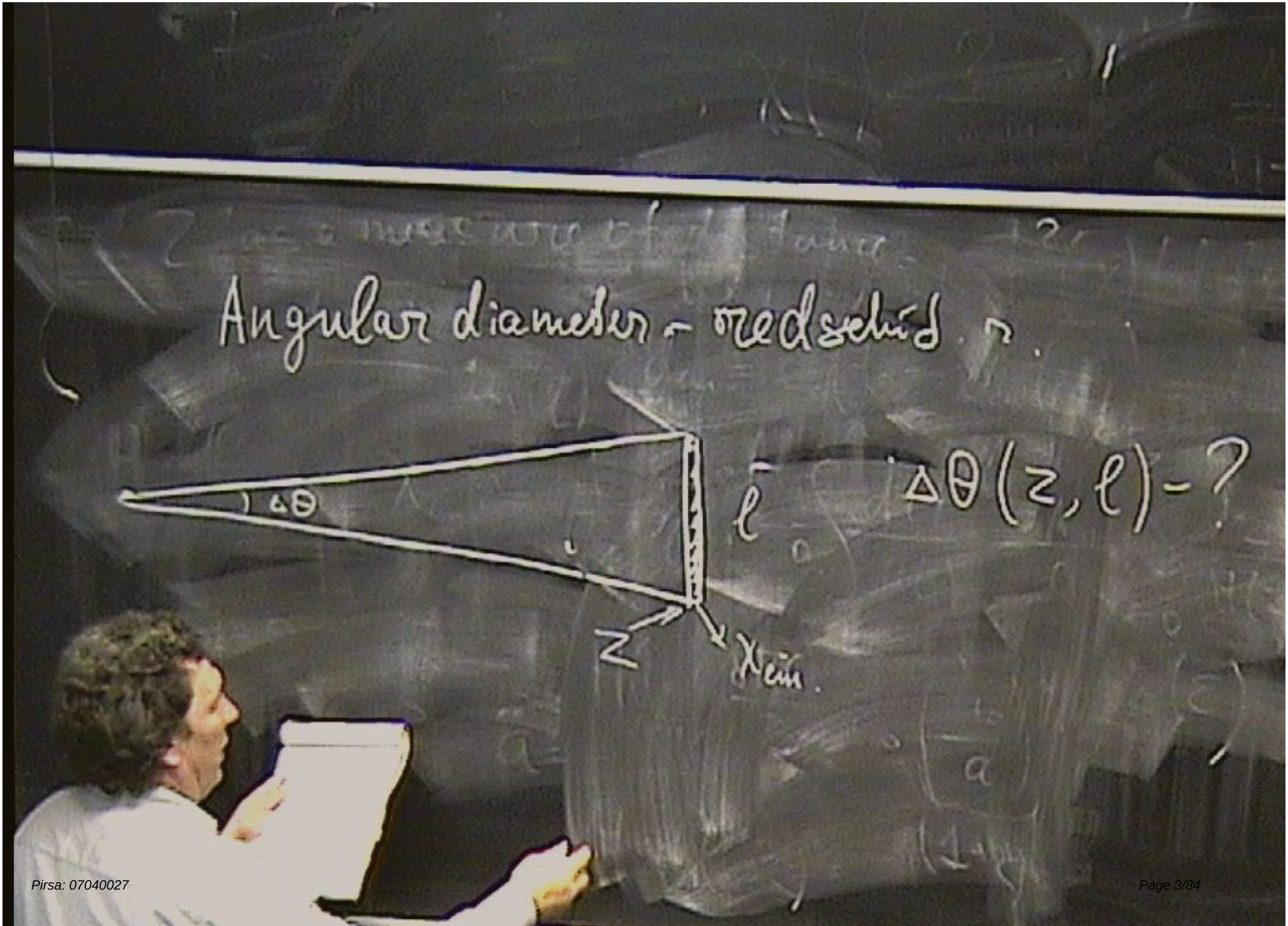
Title: Advanced Topics in Cosmology 2B

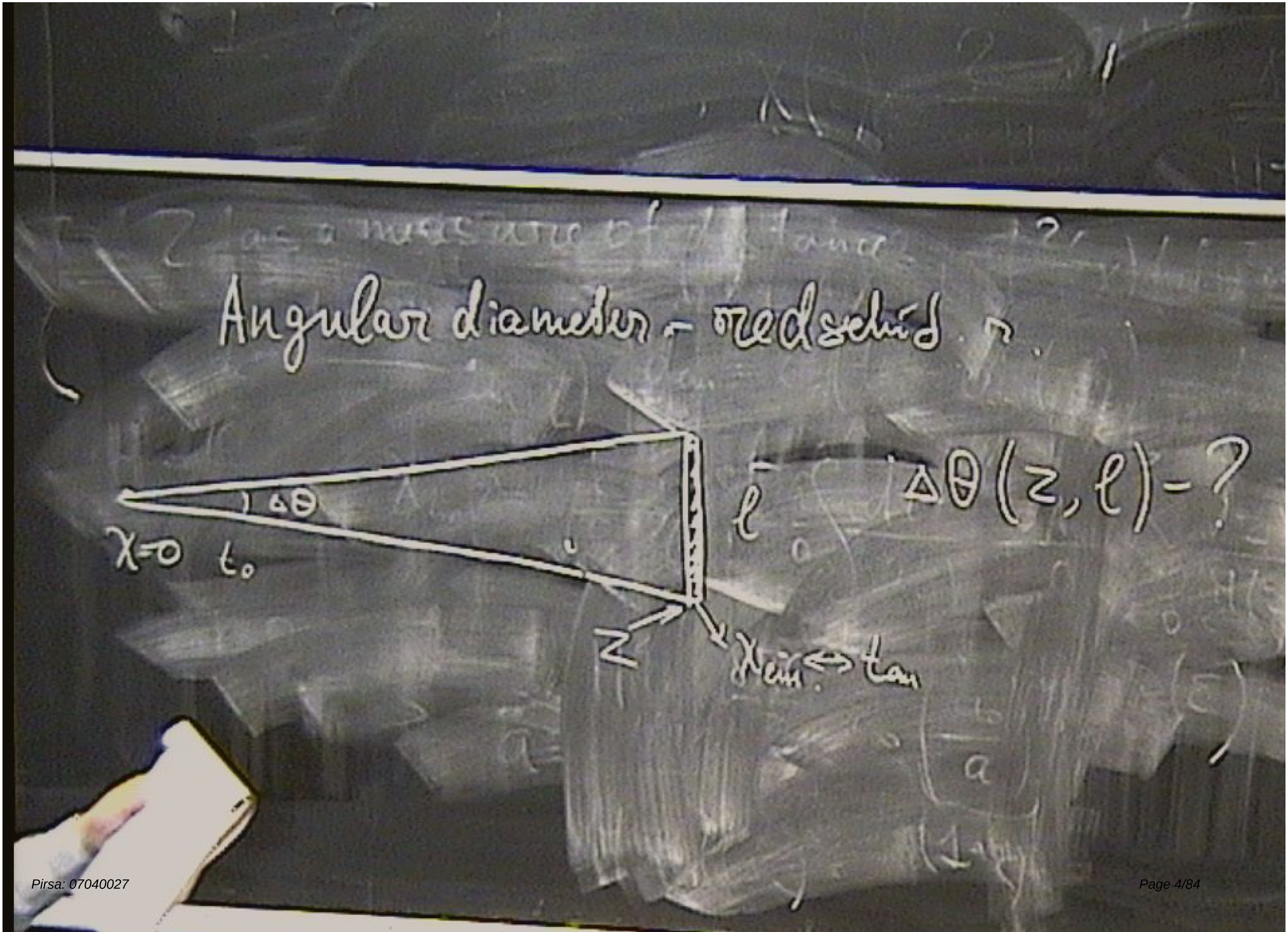
Date: Apr 26, 2007 12:00 PM

URL: http://pirsa.org/07040027

Abstract: Class 2 part 2

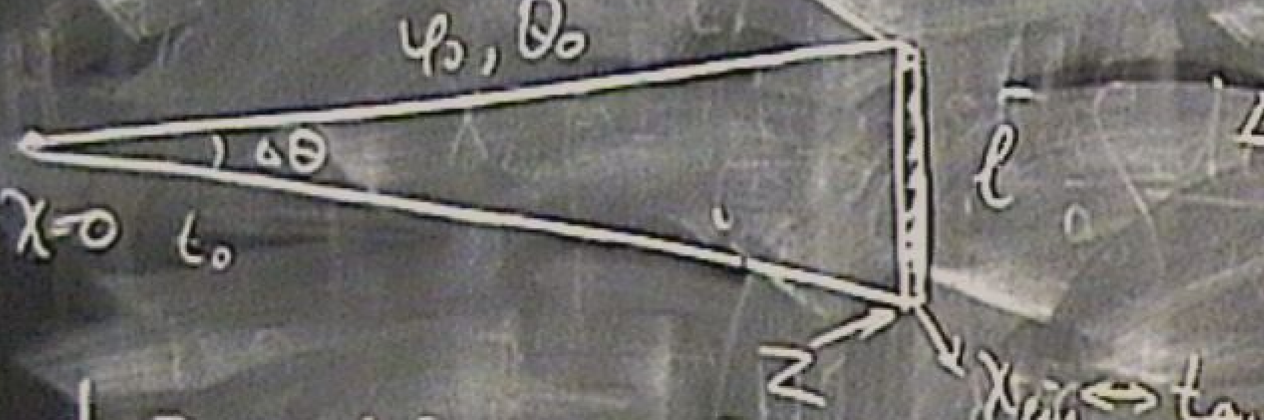
Angular diameter – redshift r.


Angular diameter - redshift ?
ΔΘ
l
ΔΘ(z, l) - ?
z
x_em.



Angular diameter – redshift r.
$\Delta\theta$
$\chi = 0$ t_0
l
$\Delta\theta(z, l) - ?$
z
$\chi_{em} \Leftrightarrow t_{em}$

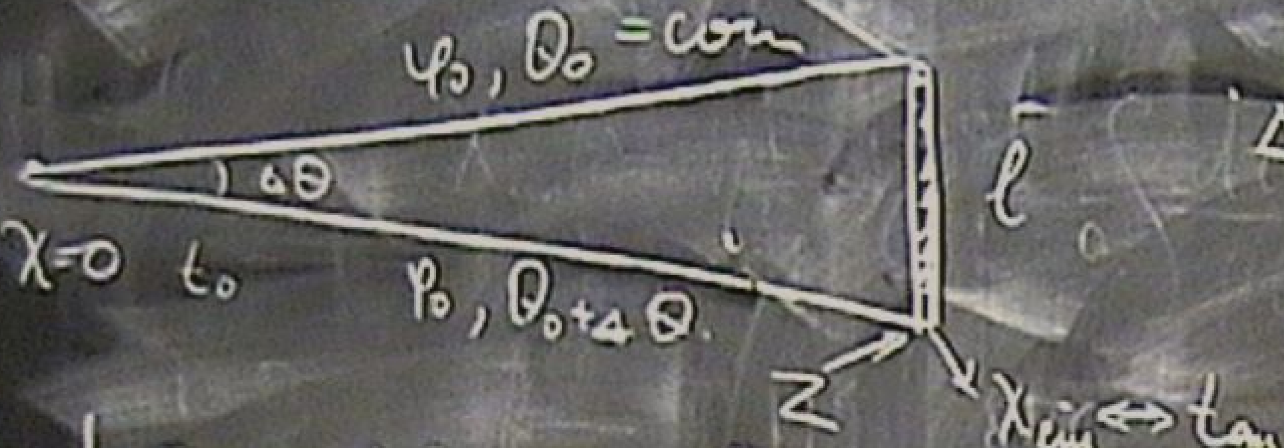
Angular diameter – redshift r.



$\Delta\theta(z, \ell) - ?$

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \Phi^2(\chi)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

Angular diameter – redshift r.

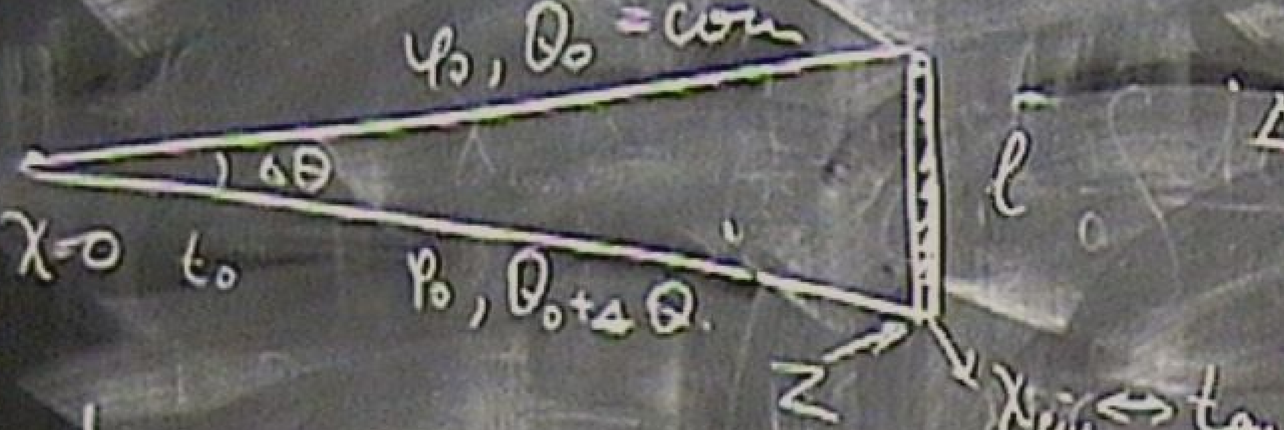


$\Delta\theta(z, l) - ?$

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \Phi^2(\chi)\left(\underbrace{d\theta^2}_{\Delta\theta^2} + \sin^2\theta\, d\varphi^2\right)\right]$$

$$l = \sqrt{-\Delta s^2} = a(t_{em})$$

Angular diameter – redshift r.

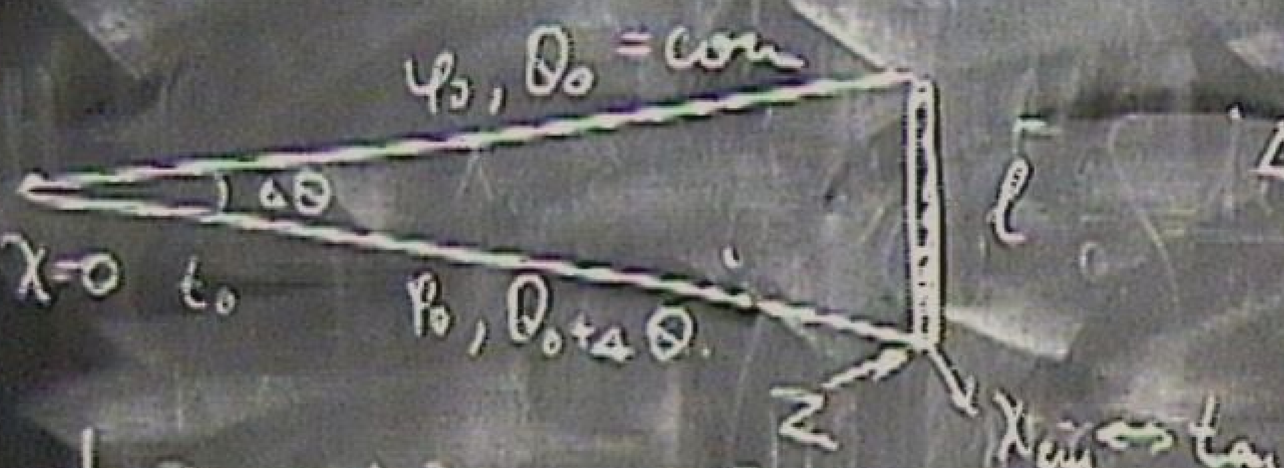


$\Delta\theta(z, \ell) - ?$

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \Phi^2(\chi)\left(\underbrace{d\theta^2}_{\Delta\theta^2} + \sin^2\theta\, d\varphi^2\right)\right]$$

$$\ell = \sqrt{-\Delta s^2} = a(t_{em})\,\Phi$$

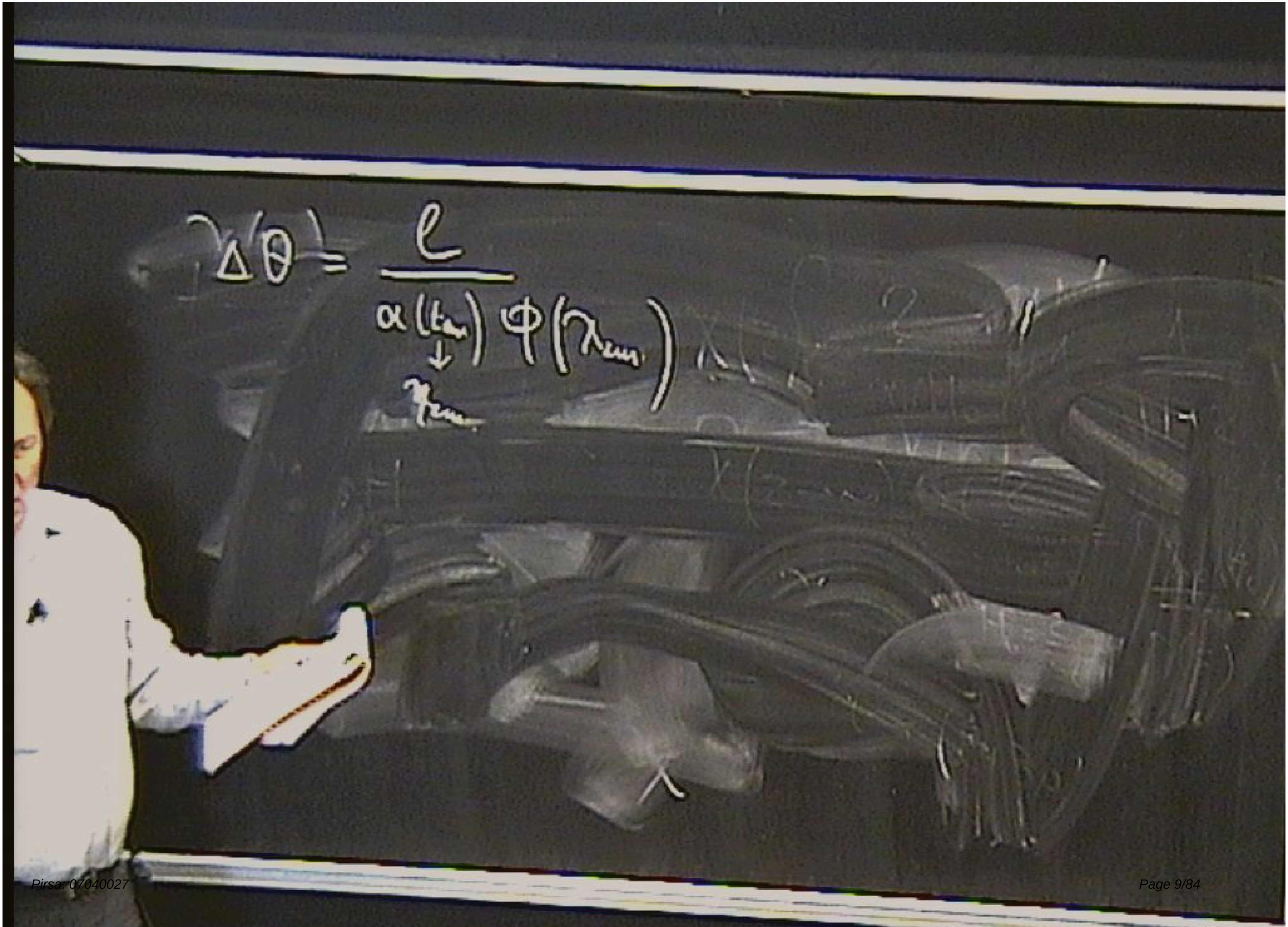
Angular diameter - redshift r.



$\Delta\theta(z, \ell) - ?$

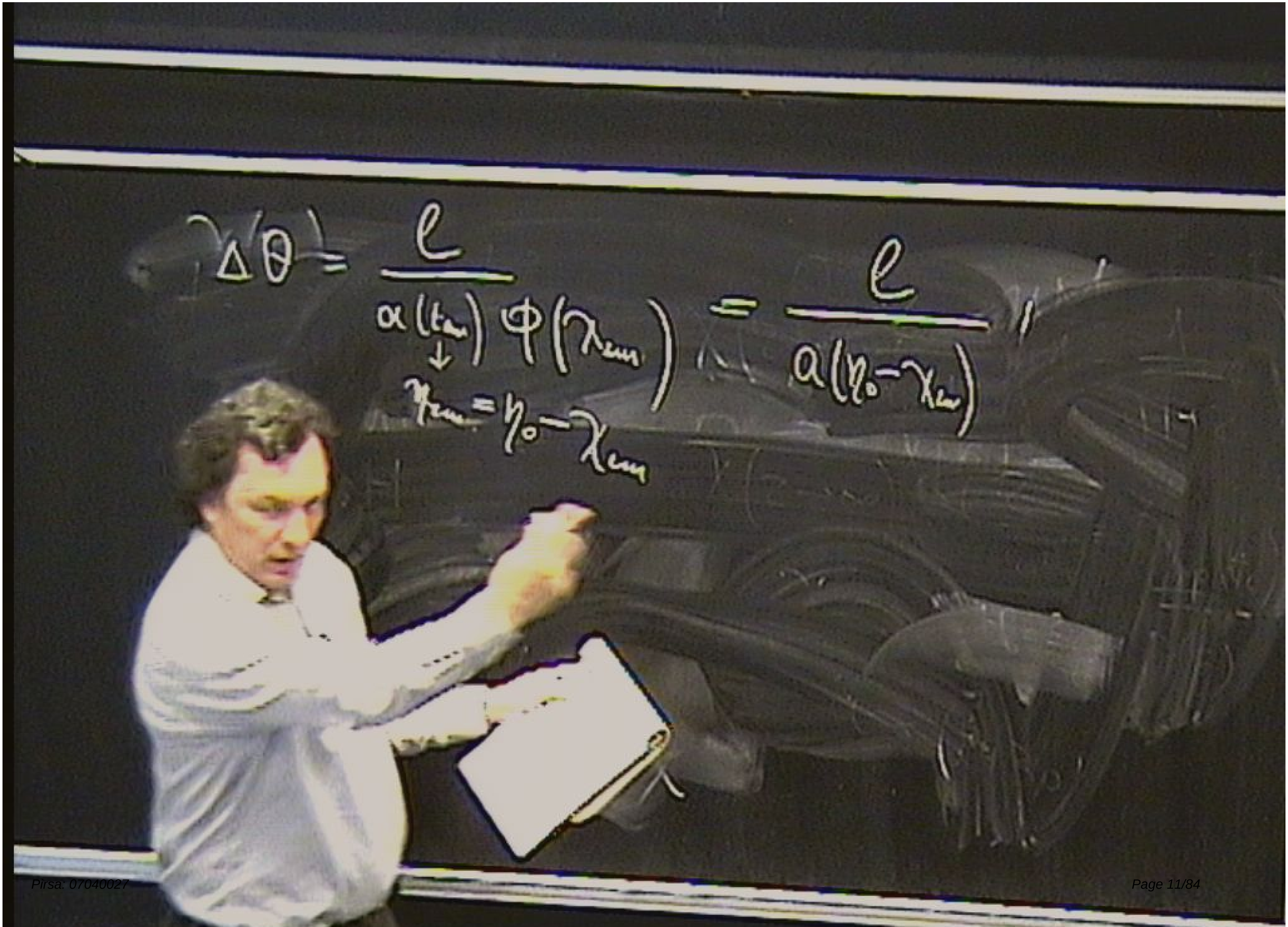
$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \Phi^2(\chi)\left(\underbrace{d\theta^2}_{\Delta\theta^2} + \sin^2\theta\, d\varphi^2\right)\right]$$

$$\ell = \sqrt{-\Delta s^2} = a(t_{em})\,\Phi(\chi_{em})\,\Delta\theta$$


$\Delta\theta = \frac{\ell}{\alpha(t_{em})\,\varphi(\eta_{em})}$
η_{em}

$$\Delta\theta = \frac{\ell}{\alpha(t_{em})\,\varphi(\eta_{em})}$$

$$\eta_{em} = \eta_0 - \chi_{em}$$


$\Delta\theta = \frac{\ell}{a(t_{em})\,\Phi(\chi_{em})} = \frac{\ell}{a(\eta_0-\chi_{em})}$
$\eta_{em} = \eta_0 - \chi_{em}$

$$\Delta\theta = \frac{\ell}{a(t_{em})\,\Phi(\chi_{em})} = \frac{\ell}{a(\eta_0-\chi_{em})\,\Phi(\chi_{em})}$$

$$t_{em} \downarrow \quad \eta_{em} = \eta_0 - \chi_{em}$$

$$\Delta\theta = \frac{\ell}{a(t_{em})\,\Phi(\chi_{em})} = \frac{\ell}{a(\eta_0-\chi_{em})\,\Phi(\chi_{em})}$$

$\eta_{em} = \eta_0 - \chi_{em}$

a) $\chi_{em} \ll \eta_0$

$$\Delta\theta = \frac{\ell}{a(t_{em})\,\Phi(\chi_{em})} = \frac{\ell}{a(\eta_0 - \chi_{em})\,\Phi(\chi_{em})}$$

$$t_{em} \downarrow \quad \eta_{em} = \eta_0 - \chi_{em}$$

a) $\chi_{em} \ll \eta_0$

$$a(\eta_0 - \chi_{em}) \approx a(\eta_0) = a_0 \qquad \Phi(\chi_{em}) \approx \chi_{em}$$

$$\Delta\theta = \frac{\ell}{D}$$

$$D = a_0\,\chi_{em}$$

b) $\eta_0 - \chi_{em} \ll \eta_0 \quad a(\eta_0 - \chi_{em}) \ll a(\eta_0)$

b) $\eta_0 - \chi_{em} \ll \eta_0$ $\quad$ $a(\eta_0 - \chi_{em}) \ll a(\eta_0)$

$$\Phi(\chi_{pm}) \to \Phi(\chi_p) = \text{cue}$$

b) $\eta_0 - \chi_{em} \ll \eta_0$ $\quad a(\eta_0 - \chi_{em}) \ll a(\eta_0)$

$$\Phi(\chi_{em}) \to \Phi(\chi_p) = cte$$

$$\Delta\Theta \propto \frac{1}{a(\eta_0 - \chi_{em})}$$

b) $\eta_0 - \chi_{em} \ll \eta_0$, $a(\eta_0 - \chi_{em}) \ll a(\eta_0)$

$$\Phi(\chi_{em}) \to \Phi(\chi_p) = ce$$

$$\Delta\Theta \propto \frac{1}{a(\eta_0 - \chi_{em})}$$

b) $\eta_0 - \chi_{em} \ll \eta_0$ $\quad a(\eta_0 - \chi_{em}) \ll a(\eta_0)$

$$\Phi(\chi_{em}) \to \Phi(\chi_p) = ce$$

$$\Delta\Theta \propto \frac{1}{a(\eta_0 - \chi_{em})} \qquad \Delta\Theta(z)$$

$\boxed{\rho = 0}$ $\boxed{k = 0}$

$$\Delta\Theta(z) =$$

b) $\eta_0 - \chi_{em} \ll \eta_0$ $\quad a(\eta_0 - \chi_{em}) \ll a(\eta_0)$

$$\Phi(\chi_{em}) \to \Phi(\chi_p) = c$$

$$\Delta\Theta \propto \frac{1}{a(\eta_0 - \chi_{em})} \qquad \Delta\Theta(z)$$

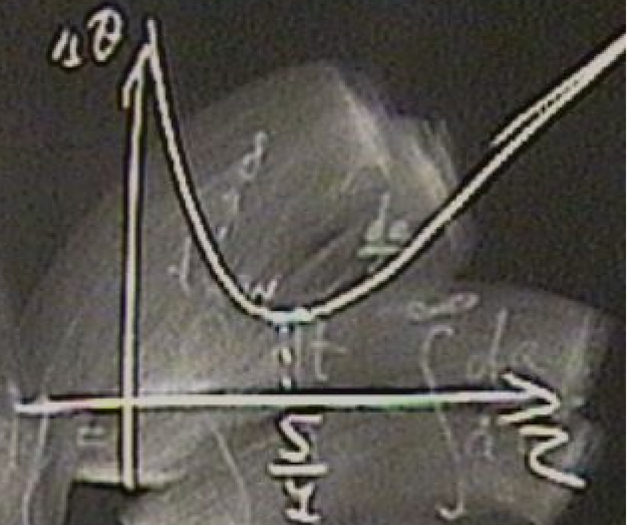
$$\chi(z) = \frac{2}{a_0 H_0}\left(1 - \frac{1}{(1+z)^{1/2}}\right)$$

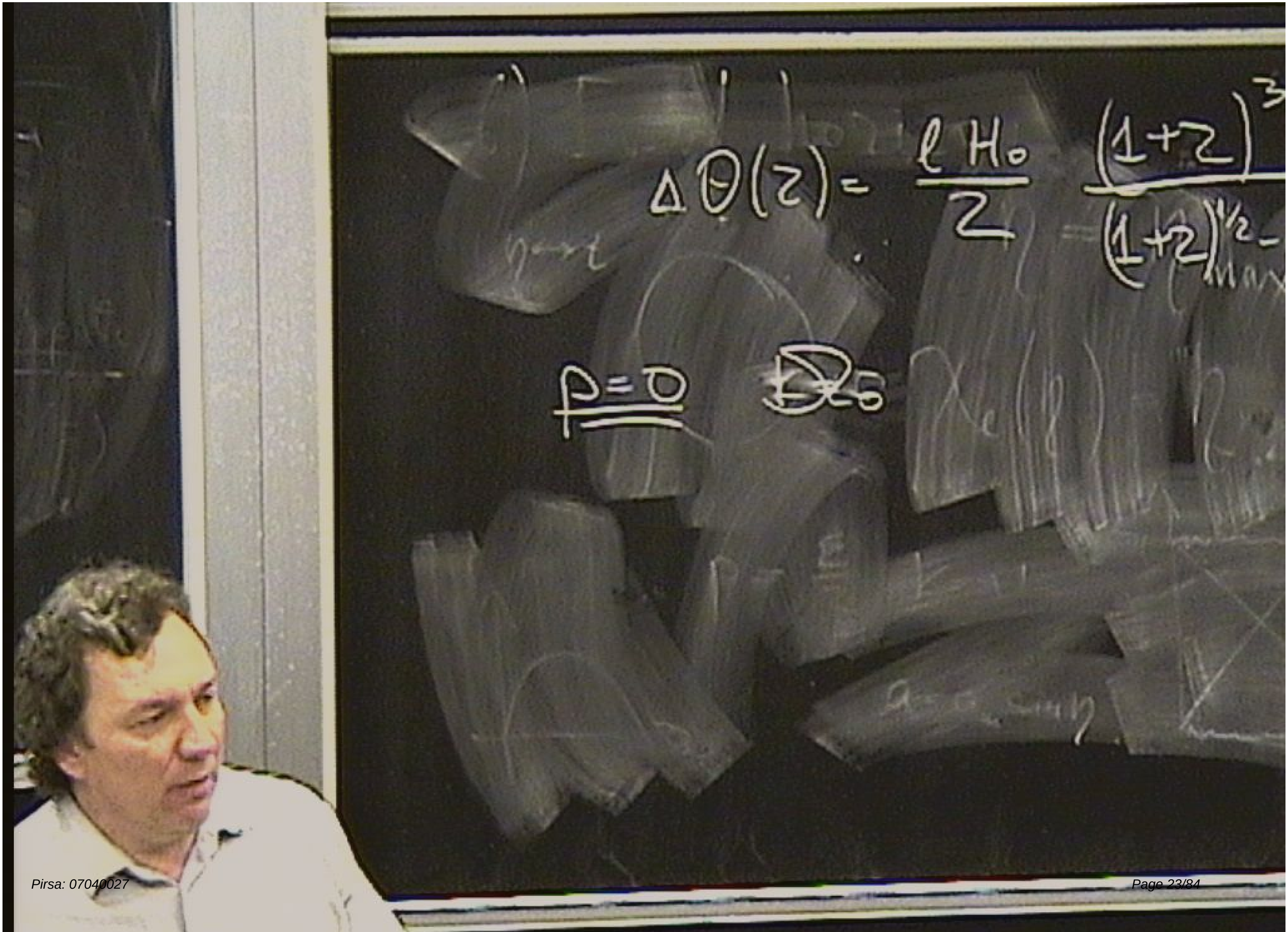
$\boxed{p=0}$ $\boxed{k=0}$

$$\Delta\Theta(z) = \frac{1}{a_0}(1+z)\frac{\ell}{\chi_{em}(z)}$$

$$\Delta\Theta(z) = \frac{\ell H_0}{2} \frac{(1+z)^{3/2}}{(1+z)^{1/2}_{max}-1}$$

$$\Delta\Theta(z) = \frac{l H_0}{2} \frac{(1+z)^{3/2}}{(1+z)^{1/2}_{max} - 1}$$

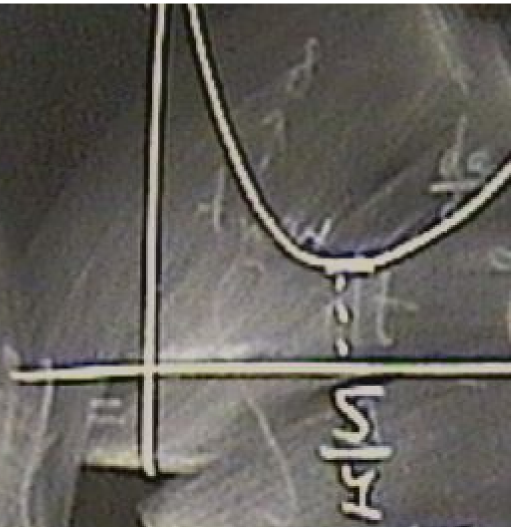




ΔΘ(z) = ℓH₀/2 (1+z)³/(1+z)^½
p = 0

$$\Delta\theta(z) = \frac{\ell H_0}{2}\,\frac{[\text{illegible}]}{(1+z)^{1/2}_{max} - 1}$$

$p = 0$

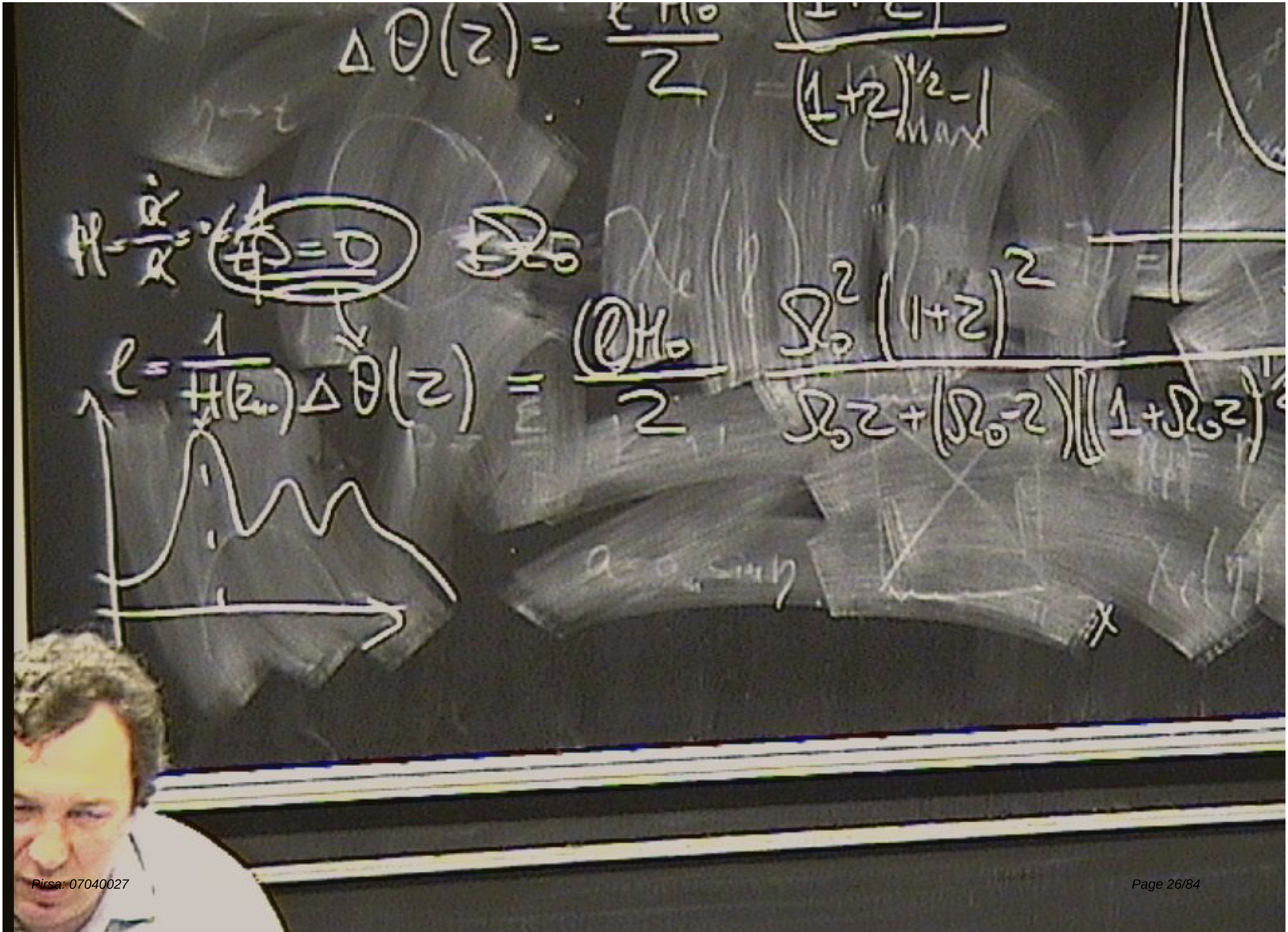


$$\Delta\theta(z) = \frac{\ell H_0}{2}\,\frac{\Omega_0^2 (1+z)^2}{\Omega_0 z + (\Omega_0 - 2)\left[(1+\Omega_0 z)^{1/2} - 1\right]}$$

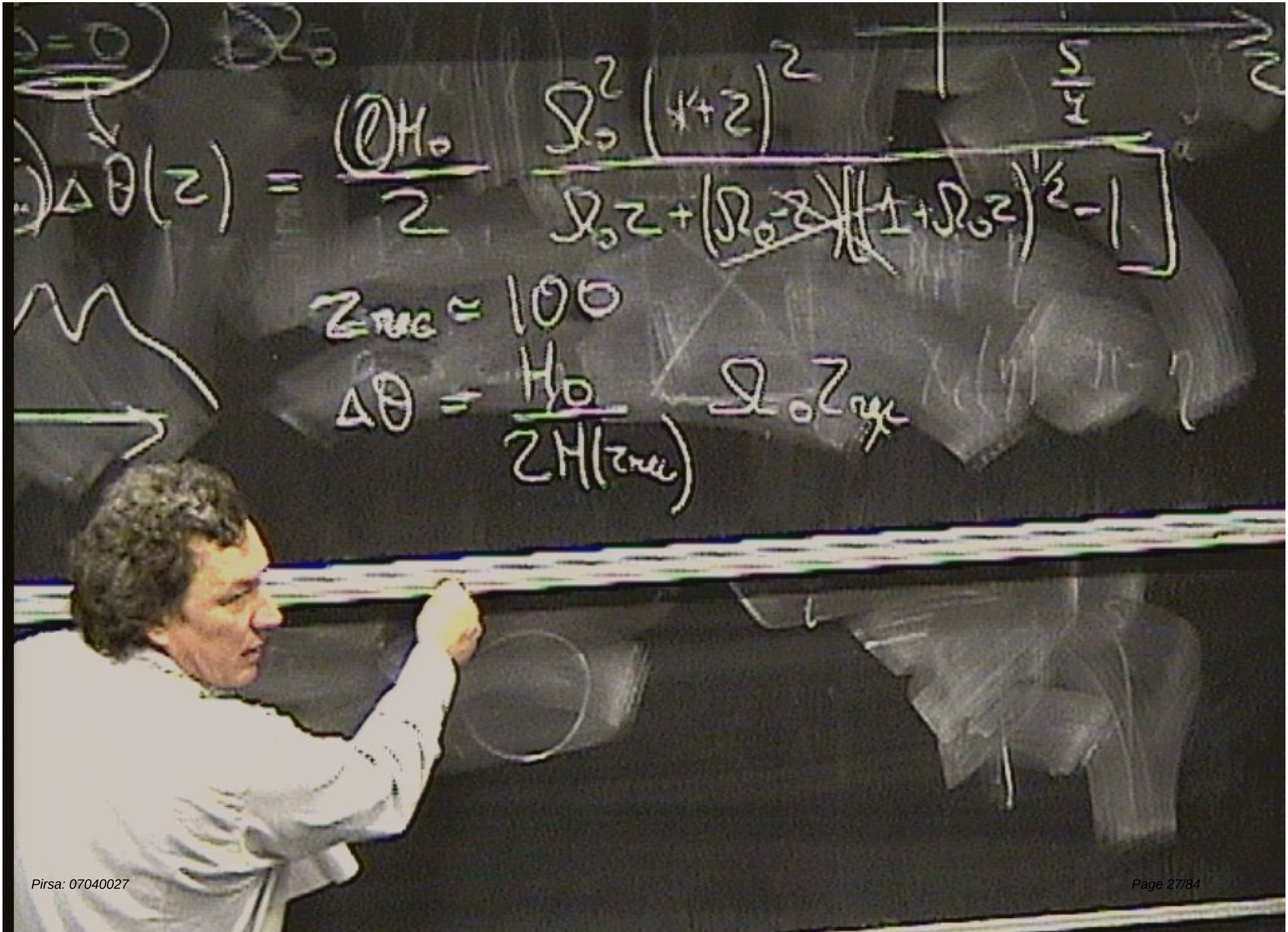
$$\Delta\Theta(z) = \frac{\ell H_0}{2} \; \frac{(1+z)}{(1+z)^{1/2}_{max} - 1}$$

$\rho = 0$ $\quad \Omega_0$

$$\Delta\theta(z) = \frac{\ell H_0}{2} \; \frac{\Omega_0^2 (1+z)^2}{\Omega_0 z + (\Omega_0 - 2)\left((1+\Omega_0 z)^{1/2} \ldots\right.}$$



$\Delta\theta(z) = \frac{l H_0}{2}$
$(1+z)^{1/2}_{max} - 1$
$H = \frac{\dot a}{a}$
$\Omega = 0$
$= \frac{l H_0}{2} \frac{\Omega_0^2 (1+z)^2}{\Omega_0 z + (\Omega_0 - 2)((1+\Omega_0 z)^{1/2}}$
$l = \frac{1}{H(z_{..})\Delta\theta(z)}$


$\Delta\theta(z) = \frac{(\ell)H_0}{2} \frac{\Omega_0^2 (1+z)^2}{\left[\Omega_0 z + (\Omega_0 - 2)\left((1+\Omega_0 z)^{1/2} - 1\right)\right]}$
$z_{rec} \simeq 100$
$\Delta\theta = \frac{H_0}{2H(z_{rec})} \Omega_0 z_{rec}$

$$\frac{\ell H_0}{2}\;\frac{\Omega_0^2 (1+z)^2}{\left[\Omega_0 z + \cancel{(\Omega_0-2)}\left((1+\Omega_0 z)^{1/2}-1\right)\right]^a}$$

$$z_{rec} \simeq 100$$

$$\Delta\Theta = \frac{H_0}{2H(z_{rec})}\,\Omega_0 z_{rec} = \frac{1}{2} z_{rec}^{-1/2}\,\Omega_0^{1/2}$$

$$\frac{3\pi c}{3}\,\Omega_0\,\varepsilon_{cr}.$$

$a \sim 1/3$

$$\Delta\theta_r \sim \frac{l_{rec}}{t_0} \approx z_{rec}^{-3/2}$$

$a \propto t^{2/3}$

$$\Delta\theta_r \sim \frac{l_{rec}}{t_0} \approx z_{rec}^{-3/2}$$

$a \propto t^{2/3}$

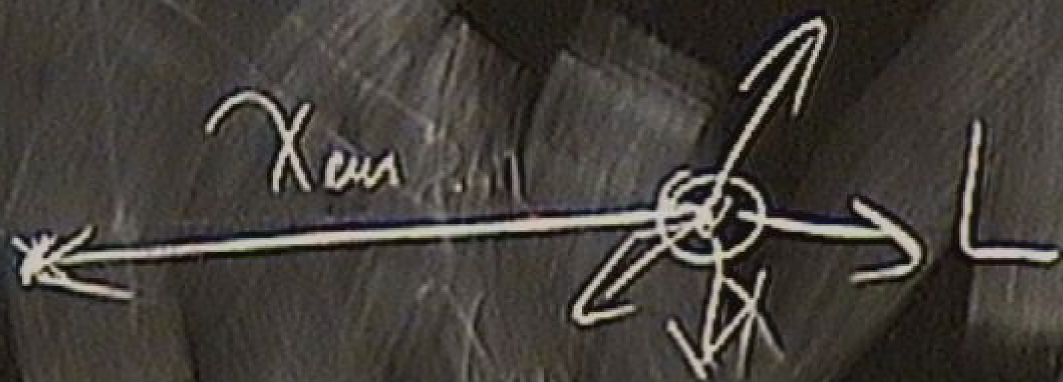
$$\Delta\theta_r \simeq \frac{t_{rec}}{t_0} \simeq z_{rec}^{-3/2}$$

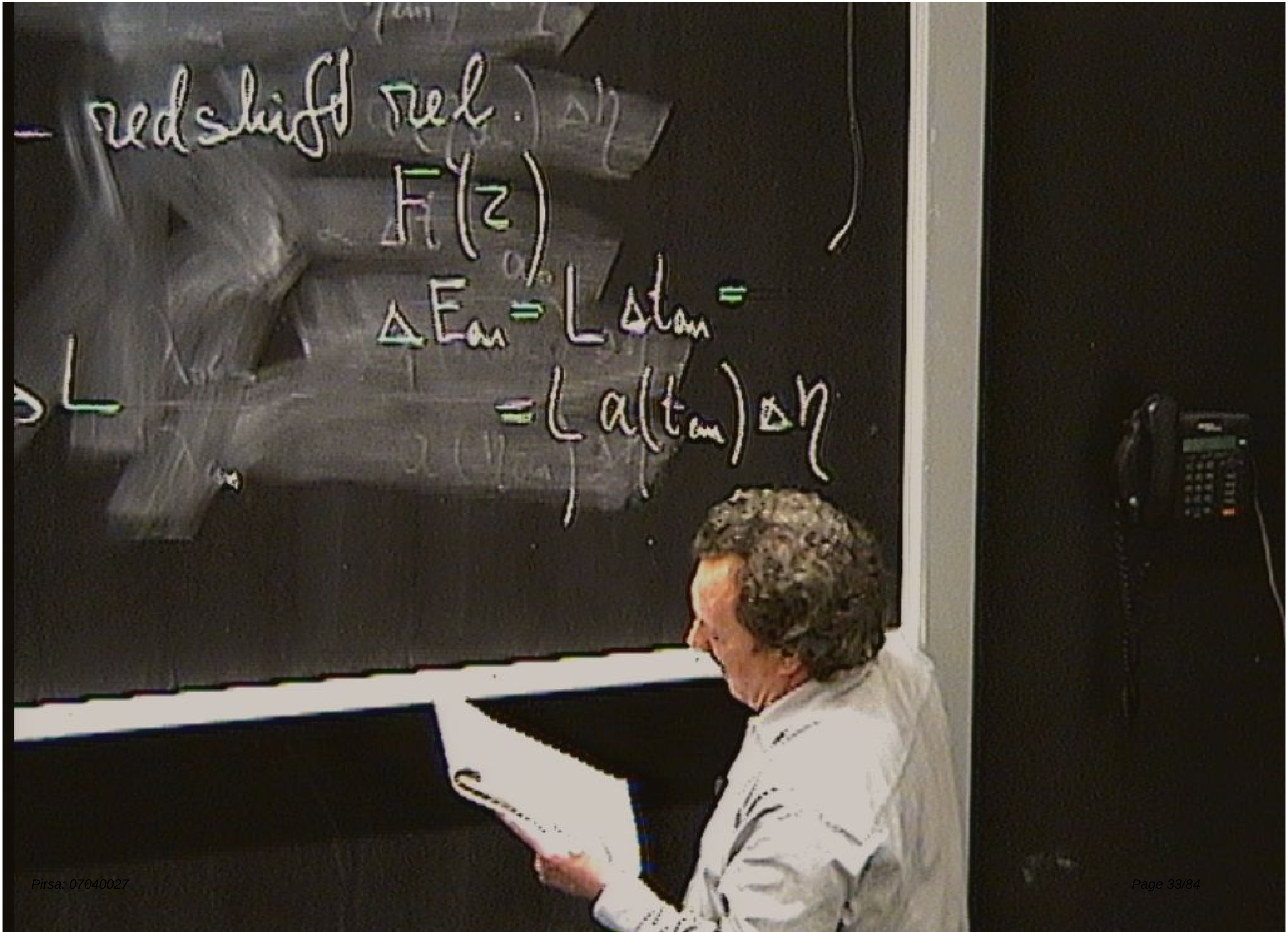
Luminosity - redshift rel.

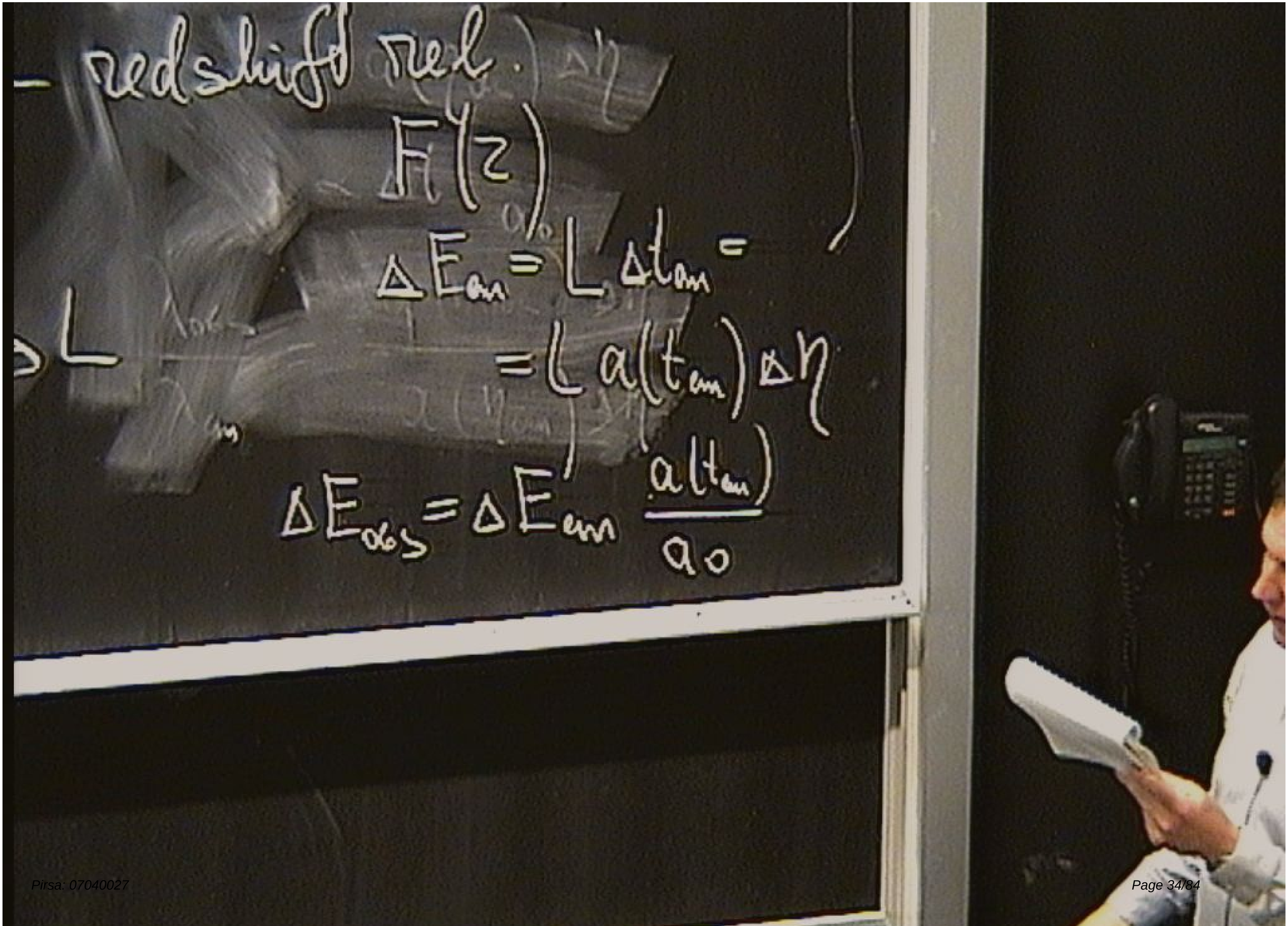
$\Delta\theta_r \approx \frac{\ldots}{t_0} \approx z_{rec}$

Luminosity - redshift rel.

$F(z)$




redshift rel
F(z)
ΔE_em = L Δt_em =
= L a(t_em) Δη

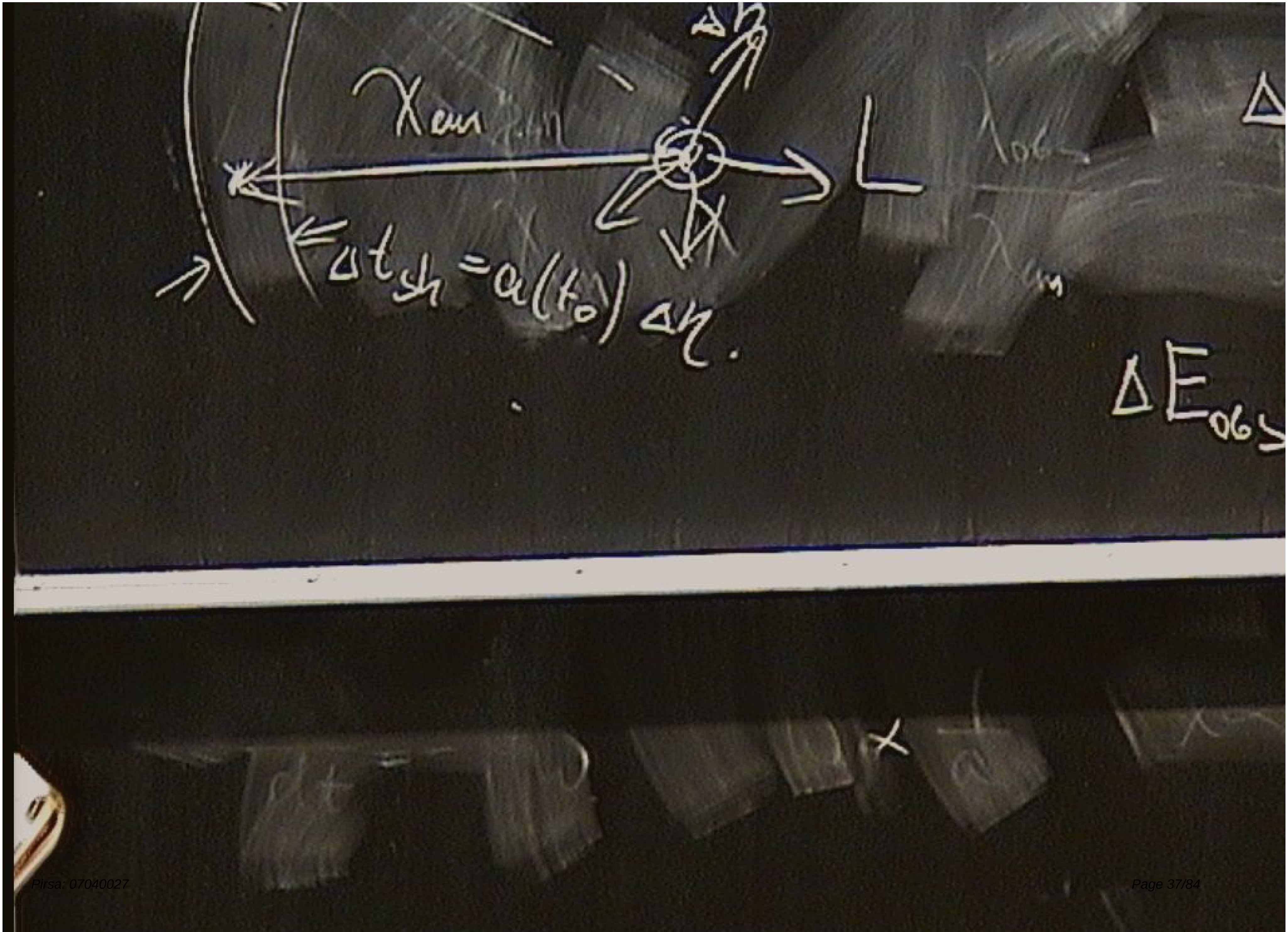

redshift rel.
F(z)
$\Delta E_{em} = L \Delta t_{em} =$
$= L\, a(t_{em}) \Delta \eta$
$\Delta E_{obs} = \Delta E_{em} \frac{a(t_{em})}{a_0}$

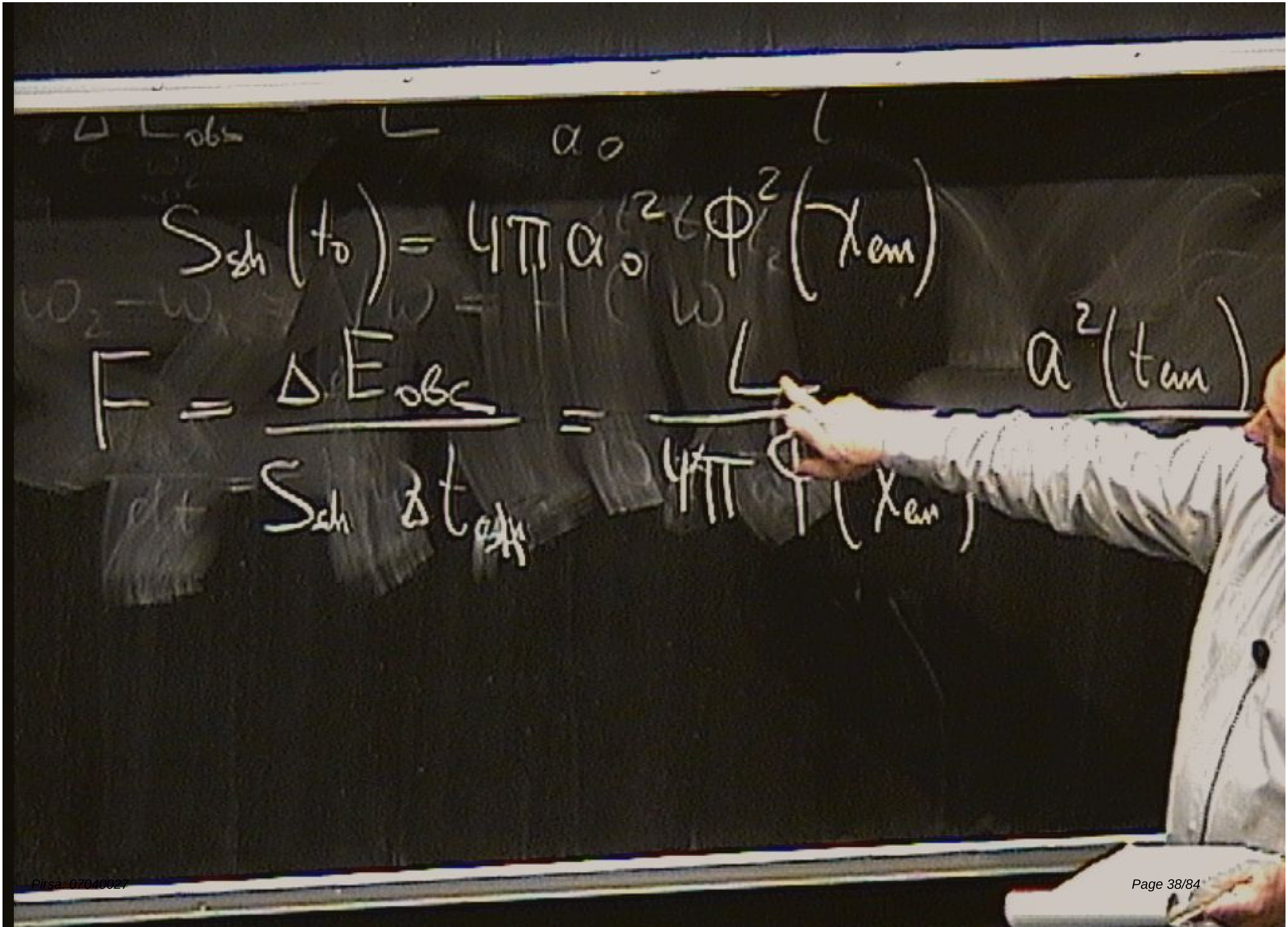
$$\Delta E_{obs} = L \, \frac{a^2(t_{em})}{a_0} \, \Delta\eta \, .$$

$$\omega \propto \frac{1}{a}$$

$$\Delta E_{obs} = L \quad a_0$$

$$S_{sh}(t_0) = 4\pi a_0^2 6 \Phi^2(\chi_{em})$$


Δη
χ_em
L
Δt_sh = a(t_0) Δη.
ΔE_obs


S_sh(t_0) = 4π a_0^2 φ^2(χ_em)
F = ΔE_obs / (S_sh Δt_obs) = L a^2(t_em) / (4π φ(χ_em))

ΔL_{obs} $\quad L \quad a_0$

$$S_{sh}(t_0) = 4\pi a_0^2 \Phi^2(\chi_{em})$$

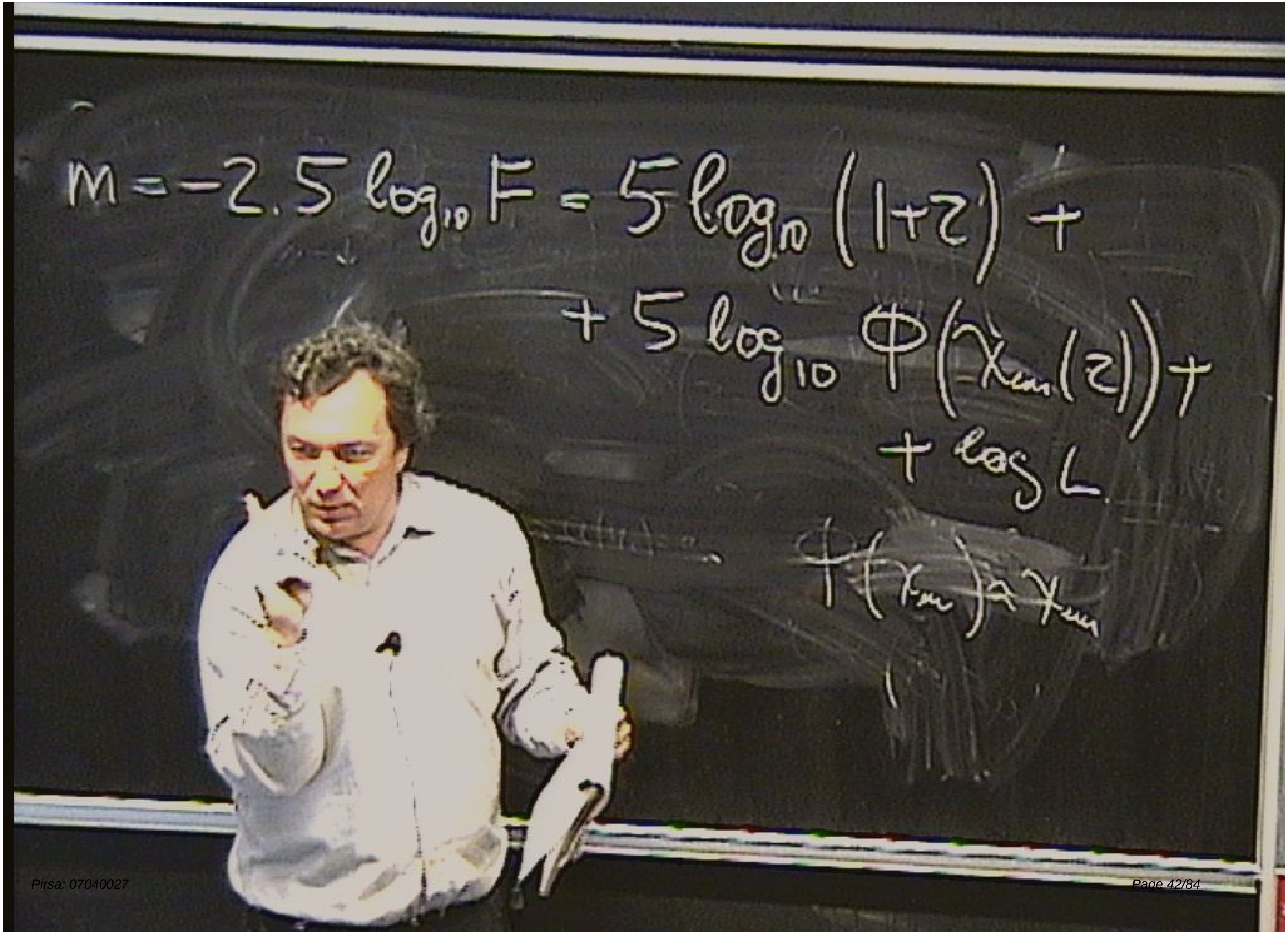
$$F = \frac{\Delta E_{obs}}{S_{sh}\,\Delta t_{obs}} = \frac{L}{4\pi\,\Phi(\chi_{em})^2}\,\frac{a^2(t_{em})}{a_0^4}$$

$$F = \frac{\Delta E_{obs}}{S_{ch}\,\Delta t_{obs}} = \frac{L}{4\pi\,\Phi(\chi_{em}(z))}\,\frac{a^2(t_{em})}{a_0^4}$$

$$F(z) = \frac{L}{4\pi a_0^2\,\Phi^2(\chi_{em}(z))\,(1+z)^2}$$

$$M = -2.5 \log_{10} F$$

$$\phi(\chi_m) \approx \chi_m$$

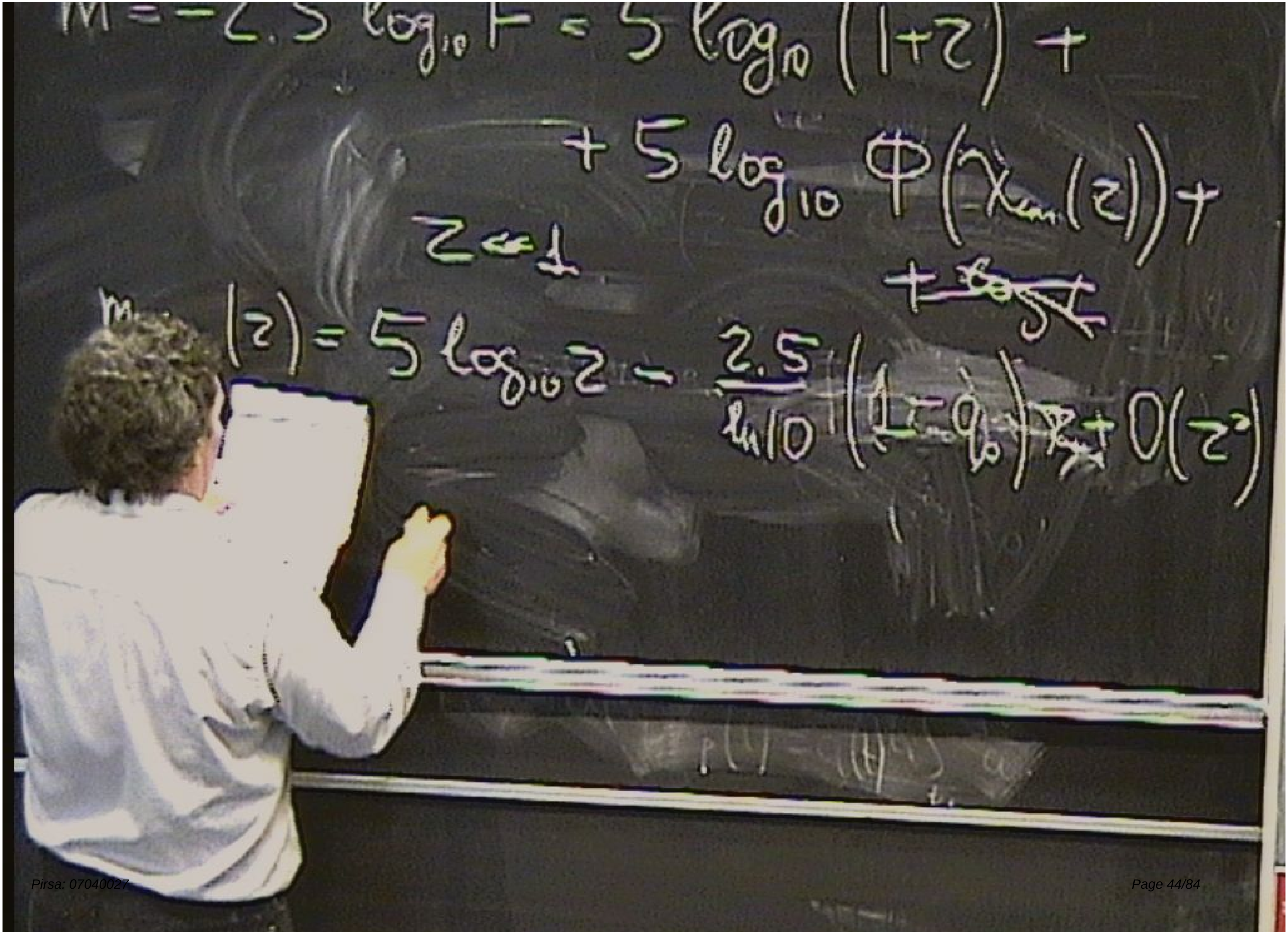

$M = -2.5 \log_{10} F = 5 \log_{10}(1+z) +$
$+ 5 \log_{10} \Phi(\chi_{em}(z)) +$
$+ \log L$
$\Phi(\chi_{em}) \approx \chi_{em}$

$$M = -2.5 \log_{10} F = 5 \log_{10} (1+z) +$$

$$+ 5 \log_{10} \Phi(\chi_{em}(z)) +$$

$$+ \cancel{\text{[illegible]}}$$

$$\Phi(\chi_{em}) \approx \chi_{em}$$


M = -2.5 log10 F = 5 log10 (1+z) +
+ 5 log10 Φ(χem(z)) +
z << 1
m(z) = 5 log10 z - 2.5/ln10 (1 - q0) z + O(z²)

$$M = -2.5 \log_{10} F = 5 \log_{10}(1+z) +$$

$$+ 5 \log_{10} \Phi(\chi_{em}(z)) +$$

$$+ \cancel{\log \ldots}$$

$$z \ll 1$$

$$m_{loc}(z) = 5 \log_{10} z - \frac{2.5}{\ln 10}(1 - q_0)\, z + O(z^2)$$

$$q_0 \equiv -\frac{\ddot{a}}{a H^2}\Big|_{t_0}$$

$$M = -2.5\log_{10} F = 5\log_{10}(1+z) +$$

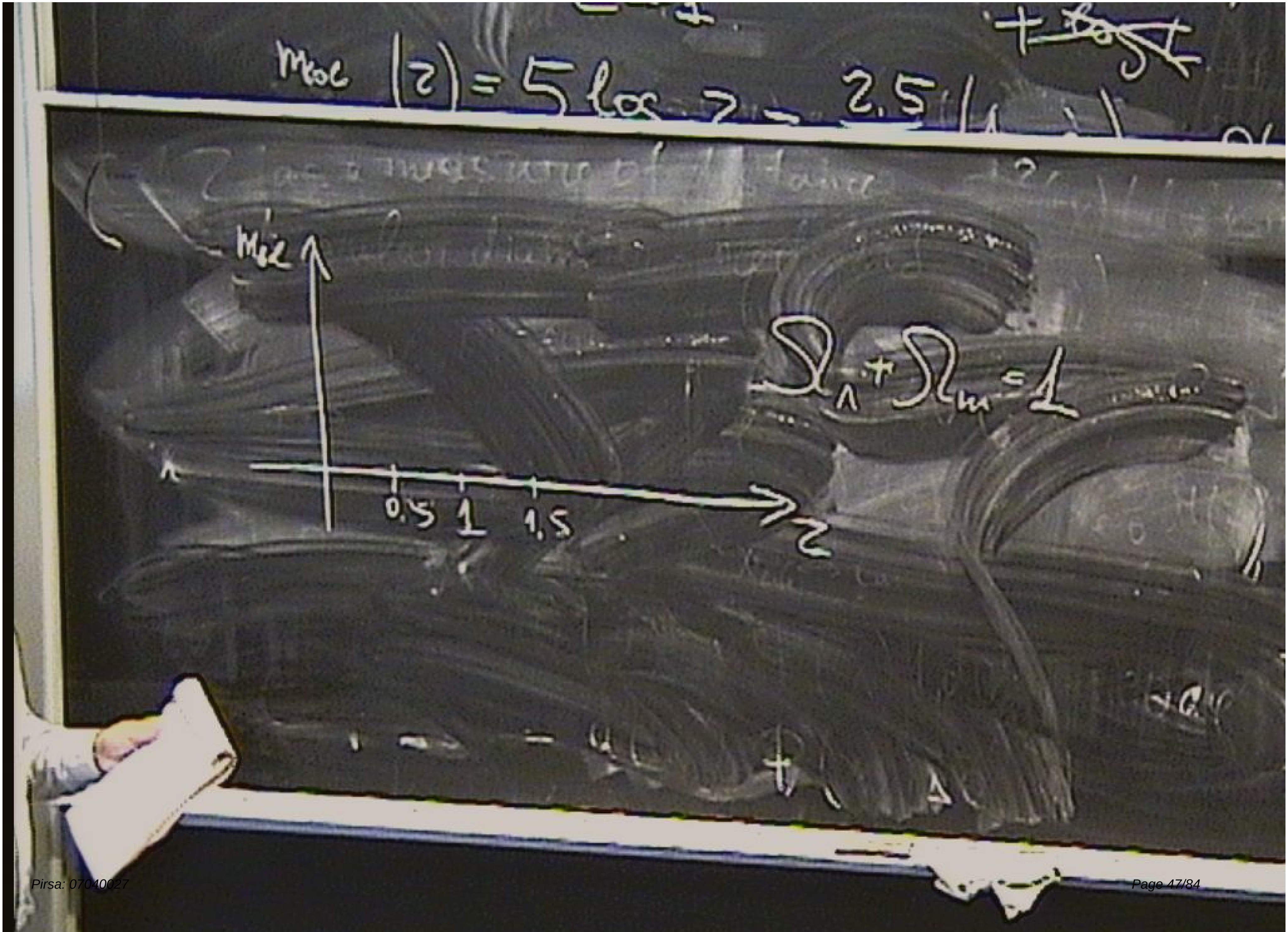
$$+ 5\log_{10}\Phi(\chi_{em}(z)) +$$

$$+ \cancel{\ldots}$$

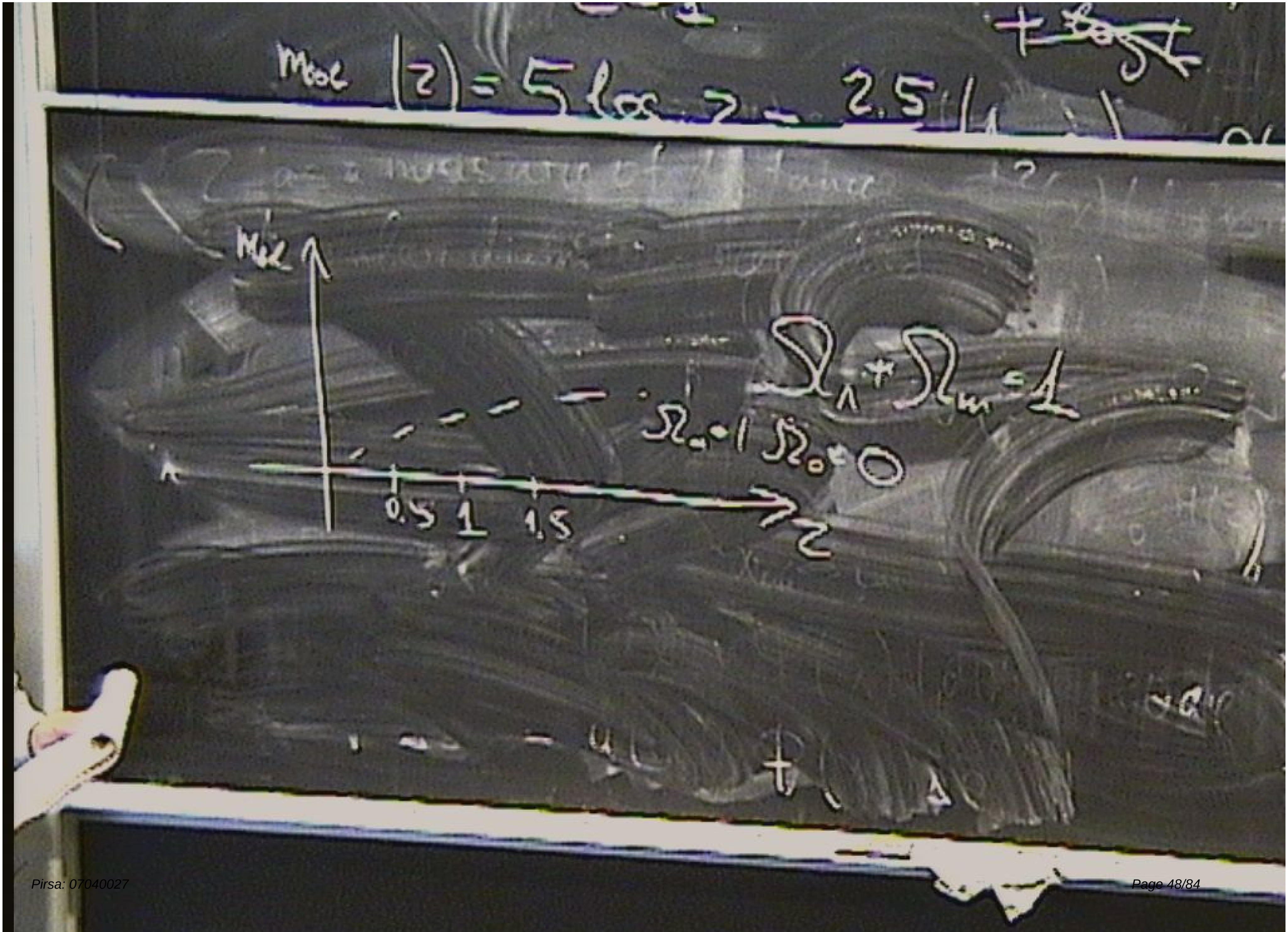
$z \ll 1$

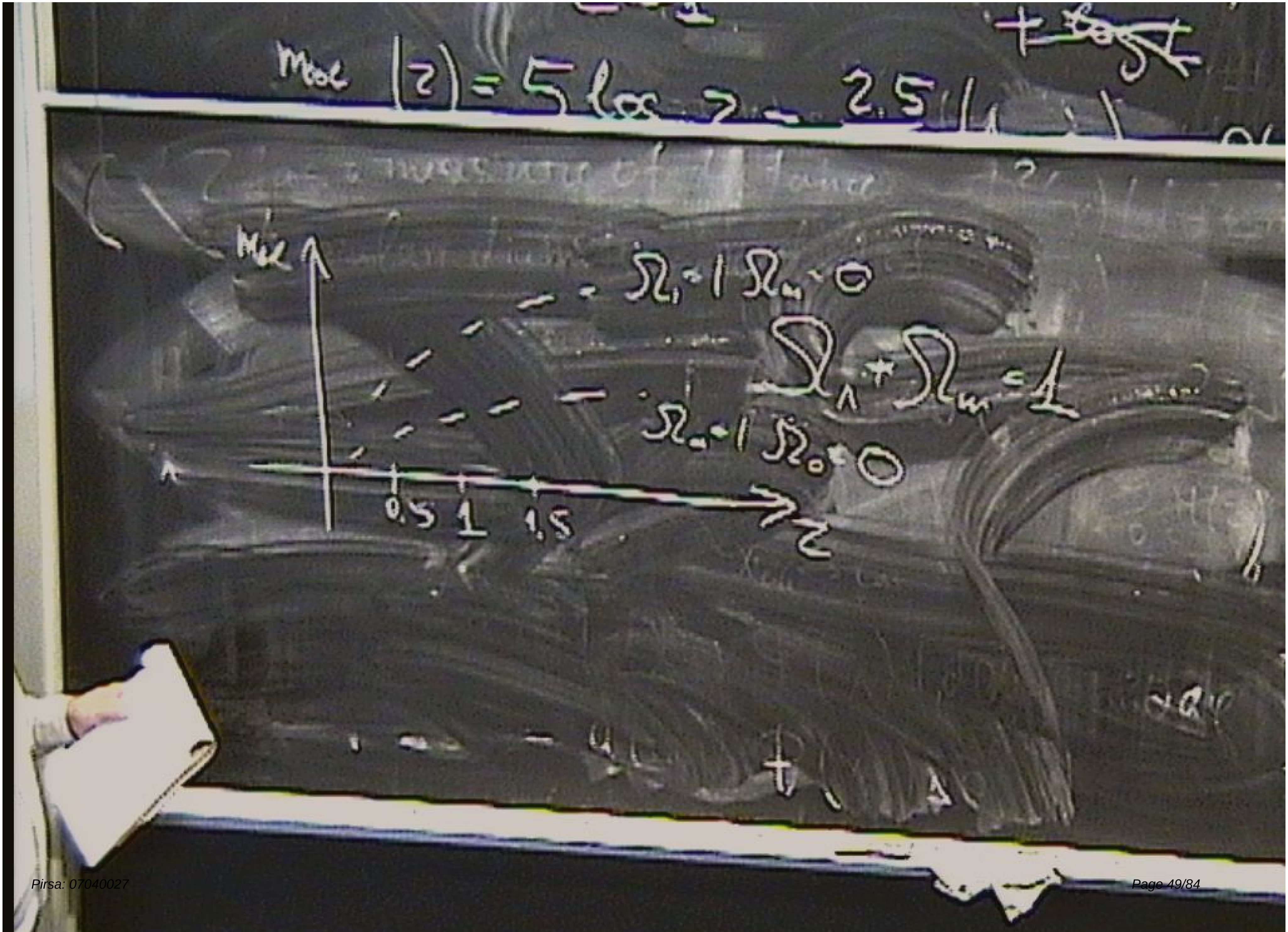
$$m_{tot}(z) = 5\log_{10} z - \frac{2.5}{\ln 10}(1 - q_0)\,z + O(z^2)$$

$$q_0 \equiv -\left.\frac{\ddot{a}}{aH^2}\right|_{t_0} = \frac{1}{2}\Omega_0\left(1 + 3\frac{p}{\varepsilon}\right)$$



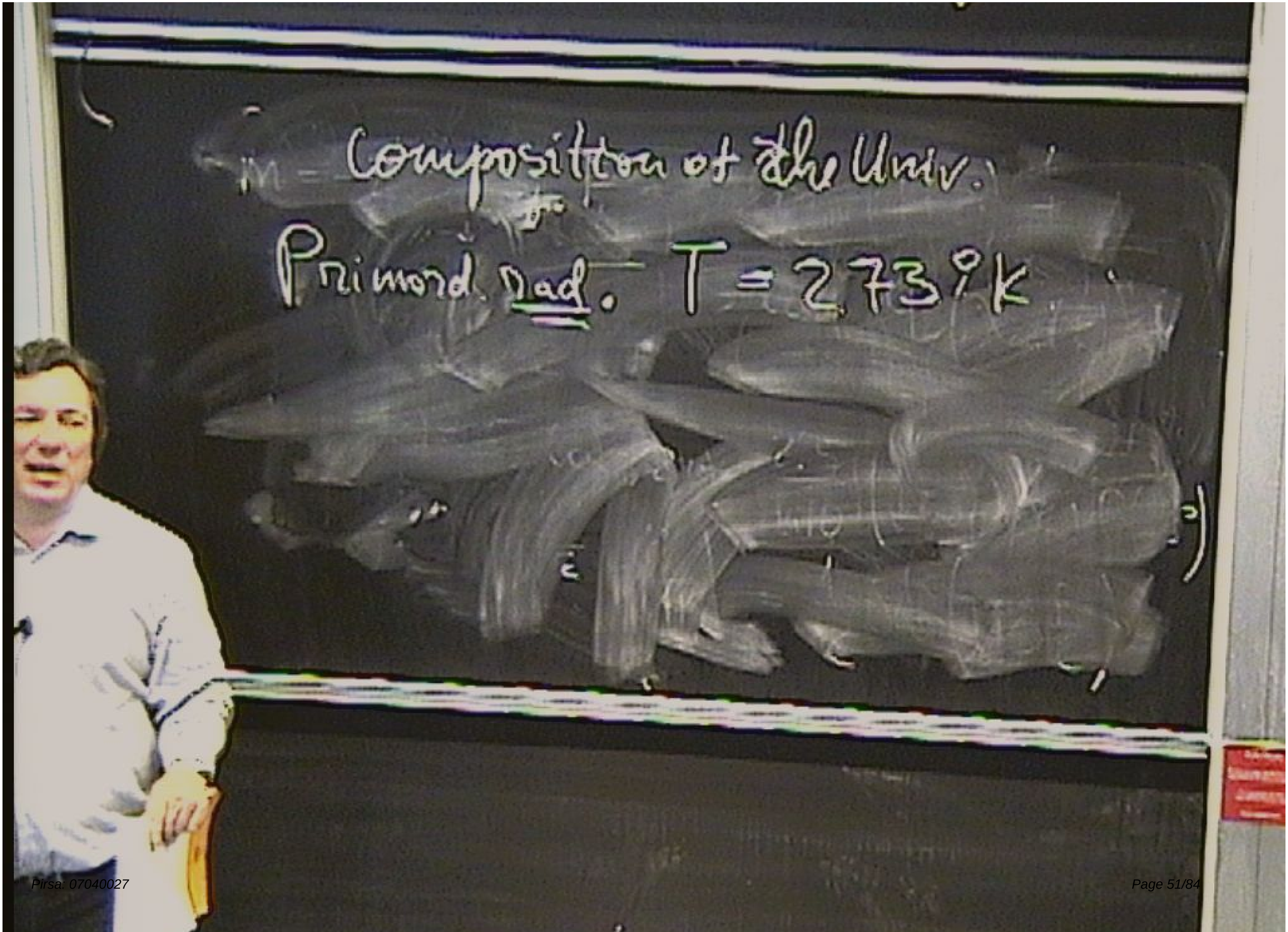
$m_{toc}(z) = 5 \log z - 2.5$
m_{toc}
0.5
1
1.5
z
$\Omega_\Lambda + \Omega_m = 1$


m_tot (z) = 5 log z - 2.5
m_tot
Ω_Λ + Ω_m = 1
Ω_m = 1 Ω_Λ = 0
0.5 1 1.5
z



m_bol (z) = 5 log z − 2.5
m_bol
Ω_Λ=1 Ω_m=0
Ω_Λ+Ω_m=1
Ω_m=1 Ω_Λ=0
0.5
1
1.5
z

Composition of the Univ.


Composition of the Univ.
Primord. rad. T = 2.73°K

Composition of the Univ.

Primord. rad. $T = 2.73^{\circ}K$

$\varepsilon_{\gamma} = 10^{-34}\ g/cm^3$

Composition of the Univ.

Primord. Rad. $T = 2.73°K$

$\varepsilon_{\gamma} = 10^{-34}\,g/cm^3$

Baryonic matter

Composition of the Univ.:

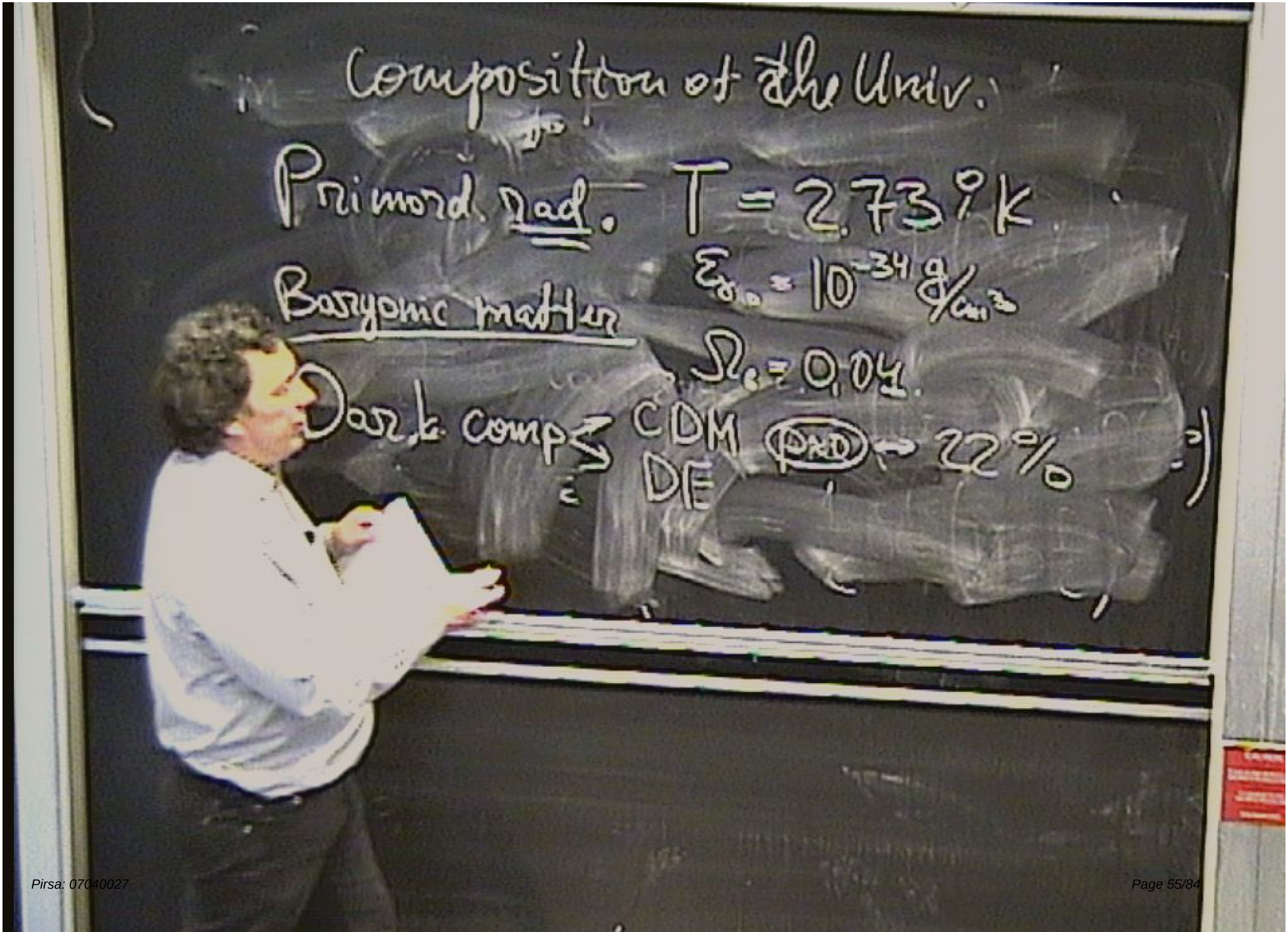
Primord. rad. $T = 2.73°K$

$\varepsilon_{\gamma_0} = 10^{-34}\, g/cm^3$

Baryonic matter

$\Omega_b = 0.04$

Dark comp.


Composition of the Univ.:
Primord. rad. T = 2.73° K
$\varepsilon_{\gamma_0} = 10^{-34}$ g/cm³
Baryonic matter
$\Omega_b = 0.04$
Dark comp. CDM → 22%
DE

Composition of the Univ.:

Primord. rad. $T = 2.73°K$

$\varepsilon_{\gamma_0} = 10^{-34}\ g/cm^3$

Baryonic matter

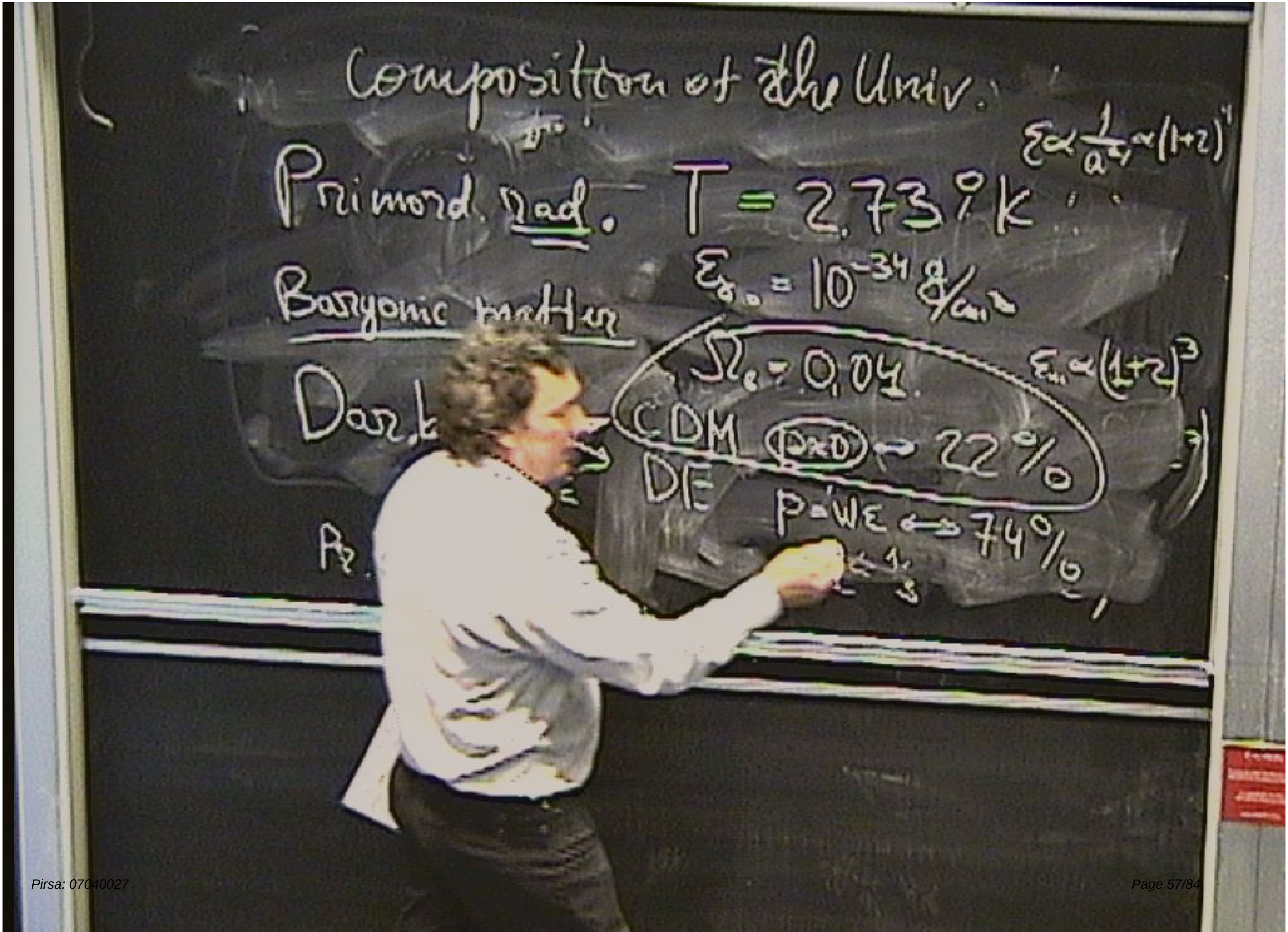
$\Omega_0 = 0.04$

Dark comp. CDM (p=0) ↔ 22%

DE $p = w\varepsilon$ ↔ 74%

$w < -\frac{1}{3}$

Pr. hr.


Composition of the Univ.
Primord. rad.
T = 2.73 K
Baryonic matter
Dark
CDM
22%
DE
p = wε
74%

Composition of the Univ.

$\varepsilon \propto \frac{1}{a^4} \propto (1+z)^4$

Primord. Rad. $T = 2.73°K$

$\varepsilon_{\gamma_0} = 10^{-34}\ g/cm^3$

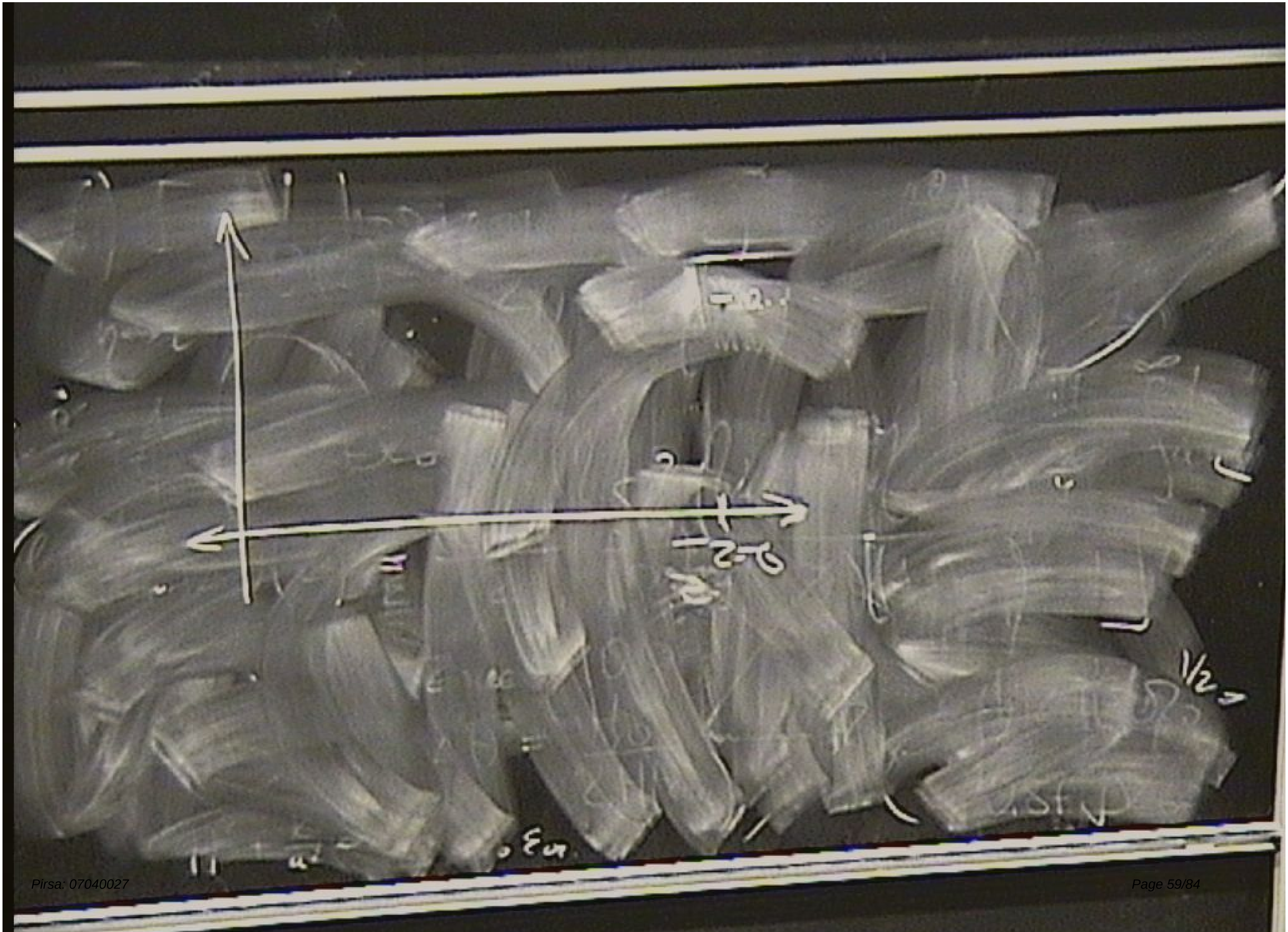
Baryonic matter $\Omega_b = 0.04$ $\varepsilon_m \propto (1+z)^3$

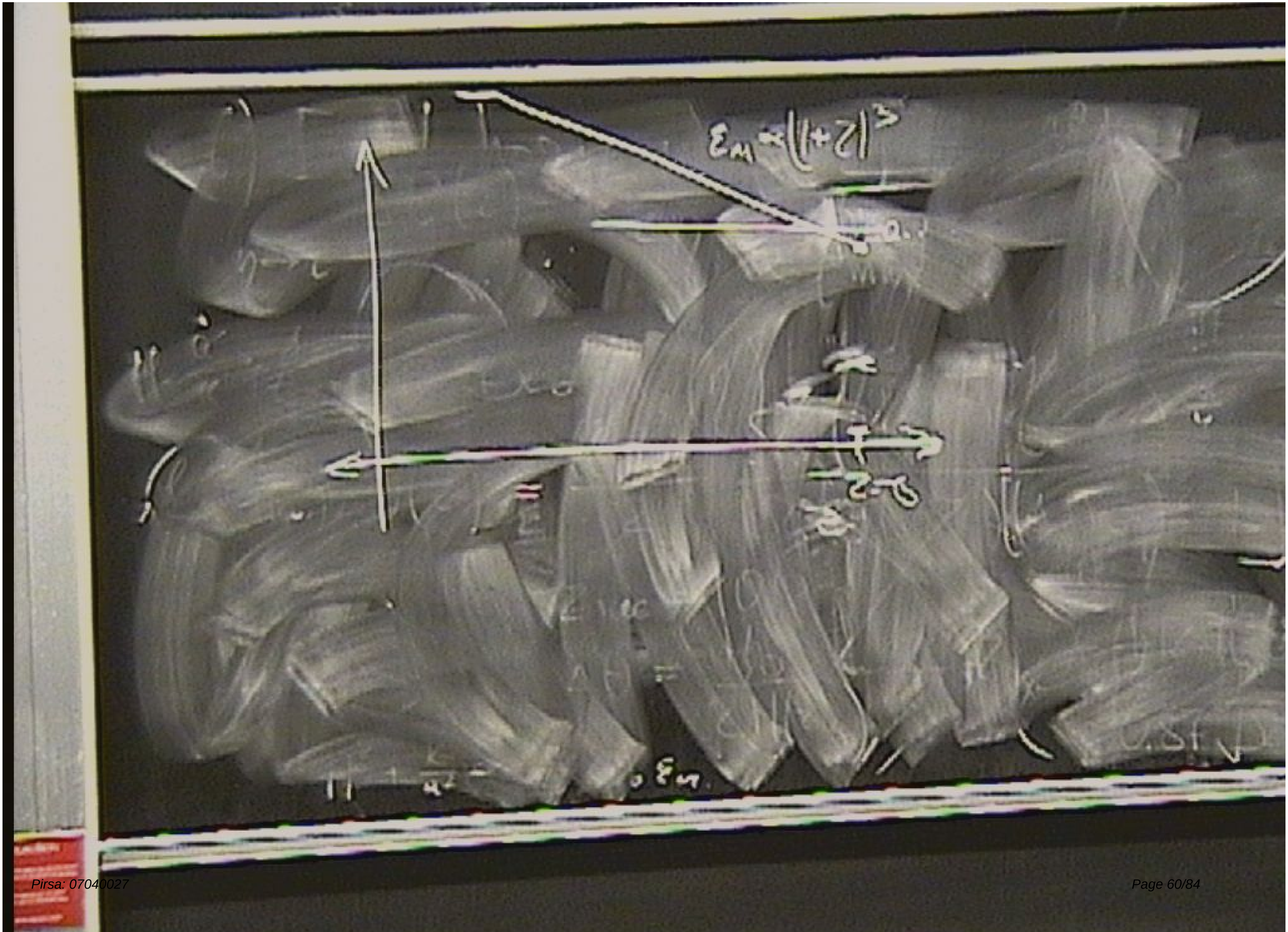
Dark comp. CDM (p=0) = 22%

DE $p = w\varepsilon$ ⟺ 74% $\varepsilon \propto (1+z)^{...}$

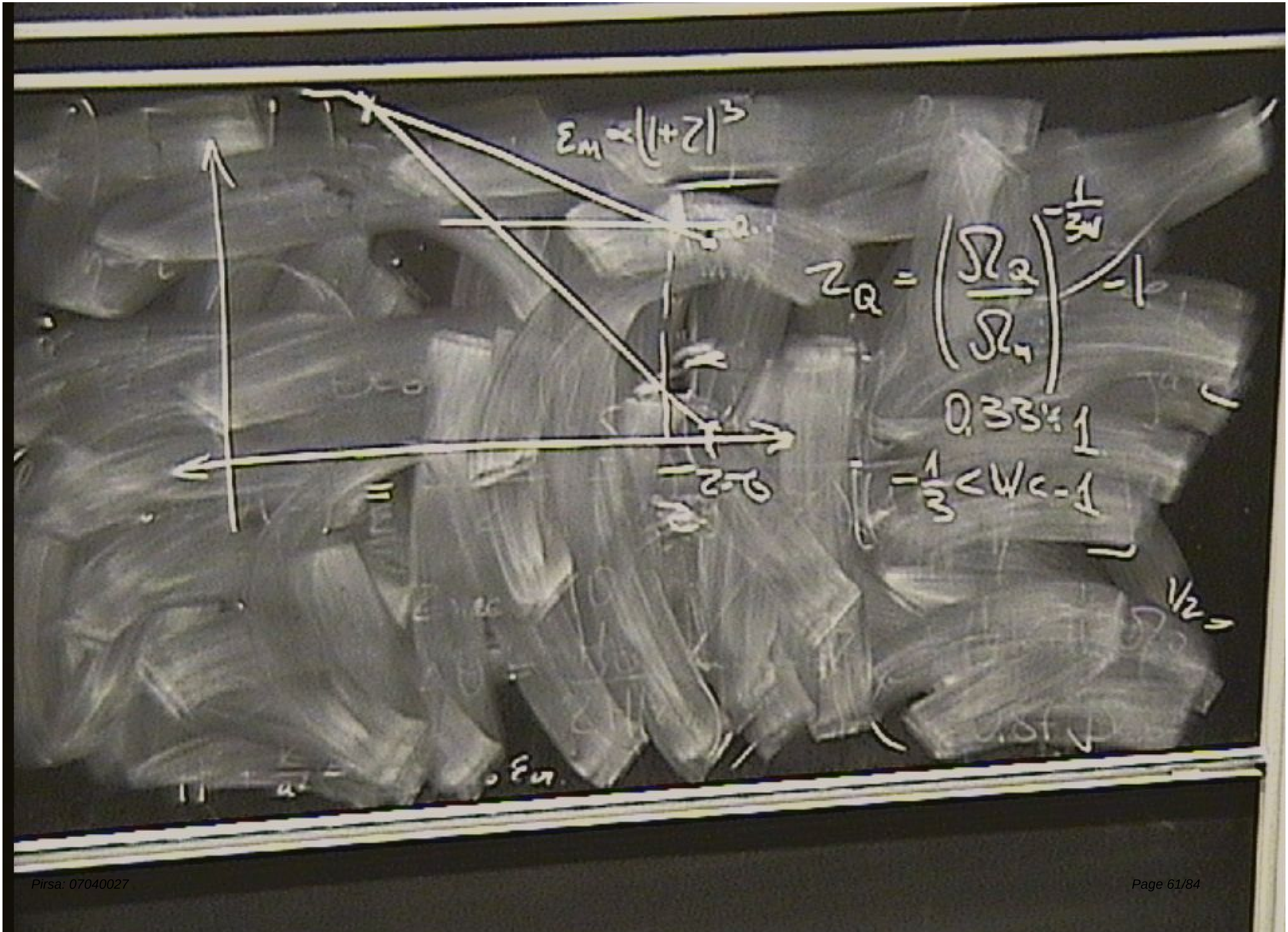
$w < -\frac{1}{3}$

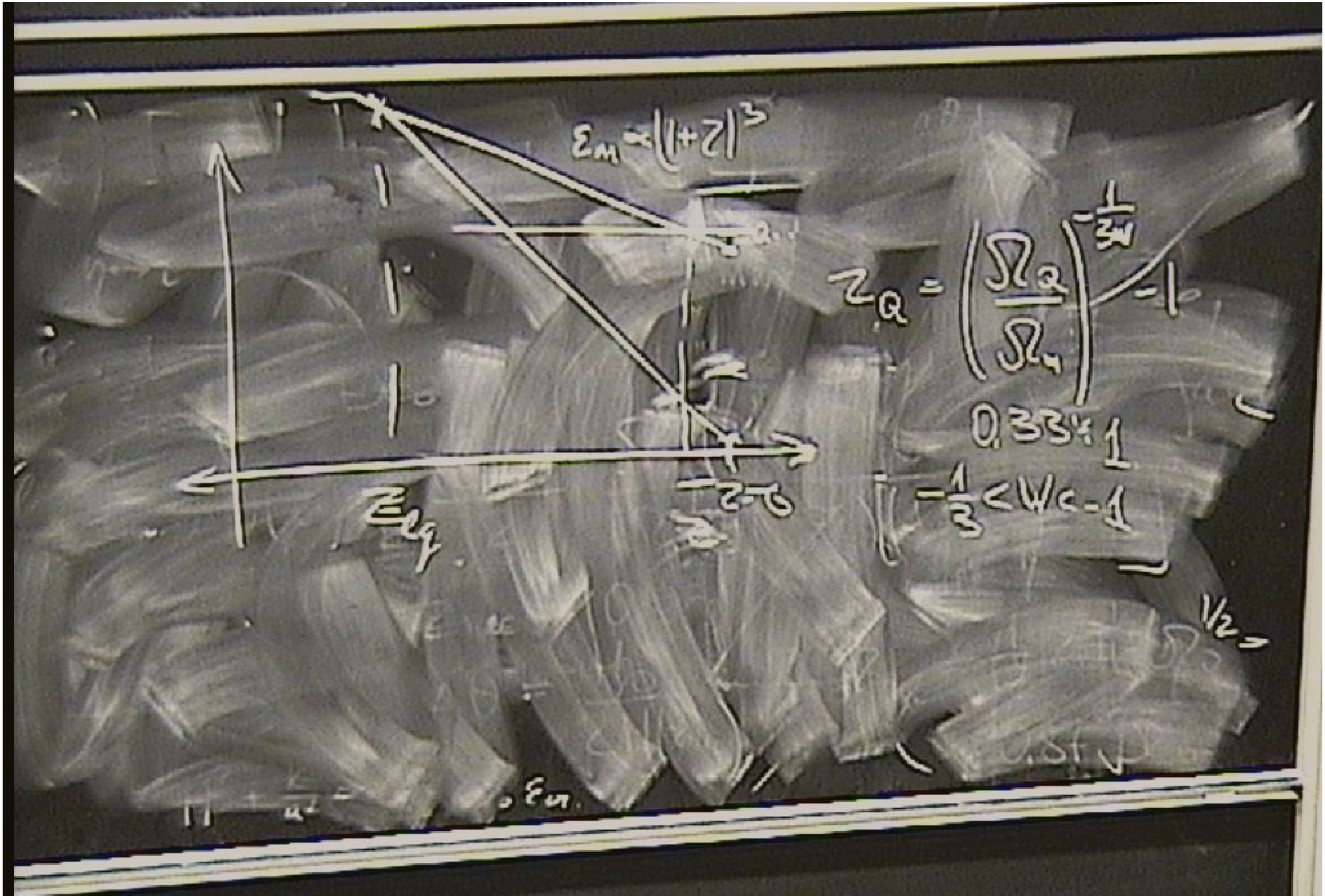
Rr. m.

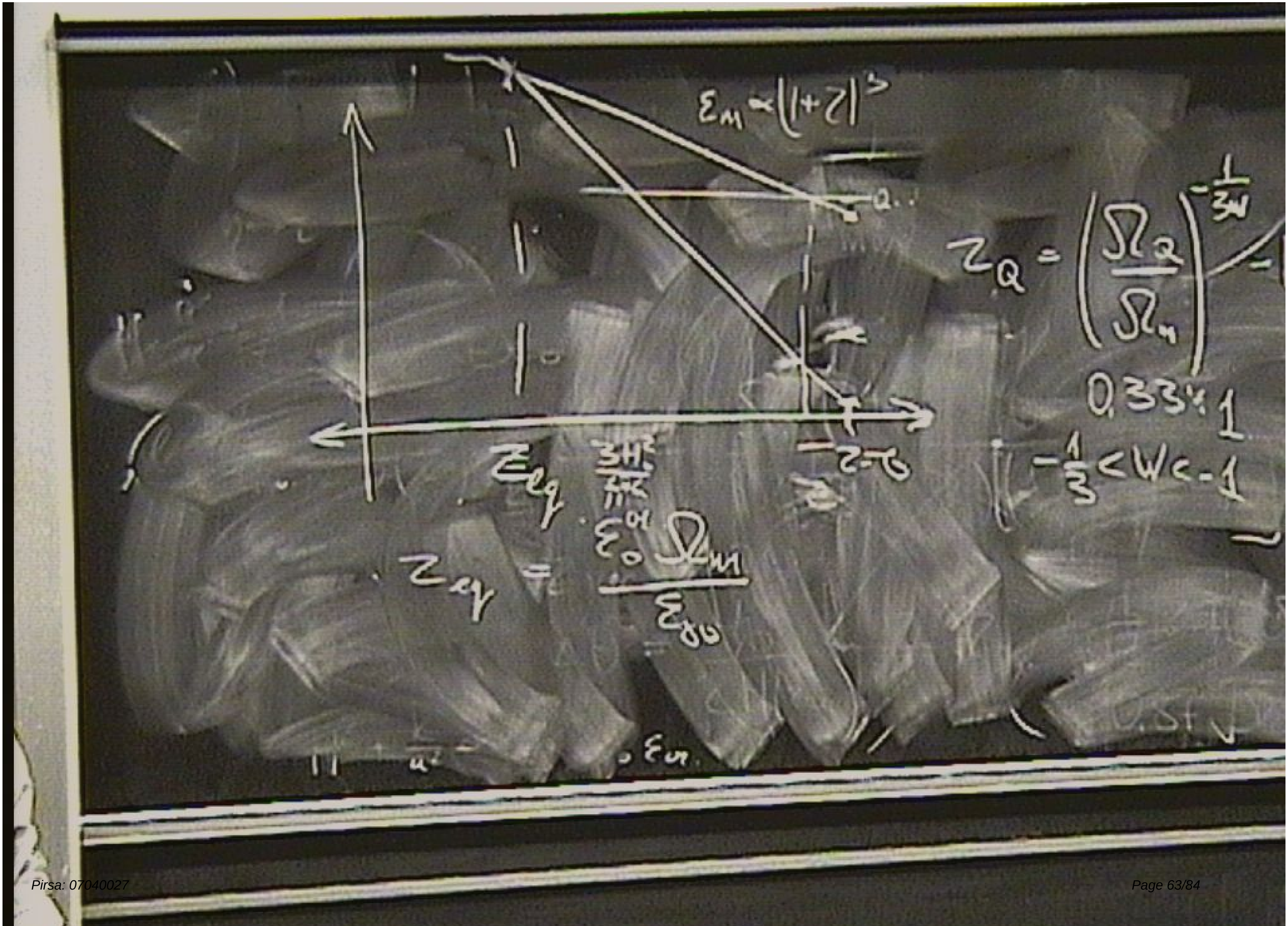


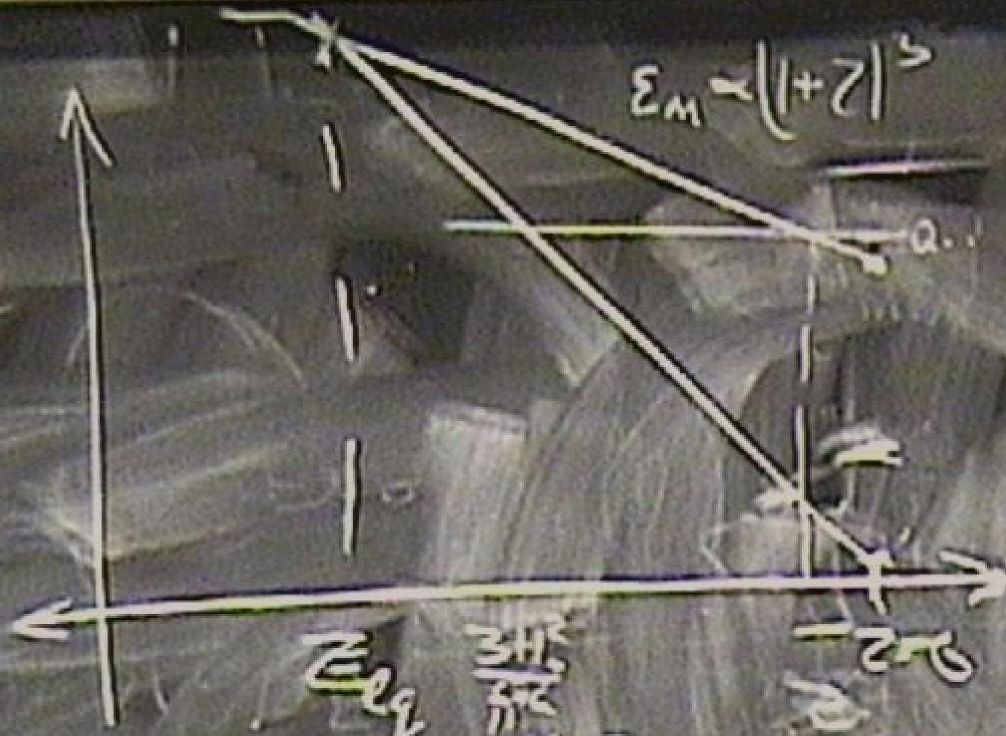


$\varepsilon_M \propto (1+z)^3$


$\varepsilon_M \propto (1+z)^3$
$z_Q = \left(\frac{\Omega_Q}{\Omega_M}\right)^{-\frac{1}{3w}} - 1$
0.33 : 1
z=0
$-\frac{1}{3} < w < -1$


$\varepsilon_m \propto (1+z)^3$
$z_Q = \left(\frac{\Omega_Q}{\Omega_M}\right)^{-\frac{1}{3w}} - 1$
0.33 : 1
$-\frac{1}{3} < w < -1$
$z = 0$
z_{eq}


εm ∝ (1+z)^3
ZQ = (ΩQ/Ωm)^(-1/3w)
0.33 : 1
-1/3 < w < -1
z=0
Zeq
3H²/8πG
Zeq = ε0 Ωm / εγ0



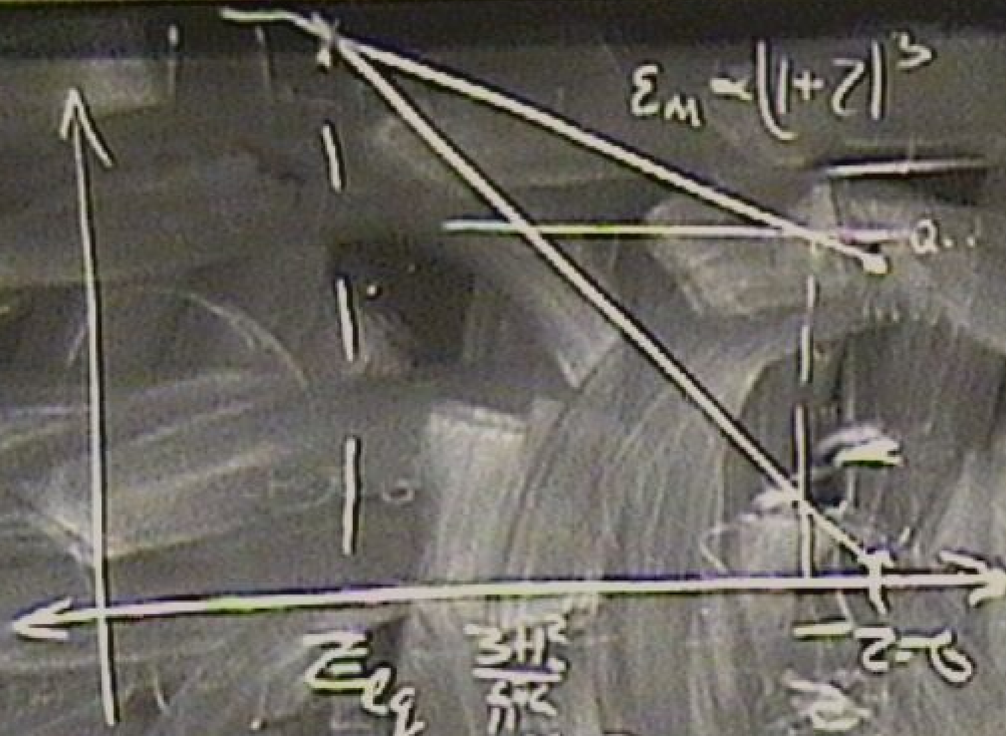
$$z_Q = \left(\frac{\Omega_Q}{\Omega_m}\right)^{-\frac{1}{3w}} - 1$$

$0.33 : 1$

$$-\frac{1}{3} < w < -\frac{1}{2}$$

$$z_{eq} = \frac{\varepsilon_0^{m}\,\Omega_m}{\varepsilon_{r0}} - 1 \approx 2.26\times10^4\,\Omega_m h_{75}^2$$

$h_{75} =$



$$\varepsilon_m \propto (1+z)^3$$

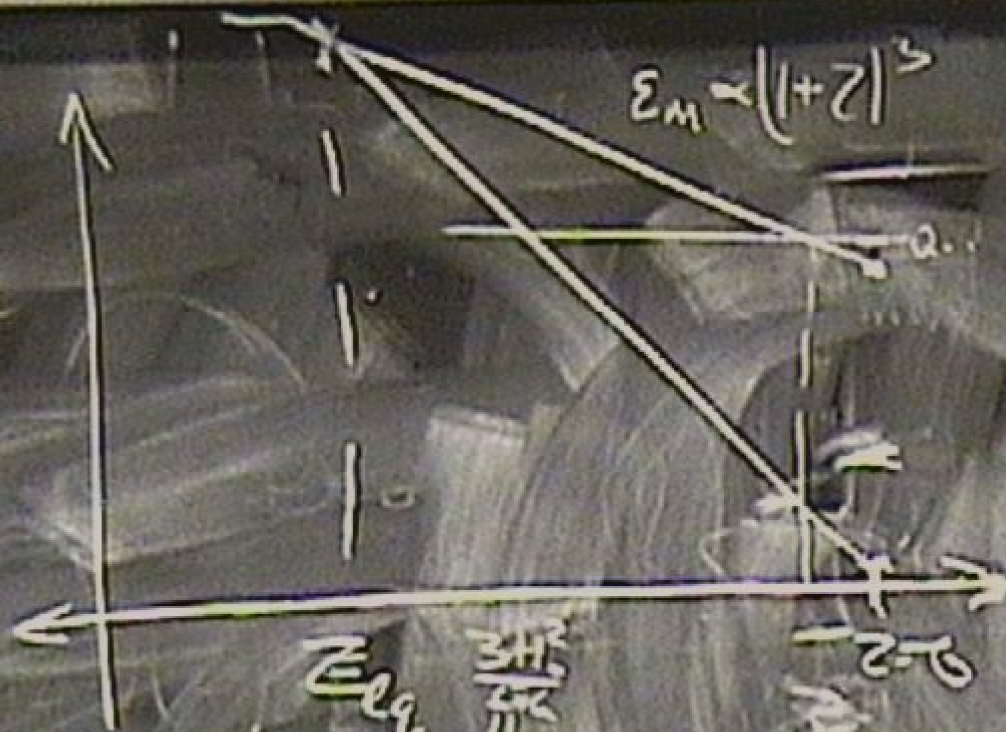
$$z_Q = \left(\frac{\Omega_Q}{\Omega_m}\right)^{-\frac{1}{3w}} - 1$$

$$0.33 : 1$$

$$-\frac{1}{3} < w < -\frac{1}{2}$$

$$z_{eq} = \frac{\varepsilon_0^{01}\,\Omega_m}{\varepsilon_{\gamma 0}} - 1 \approx 2.26\times 10^4\, \Omega_m\, h_{75}^{1/2}$$

$$h_{75} = \frac{H_0}{75}$$



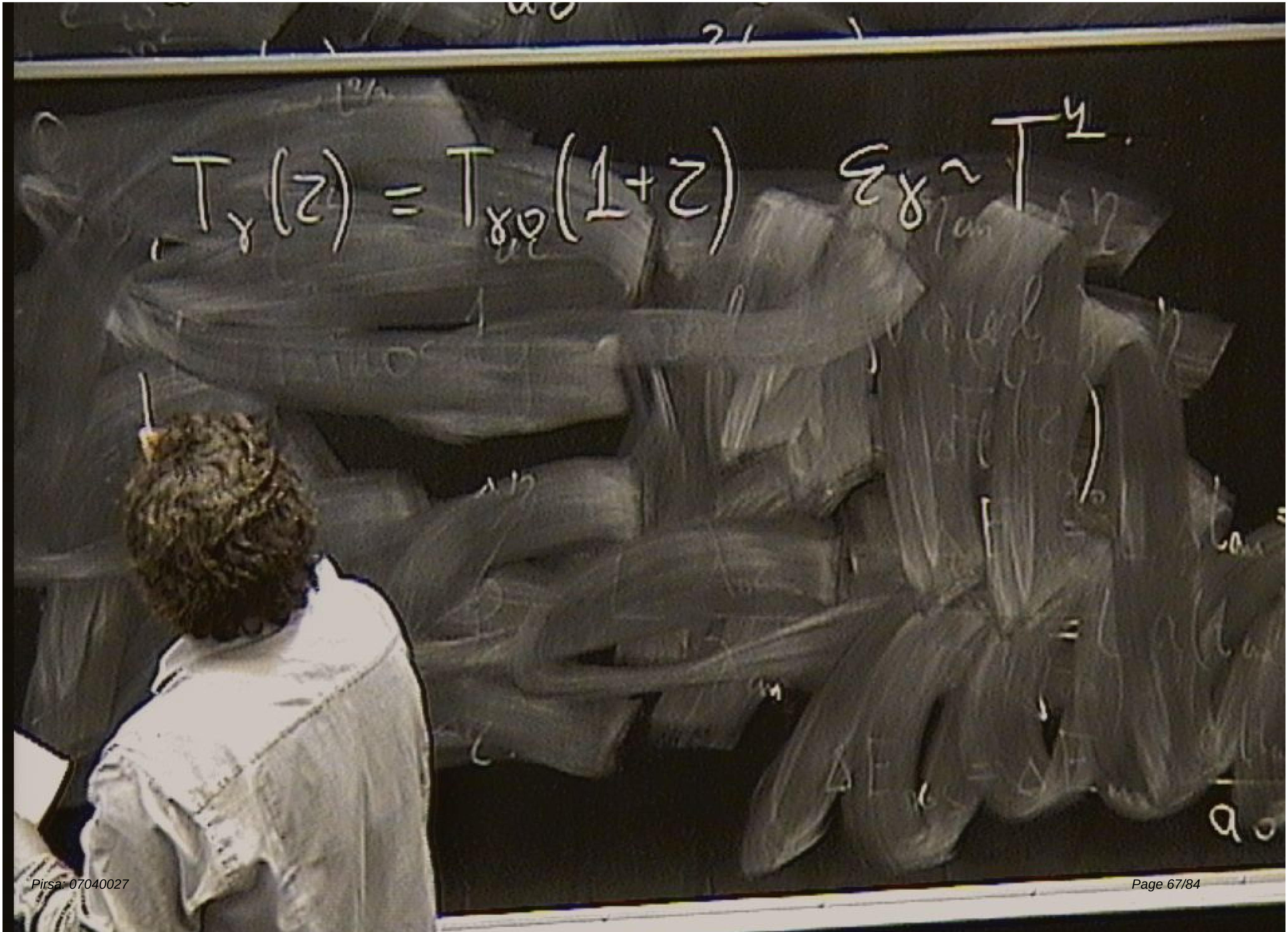
$$z_Q = \left(\frac{\Omega_Q}{\Omega_m}\right)^{-\frac{1}{3w}} - 1$$

$0.33 \div 1$

$-\frac{1}{3} < w < -\frac{1}{2}$

$$z_{eq} = \frac{\varepsilon_0 \Omega_m}{\varepsilon_{\gamma 0}} - 1 \approx 2.26 \times 10^4 \, \Omega_m \, h_{75}^{\ 1/2}$$

$$h_{75} = \frac{H_0}{75 \frac{km}{s \cdot Mpc}}$$


$T_\gamma(z) = T_{\gamma,0}(1+z)$
$\varepsilon_\gamma \sim T^4$

$$T_\gamma(z) = T_{\gamma,0}(1+z) \qquad \varepsilon_\gamma \sim T^4.$$

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{1}{4t^2} = \frac{8\pi G}{3} T_\gamma^4 \qquad T \propto \frac{1}{\sqrt{t}}$$

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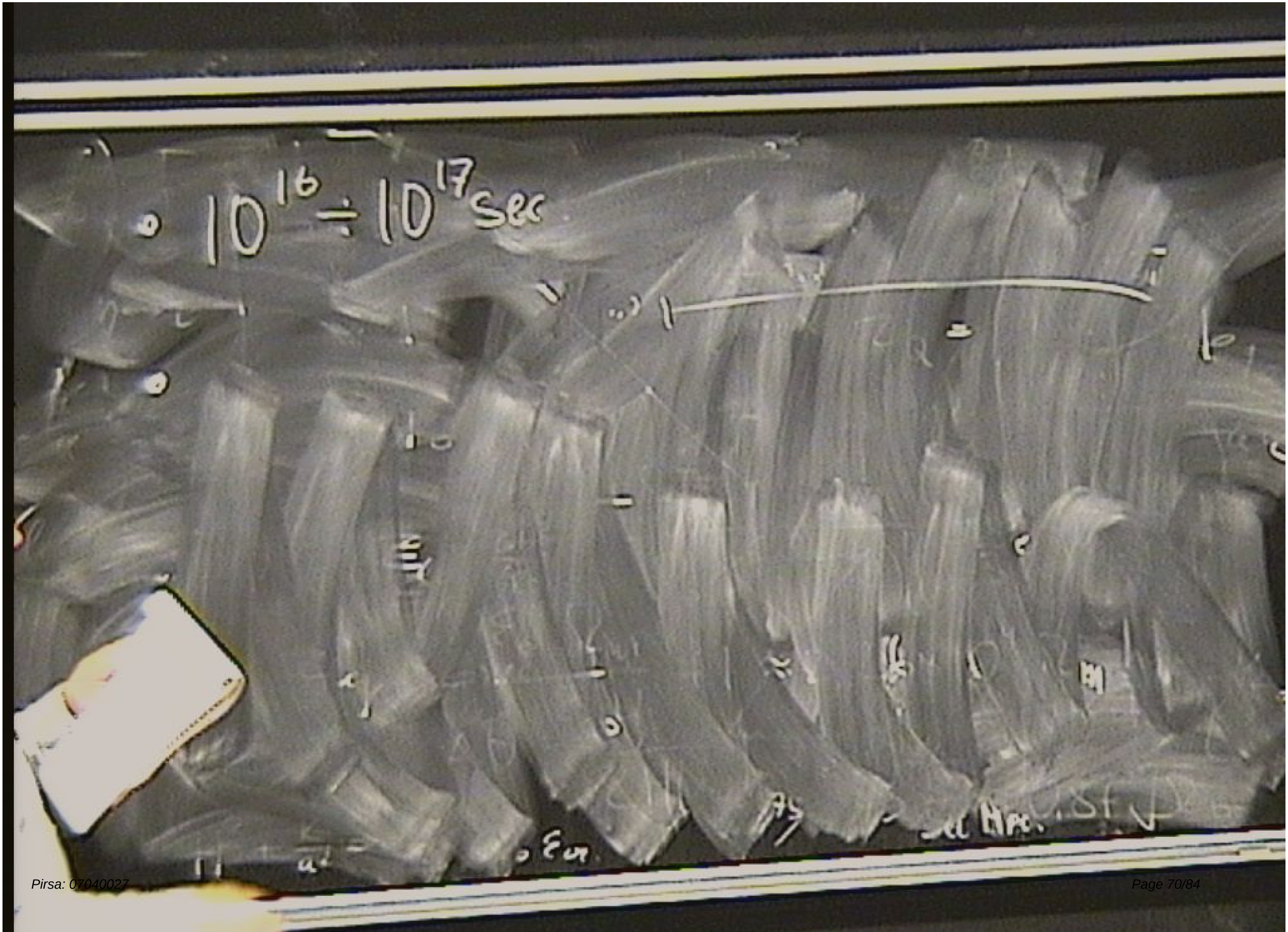
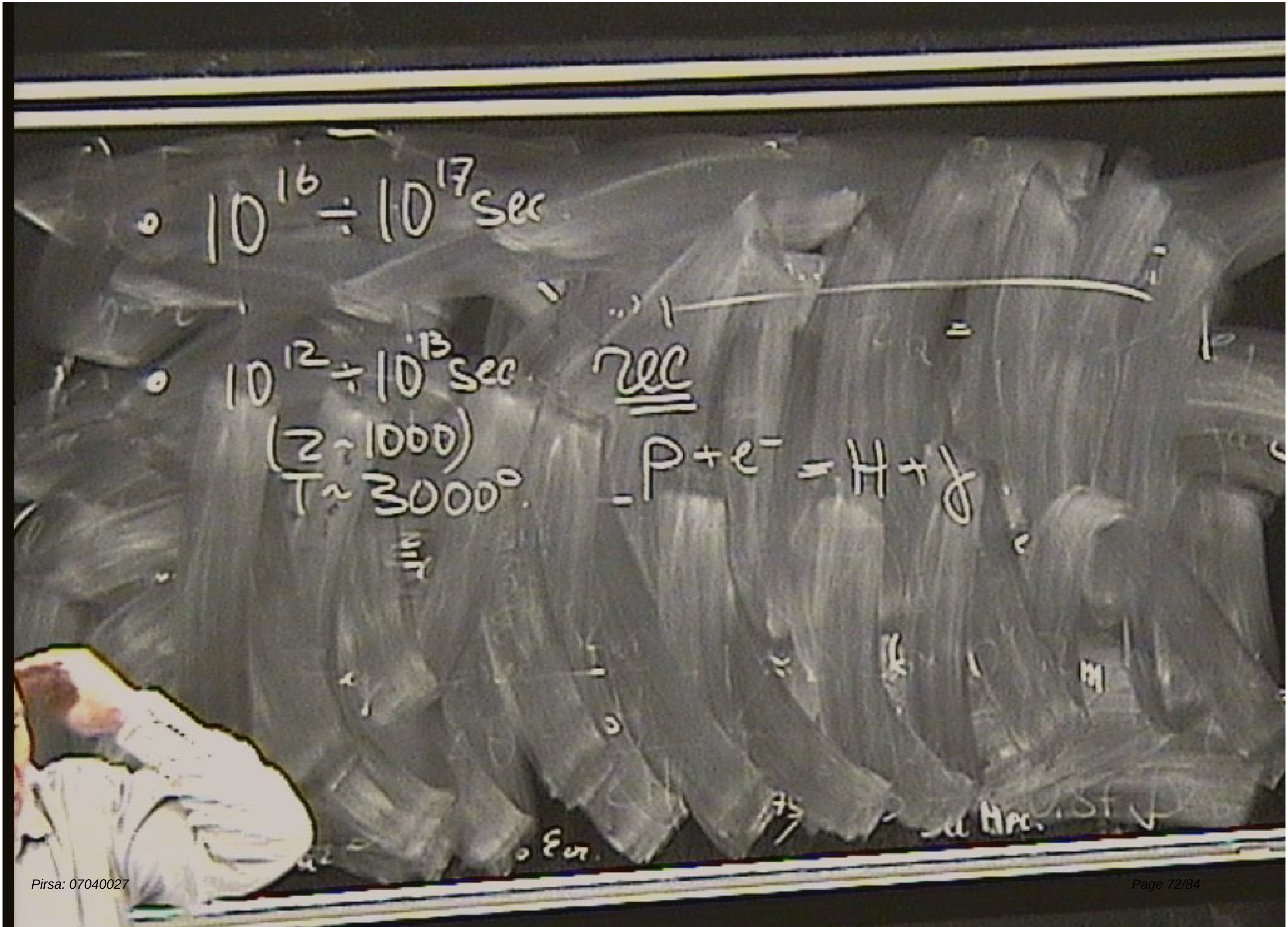
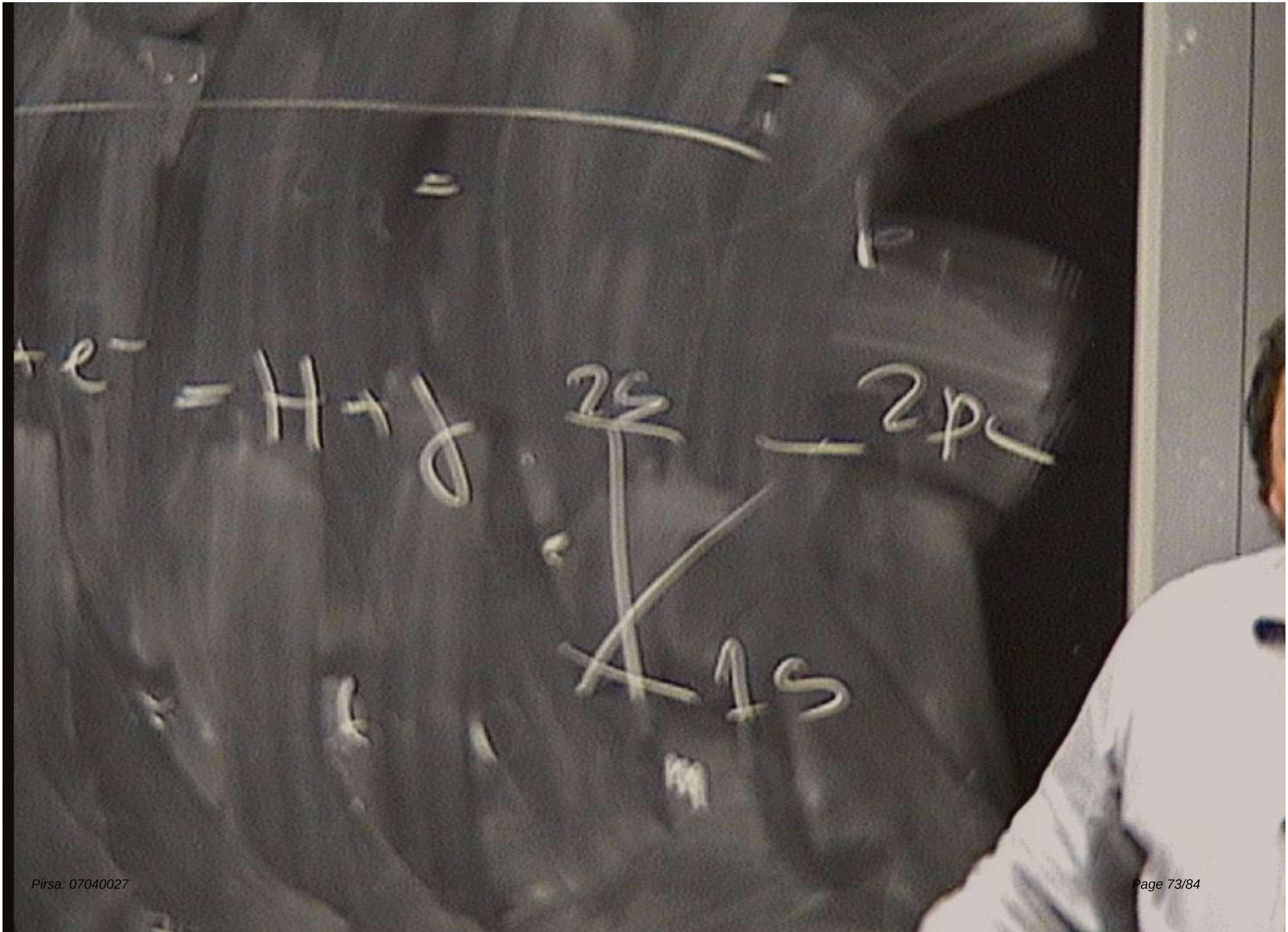
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{4t^2} = \frac{8\pi G}{3} T_\gamma^4 \qquad T_{\mathrm{MeV}} = \frac{O(1)}{\sqrt{t}}$$


$10^{16} \div 10^{17}$ sec

- $10^{16} \div 10^{17}$ sec

- $10^{12} \div 10^{13}$ sec
  $(z \sim 1000)$
  $T \sim 3000^\circ$


• $10^{16} \div 10^{17}$ sec
• $10^{12} \div 10^{13}$ sec
($z \sim 1000$)
$T \sim 3000°$
rec
$p + e^- = H + \gamma$


+ e⁻ = H + γ
2s
2p
1s

- $10^{16} \div 10^{17}$ sec

- $10^{12} \div 10^{13}$ sec rec

  $(z \sim 1000)$

  $T \sim 3000°$

  $p + e^- \rightleftharpoons H + \gamma$

- 10^{11} sec

  $\varepsilon_\gamma \sim \varepsilon_m$

  1s

$\Delta E_{dm} = L$

- $200 \div 300$ sec $(T \sim 0.05)$

• $200 \div 300$ sec $(T \sim 0.05)$

He^4, De, He^3,

• $\underline{\underline{1 \sec}}$ $(T \sim 0.5\,\text{Mev})$

• $200 \div 300$ sec $(T \sim 0.05)$

He^4, De, He^3,

• 1 sec $(T \sim 0.5\ \text{Mev})$ e^+, e^- rea...

- $200 \div 300$ sec $(T \sim 0.05)$

He^4, De, He^3, Li

- $\underline{1\,\text{sec}}$ $(T \sim 0.5\,\text{Mev})$ e^+, e^- rea

- 0.1 se $(T \sim 1 \div 2\,\text{Mev})$

• 200 ÷ 300 sec (T ~ 0.05)

He^4, De, He^3

• 1 sec (T ~ 0.5 MeV) e^+, e^- rea...

• 0.1 se (T ~ 1 ÷ 2 MeV) weak in

a) primord...

б) $n + e^+ \to p + \tilde{\nu}$

• $200 \div 300$ sec $(T \sim 0.05)$

He^4, De, He^3

• 1 sec $(T \sim 0.5\,\text{MeV})$ e^+, e^- rea

• 0.1 se $(T \sim 1 \div 2\,\text{MeV})$ weak in

a) primord

b) $n + e^+ \to p + \bar{\nu}$

• $200 \div 300$ sec ($T \sim 0.05$)

He^4, De, He^3

• 1 sec ($T \sim 0.5$ Mev) e^+, e^- rea

• 0.1 se ($T \sim 1 \div 2$ Mev) break in

a) primord

b) $n + e^+ \to p + \bar{\nu}$

$\sim 10^{-5}$ s ($T \sim 200$ Mev)

$\bar{q}$

$0.2\,se$ $(T \sim 1 \div 2\,\mathrm{Mev})$ weak

a) pri

n

$10^{-5}\,s$ $(T \sim 200\,\mathrm{Mev})$

$10^{-10}\,sec - 10^{-14}\,sc$ $(100\,\mathrm{Gev} - 10\,\mathrm{Tev})$

$$4\pi a_0^2 \Phi^2(\chi_{em}(z))(1+z)^2$$

$\Delta\ 10^{-14}\,\mathrm{sec} - 10^{-2\,4/3}\,\mathrm{sec}$

$$= \frac{\Delta E}{S_{ch}\ \Delta t_{obs}} = \frac{}{4\pi\ \Phi(\chi_{em}(z))}\ \frac{a^2(\ )}{a_0^4} =$$

$$\boxed{(\ ) = \frac{L}{4\pi a_0^2\, \Phi^2(\chi_{em}(z))\,(1+z)^2}}$$

10^{-14} sec $- 10^{-2}$ 4/3 sec

Inflation 10^{-35} sec

$$= \frac{E}{S \, \Delta t_{phys}} = \frac{}{4\pi \, \Phi(\chi_{end})} \frac{(\;)}{a_0^2} =$$

$$(\;) = \frac{L}{4\pi a_0^2 \, \Phi^2(\chi_{em}(z)) \, (1+z)^2}$$