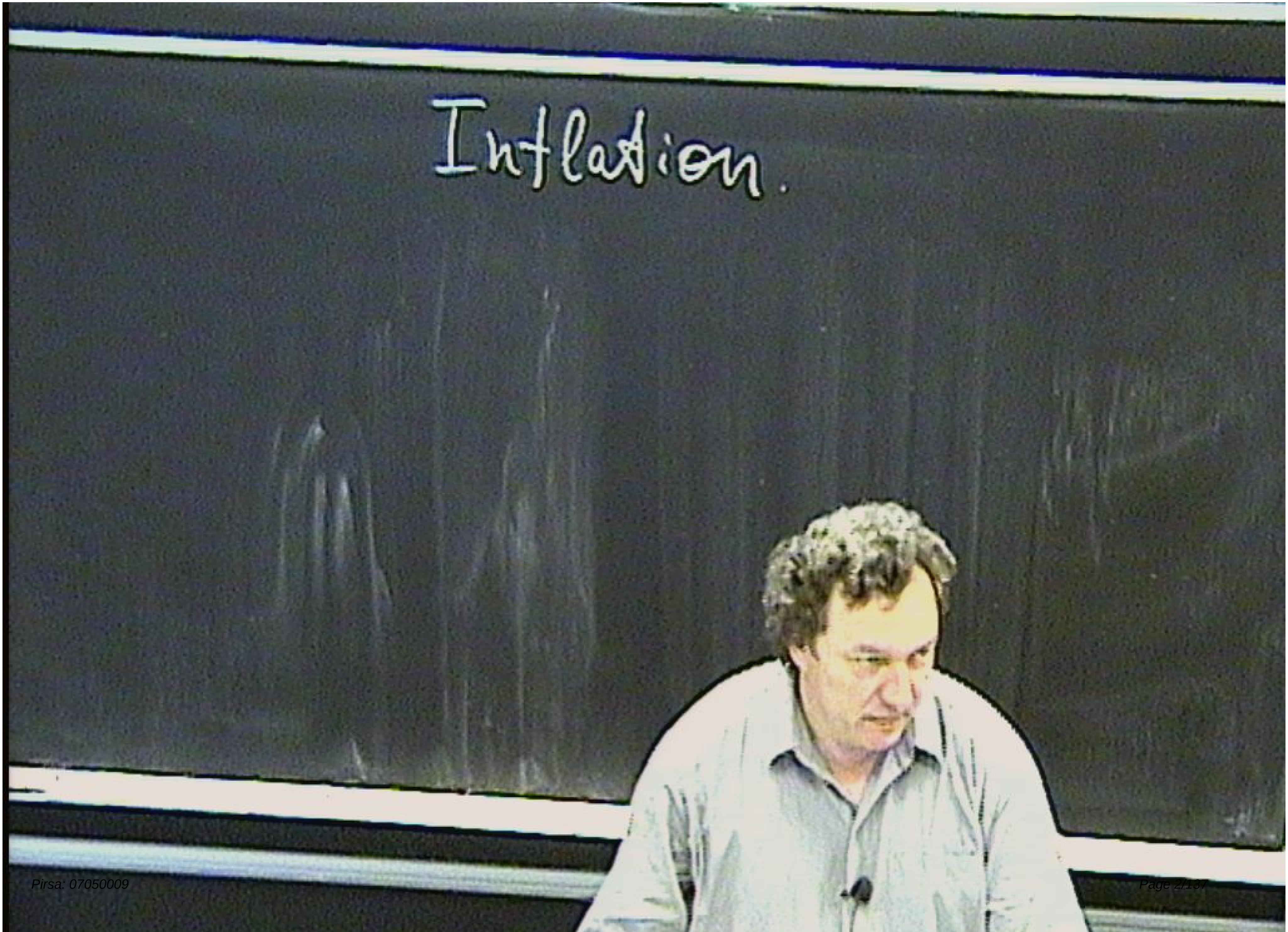
Title: Advanced Topics in Cosmology

Date: May 03, 2007 10:00 AM

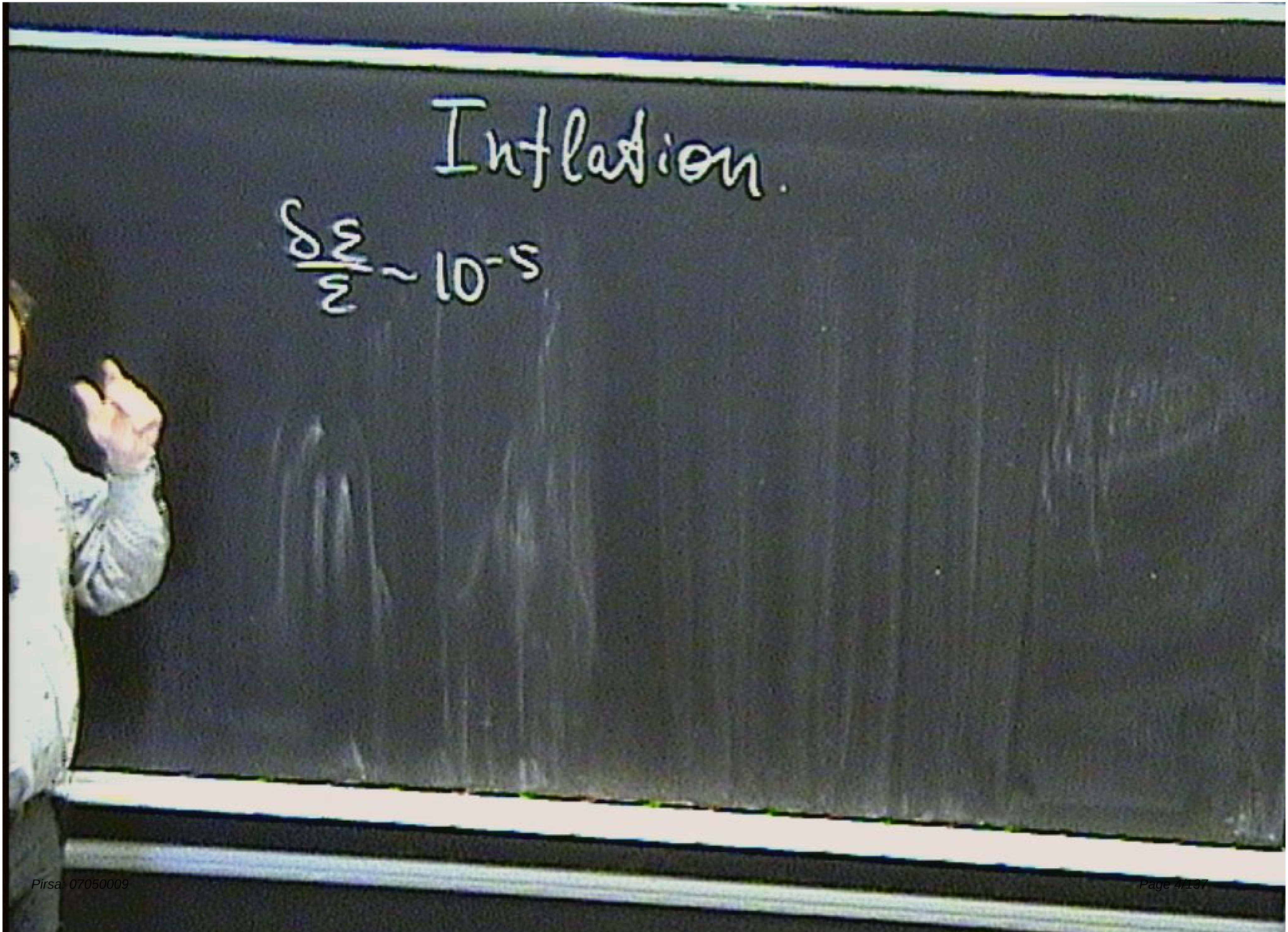
URL: http://pirsa.org/07050009

Abstract: Class 3


Inflation.

Inflation.

$\delta\varepsilon \sim 10^{-5}$


Inflation.
$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-5}$

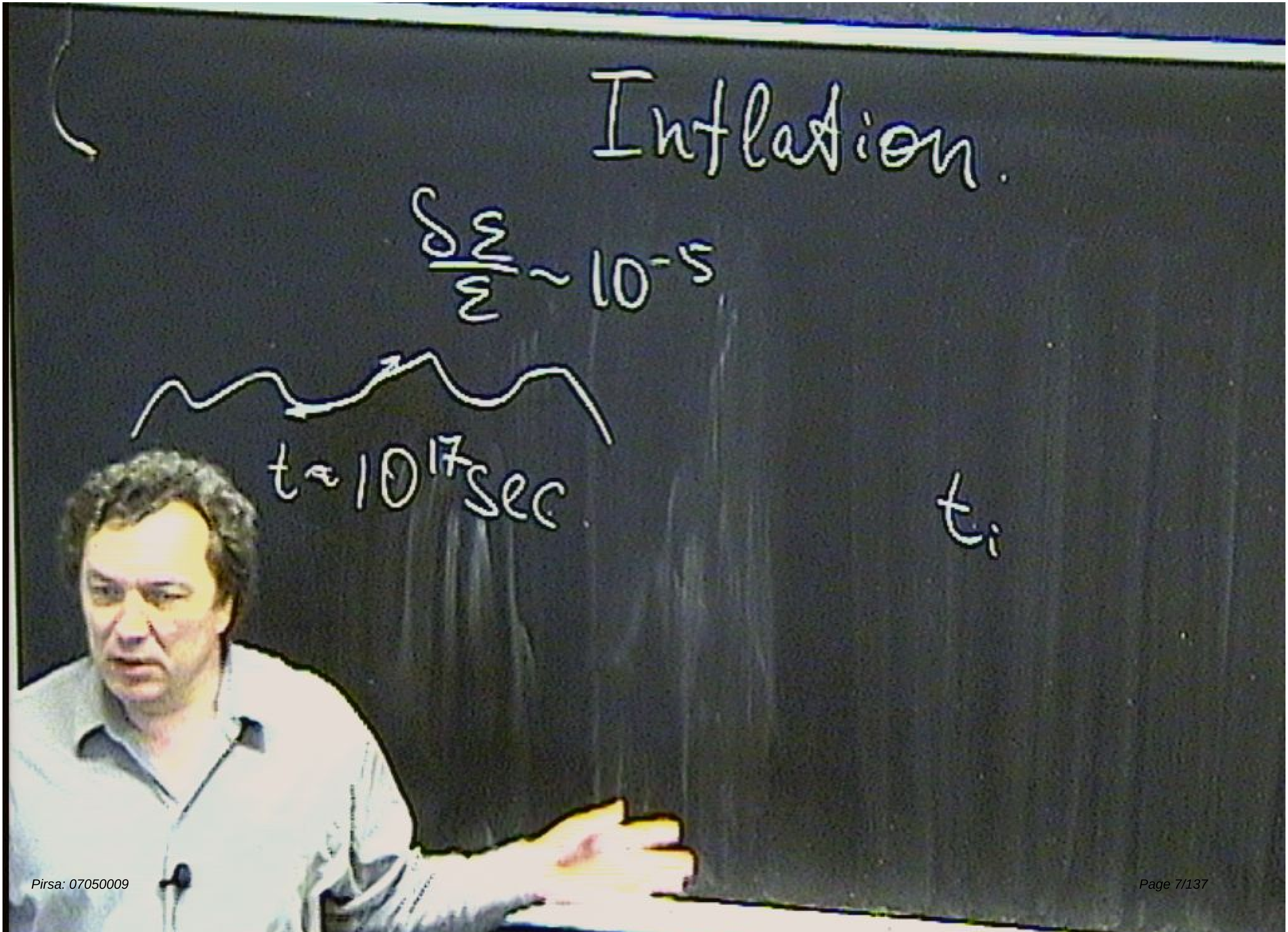
Inflation.

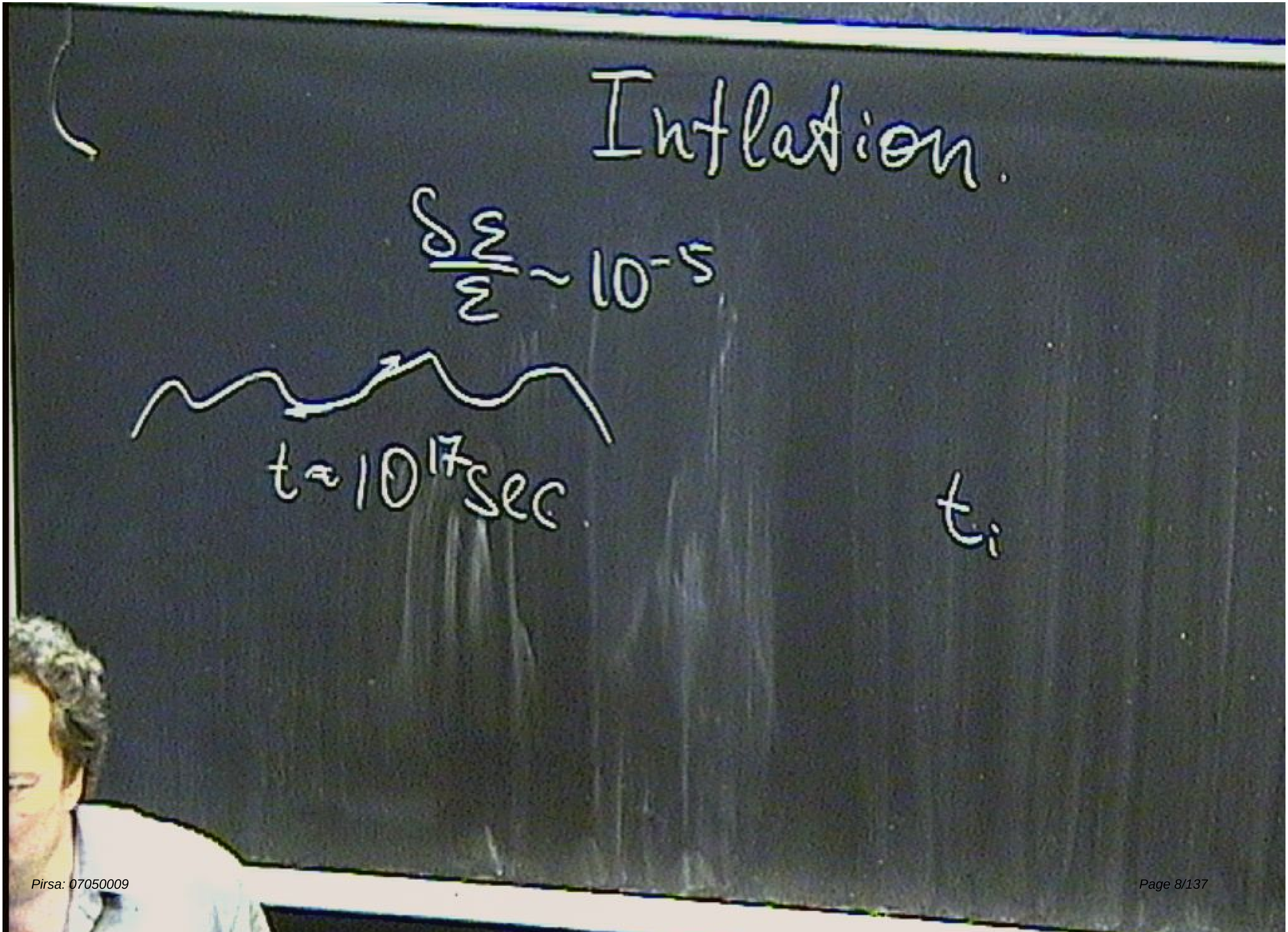
$$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-5}$$

Inflation.

$$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-5}$$

$t \approx 10^{17}$ sec


Inflation.
$\frac{\delta \varepsilon}{\varepsilon} \sim 10^{-5}$
$t \approx 10^{17}$ sec
t_i


Inflation.
$\frac{\delta \varepsilon}{\varepsilon} \sim 10^{-5}$
$t \approx 10^{17}$ sec
t_i

Inflation.

$$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-5}$$

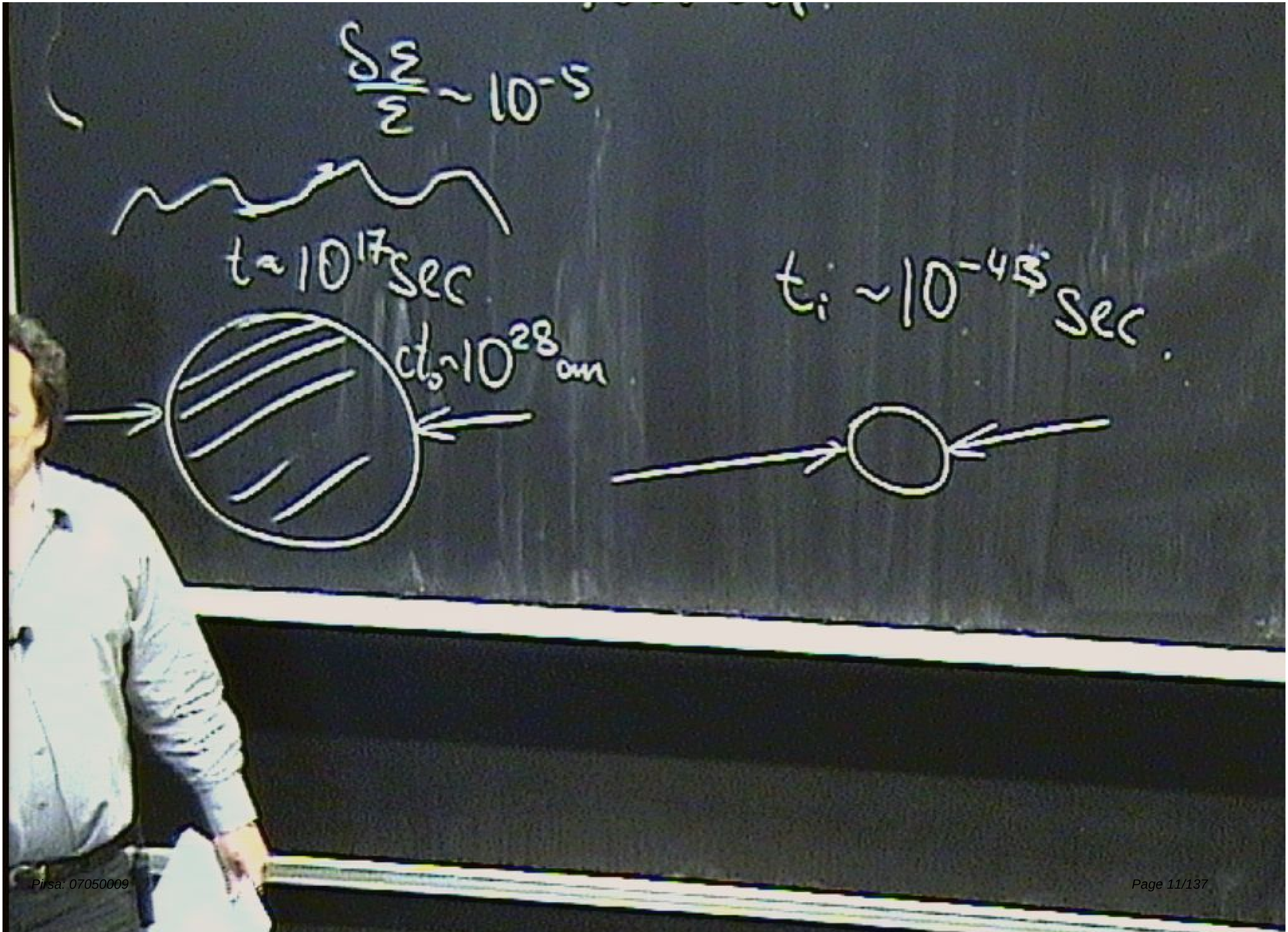
$t \sim 10^{17}\,\mathrm{sec}$

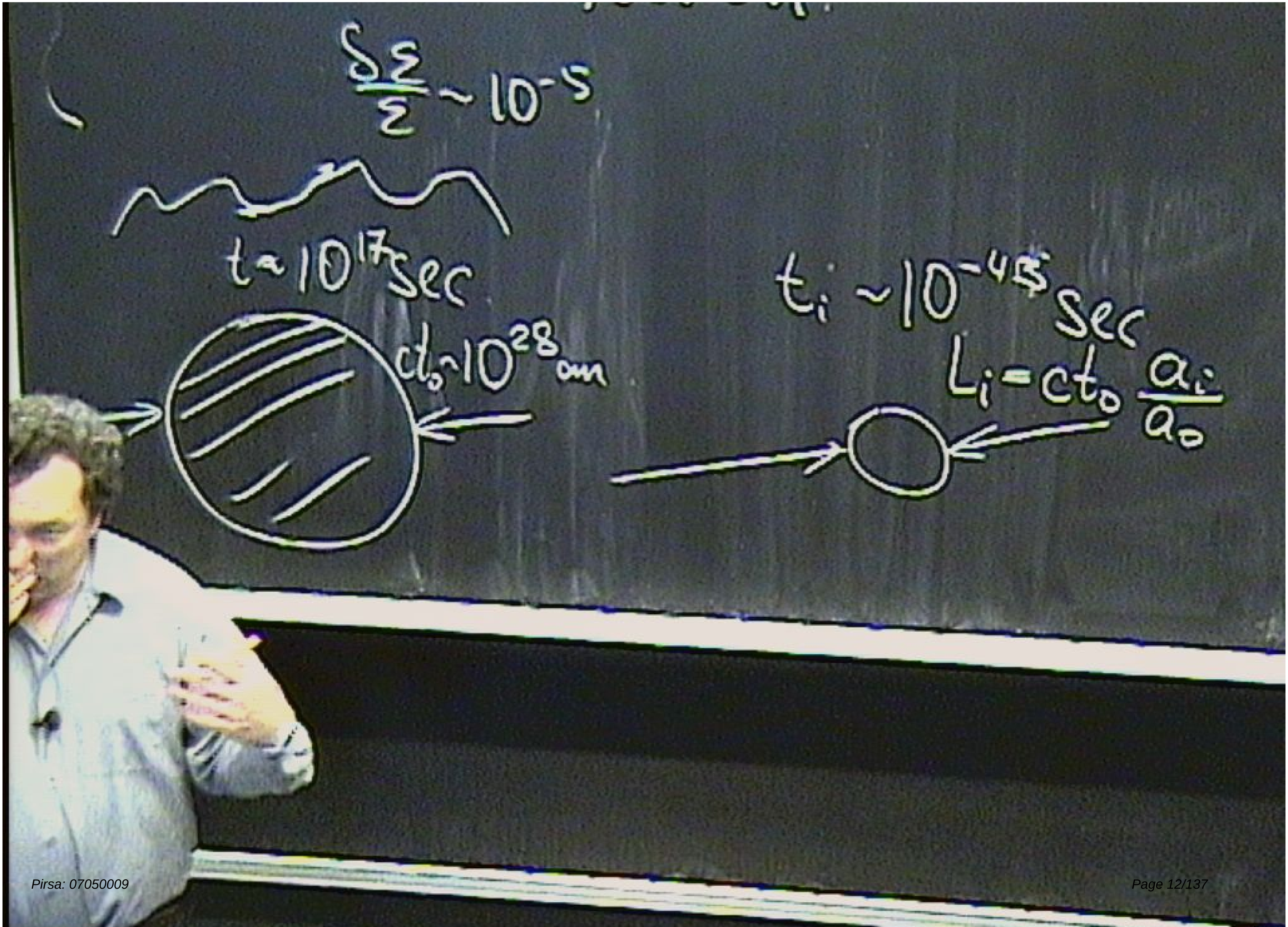
$t_i \sim 10^{-45}\,\mathrm{sec}.$

$$\frac{\delta \varepsilon}{\varepsilon} \sim 10^{-5}$$

$t \approx 10^{17}\,\mathrm{sec}$

$t_i \sim 10^{-43}\,\mathrm{sec}.$


δε/ε ~ 10^-5
t ≈ 10^17 sec
ct_0 ~ 10^28 cm
t_i ~ 10^-43 sec



$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-5}$
$t \approx 10^{17}$ sec
$ct_0 \sim 10^{28}$ cm
$t_i \sim 10^{-45}$ sec
$L_i = ct_0 \frac{a_i}{a_0}$

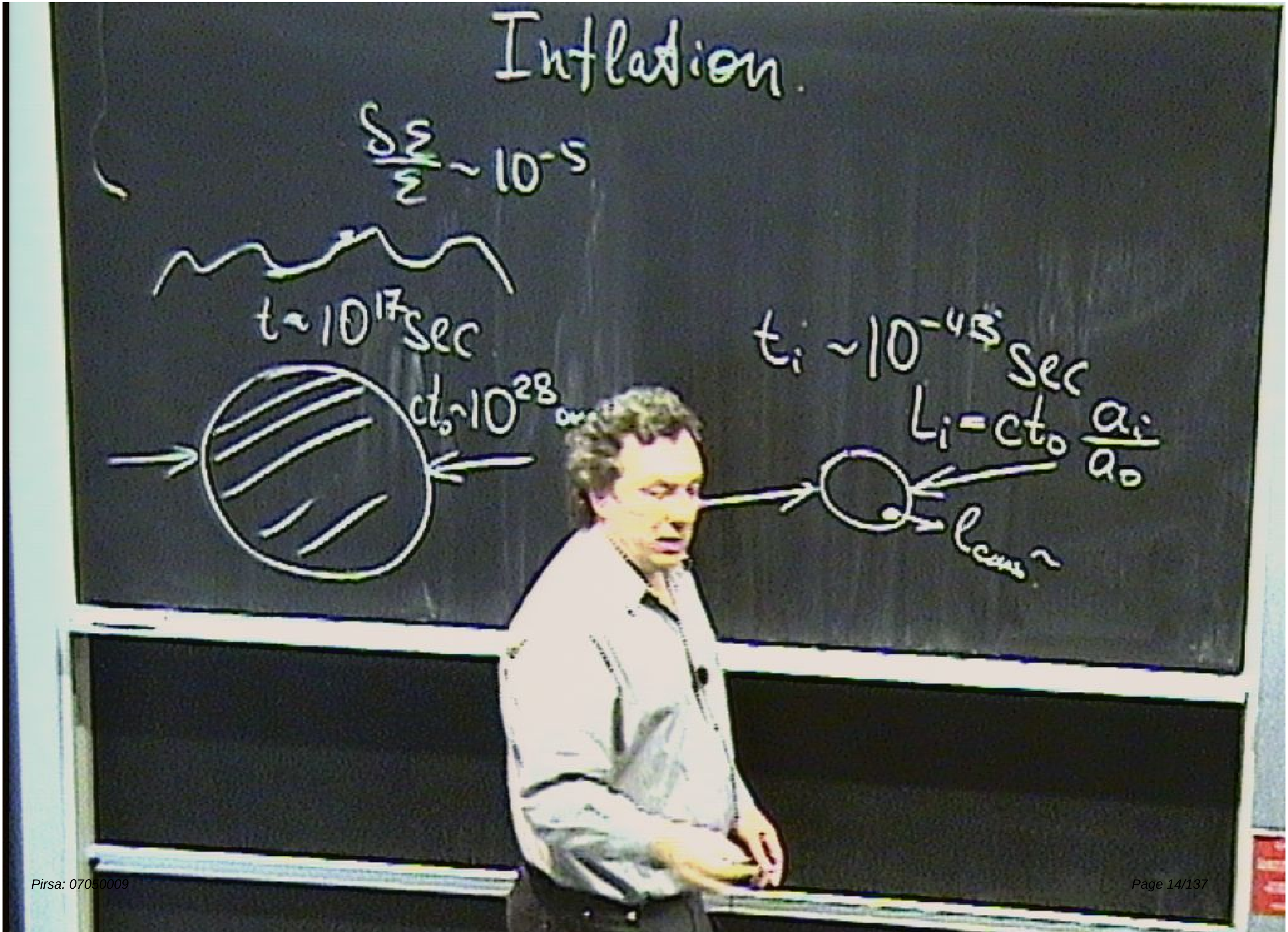
$$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-5}$$

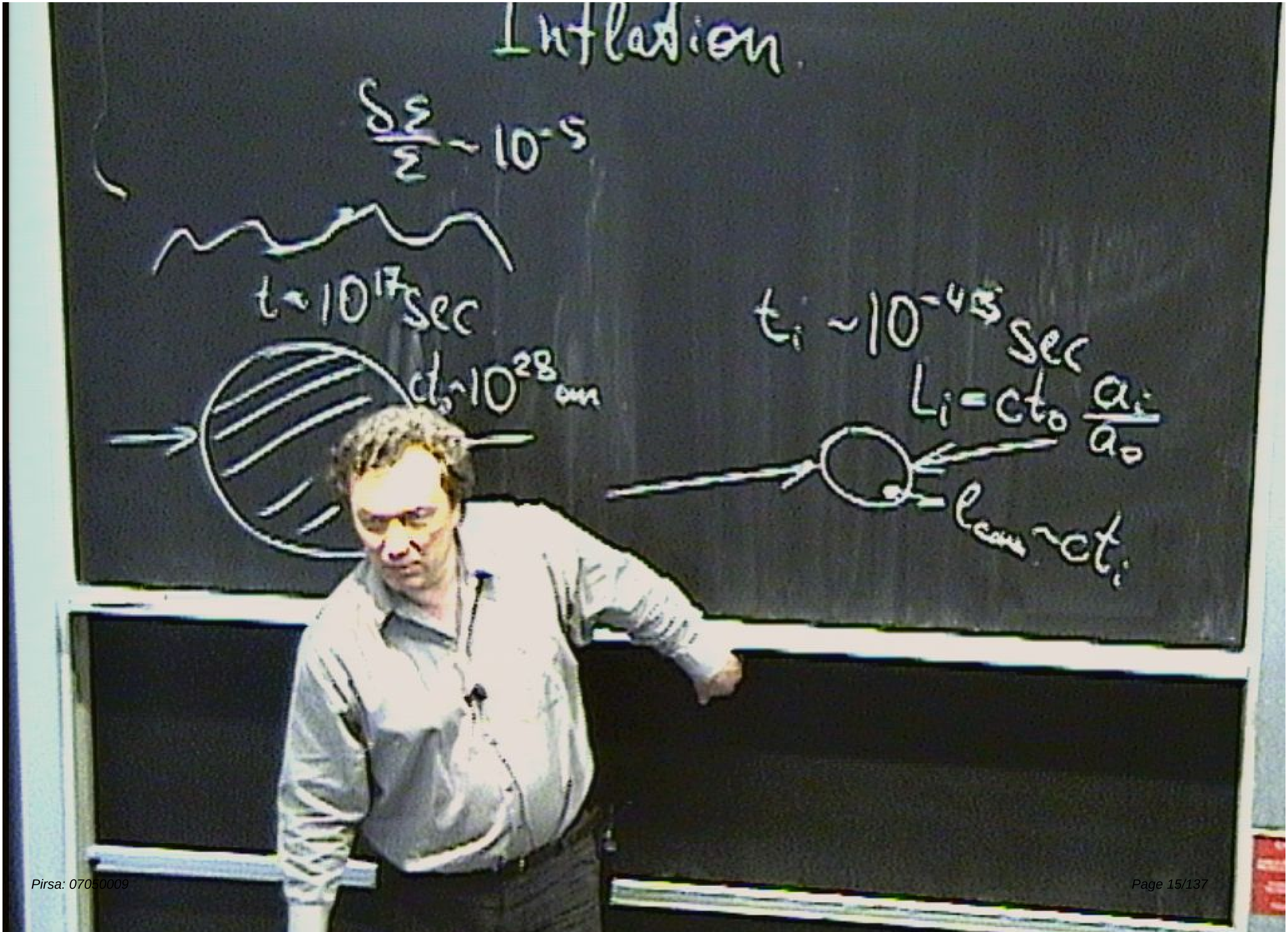
$t \approx 10^{17}\,\text{sec}$

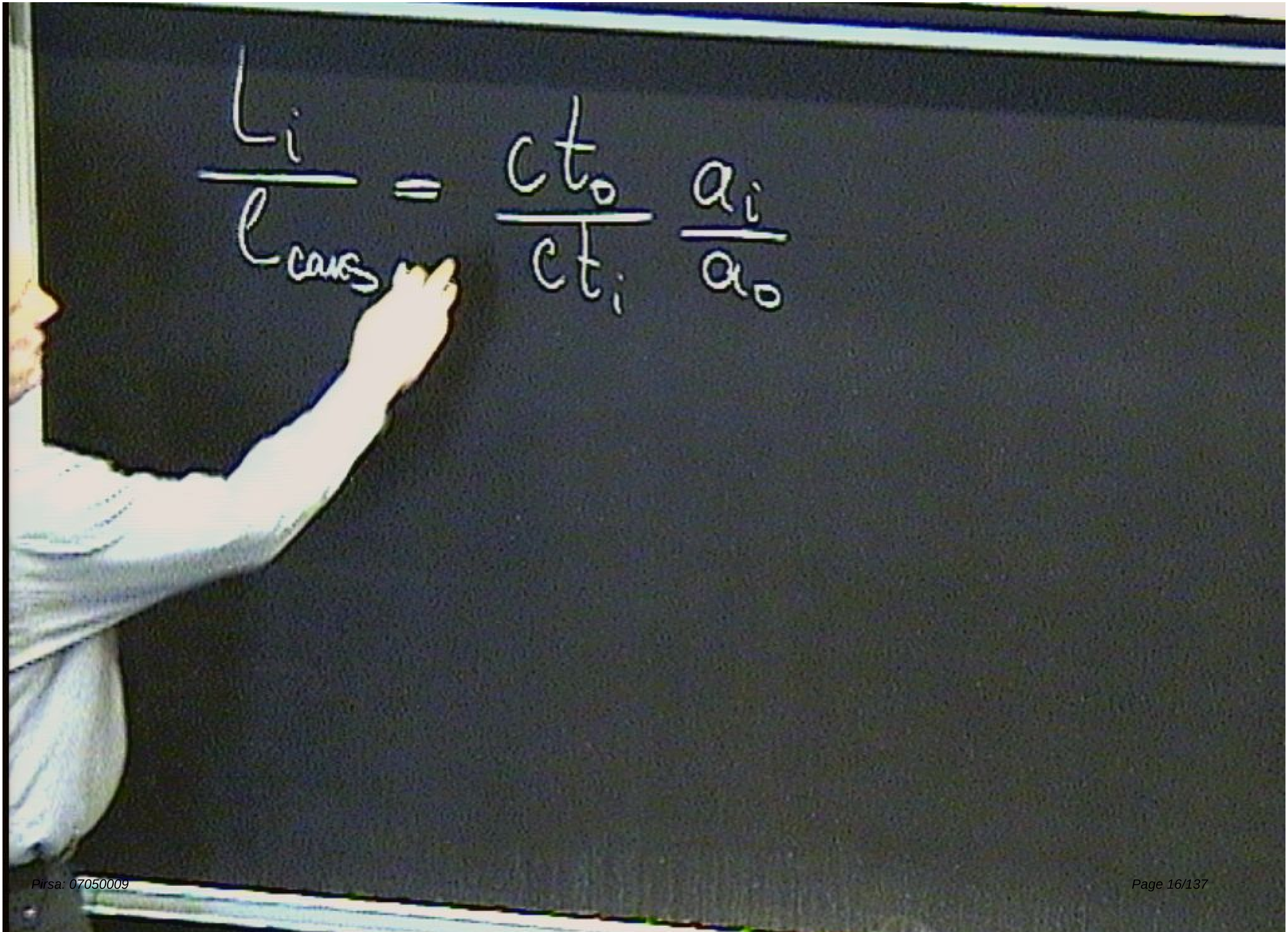
$ct_0 \sim 10^{28}\,\text{cm}$

$t_i \sim 10^{-45}\,\text{sec}$

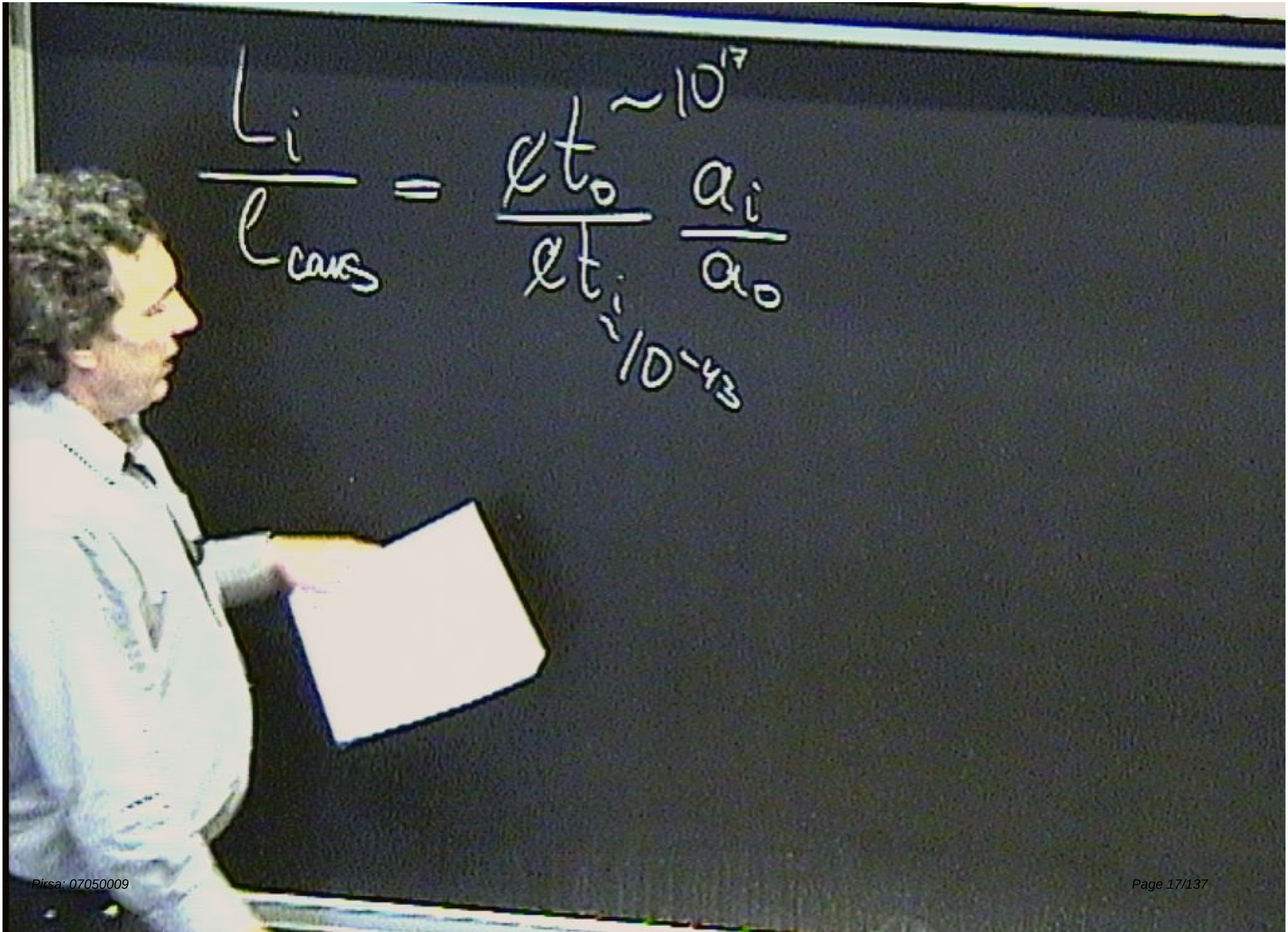
$$L_i = ct_0 \frac{a_i}{a_0}$$


Inflation.
$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-5}$
$t \sim 10^{17}$ sec
$ct_0 \sim 10^{28}$
$t_i \sim 10^{-43}$ sec
$L_i = ct_0 \frac{a_i}{a_0}$


Inflation
δε/ε ~ 10^-5
t ~ 10^17 sec
ct₀ ~ 10^28 cm
t_i ~ 10^-43 sec
L_i = ct₀ a_i/a₀
l_caus ~ ct_i

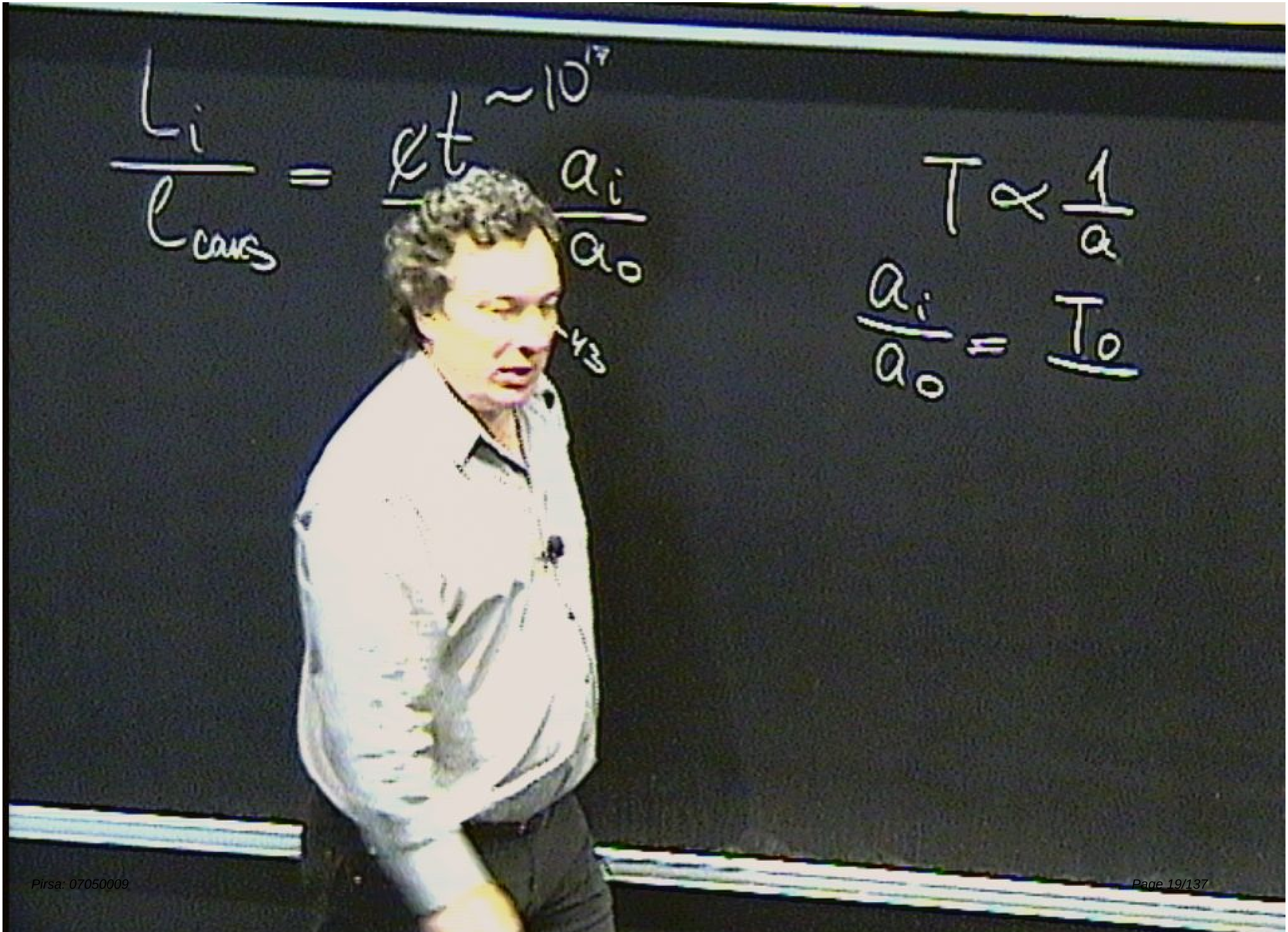


$$\frac{L_i}{\ell_{cons}} = \frac{ct_0}{ct_i}\,\frac{a_i}{a_0}$$


$\frac{L_i}{\ell_{caus}} = \frac{\overset{\sim 10^{17}}{\ell t_0}}{\underset{\sim 10^{-43}}{\ell t_i}} \frac{a_i}{a_0}$

$$\frac{L_i}{\ell_{cons}} = \frac{\ell \overset{\sim 10^{17}}{t_0}}{\ell \underset{\sim 10^{-43}}{t_i}} \frac{a_i}{a_0}$$

$$T \propto \frac{1}{a}$$



$\frac{L_i}{\ell_{cons}} = ct \sim 10^{17} \frac{a_i}{a_0}$
$T \propto \frac{1}{a}$
$\frac{a_i}{a_0} = \frac{T_0}{\ }$

$$\frac{L_i}{\ell_{cons}} = \frac{\ell t_0^{\sim 10^{17}}}{\ell t_i^{\sim 10^{-43}}} \frac{a_i}{a_0}$$

$$T \propto \frac{1}{a}$$

$$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3^\circ K}{}$$

$$\frac{L_i}{\ell_{\text{curv}}} = \frac{\ell t_0^{\sim 10^{17}}}{\ell t_i \, _{\sim 10^{-43}}} \frac{a_i}{a_0}$$

$$T \propto \frac{1}{a}$$

$$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \underline{\underline{3^\circ K}}$$

$$\frac{L_i}{\ell_{cons}} = \frac{\ell t_0^{\sim 10^{17}}}{\ell t_{i\,\sim 10^{-43}}} \frac{a_i}{a_0} \sim 10^{28} \qquad T \propto \frac{1}{a}$$

$$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3^\circ K}{10^{32}} \sim 10^{-32}$$

m

$$t_i \sim 10^{-43} \text{ sec}$$

$$10^{84} \quad L_i = c t_0 \frac{a_i}{a_0}$$

$$l_{caus} \sim c t_i$$

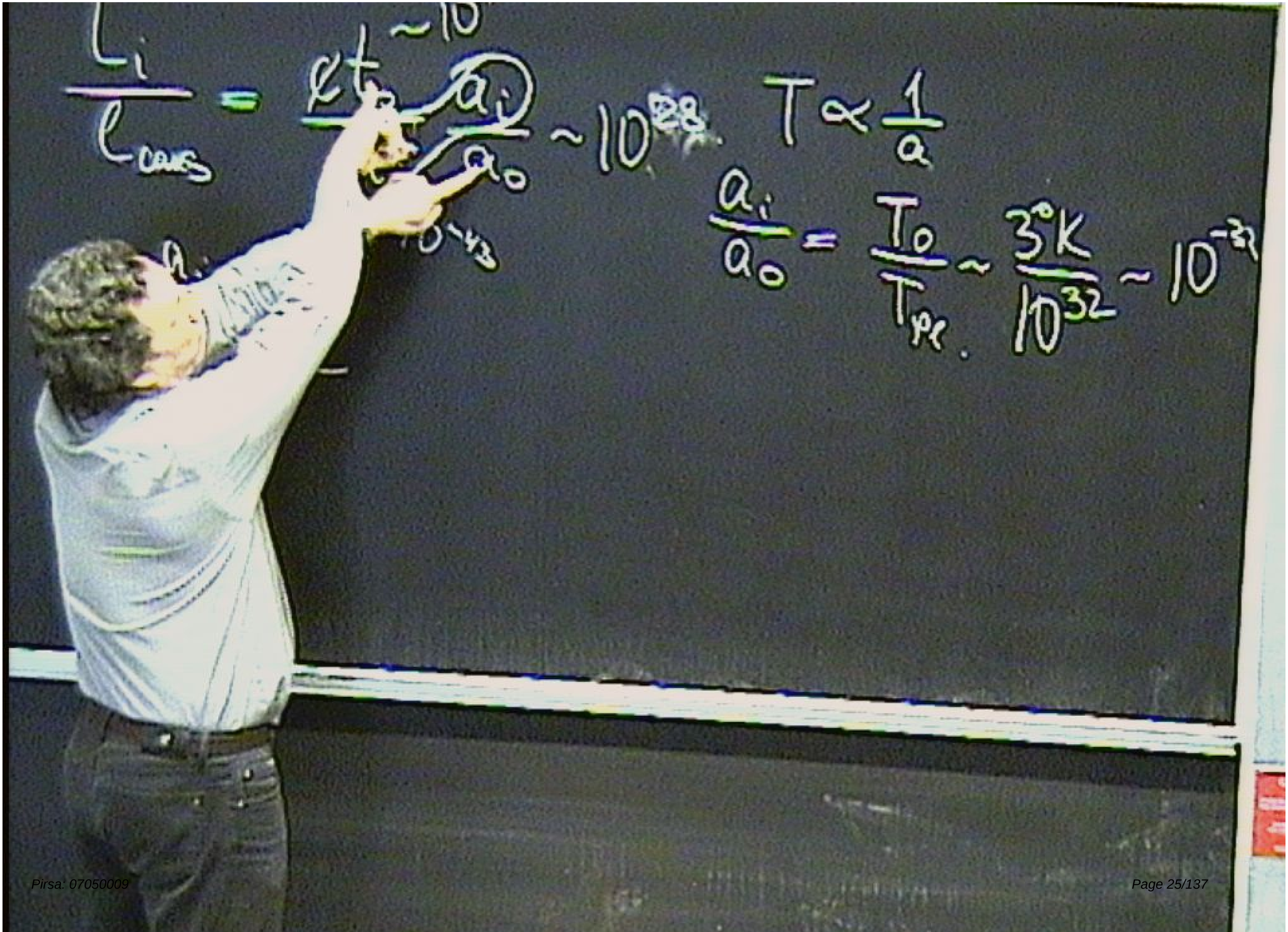
$$\frac{L_i}{\ell_{\text{caus}}} = \frac{c t_0^{\sim 10^{17}}}{c t_i} \frac{a_i}{a_0} \sim 10^{28}$$

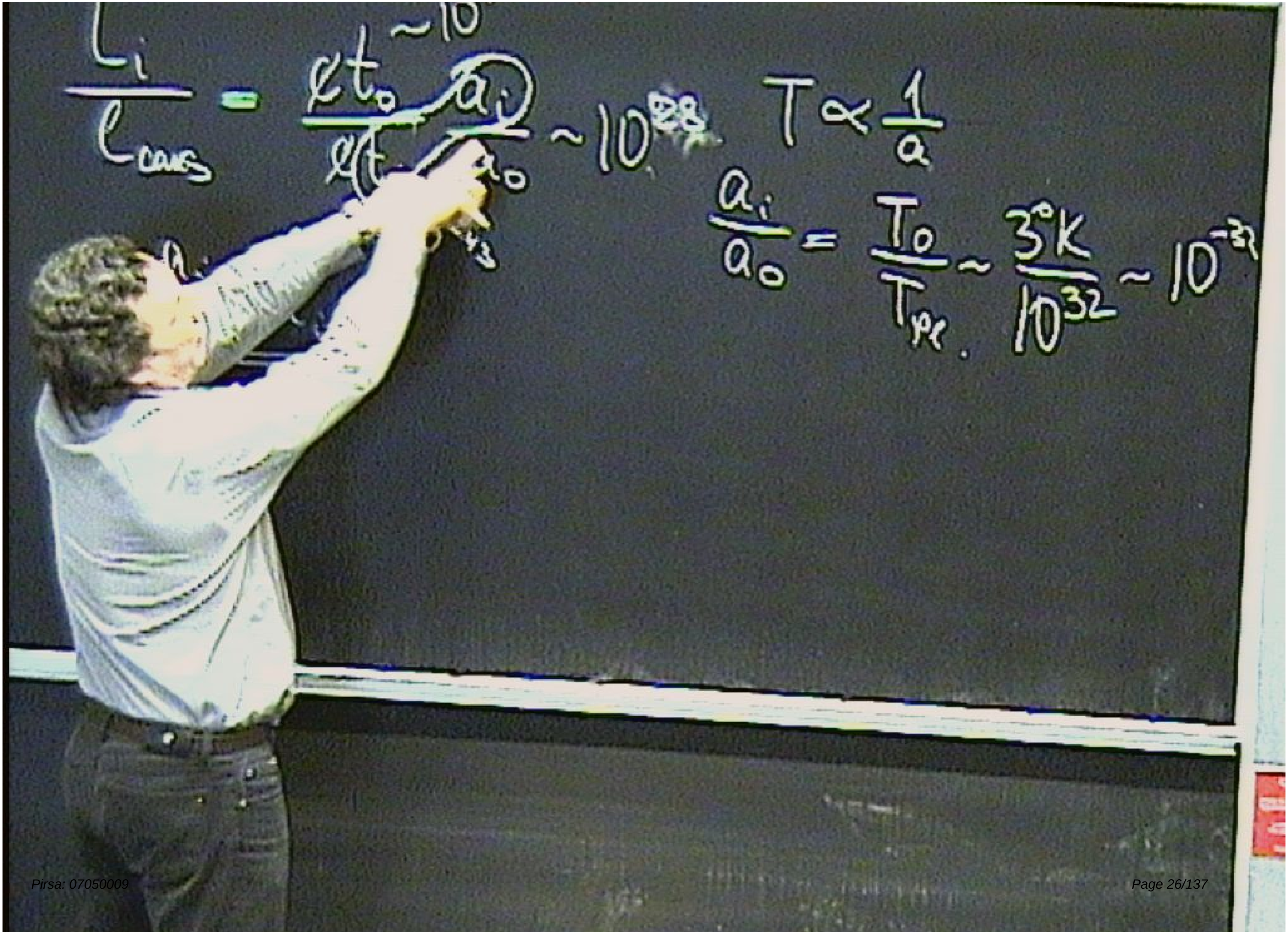
$$t_i / 10^{-43}$$

$$\frac{a_i}{t_i} \sim \dot{a}_i$$

$$T \propto \frac{1}{a}$$

$$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim$$


$\frac{l_i}{l_{cons}} = \frac{ct_i\, a_i}{a_0} \sim 10^{28}$
$T \propto \frac{1}{a}$
$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3^\circ K}{10^{32}} \sim 10^{-32}$


$T \propto \frac{1}{a}$
$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3°K}{10^{32}} \sim 10^{-32}$

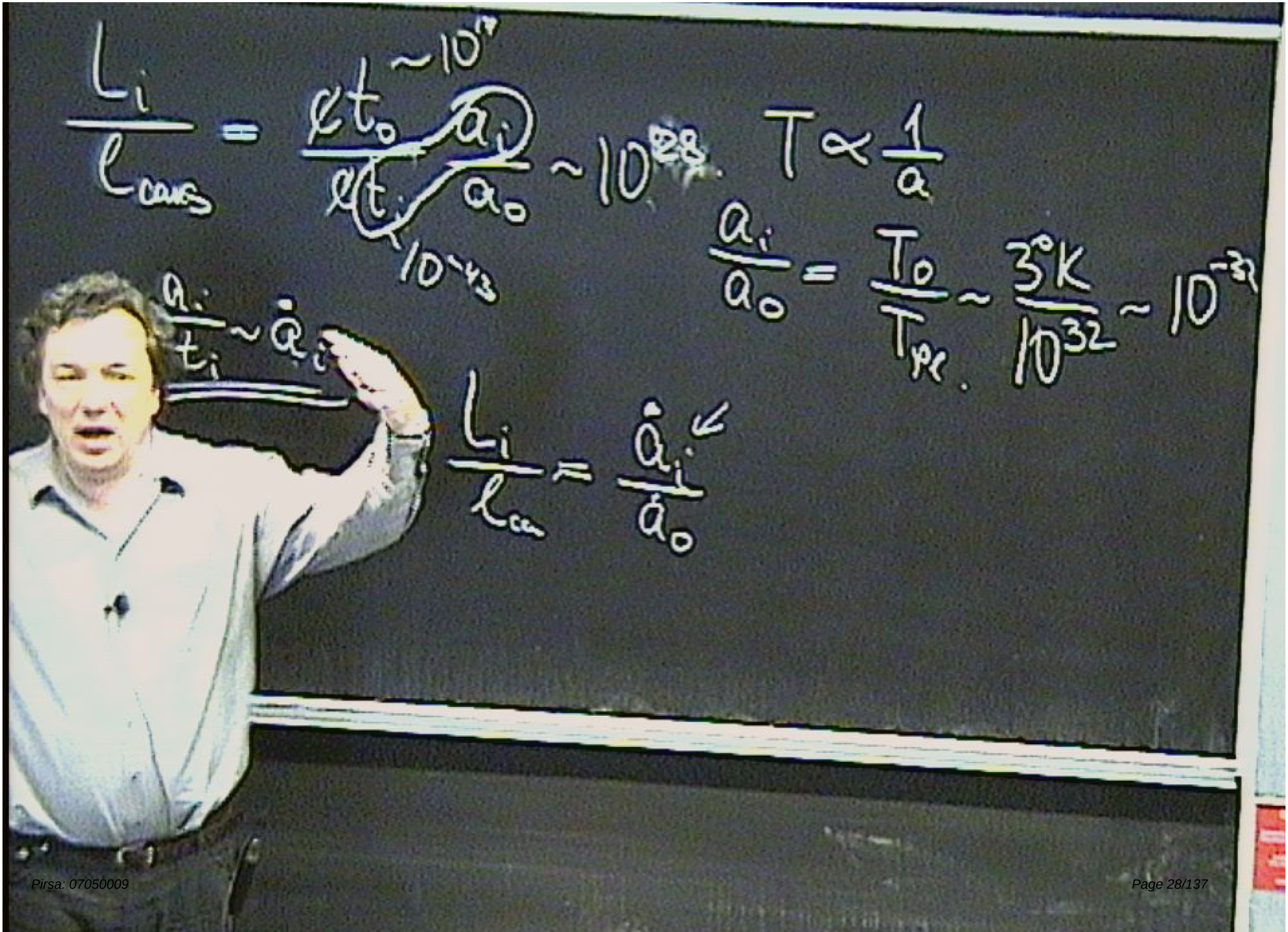
$$\frac{L_i}{\ell_{cons}} = \frac{\ell t_0 \, a_i}{\ell t_i \, a_0} \sim 10^{28}$$

$$t_0 \sim 10^{17} \qquad t_i \sim 10^{-43}$$

$$T \propto \frac{1}{a}$$

$$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3^\circ K}{10^{32}} \sim 10^{-32}$$

$$\frac{a_i}{t_i} \sim \dot{a}_i$$


$\frac{L_i}{l_{cons}} = \frac{c t_0 \, a_i}{c t_i \, a_0} \sim 10^{28}$
$\sim 10^{17}$
10^{-43}
$T \propto \frac{1}{a}$
$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3°K}{10^{32}} \sim 10^{-32}$
$\frac{a_i}{t_i} \sim \dot{a}_i$
$\frac{L_i}{l_{con}} = \frac{\dot{a}_i}{a_0}$

$$\frac{L_i}{\ell_{cons}} = \frac{\ell\, \overset{\sim 10^{17}}{t_0}\, a_i}{\ell\, \underset{10^{-43}}{t_i}\, a_0} \sim 10^{28} \qquad T \propto \frac{1}{a}$$

$$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3^\circ K}{10^{32}} \sim 10^{-32}$$

$$\underline{\underline{\frac{a_i}{t_i} \sim \dot{a}_i}}$$

$$\frac{L_i}{\ell_{co}} \approx \frac{\dot{a}_i}{a_0} > 1$$

$$\frac{L_i}{\ell_{cons}} = \frac{\ell t_0}{\ell t_i}\frac{a_i}{a_0} \sim 10^{28}$$

$t_0 \sim 10^{17}$, $t_i \sim 10^{-43}$

$$T \propto \frac{1}{a}$$

$$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3^\circ K}{10^{32}} \sim 10^{-32}$$

$$\underline{\underline{\frac{a_i}{t_i} \sim \dot{a}_i}}$$

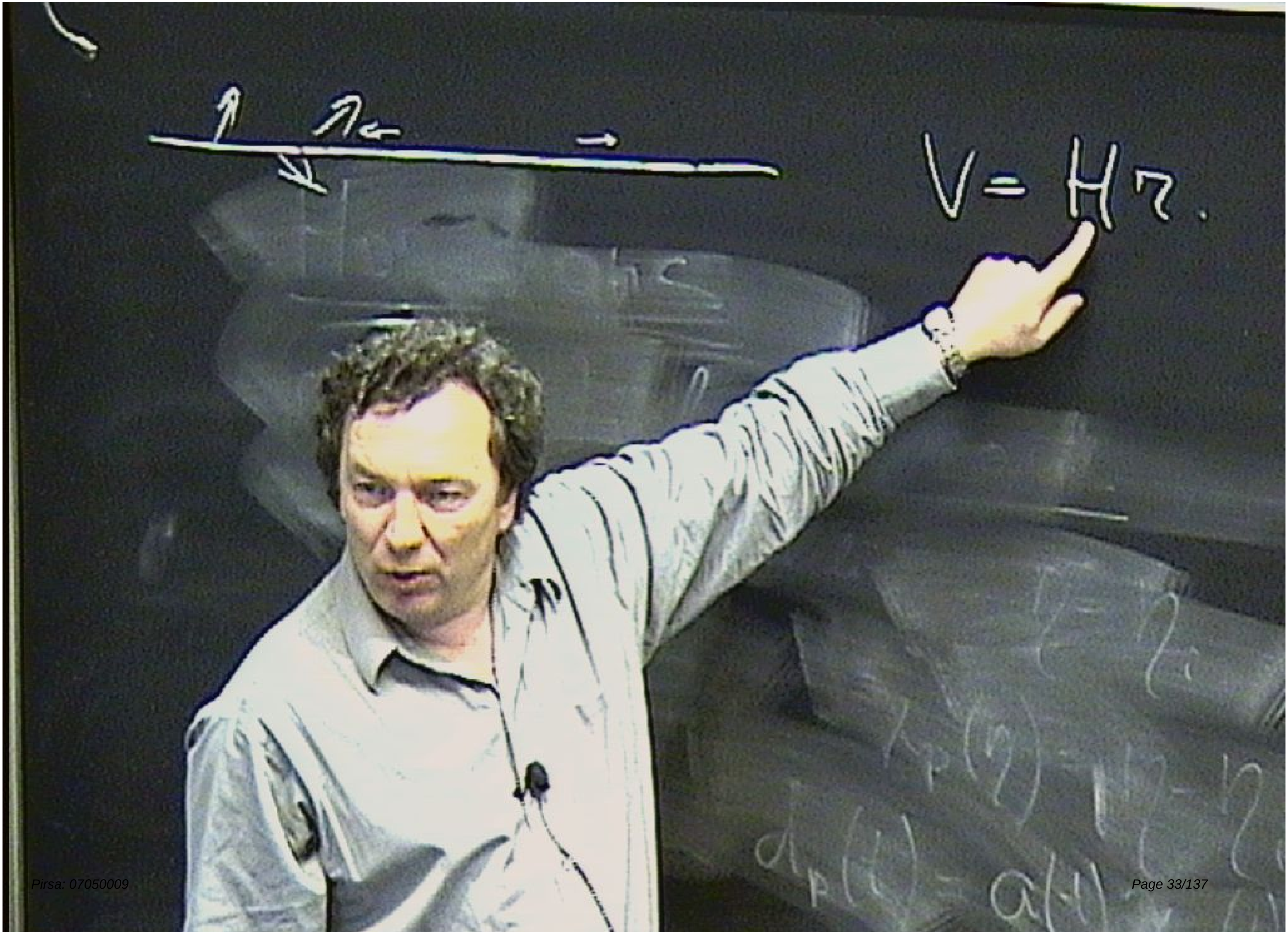
$$\frac{L_i}{\ell_{con}} \approx \frac{\dot{a}_i}{\dot{a}_0} > 1$$

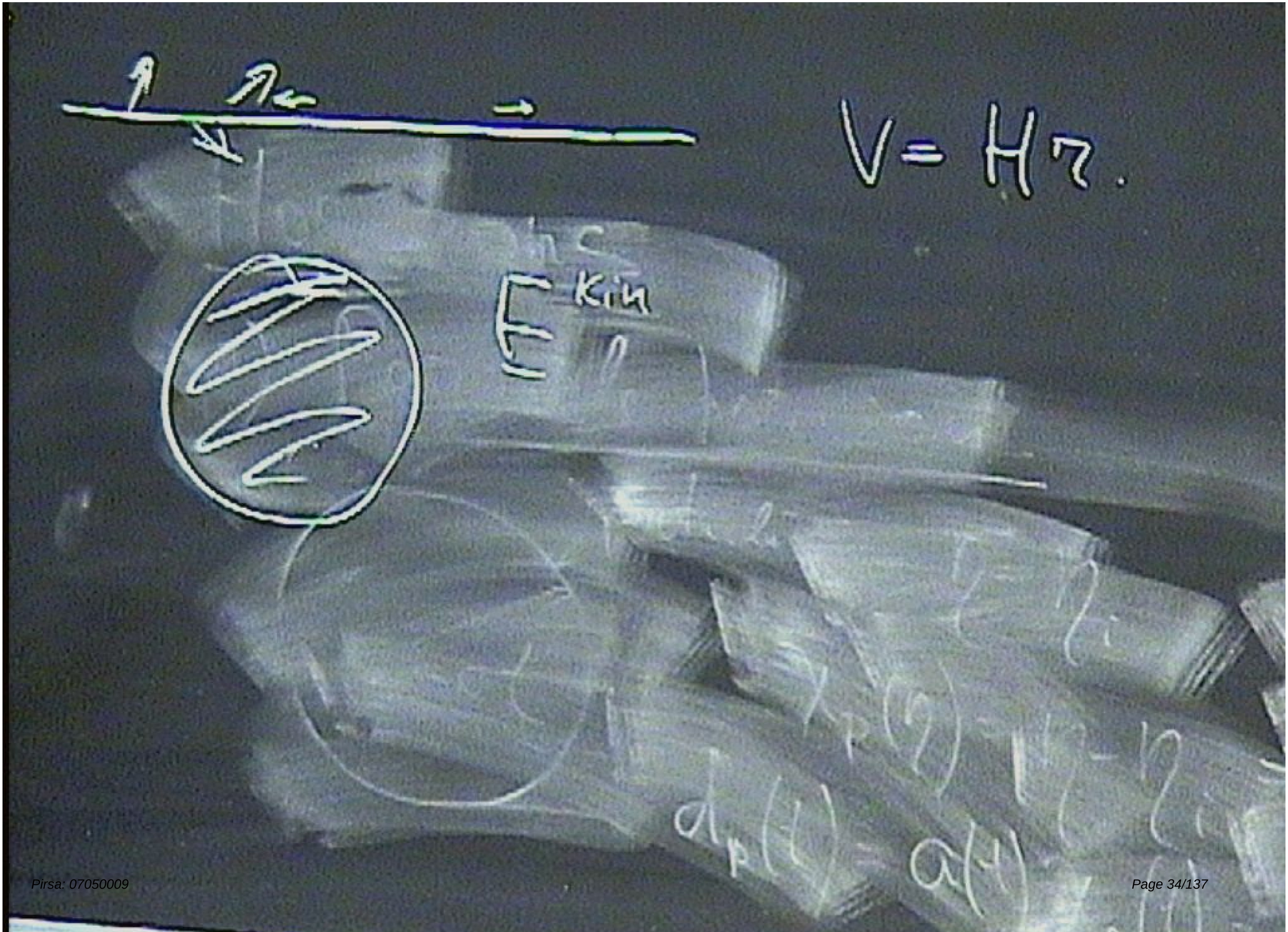
$$\frac{L_i}{\ell_{caus}} = \frac{c\,t_0^{\sim 10^{17}}\,a_i}{c\,\underbrace{t_i}_{10^{-43}}\,a_0} \sim 10^{28} \qquad T \propto \frac{1}{a}$$

$$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3^\circ K}{10^{32}} \sim 10^{-31}$$

$$\frac{a_i}{t_i} \sim \dot{a}_i$$

$$\frac{L_i}{\ell_{ca}} = \frac{\dot{a}_i}{\dot{a}_0} > 1$$

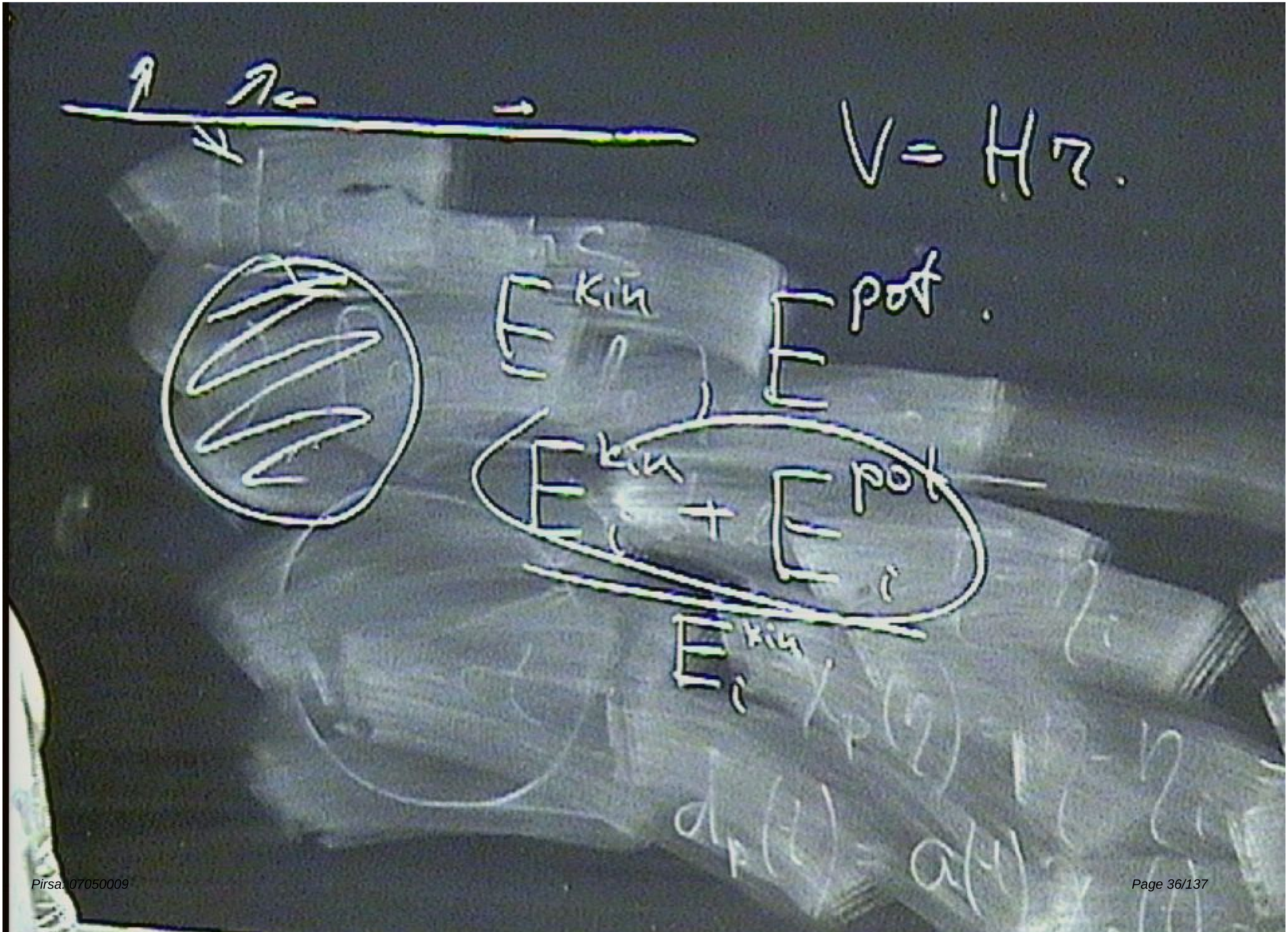

V = Hr.

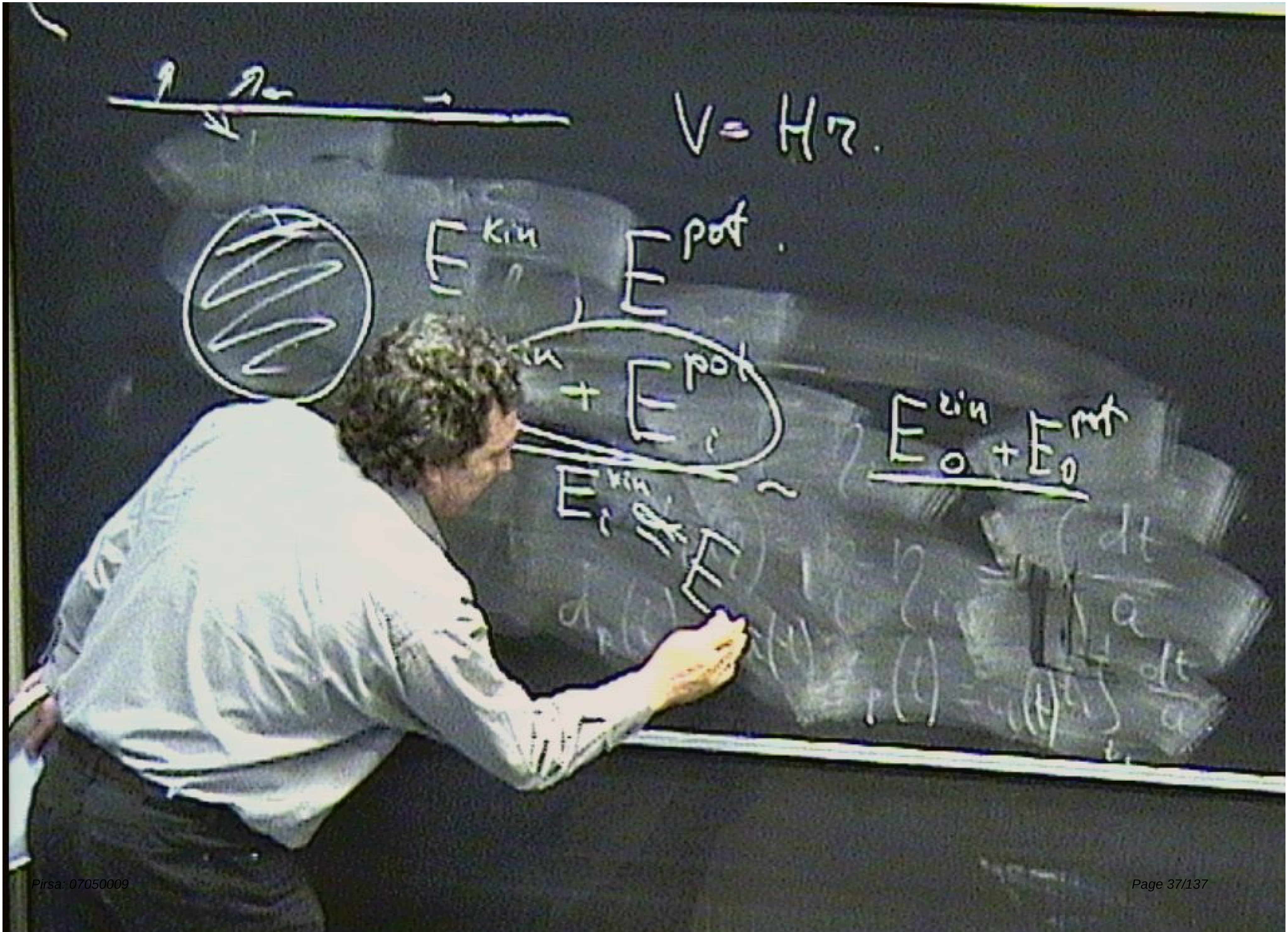

V = Hr.
E kin

$$V = Hr.$$

$$E_i^{kin},\ E_i^{pot}.$$

$$\frac{E_i^{kin} + E_i^{pot}}{E_i^{kin}}$$


$V = Hr.$
E^{kin}
$E^{pot}.$
$E_i^{kin} + E_i^{pot}$
E_i^{kin}


V = Hr.
E^{kin}
E^{pot}
$+ E_i^{pot}$
E_i^{kin}
$E_0^{kin} + E_0^{pot}$

$V = Hr.$

$E^{kin}_i \,,\; E^{pot}.$

$E^{kin}_i + E^{pot}_i \sim E^{kin}_0 + E^{pot}_0$

$E^{kin}_i \approx E^{kin}_0 \left(\frac{a_i}{a_0}\right)^2$

$V = Hr.$

$E^{kin}, E^{pot}.$

$E_i^{kin} + E_i^{pot}$

$E_i^{kin} \sim E_0^{kin}\left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2$

$\frac{E_0^{kin} + E_0^{pot}}{E_0^{kin}} \left(\dot{a}_i\right)^2 \ldots (a_i)$

$V = Hr.$

$E^{kin}, \; E^{pot}.$

$$\frac{E_i^{kin} + E_i^{pot}}{E_i^{kin}} \sim \frac{E_0^{kin} + E_0^{pot}}{E_0^{kin}} \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2$$

$$E_i^{kin} \sim E_0^{kin} \left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2$$

$$V = Hr$$

$$E^{kin} \qquad E^{pot}$$

$$E_i^{kin} + E_i^{pot} \sim \left(\frac{E_0^{kin} + E_0^{pot}}{E_0^{kin}}\right)\left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2$$

$$O(1)$$

$$E_i^{kin} \approx E_0^{kin}\left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2$$

$V = Hr.$

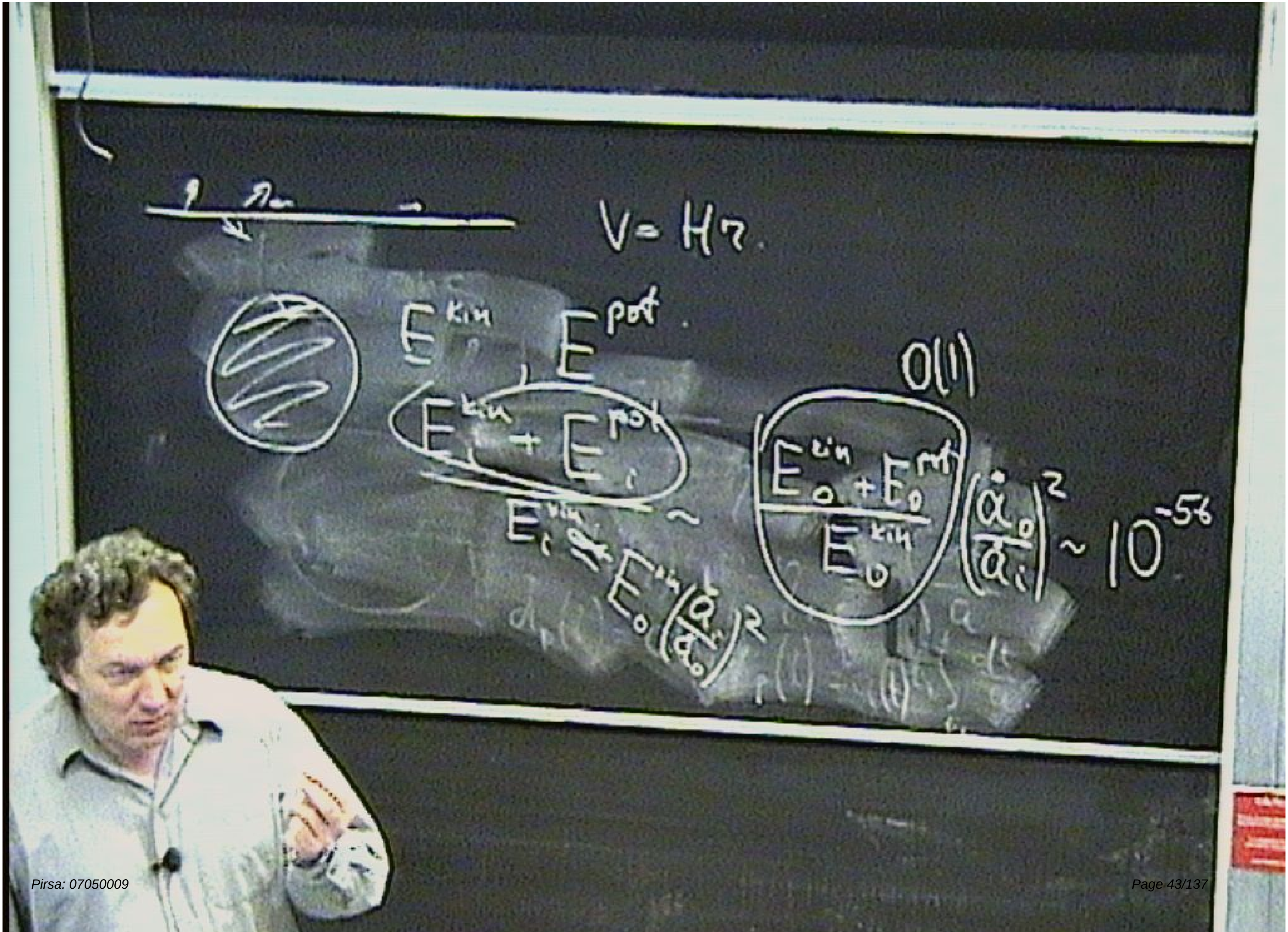
$E^{kin}, E^{pot}.$

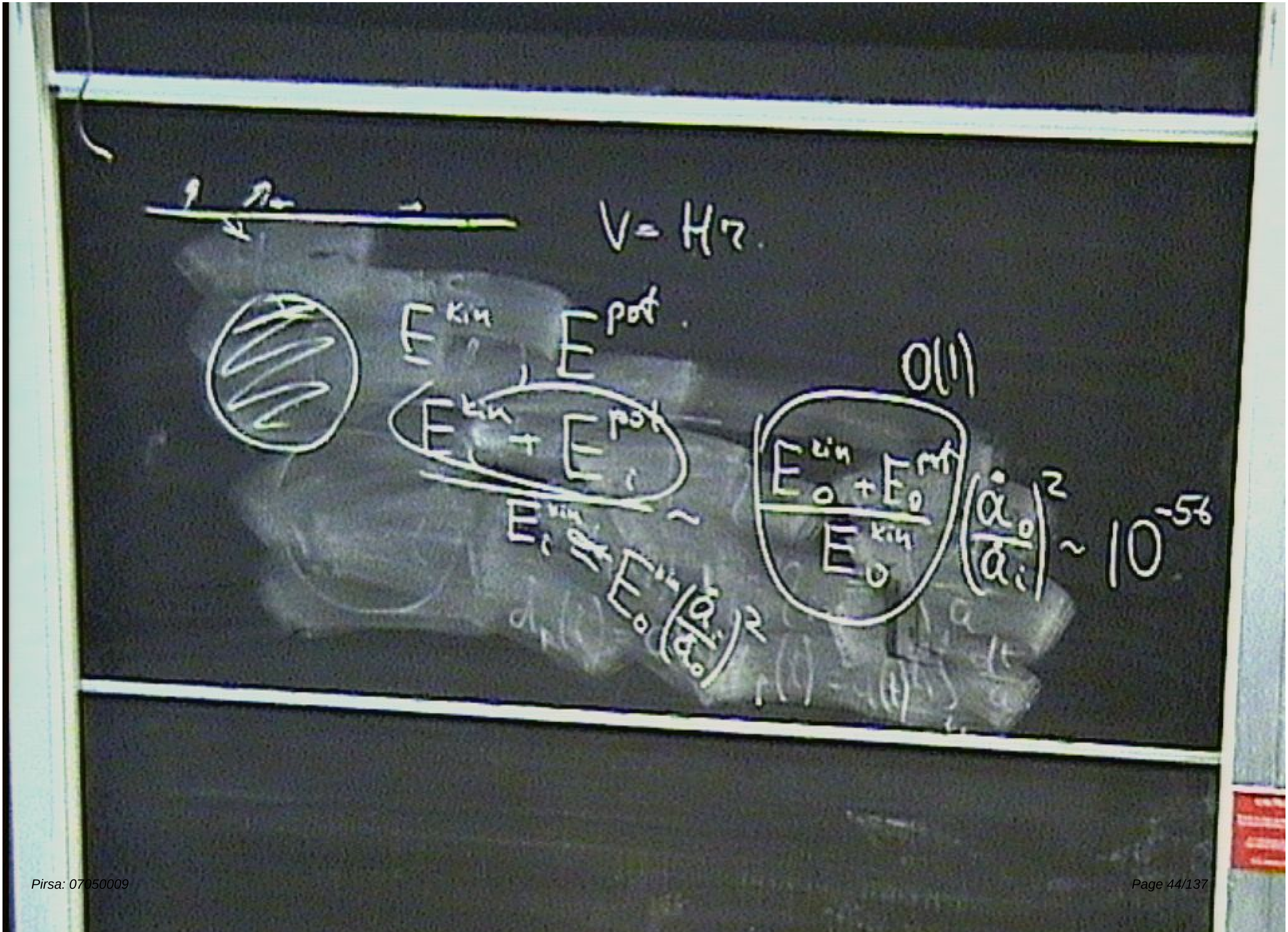
$O(1)$

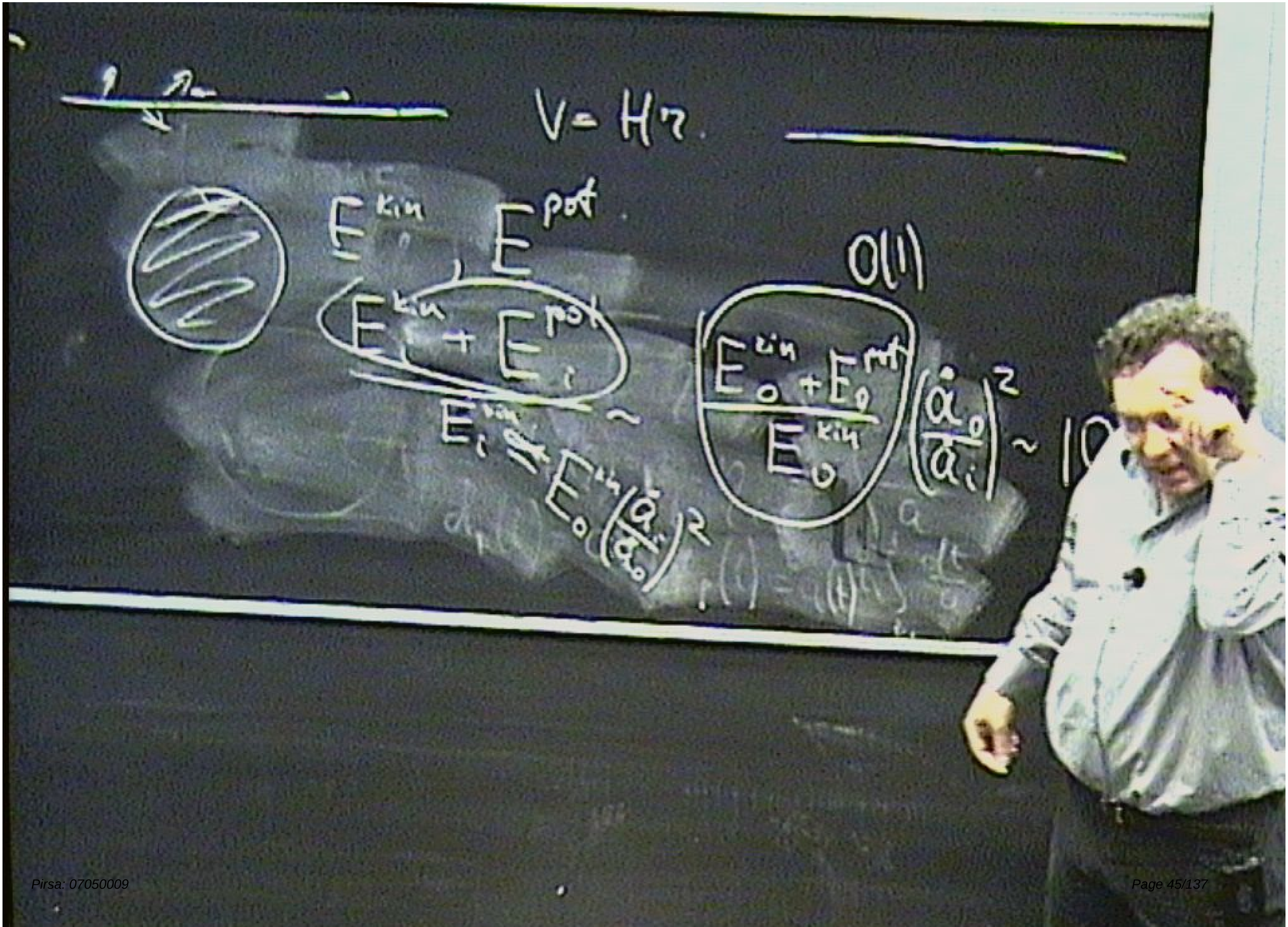
$E_i^{kin} + E_i^{pot}$

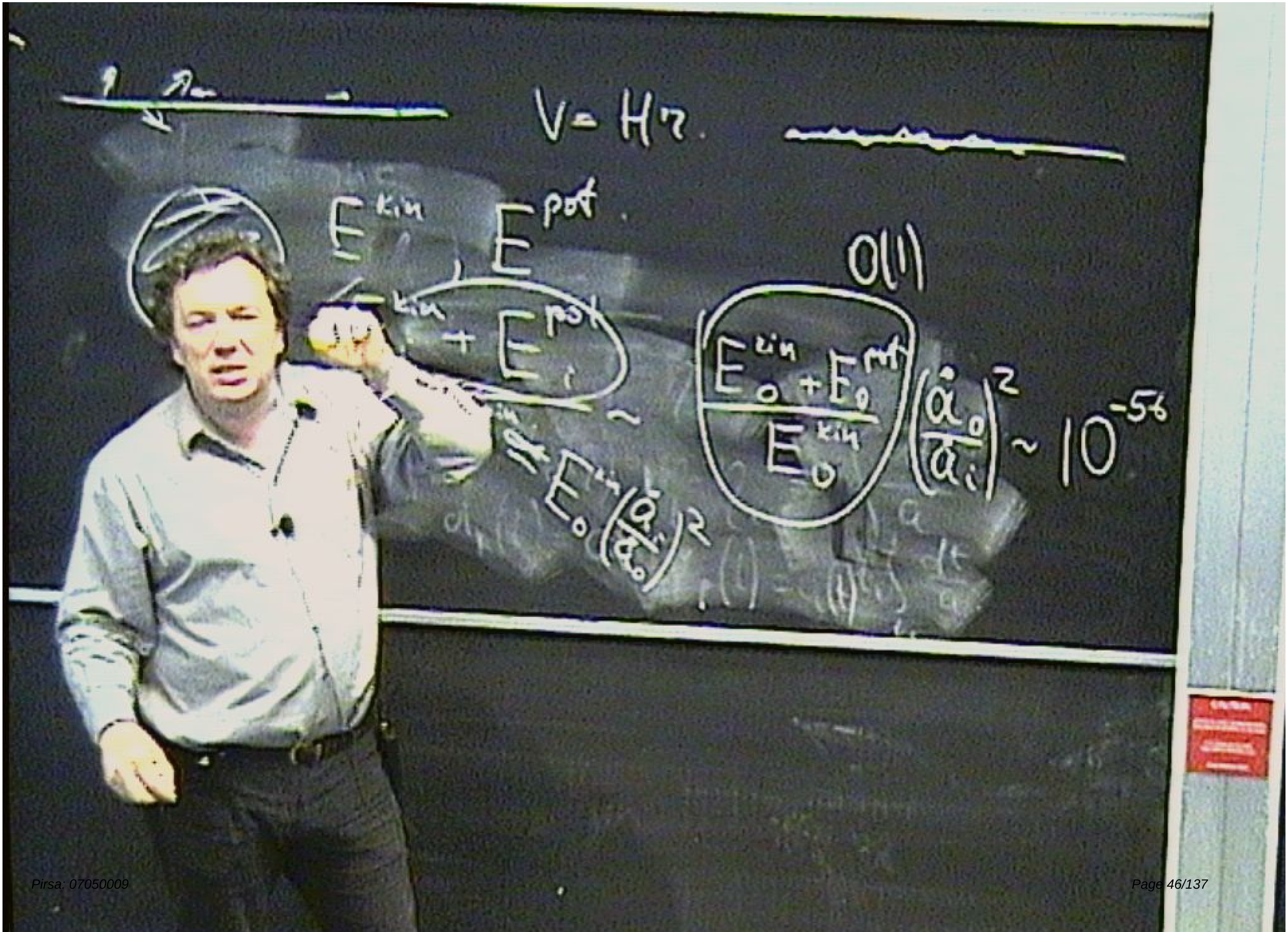
$E_i^{kin} \sim E_0^{kin} \left(\frac{\dot a_i}{\dot a_0}\right)^2 \sim$

$\frac{E_0^{kin} + E_0^{pot}}{E_0^{kin}} \left(\frac{\dot a_0}{\dot a_i}\right)^2 \sim$


V = Hr
E^kin, E^pot
O(1)
E_i^kin + E_i^pot
E_i^kin ~ E_0^kin (ȧ_0/ȧ_i)^2
(E_0^kin + E_0^pot) / E_0^kin (ȧ_0/ȧ_i)^2 ~ 10^-56


V = Hr
E^kin, E^pot
O(1)
(E_i^kin + E_i^pot)/E_i^kin ~ (E_0^kin + E_0^pot)/E_0^kin (ȧ_0/ȧ_i)^2 ~ 10^-56
E_i^kin = E_0^kin (ȧ_i/ȧ_0)^2


V = Hr
E^kin E^pot
O(1)


V = Hr
E^kin E^pot
O(1)
E_i^kin + E_i^pot ~ E_0^kin (ȧ_i/ȧ_0)^2
(E_0^kin + E_0^pot)/E_0^kin (ȧ_0/ȧ_i)^2 ~ 10^-56

$$V = Hr$$

$$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-5}$$

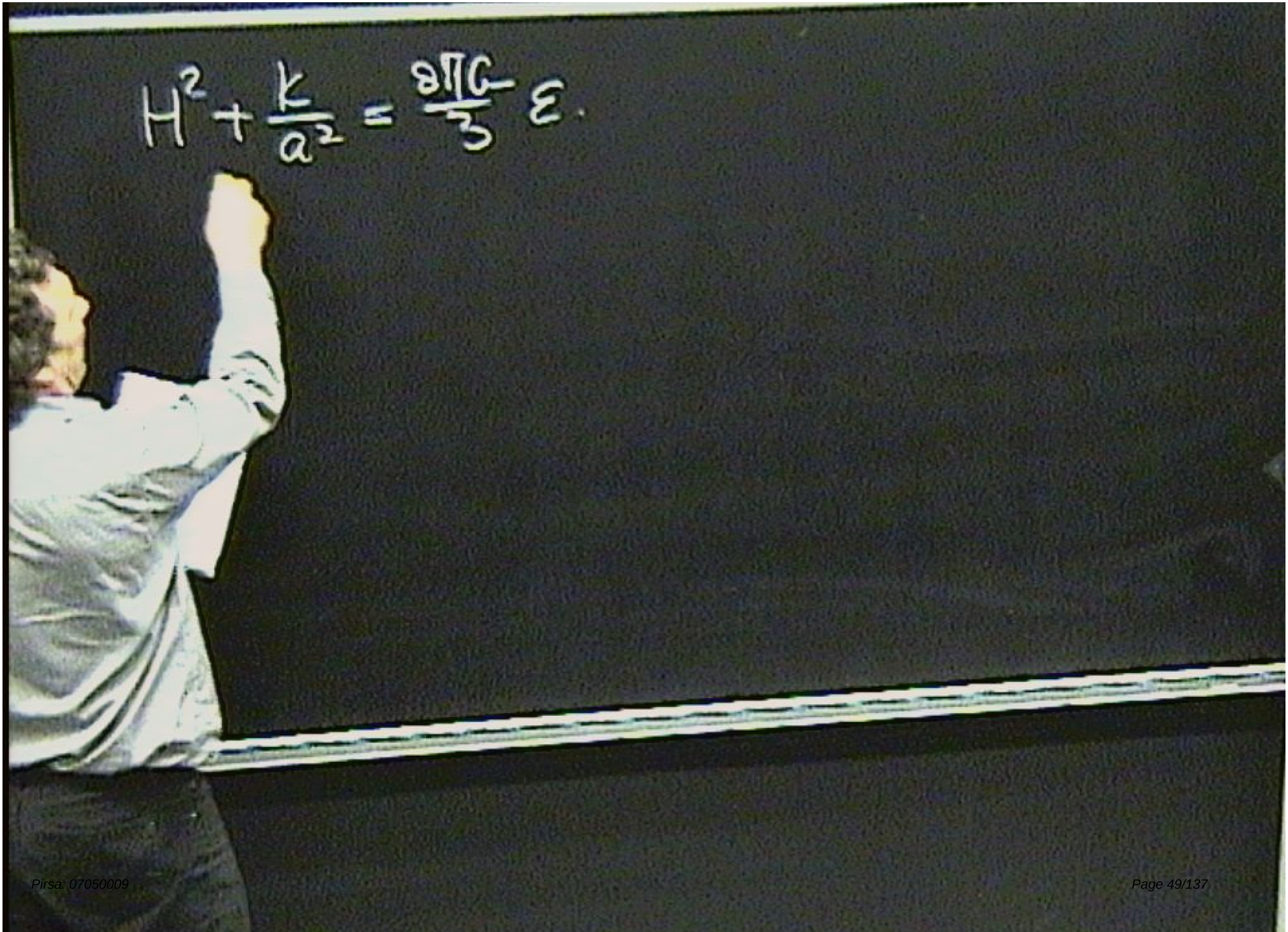
$$O(1)$$

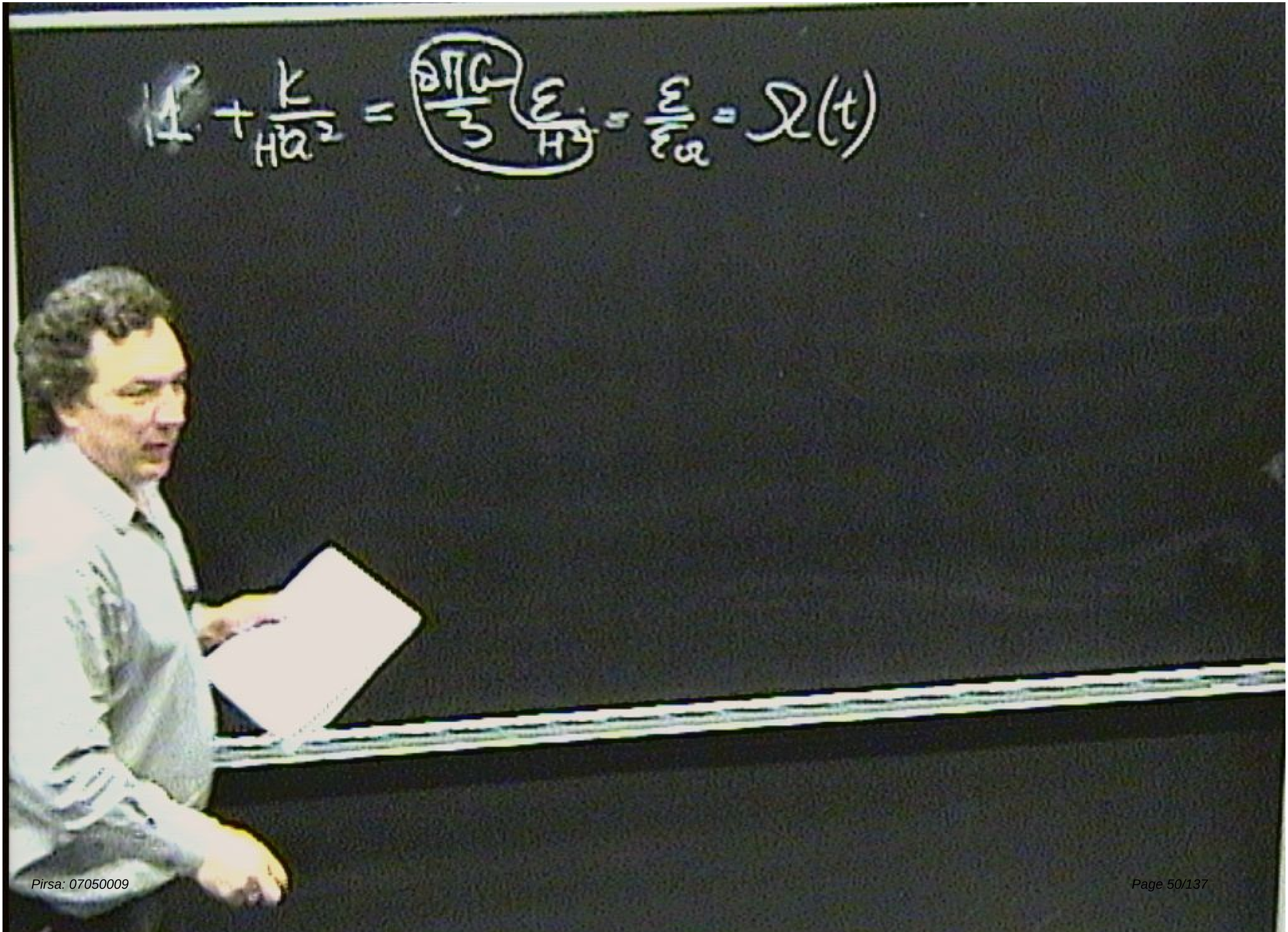
$$E^{kin}, \; E^{pot}$$

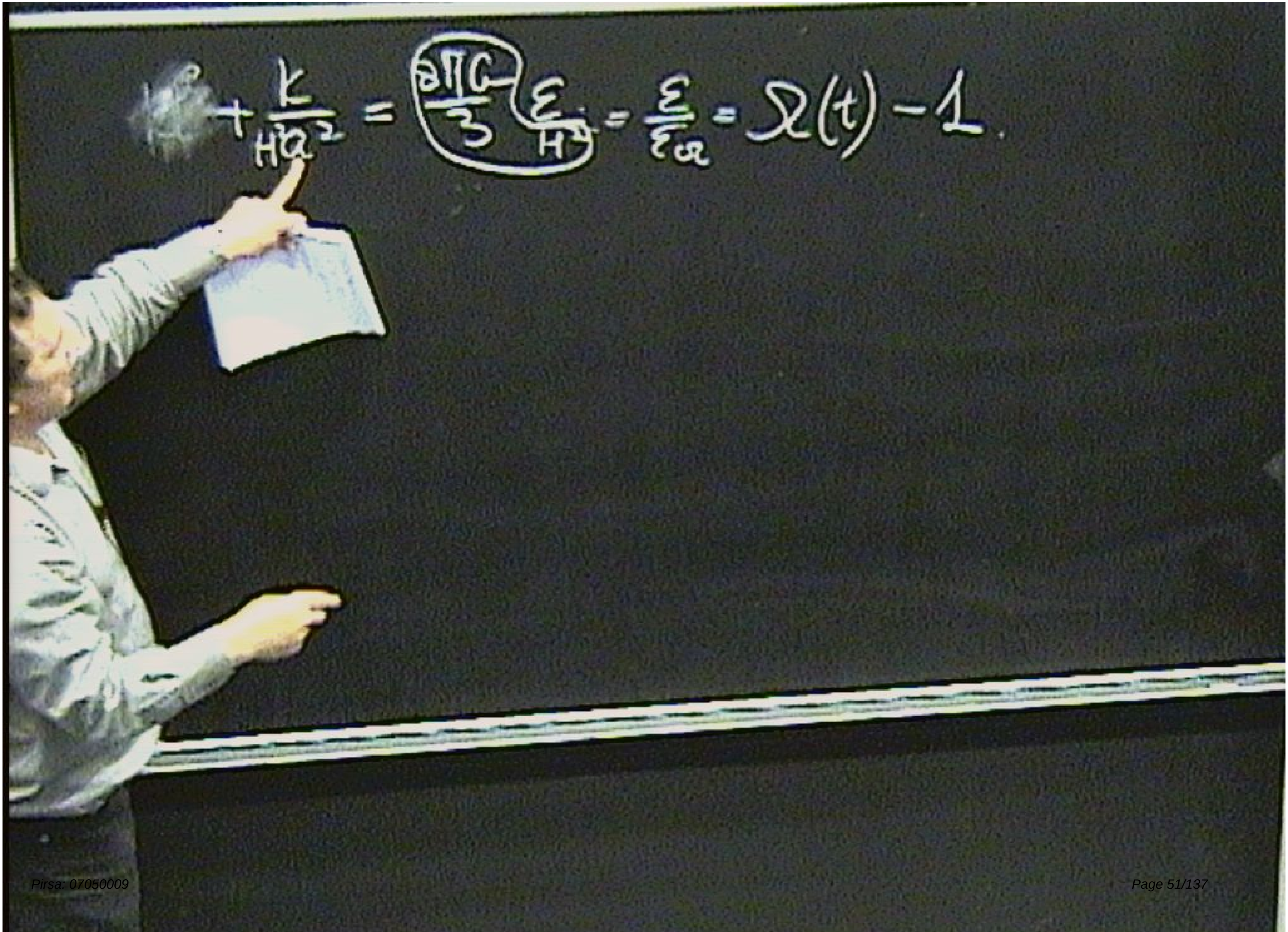
$$\frac{E_i^{kin} + E_i^{pot}}{E_i^{kin}} \sim \frac{E_0^{kin} + E_0^{pot}}{E_0^{kin}} \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 \sim 10^{-56}$$

$$E_i^{kin} \simeq E_0^{kin} \left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2$$

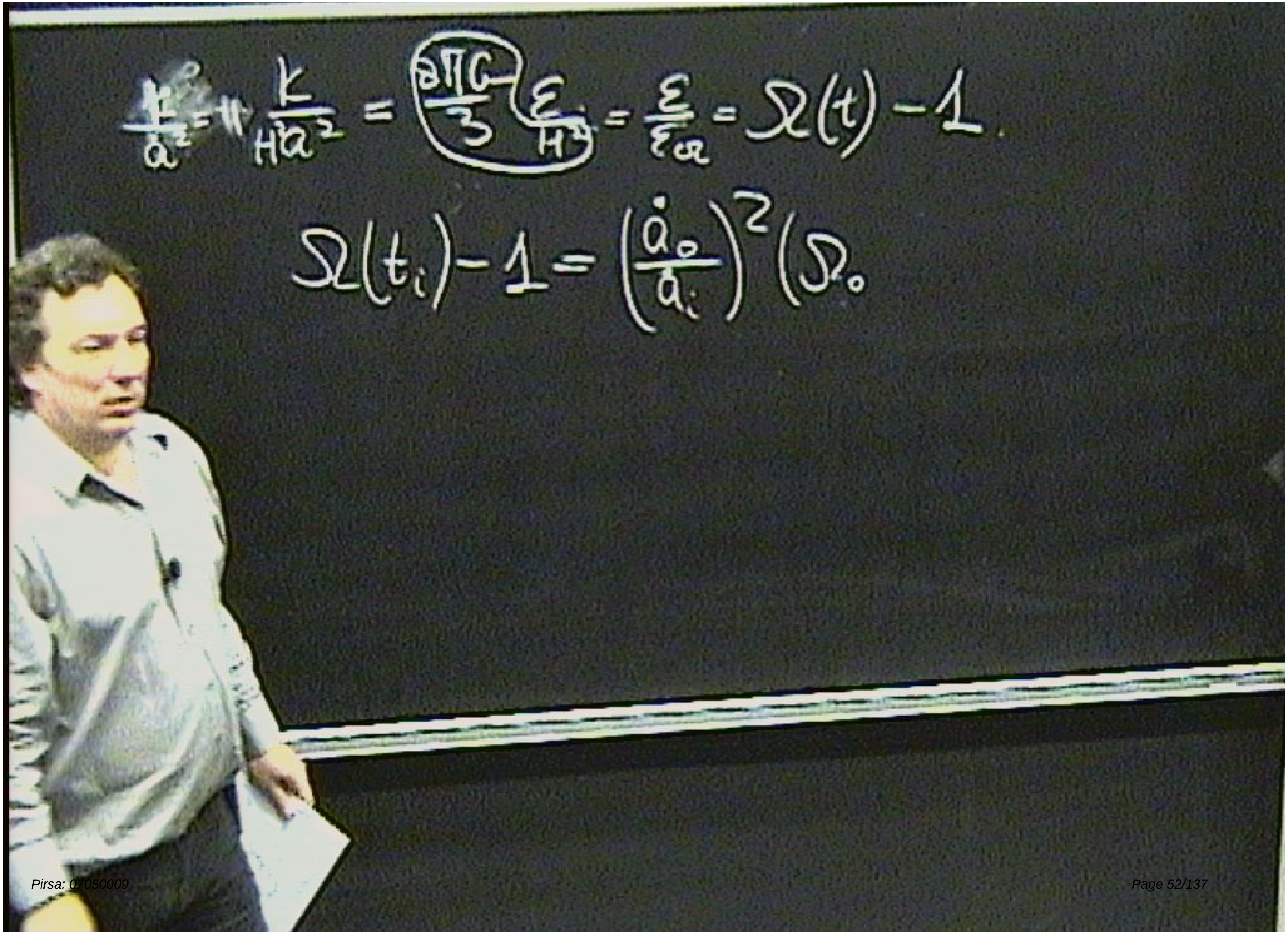
$$H^2 + \frac{k}{a^2}$$

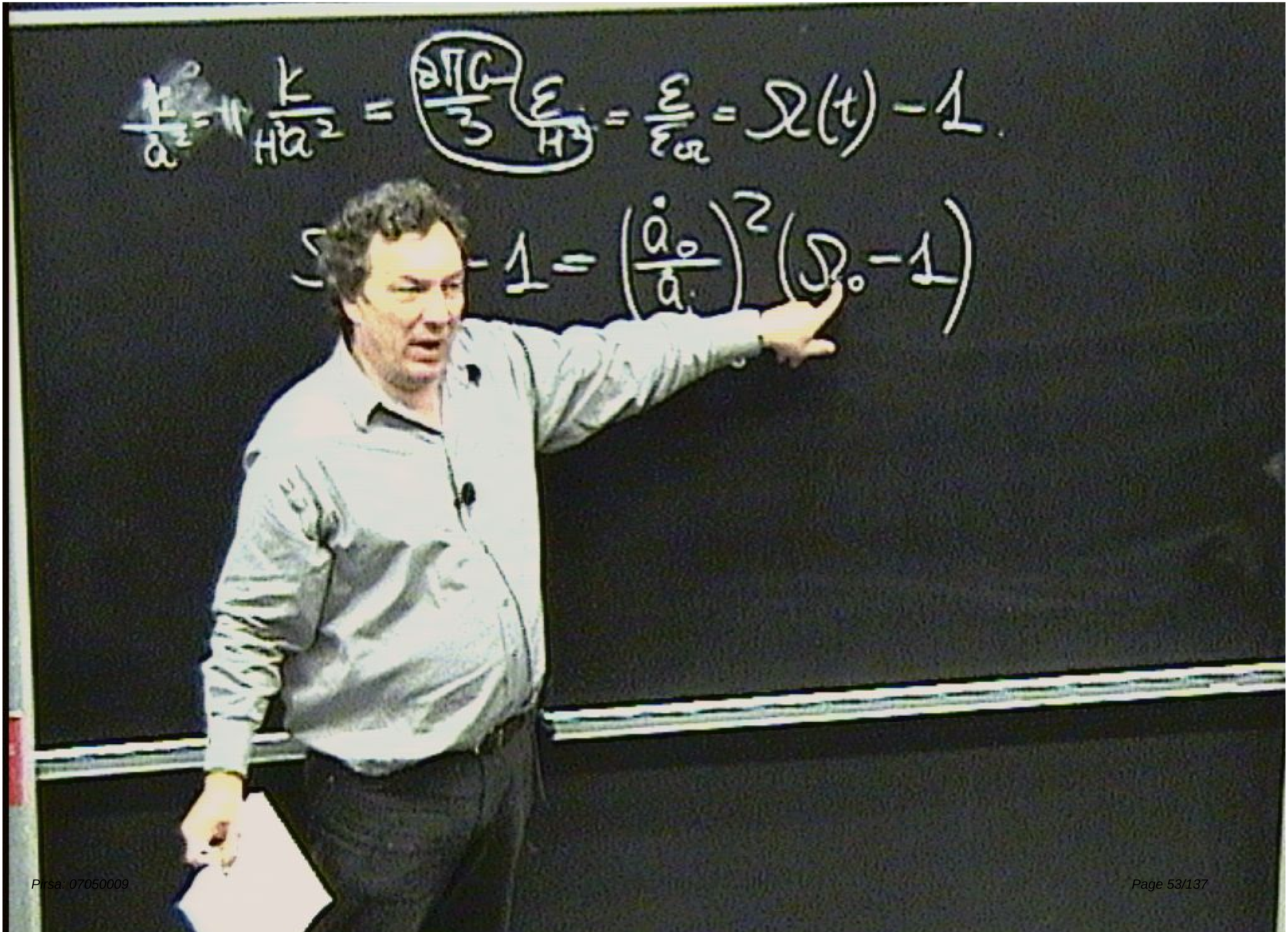

$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\varepsilon.$


$1+\frac{k}{Ha^2}=\frac{8\pi G}{3}\frac{\varepsilon}{H^2}=\frac{\varepsilon}{\varepsilon_c}=\Omega(t)$



+ k/(H²a²) = 8πG/3 · ε/H² = ε/ε_cr = Ω(t) − 1


$\frac{k}{\dot a^2} = \frac{k}{H^2 a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} - \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$
$\Omega(t_i) - 1 = \left(\frac{\dot a_0}{\dot a_i}\right)^2 (\Omega_0$


$\frac{k}{Ha^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$
$-1 = \left(\frac{\dot{a}_0}{\dot{a}}\right)^2 (\Omega_0 - 1)$

$$\frac{k}{Ha^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$$

$$\Omega(t_i) - 1 = \underset{10^{-5}}{\left(\frac{\dot{a}_0}{a_i}\right)^2}\left(\Omega_0 - 1\right)$$

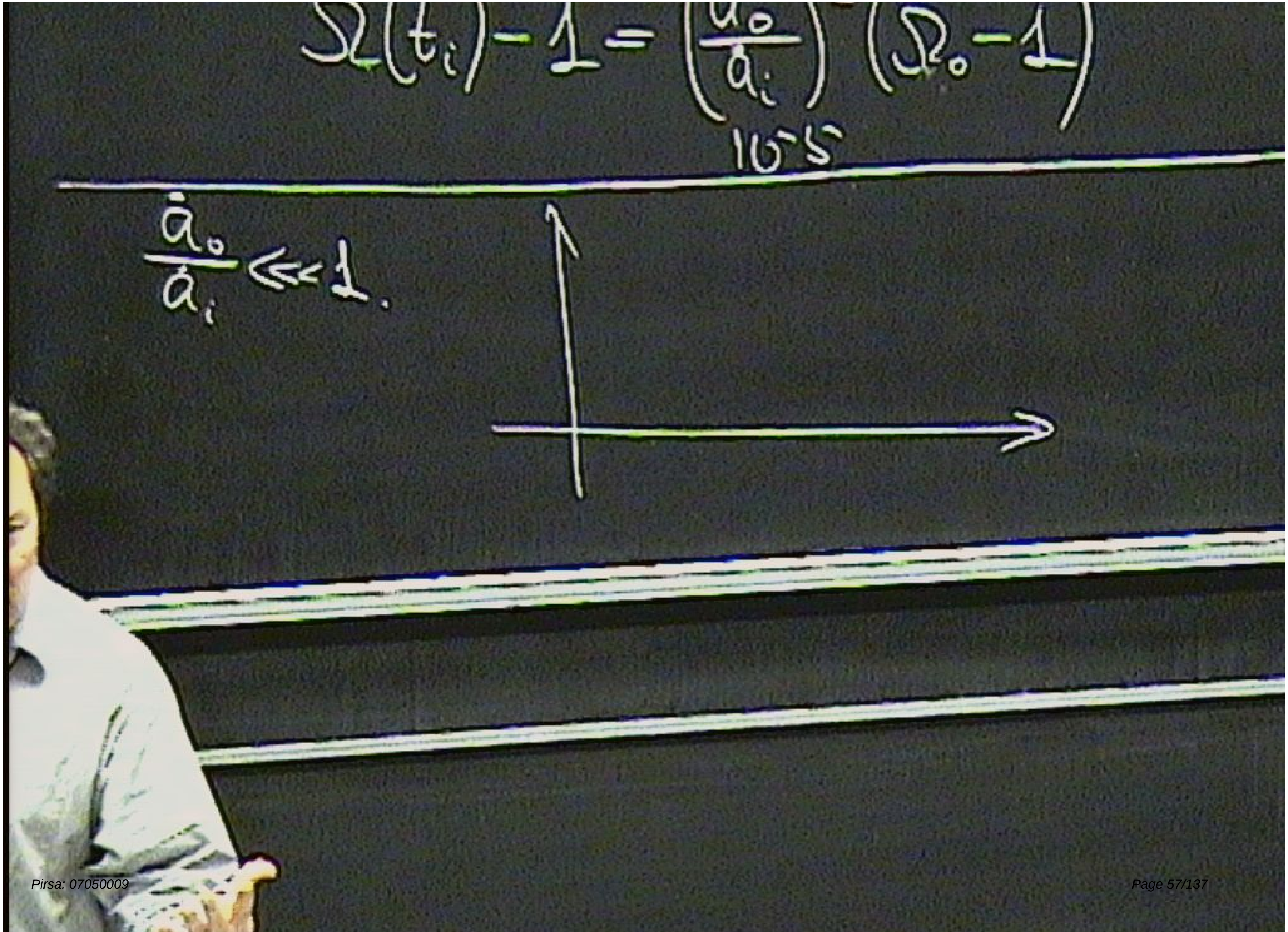
$$\frac{k}{\dot a^2} = \frac{k}{H^2 a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} - \frac{\varepsilon}{\varepsilon_c} = \Omega(t) - 1$$

$$\Omega(t_i) - 1 = \underbrace{\left(\frac{\dot a_0}{\dot a_i}\right)^2}_{10^{-5}}(\Omega_0 - 1)$$

$$\frac{k}{H^2 a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1.$$

$$\Omega(t_i) - 1 = \underbrace{\left(\frac{\dot a_0}{\dot a_i}\right)^2}_{10^{-5}} (\Omega_0 - 1)$$

$$\frac{\dot a_0}{\dot a_i} \lll 1.$$


Ω(t_i) − 1 = (a_0 / a_i) (Ω_0 − 1)
10^-5
ȧ_0 / ȧ_i <<< 1.

$$\Omega(t_i) - 1 = \left(\frac{a_0}{a_i}\right)(\Omega_0 - 1)$$

$$10^{-5}$$

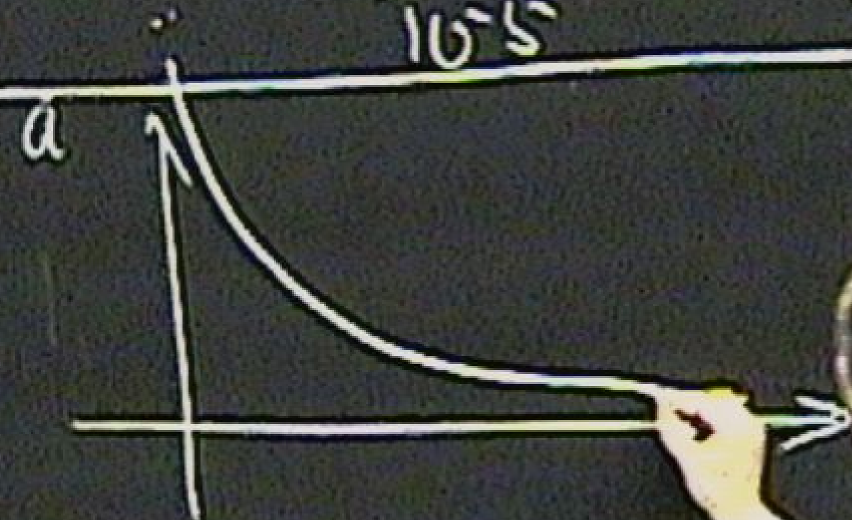
$$\frac{\dot{a}_0}{a_i} \lll 1.$$

$$\frac{k}{a^2} \cdot \frac{k}{H^2 a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$$

$$\Omega(t_i) - 1 = \left(\frac{\dot a_0}{\dot a_i}\right)^2 (\Omega_0 - 1)$$

10^{-5}

$$\frac{\dot a_0}{\dot a_i} \lll 1.$$

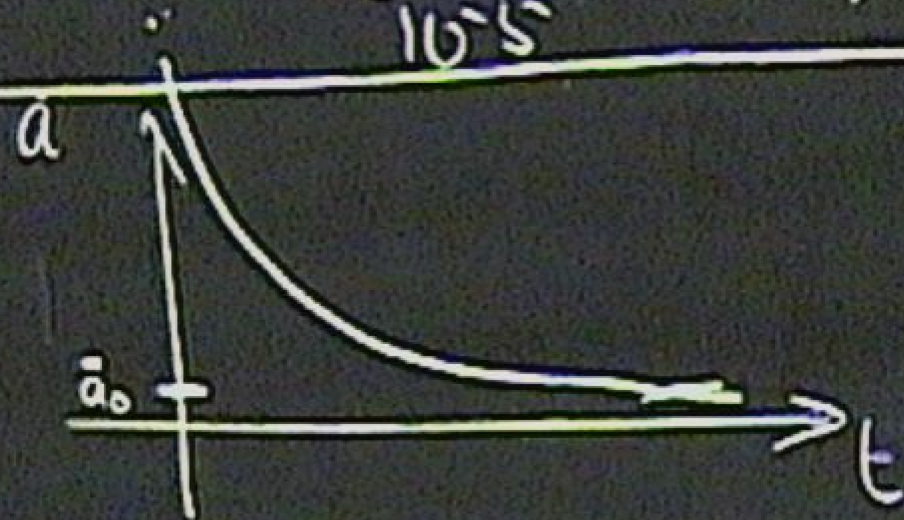


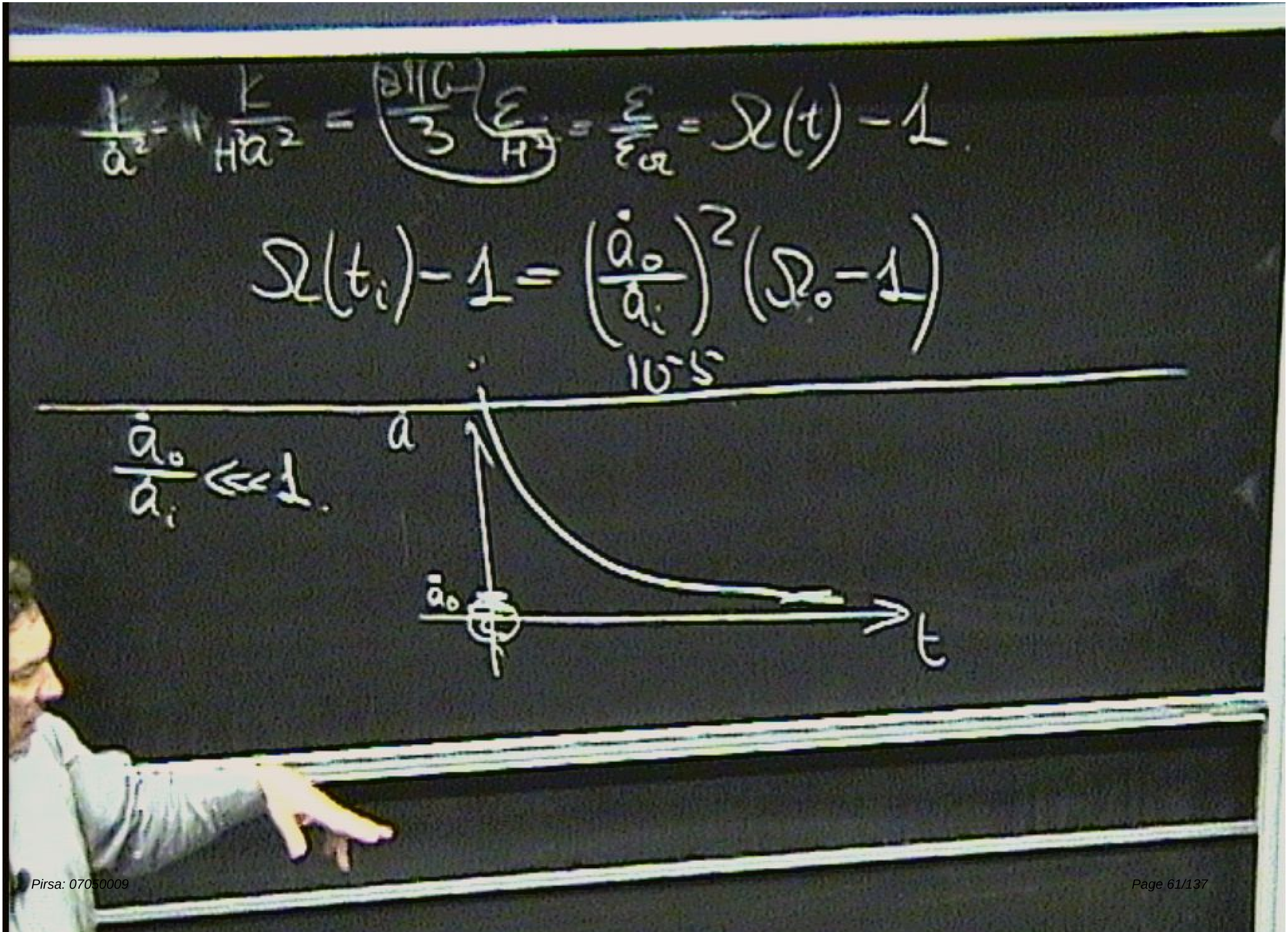
$$\frac{k}{\dot a^2} = \frac{k}{H^2 a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$$

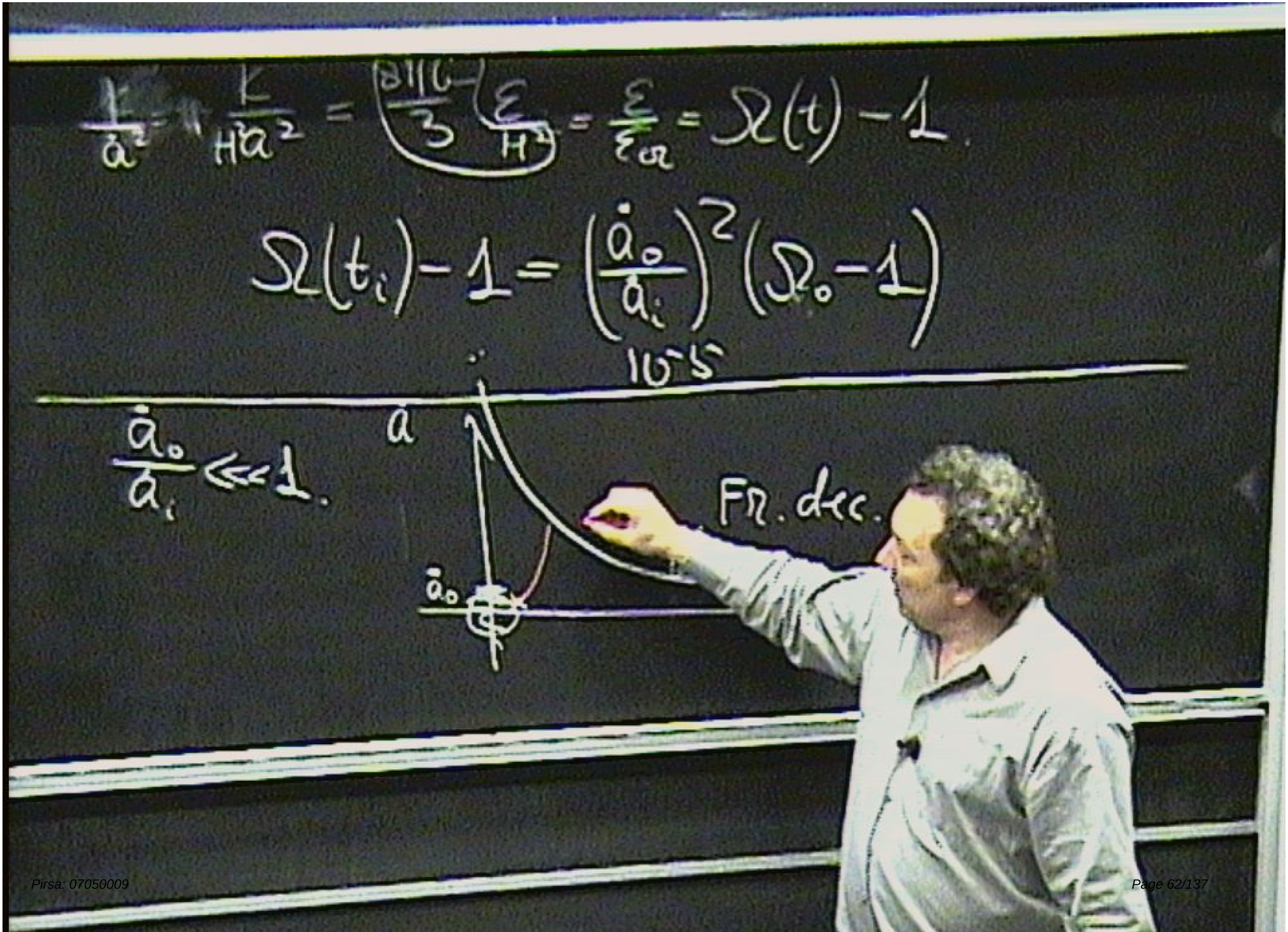
$$\Omega(t_i) - 1 = \left(\frac{\dot a_0}{\dot a_i}\right)^2 (\Omega_0 - 1)$$

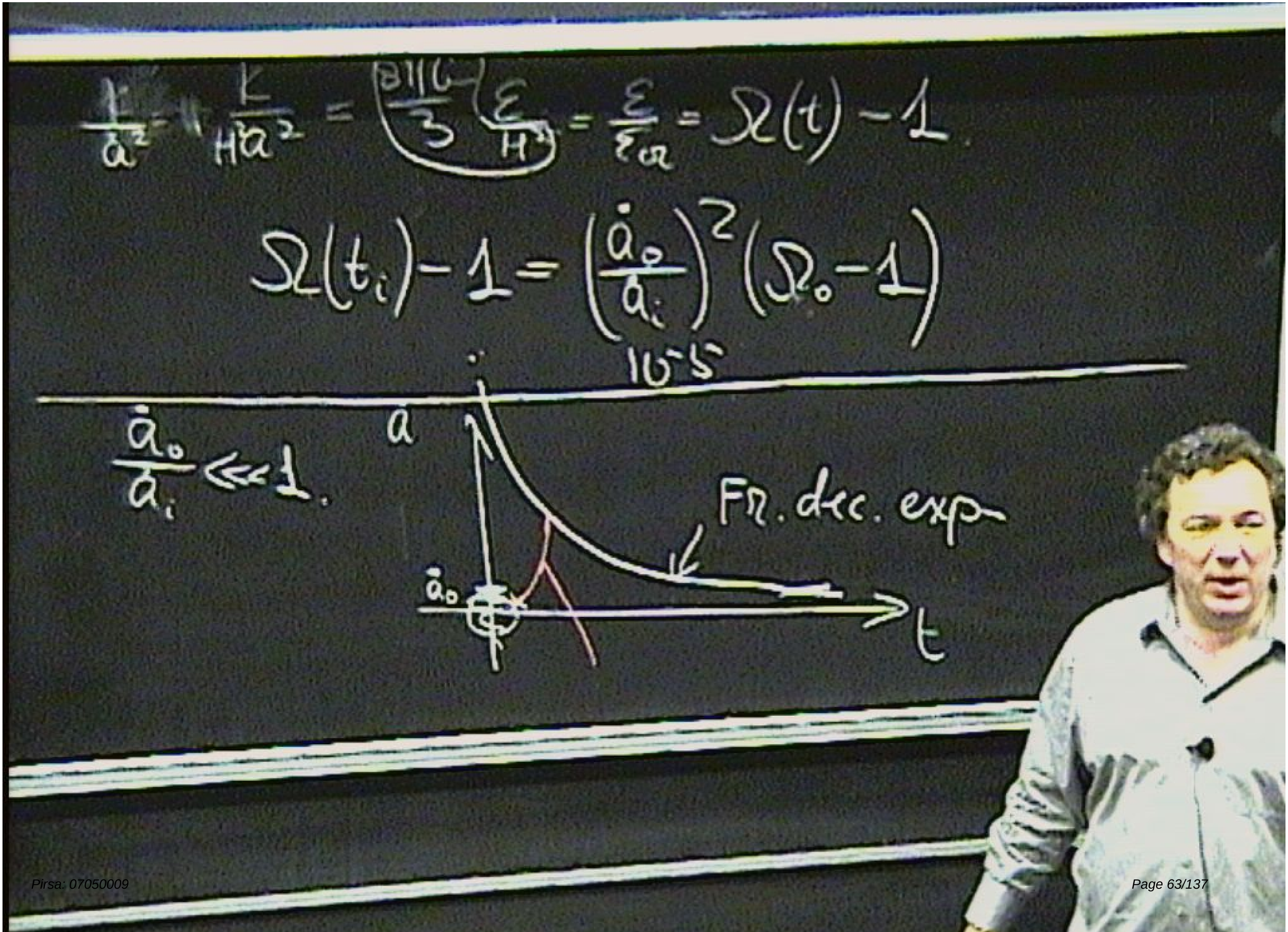
10^{-5}

$$\frac{\dot a_0}{\dot a_i} \lll 1.$$




$\frac{k}{H^2a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$
$\Omega(t_i) - 1 = \left(\frac{\dot a_0}{\dot a_i}\right)^2 (\Omega_0 - 1)$
10^{-5}
$\frac{\dot a_0}{\dot a_i} \lll 1$
$\dot a$
$\dot a_0$
t


$\frac{k}{H^2a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$
$\Omega(t_i) - 1 = \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 (\Omega_0 - 1)$
10^{-5}
$\dot{a}$
$\frac{\dot{a}_0}{\dot{a}_i} \lll 1$
$\dot{a}_0$
Fr. dec.

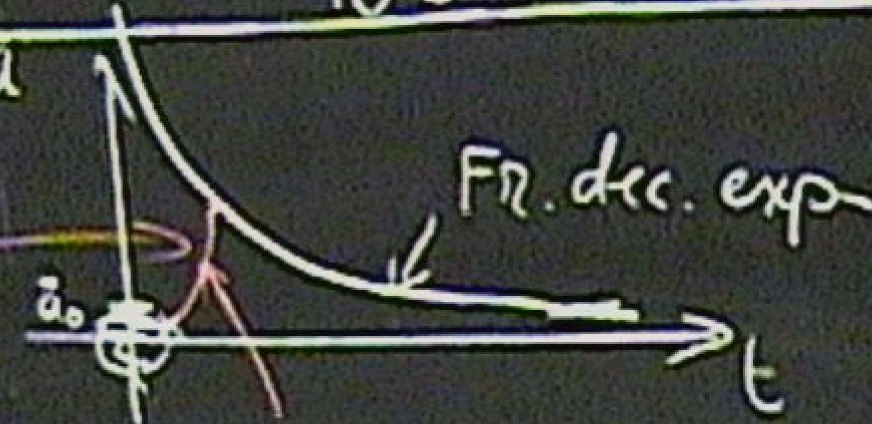

$\frac{k}{a^2} \quad \frac{k}{H^2 a^2} = \frac{8\pi G}{3} \frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$
$\Omega(t_i) - 1 = \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 (\Omega_0 - 1)$
10^{-5}
$\frac{\dot{a}_0}{\dot{a}_i} \lll 1$
$\dot{a}$
Fr. dec. exp
$\dot{a}_0$
t

$$\frac{k}{\dot{a}^2} = \frac{k}{H^2 a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$$

$$\Omega(t_i) - 1 = \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 (\Omega_0 - 1)$$

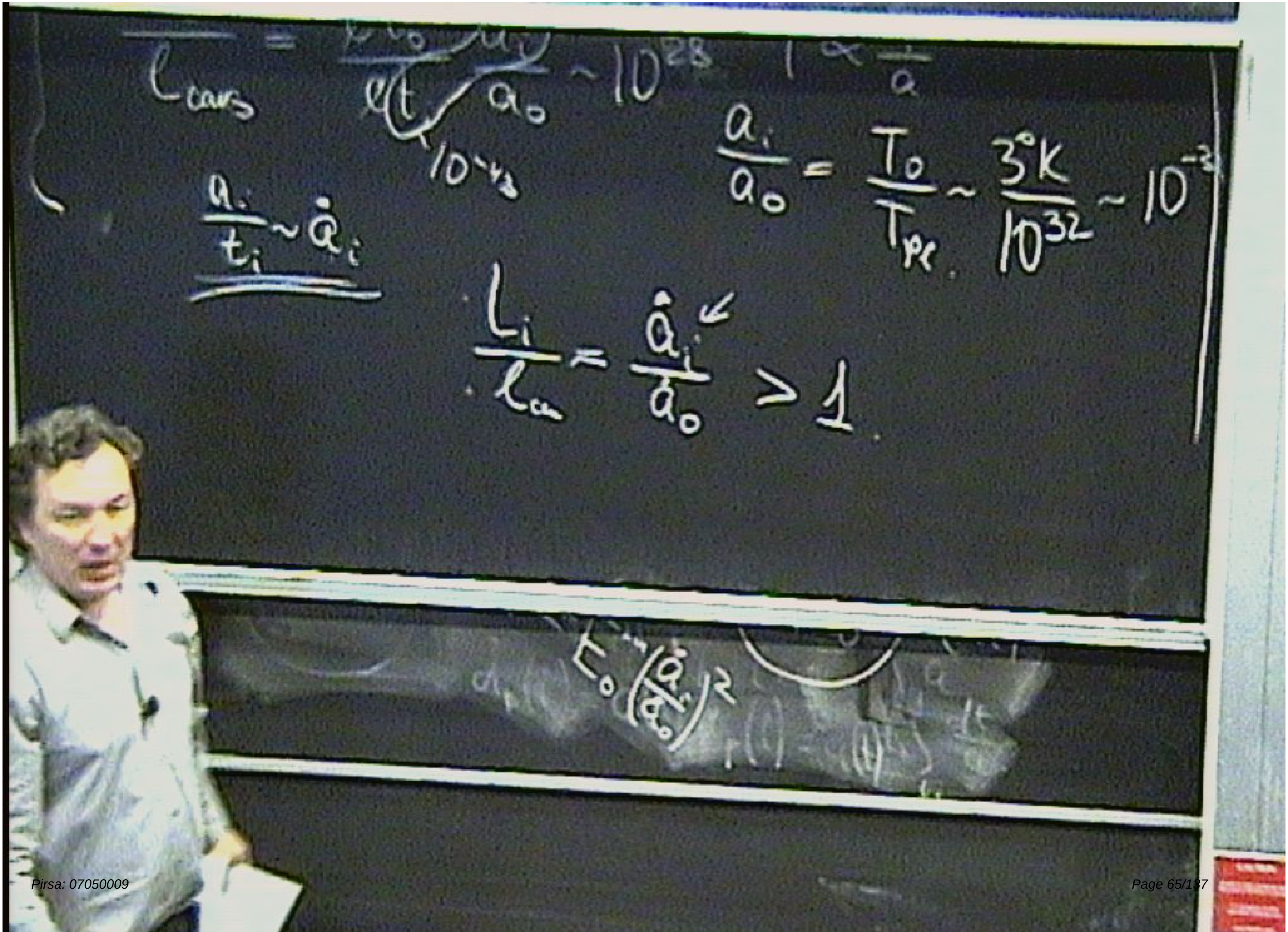
10^{-5}

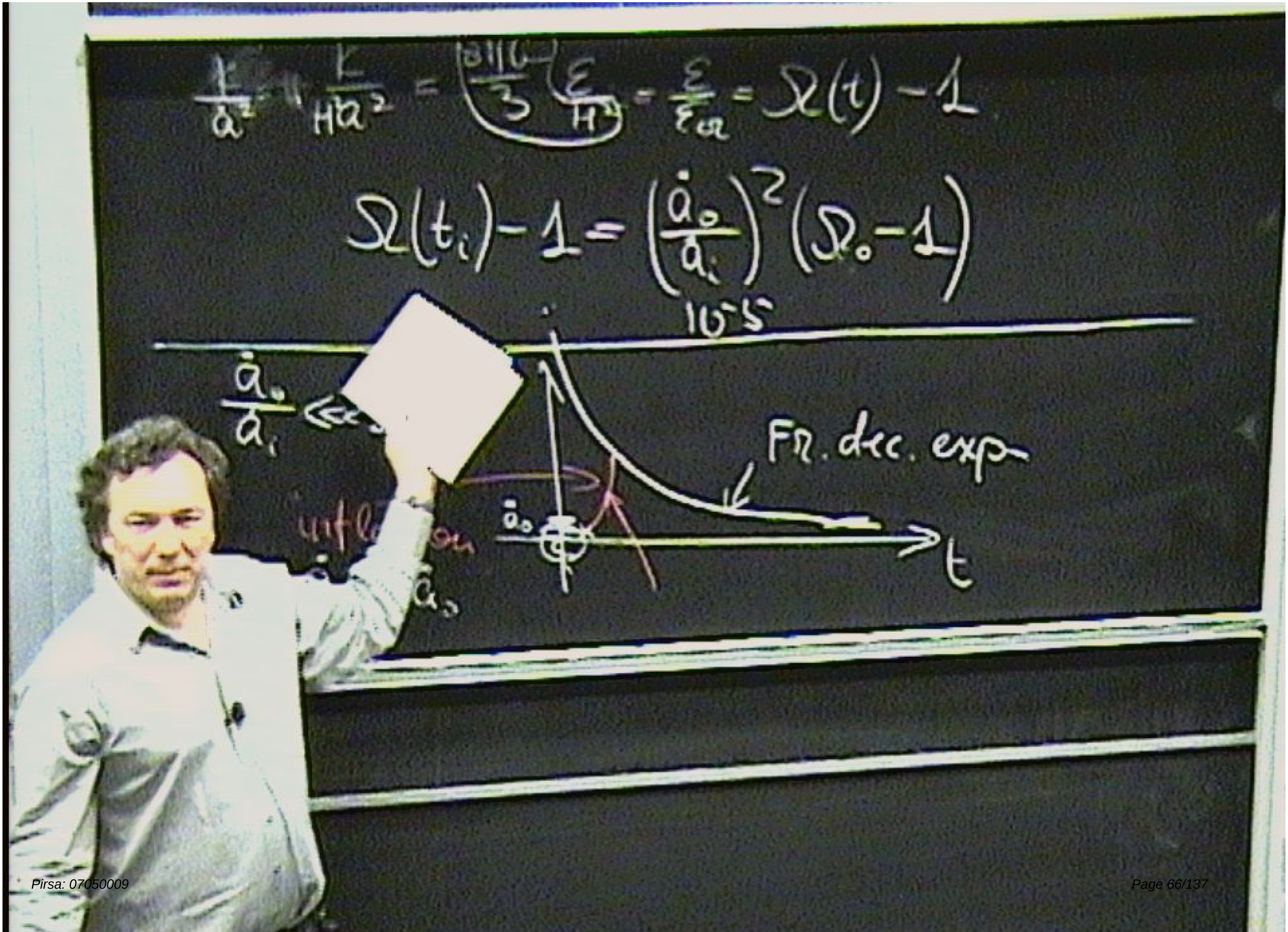
$\frac{\dot{a}_0}{\dot{a}_i} \lll 1.$



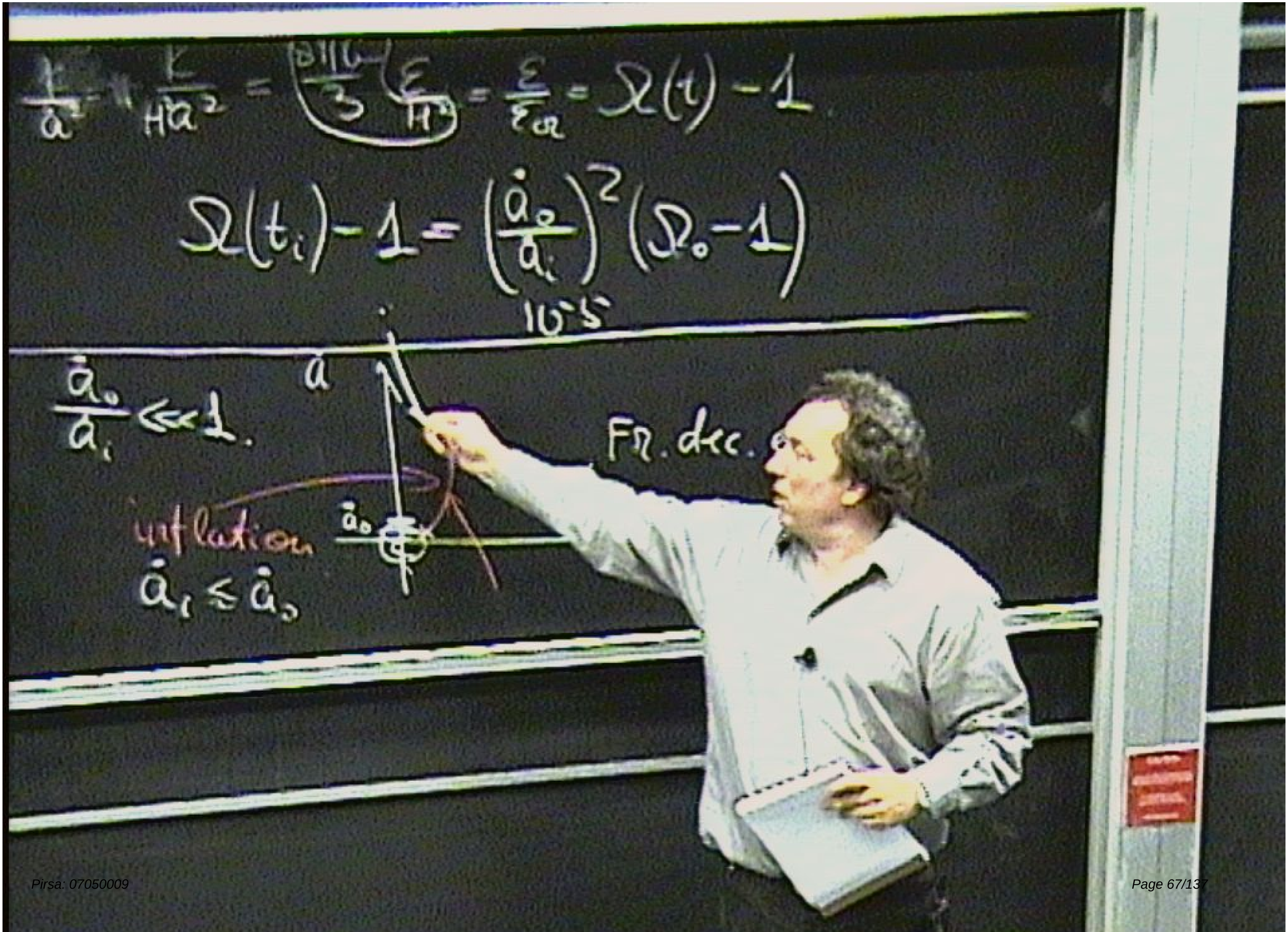
inflation

$\dot{a}_i \leq \dot{a}_0$


l_{caus}
10^{28}
$\frac{a_i}{t_i} \sim \dot{a}_i$
$\frac{a_i}{a_0} = \frac{T_0}{T_{pl}} \sim \frac{3°K}{10^{32}} \sim 10^{-3}$
$\frac{L_i}{l_{ca}} = \frac{\dot{a}_i}{\dot{a}_0} > 1$
$t_0 \left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2$



$\Omega(t) - 1$
$\Omega(t_i) - 1 = \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 (\Omega_0 - 1)$
10^{-5}
Fr. dec. exp
t



$\frac{k}{Ha^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$
$\Omega(t_i) - 1 = \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 (\Omega_0 - 1)$
10^{-5}
$\dot{a}$
$\frac{\dot{a}_0}{\dot{a}_i} \lll 1.$
Fr. dec.
inflation
$\dot{a}_0$
$\dot{a}_i \leq \dot{a}_0$

$$\frac{k}{a^2} \quad \frac{k}{H^2 a^2} = \frac{8\pi G}{3}\frac{\varepsilon}{H^2} = \frac{\varepsilon}{\varepsilon_{cr}} = \Omega(t) - 1$$

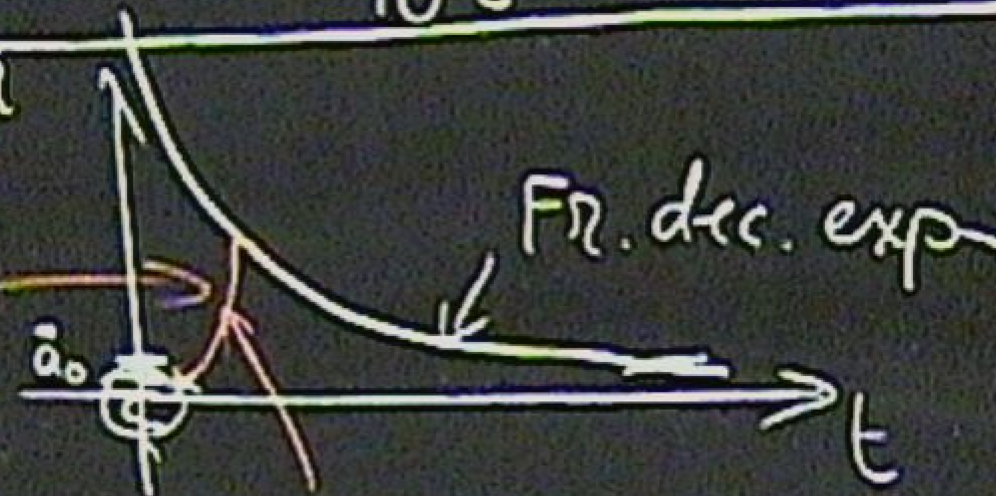
$$\Omega(t_i) - 1 = \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 (\Omega_0 - 1)$$

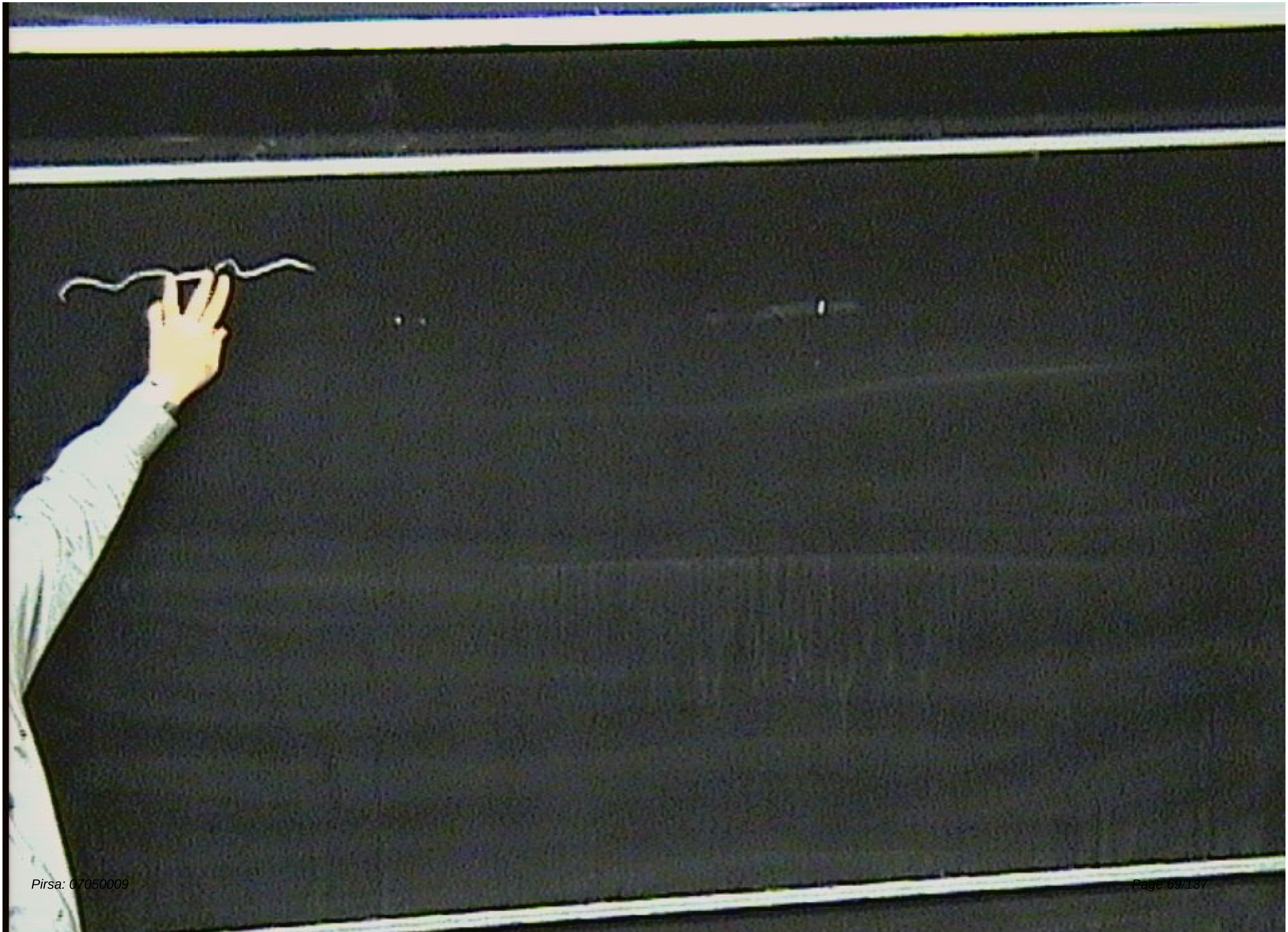
10^{-5}

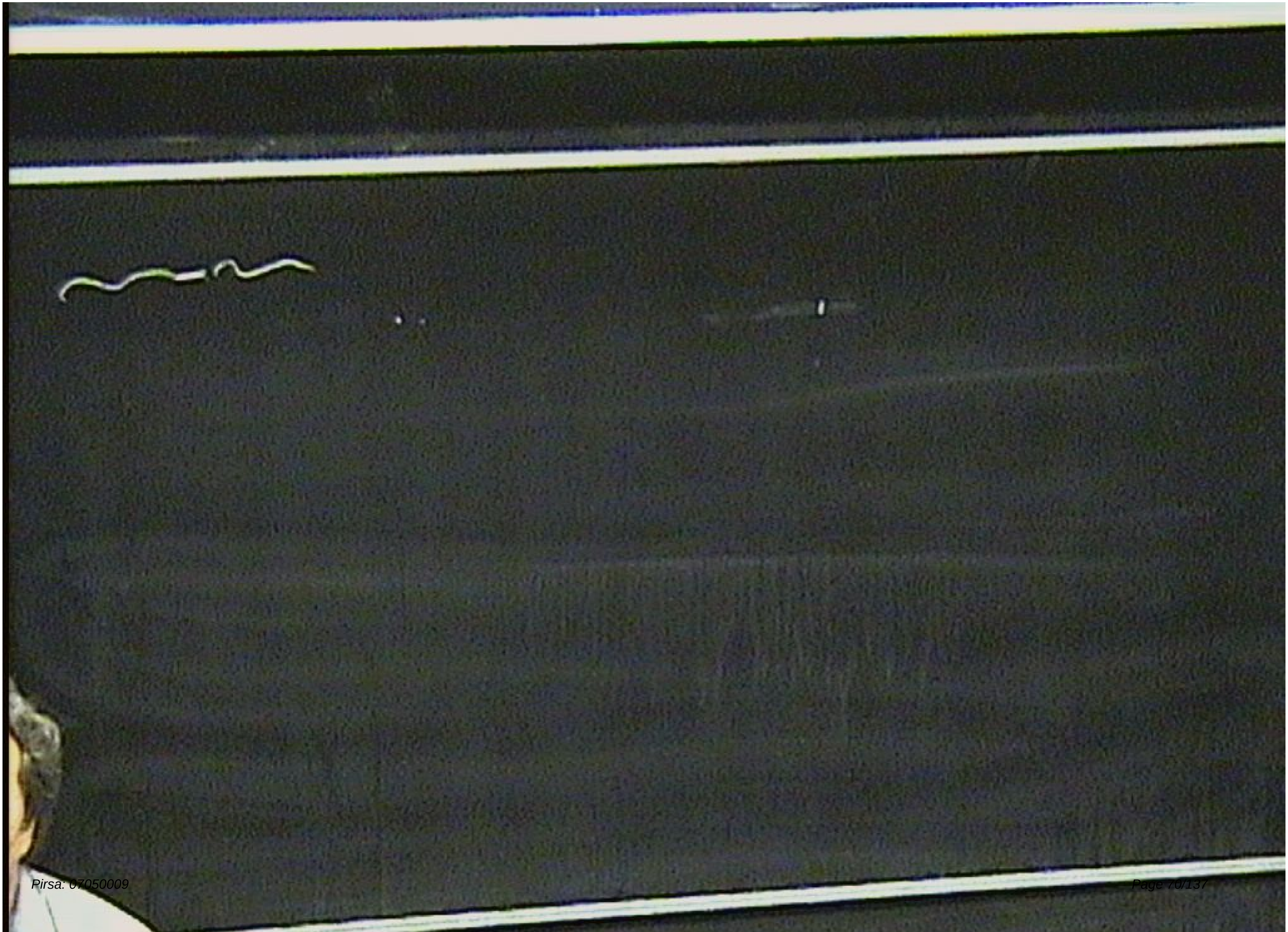
$$\frac{\dot{a}_0}{\dot{a}_i} \lll 1.$$

inflation

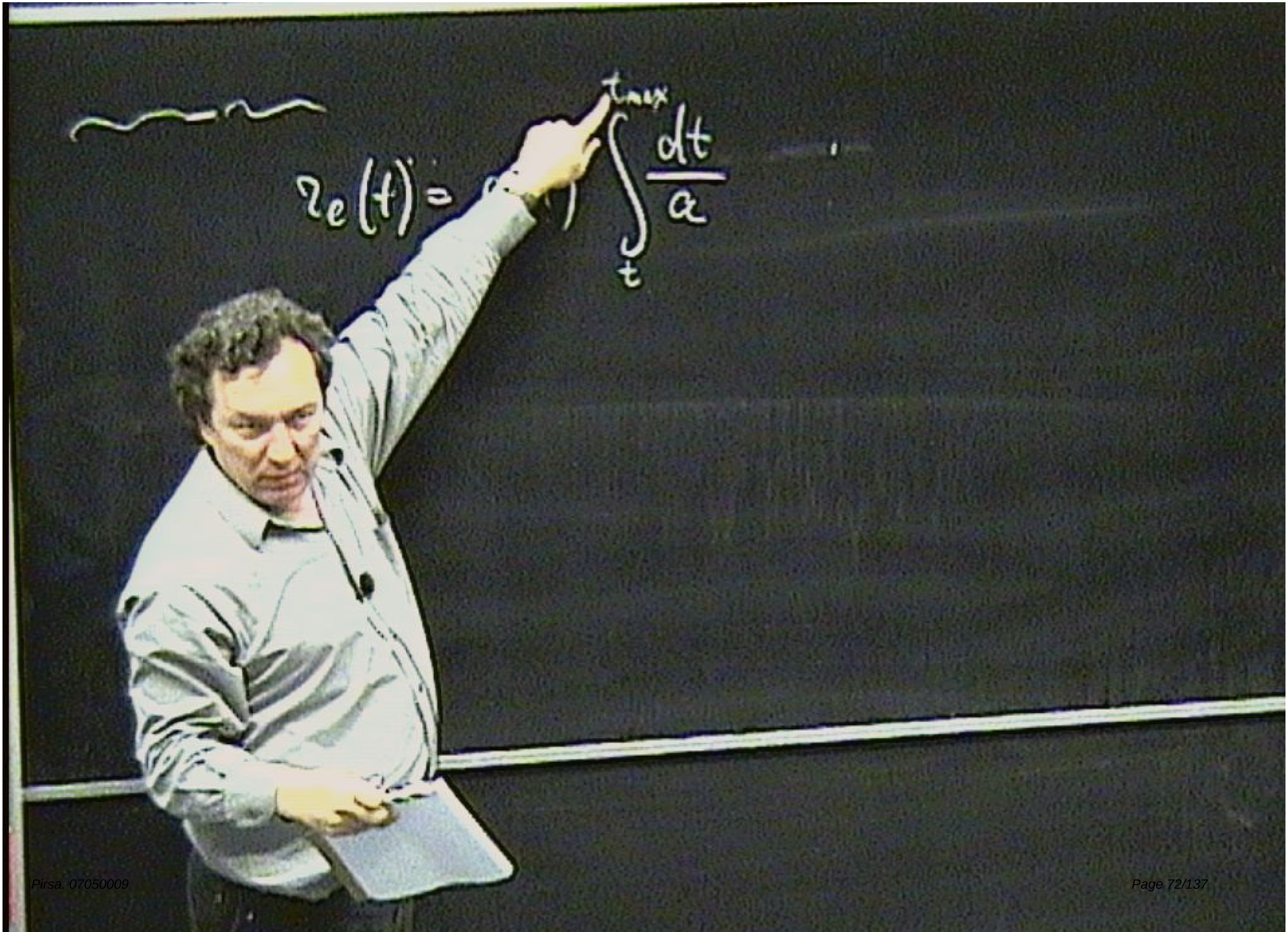
$\dot{a}_i \leq \dot{a}_0$

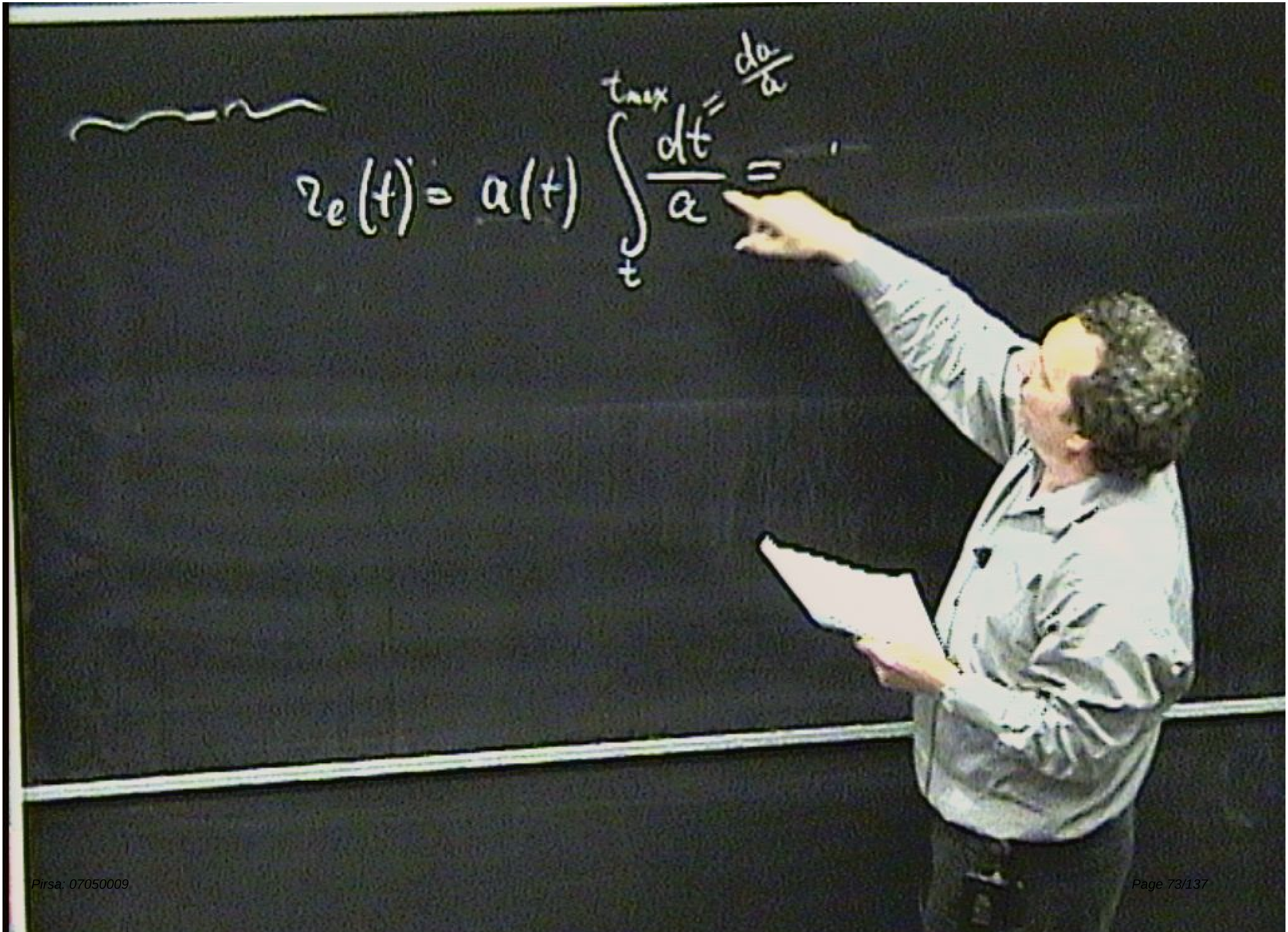


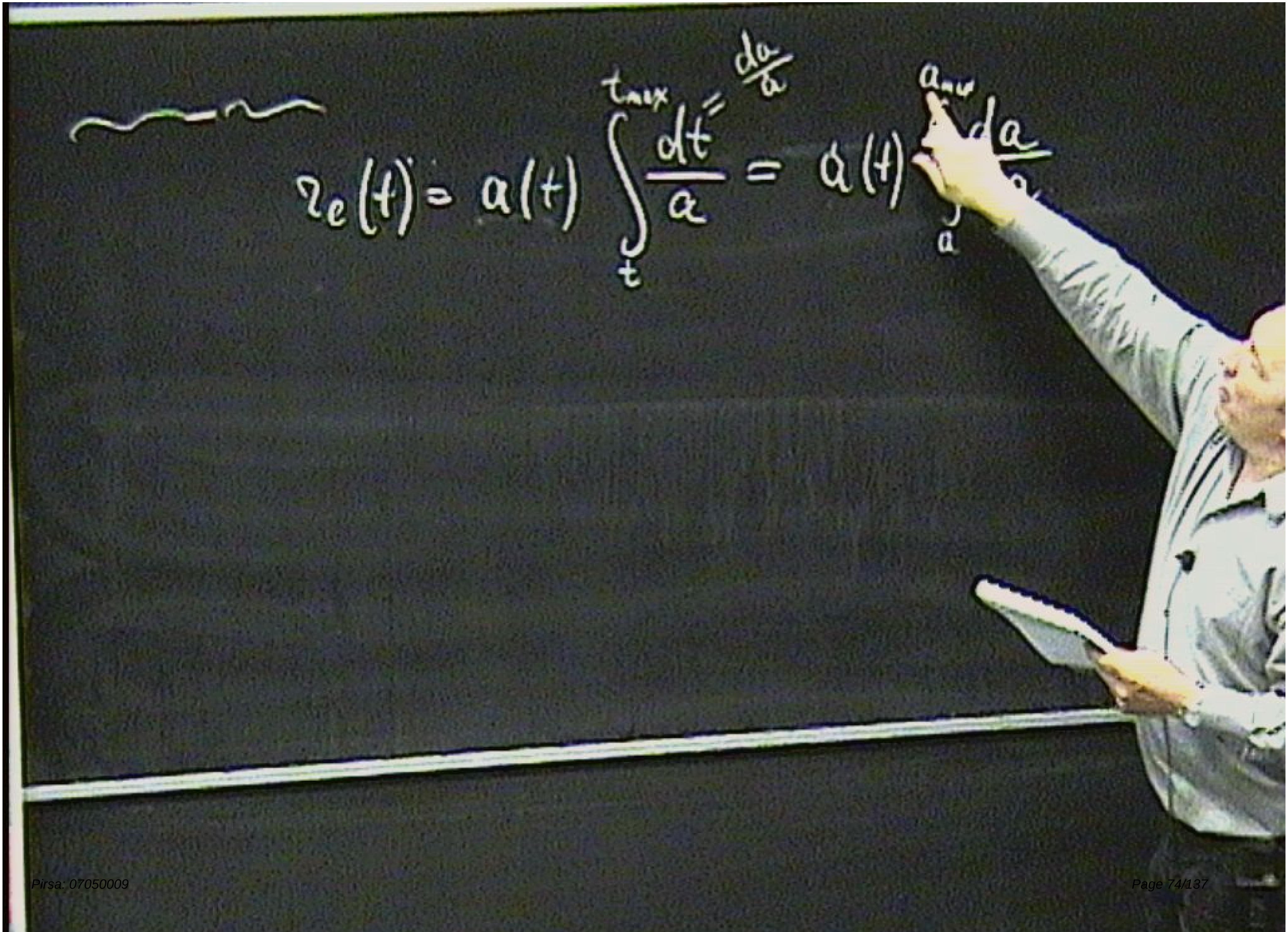




$$r_e(t) = a(t) \int$$


r_e(t) =
t_max
dt
a
t


$r_e(t) = a(t) \int_t^{t_{max}} \frac{dt'}{a} =$
$\frac{da}{\dot{a}}$


$r_e(t) = a(t) \int_t^{t_{max}} \frac{dt'}{a} = a(t) \int_a^{a_{max}} da$
$\frac{da}{a}$

$$r_e(t) = a(t)\int_t^{t_{max}} \frac{dt'}{a} = a(t)\int_a^{a_{max}\to\infty} \frac{da}{\dot{a}a}$$

$$dt' = \frac{da}{\dot{a}}$$

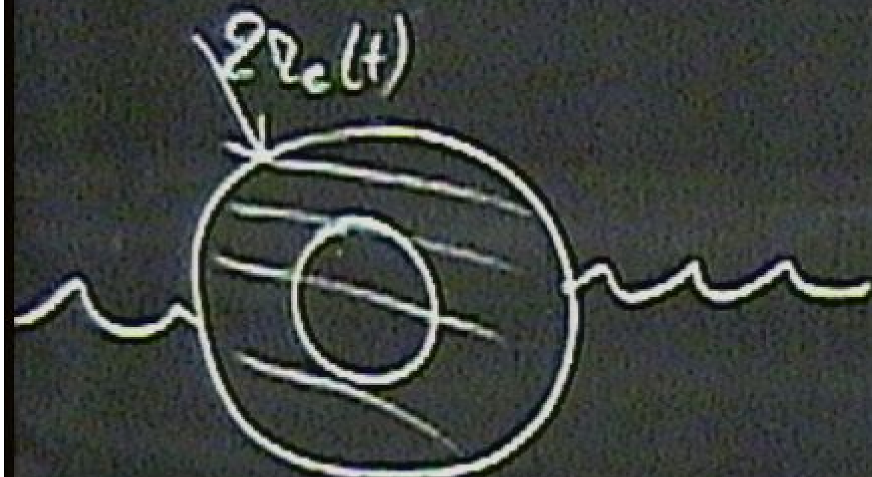
$$r_e(t) = a(t)\int_t^{t_{max}} \frac{dt'}{a} = a(t)\int_a^{a_{max}\to\infty} \frac{da}{\dot{a}a}$$

$$dt' = \frac{da}{\dot{a}}$$

$$r_e(t) = a(t) \int_t^{t_{max}} \frac{dt}{a} \overset{\frac{da}{\dot a}}{=} a(t) \int_a^{a_{max} \to \infty} \frac{da}{\dot a a} \to \text{finite.}$$

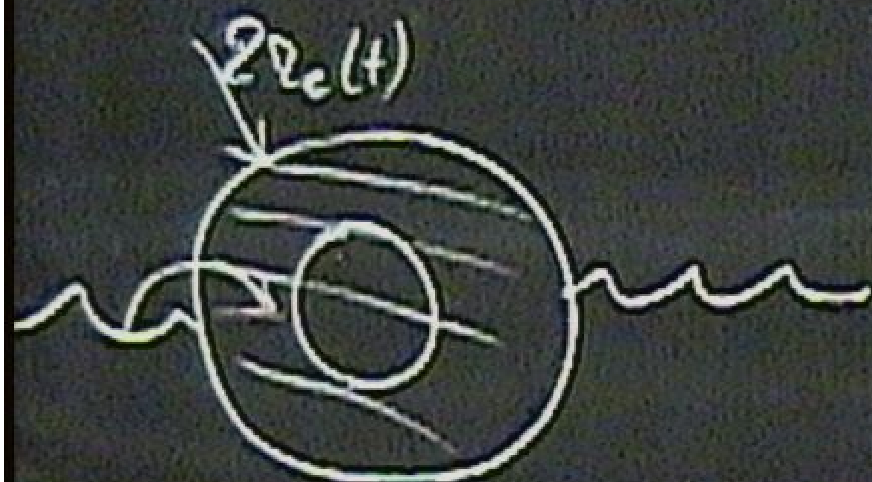
$$r_e(t) = a(t)\int_t^{t_{max}} \frac{dt'}{a} = a(t)\int_a^{a_{max}} \frac{da}{\dot{a}a} \to \text{finite}$$

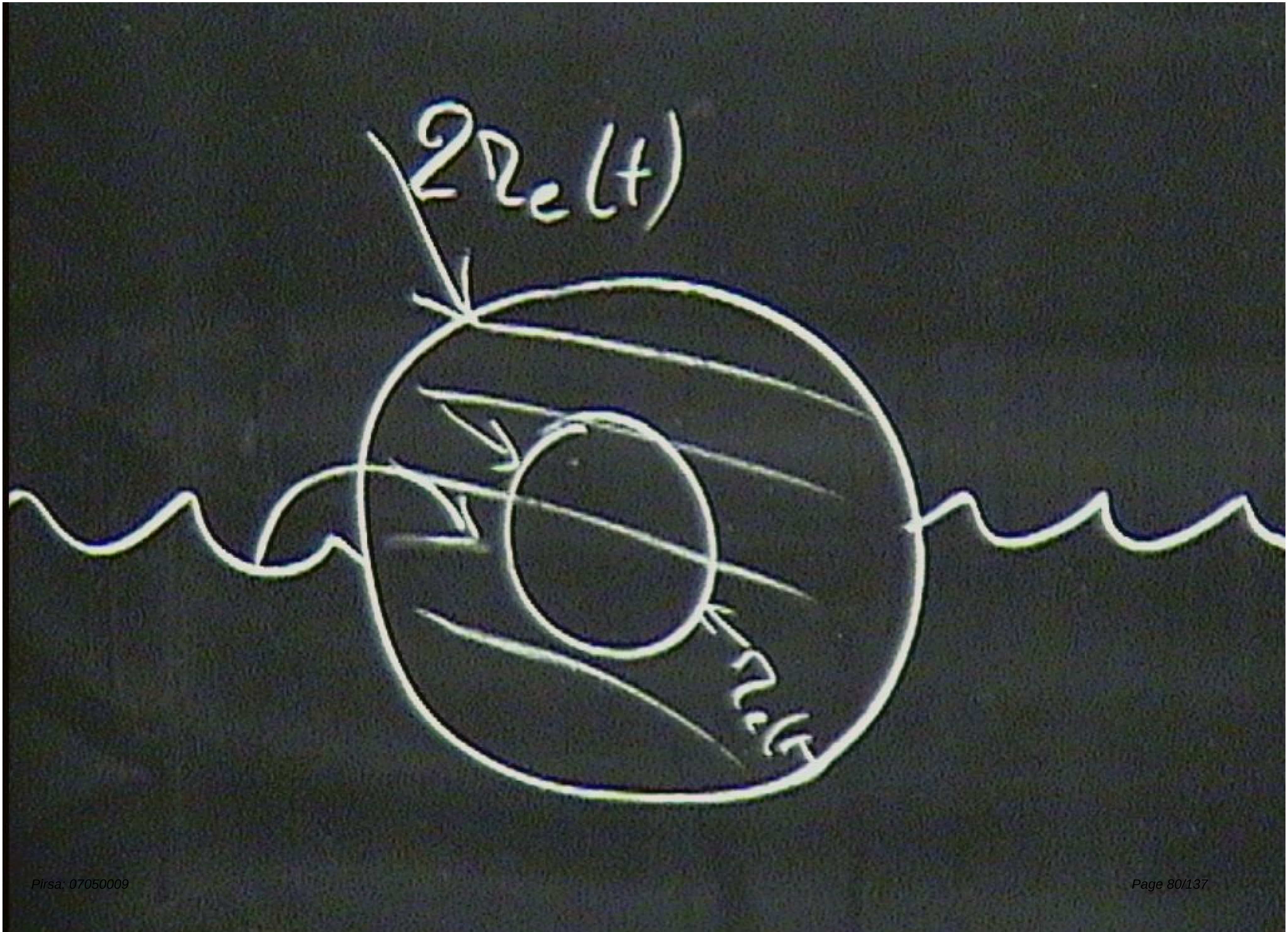
$$dt' = \frac{da}{\dot{a}}$$



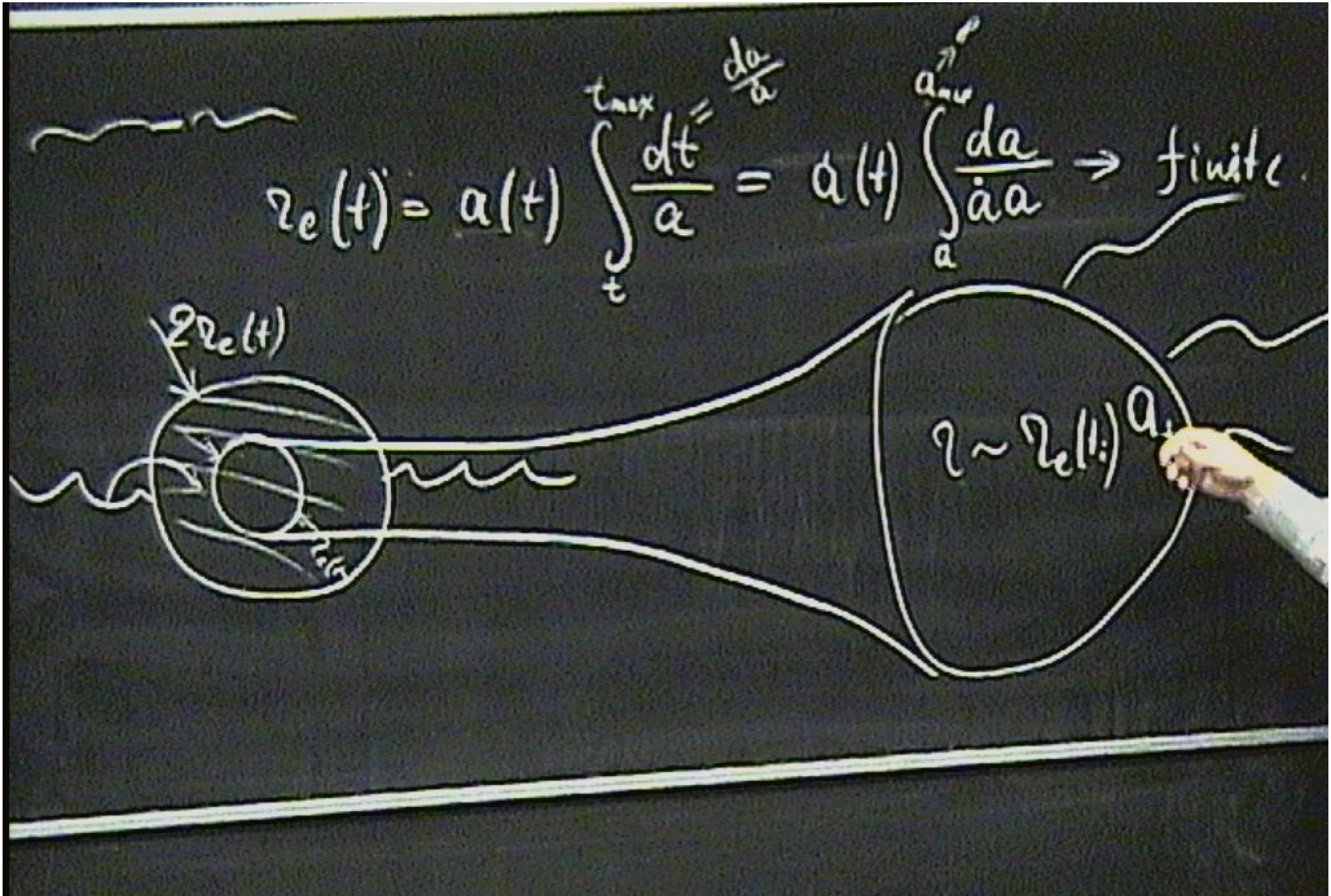
$$r_e(t) = a(t) \int_t^{t_{max}} \frac{dt'}{a} = a(t) \int_a^{a_{max}} \frac{da}{\dot{a}\, a} \Rightarrow \text{finite}$$

$$dt' = \frac{da}{\dot{a}}$$





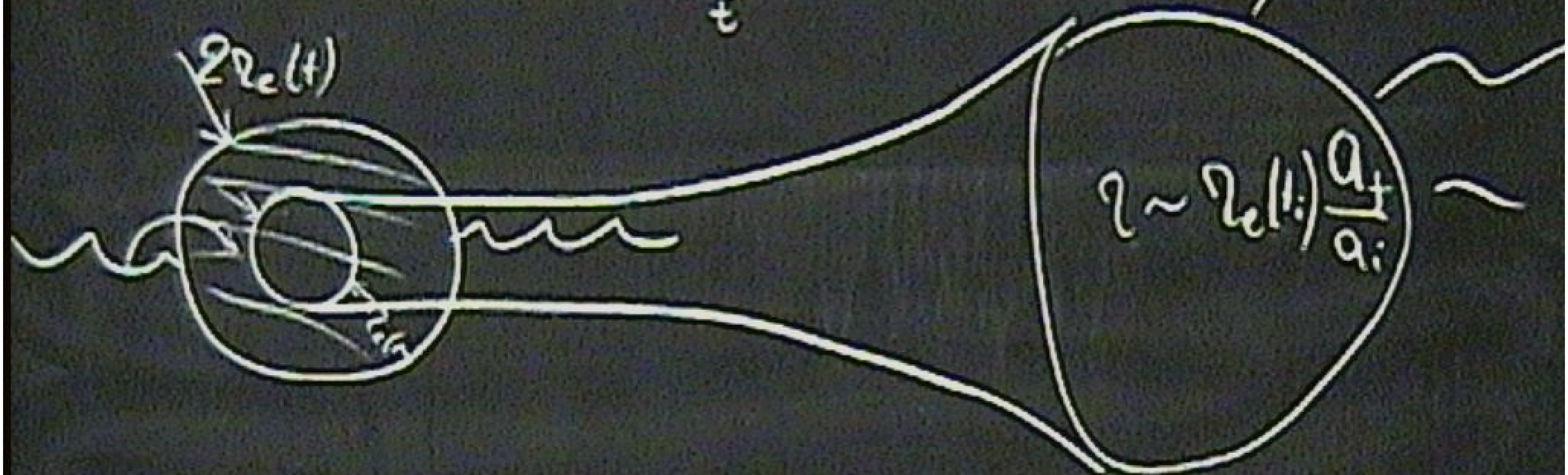
$2r_e(t)$
$r_e(t)$

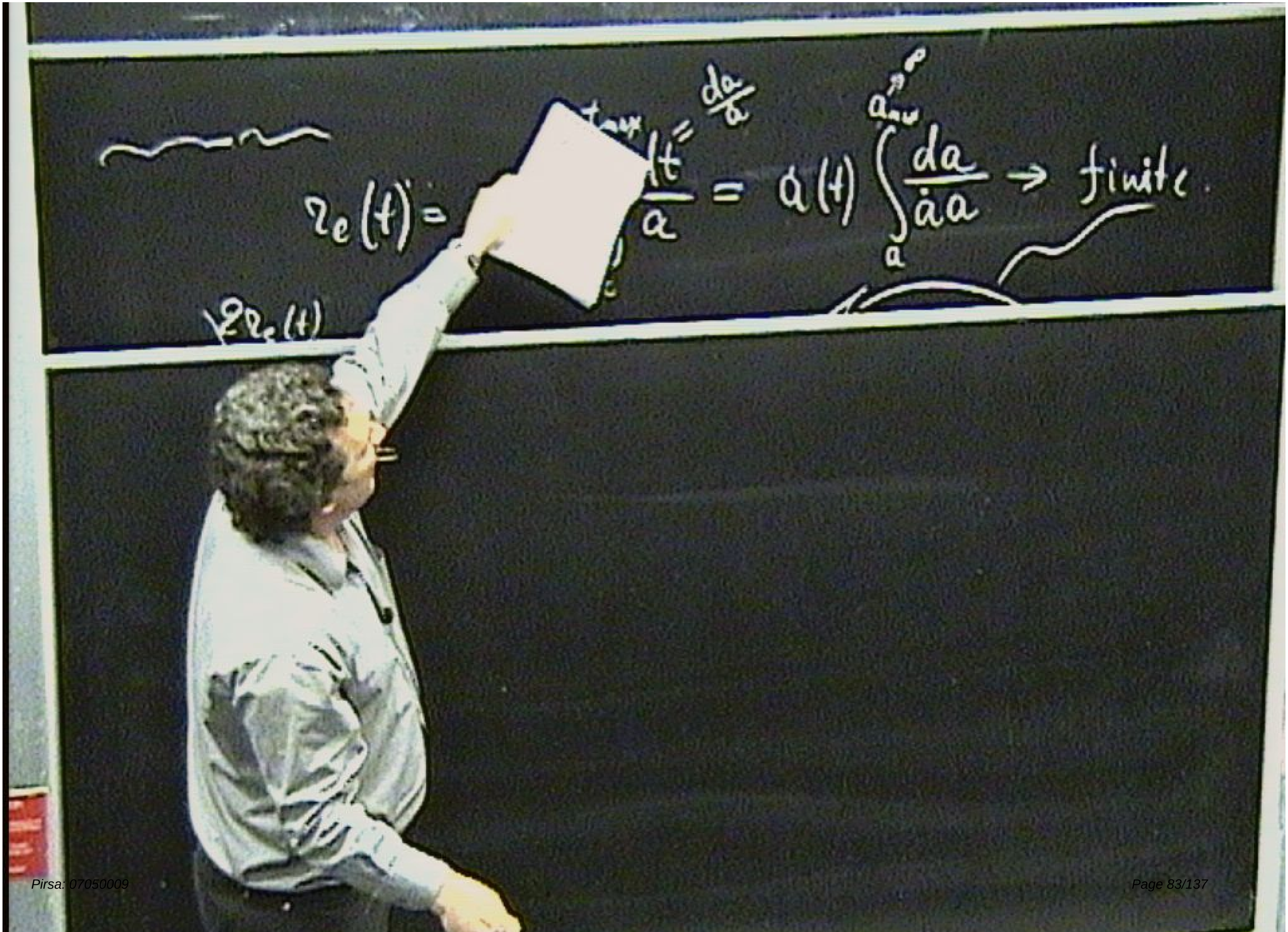


$r_e(t) = a(t) \int_t^{t_{max}} \frac{dt}{a} = a(t) \int_a^{a_{max} \to \infty} \frac{da}{\dot{a}a} \to$ finite
$dt = \frac{da}{\dot{a}}$
$2r_e(t)$
r_e
$r \sim r_e(t_i)\, a_i$

$$r_e(t) = a(t)\int_t^{t_{max}} \frac{dt}{a} = a(t)\int_a^{a_{max}\to\infty} \frac{da}{\dot{a}a} \to \text{finite}$$

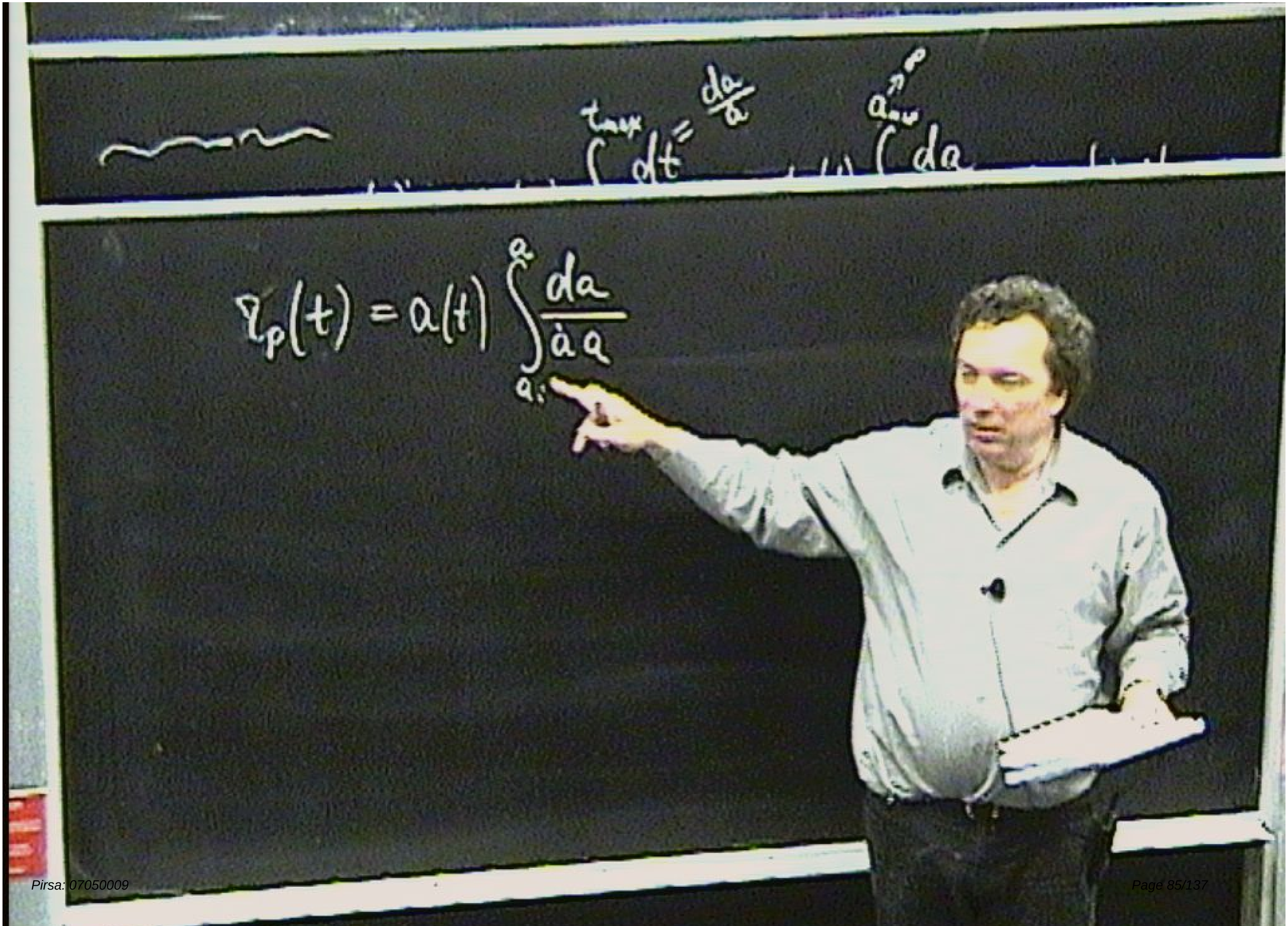
$$dt = \frac{da}{\dot{a}}$$

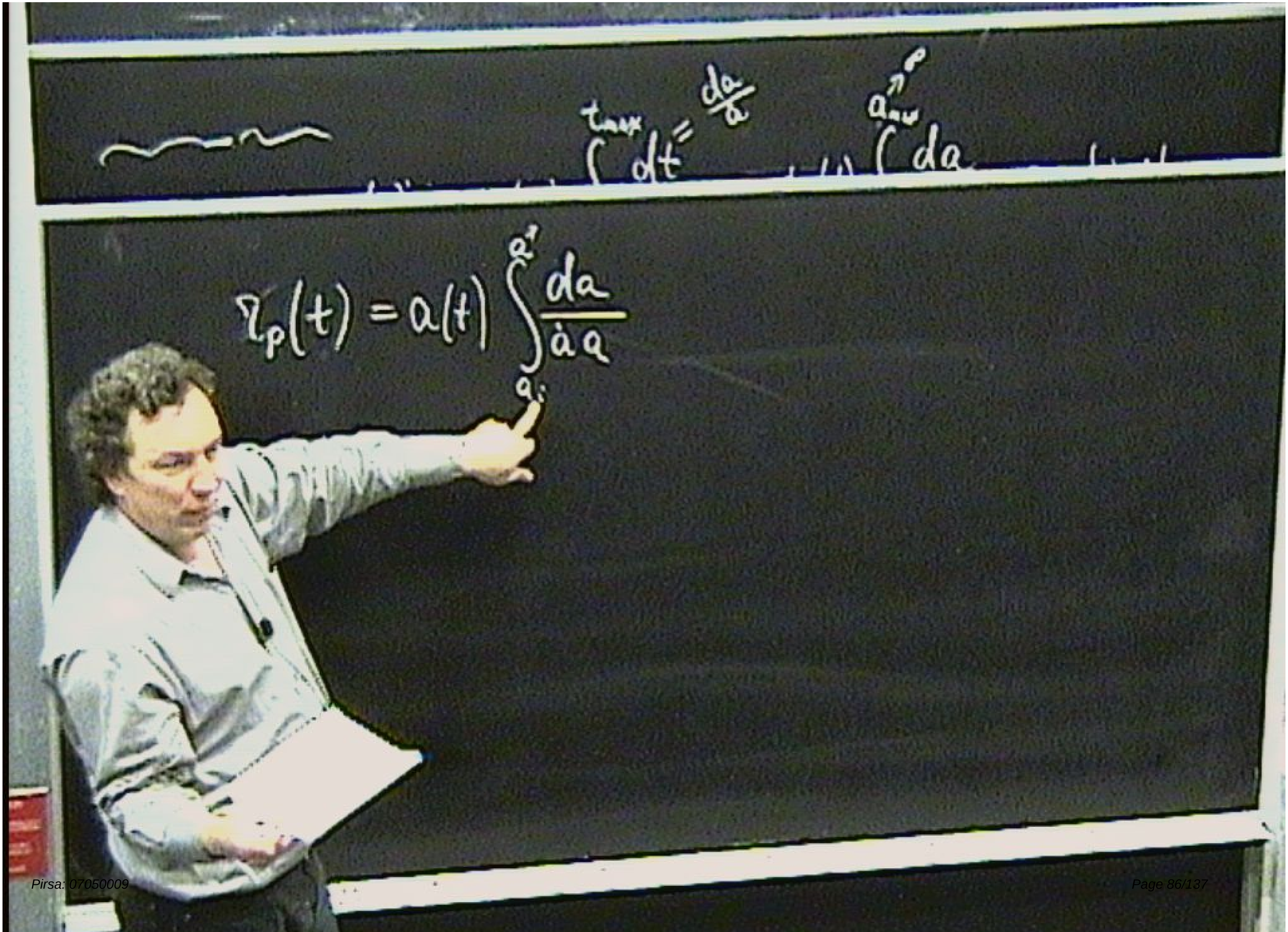



finite

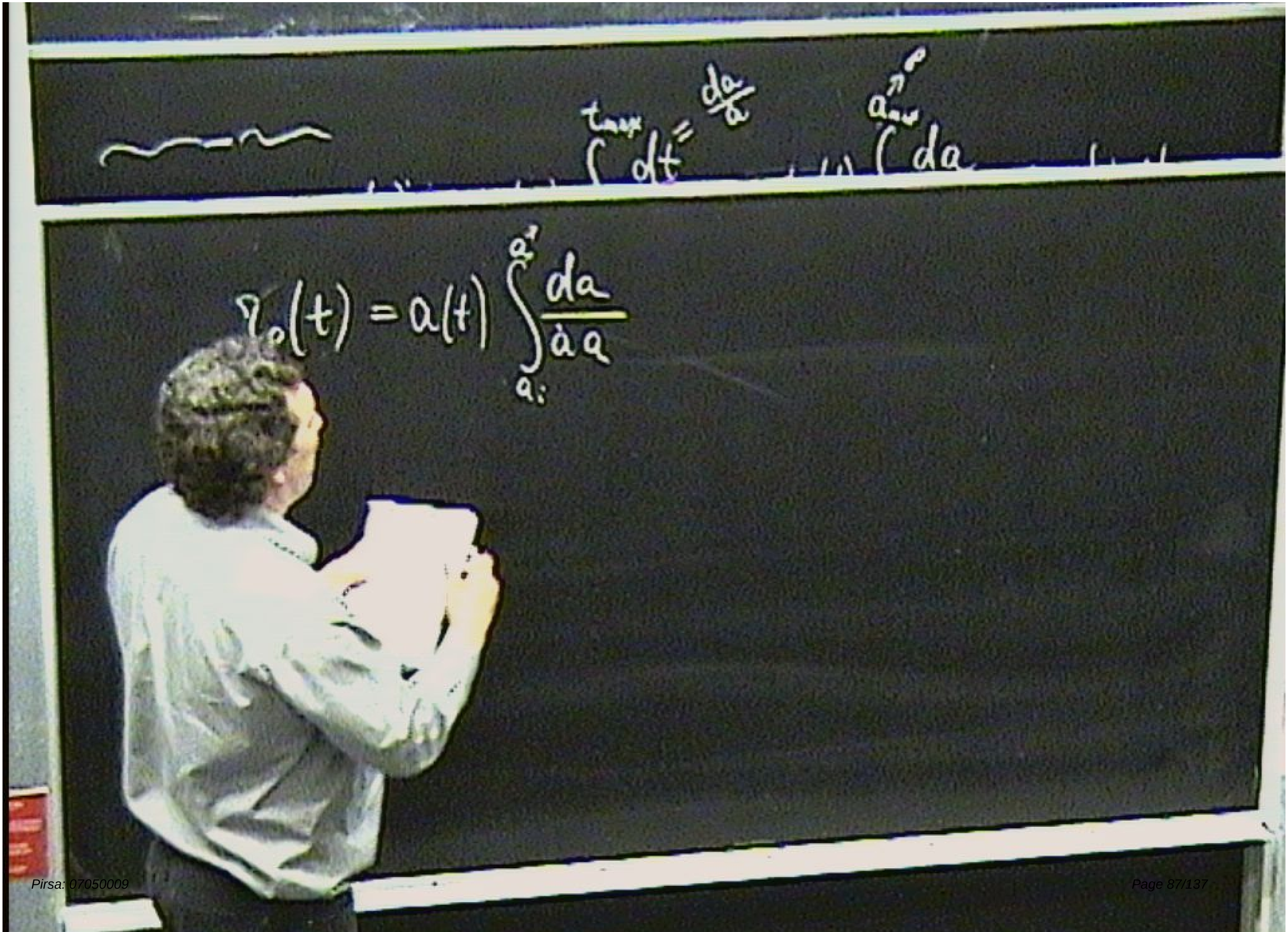
$$\int^{t_{max}} dt = \frac{da}{\dot a} \qquad \int^{a_{max} \to \infty} da$$

$$\eta_p(t) = a(t) \int_{a_i}^{a} \frac{da}{\dot a\, a}$$


$\eta_p(t) = a(t) \int_{a_i}^{a} \frac{da}{\dot{a}\, a}$



$\eta_p(t) = a(t) \int_{a_i}^{a'} \frac{da}{\dot{a}\, a}$


$\int^{t_{max}} dt = \frac{da}{\dot a}$
$\int^{a_{max}} da$
$r_e(t) = a(t) \int_{a_i}^{a'} \frac{da}{\dot a a}$

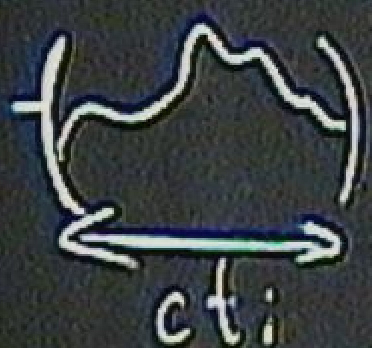
$$\int^{t_{max}} dt = \frac{da}{\dot a} \qquad \int^{a_{max}} da$$

$$\eta_p(t) = \frac{a(t)}{a_i}\, a_i \int_{a_i}^{a} \frac{da}{\dot a\, a} \approx \frac{a(t)}{a_i}\, \eta_e(t)$$

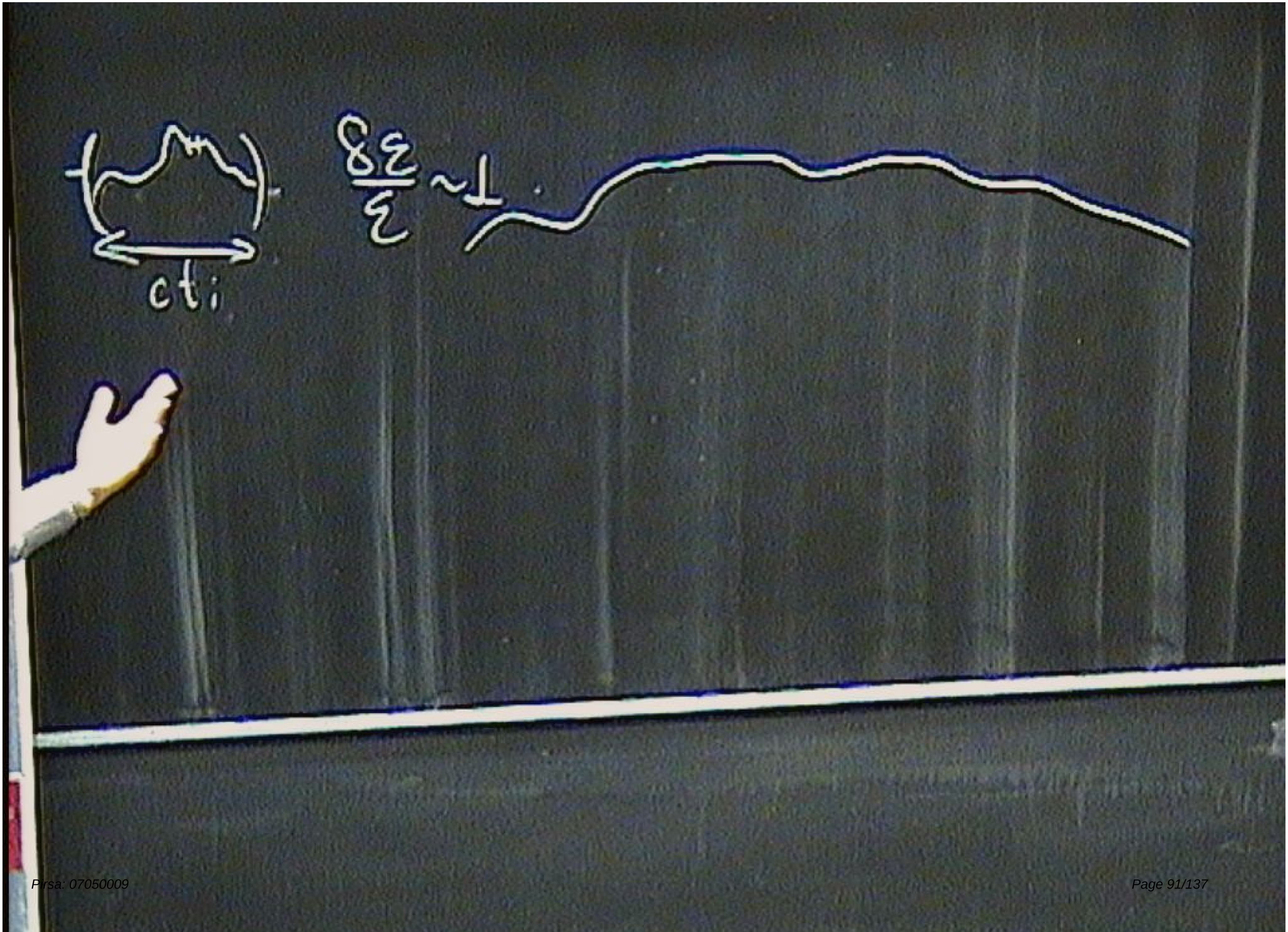
$$r_e(t) = a(t)\int_t^{t_{max}} \frac{dt}{a} = a(t)\int_a^{a_{max}\to\infty} \frac{da}{\dot a a} \to \text{finite}$$

$$dt = \frac{da}{\dot a}$$

$$r_p(t) = \frac{a(t)}{a_i}\, a_i\int_{a_i}^{a} \frac{da}{\dot a a} \approx \frac{a(t)}{a_i}\, r_e(t_i)$$

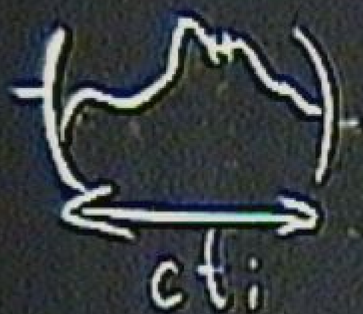


$$\frac{\delta\varepsilon}{\varepsilon} \sim 1 .$$


$\frac{\delta\varepsilon}{\varepsilon} \sim 1$
ct_i

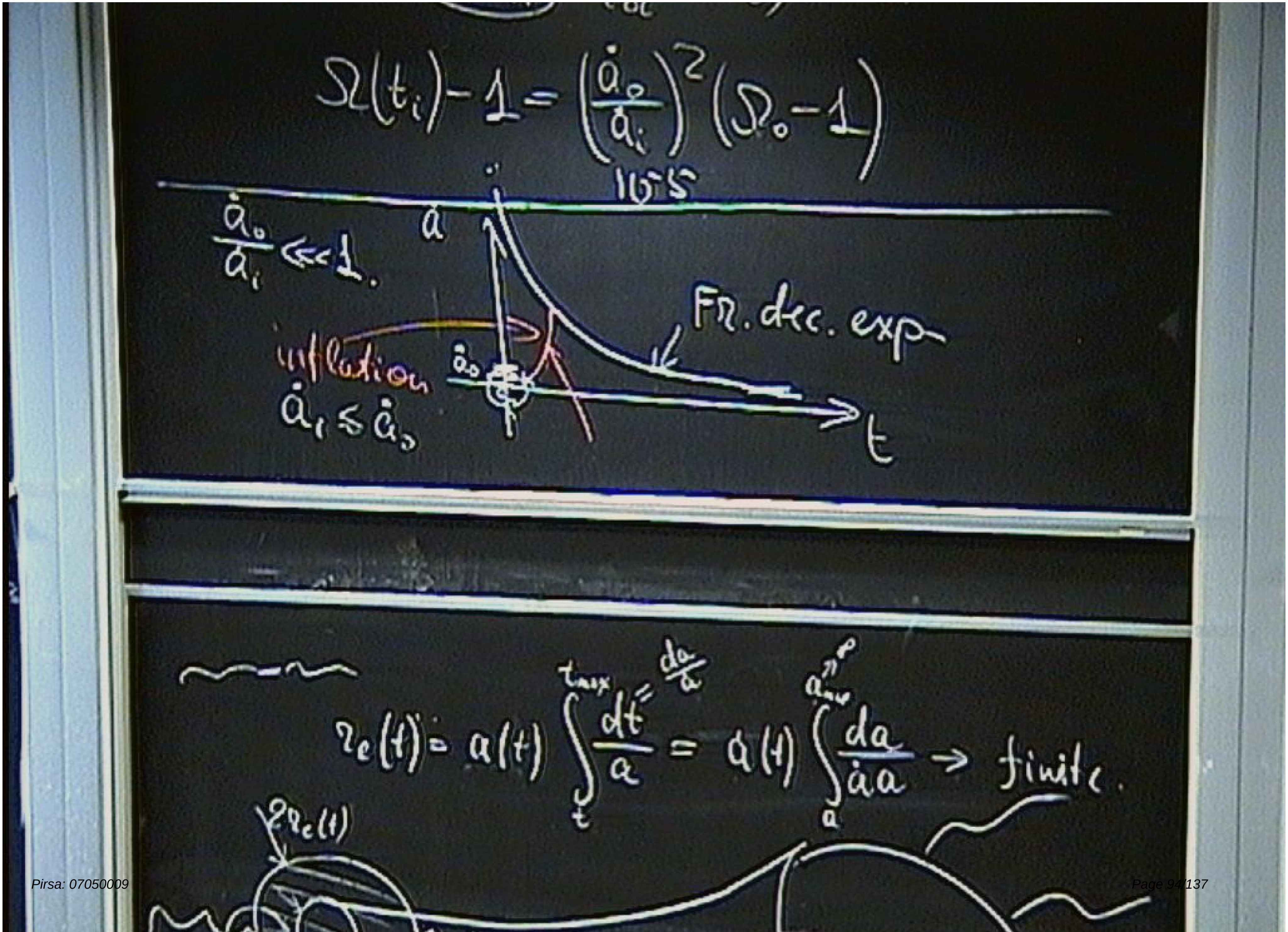
ct_i

$$\frac{\delta \varepsilon}{\varepsilon} \sim 1$$



$\frac{\delta \varepsilon}{\varepsilon} \sim 1$.

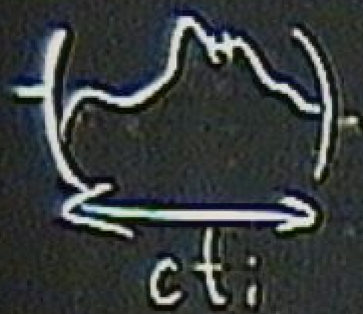
$\dot{a}_i \ll \dot{a}_0$


Ω(t_i) − 1 = (ȧ_0 / ȧ_i)^2 (Ω_0 − 1)
10^-5
ȧ_0 / ȧ_i <<< 1.
ȧ
Fr. dec. exp
inflation
ȧ_0
ȧ_1 ≤ ȧ_0
t
r_e(t) = a(t) ∫_t^{t_max} dt'/a = a(t) ∫_a^{a_max} da/(ȧ a) → finite.
2 r_e(t)

$$\frac{\delta \varepsilon}{\varepsilon} \sim 1$$

$$\boxed{\dot{a}_i \lll \dot{a}_0}$$

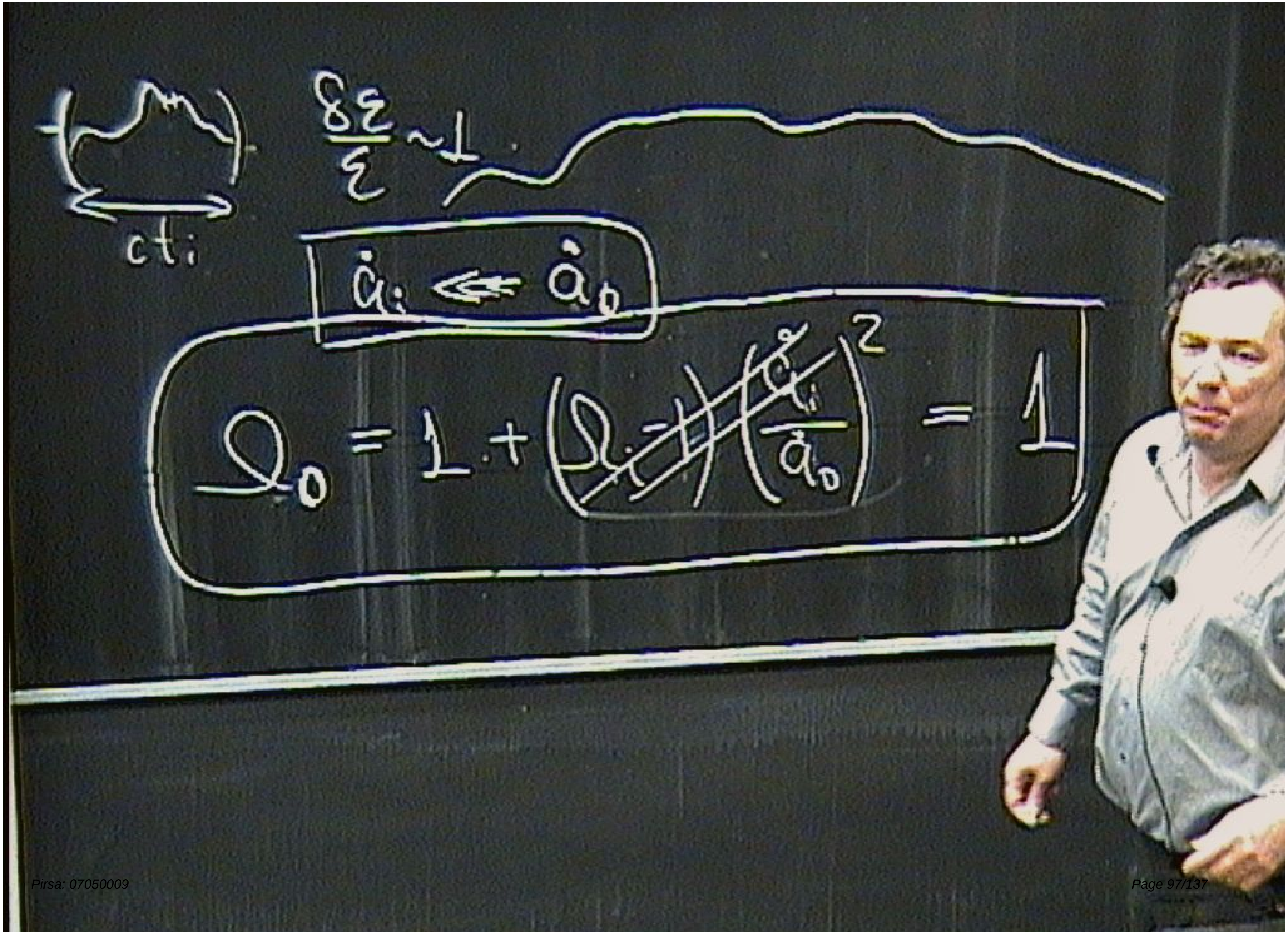
$$\Omega_0 = 1 + (\Omega_i - 1)\left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2$$

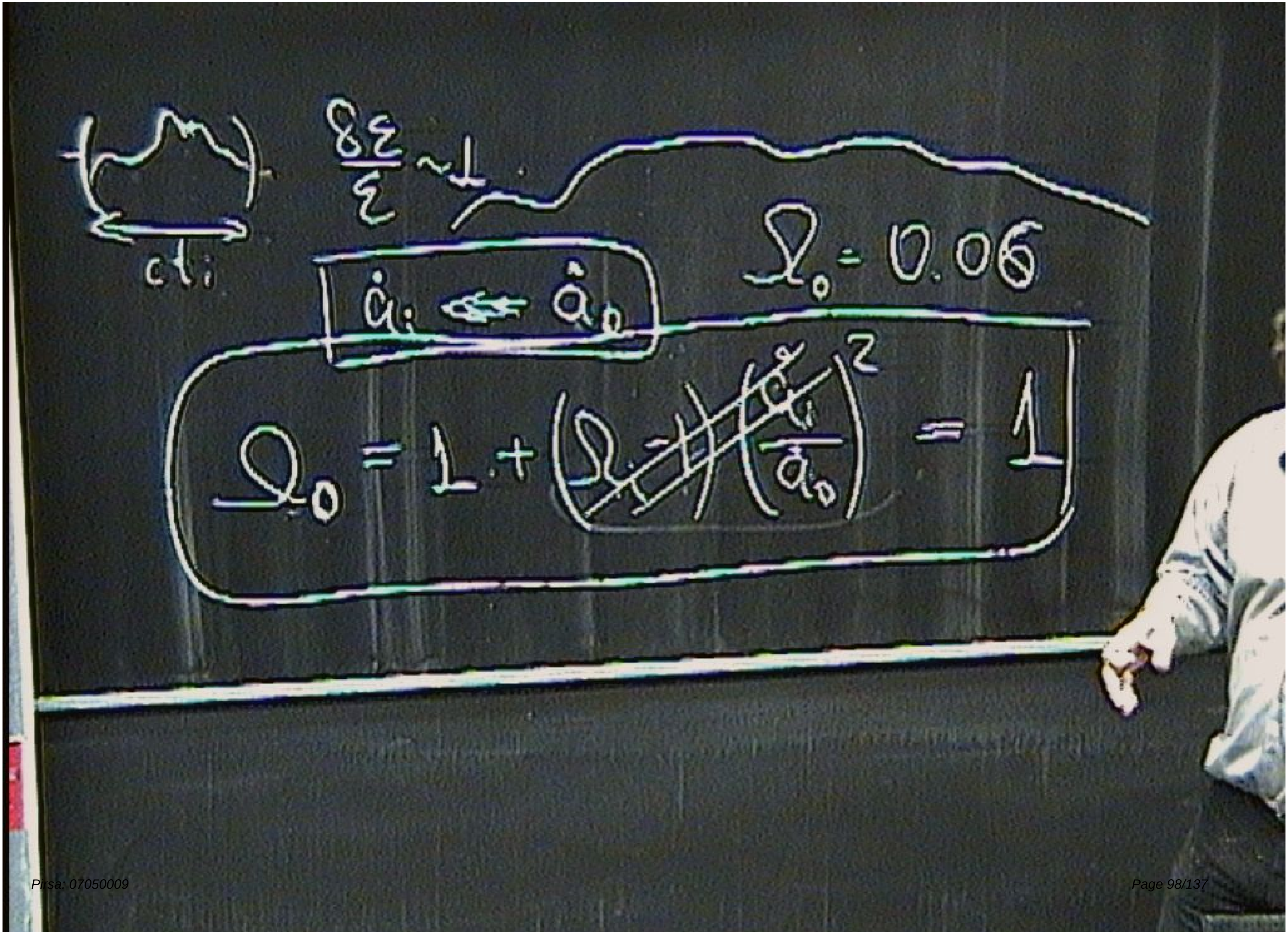


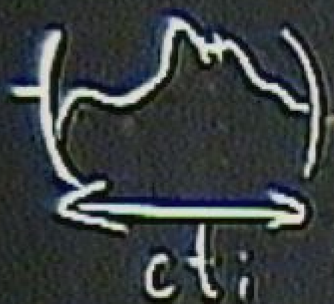
$$\frac{\delta \varepsilon}{\varepsilon} \sim 1$$

$$\boxed{\dot{a}_i \ll \dot{a}_0}$$

$$\Omega_0 = 1 + \cancel{(\Omega_i - 1)}\left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2 = 1$$


δε/ε ~ 1
ct_i
ȧ_i ⋘ ȧ_0
Ω_0 = 1 + (Ω_i − 1)(ȧ_i/ȧ_0)^2 = 1


δε/ε ~ 1
ct_i
ȧ_i ⇐ ȧ_0
Ω_0 = 0.06
Ω_0 = 1 + (Ω_i − 1)(ȧ_i/ȧ_0)^2 = 1



$\frac{\delta\varepsilon}{\varepsilon} \sim 1$

0.2–0.3

$\Omega_0 = 0.06$

$\boxed{\dot{a}_i \lll \dot{a}_0}$

$$\boxed{\Omega_0 = 1 + \cancel{(\Omega_i - 1)\left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2} = 1}$$

$\frac{\delta\varepsilon}{\varepsilon} \sim 1$

$\Omega_0 = \frac{0.2 - 0.3}{0.06}$

$\boxed{\dot{a}_i \lll \dot{a}_0}$

$$\boxed{\Omega_0 = 1 + \cancel{(\Omega_i - 1)}\left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2 = 1}$$

How can gr. become repulsive

How can gr. become repulsive

$$\ddot{a} = -\frac{4\pi G}{3}(\varepsilon + 3p)\,a$$

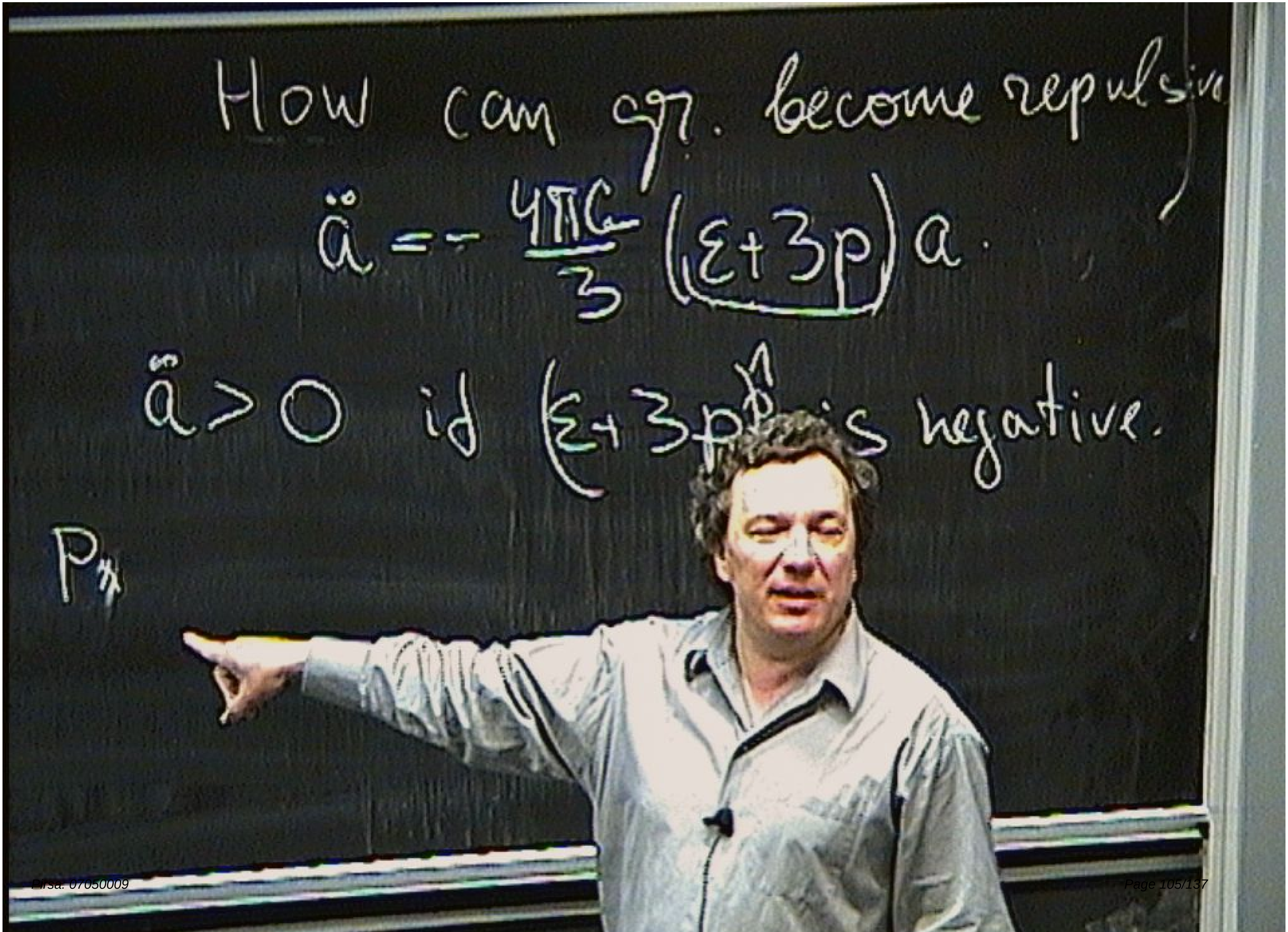
How can gr. become repulsive

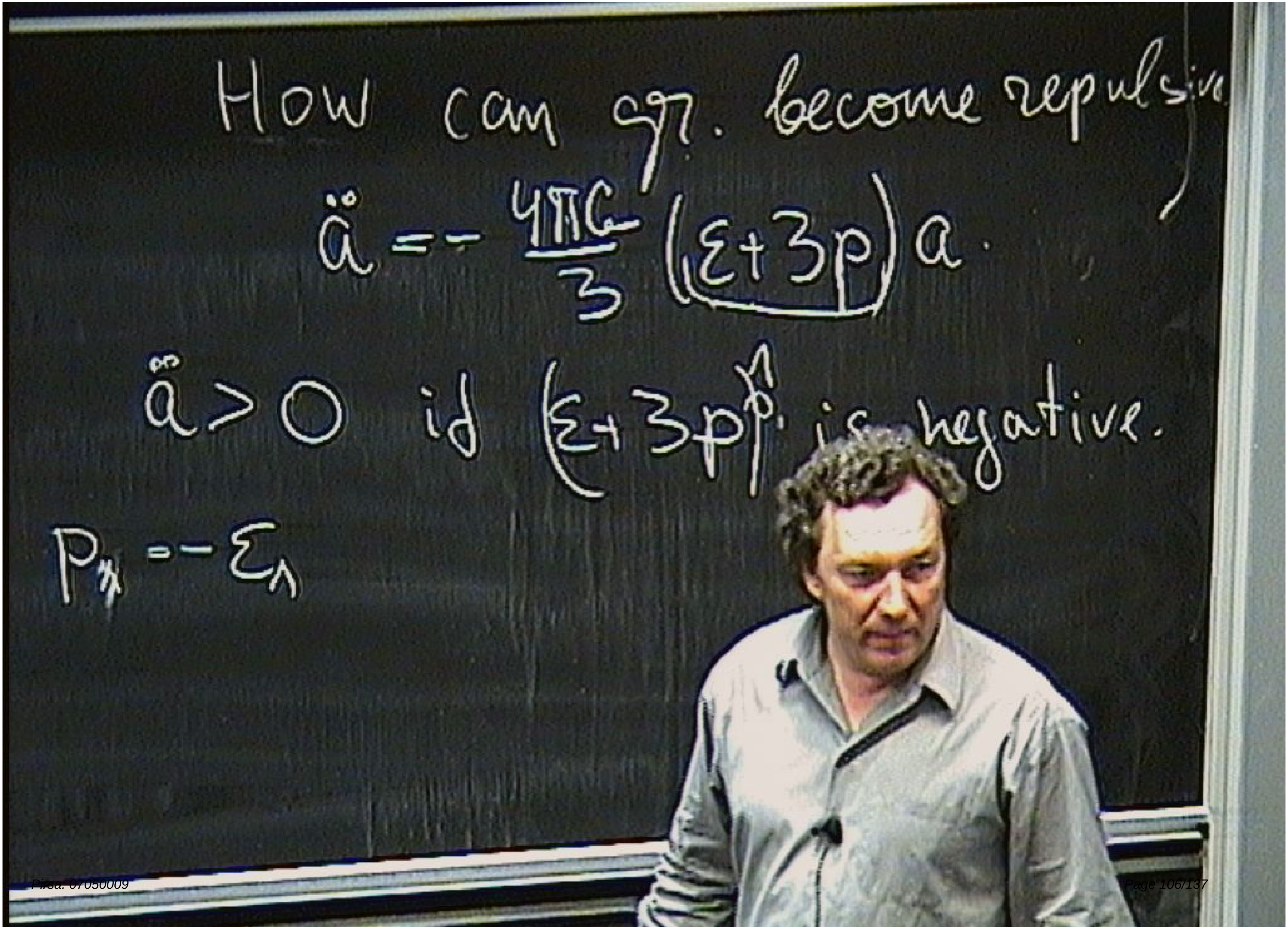
$$\ddot{a} = -\frac{4\pi G}{3}\underbrace{(\varepsilon + 3p)}_{\overset{\uparrow}{0}}a.$$

How can gr. become repulsive

$$\ddot{a} = -\frac{4\pi G}{3}\underbrace{(\varepsilon + 3p)}\,a.$$

$\ddot{a} > 0$ if $(\varepsilon + 3p)$ is negative.


How can gr. become repulsive
ä = - 4πG/3 (ε+3p) a.
ä > 0 if (ε+3p) is negative.


How can gr. become repulsive
$\ddot{a} = -\frac{4\pi G}{3}(\varepsilon + 3p)a$.
$\ddot{a} > 0$ if $(\varepsilon + 3p)$ is negative.
$p_\Lambda = -\varepsilon_\Lambda$

How can gr. become repulsive

$$\ddot{a} = -\frac{4\pi G}{3}(\varepsilon + 3p)\,a.$$

$\ddot{a} > 0$ if $(\varepsilon + 3p)$ is negative.

$p_\Lambda = -\varepsilon_\Lambda \qquad \varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda$

How can G.R. become repulsive

$$\ddot{a} = -\frac{4\pi G}{3}(\varepsilon + 3p)\,a$$

$\ddot{a} > 0$ if $(\varepsilon + 3p)$ is negative.

$p_\Lambda = -\varepsilon_\Lambda$ $\qquad \varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda < 0$

a

How can gr. become repulsive

$$\ddot{a} = -\frac{4\pi G}{3}(\varepsilon + 3p)\,a.$$

$\ddot{a} > 0$ if $(\varepsilon + 3p)$ is negative.

$p_\Lambda = -\varepsilon_\Lambda \qquad \varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda < 0$

$$a \propto e^{H_\Lambda t}$$

How can GR become repulsive

$$\ddot{a} = -\frac{4\pi G}{3}\underbrace{(\varepsilon + 3p)}\, a$$

$\ddot{a} > 0$ if $(\varepsilon + 3p)$ is negative.

$P_\Lambda = -\varepsilon_\Lambda \qquad \varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda < 0$

$a \propto e^{H_\Lambda t} \qquad \dot{a} \propto a$

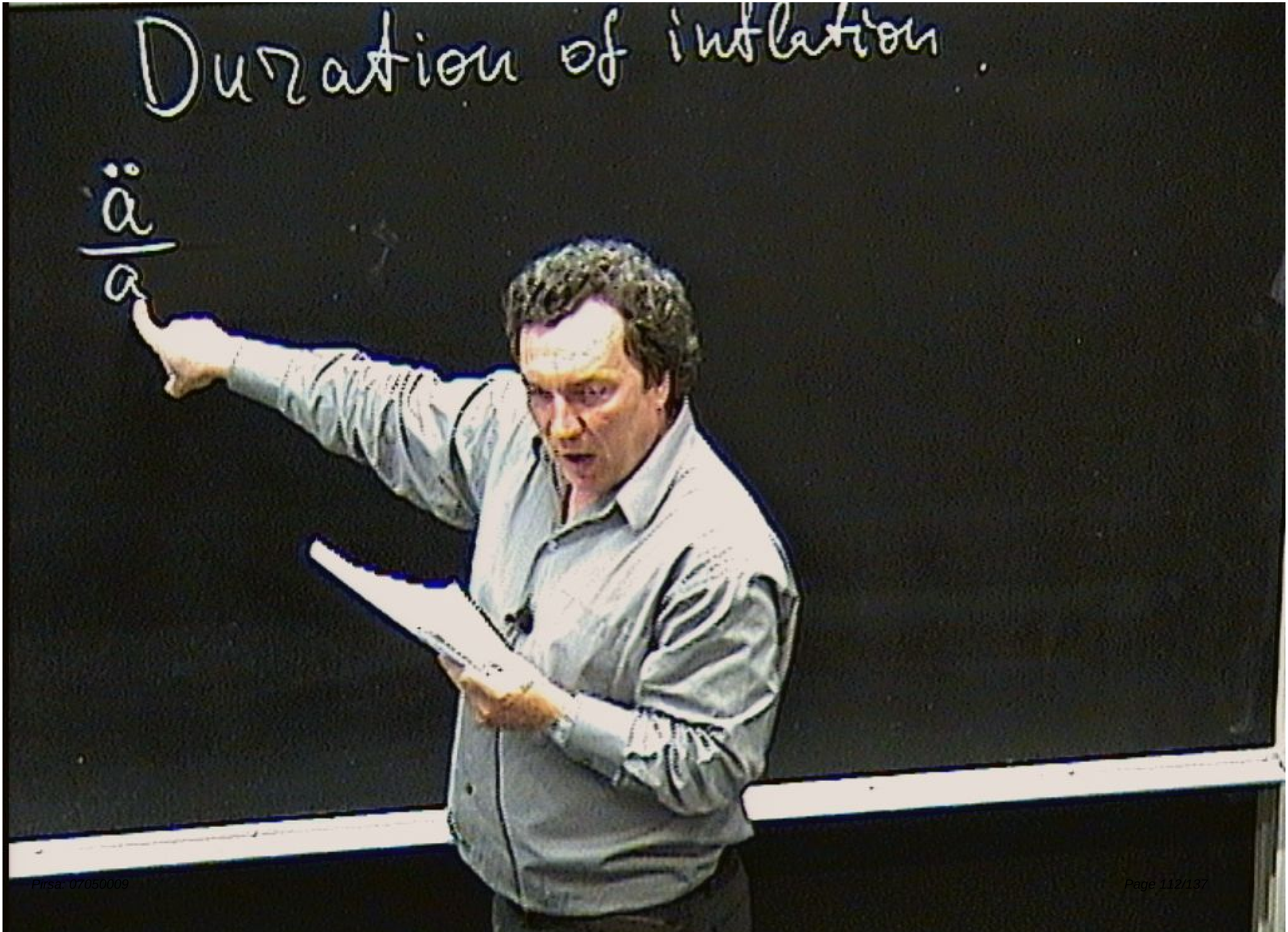
How can gr. become repulsive

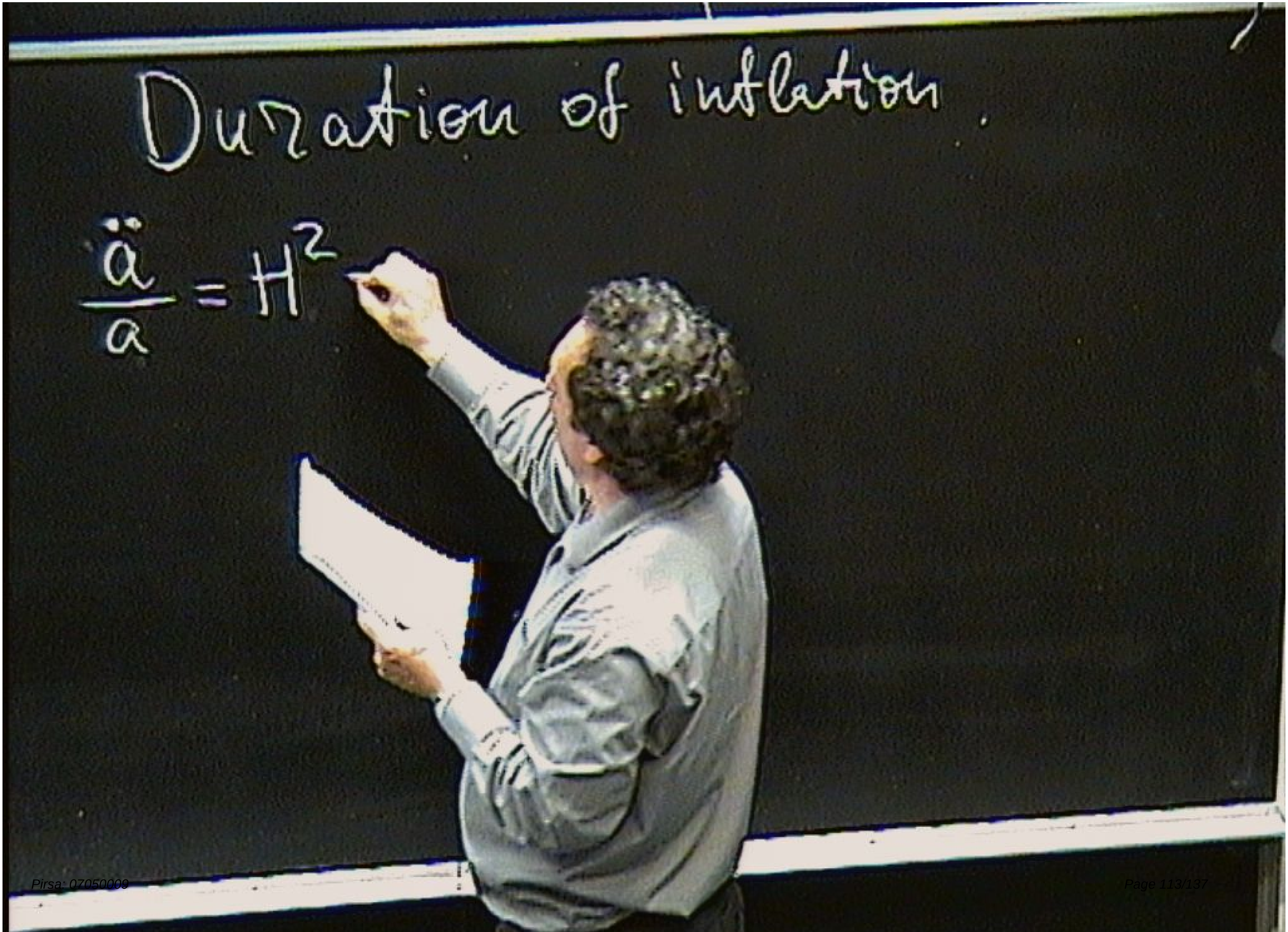
$$\ddot{a} = -\frac{4\pi G}{3}\underbrace{(\varepsilon + 3p)}\, a$$

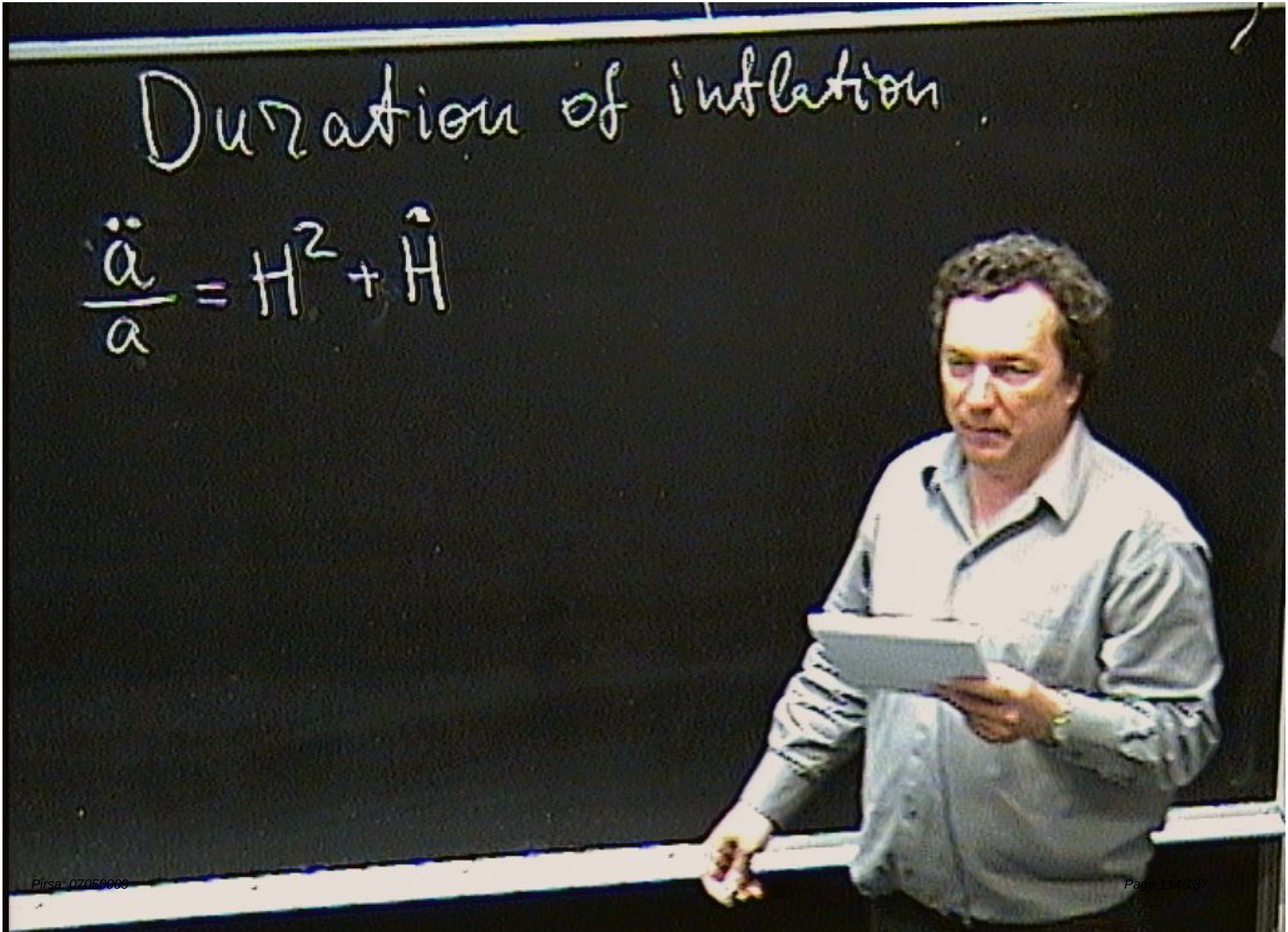
$\ddot{a} > 0$ if $(\varepsilon + 3p)$ is negative.

$p_\Lambda = -\varepsilon_\Lambda$ $\qquad \varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda < 0$

$$a \propto e^{H_\Lambda t} \qquad \dot{a} \propto a$$

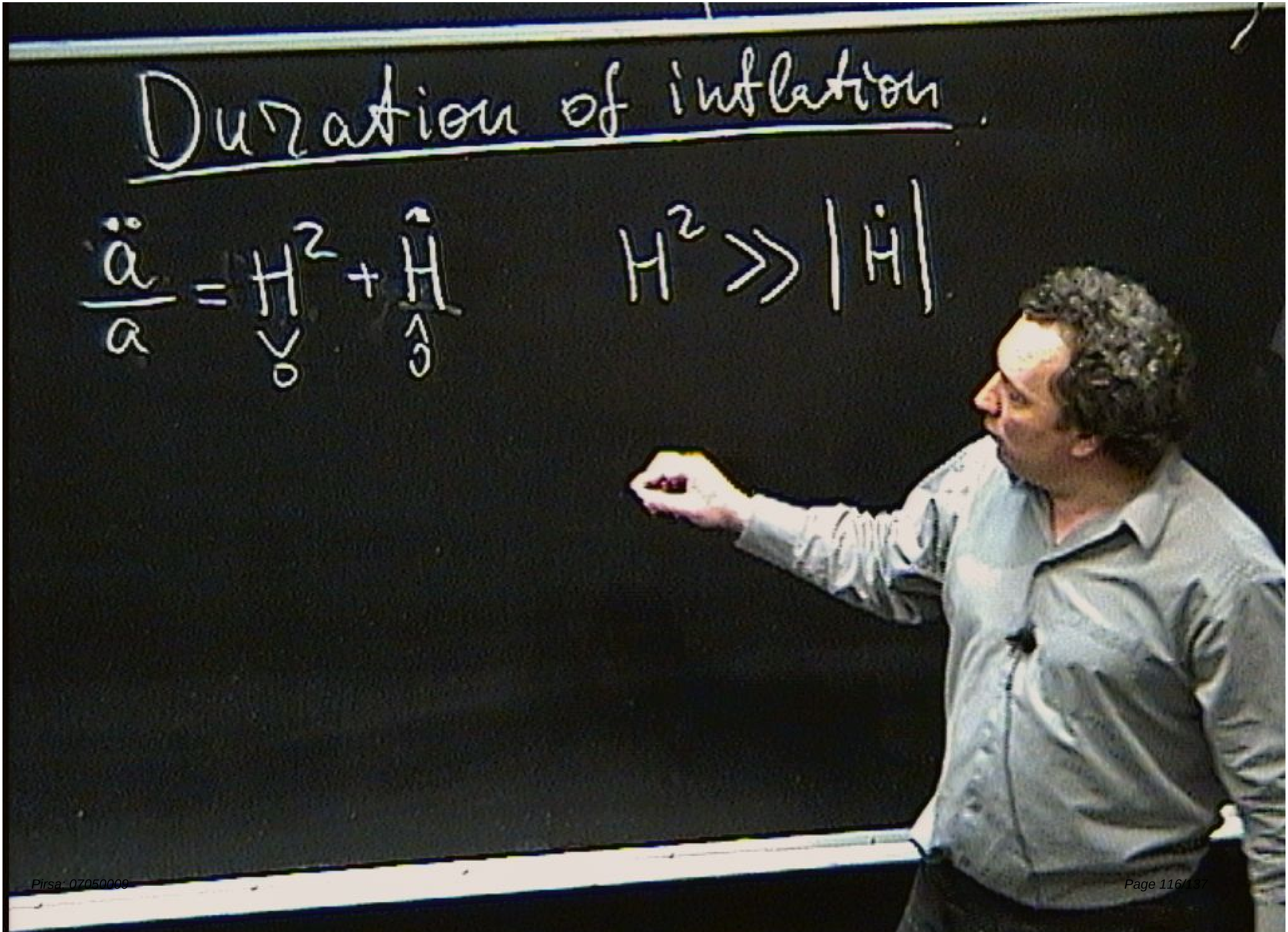

Duration of inflation.
ä
a

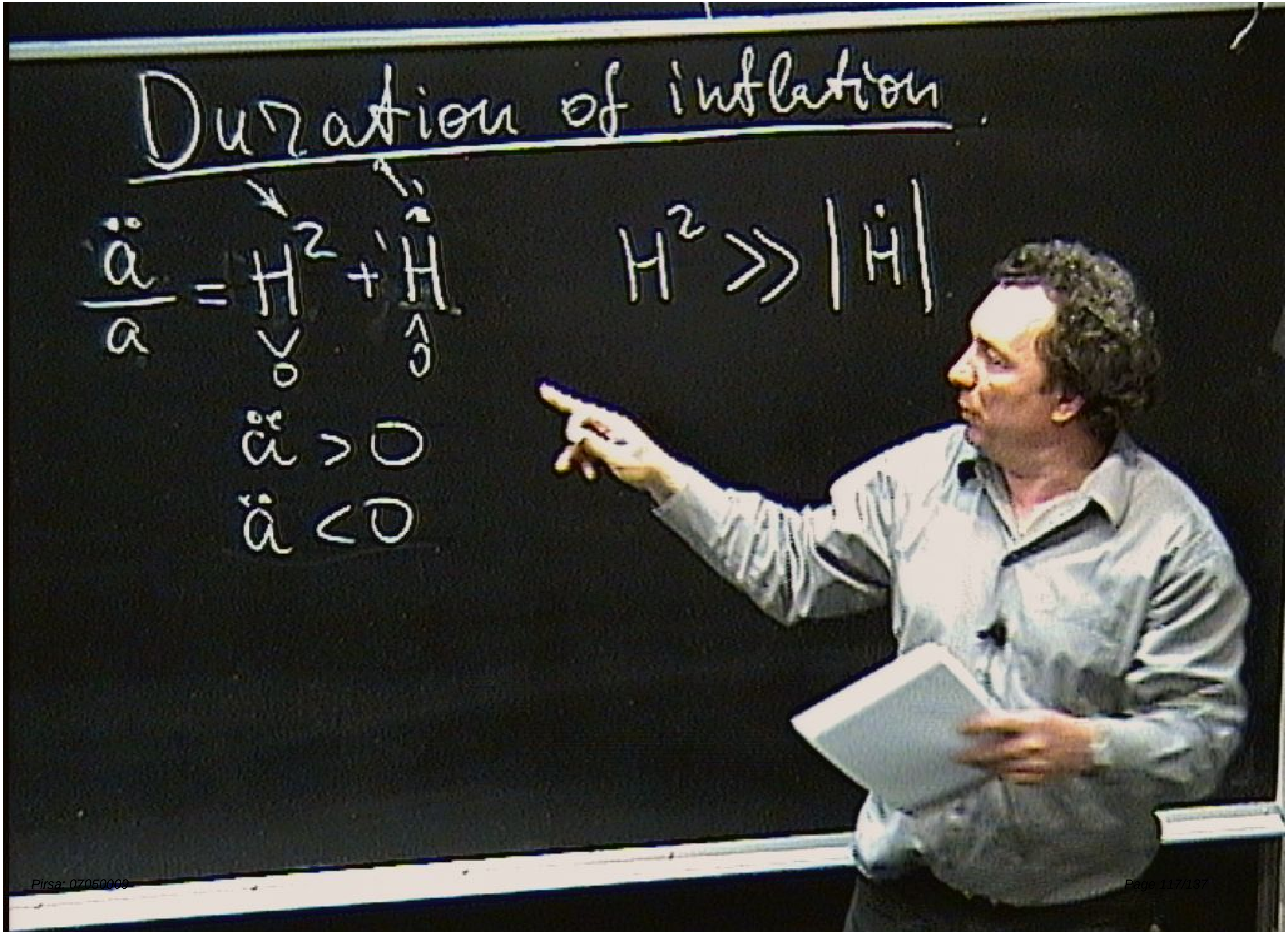

Duration of inflation.
$\frac{\ddot{a}}{a} = H^2$

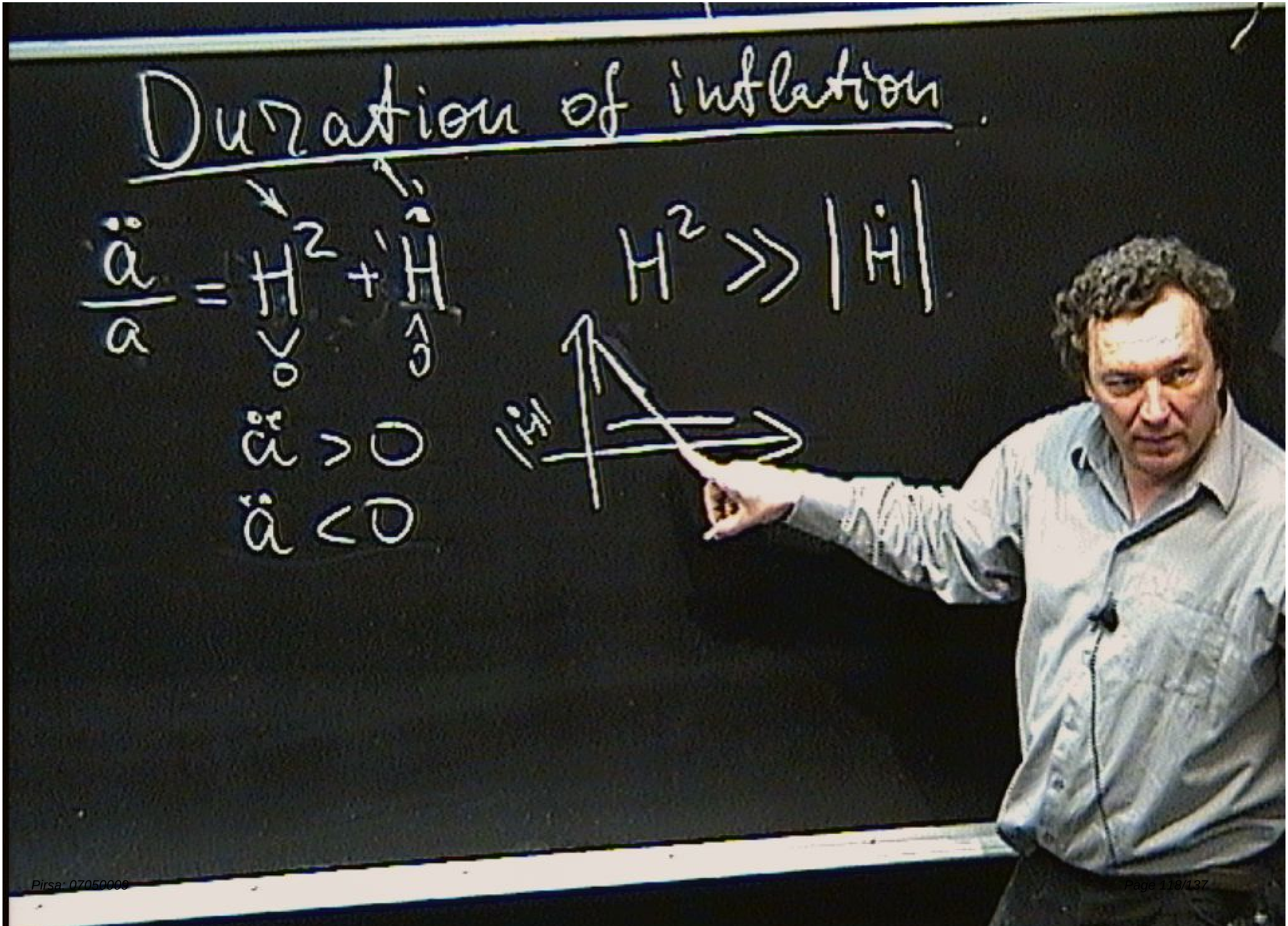

Duration of inflation
$\frac{\ddot{a}}{a} = H^2 + \dot{H}$

Duration of inflation

$$\frac{\ddot{a}}{a} = \underset{\overset{\vee}{0}}{H^2} + \dot{H}$$


Duration of inflation
$\frac{\ddot{a}}{a} = H^2 + \dot{H}$
$H^2 \gg |\dot{H}|$


Duration of inflation
ä/a = H² + Ḣ
H² >> |Ḣ|
ä > 0
ä < 0



Duration of inflation
$\frac{\ddot{a}}{a} = H^2 + \dot{H}$
$\ddot{a} > 0$
$\ddot{a} < 0$
$H^2 \gg |\dot{H}|$
$|\dot{H}|$

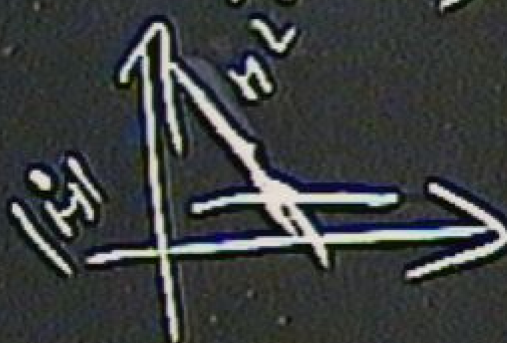
Duration of inflation

$$\frac{\ddot a}{a} = \underset{\substack{\vee \\ 0}}{H^2} + \underset{\substack{\wedge \\ 0}}{\dot H}$$

$$H^2 \gg |\dot H|$$

$\ddot a > 0$

$\ddot a < 0$



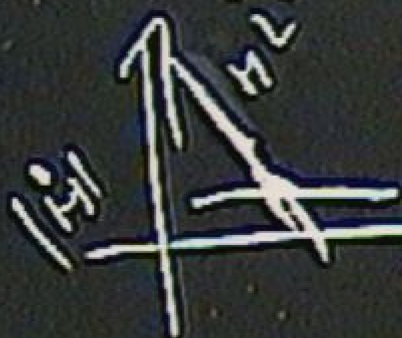
$t_f \sim$

Duration of inflation

$$\frac{\ddot a}{a} = \underset{>0}{H^2} + \underset{<0}{\dot H} \qquad H^2 \gg |\dot H|$$

$\ddot a > 0$

$\ddot a < 0$



$$\Rightarrow \quad t_f \sim \frac{H_i}{\dot H_i}$$

Duration of inflation

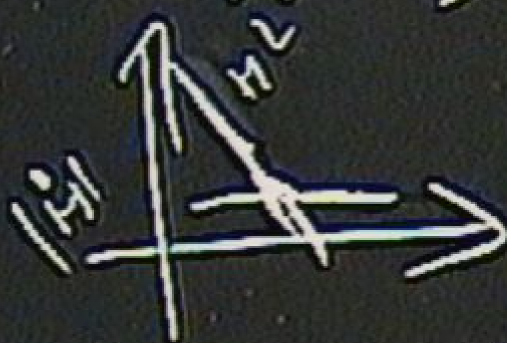
$$\frac{\ddot a}{a} = H^2 + \dot H$$

$H^2 > 0$, $\dot H < 0$

$\ddot a > 0$

$\ddot a < 0$

$$H^2 \gg |\dot H|$$



$$t_f \sim \frac{H_i}{\dot H_i}$$

$|\dot H| \ll H^2 \qquad a \propto e^{H_i t}$

Duration of inflation

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} \qquad H^2 \gg |\dot{H}|$$

$\ddot{a} > 0 \quad \Longrightarrow \quad t_f \sim \frac{H_i}{\dot{H}_i}$

$\ddot{a} < 0$

$|\dot{H}| \ll H^2 \quad a \propto e^{H_i t}$

$$\frac{a_f}{a_i}$$

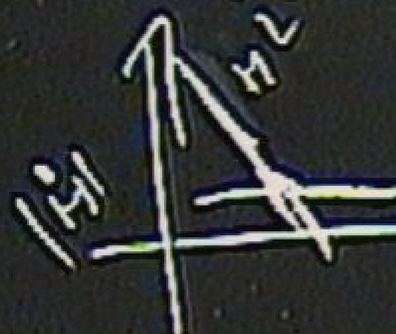
Duration of inflation

$$\frac{\ddot a}{a} = \underset{\vee \atop 0}{H^2} + \underset{\wedge \atop 0}{\dot H}$$

$\ddot a > 0$

$\ddot a < 0$

$$H^2 \gg |\dot H|$$



$$\Rightarrow \; t_f \sim \frac{H_i}{\dot H_i}$$

$|\dot H| \ll H^2 \qquad a \propto e^{H_i t}$

$$\frac{a_f}{a_i} \approx \frac{\dot a_f}{\dot a_i}$$

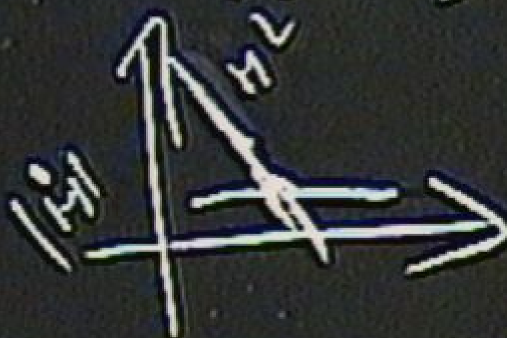
Duration of inflation

$$\frac{\ddot{a}}{a} = \underset{>0}{H^2} + \underset{\partial}{\dot{H}}$$

$\ddot{a} > 0$

$\ddot{a} < 0$

$$H^2 \gg |\dot{H}|$$



$$t_f \sim \frac{H_i}{\dot{H}_i}$$

$|\dot{H}| \ll H^2 \quad a \sim e^{H_i t}$

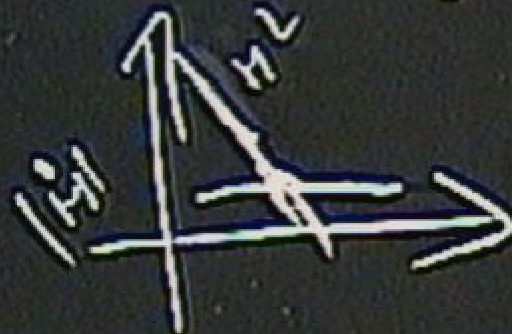
$$\frac{a_f}{a_i} \approx \frac{\dot{a}_f}{\dot{a}_i} = e^{H_i t_f}$$

Duration of inflation

$$\frac{\ddot a}{a} = \underset{\vee\atop 0}{H^2} + \underset{\wedge\atop 0}{\dot H} \qquad H^2 \gg |\dot H|$$

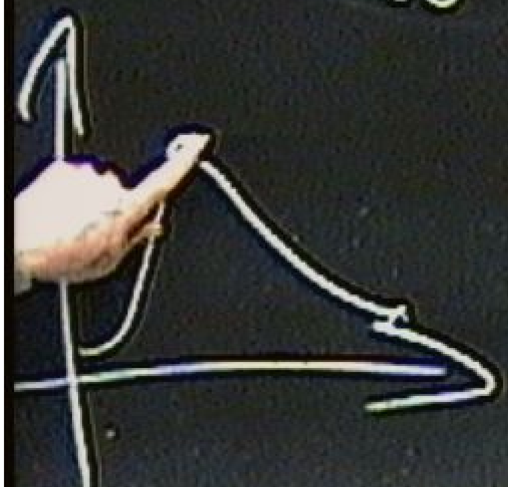
$\ddot a > 0$

$\ddot a < 0$



$$t_f \sim \frac{H_i}{\dot H_i}$$

$|\dot H| \ll H^2 \qquad a \sim e^{H_i t}$



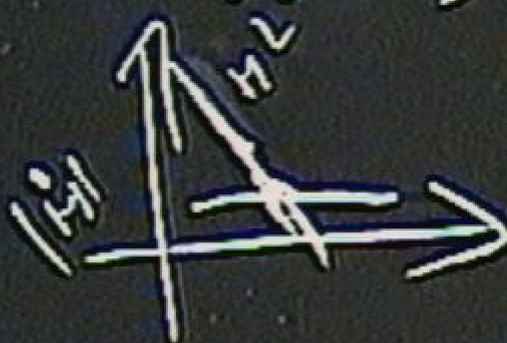
$$\frac{a_f}{a_i} \approx \frac{\dot a_f}{\dot a_i} = e^{H_i t_f}$$

Duration of inflation

$$\frac{\ddot{a}}{a} = \underset{>0}{H^2} + \underset{\gtrless 0}{\dot{H}} \qquad H^2 \gg |\dot{H}|$$

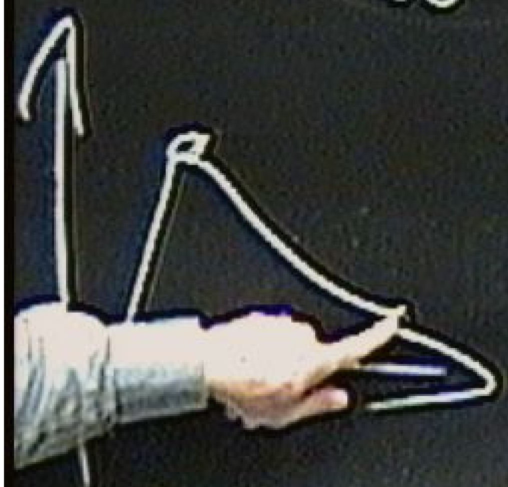
$$\ddot{a} > 0$$

$$\ddot{a} < 0$$

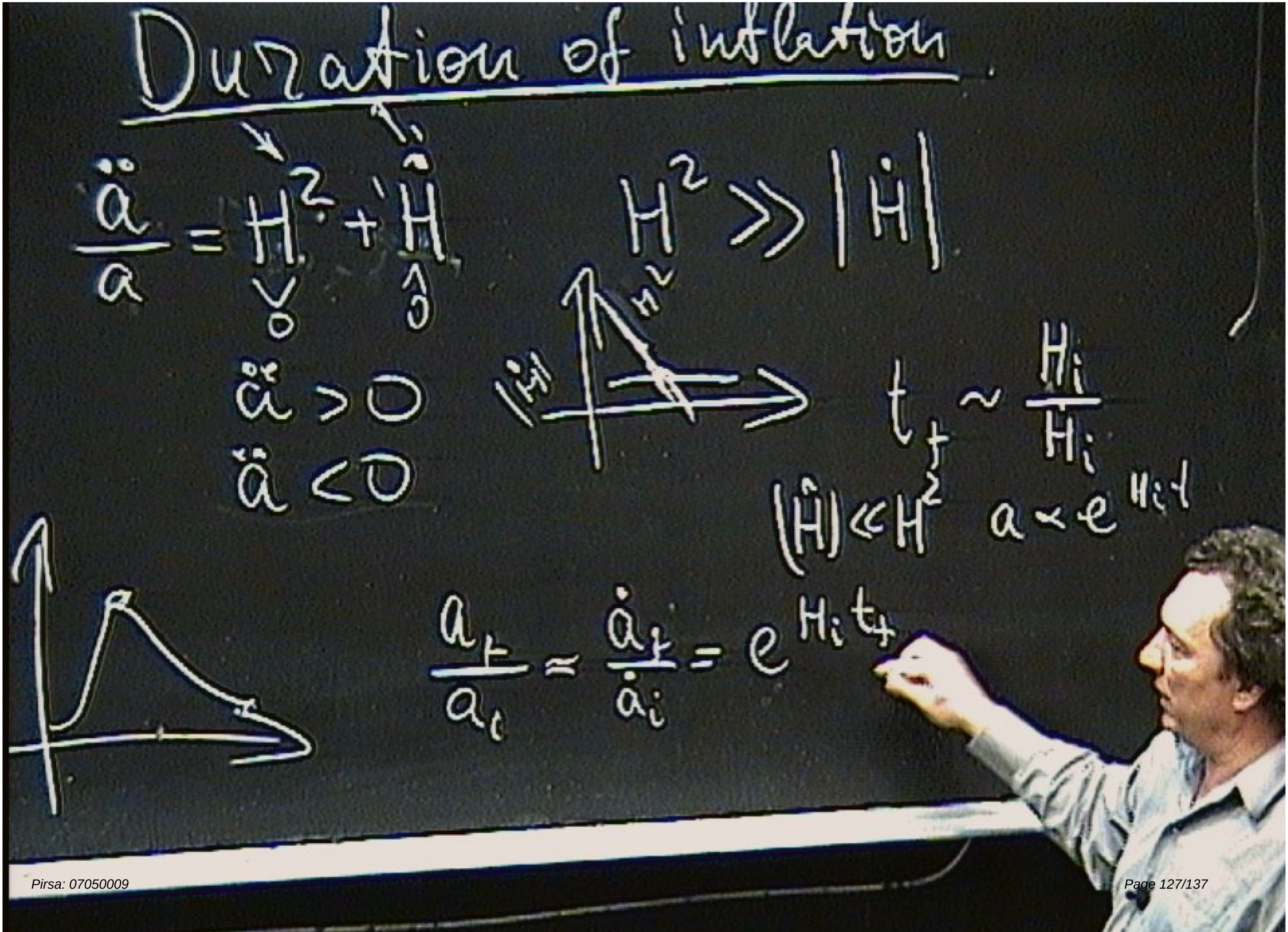


$$t_f \sim \frac{H_i}{H_i}$$

$$|\dot{H}| \ll H^2 \qquad a \propto e^{H_i t}$$



$$\frac{a_f}{a_i} \approx \frac{\dot{a}_f}{\dot{a}_i} = e^{H_i t_f}$$

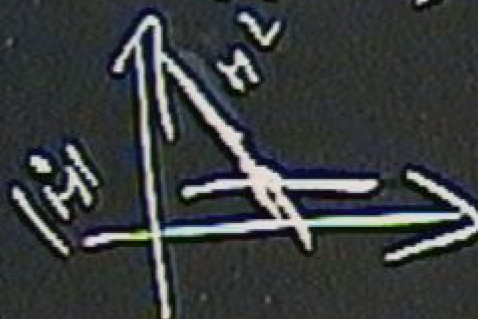

Duration of inflation
ä/a = H² + Ḣ
H² >> |Ḣ|
ä > 0
ä < 0
t_f ~ H_i/Ḣ_i
|Ḣ| << H²
a ~ e^{H_i t}
a_f/a_i ≈ ȧ_f/ȧ_i = e^{H_i t_f}

Duration of inflation

$$\frac{\ddot a}{a} = H^2 + \dot H \qquad H^2 \gg |\dot H|$$

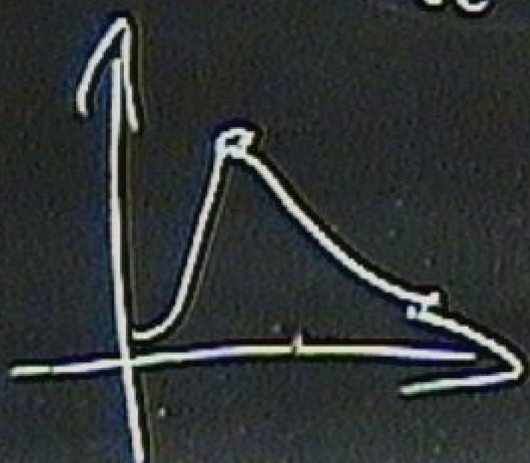
$\ddot a > 0$

$\ddot a < 0$



$$t_f \sim \frac{H_i}{\dot H_i}$$

$|\dot H| \ll H^2 \qquad a \propto e^{H_i t}$



$$\frac{a_f}{a_i} \simeq \frac{\dot a_f}{\dot a_i} = e^{H_i t_f} > 10^{28}$$

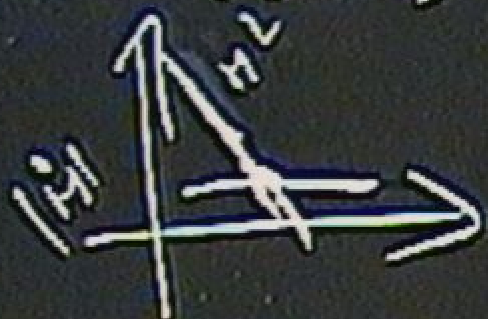
$$t_f > 70\, H_i^{-1}$$

Duration of inflation

$$\frac{\ddot a}{a} = H^2 + \dot H \qquad H^2 \gg |\dot H|$$

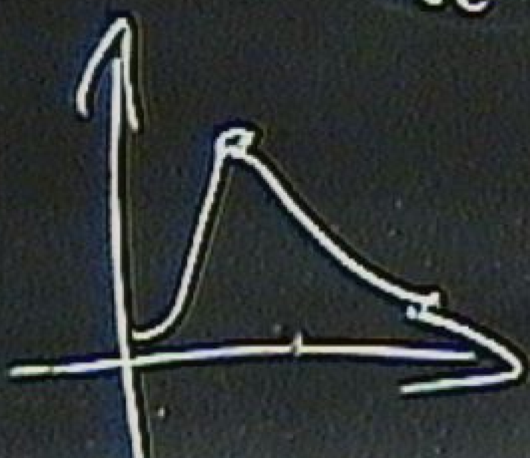
$\ddot a > 0$

$\ddot a < 0$



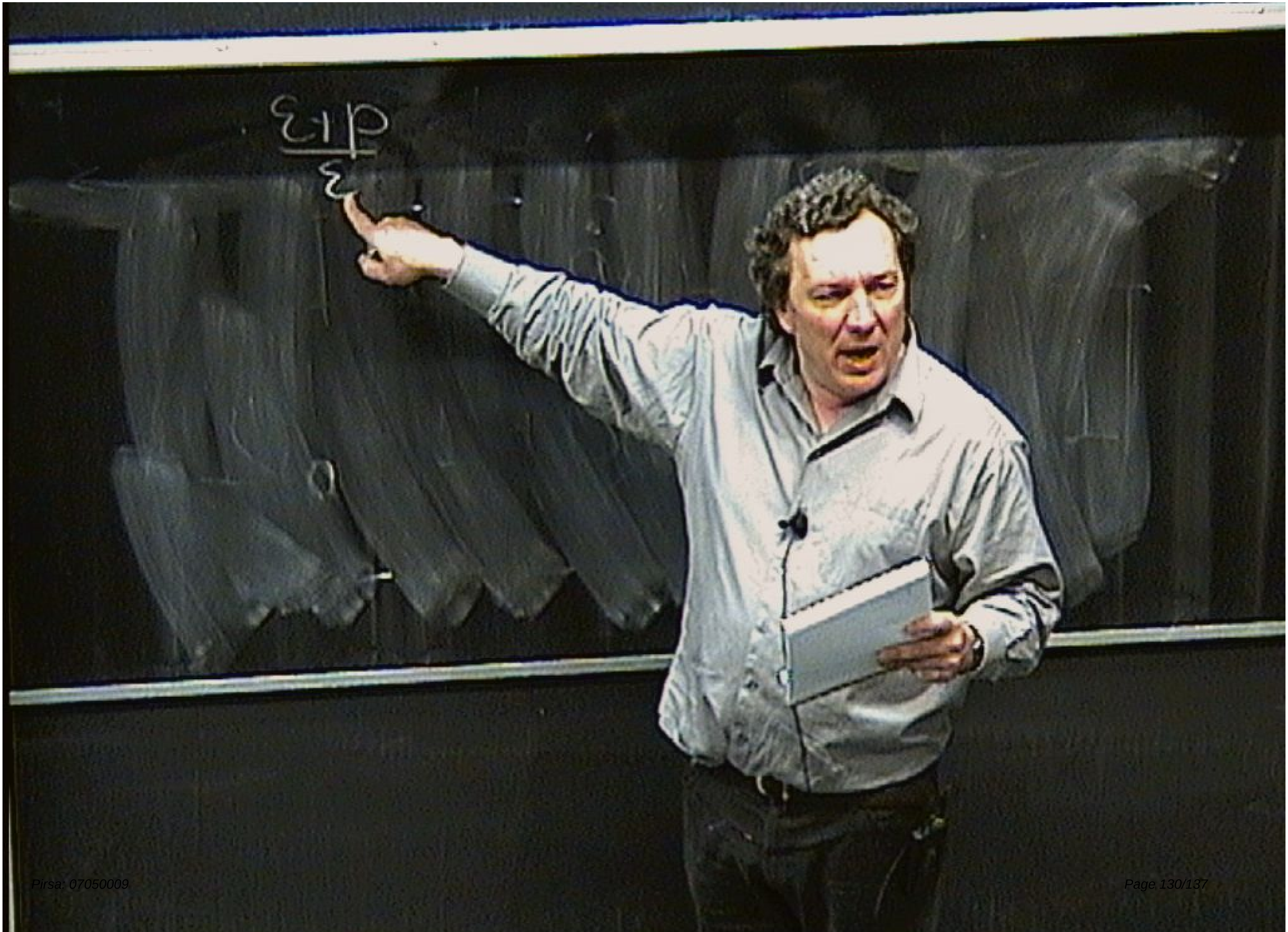
$$t_f \sim \frac{H_i}{H_i}$$

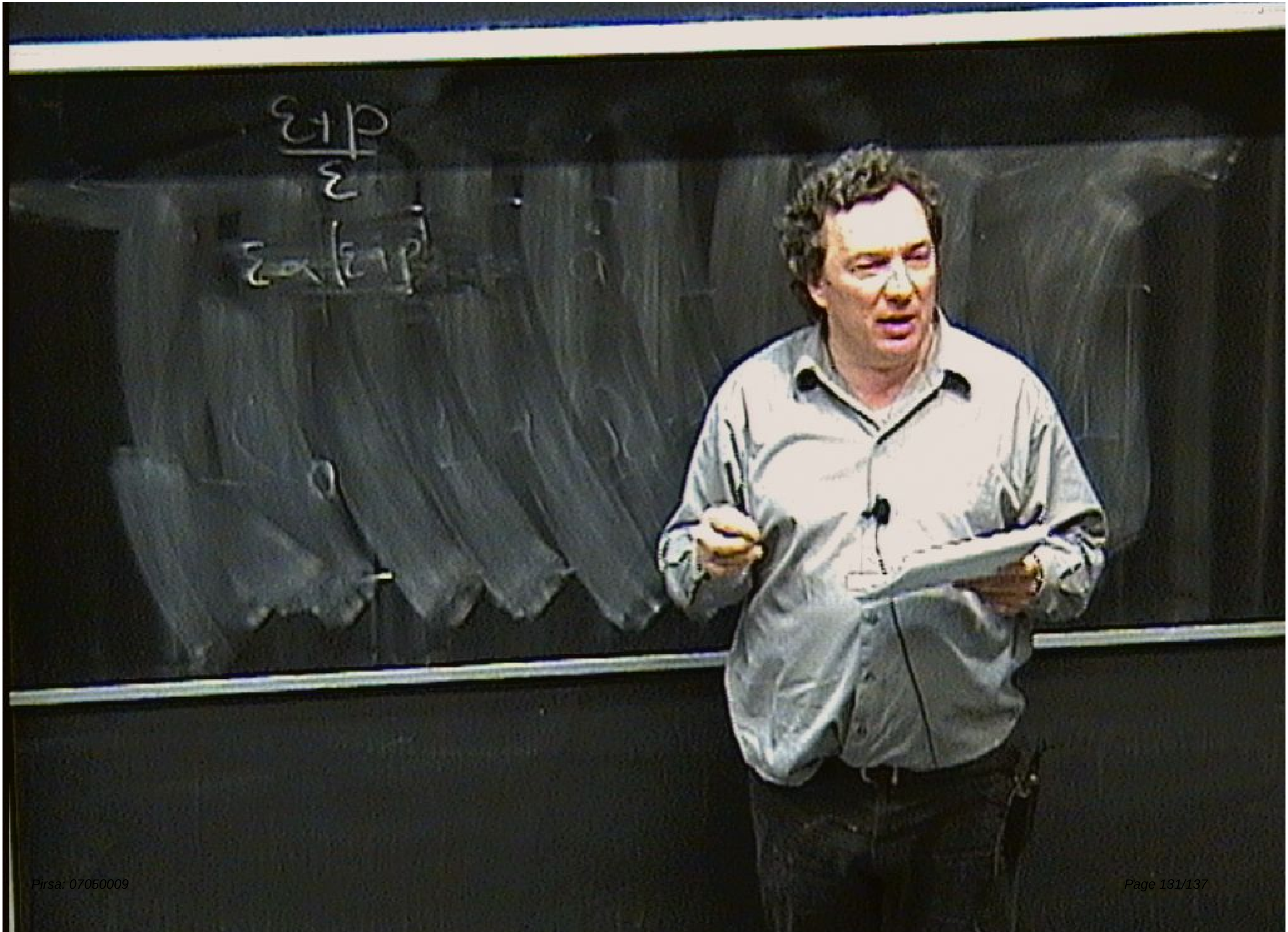
$|\dot H| \ll H^2 \qquad a \propto e^{H_i t}$

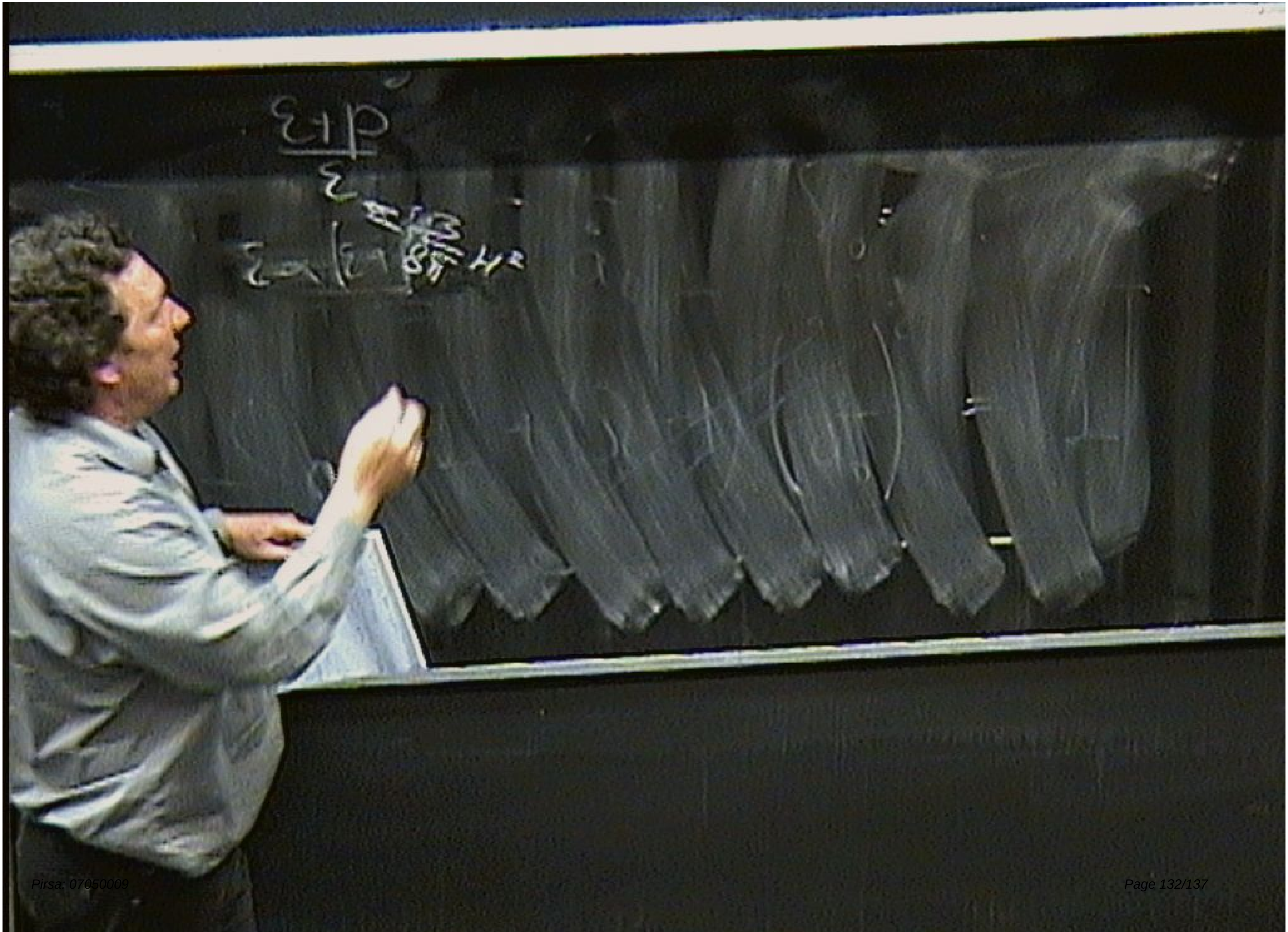


$$\frac{a_f}{a_i} \simeq \frac{\dot a_f}{\dot a_i} = e^{H_i t_f} > 10^{28}$$

$$t_f > 70\, H_i^{-1}$$


$\frac{\varepsilon \, p}{\varepsilon}$





Duration

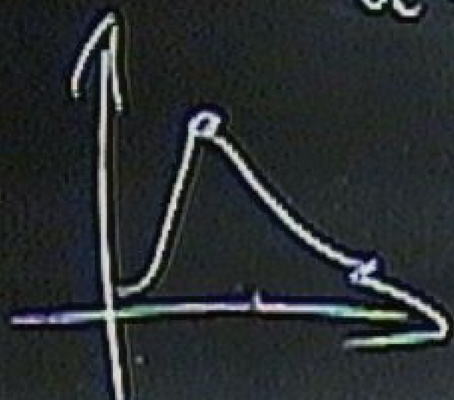
$$\frac{\ddot a}{a} = H^2 + \dot H \qquad H^2 \gg |\dot H|$$

$\ddot a > 0$

$\ddot a < 0$

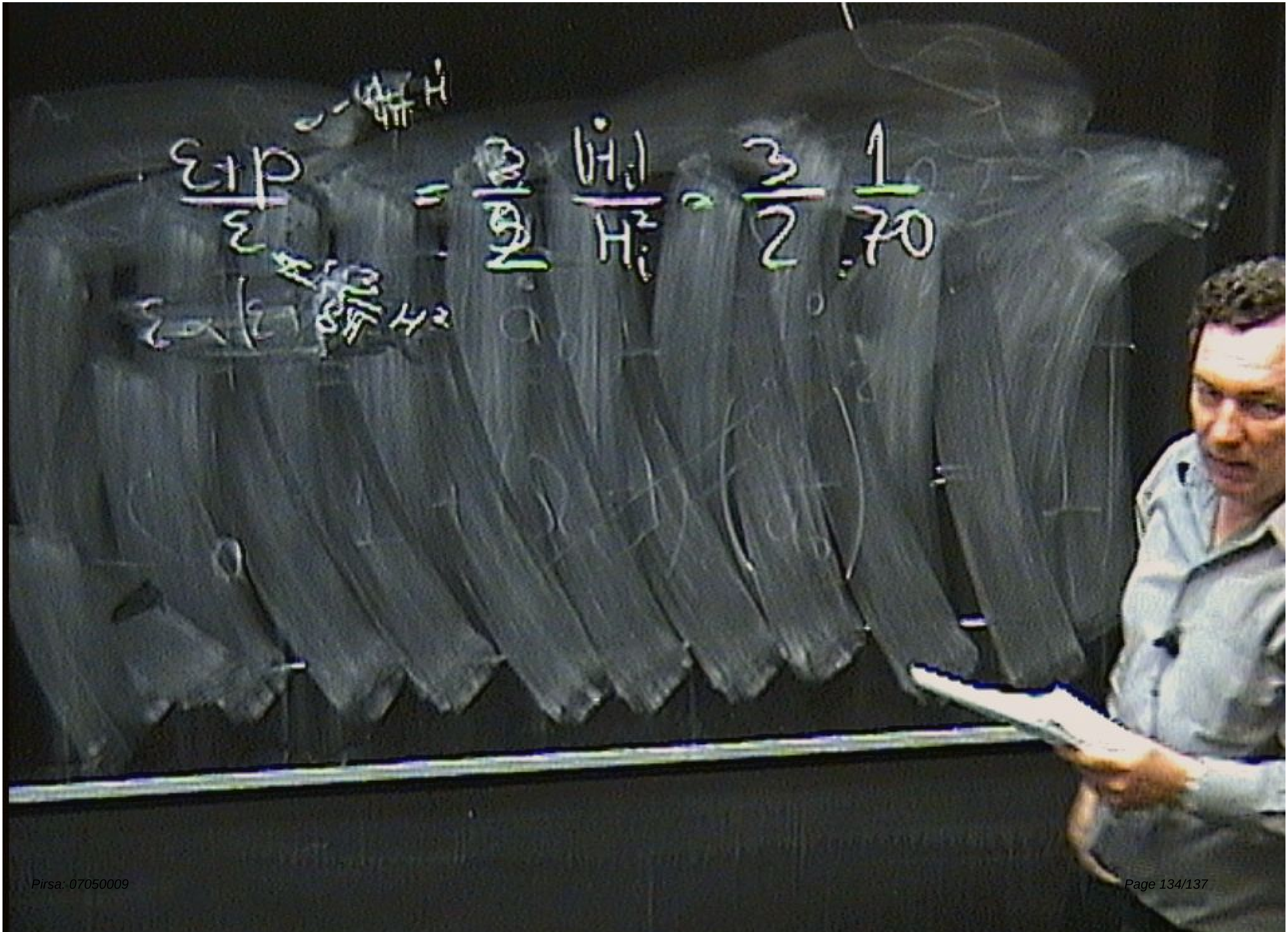
$$t_f \sim \frac{H_i}{\dot H_i} \sim 70 \frac{1}{H_i}$$

$|\dot H| \ll H^2 \qquad a \propto e^{H_i t}$



$$\frac{a_f}{a_i} \propto \frac{\dot a_f}{\dot a_i} = e^{H_i t_f} > 10^{28}$$

$$t_f > 70\, H_i^{-1}$$


$\frac{3}{2} \cdot \frac{1}{70}$

$$\frac{\varepsilon+p}{\varepsilon} = \frac{2}{3}\frac{\dot H_i}{H_i^2} \sim \frac{3}{2}\,\frac{1}{70} \sim 10^{-2}$$

$$\varepsilon \propto H^2$$

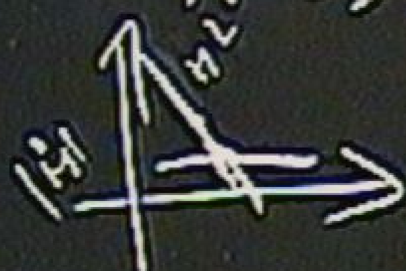
$p_\Lambda = -\varepsilon_\Lambda \qquad \varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda < 0$

Duration of inflation

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} \qquad H^2 \gg |\dot{H}|$$

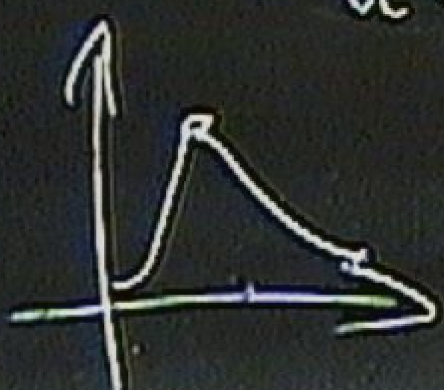
$\ddot{a} > 0$

$\ddot{a} < 0$



$$t_f \sim \frac{H_i}{\dot{H}_i} \sim 70 \frac{1}{H_i}$$

$|\dot{H}| \ll H^2 \qquad a \propto e$



$$\frac{a_f}{a_i} = \frac{\dot{a}_f}{\dot{a}_i} = e^{H_i t_f} > 10^{28}$$

$$t_f > 70\, H_i^{-1}$$

$$\frac{\ddot a}{a} = \underset{>0}{H^2} + \underset{\gtrless 0}{\dot H} \qquad H^2 \gg |\dot H|$$

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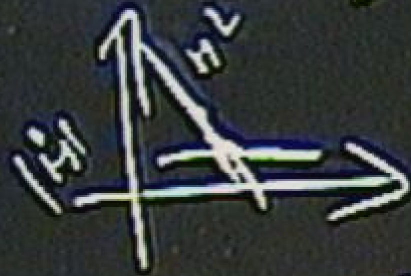
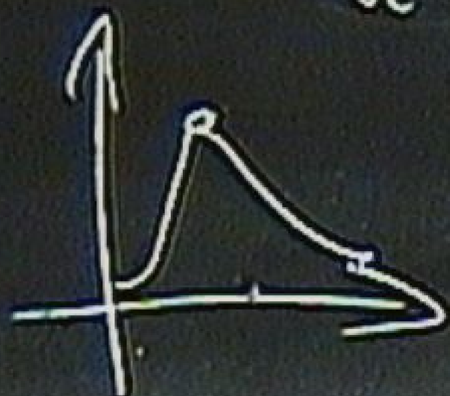


$$t_f \sim \frac{H_i^2}{H_i} \gtrsim 70 \frac{1}{H_i}$$

$$|\dot H| \ll H^2 \qquad a \propto e^{H_i t}$$



$$\frac{a_f}{a_i} \approx \frac{\dot a_f}{\dot a_i} = e^{H_i t_f} > 10^{28}$$

$$t_f > 70\, H_i^{-1}$$