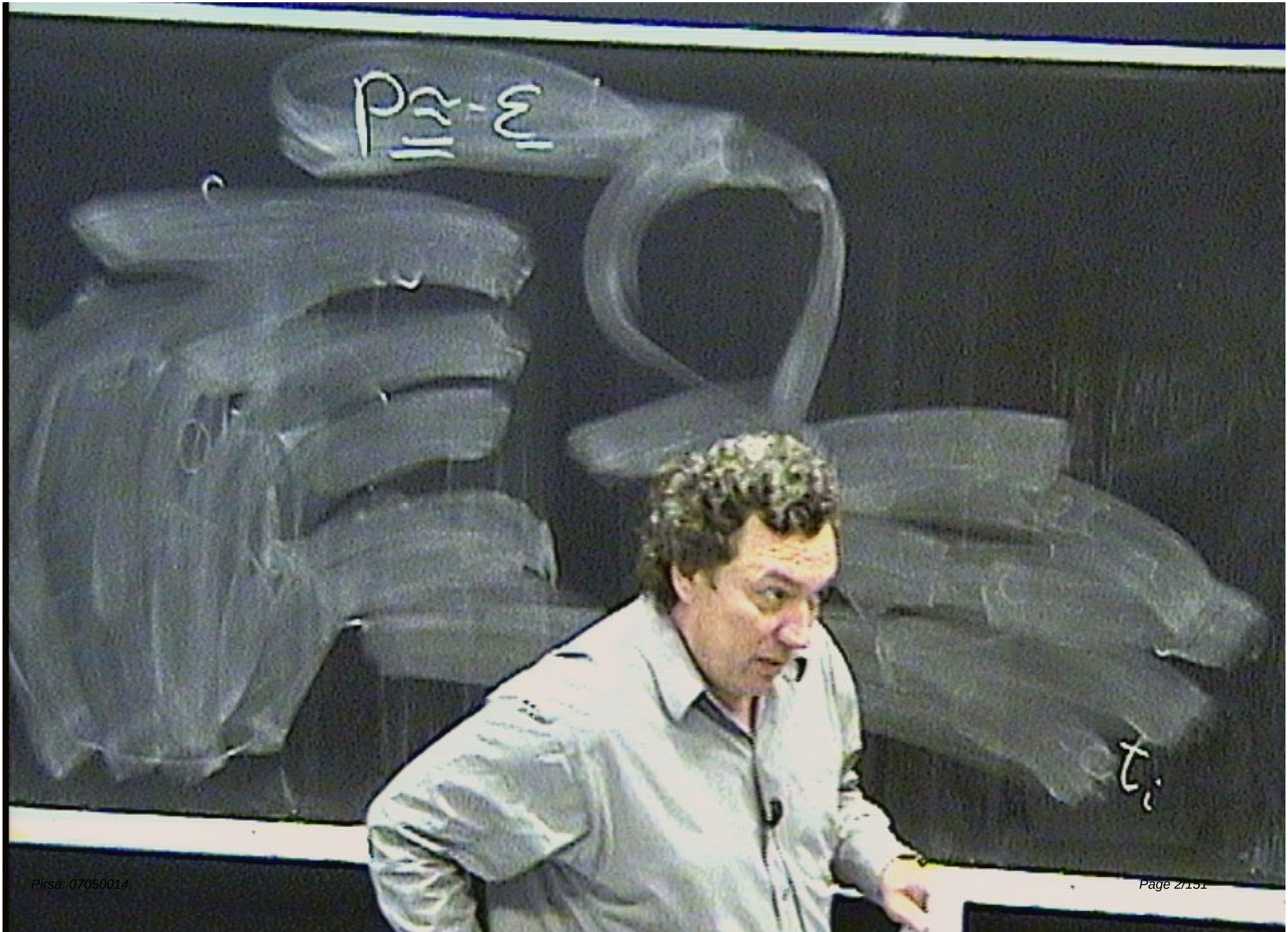
Title: Advanced Topics in Cosmology 3B

Date: May 03, 2007 12:00 PM

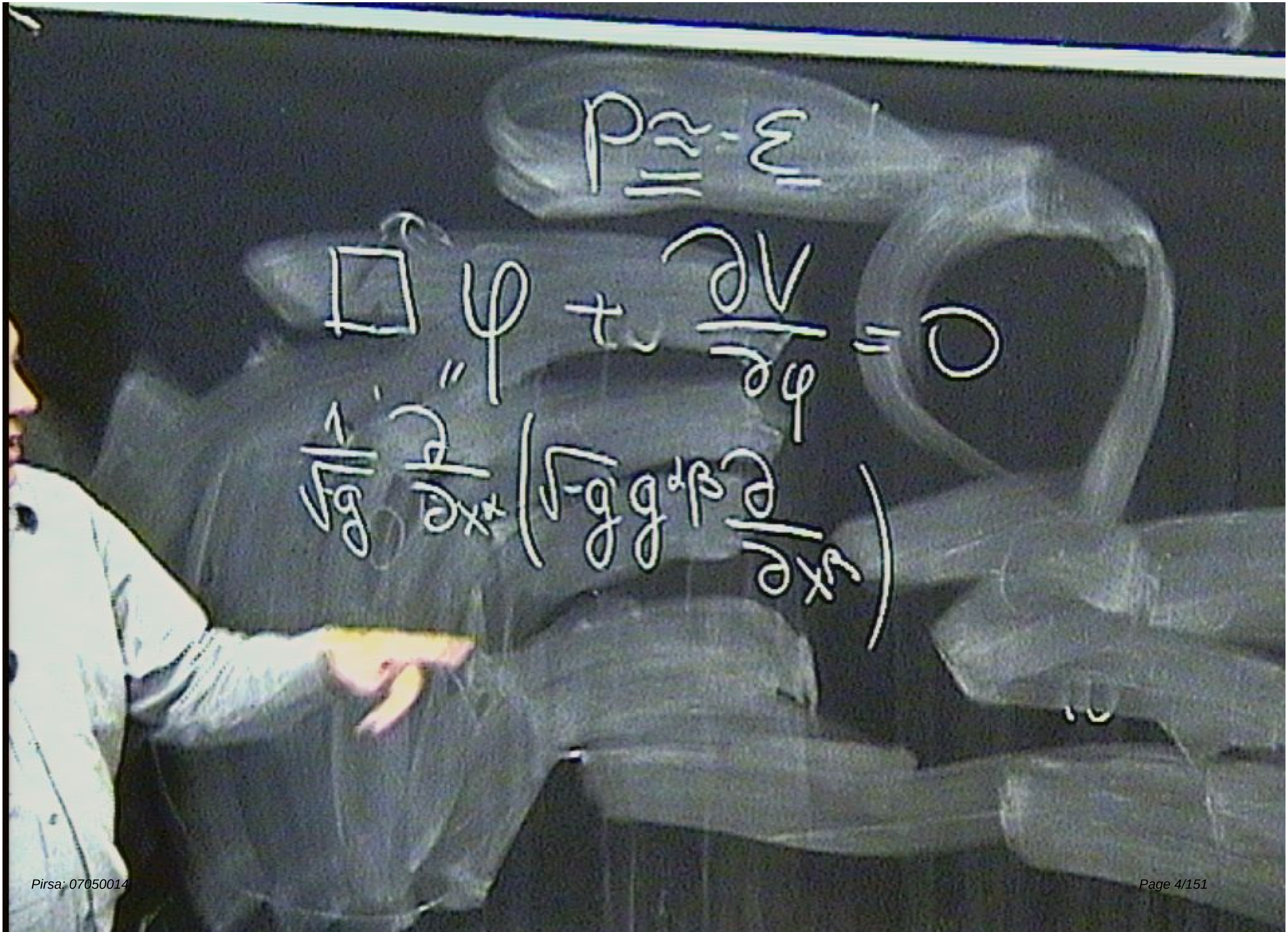
URL: http://pirsa.org/07050014

Abstract: class 3 part 2


p ≃ -ε
tᵢ

$$p \simeq -\varepsilon$$

$$\Box \varphi + \frac{\partial V}{\partial \varphi} = 0$$

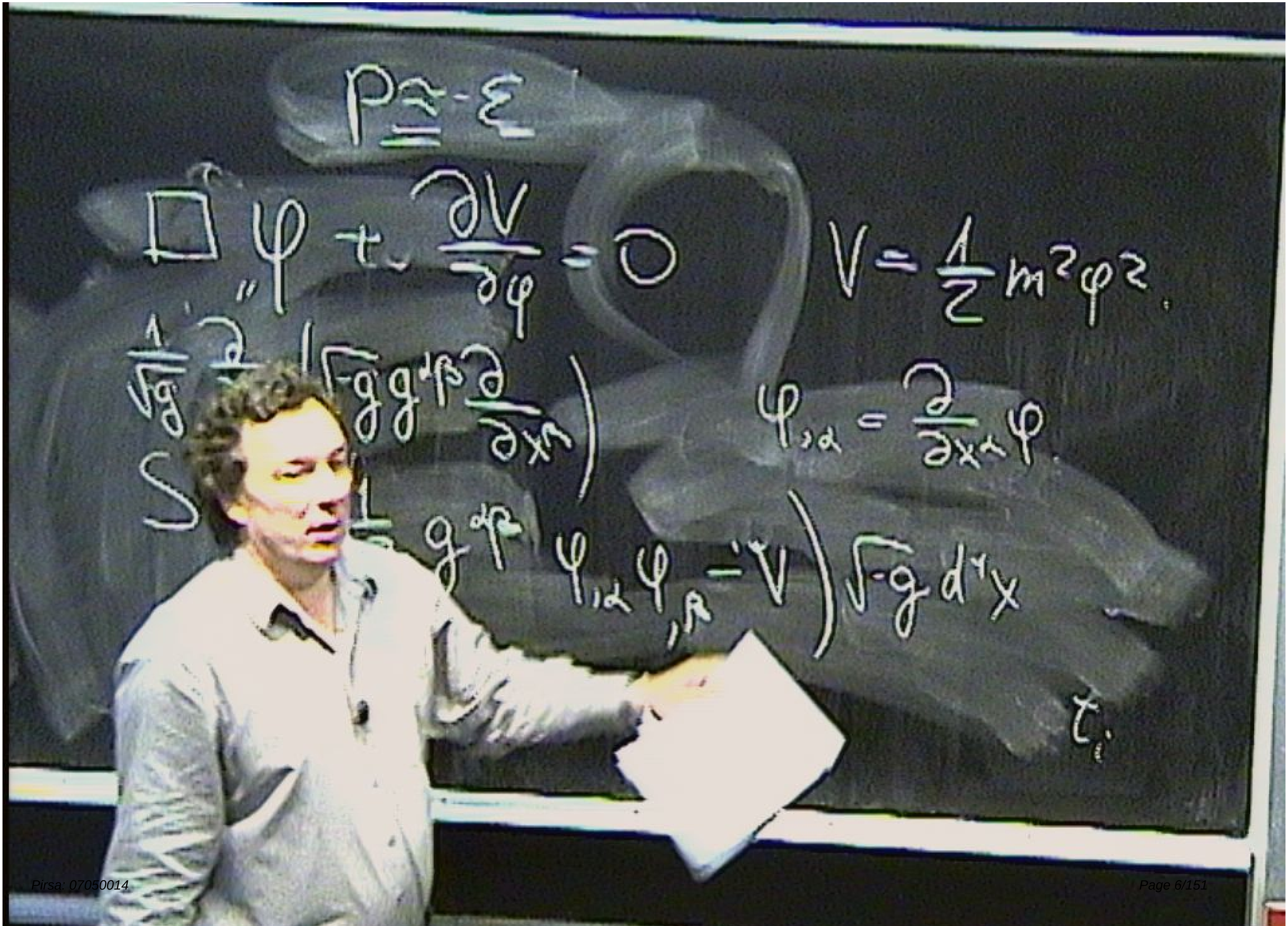

$p \cong -\varepsilon$
$\Box \varphi + \frac{\partial V}{\partial \varphi} = 0$
$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\alpha} \left(\sqrt{-g}\, g^{\alpha\beta} \frac{\partial}{\partial x^\beta} \right)$

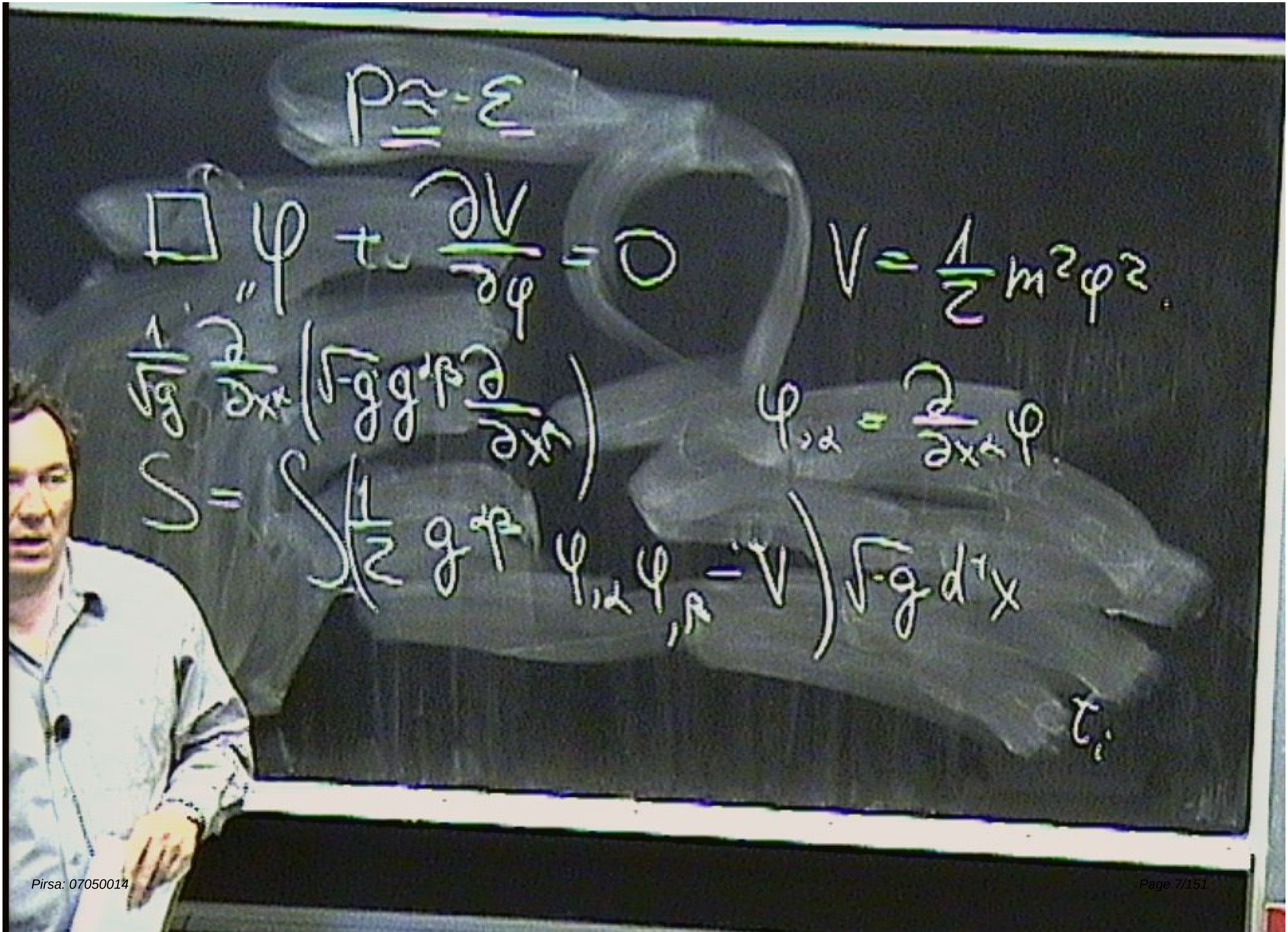
$$p \simeq -\varepsilon$$

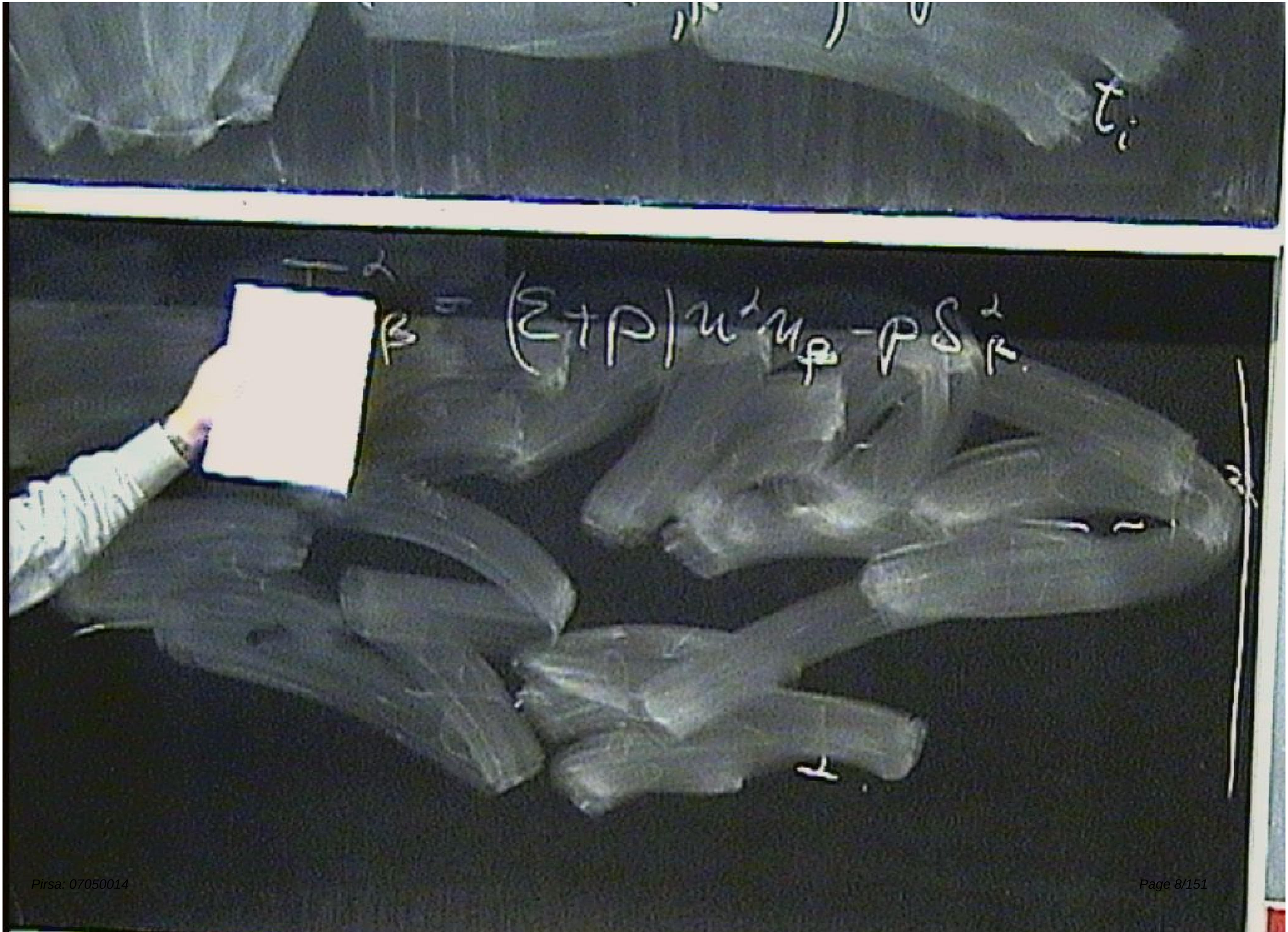
$$\Box \varphi + \frac{\partial V}{\partial \varphi} = 0$$

$$\Box = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}\, g^{\alpha\beta} \frac{\partial}{\partial x^\beta}\right)$$

$$S =$$


p ≃ -ε
□φ + ∂V/∂φ = 0
V = 1/2 m²φ²
φ,α = ∂/∂x^α φ
√-g d⁴x
t_i


$p \simeq -\varepsilon$
$\Box \varphi + \frac{\partial V}{\partial \varphi} = 0$
$V = \frac{1}{2} m^2 \varphi^2$
$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\alpha} \left(\sqrt{-g}\, g^{\alpha\beta} \frac{\partial}{\partial x^\beta} \right)$
$\varphi_{,\alpha} = \frac{\partial}{\partial x^\alpha} \varphi$
$S = \int \left(\frac{1}{2} g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} - V \right) \sqrt{-g}\, d^4x$
t_i


$T^{\alpha}_{\beta} = (\varepsilon + p) u^{\alpha} u_{\beta} - p \delta^{\alpha}_{\beta}$
t_i

$$p \simeq -\varepsilon$$

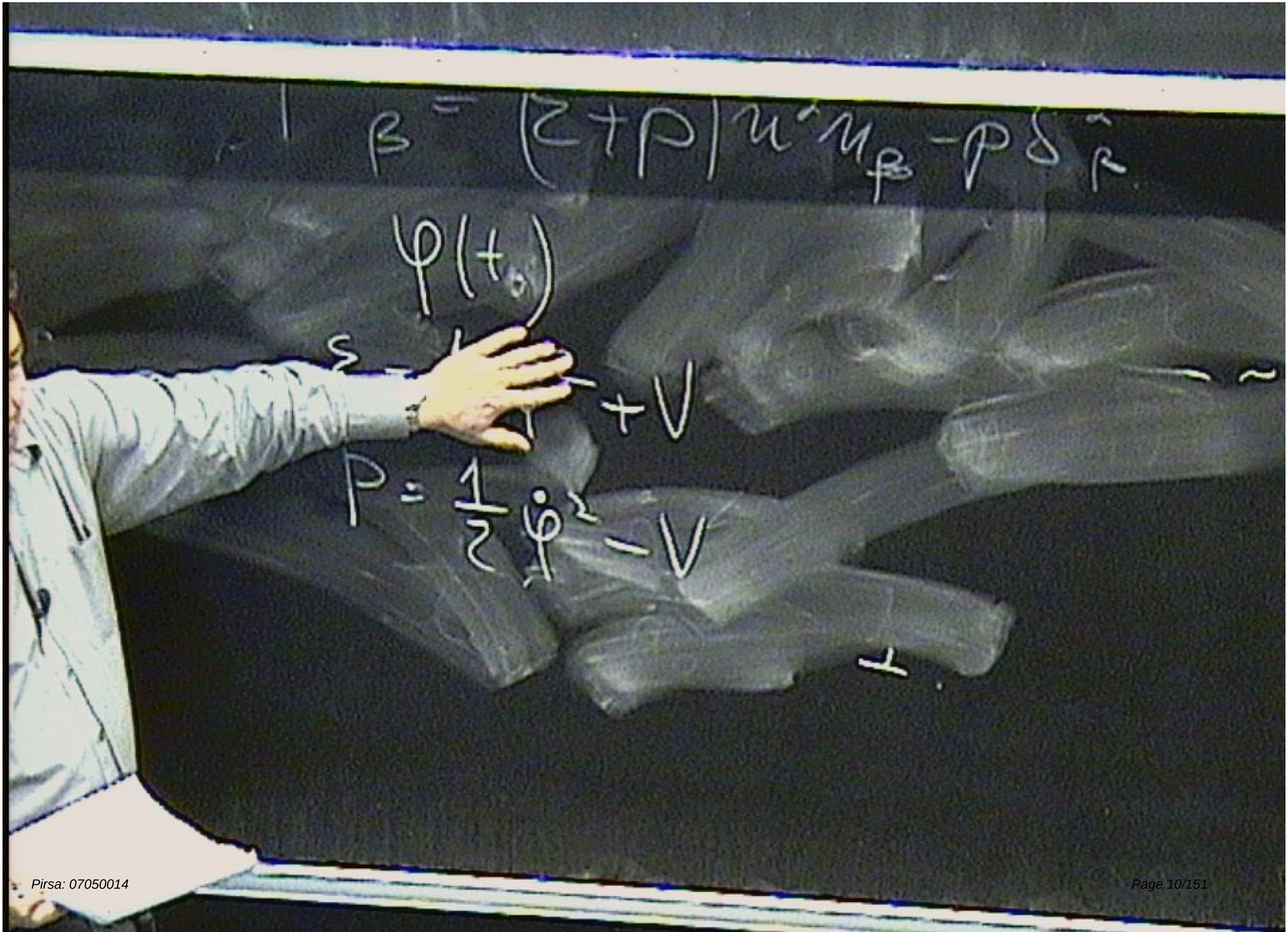
$$\Box \varphi + \frac{\partial V}{\partial \varphi} = 0 \qquad V = \frac{1}{2} m^2 \varphi^2 .$$

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\alpha} \left(\sqrt{-g}\, g^{\alpha\beta} \frac{\partial}{\partial x^\beta} \right) \qquad \varphi_{,\alpha} = \frac{\partial}{\partial x^\alpha} \varphi$$

$$S = \int \left(\frac{1}{2} g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} - V \right) \sqrt{-g}\, d^4x$$

t_i

$$T^\alpha_\beta = (\varepsilon + p) u^\alpha u_\beta - p \delta^\alpha_\beta$$


β = (ε+p)u^α u_β − p δ^α_β
φ(t₀)
+V
p = 1/2 φ̇² − V

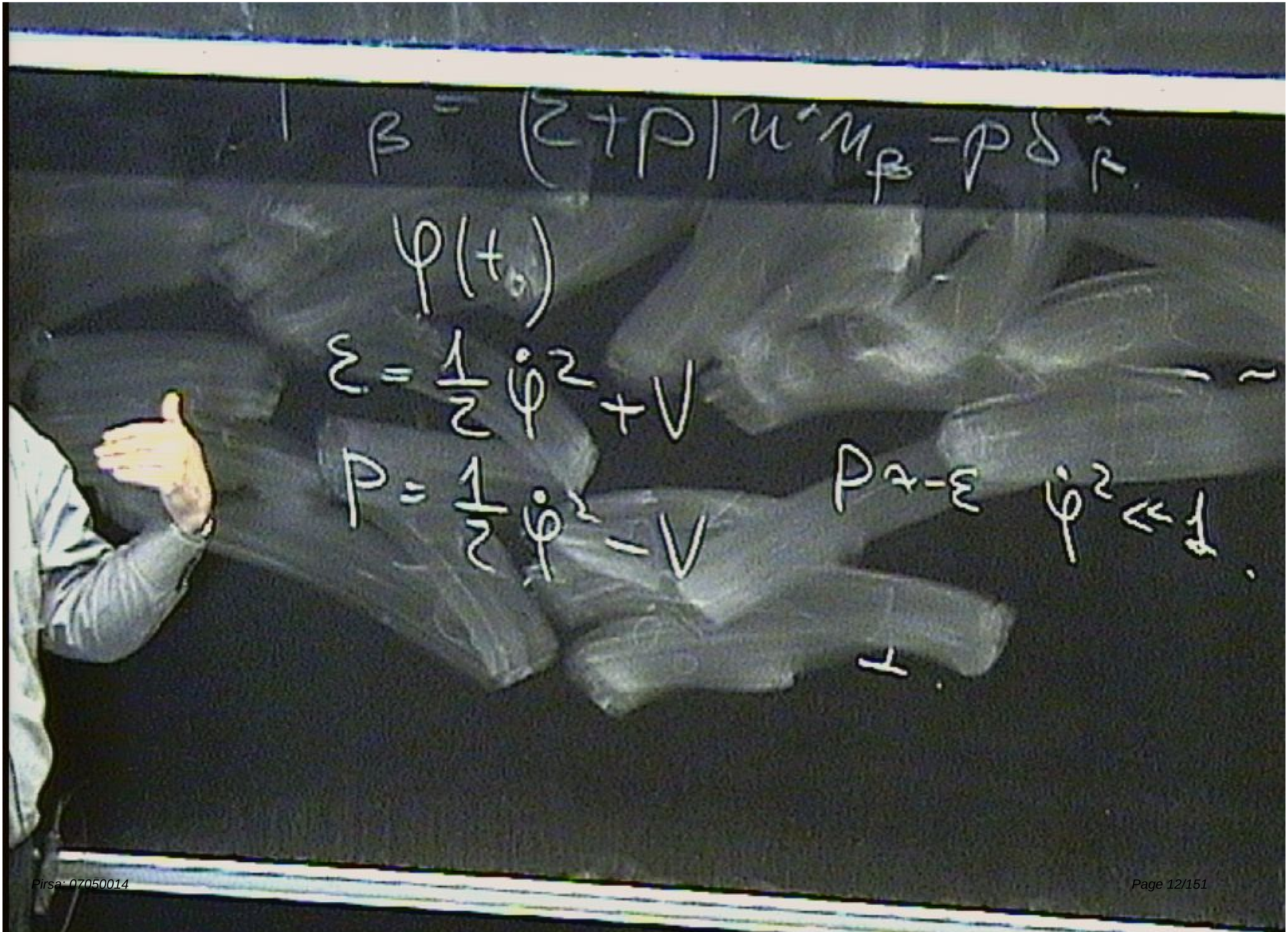
$$T^{\alpha}_{\ \beta} = (\varepsilon + p) u^{\alpha} u_{\beta} - p \delta^{\alpha}_{\beta}$$

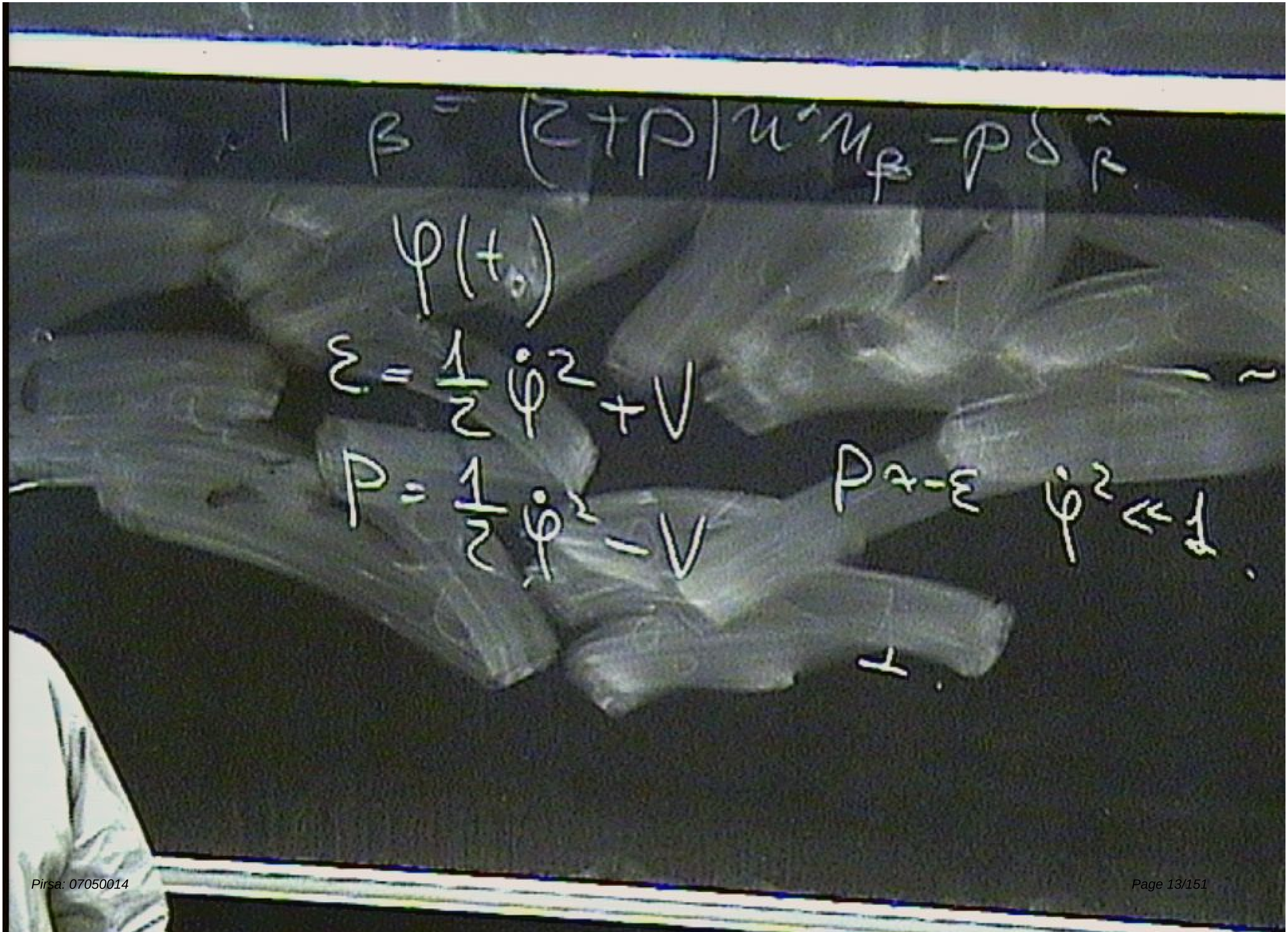
$$\varphi(t_0)$$

$$\varepsilon = \frac{1}{2}\dot{\varphi}^2 + V$$

$$p = \frac{1}{2}\dot{\varphi}^2 - V$$

$$p \approx -\varepsilon \qquad \dot{\varphi}^2 \ll 1.$$


$\beta = (\varepsilon + p) u^\alpha u_\beta - p \delta^\alpha_\beta$
$\varphi(t_0)$
$\varepsilon = \frac{1}{2}\dot{\varphi}^2 + V$
$p = \frac{1}{2}\dot{\varphi}^2 - V$
$p \approx -\varepsilon$
$\dot{\varphi}^2 \ll 1$


$T^{\alpha}_{\beta} = (\varepsilon + p) u^{\alpha} u_{\beta} - p \delta^{\alpha}_{\beta}$
$\varphi(t_0)$
$\varepsilon = \frac{1}{2}\dot{\varphi}^2 + V$
$p = \frac{1}{2}\dot{\varphi}^2 - V$
$p \approx -\varepsilon \quad \dot{\varphi}^2 \ll 1$

$T^{\alpha}_{\ \beta} = (\varepsilon + p) u^{\alpha} u_{\beta} - p \delta^{\alpha}_{\beta}$

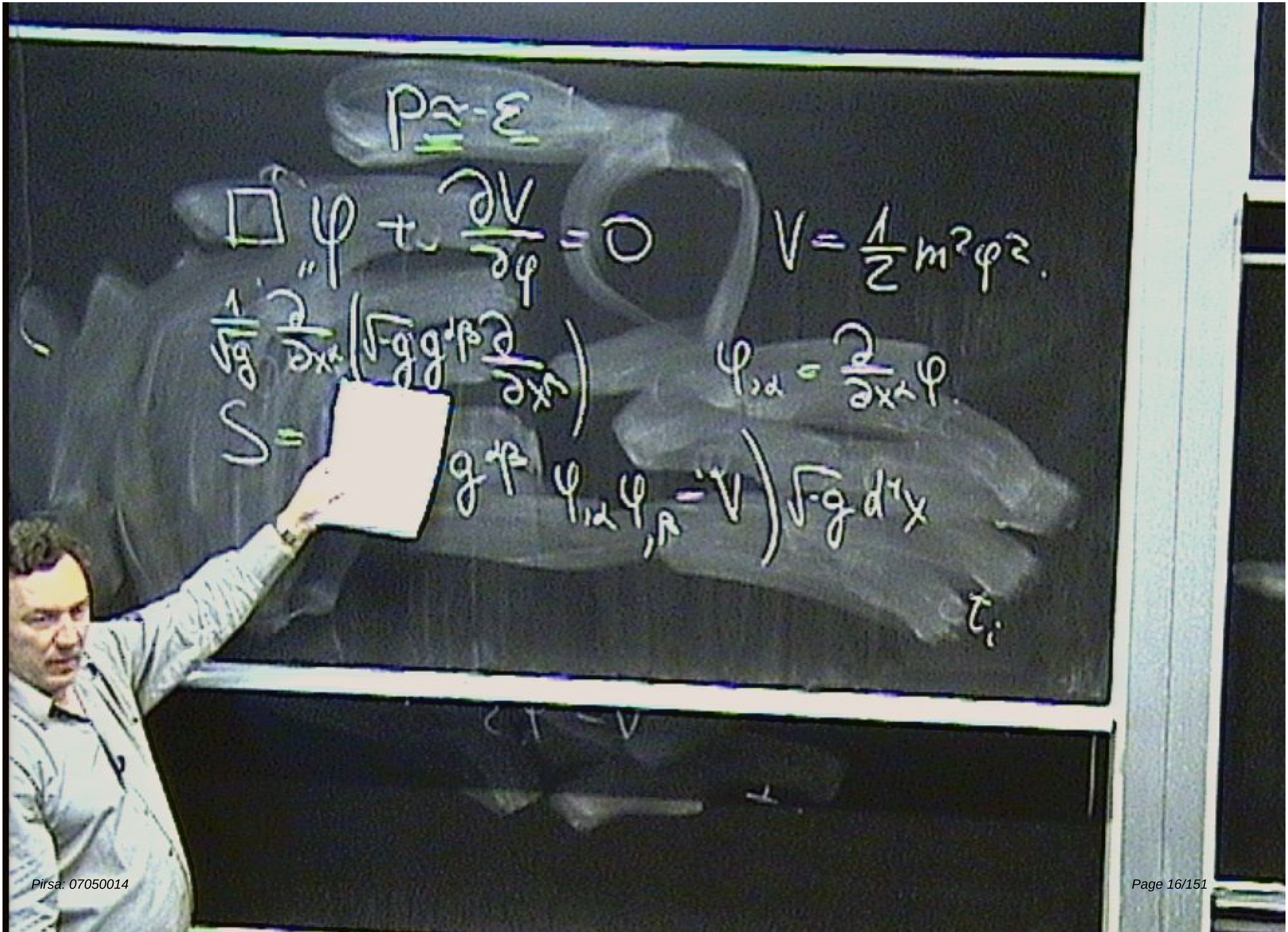
$\varphi(t_0)$

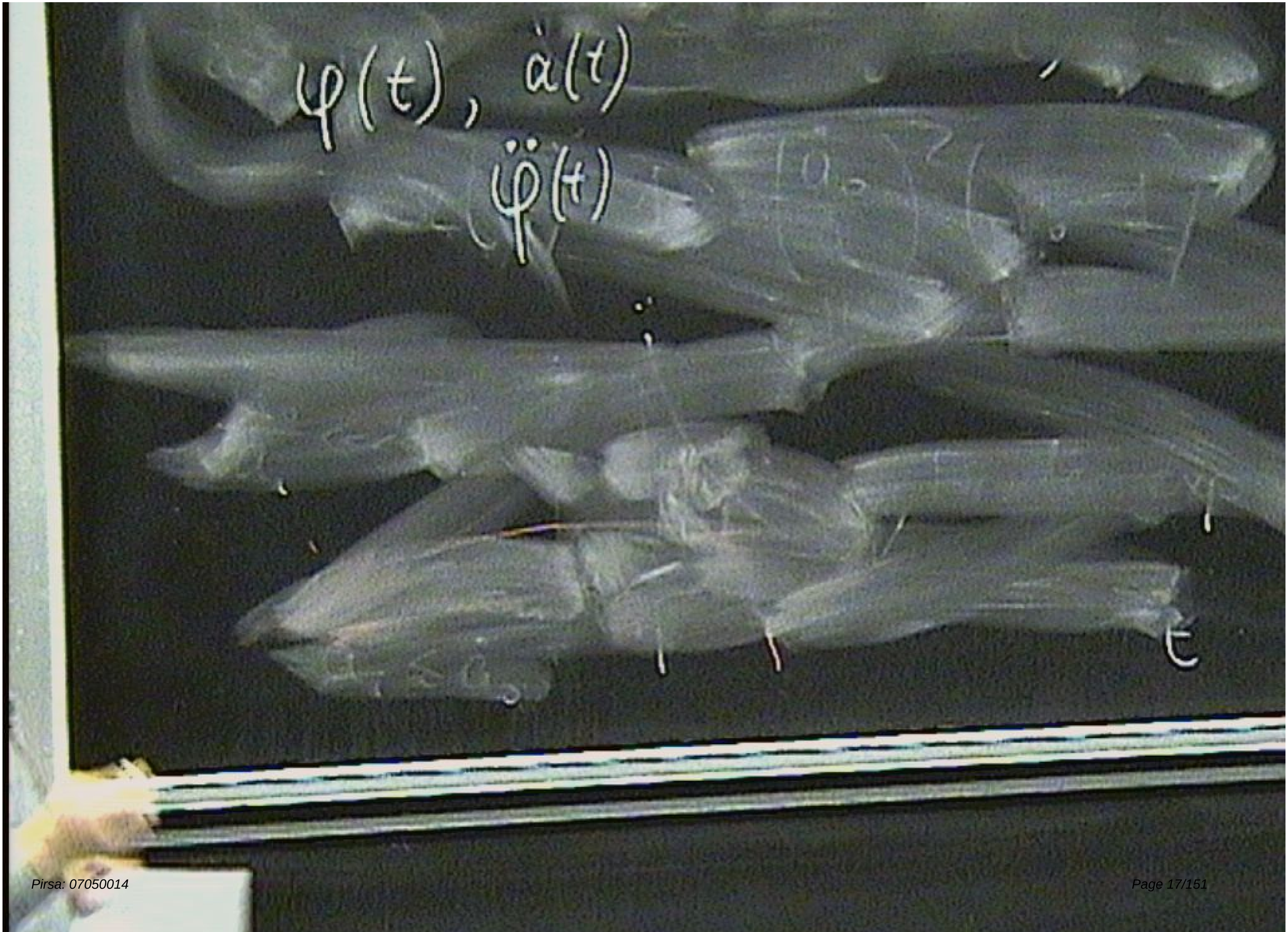
$\varepsilon = \frac{1}{2}\dot{\varphi}^2 + V$

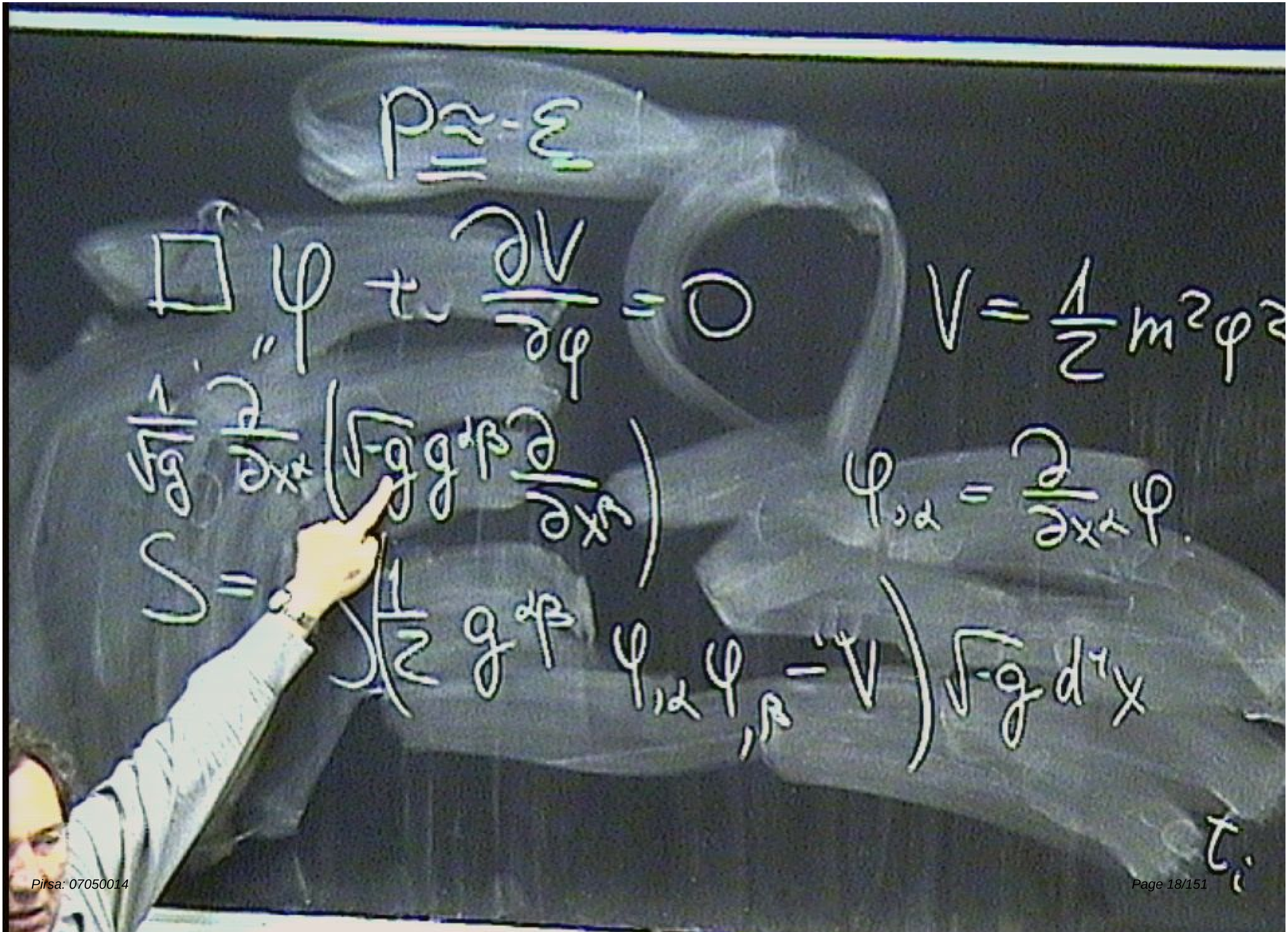
$p = \frac{1}{2}\dot{\varphi}^2 - V$

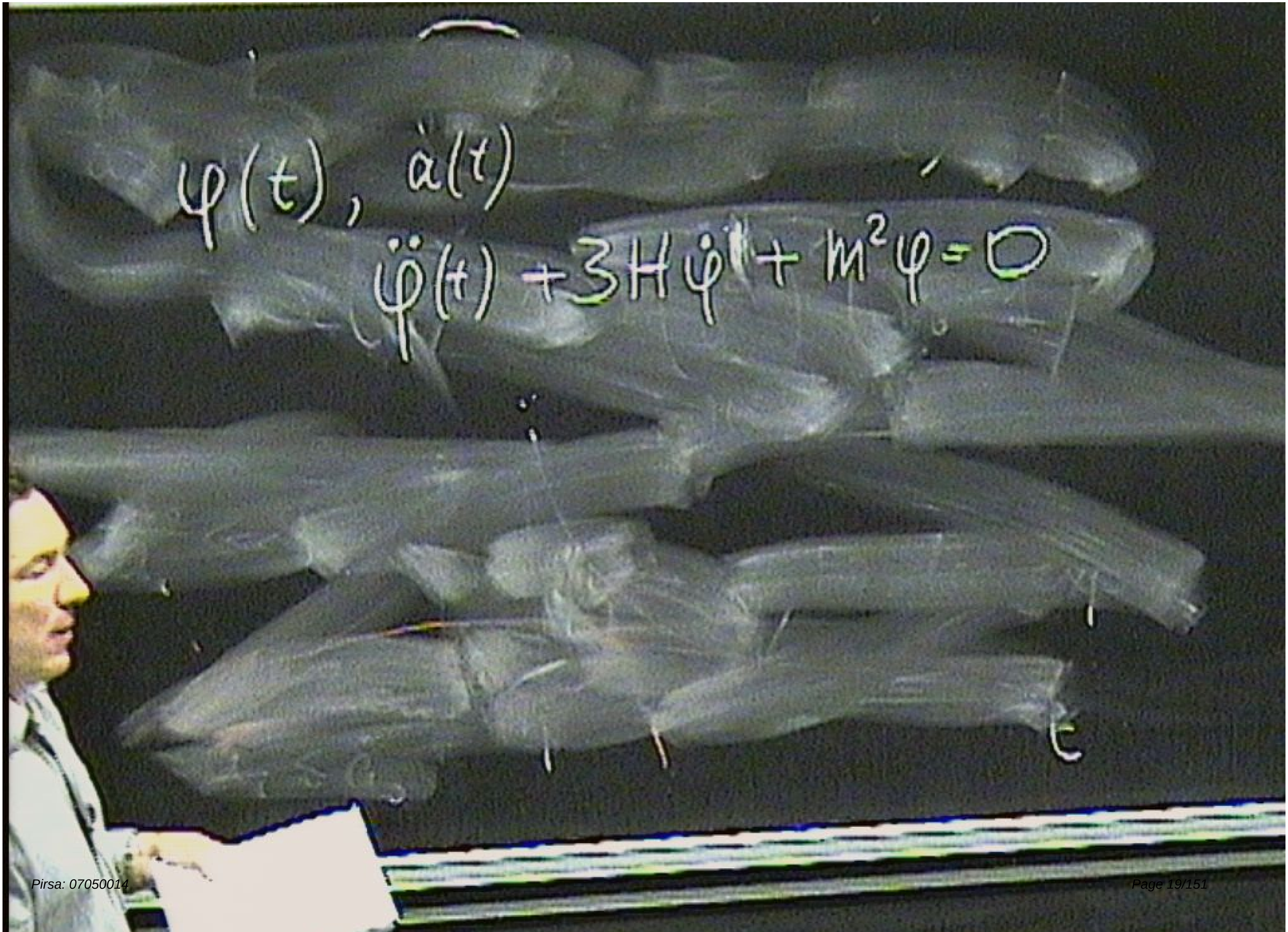
$p \approx -\varepsilon \quad \dot{\varphi}^2 \ll 1.$

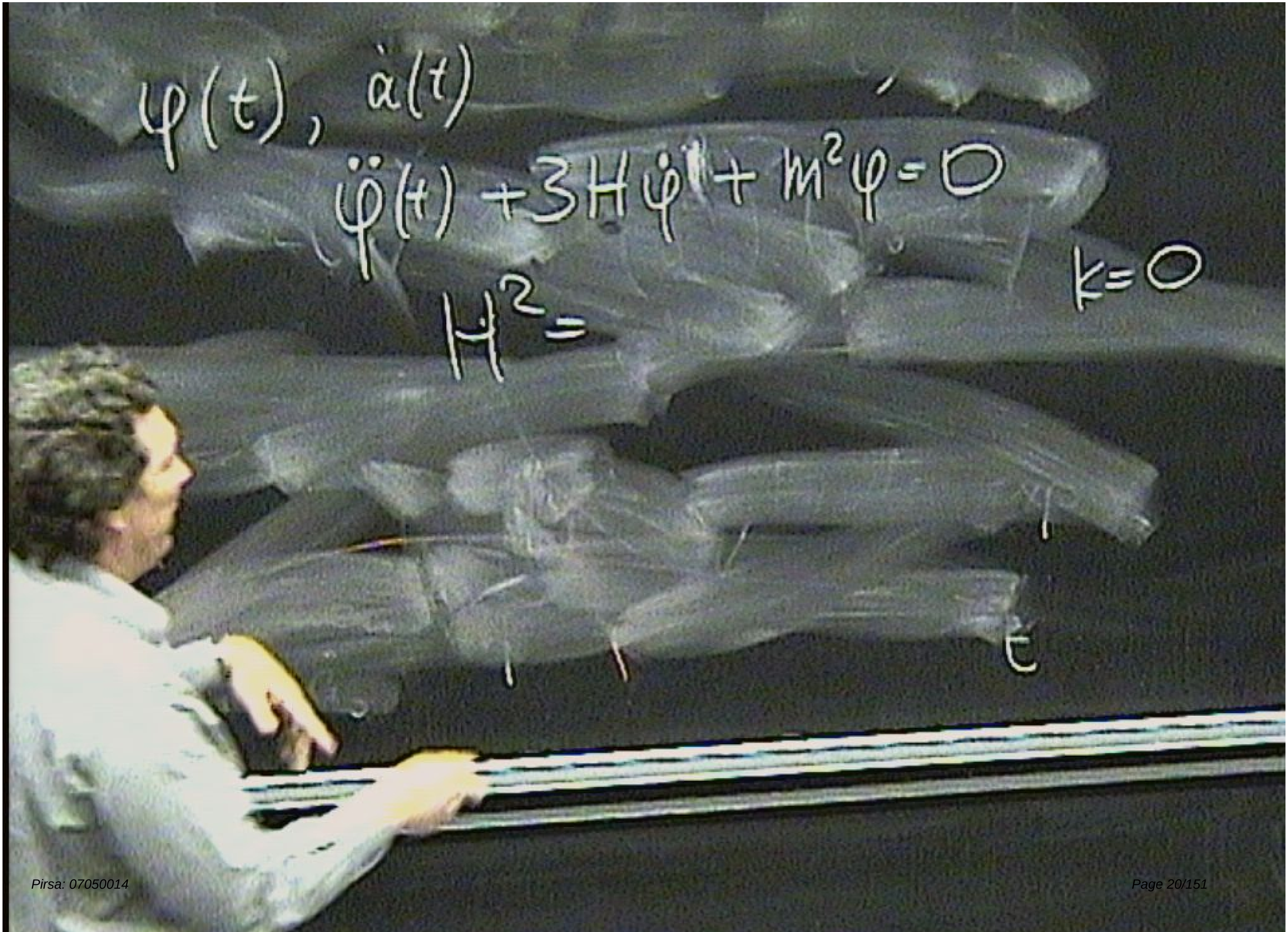
$\varphi(t), \quad \dot{a}(t)$

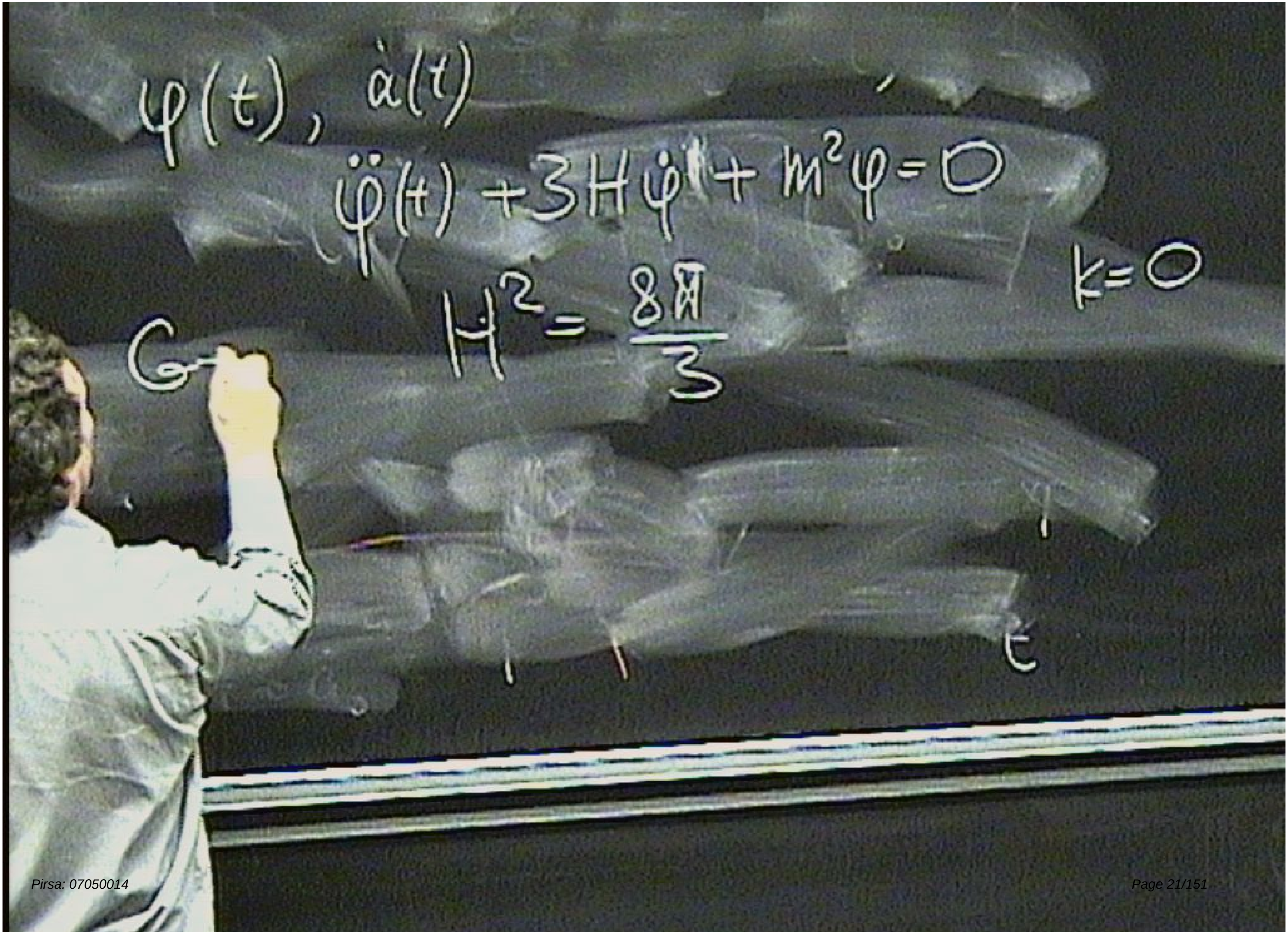

p ≈ -ε
□φ + ∂V/∂φ = 0
V = 1/2 m²φ²
1/√-g ∂/∂x^α (√-g g^αβ ∂/∂x^β)
φ,α = ∂/∂x^α φ
S =
g^αβ φ,α φ,β - V) √-g d⁴x
t_i

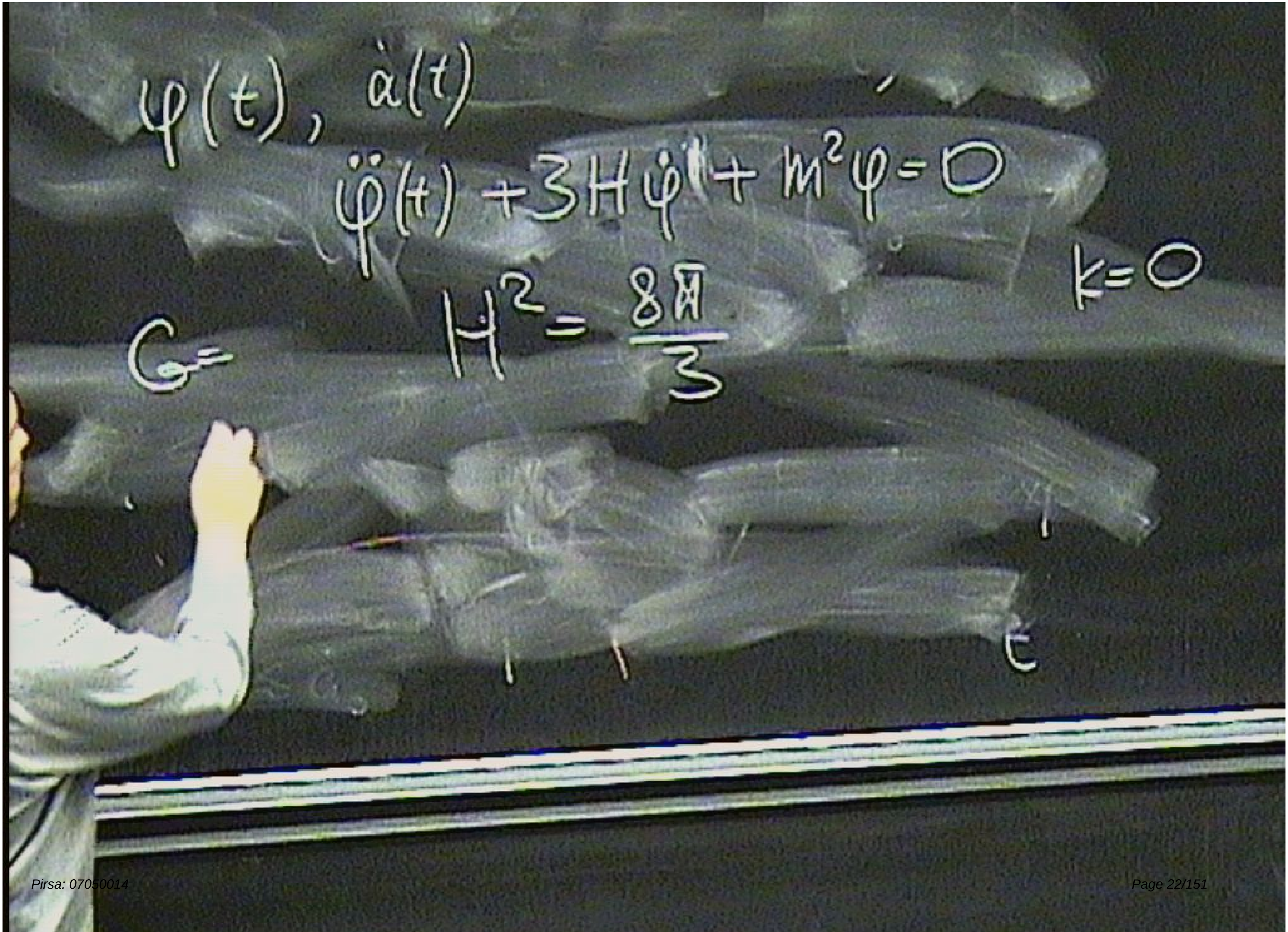

φ(t), ȧ(t)
φ̈(t)
t

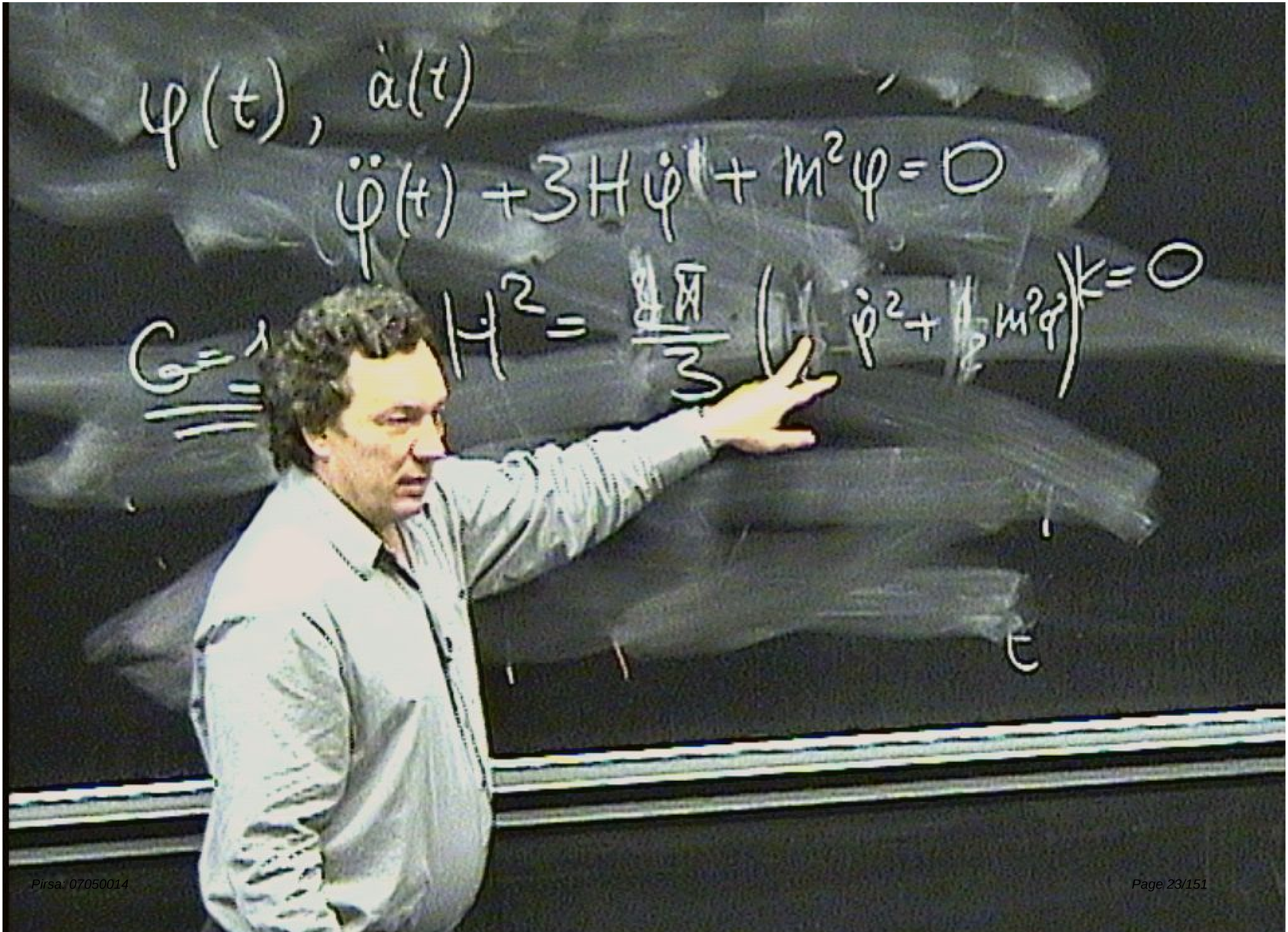

$p \approx -\varepsilon$
$\Box \varphi + \frac{\partial V}{\partial \varphi} = 0$
$V = \frac{1}{2} m^2 \varphi^2$
$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\alpha} \left(\sqrt{-g}\, g^{\alpha\beta} \frac{\partial}{\partial x^\beta} \right)$
$\varphi_{,\alpha} = \frac{\partial}{\partial x^\alpha} \varphi$
$S = \int \left(\frac{1}{2} g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} - V \right) \sqrt{-g}\, d^4x$
t_i


$\varphi(t), \ddot{a}(t)$
$\ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0$
t

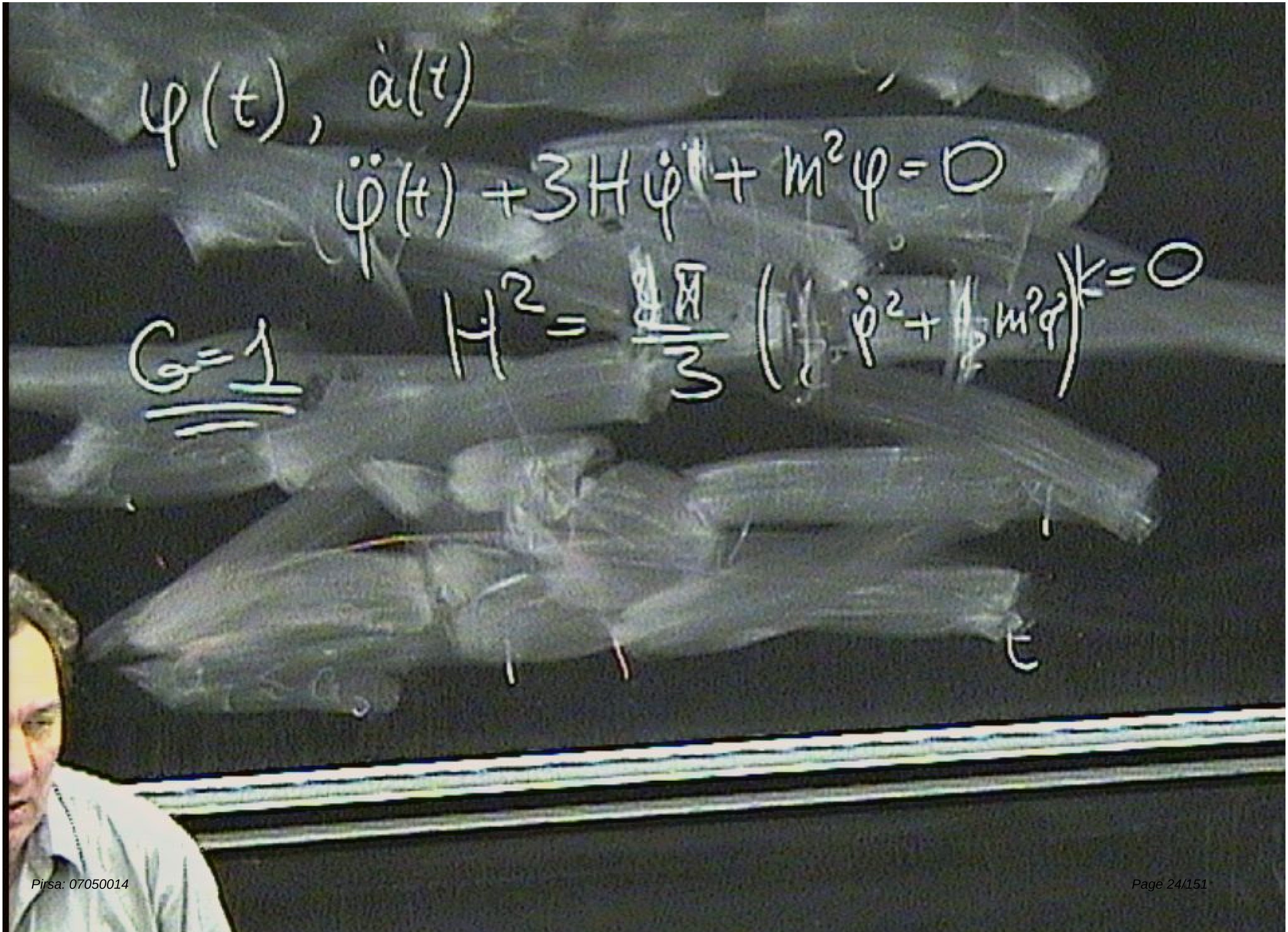

$\varphi(t), \dot{a}(t)$
$\ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0$
$k=0$
$H^2 =$
t


$\varphi(t),\ \dot{a}(t)$
$\ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0$
$k=0$
$H^2 = \frac{8\pi}{3}$
G
t


φ(t), ȧ(t)
φ̈(t) + 3Hφ̇ + m²φ = 0
k=0
H² = 8π/3
G=
t



φ(t), ȧ(t)
φ̈(t) + 3Hφ̇ + m²φ = 0
G = 1
H² = 8π/3 (½ φ̇² + ½ m²φ²)
k = 0
t

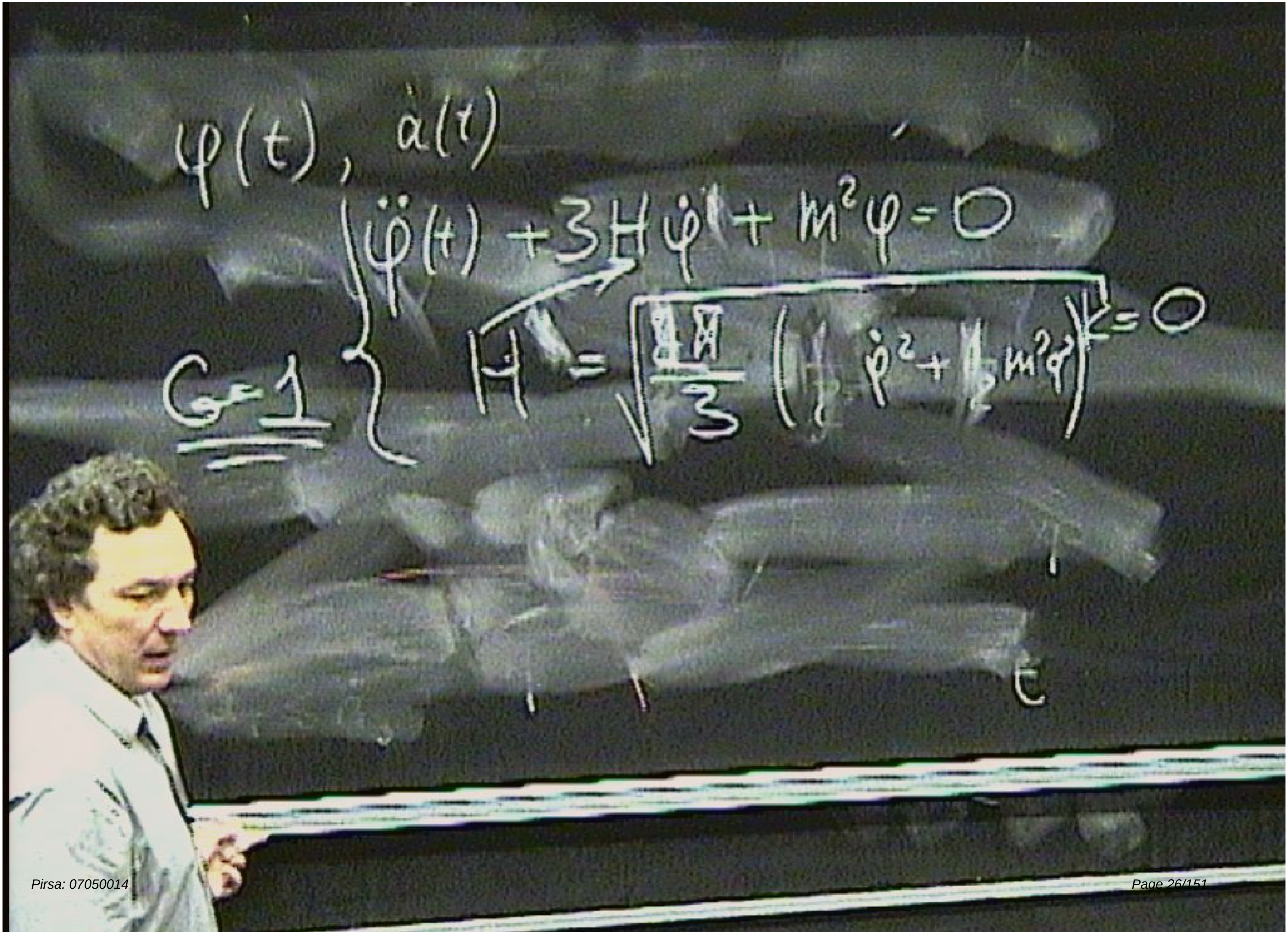

φ(t), ȧ(t)
φ̈(t) + 3Hφ̇ + m²φ = 0
k = 0
H² = 8π/3 (½ φ̇² + ½ m²φ²)
G = 1
t

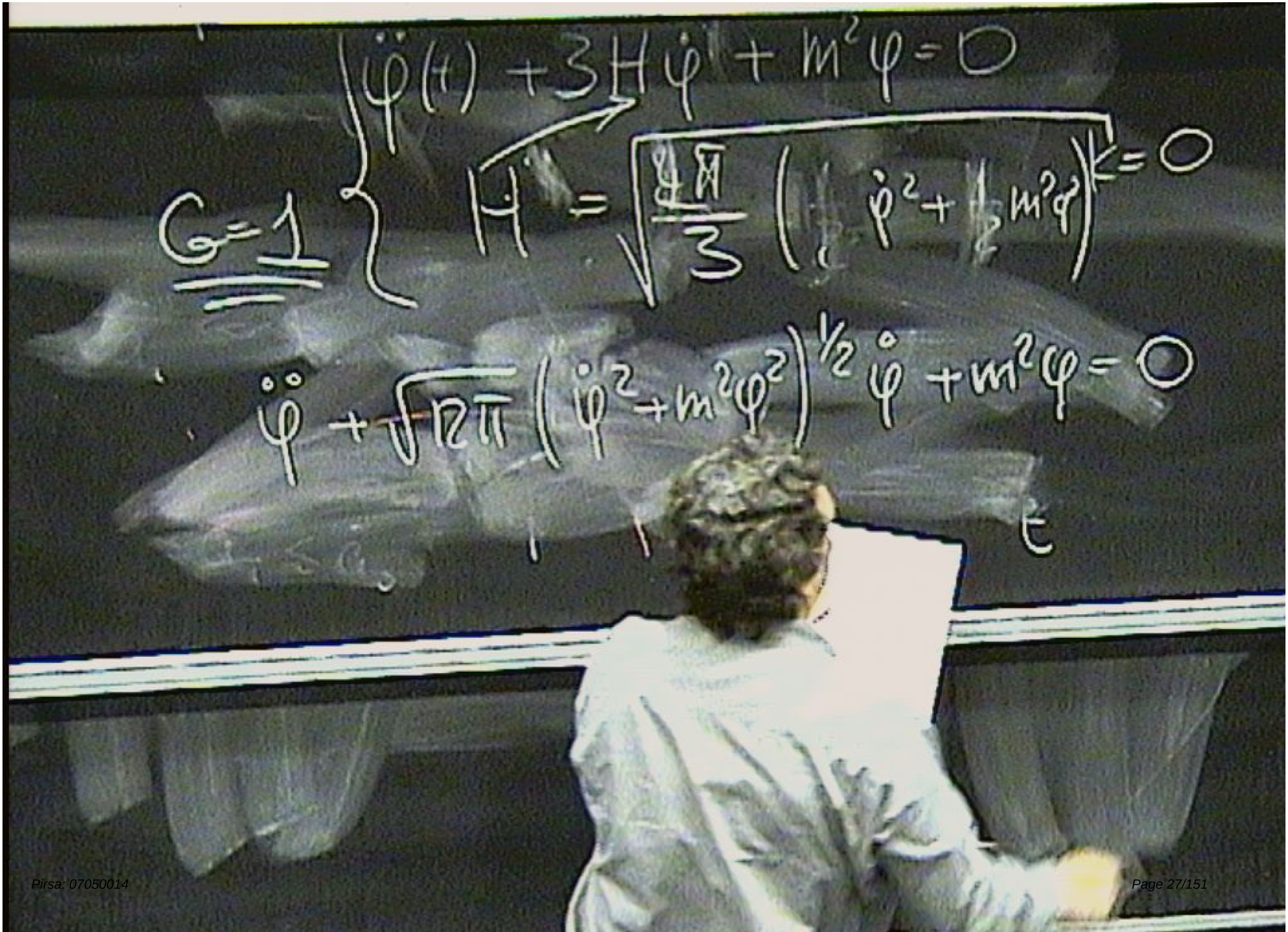
$\varphi(t), \ \dot{a}(t)$

$\ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0$

$G = 1 \quad H^2 = \frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2\right) \quad k = 0$

t


φ(t), ȧ(t)
φ̈(t) + 3Hφ̇ + m²φ = 0
H = √(8π/3 (½ φ̇² + ½ m²φ²))
G = 1



$\ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0$
$G = 1$
$H = \sqrt{\frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2\right)}$
$\ddot{\varphi} + \sqrt{12\pi}\left(\dot{\varphi}^2 + m^2\varphi^2\right)^{1/2}\dot{\varphi} + m^2\varphi = 0$
t

$$G = 1 \quad \begin{cases} \ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0 \\ H = \sqrt{\frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2\right)} \quad k = 0 \end{cases}$$

$$\ddot{\varphi} + \sqrt{12\pi}\left(\dot{\varphi}^2 + m^2\varphi^2\right)^{1/2}\dot{\varphi} + m^2\varphi = 0$$

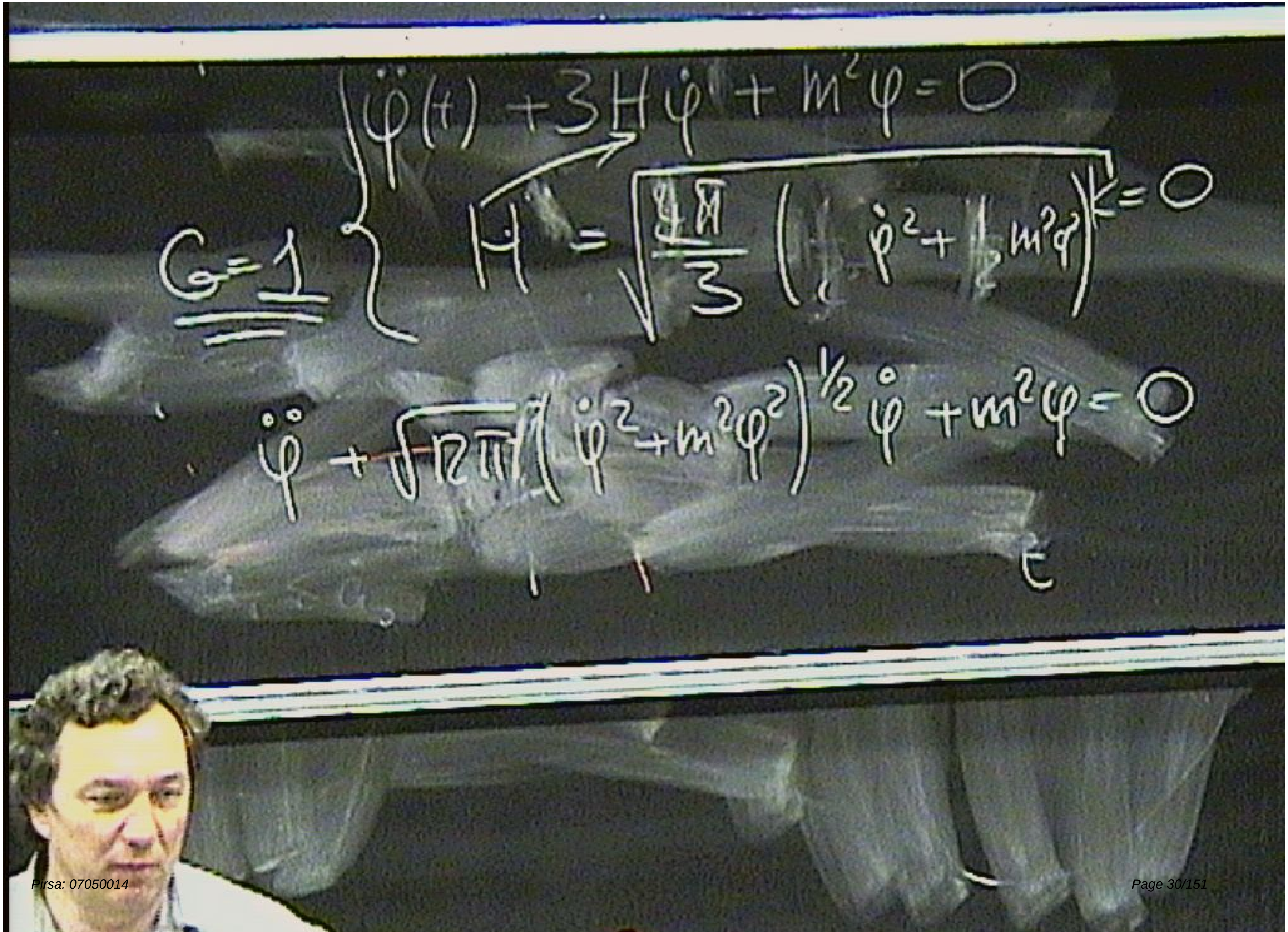
t

$G = 1$

$$\begin{cases} \ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0 \\ H = \sqrt{\frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2\right)} \quad k = 0 \end{cases}$$

$$\ddot{\varphi} + \sqrt{12\pi}\left(\dot{\varphi}^2 + m^2\varphi^2\right)^{1/2}\dot{\varphi} + m^2\varphi = 0$$

t

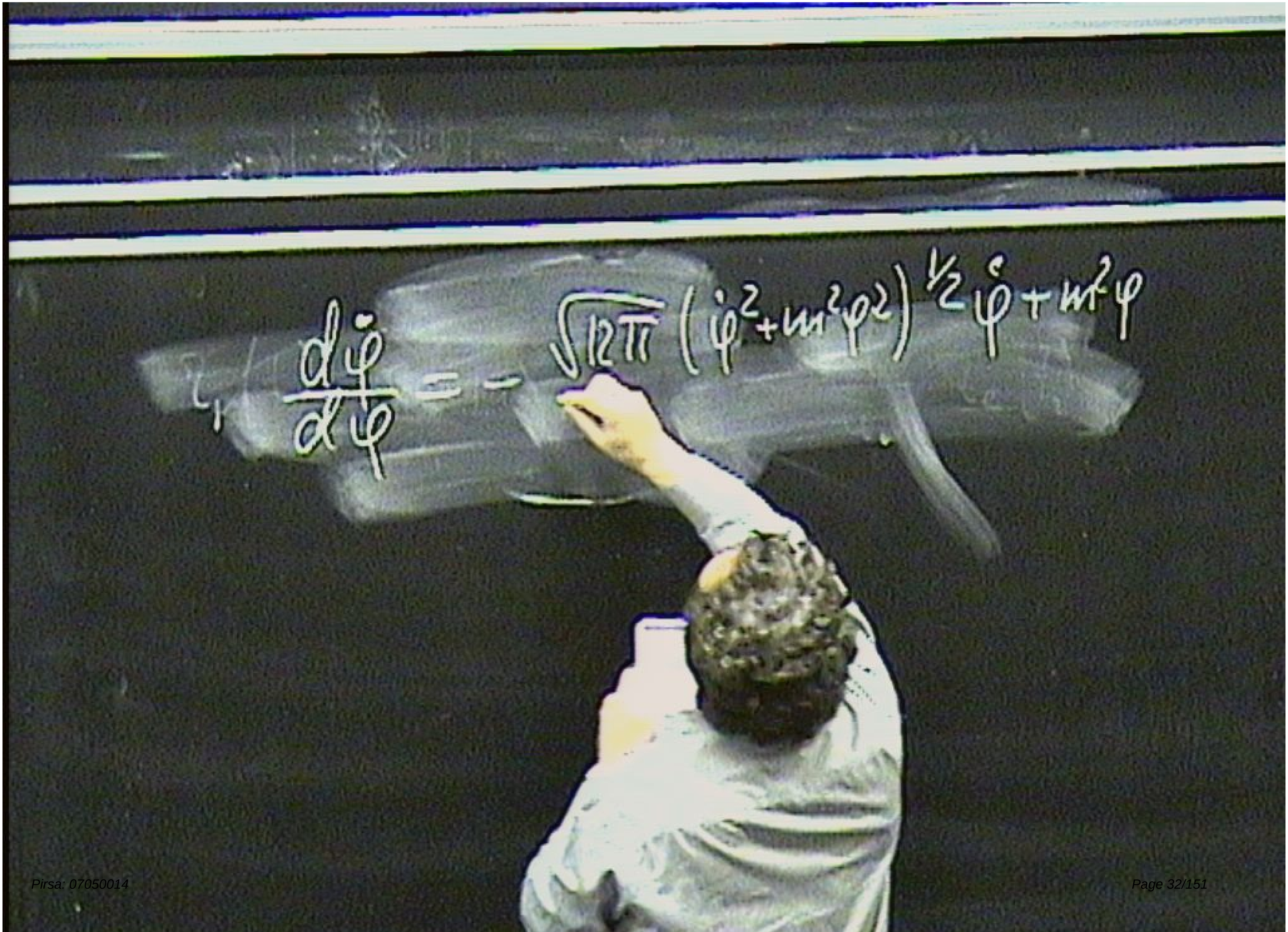


$\ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0$
$G = 1$
$H = \sqrt{\frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2\right)}$
$k = 0$
$\ddot{\varphi} + \sqrt{12\pi}\left(\dot{\varphi}^2 + m^2\varphi^2\right)^{1/2}\dot{\varphi} + m^2\varphi = 0$
t

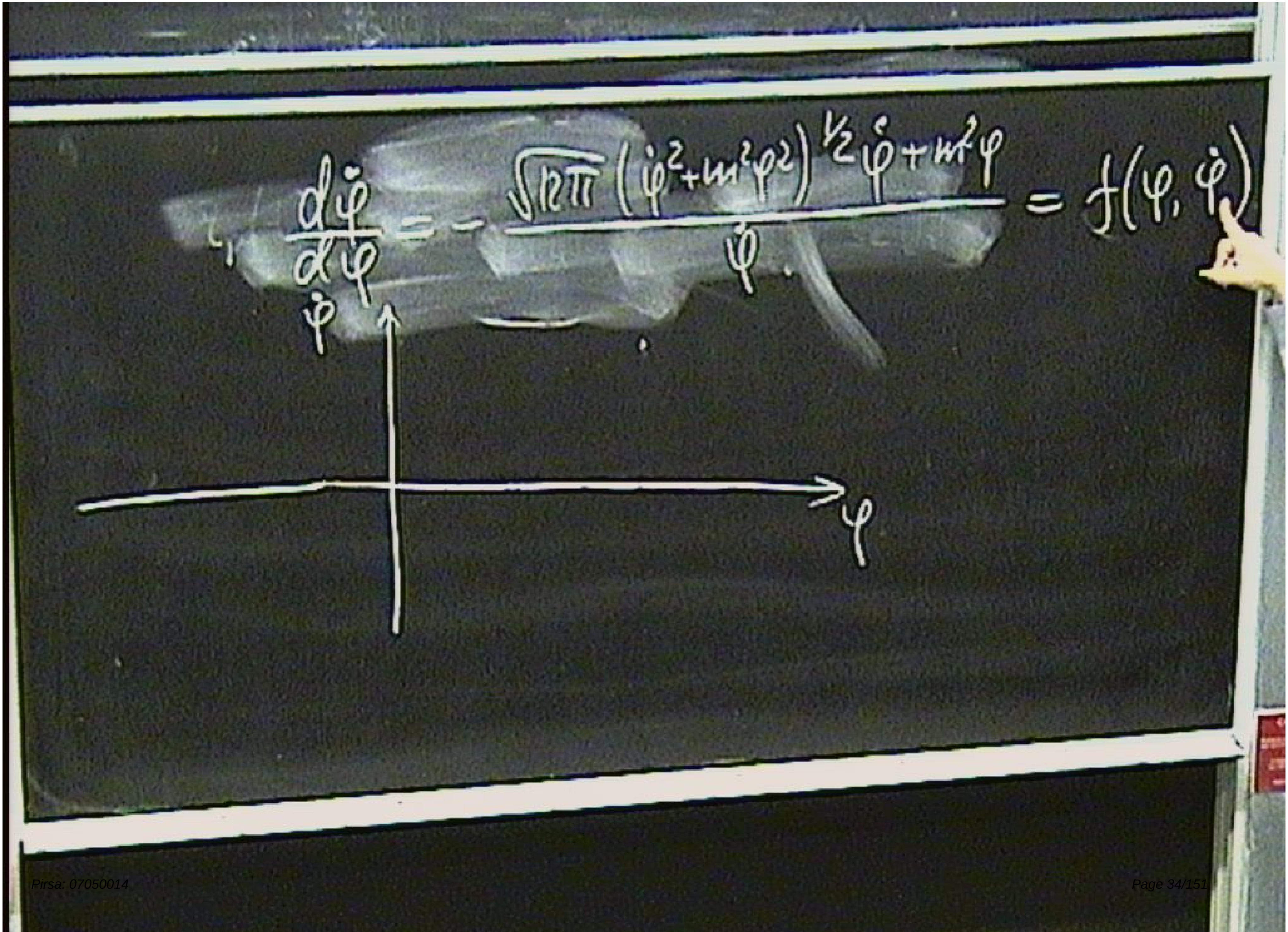
$$G = 1 \quad \begin{cases} \ddot{\varphi}(t) + 3H\dot{\varphi} + m^2\varphi = 0 \\ H = \sqrt{\frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m^2\varphi^2\right)} \quad k = 0 \end{cases}$$

$$\ddot{\varphi} + \sqrt{12\pi}\left(\dot{\varphi}^2 + m^2\varphi^2\right)^{1/2}\dot{\varphi} + m^2\varphi = 0$$

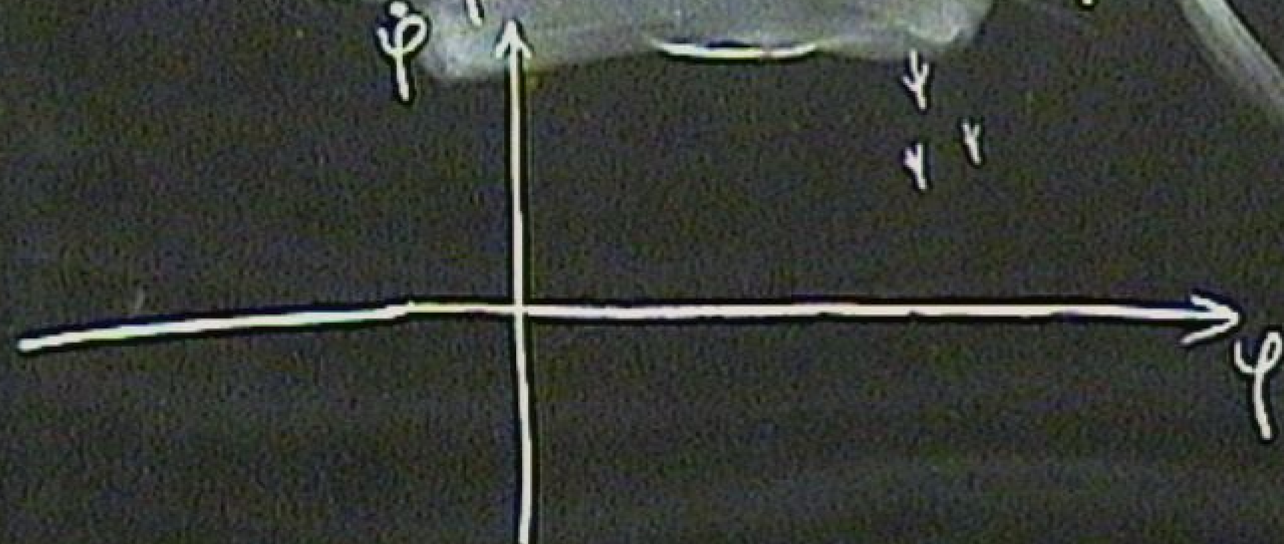
$$\dot{\varphi}(\varphi), \quad \ddot{\varphi} = \frac{d\dot{\varphi}}{dt} = \dot{\varphi}\frac{d\dot{\varphi}}{d\varphi}$$

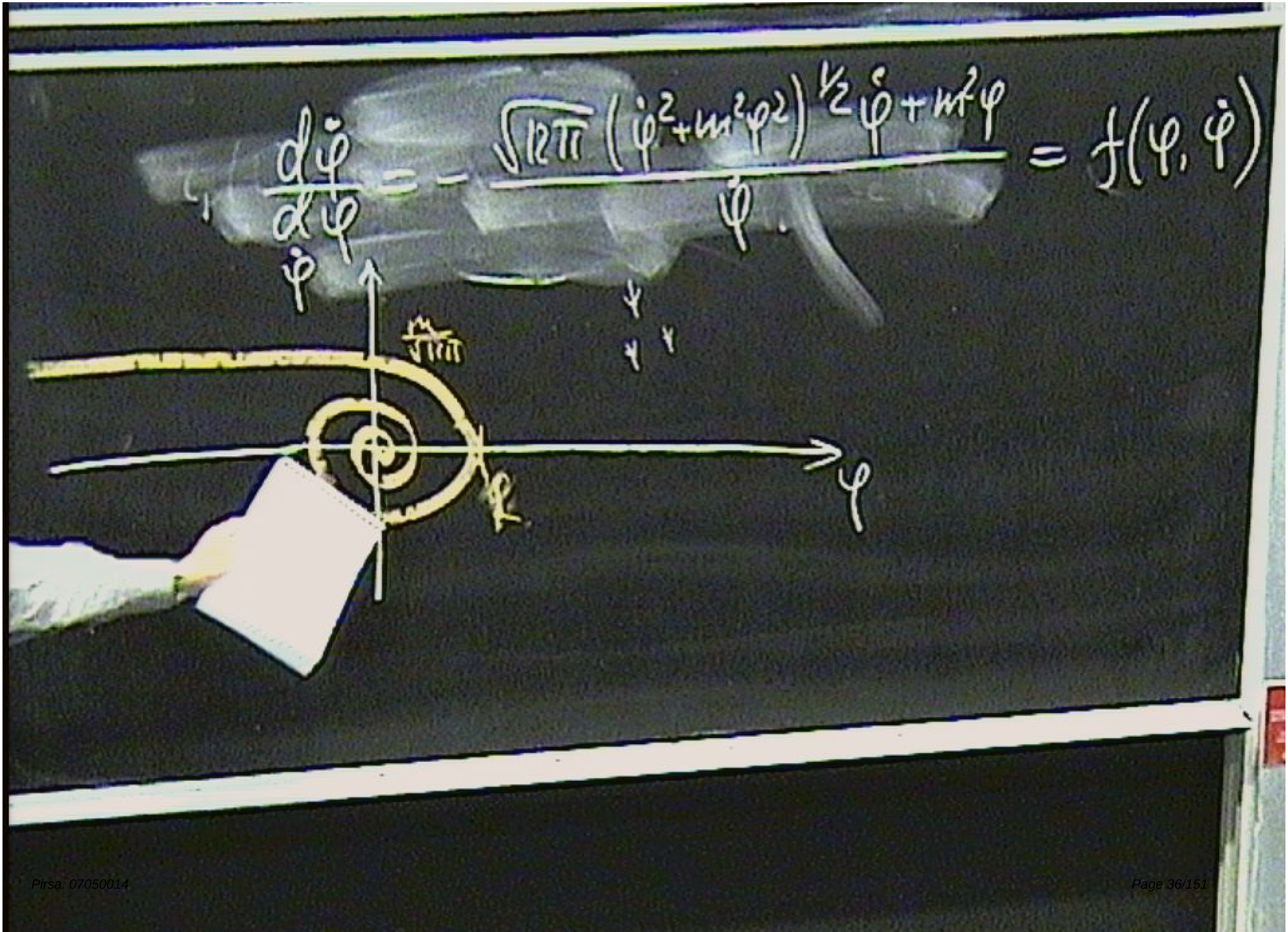

$\frac{d\dot\varphi}{d\varphi} = -\sqrt{12\pi}\,(\dot\varphi^2 + m^2\varphi^2)^{1/2}\,\dot\varphi + m^2\varphi$

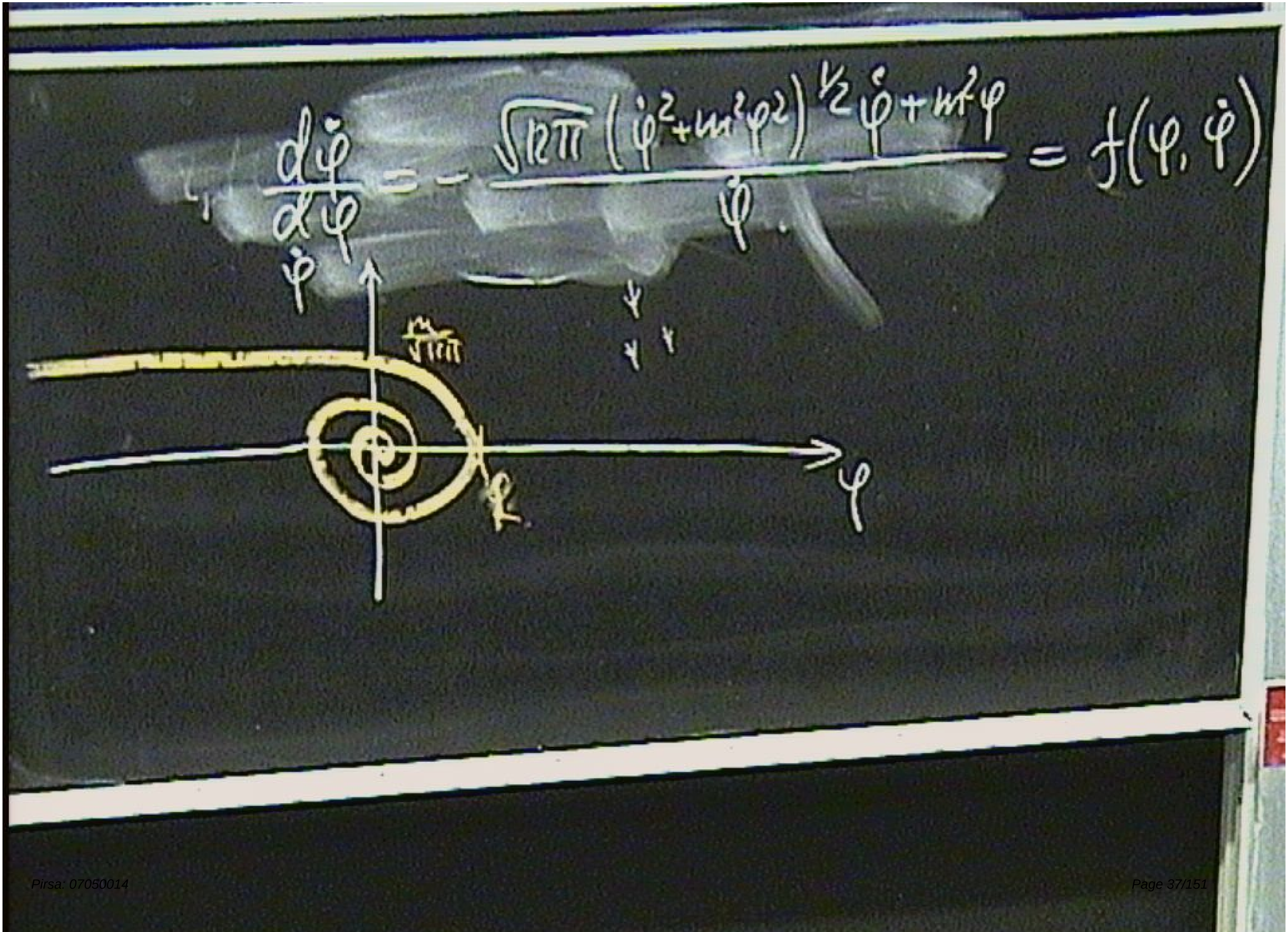
$$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\left(\dot{\varphi}^2 + m^2\varphi^2\right)^{1/2}\dot{\varphi} + m^2\varphi}{\dot{\varphi}} = f(\varphi, \dot{\varphi})$$


$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\,(\dot{\varphi}^2 + m^2\varphi^2)^{1/2}\,\dot{\varphi} + m^2\varphi}{\dot{\varphi}} = f(\varphi, \dot{\varphi})$
$\dot{\varphi}$
φ

$$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\,(\dot{\varphi}^2 + m^2\varphi^2)^{1/2}\,\dot{\varphi} + m^2\varphi}{\dot{\varphi}} = f(\varphi, \dot{\varphi})$$

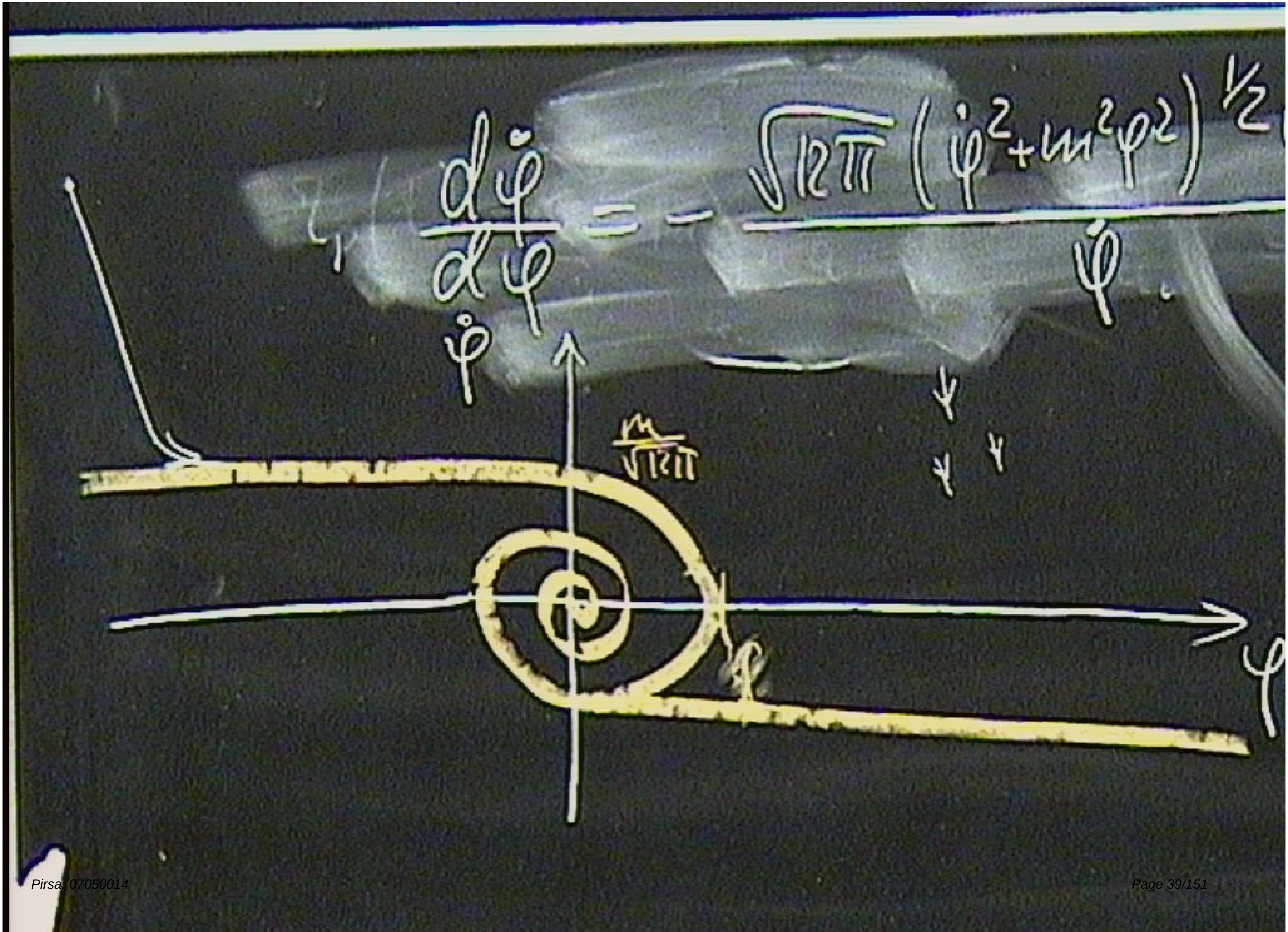



$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\,(\dot{\varphi}^2 + m^2\varphi^2)^{1/2}\,\dot{\varphi} + m^2\varphi}{\dot{\varphi}} = f(\varphi, \dot{\varphi})$
$\dot{\varphi}$
φ

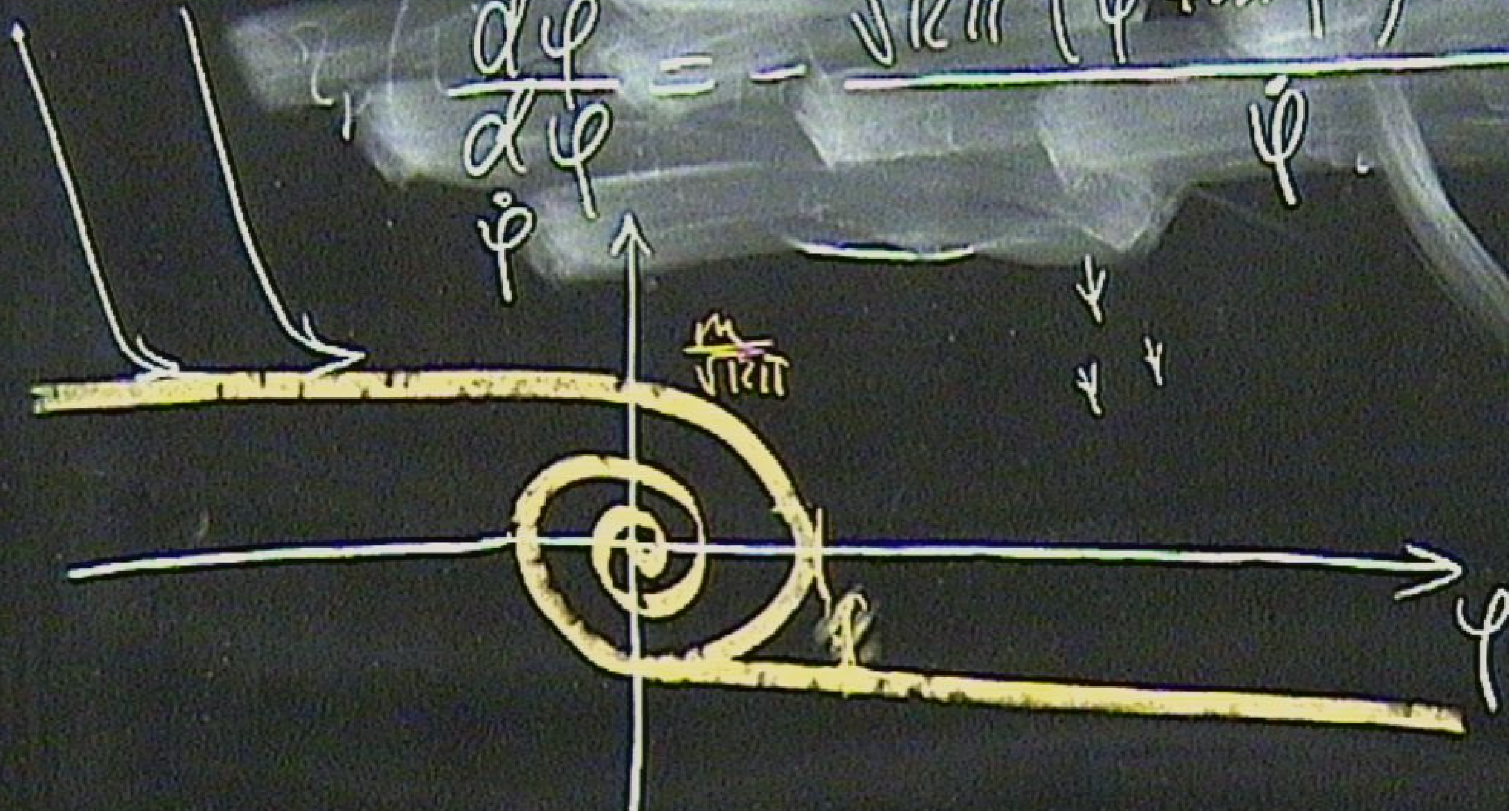


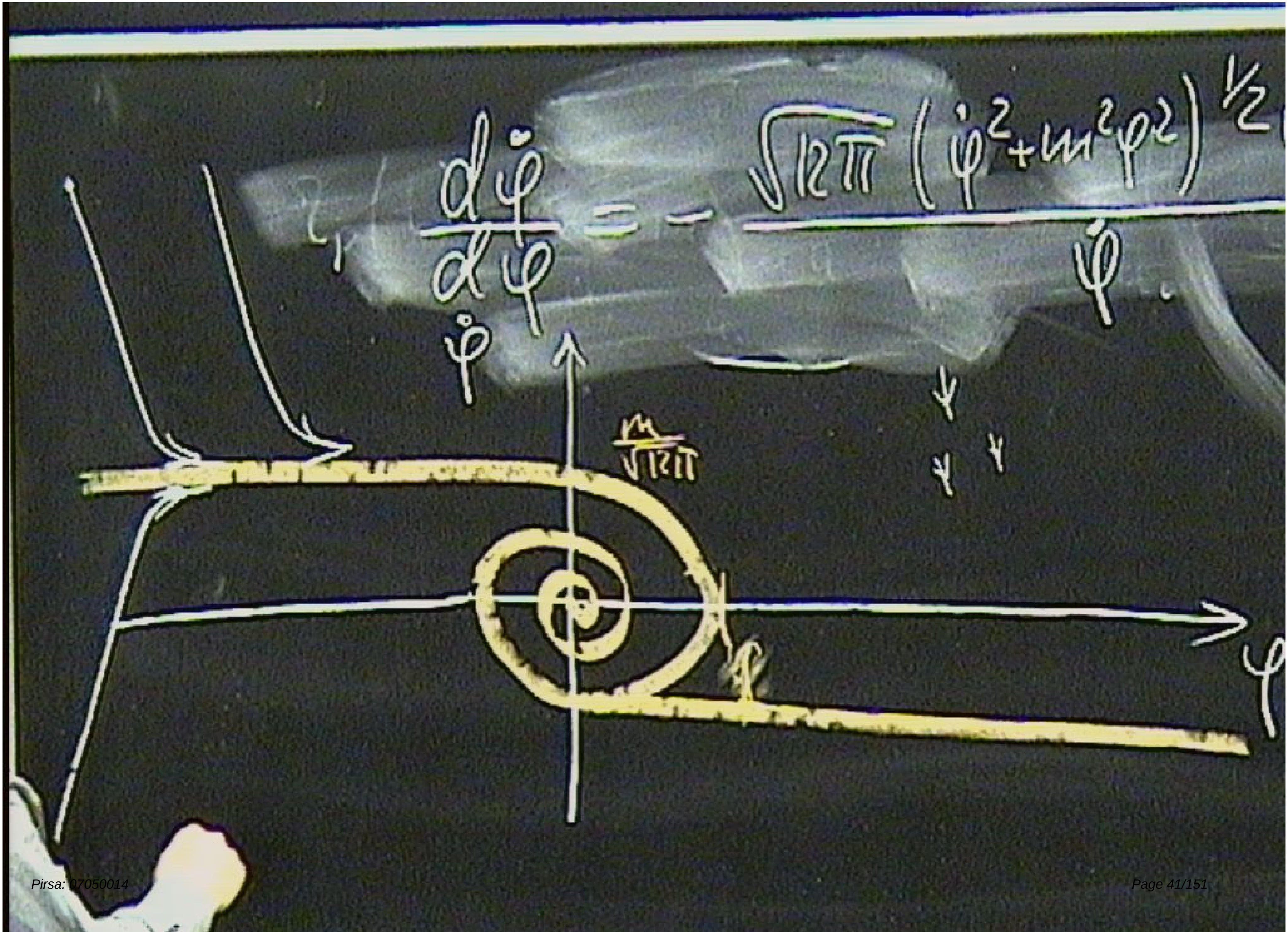
dφ̇/dφ = − (√12π (φ̇² + m²φ²)^{1/2} φ̇ + m²φ) / φ̇ = f(φ, φ̇)
φ̇
m/√12π
φ

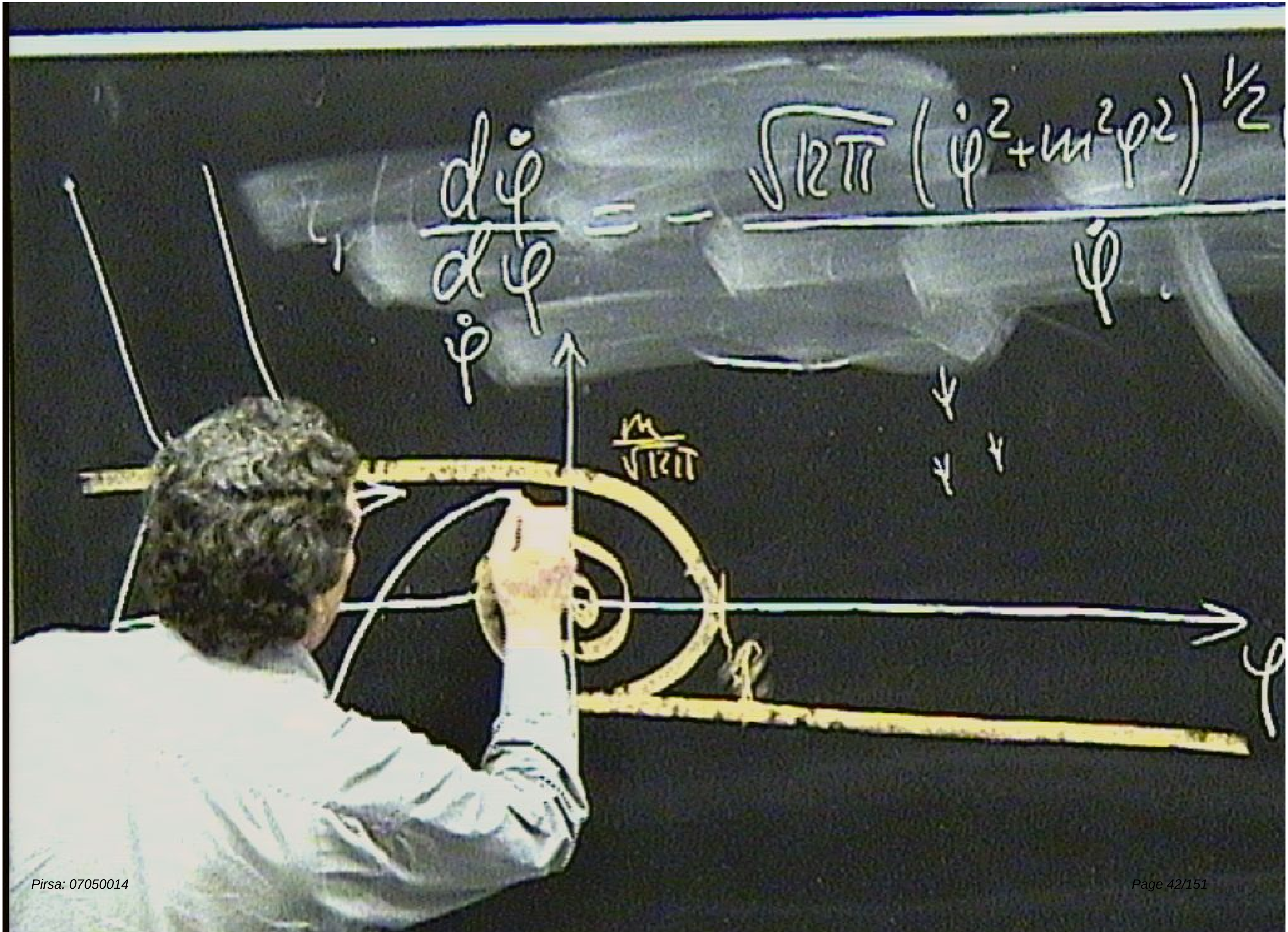

dφ
φ̇
φ
m/√12π

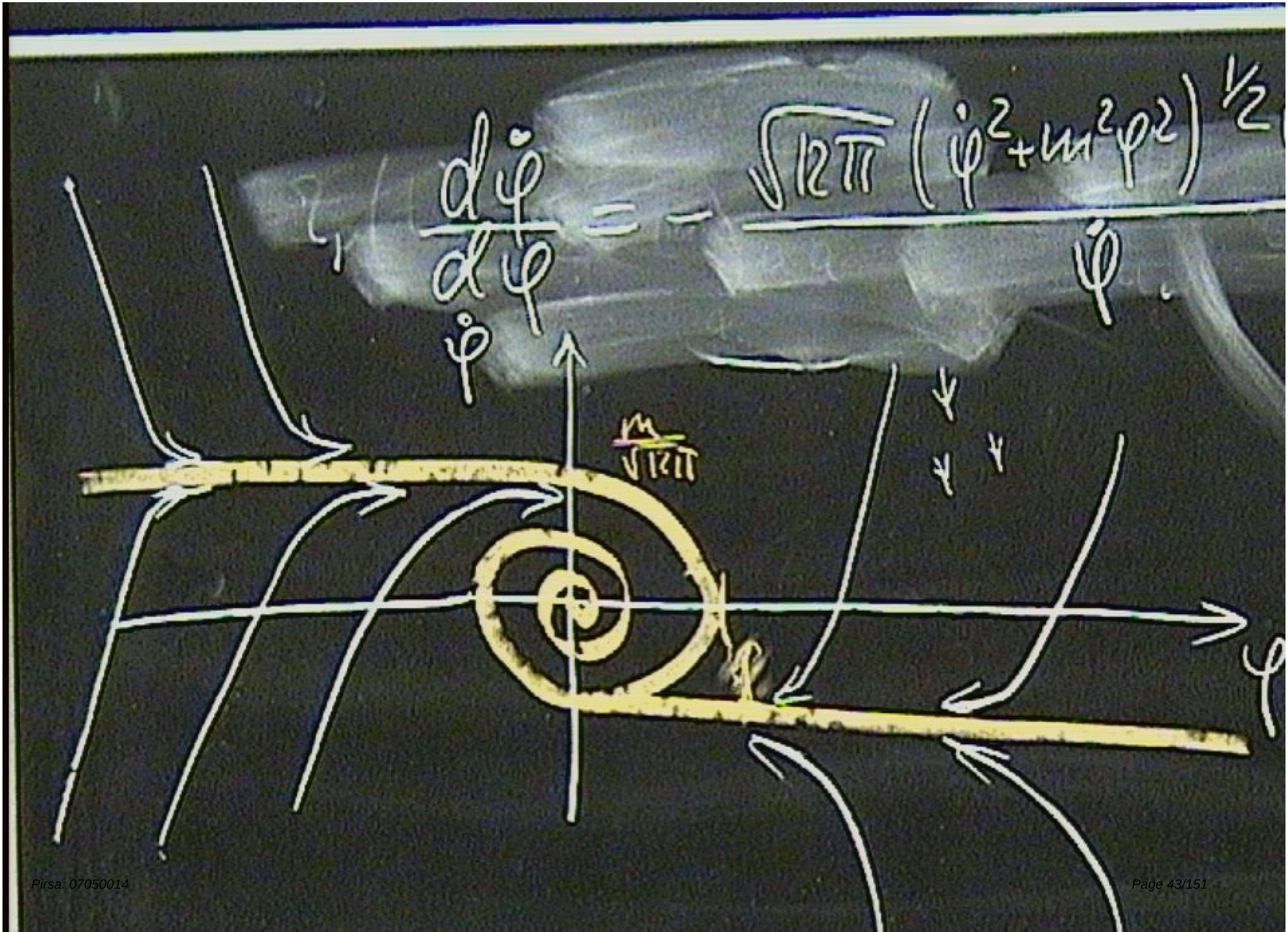

dφ̇/dφ = − √12π (φ̇² + m²φ²)^½ / φ̇
φ̇
m/√12π
φ

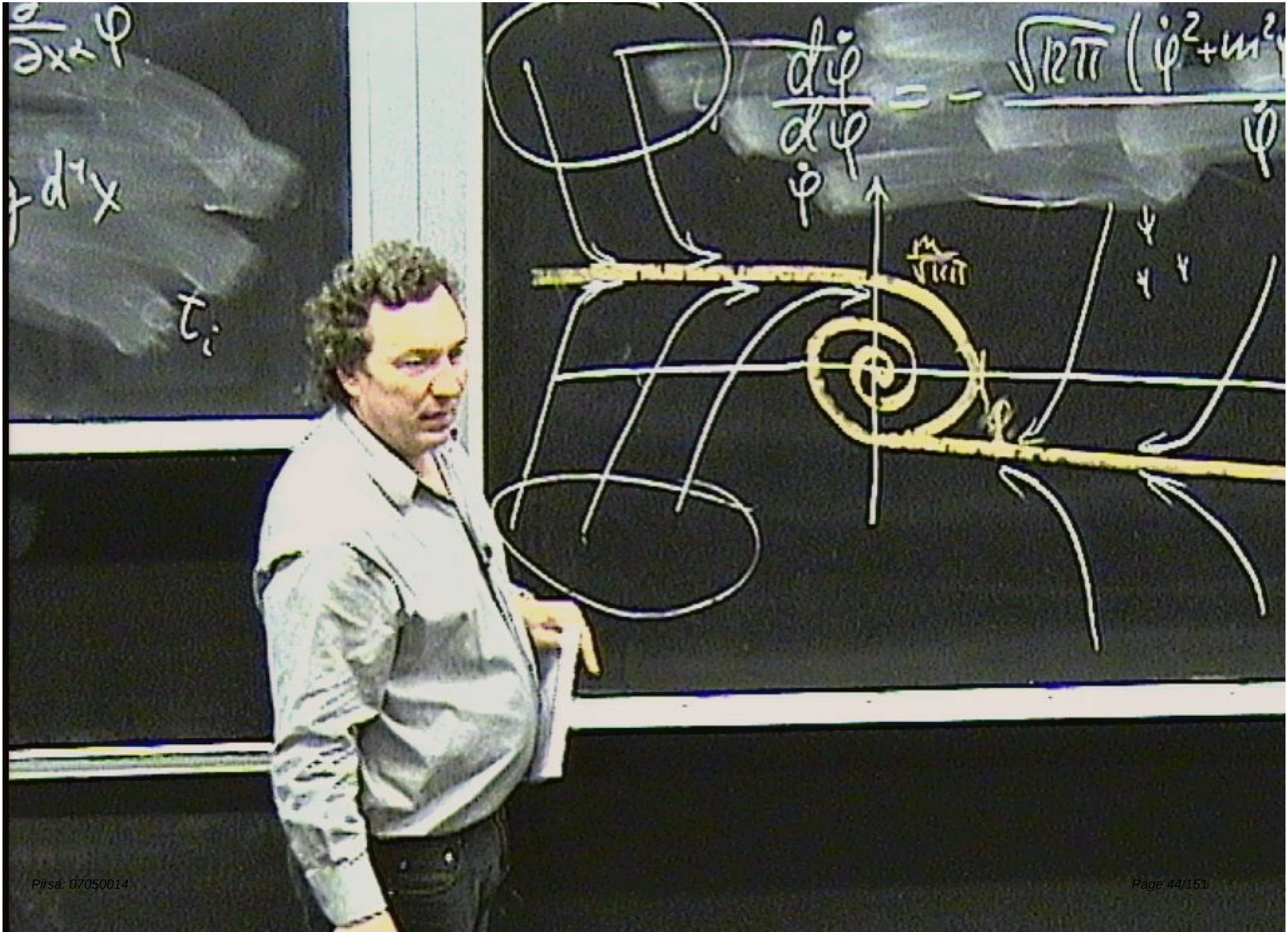
$$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\,(\dot{\varphi}^2 + m^2\varphi^2)^{1/2}}{\dot{\varphi}}$$

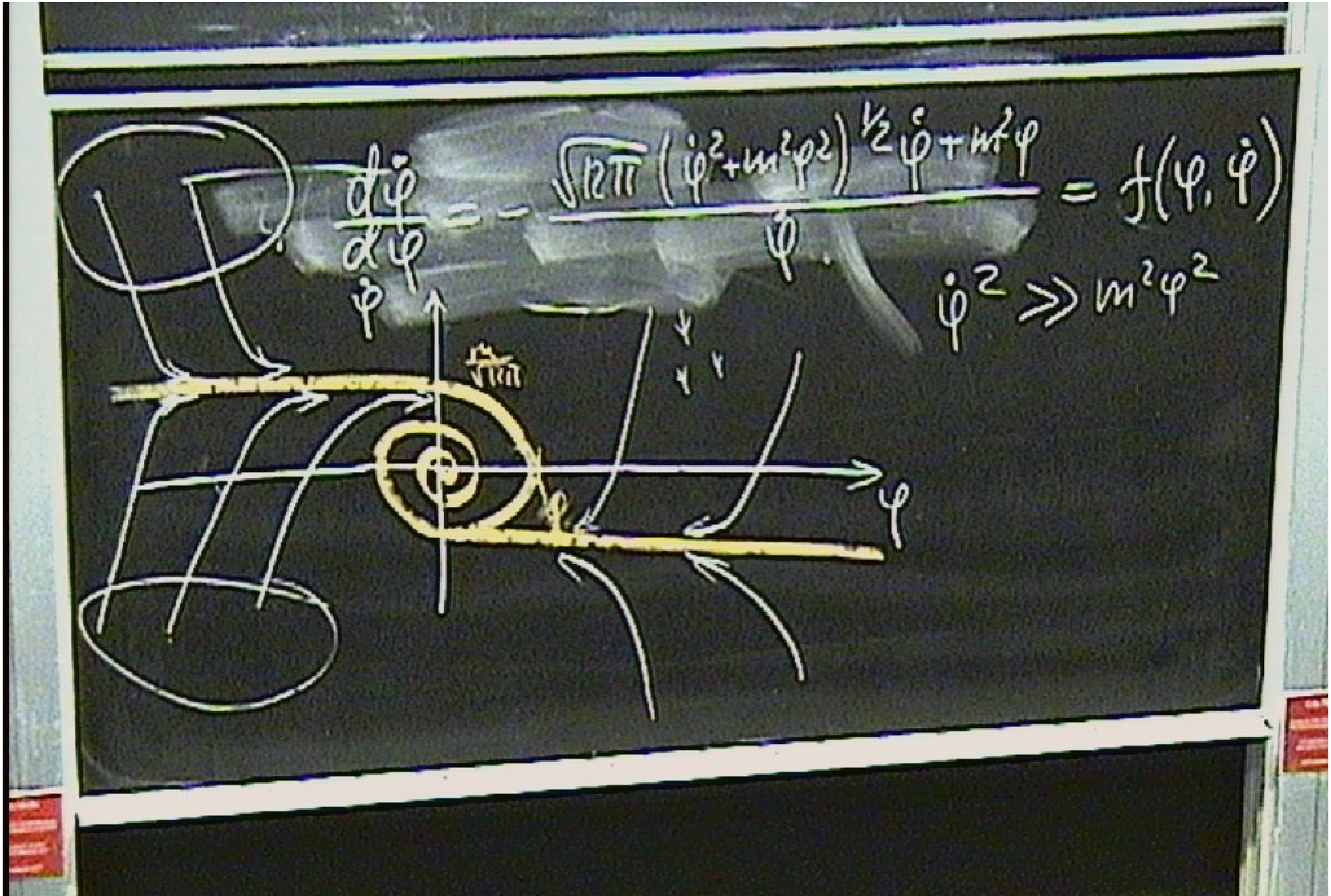


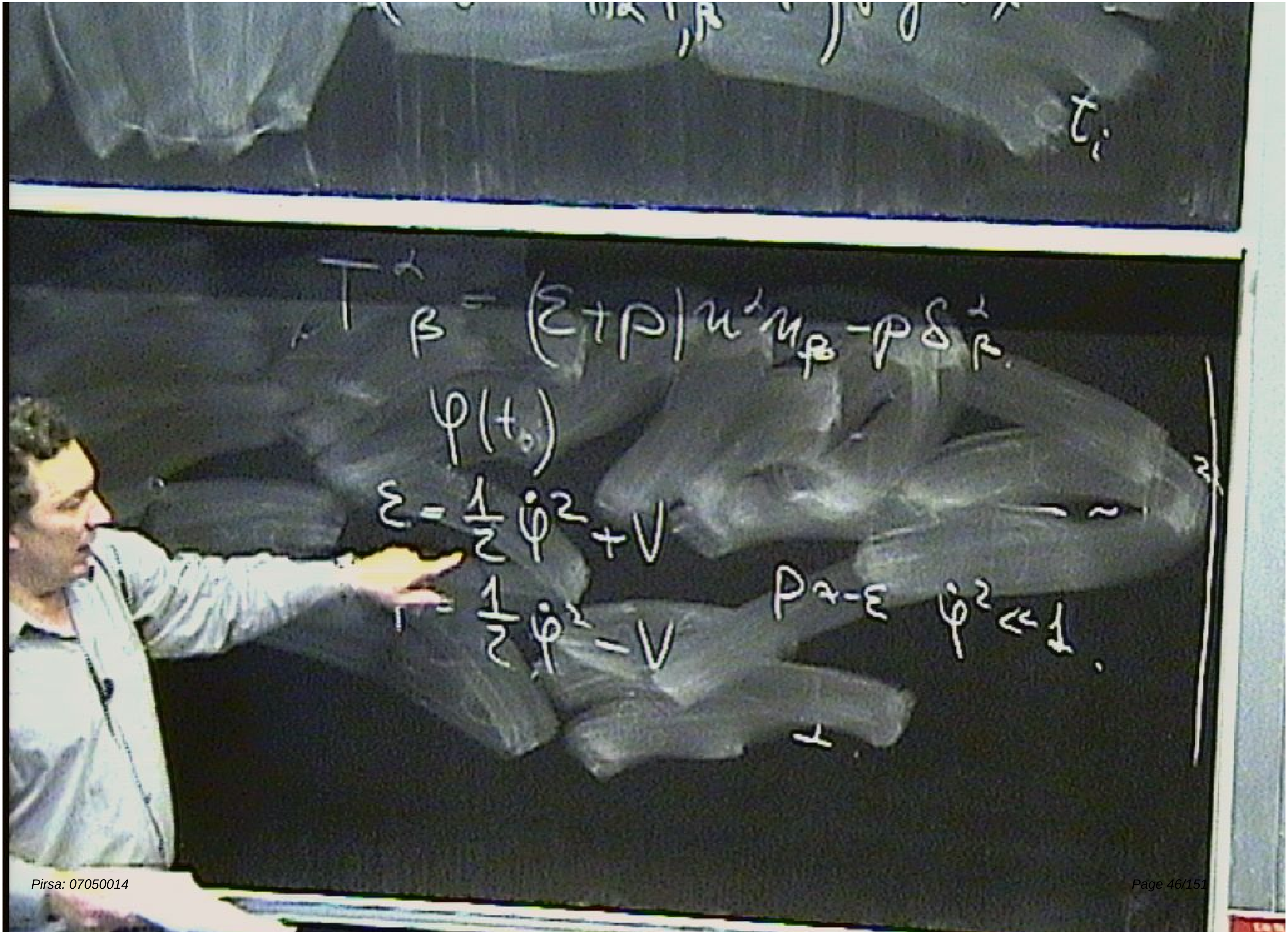

$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\,(\dot{\varphi}^2 + m^2\varphi^2)^{1/2}}{\dot{\varphi}}$
$\dot{\varphi}$
$\frac{m}{\sqrt{12\pi}}$
φ

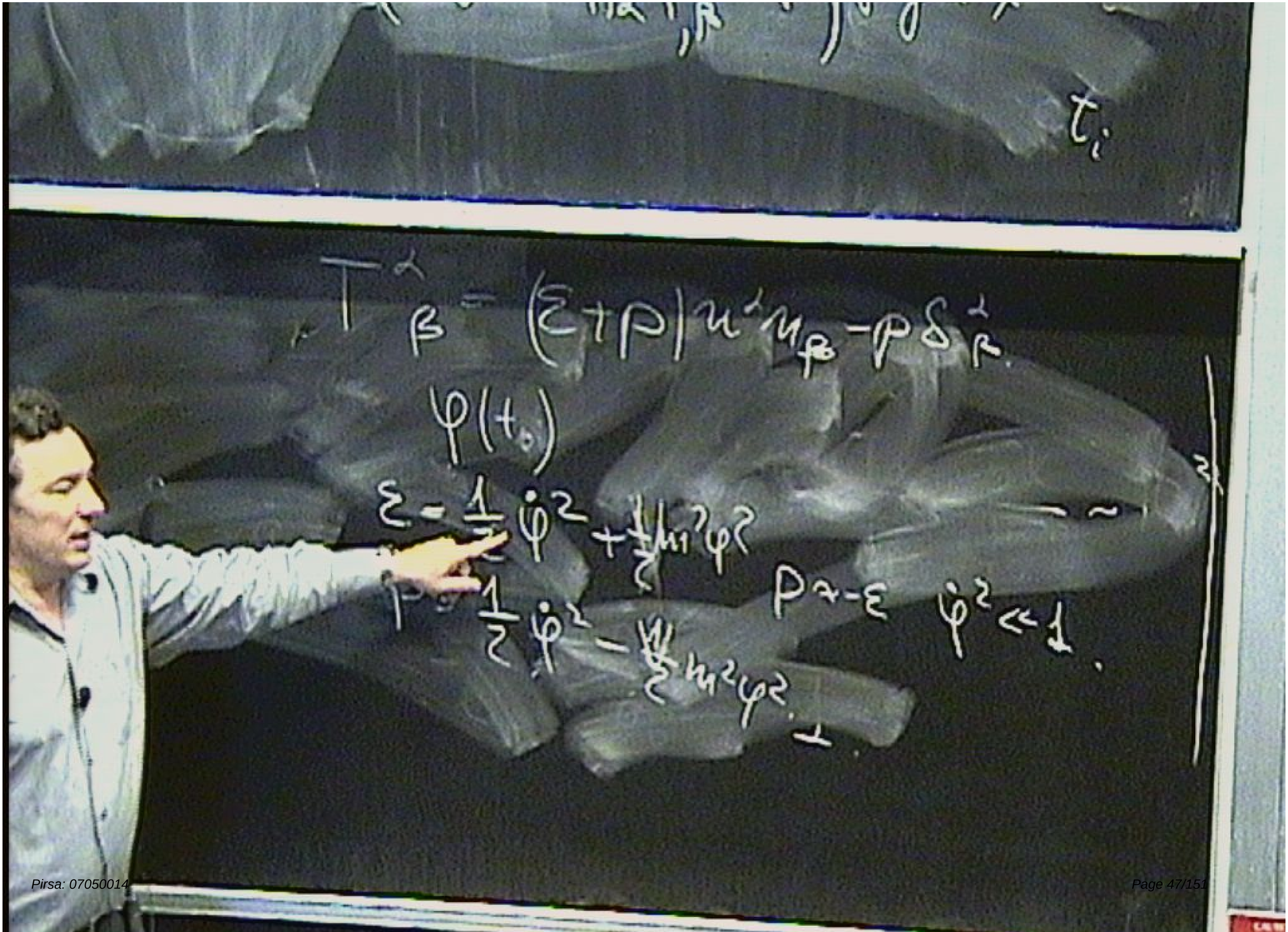

dφ̇/dφ = − √12π (φ̇² + m²φ²)^½ / φ̇
φ̇
m/√12π
φ


$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\,(\dot{\varphi}^2 + m^2\varphi^2)^{1/2}}{\dot{\varphi}}$
$\dot{\varphi}$
$\frac{m}{\sqrt{12\pi}}$
φ

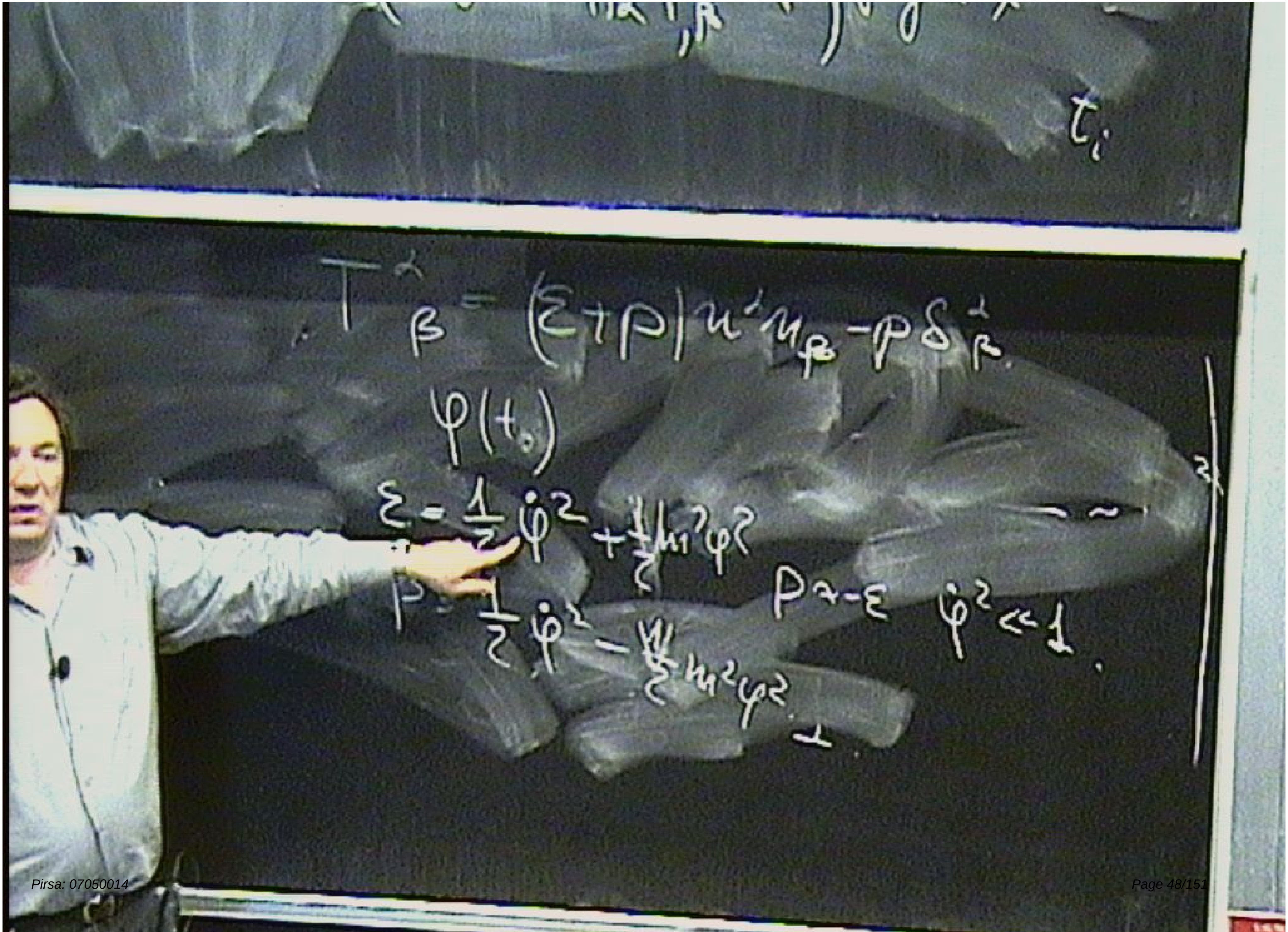



$\frac{d\dot\varphi}{d\varphi} = - \frac{\sqrt{12\pi}\,(\dot\varphi^2 + m^2\varphi^2)^{1/2}\,\dot\varphi + m^2\varphi}{\dot\varphi} = f(\varphi, \dot\varphi)$
$\dot\varphi^2 \gg m^2\varphi^2$
$\dot\varphi$
φ


t_i
$T^\alpha_\beta = (\varepsilon + p) u^\alpha u_\beta - p \delta^\alpha_\beta$
$\varphi(t_0)$
$\varepsilon = \frac{1}{2}\dot\varphi^2 + V$
$p = \frac{1}{2}\dot\varphi^2 - V$
$p \approx -\varepsilon \quad \dot\varphi^2 \ll 1$



t_i
$T^\alpha{}_\beta = (\varepsilon + p) u^\alpha u_\beta - p \delta^\alpha_\beta$
$\varphi(t_0)$
$\varepsilon = \frac{1}{2}\dot\varphi^2 + \frac{1}{2}m^2\varphi^2$
$p = \frac{1}{2}\dot\varphi^2 - \frac{1}{2}m^2\varphi^2$
$p \approx -\varepsilon \quad \dot\varphi^2 \ll 1$


t_i
$T^{\alpha}_{\beta} = (\varepsilon + p) u^{\alpha} u_{\beta} - p \delta^{\alpha}_{\beta}$
$\varphi(t_0)$
$\varepsilon = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2} m^2 \varphi^2$
$p = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2} m^2 \varphi^2$
$p \approx -\varepsilon \quad \dot{\varphi}^2 \ll 1$

t_i

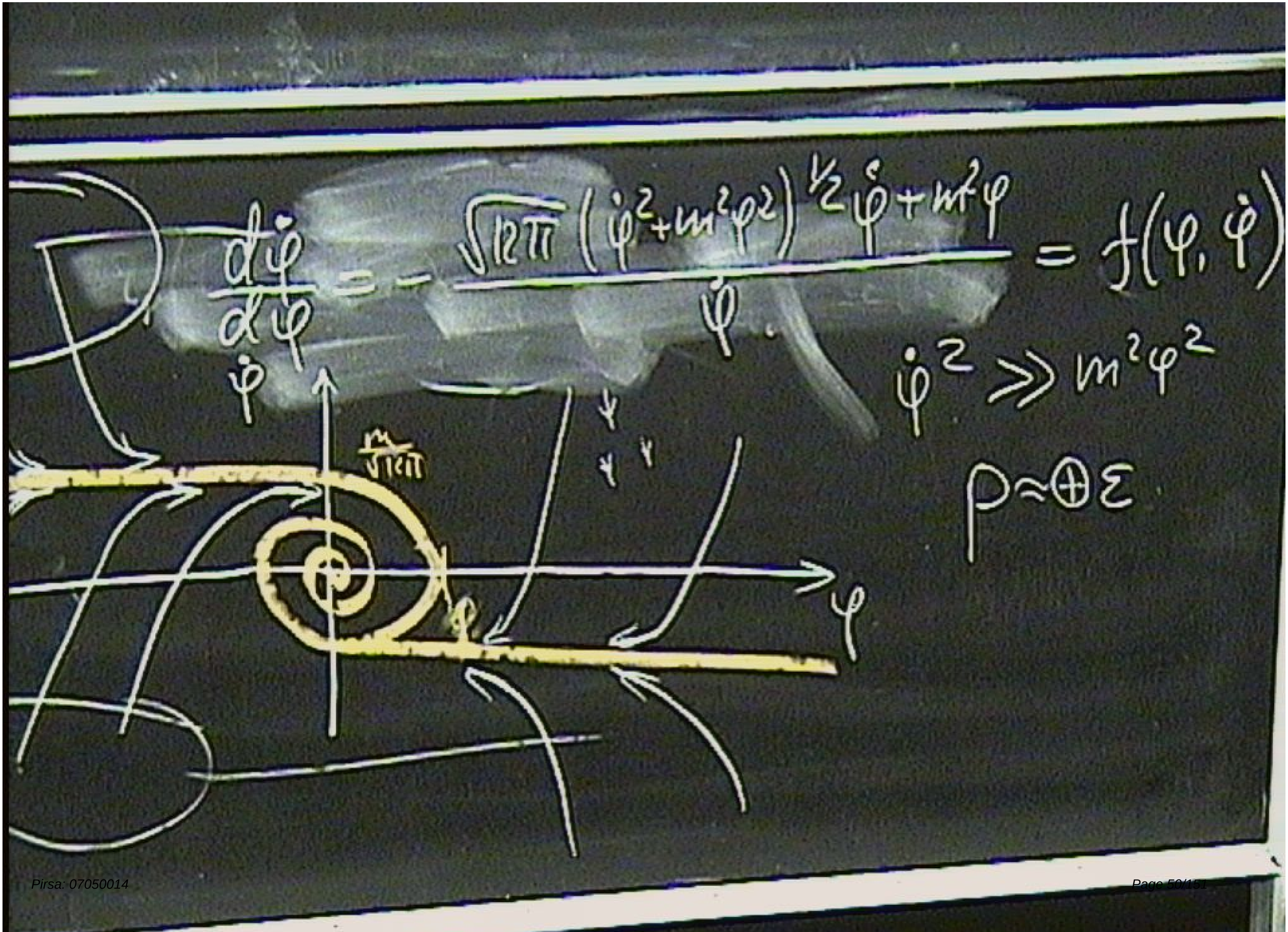
$$T^{\alpha}_{\ \beta} = (\varepsilon + p)\, u^{\alpha} u_{\beta} - p\, \delta^{\alpha}_{\beta}.$$

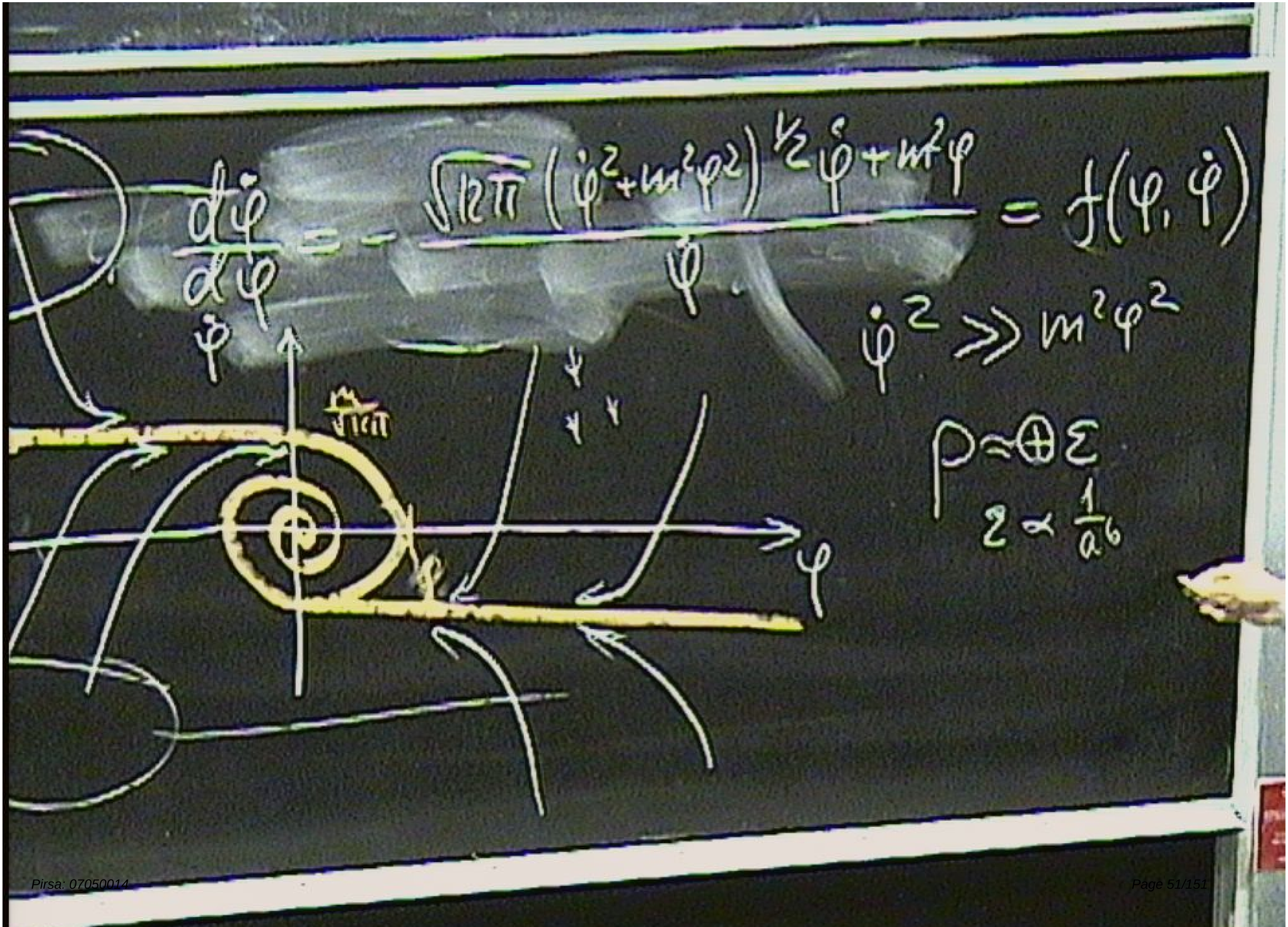
$$\varphi(t_0)$$

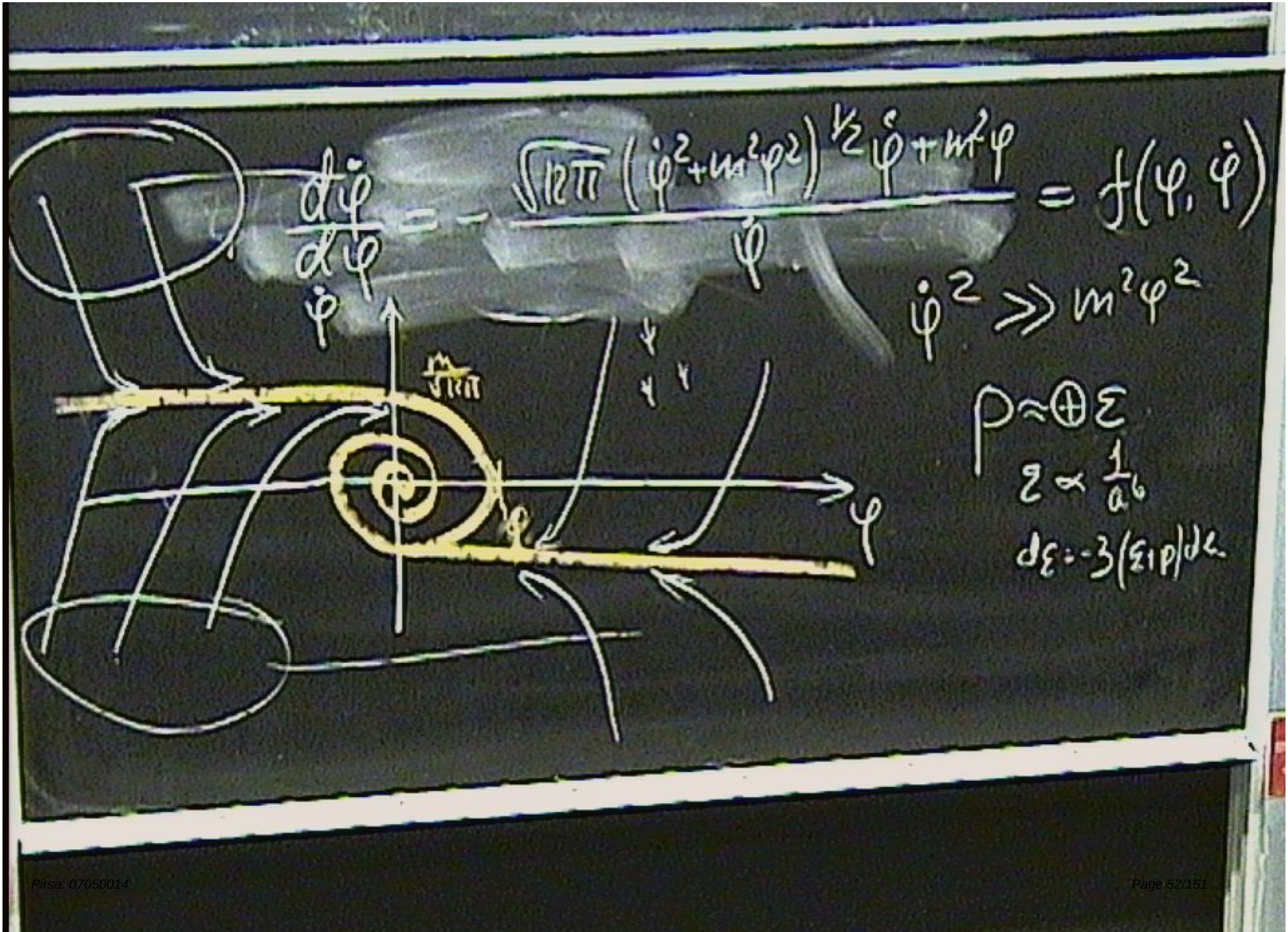
$$\varepsilon = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2} m^2 \varphi^2$$

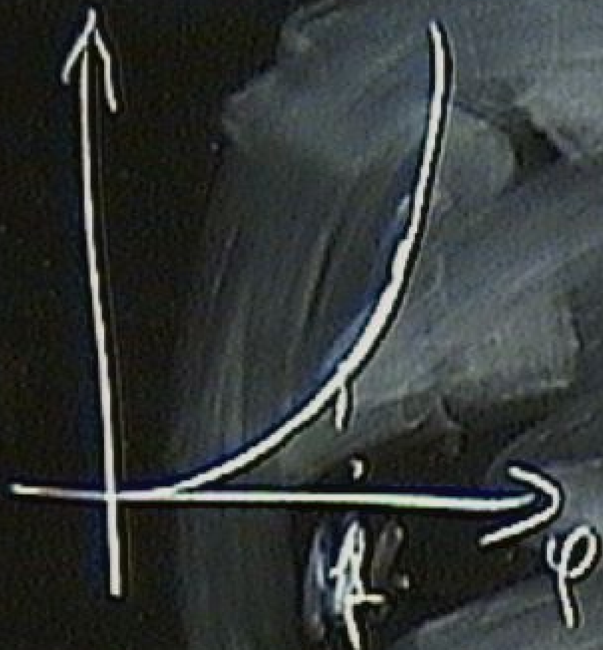
$$p = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2} m^2 \varphi^2 .$$

$p \approx -\varepsilon \quad \dot{\varphi}^2 \ll 1.$

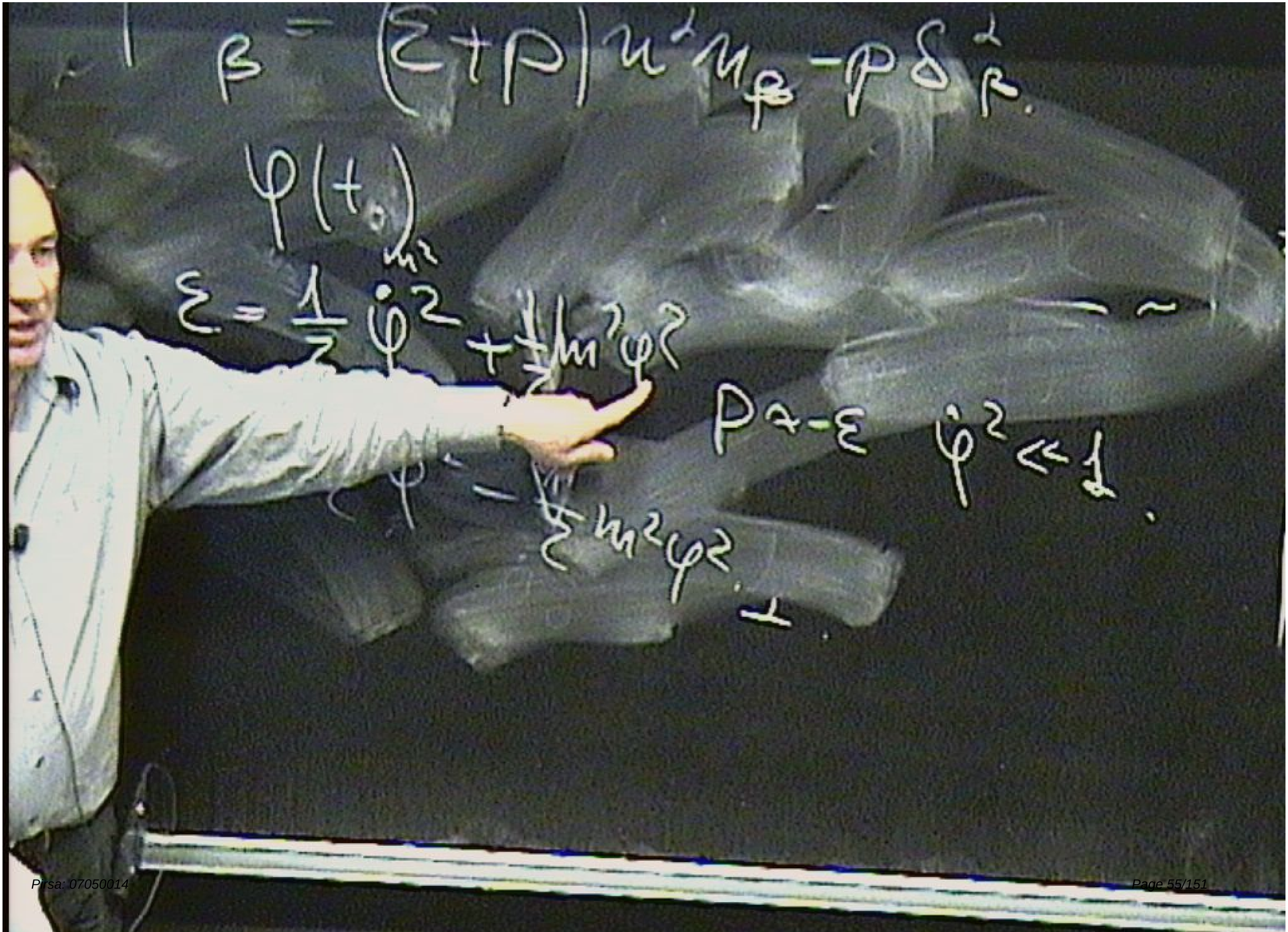

dφ̇/dφ = − (√12π (φ̇² + m²φ²)^½ φ̇ + m²φ) / φ̇ = f(φ, φ̇)
φ̇² ≫ m²φ²
φ
φ̇
m/√12π

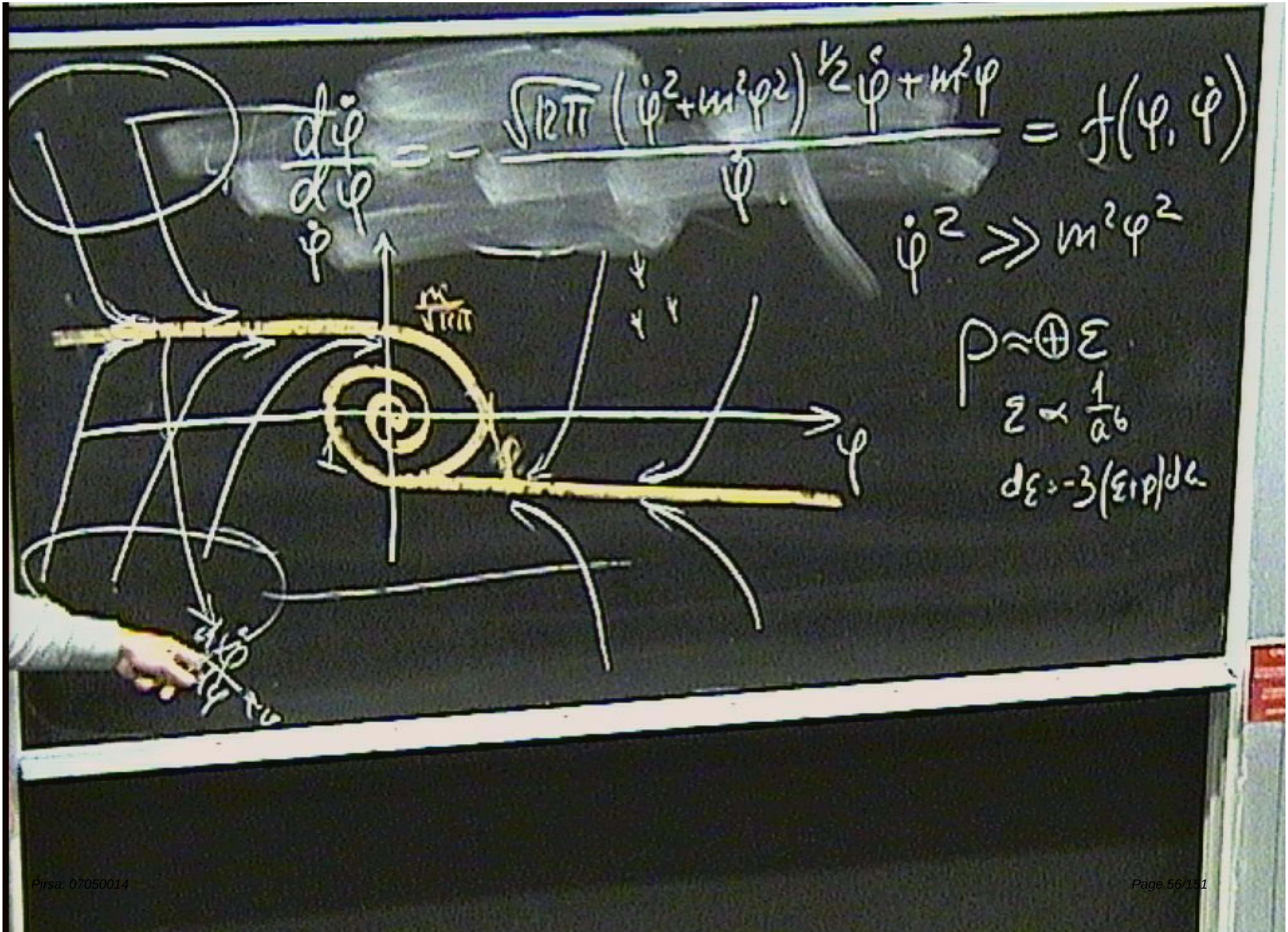

$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\sqrt{12\pi}\,(\dot{\varphi}^2 + m^2\varphi^2)^{1/2}\dot{\varphi} + m^2\varphi}{\dot{\varphi}} = f(\varphi, \dot{\varphi})$
$\dot{\varphi}^2 \gg m^2\varphi^2$
$\rho \approx \varepsilon$
$\varepsilon \propto \frac{1}{a^6}$
$\dot{\varphi}$
φ

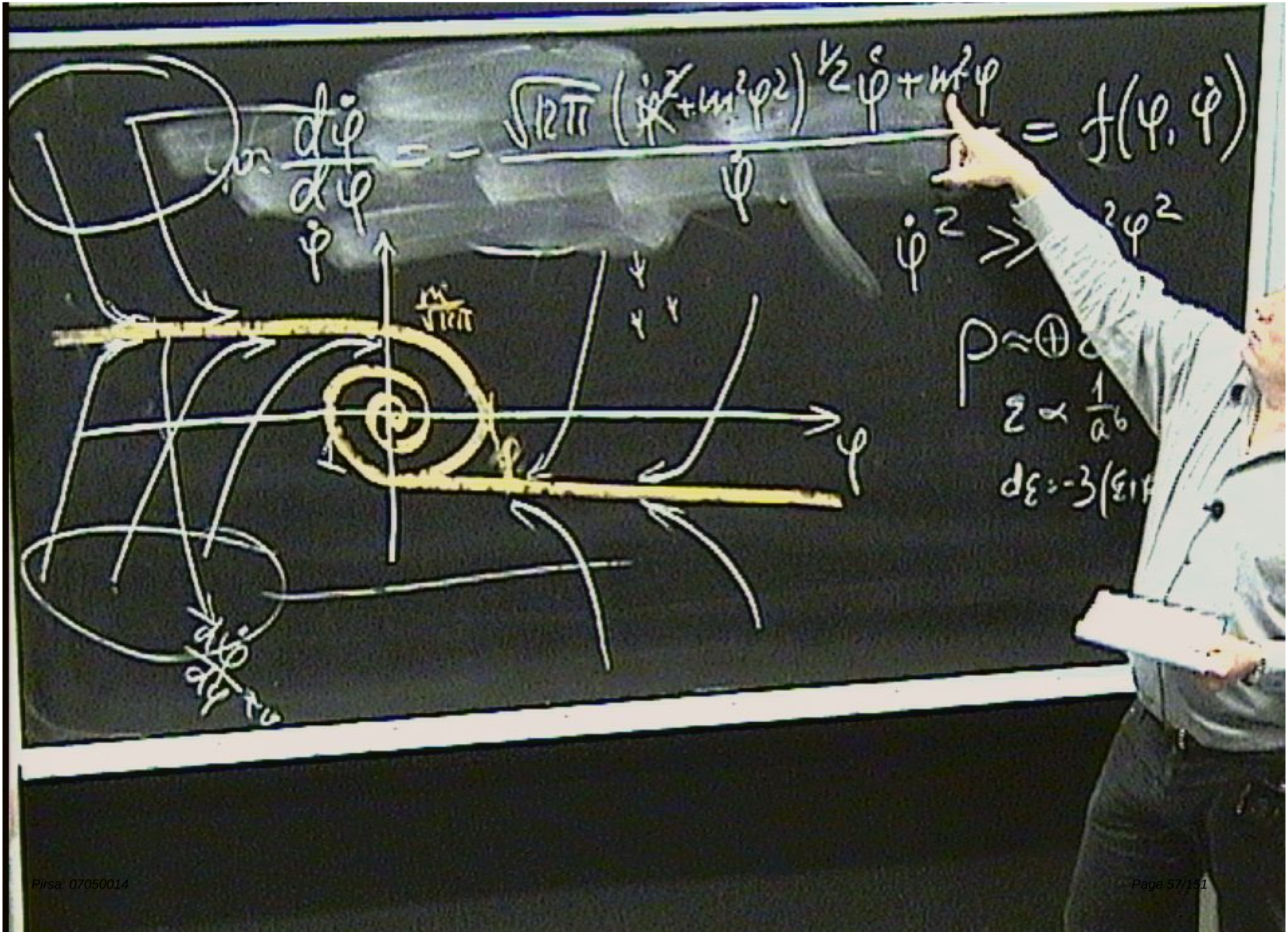

dφ̇/dφ = − (√12π (φ̇² + m²φ²)^½ φ̇ + m²φ) / φ̇ = f(φ, φ̇)
φ̇
φ
m/√12π
φ̇² ≫ m²φ²
p ≈ ⊕ε
ε ∝ 1/a⁶
dε = −3(ε+p)da

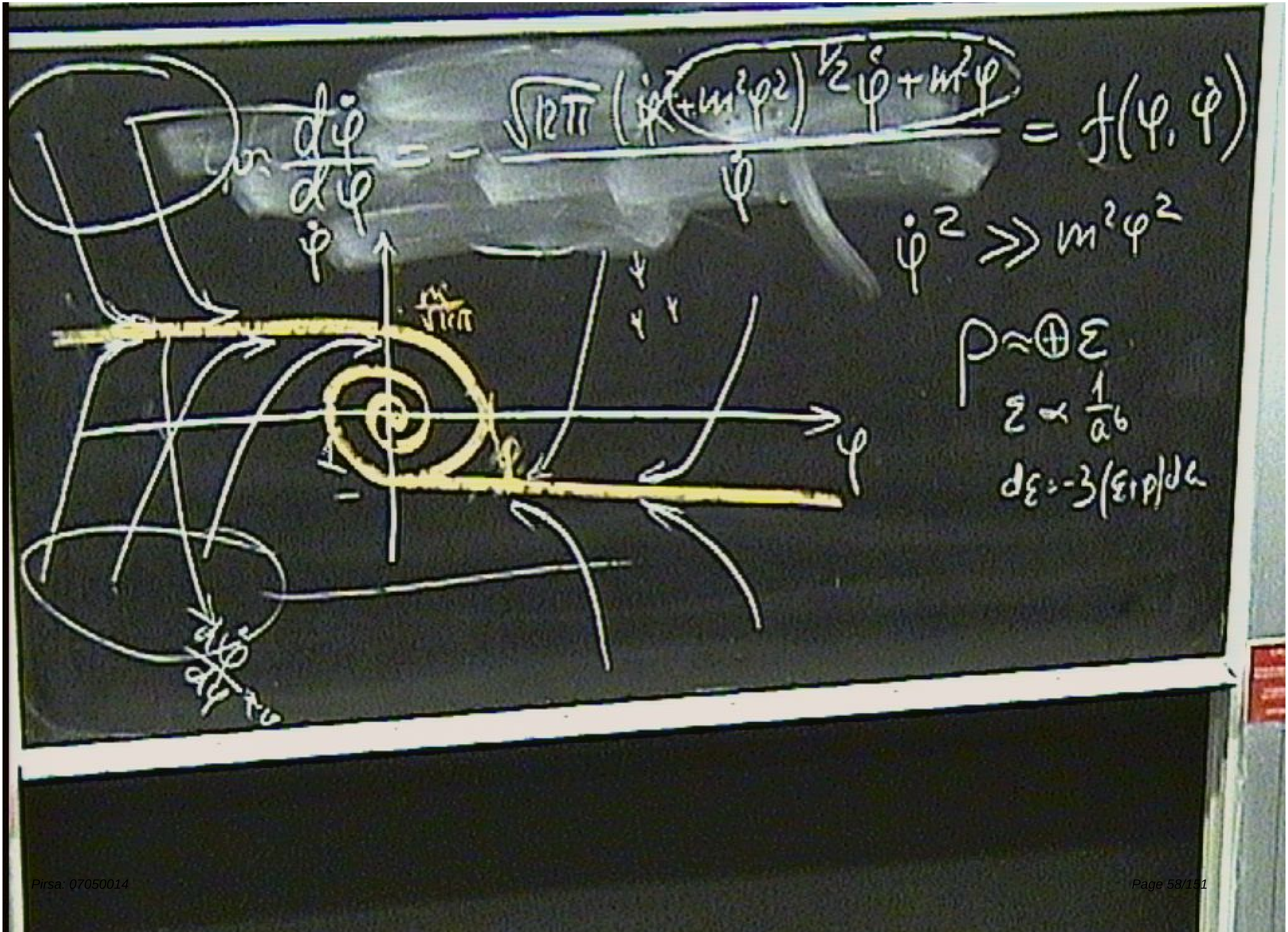

φ

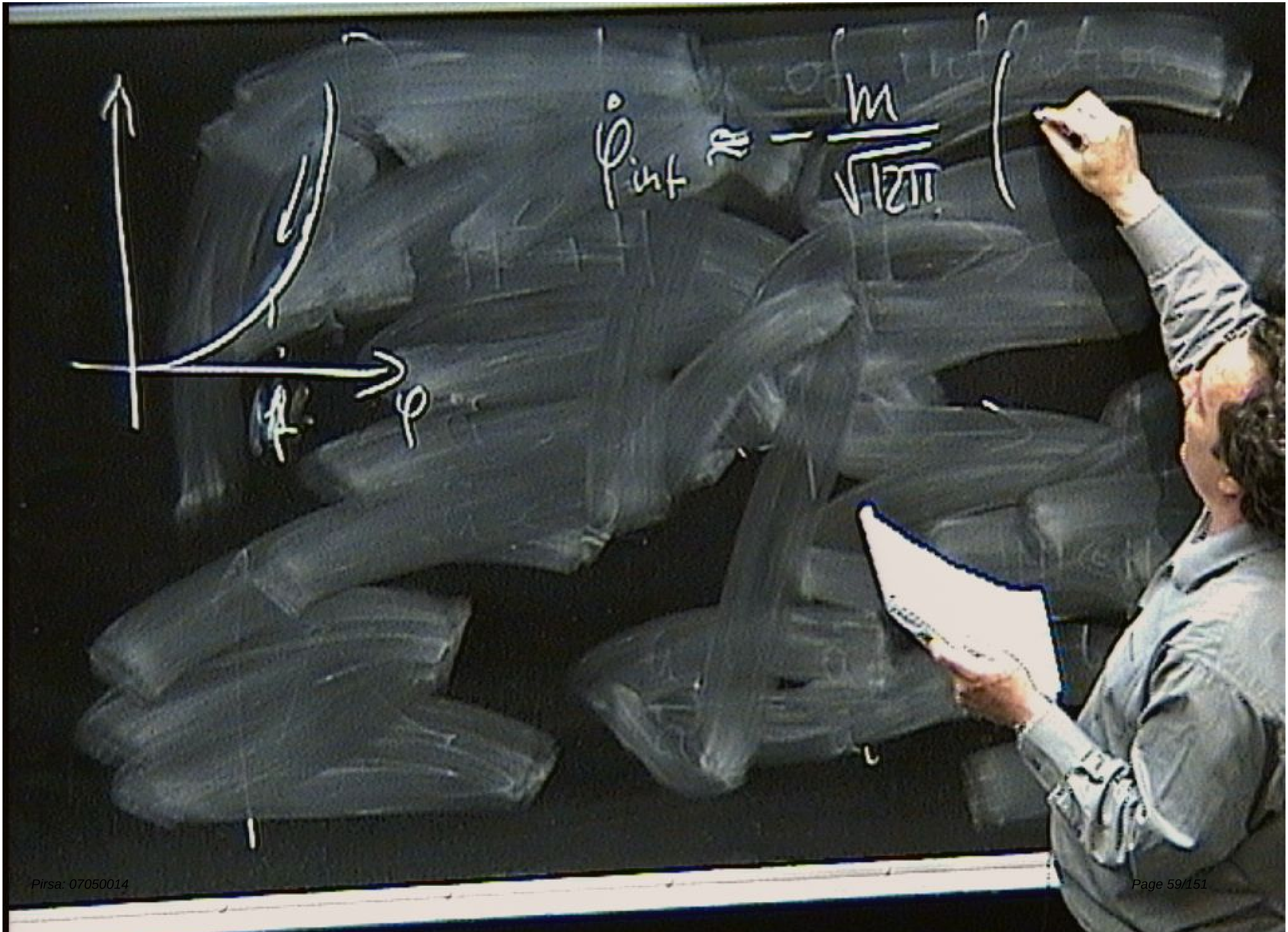

φ

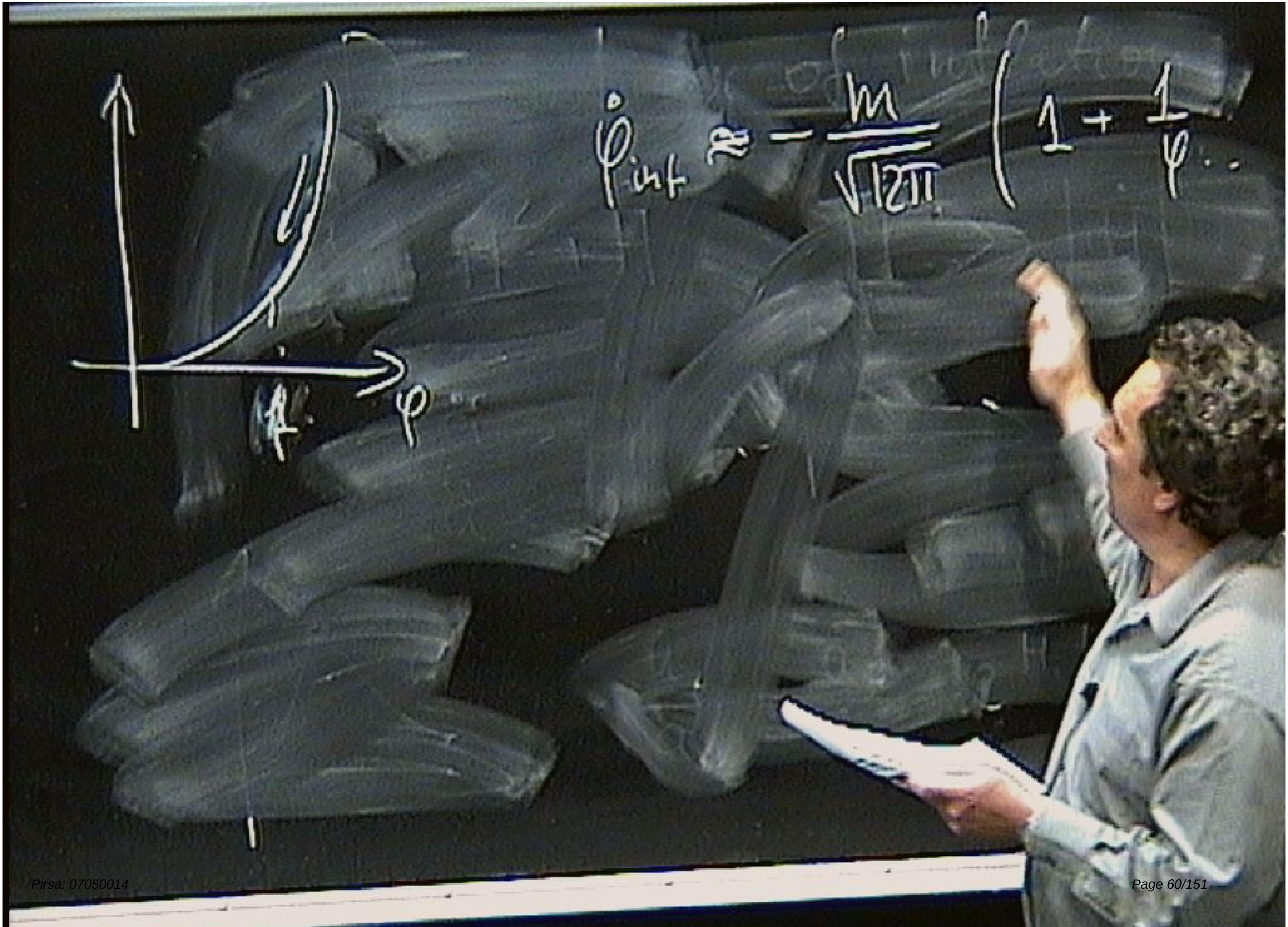

ε = 1/2 φ̇² + 1/2 m²φ²
P ≈ -ε   φ̇² ≪ 1

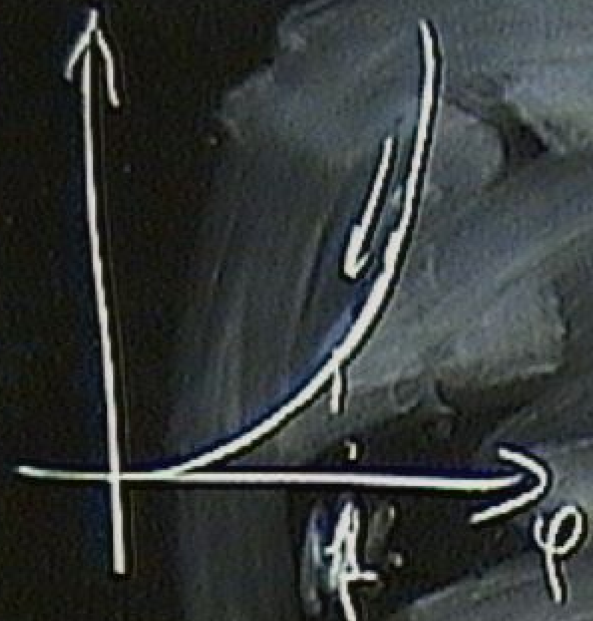

dφ̇/dφ = − (√(12π) (φ̇² + m²φ²)^{1/2} φ̇ + m²φ) / φ̇ = f(φ, φ̇)
φ̇² ≫ m²φ²
p ≈ ⊕ε
ε ∝ 1/a⁶
dε = −3(ε+p)da
φ
φ̇




φ̇² ≫ m²φ²
ε ∝ 1/a⁶
dε = -3(ε+p)da

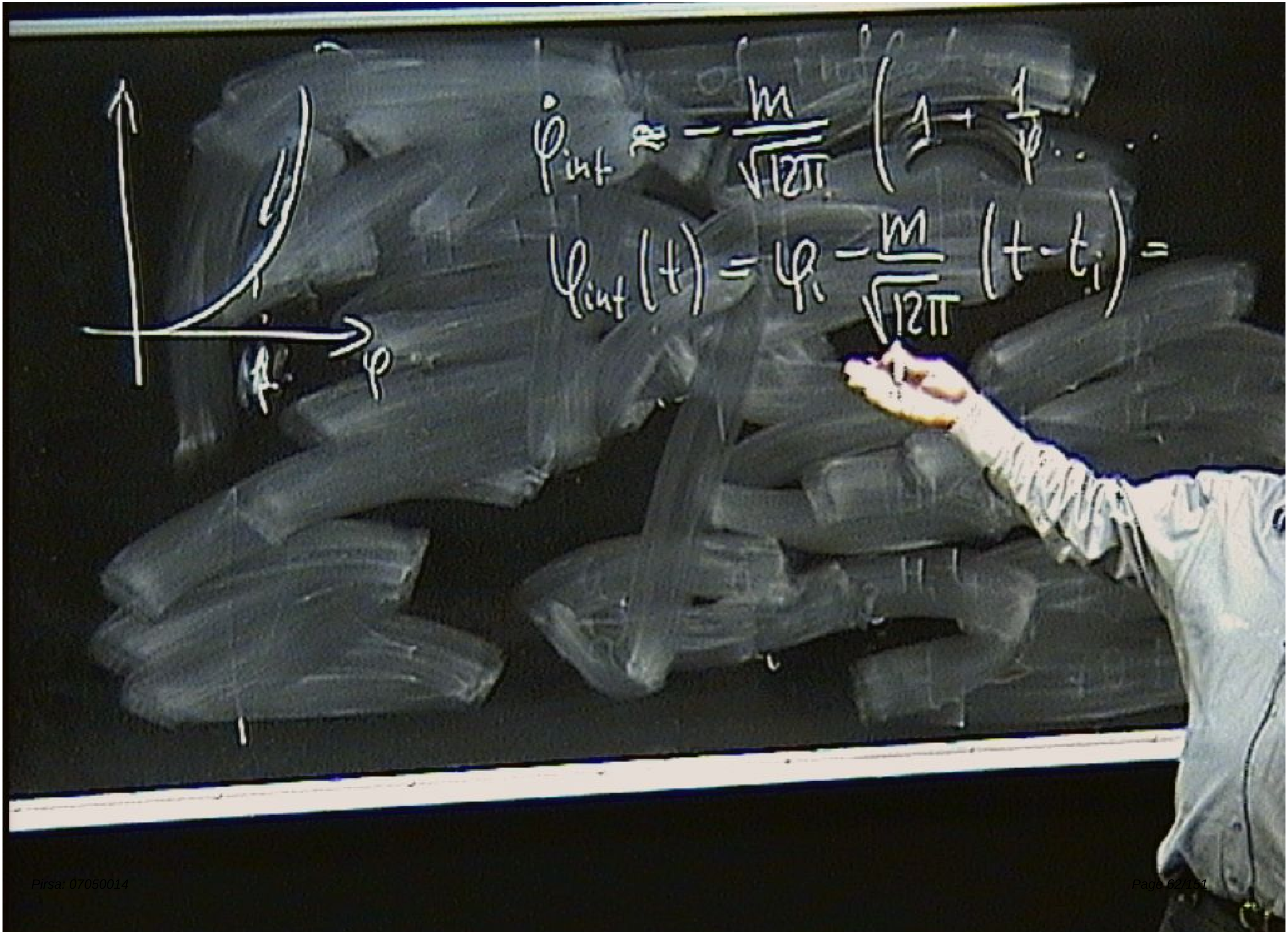

$\dot{\varphi}_{inf} \approx -\frac{m}{\sqrt{12\pi}}\,($
φ

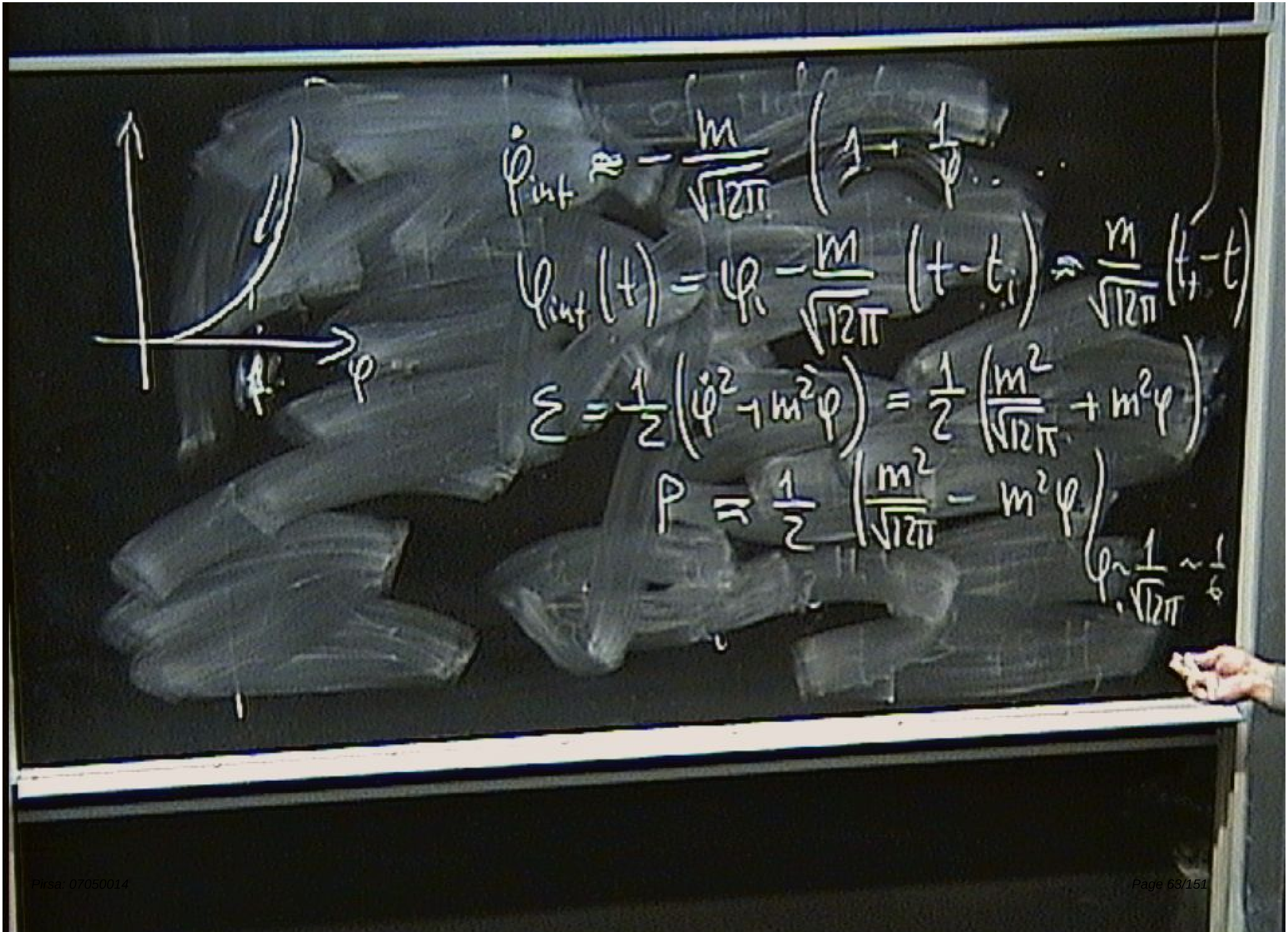

$\dot{\varphi}_{inf} \approx -\frac{m}{\sqrt{12\pi}}\left(1 + \frac{1}{\varphi}\dots\right.$
φ



$$\dot{\varphi}_{int} \approx -\frac{m}{\sqrt{12\pi}}\left(1 + \frac{1}{\varphi}\ldots\right.$$

$$\varphi_{int}(t) = \varphi_i - \frac{m}{\sqrt{12\pi}}\left(t - t_i\right)$$


φ̇_int ≈ − m/√(12π) (1 + 1/φ ...
φ_int(t) = φ_i − m/√(12π) (t − t_i) =
φ


φ̇
φ
φ̇_inf ≈ − m/√12π (1 + 1/φ ...)
φ_inf(t) = φ_i − m/√12π (t − t_i) ≈ m/√12π (t_f − t)
ε = 1/2 (φ̇² + m²φ) = 1/2 (m²/√12π + m²φ)
p ≈ 1/2 (m²/√12π − m²φ)
φ ~ 1/√12π ~ 1/6

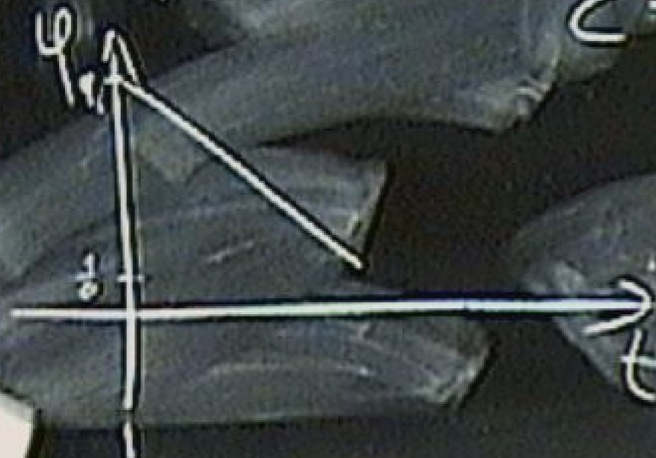
$$\dot{\varphi}_{inf} \approx -\frac{m}{\sqrt{12\pi}}\left(1 + \frac{1}{\varphi}\dots\right)$$

$$\varphi_{inf}(t) = \varphi_i - \frac{m}{\sqrt{12\pi}}(t - t_i) \approx \frac{m}{\sqrt{12\pi}}(t_f - t)$$

$$\varepsilon = \frac{1}{2}\left(\dot{\varphi}^2 + m^2\varphi\right) = \frac{1}{2}\left(\frac{m^2}{\sqrt{12\pi}} + m^2\varphi\right)$$

$$p \approx \frac{1}{2}\left(\frac{m^2}{\sqrt{12\pi}} - m^2\varphi\right)$$

$$\varphi_f \sim \frac{1}{\sqrt{12\pi}} \sim \frac{1}{6}$$



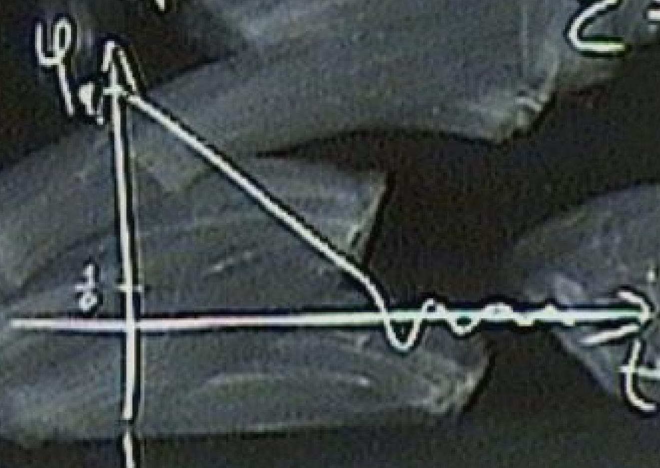
$$\dot{\varphi}_{inf} \approx -\frac{m}{\sqrt{12\pi}}\left(1 + \frac{1}{\varphi}\dots\right)$$

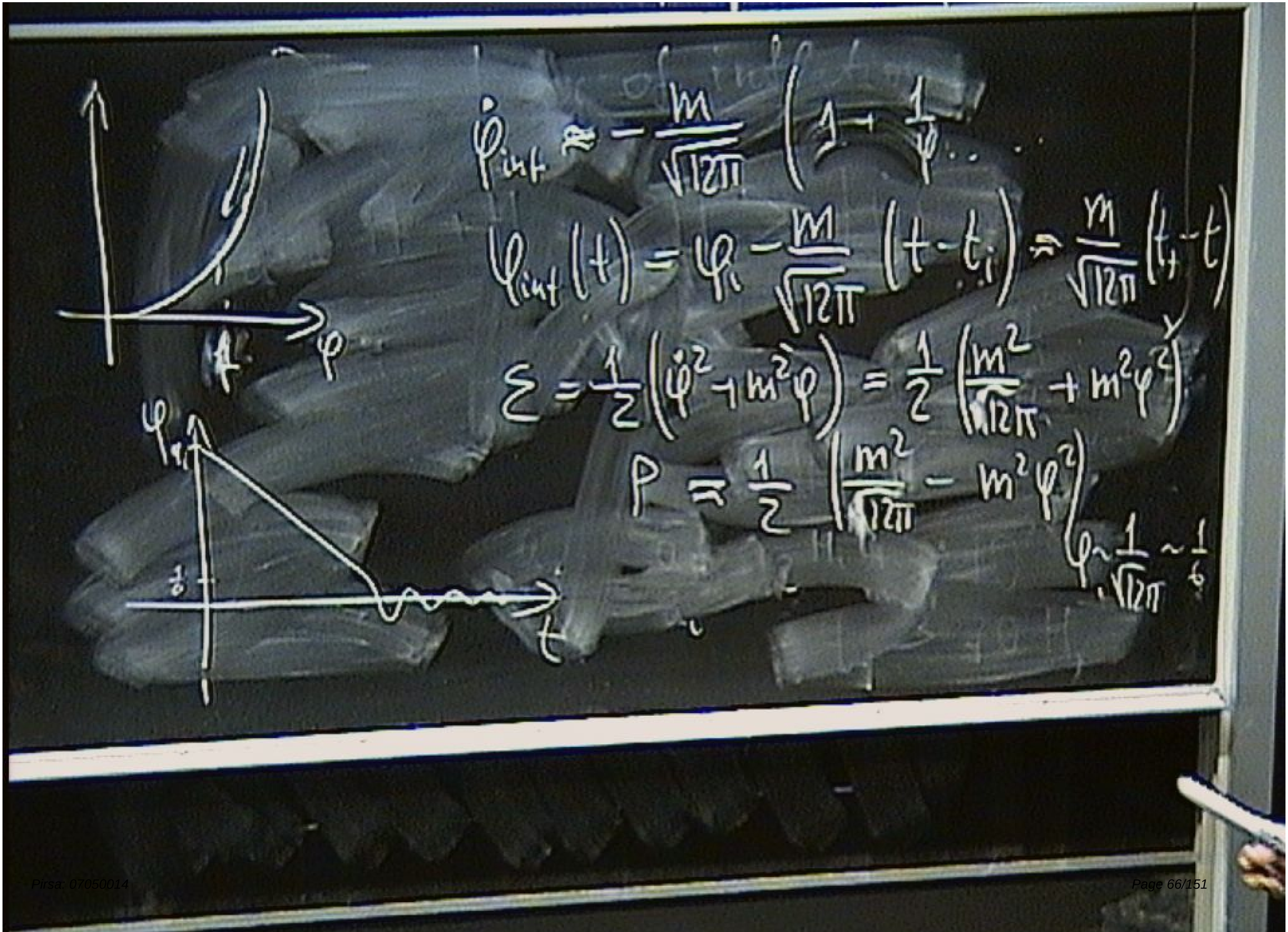
$$\varphi_{inf}(t) = \varphi_i - \frac{m}{\sqrt{12\pi}}(t - t_i) = \frac{m}{\sqrt{12\pi}}(t_f - t)$$

$$\varepsilon = \frac{1}{2}\left(\dot{\varphi}^2 + m^2\varphi\right) = \frac{1}{2}\left(\frac{m^2}{\sqrt{12\pi}} + m^2\varphi\right)$$

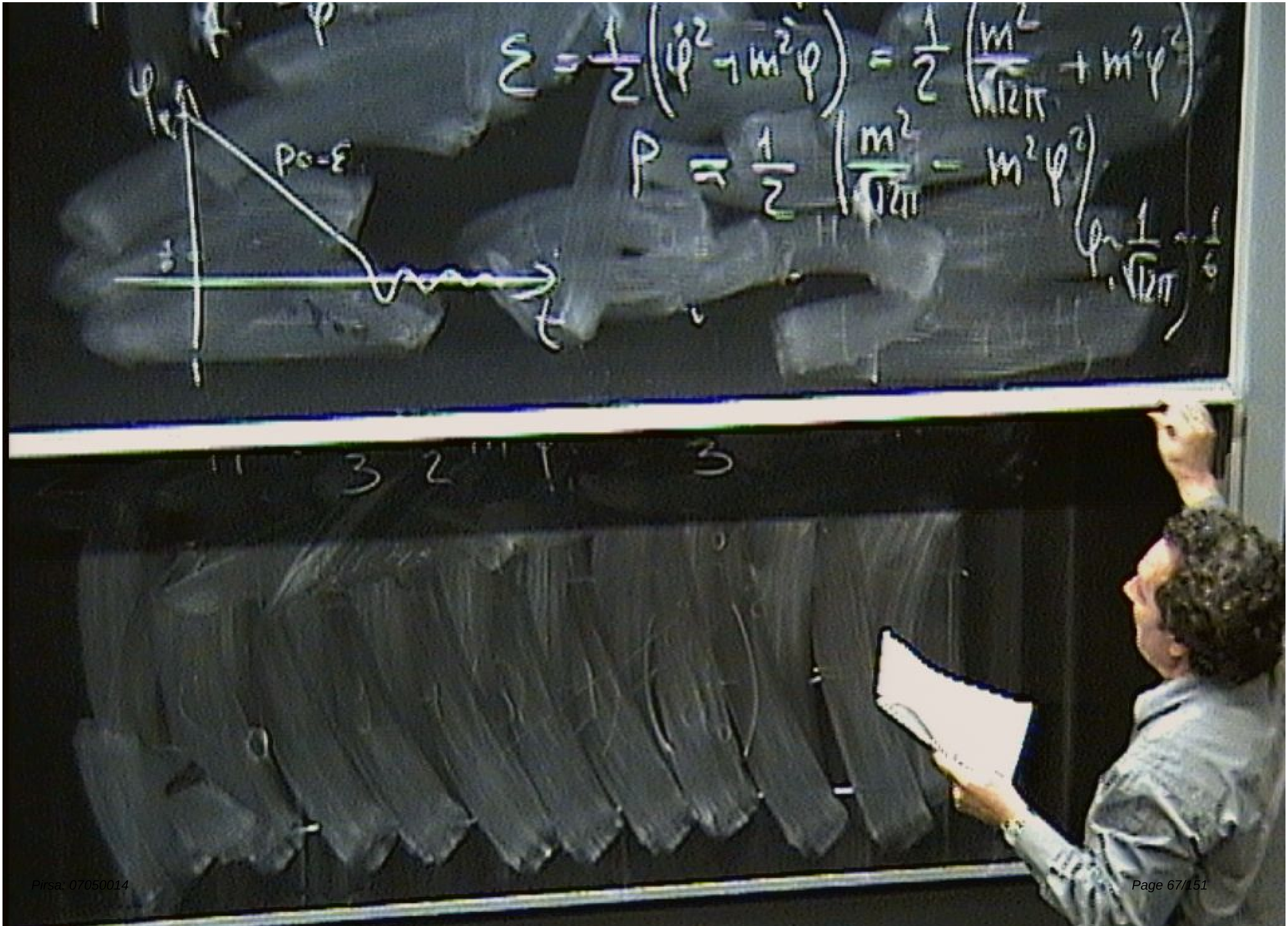
$$p = \frac{1}{2}\left(\frac{m^2}{\sqrt{12\pi}} - m^2\varphi\right)$$

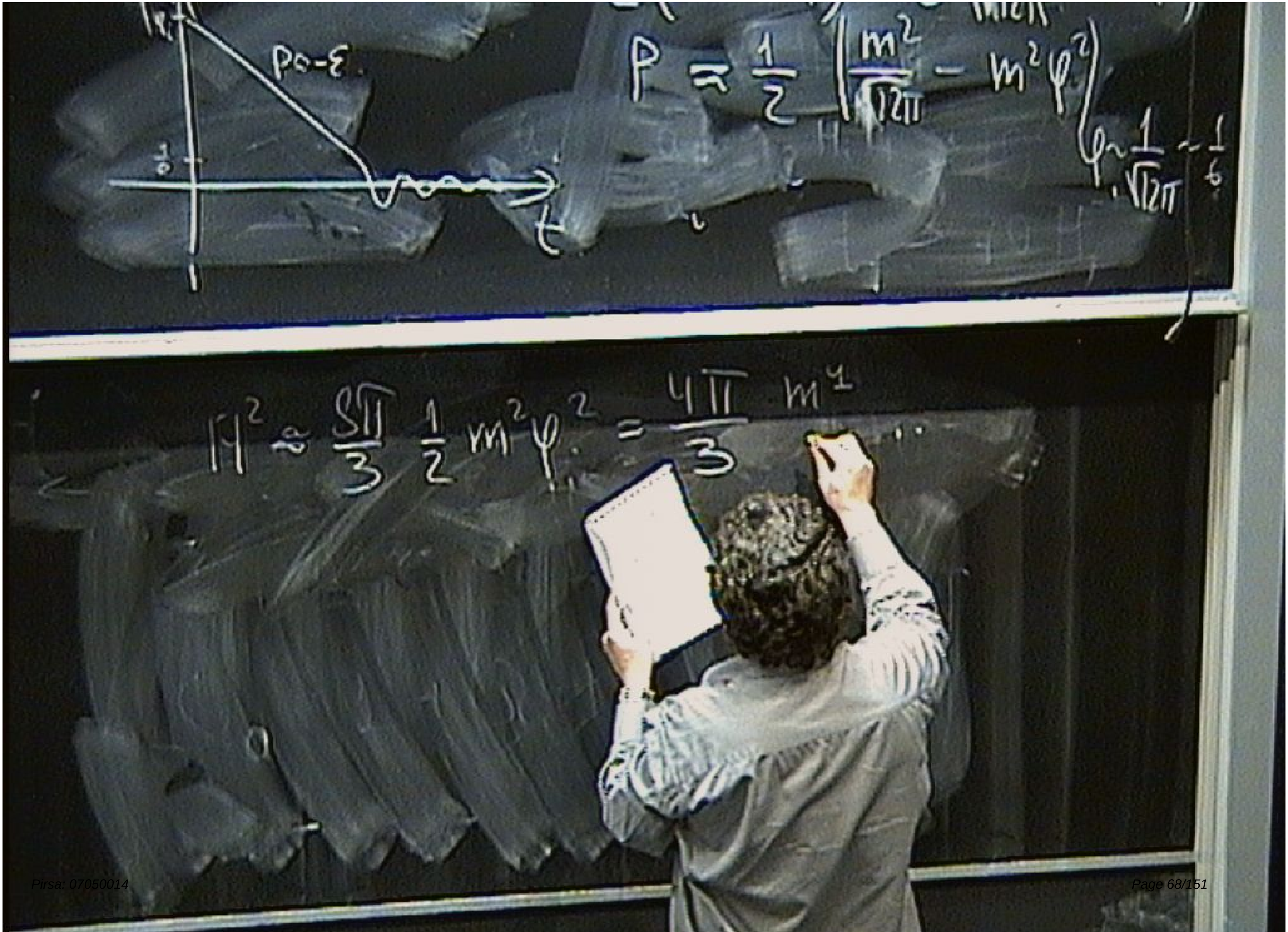
$$\varphi \sim \frac{1}{\sqrt{12\pi}} \sim \frac{1}{6}$$

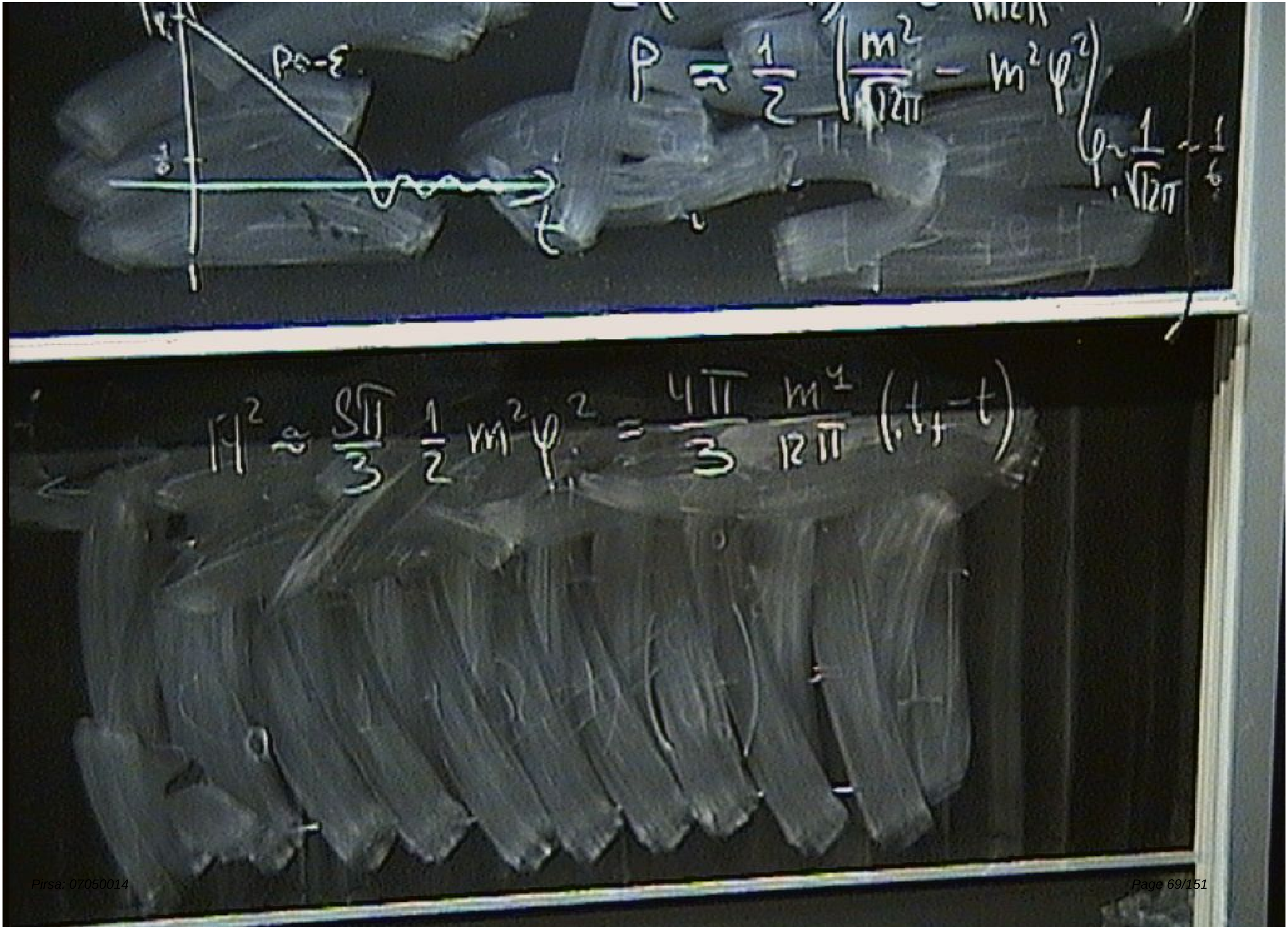


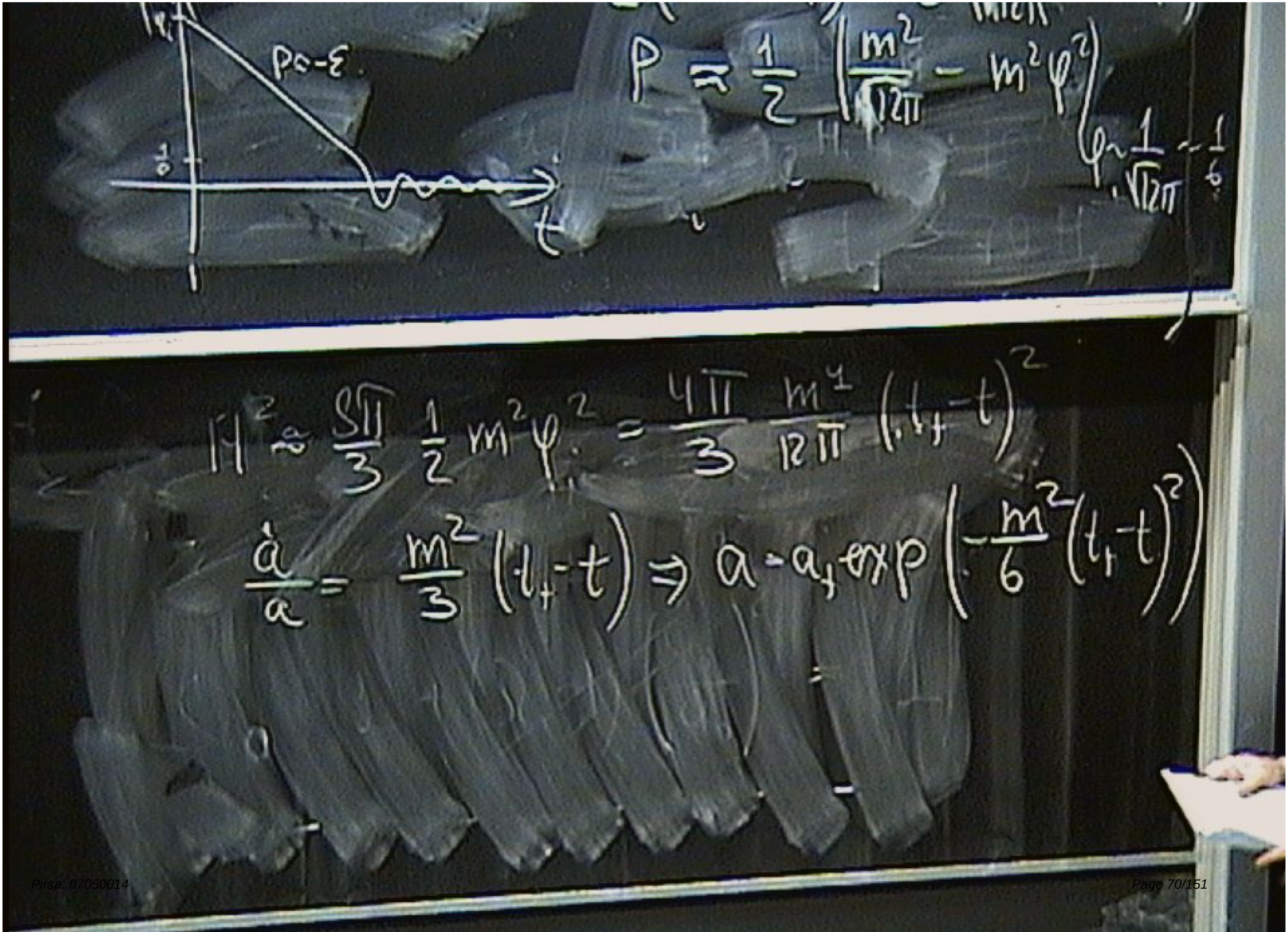


φ̇_inf ≈ − m/√(12π) (1 + 1/φ …
φ_inf(t) = φ_i − m/√(12π) (t − t_i) ≈ m/√(12π) (t_f − t)
ε = 1/2 (φ̇² + m²φ) = 1/2 (m²/(12π) + m²φ²)
p = 1/2 (m²/(12π) − m²φ²)
φ ~ 1/√(12π) ~ 1/6
φ
t

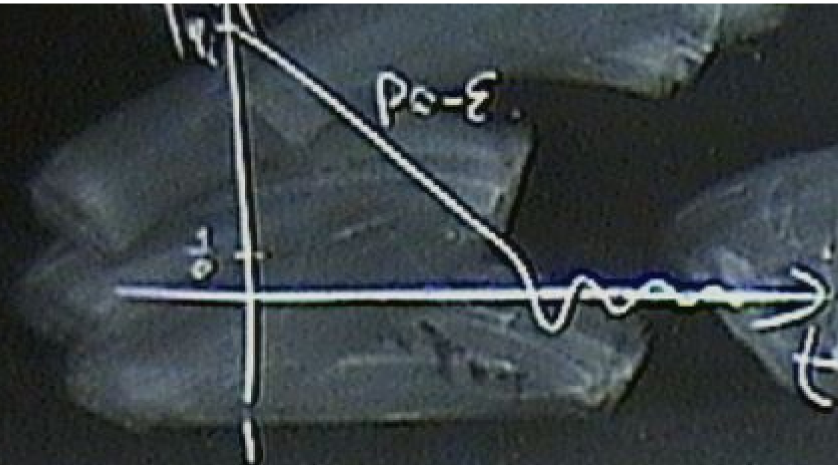



p=-ε
ρ ≈ 1/2 (m²/√12π − m²φ²)
φ ~ 1/√12π ~ 1/6
H² ≈ 8π/3 · 1/2 m²φ² = 4π/3 m⁴


p≈−ε
ρ ≈ 1/2 (m²/√12π − m²φ²)
φ ~ 1/√12π ~ 1/6
H² ≈ 8π/3 · 1/2 m²φ² = 4π/3 · m⁴/12π (t_f − t)



$p=-\varepsilon$
$\frac{1}{6}$
t
$p \approx \frac{1}{2}\left(\frac{m^2}{\sqrt{12\pi}} - m^2\varphi^2\right)$
$\varphi \sim \frac{1}{\sqrt{12\pi}} \sim \frac{1}{6}$
$H^2 \approx \frac{8\pi}{3}\frac{1}{2}m^2\varphi^2 = \frac{4\pi}{3}\frac{m^4}{12\pi}(t_f - t)^2$
$\frac{\dot{a}}{a} = \frac{m^2}{3}(t_f - t) \Rightarrow a = a_f \exp\left(-\frac{m^2}{6}(t_f - t)^2\right)$

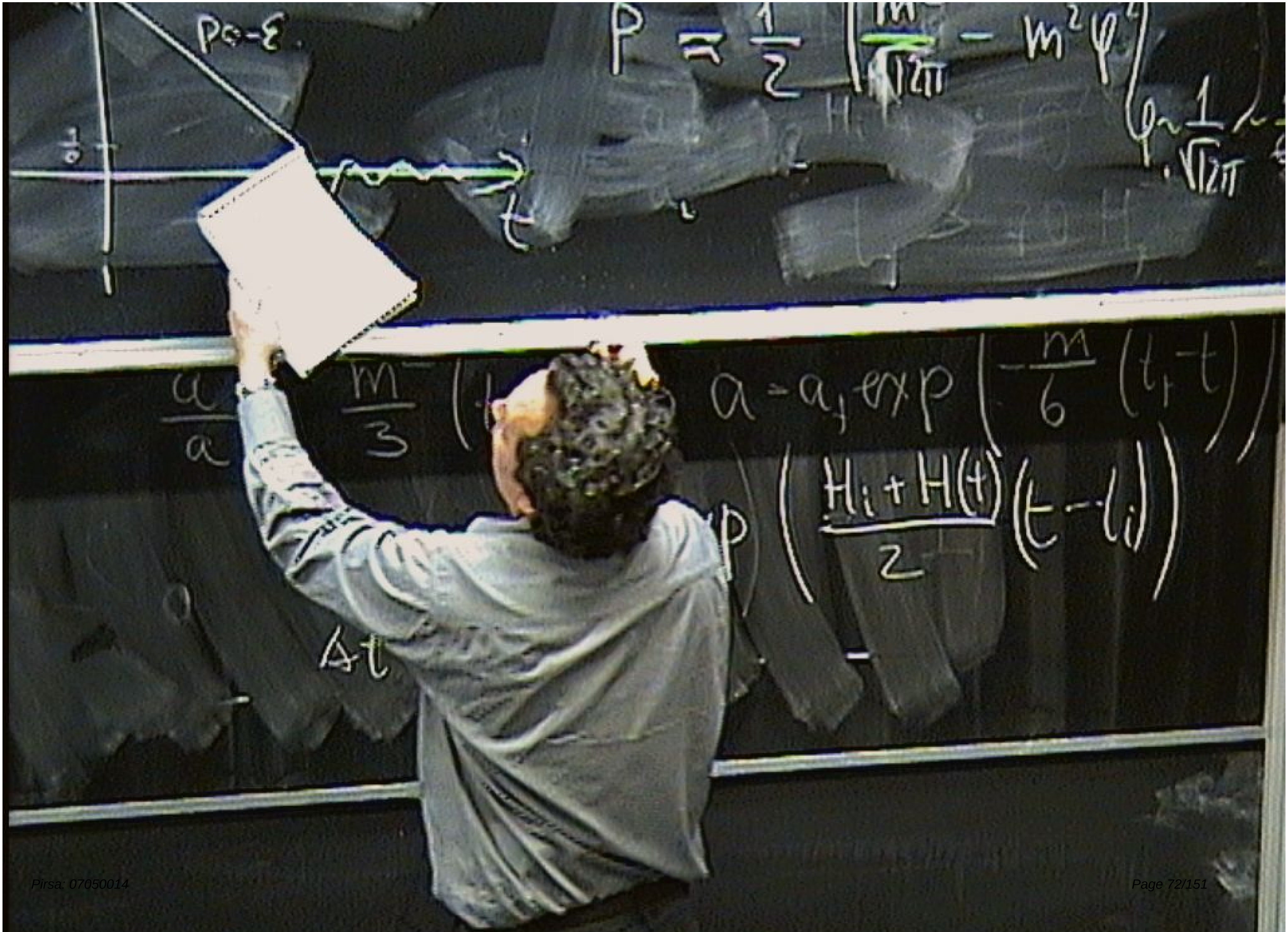


$$\rho \approx \frac{1}{2}\left(\frac{m^2}{\sqrt{12\pi}} - m^2\varphi^2\right) \qquad \varphi \sim \frac{1}{\sqrt{12\pi}} \sim \frac{1}{6}$$

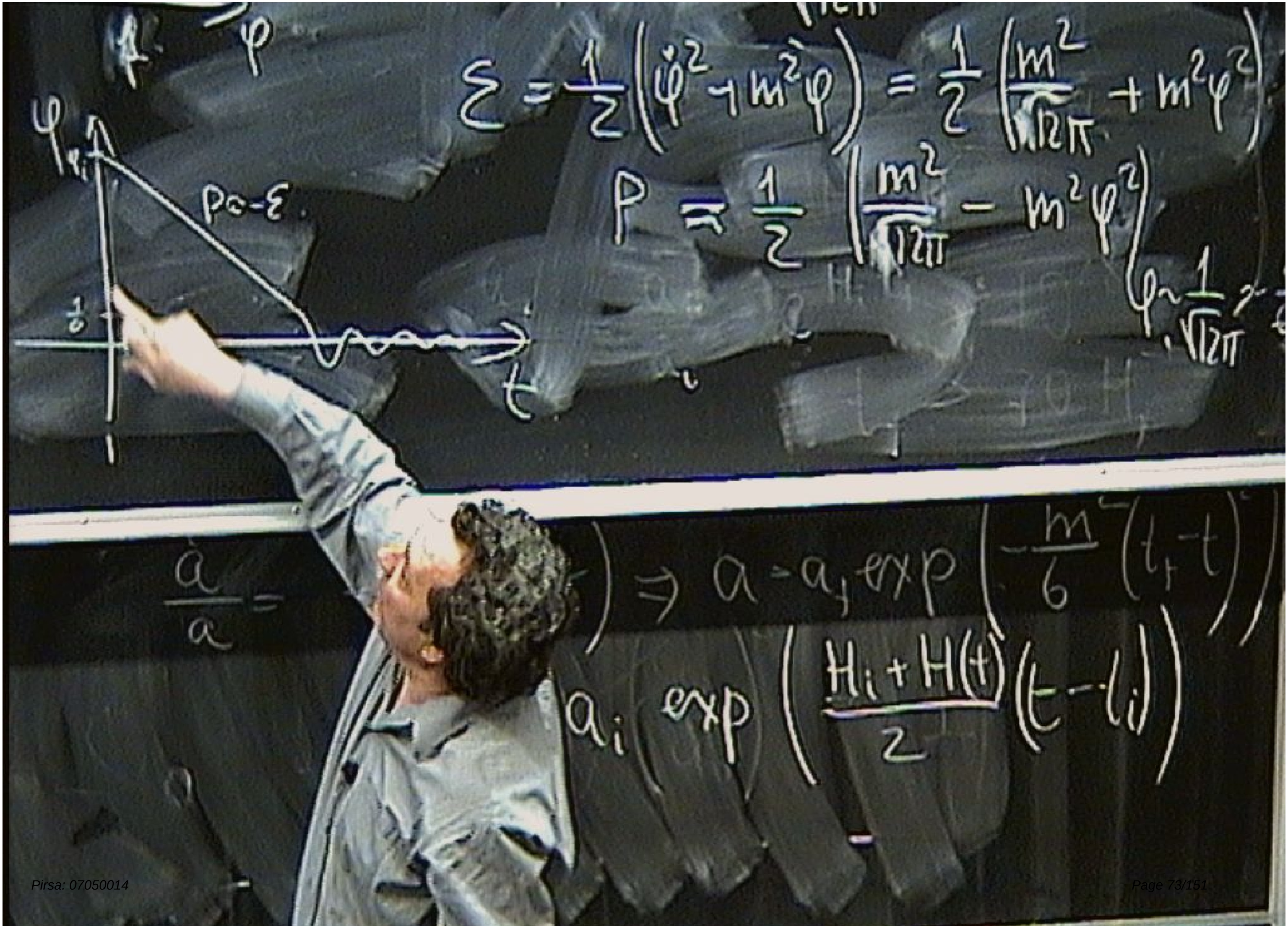
$$H^2 \approx \frac{8\pi}{3}\,\frac{1}{2}\,m^2\varphi^2 = \frac{4\pi}{3}\,\frac{m^4}{12\pi}\,(t_f - t)^2$$

$$\frac{\dot a}{a} = \frac{m^2}{3}(t_f - t) \Rightarrow a = a_f \exp\left(-\frac{m^2}{6}(t_f - t)^2\right)$$

$$= a_i \exp\left(\frac{H_i + H(t)}{2}(t - t_i)\right)$$



ρ = 1/2
a = a₁ exp(−m/6 (t₁ − t))
(H₁ + H(t))/2 (t − t₁)
Δt


ε = 1/2 (φ̇² + m²φ²) = 1/2 (m²/√12π + m²φ²)
p = 1/2 (m²/√12π − m²φ²)
p ≈ −ε
φ ~ 1/√12π
⇒ a = a_f exp(−m²/6 (t_f − t)²)
a_i exp((H_i + H(t))/2 (t − t_i))

$$H^2 \approx \frac{8\pi}{3}\frac{1}{2}m^2\varphi^2 = \frac{4\pi}{3}\frac{m^4}{12\pi}(t_f - t)^2$$

$$\frac{\dot{a}}{a} = \frac{m^2}{3}(t_f - t) \Rightarrow a = a_f \exp\left(-\frac{m^2}{6}(t_f - t)^2\right)$$

$$= a_i \exp\left(\frac{H_i + H(t)}{2}(t - t_i)\right)$$

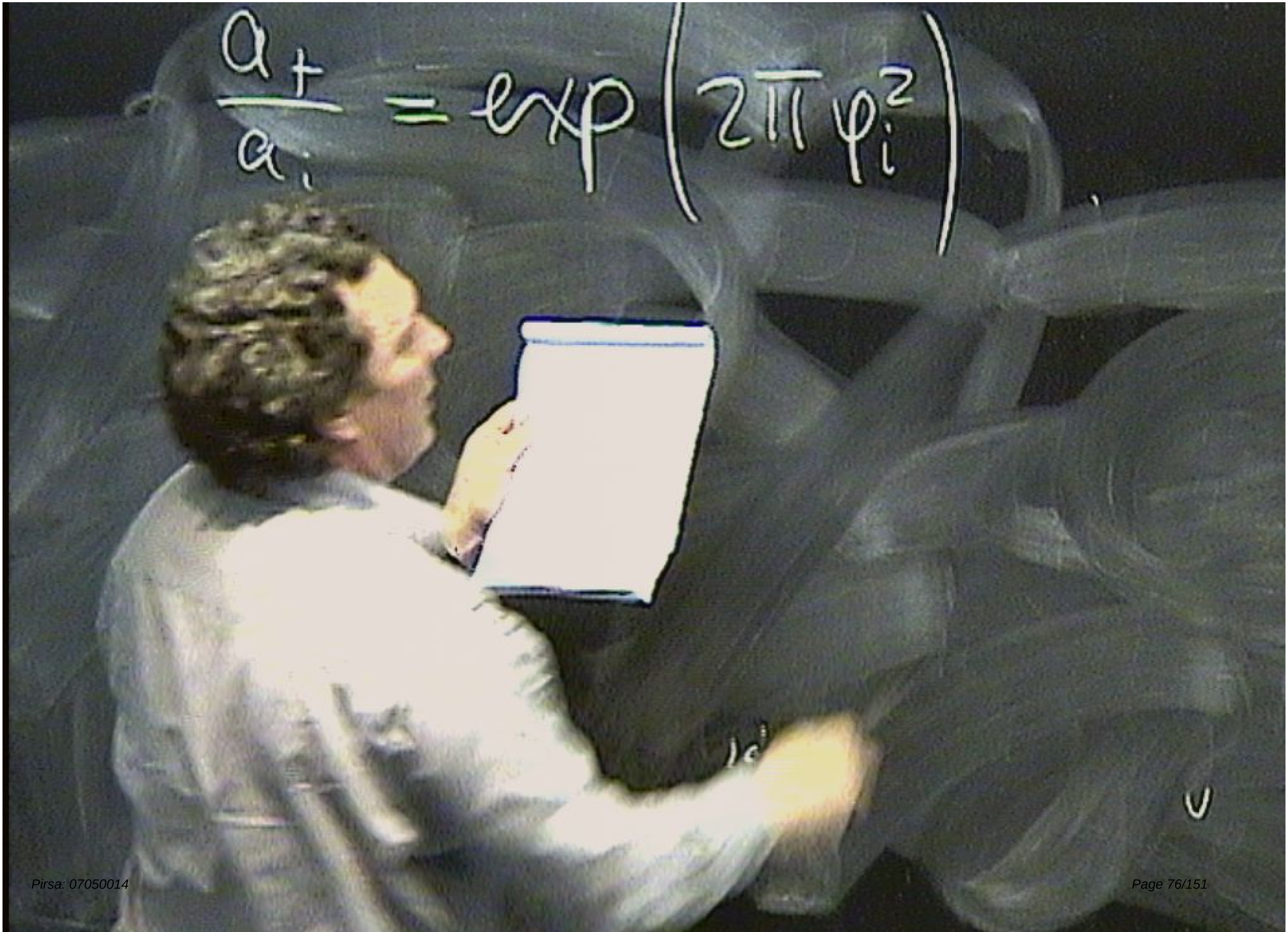
$$\Delta t \sim \sqrt{12\pi}\,\frac{\varphi_i}{m}$$

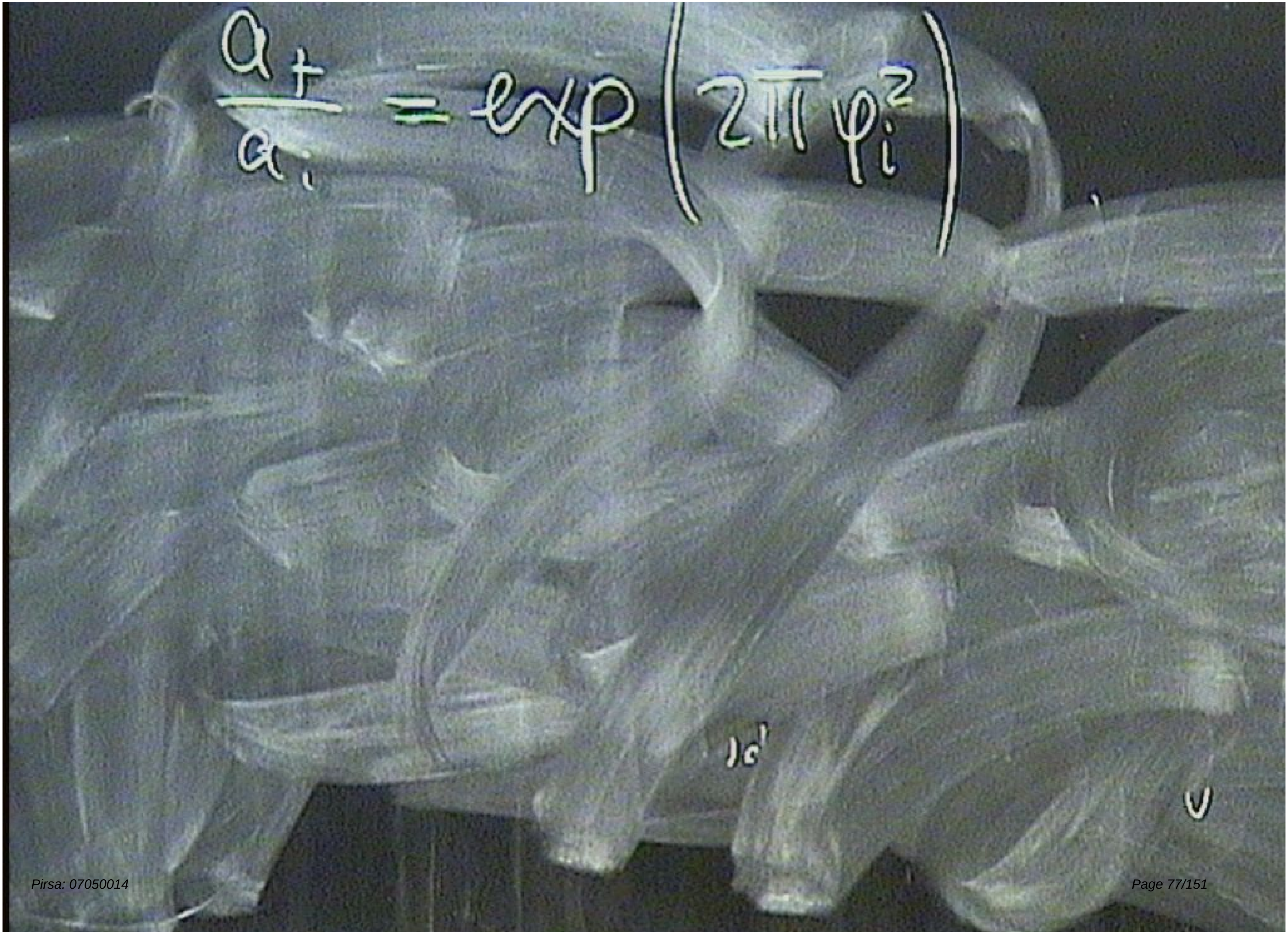
$$H^2 \approx \frac{8\pi}{3}\,\frac{1}{2}\,m^2\varphi^2 = \frac{4\pi}{3}\,\frac{m^4}{12\pi}\,(t_f - t)^2$$

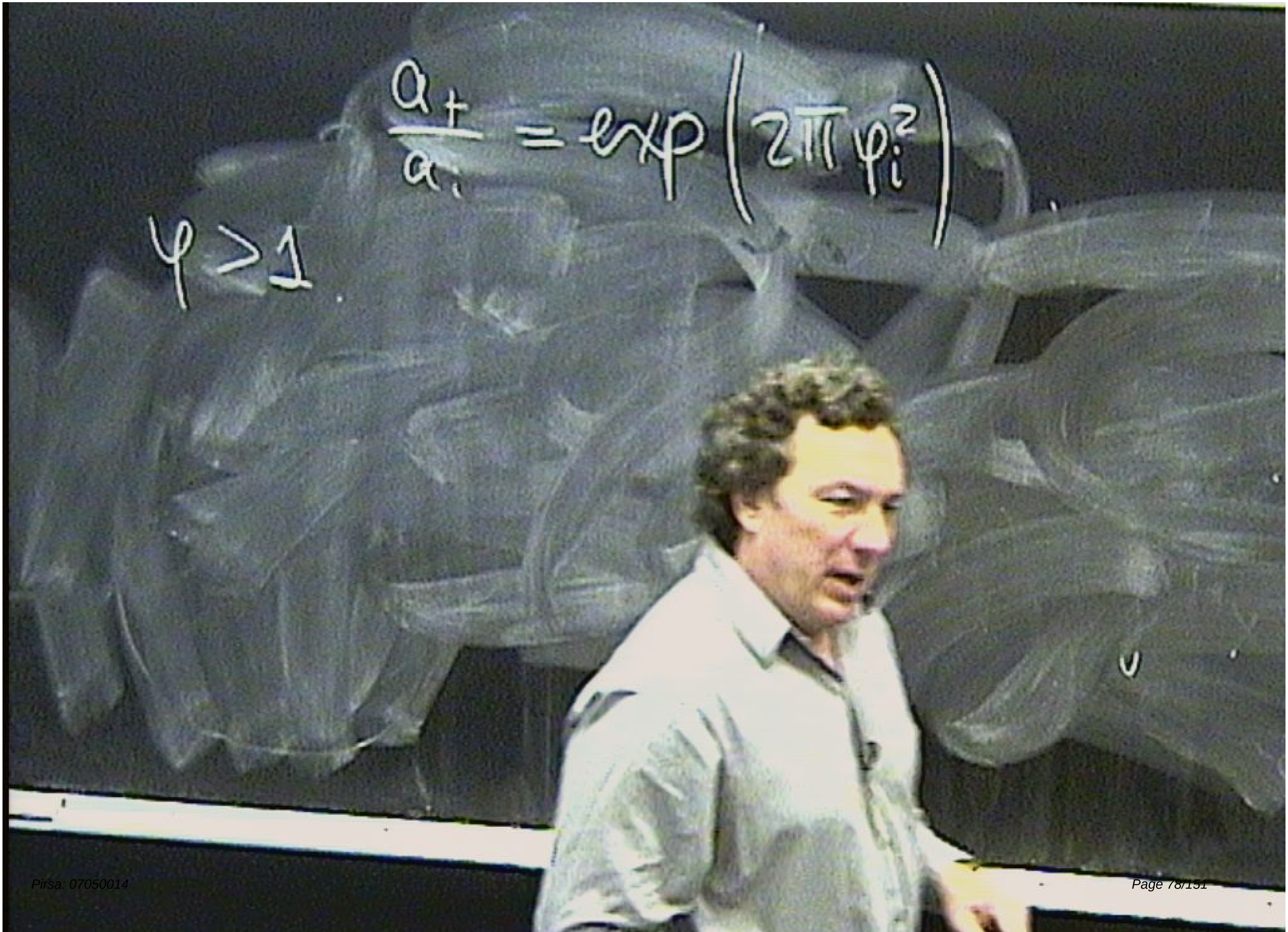
$$\frac{\dot a}{a} = \frac{m^2}{3}(t_f - t) \Rightarrow a = a_f \exp\left(-\frac{m^2}{6}(t_f - t)^2\right)$$

$$= a_i \exp\left(\frac{H_i + H(t)}{2}(t - t_i)\right)$$

$$\Delta t \sim \sqrt{12\pi}\,\frac{\varphi_i}{m}$$


$\frac{a_f}{a_i} = \exp\left(2\pi \varphi_i^2\right)$


$\frac{a_f}{a_i} = \exp\left(2\pi \varphi_i^2\right)$


$\frac{a_f}{a_i} = \exp(2\pi\varphi_i^2)$
$\varphi > 1$

$$\frac{a_f}{a_i} = \exp\left(2\pi\varphi_i^2\right)$$

$$\varphi > 1$$

$$m^2\varphi^2 < 1$$

$$\varphi_i < \frac{1}{m}$$

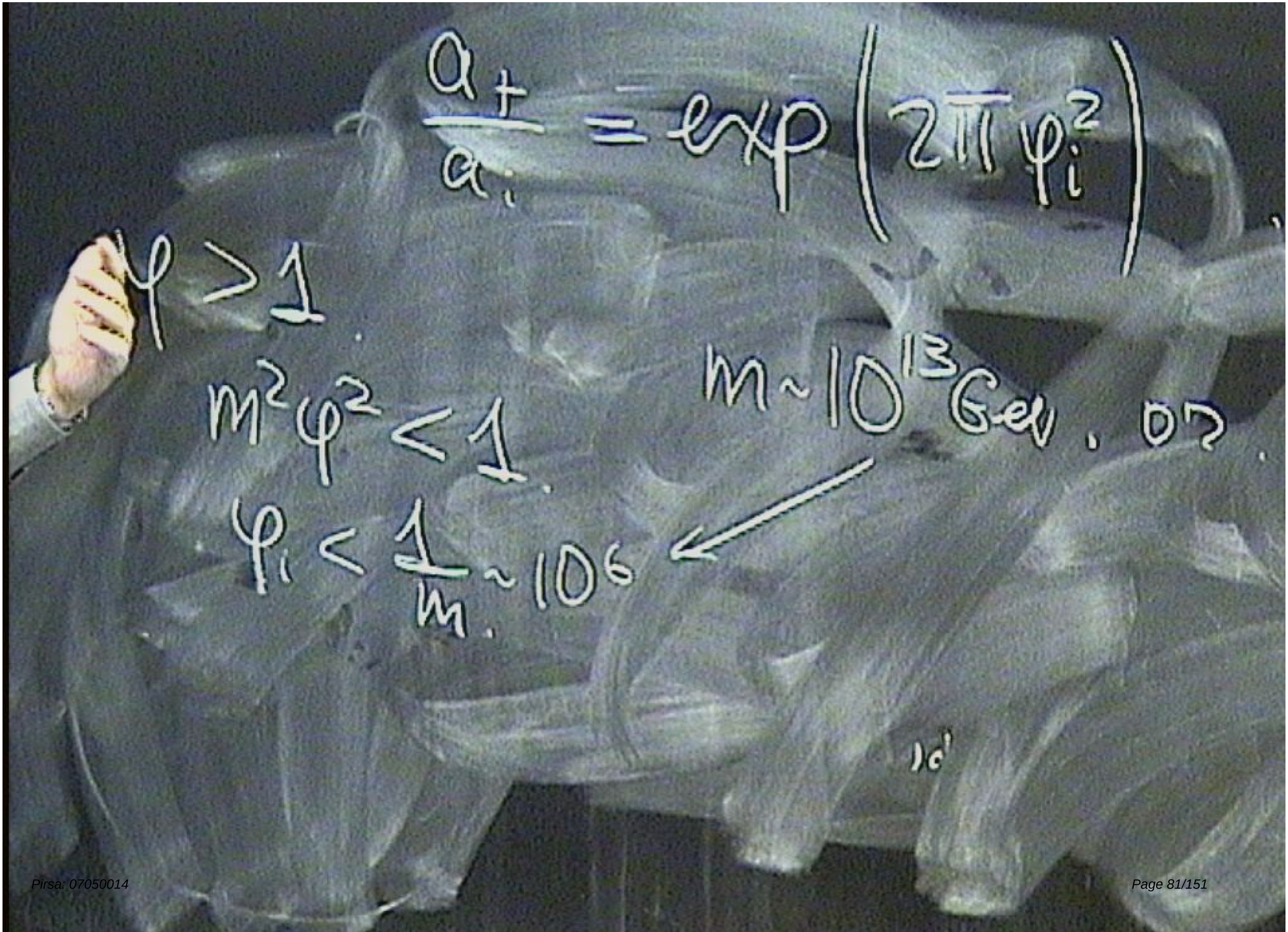
$$\frac{a_f}{a_i} = \exp\left(2\pi\varphi_i^2\right).$$

$\varphi > 1$.

$m^2\varphi^2 < 1$.

$\varphi_i < \frac{1}{m}$.

$m \sim 10^{13}\,\mathrm{GeV}$.


$\frac{a_f}{a_i} = \exp(2\pi\varphi_i^2)$
$\varphi > 1$
$m^2\varphi^2 < 1$
$\varphi_i < \frac{1}{m} \sim 10^6$
$m \sim 10^{13}$ GeV, 0?

$$\frac{a_f}{a_i} = \exp\left(2\pi \varphi_i^2\right)$$

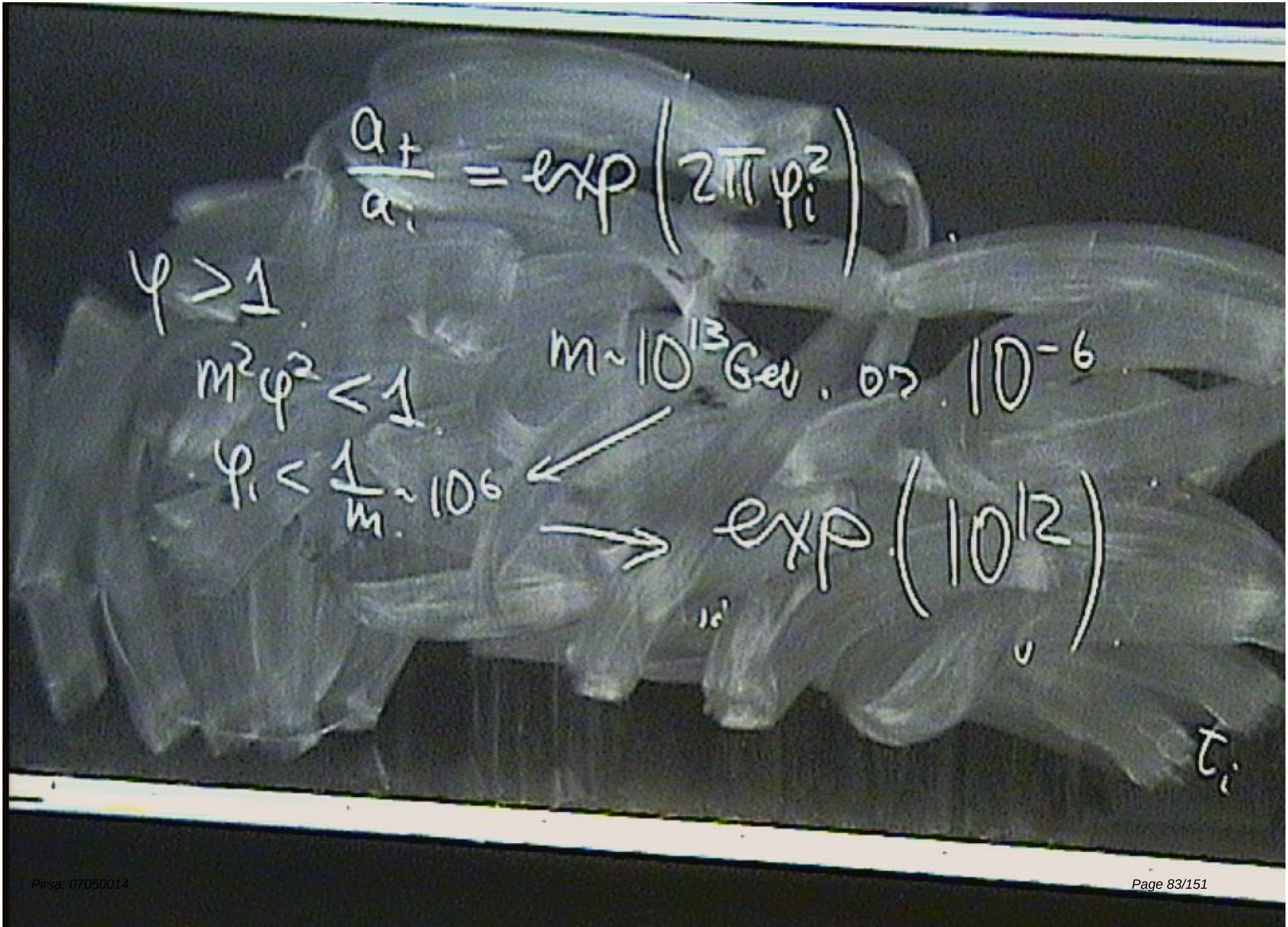
$$\varphi > 1$$

$$m^2 \varphi^2 < 1$$

$$\varphi_i < \frac{1}{m} \sim 10^6$$

$$m \sim 10^{13}\,\text{GeV}$$

$$\longrightarrow \exp$$


$\frac{a_t}{a_i} = \exp(2\pi \varphi_i^2)$
$\varphi > 1$
$m^2 \varphi^2 < 1$
$\varphi_i < \frac{1}{m} \sim 10^6$
$m \sim 10^{13}$ GeV, or 10^{-6}
$\rightarrow \exp(10^{12})$
t_i

$$\frac{a_f}{a_i} = \exp\left(2\pi \varphi_i^2\right)$$

$\varphi > 1$

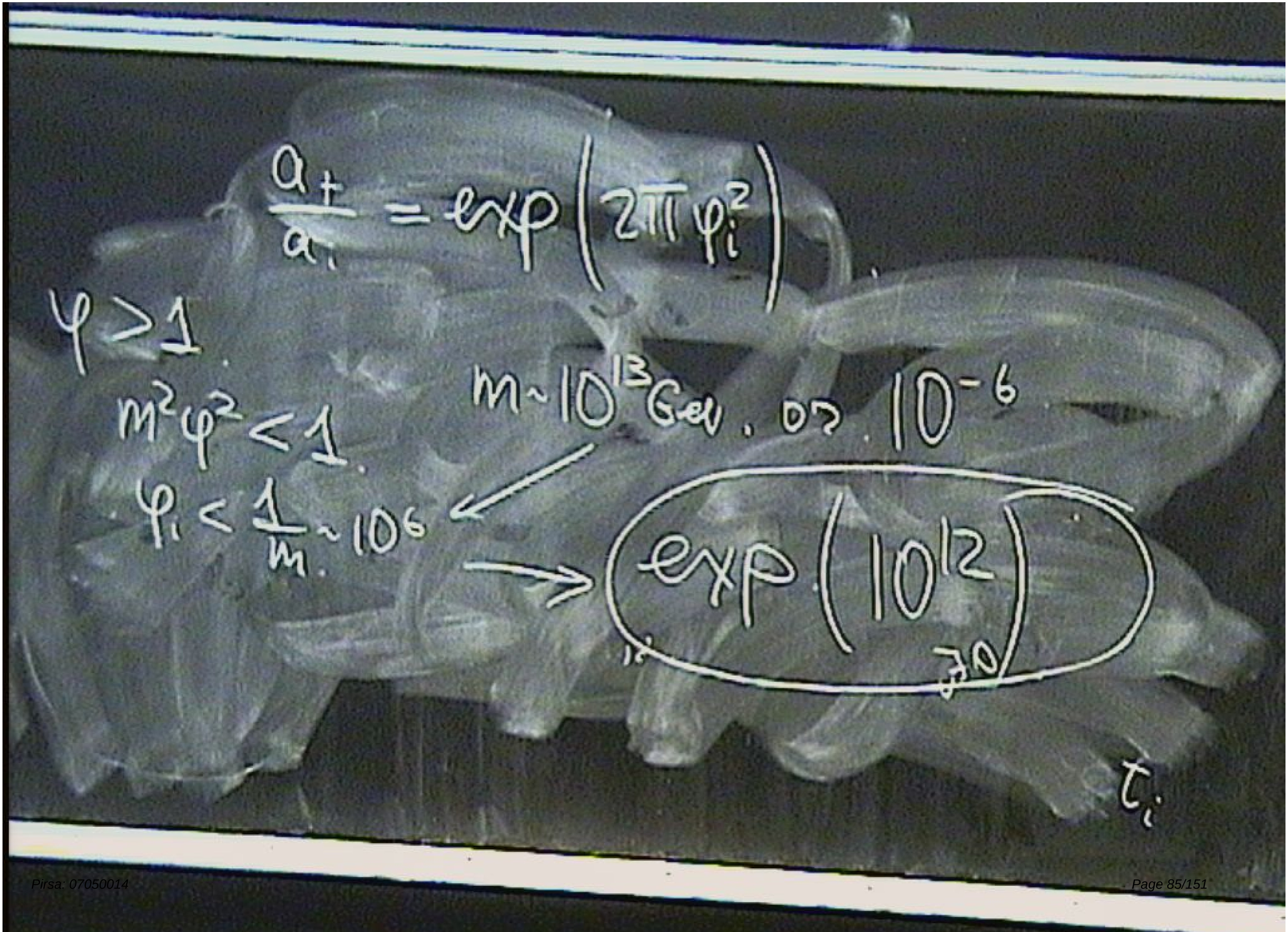
$m^2 \varphi^2 < 1$

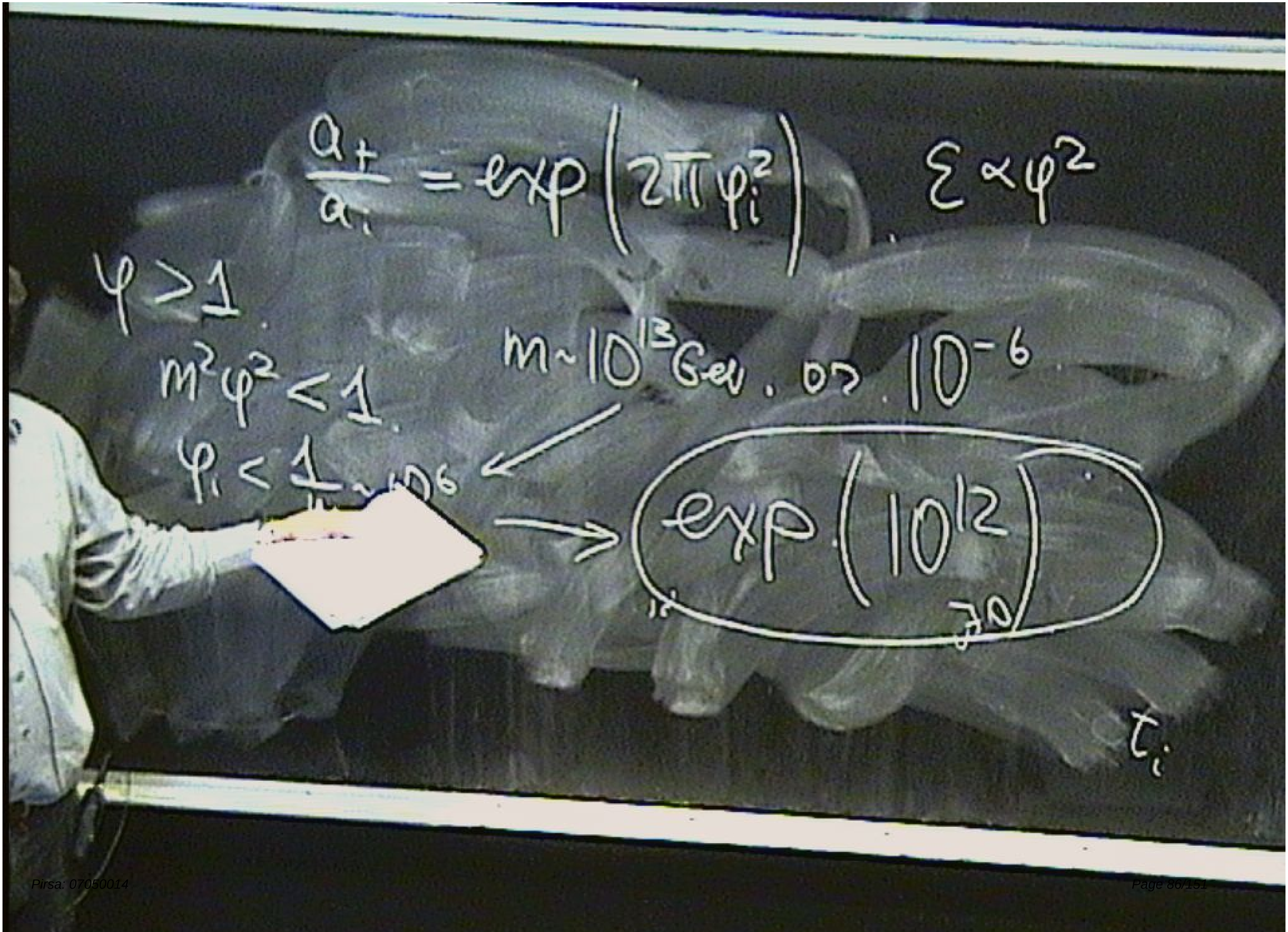
$\varphi_i < \frac{1}{m} \sim 10^6$

$m \sim 10^{13}\,\mathrm{GeV}$, or 10^{-6}

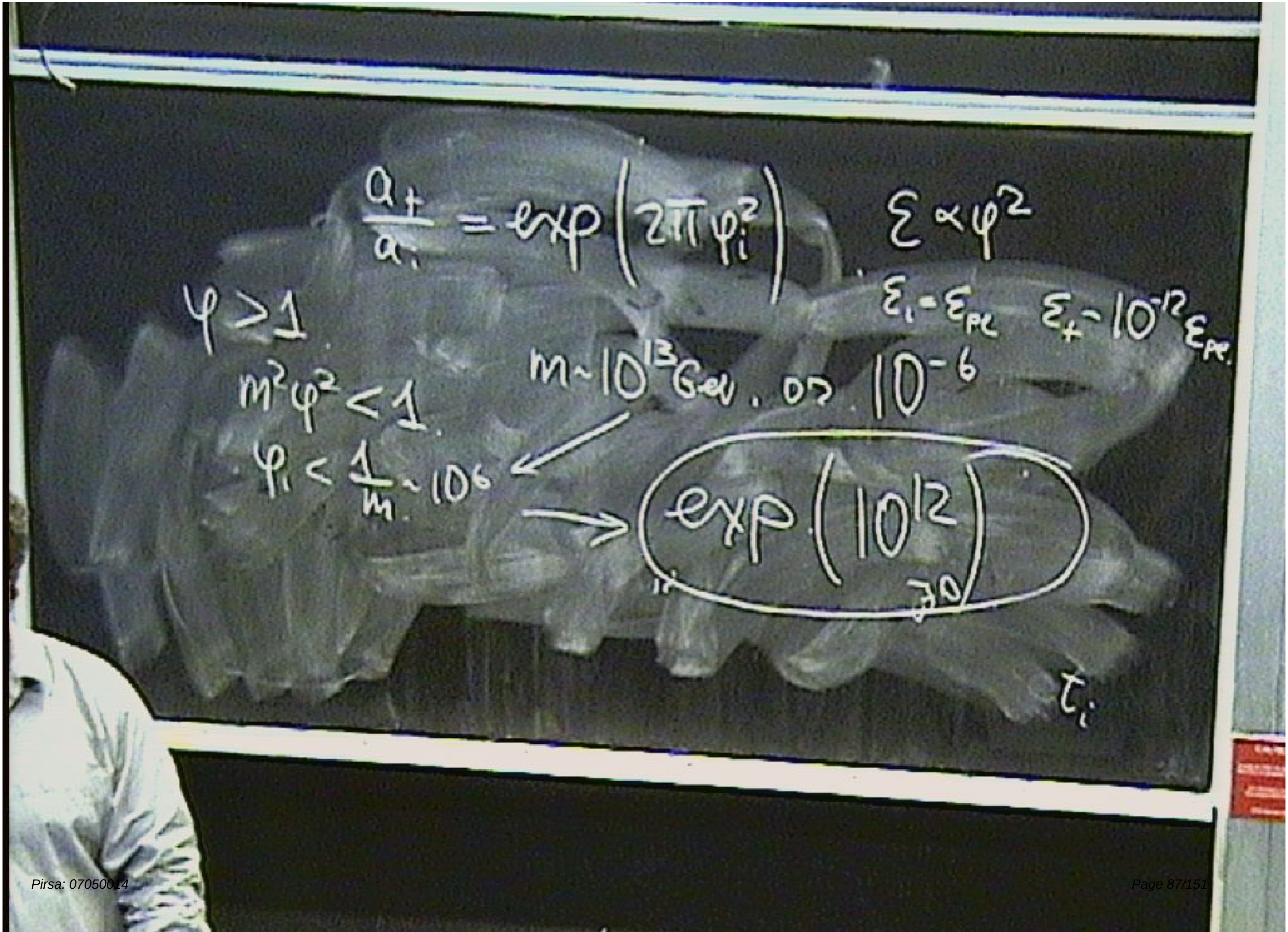
$\rightarrow \exp\left(10^{12}\right)$

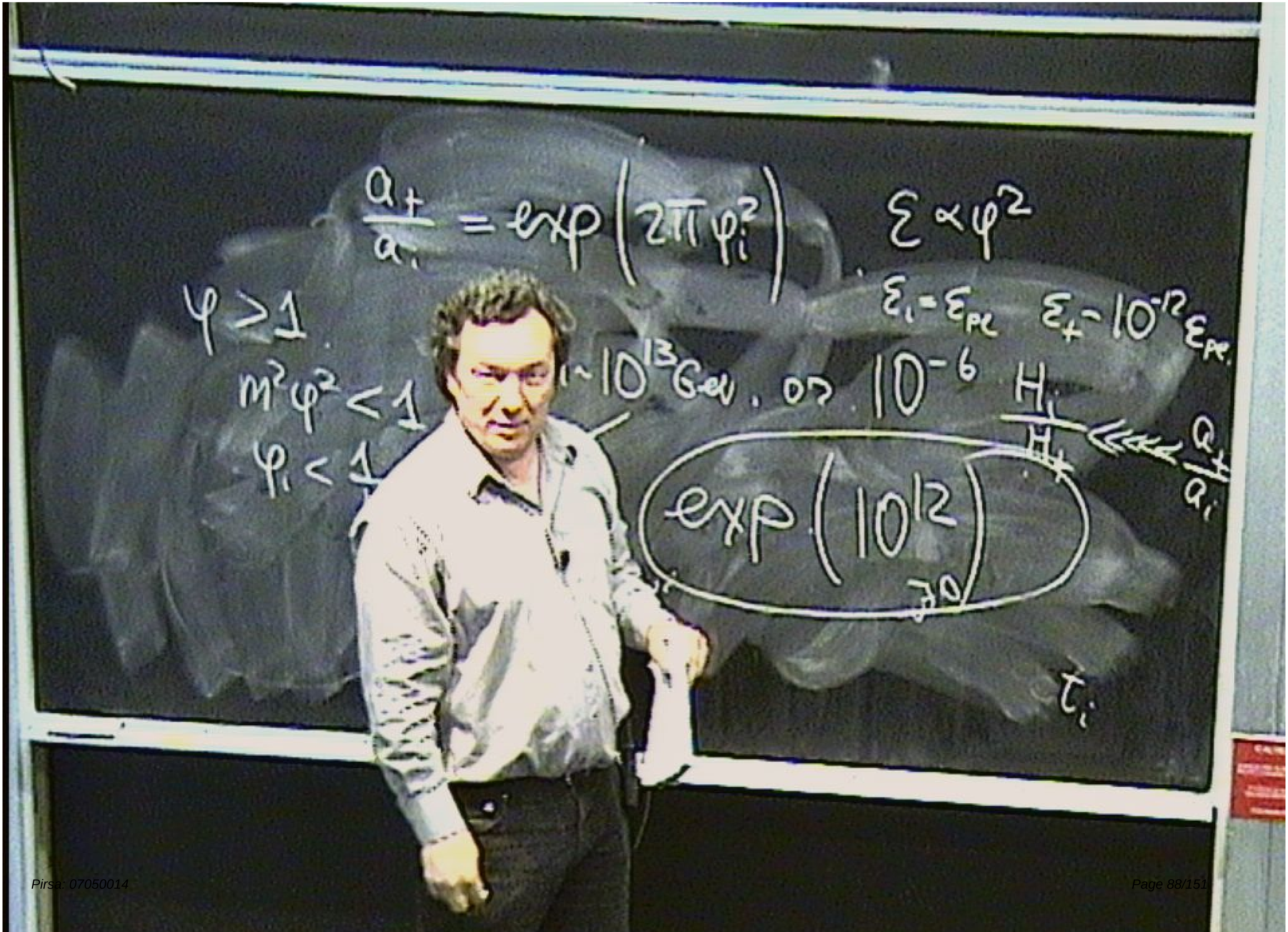
t_i

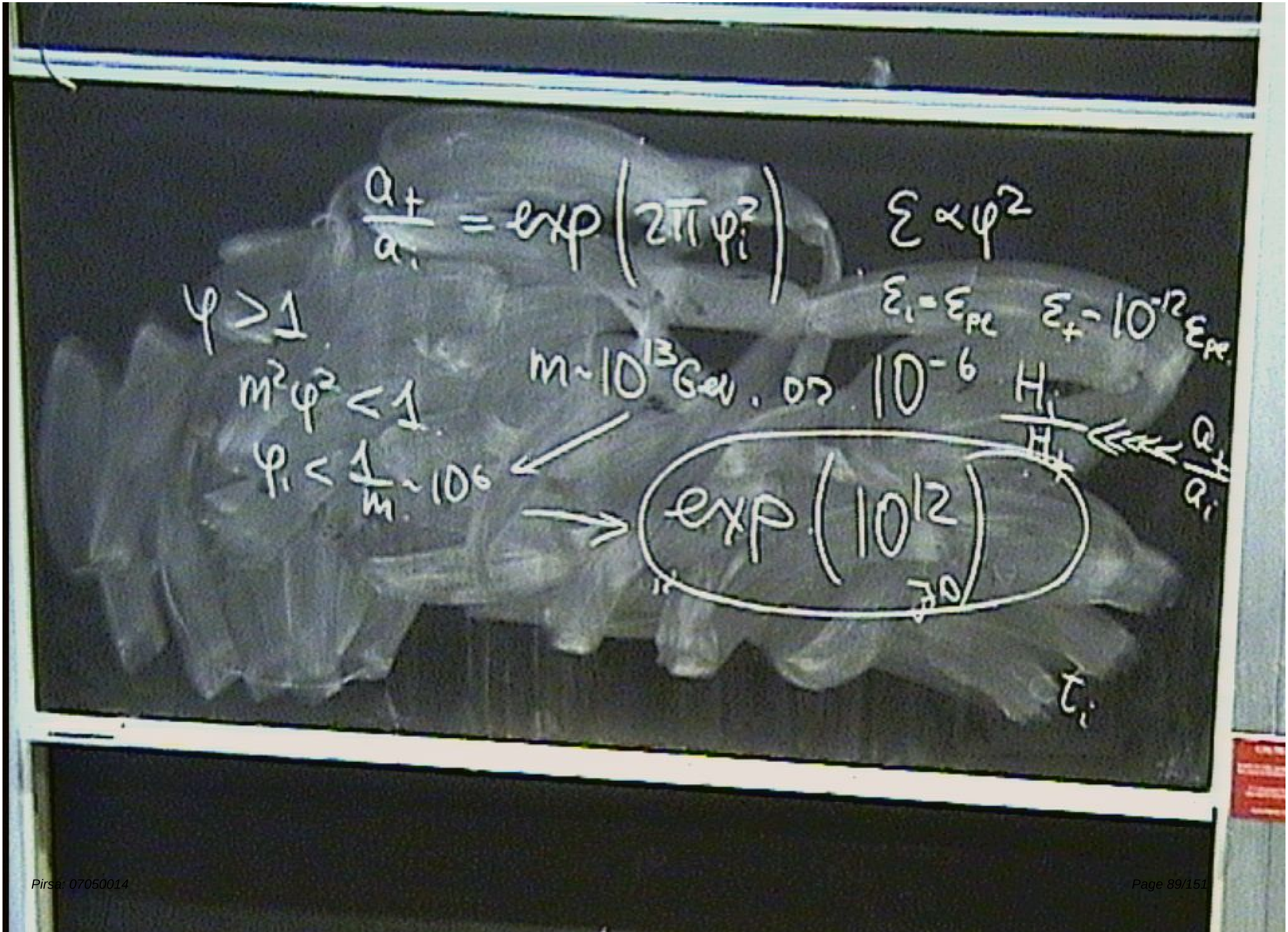

$\frac{a_t}{a_i} = \exp\left(2\pi \varphi_i^2\right)$
$\varphi > 1$
$m^2 \varphi^2 < 1$
$\varphi_i < \frac{1}{m} \sim 10^6$
$m \sim 10^{13}$ GeV or 10^{-6}
$\exp\left(10^{12}\right)$
t_i



a_t / a_i = exp(2π φ_i^2)
ε ∝ φ^2
φ > 1
m^2 φ^2 < 1
m ~ 10^13 GeV . or 10^-6
φ_i < 1
exp(10^12)
t_i


$\frac{a_f}{a_i} = \exp(2\pi \varphi_i^2)$
$\varepsilon \propto \varphi^2$
$\varphi > 1$
$\varepsilon_i = \varepsilon_{Pl}$
$\varepsilon_f \sim 10^{-12} \varepsilon_{Pl}$
$m^2 \varphi^2 < 1$
$m \sim 10^{13}$ Gev, or 10^{-6}
$\varphi_i < \frac{1}{m} \sim 10^6$
$\exp(10^{12})$
t_i


a_t / a_i = exp(2π φ_i²)
ε ∝ φ²
φ > 1
ε_i = ε_pl   ε_t ~ 10^-12 ε_pl
m²φ² < 1
~10^13 GeV, or 10^-6
H_i / H_t a_t / a_i
φ_i < 1
exp(10^12)
t_i

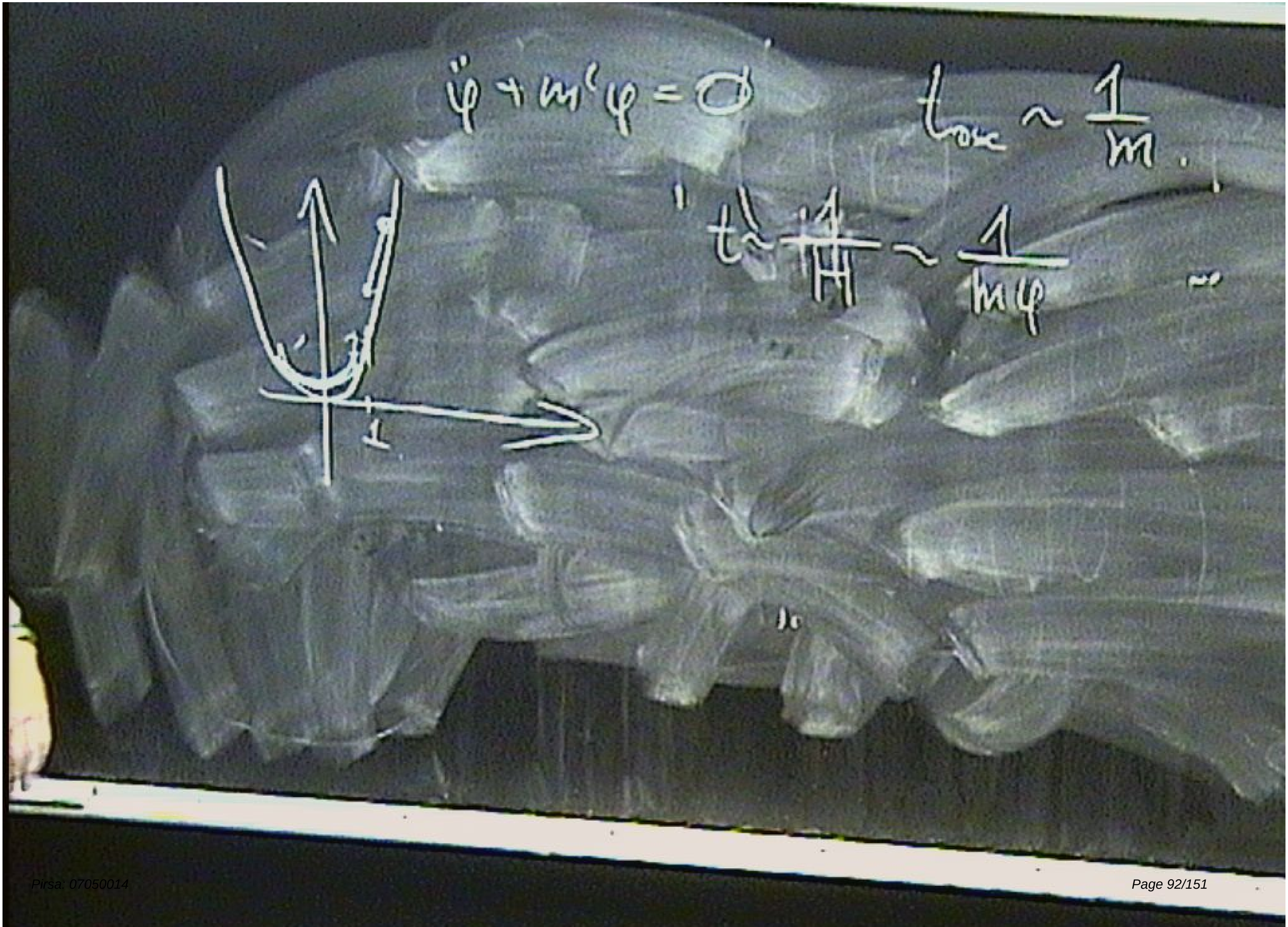

a_f / a_i = exp(2π φ_i^2)
ε ∝ φ^2
φ > 1
ε_i ~ ε_pl
ε_f ~ 10^-12 ε_pl
m^2 φ^2 < 1
m ~ 10^13 GeV, or 10^-6
H_i / H_f
a_f / a_i
φ_i < 1/m ~ 10^6
exp(10^12)
t_i

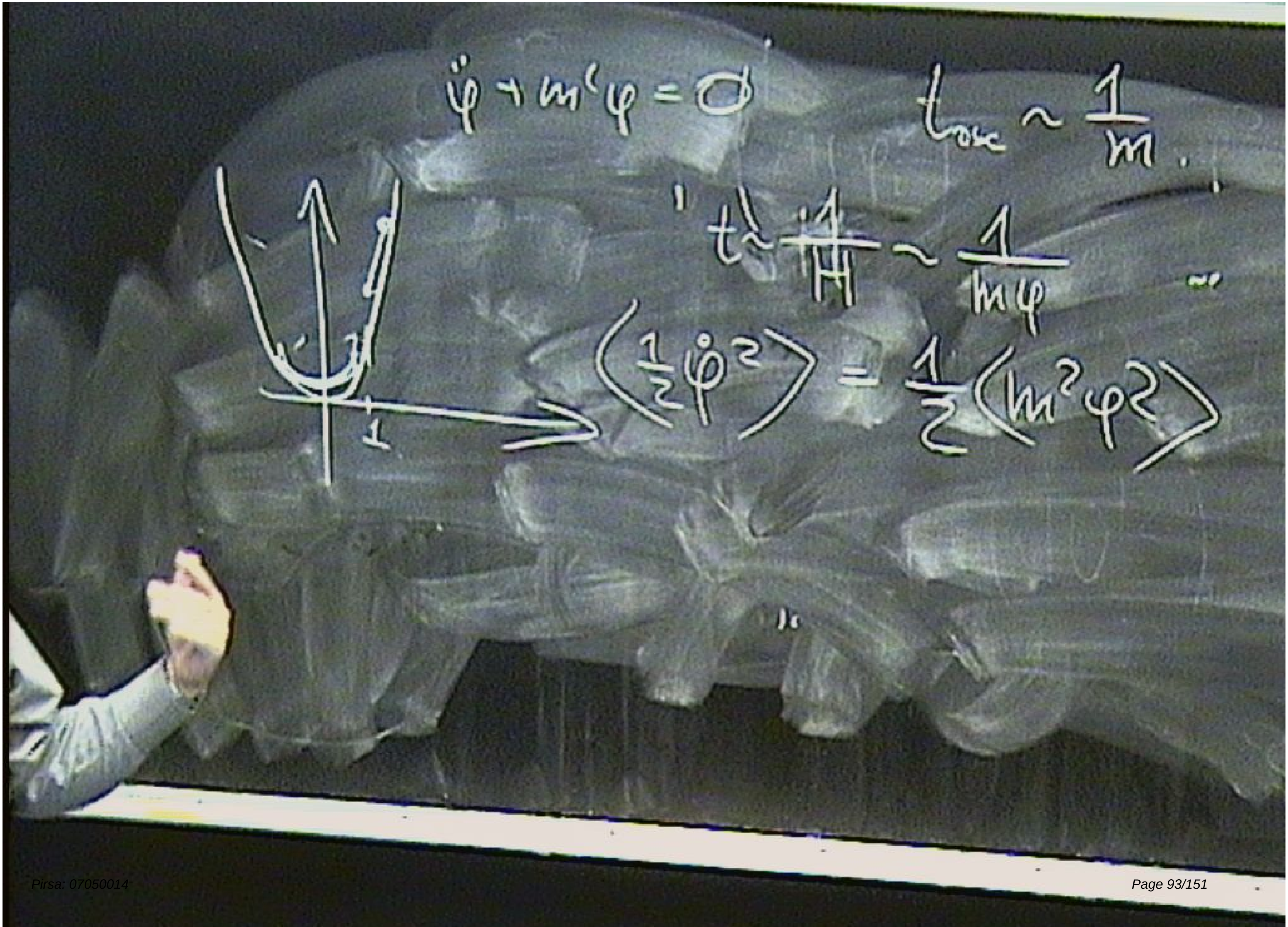
$$\ddot{\varphi} + m^2\varphi = 0 \qquad t_{osc} \sim \frac{1}{m}$$

$$t \sim \frac{1}{H}$$

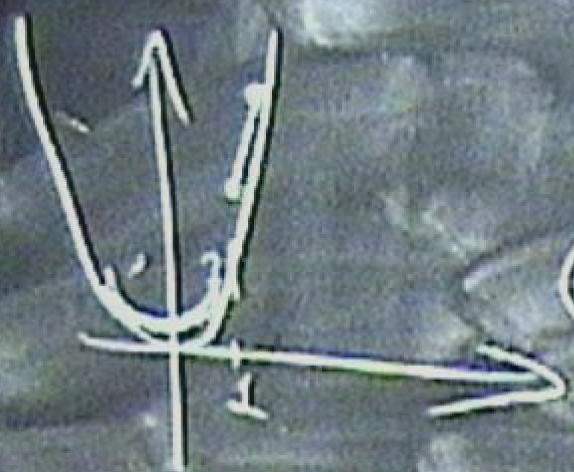
$$\ddot{\varphi} + m^2\varphi = 0 \qquad t_{osc} \sim \frac{1}{m}$$

$$t \sim \frac{1}{H} \sim \frac{1}{m\varphi}$$


$\ddot{\varphi} + m^2\varphi = 0$
$t_{osc} \sim \frac{1}{m}$
$t \sim \frac{1}{H} \sim \frac{1}{m_\varphi}$
1


$\ddot{\varphi} + m^2\varphi = 0$
$t_{osc} \sim \frac{1}{m}$.
$t \sim \frac{1}{H} \sim \frac{1}{m\varphi}$
$\langle \frac{1}{2}\dot{\varphi}^2 \rangle = \frac{1}{2}\langle m^2\varphi^2 \rangle$
1

$$\ddot{\varphi} + m^2\varphi = 0 \qquad t_{osc} \sim \frac{1}{m}.$$

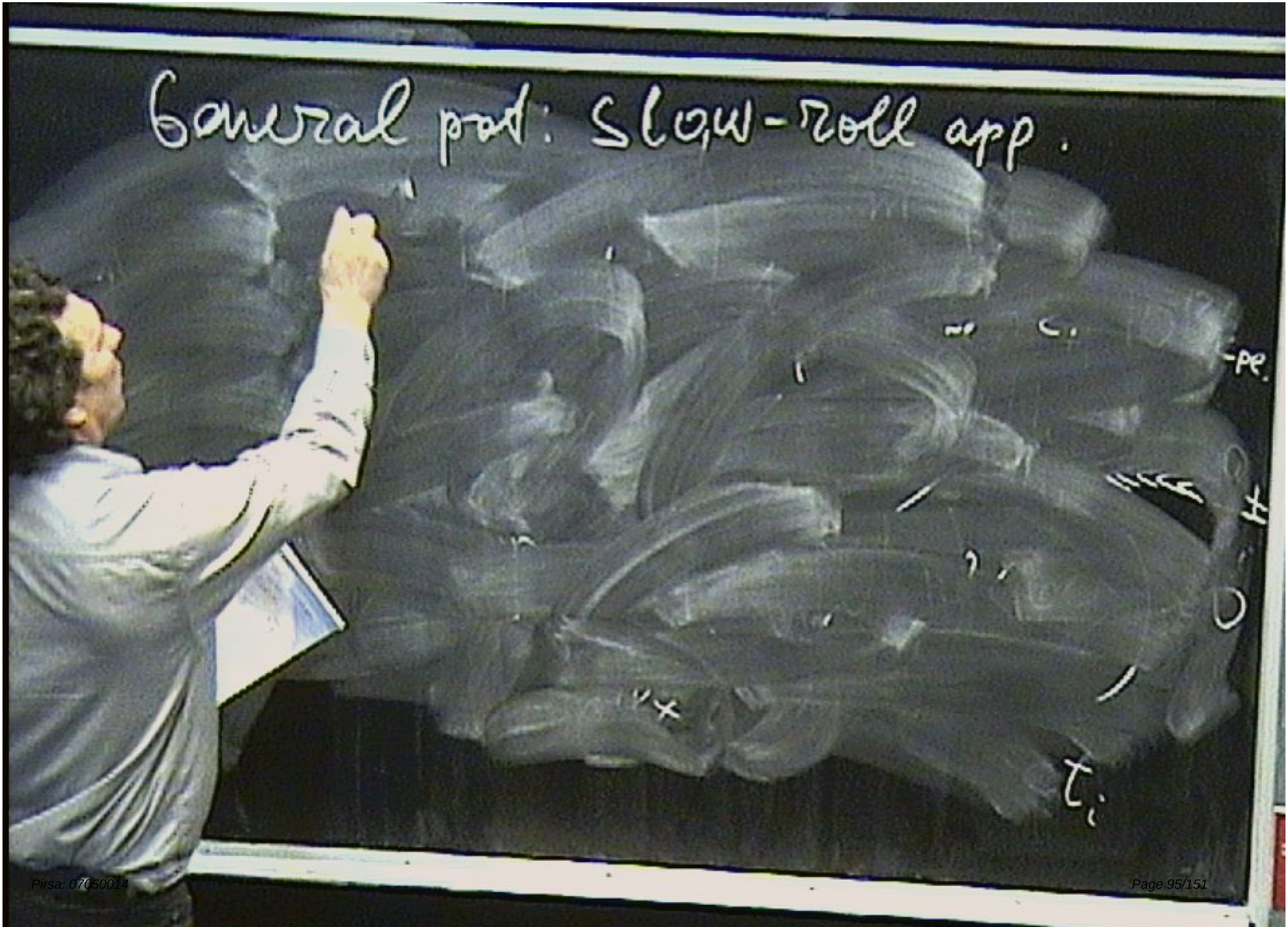


$$t \sim \frac{1}{H} \sim \frac{1}{m\varphi}$$

$$\left\langle \frac{1}{2}\dot{\varphi}^2 \right\rangle = \frac{1}{2}\left\langle m^2\varphi^2 \right\rangle$$

$$p = \frac{1}{2}\left(\langle \dot{\varphi}^2 \rangle - m^2 \langle \varphi^2 \rangle \right) = 0$$

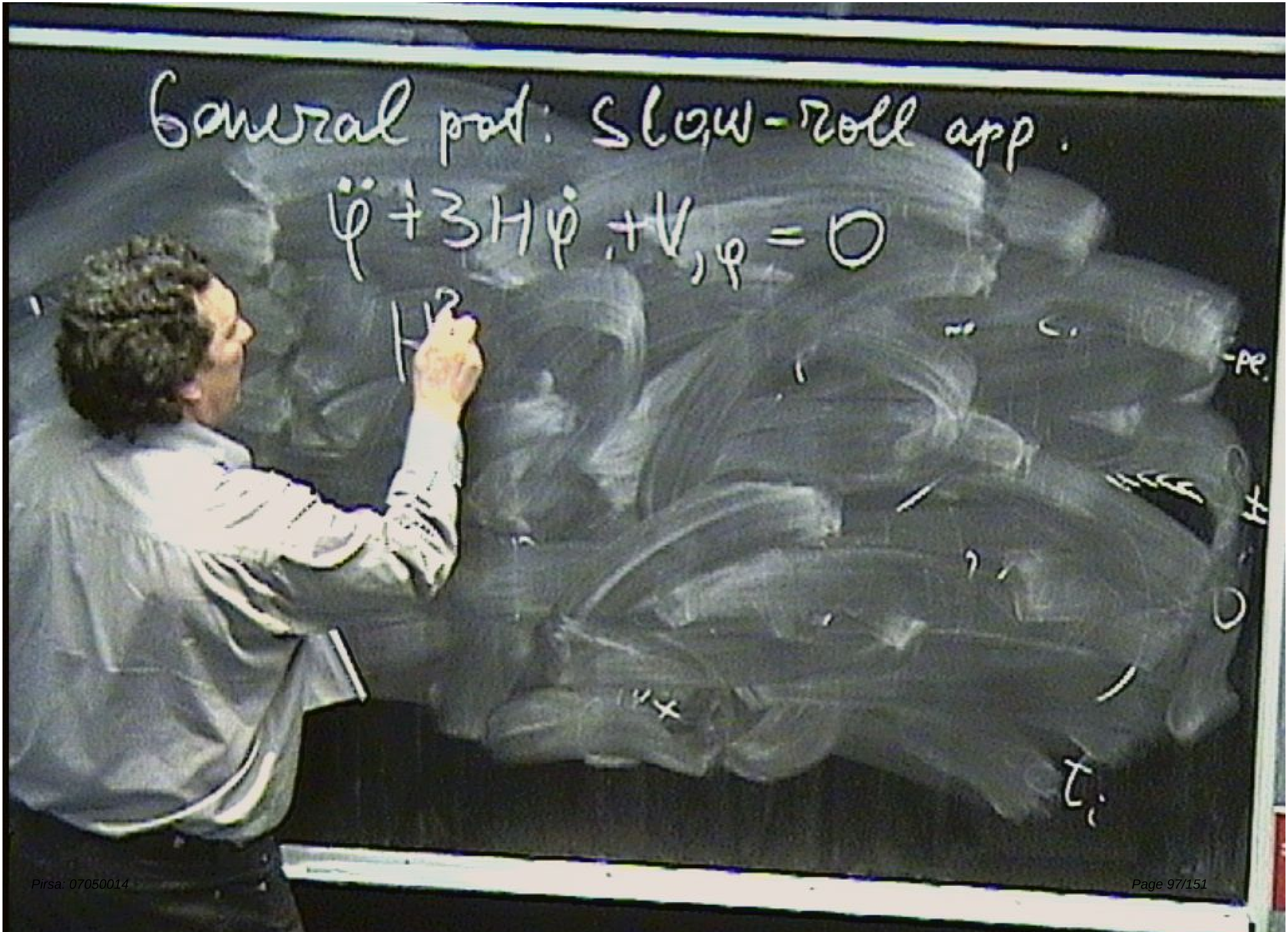
t_i


General pot: slow-roll app.

General pot: slow-roll app.

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi}$$

t_i


General pot: slow-roll app.
$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0$
H^2

General pot: slow-roll app.

$$\begin{cases} \ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0 \\ H^2 = \frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) \end{cases}$$

t_i

General pot: slow-roll app.

$$\begin{cases} 3H\dot{\varphi} + V_{,\varphi} \simeq 0 \\ H^2 \simeq \frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) \end{cases}$$

$a(\varphi)$

General pot: slow-roll app.

$$\begin{cases} 3H\dot{\varphi} + V_{,\varphi} \approx 0 \\ H^2 \approx \frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) \end{cases}$$

$a(\varphi)$

$$H = \frac{d\ln a}{dt} = \sqrt{\frac{8\pi}{3}V}$$

$$\frac{d\ln a}{dt} = \dot{\varphi}\frac{d\ln a}{d\varphi} = -\frac{V_{,\varphi}}{3H}\frac{d\ln a}{d\varphi}$$

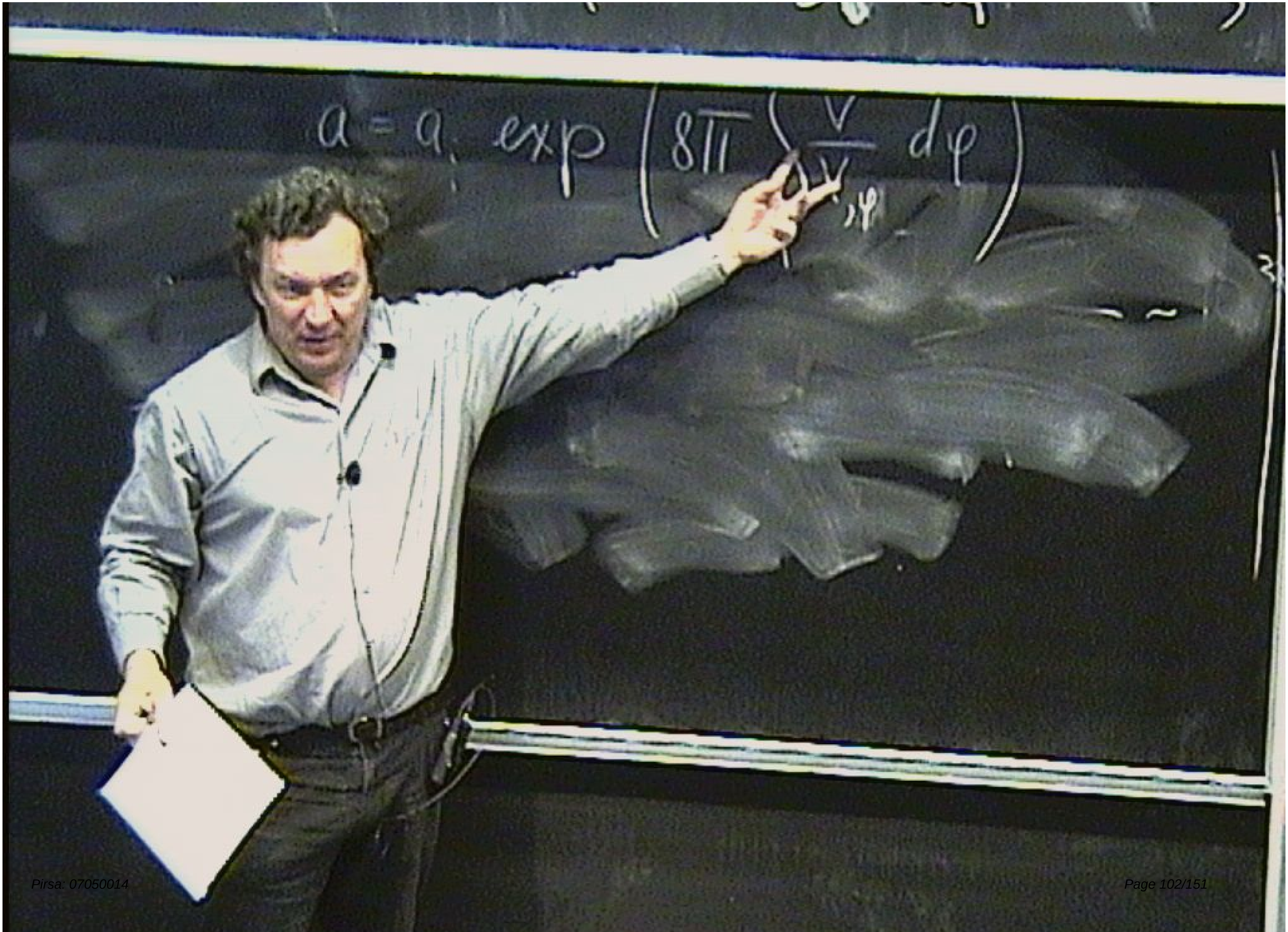
General pot: slow-roll app.

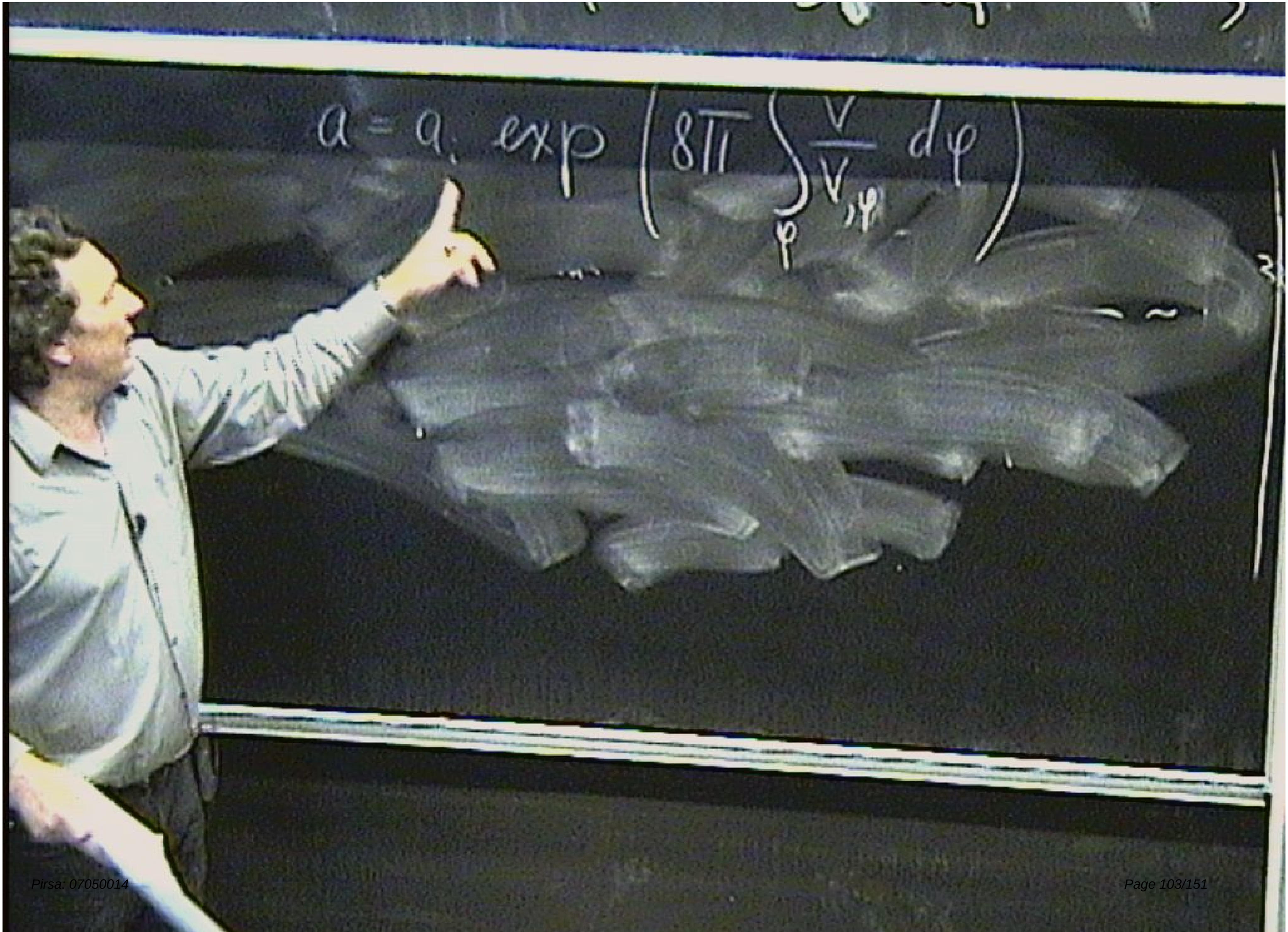
$$\begin{cases} 3H\dot{\varphi} + V_{,\varphi} \simeq 0 \\ H^2 \simeq \frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) \end{cases}$$

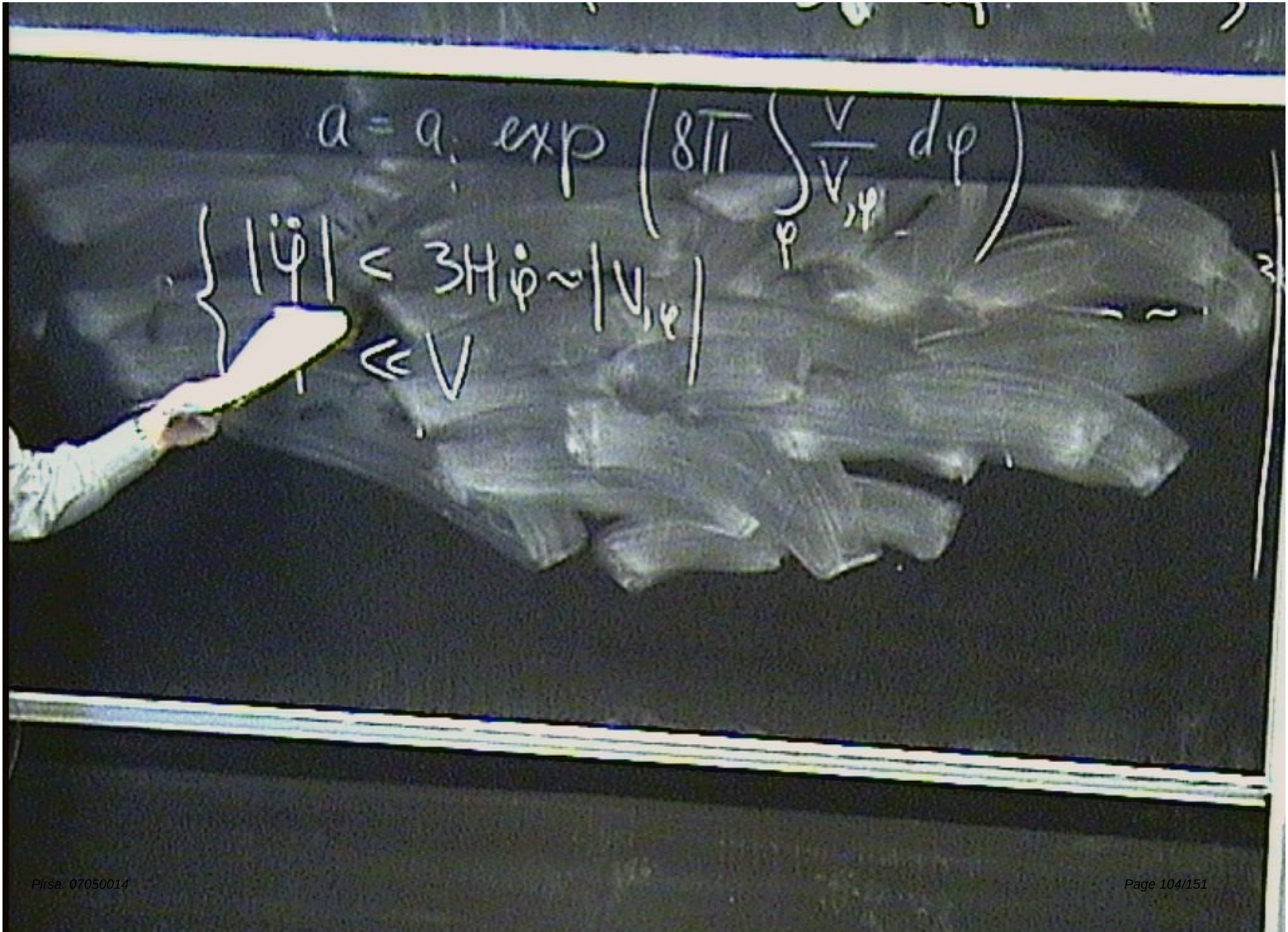
$a(\varphi)$

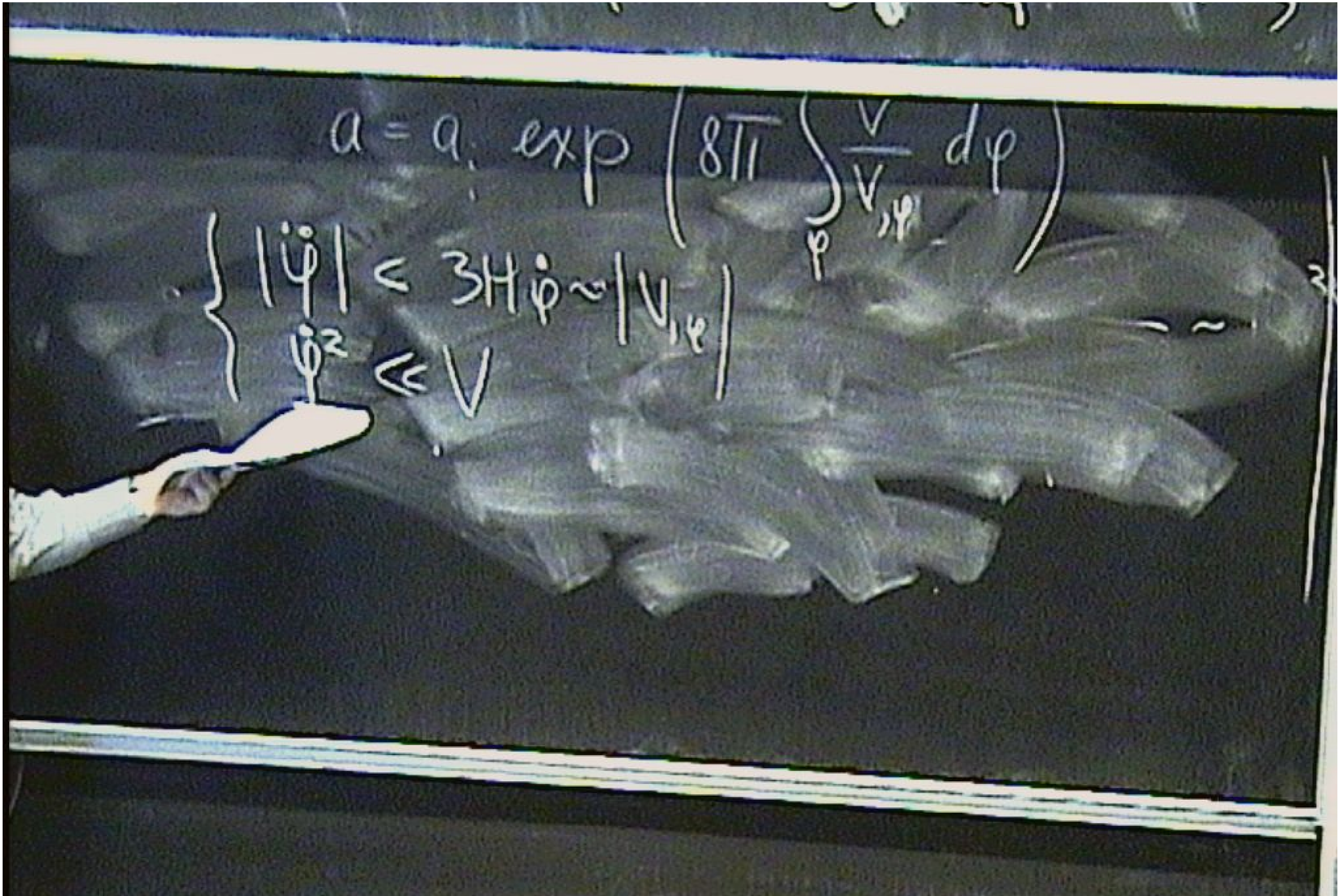
$$H = \frac{d\ln a}{dt} = \sqrt{\frac{8\pi}{3}V}$$

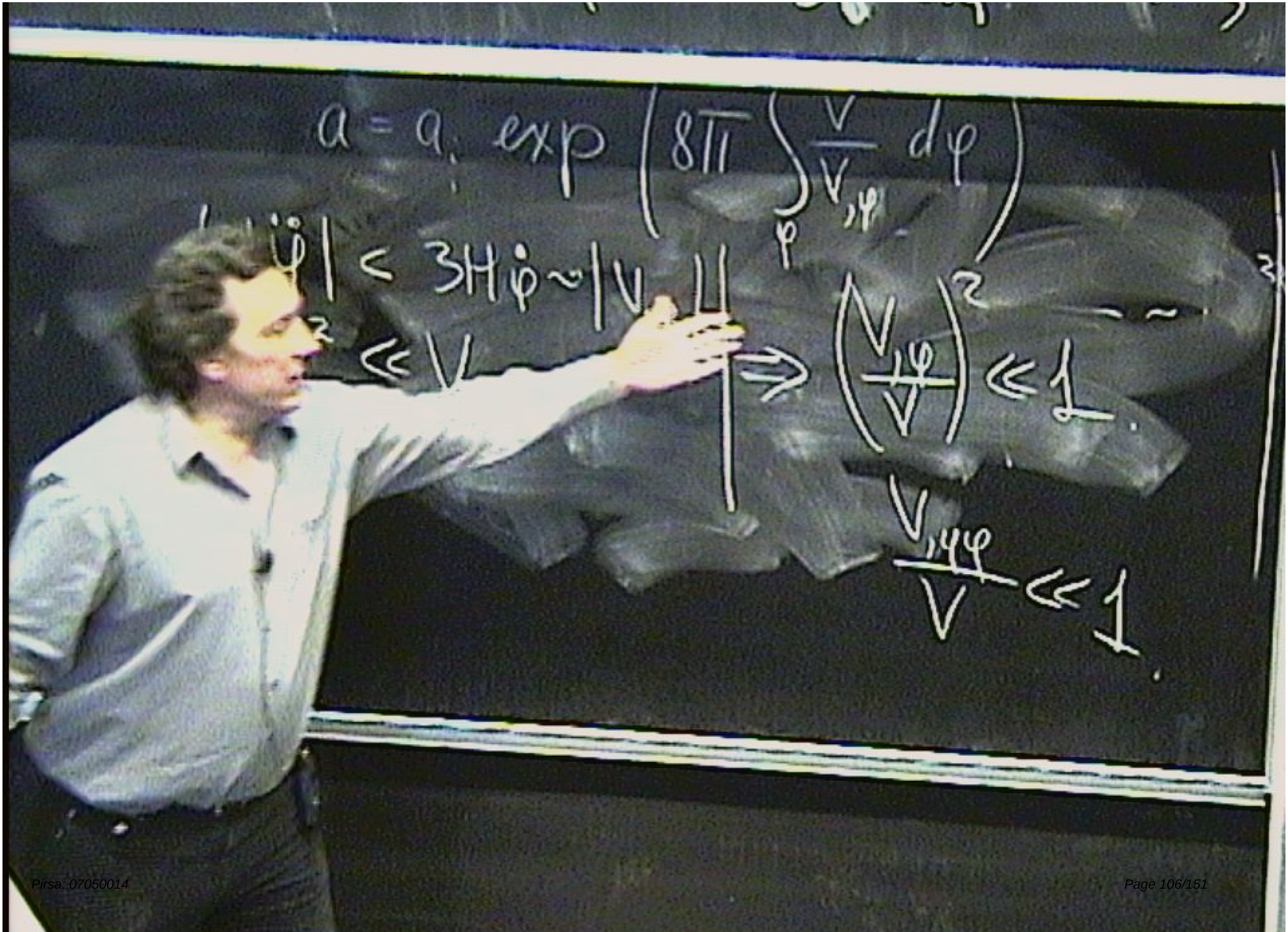
$$\frac{d\ln a}{dt} = \dot{\varphi}\frac{d\ln a}{d\varphi} = -\frac{V_{,\varphi}}{3H}\frac{d\ln a}{d\varphi} = H^2 = \frac{8\pi}{3}V$$

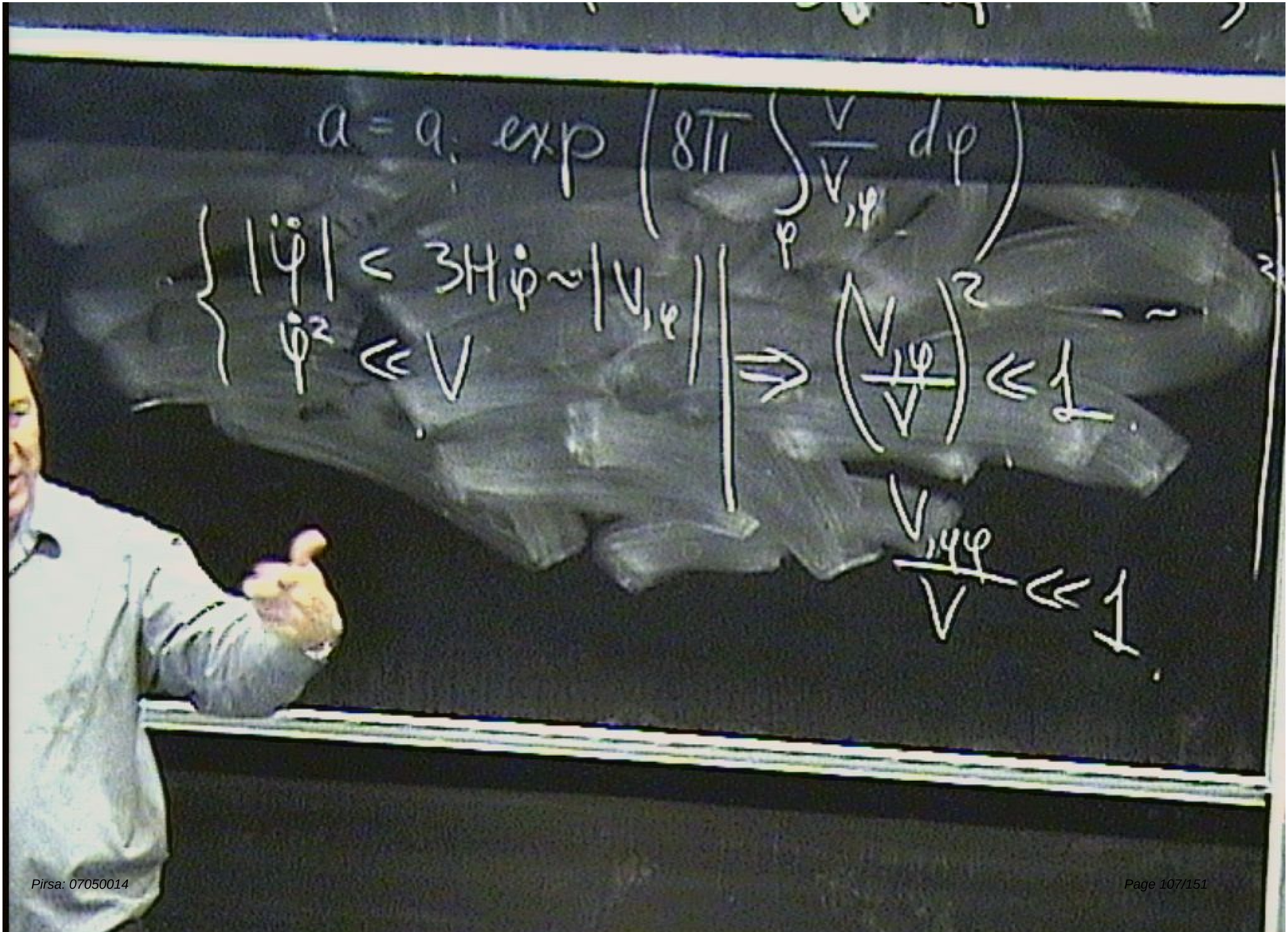

$a = a_i \exp\left(8\pi \int \frac{V}{V_{,\varphi}} d\varphi\right)$


$a = a_i \exp\left(8\pi \int_{\varphi} \frac{V}{V_{,\varphi}} d\varphi\right)$


$a = a_i \exp\left(8\pi \int_{\varphi} \frac{V}{V_{,\varphi}} d\varphi\right)$
$|\ddot{\varphi}| < 3H\dot{\varphi} \sim |V_{,\varphi}|$
$\ll V$


$a = a_i \exp\left(8\pi \int_{\varphi} \frac{V}{V_{,\varphi}} d\varphi\right)$
$|\ddot{\varphi}| < 3H\dot{\varphi} \sim |V_{,\varphi}|$
$\dot{\varphi}^2 \ll V$


a = a_i exp (8π ∫ V / V,φ dφ)
< 3Hφ̇ ~ |V,φ|
≪ V
⇒ (V,φ / V)^2 ≪ 1
V,φφ / V ≪ 1


$a = a_i \exp\left(8\pi \int_{\varphi} \frac{V}{V_{,\varphi}} d\varphi\right)$
$|\ddot{\varphi}| < 3H\dot{\varphi} \sim |V_{,\varphi}|$
$\dot{\varphi}^2 \ll V$
$\Rightarrow \left(\frac{V_{,\varphi}}{V}\right)^2 \ll 1$
$\frac{V_{,\varphi\varphi}}{V} \ll 1$

$$a = a_i \exp\left(8\pi \int_{\varphi_i}^{\varphi} \frac{V}{V_{,\varphi}}\, d\varphi\right)$$

$$\left\{ \begin{array}{l} |\ddot{\varphi}| < 3H\dot{\varphi} \sim |V_{,\varphi}| \\ \dot{\varphi}^2 \ll V \end{array} \right. \Rightarrow \left(\frac{V_{,\varphi}}{V}\right)^2 \ll 1$$

$$\frac{V_{,\varphi\varphi}}{V} \ll 1.$$

$V \propto \varphi^4 \quad \varphi \gg 1$

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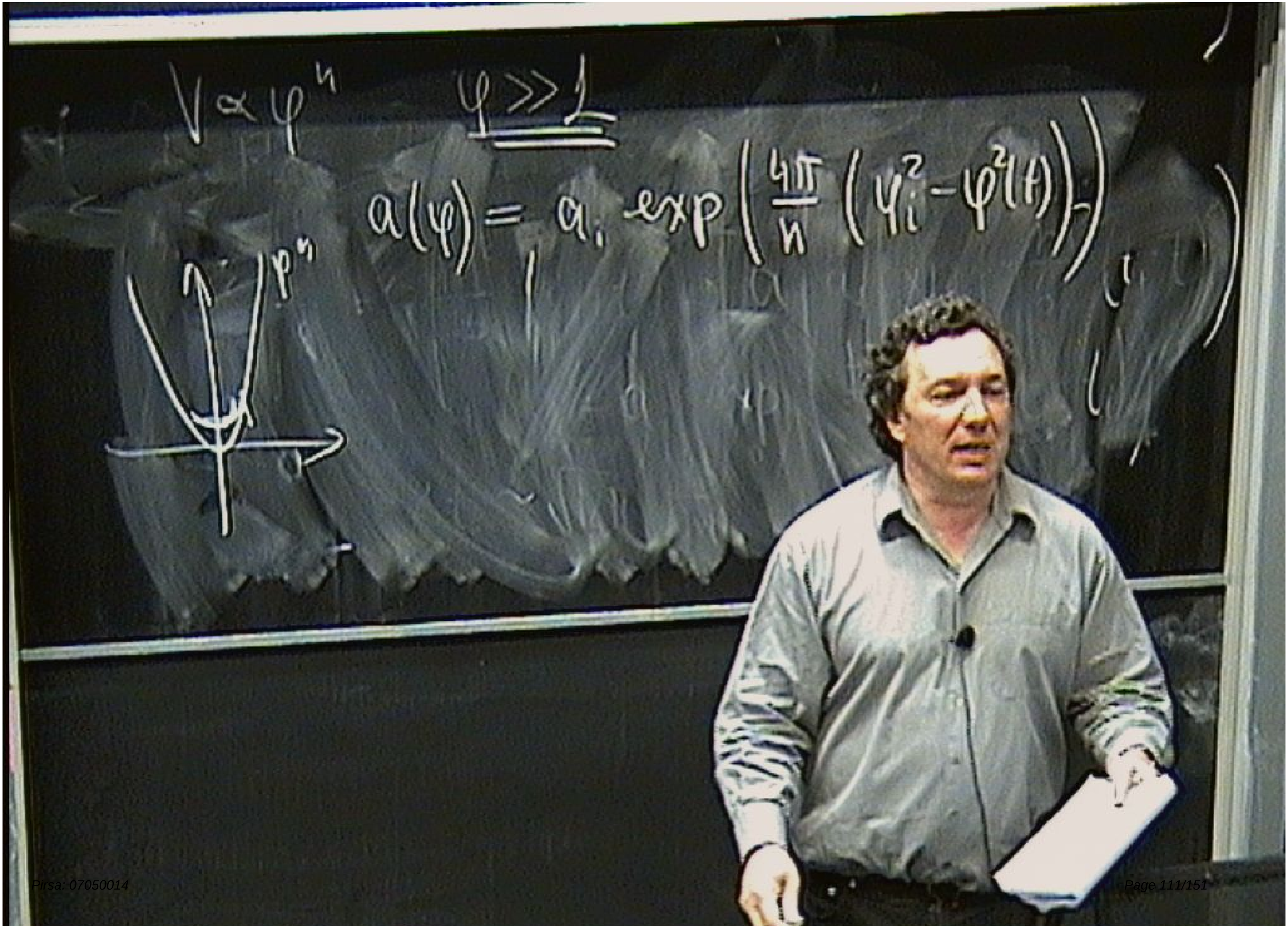
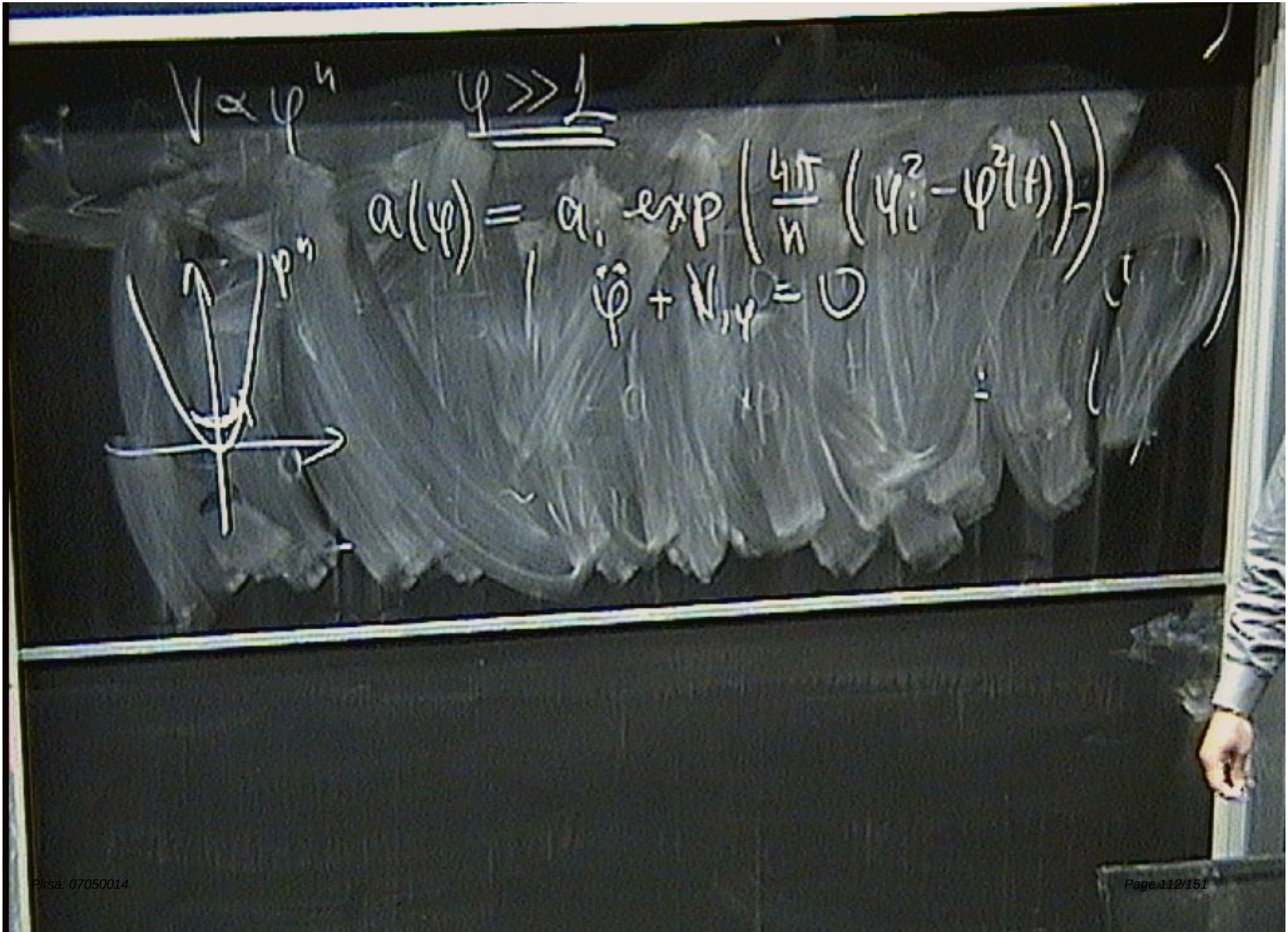
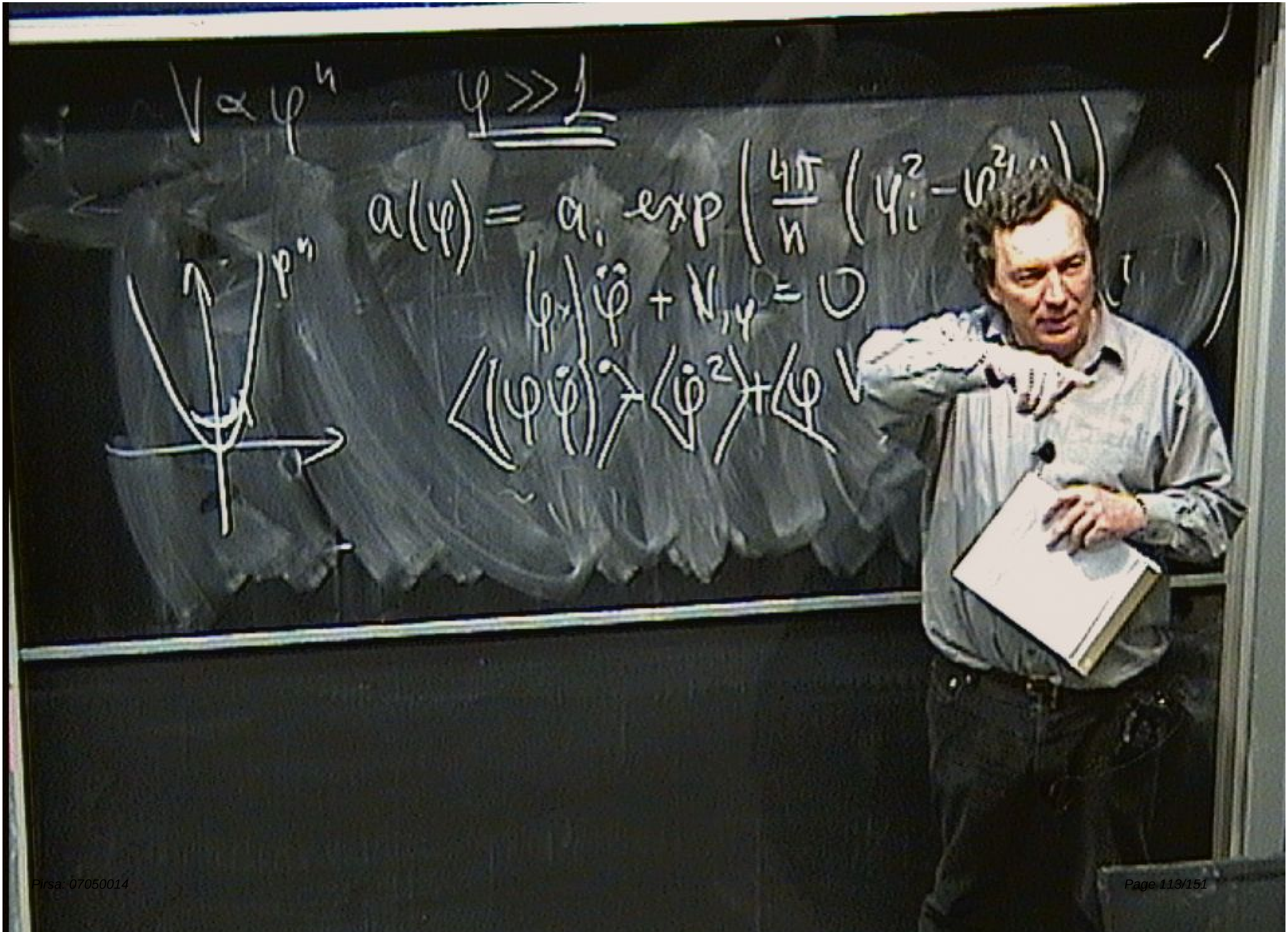
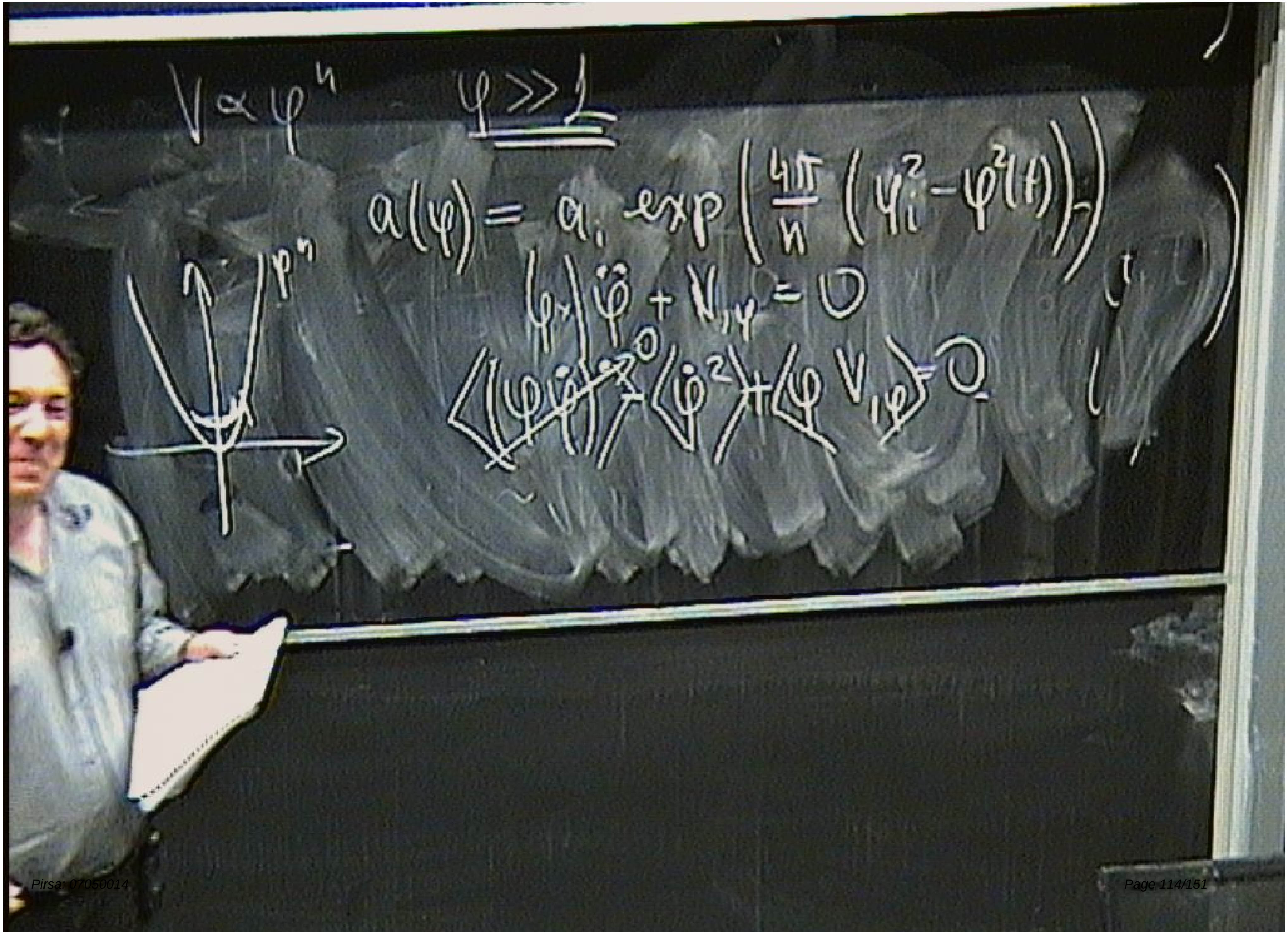
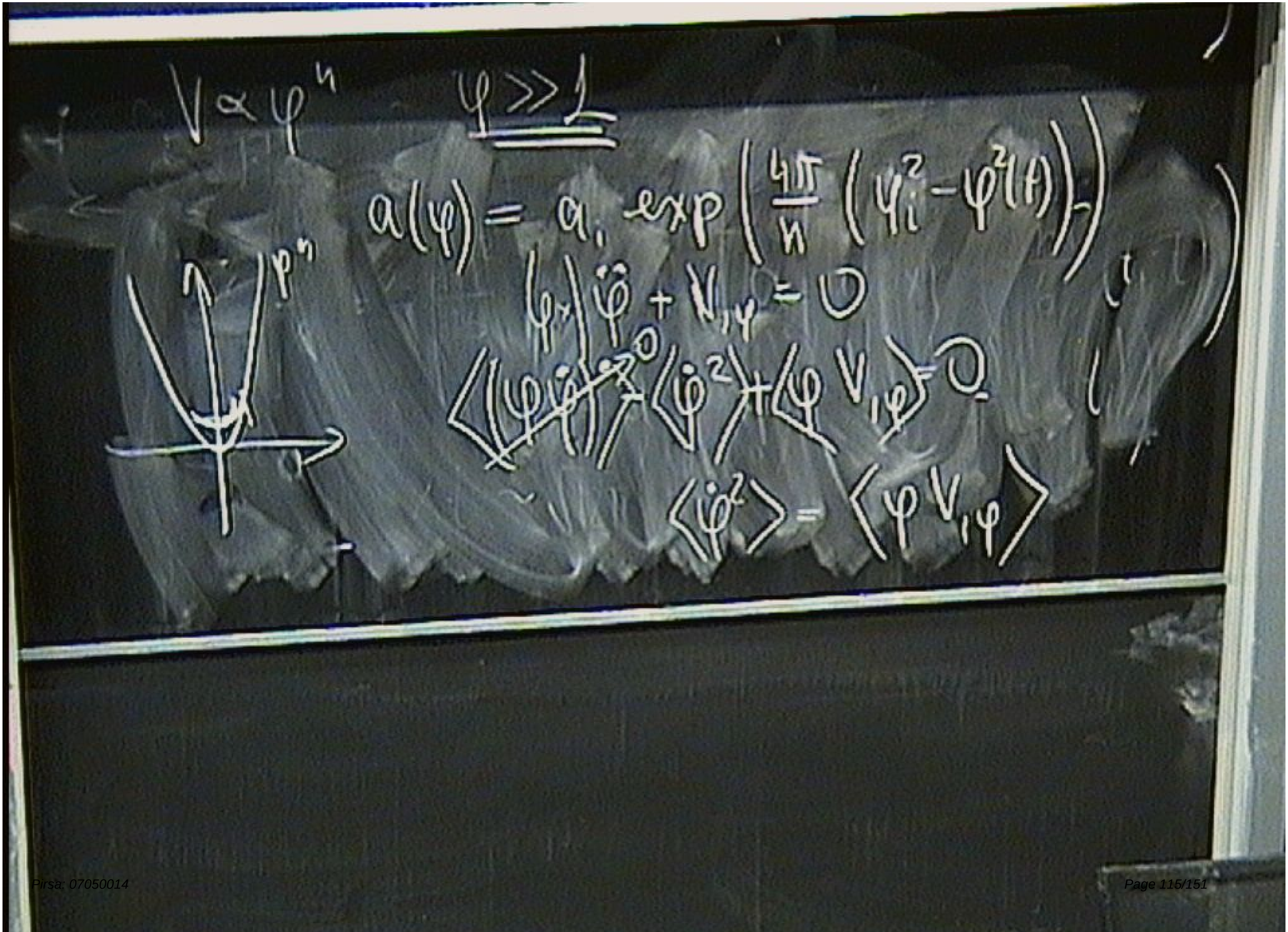
$$a(\varphi) = a_i \exp\left(\frac{4\pi}{n}\left(\varphi_i^2 - \varphi^2(t)\right)\right)$$

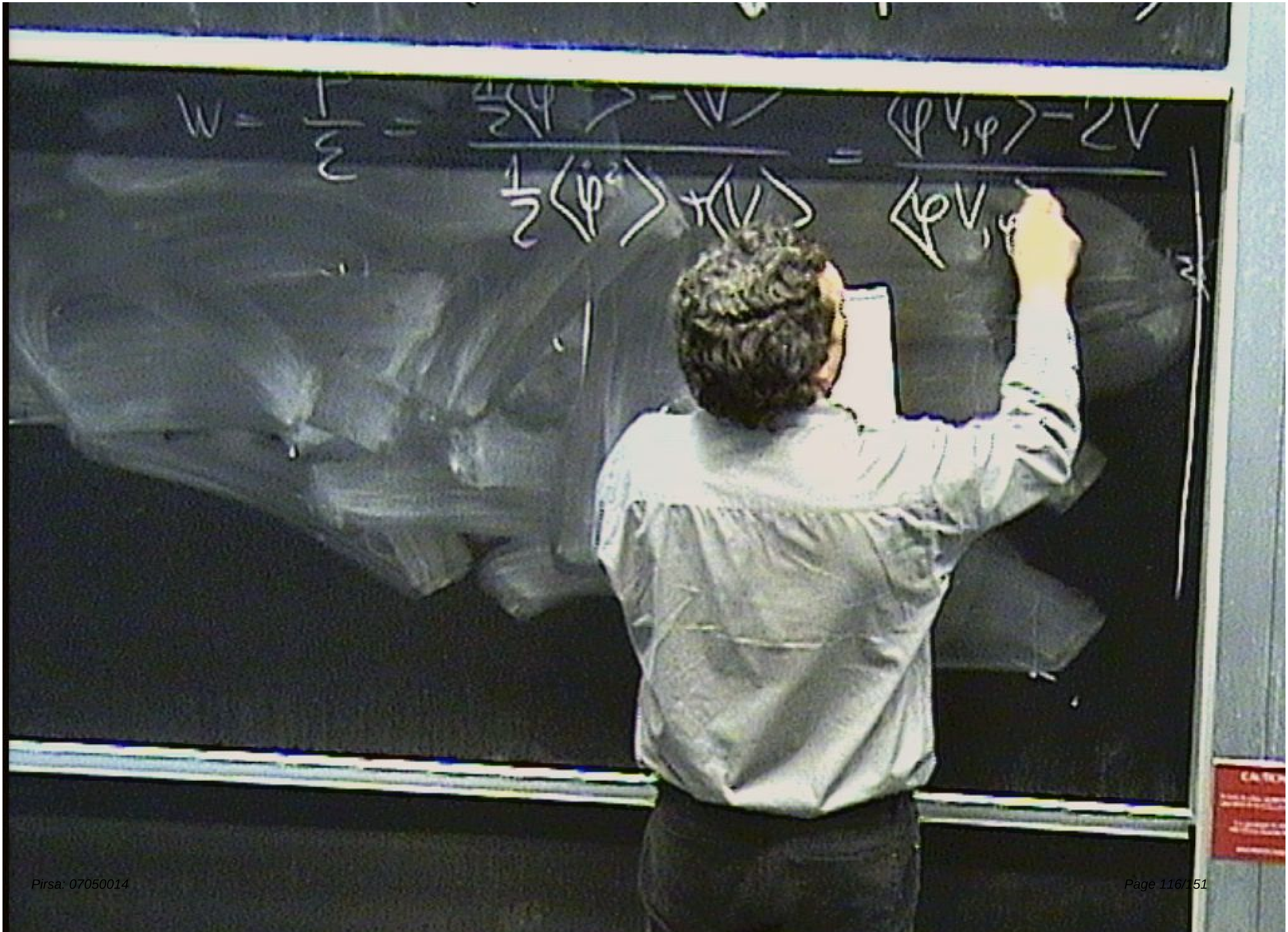

$V \propto \varphi^4$
$\varphi \gg 1$
$a(\varphi) = a_i \exp\left(\frac{4\pi}{n}\left(\varphi_i^2 - \varphi^2(t)\right)\right)$
φ^4

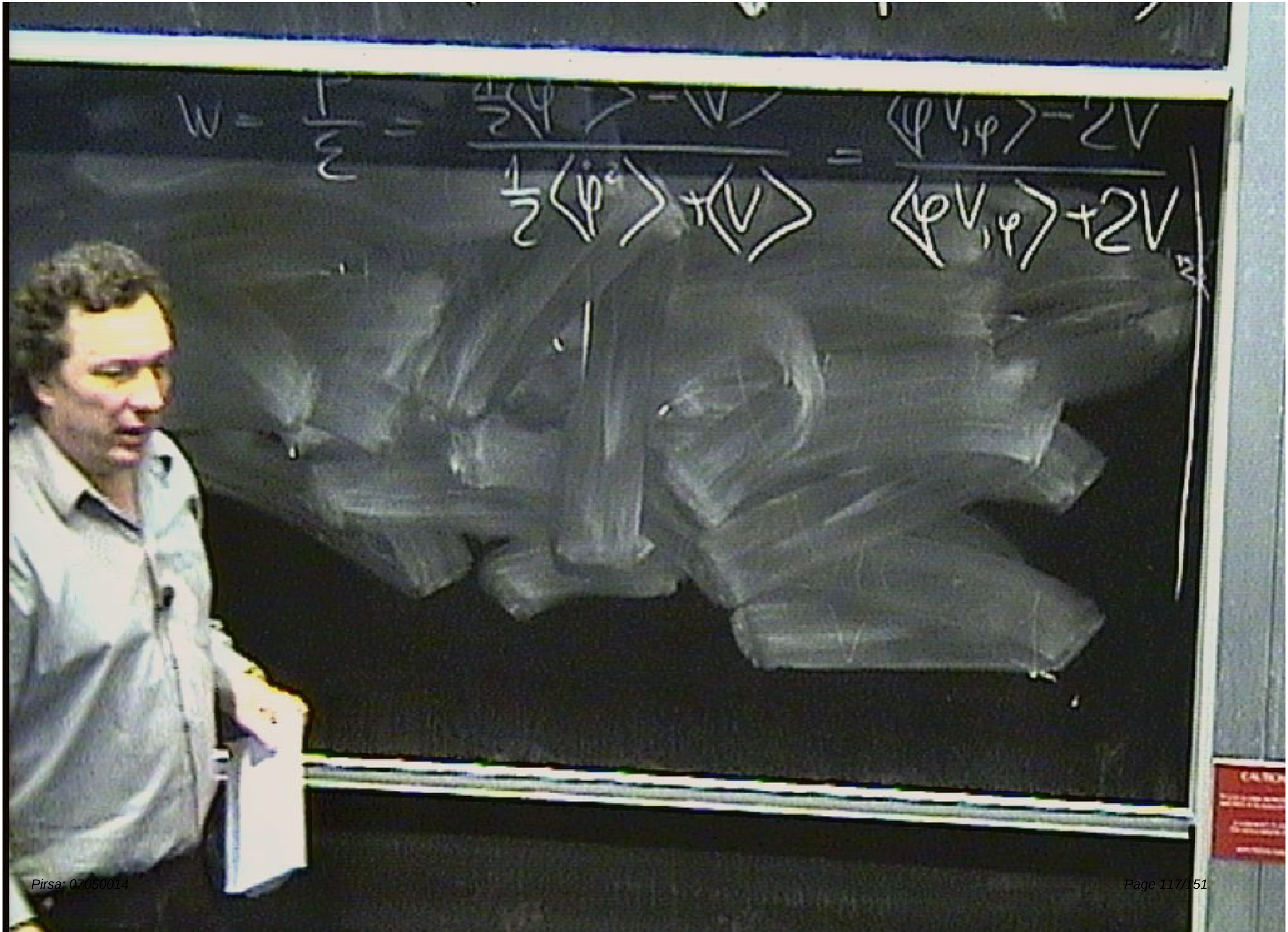

$V \propto \varphi^4$
$\varphi \gg 1$
$a(\varphi) = a_i \exp\left(\frac{4\pi}{n}\left(\varphi_i^2 - \varphi^2(t)\right)\right)$
$\ddot{\varphi} + V_{,\varphi} = 0$
φ^4

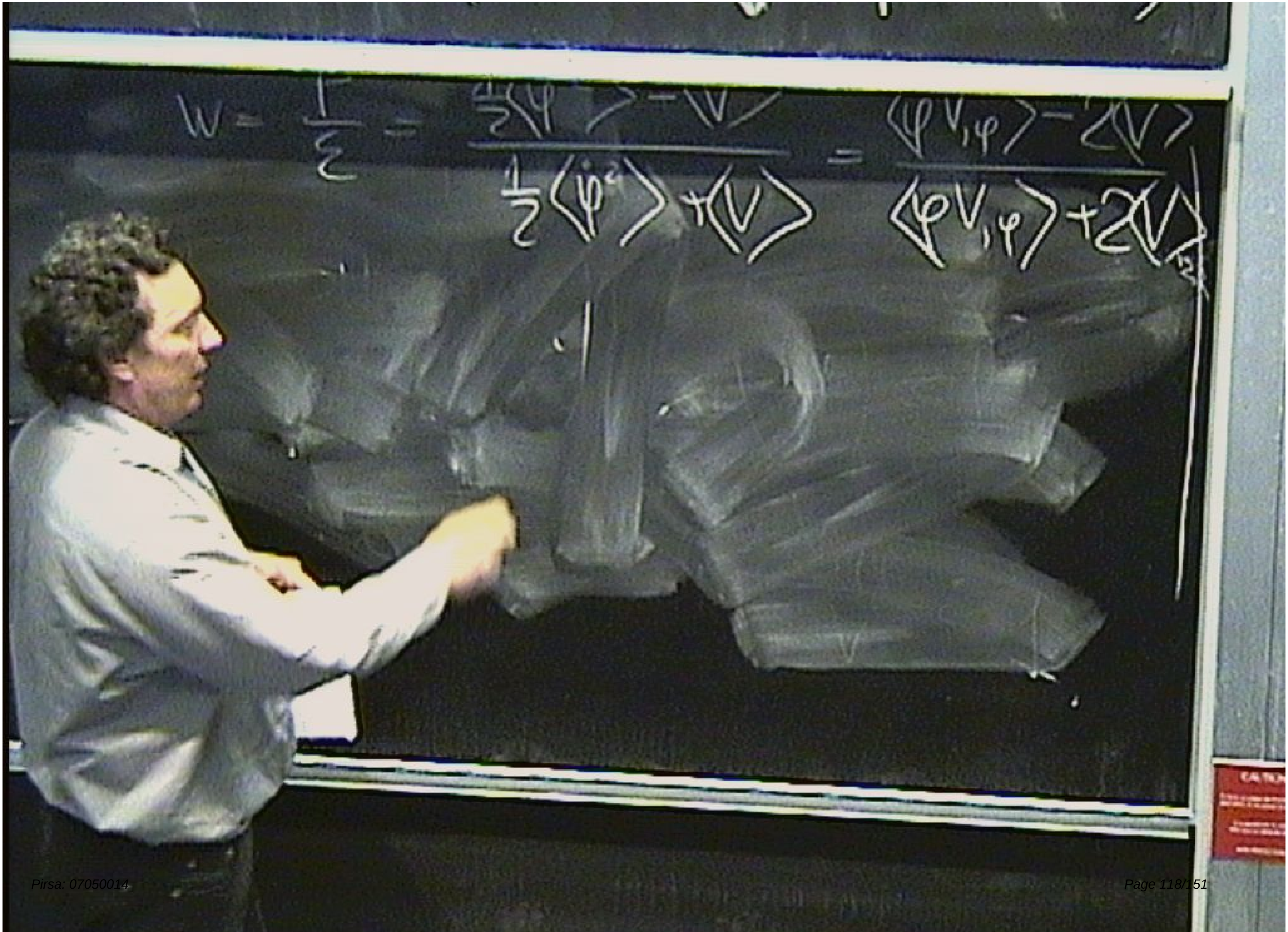


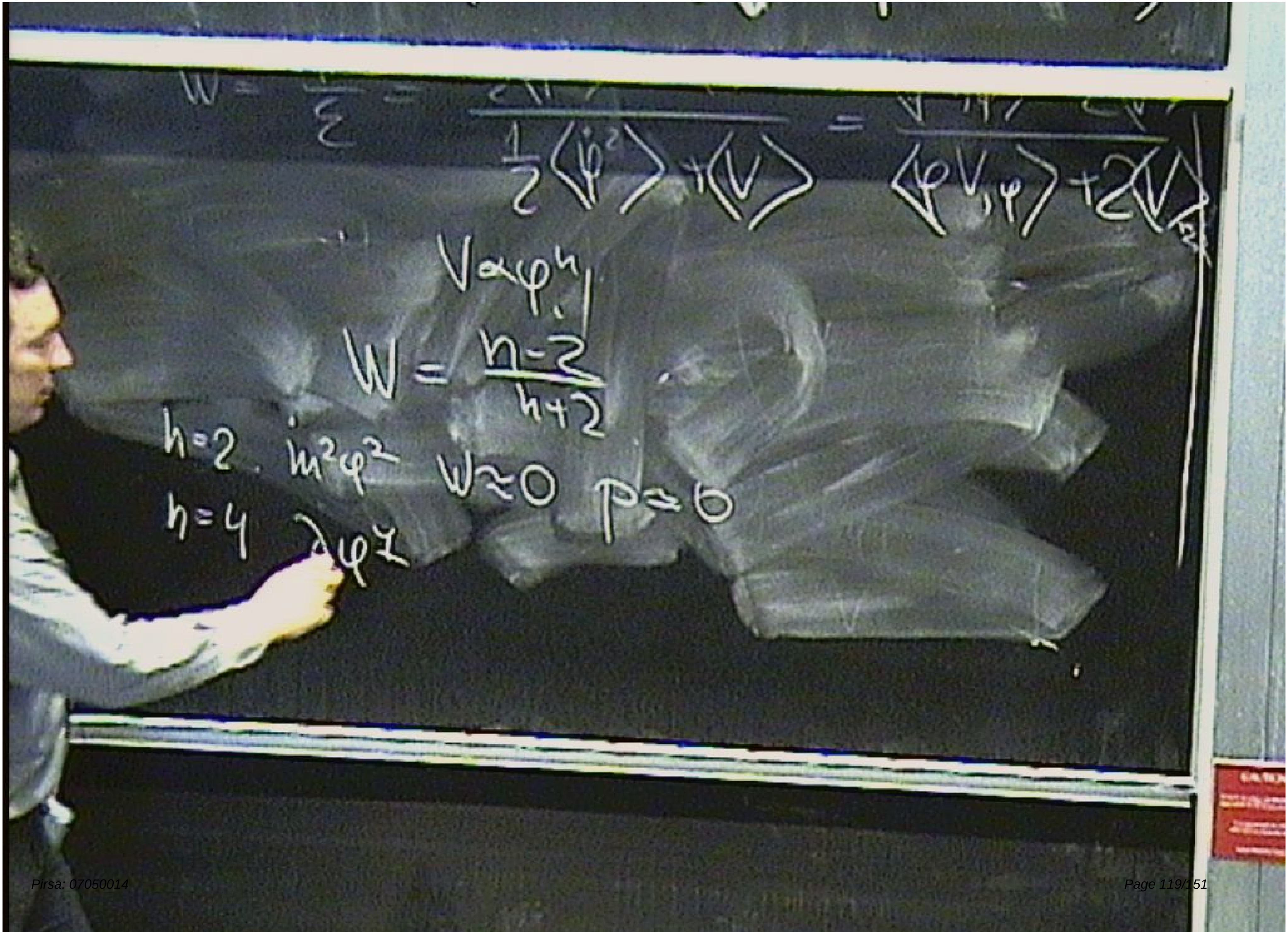

V ∝ φ^4   φ >> 1
a(φ) = a_i exp(4π/n (φ_i^2 − φ^2(t)))
φ̈ + V,φ = 0
⟨φφ̈⟩ + ⟨φ̇^2⟩ + ⟨φV,φ⟩ = 0


V ∝ φ^4
φ >> 1
a(φ) = a_i exp(4π/n (φ_i^2 − φ^2(t)))
(φ×) φ̈ + V,φ = 0
⟨(φφ̇)˙⟩ + ⟨φ̇^2⟩ + ⟨φ V,φ⟩ = 0
⟨φ̇^2⟩ = ⟨φ V,φ⟩


W = 1/ε = (½⟨φ̇²⟩ − ⟨V⟩) / (½⟨φ̇²⟩ + ⟨V⟩) = (⟨φV,φ⟩ − 2V) / ⟨φV,φ




$W = \frac{1}{\varepsilon} = \frac{\frac{1}{2}\langle\dot{\varphi}^2\rangle - \langle V\rangle}{\frac{1}{2}\langle\dot{\varphi}^2\rangle + \langle V\rangle} = \frac{\langle\varphi V_{,\varphi}\rangle - 2\langle V\rangle}{\langle\varphi V_{,\varphi}\rangle + 2\langle V\rangle}$



½⟨φ̇²⟩+⟨V⟩
⟨φV,φ⟩+2⟨V⟩
V∝φ^n
W = (n-2)/(n+2)
n=2 m²φ² W≈0 p≈0
n=4 λφ⁴

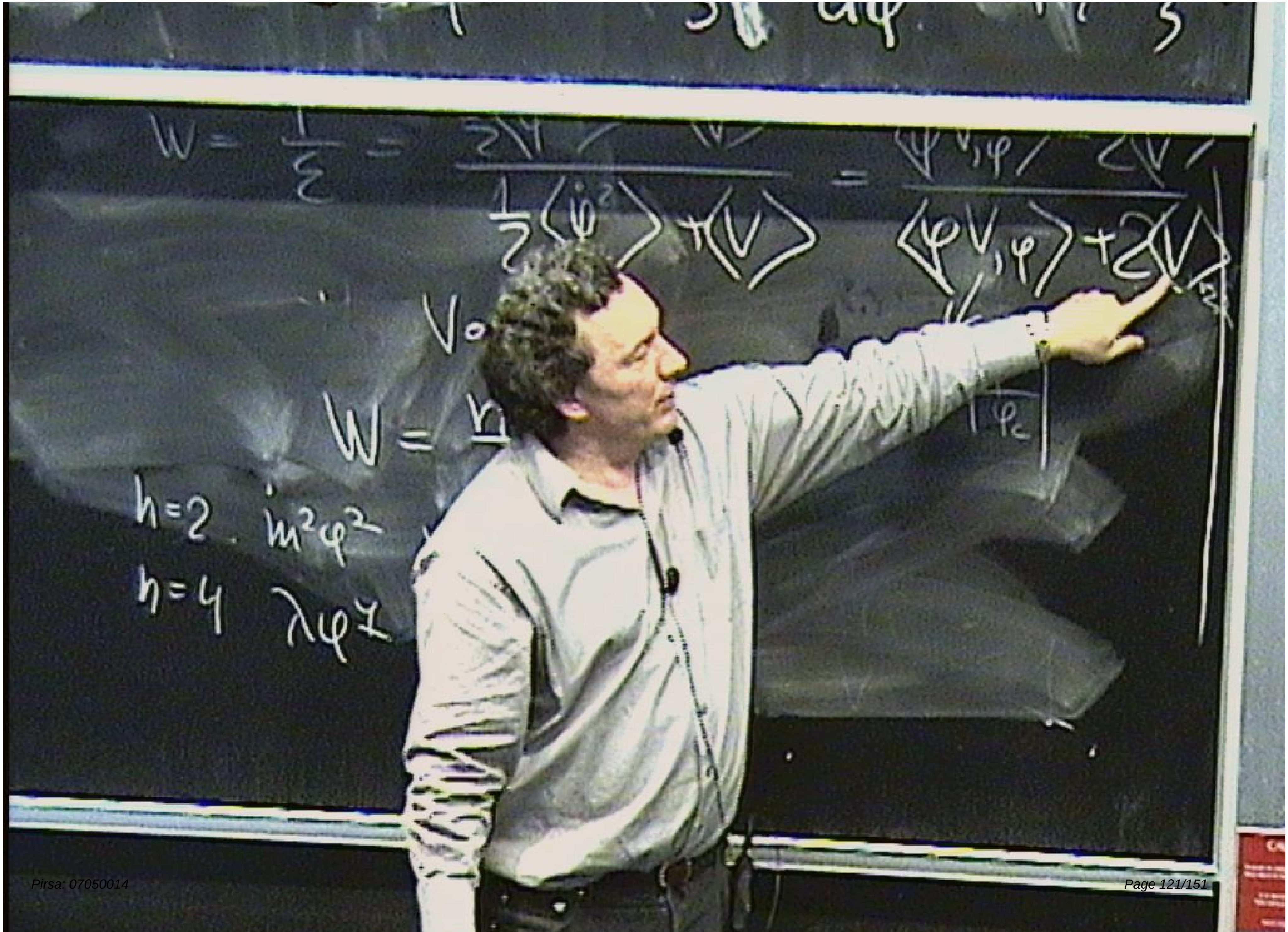
$$w = \frac{p}{\varepsilon} = \frac{\cdots}{\frac{1}{2}\langle\dot{\varphi}^2\rangle + \langle V\rangle} = \frac{\cdots}{\langle \varphi V_{,\varphi}\rangle + 2\langle V\rangle}$$

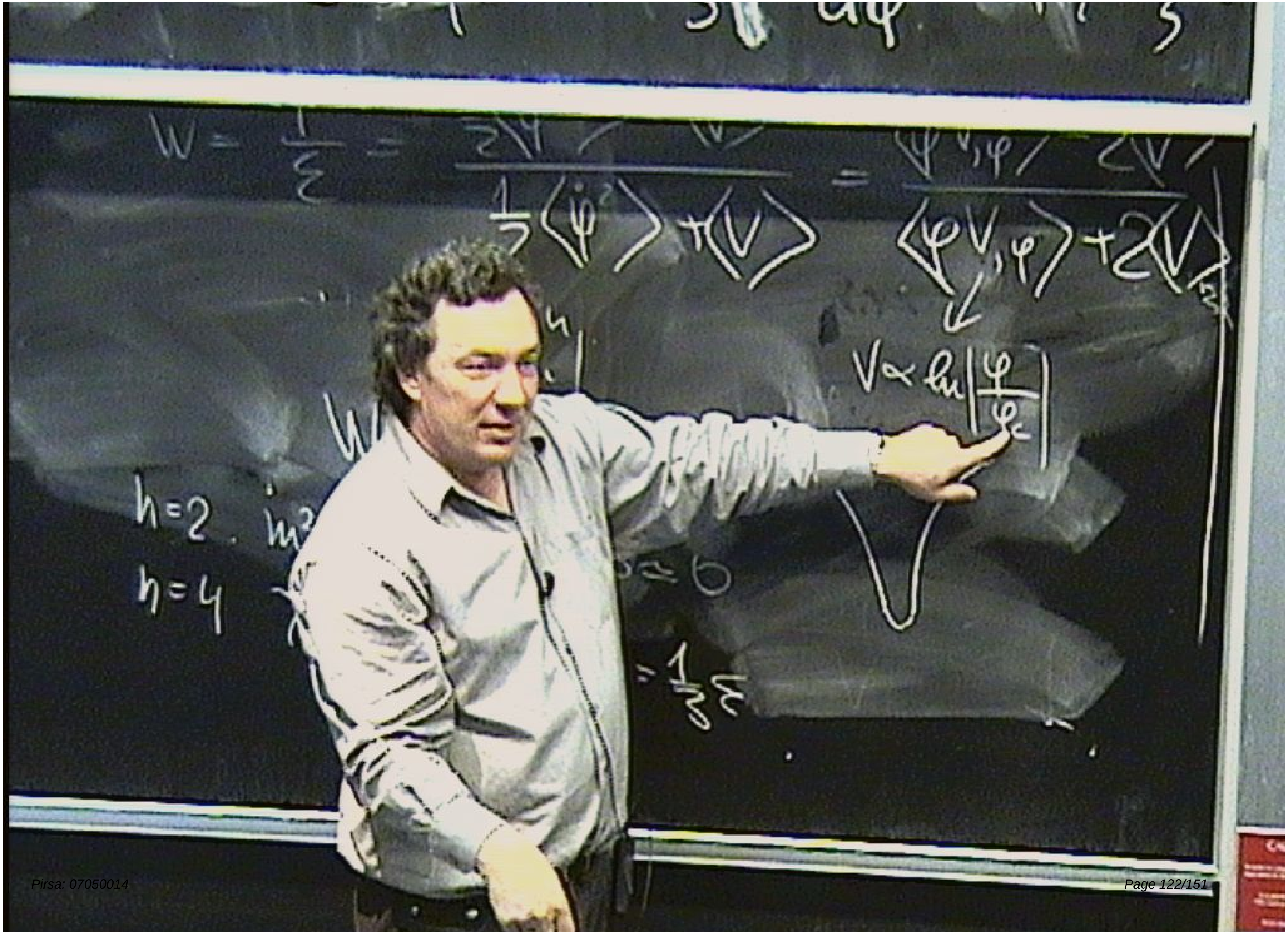
$$V \propto \varphi^n$$

$$w = \frac{n-2}{n+2}$$

$n=2$. $m^2\varphi^2$ $w \approx 0$ $p \approx 0$

$n=4$ $\lambda\varphi^4$ $w = \frac{1}{3}$ $p = \frac{1}{3}\varepsilon$


h=2 m²φ²
h=4 λφ⁴



W =
V∝ ln|φ/φc|
n=2
n=4

$$w = \frac{p}{\varepsilon} = \frac{\frac{1}{2}\langle\dot\varphi^2\rangle - \langle V\rangle}{\frac{1}{2}\langle\dot\varphi^2\rangle + \langle V\rangle} = \frac{\langle\varphi V_{,\varphi}\rangle - 2\langle V\rangle}{\langle\varphi V_{,\varphi}\rangle + 2\langle V\rangle}$$

$V \propto \varphi^n$

$$w = \frac{n-2}{n+2}$$

$n=2$ $m^2\varphi^2$ $w \simeq 0$ $p \simeq 0$

$n=4$ $\lambda\varphi^4$ $w \simeq \frac{1}{3}$ $p = \frac{1}{3}\varepsilon$

$V \propto \ln\left|\frac{\varphi}{\varphi_c}\right|$

$$W = \frac{1}{\varepsilon} = \frac{\ldots}{\frac{1}{2}\langle \dot{\varphi}^2 \rangle + \langle V \rangle} = \frac{\langle \varphi V_{,\varphi} \rangle - \langle V \rangle}{\langle \varphi V_{,\varphi} \rangle + 2\langle V \rangle}$$

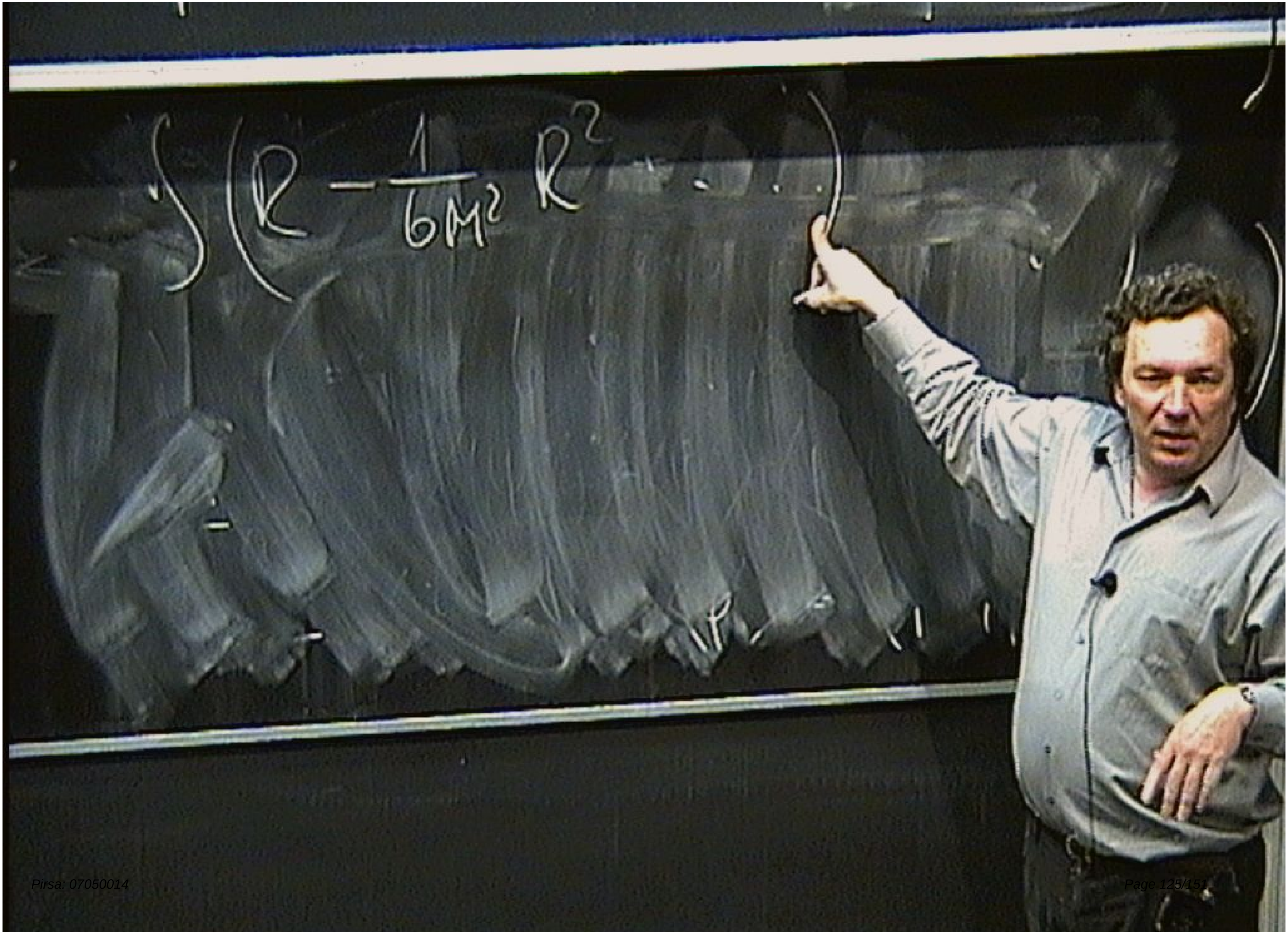
$V \propto \varphi^n$

$$W = \frac{n-2}{n+2}$$

$n=2 \quad m^2\varphi^2 \quad w \approx 0 \quad p \approx 0$

$n=4 \quad \lambda\varphi^4 \quad w \approx \frac{1}{3} \quad p = \frac{1}{3}\varepsilon$

$V \propto \ln\left|\frac{\varphi}{\varphi_c}\right|$


$\int \left(R - \frac{1}{6M^2} R^2 \ldots \right)$

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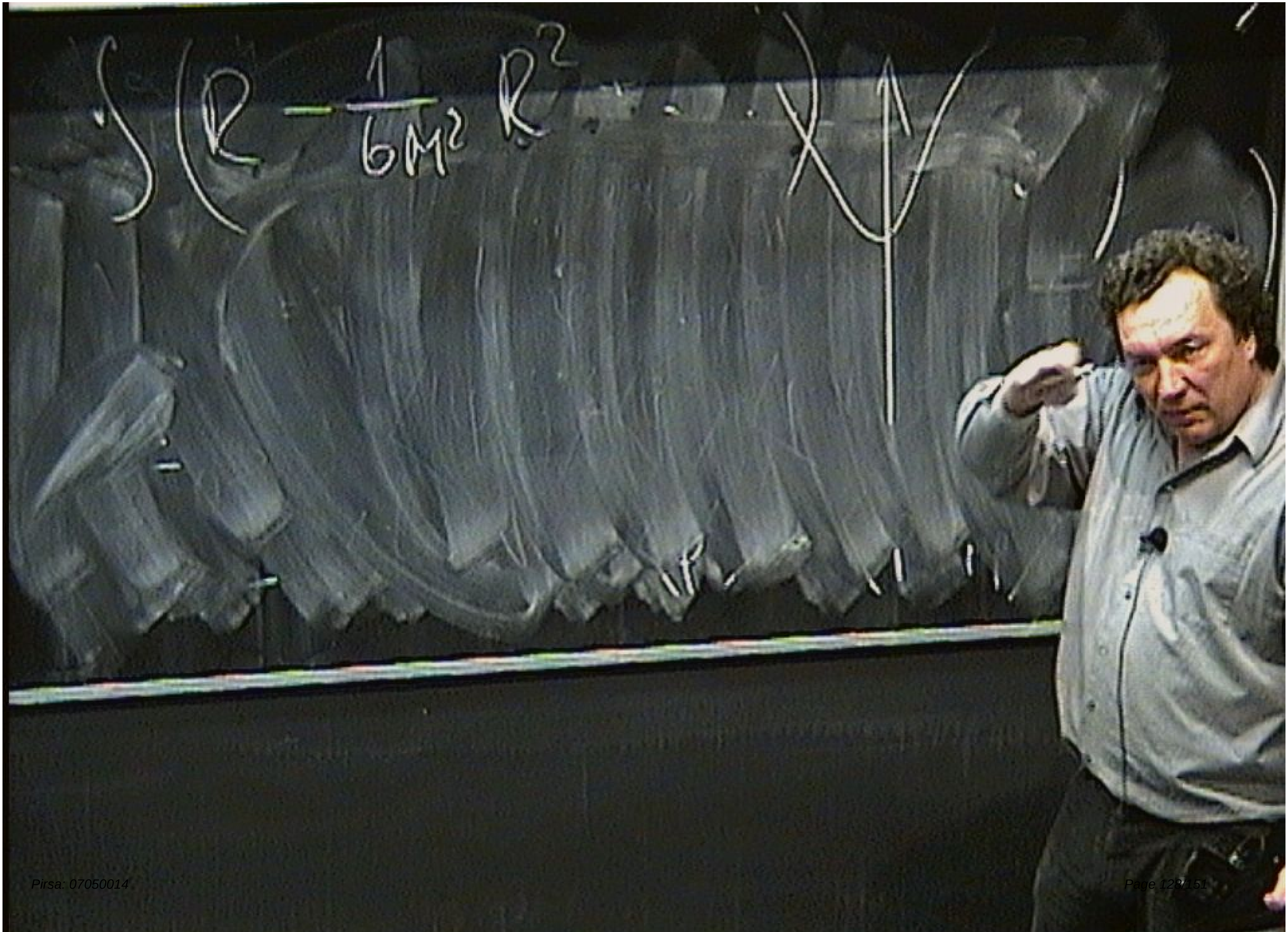
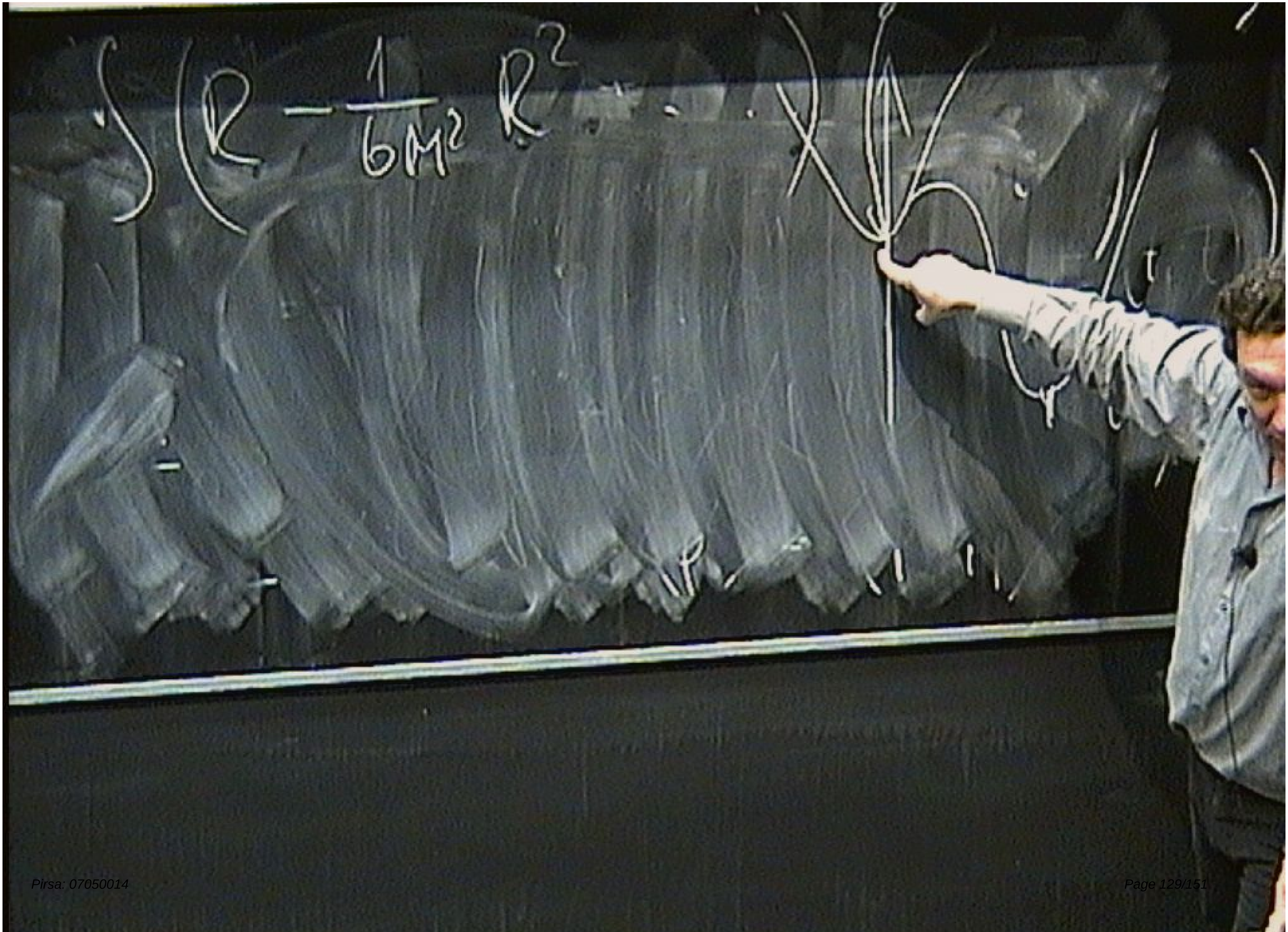
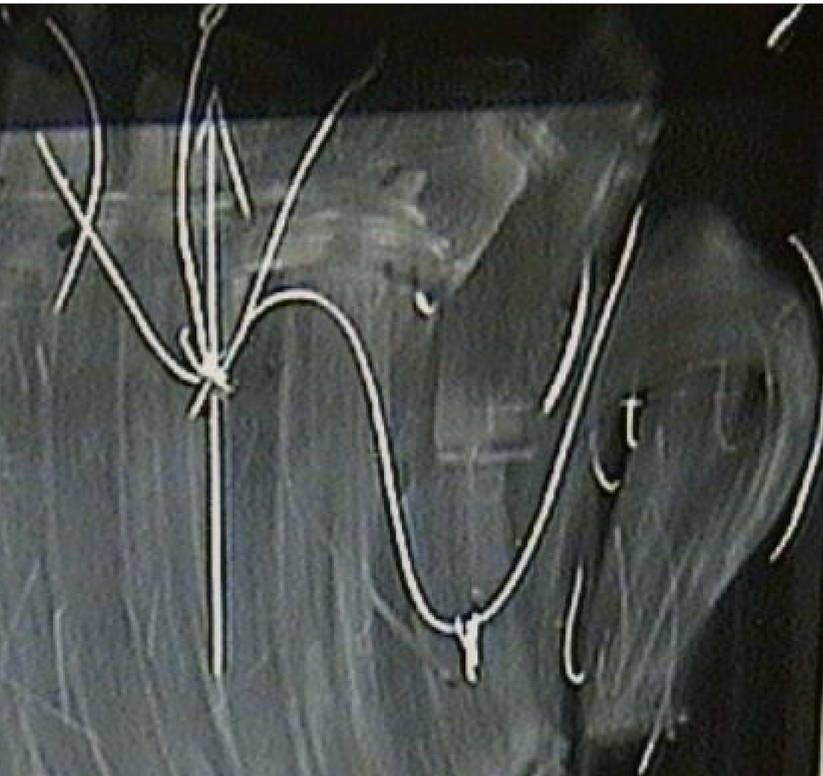
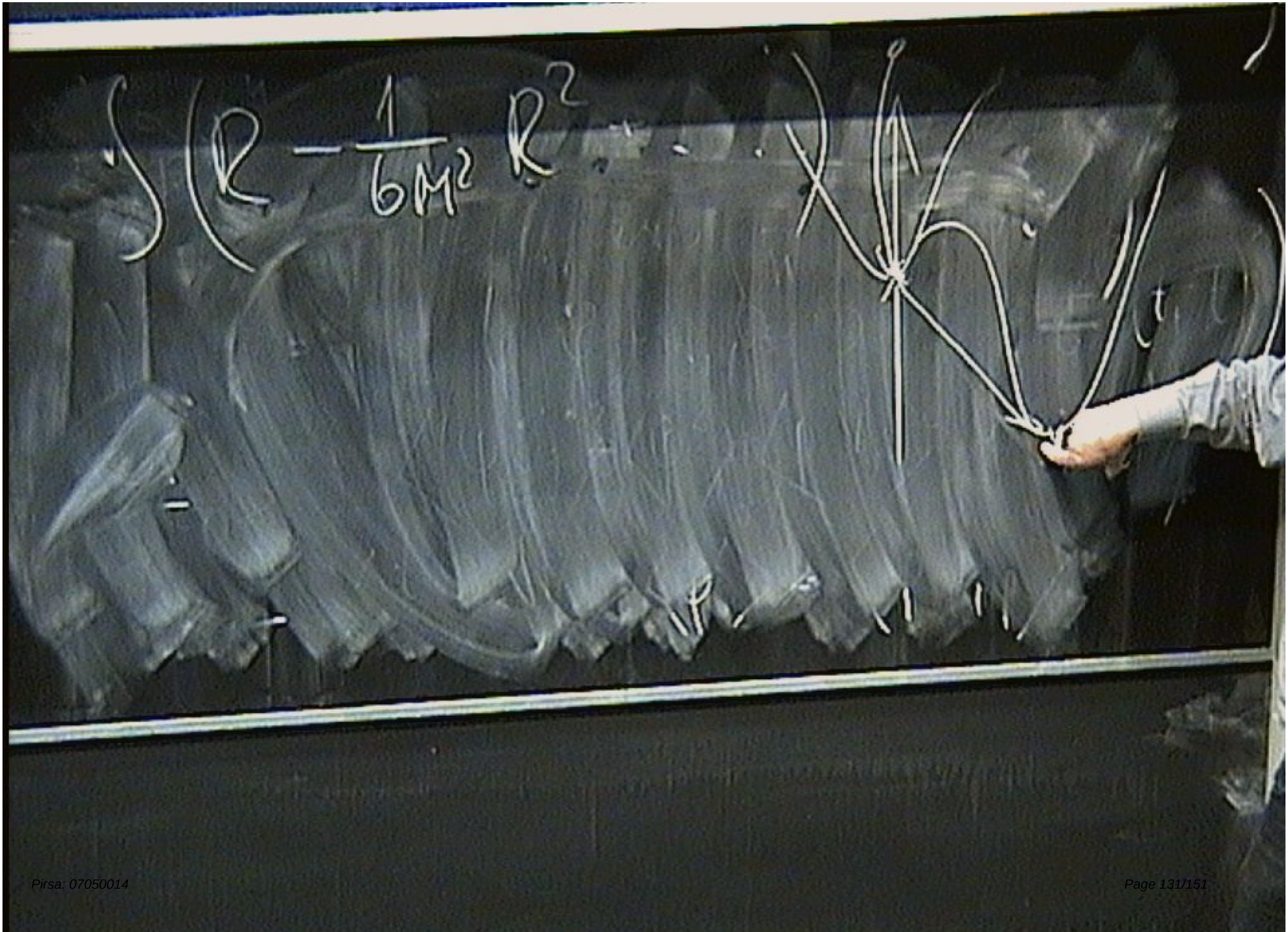
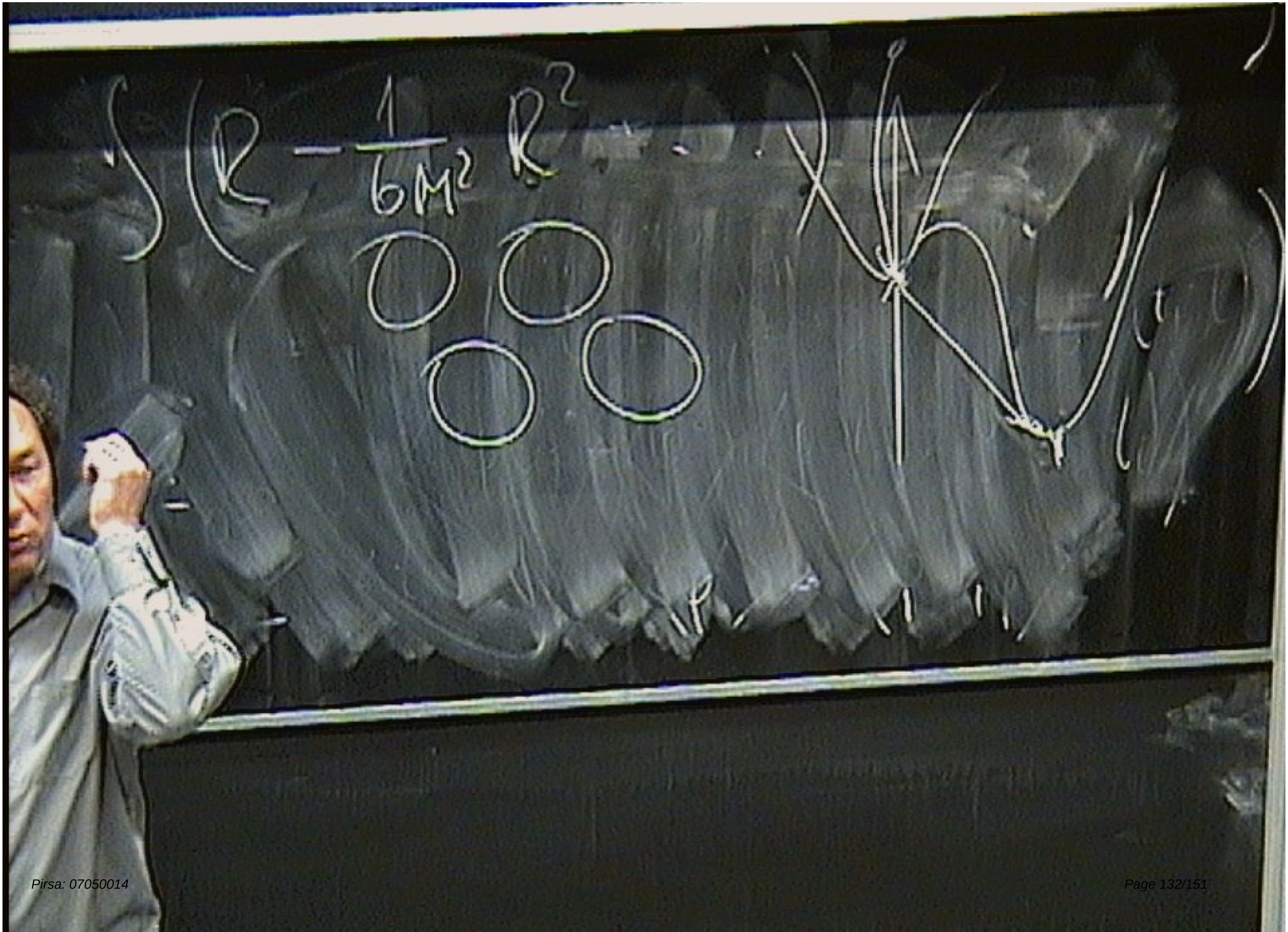

∫(R − 1/6M² R²

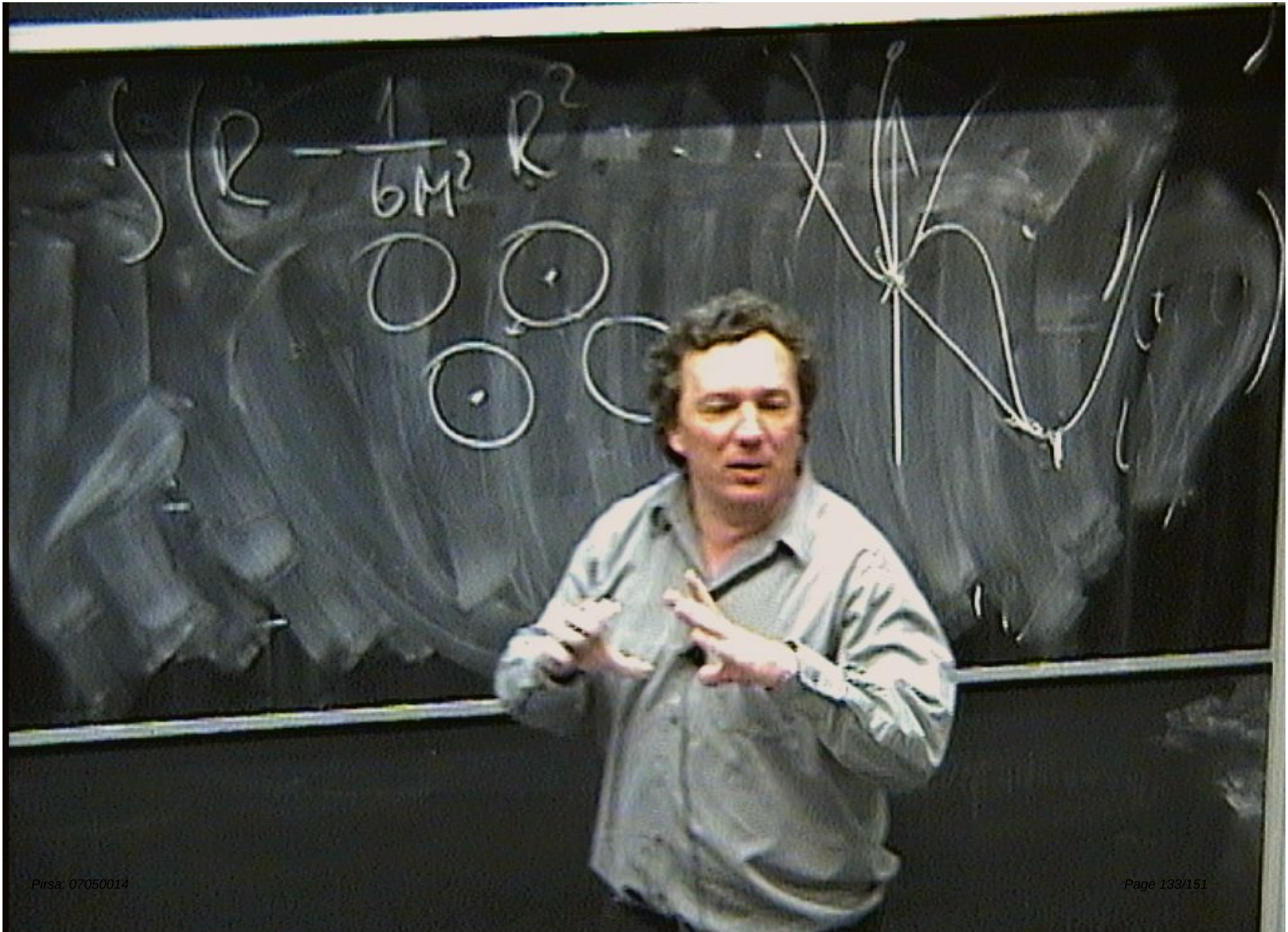

$\int \left(R - \frac{1}{6M^2} R^2 \ldots \right.$

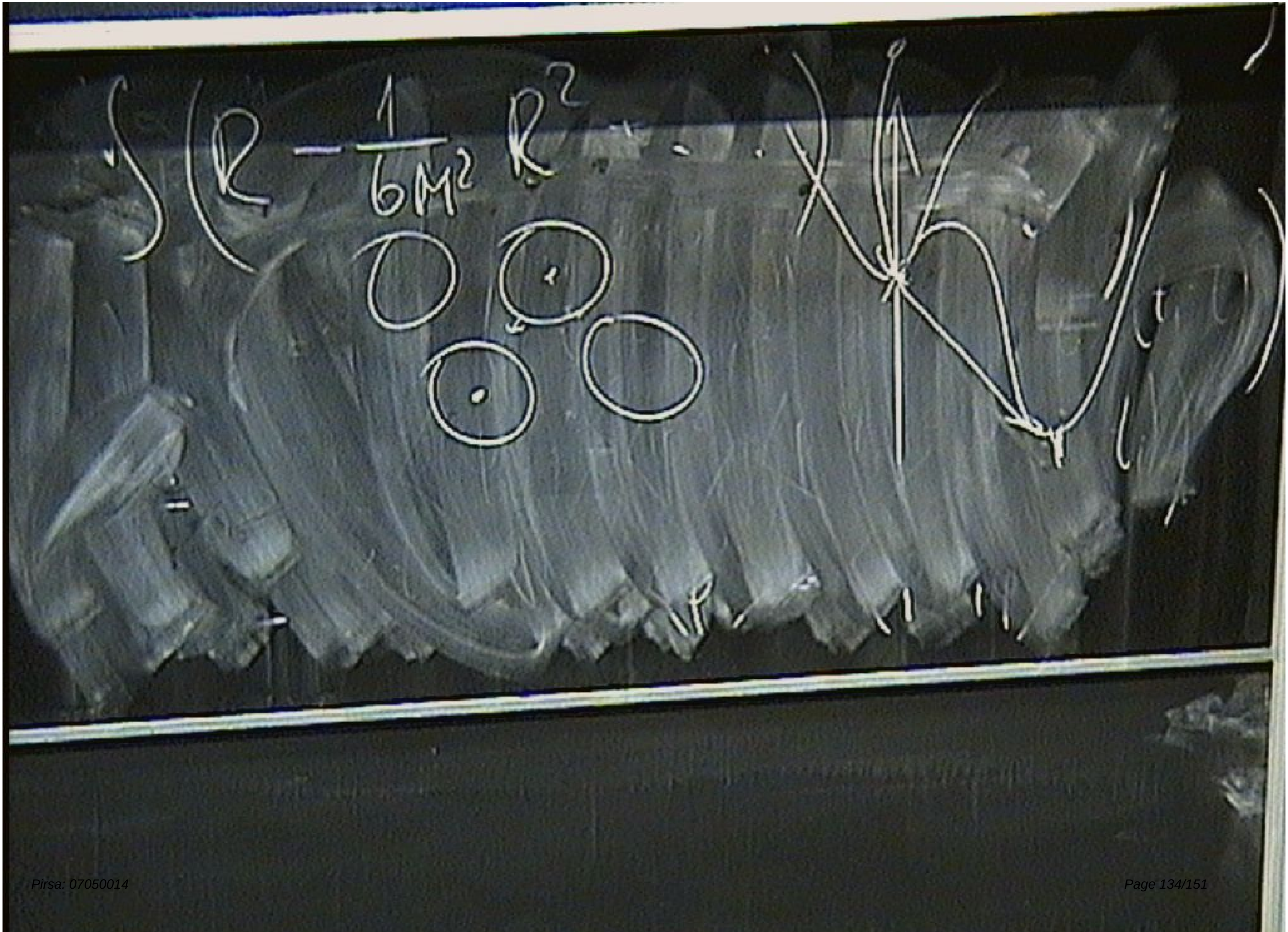
$$\int \left(R - \frac{1}{6M^2} R^2 \ldots \right)$$

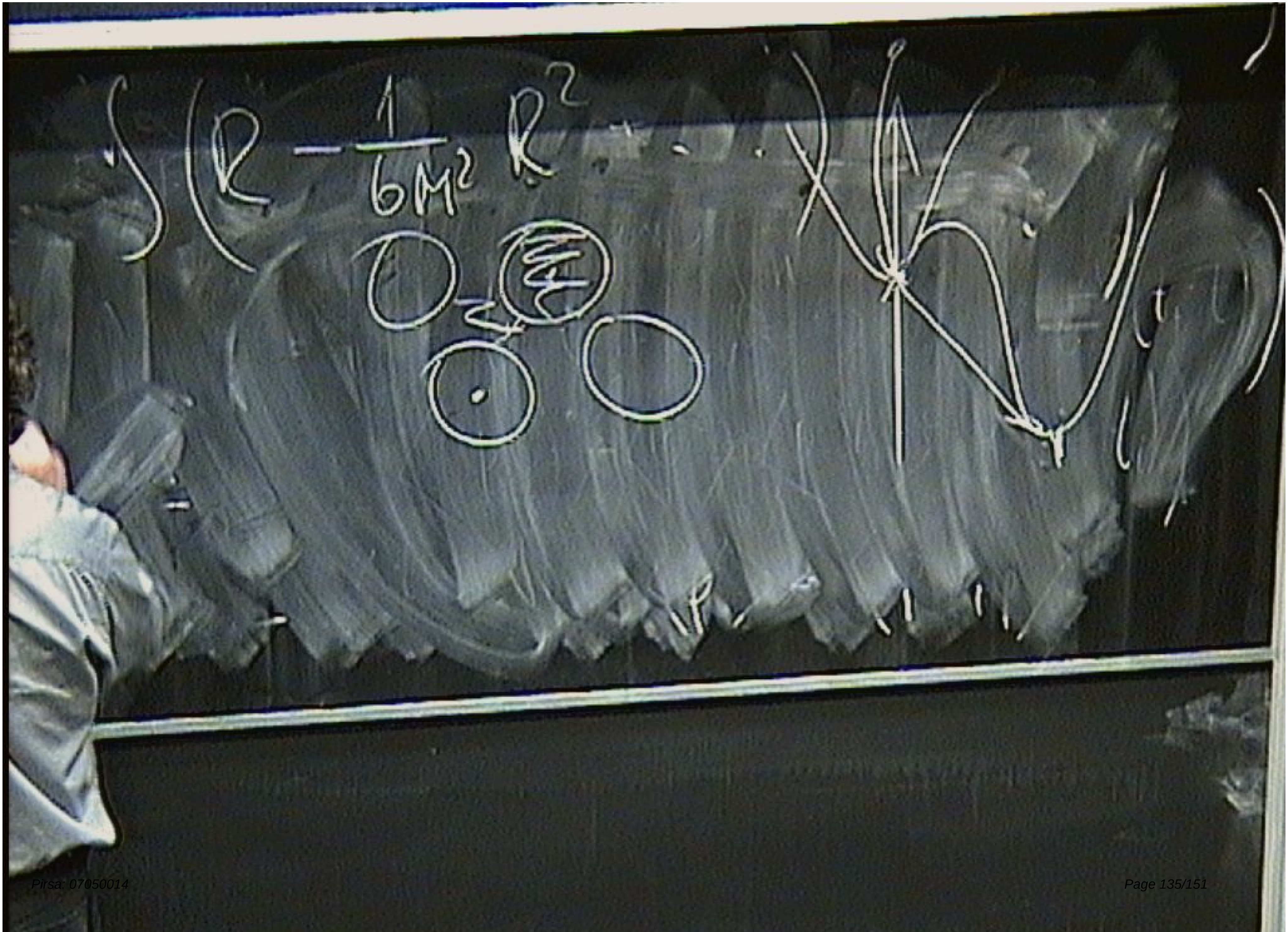


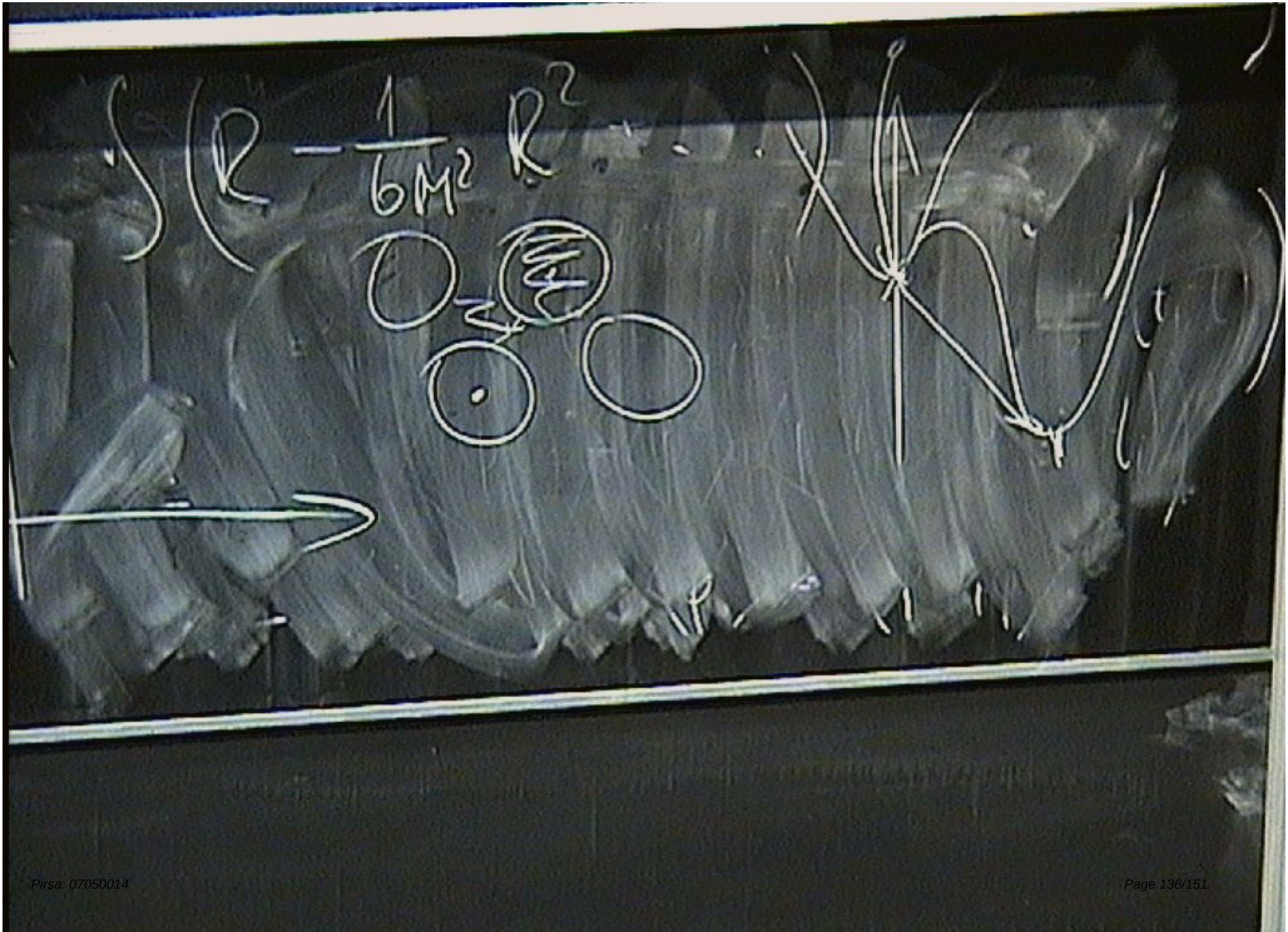

∫ (R − 1/6M² R² . . .

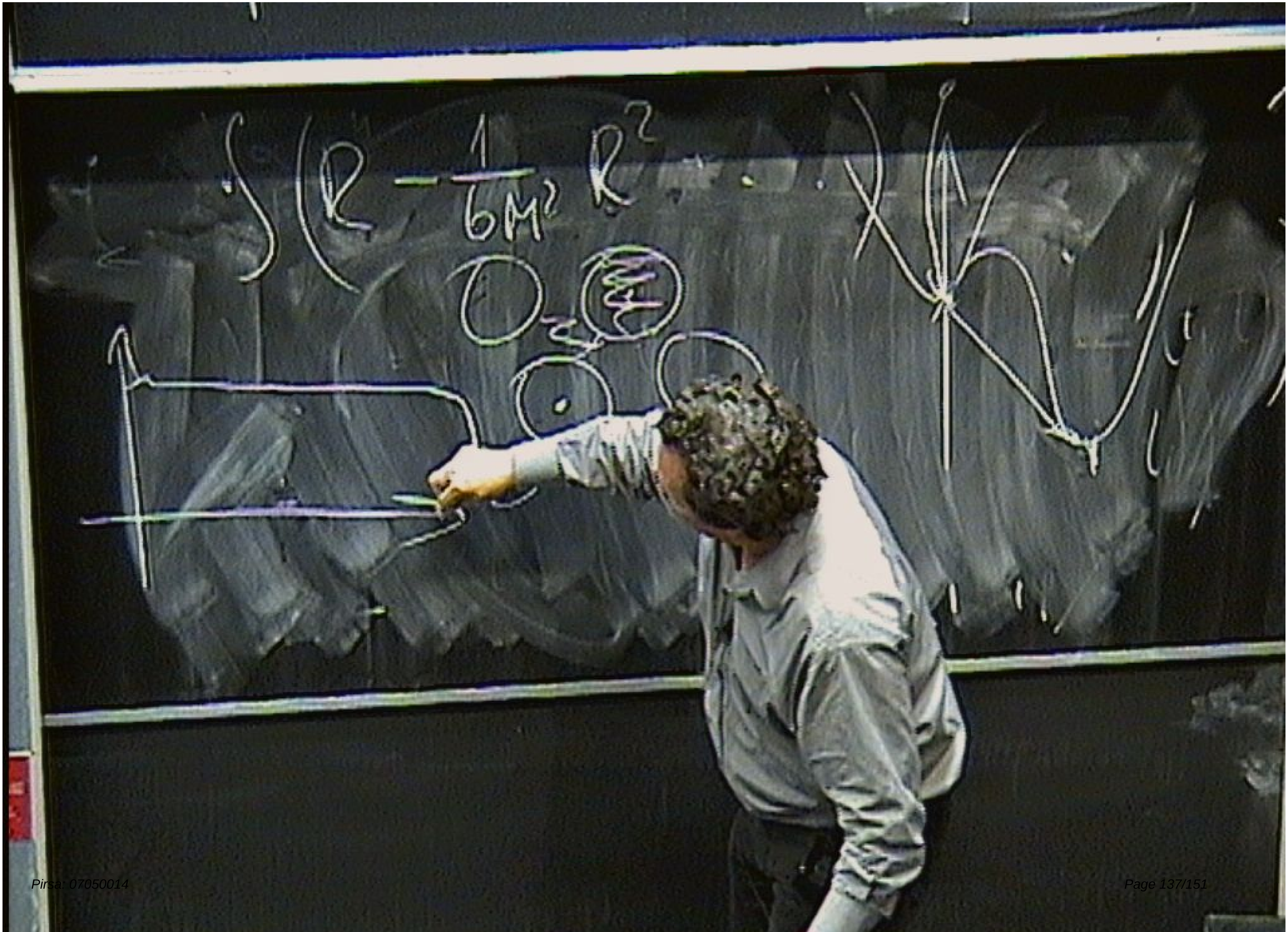

$\int \left(R - \frac{1}{6M^2} R^2 \ldots \right)$

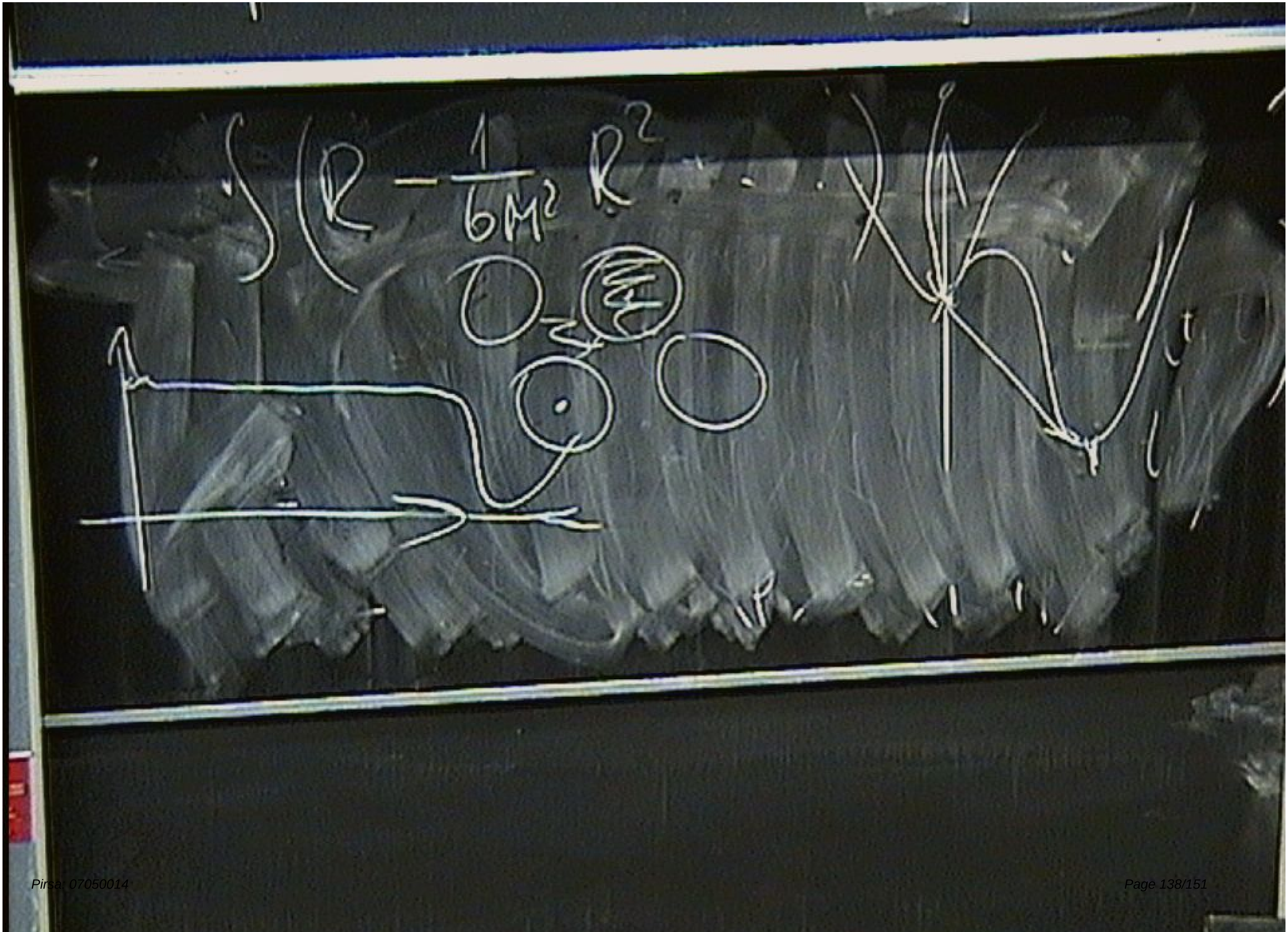

$\int \left(R - \frac{1}{6M^2} R^2 \ldots \right)$

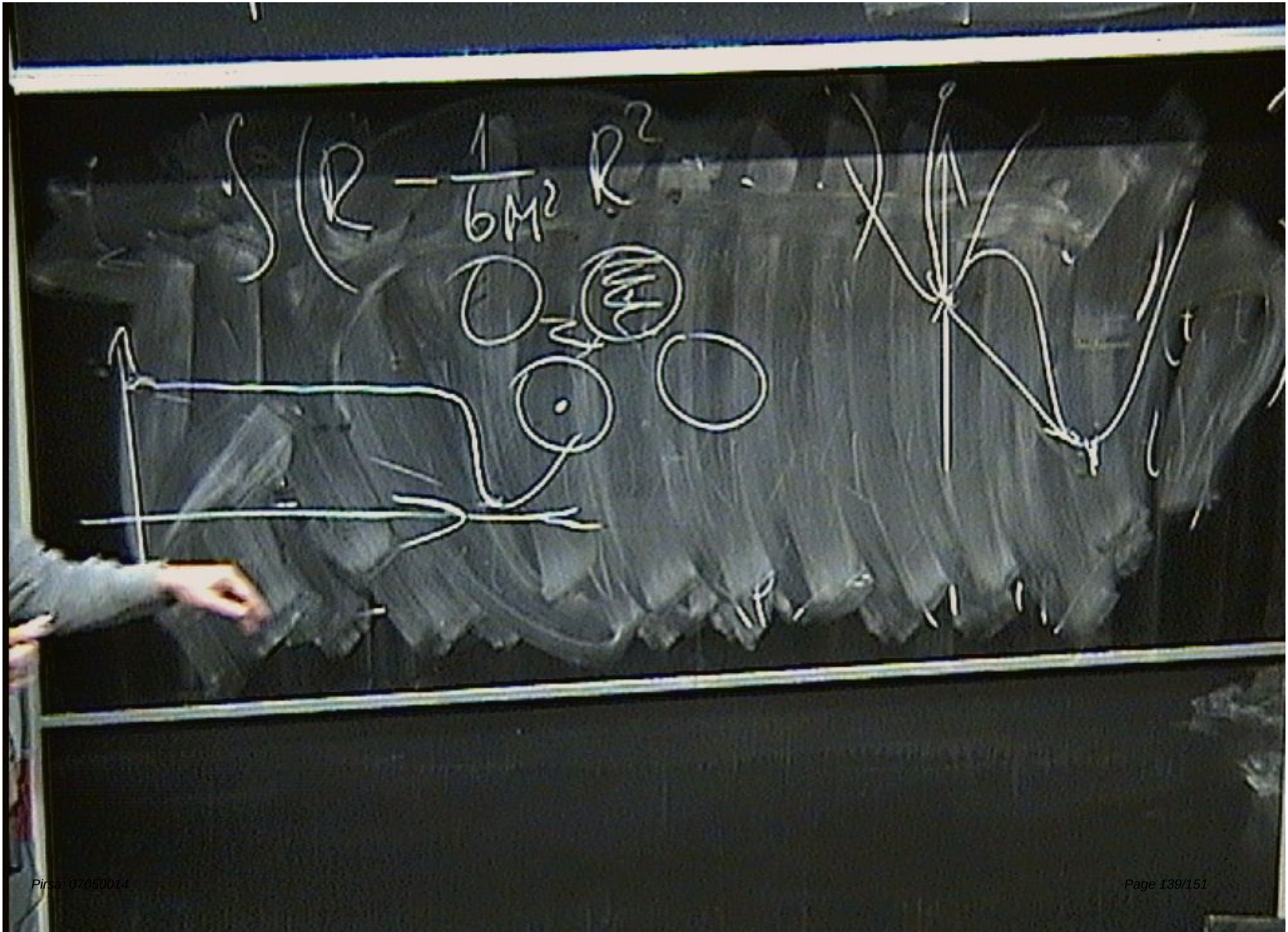

$\int \left(R - \frac{1}{6M^2} R^2 \ldots \right)$

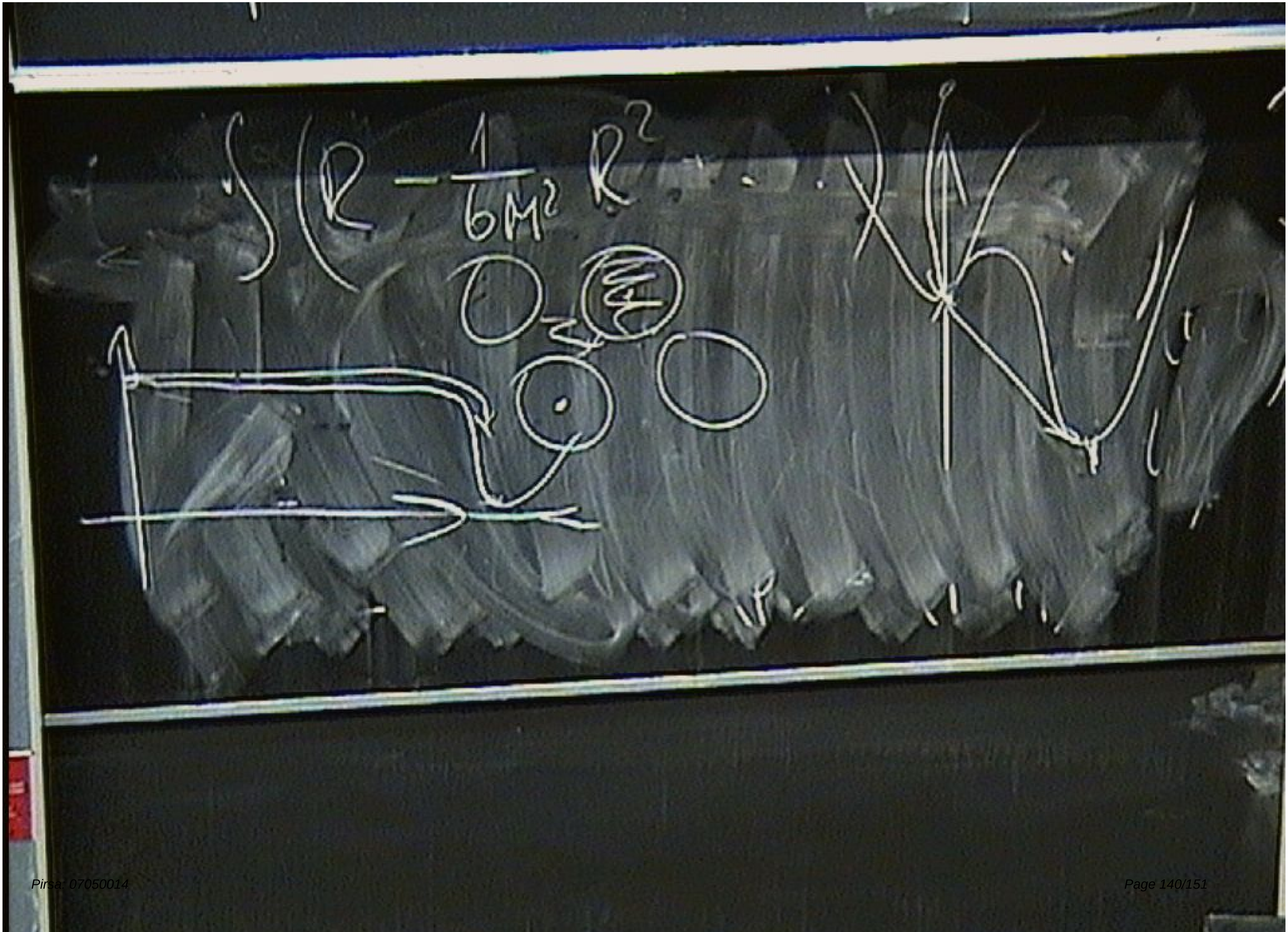


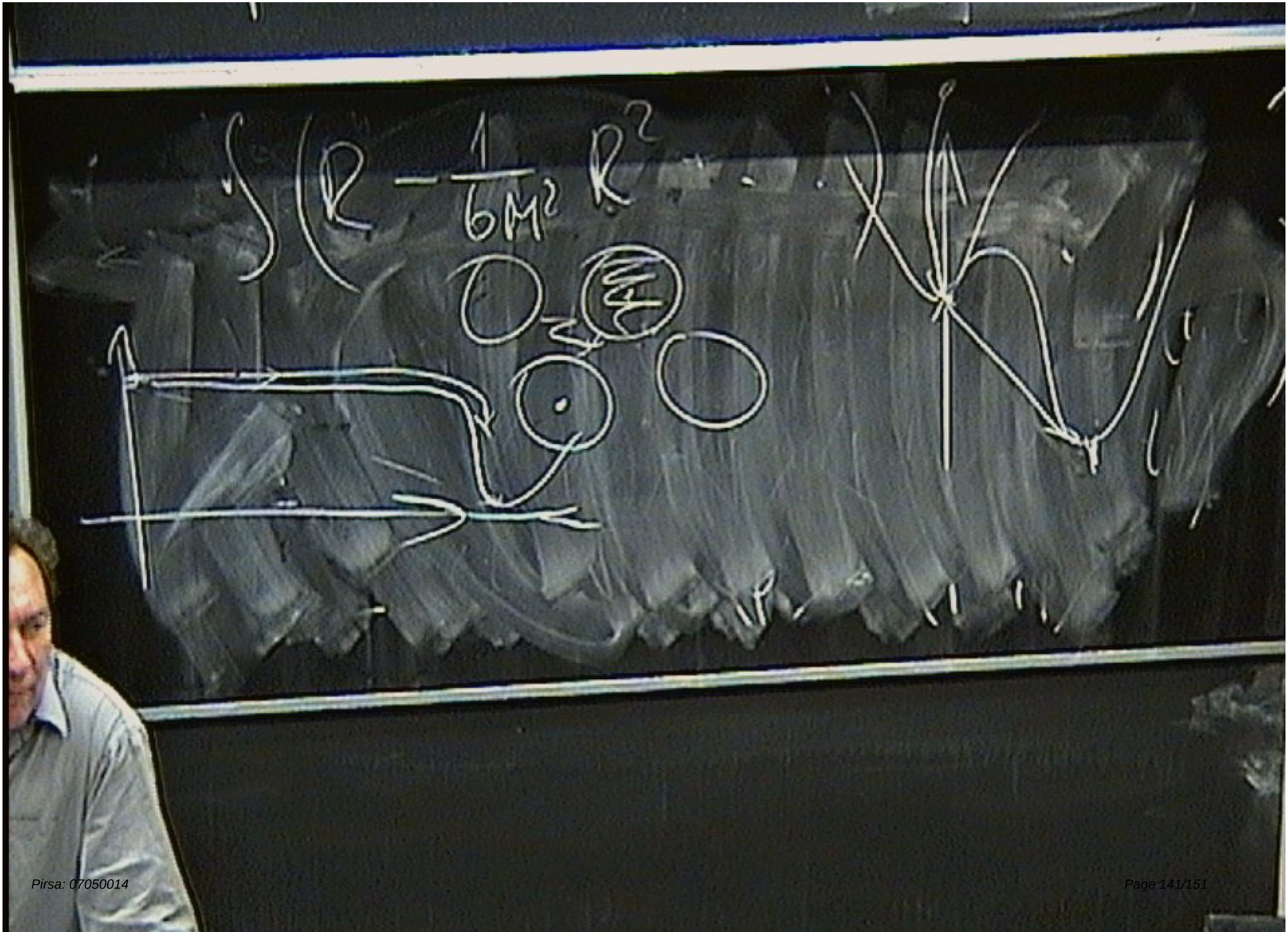

$\int \left(R - \frac{1}{6M^2} R^2 \ldots \right)$

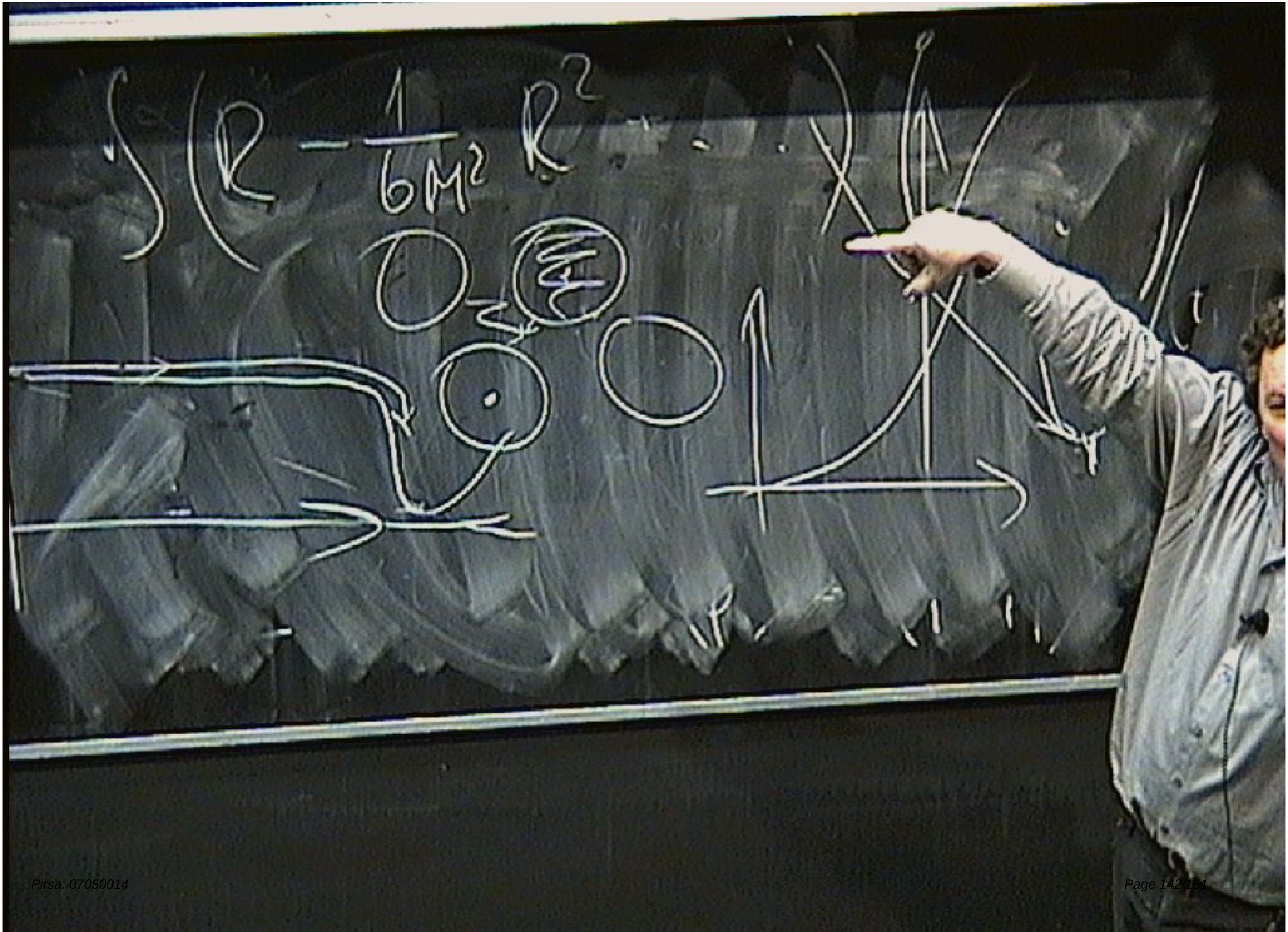

$\int (R - \frac{1}{6M^2} R^2$

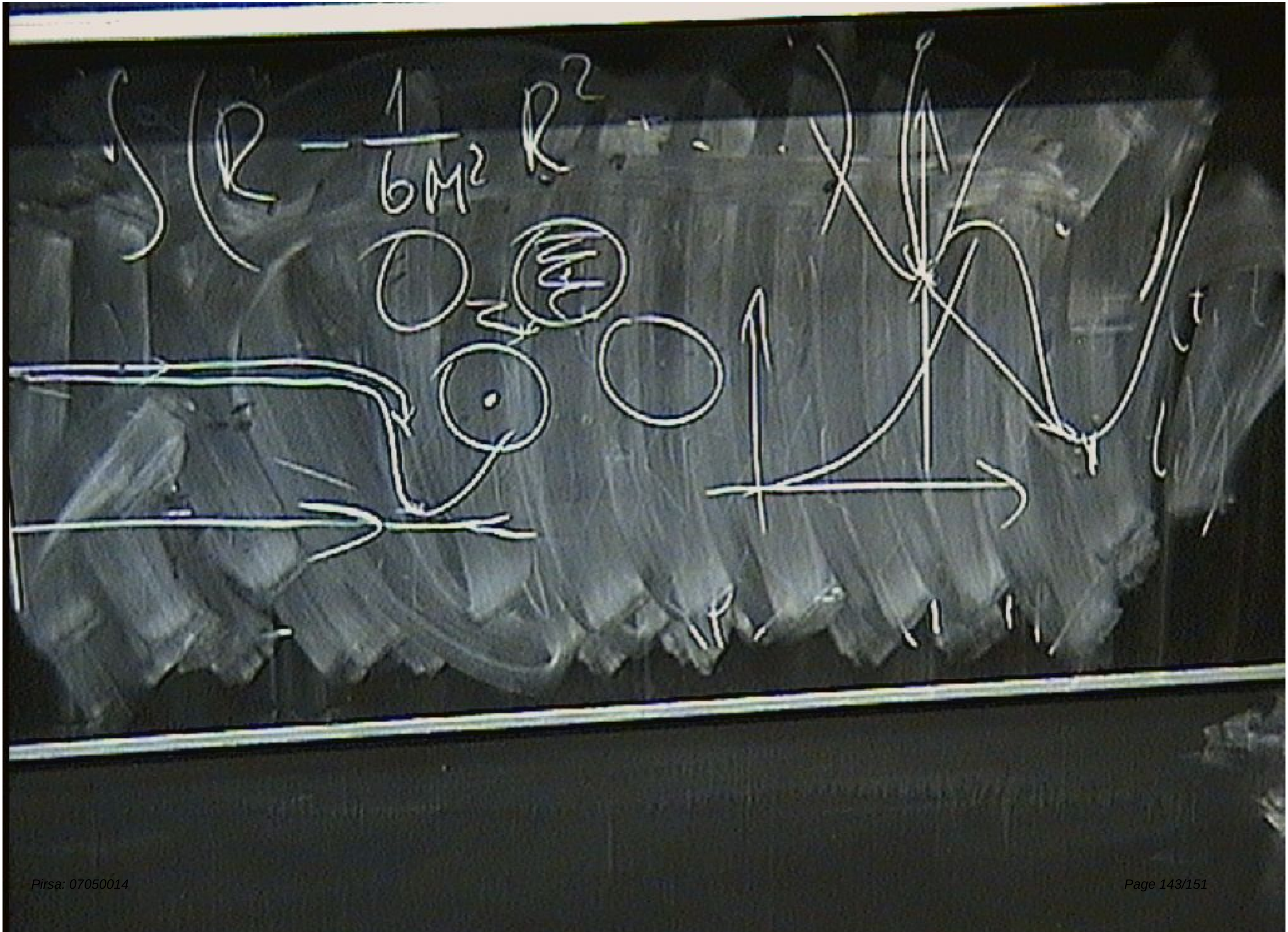

∫(R − 1/6M² R² ...)

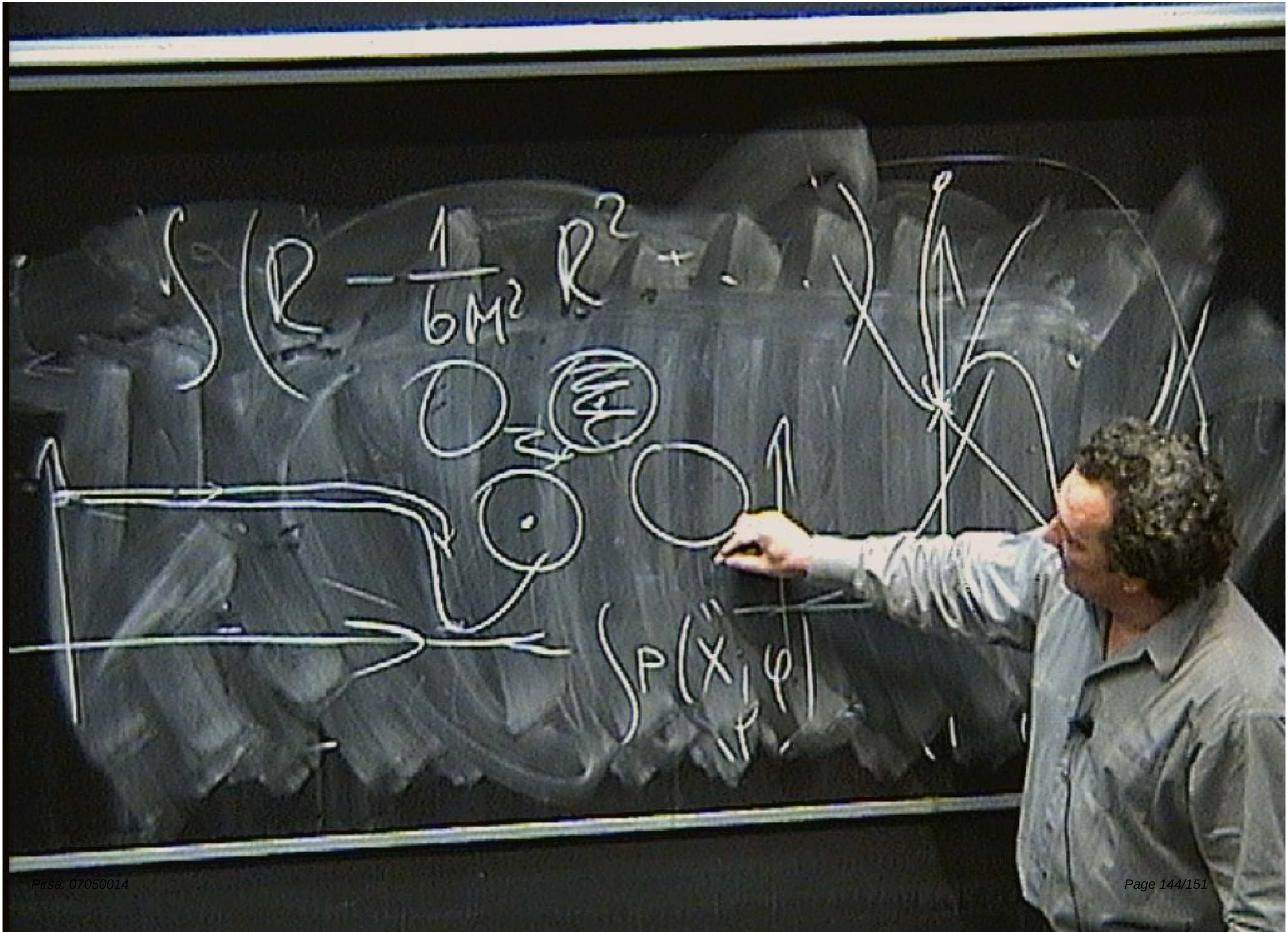

$\int (R - \frac{1}{6M^2} R^2 \ldots)$

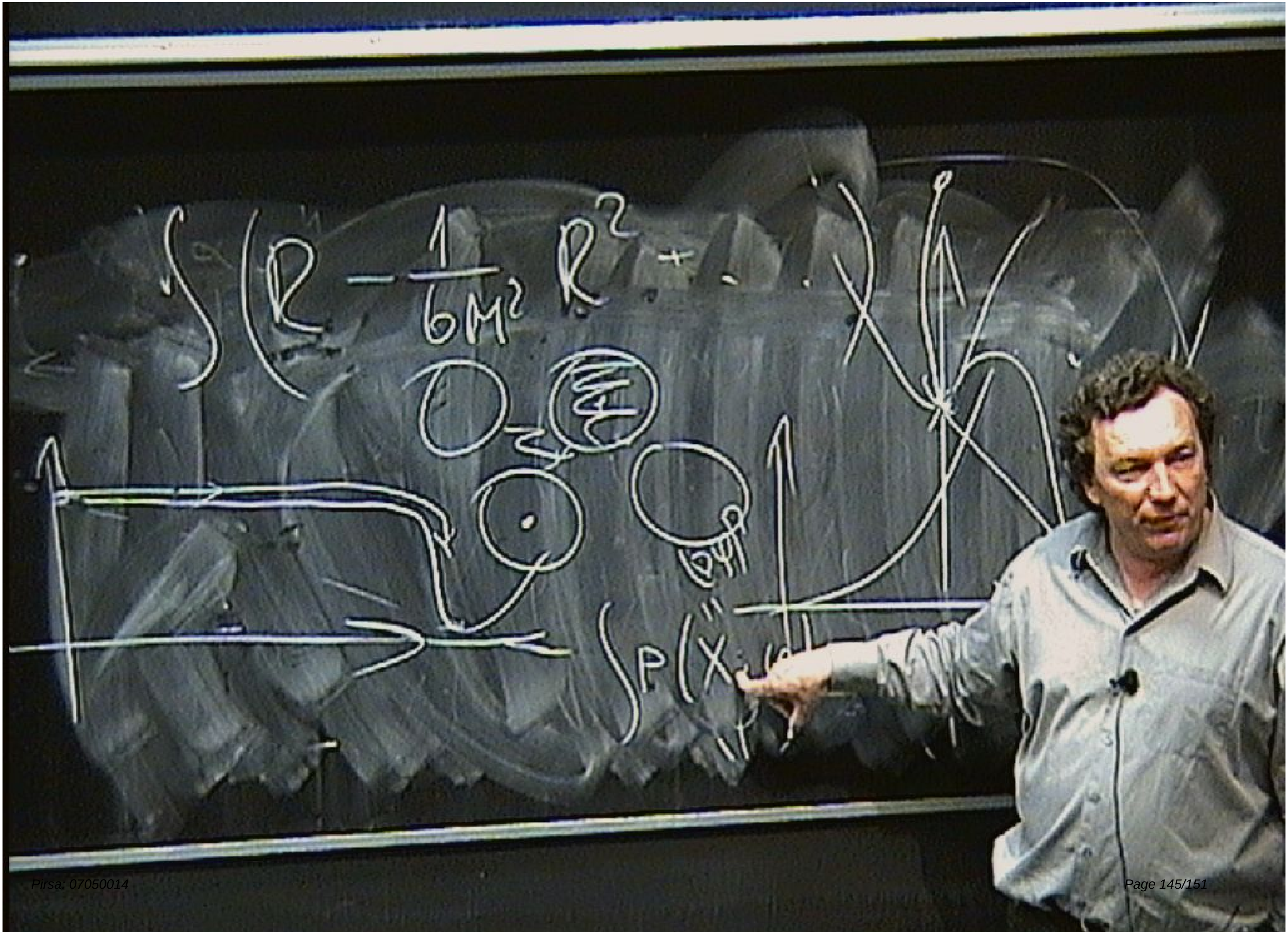

∫(R − 1/6M² R²

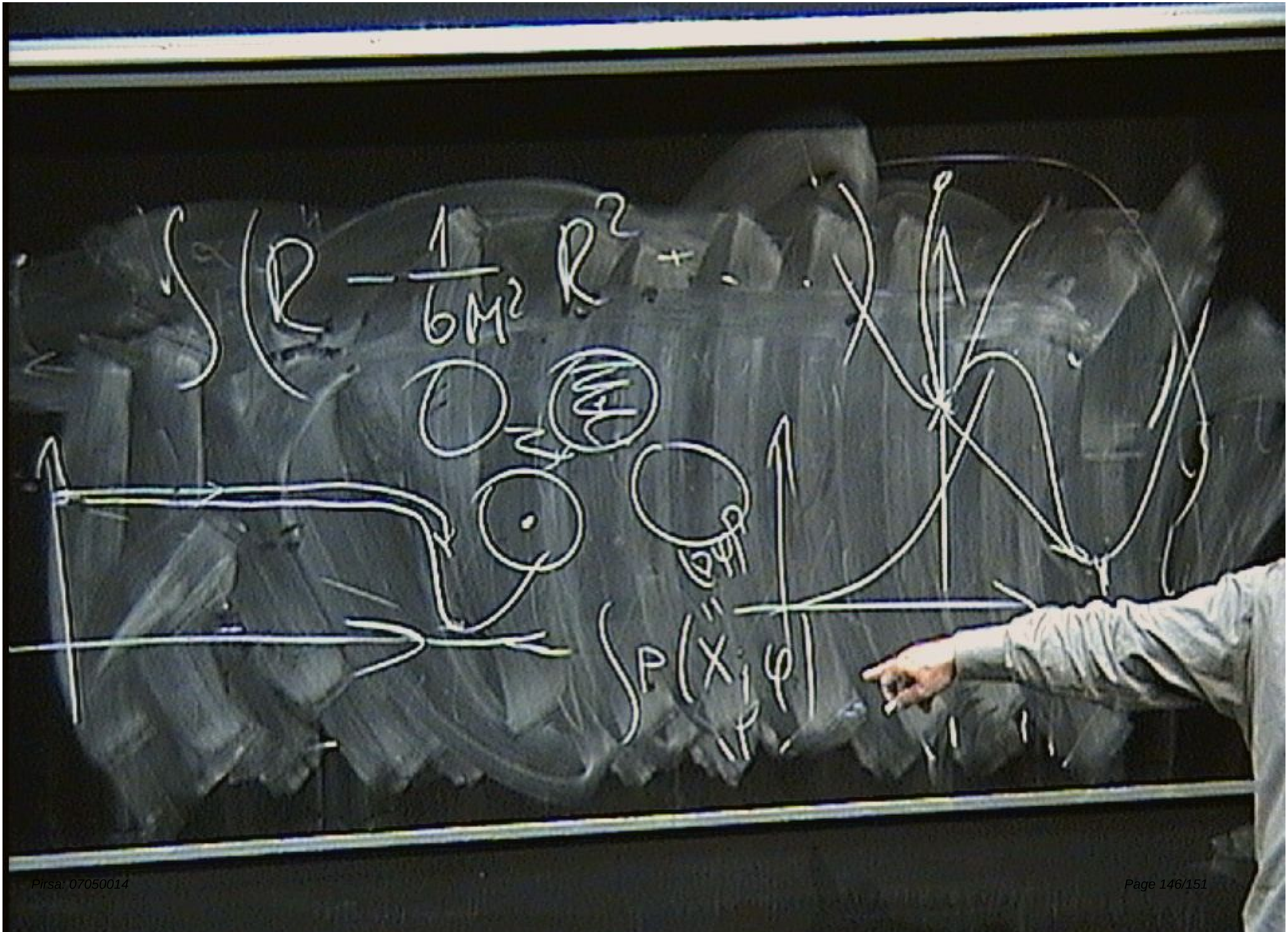

∫(R − 1/6M² R² ...)

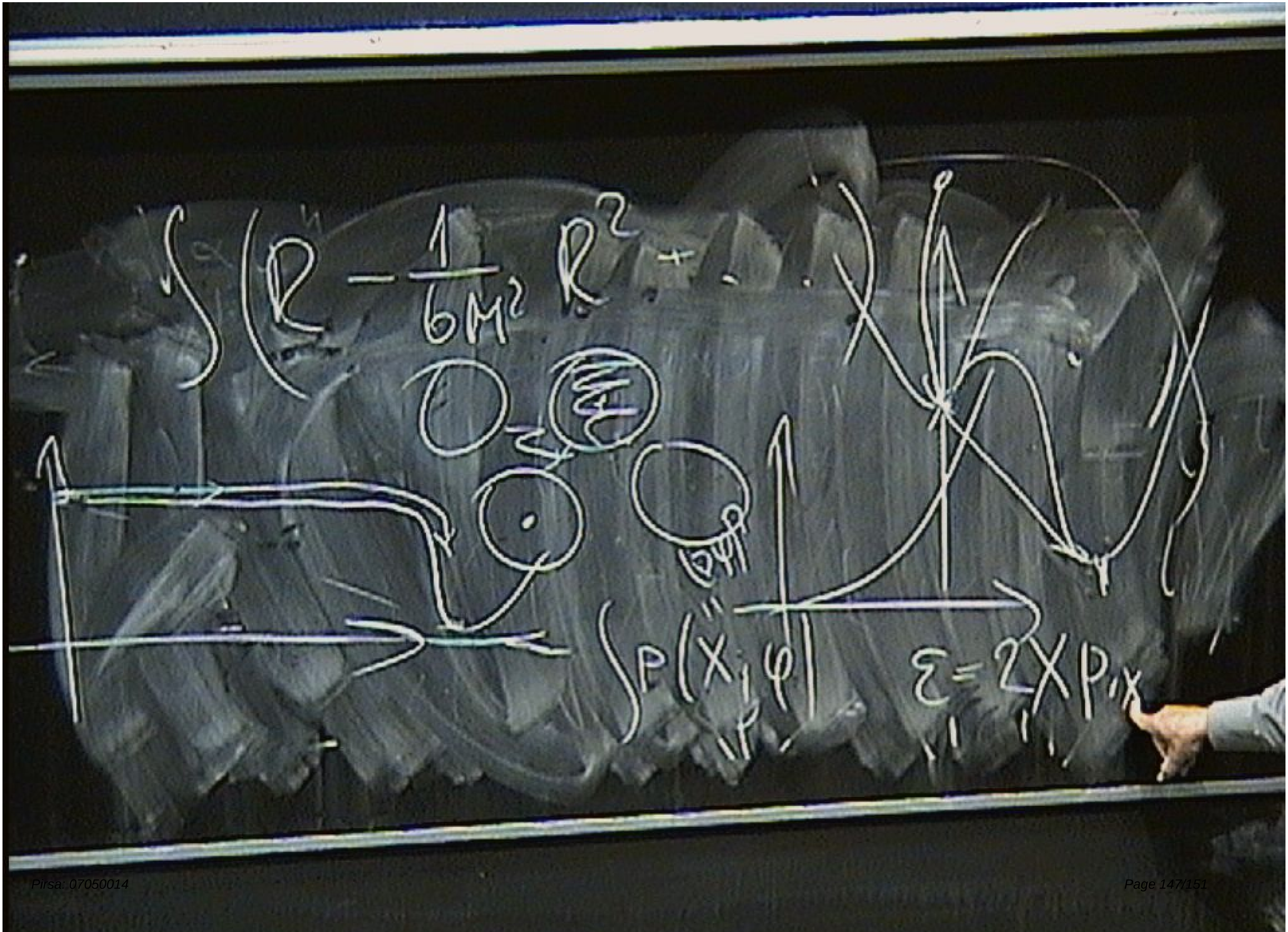

∫(R − 1/6M² R² . . .

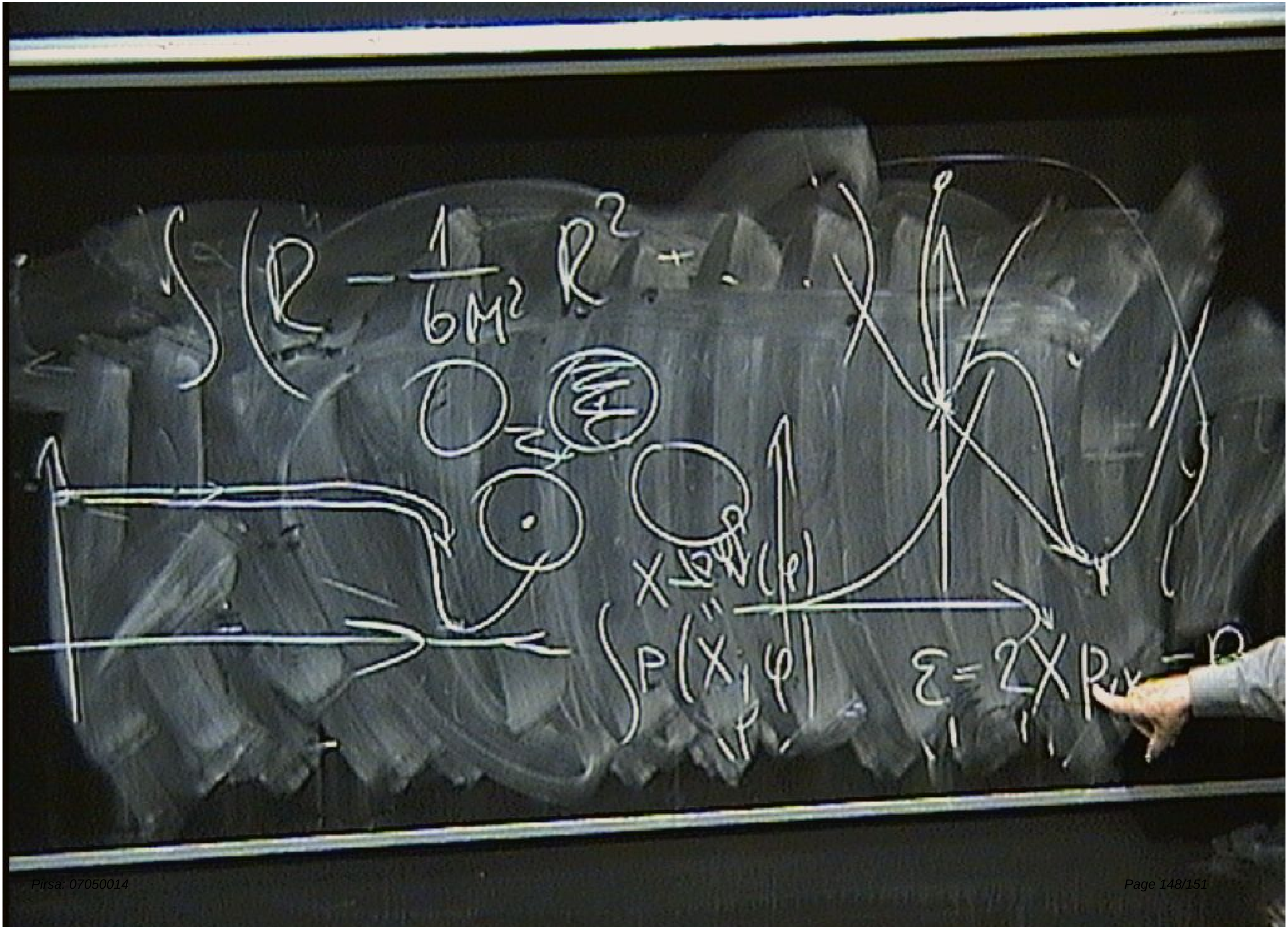

∫(R − 1/6M² R² . . .)

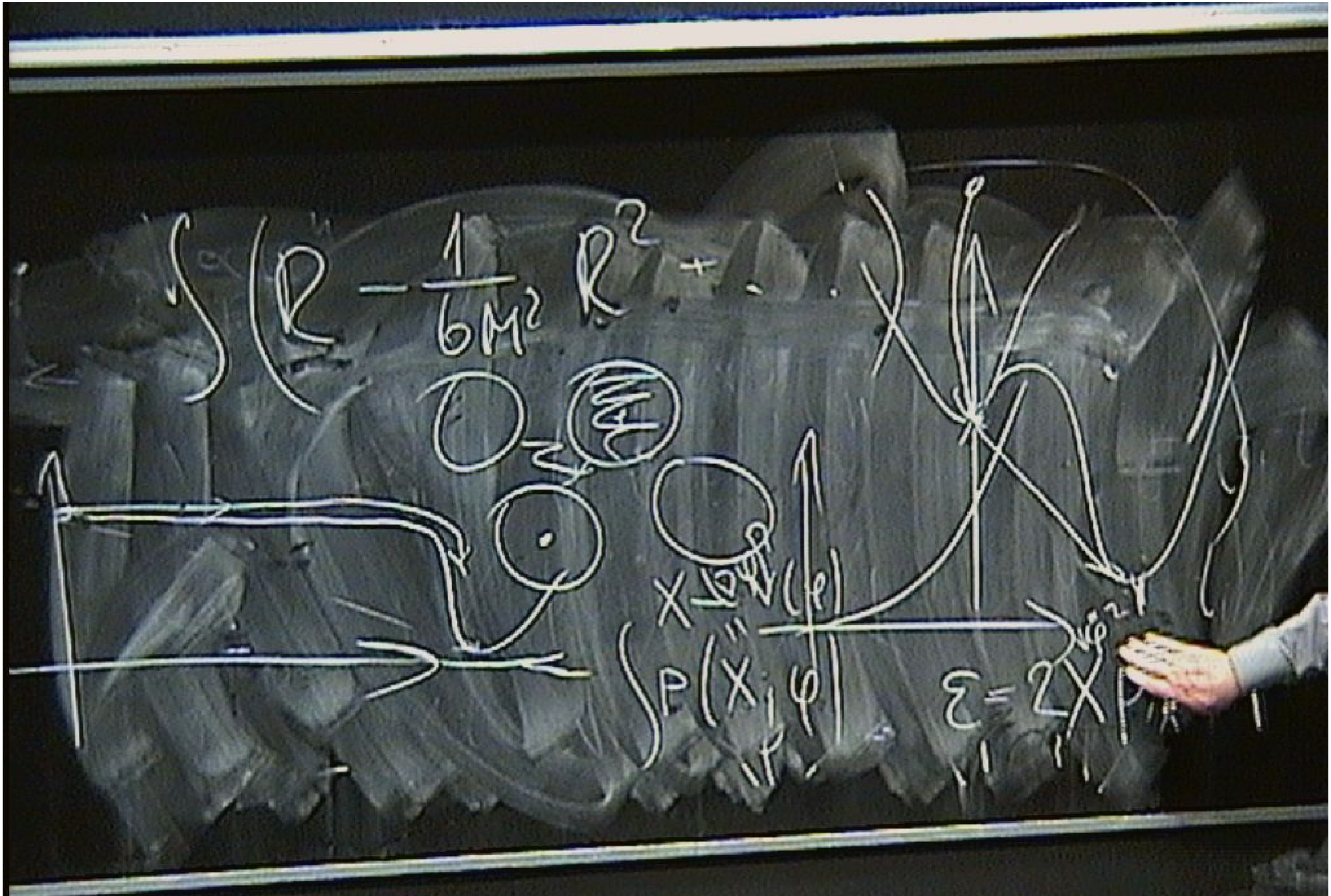

∫(R − 1/6M² R² + . .
∫P(X, φ)

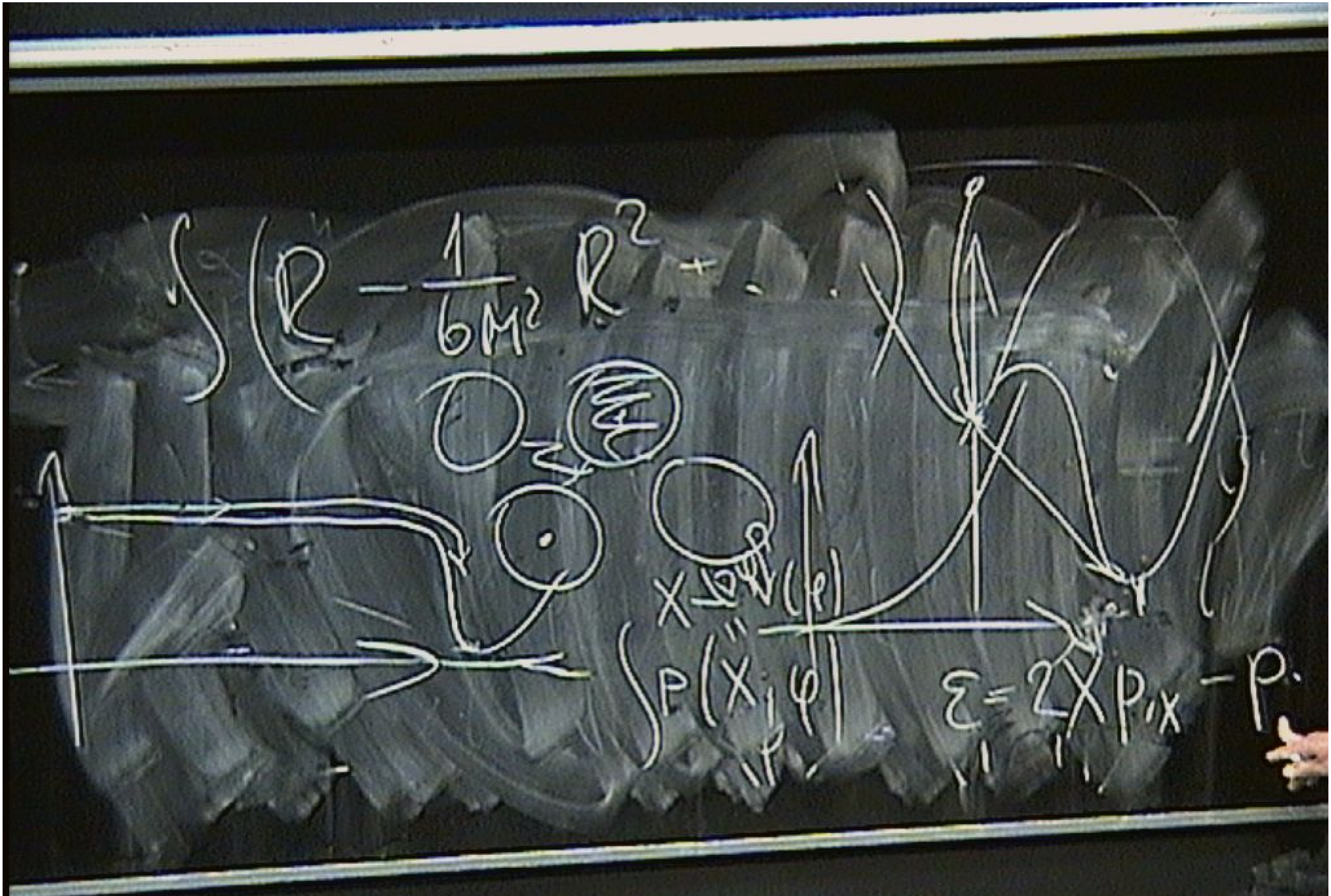


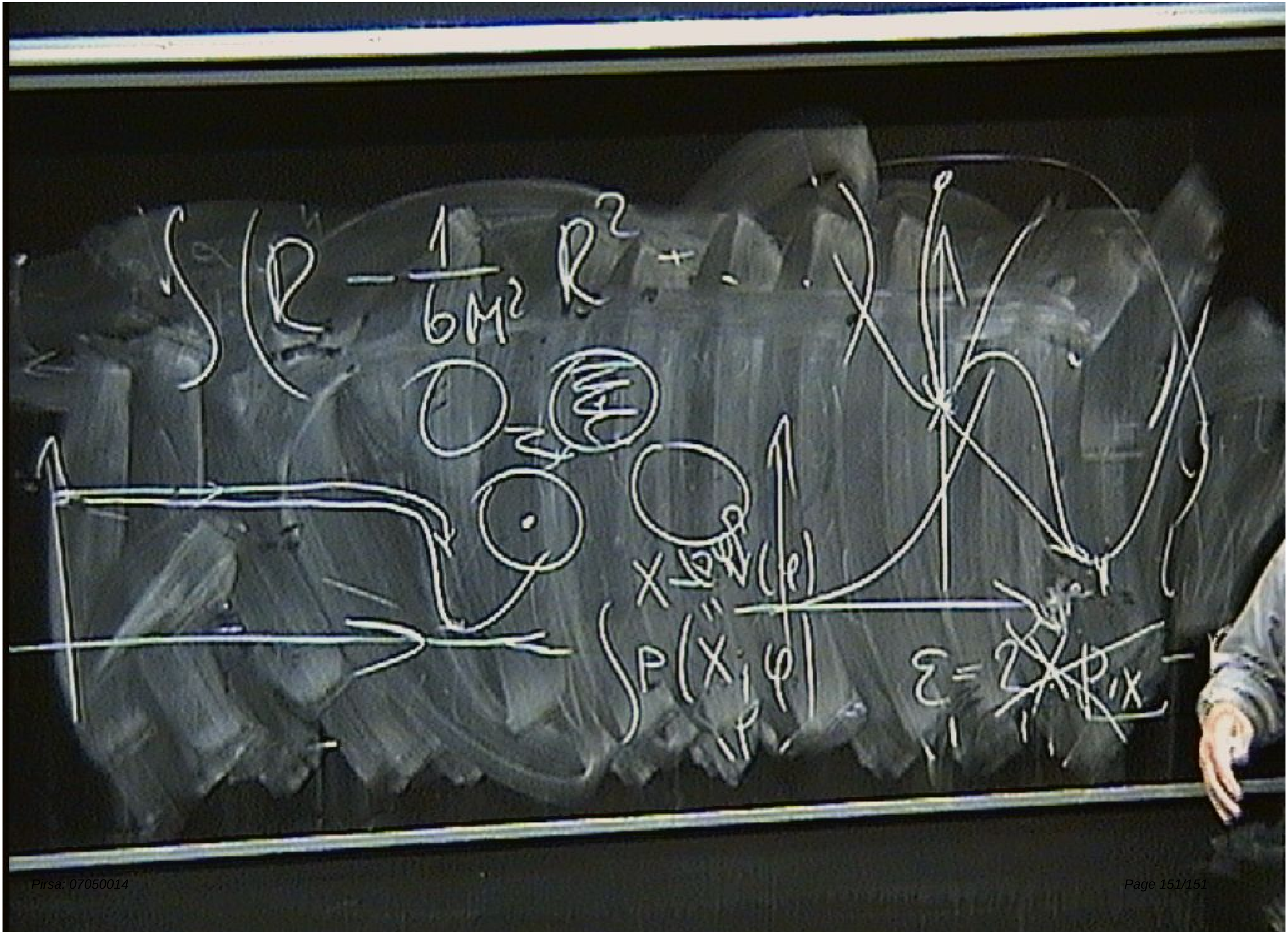



∫(R − 1/6M² R² + ...)
ε = 2XP.X






∫(R − 1/(6M²) R² +
X = ∂²V(φ)
∫p(X, φ)
ε = 2X p,x − p


∫(R − 1/6M² R² + ...)