Title: Graduate Course on Standard Model & Quantum Field Theory - 17B

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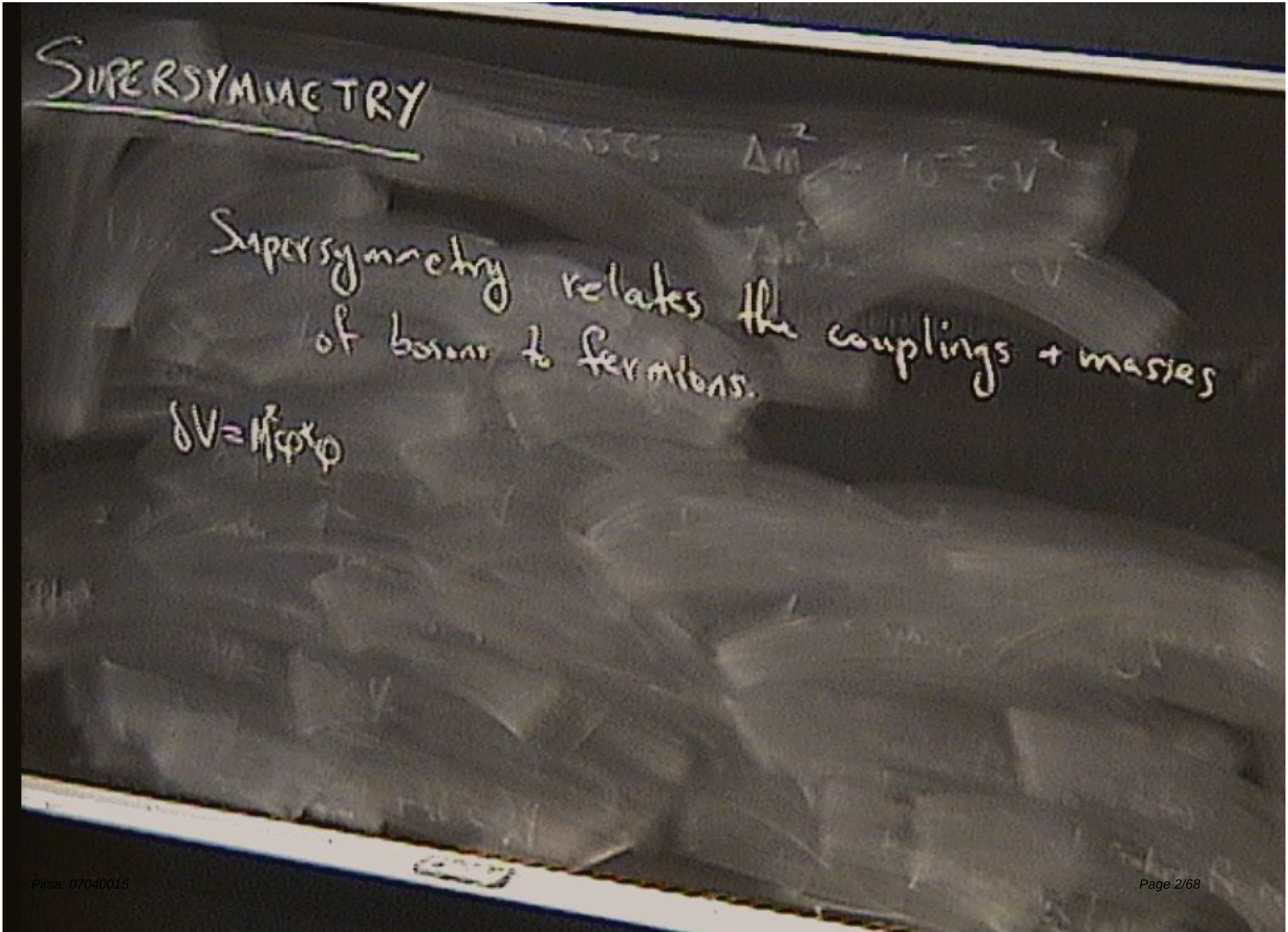
Abstract: Graduate Course on Standard Model & Quantum Field Theory


SUPERSYMMETRY
Supersymmetry relates the couplings + masses
of bosons to fermions.

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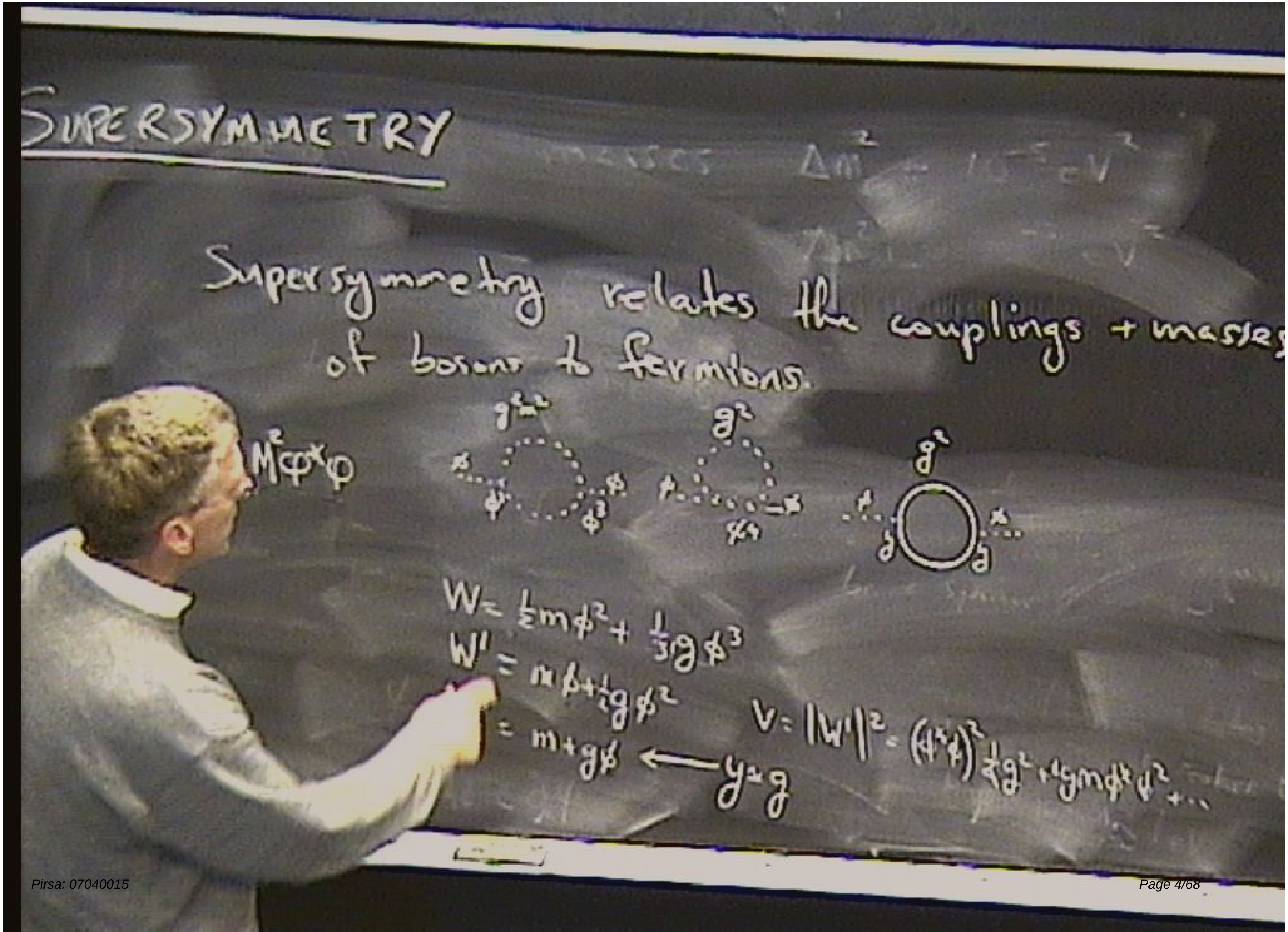
Supersymmetry relates the couplings + masses of bosons to fermions.

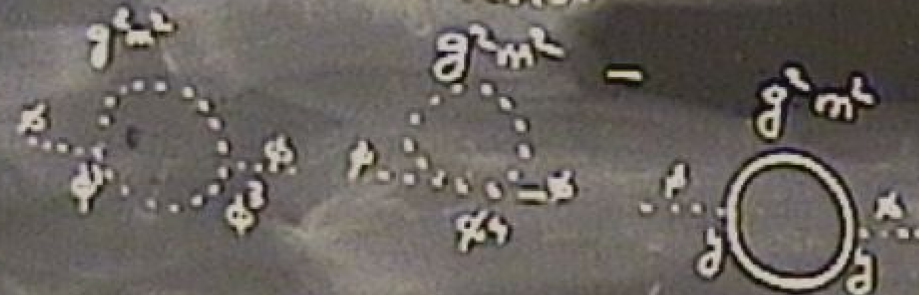
$\delta V = M^2 \phi^* \phi$


SUPERSYMMETRY
Supersymmetry relates the couplings + masses of bosons to fermions.
M²φ*φ
g²m²
g²
g²
W = ½mφ² + ⅓gφ³
W′ = mφ + ½gφ²
= m + gφ
y = g
V = |W′|² = (φ*φ)² ¼g² + ½gmφ*φ² + …

SUPERSYMMETRY

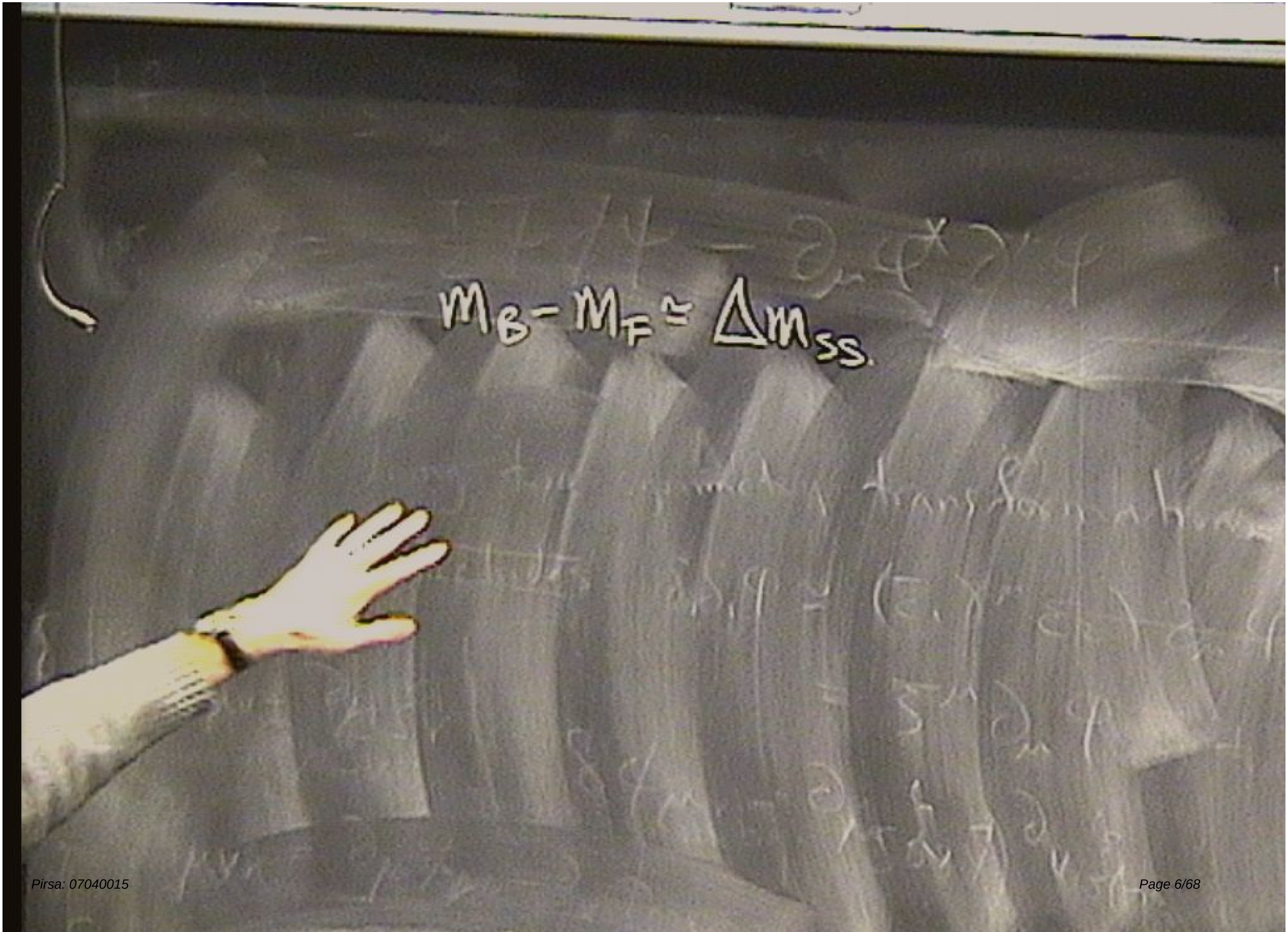
Supersymmetry relates the couplings + masses of bosons to fermions.

$$\delta V = M^2 \phi^* \phi$$



$$W = \tfrac{1}{2} m \phi^2 + \tfrac{1}{3} g \phi^3$$
$$W' = m\phi + \tfrac{1}{2} g \phi^2$$
$$W'' = m + g\phi \longleftarrow y = g$$

$$V = |W'|^2 = (\phi^*\phi)^2 \tfrac{1}{4} g^2 + \tfrac{1}{2} g m \phi^* \phi^2 + \ldots$$

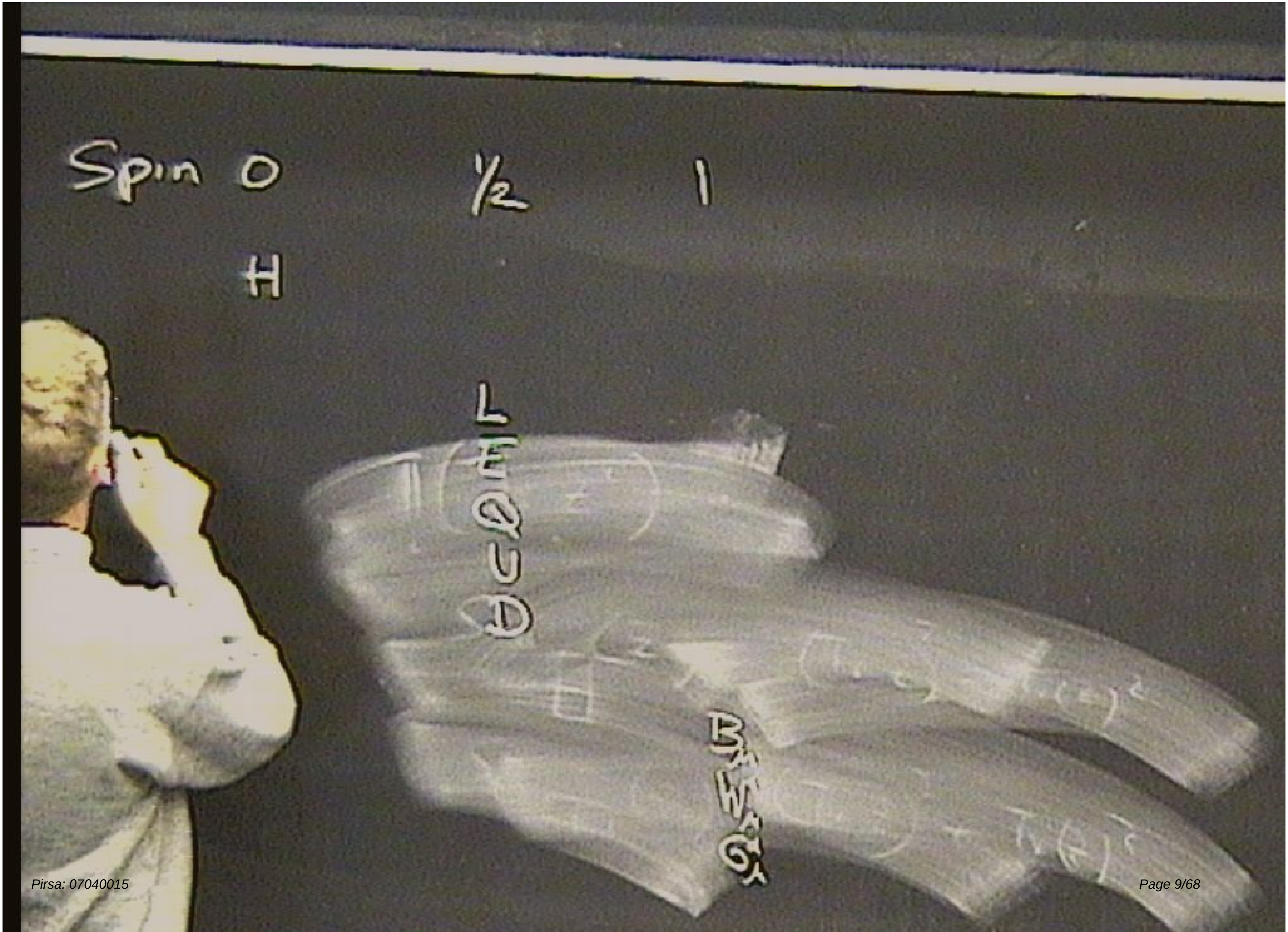

$m_B - m_F \simeq \Delta m_{SS}$

$$m_B - m_F \simeq \Delta m_{ss}$$

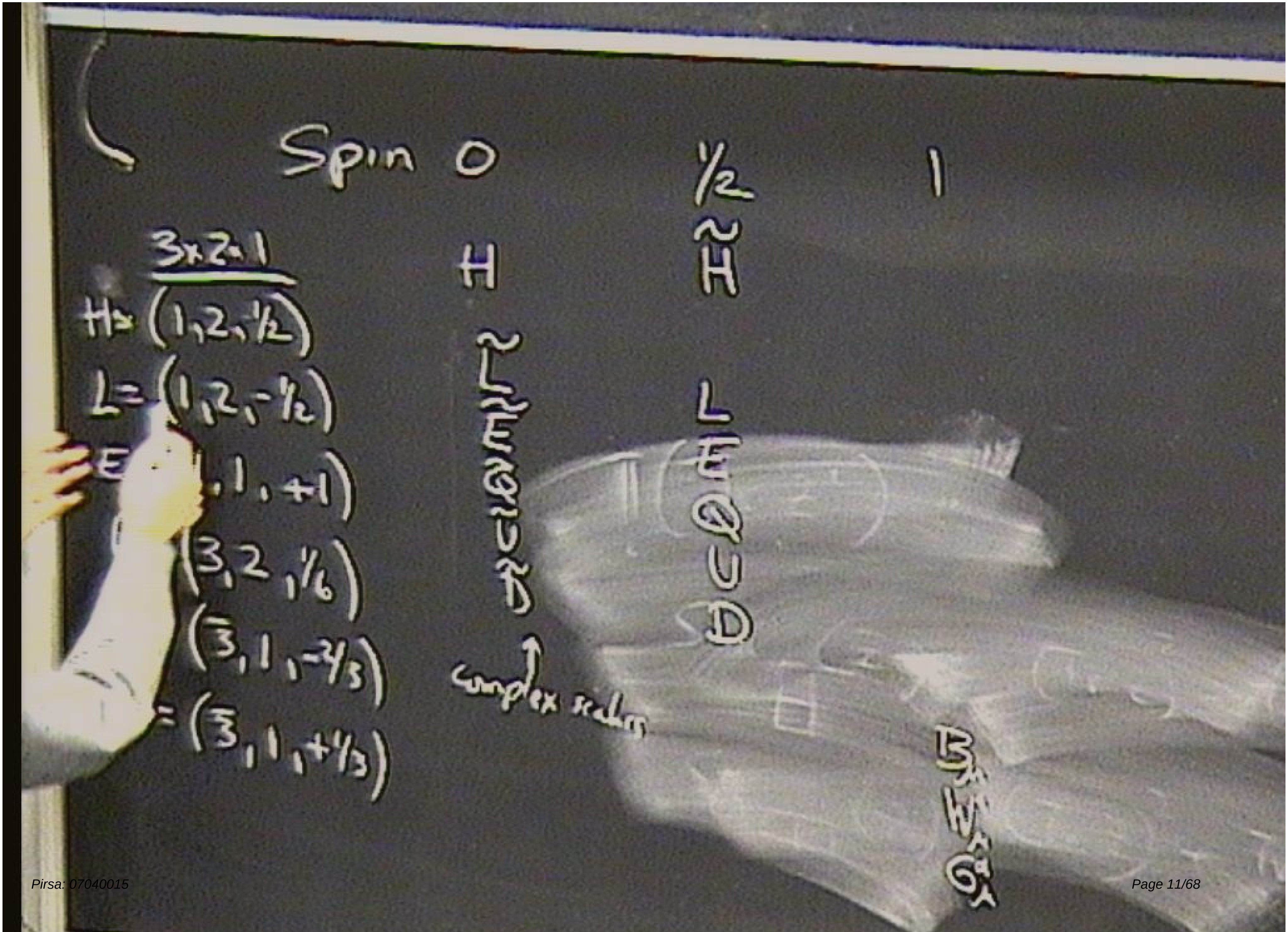
$$\delta V \simeq (\Delta m_{ss})^2 \varphi^* \varphi .$$

$$m_B - m_F \simeq \Delta m_{ss}$$
$$\simeq 1\ \text{TeV}$$

$$\delta V \simeq (\Delta m_{ss})^2\, \varphi^* \varphi .$$


Spin 0
½
1
H

	Spin 0	1/2	1
$3\times 2\times 1$	H		
$H=(1,2,\tfrac{1}{2})$			
$L=(1,2,-\tfrac{1}{2})$		L	
$E=(1,1,+1)$		E	
$Q=(3,2,\tfrac{1}{6})$		Q	
$U=(\bar{3},1,-\tfrac{2}{3})$		U	
$D=(\bar{3},1,+\tfrac{1}{3})$		D	
			B
			W
			G



Spin 0 ½ 1
3x2x1
H H̃
H=(1,2,½)
L=(1,2,-½) L̃ L
E (1,1,+1) Ẽ E
(3,2,1/6) Q̃ Q
(3̄,1,-⅔) Ũ U
(3̄,1,+⅓) D̃ D
complex scalars
B W G

	Spin 0	$\frac{1}{2}$	1
$\underline{3 \times 2 \times 1}$			
$H = (1, 2, -\frac{1}{2})$	H	$\tilde{H}$	
$L = (1, 2, -\frac{1}{2})$	$\tilde{L}$	L	
$E = (1, 1, +1)$	$\tilde{E}$	E	
$Q = (3, 2, \frac{1}{6})$	$\tilde{Q}$	Q	
$U = (\bar{3}, 1, -\frac{2}{3})$	$\tilde{U}$	U	
$D = (\bar{3}, 1, +\frac{1}{3})$	$\tilde{D}$	D	
	complex scalars	b	B
		w	W
		g	G

L
E
Q
U
D
b
w
g

B
W
G

Interactions:

W = cubic, holomorphic polynomial

which $SU_3 \times SU_2 \times U_1$ invariant polynomial

which $SU_3 \times SU_2 \times U_1$ invari

$$W = y^u Q U H$$

B_μ

W^i_μ

G^a_μ

Interactions: *renormalizable

$W =$ cubic*, holomorphic polynomial which $SU_3 \times SU_2 \times U_1$ invari

$$W = y_{ij} Q_i U_j H$$

B_μ

W^i_μ

G^a_μ

$3 \times 2 \times 1$

$H = (1, 2, 1/2)$ $\quad H \quad \tilde{H}$

$L = (1, 2, -1/2)$ $\quad \tilde{L} \quad L$

$E = (1, 1, +1)$ $\quad \tilde{E} \quad E$

$Q = (3, 2, 1/6)$ $\quad \tilde{Q} \quad Q$

$U = (\bar{3}, 1, -2/3)$ $\quad \tilde{U} \quad U$

$D = (\bar{3}, 1, +1/3)$ $\quad \tilde{D} \quad D$

complex scalars

$b \quad B_\mu$

$w \quad W_\mu^a$

$g \quad G_\mu^a$

$(\sigma_i)^* = M \sigma_i M^{-1} \quad M = (i\sigma_2)$

$\tilde{H}$

H

L

E

Q

U

D

b $\qquad B_\mu$

w $\qquad W_\mu$

g $\qquad G_\mu^a$

Interactions: *renormalizable

W = cubic*, holomorphic polynomial which $SU_3 \times SU_2 \times U_1$ invari[ant]

$$W = y_{ij} Q_i D_j H + y_{ij} L_i E_j H$$

Experiment

$$\left(\bar{\Psi}_i \gamma_L \Psi_j\right) \frac{\partial^2 W}{\partial \phi_i \, \partial \phi_j},$$

, holomorphic
polynomial
which $SU_3 \times SU_2 \times U_1$ invari[ant]

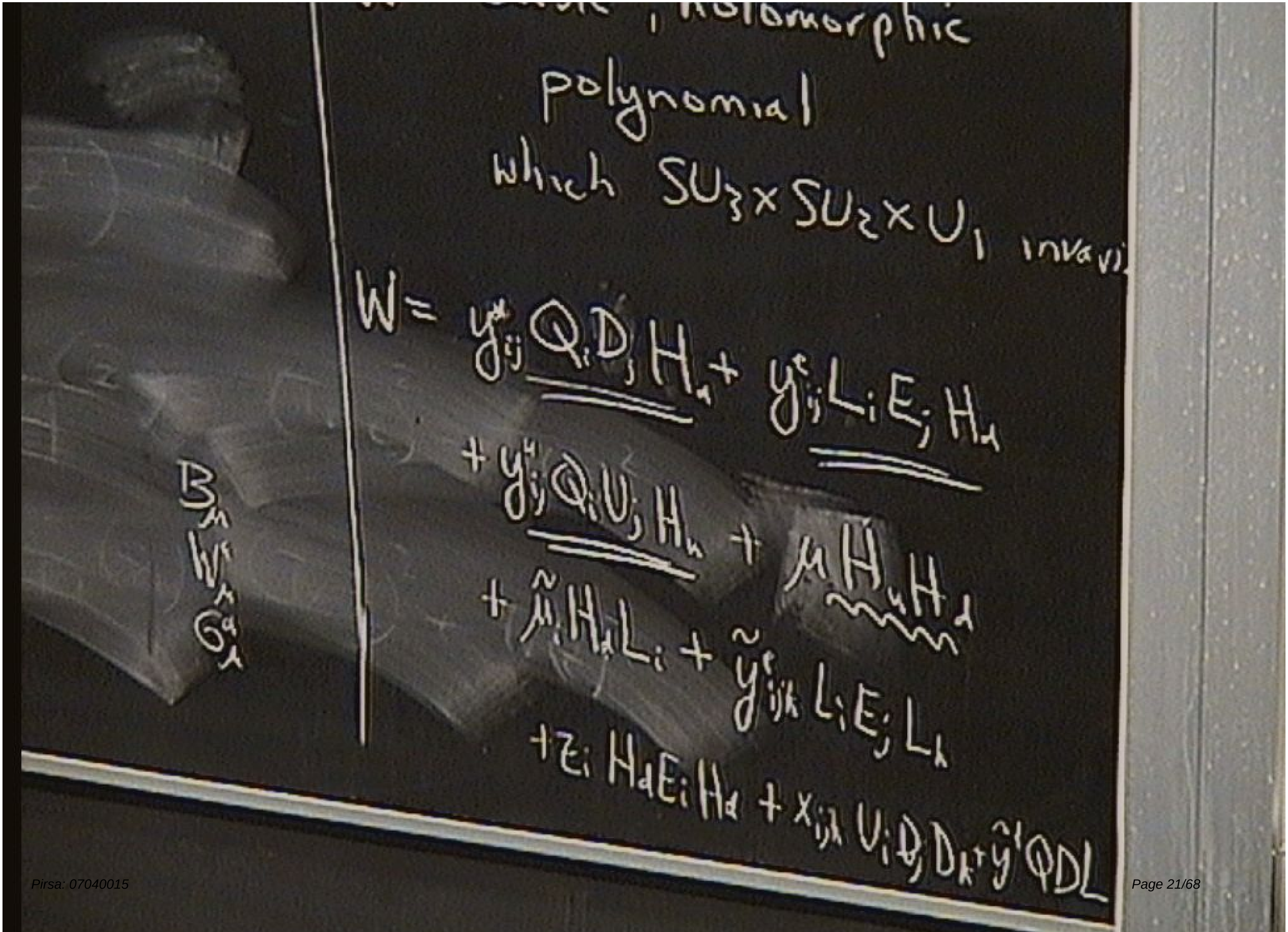
$$W = y^d_{ij} Q_i D_j H_d + y^e_{ij} L_i E_j H_d + y^u_{ij} Q_i U_j H_u + \mu H_u H_d + \tilde{\mu}_i H_u L_i + \ldots$$

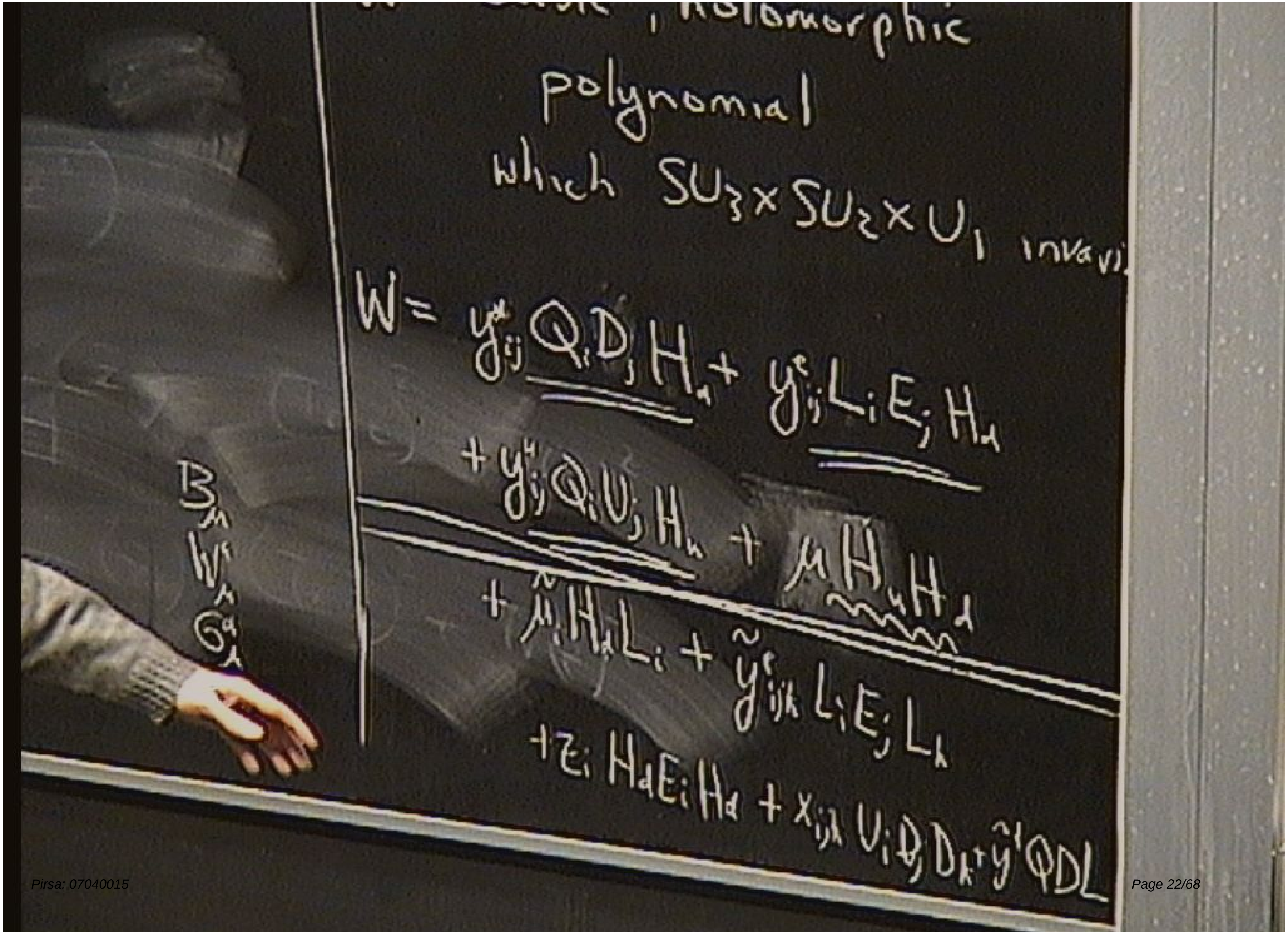
B_μ
W^i_μ
G^a_μ

..., holomorphic
polynomial
which $SU_3 \times SU_2 \times U_1$ invariant

B_μ
W^i_μ
G^a_μ

$$W = y^d_{ij} Q_i D_j H_d + y^e_{ij} L_i E_j H_d$$
$$+ y^u_{ij} Q_i U_j H_u + \mu H_u H_d$$
$$+ \tilde{\mu}_i H_d L_i + \tilde{y}^e_{ijk} L_i E_j L_k$$
$$+ z_i H_d E_i H_d$$


holomorphic
polynomial
which $SU_3 \times SU_2 \times U_1$ invari
$W = y^d_{ij} Q_i D_j H_d + y^e_{ij} L_i E_j H_d$
$+ y^u_{ij} Q_i U_j H_u + \mu H_u H_d$
$+ \tilde{\mu}_i H_d L_i + \tilde{y}^e_{ijk} L_i E_j L_k$
$+ z_i H_d E_i H_d + x_{ijk} U_i D_j D_k + \hat{y}' Q D L$
B_μ
W^a_μ
G^a_μ


holomorphic
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$W = y^d_{ij} Q_i D_j H_d + y^e_{ij} L_i E_j H_d$
$+ y^u_{ij} Q_i U_j H_u + \mu H_u H_d$
$+ \tilde{\mu}_i H_d L_i + \tilde{y}^e_{ijk} L_i E_j L_k$
$+ z_i H_d E_i H_d + x_{ijk} U_i D_j D_k + \tilde{y}' Q D L$
B_μ
W^a_μ
G^a_μ

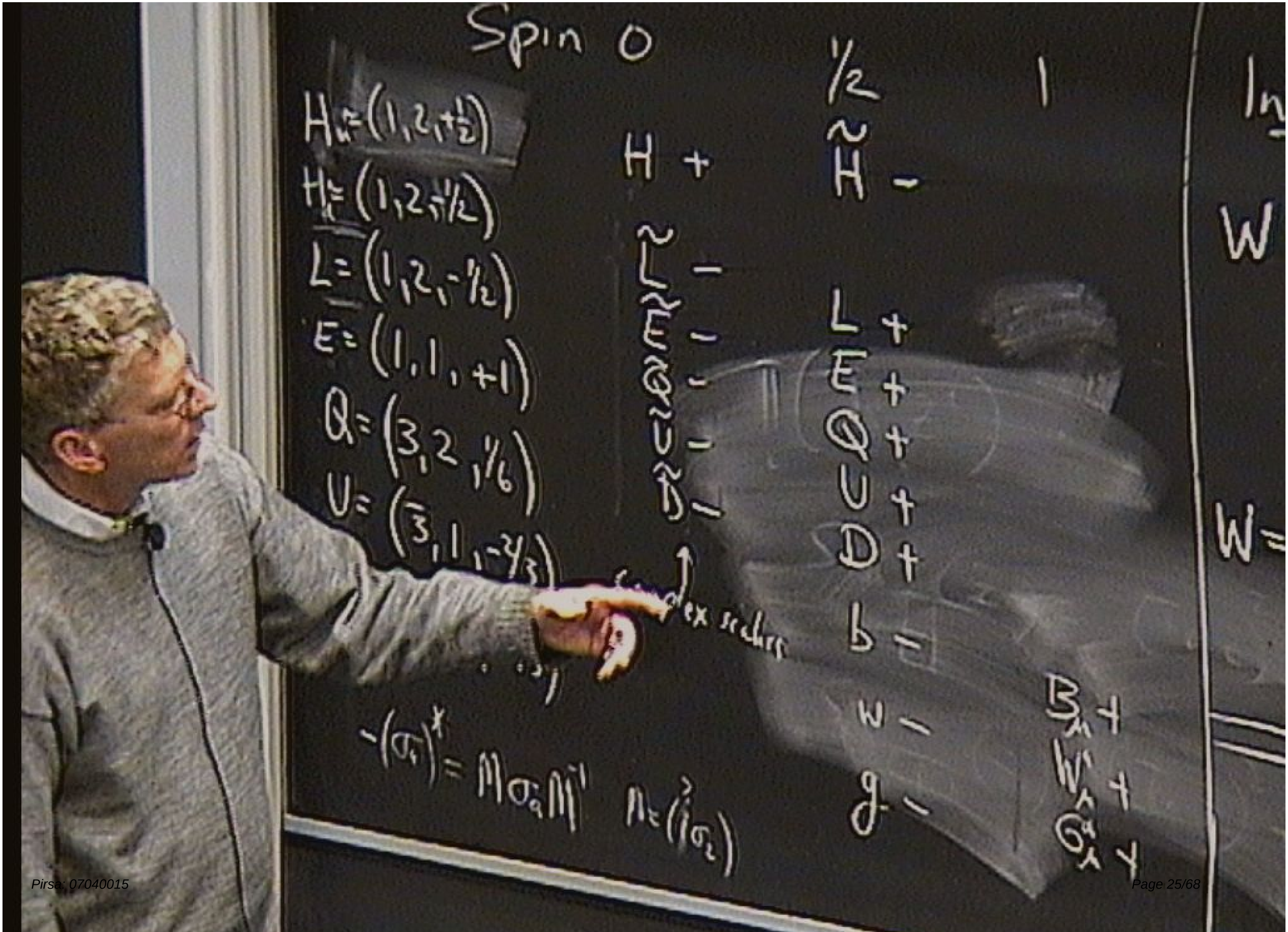
R-parity: Simple symmetry which for killing all $\not{B}$ $\not{L}$ interactions in a cubic W

$R = (-)$

R-parity: simple symmetry which for killing all $\not{B}$ $\not{L}$ interactions in a cubic

$R = (-)^x$ $\quad$ $x = 0$ for all SM ptcles

$x = +1$ " " superpartners



Spin 0
1/2
1
H_u = (1, 2, +1/2)
H_d = (1, 2, -1/2)
L = (1, 2, -1/2)
E = (1, 1, +1)
Q = (3, 2, 1/6)
U = (3̄, 1, -2/3)
H +
H̃ −
L̃ −
Ẽ −
Q̃ −
Ũ −
D̃ −
L +
E +
Q +
U +
D +
b −
w −
g −
W
W =

ν_R 1

$\tilde{H}$ −

L +

E +

Q +

U +

D +

L_i b −

w − B_μ +

g − W_μ +

G_μ +

Interactions: *renormalizable

$W =$ cubic*, holomorphic polynomial

which $SU_3 \times SU_2 \times U_1$ invari

$$W = y^d_{ij} Q_i D_j H_d + y^e_{ij} L_i E_j H_d + y^u_{ij} Q_i U_j H_u + \mu H_u H_d$$

$$+ \cancel{\tilde{\mu}_i H_u L_i} + \cancel{\tilde{y}^e_{ijk} L_i E_j L_k} + \cancel{z_i H_d E_i H_d} + \cancel{x_{ijk} U_i D_j D_k} + \cancel{y' QDL}$$

<u>R-parity</u>: Simple symmetry which

all $\cancel{B}$ $\cancel{L}$ interacti

$R = (-)^{X}$

$X = 0$ for all SM ptcle

$X = +1$ " " superpartn

$X = 3B + L + F$

...plies that the lightest superpartner (LSP) is stable.

* LSP is a natural DM candidate.

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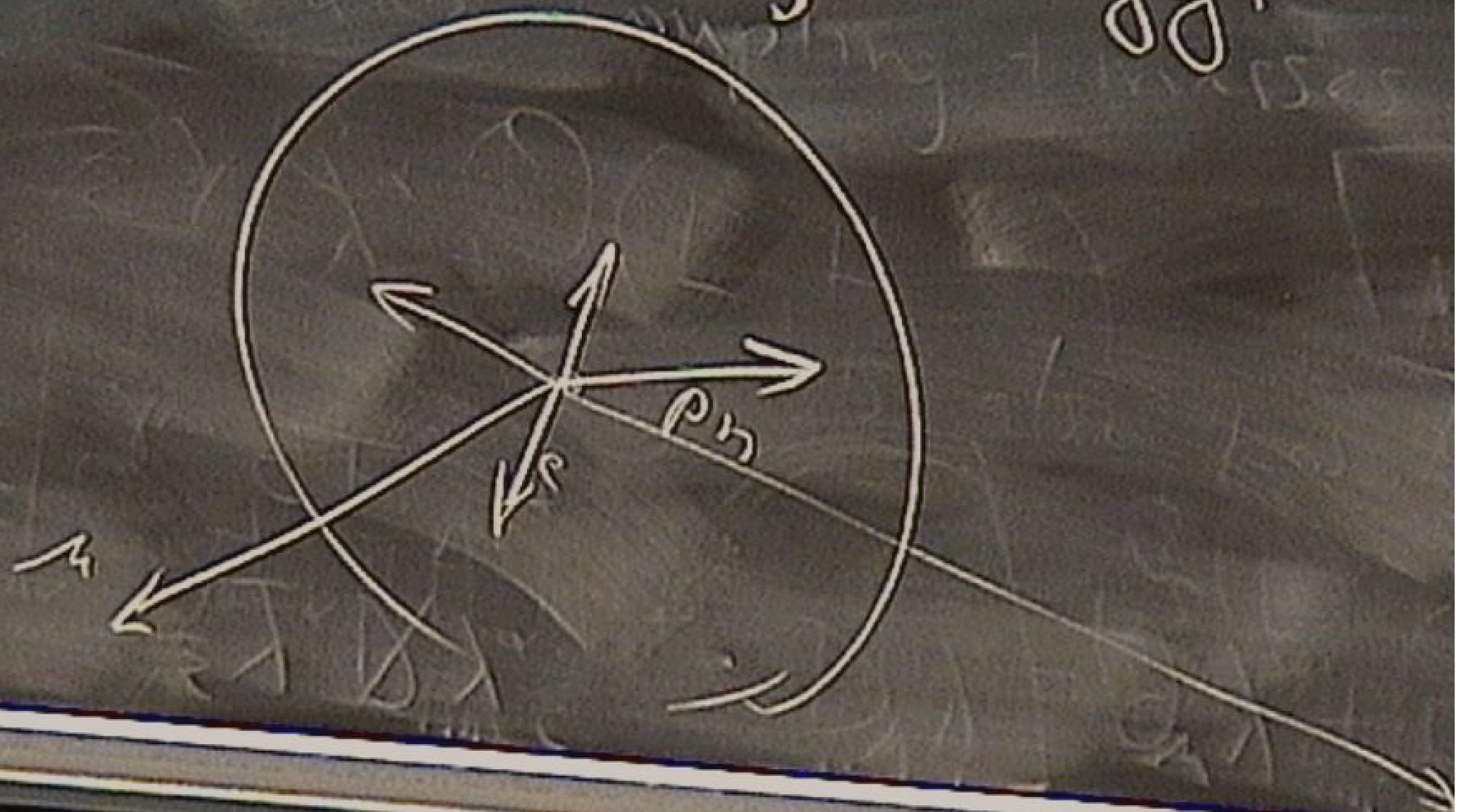
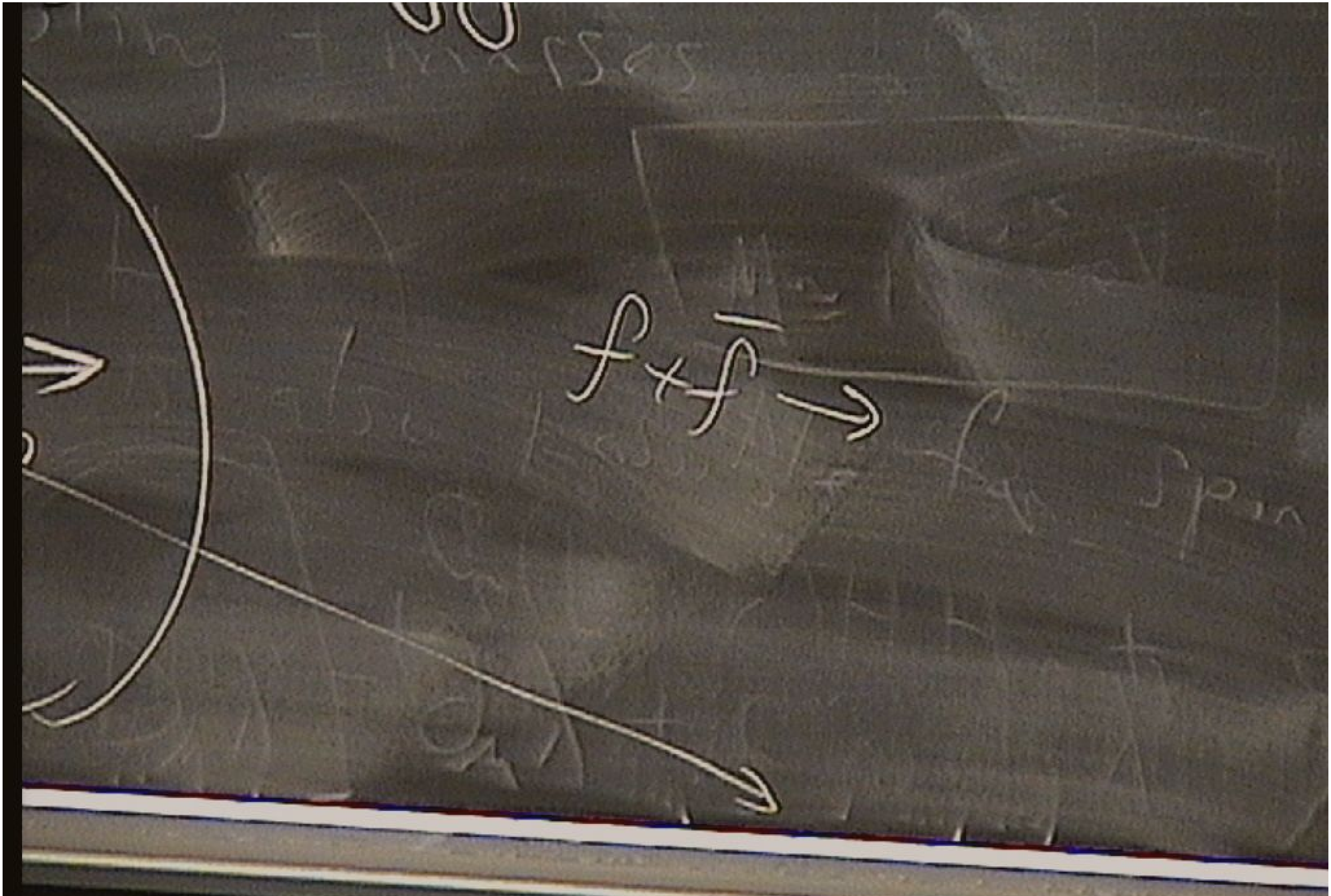
* universal experimental signature for SUSY is missing energy:

is missing energy:

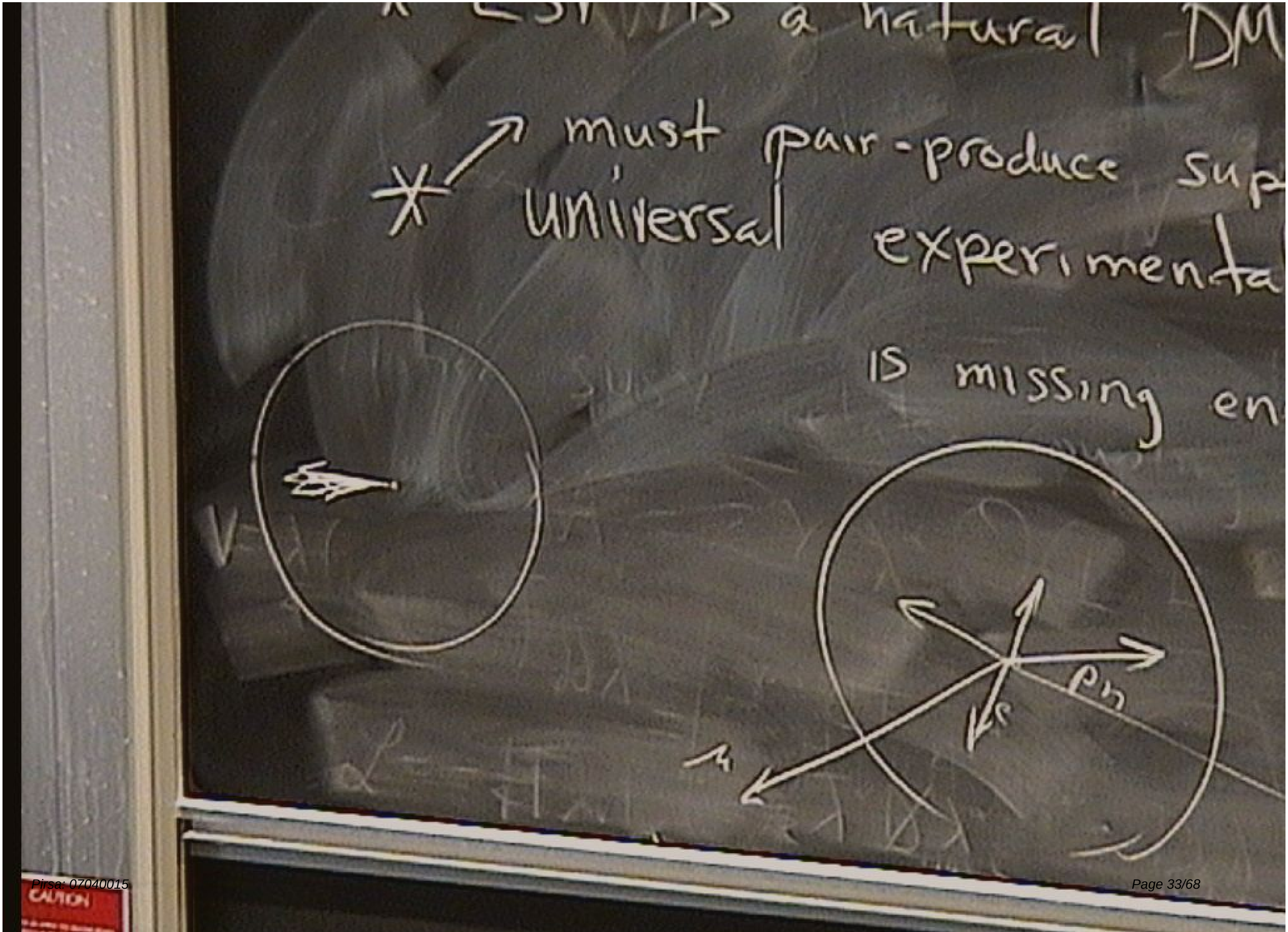



$f+\bar{f} \rightarrow$

... partner (LSP) is stable.

* LSP is a natural DM candidate.
* universal experimental signature for SUSY is missing energy:
  → must pair-produce superpartners

$$f + \bar{f} \to \tilde{\chi}\,\bar{\tilde{\chi}}, \qquad \bar{\tilde{\chi}} \to \cdots + LSP, \qquad \tilde{\chi} \to \cdots + LSP$$


a natural DM
must pair-produce sup
universal
experimenta
is missing en
CAUTION

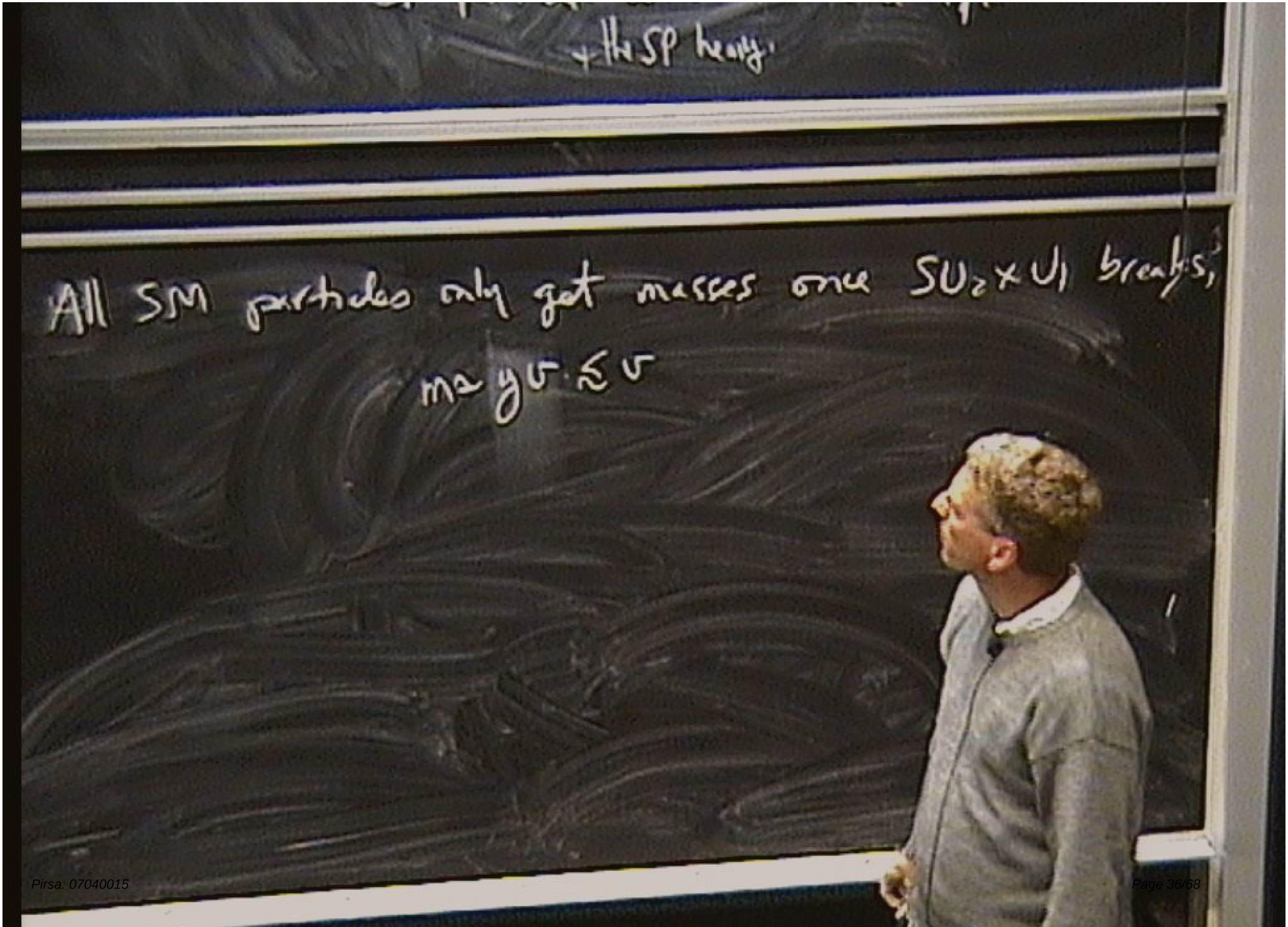
Once SUSY spontaneously breaks, the superpartners can get masses, different from their SM counterparts.

Why is it that the SM particles are light while the superpartners are heavy?

Once SUSY spontaneously breaks, the superpartners can get masses different from their SM counterparts.

is it that the SM particles are light while the superpartners are heavy?

$\gtrsim v \approx 246$ GeV then it is always the SM particles which like to be light + the SP heavy.


+ the SP heavy.
All SM particles only get masses once SU2 x U1 breaks,
m ≃ yv ≲ v

Spin 0 $\quad$ 1/2 $\quad$ 1

$H_u = (1, 2, +\tfrac{1}{2})$

$H_d = (1, 2, -\tfrac{1}{2})$

$L = (1, 2, -\tfrac{1}{2})$

$E = (1, 1, +1)$

$Q = (3, 2, \tfrac{1}{6})$

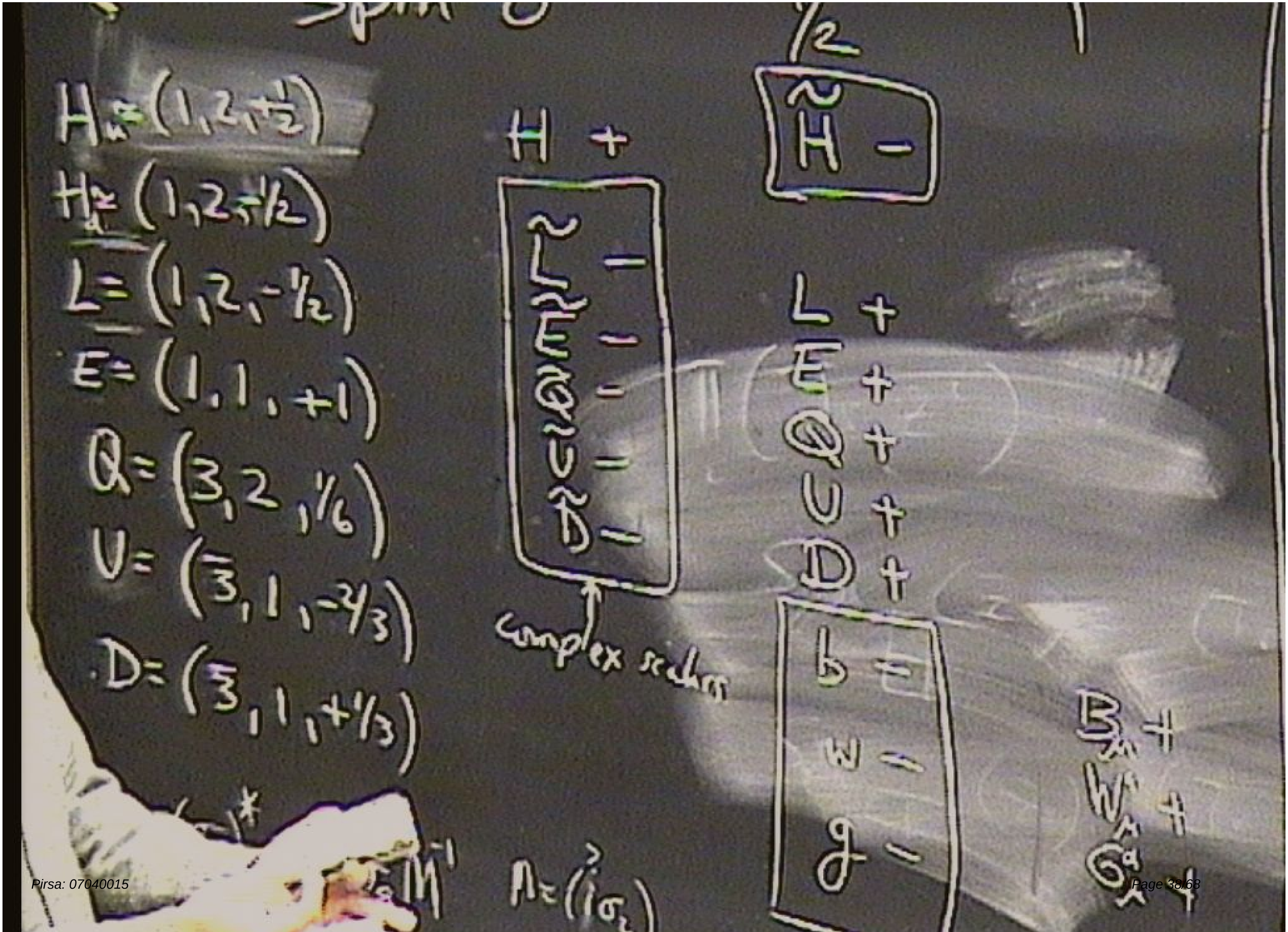
$U = (\ldots, 1, -\tfrac{4}{3})$

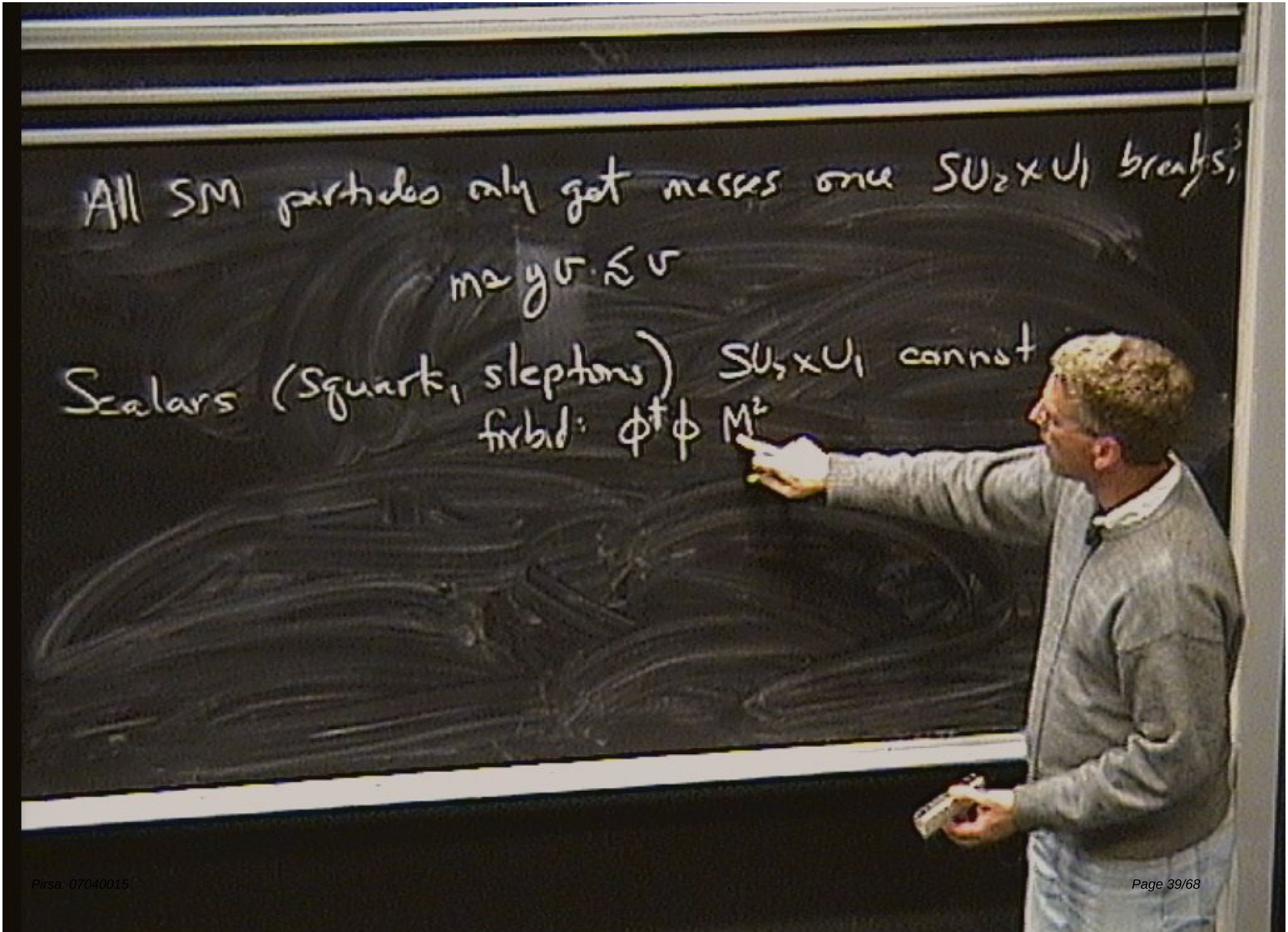
$D = (\bar{3}, 1, +\tfrac{1}{3})$

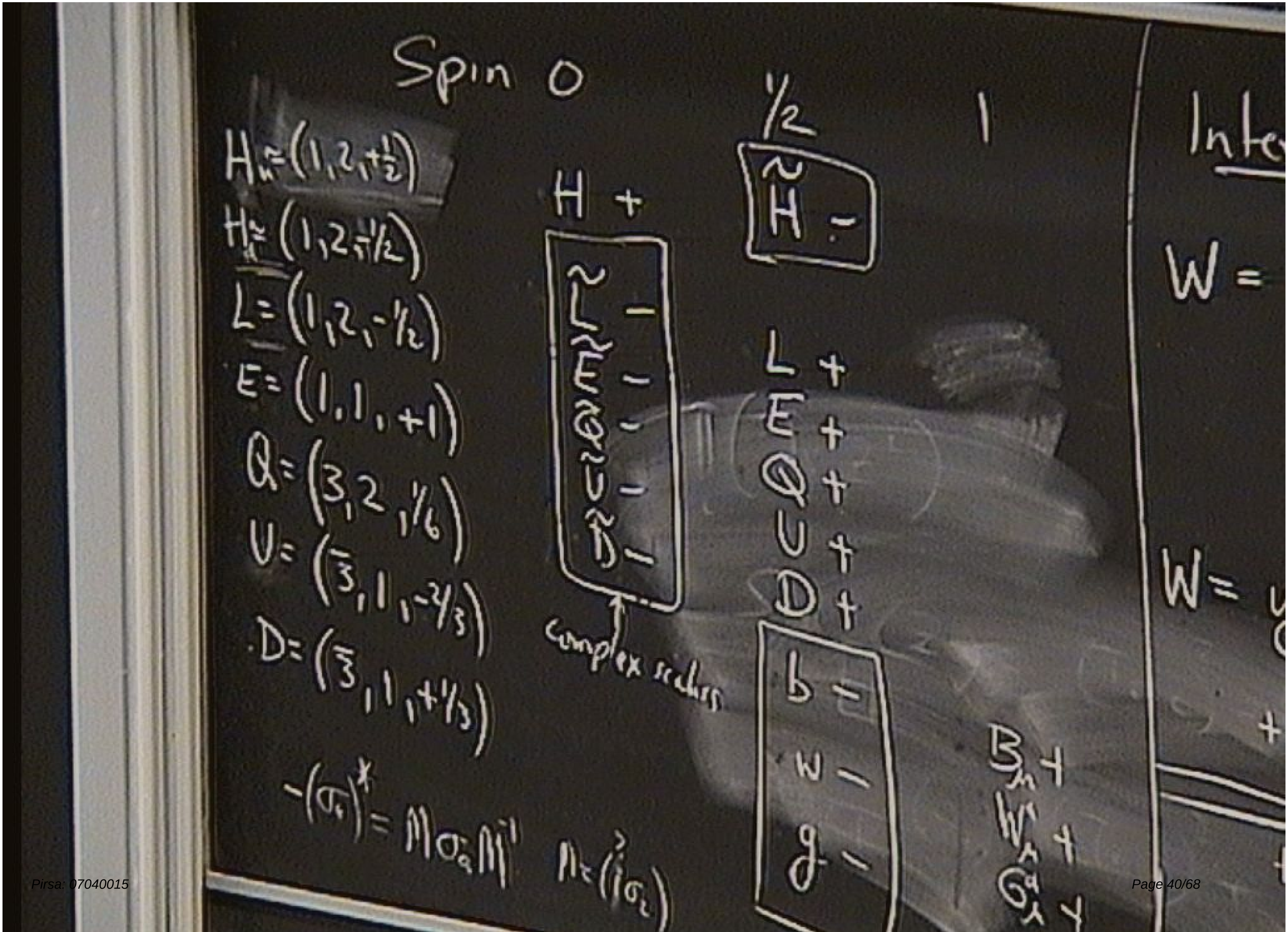
$-(\sigma_a)^* = M \sigma_a M^{-1} \qquad M = (i\sigma_2)$

Spin 0	Spin 1/2	Spin 1
H +	$\boxed{\tilde{H}\ -}$	
$\tilde{L}$ −	L +	
$\tilde{E}$ −	E +	
$\tilde{Q}$ −	Q +	
$\tilde{U}$ −	U +	
$\tilde{D}$ −	D +	
	b −	B_μ +
	w −	W^a_μ +
	g −	G^a_μ +

complex scalars


spin 0
1/2
1
H_u = (1,2,+1/2)
H_d = (1,2,-1/2)
L = (1,2,-1/2)
E = (1,1,+1)
Q = (3,2,1/6)
U = (3̄,1,-2/3)
D = (3̄,1,+1/3)
H +
H̃ −
L̃ −
Ẽ −
Q̃ −
Ũ −
D̃ −
complex scalars
L +
E +
Q +
U +
D +
b −
w −
g −
B +
W +
G +


All SM particles only get masses once $SU_2 \times U_1$ breaks;
$m \approx y v \lesssim v$
Scalars (squarks, sleptons) $SU_2 \times U_1$ cannot
forbid: $\phi^\dagger \phi M^2$


Spin 0
1/2
1
Inter
H_u = (1, 2, +1/2)
H_d = (1, 2, -1/2)
L = (1, 2, -1/2)
E = (1, 1, +1)
Q = (3, 2, 1/6)
U = (3̄, 1, -2/3)
D = (3̄, 1, +1/3)
H +
H̃ −
L̃ −
Ẽ −
Q̃ −
Ũ −
D̃ −
complex scalars
L +
E +
Q +
U +
D +
b −
w −
g −
B_μ +
W_μ +
G^a_μ +
W =
W =
-(σ_a)* = M σ_a M^-1
M = (i σ_2)

SM particles which like to be light.
+ the SP heavy.

All SM particles only get masses once $SU_2 \times U_1$ breaks,

$m \sim y v \lesssim v$

Scalars (squarks, sleptons) $SU_2 \times U_1$ cannot forbid: $\phi^\dagger \phi$

Gauginos are in adjoint rep (real) & so are also not forbidden a mass $\bar{\lambda}^a \lambda_a$

$H_u = (1, 2, +\tfrac{1}{2})$

$H_d = (1, 2, -1/2)$

$L = (1, 2, -1/2)$

$E = (1, 1, +1)$

$Q = (3, 2, 1/6)$

$U = (\bar{3}, 1, -2/3)$

$D = (\bar{3}, 1, +1/3)$

$H \; +$

$\boxed{\begin{array}{l} \tilde{L} \; - \\ \tilde{E} \; - \\ \tilde{Q} \; - \\ \tilde{U} \; - \\ \tilde{D} \; - \end{array}}$

complex scalars

$\boxed{\tilde{H} \; -}$

$L \; +$

$E \; +$

$Q \; +$

$U \; +$

$D \; +$

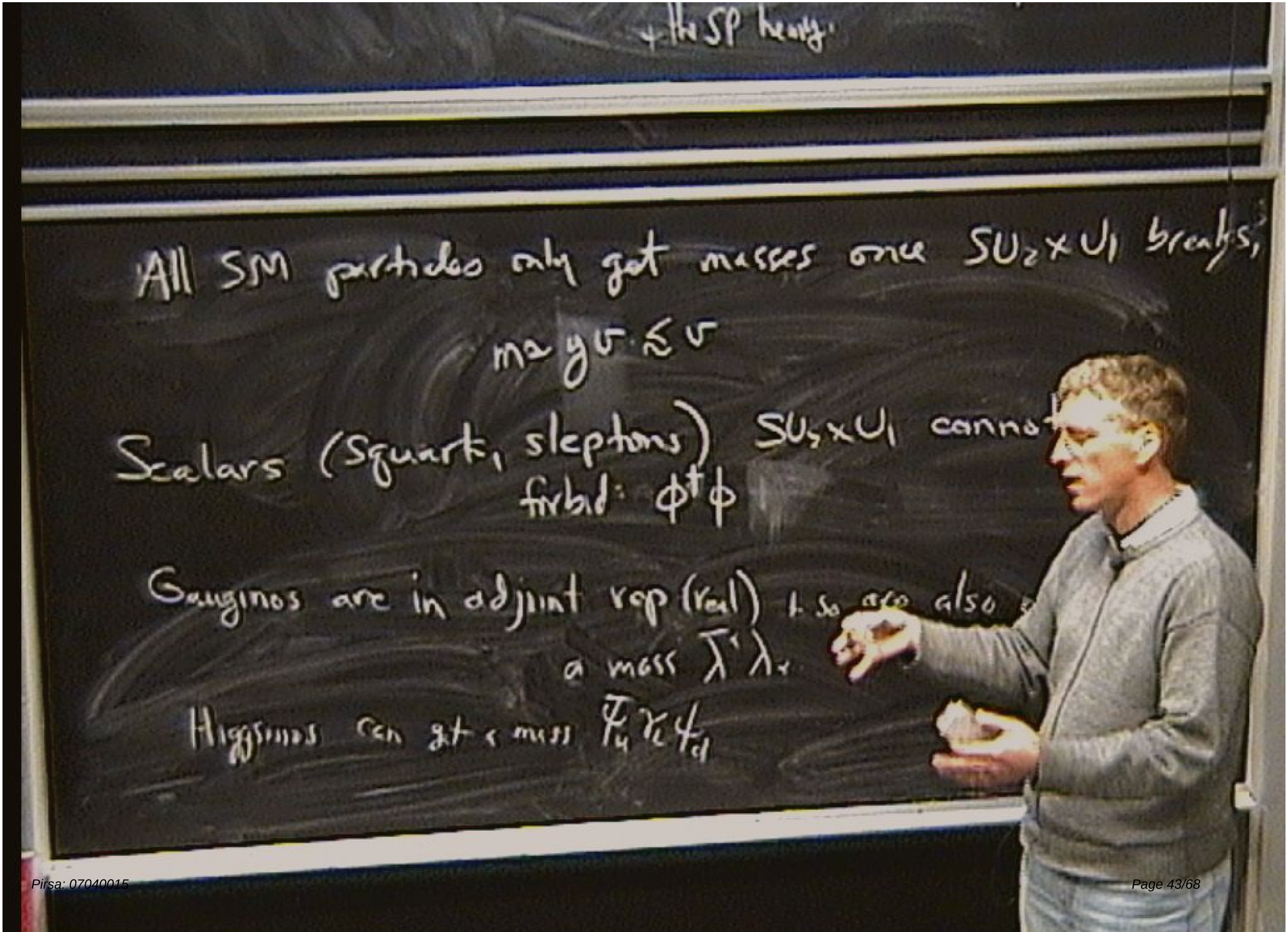
$\boxed{\begin{array}{l} b \; - \\ w \; - \\ g \; - \end{array}}$

$B_\mu \; +$

$W^i_\mu \; +$

G^a_μ

$-(\sigma_a)^* = M \sigma_a M^{-1} \qquad M = (i\sigma_2)$


+ the SP heavy.
All SM particles only get masses once SU2 x U1 breaks;
m ~ yv ≲ v
Scalars (squarks, sleptons) SU2 x U1 cannot
forbid: φ†φ
Gauginos are in adjoint rep (real) ⇒ so are also
a mass λ̄ᵃλₐ
Higgsinos can get a mass

$W = m\phi + \frac{1}{2} g \phi^2$

$W' = m + g\phi \longleftarrow y \sim g$

$V = |W'|^2 + (\phi^\dagger \phi)^2 \frac{1}{4} g^2 + \frac{1}{2} g m \phi^\dagger v^2 + \dots$

What about the other Higgs?

$\mathcal{L}_{yuk} = (\bar{Q}\, y_u\, U) H_u +$

$W = m\phi + g\phi^2$

$W' = m + g\phi \leftarrow y = g$

$V = |W'|^2 + (\phi^\dagger \phi)^2 \frac{1}{2} g^2 + y g m \phi^\dagger \phi^2 + \dots$

What about the other Higgs?

$$\mathcal{L}_{yuk} = (\bar{Q} Y_U U) H_u + (\bar{Q} Y_D D) H_d + (\bar{L} Y_E E) H_d +$$
$$(\bar{Q} Y_U U) \tilde{H}_d + (\bar{Q} Y_D D) \tilde{H}_u + (\bar{L} Y_E E) \tilde{H}_u$$

$$\mathcal{L}_{\mathrm{yuk}} = (\bar{Q}\,\gamma_L U) \ldots +$$

$$+ (\bar{Q}\,\gamma_L U)\,\tilde{A}$$

$$H_u \equiv$$

$$H_u = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$H_d = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$$

What about the other Higgs?

$$\mathcal{L}_{yuk} = (\bar{Q} Y_u U) H_u + (\bar{Q} Y_d D) H_d + (\bar{L} Y_e E) H_d$$
$$+ (\bar{Q} Y_u U) \tilde{H}_d + (\bar{Q} Y_d D) \tilde{H}_u + (\bar{L} Y_e E) \tilde{H}_u$$

$H_u = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ Check that V is minimized by

$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}$

$H_d = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$ $\langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}$

What about the other Higgs?

$$\mathcal{L}_{yuk} = (\bar{Q} Y_L U) H_u + (\bar{Q} Y_L D) H_d + (\bar{L} Y_L E) H_d$$
$$+ (\bar{Q} Y_L U) \tilde{H}_d + (\bar{Q} Y_L D) \tilde{H}_u + (\bar{L} Y_L E) \tilde{H}_u$$

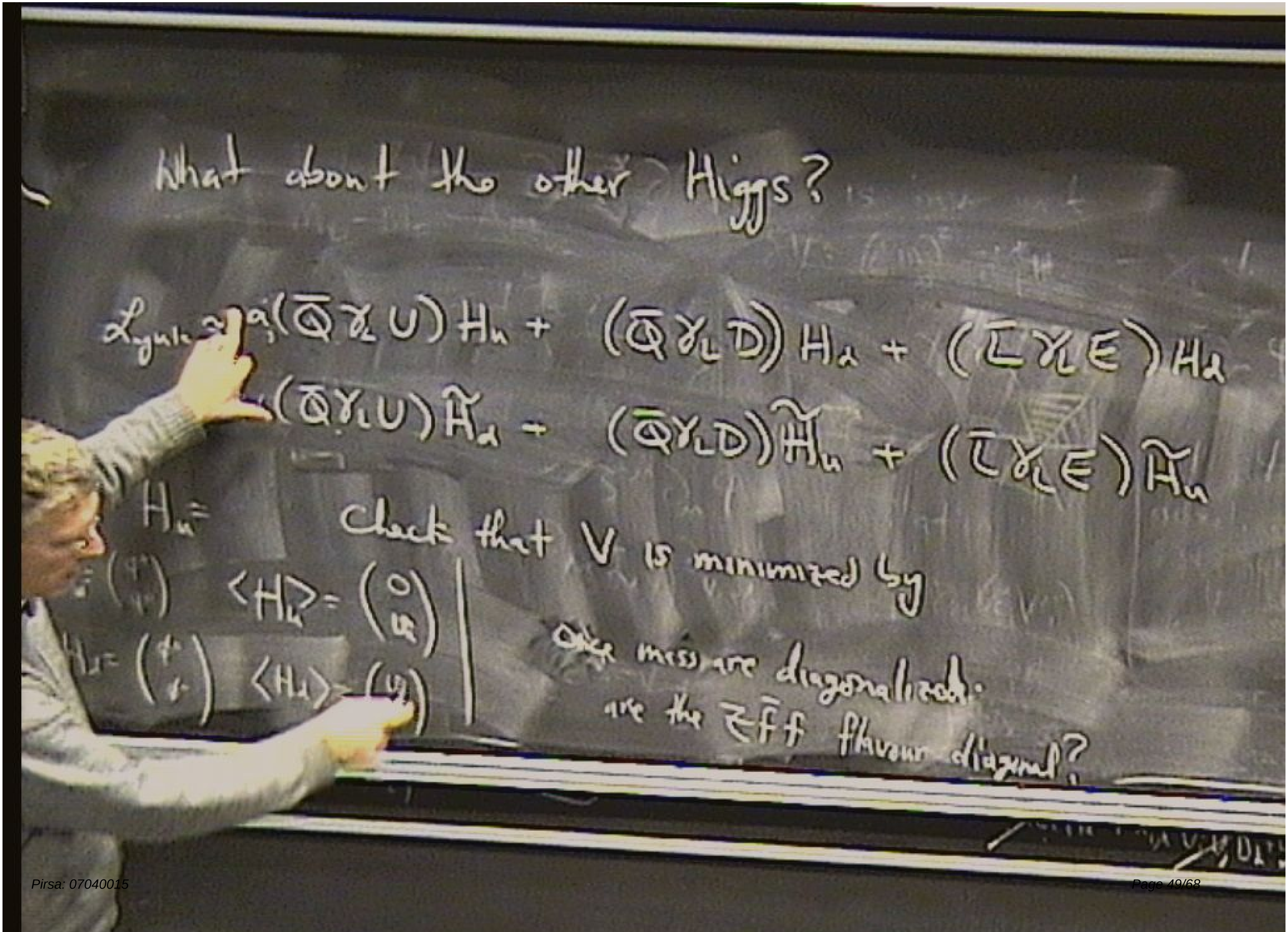
$H_u =$

Check that V is minimized by

$H_u = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ $\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}$

$H_d = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$ $\langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}$

m


What about the other Higgs?
$\mathcal{L}_{yuk} = q_i(\bar{Q} Y_U U) H_u + (\bar{Q} Y_D D) H_d + (\bar{L} Y_E E) H_d$
$(\bar{Q} Y_U U) \tilde{H}_d + (\bar{Q} Y_D D) \tilde{H}_u + (\bar{L} Y_E E) \tilde{H}_u$
$H_u =$
Check that V is minimized by
$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}$
$H_d = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
$\langle H_d \rangle = \begin{pmatrix} v_d \\ \end{pmatrix}$
Once mass are diagonalized:
are the $Z\bar{f}f$ flavour diagonal?

other Higgs?

$$\ldots H_u + (\bar{Q} Y_L D) H_d + (\bar{L} Y_L E) H_d$$

$$\ldots - (\bar{Q} Y_L D) \tilde{H}_u + (\bar{L} Y_L E) \tilde{H}_u$$

that V is minimized by

Once mass are diagonalized:

are the $Z \bar{f} f$ flavour diagonal?

Kaymakcalan Schwinger

other Higgs?

$$\ldots_u + (\bar{Q} Y_L D) H_d + (\bar{L} Y_L E) H_d$$

$$\ldots_d + (\bar{Q} Y_L D) \tilde{H}_u + (\bar{L} Y_L E) \tilde{H}_u$$

that V is minimized by

Once mass are diagonalized:
are the $Z \bar{f} f$ flavour diagonal?

$m_t g \beta \longleftarrow y_t g \qquad (\ldots) \leq g^2 \ldots y_t m_t g^2 v^2 + \ldots$

What about the other Higgs?

$$\mathcal{L}_{yuk} \ni q_1 (\bar{Q} \gamma_L U) H_u + (\bar{Q} \gamma_L D) H_d + (\bar{L} \gamma_L E) H_d$$

$H_u = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$ $\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}$

$\begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$ $\langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}$

Check that V is minimized by

Once masses are diagonalized.
are the $\bar{f} f$ flavour diagonal?

$$\mathcal{L}_{yuk} \simeq y^u_{ij}(\bar{Q}_i \gamma_L U) +$$

$$\frac{m_e}{m_t} \simeq \frac{y^e}{y_t}\,\frac{v_d}{v_u} \qquad m_u \simeq y^u v_u$$

$$H_u \simeq$$

Check

$$H = (\phi^+)$$

$) H_u + (\bar{L} Y_L E) H_d$

$y^u v_d \qquad m_\ell \simeq y^\ell v_d$

$\frac{v_u}{v_d} \equiv \tan\beta$ could be large.

nimized by

$K \to \mu^+ \mu^-$

$\bar{s}d \to \mu^+ \mu^-$

e diagonalized.

eff fl...

$$- \quad g^2 m^2 \leq 0$$

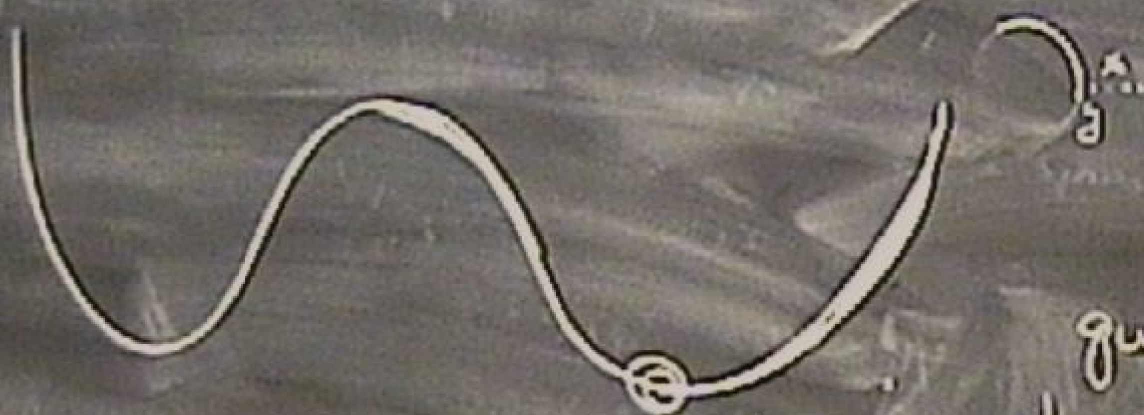
$g \ldots$

quartic terms:

turn out be proportional to g_1^2, g_2^2

SUPERSYMMETRY

expect one Higgs to be $\lesssim 130$ GeV.



quartic terms:

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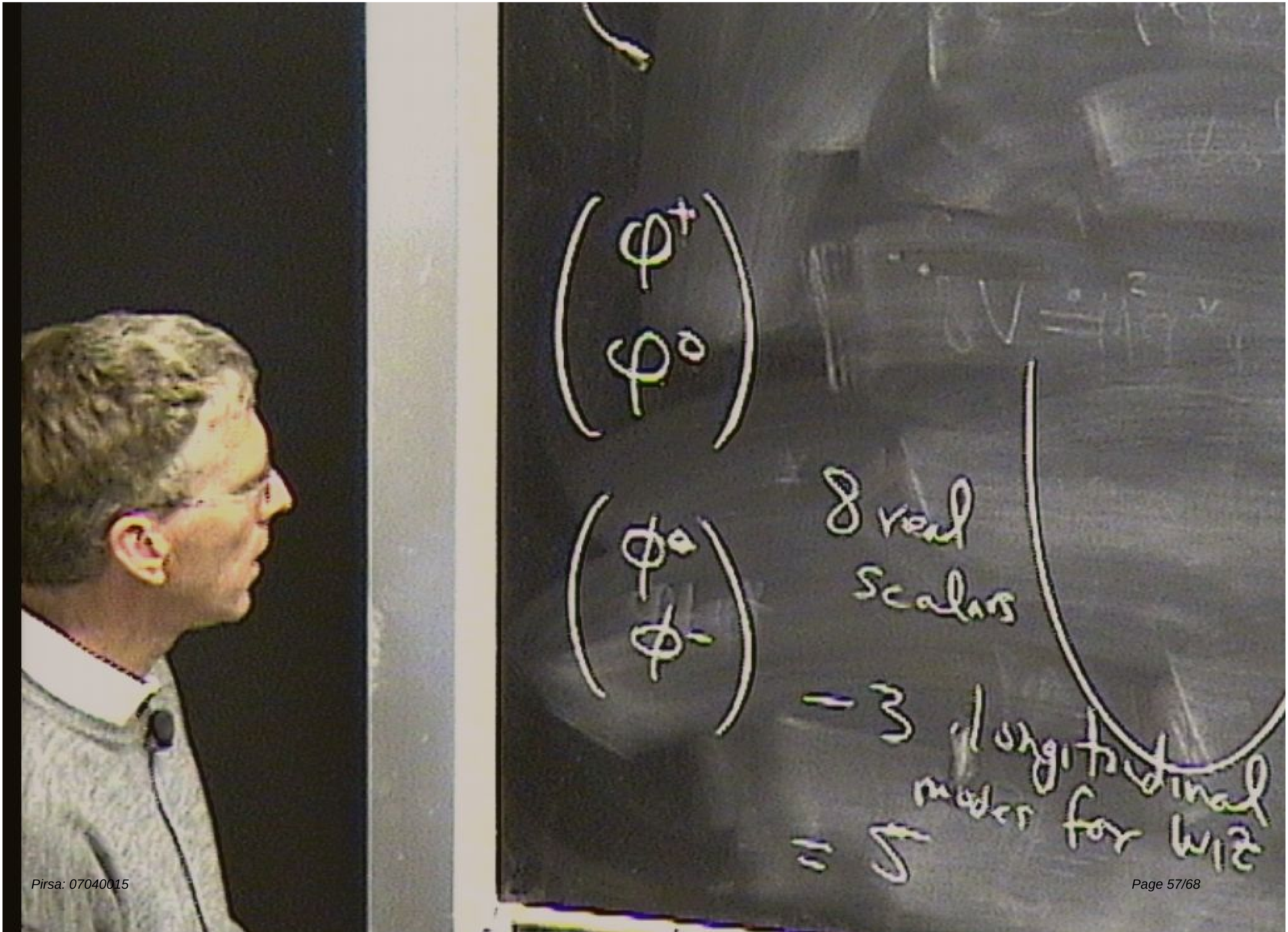

$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
$\begin{pmatrix} \phi^{0*} \\ \phi^- \end{pmatrix}$
8 real scalars
− 3 longitudinal modes for W, Z
= 5

expec

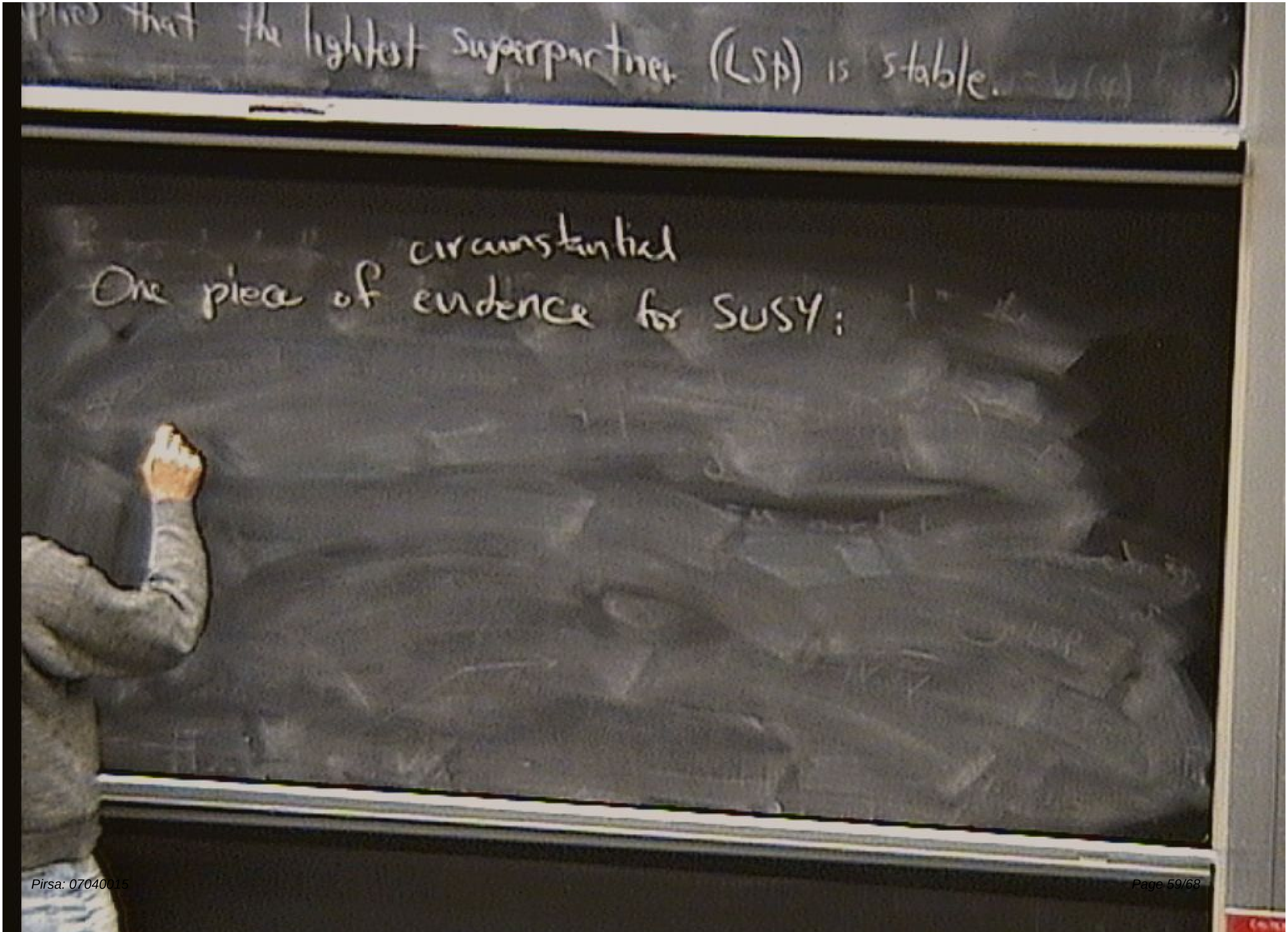
$$\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$$

$$\begin{pmatrix}\phi^0\\ \phi^-\end{pmatrix}$$

8 real scalars

− 3 longitudinal modes for W, Z

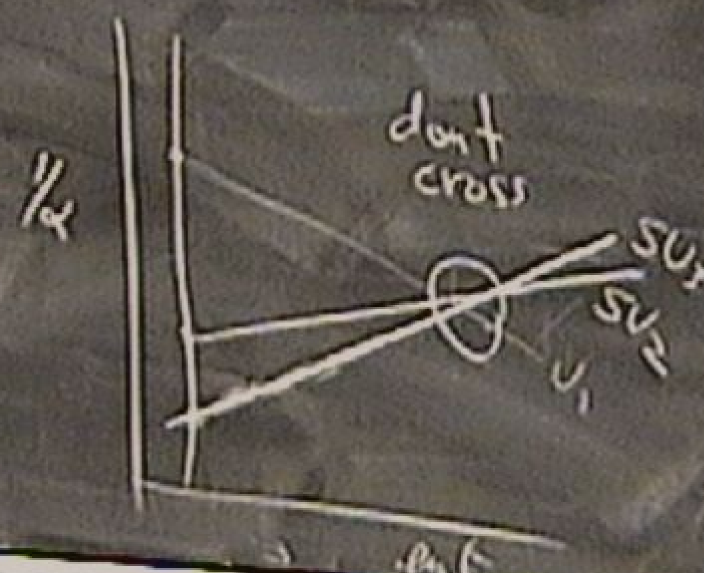
= 5 = 1 charged + 3 neutral


plies that the lightest superpartner (LSP) is stable.
circumstantial
One piece of evidence for SUSY:

One piece of circumstantial evidence for SUSY:

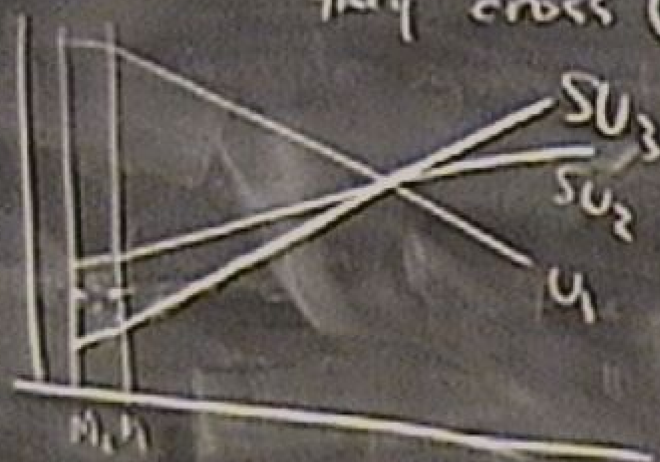
α_3
α_2 all run with energy:
α_1

$$\frac{1}{\alpha_i(E)} = \frac{1}{\alpha_i(m_Z)} + b_i \log\left(\frac{E}{m_Z}\right)$$

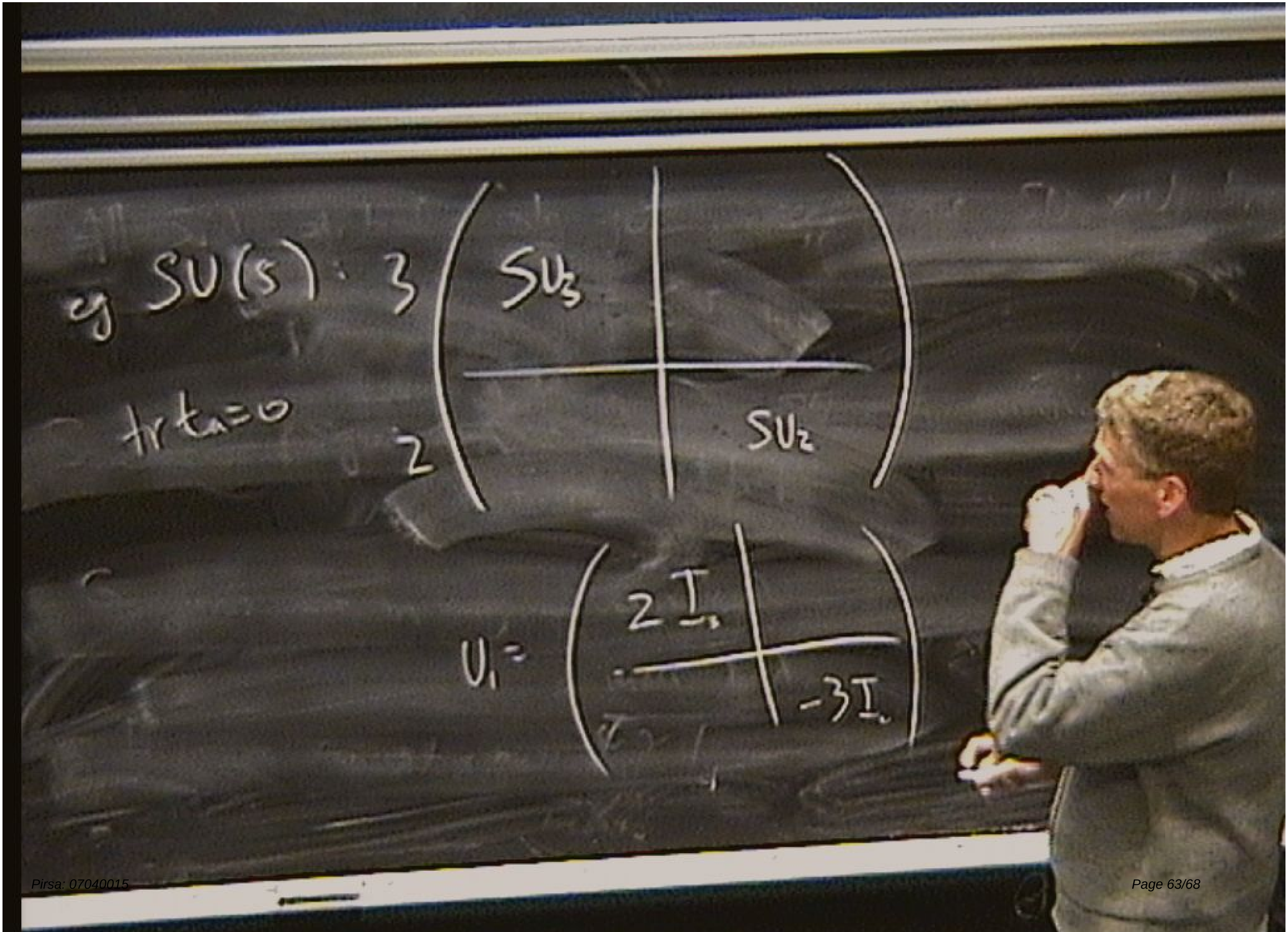


<u>But</u> if the b_i's are computed using the SS spectrum they cross (at $E \simeq 10^{14 \div 15}$ GeV)

<u>But</u> if the b_i's are computed using the SS spectrum they cross (at $E \simeq 10^{16.5}$ GeV)



This could be evidence for grand unification of interactions.

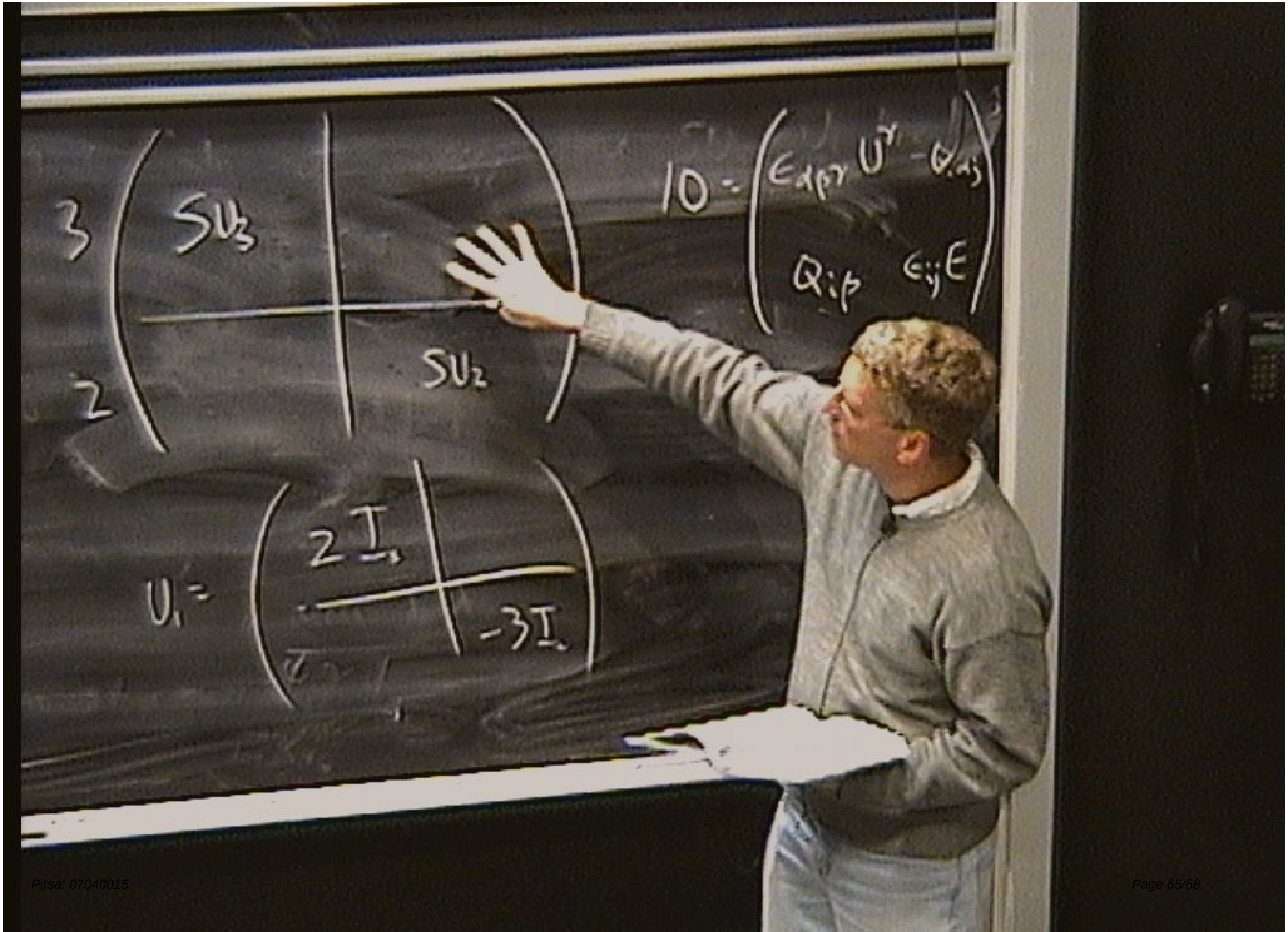

eg SU(5): 3 SU3
tr ta=0
2 SU2
U1 = 2 I3
-3 I2

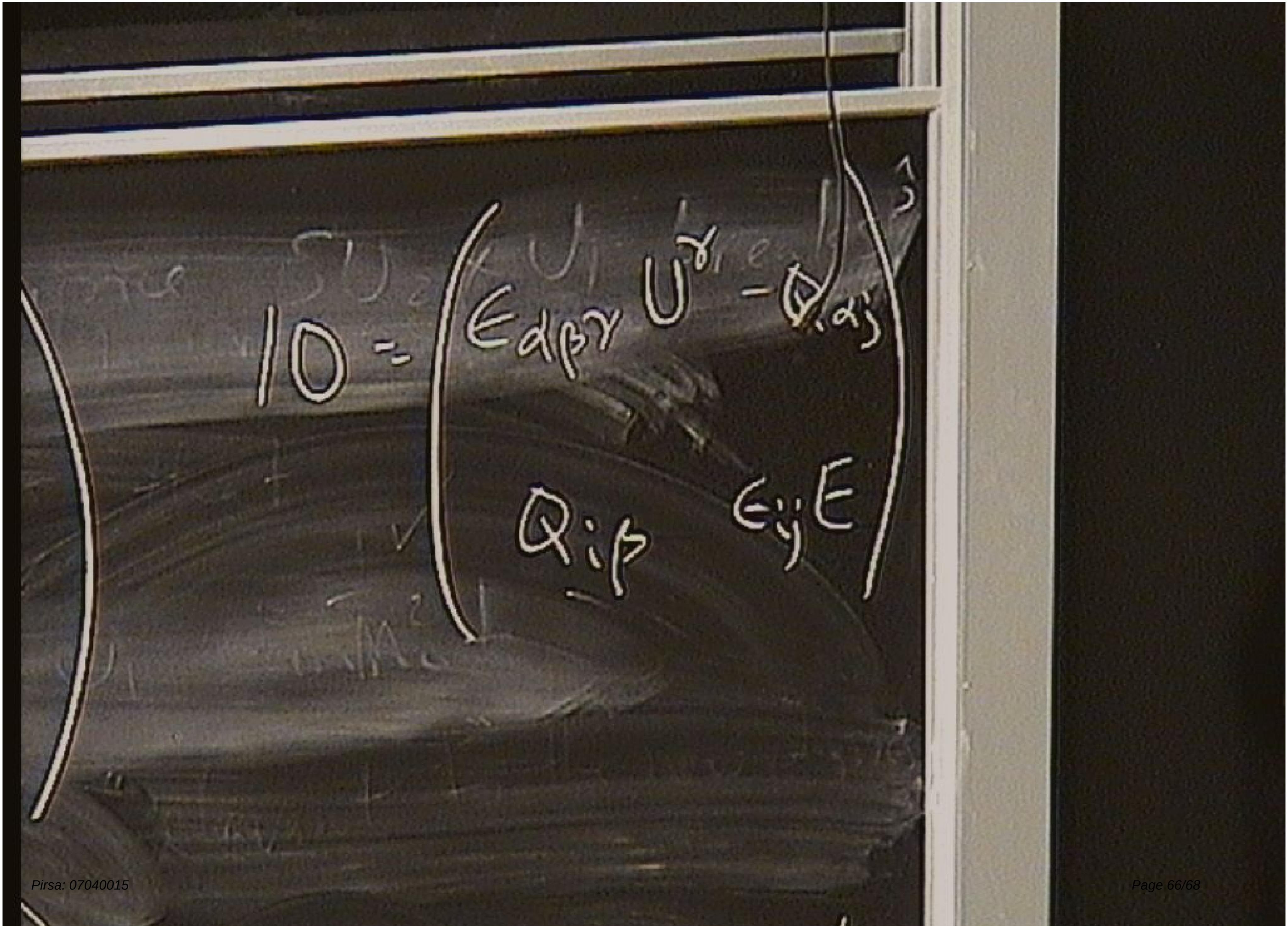
eg $SU(5)$:

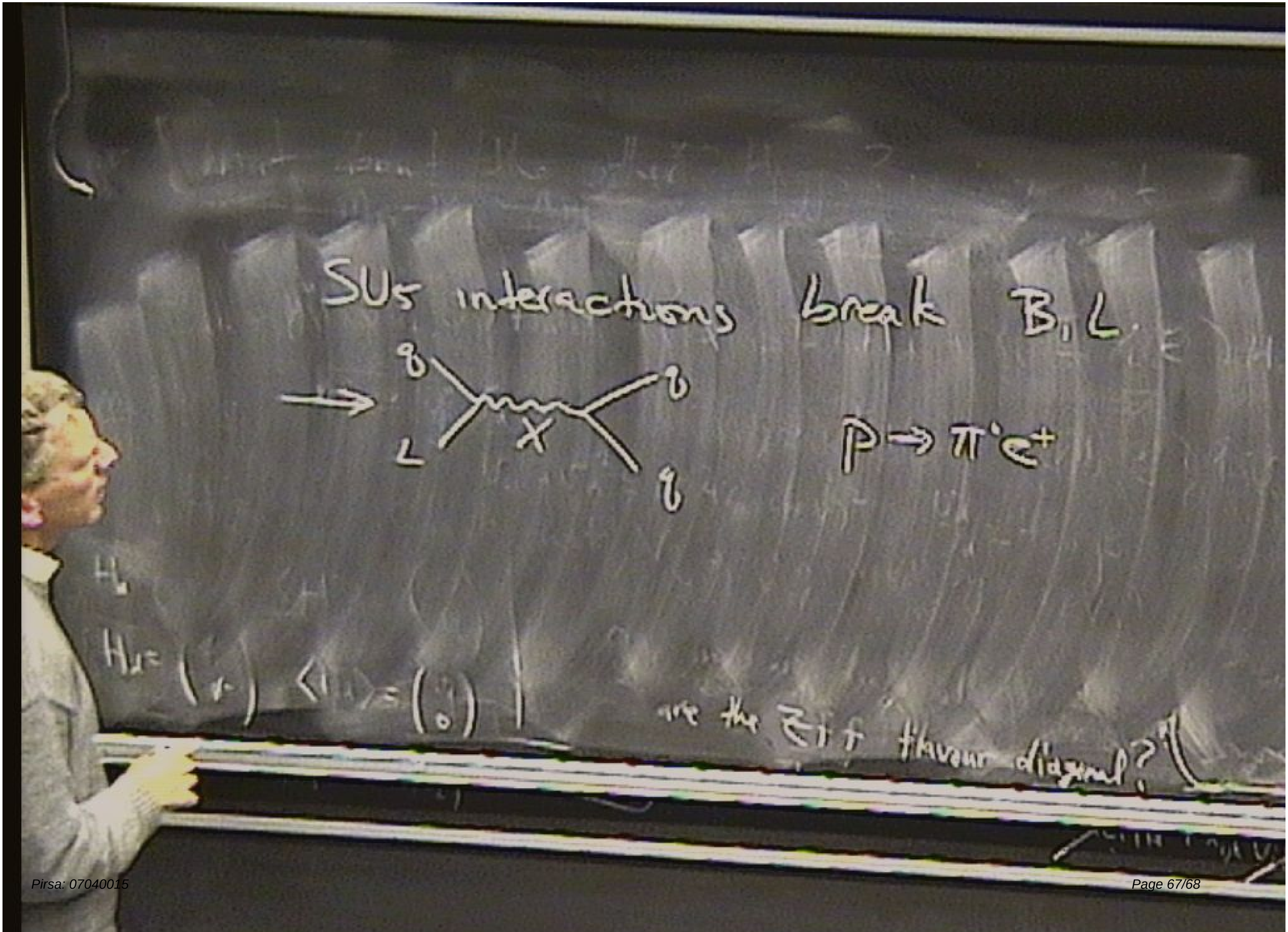
$$\begin{array}{c} 3 \\ 2 \end{array}\left(\begin{array}{c|c} SU_3 & \\ \hline & SU_2 \end{array}\right)$$

$\mathrm{tr}\, t_a = 0$

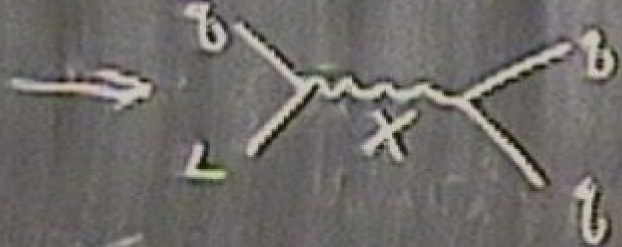
$$\begin{pmatrix} D_\alpha \\ L_i \end{pmatrix}_R \in 5 \qquad U_1 = \left(\begin{array}{c|c} 2 I_3 & \\ \hline & -3 I_2 \end{array}\right)$$


3
SU3
2
SU2
U1 =
2 I
-3 I
10 =
Q i β
ε ij E


10 = (εαβγ Uγ −Qαj
Qiβ εij E)


SU5 interactions break B, L
q
L
X
q
q
p → π°e+
are the Σ's flavour diagonal?

SU_5 interactions break B, L.



$p \to \pi^0 e^+$

$A \simeq \dfrac{g^2}{M_X^2}$ $\quad \Gamma \simeq \dfrac{g^4}{M_X^4} m_p^5$ $\quad \tau_p > \tau_{exp}$

$\to M_X \gtrsim 10^{15-16}\ \mathrm{GeV}$