Title: Cusp Anomaly, Integrability and ADS/CFT

Date: May 01, 2007 02:00 PM

URL: http://pirsa.org/07050001

Abstract: TBA

CUSP ANOMALY, INTEGRABILITY AND ADS/CFT

Sergio Benvenuti
Princeton University

ANOMALOUS DIMENSIONS AND SPIN CHAINS

HEISENBERG MODEL FOR THE SU(2) SECTOR

BETHE ANSATZ

PSU(2,2|4) SYMMETRY IN THE FULL $N=4$ SYM

BES/BHL "DRESSING" PHASE

INTEGRAL EQUATION

NUMERICAL STUDY

AdS/CFT, ANOMALOUS DIMENSIONS AND STRINGS

BASIC EXAMPLE:

$\mathcal{N}=4$ SYM $\Longleftrightarrow$ $AdS_5 \times S^5$ TYPE IIB

SCALING DIMENSION OF GAUGE INVARIANT OPERATORS $\Longleftrightarrow$ ENERGIES OF STRING STATES

$\Delta(\lambda)$ $E(\lambda)$

PERTURBATIVE IF $\lambda \ll 1$

PERTURBATIVE IF $\lambda \gg 1$

$tr(\cdot\cdot x\, y\, D^2 y\, x\, z \cdot\cdot)$

(WE WILL ALWAYS BE IN THE LARGE N LIMIT)

$\lambda = g_{YM}^2 N$

WE WANT TO COMPUTE THE ANOMALOUS DIMENSION OF OPERATORS LIKE

$$tr(x x y x y x y y)$$

FEYNMAN DIAGRAMS

WE CONSIDER THE QUANTUM MECHANICS OF J SPINS

$$\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\cdots$$

SU(2) SPIN CHAIN OF LENGTH J

HAMILTONIAN:

$$\mathcal{H} = \lambda \sum_{i=1}^{J} \left(1 - P_{i,i+1}\right) + \qquad \leftarrow \text{1-LOOP: HEISENBERG MODEL}$$

$$+ \lambda^2 \sum \left(-4\,1 + 6\,P_{i,i+1} - P_{i,i+1}\,P \;-\; P \cdot P\right)$$

$$+ \lambda^3 \;\ldots$$

FROM CFT'S TO SPIN CHAINS

WE WANT TO COMPUTE
THE ANOMALOUS DIMENSION
OF OPERATORS LIKE

$$\mathrm{tr}(xxyxyxyy)$$

FEYNMAN DIAGRAMS

WE CONSIDER THE QUANTUM MECHANICS OF J SPINS

$$\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\cdots$$

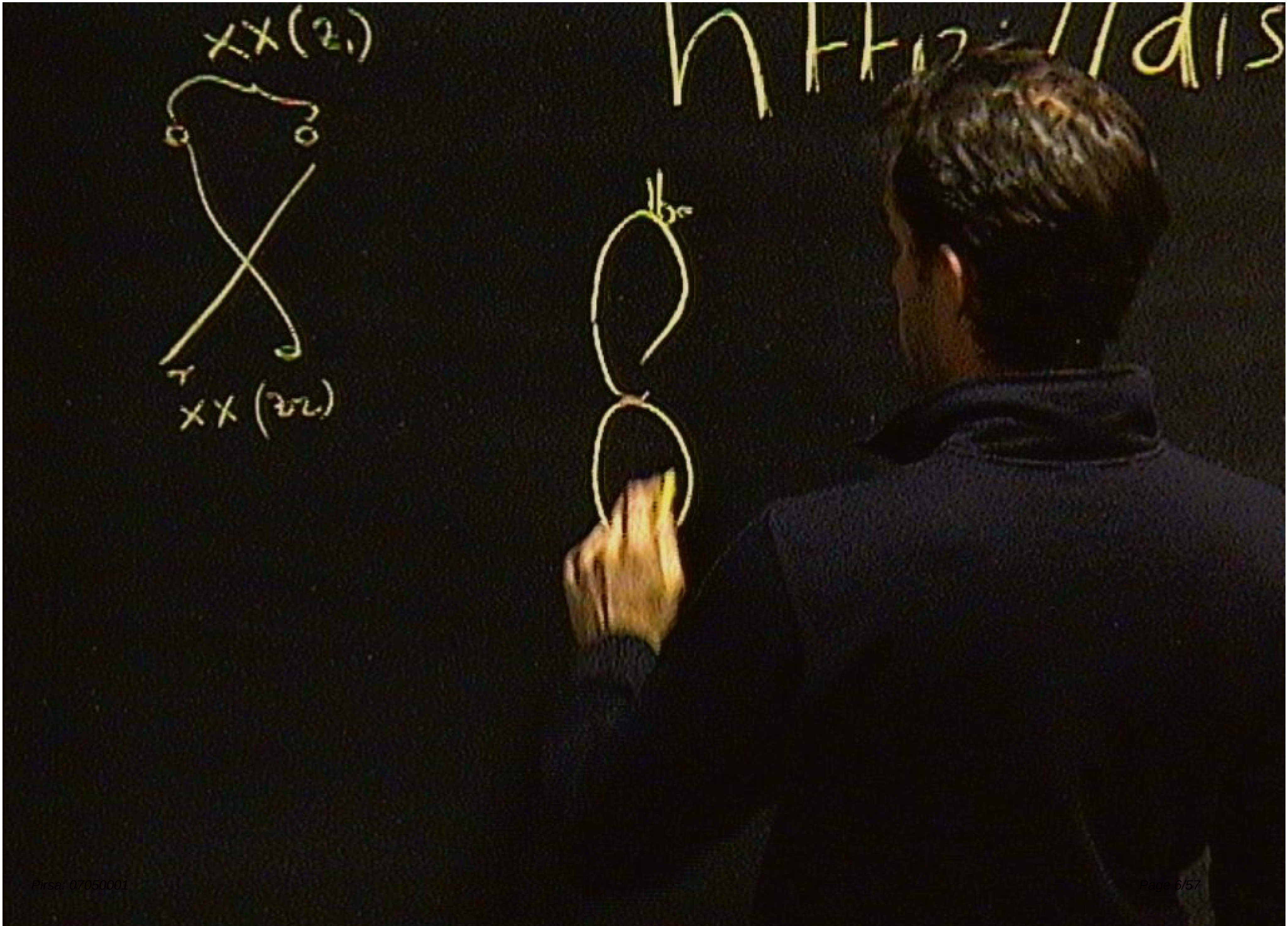
SU(2) SPIN CHAIN OF LENGTH J

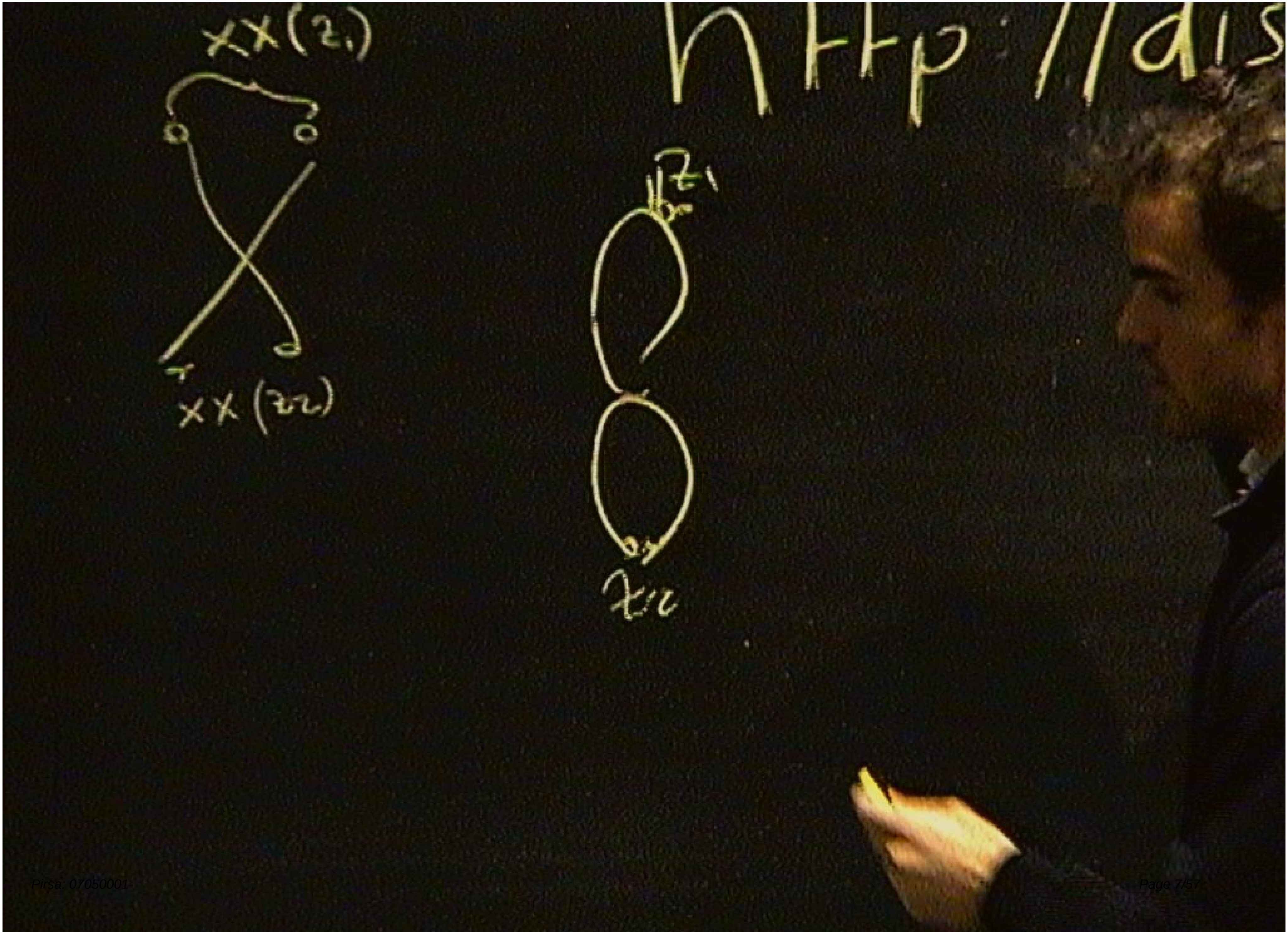
HAMILTONIAN:

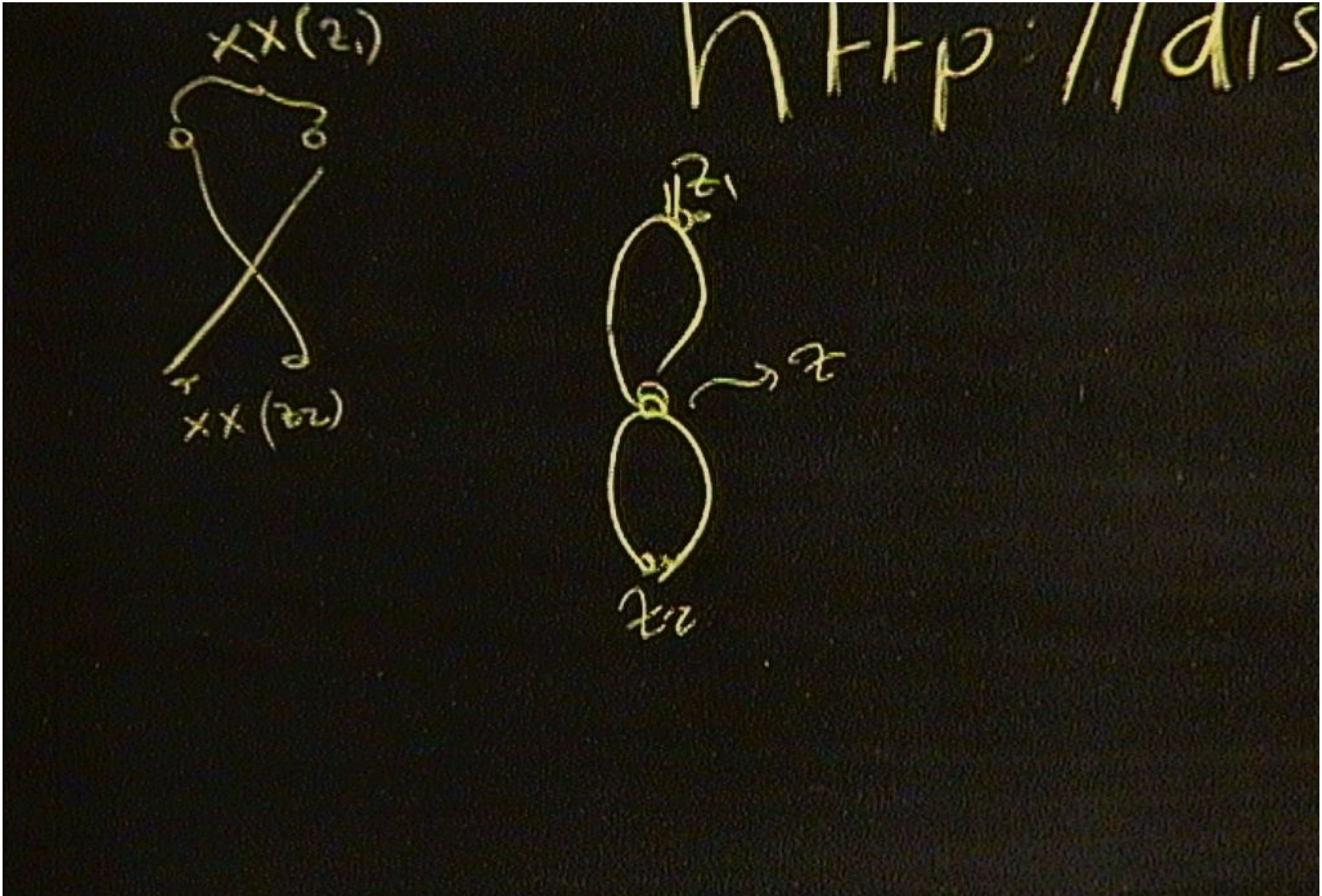
$$\mathcal{H} = \lambda \sum_{i=1}^{J} \left(\mathbb{1} - P_{i,i+1}\right) + \quad \leftarrow \text{1-LOOP: HEISENBERG MODEL}$$

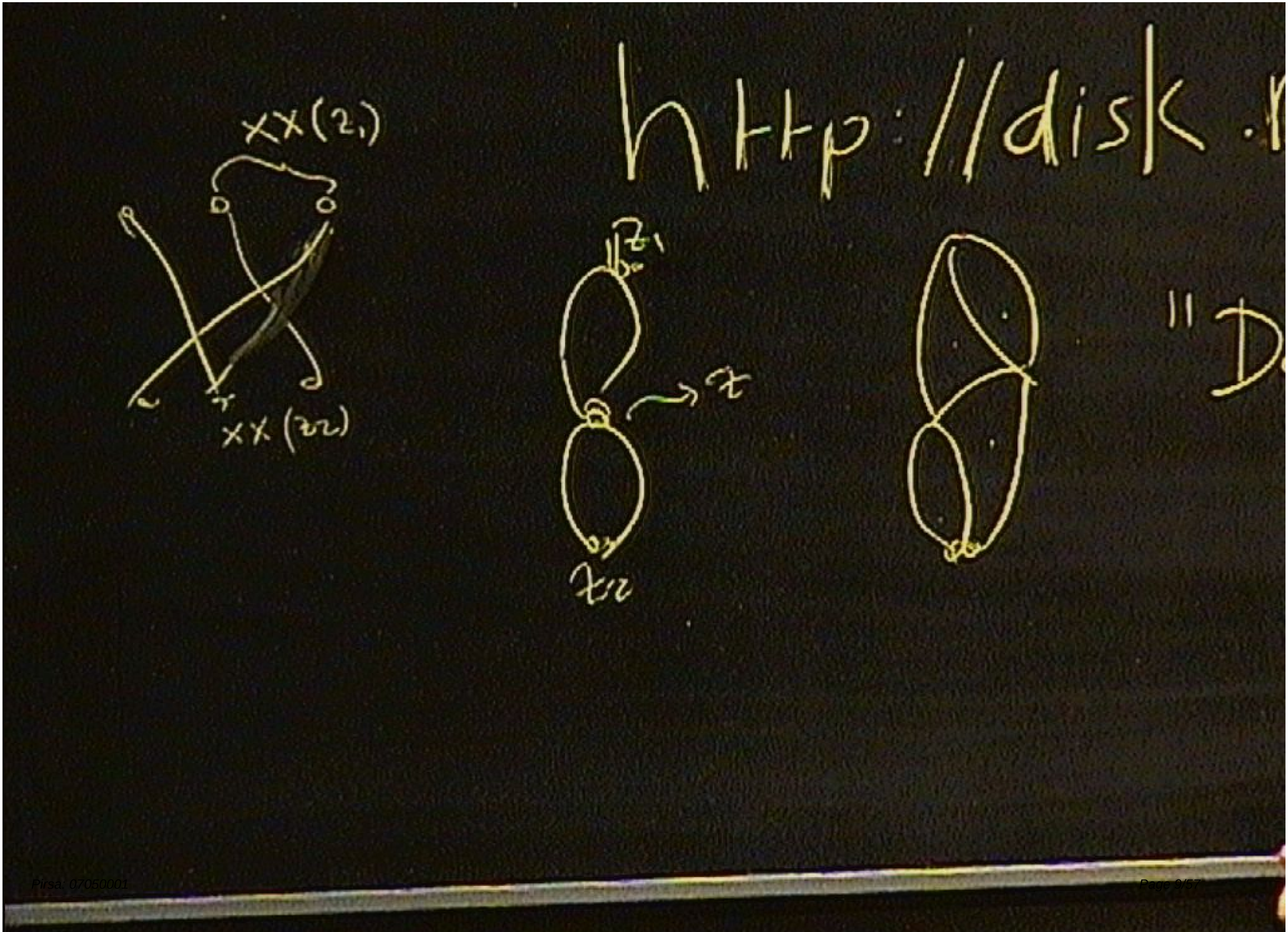
$$+ \lambda^2 \sum \left(-4\mathbb{1} + 6 P_{i,i+1} - P_{i,i+1} P - P \cdot P\right)$$

$$+ \lambda^3 \ldots$$




xx(z1)
xx(z2)
http://dis
z1
z2


http://dis
xx(z1)
xx(z2)


xx(z1)
xx(z2)
http://disk.
z1
x
x2

SPECTRUM OF SPIN CHAINS : BETHE ANSATZ

GROUND STATE: ↑↑↑↑↑↑↑

1ST EXCITATIONS : ↑↑↑↓↑↑↑ IMPURITY TRAVELS WITH MOMENTUM p
(SINGLE MAGNON)

$$E(p) = 4 \sin^2(p/2) \qquad 0 \le p < 2\pi$$

2-MAGNON STATES:

$$|\psi\rangle = \sum_{x_1, x_2} \psi(x_1, x_2) \,|\uparrow\uparrow\underset{x_1}{\downarrow}\uparrow\uparrow\uparrow\underset{x_2}{\downarrow}\downarrow\downarrow\rangle$$

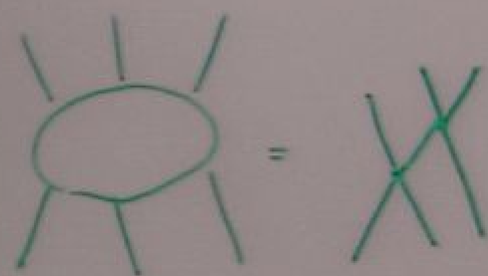
$$\psi(x_1, x_2) = e^{ip_1x_1 + ip_2x_2} + \underset{\substack{\uparrow \\ \text{S-MATRIX}}}{S(p_1,p_2)}\, e^{ip_1x_2 + ip_2x_1}$$

$$S(p_1,p_2) = \frac{2e^{ip_1} - e^{ip_1+ip_2} - 1}{2e^{ip_2} - e^{ip_1+ip_2} - 1}$$

THIS DESCRIBES 2→2 SCATTERING OF MAGNONS

MULTI-MAGNON STATES

IF THE HAMILTONIAN IS INTEGRABLE, THE SCATTERING OF MANY MAGNONS FACTORIZES IN 2→2 SCATTERING



THE ALLOWED MOMENTA ARE OBTAINED BY THE BETHE EQUATIONS

$$e^{ip_k L} = \prod_{\substack{j=1 \\ j\neq k}}^{M} S(p_k, p_j) \qquad k=1 \dots M$$

FULL $N=4$ SYM

BEISERT 05

$PSU(2,2|4)$ SYMMETRY

$16 = 8_B + 8_F$ EXCITATIONS

- DISPERSION RELATION:

$$E(p) = -1 + \sqrt{1 + 8\lambda \sin^2(p/2)}$$

VALID AT LARGE J FOR ANY λ

- S-MATRIX $(16^2 \times 16^2)$

FIXED BY SYMMETRIES
MODULO A SCALAR FACTOR

$$\sigma(p_1, p_2; \lambda) = e^{i\theta}$$

$\uparrow$
"DRESSING" PHASE

MAIN REQUIREMENT: CROSSING SYMMETRY

JANIK 06

FULL N=4 SYM

BEISERT 05

PSU(2,2|4) SYMMETRY

$16 = 8_B + 8_F$ EXCITATIONS

- DISPERSION RELATION:

$$E(p) = -1 + \sqrt{1 + 8\lambda \sin^2(p/2)}$$

VALID AL LARGE J FOR ANY λ

- S-MATRIX $(16^2 \times 16^2)$

FIXED BY SYMMETRIES MODULO A SCALAR FACTOR

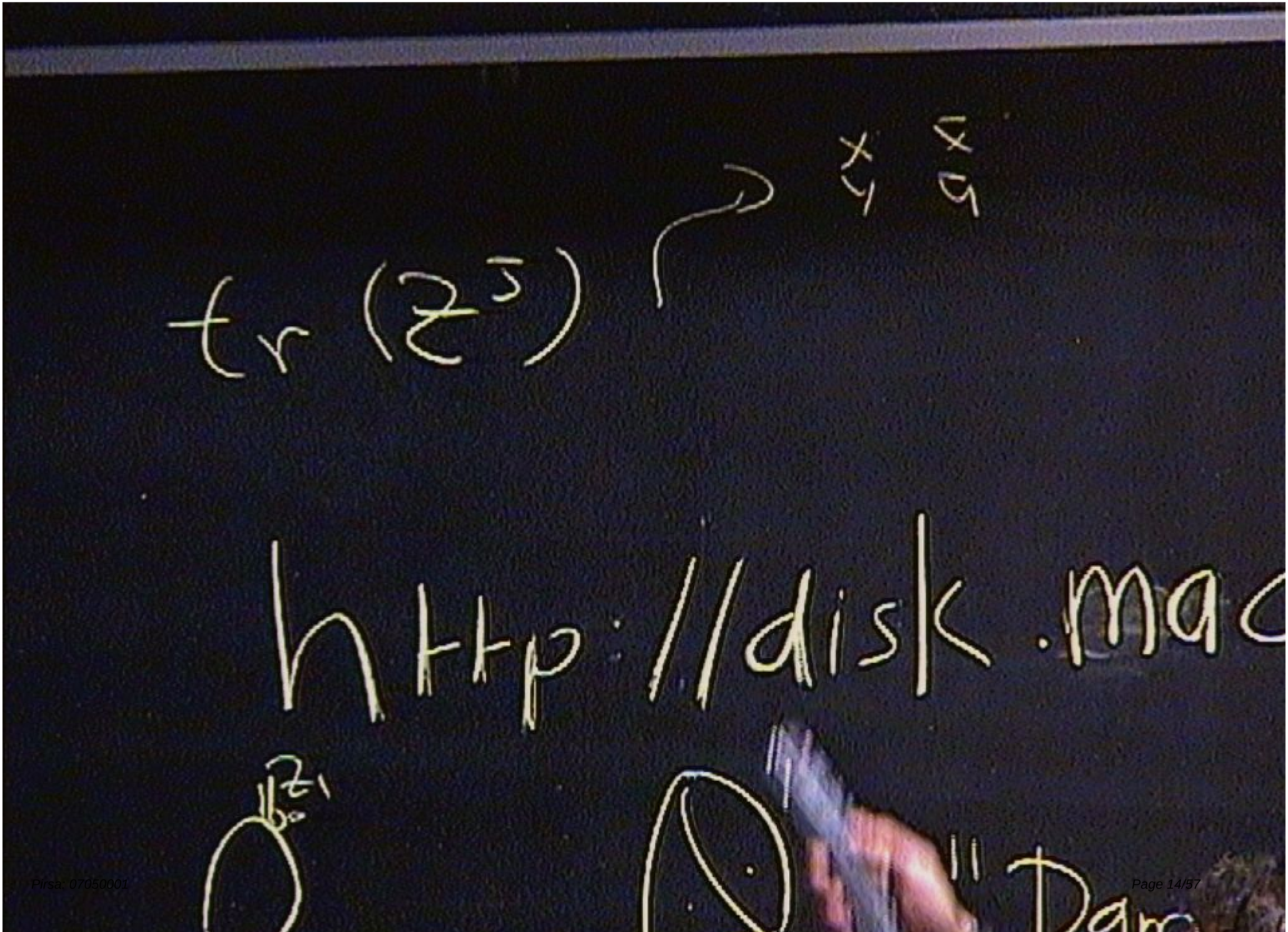
$$\sigma(p_1, p_2; \lambda) = e^{i\theta}$$

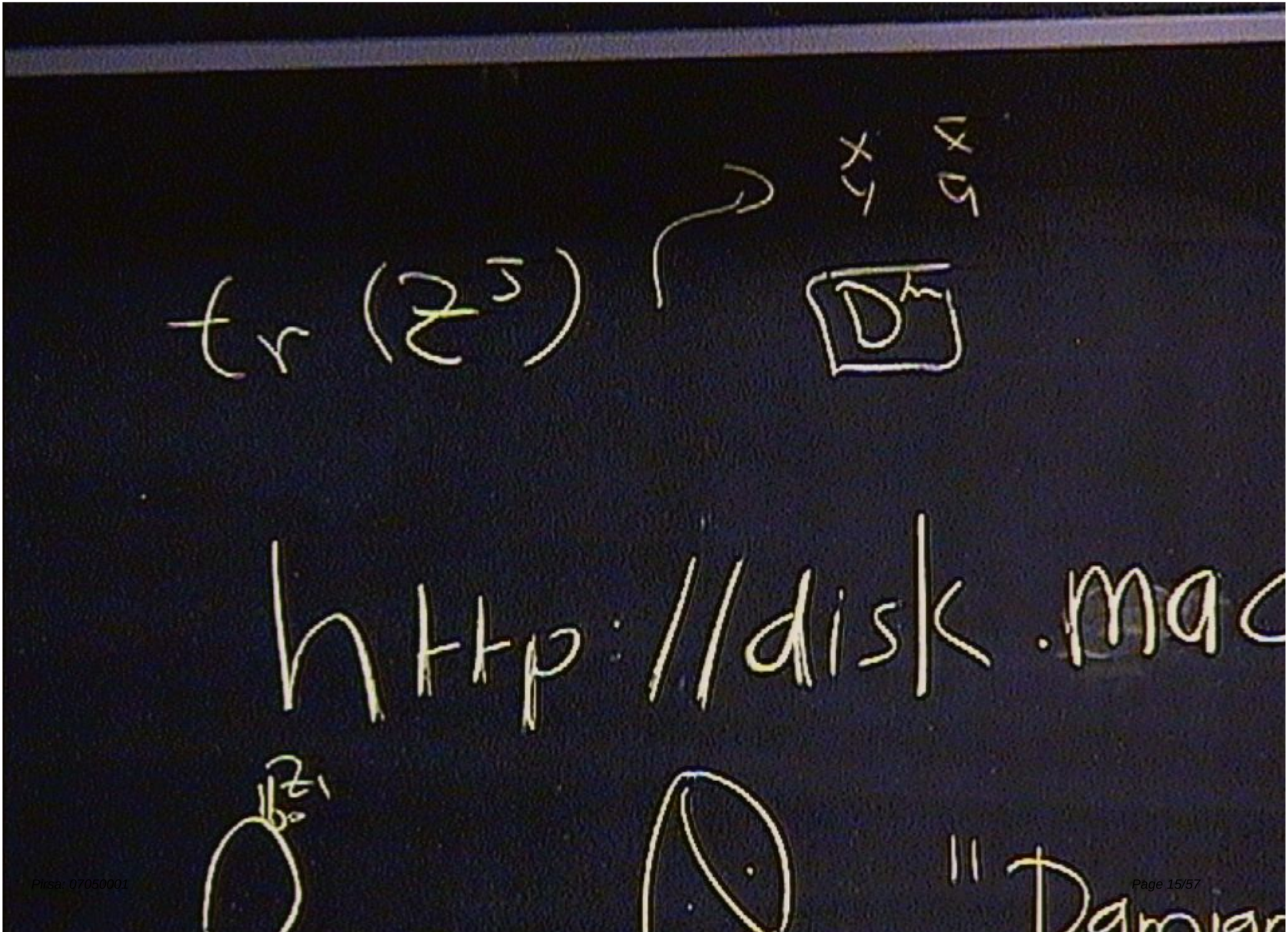
$\uparrow$

"DRESSING" PHASE

MAIN REQUIREMENT: CROSSING SYMMETRY

JANIK 06


tr (Z^5)
x y
x̄ ȳ
http://disk.mac


tr(Z^5)
D^n
http://disk.mac

FULL $N=4$ SYM

BEISERT 05

$PSU(2,2|4)$ SYMMETRY

$16 = 8_B + 8_F$ EXCITATIONS

- DISPERSION RELATION:

$$E(p) = -1 + \sqrt{1 + 8\lambda \sin^2(p/2)}$$

VALID AT LARGE J
FOR ANY λ

- S-MATRIX $(16^2 \times 16^2)$

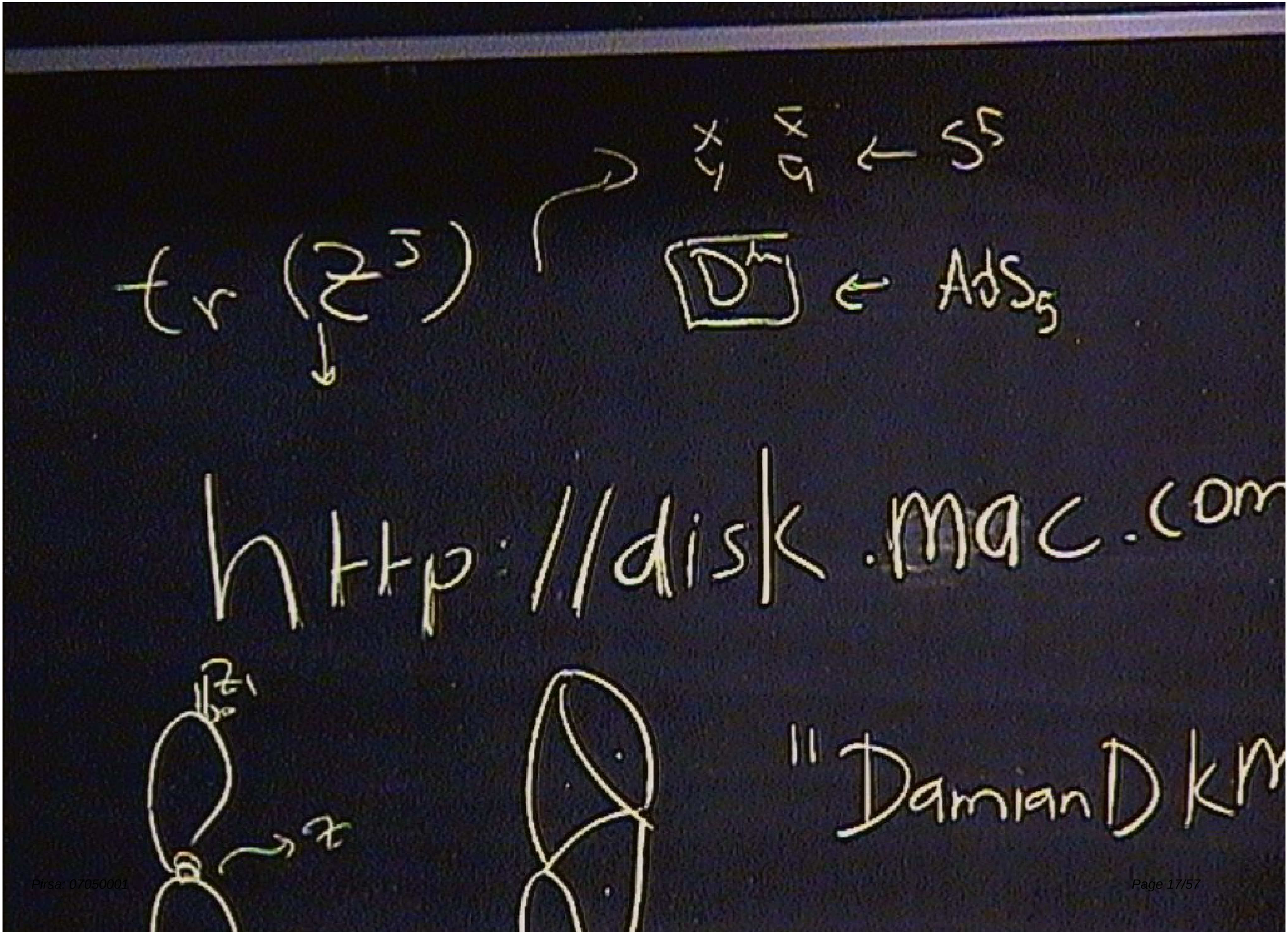
FIXED BY SYMMETRIES
MODULO A SCALAR FACTOR

$$\sigma(p_1, p_2; \lambda) = e^{i\theta}$$

$\uparrow$
"DRESSING" PHASE

MAIN REQUIREMENT: CROSSING SYMMETRY

JANIK 06


tr (Z^J)
← S5
D^n ← AdS5
http://disk.mac.com
"DamianDkm

FULL N=4 SYM

BEISERT 05

PSU(2,2|4) SYMMETRY

$16 = 8_B + 8_F$ EXCITATIONS

- DISPERSION RELATION:

$$E(p) = -1 + \sqrt{1 + 8\lambda \sin^2(p/2)}$$

VALID AT LARGE J
FOR ANY λ

- S-MATRIX $(16^2 \times 16^2)$

FIXED BY SYMMETRIES
MODULO A SCALAR FACTOR

$$\sigma(p_1, p_2; \lambda) = e^{i\theta}$$

$\uparrow$

"DRESSING" PHASE

MAIN REQUIREMENT: CROSSING SYMMETRY

JANIK 05

THE DRESSING PHASE

- UP TO 3 LOOPS: $\theta = 0$
- IN CLASSICAL STRINGS: A.F.S.

ARUTYUNOV FROLOV STAUDACHER 04

$$\theta(p_1,p_2) = 2\sum_{r=2}^{\infty} q_r(p_1)\, q_{r+1}(p_2) - q_r(p_2)\, q_{r+1}(p_1)$$

$$q_r(p) = \frac{2\sin((r-1)p/2)}{r-1}\left[\frac{-1+\sqrt{1+g^2\sin^2 p/2}}{2g\sin(p/2)}\right]^{r-1}$$

- STRING 1 LOOP — HERNANDEZ-LOPEZ 06
- CONJECTURE FOR LARGE-λ ASYMPTOTIC EXPANSION — BEISERT-HERNANDEZ-LOPEZ 06

$$\theta = \sum_{r,s} c_{r,s}(g)\left[q_r(p_1)\, q_s(p_2) - q_r(p_2)\, q_s(p_1)\right]$$

- EXACT PROPOSAL — BEISERT-EDEN-STAUDACHER 06

$$c_{r,s}(g) = 2\cos\left[\frac{\pi}{2}(s-r-1)\right](r-1)(s-1)\int_0^{\infty}\frac{J_{r-1}(2gt)\, J_{s-1}(2gt)}{t\,(e^t-1)}\,dt$$

THE CUSP ANOMALY

IN GAUGE THEORY WE CONSIDER
THE OPERATORS:

$$\mathrm{tr}\left(\phi\, D^S \phi\right)$$

THEIR ANOMALOUS DIMENSION AT LARGE S IS

$$\Delta - S = f(g)\,\ln(S) + \ldots$$

AT SMALL g

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 \ldots$$

IN STRING THEORY WE CONSIDER
A FOLDED STRING ROTATING IN AdS_5

$$ds^2 = d\rho^2 - \cosh^2\rho\, dt^2 + \sinh^2\rho\, d\theta^2 \qquad (AdS_3)$$

$$\begin{cases} t = \kappa\tau \\ \theta \simeq \kappa\tau \\ \rho \simeq \sigma \end{cases} \qquad \kappa = \frac{1}{\pi}\ln\left(\frac{S}{\sqrt{\lambda}}\right)$$

GUBSER
KLEBANOV
POLYAKOV 02

FROLOV
TSEYTLIN 02

1-LOOP

2-LOOP

THE CUSP ANOMALY

IN GAUGE THEORY WE CONSIDER THE OPERATORS:

$$\mathrm{tr}(\phi D^s \phi)$$

THEIR ANOMALOUS DIMENSION AT LARGE S IS

$$\Delta - S = f(g)\,\ln(S) + \ldots$$

AT SMALL g

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 \ldots$$

IN STRING THEORY WE CONSIDER A FOLDED STRING ROTATING IN AdS_5

$$ds^2 = d\rho^2 - \cosh^2\rho\, dt^2 + \sinh^2\rho\, d\theta^2 \qquad (AdS_3)$$

$$\begin{cases} t = \kappa\tau \\ \theta \simeq \kappa\tau \\ \rho \simeq \sigma \end{cases} \qquad \kappa = \frac{1}{\pi}\ln\left(\frac{S}{\sqrt{\lambda}}\right)$$

GUBSER KLEBANOV POLYAKOV '02

FROLOV TSEYTLIN '02

1-LOOP

2-LOOP

$$f(g) = 4g - 3\ln 2 \ldots$$

• IN CLASSICAL STRINGS: A.F.S.

ARUTYUNOV FROLOV STAUDACHER 04

$$\theta(p_1,p_2) = 2\sum_{r=2}^{\infty} q_r(p_1)\, q_{r+1}(p_2) - q_r(p_2)\, q_{r+1}(p_1)$$

$$q_r(p) = \frac{2\sin((r-1)p/2)}{r-1}\left[\frac{-1+\sqrt{1+g^2\sin^2 p/2}}{2g\sin(p/2)}\right]^{r-1}$$

• STRING 1 LOOP HERNANDEZ-LOPEZ 06

• CONJECTURE FOR LARGE-λ ASYMPTOTIC EXPANSION BEISERT-HERNANDEZ-LOPEZ 06

$$\theta = \sum_{r,s} c_{r,s}(g)\left[q_r(p_1)\, q_s(p_2) - q_r(p_2)\, q_s(p_1)\right]$$

• EXACT PROPOSAL BEISERT-EDEN-STAUDACHER 06

$$c_{r,s}(g) = 2\cos\left[\frac{\pi}{2}(s-r-1)\right](r-1)(s-1)\int_0^{\infty} \frac{J_{r-1}(2gt)\, J_{s-1}(2gt)}{t\,(e^t-1)}\, dt$$

BES EQUATION

BASED ON THE BEHLS S-MATRIX, B.E.S. STUDIED THE BETHE ANSATZ FOR THE CUSP ANOMALY.

INTEGRAL EQUATION FOR $f(g)$:

$$f(g) = 16 g^2 \hat{\sigma}(0)$$

$$\hat{\sigma}(t;g) = \frac{t}{e^t - 1}\left(K(2gt, 0) - 4g^2 \int_0^\infty dt' \, K(2gt, 2gt') \, \hat{\sigma}(t') \right)$$

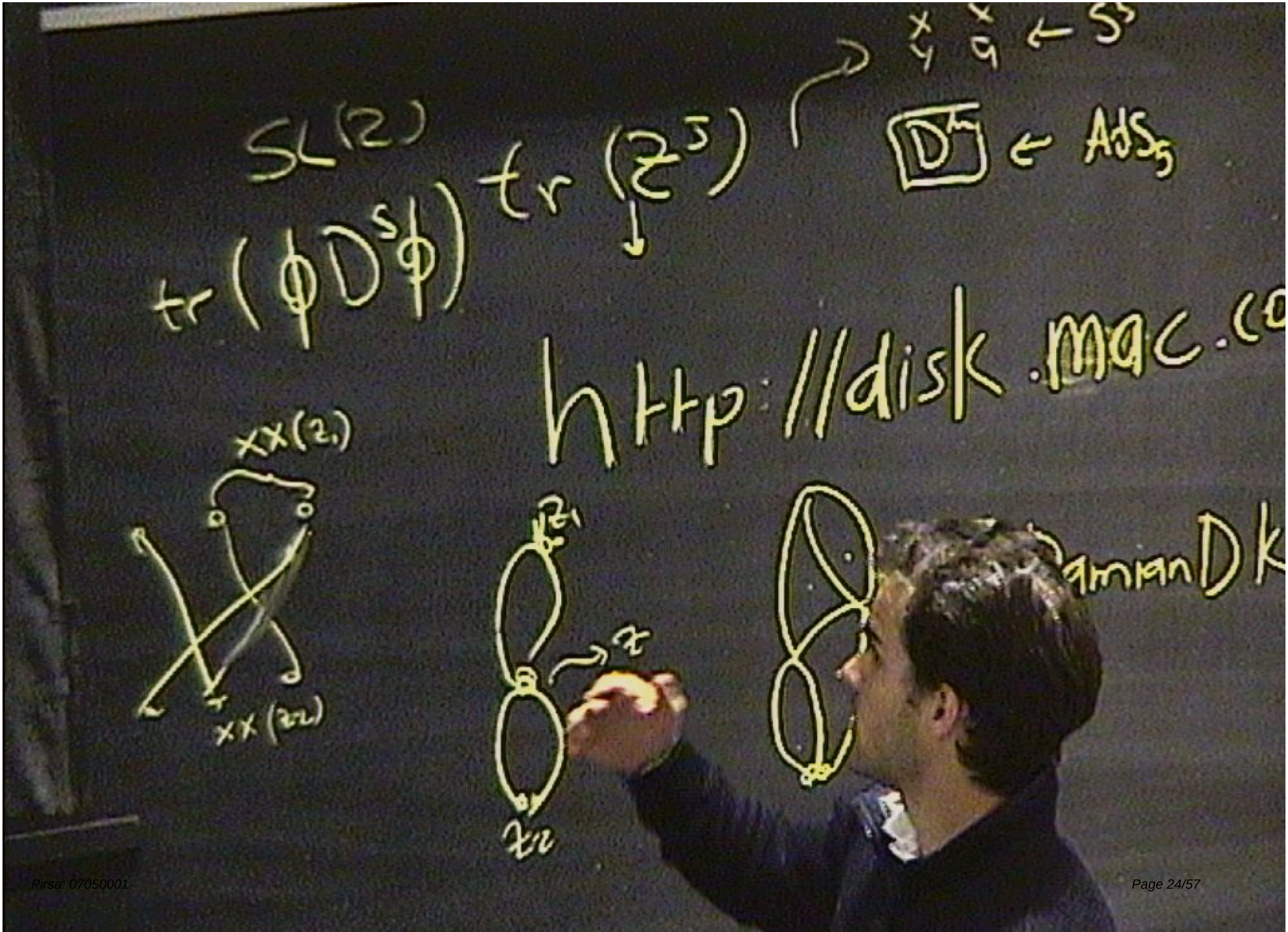
$$K(t,t') = K^m(t,t') + 2K^c(t,t')$$

$$K^m(t,t') = \frac{J_1(t) J_0(t') - J_0(t) J_1(t')}{t - t'}$$

$$K^c(t,t') = 4g^2 \int_0^\infty dt'' \, K_1(t, 2gt'') \frac{t''}{e^{t''} - 1} K_0(2gt'', t')$$

$$K_0(t,t') = \frac{t J_1(t) J_0(t') - t' J_0(t) J_1(t')}{t^2 - t'^2}$$

$$K_1(t,t') = \frac{t' J_1(t) J_0(t') - t J_0(t) J_1(t')}{t^2 - t'^2}$$


$SL(2)$
$tr(\phi D^s \phi)$
$tr(Z^s)$
$\leftarrow S^5$
$\boxed{D^5} \leftarrow AdS_5$
http://disk.mac.co
$xx(z_1)$
$xx(z_2)$
z_1
z
z_2

$x\ y\ z\ q \leftarrow S^5$

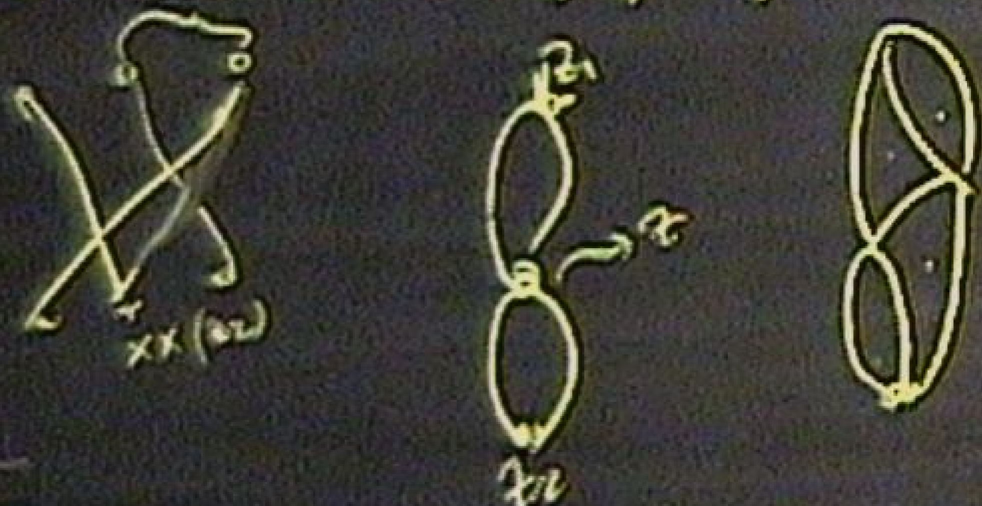
$\mathrm{SL}(2)$

$\mathrm{tr}(Z^5)$ $\boxed{D^5} \leftarrow AdS_5$

$\mathrm{tr}(\phi D^s \phi)$

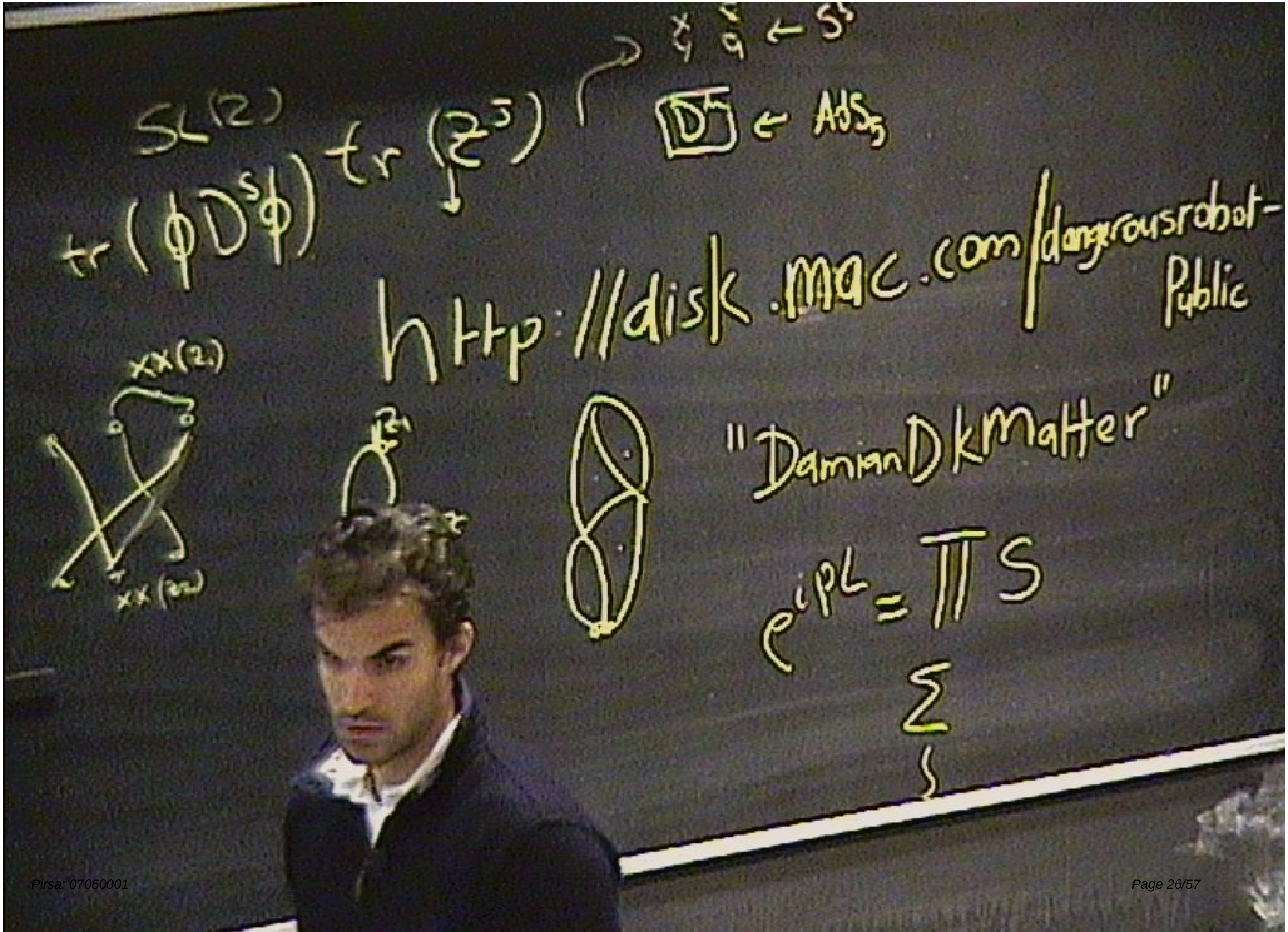
http://disk.mac.com/dangerousrobot-Public

$xx(z_1)$



"DamianDkMatter"

$$e^{ipL} = \prod S$$


SL(2)
tr(ϕD^sϕ) tr(Z^J)
AdS5
S5
http://disk.mac.com/dangerousrobot-Public
XX(2)
"DamianDkMatter"
e^ipL = ∏S
Σ

BES EQUATION

BASED ON THE BEHLS S-MATRIX, B.E.S. STUDIED THE BETHE ANSATZ FOR THE CUSP ANOMALY.

INTEGRAL EQUATION FOR $f(g)$:

$$f(g) = 16 g^2 \hat{\sigma}(0)$$

$$\hat{\sigma}(t;g) = \frac{t}{e^t - 1}\left(K(2gt, 0) - 4g^2 \int_0^\infty dt' \, K(2gt, 2gt') \, \hat{\sigma}(t')\right)$$

$$K(t,t') = K^m(t,t') + 2K^c(t,t')$$

$$K^m(t,t') = \frac{J_1(t) J_0(t') - J_0(t) J_1(t')}{t - t'}$$

$$K^c(t,t') = 4g^2 \int_0^\infty dt'' \, K_1(t, 2gt'') \frac{t''}{e^{t''} - 1} K_0(2gt'', t')$$

$$K_0(t,t') = \frac{t J_1(t) J_0(t') - t' J_0(t) J_1(t')}{t^2 - t'^2}$$

$$K_1(t,t') = \frac{t' J_1(t) J_0(t') - t J_0(t) J_1(t')}{t^2 - t'^2}$$

PERTURBATIVE EXPANSION OF $f(g)$ FROM BES EQUATION

4-LOOP:
- BERN, CZAKON, DIXON, KOSOWER, SMIRNOV 06
- CACHAZO SPRADLIN VOLOVICH 06

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6$$
$$-16\left(\frac{73}{630}\pi^6 + 4\,\zeta(3)^2\right)g^8$$
$$+32\left(\frac{887}{14175}\pi^8 + \frac{4}{3}\pi^2\,\zeta(3)^2 + 40\,\zeta(3)\,\zeta(5)\right)g^{10}$$
$$- \ldots$$

TRANSCENDENTALITY PRINCIPLE

(KOTIKOV LIPATOV)

THE TERM g^{2k} HAS TRANSCENDENTALITY $2k-2$

$$t(\pi^n) = n$$
$$t(\zeta(n)) = n$$

CONJECTURE FOR QCD:

HIGHER TRANSCENDENTALITY TERMS IN QCD

PERTURBATIVE EXPANSION OF $f(g)$

FROM BES EQUATION

4-LOOP:
- BERN, CZAKON, DIXON, KOSOWER, SMIRNOV 06
- CACHAZO SPRADLIN VOLOVICH 06

$$
\begin{aligned}
f(g) = {} & 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 \\
& -16\left(\frac{73}{630}\pi^6 + 4\,\zeta(3)^2\right) g^8 \\
& +32\left(\frac{887}{14175}\pi^8 + \frac{4}{3}\pi^2 \zeta(3)^2 + 40\,\zeta(3)\,\zeta(5)\right) g^{10} \\
& - \ldots
\end{aligned}
$$

TRANSCENDENTALITY PRINCIPLE

(KOTIKOV LIPATOV)

THE TERM g^{2k} HAS TRANSCENDENTALITY $2k-2$

$$t(\pi^n) = n$$

$$t(\zeta(n)) = n$$

CONJECTURE FOR QCD:

HIGHER TRANSCENDENTALITY TERMS IN QCD COINCIDE WITH

QUANTITATIVE STUDY OF $f(g)$ AT FINITE g.

THE BES INTEGRAL EQUATION ALLOWS, FOR THE FIRST TIME, TO COMPUTE ANOMALOUS DIMENSIONS AT FINITE VALUES OF THE COUPLING

BENNA, S.B., KLEBANOV SCARDICCHIO 06

EXPANDING

$$\hat{\sigma}(t) = \frac{t}{e^t - 1} \sum_{n=1}^{\infty} s_n(g) \frac{J_n(2gt)}{2gt}$$

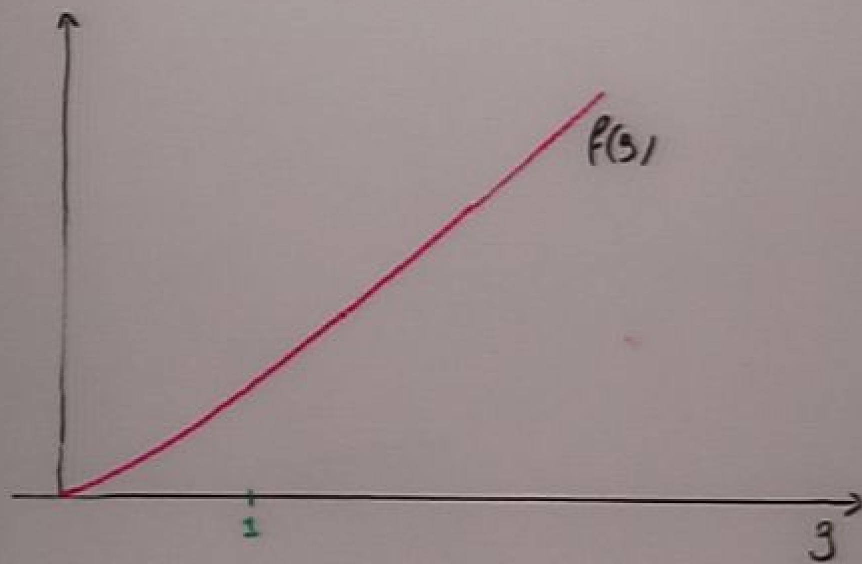
ONE OBTAINS A MATRIX EQUATION FOR $s_n(g)$

TO COMPUTE $s_n(g)$ at FINITE g IT IS ENOUGH TO CONSIDER FINITE MATRICES $K \times K$ WITH $K \sim g$

WITH THIS METHOD IT'S POSSIBLE TO COMPUTE

$$f(g) = 16\, g^2\, s_1(g)$$

NUMERICALLY



AT LARGE g:

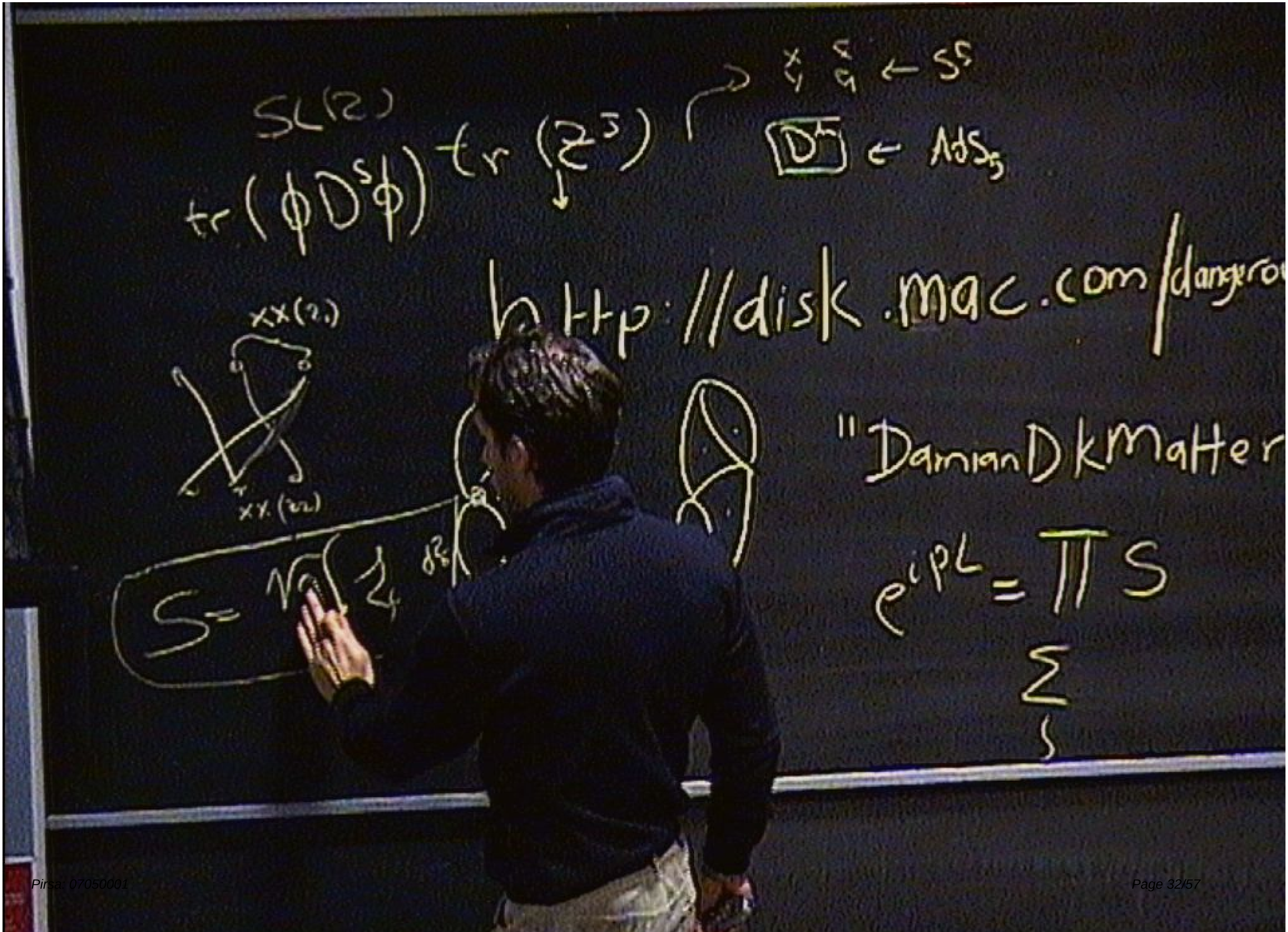
$$f(g) = (4 \pm 10^{-6})\, g + \quad \leftarrow \text{GKP}$$

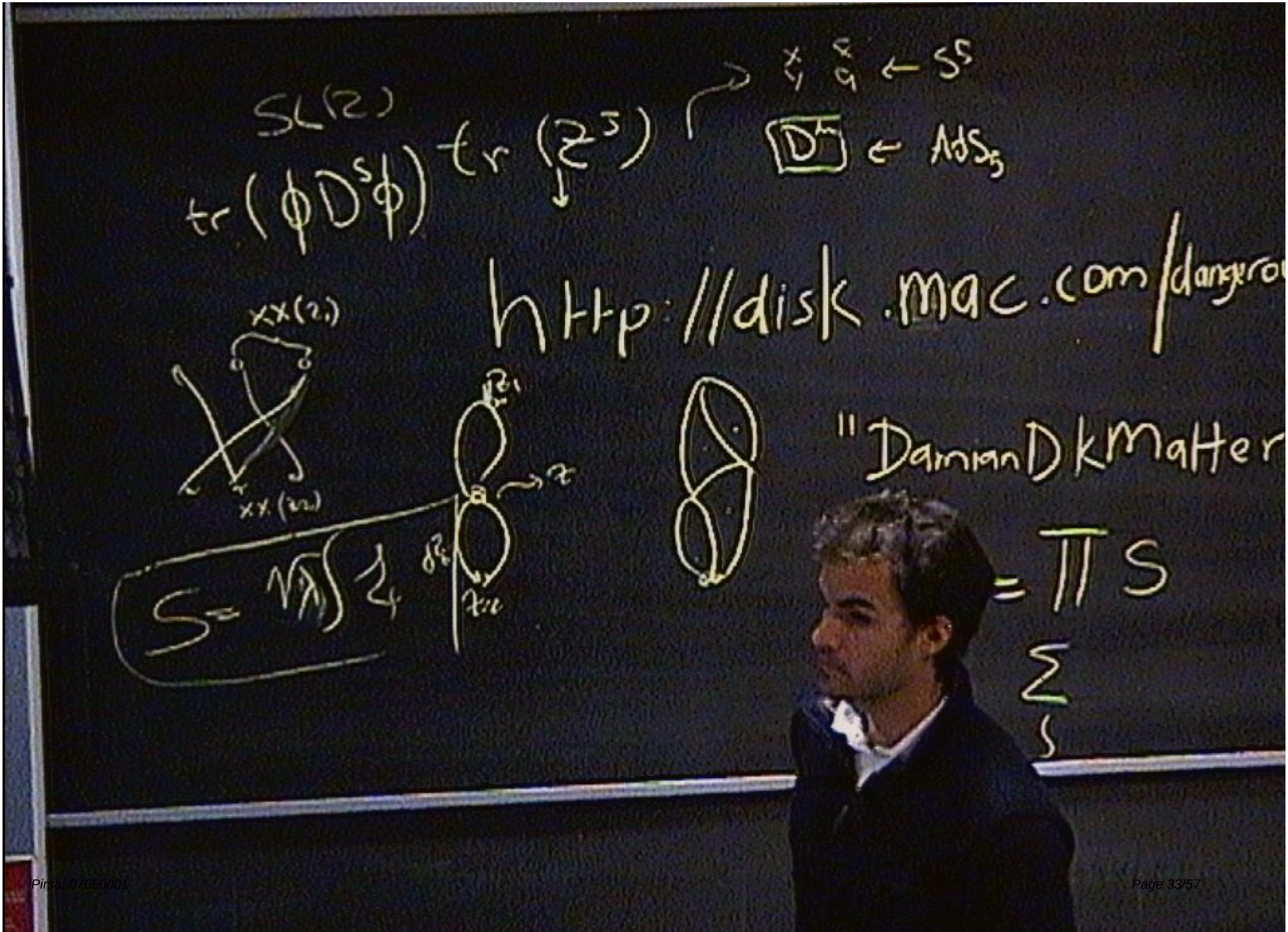
$$-(0.661907(2)) + \quad \leftarrow \text{FROLOV-TSEYTLIN } 3\,\frac{\ln(2)}{\pi}$$

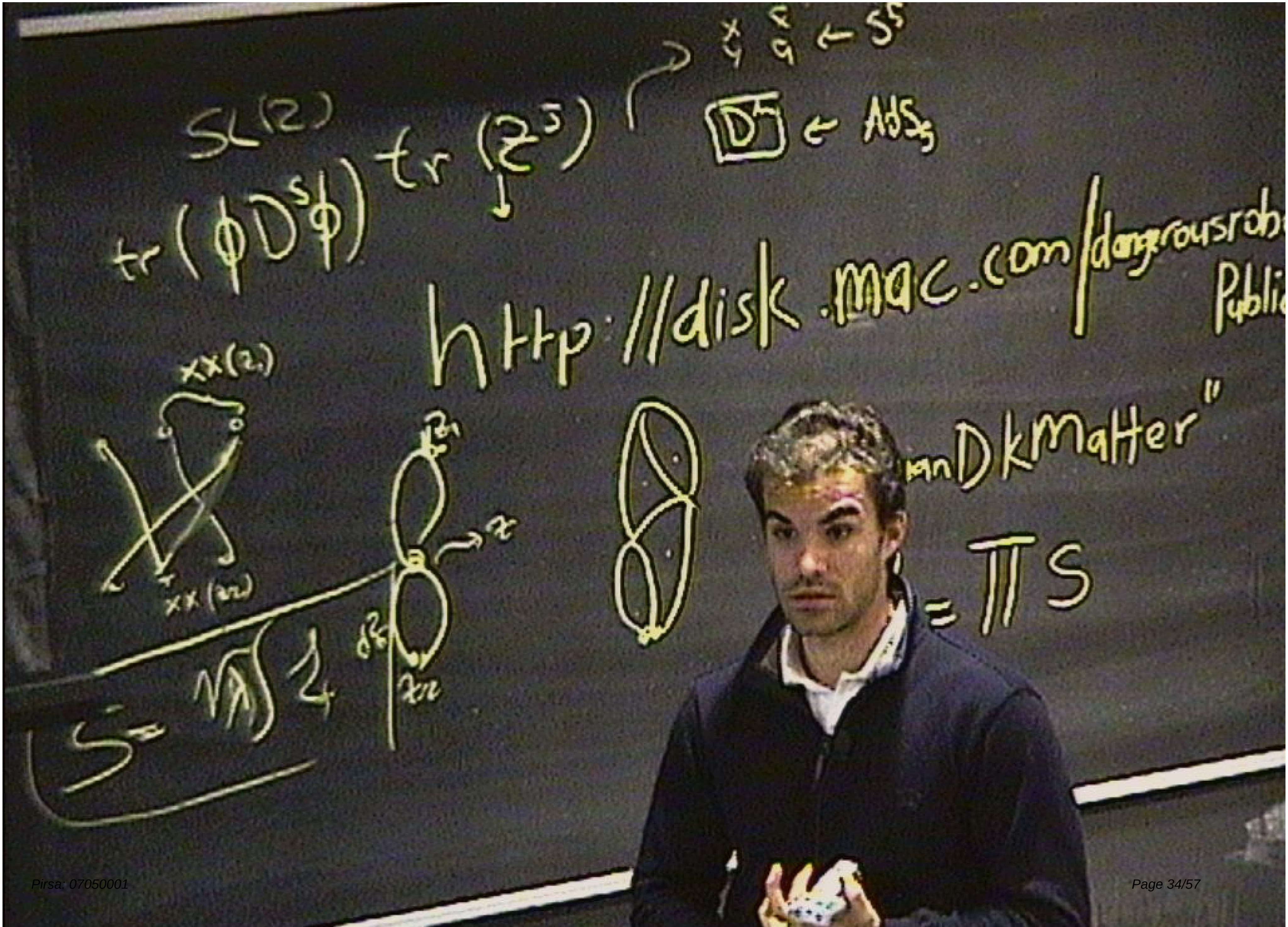
$$-(0.0232(1))\,\frac{1}{g} \quad \leftarrow \text{2 LOOP PREDICTION}$$

4g TERM OBTAINED ANALYTICALLY

ALDAY, ARUTYUNOV, BENNA, EDEN, KLEBANOV 07

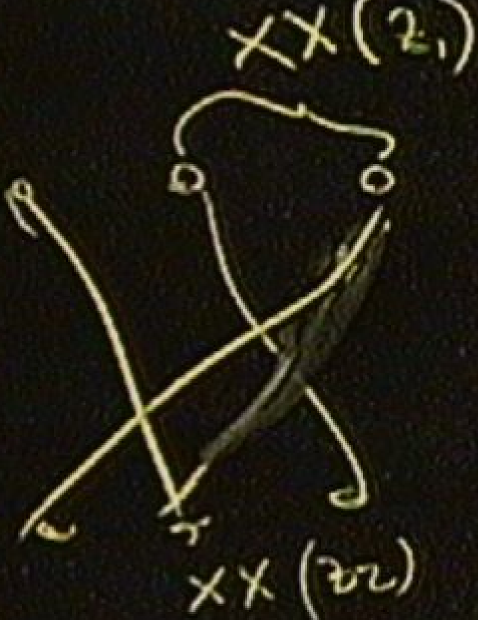

SL(2)
tr(ϕD^5ϕ) tr(Z^J)
← S^5
D^5 ← AdS_5
http://disk.mac.com/dangero
XX(2.)
"DamianDkMatter
e^{ipL} = ∏ S
Σ

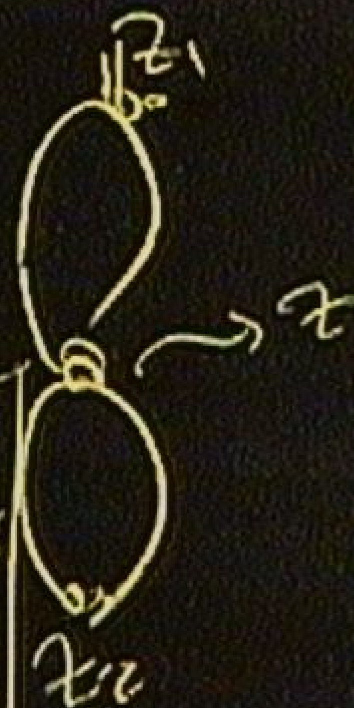

SL(2)
tr(ϕD^5ϕ) tr(Z^J)
← S^5
← AdS_5
http://disk.mac.com/dangero
XX(z_1)
XX(z_2)
"DamianDkMatter
S = √λ ∫


SL(2)
tr(φD^sφ)
tr(Z^J)
AdS5
S5
http://disk.mac.com/dangerousrob
Public
xx(2,)
DkMatter"

$\mathrm{tr}(\phi D^5 \phi)$

http://d

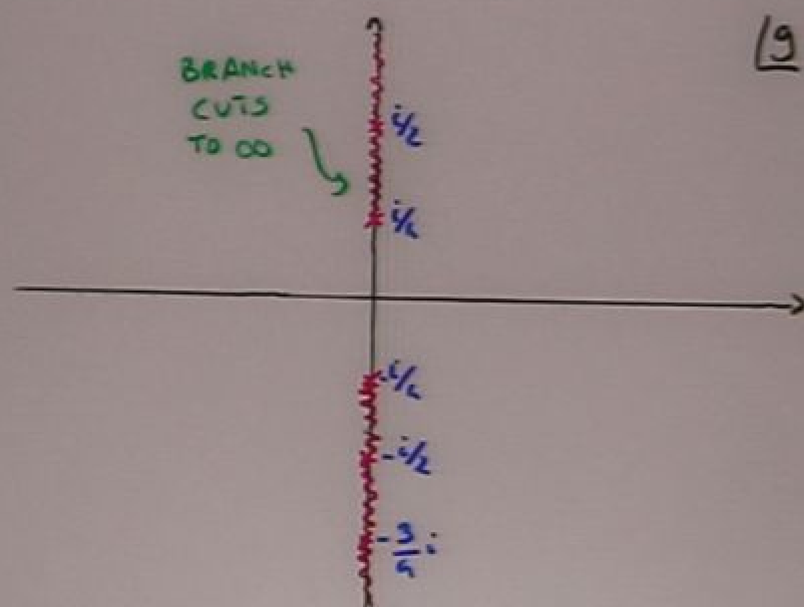




$$\boxed{S = \frac{N}{\lambda}\int \mathcal{L}_4 \, d^2\sigma}$$

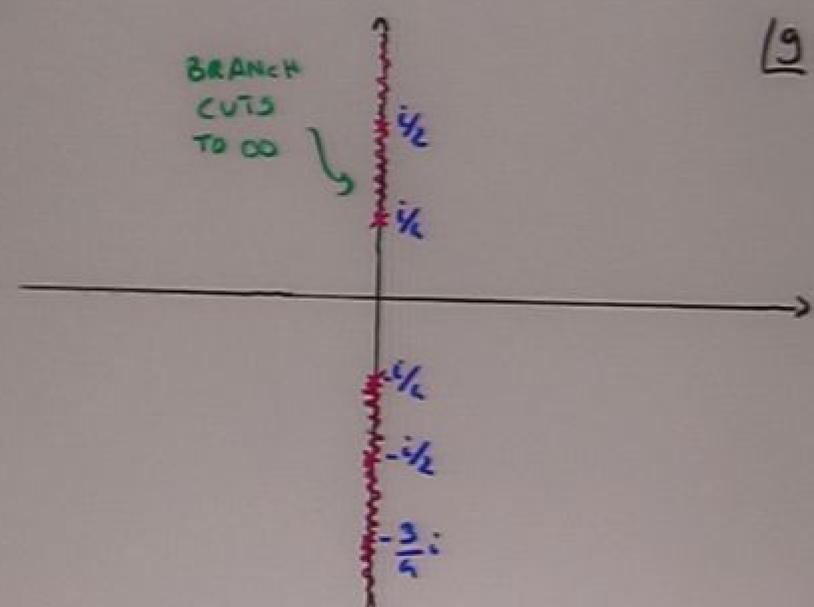
ANALYTIC STRUCTURE

- AT SMALL g, $f(g)$ HAS A FINITE RADIUS OF CONVERGENCE, $|g| < 1/4$
- AT LARGE g THE EXPANSION IS ONLY ASYMPTOTIC. ESSENTIAL SINGULARITY.



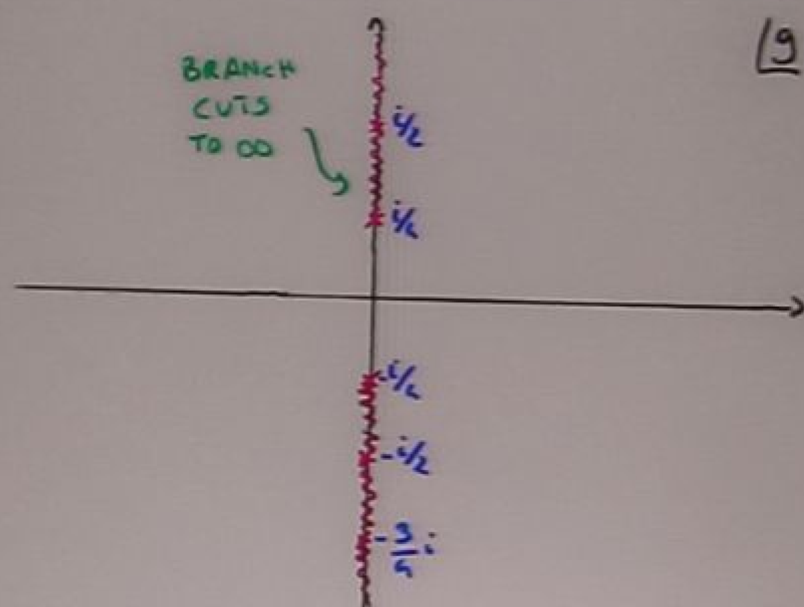
ANALYTIC STRUCTURE

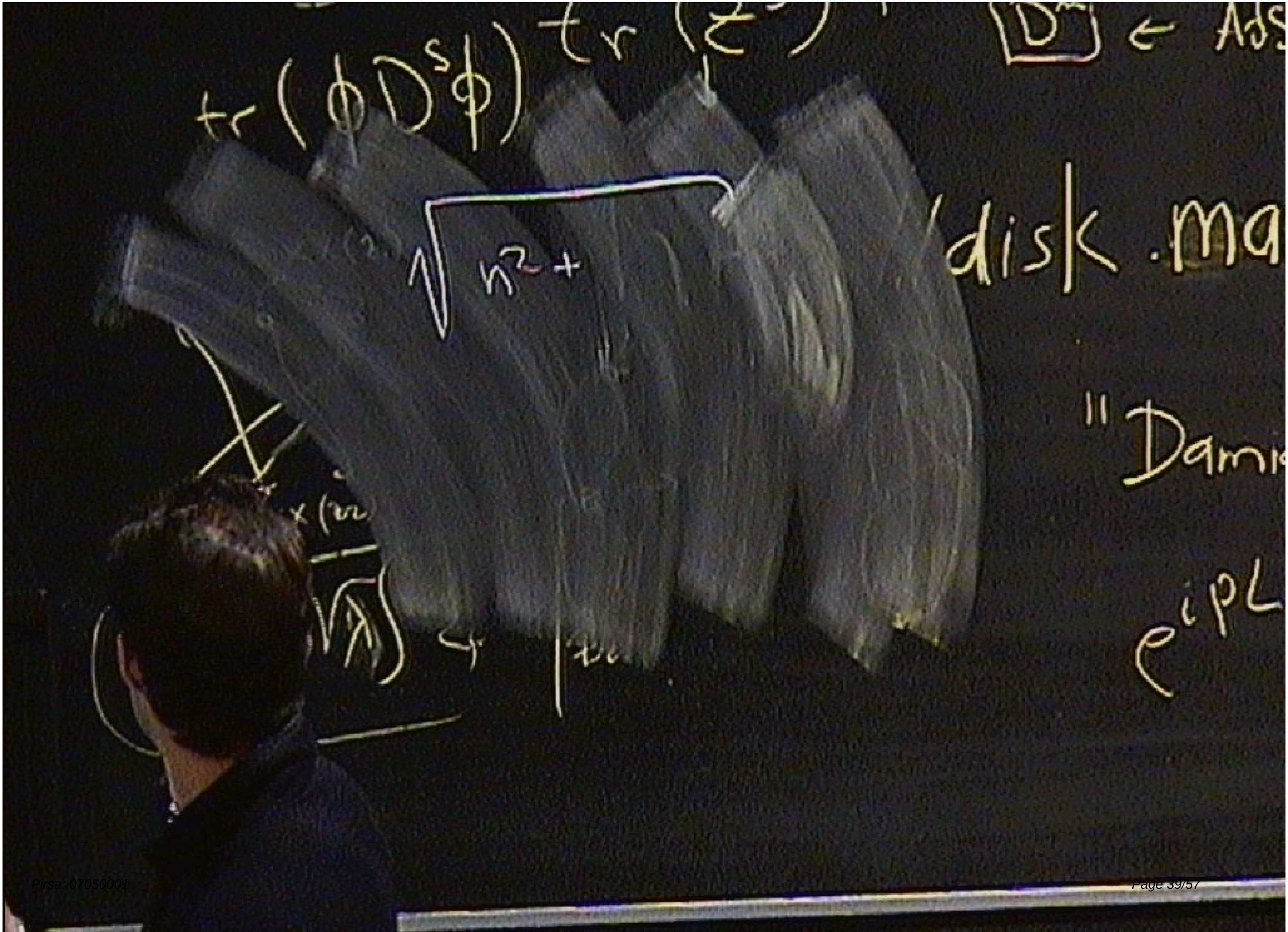
- AT SMALL g, $f(g)$ HAS A FINITE RADIUS OF CONVERGENCE, $|g| < 1/4$
- AT LARGE g THE EXPANSION IS ONLY ASYMPTOTIC. ESSENTIAL SINGULARITY.

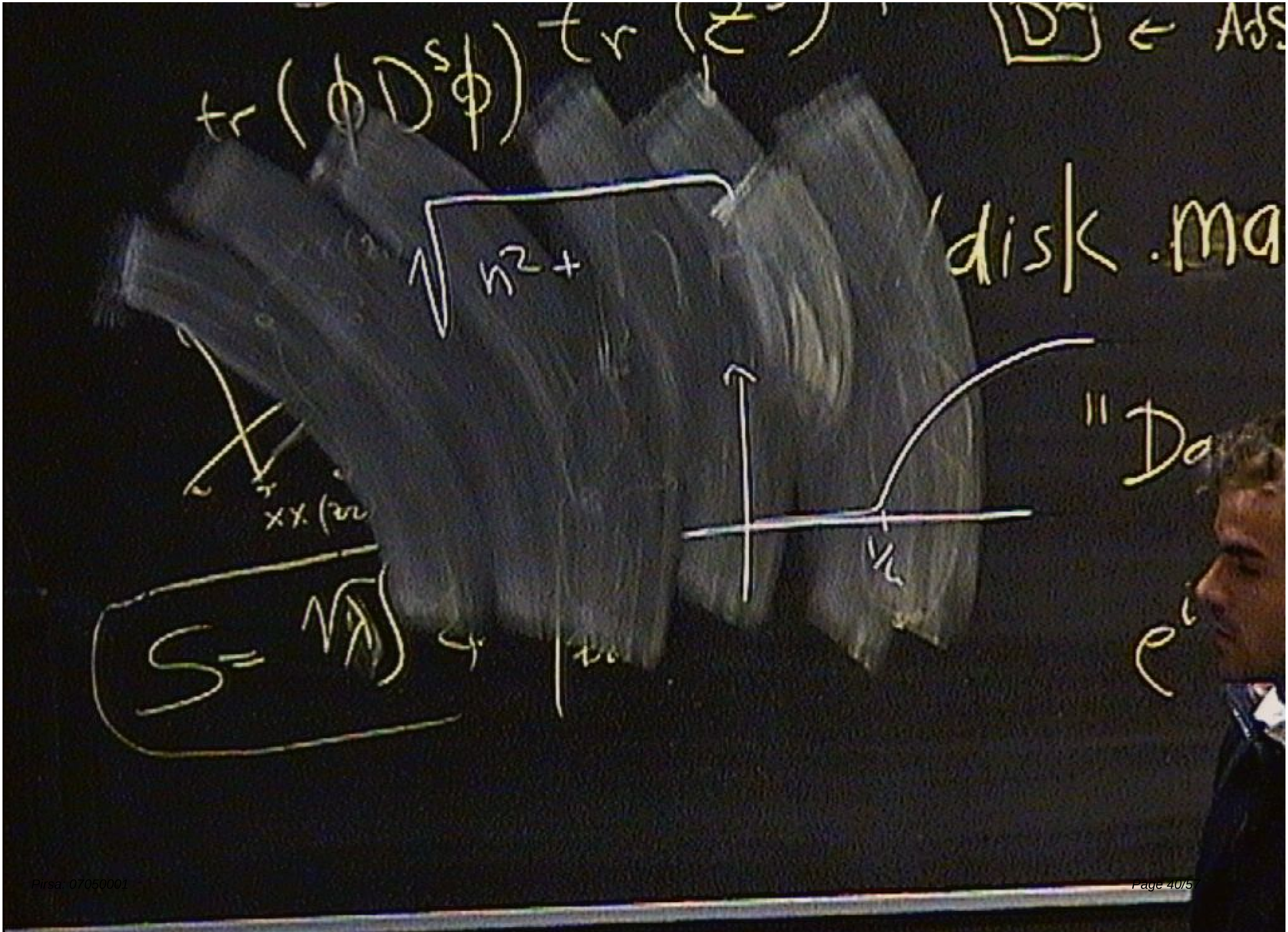


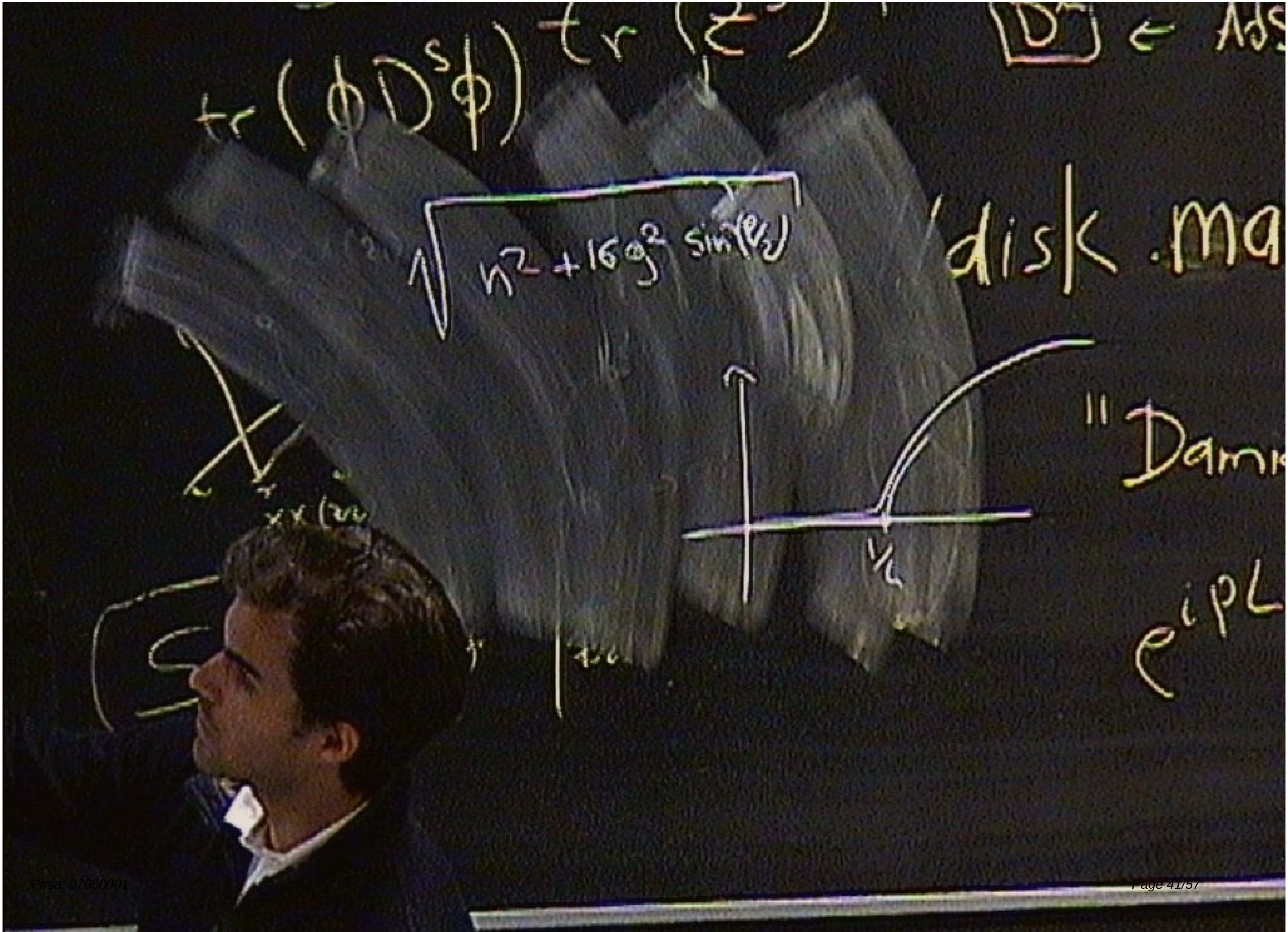
ANALYTIC STRUCTURE

- AT SMALL g, $f(g)$ HAS A FINITE RADIUS OF CONVERGENCE, $|g| < 1/4$
- AT LARGE g THE EXPANSION IS ONLY ASYMPTOTIC. ESSENTIAL SINGULARITY.



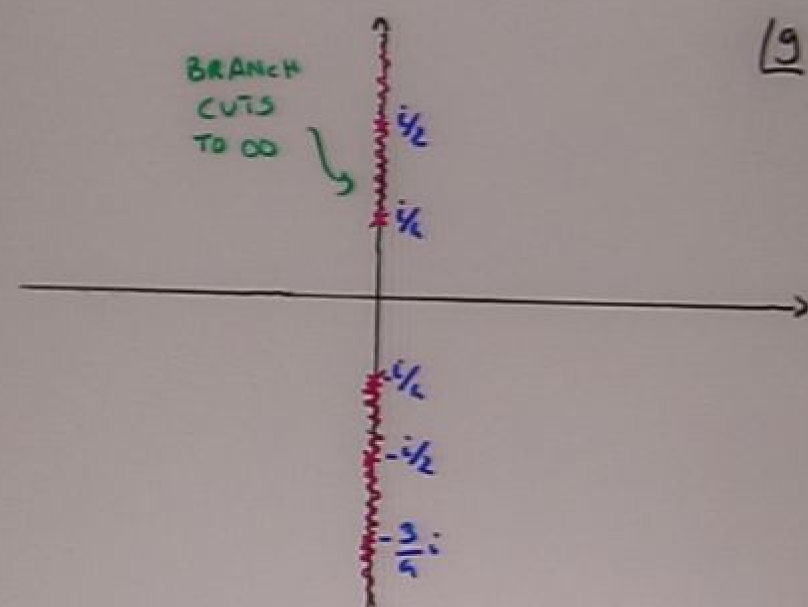

tr (ϕ D^s ϕ) tr
disk .ma
"Dami


tr (ϕ D^s ϕ) tr (
disk .ma
← AdS
"Da
S= N√λ ∫


tr(ϕD^sϕ) tr(
disk .ma
"Dam
√(n² + 16g² sin²(p/2))
e^{ipL

ANALYTIC STRUCTURE

- AT SMALL g, $f(g)$ HAS A FINITE RADIUS OF CONVERGENCE, $|g| < 1/4$
- AT LARGE g THE EXPANSION IS ONLY ASYMPTOTIC. ESSENTIAL SINGULARITY.



- IN CLASSICAL STRINGS: A.F.S.

ARUTYUNOV FROLOV STAUDACHER 04

$$\theta(p_1,p_2) = 2\sum_{r=2}^{\infty} q_r(p_1)\, q_{r+1}(p_2) - q_r(p_2)\, q_{r+1}(p_1)$$

$$q_r(p) = \frac{2\sin((r-1)p/2)}{r-1}\left[\frac{-1+\sqrt{1+g^2\sin^2 p/2}}{2g\sin(p/2)}\right]^{r-1}$$

- STRING 1 LOOP HERNANDEZ-LOPEZ 06

- CONJECTURE FOR LARGE-λ ASYMPTOTIC EXPANSION BEISERT-HERNANDEZ-LOPEZ 06

$$\theta = \sum_{r,s} c_{r,s}(g)\left[q_r(p_1)\, q_s(p_2) - q_r(p_2)\, q_s(p_1)\right]$$

- EXACT PROPOSAL BEISERT-EDEN-STAUDACHER 06

$$c_{r,s}(g) = 2\cos\left[\frac{\pi}{2}(s-r-1)\right](r-1)(s-1)\int_0^{\infty}\frac{J_{r-1}(2gt)\,J_{s-1}(2gt)}{t\,(e^t-1)}\,dt$$

BASED ON THE BEHLS S-MATRIX,

B.E.S. STUDIED THE BETHE ANSATZ

FOR THE CUSP ANOMALY.

INTEGRAL EQUATION FOR $f(g)$:

$$f(g) = 16 g^2 \hat{\sigma}(0)$$

$$\hat{\sigma}(t;g) = \frac{t}{e^t - 1}\left(K(2gt, 0) - 4g^2 \int_0^\infty dt' \, K(2gt, 2gt')\, \hat{\sigma}(t') \right)$$

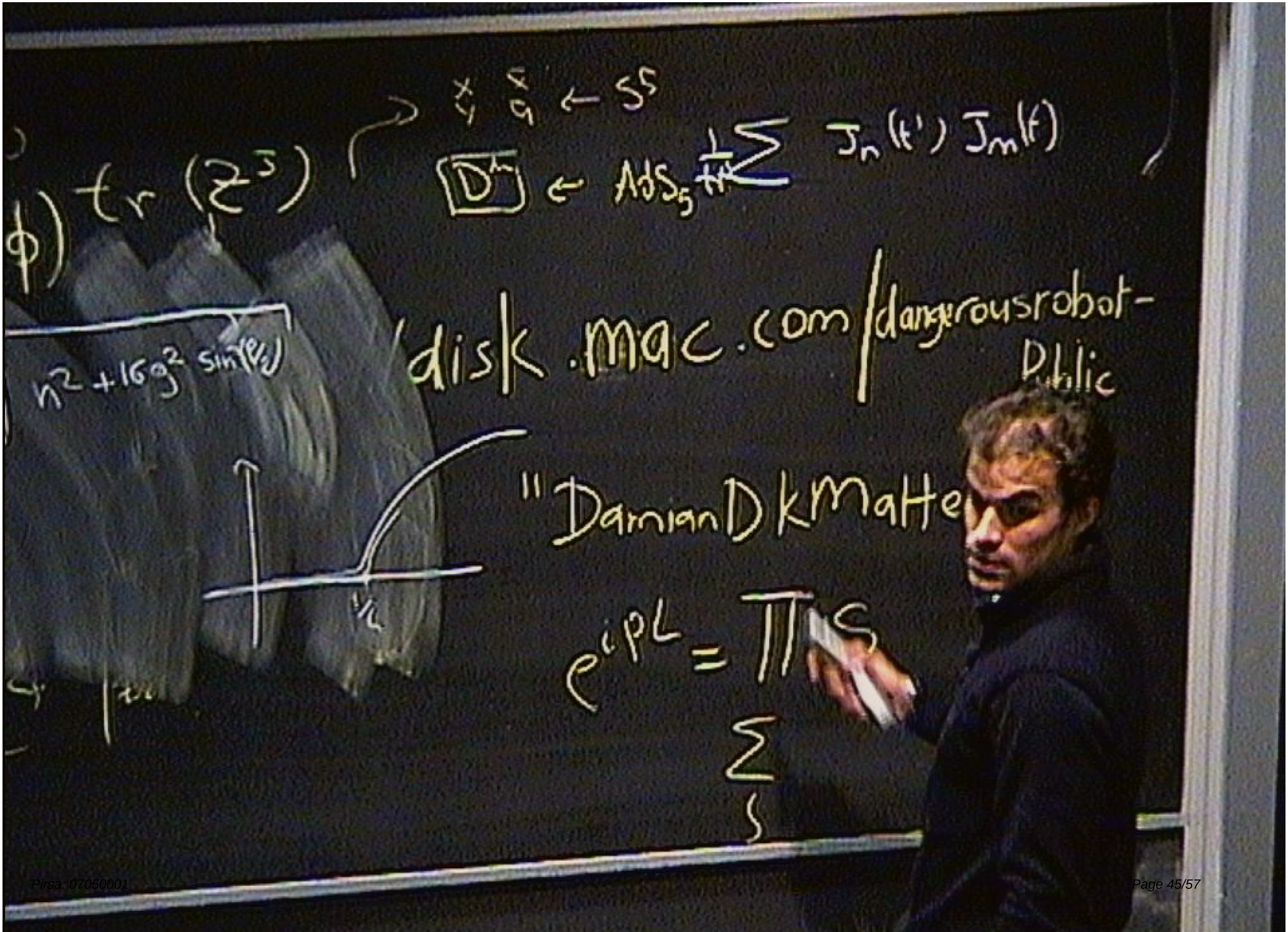
$$K(t,t') = K^m(t,t') + 2K^c(t,t')$$

$$K^m(t,t') = \frac{J_1(t) J_0(t') - J_0(t) J_1(t')}{t - t'}$$

$$K^c(t,t') = 4g^2 \int_0^\infty dt'' \, K_1(t, 2gt'') \frac{t''}{e^{t''} - 1} K_0(2gt'', t')$$

$$K_0(t,t') = \frac{t J_1(t) J_0(t') - t' J_0(t) J_1(t')}{t^2 - t'^2}$$

$$K_1(t,t') = \frac{t' J_1(t) J_0(t') - t J_0(t) J_1(t')}{t^2 - t'^2}$$


disk.mac.com/dangerousrobot-
Public
"DamianDkMatte

BASED ON THE BEHLS S-MATRIX,
B.E.S. STUDIED THE BETHE ANSATZ
FOR THE CUSP ANOMALY.

INTEGRAL EQUATION FOR $f(g)$:

$$f(g) = 16\, g^2\, \hat{\sigma}(0)$$

$$\hat{\sigma}(t;g) = \frac{t}{e^t-1}\left(K(2gt,0) - 4g^2\int_0^\infty dt'\, K(2gt,2gt')\,\hat{\sigma}(t')\right)$$

$$K(t,t') = K^m(t,t') + 2K^c(t,t')$$

$$K^m(t,t') = \frac{J_1(t)J_0(t') - J_0(t)J_1(t')}{t-t'}$$

$$K^c(t,t') = 4g^2\int_0^\infty dt''\, K_1(t,2gt'')\frac{t''}{e^{t''}-1}\, K_0(2gt'',t')$$

$$K_0(t,t') = \frac{t\,J_1(t)J_0(t') - t'\,J_0(t)J_1(t')}{t^2-t'^2}$$

$$K_1(t,t') = \frac{t'\,J_1(t)J_0(t') - t\,J_0(t)J_1(t')}{t^2-t'^2}$$

- UP TO 3 LOOPS: $\Theta = 0$
  BASED ON THE BEHLS S-MATRIX,
- IN CLASSICAL STRINGS: AFS
  B.E.S. STUDIED THE BETHE ANSATZ

ARUTYUNOV FROLOV STAUDACHER et al.

$$\Theta(p_1,p_2) \equiv \sum_{r=2}^{\infty} \left[q_r(p_1)\, q_{r+1}(p_2) - q_r(p_2)\, q_{r+1}(p_1) \right]$$

INTEGRAL EQUATION FOR $f(g)$:

$$q_r(p) = \frac{2\sin((r-1)p/2)}{r-1}\left[\frac{-1+\sqrt{1+g^2\sin^2(p/2)}}{2g\sin(p/2)}\right]^{r-1}$$

$$f(g) = 16\, g^2\, \hat{\sigma}(0)$$

- STRING 1st LOOP: $\hat{\sigma}(t) = \frac{t}{e^t-1}\left(K(2gt,0) - 4g^2\int_0^\infty dt'\, K(2gt,2gt')\,\hat{\sigma}(t')\right)$

- CONJECTURE FOR ASYMPTOTIC EXPANSION: $K(t,t') = K_0(t,t') + 2K_1(t,t')$ — HERNANDEZ LOPEZ et al.

$$K^{\sim}(t,t') = \frac{J_1(t)\,J_0(t') - J_0(t)\,J_1(t')}{t-t'}$$

$$\Theta = \sum_{r,s} c_{r,s}(g)\left[q_r(p_1)\,q_s(p_2) - q_r(p_2)\,q_s(p_1)\right]$$

- EXACT PROPOSAL — BEISERT-EDEN-STAUDACHER et al.

$$K_0(t,t') = \frac{t\,J_1(t)\,J_0(t') - t'\,J_0(t)\,J_1(t')}{t^2-t'^2}$$

$$K_1(t,t') = \frac{t'\,J_1(t)\,J_0(t') - t\,J_0(t)\,J_1(t')}{t^2-t'^2}$$

$$c_{r,s}(g) = 2\cos\left[\frac{\pi}{2}(s-r-1)\right](r-1)(s-1)\int_0^\infty \frac{J_{r-1}(2gt)\,J_{s-1}(2gt)}{t\,(e^t-1)}\,dt$$

- UP TO 3 LOOPS: $\theta = 0$

- IN CLASSICAL STRINGS: A.F.S.

ARUTYUNOV FROLOV STAUDACHER 04

$$\theta(p_1,p_2) = 2\sum_{r=2}^{\infty} q_r(p_1)\, q_{r+1}(p_2) - q_r(p_2)\, q_{r+1}(p_1)$$

$$q_r(p) = \frac{2\sin((r-1)p/2)}{r-1}\left[\frac{-1+\sqrt{1+g^2\sin^2(p/2)}}{2g\sin(p/2)}\right]^{r-1}$$

- STRING 1 LOOP — HERNANDEZ-LOPEZ 06

- CONJECTURE FOR LARGE-λ ASYMPTOTIC EXPANSION — BEISERT-HERNANDEZ-LOPEZ 06

$$\theta = \sum_{r,s} c_{r,s}(g)\left[q_r(p_1)\, q_s(p_2) - q_r(p_2)\, q_s(p_1)\right]$$

- EXACT PROPOSAL — BEISERT-EDEN-STAUDACHER 06

$$c_{r,s}(g) = 2\cos\left[\frac{\pi}{2}(s-r-1)\right](r-1)(s-1)\int_0^{\infty}\frac{J_{r-1}(2gt)\, J_{s-1}(2gt)}{t\,(e^t-1)}\, dt$$

SUMMARY

- SPECTRUM OF GAUGE THEORIES IN TERMS OF SPIN CHAINS
- BETHE ANSATZ FOR $\mathcal{N}=4$ SYM AND $AdS_5 \times S^5$
- CUSP ANOMALY: NON BPS OBSERVABLE

OPEN QUESTIONS

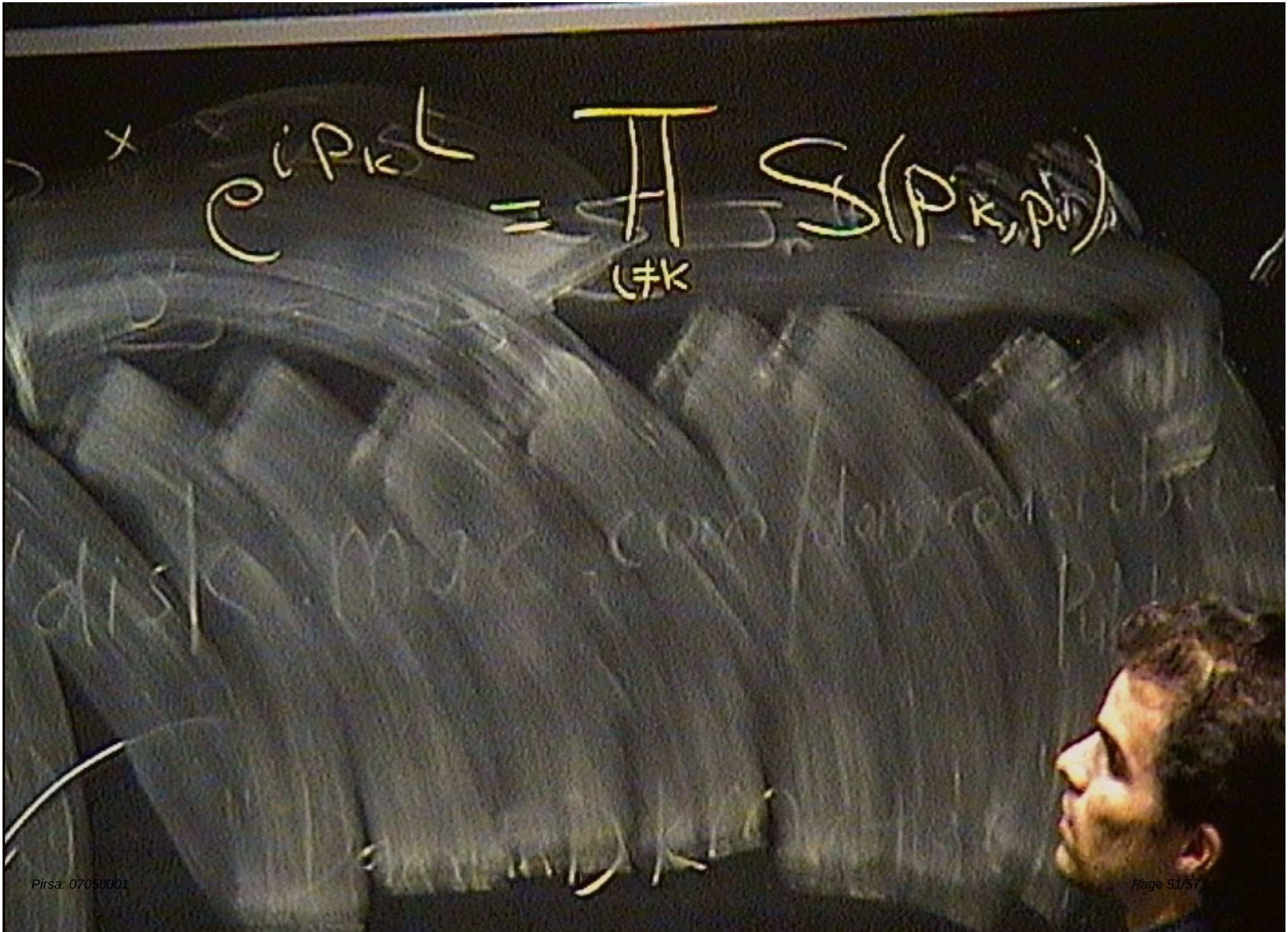
- FIND FURTHER EVIDENCE FOR THE BEHLS S-MATRIX
- STUDY ANALYTICAL STRUCTURE OF ANOMALOUS DIMENSIONS.
- FINITE-LENGTH EFFECTS: HAMILTONIAN?

SUMMARY

- SPECTRUM OF GAUGE THEORIES IN TERMS OF SPIN CHAINS
- BETHE ANSATZ FOR $\mathcal{N}=4$ SYM AND $AdS_5 \times S^5$
- CUSP ANOMALY: NON BPS OBSERVABLE

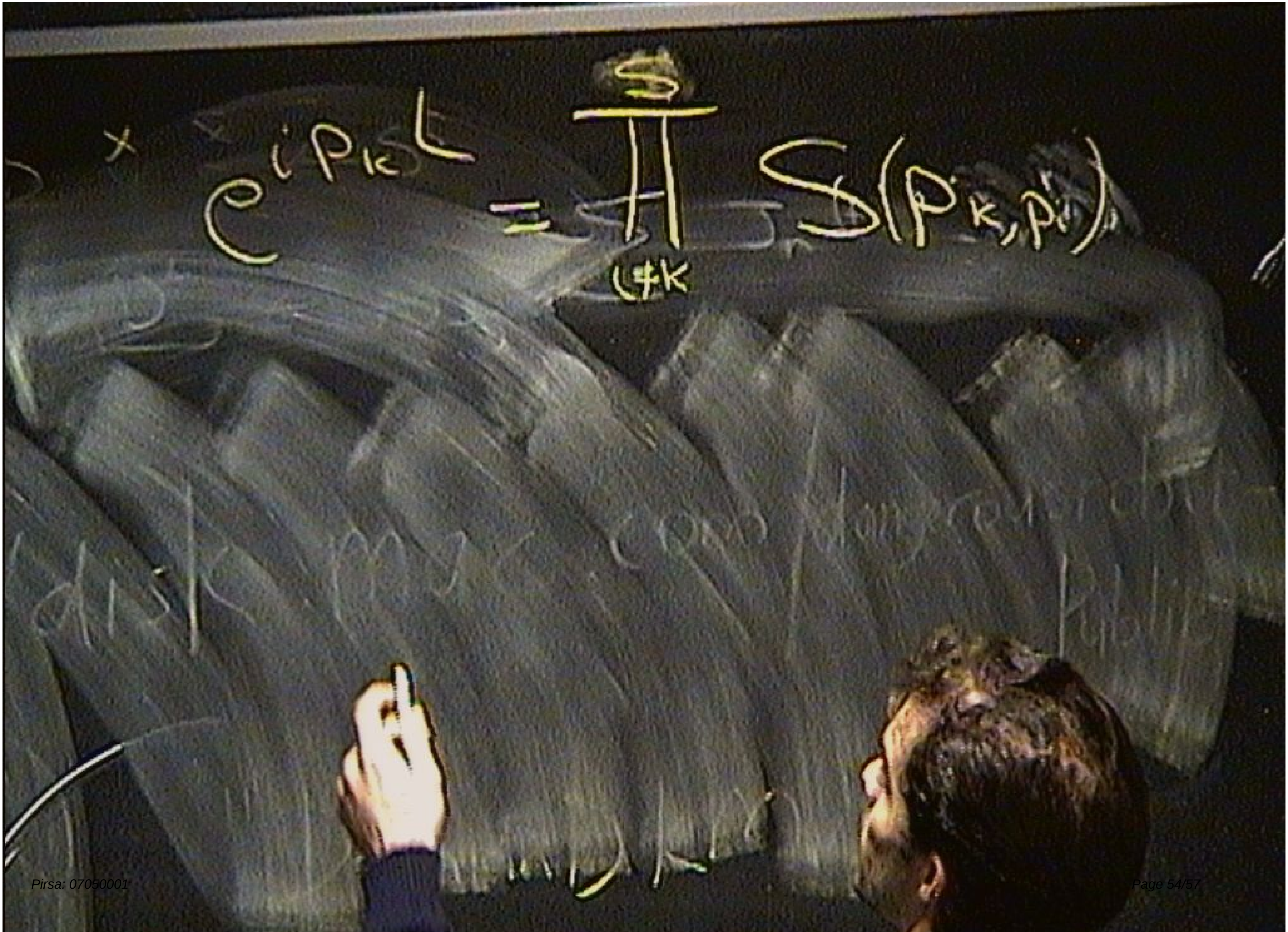
OPEN QUESTIONS

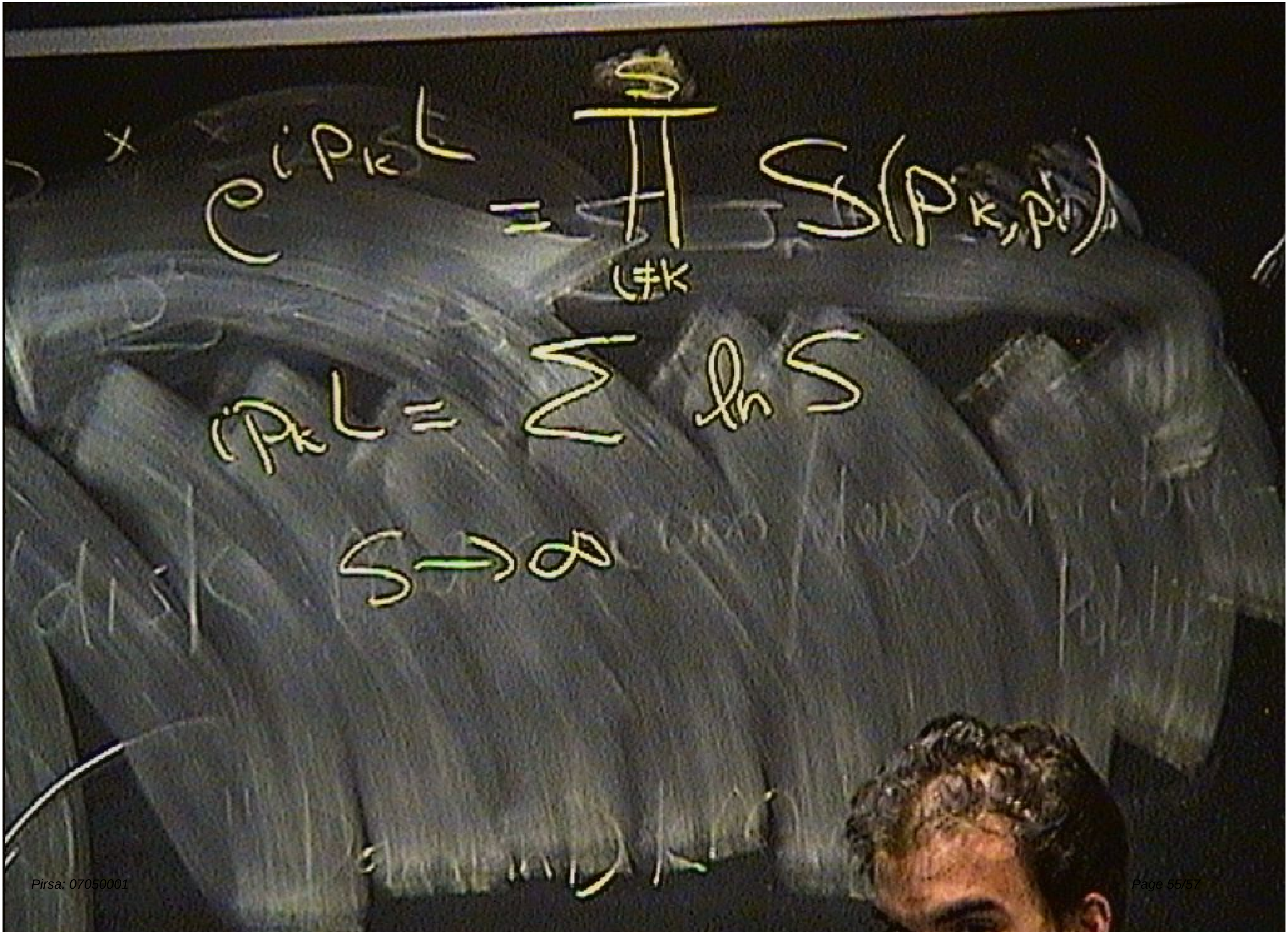
- FIND FURTHER EVIDENCE FOR THE BEHLS S-MATRIX
- STUDY ANALYTICAL STRUCTURE OF ANOMALOUS DIMENSIONS.
- FINITE-LENGTH EFFECTS: HAMILTONIAN?


$e^{ip_k L} = \prod_{l \neq k} S(p_k, p_l)$

$$e^{ip_k L} = \prod_{l \neq k}^{M} S(p_k, p_l)$$

$$e^{i p_k L} = \prod_{l \neq k}^{S} S(p_k, p_l)$$


$e^{ip_k L} = \prod_{l \neq k}^{S} S(p_k, p_l)$


$e^{ip_k L} = \prod_{l \neq k} S(p_k, p_l)$
$ip_k L = \sum \ln S$
$S \to \infty$

$$e^{ip_k L} = \prod_{l \neq k} S(p_k, p_l)$$

$$i p_k L = \sum \ln S$$

$$S \to \infty$$

SUMMARY

- SPECTRUM OF GAUGE THEORIES IN TERMS OF SPIN CHAINS
- BETHE ANSATZ FOR $\mathcal{N}=4$ SYM AND $AdS_5 \times S^5$
- CUSP ANOMALY: NON BPS OBSERVABLE

OPEN QUESTIONS

- FIND FURTHER EVIDENCE FOR THE BEHLS S-MATRIX
- STUDY ANALYTICAL STRUCTURE OF ANOMALOUS DIMENSIONS.
- FINITE-LENGTH EFFECTS: HAMILTONIAN?