Title: Coupled Flux Qubits with Controllable Interaction

Date: Apr 04, 2007 04:00 PM

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Abstract: After a brief overview of the three broad classes of superconducting quantum bits (qubits)--flux, charge and phase--I describe experiments on single and coupled flux qubits. The quantum state of a flux qubit is measured with a Superconducting QUantum Interference Device (SQUID). Single flux qubits exhibit the properties of a spin-1/2 system, including superposition of quantum states, Rabi oscillations and spin echoes. Two qubits, coupled by their mutual inductance and by screening currents in the readout SQUID, produce a ground state |0> and three excited states |1>, |2> and |3>. Microwave spectra reveal an anticrossing between the |1>and |2> energy levels. The level repulsion can be reduced to zero by means of a current pulse in the SQUID that changes its dynamic inductance and hence the coupling between the qubits. The results are in good agreement with predictions. The ability to switch the coupling between qubits on and off permits efficient realization of universal quantum logic. This work was in collaboration with T. Hime, B.L.T. Plourde, P.A. Reichardt, T.L. Robertson, A. Ustinov, K.B. Whaley, F.K. Wilhelm and C.-E. Wu, and supported by AFOSR, ARO and NSF.

Coupled Flux Qubits with Controllable Interaction

- **Introduction: Single flux qubits**
- **Controllable coupling of two flux qubits:**
  - **Theory**
  - **Experiment**
- **Concluding remarks**

Experiments:
Travis Hime
Britton Plourde
Paul Reichardt
Tim Robertson
Alexey Ustinov
Cheng-En Wu
Theory:
Birgitta Whaley
Frank Wilhelm
Jun Zhang

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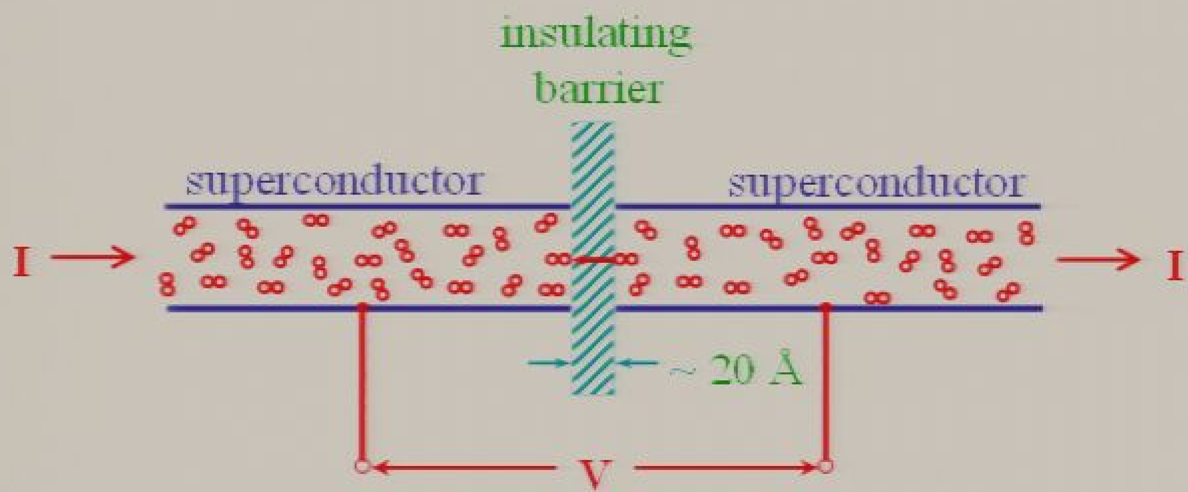
Air Force Office of Scientific Research
Army Research Office
National Science Foundation

Superconductivity

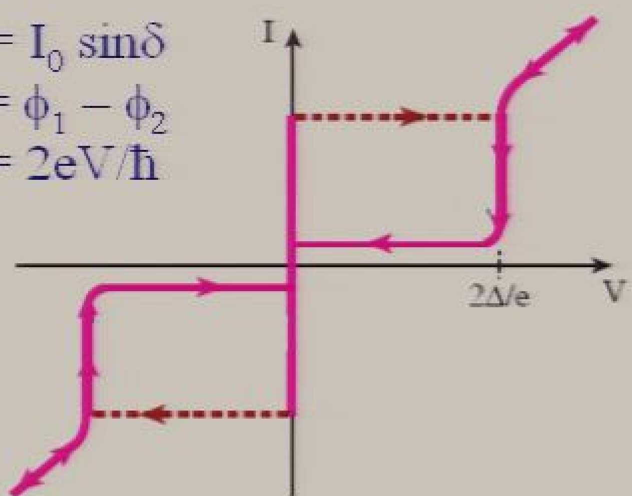
Josephson Tunneling



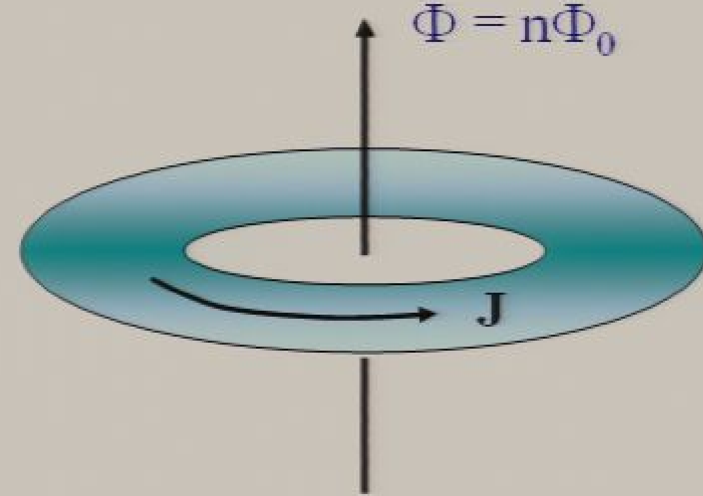
$I = I_0 \sin\delta$

$\delta = \phi_1 - \phi_2$

$d\delta/dt = 2eV/\hbar$



Flux Quantization



$\Phi = n\Phi_0$ $(n = 0, \pm 1, \pm 2, \ldots)$
where
$\Phi_0 \equiv h/2e \approx 2 \times 10^{-15}$ Wb
is the flux quantum

Superconductivity

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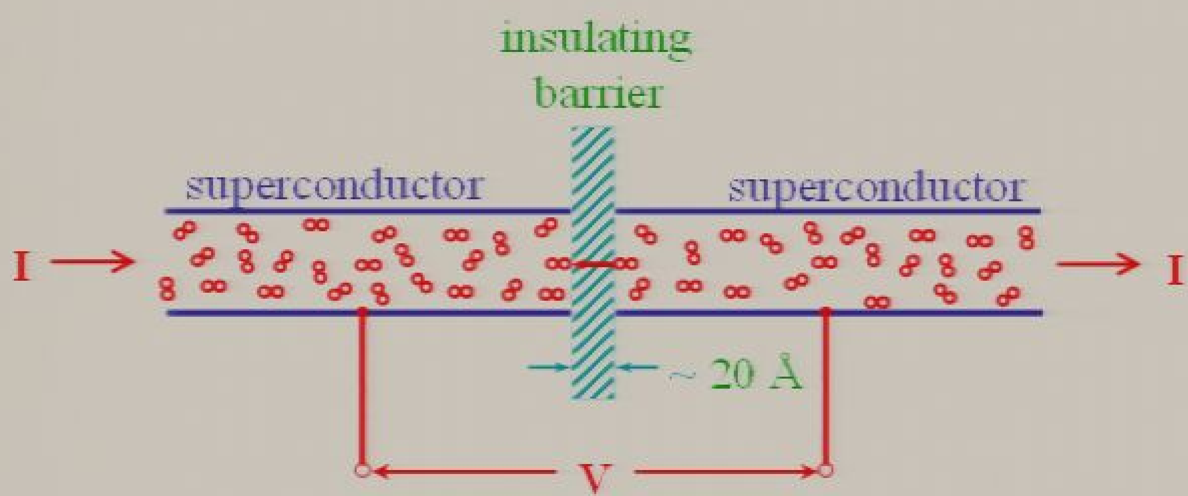

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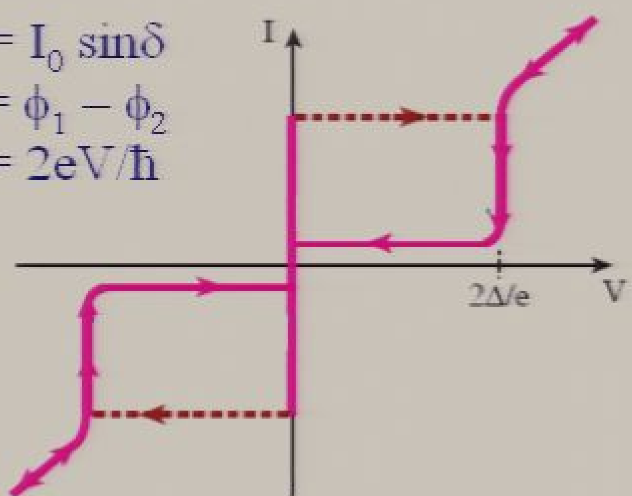

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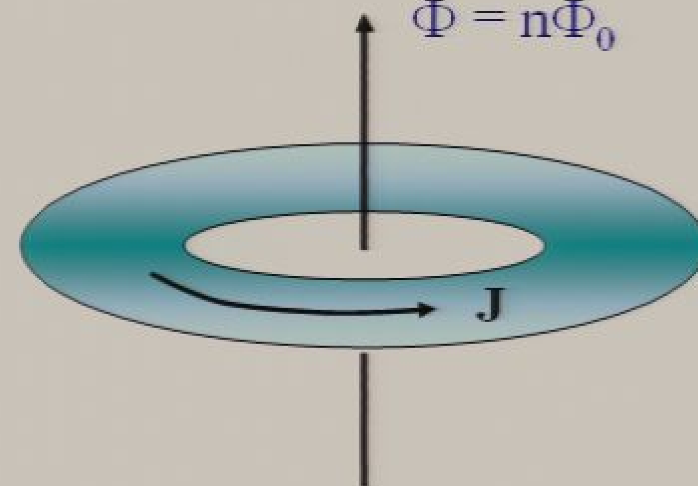

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Three-Junction Flux Qubit

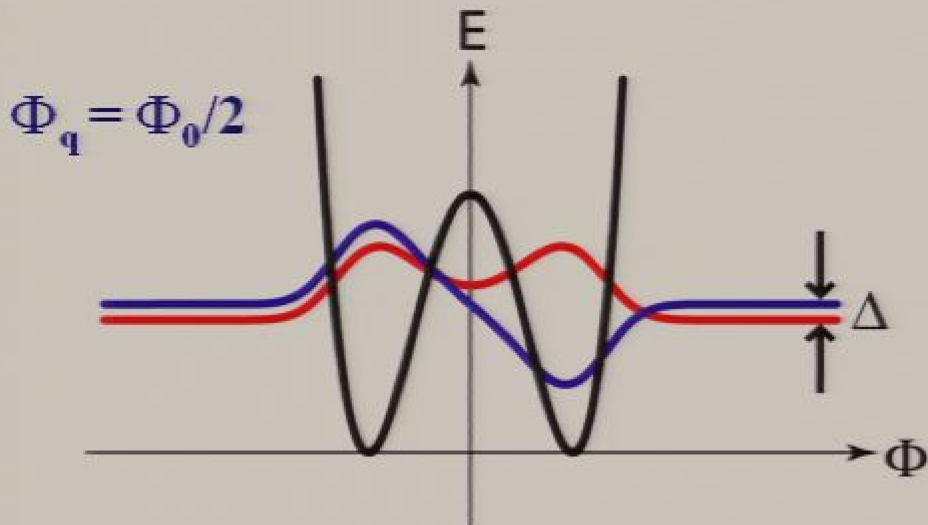
Degeneracy point: Applied flux $\Phi_q = \Phi_0/2$

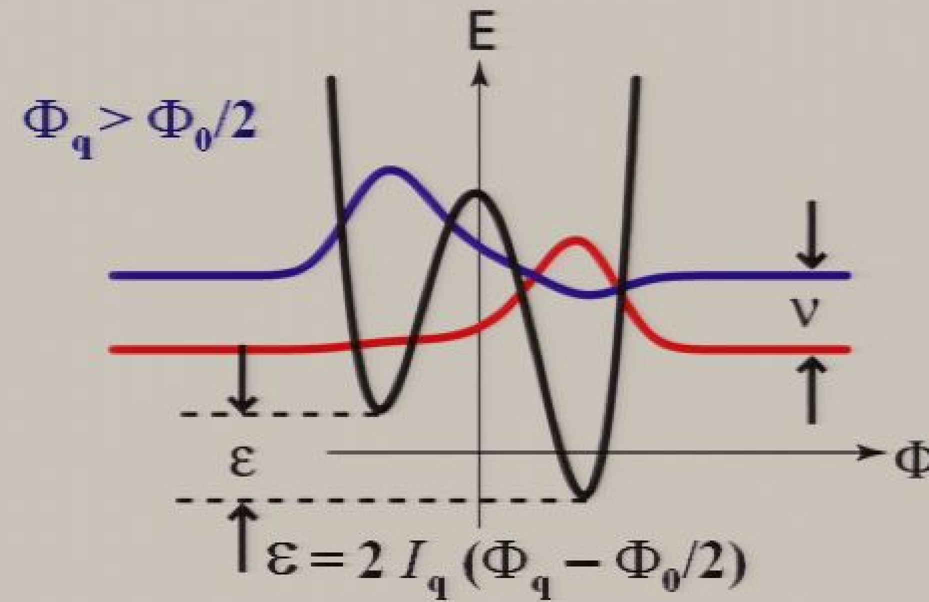
$$|\Psi\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$$

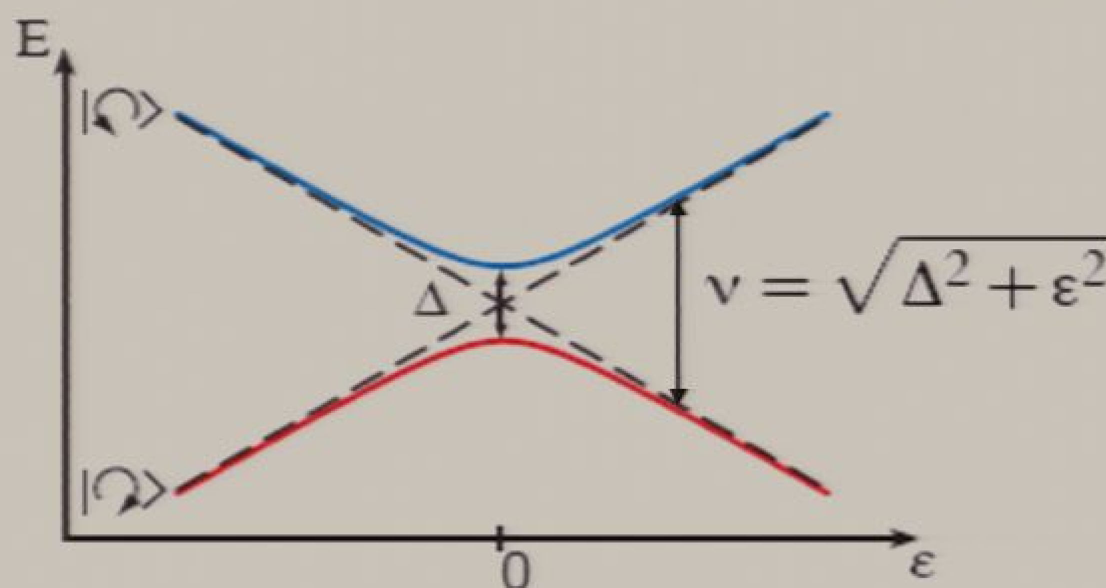
J.E. Mooij *et al.*, *Science* **285**, 1036 (1999)
C.H. Van der Wal *et al.*, *Science* **290**, 773 (2000)

- Loop inductance << Josephson inductance $\Phi_0/2\pi I_0$

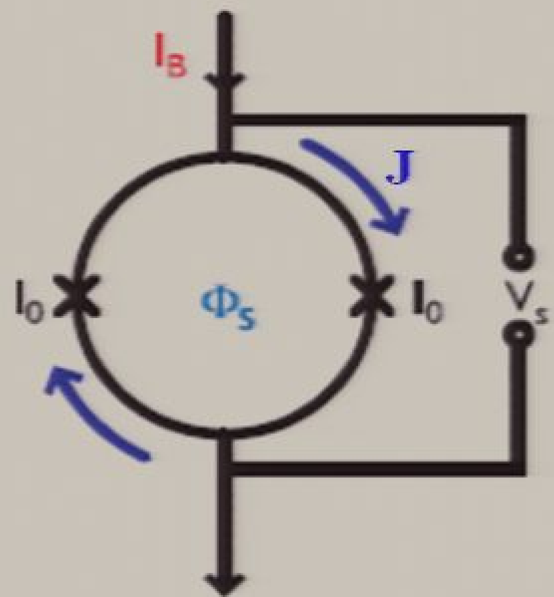
Energy of the Flux Qubit

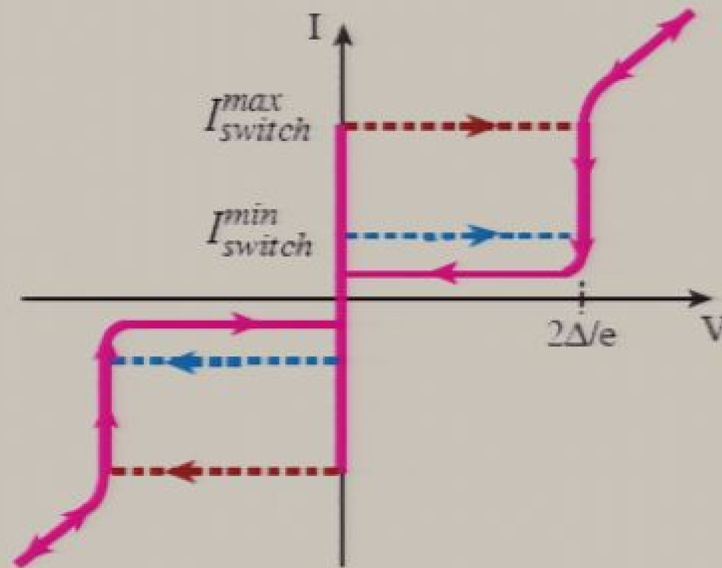


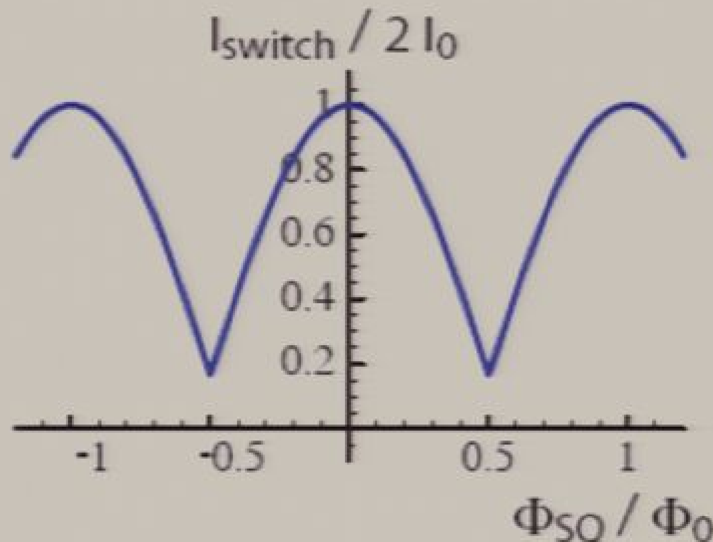




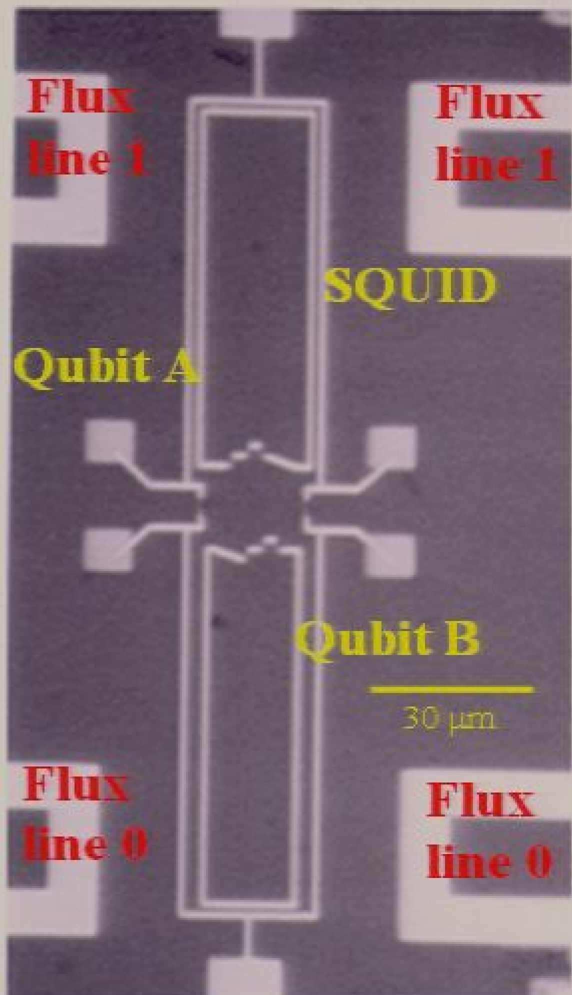
The dc Superconducting Quantum Interference Device (SQUID)

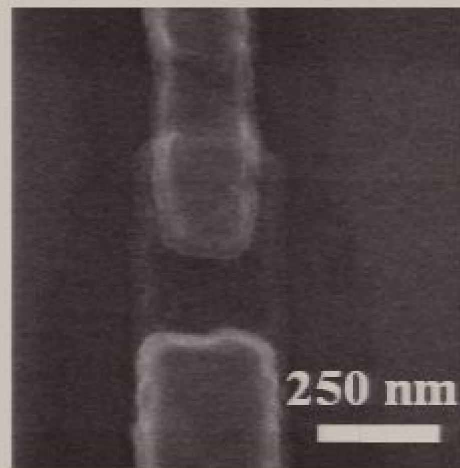






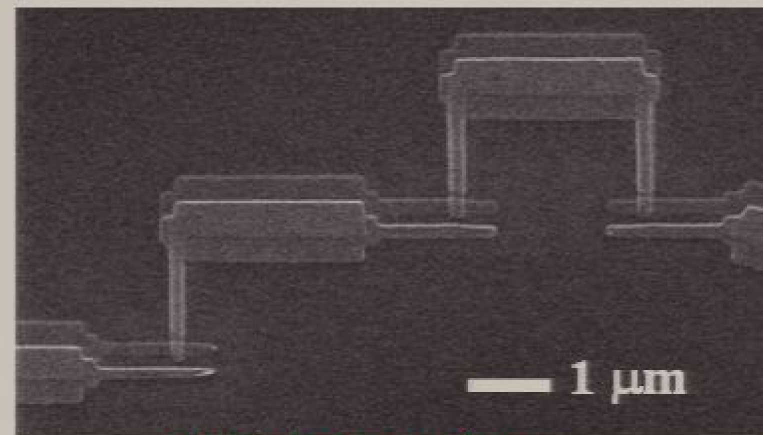
Two Flux Qubits, a SQUID and Flux Lines





SQUID junctions

215 x 250 nm² $C \approx 8.5$ fF



Qubit junctions

180x205 nm² $C_0 \approx 6.5$ fF

$L_q \sim 200$ pH

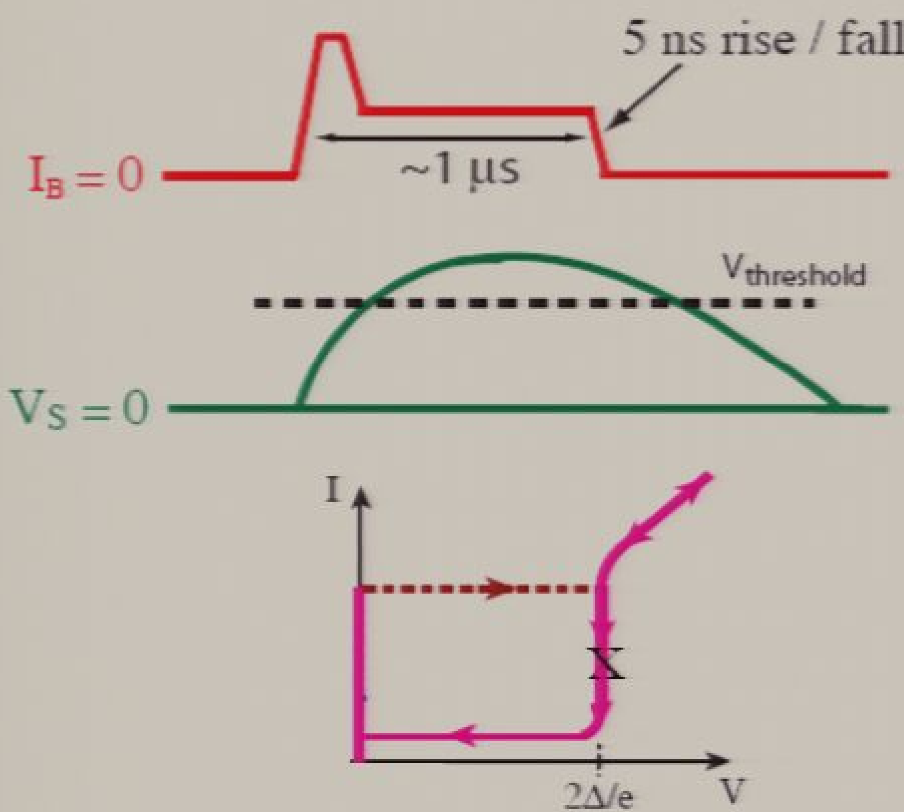
$L_J \sim 600$ pH

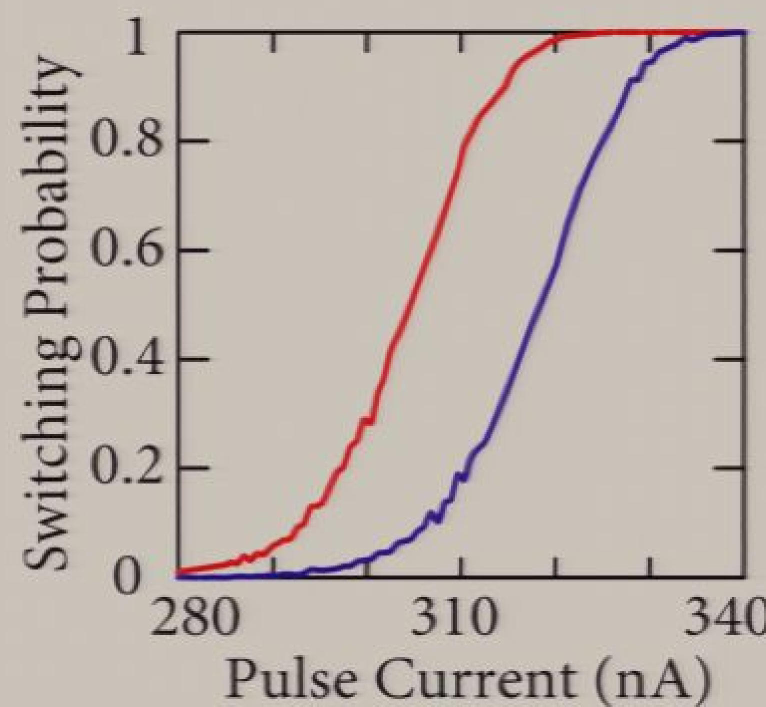
Loop inductance not negligible

- E-beam lithography, angled evaporation of Al film
- Two on-chip flux lines enable one to apply independent fluxes to any two of the three devices
- Large inductances to keep currents in flux lines small and provide adequate coupling energy
- No deposited insulating layer

B.L.T. Plourde *et al.*, *Phys. Rev.* **B72**, 060506-1 (20

SQUID Readout





- Pulse bias current: detect switching events
- Repeat (say) 1000 times to determine probability
- Increment bias current and repeat

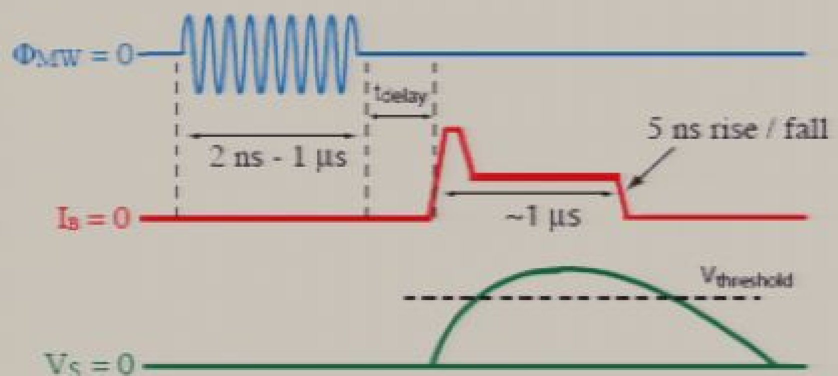
$\Phi_{QA} = 0.48\ \Phi_0$

$\Phi_{QA} = 0.52\ \Phi_0$

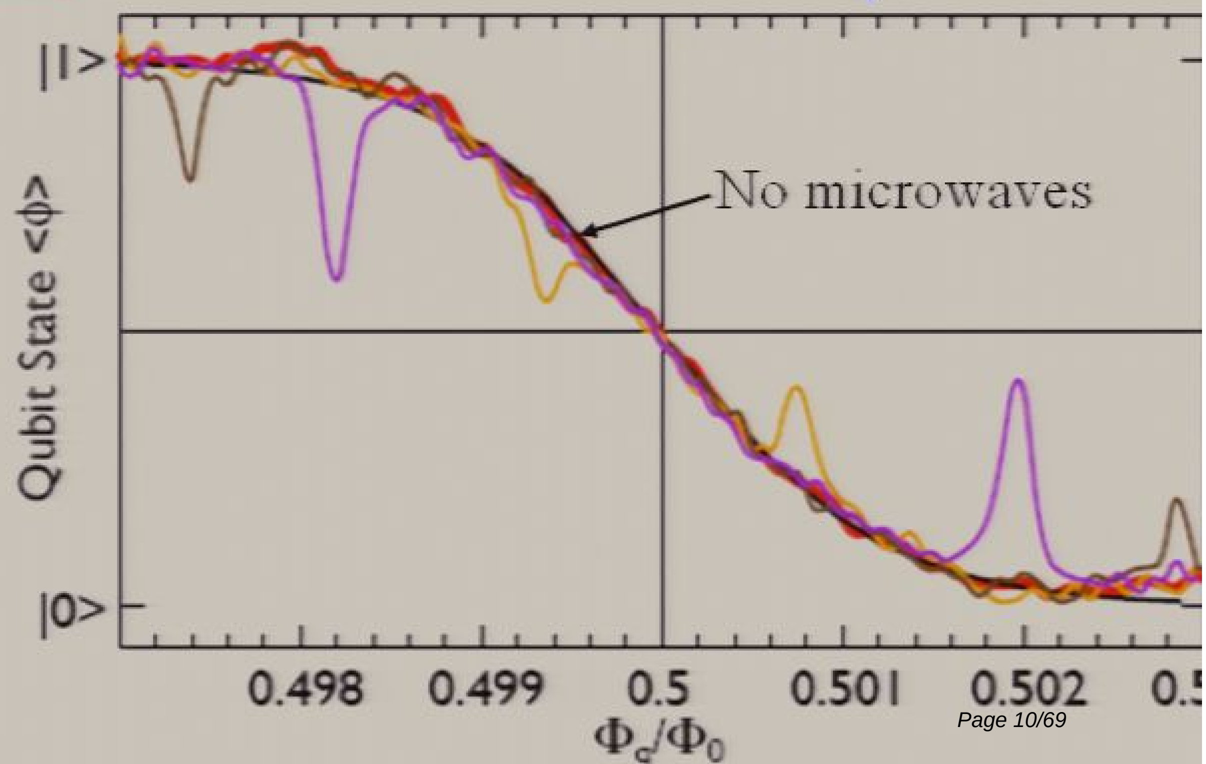
$\Phi_S\ = \text{constant}$

- Determine current $I_S^{50\%}$ for 50% switching probability

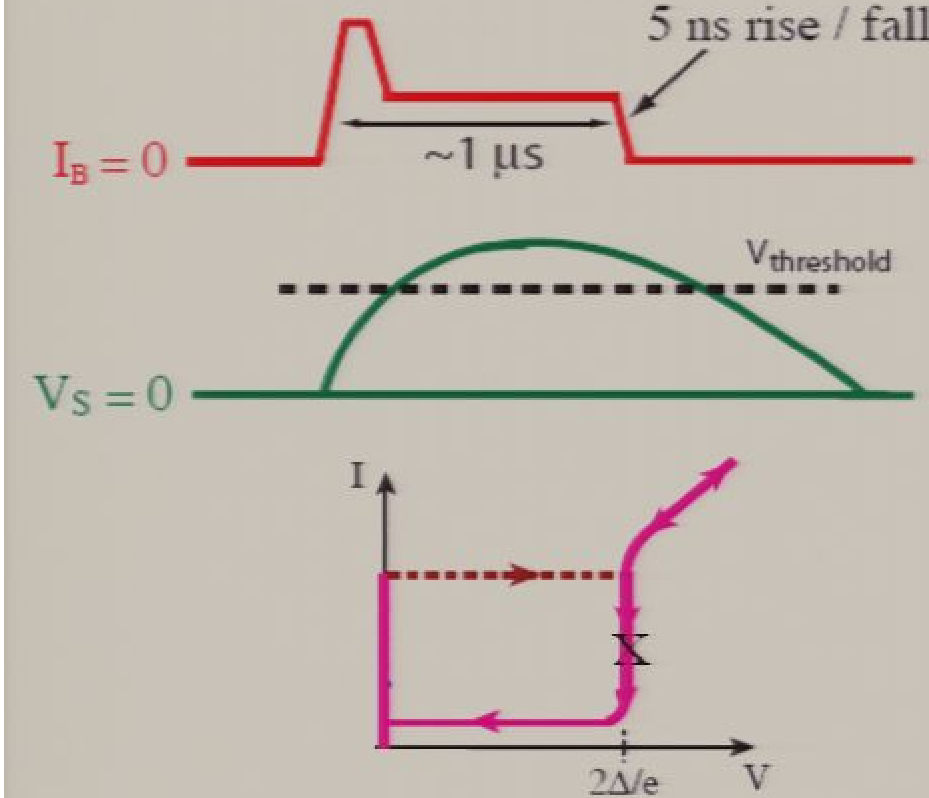
Spectroscopy

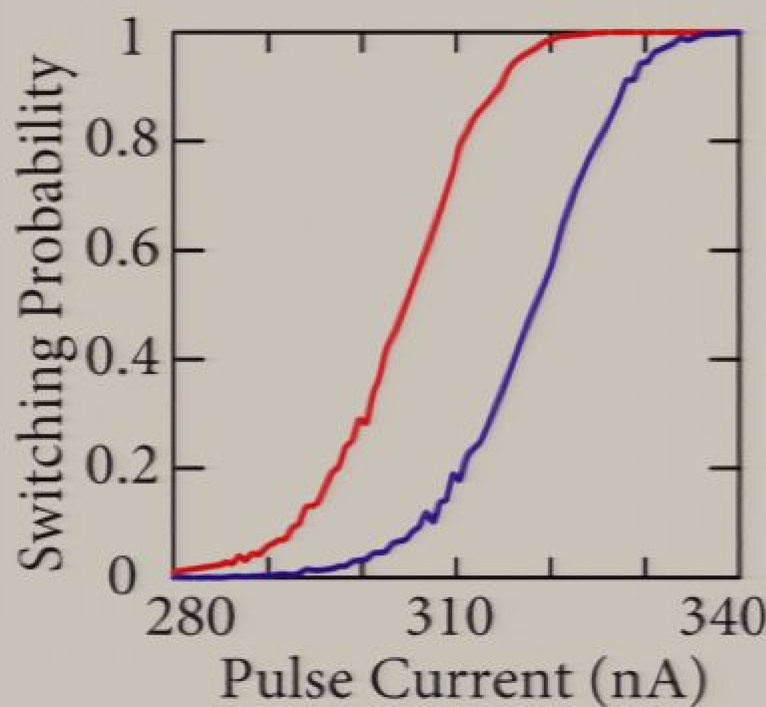


Three microwave frequencies



SQUID Readout





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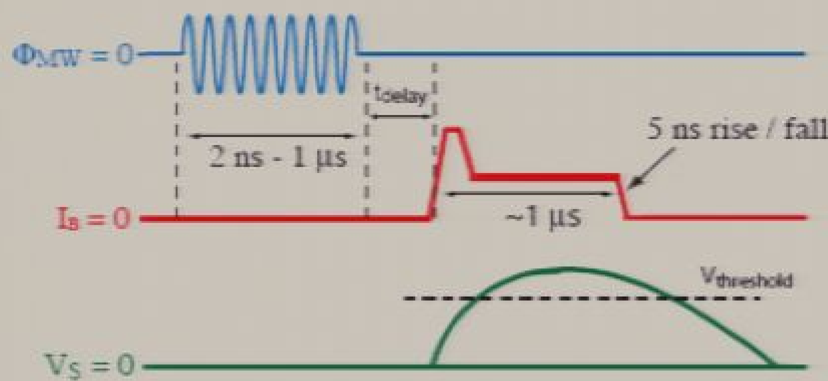

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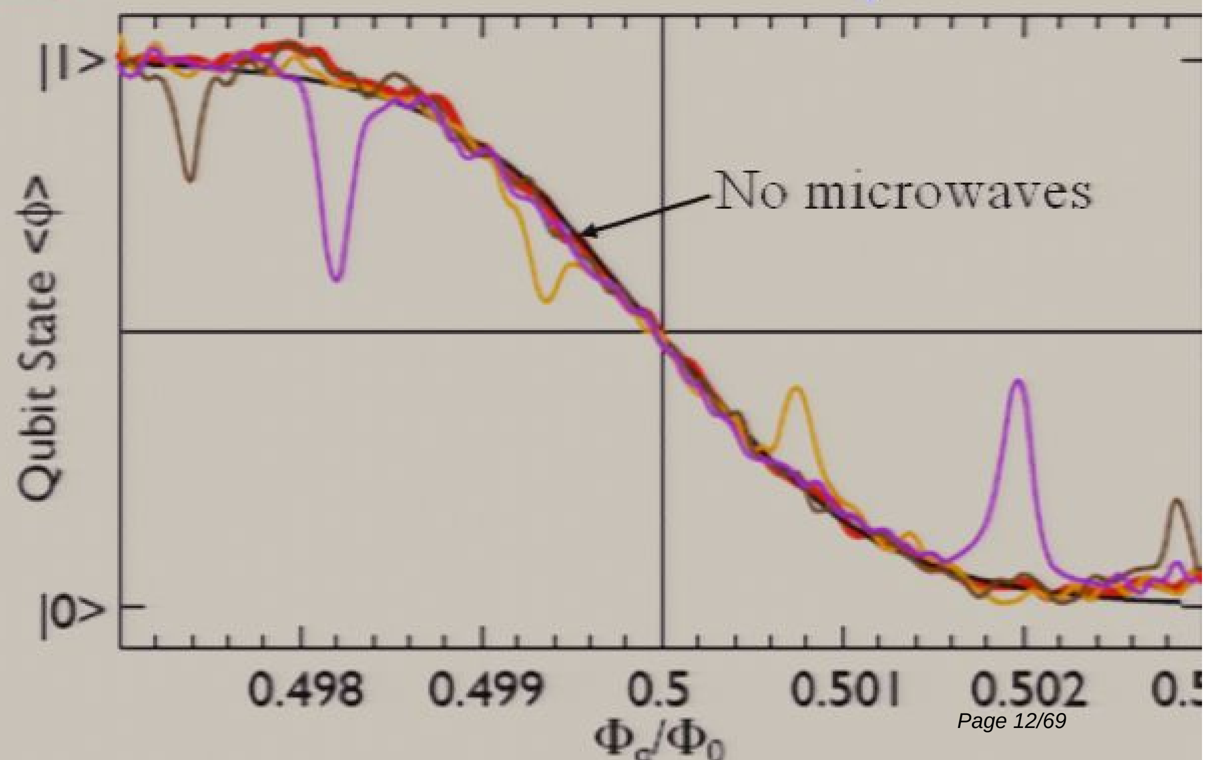
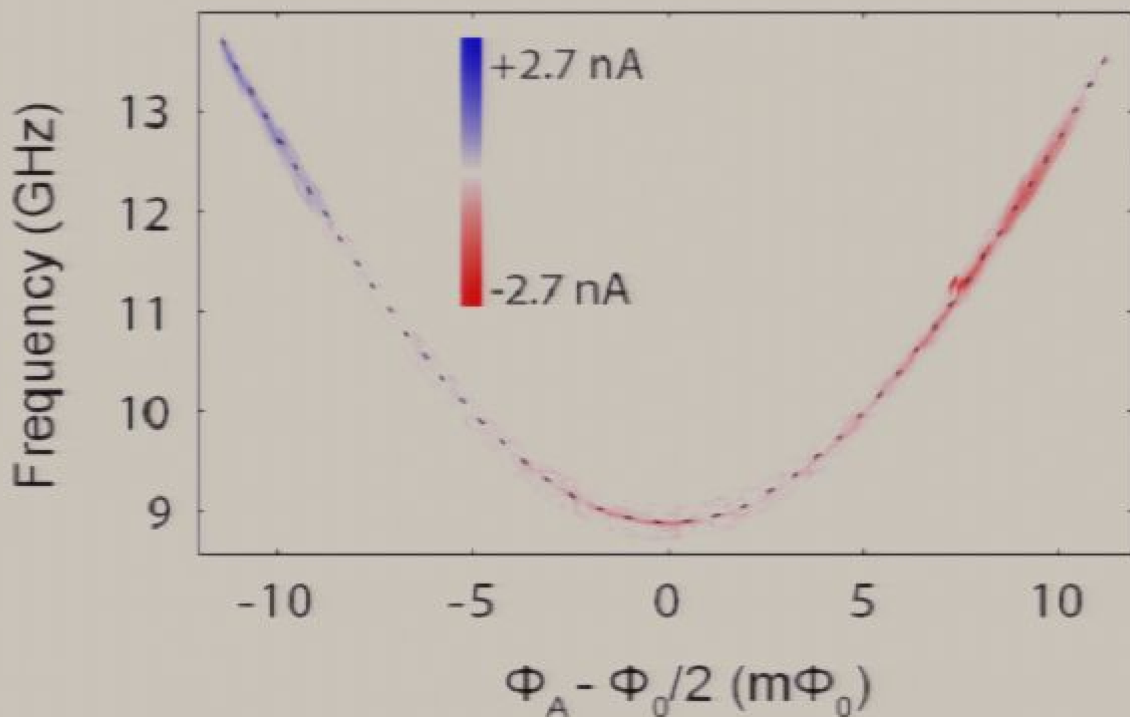


Microwave Spectroscopy of Qubit A



Dashed line is fit to $\nu = (\Delta^2 + \varepsilon^2)^{1/2}$
Δ is splitting at degeneracy point and $\varepsilon = 2I_\mathbf{q}(\Phi_\mathbf{q} - \Phi_0/2)$

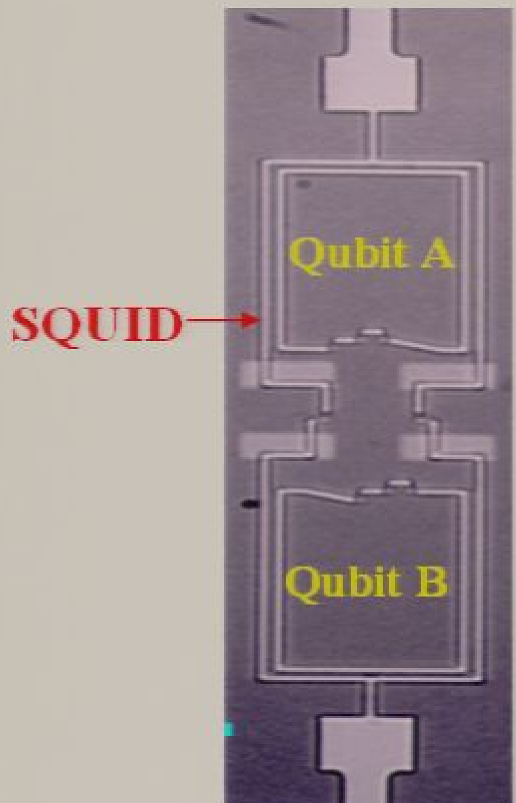
$\Delta_A/h = 8.872 \pm 0.005$ GHz
$I_\mathbf{q} = \frac{1}{2}\mathrm{d}\varepsilon/\mathrm{d}\Phi_q = 146.0 \pm 0.2$ nA

Drift in one mon
< 0.1 mΦ_0

Controllable Coupling of Two Qubits

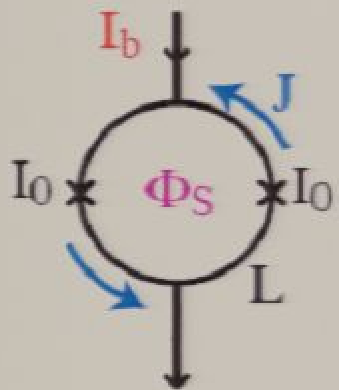
- The ability to switch the coupling of two qubits on and off is highly desirable for certain quantum computing algorithms:
  - Qubits uncoupled: prepare their quantum state independently
  - Qubits coupled: allow the qubits to entangle
  - Qubits uncoupled: read out the states of the qubits

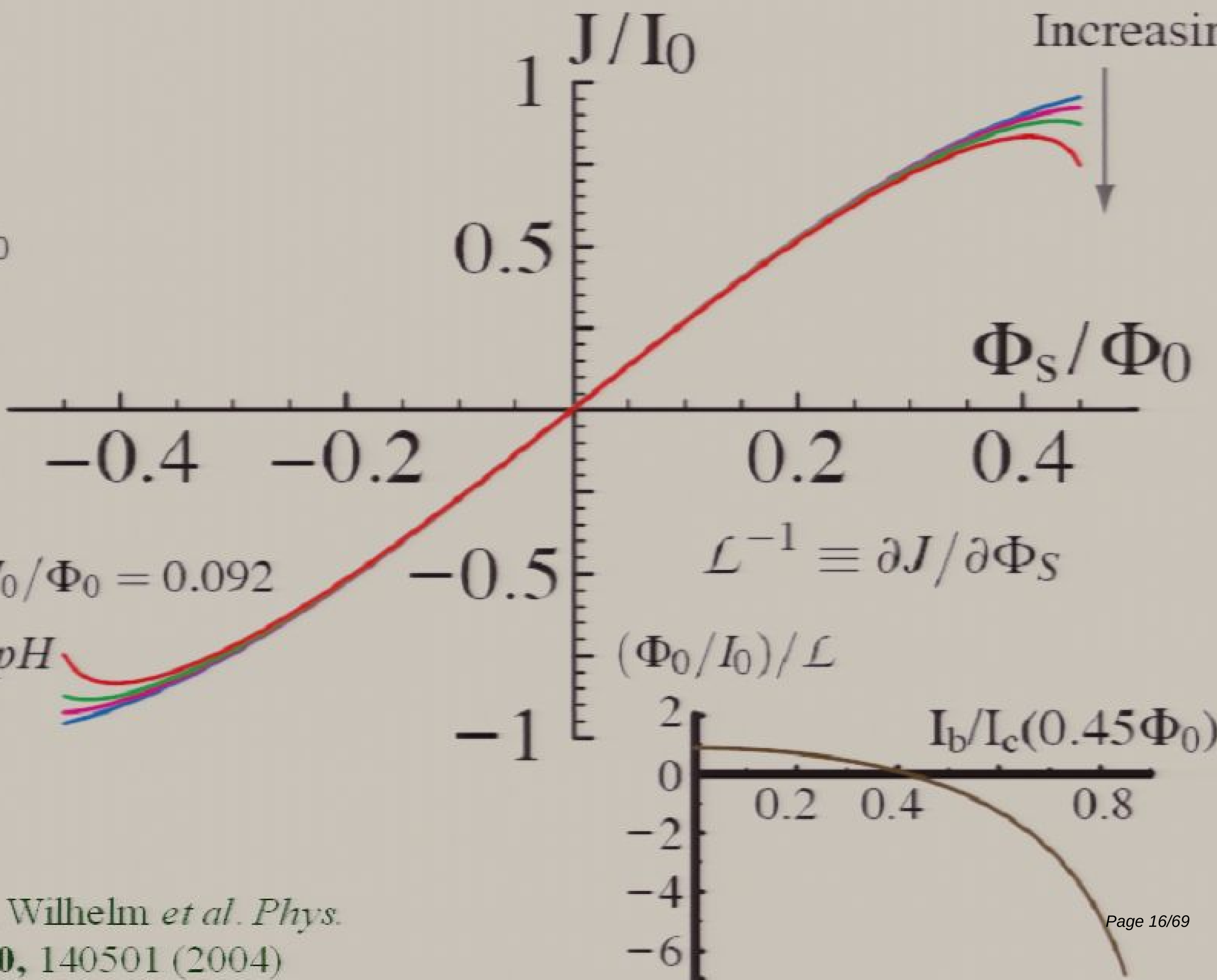
Controllable Coupling of Two Qubits: Theory



- Qubits have an interaction of the form $\sigma_{AZ}\sigma_{BZ}$, where σ is a Pauli spin matrix
- The fixed part of this qubit-qubit interaction ca be written in the form $M_{qq} I_{qA} I_{qB}$, where I_{qA} an I_{qB} are the qubit circulating currents and M_{qq} is the mutual inductance between qubits
- Qubits are also coupled via the SQUID: *this coupling depends on the SQUID current and flux biases*
- Thus, one can use the SQUID to control the total coupling energy between the qubits

Circulating Current in dc SQUID vs. Applied Flu





Plourde, Wilhelm *et al. Phys. Rev. B* **70**, 140501 (2004)

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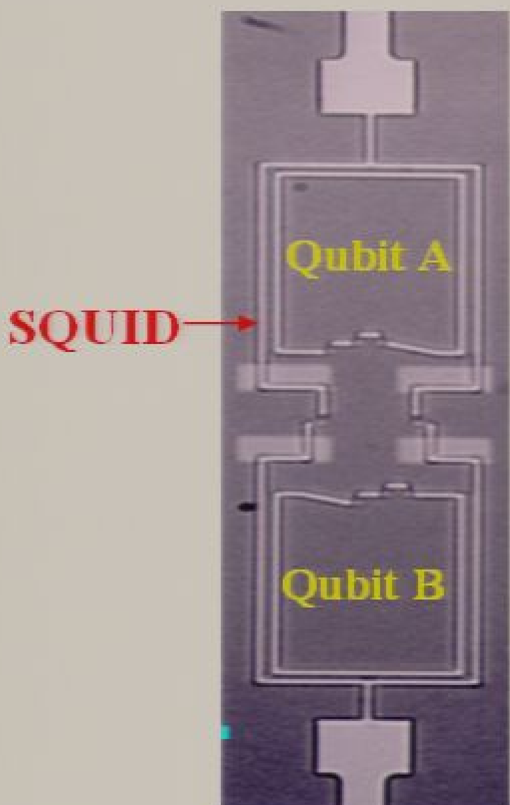

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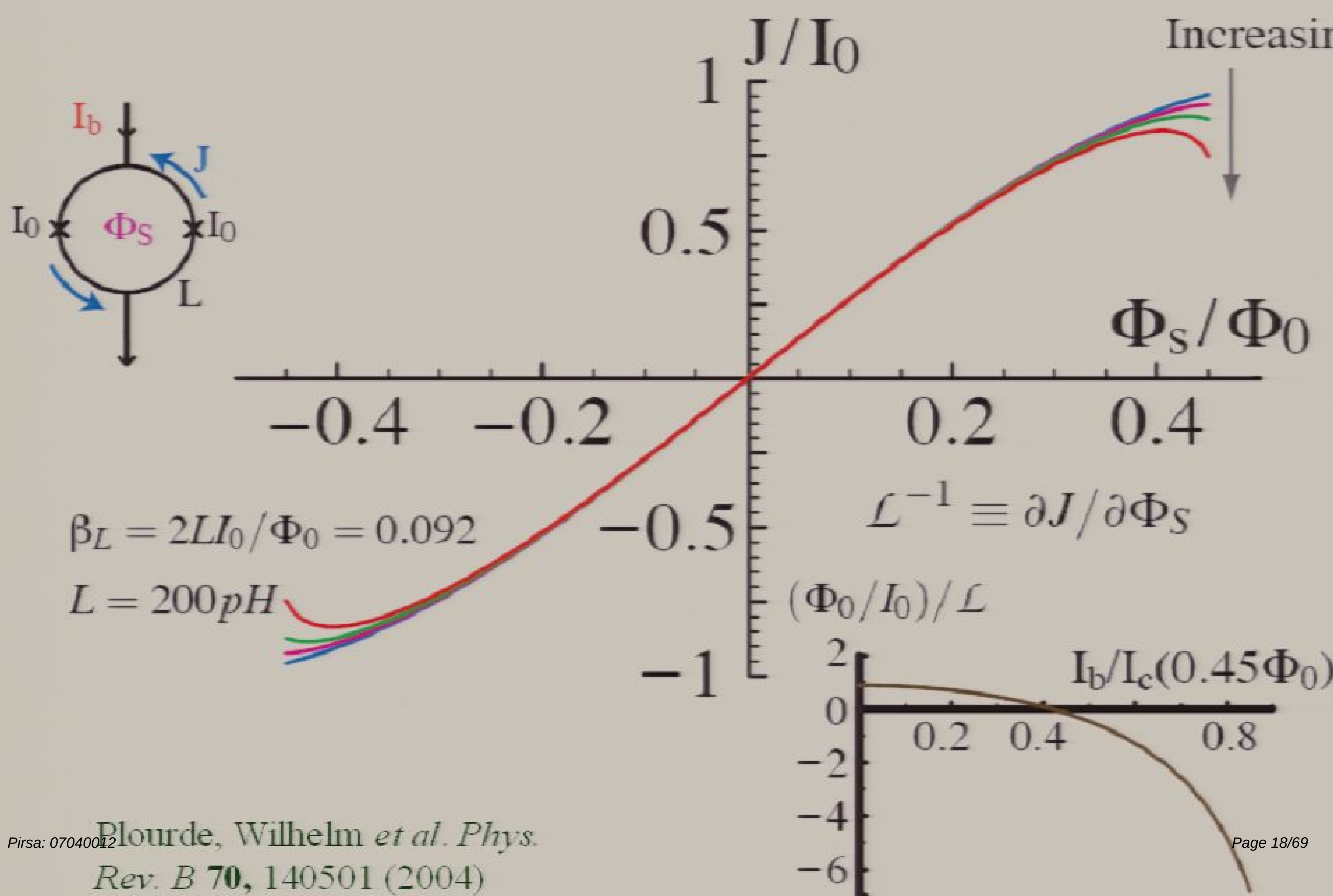
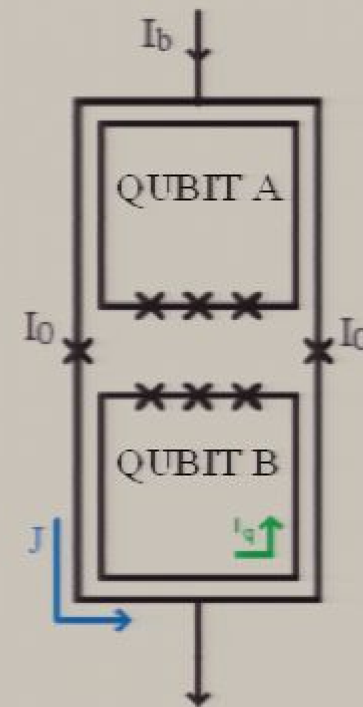


Plourde, Wilhelm *et al. Phys. Rev. B* **70**, 140501 (2004)

Controllable Flux Qubit Coupling Using a SQUID



directly coupled flux. The total flux change is

Net coupling energy is

$$\Delta\Phi_{qA} = \Delta J M_{qBs} - 2M_{qq}I_{qB} = 2I_q(-M_{qBs}M_{qAs}/L - M_{qq}).$$

$$K = I_q^2(-M_{qBs}M_{qAs}/L - M_{qq}) = K_s + K_0.$$

- The associated energy is $I_q\Delta\Phi_{qA}$. In addition a contribution $-½\, I_q\Delta\Phi_{qA}$ arises from the energy stored in the mutual inductance (Ferber and Wilhelm).

- Thus, the net coupling energy is

$$K = ½\, I_q\Delta\Phi_{qA} = I_q^2(-M_{qBs}M_{qAs}/L - M_{qq}) = K_s + K_0.$$

Numerical Values

- Qubits

  $I_q = 0.46\,\mu\text{A},\ M_{qs} = 33\,\text{pH},\ M_{qq} = 0.25\,\text{pH}$

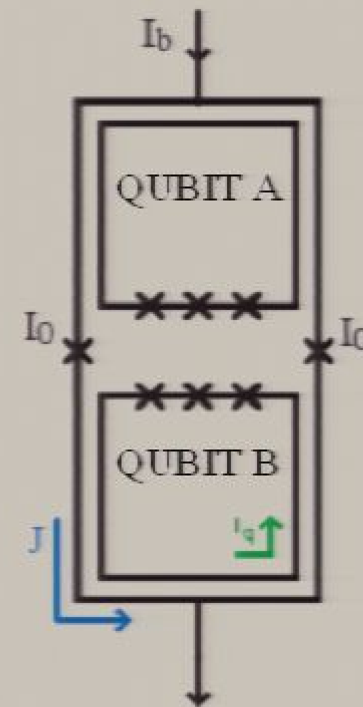
  $K_0/h = -M_{qq}I_q^2/h = \underline{-0.16\,\text{GHz}}$

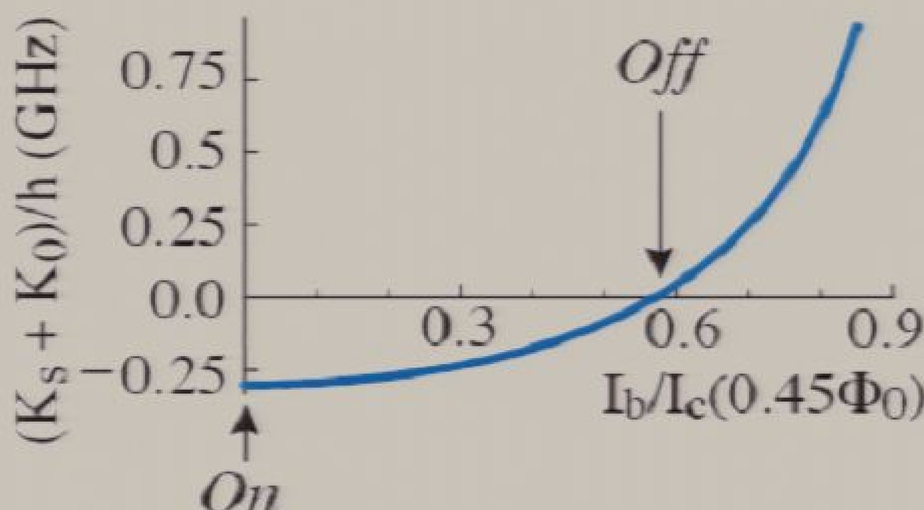
- SQUID

  $L = 200\,\text{pH},\ I_0 = 0.48\,\mu\text{A},\ \beta_L = 0.092$

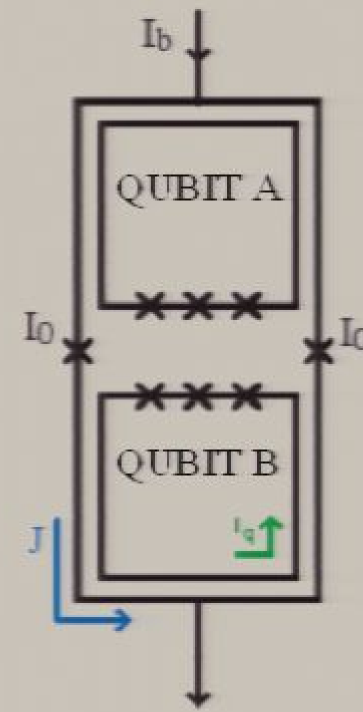
$\Phi_S = 0.45\Phi_0$ $\quad I_b = 0:\ K_s/h = \underline{-0.14\,\text{GHz}}$

$\quad I_b = 0.57 I_c(\Phi_S):\ K_s/h = \underline{0.16\,\text{GHz}}$





Controllable Flux Qubit Coupling Using a SQUID



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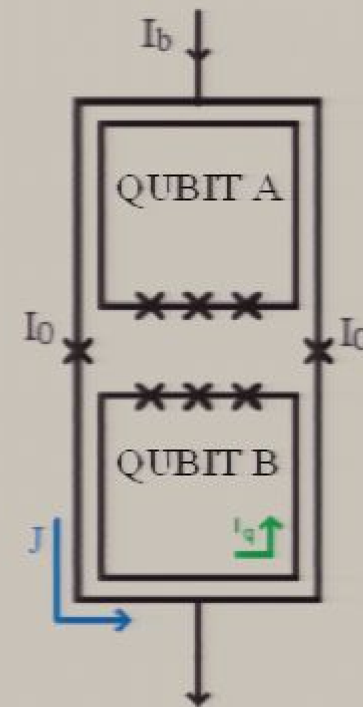

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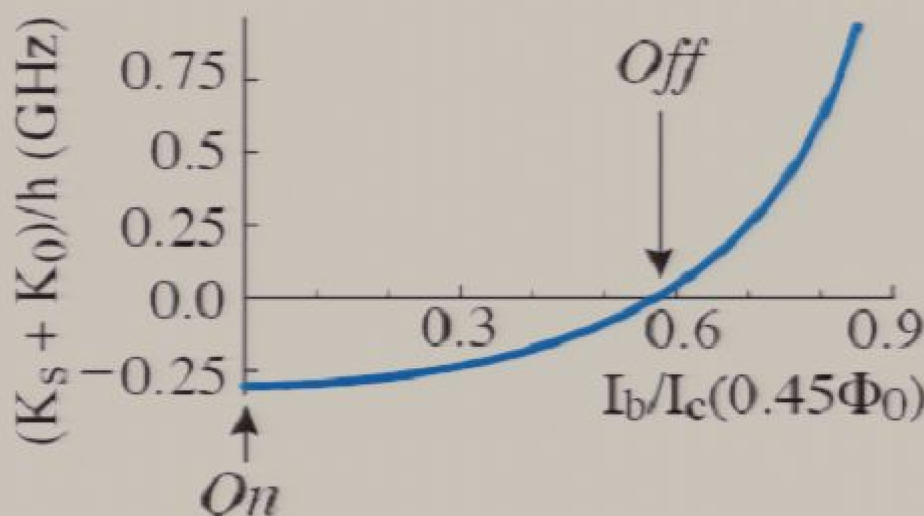

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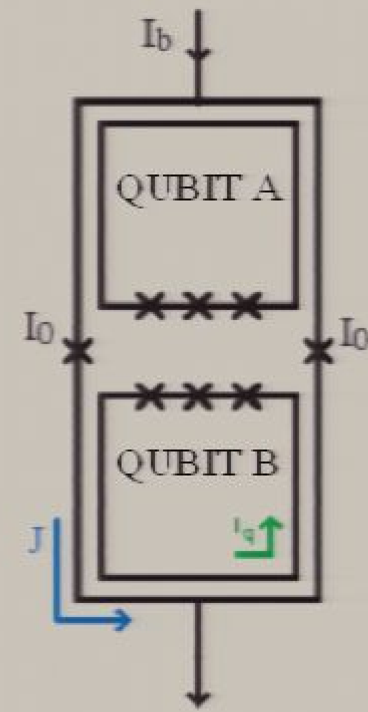

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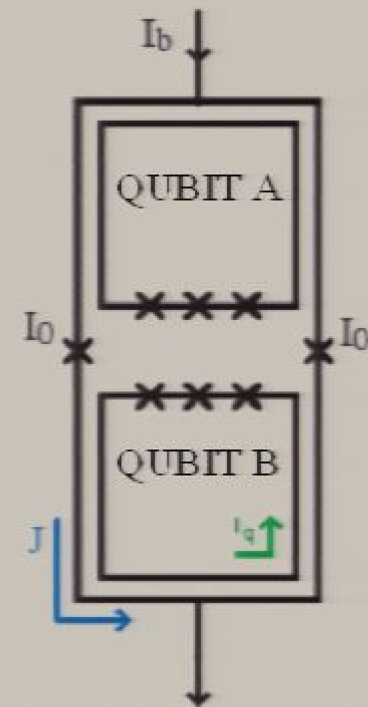

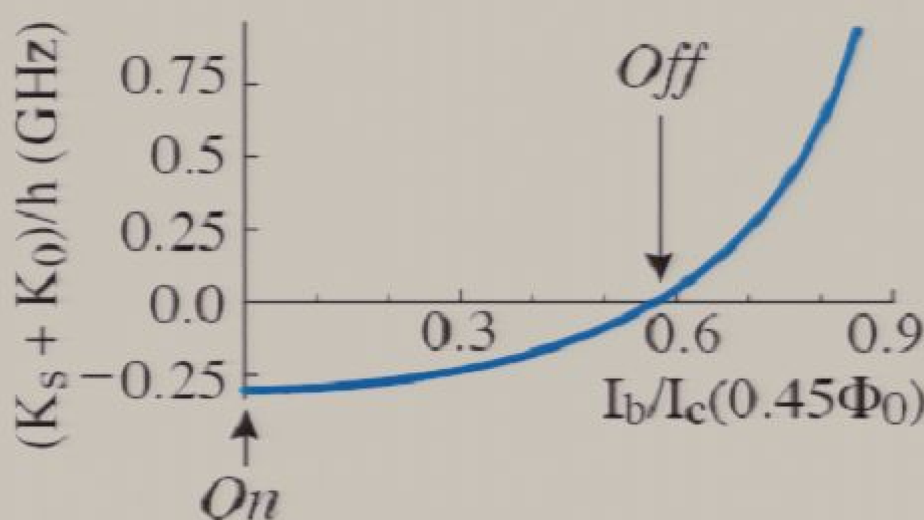

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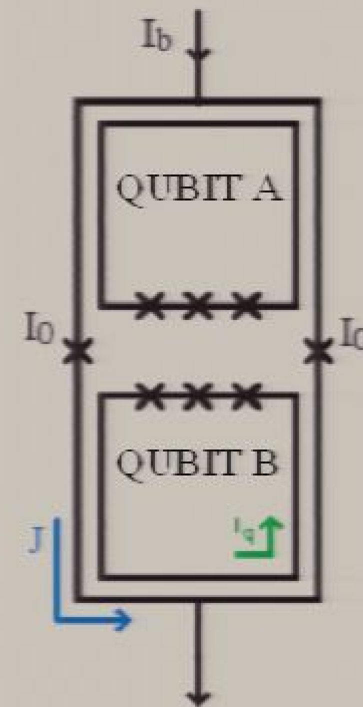

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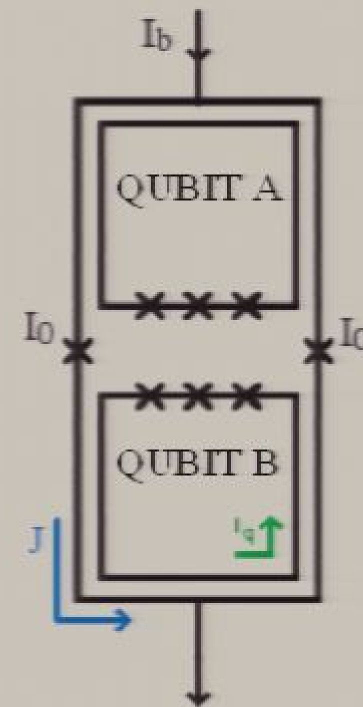

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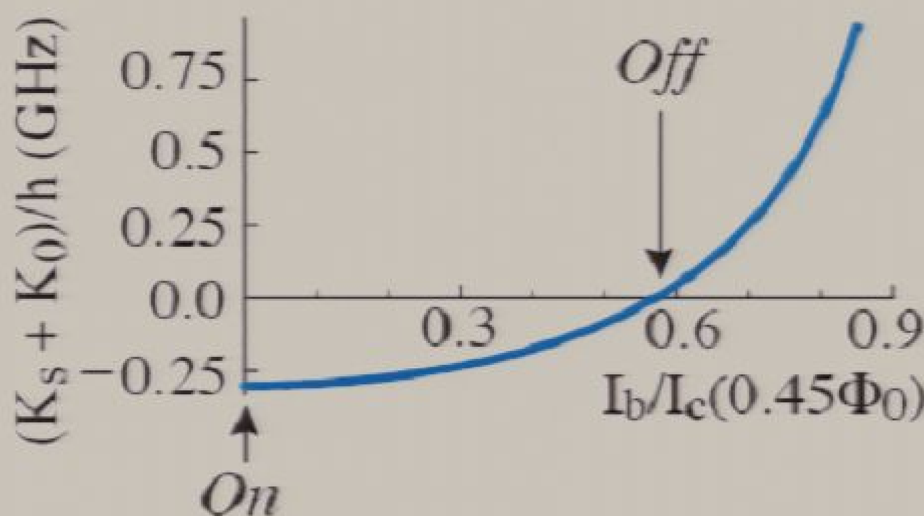


Controllable Coupling of Two Qubits: Experiment

Quantum Mechanics of Coupled Flux Qubit

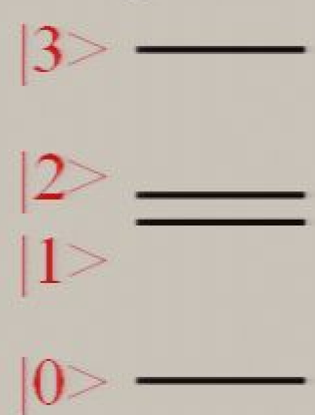
$H_{2qb} = (-{}^1\!/_2\varepsilon_A\boldsymbol{\sigma}_{Az} - {}^1\!/_2\Delta_A\boldsymbol{\sigma}_{Ax}) + (-{}^1\!/_2\varepsilon_B\boldsymbol{\sigma}_{Bz} - {}^1\!/_2\Delta_B\boldsymbol{\sigma}_{Bx}) + K\boldsymbol{\sigma}_{Az}\boldsymbol{\sigma}_{Bz}$

Basis states:

Symmetric triplet $|11\rangle$, $|S\rangle \equiv (|01\rangle + |10\rangle)/2^{1/2}$, $|00\rangle$

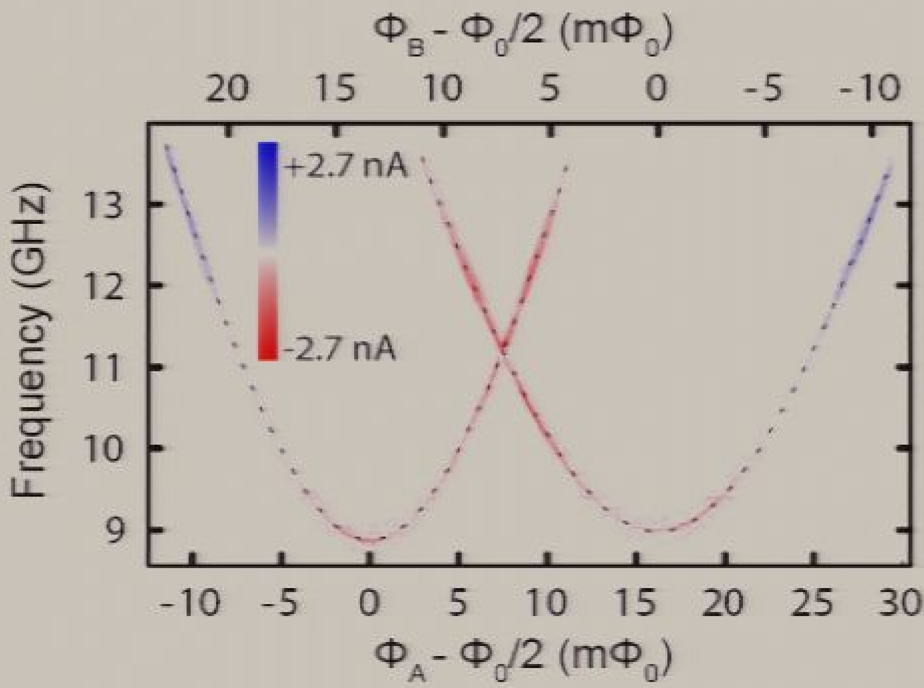
Antisymmetric singlet $|A\rangle \equiv (|01\rangle - |10\rangle)/2^{1/2}$

Eigenstates



Storcz *et al. Phys. Rev.* A **67**, 042319 (2003)

Microwave Spectroscopy of Qubits A and B



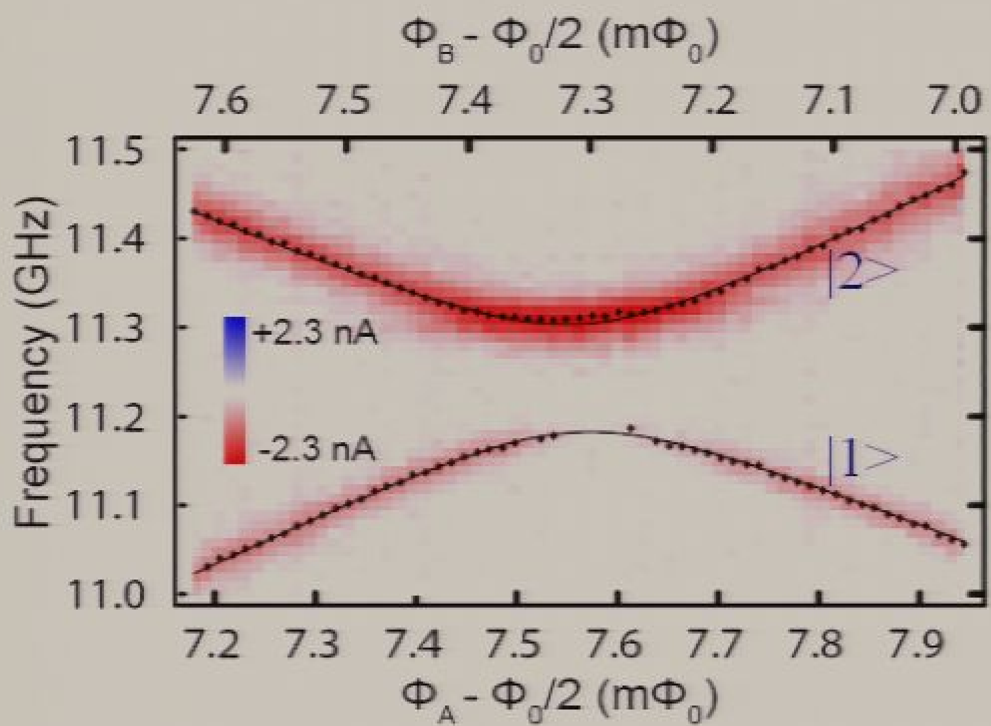
Dashed lines are fits to $\nu = (\Delta^2 + \varepsilon^2)^{1/2}$

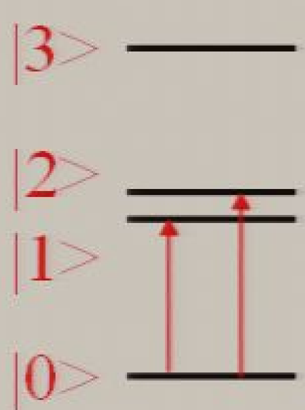
$\Delta_A/h = 8.872 \pm 0.005$ GHz
$\Delta_B/h = 8.990 \pm 0.004$ GHz

$I_{qA} = 146.0 \pm 0.2$ nA
$I_{qB} = 147.8 \pm 0.2$ nA

- Two independent flux lines enable one to move the spectra to arbitrary values of flux
- Spectra measured at a constant SQUID flux: qubit-qubit coupling is constant
- Drift in one month $< 0.1\ \mathrm{m}\Phi_0$

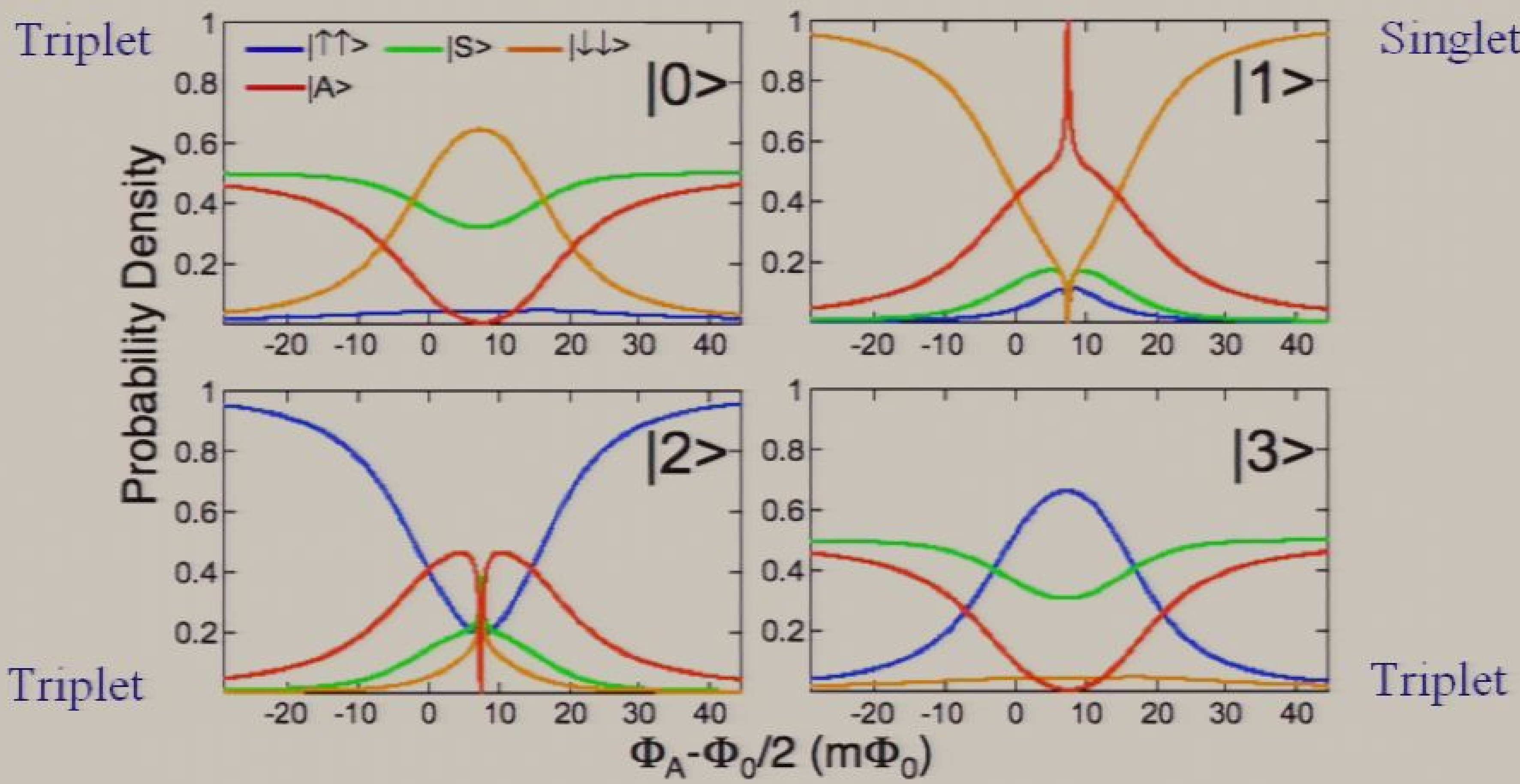
Anticrossing of |1> and |2>



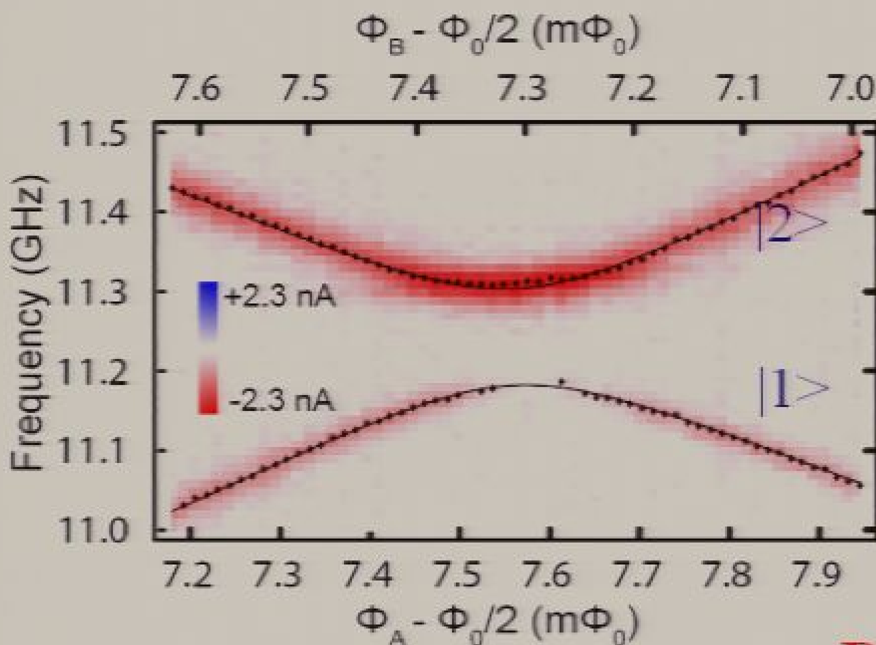


- Qubit intersection frequency 11.25 GH
- Spectra measured at constant SQUID flux, 0.28 Φ_0, and hence at constant qubit-qubit coupling strength
- Zero SQUID bias current
- Minimum splitting 122.6 ± 0.8 MHz
- Note absence of data for |1> near anticrossing

Coupled-Qubit States

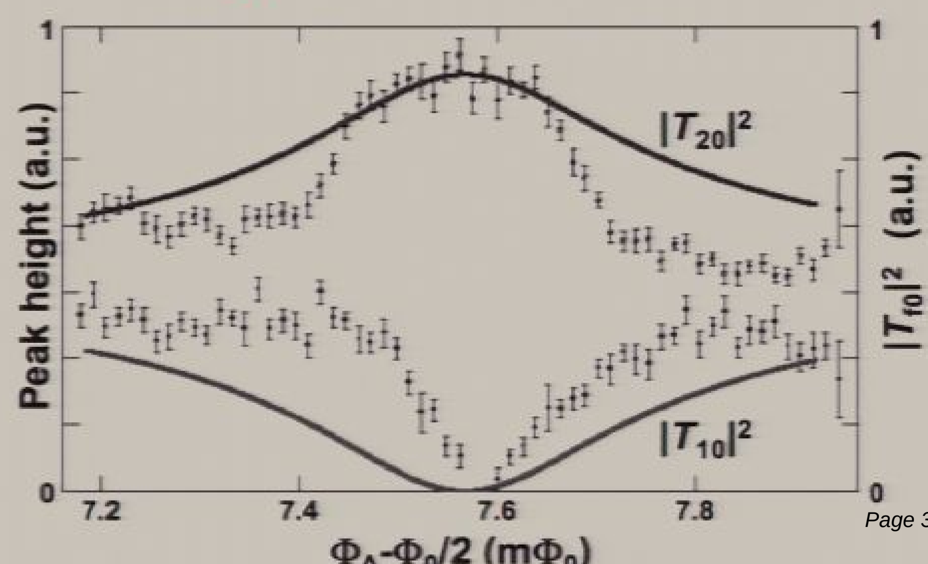


Peak Heights at Anticrossing of |1> and |2>



- Precisely at the anticrossing:
  - |0>, |2> are mixtures of triplet stat
  - |1> is pure singlet
- Transitions from |0> to |1> forbidden

Peak Heights and Matrix Elements

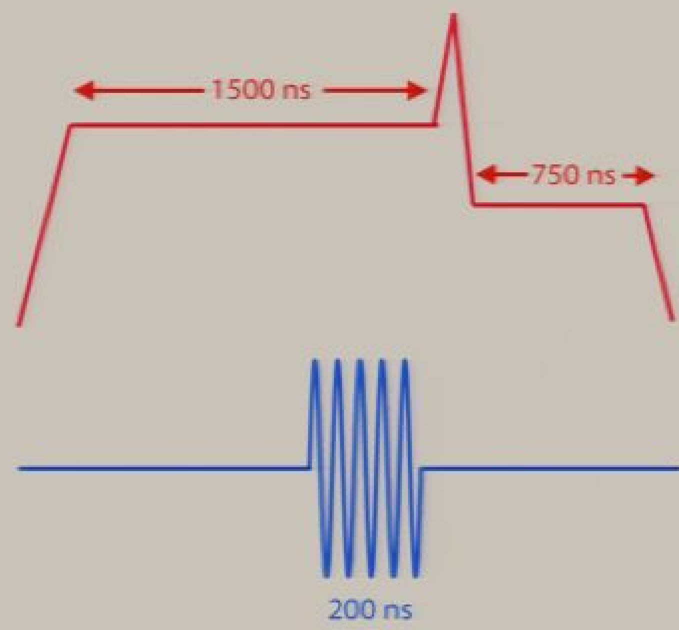


Matrix elements

$$|\langle f|\sigma_{zA} + \sigma_{zB}|0\rangle|^2$$

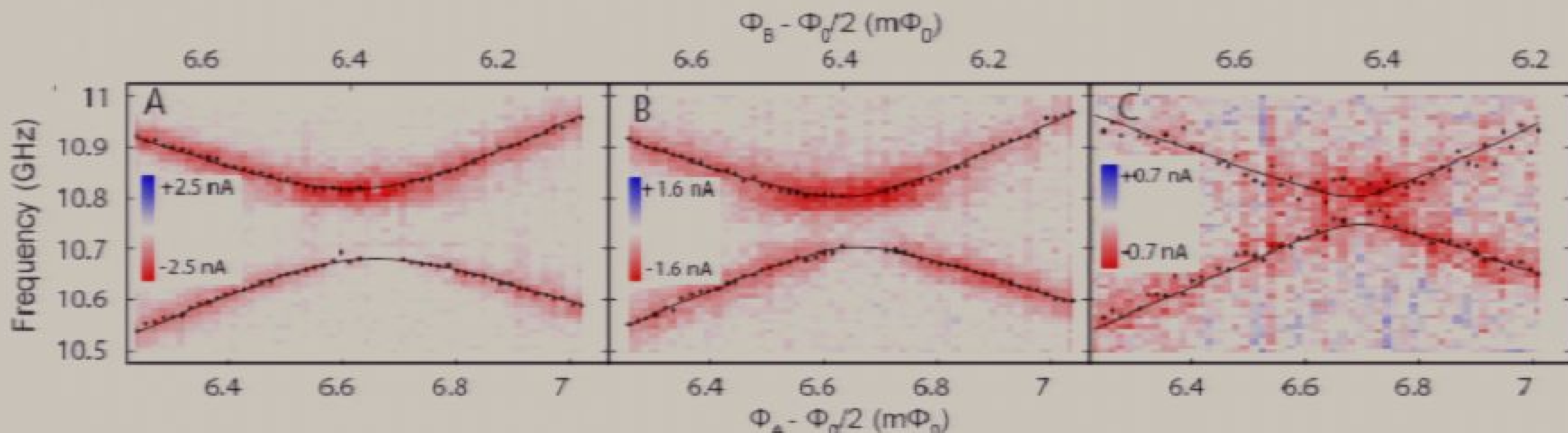
Controlling the Qubit-Qubit Coupling

• Apply bias current pulse I_b to the SQUID to change its dynamic inductance in the zero voltage state



Anticrossing of |1> and |2>: Changing the Coupling

Qubit intersection frequency: 10.75 GHz



Bias current:	0	417	557	nA
Splitting:	135 ± 2	103 ± 2	56 ± 5	MHz

Measured and Calculated Parameters

Mutual inductances

	Flux map (pH)*	Fast Henry (pH)*
M_{f0qA}	1.65	1.70
M_{f1qA}	-3.21	-3.20
M_{f0qB}	3.10	3.10
M_{f1qB}	-0.72	-0.70
M_{f0s}	7.52	7.27
M_{f1s}	-5.54	-5.54
M_{qq}	–	0.75
M_{qAs}	–	87.2
M_{qBs}	–	63.8

*Average error: 1.6 %

Self inductances

	Fast Henry (pH)
L_{qA}	194
L_{qB}	182
L_{s}	423

Qubit currents

$$d\varepsilon/d\Phi_q = 2I_{\mathrm{q}}$$

$$I_{\mathrm{qA}} = 146.0 \pm 0.2 \text{ nA}$$

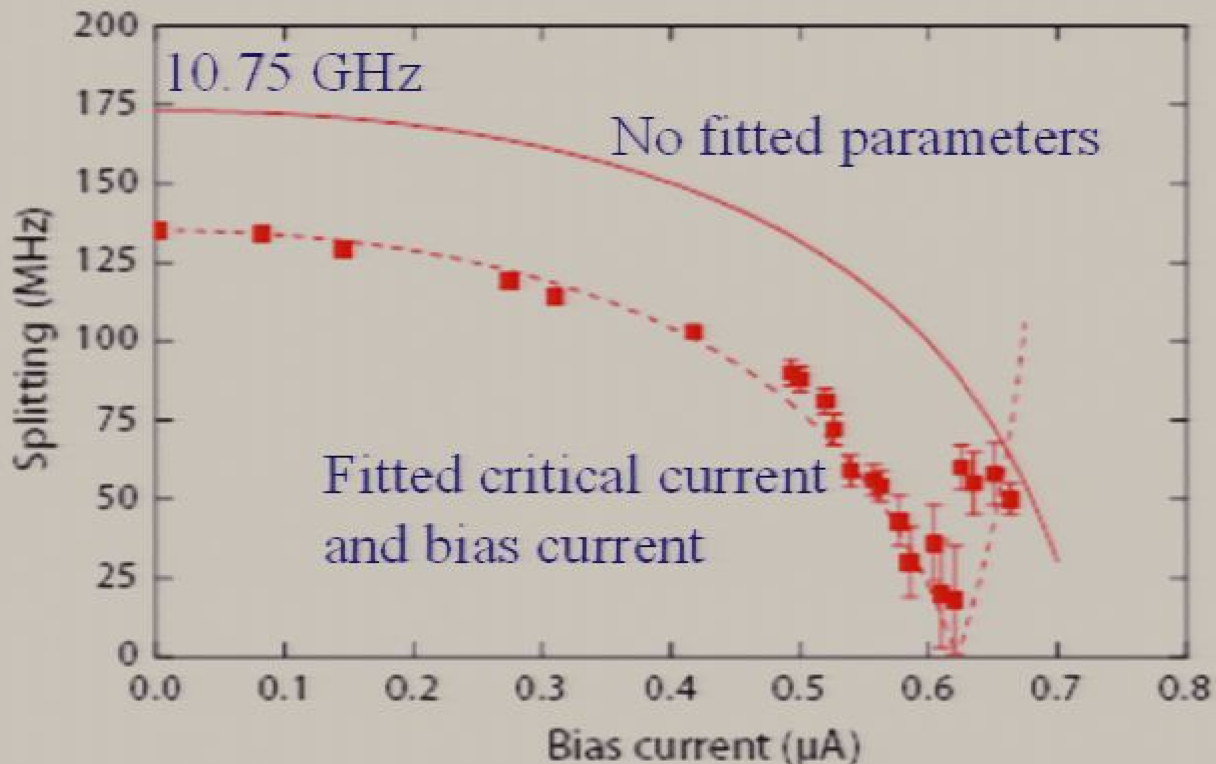
$$I_{\mathrm{qB}} = 147.8 \pm 0.2 \text{ nA}$$

SQUID critical curren

$$2I_0 = \pi\Delta/2R_{\mathrm{NN}}$$

$$= 1.21 \pm 0.054$$

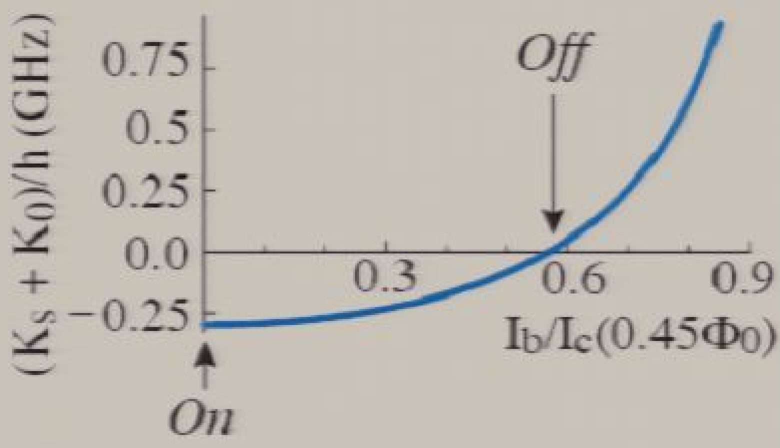
Splitting Frequency vs. SQUID Bias Current



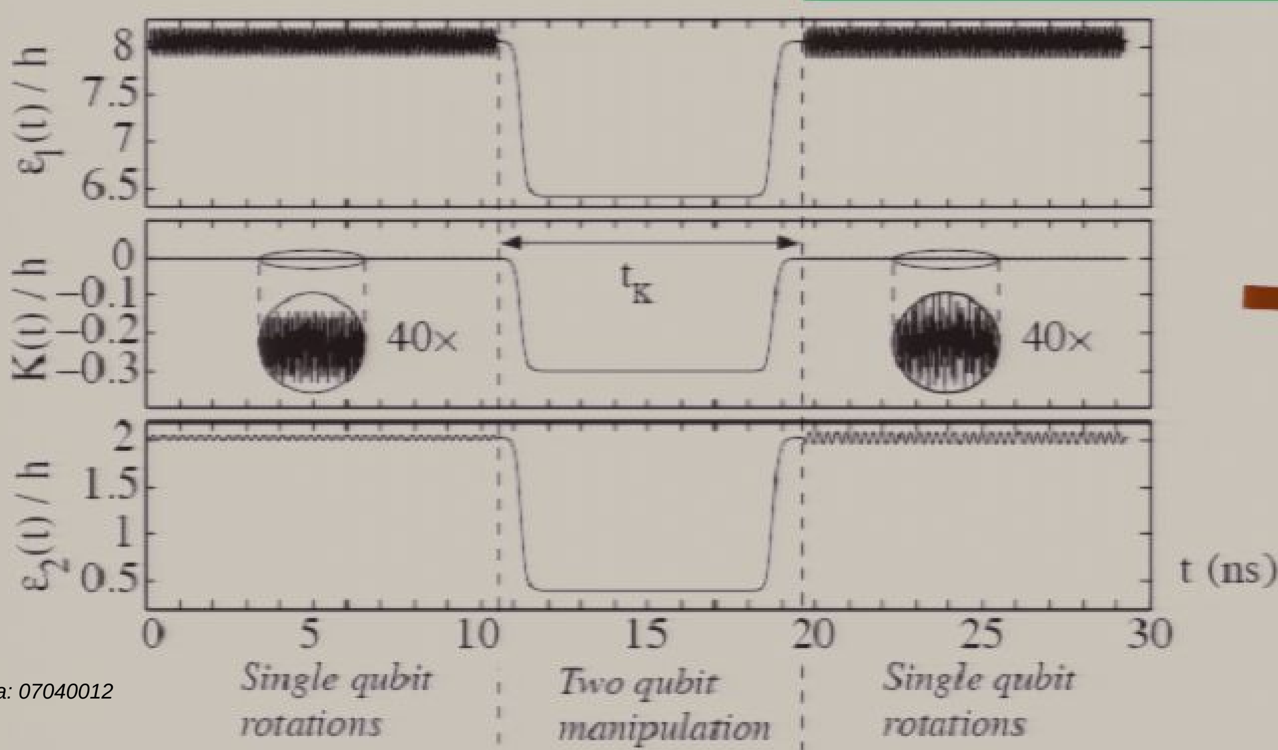
T. Hime *et al.*, *Science* **314,** 1427 (200

Entangling operation with variable coupling

For feasible parameters:



- When combined with appropriate single qubit rotations with K = 0, a single pulse of current can generate the CNOT gate in 29 ns.
- Qubit states can be determined immediately afterwards with a larger I_b pulse to measure SQUID critical current *without changing the static flux.*
- Pulse parameters can be adjusted to compensate for both crosstalk terms and finite risetime of I_b pulse.



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

CNOT

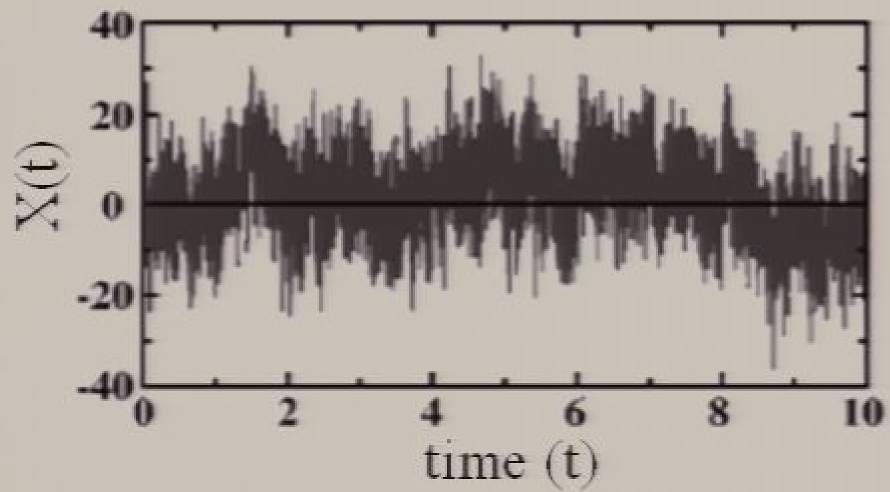
Plourde, Wilhelm *et al.*, *Ph Rev.* B **70** 140501 (R) (200

Concluding Remarks

- Spectroscopy of coupled qubits shows splitting of |1> and |2> energ
- Peak heights measured from transitions from |0> to |1> and |2> agre qualitatively with square of calculated matrix elements
- Coupling reduced to zero—and its sign even reversed—by bias current pulse in SQUID, in good agreement with prediction
- The ability to measure the quantum states of two qubits and to swit their coupling on and off with a single SQUID solely by pulsing its bias current is an efficient architecture
- Independent flux lines are the key; their fluxes are kept constant
- The quantum controlled NOT (CNOT) logic gate can be implement with this architecture, and provides the building block for scalable universal logic

BUT

The Ubiquitous 1/f Noise



Spectral density:
$S_x(f) \propto 1/f^{\beta}, \beta \sim 1$

- Vacuum tubes
- Carbon resistors
- Semiconductor devices
- Metal films
- Superconducting devices

Decoherence in Superconducting Qubits: A Model for l/f Flux Noise

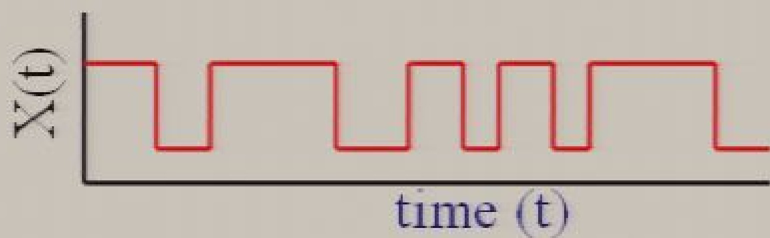
- Introduction
- 1/f flux noise in SQUIDs and qubits
- Model for l/f flux noise
- Questions & concluding remarks

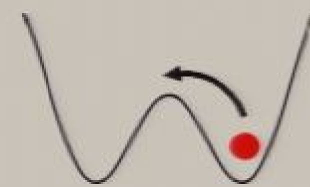
Roger Koch
David DiVincenzo
IBM Yorktown Heights

JC
UC Berkeley
LBNL

Support
Army Research Office (DD
Department of Energy (JC)

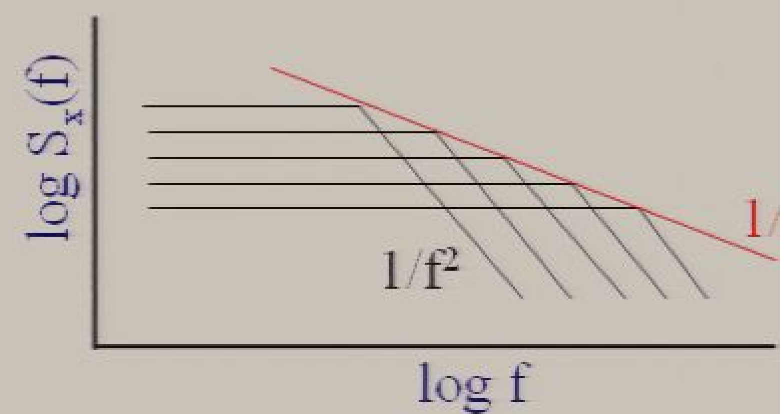
Random Telegraph Signals and l/f Noise





- For a single characteristic time τ:

$$S_{RTS}(f) \propto \tau/[1 + (2\pi f\tau)^2]$$



- The superposition of a set of Lorentzians with a broad distribution of τ yields 1/f noise (Machlup 1954)

1/f Noise in Superconducting Devices

Examples: SQUIDS, single electron transistors (SETs), charge qubits, flux qubits, phase qubits

Three kinds of noise:

Charge noise: Hopping of electrons between traps induces fluctuatin charges onto nearby films and junctions

Critical current noise: Trapping and release of electrons in tunnel barriers modify the transparency of the junction, causing its resistance and critical current to fluctuate (Wellstood, Buhrman, Van Harlingen, Martinis….). At low temperatures, the process may involve quantum tunneling and atomic rearrangement

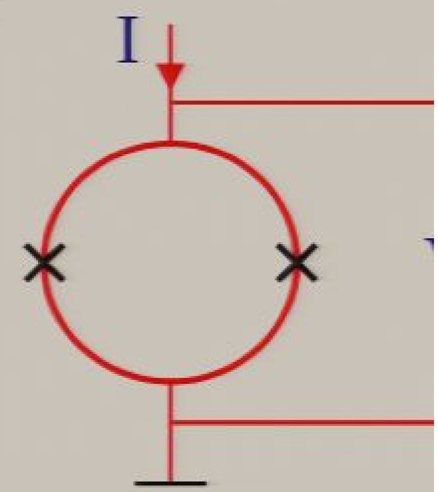
Flux noise: Flux-sensitive devices (SQUIDs, flux qubits….) exhibit flux noise of hitherto unknown origin

- All three kinds of noise generate decoherence in superconducting qubits, and are a significant barrier to scaling up to many qubits

1/f Flux Noise in SQUIDs and Qubits

1/f Flux Noise in SQUIDs

- Measurements of noise in dc SQUIDs in a flux-locked loop yield the spectral density of the *equivalent* 1/f flux noise $S_\Phi(f) \propto 1/f$
- By using two different bias reversal schemes, Koch *et al.* (1983) showed there were two independent contributions:



- **Critical current noise**

– can be removed by bias current reversal

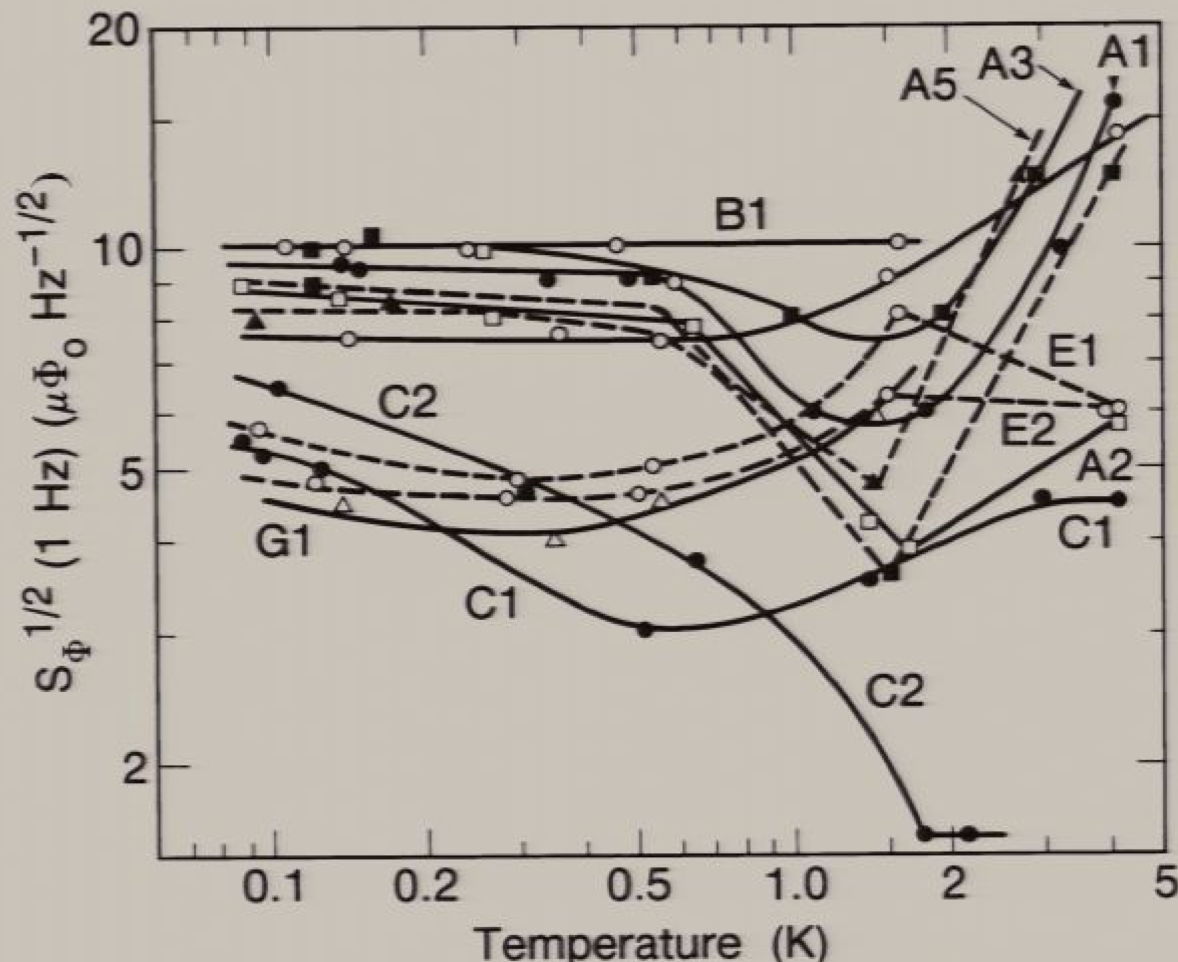
- **Flux noise**

– *cannot* be removed by bias current reversal

$S_\Phi^{1/2}$ (1 Hz) $\approx 5 - 15\ \mu\Phi_0\ \text{Hz}^{-1/2}$ for SQUID areas $6 \times 10^{-6} - 7\ \text{mm}^2$

Quartz, glass and Si substrates

DC SQUIDs: $S_\Phi^{1/2}$(1Hz) vs. T



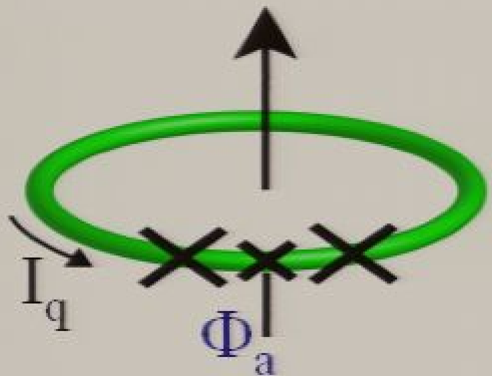
Loop material
Nb (A1-4, F1)
Pb (C2)
PbIn (A5, B1,
C1, E1,2)

Loop effective
areas:
1,400 μm²–
200,000 μm²

At low T: $S_\Phi^{1/2}(1\text{Hz}) \approx 7 \pm 3\ \mu\Phi_0\text{Hz}^{-1/2}$

Wellstood, Urbina and Clarke (1987)

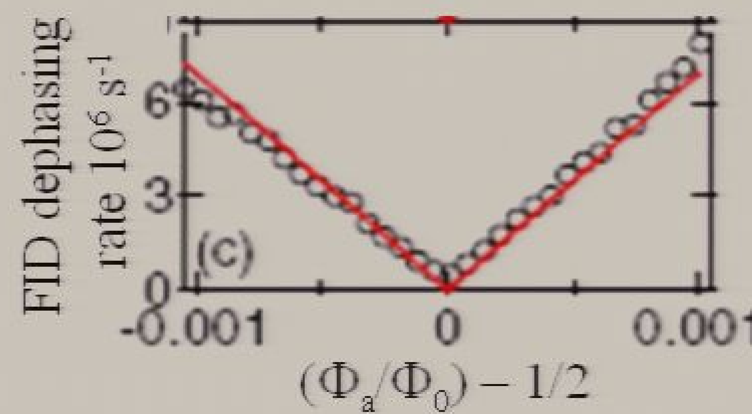
l/f Flux Noise in Flux Qubits



- Decoherence due to flux noise enters via $\partial\nu/\partial\Phi_a$, where $\nu = (\Delta^2 + \varepsilon^2)^{1/2}$ and $\varepsilon = 2I_q(\Phi_a - \Phi_0/2)$
- Hence decoherence due to flux noise vanishes at $\Phi_a = \Phi_0/2$

Yoshihara *et al.* (2006)

- Measured 5 flux qubits:
  $S_\Phi^{1/2}$ (1 Hz) = 0.9 – 2$\mu\Phi_0$ Hz$^{-1/2}$
- Showed that decoherence was *not* due to critical current fluctuations
- Loop area of qubit ≈ 3 μm^2



- 1/f flux noise also observed in:
  - Quantronium (charge-phase hybrid) (Ithier *et al.* 2005)
  - Phase qubits (Martinis *et al.*, private communication)

Area Dependence of l/f Flux Noise

	Area (μm^2)	$S_\Phi^{1/2}$ (1 Hz) ($\mu\Phi_0$ $Hz^{-1/2}$)
Wellstood *et al.* (SQUIDs)	~2 x 10^5	5 – 10
Yoshihara *et al.* (flux qubits)	~3	1 – 2

- Measurements performed at millikelvin temperatures
- Experiments heavily shielded against environmental magnetic field noise
- Weak tendency for flux noise to increase with area
- Data rule out a "universal magnetic field noise"

Model for 1/f Flux Noise

Model for 1/f Flux Noise

- Single electrons hop on and off defect centers by thermal activation (as with charge and critical current noise)
- The spin of an electron is ***locked in direction*** while the electron occupies a given trap. This direction varies randomly from trap to trap. The flux coupled to the qubit by an electron at a given position depends on the orientation of the spin.
- Two issues:
  - What is the mechanism for "spin locking"? The locking must persist for times at least as long as the inverse of the lowest frequency at which 1/f noise is observed.
  - What is the magnitude of the 1/f flux noise generated by an ensemble of these RTSs?

Spin Locking I: The Kramers Degeneracy*

- For an odd number of fermions with spin ½ in zero magnetic field, Kramers showed that the ground state is doubly degenerate. It was shown subsequently that this result is a consequence of symmetry. The two states have oppositely directed momenta.
- For an electron with nonzero orbital momentum (L > 0), the magnetic moment $\hat{\mathbf{M}} = \mu_B(\hat{\mathbf{L}} + 2\hat{\mathbf{S}})$ produced by spin-orbit coupling is locked to the direction of the crystal field (electric).
- If there is no orbital angular momentum (for example, if it is quenched), the remaining angular momentum is due solely to the electron, which does not couple to the crystal field and will thus not be locked

*H.A. Kramers, Koninkl. Ned. Akad. Wetenschap., Proc. **33**, 959 (1930)

Spin Locking II: Van Vleck Cancellation

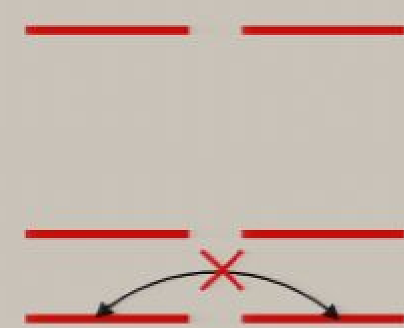
MARCH 1, 1940 PHYSICAL REVIEW VOLUME 57

Paramagnetic Relaxation Times for Titanium and Chrome Alum

J. H. VAN VLECK
Harvard University, Cambridge, Massachusetts
(Received December 26, 1939)

- Van Vleck showed that in zero magnetic field matrix elements for direct transitions between the two states vanish. Thus, spin flip processes are forbidden, and the electron spin remains locked in the state it occupies.

The 6 states of a p-electron split by a crystal field



Spin Locking III: Second-Order Processe

PHYSICAL REVIEW VOLUME 107, NUMBER 2 JULY 15, 1957

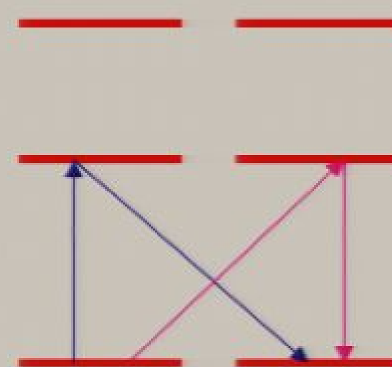
Donor Electron Spin Relaxation in Silicon*

ELIHU ABRAHAMS
Department of Physics, Rutgers University, New Brunswick, New Jersey, and Bell Telephone Laboratories, Murray Hill, New Jersey
(Received April 15, 1957)

Second-order processes are in principle allowed. However, for the Raman process Abrahams showed that the lifetime is

$$\tau \propto 1/T^{13}$$

Thus, transition rates for such processes are utterly negligible at low temperatures

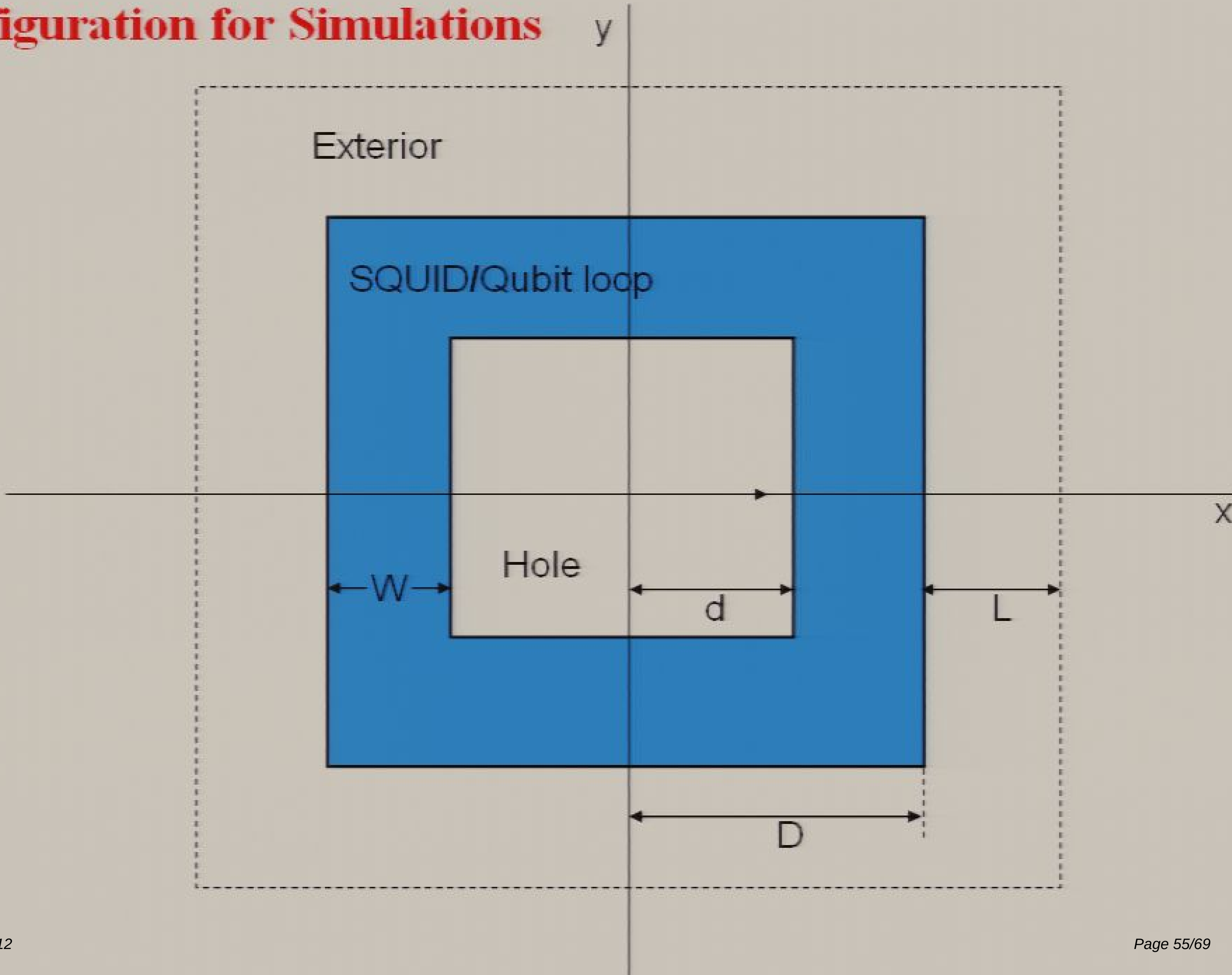


Second-order spin-flip processes

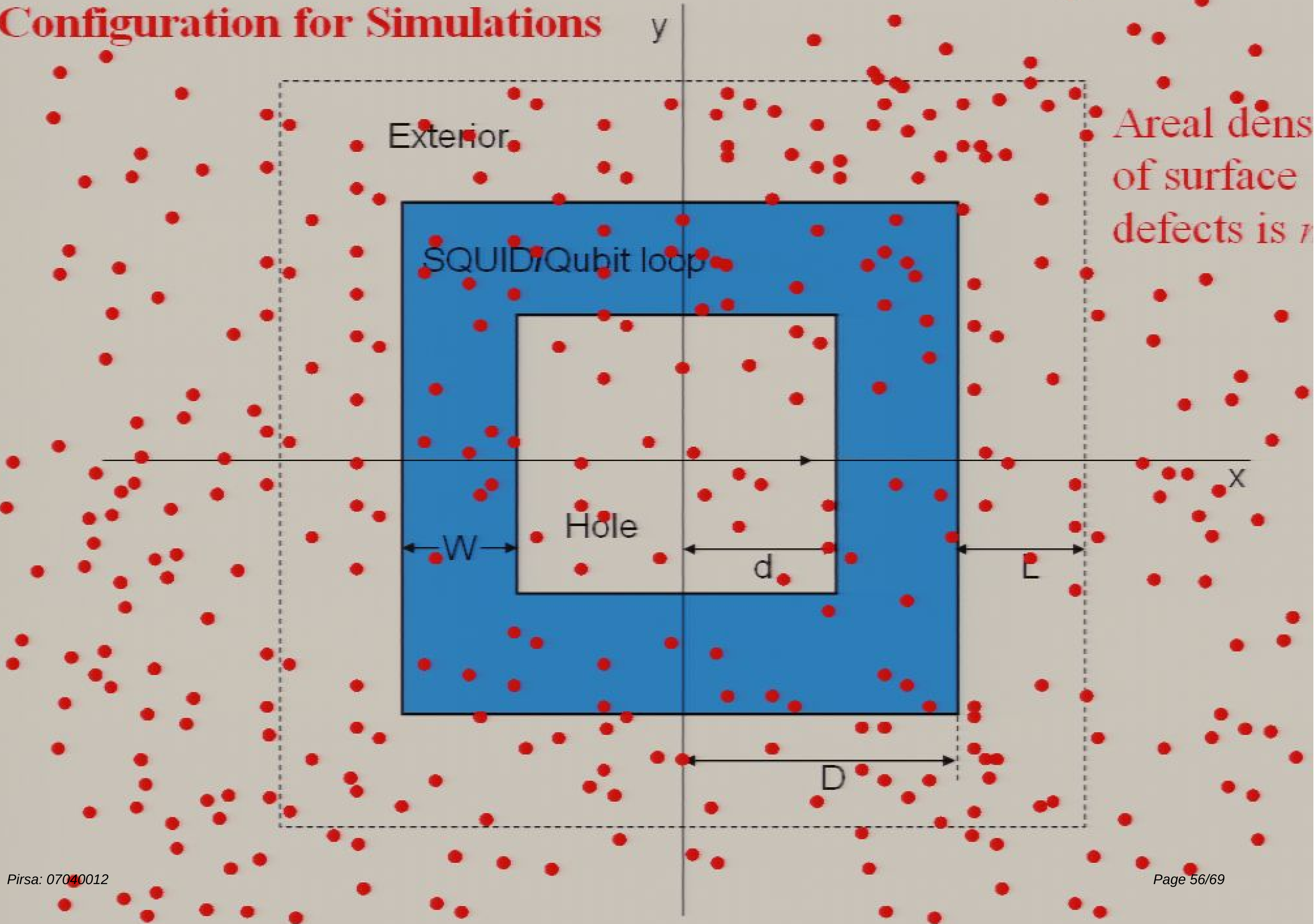
- Abrahams wrote this paper to explain the absence of electron spin resonance in donors in Si at low temperatures. In this context, spin locking is well established.

Calculation of 1/f Noise Magnitude

Configuration for Simulations



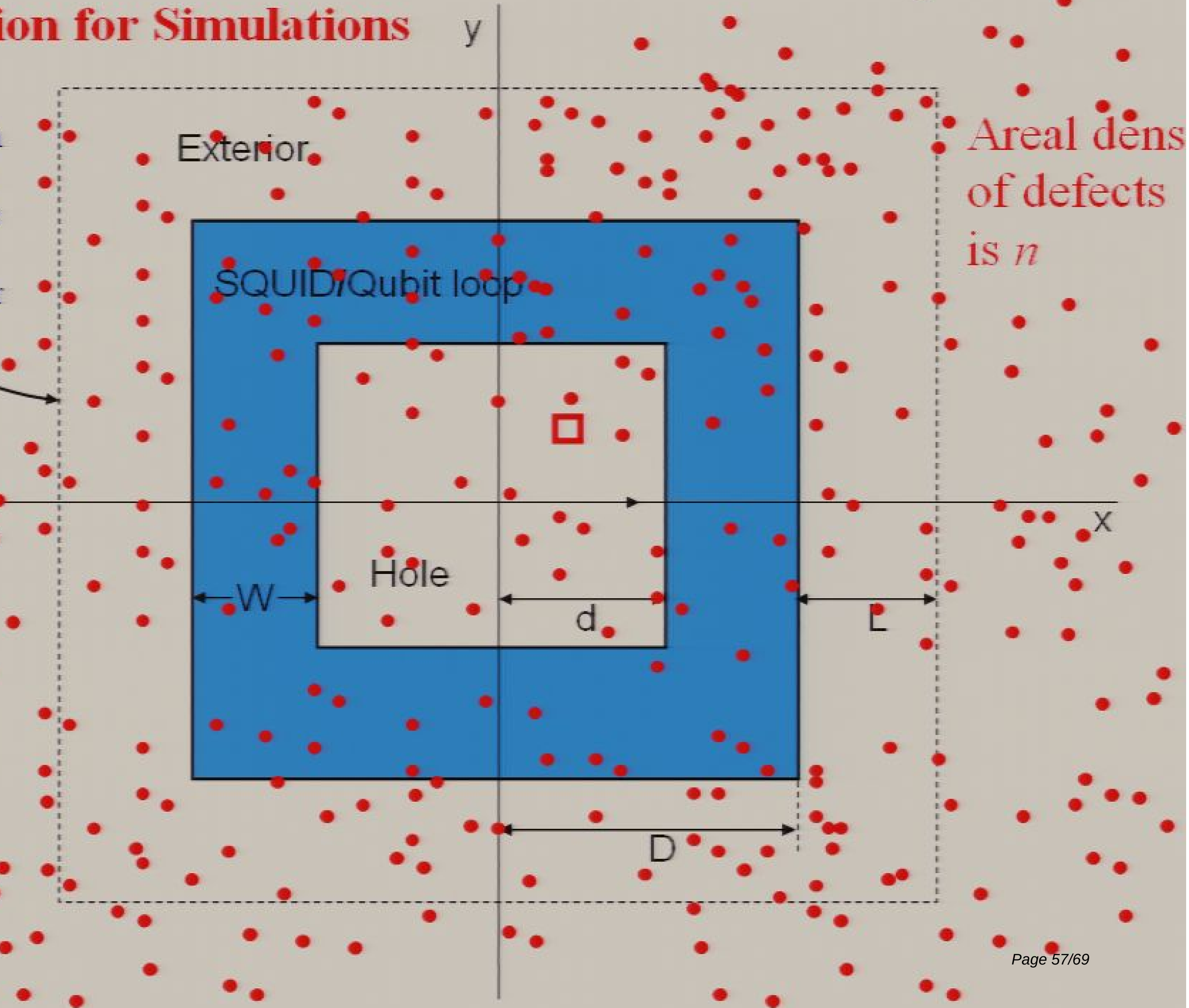
Configuration for Simulations



Areal dens
of surface
defects is *n*

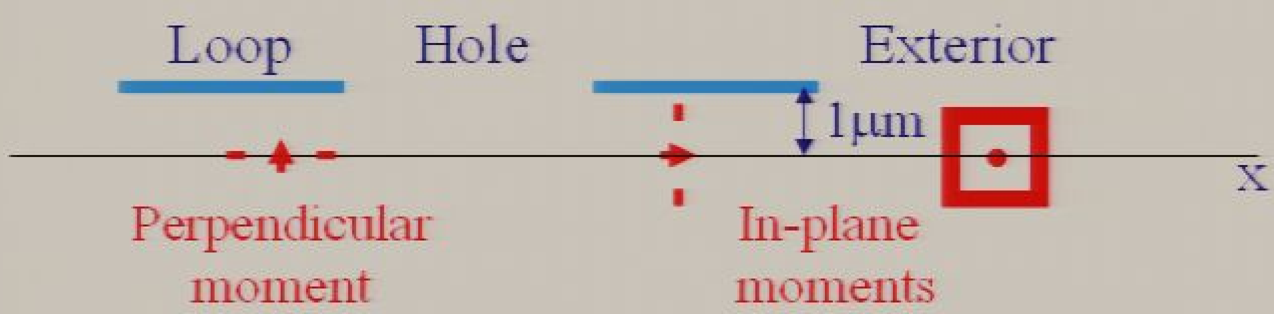
Configuration for Simulations

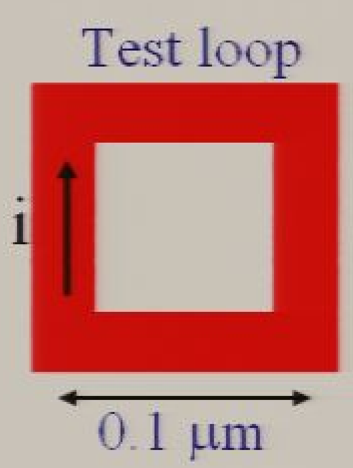
Boundary at which flux coupled from current loop is less than 1% of flux from loop at center



Areal dens of defects is n

Simulation Scheme for One Spin





Area A

Magnetic moment $Ai = \mu_B = 9.27 \times 10^{-24}$ J/T

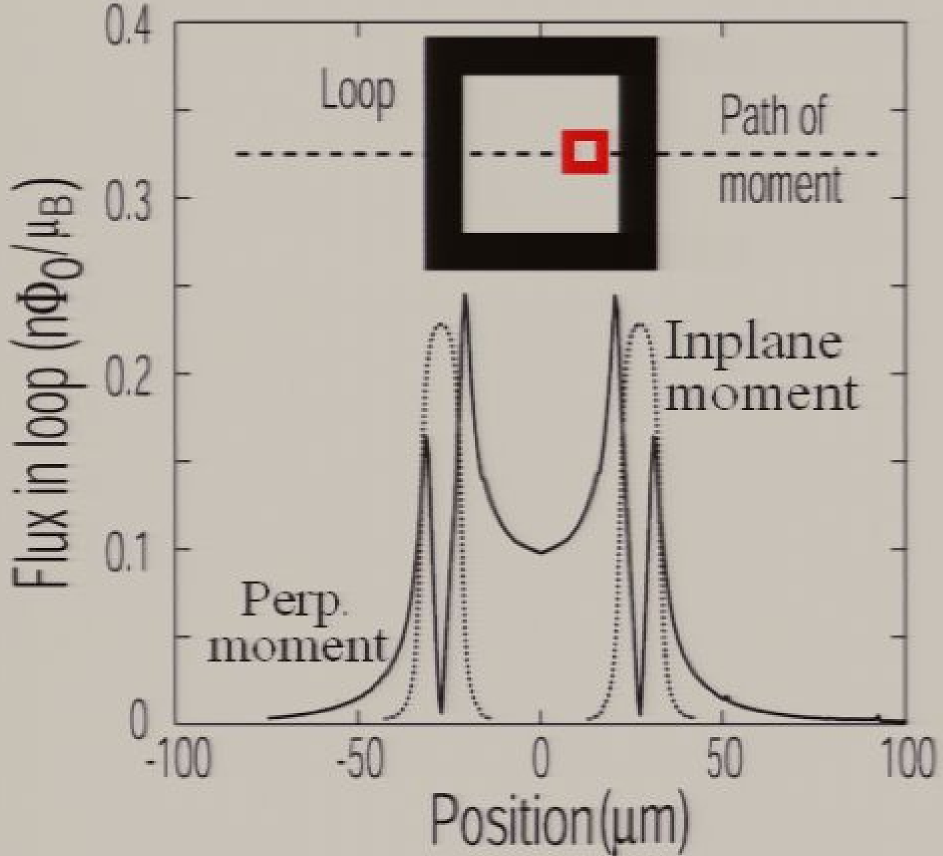
Mutual inductance to SQUID loop M(x,y)

Calculate M using superconducting FastHenry

Flux coupled to SQUID loop $\Phi_s = M(x,y)\mu_B/A$

Plot $\Phi_s/\mu_B = M(x,y)/A$ versus position

Flux Coupled to SQUID Loop from Current Loo



Flux from perpendicular moment
- Local minimum at the middle
- Peaks at edges of film
- Vanishes at midpoint of the film

Flux from in-plane moment parall
to the path
- Peaks at midpoint of film
- Falls off rapidly away from film

Mean Square Flux Noise Coupled to SQUID Loop by Ensemble of Spins

For one spin at (x,y), the flux coupled to the SQUID loop is

$$\Phi_s = M(x,y)\,\mu_B\,/A$$

Mean square value $\langle M^2(x,y)\rangle = (M_x^2 + M_y^2 + M_z^2)/3$

Mean square flux noise is

$$\langle(\delta\Phi_s)^2\rangle = 8n\,\mu_B^2\int_0^{(D+L)}dx\int_0^{x}dy\,\langle M^2(x,y)\rangle/A^2$$

Spectral Density of Flux Noise

Set the spectral density $S_\Phi(f) = \alpha/f$, where α is to be determined

Introduce lower and upper cut-off frequencies f_1 and f_2

Then $<(\delta\Phi_s)^2> = \alpha\int_{f_1}^{f_2} df/f = \alpha \ln(f_2/f_1)$

Assuming $f_1 = 10^{-4}$ Hz and $f_2 = 10^9$ Hz

We find $S_\Phi(f)/\Phi_0^2 \approx <(\delta\Phi_s/\Phi_0)^2>/30f$

Note that this result is only very weakly dependent on f_1 and f_2

Fit of Areal Density of Defects *n*

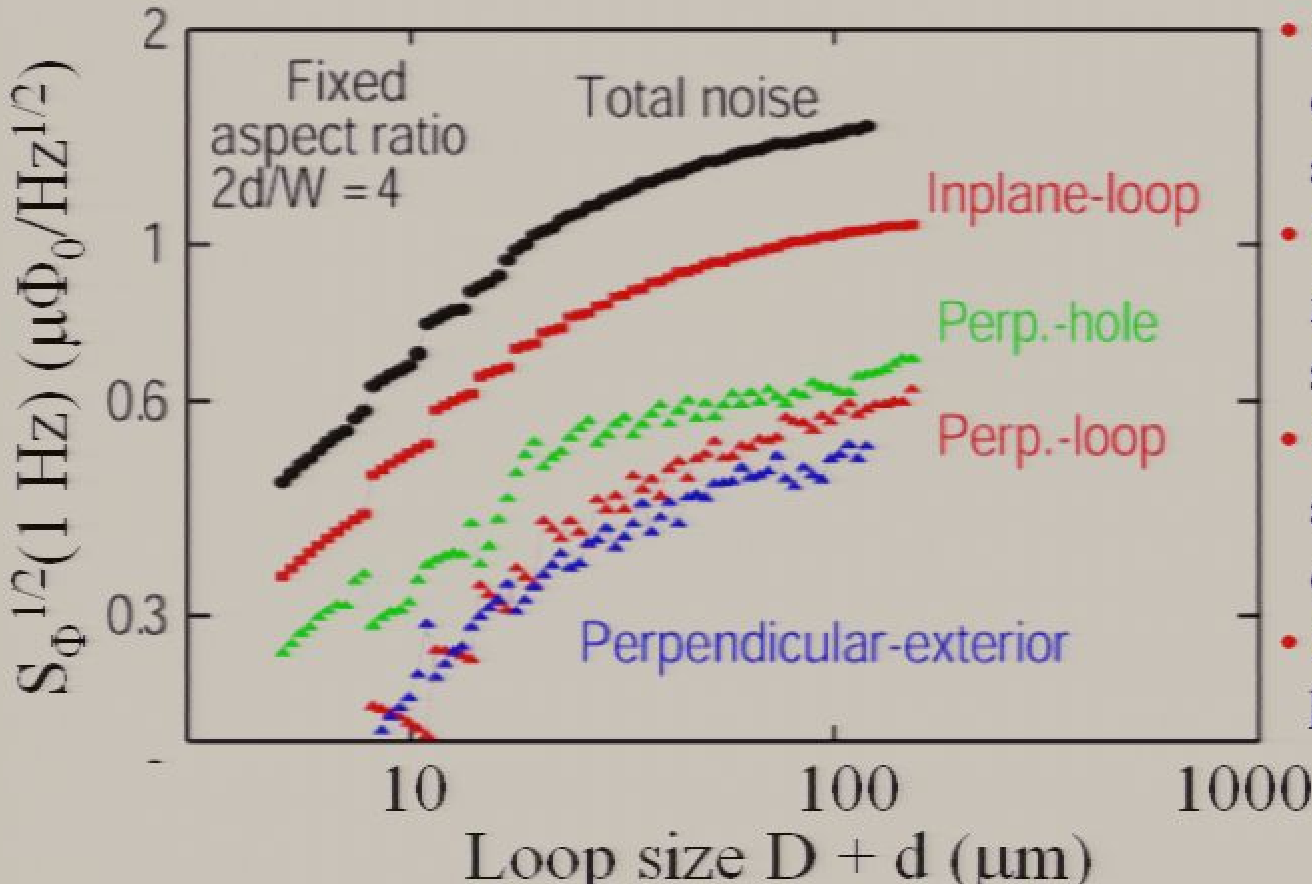
We regard *n* as a fitting parameter to produce values of $S_\Phi(1\ \text{Hz})$ comparable to those observed experimentally:

$$n = 5 \times 10^{17}\ \text{m}^{-2}.$$

We assume these are surface defects in the SiO_2 and contaminants produced by exposure to chemicals and the atmosphere. For a 10-nm layer, this value corresponds to about 1 defect per 10^3 atoms.

Koch and Hamers (1987): Performed STM measurements on ultra clean Si surface exposed to 3×10^{-7} torr of O_2 for 20 s in UHV chamber. They found about 8 two-level systems in $6.5 \times 6.5\ \text{nm}^2$ are in a bandwidth of 10-500 Hz, corresponding to an areal density of $2 \times 10^{17}\ \text{m}^{-2}$. This represents an areal defect density of 21×10^{17} m over 13 decades of frequency.

Flux Noise Versus Loop Size for Fixed Aspect Ratio



- Noise increases by a factor of about 4 for increase in area of about 200.
- The dominant noise is from inplane moments under the superconductor.
- This noise could equally w arise from inplane moment on the superconducting fil
- Inplane noise is negligible hole and exterior.

Could the Noise be Generated by Nuclear Spins

Model predicts that noise power scales as $\mu^2 n$

Superconducting film

- ^{52}Nb: 5.56×10^{28} m^{-3}, abundance 100%, $\mu = 0.00336\ \mu_B$
- ^{207}Pb: 3.30×10^{28} m^{-3}, abundance 22%, $\mu = 0.00032\ \mu_B$
- Nb noise power/Pb noise power $\approx$ 850
- Wellstood *et al.* measured essentially the same 1/f noise in SQUIDs with Nb and Pb loops

Substrate

- ^{29}Si: 5.0×10^{28} m^{-3}, abundance 5%, $\mu = 0.00030\ \mu_B$
- Sapphire ^{27}Al: $\approx 3 \times 10^{28}$ m^{-3}, abundance 100%, $\mu = 0.0020\ \mu_B$
- Sapphire noise power/Si noise power $\approx$ 500
- Martinis *et ali.* (private comm.) find no significant difference

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- Sapphire noise power/Si noise power ≈ 500
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We have no model for 1/f noise from nuclei

Questions

- What kinds of defects are involved?
- How might one measure the density of two level systems?
  - Variable temperature STM to determine the density of defect[s] as a function of temperature
- How might one verify the scaling with dimensions?
  - Fabricate SQUIDs with a wide range of dimensions in a singl[e] process
- How might one reduce the magnitude of 1/f flux noise?
  - Measure the flux noise in a device at low temperature, attemp[t] to remove the surface contamination with *in situ* ion bombardment, and remeasure the noise (heroic and nonscalabl[e]
  - Surface passivation

    Two sets of IBM SQUIDs which were passivated showed lower noise:

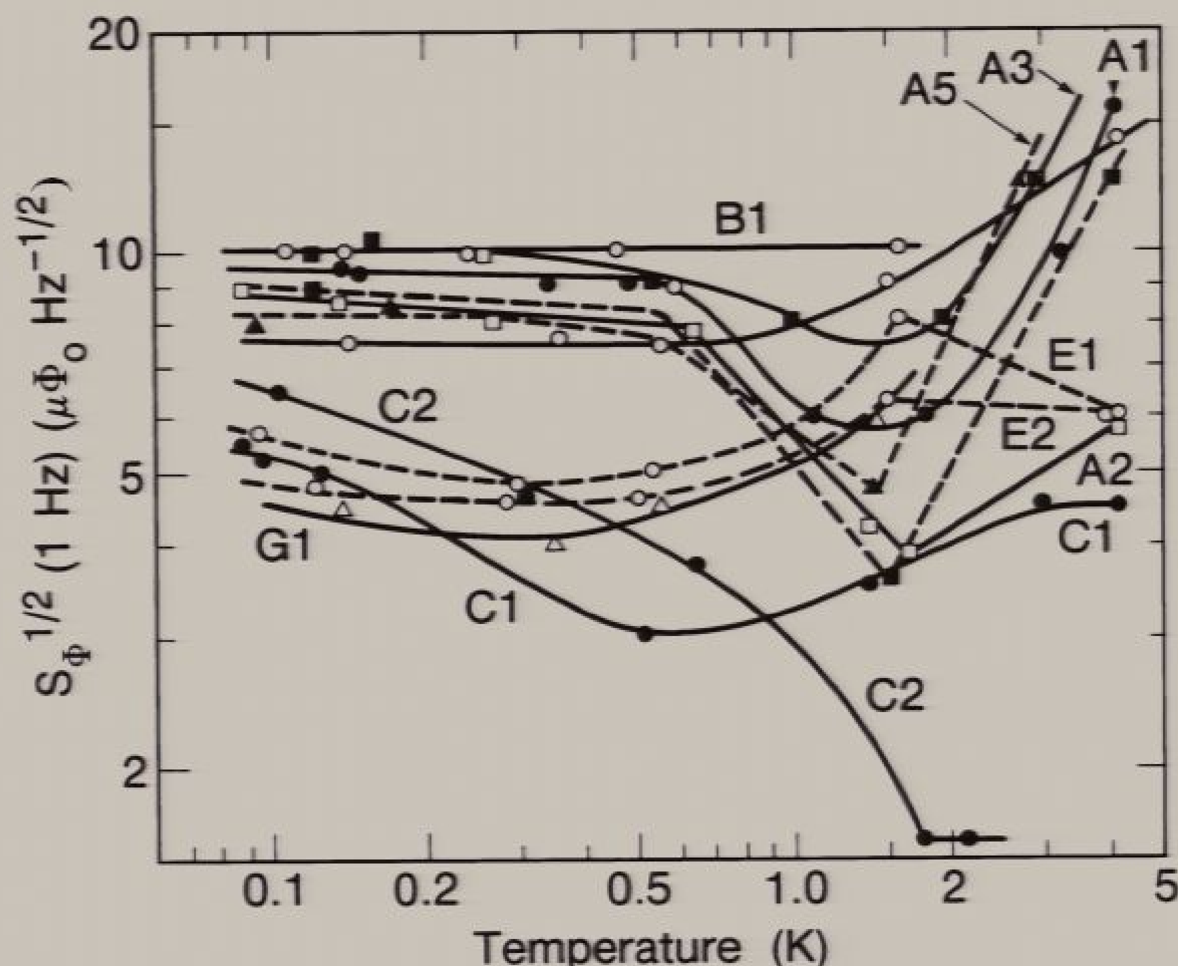
    Foglietti *et al.* $S_\Phi^{1/2}$ (1 Hz) $\approx 0.5 \times 10^{-6}\ \Phi_0\ \mathrm{Hz}^{-1/2}$

    Tesche *et al.* $S_\Phi^{1/2}$ (1 Hz) $\approx 0.2 \times 10^{-6}\ \Phi_0\ \mathrm{Hz}^{-1/2}$

Concluding Remarks

- Flux noise model based on the hopping of electrons between traps in which their spins have fixed, random directions
- Crucial underlying physics of "spin locking" is the two-fold degeneracy—the Kramers degeneracy—and that transitions between these states do not occur at low temperature
- Traps have a broad range of energies, and hence trapping times, so that uncorrelated processes lead to 1/f noise
- A trap areal density of $5 \times 10^{17} m^{-2}$ is required to account for the observed levels of 1/f noise: this appears to be reasonable for heavily contaminated surfaces
- Noise amplitude increases slowly as the device dimensions are increased
- The model does not discriminate between defects on the substrate and on the superconductor
- This model unifies the concepts of charge, critical current and flux noise

DC SQUIDs: $S_{\Phi}^{1/2}$(1Hz) vs. T



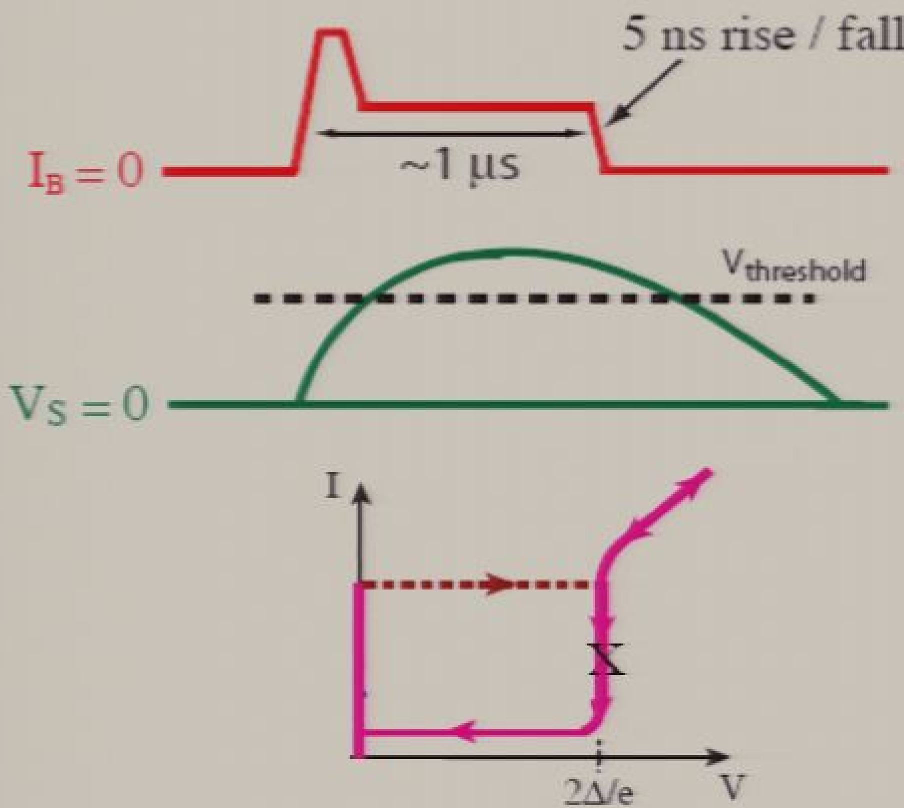
Loop material
Nb (A1-4, F1)
Pb (C2)
PbIn (A5, B1,
C1, E1,2)

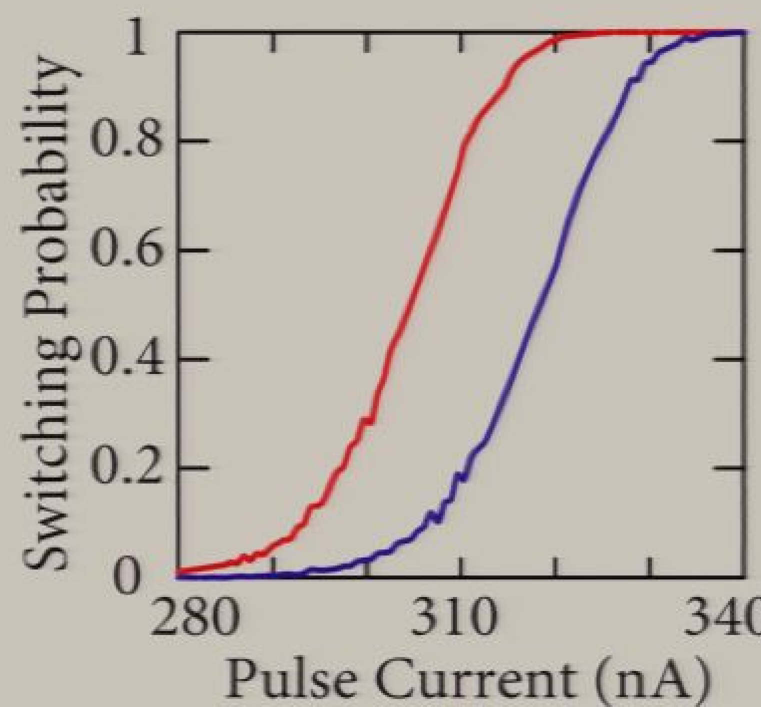
Loop effective
areas:
1,400 μm^2–
200,000 μm^2

At low T: $S_{\Phi}^{1/2}(1\text{Hz}) \approx 7 \pm 3\ \mu\Phi_0 \text{Hz}^{-1/2}$

Wellstood, Urbina and Clarke (1987)

SQUID Readout





- Pulse bias current: detect switching events
- Repeat (say) 1000 times to determine probability
- Increment bias current and repeat

$\Phi_{QA} = 0.48\ \Phi_0$

$\Phi_{QA} = 0.52\ \Phi_0$

$\Phi_S\ = \text{constant}$

- Determine current $I_S^{50\%}$ for 50% switching probability