Title: Constraining Inverse Curvature Gravity with Supernovae

Date: Apr 17, 2007 11:00 AM

URL: http://pirsa.org/07040009

Abstract: We show that the current accelerated expansion of the Universe can be explained without resorting to dark energy. Models of generalized modified gravity, with inverse powers of the curvature can have late time accelerating attractors without conflicting with solar system experiments. We have solved the Friedman equations for the full dynamical range of the evolution of the Universe. This allows us to perform a detailed analysis of Supernovae data in the context of such models that results in an excellent fit. Hence, inverse curvature gravity models represent an example of phenomenologically viable models in which the current acceleration of the Universe is driven by curvature instead of dark energy. If we further include constraints on the current expansion rate of the Universe from the Hubble Space Telescope and on the age of the Universe from globular clusters, we obtain that the matter content of the Universe is 0.07 <= omega_m <= 0.21 (95% Confidence). Hence the inverse curvature gravity models considered can not explain the dynamics of the Universe just with a baryonic matter component.

Constraining Inverse Curvature Gravity with Supernovae

O. Mena, J. Santiago and JW

PRL, 96, 041103, 2006

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Jochen Weller

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The Cosmic Pie

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- Combining cosmic microwave background, Supernovae and large scale structure observations: WMAP, SCP, High-z, SNLS,..., SDSS, 2dF, ...

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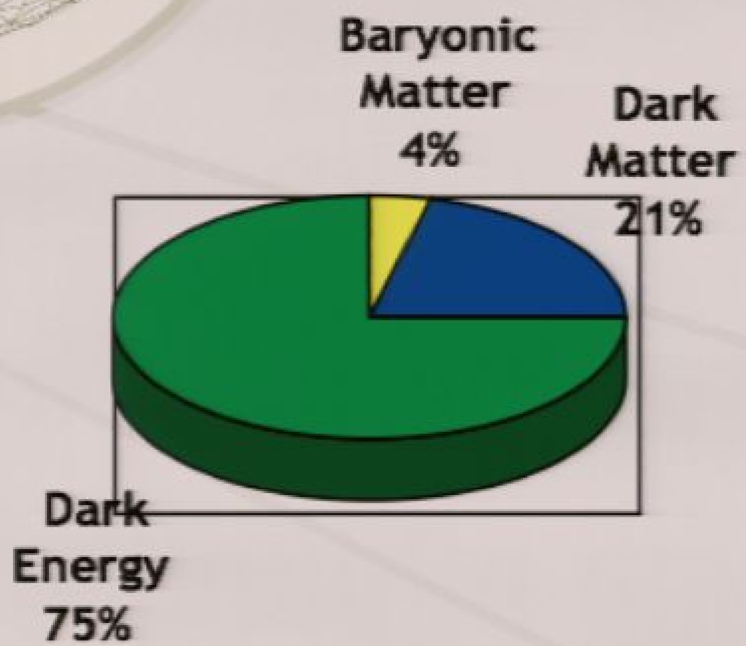

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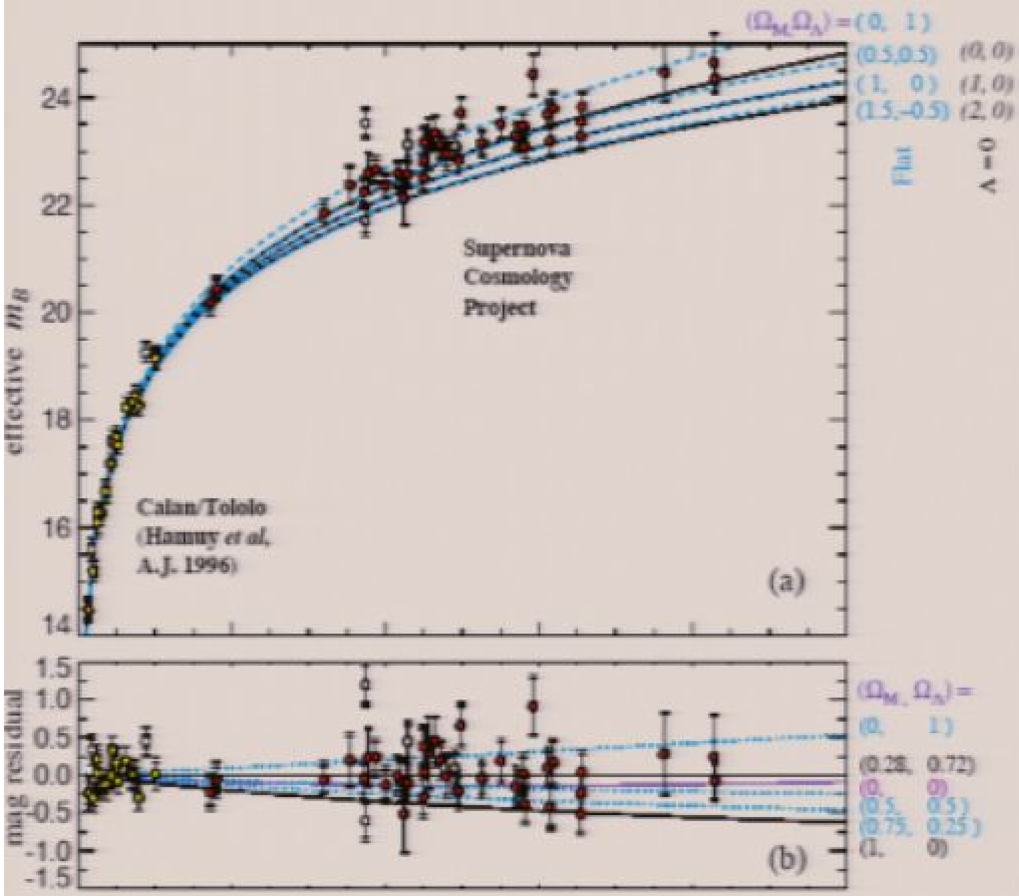
Supernovae Measurements



- SNe allow measurement of distance - redshift relation at large redshifts: **The expansion of the Universe is accelerating !**
- Perlmutter et al.; Riess et al.; Knop et al.; Astier et al.

Cosmological Constant

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Dark Energy

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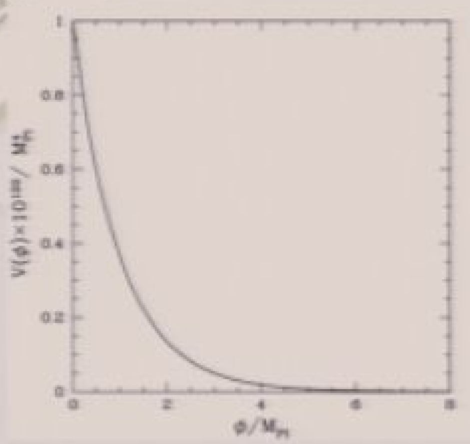

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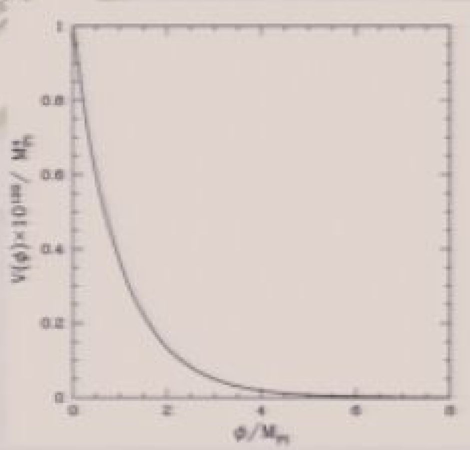

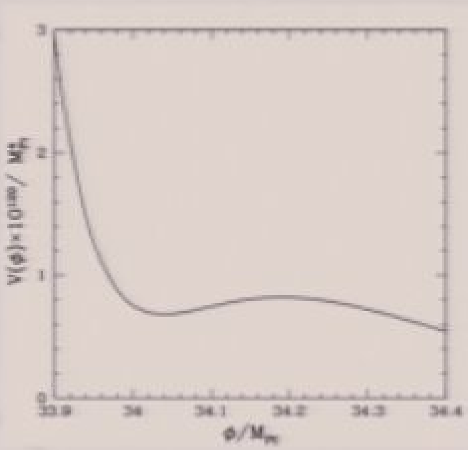

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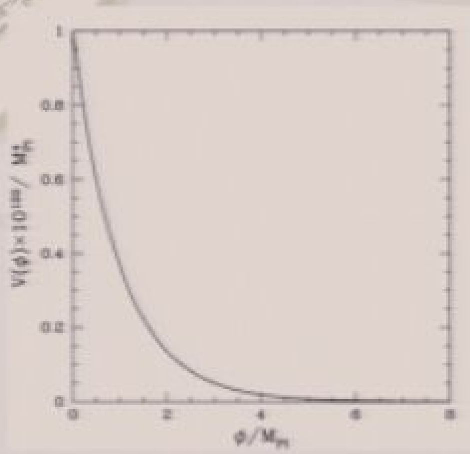
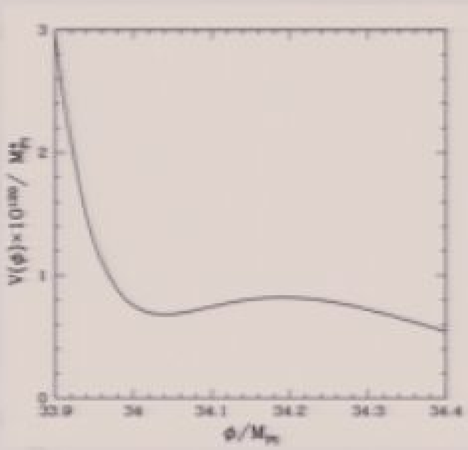


Dynamical dark energy

Equation of state of scalar field:

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$



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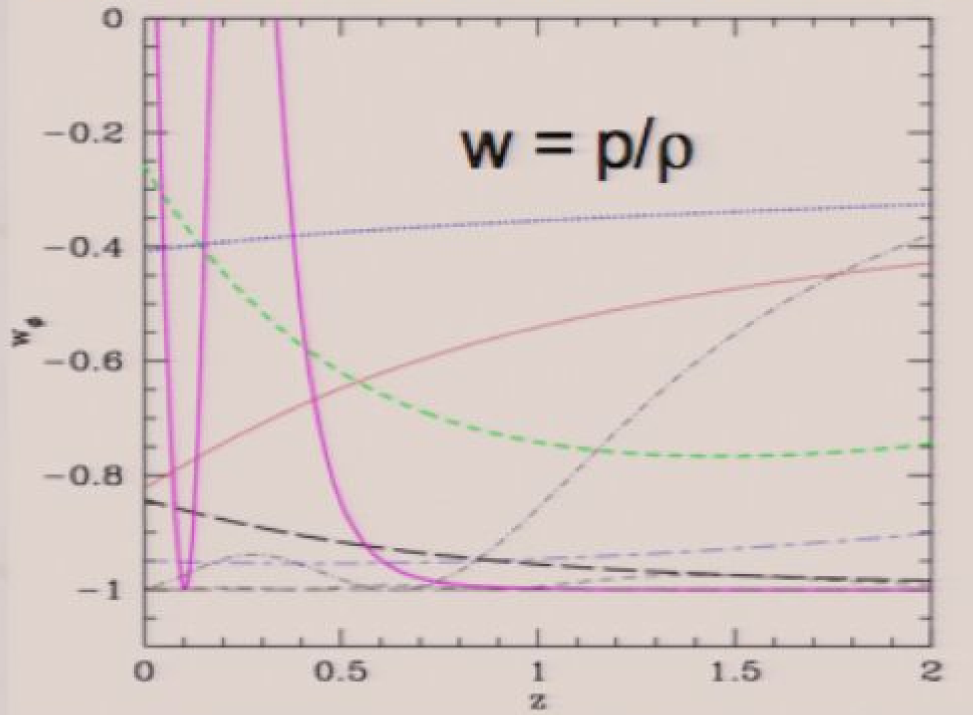
2nd try (Steinhardt, Caldwell et al. 1998):

$$V(\phi) = M^4 e^{-M_{\rm pl}/\phi} \quad ; \quad V(\phi) = M^{4+\alpha}/\phi^{\alpha}$$

Different Quintessence Models

scalar field dark energy models (quintessence)



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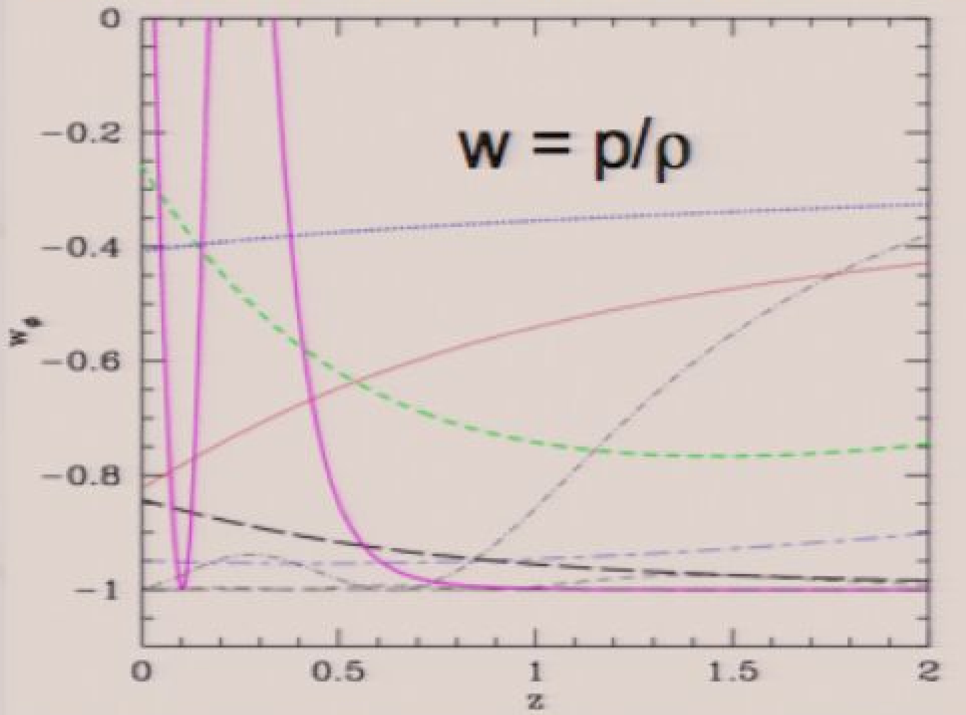

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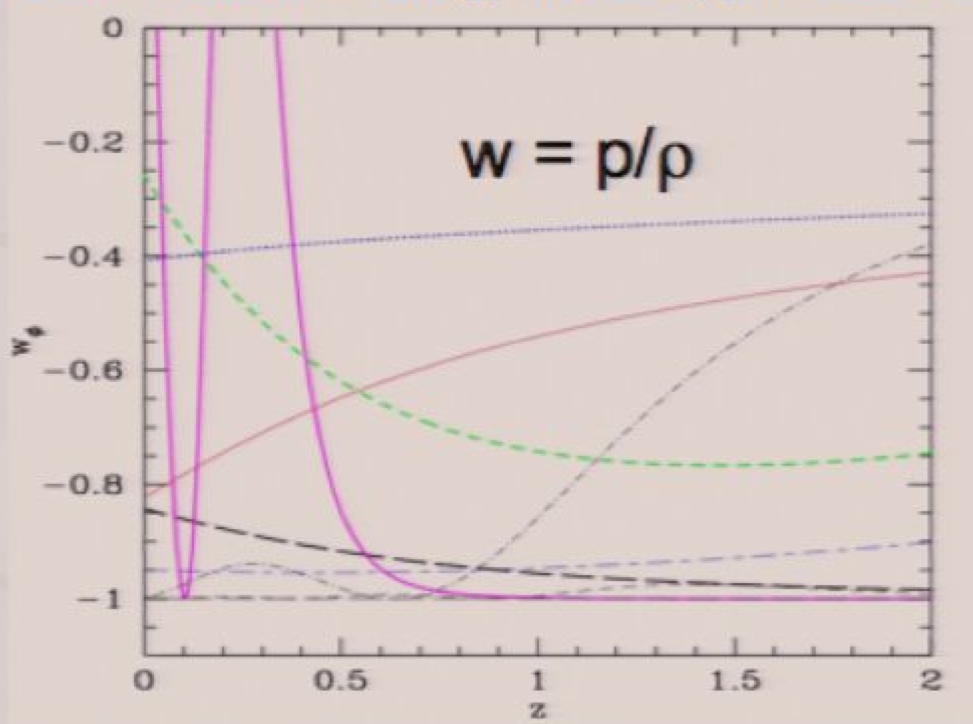

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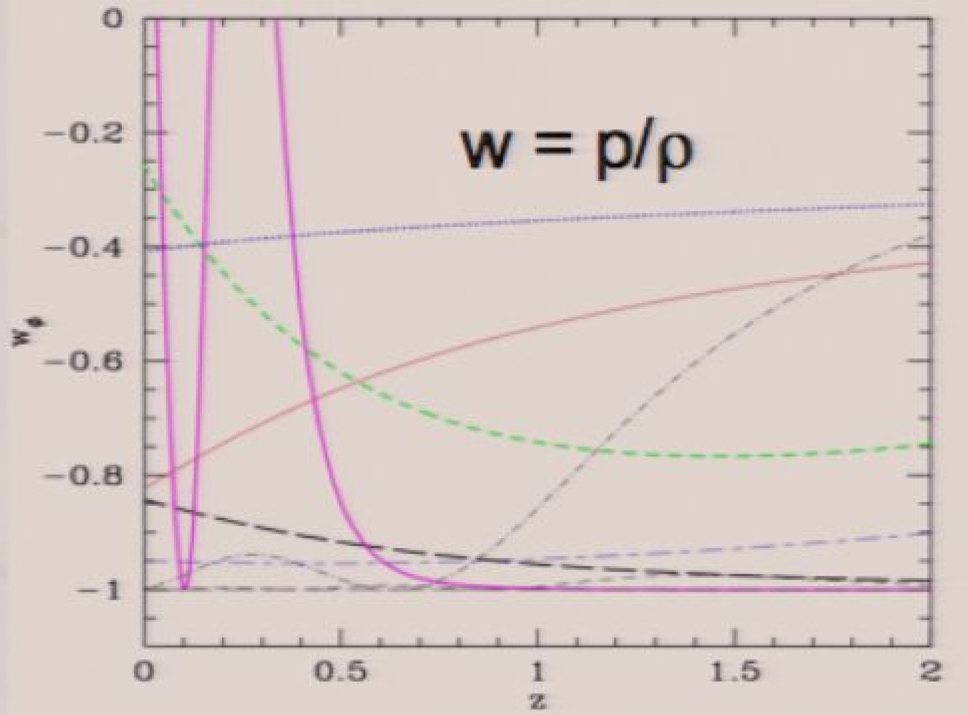

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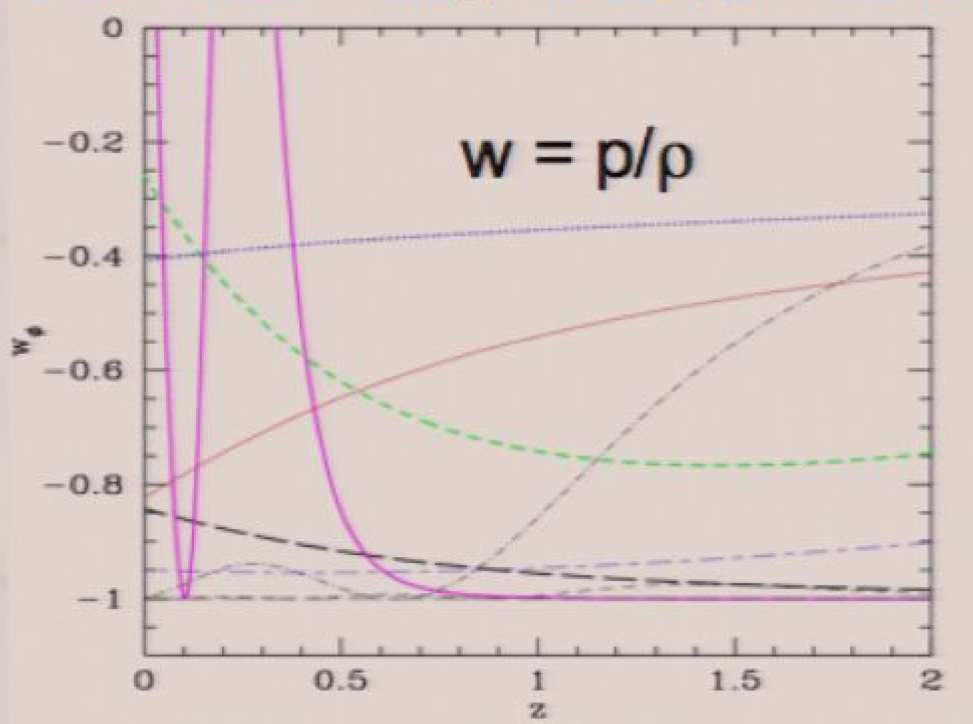

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but maybe something completely different ...

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Maybe gravity is standard at short distances...

but gets modified on large distances ...

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- purely gravitational alternative to dark energy

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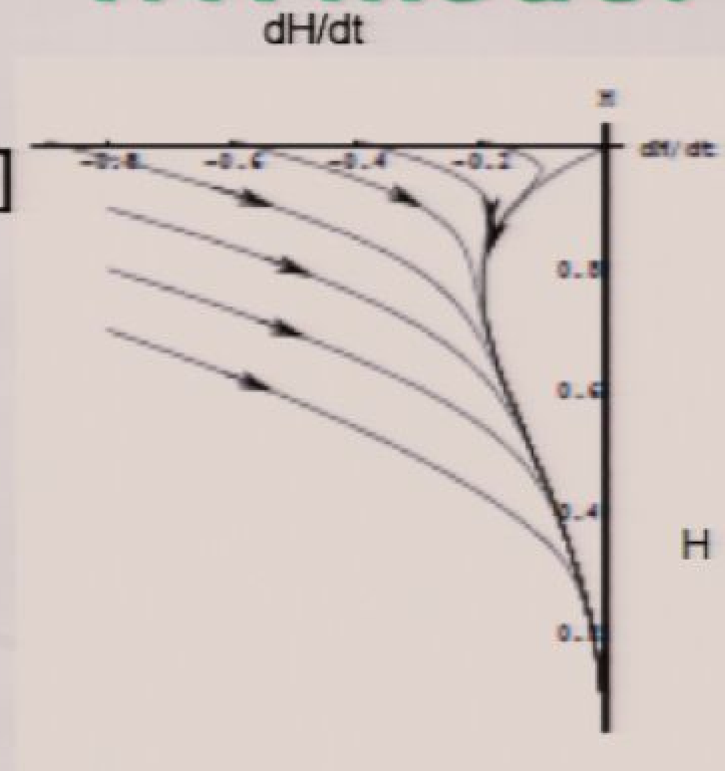

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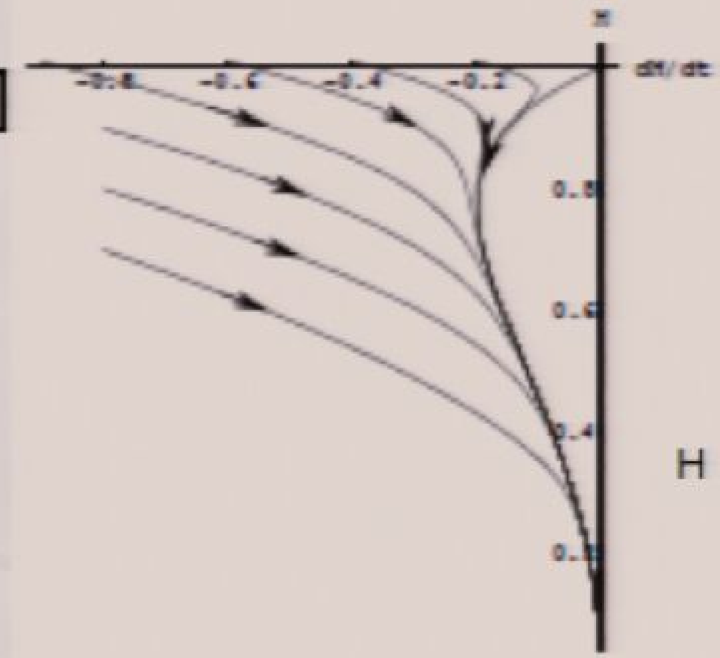

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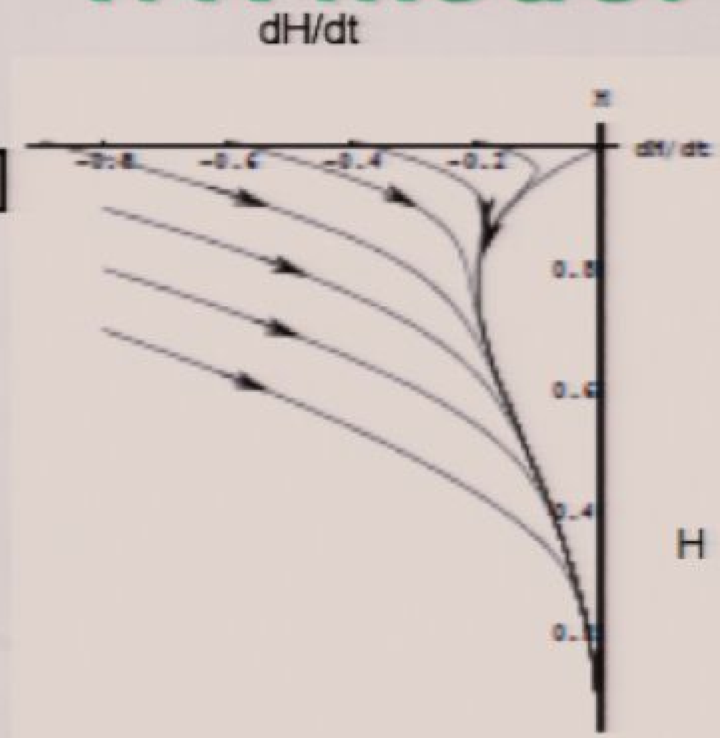

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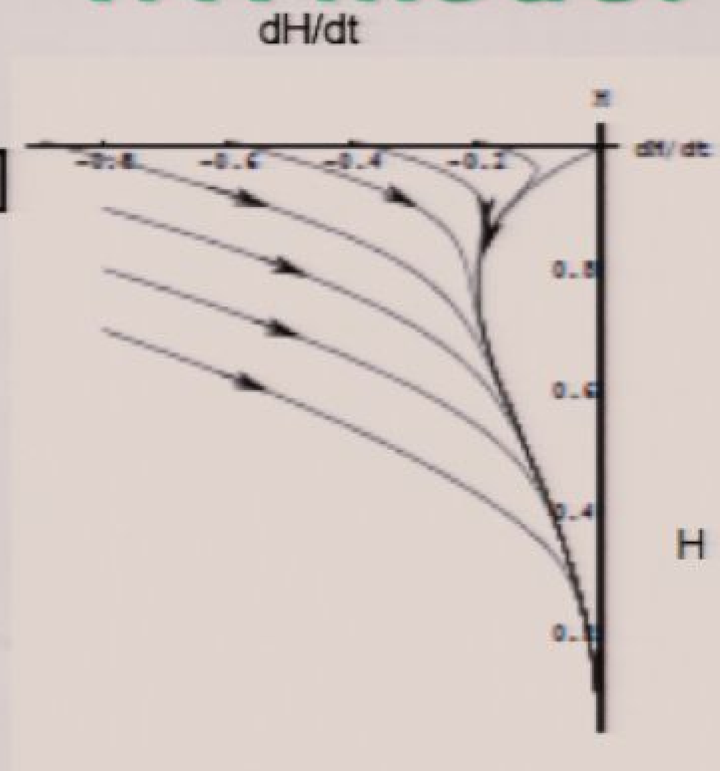

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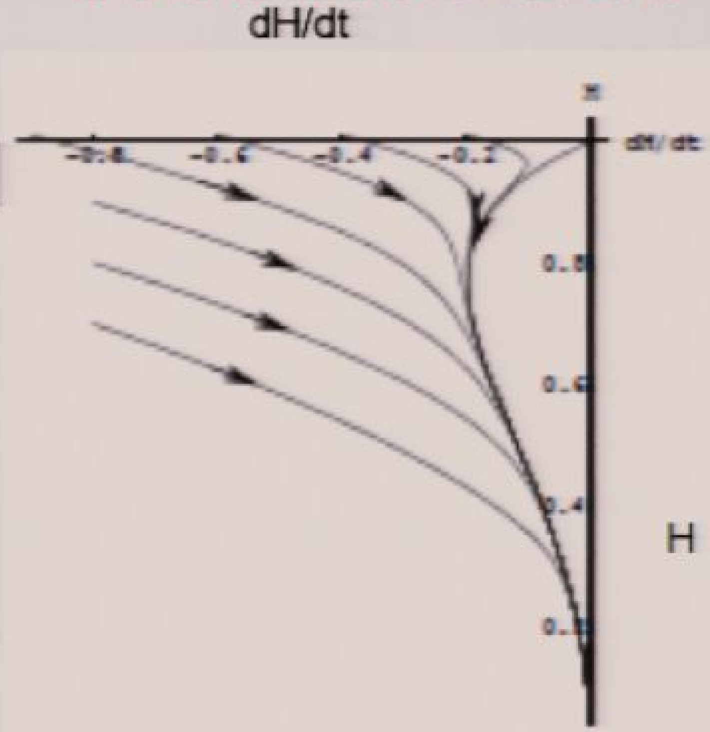

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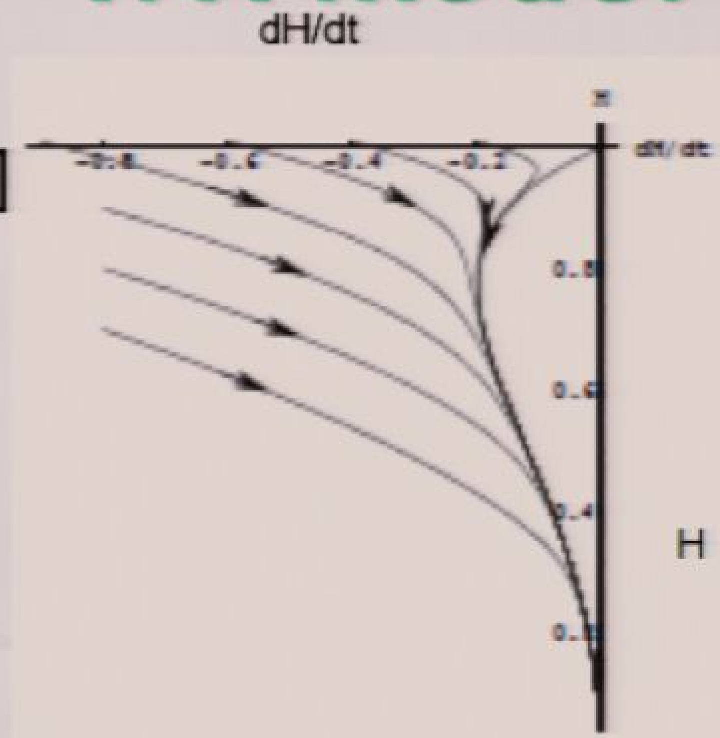

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- Observational consequences similar to dark energy with $w = -2/3$

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- Simplest model ($\propto 1/R^n$) ruled out by observations of distant Quasars and the deflection of their light by the sun with VLBI: $\omega > 35000$ [Chiva ('03), Soussa, Woodard ('03),...]

The New Model

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[CDDTT'04]

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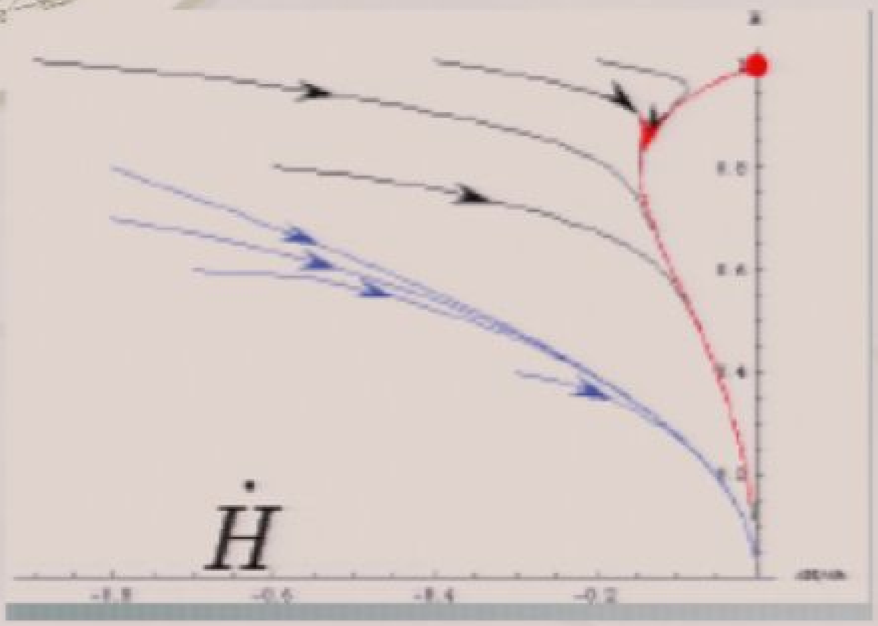
Example $1/R_{\mu\nu}R^{\mu\nu}$ Model



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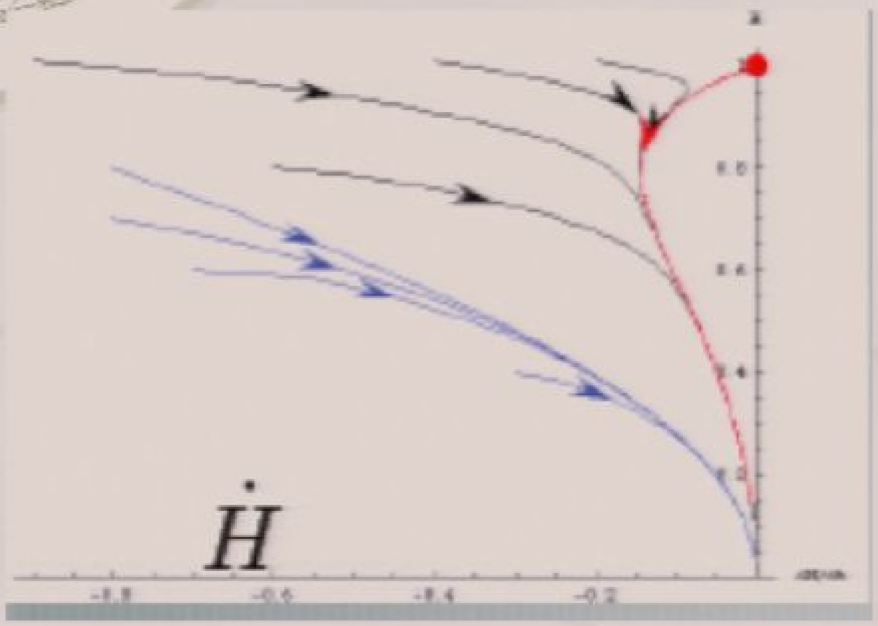

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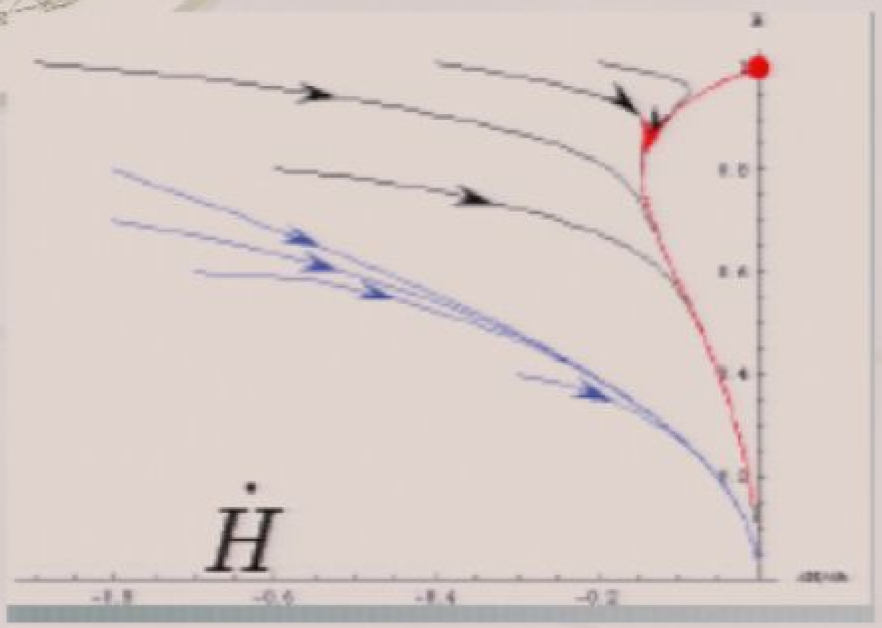

- Unstable de Sitter Point (H=const)
- Two late time attractors:
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Afraid of Ghosts ?

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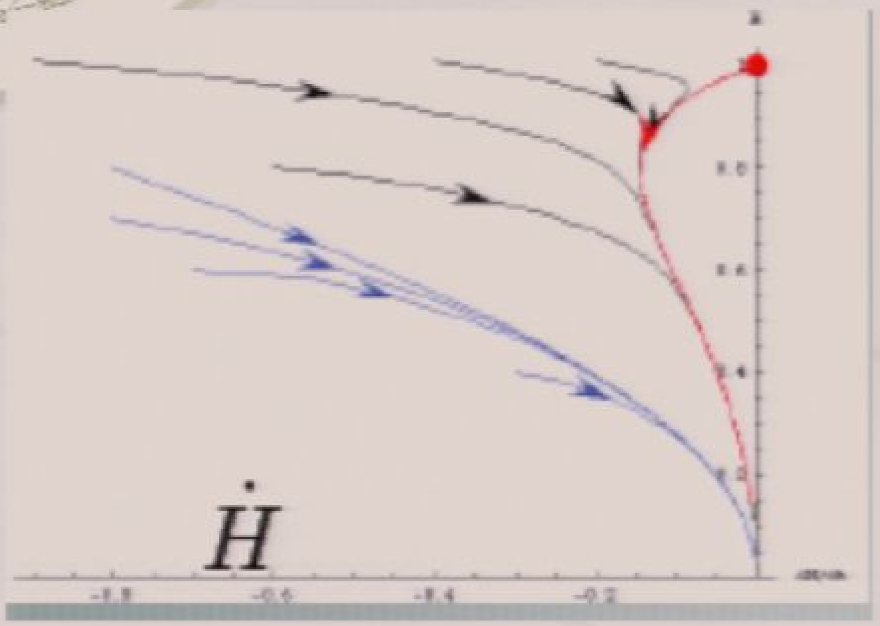

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$$P = R_{\mu\nu} R^{\mu\nu}$$

$$Q = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

$$F(R, Q-4P)$$

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- In general $F(R,Q-4P)$ with $Q=R_{\mu\nu}R^{\mu\nu}$ and $P=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ has no ghosts, however…

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- Also have to check 2nd order derivatives for finite propagation speeds (De Felice et al. astro-ph/0604154)

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- For higher inverse powers $1/(aR^2+bP+cQ)^n$ there is hope !

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$$\phi(r) \simeq -\left[1 - \frac{\alpha}{2}\left(\frac{r}{r_c}\right)^{6n+4}\right]\frac{GM}{r}$$

Navarro et al. 2005

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Non-Cosmological Tests

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$$\alpha=\frac{12a+4b+4c}{12a+3b+2c} \quad \hat{\mu}=\frac{\mu}{[12a+3b+2c]^{1/6}} \quad \sigma=\mathrm{sign}\,(12a+3b+2c)$$

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$\bar{\omega}_m$

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- For $\alpha_3 < \alpha < \alpha_4$: no longer stable attractor and singularity is reached in the future through an accelerated phase. For small $\bar{\omega}_m$ this appears in the past.

Solving the Friedman Equation for n=1

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Perturbative Solution for α=1

$$u = \log H \qquad x = \log a$$

$$6\ddot{u} + 15\dot{u}^2 + 34\dot{u} + 8 + 18\Delta e^{4u}(2+\dot{u})^6\left[e^{2(\bar{u}-u)} - 1\right] = 0$$

$$\Delta \equiv= 12a + 3b + 4c = 4(3a - c)$$ for $\alpha = 1$

$$\bar{u} = \log \bar{H} = \log\left(H_0\sqrt{\Omega_r e^{-4x} + \Omega_m e^{-3x}}\right)$$

good approximation in the past

$$H = \bar{H}(1+\epsilon) \qquad u = \bar{u} + \log(1+\epsilon) \approx \bar{u} + \epsilon$$

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Solution and Conditions

$$\epsilon = -\frac{1}{9H_0^6\Delta\Omega_m}\frac{40\Omega_r + 37\Omega_m e^x}{\Omega_r + \Omega_m e^x}e^{9x}$$

$$\epsilon \ll 1$$

$$6\frac{\ddot{\epsilon}}{1+\epsilon} + 9\left(\frac{\dot{\epsilon}}{1+\epsilon}\right)^2 + 34\frac{\dot{\epsilon}}{1+\epsilon} \ll \ddot{u} + 15\dot{u}^2 + 34\dot{u} + 8$$

$$\frac{\dot{\epsilon}}{1+\epsilon} \ll 2 + \dot{u}$$

Specific Conditions

$$a \ll \left(\frac{-9 H_0^6 \Delta \Omega_m^3}{37} \right)^{1/9} \sim \mathcal{O}(1)$$

For example with Δ=-4 at
a=0.2:

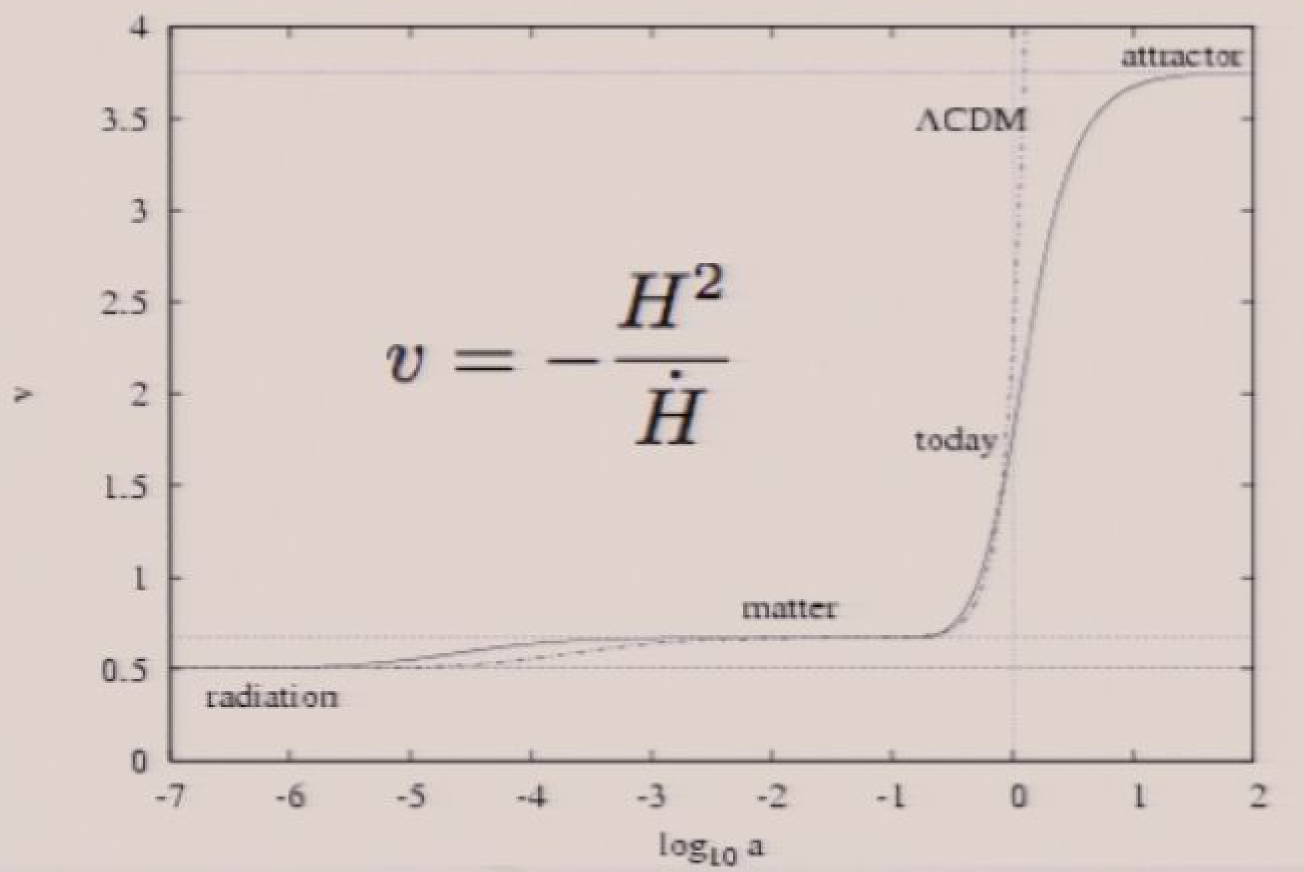
$$1.2 \times 10^{-3} \ll 1$$

$$0.99 \ll 9.24$$

$$0.011 \ll 0.5$$

In general all 3 conditions break down at a > 0.1-0.2

Dynamics of best fit model



$$a \sim t^p \quad \rightarrow \quad v = p$$

Approximation and Numerical Solution

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- Very accurate for $z \geq$ few (7), better than 0.1% with $H_E^2 = 8\pi G/\rho$ the standard Einstein gravity solution at early times.

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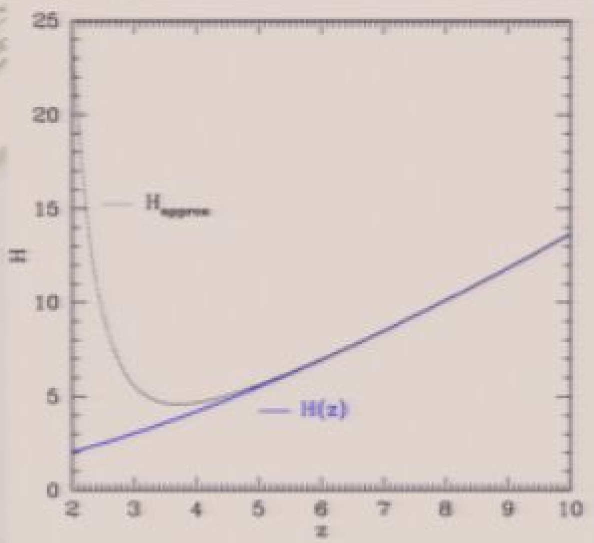

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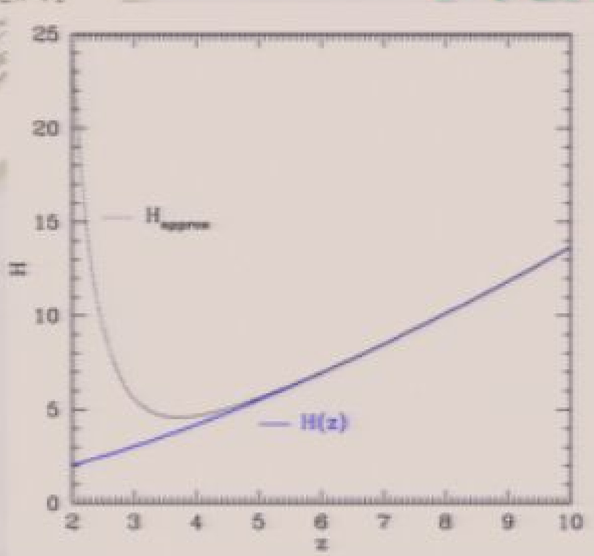

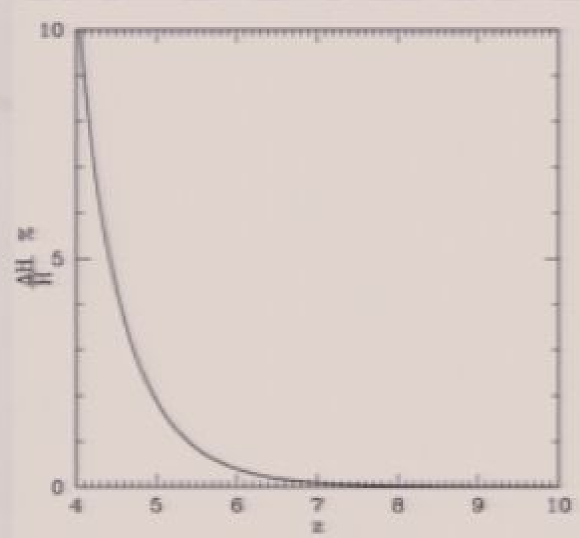

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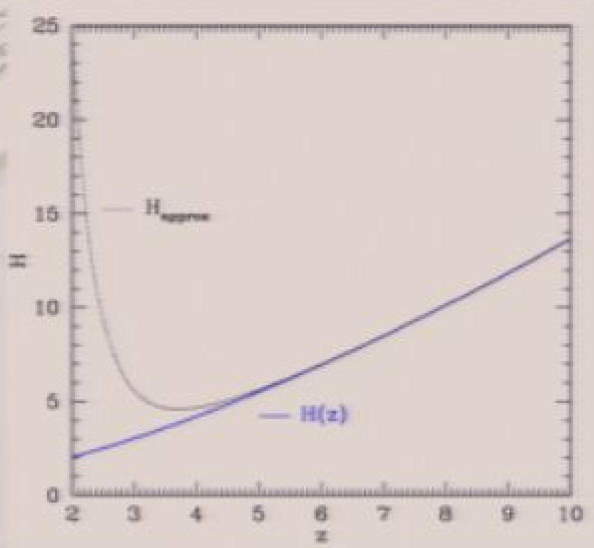

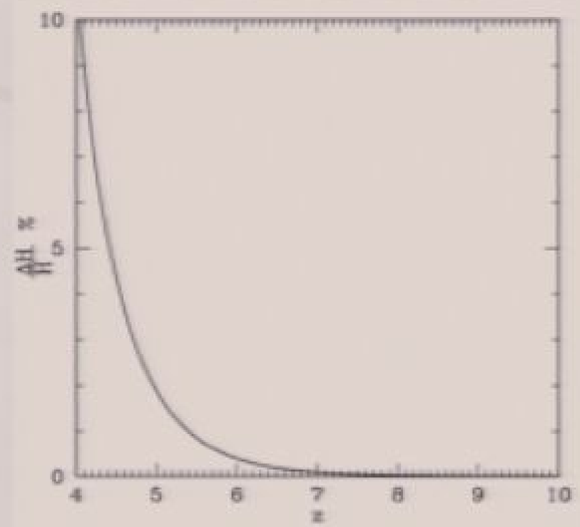

- Very accurate for z ≥ few (7), better than 0.1% with $H_E^2 = 8\pi G/\rho$ the standard Einstein gravity solution at early times.

- Use approximate solution as initial condition at z=few (7) for numerical solution (approximation very accurate and numerical codes can cope)

Fit to Supernovae Data

$\hat{\mu}$

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- Include intrinsic magnitude of Supernovae as free parameter: Degenerate with value of H_0 or better absolute scale of $H(z)$. Measure all dimensionful quantities in units of $\hat{\mu}$

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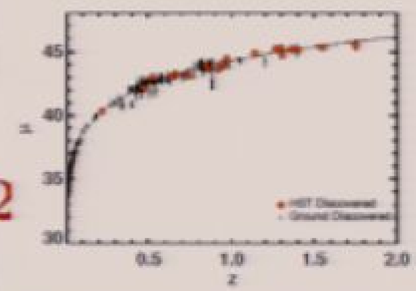
$\sigma = -1\,,$	$0.89 \leq \alpha \leq 1.10$	low
$\sigma = +1\,,$	$1.10 \leq \alpha \leq 2.16$	high

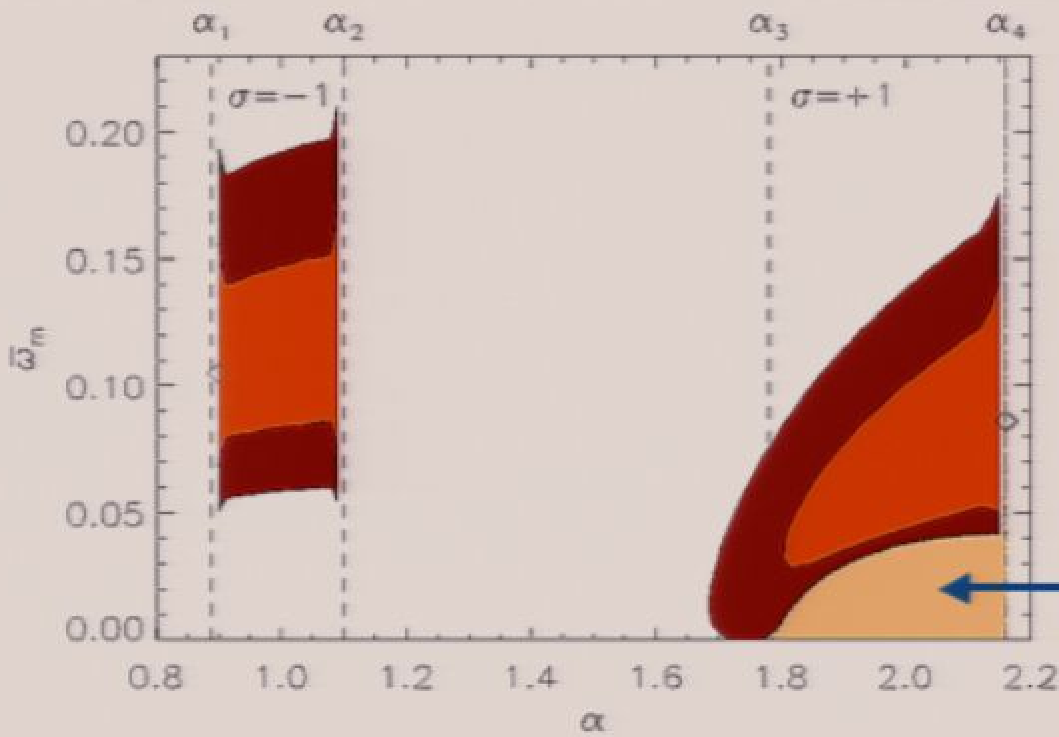
- Fit to Riess et al (2004) gold sample; a compilation of 157 high confidence Type Ia SNe data.



$$\alpha = 0.9\,, \qquad \bar{\omega}_m = 0.105\,, \chi^2 = 184.9$$

$$\alpha = 2.15\,, \qquad \bar{\omega}_m = 0.085\,, \chi^2 = 185.2$$

- very good fits, similar to ΛCDM ($\chi 2$ = 183.3)



Universe hits singularity in the past

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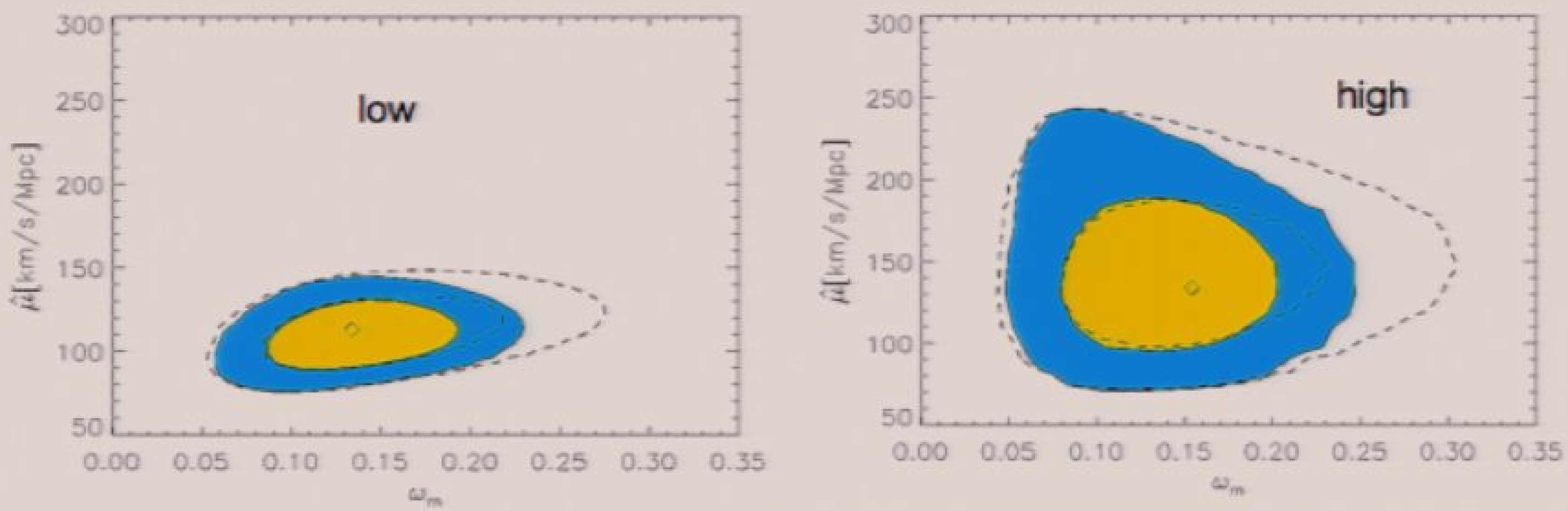

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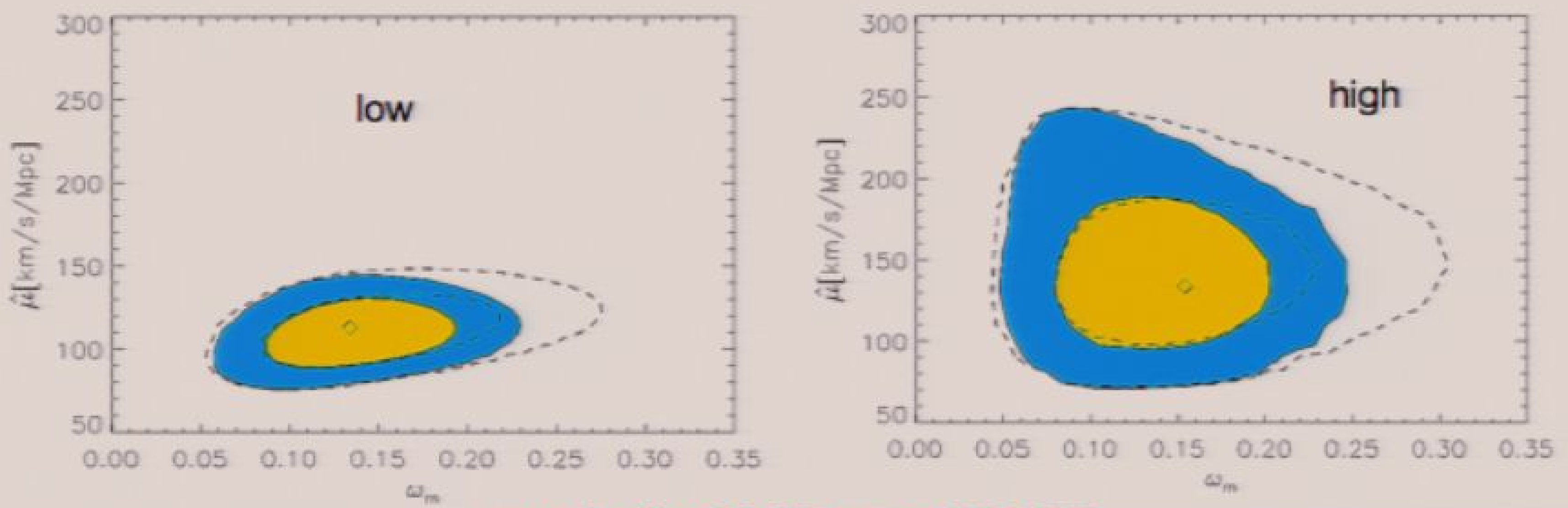

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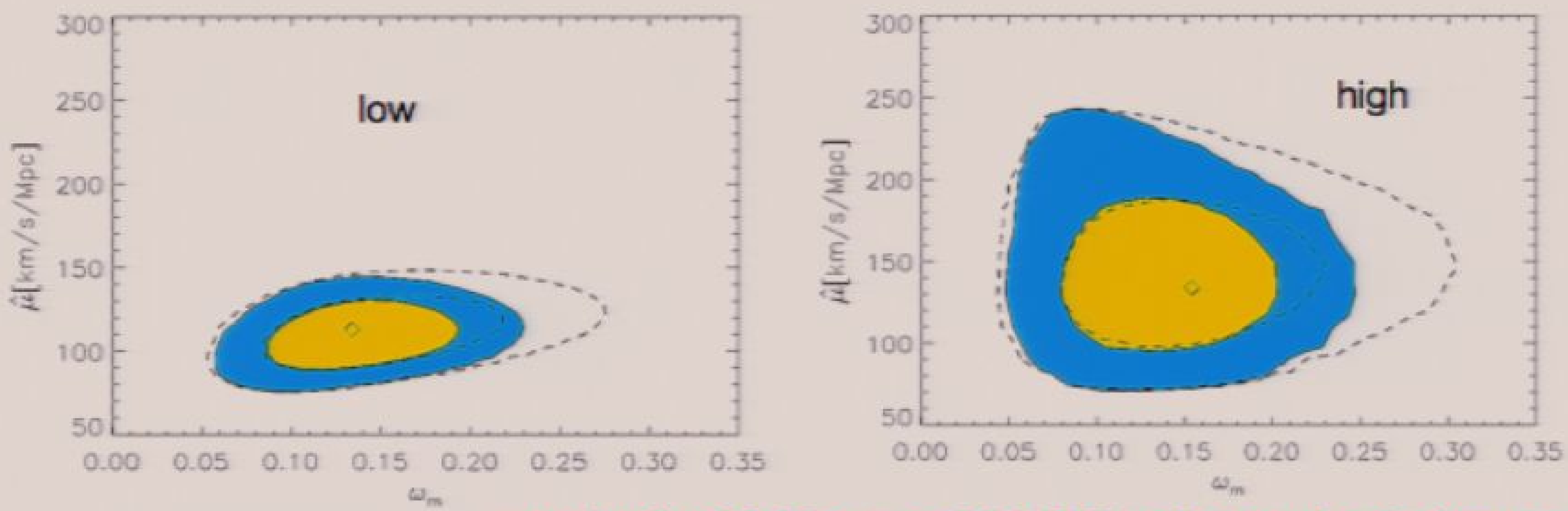

marginalized 0.07 < ω_m < 0.21 (95% c.l.); require dark matt

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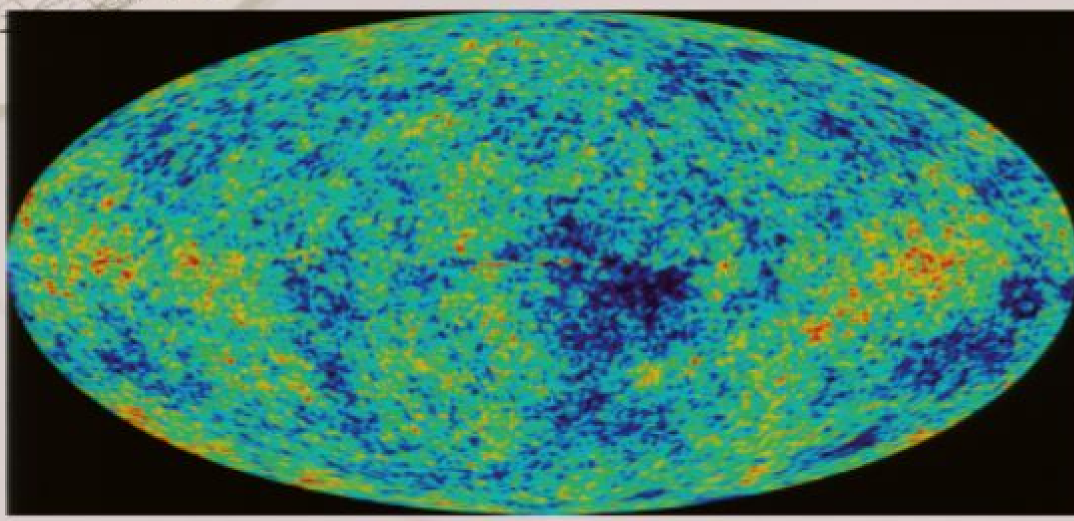
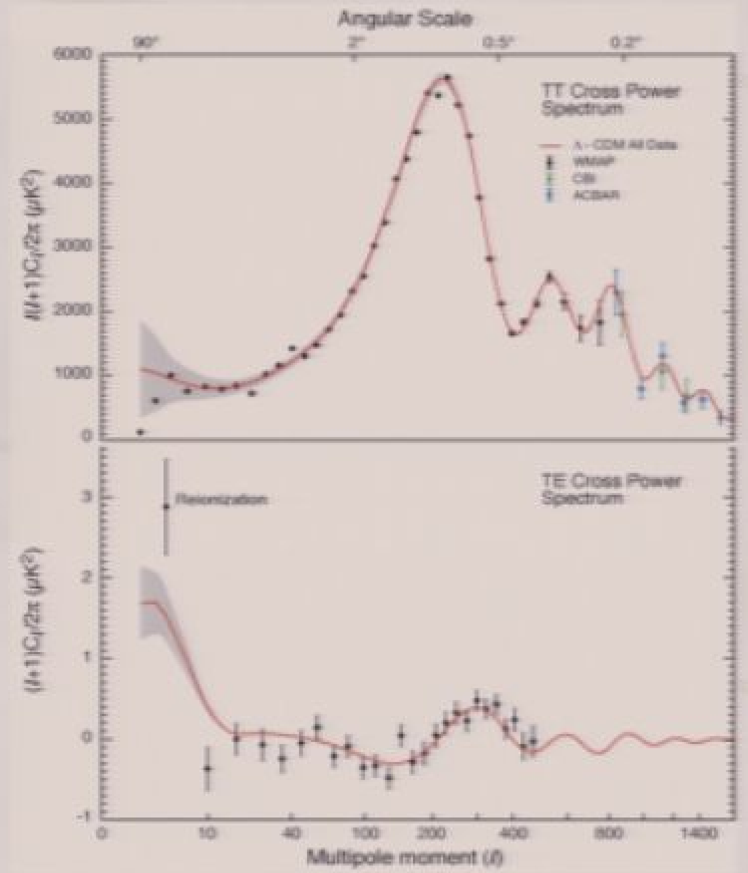
Combining with the Cosmic Microwave Background ?

WMAP





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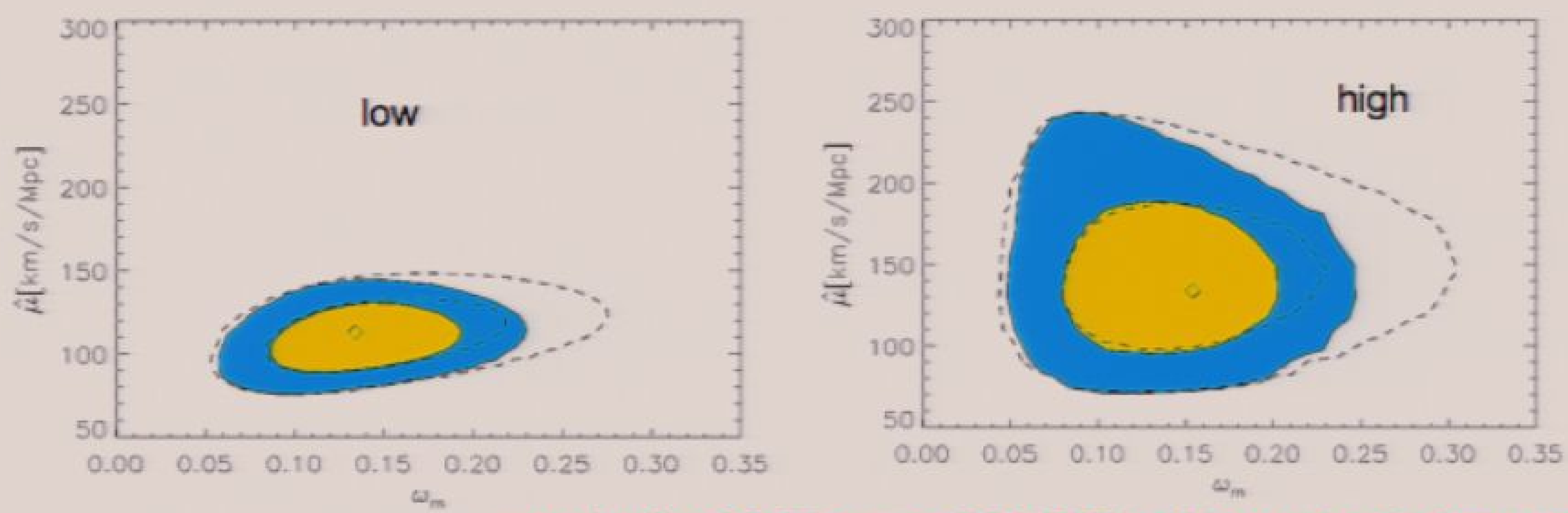

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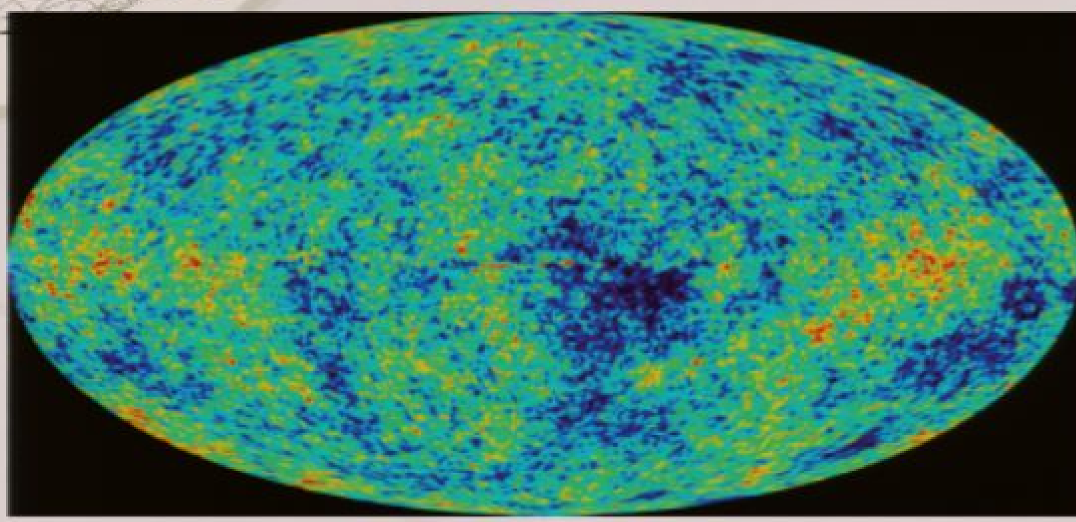

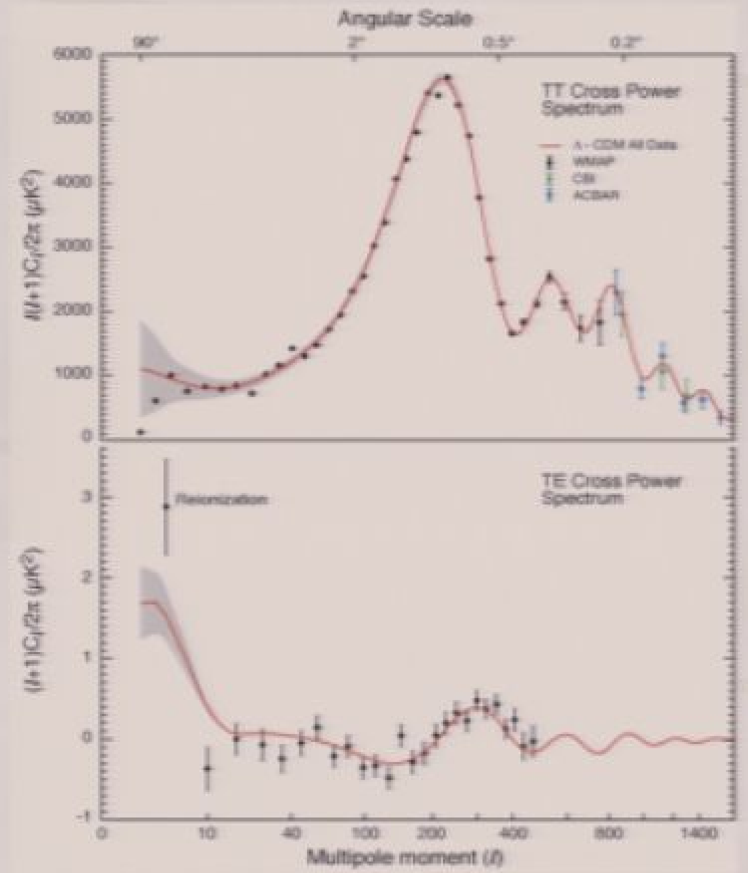

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CMB for the Brave

Small scale CMB anisotropies are mainly affected by the physical cold dark matter and baryon densities and the angular diameter distance to last scattering

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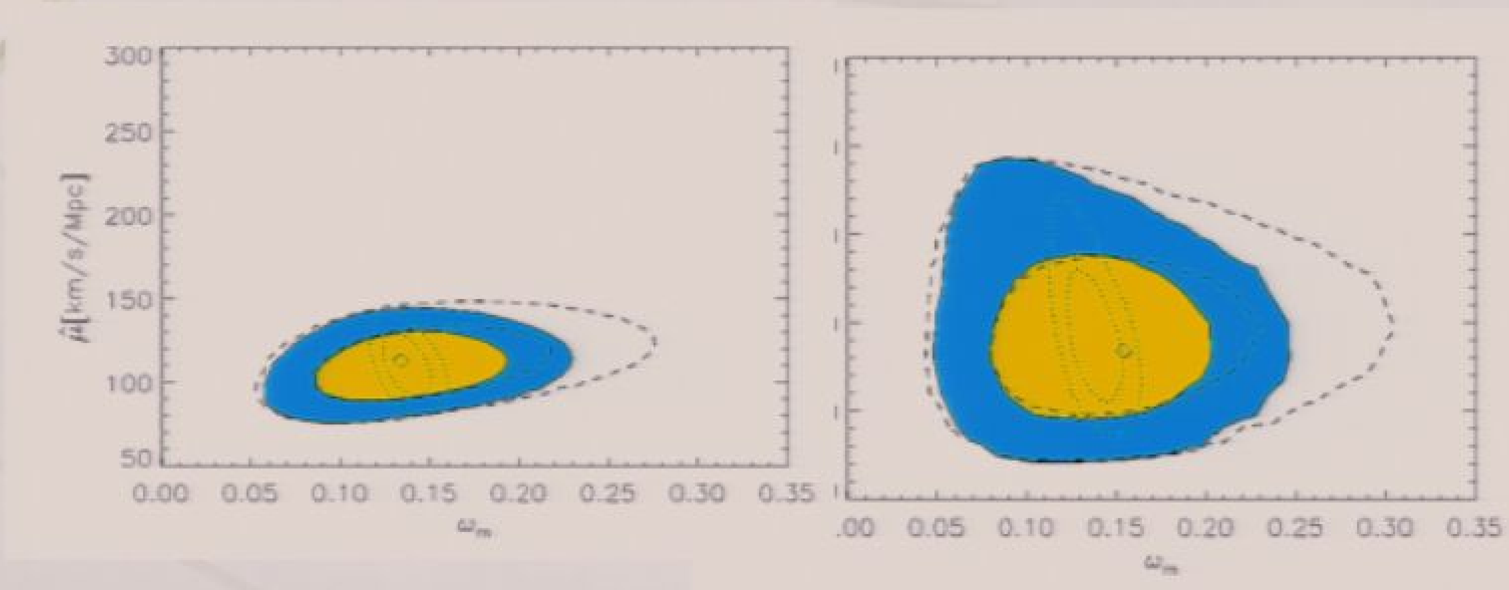
Angular Diameter Distance to Last Scattering



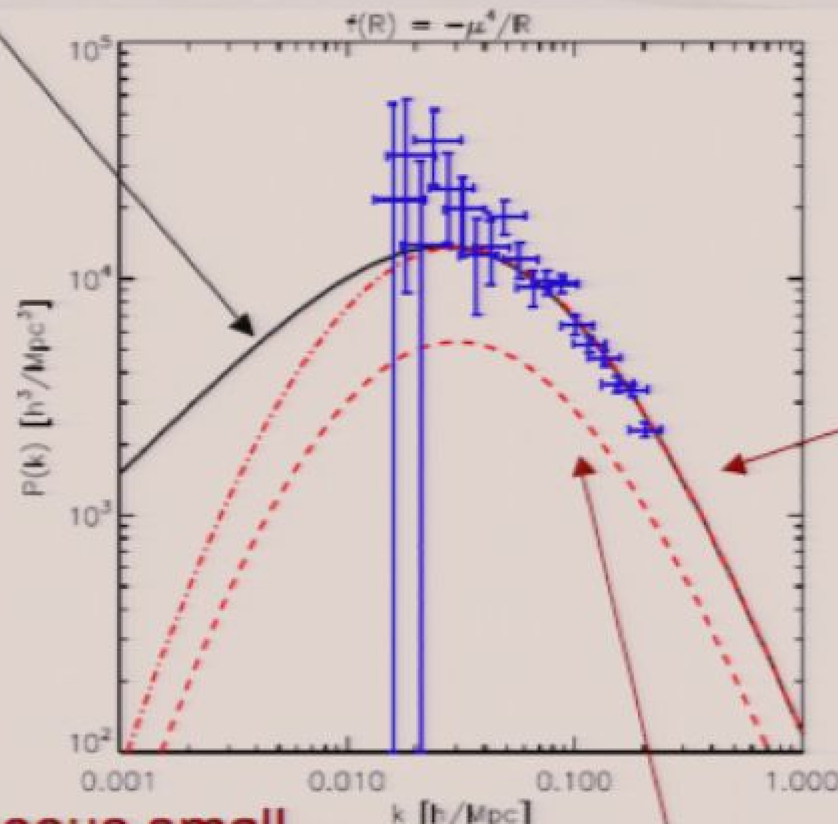
For the brave:
Angular diameter distance to last last scattering with WMAP data - might as well be bogus !

Including Perturbations in 1/R modes

ΛCDM

Bean et al. 2006

SDSS data



1/R shifted to fit small scales

no simultaneous small scale agreement and CMB

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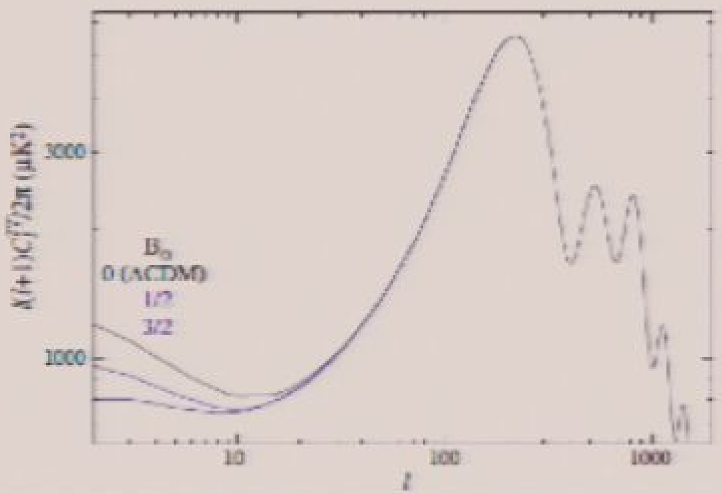
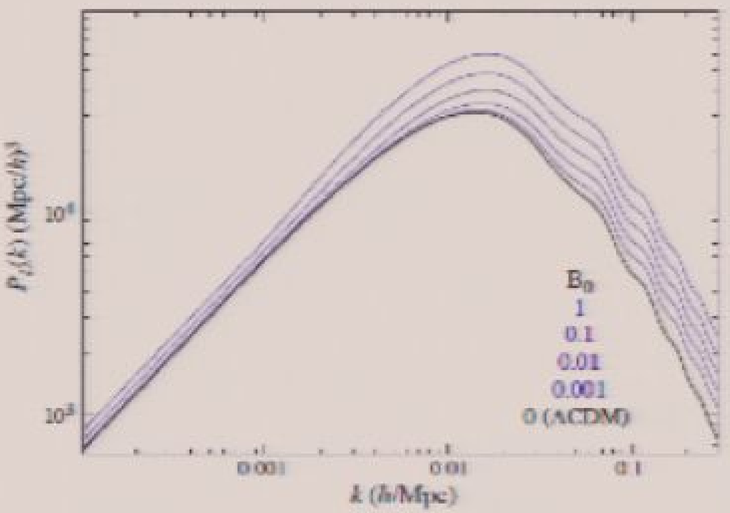
But also, ...

Song, Hu, Sawicki 2006





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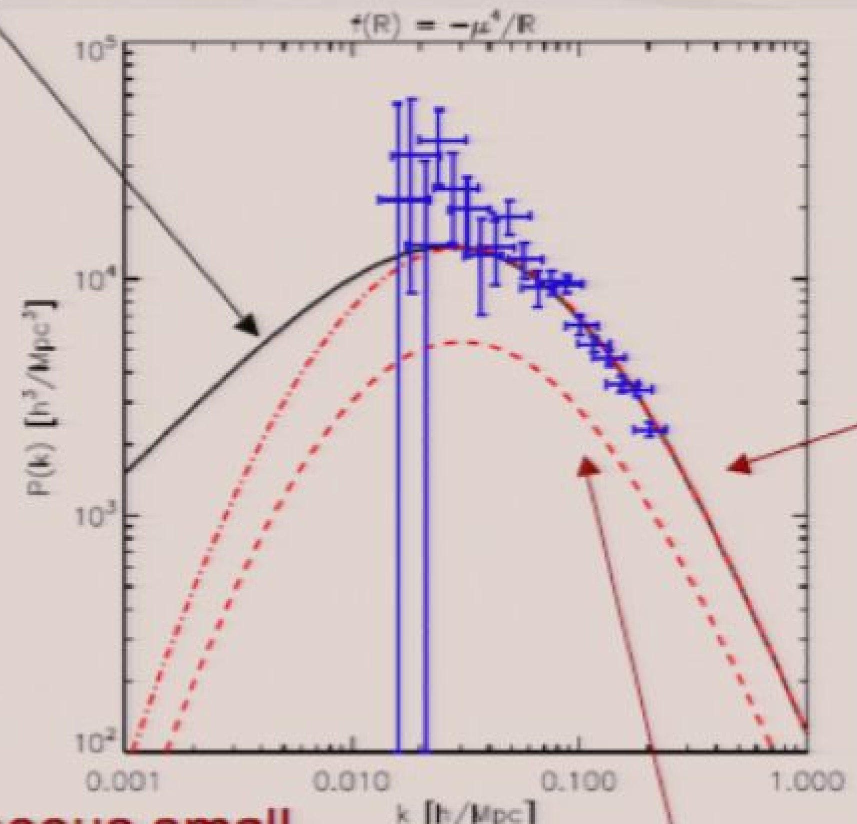

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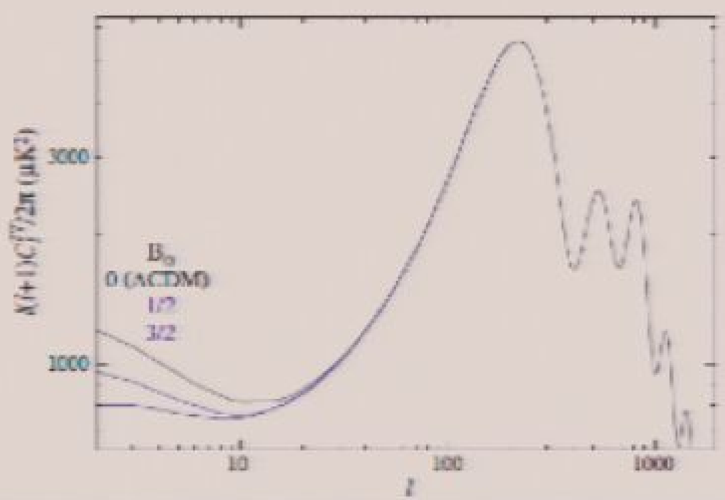

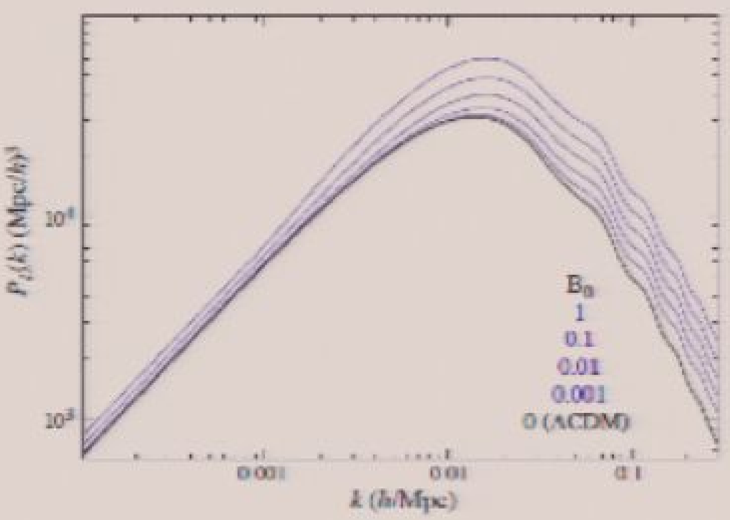

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Conclusions

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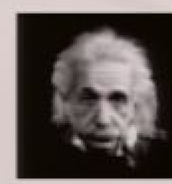

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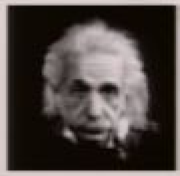

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