Title: DARK ENERGY AS A CURVATURE EFFECT IN NON-LINEAR THORIES OF GRAVITATION

Date: Apr 24, 2007 11:00 AM

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Abstract: Dark matter and dark energy can be explained without resorting to exotic fields if one accepts that the geometry of spacetime is governed by suitable generalized gravitational theories based on Lagrangians that are non-linear in the curvature of a metric and/or a torsionless linear connection, i.e. in second order and first order formalisms. A convenient choice of nonquadratic Lagrangians can fit well most of the astrophysical, cosmological and solar system requirements imposed by experimental results, without drastic modifications of Einstein field equations and with FRW Cosmologies preserved as a good approximation of Nature at a global scale.

DARK ENERGY

AS

A NON-LINEAR CURVATURE EFFECT

OF EXTENDED GRAVITY

Perimeter Institute, Canada, April 24, 2007

CRM, Mac Gills University, Canada, April 26, 2007

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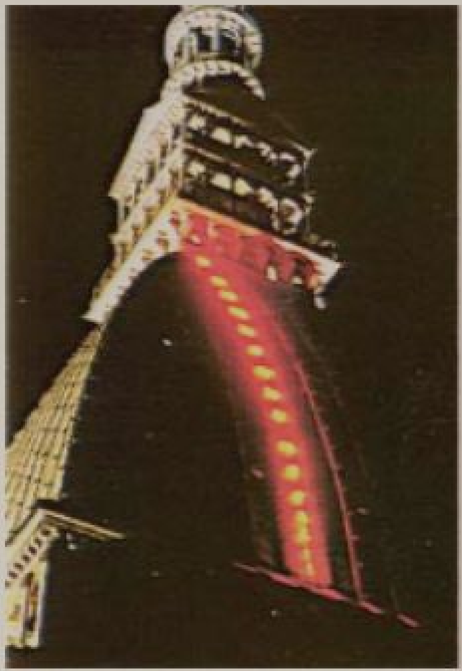

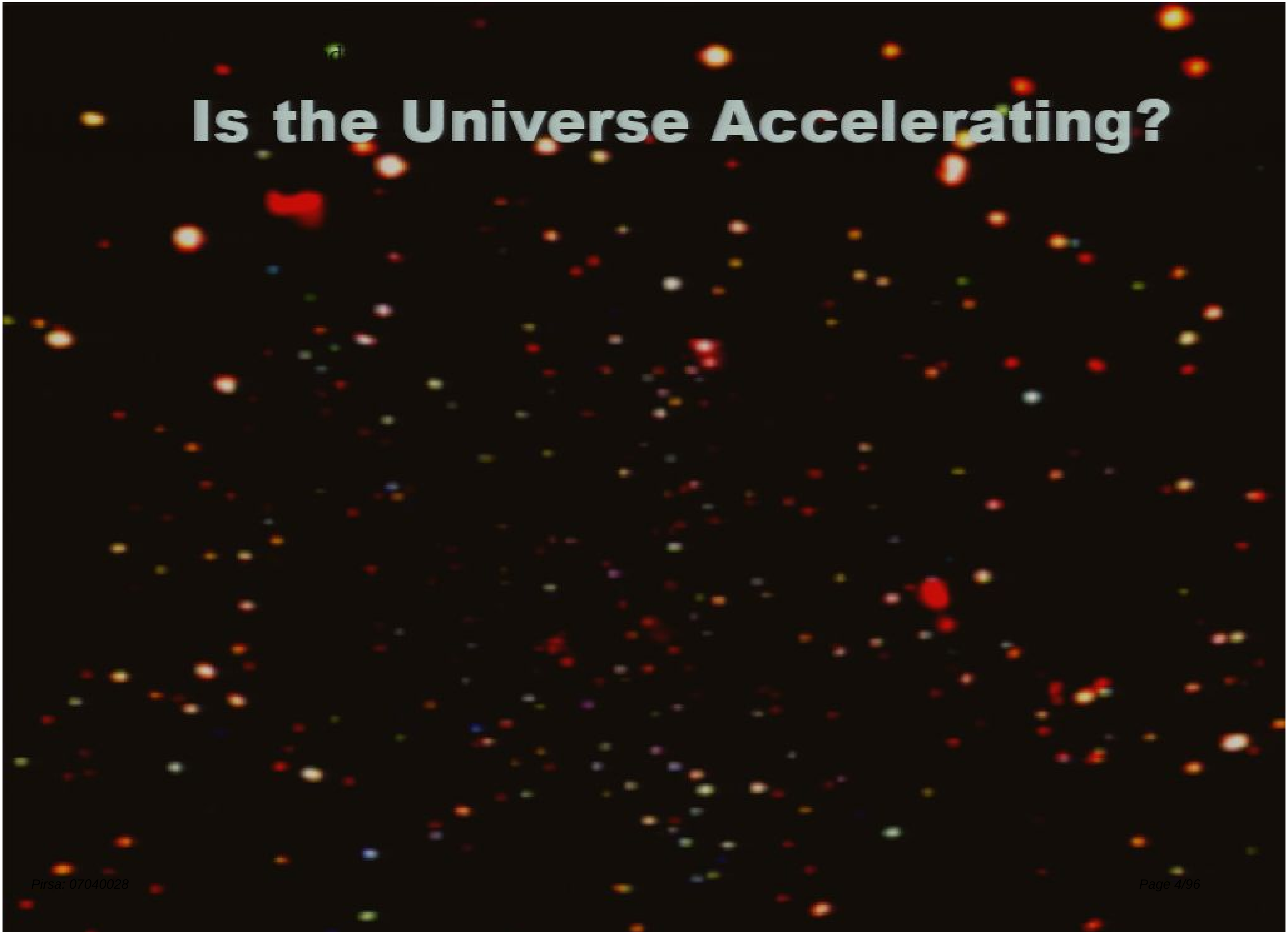
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- Ia-type Supernovae: Perlmutter [1999], Riess [1998]
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→ **Theoretical Models for Cosmological acceleration**

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24 April 2007

Standard Cosmological Model

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$$g = -dt^2 + a^2 \left[\frac{1}{1 - Kr^2} dr^2 + r^2 \left(d\theta^2 + \sin^2(\theta) d\varphi^2 \right) \right]$$

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- $w = -1$ vacuum
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$q(t, w = 0, \frac{1}{3}) > 0$
$w = w_{eff} > -1$

↔

$q^{exp} < 0$
$w^{exp}_{eff} \in [-1.45, -0.74]$

[STANDARD MODEL] [Spergel 2003, Perlmutter 1999]

Modified Cosmological Models

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$$G_{\mu\nu} = (8\pi G)\, T_{\mu\nu}$$

$$T_{\mu\nu} \Rightarrow \tilde{T}_{\mu\nu}$$

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Curvature invariants should be taken into account

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We do not suitably understand gravity at large scales

f(R) Gravity

[Buchdahl, 1960]

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$$A = A^f_{\mathrm{grav}} + A_{\mathrm{mat}} = \int (\sqrt{g} f(R) + 2\kappa L_{\mathrm{mat}}) d^4x$$

Action functional

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Higher order Gravity (4th)!

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- Polynomial Lagrangian are realistic models [Odintsov & Nojiri, 2003]
- Possible cut-off [Carroll, 2004]

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Palatini Formalism in f(R) Gravity

[Vollick,Phys.Rev.D68, 2003]

[Flanagan,CQG 21, 2003]

[Meng&Wang,CQG 20, 2003]

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f(R) Gravity

[Buchdahl, 1960]

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Action functional

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Properties and problems:

- $f(R)=R+AR^{-1}$ $\begin{cases} q(t)<0 & \text{[Capozziello, 2002]} \\ w_{eff}<-1 & \text{[Carroll, 2003]} \end{cases}$
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- Possible cut-off

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$$P_\mu^\nu = \frac{1}{2}\frac{f(R)}{f'(R)}\delta_\mu^\nu + \frac{\kappa}{f'(R)}g^{\nu\alpha}T_{\alpha\mu}$$

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- Second order field equations;
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- Nice accordance with experimental Data (Acceleration of the Universe);
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- Cosmological perturbations and formation of large scale inomogeneities; [Koivisto 2005]
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It is worth to continue investigating Palatini formalism

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- Pure powers and polynomial Lagrangians

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$$f(R) = R - \frac{6\alpha}{\sinh(R)}$$

$$\longrightarrow \begin{cases} H^2 \simeq \tau^{-1} \\ w_{eff} = -2 - w \end{cases}$$

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Ricci-Squared Lagrangians

(Palatini formalism)

$$A = A_{\text{grav}} + A_{\text{mat}} = \int (\sqrt{\det g} f(S) + 2\kappa L_{\text{mat}}(\Psi)) d^4x \quad \begin{cases} S \equiv S(g, \Gamma) = g^{\mu\alpha} R_{(\alpha\nu)}(\Gamma) g^{\nu\beta} R_{(\beta\mu)}(\Gamma) \\ R_{\mu\nu} = R_{\mu\nu}(\Gamma) \end{cases}$$

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$$A = A_{\text{grav}} + A_{\text{mat}} = \int (\sqrt{\det g} f(S) + 2\kappa L_{\text{mat}}(\Psi)) d^4x$$

- Action functional

FIELD EQUATIONS

$$\begin{cases} 2f'(S) g^{\alpha\beta} R_{(\mu\alpha)}(\Gamma) R_{(\beta\nu)}(\Gamma) - \frac{1}{2} f(S) g_{\mu\nu} = \kappa T_{\mu\nu} \\ \nabla_\sigma^\Gamma (\sqrt{\det g} f'(S) g^{\mu\alpha} R_{(\alpha\beta)}(\Gamma) g^{\beta\nu}) = 0 \end{cases}$$

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$$h_{\mu\nu} = b(t) \text{Diag}\left(-\epsilon_0 c(t), \frac{\epsilon_1 a^2}{1 - Kr^2}, \epsilon_2 r^2 a^2, \epsilon_3 r^2 a^2 \sin^2(\theta)\right)$$

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$$\hat{H}^2 = \epsilon_0 \left[\frac{f(\tau) + \kappa\tau + 2\kappa\rho}{6\sqrt{f'(\tau)[f(\tau) - 2\kappa\rho]}} - \epsilon_1 \frac{Kc}{a^2}\right]$$

Signature Changing

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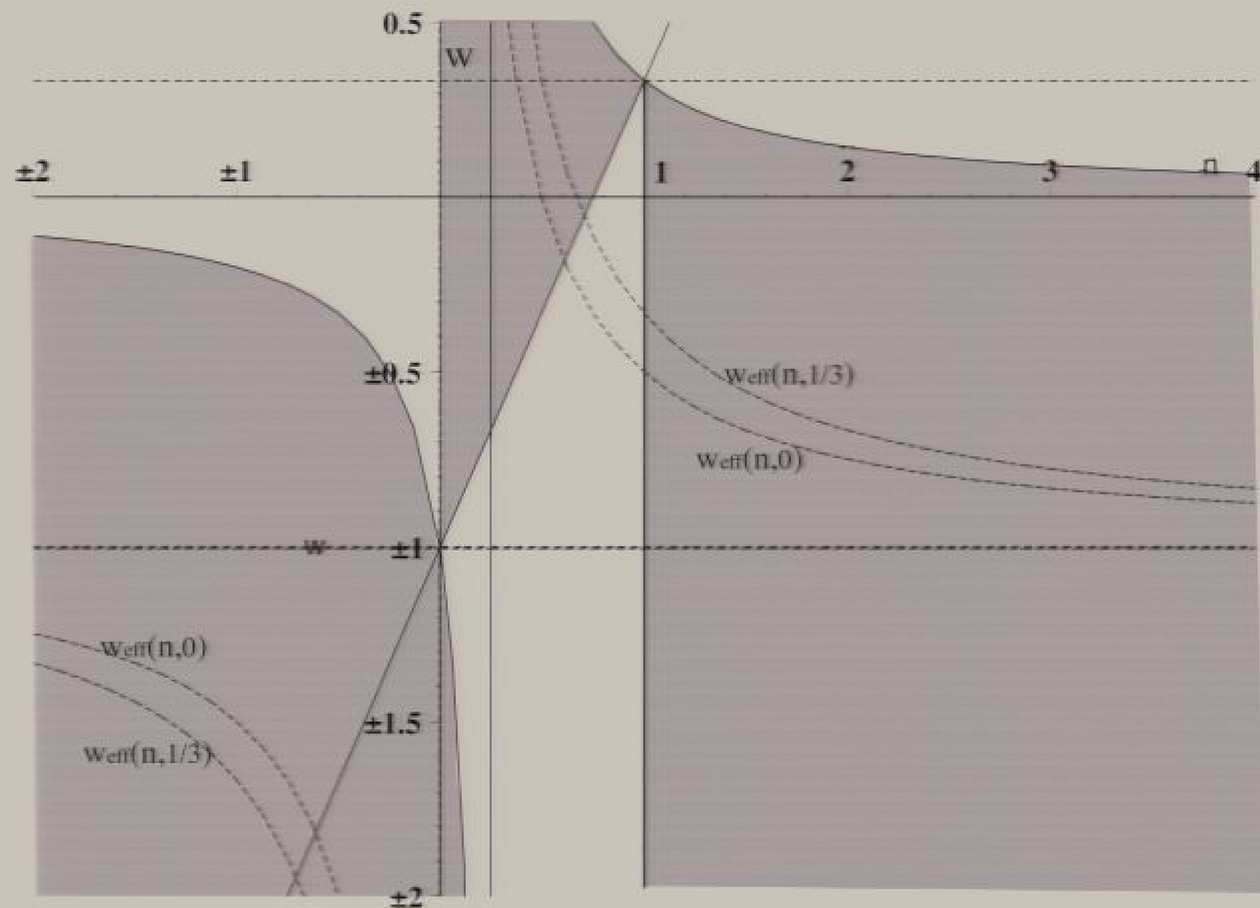
Cosmological Models

- Pure power Lagrangians

$$f(S) = \beta S^n$$

$$\begin{cases} b(t) \simeq a^{-3(1+w)\left(1-\frac{1}{2n}\right)} \\ c(t) = \sqrt{\frac{4wn-w-1}{3w+3-4n}} \end{cases}$$



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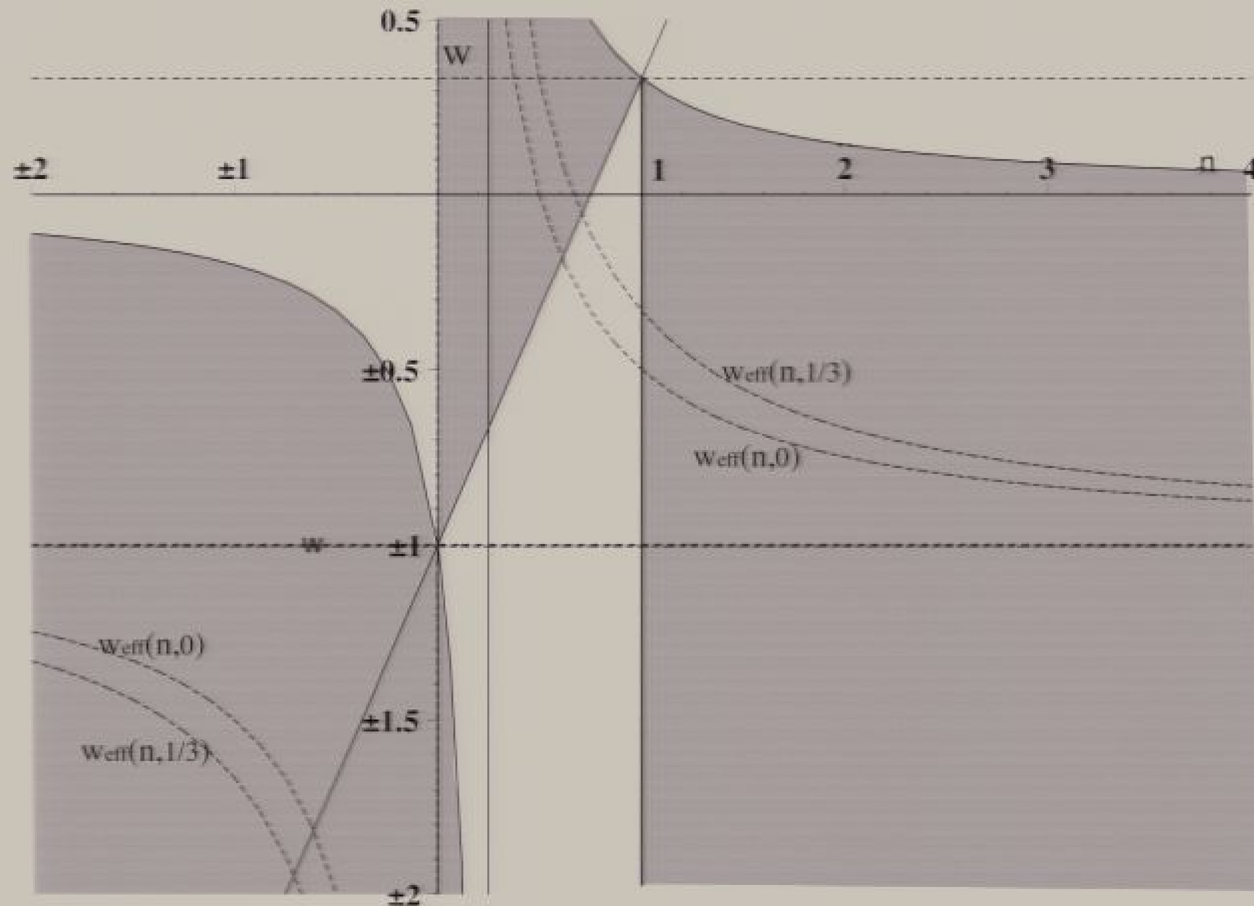

- MFE and cosmological parameters

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \epsilon_0 P(n,w)\eta^{\frac{1}{2n}} a^{-\frac{3(1+w)}{2n}} - \epsilon_0\epsilon_1 Q(n,w) K a^{-2}$$

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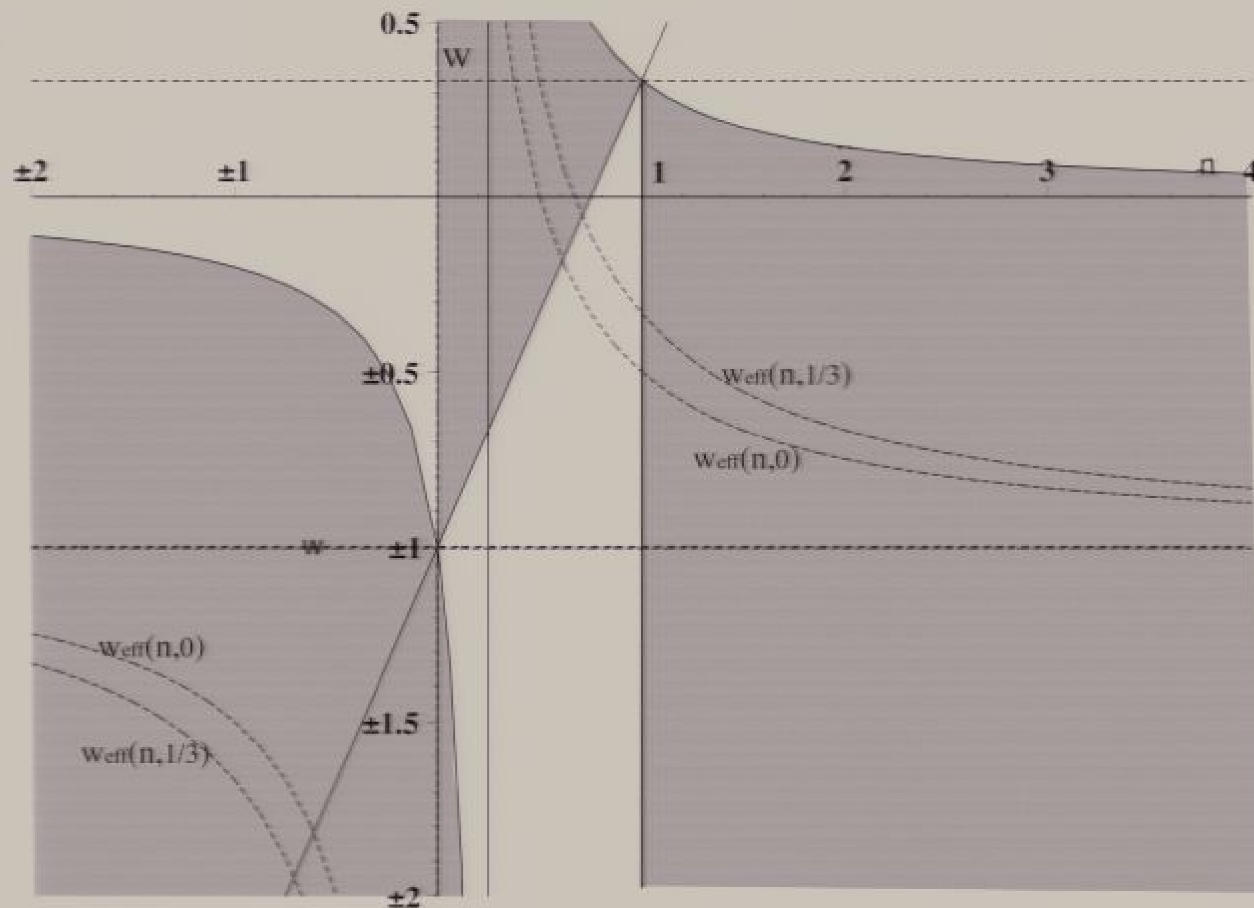

- MFE and cosmological parameters

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Conformal Transformations

- Scalar tensor theories (NMC)

$$\mathcal{A} = \int d^4x \sqrt{-g}\left[F(\phi)R + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi)\right]$$

$$\bar{g}_{\mu\nu} = e^{2\omega}Fg_{\mu\nu} = -2Fg_{\mu\nu}$$ ← **Scalar factor**

$$\sqrt{-g}\left(FR + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V\right) = \sqrt{-\bar{g}}\left(-\frac{1}{2}\bar{R} + \frac{1}{2}\bar{\phi}_{;\alpha}\bar{\phi}^{\alpha}_{;} - \bar{V}\right)$$ **Einstein frame**

- Alternative theories of Gravity (FOG)

$$\mathcal{A} = \int d^4x \sqrt{-g}f(R).$$

$\omega = \frac{1}{2}\ln|f'(R)|$ ← **Scalar factor**

$$\sqrt{-g}f(R) = \sqrt{-\bar{g}}\left(-\frac{1}{2}\bar{R} + \frac{1}{2}\bar{\phi}_{;\alpha}\bar{\phi}^{\alpha}_{;} - \bar{V}\right)$$ **Einstein frame**

$$\begin{array}{ccccc}
\mathcal{L}_{FOG} & \longleftrightarrow & \mathcal{L}_{NMC} & \longleftrightarrow & \mathcal{L}_{HE} \\
\updownarrow & & \updownarrow & & \updownarrow \\
\text{FOG Eqs.} & \longleftrightarrow & \text{NMC Eqs.} & \longleftrightarrow & \text{Einstein Eqs.} \\
\updownarrow & & \updownarrow & & \updownarrow \\
\text{FOG Solutions} & \longleftrightarrow & \text{NMC Solutions} & \longleftrightarrow & \text{Einstein Solutions}
\end{array}$$

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Palatini Formalism and Conformal Transformations

- Palatini transformations for non minimally coupled theories

$$A_3 = \int \sqrt{-g}\ [K(\phi, R) + \frac{\varepsilon}{2} \overset{g}{\nabla}_\mu \phi \overset{g}{\nabla}{}^\mu \phi - V(\phi) + \kappa L_{\mathrm{mat}}(\Psi, \overset{g}{\nabla} \Psi)] d^4x$$

Bi-metric struct.

$$h_{\mu\nu} = \frac{\partial K(\phi, R)}{\partial R} g_{\mu\nu}$$

FIELD EQUATIONS

$$\begin{cases} \left[\frac{\partial K(\phi,R)}{\partial R}\right] R_{(\mu\nu)} - \frac{1}{2} K(\phi, R) g_{\mu\nu} = \kappa [T^{\phi}_{\mu\nu} + T^{\mathrm{mat}}_{\mu\nu}] \\ \nabla^{\Gamma}_{\alpha} \left(\sqrt{-g} \left[\frac{\partial K(\phi,R)}{\partial R}\right] g^{\mu\nu}\right) = 0 \end{cases}$$

R ➔ 0 Limit

$$h_{\mu\nu} = K_1(\phi) g_{\mu\nu}$$

- Einstein frame for cosmological applications: FRW metric

$$L_t = 6F(\phi) a\dot{a}^2 + 6F_\phi(\phi) a^2 \dot{a}\dot{\phi} - 6F(\phi) aK + \frac{1}{2} a^3 \dot{\phi}^2 - a^3 V(\phi).$$

$$\begin{cases} \bar{a} = \sqrt{-2F(\phi)}\, a \\ \frac{d\bar{\phi}}{dt} = \sqrt{\frac{3F_\phi{}^2 - F}{2F^2}} \frac{d\phi}{dt} \\ d\bar{t} = \sqrt{-2F(\phi)}\, dt. \end{cases}$$

Intrinsically defined in the Palatini formalism

$$\frac{1}{\quad} L_t = -3\bar{a}\dot{\bar{a}}^2 + 3K\bar{a} + \frac{1}{2}\bar{a}^3 \dot{\bar{\phi}}^2 - \bar{a}^3 \bar{V}(\bar{\phi}) = \bar{L}_t$$

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Conclusions

- Alternative gravitational theories can provide an explanation to current acceleration of universe and other effects.
- Cosmological parameters are in accordance (fitting) with the experimental results (also for rotation curves of galaxies).
- Palatini formalism prevents from instability problems and has a deeper insight in the geometry of spacetime.

Problems

Conclusions

- Alternative gravitational theories can provide an explanation to current acceleration of universe and other effects.
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Problems

- Why this contributions are so important at the present epoch.
- Dark energy models are still necessary?
- This models really come from QG?

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References

1. *Accelerated cosmological models in first-order nonlinear gravity*
Gianluca Allemandi, Andrzej Borowiec, Mauro Francaviglia
Phys. Rev. D 70 (4) 043524 (2004) - hep-th/0403264

2. *Accelerated Cosmological Models in Ricci squared Gravity*
Gianluca Allemandi, Andrzej Borowiec, Mauro Francaviglia
Phys. Rev. 70 (10) 103503 (2004) - hep-th/0407090

3. *Conformal Aspects of Palatini Approach in Extended Theories of Gravity*
Gianluca Allemandi, Monica Capone, Salvatore Capozziello, Mauro Francaviglia
Journ. Gen. Rel. & Grav.38 (1) 33-60 (2006) - hep-th/0409198

4. *Dark Energy Dominance and Cosmic Acceleration in First Order Gravity*
Gianluca Allemandi, Andrzej Borowiec, Mauro Francaviglia, Sergei Odintsov
Phys. Rev. 71 (9) 063505 (2005)

5. *Post-Newtonian Parameters from Alternative Theories of Gravity*
Gianluca Allemandi, Mauro Francaviglia, Matteo Luca Ruggiero, Angelo Tartaglia
Journ. Gen. Rel. & Grav.37 (1) 1891-1904 (2005) - gr-qc/0506123

6. *F(R) Theories of Gravity in Palatini Approach Matched with Observations*
Vincenzo Cardone, Salvatore Capozziello, Mauro Francaviglia
Journ. Gen. Rel. & Grav.38 (2) 1-24 (2006)

$$R^{1+\varepsilon}$$

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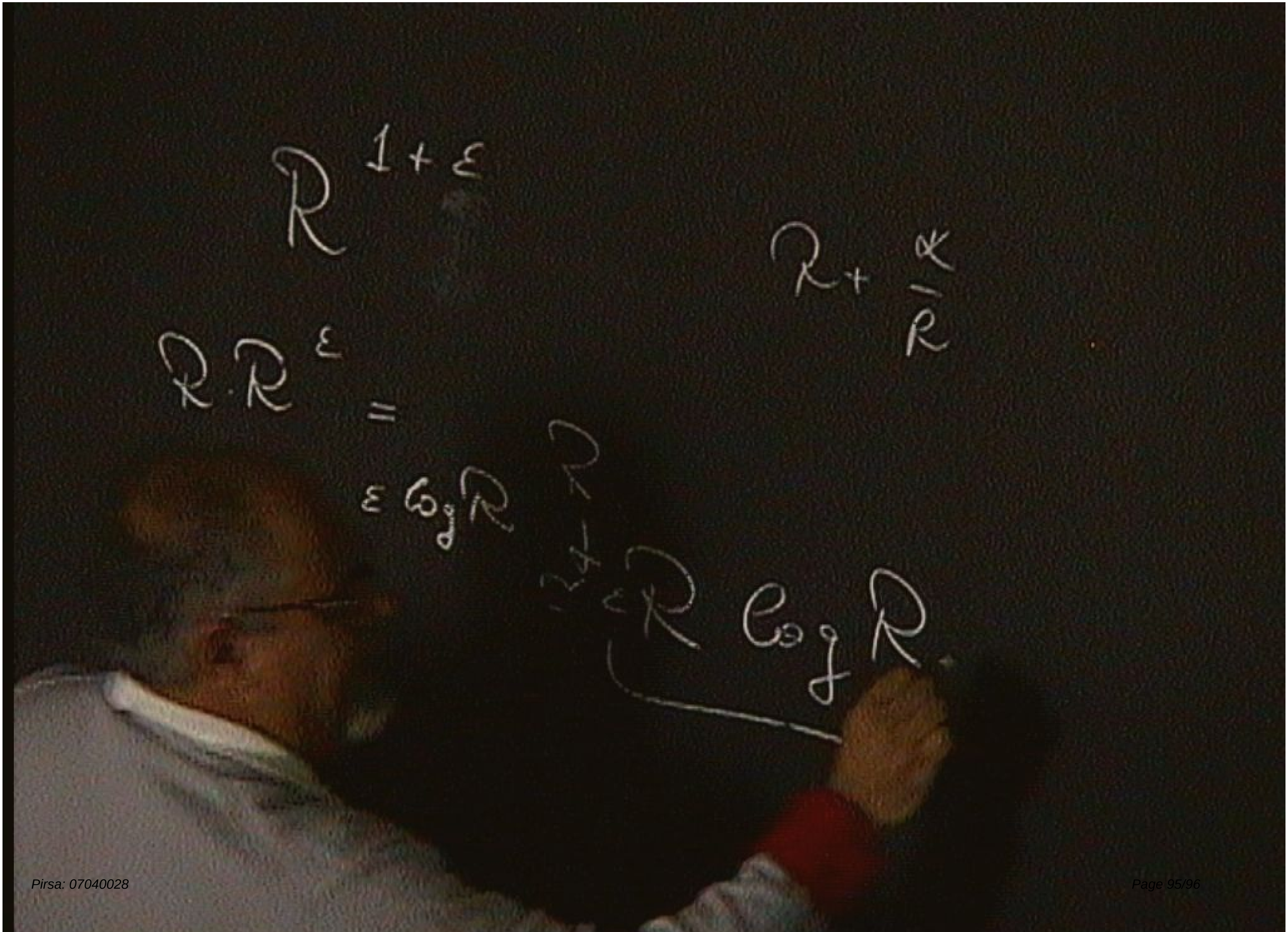
$$R + \frac{\alpha}{R}$$

$$R \cdot R^{\varepsilon} =$$

$$= R\, e^{\varepsilon \log R} \simeq \varepsilon R \log R$$


R 1+ε
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