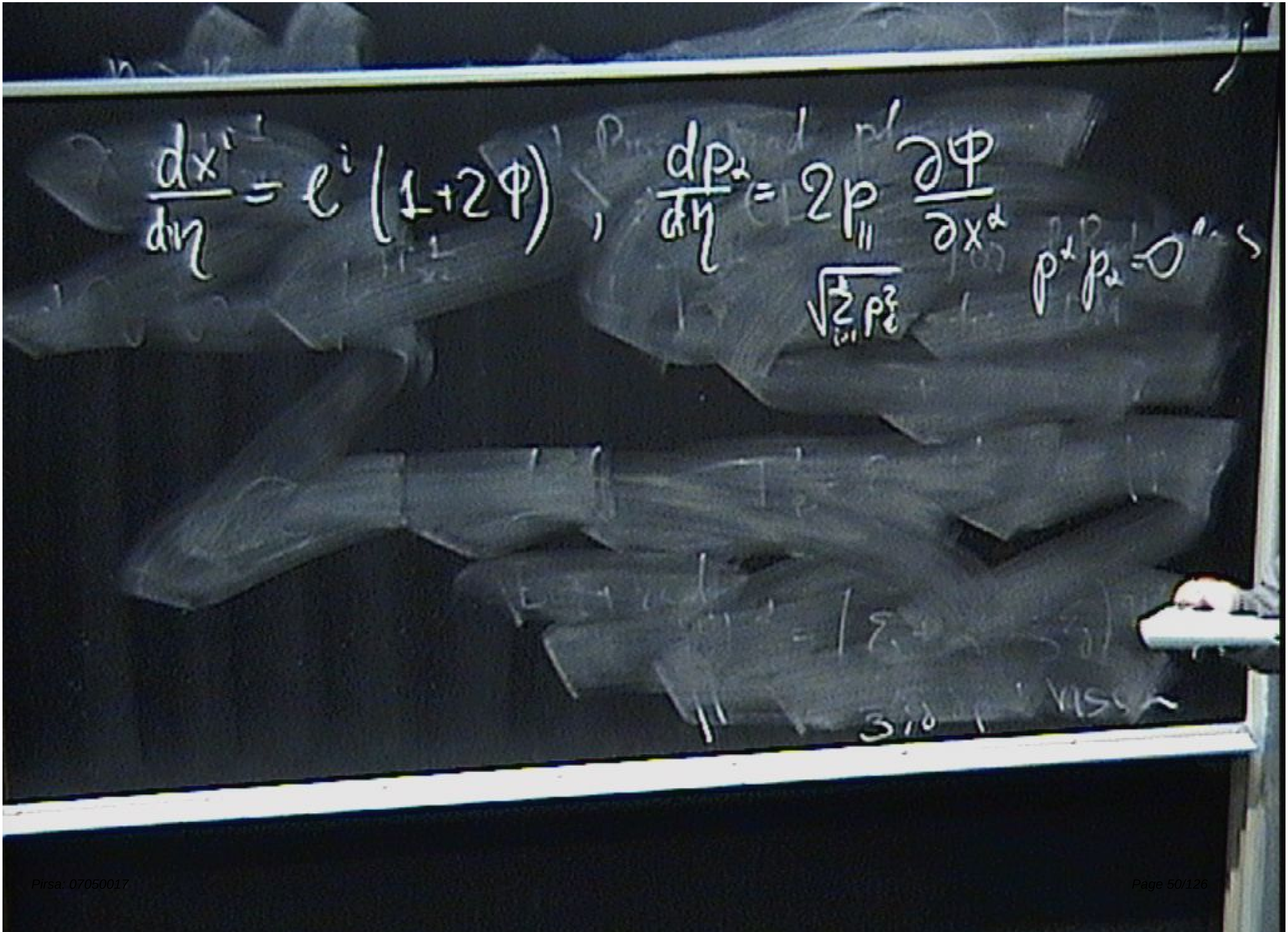
$$u_\alpha u^\alpha = g_{00} u^{0\,2} \Rightarrow u^0 = \frac{1}{\sqrt{g_{00}}}$$

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - (1-2\Psi)\delta_{ir}\,dx^i dx^r\right] g_{00}$$


$\frac{dx^i}{d\eta} = e^i(1+2\Phi)$,

$$\frac{dx^i}{d\eta} = e^i\left(1+2\Phi\right), \quad \frac{dp_\alpha}{d\eta} = 2p_{\parallel}\frac{\partial\Phi}{\partial x^\alpha}$$

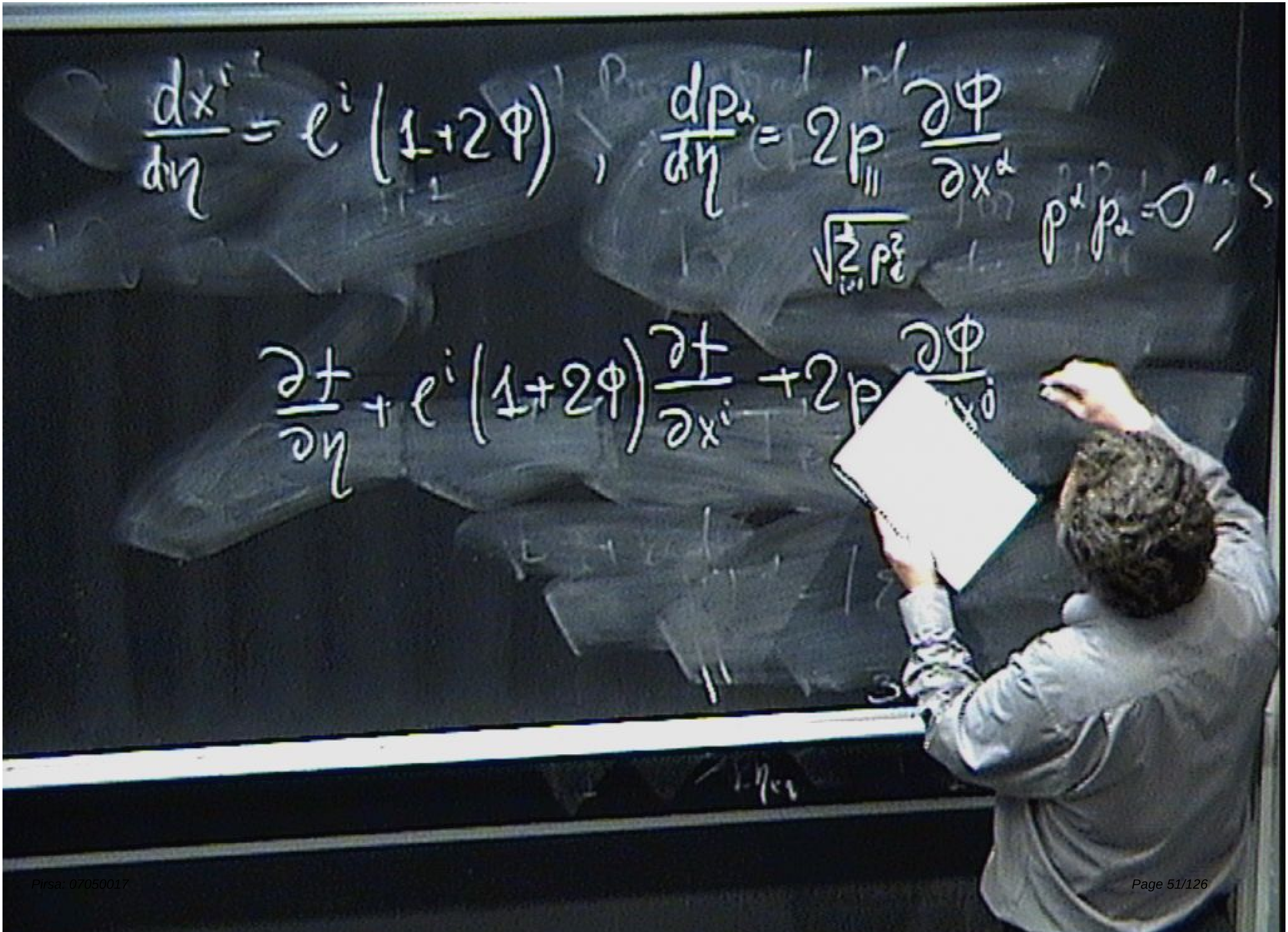
$$\sqrt{\sum_{i=1}^{3} p_i^2}$$



$$\frac{dx^i}{d\eta} = e^i\left(1+2\Phi\right), \quad \frac{dp_\alpha}{d\eta} = 2p_{\parallel}\frac{\partial\Phi}{\partial x^\alpha}$$

$$p_{\parallel} = \sqrt{\sum_{i=1}^{3} p_i^2}$$

$$p^\alpha p_\alpha = 0$$



$$\frac{dx^i}{d\eta} = e^i\left(1+2\phi\right), \quad \frac{dp_\alpha}{d\eta} = 2p_\parallel \frac{\partial\Phi}{\partial x^\alpha}$$

$$\sqrt{\sum_{i=1}^{3} p_i^2}$$

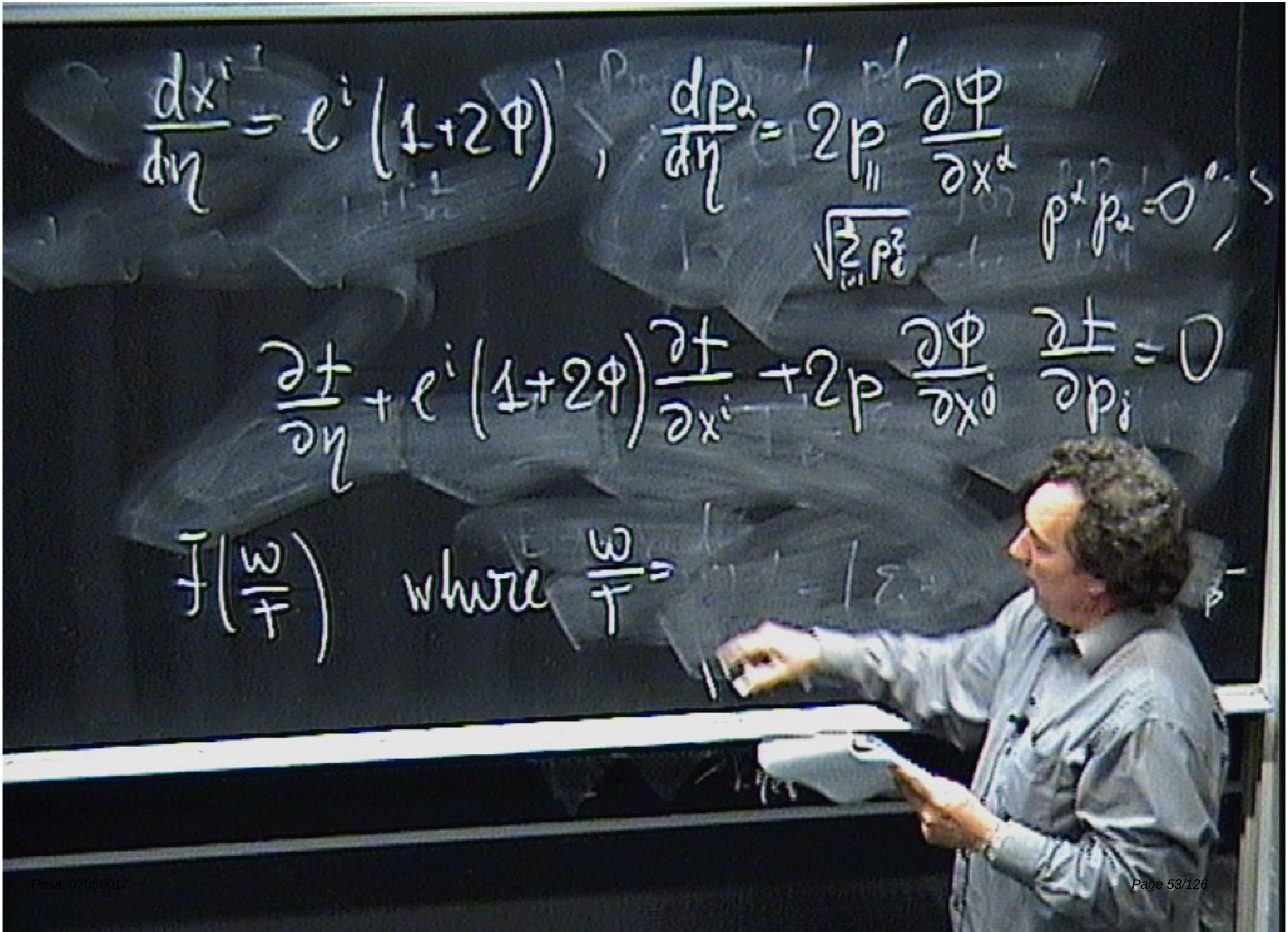
$$p^\alpha p_\alpha = 0$$

$$\frac{\partial f}{\partial\eta} + e^i\left(1+2\phi\right)\frac{\partial f}{\partial x^i} + 2p \frac{\partial\Phi}{\partial x^j}$$

$$\frac{dx^i}{d\eta} = \ell^i(1+2\phi) \; , \quad \frac{dp_\alpha}{d\eta} = 2p_{\parallel}\frac{\partial\Phi}{\partial x^\alpha}$$

$$\sqrt{\sum_{i=1}^{3} p_i^2} \qquad p^\alpha p_\alpha = 0$$

$$\frac{\partial f}{\partial\eta} + \ell^i(1+2\phi)\frac{\partial f}{\partial x^i} + 2p\frac{\partial\phi}{\partial x^j}\frac{\partial f}{\partial p_j} = 0$$

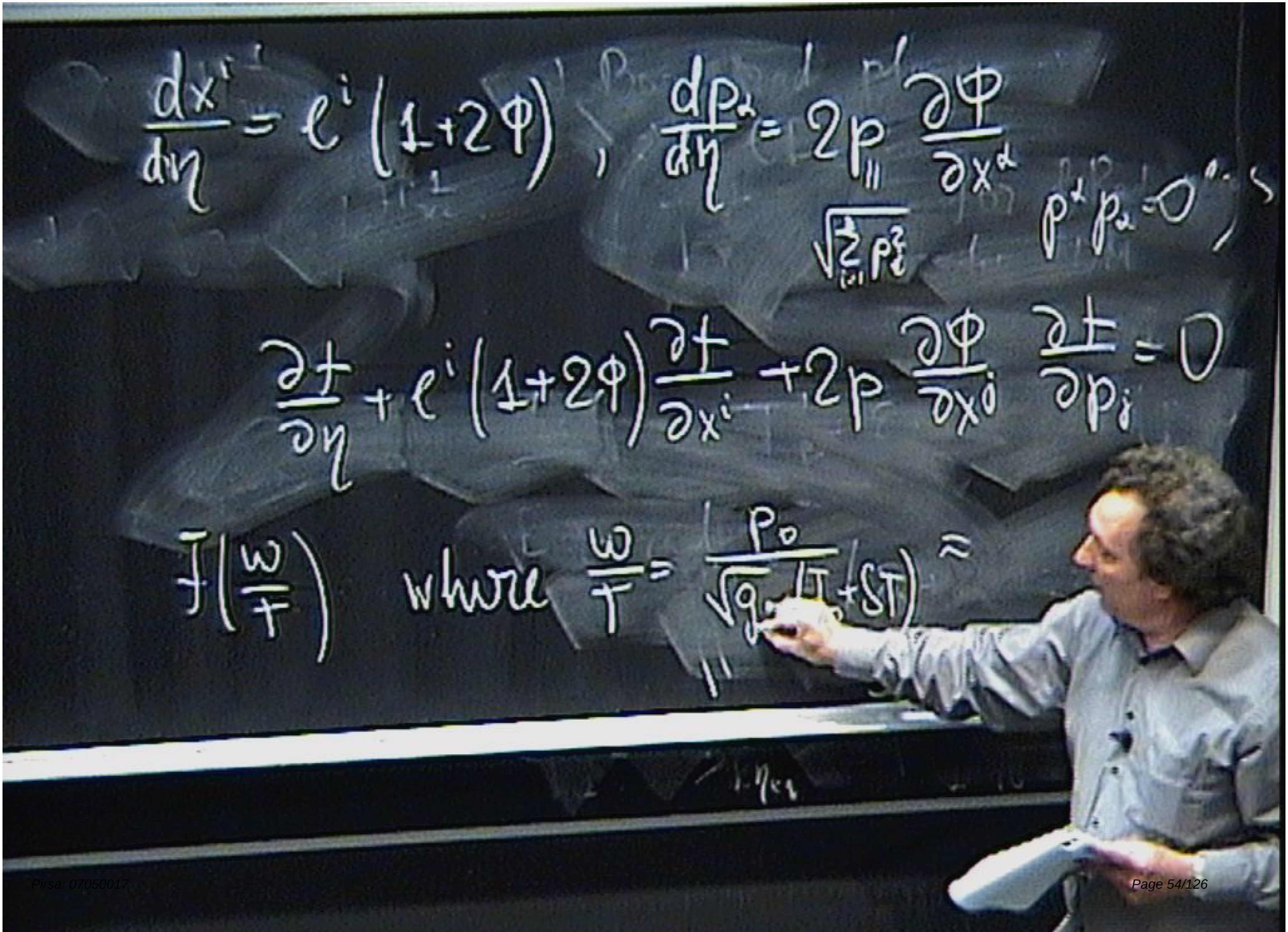


$$\frac{dx^i}{d\eta} = e^i (1+2\phi) \; , \quad \frac{dp_\alpha}{d\eta} = 2p_{\parallel} \frac{\partial \Phi}{\partial x^\alpha} \qquad p^\alpha p_\alpha = 0 \; ;$$

$$\sqrt{\textstyle\sum_{i=1}^{3} p_i^2}$$

$$\frac{\partial f}{\partial \eta} + e^i (1+2\phi) \frac{\partial f}{\partial x^i} + 2p \frac{\partial \Phi}{\partial x^j} \frac{\partial f}{\partial p_j} = 0$$

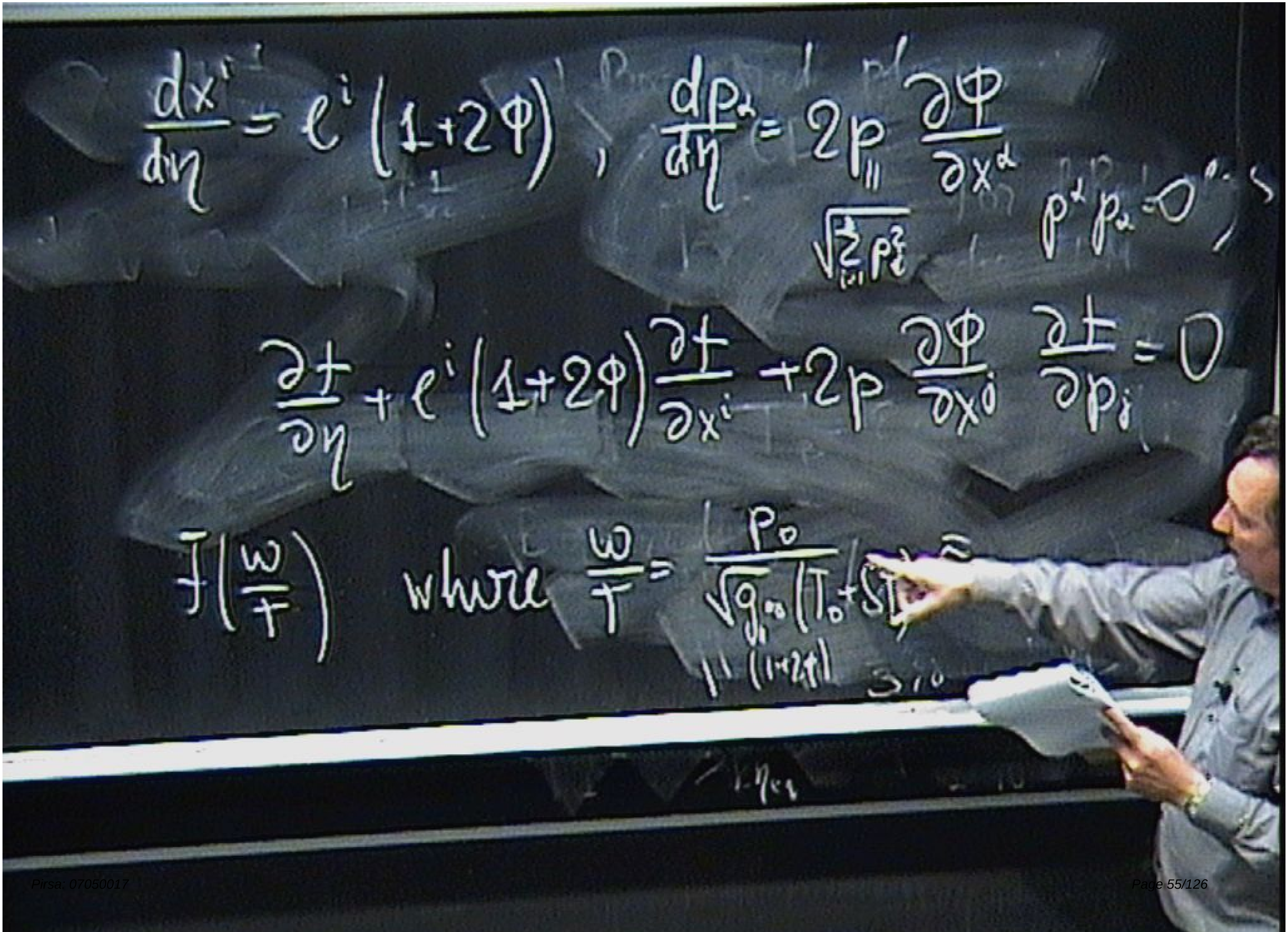
$f\left(\frac{w}{T}\right)$ where $\frac{w}{T} =$



$$\frac{dx^i}{d\eta} = e^i(1+2\phi)\ , \quad \frac{dp_\alpha}{d\eta} = 2\underset{\underset{\sqrt{\sum_{i=1}^{3} p_i^2}}{\|}}{p_{\|}}\,\frac{\partial\phi}{\partial x^\alpha} \qquad \text{“}p^\alpha p_\alpha = 0\text{”}$$

$$\frac{\partial f}{\partial\eta} + e^i(1+2\phi)\frac{\partial f}{\partial x^i} + 2p\,\frac{\partial\phi}{\partial x^j}\,\frac{\partial f}{\partial p_j} = 0$$

$f\left(\frac{w}{T}\right)$ where $\frac{w}{T} = \frac{p_0}{\sqrt{g_{00}}\,(T+\delta T)} \approx$



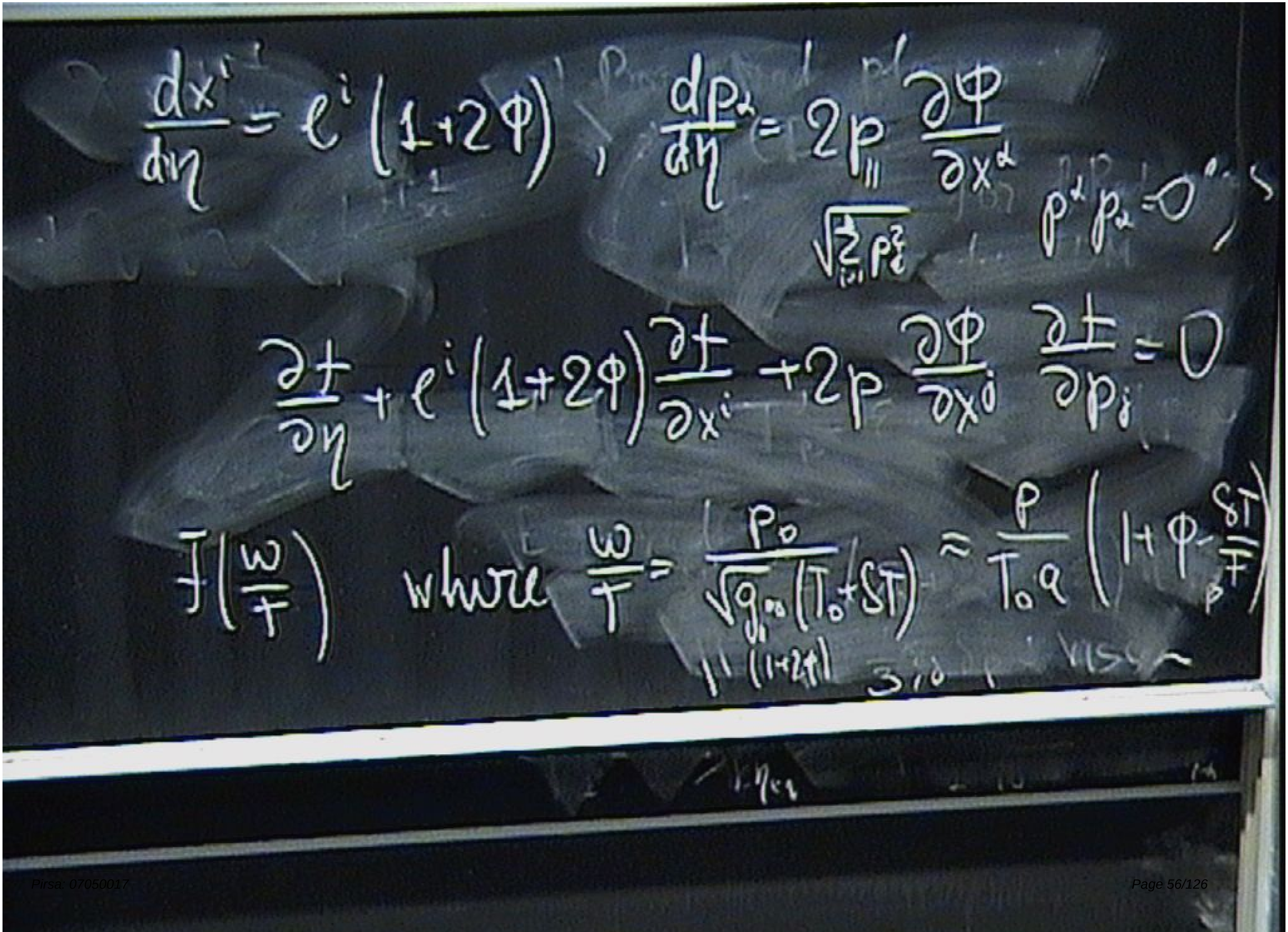
$$\frac{dx^i}{d\eta} = e^i(1+2\phi) \,, \qquad \frac{dp_\alpha}{d\eta} = 2p_{\parallel}\frac{\partial\phi}{\partial x^\alpha}$$

$$p_{\parallel} = \sqrt{\sum_{i=1}^{3} p_i^2} \qquad \text{“} p^\alpha p_\alpha = 0 \text{”} \,)$$

$$\frac{\partial f}{\partial\eta} + e^i(1+2\phi)\frac{\partial f}{\partial x^i} + 2p\frac{\partial\phi}{\partial x^j}\frac{\partial f}{\partial p_j} = 0$$

$$f\left(\frac{w}{T}\right) \quad \text{where } \frac{w}{T} = \frac{p_0}{\sqrt{g_{00}}\,(T_0+\delta T)} \approx$$

$$\sqrt{g_{00}} = (1+2\phi)$$



$$\frac{dx^i}{d\eta} = e^i\left(1+2\phi\right), \quad \frac{dp_\alpha}{d\eta} = 2p_{\parallel}\frac{\partial\Phi}{\partial x^\alpha}$$

$$\sqrt{\sum_{i=1}^{3} p_i^2} \qquad p^\alpha p_\alpha = 0$$

$$\frac{\partial f}{\partial\eta} + e^i\left(1+2\phi\right)\frac{\partial f}{\partial x^i} + 2p\frac{\partial\Phi}{\partial x^j}\frac{\partial f}{\partial p_j} = 0$$

$$f\left(\frac{\omega}{T}\right) \quad \text{where} \quad \frac{\omega}{T} = \frac{p_0}{\sqrt{\underset{(1+2\phi)}{g_{00}}}\left(T_0+\delta T\right)} \approx \frac{p}{T_0 a}\left(1+\phi-\frac{\delta T}{T}\right)$$

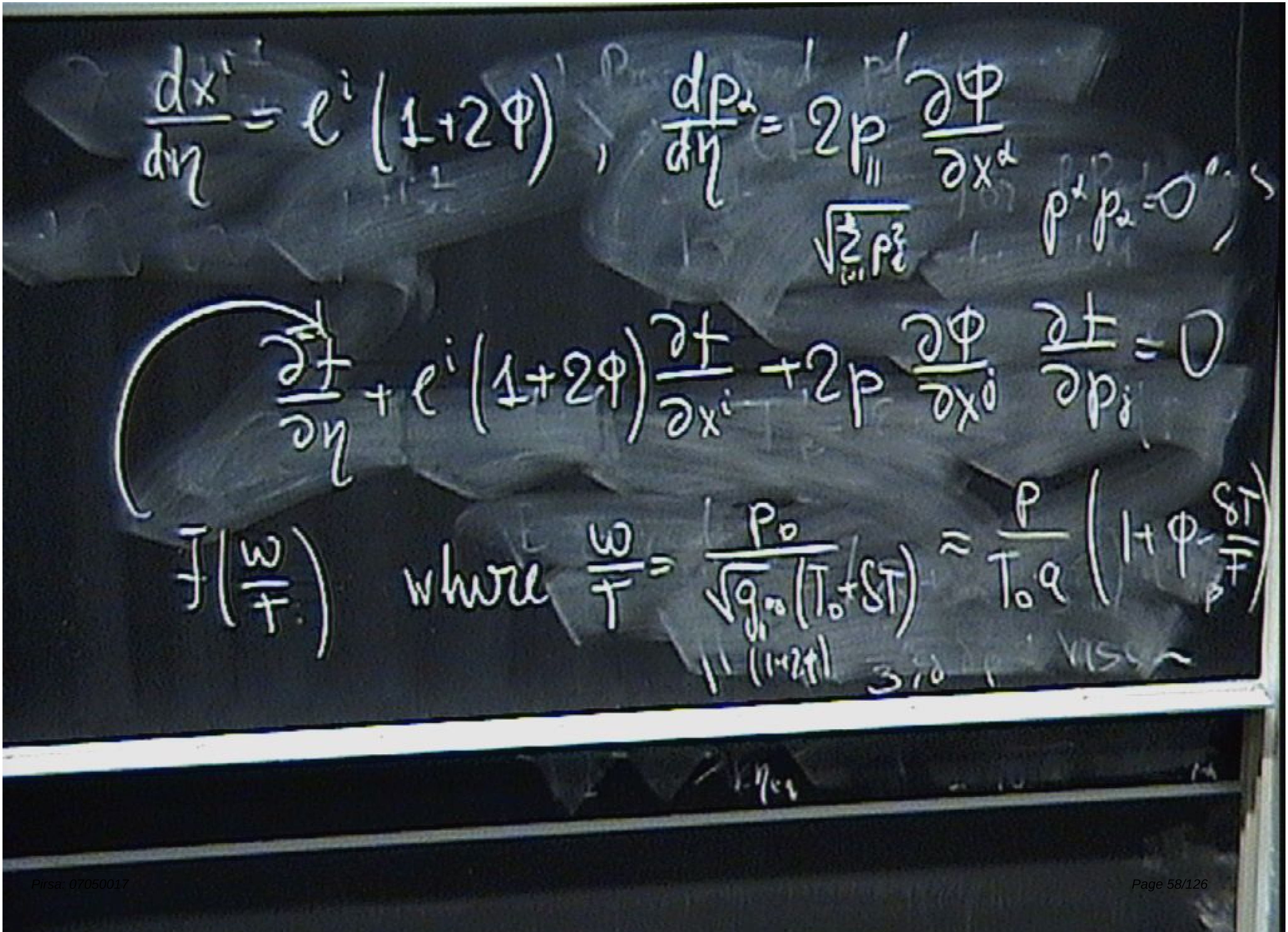
$$\frac{dx^i}{d\eta} = \ell^i (1+2\phi) \, , \quad \frac{dp_\alpha}{d\eta} = 2p_{\parallel} \frac{\partial\phi}{\partial x^\alpha}$$

$$\sqrt{\sum_{i=1}^{3} p_i^2} \qquad p^\alpha p_\alpha = 0 \, ;$$

$$\frac{\partial f}{\partial\eta} + \ell^i (1+2\phi) \frac{\partial f}{\partial x^i} + 2p \frac{\partial\phi}{\partial x^j} \frac{\partial f}{\partial p_j} = 0$$

$f\left(\frac{\omega}{T}\right)$ where $\frac{\omega}{T} = \frac{p_0}{\sqrt{g_{00}}\,(T_0+\delta T)} \approx \frac{p}{T_0 a}\left(1+\phi-\frac{\delta T}{T}\right)$

$(1+2\phi)$



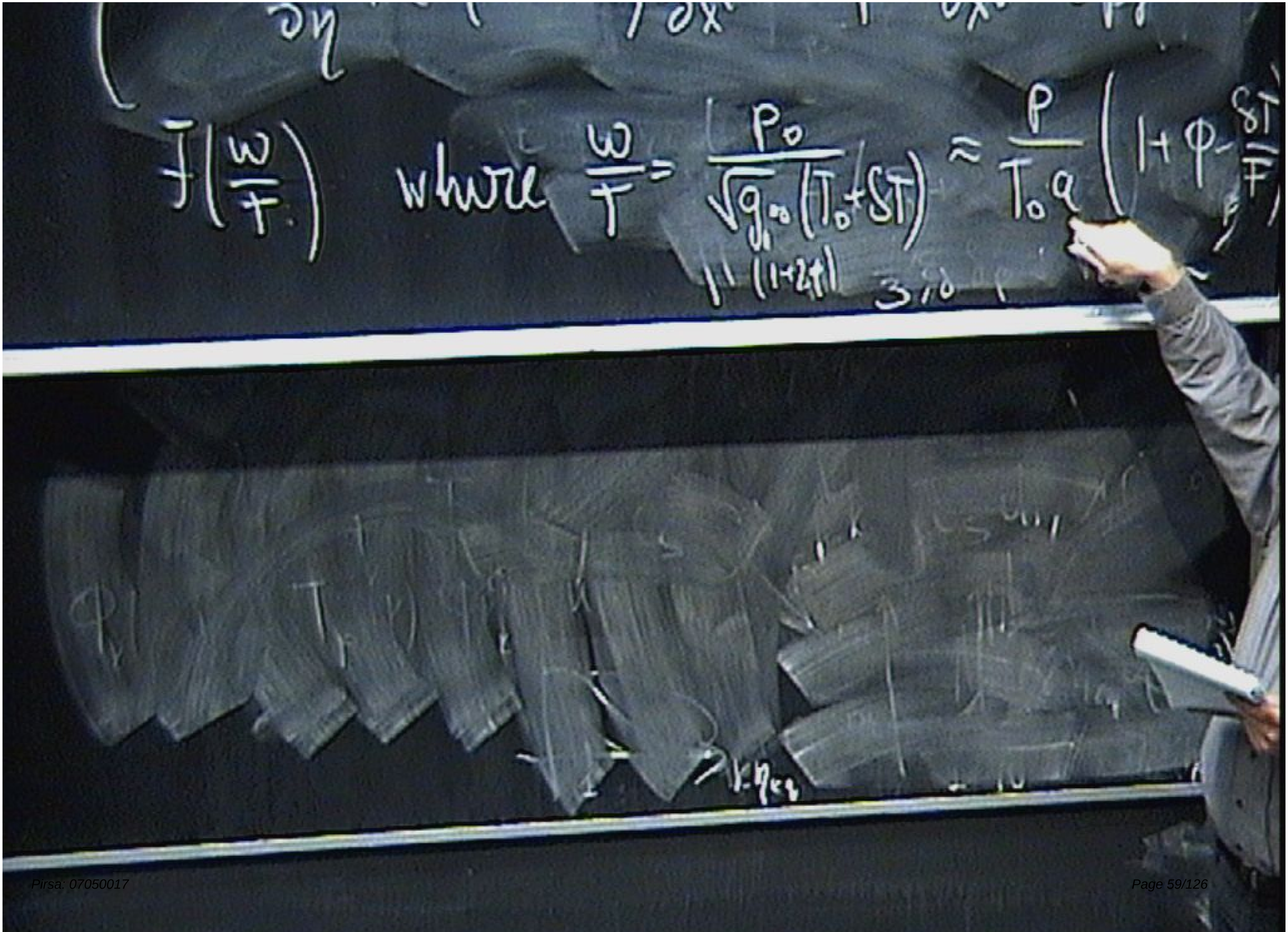
$$\frac{dx^i}{d\eta} = \ell^i (1+2\phi) \ , \quad \frac{dp_\alpha}{d\eta} = 2p \frac{\partial \Phi}{\partial x^\alpha}$$

$$p = \sqrt{\sum_{i=1}^{3} p_i^2} \qquad p^\alpha p_\alpha = 0$$

$$\frac{\partial f}{\partial \eta} + \ell^i (1+2\phi) \frac{\partial f}{\partial x^i} + 2p \frac{\partial \Phi}{\partial x^j} \frac{\partial f}{\partial p_j} = 0$$

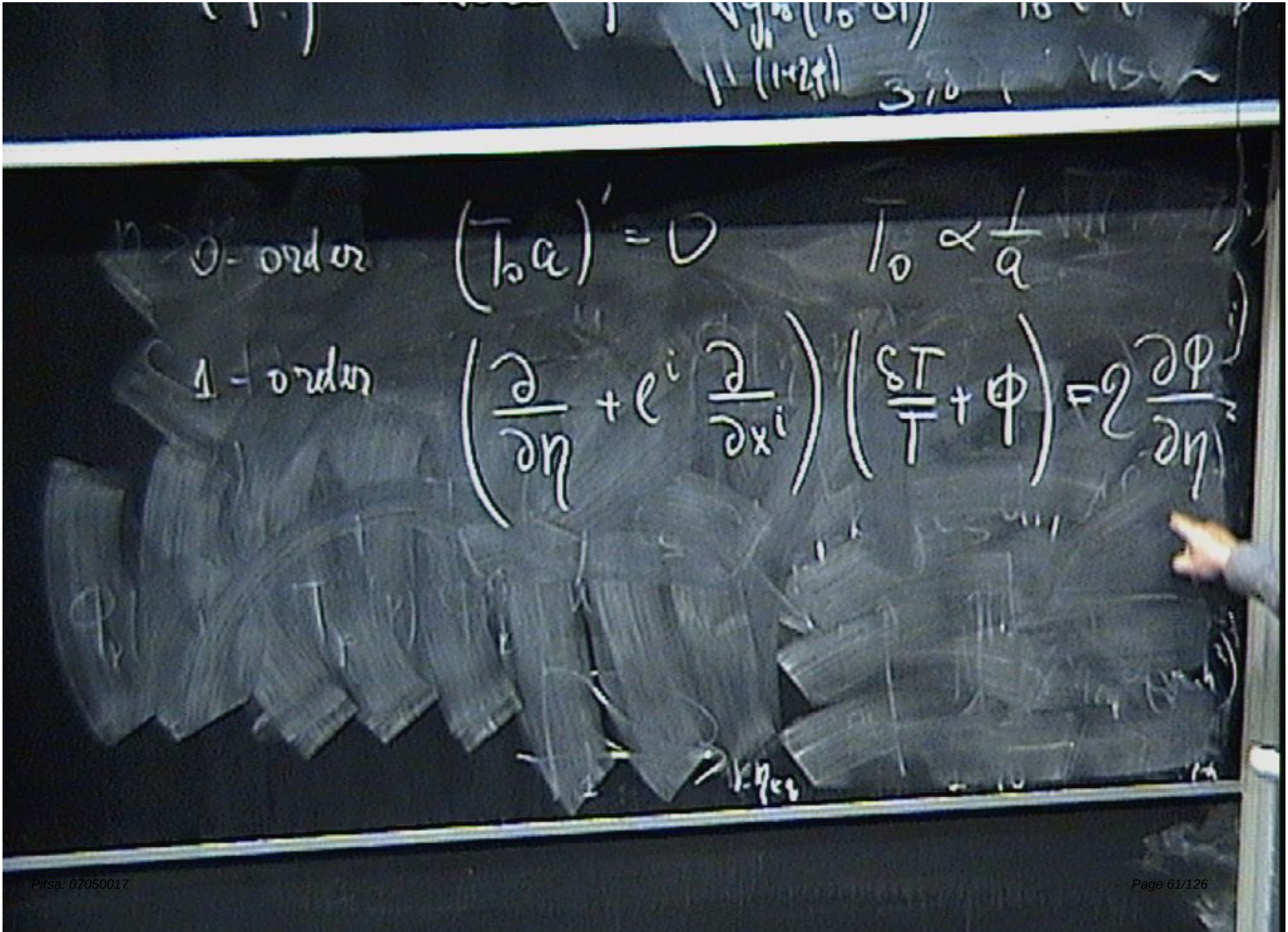
$f\left(\frac{\omega}{T}\right)$ where $\frac{\omega}{T} = \frac{p_0}{\sqrt{g_{00}}\,(T_0 + \delta T)} \approx \frac{p}{T_0 a}\left(1 + \phi - \frac{\delta T}{T}\right)$

$(1+2\phi)$



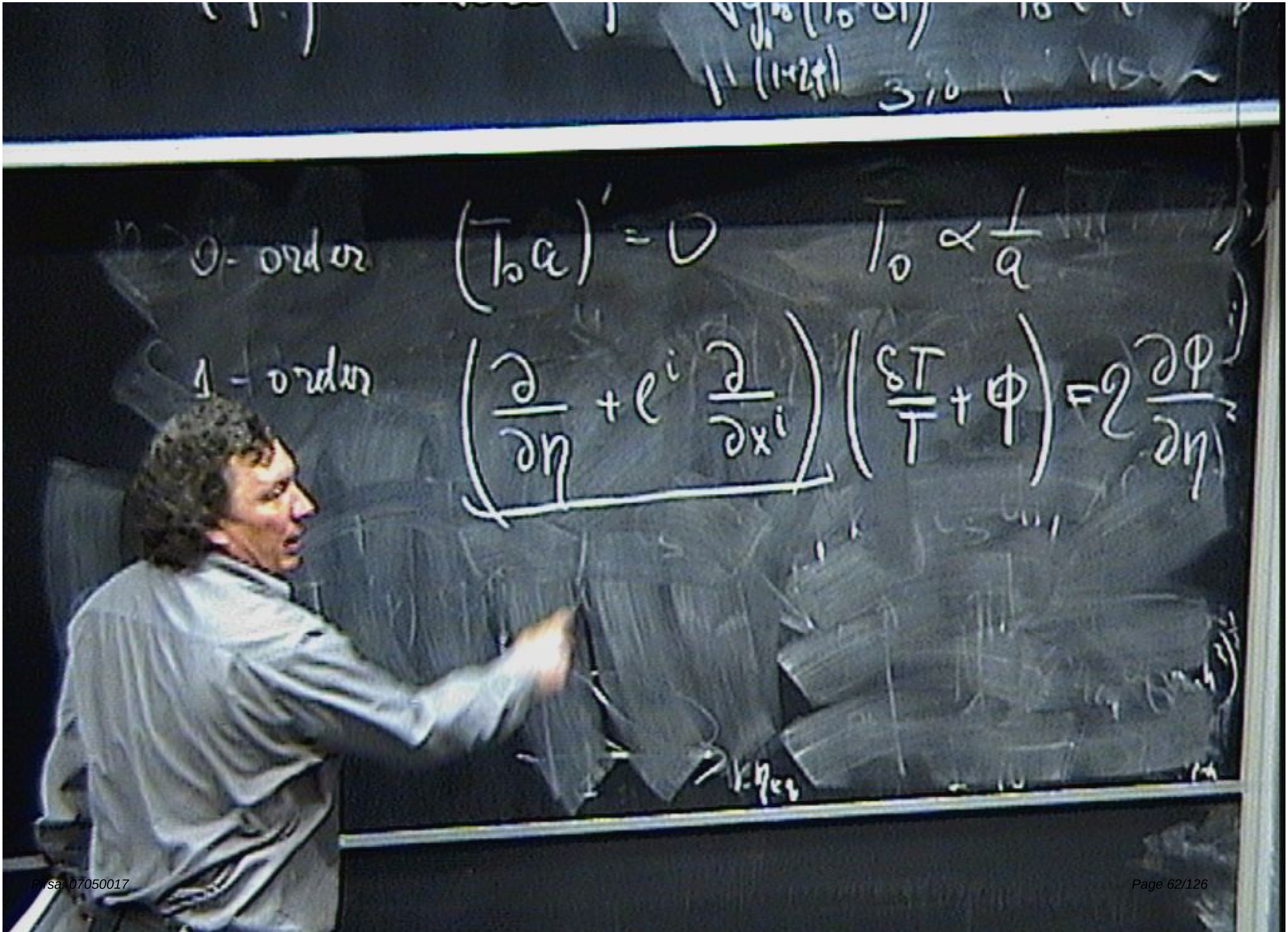
$$\mathcal{F}\left(\frac{w}{T}\right) \quad \text{where} \quad \frac{w}{T} = \frac{\rho_0}{\sqrt{\underset{(1+2\varphi)}{g_{00}}}\,(T_0+\delta T)} \approx \frac{\rho}{T_0\, a}\left(1+\varphi-\frac{\delta T}{T}\right)$$

0- order $(T_0 a)' = 0$ $T_0 \propto \frac{1}{a}$



0-order $(T_0 a)' = 0$ $\quad T_0 \propto \frac{1}{a}$

1-order $\left(\frac{\partial}{\partial\eta} + e^i \frac{\partial}{\partial x^i}\right)\left(\frac{\delta T}{T} + \Phi\right) = 2\frac{\partial\Phi}{\partial\eta}$



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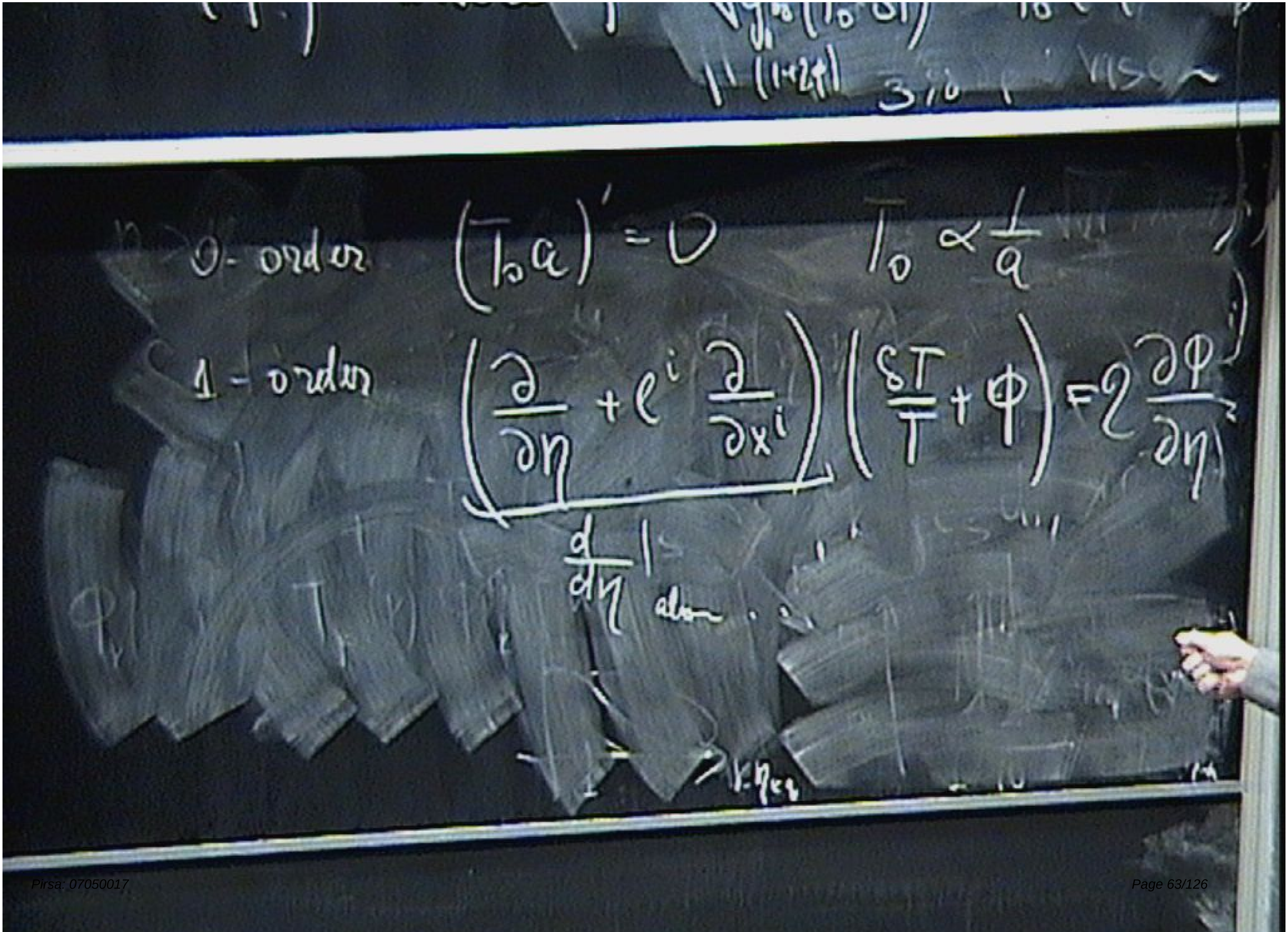


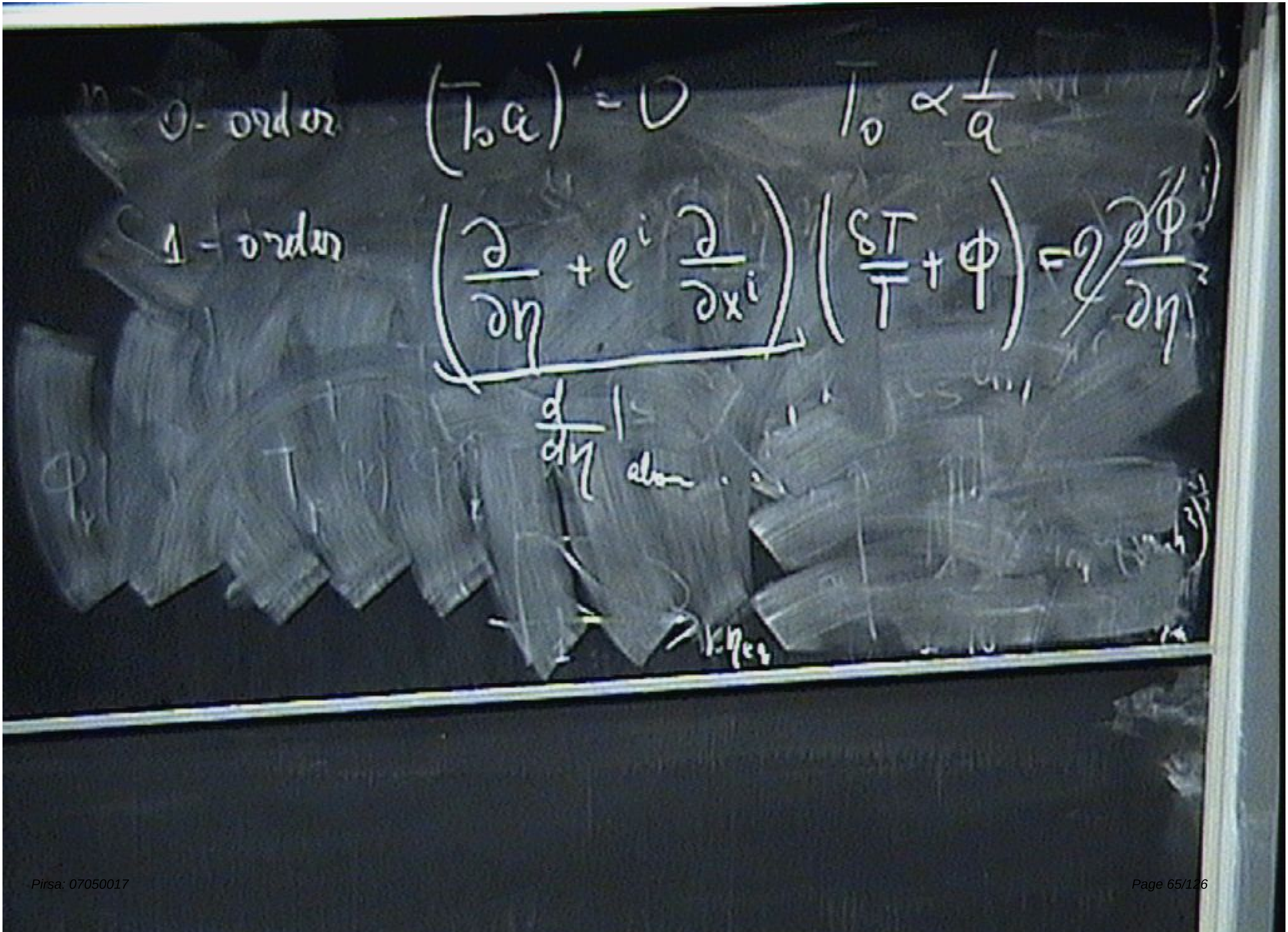
0- order $(T_0 a)' = 0$ $\qquad T_0 \propto \frac{1}{a}$

1 - order $$\underbrace{\left(\frac{\partial}{\partial\eta} + e^i\frac{\partial}{\partial x^i}\right)}_{\left.\frac{d}{d\eta}\right|_{\text{alon}}}\left(\frac{\delta T}{T} + \Phi\right) = 2\frac{\partial\Phi}{\partial\eta}$$

0- order $\quad (T_0 a)' = 0 \qquad T_0 \propto \frac{1}{a}$

1 - order $\quad \underline{\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)}\left(\frac{\delta T}{T} + \phi\right) = \cancel{2}\frac{\cancel{\partial}\phi}{\partial\eta}$

$$\left.\frac{d}{d\eta}\right|_{\text{along}} \ldots$$

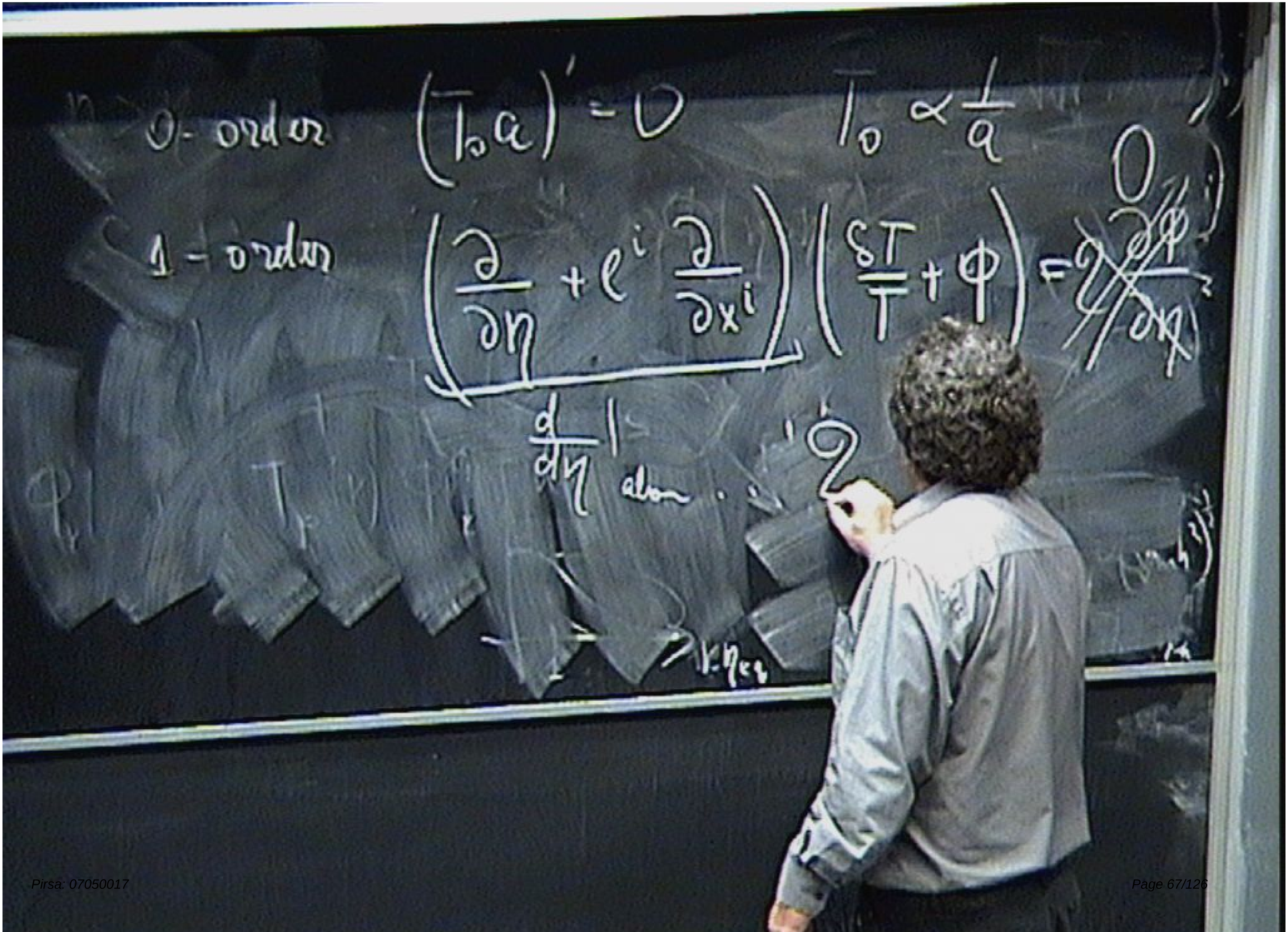


0- order $(T_0 a)' = 0$ $\qquad T_0 \propto \frac{1}{a}$

1 - order $\underbrace{\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)}_{\left.\frac{d}{d\eta}\right|_{\text{along}}} \left(\frac{\delta T}{T} + \Phi\right) = \cancel{2}\frac{\partial\Phi}{\partial\eta}$

0- order $(T_0 a)' = 0$ $T_0 \propto \frac{1}{a}$

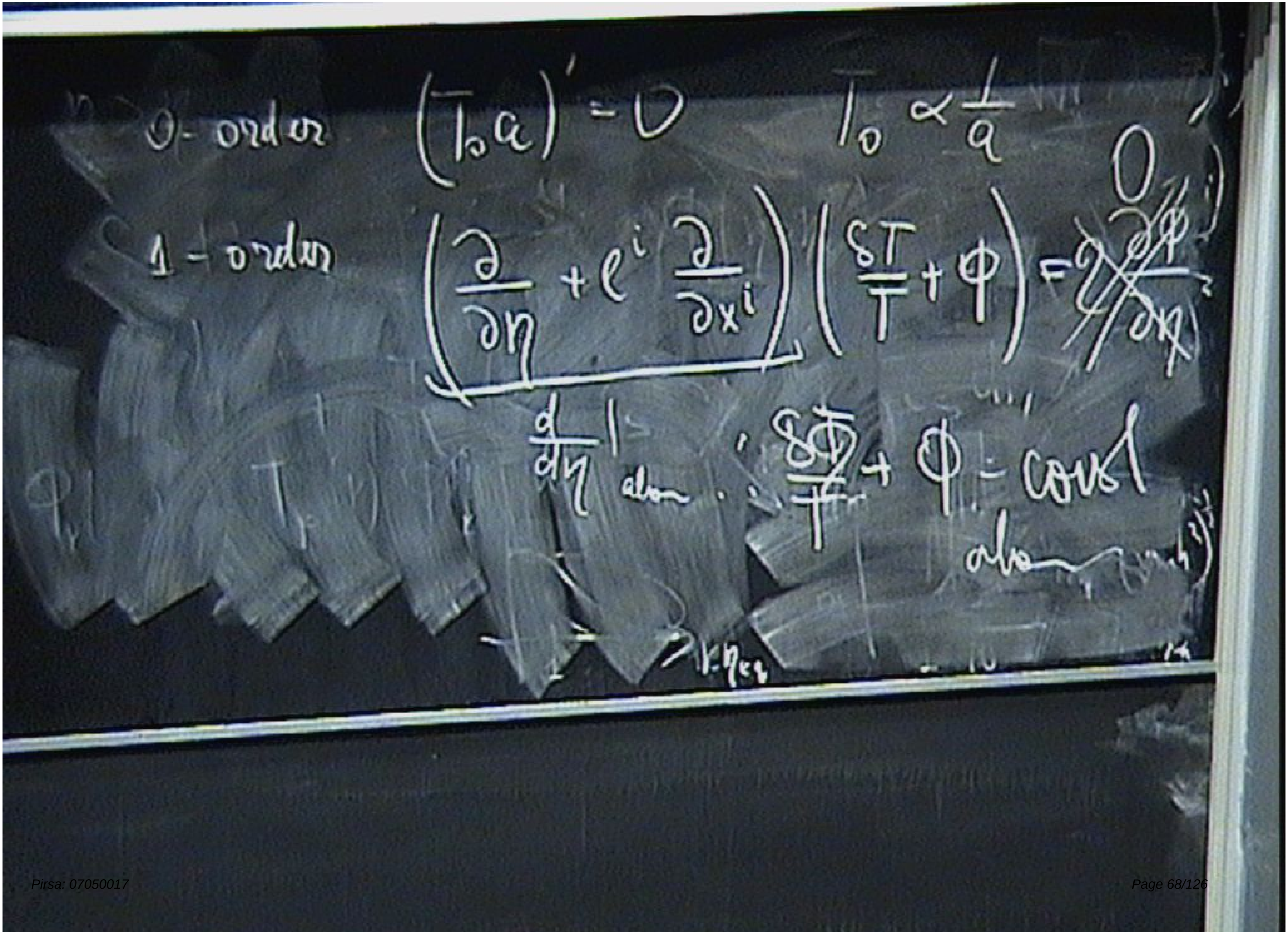
1 - order $\underbrace{\left(\frac{\partial}{\partial\eta} + e^i \frac{\partial}{\partial x^i}\right)}_{\left.\frac{d}{d\eta}\right|_{\text{along}}}\left(\frac{\delta T}{T} + \phi\right) = \cancel{\frac{\partial\psi}{\partial\eta}}$



0- order $(T_0 a)' = 0$ $T_0 \propto \frac{1}{a}$

1 - order $$\underbrace{\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)}_{\frac{d}{d\eta}\big|_{\text{along}}}\left(\frac{\delta T}{T} + \phi\right) = \overset{0}{\cancel{2\frac{\partial\phi}{\partial\eta}}}$$

2

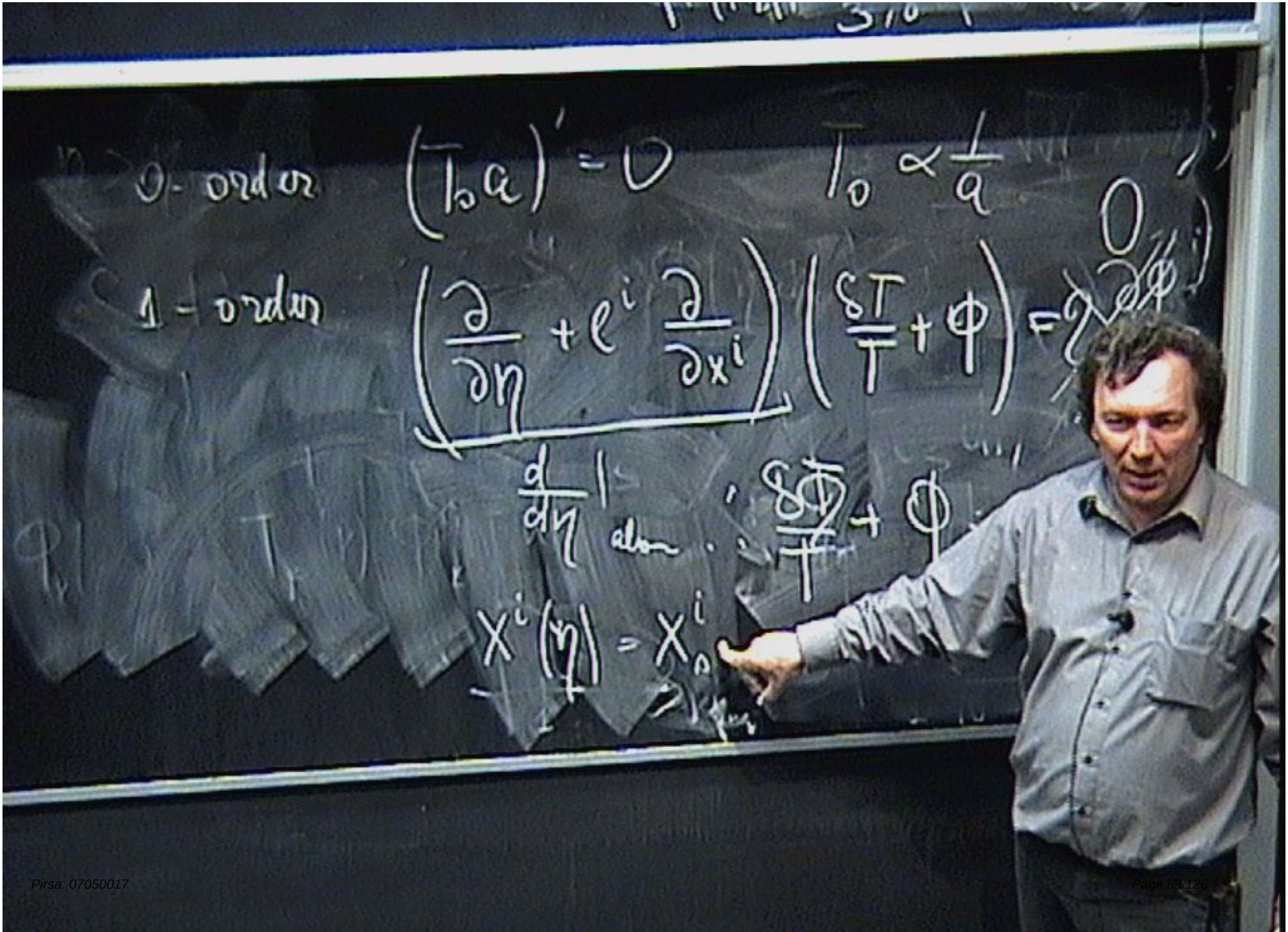


0- order $(T_0 a)' = 0$ $\qquad T_0 \propto \frac{1}{a}$

1 - order $$\underbrace{\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)}_{\left.\frac{d}{d\eta}\right|_{\text{along}}}\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{\cancel{2\frac{\partial\Phi}{\partial\eta}}}$$

$$\therefore \frac{\delta T}{T} + \Phi = \text{const}$$

along

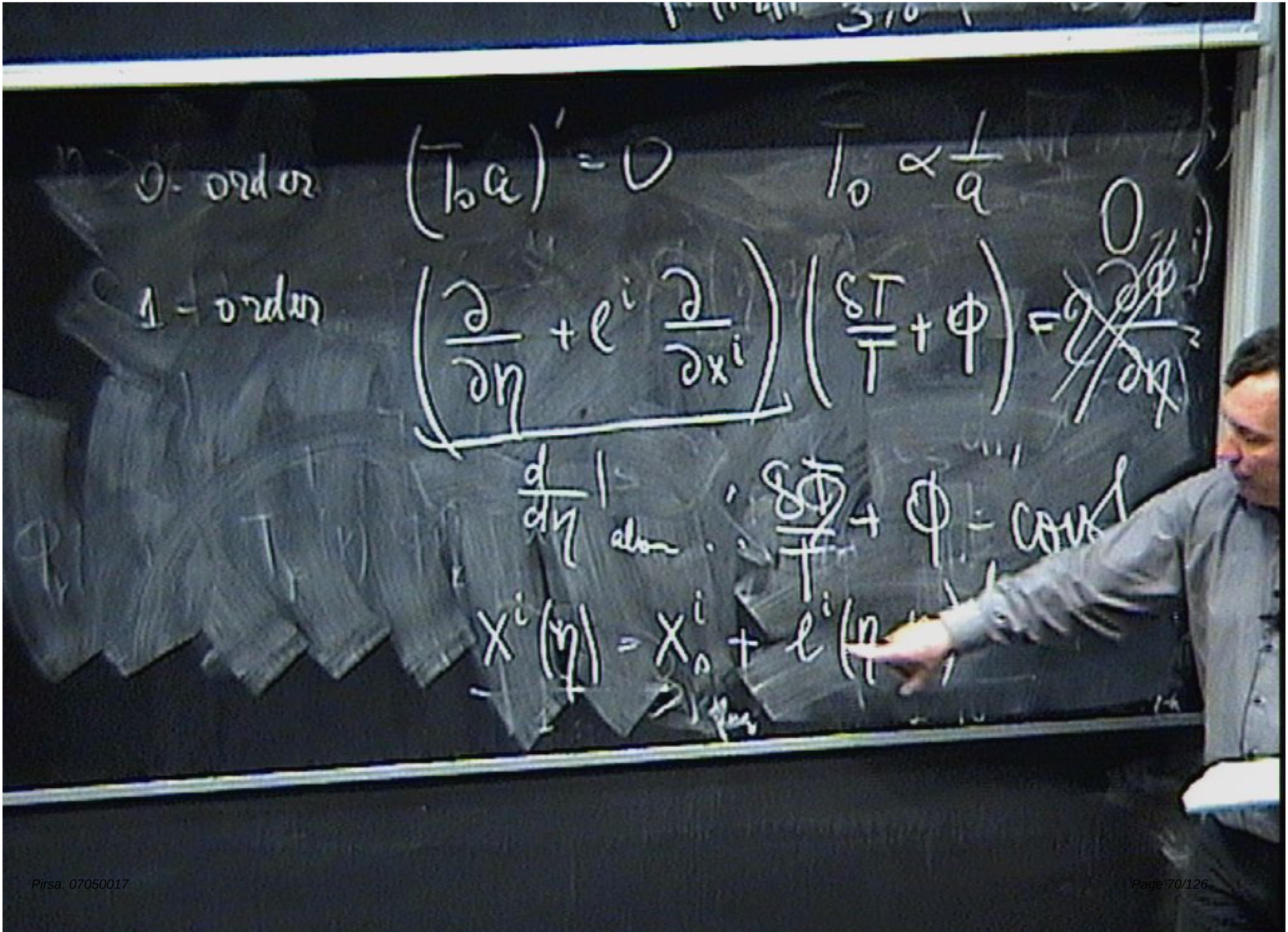


0- order $(T_0 a)' = 0$ $T_0 \propto \frac{1}{a}$

1 - order $\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{2\,\partial\Phi}$

$\frac{d}{d\eta}\Big|_{along}$: $\frac{\delta\Phi}{T} + \Phi$

$x^i(\eta) = x_0^i$

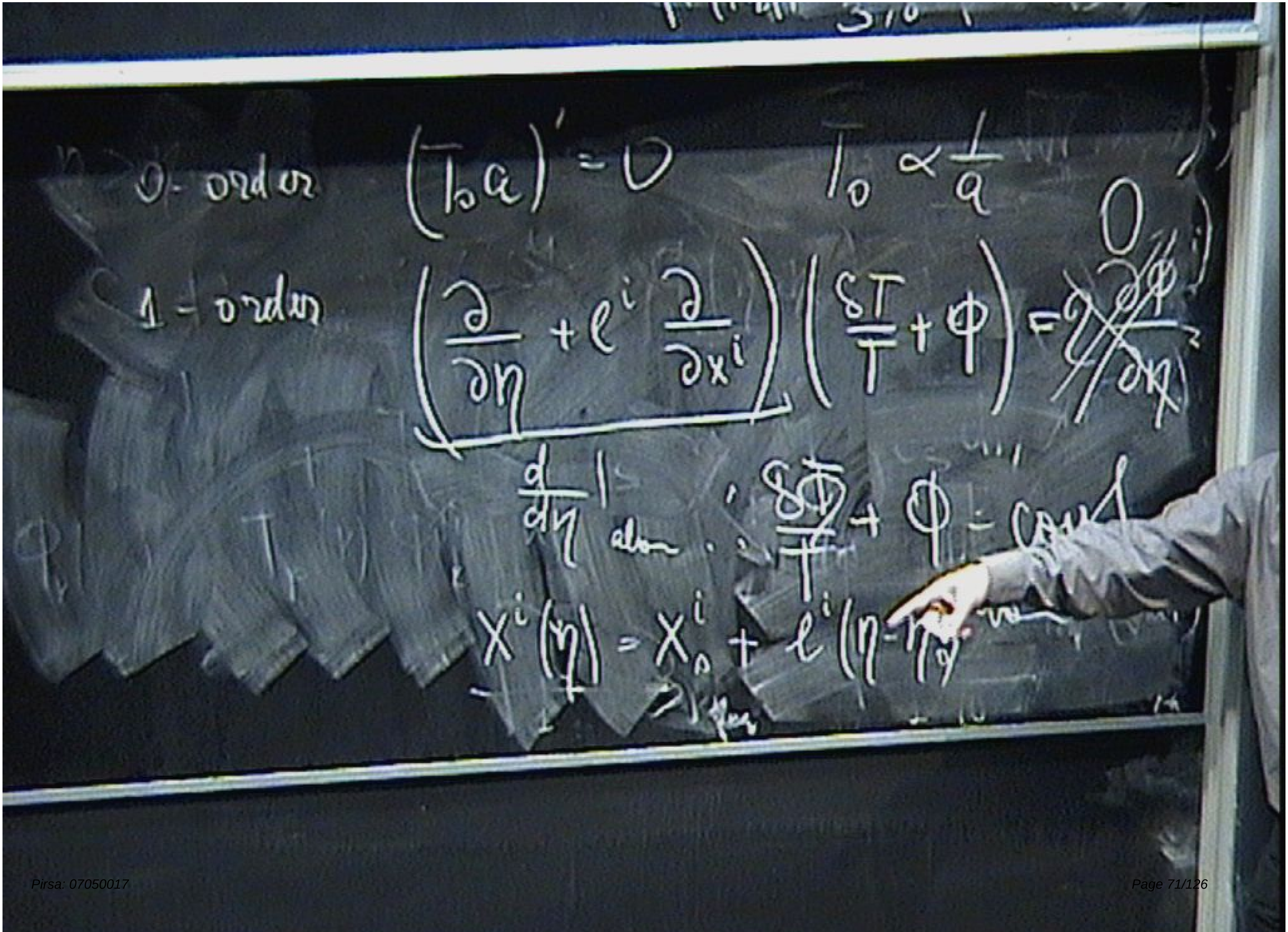


0- order $(T_0 a)' = 0$ $\qquad T_0 \propto \frac{1}{a}$

1 - order $\underbrace{\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)}_{\frac{d}{d\eta}\big|_{along}}\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{\cancel{2\frac{\partial\Phi}{\partial\eta}}}$

$\frac{d}{d\eta}\Big|_{along}$; $\frac{\delta T}{T} + \Phi = const$

$x^i(\eta) = x^i_0 + l^i(\eta$

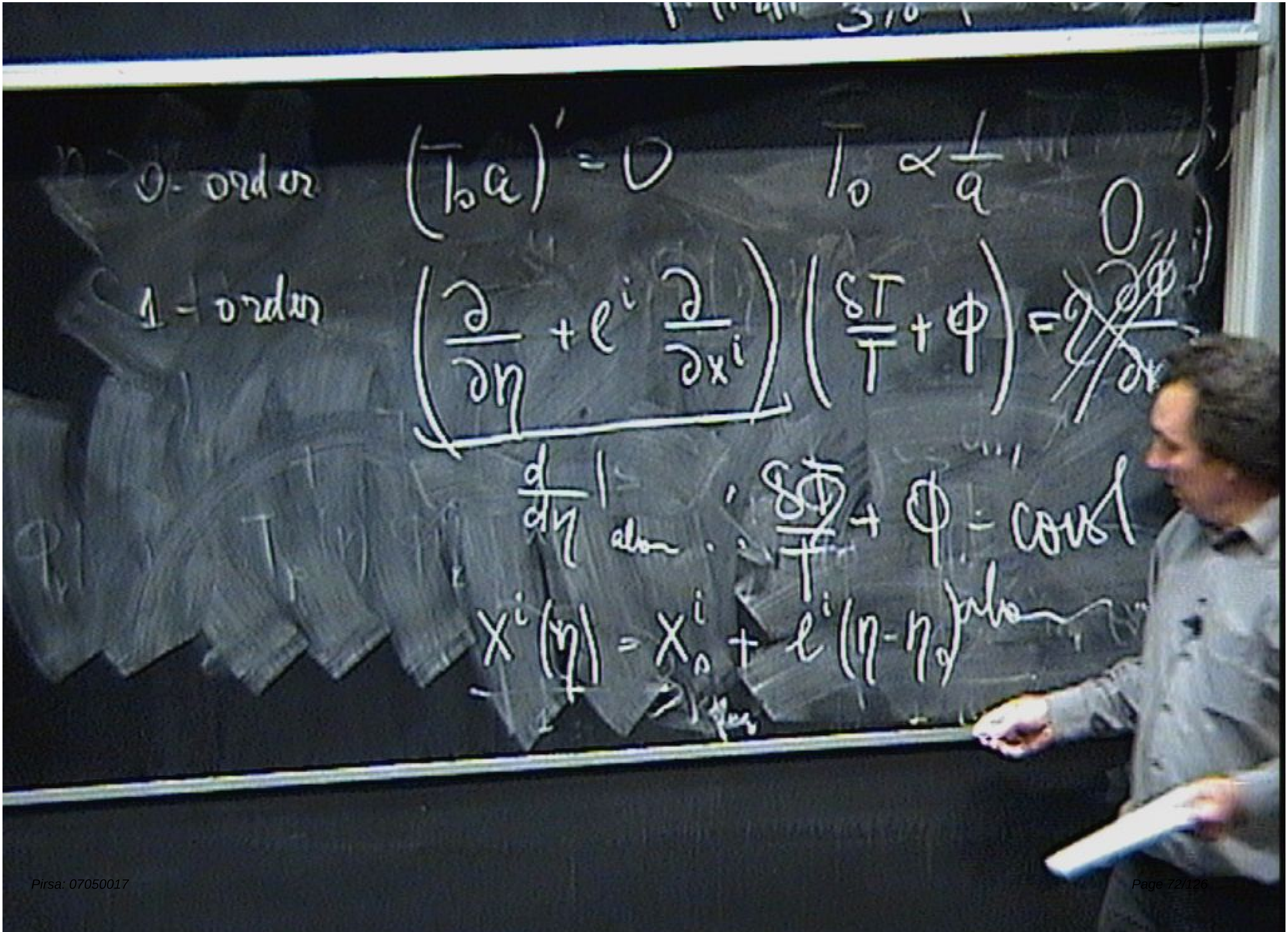


0-order $(T_0 a)' = 0$ $\quad T_0 \propto \frac{1}{a}$

1-order $\underline{\left(\frac{\partial}{\partial \eta} + l^i \frac{\partial}{\partial x^i}\right)}\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{\cancel{2\frac{\partial \Phi}{\partial \eta}}}$

$\frac{d}{d\eta}\Big|_{\text{along}}$; $\frac{\delta T}{T} + \Phi = \text{const}$

$x^i(\eta) = x_0^i + l^i(\eta - \eta_0)$



0- order $(T_0 a)' = 0$ $T_0 \propto \frac{1}{a}$

1 - order $\underline{\left(\frac{\partial}{\partial \eta} + l^i \frac{\partial}{\partial x^i}\right)}\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{\cancel{2\frac{\partial \Phi}{\partial \eta}}}$

$\frac{d}{d\eta}\Big|_{\text{along}}$; $\frac{\delta T}{T} + \Phi = \text{const}$

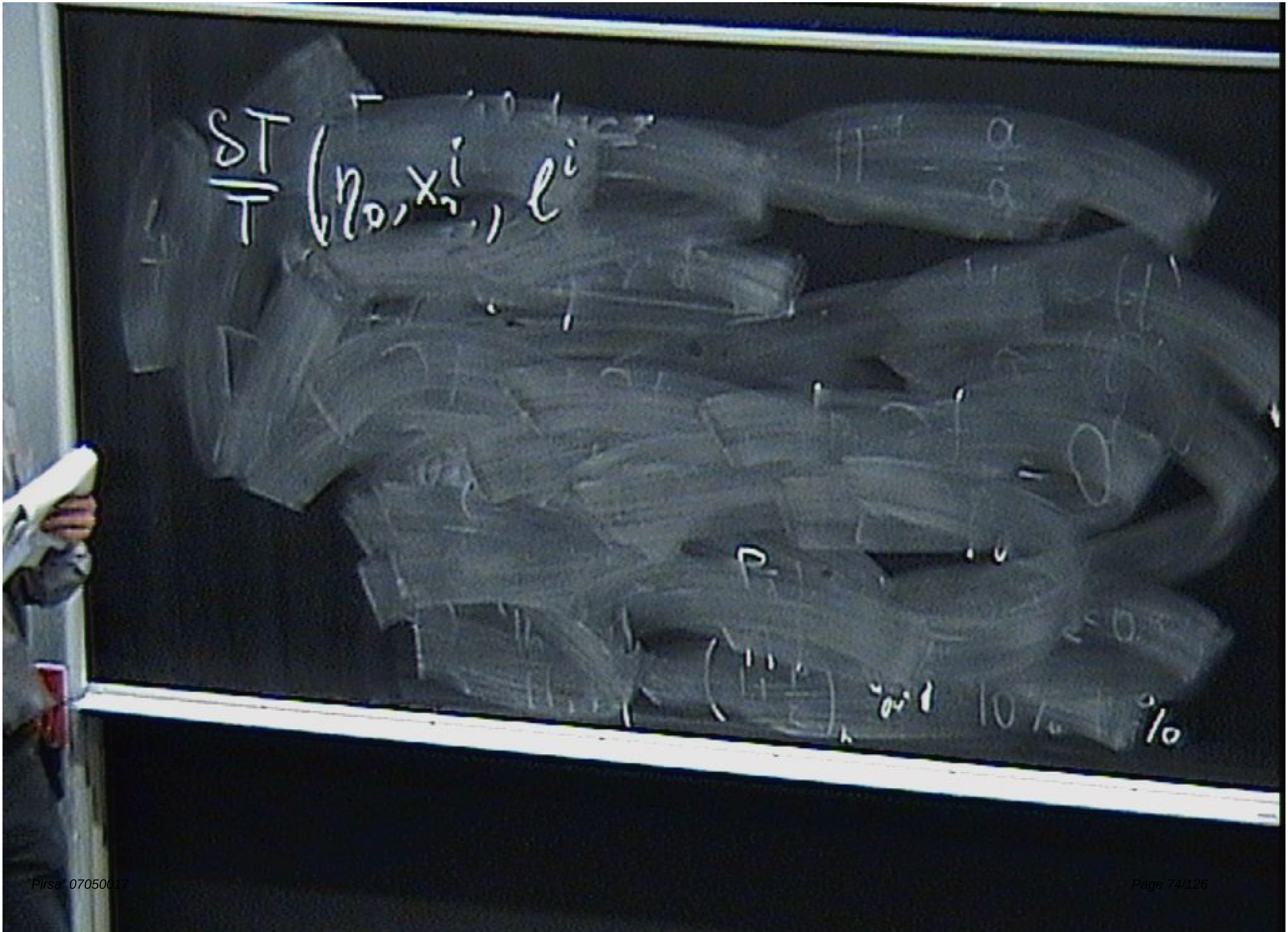
$x^i(\eta) = x_0^i + l^i(\eta - \eta_0)$ along

0- order $(T_0 a)' = 0$ $T_0 \propto \frac{1}{a}$

1 - order $\underline{\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)}\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{\cancel{2\frac{\partial\Phi}{\partial\eta}}}$

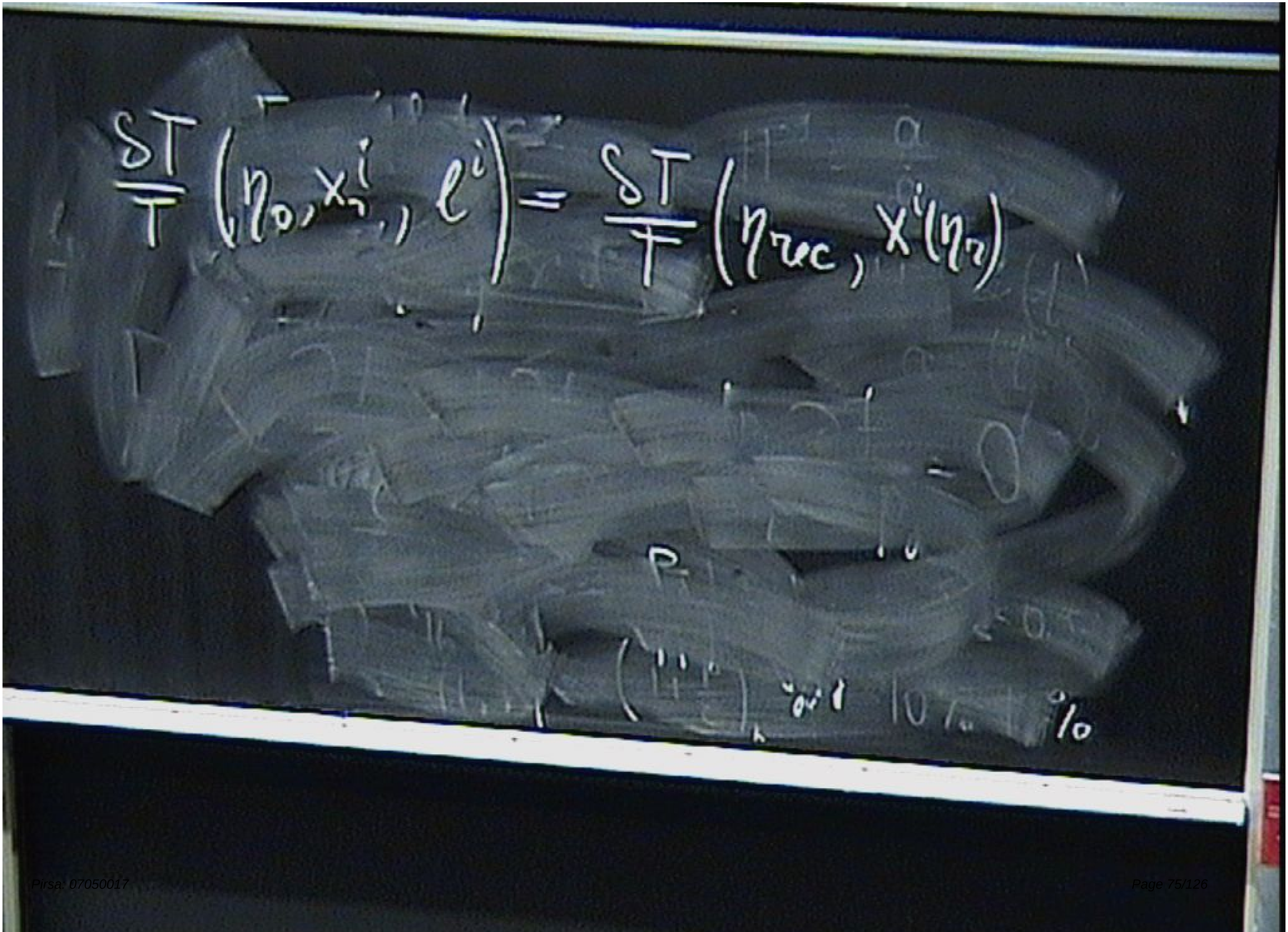
$\frac{d}{d\eta}\Big|_{\text{along}}$; $\frac{\delta T}{T} + \Phi = \text{const}$

$\underline{x^i(\eta) = x_0^i + l^i(\eta - \eta_0)}$ along

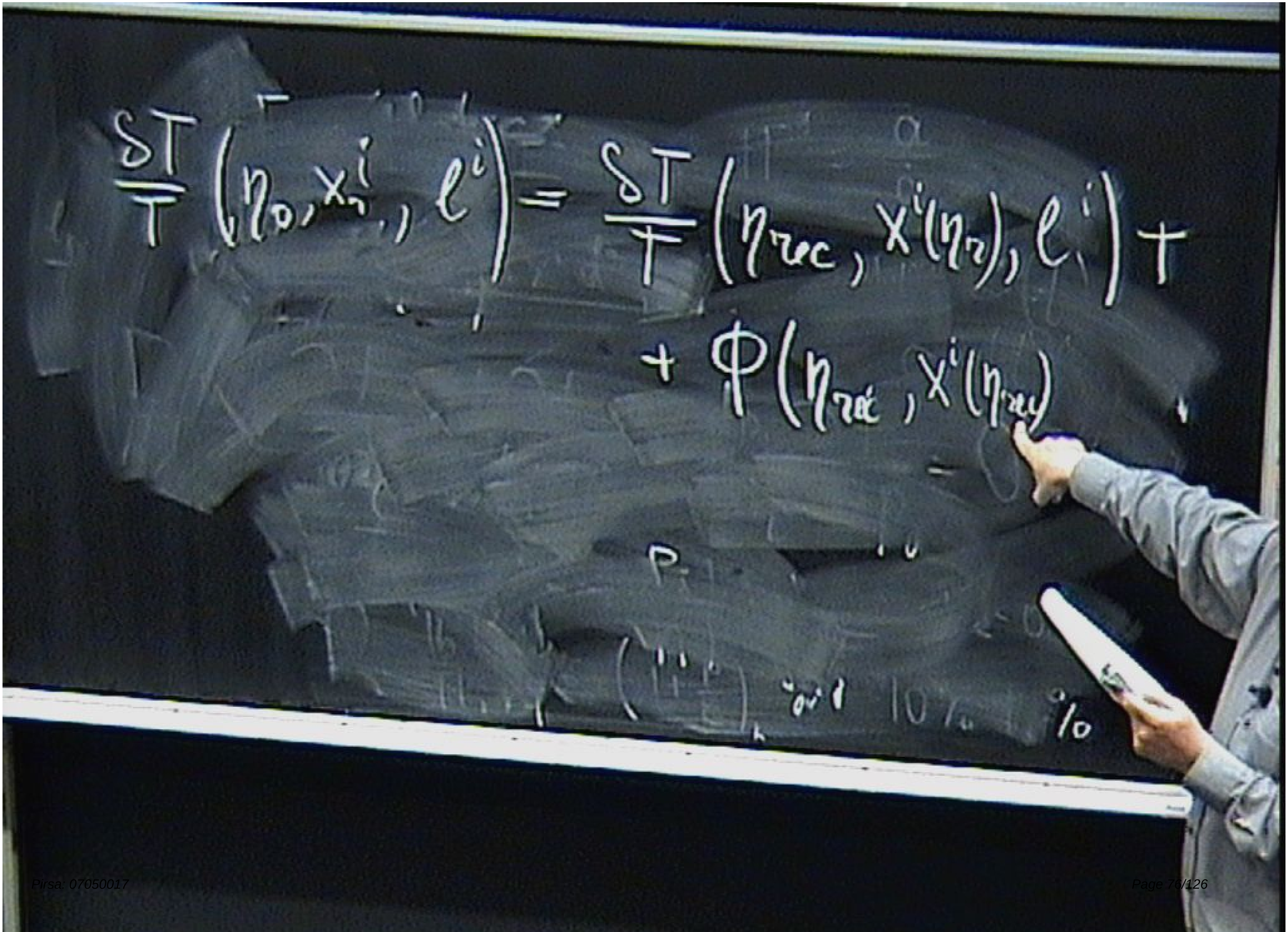


$\frac{\delta T}{T}\left(\eta_0, x^i_\eta, l^i\right.$

10 % %



$$\frac{\delta T}{T}\left(\eta_0, x_0^i, l^i\right) = \frac{\delta T}{T}\left(\eta_{rec}, x^i(\eta_r)\right)$$



$$\frac{\delta T}{T}\left(\eta_0, x^i_r, l^i\right) = \frac{\delta T}{T}\left(\eta_{rec}, x^i(\eta_r), l^i\right) +$$

$$+ \Phi\left(\eta_{rec}, x^i(\eta_{rec})\right)$$

10 % %

$$\frac{\delta T}{T}\left(\eta_0, x_0^i, l^i\right) = \frac{\delta T}{T}\left(\eta_{rec}, x^i(\eta_r), l^i\right) +$$

$$+ \Phi\left(\eta_{rec}, x^i(\eta_{rec})\right) -$$

$$= \Phi\left(\eta_0, x_0^i\right)$$

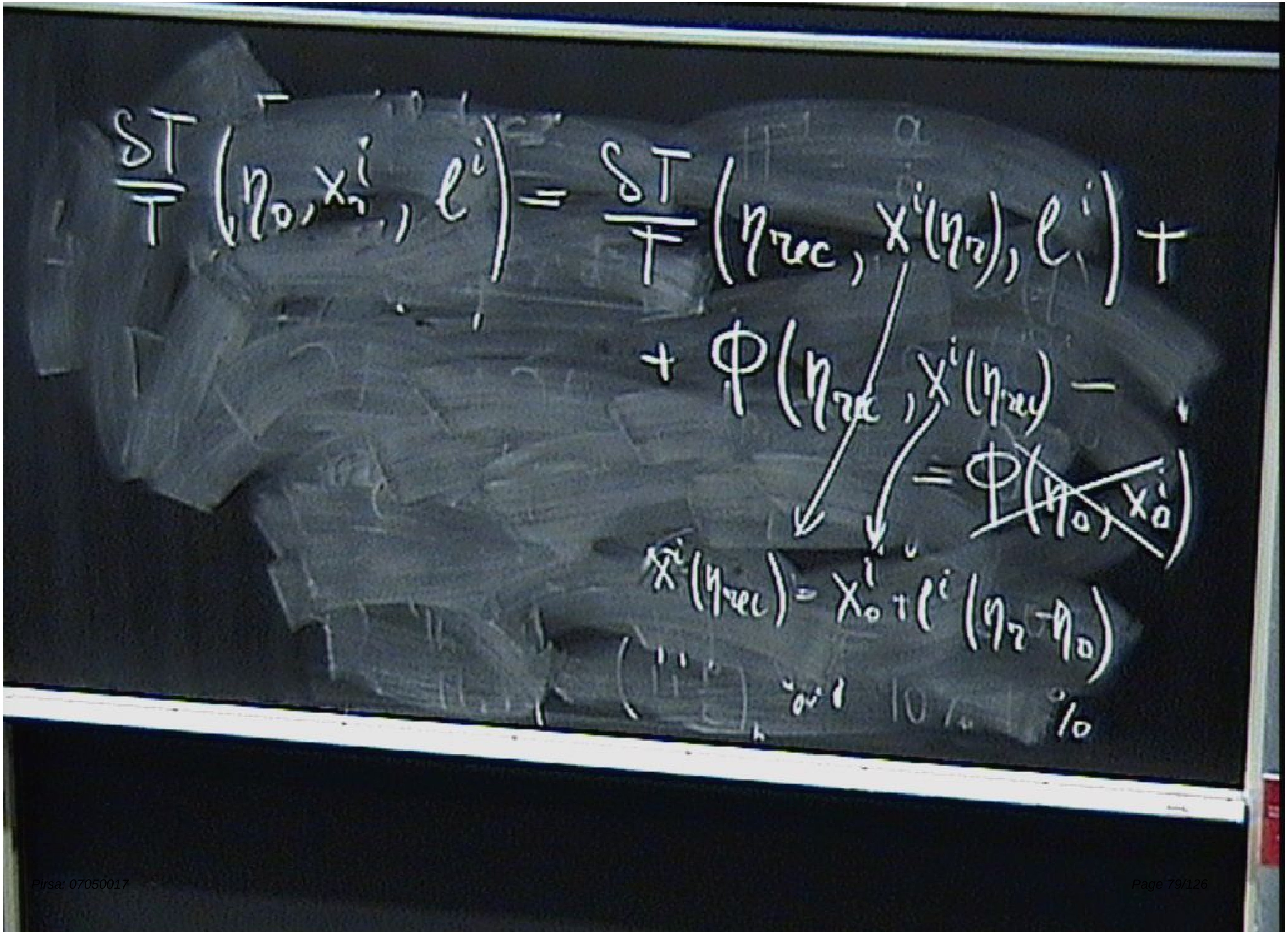
$$\frac{\delta T}{T}\left(\eta_0, x_0^i, l^i\right) = \frac{\delta T}{T}\left(\eta_{rec}, x^i(\eta_r), l^i\right) +$$

$$+ \Phi\left(\eta_{rec}, x^i(\eta_{rec})\right) -$$

$$= \Phi\left(\eta_0, x_0^i\right)$$

$$x^i(\eta_{rec}) = x_0^i + l^i\left(\eta_r - \eta_0\right)$$

%



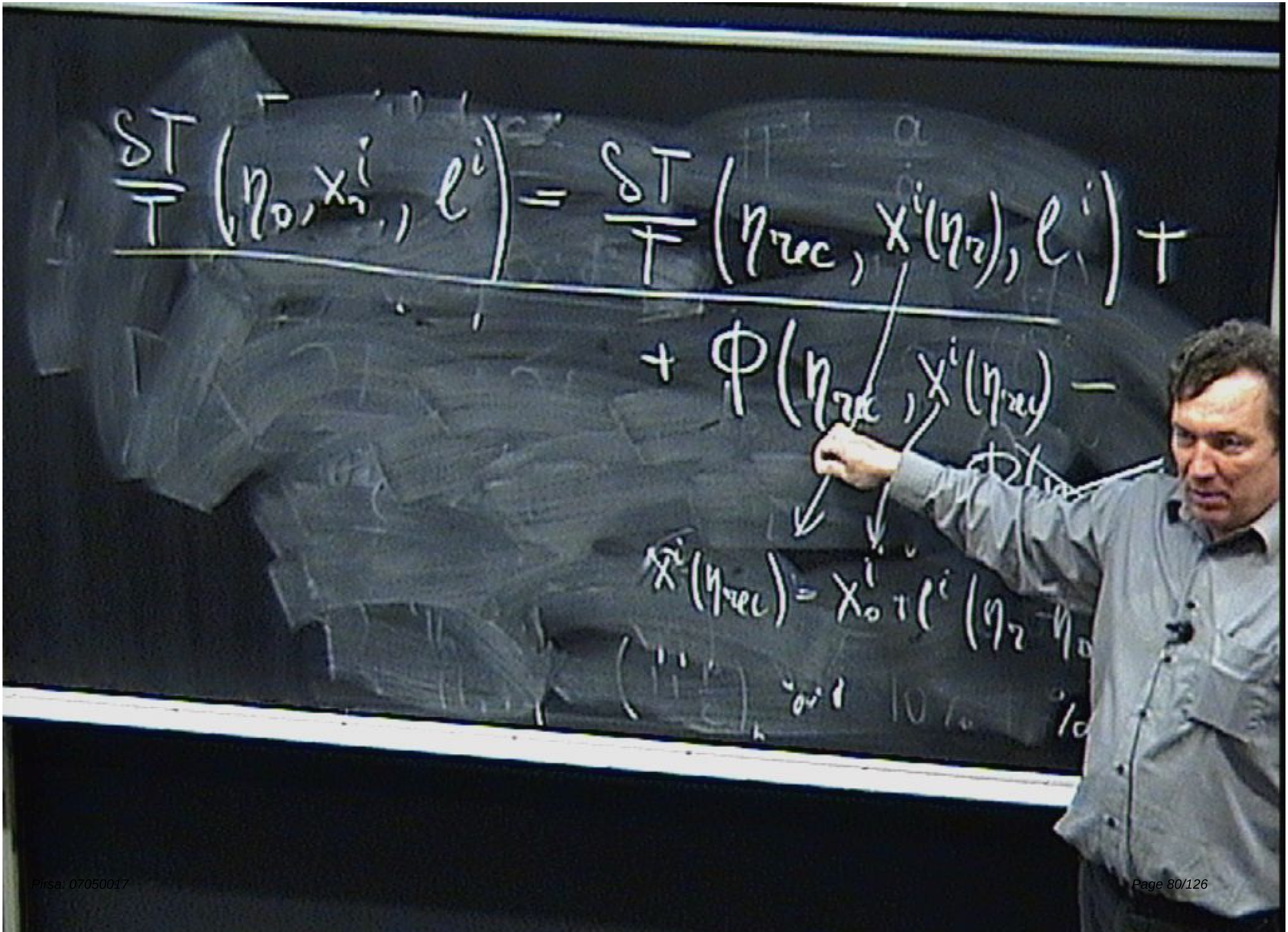
$$\frac{\delta T}{T}(\eta_0, x^i_?, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_r), l^i) +$$

$$+ \Phi(\eta_{rec}, x^i(\eta_{rec}) -$$

$$= \Phi\cancel{(\eta_0, x^i_0})$$

$$x^i(\eta_{rec}) = x^i_0 + l^i(\eta_r - \eta_0)$$

%



$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_r), l^i) +$$

$$+ \Phi(\eta_{rec}, x^i(\eta_{rec}) -$$

$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_r - \eta_0$$

$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_r), l^i) +$$

$$+\ \Phi(\eta_{rec}, x^i(\eta_{rec})) -$$

$$= \cancel{\Phi(\eta_0, x_0^i})$$

$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_r - \eta_0)$$

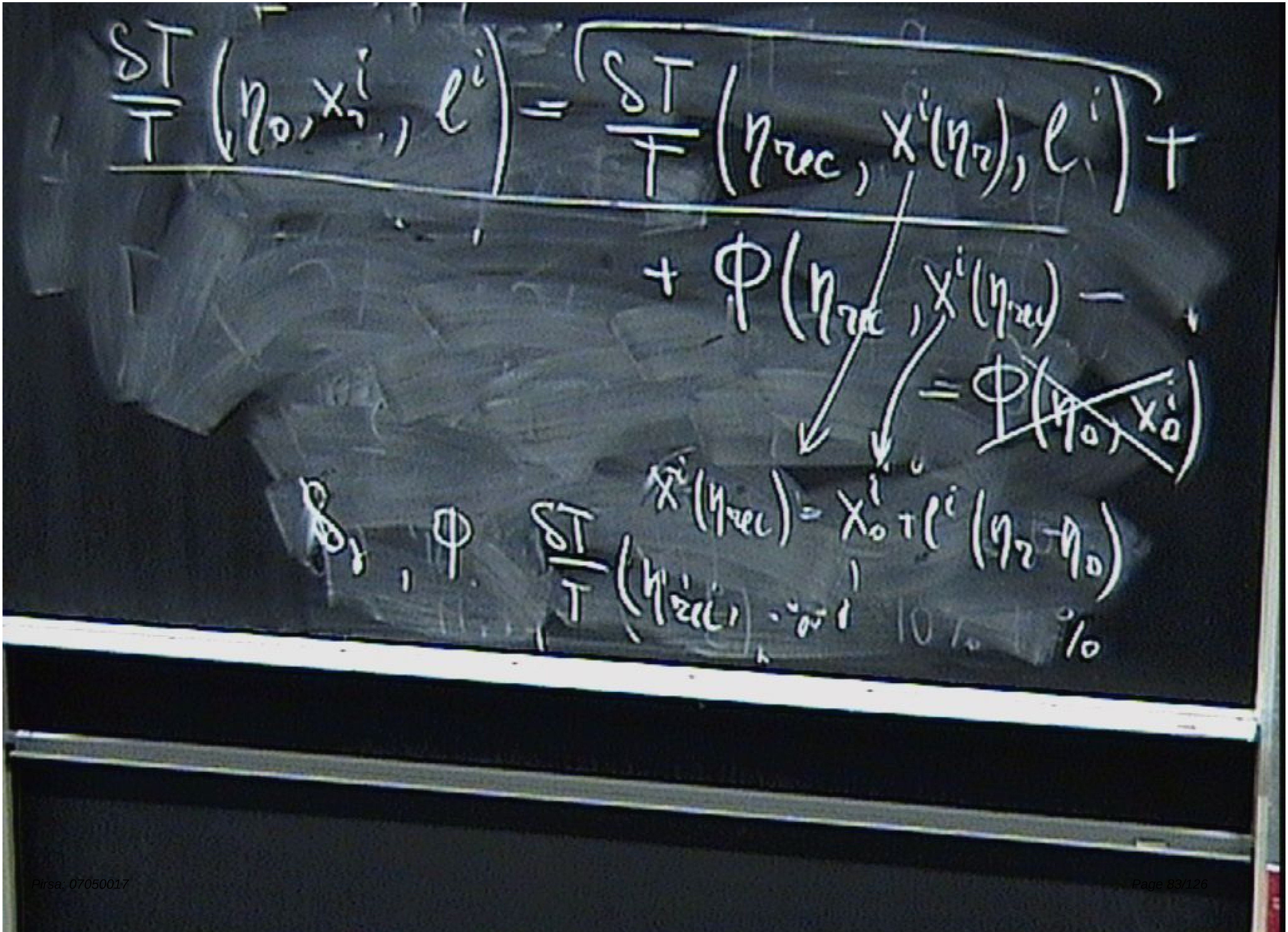
$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_r), l^i) +$$

$$+ \Phi(\eta_{rec}, x^i(\eta_{rec})) -$$

$$- \cancel{\Phi(\eta_0, x_0^i)}$$

$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_r - \eta_0)$$

δ_γ , Φ $\frac{\delta T}{T}$ ([illegible] %



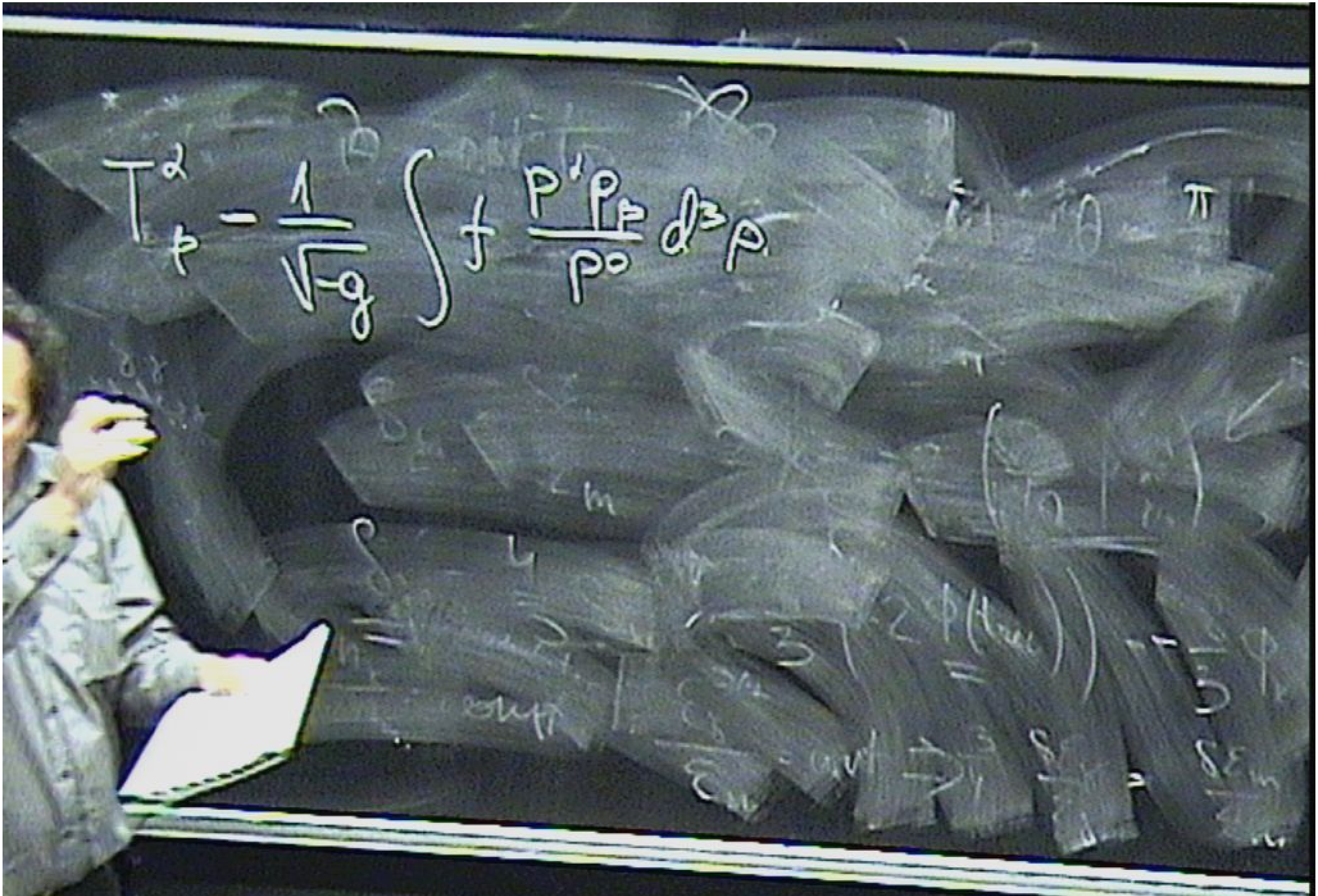
$$\frac{\delta T}{T}(\eta_0, x^i_0, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_r), l^i) +$$

$$+ \Phi(\eta_{rec}, x^i(\eta_{rec})) - $$

$$= \cancel{\Phi(\eta_0, x^i_0)}$$

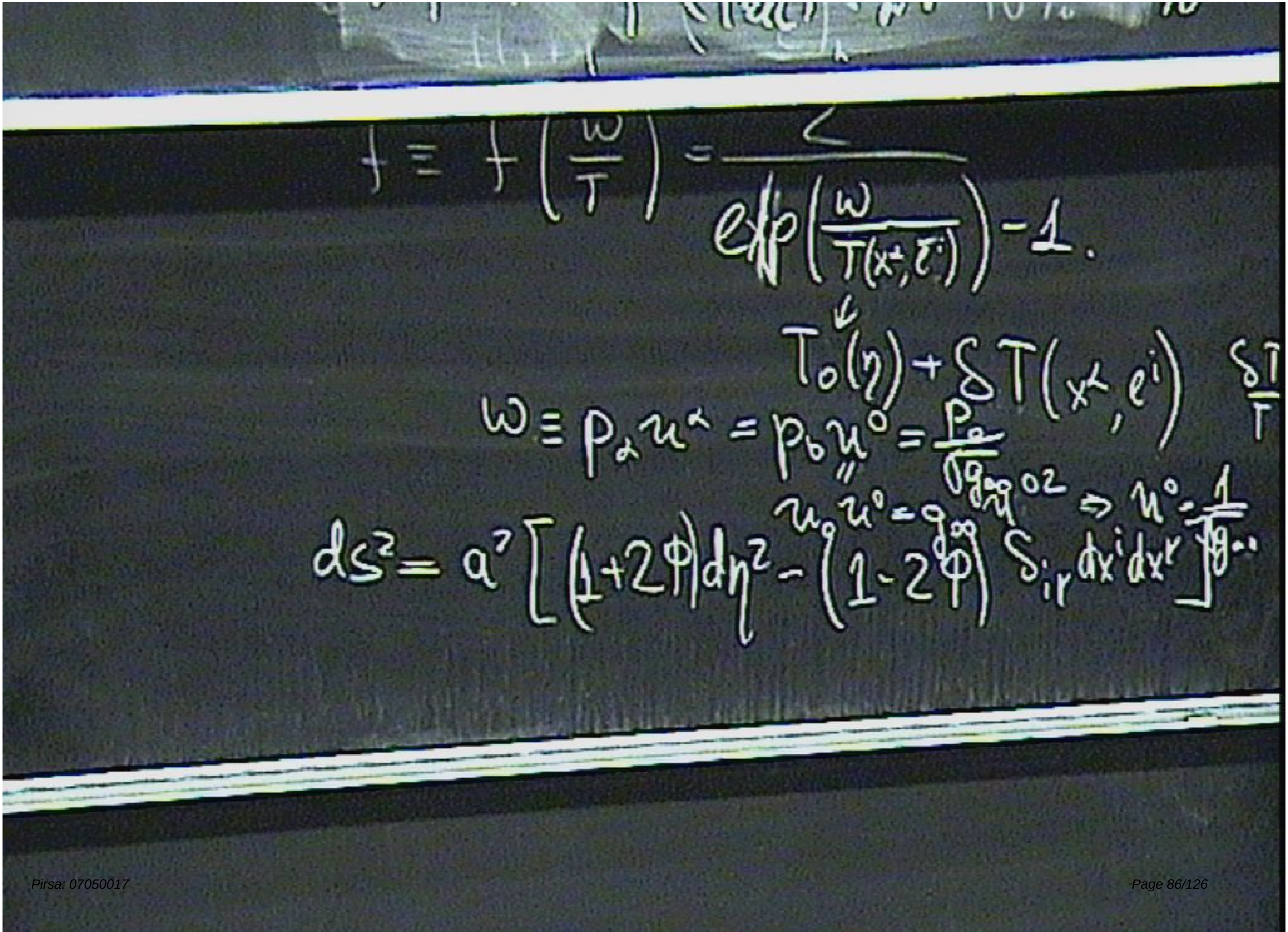
$$x^i(\eta_{rec}) = x^i_0 + l^i(\eta_r - \eta_0)$$

δ_γ, Φ, $\frac{\delta T}{T}(\eta_{rec}, \ldots)$ 10% %



$$T^{\alpha}{}_{\beta} - \frac{1}{\sqrt{-g}} \int f \, \frac{p^{\alpha} p_{\beta}}{p^{0}} \, d^{3}p$$

$$T^{\alpha}_{\;\beta} = \frac{1}{\sqrt{-g}} \int f \, \frac{p^{\alpha} p_{\beta}}{p^{0}} \, d^3 p$$



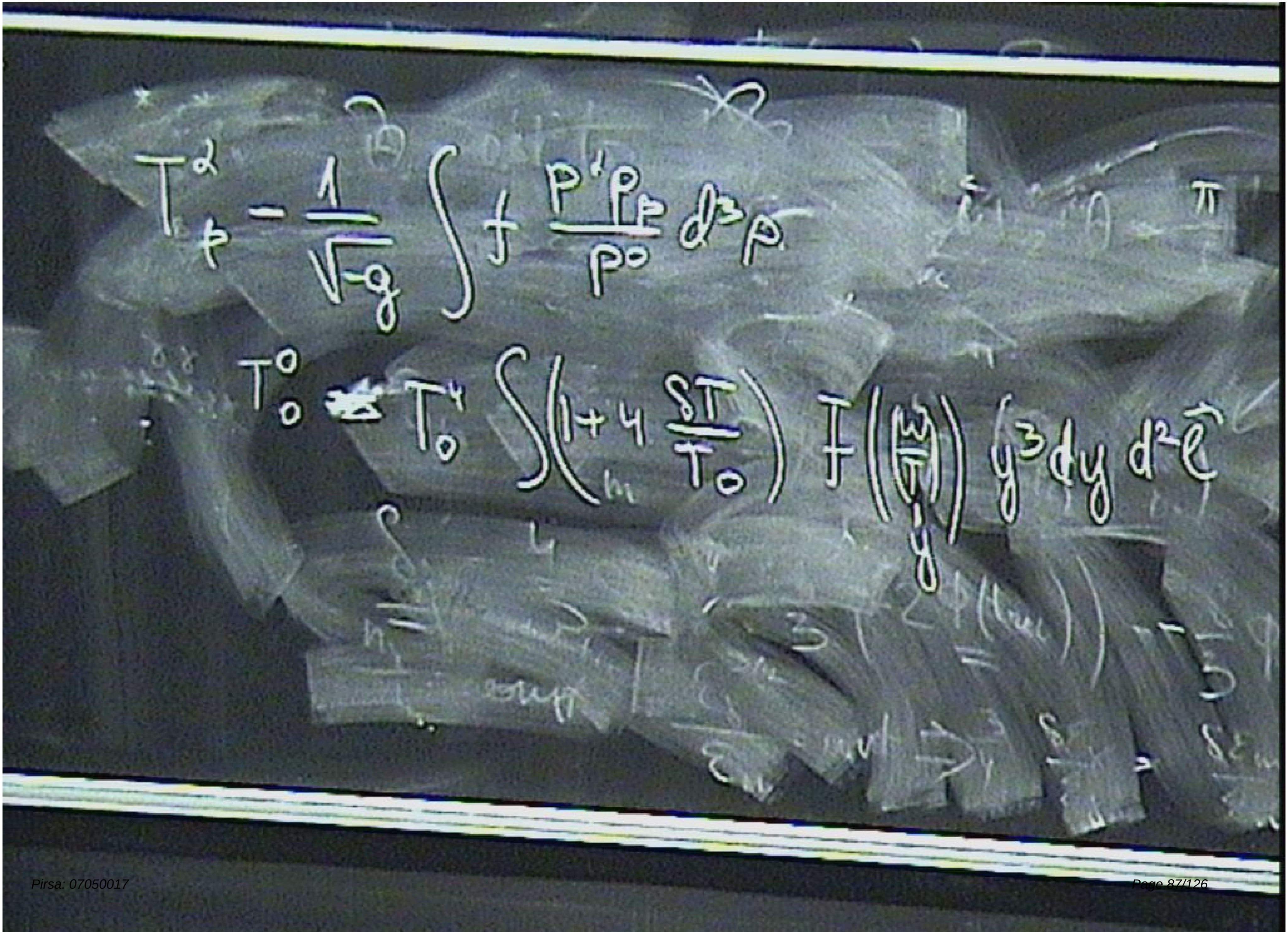
$$f \equiv f\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^\alpha, e^i)}\right) - 1}.$$

$$T_0(\eta) + \delta T(x^\alpha, e^i) \qquad \frac{\delta T}{T}$$

$$\omega \equiv p_\alpha u^\alpha = p_0 u^0 = \frac{p_0}{\sqrt{g_{00}}}$$

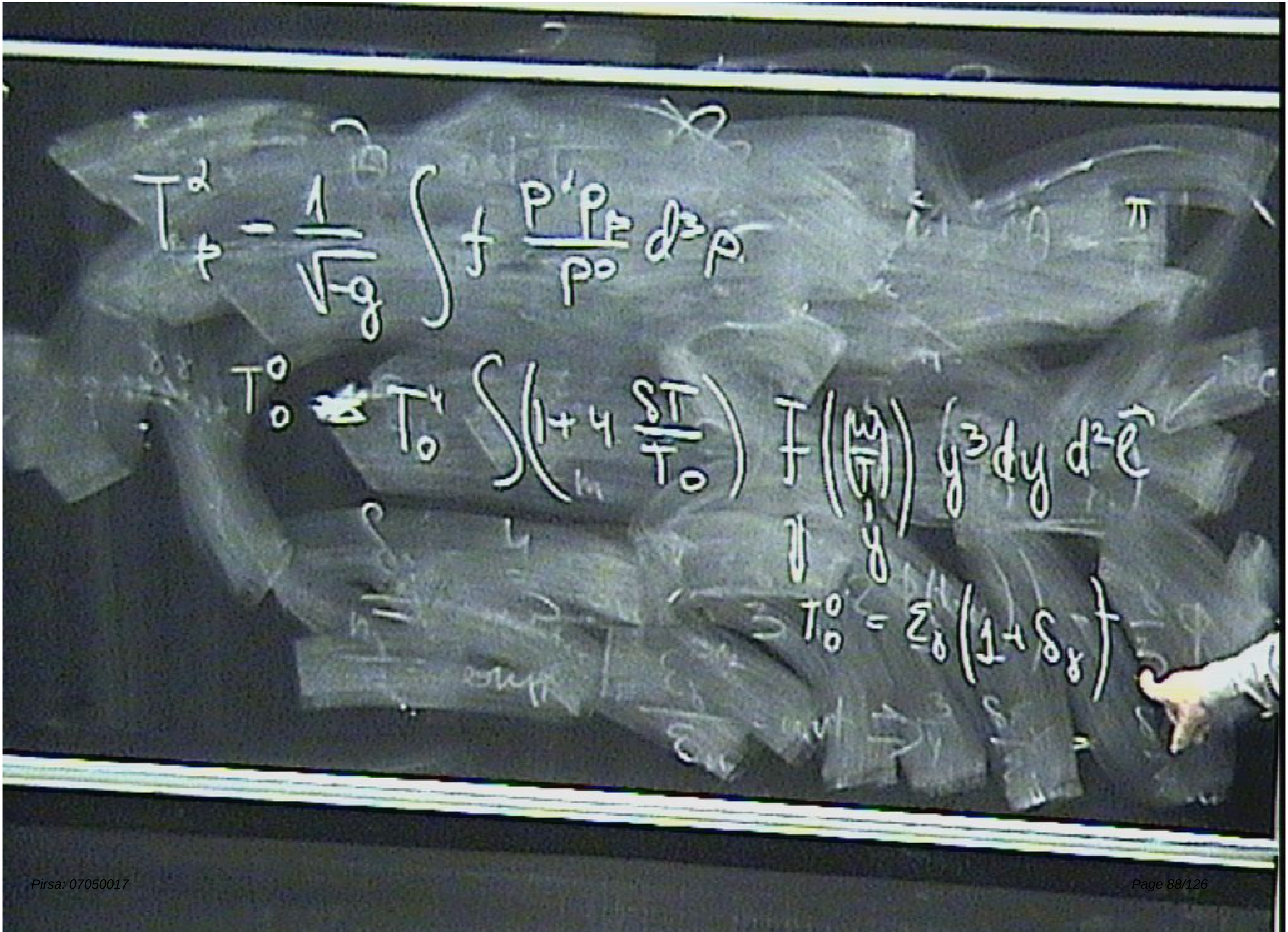
$$u_0 u^0 = g_{00} u^{0\,2} \Rightarrow u^0 = \frac{1}{\sqrt{g_{00}}}$$

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - (1-2\Phi)\delta_{ir}dx^i dx^r\right]$$



$$T^{\alpha}_{\beta} = \frac{1}{\sqrt{-g}} \int f \, \frac{p^{\alpha} p_{\beta}}{p^{0}} \, d^3p$$

$$T^{0}_{0} = T_0^4 \int \left(1 + 4\,\frac{\delta T}{T_0}\right)_{n} F\left(\underset{y}{\left(\frac{\omega}{T}\right)}\right) y^3 \, dy \, d^2\vec{e}$$



$$T^\alpha_\beta = \frac{1}{\sqrt{-g}} \int f \, \frac{p^\alpha p_\beta}{p^0} \, d^3p$$

$$T^0_0 = T_0^4 \int \left(1 + 4\,\frac{\delta T}{T_0}\right)_{m} \underset{\Downarrow}{F}\left(\underset{\substack{\Downarrow \\ y}}{\frac{\omega}{T}}\right) y^3 \, dy \, d^2\vec{e}$$

$$T^0_0 = \varepsilon_0 \left(1 + \delta_\gamma\right)$$

$$\rho_q \qquad \frac{1}{\sqrt{g_{44}}} \qquad \delta \quad \varepsilon$$

$$\rho = \frac{1}{\sqrt{-g}} \int f \frac{p}{p^0} d^3 p$$

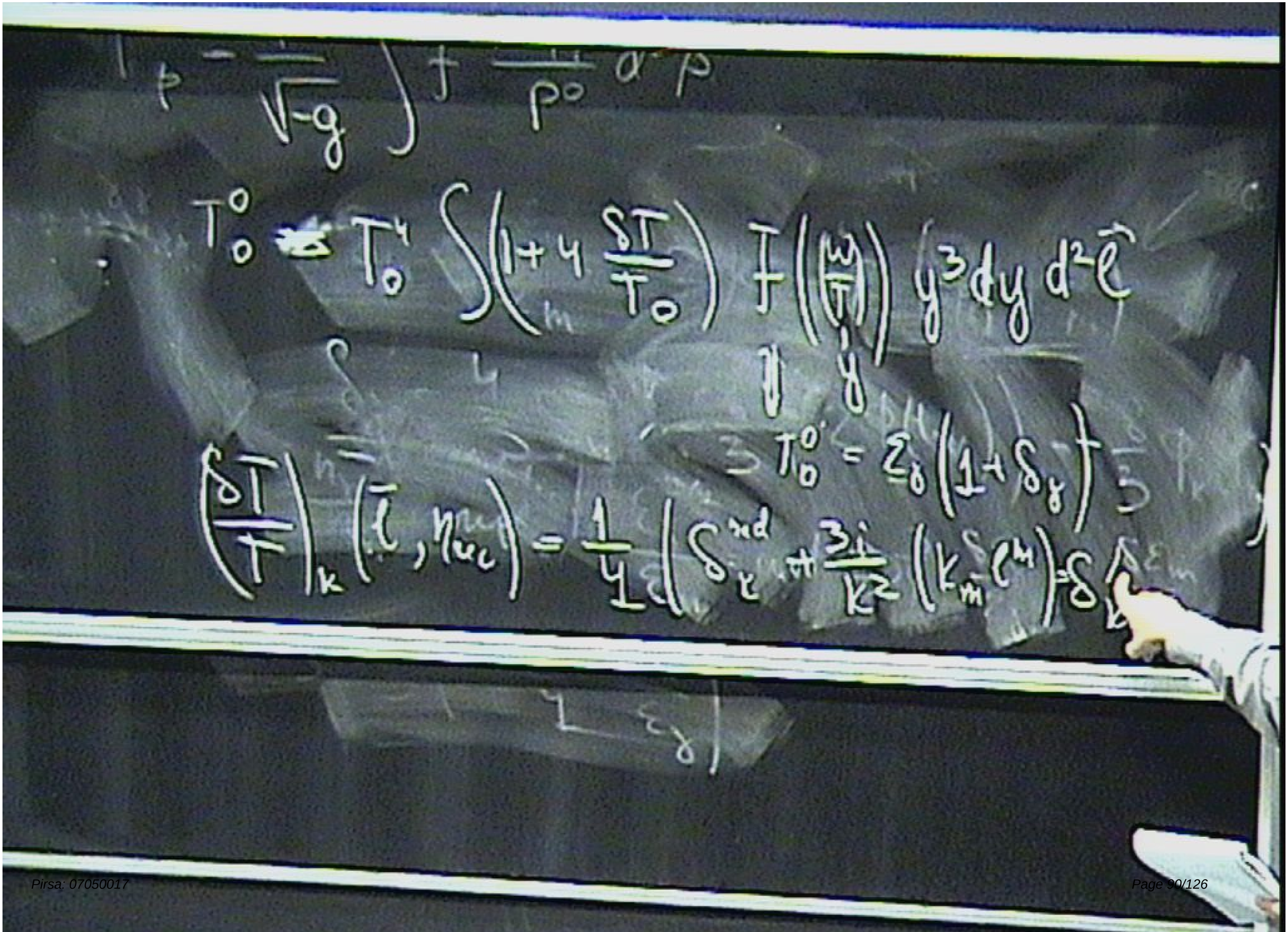
$$T^0_0 = T_0^4 \int \left(1 + 4\frac{\delta T}{T_0}\right)_m F\left(\frac{\omega}{T}\right) y^3 dy\, d^2\vec{e}$$

$$\Downarrow \qquad y$$

$$T^0_0 = \varepsilon_\gamma \left(1 + \delta_\gamma\right)$$

$$\left(\frac{\delta T}{T}\right)_k$$

$$\varepsilon_\gamma$$



$$T_p = \frac{1}{\sqrt{-g}} \int f \frac{\ }{p^0} d^3 p$$

$$T^0_0 = T_0^4 \int \left(1 + 4\frac{\delta T}{T_0}\right) f\left(\frac{\omega}{T}\right) y^3 dy\, d^2\vec{e}$$

$$T^0_0 = \varepsilon_\gamma \left(1 + \delta_\gamma\right)$$

$$\left(\frac{\delta T}{T}\right)_k \left(\vec{e}, \eta_{rec}\right) = \frac{1}{4}\left(\delta_\gamma^{rad} + \frac{3i}{k^2}\left(k_m e^m\right) \delta\right.$$

$$\;_p - \frac{1}{\sqrt{-g}} \int f \frac{}{p^0} d^3 p$$

$$T^0_0 = T_0^4 \int \left(1 + 4\frac{\delta T}{T_0}\right)_m F\left(\underset{\underset{y}{\Downarrow}}{\frac{\omega}{T}}\right) y^3 dy\, d^2\hat{e}$$

$$T^0_0 = \varepsilon_\gamma \left(1 + \delta_\gamma\right)$$

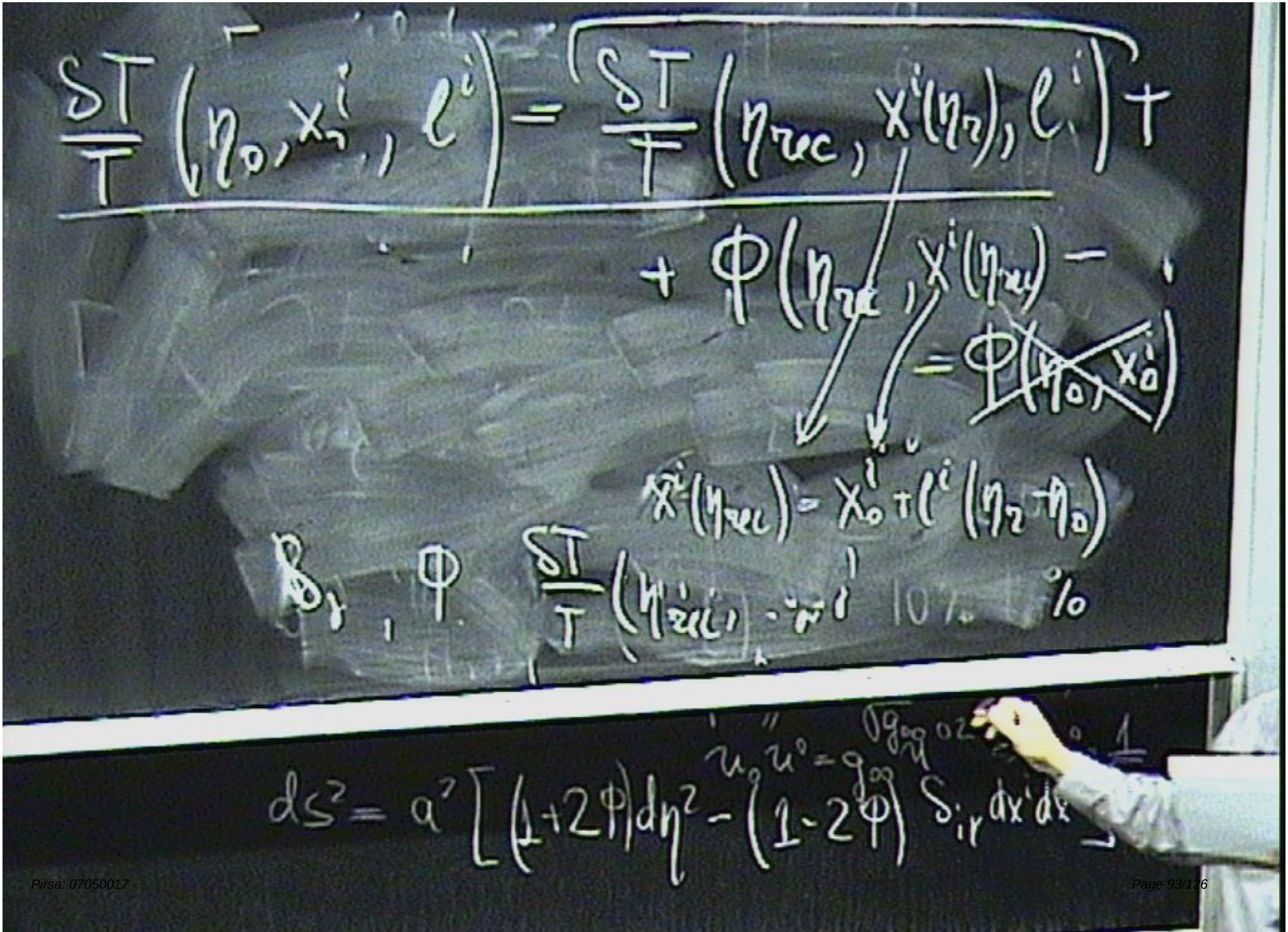
$$\left(\frac{\delta T}{T}\right)_k \left(\hat{e}, \eta_{rec}\right) = \frac{1}{4}\left(\delta_k^{rad} + \frac{3i}{k^2}\left(k_m e^m\right)\delta_k\right)_{rec}$$

$$T_{\beta} = \frac{}{\sqrt{-g}} \int f \frac{}{p^0} d^3 p$$

$$T^0_0 \approx T_0^4 \int_m \left(1 + 4\,\frac{\delta T}{T_0}\right) F\Big(\underset{y}{\left(\frac{\omega}{T}\right)}\Big)\, y^3\, dy\, d^2\vec{e}$$

$$T^0_0 = \varepsilon_0 \left(1 + \delta_\gamma\right)$$

$$\left(\frac{\delta T}{T}\right)_k \left(\vec{e}, \eta_{rec}\right) = \frac{1}{4}\left(\delta_k^{rad} + \frac{3i}{k^2}\left(k_m e^m\right)\delta_k \ldots\right)$$



$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_r), l^i) +$$

$$+ \Phi(\eta_{rec}, x^i(\eta_{rec})) -$$

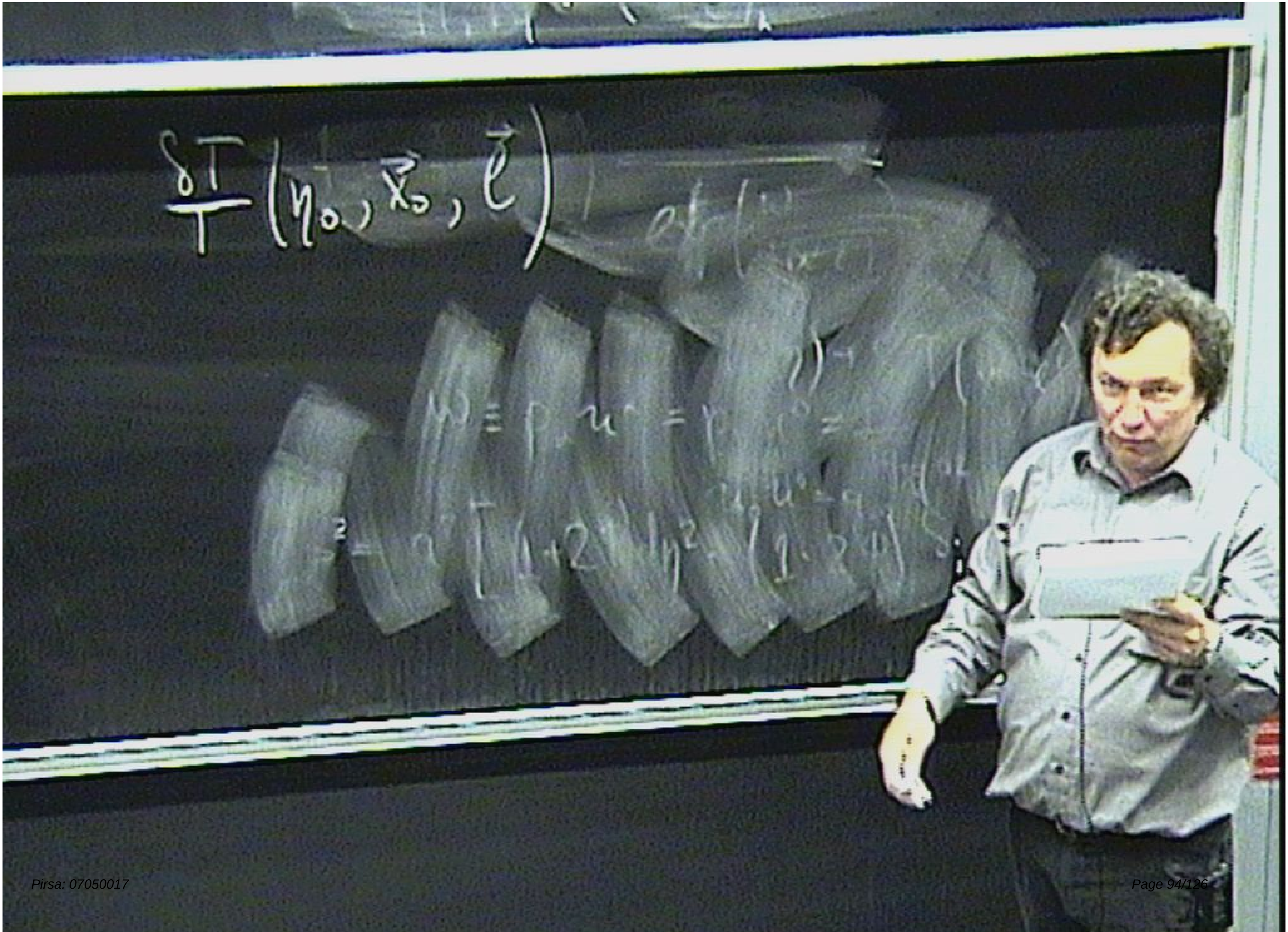
$$= \cancel{\Phi(\eta_0, x_0^i)}$$

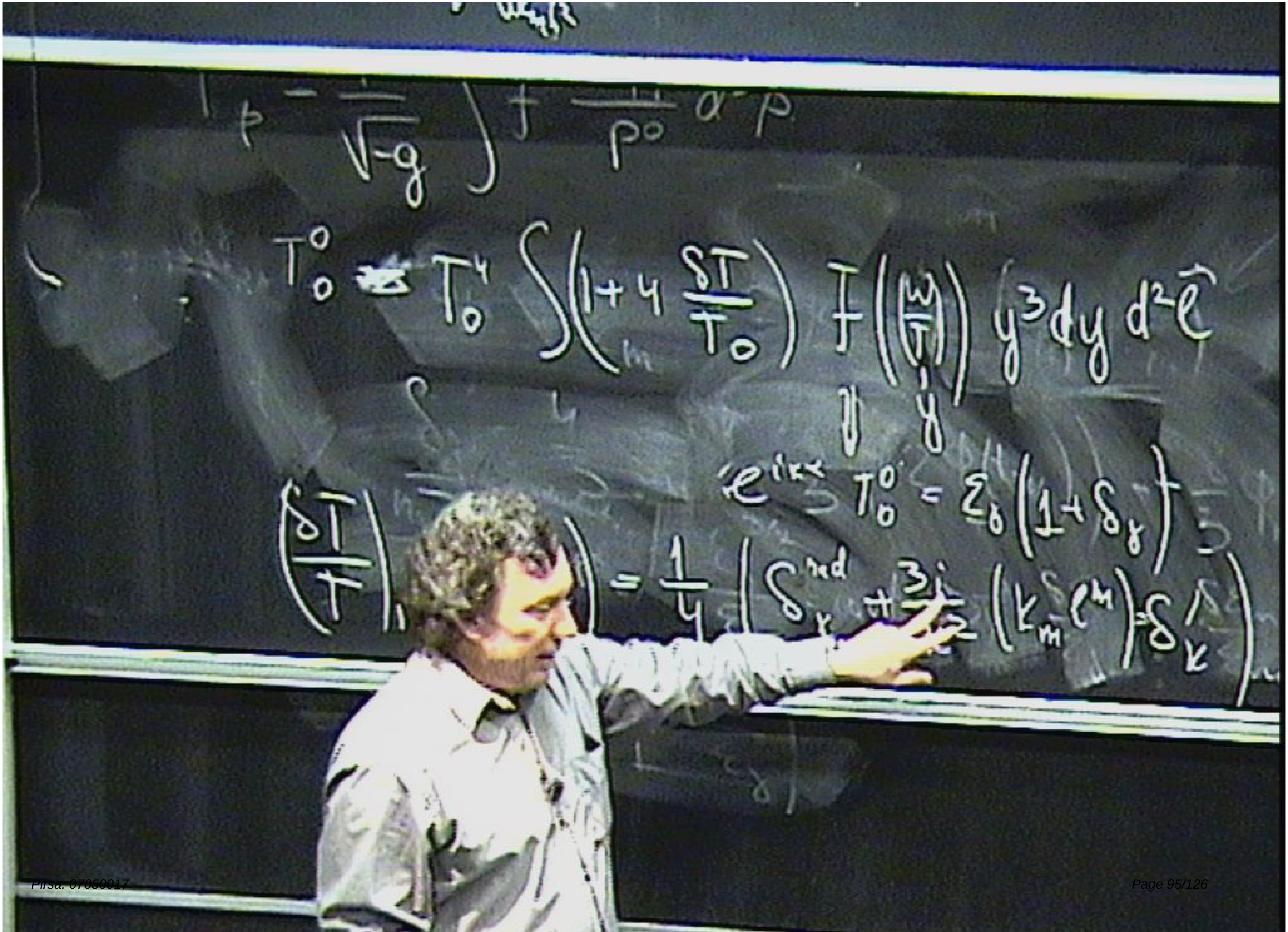
$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_r - \eta_0)$$

δ_γ , Φ $\frac{\delta T}{T}(\eta_{rec}, \ldots$ 10% %

$u_0 u^0 = g_{00}$ $\sqrt{g_{00}}$ 1

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$


$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{l})$

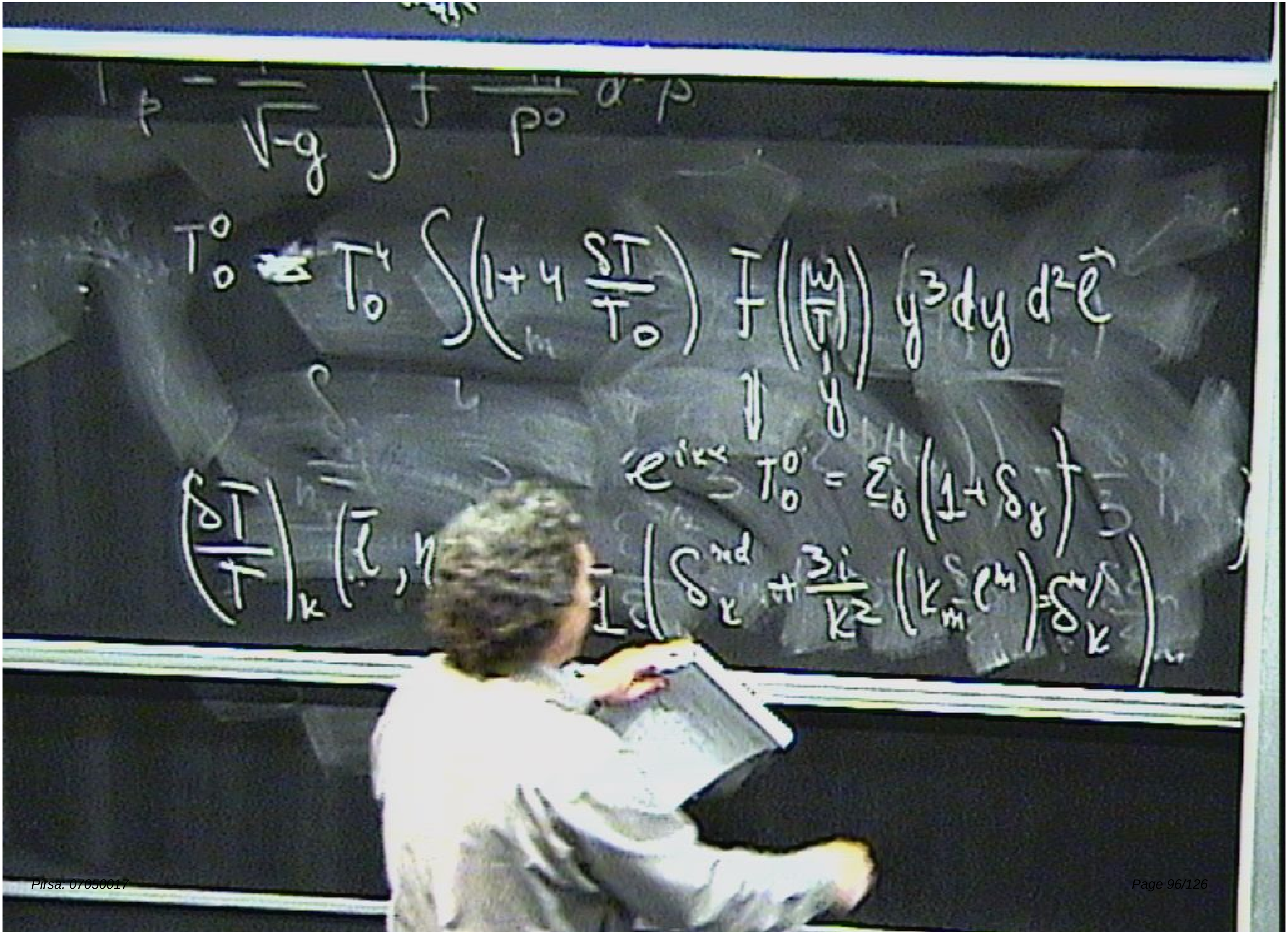


$$T_{\ p} = \frac{1}{\sqrt{-g}} \int f \frac{\quad}{p^0} d^3p$$

$$T^0_0 = T_0^4 \int \left(1 + 4\frac{\delta T}{T_0}\right) \mathcal{F}\left(\frac{\omega}{T}\right) y^3 dy\, d^2\vec{e}$$

$$e^{ikx}\ T^0_0 = \varepsilon_0 \left(1 + \delta_\gamma\right)$$

$$\left(\frac{\delta T}{T}\right)_{,} = \frac{1}{4}\left(\delta_k^{red} + \frac{3i}{2}\left(k_m e^m\right)\delta_k^{\wedge}\right)$$



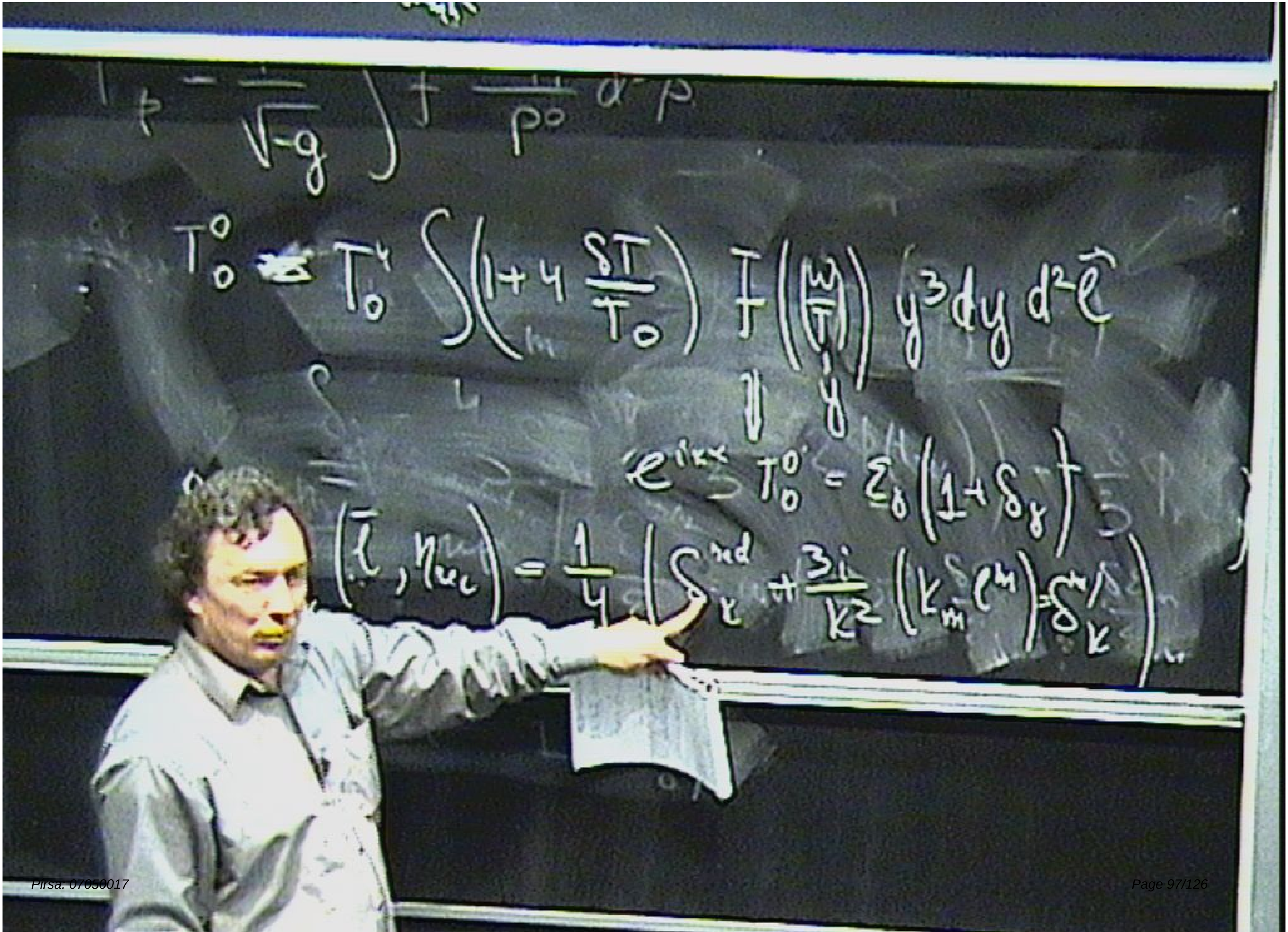
$$T_{\mu} = \frac{1}{\sqrt{-g}} \int f \frac{}{p^0} d^3 p$$

$$T^0_0 = T_0^4 \int \left(1 + 4\frac{\delta T}{T_0}\right)_m F\left(\frac{w}{T}\right) y^3 dy\, d^2\vec{e}$$

$$\Downarrow \quad y$$

$$e^{ikx} T^0_0 = \varepsilon_\gamma \left(1 + \delta_\gamma\right)$$

$$\left(\frac{\delta T}{T}\right)_k \left(\vec{e}, \eta \right. \quad \frac{1}{4}\left(\delta_k^{nd} + \frac{3i}{k^2}\left(k_m e^m\right)\delta_k^{m}\right)$$



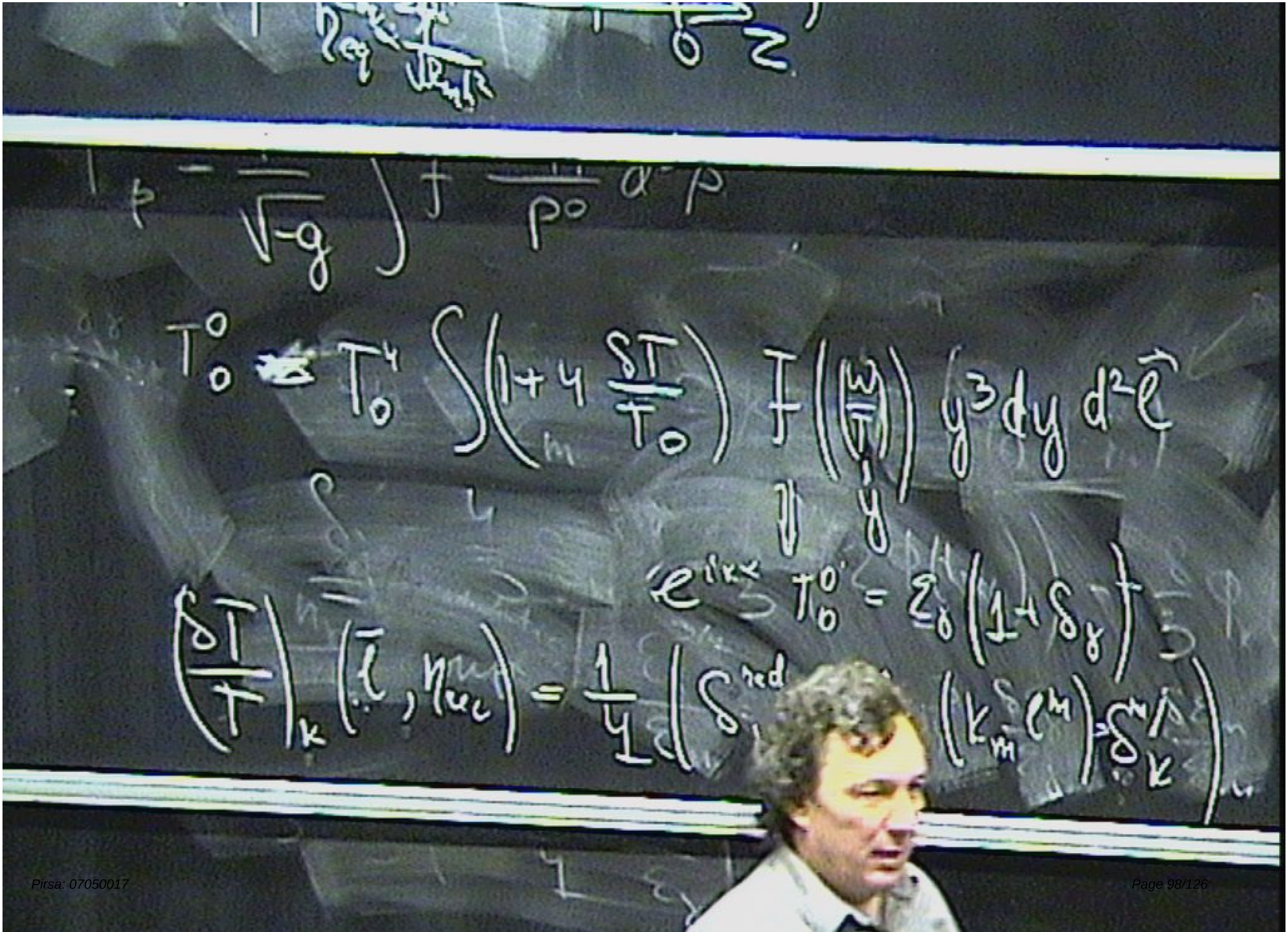
$$T_\mu = \frac{1}{\sqrt{-g}} \int f \frac{\quad}{p^0} d^3 p$$

$$T^0_0 = T_0^4 \int \left(1 + 4\frac{\delta T}{T_0}\right) f\left(\frac{w}{T}\right) y^3 dy\, d^2\vec{e}$$

$$\Downarrow \quad y$$

$$e^{ikx}\, T^{0\prime}_0 = \varepsilon_\gamma (1+\delta_\gamma)$$

$$(\vec{l}, \eta_{rec}) = \frac{1}{4}\left(\delta_k^{ind} + \frac{3i}{k^2}(k_m e^m)\delta_k^m \right)$$



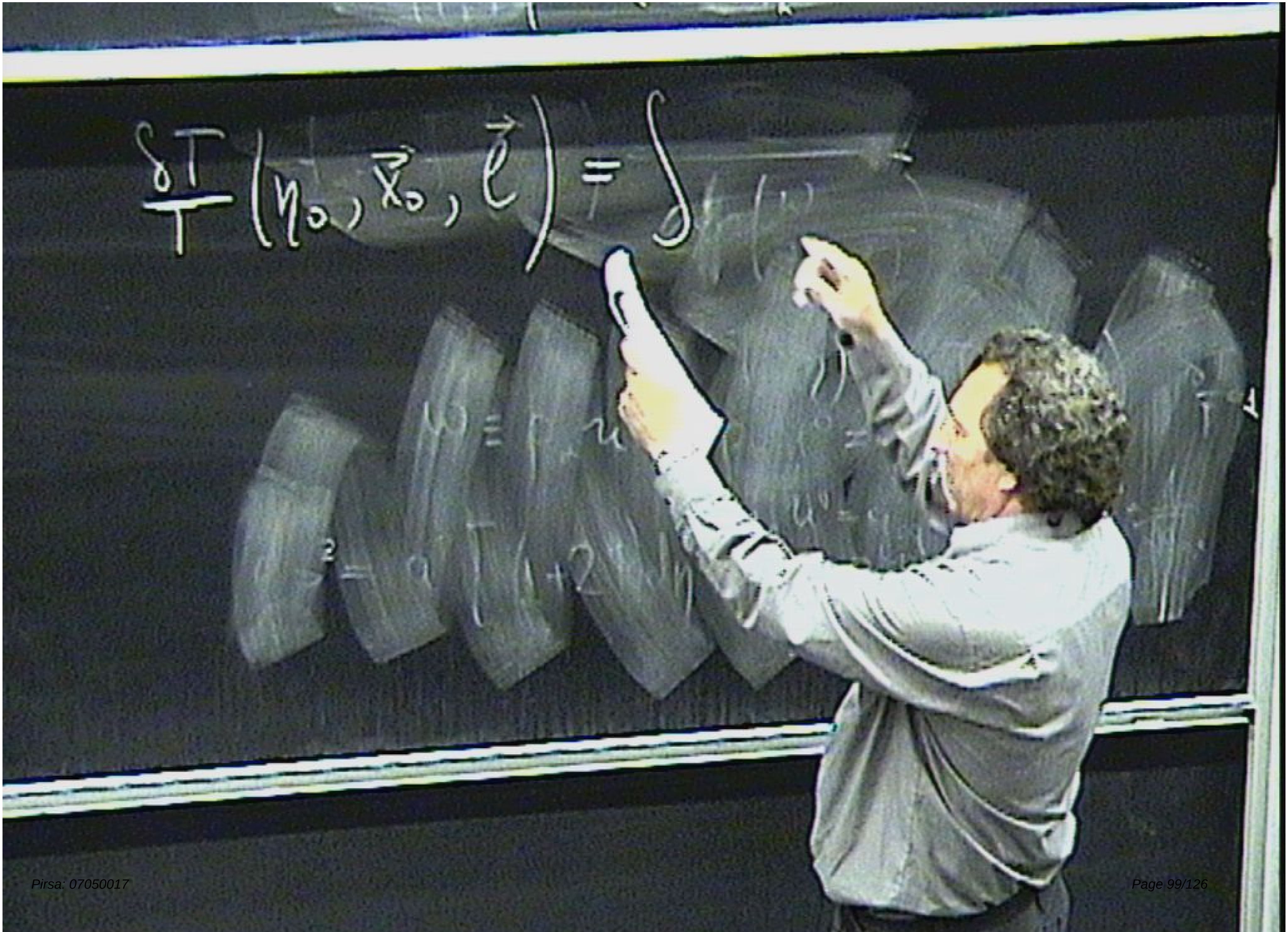
$$\frac{1}{\sqrt{-g}} \Big) + \frac{1}{p^0} a^\mu p$$

$$T^0_0 = T_0^4 \int \Big(1 + 4\frac{\delta T}{T_0}\Big) f\Big(\frac{\omega}{T}\Big) y^3 dy\, d^2 \vec{e}$$

$$\frac{\omega}{T} = y$$

$$e^{ikx}\; T^{0\,\prime}_0 = \varepsilon_0 (1 + \delta_\gamma)$$

$$\Big(\frac{\delta T}{T}\Big)_k (\vec{e}, \eta_{rec}) = \frac{1}{4}\Big(\delta^{rad} \ldots (k_m e^m)\, \delta^m_k \Big)$$

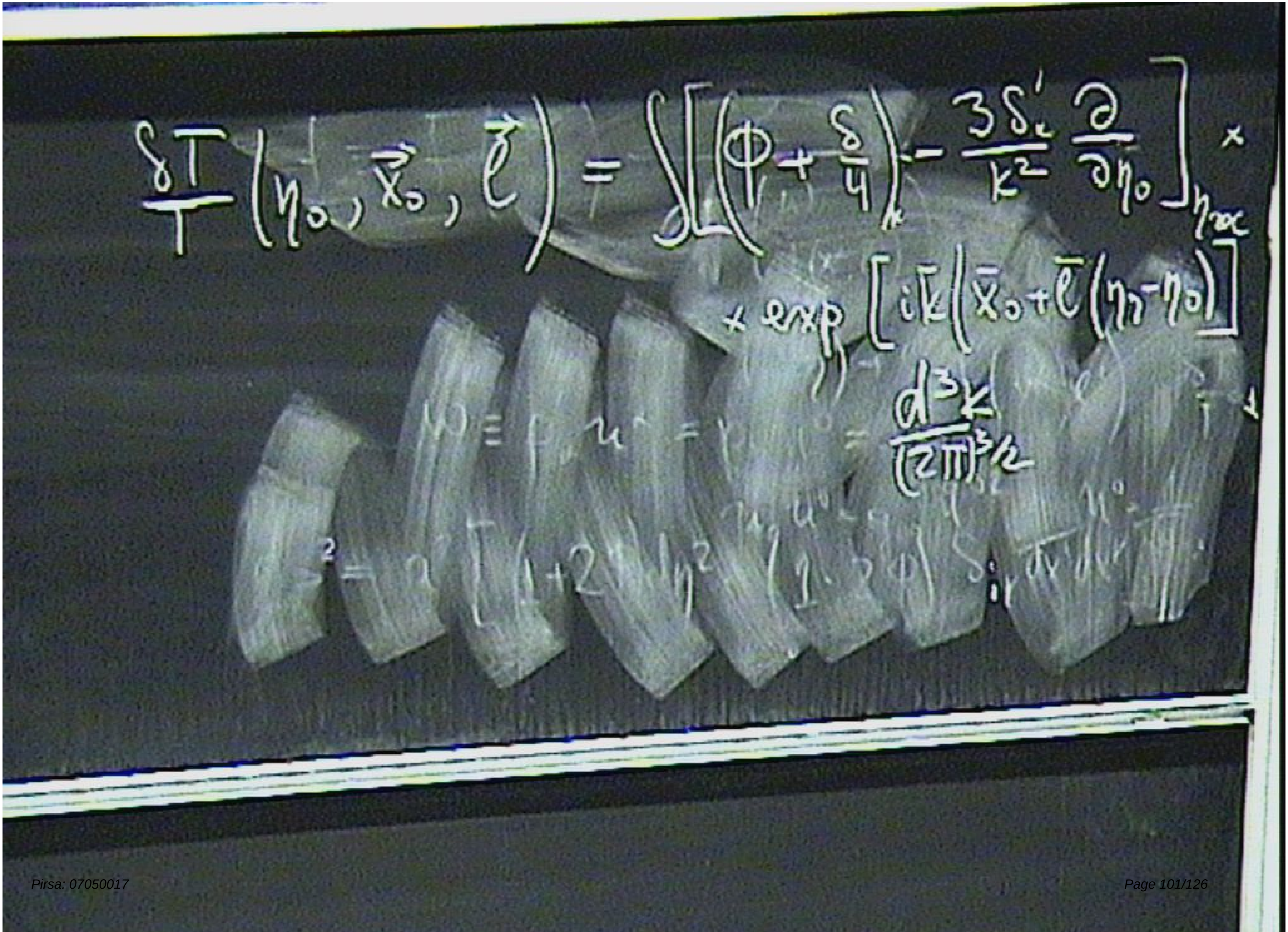


$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{e}) = \int$$

$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{\ell}) = \int \left(\Phi + \frac{\delta}{4}\right)_*$$

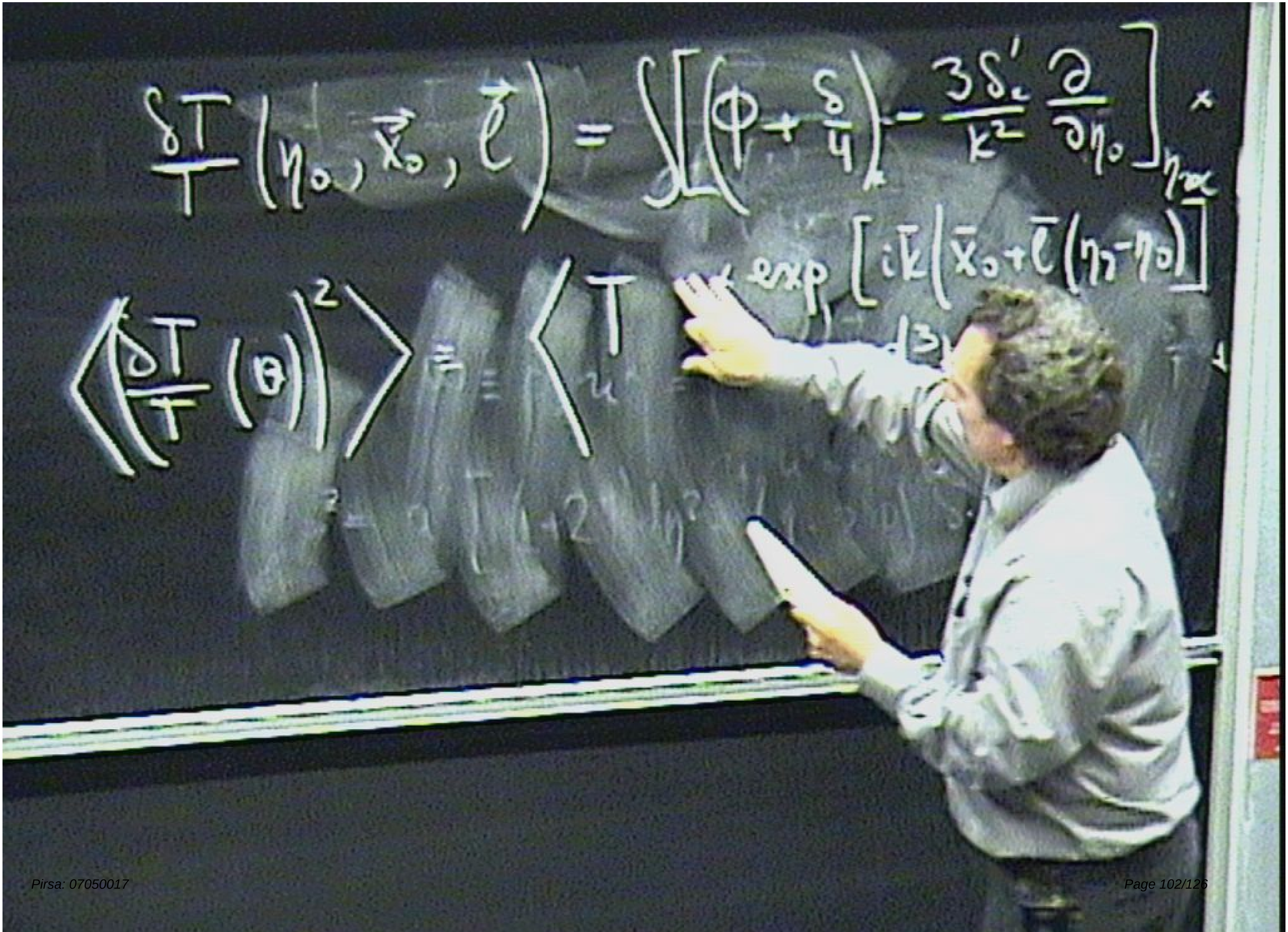
[illegible]

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$



$$\frac{\delta T}{T}\left(\eta_0, \vec{x}_0, \vec{\ell}\right) = \int \left[\left(\Phi + \frac{\delta}{4}\right)_k - \frac{3\delta_c'}{k^2}\frac{\partial}{\partial \eta_0}\right]_{\eta_{rec}} \times$$

$$\times \exp\left[i\bar{k}\left(\bar{x}_0 + \bar{\ell}\left(\eta_r - \eta_0\right)\right)\right] \frac{d^3k}{(2\pi)^{3/2}}$$

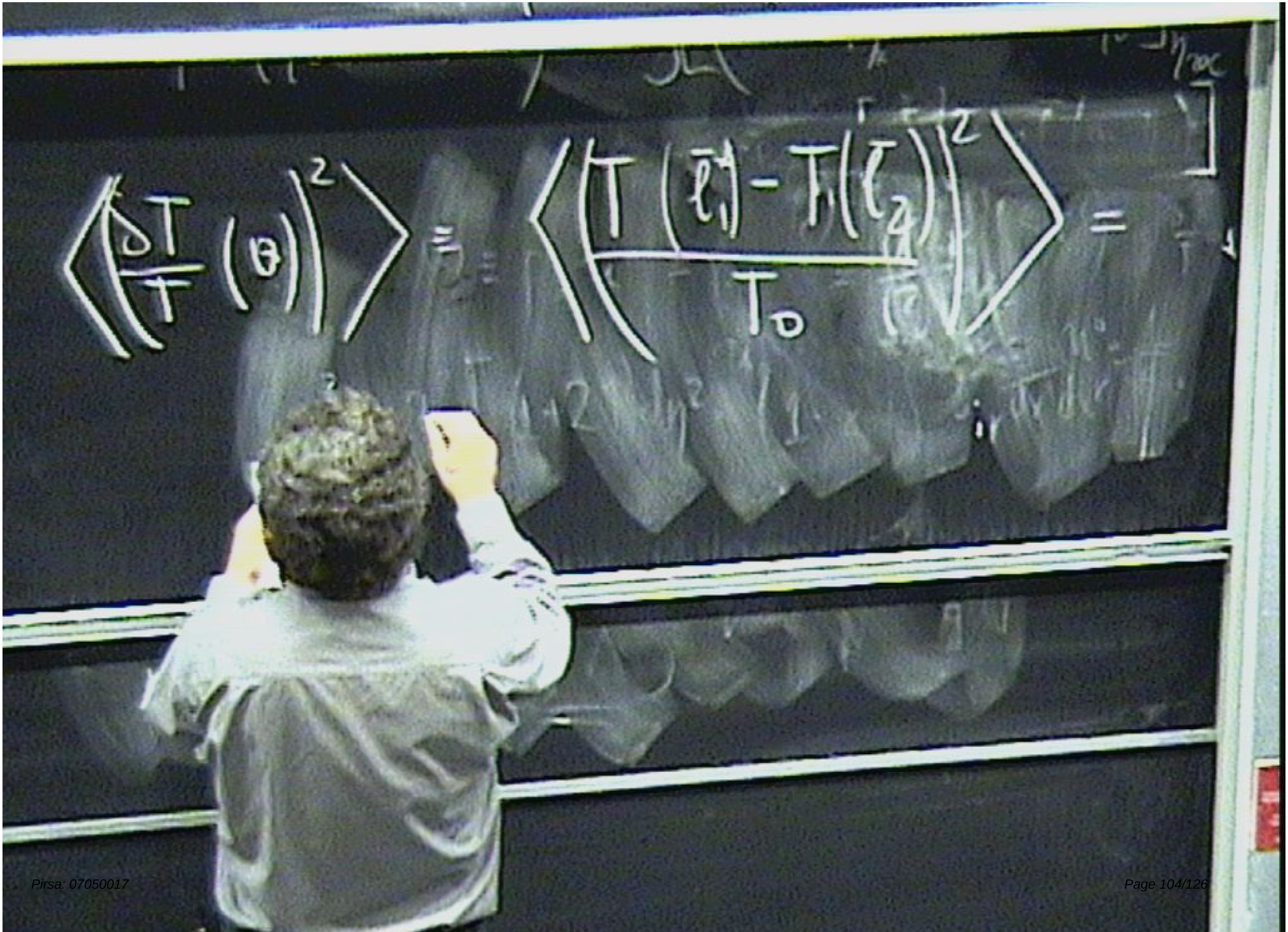


$$\frac{\delta T}{T}\left(\eta_0, \vec{x}_0, \vec{\ell}\right) = \int \left[\left(\Phi + \frac{\delta}{4}\right)_k - \frac{3\delta_c'}{k^2}\frac{\partial}{\partial \eta_0}\right]_{\eta_{rec}} \times \exp\left[i\vec{k}\left(\vec{x}_0 + \vec{\ell}\left(\eta_r - \eta_0\right)\right]\right.$$

$$\left\langle \left(\frac{\delta T}{T}(\theta)\right)^2 \right\rangle = \left\langle T_u \right.$$

$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{\ell}) = \int \left[\left(\Phi + \frac{\delta}{4}\right) - \frac{3\delta_c'}{k^2}\frac{\partial}{\partial\eta_0}\right]_{\eta_*} \times$$

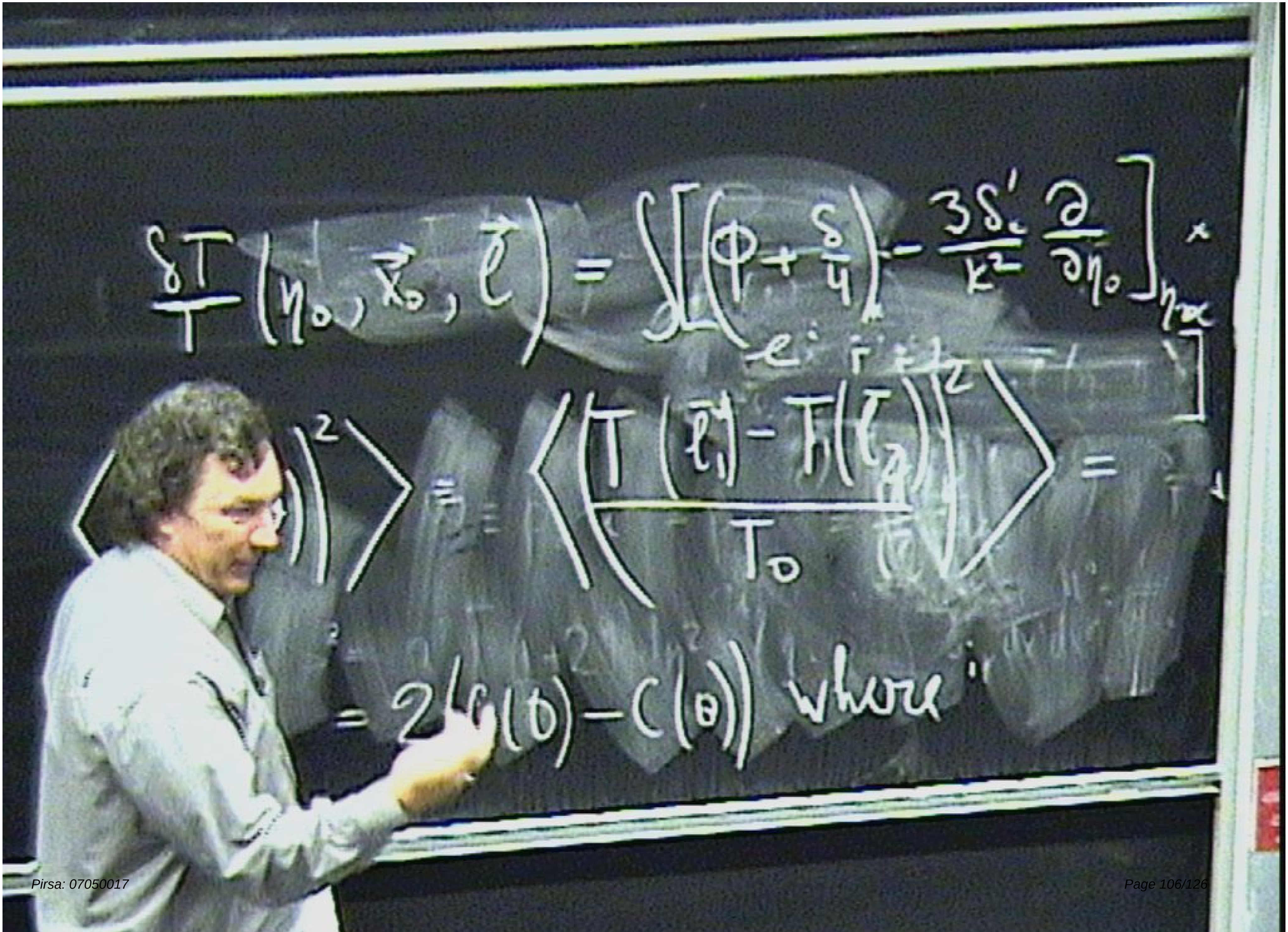
$$\left\langle \left(\frac{\delta T}{T}(\theta)\right)^2 \right\rangle = \left\langle \left(\frac{T(\vec{\ell}_1) - T(\vec{\ell}_2)}{T_0}\right)^2 \right\rangle$$



$$\left\langle \left(\frac{\Delta T}{T}(\theta)\right)^2 \right\rangle = \left\langle \left(\frac{T(\bar{e}_1) - T(\bar{e}_2)}{T_0}\right)^2 \right\rangle =$$

$$= 2\left(C(0) - C(\theta)\right) \text{ where:}$$

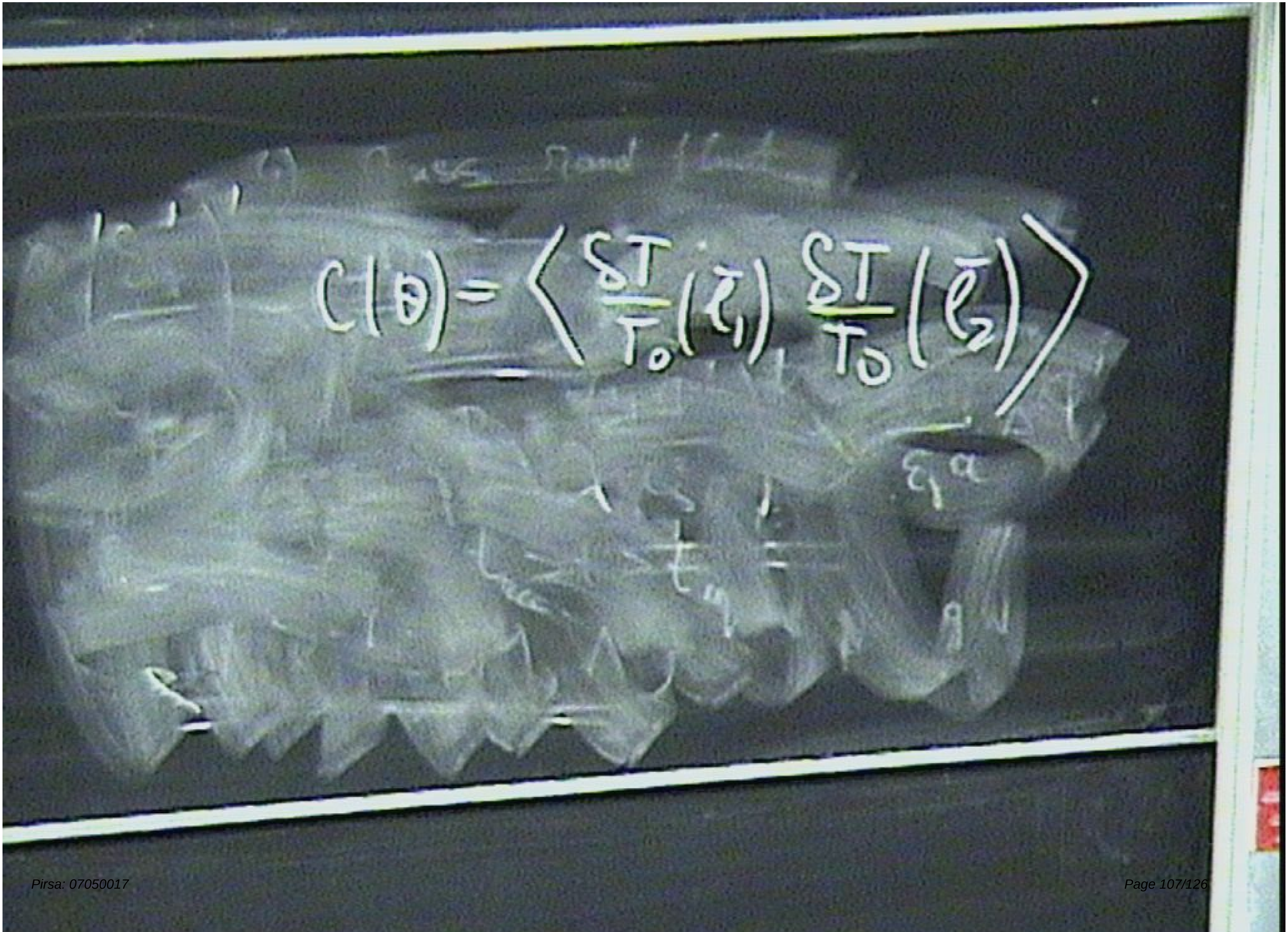
$$C(\theta) = \left\langle \frac{\delta T}{T_0}(\bar{e}_1)\, \frac{\delta T}{T_0}(\bar{e}_2) \right\rangle$$



$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{\ell}) = \int \left[\left(\Phi + \frac{\delta}{4} \right) - \frac{3\delta_c'}{k^2} \frac{\partial}{\partial \eta_0} \right]_{\eta_{rec}}$$

$$\left\langle \left(\quad \right)^2 \right\rangle = \left\langle \left(\frac{T(\vec{\ell}_1) - T(\vec{\ell}_2)}{T_0} \right)^2 \right\rangle =$$

$$= 2(C(0) - C(\theta)) \text{ where :}$$



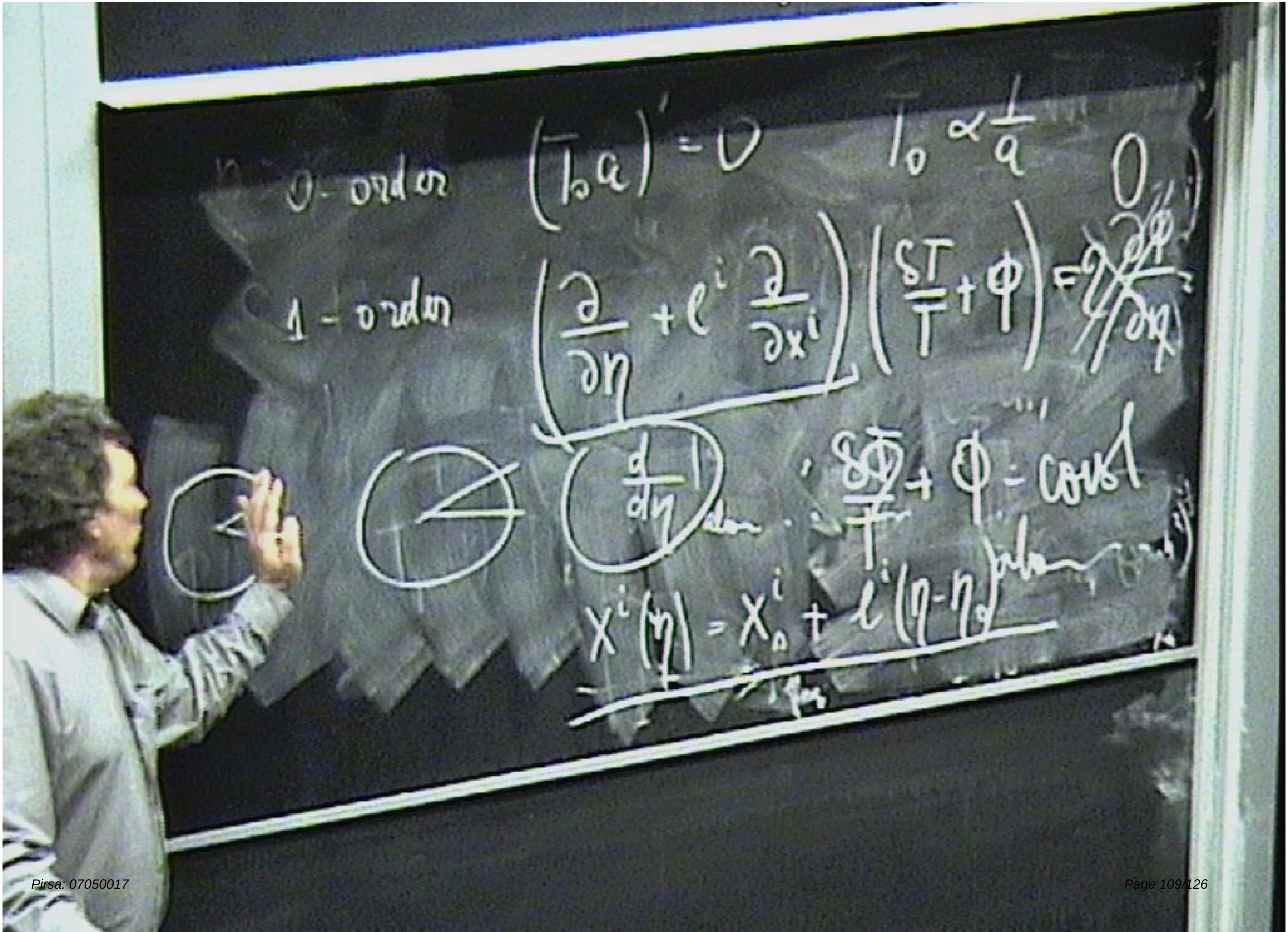
$$C(\theta) = \left\langle \frac{\delta T}{T_0}(\bar{e}_1)\, \frac{\delta T}{T_0}(\bar{e}_2) \right\rangle$$

0 - order $(T_0 a)' = 0$ $\quad T_0 \propto \frac{1}{a}$

1 - order $$\underline{\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)}\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{\cancel{2\frac{\partial\Phi}{\partial\eta}}}$$

$$\frac{d}{d\eta}\Big|_{\text{along}} \, ; \quad \frac{\delta\Phi}{T} + \Phi = \text{const} \Big)_{\text{along}}$$

$$\underline{x^i(\eta) = x_0^i + l^i(\eta - \eta_0)}$$



0- order $(T_0 a)' = 0$ $T_0 \propto \frac{1}{a}$

1 - order $\left(\frac{\partial}{\partial\eta} + l^i \frac{\partial}{\partial x^i}\right)\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{\cancel{2\frac{\partial\Phi}{\partial\eta}}}$

$\frac{d}{d\eta}\Big|_{along}$; $\frac{\delta\Phi}{T} + \Phi = const$

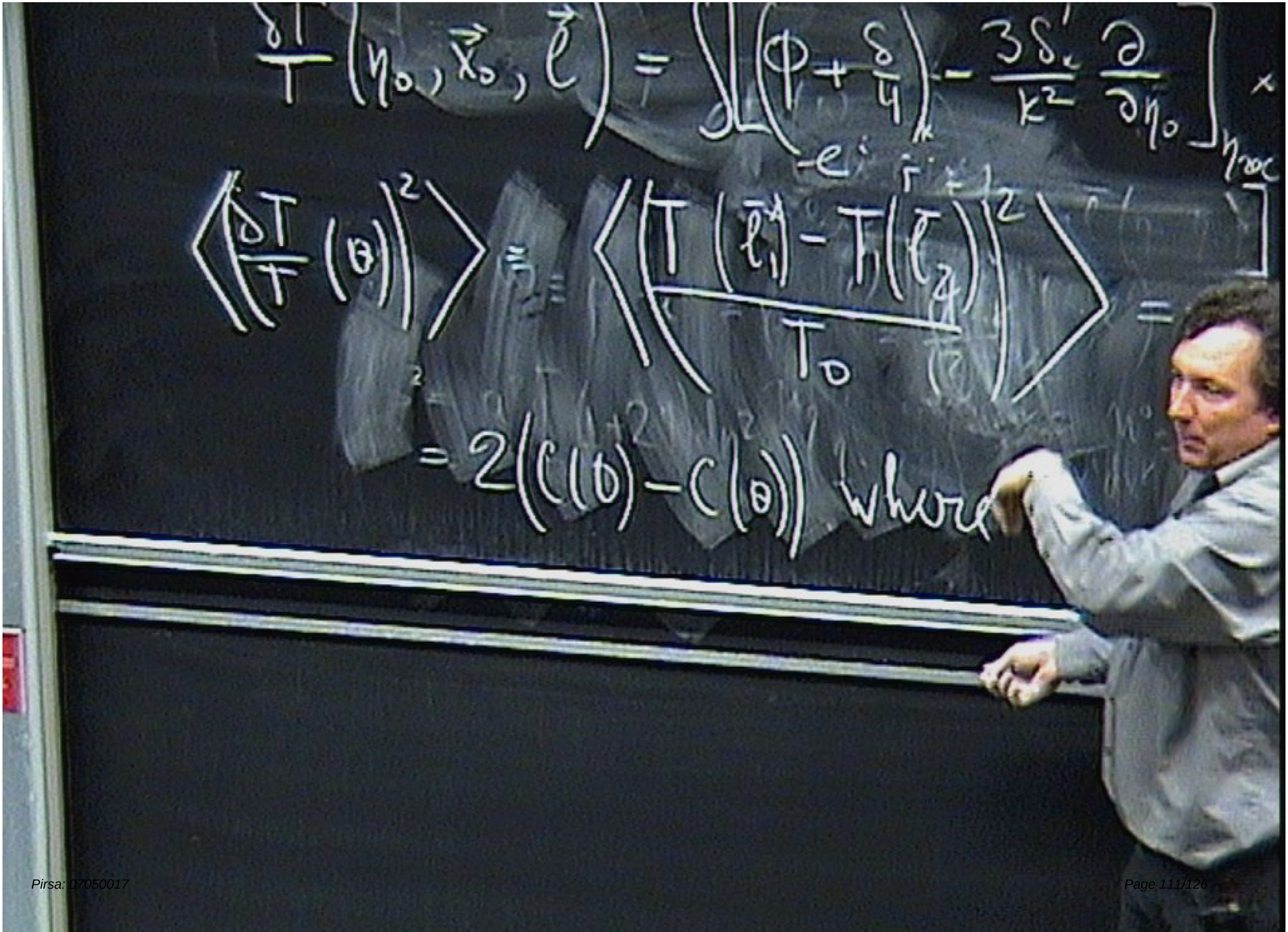
$x^i(\eta) = x_0^i + l^i(\eta - \eta_0)$ along

0 - order $\quad (T_0 a)' = 0 \qquad T_0 \propto \frac{1}{a}$

1 - order $\quad \left(\frac{\partial}{\partial \eta} + l^i \frac{\partial}{\partial x^i}\right)\left(\frac{\delta T}{T} + \Phi\right) = \overset{0}{\cancel{2\frac{\partial \Phi}{\partial \eta}}}$

$\left.\frac{d}{d\eta}\right|_{\text{along}}$, $\frac{\delta T}{T} + \Phi = \text{const}$

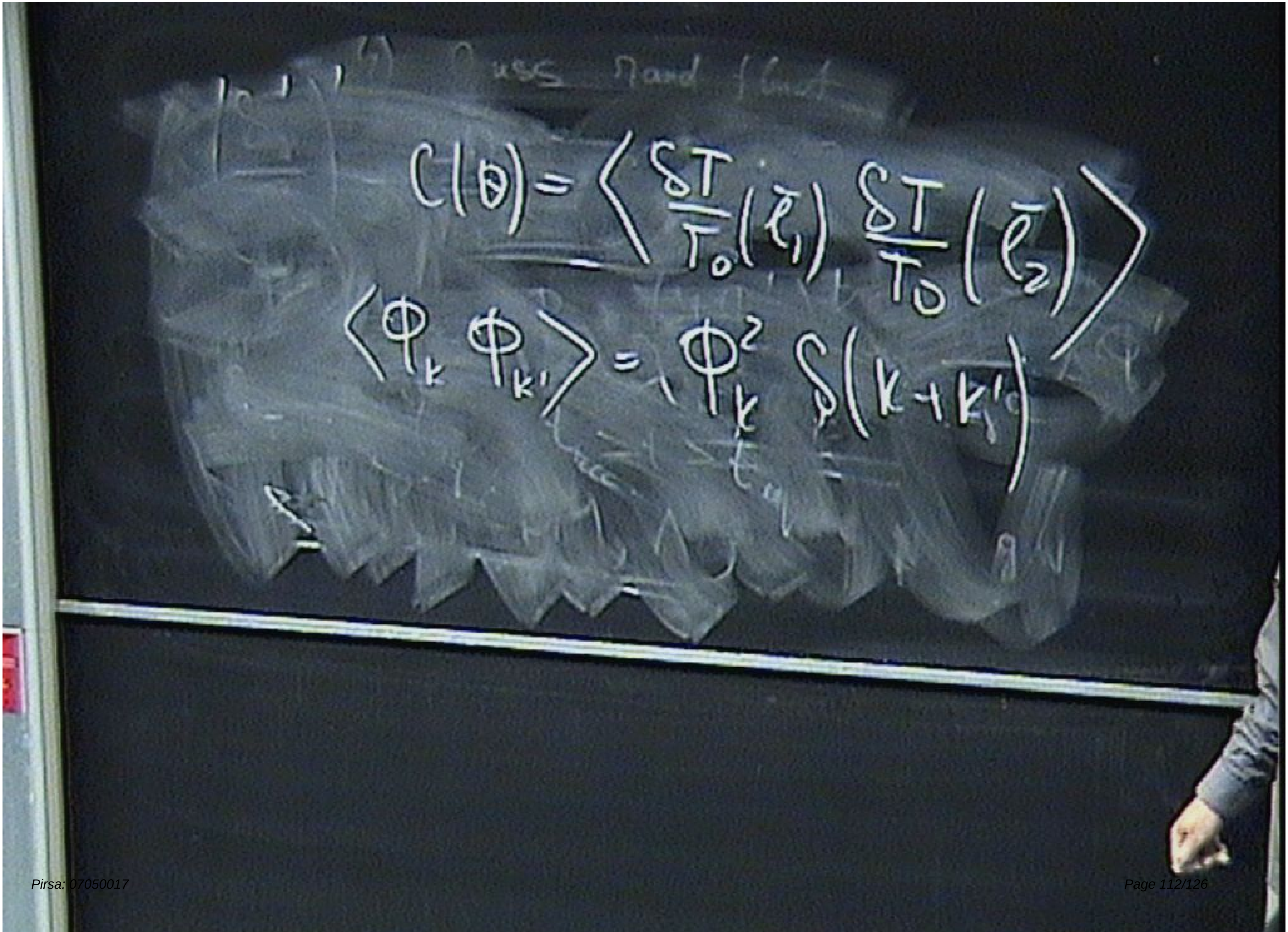
$x^i(\eta) = x_0^i + l^i(\eta - \eta_0)$ along



$$\frac{\delta T}{T}\left(\eta_0, \vec{x}_0, \vec{\ell}\right) = \int d\left[\left(\Phi + \frac{\delta}{4}\right)_k - \frac{3\delta_k'}{k^2}\frac{\partial}{\partial \eta_0}\right]_{\eta_{rec}} \times$$

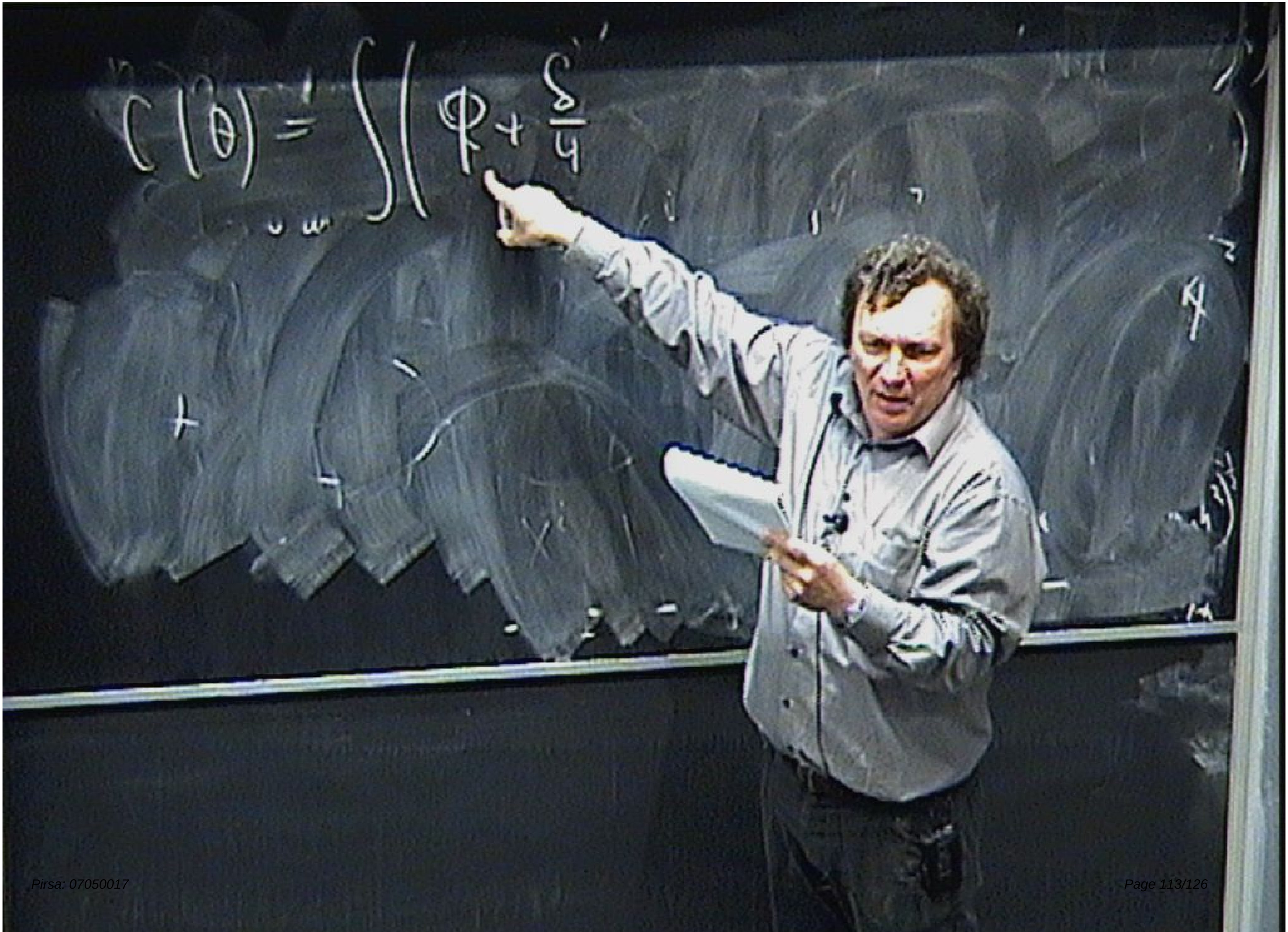
$$\left\langle \left(\frac{\delta T}{T}(\theta)\right)^2 \right\rangle = \left\langle \left(\frac{T(\vec{\ell}_1) - T(\vec{\ell}_2)}{T_0}\right)^2 \right\rangle =$$

$$= 2\left(C(0) - C(\theta)\right) \text{ where}$$

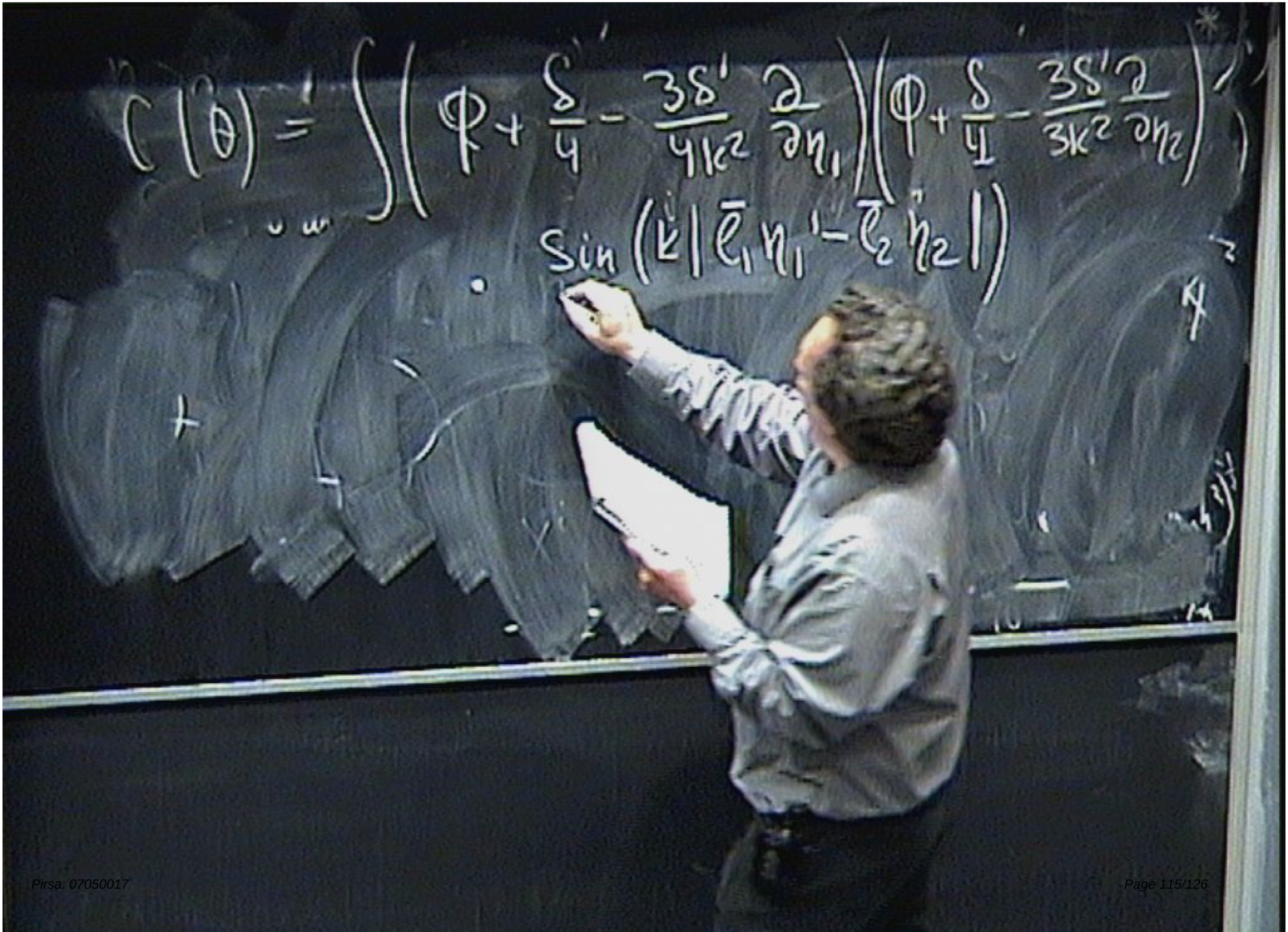


$$C(\theta) = \left\langle \frac{\delta T}{T_0}(\bar{e}_1)\, \frac{\delta T}{T_0}(\bar{e}_2) \right\rangle$$

$$\langle \phi_k \, \phi_{k'} \rangle = \phi_k^2 \, \delta(k+k')$$

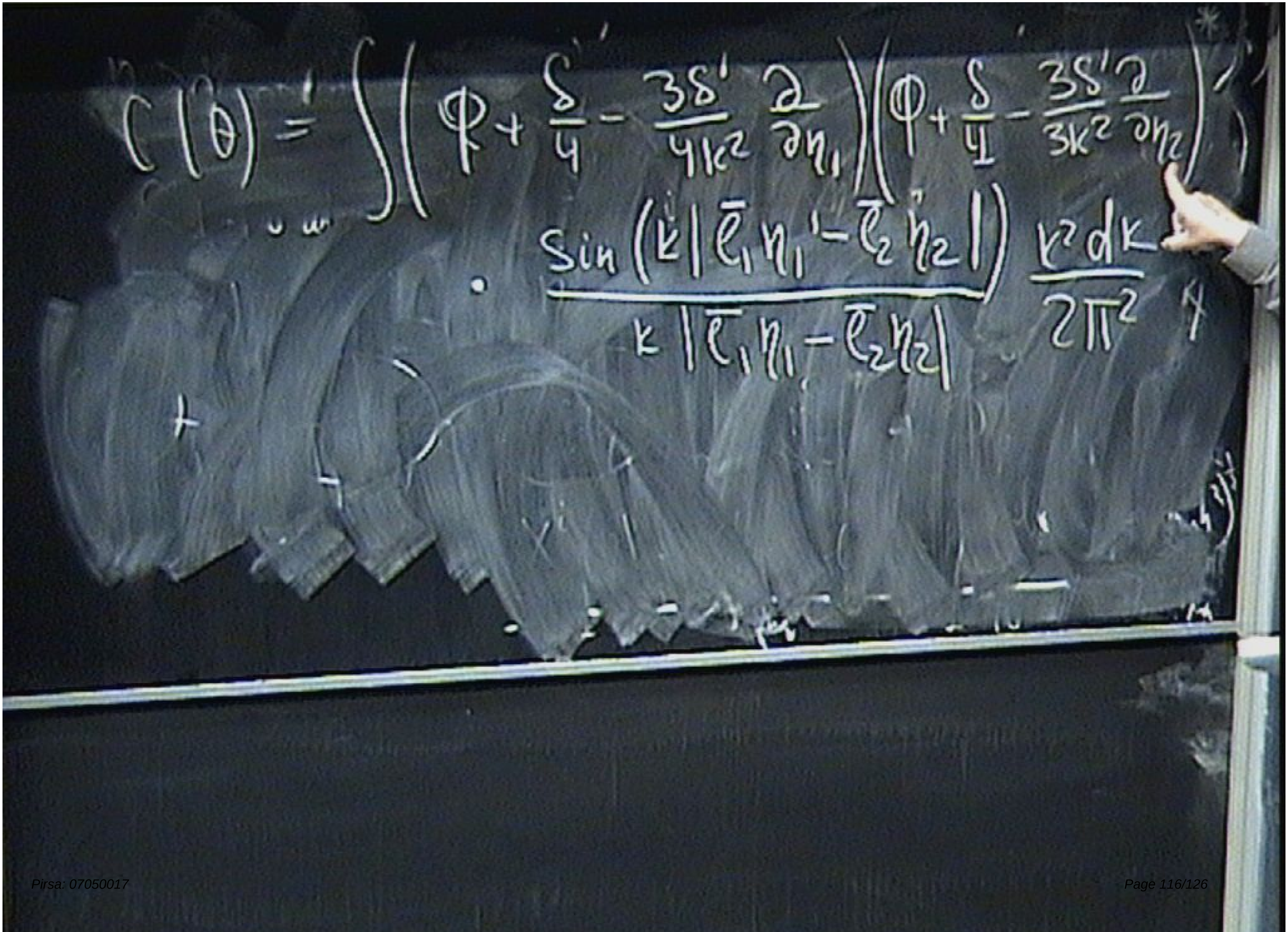

$C(\theta) = \int (\Phi + \frac{\delta}{4}$

$$C(\theta) = \int \left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right.$$

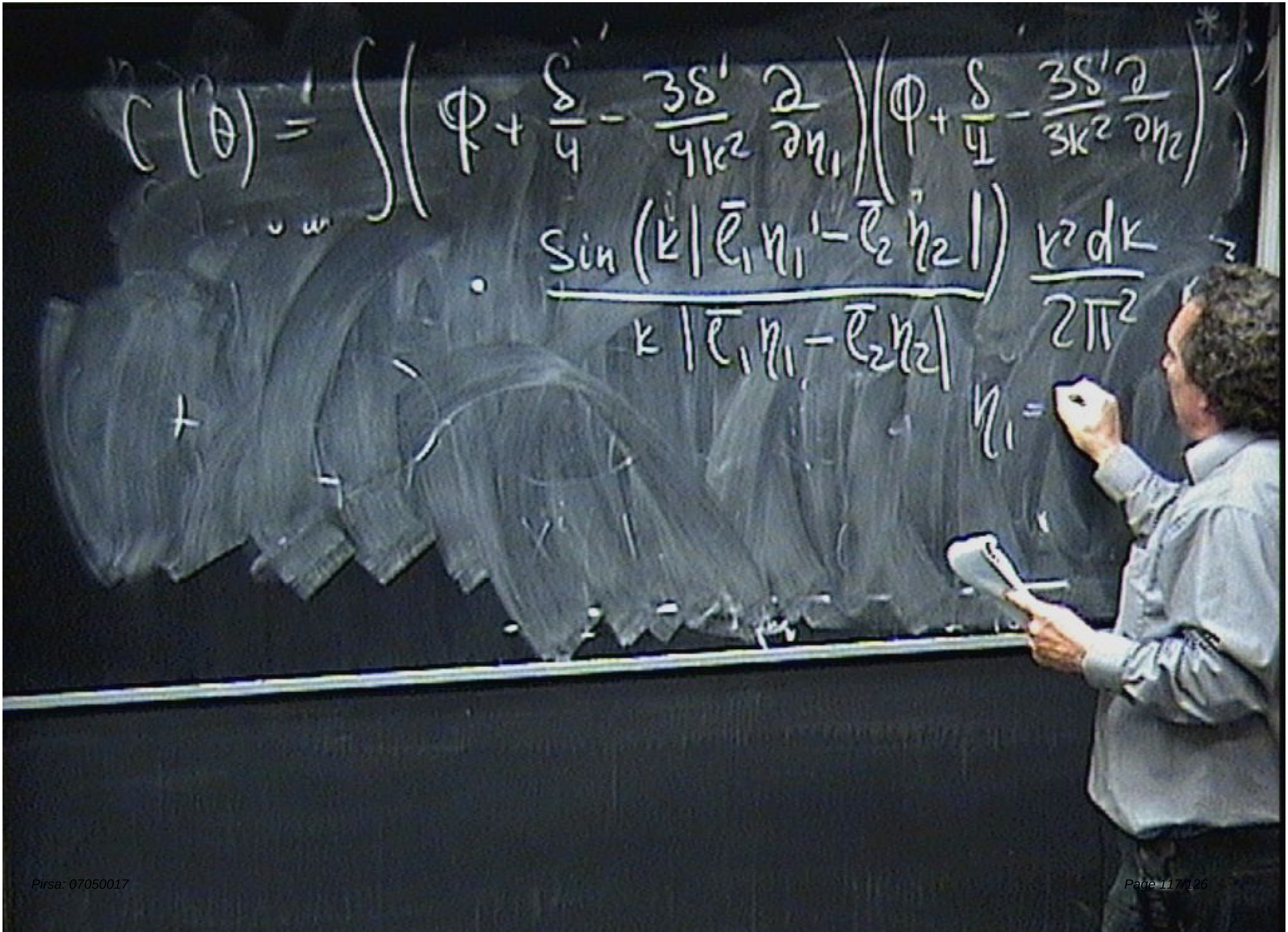


$$C(\theta) = \int \left(\varphi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left(\varphi + \frac{\delta}{4} - \frac{3\delta'}{3k^2} \frac{\partial}{\partial \eta_2} \right)$$

$$\sin\left(k \left| \bar{e}_1 \eta_1 - \bar{e}_2 \eta_2 \right| \right)$$

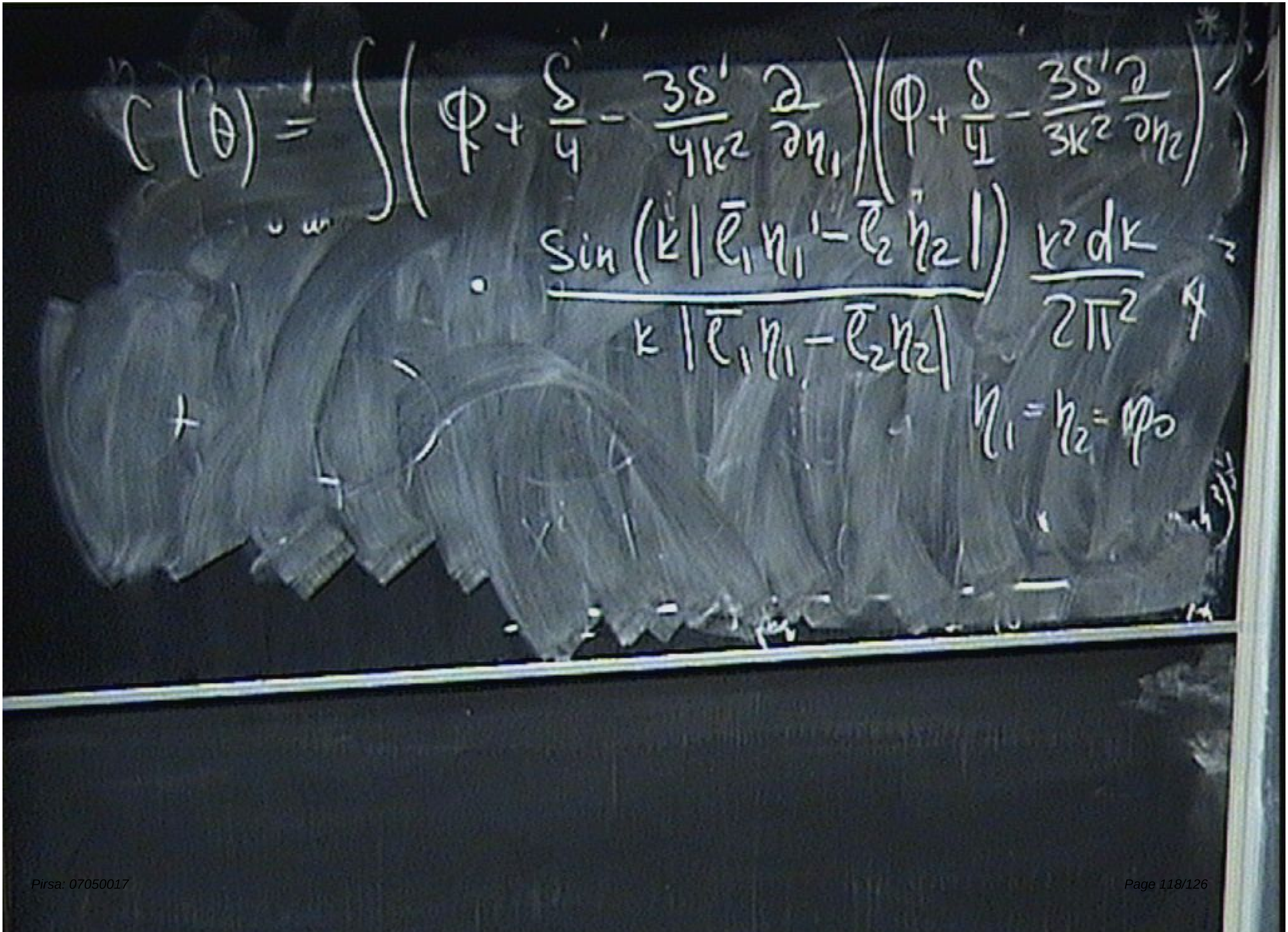


$$C(\theta) = \int \left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial \eta_1} \right)\left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{3k^2}\frac{\partial}{\partial \eta_2} \right)^{*} \cdot \frac{\sin\left(k\,|\bar{e}_1\eta_1 - \bar{e}_2\eta_2|\right)}{k\,|\bar{e}_1\eta_1 - \bar{e}_2\eta_2|}\,\frac{k^2\,dk}{2\pi^2}$$



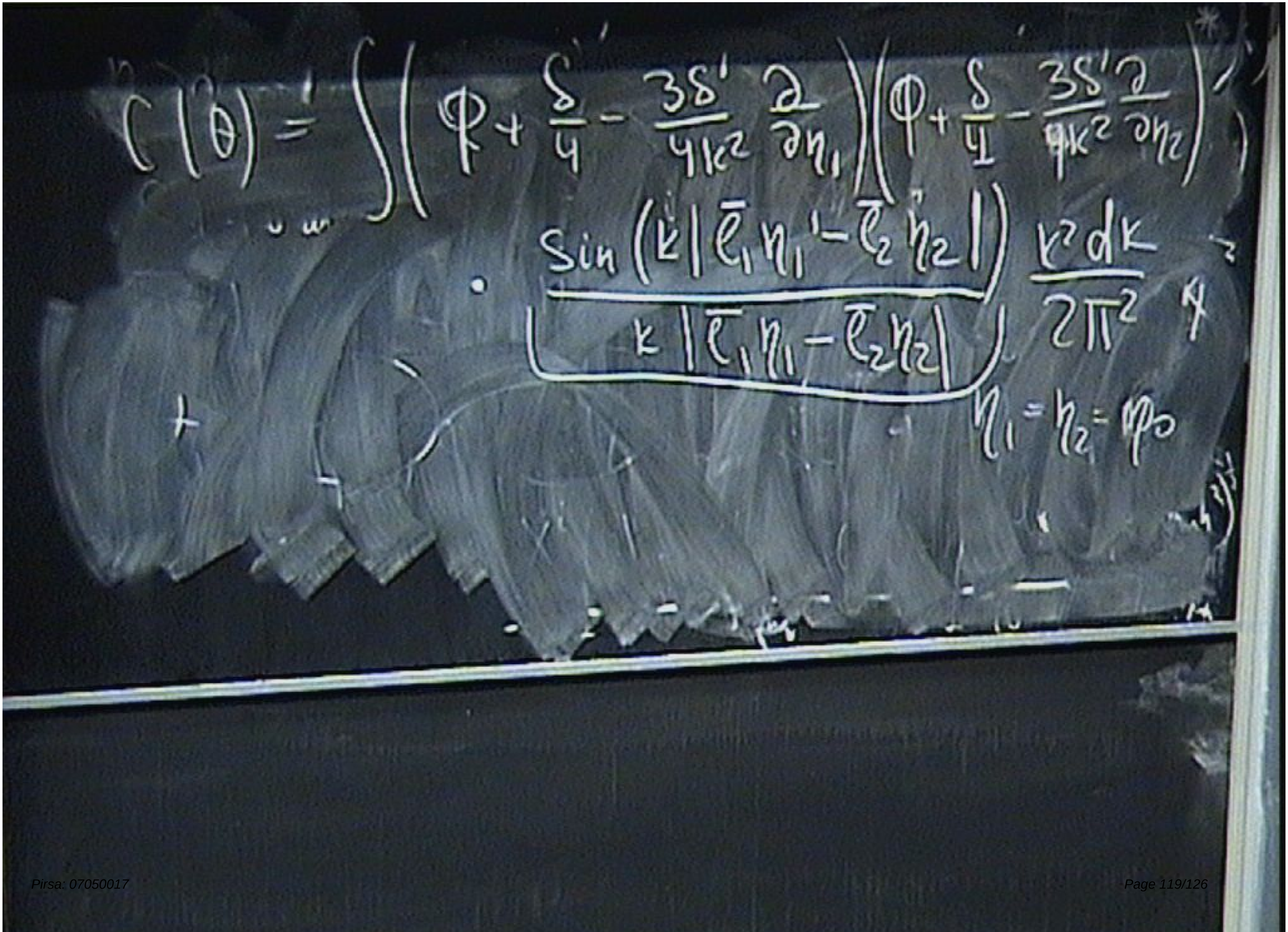
$$C(\theta) = \int \left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial \eta_1}\right)\left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{3k^2}\frac{\partial}{\partial \eta_2}\right)^* \cdot \frac{\sin\left(k\,|\bar{e}_1\eta_1 - \bar{e}_2\eta_2|\right)}{k\,|\bar{e}_1\eta_1 - \bar{e}_2\eta_2|}\,\frac{k^2\,dk}{2\pi^2}$$

$\eta_1 =$



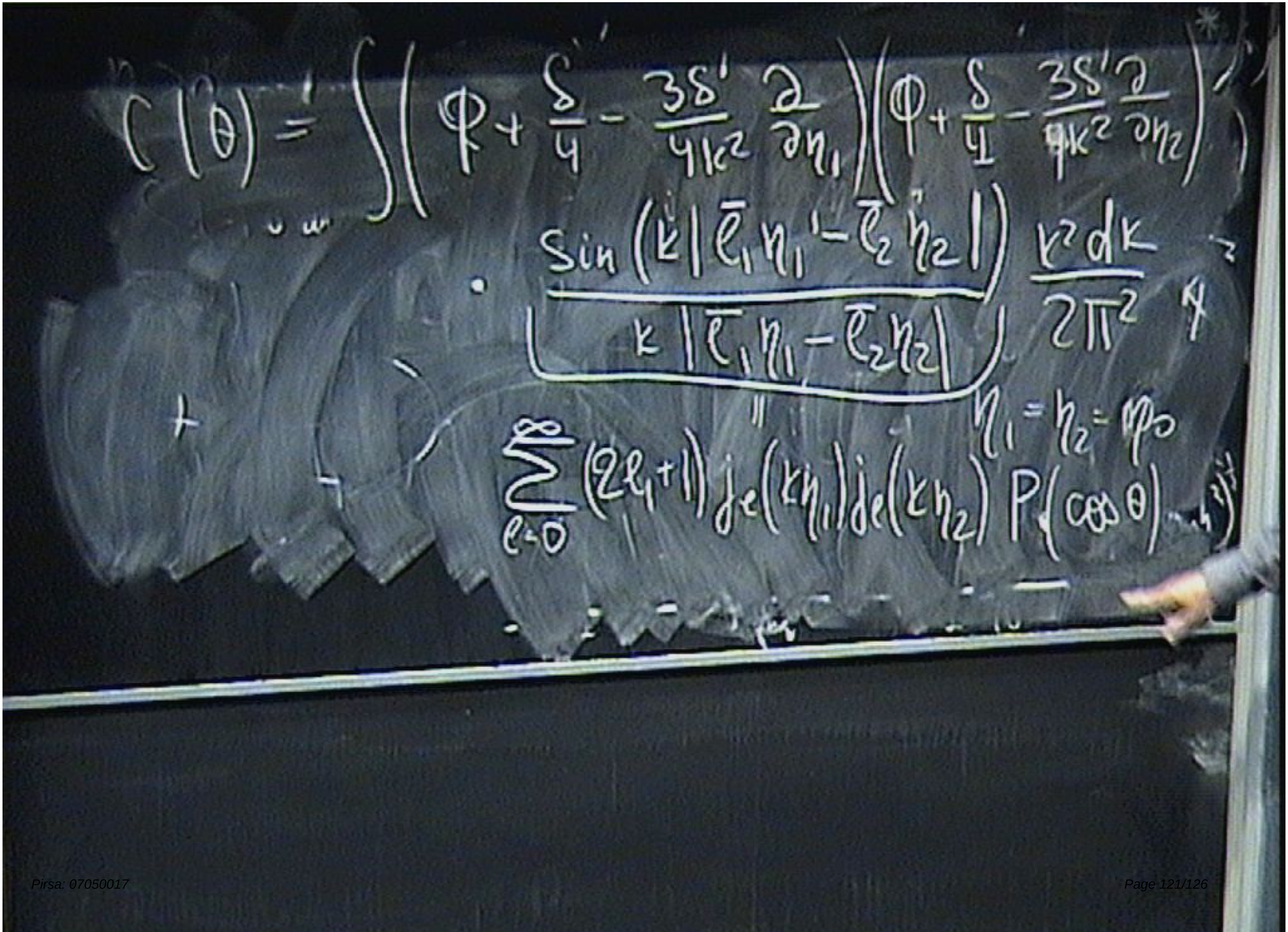
$$C(\theta) = \int \left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{3k^2} \frac{\partial}{\partial \eta_2} \right) \cdot \frac{\sin\left(k \left| \bar{e}_1 \eta_1 - \bar{e}_2 \eta_2 \right| \right)}{k \left| \bar{e}_1 \eta_1 - \bar{e}_2 \eta_2 \right|} \frac{k^2 dk}{2\pi^2}$$

$$\eta_1 = \eta_2 = \eta_0$$



$$C(\theta) = \int \left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial \eta_1}\right)\left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial \eta_2}\right)^* \cdot \left.\frac{\sin\left(k\left|\bar{e}_1\eta_1 - \bar{e}_2\eta_2\right|\right)}{k\left|\bar{e}_1\eta_1 - \bar{e}_2\eta_2\right|}\right|_{\eta_1 = \eta_2 = \eta_0} \frac{k^2\,dk}{2\pi^2}$$

$$C(\theta) = \int \left(\varphi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial \eta_1} \right)\left(\varphi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial \eta_2} \right)^{*} \cdot \underbrace{\frac{\sin\left(k\left|\bar{e}_1\eta_1 - \bar{e}_2\eta_2\right|\right)}{k\left|\bar{e}_1\eta_1 - \bar{e}_2\eta_2\right|}}_{\eta_1 = \eta_2 = \eta_0} \frac{k^2\,dk}{2\pi^2}$$



$$C(\theta) = \int \left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial\eta_1}\right)\left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial\eta_2}\right)^*$$

$$\cdot \underbrace{\frac{\sin\left(k\left|\bar{e}_1\eta_1 - \bar{e}_2\eta_2\right|\right)}{k\left|\bar{e}_1\eta_1 - \bar{e}_2\eta_2\right|}}_{} \frac{k^2\,dk}{2\pi^2}$$

$$\eta_1 = \eta_2 = \eta_0$$

$$= \sum_{\ell=0}^{\infty}(2\ell_1+1)\, j_\ell(k\eta_1)\, j_\ell(k\eta_2)\, P_\ell(\cos\theta)$$

$$C(\theta) = \frac{1}{\,}\int \left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial\eta_1}\right)\left(\Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2}\frac{\partial}{\partial\eta_2}\right)$$

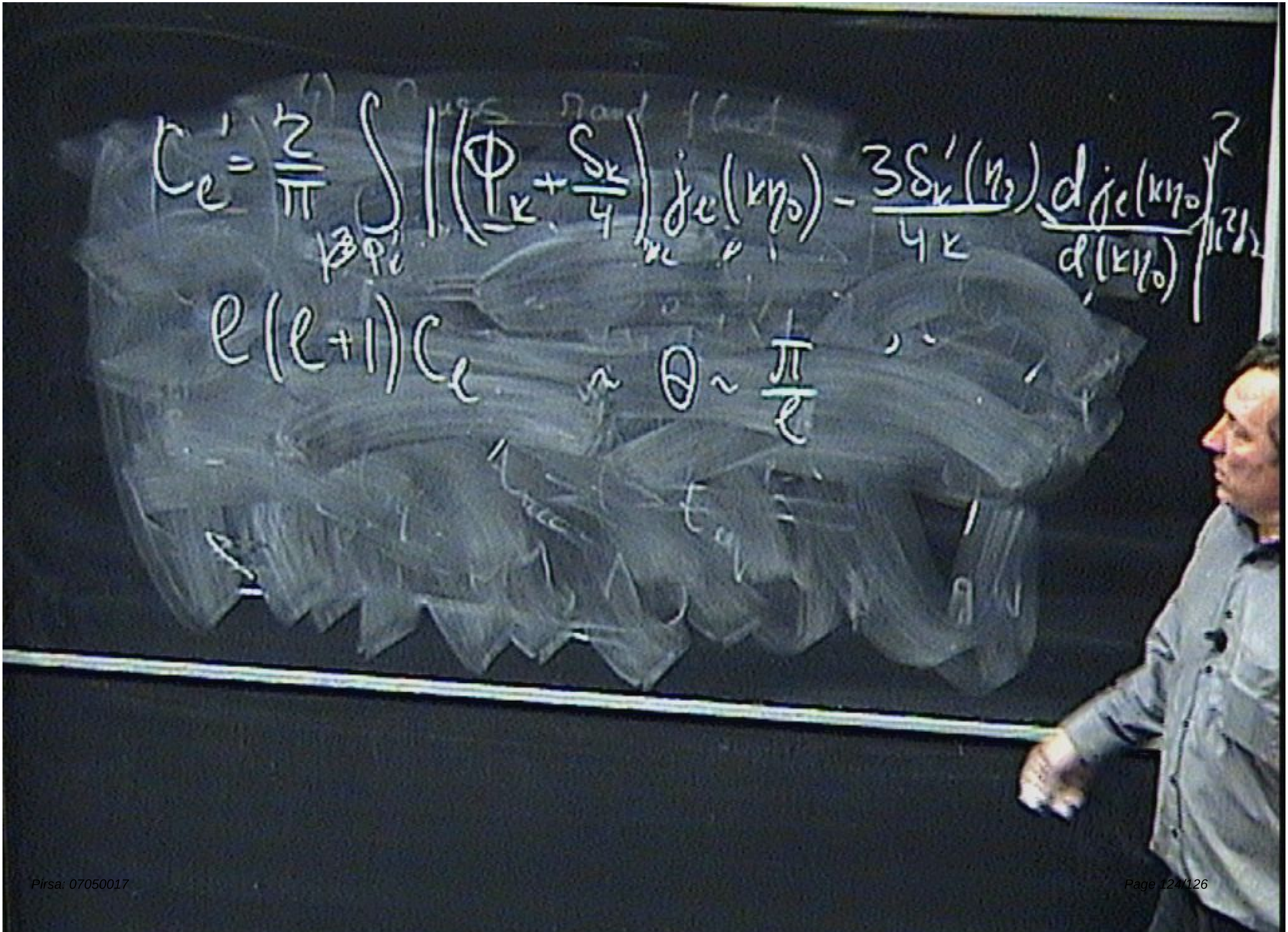
$$\cdot \underbrace{\frac{\sin\left(k\left|\bar{e}_1\eta_1 - \bar{e}_2\eta_2\right|\right)}{k\left|\bar{e}_1\eta_1 - \bar{e}_2\eta_2\right|}}_{=\ \sum_{\ell=0}^{\infty}(2\ell_1+1)\, j_\ell(k\eta_1)\, j_\ell(k\eta_2)\, P_\ell(\cos\theta)} \frac{k^2\,dk}{2\pi^2} \times$$

$$\eta_1 = \eta_2 = \eta_0$$

$$+$$

$$\Rightarrow \frac{1}{4\pi}\sum (2\ell+1)\, C_\ell\, P_\ell(\theta)$$

$$C_\ell = \frac{2}{\pi} \int \left| \left(\Phi_k + \frac{\delta_k}{4} \right)_{\eta_*} j_\ell(k\eta_0) - \frac{3\delta_k'(\eta_*)}{4k} \frac{d j_\ell(k\eta_0)}{d(k\eta_0)} \right|^2_{\eta_*}$$



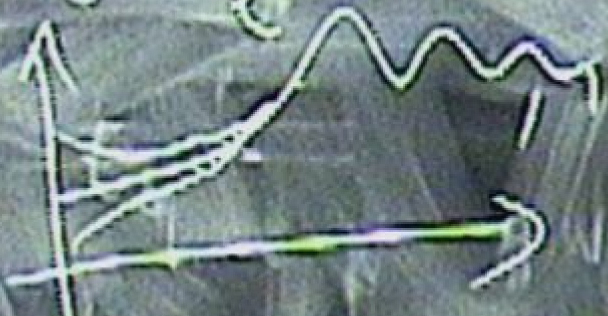
$$C_\ell' = \frac{2}{\pi}\int_{k^3 \varphi_\ell} \left|\left(\Phi_k + \frac{\delta_k}{4}\right)_{\eta_*} j_\ell(k\eta_0) - \frac{3\delta_k'(\eta_*)}{4k}\frac{d\,j_\ell(k\eta_0)}{d(k\eta_0)}\right|^2_{k^2\eta_*}$$

$$\ell(\ell+1)C_\ell \sim \Theta \sim \frac{\pi}{\ell}$$

$$C_\ell' = \frac{2}{\pi}\int_{k^2 P_{\Phi_i}} \left|\left(\Phi_k + \frac{\delta_k}{4}\right)_{\eta_*} j_\ell(k\eta_0) - \frac{3\delta_k'(\eta_*)}{4k}\frac{d\, j_\ell(k\eta_0)}{d(k\eta_0)}\right|^2_{k^3\eta_*}$$

$$\underline{\underline{\ell(\ell+1)C_\ell}} \sim \quad \theta \sim \frac{\pi}{\ell}$$

$$\frac{\delta T}{T} \sim \frac{1}{\sqrt{5}}$$



$$C_\ell' = \frac{2}{\pi}\int_{k^3 P_i} \left| \left(\Phi_k + \frac{\delta_k}{4} \right) j_\ell(k\eta_0) - \frac{3\delta_k'(\eta_*)}{4k} \frac{d\, j_\ell(k\eta_0)}{d(k\eta_0)} \right|^2$$

$$\underline{\underline{\ell(\ell+1)C_\ell}} \quad \theta \sim \frac{\pi}{\ell}$$

$$\frac{\delta T}{T} \sim \frac{1}{\sqrt{5}}$$