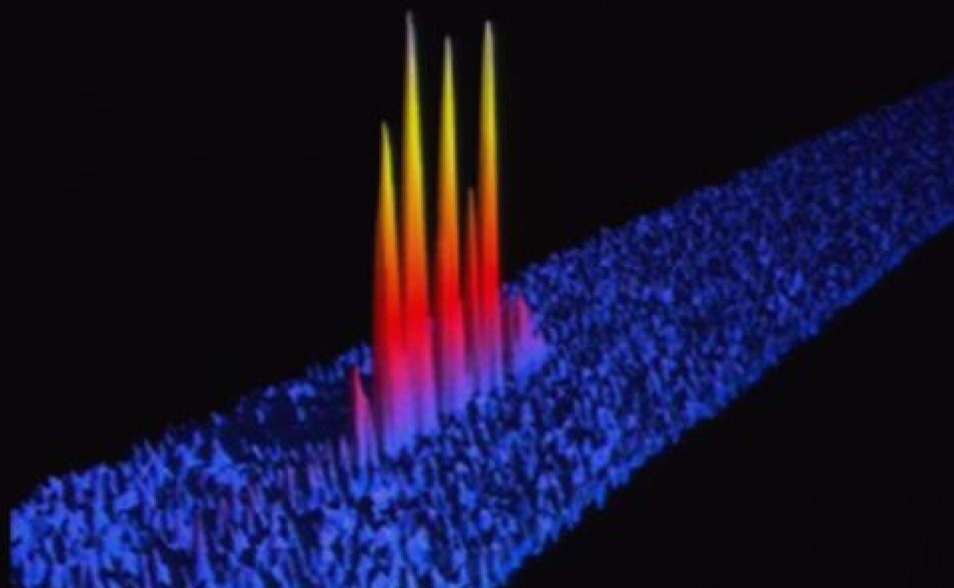
Title: Models for multimode Bose-Einstein condensates with exact analytical solutions

Date: Apr 11, 2007 04:00 PM

URL: http://pirsa.org/07050005

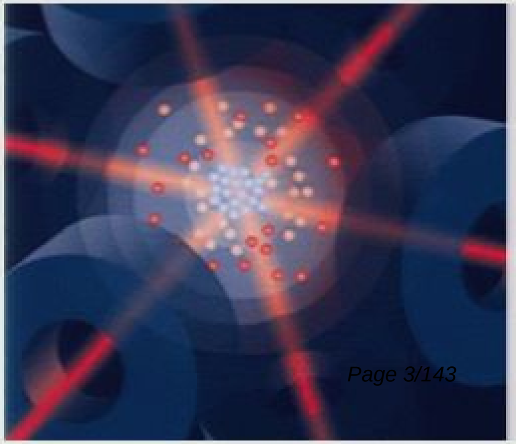
Abstract: Inelastic collisions occur in Bose-Einstein condensates, in some cases, producing particle loss in the system. Nevertheless, these processes have not been studied in the case when particles do not escape the trap. We show that such inelastic processes are relevant in quantum properties of the system such as the evolution of the relative population and entanglement. Moreover, including inelastic terms in the models of multimode condensates allows for an exact analytical solution.Â

Models for multimode Bose-Einstein condensates with exact analytical solutions



Ivette Fuentes-Schuller
(nee Fuentes-Guridi)
with Pablo Barberis-Blostein
Universidad Nacional Autonoma de Mexico

outline



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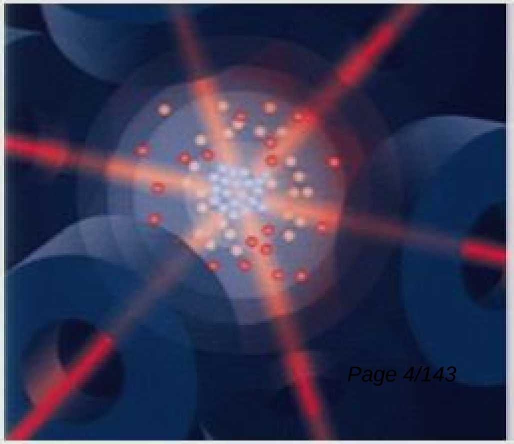

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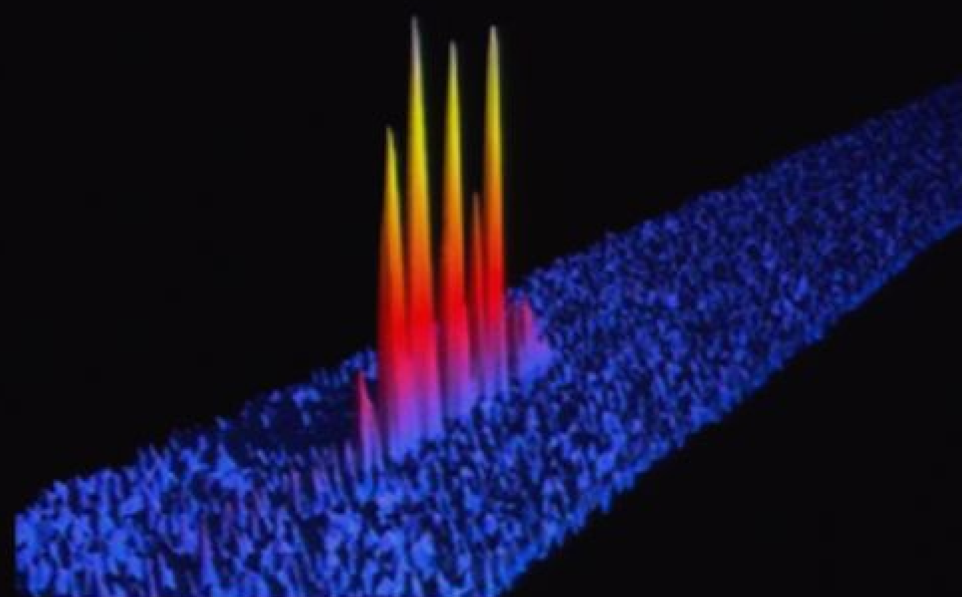

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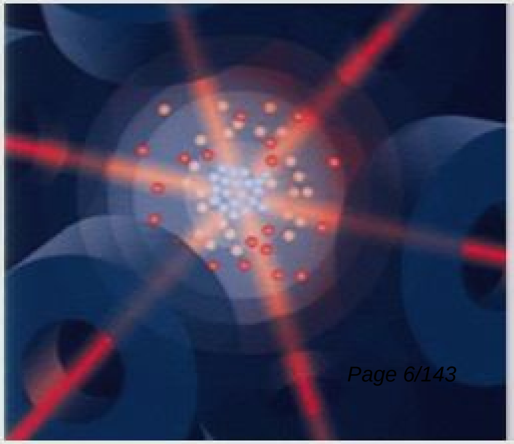
- The two-mode Bose-Einstein condensate



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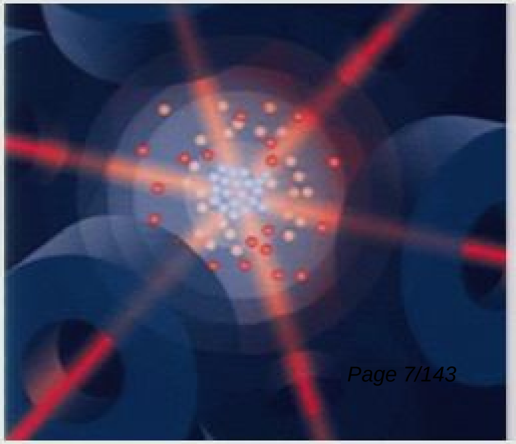
- The two-mode Bose-Einstein condensate
- Inelastic collisions in BECs



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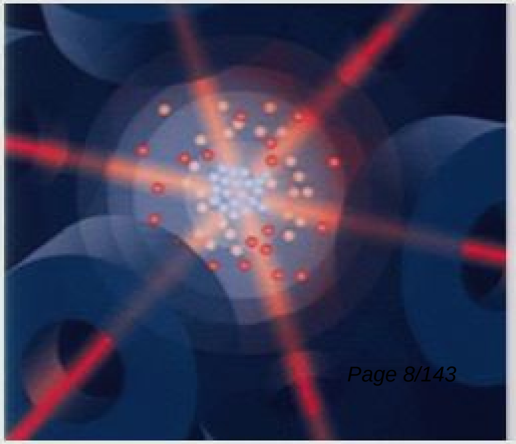

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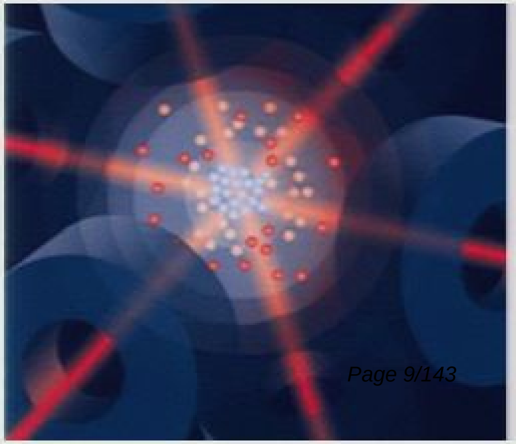

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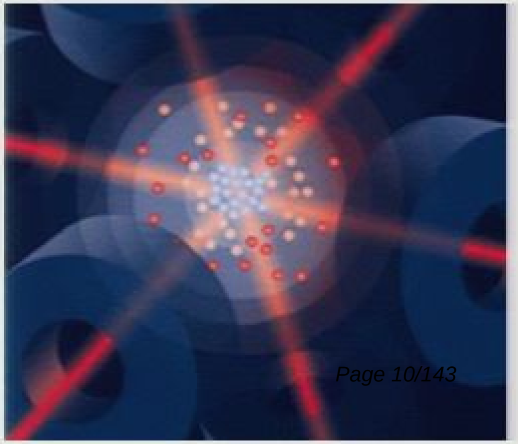

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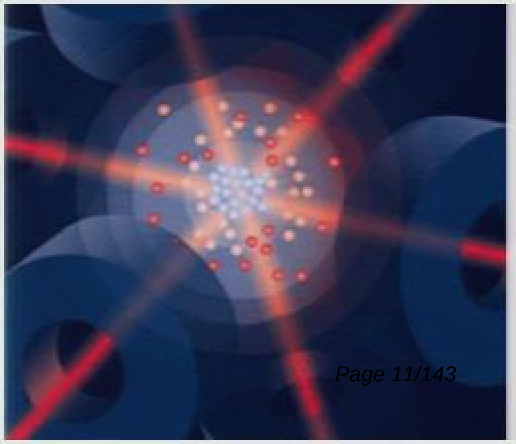

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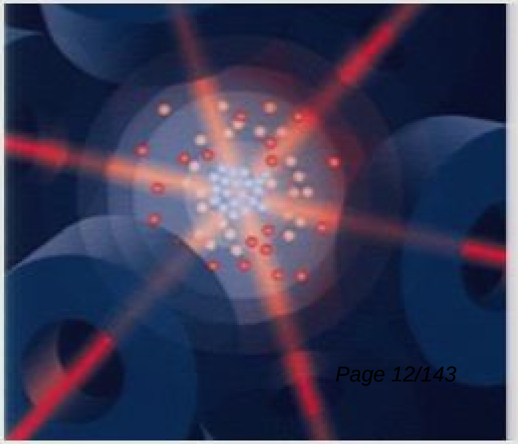

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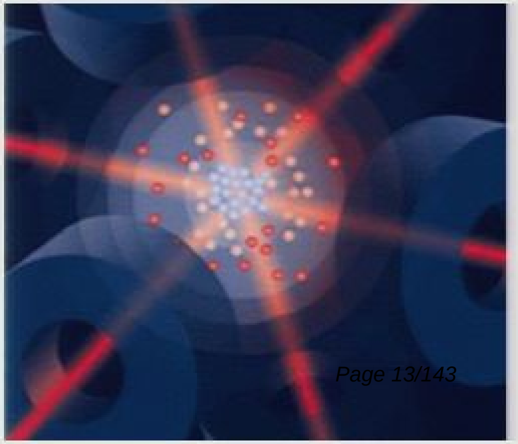
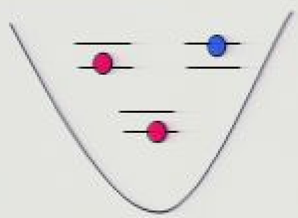
  - Multimode condensates



The two-mode condensate

TM BEC: internal degrees of freedom

N atoms with two internal degrees of freedom



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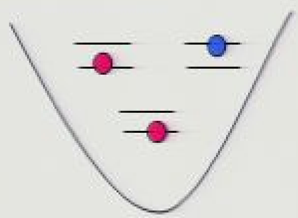
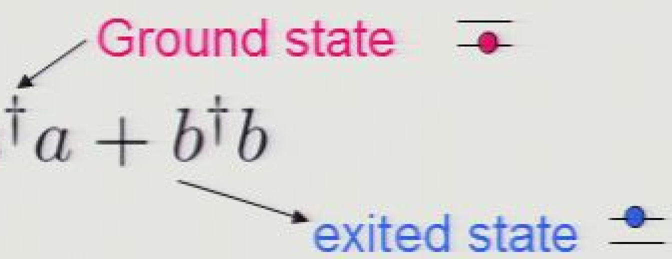


In the two-mode approximation

$$N = a^\dagger a + b^\dagger b$$



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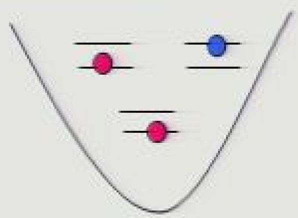

In the whole approximation

Ground state

$$N = a^\dagger a + b^\dagger b$$

Excited state

TM BEC: internal degrees of freedom

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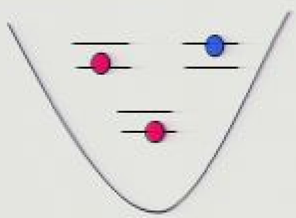

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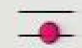
Ground state 

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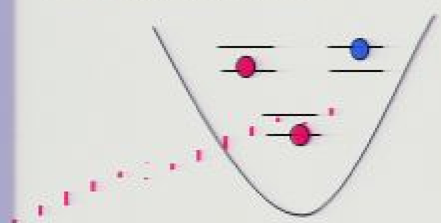


In the two-mode approximation

Ground state

$$N = a^\dagger a + b^\dagger b$$

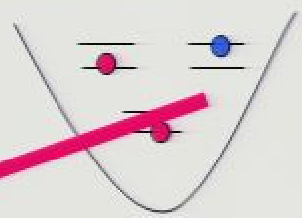
exited state

Relative population: $m = a^\dagger a - b^\dagger b$

Main interactions:

TM BEC: internal degrees of freedom

N atoms with two internal degrees of freedom



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Josephson-type interaction

$ab^\dagger$ Ground to excited state

$a^\dagger b$ Excited to ground state

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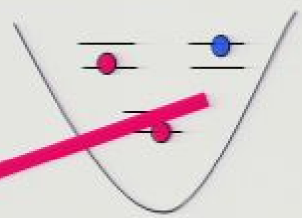

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Two body elastic collisions

$$a^\dagger a a^\dagger a$$

$$b^\dagger b b^\dagger b$$

$$a^\dagger b^\dagger a b$$

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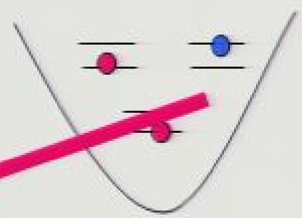

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Ground state ($a^\dagger a$)

exited state ($b^\dagger b$)

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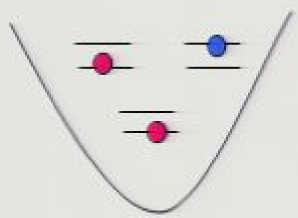
$b^\dagger b b^\dagger b$

$a^\dagger b^\dagger a b$

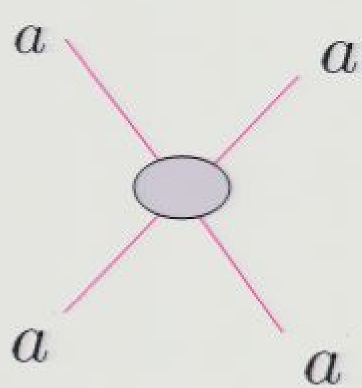
Elastic collisions



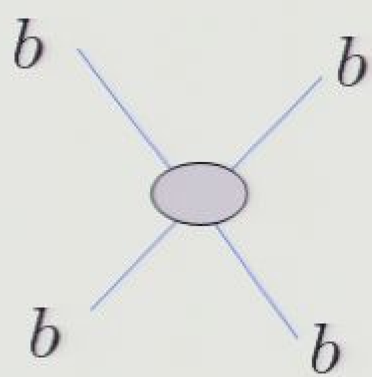
Two body elastic collisions

Number of particles in each mode: conserved

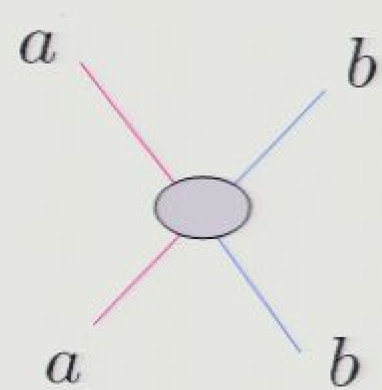
$$a^\dagger a a^\dagger a$$



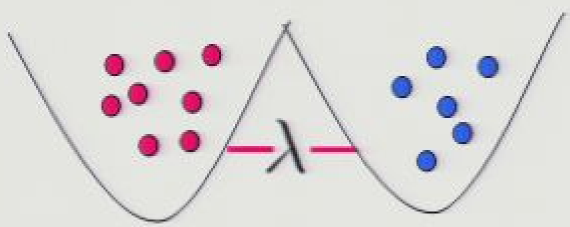
$$b^\dagger b b^\dagger b$$



$$a^\dagger b^\dagger a b$$



TM BEC: double well



$a^\dagger a$ $b^\dagger b$

$a^\dagger a a^\dagger a$ $b^\dagger b b^\dagger b$

Josphson-type interaction:

Tunneling barrier

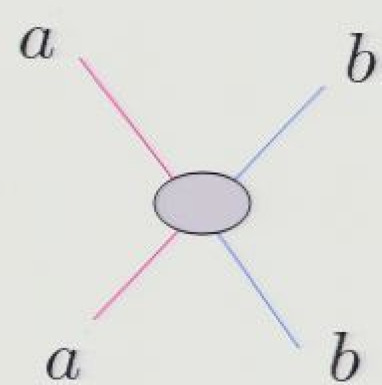
$ab^\dagger$ Ground to excited state

$a^\dagger b$ Excited to ground state

Two body elastic collisions

In the region where the wave functions overlap:

$$a^\dagger b^\dagger ab$$



Canonical Two-mode BEC model

Free energy particles in the trap

Interaction with laser/tunnelin barrier

Josphson-type ineraction

$$
\begin{aligned}
H_2 \;=\; & \omega_a a^\dagger a + \omega_a b^\dagger b + \lambda(e^{i\phi} a^\dagger b + e^{-i\phi} a b^\dagger) \\
+ \; & \mathcal{U}_a a^\dagger a^\dagger a a + \mathcal{U}_b b^\dagger b^\dagger b b + \mathcal{U}_{ab} a^\dagger b^\dagger a b.
\end{aligned}
$$

Elastic collisions

Canonical Two-mode BEC model

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\end{aligned}
$$

Elastic collisions

$$
\begin{aligned}
A_0 &= \Omega N + U N^2 \\
&= \Omega(a^\dagger a + b^\dagger b) + \mathcal{U}(a^\dagger a^\dagger a a + b^\dagger b^\dagger b b + 2 a^\dagger b^\dagger a b) \\
\omega_a &= \Omega + \delta\omega \\
\omega_b &= \Omega - \delta\omega
\end{aligned}
$$

$$\delta\omega = \omega_a - \omega_b$$

$$
\begin{aligned}
U &= \mathcal{U}_a = \mathcal{U}_b \\
\mathcal{U}_{ab} &= \mathcal{U} + 2U
\end{aligned}
$$

Canonical Two-mode BEC model

$$H_{two} = \delta\omega(a^\dagger a - b^\dagger b) + \lambda(e^{i\phi}a^\dagger b + e^{-i\phi}ab^\dagger) + U\,a^\dagger b^\dagger ab$$

G. J. Milburn, J. Corney, E. M. Wright, and D. F. Walls, Phys. Rev. A 55, 4318 (1997).
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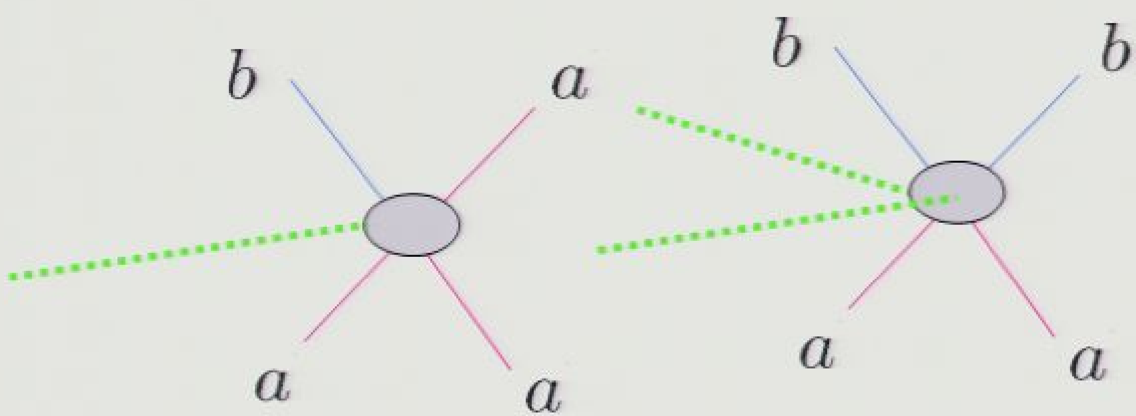
No analytical solution

Hamiltonian diagonalized exactly by numerical means

Ground state and first excited state found by Bethe ansatz

Inelastic collisions?

$a^\dagger a^\dagger a b$ $\qquad$ $a^\dagger a^\dagger b b$



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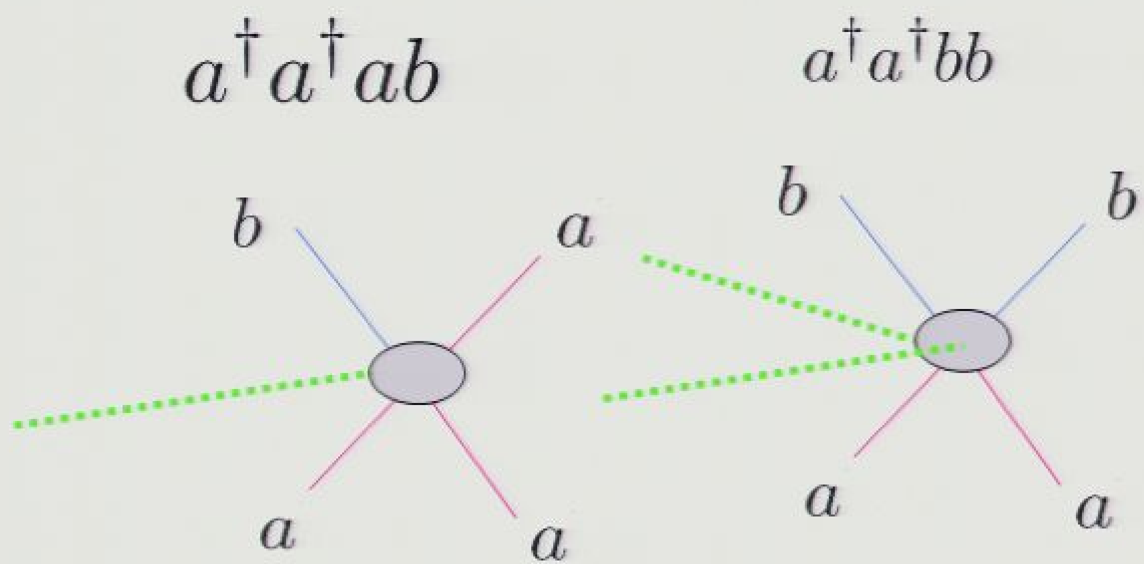
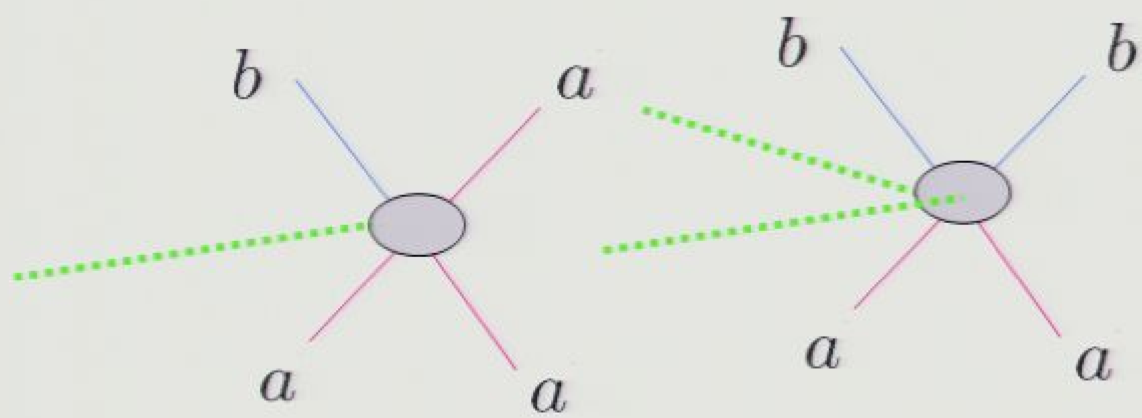


Particles change internal state
after the collision

Inelastic collisions?

$a^\dagger a^\dagger ab$ $\qquad$ $a^\dagger a^\dagger bb$



Particles change internal state after the collision

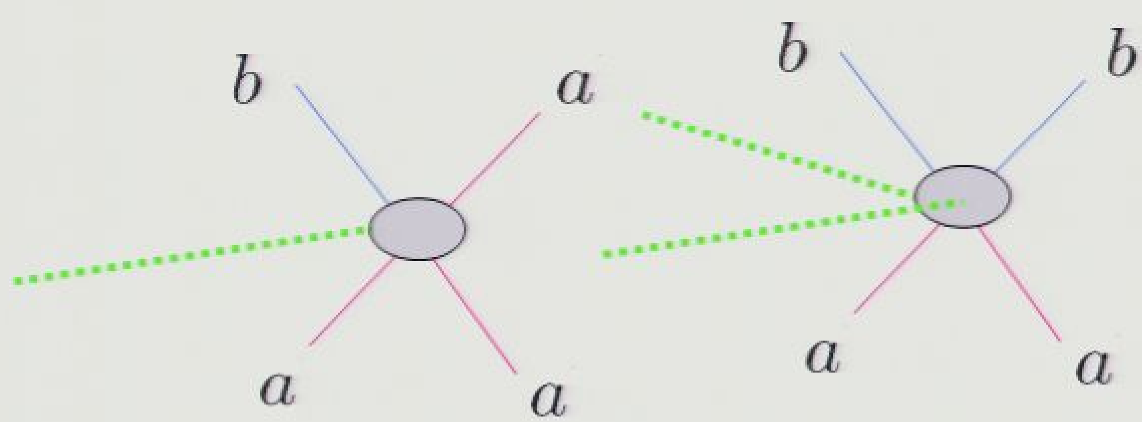
In the overlapping region two particles from one well collide and one or both of then end up in the other well.

Inelastic collisions

Inelastic collisions?

Complicate the Hamiltonian?

$$a^\dagger a^\dagger a b \qquad a^\dagger a^\dagger b b$$



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- The internal degree of freedom is changed by then collision to a state not trapped by the potential
- Spin-exchange: particle change internal state after collision: If recombination energy larger that trap potential the particle is lost.
- Dipole-relaxation: particles change internal state during interaction due to dipole moment: Excess of energy transforms into momentum which can make particle escape the trap.

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But what happens if the excess energy is not enough for the particle to escape the trap?

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How can these processes be observed?

What are their effects in the system?

Bosons: Two-mode BEC

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\begin{aligned}
H_{two} &= \delta\omega(a^\dagger a - b^\dagger b) + \lambda(e^{i\phi}a^\dagger b + e^{-i\phi}ab^\dagger) \\
&+ U\, a^\dagger b^\dagger ab \\
&+ \Lambda(e^{2i\phi}a^\dagger a^\dagger bb + h.c.) \\
&+ \mu((a^\dagger a^\dagger ab - b^\dagger a^\dagger ab)e^{i\phi} + h.c.),
\end{aligned}
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Bosons: Two-mode BEC

Free energy particles in the trap

Josphson-type interaction

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Interaction with laser/ barrier

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Elastic collisions

Interaction with laser/ barrier

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Two particles change state/well ← inelastic collisions

$$+ \; \Lambda(e^{2i\phi} a^\dagger a^\dagger b b + h.c.)$$

One particle changes state/well

$$+ \; \mu((a^\dagger a^\dagger a b - b^\dagger a^\dagger a b) e^{i\phi} + h.c.),$$

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$$+ \; \mu((a^\dagger a^\dagger a b - b^\dagger a^\dagger a b) e^{i\phi} + h.c.),$$

Bosons: Two-mode BEC

Free energy particles in the trap

Josphson-type interaction

$$H_{two} = \delta\omega(a^\dagger a - b^\dagger b) + \lambda(e^{i\phi} a^\dagger b + e^{-i\phi} a b^\dagger)$$

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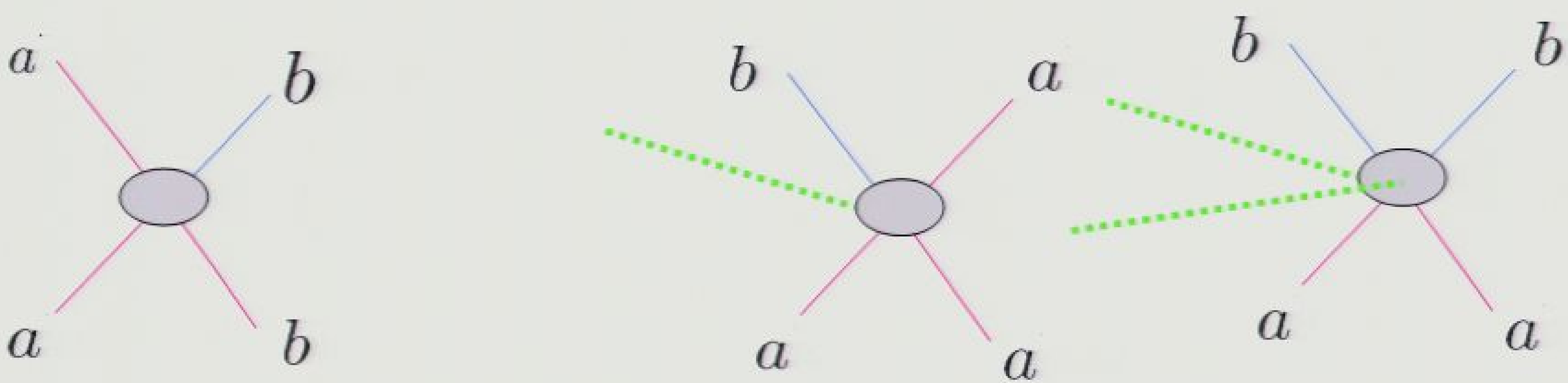


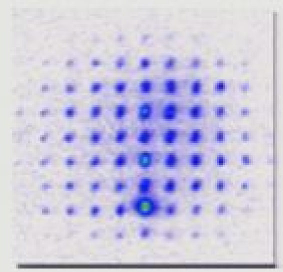
Analytical solution!!!

Comment on analytical solutions in many-body systems…

Many-body systems are relevant in most areas in physics

Implementation of quantum information

Ion traps, NMR, Optical lattices, spin chains



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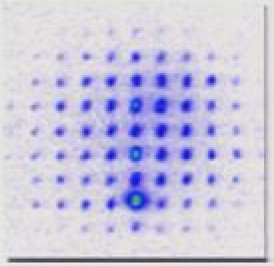

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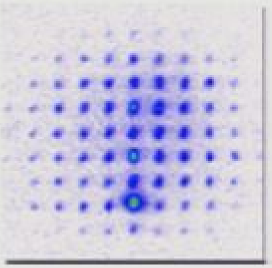

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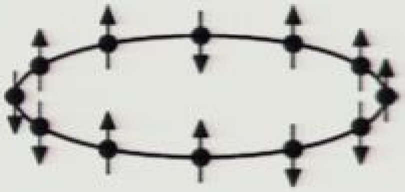

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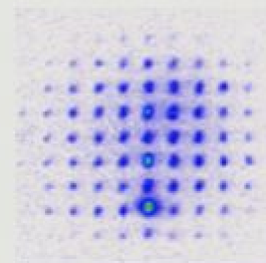

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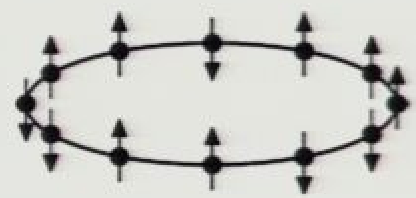
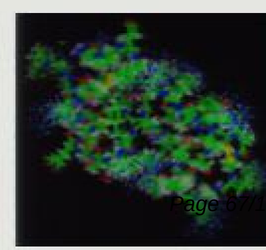
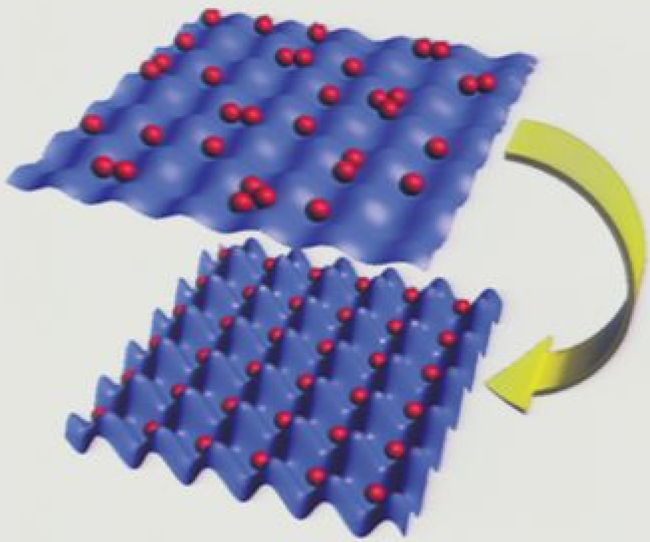


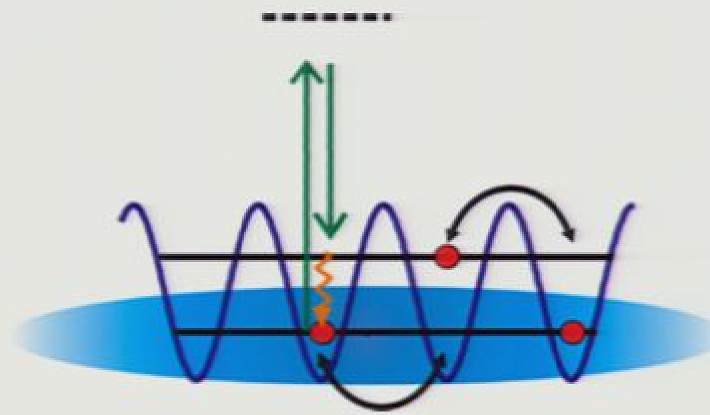
More dimensions: Numerical and approximate solutions

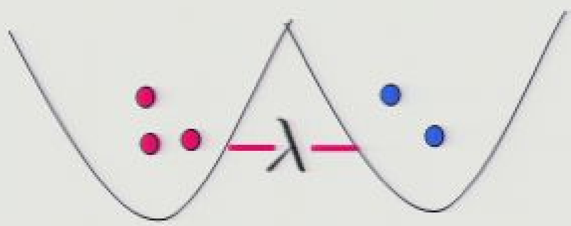
Restricted by the growing degrees of freedom



Quantum Computation in optical lattices







Our model in the case of a few number of particles:

1 and 2 qubit gates

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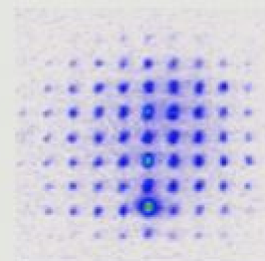

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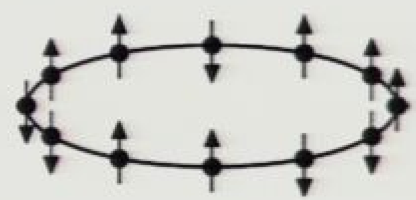

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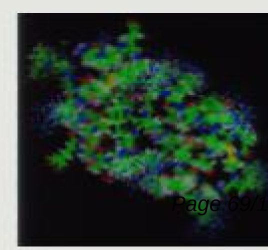

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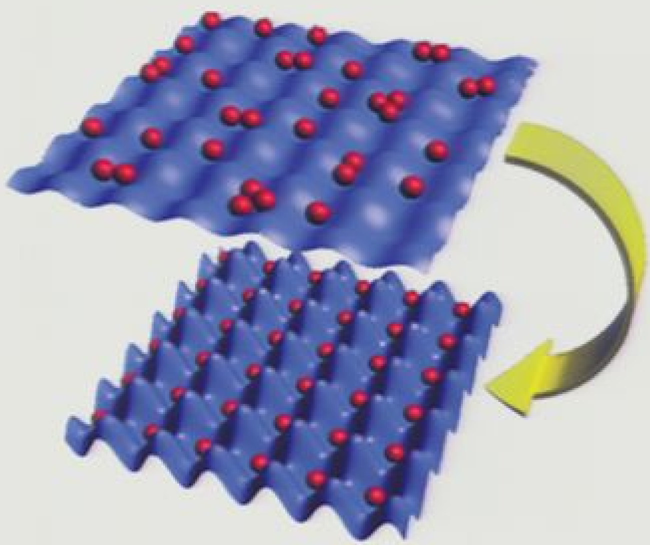

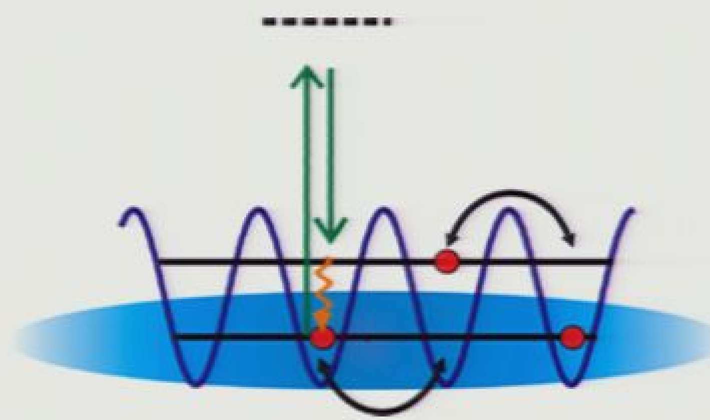

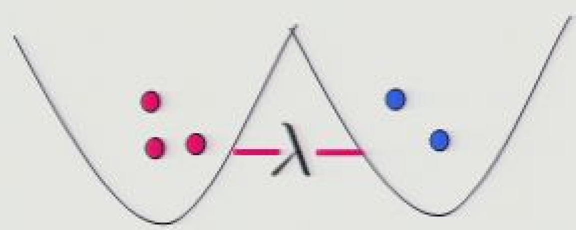

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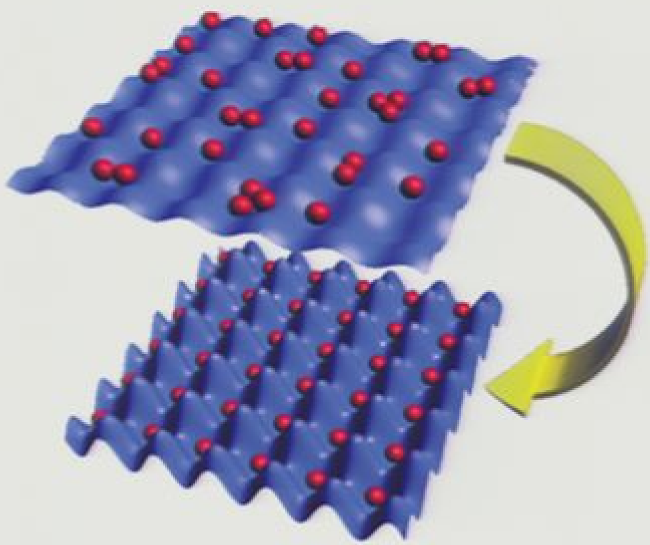

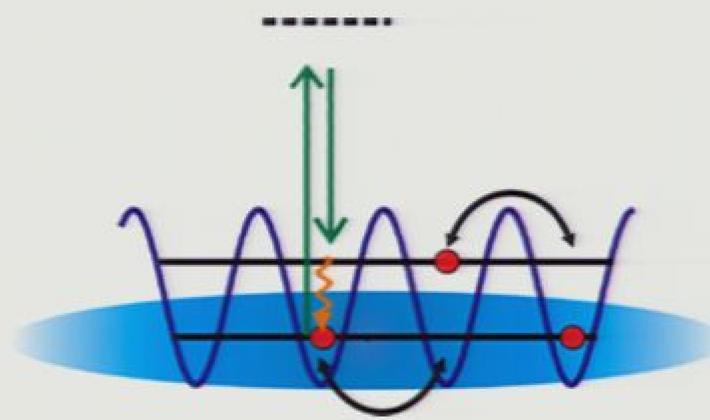

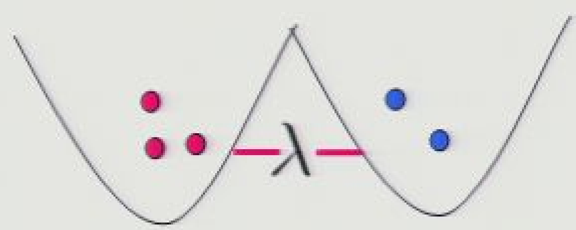

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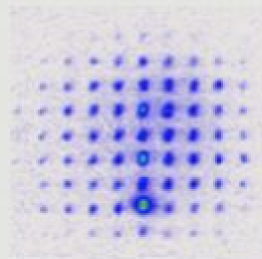

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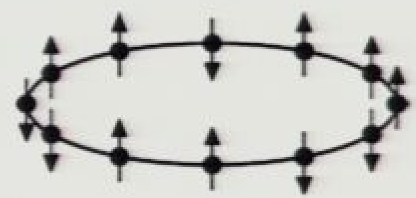

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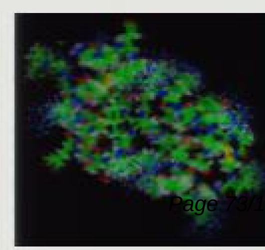

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$m = a^\dagger a - b^\dagger b$ relative population $m = -N, ..., N$

$U = e^{\xi a^\dagger b - \xi^* a b^\dagger}$ Two-mode displacement operator $\xi = \frac{\theta}{2} e^{i\phi}$

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There are three free parameters

Inelastic collisions depend on:

Elastic scattering rate

Energy difference between modes

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The ground state $U|N, m_0\rangle$ is now trivially found

by minimizing $\mathcal{E}_m$ with respect to m. $\mathcal{E}_m = A_1 m + A_2 m^2$

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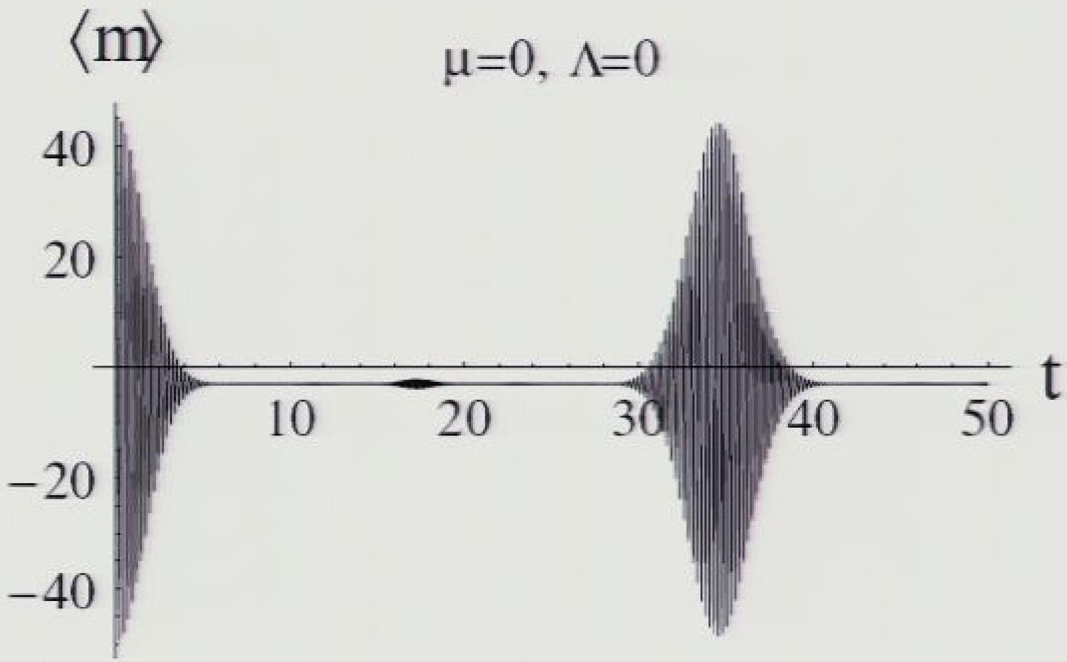
$U|N, -N\rangle$ Found before to describe well the behavior of the condensate!

Coherent state

Time evolution of relative population

Starting with the state $U|N,-N\rangle$



Canonical model: no inelastic collisions

Collapse and revivals of Rabi Oscillations

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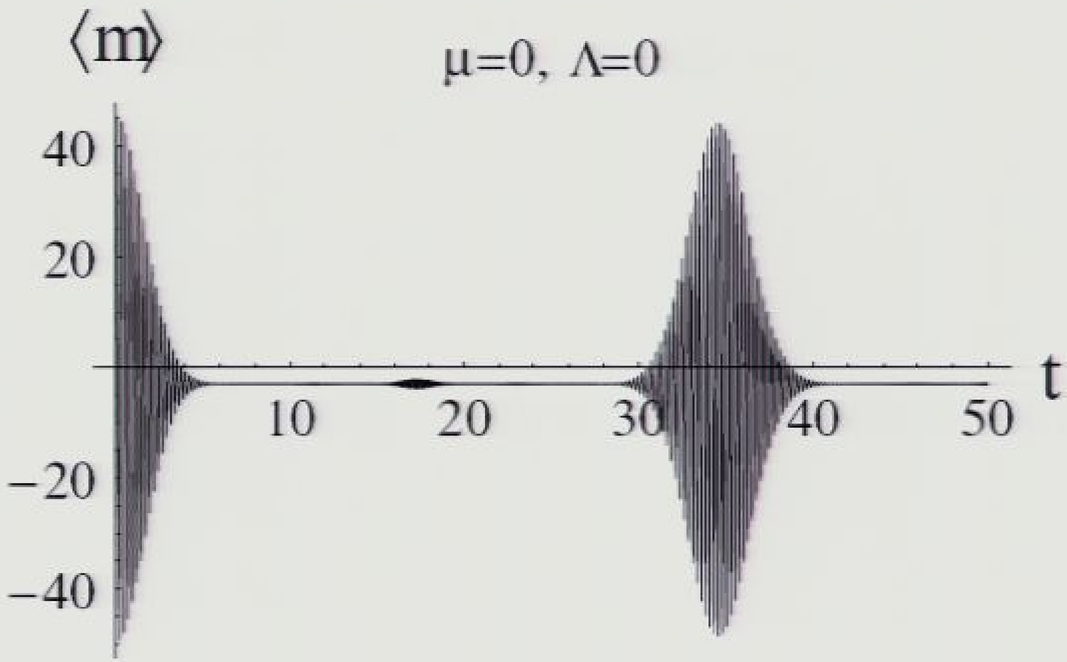

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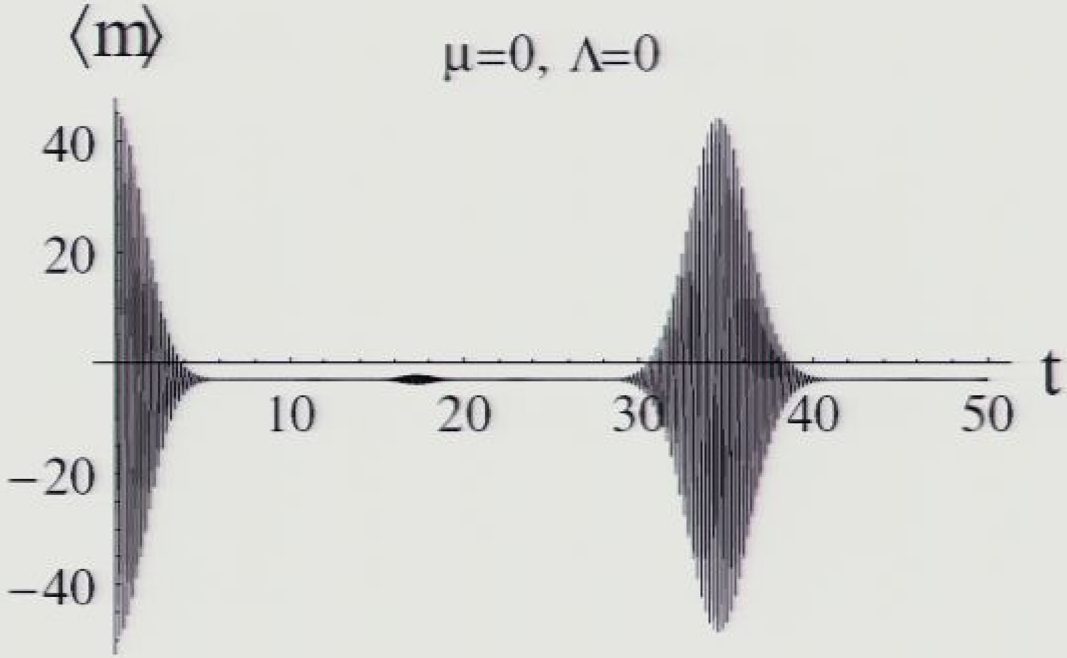

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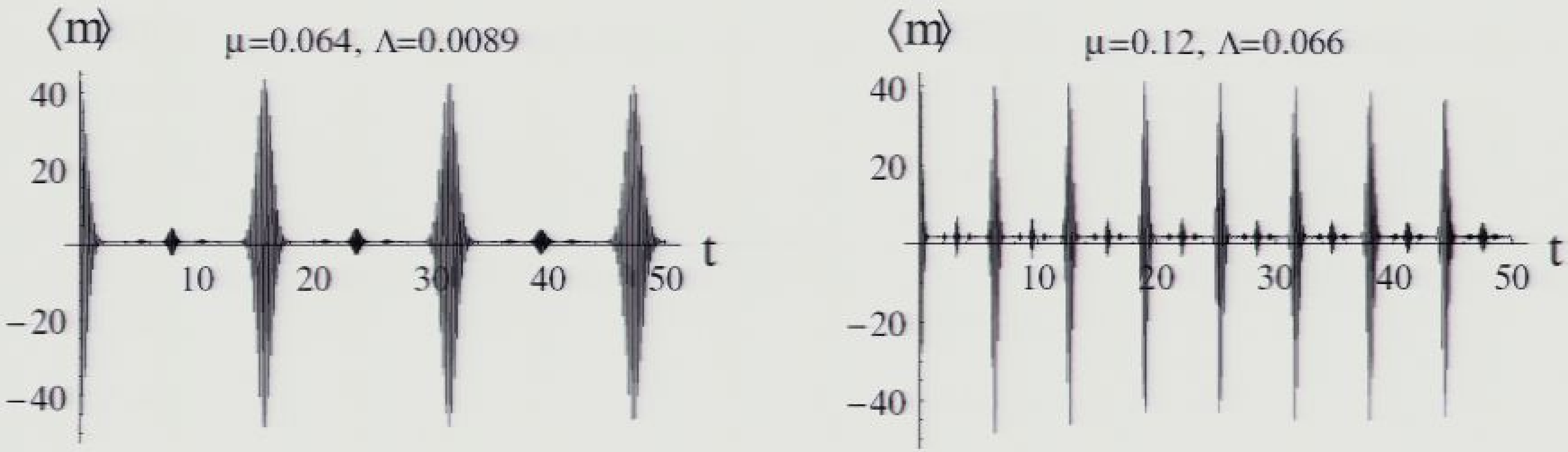
Our model: with inelastic collisions:

Numerical solution



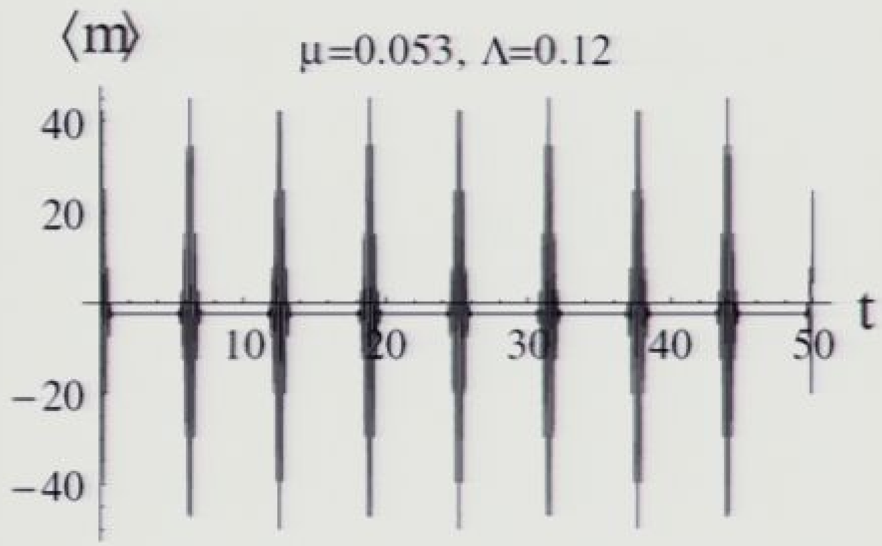
Inelastic collisions have an important effect in the evolution

Time evolution of relative population

Analytical solution

$$\langle a^\dagger a - b^\dagger b\rangle = \cos\theta \sum_{-N}^{N} m\,|C_m|^2 - \sin\theta \sum_{-N+1}^{N} C_m C_{m-1} L_m$$

$$L_m = \cos(\phi + (E_{m-1} - E_m)\,t)(N(N+1) - m(m-1))$$



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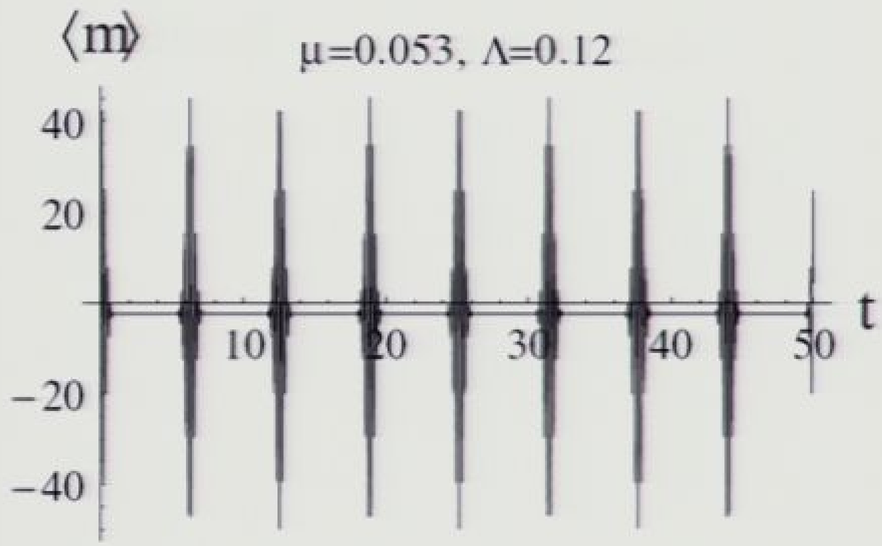

We can predict the collapse time and periodicity of the pattern

$$t_r = (2n+1)\pi/(2A_2)$$

$$(-A_1 - A_2(2m-1))t_1 = 2\pi n_m$$

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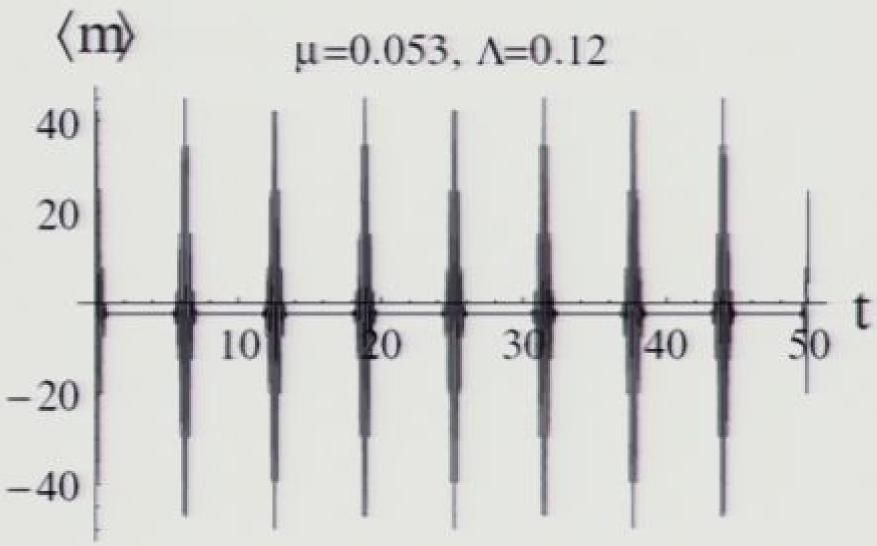

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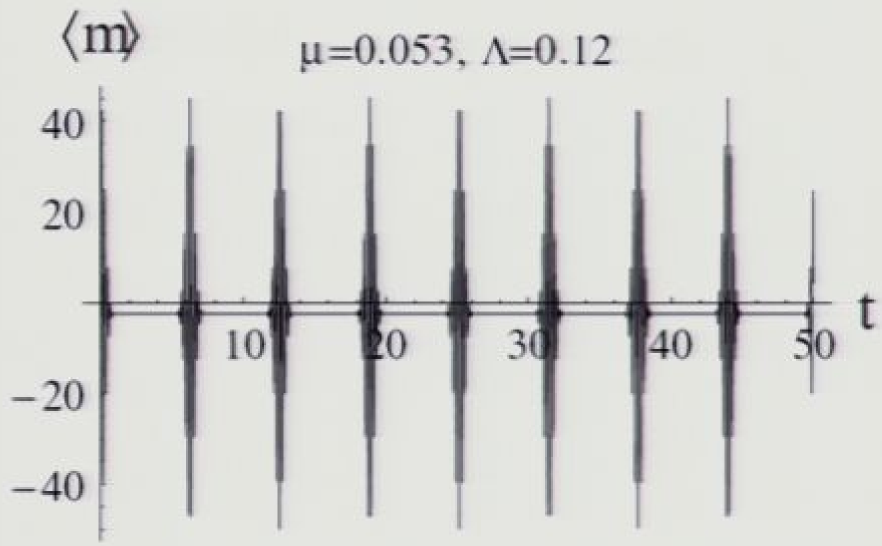

We can predict the collapse time

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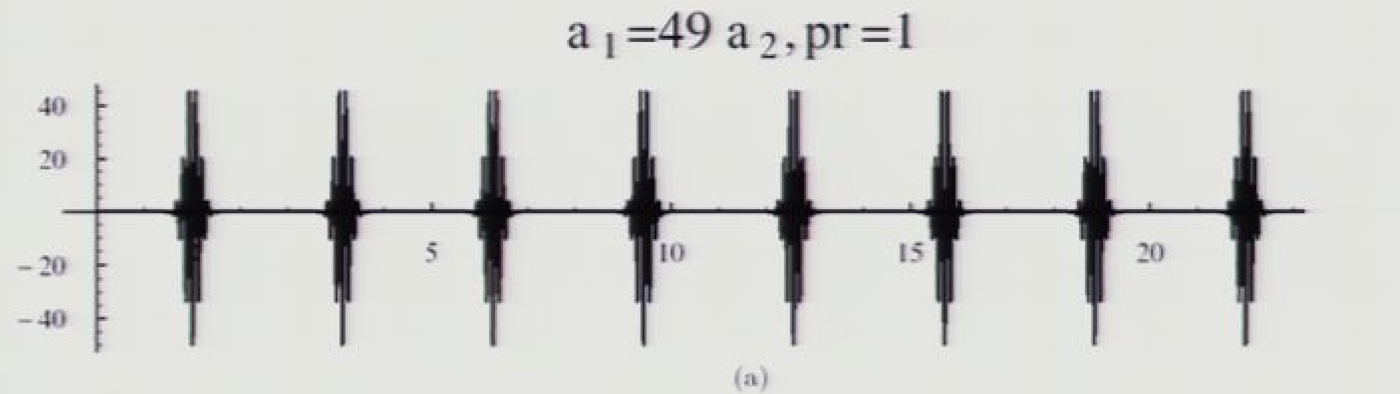
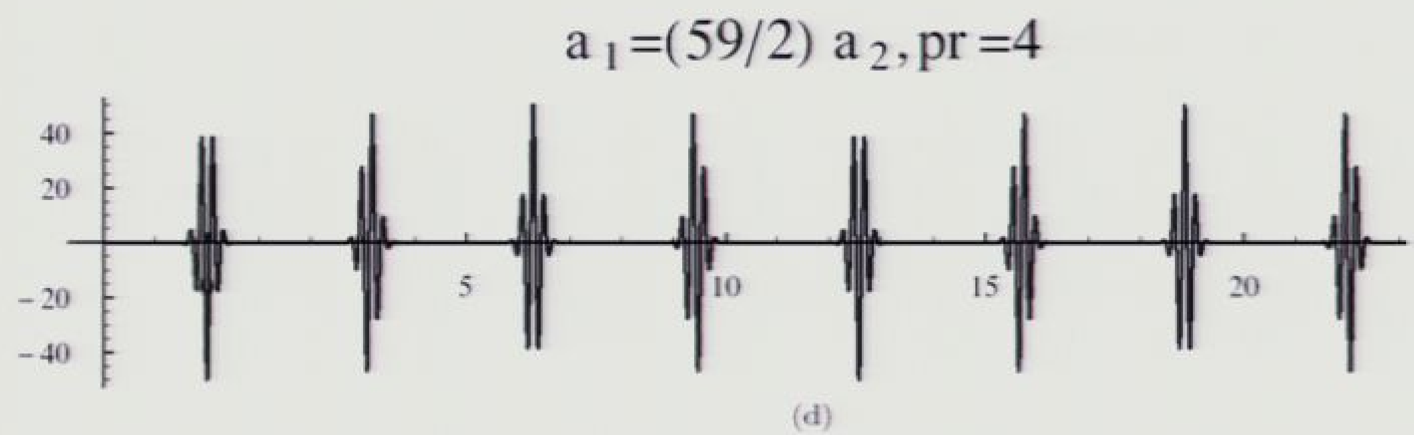
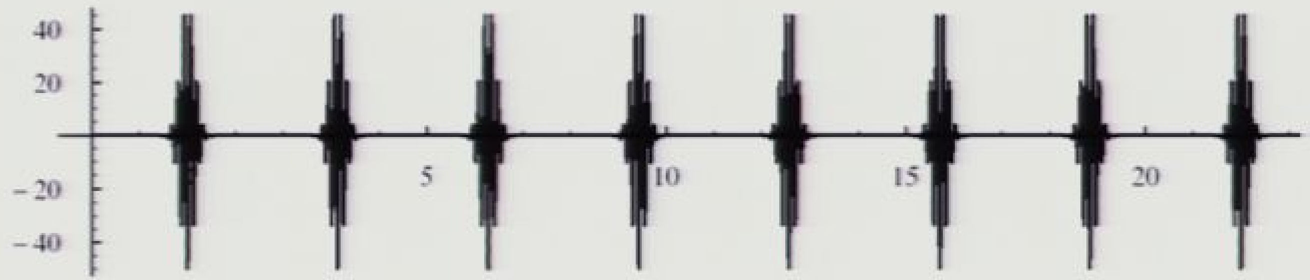




Time evolution of relative population

$a_1 = 49\, a_2, pr = 1$

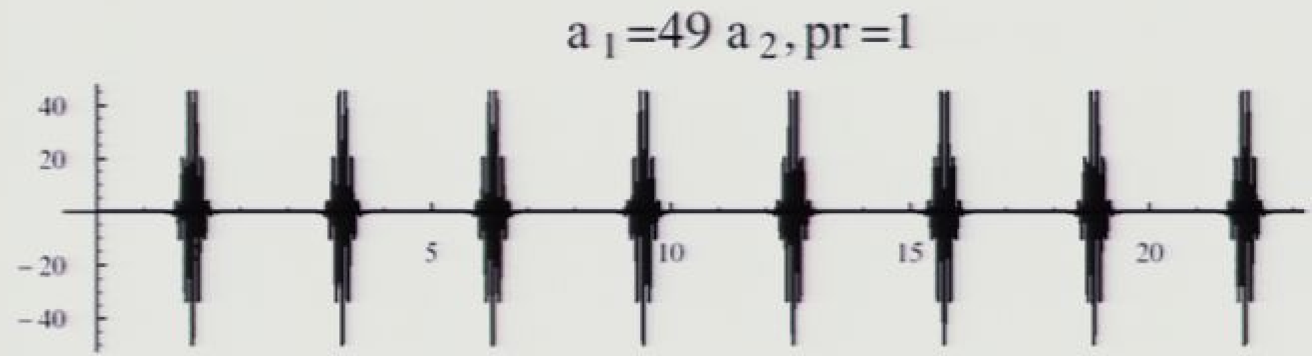


(a)

$a_1 = (59/2)\, a_2, pr = 4$

(d)

Time evolution of relative population



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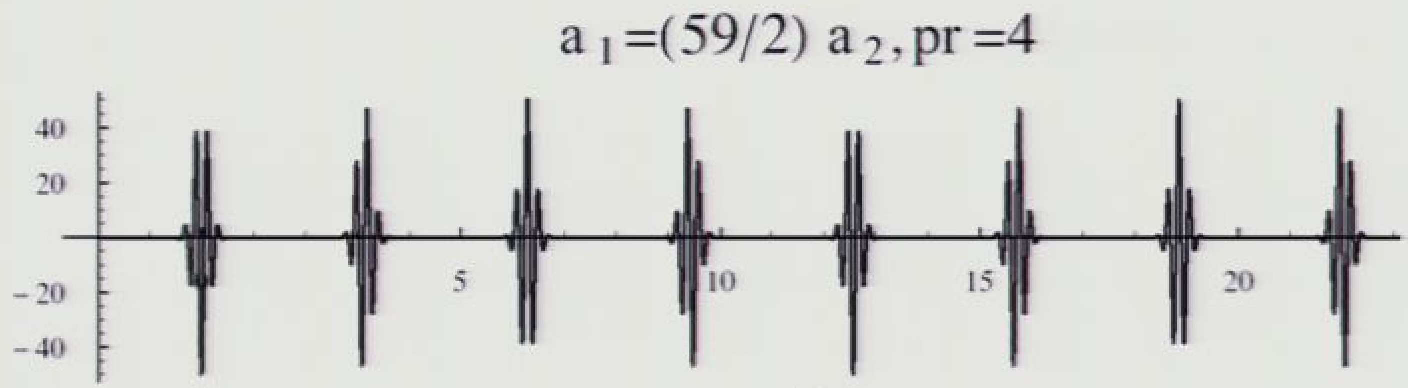

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J. I. Cirac, M. Lewenstein, K. Mølmer and P. Zoller, Phys. Rev. A 57, 1208 (1998).

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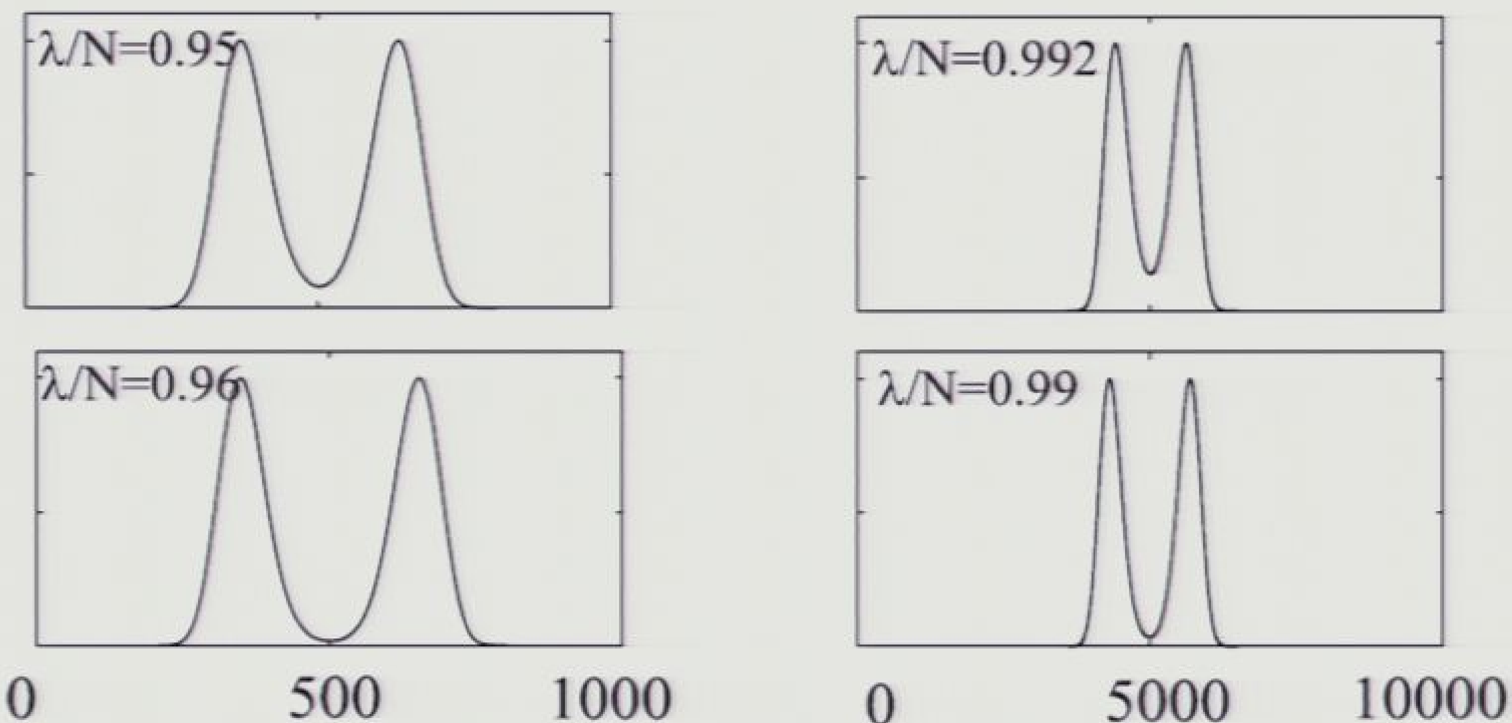
Cat state: probability distribution for the number state is binomial



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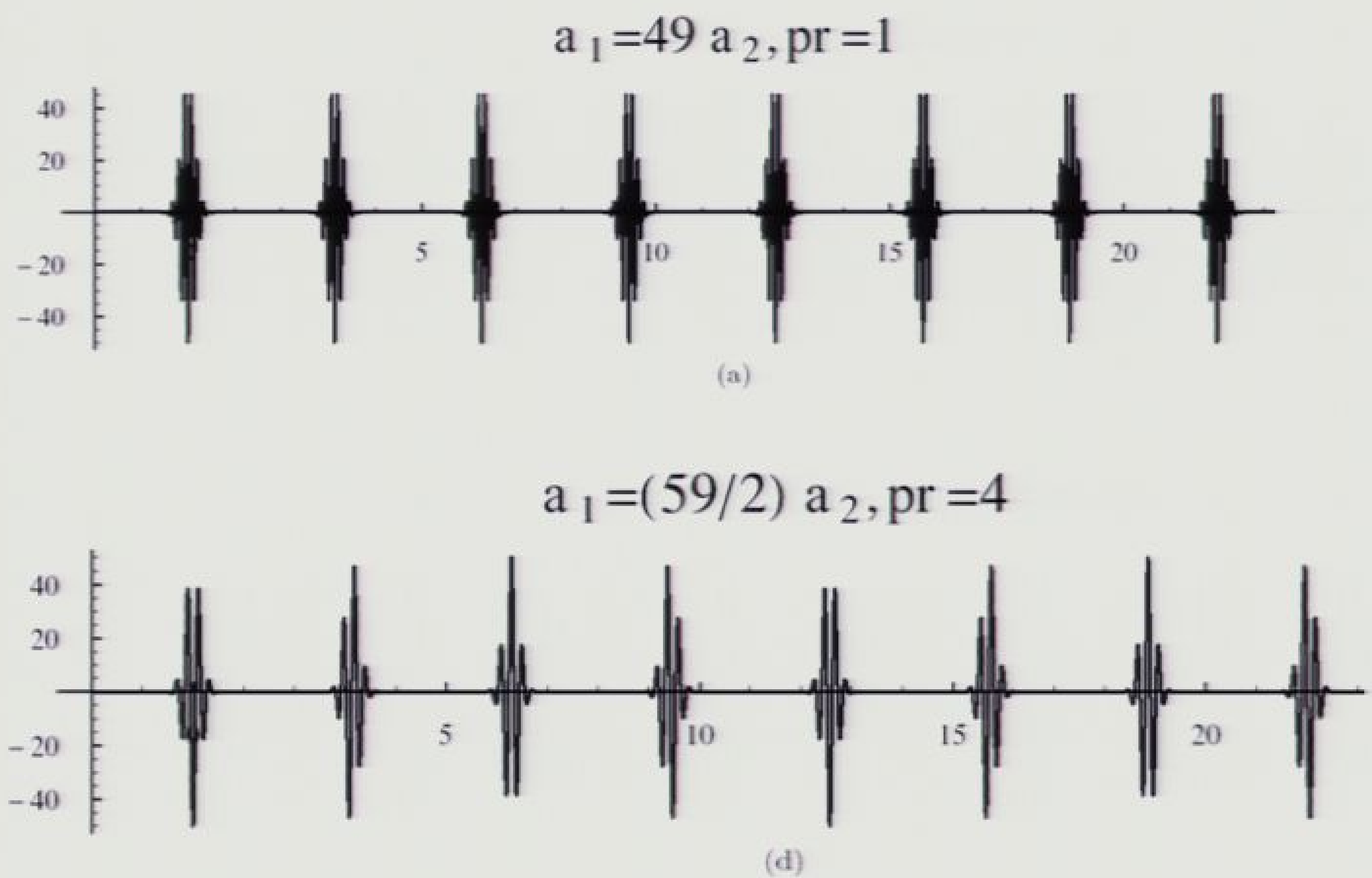

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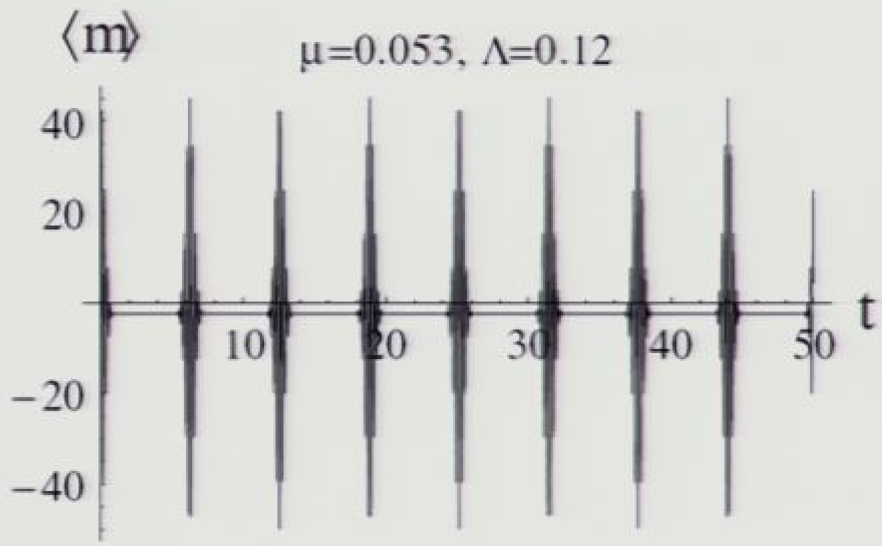

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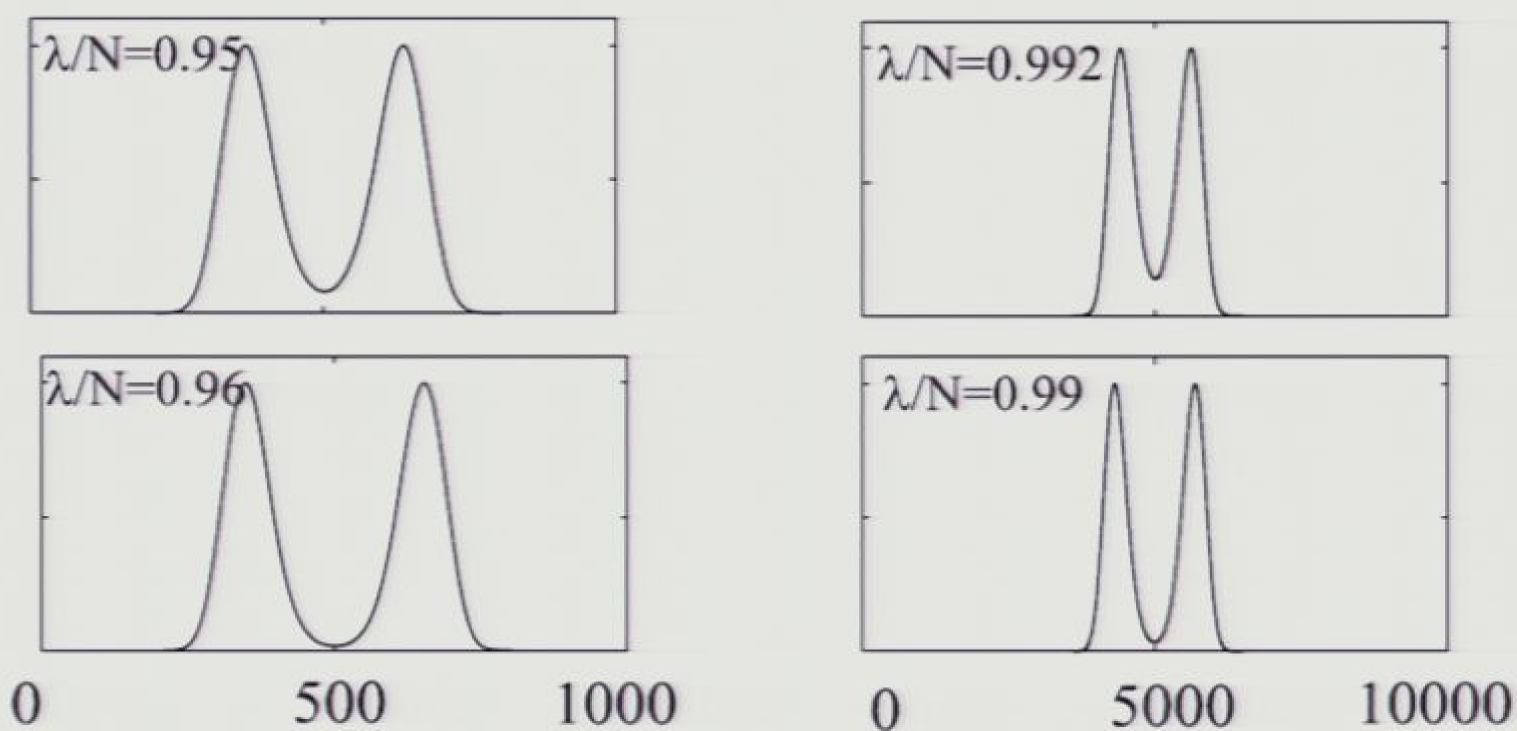

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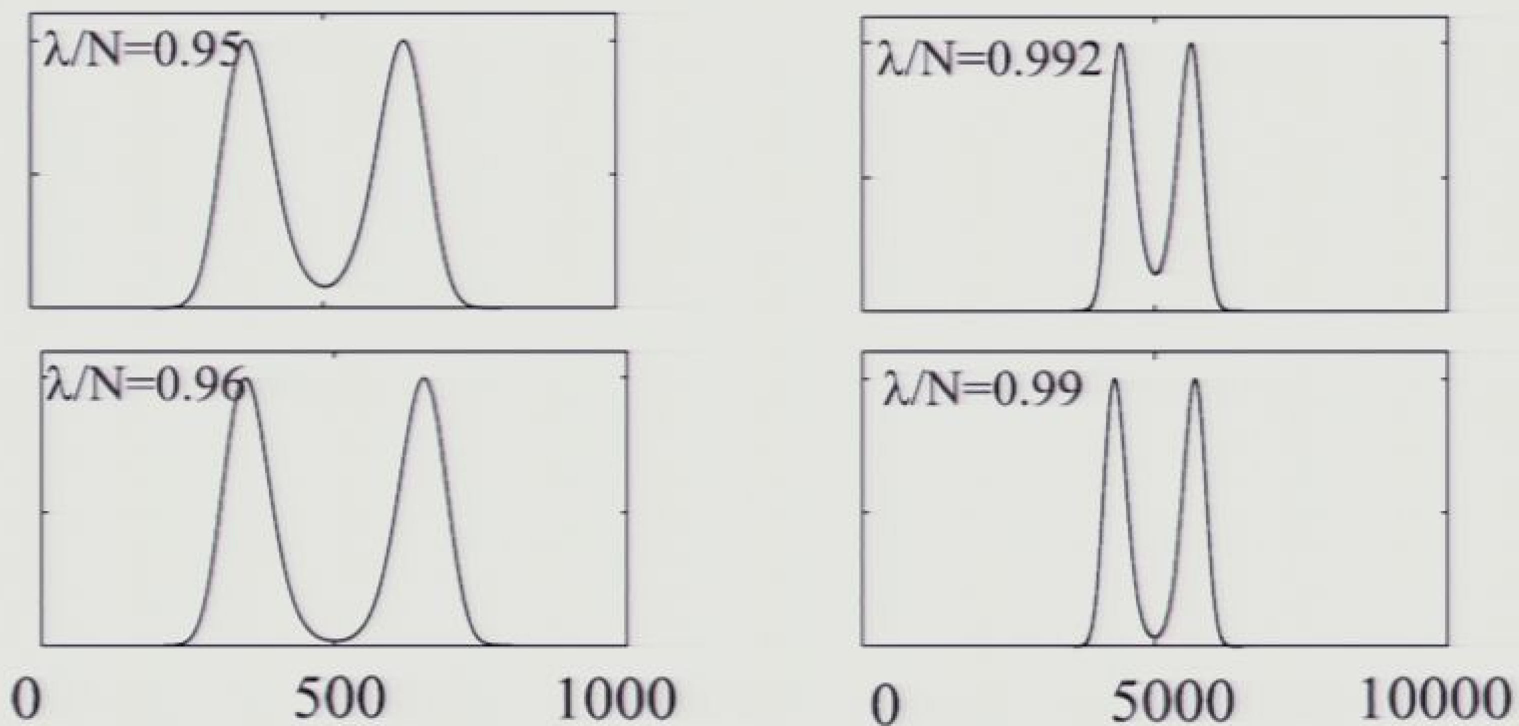

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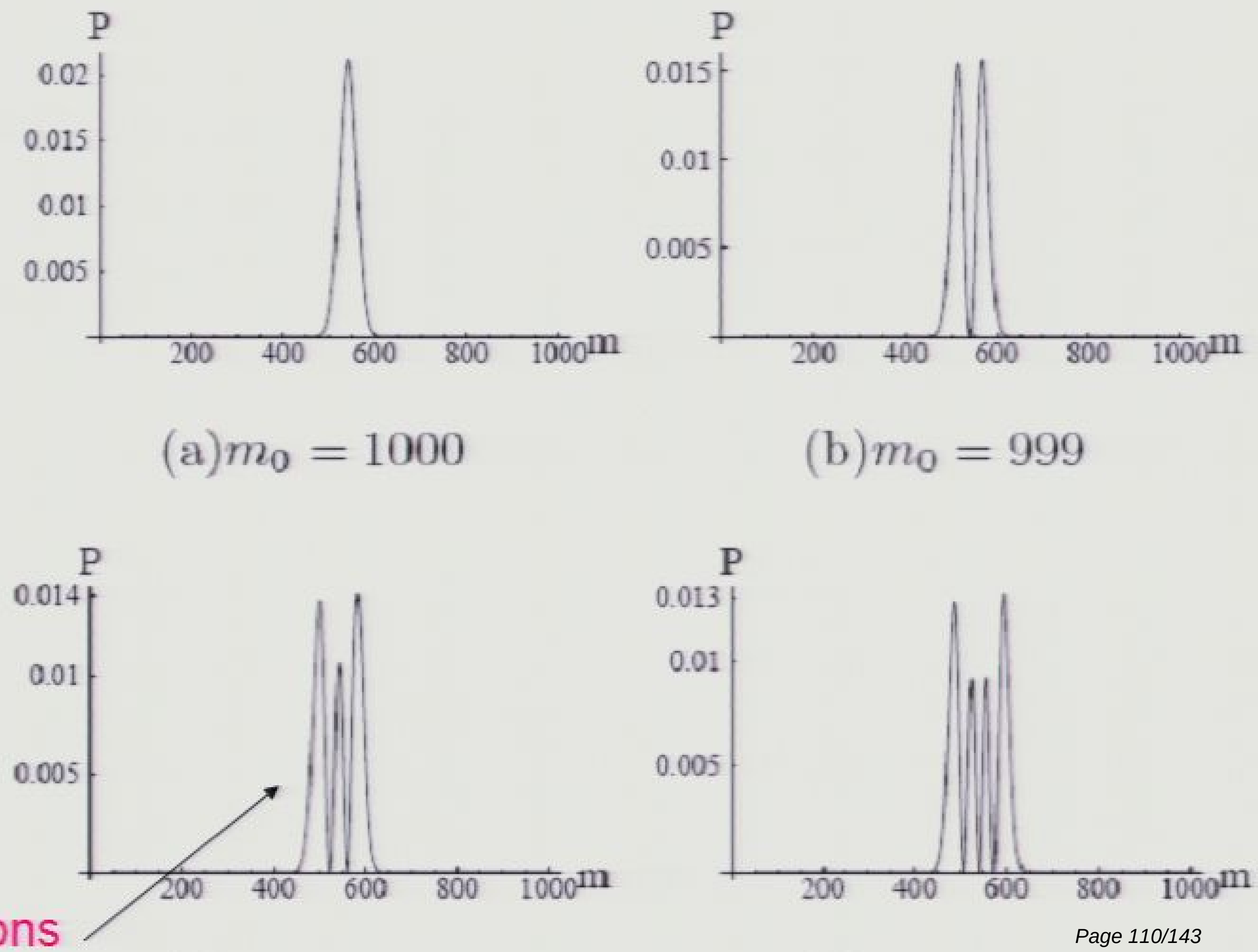
Probability distribution

for different ground

States in our model

Abrupt transition!



(a) $m_0 = 1000$ (b) $m_0 = 999$

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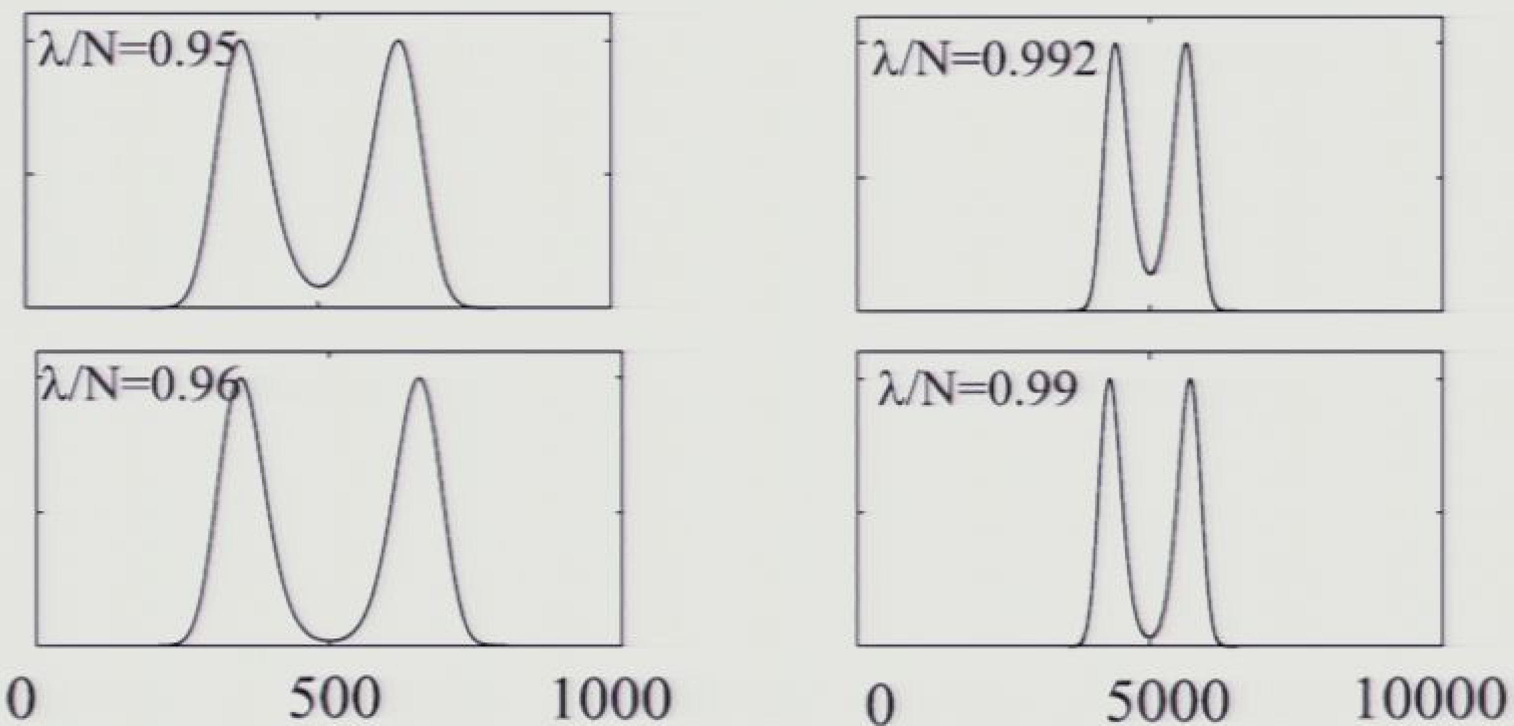

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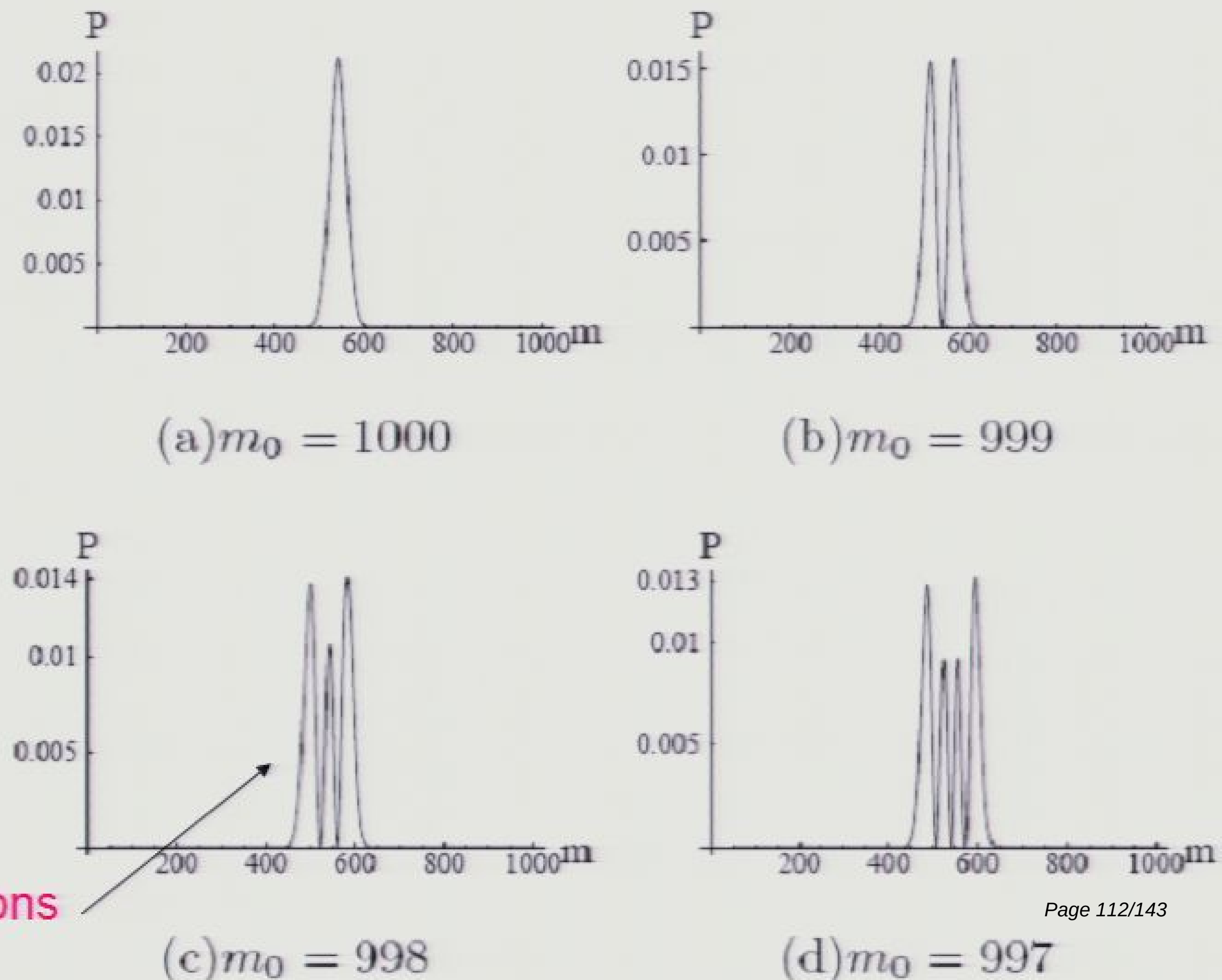

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Effect of inelastic collisions

Entanglement

$$S(\rho_a) = -tr(\rho_a \log_2(\rho_a)) \;\; = -\sum m |d^N_{m_0,m}(\theta)|^2 \log_2 |d^N_{m_0,m}(\theta)|^2$$

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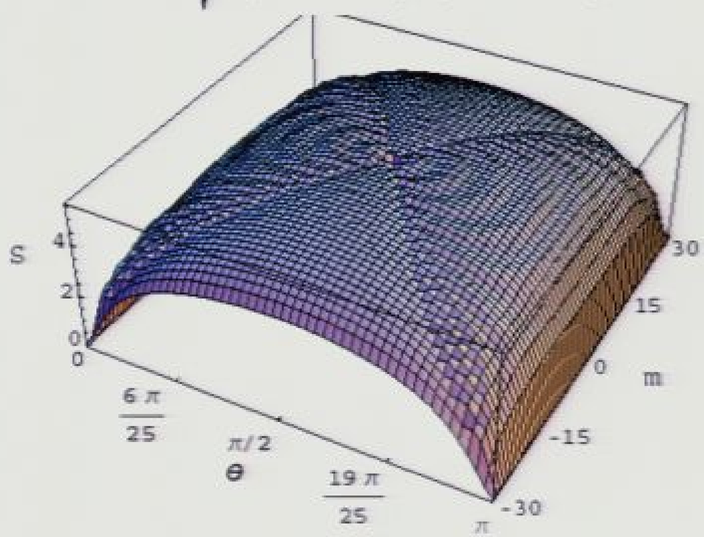

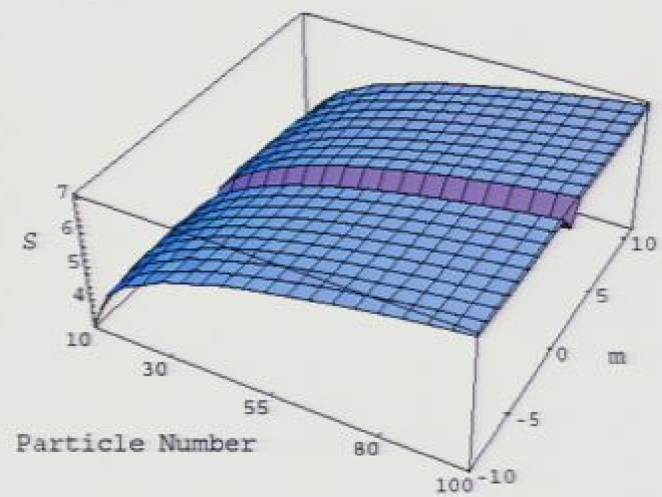

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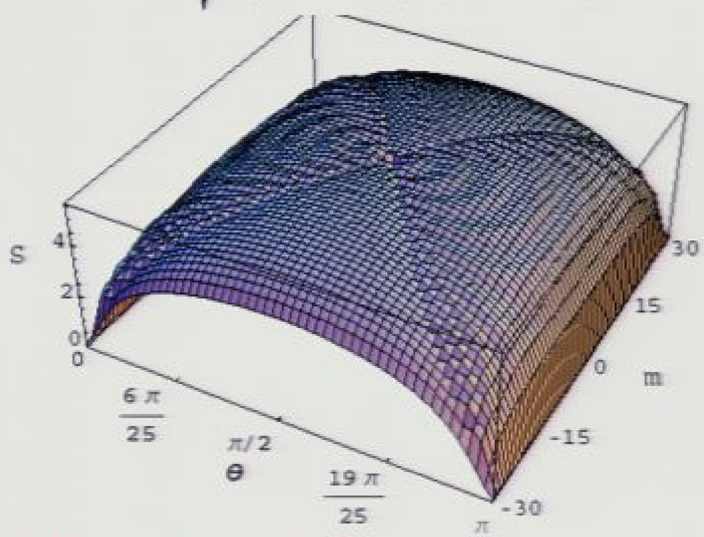
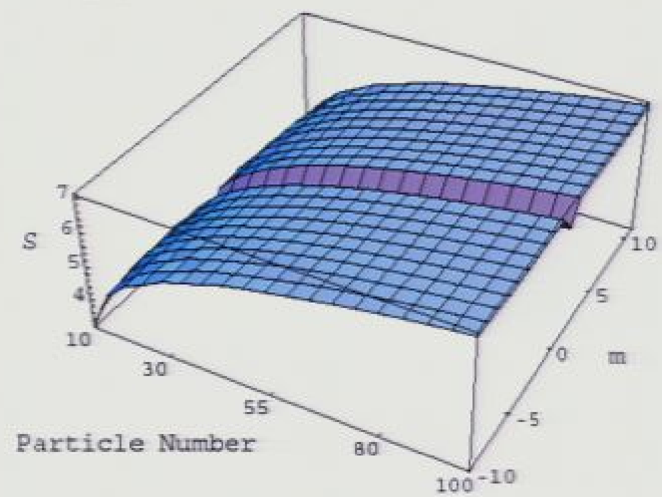




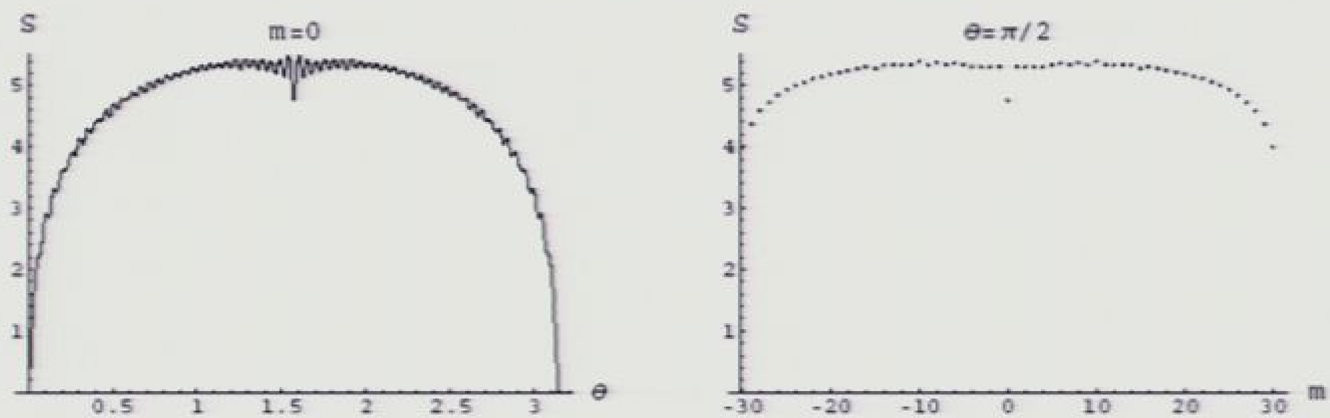
Maximally entangled states:

Large N, collision rate comparable to the natural frequency.

Large laser coupling with small detuning, or

Low barrier and symmetric well

Roll of collisions on the generation of entanglement



Generalizations

Many-body collisions

Higher order Hamiltonians

$$H = U^\dagger H_0 U$$

Many-body collisions

Higher order Hamiltonians

3-body elastic and inelastic collisions

$$H = U^\dagger H_0 U$$

$$H_0 \;=\; A_1(a^\dagger a - b^\dagger b) + A_2(a^\dagger a - b^\dagger b)^2 + A_3(a^\dagger a - b^\dagger b)^3$$

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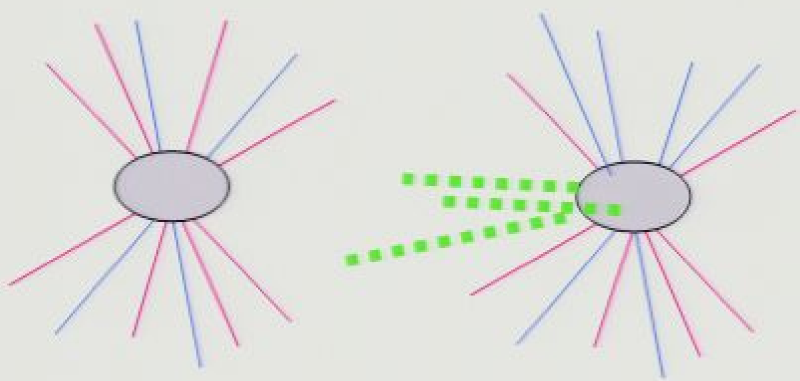
n-body elastic and inelastic collisions



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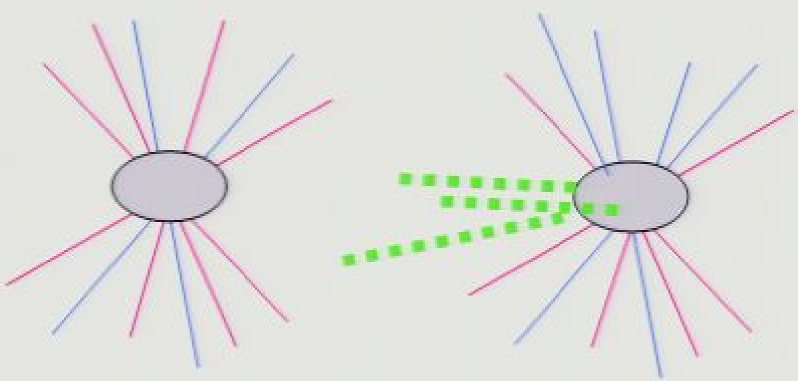

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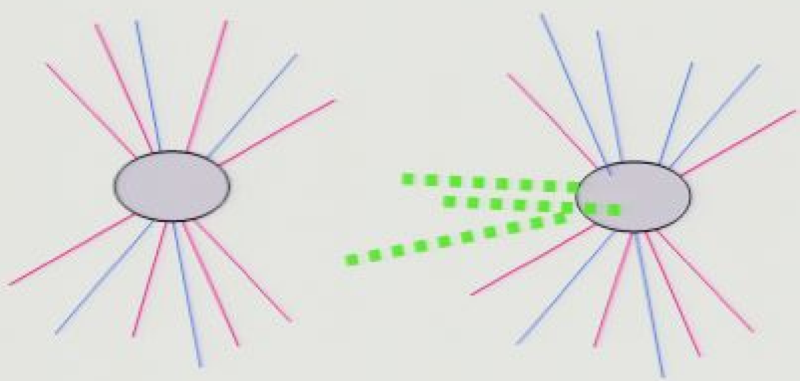

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$$U^\dagger |j, m\rangle$$

Exact analytical solution

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Multimode

Two-mode BEC SU(2)

$$J_z = a^\dagger a - b^\dagger b$$

$$J_+ = a^\dagger b \quad J_- = ab^\dagger$$

Schwinger representation

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Diagonal generators

$$U = e^{i\theta a^\dagger b} e^{i\phi b^\dagger c} e^{i\alpha a^\dagger c} \ldots$$

Most general rotation in SU(3)

Spin-1/2 systems

The model also can be used to describe the n-body interaction of N spin-1/2 particles interacting with a laser

(a generalization of the Lipkin-Meshkov-Glick model with analytical solution)

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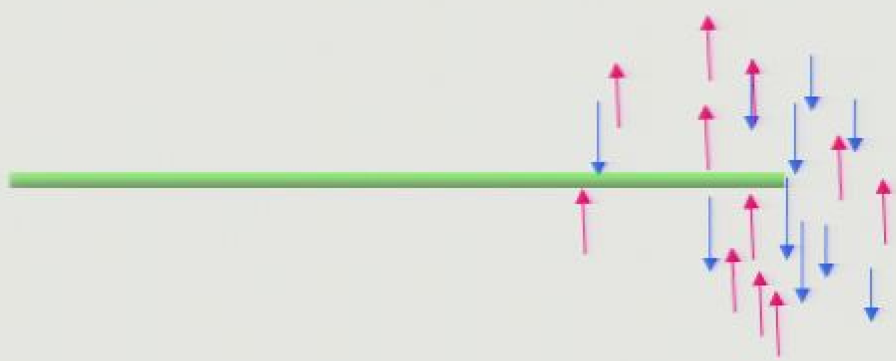

System useful in the implementation of quantum computation

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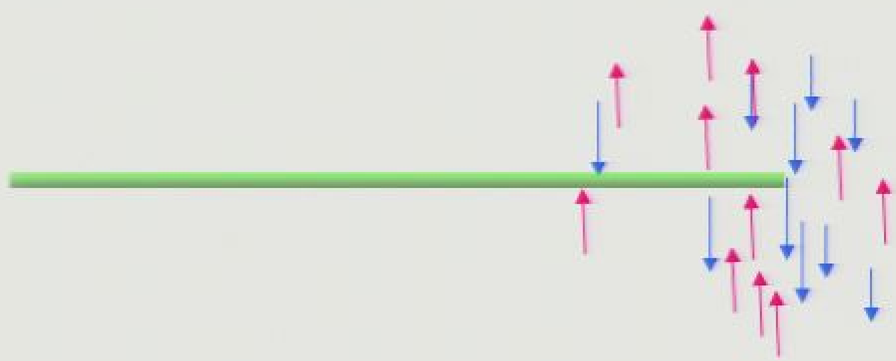

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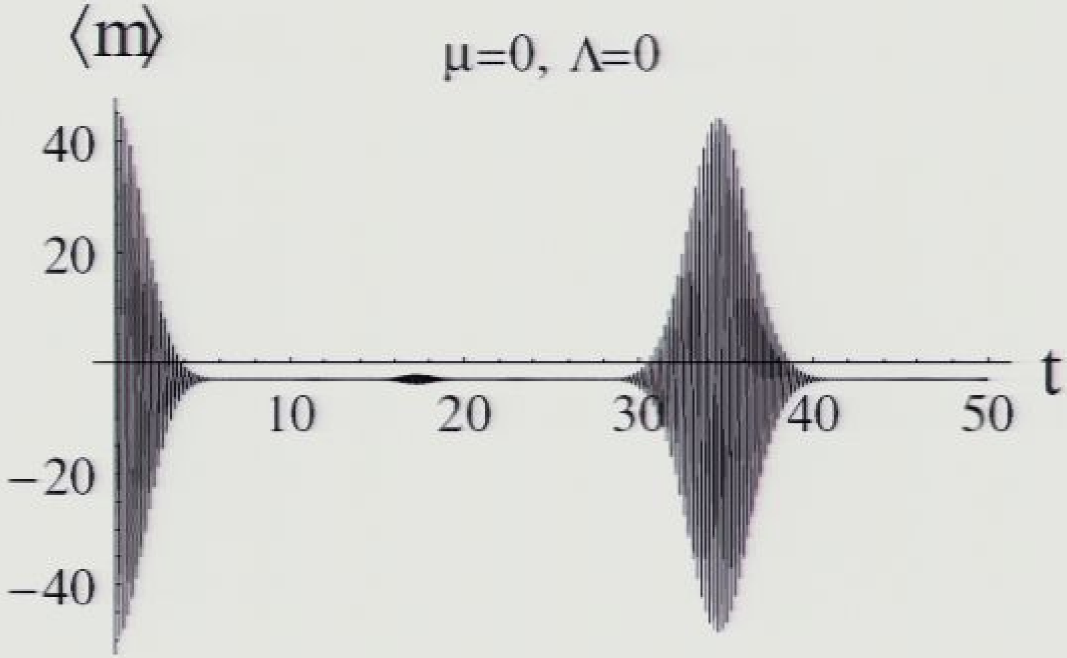
Time evolution of relative population

Starting with the state $U|N, -N\rangle$



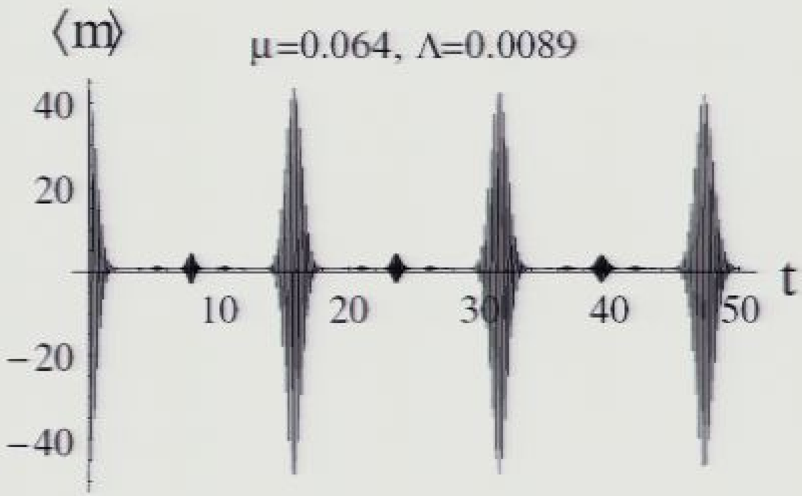
Canonical model: no inelastic collisions

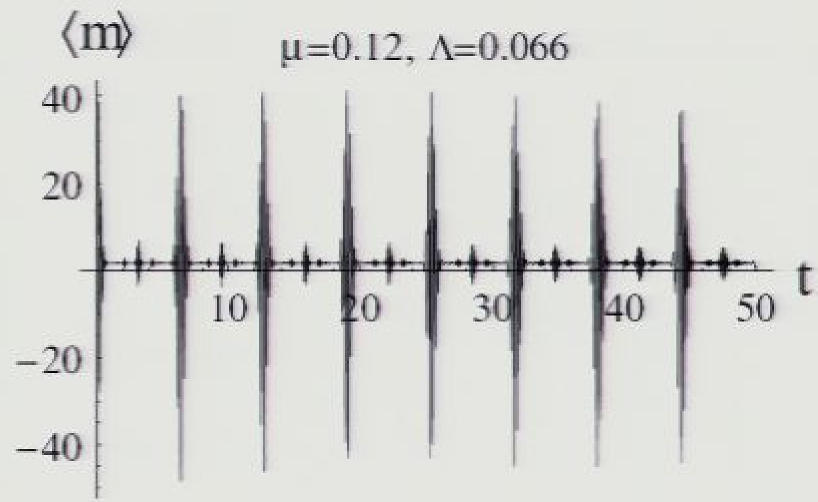
Collapse and revivals of Rabi Oscillations

Time evolution of relative population

Our model: with inelastic collisions:

Numerical solution





Inelastic collisions have an important effect in the evolution

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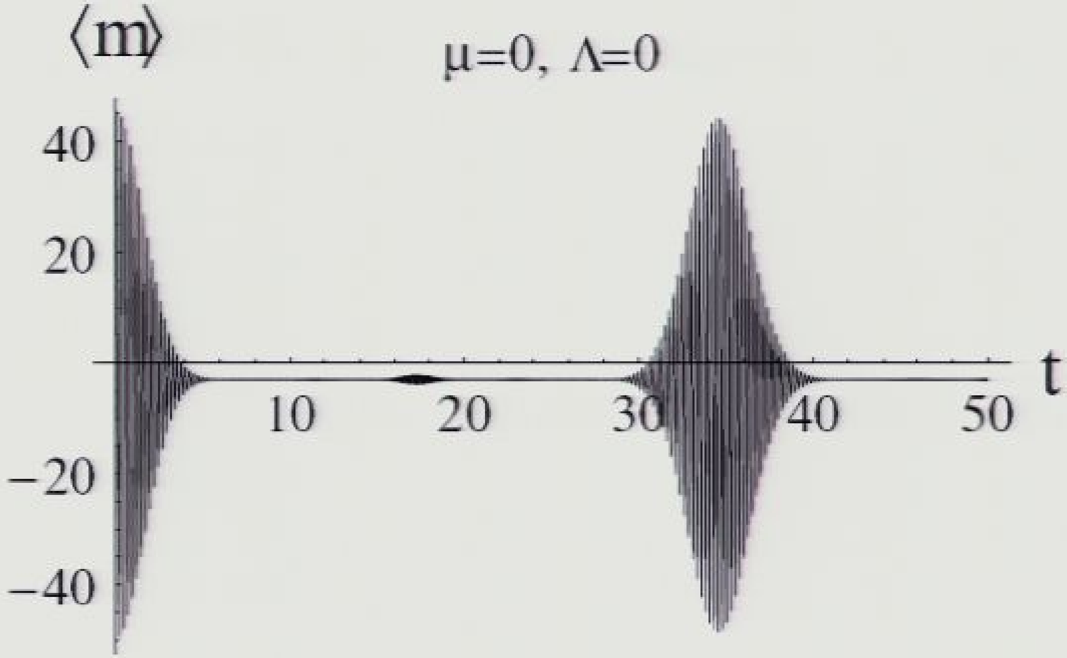

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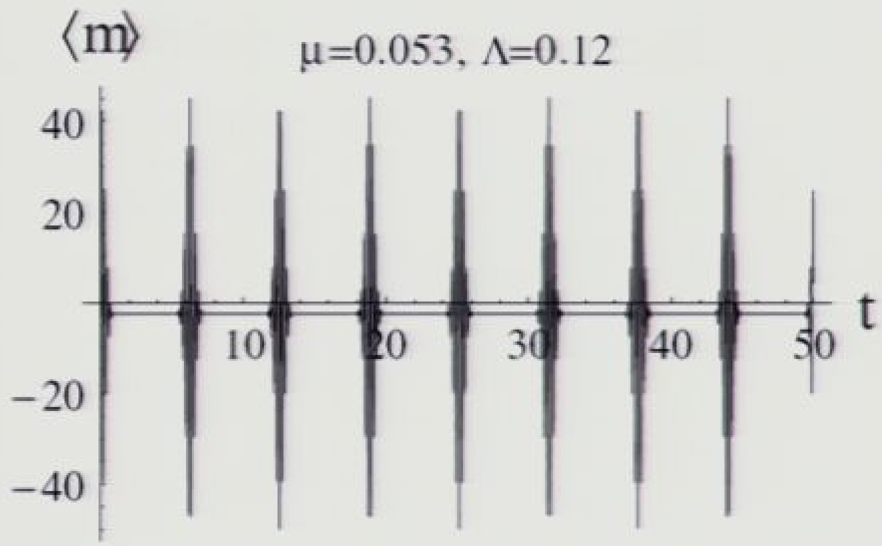

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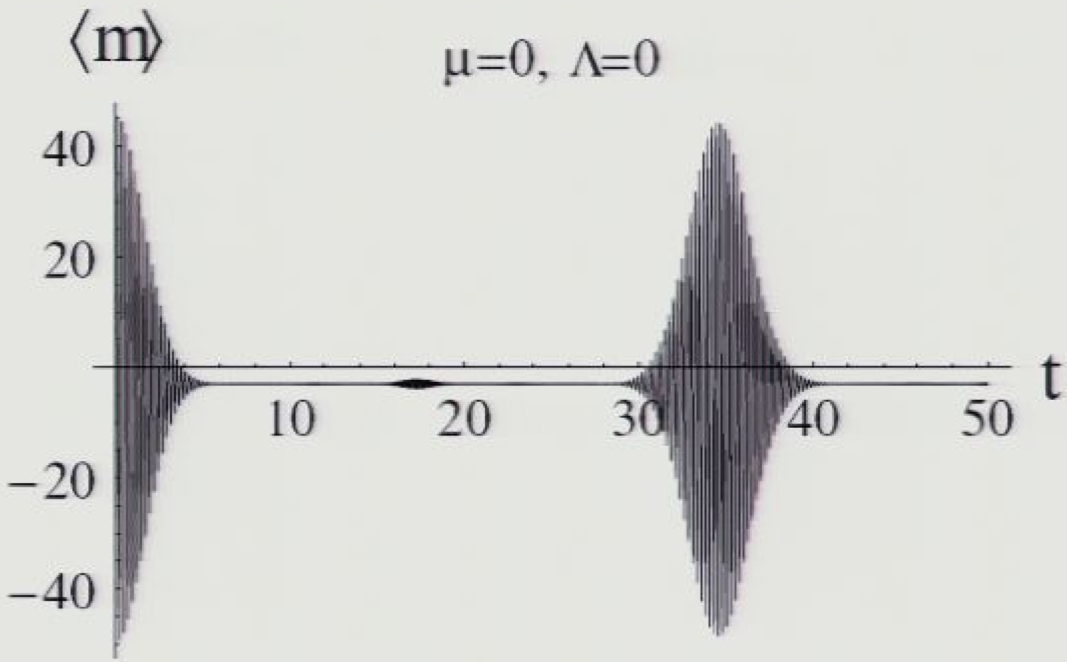

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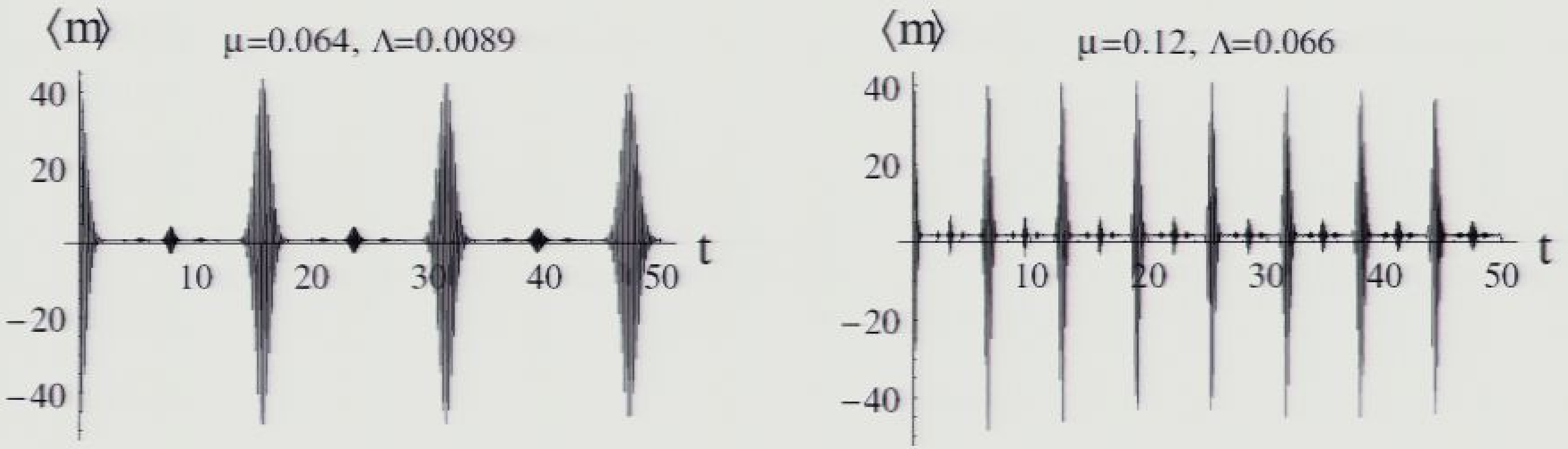

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