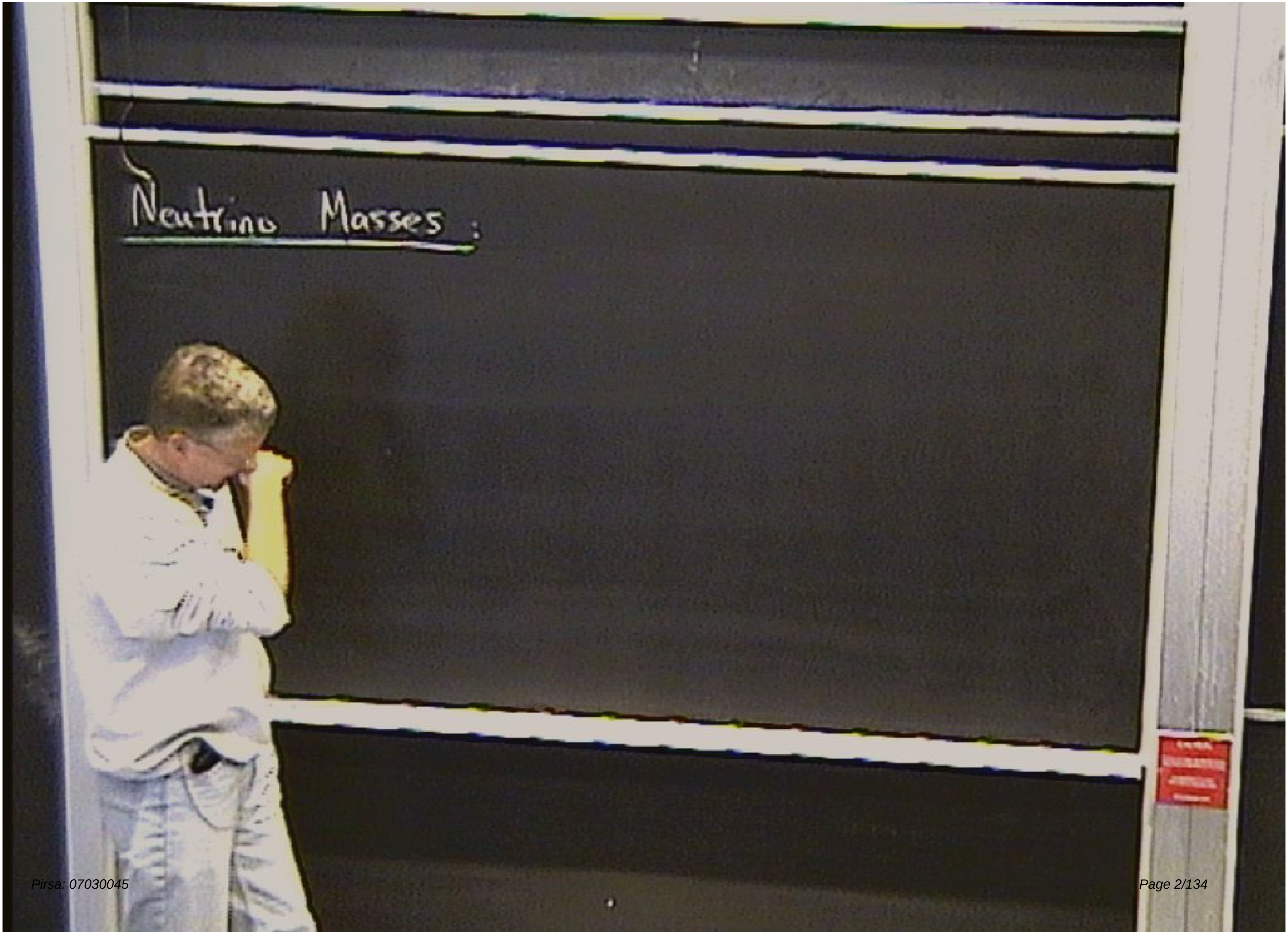
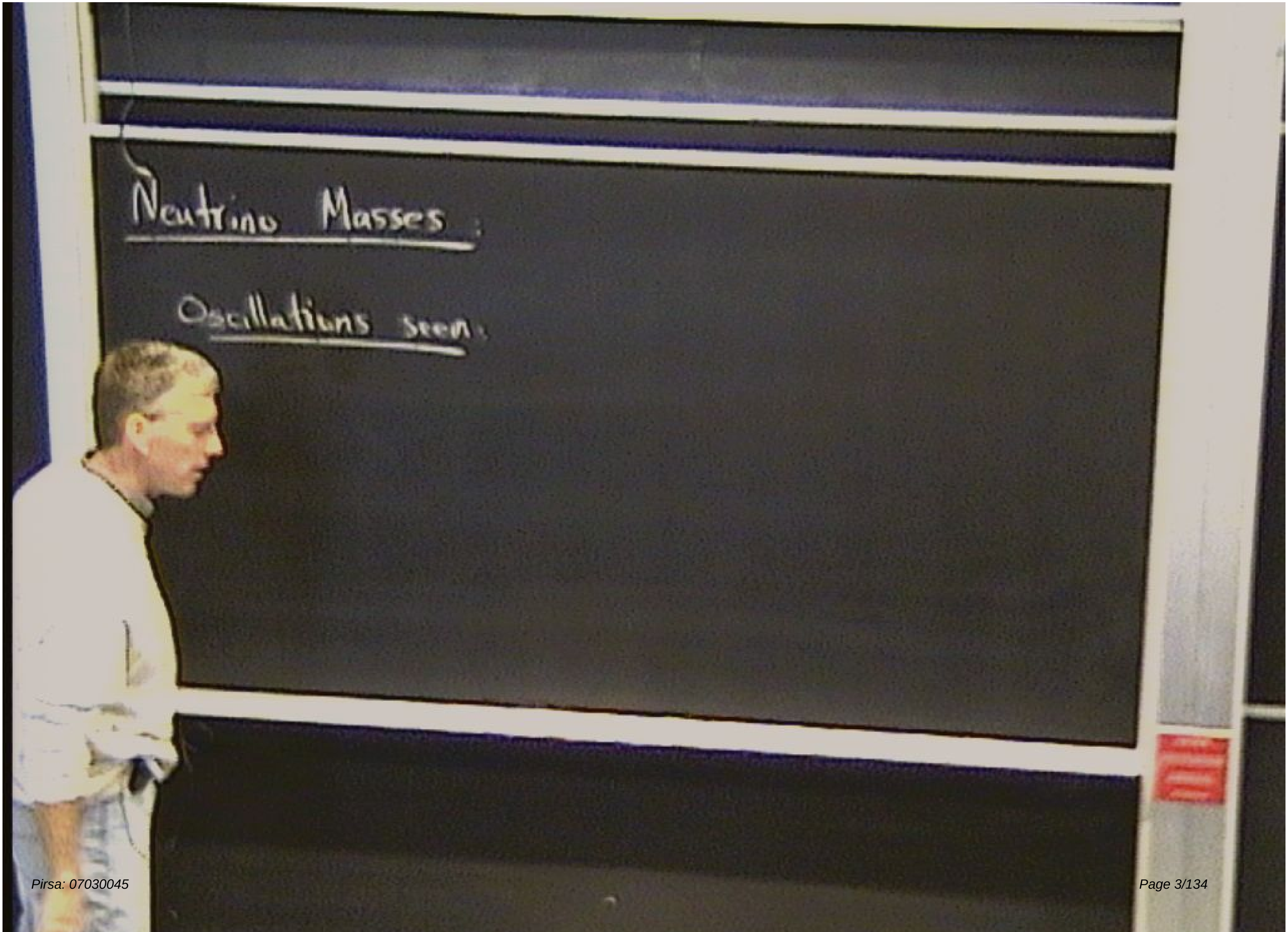
Title: Graduate Course on Standard Model & Quantum Field Theory - 16A

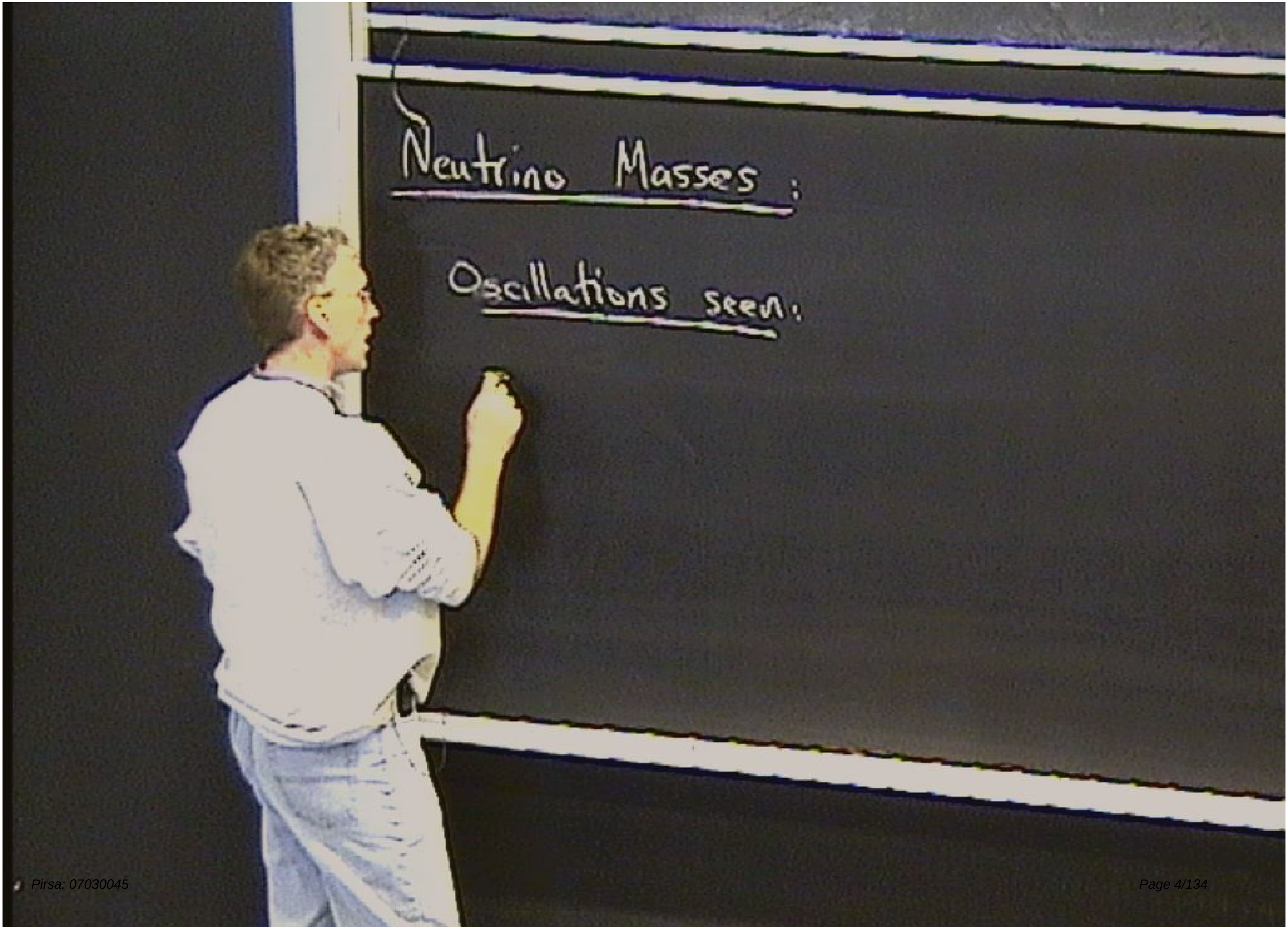
Date: Mar 28, 2007 11:00 AM

URL: http://pirsa.org/07030045

Abstract: Graduate Course on Standard Model & Quantum Field Theory


Neutrino Masses :


Neutrino Masses:
Oscillations seen: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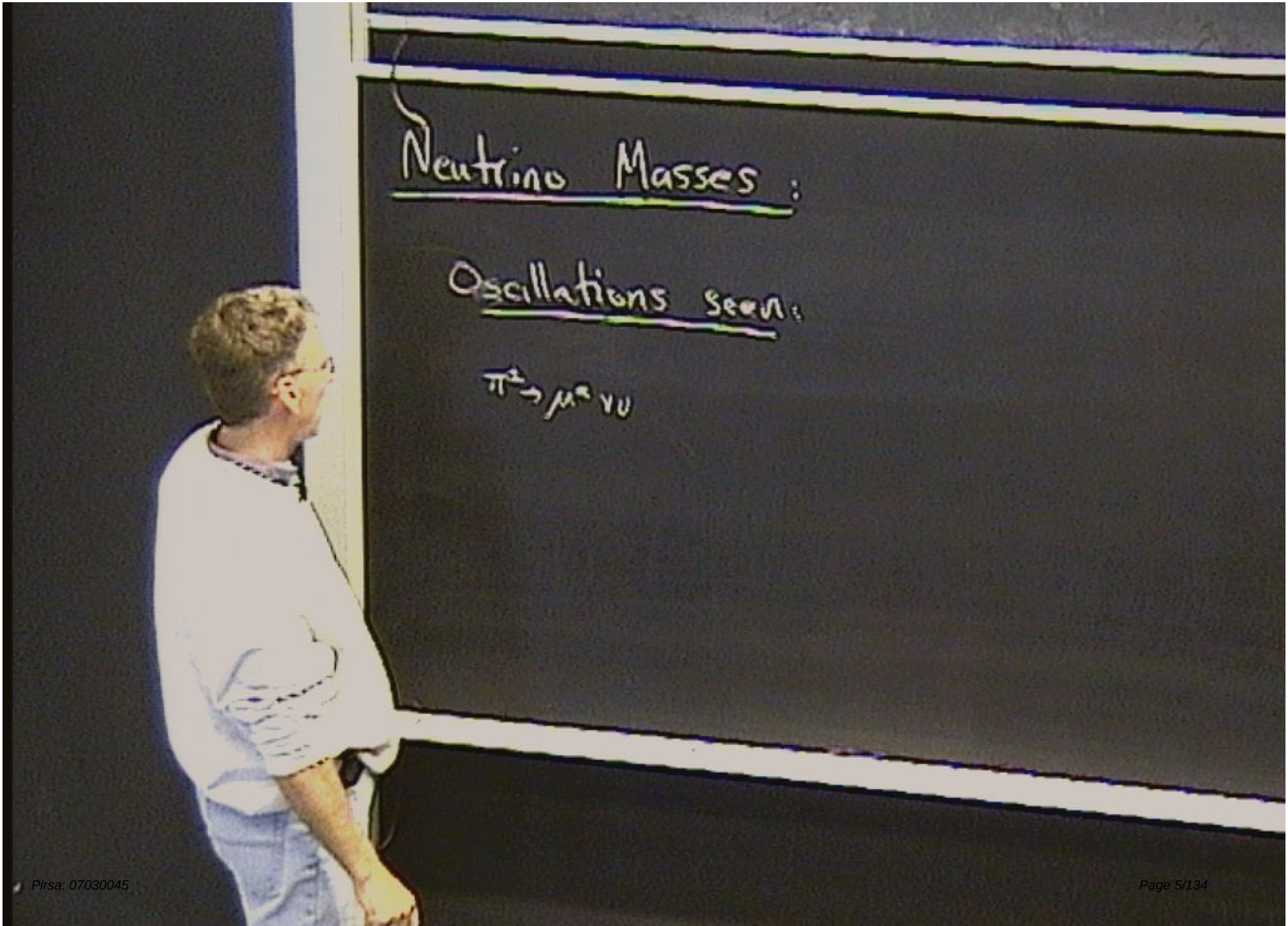
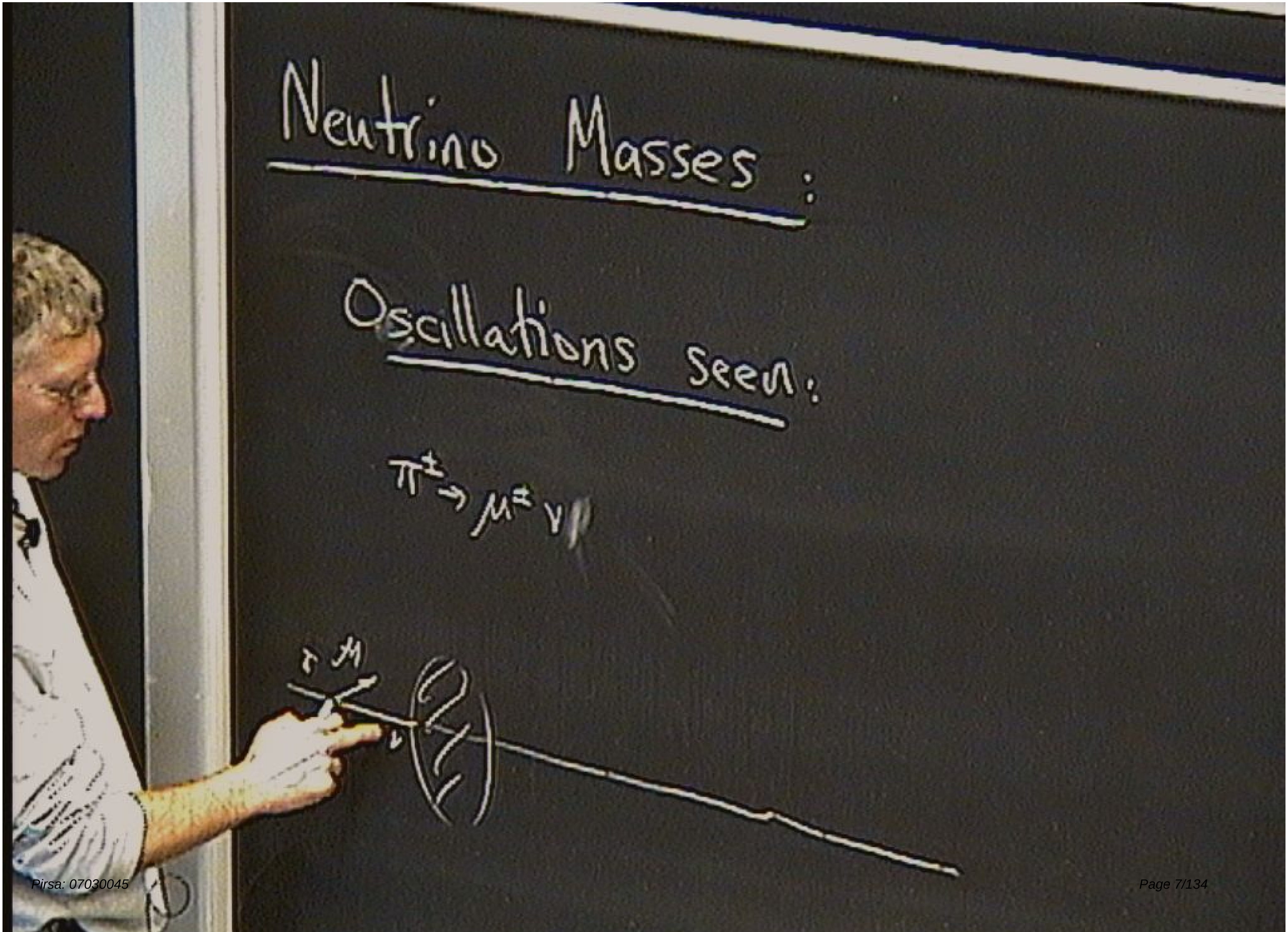

Neutrino Masses :
Oscillations seen:
π± → μ± νμ

Neutrino Masses:

Oscillations seen:

$\pi^{\pm} \to \mu^{\pm} \nu$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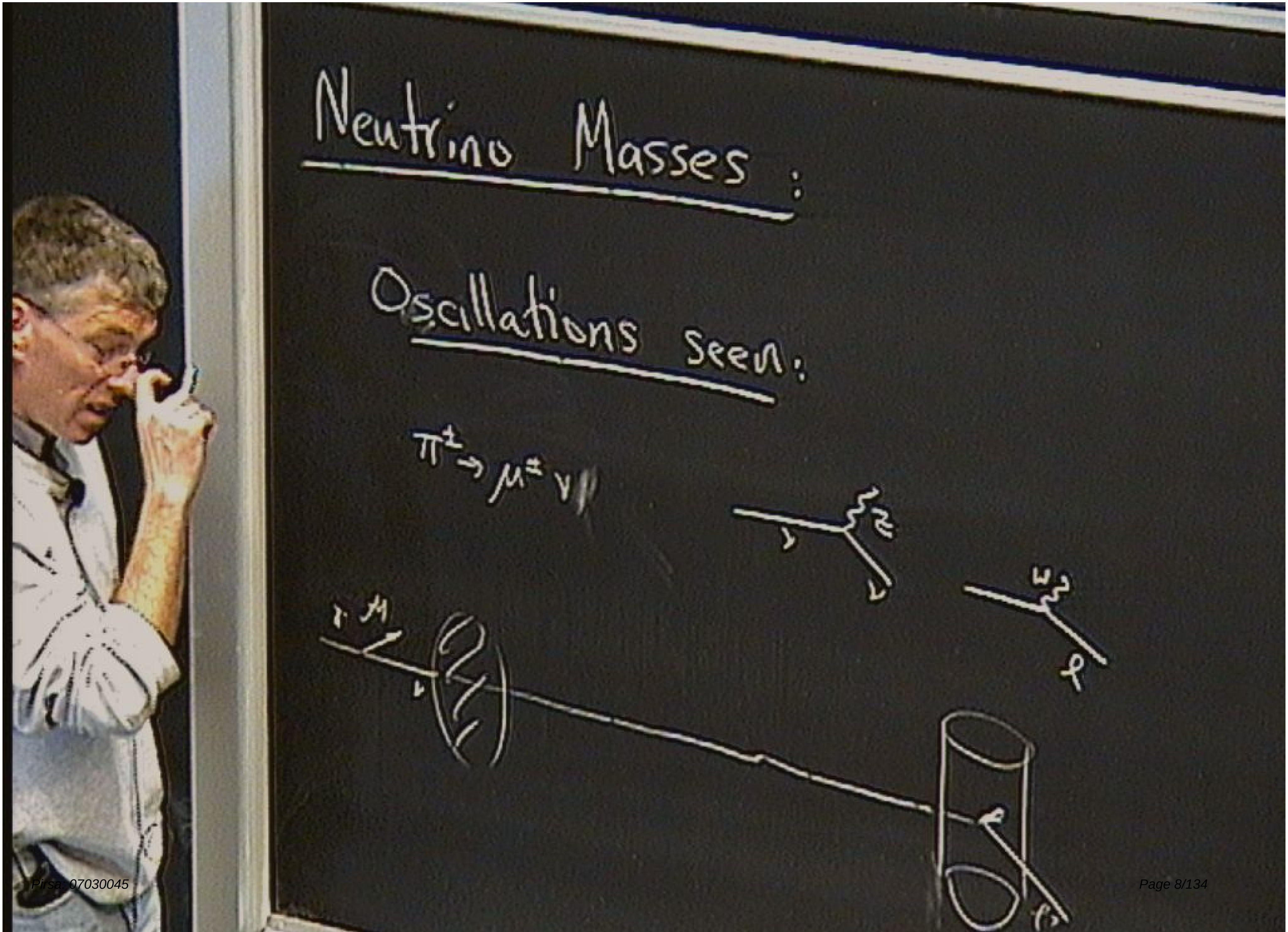
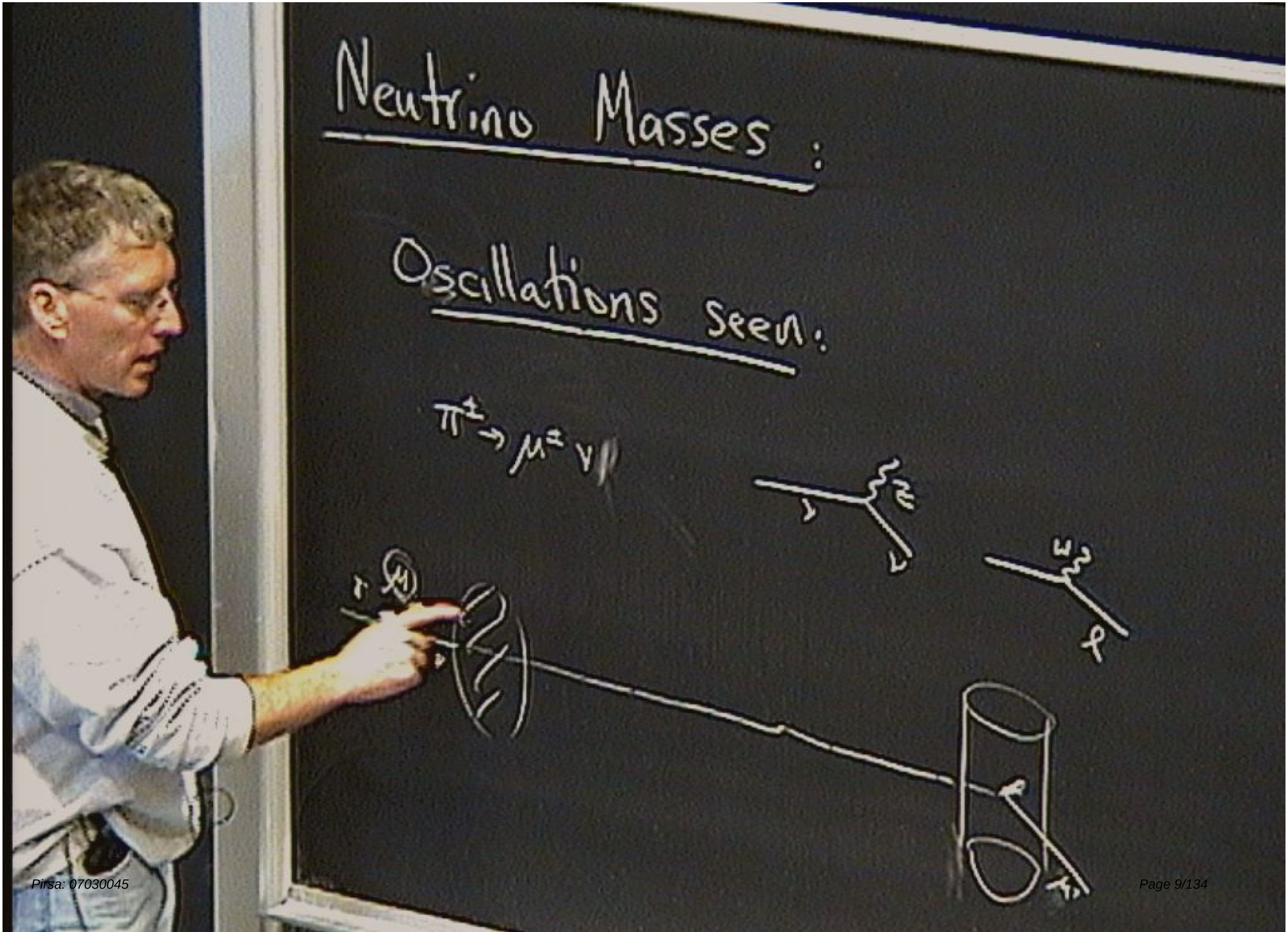
Oscillations seen:
$\pi^{\pm} \to \mu^{\pm} \nu$
π μ
ν


Neutrino Masses :
Oscillations seen:
$\pi^\pm \to \mu^\pm \nu$
Z
W
ν
ℓ
μ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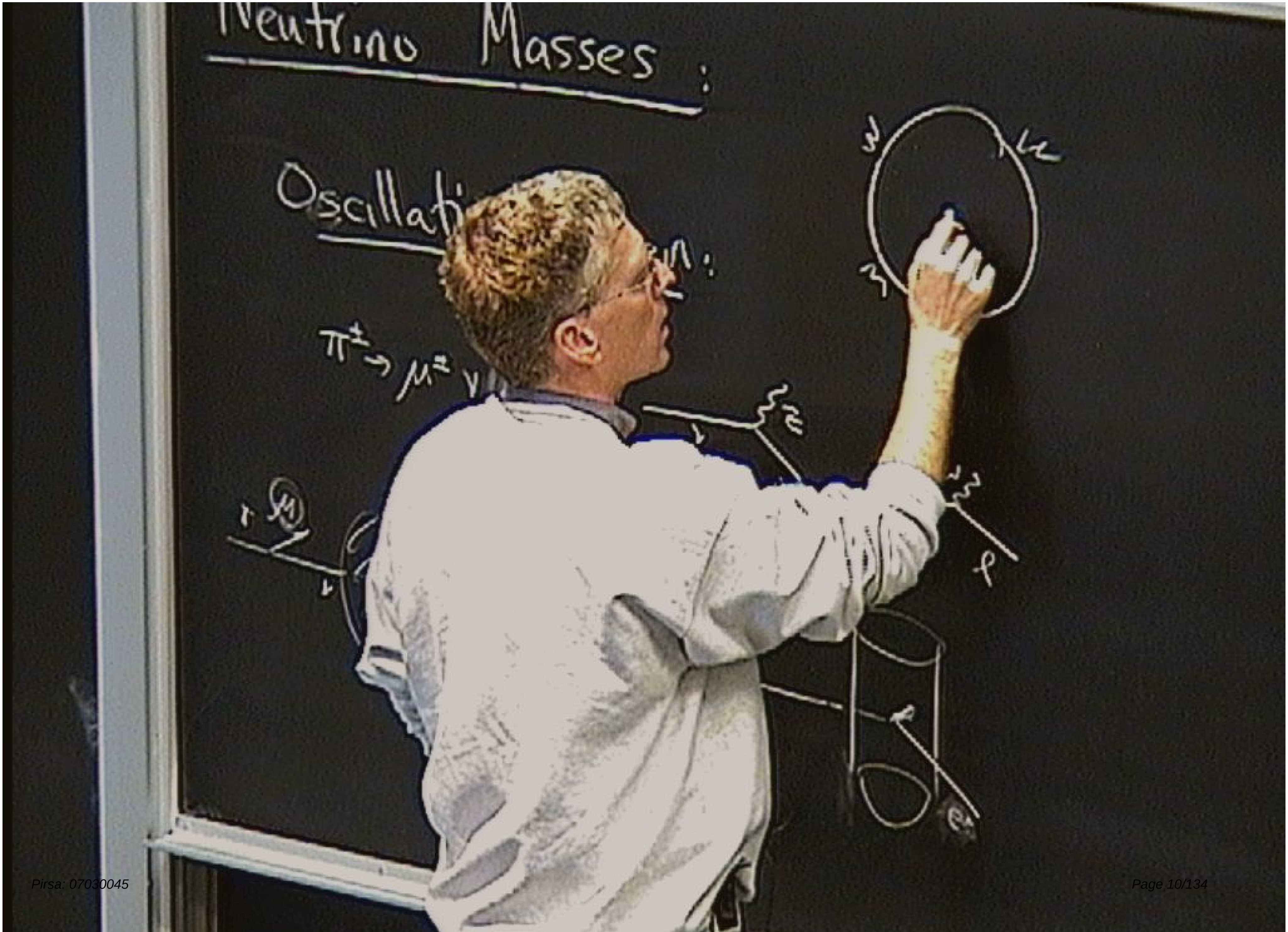
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Oscillati... ...n:
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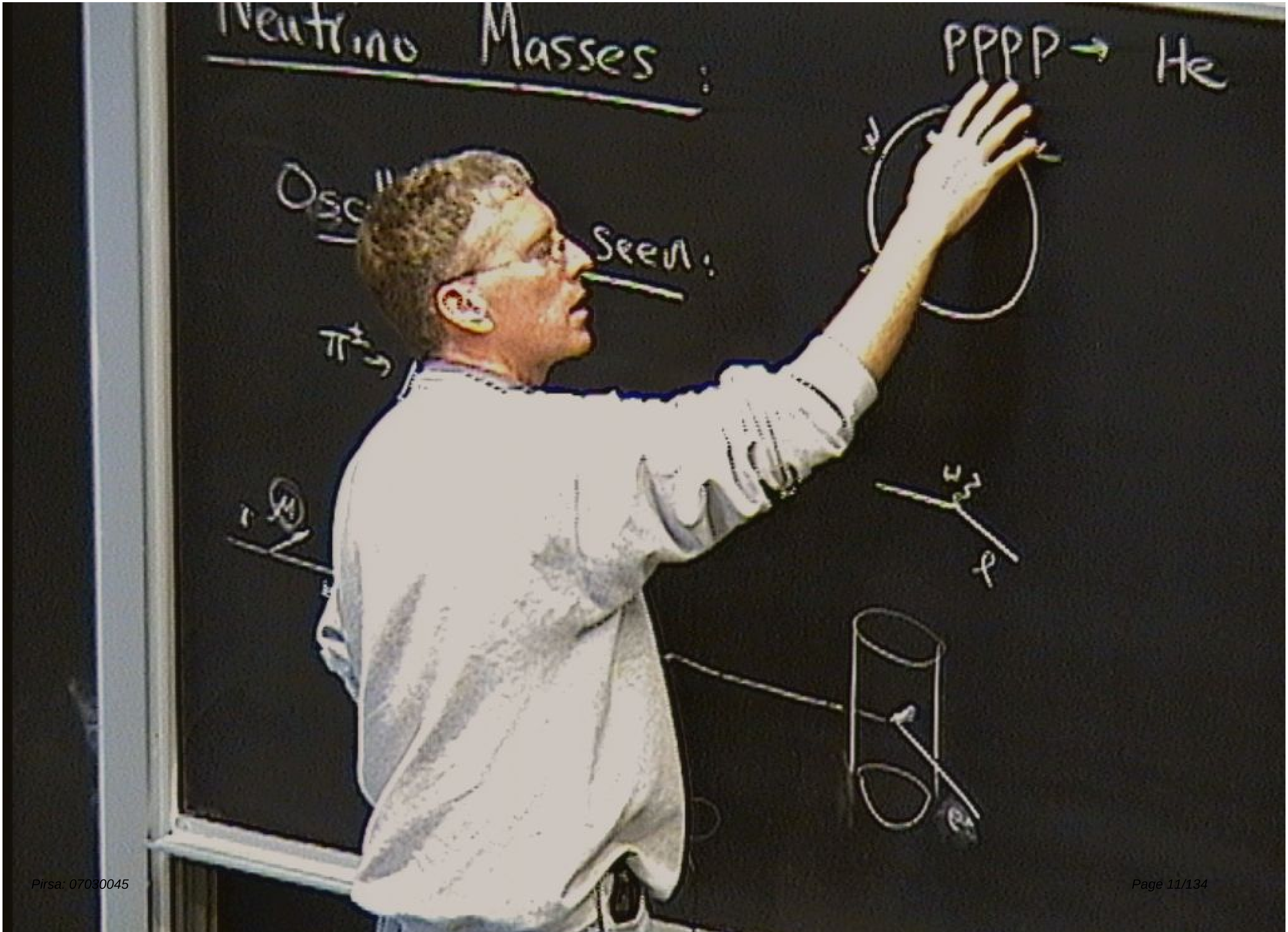
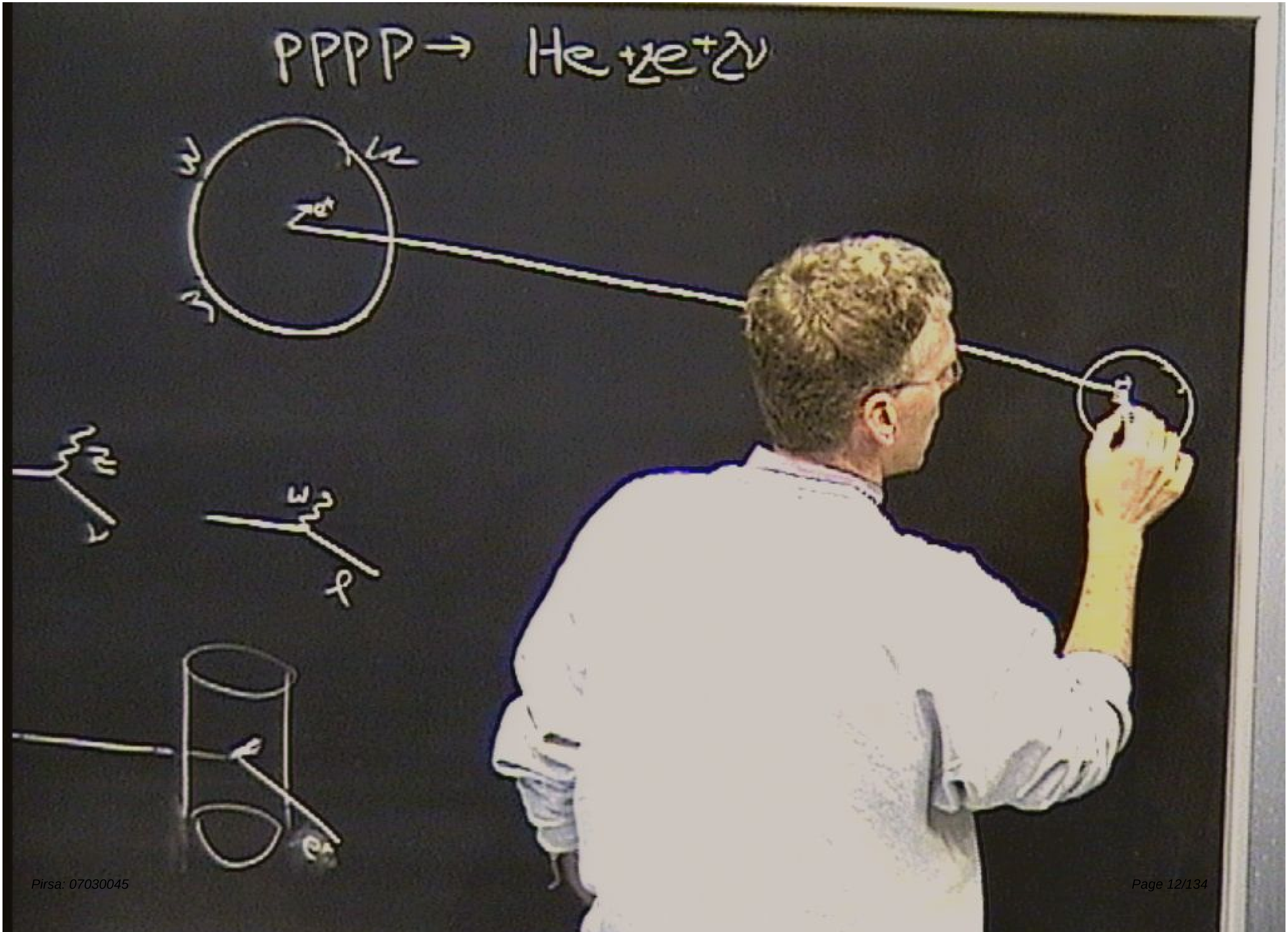
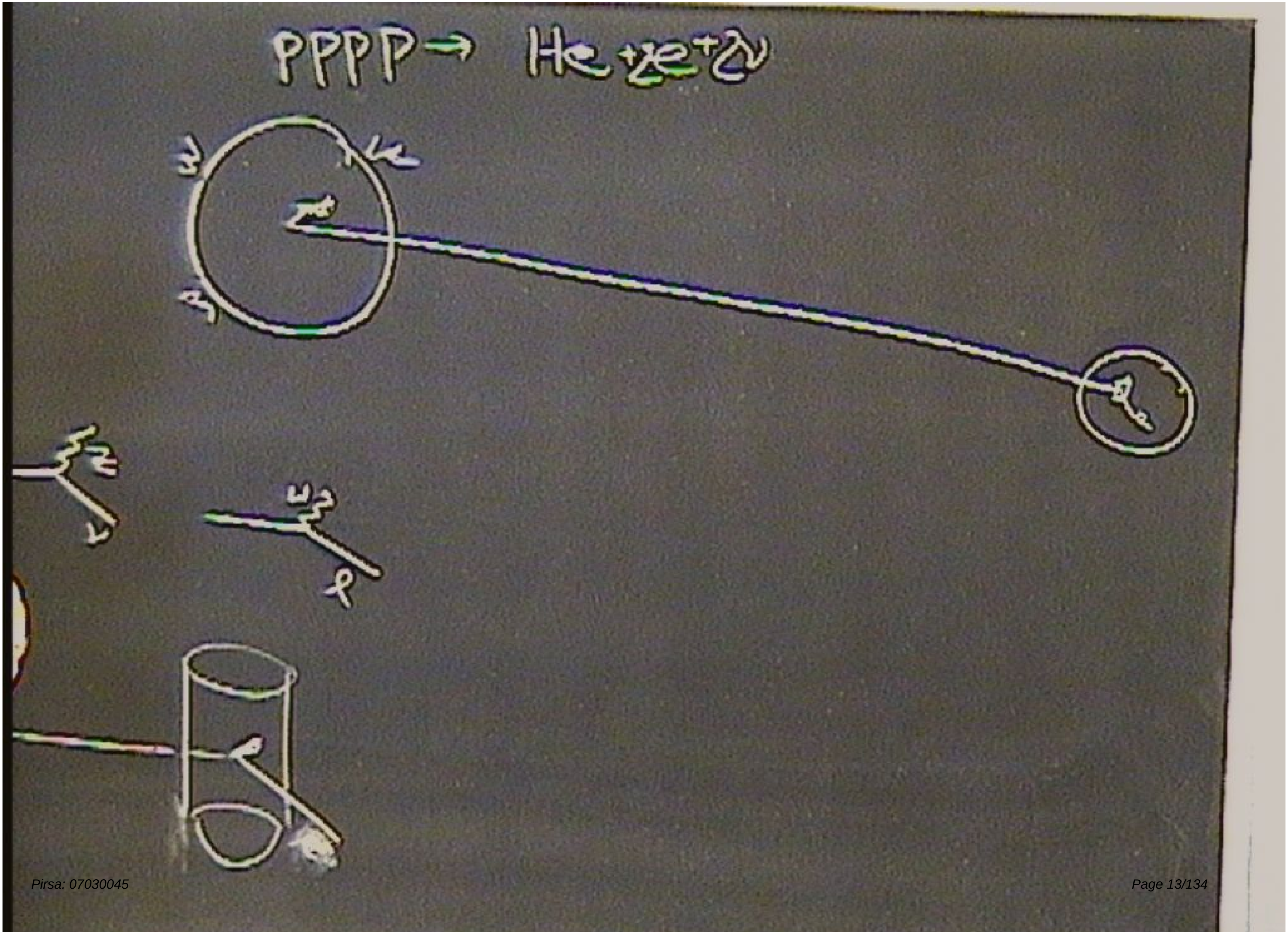

Neutrino Masses :
PPPP → He
Seen.


PPPP → He + 2e⁺ + 2ν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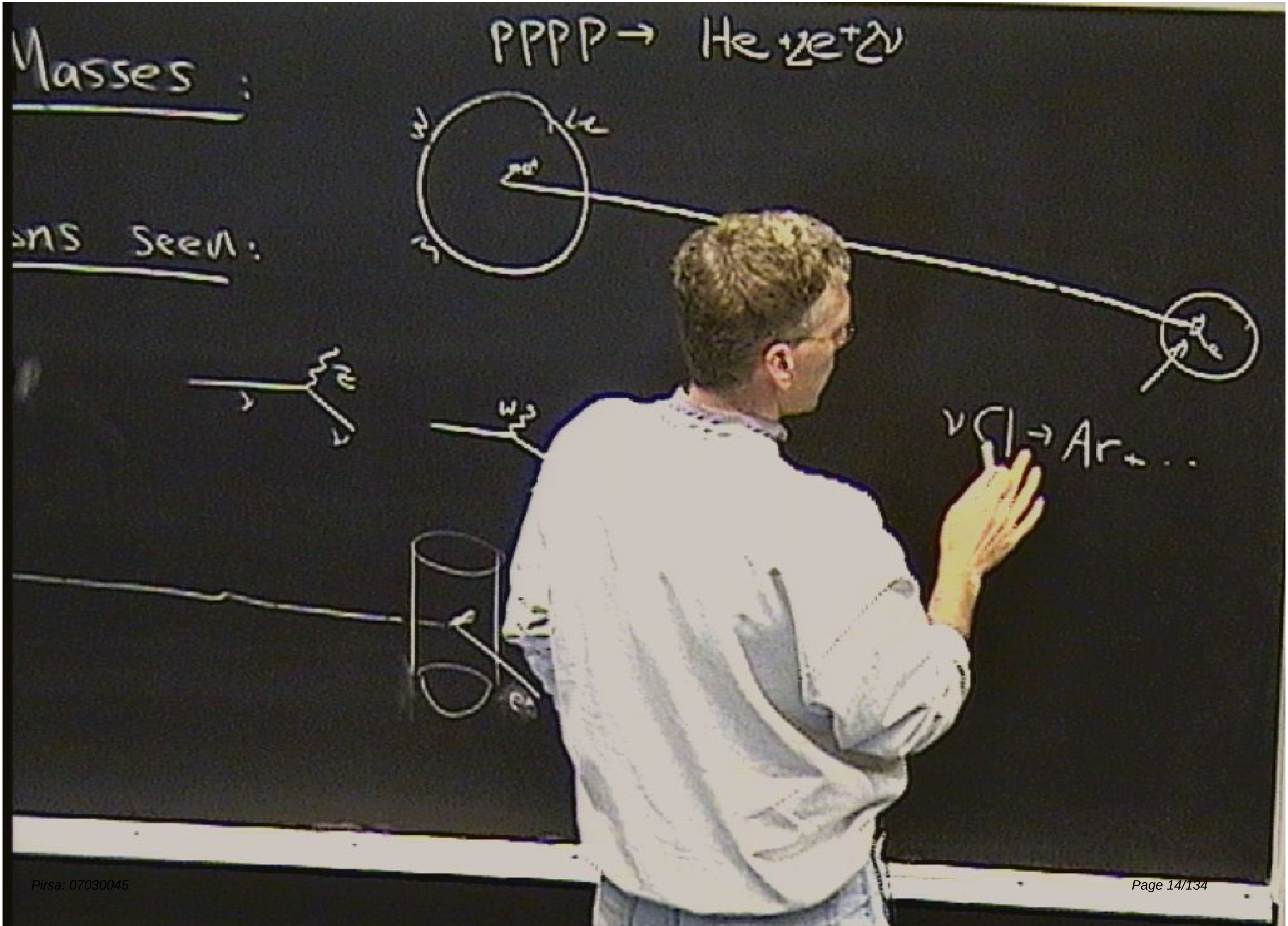
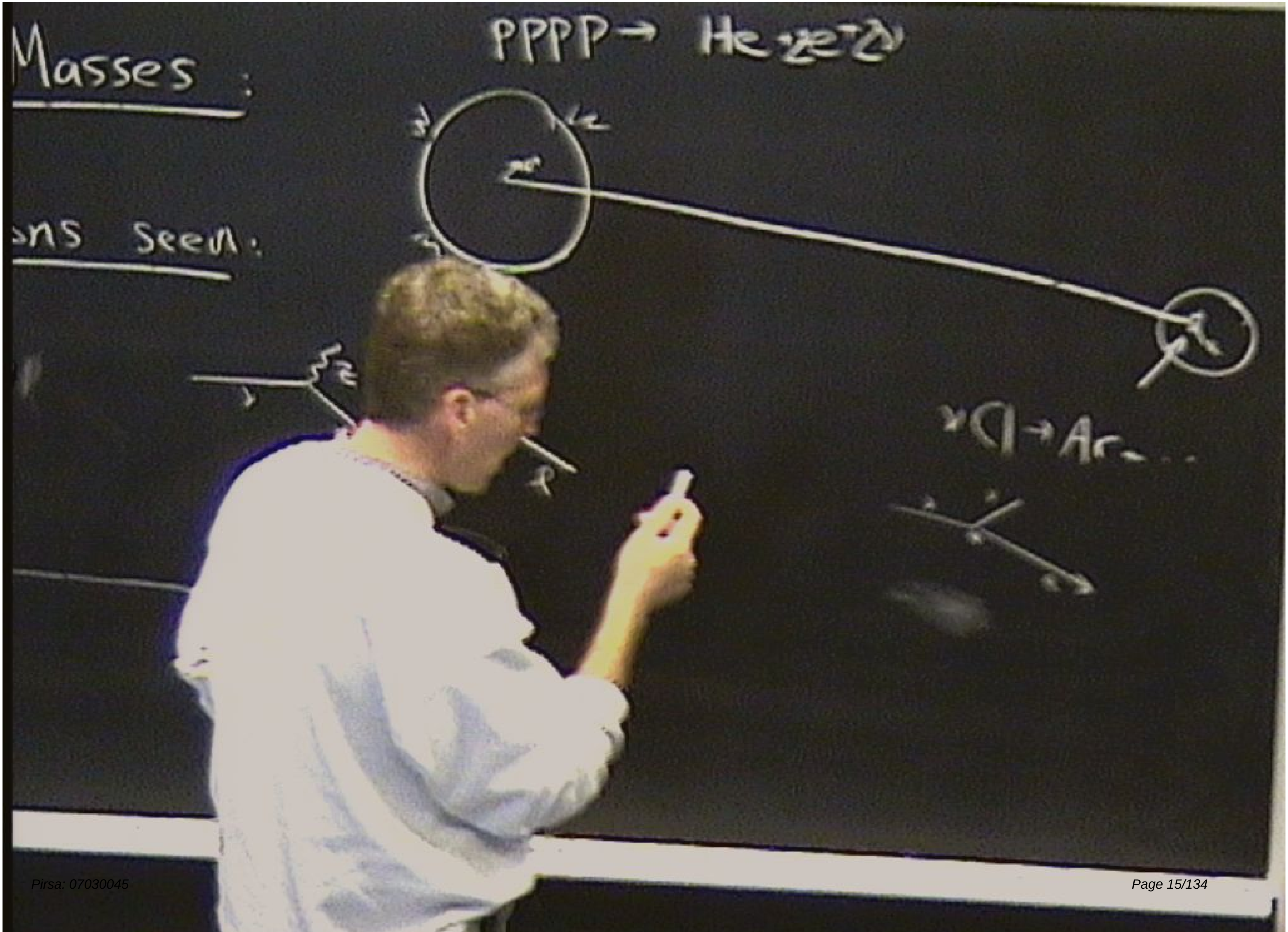

Masses :
ons Seen:
PPPP → He +2e+2ν
νCl → Ar + . .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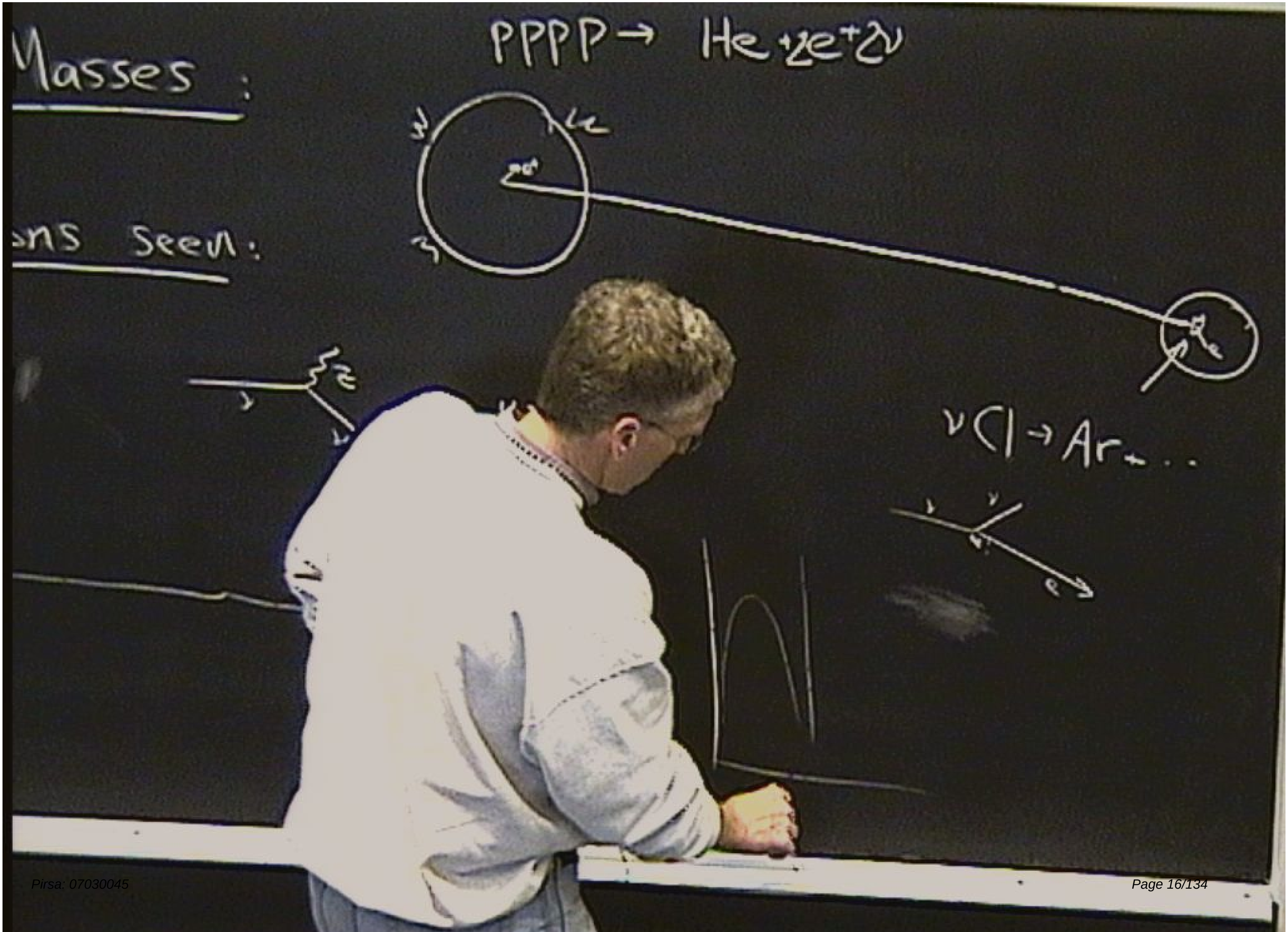

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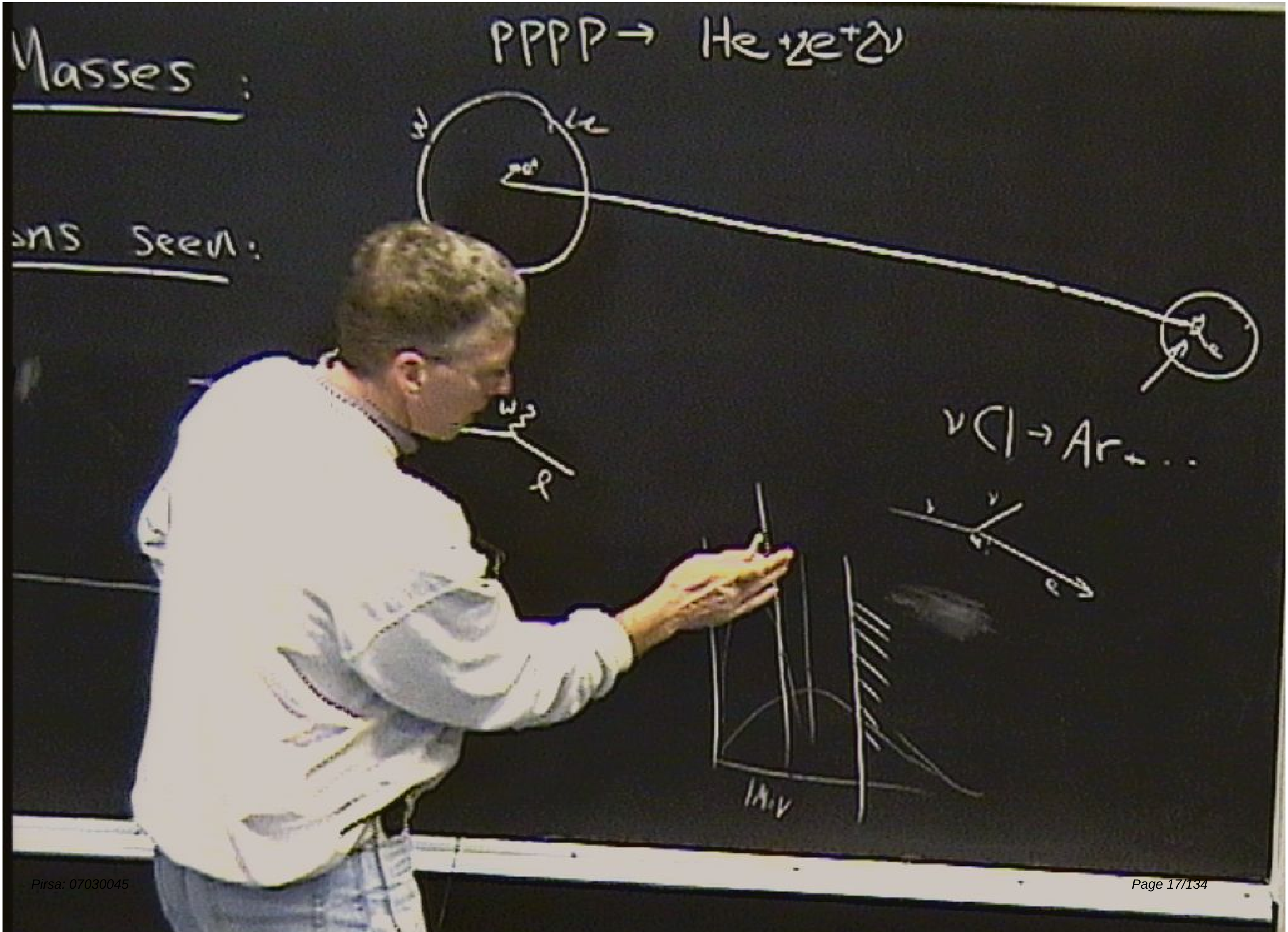
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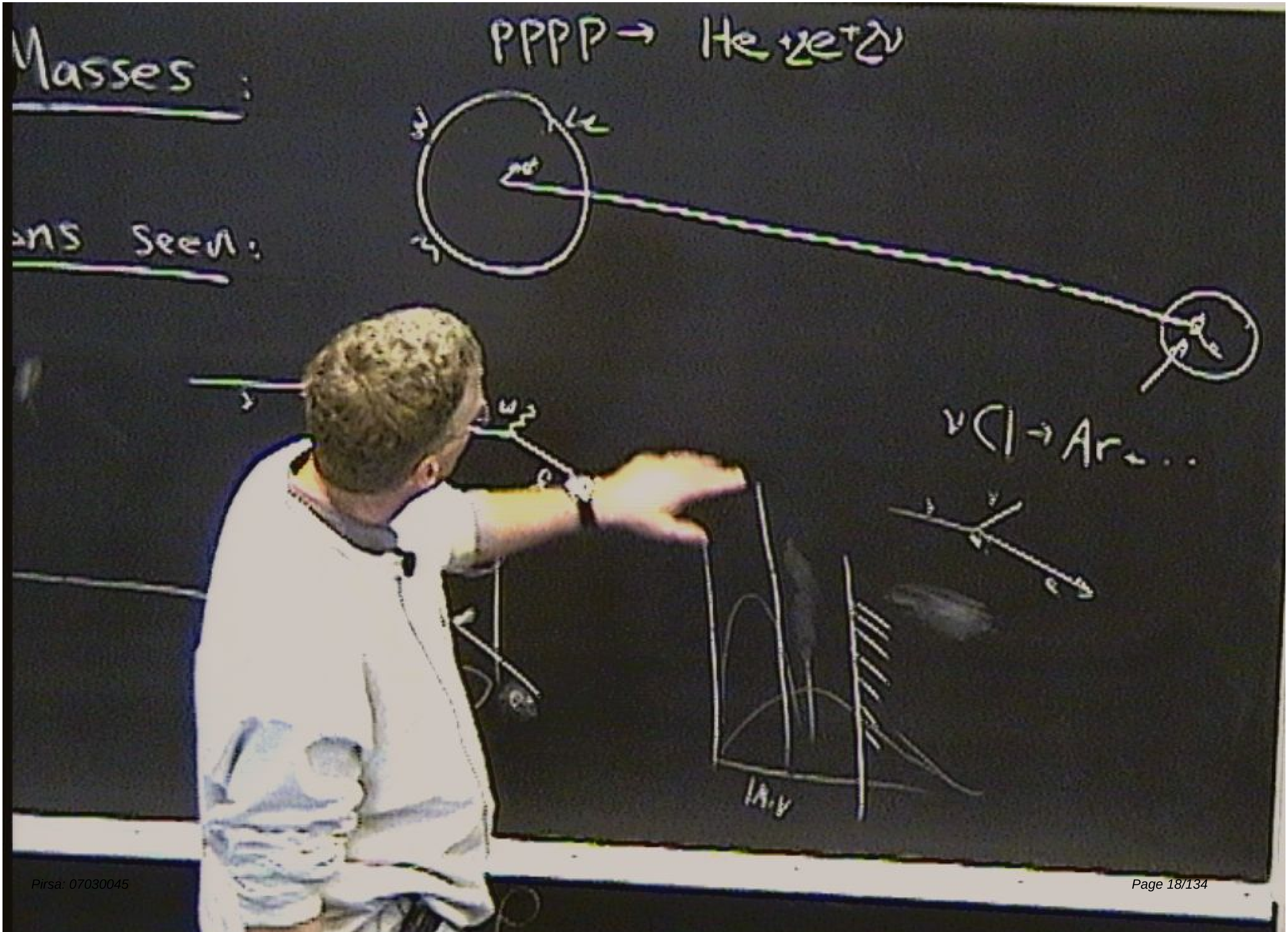
1MeV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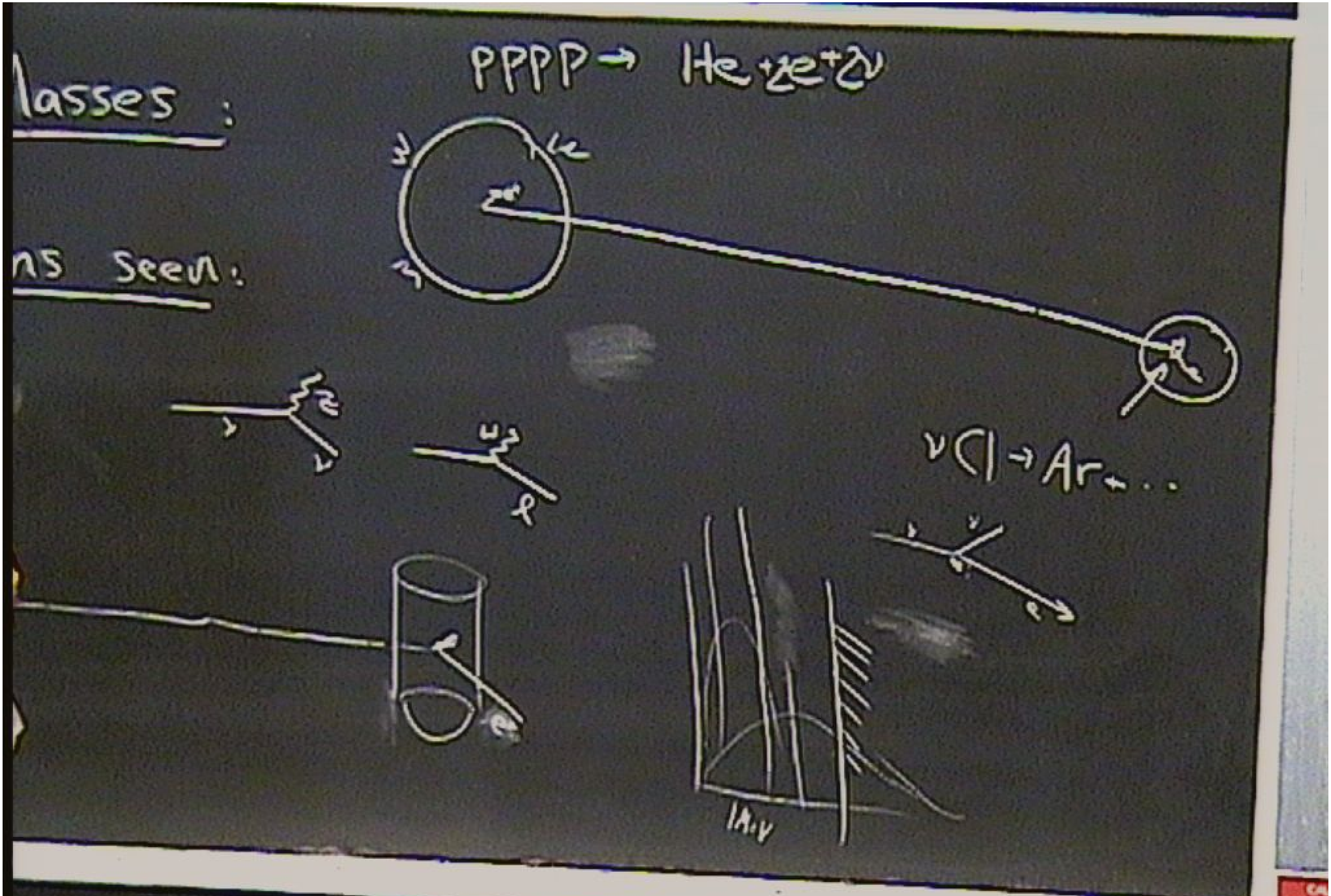
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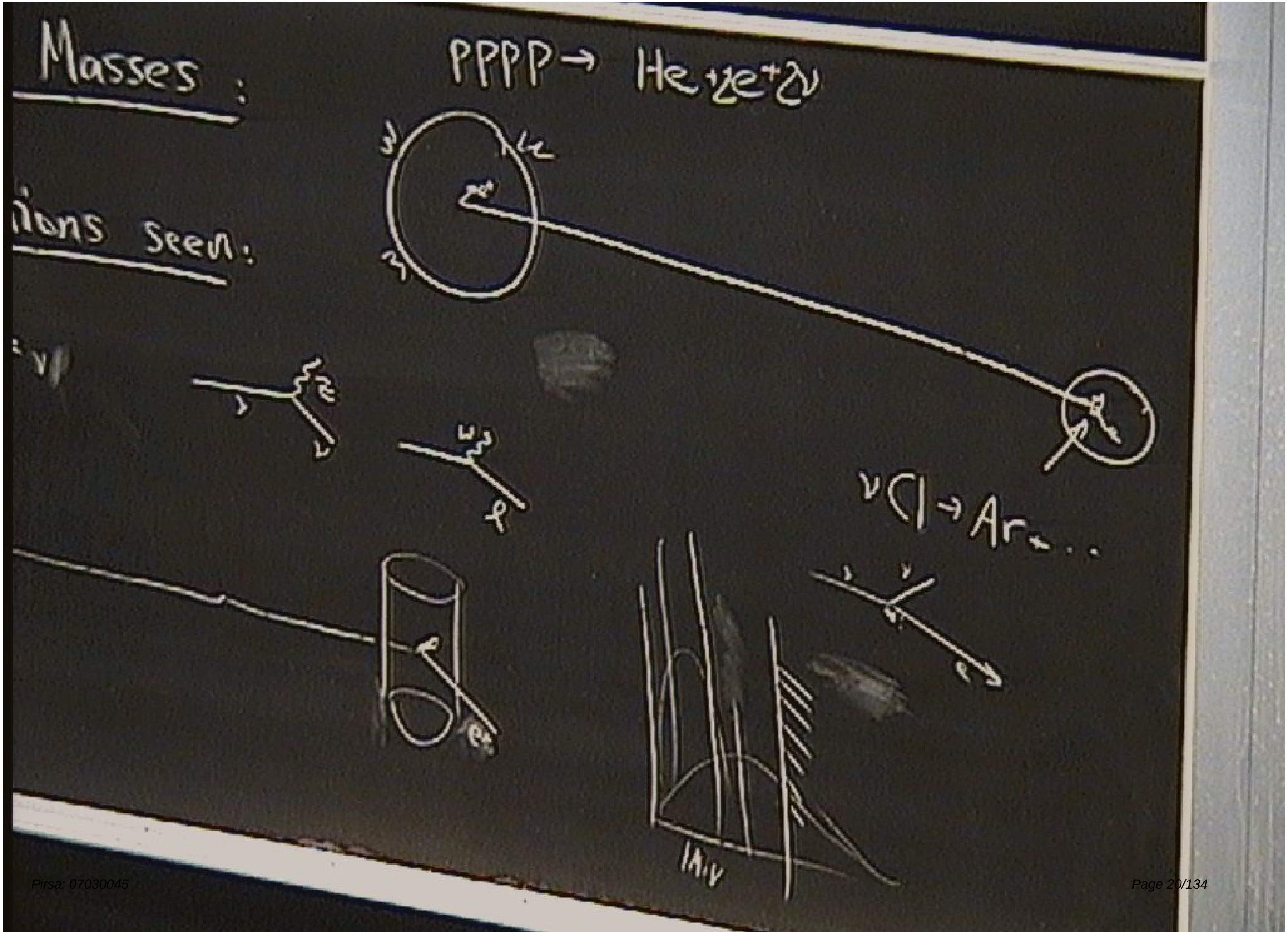
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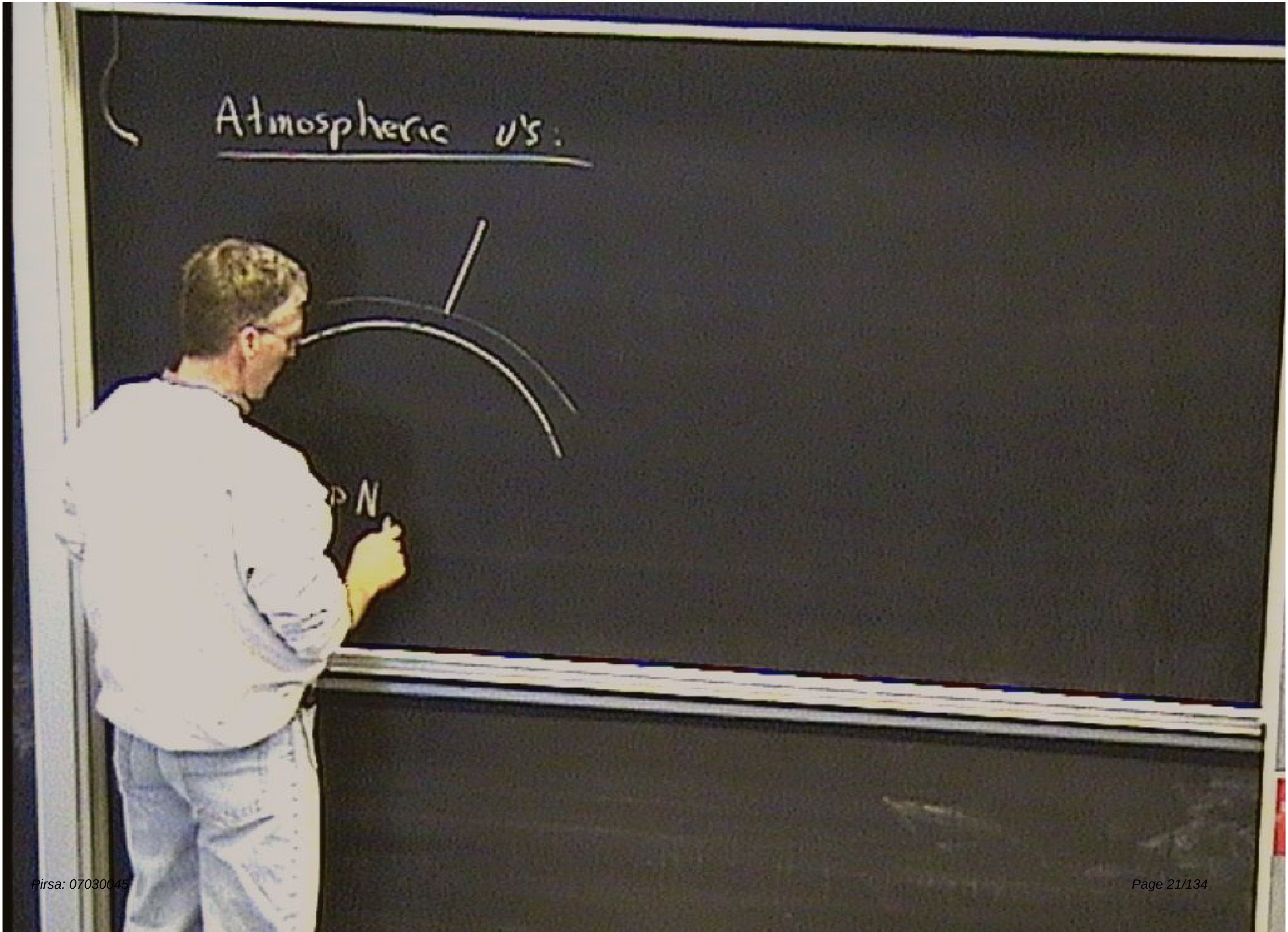
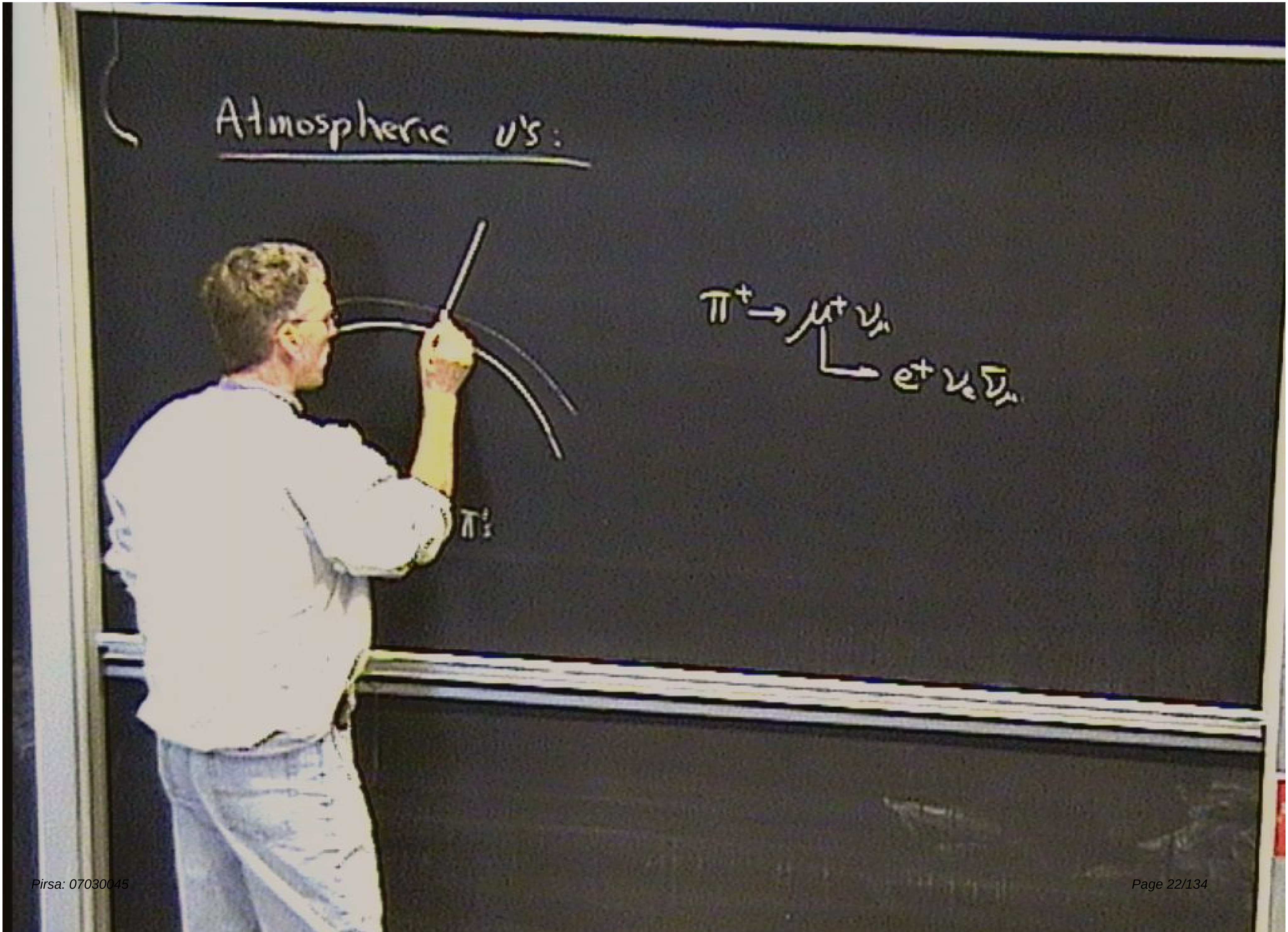
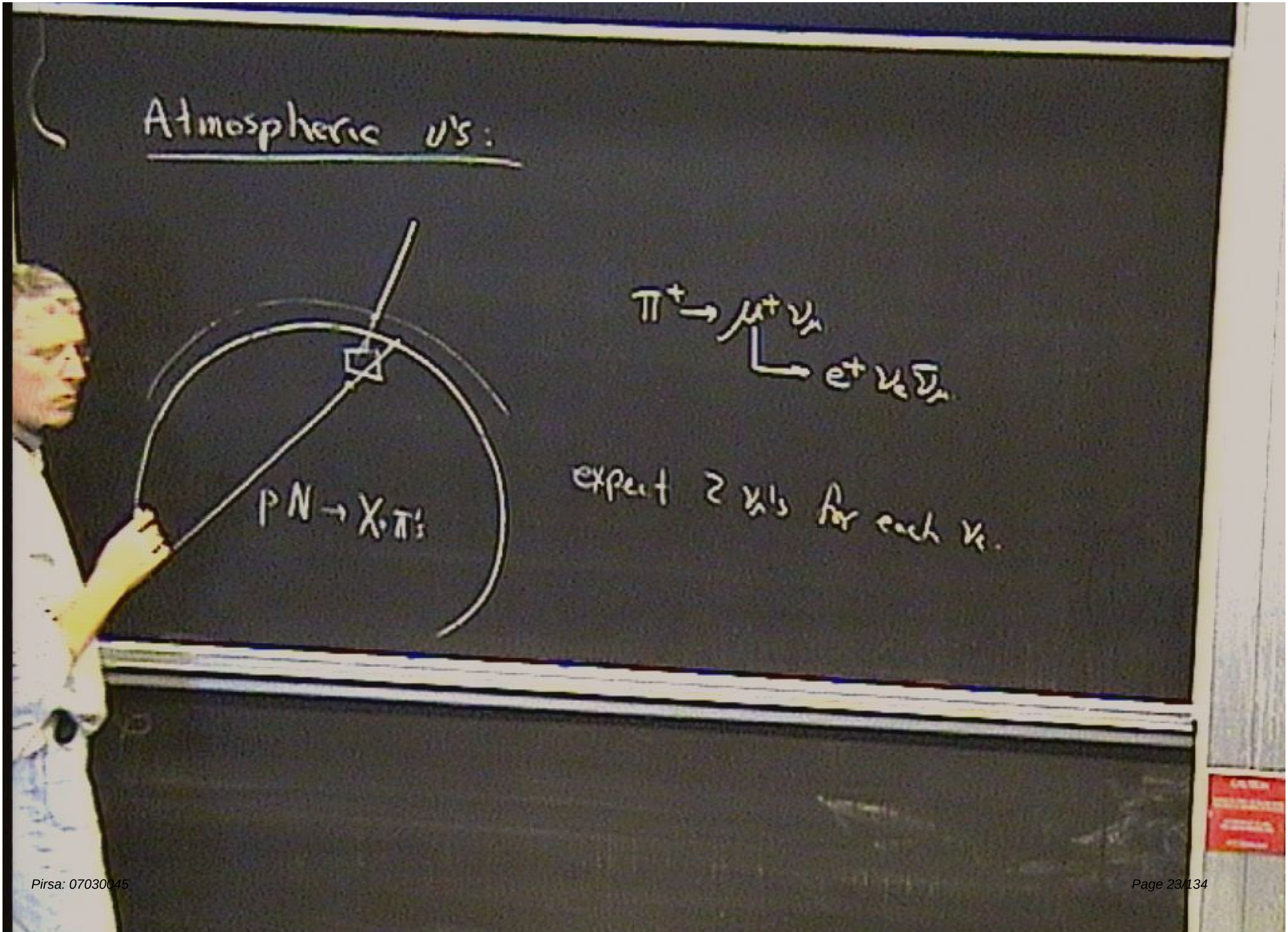
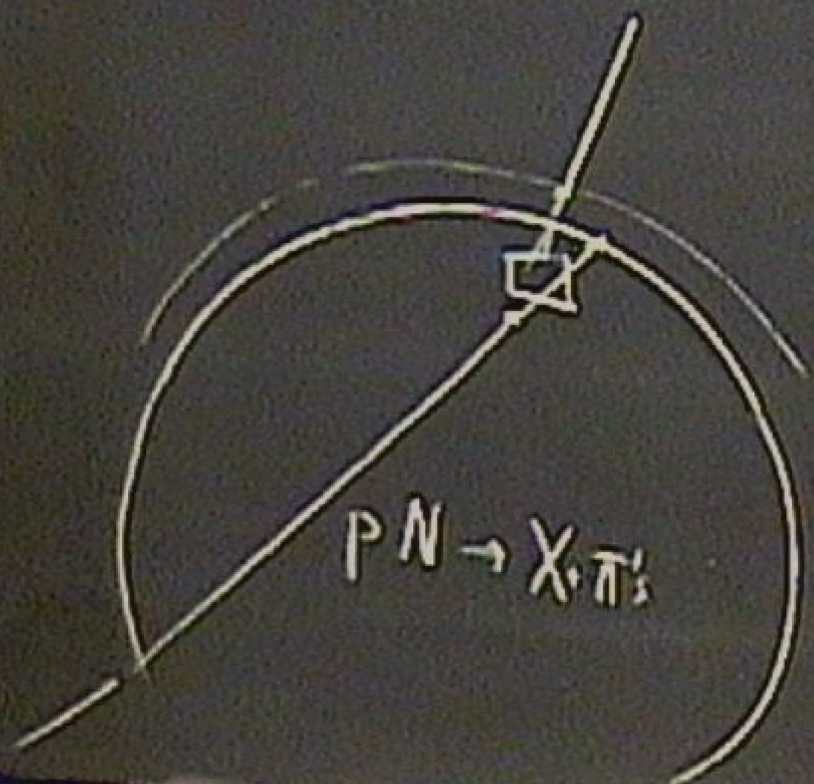

Atmospheric ν's:
N


Atmospheric ν's:
π⁺ → μ⁺ ν_μ
e⁺ ν_e ν̄_μ
π's


Atmospheric ν's:
$\pi^+ \to \mu^+ \nu_\mu$
$\to e^+ \nu_e \bar{\nu}_\mu$
$pN \to X + \pi$'s
expect 2 ν_μ's for each ν_e.

Atmospheric ν's:



$\pi^+ \to \mu^+ \nu_\mu$

$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$

expect 2 ν_μ's for each ν_e.

find: 1 ν_μ for each ν_e

Resonant oscillations (MSW)

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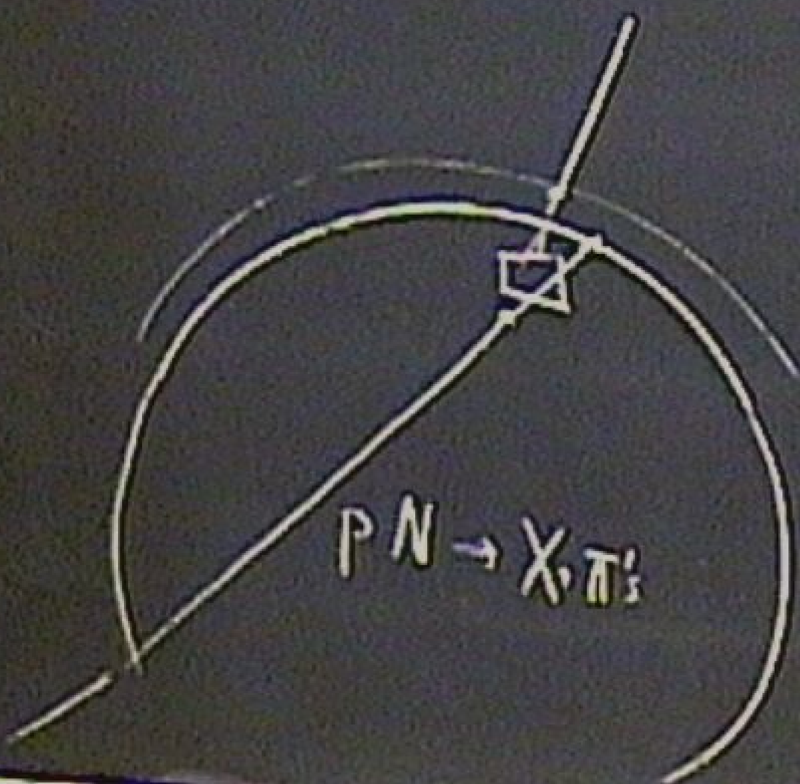
Neutrino Oscillations

Atmospheric ν's:



$$\pi^+ \to \mu^+ \nu_\mu$$
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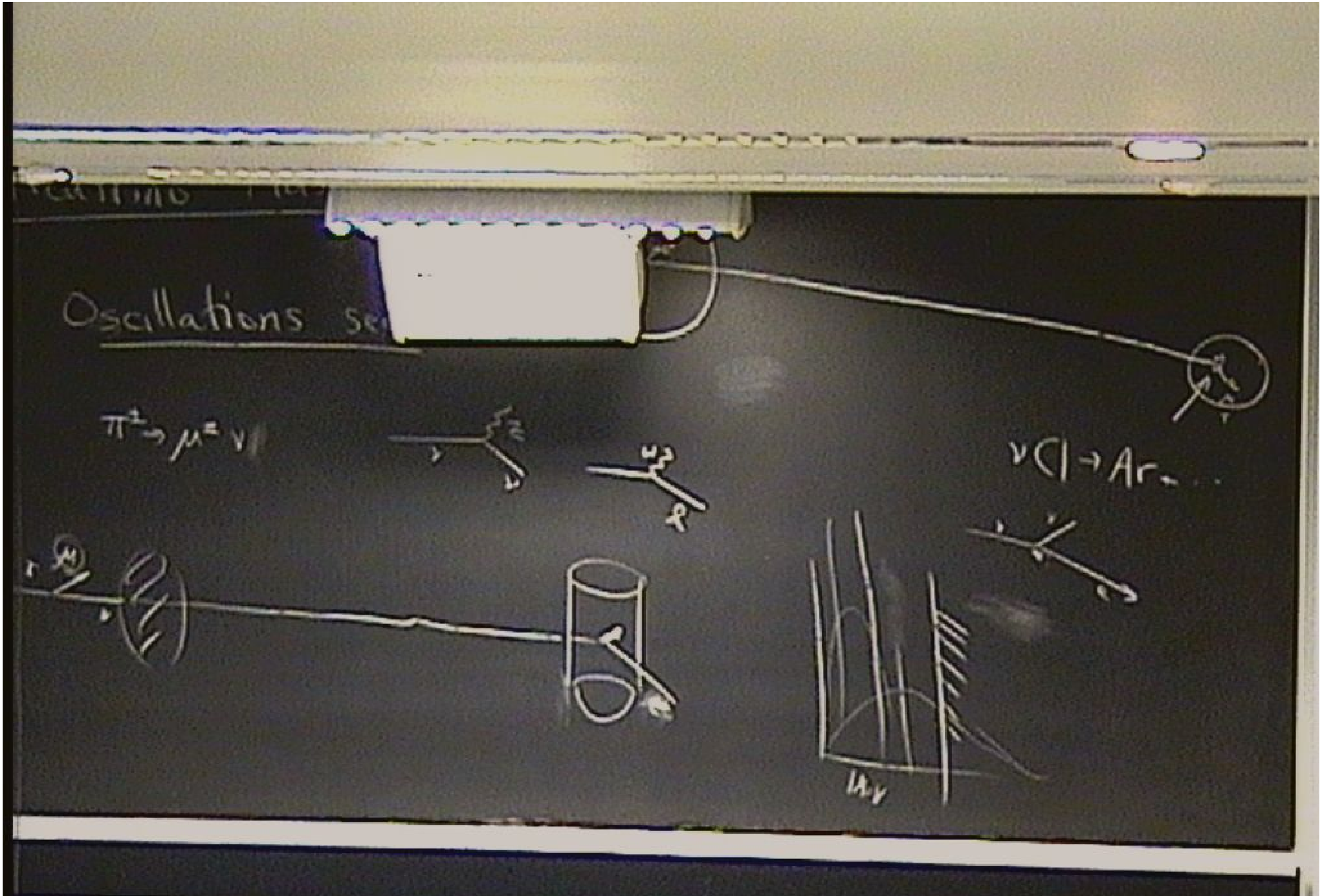
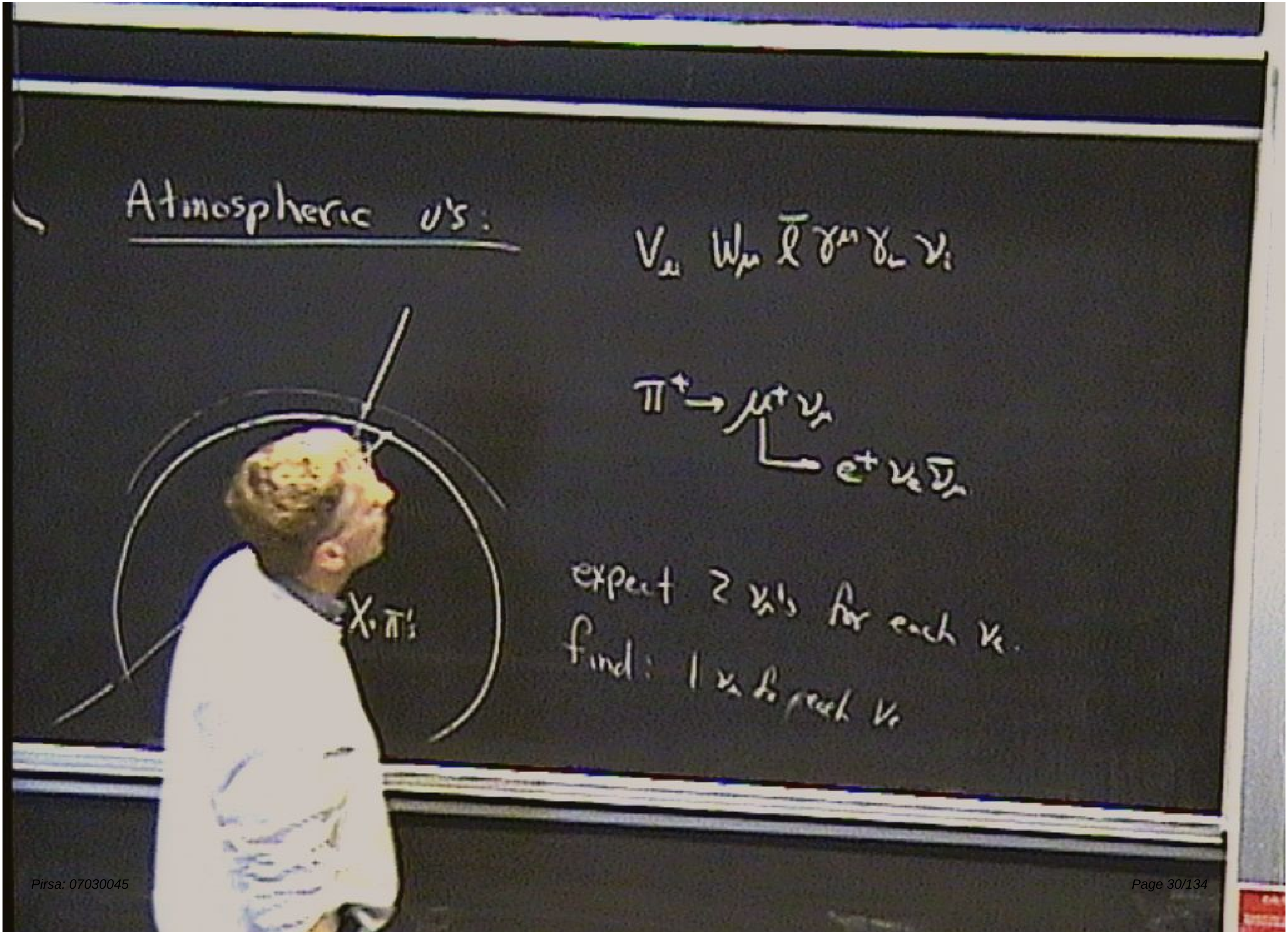


Oscillations se
$\pi^{\pm} \to \mu^{\pm} \nu$
$\nu Cl \to Ar + \ldots$


Atmospheric ν's:
$V_\mu W_\mu \bar{\ell} \gamma^\mu \gamma_- \nu_i$
$\pi^+ \to \mu^+ \nu_\mu$
$e^+ \nu_e \bar{\nu}_\mu$
expect 2 ν_μ's for each ν_e.
find: 1 ν_μ for each ν_e
X, π's

Neutrino Oscillations If there were a mass term for ν's, if it mixed ν flavours it would induce a CKM-like matrix in the cc interactions of leptons.

(PMNS matrix)

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For Quarks $Q=\begin{pmatrix} u \\ d \end{pmatrix}$ U D

leptons $L=\begin{pmatrix} \nu \\ e \end{pmatrix}$ $\underset{?}{N}$ E

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what if there were a RH neutrino.

Neutrino Oscillations

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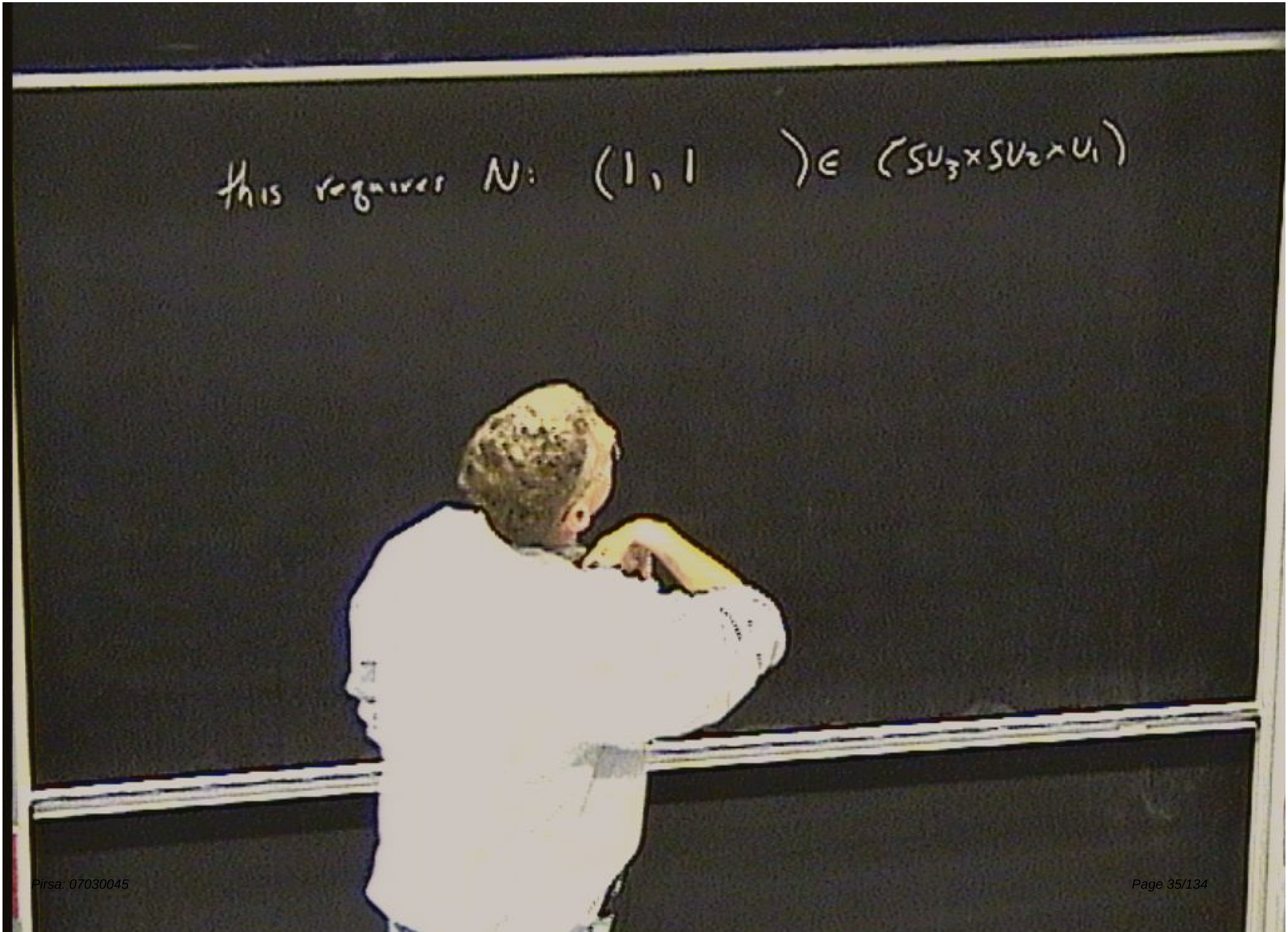
For Quarks $Q = \begin{pmatrix} u \\ d \end{pmatrix}$ U D

leptons $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$ $\underset{?}{N}$ E

what if there were a RH neutrino.
N has QN s.t. LNH is allowed

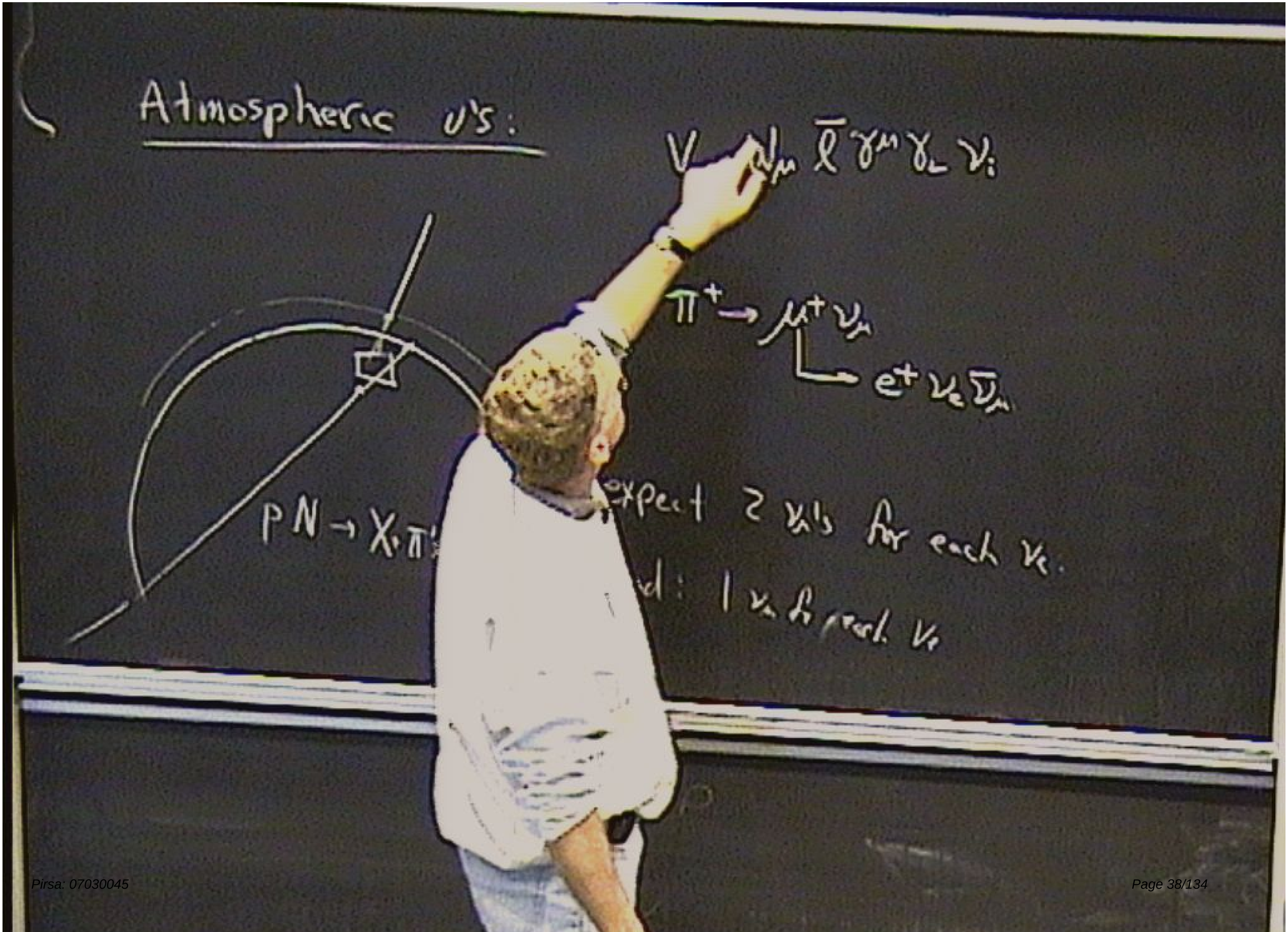

this requires N: (1, 1) ∈ (SU3 × SU2 × U1)

this requires N: $\underline{(1,1,0)} \in (SU_3 \times SU_2 \times U_1)$

"sterile"

this requires N: $\underbrace{(1,1,0)}_{\text{"sterile"}} \in (SU_3 \times SU_2 \times U_1)$

easy to miss.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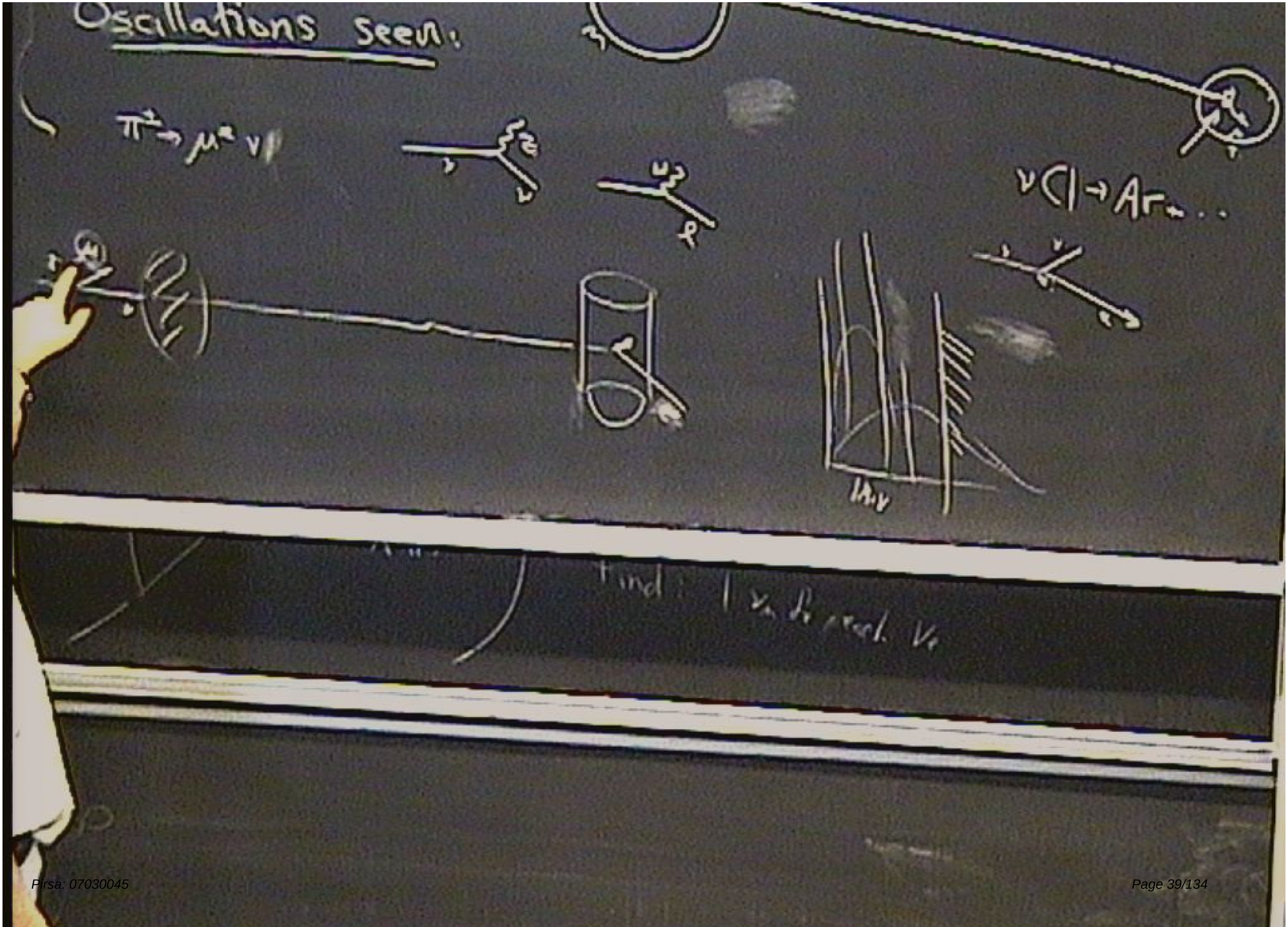
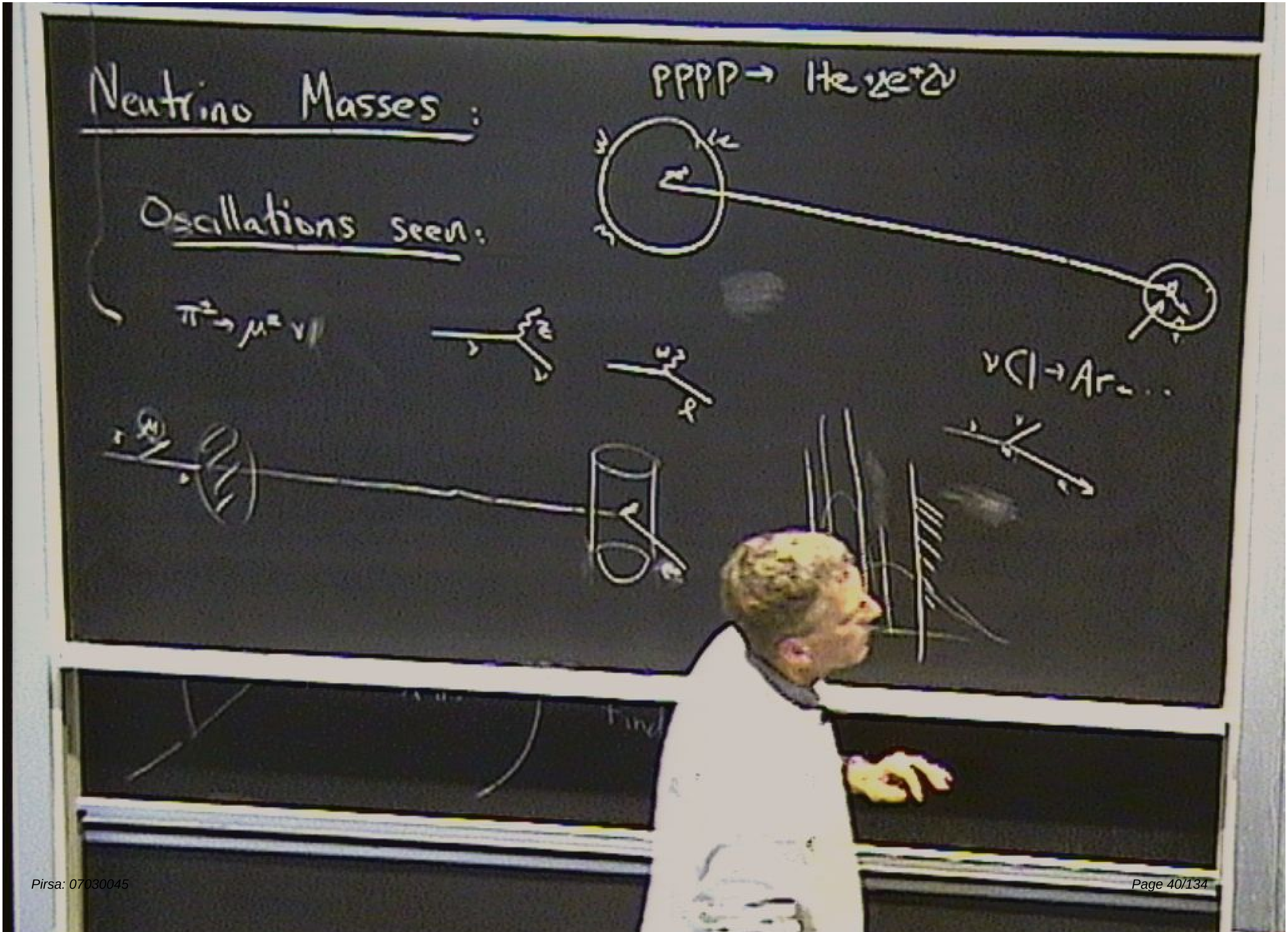
π+ → μ+ νμ
e+ νe ν̄μ
pN → X, π's
expect 2 νμ's for each νe.
1 νμ for each νe


Oscillations seen:
π± → μ± ν
νCl → Ar
Find:


Neutrino Masses:
Oscillations seen:
PPPP → He 2e+ 2ν
νCl → Ar...

this requires N: $\underline{\underline{(1,1,0)}} \in (SU_3 \times SU_2 \times U_1)$

"Sterile"

easy to miss. but also their existence is constrained

from: - SNO measurements of solar neutrinos

- energy losses in Supernovae, stars, etc.

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ex: $\mathcal{L} = \mathcal{L}_{SM} - \bar{N}_i (\not\partial \delta_{ij} + M_{ij}) N_j - \left(y_{ij} (\bar{L}_i \gamma_L N_j) H + c.c. \right)$

this requires N: $\underline{\underline{(1,1,0)}} \in (SU_3 \times SU_2 \times U_1)$

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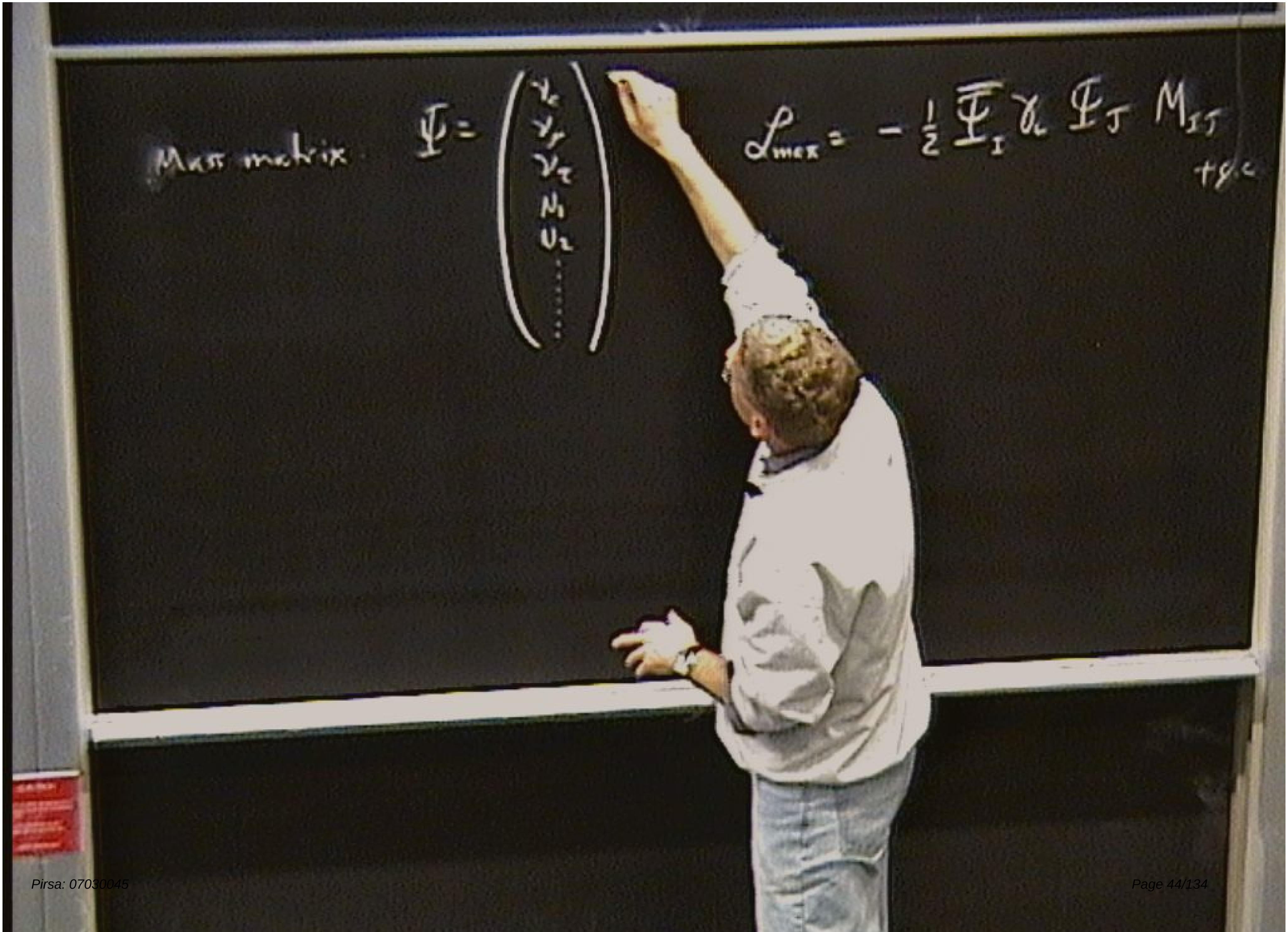
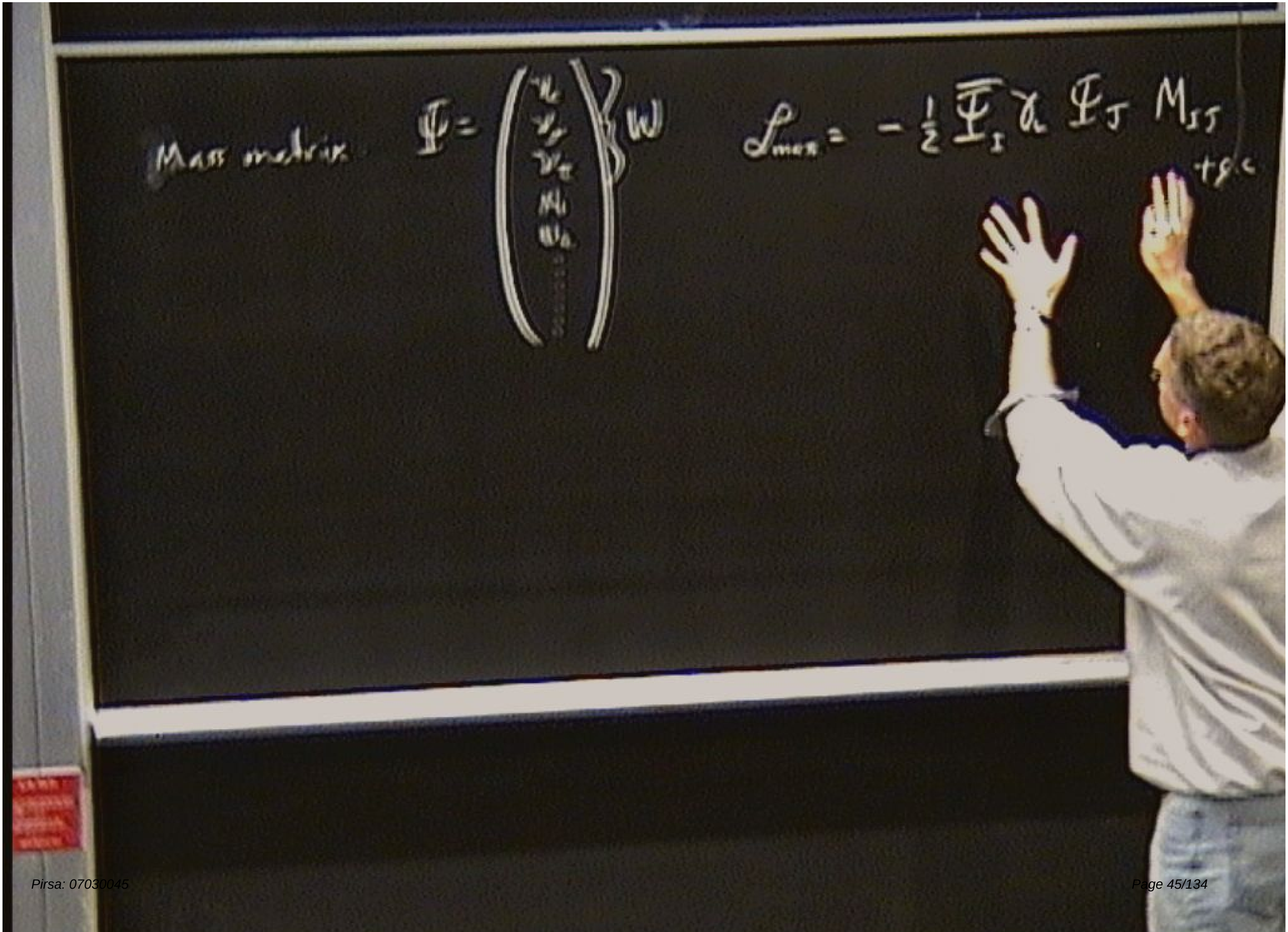
$(\nu_i, \gamma_i N_i)_L \quad H = \binom{\cdot}{\cdot}$

if N_i's exist: $\mathcal{L} = \mathcal{L}_{SM} - \bar{N}_i (\not{\partial} \delta_{ij} + M_{ij}) N_j - (y_{ij} (\bar{L}_i \gamma_L N_j) H + c.c.)$


Mass matrix
$\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$
$\mathcal{L}_{mass} = -\frac{1}{2} \bar{\Psi}_I \gamma_c \Psi_J M_{IJ}$
$+ h.c.$


Mass matrix
$\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ W
$\mathcal{L}_{mass} = -\frac{1}{2} \overline{\Psi}_I \gamma_L \Psi_J M_{IJ}$
$+ h.c.$

Mass matrix

$$\Psi = \left.\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}\right\} W \qquad \mathcal{L}_{mass} = -\frac{1}{2} \bar{\Psi}_I \gamma_L \Psi_J M_{IJ} + h.c.$$

$$M = \left(\begin{array}{c|c} 0 & y_j v \\ \hline y_j v & M_{ij} \end{array}\right)$$

Mass matrix:

$$\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix} \quad \left.\begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix}\right\} W$$

$$\mathcal{L}_{mass} = -\frac{1}{2}\,\bar{\Psi}_I\,\gamma_L\,\Psi_J\,M_{IJ} + h.c.$$

$$M = \begin{array}{c|c} & \begin{matrix} e\ \mu\ \tau & \quad j \end{matrix} \\ \begin{matrix} a \\ \\ i \end{matrix} & \left(\begin{array}{c|c} 0 & y_{aj}v \\ \hline y_{ia}v & M \end{array}\right) \end{array}$$

$v \simeq 246$ GeV

$y_{ai} \ll 1$

Mass matrix: $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ $\left.\begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix}\right\} W$

$$\mathcal{L}_{mass} = -\frac{1}{2} \bar{\Psi}_I \gamma_L \Psi_J M_{IJ} + h.c.$$

$$M = \begin{array}{c} \\ a \\ i \end{array} \begin{array}{c} \begin{array}{cc} a = e\,\mu\,\tau & j \end{array} \\ \left(\begin{array}{c|c} 0 & y_{aj} v \\ \hline y_{ia} v & M_{ij} \end{array}\right) \end{array}$$

$v \approx 246$ GeV

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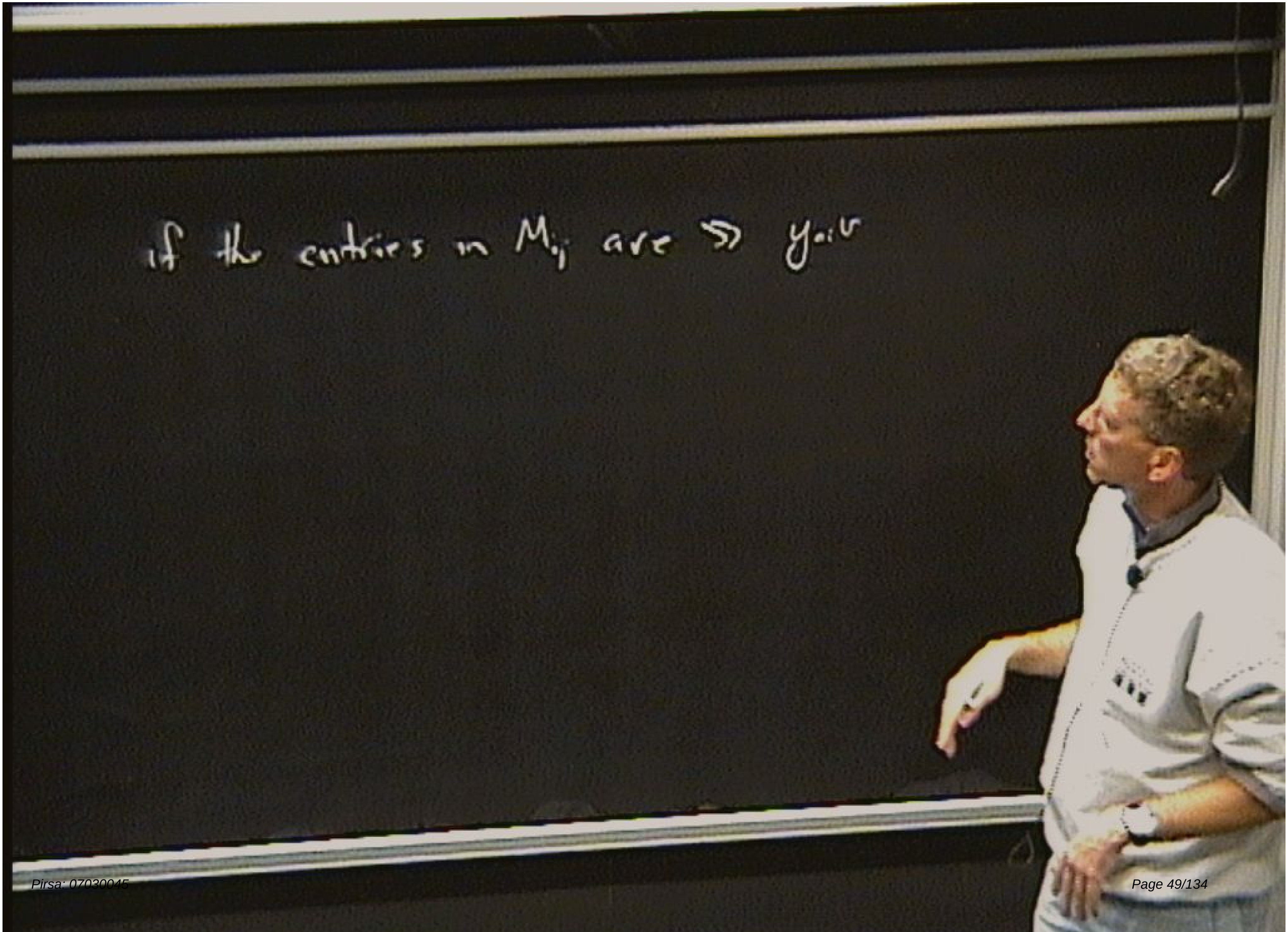
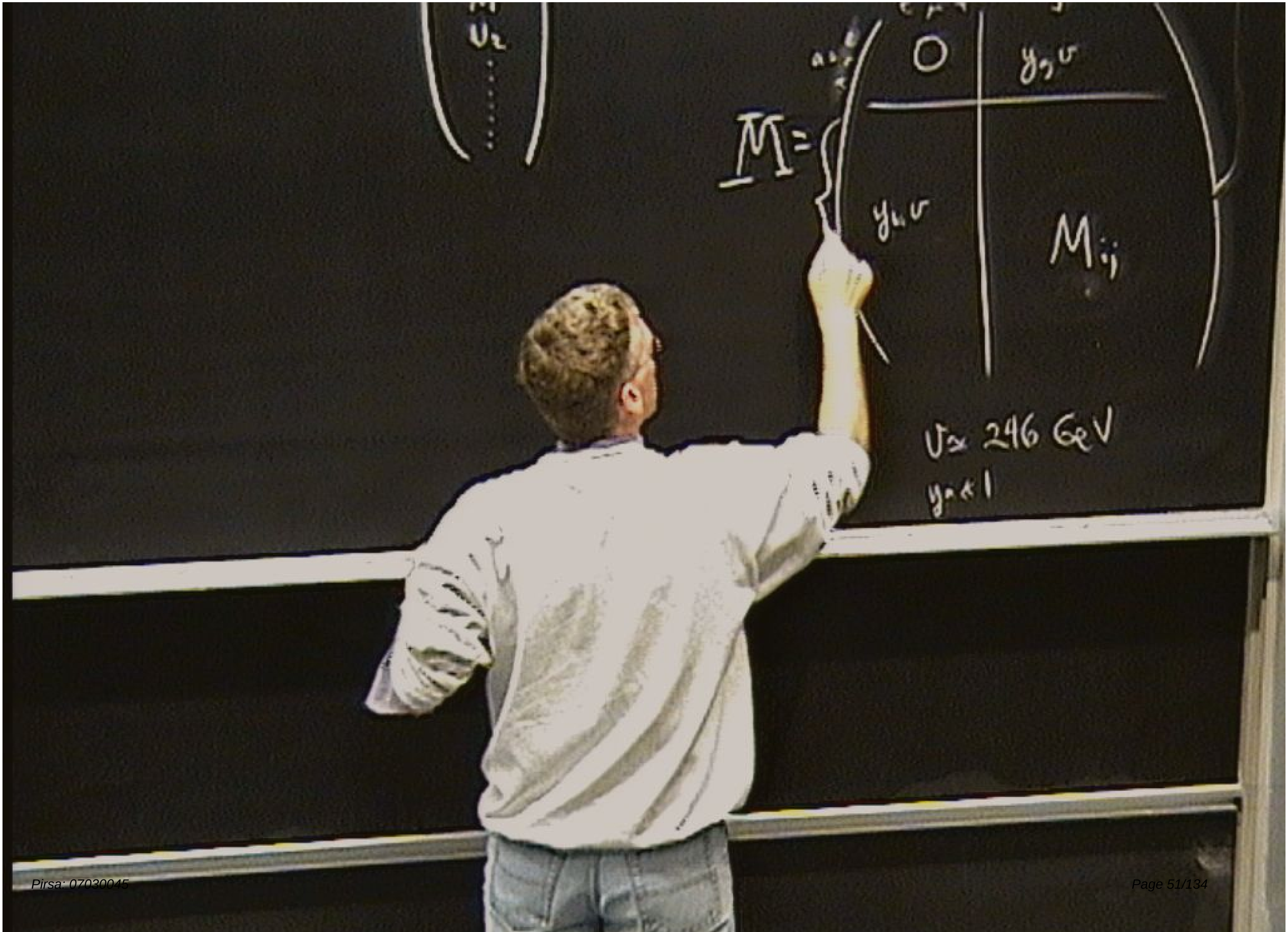
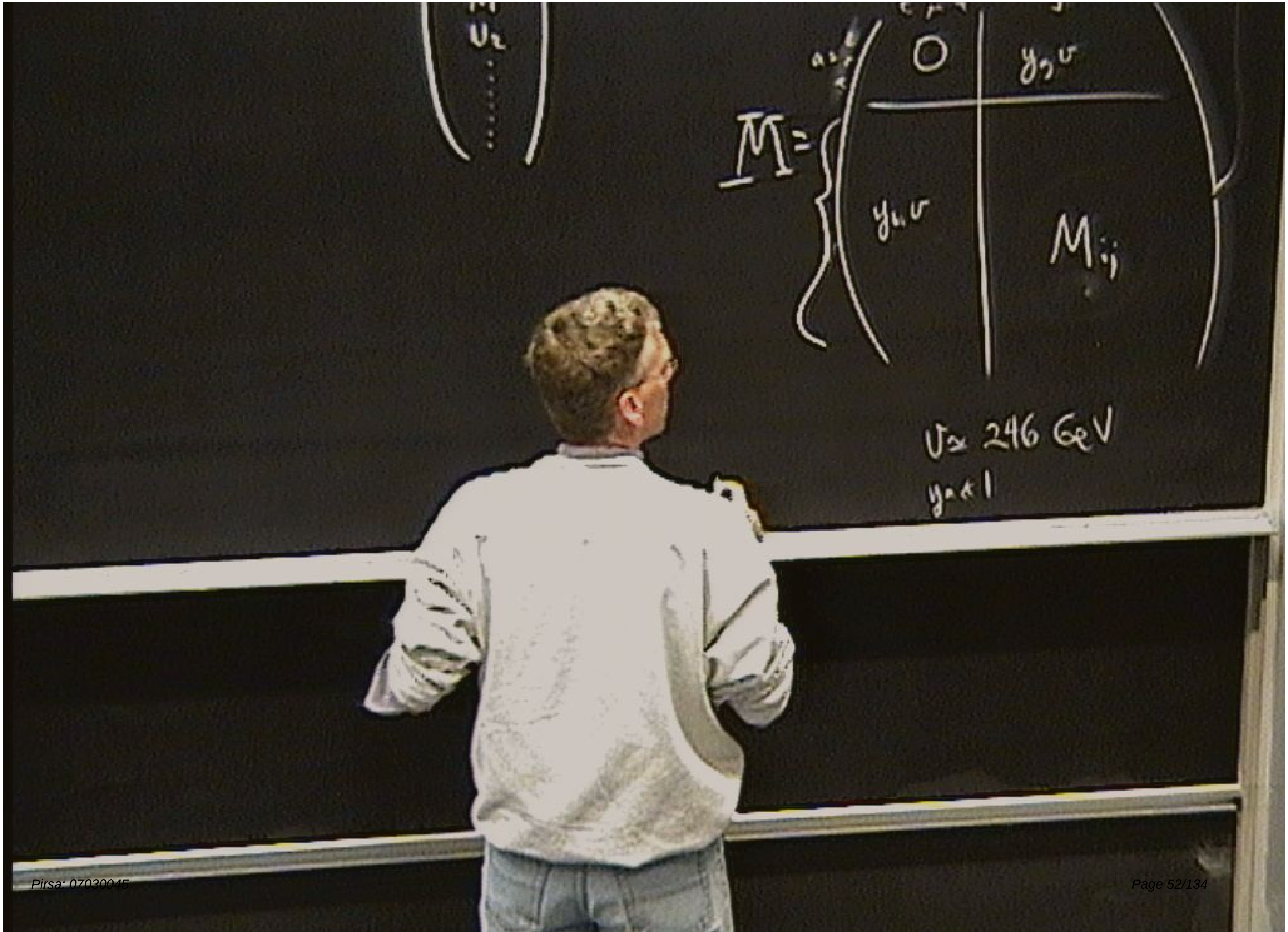
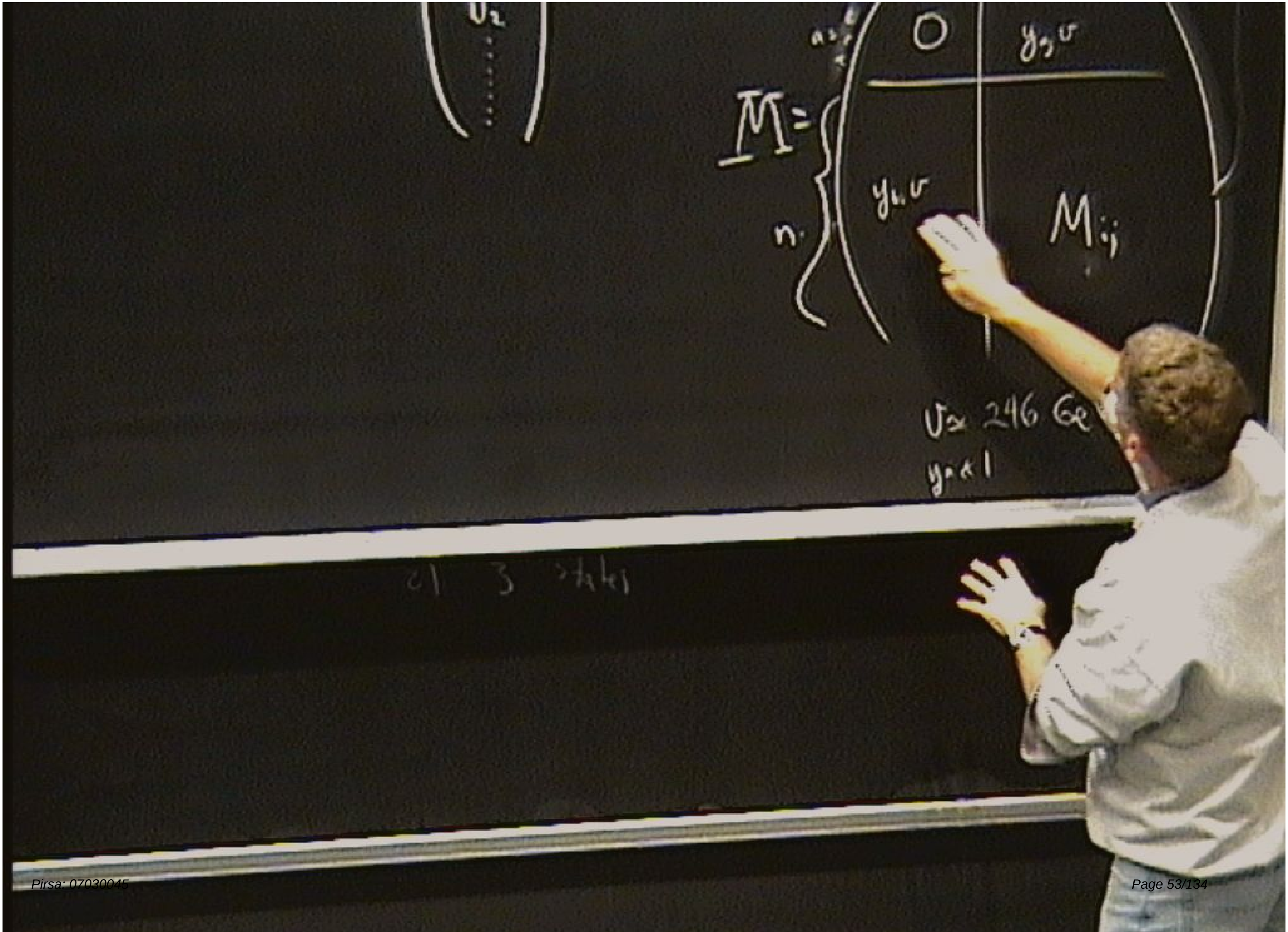

if the entries in M_{ij} are $\gg$ your

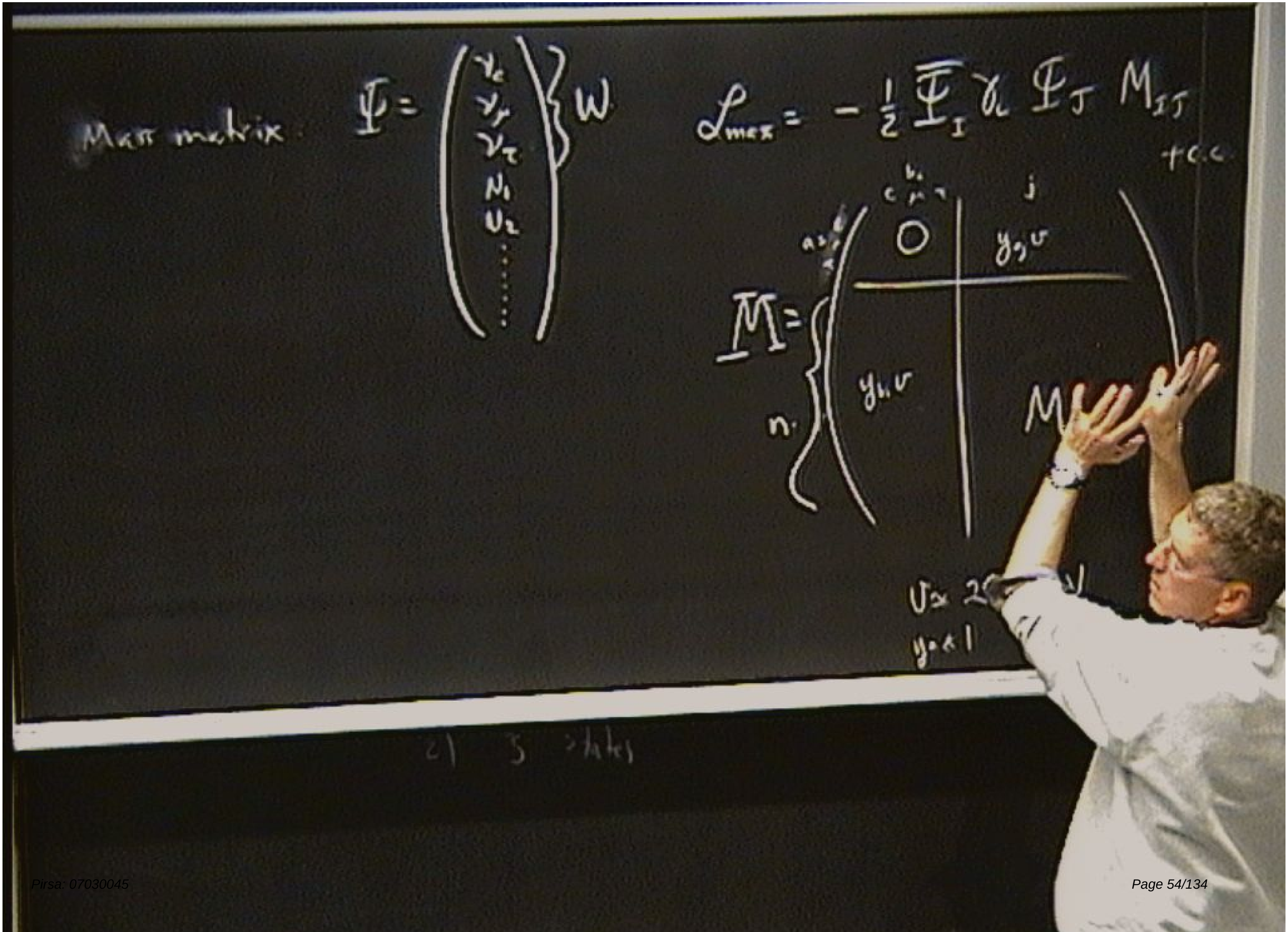
if the entries in M_{ij} are $\gg y_{ai}v$ (no zero eigenvalues etc)


v_2
$M =$
0
$y_\nu v$
$y_\nu v$
M_{ij}
$v \approx 246$ GeV
$y_\nu \ll 1$



M=
O
$y_\nu v$
$y_\nu v$
M_{ij}
$v \simeq 246$ GeV
$y_\nu \ll 1$


M =
O
yν v
yν v
Mij
v ≈ 246 Ge
yν ≪ 1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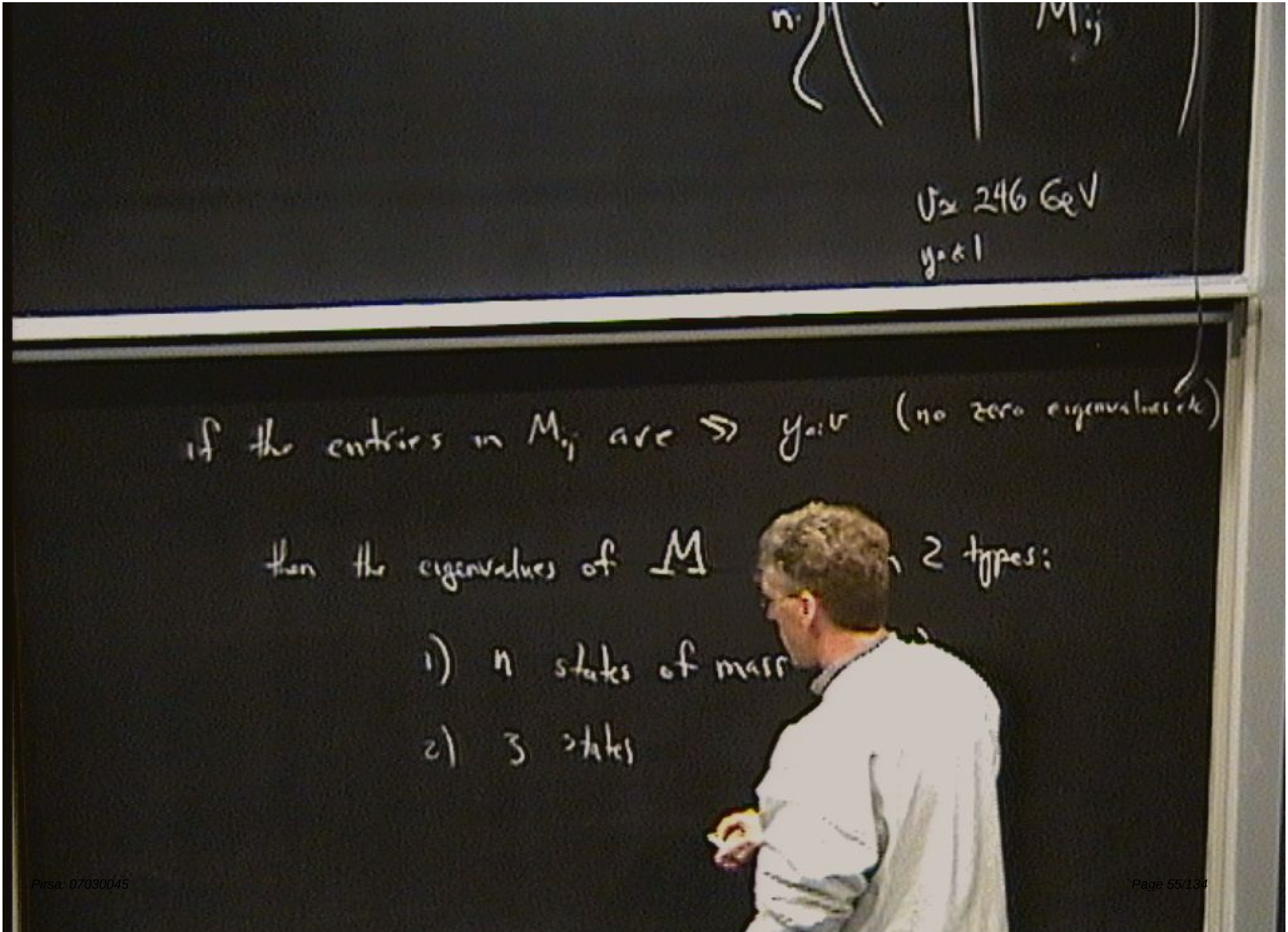
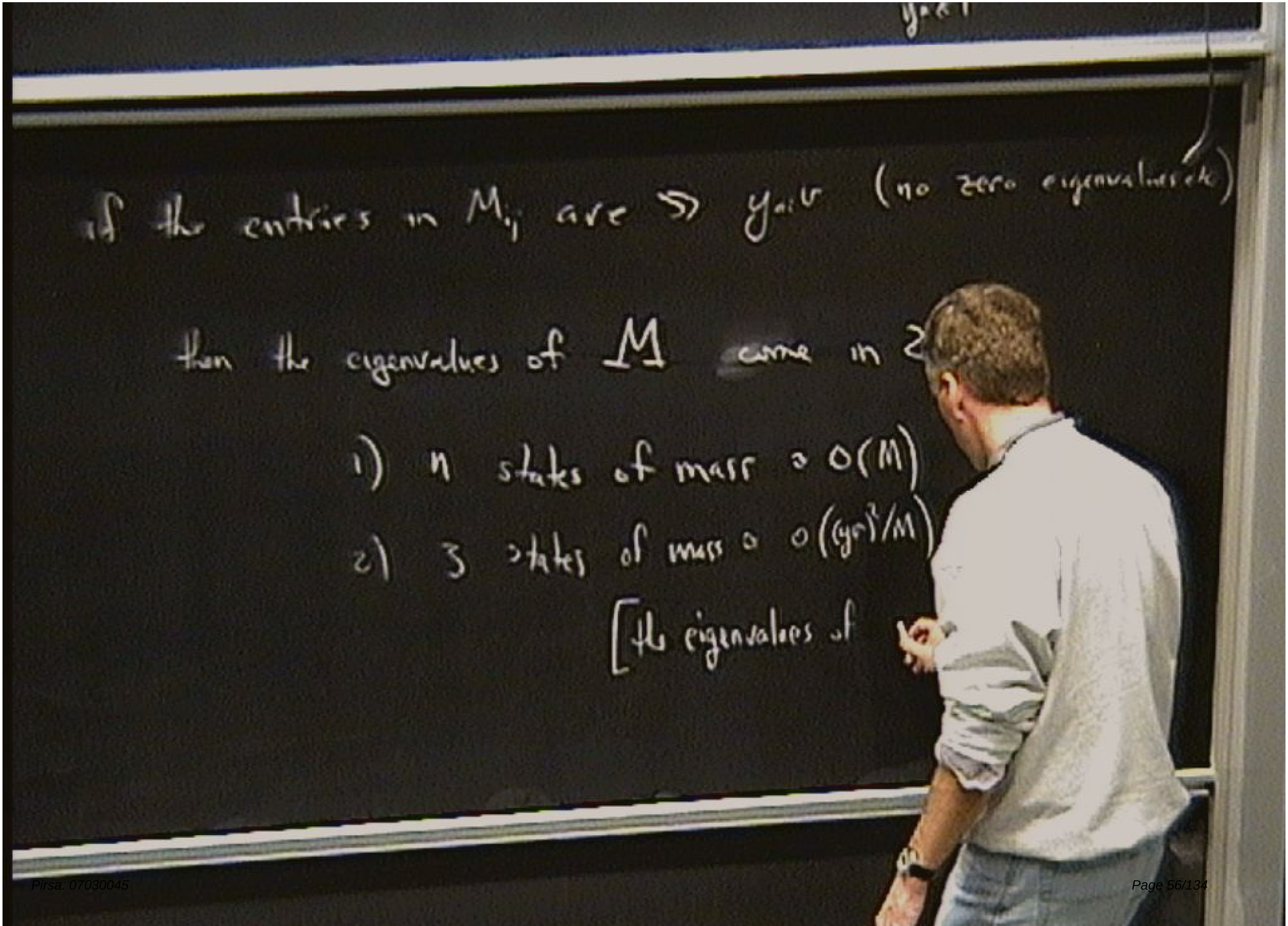
$\mathcal{L}_{mass} = -\frac{1}{2} \bar{\Psi}_I \gamma_5 \Psi_J M_{IJ}$ + c.c.
$M = \begin{pmatrix} 0 & y_{ij} v \\ y_{ji} v & M \end{pmatrix}$
n
$y_0 \ll 1$


M_{ij}
$v \approx 246$ GeV
$y_a \ll 1$
if the entries in M_{ij} are $\gg y_a v$ (no zero eigenvalues etc)
then the eigenvalues of $\mathcal{M}$ 2 types:
1) n states of mass
2) 3 states


if the entries in M_ij are >> y_a v (no zero eigenvalues etc)
then the eigenvalues of M come in 2
1) n states of mass ≈ O(M)
2) 3 states of mass ≈ O((yv)^2/M)
[the eigenvalues of

if the entries in M_{ij} are $\gg y_a v$ (no zero eigenvalues etc)

then the eigenvalues of $\mathcal{M}$ come in 2 types:

1) n states of mass $\approx O(M)$

2) 3 states of mass $\approx O\left((y_a v)^2/M\right)$

[the eigenvalues of $(y_a v) M^{-1}_{ij}$

if the entries in M_{ij} are $\gg y_{ai}v$ (no zero eigenvalues etc)

then the eigenvalues of $\mathcal{M}$ come in 2 types:

1) n states of mass $\approx O(M)$

2) 3 states of mass $\approx O((yv)^2/M)$

[the eigenvalues of $(y_{ai}v) M^{-1}_{ij} (y_{bj}v) \equiv m_{ab}$

if the entries in M_{ij} are $\gg y_{ai} v$ (no zero eigenvalues etc.)

then the eigenvalues of $\mathcal{M}$ come in 2 types:

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2) 3 states of mass $\approx O((yv)^2/M)$

[the eigenvalues of $(y_{ai} v) M^{-1}_{ij} (y_{bj} v) \equiv m_{ab}$]

ass matrix: $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ $\left.\begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix}\right\} W$

$$\mathcal{L}_{mass} = -\frac{1}{2} \bar{\Psi}_I \gamma_L \Psi_J M_{IJ} + c.c.$$

$$\begin{array}{cc} & \begin{array}{cc} e\ \mu\ \tau & j \end{array} \\ \begin{array}{c} a \\ n \end{array} & M = \left(\begin{array}{c|c} 0 & y_{aj} v \\ \hline y_{ia} v & M_{ij} \end{array}\right) \end{array}$$

$v \approx 246$ GeV

$y_a \lesssim 1$

... come in 2 types:

... of mass $\approx O(M)$

this requires N: $\underline{\underline{(1,1,0)}} \in (SU_3 \times SU_2 \times U_1)$

"Sterile"

easy to miss. but also their existence is constrained from: - SNO measurements of solar neutrinos

- energy losses in Supernovae, stars, etc.

If N_i's exist: $\mathcal{L} = \mathcal{L}_{SM} - \bar{N}_i (\not{\partial}_i + M_{ij}) N_j - (y_{ai} (\bar{L}_a \gamma_L N_i) H + c.c.)$ $(\nu, \ell, N_i)_L$ $H = \binom{0}{v}$

Mass matrix:

$$\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix} \Big\} W$$

$$\mathcal{L}_{mass} = -\frac{1}{2}\,\overline{\Psi}_I\,\gamma_c\,\Psi_J\,M_{IJ} + c.c.$$

$$\mathbb{M} = \begin{array}{c} \\ 3 \\ n \end{array} \begin{array}{c} \begin{array}{cc} e\ \mu\ \tau & \quad j \end{array} \\ \begin{pmatrix} 0 & y_j v \\ y_i v & M_{ij} \end{pmatrix} \end{array}$$

$v \simeq 246$ GeV

$y_i \lesssim 1$

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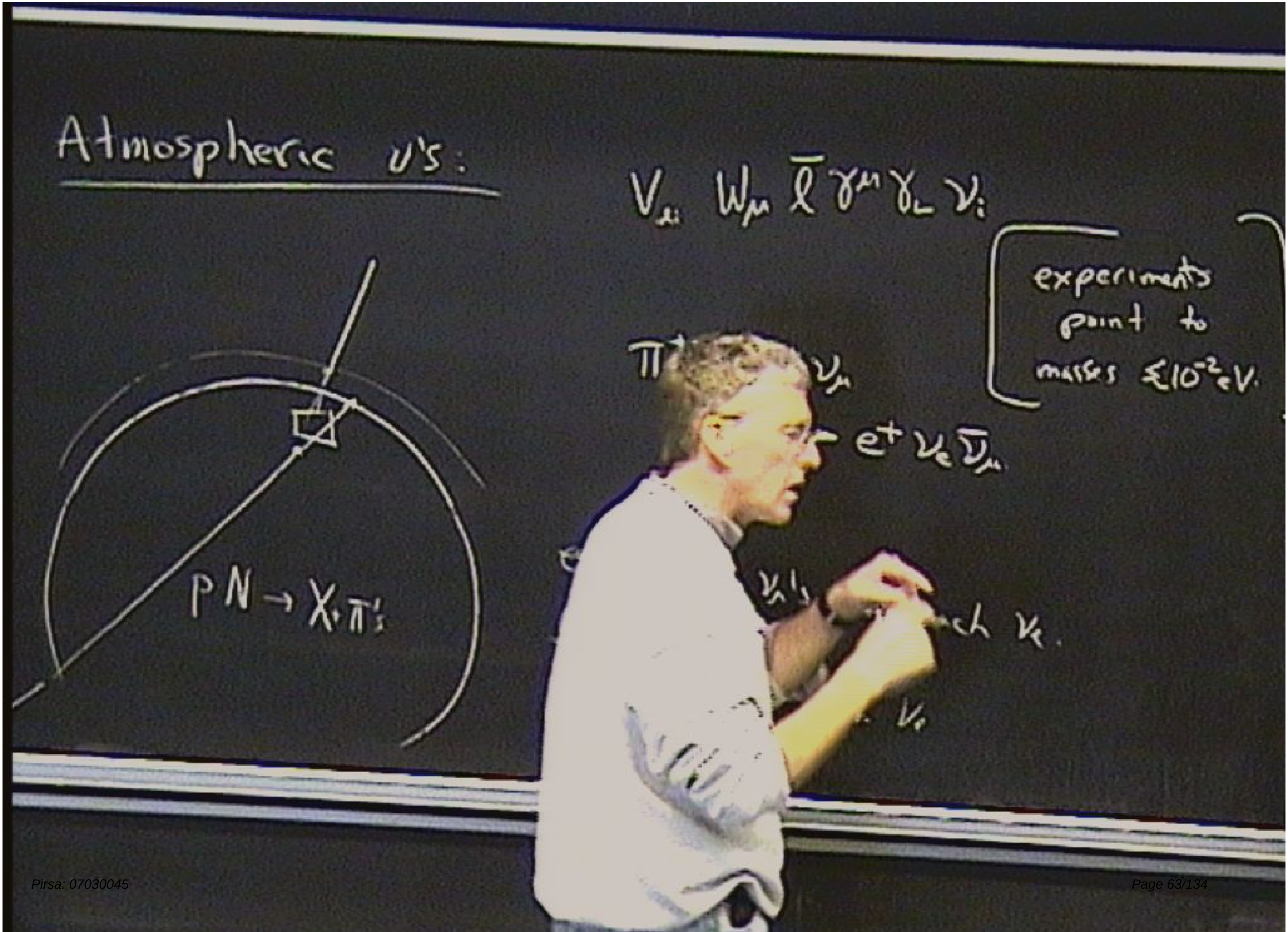
Atmospheric ν's:
V_ei W_μ l̄ γ^μ γ_L ν_i
experiments
point to
masses ≲ 10^-2 eV.
pN → X + π's
e^+ ν_e ν̄_μ

atrix $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$

$$-\frac{1}{2}\,\bar{\Psi}_I\,\gamma_L\,\Psi_J\,M_{IJ} + \text{c.c.}$$

$$M = \begin{array}{cc} & \begin{array}{cc} \nu_a \; (e\ \mu\ \tau) & j \end{array} \\ \begin{array}{c} a=e,\mu,\tau \\ n \end{array} & \left(\begin{array}{c|c} 0 & y_{aj} v \\ \hline y_{ia} v & M_{ij} \end{array}\right) \end{array}$$

$v \simeq 246$ GeV

$y_a \lesssim 1$

$n \cdot \Big(\quad \Big| \quad M_{ij} \Big)$

$v \simeq 246$ GeV

$y_{ai} \ll 1$

1) n states of mass $\simeq O(M)$

2) 3 states of mass $\simeq O((yv)^2/M)$

[the eigenvalues of $(y_{ai} v) M^{-1}_{ij} (y_{bj} v) \equiv m_{ab}$]

naturally very light.

Mass matrix: $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ $\Big\}\, W$

$$\mathcal{L}_{mass} = -\frac{1}{2}\, \bar{\Psi}_I\, \gamma_L\, \Psi_J\, M_{IJ} \quad + \text{c.c.}$$

$$\begin{array}{cc} & \begin{array}{cc} e\,\mu\,\tau & j \end{array} \\ \mathbb{M} = \begin{array}{c} a = e,\mu,\tau \\ \\ n \end{array} & \left(\begin{array}{c|c} 0 & y_{aj} v \\ \hline y_{ia} v & M_{ij} \end{array}\right) \end{array}$$

$v \simeq 246$ GeV

$y_a \lesssim 1$

if $\dfrac{y^2 v^2}{M} \simeq 10^{-2}\,\text{eV}$

$y \simeq O(1)$ $\qquad v \simeq 10^2\,\text{GeV} \simeq 10^{11}\,\text{eV}$

1) n states of mass $\simeq O(M)$

Mass matrix: $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ W

$$\mathcal{L}_{mass} = -\frac{1}{2}\,\bar{\Psi}_I\,\gamma_c\,\Psi_J\,M_{IJ} + c.c.$$

$$\underline{M} = \begin{pmatrix} 0 & y_{ij}v \\ y_{ij}v & M_{ij} \end{pmatrix}$$

if $\dfrac{y^2 v^2}{M} \simeq 10^{-2}\,eV$

$O(1)$ $\quad v \simeq 10^2\,GeV \simeq 10^{11}\,eV$

$M \simeq$

$v \simeq 246\,GeV$

$y_0 \ll 1$

1) n states of mass $\simeq O(M)$

2) ... $O((yv)^2/M)$

Mass matrix. $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ $\big\}\, W$

$$\mathcal{L}_{mass} = -\frac{1}{2}\, \overline{\Psi}_I\, \gamma_L\, \Psi_J\, M_{IJ} + c.c.$$

$$\underline{M} = \begin{array}{c|c|c} & e\ \mu\ \tau & j \\ \hline 3 & 0 & y_{ij} v \\ n & y_{ij} v & M_{ij} \end{array}$$

if $\dfrac{y^2 v^2}{M} \simeq 10^{-2}\,eV$

$y \simeq O(1)$ $\quad v \simeq 10^2\,GeV \simeq 10^{11}\,eV$

$10^{22}/10^{-2}\,eV \simeq$

$v \simeq 246\,GeV$

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n states of mass $\simeq O(M)$

$\simeq O\left((yv)^2/M\right)$

Mass matrix: $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ $\left.\begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix}\right\} W$

$$\mathcal{L}_{mass} = -\frac{1}{2} \bar{\Psi}_I \gamma_L \Psi_J M_{IJ} + c.c.$$

$$\mathbb{M} = \begin{array}{cc} & \begin{array}{cc} e\,\mu\,\tau & j \end{array} \\ \begin{array}{c} \alpha \\ n \end{array} & \begin{pmatrix} 0 & y_{\alpha j} v \\ y_{i\alpha} v & M_{ij} \end{pmatrix} \end{array}$$

if $\frac{y^2 v^2}{M} \simeq 10^{-2}\,\text{eV}$

$y \simeq O(1)$ $\quad v \simeq 10^2\,\text{GeV} \simeq 10^{11}\,\text{eV}$ $\qquad v \simeq 246\,\text{GeV}$

$M \simeq 10^{22}/10^{-2}\,\text{eV} \simeq 10^{24}\,\text{eV} \simeq 10^{15}\,\text{GeV}$ $\qquad y_e \ll 1$

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Mass matrix: $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ $\left.\begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix}\right\} W$

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$$\underline{M} = \begin{array}{c|c} \begin{matrix} e & \mu & \tau \end{matrix} & j \\ \begin{pmatrix} 0 & y_{aj} v \\ y_{ia} v & M_{ij} \end{pmatrix} \end{array}$$

n

if $\frac{y^2 v^2}{M} \simeq 10^{-2}\,\text{eV}$

$v \simeq 10^2\,\text{GeV} \simeq 10^{11}\,\text{eV}$ $\quad$ $v \simeq 246\,\text{GeV}$

$M \simeq y^2 v^2 / 10^{-2}\,\text{eV} \simeq y^2 10^{24}\,\text{eV} \simeq y^2 10^{15}\,\text{GeV}$ $\quad$ $y_t \simeq 1$

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if $\dfrac{y^2 v^2}{M} \simeq 10^{-2}\,\mathrm{eV}$

$v \simeq 10^2\,\mathrm{GeV} \simeq 10^{11}\,\mathrm{eV}$

$v \simeq 246\,\mathrm{GeV}$

$y \approx 1$

$M \simeq y^2 v^2 / 10^{-2}\,\mathrm{eV} \simeq y^2\, 10^{24}\,\mathrm{eV} \simeq y^2\, 10^{15}\,\mathrm{GeV}$

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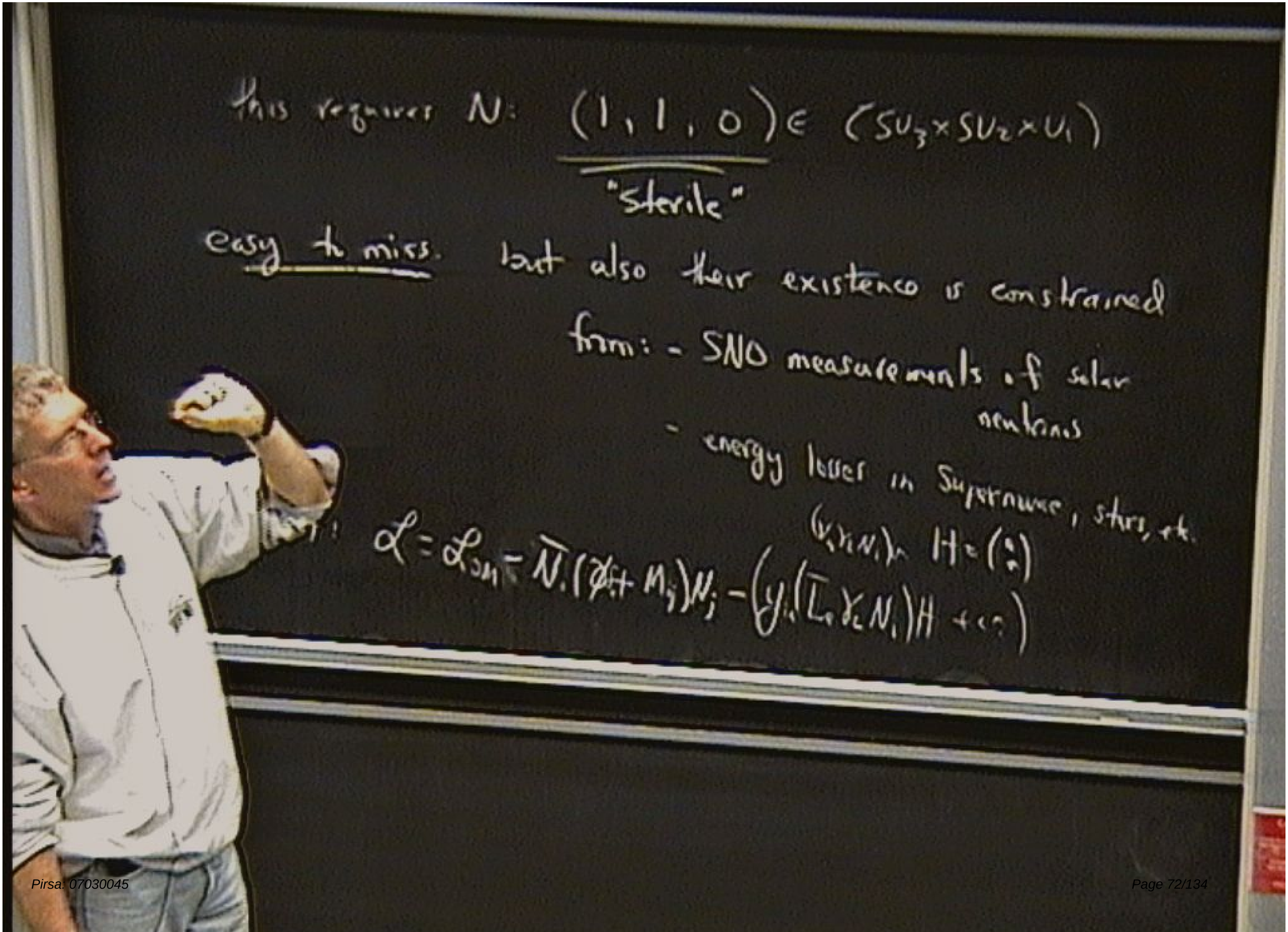
this requires N: (1, 1, 0) ∈ (SU3 × SU2 × U1)
"Sterile"
easy to miss. but also their existence is constrained
from: - SNO measurements of solar neutrinos
- energy losses in Supernovae, stars, etc.
L = L_SM + N̄_i (∂̸ + M_ij) N_j − (y_ia (L̄_a γ_c N_i) H + c.c.)

Mass matrix: $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ $\}W$

$$\mathcal{L}_{mass} = -\frac{1}{2} \bar{\Psi}_I \gamma_L \Psi_J M_{IJ} + c.c.$$

$$\mathbb{M} = \begin{array}{c|c} & \begin{array}{cc} e\ \mu\ \tau & j \end{array} \\ \begin{array}{c} a{:}3 \\ n \end{array} & \left(\begin{array}{c|c} 0 & y_{aj} v \\ \hline y_{ia} v & M_{ij} \end{array}\right) \end{array}$$

"See saw" mechanism

if $\dfrac{y^2 v^2}{M} \simeq 10^{-2}\,eV$

$v \simeq 10^2\,GeV \simeq 10^{11}\,eV$

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[the eigenvalues of $(y_{ai} v) M^{-1}_{ij} (y_{bj} v) \equiv m_{ab}$]

naturally very light.

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$y_i < 1$

if the entries in M_{ij} are $\gg y_{ai} v$ (no zero eigenvalues etc)

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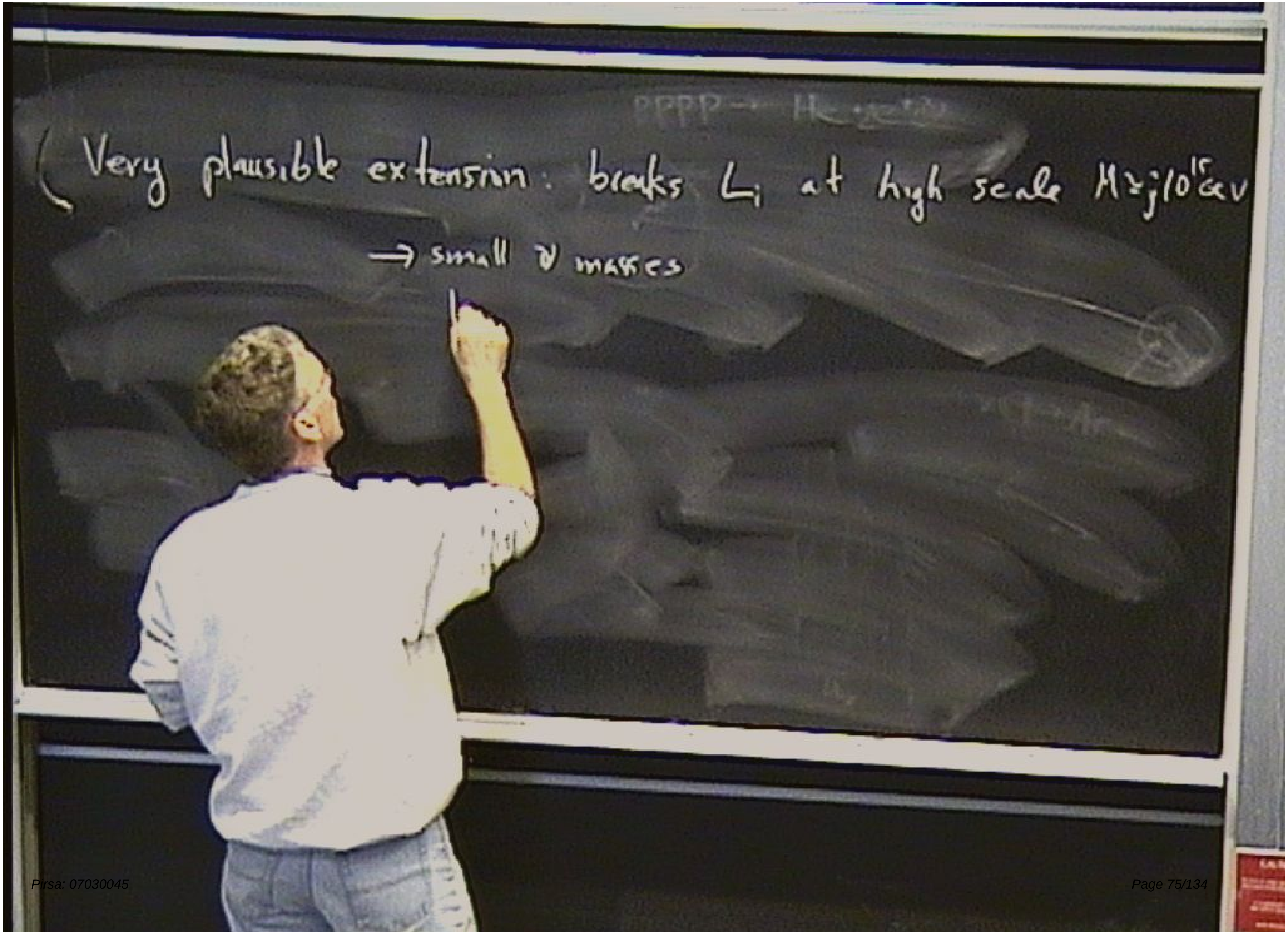
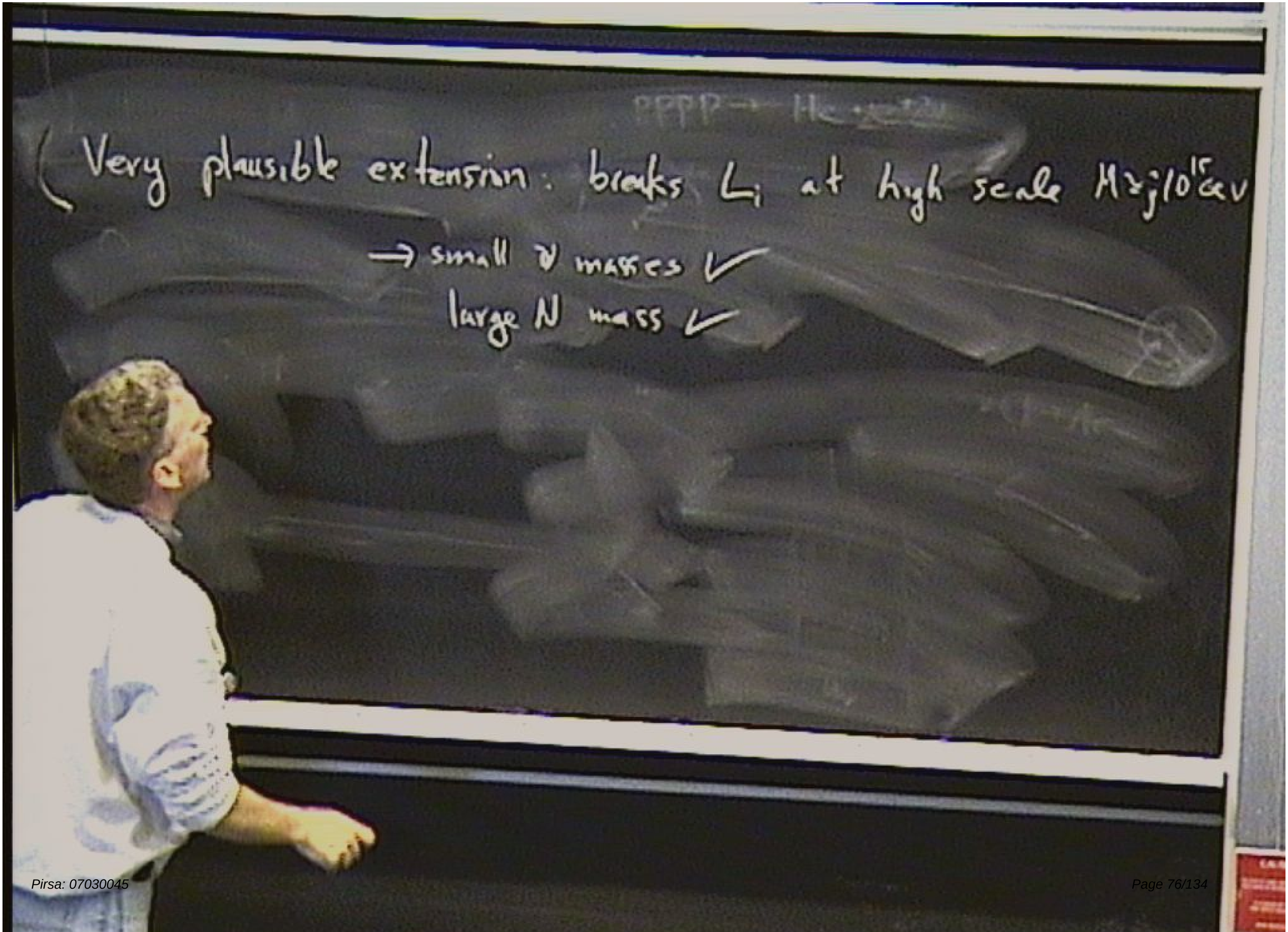

Very plausible extension: breaks L_i at high scale $M \gtrsim 10^{15}$ GeV
→ small ν masses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→ small ν masses ✓ $O(y^2 v^2/M)$

large N mass ✓ $O(M)$

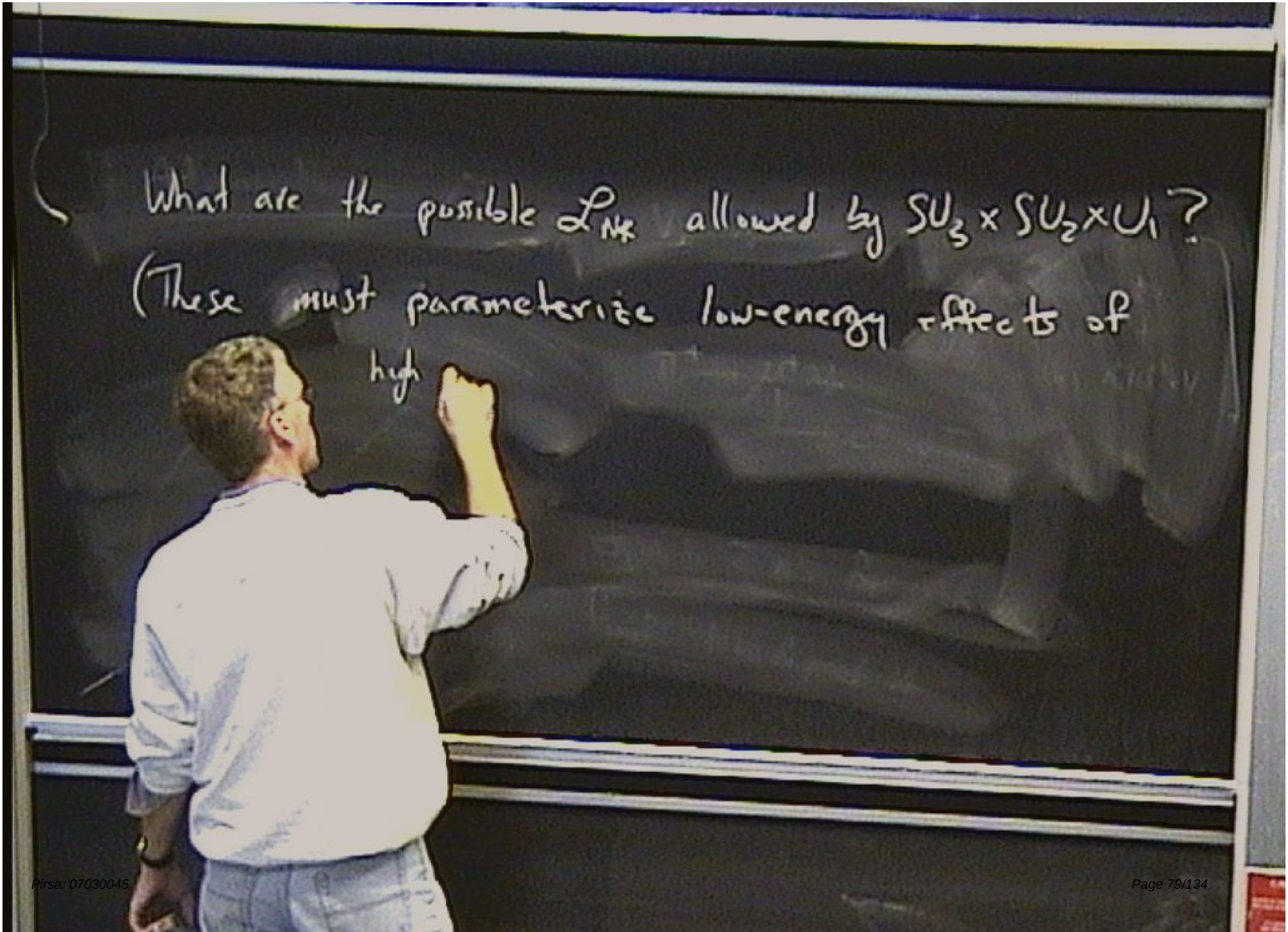
(Very plausible extension: breaks L_i at high scale $M \gtrsim 10^{15}$ GeV

$\rightarrow$ small ν masses ✓ $\quad O(y^2 v^2/M)$

large N mass ✓ $\quad O(M)$

If all the N's are heavy, they can be "integrated out" + their effects at low energies ($\ll M$) is described by

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \mathcal{L}_{NR}$$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What are the possible $\mathcal{L}_{NR}$ allowed by $SU_3 \times SU_2 \times U_1$?

(These <u>must</u> parameterize low-energy effects of high energy modifications to the SM,

<u>if</u> no new particles are light.

Mass matrix $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$

$-\frac{1}{2} \bar{\Psi}_I \gamma_L \Psi_J M_{IJ}$ + c.c.

$$M = \begin{pmatrix} 0 & y_j v \\ y_{ai} v & M_{ij} \end{pmatrix}$$

"See saw" mechanism

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if the entries in M_{ij} are $\gg y_{ai} v$ (no zero eigenvalues etc.)

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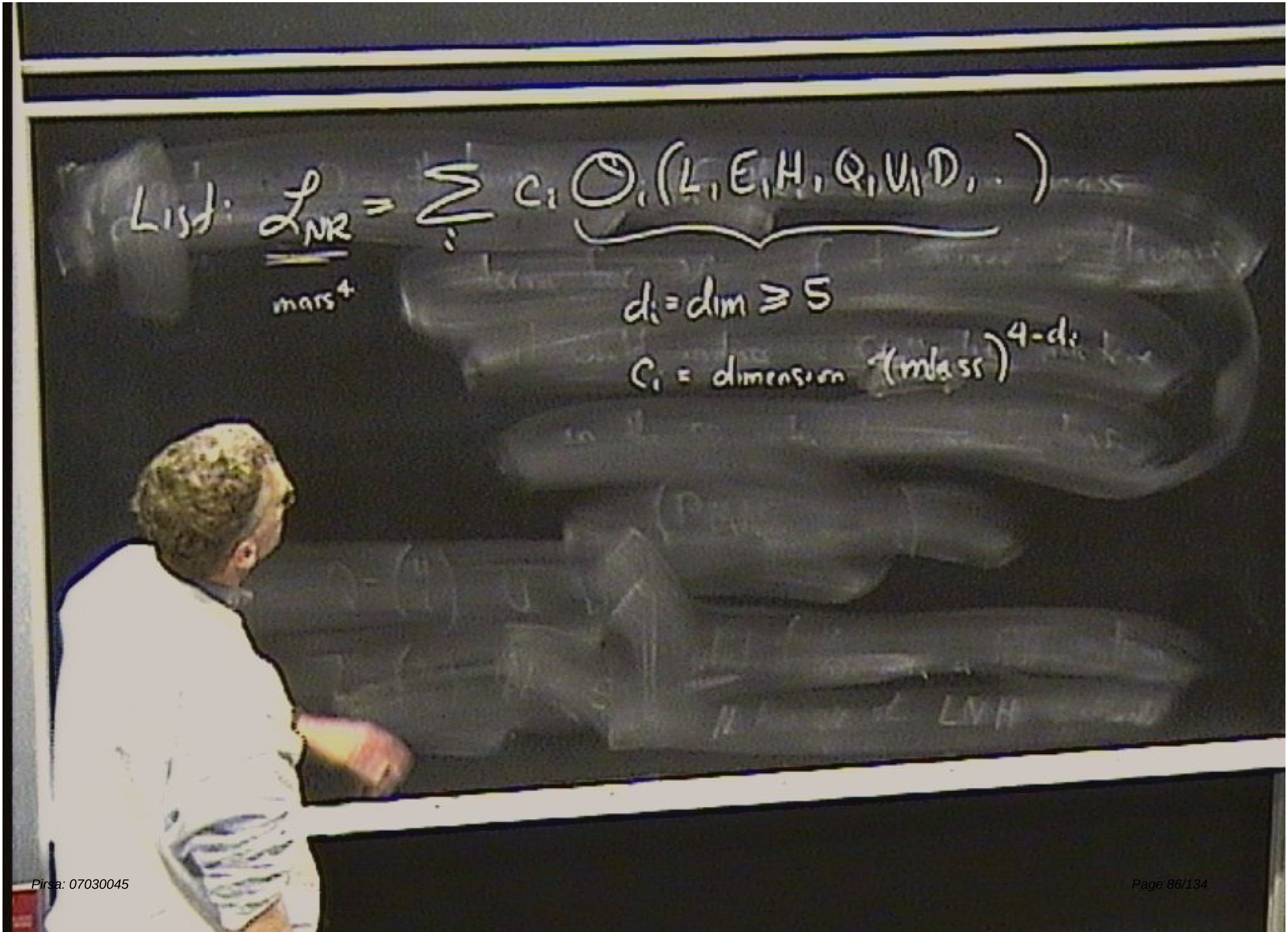
List: $\mathcal{L}_{NR} =$

mass^4


List: $\mathcal{L}_{NR} = \sum_i c_i \underbrace{O_i(L, E, H, Q, U, D, \ldots)}$
mass⁴
$d_i = \dim \geq 5$
c_i = dimension (mass)$^{4-d_i}$

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...minant coefficients: dimension 5: $\mathcal{O}_i$ for which $d_i = 5$.

...dom. " " 6: 6

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Sub dom. " " 6: 6

<u>Claim</u>:

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Sub dom. " " 6:

<u>Claim</u>: there is a unique dim 5 interaction built from only SM fields which is $SU_3 \times SU_2 \times U_1$ invariant.

this requires N: $(1, 1, 0) \in (SU_3 \times SU_2 \times U_1)$

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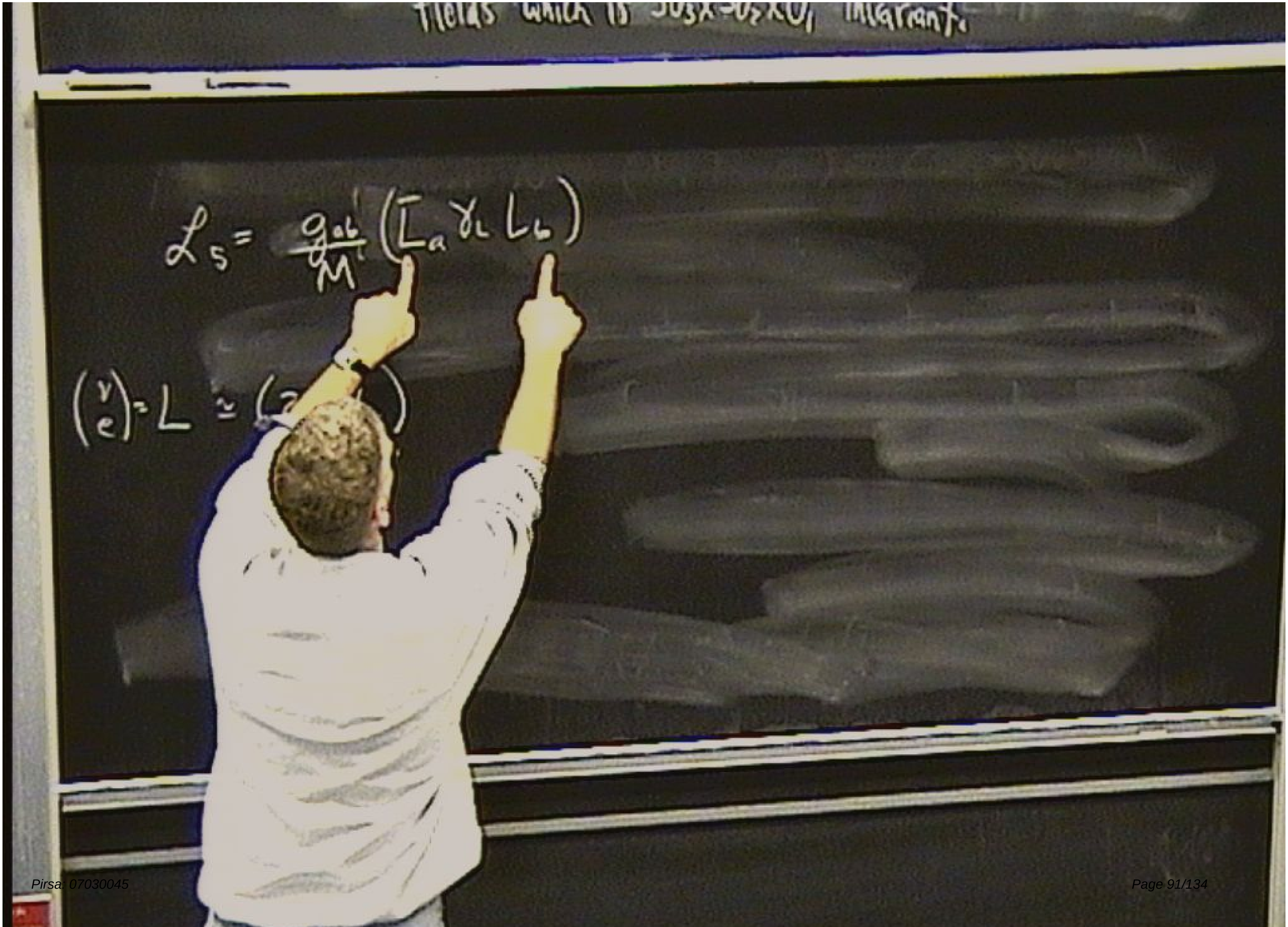
fields which is SU3×SU2×U1 invariant.
$\mathcal{L}_5 = \frac{g_{ab}}{M}(\bar{L}_a \gamma_L L_b)$
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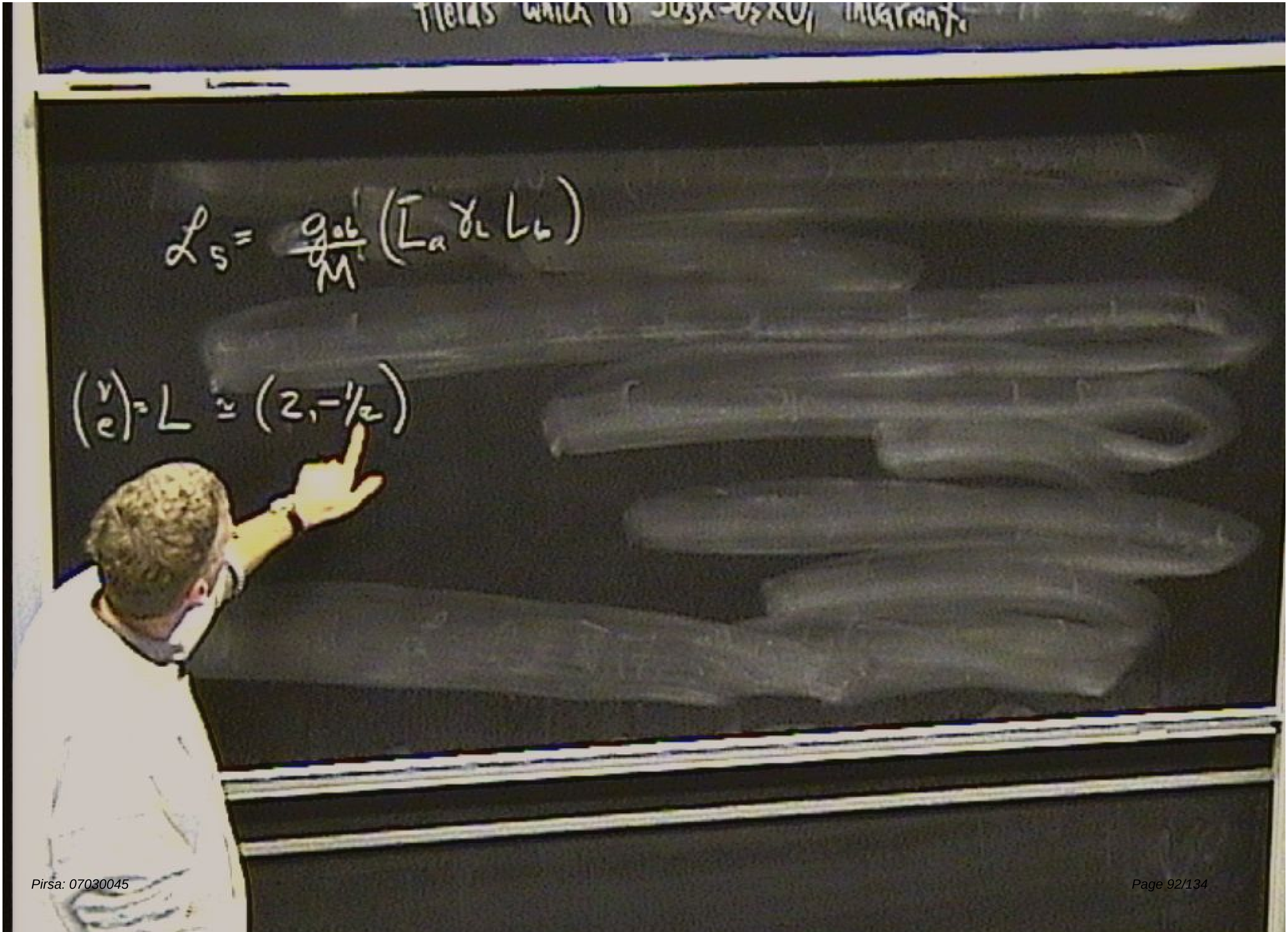
fields which is SU3×SU2×U1 invariant.
L_5 = g_ab/M (L̄_a γ_L L_b)
(ν e) = L ≃ (2, −1/2)

fields which is $SU_3 \times SU_2 \times U_1$ invariant.

$$\mathcal{L}_5 = \frac{g_{ab}}{M}\left(\bar{L}_a \gamma_L L_b\right)(HH)$$

$$\begin{pmatrix}\nu \\ e\end{pmatrix} = L \simeq \left(2, -\tfrac{1}{2}\right)$$

$$H \simeq \left(2, +\tfrac{1}{2}\right)$$

fields which is $SU_3 \times SU_2 \times U_1$ invariant.

$$\mathcal{L}_5 = \frac{g_{ab}}{M}\left(\bar{L}_a \gamma_L L_b\right)(HH)$$

flavor indices

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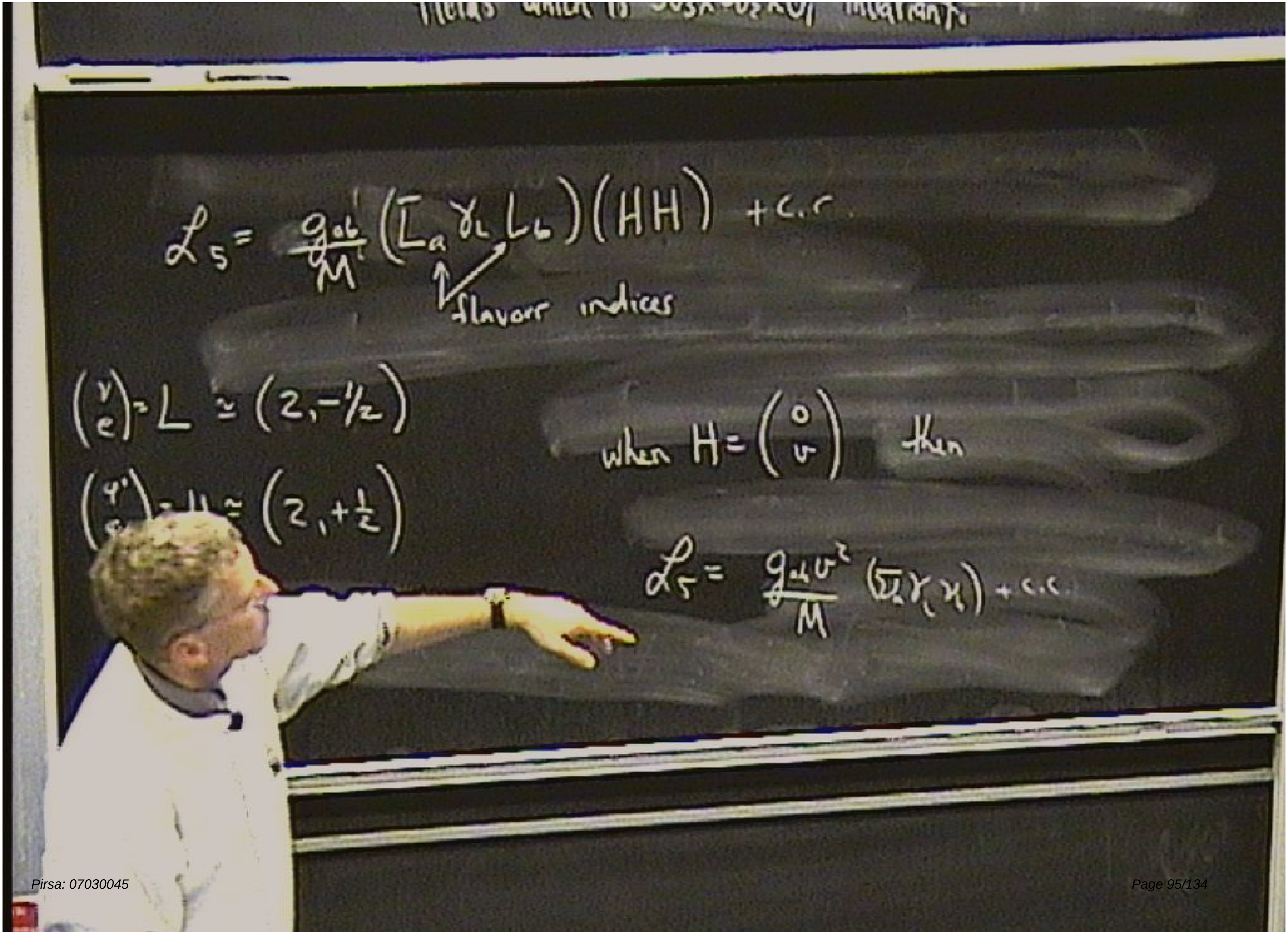
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flavor indices
$\begin{pmatrix}\nu\\ e\end{pmatrix} = L \simeq (2, -\tfrac{1}{2})$
$\simeq (2, +\tfrac{1}{2})$
when $H = \begin{pmatrix}0\\ v\end{pmatrix}$ then
$\mathcal{L}_5 = \frac{g_{ab} v^2}{M}(\bar{\nu}_a \gamma_L \nu_b) + c.c.$

$M \simeq \ldots \, eV \simeq 3 \times 10^{27} \, eV \simeq 3 \times 10^{18} \, GeV$

if the entries in M_{ij} are $\gg y_{ai} v$ (no zero eigenvalues)

then the eigenvalues of $\underline{M}$ come in 2 types:

1) n states of mass $\approx O(M)$

2) 3 states of mass $\approx O(\frac{(yv)^2}{M})$

[the eigenvalues of $(y v) M^{-1}_{ij} (y_j v) \approx$ ...]

naturally very light.

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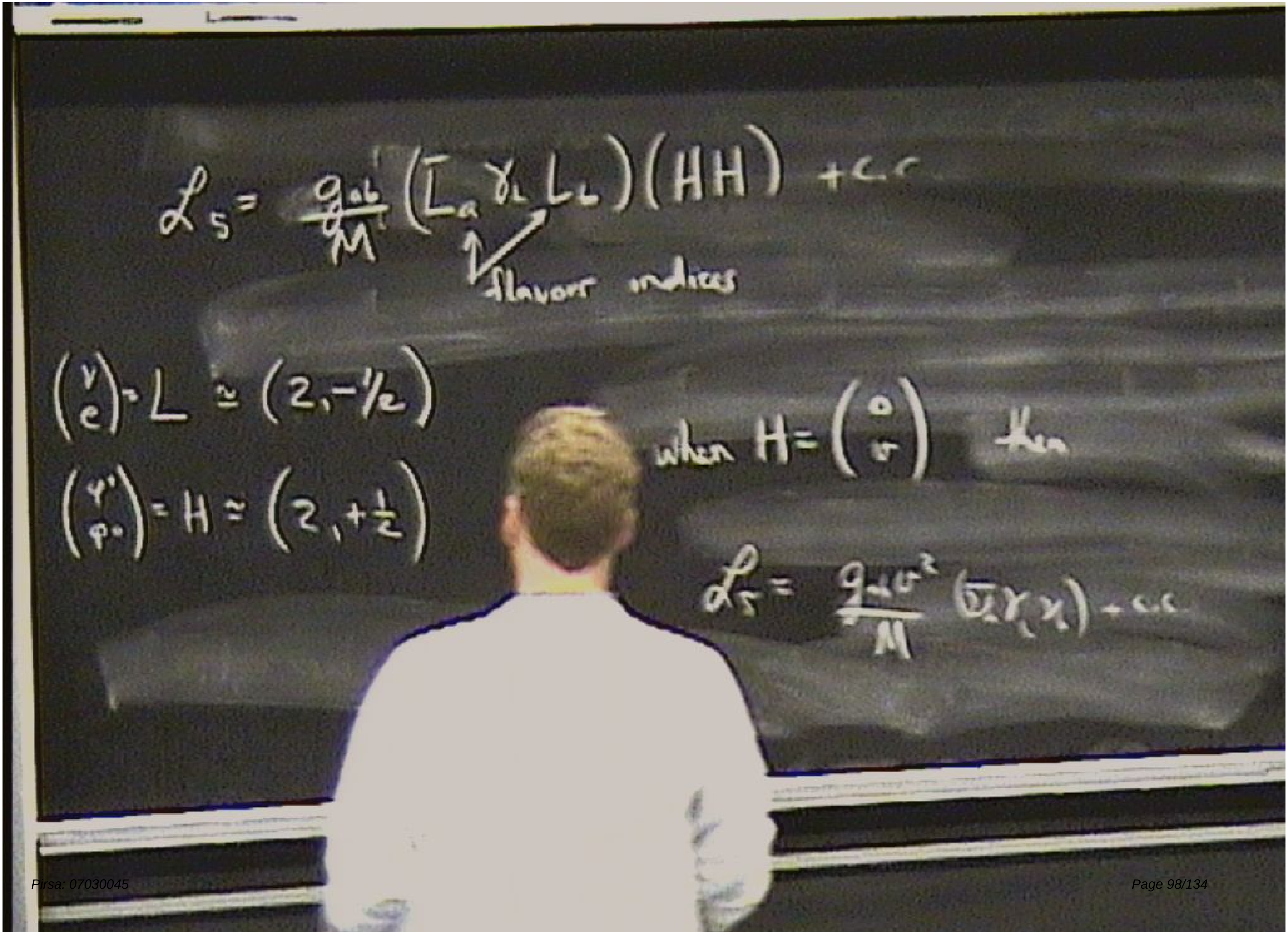
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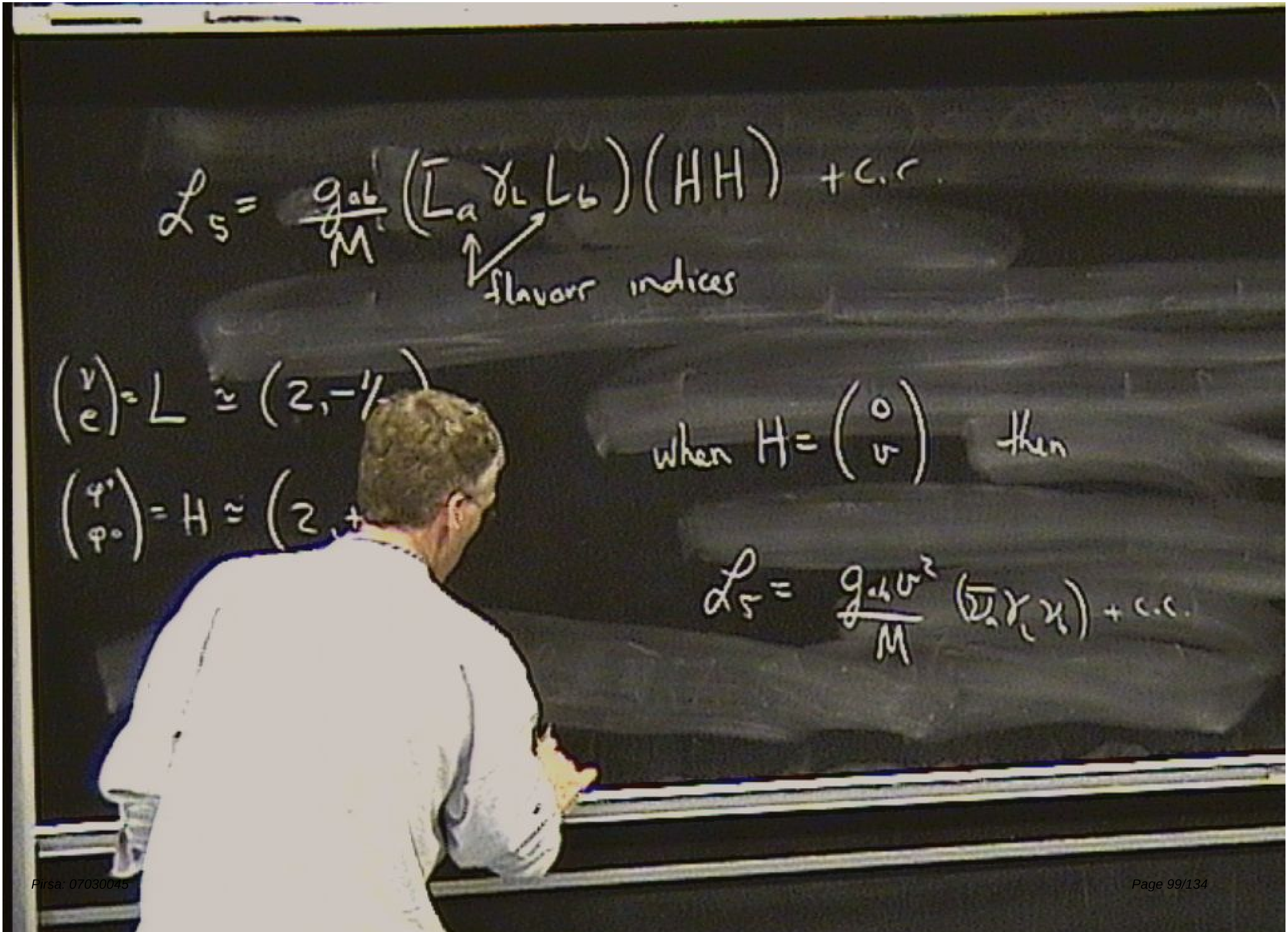
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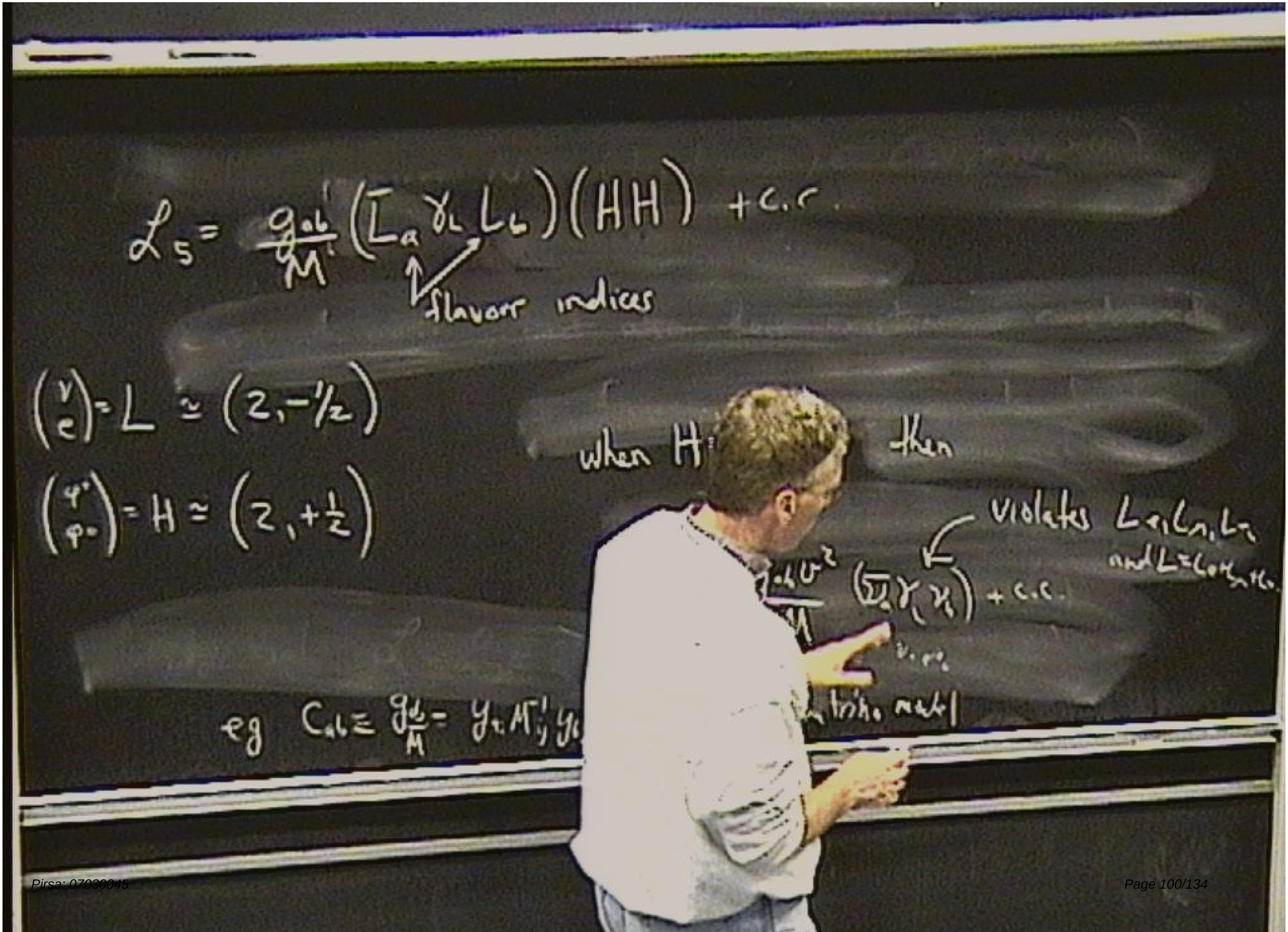

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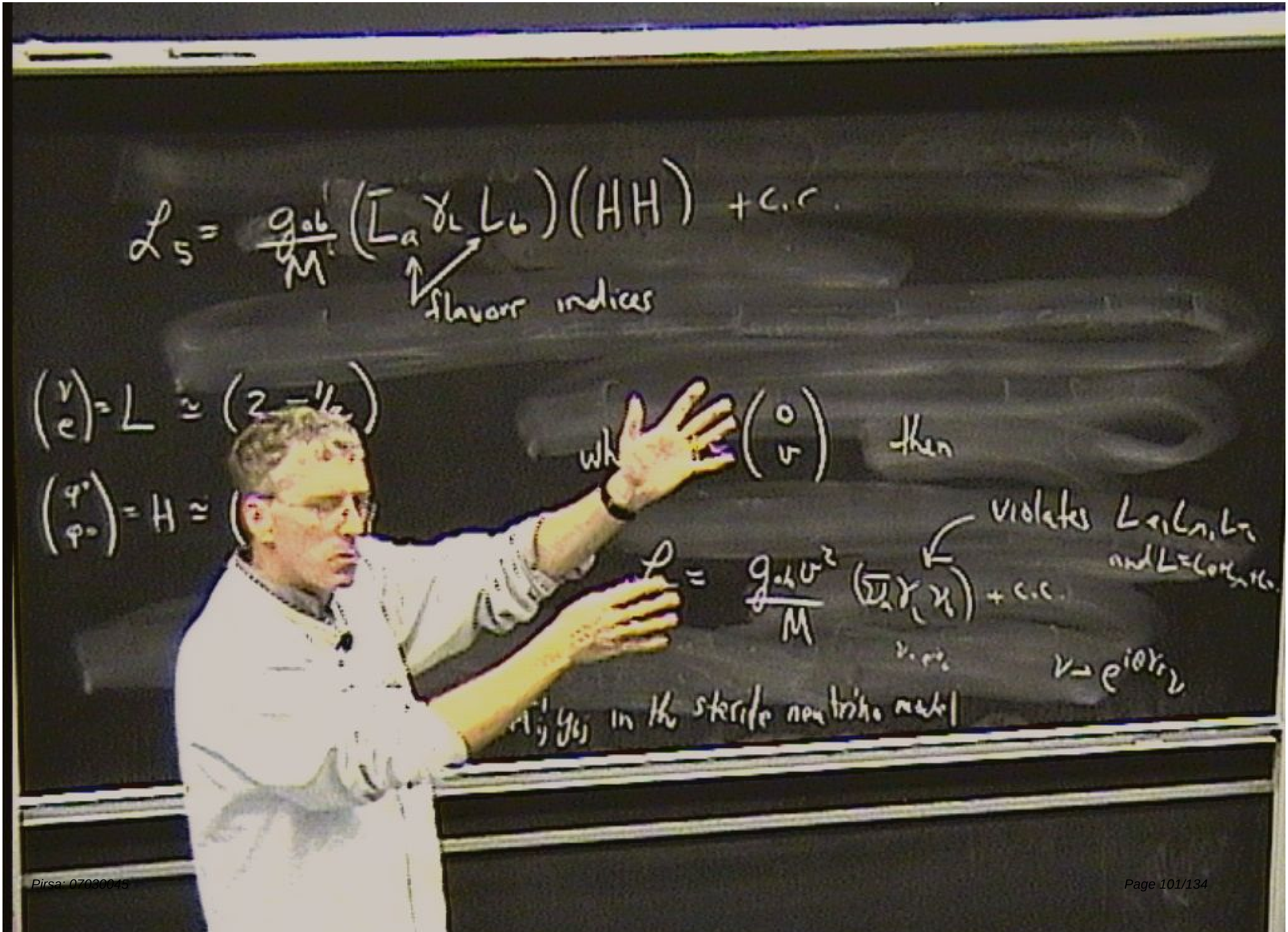
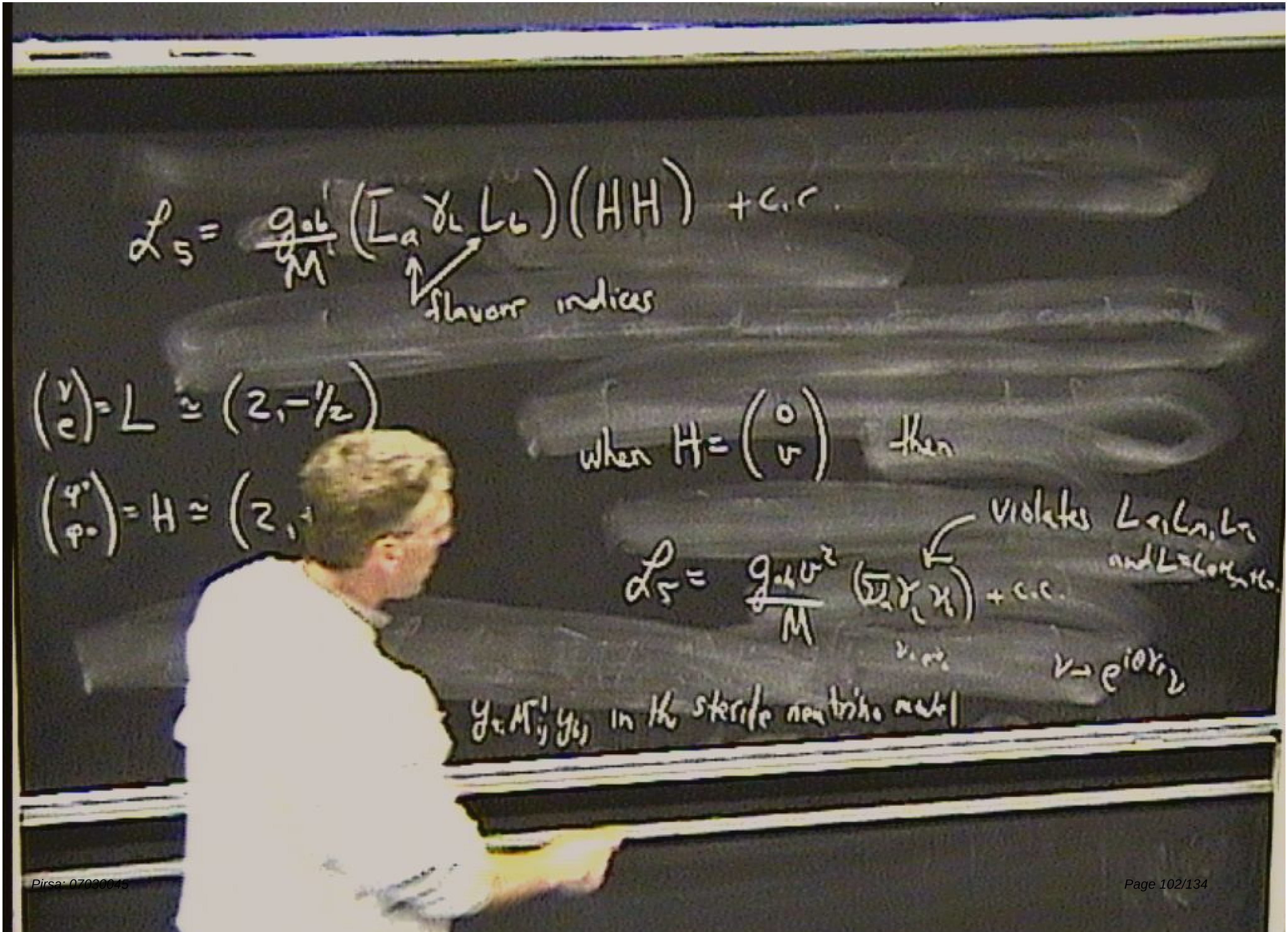
when H
then
violates L_e, L_μ, L_τ
and $L = L_e + L_\mu + L_\tau$
$\frac{g_{ab} v^2}{M}(\bar{\nu}_a \gamma_L \nu_b) + c.c.$
eg $C_{ab} = \frac{g_{ab}}{M} =$



L5 = gab/M (La γL Lb)(HH) + c.c.
flavor indices
(ν e) = L ≈ (2, -1/2)
(φ+ φ0) = H ≈ (
then
violates Le, Lμ, Lτ
and L = Le + Lμ + Lτ
L = gab v²/M (νa γL νb) + c.c.
in the sterile neutrino model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$\nu \to e^{i\theta} \nu$
$y_{ai} M_{ij}^{-1} y_{bj}$ in the sterile neutrino model

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violates L_e, L_μ, L_τ and $L = L_e + L_\mu + L_\tau$

L breaking scale $\rightarrow M$

$\nu \to e^{i\theta\gamma_5/2}\nu$

eg $C_{ab} = \frac{g_{ab}}{M} = y_{ai} M^{-1}_{ij} y_{bj}$ in the sterile neutrino model

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L breaking scale $\rightarrow M$

$\nu_a \nu_b$

$\nu \rightarrow e^{i\theta/2}\nu$

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$$\mathcal{L}_5 = \frac{g_{ab}}{M}(\bar{L}_a \gamma_L L_b)(HH) + c.c.$$

flavor indices

$$\begin{pmatrix} \nu \\ e \end{pmatrix} = L \simeq (2, -1/2)$$

$$\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = H \simeq (2, +\tfrac{1}{2})$$

when $H = \begin{pmatrix} 0 \\ v \end{pmatrix}$ then

violates L_e, L_μ, L_τ and $L = L_e + L_\mu + L_\tau$

$$\mathcal{L}_5 = \frac{g_{ab} v^2}{M} (\bar{\nu}_a \gamma_L \nu_b) + c.c.$$

L breaking scale

$\nu_L \nu_L$ $\quad \nu \to e^{i\theta \gamma_5 / 2} \nu$

eg $C_{ab} \equiv \frac{g_{ab}}{M} = y_{ai} M_{ij}^{-1} y_{bj}$ in the sterile neutrino model fields which is $SU_3 \times SU_2 \times U_1$ invariant

$$\mathcal{L}_5 = \frac{g_{ab}}{M}\left(\bar{L}_a \gamma_L L_b\right)(HH) + \text{c.c.}$$

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L breaking scale $\longrightarrow M$

$\nu_a \rho_a$ $\qquad$ $\nu_a = e^{i\theta\gamma_5/2}$

eg $C_{ab} = \frac{g_{ab}}{M} = y_i M_{ij}^{-1} y_{j}$ in the sterile neutrino model

What are the possible $\mathcal{L}_{NR}$ allowed by $SU_3 \times SU_2 \times U_1$?

(These must parameterize low-energy effects of high energy modifications to the SM,

if no new particles are light,

and if the Higgs particle is really light enough to be in the low-energy theory.

Mass matrix $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$ $\qquad -\frac{1}{2} \overline{\Psi}_I \gamma_c \Psi_J M_{IJ}$ + c.c.

$$\mathcal{M} = \begin{array}{c} 3 \\ \\ n \end{array} \begin{pmatrix} 0 & y_{ji} v \\ y_{ij} v & M_{ij} \end{pmatrix}$$

"See saw" mechanism

if $\frac{y^2 v^2}{M} \simeq 10^{-2}$ eV

$v \simeq 10^2$ GeV $\simeq 10^{11}$ eV

$M \simeq y^2 v^2 / 10^{-2}$ eV $\simeq y^2 10^{24}$ eV $\simeq y^2 10^{15}$ GeV

$v \simeq 246$ GeV

$y_{ij} \ll 1$

if the entries in M_{ij} are $\gg y_{ij} v$ (no zero eigenvalues ...)

then the eigenvalues of $\mathcal{M}$ come in 2 types:

$$\mathcal{L}_5 = \frac{g_{ab}}{M}\left(\bar{L}_a \gamma_L L_b\right)(HH) + c.c.$$

flavor indices

$$\begin{pmatrix}\nu\\ e\end{pmatrix} = L = (2, -\tfrac{1}{2})$$

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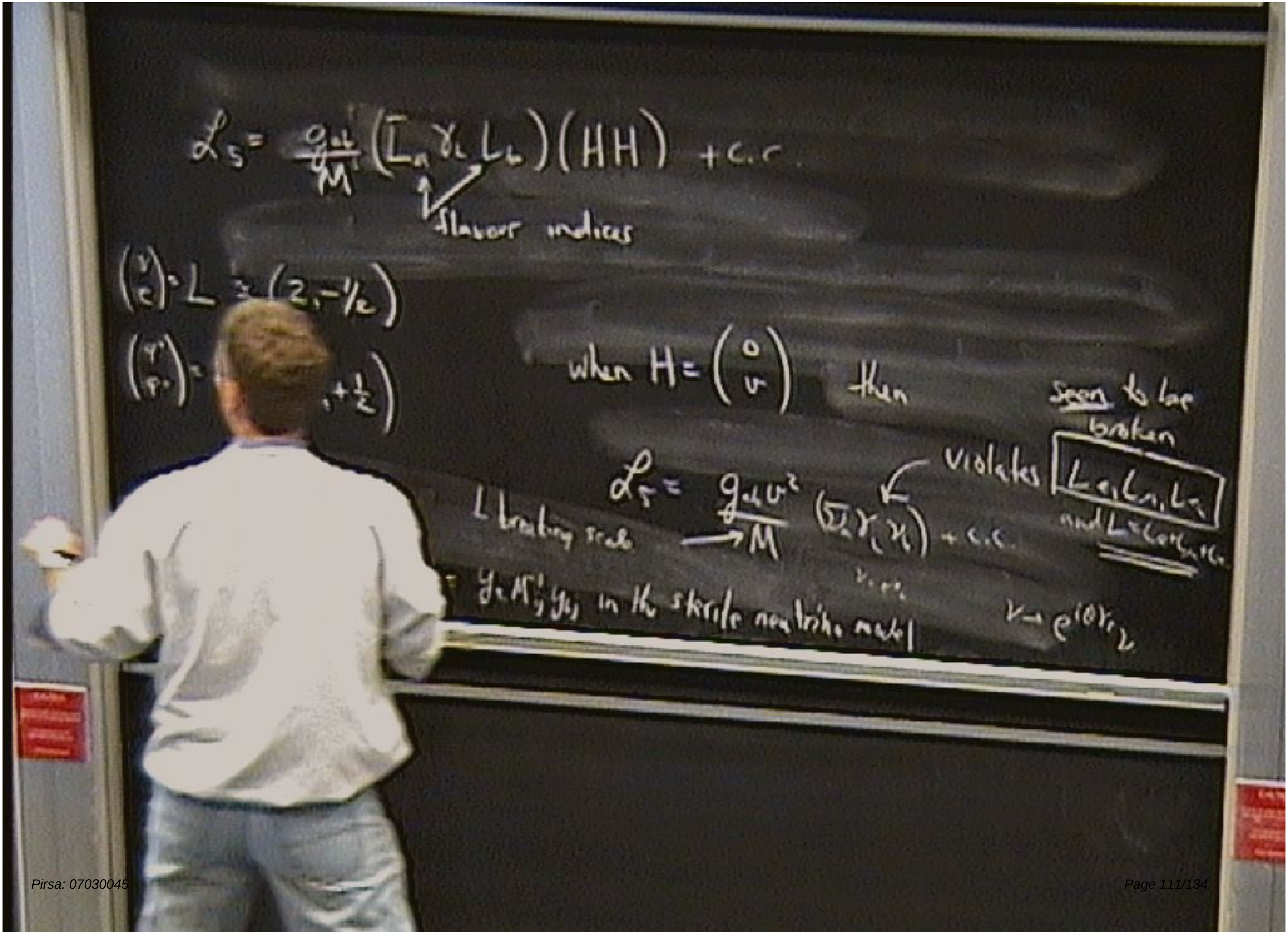
L breaking scale $\longrightarrow M$

$\nu_a \, \nu_b$

eg $C_{ab} = \frac{g_{ab}}{M} = y_a M_{ij}^{-1} y_{bj}$ in the sterile neutrino model

$\nu \to e^{i\theta \gamma_5}\nu$



flavor indices
when H =
then
seen to be broken
violates
L breaking scale
+ c.c.
in the sterile neutrino model

$$\mathcal{L}_5 = \frac{g_{ab}}{M}(\bar{L}_a \gamma_\mu L_b)(HH) + c.c.$$

flavor indices

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Seen to be broken

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$$\mathcal{L}_5 = \frac{g_{ab}}{M} \left(\bar{L}_a \gamma_? L_b \right) \left(H H \right) + \text{c.c.}$$

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L breaking scale $\rightarrow M$

ν mass

violates $\boxed{L_e, L_\mu, L_\tau}$ Seen to be broken

and $L = L_e + L_\mu + L_\tau$

is this broken?

$v \to e^{i\theta Y_{1/2}}$

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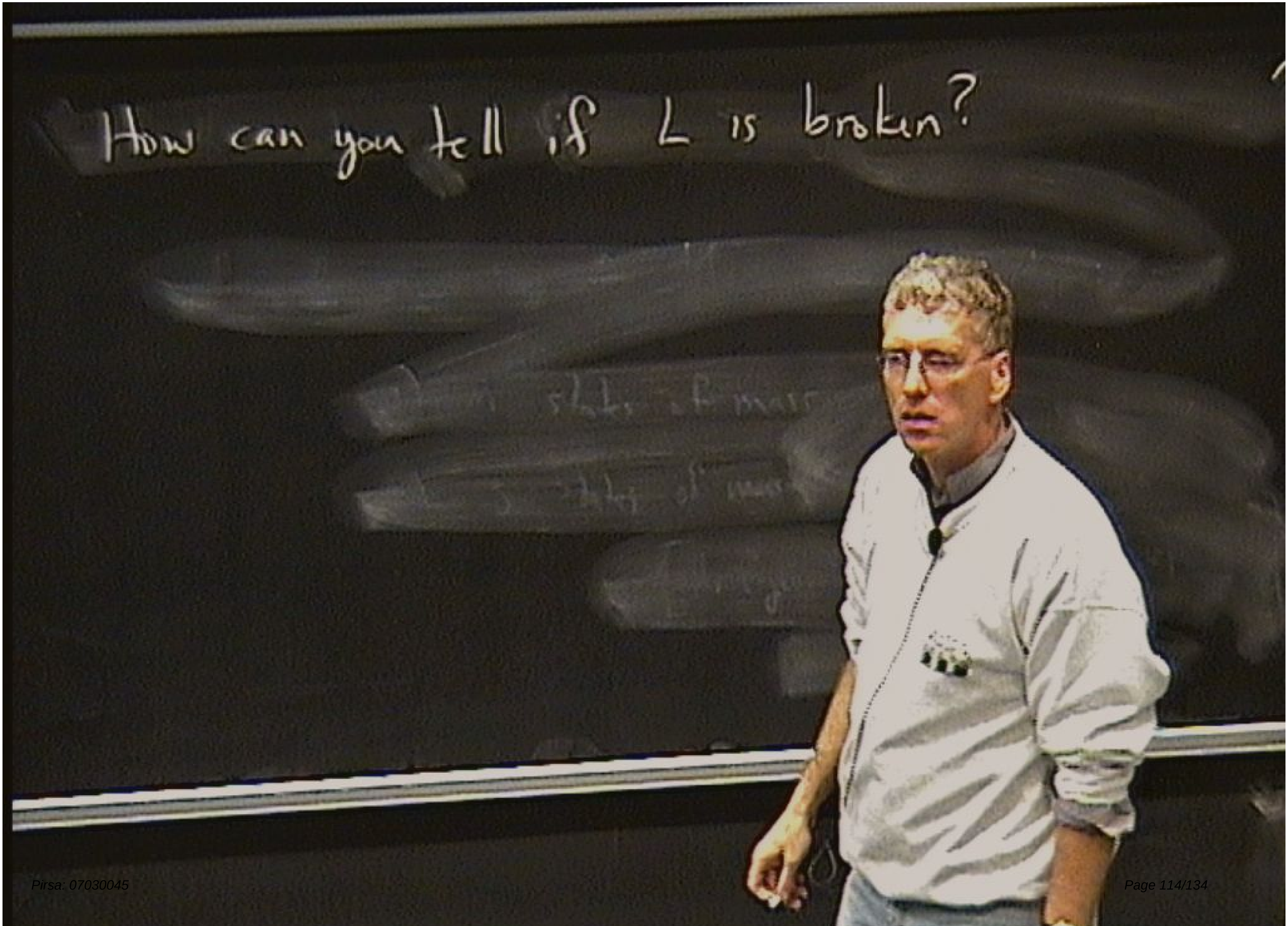
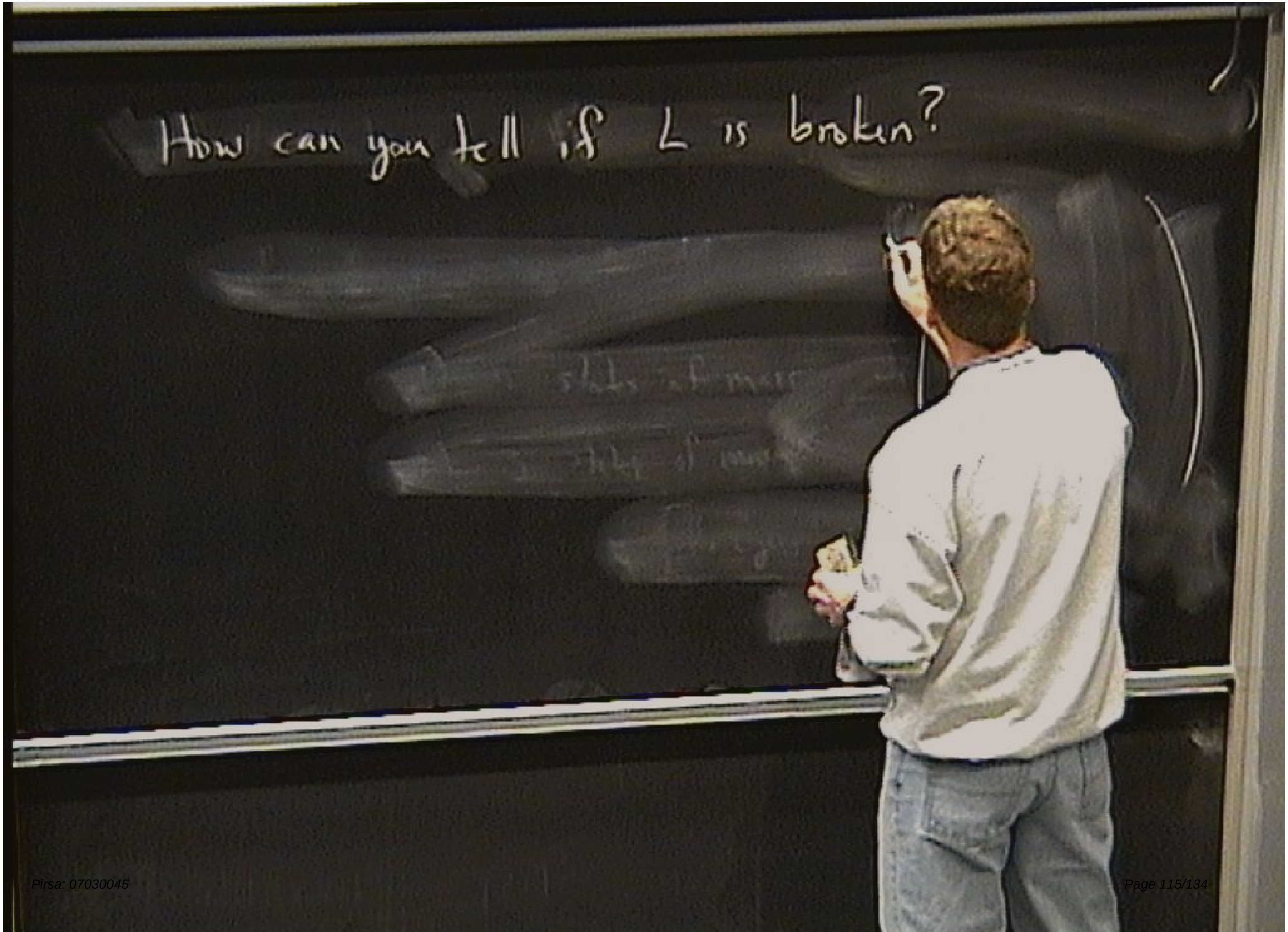

How can you tell if L is broken?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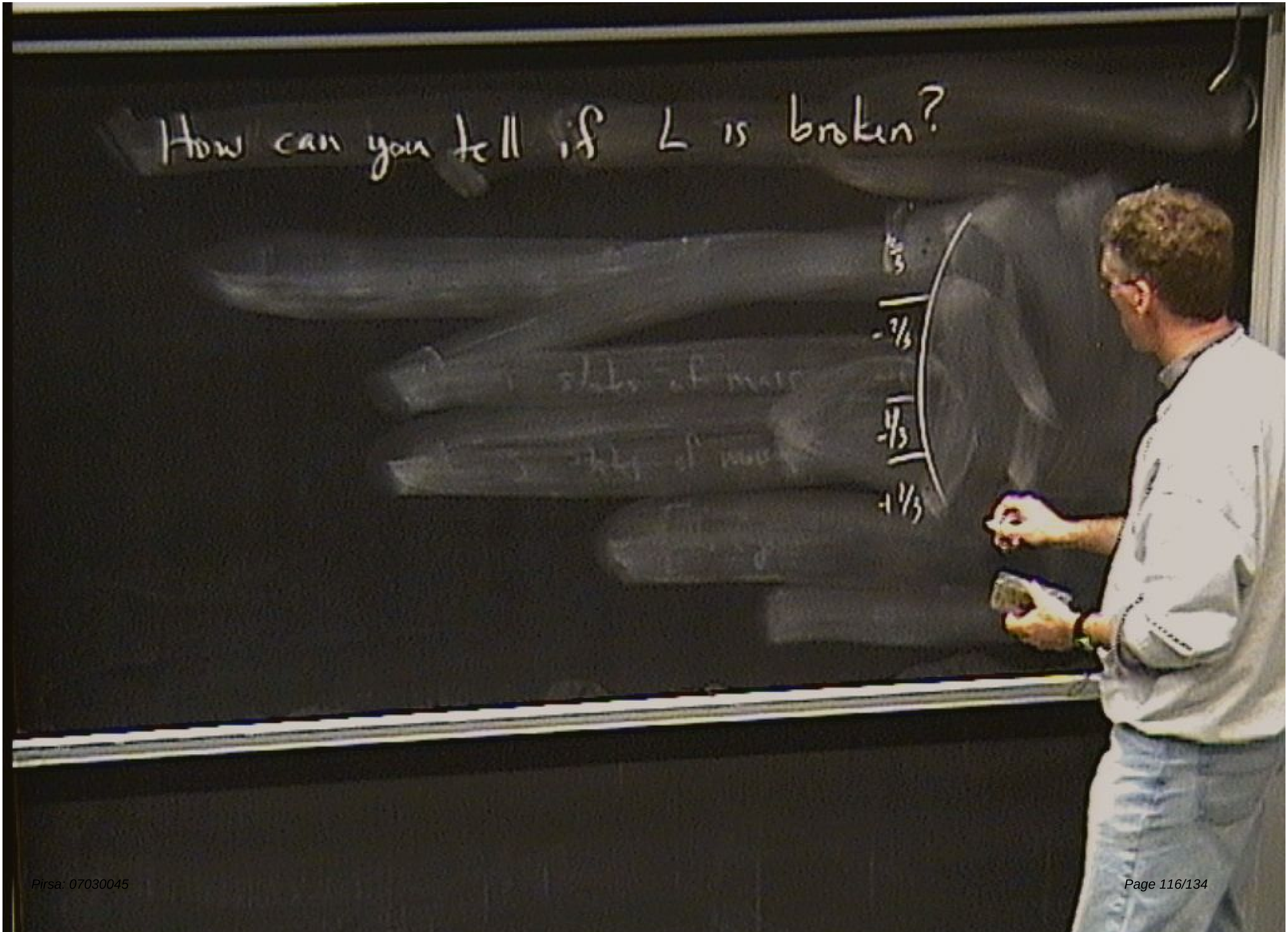
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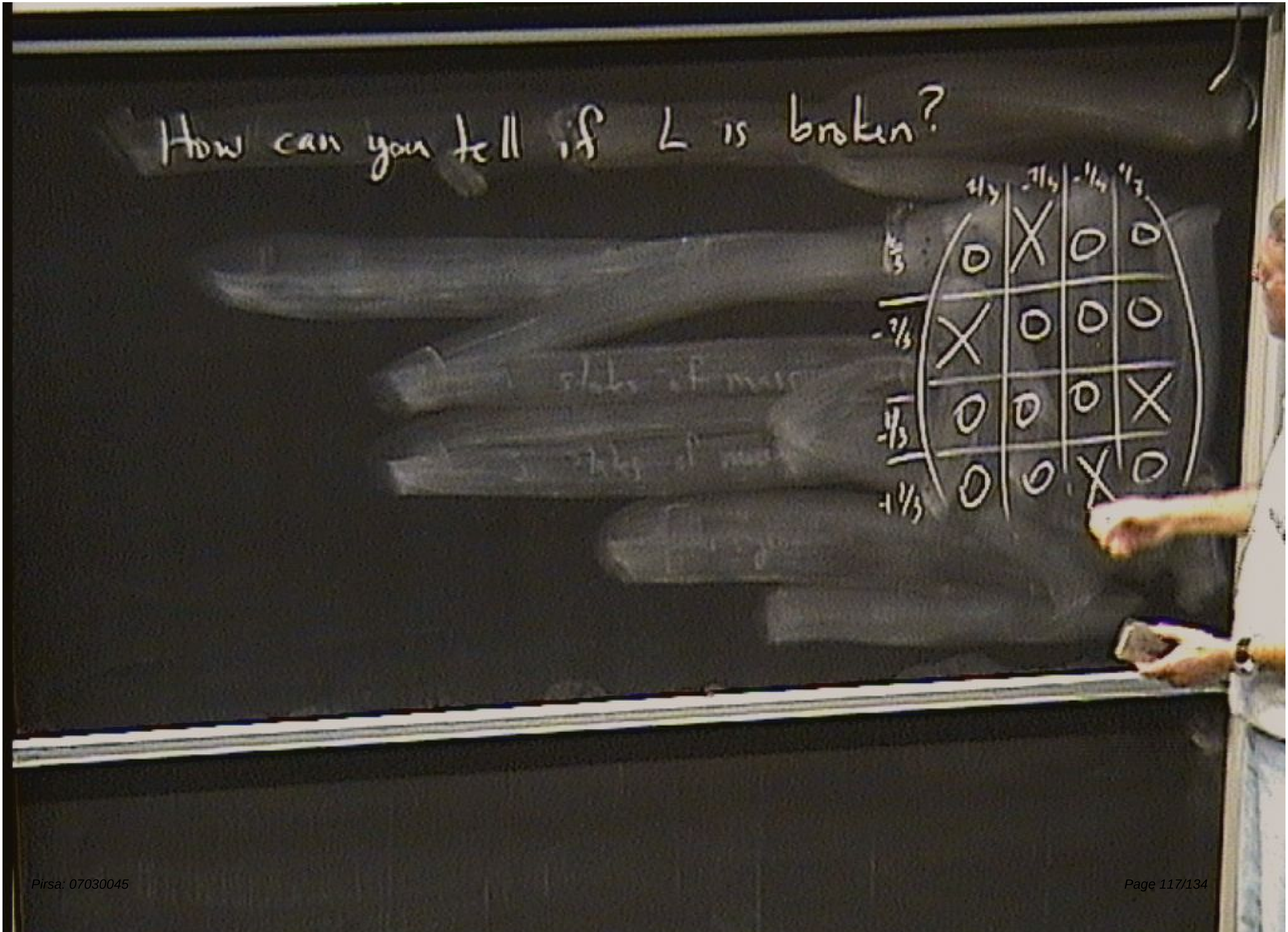
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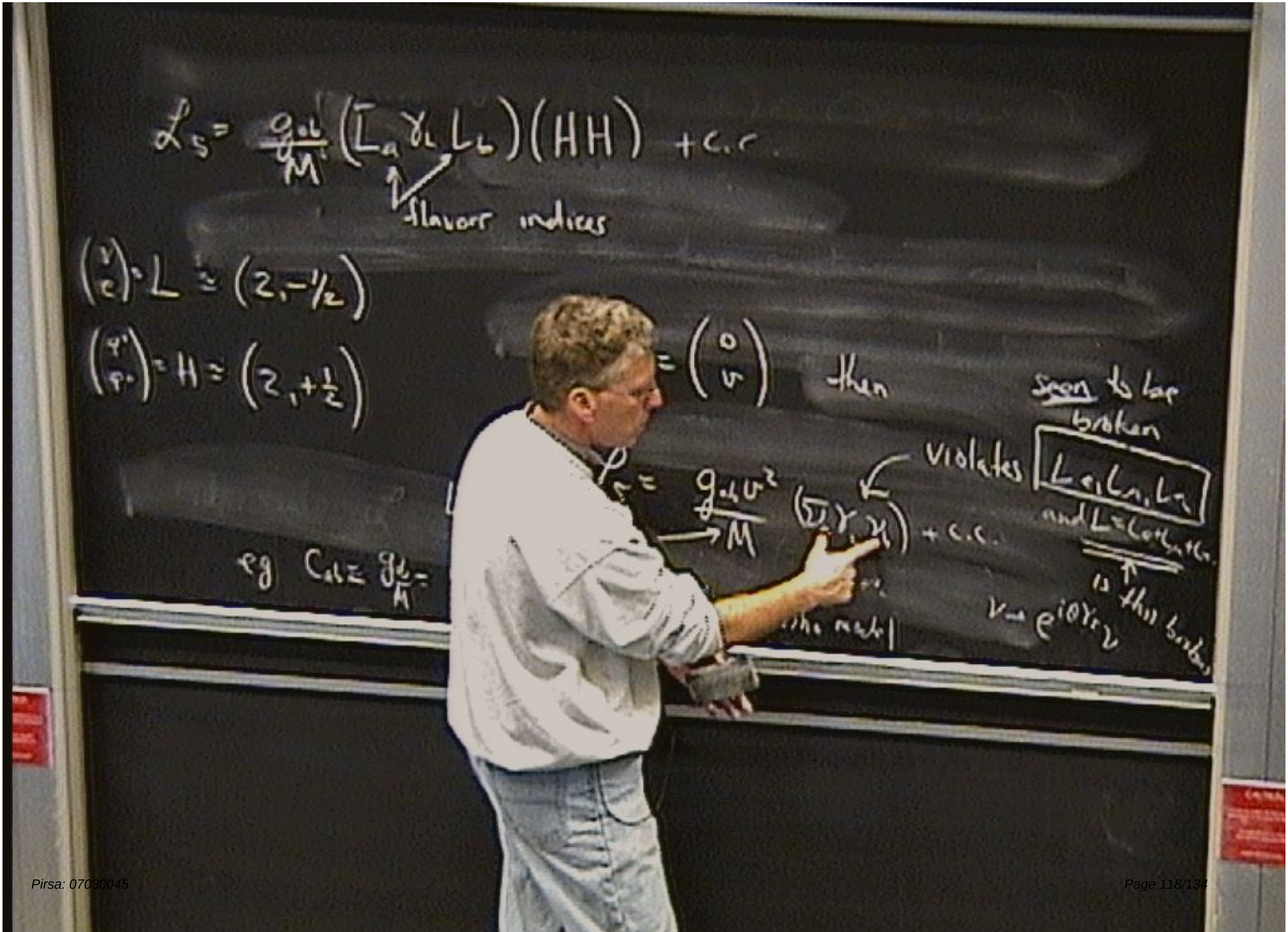
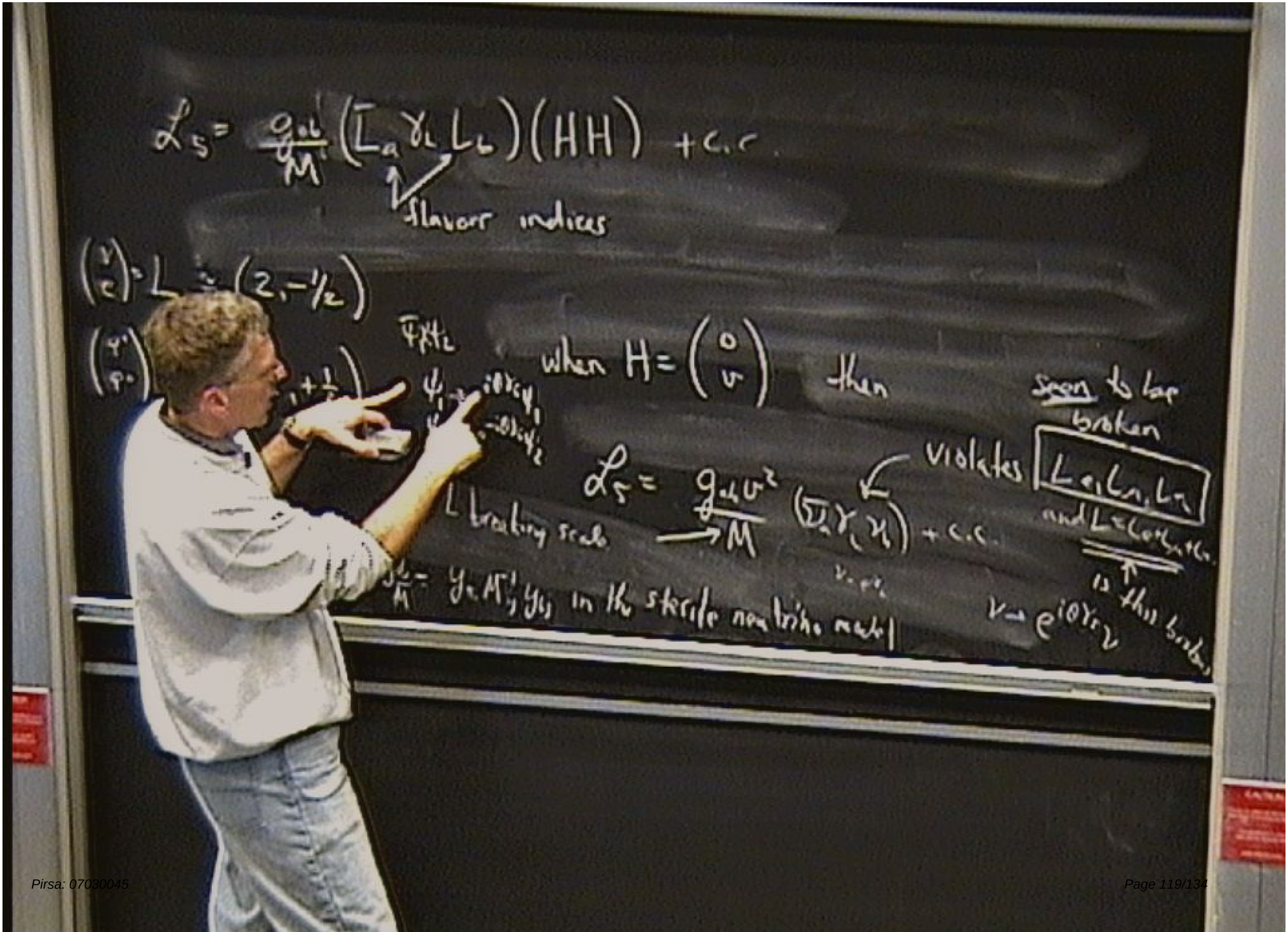

flavor indices
then
seen to be broken
violates
+ c.c.


flavor indices
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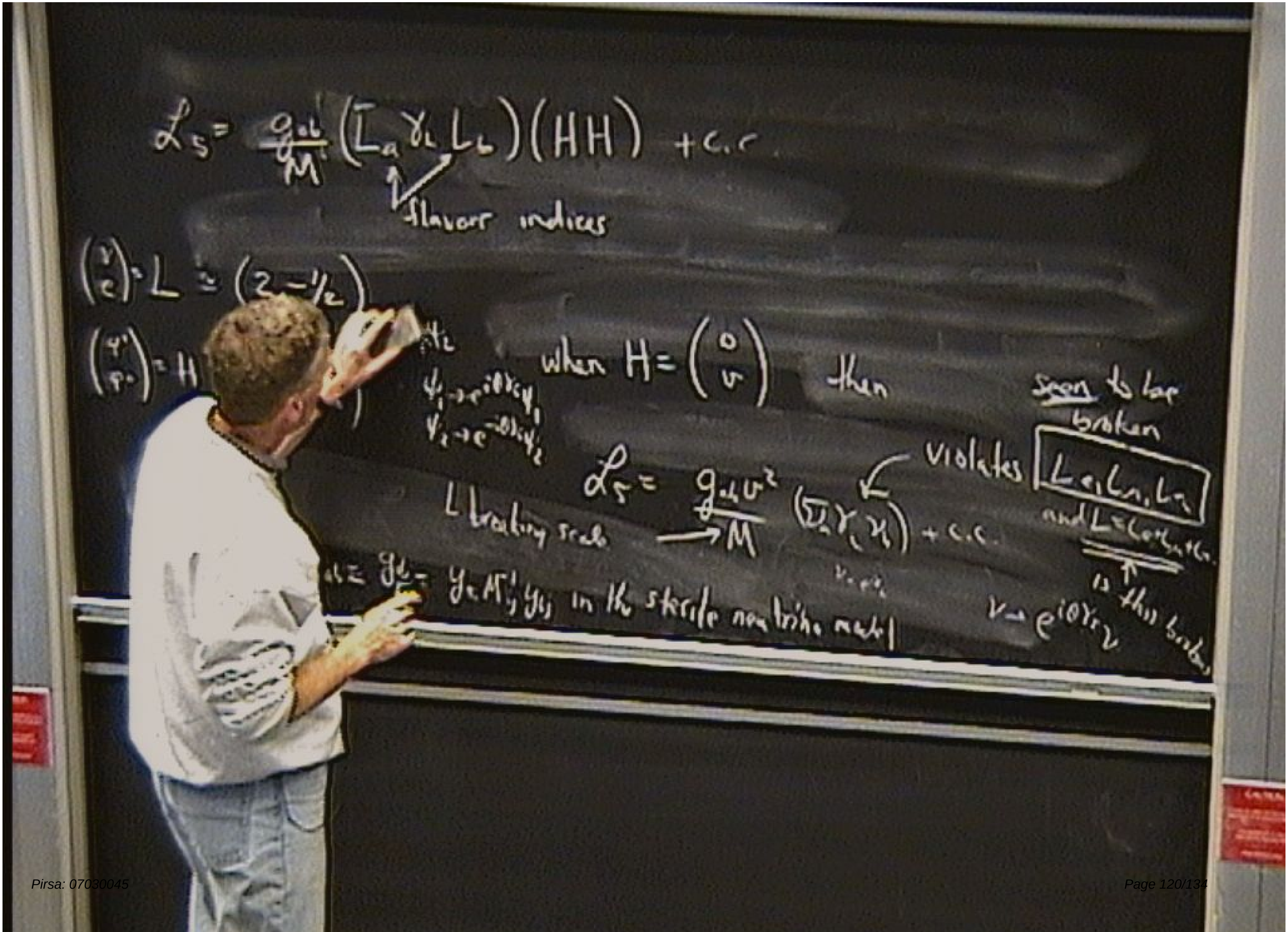
flavor indices
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L breaking scale $\rightarrow M$

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Mass matrix: $\Psi = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_1 \\ N_2 \\ \vdots \end{pmatrix}$

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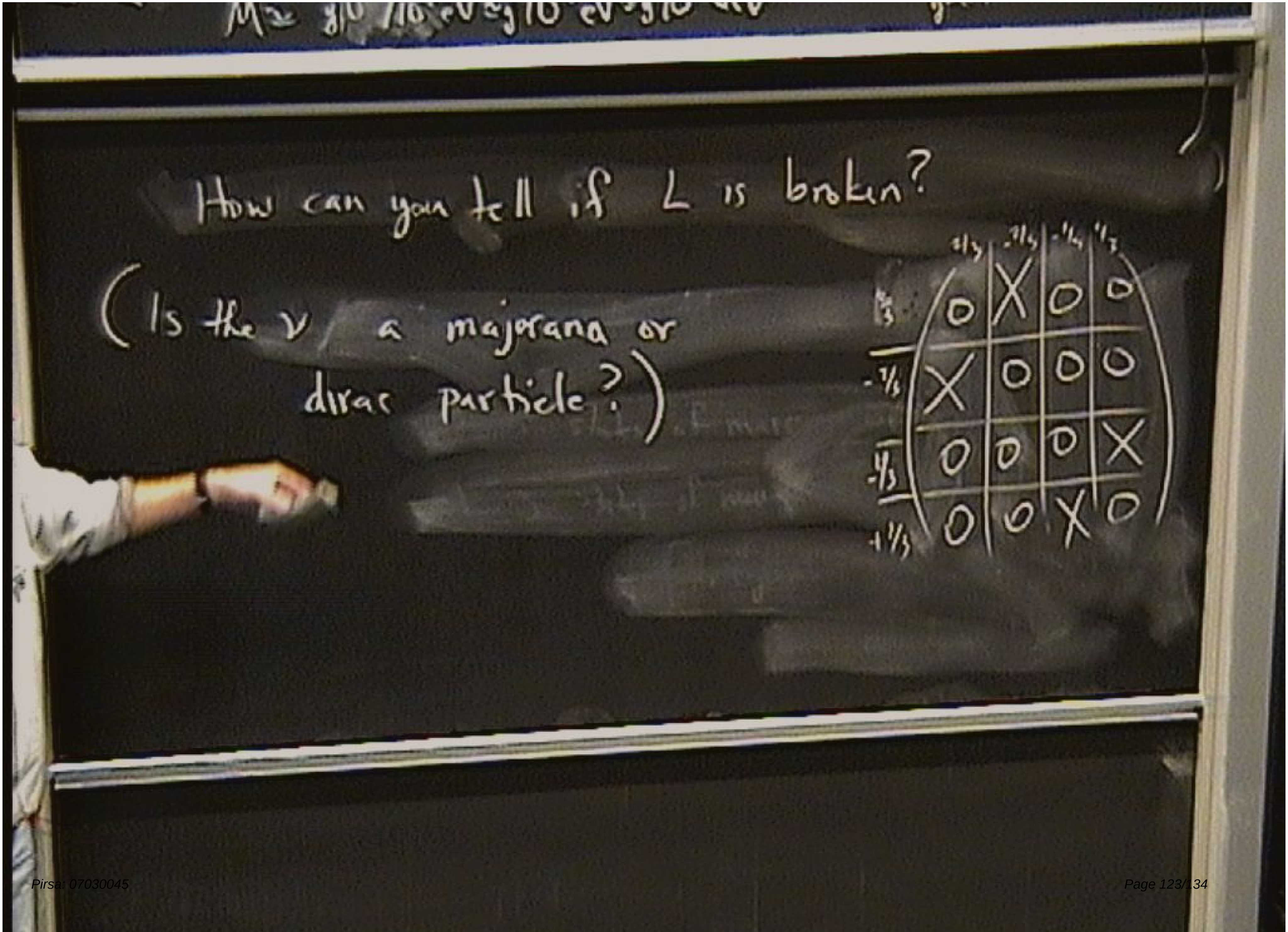
$M \simeq y_{ij} v^2 / 10^{-2}\,\text{eV} \simeq y^2 10^{24}\,\text{eV} \simeq y^2 10^{15}\,\text{GeV}$

$v \simeq$ [illegible] $16\,\text{GeV}$

$y \ll 1$

How can you tell if L is broken?


How can you tell if L is broken?
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(Is the ν a majorana or dirac particle?)

(or: is ν distinguishable from $\bar{\nu}$?)

$$\begin{array}{c|cccc} & 2/3 & -1/3 & -1/3 & 2/3 \\ \hline 2/3 & & & & \\ -1/3 & & & & \\ -1/3 & & & & \\ 2/3 & & & & \end{array}\left(\begin{array}{cccc} O & X & O & O \\ X & O & O & O \\ O & O & O & X \\ O & O & X & O \end{array}\right)$$

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(Is the ν a majorana or
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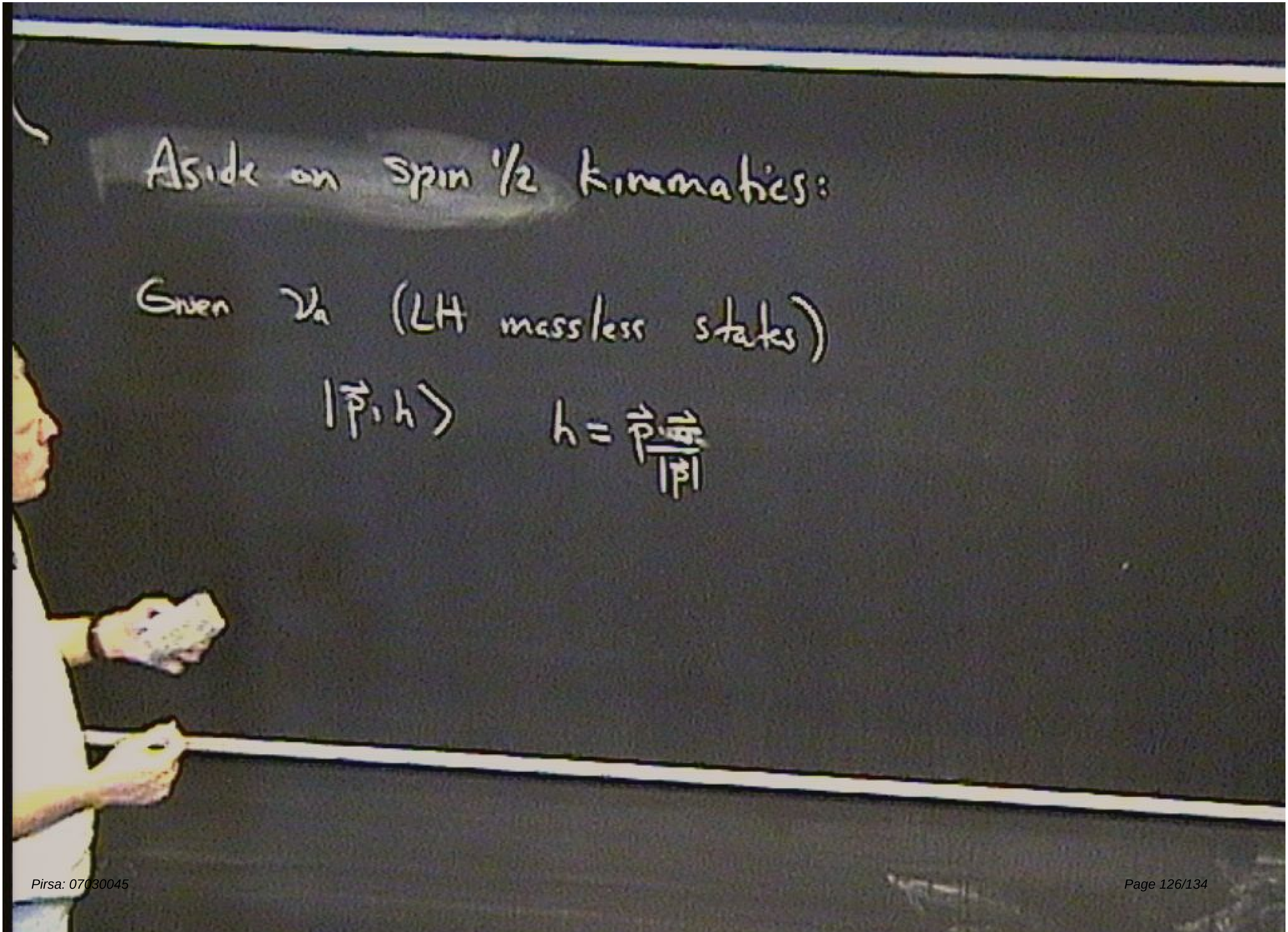
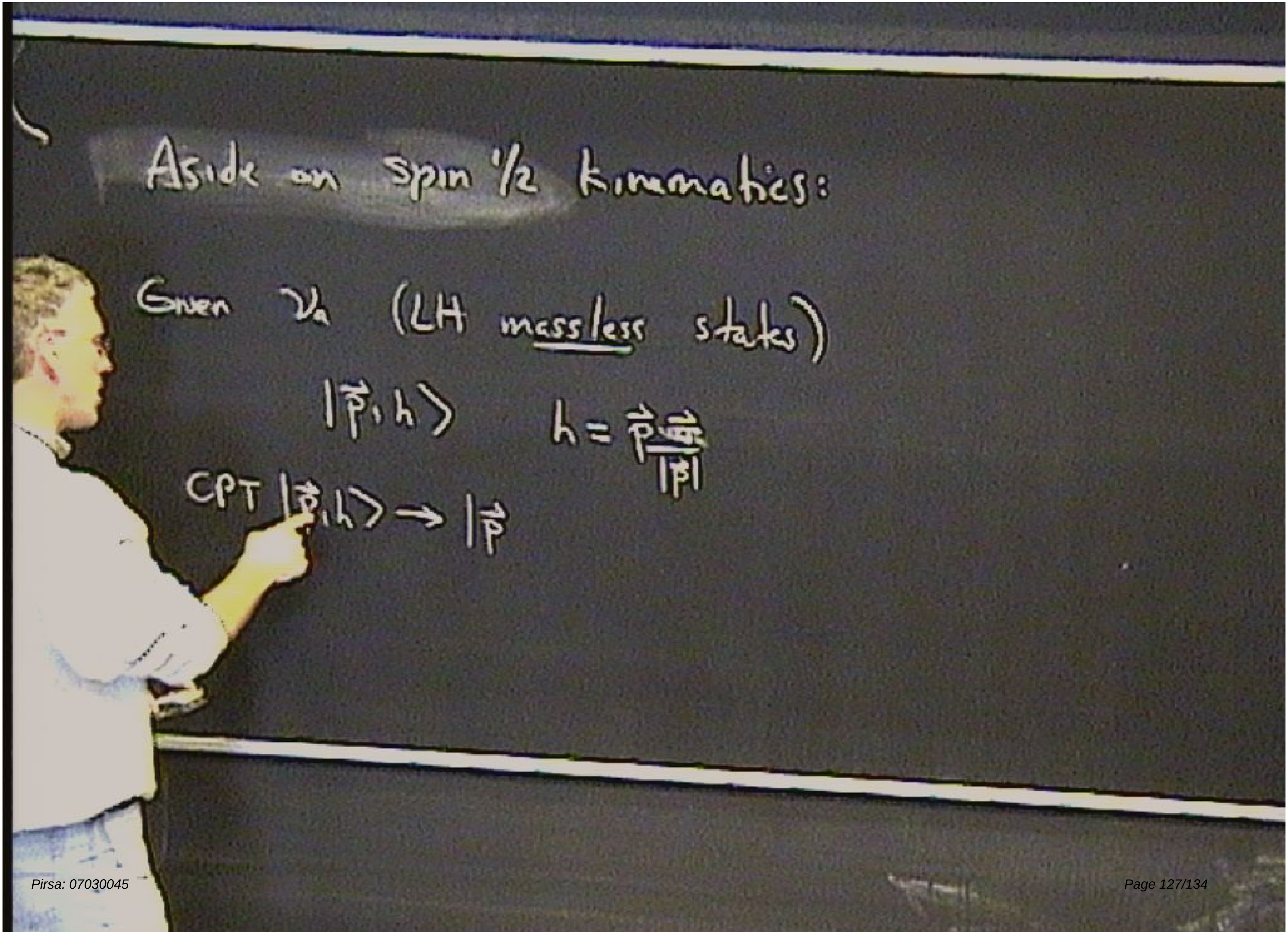
$$\begin{array}{c|cccc} & 2/3 & -2/3 & -1/3 & 1/3 \\ \hline 2/3 & O & X & O & O \\ -2/3 & X & O & O & O \\ \overline{-1/3} & O & O & O & X \\ 1/3 & O & O & X & O \end{array}$$



Aside on spin 1/2 kinmatics:
Given ν_a (LH massless states)
$|\vec{p}, h\rangle$
$h = \frac{\vec{p}}{|\vec{p}|}$


Aside on spin ½ kinematics:
Given ν_a (LH massless states)
$|\vec{p}, h\rangle$ $h = \vec{p}\cdot\vec{s}/|\vec{p}|$
CPT $|\vec{p}, h\rangle \rightarrow |\vec{p}$

Aside on spin 1/2 kinematics:

Given ν_a (LH massless states)

$|\vec{p}, h\rangle \qquad h = \frac{\vec{p}\cdot\vec{\sigma}}{|\vec{p}|}$

CPT $|\vec{p}, h\rangle \to |\vec{p}$

Aside on spin 1/2 kinematics:

Given ν_a (LH massless states)

$|\vec{p}, h\rangle \qquad h = \frac{\vec{p}\cdot\vec{S}}{|\vec{p}|}$

CPT $|\vec{p}, h\rangle \rightarrow |\vec{p}, -h\rangle$

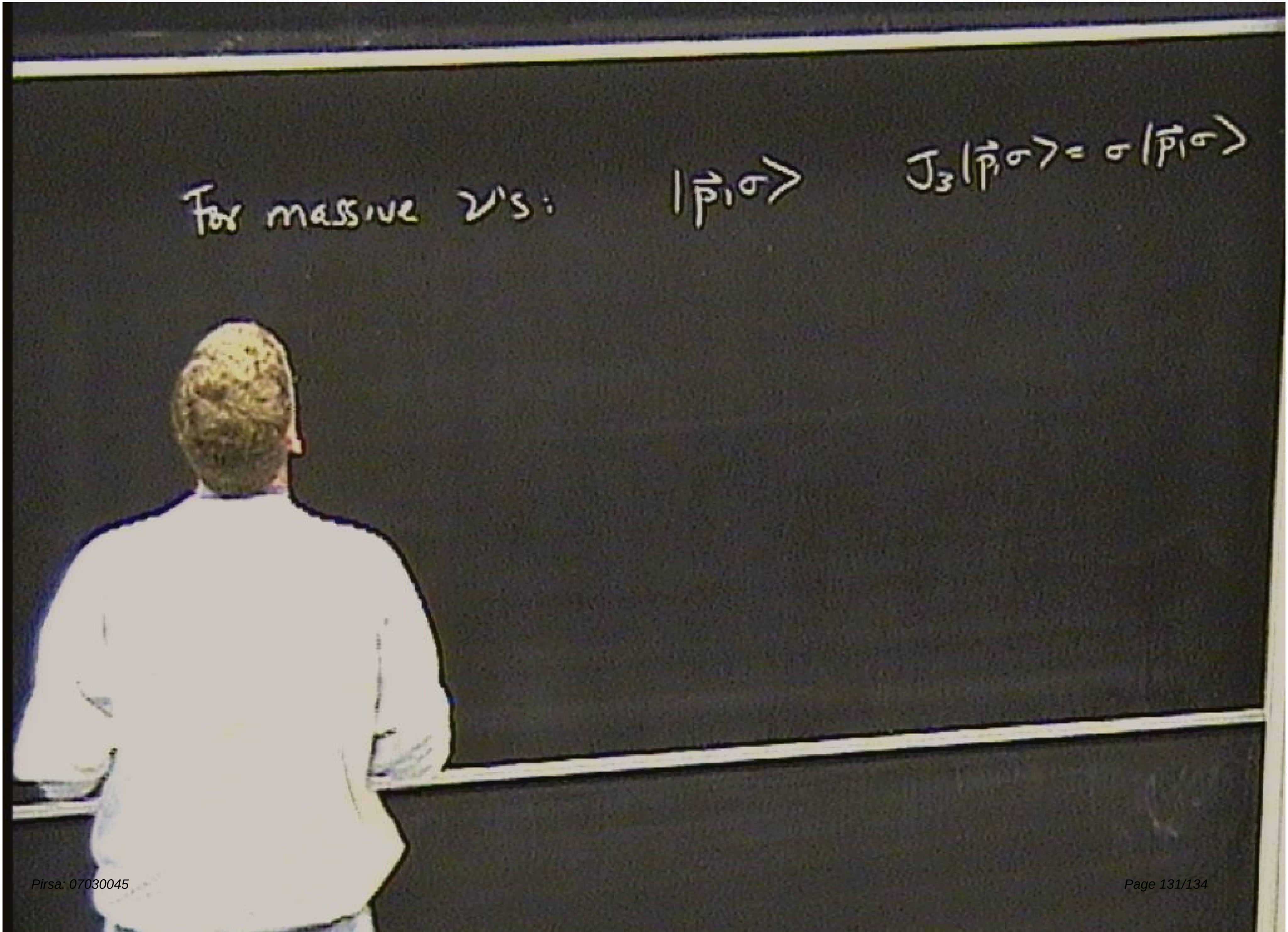
$\vec{J} = \vec{r} \times \vec{p}$

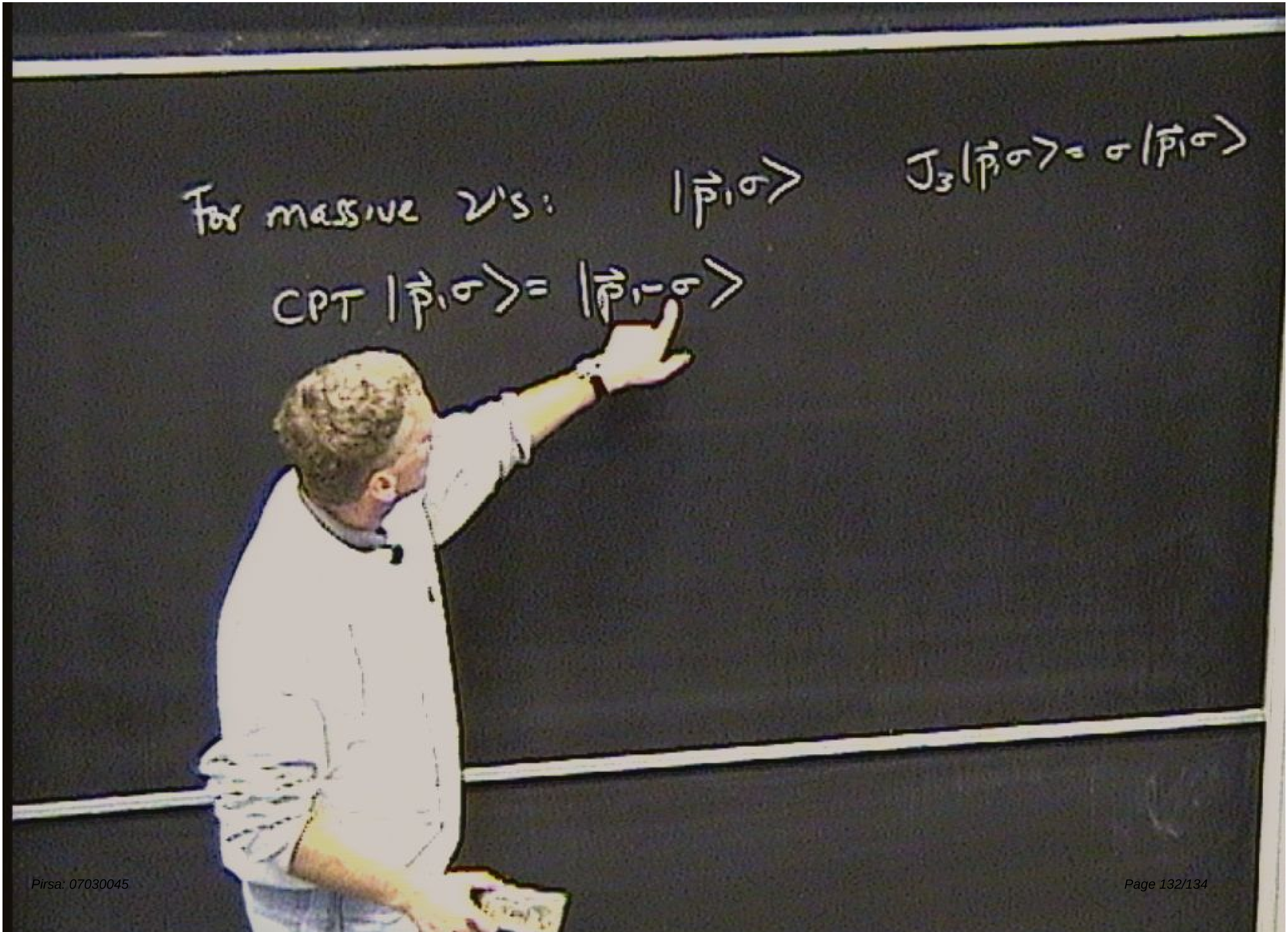
Aside on spin 1/2 kinematics:

Given ν_a (LH massless states)

$$|\vec{p}, h\rangle \qquad h = \frac{\vec{p}\cdot\vec{\sigma}}{|\vec{p}|}$$

antinus: CPT $|\vec{p}, h\rangle \rightarrow |\vec{p}, -h\rangle$ are distinguishable.


For massive ν's:
$|\vec{p},\sigma\rangle$
$J_3|\vec{p},\sigma\rangle = \sigma|\vec{p},\sigma\rangle$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For massive ν's: $|\vec{p},\sigma\rangle$ $\quad J_3|\vec{p},\sigma\rangle = \sigma|\vec{p},\sigma\rangle$

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For massive ν's: $|\vec{p}, \sigma\rangle$ $\qquad J_3|\vec{p},\sigma\rangle = \sigma|\vec{p},\sigma\rangle$

$CPT\,|\vec{p},\sigma\rangle = |\vec{p},-\sigma\rangle \qquad \sigma = \pm 1/2.$

if $CPT\,|\vec{p}, 1/2\rangle$ is the same as $|\vec{p}, -1/2\rangle$

then ptcle + antiptcle are indistinguishable "majorana".

BUT if $L|\vec{p},\sigma\rangle = \quad |p,\sigma\rangle$ then $CPT|p,\sigma\rangle$ is different than $|\vec{p},-\sigma\rangle$ "Dirac".