

Title: The deformed conifold, baryonic branch, and cosmological applications.

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Abstract: TBA

# The deformed conifold, baryonic branch, and IR dynamics

Anatoly Dymarsky

Princeton University

Perimeter Institute, April 17, 2007

# Outline

- Warped deformed conifold (KS) solution and dual gauge theory
- Family of solutions dual to baryonic branch of gauge theory
- Non-perturbative objects on baryonic branch
- Spectrum of low-lying glueballs via SUGRA fluctuations

# Warped deformed conifold (KS) solution and dual gauge theory

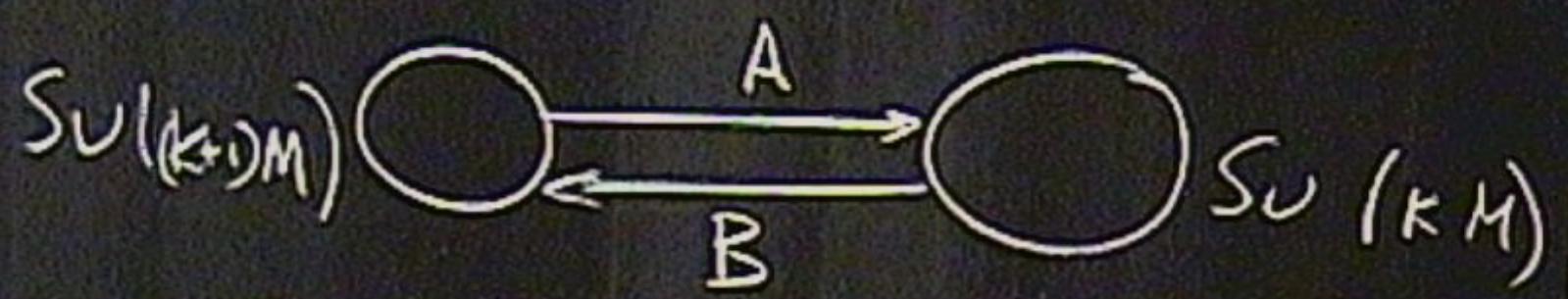
# Field theory and deformed conifold

ST on warped deformed conifold is dual to  
N=1  $SU(N+M) \times SU(N)$  SUSY gauge theory

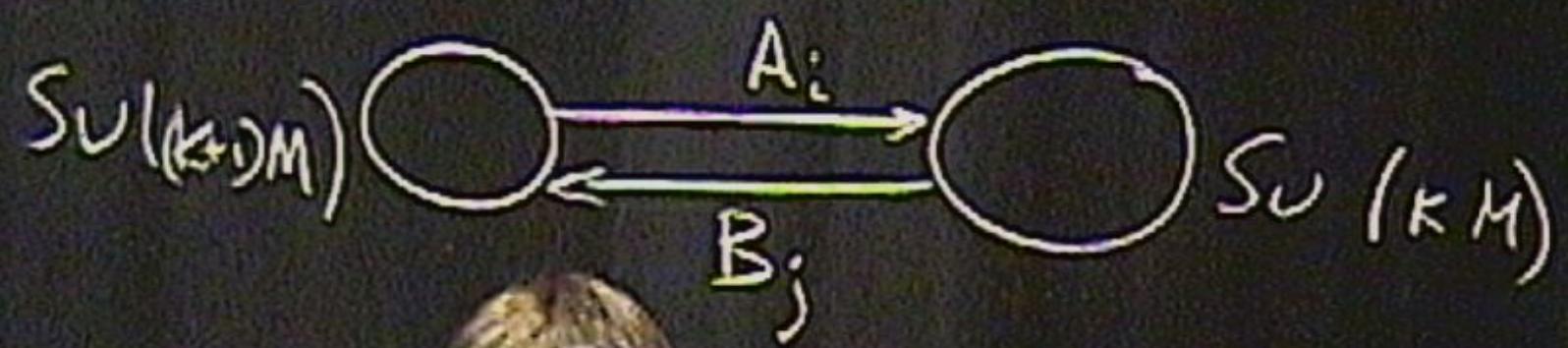
- For  $N \sim kM$  the RG cascade turns  $k$  step into  $k-1$

$$\dots \rightarrow SU((k+1)M) \times SU(kM) \rightarrow SU(kM) \times SU((k-1)M) \rightarrow \dots$$
$$\dots \rightarrow SU(2M) \times SU(M)$$

- Bi-fundamental fields  $A_{i\beta}^{\alpha}$  and  $B_{j\alpha}^{\beta}$   
 $\alpha = 1..(k+1)M$   
 $\beta = 1..kM$   
 $i, j = 1..2$
- $Z_2$  symmetry followed by c.c. exchanges  
 $A_{i\beta}^{\alpha}$  and  $B_{j\alpha}^{\beta}$
- $SU(2) \times SU(2)$  symmetry acts on  $i, j$  indexes



$i, j = 1, 2$



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# Field theory and deformed conifold

Low-energy field content at the last of the cascade - mesons and baryons

- Matrix of mesons
- Baryonic fields

$$M_{ij\beta_2}^{\beta_1} = A_{i\beta_2}^{\alpha} B_{j\alpha}^{\beta_1}$$

$$A = \varepsilon_{i_1 \dots i_{2M}} A_{11}^{i_1} \dots A_{1M}^{i_M} A_{21}^{i_{M+1}} \dots A_{2M}^{i_{2M}}$$

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# Emergent deformed conifold

## Moduli space

- Superpotential

$$W = \lambda \text{Tr} \det M + X(Det \det M + AB + \Lambda_{2M}^{4M})$$

- Mesonic branch

$$X=A=B=0$$

$$\det \det M = -\Lambda_{2M}^{4M}$$

$$\Leftrightarrow$$

$$\sum z_i^2 = -\frac{\varepsilon^2}{2}$$

$$(Det M)_{ij} \sim \begin{pmatrix} z_1 + iz_2 & z_3 - iz_4 \\ z_3 + iz_4 & -z_1 + iz_2 \end{pmatrix}$$

- Baryonic branch

$$M=0$$

$$AB = -\Lambda_{2M}^{4M}$$

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$Z_2$  symmetry  
inverts

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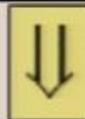
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# Deformed conifold solution

Solution by Klebanov and Strassler

$$ds^2 = h^{-\frac{1}{2}}(Y)g_{\mu\nu}dx^\mu dx^\nu + h^{\frac{1}{2}}(Y)g_{ij}dY^i dY^j$$

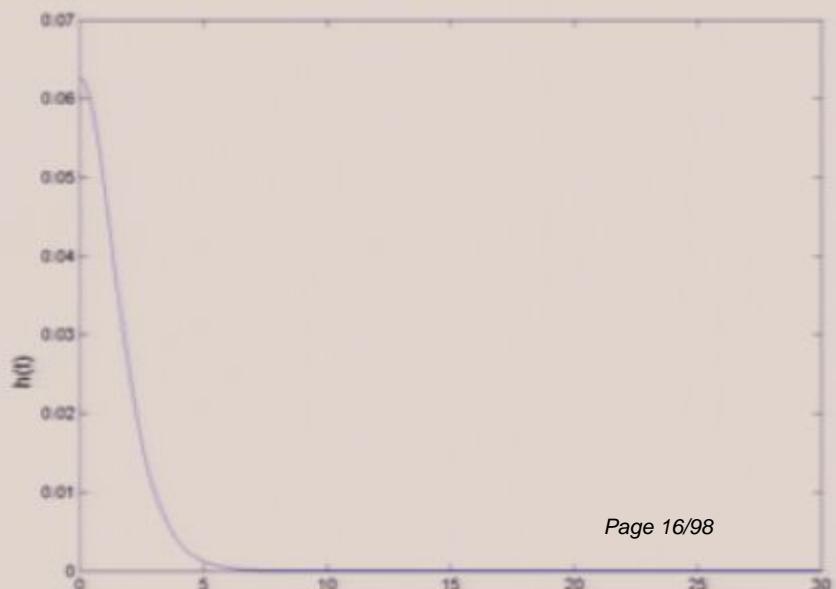


$$ds^2 = h^{-\frac{1}{2}}(t)dx^2 + h^{\frac{1}{2}}(t)dS_{DC}^2$$

- $dS_{DC}^2$  is a Ricci-flat metric on the deformed conifold

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- warped factor  $h(t)$  is finite at  $t = 0$  - confinement



# Geometry of KS solution

$$\frac{1}{4\pi^2 \alpha'} \int_{S^3} F_3 = M$$

$$\frac{1}{(4\pi^2 \alpha')^2} \int_{T^{1,1}} F_5 = N$$

$S^3$

IR

$$ds^2 = h_0^{-\frac{1}{2}} dx^2 + R_0^2 ds_{S^3}^2$$

$$R_0 \sim \varepsilon$$

KS solution

$S^3$

$T^{1,1}$

$S^2$

UV

$$ds^2 = h^{\frac{1}{2}}(r) dx^2 + h^{\frac{1}{2}}(r) (dr^2 + r^2 d\Omega_{T^{1,1}}^2)$$

$$r \simeq e^{t/3}$$

Klebanov-Tseytlin, Klebanov-Strassler

$$T^I = \frac{SU(2) \times SU(2)}{U(1)}$$

$i, j = 1, 2$

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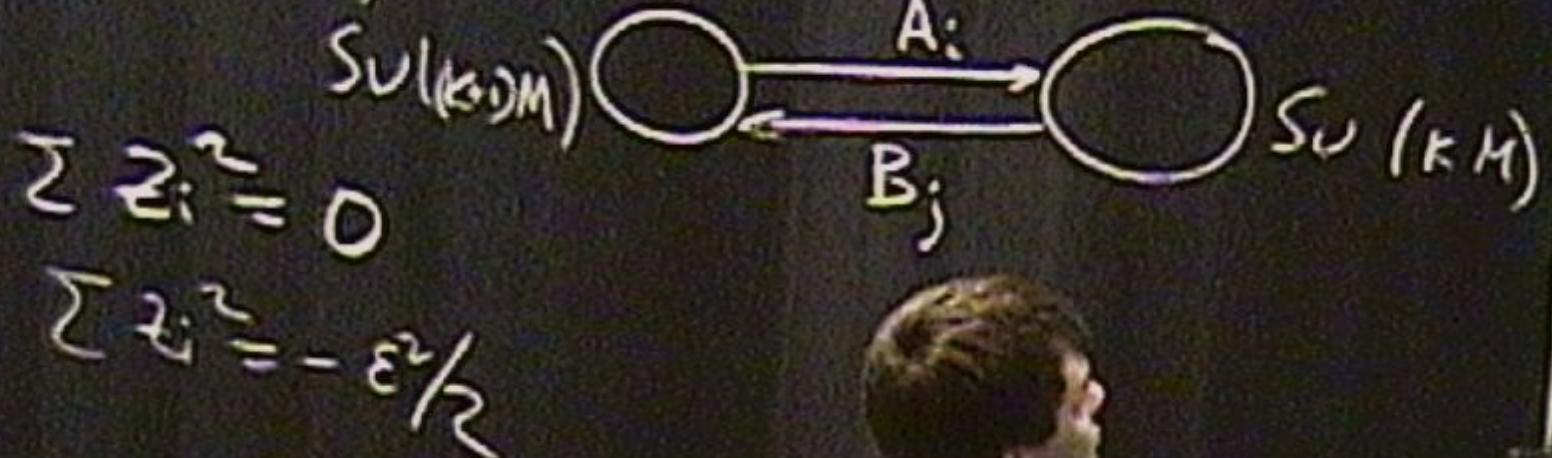
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$$\sum z_i^2 = -\varepsilon/k$$

# Properties of KS solution

- smooth everywhere, including tip
- $SU(2) \times SU(2)$  and  $Z_2$  symmetric
- imaginary self-dual  $G_3 = i *_6 G_3$  with constant dilaton  $\phi = 0$
- manifold of compactification is (conformally) CY
- nontrivial flux  $F_3$  through 3-cycle
- nonzero NSNS  $H_3$  and  $F_5$  RR fluxes
- vanishing RR scalar  $C = 0$

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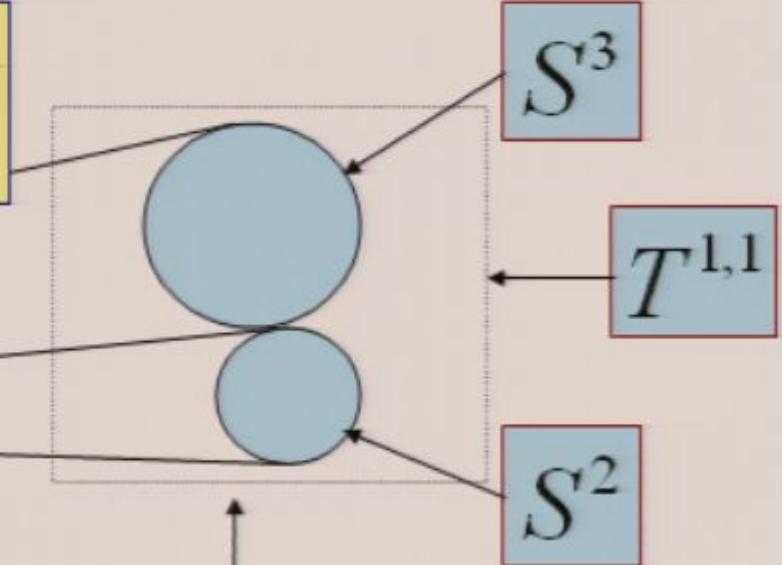
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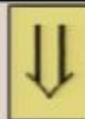
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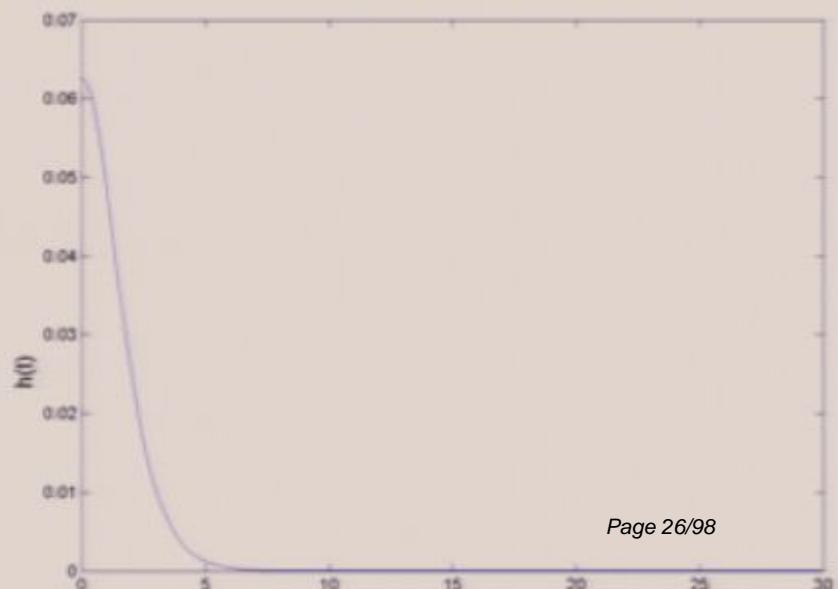


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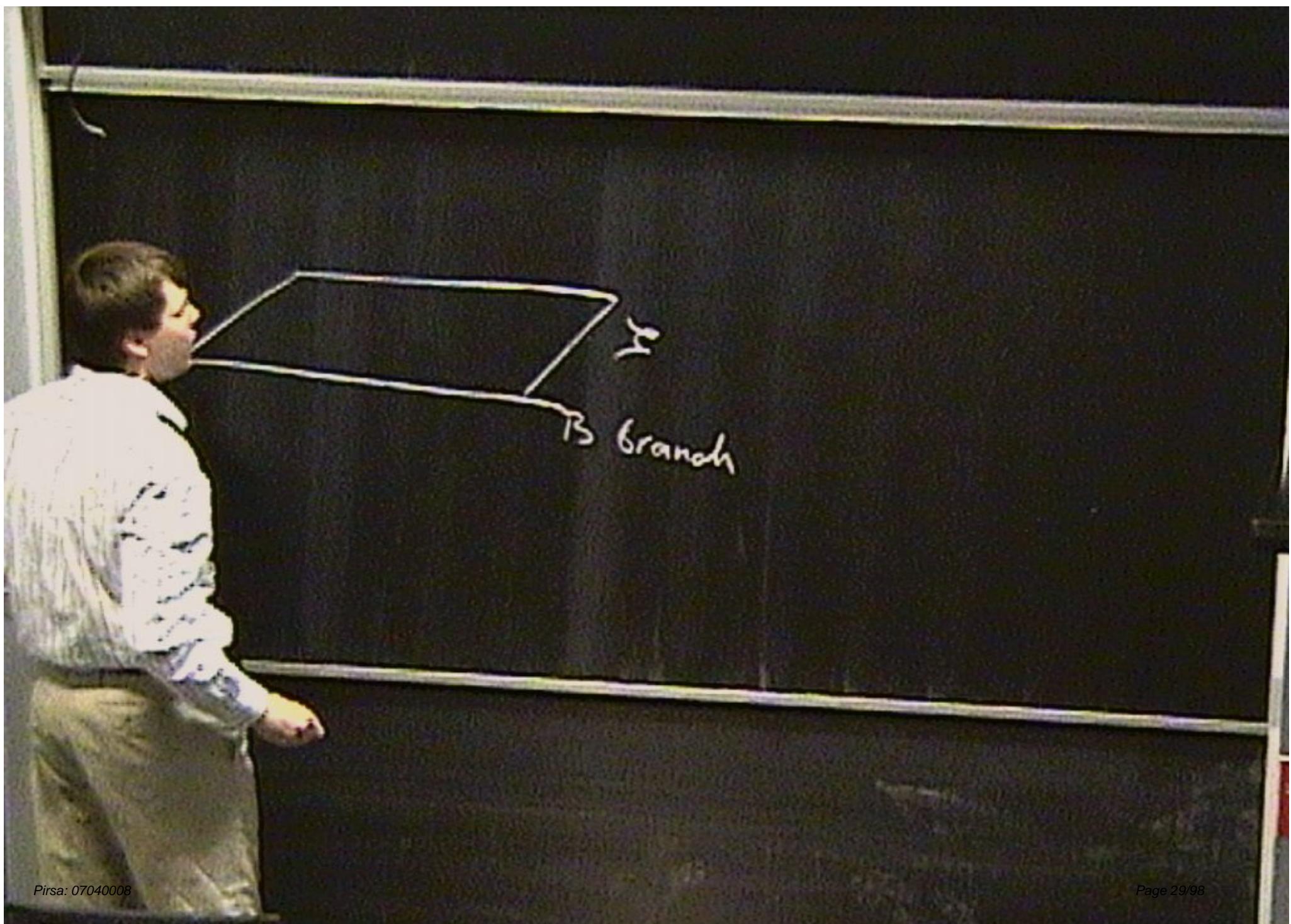
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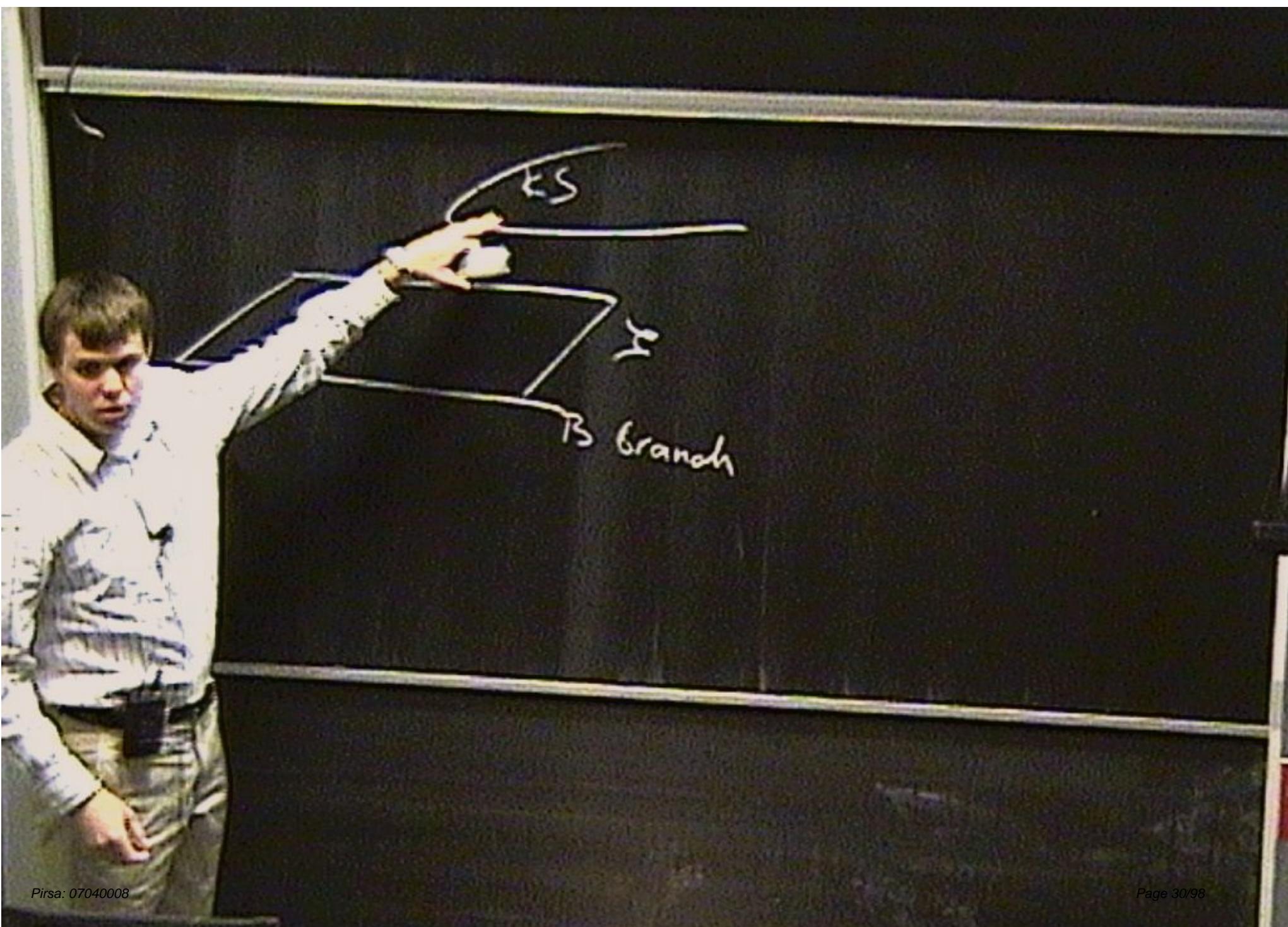
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- KS corresponds to  $\zeta=1$  because of unbroken  $Z_2$

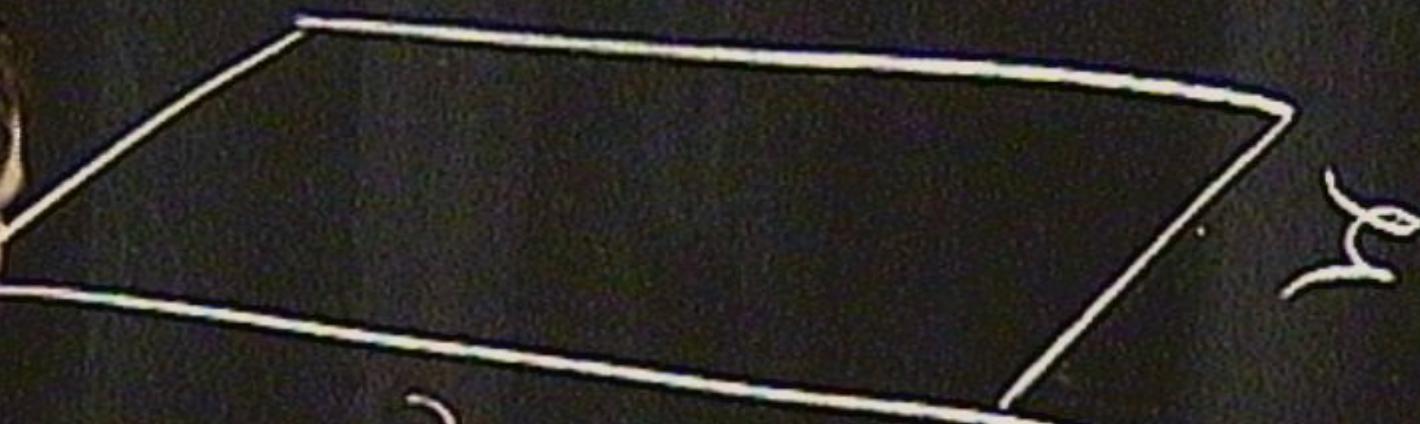
- $U(1)_B$  symmetry of gauge theory  
(not of geometry)

What is gravity dual for  $\zeta \neq 1$  ?





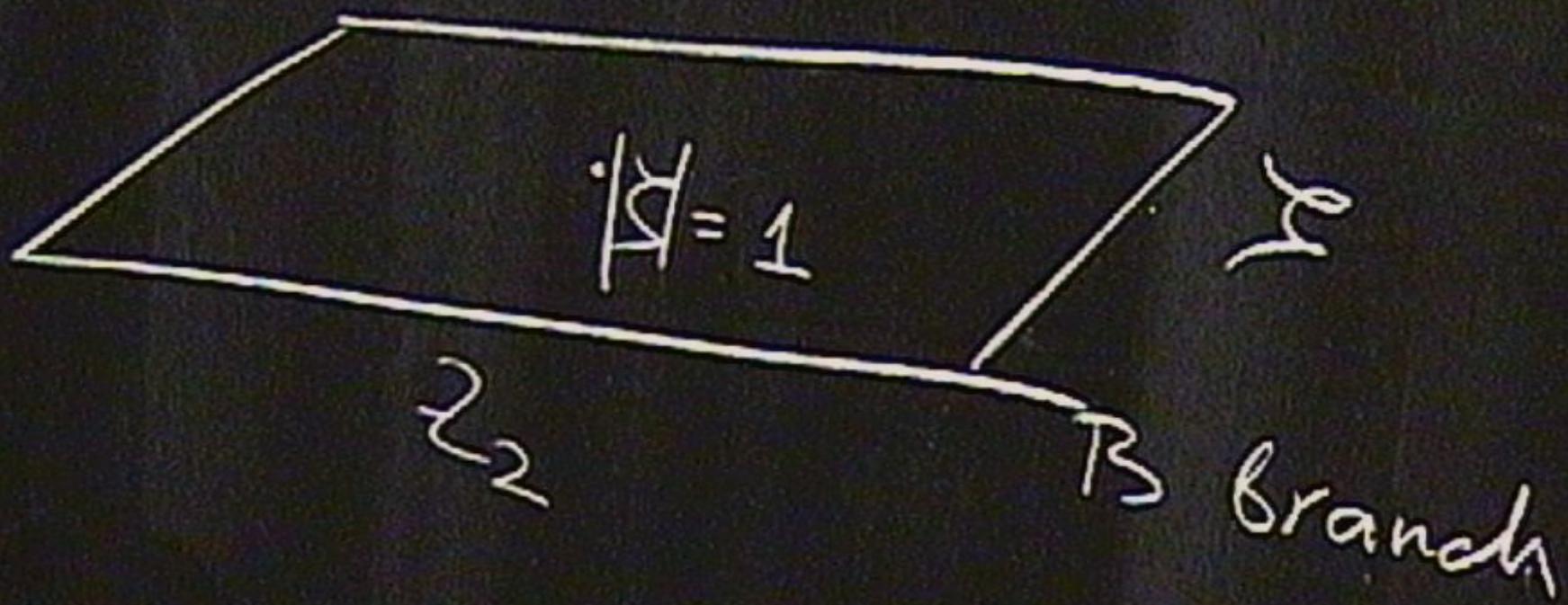
ES      Z<sub>2</sub>



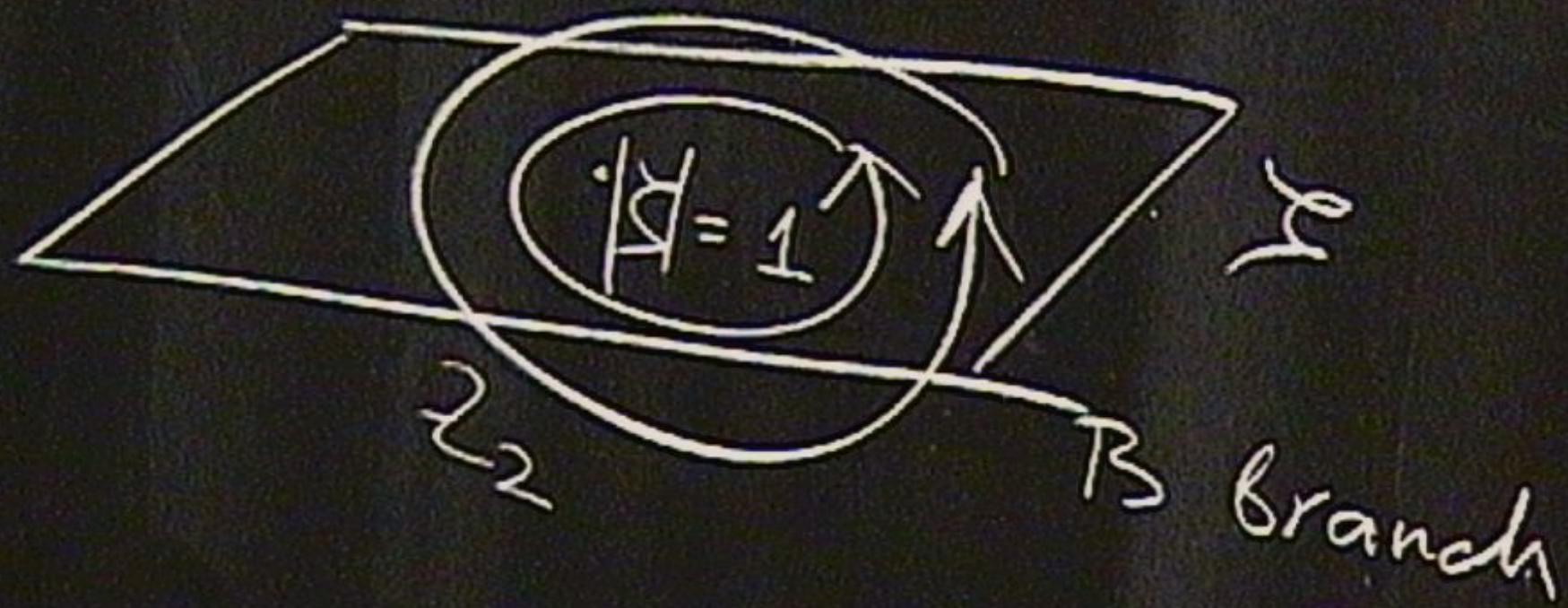
C

B branch

ES Z<sub>2</sub>



ES  $Z_2$



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# **Family of SUGRA solutions dual to baryonic branch**

Gubser, Herzog, Klebanov

Butti, Grana, Minasian, Petrini, Zaffaroni

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# Baryonic branch away from KS

Papadopoulos and Tseytlin (PT) ansatz

general  $SU(2) \times SU(2)$  symmetric ansatz for metric and fluxes

Metric:  $a, v, \phi, x, g, A(t)$

Fluxes:  $h_1, h_2, \chi, b, K(t)$

$$ds^2 = e^{2A} dx^2 + ds_6^2$$

$$ds_6^2 = e^x \left( v^{-1} (dt^2 + g_5^2) + (e^g + a^2 e^{-g})(e_1^2 + e_2^2) + e^{-g} (\varepsilon_1^2 + \varepsilon_2^2 - 2a(e_1 \varepsilon_1 + e_2 \varepsilon_2)) \right)$$

$e_1, e_2$

$\varepsilon_1, \varepsilon_2, \varepsilon_3(g_5)$

$S^2$

$S^3$

invariant forms on

# Baryonic branch away from KS

Butti, Grana, Minasian, Petrini, Zaffaroni employed **method of  $SU(3)$  structure** and found the solution

Method of  $SU(3)$  structure:  
consider Killing equation instead of E.O.M.

$$\delta\lambda = \partial_A \phi \Gamma^A \Psi^* + \frac{1}{3!} G_{3ABC} \Gamma^{ABC} \Psi = 0$$

$$\delta\Psi_A = D_A \Psi + "F_{5A} \Psi + (H_3 + F_3)_A \Psi^* " = 0$$

Killing spinor  $\Psi = \alpha \Psi_0 + \beta^* \Psi_0^*$  is expressed through  
a pure spinor  $\Psi_0$  invariant under action of  $SU(3)$

## BGMPZ solution

$a(t, U)$

$v(t, U)$

Baryonic branch parameter  $U$

determines boundary conditions

$$\dot{a} = \dot{a}(a, v, t) \quad \dot{v} = \dot{v}(a, v, t)$$

Analytic solution is not known unless for KS ( $U=0$ )

All other fields are constructed through  $a(t), v(t)$

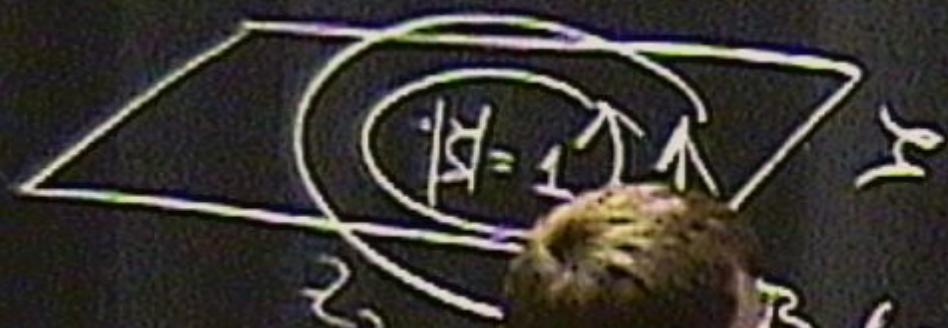
$$\dot{\phi} = \dot{\phi}(a, v, t) \Rightarrow$$

$$x, g, \dot{A}, h_1, h_2, \dot{\chi}, b, K = f(a, v, \phi, t)$$

Ambiguities in  $\phi \rightarrow \phi + \text{const}$  and  $A \rightarrow A + \text{const}$   
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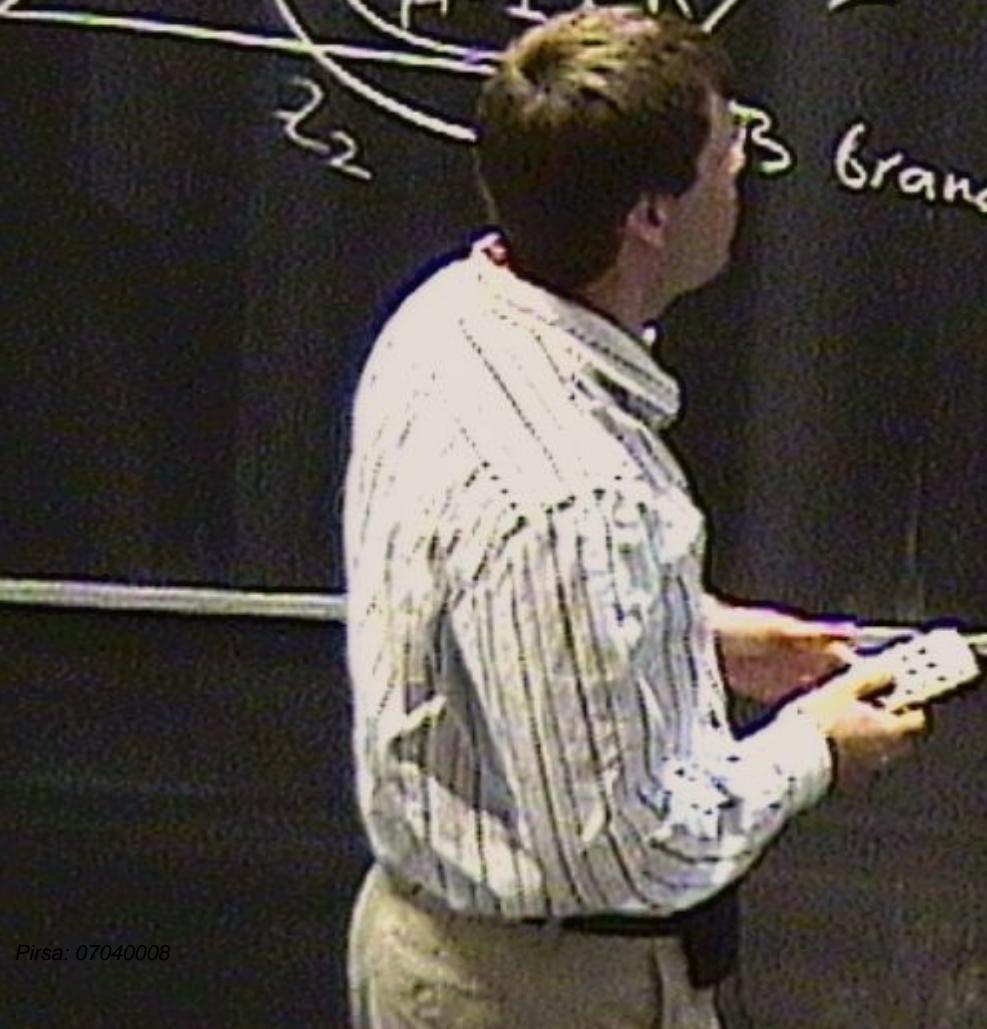
(AD, Klebanov, Seiberg)

KS Z<sub>2</sub>

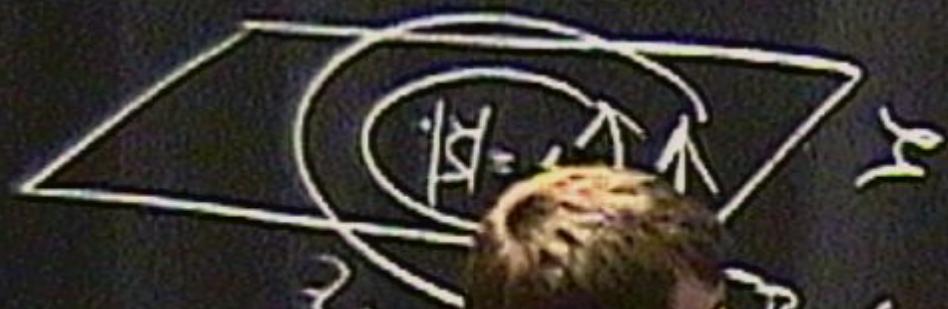


$\beta$  branch

$$ds_{10}^2 = e^{2A} dx^2 + \dots$$

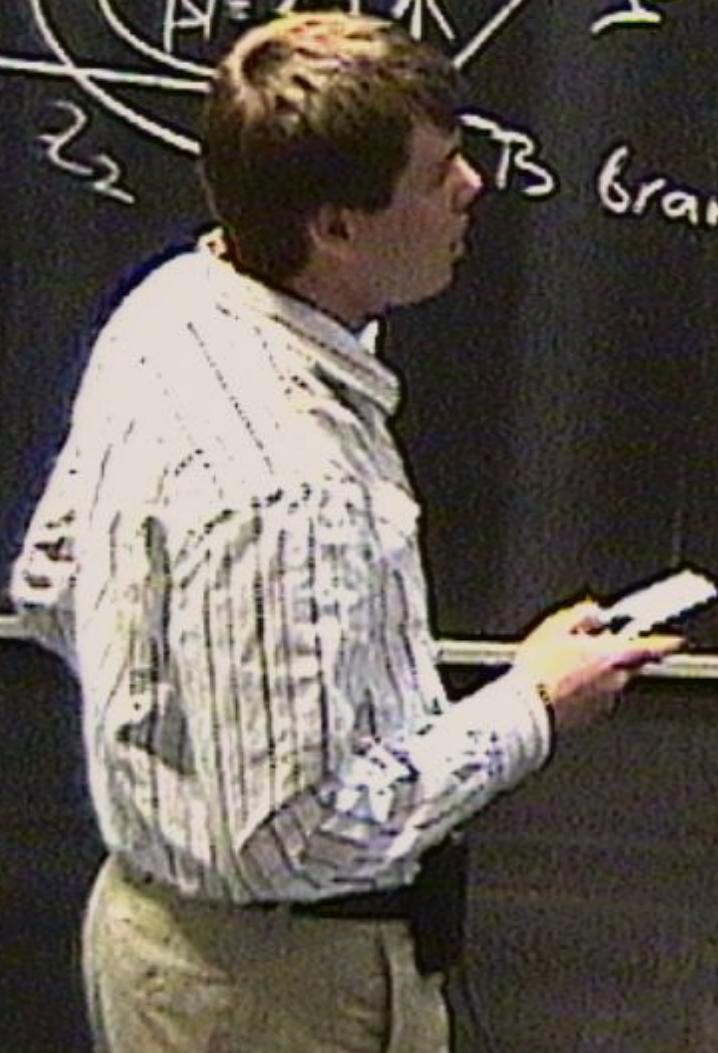


CS  $Z_2$

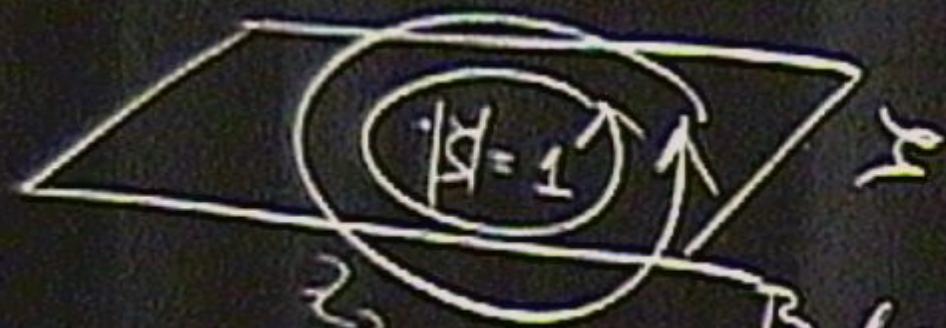


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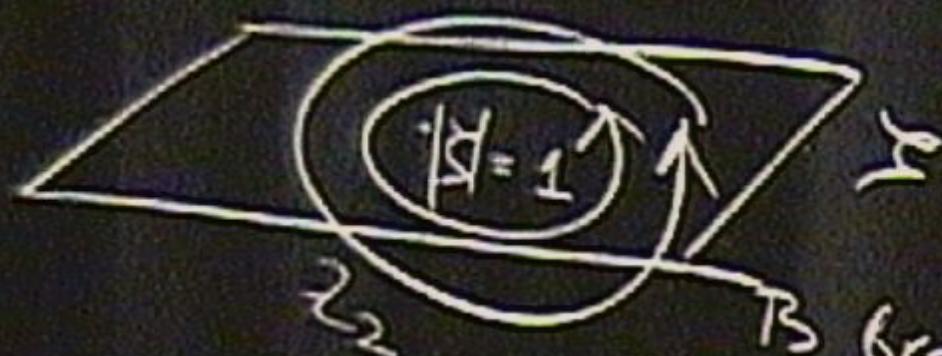
KS     $Z_2$



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CS  $\mathbb{Z}_2$



$\gamma_1$   
 $\gamma_2$   
branch

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## BGMPZ solution

UV asymptotic should be universal for all  $U$

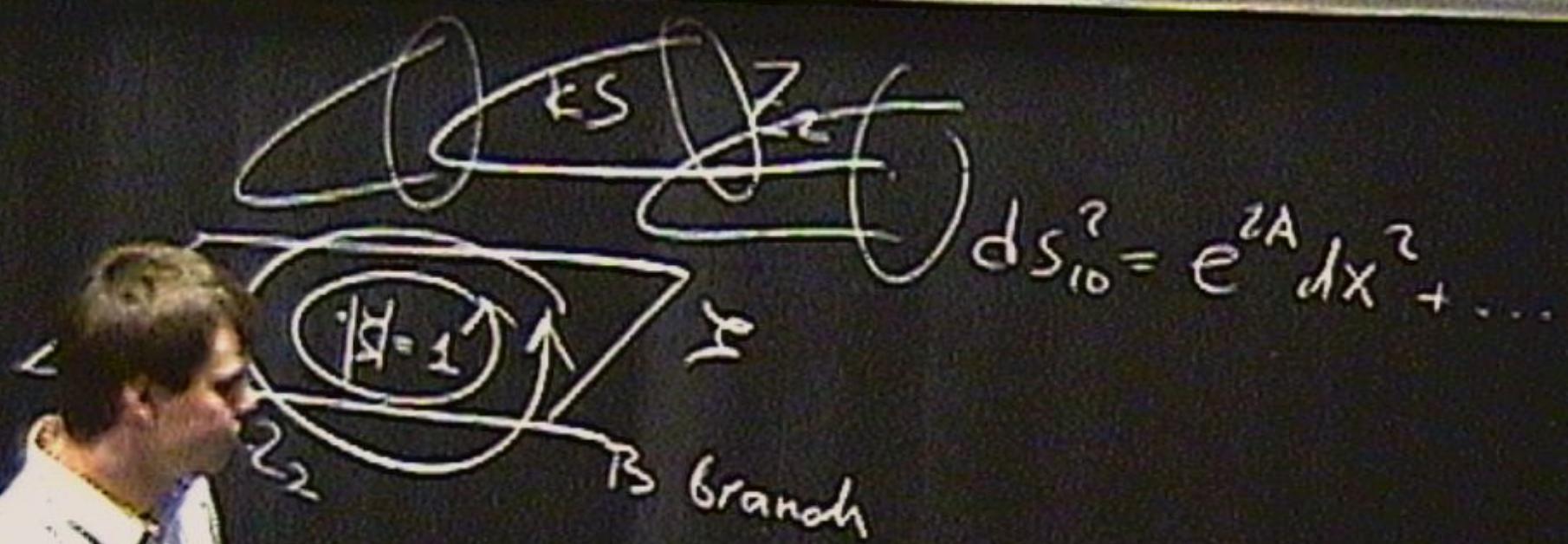
$$\phi(t = \infty, U) = 0$$

Equation for  $A$  can be integrated explicitly (DKS)

$$e^{4A} = \frac{U^2}{e^{-2\phi} - 1}$$

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(AD, Klebanov, Tachikawa; work in progress)

$$e^{4\phi} = \frac{-64v(a \cosh(t) + 1)^3 \sinh(t)^5}{3U^3(-1 - a^2 - 2a \cosh(t))^{\frac{3}{2}}(t \cosh(t) - \sinh(t))^3}$$



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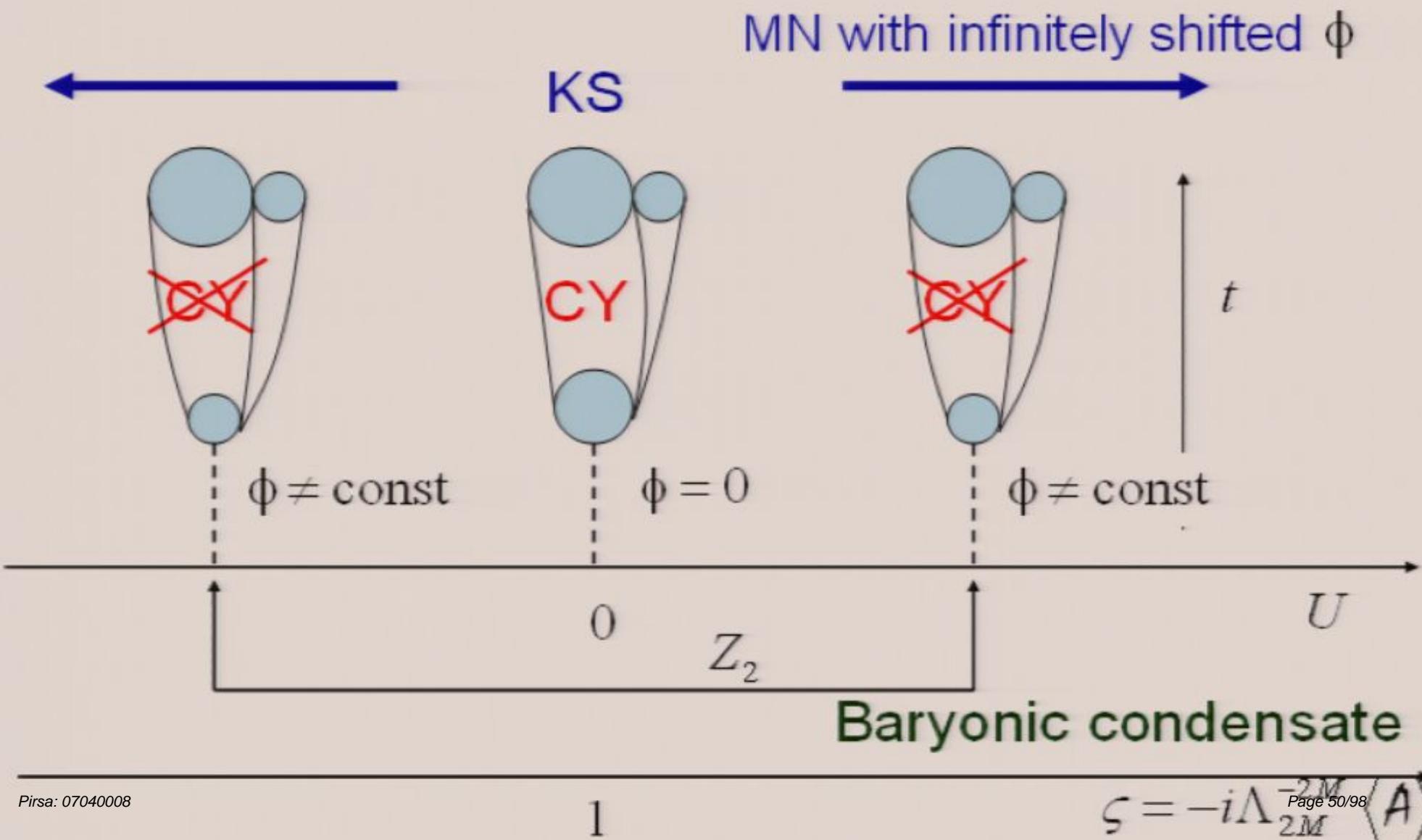
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# Baryonic branch in the vicinity of KS

Gubser, Herzog, Klebanov

- $U(1)_B$  symmetry is spontaneously broken.  
 $\varphi$  is a massless goldstone (pseudoscalar)
- $\varphi$  can not condense  $\delta F_3 = {}^*_4 d\varphi + \dots$
- saxion S (superpartner of  $\varphi$ ) is responsible for motion along baryonic branch  $\delta\zeta$
- explicit metric is known for zero momentum  $\delta\zeta = \text{const}$   
the  $Z_2$  symmetry is broken

# BGMPZ (baryonic branch) solution



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- saxion S (superpartner of  $\varphi$ ) is responsible for motion along baryonic branch  $\delta\zeta$
- explicit metric is known for zero momentum  $\delta\zeta = \text{const}$   
the  $Z_2$  symmetry is broken

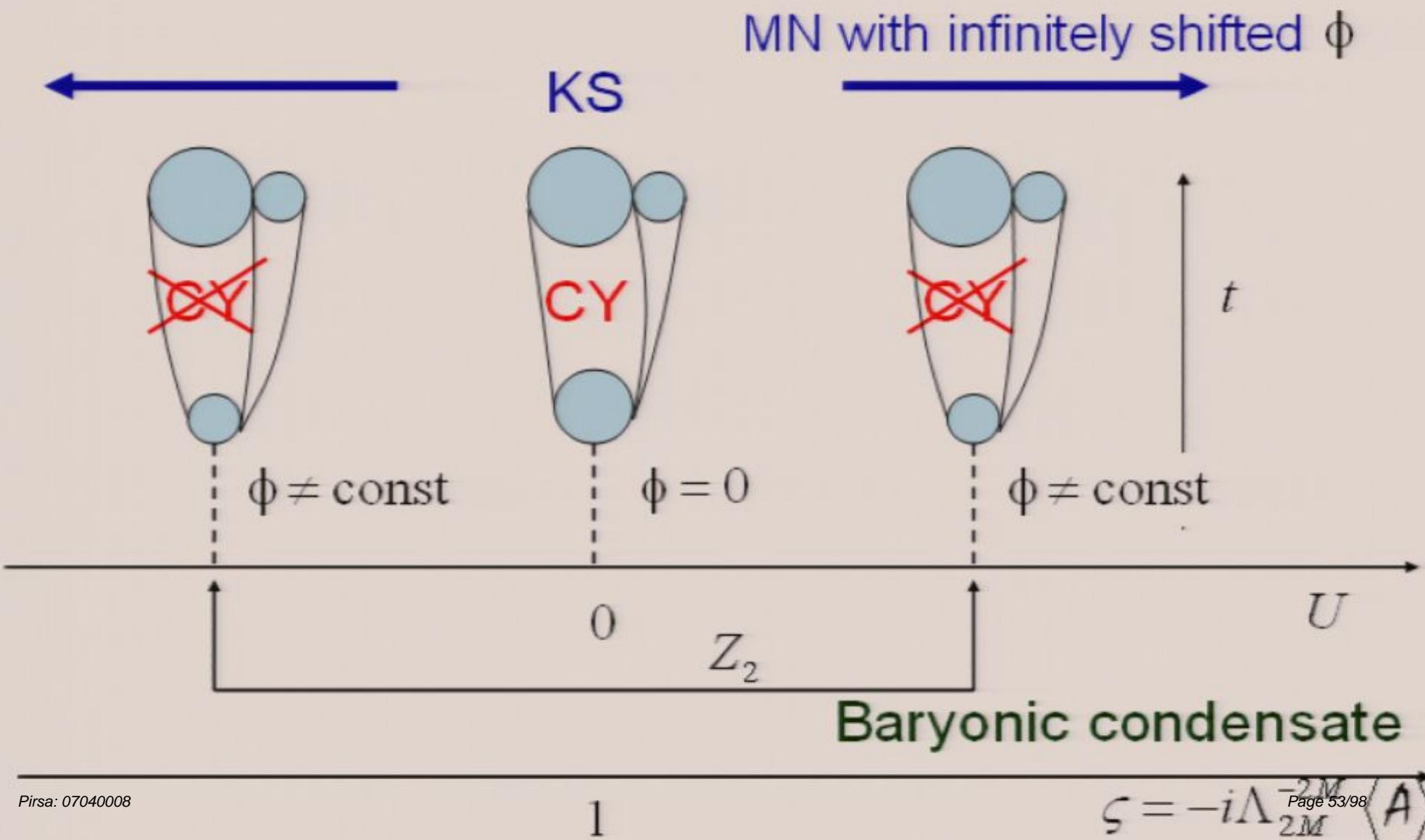
# Compactification effects

Gubser, Herzog, Klebanov

- Compactification lifts the branch and fixes the value of baryonic condensate  $U$
- $U(1)_B$  is gauged and axion is “eaten” by vector boson
- Saxon also acquires mass, which is parametrically small because of large CY volume

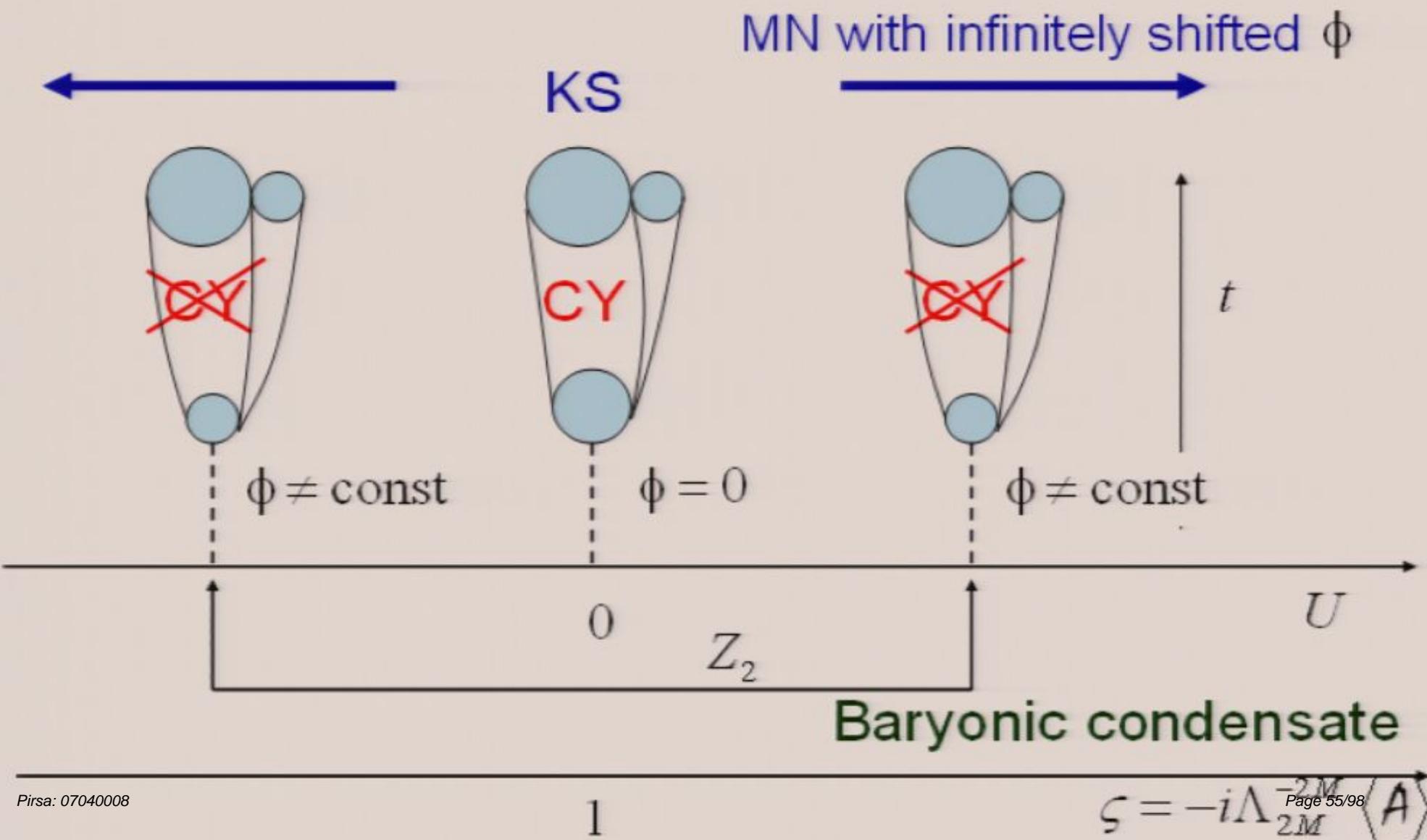
This is a promising mechanism for cosmological models

# BGMPZ (baryonic branch) solution



# Branes on the conifold

# BGMPZ (baryonic branch) solution



# Baryonic condensate

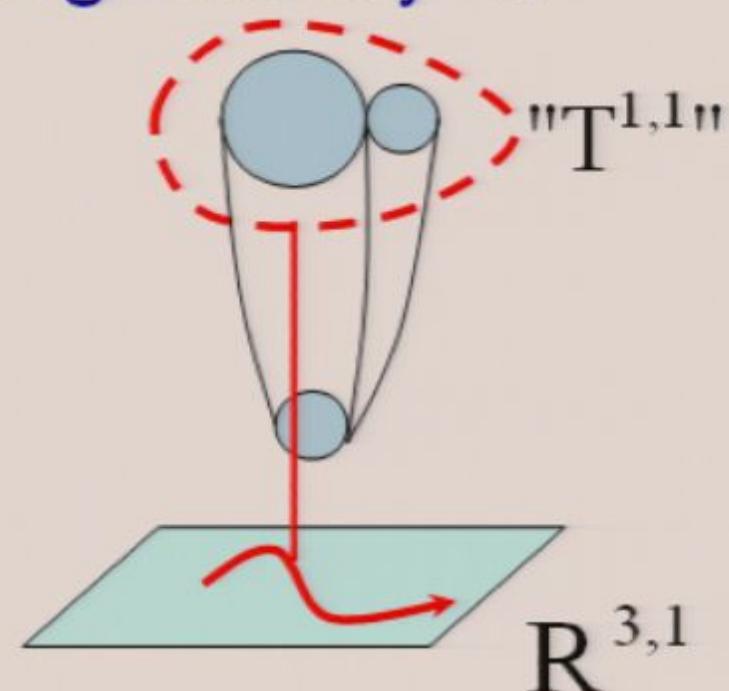
Benna, AD, Klebanov

Objective: to measure baryonic condensate  $\langle A \rangle$   
and express it in terms of dual geometry  $U$

Baryonic operator  $A(x)$

is uncharged under

$SU(2) \times SU(2)$  and  
is dual to D5 wrapping  
base of the cone



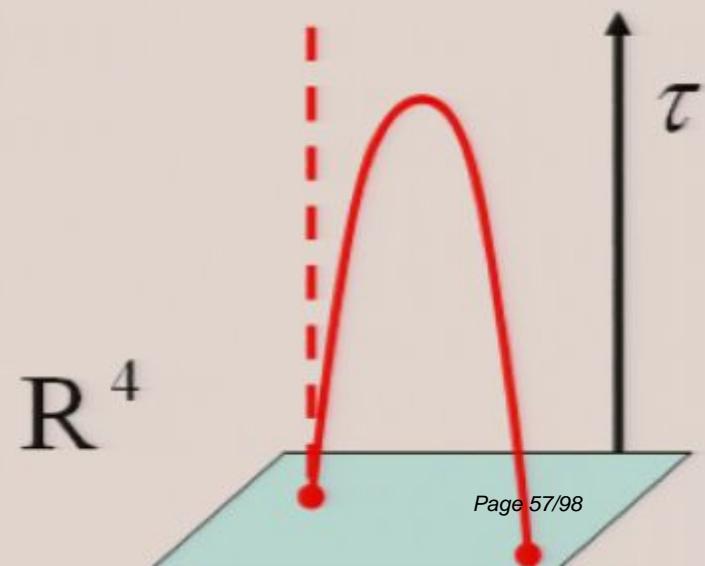
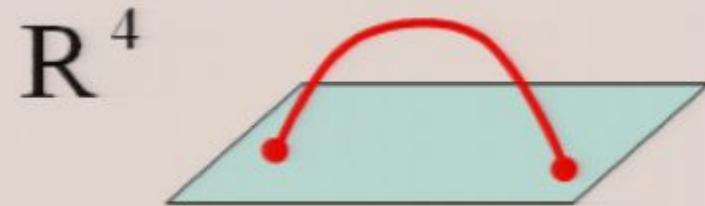
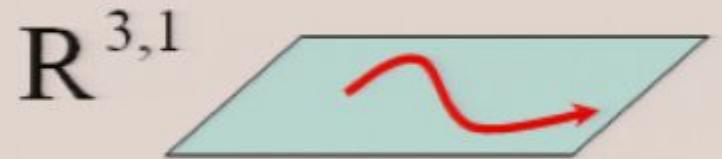
Measuring condensate requires  
rotation to Euclidean space

# Baryonic condensate

Example: measuring two-point functions in AdS

- local operator  $O(x^\mu)$
- propagator  $\langle O(x)O(x') \rangle$  is given by classical trajectory stretched into AdS
- expectation of condensate is measured by taking  $|x-x'| \rightarrow \infty$  or stretching string further “up” to infinity (dashed line) - rotation to Euclidean space

$$x^0 \rightarrow \tau$$



# Baryonic branch away from KS

Papadopoulos and Tseytlin (PT) ansatz

general  $SU(2) \times SU(2)$  symmetric ansatz for metric and fluxes

Metric:  $a, v, \phi, x, g, A(t)$

Fluxes:  $h_1, h_2, \chi, b, K(t)$

$$ds^2 = e^{2A} dx^2 + ds_6^2$$

$$ds_6^2 = e^x \left( v^{-1} (dt^2 + g_5^2) + (e^g + a^2 e^{-g})(e_1^2 + e_2^2) + e^{-g} (\varepsilon_1^2 + \varepsilon_2^2 - 2a(e_1 \varepsilon_1 + e_2 \varepsilon_2)) \right)$$

$e_1, e_2$

$\varepsilon_1, \varepsilon_2, \varepsilon_3(g_5)$

$S^2$

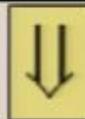
$S^3$

invariant forms on

# Deformed conifold solution

Solution by Klebanov and Strassler

$$ds^2 = h^{-\frac{1}{2}}(Y)g_{\mu\nu}dx^\mu dx^\nu + h^{\frac{1}{2}}(Y)g_{ij}dY^i dY^j$$

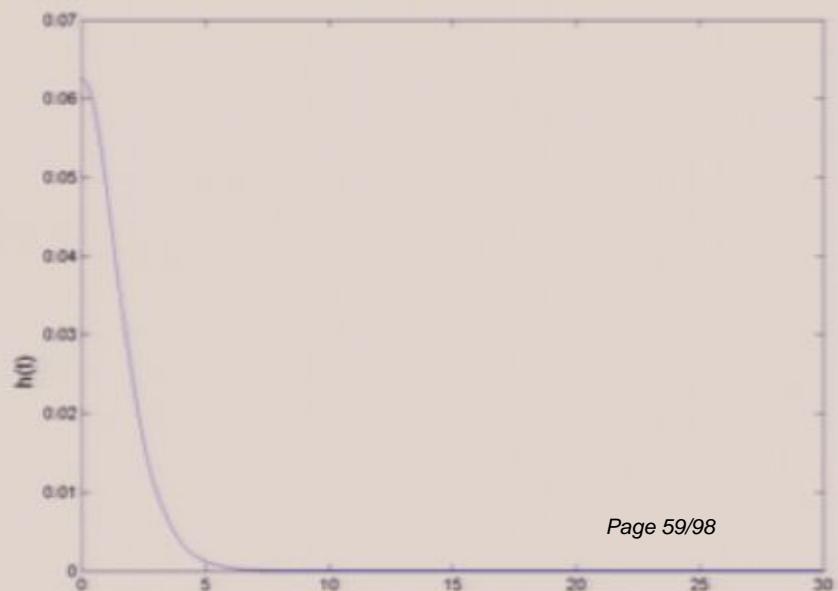


$$ds^2 = h^{-\frac{1}{2}}(t)dx^2 + h^{\frac{1}{2}}(t)dS_{DC}^2$$

- $dS_{DC}^2$  is a Ricci-flat metric on the deformed conifold

$$\sum z_i^2 = -\frac{\varepsilon^2}{2}$$

- warped factor  $h(t)$  is finite at  $t = 0$  - confinement



# Field theory and deformed conifold

Low-energy field content at the last of the cascade - mesons and baryons

- Matrix of mesons
- Baryonic fields

$$M_{ij\beta_2}^{\beta_1} = A_{i\beta_2}^{\alpha} B_{j\alpha}^{\beta_1}$$

$$A = \varepsilon_{i_1 \dots i_{2M}} A_{11}^{i_1} \dots A_{1M}^{i_M} A_{21}^{i_{M+1}} \dots A_{2M}^{i_{2M}}$$

$$B = \varepsilon^{i_1 \dots i_{2M}} B_{1i_1}^1 \dots B_{1i_M}^M B_{2i_{M+1}}^1 \dots B_{2i_{2M}}^M$$

# BGMPZ solution

$a(t, U)$

$v(t, U)$

Baryonic branch parameter  $U$

determines boundary conditions

$$\dot{a} = \dot{a}(a, v, t) \quad \dot{v} = \dot{v}(a, v, t)$$

Analytic solution is not known unless for KS ( $U=0$ )

All other fields are constructed through  $a(t), v(t)$

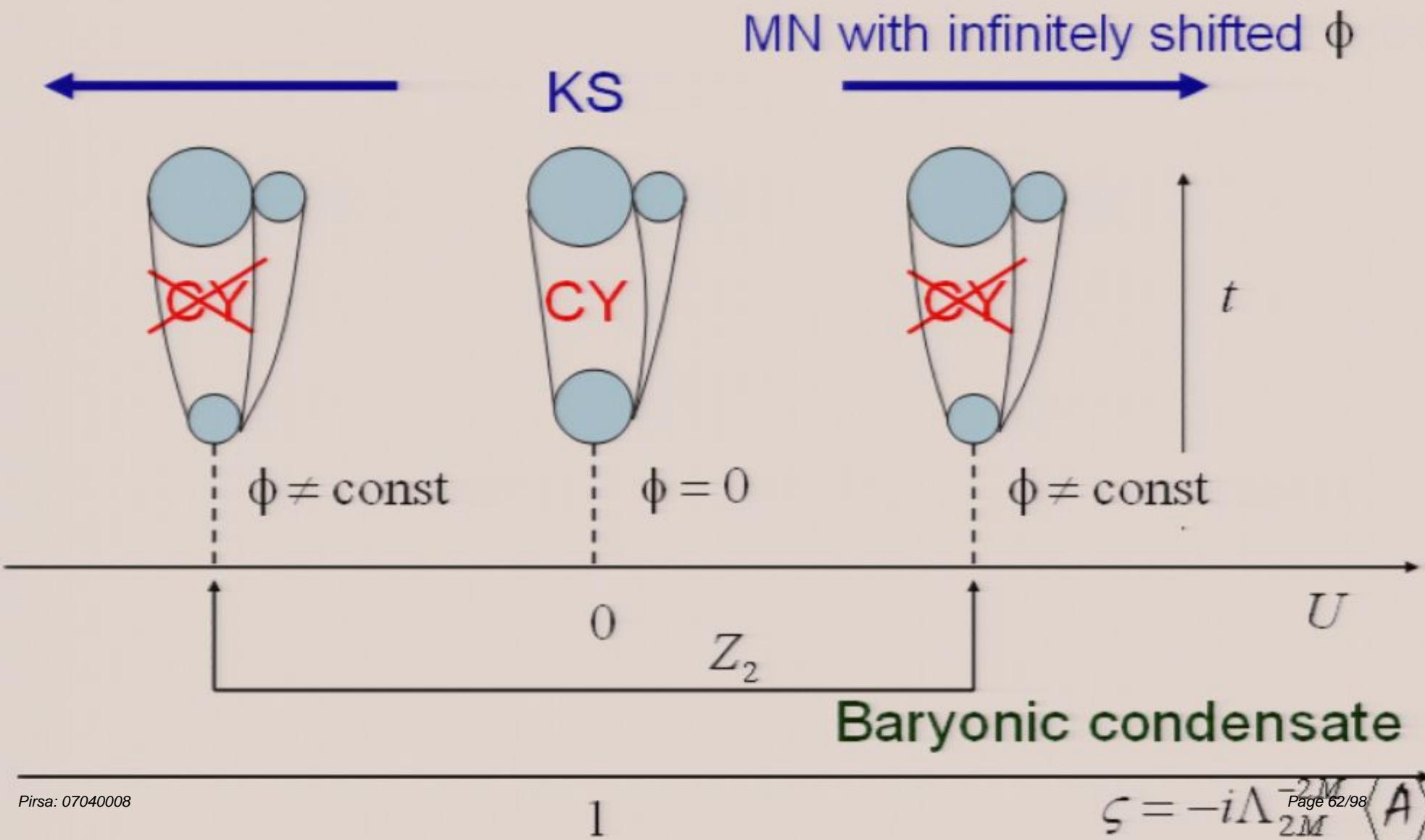
$$\dot{\phi} = \dot{\phi}(a, v, t) \Rightarrow$$

$$x, g, \dot{A}, h_1, h_2, \dot{\chi}, b, K = f(a, v, \phi, t)$$

Ambiguities in  $\phi \rightarrow \phi + \text{const}$  and  $A \rightarrow A + \text{const}$   
must be fixed by boundary conditions

(AD, Klebanov, Seiberg)

# BGMPZ (baryonic branch) solution



# Baryonic condensate

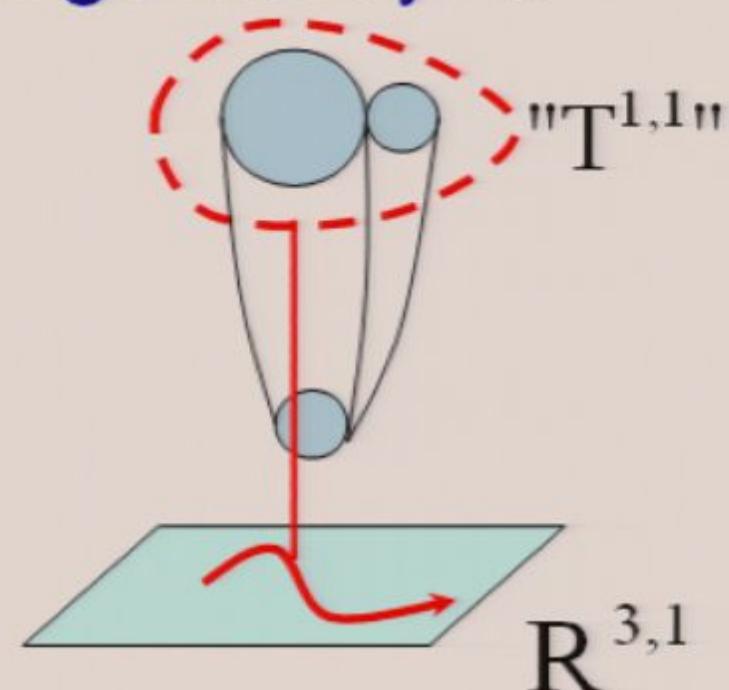
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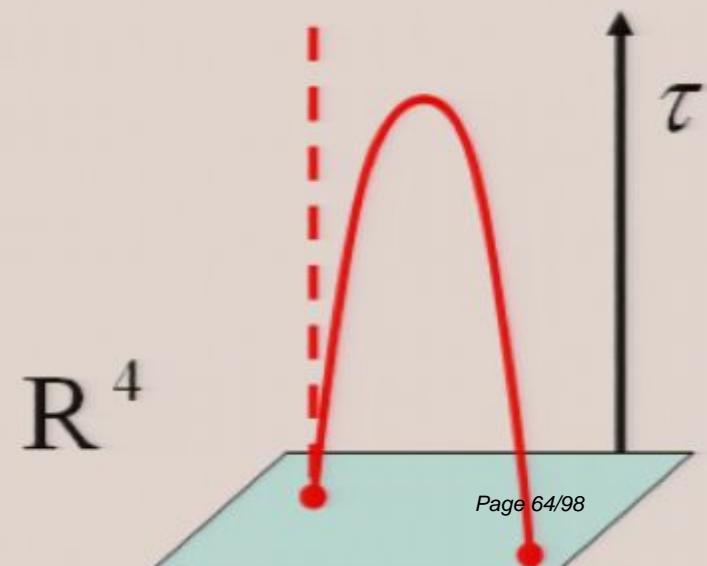
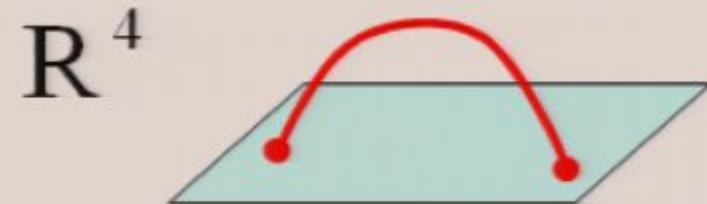
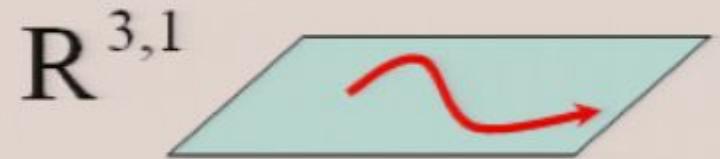
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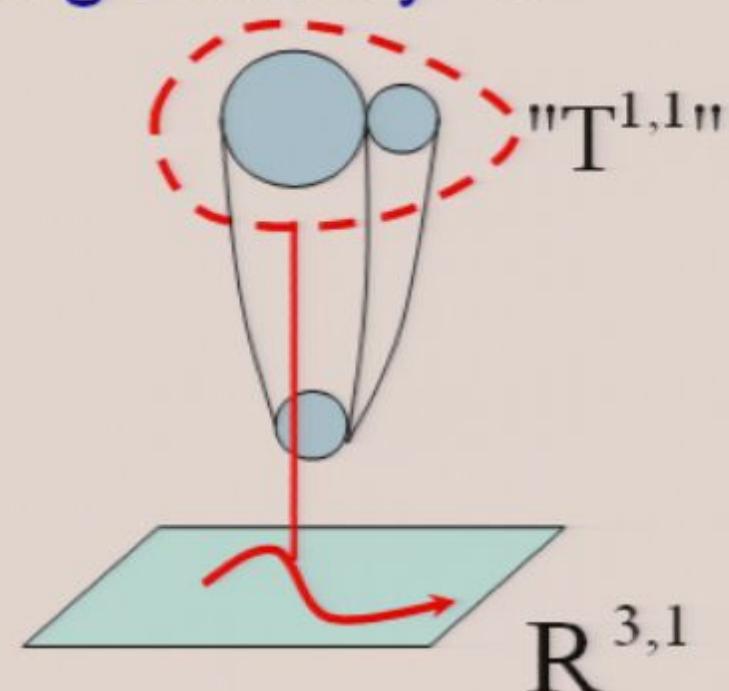
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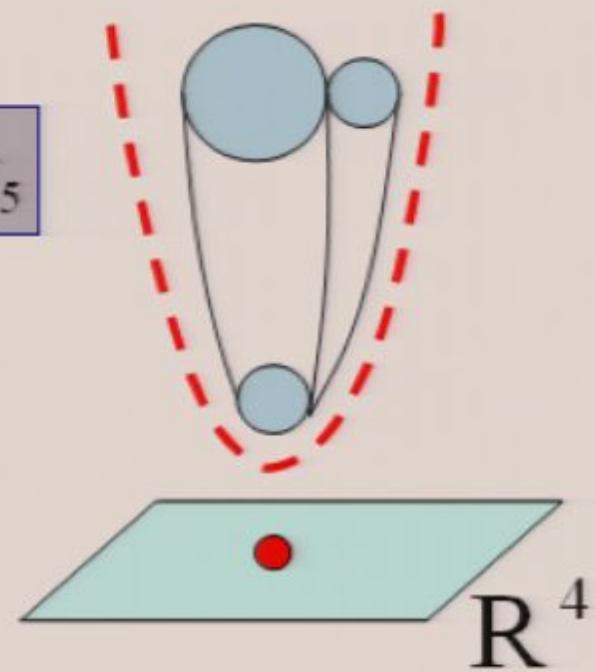
# Baryonic condensate

Baryonic vev is measured by Euclidean D5 wrapping the entire conifold

- in UV embedding ends on warped D5 with dissolved D3 (gauge field) Aharony
- induced gauge field has to be  $SU(2) \times SU(2)$  invariant  $A = \xi(t) g_5$

Kappa-symmetry in Euclidean space is tricky. We found BPS equation by taking square root of E.O.M.

$$\dot{\xi} = f(\xi, t, U)$$



## Baryonic condensate

There are 3 real solutions  $\xi(t)$

DBI action is divergent. We calculate it as a function of UV cut-off. For exactly 2 solutions action diverges logarithmically which admits AdS/CFT interpretation. The third one is unphysical

- two different solutions  $\xi_A, \xi_B$  correspond to  $A, B$
- the same solutions with opposite orientation of CS term correspond to anti-baryons  $\bar{A}, \bar{B}$ . This is consistent with  $U(1)_B$  charges of the baryons

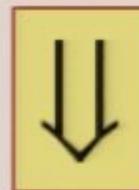
# Baryonic condensate

Both solutions  $\xi_A, \xi_B$  lead to the same divergence in DBI action

$$S[\xi_{A,B}, r_{\text{UV}}] = \frac{9 g_s^2 M^3}{16\pi^2} \left( \log(r_{\text{UV}})^3 + O(\log(r_{\text{UV}})^2) \right)$$

which can be interpreted as the dimension

$$\varphi(r) \sim e^{-S(r)}$$



$$-\frac{d\varphi}{d \log r} = \Delta\varphi$$

$$\Delta_{A,B} = \frac{27 g_s^2 M^3}{16\pi^2} \left( \log(r_{\text{UV}})^2 + O(\log(r_{\text{UV}})^1) \right) = \frac{3}{4} M k(k+1)$$

# Baryonic condensate

Expectation of baryonic operator is given (up to a multiplicative factor) by a renormalized action

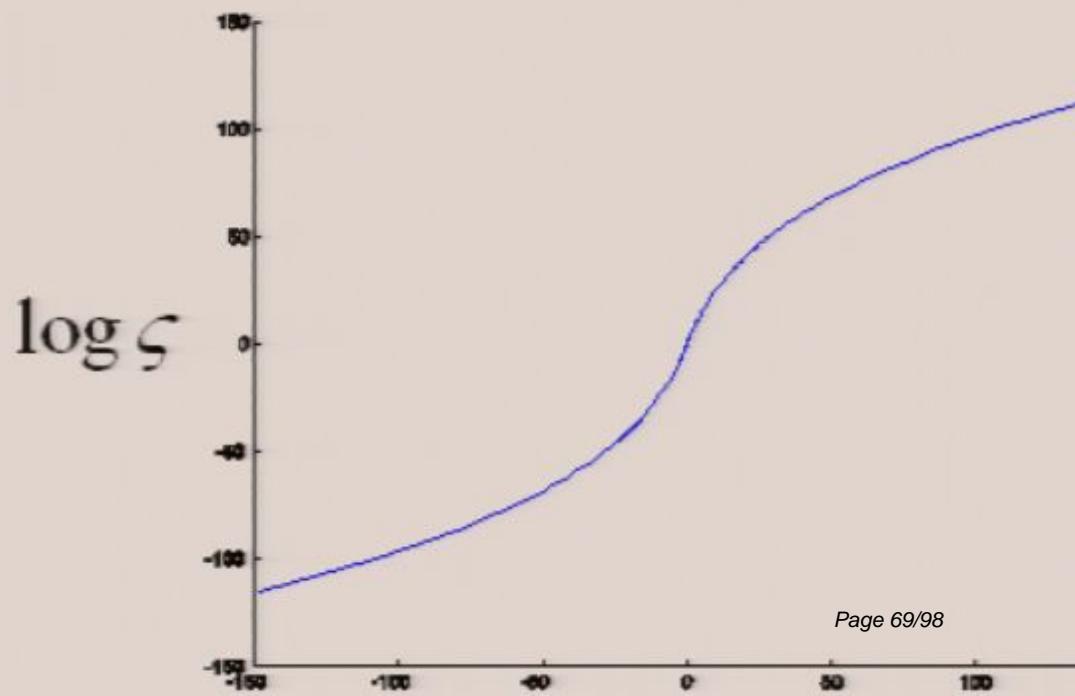
$$\langle A \rangle(U) \sim e^{-(S_A[U, r_{UV}] - \text{divergence}(r_{UV}))}$$

As an extra evidence of our approach we find

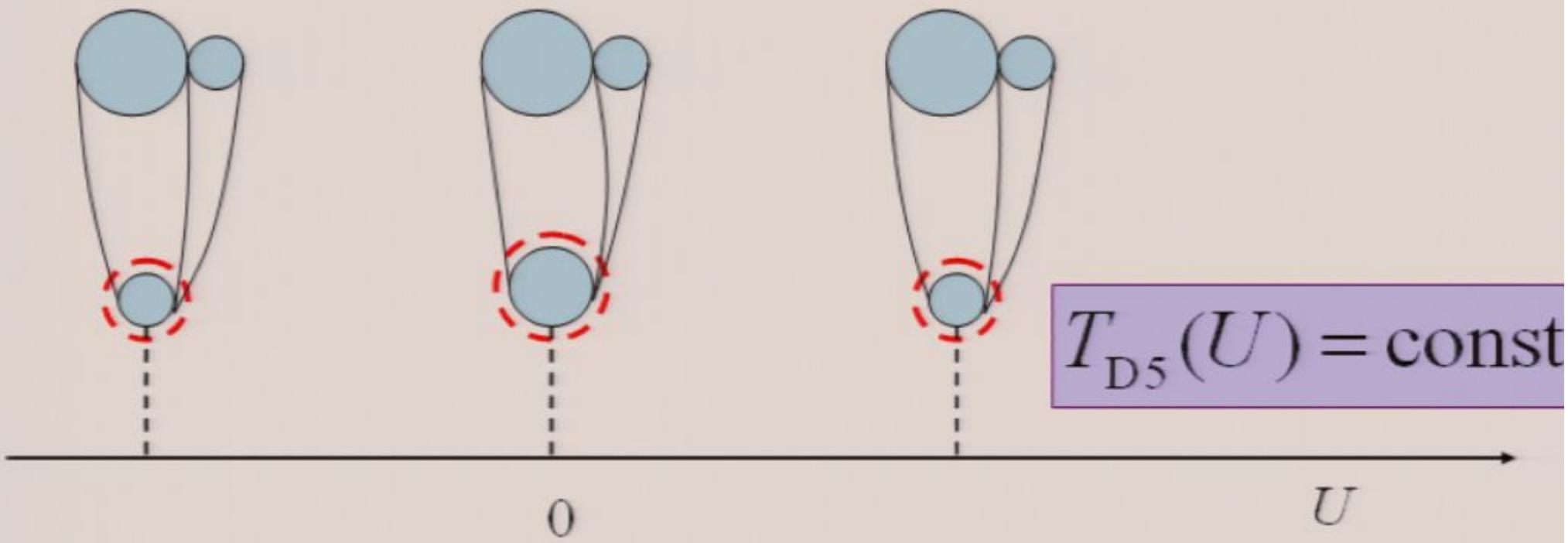
$$\langle A \rangle \langle B \rangle(U) = \text{const}$$



$$AB = -\Lambda_{2M}^{4M}$$



# BPS domain wall (D5 brane)



- D5 is dual to BPS domain wall separating different vacua in gauge theory
- it wraps minimal  $S^3$  at the tip of the cone

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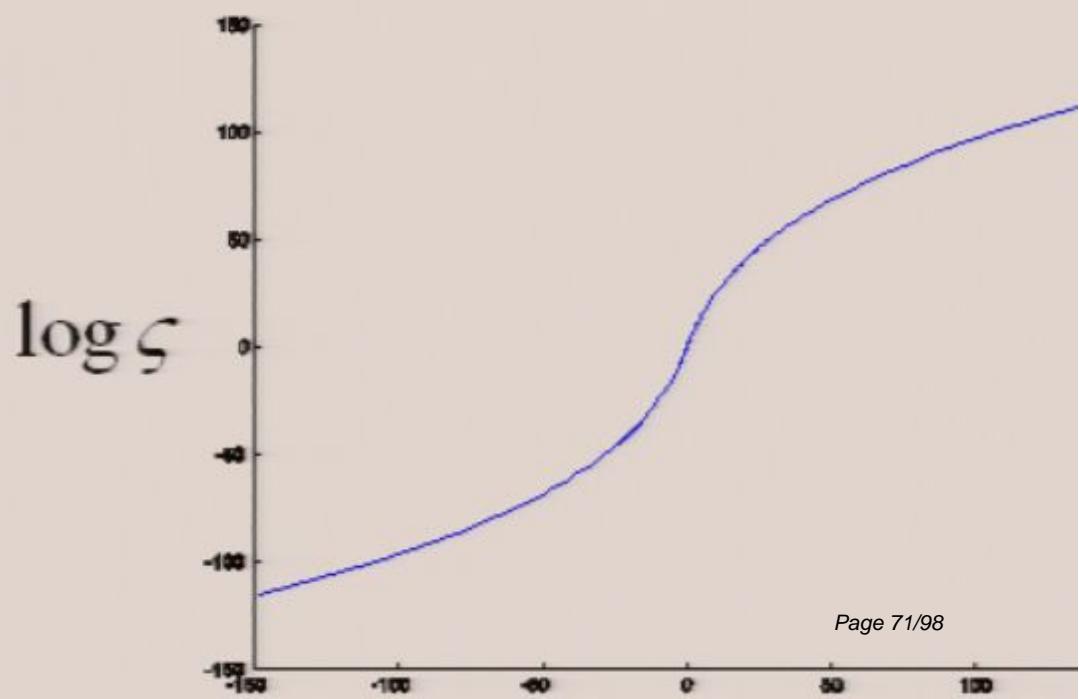
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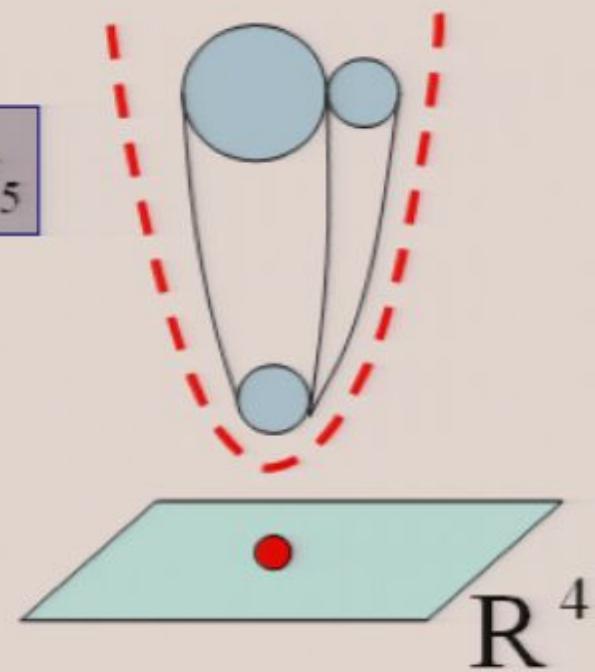
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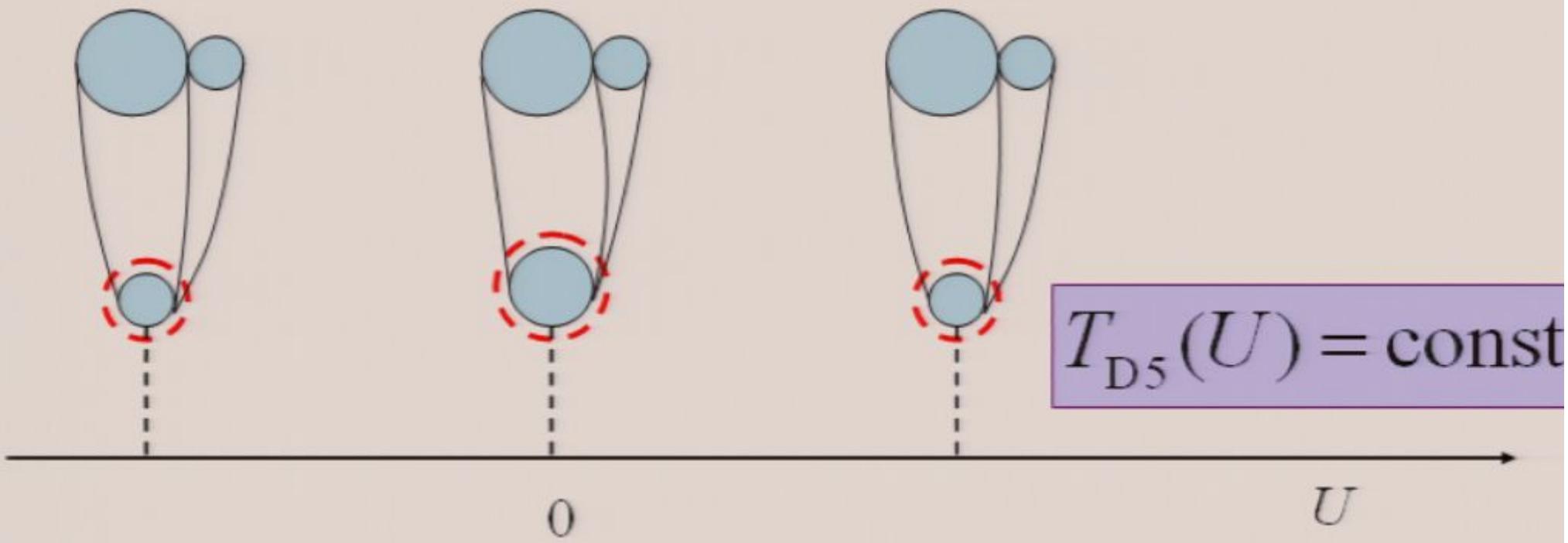
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## BPS domain wall (D5 brane)

tension of D5 does not depend on the value of  
baryonic condensate  $U$

AD, Klebanov, Seiberg

- gauge theory argument: tension of BPS can not depend on the value of condensate
- gravity side: constancy was used to check (numerically) the choice of boundary conditions on dilaton  $\phi$  and warped factor  $A$

# BPS domain wall (D5 brane)

Rigorous proof on the gravity side is based on  
calibration argument

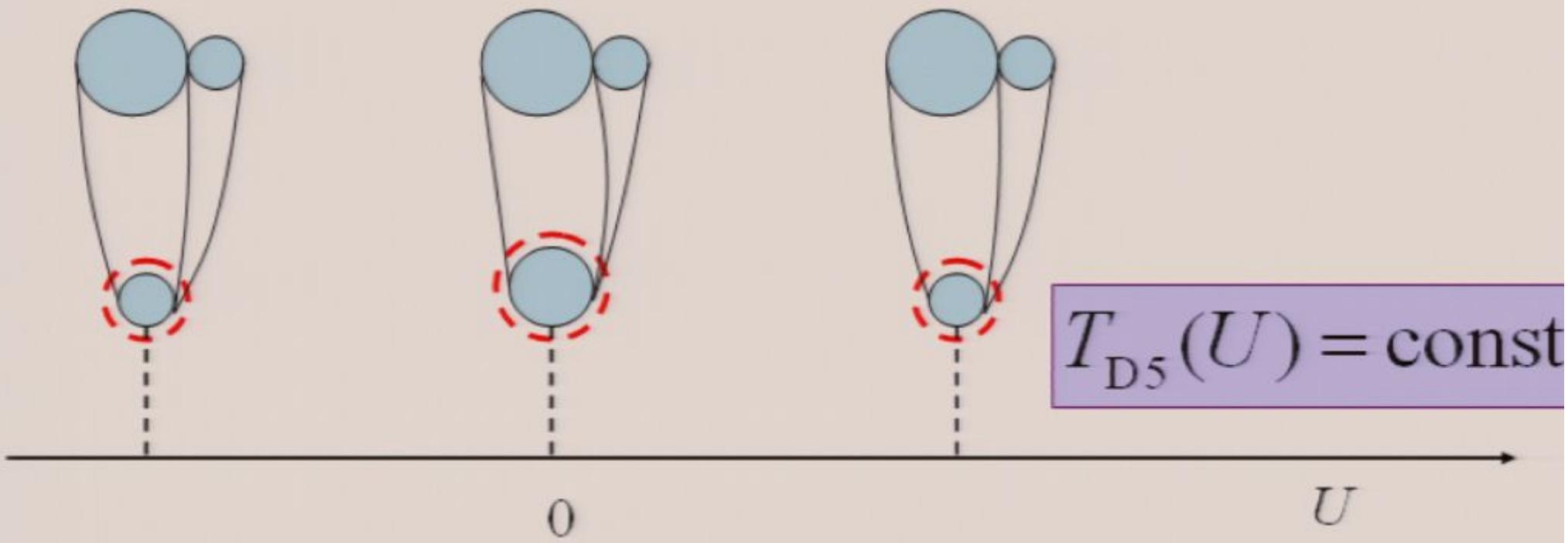
Martucci, Smyth; AD, Klebanov, Tachikawa, in progress

- DBI action of D5 is calibrated by holomorphic (0,3) form on CY

$$T = \int e^{3A-\phi} \sqrt{\det(g+M)} \geq \int \Omega$$

- the D5 is SUSY if inequality is satisfied
- 3-cycle can be pulled to infinity because  $d\Omega = 0$
- geometry at infinity is universal ( $U$  independent)

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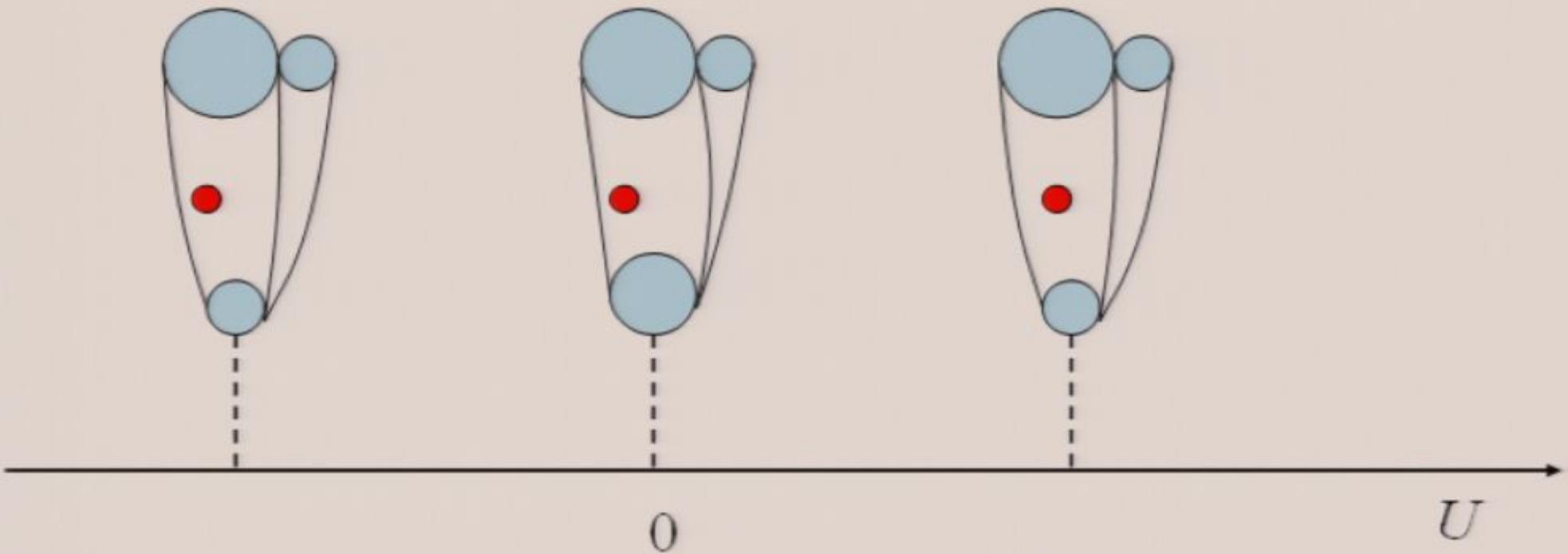
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# D3 and anti-D3 on the Bbranch



$$V_{D3(\bar{a}D3)} = V_{D3(\bar{a}D3)}(U, t)$$

# D3 and Anti-D3 on Bbranch

Killing spinor

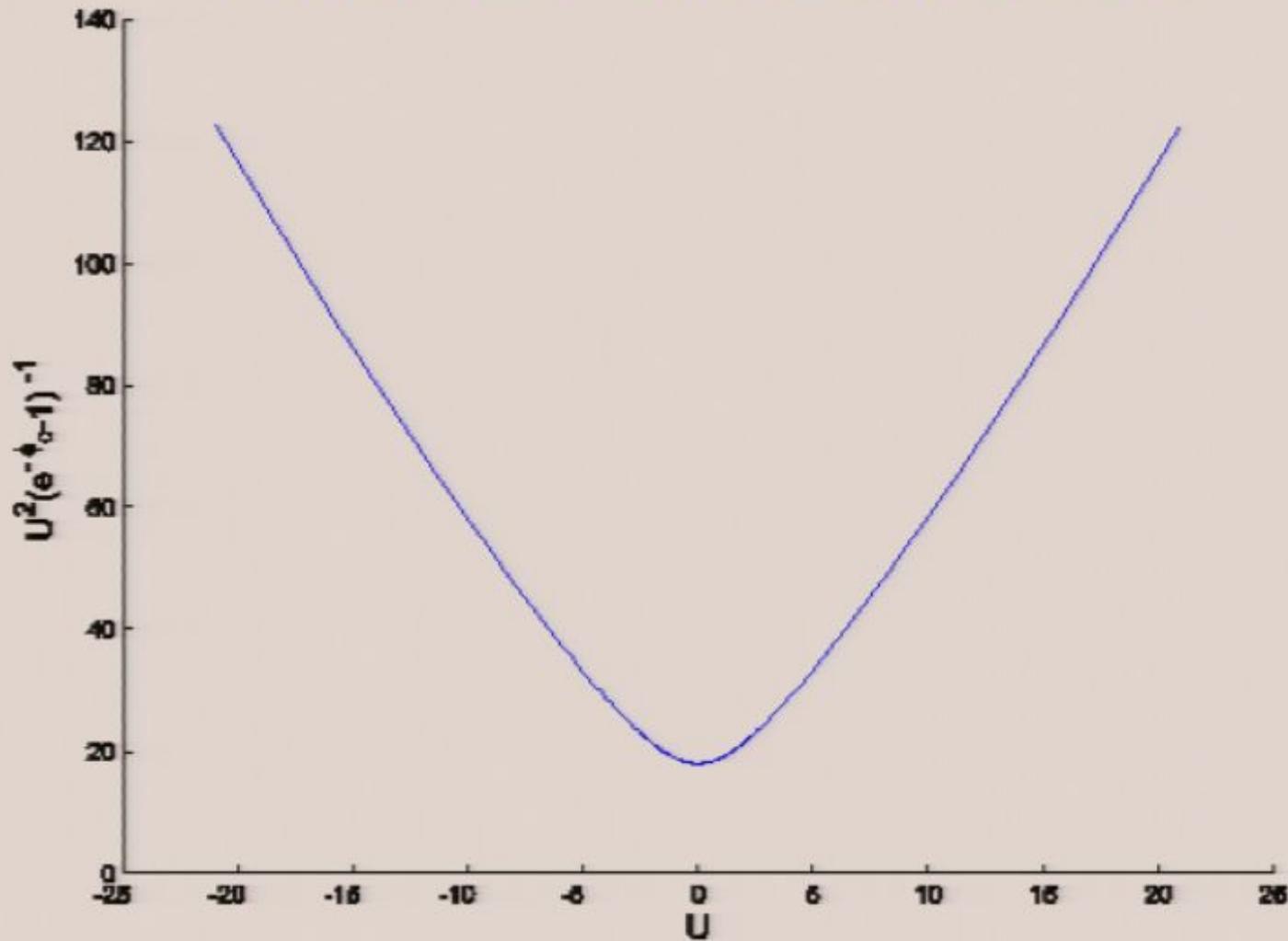
$$\Psi = \alpha \Psi_0 + \beta^* \Psi_0^*$$

$$\alpha = \frac{U^{1/4} e^{\phi/4} (1+e^\phi)^{3/8}}{(1-e^\phi)^{1/8}} \quad \beta = i \frac{U^{1/4} e^{\phi/4} (1-e^\phi)^{3/8}}{(1+e^\phi)^{1/8}}$$

$\beta = 0$  at KS and  $\beta \neq 0$  away from KS

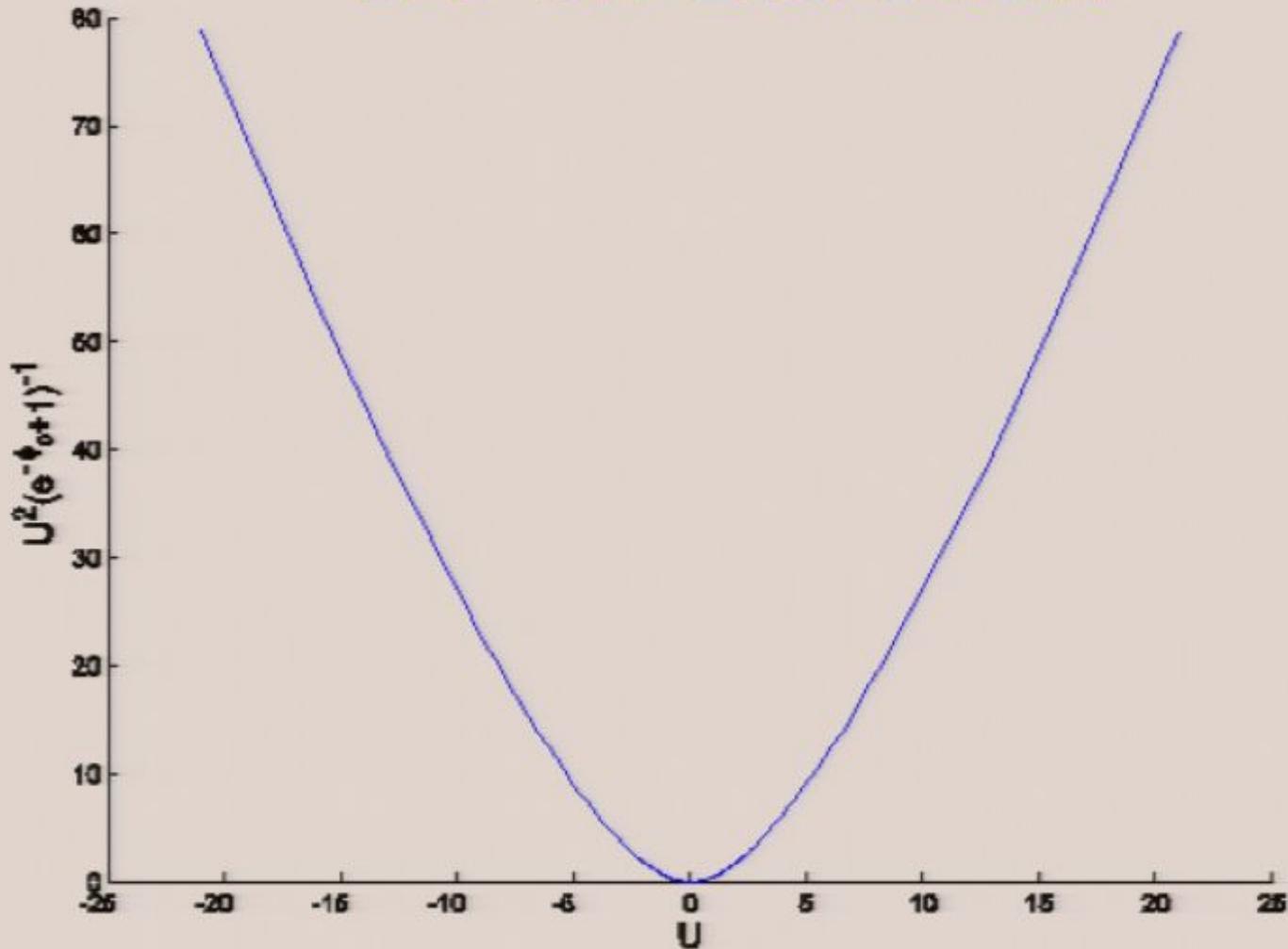
- D3 is SUSY at  $U = \phi = 0$  (KS) and breaks SUSY elsewhere
- Anti-D3 breaks SUSY everywhere

# Anti-D3 on Bbranch



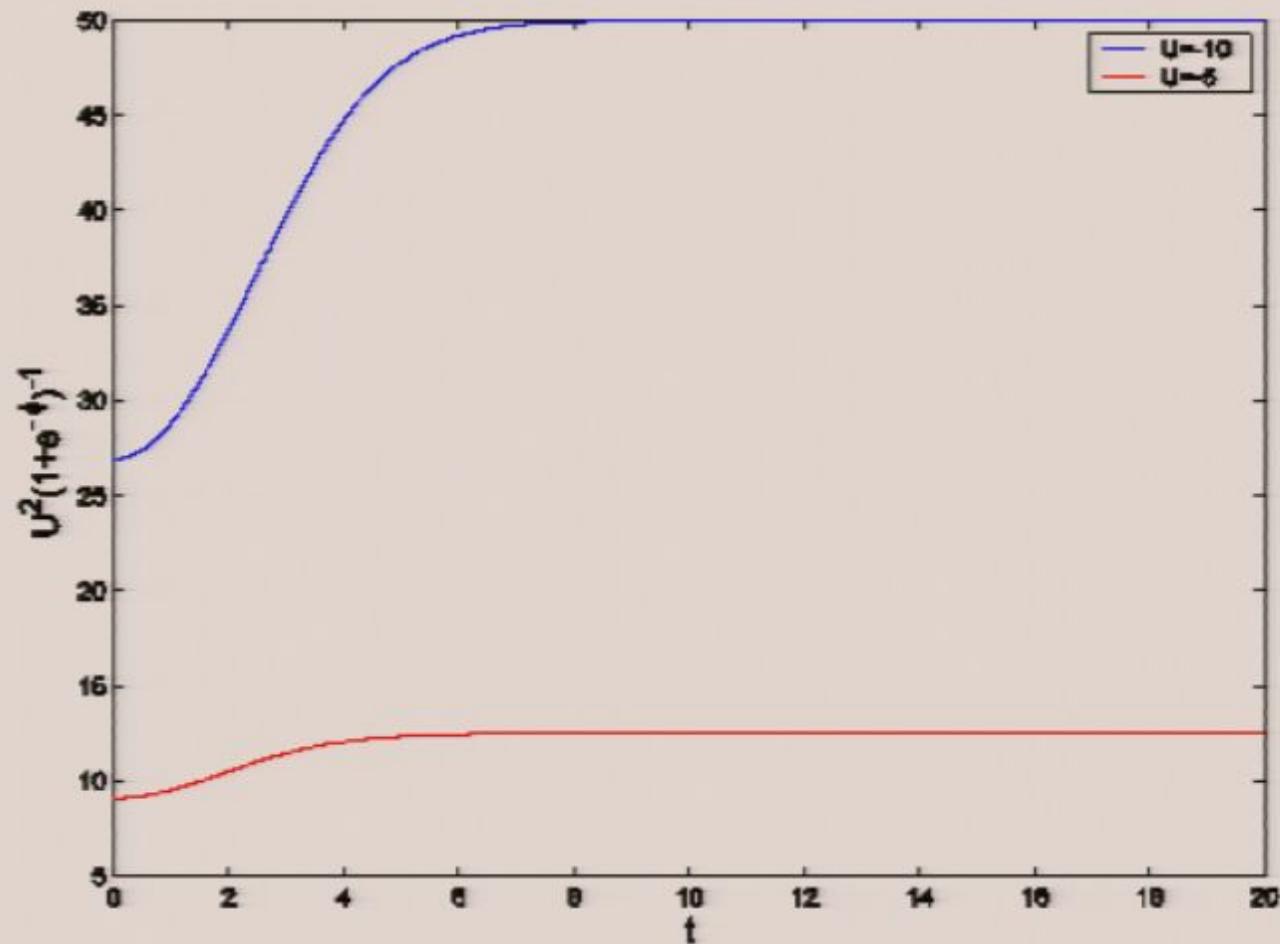
$$V_{aD3}(t=0, U) \sim \frac{U^2}{e^{-\phi_0} - 1}$$

# D3 on Bbranch



$$V_{D3}(t=0, U) \sim \frac{U^2}{e^{-\phi_0} + 1}$$

# D3 on Bbranch



$$V_{D3}(t, U) \sim \frac{U^2}{e^{-\phi(t)} + 1}$$

$$ds_{10}^2 = e^{2A} dx^2 + \dots$$

$$\frac{1}{\rho^2}$$

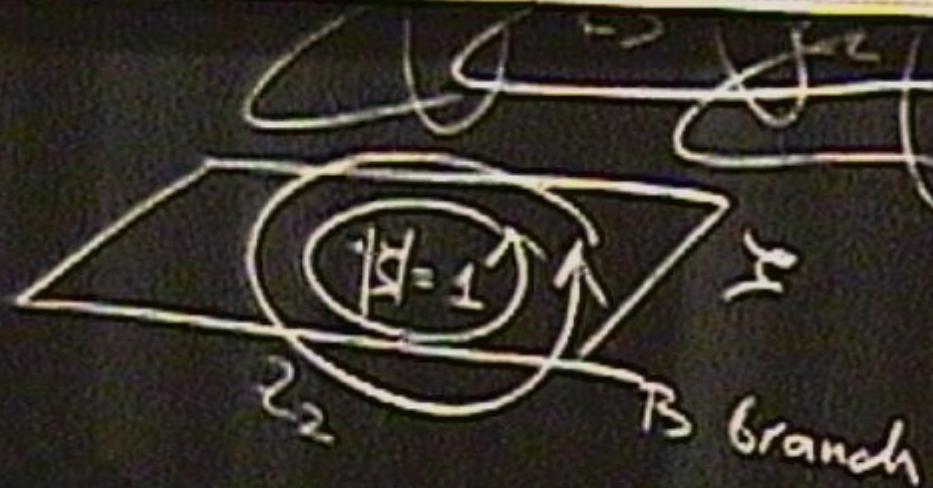
$$\phi^2$$

$\mathcal{F}$

$$ds_{10}^2 = e^{2A} dx^2 + \dots$$
$$\frac{N^2}{P^2} = D\text{-term}$$

anach

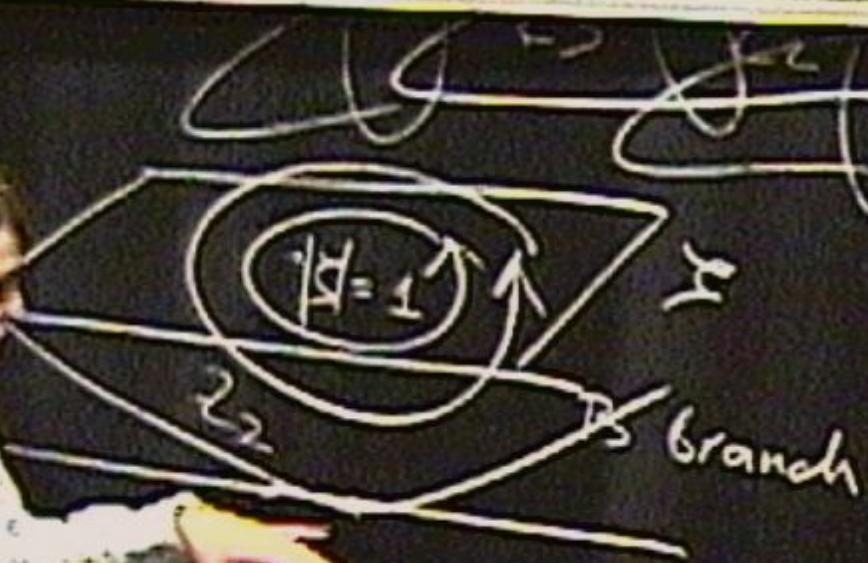
$$\dot{\varphi}^2$$



$$ds_{10}^2 = e^{2A} dx^2 + \dots$$

$$\frac{u^2}{\rho^2} = D - \text{term}$$

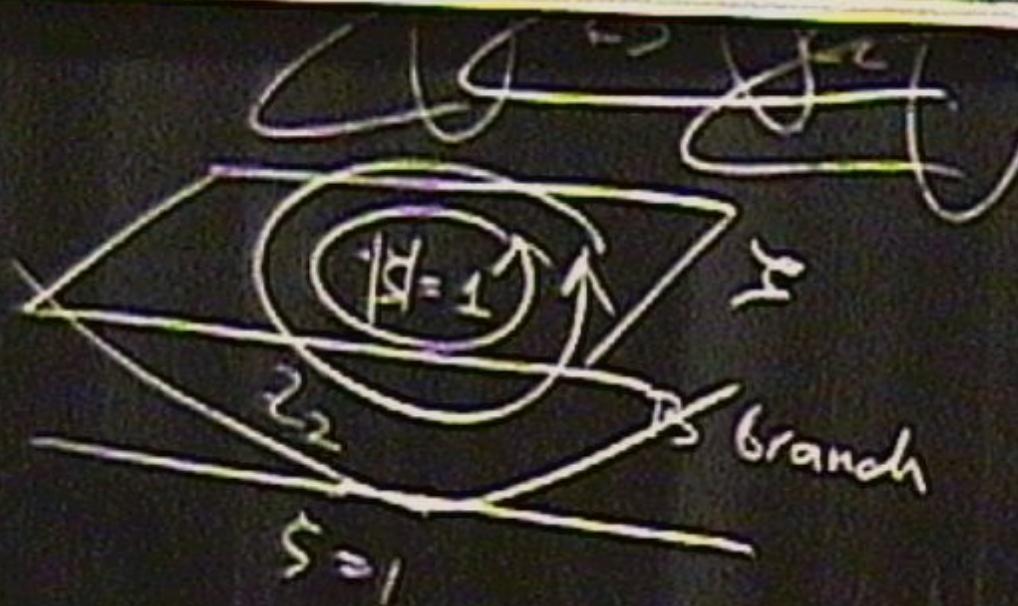
$$\rho^2$$



$$ds_{10}^2 = e^{2A} dx^2 + \dots$$

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$$ds_{10}^2 = e^{2A} dx^2 + \dots$$

$$\frac{u^2}{\rho^2} = D - \text{term}$$

$$\dot{\rho}^2$$

# Masses of low-lying glueballs

Glueballs in gauge theory/ operators in gauge theory       $\longleftrightarrow$       fluctuation of SUGRA

Simplest example: traceless components of stress-energy tensor

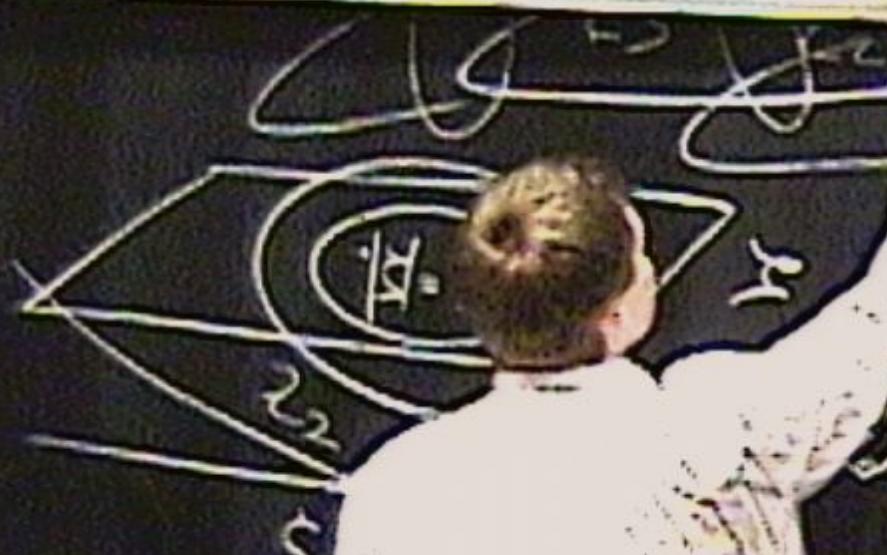
$$T_{\langle\mu\nu\rangle} \sim \delta g_{\langle\mu\nu\rangle} = e^{2A-\phi/2} \xi_{\langle\mu\nu\rangle} \varphi$$

$$ds_{10}^2 = e^{2A-\phi/2} dx^2 + e^{-\phi/2} ds_6^2 \Rightarrow ds_5^2 = e^{2T} dx^2 + dr^2$$

$$\partial_r e^{2T} \partial_r \varphi + m^2 \varphi = 0$$

obey minimal scalar equation

$$\tilde{s}_{\mu\nu} \cdot e^{ikx}$$

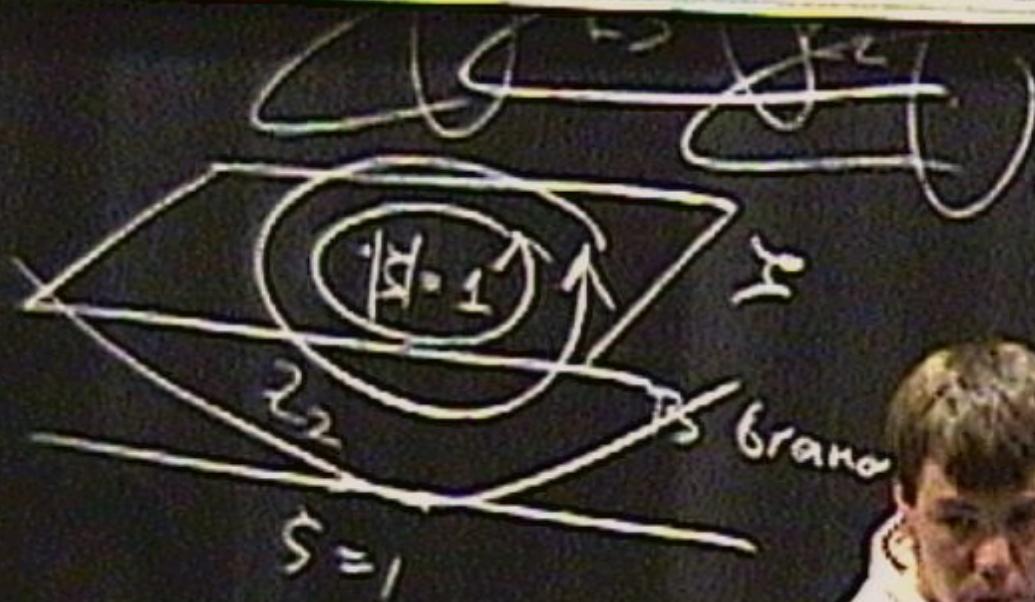


$$ds_{10}^2 = e^{2A} dx^2 +$$

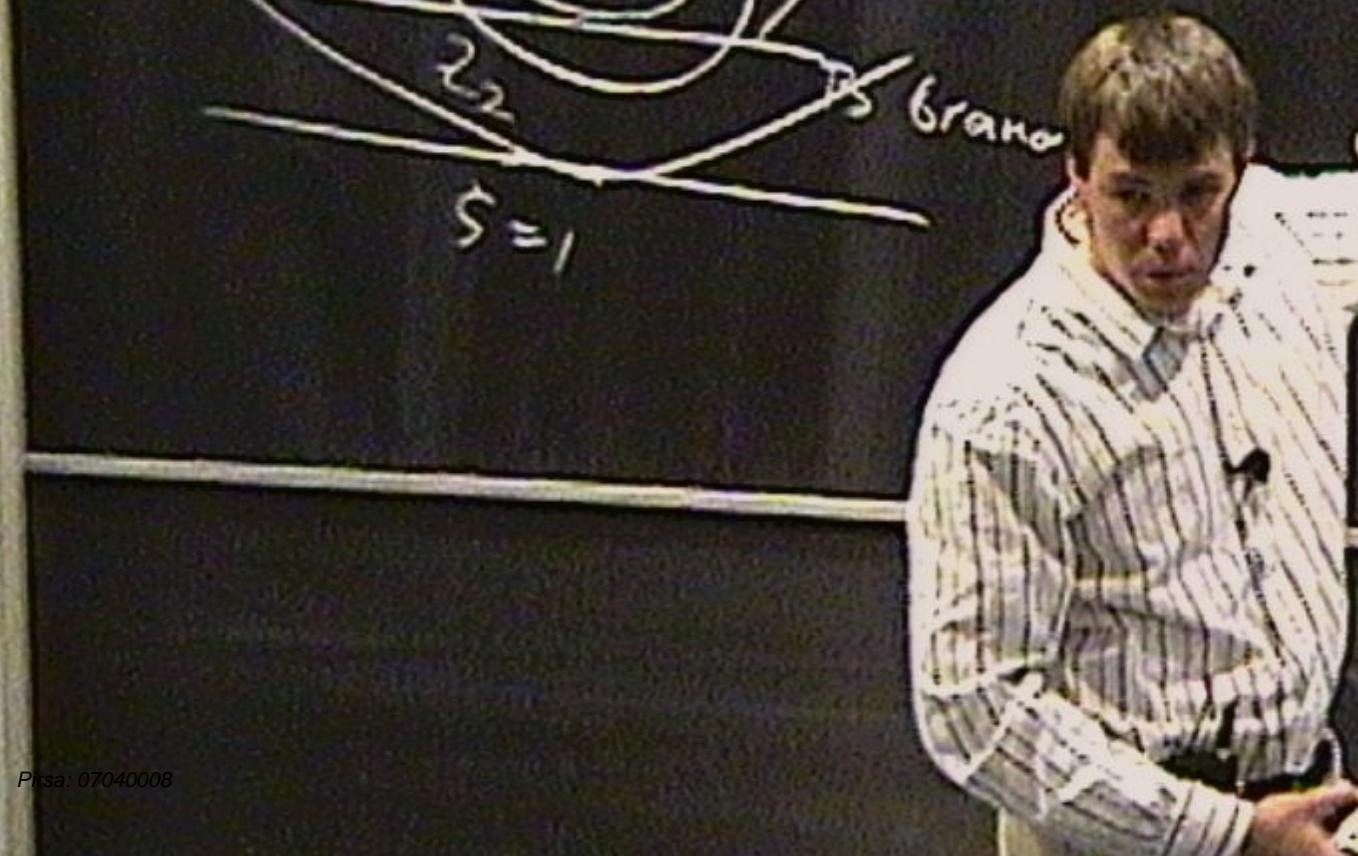
$$\frac{r^2}{\rho^2} = D\text{-term}$$

$$\rho^2$$

$$\tilde{g}_{\mu\nu} \cdot e^{i\kappa x} \\ r^2 = \bar{\kappa}^2$$



$$ds_{10}^2 = e^{2A} dx^2 + \dots$$
$$\frac{r^2}{b^2} - D\text{-term}$$



# Gravity multiplet

$$T_{\langle\mu\nu\rangle}$$

- traceless components of stress-energy tensor – spin 2

$$\psi_\mu$$

- superpartner – spin 3/2

$$J_\mu^\perp$$

- vector (transversal) component of anomalous chiral current – spin 1

$$J_\mu^\perp \sim A_\mu$$

Chiral anomaly in gauge theory – spontaneous breaking of U(1)  
 $\psi \rightarrow \psi + \text{const}$  symmetry

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# Gravity multiplet

## SUGRA fluctuation ansatz

$$d\psi \rightarrow d\psi + A_\mu dx^\mu$$

Microscopic equation for  $A_\mu$  valid in far IR  
A.Dymarsky, D.Melnikov

Coincides with equation derived from 5d SUGRA  
M. Bianchi, O. DeWolfe, D. Freedman, and K. Pilch

$$A_\mu \sim \xi_\mu \varphi, \quad \partial_\mu A_\mu = 0$$

$$\partial_r e^{2T} \partial_r \varphi + m^2 \varphi + 2e^{2T} \frac{\partial^2 T}{\partial r^2} \varphi = 0$$

Form SUSY QM  
with minimal  
scalar equation

# Gravity multiplet

Evaluating spectrum numerically for the entire branch

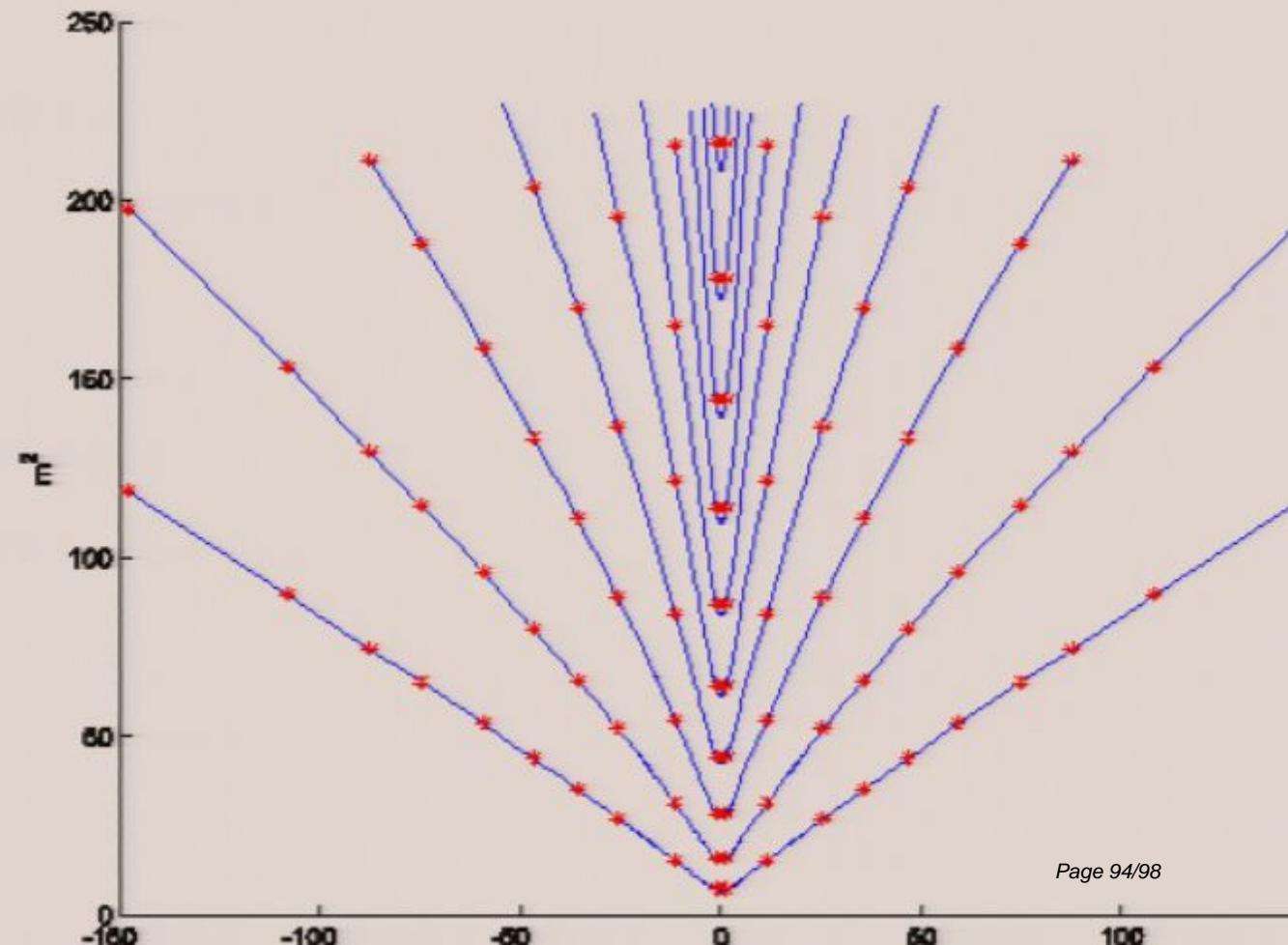
Use both equations to estimate the error bar

$m_n(U)$  scales  
as  $U^{4/3}$

Spectrum is  
linear in  $n$  with  
good accuracy

$$m_n \sim n$$

Error bar is of  
order 1%



# Anomaly multiplet

G. Arutyunov, S. Frolov, and S. Theisen  
and  
O. DeWolfe and D. Freedman

Derived equations for gravity-scalar fluctuations in  
5d SUGRA. Solutions coincide in the examples  
considered.

These equations form SUSY QM (A.Dymarsky,  
D.Melnikov):

one corresponds to the trace of S.E. tensor  $T_{\mu\mu}$

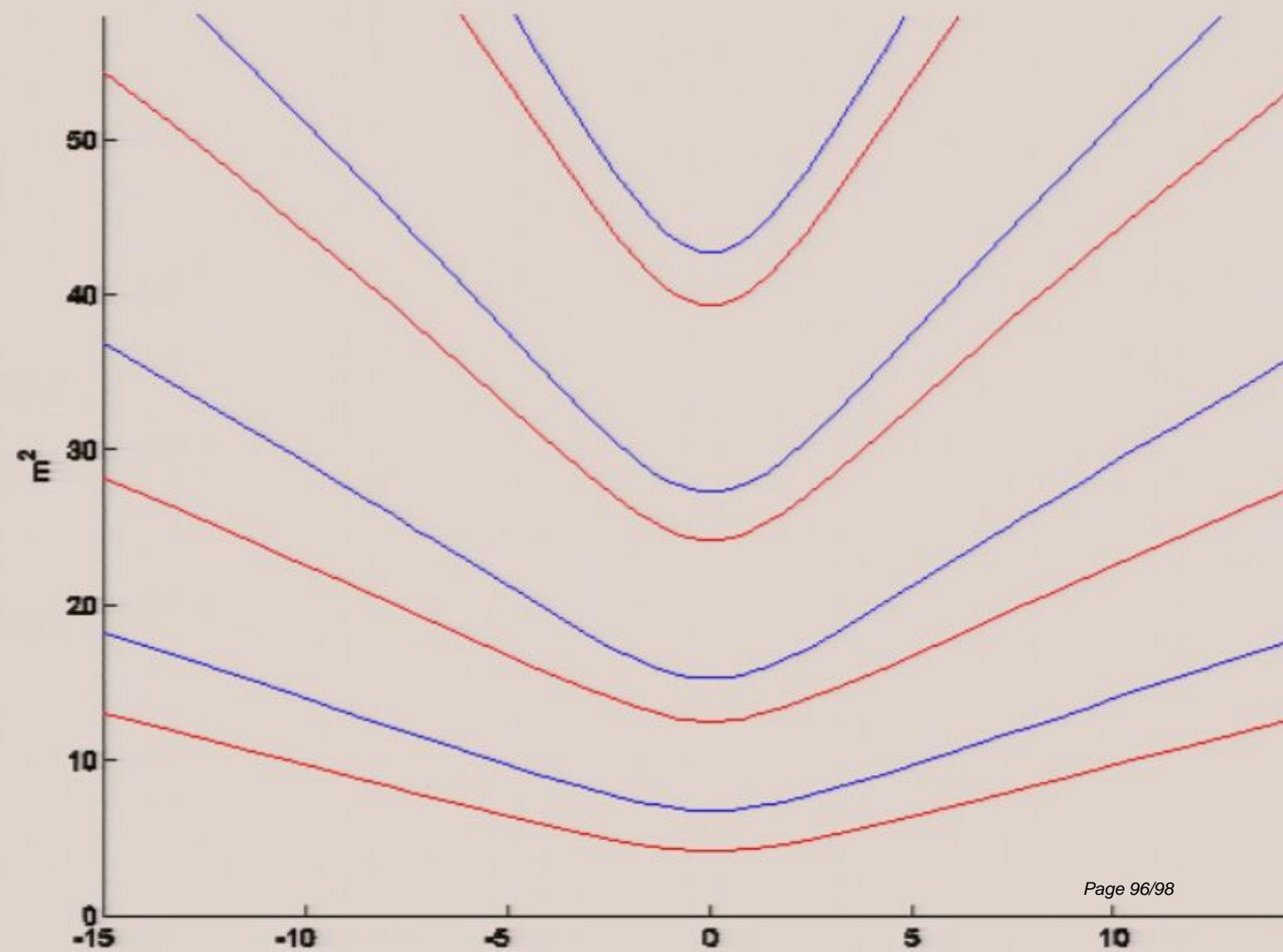
another to longitudinal component of chiral  
current

$$J_\mu^{\parallel} \sim \partial_\mu \phi$$

# Spectrum: gravity vs anomaly multiplet

Anomaly multiplet is lighter.

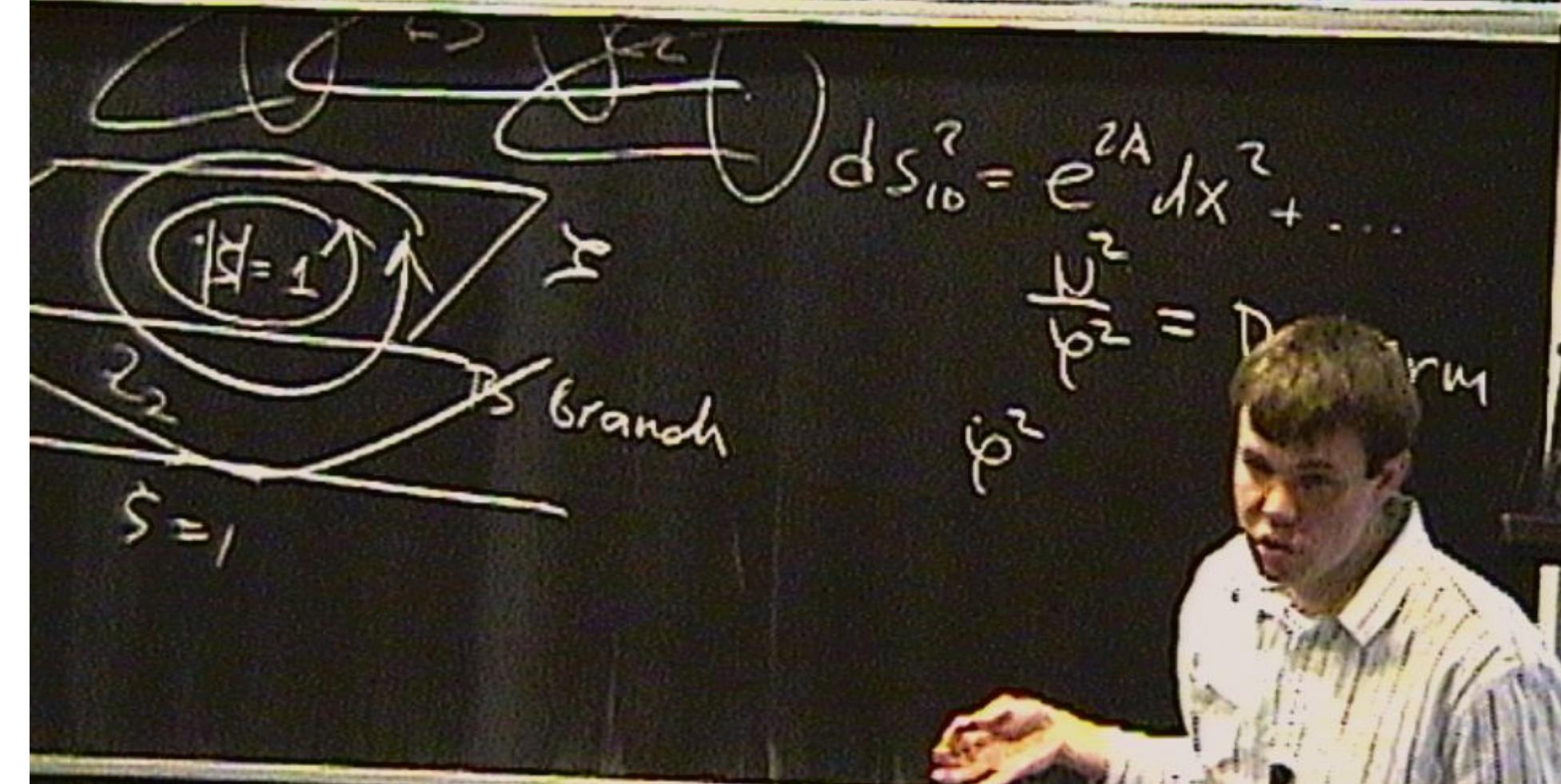
There is not degeneracy between multiplets at any  $U$



$$\xi_{\mu\nu} \cdot e^{ikx}$$

$$t^2 = \bar{m}$$

$$\frac{m_n^a - m_n^g}{m_n^r}$$



## Conclusion and future directions

- Family of BGMPZ solutions provides gravity dual description of  $N=1$  SYM on baryonic branch
- This theory exhibits rich IR dynamics
- Compactification effects: masses of axion/saxion, fixing  $U$
- Breaking SUSY
- Applications to cosmology