Title: Advanced Topics in Cosmology

Date: Apr 26, 2007 10:00 AM

URL: http://pirsa.org/07040024

Abstract: Class 2

$$\underline{\underline{\kappa=1}} \quad ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \Phi^2(\chi)\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

$$\overset{\eta = \int \frac{dt}{a}}{=} a^2(\eta)\left[d\eta^2 - d\chi^2 - \Phi^2(\chi)\, d\Omega^2\right]$$

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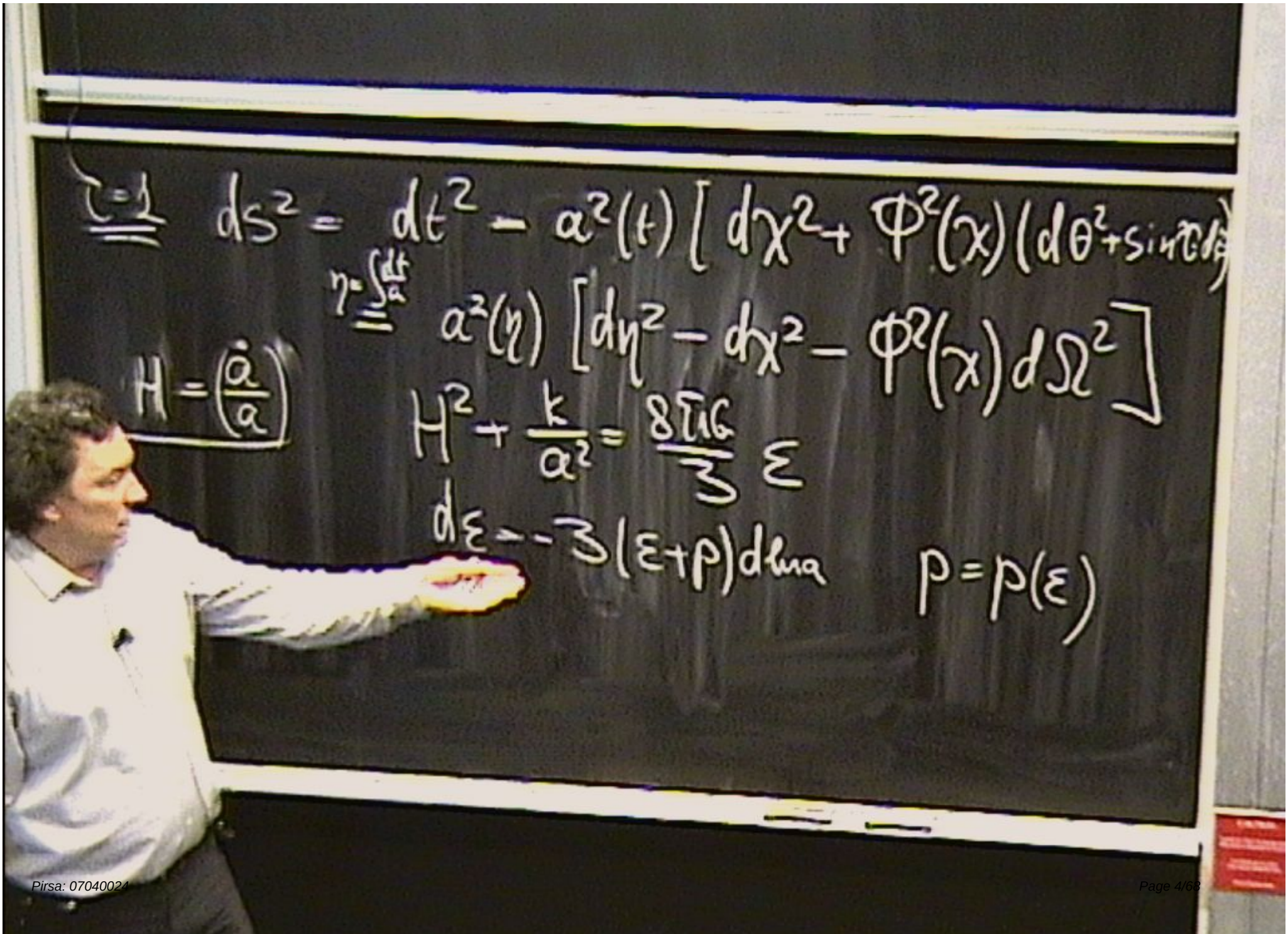

$\underline{\underline{\tau=1}}$ $ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \Phi^2(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$
$\eta = \int \frac{dt}{a}$
$= a^2(\eta)\left[d\eta^2 - d\chi^2 - \Phi^2(\chi)\, d\Omega^2\right]$
$H = \left(\frac{\dot{a}}{a}\right)$
$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\varepsilon$
$d\varepsilon = -3(\varepsilon + p)\, d\ln a$
$p = p(\varepsilon)$

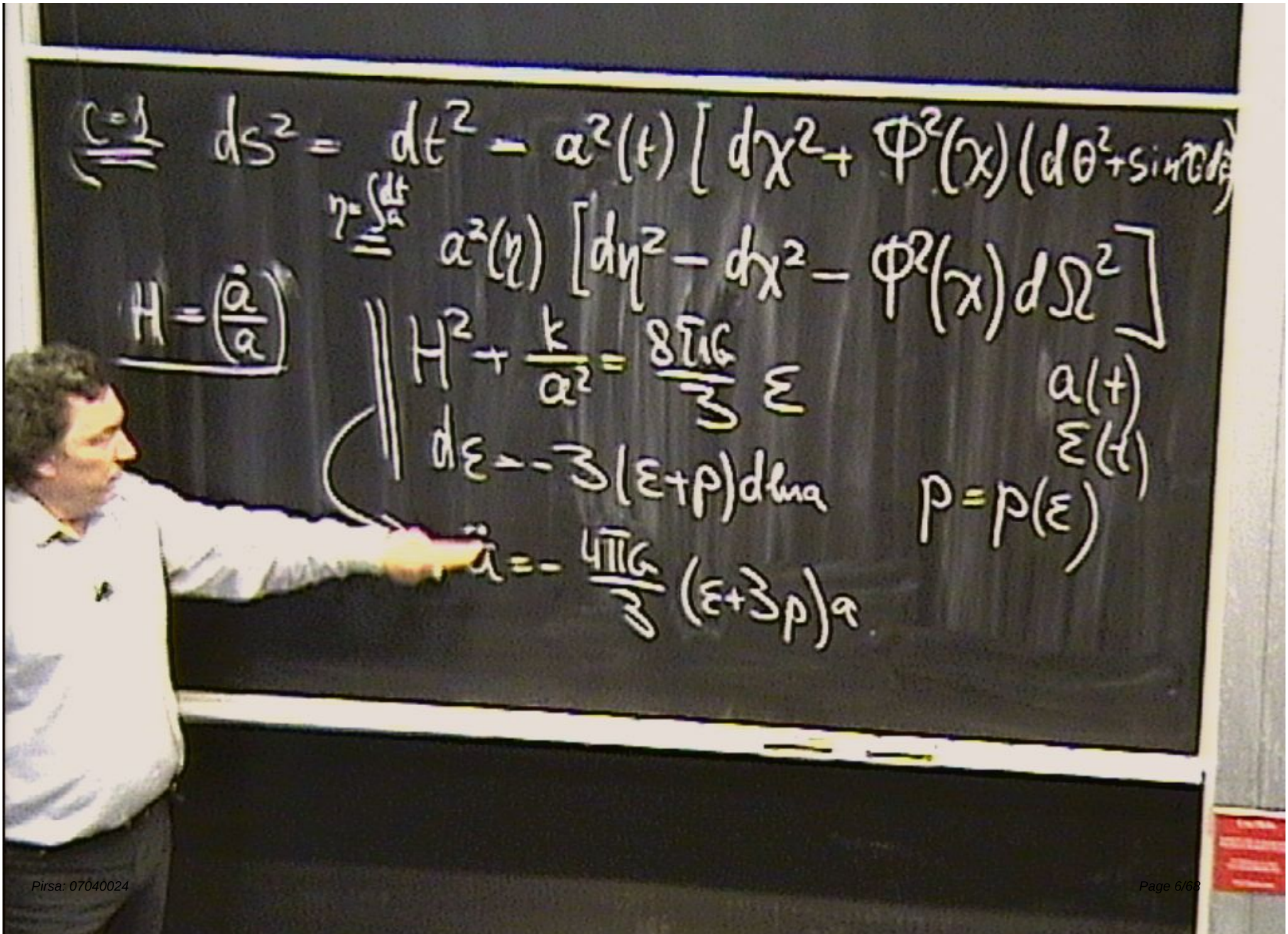
$\underline{\underline{c=1}}$ $\quad ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \Phi^2(\chi)(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$

$$\overset{\eta = \int \frac{dt}{a}}{=} a^2(\eta)\left[d\eta^2 - d\chi^2 - \Phi^2(\chi)\, d\Omega^2\right]$$

$\underline{H = \left(\frac{\dot a}{a}\right)}$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\varepsilon \qquad a(t)$$

$$d\varepsilon = -3(\varepsilon + p)\, d\ln a \qquad \varepsilon(t) \qquad p = p(\varepsilon)$$


c=1
ds² = dt² − a²(t) [dχ² + Φ²(χ) (dθ² + sin²θ dφ²)
η = ∫ dt/a
= a²(η) [dη² − dχ² − Φ²(χ) dΩ²]
H = (ȧ/a)
H² + k/a² = 8πG/3 ε
dε = −3(ε+p) d ln a
ä = − 4πG/3 (ε+3p) a
a(t)
ε(t)
p = p(ε)

$\underline{c=1}$ $\quad ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \Phi^2(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$

$$\overset{\eta = \int \frac{dt}{a}}{=} a^2(\eta)\left[d\eta^2 - d\chi^2 - \Phi^2(\chi)\, d\Omega^2\right]$$

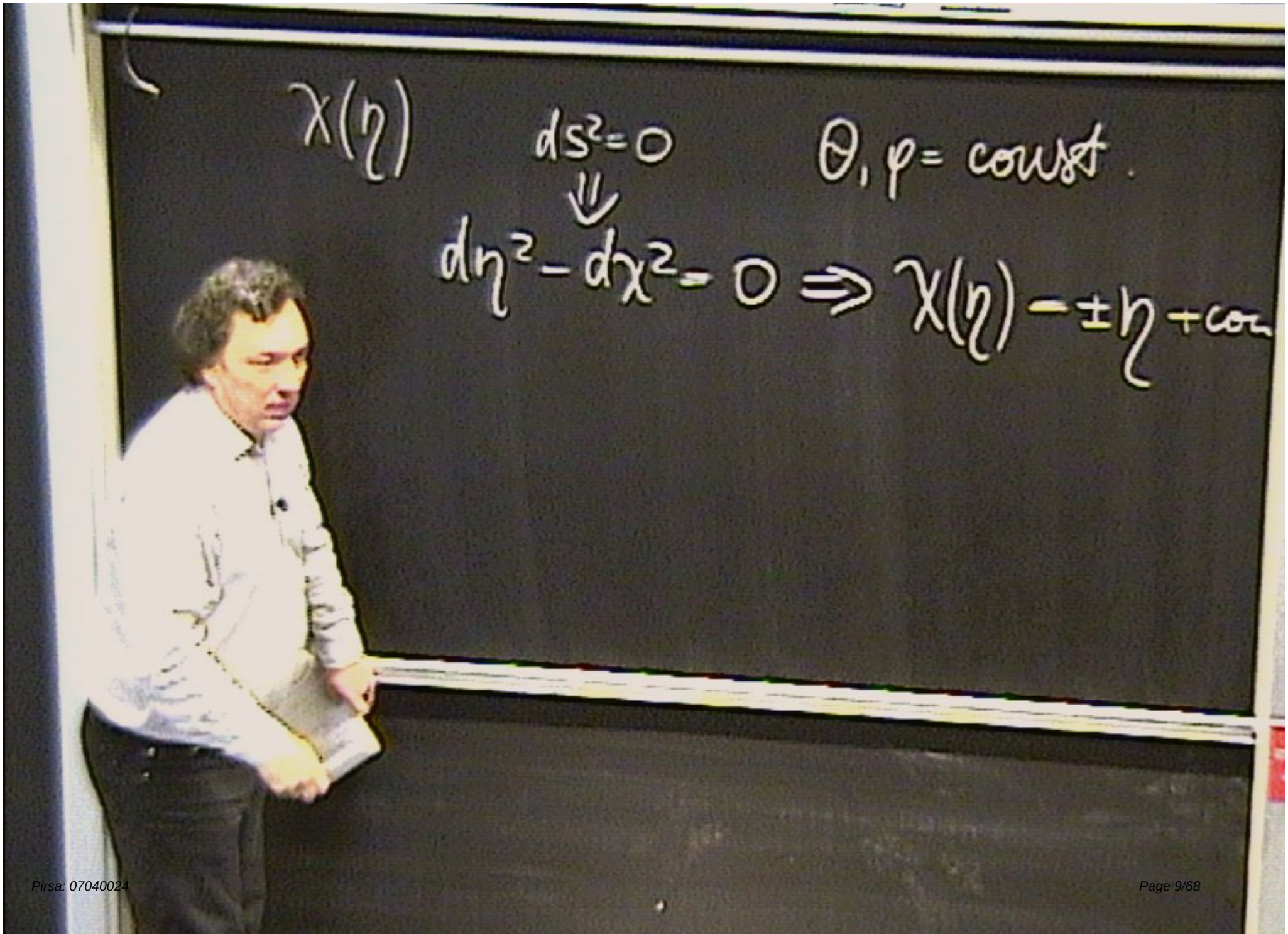
$$\underline{H = \left(\frac{\dot a}{a}\right)}$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\varepsilon \qquad a(t)$$

$$d\varepsilon = -3(\varepsilon + p)\, d\ln a \qquad \varepsilon(t) \qquad p = p(\varepsilon)$$

$$\ddot a = -\frac{4\pi G}{3}(\varepsilon + 3p)\, a$$

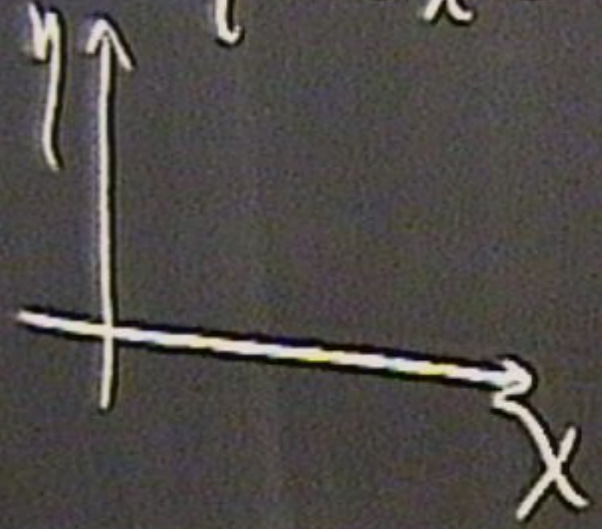
$x(\eta)$ $\quad ds^2 = 0 \quad$ $\theta, \varphi = \text{const}.$

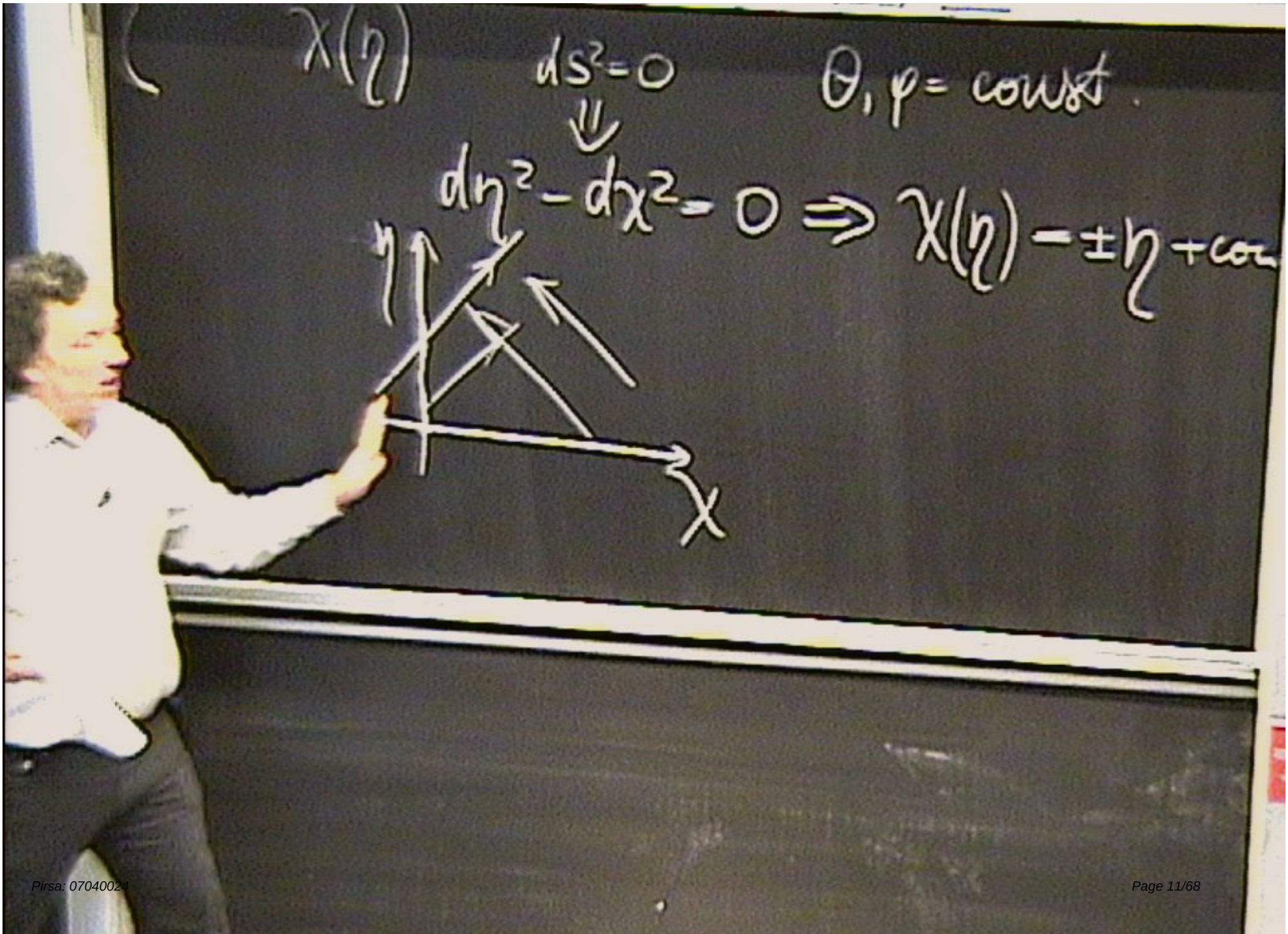

$x(\eta)$
$ds^2 = 0$
$\Downarrow$
$d\eta^2 - dx^2 = 0 \Rightarrow x(\eta) = \pm\eta + \text{con}$
$\theta, \varphi = \text{const}.$

$x(\eta)$ $\quad ds^2 = 0 \quad \theta, \varphi = \text{const}.$

$$\Downarrow$$

$$d\eta^2 - dx^2 = 0 \Rightarrow x(\eta) = \pm\eta + \text{const}$$




$x(\eta)$
$ds^2 = 0$
$\Downarrow$
$d\eta^2 - dx^2 = 0 \Rightarrow x(\eta) = \pm\eta + \text{const}$
$\theta, \varphi = \text{const}$.
η
x

$$-dx^2 = 0 \Rightarrow x(\eta) = \pm\eta + \text{con}$$

$$\underset{-\infty\ +\infty}{x} \to \tilde{x} = \operatorname{artg} x$$

x

Horizons

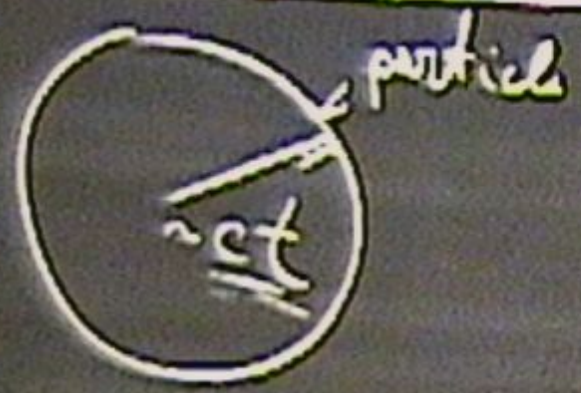
a) Particle horizon.

Horizons

a) Particle horizon.

Horizons

a) Particle horizon.



$\eta = \eta_i$

χ

Horizons

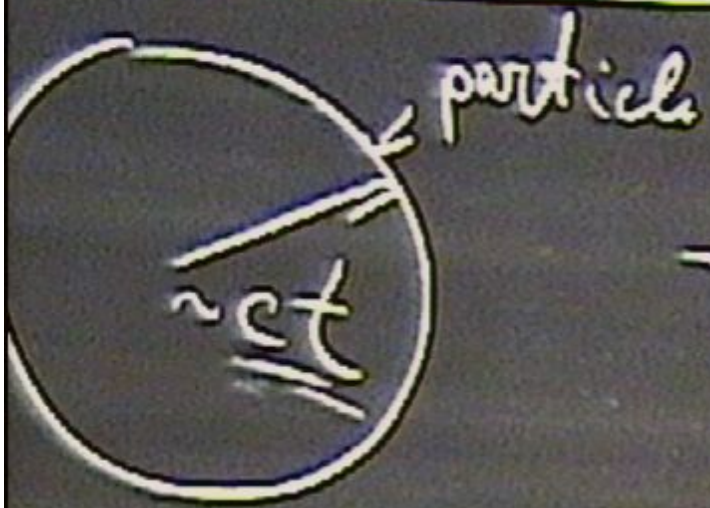
a) Particle horizon.

$$\eta = \eta_i$$

$$\chi_p(\eta) = \eta - \eta_i$$

Horizons

Particle horizon.

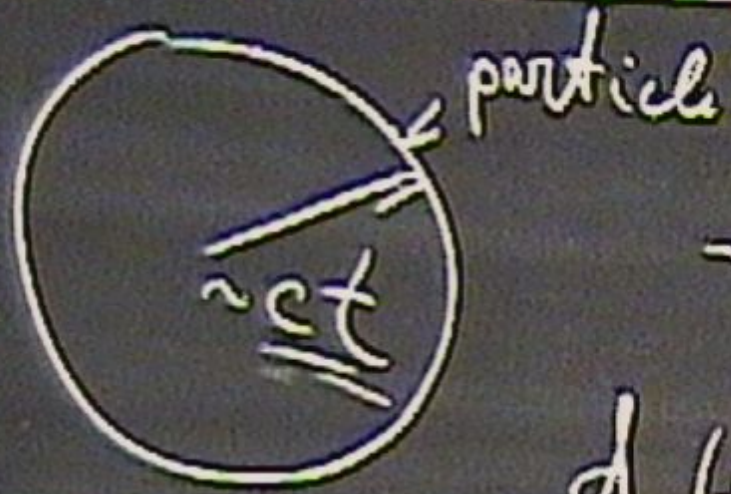


$$\eta = \eta_i$$

$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a}.$$

Horizons

a) Particle horizon.



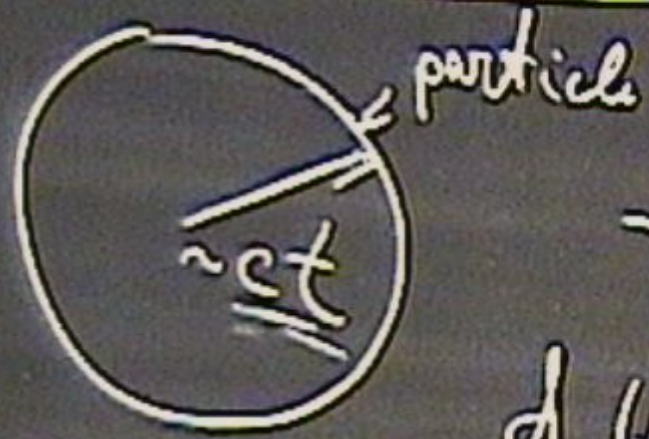
$$\eta = \eta_i$$

$$\chi_p(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a}.$$

$$d_p(t) = a(t)\,\chi_p$$

Horizons

a) Particle horizon.



$$\eta = \eta_i$$

$$\chi_p(\eta) = \eta - \eta_i = \int^t \frac{dt}{a}$$

$$d_p(t) = a(t)\,\chi_p(t) = a(t)\int_{t_i}^{t} \frac{dt}{a}.$$

$p=0,\ k=0 \quad a \propto t^{2/3} \qquad d_p(t) = 3\ t$

$p=0,\ k=0 \quad a \propto t^{2/3} \qquad d_p(t) = 3ct$

$p=\frac{5}{3} \quad a \propto t^{1/2} \qquad d_p = 2ct.$

$p=0,\ k=0 \quad a \propto t^{2/3} \qquad d_p(t) = 3ct$

$p=\frac{2}{3} \quad a \propto t^{1/2} \qquad d_p = 2ct.$

$\chi_p = \eta - \eta_i$

$p=0,\ k=0 \quad a \propto t^{2/3}$

$p=\frac{4}{3} \quad a \propto t^{1/2}$

$\chi_p = \eta - \eta_i$

$d_p(t) = 3ct$

$d_p = 2ct.$

$a = \frac{1}{H\eta}$

$p=0,\ k=0 \quad a \propto t^{2/3}$

$p=\frac{2}{3} \quad a \propto t^{1/2}$

$\chi_p = \eta - \eta_i$

$d_p(t) = H_\Lambda^{-1}$

$d_p(t) = 3ct$

$d_p = 2ct.$

$a \propto \frac{1}{H_\Lambda \eta} \qquad a(t) \propto e^{H_\Lambda t}.$

$p=0,\ k=0 \quad a \propto t^{2/3} \qquad d_p(t) = 3c\,t$

$p=\frac{\xi}{3} \quad a \propto t^{1/2} \qquad d_p = 2ct.$

$\chi_p = \eta - \eta_i \qquad a = -\frac{1}{H_\Lambda \eta} \qquad a(t) \propto e^{H_\Lambda t}$

$$d_p(t) = H_\Lambda^{-1} \exp(H_\Lambda t) \int_{t_i}^{t} \frac{dt}{H_\Lambda^{-1} \exp(H_\Lambda t)} =$$

$$= H_\Lambda^{-1} \left[\exp\left(H_\Lambda (t - t_i)\right) - 1 \right]$$

$p=0,\ k=0 \quad a \propto t^{2/3} \qquad d_p(t) = 3c\,t$

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$$= H_\Lambda^{-1} \left[\exp\left(H_\Lambda (t - t_i)\right) - 1 \right]$$

b) Event horizon.

$\eta \to t$

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$\eta \leftrightarrow t$

$\eta = \eta_{max}$

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b) Event horizon.

$\eta \leftrightarrow t$

$$\eta = \eta_{max}$$

$$\chi_e(\eta) = \eta_{max} - \eta = \int_t^{t_{max}} \frac{dt}{a} = \int_a^{\infty} \frac{da}{\dot{a}\, a}$$

$t_{max} \to \infty$, $\frac{dt}{a} = \frac{da}{\dot{a}}$

b) Event horizon.

$\eta \leftrightarrow t$

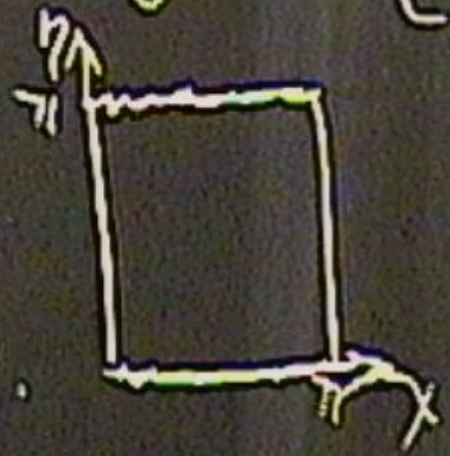
$$\eta = \eta_{max}$$

$$\chi_e(\eta) = \eta_{max} - \eta = \int_t^{t_{max}} \frac{dt}{a} = \int_a^{\infty} \frac{da}{\dot{a}\, a}$$

$\frac{dt}{a} = \frac{da}{\dot{a}}$

$p = \frac{\varepsilon}{3}$ $k=+1$

$a = a_m \sin\eta$.



b) Event horizon.

$\eta \leftrightarrow t$

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$$\chi_e(\eta) = \eta_{max} - \eta = \int_t^{t_{max}} \frac{dt}{a} = \int_a^{\infty} \frac{da}{\dot{a}a}$$

$\frac{dt}{a} = \frac{da}{\dot{a}}$

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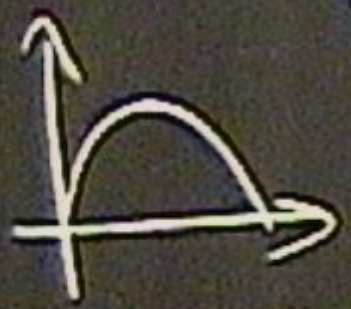


$a = a_m \sin\eta$.

$\chi = \eta$

b) Event horizon.

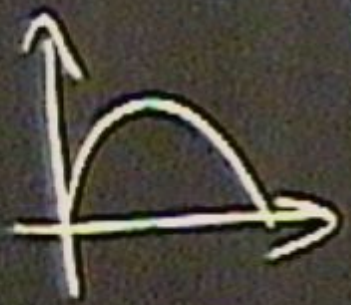
$\eta \to t$

$$\eta = \eta_{max}$$

$$\chi_e(\eta) = \eta_{max} - \eta = \int_t^{t_{max}} \frac{dt}{a} = \int_a^{\infty} \frac{da}{\dot{a}\, a}$$

$\dot{a} = \frac{da}{a}$, $t_{max} \to \infty$

$p = \frac{\varepsilon}{3}$ $k \to 1$



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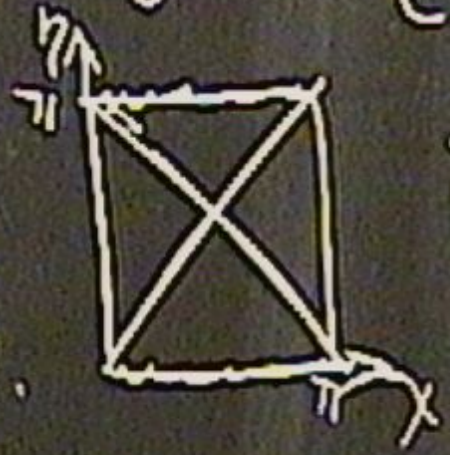
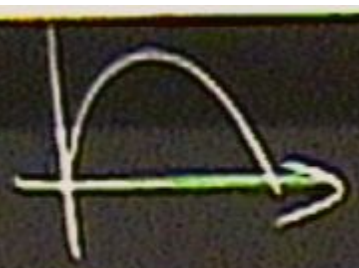
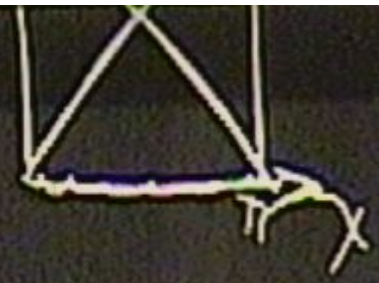


解 η

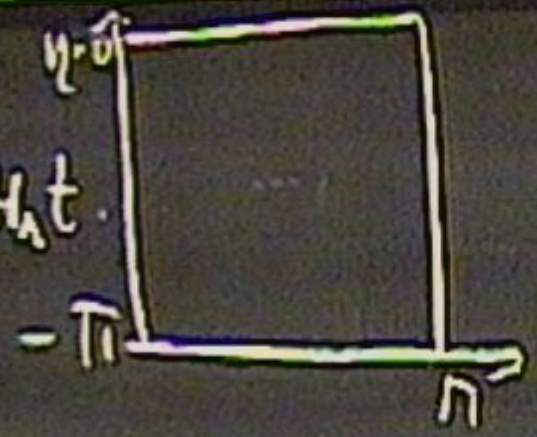
$\chi_e(\eta) = \pi - \eta$.



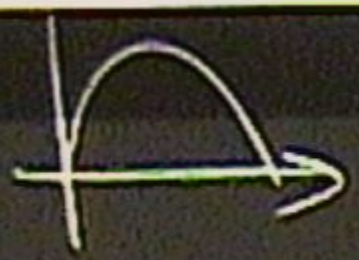
$a = a_m \sin\eta$.



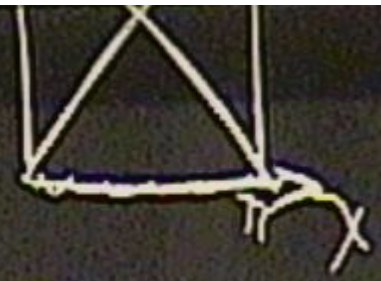
$\chi_c(\eta) = \pi$



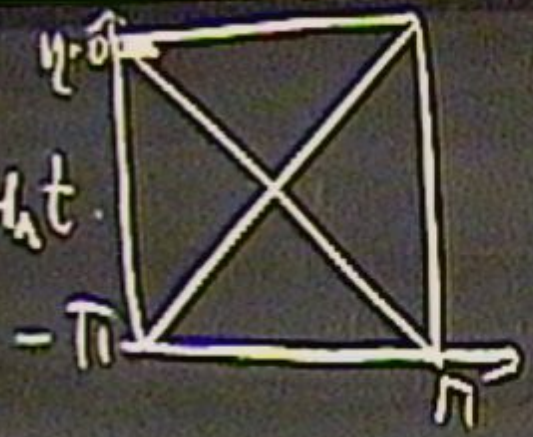
$a \propto \mathrm{ch}\, H_h t$.



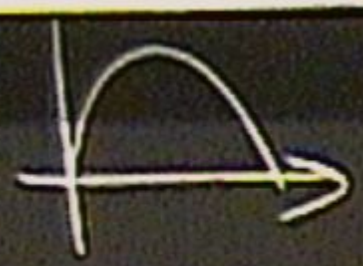
$a = a_m \sin\eta$.



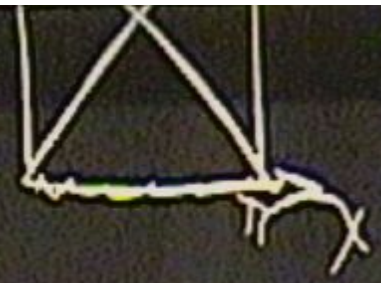
$\chi_c(\eta) = \pi$



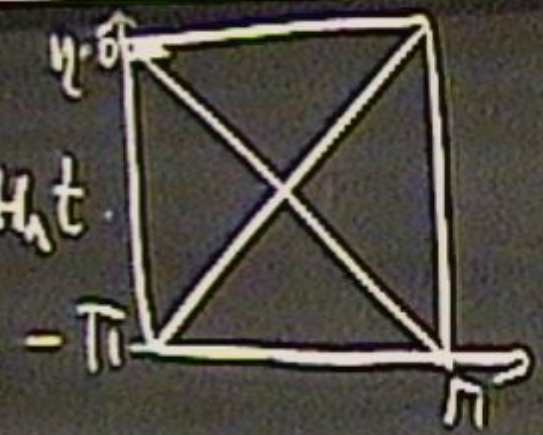
$a \propto \mathrm{ch}\, H_0 t$.



$a = a_m \sin\eta$



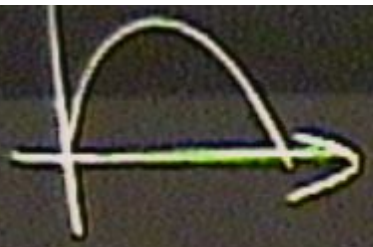
$\chi_c(\eta) = T$



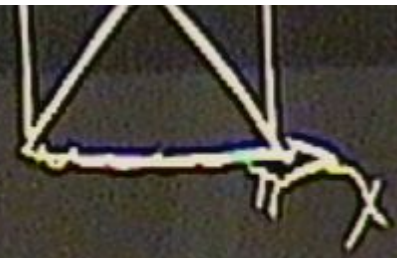
$a \propto d H_\Lambda t$

$a(t) \propto e^{H_\Lambda t}$

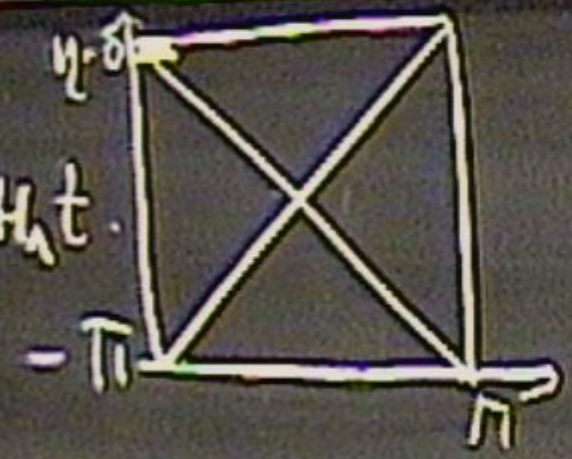
$d_c(t) = e^{H_\Lambda t} \int_t^\infty \frac{dt}{e^{H_\Lambda t}} = H_\Lambda^{-1}$



$a = a_m \sin\eta$.



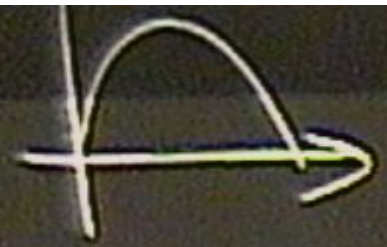
$\chi_c(\eta) = \pi$



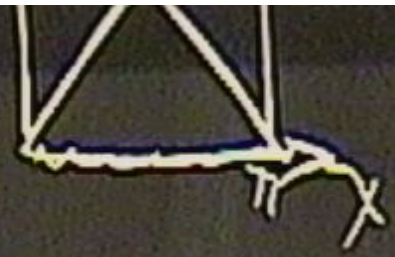
$a \propto \cosh H_\Lambda t$.

$a(t) \propto e^{H_\Lambda t}$.

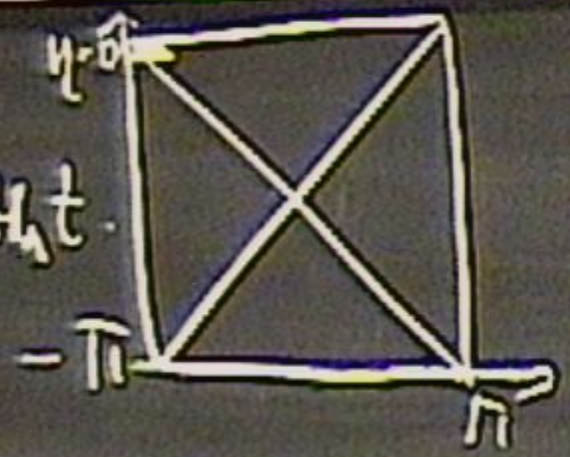
$d_c(t) = e^{H_\Lambda t} \int_t^\infty \frac{dt}{e^{H_\Lambda t}} = H_\Lambda^{-1}$



$a = a_m \sin\eta$.



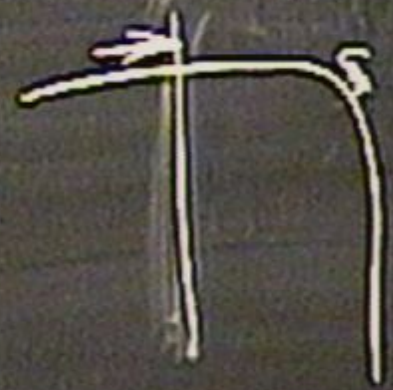
$\chi_c(\eta) = \pi -$

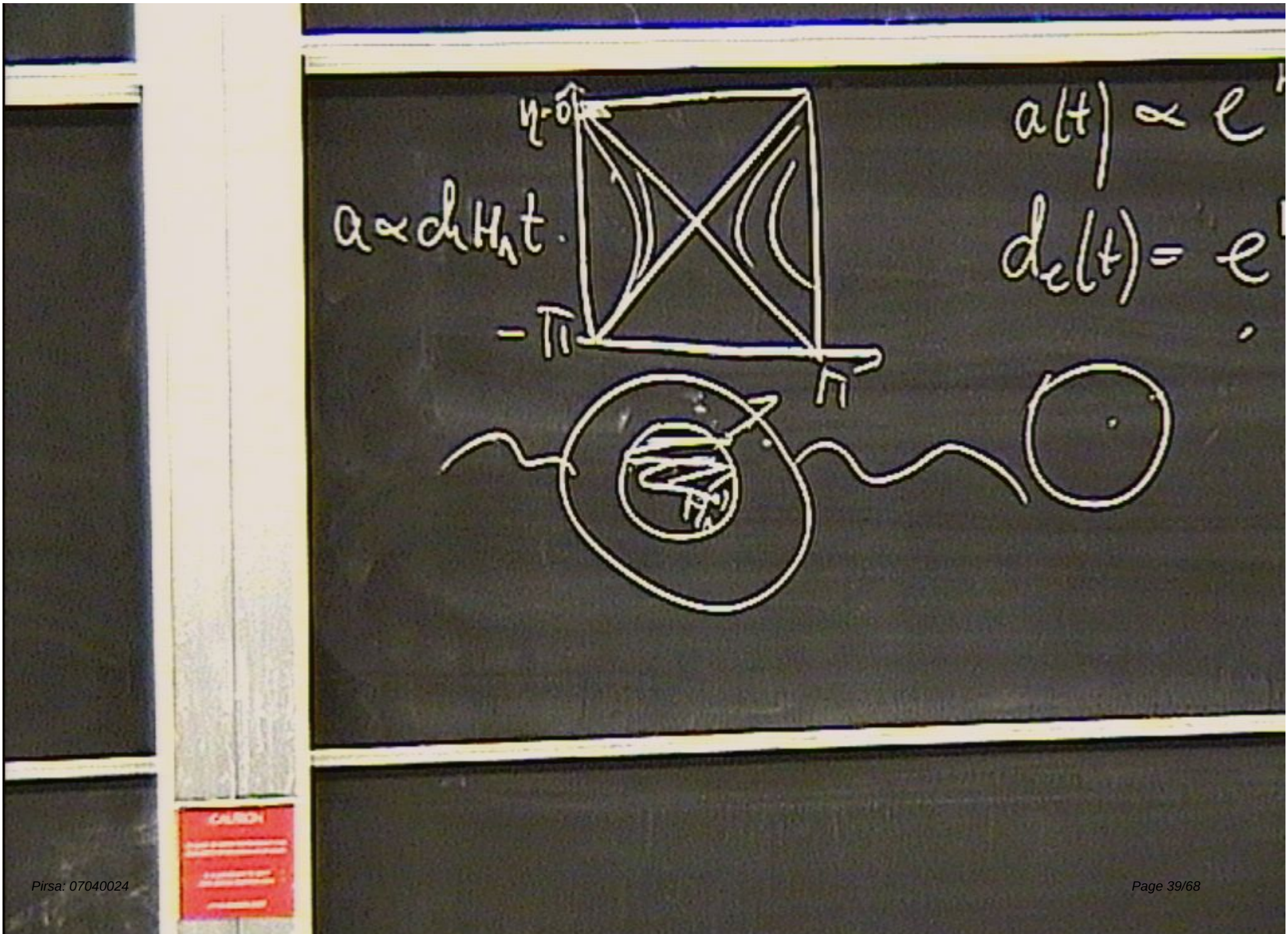


$a \propto \cosh H_\Lambda t$.

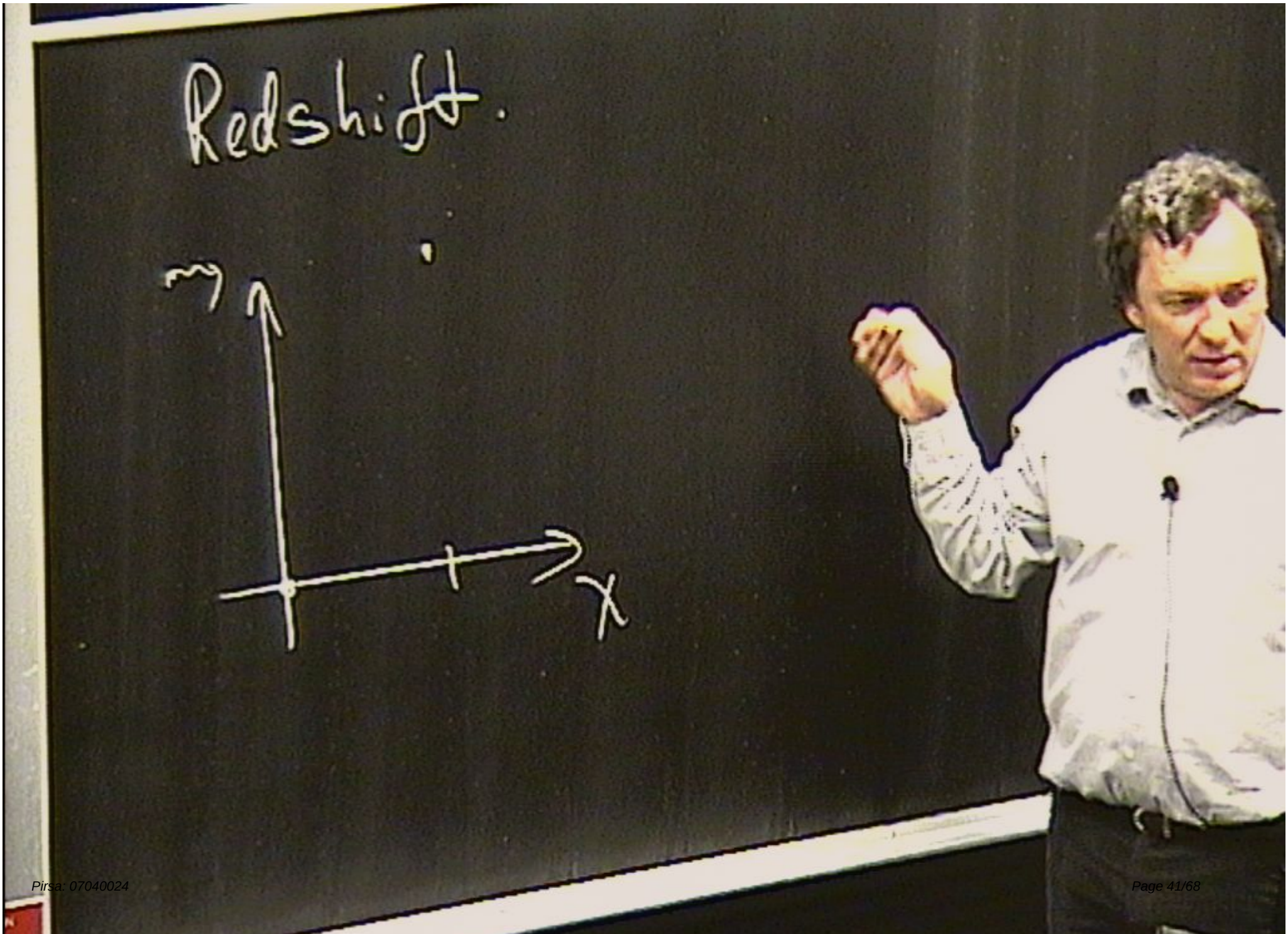
$a(t) \propto e^{H_\Lambda t}$.

$d_c(t) = e^{H_\Lambda t} \int_t^\infty \frac{dt}{e^{H_\Lambda t}} = H_\Lambda^{-1}$

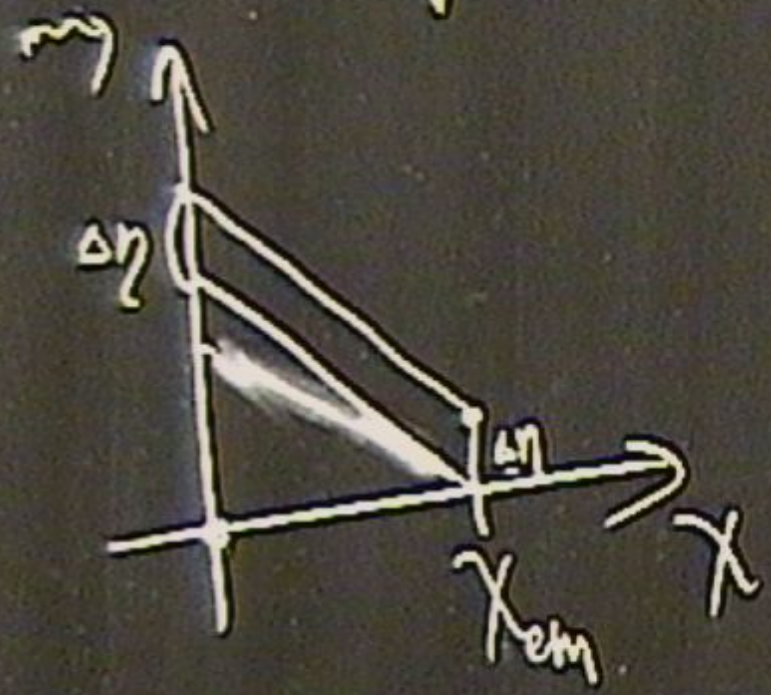



$\eta = 0$
$a \propto ch H_1 t$.
$-\pi$
π
$a(t) \propto e$
$d_c(t) = e$

Redshift.


Redshift.
x

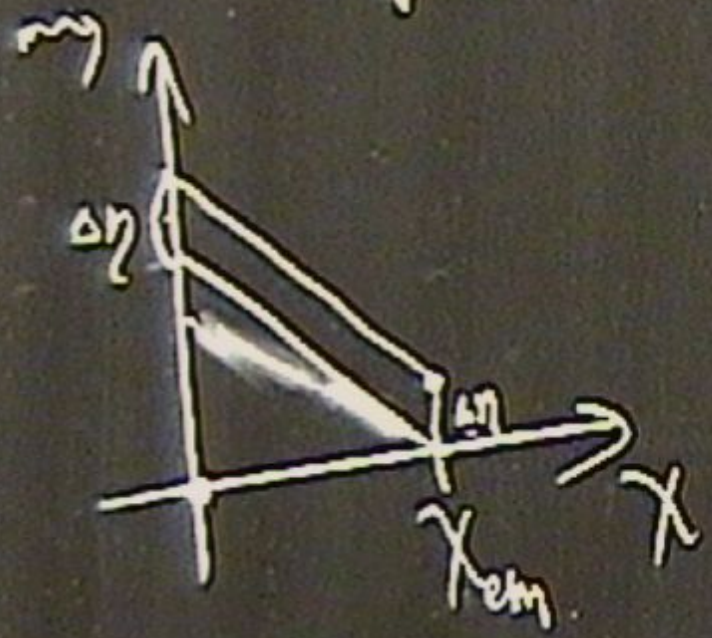
Redshift.



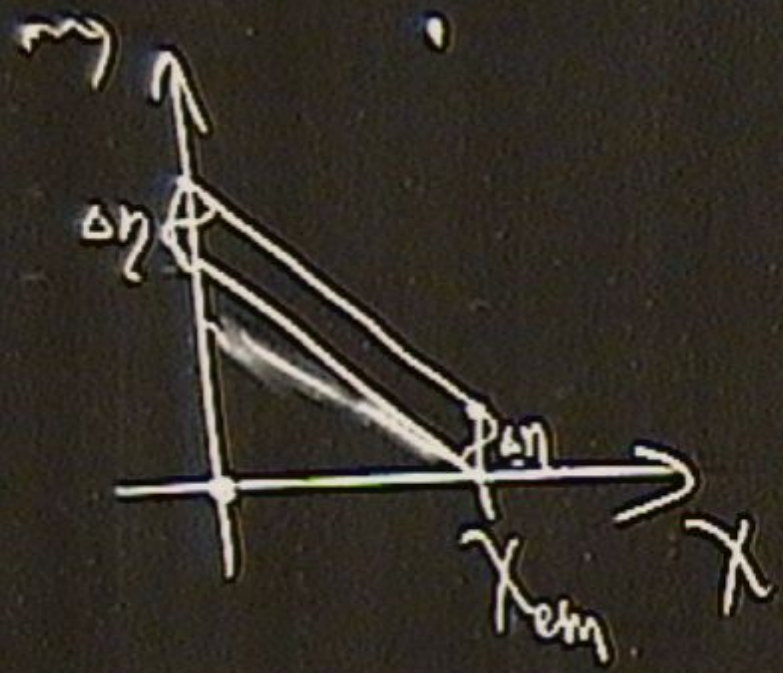
Redshift.

$$\Delta t_{em} = a(\eta_{em})\,\Delta\eta$$

$$\Delta t_{obs} = a(\eta_{obs})\,\Delta\eta$$



Redshift.

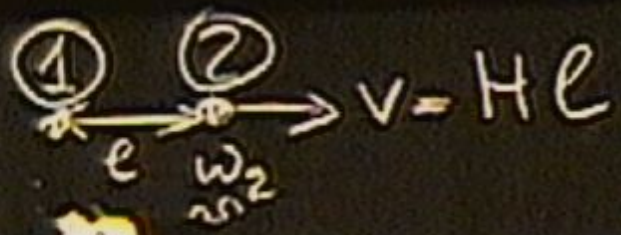


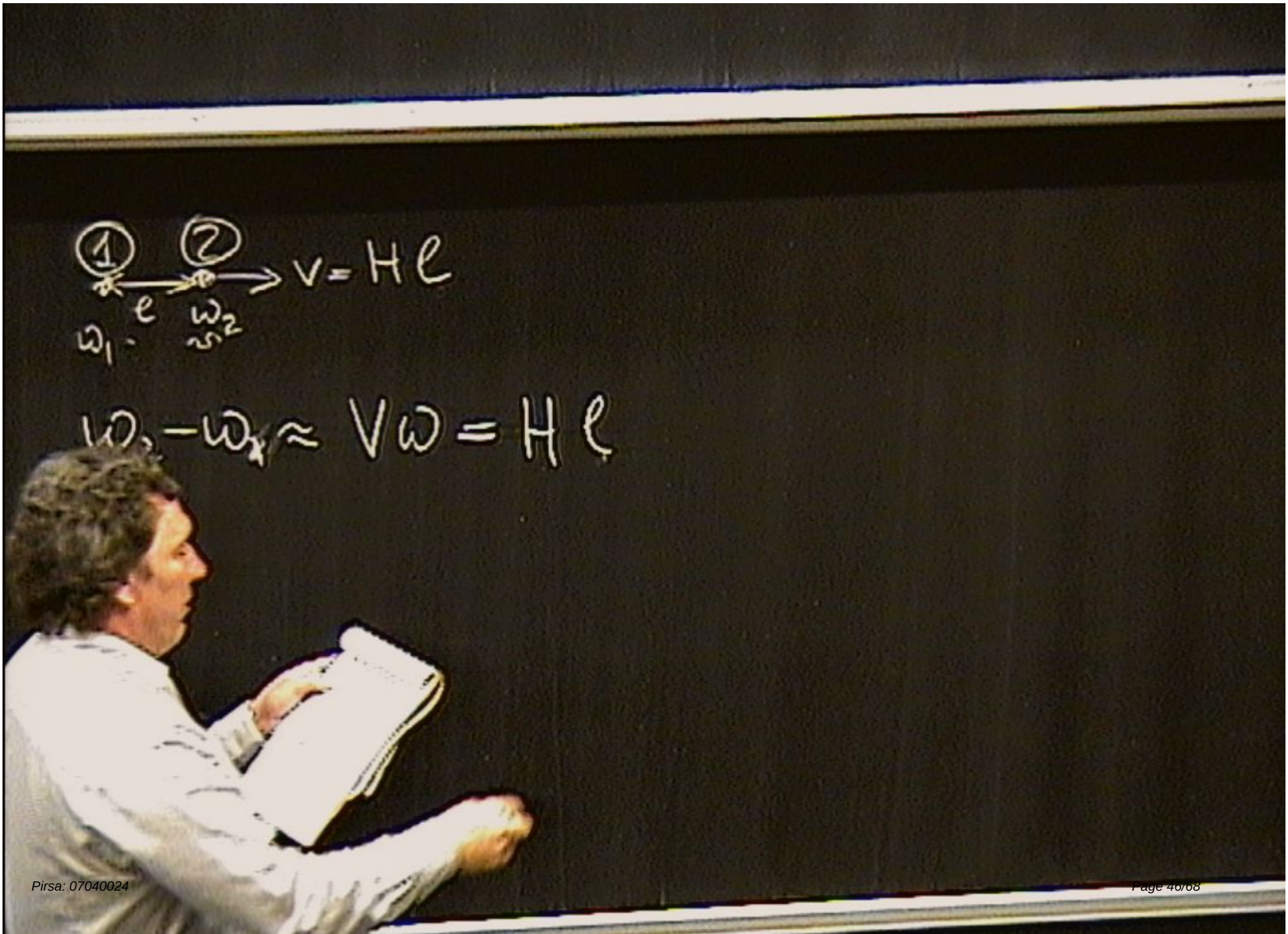
$$\Delta t_{em} = a(\eta_{em})\,\Delta\eta$$

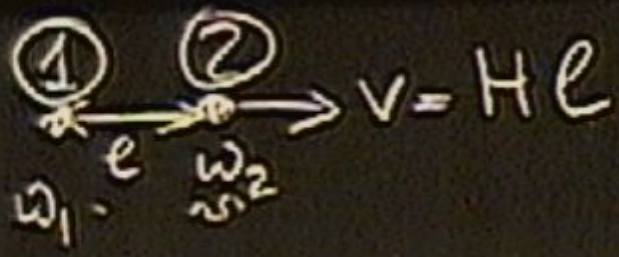
$$\Delta t_{obs} = a(\eta_{obs})\,\Delta\eta$$

$$\lambda \propto \Delta t$$

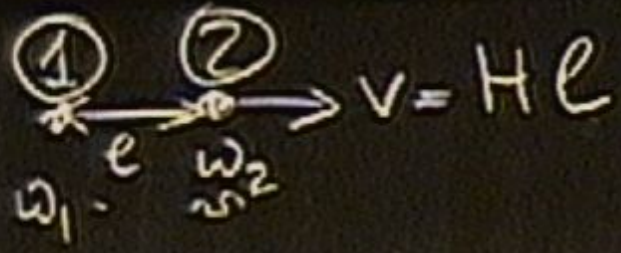
$$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{a(\eta_{obs})\,\Delta\eta}{a(\eta_{em})\,\Delta\eta}$$


1
2
v = Hℓ
ℓ
ω2


1
2
v = H ℓ
ℓ
ω1
ω2
ω2 − ω1 ≈ V ω = H ℓ



$$\omega_2 - \omega_1 \approx V\omega = H\overset{\Delta t = t_1 - t_2}{\ell}\,\omega$$



$$\omega_2 - \omega_1 \approx V\omega = H \overbrace{\ell}^{\Delta t = t_1 - t_2} \omega$$

$$\frac{d\omega}{dt} = -\overset{\frac{\dot{a}}{a}}{H}\omega \qquad \omega \propto \frac{1}{a}.$$

$$\overset{1}{\bullet} \xrightarrow{\ell} \overset{2}{\bullet} \longrightarrow v = H\ell$$

ω_1 $\quad$ ω_2

$$\Delta t = t_1 - t_2$$

$$\omega_2 - \omega_1 \approx V\omega = H\ell\,\omega$$

$$\frac{d\omega}{dt} = -H\omega \qquad \omega \propto \frac{1}{a}. \qquad \lambda \propto a.$$

Redshift. pur. as.

$z =$

Redshift pure as

$$z = \frac{\lambda(t_0) - \lambda(t_{em})}{\lambda(t_{em})}$$

Redshift pur: as

$$z = \frac{\lambda(t_0) - \lambda(t_{em})}{\lambda(t_{em})} \Rightarrow 1 + z = \frac{a_0}{a(t)}$$

Redshift. pure: as

$$z = \frac{\lambda(t_0) - \lambda(t_{em})}{\lambda(t_{em})} \Rightarrow 1 + z = \frac{a_0}{a(t)}$$

$$t \leftrightarrow z$$

Redshift pur: as

$$z = \frac{\lambda(t_0) - \lambda(t_{em})}{\lambda(t_{em})} \Rightarrow 1 + z = \frac{a_0}{a(t)}$$

$$t \leftrightarrow z \quad t(z) - ?$$

Redshift. pur. as.

$$z = \frac{\lambda(t_0) - \lambda(t_{em})}{\lambda(t_{em})} \Rightarrow \underbrace{1 + z = \frac{a_0}{a(t)}}$$

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$$dz = -\frac{a_0}{a^2(t)}\,\dot{a}\,dt = -H(1+z)\,dt.$$

Redshift. pur. as.

$$z = \frac{\lambda(t_0) - \lambda(t_{em})}{\lambda(t_{em})} \Rightarrow \underbrace{1+z = \frac{a_0}{a(t)}}$$

$t \leftrightarrow z$. $t(z) - ?$

$$dz = -\frac{a_0}{a^2(t)}\,\dot{a}\,dt = -H(1+z)\,dt$$

$$\Delta t = \int_z^{\infty} \frac{dz}{H(z)(1+z)}$$

Z as a measure of distance.

$d_{cm} \leftrightarrow x_{cm}$

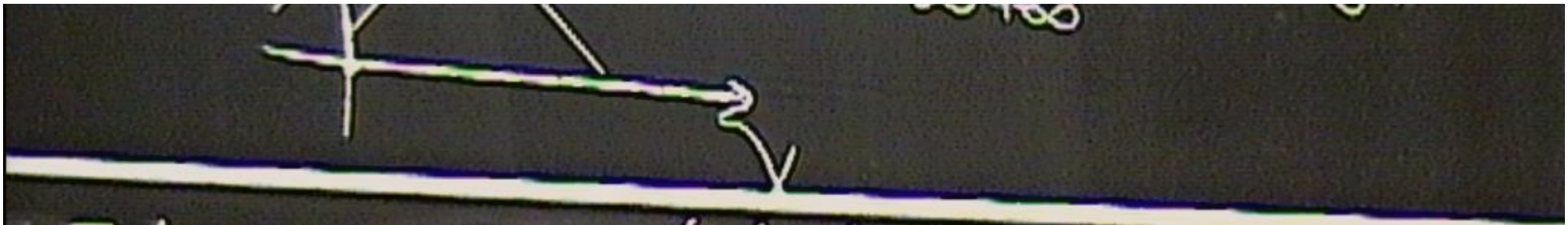
d_s ⟷ c_m

z as a measure of distance

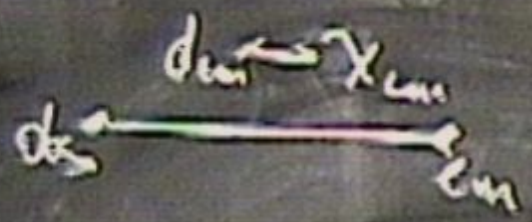
$$d_{em} \xrightarrow{\quad} x_{em}$$

$$d_{os} \longleftarrow e_m$$

$$d_{em} = c\left(t_0 - t_{em}(z)\right) = d_{em}(z)$$



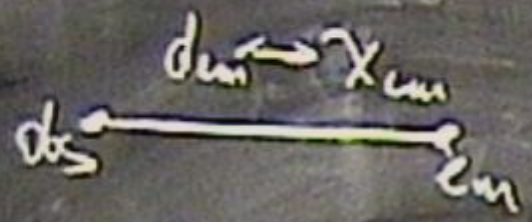
z as a measure of distance.



$$d_{em} = c\left(t_0 - t_{em}(z)\right) = d_{em}(z)$$

$$\chi_{em}(z) = \eta_0 - \eta_{em}$$

z as a measure of distance.



$$d_{em} = c\left(t_0 - t_{em}(z)\right) \equiv d_{em}(z)$$

$$\chi_{em}(z) = \eta_0 - \eta_{em} = \int \frac{dt}{a(t)}$$

z as a measure of distance.

$$d_{em} \leftrightarrow \chi_{em}$$

$$d_{em} = c\left(t_0 - t_{em}(z)\right) = d_{em}(z)$$

$$\chi_{em}(z) = \eta_0 - \eta_{em} = \frac{1}{a_0}\int_{t_{em}}^{t_0} \frac{dt}{a(t)} = \frac{1}{a_0}\int_0^z \frac{dz}{H(z)}$$

$$d_{em} = c\left(t_0 - t_{em}(z)\right) \equiv d_{em}(z)$$

$$\chi_{em}(z) = \eta_0 - \eta_{em} = \frac{1}{a_0}\int_{t_{em}}^{t_0} \frac{dt}{a(t)} = \frac{1}{a_0}\int_0^z \frac{dz}{H(z)}$$

$$k=0,\ p=0 \qquad H^2 = \frac{8\pi G}{3}\,\varepsilon_0 \left(\frac{a_0}{a}\right)^n$$

$$H \propto (1+z)^{3/2}, \quad (1+z)^3$$

$$d_p(t) = a(t)\,\chi_p(t) = a(t)\int_{t_i}^{t} \frac{dt}{a}.$$

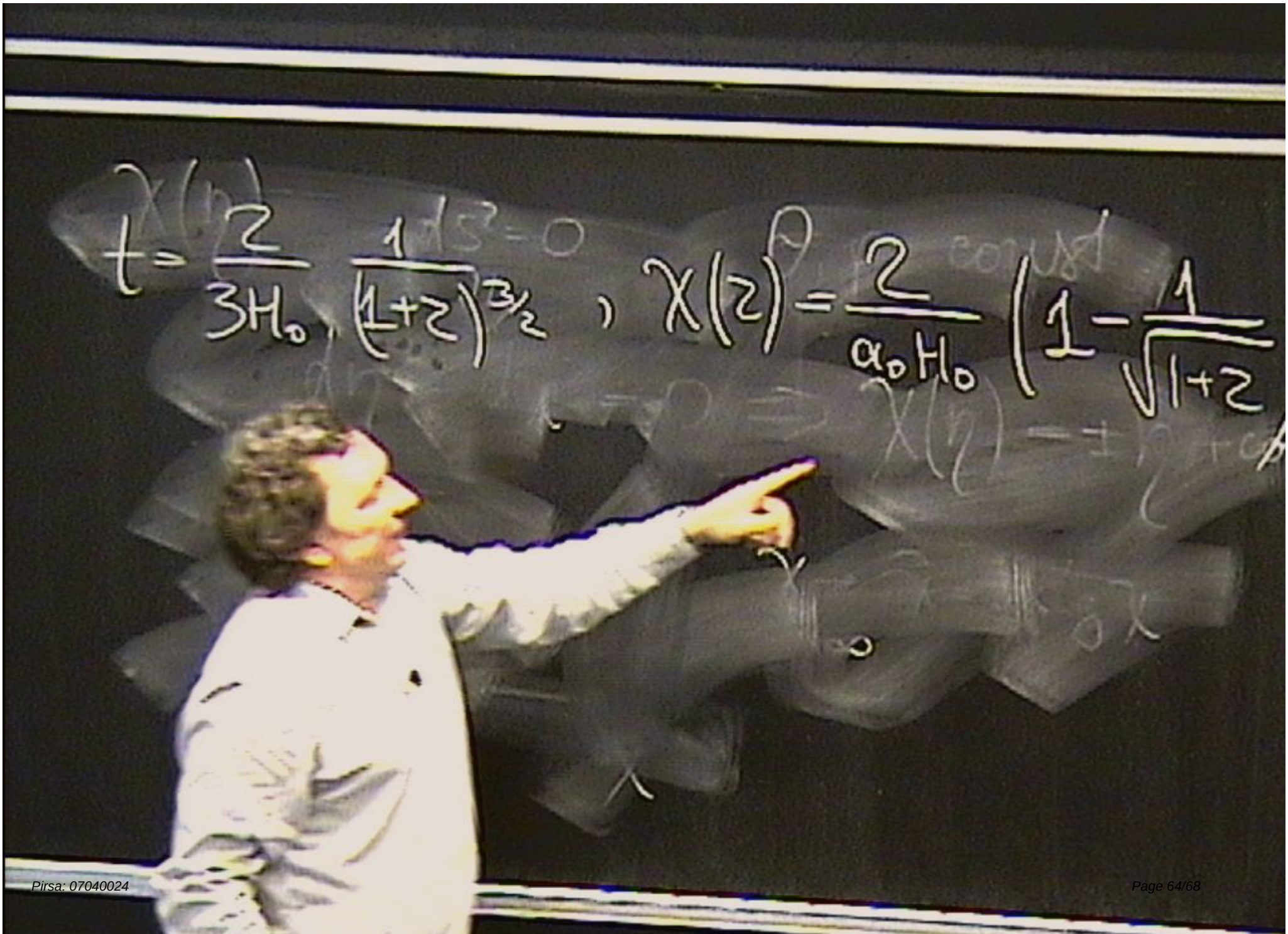
$$t = \frac{2}{3H_0} \frac{1}{(1+z)^{3/2}}$$

$$ds^2 = 0$$

$$\theta$$

$$\chi$$

$$\infty$$


$t = \frac{2}{3H_0} \frac{1}{(1+z)^{3/2}}$, $\chi(z) = \frac{2}{a_0 H_0} \left(1 - \frac{1}{\sqrt{1+z}}\right.$

$$t = \frac{2}{3H_0}\frac{1}{(1+z)^{3/2}}, \quad \chi(z) = \frac{2}{a_0 H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

$$\chi(z \to \infty) \to \frac{2}{a_0 H_0}$$

$$t = \frac{2}{3H_0}\frac{1}{(1+z)^{3/2}}, \quad \chi(z) = \frac{2}{a_0 H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

$$a_0\chi(z\to\infty) \Rightarrow \frac{2}{a_0 H_0} = \underline{\underline{\chi_p(t_0)}}$$

$$H_0 = \frac{2}{3t_0}$$

$$t = \frac{2}{3H_0} \frac{1}{(1+z)^{3/2}}, \quad \chi(z) =$$

$$ds^2 = 0$$

$$a_0 \chi(z \to \infty$$

$$\Omega_0 = \frac{\varepsilon}{\varepsilon^{cr}}$$

$$\gamma$$

$$\Big)^{3/2}, \quad \chi(z) = \frac{c}{a_0 H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

$$a_0\,\chi(z\to\infty) \Rightarrow \frac{c}{a_0 H_0} \equiv \underline{\underline{\chi_p}}(t_0)$$

$$H_0 = \frac{2}{3t_0}$$

$$p(\varepsilon)$$

$$\ddot{a} \propto (\varepsilon + 3p)$$