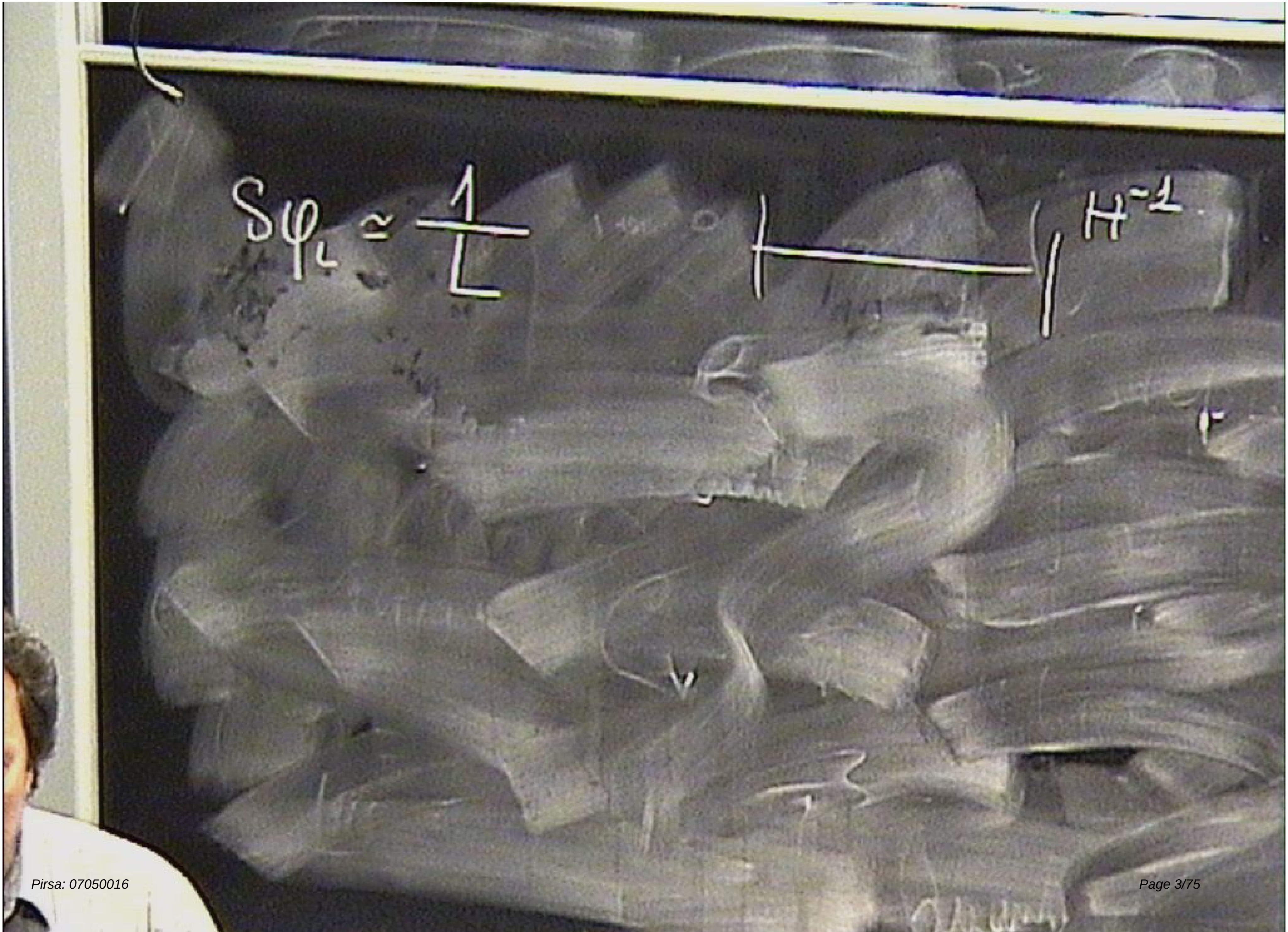
Title: Advanced Topics in Cosmology

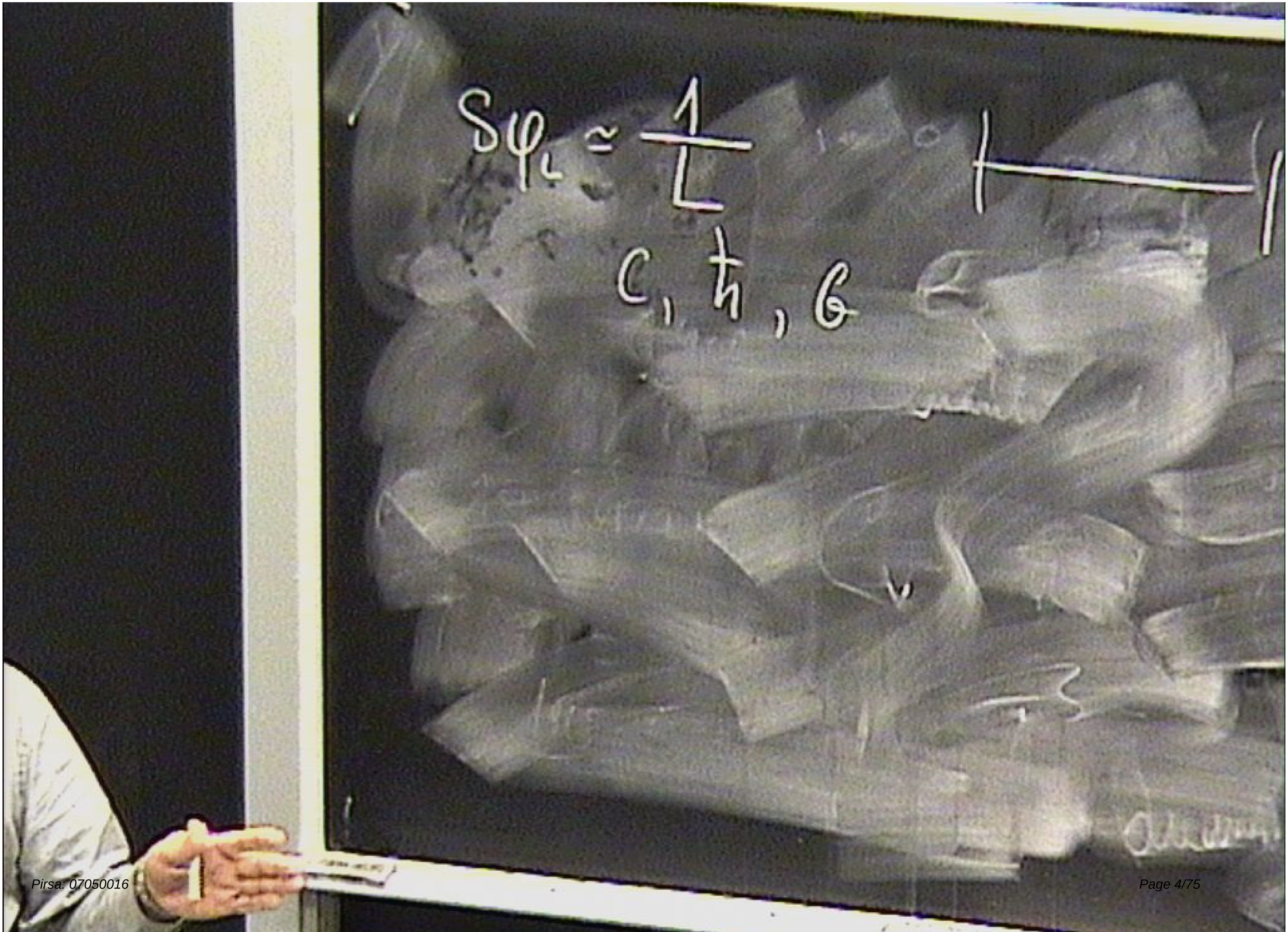
Date: May 17, 2007 10:00 AM

URL: http://pirsa.org/07050016

Abstract: Class 5 part 2

$$\delta\varphi_L \simeq \frac{1}{L}$$


$\delta\varphi_L \simeq \frac{1}{L}$
H^{-1}


$\delta\varphi_i \simeq \frac{1}{L}$
$c, \hbar, G$

$$\frac{\delta \varphi_L}{\varphi_{pl}} \simeq \frac{1}{\left(L/\ell_{pl}\right)}$$

$c, \hbar, G$

$$\delta\varphi_L \simeq \frac{1}{\left(L/\ell_{pl}\right)}$$

H^{-1}

$c,\ \hbar,\ G$

$$^{(T)}h^{i}_{i} = 0 \qquad ^{(T)}h^{i}_{k,i} = 0$$

$c, \hbar, G$

$$(T) \quad h^{i}_{i} = 0 \qquad (T) \quad h^{i}_{k,i} = 0$$

$$h''_{ik} + 2\frac{a'}{a} h'_{ik} - \Delta h_{ik} = \varepsilon$$

$$^{(T)}h^{i}_{i} = 0 \qquad ^{(T)}h^{i}_{k,i} = 0$$

$$h''_{ik} + 2\frac{a'}{a}h'_{ik} - \Delta h_{ik} = 0$$

$$h_{ik} = \frac{\mu}{a}e_{ik}$$

$$^{(T)}h_i^i = 0 \qquad ^{(T)}h_{k,i}^i = 0$$

$$h''_{ik} + 2\frac{a'}{a}h'_{ik} - \Delta h_{ik} = 0$$

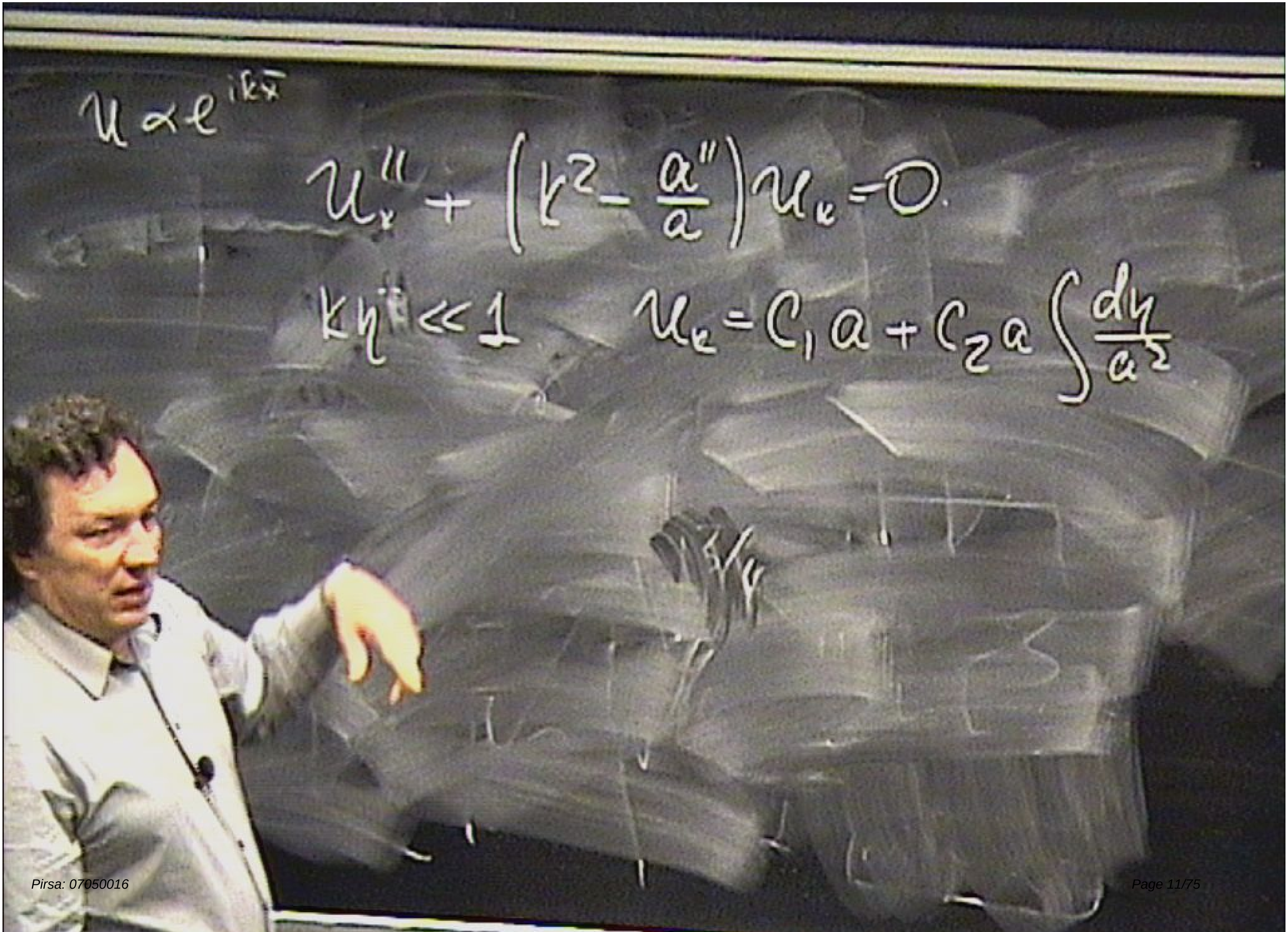
$$h_{ik} = \frac{u}{a} e_{ik}. \qquad k^i e_{ik} = 0$$

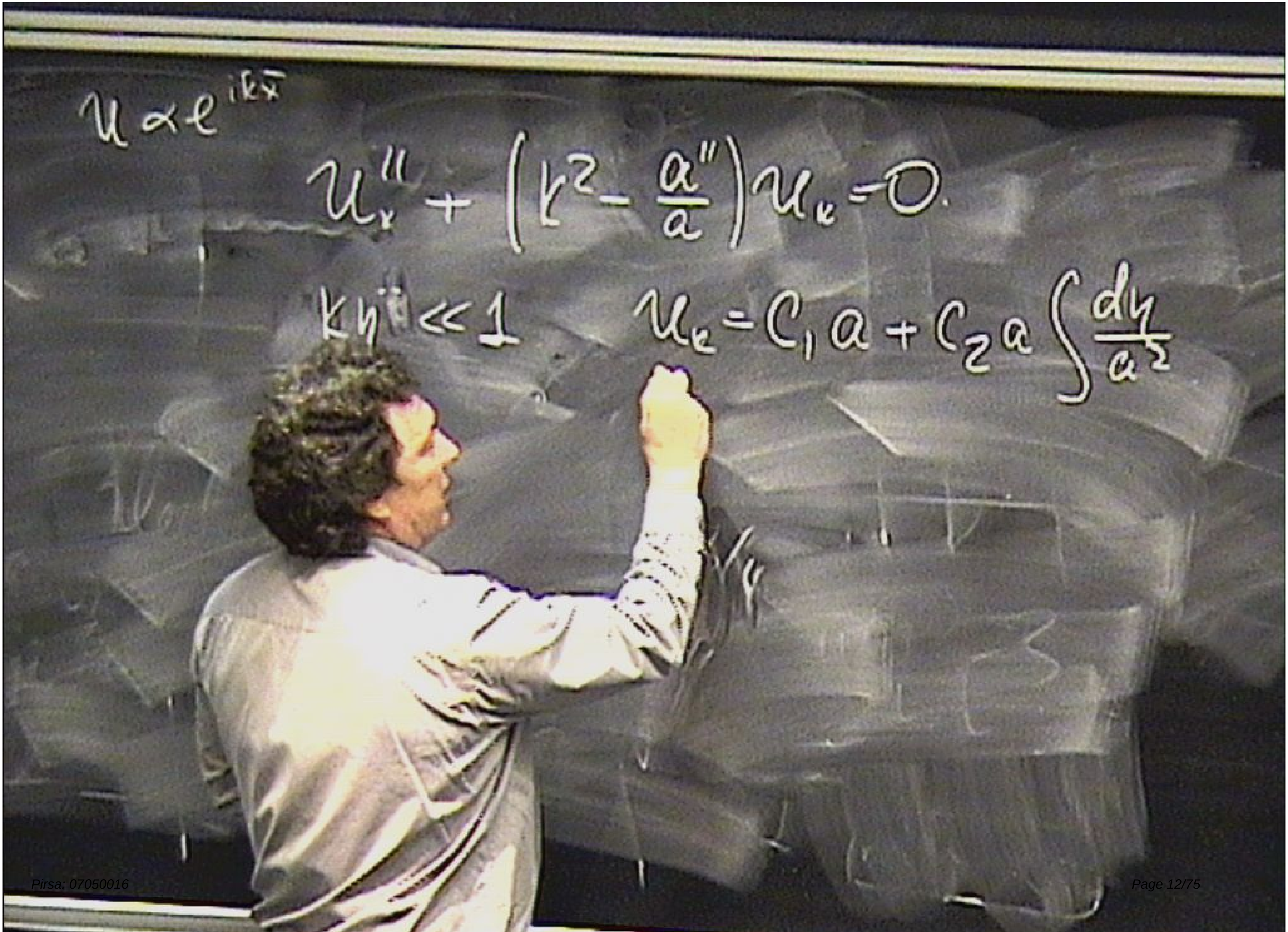
$$e_i^i = 0$$

$u \propto e^{ik\bar{x}}$

$$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0.$$

$3/4$


$u \propto e^{ik\bar{x}}$
$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0$
$k\eta \ll 1$
$u_k = C_1 a + C_2 a \int \frac{d\eta}{a^2}$


$u \propto e^{ik\bar{x}}$
$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0.$
$k\eta \ll 1$
$u_k = C_1 a + C_2 a \int \frac{d\eta}{a^2}$

$$u \propto e^{i\vec{k}\vec{x}}$$

$$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0.$$

$k\eta \ll 1$

$$u_k = C_1 a + C_2 a \int \frac{d\eta}{a^2}$$

$$h_k \propto C_1 + C_2 \int \frac{d\eta}{a^2}$$

$$a \propto e^{Ht} \propto \frac{1}{H\eta}$$

$$-\infty < \eta < 0$$

$$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0$$

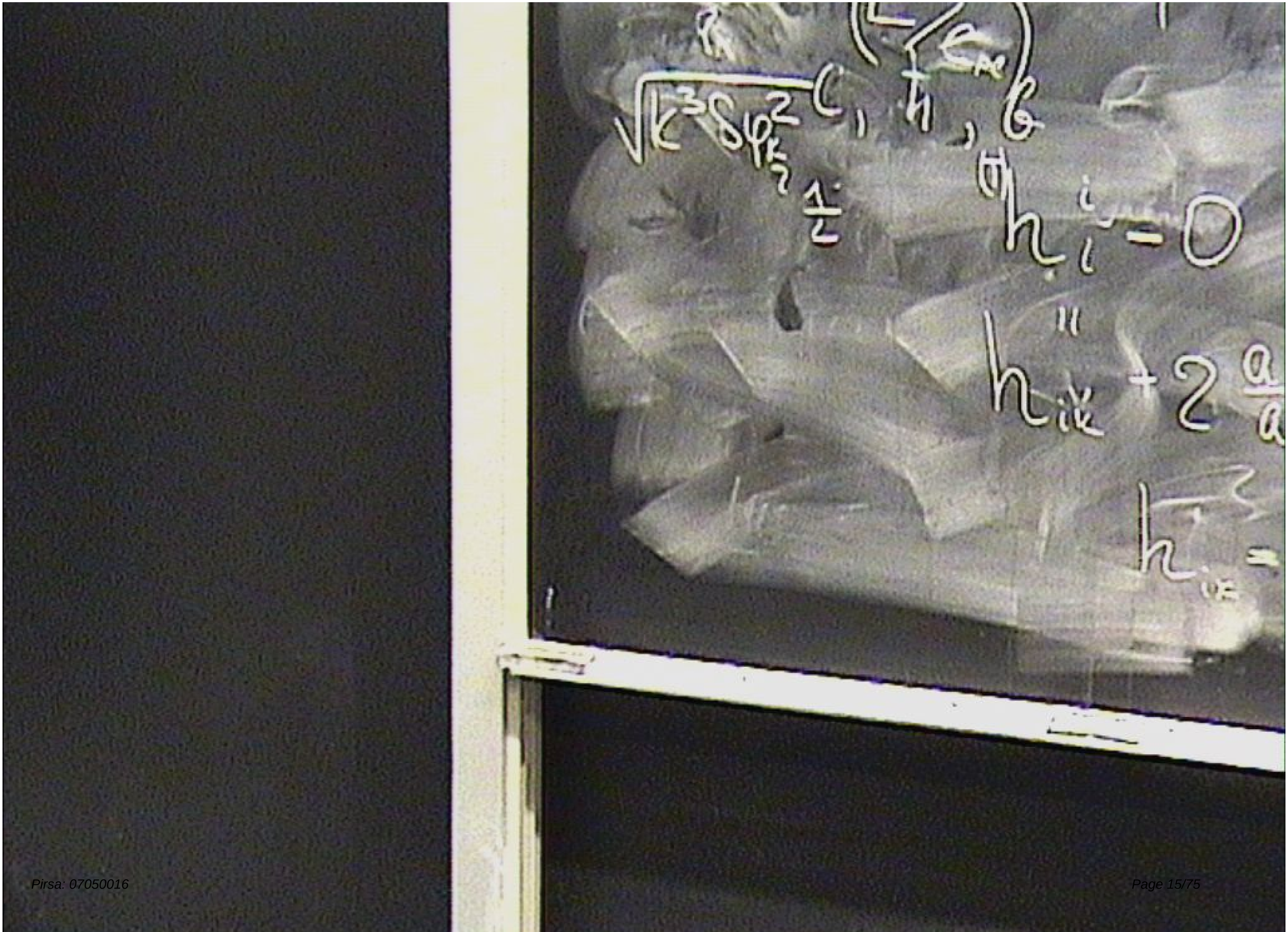
$k\eta \ll 1$ $\quad u_k = C_1 a + C_2 a \int \frac{d\eta}{a^2}$

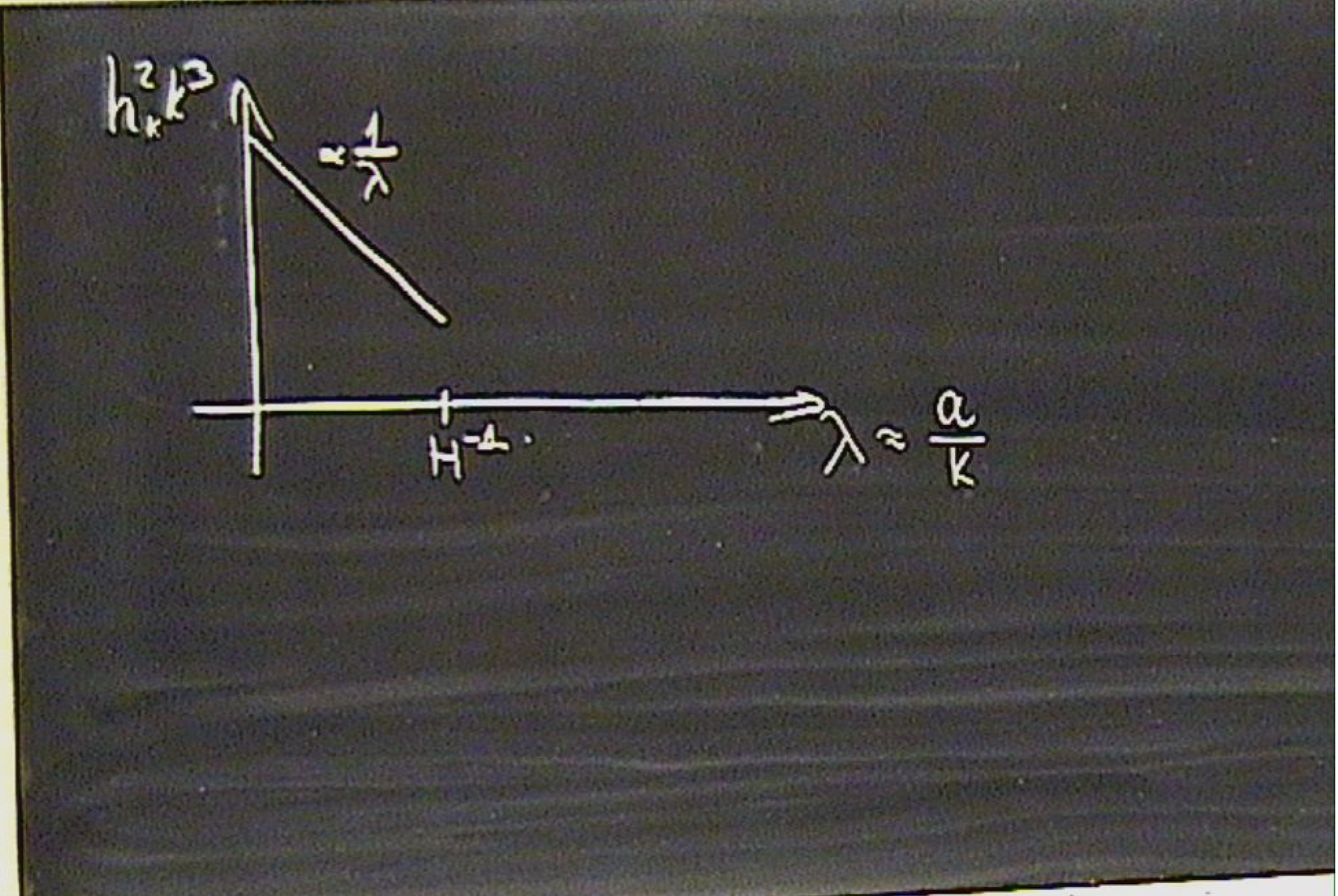
$$h_k \propto C_1 + C_2 \int \frac{d\eta}{a^2}$$

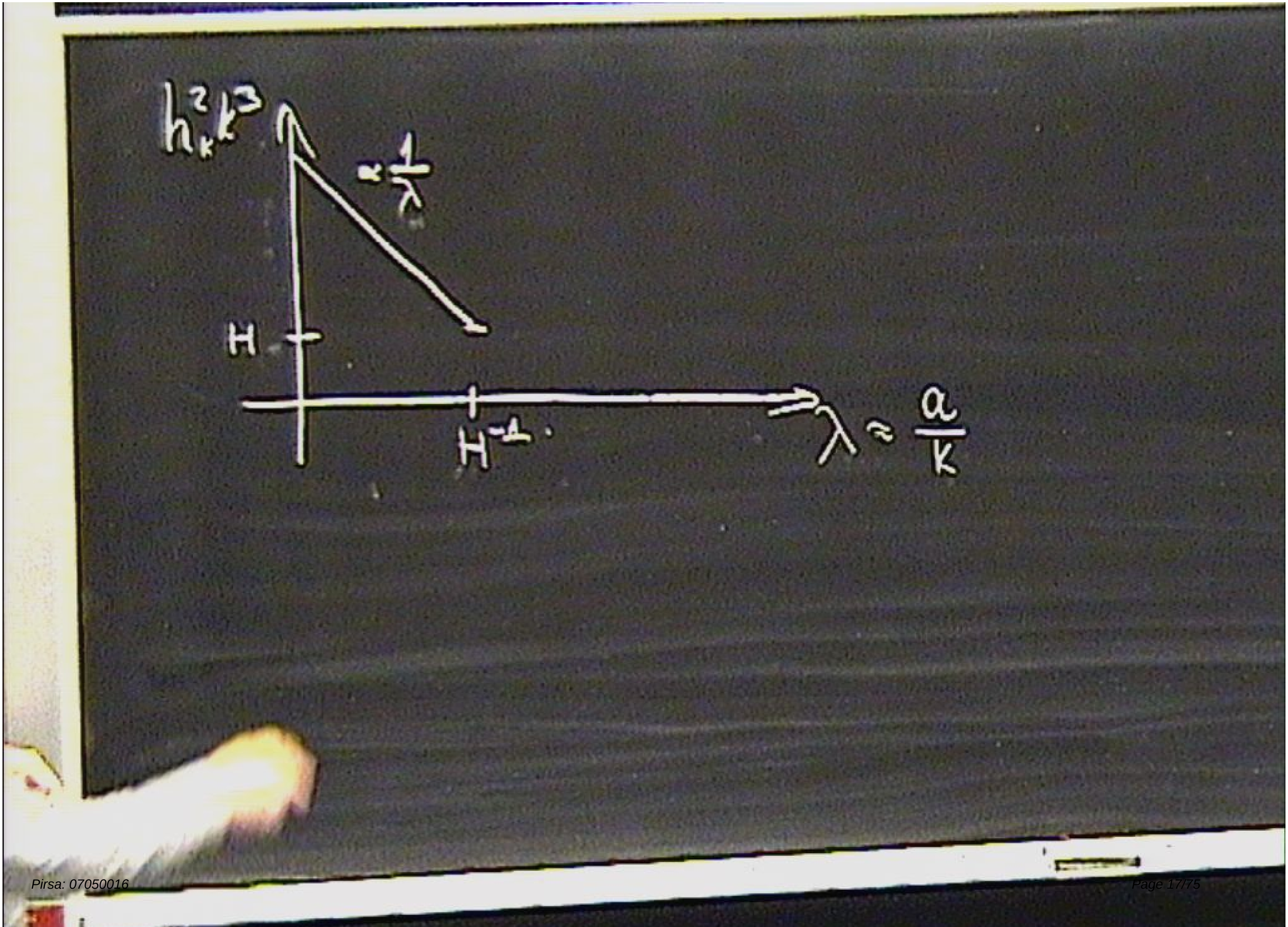
$$a \propto e^{Ht} \quad \frac{1}{H}$$

$k\eta \gg 1.$ $\quad u \propto e^{ik\eta}$ $\quad -\infty < \eta < 0$

$$h \propto \frac{1}{a} e^{ik\eta}$$


$\sqrt{k^3 \delta\varphi_k^2}$
$\zeta,\ h,\ \sigma$
$h_i^i = 0$
$h_{ik}'' + 2\frac{a'}{a}$


$h_k^2 k^3$
$\propto \frac{1}{\lambda}$
H^{-1}
$\lambda \approx \frac{a}{k}$



$h_k^2 k^3$
$\propto \frac{1}{\lambda}$
H
H^{-1}
$\lambda \approx \frac{a}{k}$

H^{-2}

$$\delta\varphi_k \sim \frac{1}{\sqrt{k}}\, e^{\cdots}\, a^{\pm}$$

$$= 0 \qquad h^{(T)\,i}_{k\,i} = 0$$

$$+ 2\frac{a'}{a} h' \ldots \Delta h_{ik} = 0$$

$$\frac{u}{a} e_{ik} \qquad k^i e_{ik} = 0$$

$$e^i_i = 0$$

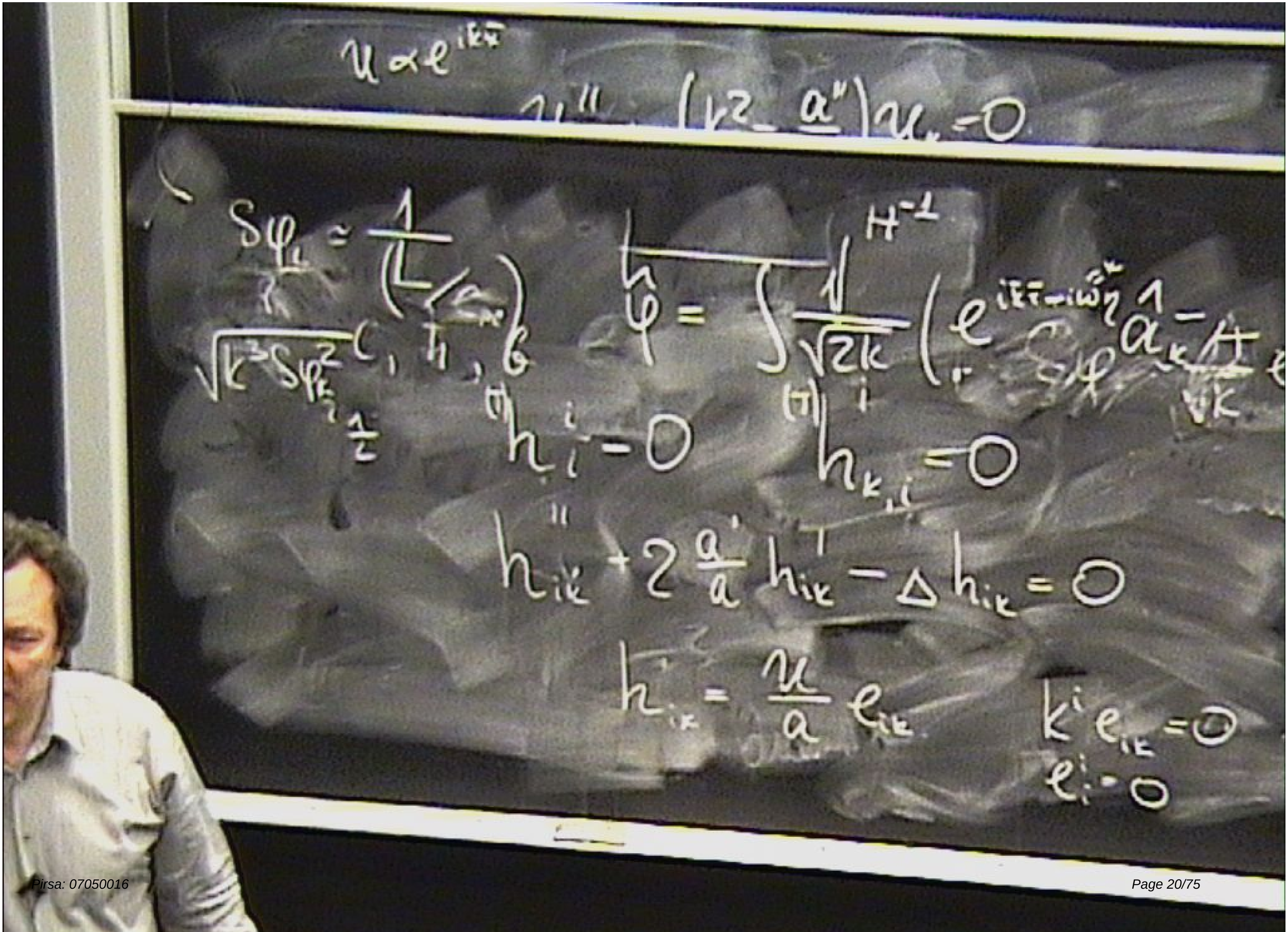
$$\delta\varphi_k \sim \frac{1}{\sqrt{k}} e^{\cdots}$$

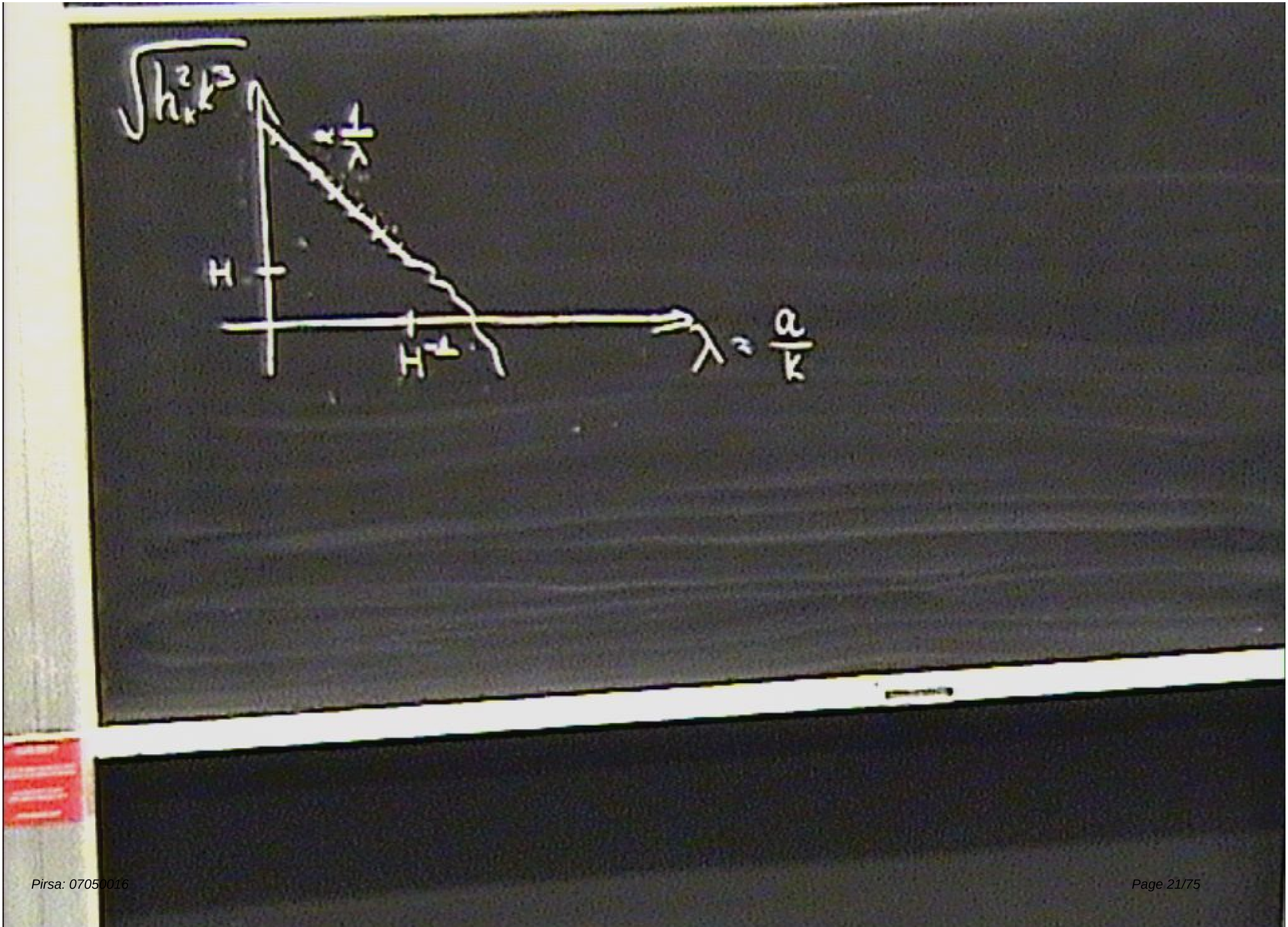
$$= 0 \qquad h_{k,i} = 0$$

$$+ 2\frac{a'}{a} h_{ik}' - \Delta h_{ik} = 0$$

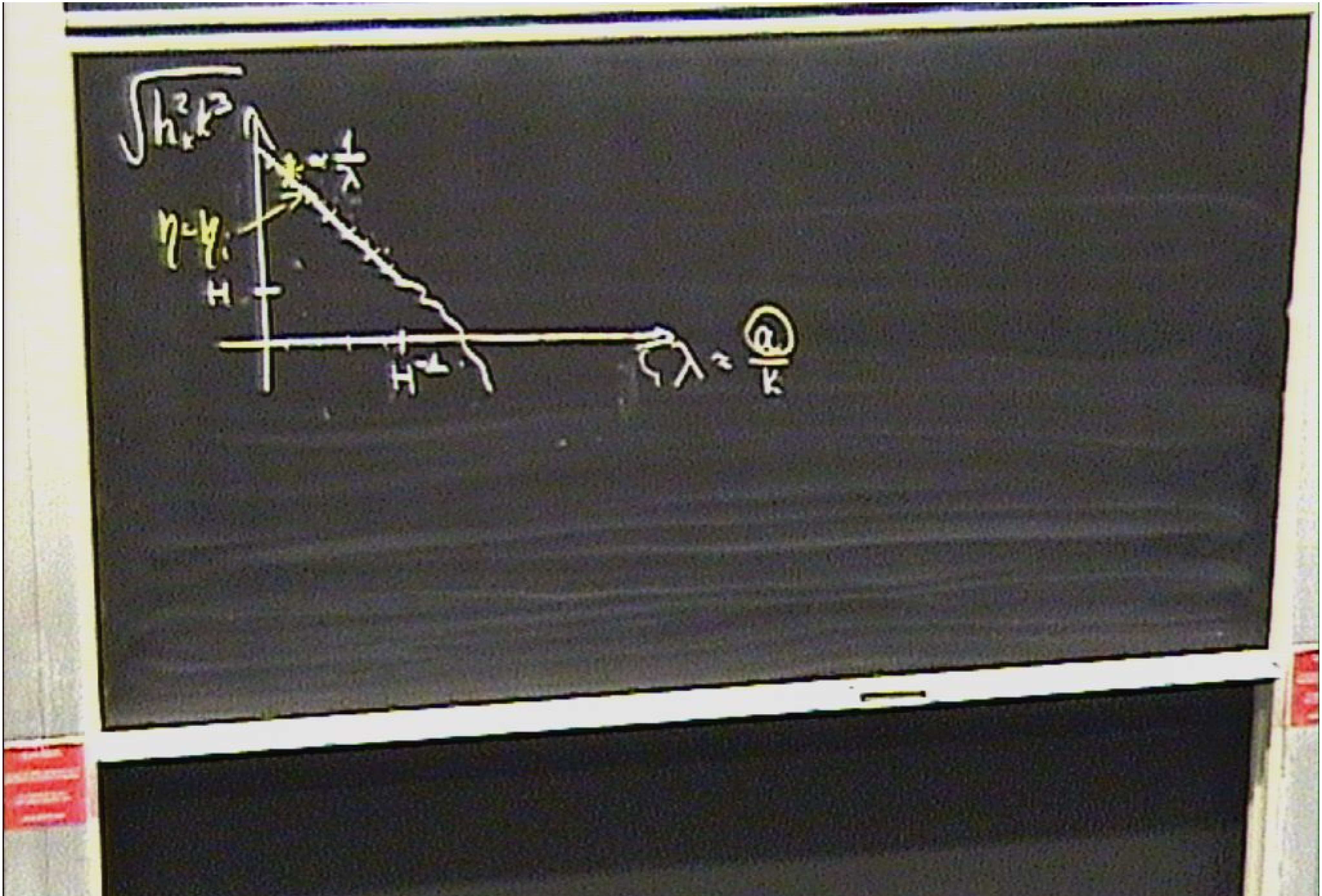
$$h_{ik} = \frac{u}{a} e_{ik} \qquad k^i e_{ik} = 0$$

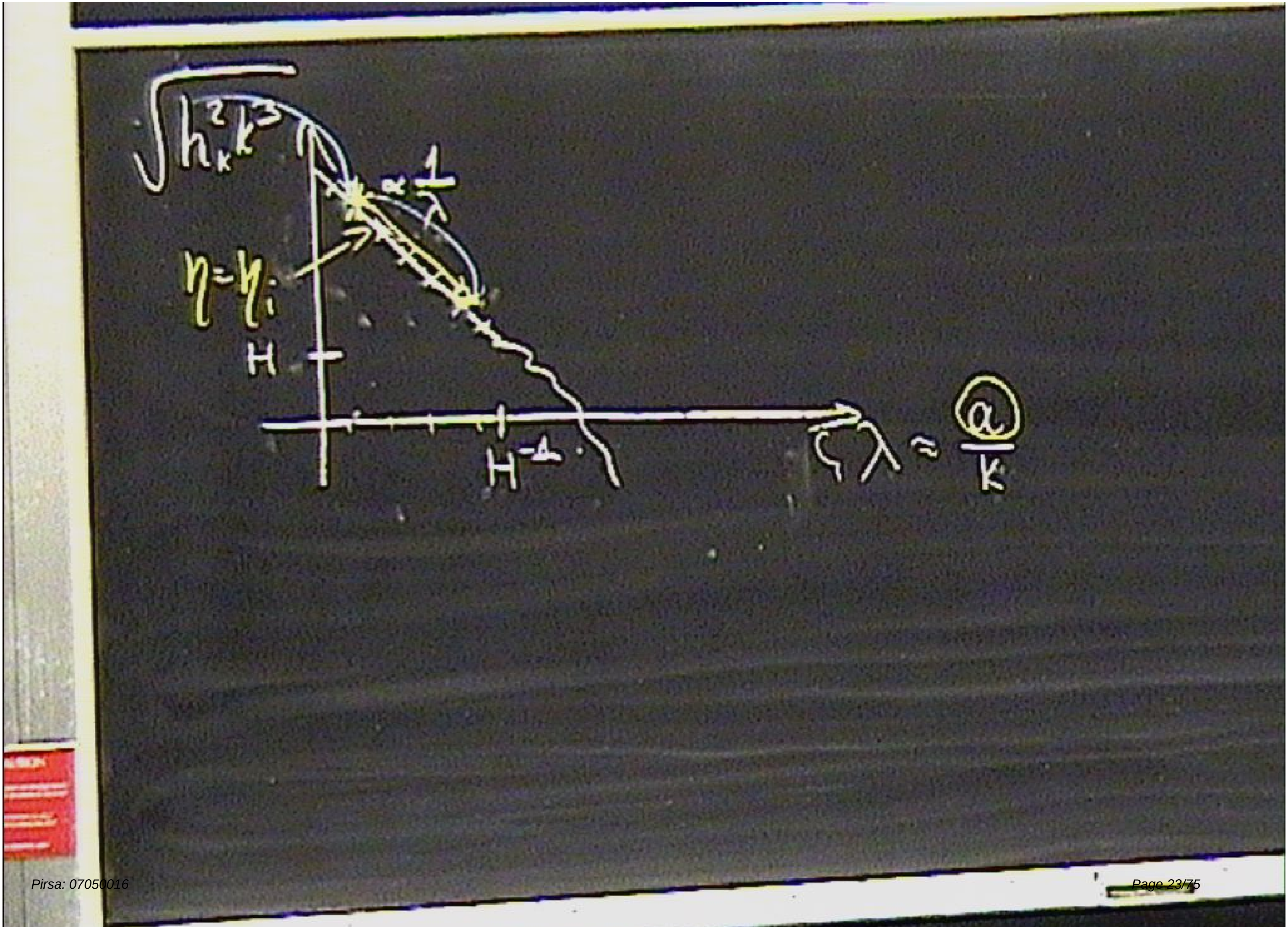
$$e^i_i = 0$$

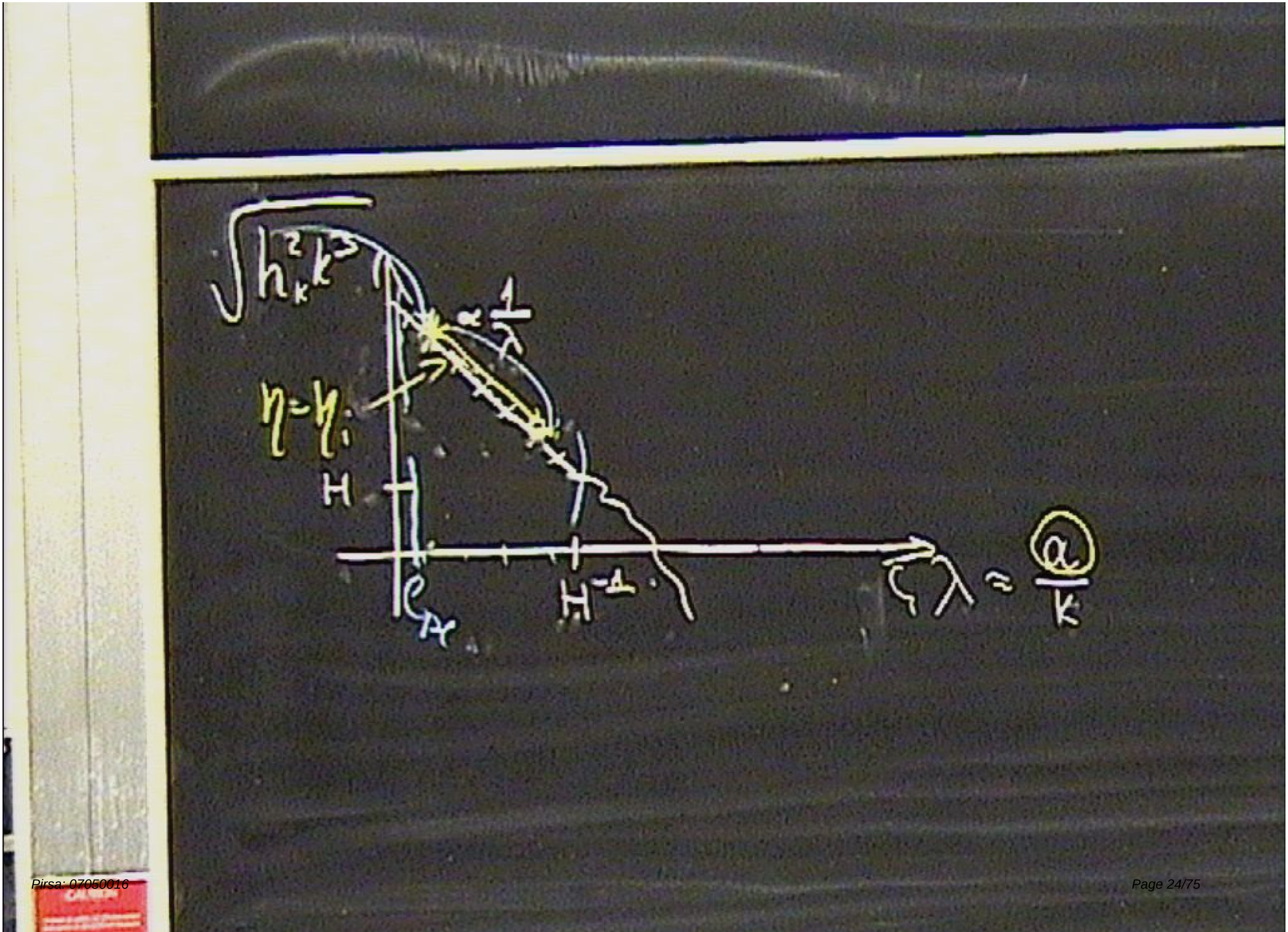

$u \propto e^{ikx}$
$u'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0$
$\delta\varphi_k \simeq \frac{1}{(L/a_{max})}$
$\sqrt{k^3 \delta\varphi_k^2}$
$\hat{\varphi} = \int \frac{1}{\sqrt{2k}} \left(e^{ikx - i\omega\eta} \hat{a}_k - \dots \right)$
$h^{(T)i}_{i} = 0$
$h^{(T)}_{k,i} = 0$
$h''_{ik} + 2\frac{a'}{a} h'_{ik} - \Delta h_{ik} = 0$
$h_{ik} = \frac{u}{a} e_{ik}$
$k^i e_{ik} = 0$
$e^i_i = 0$



$\sqrt{h_k^2 k^3}$
$\propto \frac{1}{\lambda}$
H
H^{-1}
$\lambda = \frac{a}{k}$


$\sqrt{h_k^2 k^3}$
$\sim \frac{1}{\lambda}$
$\eta = \eta_i$
H
H^{-1}
$\lambda = \frac{a}{k}$


$\sqrt{h_k^2 k^3}$
$\propto \frac{1}{\lambda}$
$\eta = \eta_i$
H
H^{-1}
$\lambda \approx \frac{a}{k}$



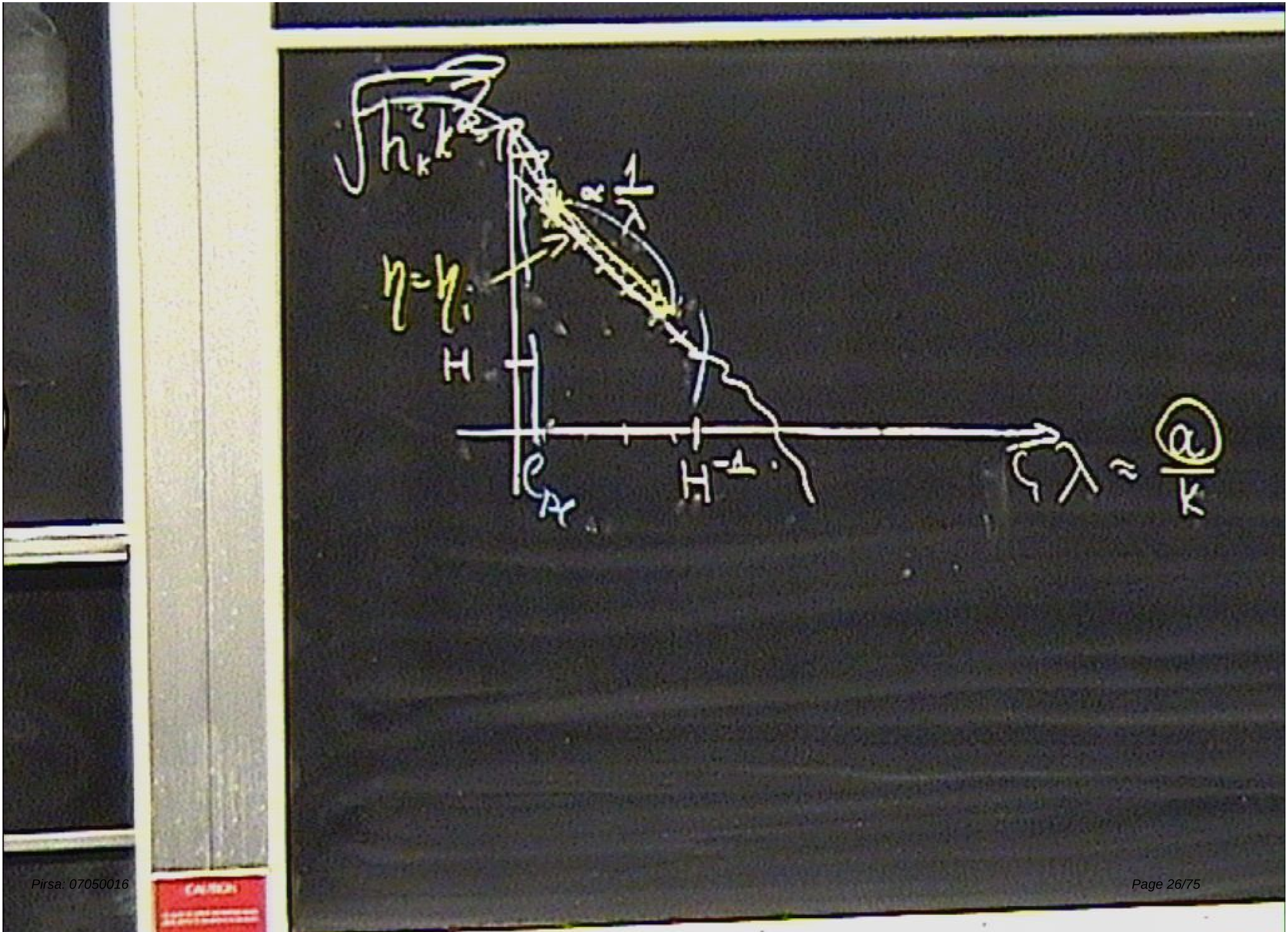
$\sqrt{h_k^2 k^3}$
$\propto \frac{1}{\lambda}$
$\eta = \eta_i$
H
H^{-1}
$\lambda = \frac{a}{k}$

$a \propto e^{Ht} = -\frac{1}{H\eta}$

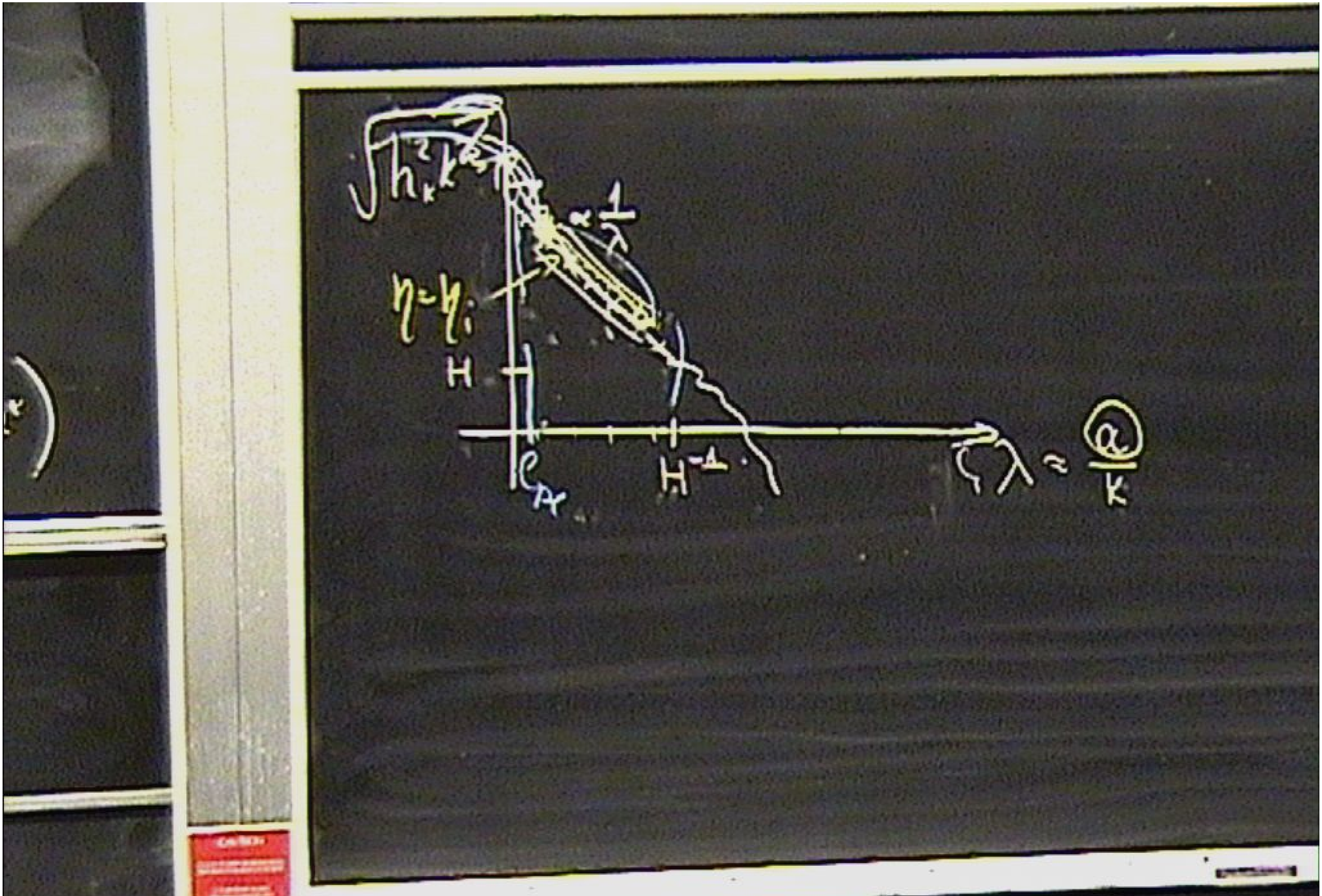
$k\eta \gg 1. \quad u \propto e^{ik\eta} \qquad -\infty < \eta < 0$

$h \propto \frac{1}{a} e^{ik\eta}$

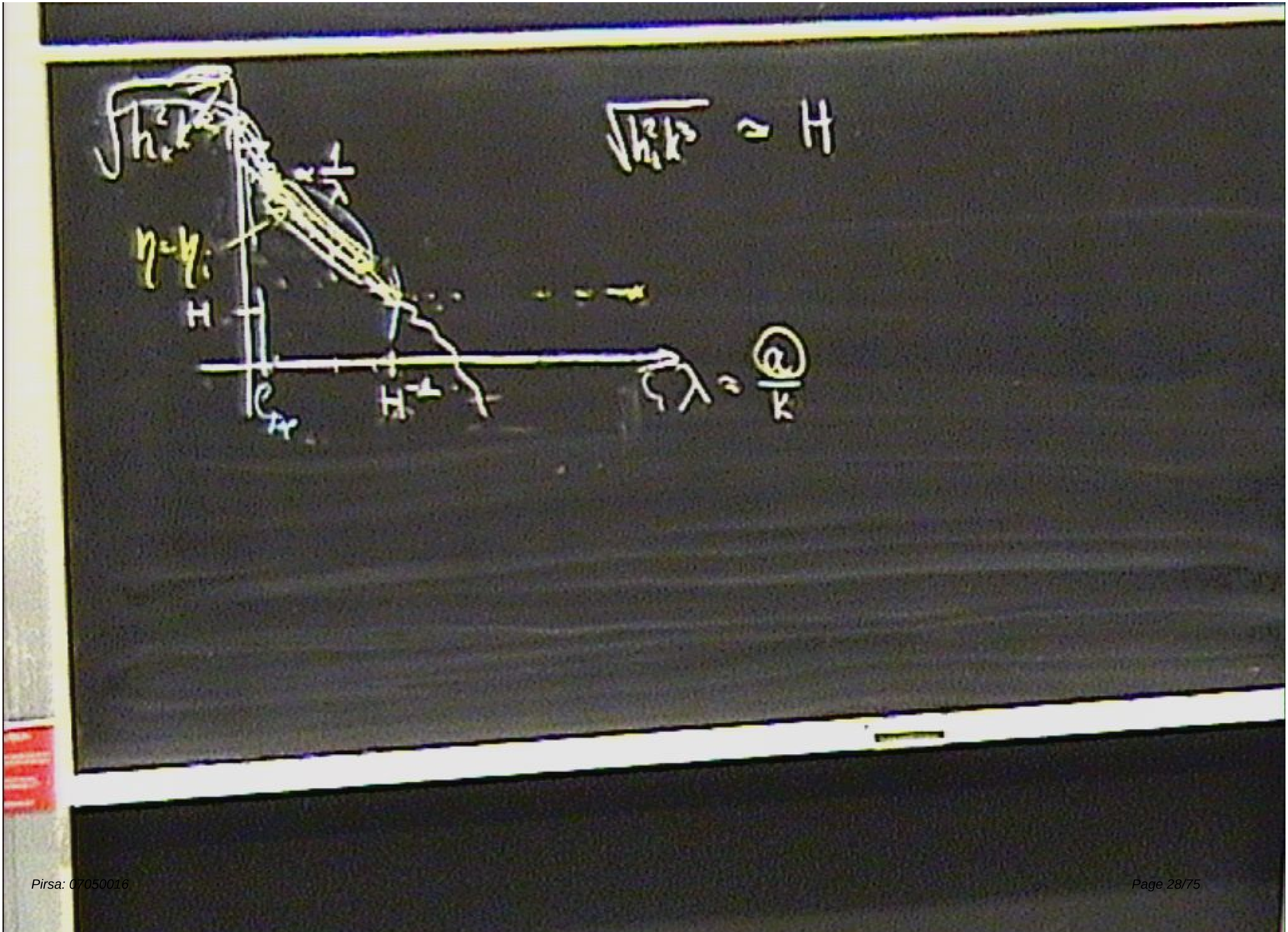
$ds^2 = a^2(\eta)\left(d\eta^2 - (\delta_{ik} + h_{ik})\,dx^i dx^k\right)$

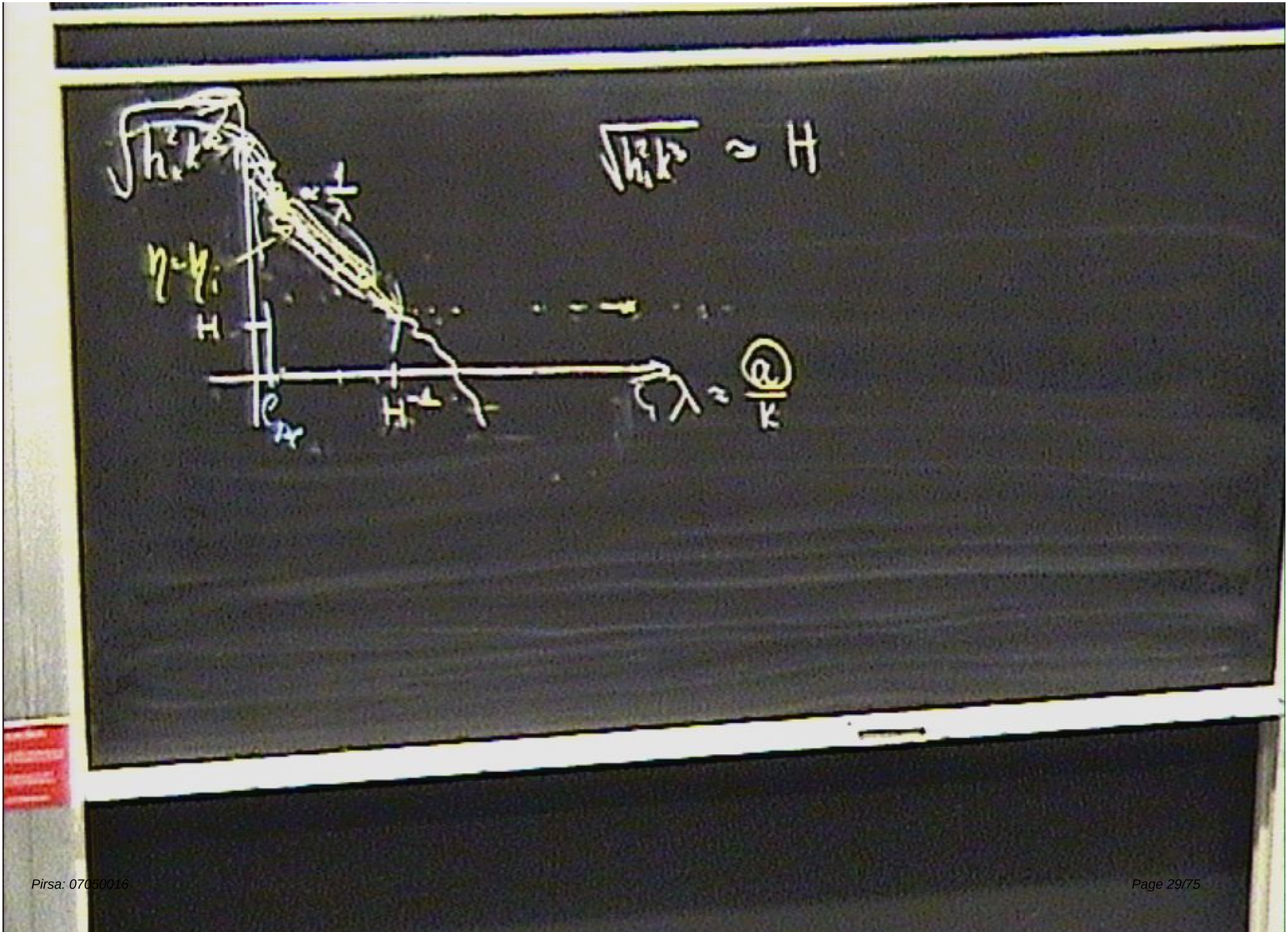


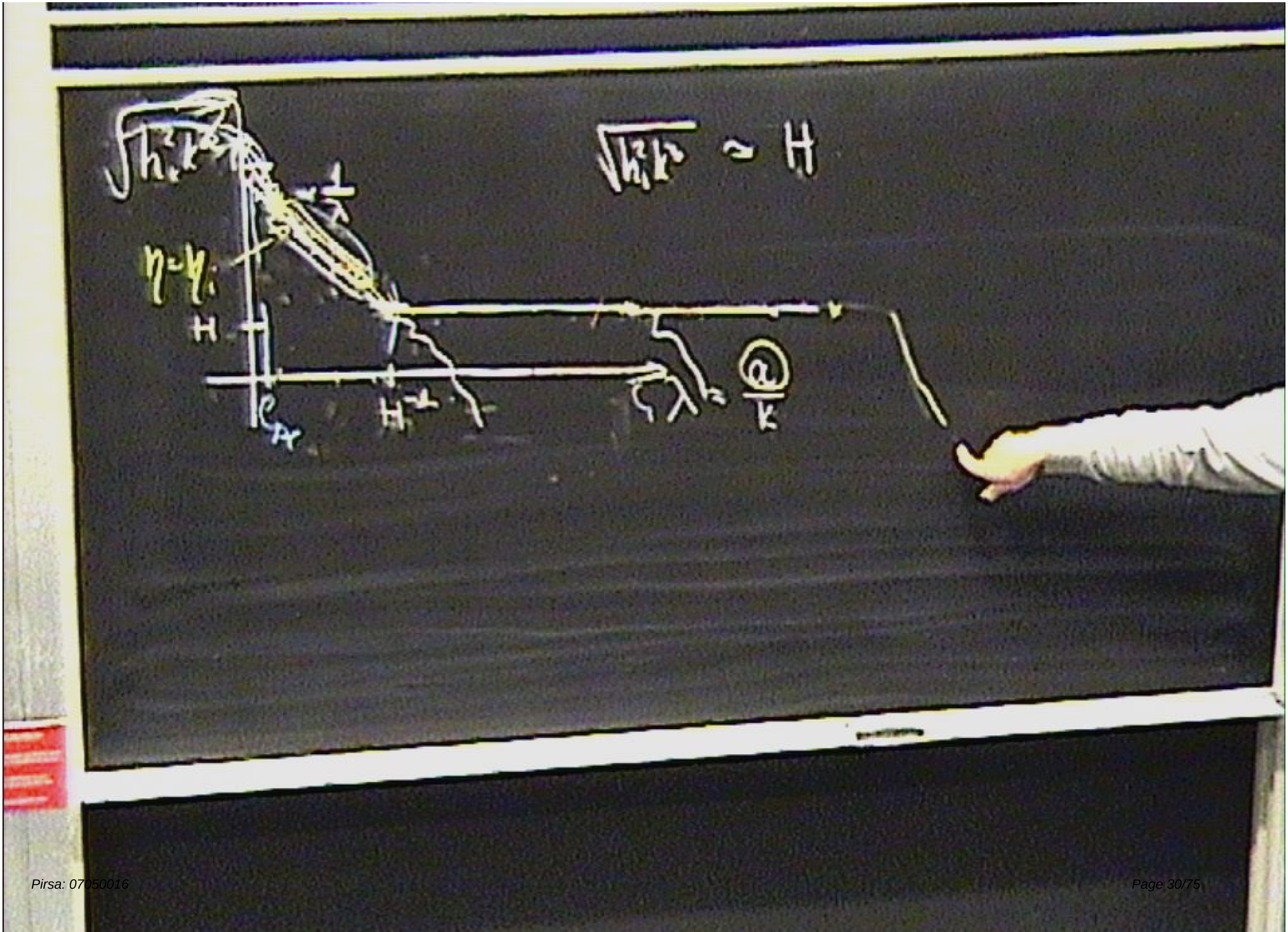
$\int h_k^2 k^2$
$\propto \frac{1}{\lambda}$
$\eta = \eta_i$
H
ℓ_{Pl}
H^{-1}
$\lambda \approx \frac{a}{k}$



$\int h_k^2 k^2$
$\propto \frac{1}{\lambda}$
$\eta = \eta_i$
H
ℓ_{Pl}
H^{-1}
$\lambda \approx \frac{a}{k}$


$\sqrt{h_k^2 k^3} \simeq H$
H
H^{-1}
$\lambda = \frac{a}{k}$

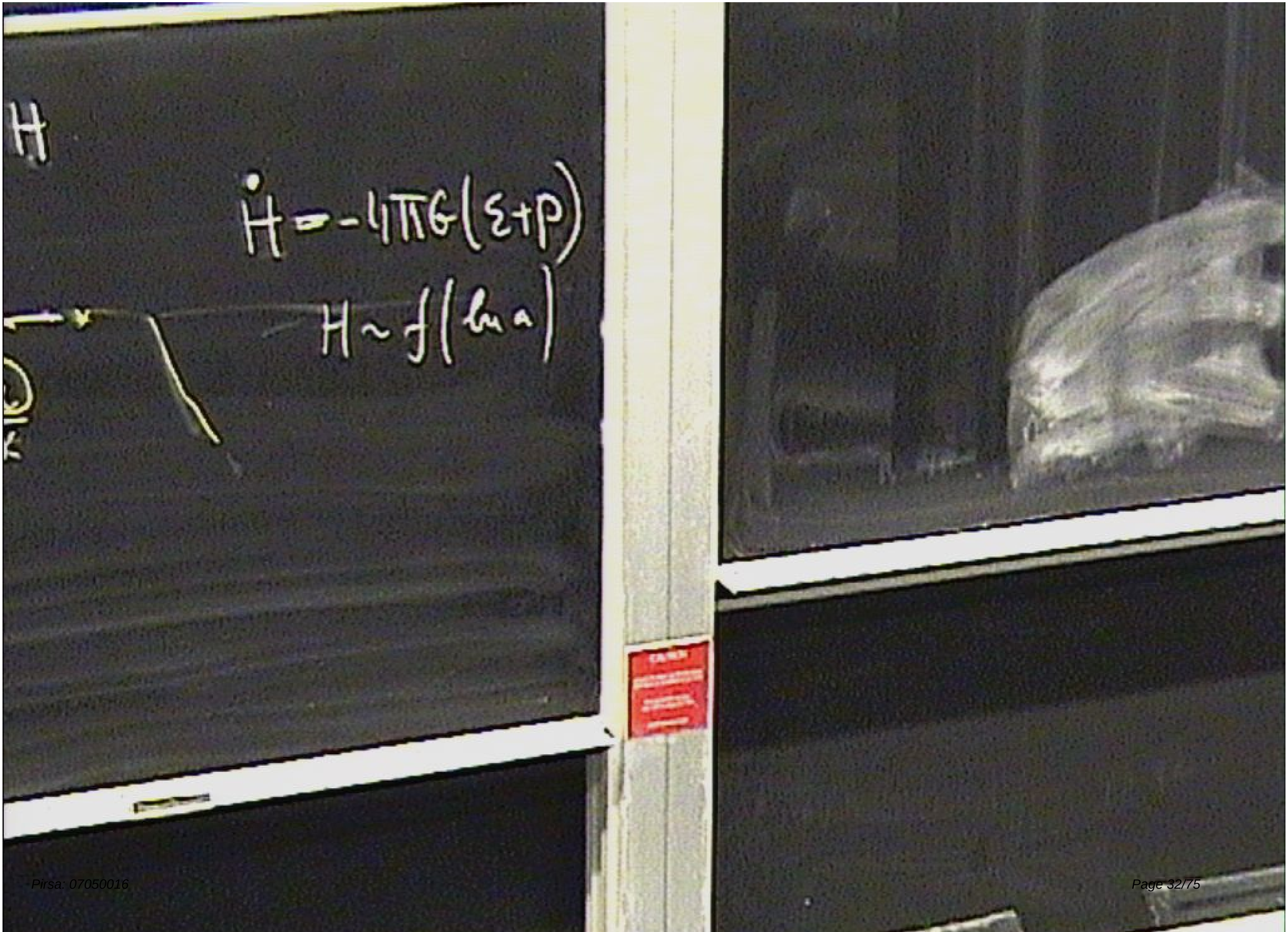

√(h_k^2 k^3) ~ H
η = η_i
H
H^-1
λ = a/k



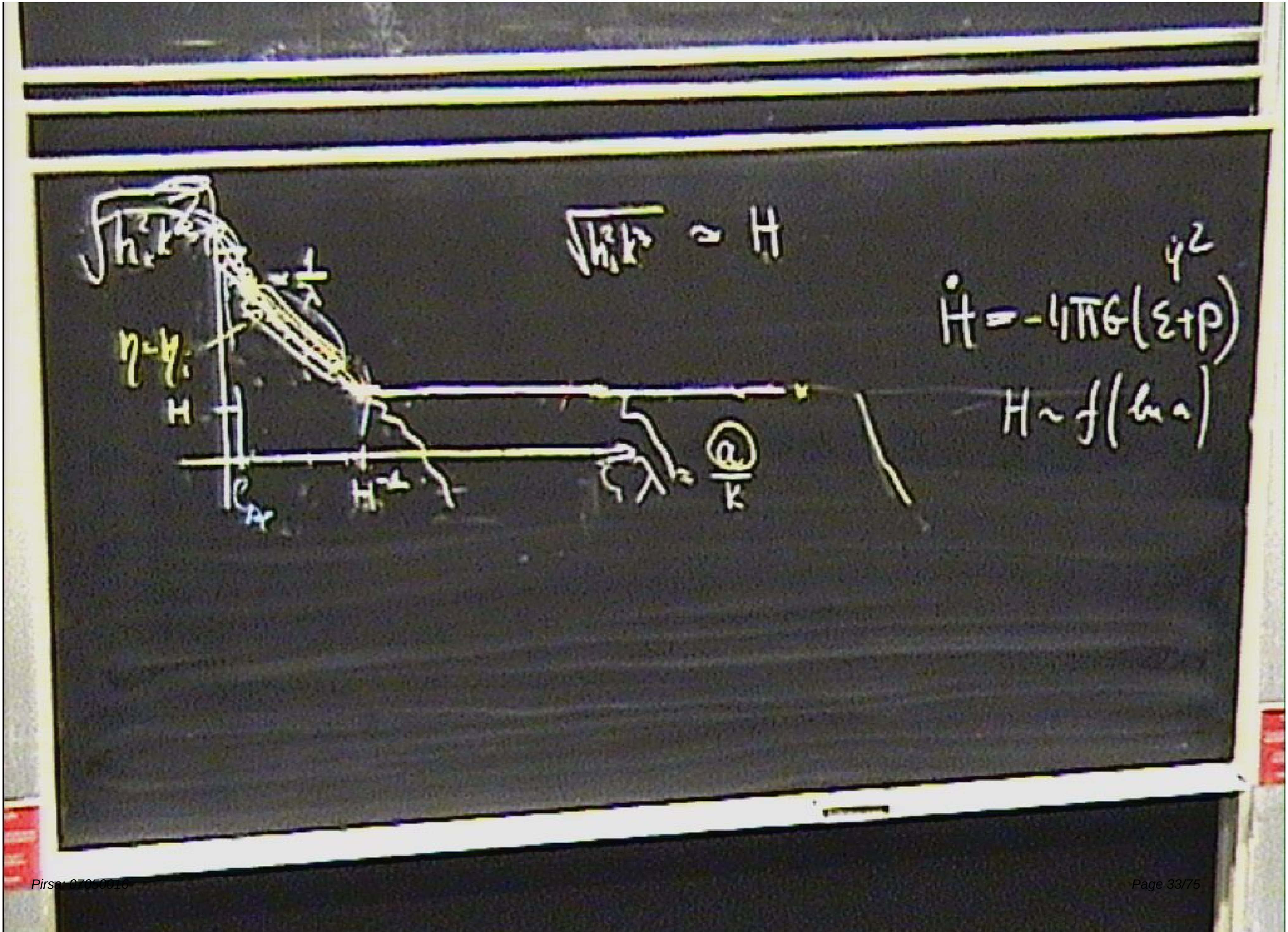
$\simeq H$

$$\dot{H} = -4\pi G(\varepsilon + p)$$

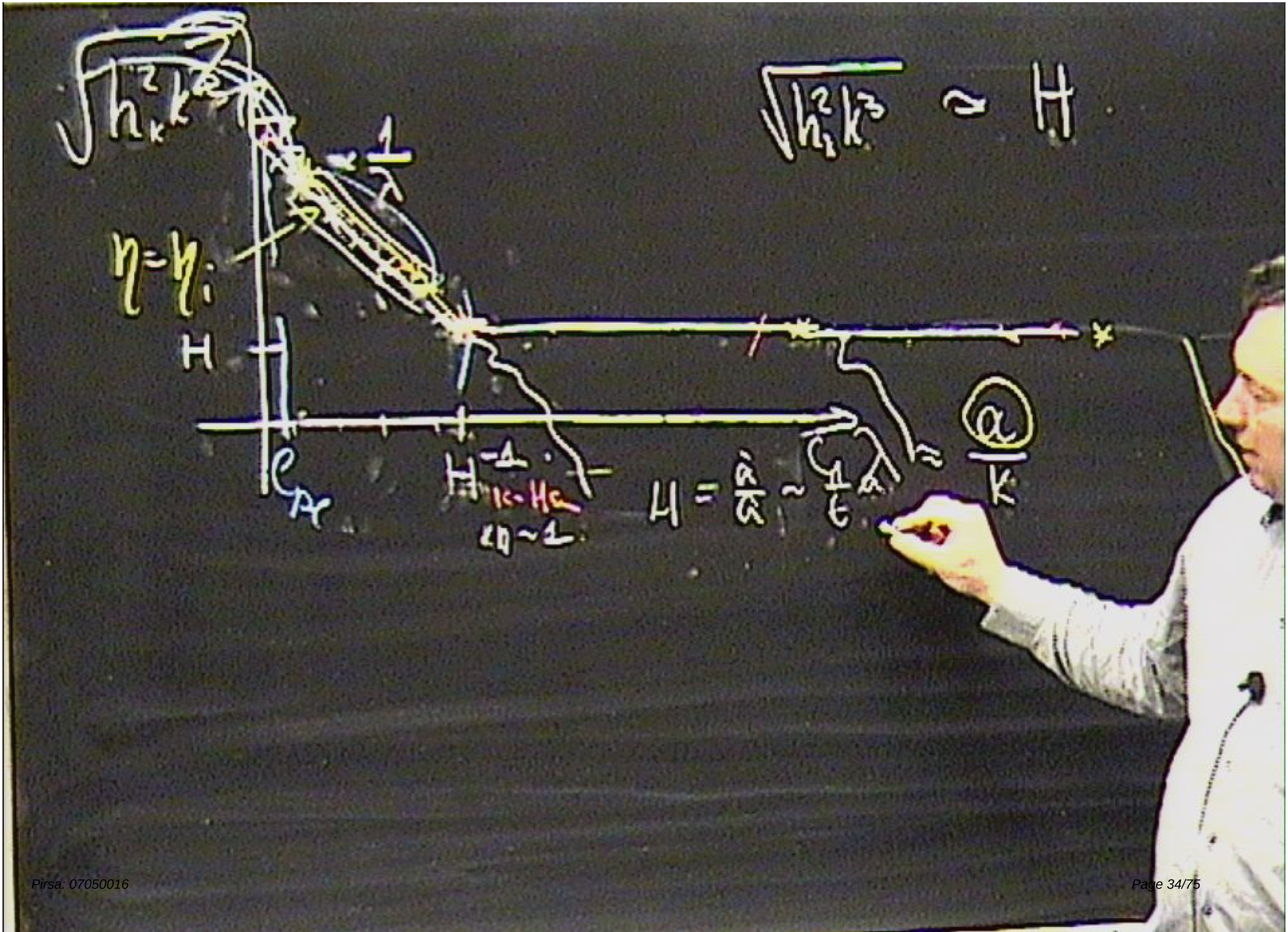
$\simeq \dfrac{a}{k}$



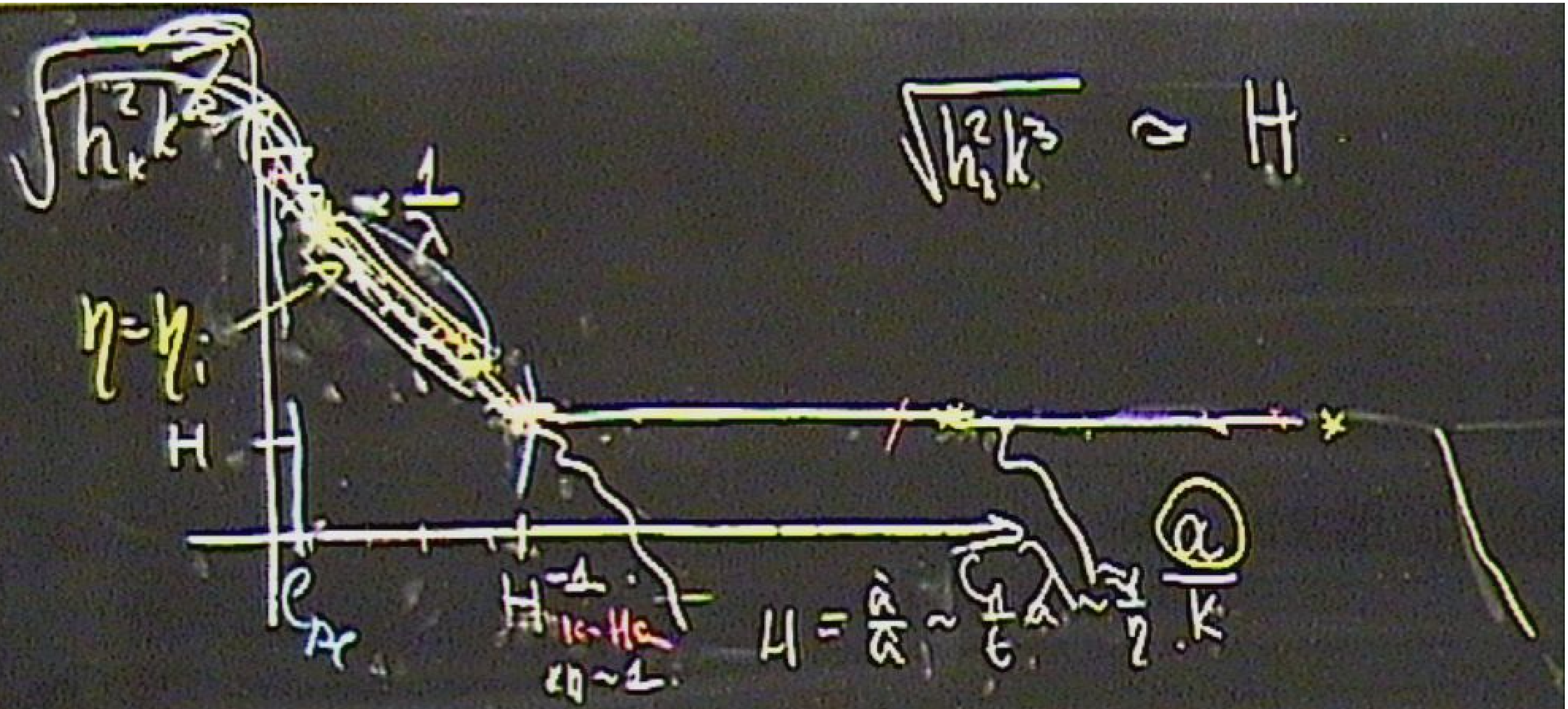
H
$\dot{H} = -4\pi G(\varepsilon + p)$
$H \sim f(\ln a)$

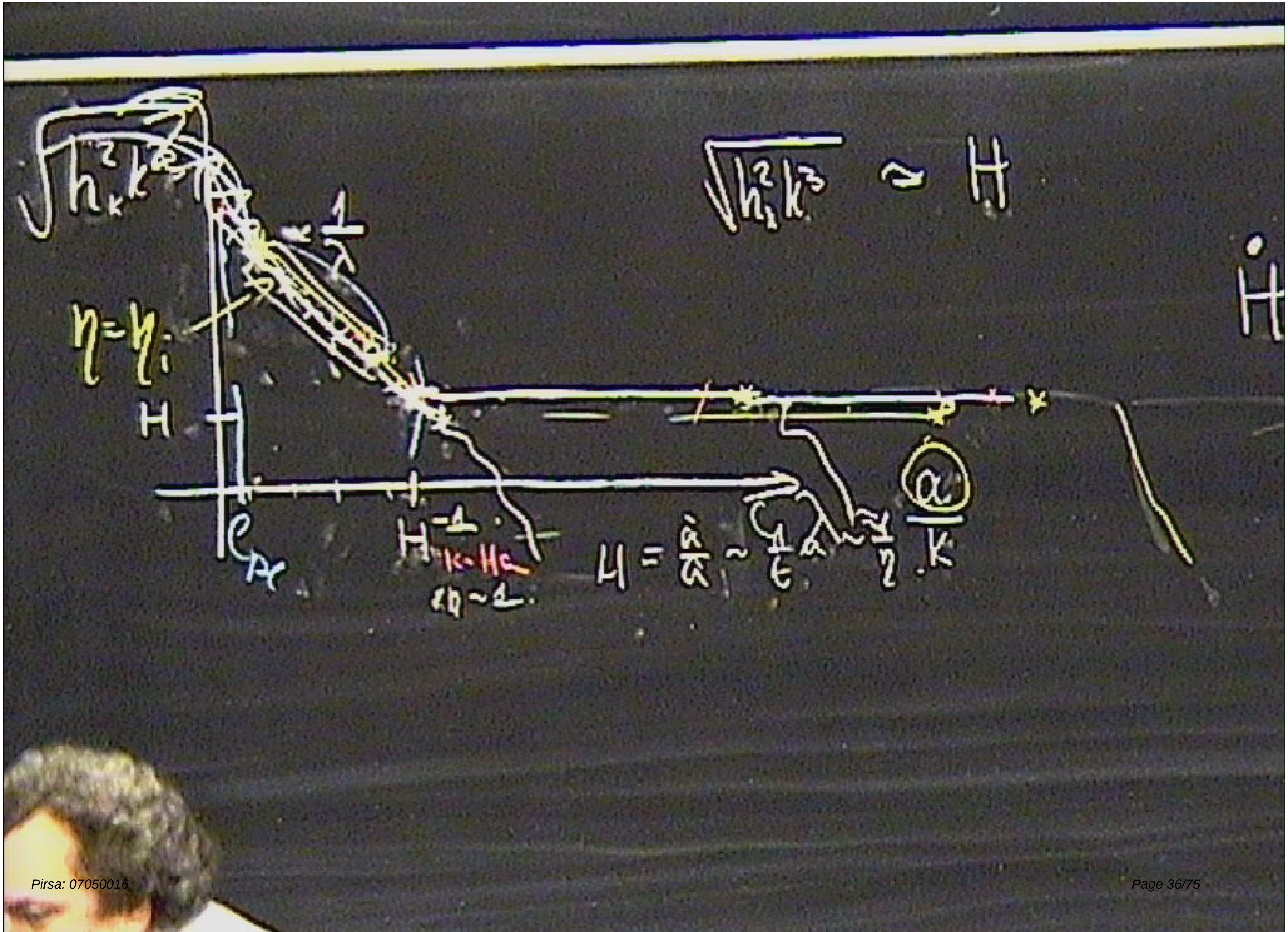


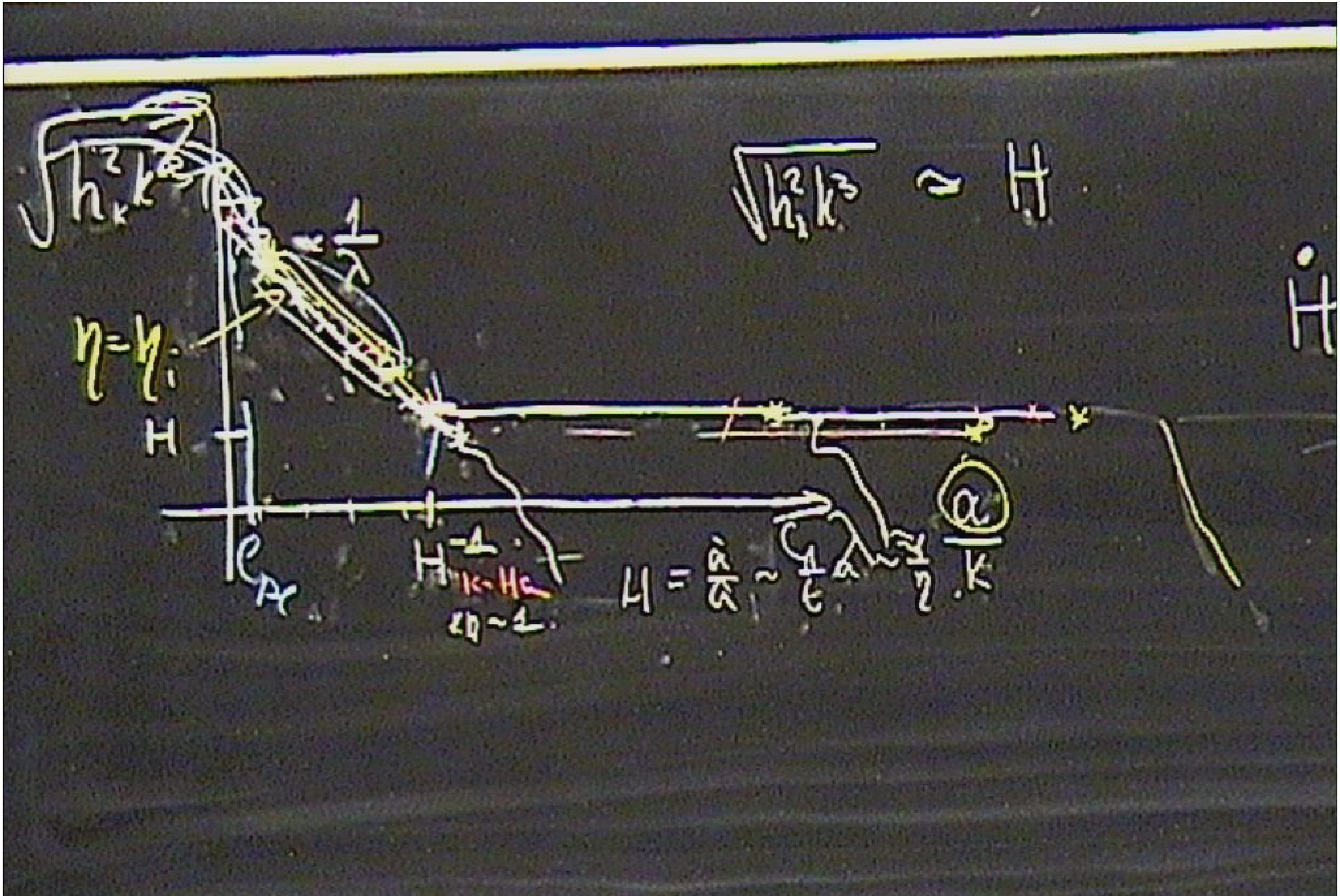
$\sqrt{h_i^2 k^3} \sim H$
$\dot{\psi}^2$
$\dot{H} = -4\pi G(\varepsilon + p)$
$H \sim f(\ln a)$
$\eta - \eta_i$
H
H^{-1}
$\lambda = \frac{a}{k}$


√(h_k² k³) ~ H
η=η_i
H
H⁻¹
k~Ha
kη~1
H = ȧ/a ~ 1/t
a/k

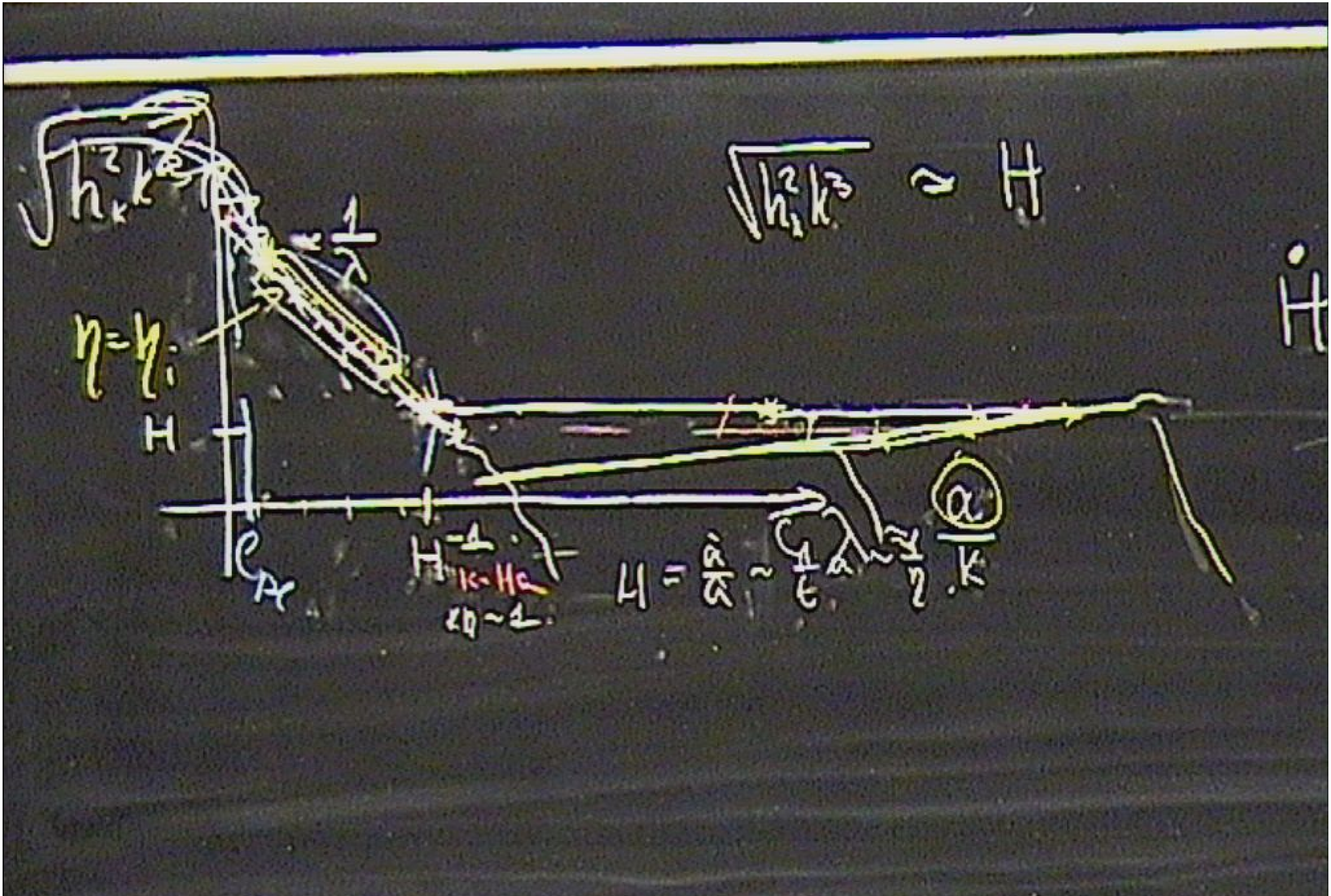
$\sqrt{h_k^2 k^3} \simeq H$

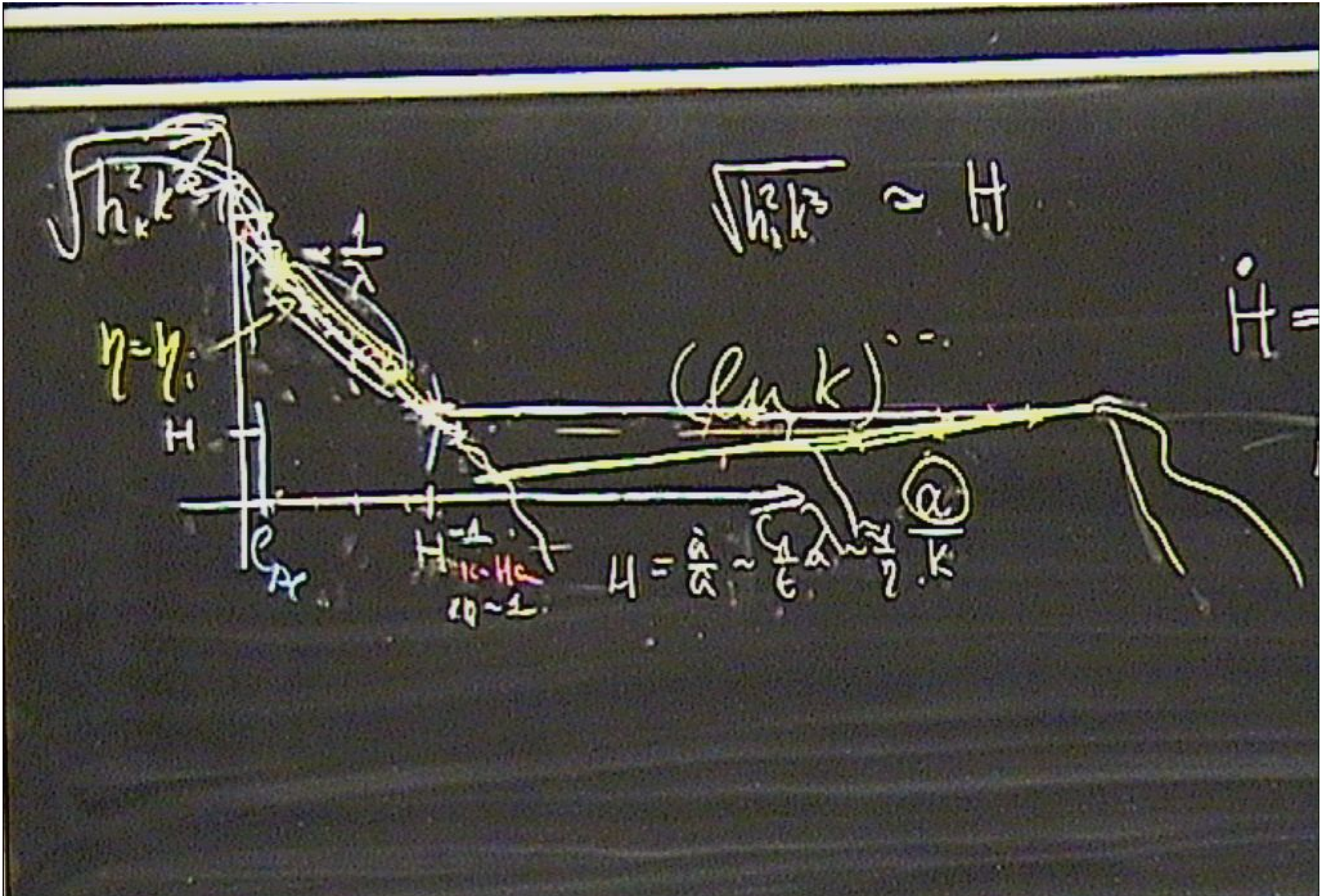



$\sqrt{h_k^2 k^3}$
$\frac{1}{\lambda}$
$\eta = \eta_i$
H
H^{-1}
$k = Ha$
$k\eta \sim 1$
$\sqrt{h_k^2 k^3} \sim H$
$H = \frac{\dot a}{a} \sim \frac{1}{t}$
$\frac{a}{k}$



$\sqrt{h_k^2 k^3} \sim H$
$\eta = \eta_i$
H
H^{-1}
$k \sim Ha$
$k\eta \sim 1$
$H = \frac{\dot a}{a} \sim \frac{1}{t}$
$\frac{a}{k}$


$\sqrt{h_k^2 k^3}$
$\sim \frac{1}{\eta}$
$\eta = \eta_i$
H
H^{-1}
$k = Ha$
$k\eta \sim 1$
$H = \frac{\dot{a}}{a} \sim \frac{1}{t}$
$\lambda \sim \frac{1}{\eta}$
$\frac{a}{k}$
$\sqrt{h_k^2 k^3} \sim H$
$\dot{H}$


$\sqrt{h_k^2 k^3} \sim H$
$\eta \sim \eta_i$
H
H^{-1}
ℓ_{Pl}
$(\ln k)$
$H = \frac{\dot a}{a} \sim \frac{1}{t}$
$\frac{a}{k}$
$\dot H =$

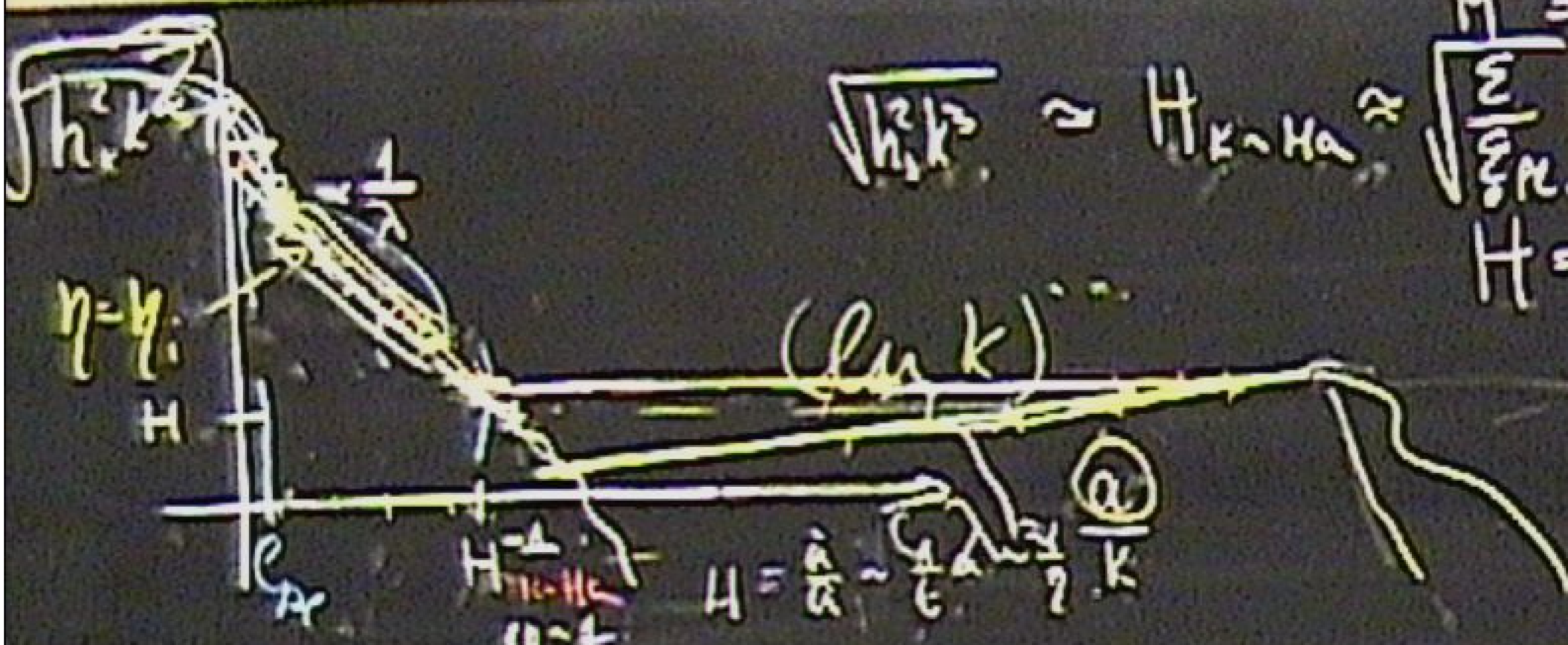
$$H^2 = \frac{8\pi G}{3}\varepsilon$$

$$\sqrt{h_k^2 k^3} \sim H_{k\sim Ha} \approx \sqrt{\frac{\varepsilon}{\varepsilon_{Pl}}}\Bigg|_{k\sim Ha}$$

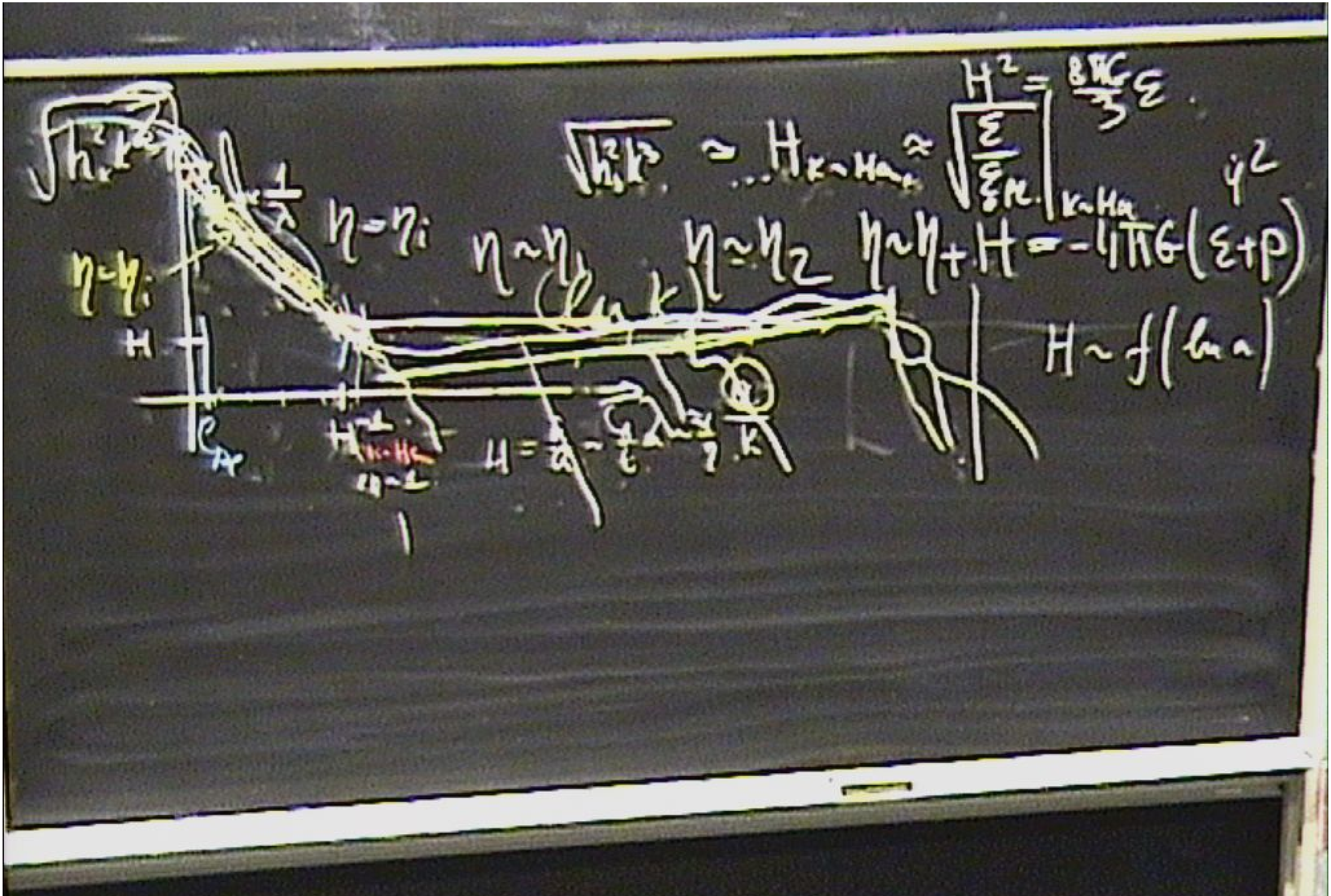
$$\dot\varphi^2$$

$$\dot H = -4\pi G(\varepsilon + p)$$

$$H \sim f(\ln a)$$



$$H = \frac{\dot a}{a} \sim \frac{1}{t}\,a \sim \frac{1}{\eta}\,\frac{a}{k}$$



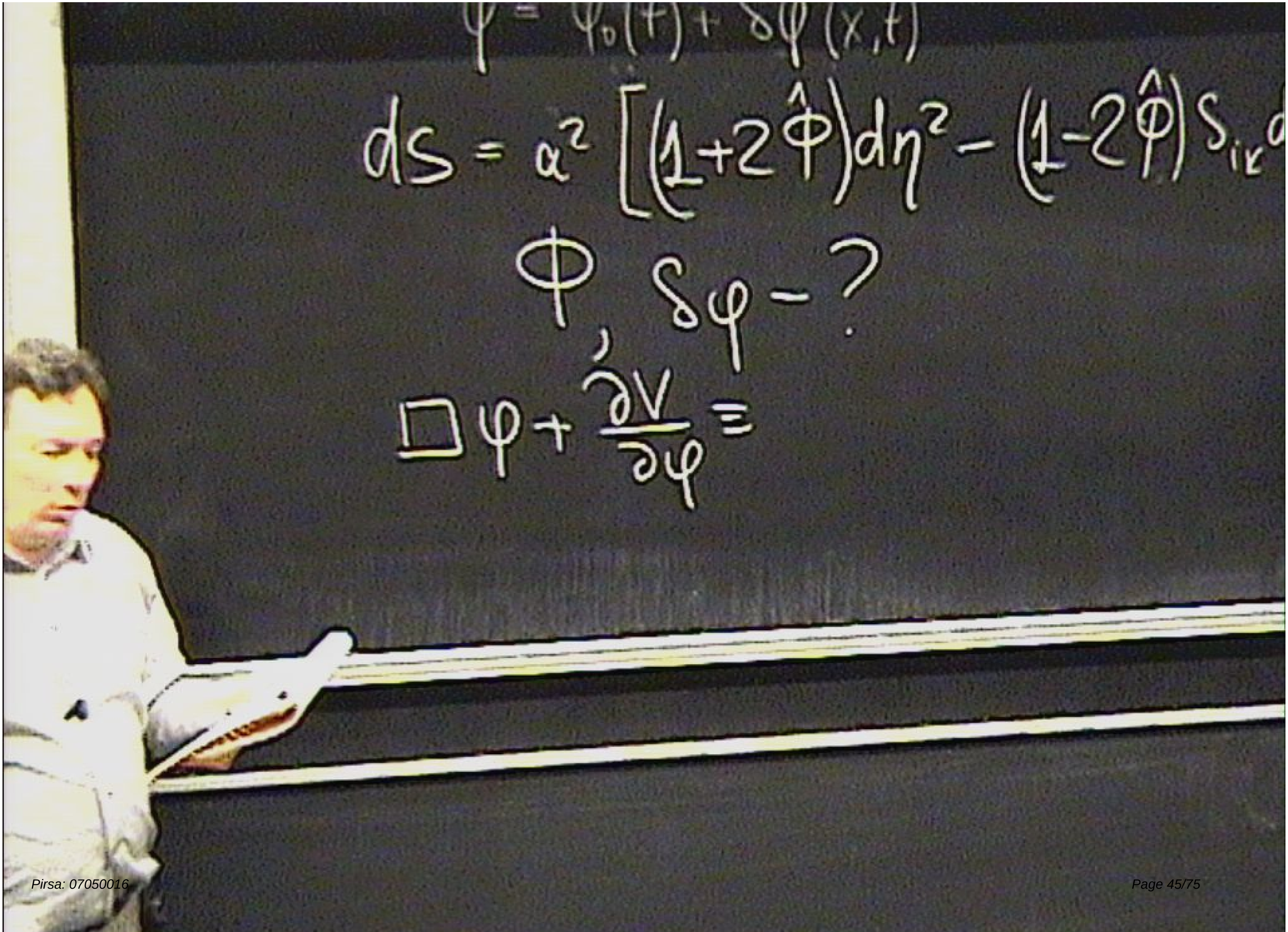
H^2 = 8πG/3 ε
√(h_k^2 k^3) ~ H_{k~Ha} ≈ √(ε/ε_Pl) |_{k~Ha}
η = η_i
η ~ η_1
η ~ η_2
η ~ η_+
Ḣ = −4πG(ε+p)
H ~ f(ln a)
H = 1/a ~ 1/t ~ 1/η

$$\varphi = \varphi_0(t)$$

$$\varphi = \varphi_0(t) + \delta\hat{\varphi}(x,t)$$

$$\varphi = \varphi_0(t) + \delta\varphi(x,t)$$

$$ds = a^2 \left[(1+2\Phi)\, d\eta^2 - (1-2\Phi)\, \delta_{ik}\, dx^i dx^k \right]$$



$\varphi = \varphi_0(t) + \delta\varphi(x,t)$
$ds = a^2 [(1+2\hat{\Phi})d\eta^2 - (1-2\hat{\Phi})S_{ik}d$
$\Phi, \delta\varphi - ?$
$\Box\varphi + \frac{\partial V}{\partial\varphi} =$

$$ds^2 = a^2\left[(1+2\hat{\Phi})d\eta^2 - (1-2\hat{\Phi})\delta_{ik}dx^i dx^k\right]$$

$$\Phi,\ \delta\varphi - ?$$

$$\Box\varphi + \frac{\partial V}{\partial\varphi} \equiv \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}\, g^{\alpha\beta}\frac{\partial\varphi}{\partial x^\beta}\right) + V_{,\varphi} = 0$$

$$\varphi = \varphi_0(t) + \delta\varphi(x,t)$$

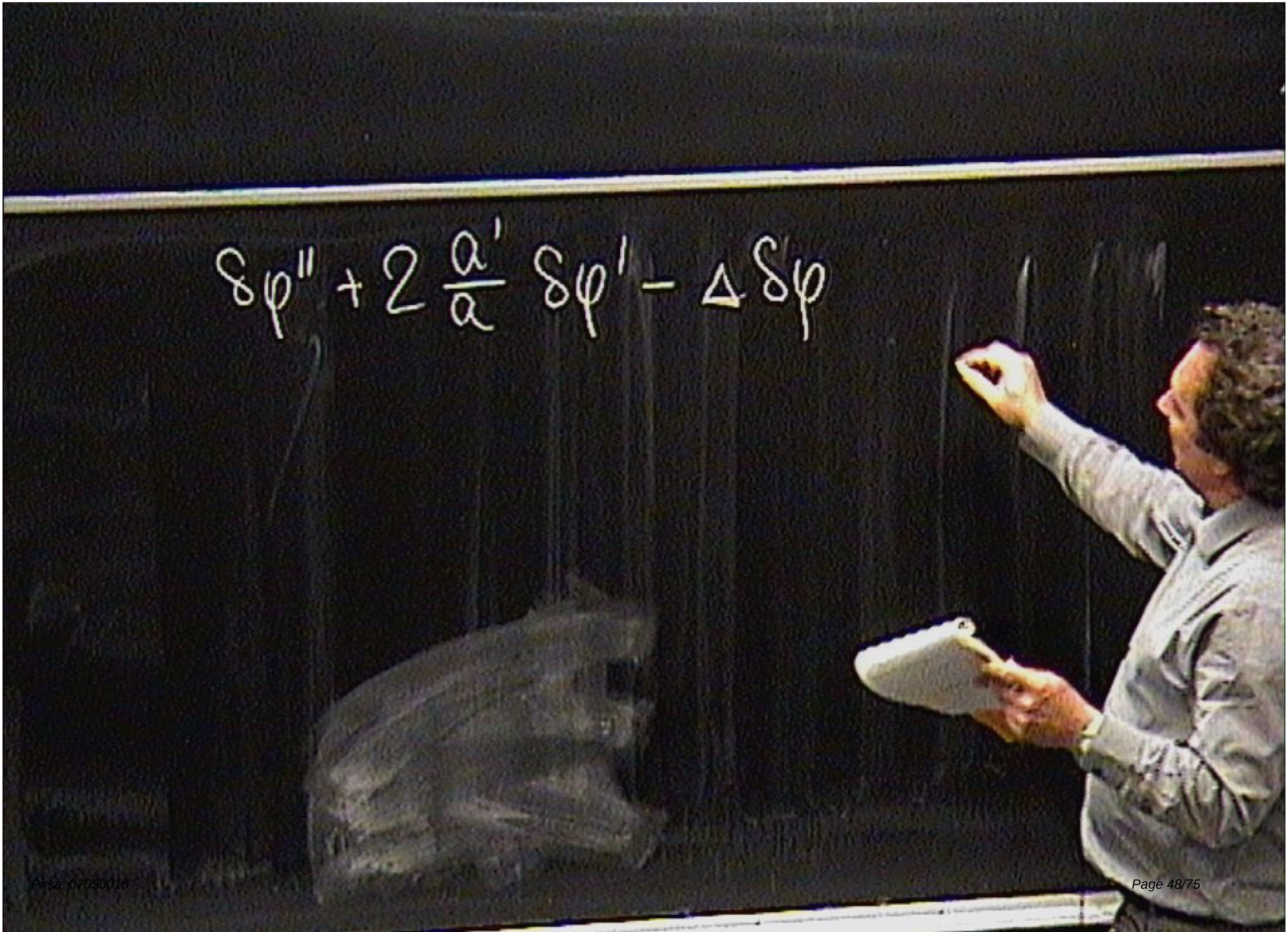
$$ds = a^2\left[(1+2\hat{\Phi})\,d\eta^2 - (1-2\hat{\Phi})\,\delta_{ik}\,dx^i dx^k\right]$$

$$\Phi,\ \delta\varphi - ?$$

$\varphi_0 + \delta\varphi$

$$\Box\varphi + \frac{\partial V}{\partial\varphi} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}\, g^{\alpha\beta}\frac{\partial\varphi}{\partial x^\beta}\right) + V_{,\varphi} = 0$$

$$G^0_i = 8\pi G\, T^0_i$$


$\delta\varphi'' + 2\frac{a'}{a}\delta\varphi' - \Delta\delta\varphi$

$$\delta\varphi'' + 2\frac{a'}{a}\delta\varphi' - \Delta\delta\varphi + a^2 V_{,\varphi\varphi}$$

$$\delta\varphi'' + 2\frac{a'}{a}\delta\varphi' - \Delta\delta\varphi + a^2 V_{,\varphi\varphi}\delta\varphi$$

$$\delta\varphi'' + 2\frac{a'}{a}\delta\varphi' - \Delta\delta\varphi + a^2 V_{,\varphi\varphi}\delta\varphi -$$

$$- 4\varphi_0'\Phi' + 2a^2 V_{,\varphi}\Phi = 0$$

$$\delta\varphi'' + 2\frac{a'}{a}\delta\varphi' - \Delta\delta\varphi + a^2 V_{,\varphi\varphi}\,\delta\varphi -$$

$$-4\varphi_0'\,\Phi' + 2a^2 V_{,\varphi}\,\Phi = 0$$

$\mathcal{H} = \frac{a'}{a}$

$$\Phi' + \mathcal{H}\,\Phi = 4\pi$$

$$\lambda_{ph} = \frac{\pi a}{k} \ll H^{-1} \equiv k\eta \gg 1$$

$$\lambda_{ph} = \frac{\pi a}{k} \ll H^{-1} \equiv k\eta \gg 1$$

$$\phi' \sim k\phi \qquad \mathcal{H}\phi \sim \phi/\eta$$

$$\lambda_{ph} = \frac{\pi a}{k} \ll H^{-1} \equiv k\eta \gg 1.$$

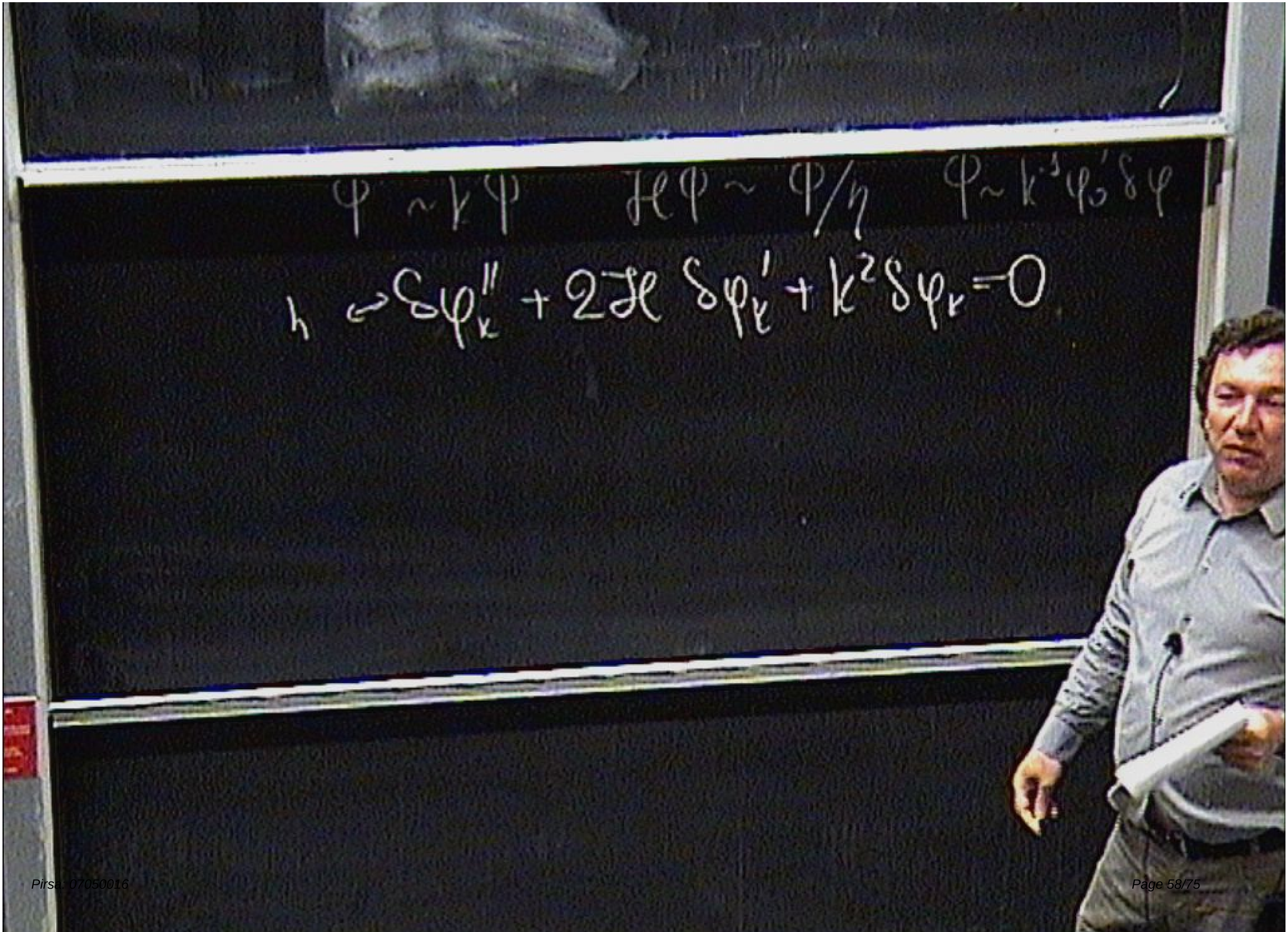
$$\Phi' \sim k\Phi \qquad \mathcal{H}\Phi \sim \Phi/\eta \qquad \Phi \sim k^{-1}\varphi_0'\,\delta\varphi.$$

$V'' \simeq m^2$

$$\mathcal{H} = \frac{a'}{a} \qquad \begin{cases} \underbrace{\delta\varphi'' + 2\frac{a'}{a}\delta\varphi' - \Delta\delta\varphi}_{} + a^2 V_{,\varphi\varphi}\,\delta\varphi - 4\varphi_0'\Phi' + 2a^2 V_{,\varphi}\Phi = 0 \\ \Phi' + \mathcal{H}\Phi = 4\pi\varphi_0'\delta\varphi \end{cases}$$

$$\Phi' \sim k\Phi \qquad \mathcal{H}\varphi \sim \Phi/\eta \qquad \Phi \sim k^{-1}\varphi_0'\,\delta\varphi.$$

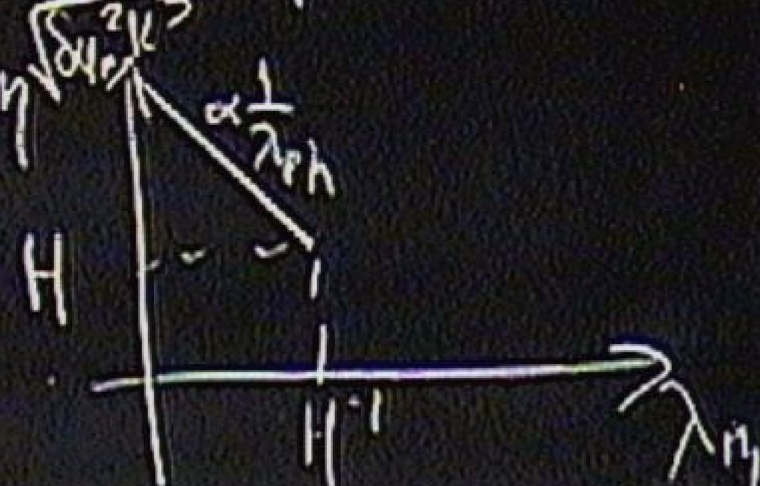
$$\delta\varphi_k'' + 2\mathcal{H}\,\delta\varphi_k' + k^2\delta\varphi_k = 0$$


$\Phi \sim k\varphi \qquad \mathcal{H}\varphi \sim \Phi/\eta \qquad \Phi \sim k^{-3}\varphi_0'\,\delta\varphi$
$\eta \leftrightarrow \delta\varphi_k'' + 2\mathcal{H}\,\delta\varphi_k' + k^2\delta\varphi_k = 0$

$\Phi' \sim k\Phi \qquad \mathcal{H}\Phi \sim \Phi/\eta \qquad \Phi \sim k^{-1}\varphi_0'\delta\varphi$

$$h \leftrightarrow \delta\varphi_k'' + 2\mathcal{H}\,\delta\varphi_k' + k^2\delta\varphi_k = 0$$

$$\delta\varphi_k \propto \frac{C_1}{a}\, e^{ik\eta}$$



ϕ

$\varphi_0 + \delta\varphi$

$\Box \varphi +$

$H \propto m\varphi.$

$$\Phi' \sim k\Phi \qquad \mathcal{H}\Phi \sim \Phi/\eta \qquad \Phi \sim k^{-1}\varphi_0'\delta\varphi$$

$$\omega \propto \sqrt{k^2 \cdots}$$

$$h \leftrightarrow \delta\varphi_k'' + 2\mathcal{H}\,\delta\varphi_k' + k^2\,\delta\varphi_k = 0$$

$$\delta\varphi_k \propto \frac{C_1}{a}\, e^{ik\eta}$$

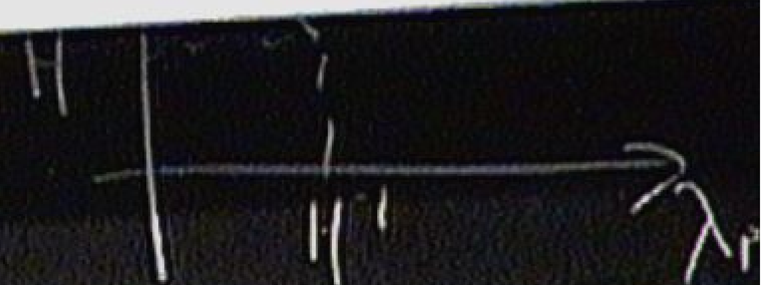
$$\frac{a'}{a}$$

$$-4\varphi_0'\Phi' + 2a^2 V_{,\varphi}\Phi = 0$$

$$\Phi' + \mathcal{H}\Phi = 4\pi\varphi_0'\delta\varphi$$

$$\cancel{\ddot{\varphi}} + 3H\dot{\varphi} + V_{,\varphi} = 0$$

$$H^2 = \frac{8\pi}{3}\left(\cancel{\frac{1}{2}\dot{\varphi}^2} + V\right)$$



$$\eta \to t = \int a\, d\eta \qquad \dot{} \equiv \frac{d}{dt}$$

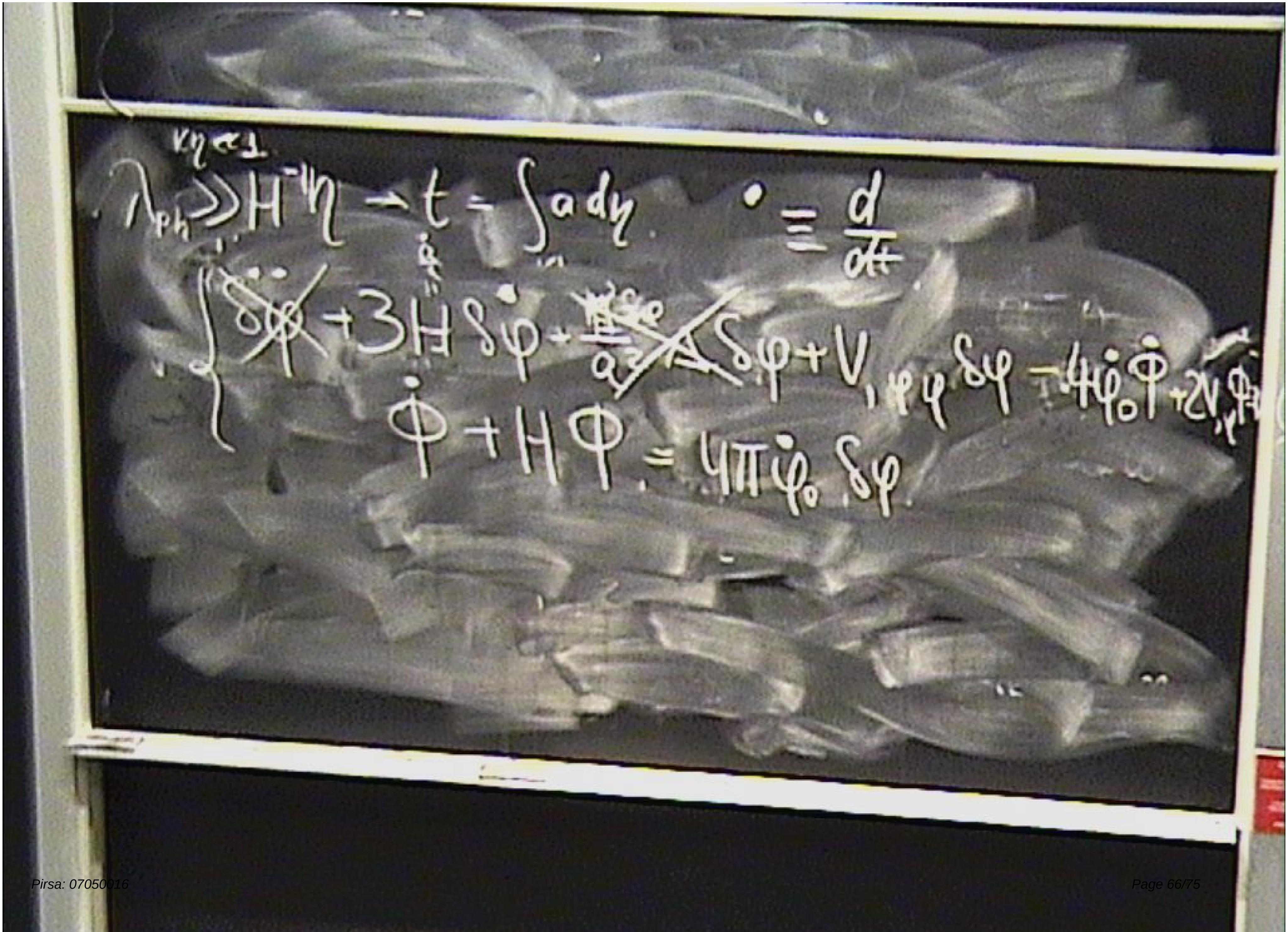
$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} - \frac{1}{a^2}\Delta\delta\varphi + V_{,\varphi\varphi}\delta\varphi = 4\dot{\varphi}_0\dot{\Phi} + 2V_{,\varphi}\Phi$$

$$\eta \to t = \int a\, d\eta \qquad \dot{} \equiv \frac{d}{dt}$$

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} + \frac{1}{a^2}\Delta\delta\varphi + V_{,\varphi\varphi}\delta\varphi = 4\dot{\varphi}_0\dot{\Phi} + 2V_{,\varphi}\Phi$$

$$\eta \to t = \int a\, d\eta \qquad \dot{} \equiv \frac{d}{dt}$$

$$\begin{cases} \ddot{\delta\varphi} + 3H\,\dot{\delta\varphi} + \frac{1}{a^2}\Delta\delta\varphi + V_{,\varphi\varphi}\,\delta\varphi = 4\dot{\varphi}_0\dot{\Phi} + 2V_{,\varphi}\Phi \\ \dot{\Phi} + H\Phi = 4\pi\dot{\varphi}_0\,\delta\varphi \end{cases}$$


$k\eta \ll 1$
$\lambda_{ph} \gg H^{-1}\eta \to t = \int a\, d\eta \qquad \dot{} \equiv \frac{d}{dt}$
$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} + \frac{k^2}{a^2}\delta\varphi + V_{,\varphi\varphi}\delta\varphi = 4\dot{\varphi}_0\dot{\Phi} + 2V_{,\varphi}\Phi$
$\dot{\Phi} + H\Phi = 4\pi\dot{\varphi}_0\delta\varphi$

$k\eta \ll 1$

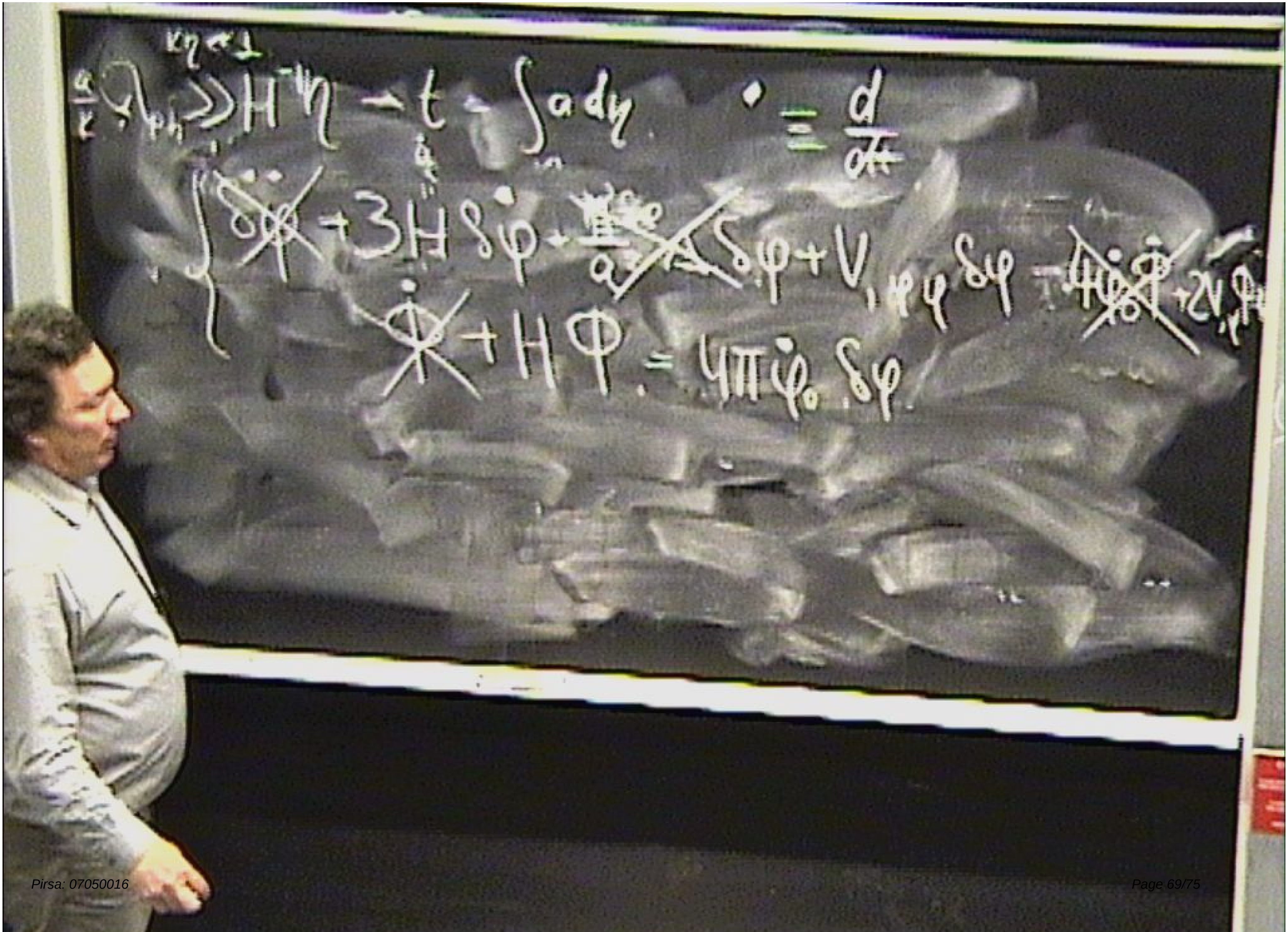
$\frac{a}{k} \sim \lambda_{ph} \gg H^{-1} \qquad \eta \to t = \int a\, d\eta \qquad \dot{} \equiv \frac{d}{dt}$

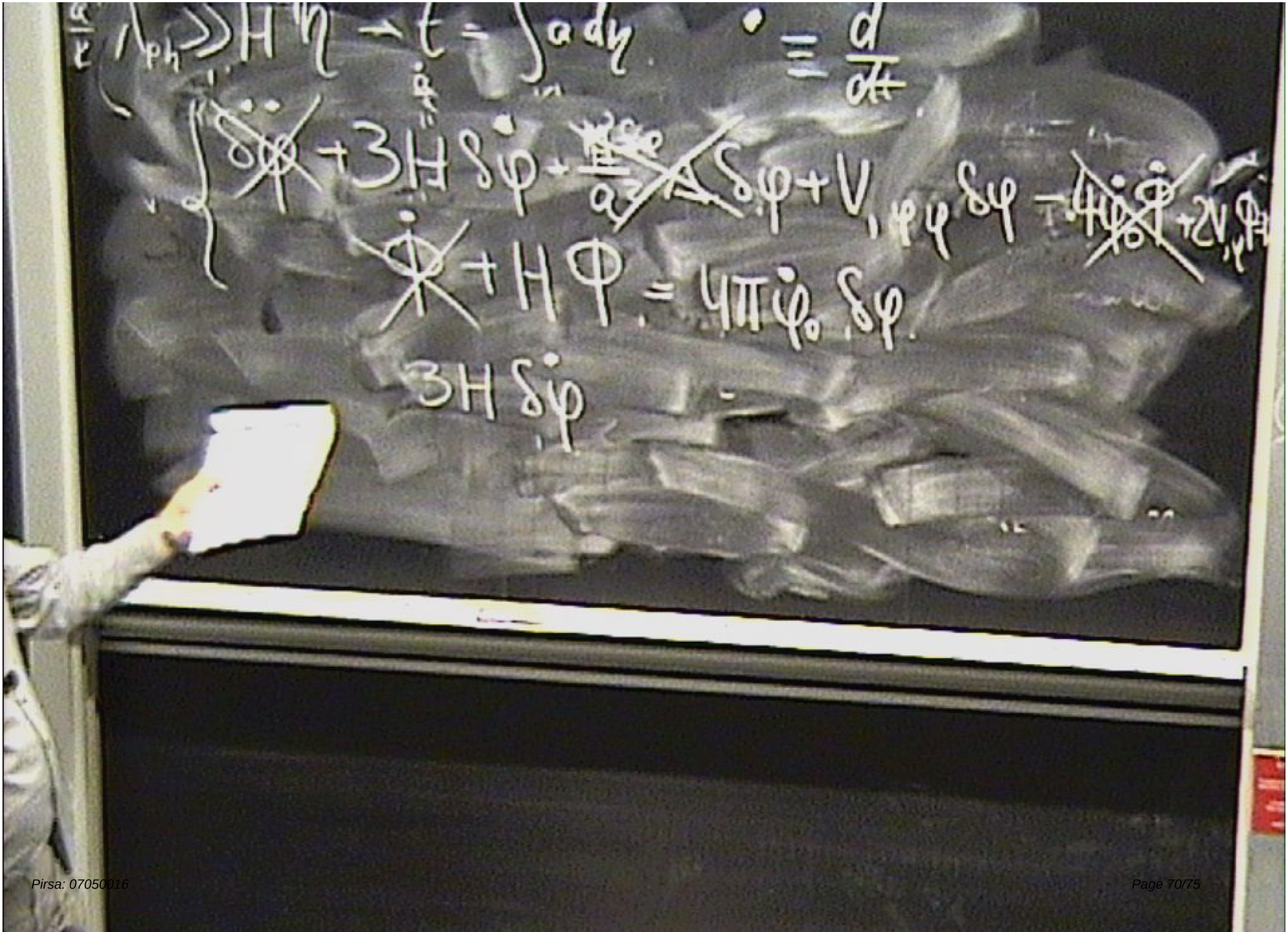
$$\begin{cases} \cancel{\ddot{\delta\varphi}} + 3H\dot{\delta\varphi} + \cancel{\frac{1}{a^2}\Delta\delta\varphi} + V_{,\varphi\varphi}\delta\varphi - 4\dot{\varphi}_0\dot{\Phi} + 2V_{,\varphi}\Phi \\ \dot{\Phi} + H\Phi = 4\pi\dot{\varphi}_0\,\delta\varphi \end{cases}$$

$k\eta \ll 1$

$\frac{a}{k}\lambda_{ph} \gg H^{-1}\eta \to t = \int a\, d\eta \qquad \dot{} \equiv \frac{d}{dt}$

$$\begin{cases} \cancel{\delta\ddot\varphi} + 3H\,\delta\dot\varphi + \cancel{\frac{k^2}{a^2}}\,\delta\varphi + V_{,\varphi\varphi}\,\delta\varphi - \cancel{4\dot\varphi_0\dot\Phi} + 2V_{,\varphi}\Phi = 0 \\ \dot\Phi + H\Phi = 4\pi\dot\varphi_0\,\delta\varphi \end{cases}$$


kη ≪ 1
η → t = ∫ a dη
˙ ≡ d/dt
δφ̈ + 3H δφ̇ + Δδφ + V,φφ δφ
Φ̇ + HΦ = 4π φ̇₀ δφ



≡ d/dt
+ 3H δφ̇
+ H Φ = 4π φ̇₀ δφ
3H δφ̇

$$\frac{a}{k} \quad \lambda_{ph} \gg H^{-1} \qquad \eta \to t = \int a\, d\eta \qquad \dot{} \equiv \frac{d}{dt}$$

$$\left\{ \begin{array}{l} \cancel{\ddot{\delta\varphi}} + 3H\dot{\delta\varphi} - \cancel{\frac{1}{a^2}\Delta\delta\varphi} + V_{,\varphi\varphi}\delta\varphi - \cancel{4\dot{\varphi}_0\dot{\Phi}} + 2V_{,\varphi}\Phi \\ \cancel{\dot{\Phi}} + H\Phi = 4\pi\dot{\varphi}_0\delta\varphi \end{array} \right.$$

$$\Phi = \frac{4\pi\dot{\varphi}}{H}\delta\varphi$$

$k\eta \ll 1$

$\frac{a}{k}$ $\lambda_{ph} \gg H^{-1}$ $\eta \to t = \int a\, d\eta$ $\quad \dot{} \equiv \frac{d}{dt}$

$$\begin{cases} \cancel{\ddot{\delta\varphi}} + 3H\dot{\delta\varphi} + \cancel{\frac{1}{a^2}\Delta}\delta\varphi + V_{,\varphi\varphi}\delta\varphi - \cancel{4\dot{\varphi}_0\dot{\Phi}} + 2V_{,\varphi}\Phi \\ \cancel{\dot{\Phi}} + H\Phi = 4\pi\dot{\varphi}_0\delta\varphi \end{cases}$$

$$\Phi = \frac{4\pi\dot{\varphi}}{H}\delta\varphi = -\frac{4\pi V_{,\varphi}}{3H^2}$$

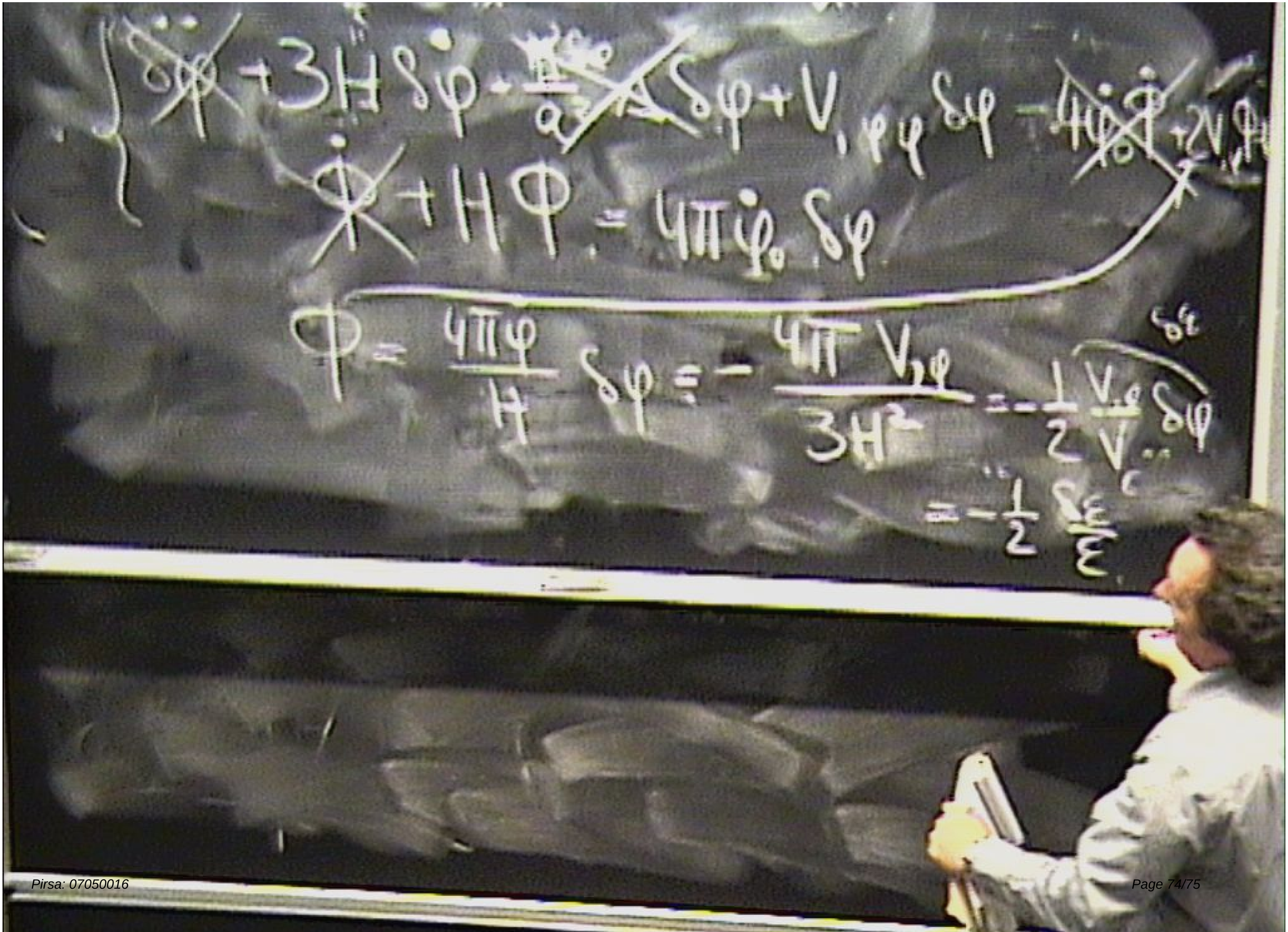
$k\eta \ll 1$

$\lambda_{ph} \gg H^{-1}$ $\quad \eta \to t = \int a\, d\eta \qquad \dot{} \equiv \frac{d}{dt}$

$$\begin{cases} \cancel{\ddot{\delta\varphi}} + 3H\dot{\delta\varphi} + \cancel{\frac{1}{a^2}\Delta\delta\varphi} + V_{,\varphi\varphi}\,\delta\varphi - \cancel{4\dot{\Phi}\dot{\varphi}_0} + 2V_{,\varphi}\Phi \\ \cancel{\dot{\Phi}} + H\Phi = 4\pi\dot{\varphi}_0\,\delta\varphi \end{cases}$$

$$\Phi = \frac{4\pi\dot{\varphi}}{H}\delta\varphi = -\frac{4\pi V_{,\varphi}}{3H^2} = -\frac{1}{2}\overbrace{\frac{V_{,\varphi}}{V_{\varepsilon}}\delta\varphi}^{\delta\varepsilon}$$

$$= -\frac{1}{2}\frac{\delta\varepsilon}{\varepsilon}$$


$\dot{\Phi} + H\Phi = 4\pi\dot{\varphi}_0\,\delta\varphi$
$\Phi = \frac{4\pi\dot{\varphi}}{H}\delta\varphi = -\frac{4\pi V_{,\varphi}}{3H^2} = -\frac{1}{2}\frac{V_{,\varphi}}{V}\delta\varphi$
$= -\frac{1}{2}\frac{\delta\varepsilon}{\varepsilon}$

$$3H\delta\dot{\varphi} + \left(V_{,\varphi\varphi} - \frac{V_{,\varphi}^2}{V}\right)\delta\varphi = 0$$