

Title: Advanced Topics in Cosmology

Date: May 24, 2007 12:00 PM

URL: <http://pirsa.org/07050017>

Abstract: Class 6 part 2

$\Phi(\kappa, \eta_{rec})$ ,  $\delta_{\delta_{or}}(\kappa, \eta_{rec})$  in terms

$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_{\sigma}}(k, \eta_{rec})$  in terms.

$\Phi_{in}$

$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_{\sigma}}(k, \eta_{rec})$  in terms.

$\epsilon, \mu/\epsilon$

CMB

$\Phi_{in}$

$\delta_{\delta_{\sigma}}$

$(\tau, \eta_0) - ?$

$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_{\sigma}}(k, \eta_{rec})$  in terms.

$\epsilon, \mu(\epsilon)$

CMB

$\Phi_{in}$

$\delta_{\sigma}$

$\delta_{\sigma}(\tau, \eta_0) - ?$

$\epsilon$

$z$

$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_0}(k, \eta_{rec})$  in terms.

$\epsilon, \mu(\epsilon)$

CMB

$\Phi_{\vec{k}}^{in}$

$\delta_{\delta_0}(\vec{k}, \eta_0) - ?$

$\epsilon$

$\propto (1+z)^4$   
 $\vec{z}$

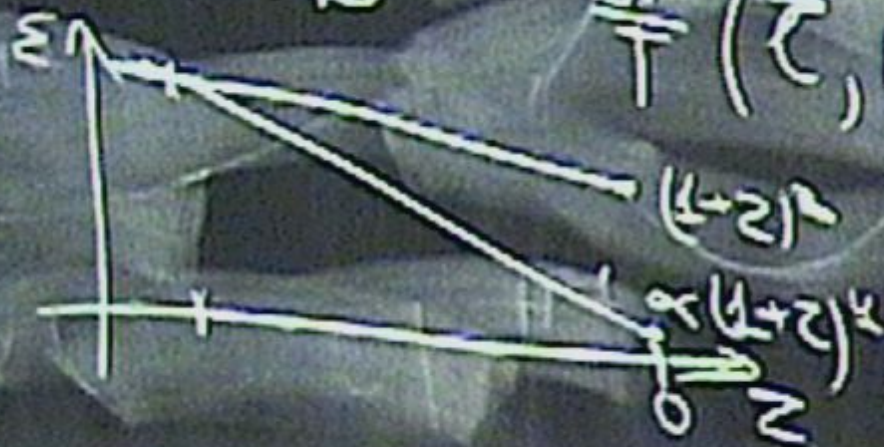
$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_0}(k, \eta_{rec})$  in terms.

$\epsilon, \Phi(\epsilon)$

CMB

$\Phi_{in}$

$\delta_T(\tau, \eta_0) - ?$



$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta}^c(k, \eta_{rec})$  in terms.

$\epsilon, \rho(\epsilon)$

CMB

$\Phi_{in}$

$\delta_{\delta}^c(\tau, \eta_0) - ?$

$$h = \frac{H_0}{75 \frac{km}{s Mpc}} \Omega_m h^2$$

$$\rho_{eq} = \frac{1}{12\pi^2}$$





$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_{ot}}(k, \eta_{rec})$  in terms.

$\epsilon, \rho(\epsilon)$

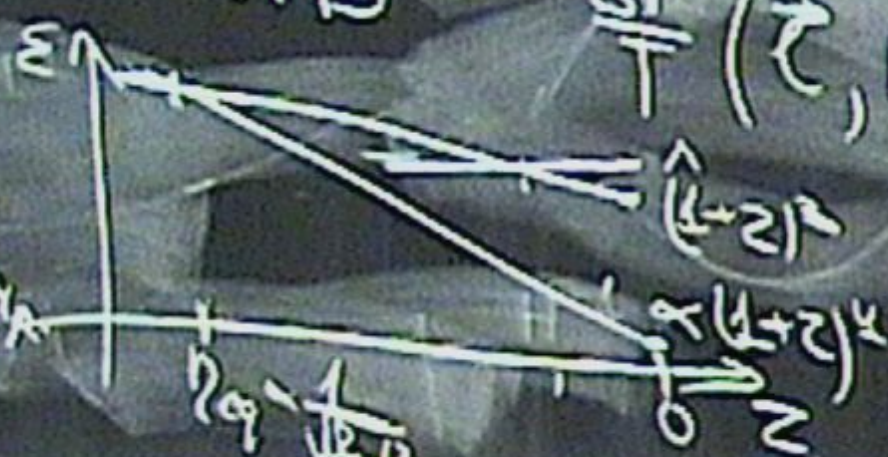
CMB

$\Phi_{in}$   
 $\delta_T$

$(\tau, \eta_0) - ?$

$$h = \frac{H_0}{75 \frac{\text{km}}{\text{Sec Mpc}}}$$

$$\eta_0 = \frac{1}{\sqrt{2\epsilon}}$$



$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta}^{\sigma}(k, \eta_{rec})$  in terms.

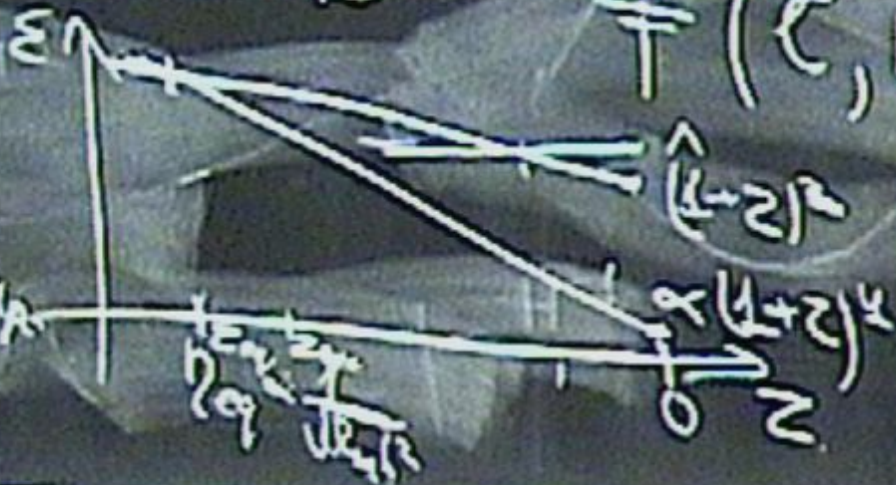
$(E, p(E))$

CMB

$\Phi^{in}$

$\delta_T(\vec{\ell}, \eta_0) - ?$

$$h = \frac{H_0}{75 \frac{km}{s Mpc}}$$



$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta}^{\sigma}(k, \eta_{rec})$  in terms.

$E, P(E)$

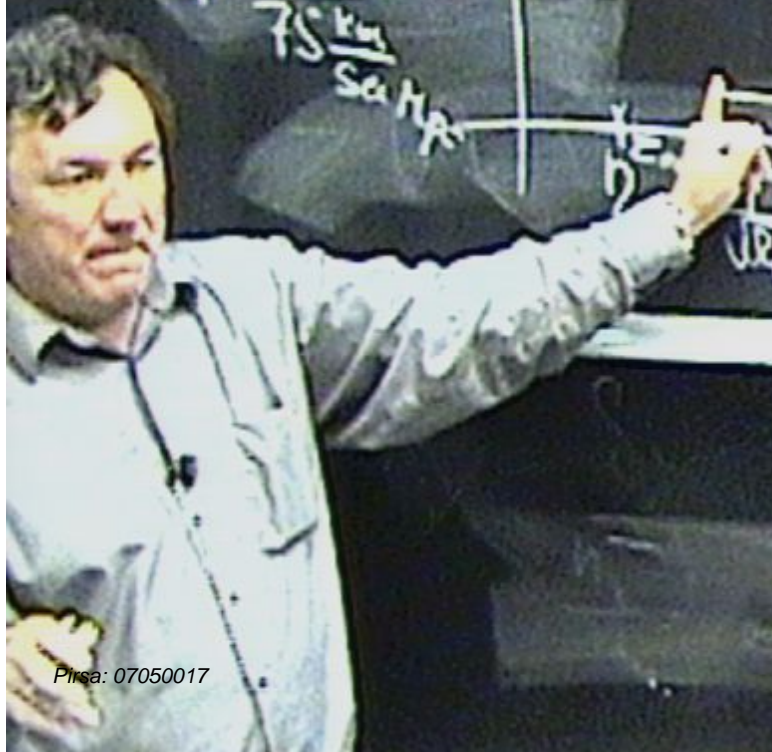
CMB

$\Phi^{in}$

$\delta_T$

$(\tau, \eta_0) - ?$

$$h = \frac{H_0}{75 \frac{km}{s Mpc}}$$



$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta}^{\sigma}(k, \eta_{rec})$  in terms.

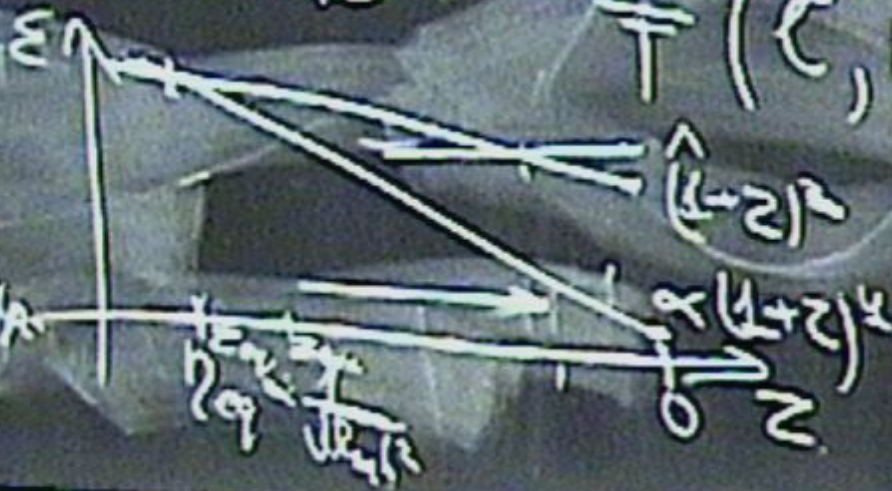
$\epsilon, -p(\epsilon)$

CMB

$\Phi_{\vec{k}}^{in}$

$\delta_{\delta}^{\sigma}(\vec{k}, \eta_0) - ?$

$$h = \frac{H_0}{75 \frac{km}{s Mpc}}$$



$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_{\sigma}}(k, \eta_{rec})$  in terms.

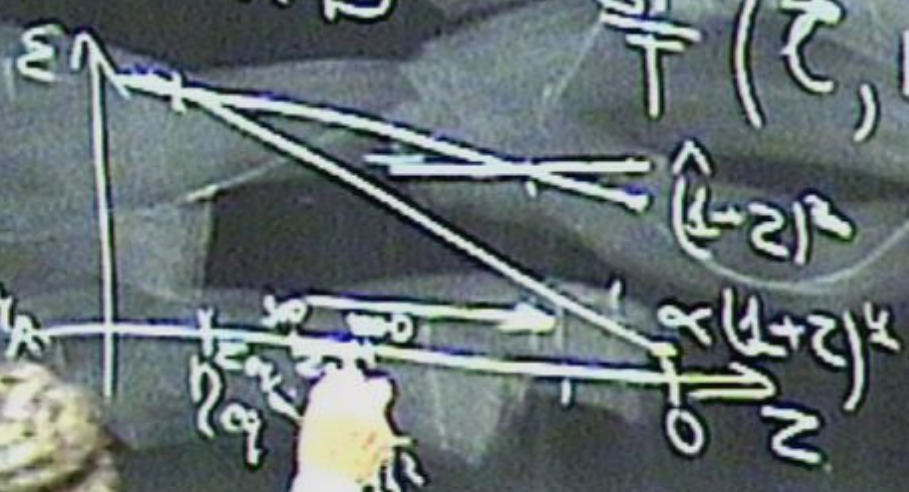
$\epsilon, \rho(\epsilon)$

CMB

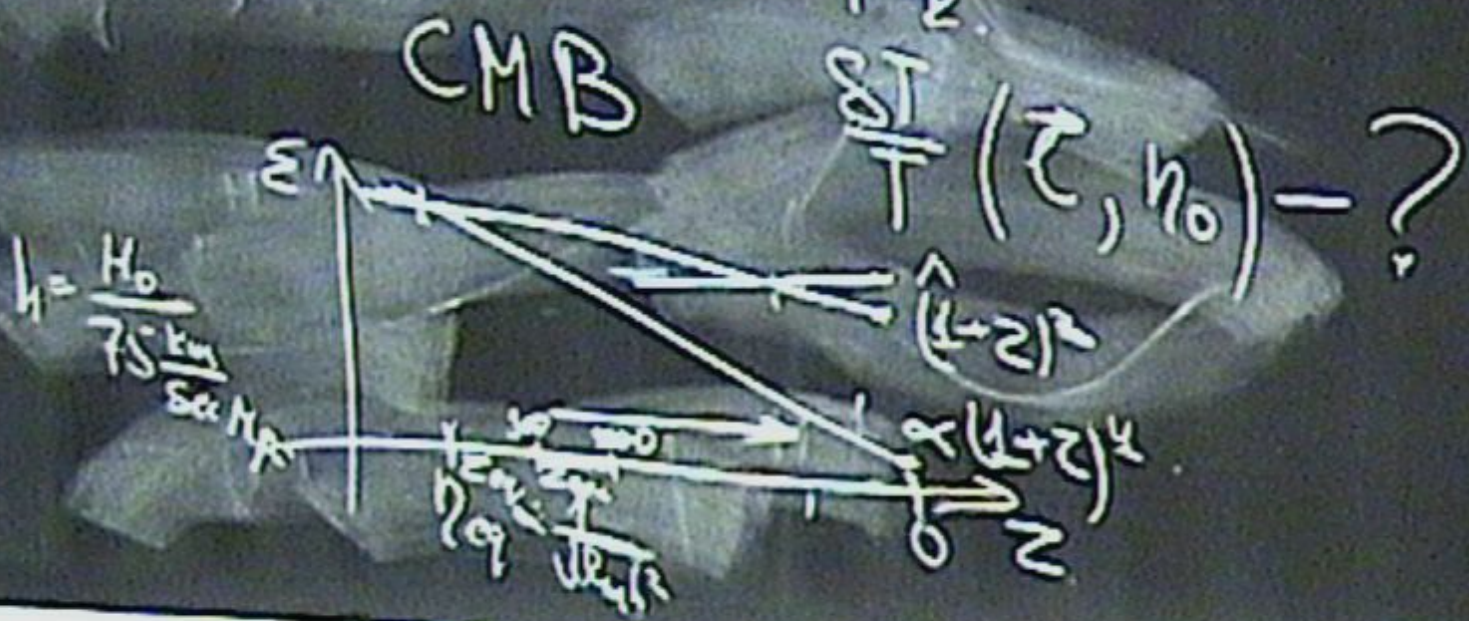
$\Phi_{in}$

$\delta_T(\vec{\ell}, \eta_0) - ?$

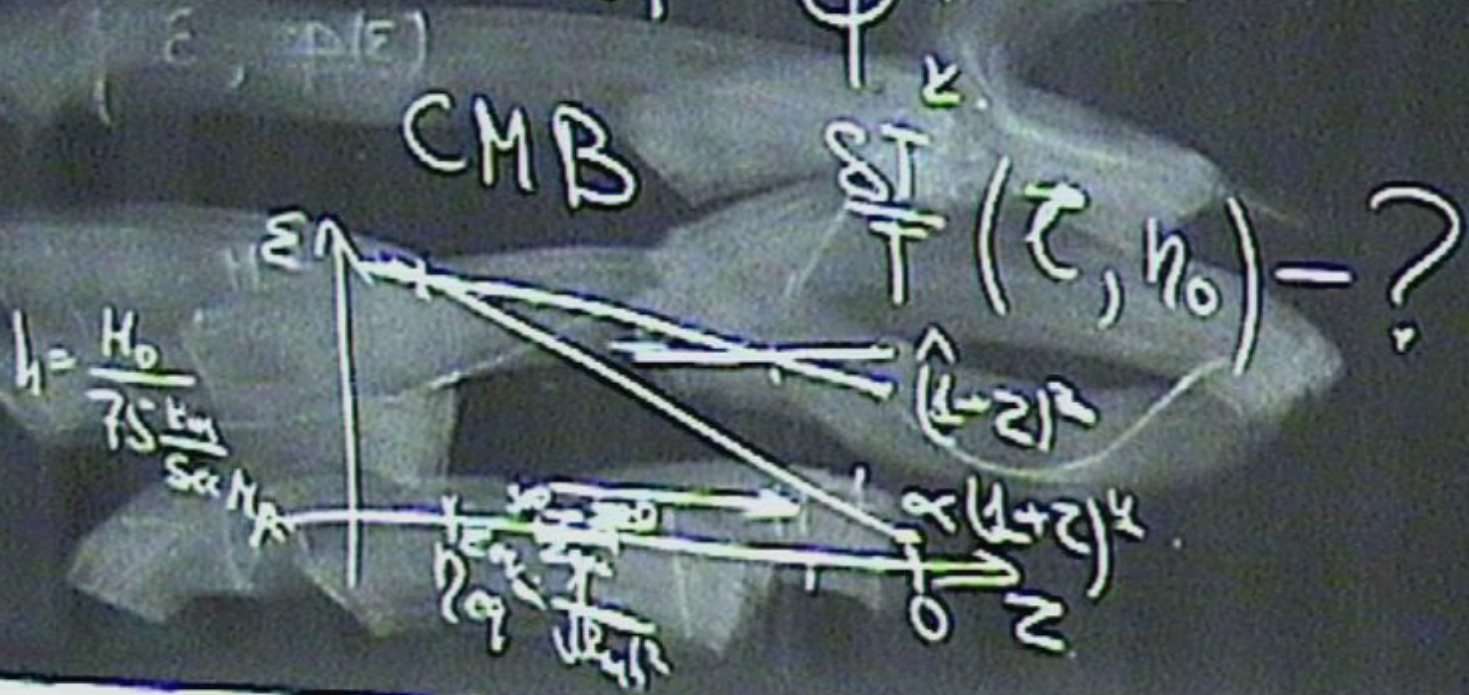
$$h = \frac{H_0}{75 \frac{\text{km}}{\text{Sec Mpc}}}$$

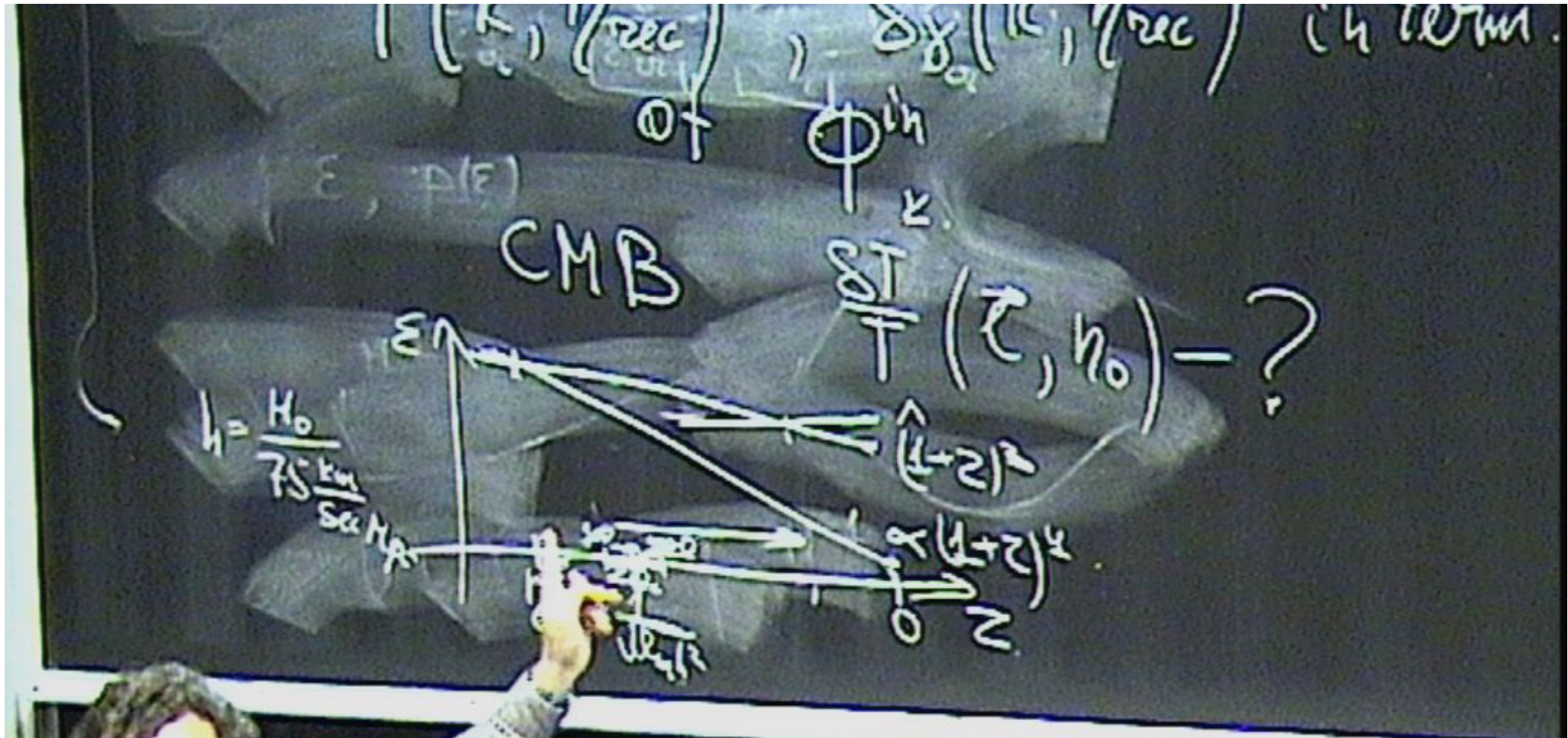


$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta}^{\infty}(k, \eta_{rec})$  in term.



$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_{\sigma}}(k, \eta_{rec})$  in terms.







$\Phi(k, \eta_{rec})$ ,  $\delta_{\delta_{\sigma}}^{\delta_{\sigma}}(k, \eta_{rec})$  in terms.

$\epsilon, \rho(\epsilon)$

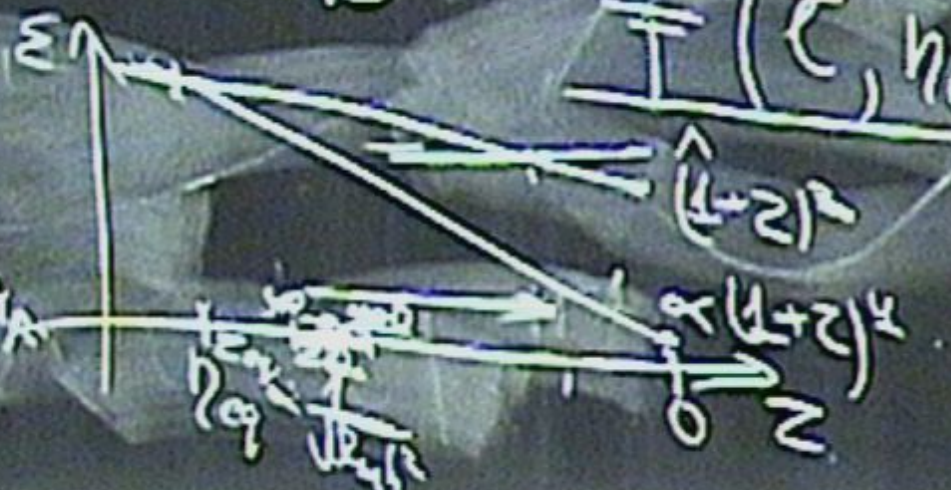
CMB

$\Phi_{in}$

$\delta_T$

$(\tau, \eta_0) - ?$

$$h = \frac{H_0}{75 \frac{km}{s Mpc}}$$





$\eta_0 \chi_0$

$\delta \delta$

$\delta$

$\delta$

$\delta$

$\frac{1}{3}$

$(-2\phi(H_{\text{ave}}))$

$\delta$

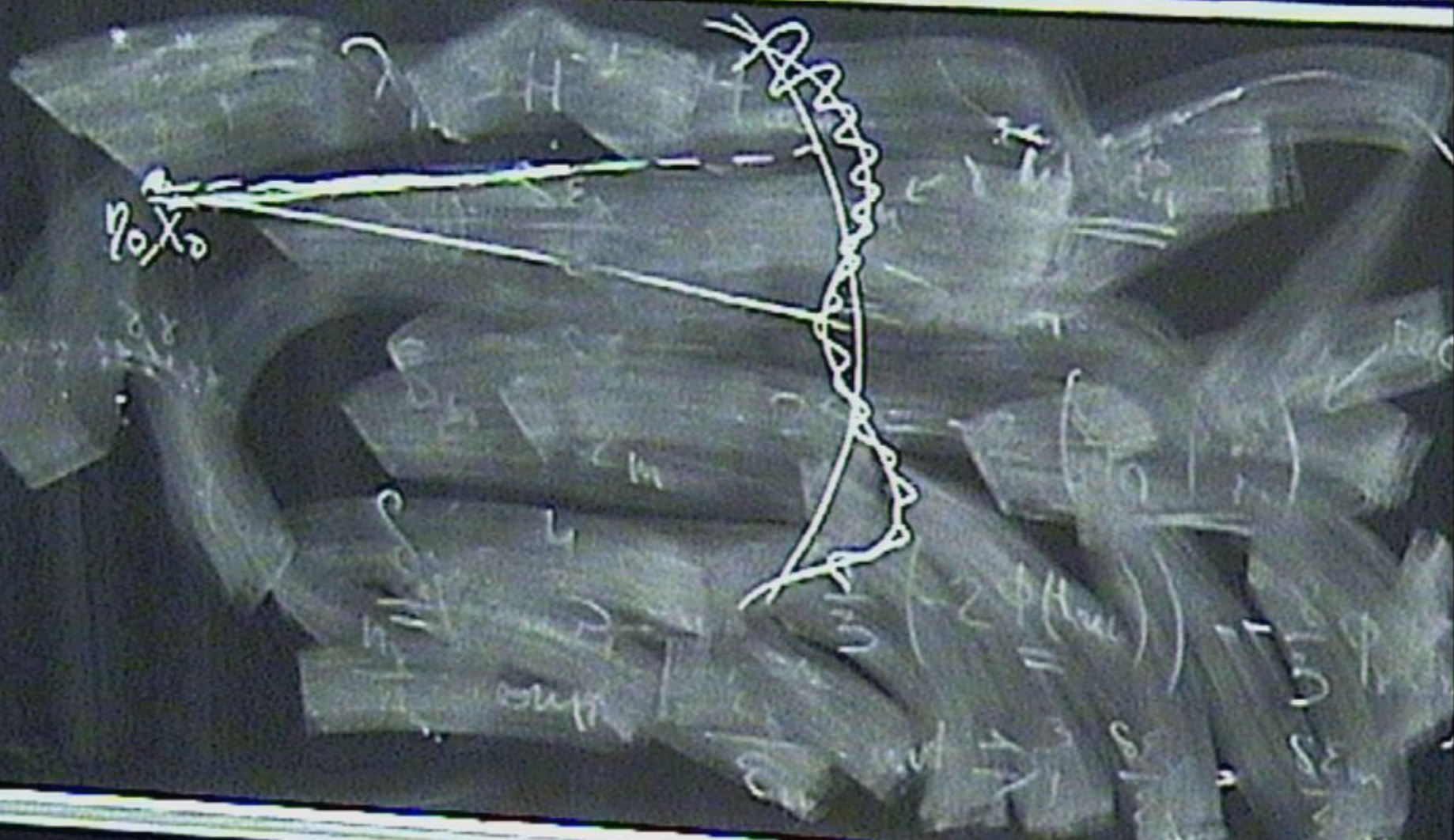
$\delta$

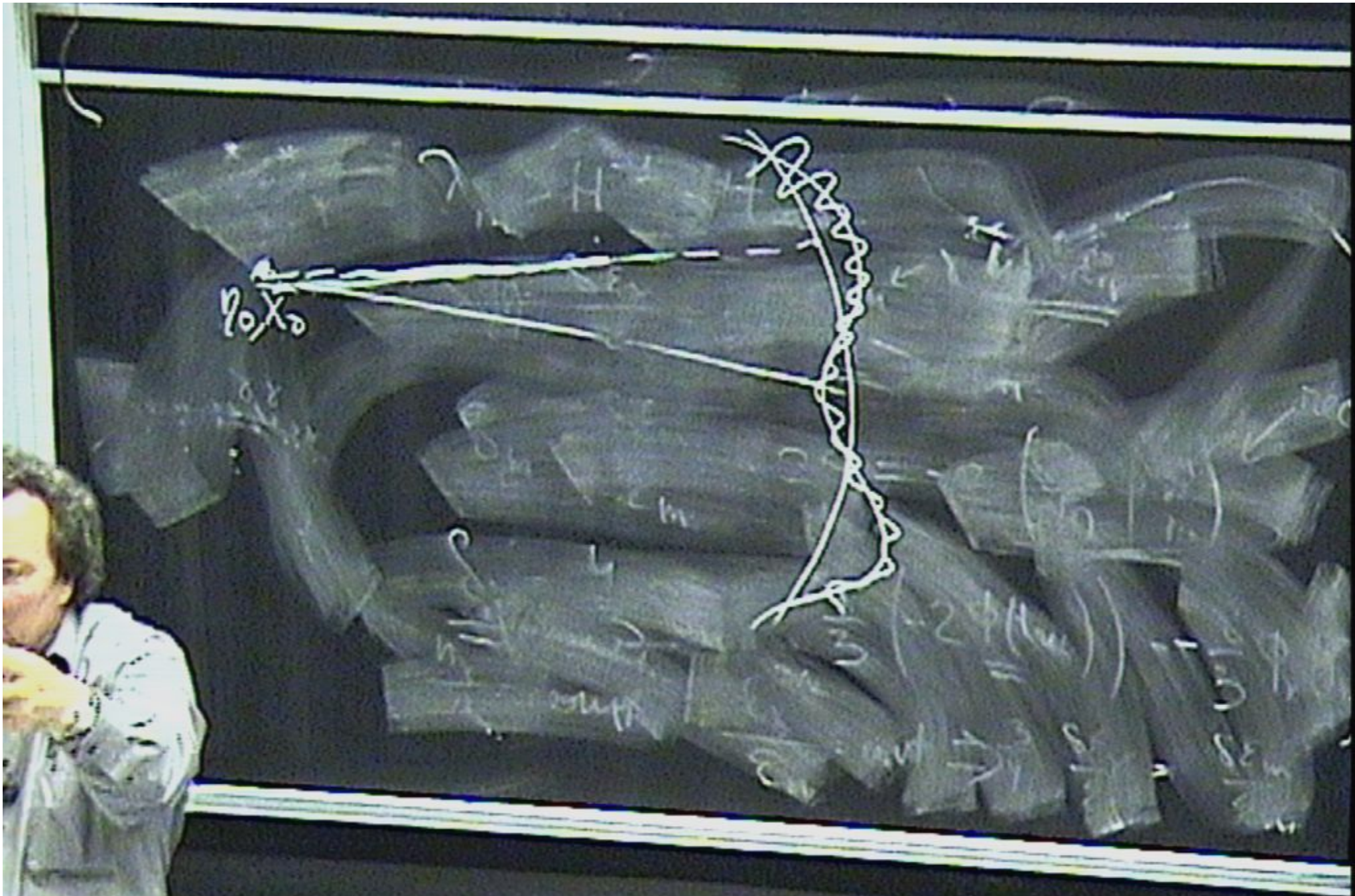
$\delta$

$\delta$



$\eta_0 \chi_0$





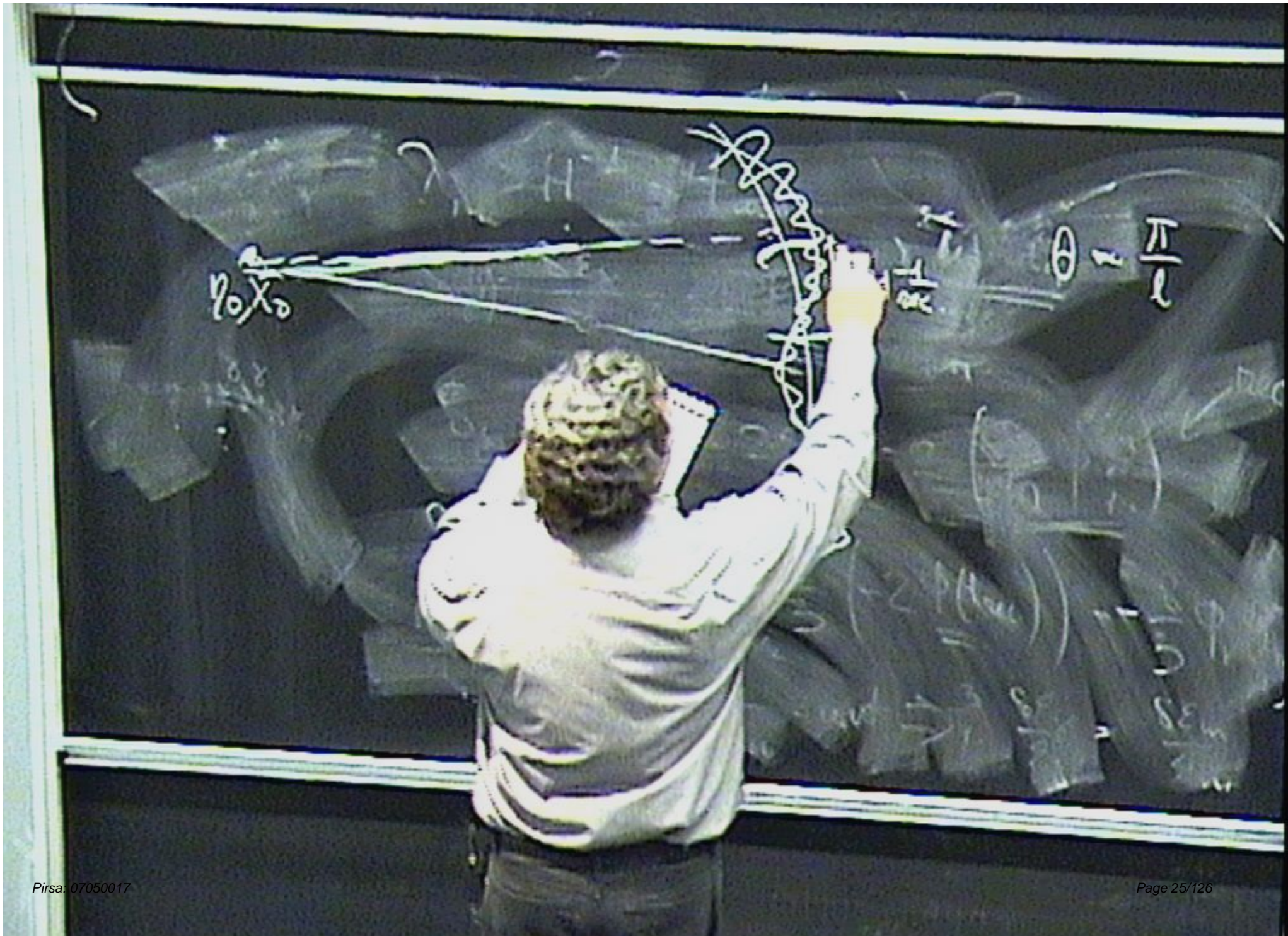


$\gamma_0 \times \gamma_0$



$H_{18}$







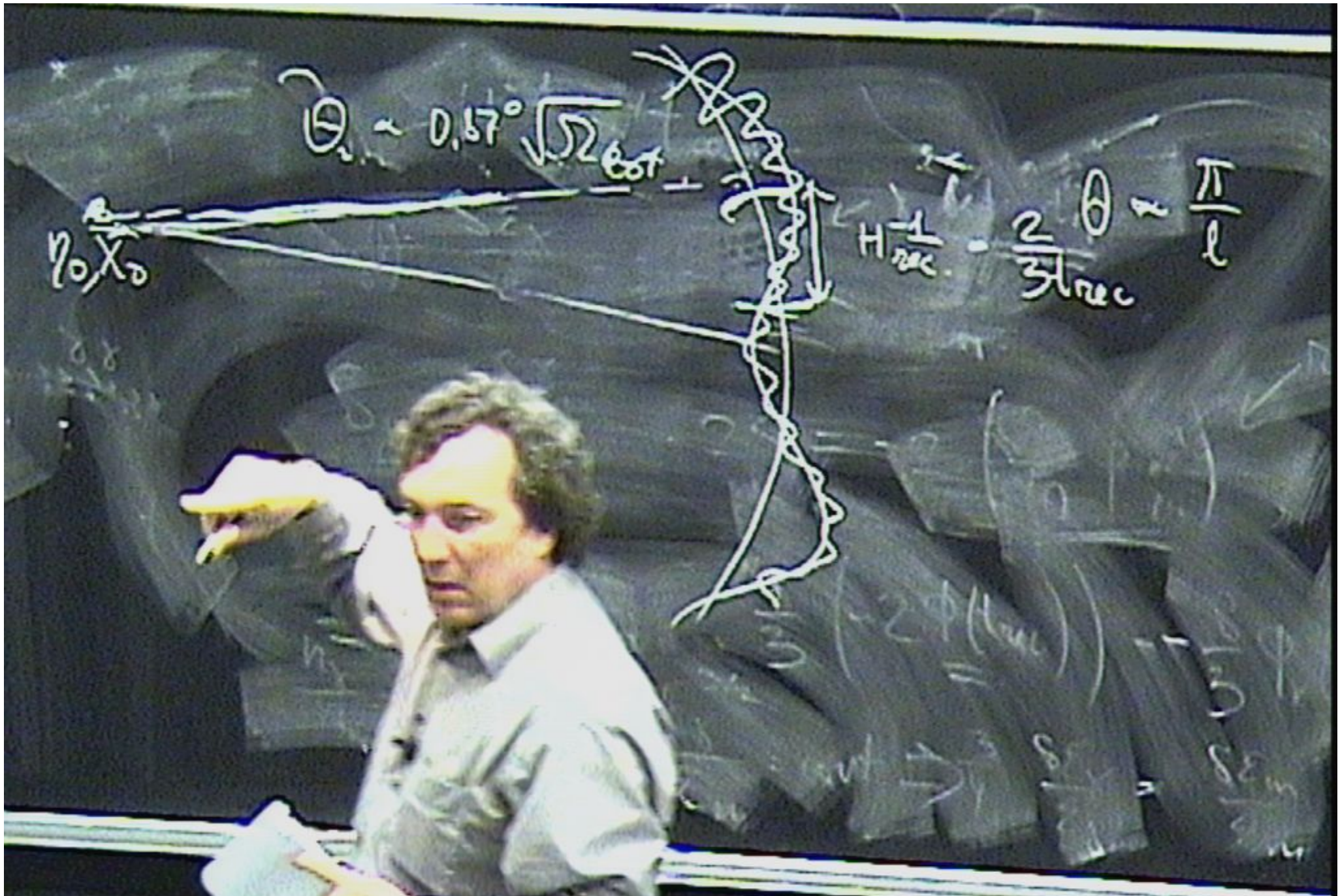
Free photons.

$$F = \frac{a}{a}$$

$$F = \frac{H}{H_0} \left( \frac{t}{t_0} \right)^2$$

$$H_0 = 0.5$$

$$\left( \frac{1-p}{2} \right) = 10\% \quad \%$$



$$\theta_r \approx 0.157^\circ \sqrt{L} e^{\alpha l}$$

$$\gamma_0 X_0$$

$$H_{rec} \approx \frac{2 \theta}{\pi l} \approx \frac{\pi}{l}$$

Free photons.

$$dN = f(\mathbf{x}, \mathbf{p}) d^3x d^3p$$

Free photons.

$$dN = f(\hbar, x, p) d^3x d^3\underline{p}$$

Free photons:

$$dN = f(\eta, x, p_i) d^3x d^3p_i$$

Free photons:

$$dN = f(\mathbf{x}, \mathbf{p}) d^3x d^3p$$

$\frac{\partial}{\partial t}$

$$h_c < 0.5$$

$$10\% \text{ } \%$$



Free photons:

$$dW = f(\eta, x^i, p_i) d^3x^i d^3p_i$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial t} + \frac{dx^i}{dt} \frac{\partial f}{\partial x^i}$$

Free photons.

$$dW = f(\eta, x^i, p_j) d^3x^i d^3p_j$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta} \frac{\partial f}{\partial x^i} + \frac{dp_j}{d\eta} \frac{\partial f}{\partial p_j} = 0$$

# Free photons:

$$dW = f(\eta, x^i, p_i) d^3x^i d^3p_i$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta} \frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta} \frac{\partial f}{\partial p_i} = 0$$

$$\vec{\ell} \Leftrightarrow \ell^i \quad \text{---} \quad \frac{p_i}{\hbar}$$

Free photons:

$$dW = f(\eta, x^i, p_i) d^3x^i d^3p_i$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta} \frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta} \frac{\partial f}{\partial p_i} = 0$$

$$\vec{\ell} \Leftrightarrow \ell^i = - \frac{p_i}{|P|} = \sqrt{\sum_i p_i^2}$$

Free photons.

$$dW = f(\eta, x^i, p_i) d^3x^i d^3p_i$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta} \frac{\partial f}{\partial x^i} + \frac{dp_i}{d\eta} \frac{\partial f}{\partial p_i} = 0$$

$$\Leftrightarrow \ell^i = - \frac{p_i}{|p|} = \frac{\sqrt{2} p_i}{\sqrt{2} |p|}$$

10% %

$$f \equiv F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^+)}\right)}$$



$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^+, \bar{e}^i)}\right) - 1}.$$

$$f \equiv F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^+, \bar{c}^*)}\right) - 1}$$

$\downarrow$   
 $T_0(\eta)$



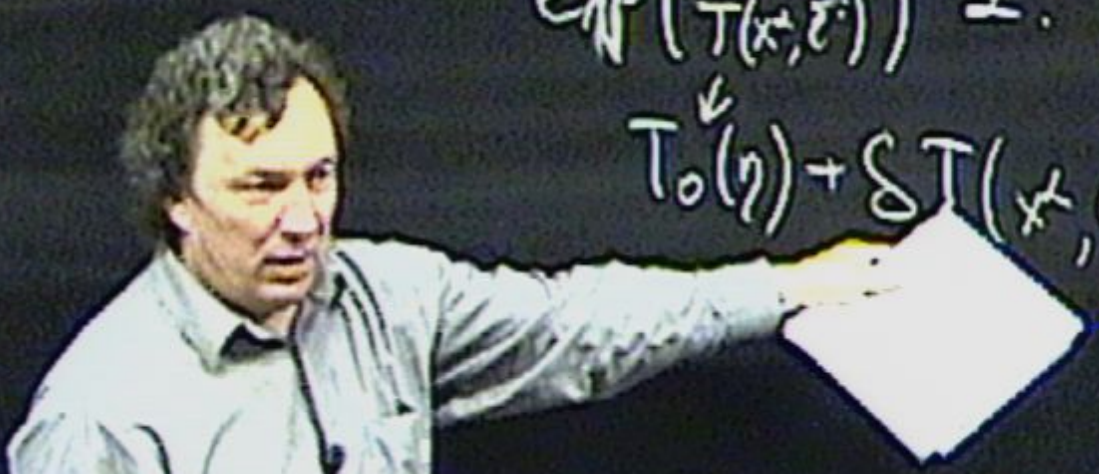
$$dW = \sum_i p_i dx_i + \sum_j p_j dp_j$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial t} + \frac{dx_i}{dt} \frac{\partial f}{\partial x_i} + \frac{dp_j}{dt} \frac{\partial f}{\partial p_j} = 0$$

$$\vec{e}^i \Leftrightarrow e^i = \frac{p_i}{|\vec{p}|} = \frac{p_i}{\sqrt{\sum_j p_j^2}}$$

$$f = F\left(\frac{w}{T}\right) = \frac{2}{\exp\left(\frac{w}{T(x^i, e^i)}\right) - 1}$$

$$T_0(\eta) + \delta T(x^i, e^i) \quad \frac{\delta T}{T}$$



$$dW = \sum_i p_i dx^i + u dp_i$$

$$\frac{Df}{dt} = \frac{\partial f}{\partial t} + \frac{dx^i}{dt} \frac{\partial f}{\partial x^i} + \frac{dp_i}{dt} \frac{\partial f}{\partial p_i} = 0$$

$$\vec{e}^i \Leftrightarrow e^i = \frac{p_i}{|p|} = \frac{p_i}{\sqrt{\sum_j p_j^2}}$$

$$f = F\left(\frac{w}{T}\right) = \frac{2}{\exp\left(\frac{w}{T(x^i, e^i)}\right) - 1}$$

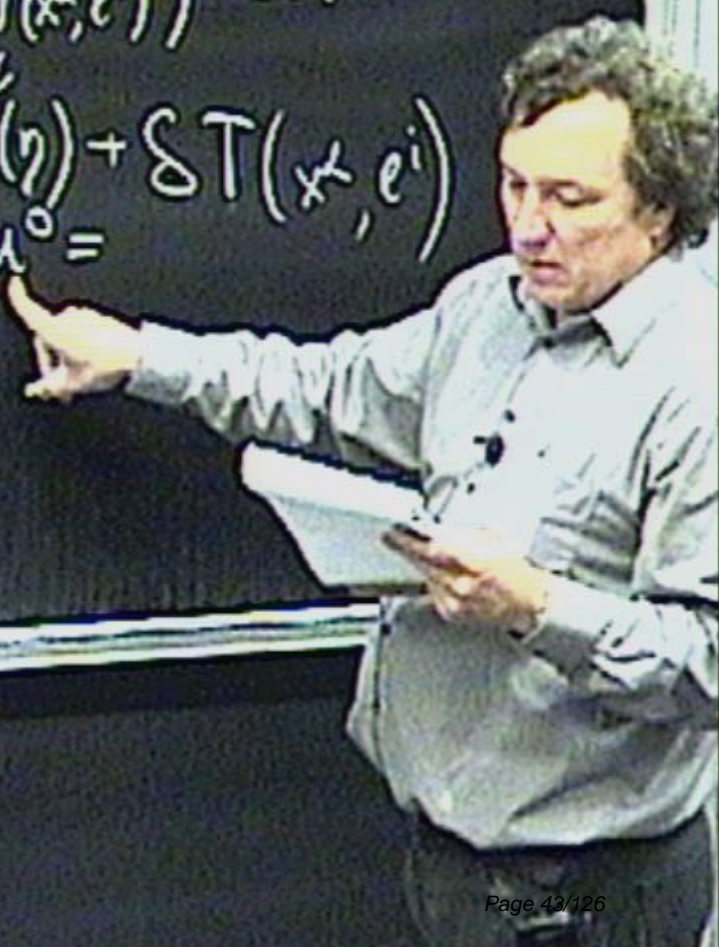
$$T_0(\eta) + \delta T(x^i, e^i) \quad \frac{\delta T}{T}$$

$$w = p_\mu x^\mu$$

$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^k, e^i)}\right) - 1}$$

$$\omega \equiv p_a u^a = p_b u^b =$$

$\downarrow$   
 $T_0(\eta) + \delta T(x^k, e^i)$



$$\left(\frac{1}{\beta}\right) = \left(\sum_i p_i^2\right)^{-1} \quad 107.1\%$$

$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^*, e^*)}\right) - 1}$$

$$\omega \equiv p_a u^a = p_b u^b =$$

$\downarrow$   
 $T_0(\eta) + \delta T(x^*, e^i) \quad \frac{\delta T}{T_0}$

$$u_0 u^0 = g_{00} u^{02} =$$

$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^{\pm}, e^i)}\right) - 1}$$

$$\omega \equiv p_x u^x = p_0 u^0 =$$

$$T_0(\eta) + \delta T(x^{\pm}, e^i) \quad \frac{\delta T}{T}$$

$$u_0 u^0 = g_{00} u^{02} \Rightarrow u^0 = \frac{1}{\sqrt{g_{00}}}$$

$$f = F\left(\frac{\omega}{T}\right) = \frac{2}{\exp\left(\frac{\omega}{T(x^t, e^i)}\right) - 1}$$

$$\omega \equiv p_a u^a = p_b u^b = \frac{p}{\rho} \quad \begin{matrix} \downarrow \\ T_0(\eta) + \delta T(x^t, e^i) \end{matrix} \quad \frac{\delta T}{T_0}$$

$$ds^2 = a^2 \left[ (1+2\Phi) d\eta^2 - (1-2\Phi) \delta_{ij} dx^i dx^j \right]$$

$$\frac{d\eta}{dt} = \frac{\partial \eta}{\partial t} + \frac{\partial x}{\partial \eta} \frac{\partial \eta}{\partial x^i} + \frac{\partial \eta}{\partial p_i} \frac{\partial \eta}{\partial p_i} = 0$$

$$\vec{e}^i \Leftrightarrow e^i = \frac{p_i}{|P|} = \frac{p_i}{\sqrt{\sum p_i^2}}$$

$$\omega \equiv p_\alpha u^\alpha = p_0 u^0 = \frac{p_0}{\sqrt{g_{00}}} = \frac{p_0}{\sqrt{1-2\phi}} \Rightarrow u^0 = \frac{1}{\sqrt{1-2\phi}}$$

$$ds^2 = a^2 \left[ (1+2\phi) d\eta^2 - (1-2\phi) \delta_{ij} dx^i dx^j \right]$$

$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi)$$

Pressure and plasma

for BRad waves  
in CDM



$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi), \quad \frac{dp_\alpha}{d\eta} = 2p_{\parallel} \frac{\partial \Phi}{\partial x^\alpha}$$

$\sqrt{\sum_{i=1}^3 p_i^2}$

$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi), \quad \frac{dp_a}{d\eta} = 2p_a \frac{\partial \Phi}{\partial x^a}$$

$$\sqrt{\sum_{i=1}^3 p_i^2} \quad p^a p_a = 0$$

$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi), \quad \frac{dp_\alpha}{d\eta} = 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha}$$

$p^\alpha p_\alpha = 0$

$$\frac{\partial t}{\partial \eta} + e^i (1 + 2\Phi) \frac{\partial t}{\partial x^i} + 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha}$$



$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi), \quad \frac{dp_\alpha}{d\eta} = 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha}$$

$\sqrt{\sum_{i=1}^3 p_i^2}$        $p^\alpha p_\alpha = 0$

$$\frac{\partial t}{\partial \eta} + e^i (1 + 2\Phi) \frac{\partial t}{\partial x^i} + 2p_i \frac{\partial \Phi}{\partial x^i} \frac{\partial t}{\partial p_i} = 0$$

$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi), \quad \frac{dp_\alpha}{d\eta} = 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha}$$

$\sqrt{\sum_{i=1}^3 p_i^2} \quad p^1 p_1 = 0$

$$\frac{\partial \pm}{\partial \eta} + e^i (1 + 2\Phi) \frac{\partial \pm}{\partial x^i} + 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha} \frac{\partial \pm}{\partial p_\alpha} = 0$$

$\tilde{f}\left(\frac{\omega}{\pm}\right)$  where  $\frac{\omega}{\pm} =$



$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi), \quad \frac{dp_i}{d\eta} = 2p_i \frac{\partial \Phi}{\partial x^i} \quad p^i p_i = 0$$

$$\sqrt{\sum_{i=1}^3 p_i^2}$$

$$\frac{\partial t}{\partial \eta} + e^i (1 + 2\Phi) \frac{\partial t}{\partial x^i} + 2p_i \frac{\partial \Phi}{\partial x^i} \frac{\partial t}{\partial p_i} = 0$$

$$\bar{J}\left(\frac{\omega}{T}\right) \quad \text{where} \quad \frac{\omega}{T} = \frac{p_0}{\sqrt{g_{ij} (T + \delta T)}} \approx$$



$$\frac{dx^i}{d\eta} = e^i (1+2\Phi), \quad \frac{dp_i}{d\eta} = 2p_i \frac{\partial \Phi}{\partial x^i} \quad p^i p_i = 0$$

$$\sqrt{\sum_{i=1}^3 p_i^2}$$

$$\frac{\partial t}{\partial \eta} + e^i (1+2\Phi) \frac{\partial t}{\partial x^i} + 2p_i \frac{\partial \Phi}{\partial x^i} \frac{\partial t}{\partial p_i} = 0$$

$$f\left(\frac{\omega}{T}\right) \quad \text{where} \quad \frac{\omega}{T} = \frac{p_0}{\sqrt{g_{00}} (T_0 + S T_0)}$$



$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi), \quad \frac{dp_\alpha}{d\eta} = 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha}$$

$\sqrt{\sum_{i=1}^3 p_i^2} \quad p^\alpha p_\alpha = 0$

$$\frac{\partial \pm}{\partial \eta} + e^i (1 + 2\Phi) \frac{\partial \pm}{\partial x^i} + 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha} \frac{\partial \pm}{\partial p_\alpha} = 0$$

$$f\left(\frac{\omega}{\pm}\right) \quad \text{where} \quad \frac{\omega}{\pm} = \frac{p_0}{\sqrt{g_{00}}(T_0 + \delta T)} \approx \frac{p}{T_0 a} \left(1 + \Phi - \frac{\delta T}{T_0}\right)$$



$$\frac{dx^i}{d\eta} = e^i (1 + 2\Phi), \quad \frac{dp_\alpha}{d\eta} = 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha}$$

$\sqrt{\sum_i p_i^2} \quad p^\alpha p_\alpha = 0$

$$\frac{\partial t}{\partial \eta} + e^i (1 + 2\Phi) \frac{\partial t}{\partial x^i} + 2p_\alpha \frac{\partial \Phi}{\partial x^\alpha} \frac{\partial t}{\partial p_\alpha} = 0$$

$$f\left(\frac{\omega}{T}\right) \quad \text{where} \quad \frac{\omega}{T} = \frac{p_0}{\sqrt{g_{00}}(T_0 + \delta T)} \approx \frac{p}{T_0 a} \left(1 + \Phi - \frac{\delta T}{T_0}\right)$$

$$\frac{dx^i}{d\eta} = e^i (1+2\Phi), \quad \frac{dp_\alpha}{d\eta} = 2p_\parallel \frac{\partial \Phi}{\partial x^\alpha} \quad p^\alpha p_\alpha = 0$$

$$\sqrt{\sum_i p_i^2}$$

$$\frac{\partial f}{\partial \eta} + e^i (1+2\Phi) \frac{\partial f}{\partial x^i} + 2p_\parallel \frac{\partial \Phi}{\partial x^i} \frac{\partial f}{\partial p_i} = 0$$

$$f\left(\frac{\omega}{T}\right) \quad \text{where} \quad \frac{\omega}{T} = \frac{p_0}{\sqrt{g_{00}}(T_0 + \delta T)} \approx \frac{p}{T_0} \left(1 + \Phi - \frac{\delta T}{T_0}\right)$$

$f\left(\frac{\omega}{T}\right)$  where  $\frac{\omega}{T} = \frac{P_0}{\sqrt{g_{00}(T_0 + \delta T)}} \approx \frac{P}{T_0 a} \left(1 + \phi - \frac{\delta T}{T_0}\right)$

0-order  $(T_0 a)' = 0$

$$T_0 \propto \frac{1}{a}$$

$n > 0$ - order  $(T_0 a)' = 0$

$$T_0 \propto \frac{1}{a}$$

$1$ - order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 2 \frac{\partial \Phi}{\partial \eta}$$

0-order  $(T_0 a)' = 0$   $T_0 \propto \frac{1}{a}$

1-order  $\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 2 \frac{\partial \Phi}{\partial \eta}$

0-order

$$(\bar{T}_0 a)' = 0$$

$$\bar{T}_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 2 \frac{\partial \Phi}{\partial \eta}$$

$\frac{d}{d\eta}$  along

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 2 \frac{\partial \Phi}{\partial \eta}$$

$$\frac{d}{d\eta} \Big|_{\text{along}}$$



0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 2 \frac{\partial \Phi}{\partial \eta}$$

$\frac{d}{d\eta}$  along

$\eta_{\text{ret}}$

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = \cancel{\frac{\partial}{\partial \eta}}$$

$\frac{d}{d\eta}$  along

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = \cancel{\frac{\partial \Phi}{\partial x^i}}$$

$$\frac{d}{d\eta} \Big|_{\text{along } \dots}$$

$\eta$

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 0$$

$$\frac{d}{d\eta} \Big|_{\text{along}} \left( \frac{\delta T}{T} + \Phi \right) = \text{const}$$

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 0$$

$\frac{d}{d\eta}$  along

$$\frac{\delta T}{T} + \Phi$$

$$x^i(\eta) = x^i_0$$

0. order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1. order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 0$$

$\frac{d}{d\eta}$  along  $\frac{\delta \Phi}{T} + \Phi = \text{const}$

$$x^i(\eta) = x_0^i + e^i \eta$$

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 0$$

$\frac{d}{d\eta}$  along  $\frac{\delta T}{T} + \Phi = \text{const}$

$$x^i(\eta) = x_0^i + e^i (\eta - \eta_0)$$

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 0$$

$\frac{d}{d\eta}$  along  $\dots$   $\frac{\delta T}{T} + \Phi = \text{const}$

$$x^i(\eta) = x_0^i + e^i (\eta - \eta_0)$$



0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 0$$

$\frac{d}{d\eta}$  along  $\dots$   $\frac{\delta T}{T} + \Phi = \text{const}$

$$x^i(\eta) = x_0^i + e^i (\eta - \eta_0)$$

$$\frac{\delta T}{T}(\eta_0, x_i^i, l^i)$$

$$\frac{\delta T}{T} = \frac{\delta a}{a}$$

$$\left( \frac{\delta T}{T} \right)_{\text{total}} = 10\% + 1\%$$

$$\frac{\delta T}{T}(\eta_0, x_i^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_2))$$

$\rho$   
 $\left( \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right)$

$$\frac{\delta T}{T}(\eta_0, x_i^i, l^i) = \frac{\delta T}{T}(\eta_{rac}, x^i(\eta_{rac}), l^i) + \Phi(\eta_{rac}, x^i(\eta_{rac}))$$



$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rac}, x^i(\eta_{rac}), l^i) + \Phi(\eta_{rac}, x^i(\eta_{rac})) - \Phi(\eta_0, x_0^i)$$

$\left( \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right)$  10% 10%

$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{\text{ref}}, x^i(\eta_{\text{ref}}), l^i) + \Phi(\eta_{\text{ref}}, x^i(\eta_{\text{ref}})) - \Phi(\eta_0, x_0^i)$$

$$x^i(\eta_{\text{ref}}) = x_0^i + l^i(\eta_{\text{ref}} - \eta_0)$$

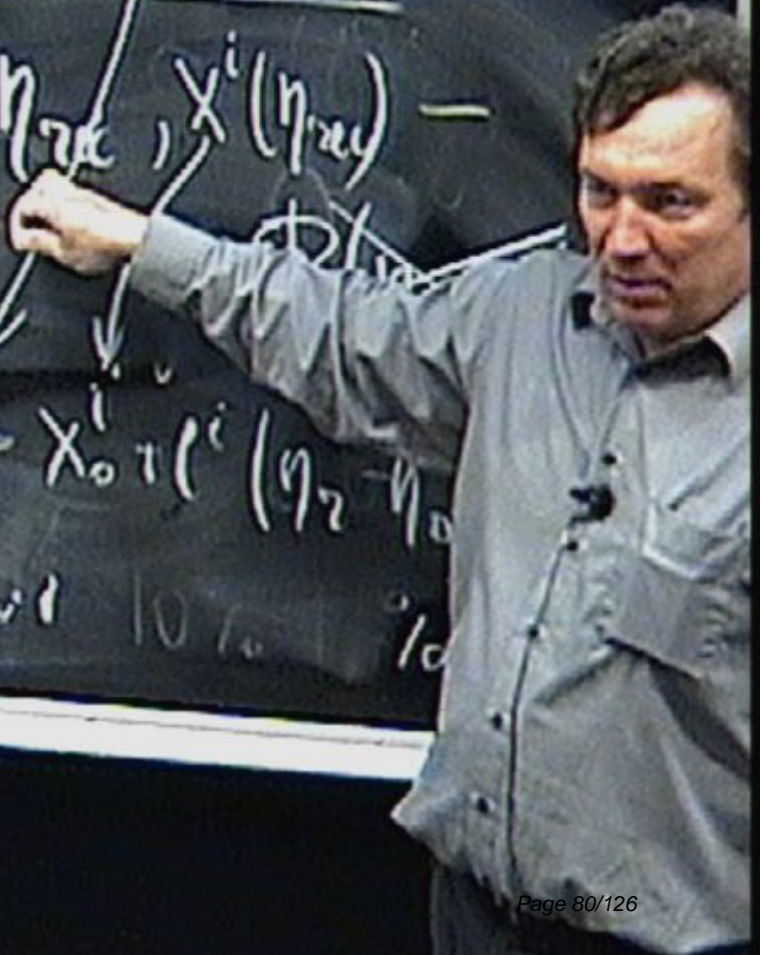
(11) and 10% 10%

$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_{rec}), l^i) + \Phi(\eta_{rec}, x^i(\eta_{rec}) - x_0^i)$$

$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_{rec} - \eta_0)$$

$$\frac{\delta T}{T}(\eta_0, x^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_{rec}), l^i) + \Phi(\eta_{rec}, x^i(\eta_{rec})) - \Phi(\eta_0, x^i(\eta_0))$$

$$x^i(\eta_{rec}) = x_0^i + l^i (\eta_{rec} - \eta_0)$$





$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_{rec}), l^i) +$$

$$+ \Phi(\eta_{rec}, x^i(\eta_{rec}) - \cancel{\Phi(\eta_0, x_0^i)})$$

$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_{rec} - \eta_0)$$

(111) 10%

$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_{rec}), l^i) +$$

$$+ \Phi(\eta_{rec}, x^i(\eta_{rec}) - \cancel{\Phi(\eta_0, x_0^i)})$$

$\delta, \Phi$

$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_{rec} - \eta_0)$$

$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_{rec}), l^i) + \Phi(\eta_{rec}, x^i(\eta_{rec})) - \Phi(\eta_0, x_0^i)$$

$\delta, \Phi$

$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_{rec} - \eta_0)$$

$$T_p^2 = \frac{1}{\sqrt{g}} \int f \frac{P' P_P}{P_0} d^3 P$$



The chalkboard contains several other faint, mostly illegible handwritten notes and equations. Some visible fragments include:
 

- $\pi$  (written multiple times)
- $\phi$  (written multiple times)
- $\psi$  (written multiple times)
- $\rho$  (written multiple times)
- $\rho_0$  (written multiple times)
- $\rho_P$  (written multiple times)
- $P'$  (written multiple times)
- $P_P$  (written multiple times)
- $P_0$  (written multiple times)
- $d^3 P$  (written multiple times)
- $\int$  (written multiple times)
- $\frac{1}{\sqrt{g}}$  (written multiple times)
- $\frac{P' P_P}{P_0}$  (written multiple times)
- $T_p^2$  (written multiple times)
- $\frac{1}{\rho_0}$  (written multiple times)
- $\frac{1}{\rho_P}$  (written multiple times)
- $\frac{1}{\rho}$  (written multiple times)
- $\frac{1}{\rho_0 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0 \rho}$  (written multiple times)
- $\frac{1}{\rho_P \rho}$  (written multiple times)
- $\frac{1}{\rho_0 \rho_P \rho}$  (written multiple times)
- $\frac{1}{\rho_0^2 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0 \rho_P^2}$  (written multiple times)
- $\frac{1}{\rho_0^2 \rho_P^2}$  (written multiple times)
- $\frac{1}{\rho_0^3 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0^2 \rho_P^3}$  (written multiple times)
- $\frac{1}{\rho_0^3 \rho_P^3}$  (written multiple times)
- $\frac{1}{\rho_0^4 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0^3 \rho_P^4}$  (written multiple times)
- $\frac{1}{\rho_0^4 \rho_P^4}$  (written multiple times)
- $\frac{1}{\rho_0^5 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0^4 \rho_P^5}$  (written multiple times)
- $\frac{1}{\rho_0^5 \rho_P^5}$  (written multiple times)
- $\frac{1}{\rho_0^6 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0^5 \rho_P^6}$  (written multiple times)
- $\frac{1}{\rho_0^6 \rho_P^6}$  (written multiple times)
- $\frac{1}{\rho_0^7 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0^6 \rho_P^7}$  (written multiple times)
- $\frac{1}{\rho_0^7 \rho_P^7}$  (written multiple times)
- $\frac{1}{\rho_0^8 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0^7 \rho_P^8}$  (written multiple times)
- $\frac{1}{\rho_0^8 \rho_P^8}$  (written multiple times)
- $\frac{1}{\rho_0^9 \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0^8 \rho_P^9}$  (written multiple times)
- $\frac{1}{\rho_0^9 \rho_P^9}$  (written multiple times)
- $\frac{1}{\rho_0^{10} \rho_P}$  (written multiple times)
- $\frac{1}{\rho_0^9 \rho_P^{10}}$  (written multiple times)
- $\frac{1}{\rho_0^{10} \rho_P^{10}}$  (written multiple times)

$$T_p^\alpha = \frac{1}{\sqrt{g}} \int + \frac{P' P_P}{P_0} d^3 P$$

$$t \equiv t \left( \frac{w}{T} \right) = \frac{c}{\exp \left( \frac{w}{T(x^{\mu}, e^i)} \right) - 1}$$

$$w \equiv p_{\alpha} u^{\alpha} = p_0 u^0 = \frac{p}{\rho} \left( T_0(\eta) + \delta T(x^{\mu}, e^i) \right) \frac{\delta T}{T}$$

$$ds^2 = a^2 \left[ (1+2\Phi) d\eta^2 - (1-2\Phi) \delta_{ij} dx^i dx^j \right]$$

$$T^2 = \frac{1}{\sqrt{g}} \int f \frac{P^i P_i}{\rho_0} d^3 p$$

$$T^0_0 = \int \left( 1 + 4 \frac{S^i S_i}{T^0_0} \right) T \left( \frac{3T}{4} \right) y^3 dy d^2 \vec{e}$$

$$T_p^2 = \frac{1}{\sqrt{g}} \int \frac{P' P_F}{\rho_0} d^3 p$$

$$T_0^0 = T_0^4 \int \left( 1 + 4 \frac{\delta T}{T_0} \right) F \left( \frac{y}{T_0} \right) y^3 dy d^2 \vec{e}$$

$$\Rightarrow T_0^0 = \sum_{\delta} (1 + \delta_{\delta})$$



$$p = \frac{1}{\sqrt{g}} \left( \frac{1}{\rho_0} \right) d^3 p$$

$$T_0 = T_0 \int \left( 1 + 4 \frac{\delta T}{T_0} \right) F \left( \frac{w}{T_0} \right) y^3 dy d^2 \vec{e}$$

$$\left( \frac{\delta T}{T_0} \right)_k$$

$$T_0 = \sum_0 (1 + \delta_8)$$

$$p = \frac{1}{\sqrt{g}} \left( \frac{1}{\rho_0} \frac{d\rho}{dz} \right)$$

$$T_0' = T_0 \int \left( 1 + 4 \frac{\delta T}{T_0} \right) T \left( \frac{w}{T} \right) y^3 dy d^2 \vec{e}$$

$$\left( \frac{\delta T}{T} \right)_k (\vec{r}, \eta_{cc}) = \frac{1}{4} \left( \delta_{\nu}^{nd} + \frac{3i}{k^2} (k_m^S c^M) \right) \delta \Delta_{\nu}^{\mu}$$

$$p = \frac{1}{\sqrt{g}} \left( \frac{1}{\rho_0} \alpha - p \right)$$

$$T_0 \rightarrow T_0' = \int \left( 1 + 4 \frac{\delta T}{T_0} \right) T \left( \frac{W}{T} \right) y^3 dy d^2 \hat{e}$$

$$\left. \frac{\delta T}{T} \right|_k (\vec{l}, \eta_{cc}) = \frac{1}{4} \left( \delta_{\vec{k}}^{nd} + \frac{3i}{k^2} (k_{\vec{m}}^S)^M \delta_{\vec{k}}^{\wedge} \right)$$

$$T_0' = \sum_0 (1 + \delta_{\gamma})$$

$$P = \frac{1}{\sqrt{g}} \left( \frac{1}{\rho_0} \frac{\partial p}{\partial t} \right)$$

$$T_0 \rightarrow T_0 \int \left( 1 + 4 \frac{\delta T}{T_0} \right) T \left( \frac{3T}{T_0} \right) y^3 dy d^2 \vec{e}$$

$$\left( \frac{\delta T}{T_0} \right)_k (\vec{r}, \eta_{\text{rec}}) = \frac{1}{4} \left( \delta_{\text{red}}^{\text{nd}} + \frac{3i}{k^2} (k_{\text{m}}^{\text{M}}) \delta_{\text{v}}^{\text{nd}} \right)$$

$$\frac{\delta T}{T}(\eta_0, x_0^i, l^i) = \frac{\delta T}{T}(\eta_{rec}, x^i(\eta_{rec}), l^i) +$$

$$+ \Phi(\eta_{rec}, x^i(\eta_{rec})) - \Phi(\eta_0, x_0^i)$$

$$x^i(\eta_{rec}) = x_0^i + l^i(\eta_{rec} - \eta_0)$$

$$\delta, \Phi \quad \frac{\delta T}{T}(\eta_{rec}, l^i) \quad \eta_0 \quad \eta_{rec}$$

$$ds^2 = a^2 \left[ (1+2\Phi) d\eta^2 - (1-2\Phi) \delta_{ij} dx^i dx^j \right]$$

$u^0 = \frac{\partial}{\partial \eta} = g_{00}^{1/2} \frac{\partial}{\partial \eta}$

$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{e})$$

et (1)

$$u) \equiv p, u = \gamma, p =$$

$$T(1+2) \gamma^2 (1-2) \delta$$



$$p = \frac{1}{\sqrt{g}} \left( \frac{1}{\rho_0} \alpha \rho \right)$$

$$T_0 \rightarrow T_0 \int \left( 1 + 4 \frac{\delta T}{T_0} \right) F \left( \frac{W}{T} \right) y^3 dy d^2 \vec{e}$$

$$\left( \frac{\delta T}{T} \right)$$

$$e^{ikx} T_0 = \sum_0 (1 + \delta_T)$$

$$= \frac{1}{4} \left( \delta_{\gamma}^{\text{red}} + \frac{3i}{2} (k_m^{\delta} c^m) = \delta_{\gamma}^{\delta} \right)$$

$$P = \frac{1}{\sqrt{g}} \int \frac{1}{\rho_0} \rho$$

$$T_0 \rightarrow T_0 \int \left(1 + 4 \frac{\delta T}{T_0}\right) F\left(\frac{\omega}{T_0}\right) y^3 dy d^2 \vec{e}$$

$$e^{i\vec{k}\cdot\vec{x}} T_0 = \sum_{\delta} \left(1 + \delta_{\delta}\right) \frac{1}{k} \left( \delta_{\vec{k}}^{\text{nd}} + \frac{3i}{k^2} (k_{\text{M}}^{\text{S}} e^{\text{M}}) = \delta_{\vec{k}}^{\text{M}} \right)$$





$$p = \frac{1}{\sqrt{g}} \left( \frac{1}{\rho_0} \rho - p \right)$$

$$T_0 \rightarrow T_0 \int \left( 1 + 4 \frac{\delta T}{T_0} \right) T \left( \frac{W}{T} \right) y^3 dy d^2 \vec{e}$$

$$e^{i k x} \rightarrow T_0 = \Sigma_0 (1 + \delta_\gamma)$$

$$(\vec{l}, \eta_{rec}) = \frac{1}{4} \left( \Sigma_{\vec{k}}^{red} + \frac{3i}{k^2} (k_m^S e^m) = \delta_{\vec{k}} \right)$$

$$T_0 \rightarrow T_0 \int \left(1 + 4 \frac{\delta T}{T_0}\right) F\left(\frac{3H}{4\pi}\right) y^3 dy d^2\vec{e}$$

$$\left(\frac{\delta T}{T}\right)_k(\vec{l}, \eta_{rec}) = \frac{1}{4} \left( \delta_{red} \left( k_m^S e_m \right) \delta_{\vec{k}}$$

$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{t}) = \int$$



$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{e}) = \int \left( \Phi + \frac{\delta}{4} \right)$$

~~Handwritten notes and equations, including  $\omega = p \cdot u$  and  $s = \int (1 + 2\eta^2)$ , are heavily obscured by diagonal black scribbles.~~

$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{e}) = \int \left[ \left( \Phi + \frac{\delta}{4} \right) - \frac{3S'_c}{k^2} \frac{\partial}{\partial \eta_0} \right]_{\eta_{\text{rec}}} \times$$

$$\times \exp \left[ i\vec{k} \left( \vec{x}_0 + \vec{e}(\eta - \eta_0) \right) \right]$$

$$\frac{d^3 k}{(2\pi)^{3/2}}$$

$$\omega = \frac{c}{a} k$$

$$^2$$

$$= 2 \int (1 + 2\eta^2)$$

$$1$$

$$2\eta^2$$

$$1$$

$$2\eta^2$$

$$1$$

$$2\eta^2$$

$$1$$

$$2\eta^2$$

$$1$$

$$2\eta^2$$

$$1$$

$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{\ell}) = \int \left[ \left( \Phi + \frac{\delta}{4} \right) - \frac{3\delta'_c}{k^2} \frac{\partial}{\partial \eta_0} \right]_{\eta_{\text{rec}}}^*$$

$$\langle \left( \frac{\delta T}{T}(\theta) \right)^2 \rangle = \langle T \rangle \exp \left[ i\vec{k} \cdot (\vec{x}_0 + \vec{\ell}(\eta_1 - \eta_0)) \right]$$

$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{e}) = \int \left[ \left( \Phi + \frac{\delta}{4} \right) - \frac{3\delta'_c}{k^2} \frac{\partial}{\partial \eta_0} \right]_{\eta_{\text{sc}}}^*$$

$$\left\langle \left( \frac{\delta T}{T}(\theta) \right)^2 \right\rangle = \left\langle \frac{\left( T(\vec{x}_1) - T(\vec{x}_2) \right)^2}{T_0} \right\rangle$$

$$\left\langle \left( \frac{\Delta T}{T} \right)^2 \right\rangle = \left\langle \frac{(T(\bar{t}_1) - T(\bar{t}_2))^2}{T_0} \right\rangle =$$





$$= 2(C(\theta) - C(\theta)) \text{ where}$$

$$C(\theta) = \left\langle \frac{\delta T}{T_0}(\vec{r}_1), \frac{\delta T}{T_0}(\vec{r}_2) \right\rangle$$

$$\frac{\delta T}{T}(\eta_0, \vec{x}_0, \vec{t}) = \int \left[ \left( \Phi + \frac{\delta}{4} \right) - \frac{3\delta'_c}{k^2} \frac{\partial}{\partial \eta_0} \right]_{\eta_{oc}}$$

$$\langle \left( \frac{T(\vec{x}_1) - T(\vec{x}_2)}{T_0} \right)^2 \rangle =$$

$$= 2(c(t) - c(0)) \text{ where}$$

Case: Grand Canonical

$$C(\theta) = \left\langle \frac{\delta T}{T_0}(\vec{r}_1), \frac{\delta T}{T_0}(\vec{r}_2) \right\rangle$$

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = \frac{\partial \Phi}{\partial \eta}$$



$$\frac{d}{d\eta}$$

$$\frac{\delta \Phi}{T} + \Phi = \text{const}$$

$$x^i(\eta) = x_0^i + e^i (\eta - \eta_0)$$

0-order

$$(T_0 a)' = 0 \quad T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 0$$



$$\frac{d}{d\eta}$$

$$\frac{\delta \Phi}{T} + \Phi = \text{const}$$

$$x^i(\eta) = x_0^i + e^i (\eta - \eta_0)$$

0-order

$$(T_0 a)' = 0$$

$$T_0 \propto \frac{1}{a}$$

1-order

$$\left( \frac{\partial}{\partial \eta} + e^i \frac{\partial}{\partial x^i} \right) \left( \frac{\delta T}{T} + \Phi \right) = 0$$



$$\frac{d}{d\eta}$$

$$\frac{\delta \Phi}{T} + \Phi = \text{const}$$

$$X^i(\eta) = X_0^i + e^i (\eta - \eta_0)$$

$$\frac{\delta I}{T}(\eta_0, \vec{x}_0, \vec{\ell}) = \int \left[ \left( \Phi + \frac{\delta}{4} \right) - \frac{3\delta'}{k^2} \frac{\partial}{\partial \eta_0} \right]_{\eta_{\text{oc}}} \times e^{i \vec{r} \cdot \vec{\ell}}$$

$$\left\langle \left( \frac{\delta T}{T}(\theta) \right)^2 \right\rangle = \left\langle \left( \frac{T(\vec{\ell}_1) - T(\vec{\ell}_2)}{T_0} \right)^2 \right\rangle = 2(c(b) - c(\theta)) \text{ where}$$



10.1.1)  $\rho_{SS}$  Rand (best)

$$C(t) = \left\langle \frac{\delta T}{T_0}(\vec{r}_1) \frac{\delta T}{T_0}(\vec{r}_2) \right\rangle$$

$$\langle \Phi_k \Phi_{k'} \rangle = \Phi_k^2 \delta(k+k')$$



$$C(\theta) = \int (\Phi + \frac{\sigma}{\sqrt{5}})$$



$$C(\theta) = \int \left( \phi + \frac{\delta}{4} - \frac{3\delta'}{4lc^2} \frac{\partial}{\partial \eta_1} \right)$$

$$C(\theta) = \int \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_2} \right) \sin(k|\bar{\ell}_1 \eta_1 - \bar{\ell}_2 \eta_2|)$$

$$C(\theta) = \int \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_2} \right) \frac{\sin(k|\bar{\epsilon}_1 \eta_1 - \bar{\epsilon}_2 \eta_2|)}{k|\bar{\epsilon}_1 \eta_1 - \bar{\epsilon}_2 \eta_2|} \frac{k^2 dk}{2\pi^2}$$

$$C(\theta) = \int \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_2} \right) \frac{\sin(k|\bar{\epsilon}_1 \eta_1 - \bar{\epsilon}_2 \eta_2|)}{k|\bar{\epsilon}_1 \eta_1 - \bar{\epsilon}_2 \eta_2|} \frac{k^2 dk}{2\pi^2}$$



$$C(\theta) = \int \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_2} \right)$$

$$\cdot \frac{\sin(k|\bar{e}_1 \eta_1 - \bar{e}_2 \eta_2|)}{k|\bar{e}_1 \eta_1 - \bar{e}_2 \eta_2|} \frac{k^2 dk}{2\pi^2}$$

$$\eta_1 = \eta_2 = \eta_0$$

$$C(\theta) = \int \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_2} \right) \frac{\sin(k|\bar{e}_1 \eta_1 - \bar{e}_2 \eta_2|)}{k|\bar{e}_1 \eta_1 - \bar{e}_2 \eta_2|} \frac{k^2 dk}{2\pi^2}$$

$$\eta_1 = \eta_2 = \eta_0$$

$$C(\theta) = \int \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_2} \right)$$

$$\cdot \frac{\sin(k|\bar{e}_1 \eta_1 - \bar{e}_2 \eta_2|)}{k|\bar{e}_1 \eta_1 - \bar{e}_2 \eta_2|} \frac{k^2 dk}{2\pi^2}$$

$$\eta_1 = \eta_2 = \eta_0$$



$$C(\theta) = \int \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_2} \right) \frac{\sin(k|\bar{\ell}_1 \eta_1 - \bar{\ell}_2 \eta_2|)}{k|\bar{\ell}_1 \eta_1 - \bar{\ell}_2 \eta_2|} \frac{k^2 dk}{2\pi^2}$$

$$+ \sum_{\ell=0}^{\infty} (2\ell+1) j_{\ell}(k\eta_1) j_{\ell}(k\eta_2) P_{\ell}(\cos\theta)$$

$\eta_1 = \eta_2 = \eta_0$

$$C(\theta) = \int \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_1} \right) \left( \Phi + \frac{\delta}{4} - \frac{3\delta'}{4k^2} \frac{\partial}{\partial \eta_2} \right)$$

$$\cdot \frac{\sin(k|\bar{e}_1 \eta_1 - \bar{e}_2 \eta_2|)}{k|\bar{e}_1 \eta_1 - \bar{e}_2 \eta_2|} \frac{k^2 dk}{2\pi^2}$$

$$\sum_{\ell=0}^{\infty} (2\ell+1) j_{\ell}(k\eta_1) j_{\ell}(k\eta_2) P_{\ell}(\cos \theta)$$

$\eta_1 = \eta_2 = \eta_0$

$$\stackrel{\frac{1}{4\pi}}{\equiv} \sum_{\ell=0}^{\infty} (2\ell+1) C_{\ell} P_{\ell}(\theta)$$

Suss. Rand. f. last

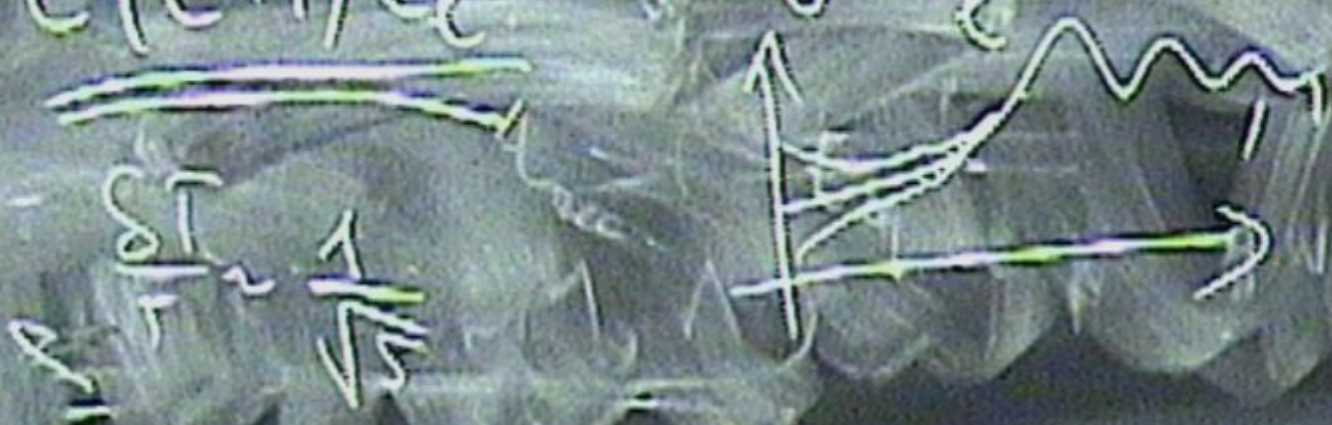
$$C_e = \frac{2}{\pi} \int \left| \left( \Phi_k + \frac{S_k}{4} \right) j_e(k\eta_0) - \frac{3S'_k(\eta_0)}{4k} \frac{d j_e(k\eta_0)}{d(k\eta_0)} \right|^2 dk$$

$$C_e' = \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\pi} \left| \left( \Phi_k + \frac{S_k}{4} \right) j_e(k\eta_0) - \frac{3S_k'(k_0)}{4k} \frac{d j_e(k\eta_0)}{d(k\eta_0)} \right|^2 dk$$

$$e(e+1)C_e \approx \theta \sim \frac{\pi}{e}$$

$$C_e = \frac{2}{\pi} \int_{\Phi_k} \left| \left( \Phi_k + \frac{S_k}{4} \right) j_e(k\eta_0) - \frac{3S_k'(k_0)}{4k} \frac{d j_e(k\eta_0)}{d(k\eta_0)} \right|^{1/2} dk$$

$$e(e+1)C_e \sim \theta \sim \frac{\pi}{2}$$



1) Mass and flux

$$C_e = \frac{2}{\pi} \int_{\Phi_k} \left| \left( \Phi_k + \frac{S_k}{4} \right) j_e(k, \eta_0) - \frac{3S_k'( \eta_0 )}{4k} \frac{d j_e(k, \eta_0)}{d(k \eta_0)} \right|^2$$

$$e(e+1)C_e \approx \theta \sim \frac{\pi}{2}$$

