Title: Ekpyrotic Perturbations & a Holographic Big Bang

Date: May 08, 2007 11:00 AM

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Abstract: TBA

Ekpyrotic Perturbations and A Holographic Big Bang

- An alternative to inflation
- Scale-invariant curvature perturbations
- Non-perturbative bounce in M theory
- "Scale invariance from Scale Invariance"

Ekpyrotic Perturbations and A Holographic Big Bang

- An alternative to inflation
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work with:

- Jean-Luc Lehners,
  Paul McFadden,
  Paul Steinhardt.

- Ben Craps,
  Thomas Hertog.

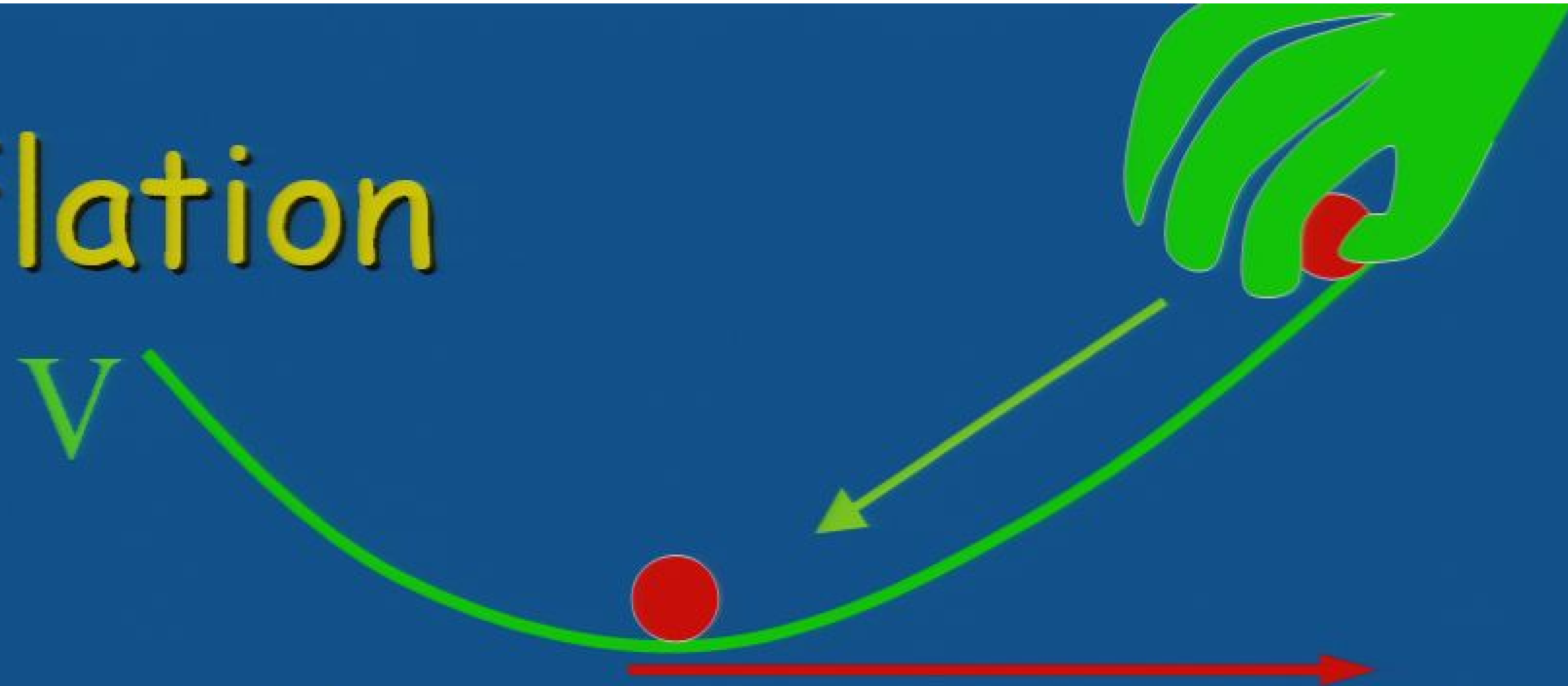
So far, observations are consistent with a spatially flat Universe, and the simplest possible perturbations:

- Gaussian
- Linear, growing mode
- Adiabatic
- Scalar
- Scale-Invariant

-as predicted by simple inflationary models,

BUT ...

Inflation



- Assumes start in a super-dense, $P=-\rho$ state: why?
- Cosmic singularity unresolved
- Requires fine tuned potentials $\lambda < 10^{-10}$

$$\rho_{DE} \sim 10^{-100}\, \rho_{INF}$$

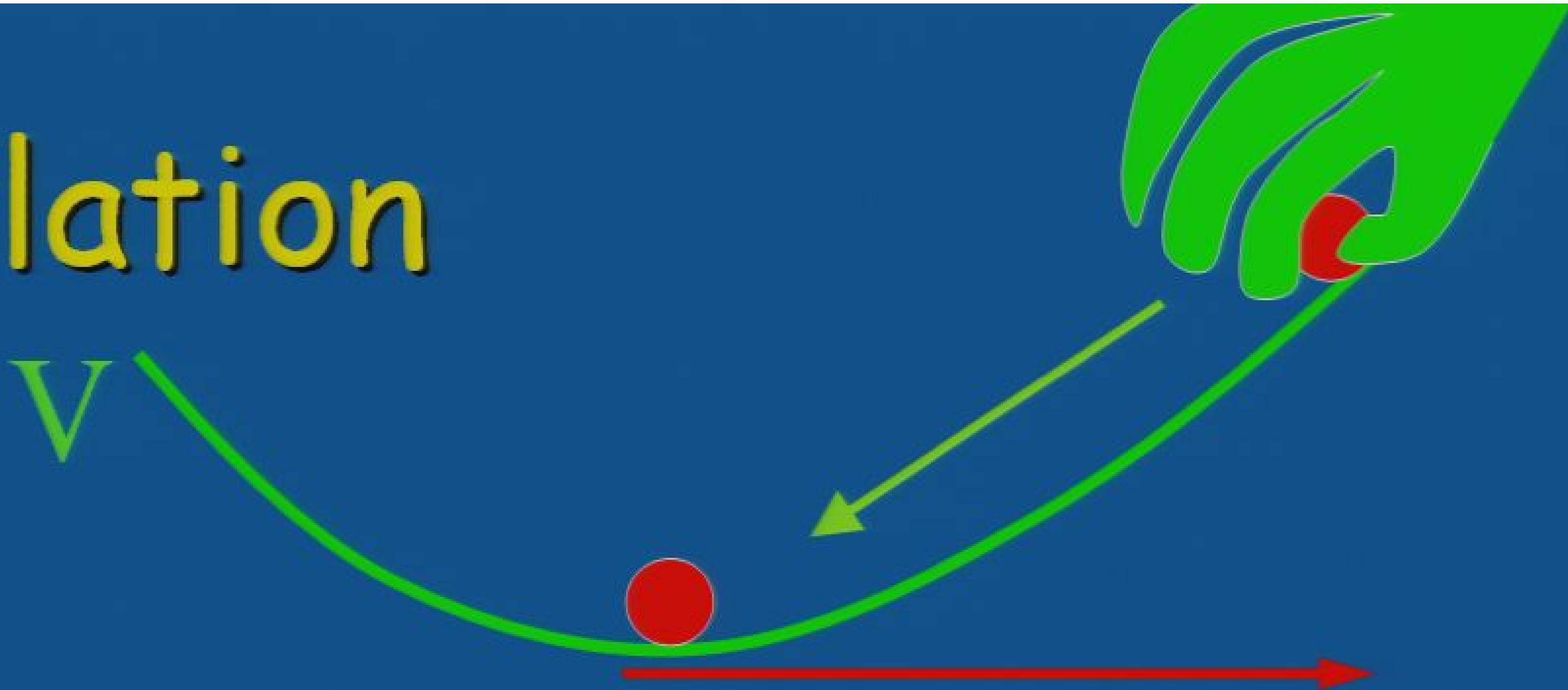
- Strange empty future
- Measure problem: canonical measure, with random ICs ~> $P(N) \sim e^{-3N}$ Gibbons+NT 2006

Inflation's most specific signature
- primordial tensor modes -
has not yet been seen

Motivations for a radical alternative

1. The dark energy puzzle: what is its role?
2. The idea that today's universe is in a dynamical, metastable state
3. String and M theory must deal with the singularity: since all we see traces back to it, it is surely crucial to determining the physical selection of states.
4. Either time began at the singularity, or it didn't. Lets consider both options.

Inflation



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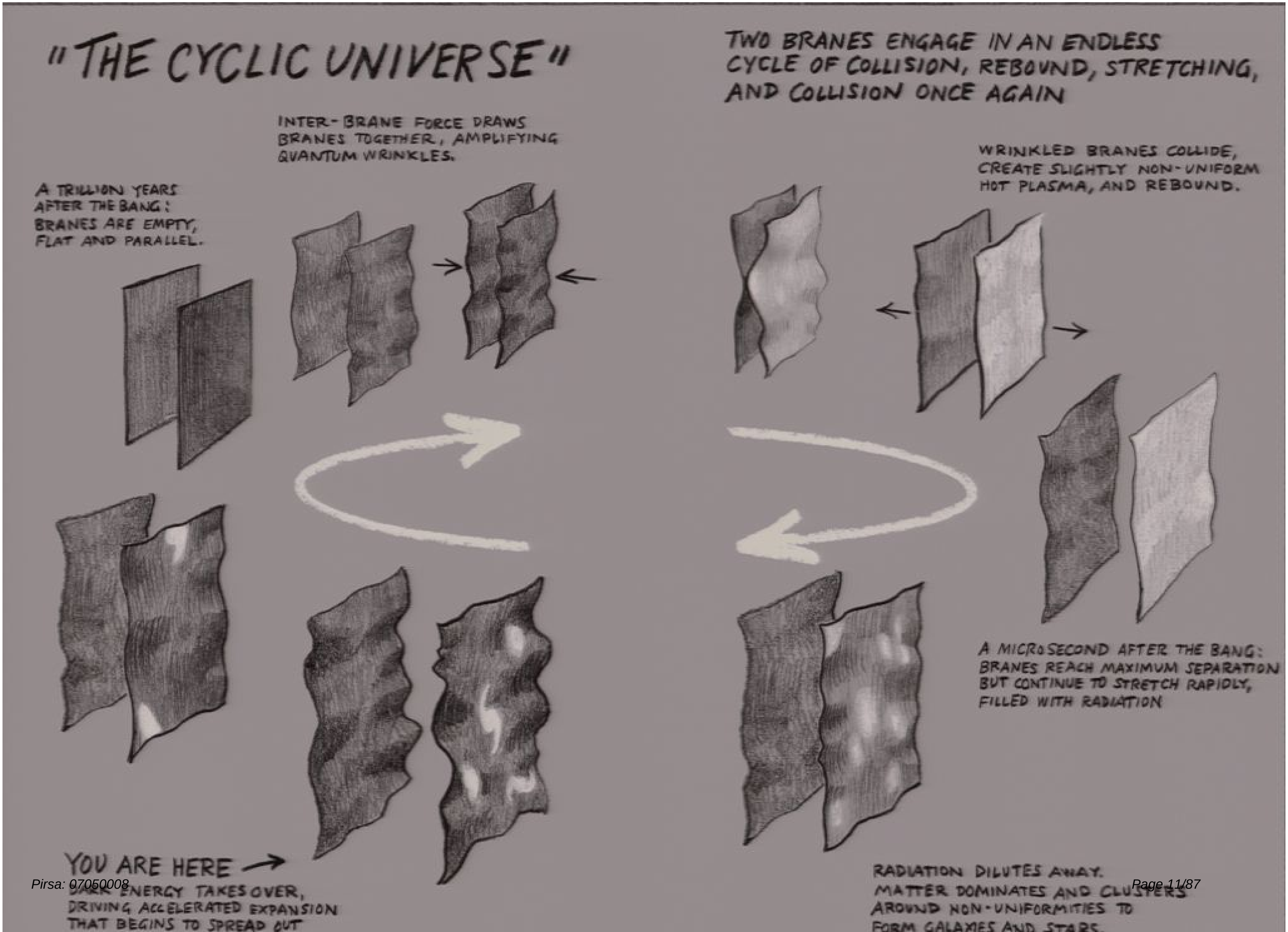
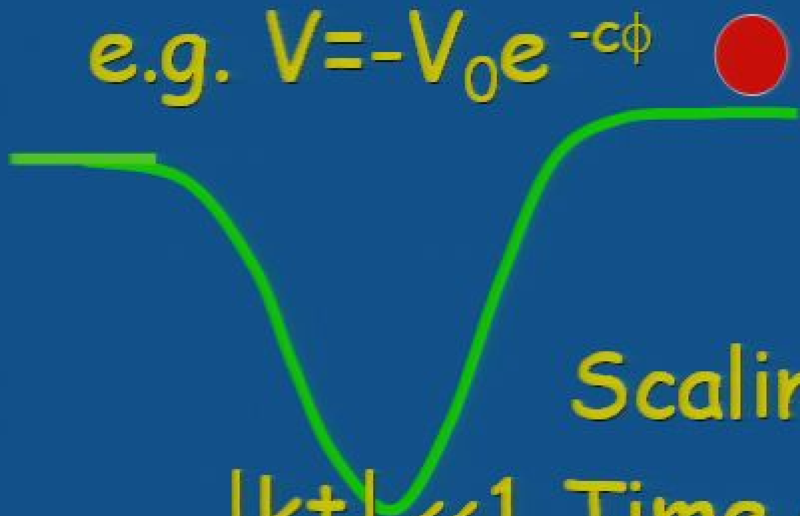

"THE CYCLIC UNIVERSE"
TWO BRANES ENGAGE IN AN ENDLESS CYCLE OF COLLISION, REBOUND, STRETCHING, AND COLLISION ONCE AGAIN
INTER-BRANE FORCE DRAWS BRANES TOGETHER, AMPLIFYING QUANTUM WRINKLES.
A TRILLION YEARS AFTER THE BANG: BRANES ARE EMPTY, FLAT AND PARALLEL.
WRINKLED BRANES COLLIDE, CREATE SLIGHTLY NON-UNIFORM HOT PLASMA, AND REBOUND.
A MICROSECOND AFTER THE BANG: BRANES REACH MAXIMUM SEPARATION BUT CONTINUE TO STRETCH RAPIDLY, FILLED WITH RADIATION
YOU ARE HERE
DARK ENERGY TAKES OVER, DRIVING ACCELERATED EXPANSION THAT BEGINS TO SPREAD OUT
RADIATION DILUTES AWAY. MATTER DOMINATES AND CLUSTERS AROUND NON-UNIFORMITIES TO FORM GALAXIES AND STARS.

Ekpyrotic perturbations

Khoury, Ovrut, Steinhardt, NT 2001

e.g. $V = -V_0 e^{-c\phi}$ ϕ (radion)



Scale symm: $x^\mu \to e^\lambda x^\mu$, $\phi \to \phi + 2\lambda/c$, $\hbar \to e^{2\lambda}\hbar$

Scaling solution: $\dot{\phi} \sim t^{-1}$

$|kt| \ll 1$ Time delay mode: $\delta\phi \sim \dot{\phi} \sim t^{-1}$

Scaling symmetry -> $\langle\delta\phi^2\rangle \sim \hbar\, t^{-2} \int d^3k/k^3$

cf Massless scalar in de Sitter;

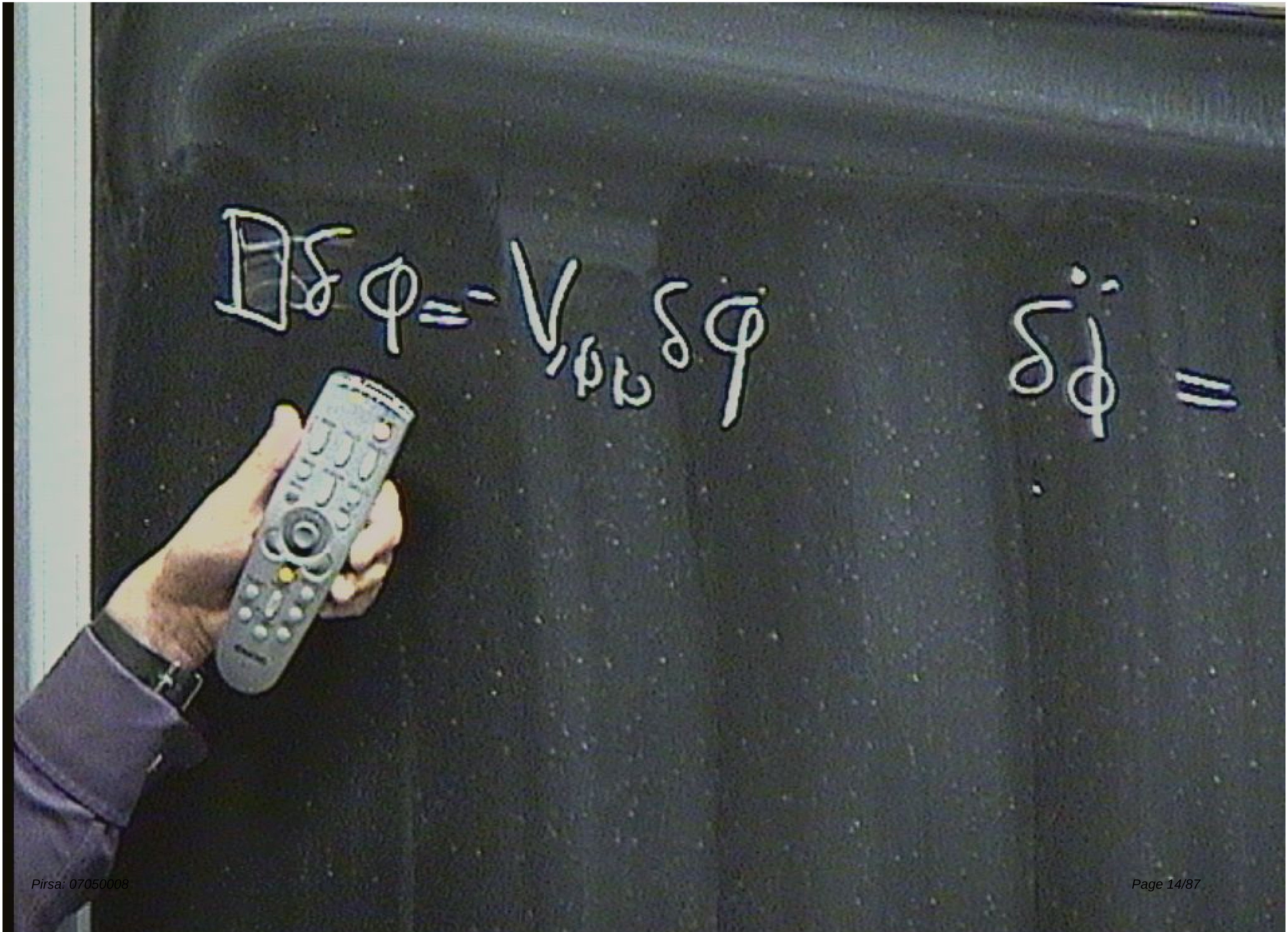
scaling background soln $ds^2 = (-dt^2 + dx^2)/(Ht)^2$

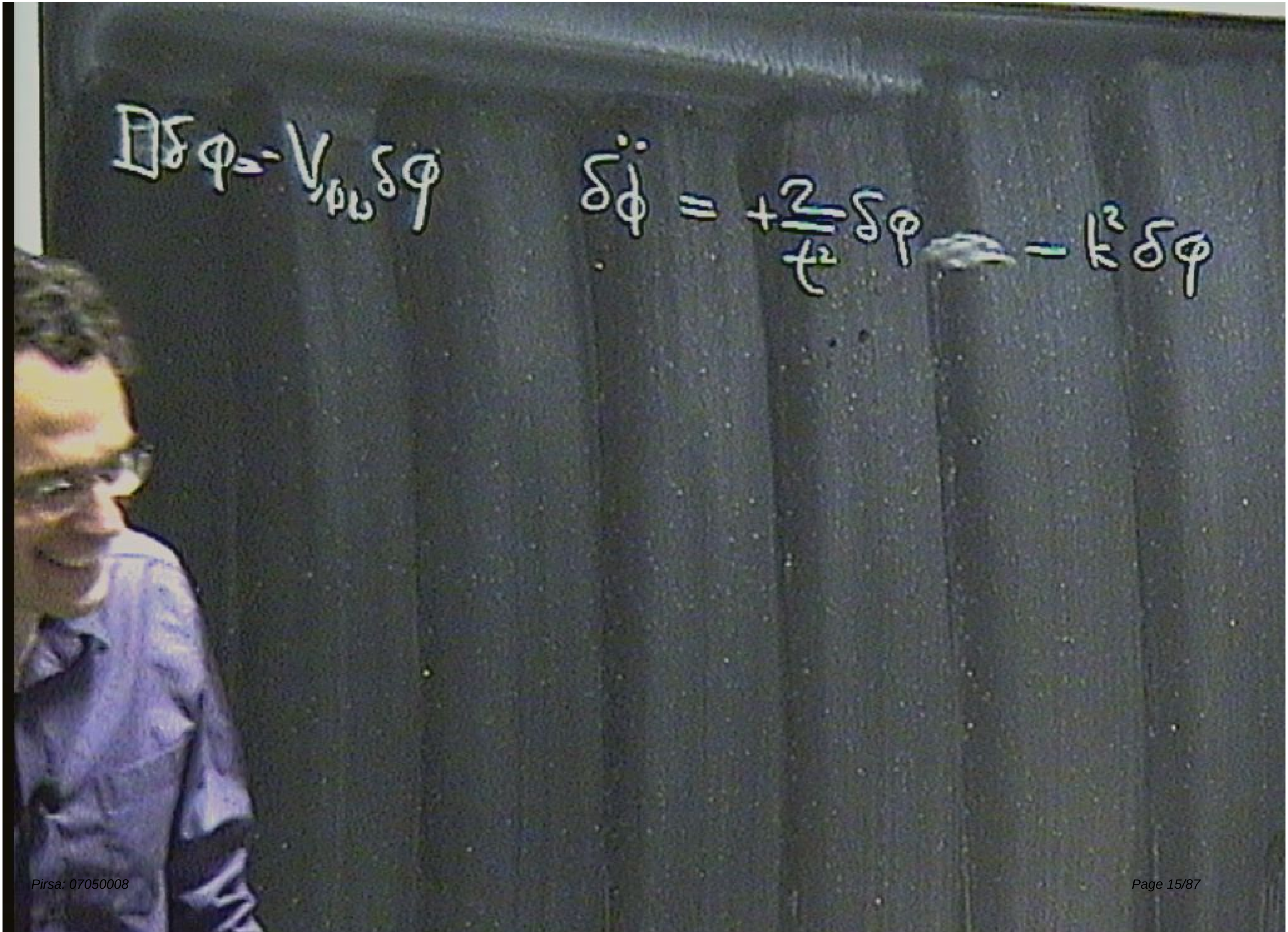
scale symmetry $x^\mu \to \lambda x^\mu$

shift mode $\phi \to \phi + c$, c constant

Hence, $\langle\delta\phi^2\rangle \sim \hbar\, H^2 \int d^3k/k^3$

$$\delta\ddot{\phi} = +\frac{2}{t^2}\delta\phi - k^2\delta\phi$$


$\Box\delta\phi = -V_{,\phi\phi}\delta\phi$
$\delta\dot{\phi} =$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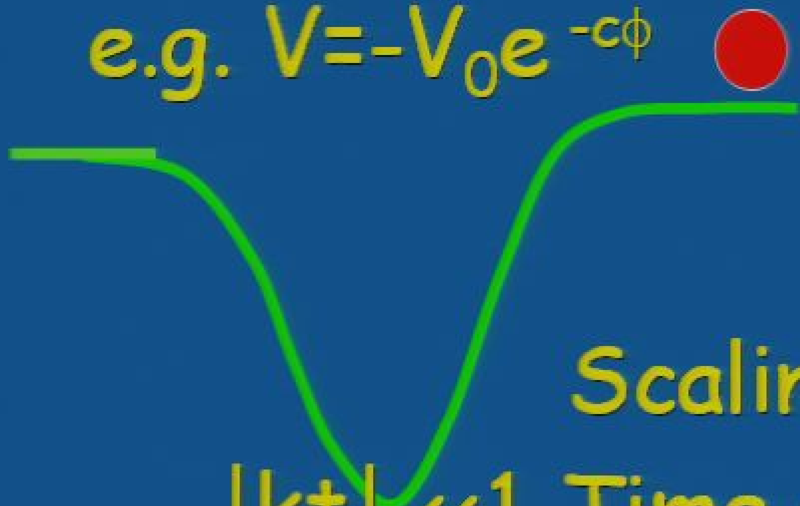
e.g. $V=-V_0 e^{-c\phi}$  ϕ (radion)

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Now include gravity

$$ds^2 = -dt^2(1+2\Phi) + a^2(t)\mathbf{dx}^2(1-2\Phi)$$

Long λ, Quasi-gauge modes

$$\delta t = \frac{\alpha_1(\mathbf{x})}{a} - \frac{\alpha_2(\mathbf{x})}{a}\int^t dt' a(t'), \qquad \delta x^i = (1+\alpha_2(\mathbf{x}))\, x^i$$

$$\Phi = \alpha_1(\mathbf{x})\frac{\dot a}{a^2} + \alpha_2(\mathbf{x})\left(1 - \frac{\dot a}{a^2}\int^t dt' a(t')\right)$$

	Local time delay	Local dilatation: "Curvature pertn. *R*"
Expanding U	Decaying	Growing
Contracting U	Growing	Decaying

How can a local time delay match on to a local spatial dilation?

Creminelli et al, Lyth, Huang…

A. 5d effects near bounce (warping of 5^{th} dimension):

Tolley et al., Battefeld et al., McFadden et al.

B. Additional light dofs in 4dET driven unstable:

Lehners, McFadden, Steinhardt, NT
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Flurry of papers 2007

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Assume two scalar fields, ϕ_1 and ϕ_2, with independent, negative, steeply flattening potentials

➡ relative pertn $\frac{\delta\phi_1}{\dot{\phi}_1} - \frac{\delta\phi_2}{\dot{\phi}_2}$ scale-invariant on long wavelengths

but this converts easily to R

General result:

$$\dot{\mathcal{R}} = -\frac{H}{\dot{H}} g_{IJ}(\phi) \frac{D^2 \phi^I}{Dt^2} s^J + \frac{H}{\dot{H}} \frac{k^2 \Psi}{a^2}$$

where the entropy perturbation is

$$s^I = \delta\phi^I - \dot{\phi}^I \frac{g_{JK}(\phi)\dot{\phi}^J \delta\phi^K}{g_{LM}(\phi)\dot{\phi}^L \dot{\phi}^M}$$

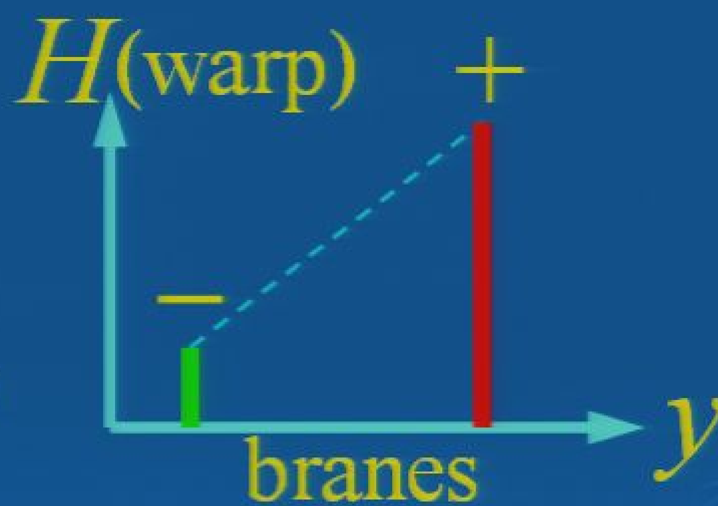
Heterotic M Theory

$$\int_5 \left(\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - Ce^{-2\phi}\right) - \sum_i \mu_i \int_4 e^{-\phi}$$

Two moduli:

radion and $V_{CY}=e^{\phi}$

Both can pick up scale-invariant perts pre-bang -> entropy perts



Before and after boundary brane collision, minus brane hits zero of *H* and bounces back. This bounce converts entropy to curvature!

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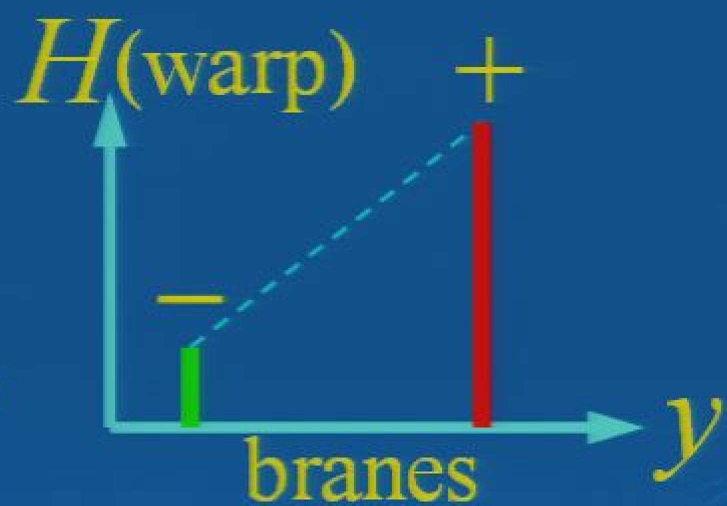

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5d solution

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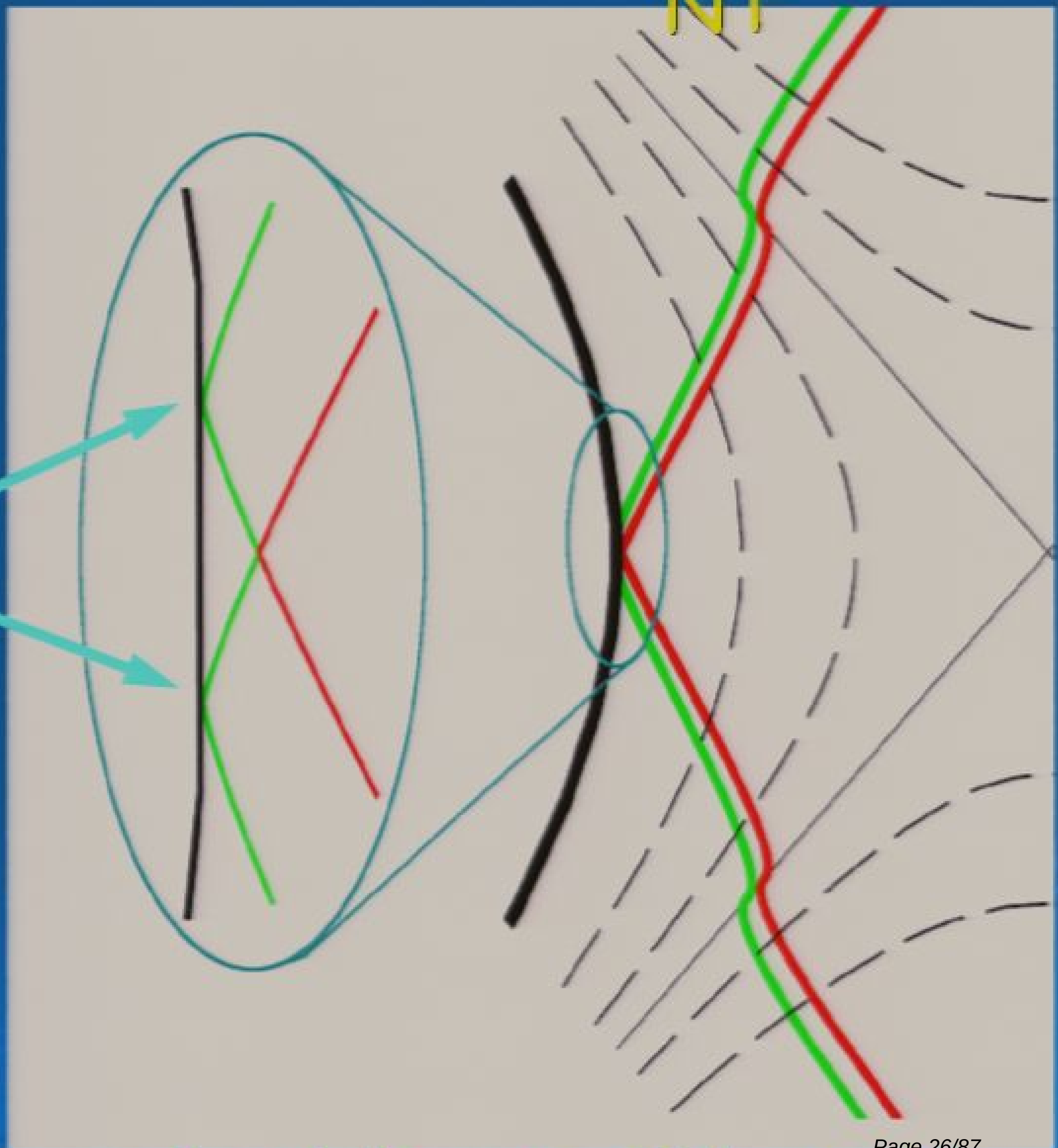
Trajectory tangential to singularity



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embedding in 5d static

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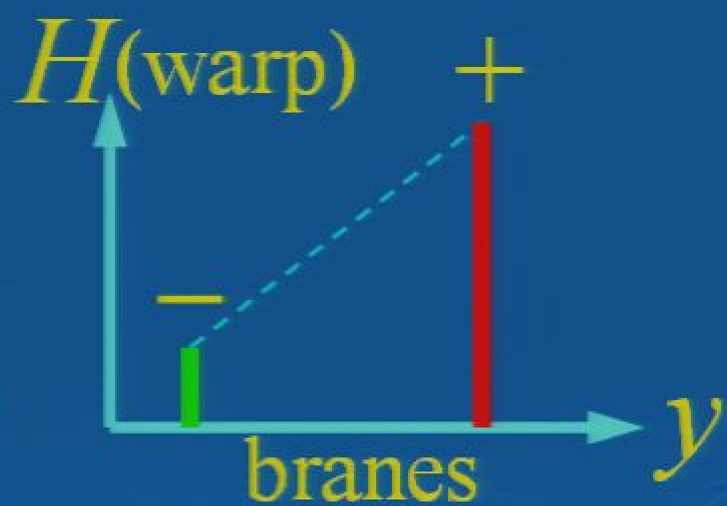

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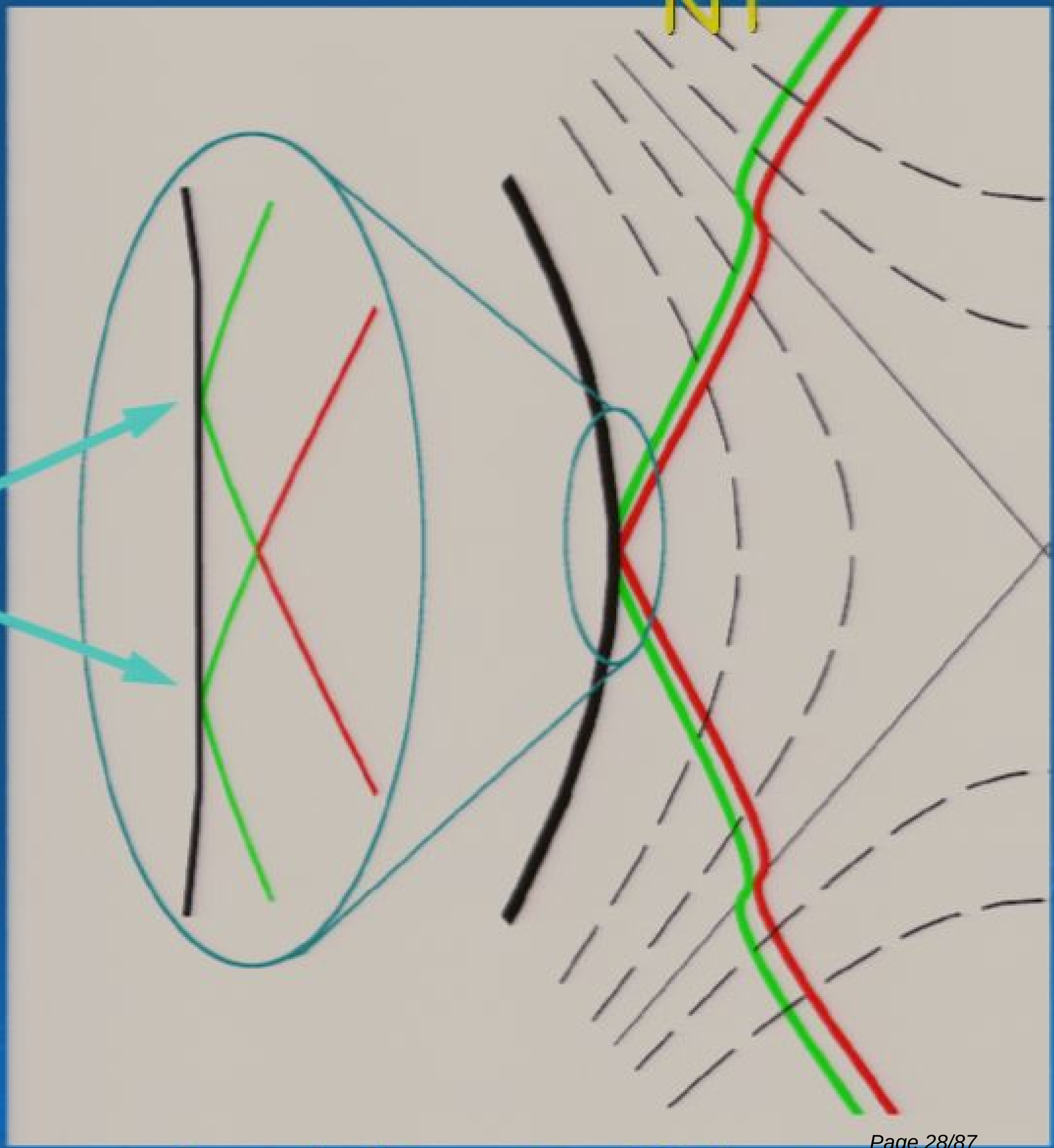

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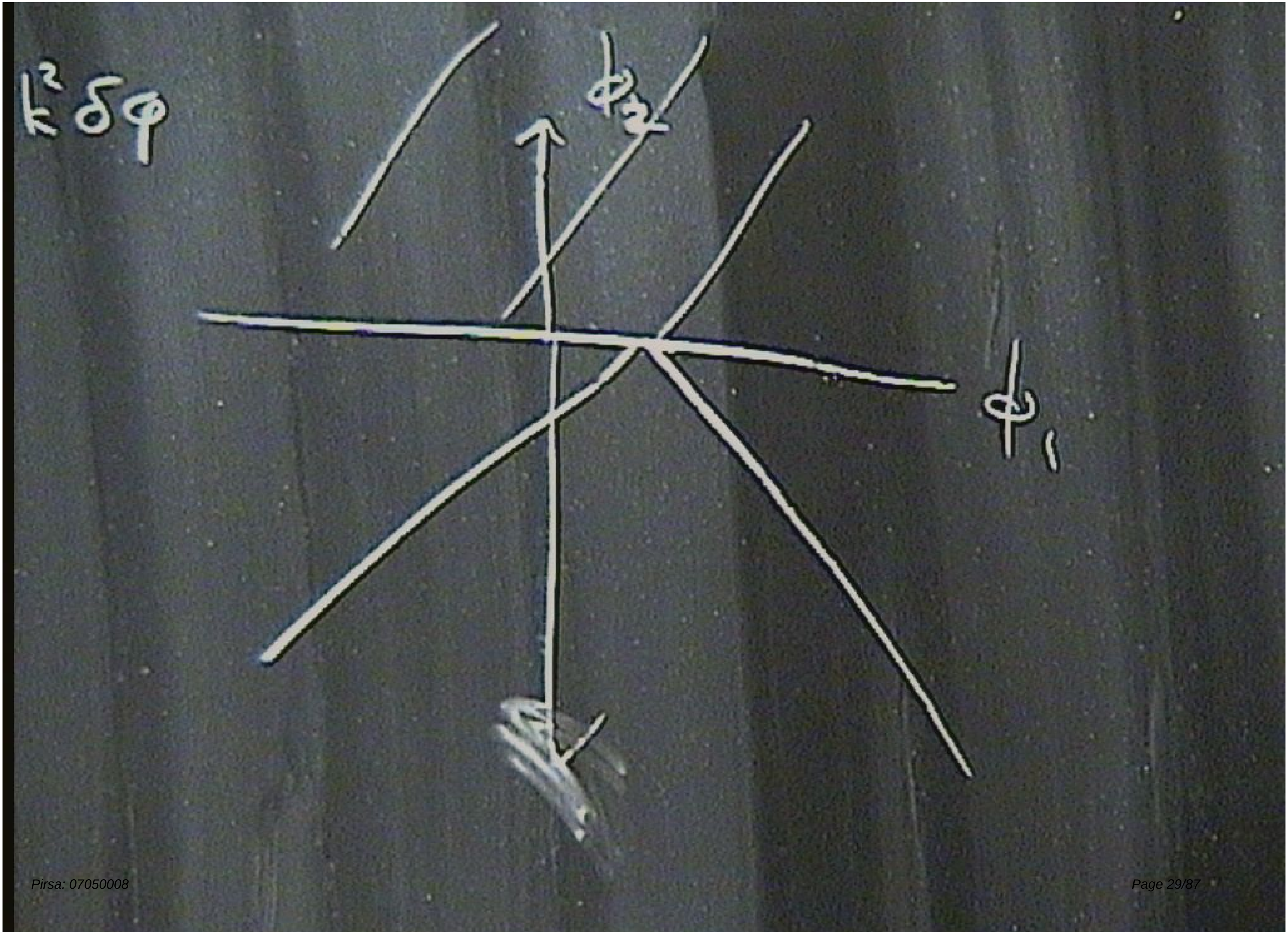
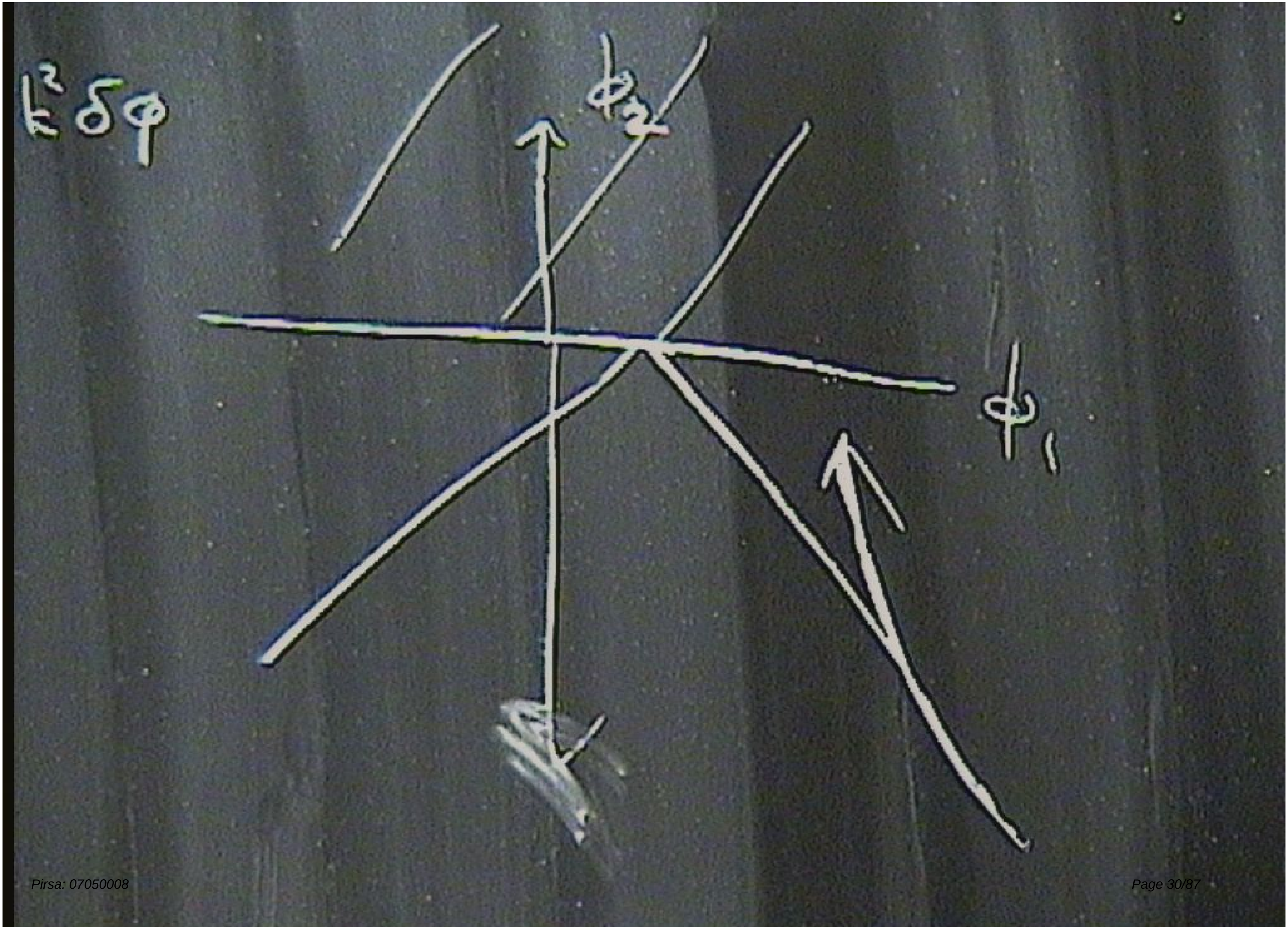
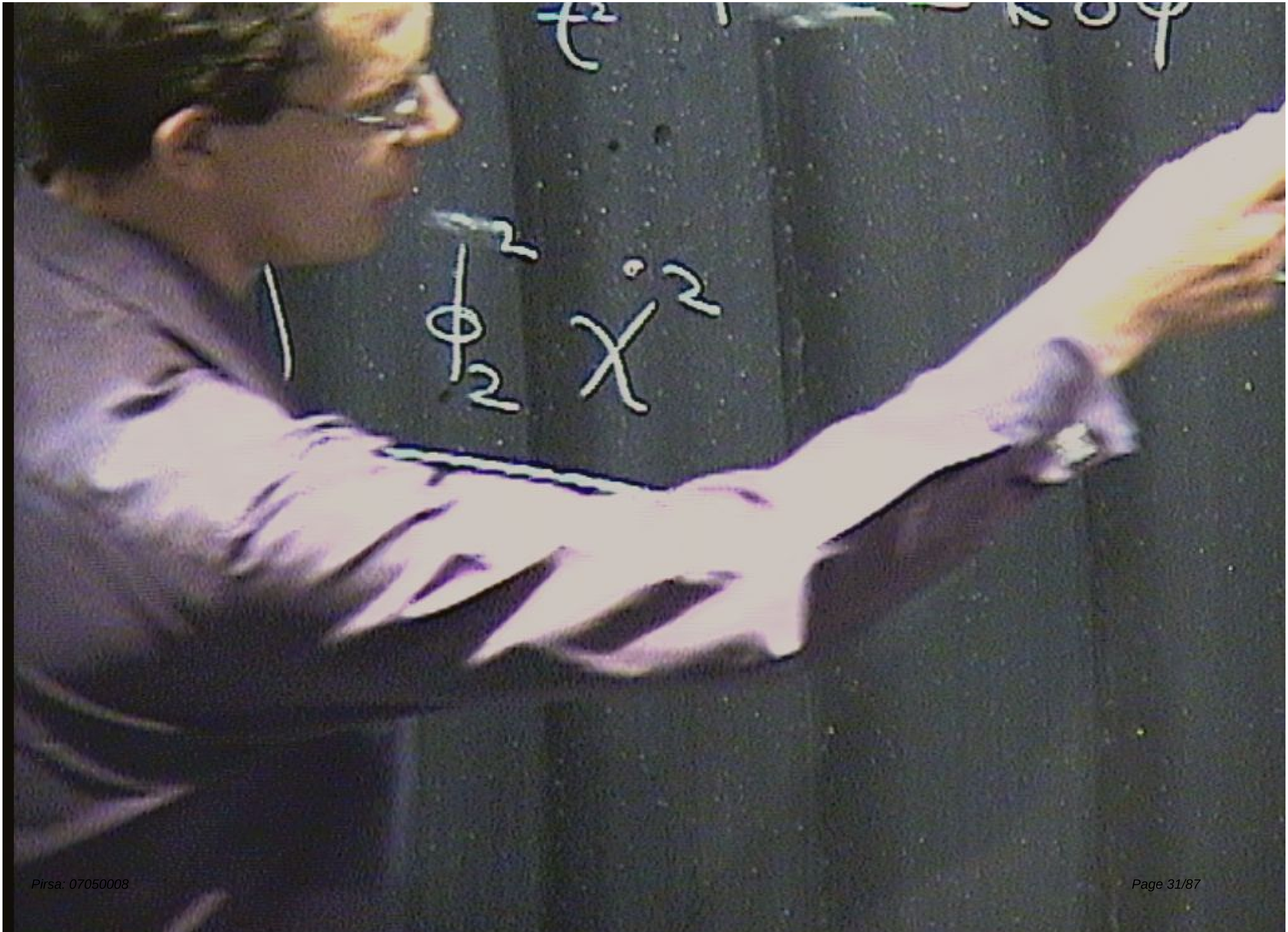
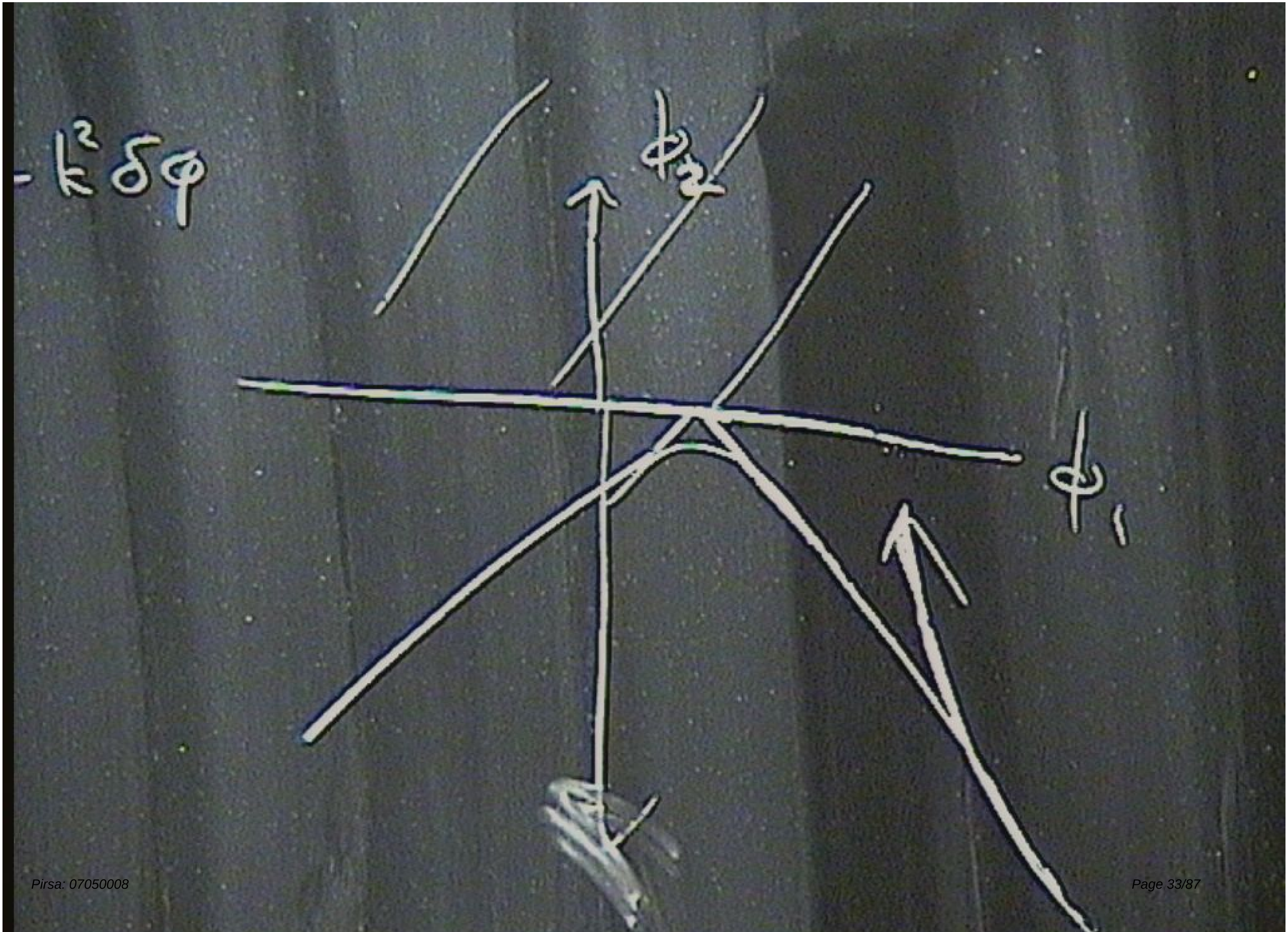

k² δφ
φ₂
φ₁



$k^2 \delta\varphi$
ϕ_2
ϕ_1



$$\int \dot{\phi}_2^2 \, \dot{\chi}^2$$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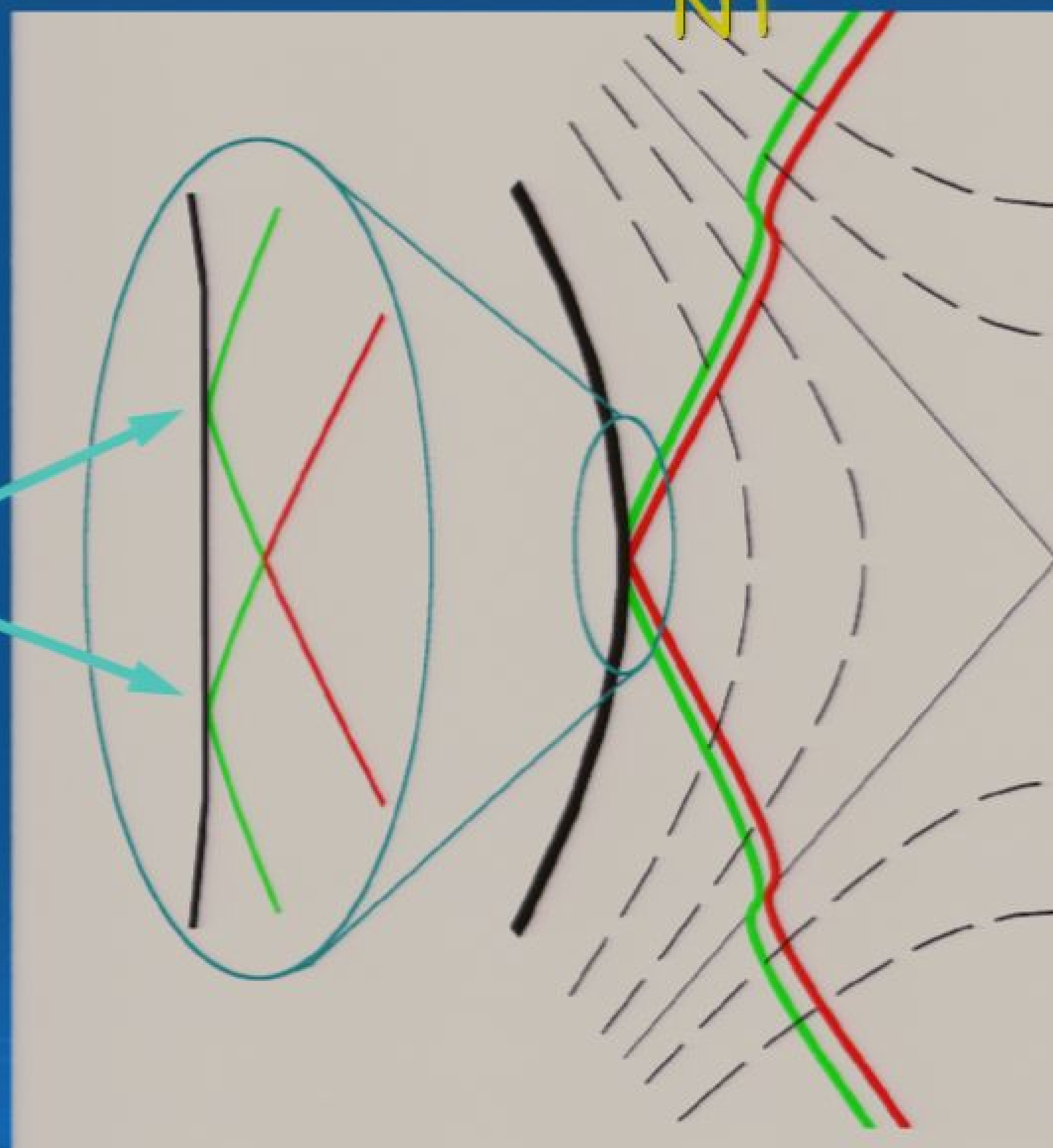

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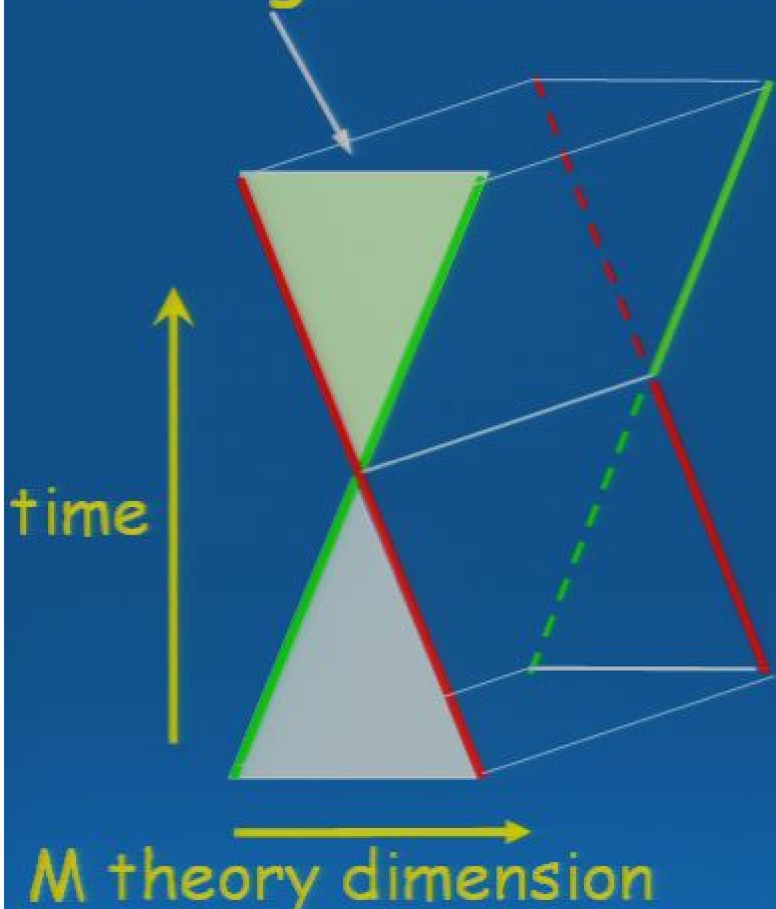
M-theory model for the bang

Perry, Steinhardt & NT, 2004
Berman & Perry, 2006
Niz+NT 2007

Winding M2 branes=Strings:



No blue-shift for winding membranes: describe perturbative string states including gravity

Weak coupling at singularity

Classical evolution of string is regular across t=0

Calculable T_{HBB} due to string creation

BUT: what about KK modes i.e. nonpert string states?

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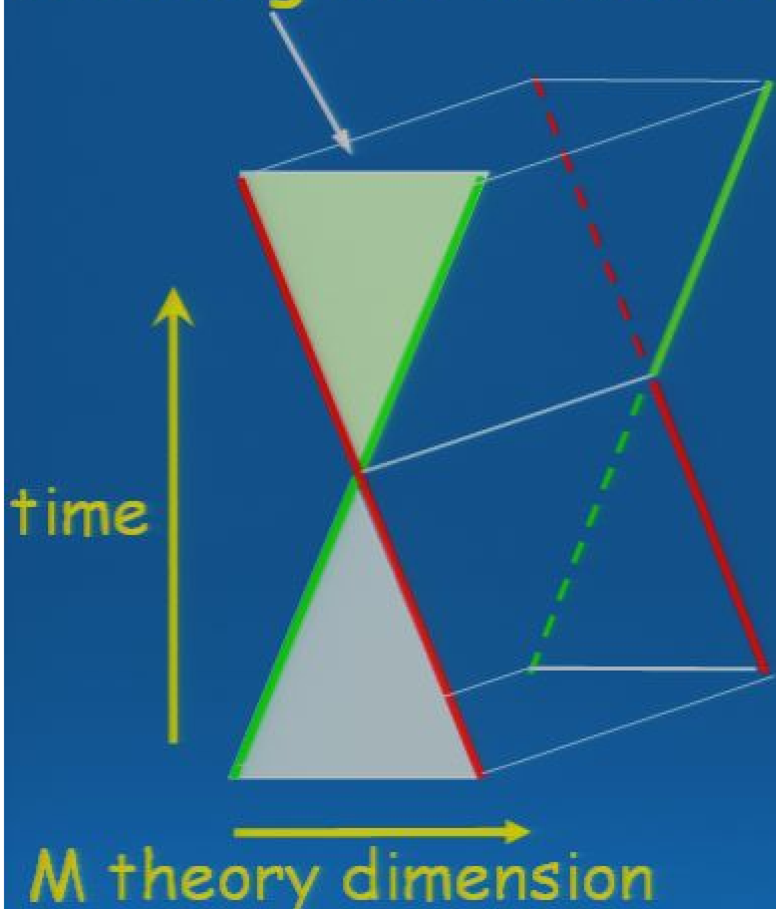

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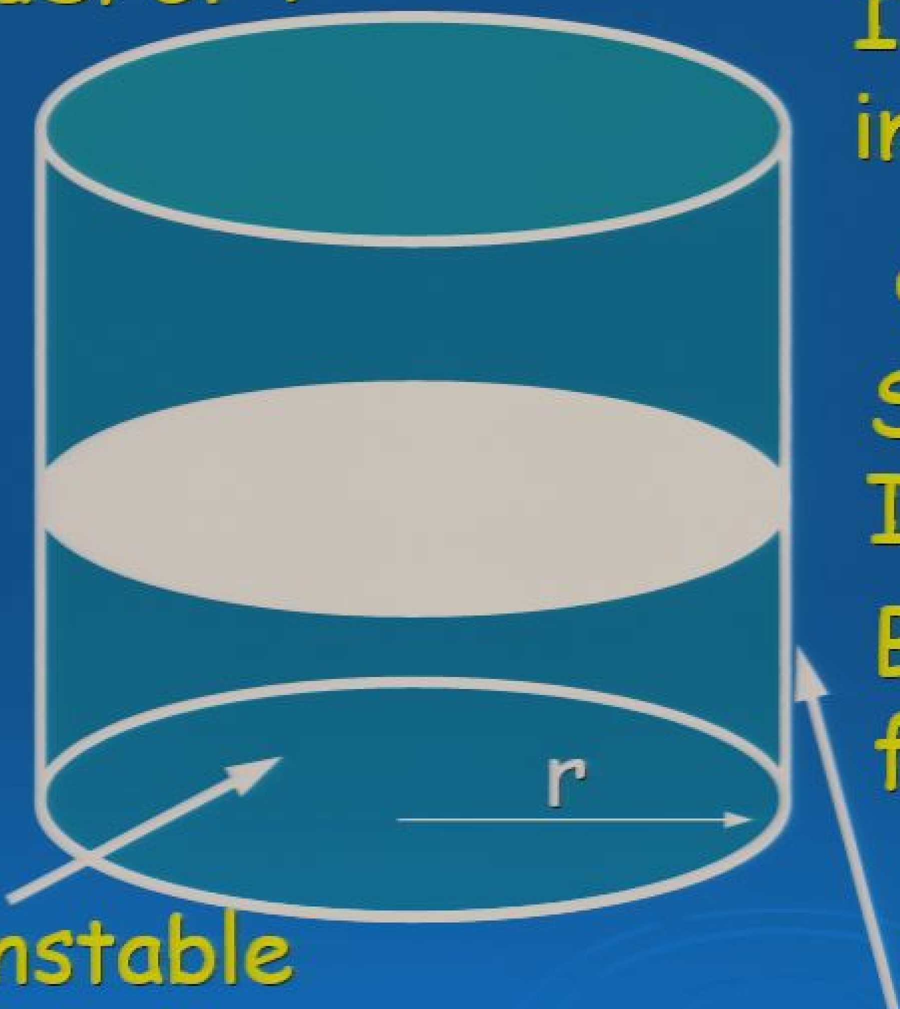
A Holographic Big Bang

AdS/CFT



IIB SUGRA on $S^5 \times AdS^5$ includes $m^2=-4$ BF scalar

$\phi \sim \alpha\, r^{-2} \ln r + \beta\, r^{-2}$

SUSY-> $\alpha=0$ no dynamics

If $\alpha=\alpha(\beta)$ -> dynamics

Bulk collapses to a finite-time singularity

Hertog+Horowitz

Unstable 5d bulk

Deformed CFT on $R \times S^3$

Also unstable :

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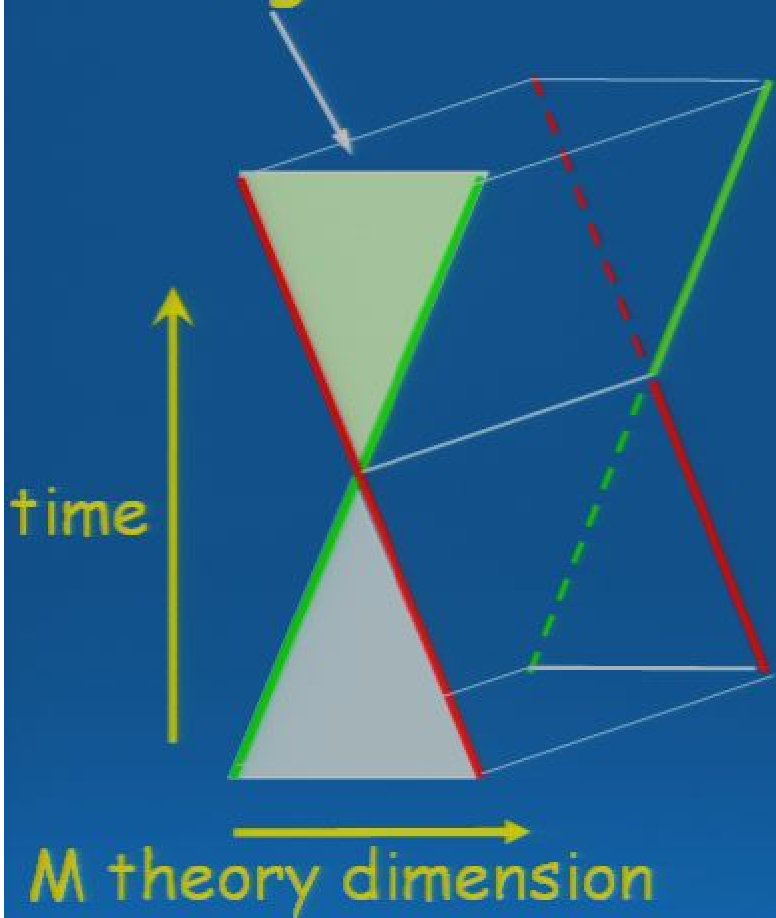

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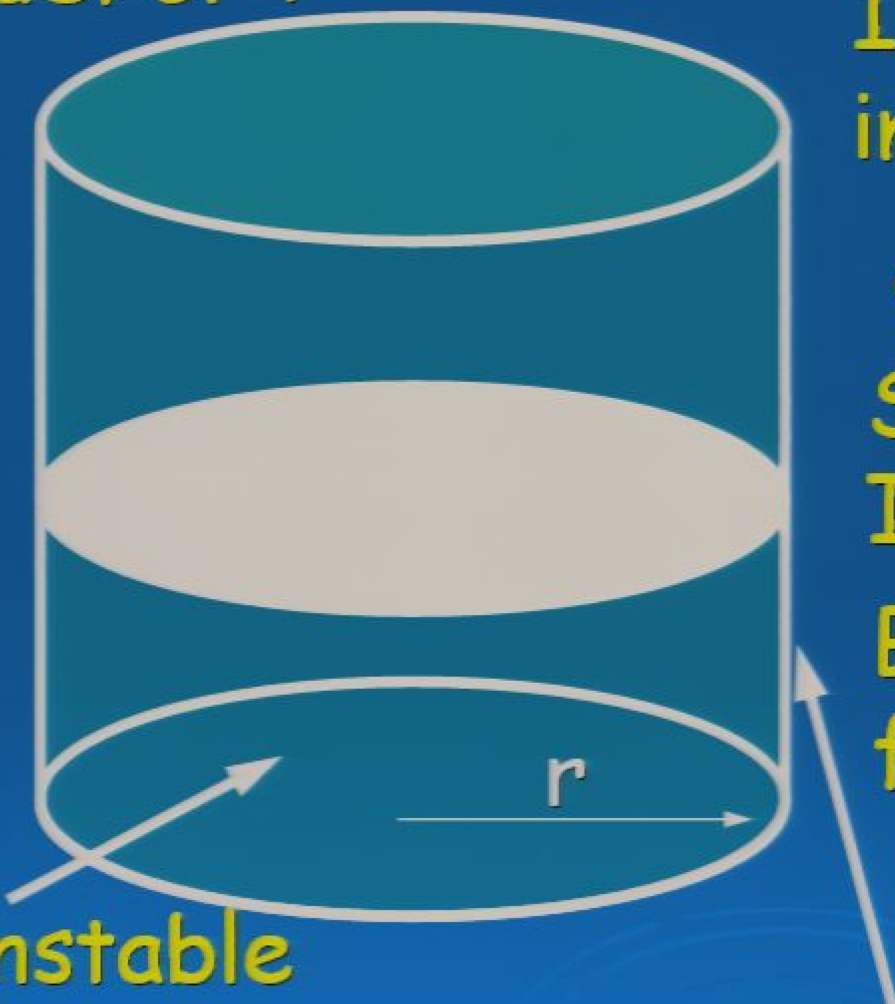

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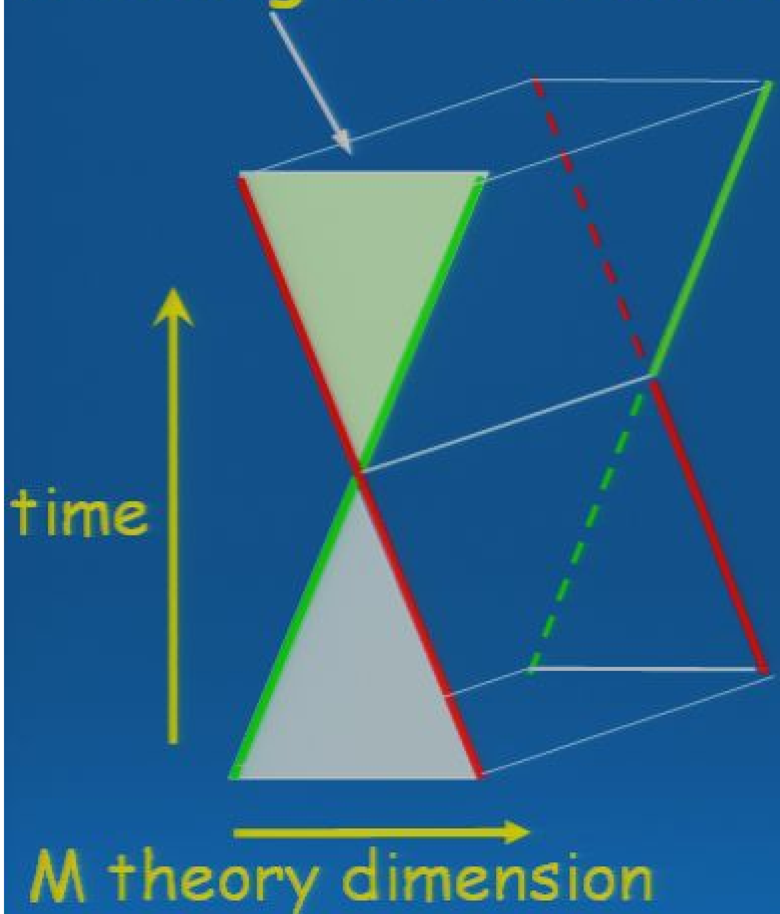

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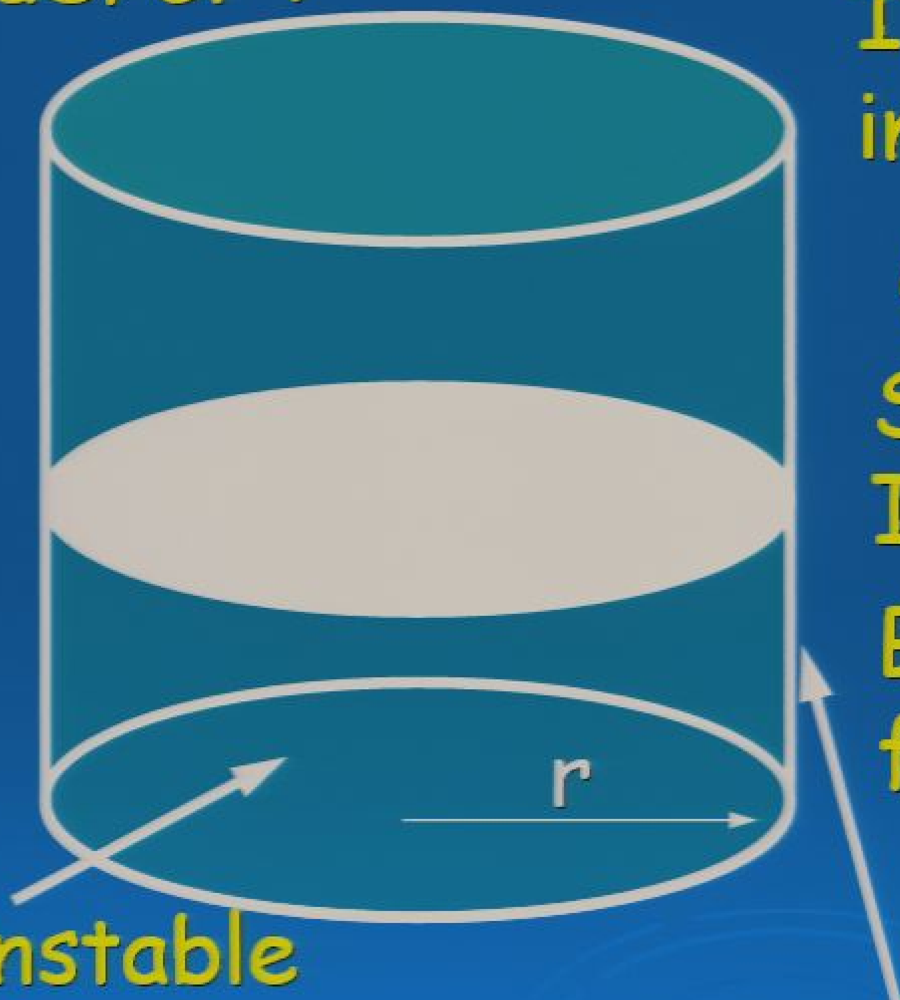

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Witten

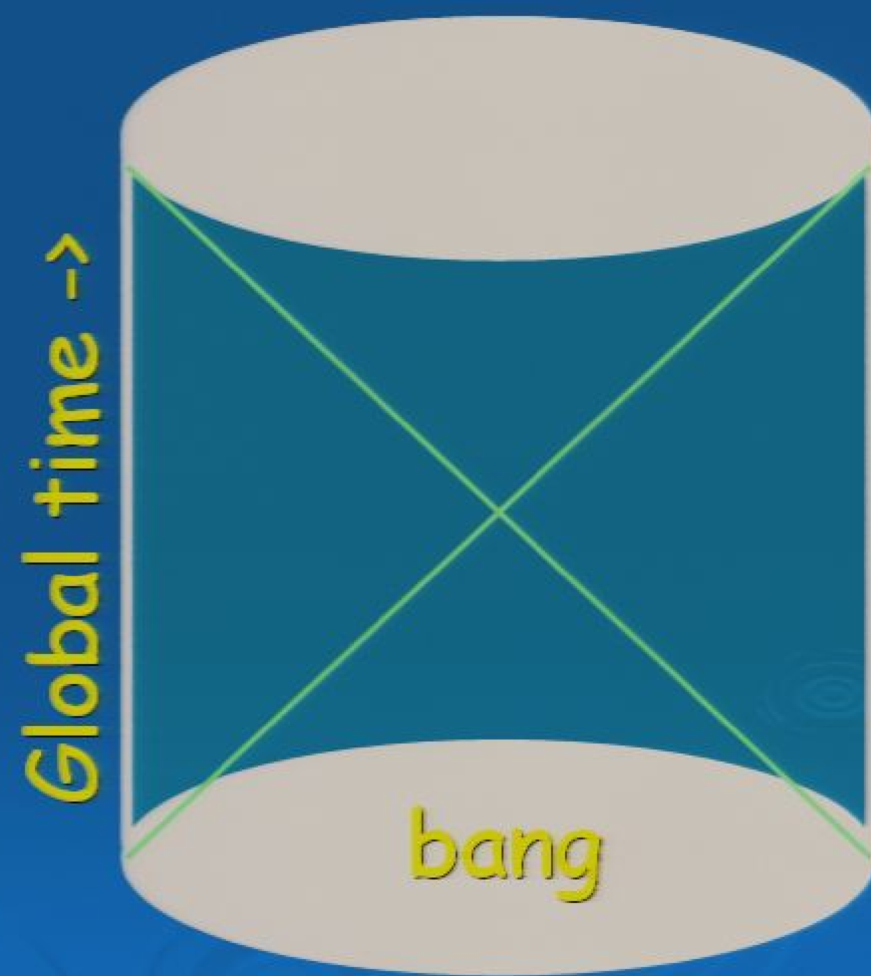
$\alpha=\lambda\,\beta$ corresponds deformation $-\lambda\phi^4$ of CFT -> instability

$(\phi^2 = Tr(\phi_1^2 - \phi_2^2))$

λ is symptotically free

$$V(\phi) = -\frac{16\,\pi^2\phi^4}{3\ln(\phi/M)}$$

large N-> β fn is 1-loop exact, V under good perturbative control



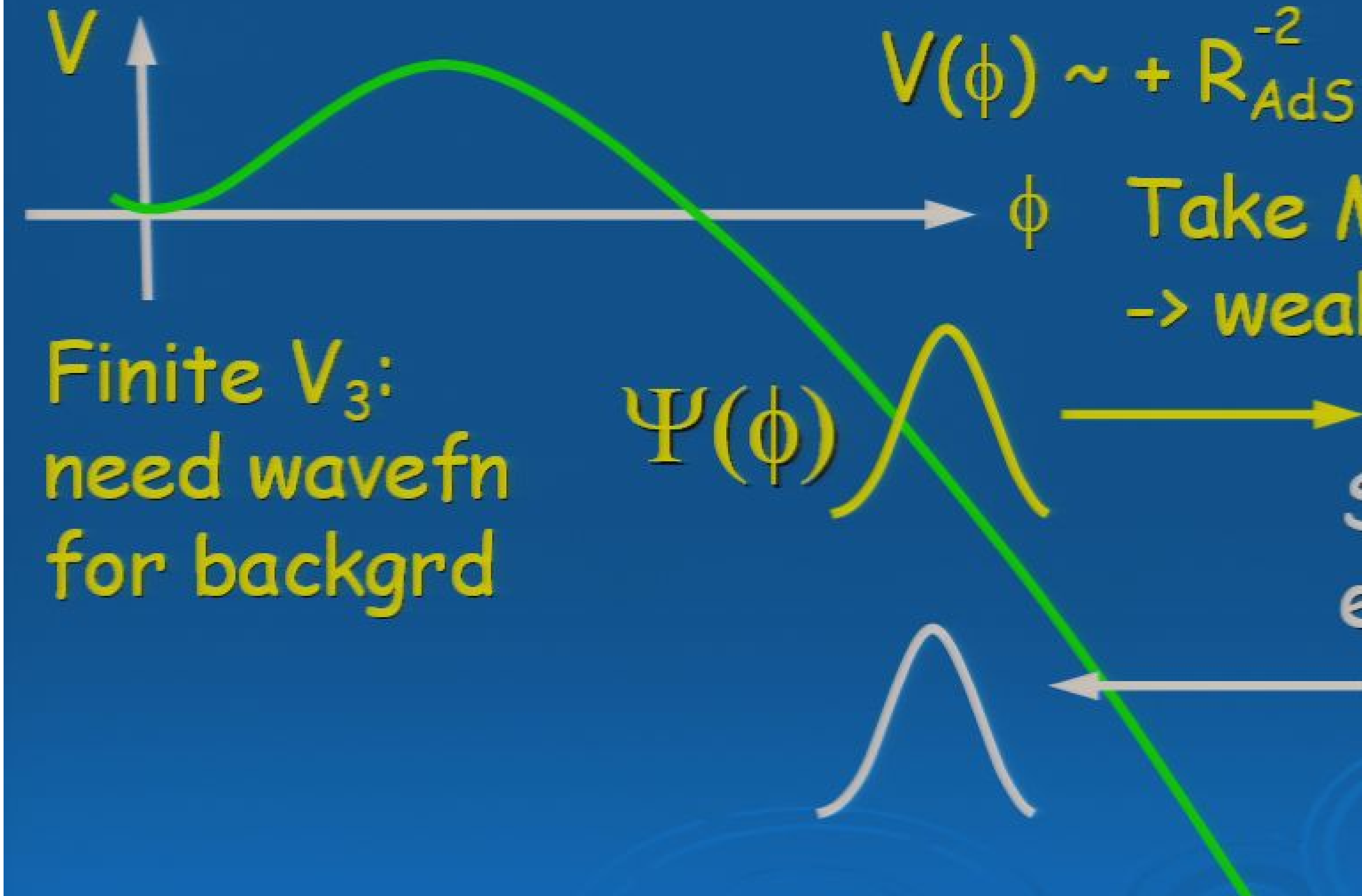
Unstable CFT

Conformal coupling

$$V(\phi) \sim + R_{AdS}^{-2}\,\phi^2 - \lambda\phi^4$$

Take $M \ll R_{AdS}^{-1}$

-> weak coupling



Finite V_3:
need wavefn
for backgrd

Self-adjoint
extension

Key Points

* No gravity in CFT

* Finite time singularity -> Ultralocality Quantum mechanics -> natural resolution of singularity via "self-adjoint extension"

* Asymptotic freedom

* Finite V_3 ~> entire background becomes quantum around singularity

* CFT is (nearly) scale invariant ->
Automatically get scale-invariant pertns

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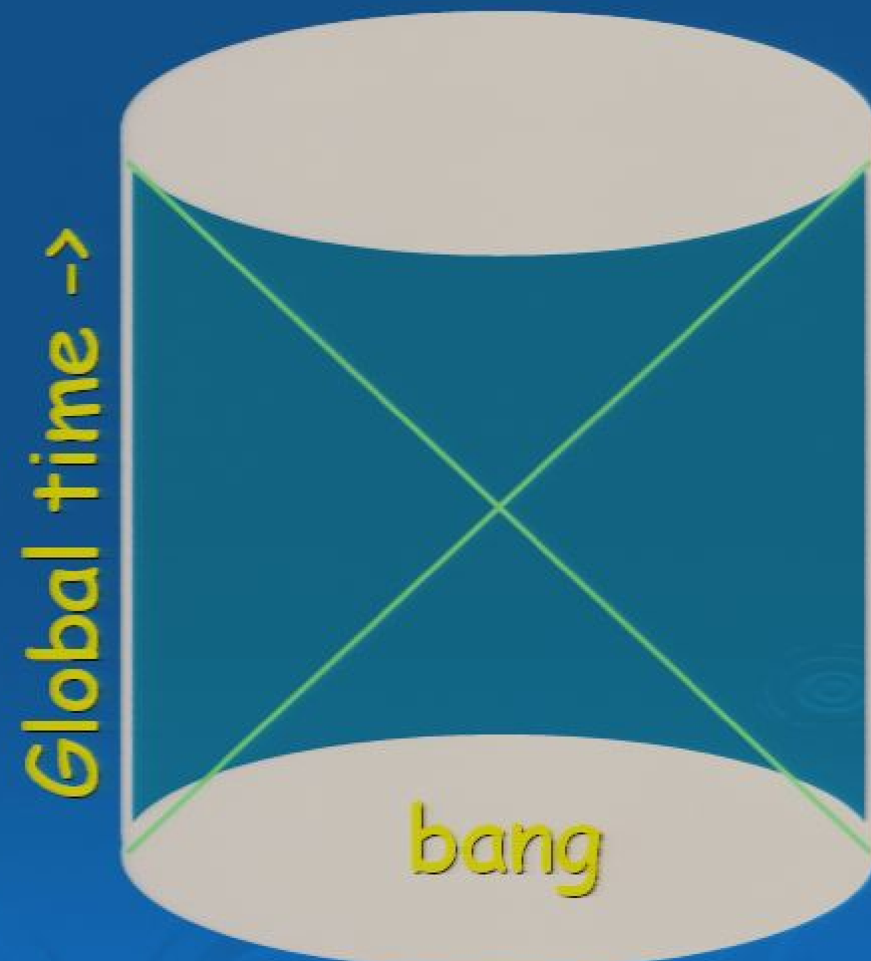

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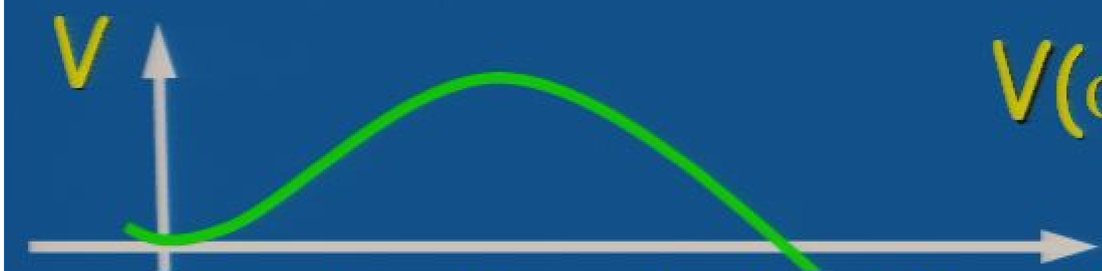

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singularity via "self-adjoint extension"

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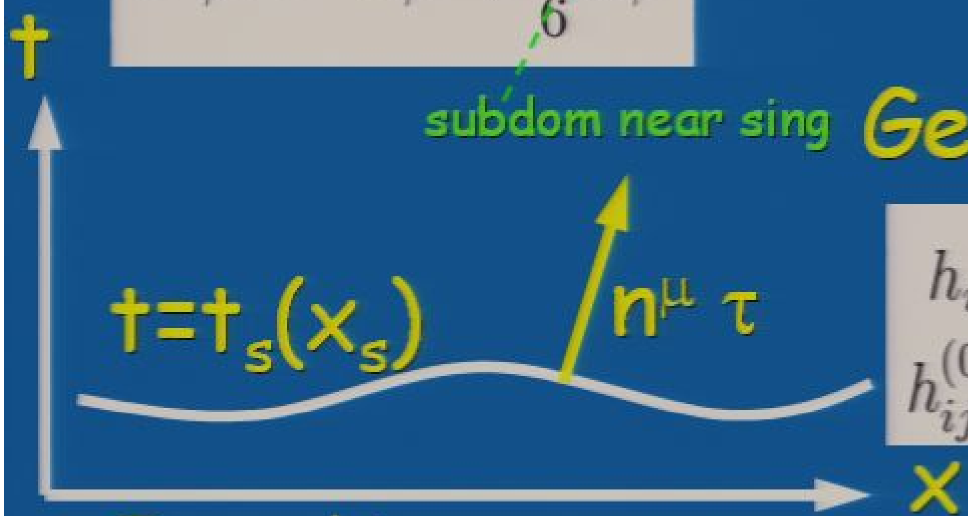
1. Ultralocality

$$\partial^2\phi = -\lambda\phi^3 + \frac{1}{6}R\phi$$

subdom near sing

zero E bg soln: $\phi = \sqrt{\frac{2}{\lambda}}\frac{1}{t-t_s}$

Gen soln: $-d\tau^2 + h_{ij}dx_s^i dx_s^j,$



$$h_{ij} = h_{ij}^{(0)} + 2K_{ij}\tau + K_{ik}h^{kl}_{(0)}K_{lj}\tau^2$$

$$h_{ij}^{(0)} \equiv \delta_{ij} - \partial_i t_s \partial_j t_s, \qquad K_{ij} \equiv \gamma\partial_i\partial_j t_s.$$

Expand in

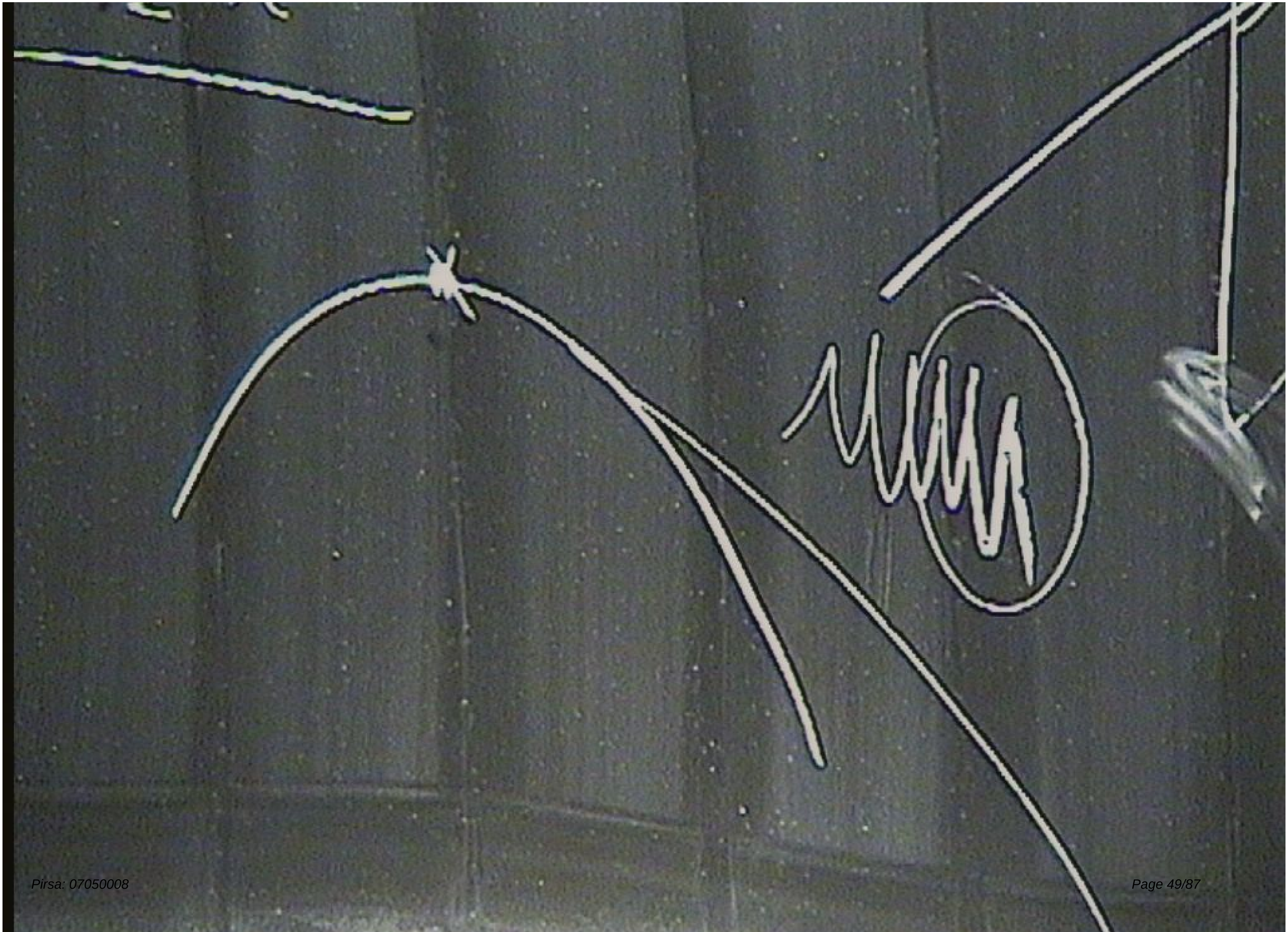
$\tau\nabla_s$

Define

$\chi = \phi^{-1}$

$$\begin{aligned}
\chi &= \left(\frac{\lambda}{2}\right)^{\frac{1}{2}}(\tau + \frac{1}{6}K_1\tau^2 + \frac{1}{18}(K_1^2 - 3K_2)\tau^3 \\
&+ \frac{1}{4}(K_3 - \frac{13}{18}K_2K_1 + \frac{7}{54}K_1^3 - \frac{1}{6}\nabla K_1)\tau^4 \\
&+ \ln\tau(\frac{1}{5}(-7K_4 + \frac{29}{3}K_1K_3 + \frac{11}{3}K_2^2 - \frac{67}{9}K_1^2K_2 + \frac{34}{27}K_1^4 - \frac{1}{18}(\partial K_1)^2 \\
&+ \frac{2}{9}K_1\nabla K_1 - \frac{1}{6}\nabla K_2)\tau^5 + C_6\tau^6 + \dots) + \rho(x_s)\tau^5 + D_6\tau^6 + \dots),
\end{aligned}$$

2 arbitrary functions: $t=t_s(x_s)$, $\rho(x_s)$



Interpretation in linearized theory

$$\delta\chi(t,\mathbf{x}) = \sqrt{\frac{\lambda}{2}}\left(-t_s(\mathbf{x}) + \frac{1}{6}t^2\nabla^2 t_s - \frac{1}{24}t^4(\nabla^4 t_s) + \ldots + \rho(\mathbf{x}_s)t^5 + \ldots\right.$$

Time delay $\delta\phi/\dot{\phi}$

Hamiltonian density $\delta\mathcal{H} = \dot{\phi}\dot{\delta\phi} + V_{,\phi}\delta\phi = \partial_t\left(\frac{\delta\phi}{\dot{\phi}}\right)\dot{\phi}^2$

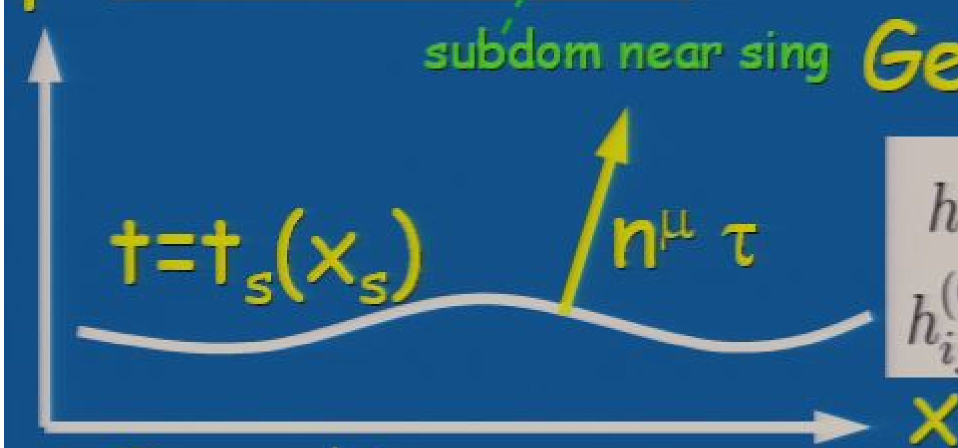
As gradients become unimportant, different spatial points decouple -> QM

Self-adjoint extension matches local time delay and energy density across singularity

1. Ultralocality

$$\partial^2\phi = -\lambda\phi^3 + \frac{1}{6}R\phi$$

zero E bg soln: $\phi = \sqrt{\frac{2}{\lambda}}\frac{1}{t-t_s}$



Gen soln: $-d\tau^2 + h_{ij}dx_s^i dx_s^j,$

$$\begin{aligned} h_{ij} &= h_{ij}^{(0)} + 2K_{ij}\tau + K_{ik}h_{(0)}^{kl}K_{lj}\tau^2 \\ h_{ij}^{(0)} &\equiv \delta_{ij} - \partial_i t_s \partial_j t_s, \qquad K_{ij} \equiv \gamma\partial_i\partial_j t_s. \end{aligned}$$

Expand in $\tau\nabla_s$

Define $\chi = \phi^{-1}$

$$\begin{aligned} \chi &= \left(\frac{\lambda}{2}\right)^{\frac{1}{2}} (\tau + \frac{1}{6}K_1\tau^2 + \frac{1}{18}(K_1^2 - 3K_2)\tau^3 \\ &+ \frac{1}{4}(K_3 - \frac{13}{18}K_2K_1 + \frac{7}{54}K_1^3 - \frac{1}{6}\nabla K_1)\tau^4 \\ &+ \ln\tau(\frac{1}{5}(-7K_4 + \frac{29}{3}K_1K_3 + \frac{11}{3}K_2^2 - \frac{67}{9}K_1^2K_2 + \frac{34}{27}K_1^4 - \frac{1}{18}(\partial K_1)^2 \\ &+ \frac{2}{9}K_1\nabla K_1 - \frac{1}{6}\nabla K_2)\tau^5 + C_6\tau^6 + \ldots) + \rho(x_s)\tau^5 + D_6\tau^6 + \ldots), \end{aligned}$$

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$$\Box\delta\phi = -V_{\phi\phi}\,\delta\varphi$$

$$\delta\ddot{\phi} = +\frac{2}{t^2}\,\delta\varphi \; -$$

$$\int \phi_2^2\,\dot{\chi}^2$$

$$\delta\ddot{\chi} = \frac{4}{t^2}\,\delta\dot{\chi} - k^2\,\delta\chi$$

$$\delta \ddot{X} = \frac{4}{t^2}\,\delta \dot{X} - k^2\,\delta X$$

$$\delta X \sim \text{const} \ \text{ or } \ t^5$$

1. Linear terms in t_s and ρ completely regular (even/odd in t) : match unambiguously across t=0
2. Nonlinear parts are then completely determined

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2. WKB, SA extension

$p_\phi \sim \sqrt{2(E-V)} \sim \lambda^{1/2}\phi^2 V_3$

WKB condn $p_\phi^{-2} dp_\phi^{-1/2}/d\phi \sim_1 \lambda\, \phi^{-3} V_3$ ~> 0, large ϕ

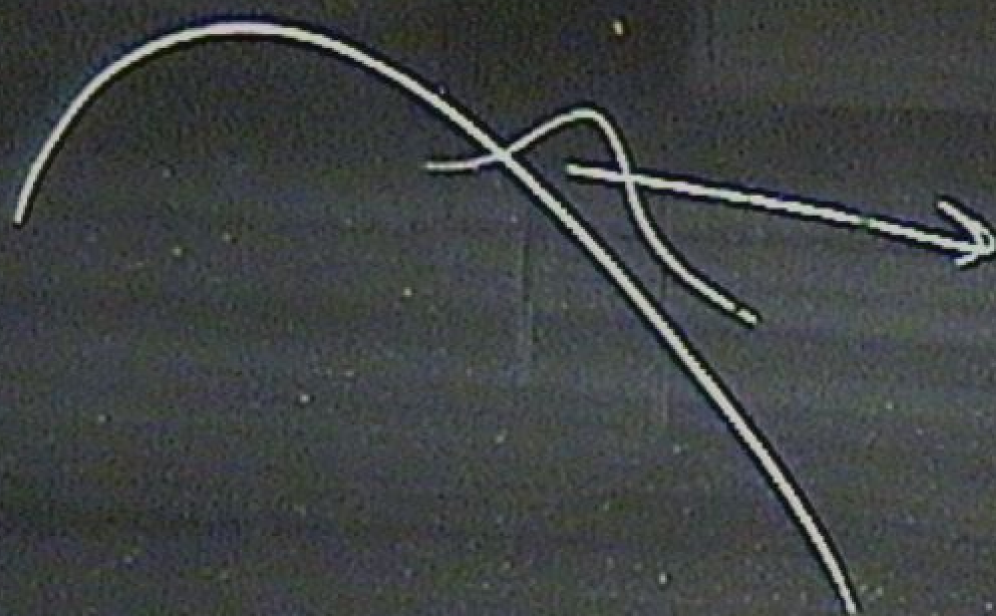
Self-Adjoint extension: Reed+Simon 70's

$\Psi \sim e^{-iET}\, p_\phi^{-1/2}\, \left(e^{i\int^\phi p_\phi d\phi} + e^{i\theta}\, e^{-i\int^\phi p_\phi d\phi}\right)$

$p_\phi \sim \phi^2 \;\text{->}\; |\Psi|^2 \sim \phi^{-2}$ normalisable

Halve Hilbert space -> unitary evolution, no probability lost at infinity

$$V = -\lambda \phi^4$$



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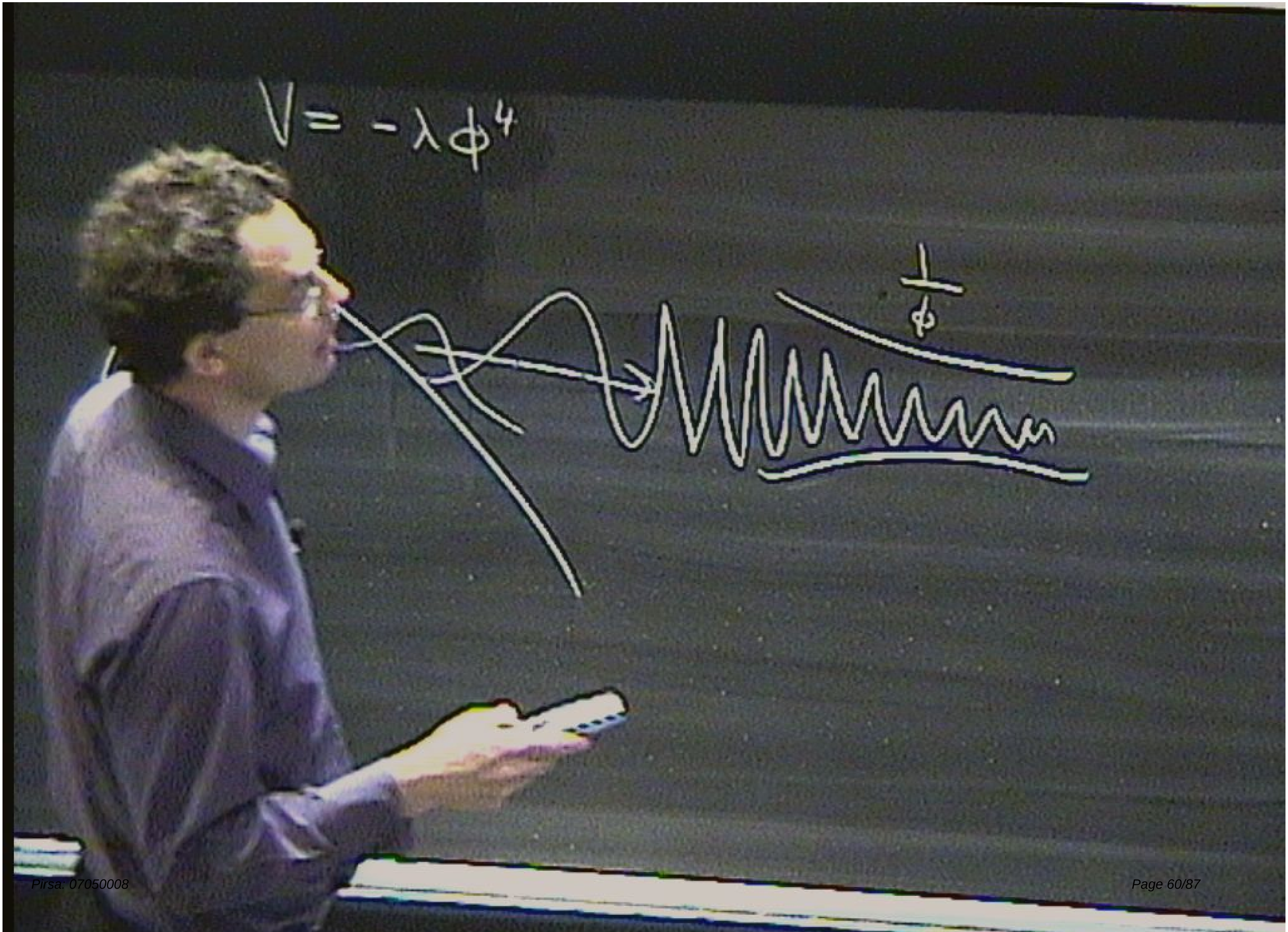


$V = -\lambda\phi^4$
$\frac{1}{\phi}$

Large ϕ at small time

Wavefunction may be calculated using complex classical solutions

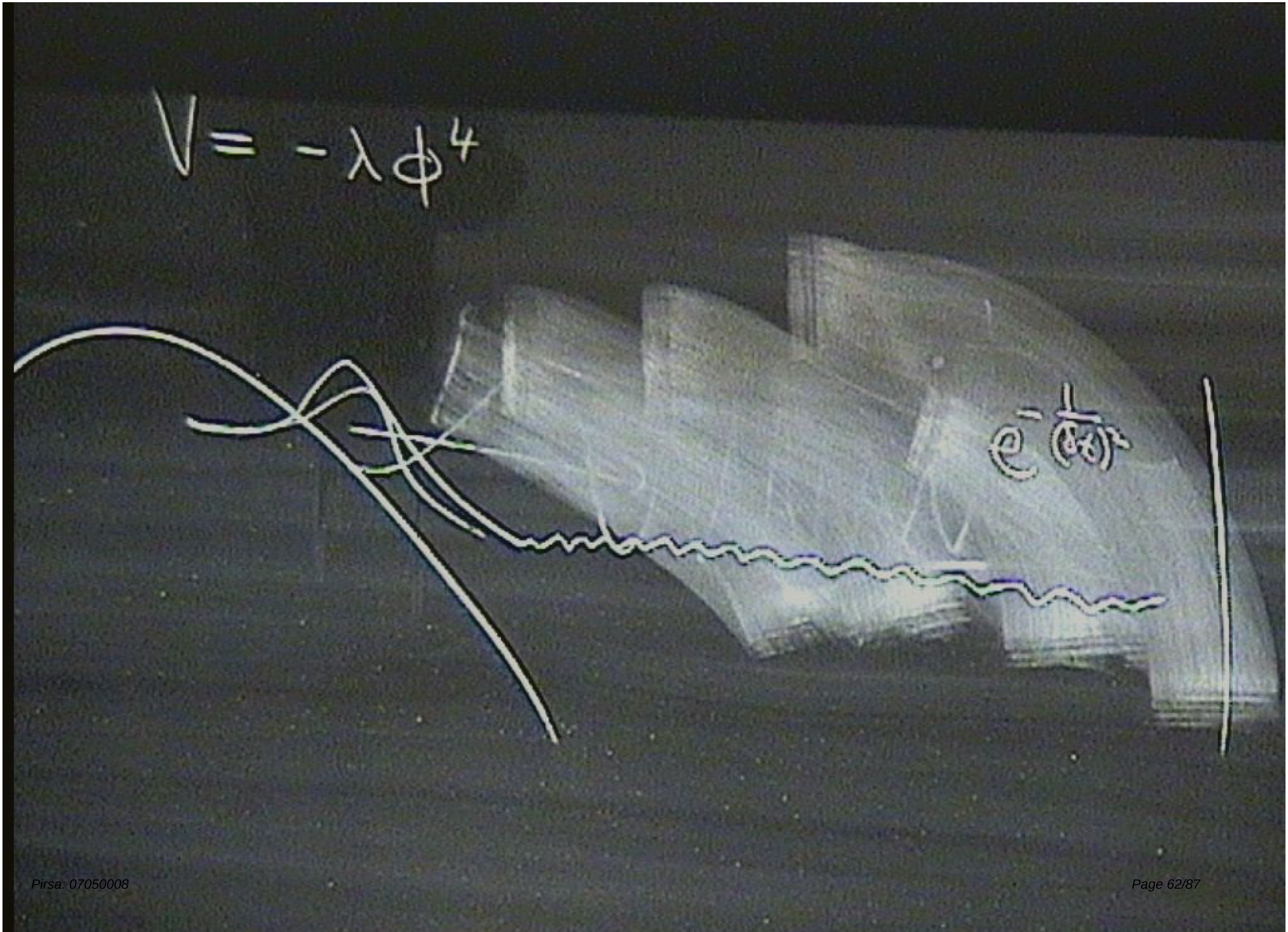
$$p + 2i\phi\, l^2 = p_0 + 2i\,\phi_0\, l^2$$

$$\Psi \sim \left(e^{iS_1} + e^{i\theta}\, e^{iS_2}\right)$$

$$\sim e^{-(\phi^2/2\, l^2)} \qquad \phi < \lambda/\delta t$$

$$\sim e^{-(1/l^2 \lambda\, \delta t^2)}\, \phi^{-1} \cos(\phi^3 + \theta) \qquad \phi > \lambda^{-1/2}/\delta t$$

$\langle\phi\rangle$ is infinite -> classical bg never exists!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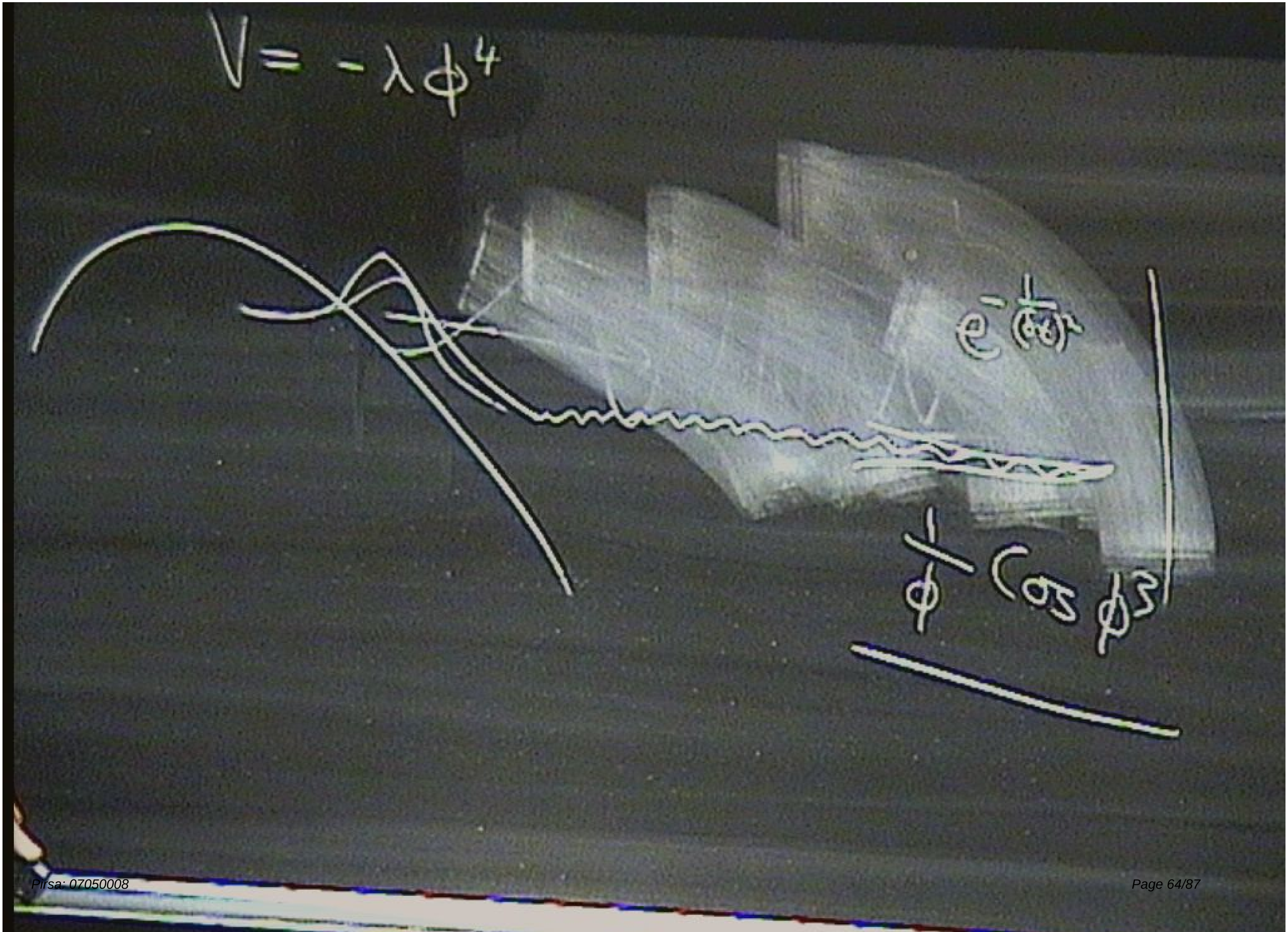
$V = -\lambda\phi^4$
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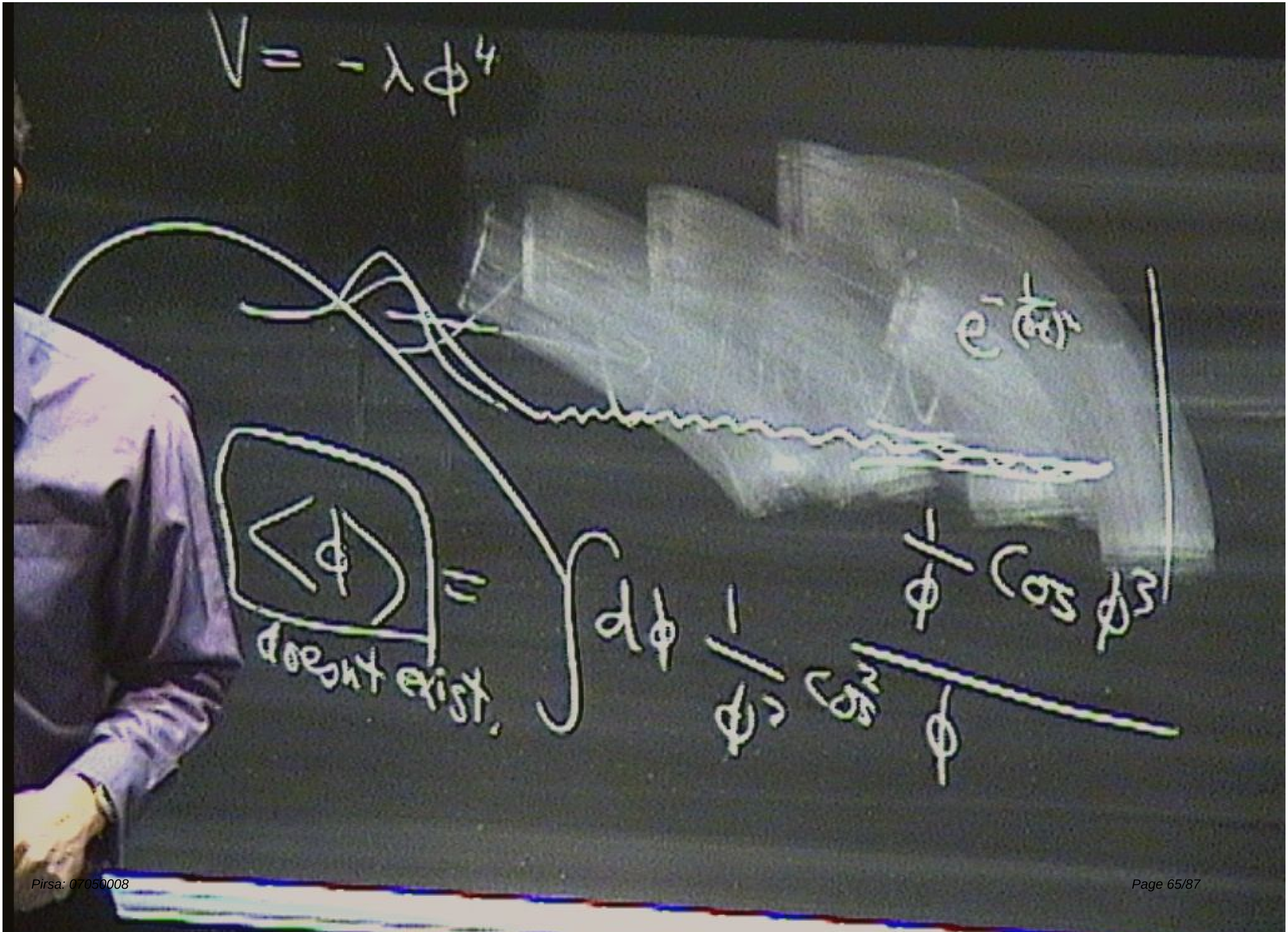
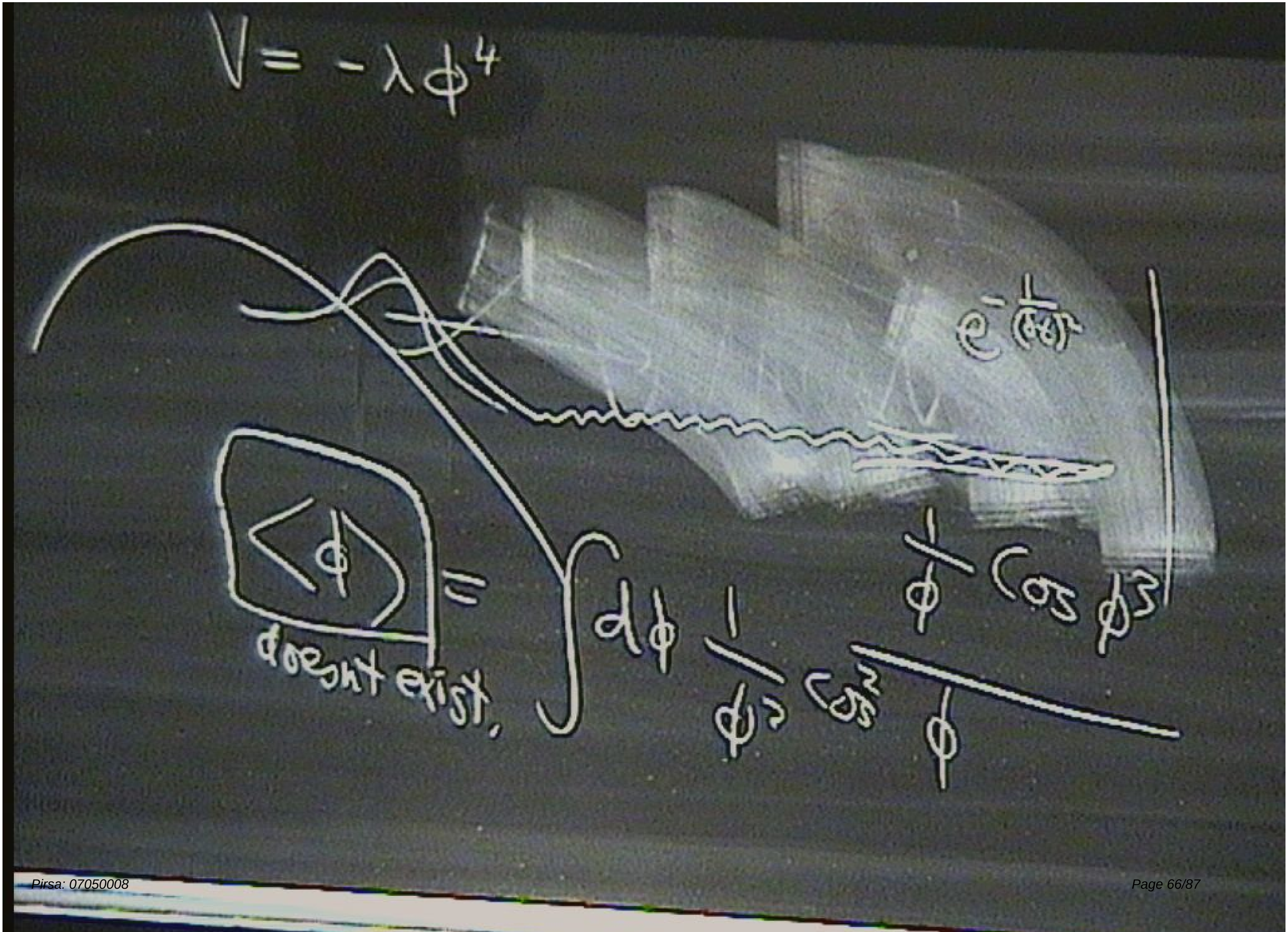
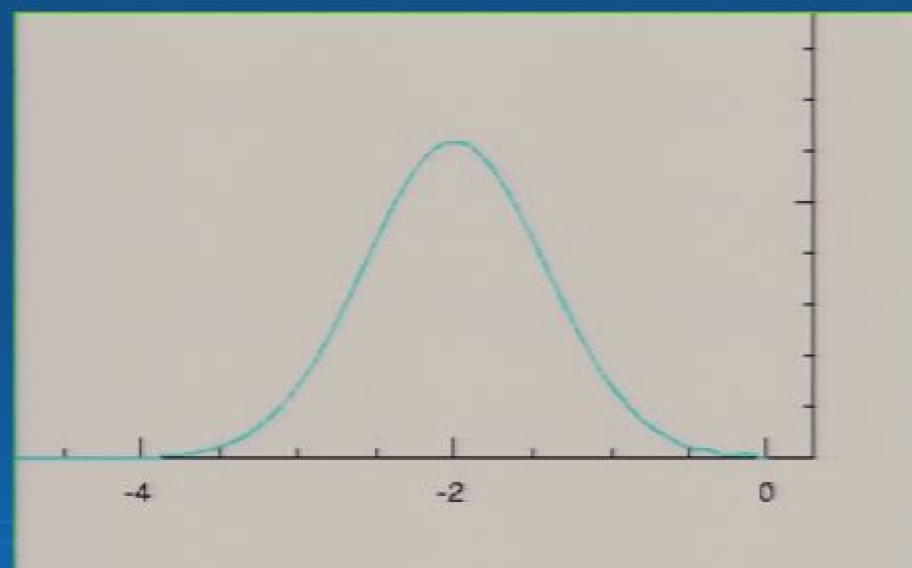

V = - λφ4
⟨φ⟩ =
∫dφ 1/φ2 cos
φ
φ cos φ3
doesnt exist.


V = -λφ⁴
⟨φ⟩ =
doesnt exist.
∫dφ 1/φ² cos²
φ
1/φ cos φ³

But for an initially localized wavepacket, large ϕ tail unimportant except near singularity, $|t-t_s| \sim \lambda^{1/2} R_{AdS}$

What happens at the singularity?

Example: free particle, incoming Gaussian wavepacket hits brick wall



χ

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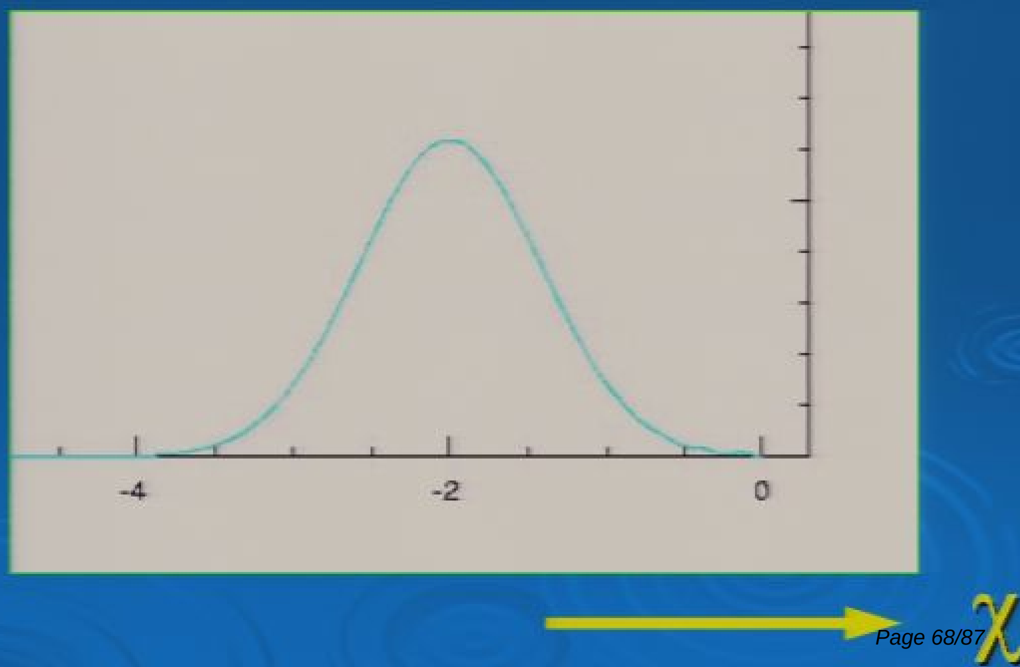

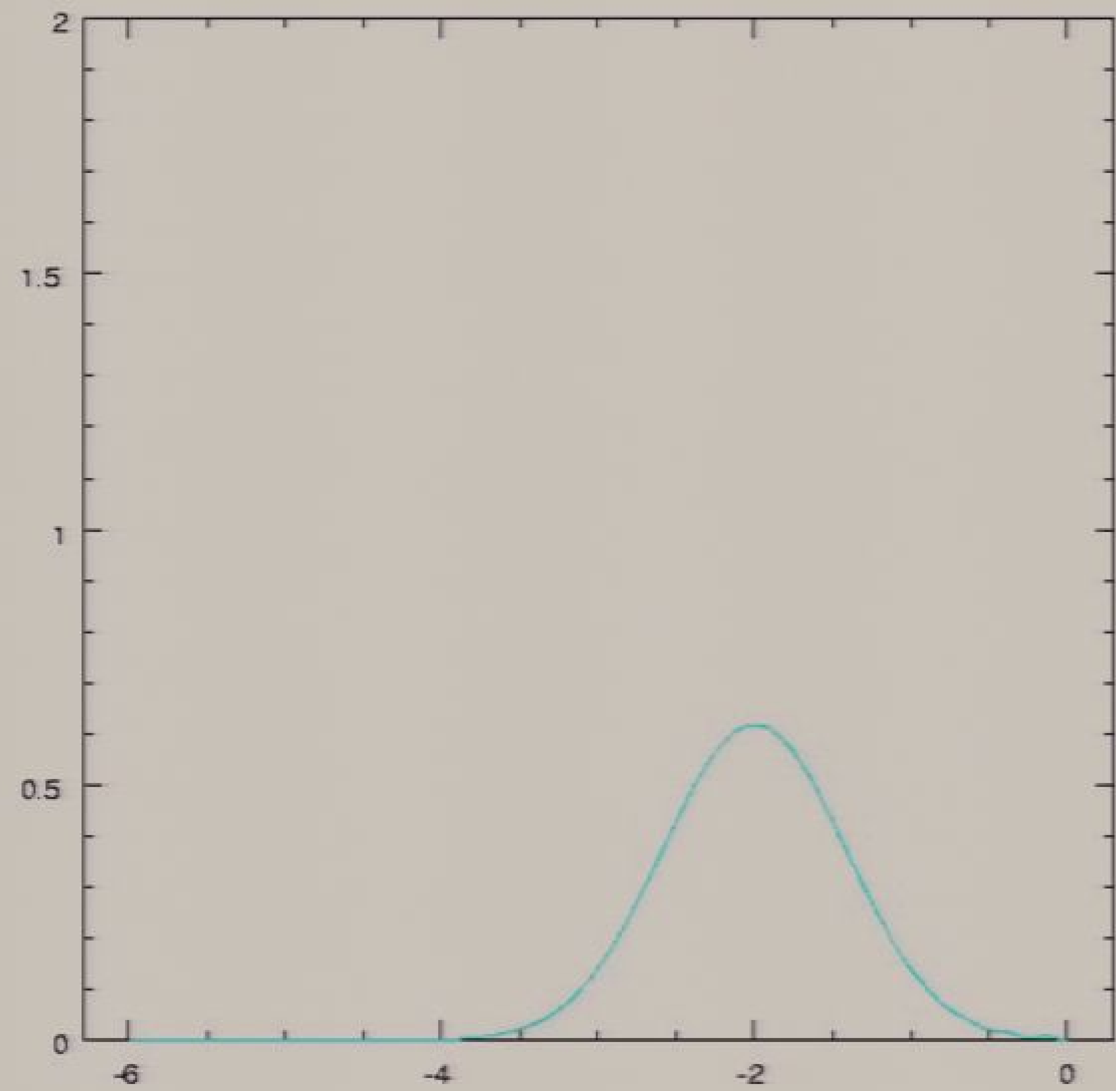
2
1.5
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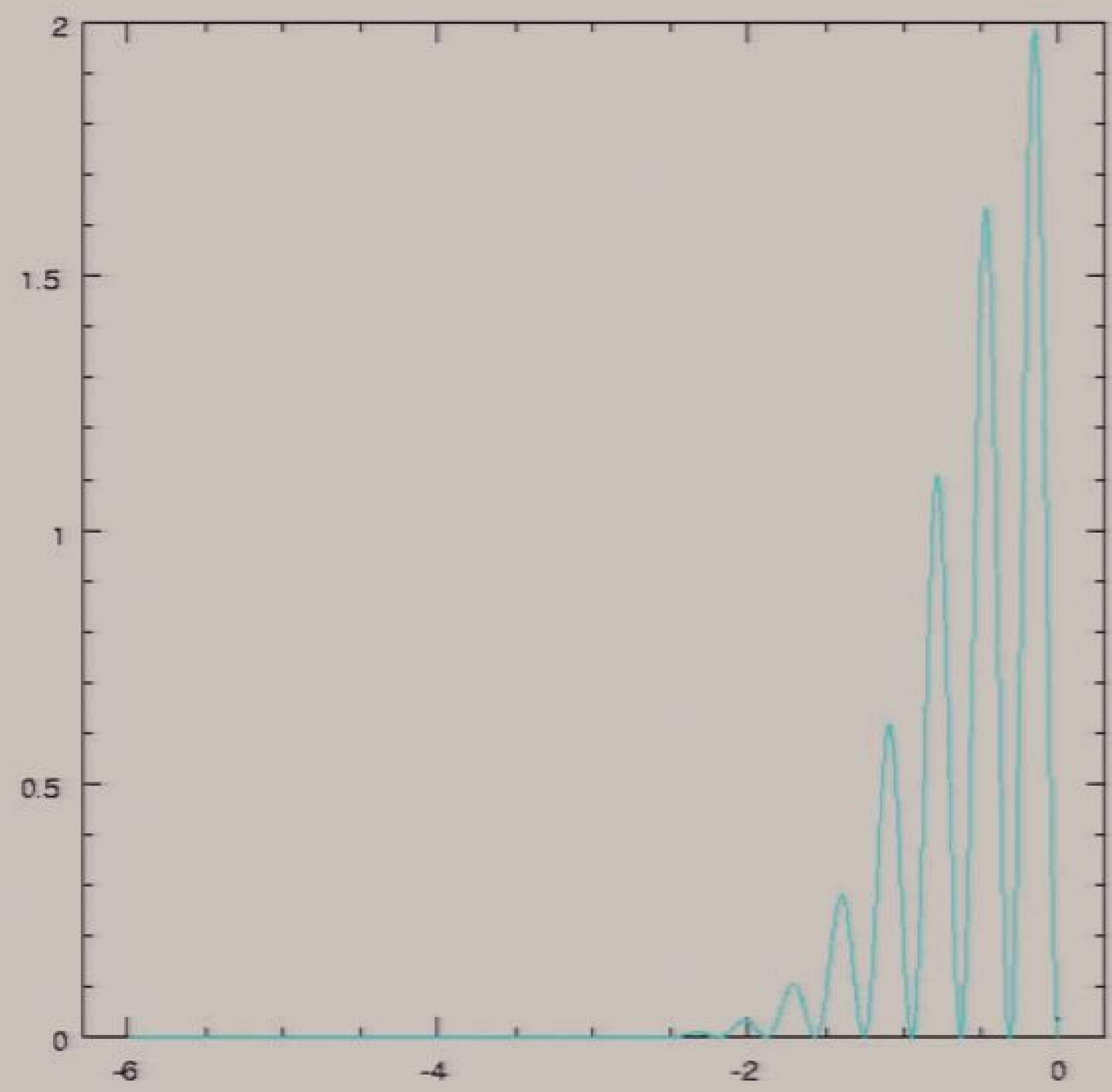
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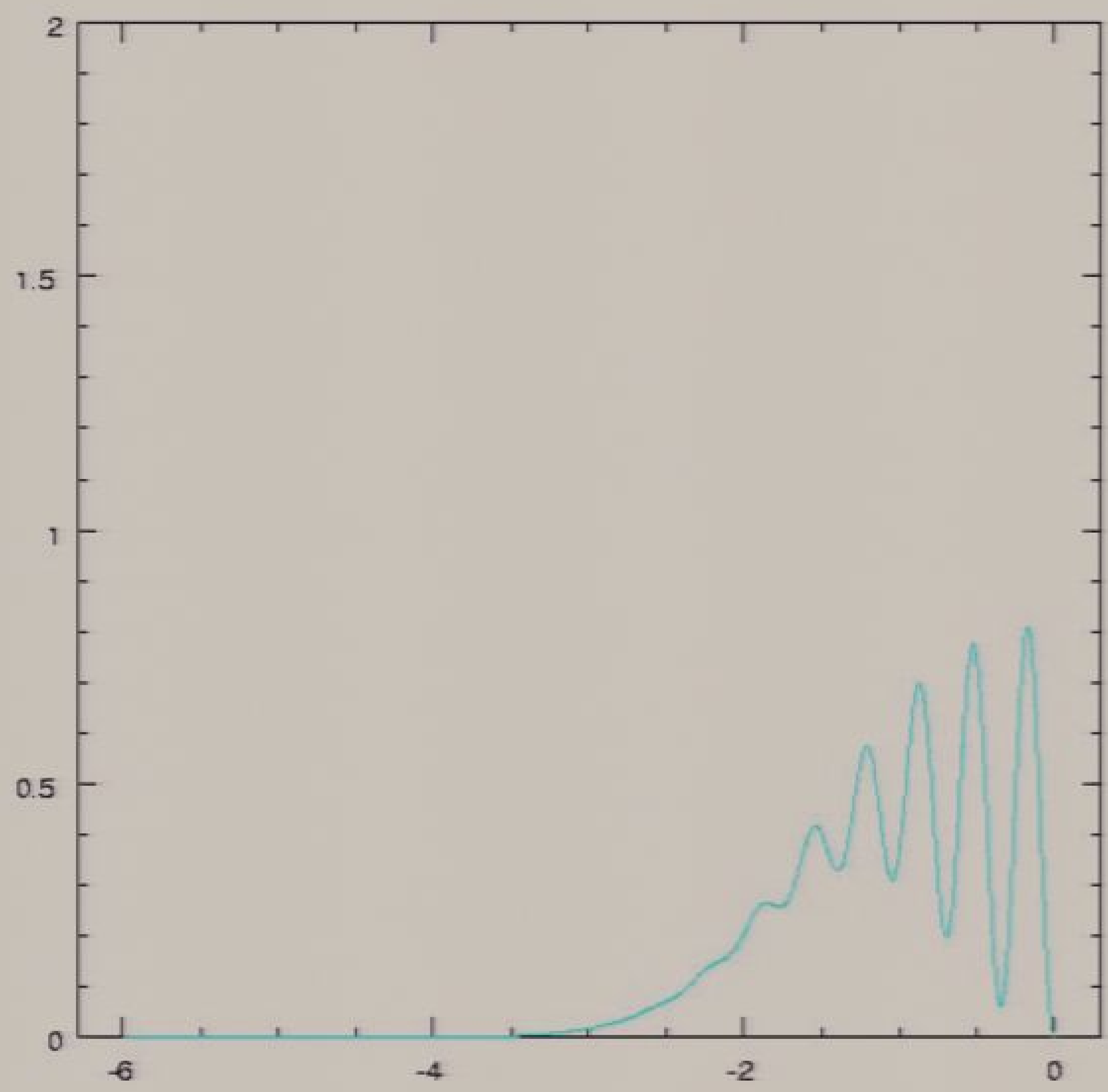
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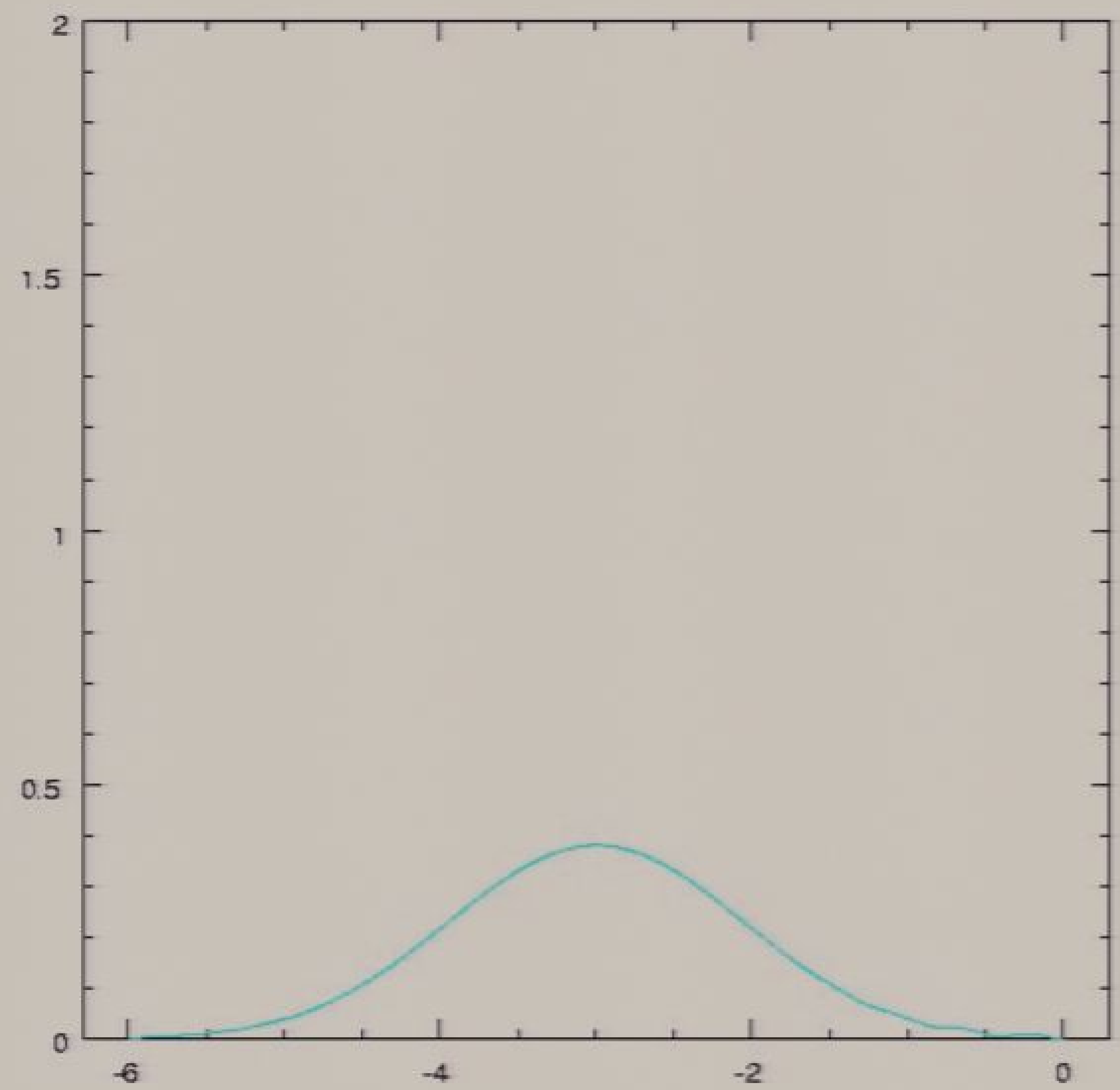
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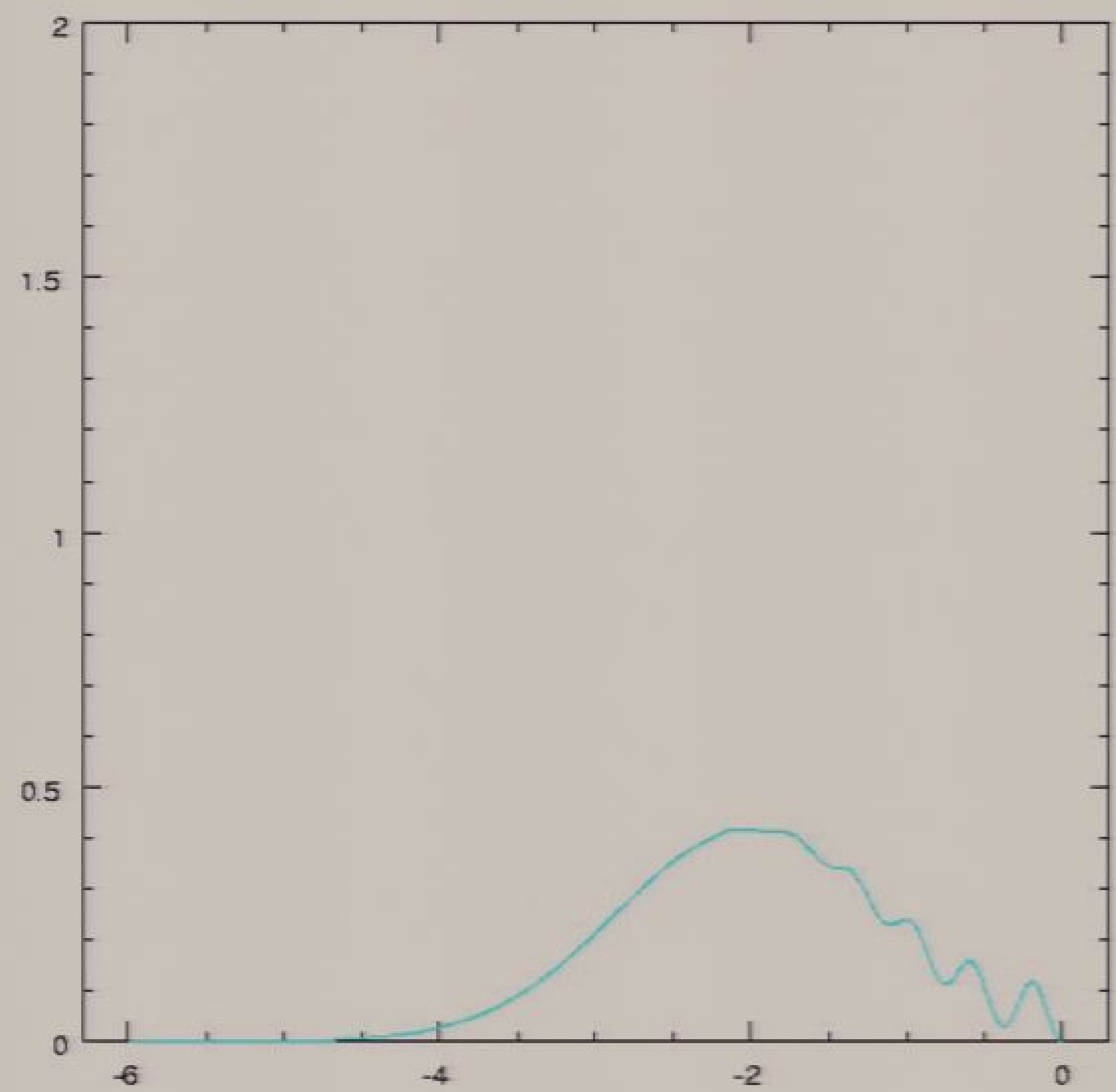

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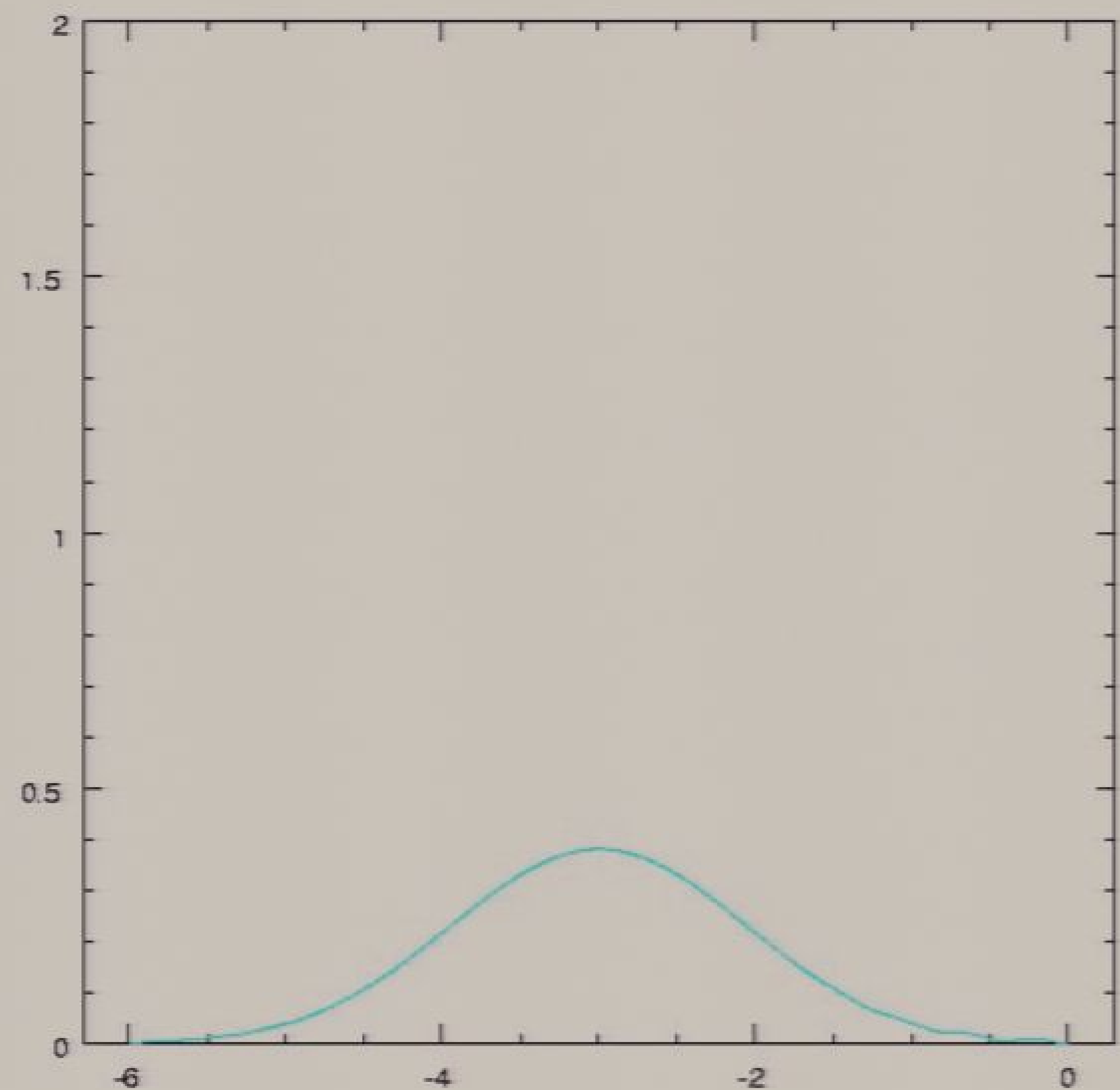
The bg/flucn split in ϕ fails totally near the singularity, but $\chi_c = \langle\chi\rangle$ is convergent at large ϕ so a bg/flucn split in χ is reasonable

$\chi = \phi^{-1}$ -> $\chi\partial^2\chi - 2(\partial\chi)^2 = \lambda$

Let $\chi = \langle\chi\rangle + \delta\chi$ -> $\delta\ddot{\chi} - 4\frac{\langle\dot{\chi}\rangle}{\langle\chi\rangle}\delta\chi = -k^2\delta\chi$

But $\langle\chi\rangle$ finite for all t (QM reflection)

-> particle creation in $\delta\chi$ is exponentially suppressed in UV, i.e. for $k > \delta t_s^{-1} \sim \lambda^{-1/2} R_{AdS}^{-1}$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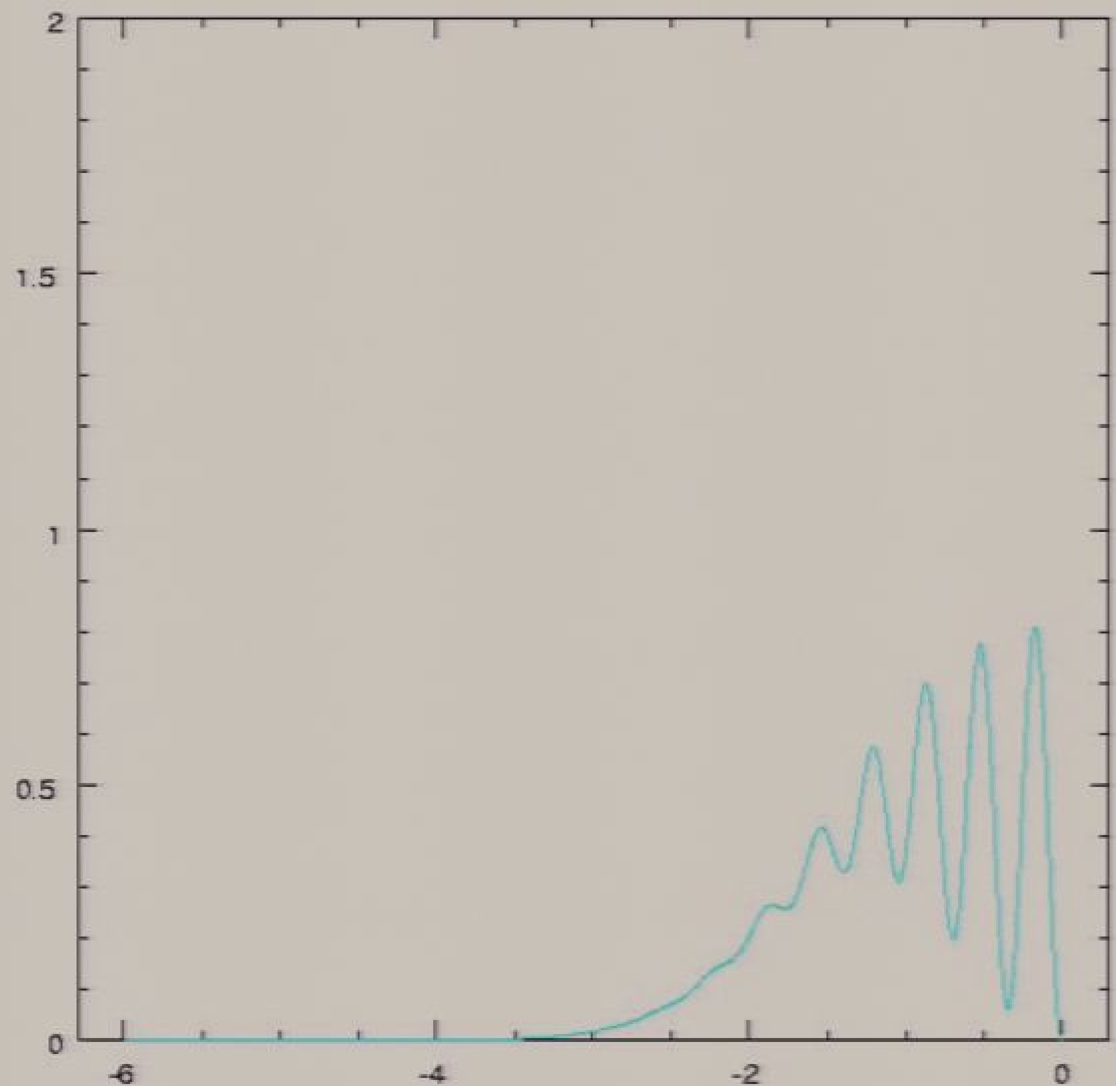
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Initial Conditions

$\phi \sim \lambda^{-1/2} R_{AdS}^{-1}$
$t_s \sim R_{AdS}$

zero energy start

QM spreading: e.g. free pticle

$\delta\phi^2 \sim \ell^2 + (\delta p/m)^2 t^2 \sim \ell^2 + (\hbar/m\ell)^2 t^2$

Minimise for given t: $\ell^2 \sim \hbar/mt$

In our case, minimal spread achieved by

$\delta\phi \sim R_{AdS}$: time delay $\delta t_s \sim \lambda^{1/2} R_{AdS}$

Away from sing$^{y.}$ $\phi = \phi_c + \delta\phi$ is reasonable

$$S \approx \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + \lambda\phi^4 \right)$$

$$\lambda = \frac{16\pi^2}{3\ln(\phi/M)} \equiv \frac{\lambda_0}{l}$$

Zero Energy soln (attractor)

$$\phi = \frac{l^{\frac{1}{2}}}{\sqrt{2\lambda_0}} \frac{1}{(-t)} \left(1 + \frac{1}{2l} - \frac{1}{4l^2} \dots \right)$$

Pertns

$$\ddot{\delta\phi} = \frac{6}{t^2} \left(1 + \frac{5}{12l} - \frac{2}{3l^2} \dots \right) \delta\phi - k^2 \delta\phi.$$

Evolve incoming modes until they become ultralocal (\` frozen'), then match across singularity using QM SA extension

3. Mode Mixing, Particle Creation

At leading order in log, no mode mixing and no particle creation. But at next order,...

Mode Evolution

$$\delta\phi^{(1)} = l^{\frac{1}{2}} f^{(1)}(kt) + l^{-\frac{1}{2}} g^{(1)}(kt) + \ldots,$$

$$\delta\phi^{(2)} = l^{-\frac{1}{2}} f^{(2)}(kt) + l^{-\frac{3}{2}} g^{(2)}(kt) + \ldots,$$

$$f^{(1)} = \cos kt \left(1 - \frac{3}{(kt)^2}\right) - 3\frac{\sin kt}{kt},$$

$$f^{(2)} = \sin kt \left(1 - \frac{3}{(kt)^2}\right) + 3\frac{\cos kt}{kt}$$

Evolve incoming pos freq mode, match across t=0, compute Bog. coeffft

$$\beta \approx -i\frac{\pi}{\ln(k/\sqrt{\lambda}M)}$$

Particle Production

Density of created particles $\rho_c = \int \frac{d^3\mathbf{k}}{(2\pi)^3} k|\beta|^2$ $\sim R_{AdS}^{-4}$

A small perturbation on V where UV cutoff kicks in $V_m \sim -\lambda^{-3} R_{AdS}^{-4}$

-> ϕ returns close to its original value

After N bounces $V(\phi_{min}) = -N R_{AdS}^{-4}$

This falls to the point where QFT fails, after $N \sim \lambda_m^{-3}$ bounces

Scale-Invariant Perturbations

"improved"

$T_{\mu\nu}$

$$\langle \mathcal{O} \rangle \equiv \langle 0, \mathrm{in} | \mathcal{O} | 0, \mathrm{in} \rangle - \langle 0, \mathrm{out} | \mathcal{O} | 0, \mathrm{out} \rangle$$

$$\langle \delta T_{00}(r,t) \delta T_{00}(0,t) \rangle \sim \frac{1}{\ln^2(1/Mr)} \frac{1}{t^2 r^6}$$

$$\langle \delta T_{0i}(r,t) \delta T_{0i}(0,t) \rangle \sim \frac{1}{\ln^2(1/Mr)} \left(\frac{1}{t^2 r^6} + \frac{1}{t^4 r^4} \right)$$

$$\langle \delta \bar{T}_{ij}(r,t) \delta \bar{T}_{ij}(0,t) \rangle \sim \frac{1}{\ln^2(1/Mr)} \frac{1}{t^6 r^2}$$

i.e.

$$\langle \frac{\delta\rho(r,t)}{P+\rho} \frac{\delta\rho(0,t)}{P+\rho} \rangle \sim \frac{1}{\ln^2(1/Mr) \ln(1/Mt)} f(r/t)$$

These will determine bulk correlators and hence cosmological perturbations

Amplitude ~ λ^3 naturally small

Tilt: red, from running of λ

Gaussian (NG ~ λ)

Scalar, Adiabatic

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Scalar, Adiabatic

- Finite density of radiation produced
- GLASS perturbations without tuning

In progress:

- Translation of perturbations into bulk
- Model with 4d bulk, 3d CFT
- Glue onto positive dark energy phase to get realistic cyclic model

Summary

* The cyclic model is (an attempt at) a more complete cosmological model than inflation, incorporating dark energy, dealing with singularity

* Possible to generate realistic curvature perturbations before the bang, even within 4dET

* Main phenomenological difference: inflation -> scale-invariant tensors