Title: New Tools for Understanding the Strong Interactions

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Abstract: The theory of strong interactions is an elegant quantum field theory known as Quantum Chromodynamics (QCD). QCD is deceptively simple to formulate, but notoriously difficult to solve. This simplicity belies the diverse set of physical phenomena that fall under its domain, from nuclear forces and bound hadrons, to high energy jets and gluon radiation. In this talk I show how systematic limits of QCD, known as effective field theories, provide a means of isolating the essential degrees of freedom for a particular problem while at the same time supplying a powerful tool for quantitative computations. The adventure will take us from the fine structure of hydrogen, to weak decays of B-mesons, to the behavior of energetic hadrons and jets in QCD.

New Tools for Understanding the Strong Interactions

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MIT

Perimeter Institute for Theoretical Physics
Apr. 2007

Outline

- Effective Field Theory, QED, Hydrogen
- Introduction to QCD, $\alpha_s(\mu)$
- Soft-Collinear Effective Theory & Energetic Particles
- Weak Decays of B mesons
- Outlook

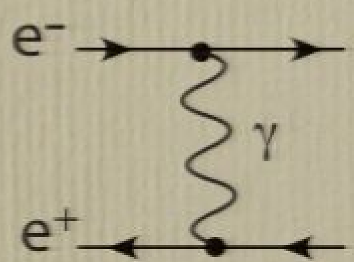
Introduction to QED

(quantum electromagnetism)

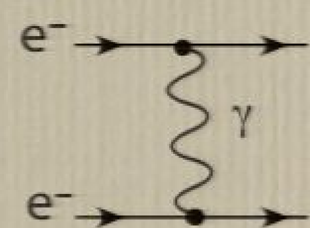
QED { Special Relativity: spacetime, $v \leq c$
Quantum Mechanics: quantization, $\Delta x \Delta p \geq \frac{\hbar}{2}$

⇒ antiparticles, spin, gauge-theory
⇒ parameters: charge & masses

Interactions

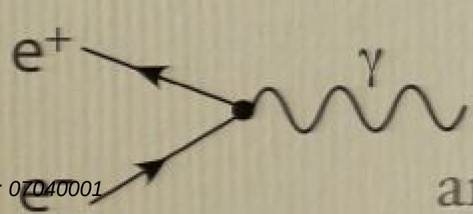


$$V = -\frac{e^2}{r}$$

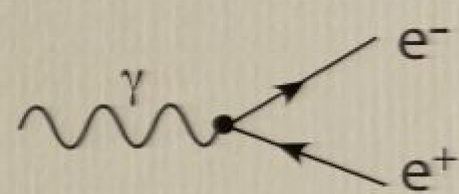


$$V = +\frac{e^2}{r}$$

two factors of the coupling



pair annihilation



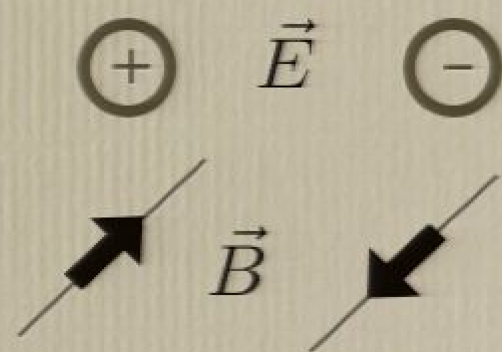
pair creation

The Standard Model Interactions

(leave out gravity and the higgs)

	Strong	Electromagnetism	Weak
	QCD	QED	
mediator:	gluons	photons	$W^{\pm}, Z^0$
typical strength:	~ 1	$\sim 10^{-2}$	$\sim 10^{-6}$
range:	~ 1 fm	∞	$\frac{1}{m_W} \to \sim 10^{-3}$ fm

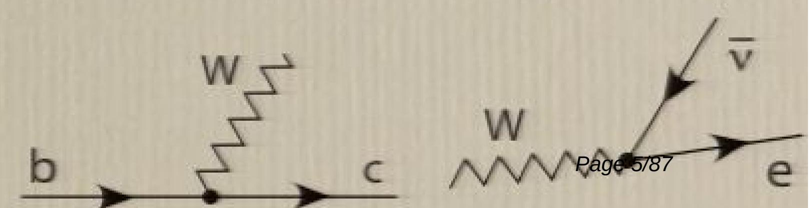
proton



$n \to pe\bar{\nu}$,

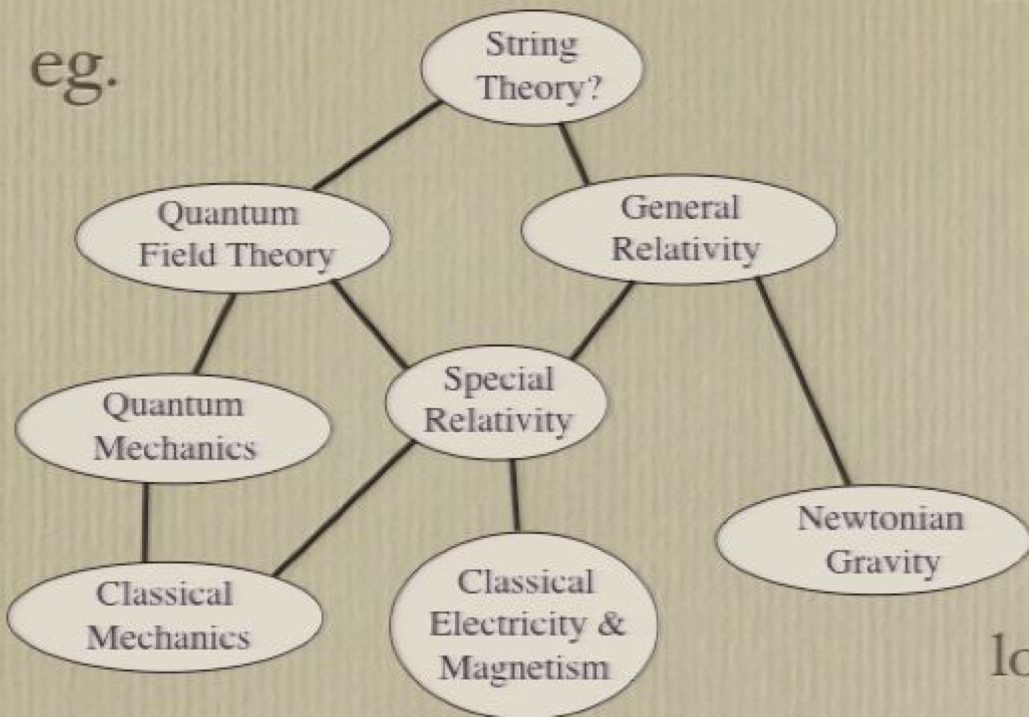
radioactive decay

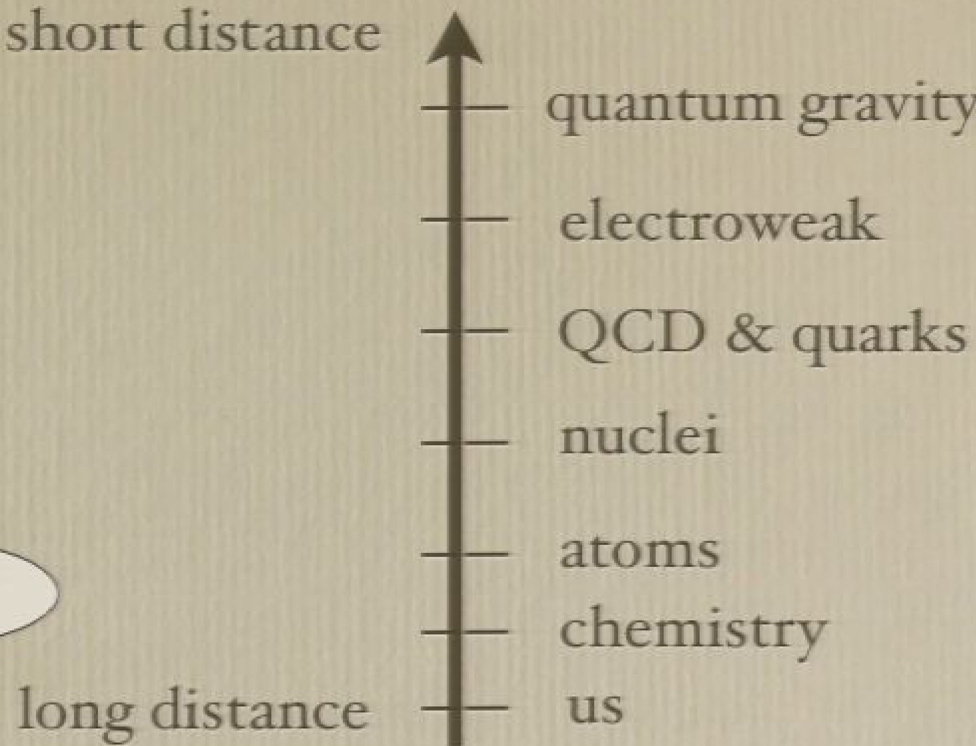
Other forces can (in principle) be derived from these



Physics compartmentalized

eg.





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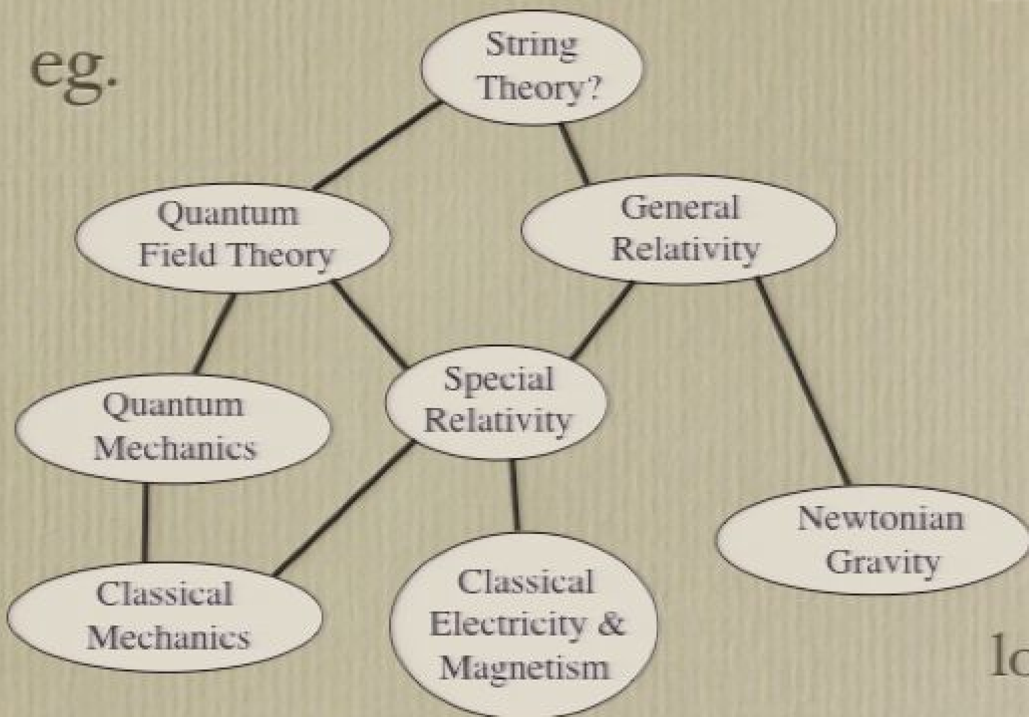
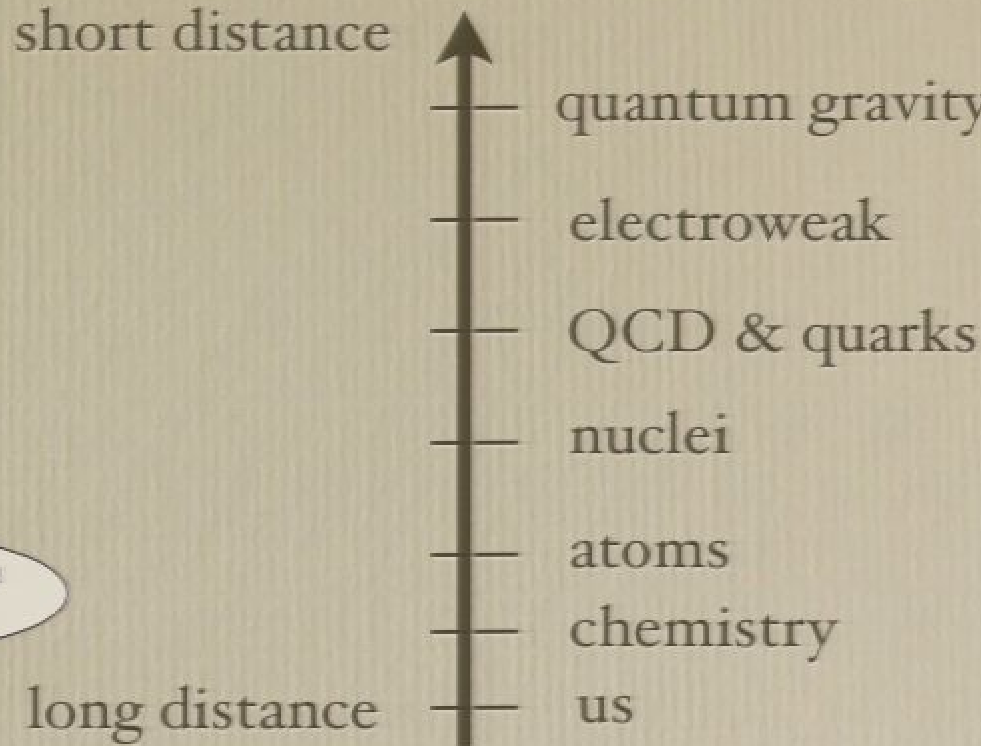
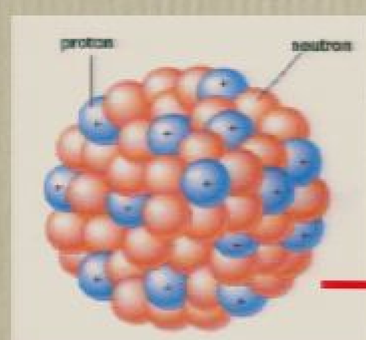




But, one doesn't need nuclear physics to build a boat



Generality
vs.
Precision

Dynamics at long distance does not depend on the details of what happens at short distance

In the quantum realm, $\lambda \sim \frac{1}{p}$, wavelength and momentum are related, so

Low energy interactions do not depend on the details of high energy interactions

Bad:

- we have to work harder to probe the interesting physics at short distances

Good:

- we can focus on the relevant interactions & degrees of freedom

- calculations are simpler

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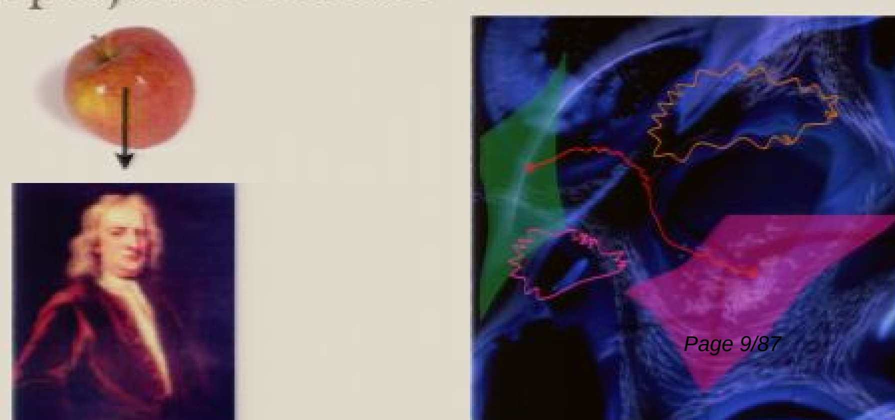
Newton didn't need quantum gravity for projectile motion



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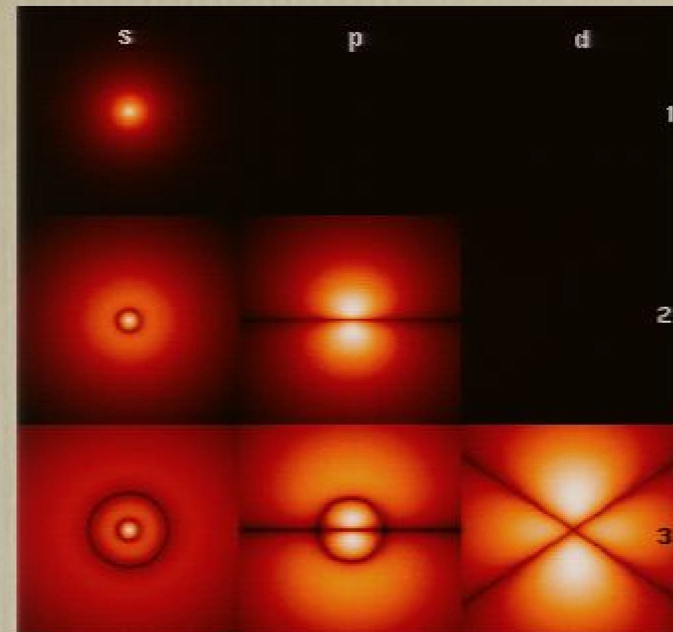
Example: Hydrogen

non-relativistic quantum mechanics

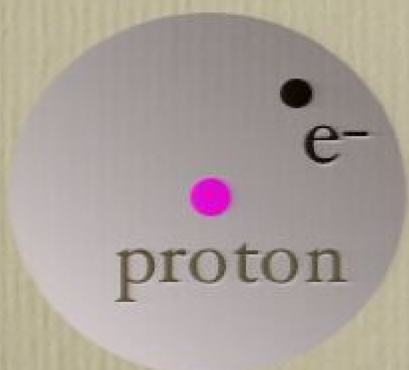


parameters: mass m_e

charges $Q_e \, , Q_p$

coupling $\alpha = \dfrac{1}{137}$

degrees of freedom:



scales: $m_p = 938\,\mathrm{MeV} \quad \to \infty$

$$m_e = 0.511\,\mathrm{MeV}$$

$$p \sim m_e \alpha = 3.7\,\mathrm{keV} \qquad \sim (a_{\mathrm{Bohr}})^{-1}$$

$$E_n = -\frac{m_e \alpha^2}{2n^2} = -\frac{13.6\,\mathrm{eV}}{n^2}$$

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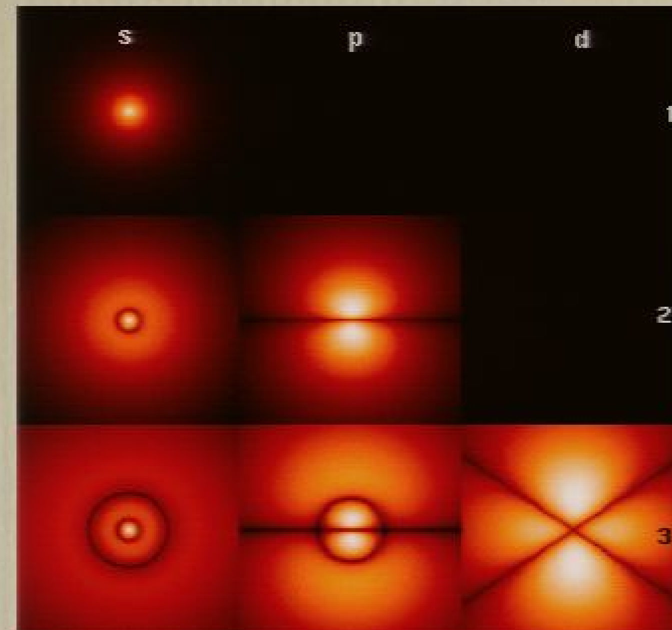

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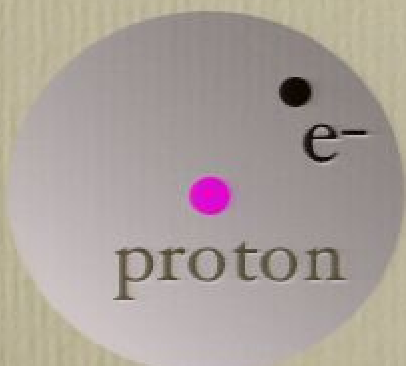

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Why not quarks? QCD? b-quark charge? e^+? weak force?

m_{proton} ? spin?

Effective Field Theory Idea

QED — short distance theory is more general

↓ expand in $\frac{p}{m_e}, \ \frac{m_e}{m_p}, \ \alpha$

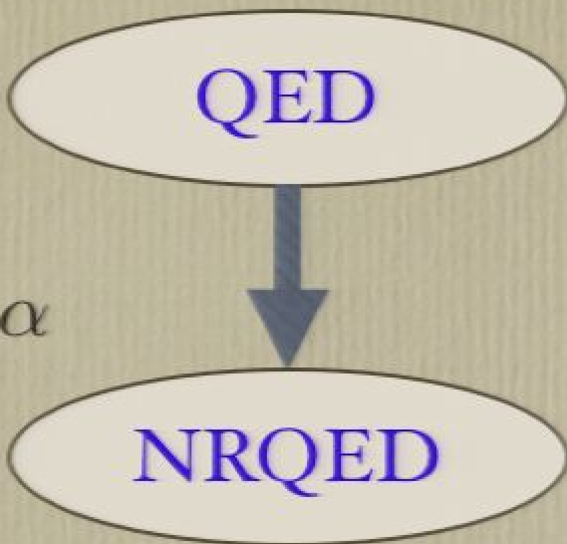
NRQED — long distance theory where its easier to compute

$$H = H_0 + \sum_{m=1}^{\infty} \epsilon^m \ H_m$$

exact answer is irrelevant, work to the desired level of precision

H_0: Nonrelativistic Quantum Mechanics

Effective Field Theory Idea



short distance theory is more general

expand in $\frac{p}{m_e}, \frac{m_e}{m_p}, \alpha$

long distance theory where its easier to compute

$$H = H_0 + \sum_{m=1}^{\infty} \epsilon^m \; H_m$$

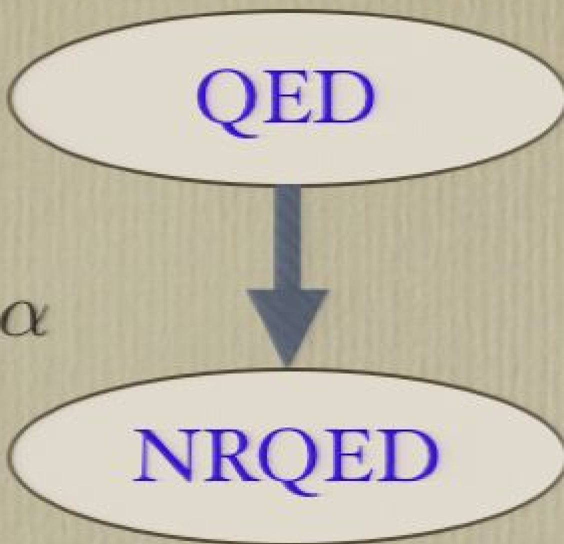
exact answer is irrelevant, work to the desired level of precision

Comments: Degrees of freedom can change

$$e^+ \longrightarrow \text{no} \;\; e^+$$

QCD, quarks $\longrightarrow$ proton

Effective Field Theory Idea



short distance theory is more general

expand in $\frac{p}{m_e}$, $\frac{m_e}{m_p}$, α

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exact answer is irrelevant, work to the desired level of precision

Comments: Symmetries of QED constrain the form of NRQED

Charge conjugation ($e^+ \leftrightarrow e^-$)
Parity ($\vec{x} \rightarrow -\vec{x}$)
Time-Reversal ($t \rightarrow -t$)

→ constrain the H_m's

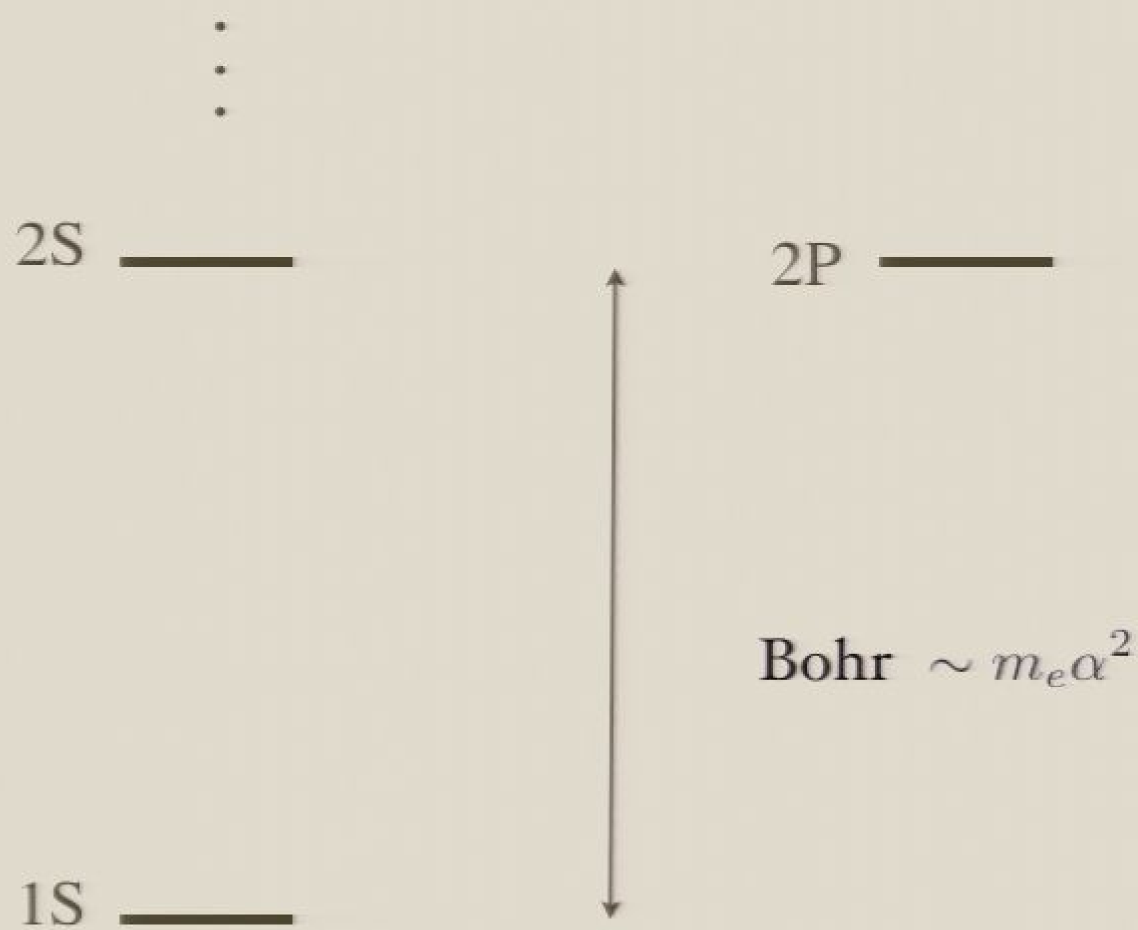
NRQED

Effective Field Theory for Non-relativistic bound states

nL_J

$F = J + S_p$

spectrum



NRQED

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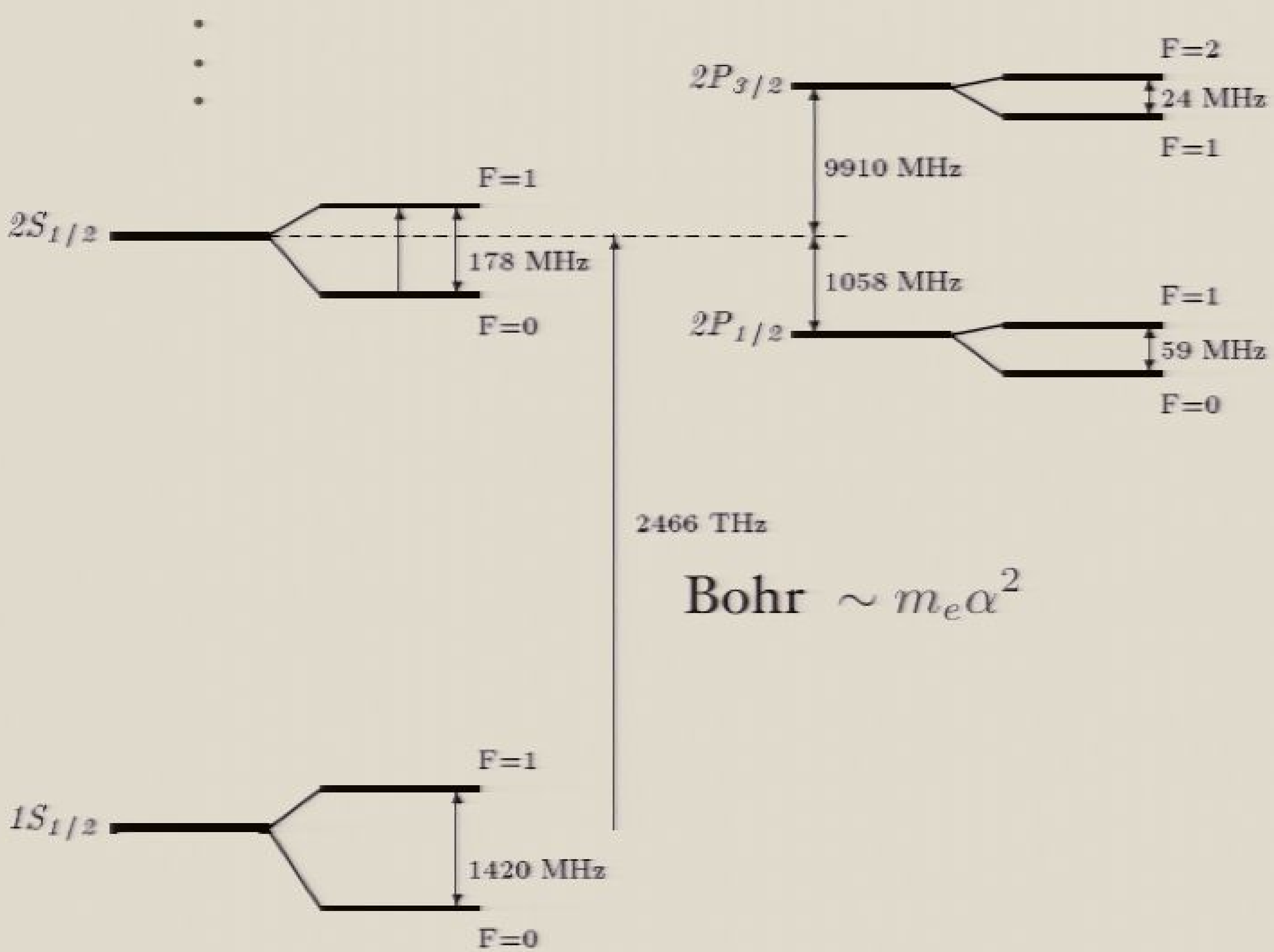

Bohr $\sim m_e \alpha^2$

NRQED

Effective Field Theory for Non-relativistic bound states

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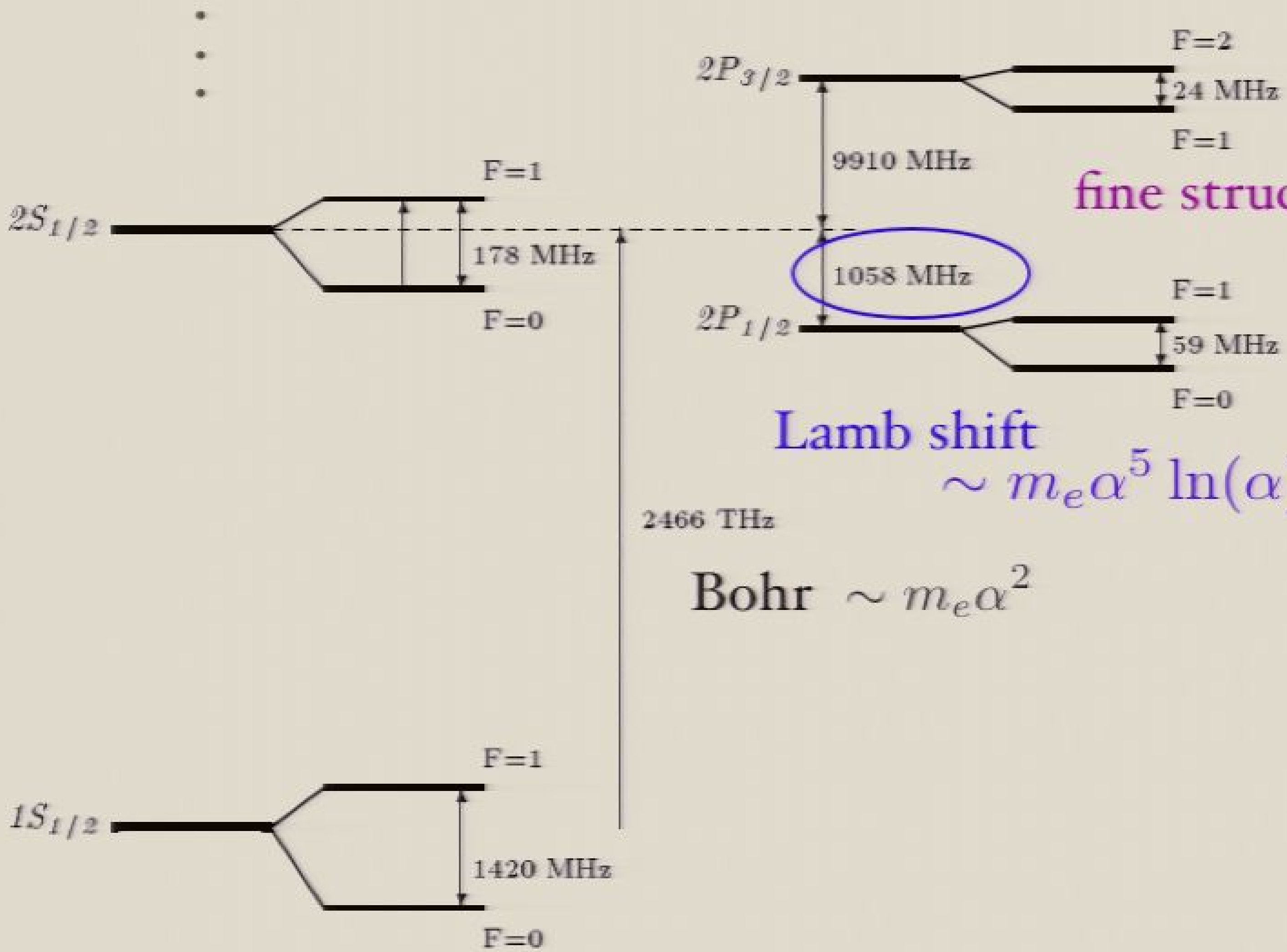

fine structure $\sim m_e \alpha^4$

Lamb shift $\sim m_e \alpha^5 \ln(\alpha)$ ← parameter fixed by QED

Bohr $\sim m_e \alpha^2$

NRQED

Effective Field Theory for Non-relativistic bound states

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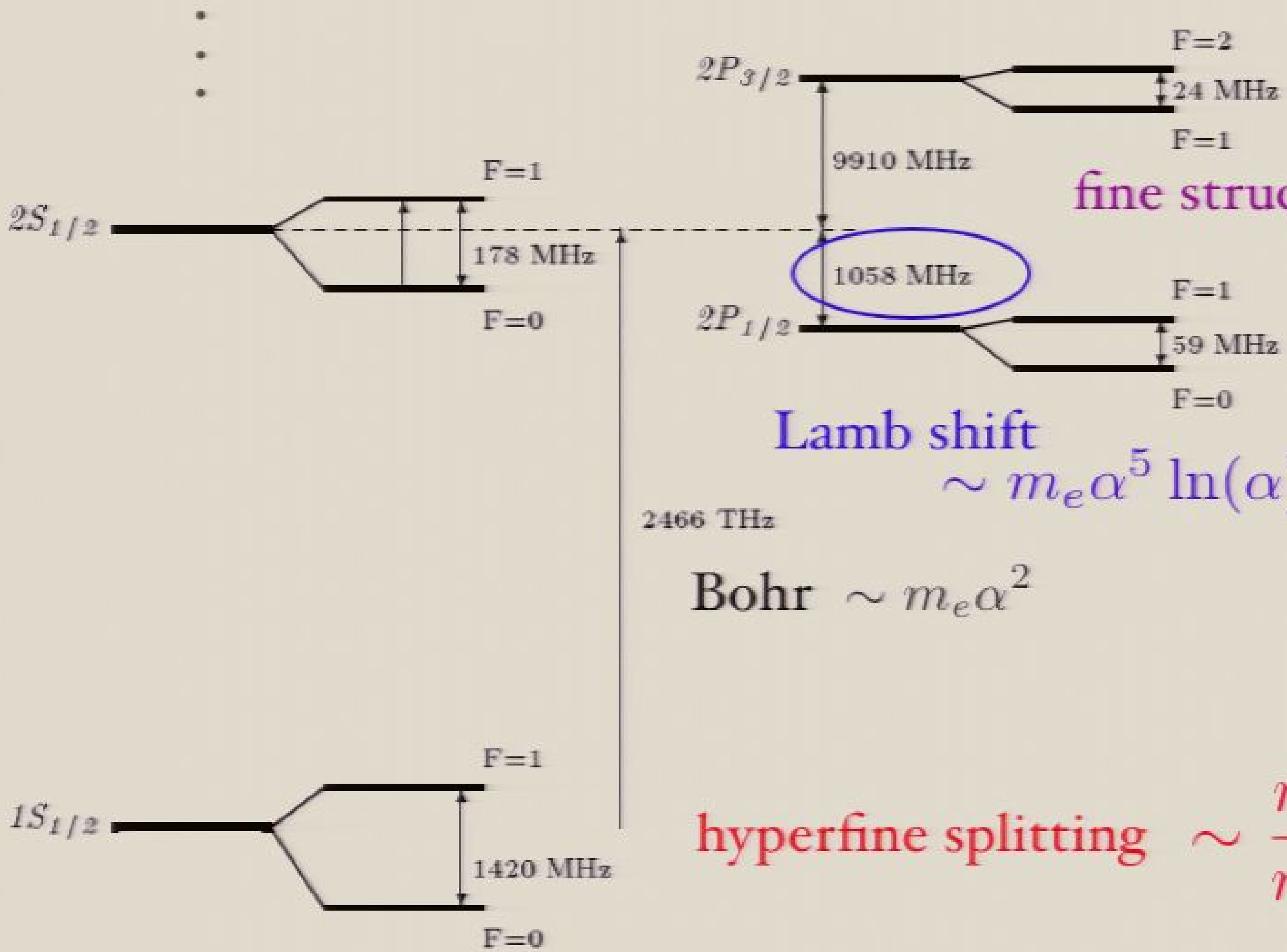


hyperfine

fine structure $\sim m_e \alpha^4$

hyperfine

Lamb shift $\sim m_e \alpha^5 \ln(\alpha)$ ← parameter fixed by QED

Bohr $\sim m_e \alpha^2$

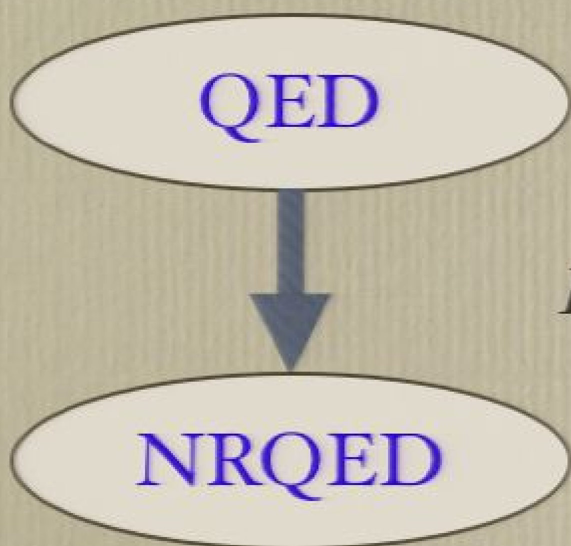
hyperfine splitting $\sim \frac{m_e^2}{m_p} \mu_e \mu_p \alpha^4$

QCD

Compute the H_m by "Matching"

Relativity: $\frac{p^4}{8m_e^3} + \ldots$

QED: $\mu_e \, , \quad \vec{L} \cdot \vec{S} \quad , \ldots$ (coefficients determined by $\alpha \, , m_e$)



$$H = H_0 + \sum_{m=1}^{\infty} \epsilon^m \, H_m$$

What about quarks?

$$\text{size} \sim 1\,\text{fm} \rightarrow 200\,\text{MeV} \gg p_\gamma$$

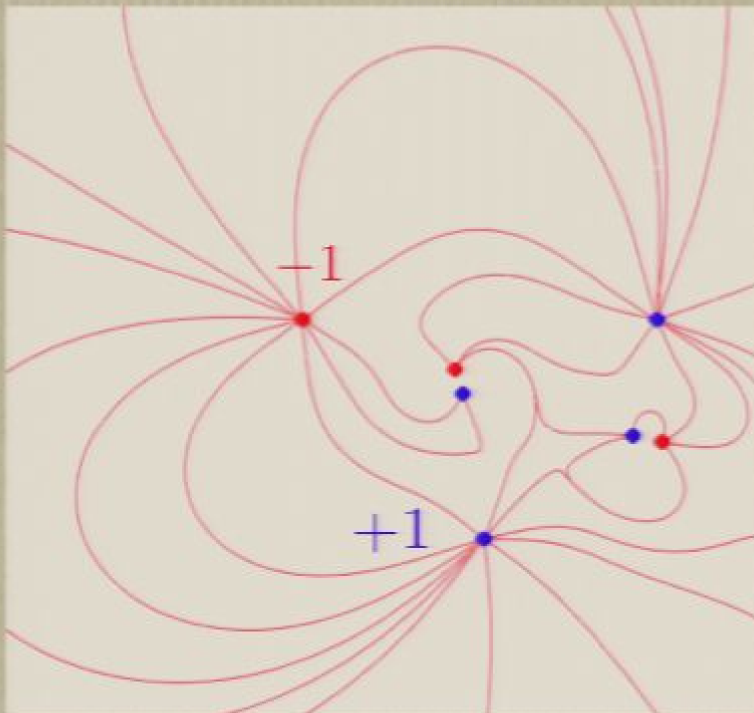
$Q_u = +2/3$

$Q_d = -1/3$

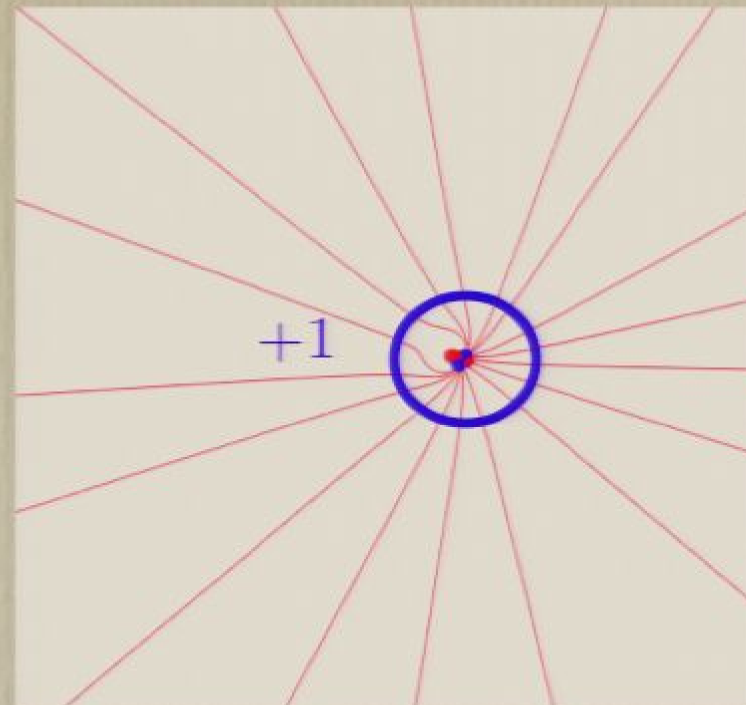
low momentum photons do not resolve the quarks, they see the proton charge

When matching couplings change too: $Q_{u,d} \rightarrow Q_p$

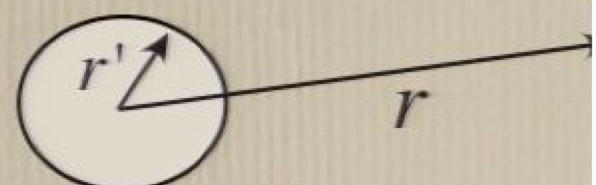
short distance



long distance



This is just an application of the multipole expansion, familiar from electromagnetism:



$$\mathcal{V}(\vec{r}) = \frac{1}{r}\underbrace{\int \rho \, d^3r'}_{\text{total charge}} + \frac{1}{r^2}\int r' cos\theta \rho \, d^3r' + \ldots$$

$$200\,\mathrm{MeV} \gg p_\gamma \;\Leftrightarrow\; r' \ll r$$

keV

What about quarks?

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When matching couplings change too: $Q_{u,d} \to Q_p$

→ other parameters: $m_p, \mu_p\ , \ldots$

in principle fixed by QCD, but it is more accurate to use experimental measurements

measure a parameter in one place, then use it in others!

= universality

Resolution μ

Resolution

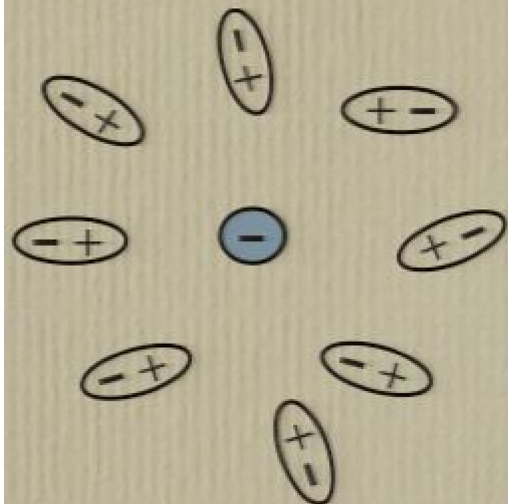
Resolution

Resolution

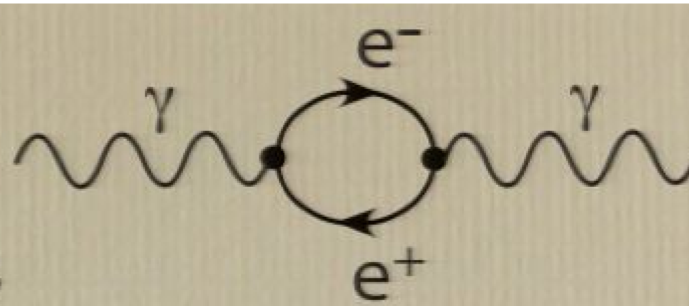
Resolution

Resolution

Vacuum Polarization



like a dielectric, gives screening



$$\alpha = \frac{e^2}{4\pi}$$

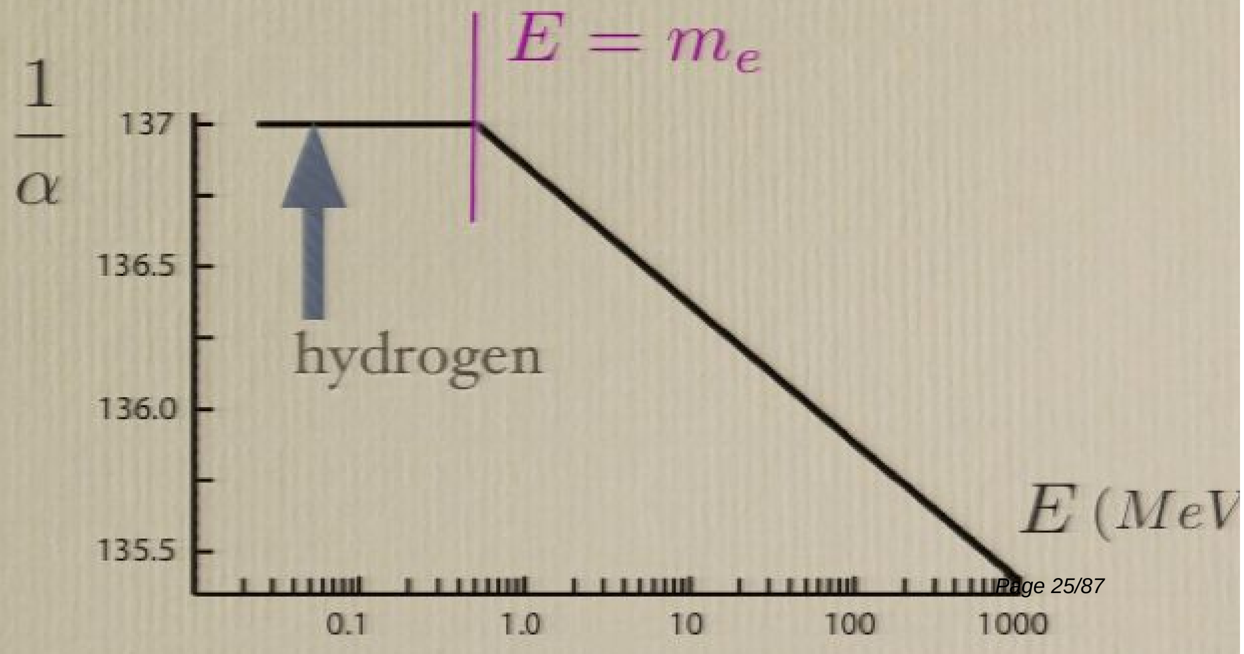
coupling is renormalized

resolution $\mu = E$

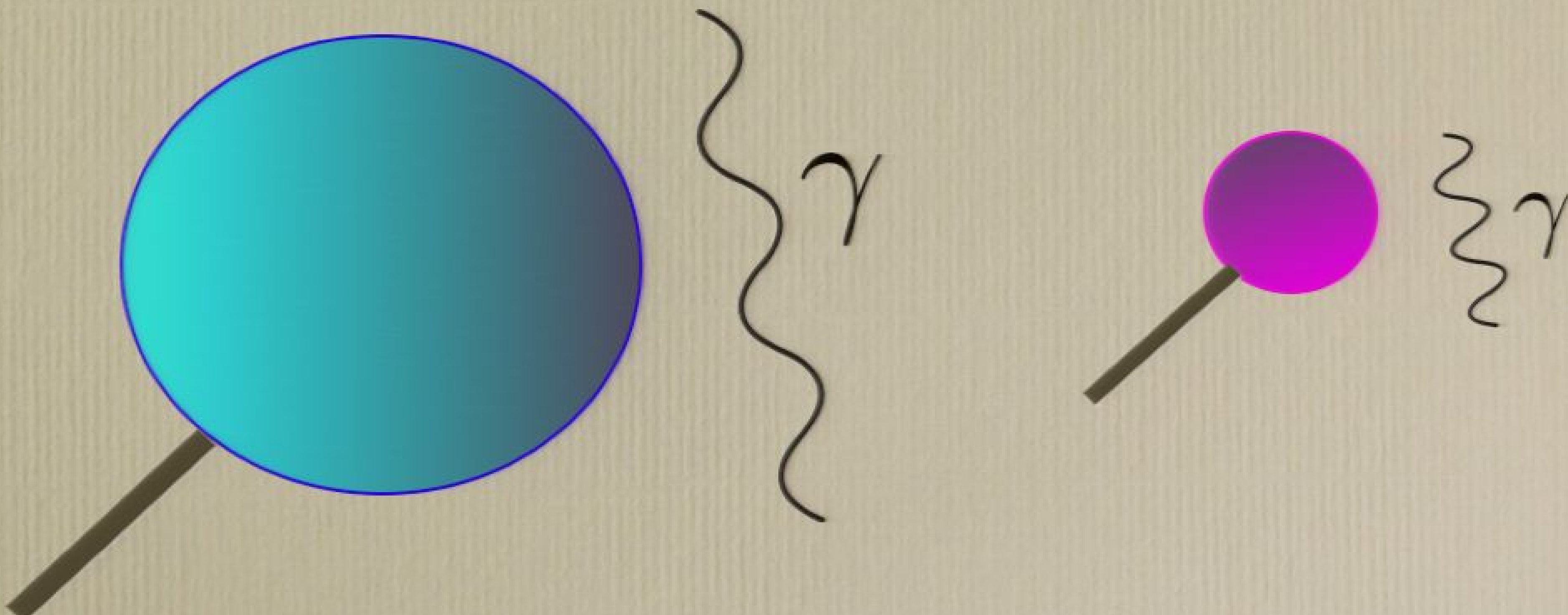
$$\mu \frac{d}{d\mu} \alpha(\mu) = \frac{2}{3\pi} \alpha^2(\mu)$$

at larger energy E, we probe shorter distances and see a larger charge

$$\alpha(E) = \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi} \ln\left(\frac{E^2}{m_e^2}\right)}$$

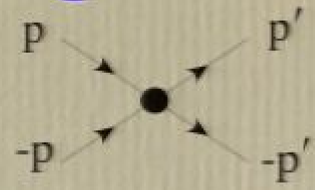


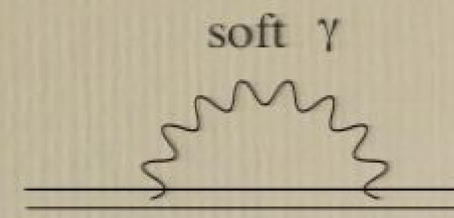
Long versus Short Distance



Lamb Shift in NRQED

Two parts:





i) effective potentials (short distance)

ii) radiation in the bound state (long distance)

$$\delta E_n = \left[\frac{4\alpha^2}{3m_e^2}|\psi_n(0)|^2 \ln\left(\frac{\mu}{m_e}\right) + \ldots\right] + \left[\frac{1}{m_e^2}\sum_{k\neq n} |\langle n|\hat{p}|k\rangle|^2 (E_k - E_n) \ln\left(\frac{\mu}{|E_n - E_k|}\right) + \ldots\right]$$

μ dependence cancels, but allows us to give separate meaning to the two pieces

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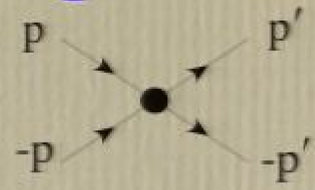

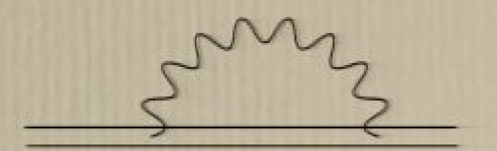

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History:

- 1947 Bethe computed ii), with $\mu = m_e$

  → large log: $\sim \ln\left(\frac{m_e}{m_e\alpha^2}\right) = -2\ln(\alpha)$

- 1949 French & Weisskopf
  Lamb & Kroll
  (Feynman, Schwinger)

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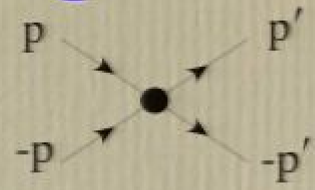

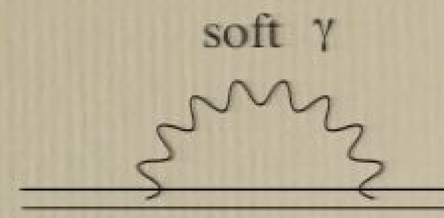

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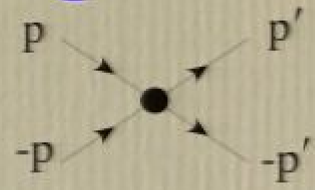

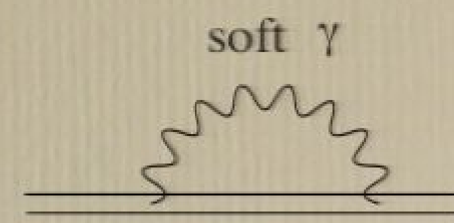

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  → large log: $\sim \ln\left(\frac{m_e}{m_e\alpha^2}\right) = -2\ln(\alpha)$

  $\Delta E(2S-2P) = 1040\,\text{MHz}$ close to the $1058\,\text{MHz}$ answer

- 1949 French & Weisskopf, Lamb & Kroll (Feynman, Schwinger) computed i) in QED and combined with ii)

  $\Delta E(2S-2P) = 1051\,\text{MHz}$

The structure of QED logs can be derived from a non-relativistic renormalization group

Luke, Manohar, Rothstein, I.S.

$$E = \frac{p^2}{2m}$$

energy resolution μ_E

momentum resolution μ_p

$$\mu_E \sim \frac{\mu_p^2}{m}$$

Correction	Observable	System	Comparison
$\alpha^8 \ln^3 \alpha$	Lamb shift	H	agrees*
		$\mu^+ e^-$, $e^+ e^-$	new
	(no h.f.s., no $\Delta\Gamma/\Gamma$)		
$\alpha^7 \ln^2 \alpha$	h.f.s.	H, $\mu^+ e^-$, $e^+ e^-$	agrees
	Lamb shift	H, $\mu^+ e^-$, $e^+ e^-$	agrees
$\alpha^3 \ln^2 \alpha$	$\Delta\Gamma/\Gamma$	$e^+ e^-$ ortho and para	agrees
$\alpha^6 \ln \alpha$	Lamb shift	H, $\mu^+ e^-$, $e^+ e^-$	agrees
	h.f.s.	H, $\mu^+ e^-$, $e^+ e^-$	agrees
$\alpha^2 \ln \alpha$	$\Delta\Gamma/\Gamma$	$e^+ e^-$ ortho and para	agrees
$\alpha^5 \ln \alpha$	Lamb shift	H, $\mu^+ e^-$, $e^+ e^-$	agrees

all from one equation

LO anomalous dimension: $\alpha^4 (\alpha \ln \alpha)^k$ stops at $k = 1$

NLO anomalous dimension: $\alpha^5 (\alpha \ln \alpha)^k$ stops at $k = 3$

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energy resolution μ_E

momentum resolution μ_p

$$\mu_E \sim \frac{\mu_p^2}{m}$$

NRQED methods are also used for the non-logarithmic terms

		Expt.(MHz)	Theory(MHz)	Agree?
H	Lamb	1057.845(9)	1057.85(1)	$< r_p^2 >$
	h.f.s	1420.405751768(1)	1420.399(2)	G_E, G_M
$\mu^+ e^-$	h.f.s	4463.30278(5)	4463.30288(55)	m_e/m_μ
$e^+ e^-$	Lamb	13012.4(1)	13012.41(8)	agree
	h.f.s	203389.10(74)	203391.70(50)	3σ
	$\Gamma_{\rm para}$	7990.9(1.7) μs^{-1}	7989.62(4) μs^{-1}	agree
	$\Gamma_{\rm ortho}$	7.0404(13) μs^{-1}	7.03996(2) μs^{-1}	agree

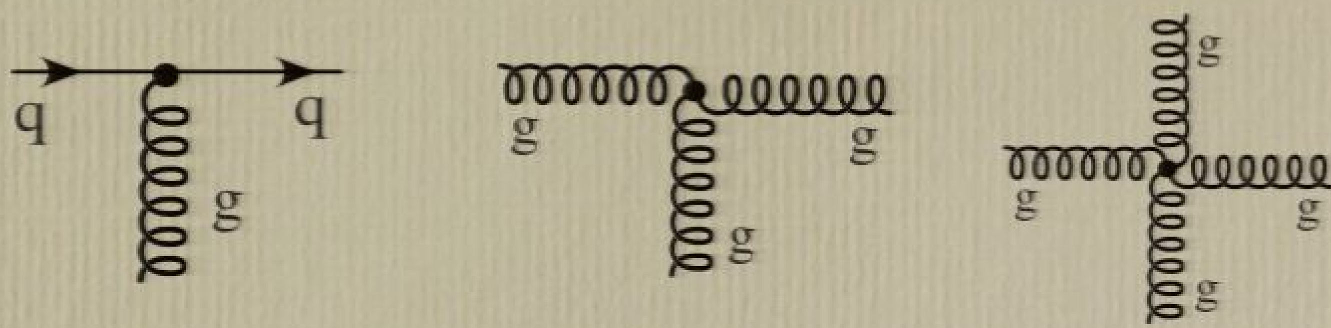
The ideas we've discussed in QED:

- resolution μ
- changes in degrees of freedom & couplings
- expansions, multiple scales
- universality

become even more crucial for QCD

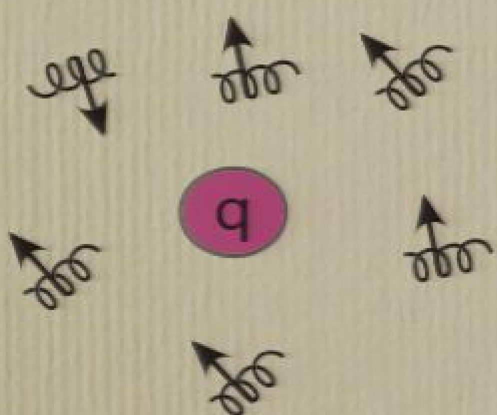
QCD Interactions are more complicated than QED:

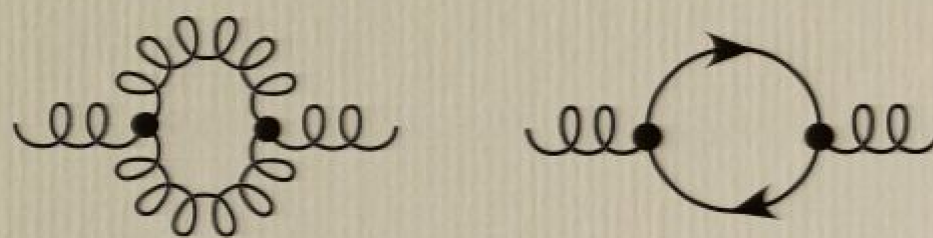
strong coupling: $g(\mu)$ $\alpha_s(\mu) = \frac{g(\mu)^2}{4\pi}$



these interactions involve the same coupling (gauge symmetry)

Vacuum response?





gluons have spin, carry color charge

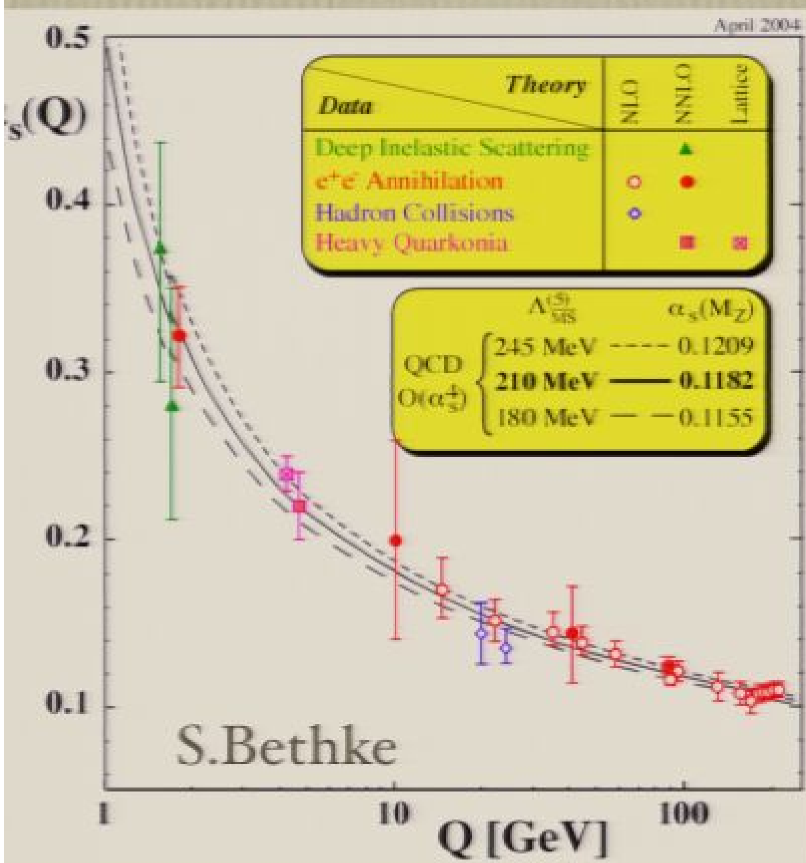
behave like a permanent magnet
anti-screen the charge

$$\beta(g) = \mu \frac{d}{d\mu} g(\mu) = -\frac{g(\mu)^3}{16\pi^2}\left(11 - \frac{2}{3}n_f\right) \quad < 0$$

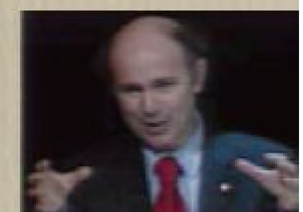
In QCD, the coupling , $g(\mu)$, behaves in the opposite way to QED, it gets weaker at short distances

slope is negative

$$\alpha_s(\mu) = \frac{g(\mu)^2}{4\pi} \qquad \beta(g) = \mu \frac{d}{d\mu} g(\mu) <$$



Gross, Politzer, Wilczek



Nobel Prize, 200

Asymptotic freedom

large $\mu = Q$, small α_s , free quarks

Infrared slavery

as $\mu = Q$ approaches a few $100\,\text{MeV}$ $(r \rightarrow 1\,\text{fm})$, the coupling gets large

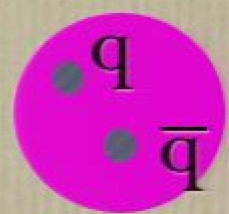
large change in the value

an expansion in $\alpha_s(\mu < 1\,\mathrm{GeV})$ is no good

- coupling gets so strong that quarks never escape unless they form a color singlet (bound) state with other quarks, ie. they are confined

Mesons

$\pi,\, K,\, \rho,\, \ldots$



Baryons

$p,\, n,\, \Sigma,\, \Delta,\, \ldots$

$r = \Lambda_{\mathrm{QCD}}^{-1}$

degrees of freedom change

an expansion in $\alpha_s(\mu < 1\,\mathrm{GeV})$ is no good

→ coupling gets so strong that quarks never escape unless they form a color singlet (bound) state with other quarks, ie. they are confined

Mesons

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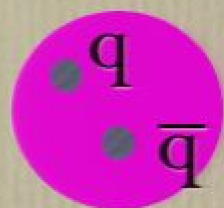

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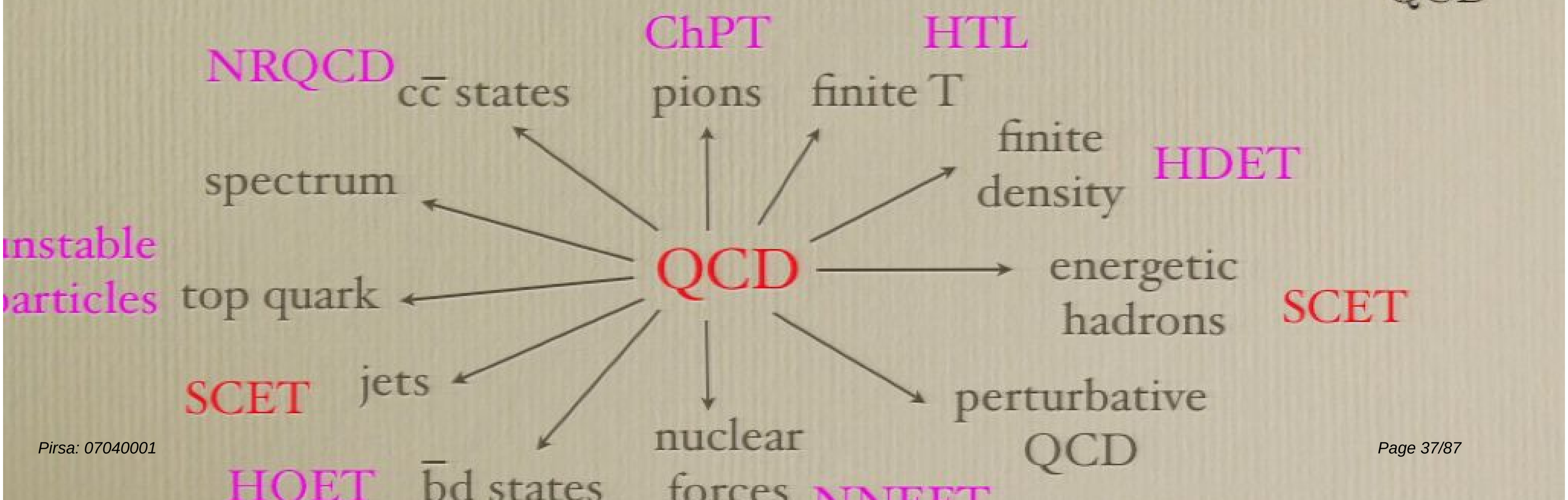
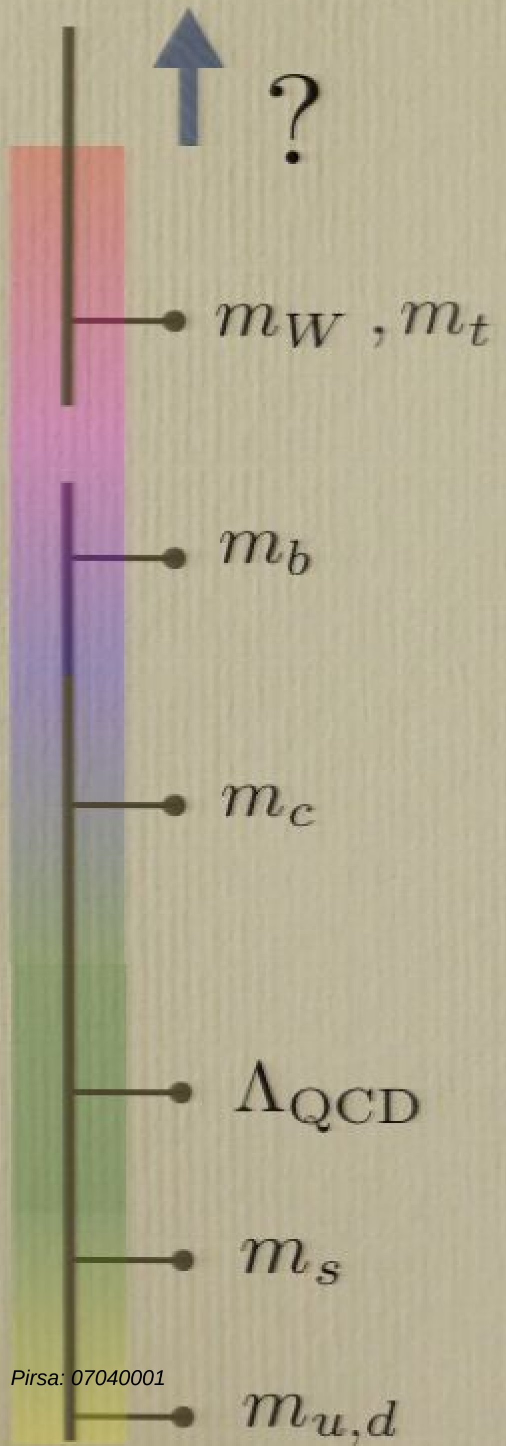
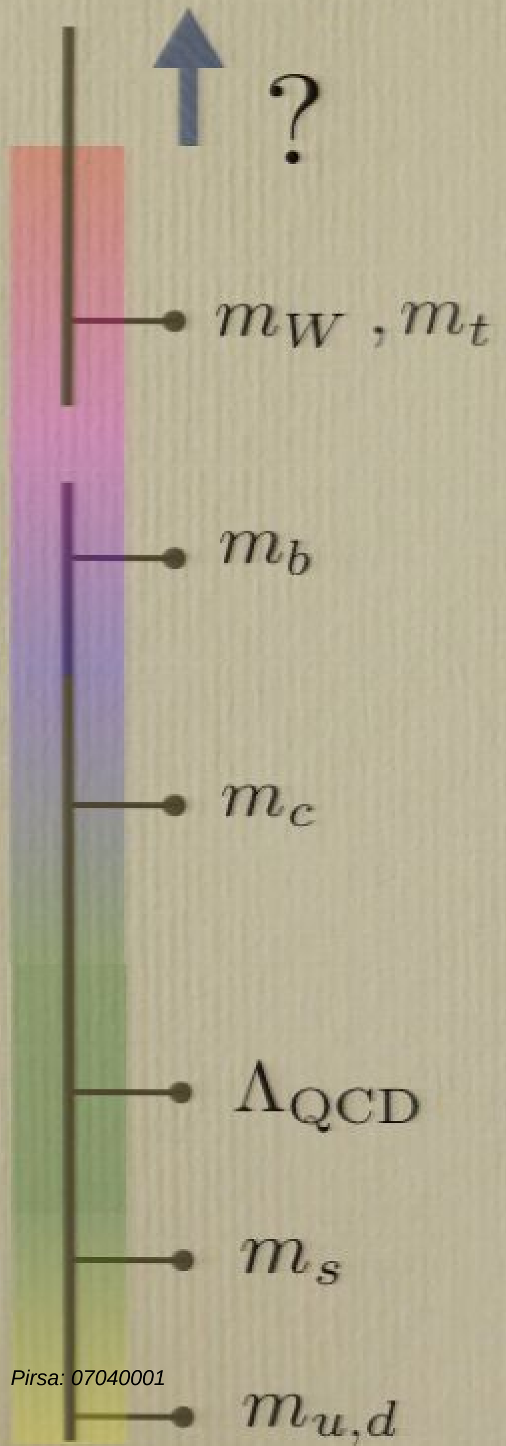
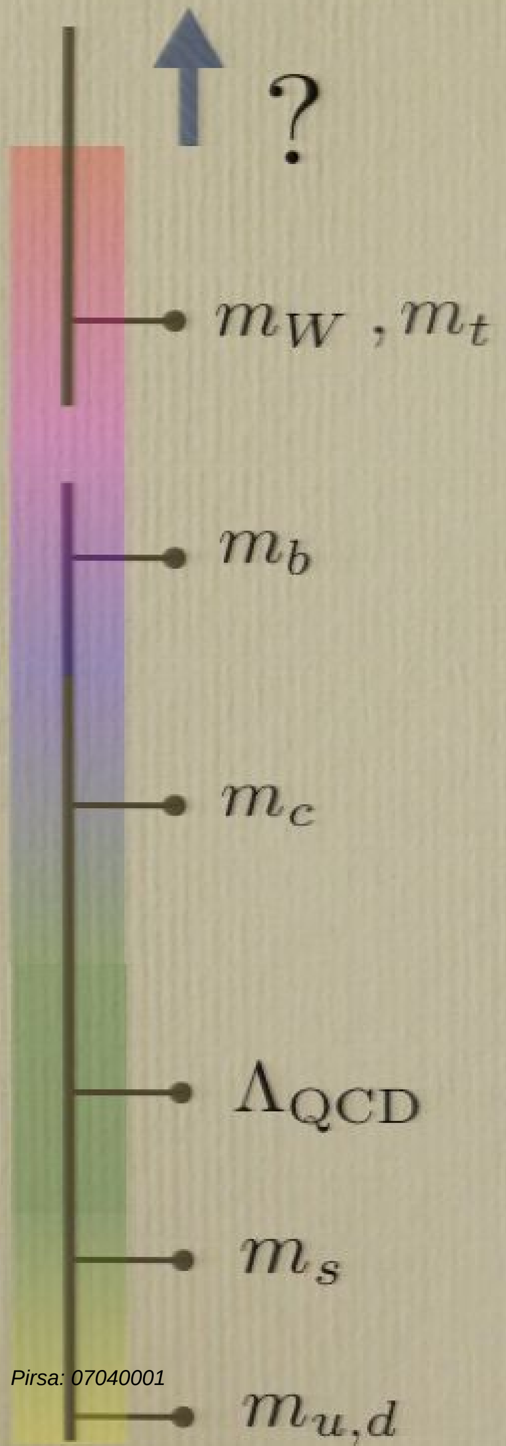



?
m_W , m_t
m_b
m_c
Λ_{QCD}
m_s
$m_{u,d}$

Is there a "Hydrogen Atom" for QCD?



Is there a "Hydrogen Atom" for QCD?



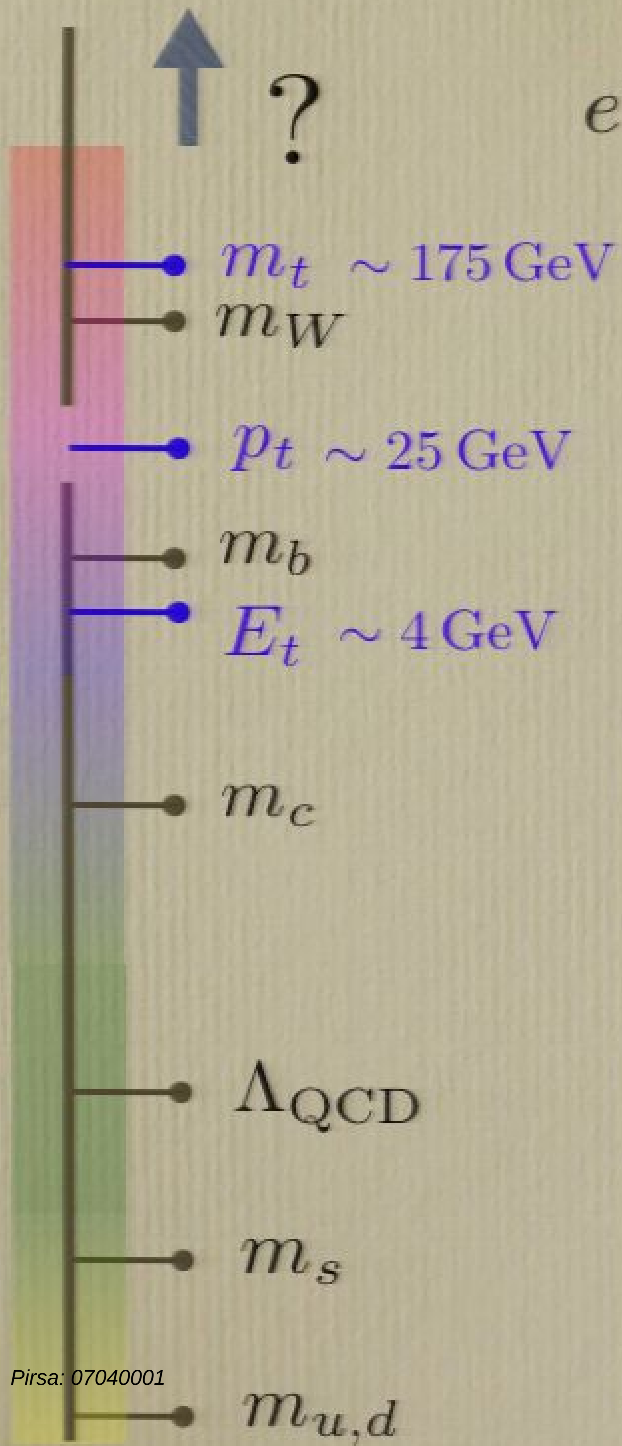
candidates: i) top quarks: $t\,\bar{t}$

ii) proton

iii) B mesons

$$B = (\bar{b}d)$$

Nonrelativistic QCD bound states?



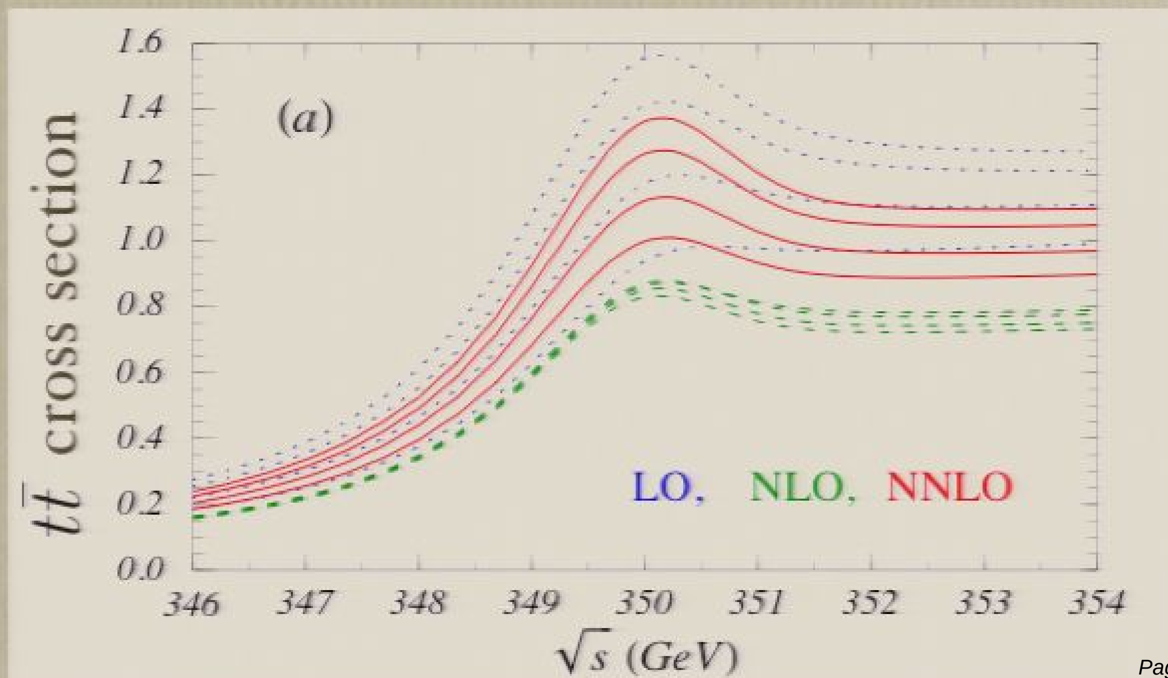
$$e^+e^- \to t\bar{t}$$

$$\Gamma_t = 1.4\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}}$$

top decays before it hadronizes

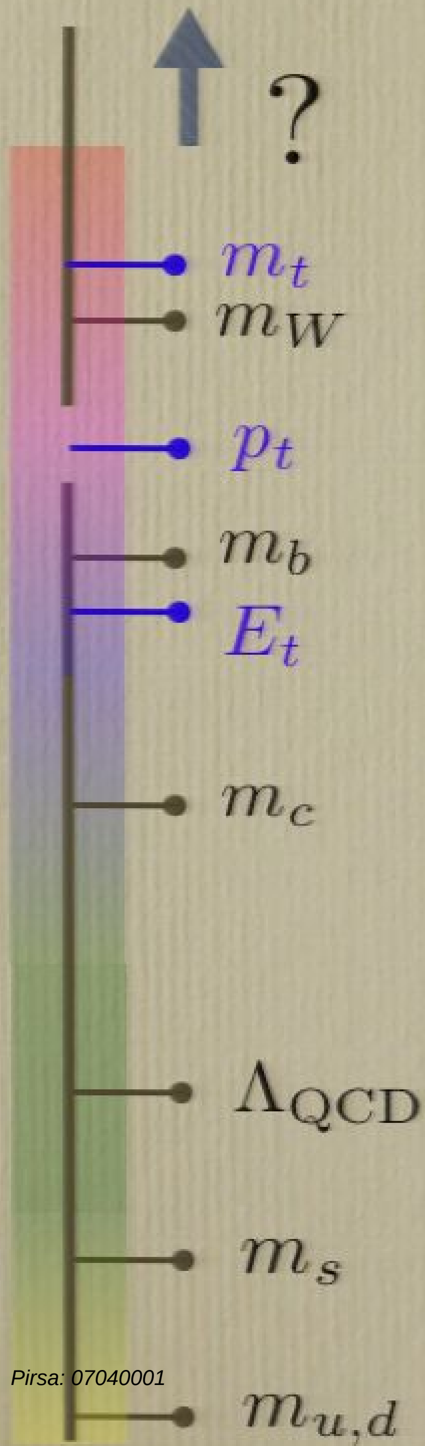
Coulombic, expansion in $\alpha_s(\mu)$:

$$\mathrm{LO} + \mathrm{NLO} + \mathrm{NNLO} + \dots$$



vary μ

$\mu = m_t, p_t, E_t$



$$e^+e^- \to t\bar{t}$$

Nonrelativistic QCD bound states?

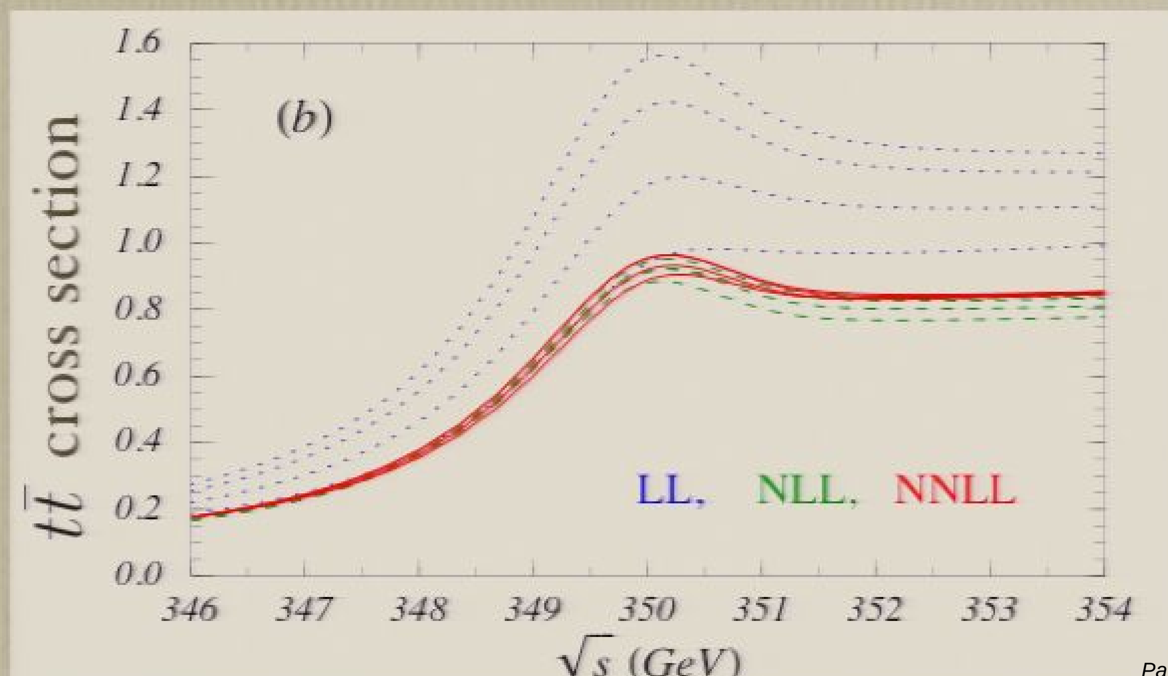
$$\Gamma_t = 1.4\,\mathrm{GeV} \gg \Lambda_{QCD}$$

top decays before it hadronizes

$$\mu \frac{d}{d\mu} C_i(\mu) = \ldots$$

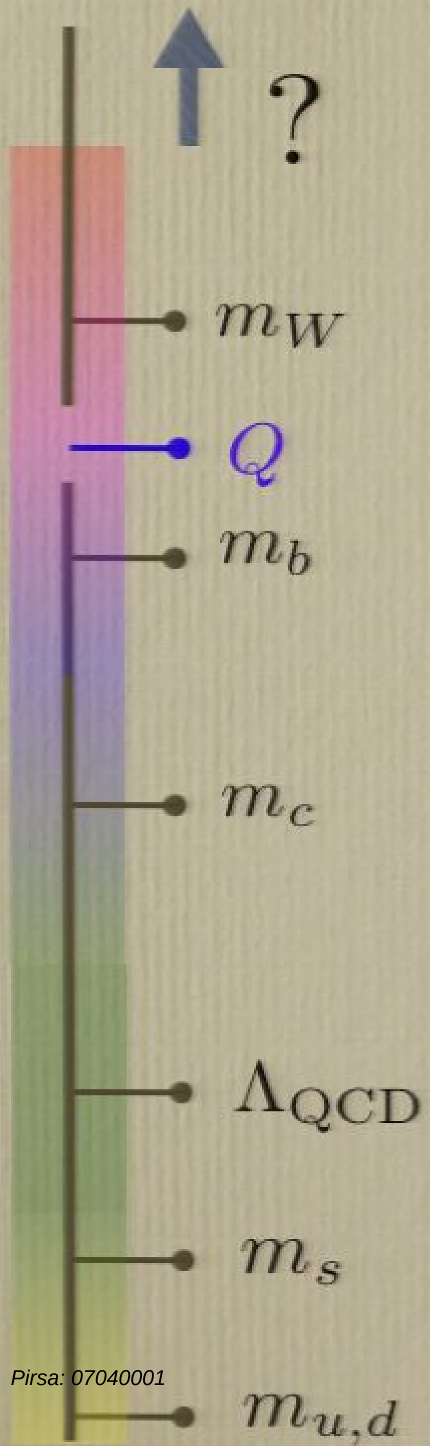
Determine the right scales

Hoang, Manohar
I.S., Teubner



vary μ

$\to m_t, y_t, \Gamma_t$



$$e^- p \to e^- X$$

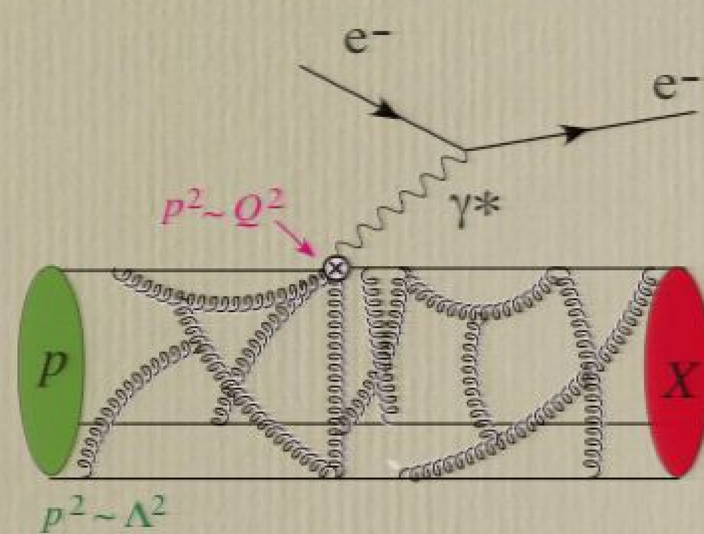
Deep Inelastic Scattering on a proton

A factorization theorem

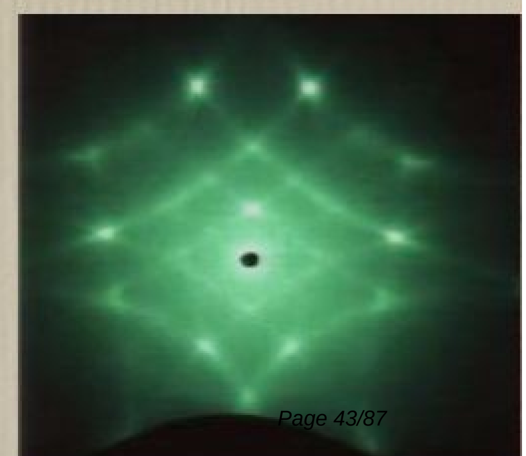
$$F_1(x, Q^2) = \frac{1}{x}\int_x^1 d\xi \; H(\xi/x, Q, \mu) \; f_{i/p}(\xi, \mu)$$

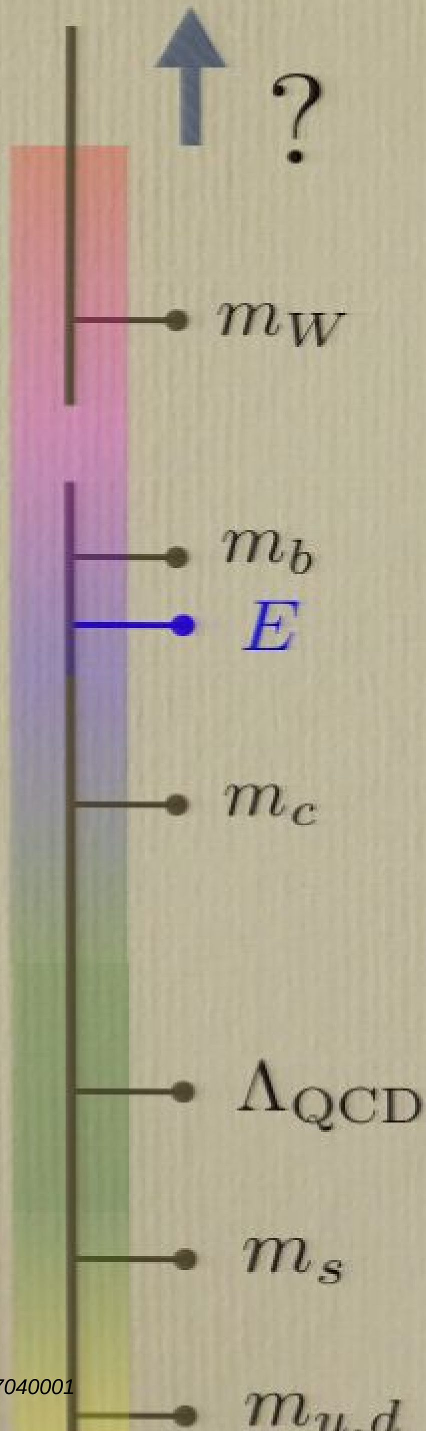
short distance process $p^2 \sim Q^2$

universal nonperturbative function $p^2 \sim \Lambda^2_{QCD}$

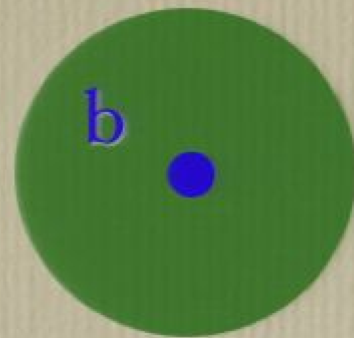


analogy: Bragg scattering of X-rays on a crystal, for this time scale the atoms are at rest





B-meson



$m_b \gg \Lambda_{QCD}$

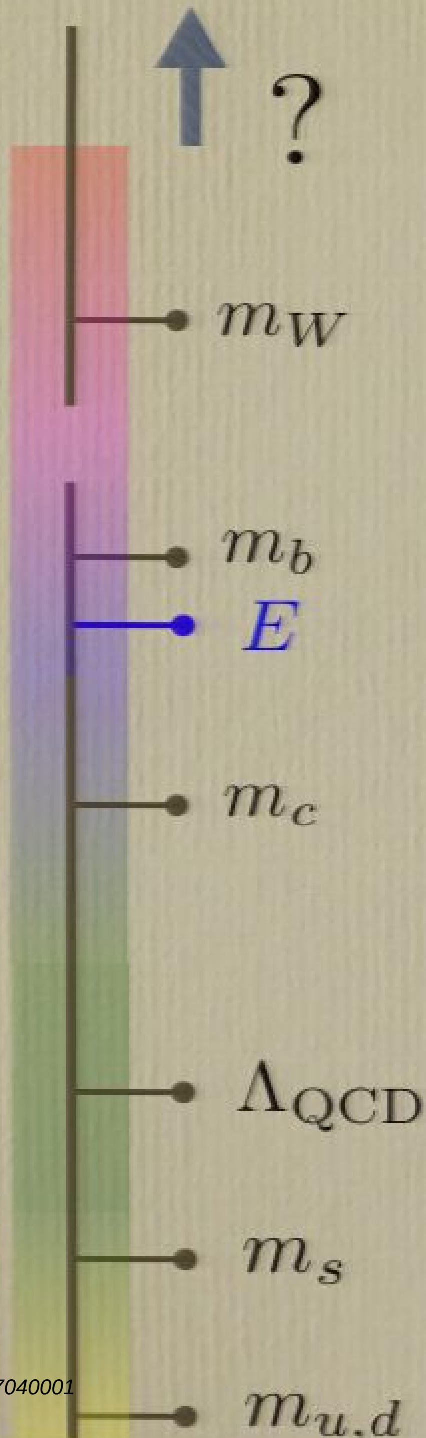
heavy quark symmetry

Isgur & Wise

Decay by weak interactions; long lived

Precision studies are sensitive to scales > m_W

The B is heavy, so many of its decay products are energetic, E



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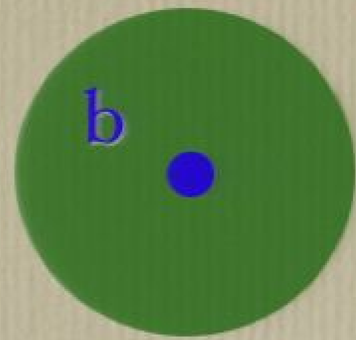

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Isgur & Wise

Decay by weak interactions; long lived

$B \to X_u \ell \bar{\nu}$ $\quad B \to D\pi$

$B \to \pi \ell \bar{\nu}$ $\quad B \to X_s \gamma$

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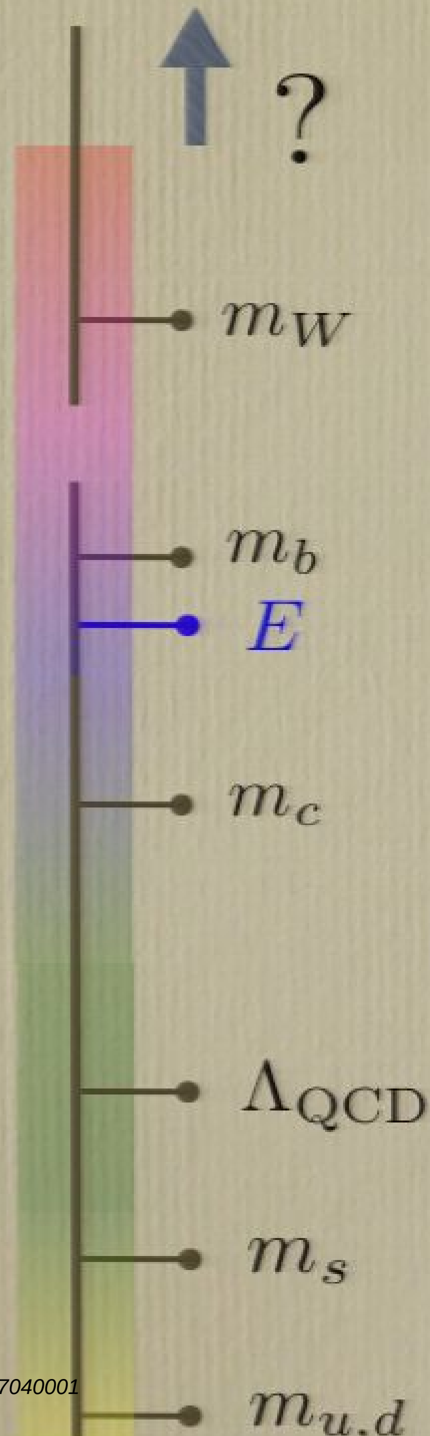

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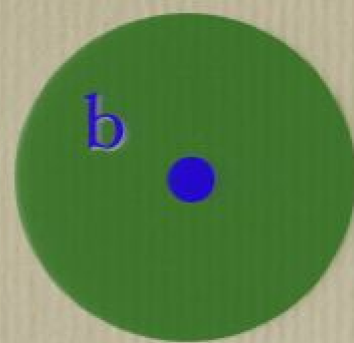

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$B \to X_u \ell \bar{\nu}$ $\quad B \to D\pi$ $\quad B \to K^* \gamma$

$B \to X_s \gamma$ $\quad B \to \rho\gamma$

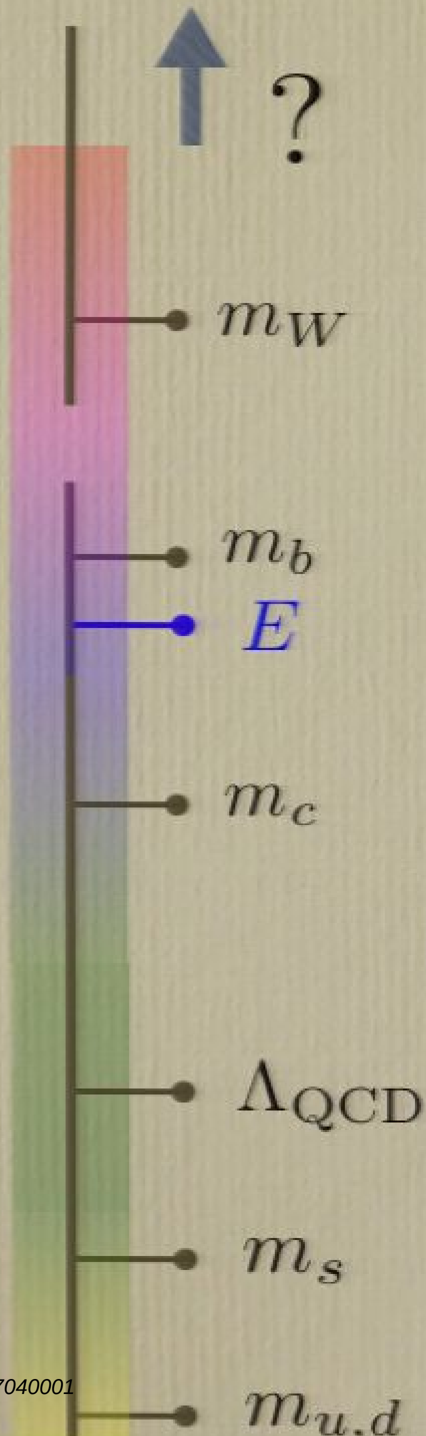
$B \to \pi \ell \bar{\nu}$

$B \to \rho\rho$ $\quad B \to K\pi$ $\quad B \to \pi\pi$

$B \to D^* \eta'$

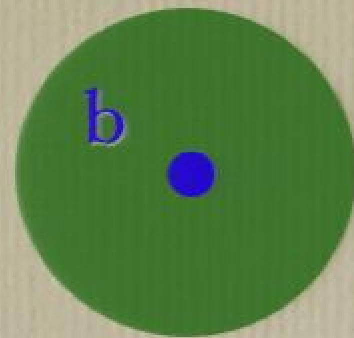
Precision studies are sensitive to scales > $\mathrm{m_W}$

The B is heavy, so many of its decay products are energetic, E



B-meson

$m_b \gg \Lambda_{QCD}$



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$B \to D^* \eta'$ $\quad B \to \rho\rho$ $\quad B \to \pi\pi$ $\quad B \to \gamma \ell \bar{\nu}$

$B \to K\pi$

Precision studies are sensitive to scales > m_W

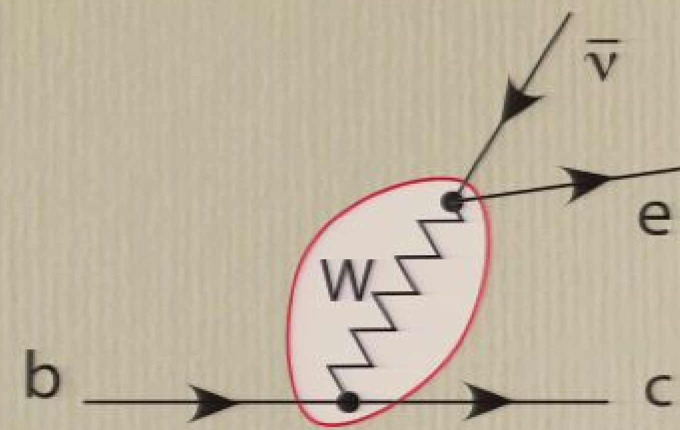
The B is heavy, so many of its decay products are energetic, E

eg. $B \to D\, e\, \bar{\nu}, \qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

1) Short Distance

$\mu = m_W \simeq 80\,\mathrm{GeV}$

gluons perturbative

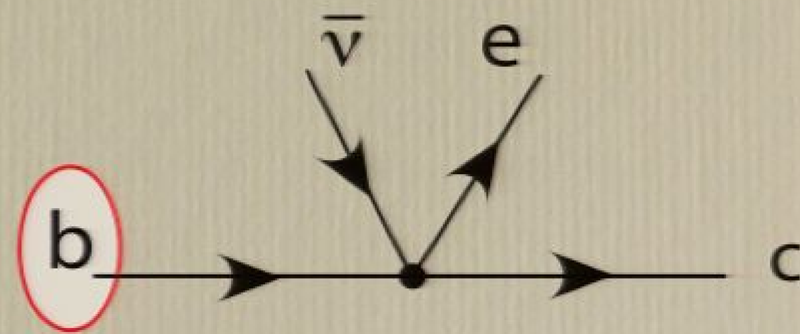


eg. $B \to D\, e\, \bar{\nu}$, $\qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

2) Intermediate Distance

$\mu = m_b \simeq 5\,\mathrm{GeV}$

gluons perturbative

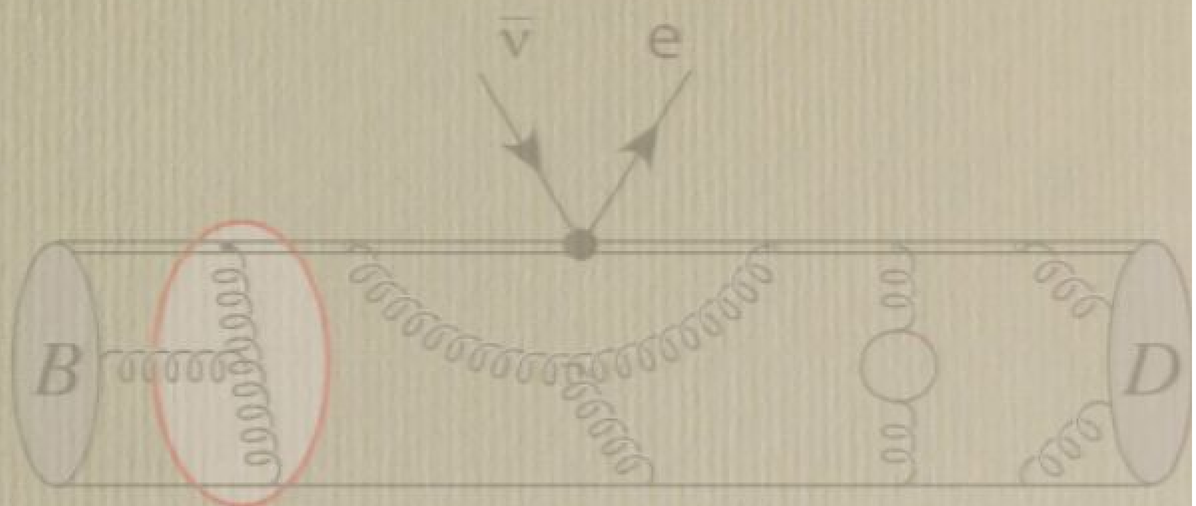


eg. $B \to D\, e\, \bar{\nu}$, $\qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

Long Distance

$= \Lambda \simeq 0.5\,\mathrm{GeV}$

uons nonperturbative

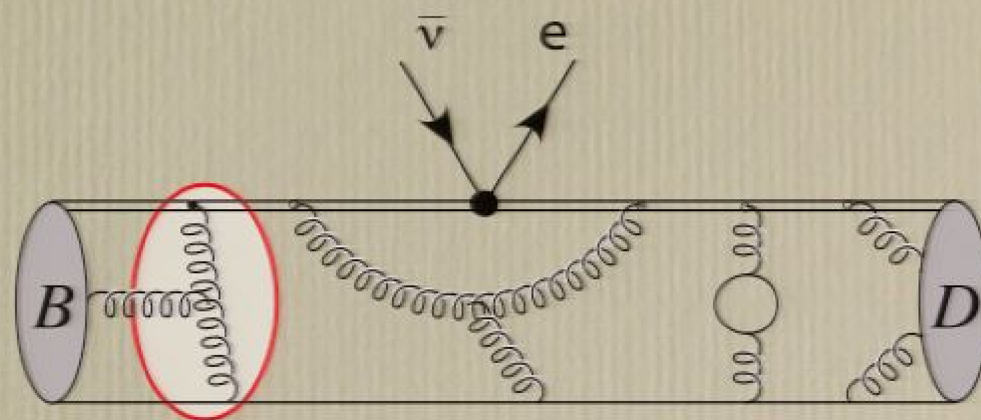


eg. $B \to D\, e\, \bar{\nu}, \qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

3) Long Distance

$\mu = \Lambda \simeq 0.5\,\mathrm{GeV}$

gluons nonperturbative

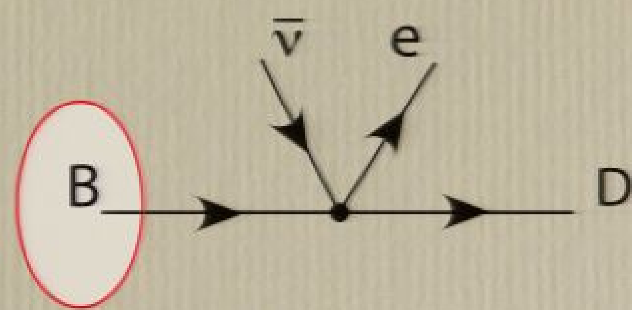


eg. $B \to D\, e\, \bar{\nu}, \qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

4) Very Long Distance

$\mu \ll \Lambda$

no gluons

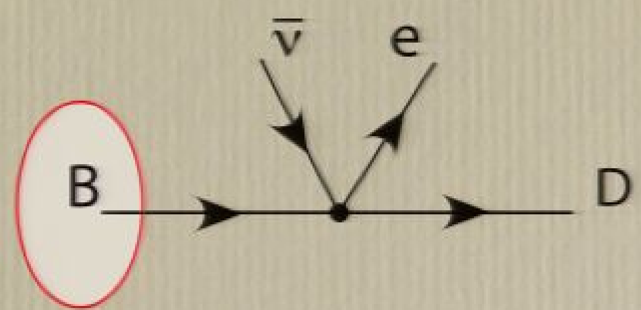


eg. $B \to D\, e\, \bar{\nu}$, $M_W^2 \gg m_b^2 \gg \Lambda^2$

4) Very Long Distance

$\mu \ll \Lambda$

no gluons



- Each of these pictures can be described by a field theory
- These theories can be matched together $H_1 \to H_2 \to H_3 \to H_4$
- At each μ we capture the most important physics

expansion parameters $\frac{m_b^2}{m_W^2} \simeq \frac{1}{250}$, $\alpha_s(m_b) \simeq 0.2$, $\frac{\Lambda}{m_b} \simeq 0.1$

Soft - Collinear Effective Theory

Bauer, Pirjol, I.S.
Fleming, Luke

An effective field theory for energetic hadrons & jets

$$E \gg \Lambda_{\rm QCD}$$

Analogy:

QED $\longleftrightarrow$ Quantum Mechanics (NRQED)

QCD $\longleftrightarrow$ SCET

Soft Collinear Effective Theory (SCET)

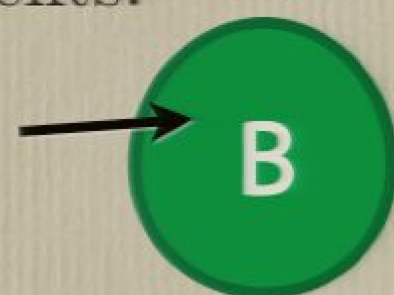
eg.

$E_\pi = 2.6\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}} \sim 0.3\,\mathrm{GeV}$

$m_B = 2E_\pi$

B has Soft constituents:

$p_s^\mu \sim \Lambda_{\mathrm{QCD}}$



Soft Collinear Effective Theory (SCET)

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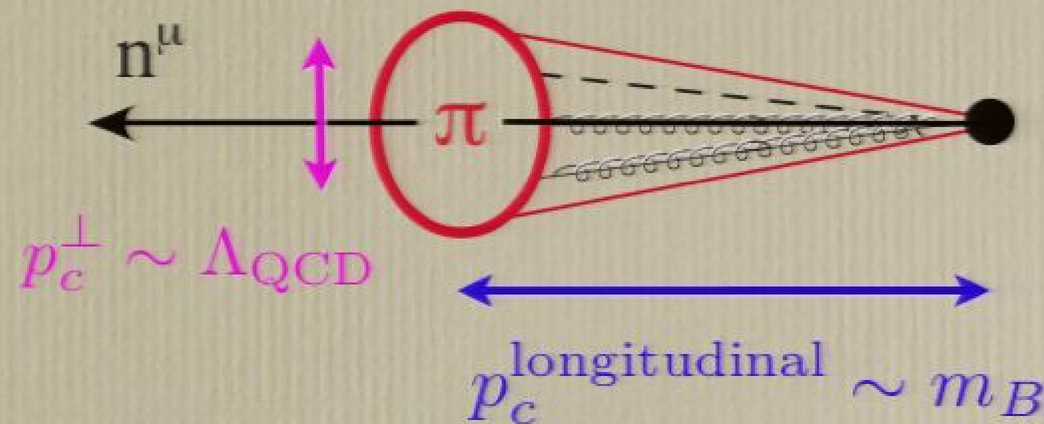

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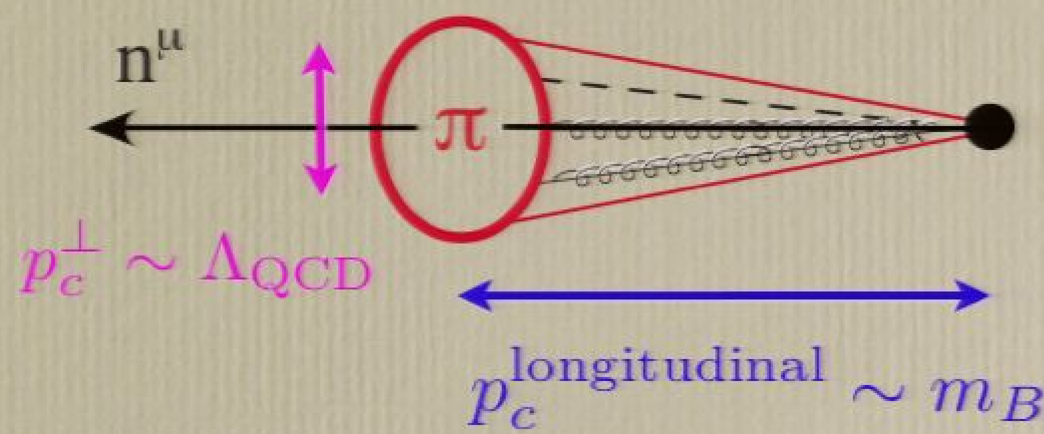

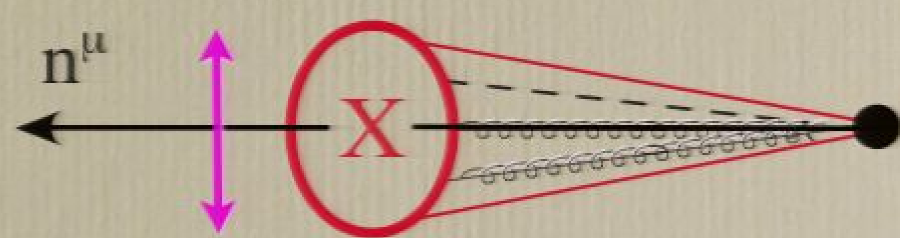

or replace π by a je
of many hadrons

$\Lambda_{\mathrm{QCD}} \ll p_c^\perp \ll m_B$

Soft Collinear Effective Theory (SCET)

eg.

$E_\pi = 2.6\,\mathrm{GeV} \gg \Lambda_{\rm QCD} \sim 0.3\,\mathrm{GeV}$ $m_B = 2E_\pi$

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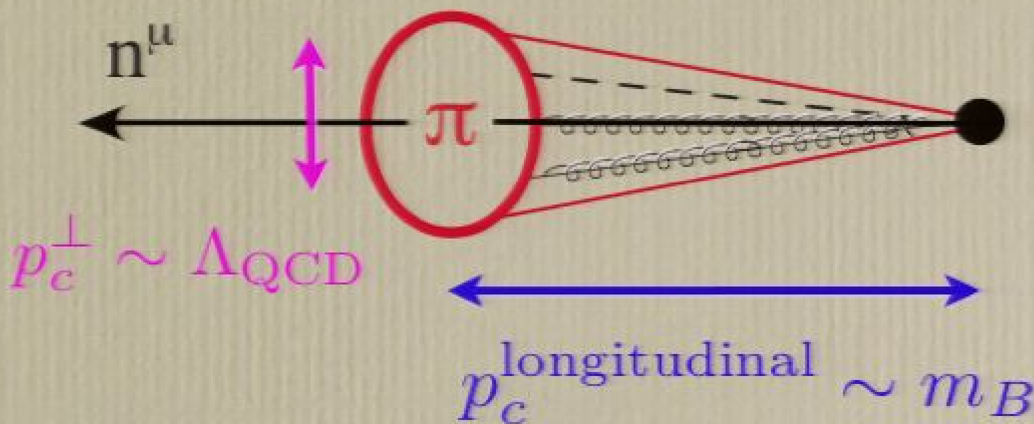

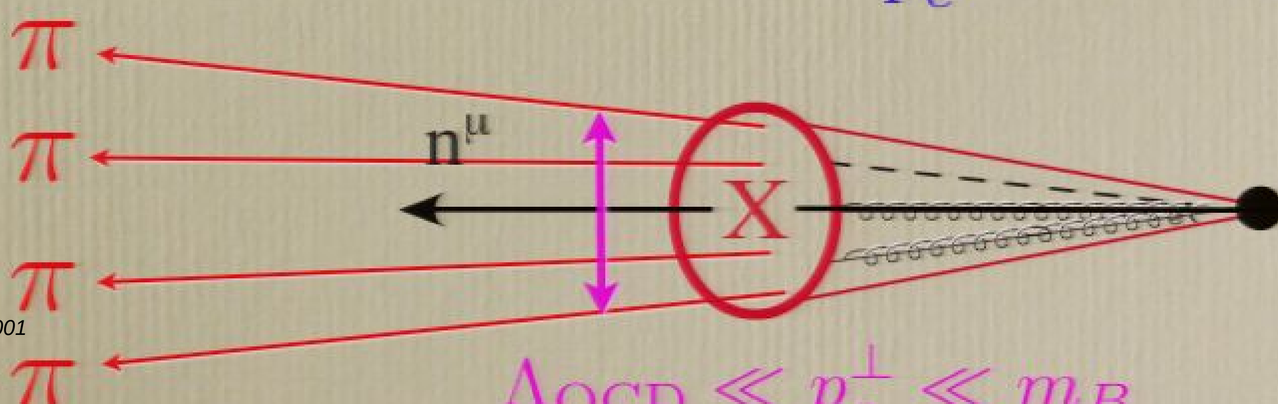

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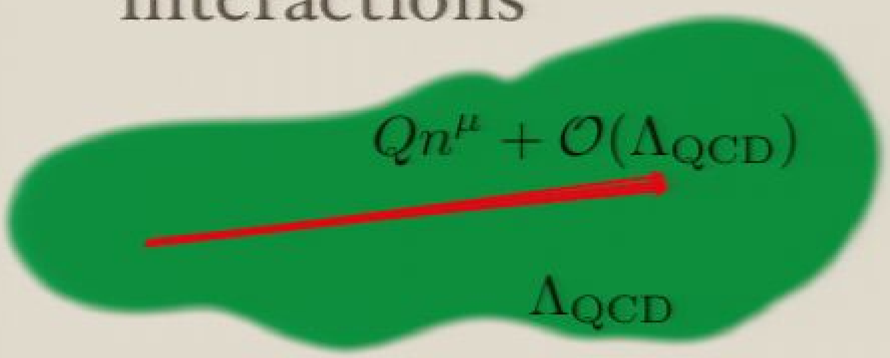
Soft Collinear Effective Theory (SCET)

eg.

$E_\pi = 2.6\,\text{GeV} \gg \Lambda_{\text{QCD}} \sim 0.3\,\text{GeV}$ $\qquad m_B = 2E_\pi$

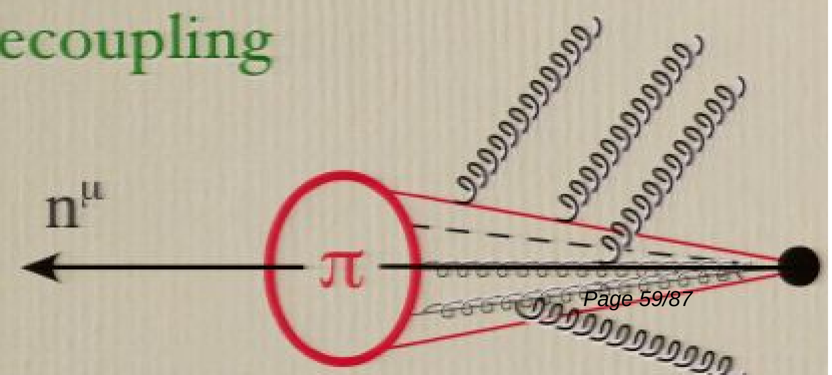
A field theory for
Soft & Collinear
interactions



organizes the interactions in a series expansion in $\frac{\Lambda_{\text{QCD}}}{E}$

(analog of the non-relativistic expansion in Q.M.)

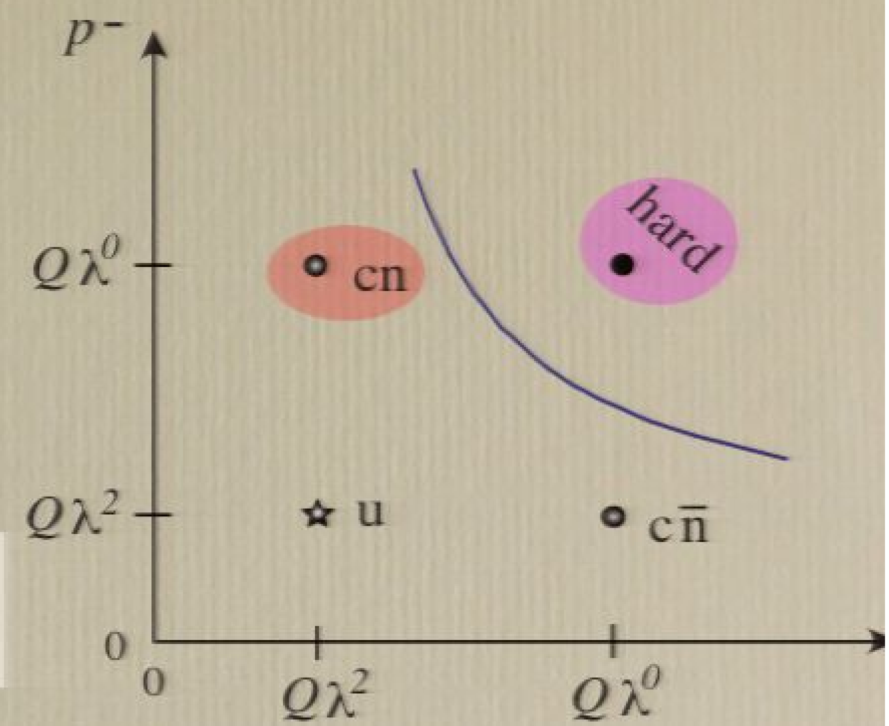
decoupling



SCET is a field theory which:

- explains how these degrees of freedom communicate with each other, and with hard interactions

$$F_1(x, Q^2) = \frac{1}{x}\int_x^1 d\xi\ H(\xi/x, Q, \mu)\ f_{i/p}(\xi, \mu)$$



SCET is a field theory which:

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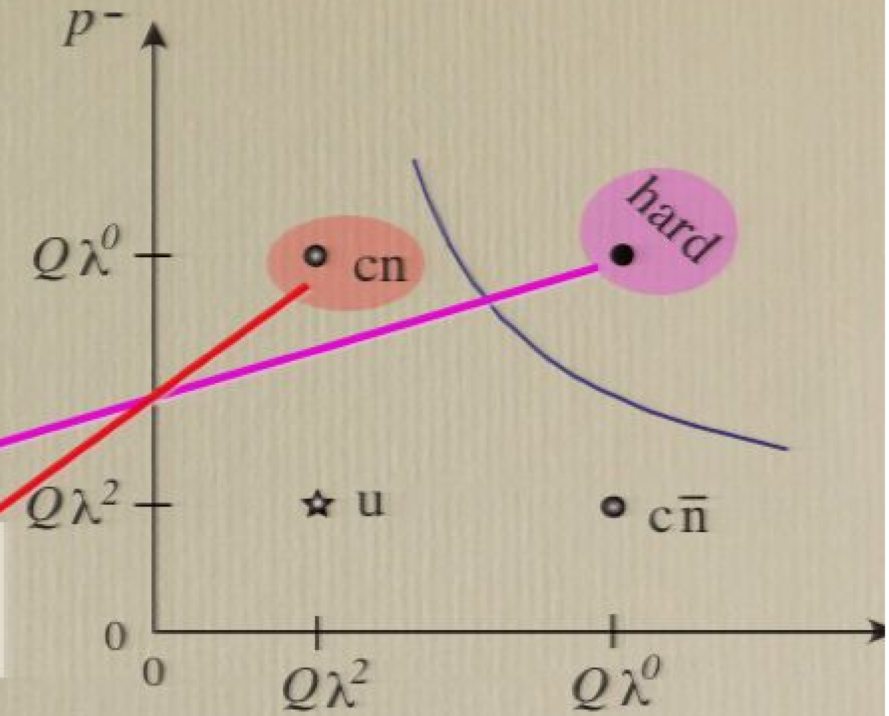

$$F_1(x, Q^2) = \frac{1}{x}\int_x^1 d\xi \; H(\xi/x, Q, \mu) \; f_{i/p}(\xi, \mu)$$

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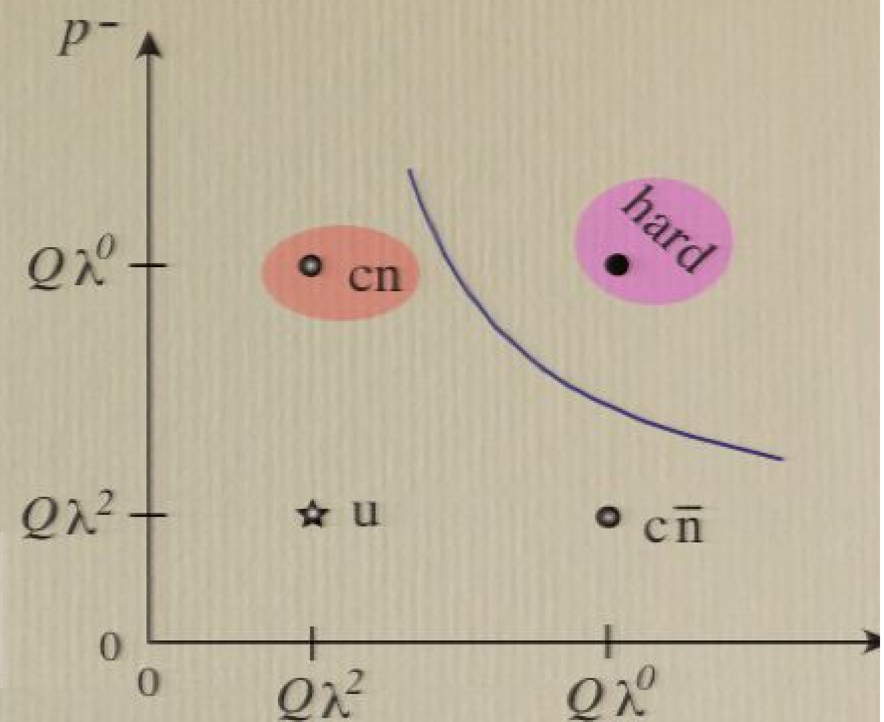

$$F_1(x, Q^2) = \frac{1}{x}\int_x^1 d\xi\, H(\xi/x, Q, \mu)\; f_{i/p}(\xi, \mu)$$

SCET is a field theory which:

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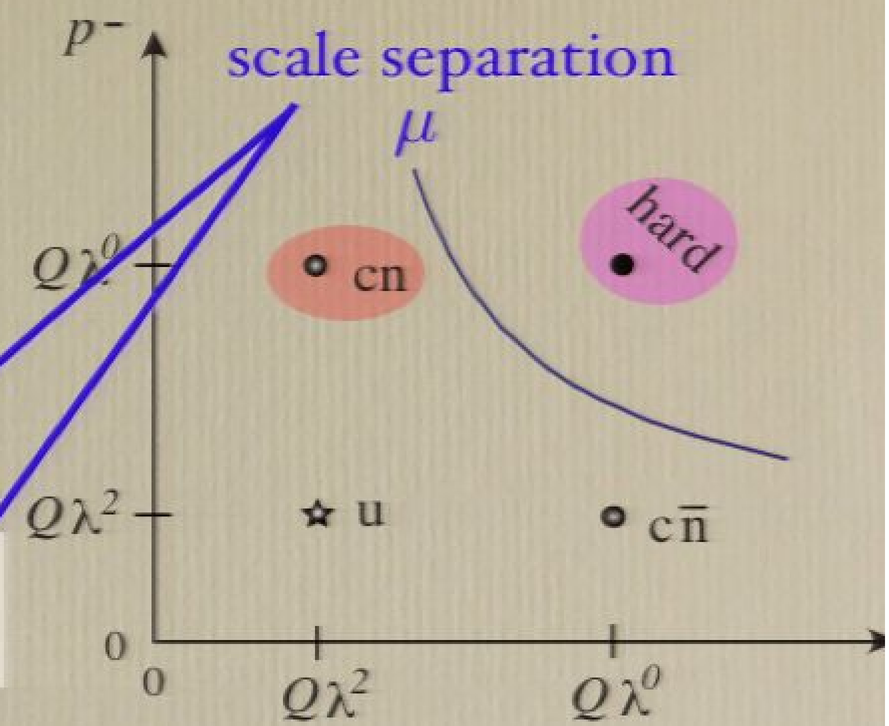

SCET is a field theory which:

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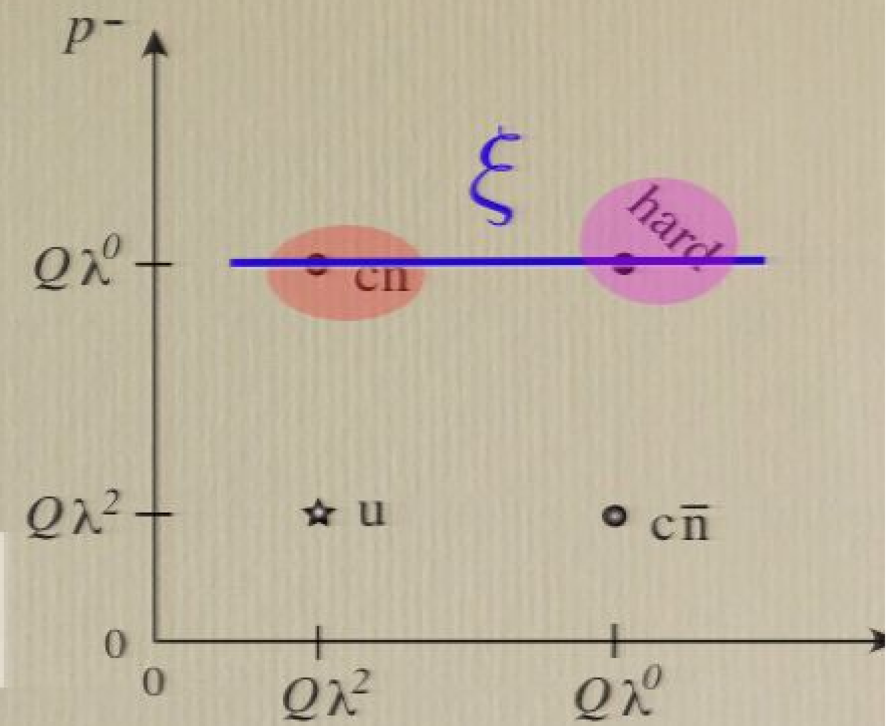


communicate by integrals

$$F_1(x, Q^2) = \frac{1}{x}\int_x^1 d\xi \; H(\xi/x, Q, \mu) \; f_{i/p}(\xi, \mu)$$

- provides a simple operator language to derive factorization theorems in fairly general circumstances

eg. unifies the treatment of factorization for exclusive and inclusive QCD processes

How is SCET used?

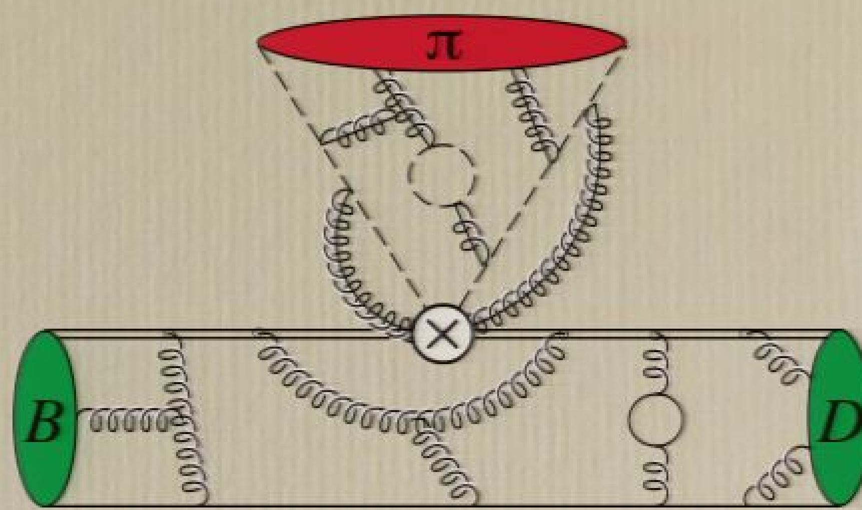
- cleanly separate short and long distance effects in QCD
  - ➡ derive new factorization theorems
  - ➡ find universal hadronic functions, exploit symmetries & relate different processes
- model independent, systematic expansion
  - ➡ study power corrections
- keep track of μ dependence
  - ➡ sum logarithms, reduce uncertainties

Factorization Example

$\bar{B}^0 \to D^+\pi^- \ , \ B^- \to D^0\pi^-$

B, D are soft , π collinear

$$\langle D\pi | H_{\rm weak} | B\rangle = N\xi(v\cdot v') \int_0^1 dx\ T(x,\mu)\ \phi_\pi(x,\mu)$$



SCET gives Universal functions
(analog of wavefunctions in Q.M.)

$\mathcal{L}_{\rm SCET} = \mathcal{L}_s^{(0)} + \mathcal{L}_c^{(0)}$ Factorization if $H_{\rm weak} = O_s \times O_c$

$\langle D^{(*)} | O_s | B\rangle = \xi(v\cdot v')$

$\langle \pi | O_c(x) | 0\rangle = f_\pi \phi_\pi(x)$

Calculate T, $\alpha_s(Q)$

$Q = E_\pi, m_b, m_c$

corrections will be $\Lambda/m_c \sim 30\%$

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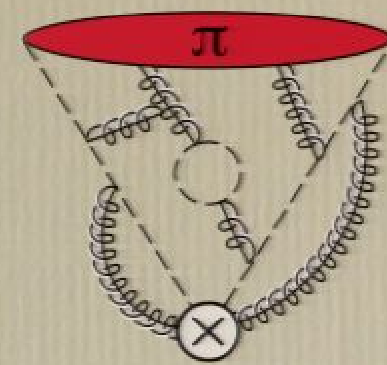

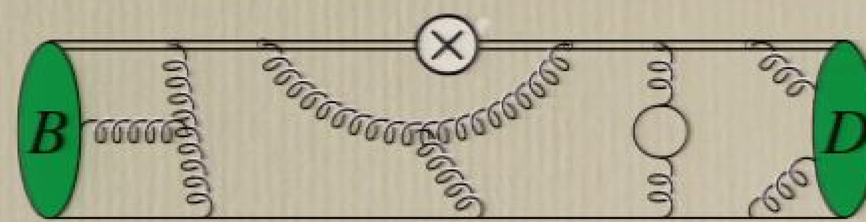

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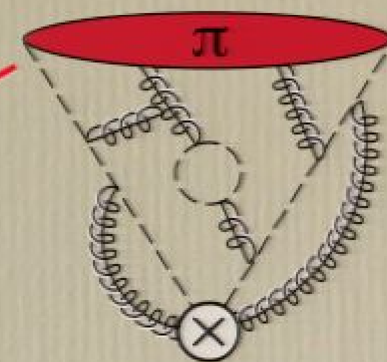

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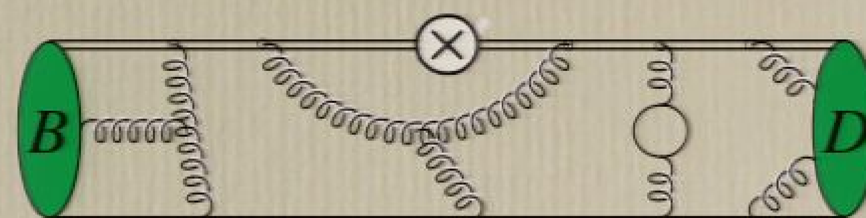

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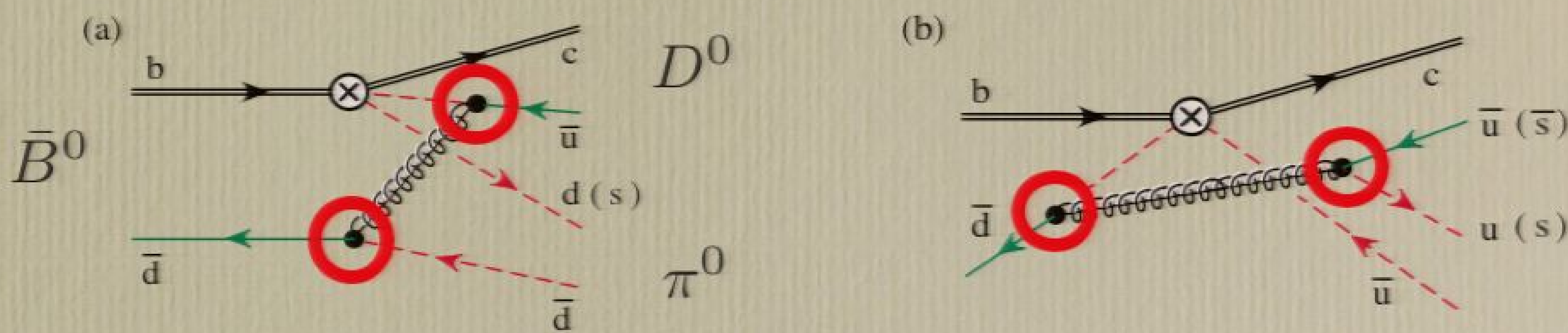
Color Suppressed Decays

Mantry, Pirjol, I.S.

$\bar{B}^0 \to D^0\pi^0$ Intractable without SCET

subleading interaction



$$A_{00}^{D^{(*)}} = N_0^{(*)} \int dx\, dz\, dk_1^+ dk_2^+\, T^{(i)}(z)\, J^{(i)}(z,x,k_1^+,k_2^+)\, S^{(i)}(k_1^+,k_2^+)\, \phi_M(x)$$

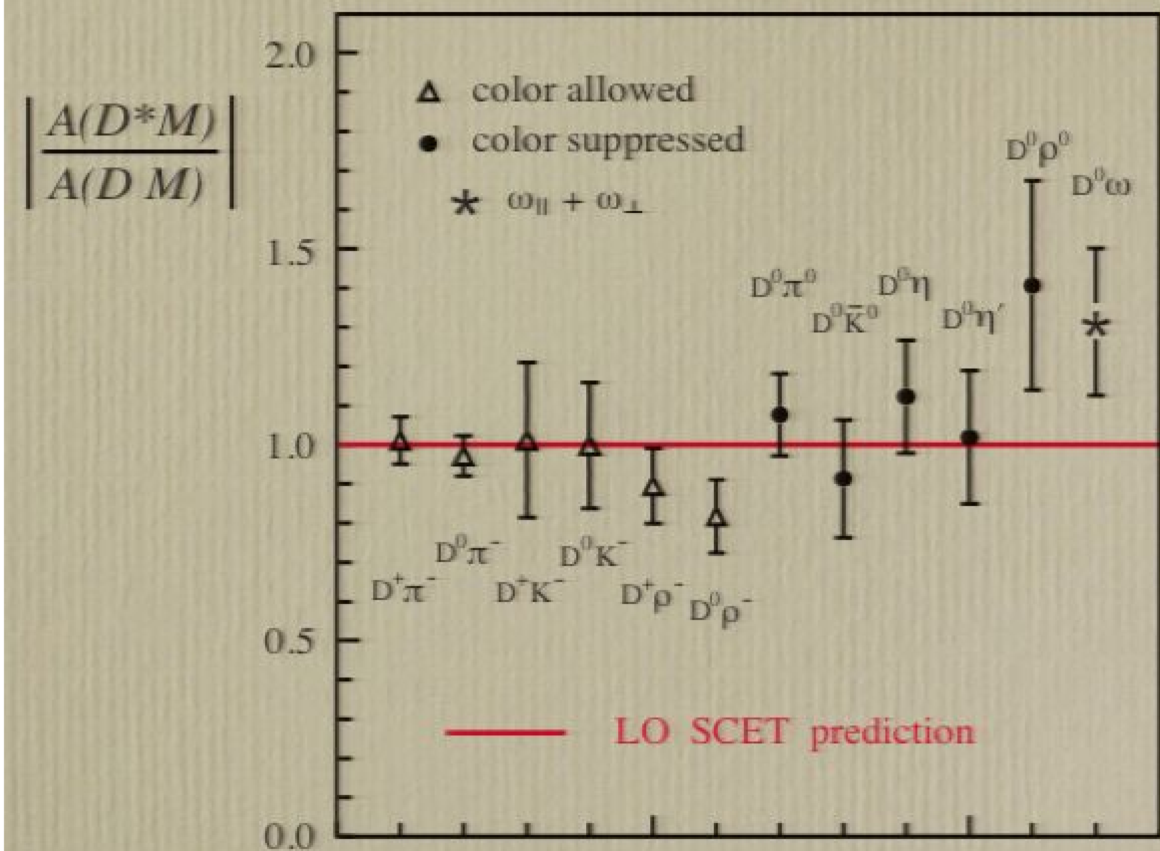
$$Q^2 \gg Q\Lambda \gg \Lambda^2$$

$$Q = m_b, E_\pi, m_c$$

prove S is same for D and D*

Comparison to Data

(Cleo, Belle, Babar)

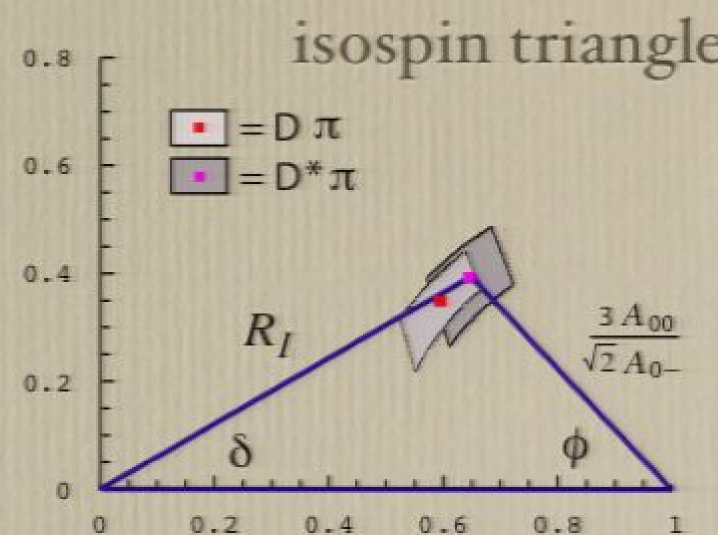


Extension to isosinglets:

Blechman, Mantry, I.S.

Extension to baryons (Λ_b) :

Leibovich, Ligeti, I.S., Wise

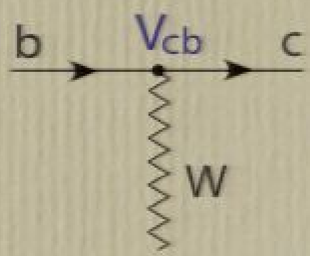


$$\delta(D\pi) = 30.4 \pm 4.8^\circ$$
$$\delta(D^*\pi) = 31.0 \pm 5.0^\circ$$

Not yet tested:

- $Br(D^*\rho^0_{\parallel}) \gg Br(D^*\rho^0_{\perp})$, $Br(D^{*0}K^{*0}_{\parallel}) \sim Br(D^{*0}K^{*0}_{\perp})$
- equal ratios $D^{(*)}K^*$, $D_s^{(*)}K$, $D_s^{(*)}K^*$; triangles for $D^{(*)}\rho$, $D^{(*)}K$

$B \to \pi\pi$ Decays & Weak Interactions



CKM Matrix

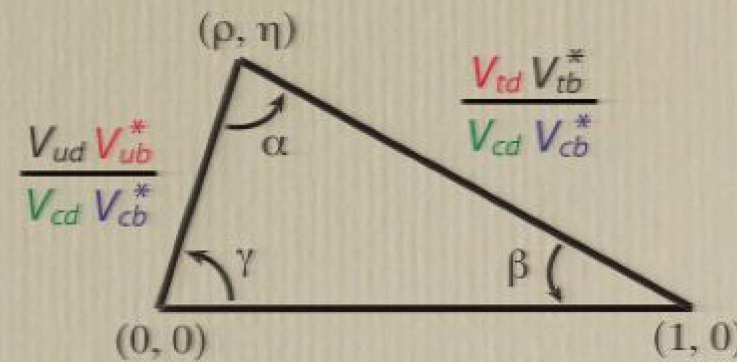
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Violate

C: exchange of particles & antiparticles

P: parity $\vec{x} \to -\vec{x}$

CP:



Can use CP-violating observables in $B \to \pi\pi$ to measure γ ,

but need to control QCD interactions

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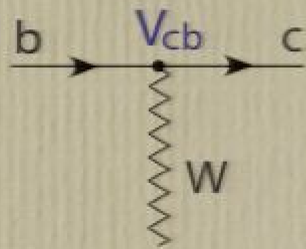

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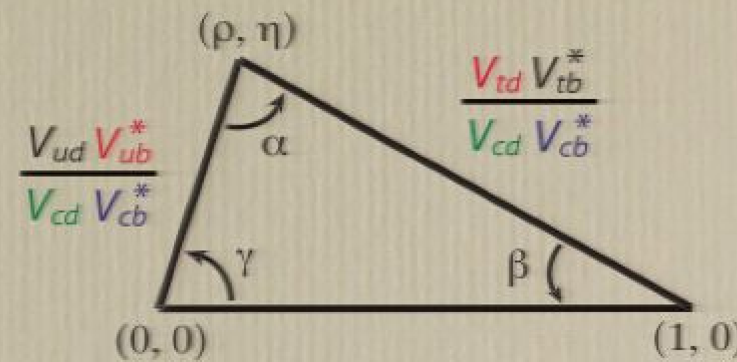

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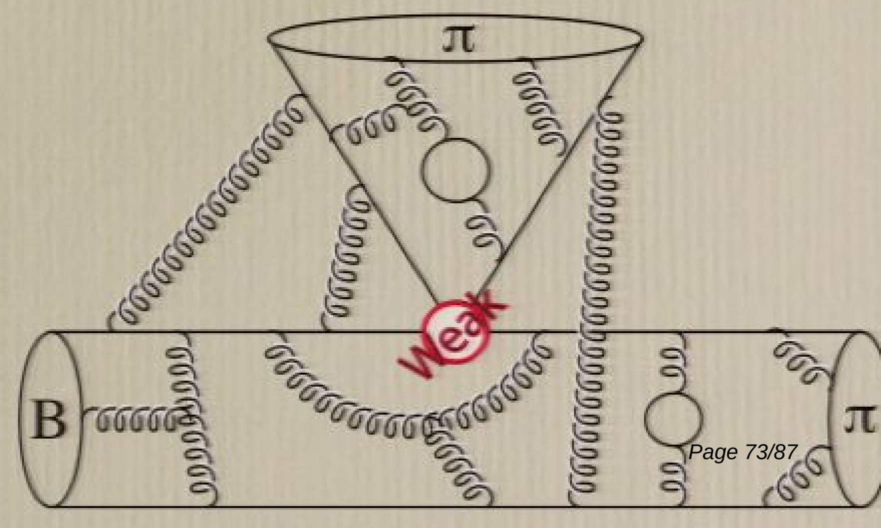
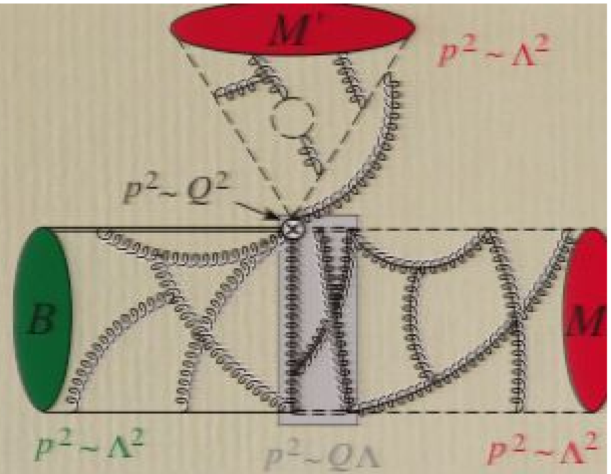


Factorization with SCET

Bauer, Pirjol, Rothstein, I.S.;

Beneke, Buchalla, Neubert, Sachrajda



Resolution $\mu = m_b$

Nonleptonic $B \to M_1 M_2$ ($\sim$ 120 channels)

$$A(B \to M_1 M_2) = A^{c\bar{c}} + N\Big\{ f_{M_2} \zeta^{BM_1} \int du\, T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_2} \int du dz\, T_{2J}(u,z) \zeta_J^{BM_1}(z) \phi^{M_2}(u) + (1 \leftrightarrow 2) \Big\}$$

Form Factors

$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E)$$

$$+\, C(E)\, \zeta^{BM}(E)$$

→ universality at $E\Lambda$

$B \to \pi \ell \bar{\nu}$,
$B \to K^* \ell^+ \ell^-$,
$B \to \rho \gamma$, ...

Resolution $\mu = \sqrt{E\Lambda}$, expansion in $\alpha_s(\sqrt{E\Lambda})$

$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z,x,k^+,E) \phi_M(x) \phi_B(k^+)$$

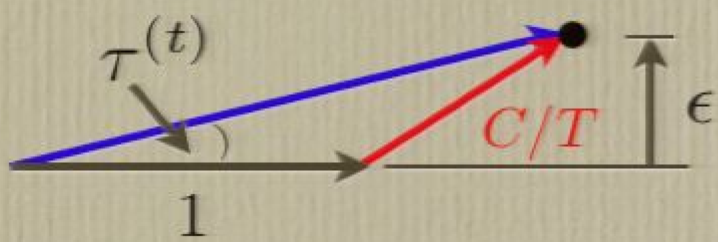
ζ^{BM} left as a form factor

$B \to \pi\pi$

$$\bar{B}^0 \to \pi^+\pi^-, \quad B^- \to \pi^0\pi^-, \; \bar{B}^0 \to \pi^0\pi^0,$$
$$B^0 \to \pi^+\pi^-, \; B^0 \to \pi^0\pi^0$$

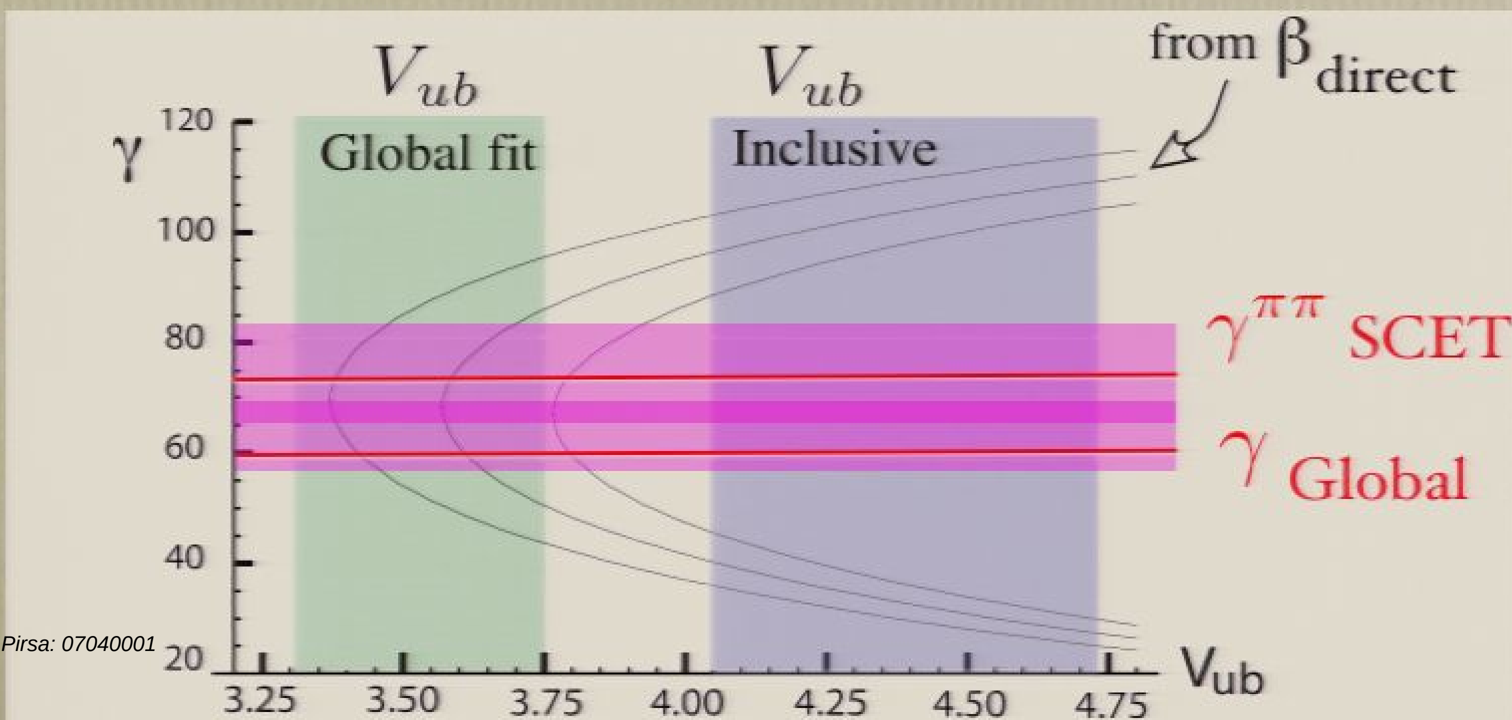
(Belle & Babar)

- $C_{\pi^0\pi^0} = -0.28 \pm 0.39$, uncertainty precludes measuring γ without input from QCD
- Factorization predicts a small relative phase for two amplitudes

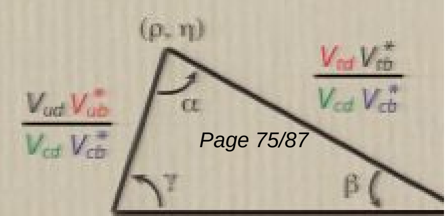


$\epsilon \sim 0, \; \tau^{(t)} \sim 0 \qquad \frac{\Lambda_{QCD}}{E_\pi} \ll 1$

Bauer, Rothstein, I.S.



expt. errors dominate



B-decays with one Jet

$B \to X_s \gamma$

$$Br(B \to X_s\gamma)^{\text{expt}}_{E_\gamma > 1.6\,\text{GeV}} = (3.55 \pm 0.26) \times 10^{-4}$$

$$Br(B \to X_s\gamma)^{\text{theory}}_{E_\gamma > 1.6\,\text{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$ Misiak et a

-0.17 Becher, Neuber

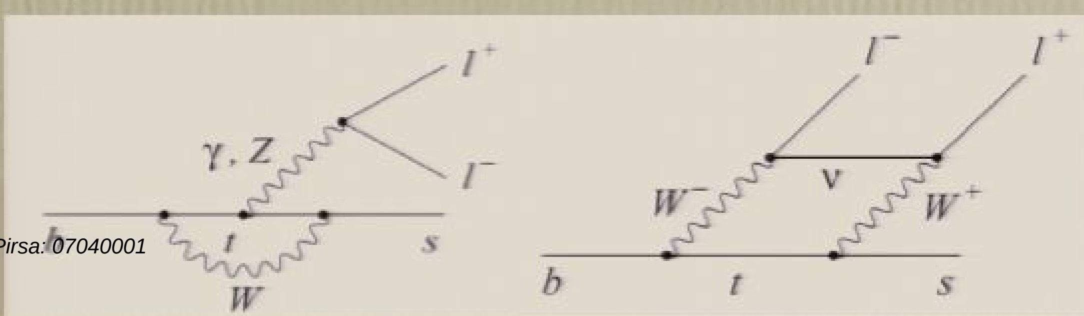
Cuts force the Xs to be jet-like and are important for comparison to the standard model

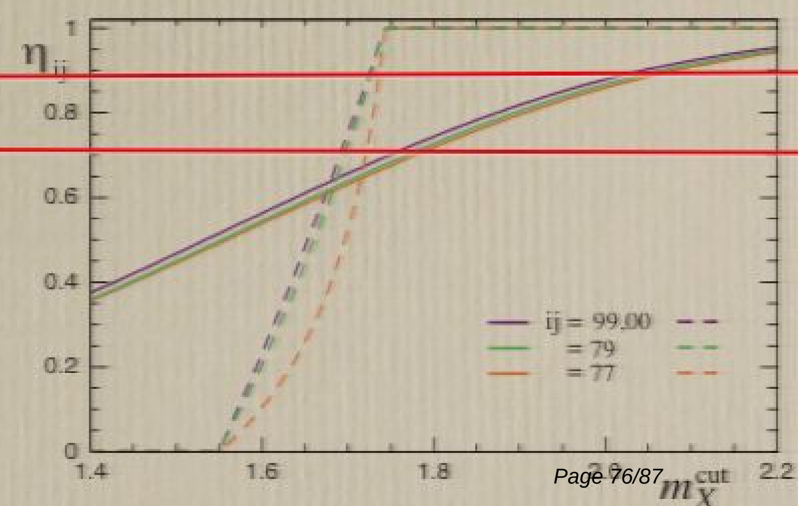
$B \to X_s \ell^+ \ell^-$

Again the cuts give a jet, and modify the standard model prediction

Lee, Ligeti, Stewart, Tackmann

10-30% reduction in the decay rate





Babar, Belle

- For many channels, control of hadronic uncertainties is crucial to test standard model & look for new physics.

$$B \to X_s \ell^+ \ell^-, B \to \pi\pi, B \to K\pi, B \to \rho\pi, \ldots$$
$$B \to \rho\gamma, B \to K^*\gamma, B \to \phi K_s, B \to \eta' K_s$$

CDF, DØ

- Test standard model / new physics in $B_s,\ \Lambda_b,\ \ldots$
- Heavy quark production, jets, ...

Immediate future:

Babar, Belle

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CDF, DØ

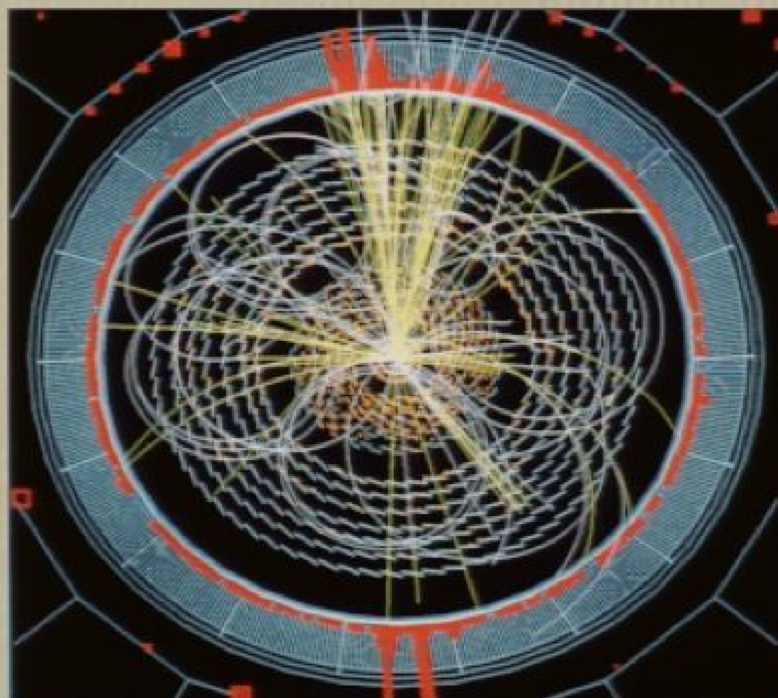
- Test standard model / new physics in $B_s, \Lambda_b, \ldots$
- Heavy quark production, jets, ...

SCET has been applied to many processes

Process	Non-Pert. functions	Utility
$\bar{B}^0 \to D^+\pi^-, \ldots$	$\xi(w)$, ϕ_π	study QCD
$\bar{B}^0 \to D^0\pi^0, \ldots$	$S(k_j^+)$, ϕ_π	study QCD
$B \to X_s^{endpt}\gamma$	$f(k^+)$	new physics, measure f
$B \to X_u^{endpt}\ell\nu$	$f(k^+)$	measure $\|V_{ub}\|$
$B \to \pi\ell\nu, \ldots$	$\phi_B(k^+)$, $\phi_\pi(x)$, $\zeta_\pi(E)$	measure $\|V_{ub}\|$, study QCD
$B \to \gamma\ell\nu$, $\gamma\ell^+\ell^-$	ϕ_B	measure ϕ_B, new physics
$B \to \pi\pi$, $K\pi$, ...	ϕ_B, ϕ_π, $\zeta_\pi(E)$ $\phi_{\bar{K}}$, $\zeta_K(E)$	new physics, CP violation, γ study QCD
$B \to K^*\gamma$, $\rho\gamma$	ϕ_B, ϕ_K, $\zeta_{K^*}^\perp(E)$ ϕ_ρ, $\zeta_\rho^\perp(E)$	measure $\|V_{td}/V_{ts}\|$, new physics
$B \to X_s\ell^+\ell^-$	$f(k^+)$	new physics
$e^-p \to e^-X$	$f_{i/p}(\xi)$, $f_{g/p}(\xi)$	study QCD , measure p.d.f's
$p\bar{p} \to X\ell^+\ell^-$	$f_{i/p}(\xi)$, $f_{g/p}(\xi)$	study QCD
$e^-\gamma \to e^-\pi^0$	ϕ_π	measure ϕ_π
$\gamma^*M \to M'$	ϕ_M, $\phi_{M'}$	study QCD
$e^+e^- \to j_1 + \text{jets}$	$\tilde{S}(k^+)$	event shapes & universality
$e^+e^- \to J/\Psi X$	$S^{(8,n)}(k^+)$	study QCD
$\Upsilon \to X\gamma$	$S^{(8,n)}(k^+)$	study QCD
$\vdots$	$\vdots$	$\vdots$

Who needs to understand QCD?

LHC



pp collider with $E_{cm} = 14\,\text{TeV}$

scales: $m_W, m_t, E_T^{\text{jet}}$

→ Energetic QCD (SCET)

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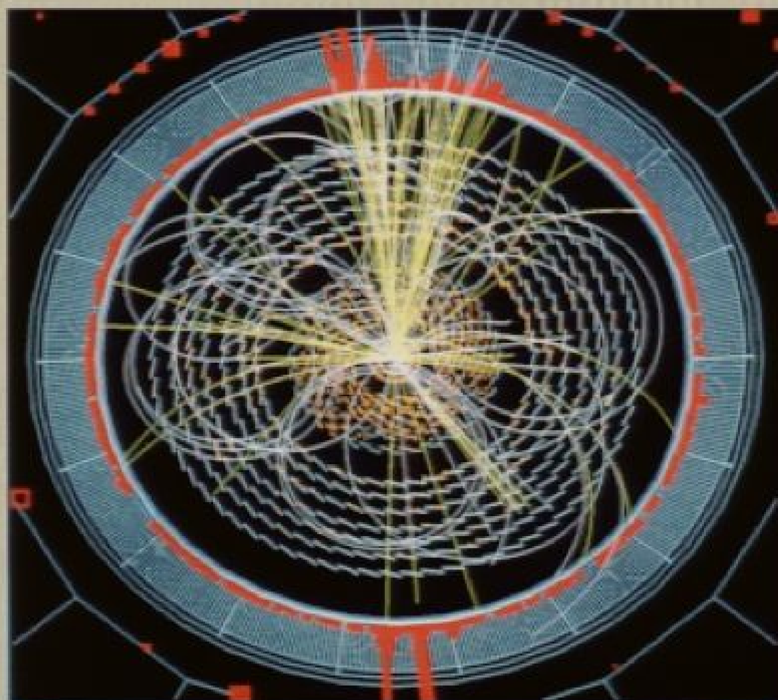

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→ Energetic QCD (SCET)

QCD at the LHC:

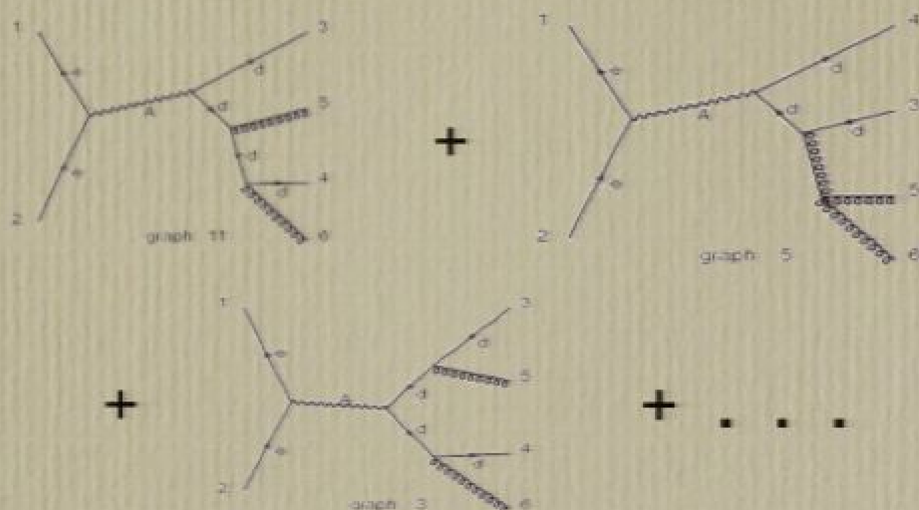
- Higher order calculations (loops and legs)

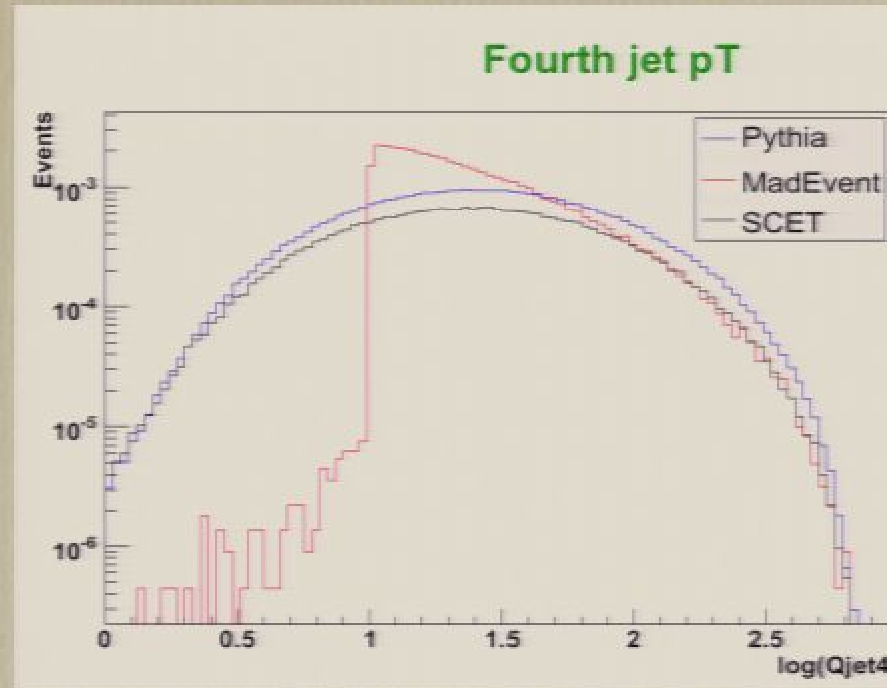
- Summation of large logs

- Understand Standard Model background.
  It is important to improve our understanding of:
  Jets,
  Parton Showering,
  Soft radiation, ...

Parton Showers

Bauer, Schwartz





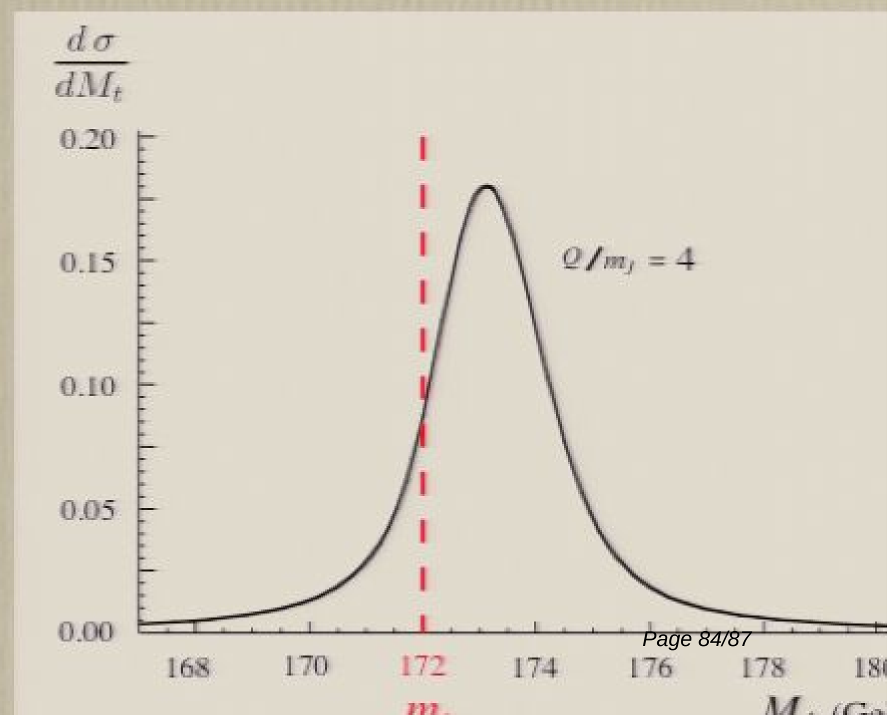
Top-Quark Mass

Fleming, Hoang, Mantry, I.S.

$m_t = 170.9 \pm 1.8\,\mathrm{GeV}$ (CDF, DØ)

what mass is it?

how do soft radiation, and the choice of the observable affect the uncertainties?



Concluding Remarks

- QED fundamental parameters & precision quantum field theory
- QCD today is as rich & diverse as ever

  many subfields which focus on different degrees of freedom and different relevant interactions

- SCET a new approach to derive factorization theorems and treat power corrections for energetic hadrons & jets

Nonleptonic B-decays

➡ universal hadronic parameters, strong phases

➡ γ (or α) from individual $B \to M_1 M_2$ channels

QCD at the LHC ➡ precise control of strong interaction effects

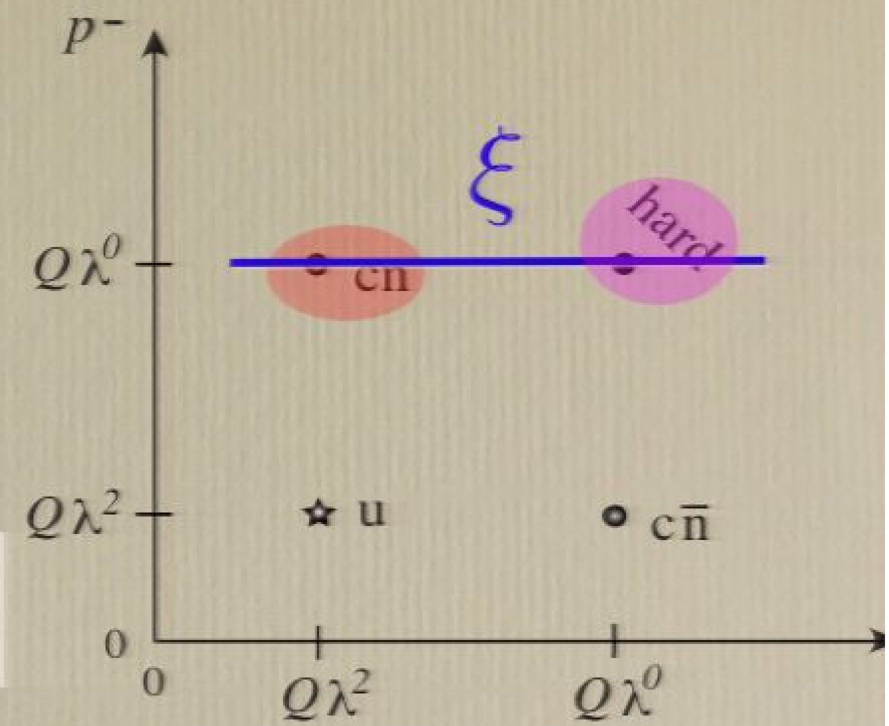
- A lot of theory and phenomenology left to study !

SCET is a field theory which:

- explains how these degrees of freedom communicate with each other, and with hard interactions

communicate by integrals

$$F_1(x, Q^2) = \frac{1}{x} \int_x^1 d\xi \; H(\xi/x, Q, \mu) \; f_{i/p}(\xi, \mu)$$



- provides a simple operator language to derive factorization theorems in fairly general circumstances

  eg. unifies the treatment of factorization for exclusive and inclusive QCD processes

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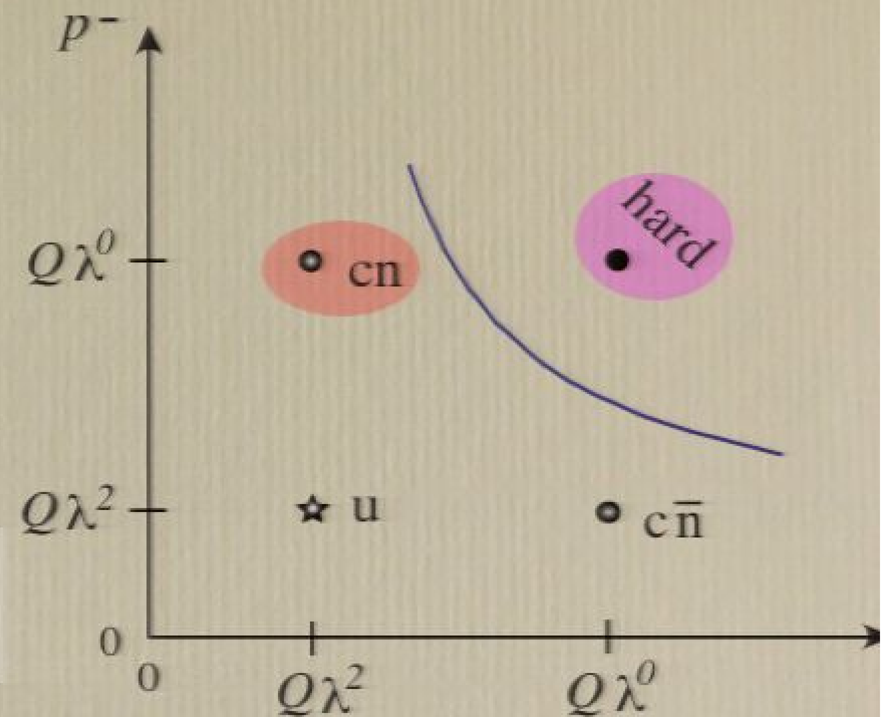

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