

Title: DARK ENERGY AS A CURVATURE EFFECT IN NON-LINEAR THORIES OF GRAVITATION

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Abstract: Dark matter and dark energy can be explained without resorting to exotic fields if one accepts that the geometry of spacetime is governed by suitable generalized gravitational theories based on Lagrangians that are non-linear in the curvature of a metric and/or a torsionless linear connection, i.e. in second order and first order formalisms. A convenient choice of nonquadratic Lagrangians can fit well most of the astrophysical, cosmological and solar system requirements imposed by experimental results, without drastic modifications of Einstein field equations and with FRW Cosmologies preserved as a good approximation of Nature at a global scale.

DARK ENERGY

AS

A NON-LINEAR CURVATURE EFFECT OF EXTENDED GRAVITY

Perimeter Institute, Canada, April 24, 2007

CRM, Mac Gills University, Canada, April 26, 2007



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Is the Universe Accelerating?

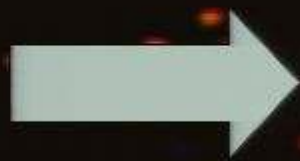


Is the Universe Accelerating?

- * Ia-type Supernovae: Perlmutter [1999], Riess [1998]
- * CMB: Spergel [2003], Bennett [2003], Melchiorri [2000]
- * Large-scale structure spectrum: Verde [2002]

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**Theoretical Models for
Cosmological acceleration**

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Standard Cosmological Model

[FRIEDMANN 1922]

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Stress-energy tensor of perfect fluid

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu}$$

- p : pressure
- ρ : density of matter
- u^{μ} : 4-velocity of co-moving fluid vector

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$$g = -dt^2 + a^2 \left[\frac{1}{1 - Kr^2} dr^2 + r^2 (d\theta^2 + \sin^2(\theta)d\varphi^2) \right]$$

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$$p = w\rho, \quad \rho = \eta a^{-3(1+w)}$$

- $w = -1$ vacuum
- $w = 0$ dust
- $w = \frac{1}{3}$ radiation

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$$\left\{ \begin{array}{l} q(t, w = 0, \frac{1}{3}) > 0 \\ w = w_{eff} > -1 \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{l} q^{exp} < 0 \\ w_{eff}^{exp} \in [-1.45, -0.74] \end{array} \right\}$$

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24 April 2007

Modified Cosmological Models

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$$G_{\mu\nu} = (8\pi G) T_{\mu\nu} \quad T_{\mu\nu} \Rightarrow \tilde{T}_{\mu\nu}$$


Dark Energy and Λ

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Dark Energy and Λ

- o Cosmological constant (Λ)
- o Time varying Λ
- o Scalar field theories
- o Phantom fields
- o Phenomenological Theories
- o Exotic matter

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$$G_{\mu\nu} \Rightarrow \tilde{G}_{\mu\nu} \quad \text{---} \quad G_{\mu\nu} = (8\pi G) T_{\mu\nu} \quad \text{---} \quad T_{\mu\nu} \Rightarrow \tilde{T}_{\mu\nu}$$

Alternative Gravitational
Theories

[Starobinsky, 1980], [Capozziello 2002], [Carroll 2003]

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WHY?

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- o *String/M-theory corrections*
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$R, R^{\mu\nu}R_{\mu\nu}, R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}, R \square^l R,$

Curvature invariants should be taken into account

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We do not suitably understand gravity at large scales

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24 April 2007

$f(R)$ Gravity

[Euchdahl, 1960]

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$$A = A_{\text{grav}}^f + A_{\text{mat}} = \int (\sqrt{g} f(R) + 2\kappa L_{\text{mat}}) d^4x$$

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Higher order Gravity (4th)!

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- **Instability problems of gravitational field** [Dolgov, Kawasaki, Phys. Lett. B573 (2003)]

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- Polynomial Lagrangian are realistic models [Odintsov & Nojiri, 2003]

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Palatini Formalism in $f(R)$ Gravity

[Vollick, Phys. Rev. D68, 2003]

[Flanagan, CQG 21, 2003]

[Meng & Wang, CQG 20, 2003]

Page 42/96

[A. B. F., Phys. Rev. D, 2004]

Palatini Formalism in $f(R)$ Gravity

$$\begin{cases} f'(R)R_{(\mu\nu)}(\Gamma) - \frac{1}{2}f(R)g_{\mu\nu} = \kappa T_{\mu\nu} & \longleftrightarrow \frac{\delta}{\delta g} \\ \nabla_{\alpha}^{\Gamma}(\sqrt{g}f'(R)g^{\mu\nu}) = 0 & \longleftrightarrow \frac{\delta}{\delta \Gamma} \end{cases}$$

**Field
equations**

[Vollick, Phys. Rev. D 68, 2003]

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Palatini Formalism in $f(R)$ Gravity

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[Vollick, Phys. Rev. D68, 2003]

[Flanagan, CQG 21, 2003]

[Meng & Wang, CQG 20, 2003]

Page 44/96

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[Buchdahl, 1960]

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$$A = A_{\text{grav}}^f + A_{\text{mat}} = \int (\sqrt{g} f(R) + 2\kappa L_{\text{mat}}) d^4x \quad \text{Action functional}$$

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Properties and problems:

- $f(R) = R + AR^{-1}$ $\left\{ \begin{array}{l} q(t) < 0 \quad [\text{Capozziello, 2002}] \\ w_{\text{eff}} < -1 \quad [\text{Carroll, 2003}] \end{array} \right.$
- Newtonian limit is recovered
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[Vollick, Phys. Rev. D68, 2003]

[Flanagan, CQG 21, 2003]

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Page 52/96

[A. B. F., Phys. Rev. D, 2004]

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Page 54/96

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[Koivisto 2005]
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$$f(R) = R - \frac{6\alpha}{\sinh(R)}$$

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Ricci-Squared Lagrangians

(Palatini formalism)

$$A = A_{\text{grav}} + A_{\text{mat}} = \int (\sqrt{\det g} f(S) + 2\kappa L_{\text{mat}}(\Psi)) d^4x \quad \begin{cases} S \equiv S(g, \Gamma) = g^{\mu\alpha} R_{(\alpha\nu)}(\Gamma) g^{\nu\beta} R_{(\beta\mu)}(\Gamma) \\ R_{\mu\nu} = R_{\mu\nu}(\Gamma) \end{cases}$$

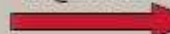
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ric structure

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$$\left\{ \begin{array}{l} 2f'(S)g^{\alpha\beta}R_{(\mu\alpha)}(\Gamma)R_{(\beta\nu)}(\Gamma) - \frac{1}{2}f(S)g_{\mu\nu} = \kappa T_{\mu\nu} \\ \nabla_{\sigma}^{\Gamma}(\sqrt{\det g} f'(S)g^{\mu\alpha}R_{(\alpha\beta)}(\Gamma)g^{\beta\nu}) = 0 \end{array} \right.$$

• Action functional

• Bi-metric structure

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Ricci-Squared Lagrangians

(Palatini formalism)

$$A = A_{\text{grav}} + A_{\text{mat}} = \int (\sqrt{\det g} f(S) + 2\kappa L_{\text{mat}}(\Psi)) d^4x$$

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structural equation

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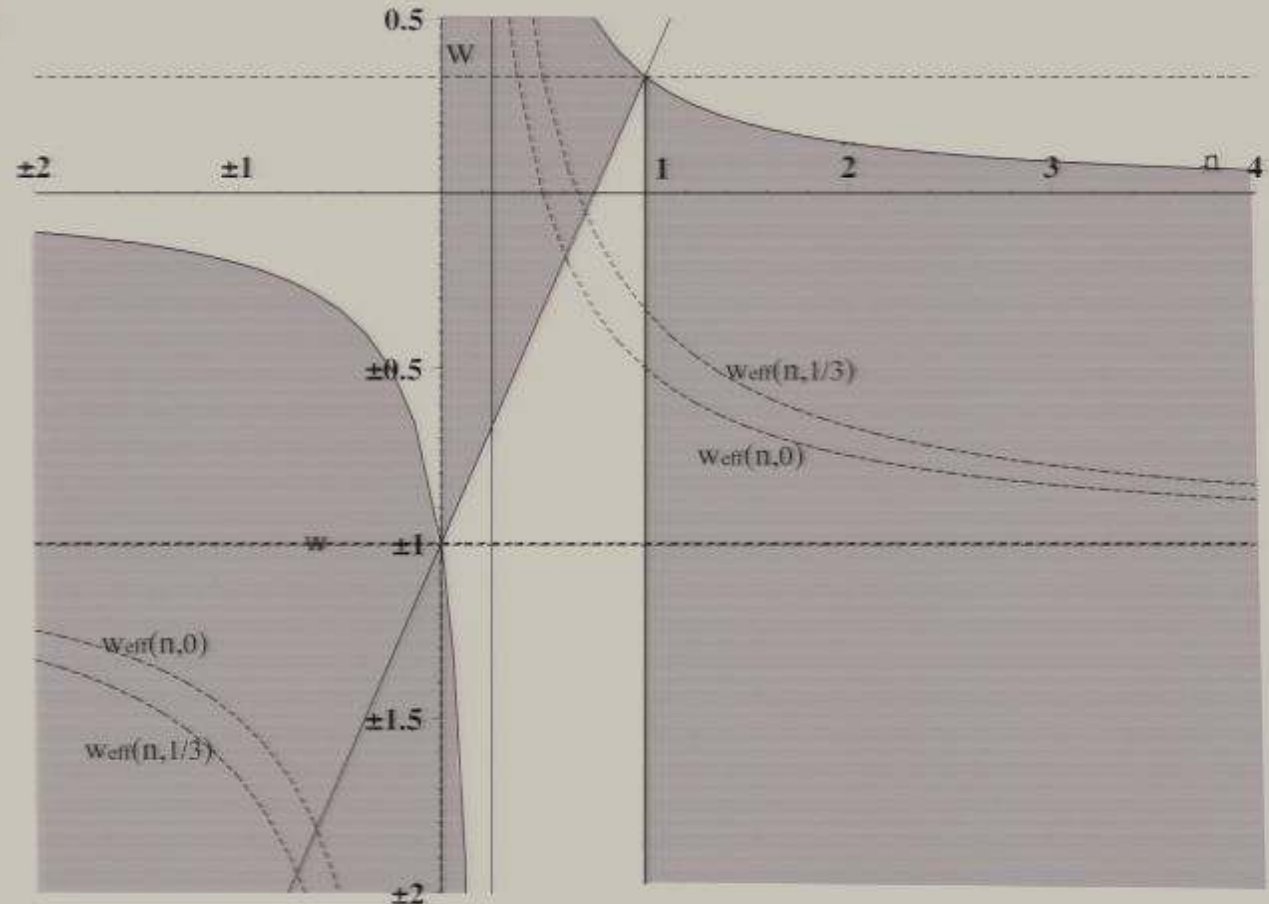
$$\hat{H}^2 = \epsilon_0 \left[\frac{f(\tau) + \kappa\tau + 2\kappa\rho}{6\sqrt{f'(\tau)[f(\tau) - 2\kappa\rho]}} - \epsilon_1 \frac{Kc}{a^2} \right]$$

Cosmological Models

- Pure power Lagrangians

$$f(S) = \beta S^n$$

$$\begin{cases} b(t) \simeq a^{-3(1+w)(1-\frac{1}{2n})} \\ c(t) = \sqrt{\frac{4wn-w-1}{3w+3-4n}} \end{cases}$$

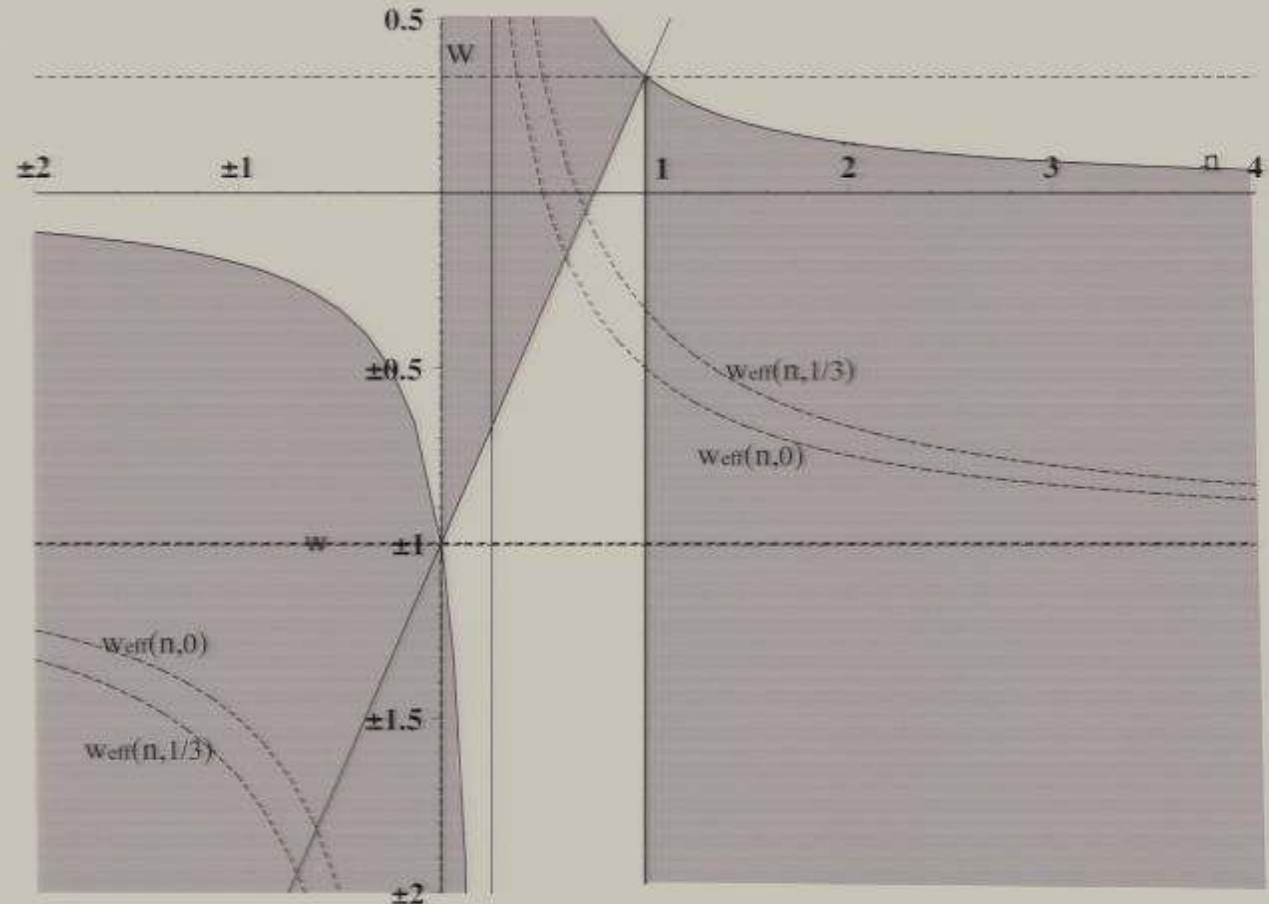


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- MFE and cosmological parameters

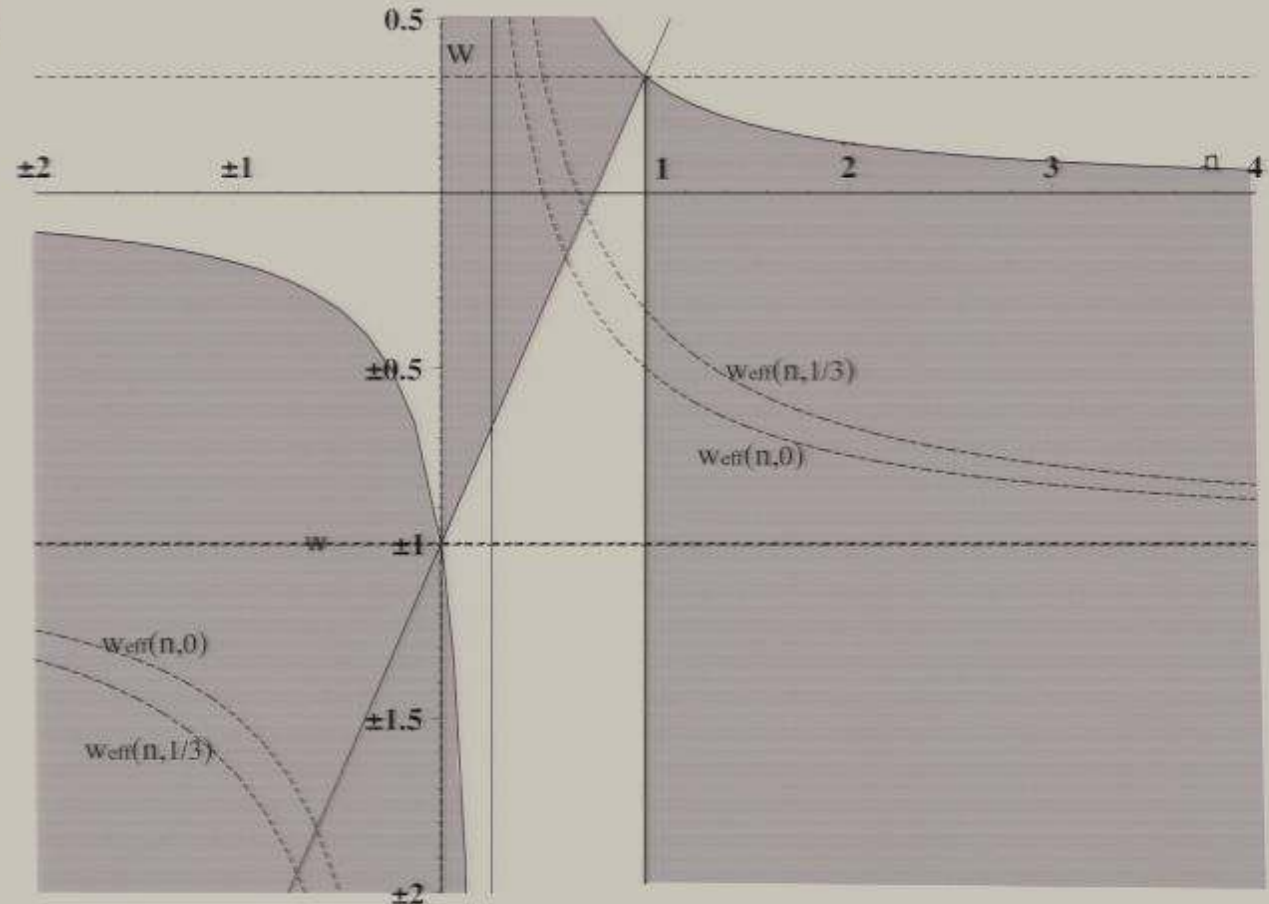
$$H_{\text{Pirsa: 07040028}}^2 = \left(\frac{\dot{a}}{a}\right)^2 = \epsilon_0 P(n, w) \eta^{\frac{1}{2n}} a^{-\frac{3(1+w)}{2n}} - \epsilon_0 \epsilon_1 Q(n, w) K a^{-2}$$

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Conformal Transformations

- Scalar tensor theories (NMC)

$$A = \int d^4x \sqrt{-g} \left[F(\phi)R + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi) \right]$$

$$\bar{g}_{\mu\nu} = e^{2\omega} F g_{\mu\nu} = -2F g_{\mu\nu} \quad \leftarrow \text{Scalar factor}$$

$$\sqrt{-g} \left(FR + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V \right) = \sqrt{-\bar{g}} \left(-\frac{1}{2}\bar{R} + \frac{1}{2}\bar{\phi}_{;\alpha}\bar{\phi}_{;\alpha} - \bar{V} \right) \quad \text{Einstein frame}$$

- Alternative theories of Gravity (FOG)

$$A = \int d^4x \sqrt{-g} f(R)$$

$$\omega = \frac{1}{2} \ln |f'(R)| \quad \leftarrow \text{Scalar factor}$$

$$\sqrt{-g} f(R) = \sqrt{-\bar{g}} \left(-\frac{1}{2}\bar{R} + \frac{1}{2}\bar{\phi}_{;\alpha}\bar{\phi}_{;\alpha} - \bar{V} \right) \quad \text{Einstein frame}$$



Palatini Formalism and Conformal Transformations

- Palatini transformations for non minimally coupled theories

$$A_3 = \int \sqrt{-g} [K(\phi, R) + \frac{\varepsilon}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) + \kappa L_{\text{mat}}(\Psi, \nabla \Psi)] d^4x$$



Bi-metric struct.

$$h_{\mu\nu} = \frac{\partial K(\phi, R)}{\partial R} g_{\mu\nu}$$

FIELD EQUATIONS

$$\begin{cases} \left[\frac{\partial K(\phi, R)}{\partial R} \right] R_{(\mu\nu)} - \frac{1}{2} K(\phi, R) g_{\mu\nu} = \kappa [T_{\mu\nu}^\phi + T_{\mu\nu}^{\text{mat}}] \\ \nabla_\alpha^\Gamma \left(\sqrt{-g} \left[\frac{\partial K(\phi, R)}{\partial R} \right] g^{\mu\nu} \right) = 0 \end{cases}$$

R → 0 Limit

$$h_{\mu\nu} = K_1(\phi) g_{\mu\nu}$$

- Einstein frame for cosmological applications: FRW metric

$$L_t = 6F(\phi) a \dot{a}^2 + 6F_\phi(\phi) a^2 \dot{a} \dot{\phi} - 6F(\phi) a K + \frac{1}{2} a^3 \dot{\phi}^2 - a^3 V(\phi).$$

$$\begin{cases} \bar{a} = \sqrt{-2F(\phi)} a \\ \frac{d\bar{\phi}}{d\bar{t}} = \sqrt{\frac{3F_\phi^2 - F}{2F^2}} \frac{d\phi}{dt} \\ d\bar{t} = \sqrt{-2F(\phi)} dt. \end{cases}$$



Intrinsically defined in the Palatini formalism



$$\frac{1}{\sqrt{-2F(\phi)}} L_t = -3\bar{a} \dot{\bar{a}}^2 + 3K\bar{a} + \frac{1}{2} \bar{a}^3 \dot{\bar{\phi}}^2 - \bar{a}^3 \bar{V}(\bar{\phi}) = \bar{L}_t$$

Conclusions

- Alternative gravitational theories can provide an explanation to current acceleration of universe and other effects.
- Cosmological parameters are in accordance (fitting) with the experimental results (also for rotation curves of galaxies).
- Palatini formalism prevents from instability problems and has a deeper insight in the geometry of spacetime.

Problems

Conclusions

- Alternative gravitational theories can provide an explanation to current acceleration of universe and other effects.
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Problems

- Why this contributions are so important at the present epoch.
- Dark energy models are still necessary?
- This models really come from QG?

References

1. Accelerated cosmological models in first-order nonlinear gravity

Gianluca Allemandi, Andrzej Borowiec, Mauro Francaviglia

Phys. Rev. D **70** (4) 043524 (2004) - [hep-th/0403264](#)

2. Accelerated Cosmological Models in Ricci squared Gravity

Gianluca Allemandi, Andrzej Borowiec, Mauro Francaviglia

Phys. Rev. **70** (10) 103503 (2004) - [hep-th/0407090](#)

3. Conformal Aspects of Palatini Approach in Extended Theories of Gravity

Gianluca Allemandi, Monica Capone, Salvatore Capozziello, Mauro Francaviglia

Journ. Gen. Rel. & Grav. **38** (1) 33-60 (2006) - [hep-th/0409198](#)

4. Dark Energy Dominance and Cosmic Acceleration in First Order Gravity

Gianluca Allemandi, Andrzej Borowiec, Mauro Francaviglia, Sergei Odintsov

Phys. Rev. **71** (9) 063505 (2005)

5. Post-Newtonian Parameters from Alternative Theories of Gravity

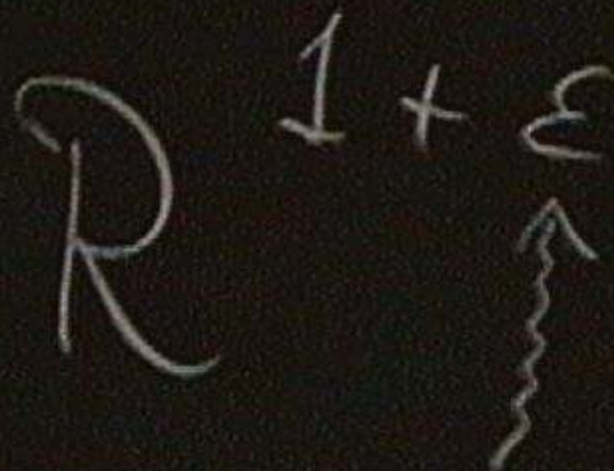
Gianluca Allemandi, Mauro Francaviglia, Matteo Luca Ruggiero, Angelo Tartaglia

Journ. Gen. Rel. & Grav. **37** (1) 1891-1904 (2005) - [gr-qc/0506123](#)

6. F(R) Theories of Gravity in Palatini Approach Matched with Observations

Vincenzo Cardone, Salvatore Capozziello, Mauro Francaviglia

Journ. Gen. Rel. & Grav. **38** (2) 1-24 (2006)

$$R^{1+\varepsilon}$$


$$R^{1+\varepsilon}$$

$$R + \frac{\kappa}{R}$$

$$R \cdot R^\varepsilon =$$

$$= R e^{\varepsilon \log R}$$

$$\approx R \log R$$

$$R^{1+\varepsilon}$$

$$R + \frac{\varepsilon}{R}$$

$$R \cdot R^\varepsilon = \varepsilon \log R$$

134-10

$$R \log R$$

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$$R \cdot R^\varepsilon =$$

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134-70

$$\left(R e^{\varepsilon \log R} + o(m^2) \right)$$