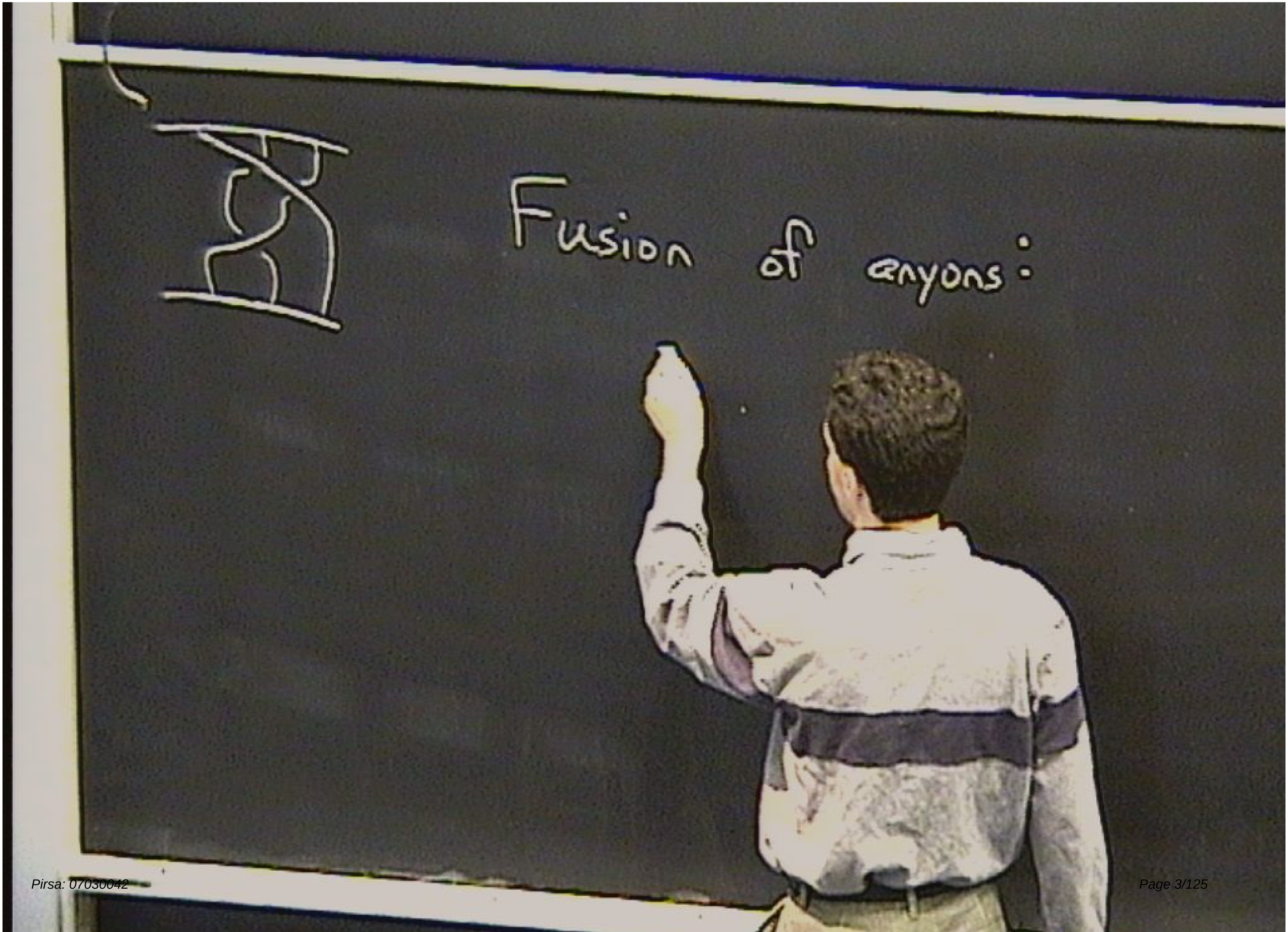
Title: Quantum Error Correction 12A

Date: Mar 27, 2007 03:30 PM

URL: http://pirsa.org/07030042

Abstract: Non-Abelian anyons (charges, fusion rules, F and R matrices, pentagon and hexagon equations), Fibonacci anyons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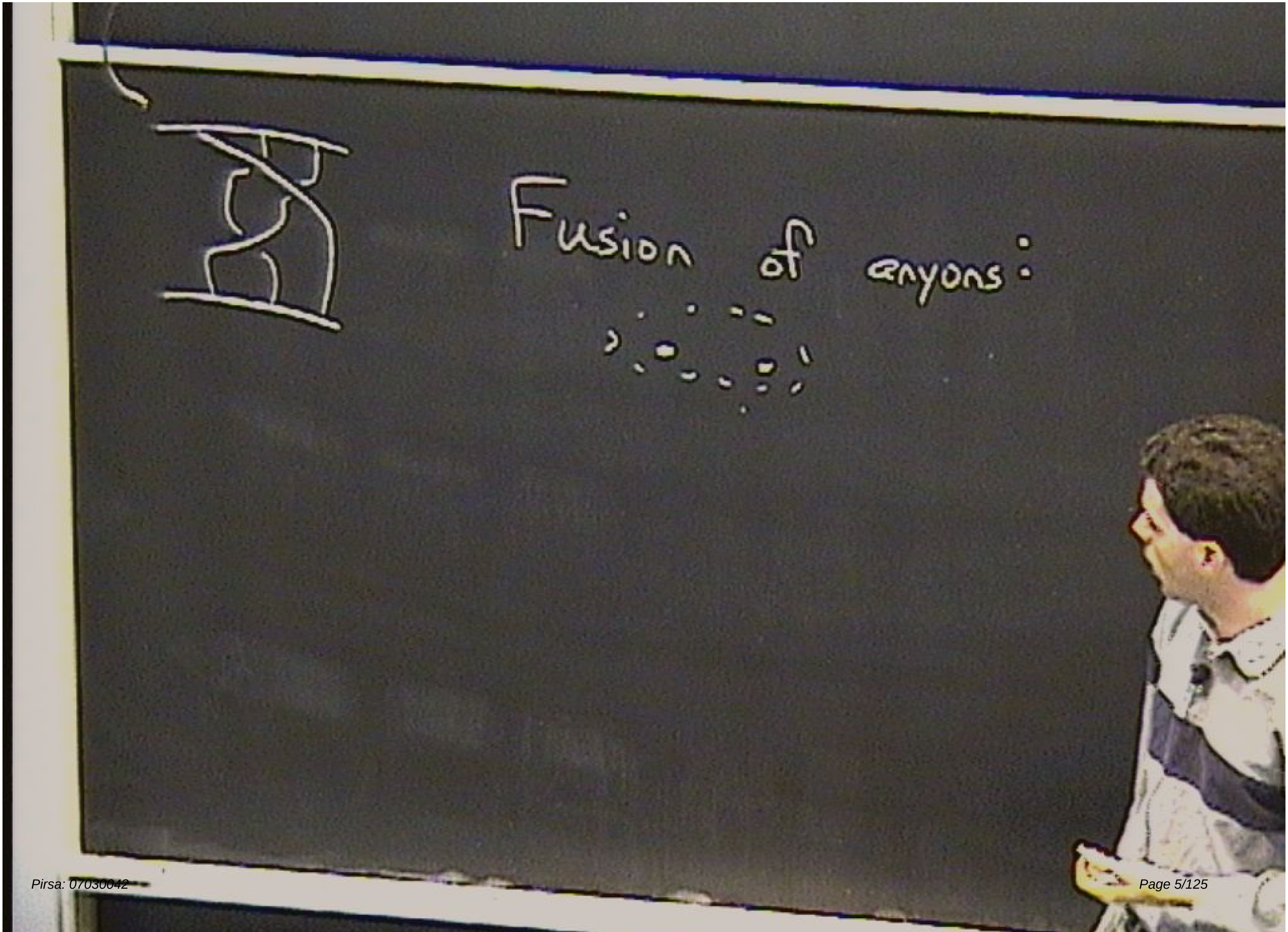
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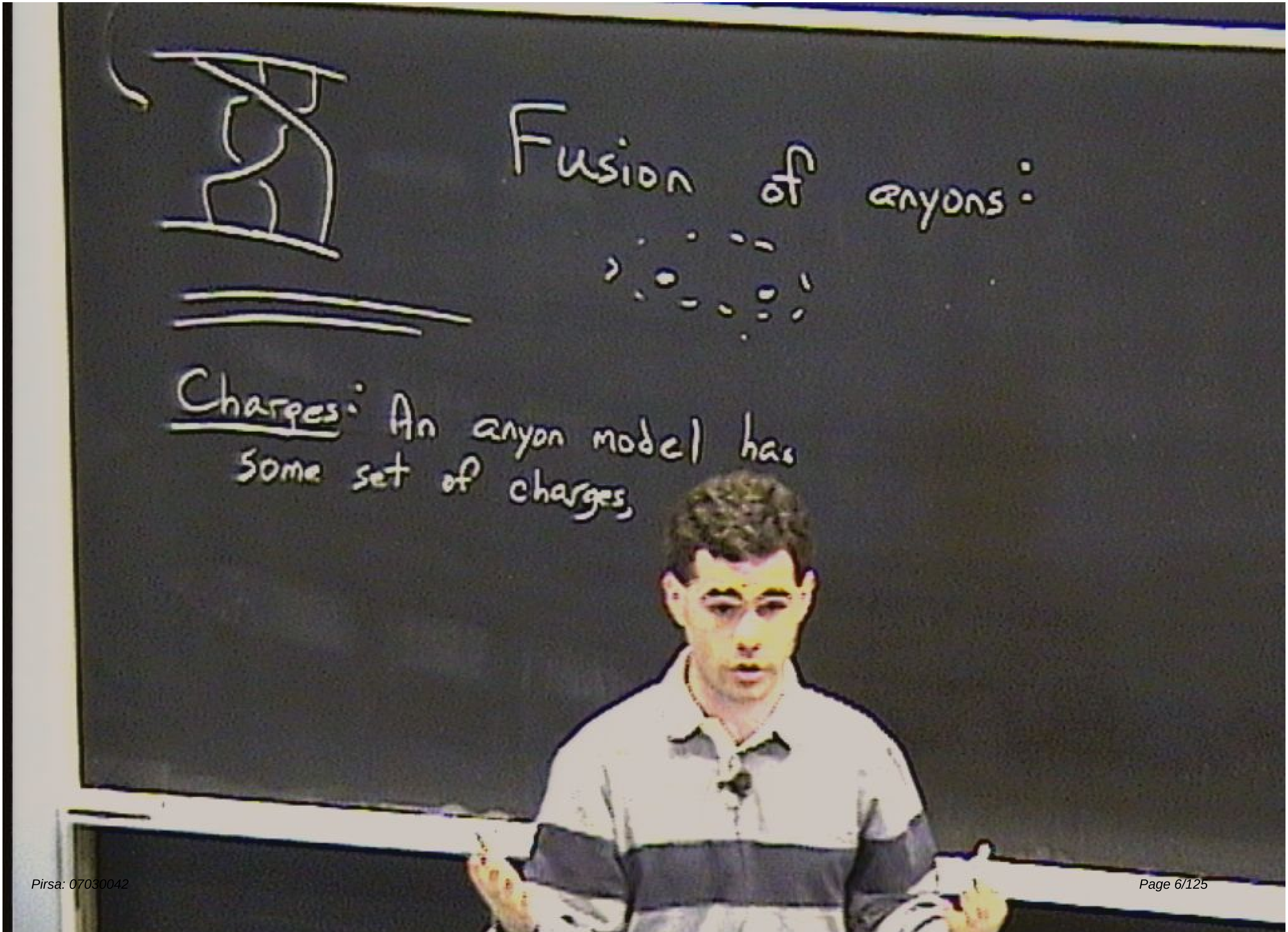
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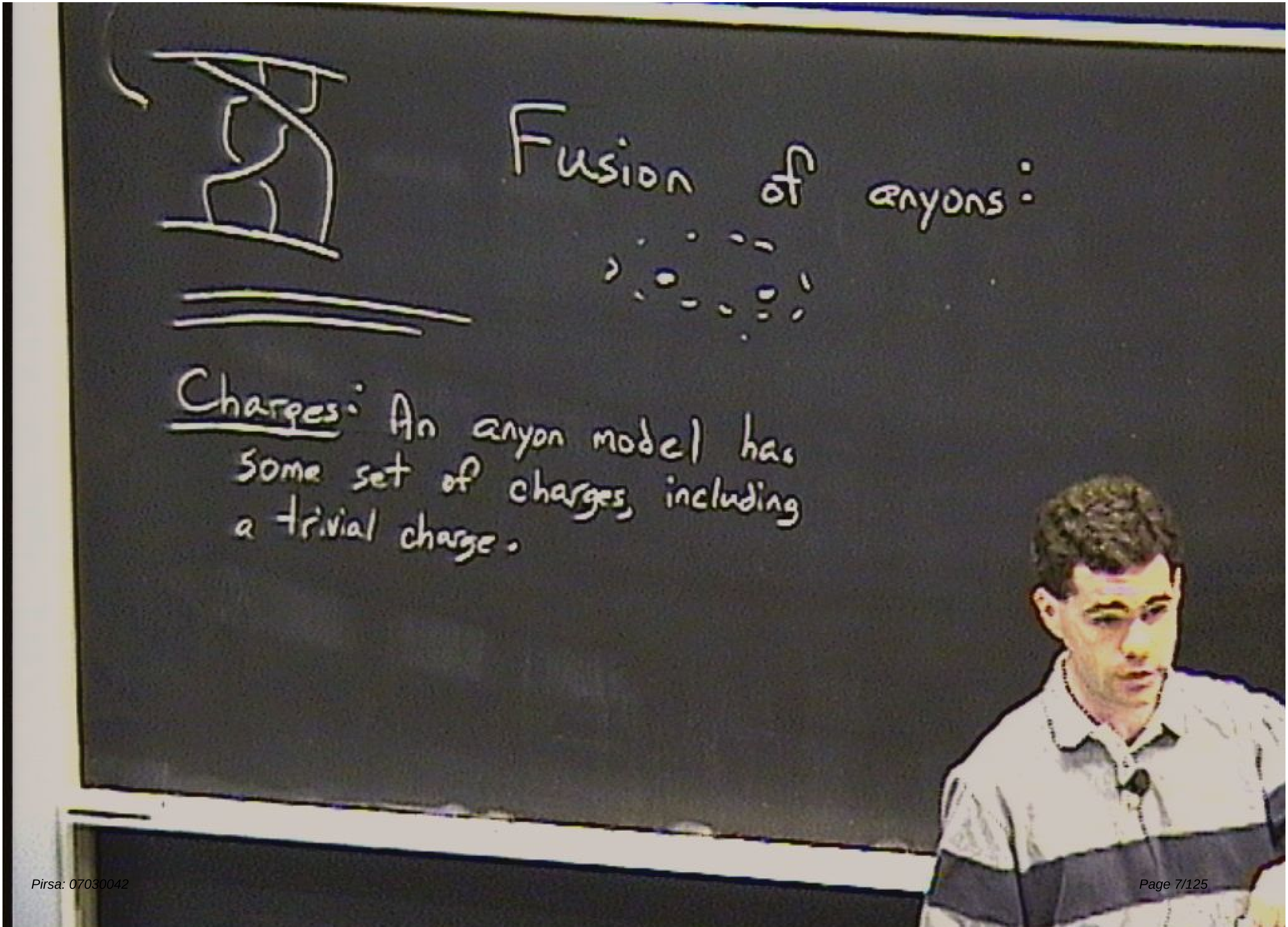
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<u>Fibonacci anyons</u>
(Yang-Lee model)

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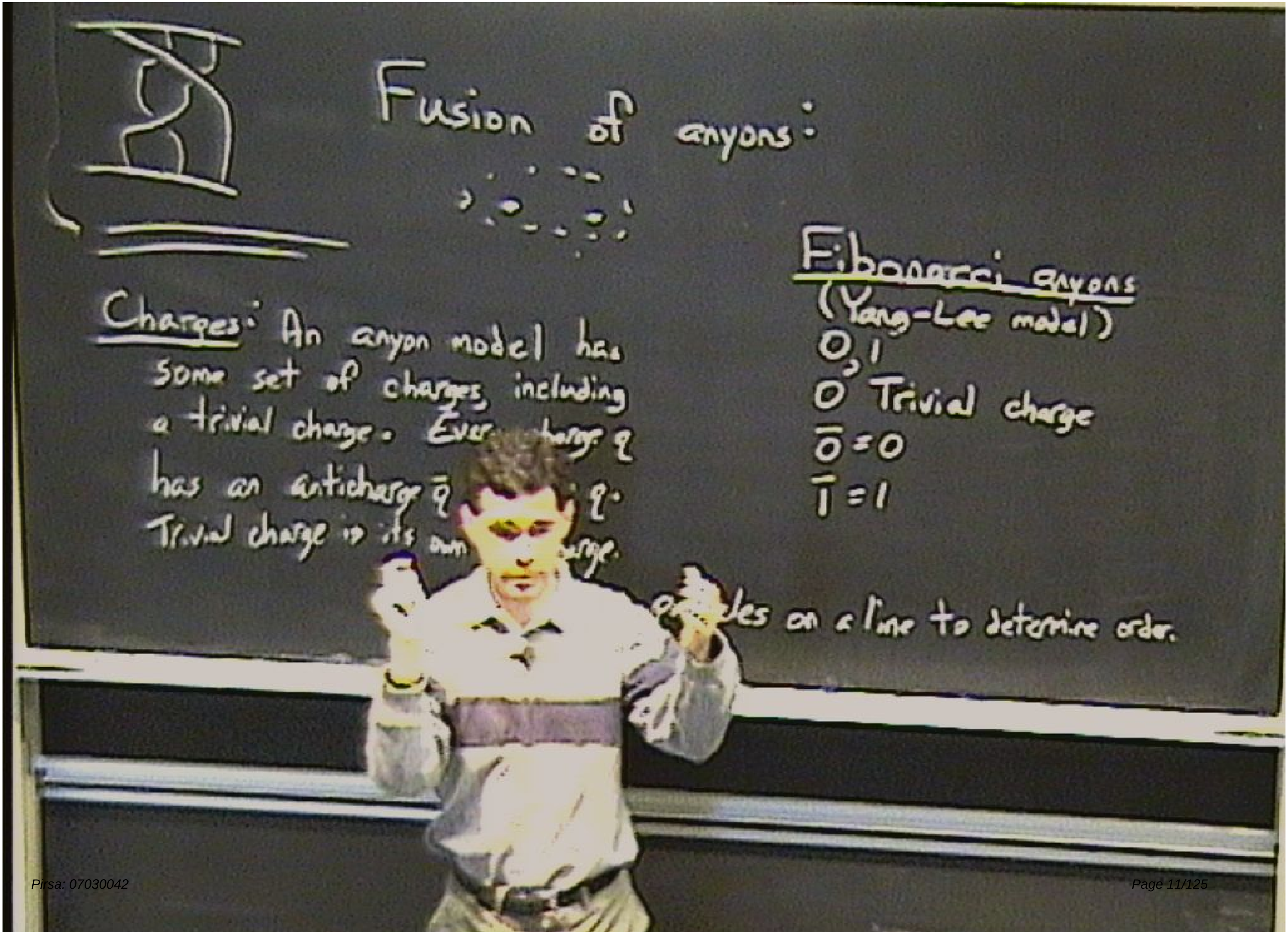
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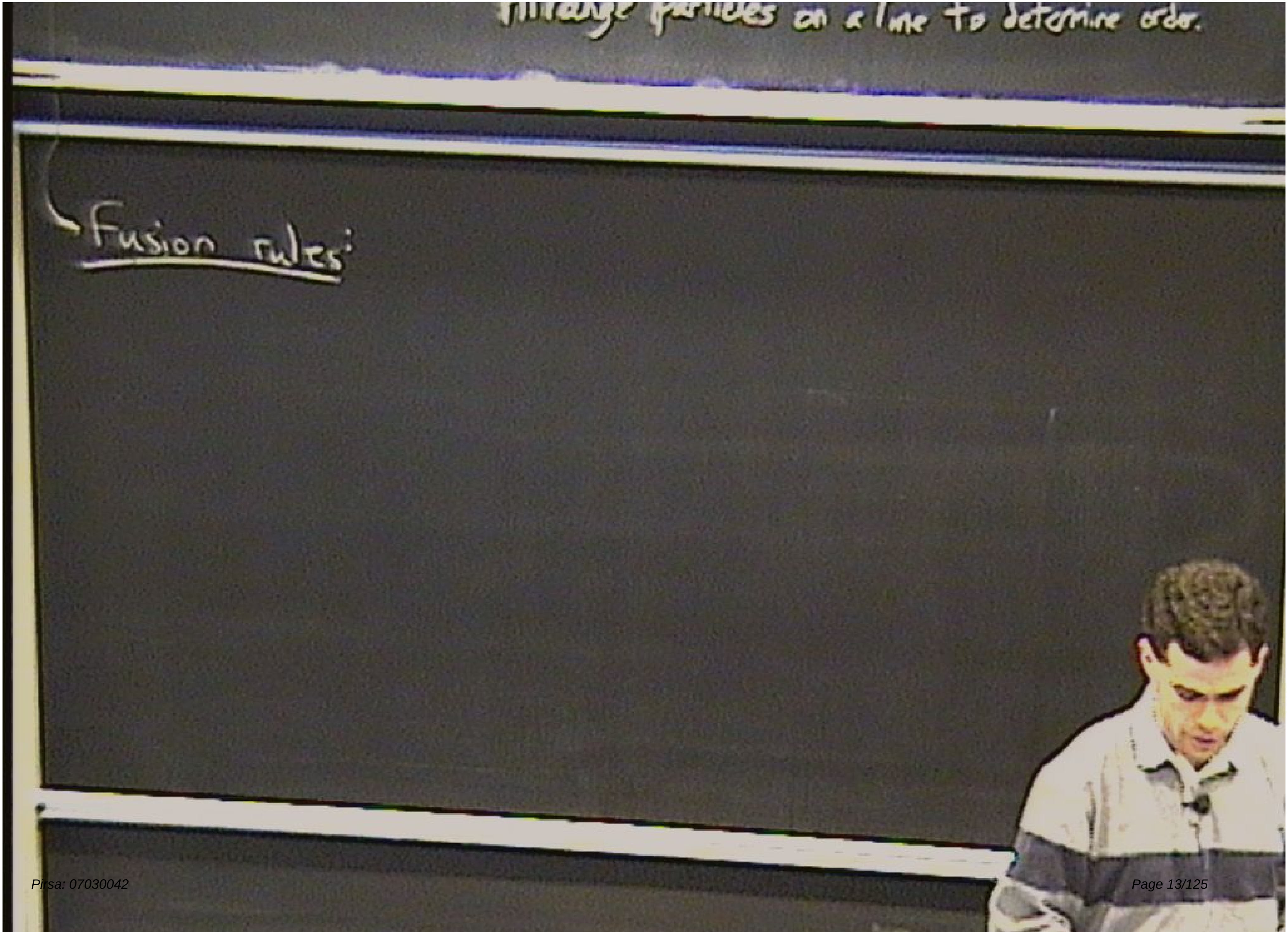
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Fusion rules:

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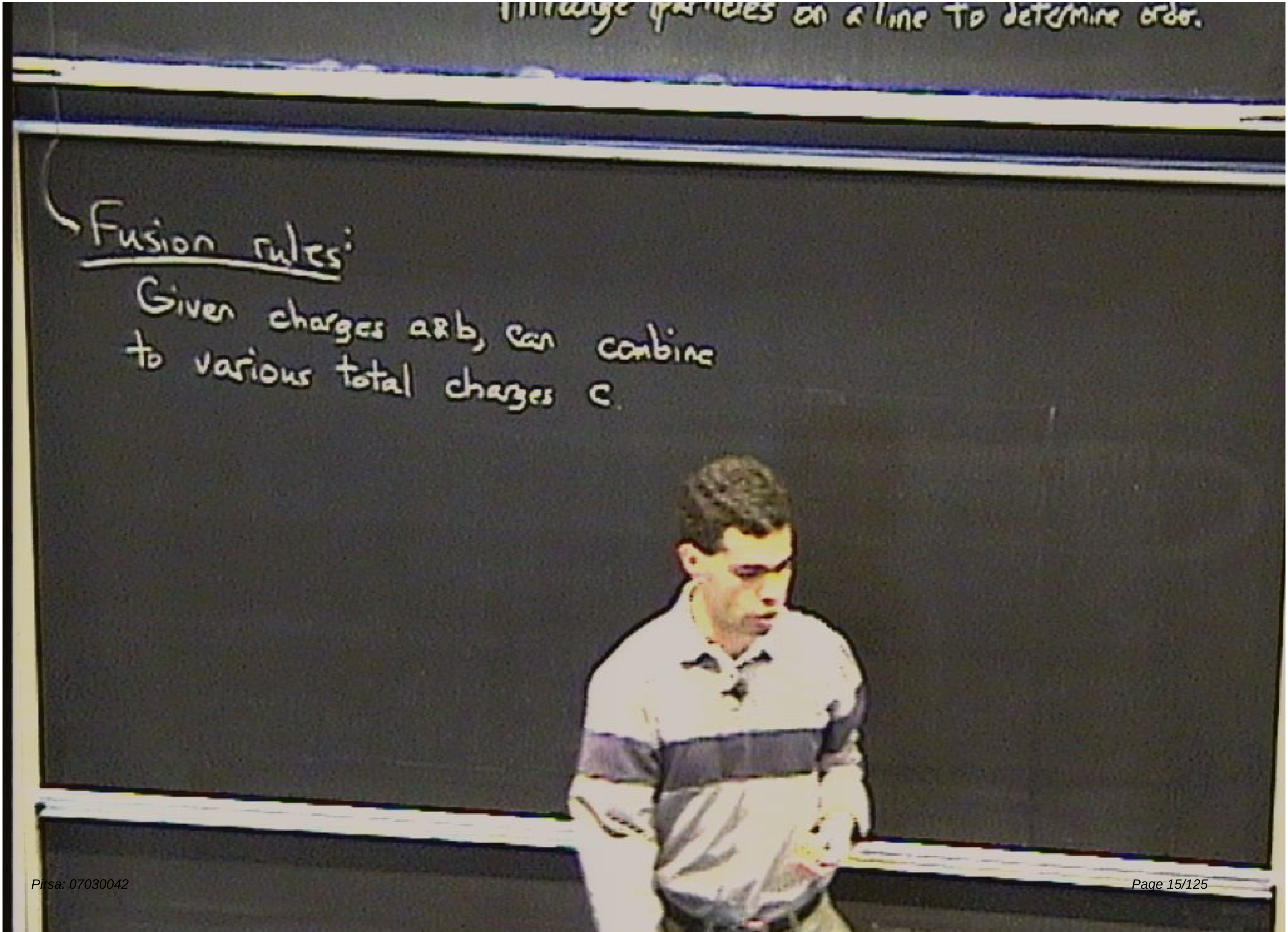
Given charges a & b, can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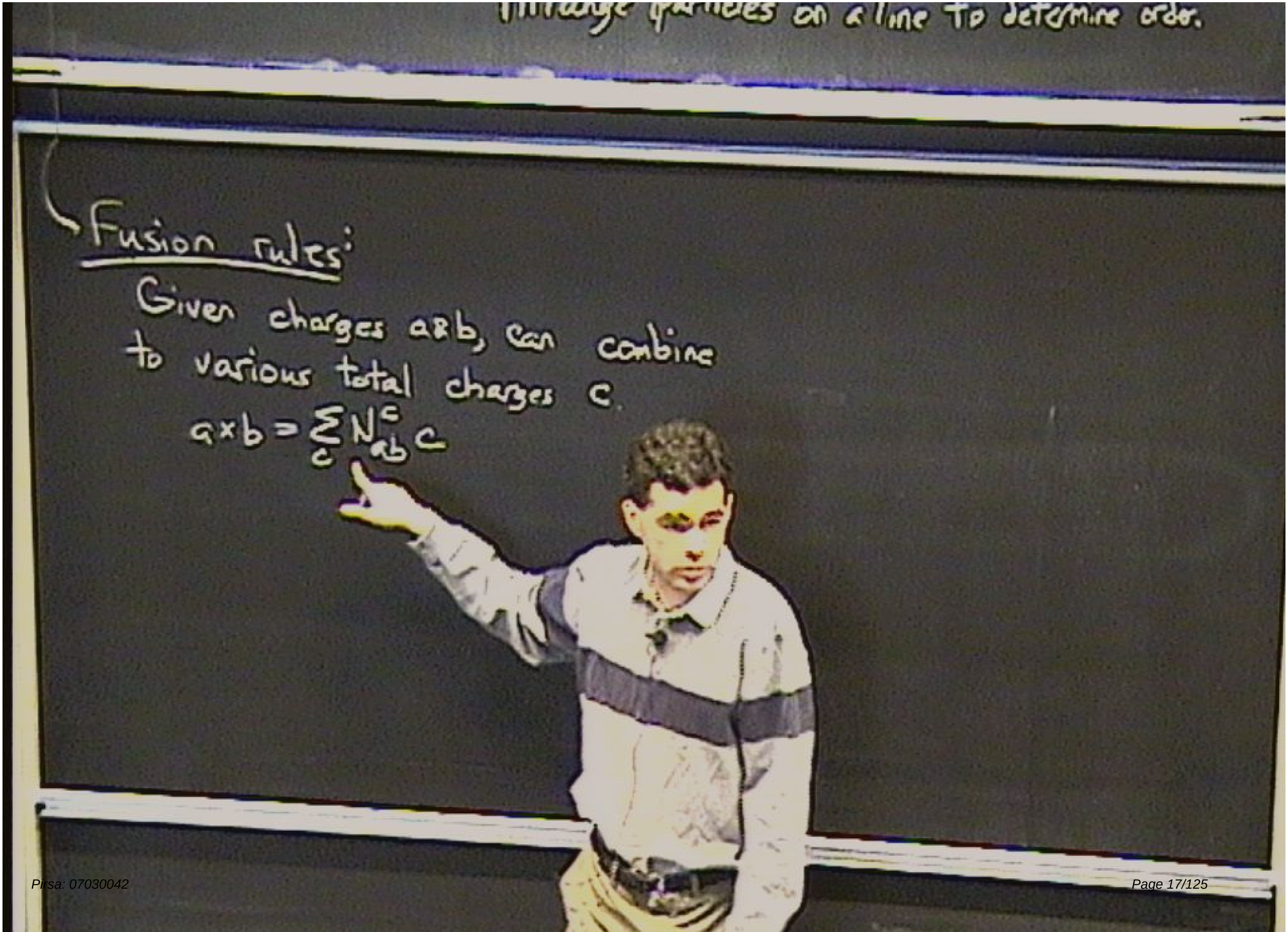
$$a \times b = \sum_c N_{ab}^c \, c$$



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Trivial fused w/ anything

Arrange particles on a line to determine order.

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Trivial fused w/ anything a, always gives a

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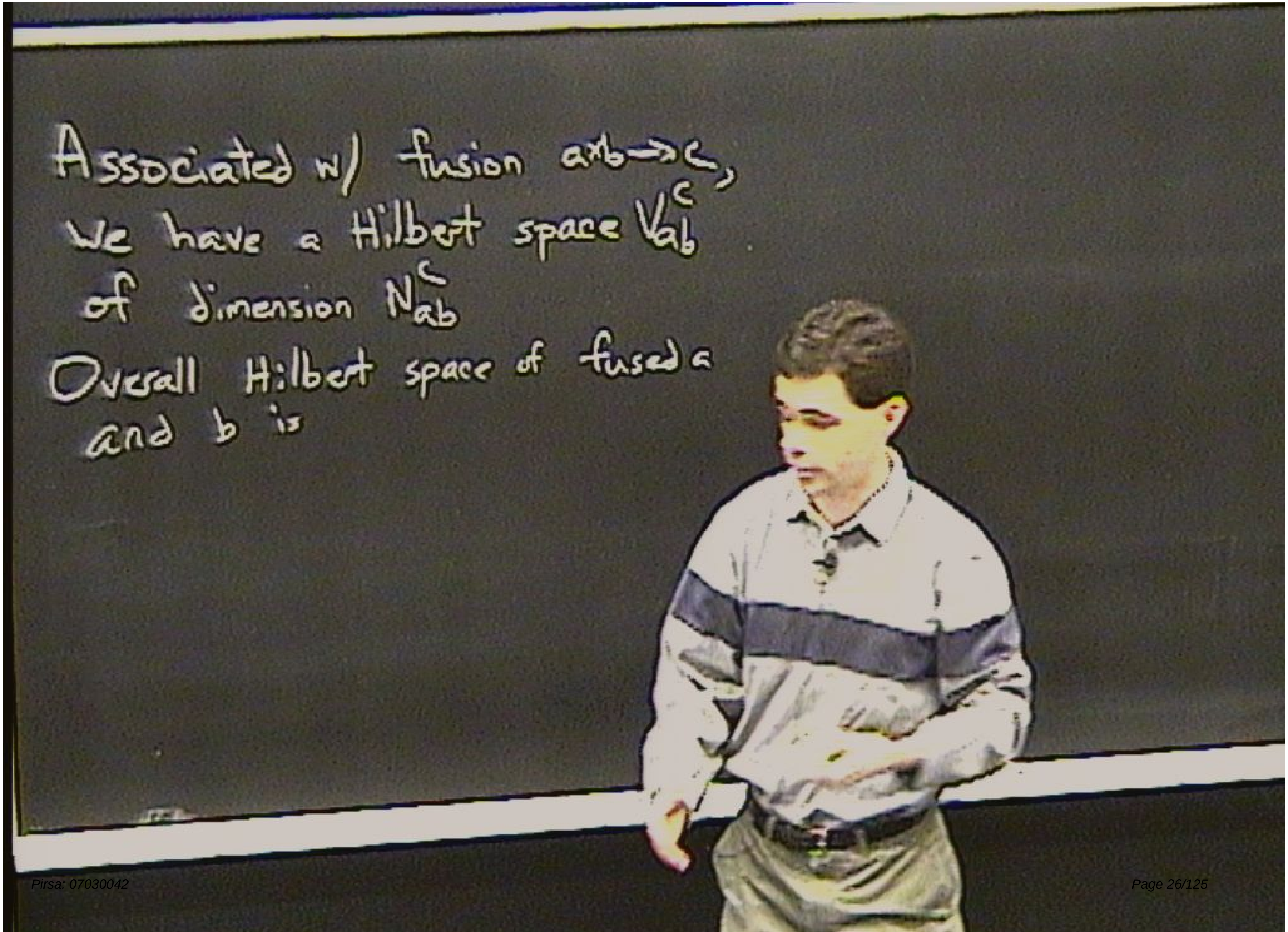
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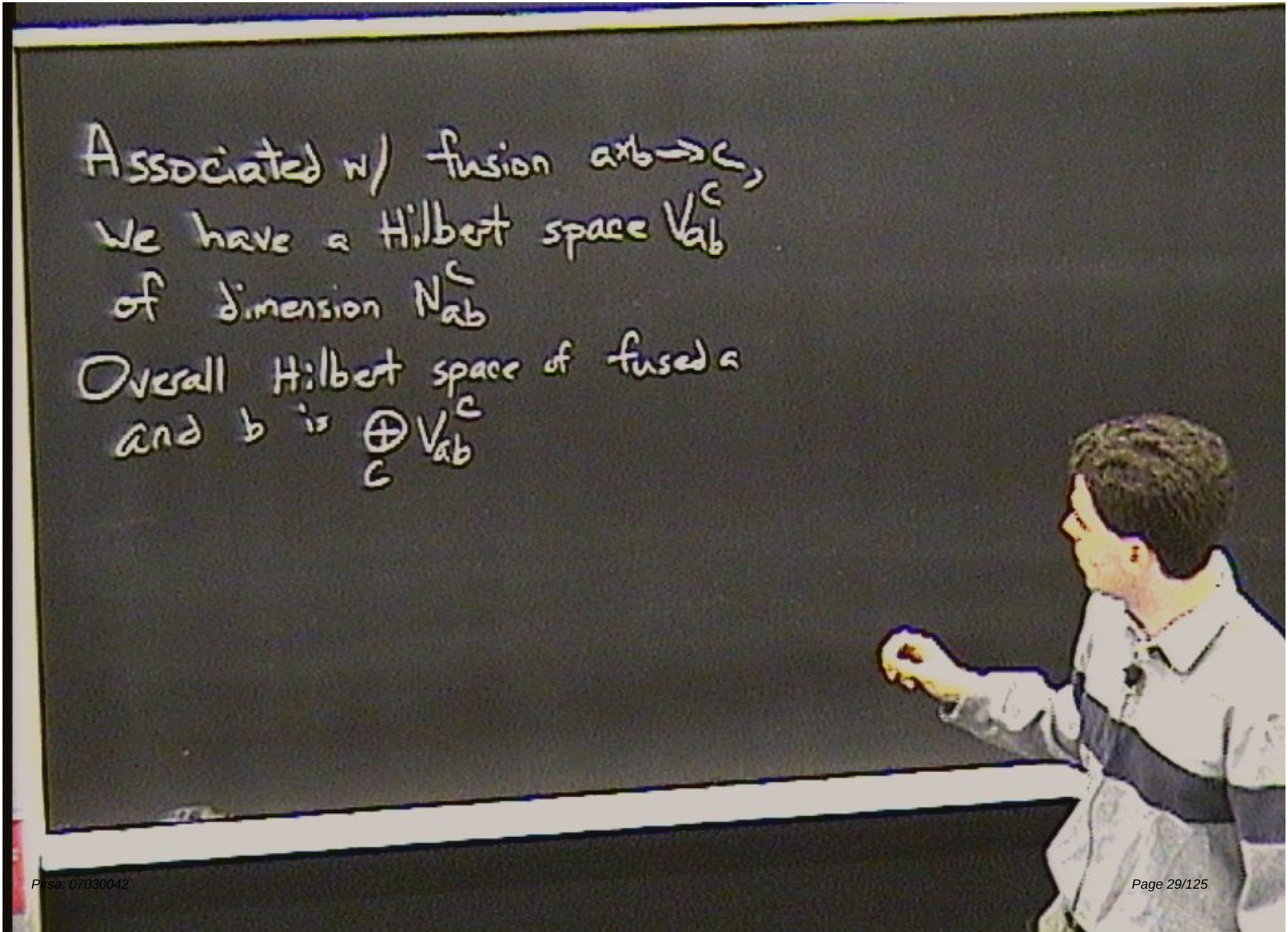
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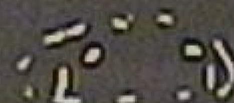 is 2-dimensional

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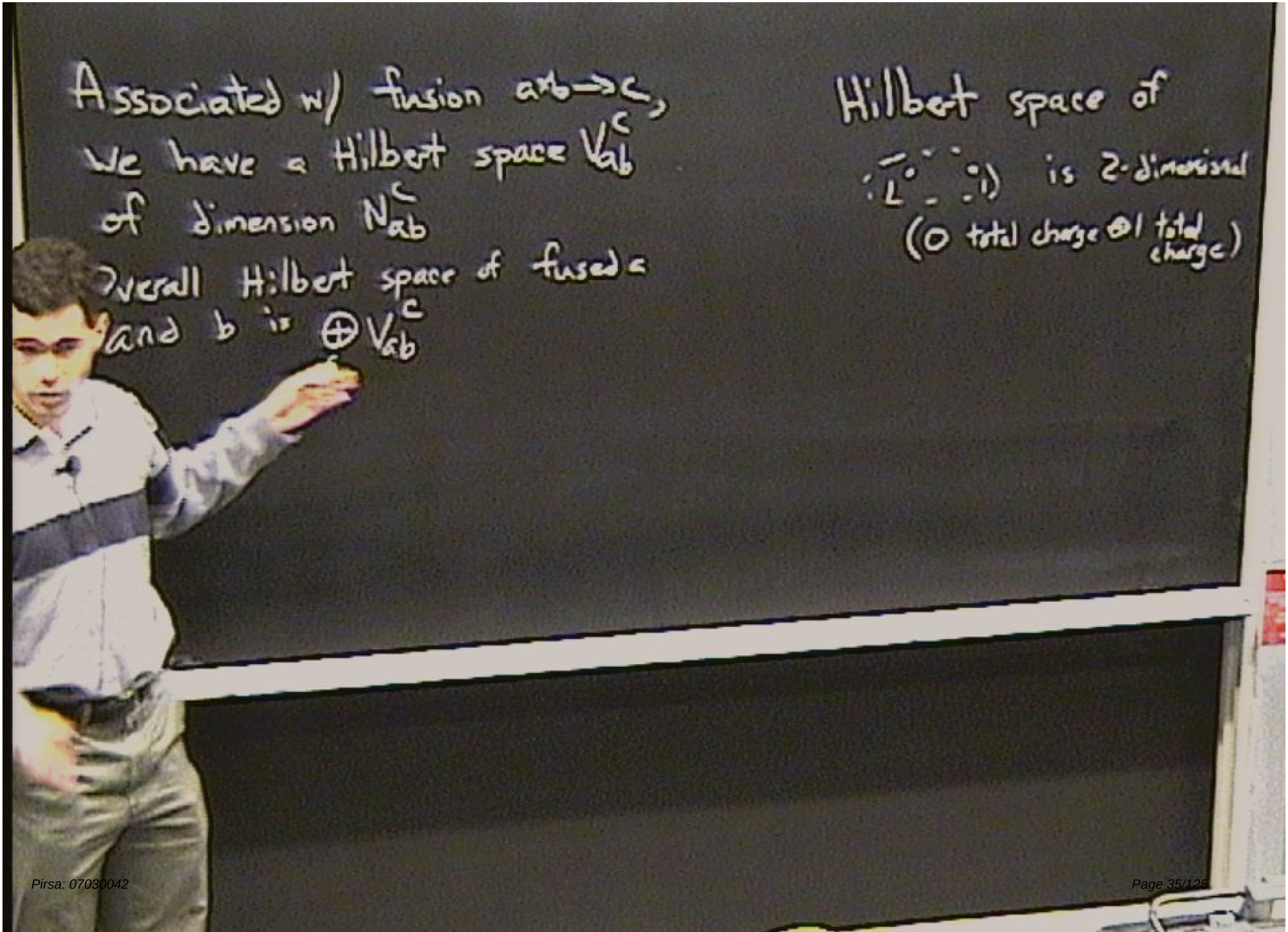
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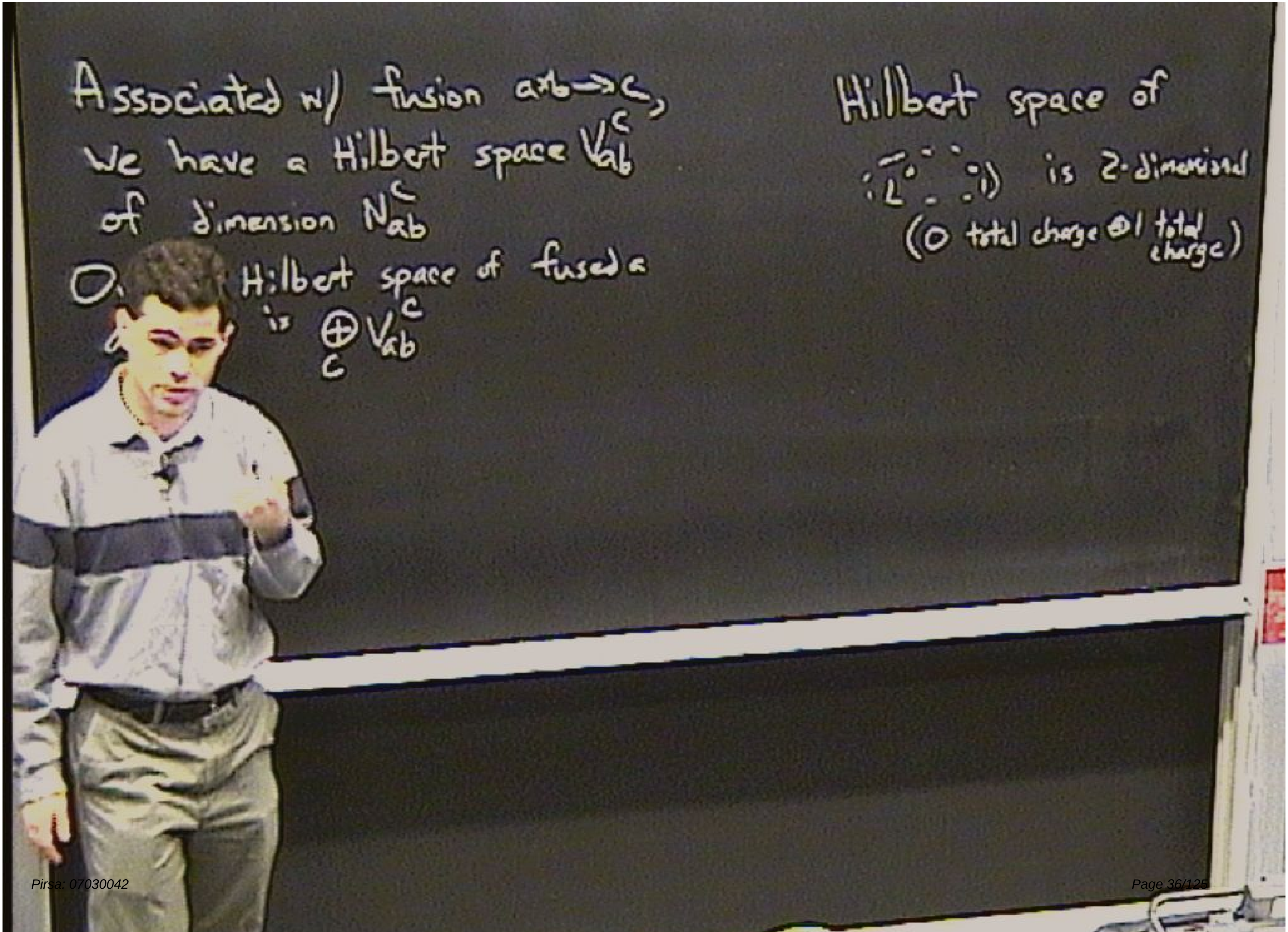
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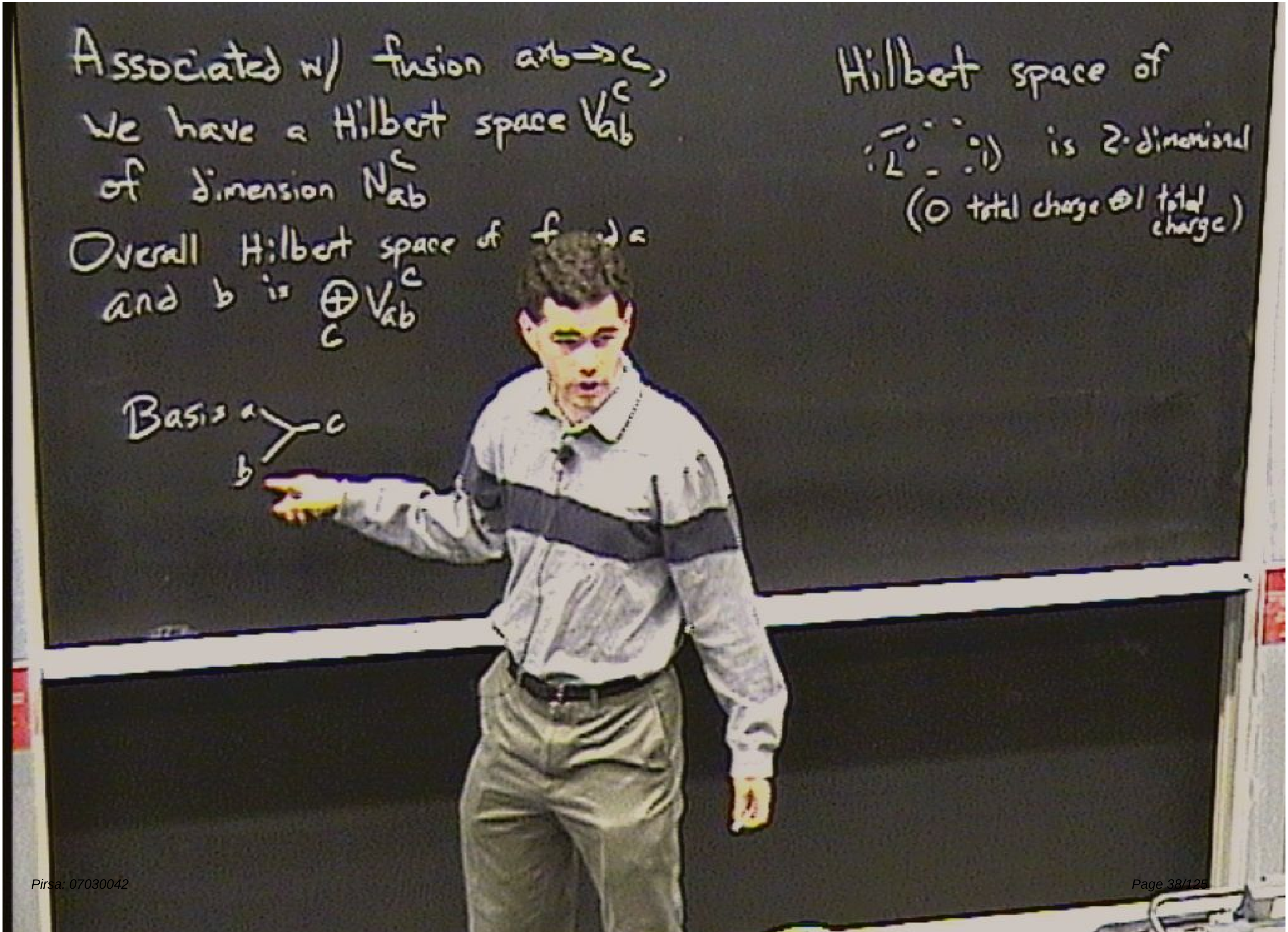
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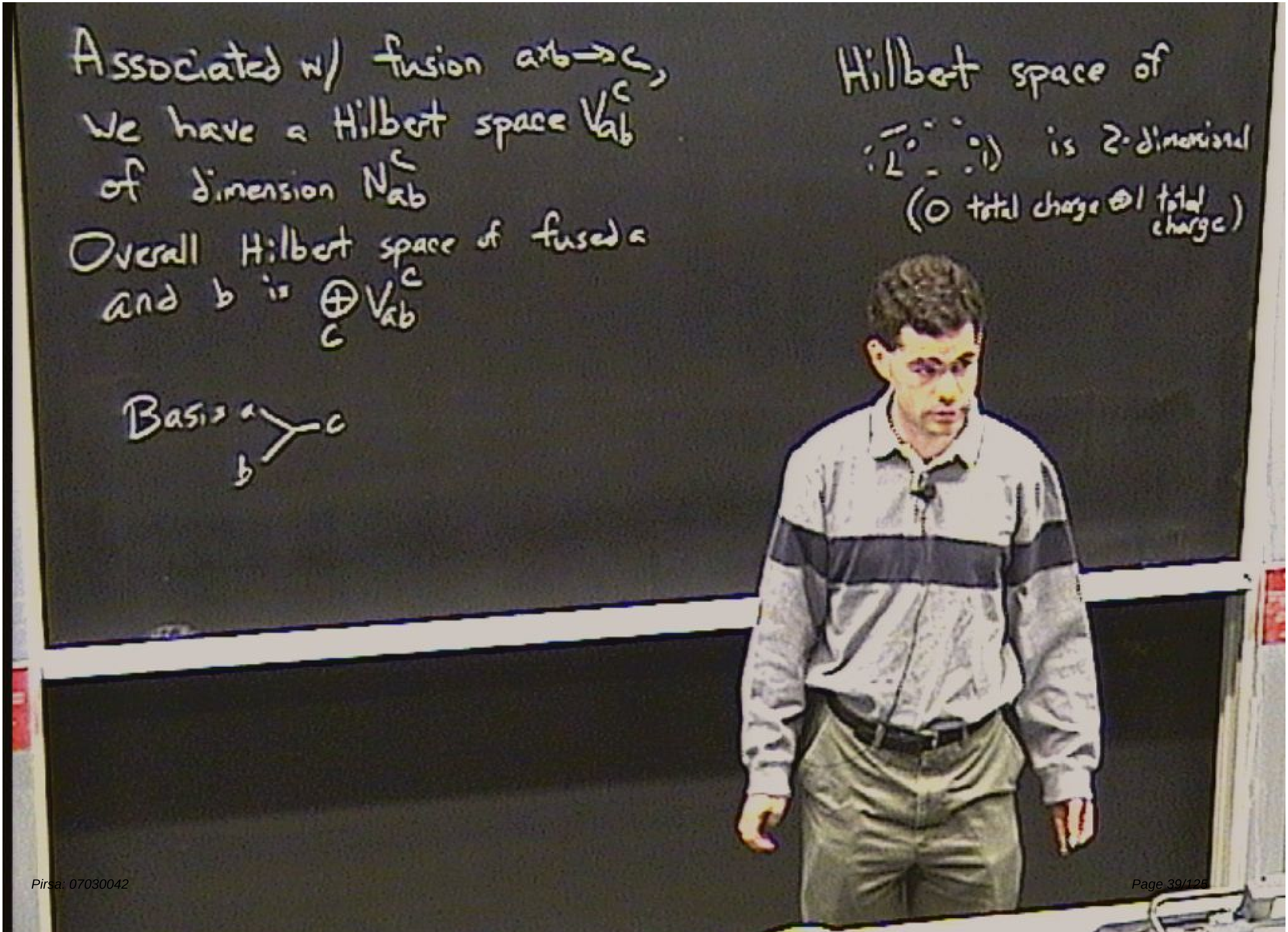
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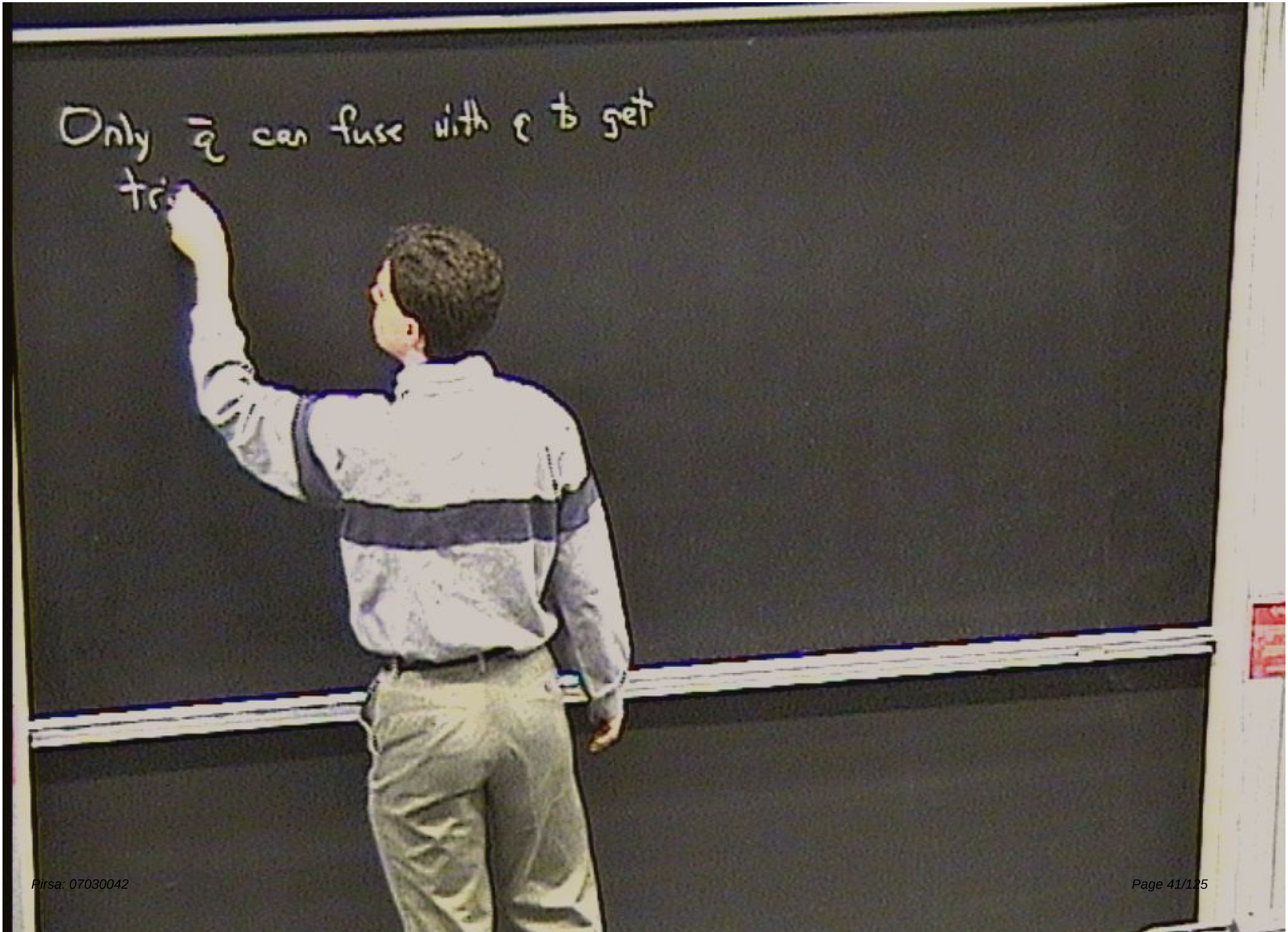
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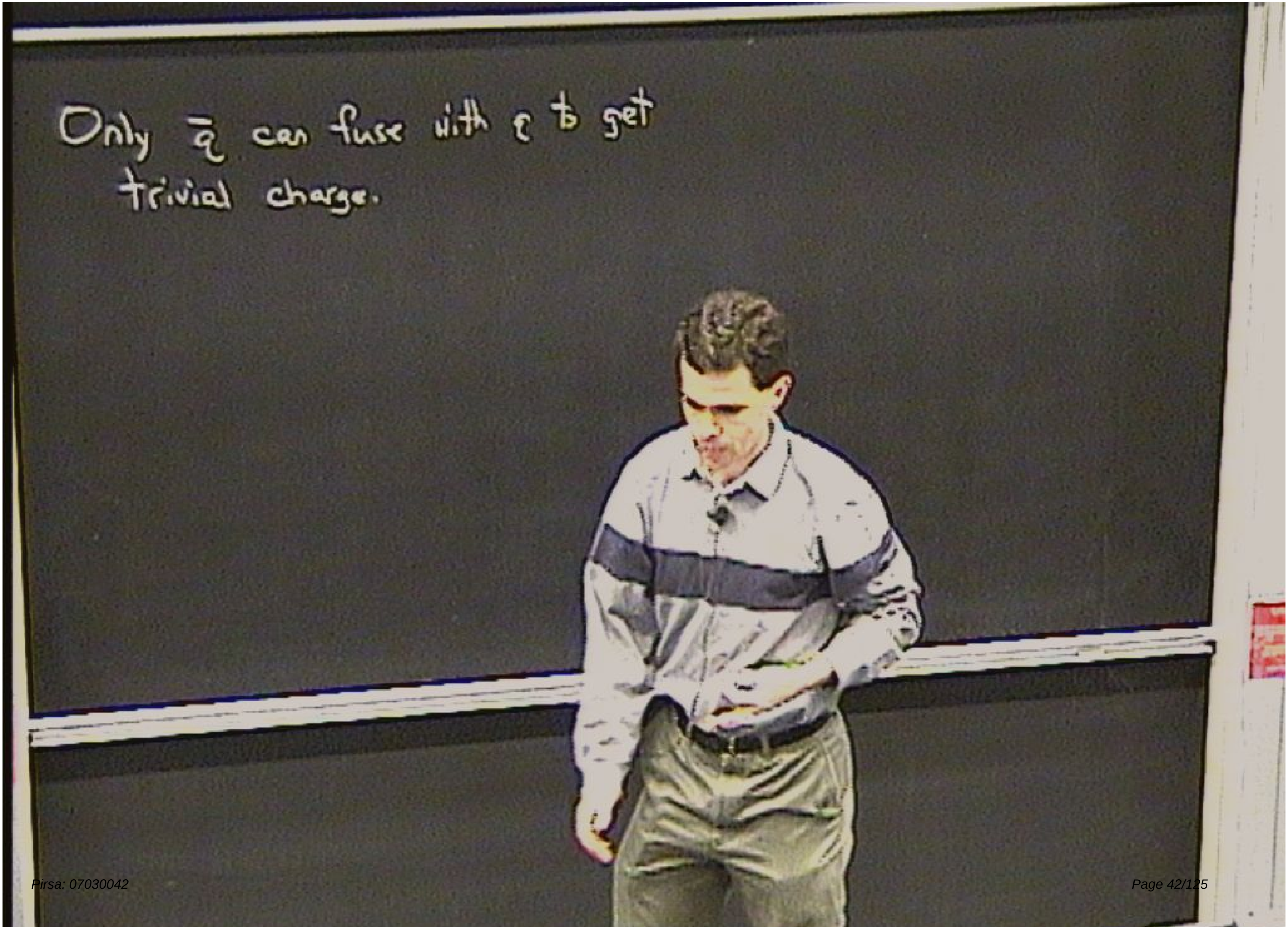
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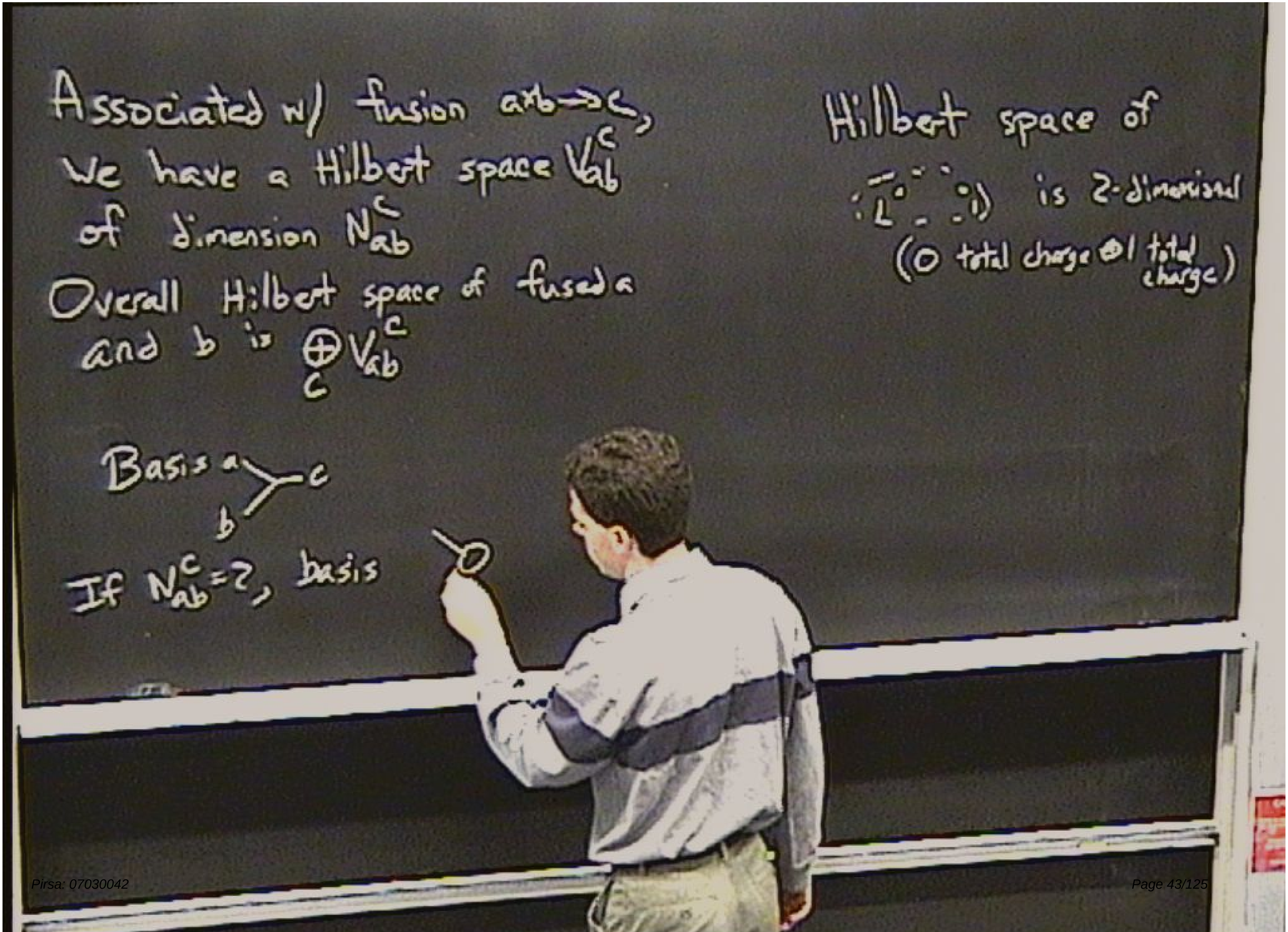
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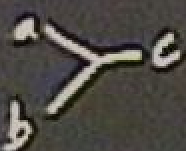
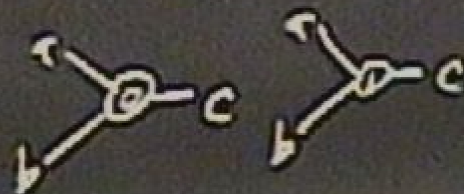
Basis 

If $N_{ab}^c = 2$, basis 

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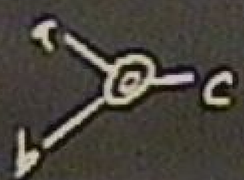
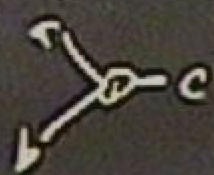
If $N_{ab}^{c} = 2$, basis , 

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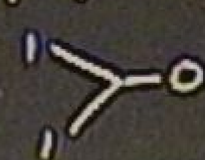
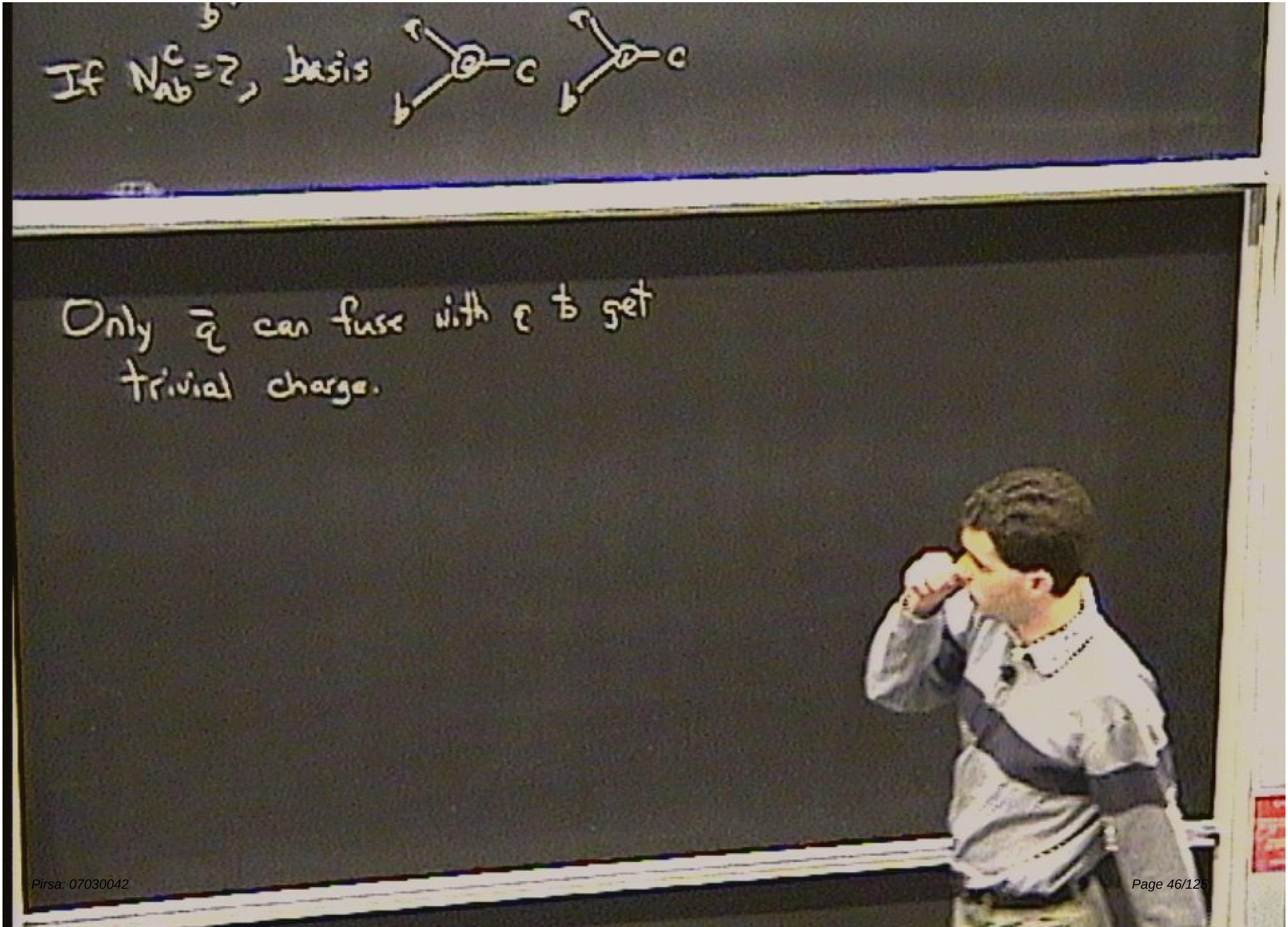
1×1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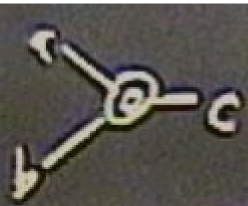
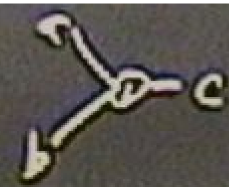
If $N_{ab}^{c}=2$, basis 

, 

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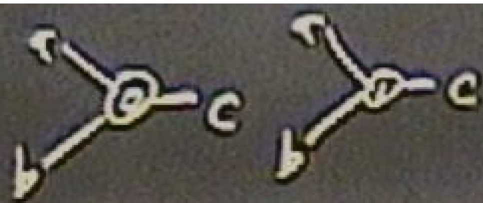
Can raise & lower indices on N_{ab}^{c} by switching charge & anticharge

E.g: $N_{ab}^{c} = N_{a\bar{c}}^{\bar{b}}$

If $N_{ab}^{c}=2$, basis 

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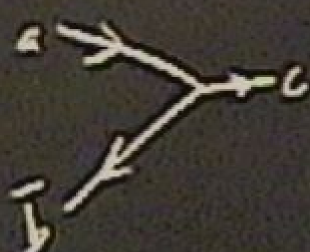

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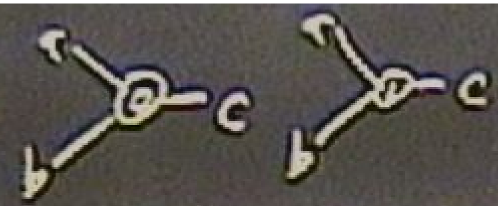

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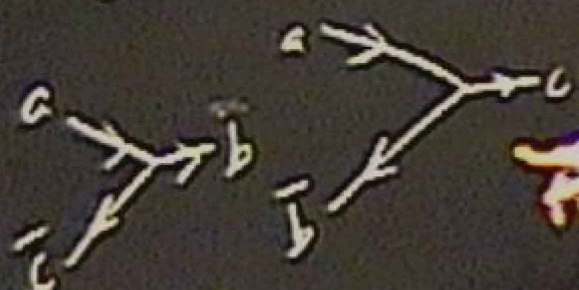
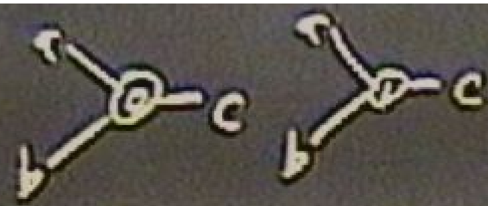


If $N_{ab}^{c}=2$, basis 

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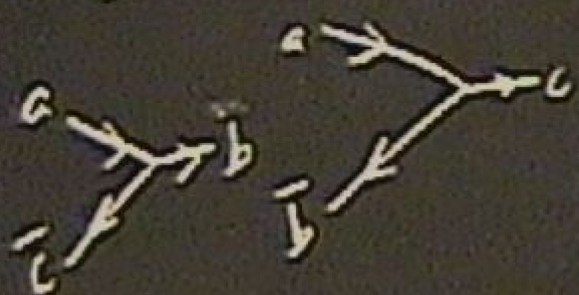
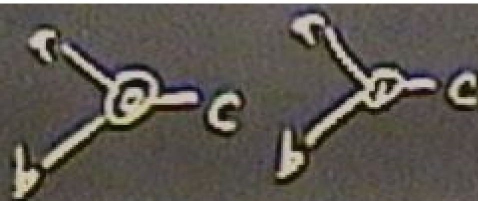


Also have splitting rules

If $N_{ab}^c = 2$, basis 

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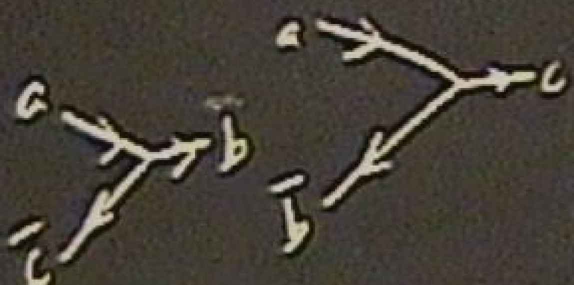

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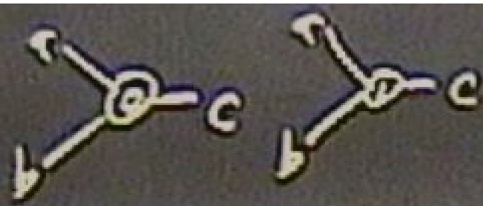
Splitting rule $c = \sum_{a,b} N_c^{ab}\, a \times b$

If $N_{ab}^{c} = 2$, basis 

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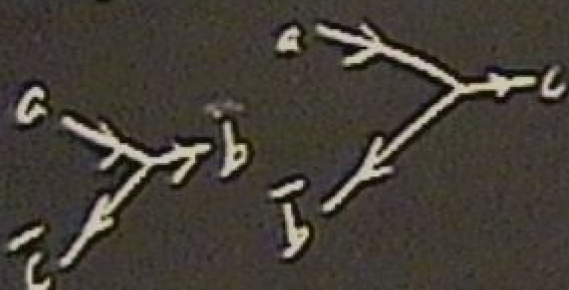

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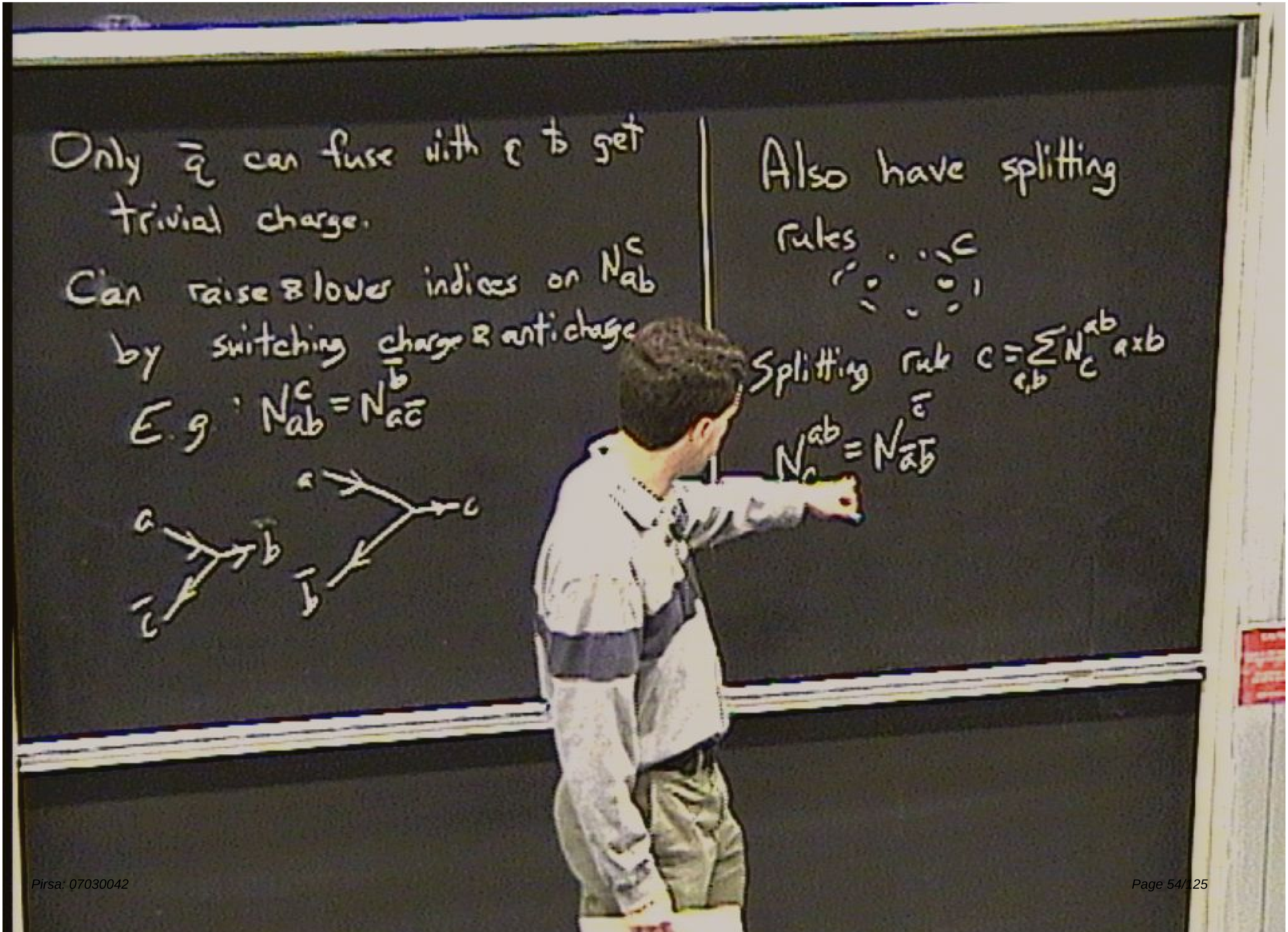
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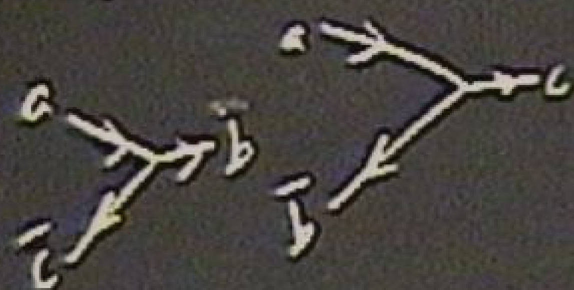
Only $\bar{a}$ can fuse with a to get trivial charge.

Can raise & lower indices on N_{ab}^{c} by switching charge & anticharge

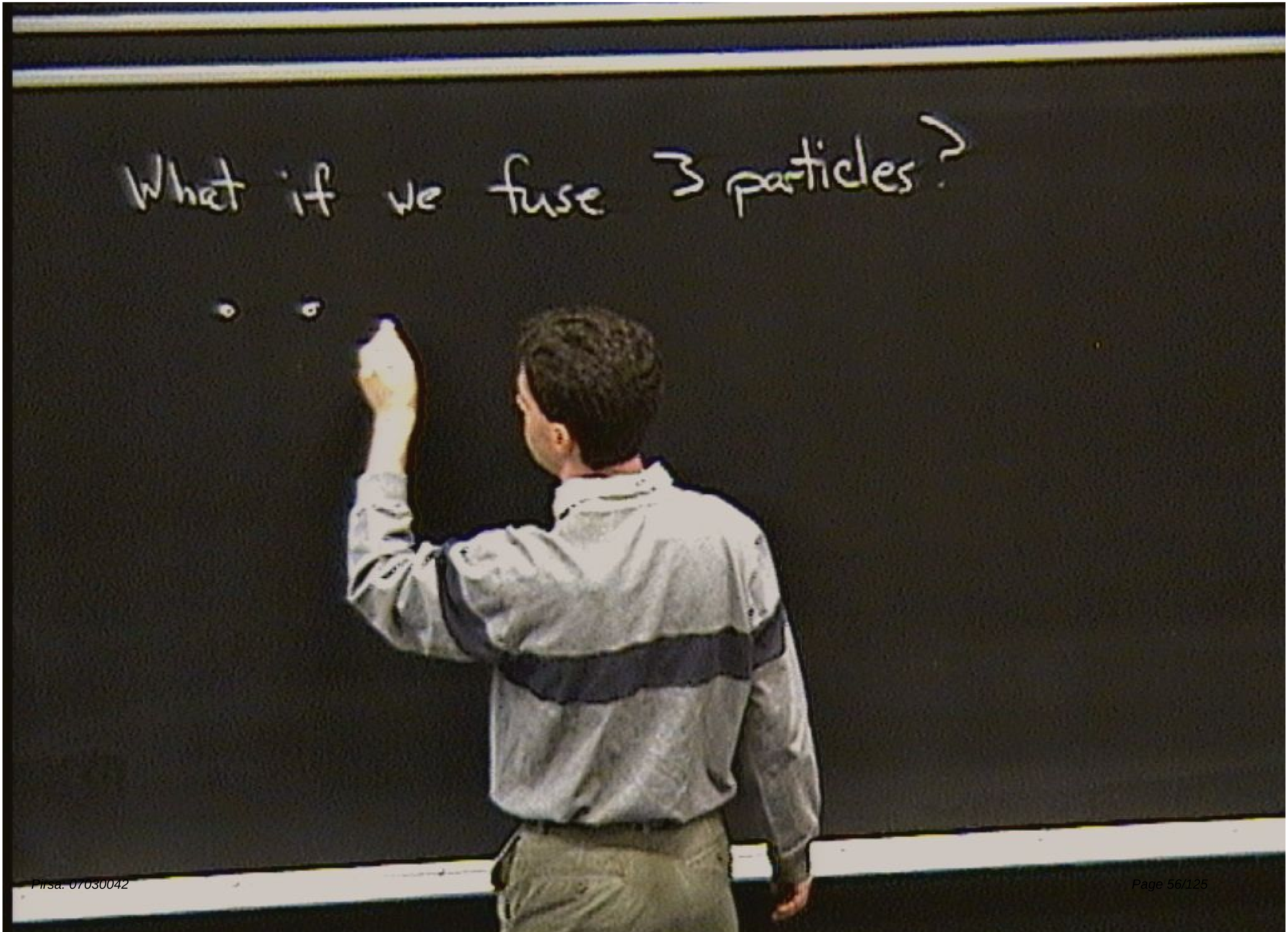
E.g: $N_{ab}^{c} = N_{a\bar{c}}^{\bar{b}}$

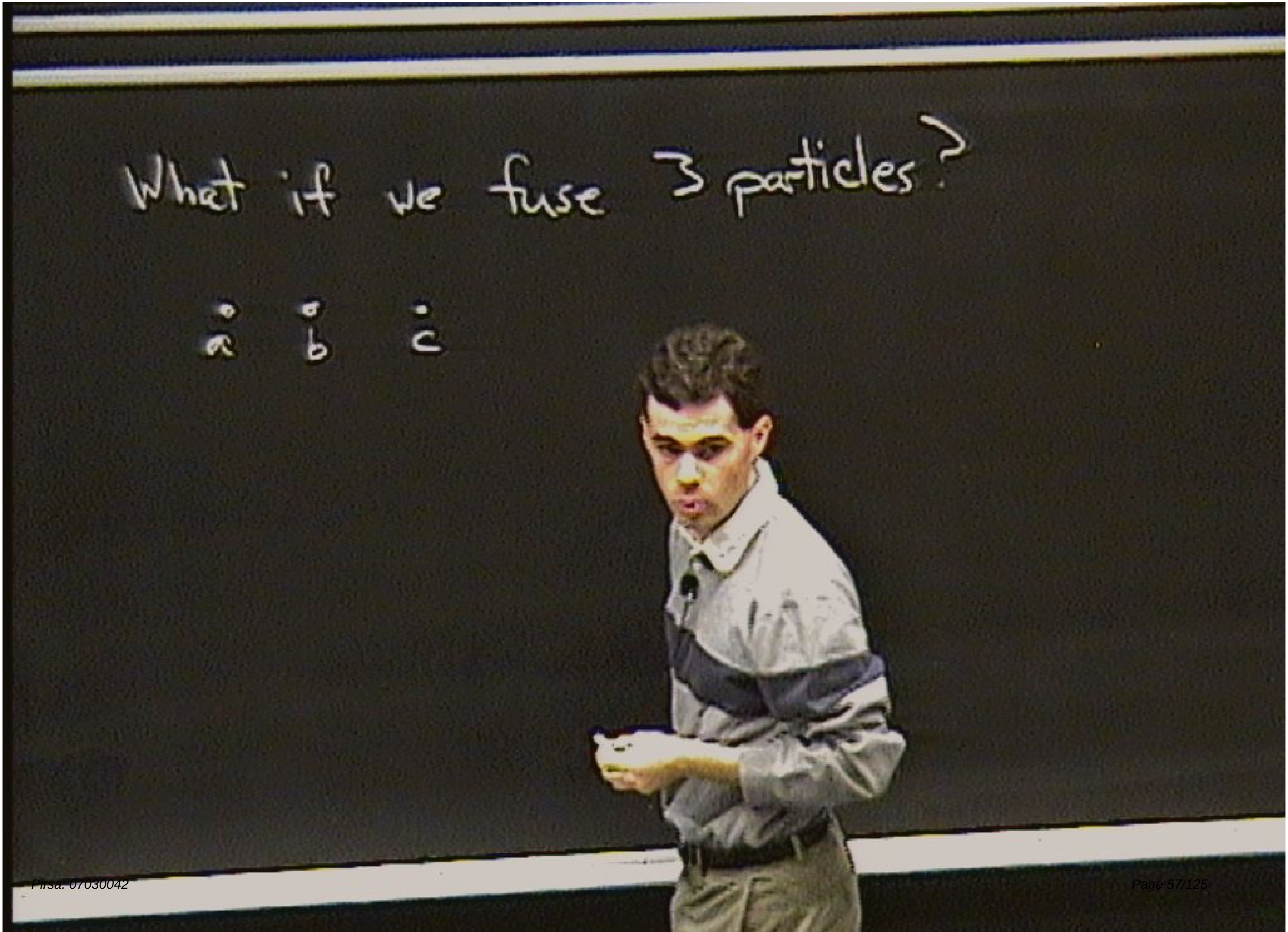


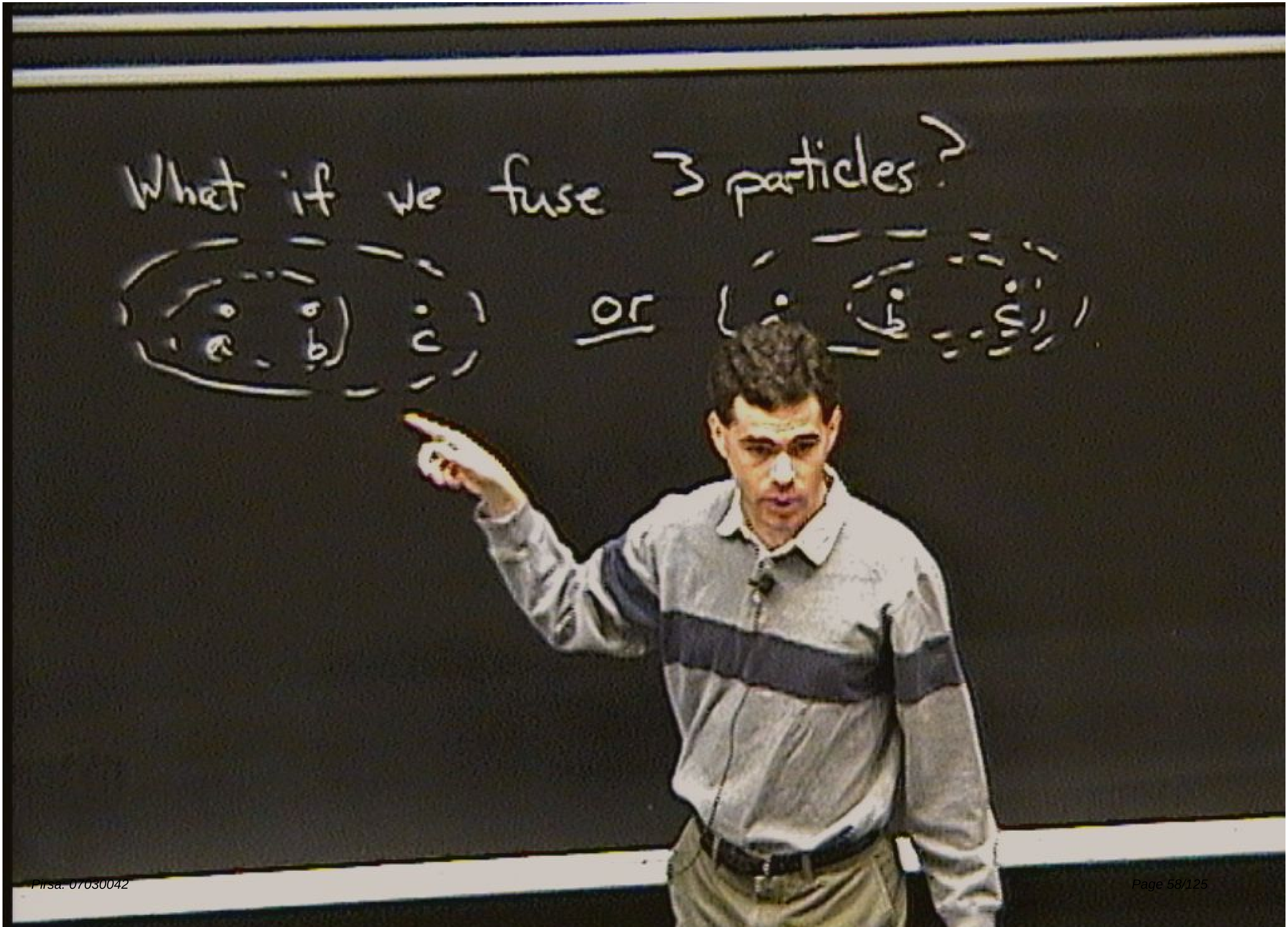
Also have splitting rules

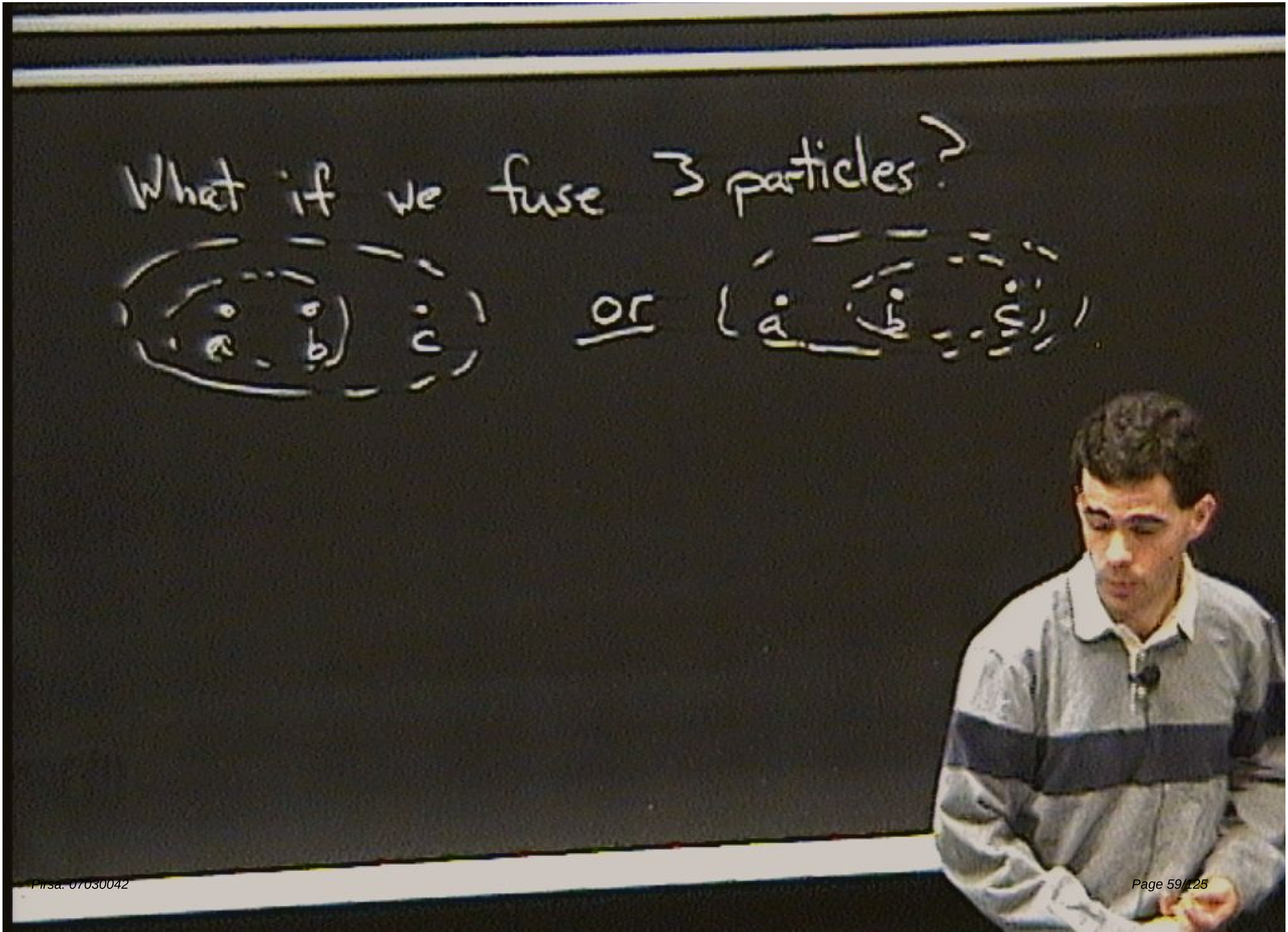
Splitting rule $c = \sum_{a,b} N_{c}^{ab}\, a \times b$

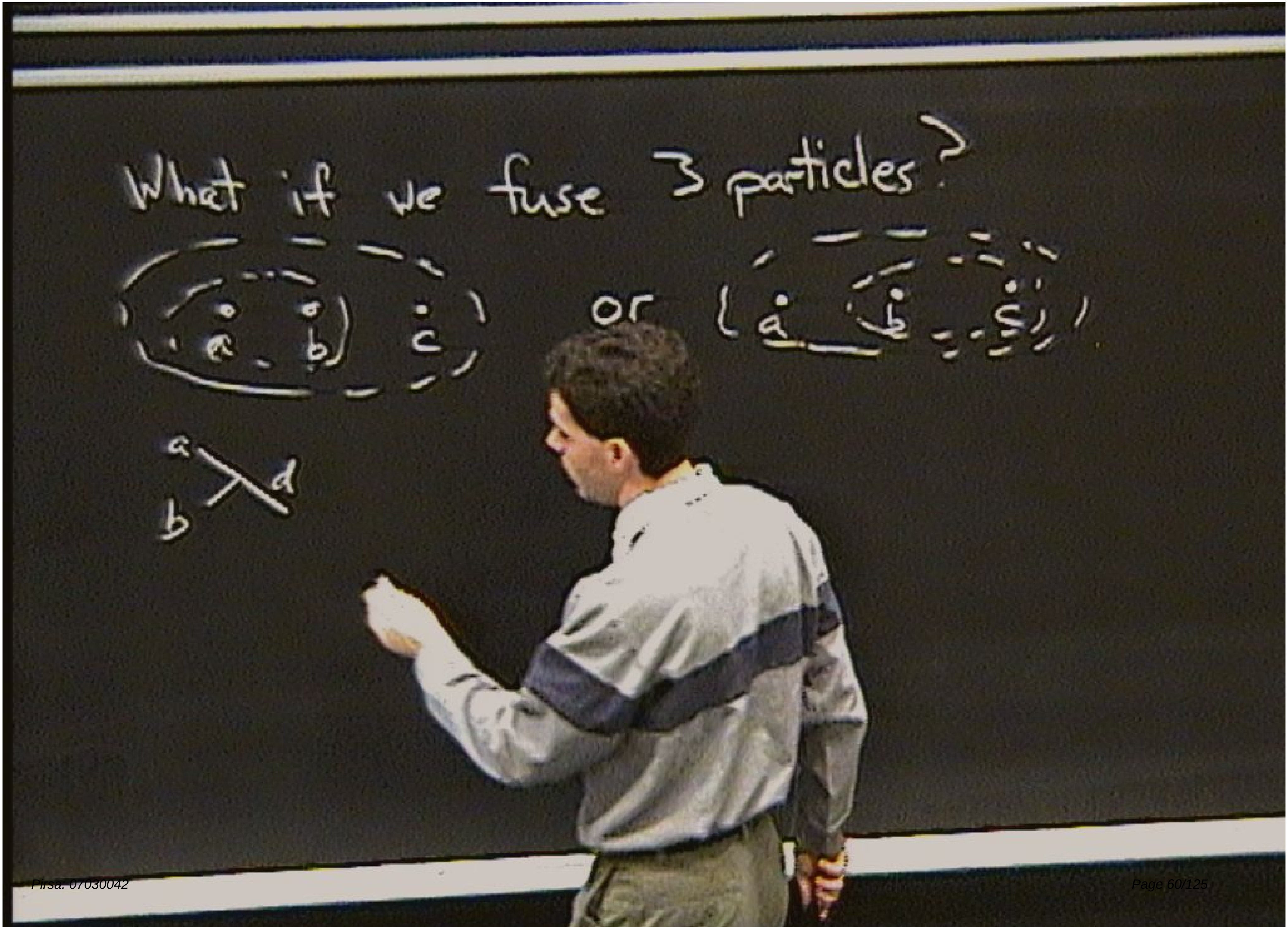
$N_{c}^{ab} = N_{\bar{a}\bar{b}}^{\bar{c}}$

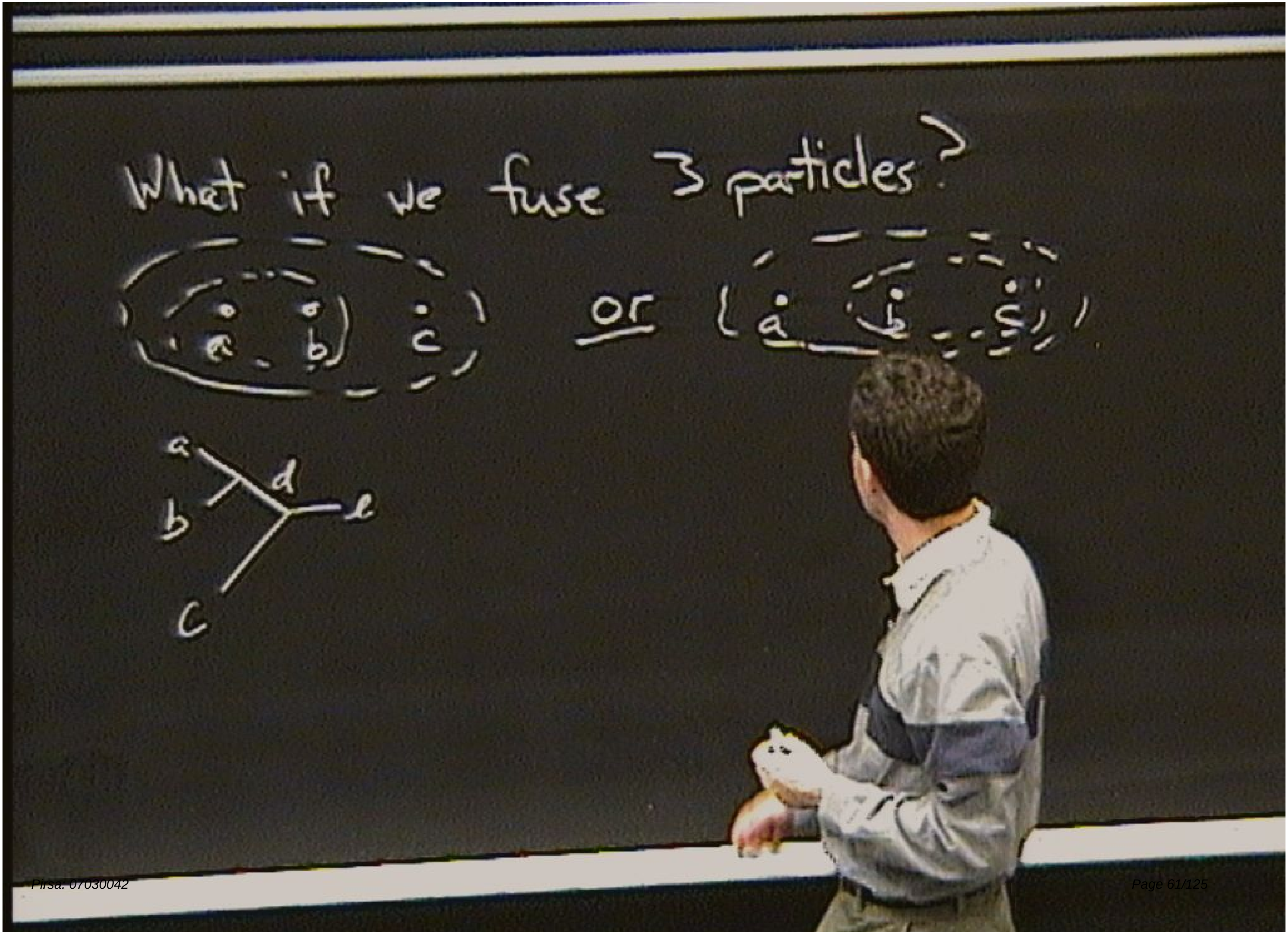

What if we fuse 3 particles?

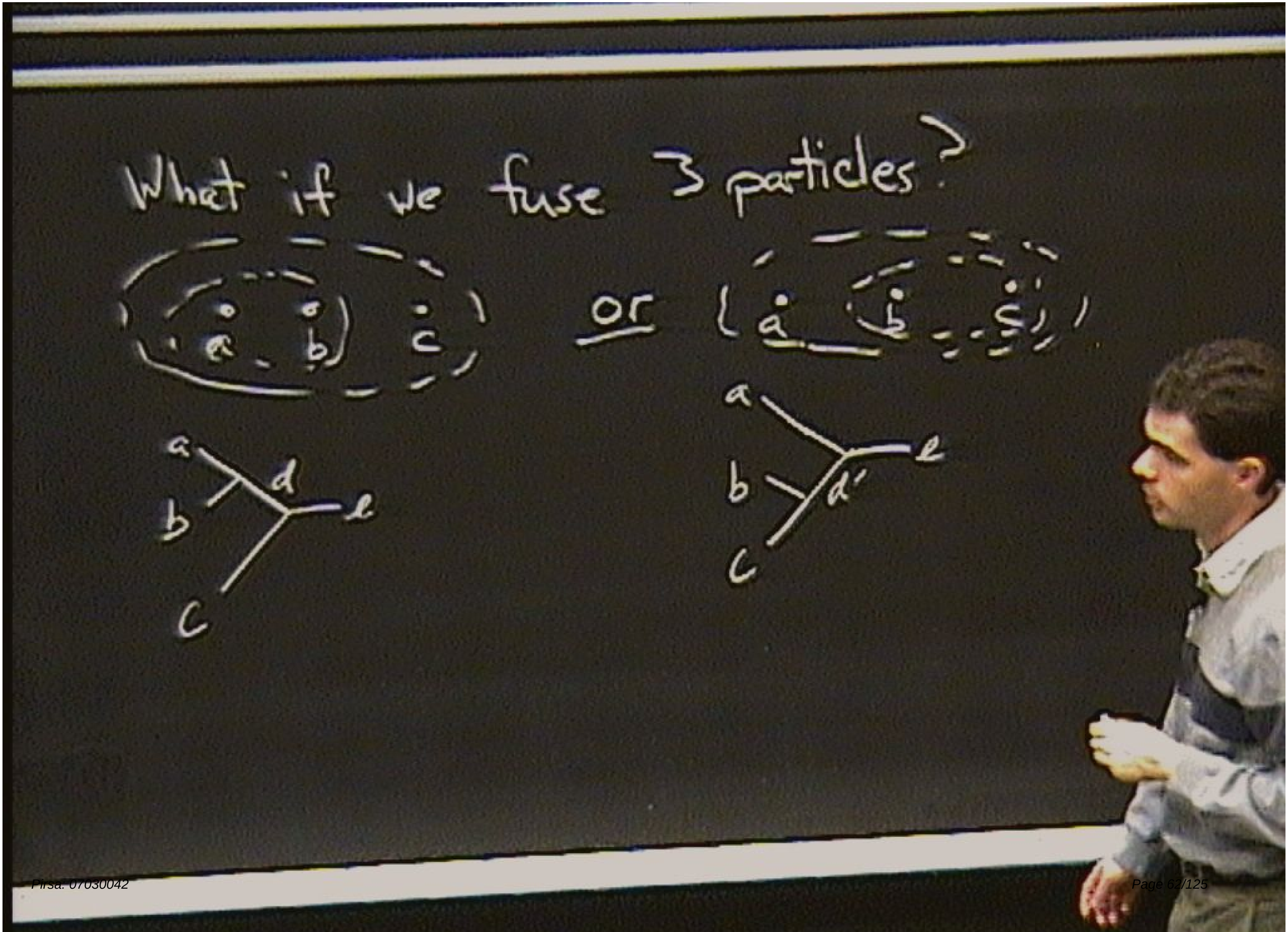

What if we fuse 3 particles?
a b c

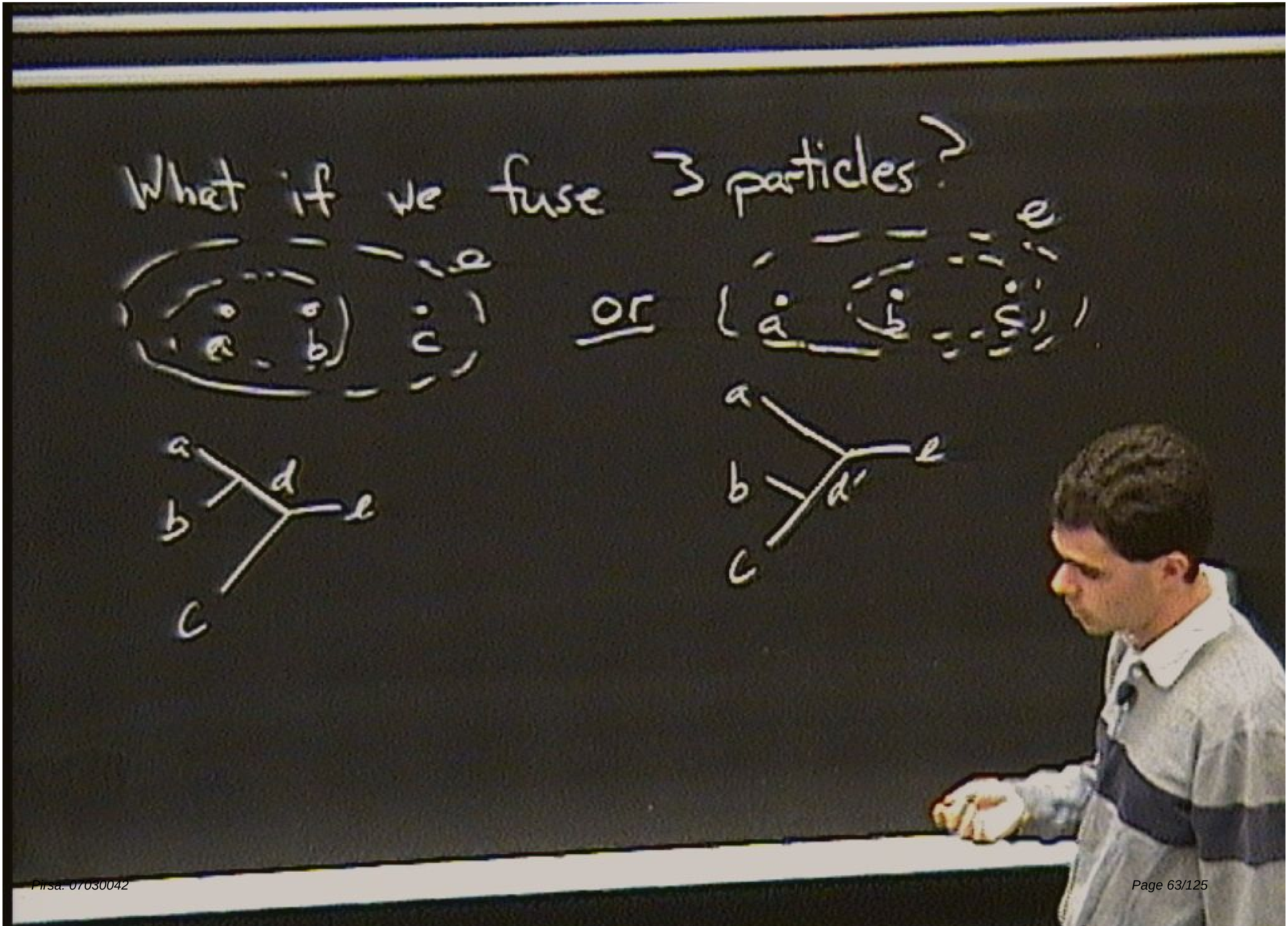

What if we fuse 3 particles?
a b c
or
a b c

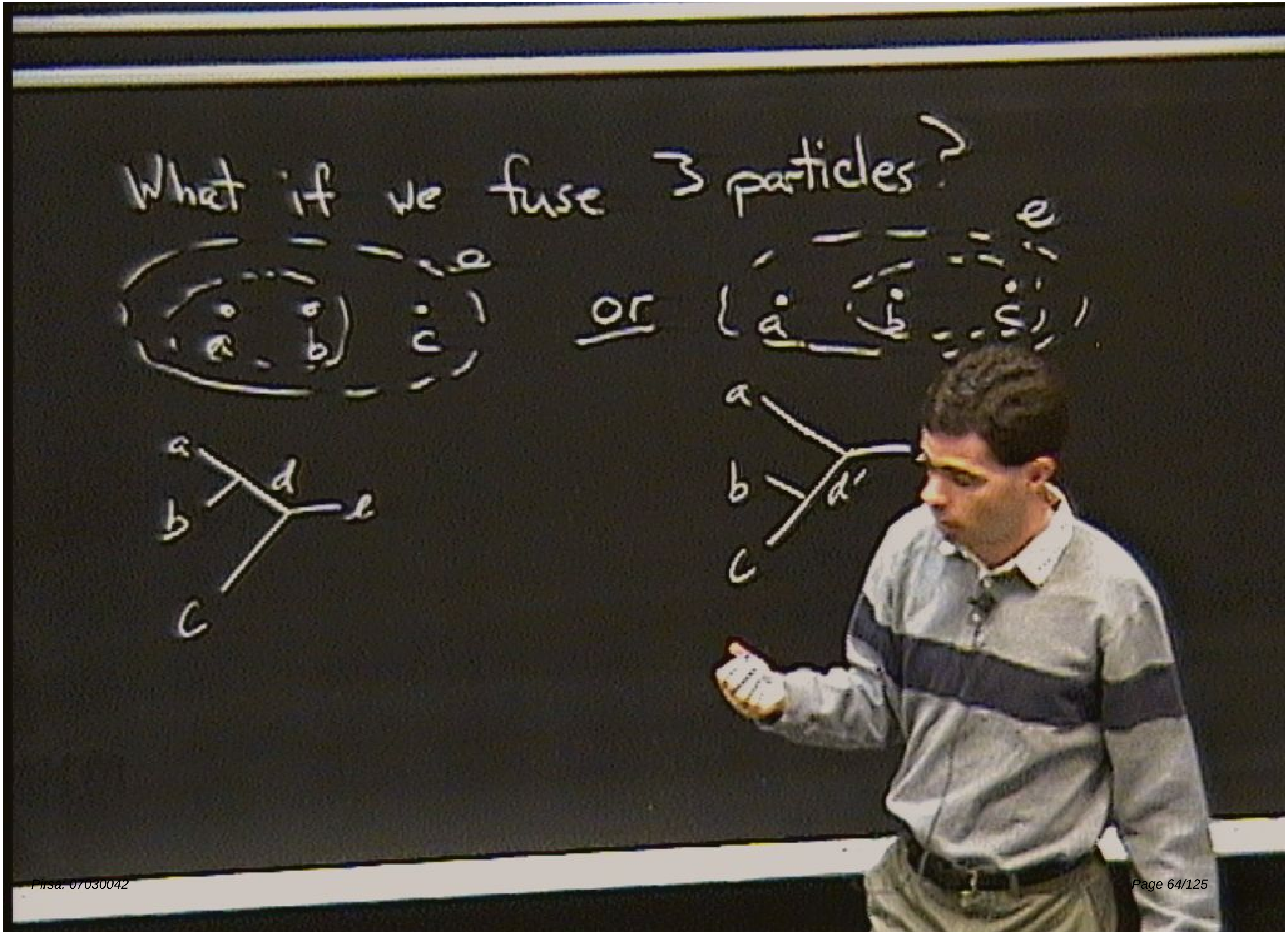

What if we fuse 3 particles?
a b c
or
a b c

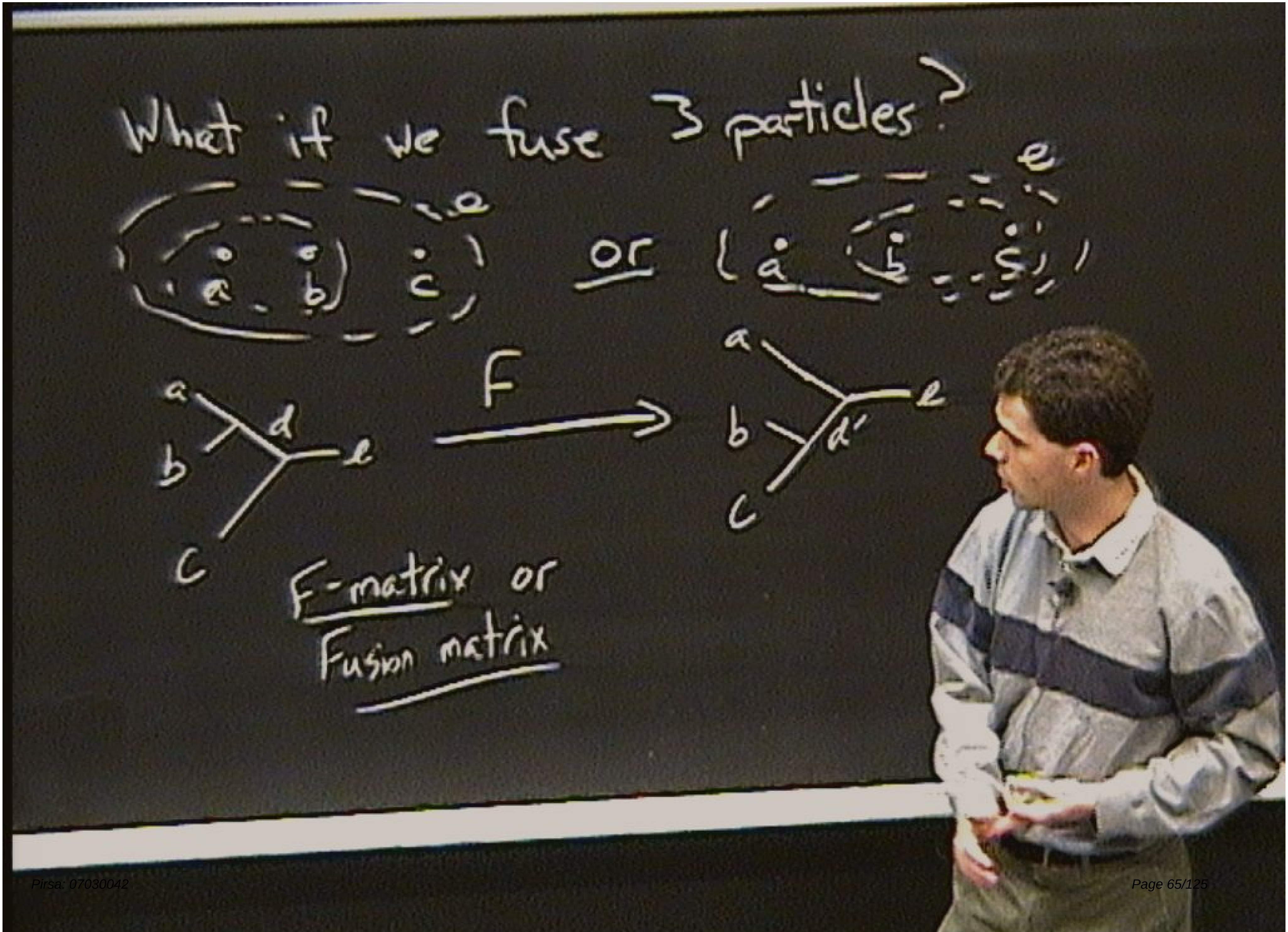

What if we fuse 3 particles?
a b c
or
a b c
a
b
d


What if we fuse 3 particles?
a b c
or
a b c
a
b
c
d
e

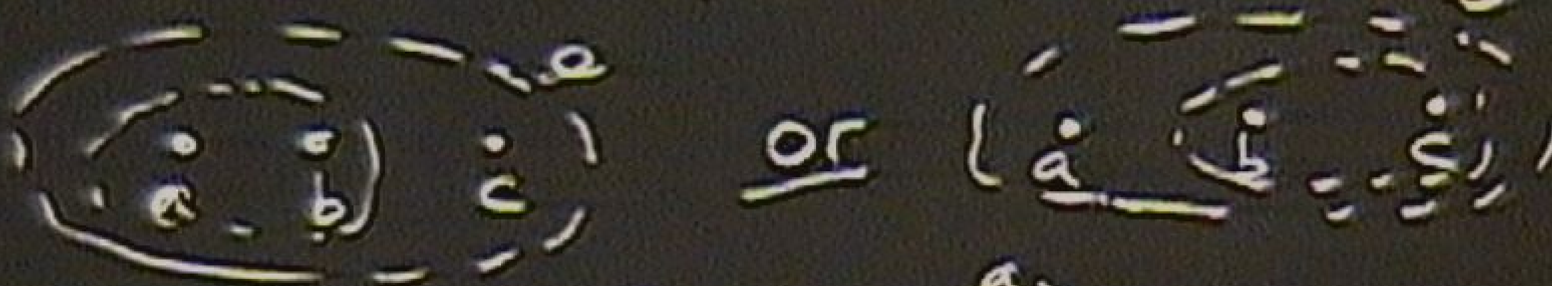

What if we fuse 3 particles?
a b c
or
a b c
a b c d e
a b c d' e

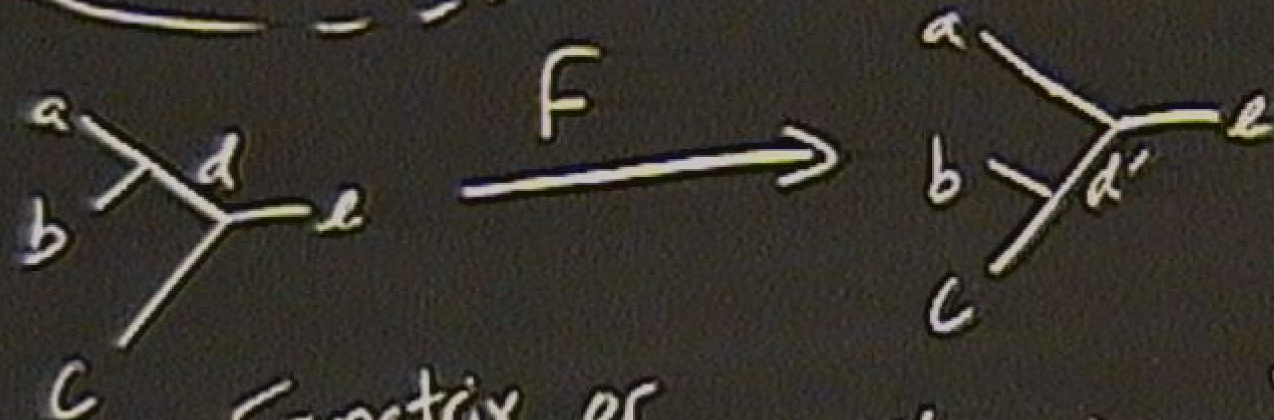

What if we fuse 3 particles?
a
b
c
e
or
a
b
c
e
a
b
d
e
c
a
e
b
d'
c


What if we fuse 3 particles?
a b c e
a b d e c
or
a b c e
a b d' c

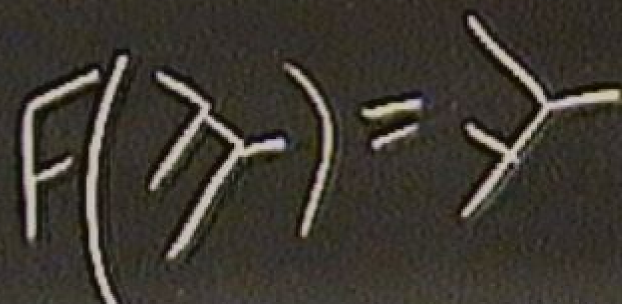

What if we fuse 3 particles?
e
a b c
or
e
a b c
a
b
d
e
c
F
a
e
b
d'
c
F-matrix or
Fusion matrix

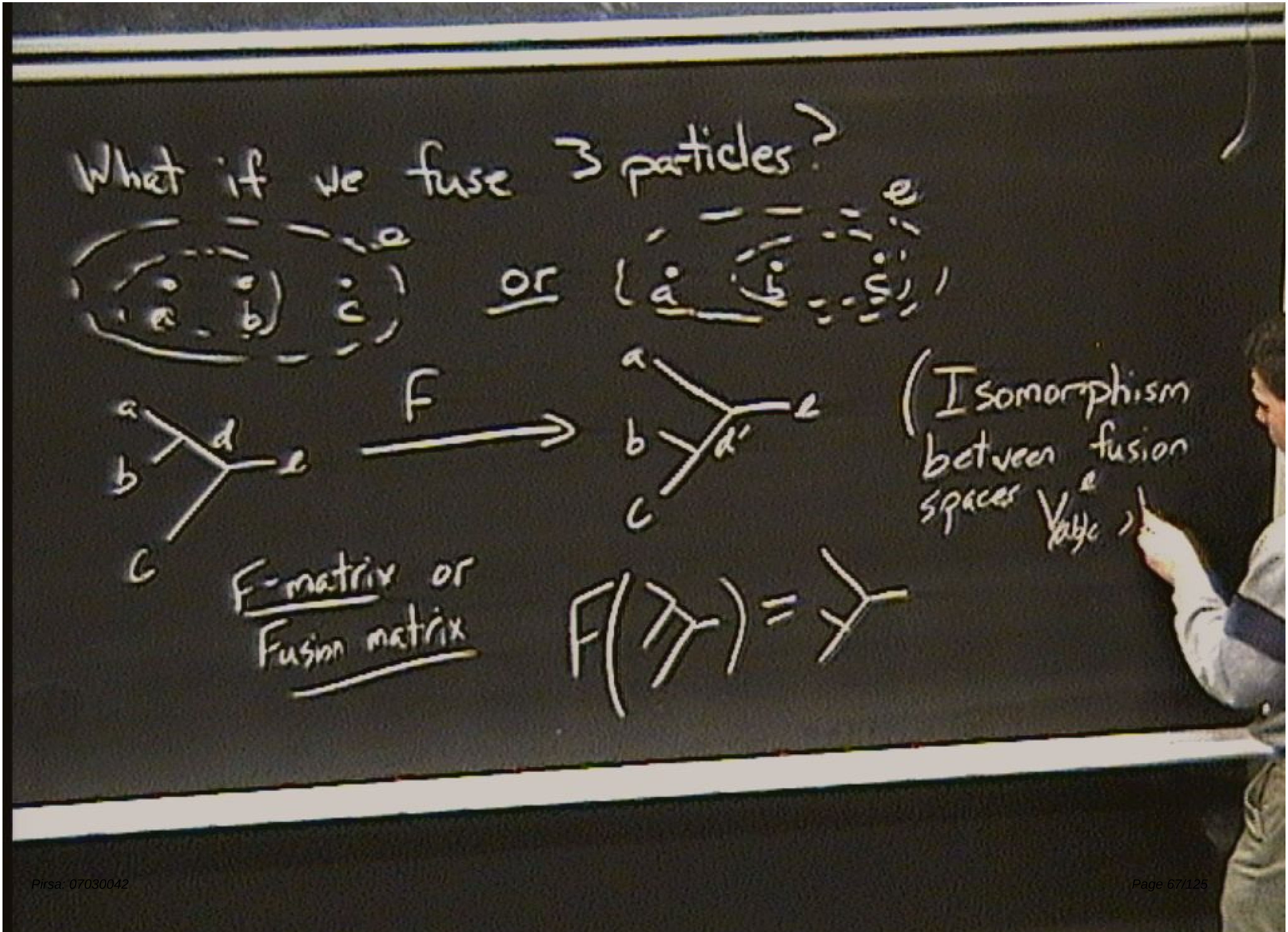
What if we fuse 3 particles?





F-matrix or Fusion matrix

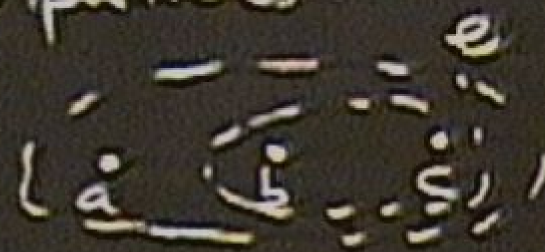


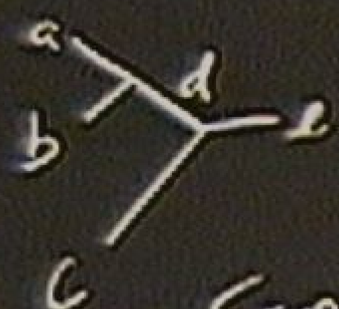


What if we fuse 3 particles?
or
F
F-matrix or
Fusion matrix
(Isomorphism
between fusion
spaces V_{abc}^{e})

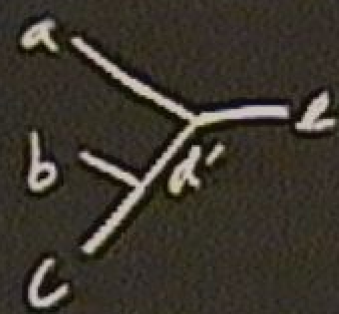
What if we fuse 3 particles?

or



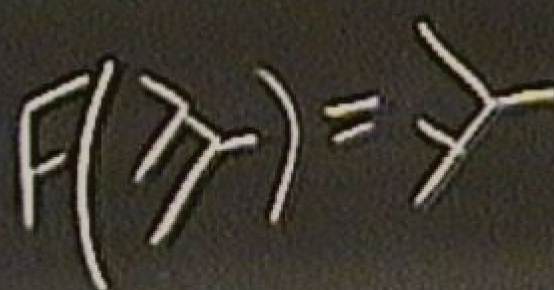


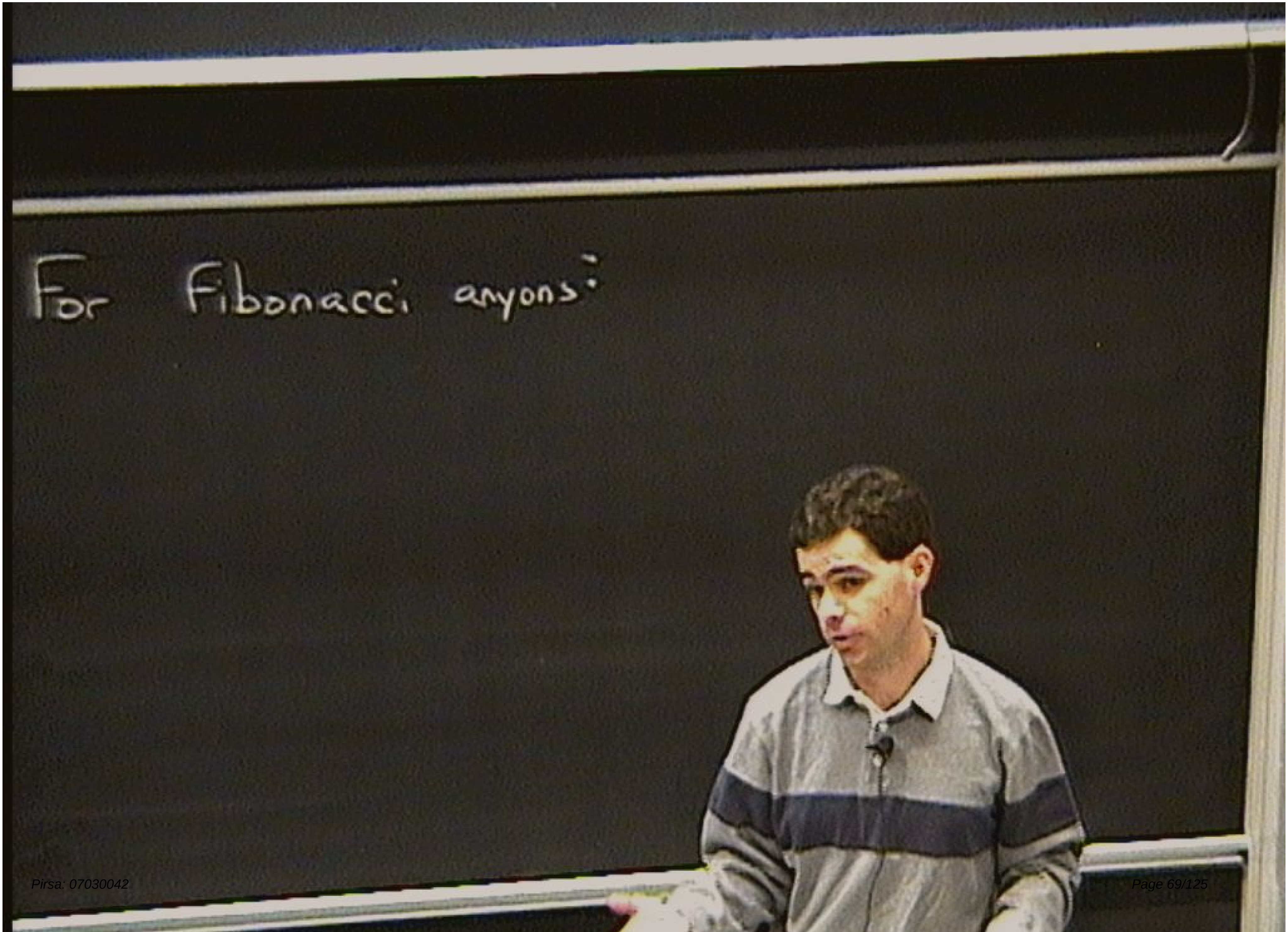
$\xrightarrow{F}$

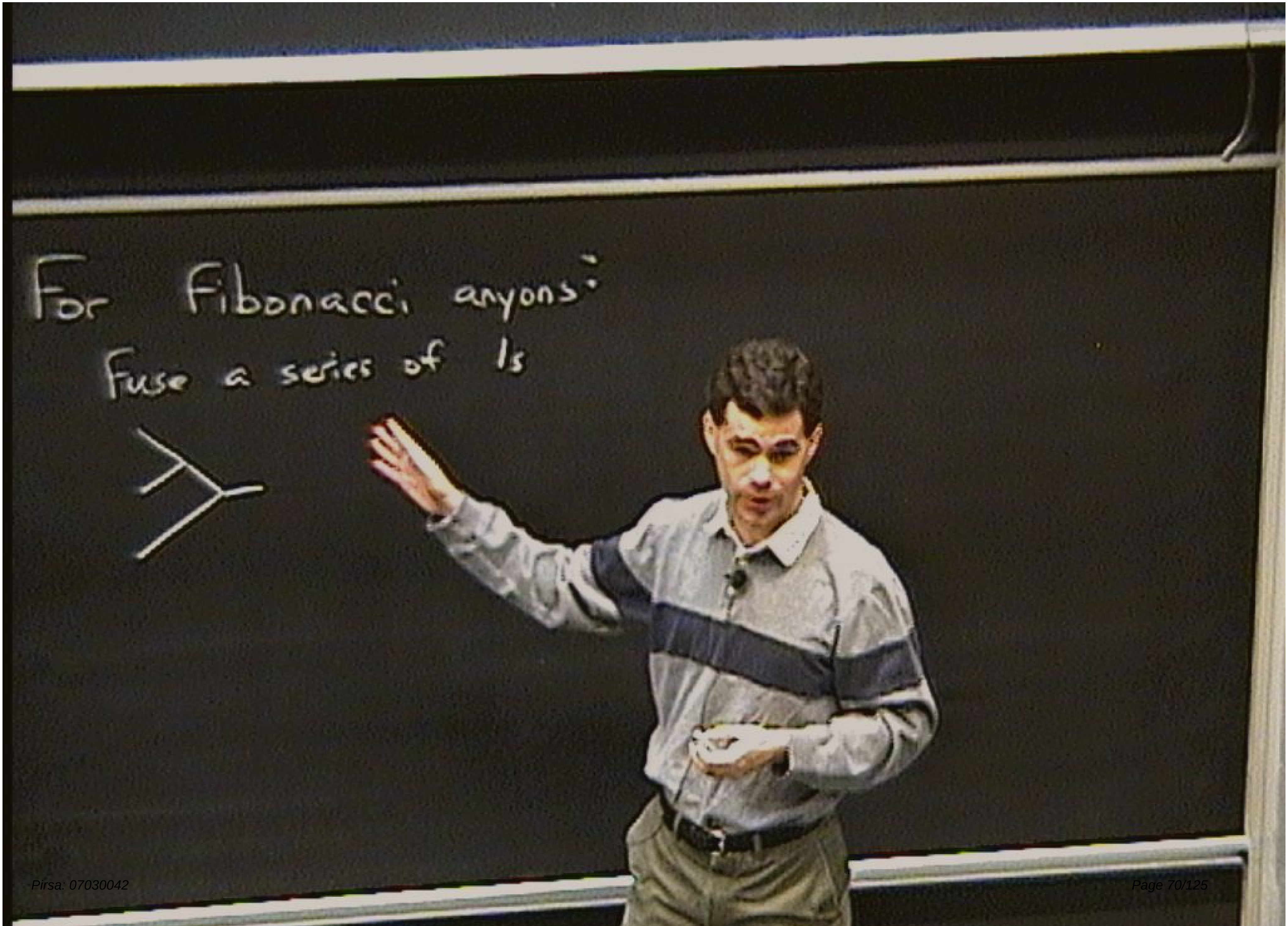


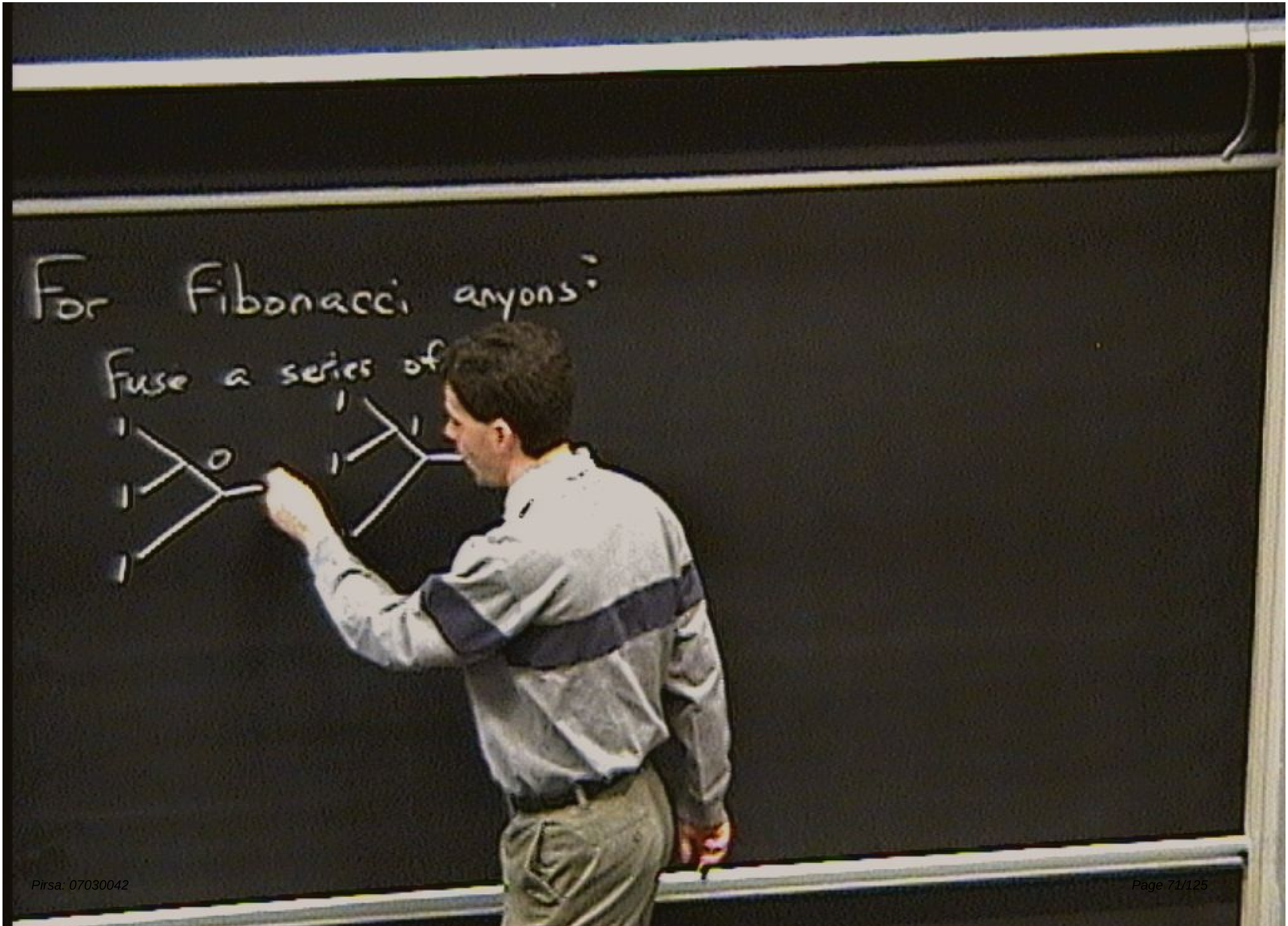
(Isomorphism between fusion spaces $V^e_{(ab)c}$, $V^e_{a(bc)}$

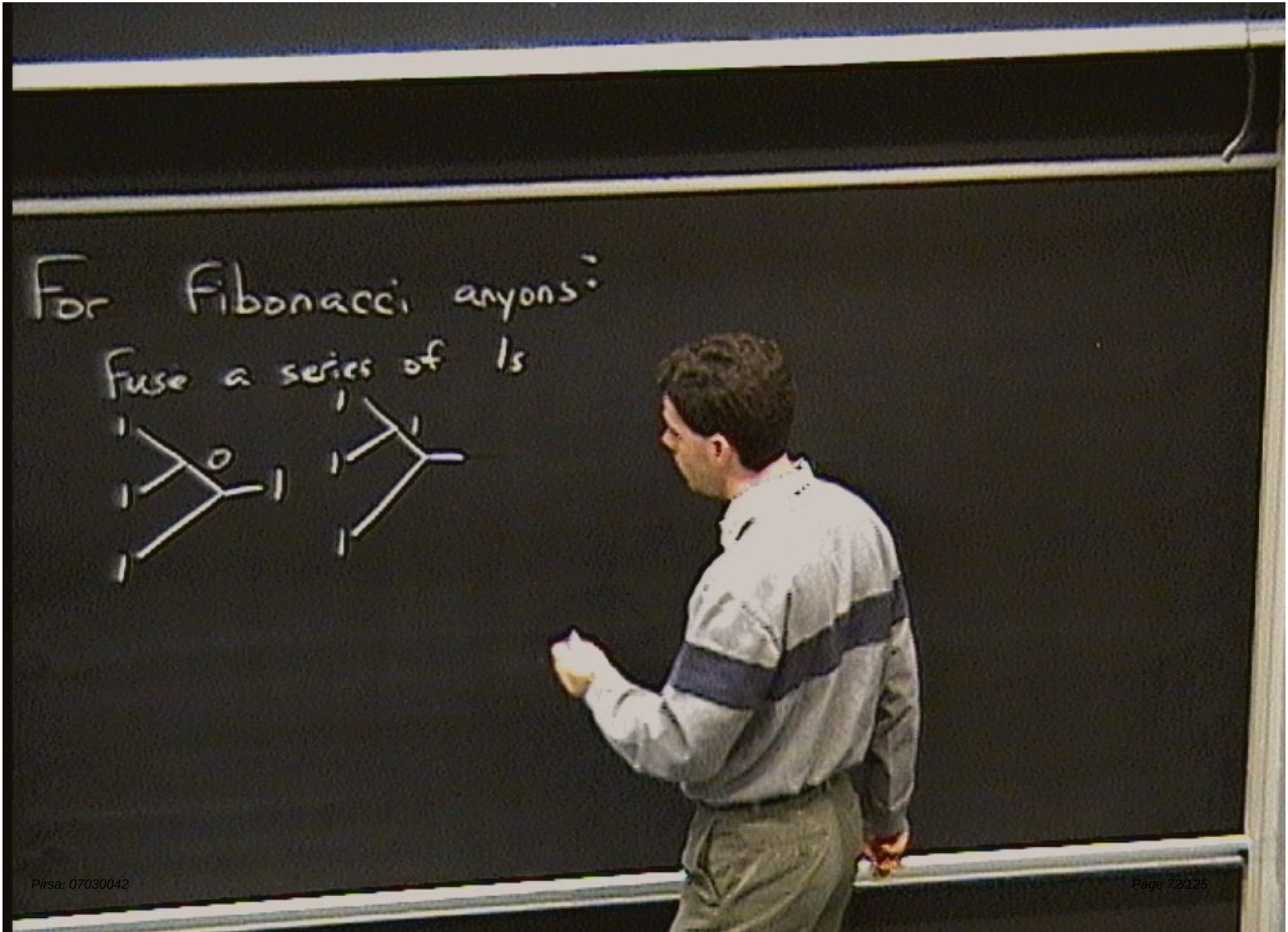
F-matrix or Fusion matrix

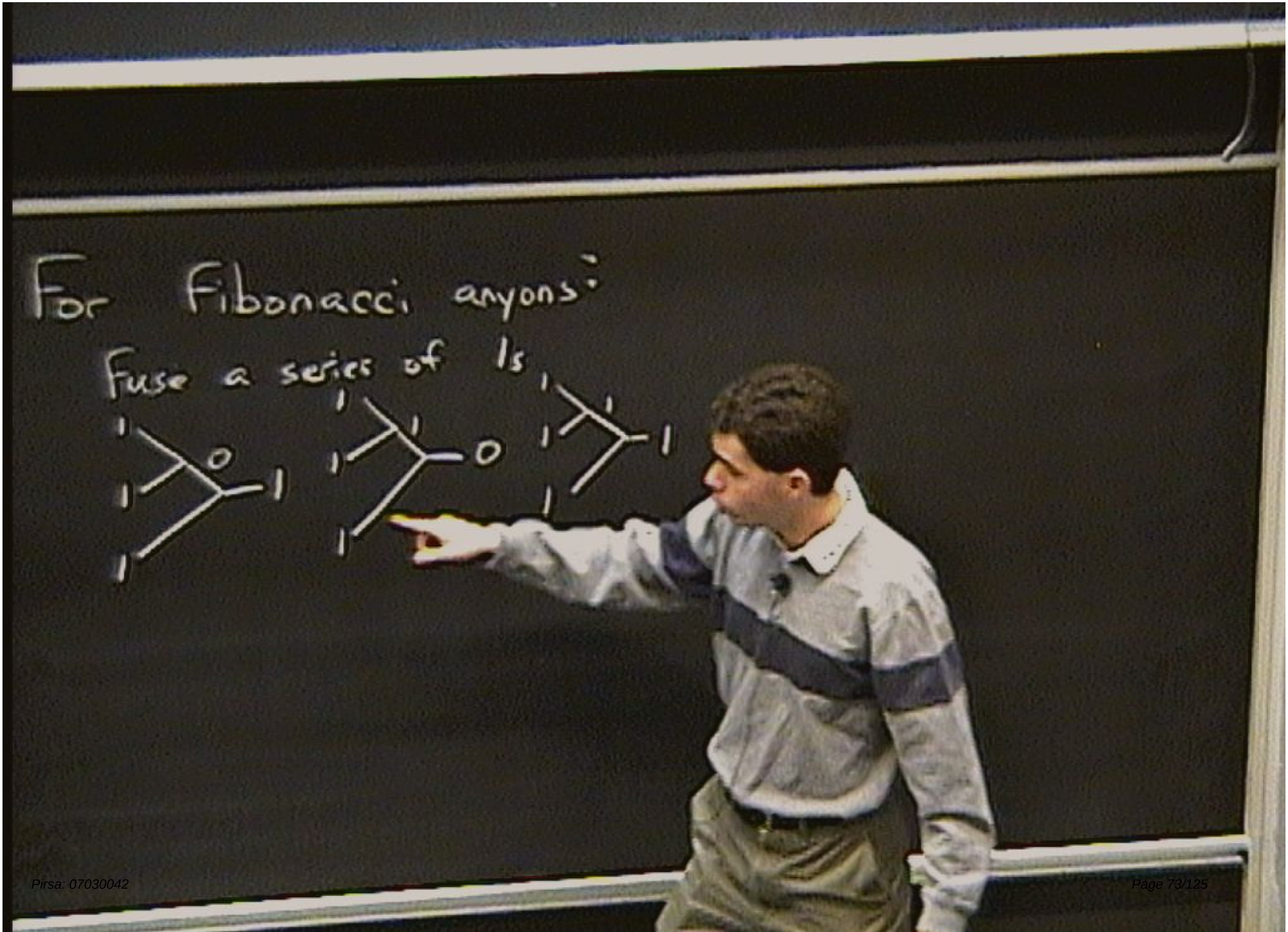



For Fibonacci anyons:


For Fibonacci anyons:
Fuse a series of 1s

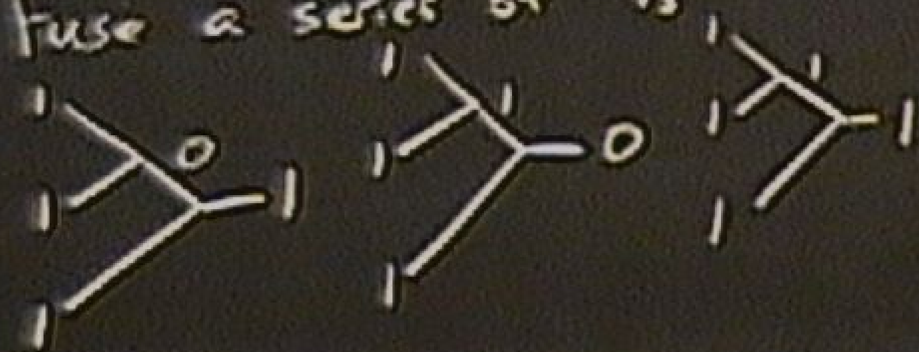

For Fibonacci anyons:
Fuse a series of


For Fibonacci anyons:
Fuse a series of 1s
1
1
1
0
1
1
1
1
1

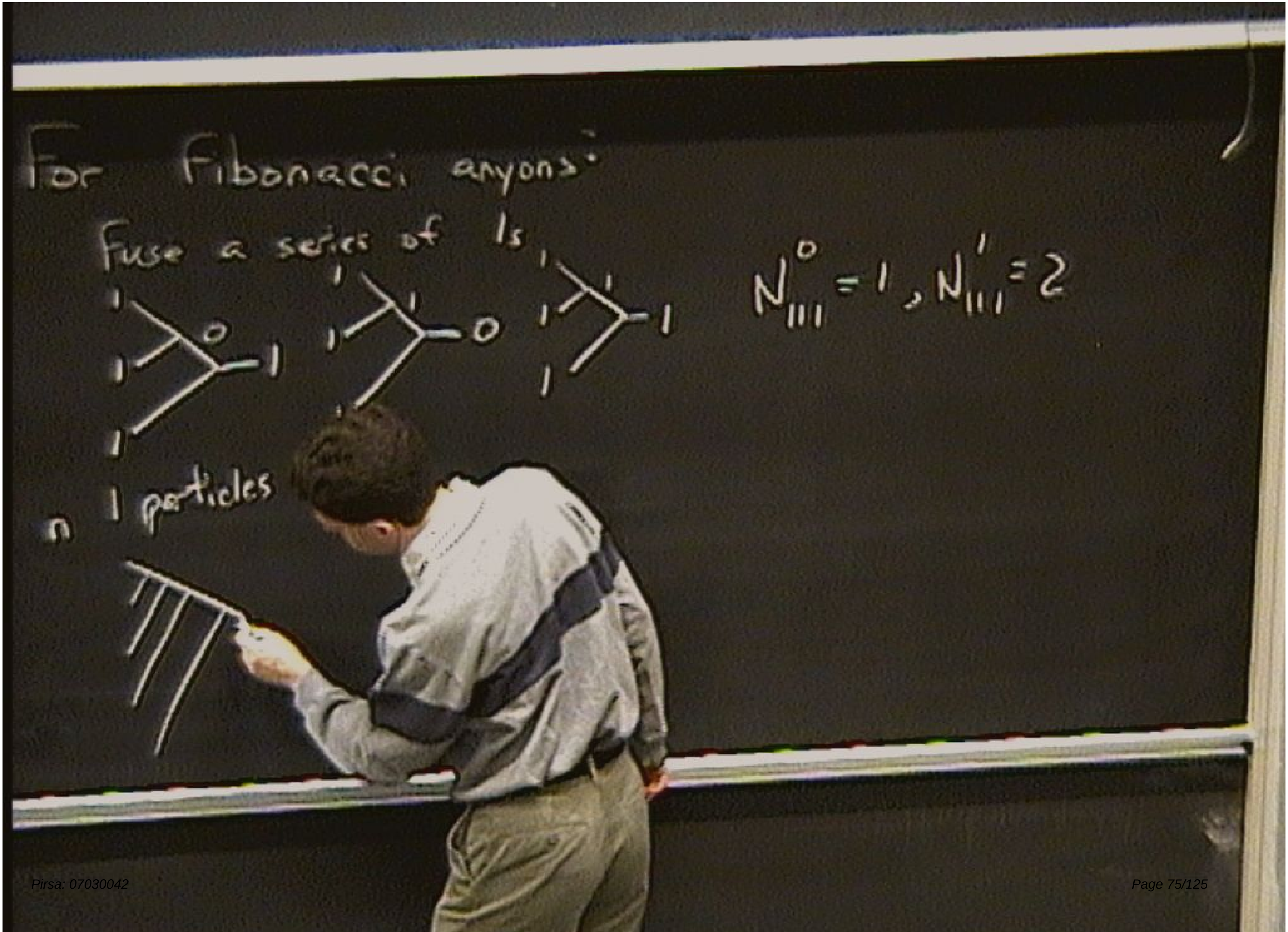

For Fibonacci anyons:
Fuse a series of 1s

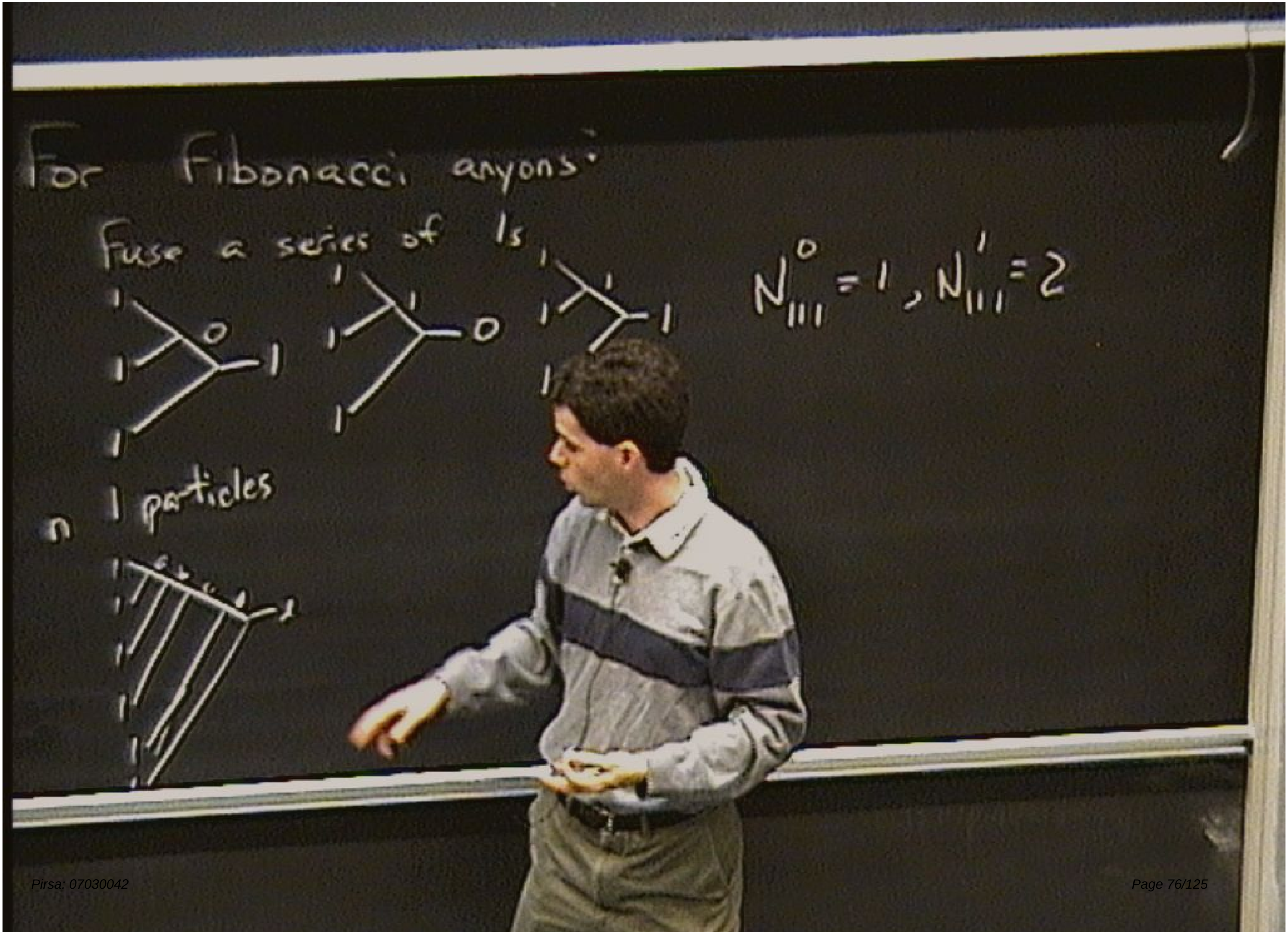
For Fibonacci anyons:

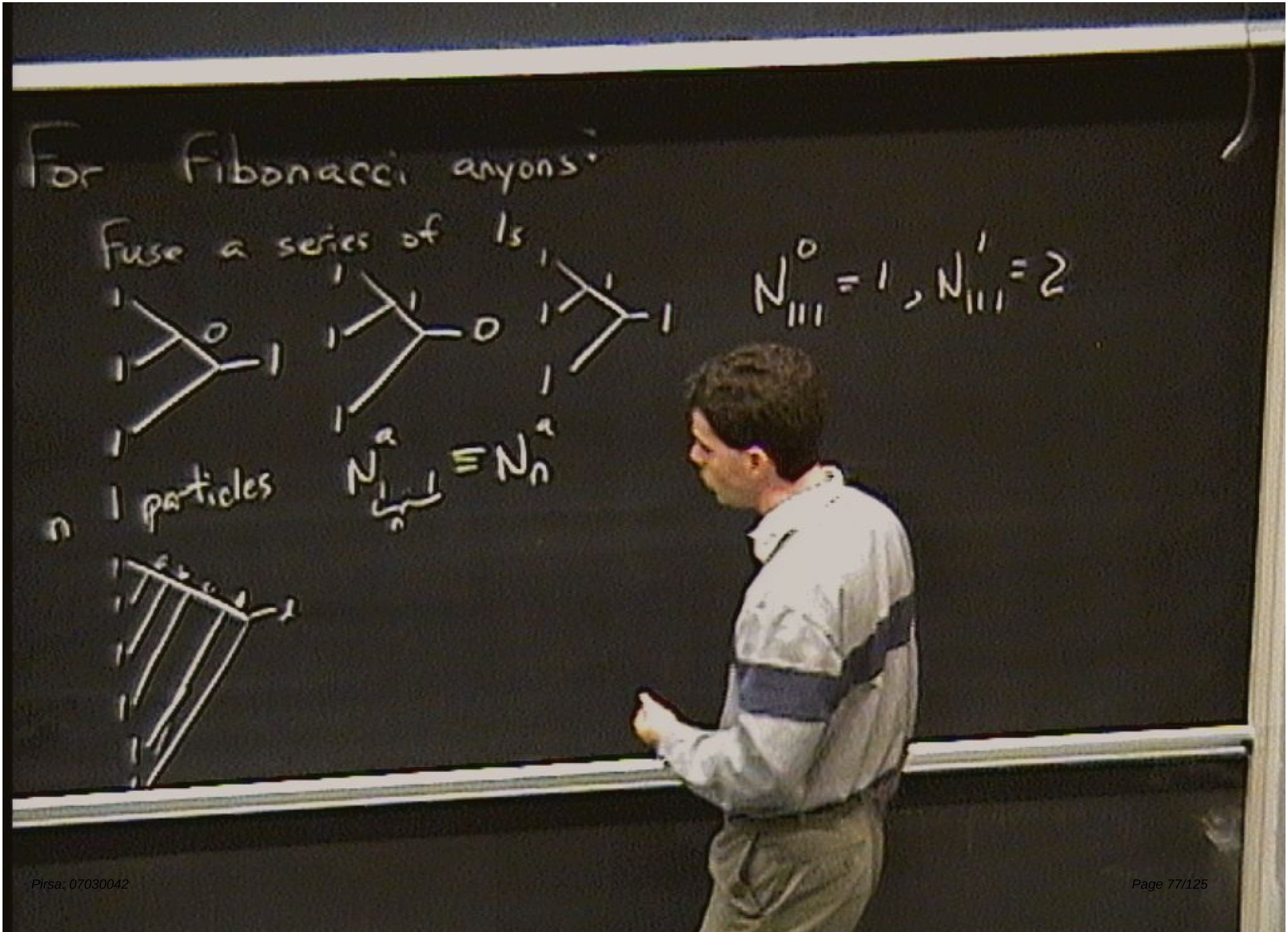
Fuse a series of 1s

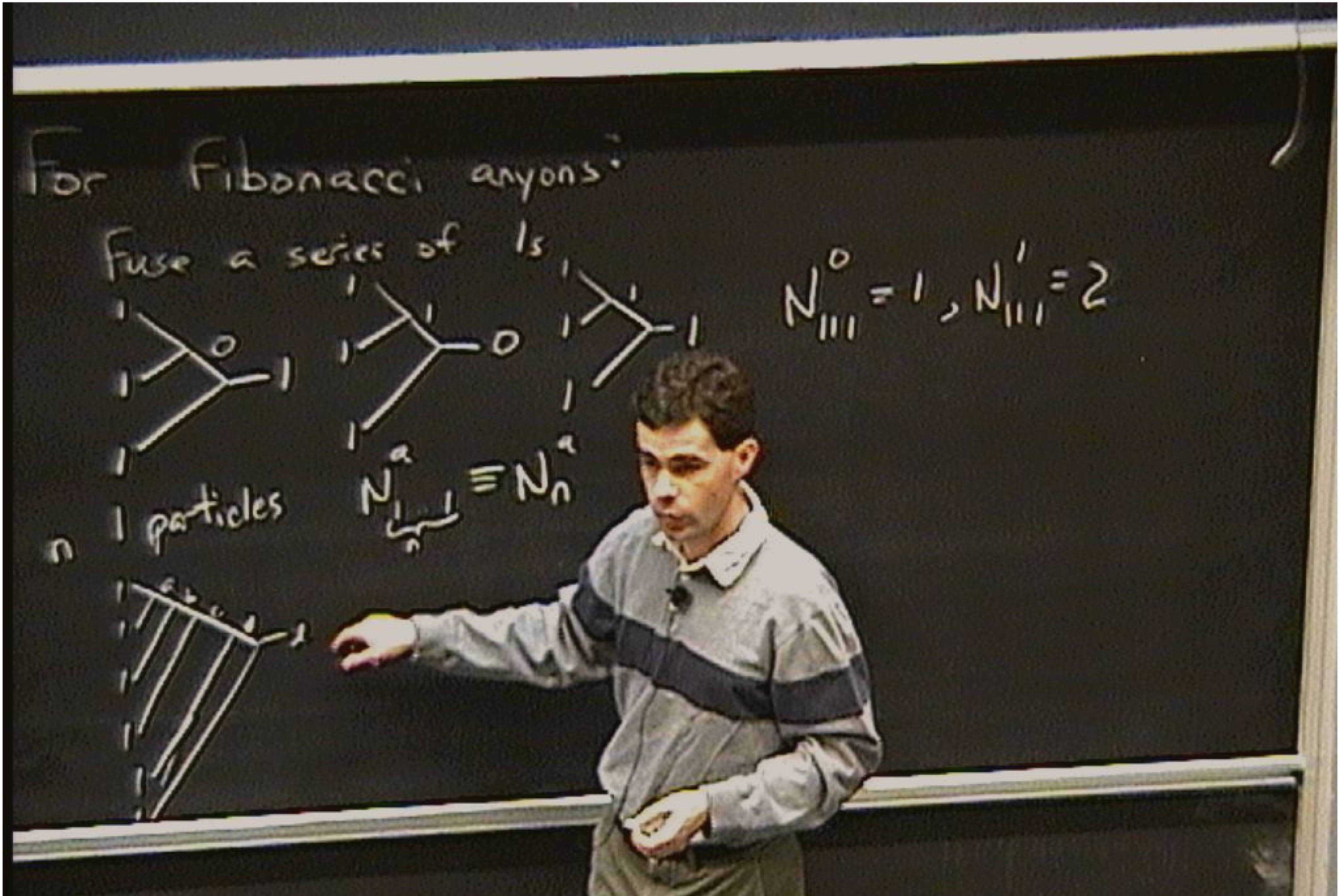


$N_{111}^{0} = 1, \; N_{111}^{1} = 2$


For Fibonacci anyons:
Fuse a series of 1s
$N_{111}^{0}=1$, $N_{111}^{1}=2$
n 1 particles


For Fibonacci anyons:
Fuse a series of 1s
N^0_111 = 1 , N^1_111 = 2
n 1 particles


For Fibonacci anyons:
Fuse a series of 1s
$N_{111}^{0} = 1, N_{111}^{1} = 2$
n 1 particles
$N^{a}_{\underbrace{1\ldots1}_{n}} \equiv N_n^{a}$


For Fibonacci anyons:
Fuse a series of 1s
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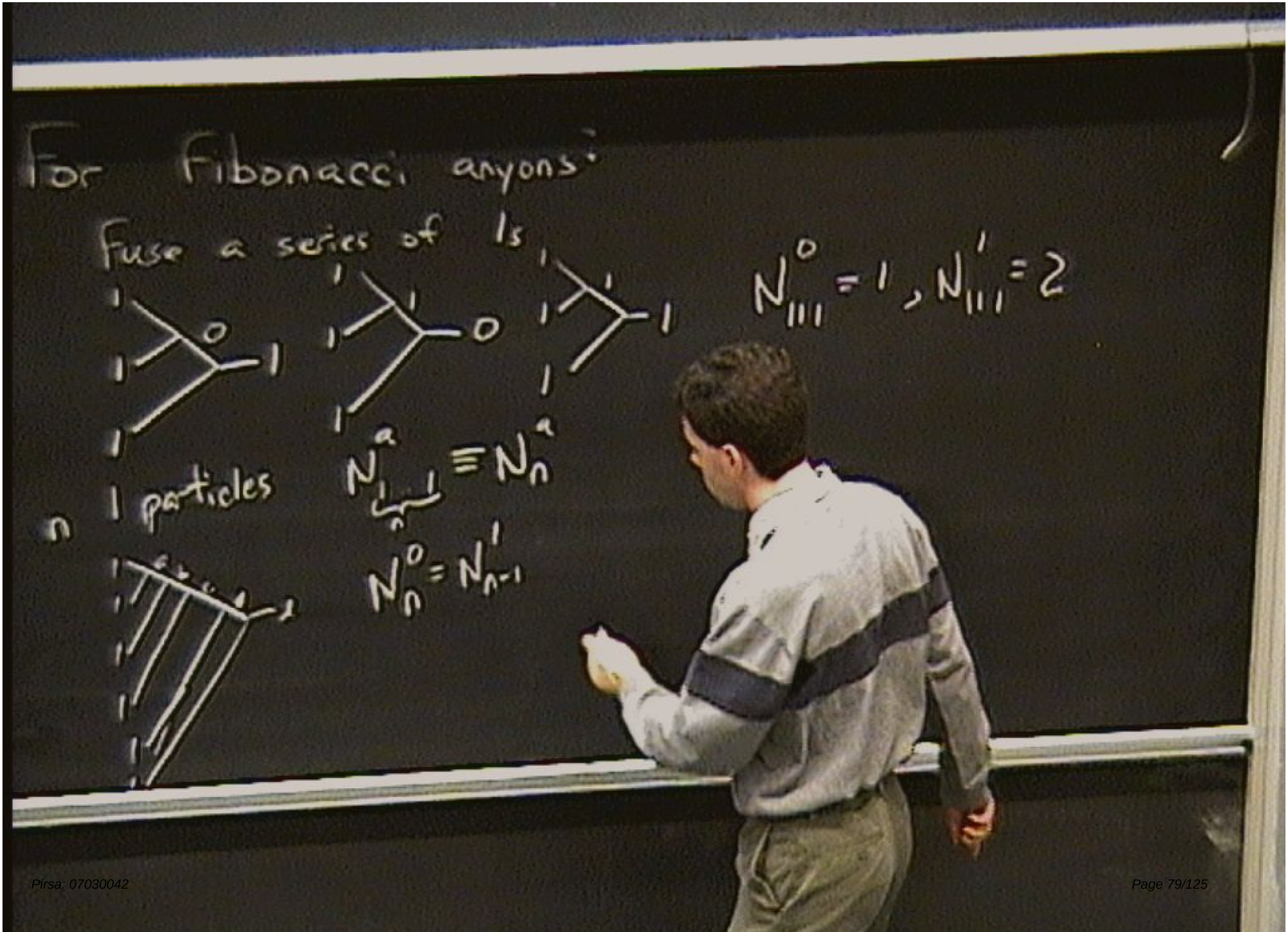
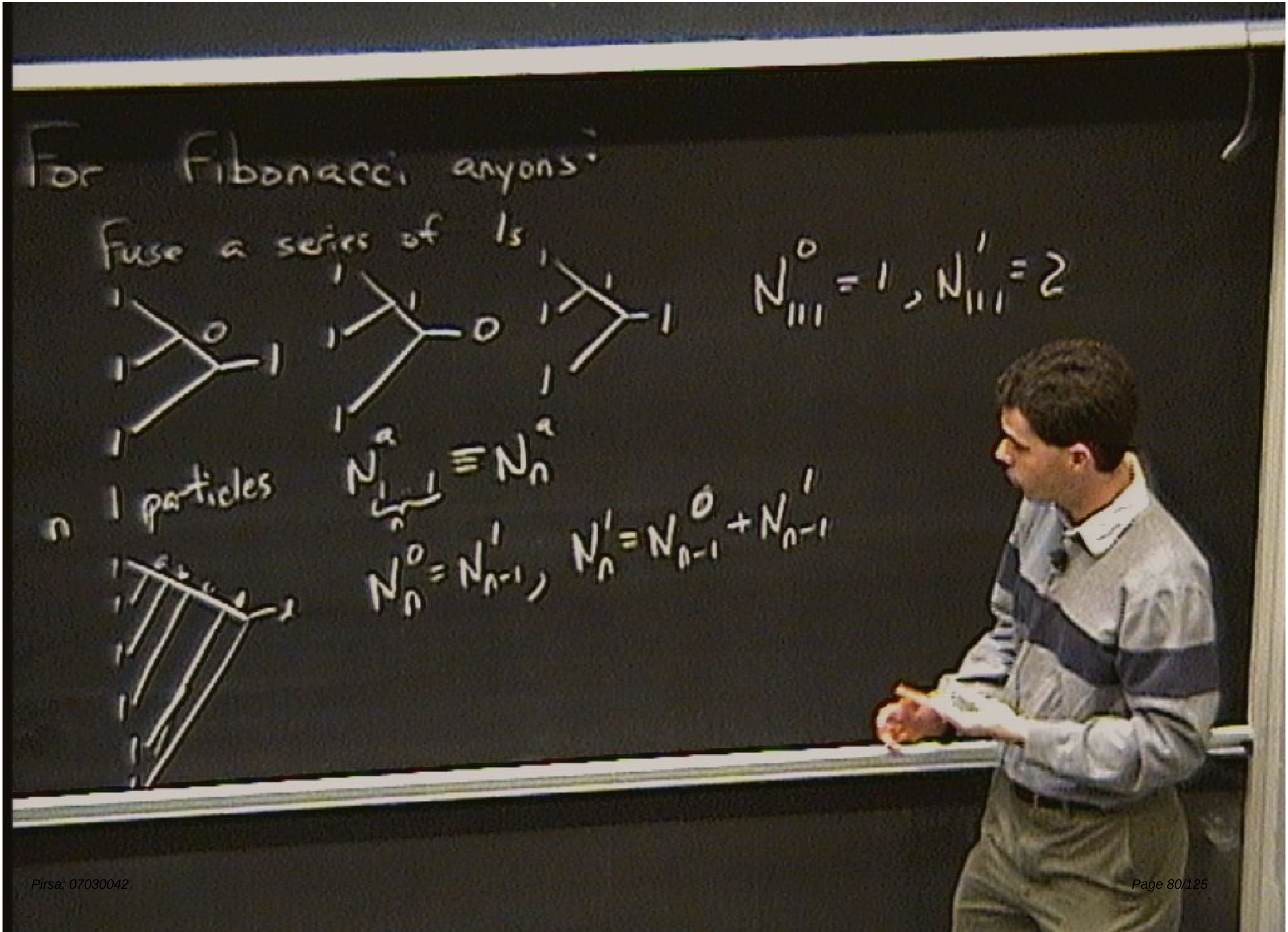
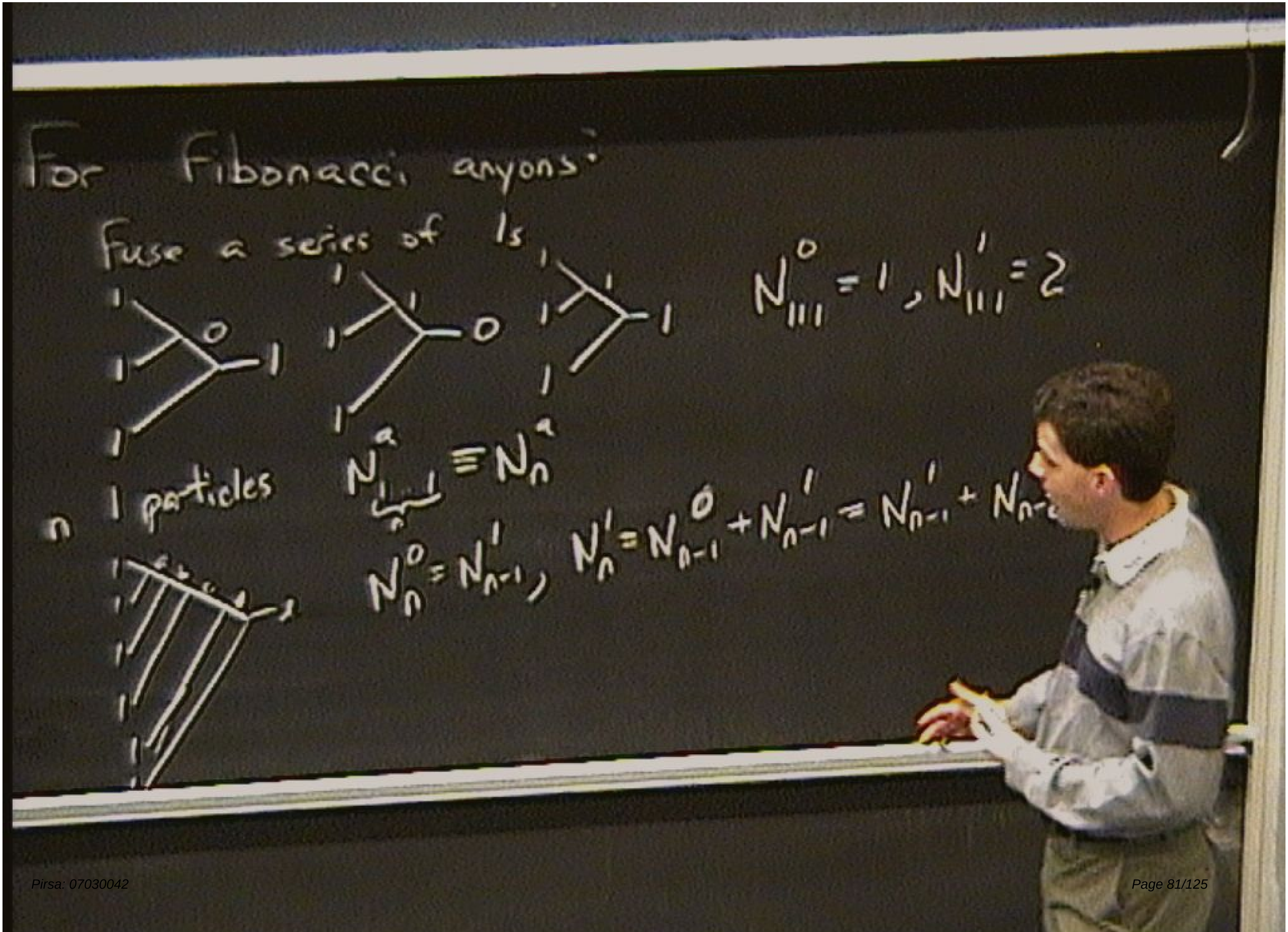
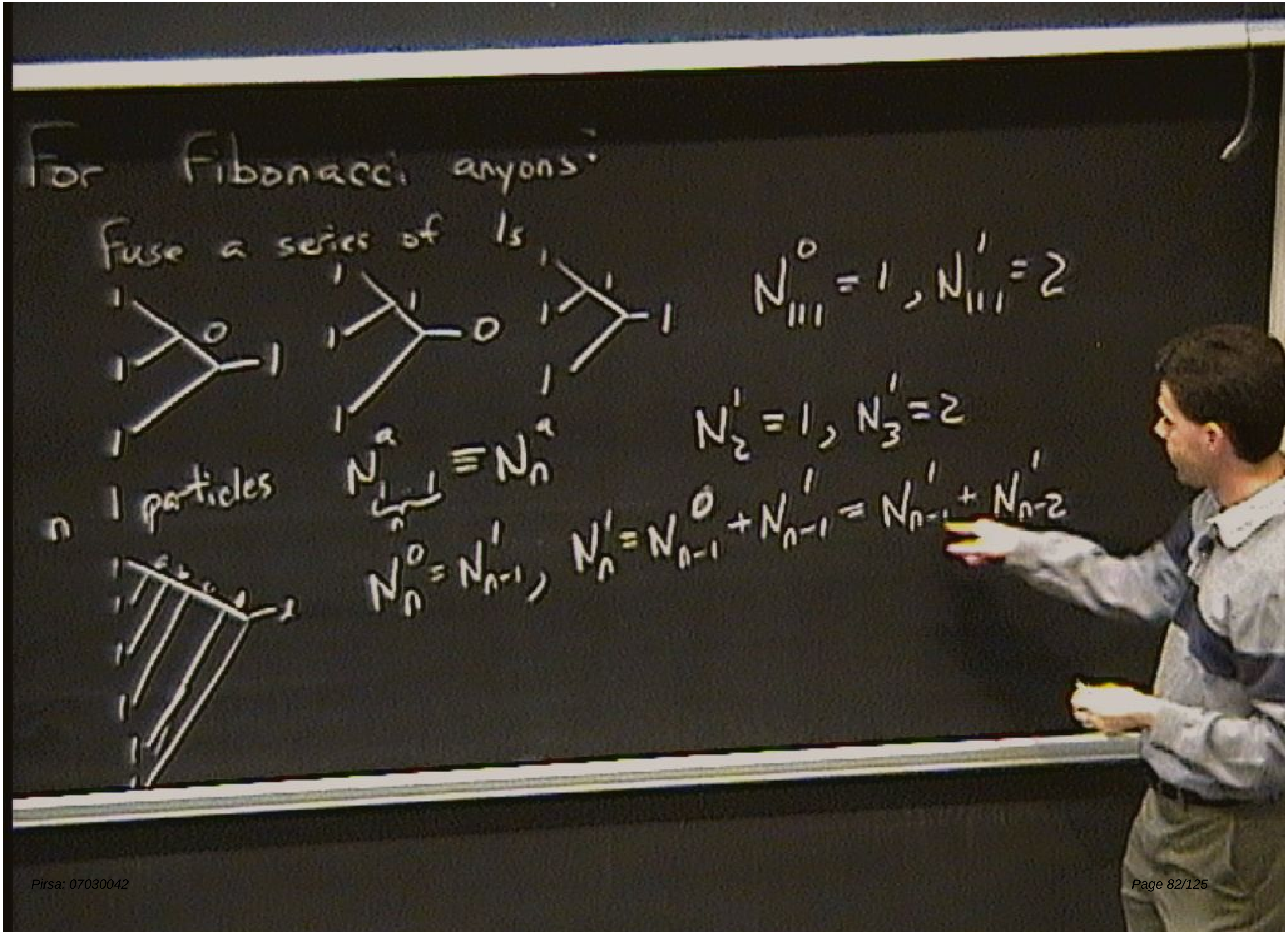
$N^{a}_{\underbrace{1\ldots1}_{n}} \equiv N_{n}^{a}$


For Fibonacci anyons:
Fuse a series of 1s
N^0_111 = 1 , N^1_111 = 2
n 1 particles
N^a_1...1 ≡ N^a_n
N^0_n = N^1_n-1


For Fibonacci anyons:
Fuse a series of 1s
$N_{111}^{0} = 1$, $N_{111}^{1} = 2$
n 1 particles
$N^{a}_{\underbrace{1\cdots1}_{n}} \equiv N_n^a$
$N_n^0 = N_{n-1}^1$, $N_n^1 = N_{n-1}^0 + N_{n-1}^1$

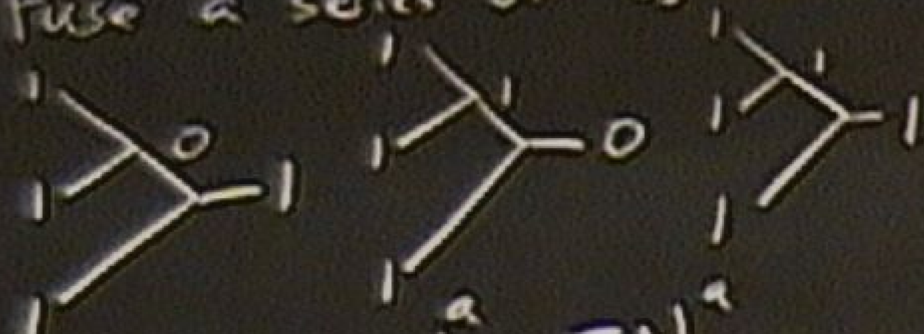

For Fibonacci anyons:
Fuse a series of 1s
$N^0_{111} = 1$, $N^1_{111} = 2$
n 1 particles
$N^a_{\underbrace{1\ldots1}_{n}} \equiv N^a_n$
$N^0_n = N^1_{n-1}$, $N^1_n = N^0_{n-1} + N^1_{n-1} = N^1_{n-1} + N_{n-2}$



For Fibonacci anyons:
Fuse a series of 1s
$N_{111}^{0} = 1$, $N_{111}^{1} = 2$
$N_2^1 = 1$, $N_3^1 = 2$
n 1 particles $N^a_{\underbrace{1 \ldots 1}_{n}} \equiv N_n^a$
$N_n^1 = N_{n-1}^0 + N_{n-1}^1 = N_{n-1}^1 + N_{n-2}^1$
$N_n^0 = N_{n-1}^1$,

For Fibonacci anyons:

Fuse a series of 1s



$N^0_{111}=1,\ N^1_{111}=2$

$N^1_2=1,\ N^1_3=2$

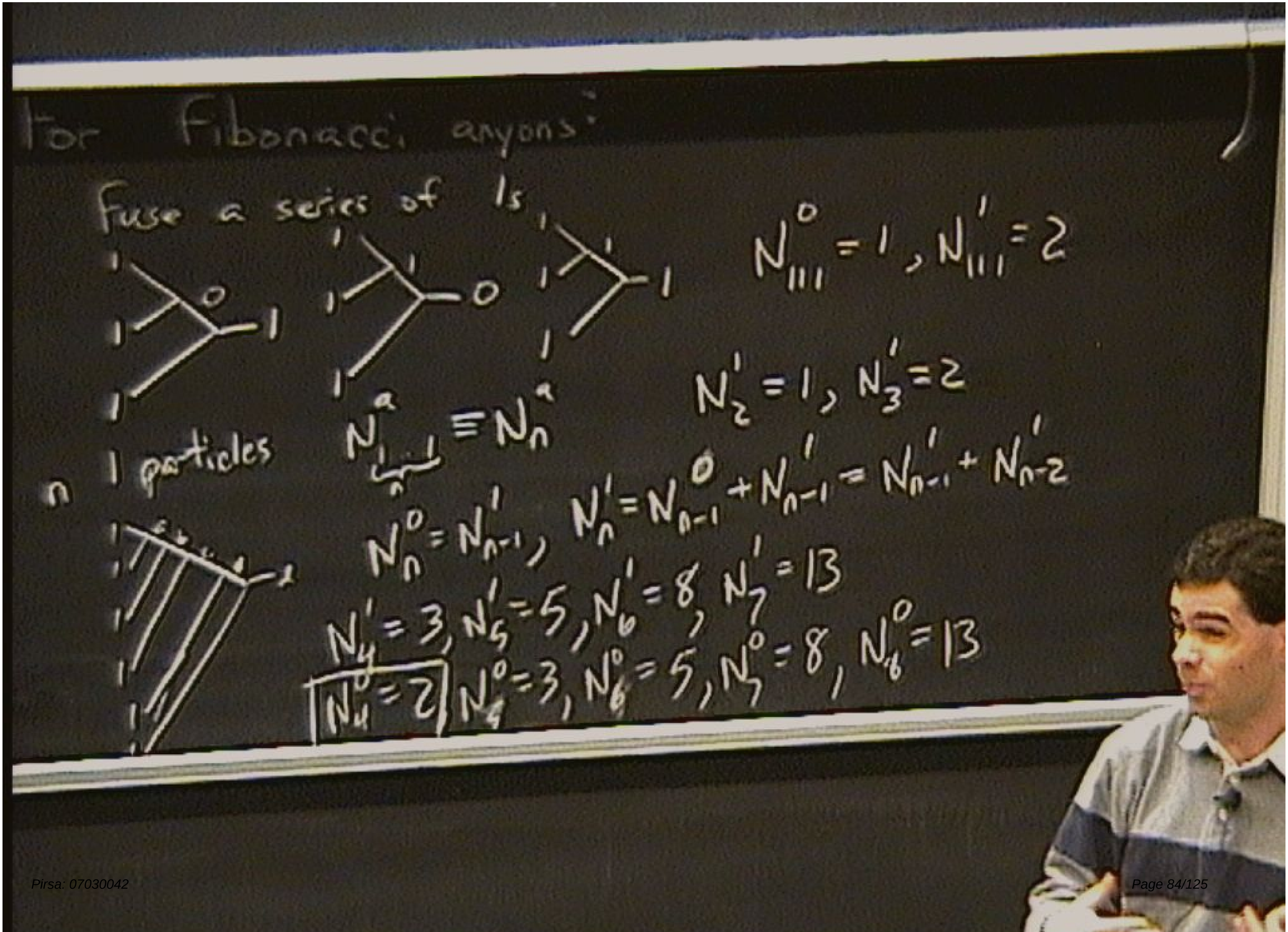
$N^a_{\underbrace{1\ldots1}_n} \equiv N^a_n$

n 1 particles

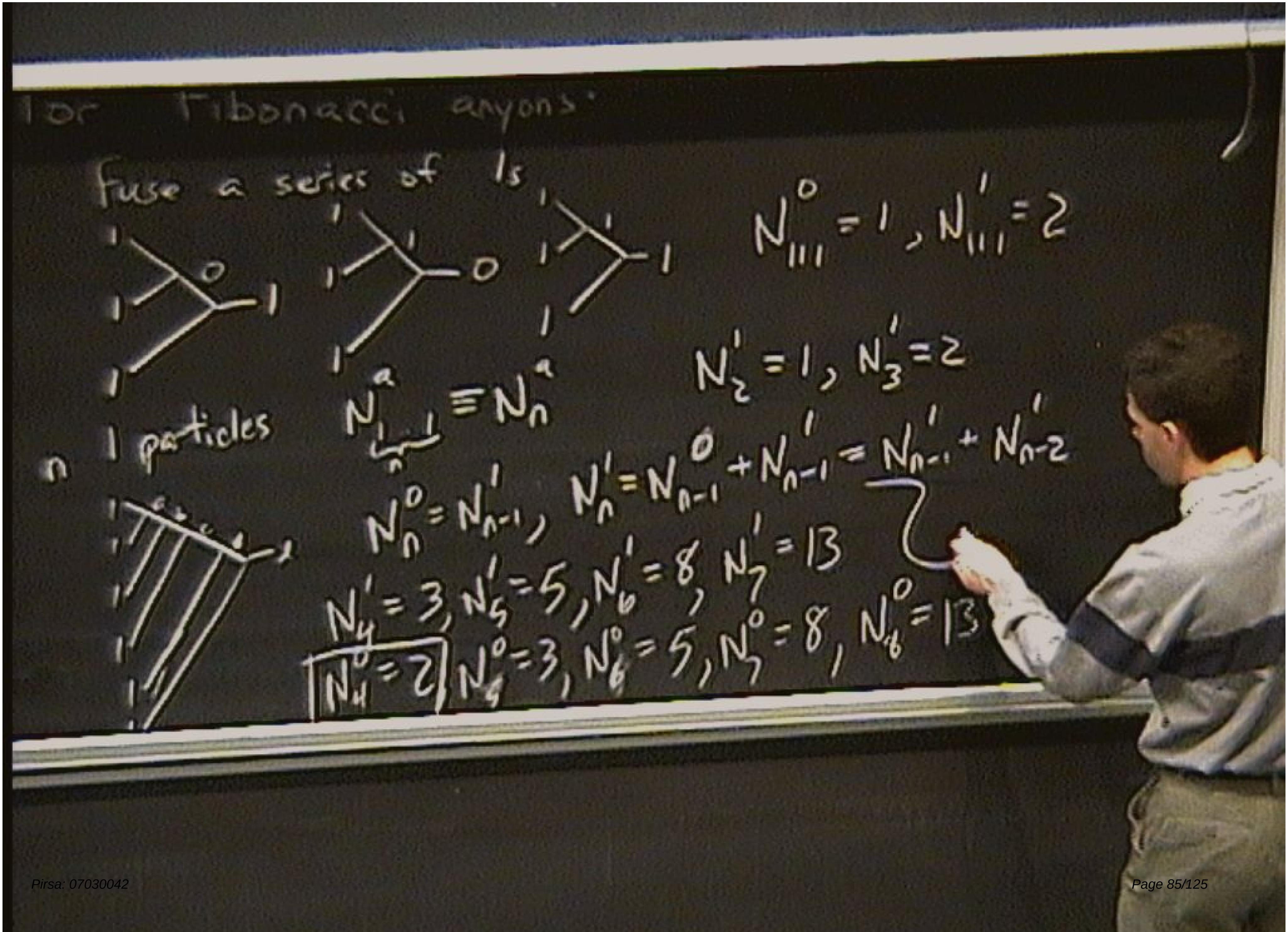
$N^0_n = N^1_{n-1},\ N^1_n = N^0_{n-1} + N^1_{n-1} = N^1_{n-1} + N^1_{n-2}$

$N^1_4=3,\ N^1_5=5,\ N^1_6=8,\ N^1_7=13$

$N^0_4=2,\ N^0_5=3,\ N^0_6=5,\ N^0_7=8,\ N^0_8=13$

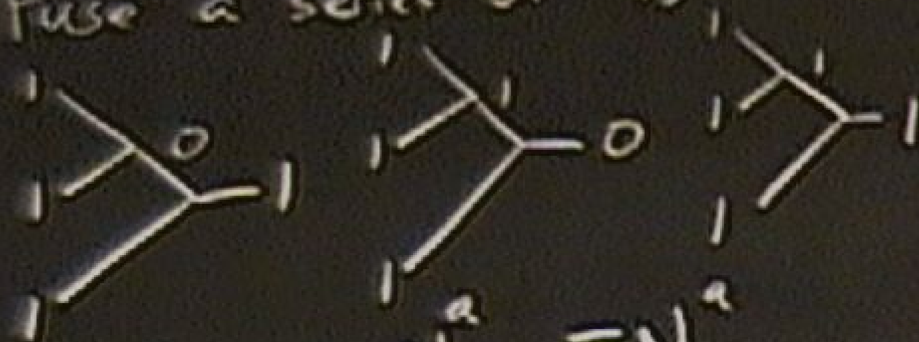


For fibonacci anyons:
Fuse a series of 1s
$N^0_{111} = 1$, $N^1_{111} = 2$
$N^1_2 = 1$, $N^1_3 = 2$
$N^a_{\underbrace{1\ldots1}_n} \equiv N^a_n$
n 1 particles
$N^1_n = N^0_{n-1} + N^1_{n-1} = N^1_{n-1} + N^1_{n-2}$
$N^0_n = N^1_{n-1}$,
$N^1_4 = 3$, $N^1_5 = 5$, $N^1_6 = 8$, $N^1_7 = 13$
$\boxed{N^0_4 = 2}$ $N^0_5 = 3$, $N^0_6 = 5$, $N^0_7 = 8$, $N^0_8 = 13$


For Fibonacci anyons:
Fuse a series of 1s
$N_{111}^{0} = 1,\ N_{111}^{1} = 2$
$N_2^1 = 1,\ N_3^1 = 2$
n 1 particles
$N_{\underbrace{1\cdots1}_{n}}^{a} \equiv N_n^a$
$N_n^0 = N_{n-1}^1,\ N_n^1 = N_{n-1}^0 + N_{n-1}^1 = N_{n-1}^1 + N_{n-2}^1$
$N_4^1 = 3,\ N_5^1 = 5,\ N_6^1 = 8,\ N_7^1 = 13$
$\boxed{N_4^0 = 2},\ N_5^0 = 3,\ N_6^0 = 5,\ N_7^0 = 8,\ N_8^0 = 13$

For Fibonacci anyons:

Fuse a series of 1s



$N_{111}^0 = 1$, $N_{111}^1 = 2$

$N_2^1 = 1$, $N_3^1 = 2$

n 1 particles $N^a_{\underbrace{1\cdots 1}_n} \equiv N_n^a$

$N_n^0 = N_{n-1}^1$, $N_n^1 = N_{n-1}^0 + N_{n-1}^1 = N_{n-1}^1 + N_{n-2}^1$

$N_n^0 = \frac{1}{\sqrt{5}}\left[\phi^n - \left(-\frac{1}{\phi}\right)^n\right]$

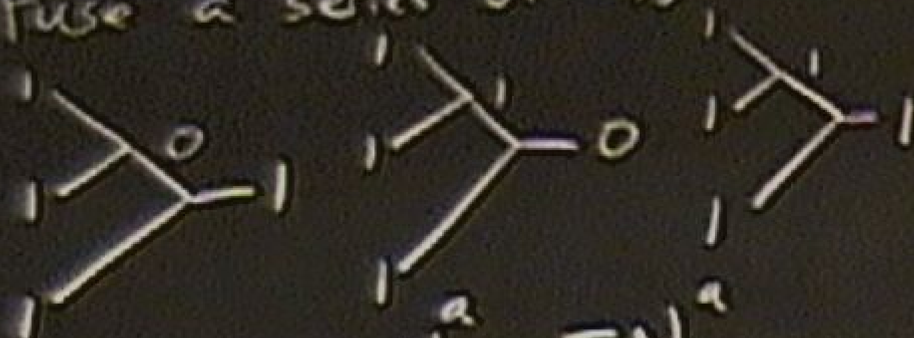
$\phi = \frac{1+\sqrt{5}}{2}$

$N_4^1 = 3$, $N_5^1 = 5$, $N_6^1 = 8$, $N_7^1 = 13$

$\boxed{N_4^0 = 2}$, $N_5^0 = 3$, $N_6^0 = 5$, $N_7^0 = 8$, $N_8^0 = 13$

For Fibonacci anyons:

fuse a series of 1s



$N_{111}^{0} = 1, \quad N_{111}^{1} = 2$

$N_2^{1} = 1, \quad N_3^{1} = 2$

n 1 particles $\quad N^{a}_{\underbrace{1\cdots 1}_{n}} \equiv N_n^{a}$

$N_n^{0} = N_{n-1}^{1}, \quad N_n^{1} = N_{n-1}^{0} + N_{n-1}^{1} = N_{n-1}^{1} + N_{n-2}^{1}$

$N_n^{0} = \frac{1}{\sqrt{5}}\left(\phi^n + \left(-\frac{1}{\phi}\right)\ldots\right.$

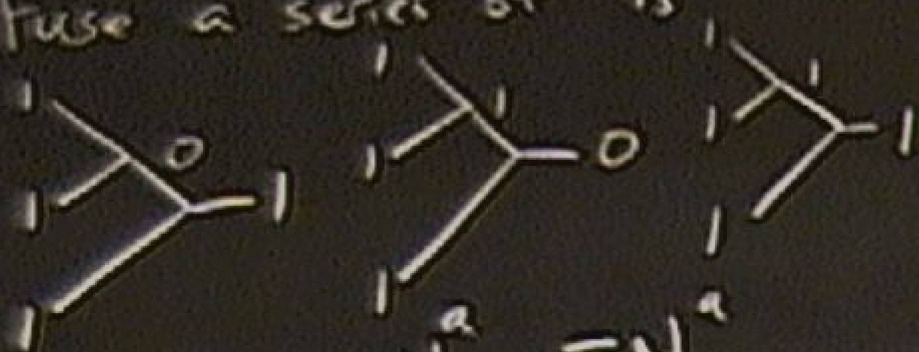
$\phi = \frac{1+\sqrt{5}}{2}$

$N_4^{1} = 3, N_5^{1} = 5, N_6^{1} = 8, N_7^{1} = 13$

$\boxed{N_4^{0} = 2}, N_5^{0} = 3, N_6^{0} = 5, N_7^{0} = 8, N_8^{0} = 13$

For Fibonacci anyons:

fuse a series of 1s



$N_{111}^{0} = 1$, $N_{111}^{1} = 2$

$N_2^1 = 1$, $N_3^1 = 2$

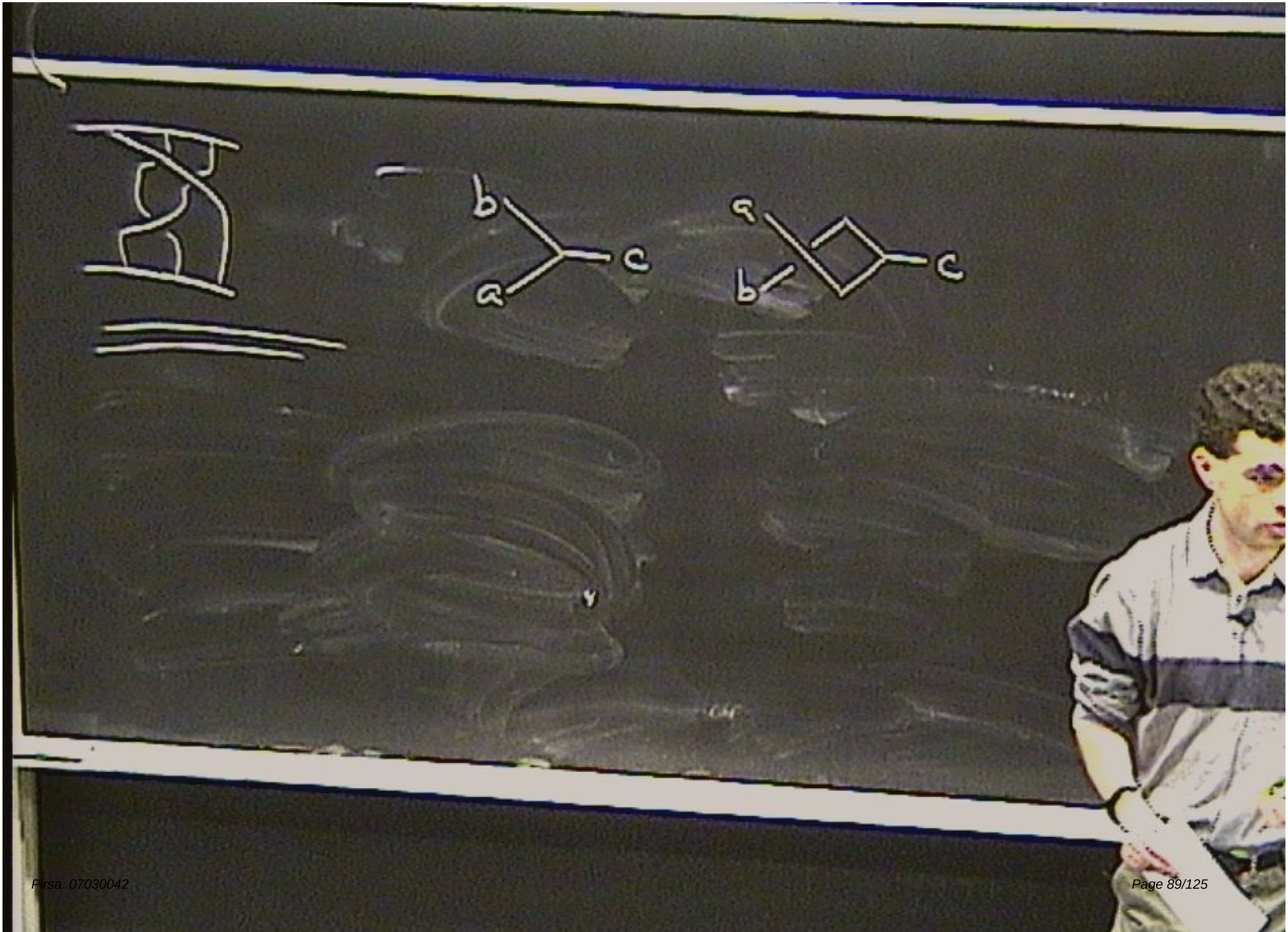
n 1 particles $N^{a}_{\underbrace{1\ldots1}_{n}} \equiv N_n^a$

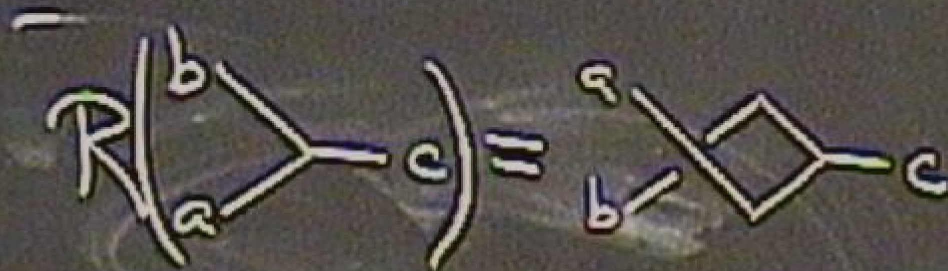
$N_n^0 = N_{n-1}^1$, $N_n^1 = N_{n-1}^0 + N_{n-1}^1 = N_{n-1}^1 + N_{n-2}^1$

$N_n^0 = \frac{1}{\sqrt{5}}\left[\phi^n + \left(-\frac{1}{\phi}\right)^n\right]$, $\phi = \frac{1+\sqrt{5}}{2}$

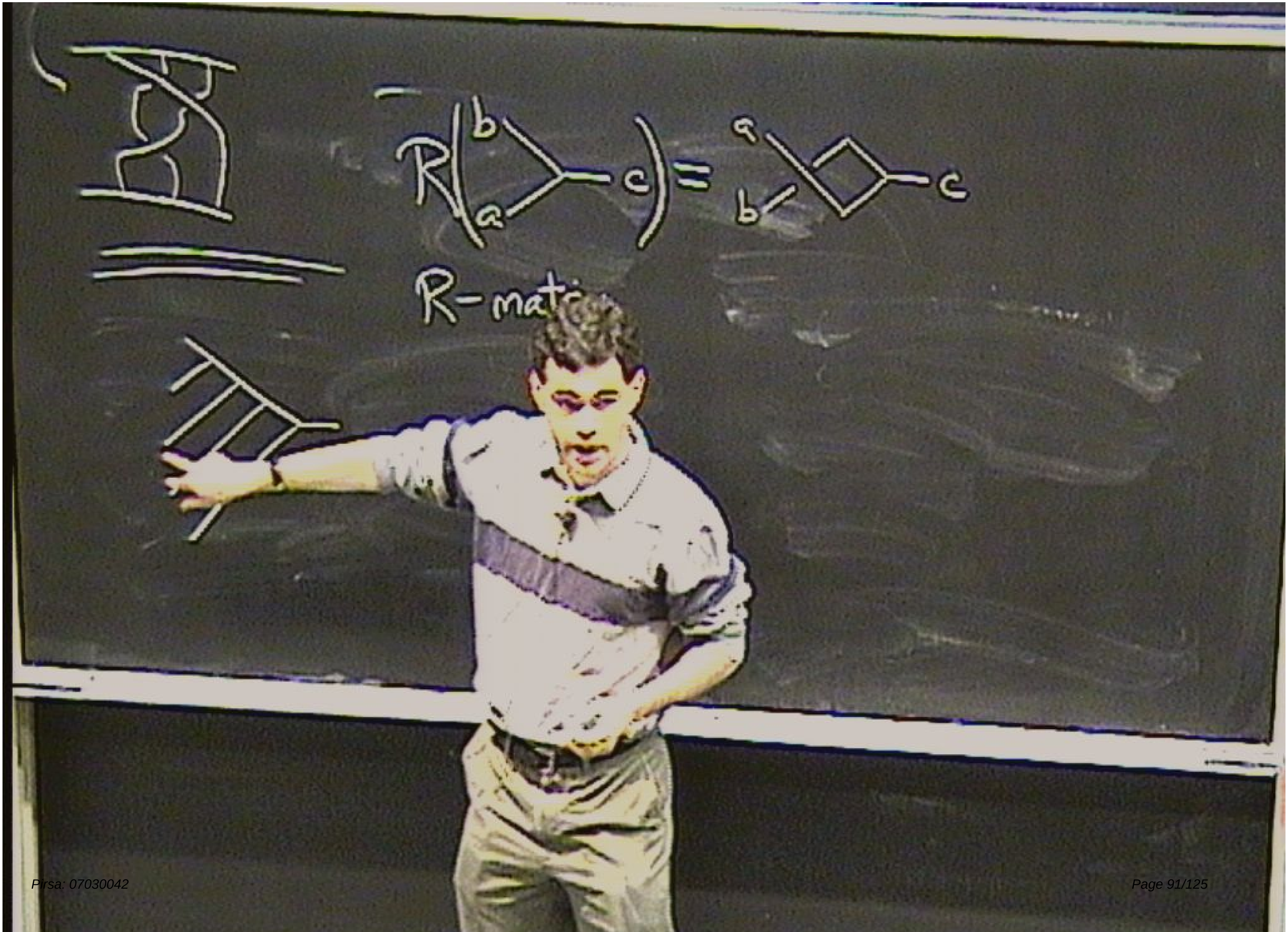
$N_4^1 = 3$, $N_5^1 = 5$, $N_6^1 = 8$, $N_7^1 = 13$

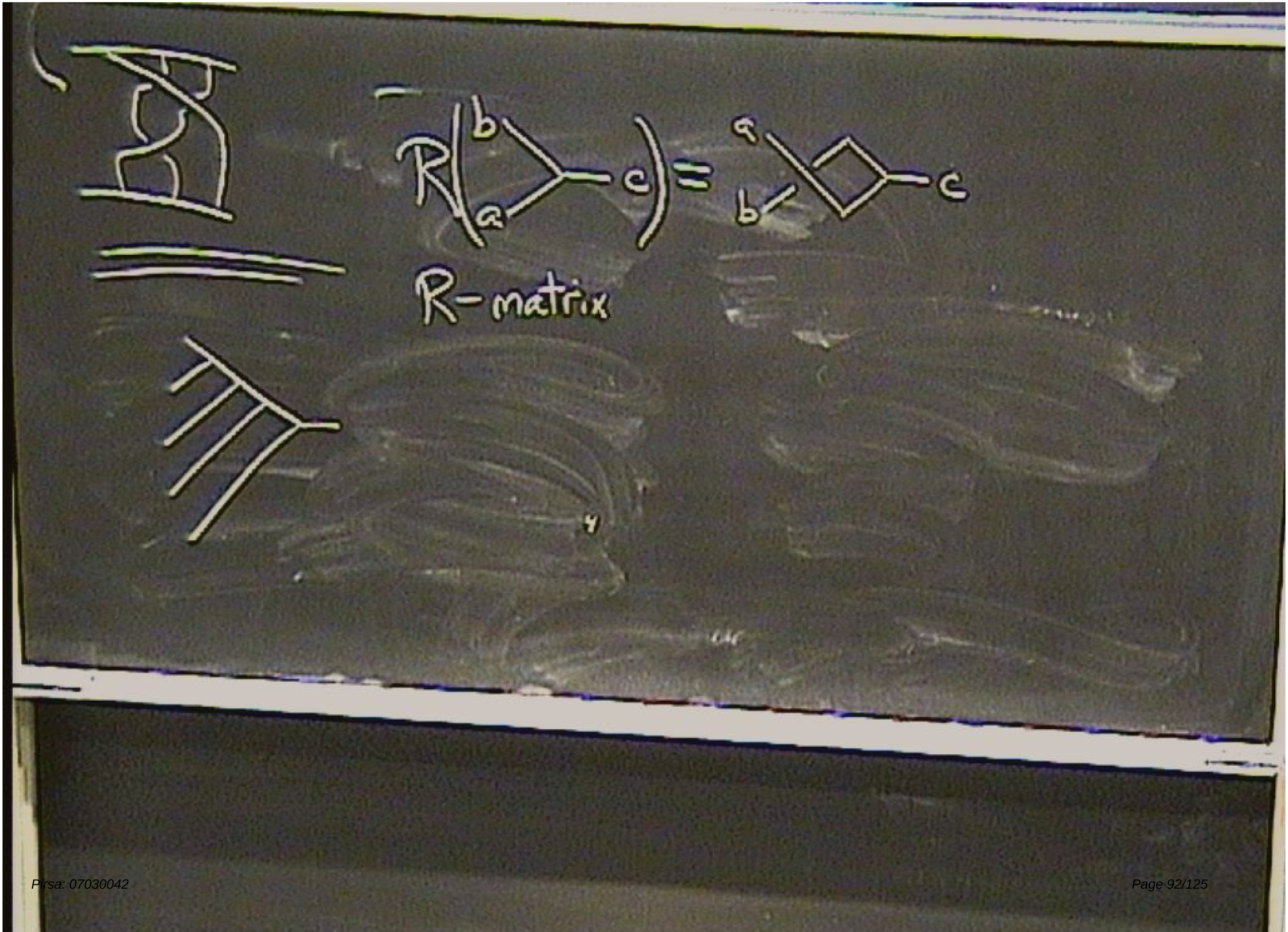
$\boxed{N_4^0 = 2}$, $N_5^0 = 3$, $N_6^0 = 5$, $N_7^0 = 8$, $N_8^0 = 13$

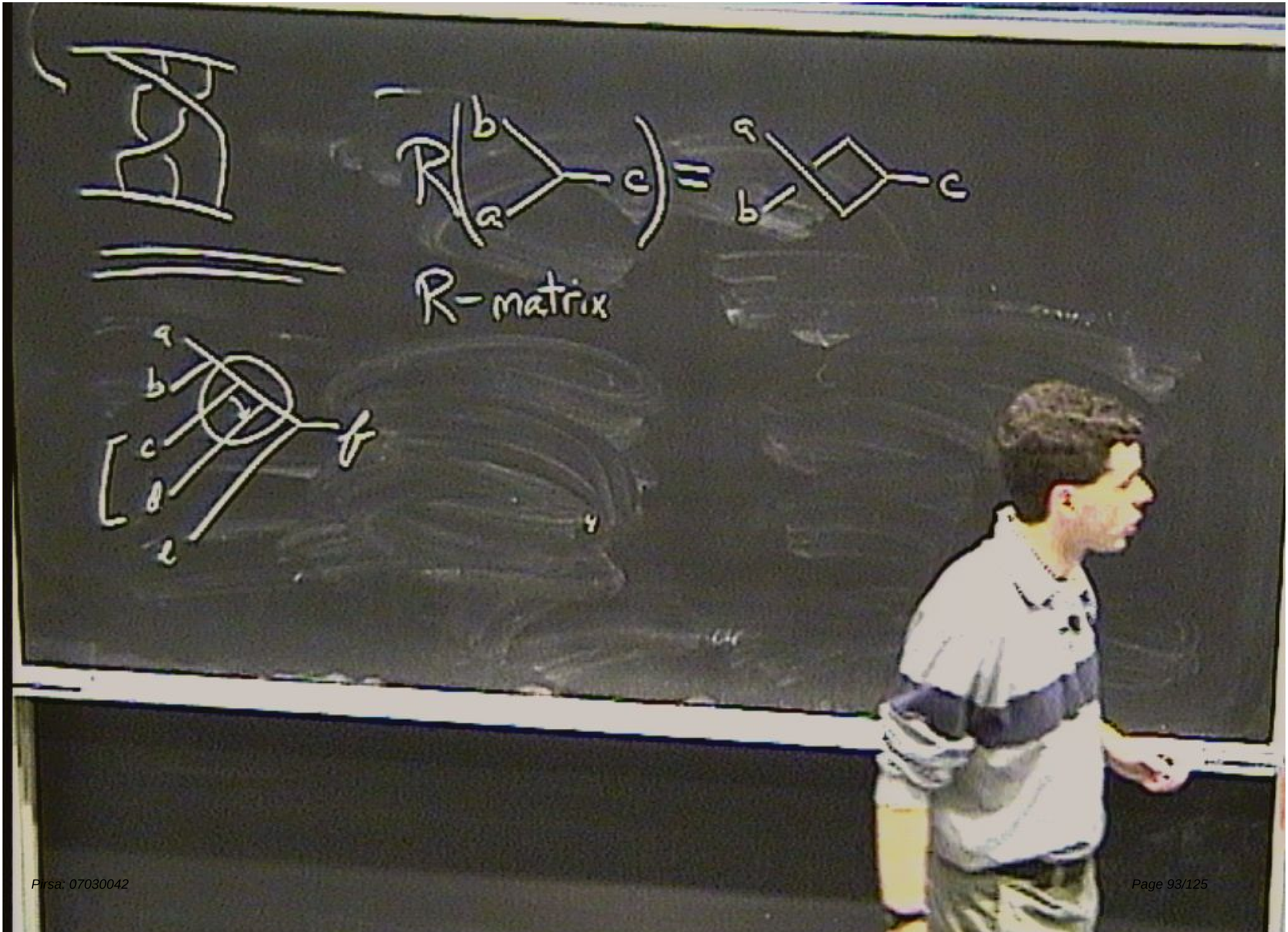

b
a
c
a
b
c

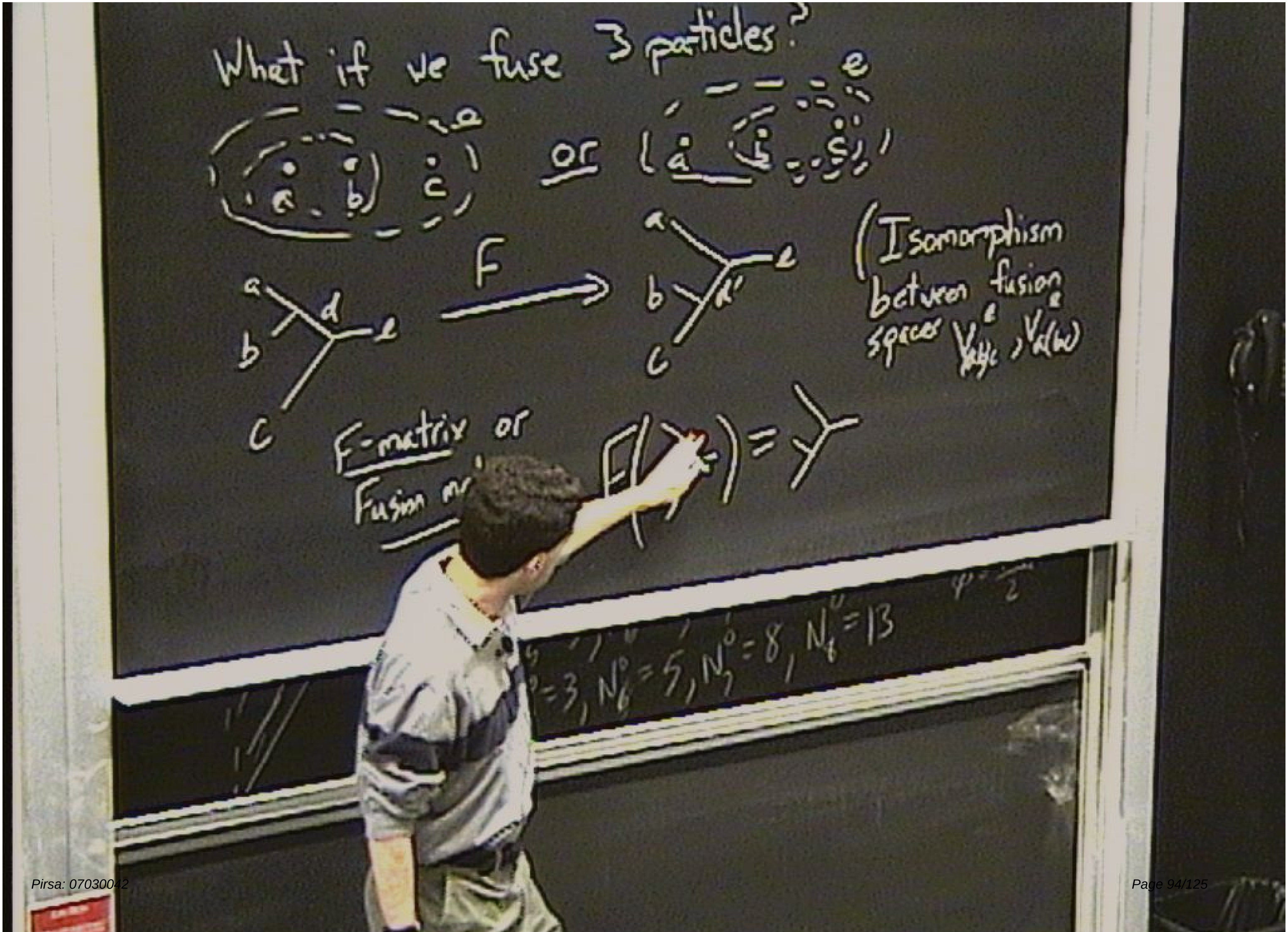


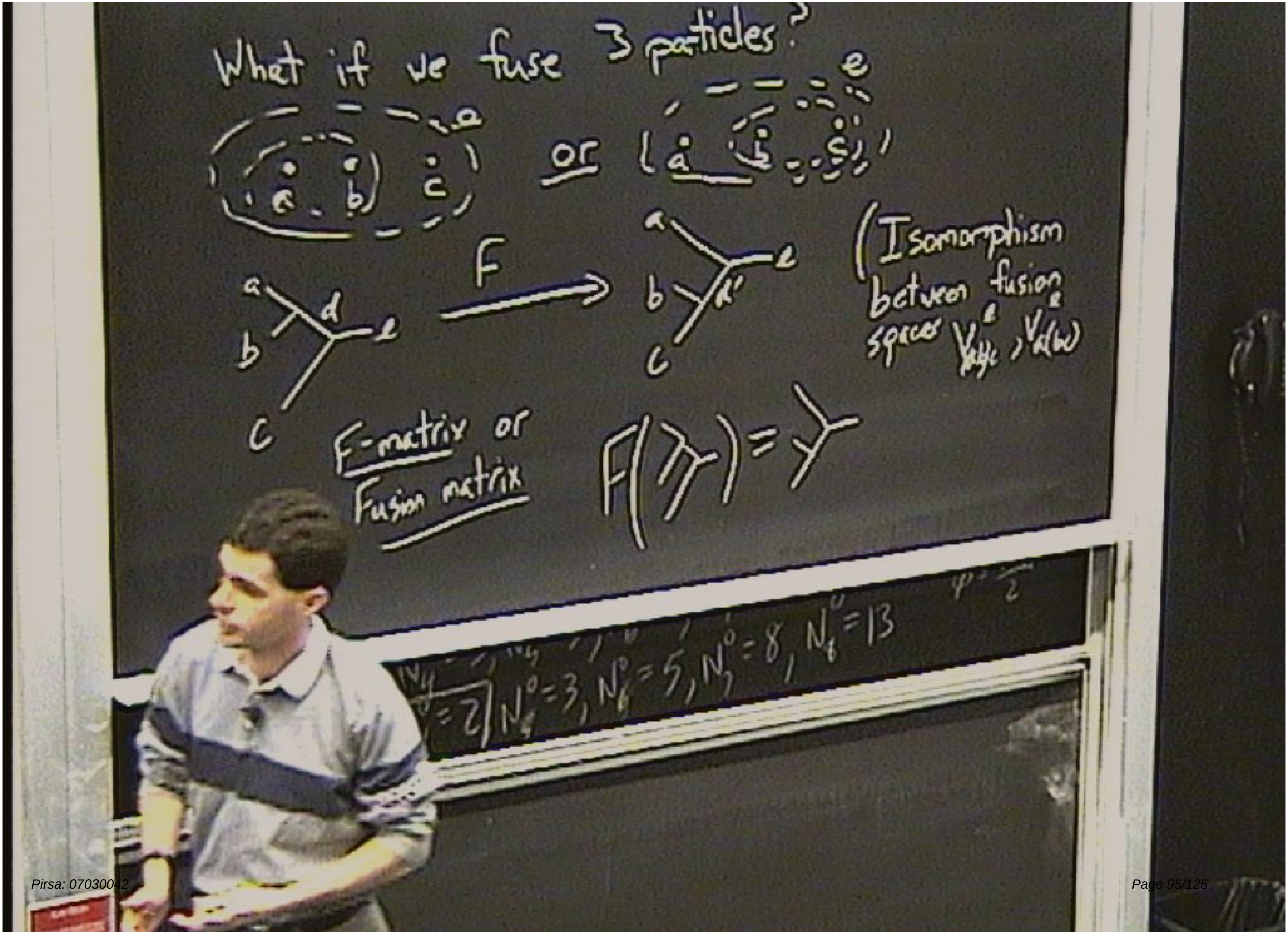
R-matrix

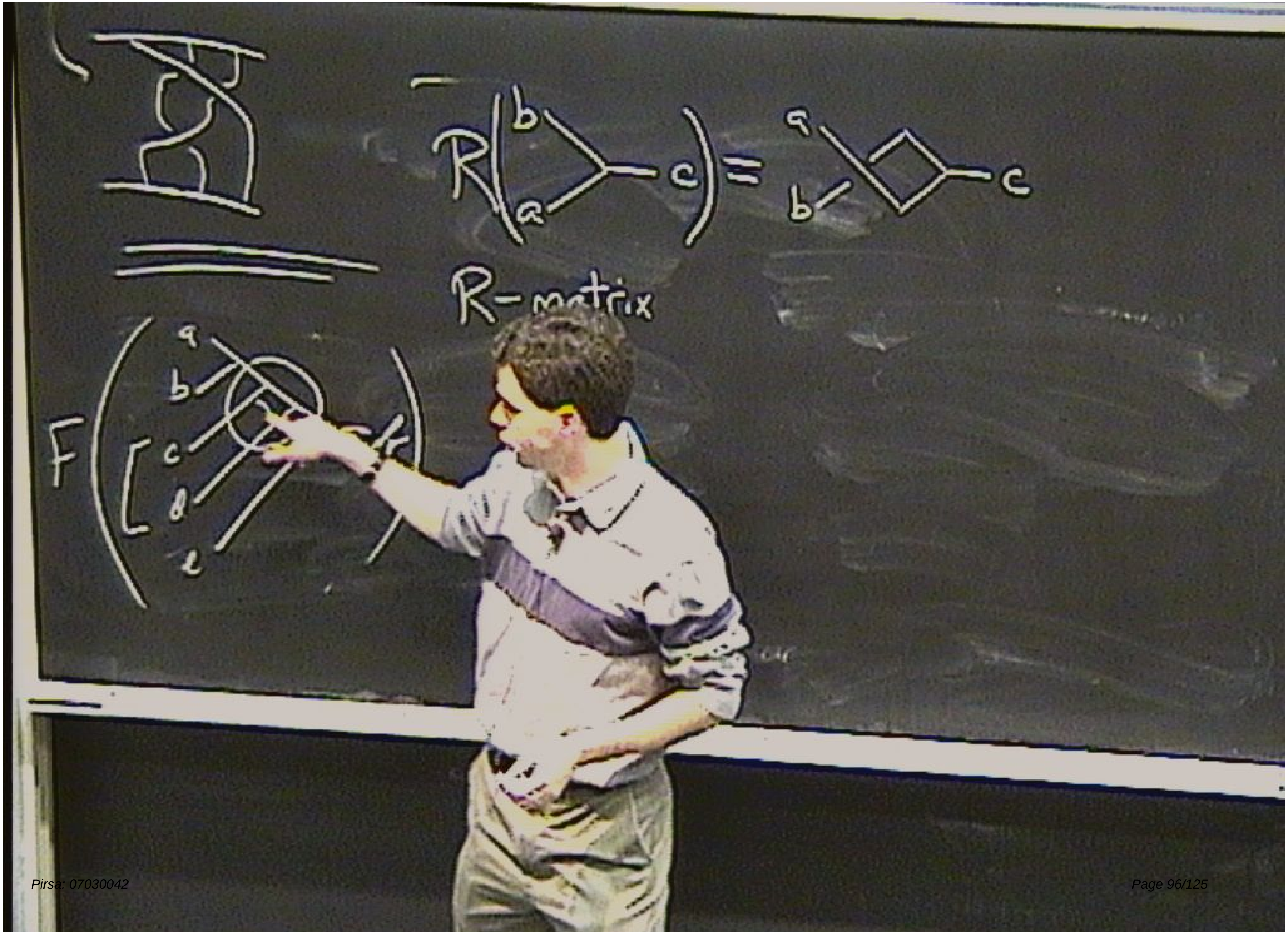


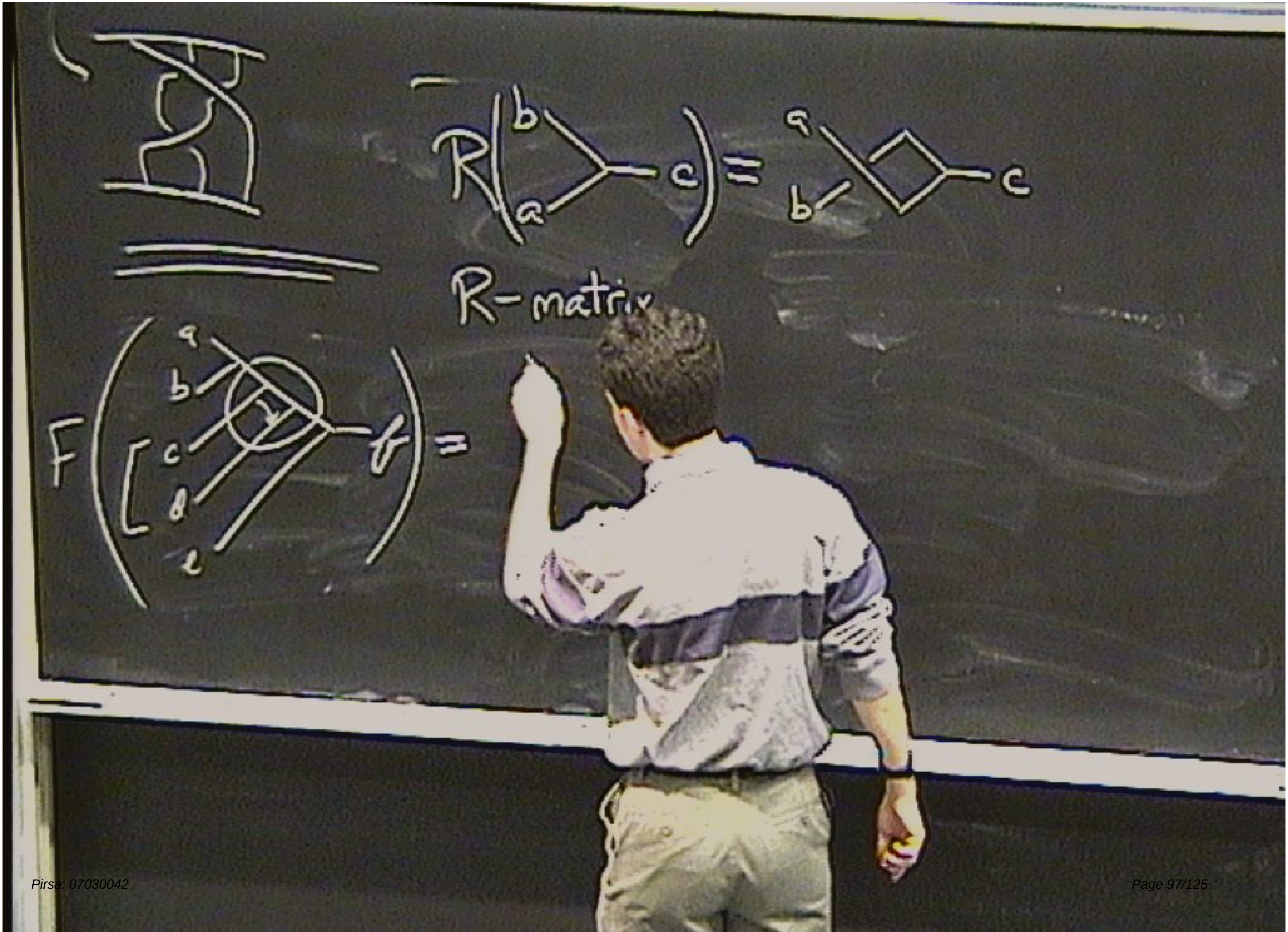

R-matrix

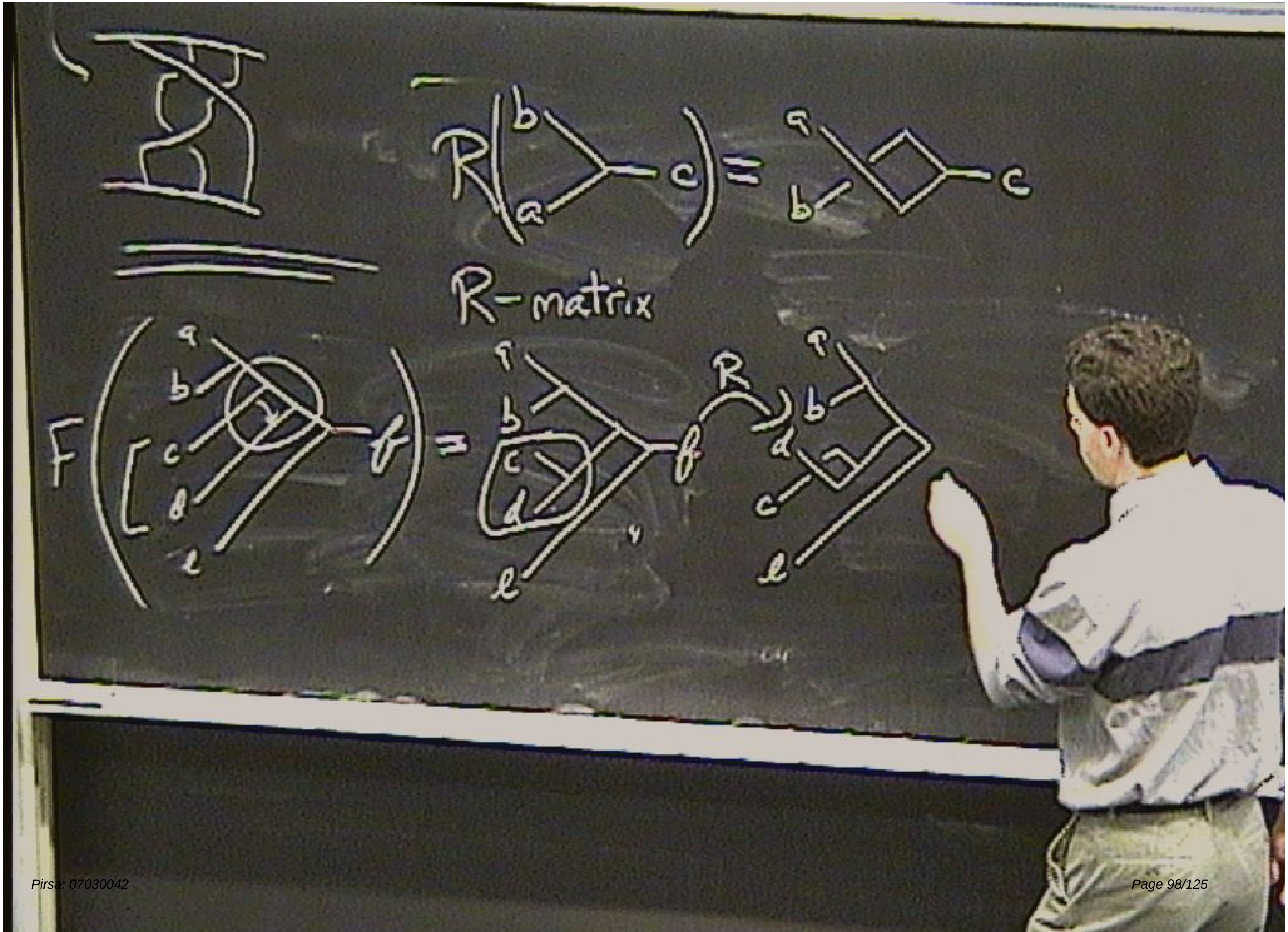

R
b
a
c
=
a
b
c
R-matrix
a
b
c
d
e
f

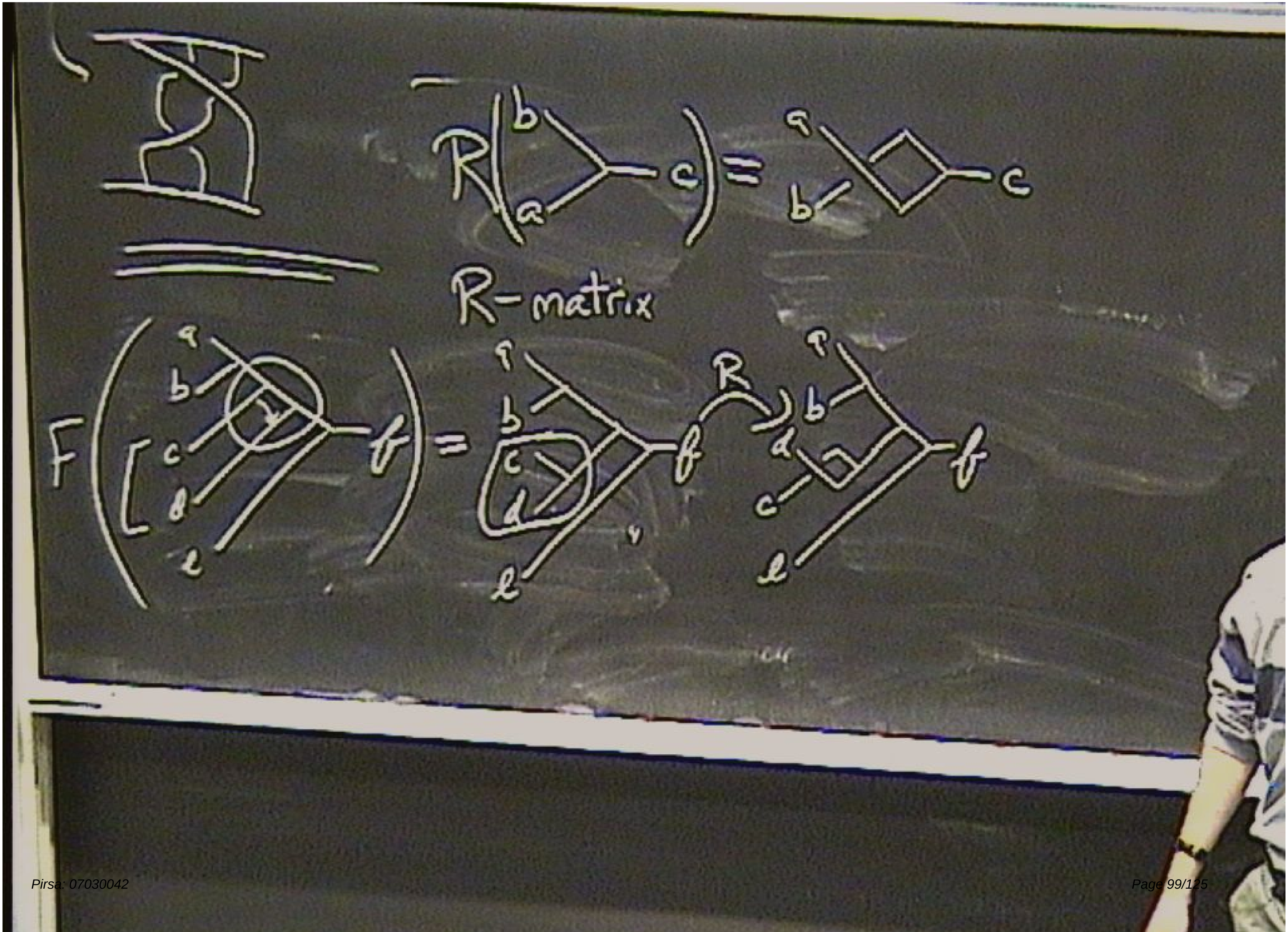

What if we fuse 3 particles?
or
F
Isomorphism between fusion spaces
F-matrix or

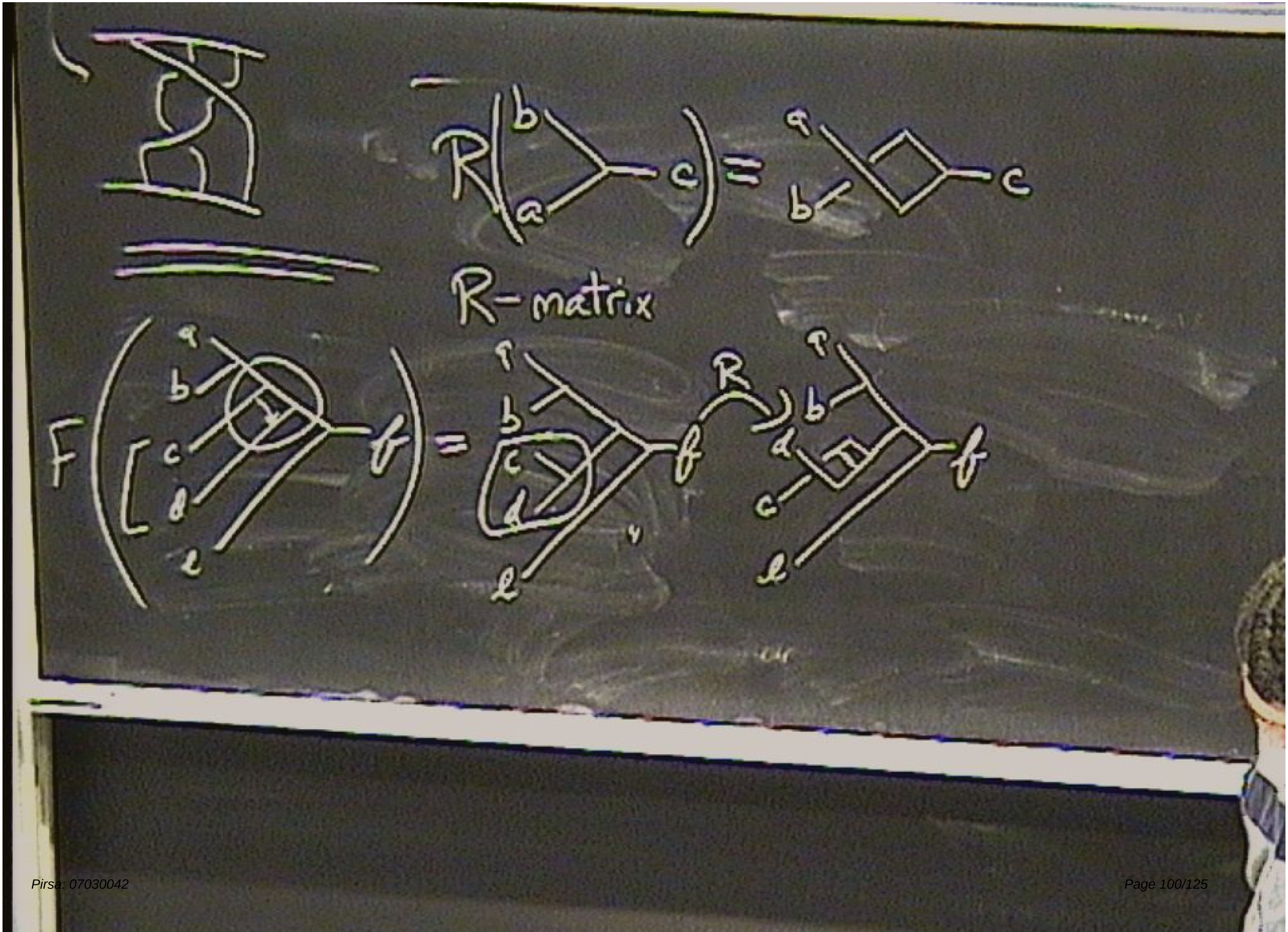

What if we fuse 3 particles?
or
F
(Isomorphism between fusion spaces
F-matrix or Fusion matrix
N_4^0 = 3, N_6^0 = 5, N_7^0 = 8, N_8^0 = 13

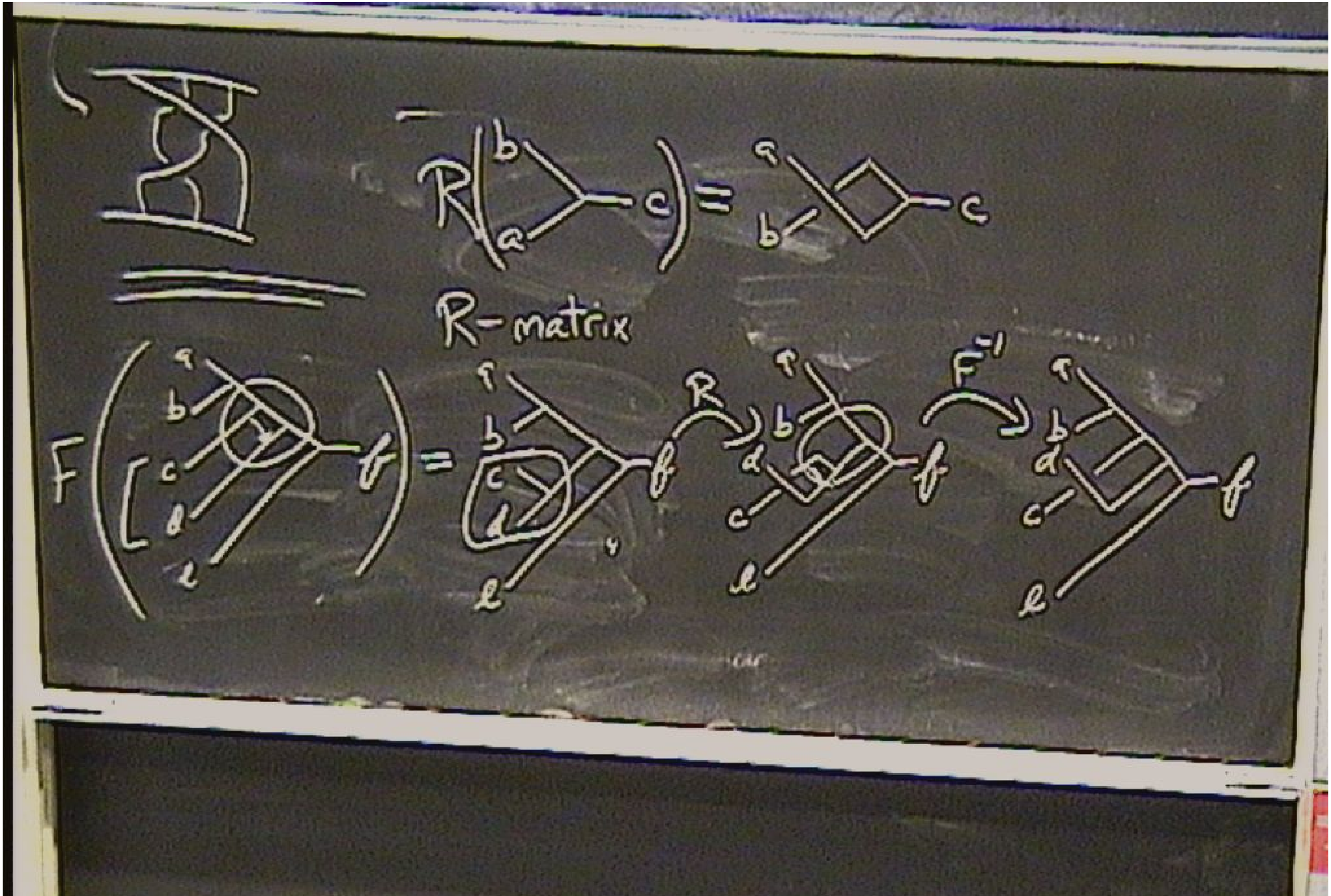

R-matrix

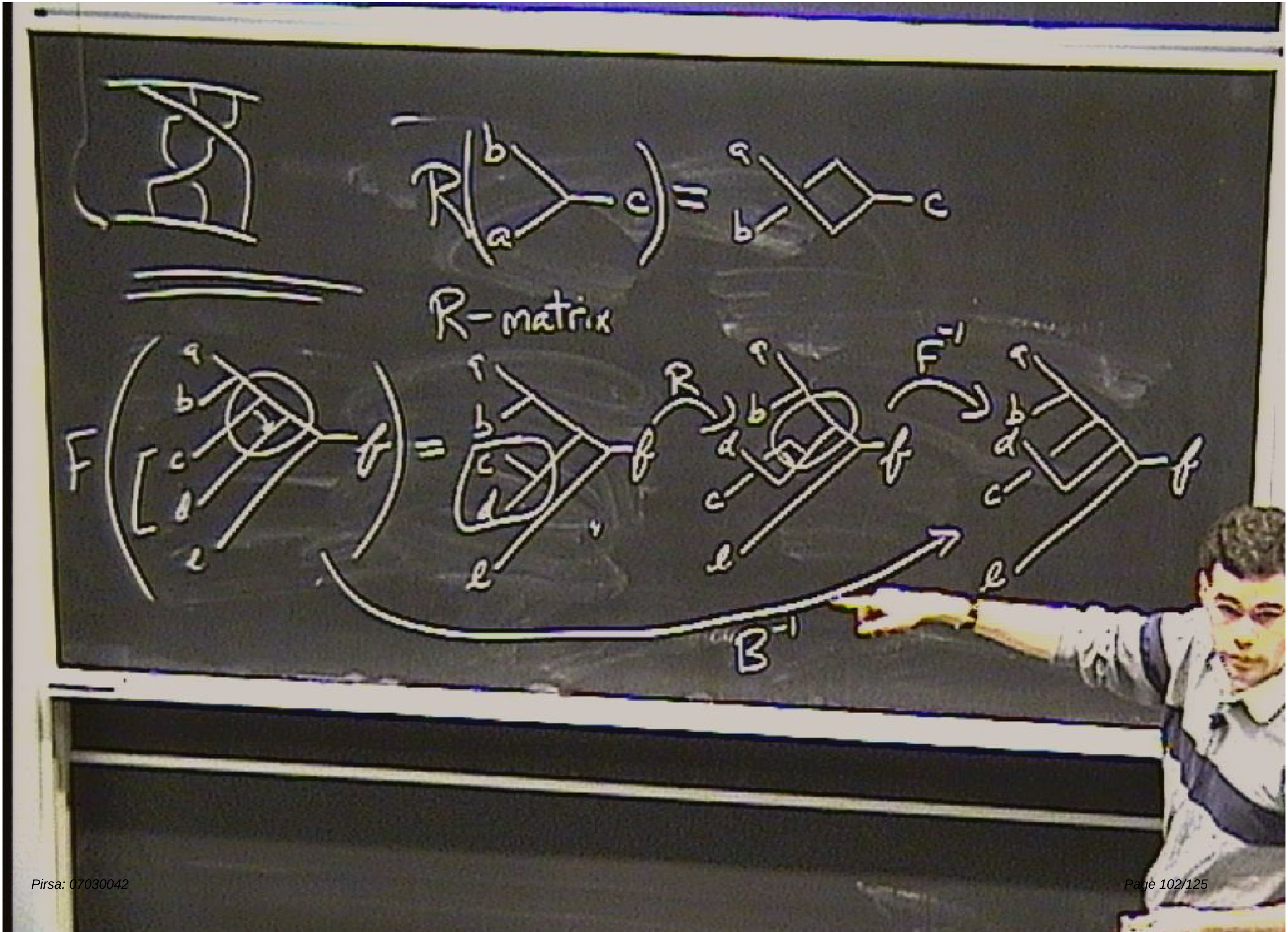

R-matri

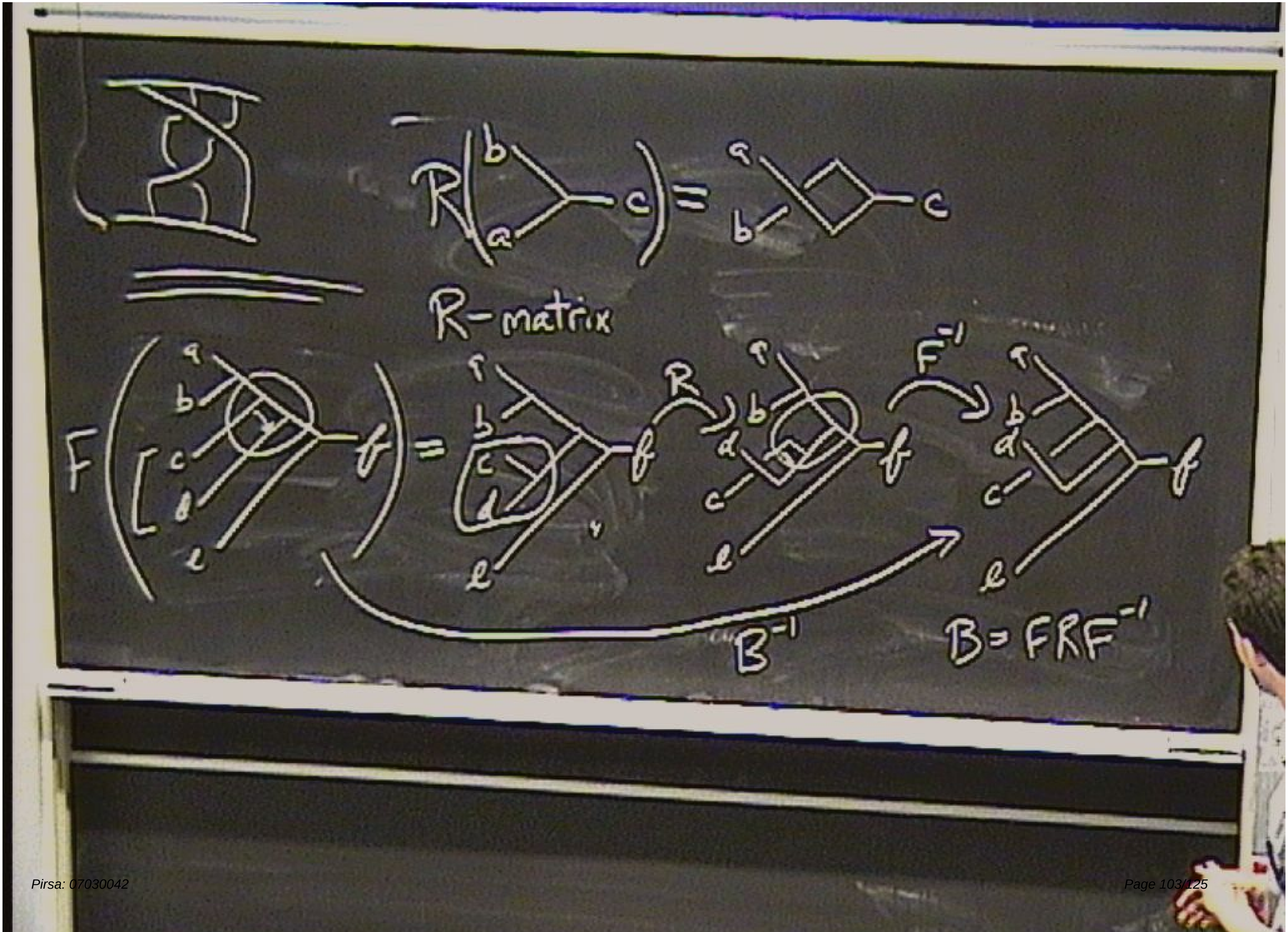

R
b
a
c
=
a
b
c
R-matrix
F
a
b
c
d
e
f
=
a
b
c
d
e
f
R
a
b
d
c
e

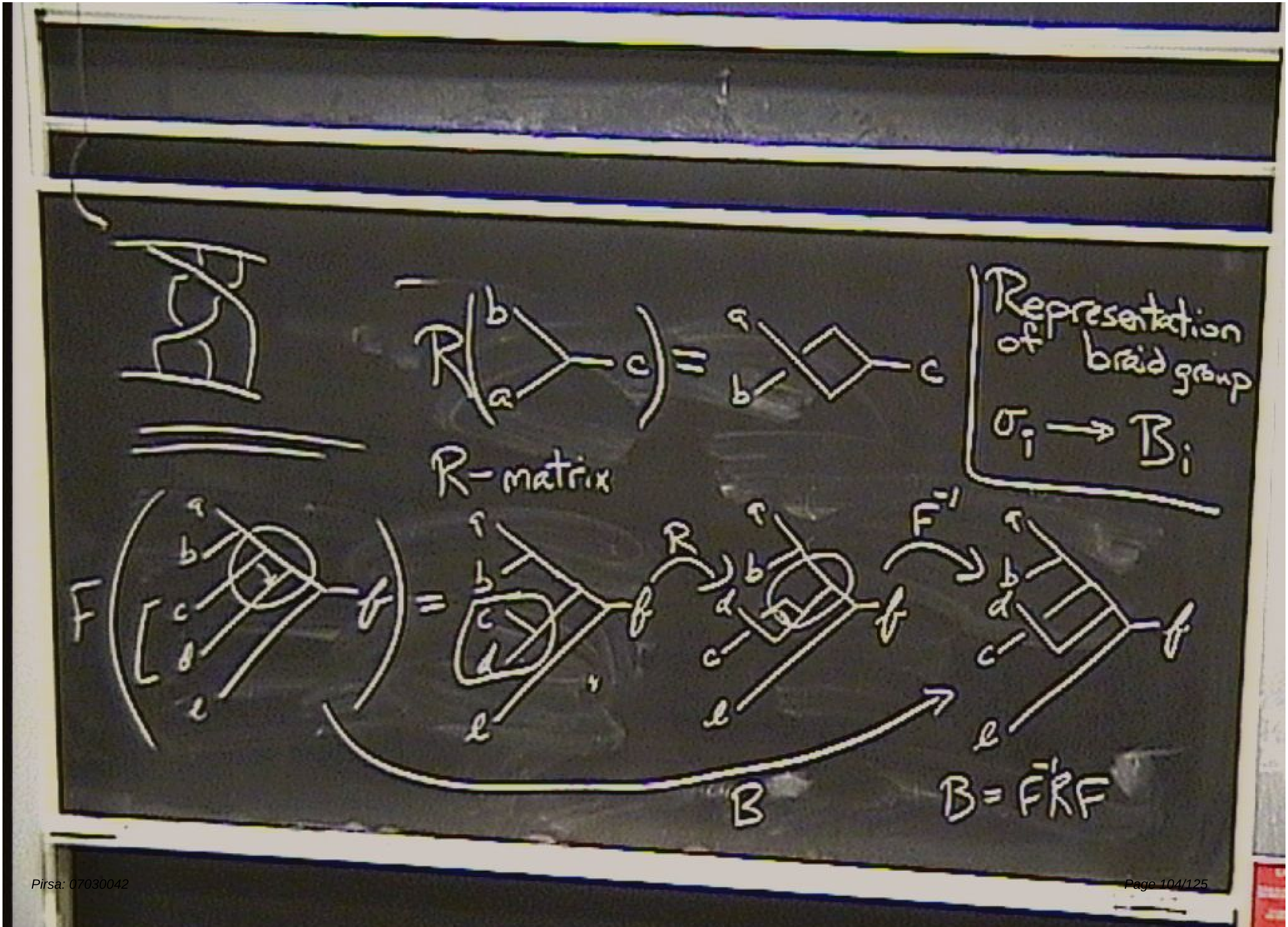

R-matrix


R-matrix

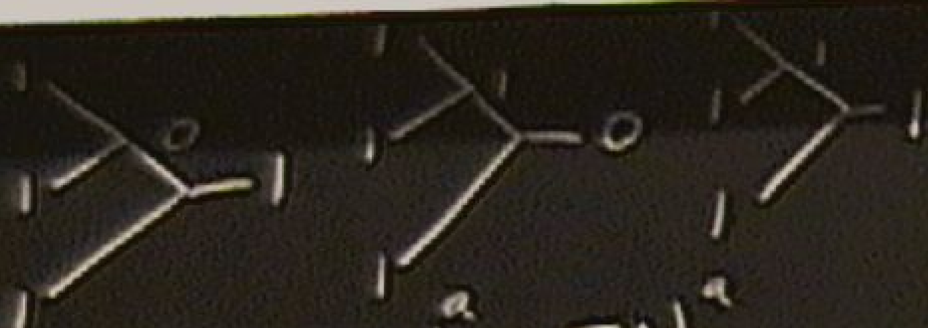

R-matrix


R-matrix
R
F
F^-1
B^-1
a
b
c
d
e
f


R-matrix
R
F⁻¹
B⁻¹
B = FRF⁻¹


Representation of braid group
$\sigma_i \to B_i$
R-matrix
R
F^{-1}
B
$B = F^{-1}RF$

Fusion matrix



$N_{111}^{0} = 1,\ N_{111}^{1} = 2$

$N_2^1 = 1,\ N_3^1 = 2$

n 1 particles $\quad N^a_{\underbrace{1\ldots1}_{n}} \equiv N_n^a$

$N_n^0 = N_{n-1}^1,\quad N_n^1 = N_{n-1}^0 + N_{n-1}^1 = N_{n-1}^1 + N_{n-2}^1$

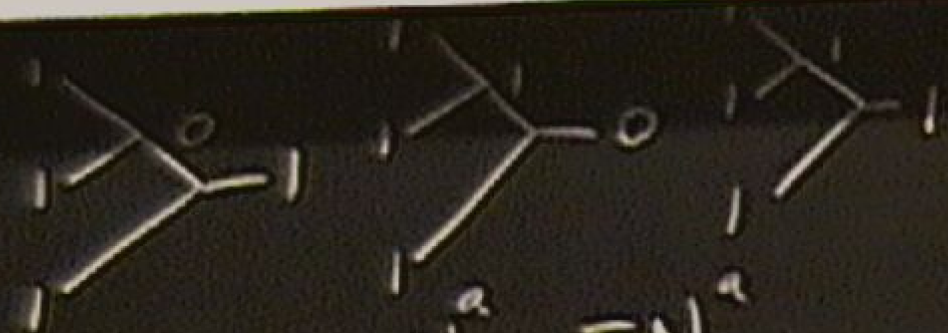
$N_n^0 = \frac{1}{\sqrt{5}}\left[\phi^n + \left(-\frac{1}{\phi}\right)^n\right]$

$\phi = \frac{1+\sqrt{5}}{2}$

$N_4^1 = 3,\ N_5^1 = 5,\ N_6^1 = 8,\ N_7^1 = 13$

$\boxed{N_4^0 = 2},\ N_5^0 = 3,\ N_6^0 = 5,\ N_7^0 = 8,\ N_8^0 = 13$

Fusion matrix



$N_{111}^{0} = 1, \quad N_{111}^{1} = 2$

$N_2^1 = 1, \quad N_3^1 = 2$

n 1 particles $\quad N_{1\cdots1}^{a} \equiv N_n^{a}$

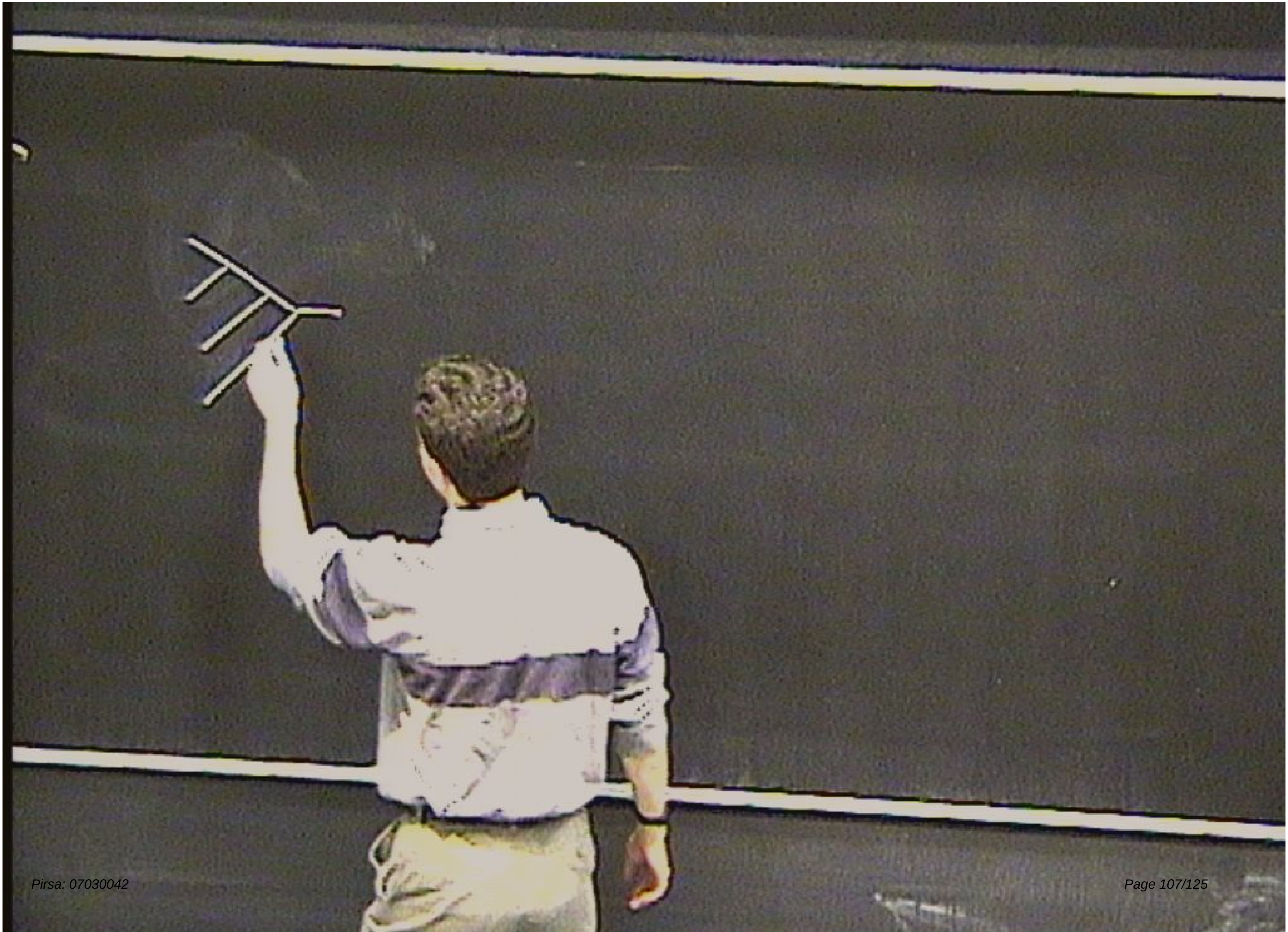
$N_n^0 = N_{n-1}^1, \quad N_n^1 = N_{n-1}^0 + N_{n-1}^1 = N_{n-1}^1 + N_{n-2}^1$

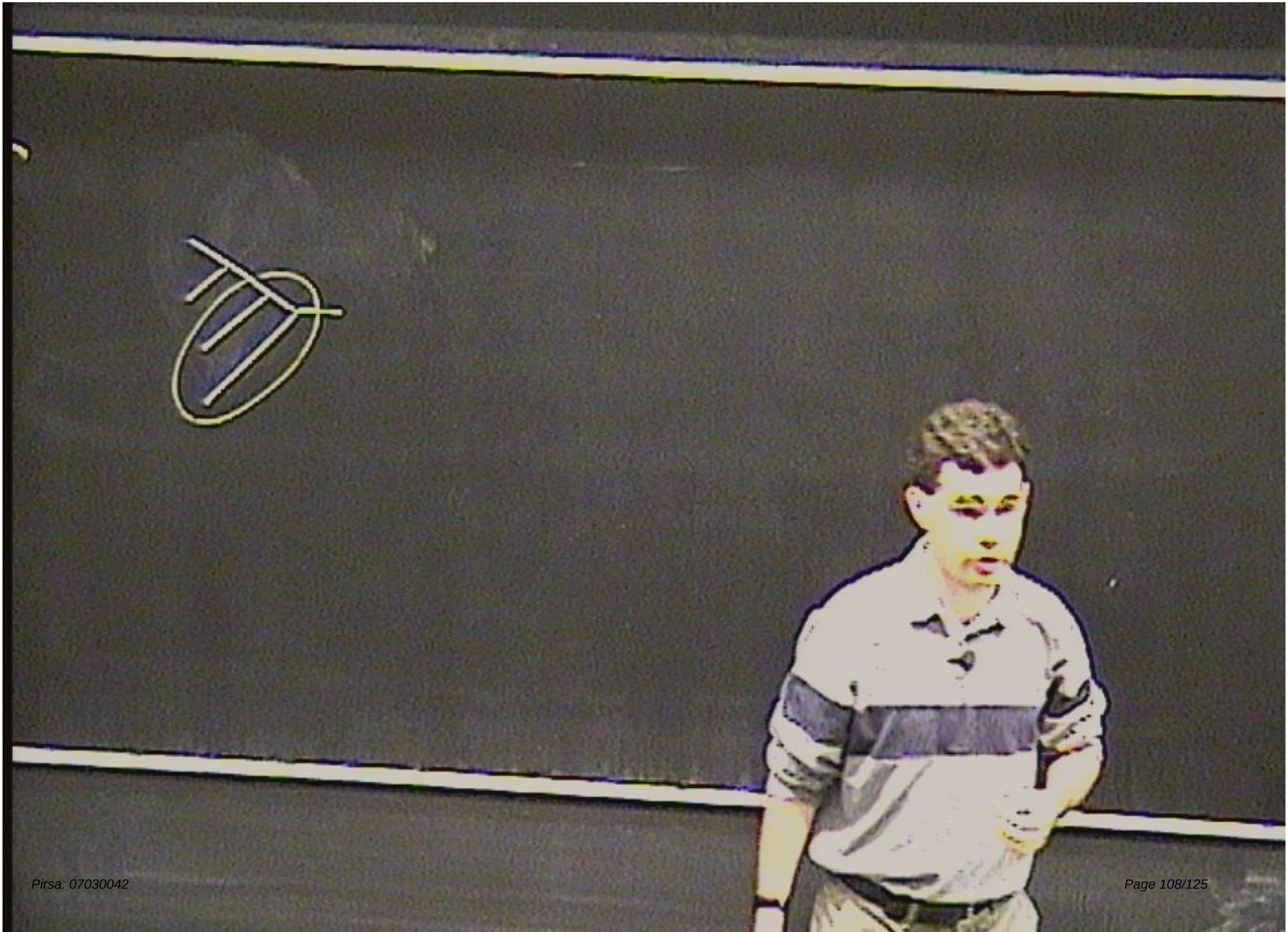
$N_n^0 = \frac{1}{\sqrt{5}}\left[\phi^n + \left(-\frac{1}{\phi}\right)^n\right]$

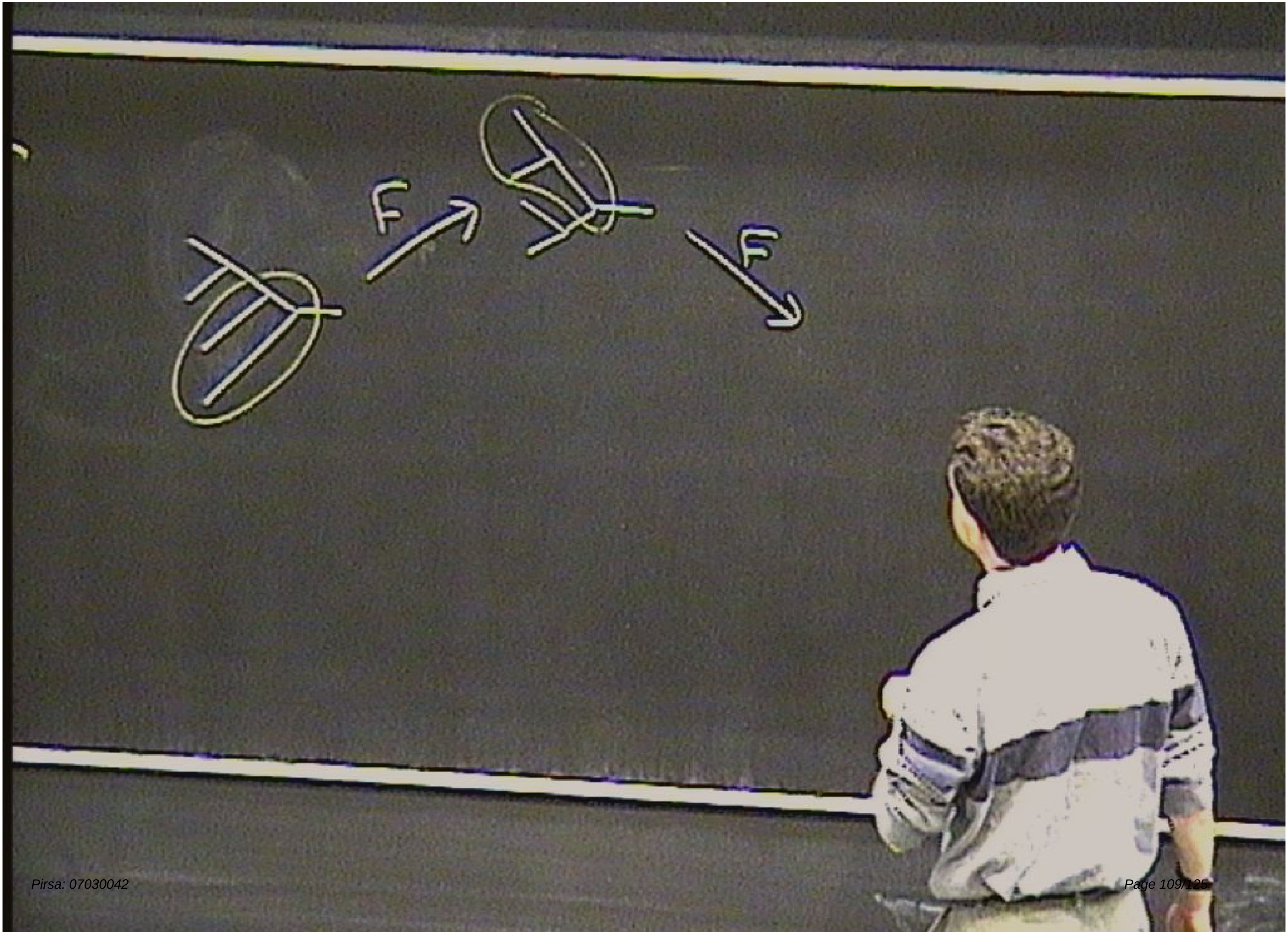
$\phi = \frac{1+\sqrt{5}}{2}$

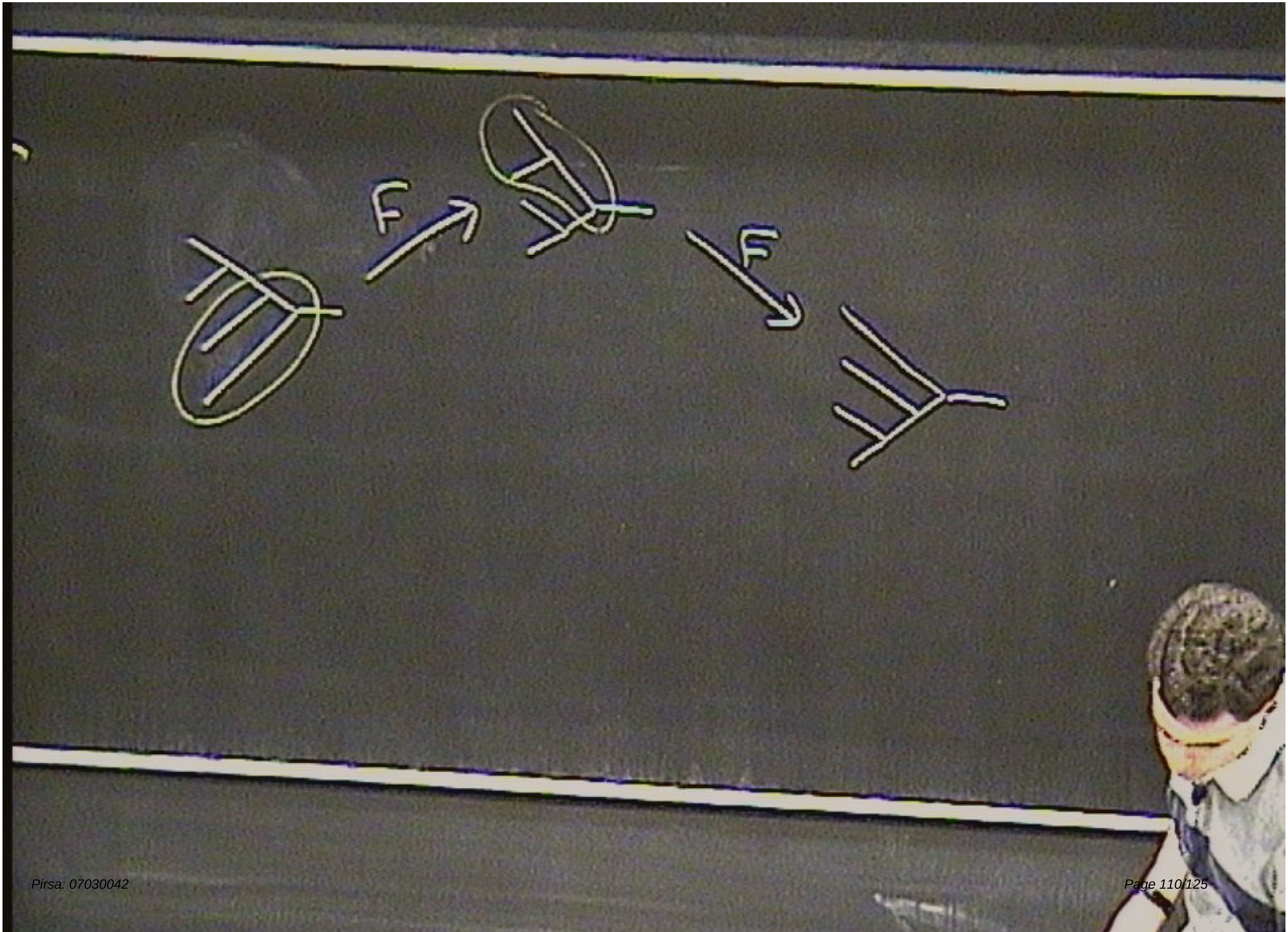
$N_4^1 = 3, \; N_5^1 = 5, \; N_6^1 = 8, \; N_7^1 = 13$

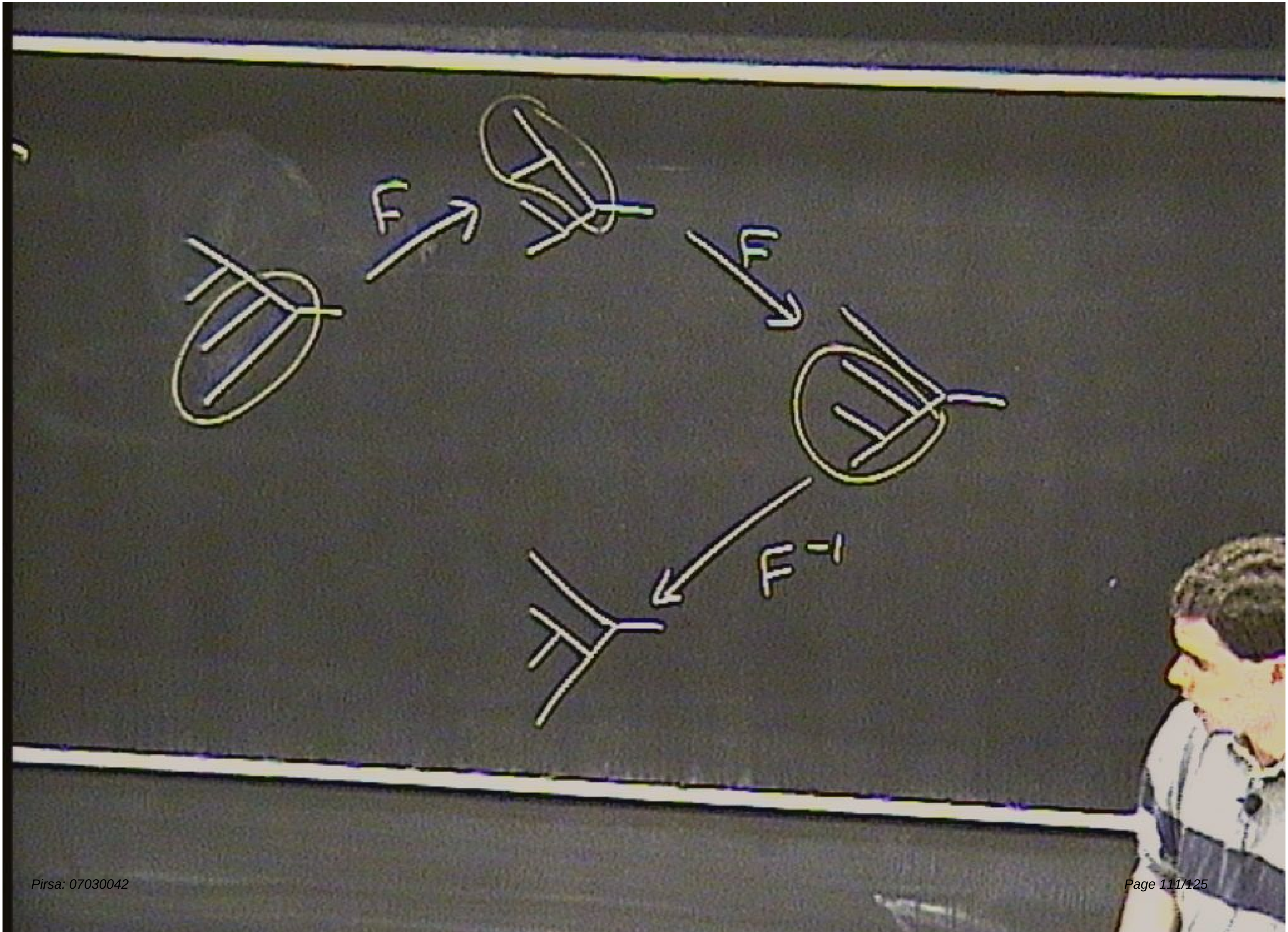
$\boxed{N_4^0 = 2}, \; N_5^0 = 3, \; N_6^0 = 5, \; N_7^0 = 8, \; N_8^0 = 13$

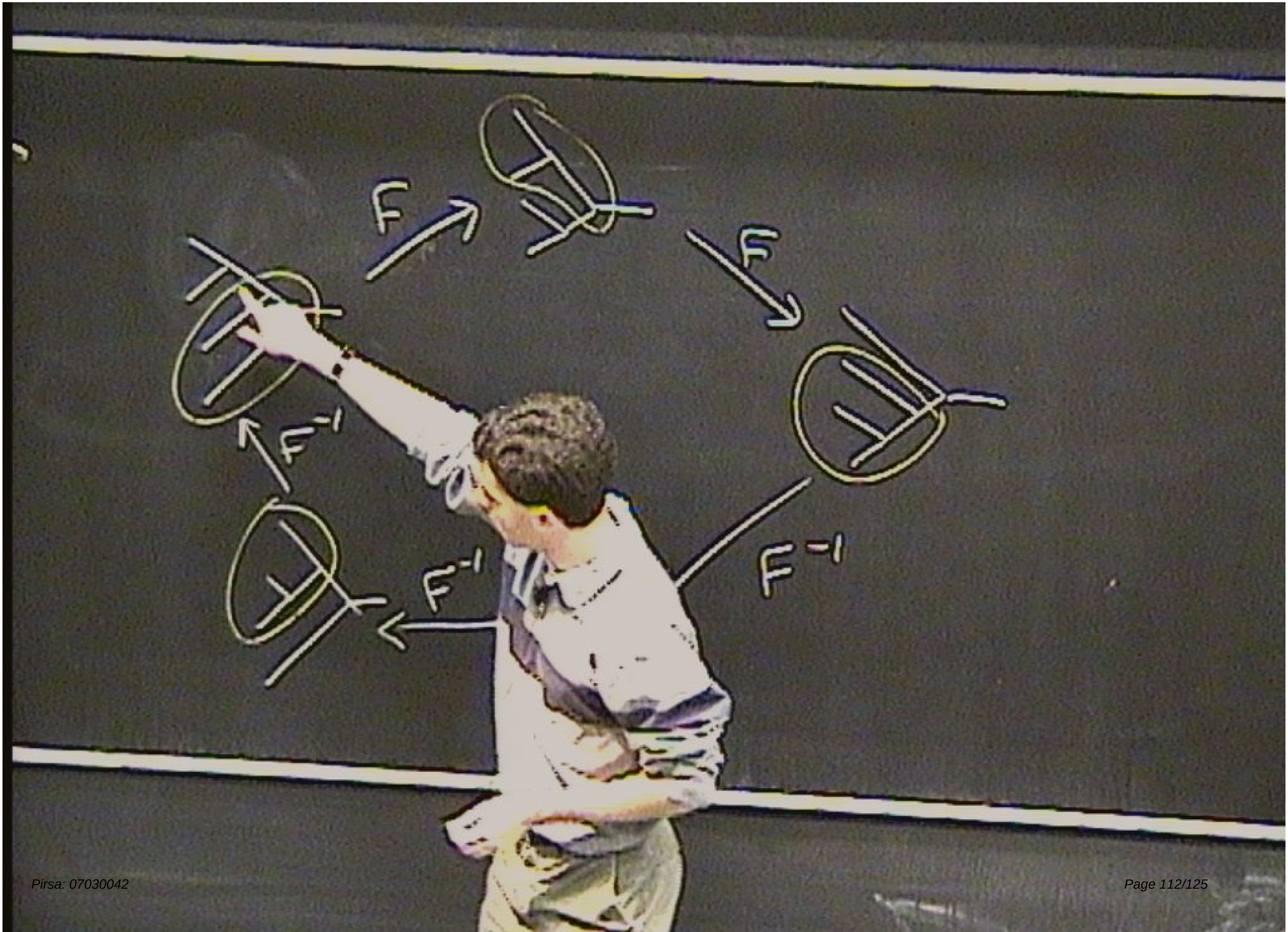


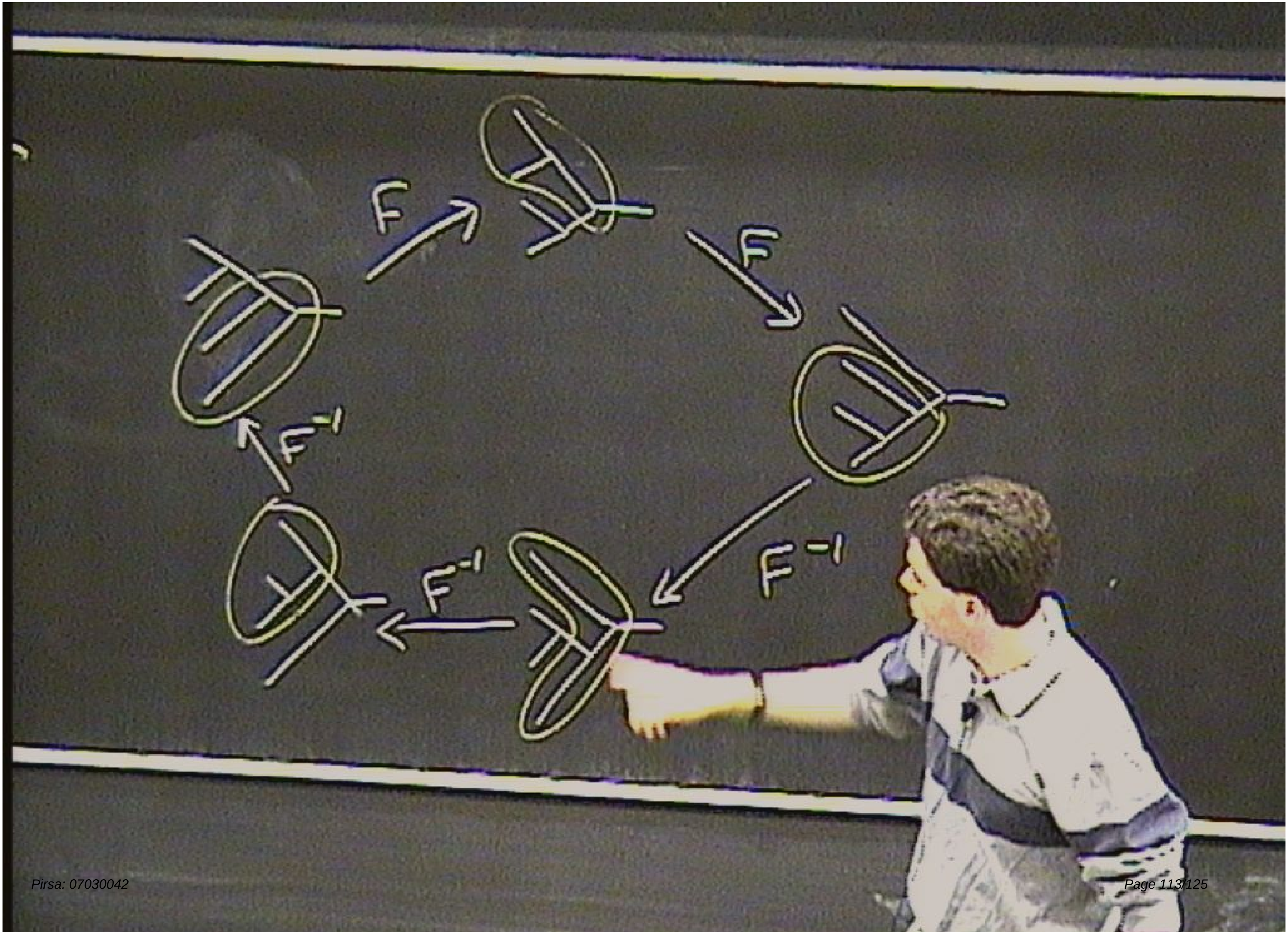



F
F

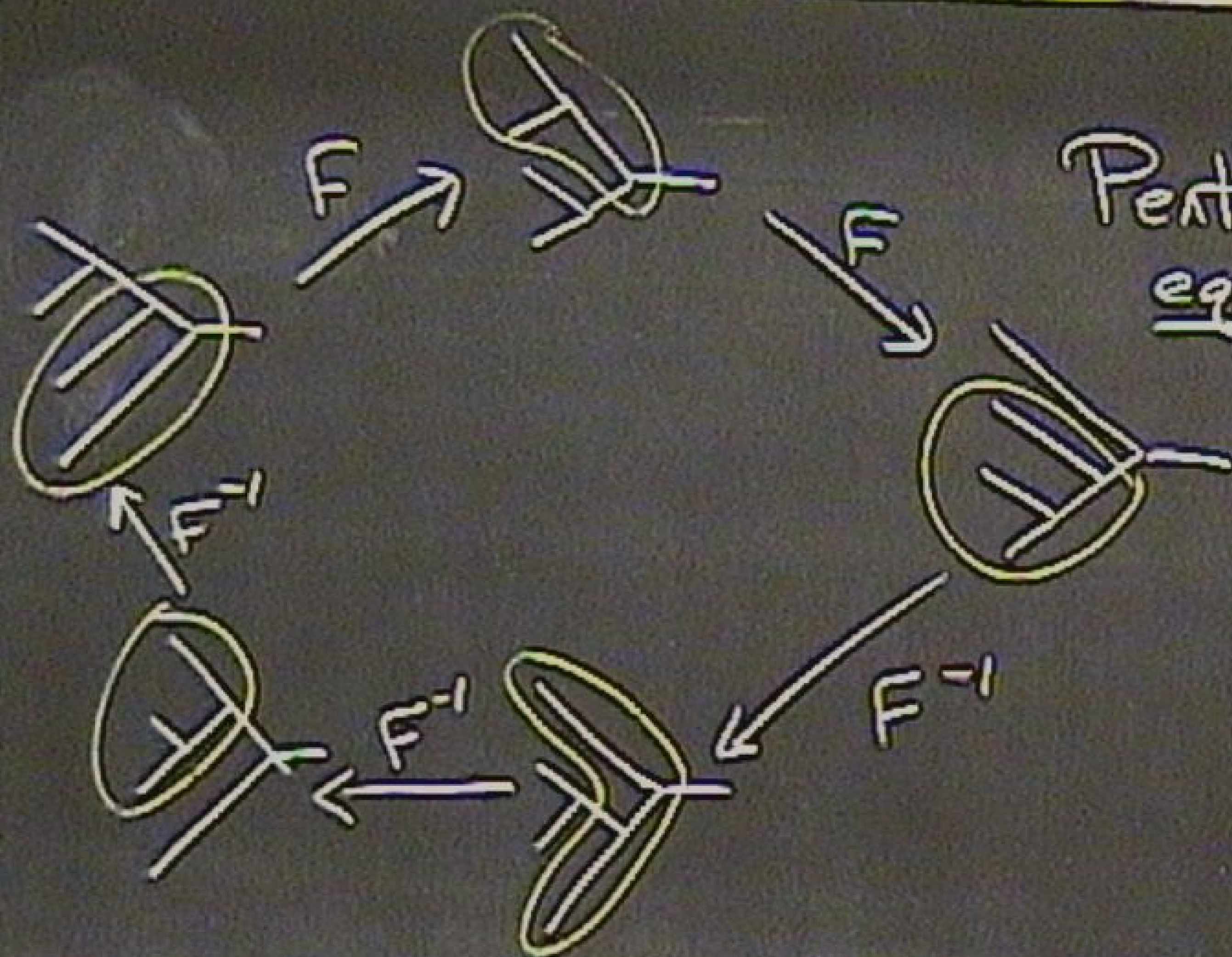

F
F


F
F
F^{-1}


F
F
F⁻¹
F⁻¹
F⁻¹

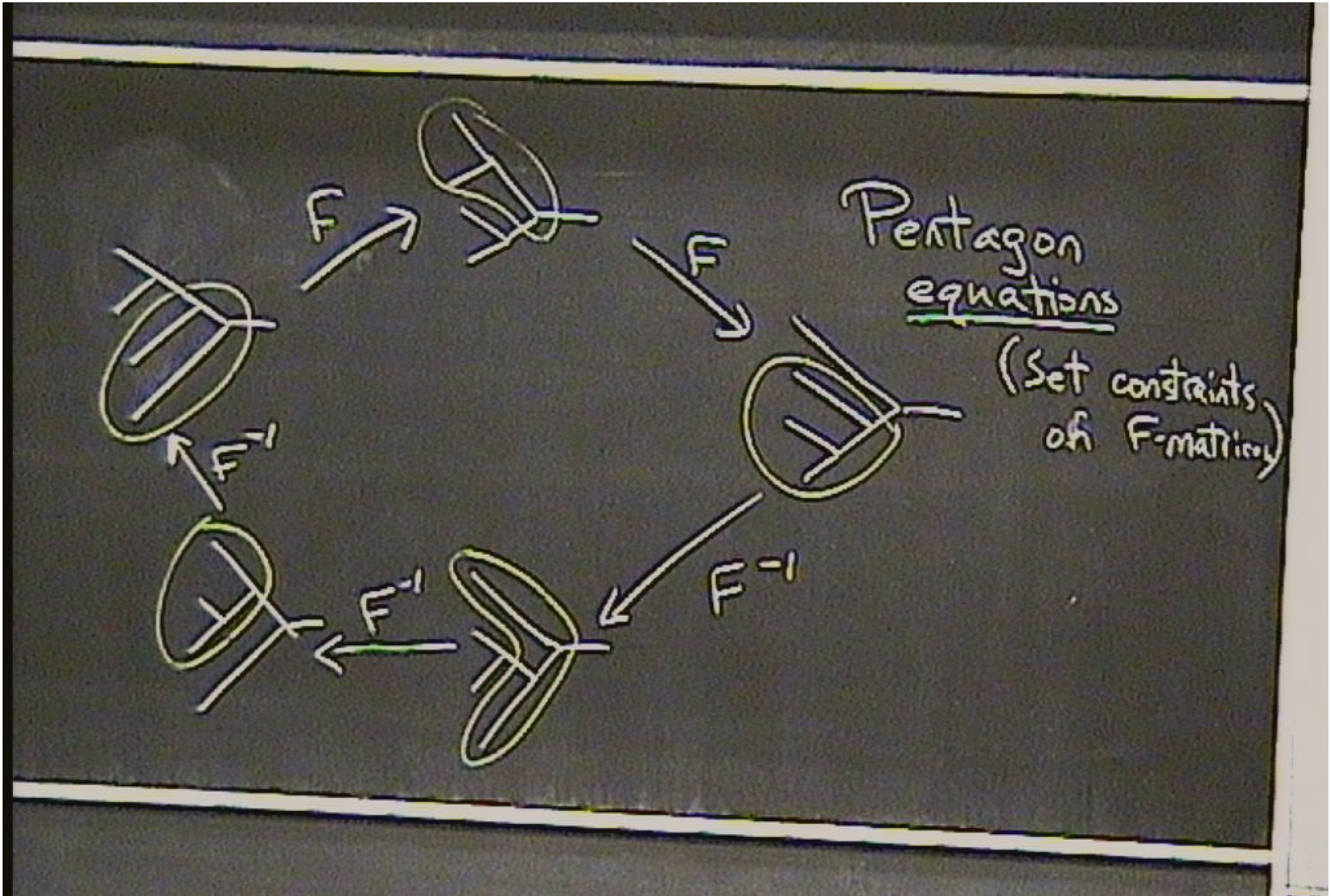


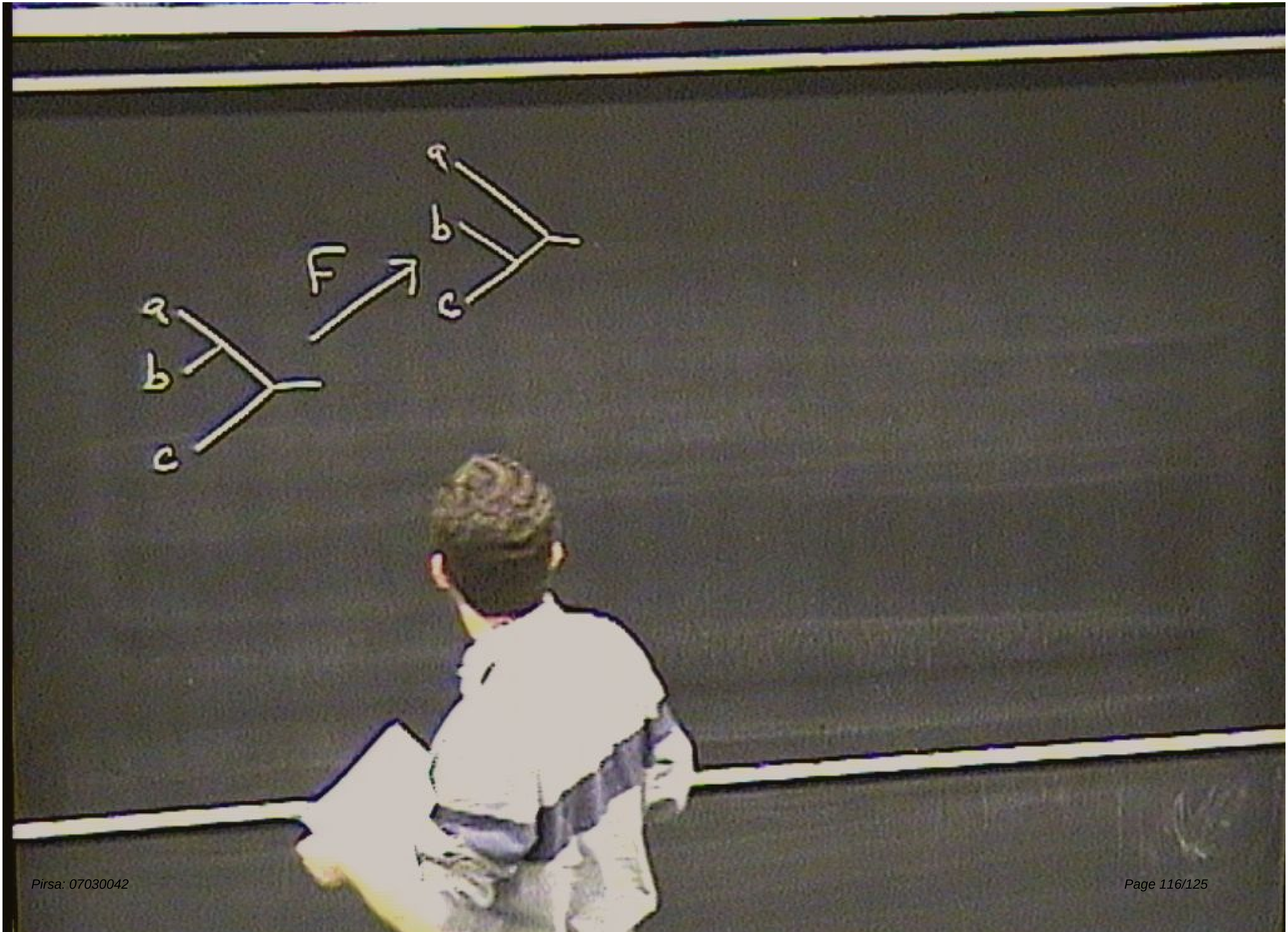
F
F
F^{-1}
F^{-1}
F^{-1}



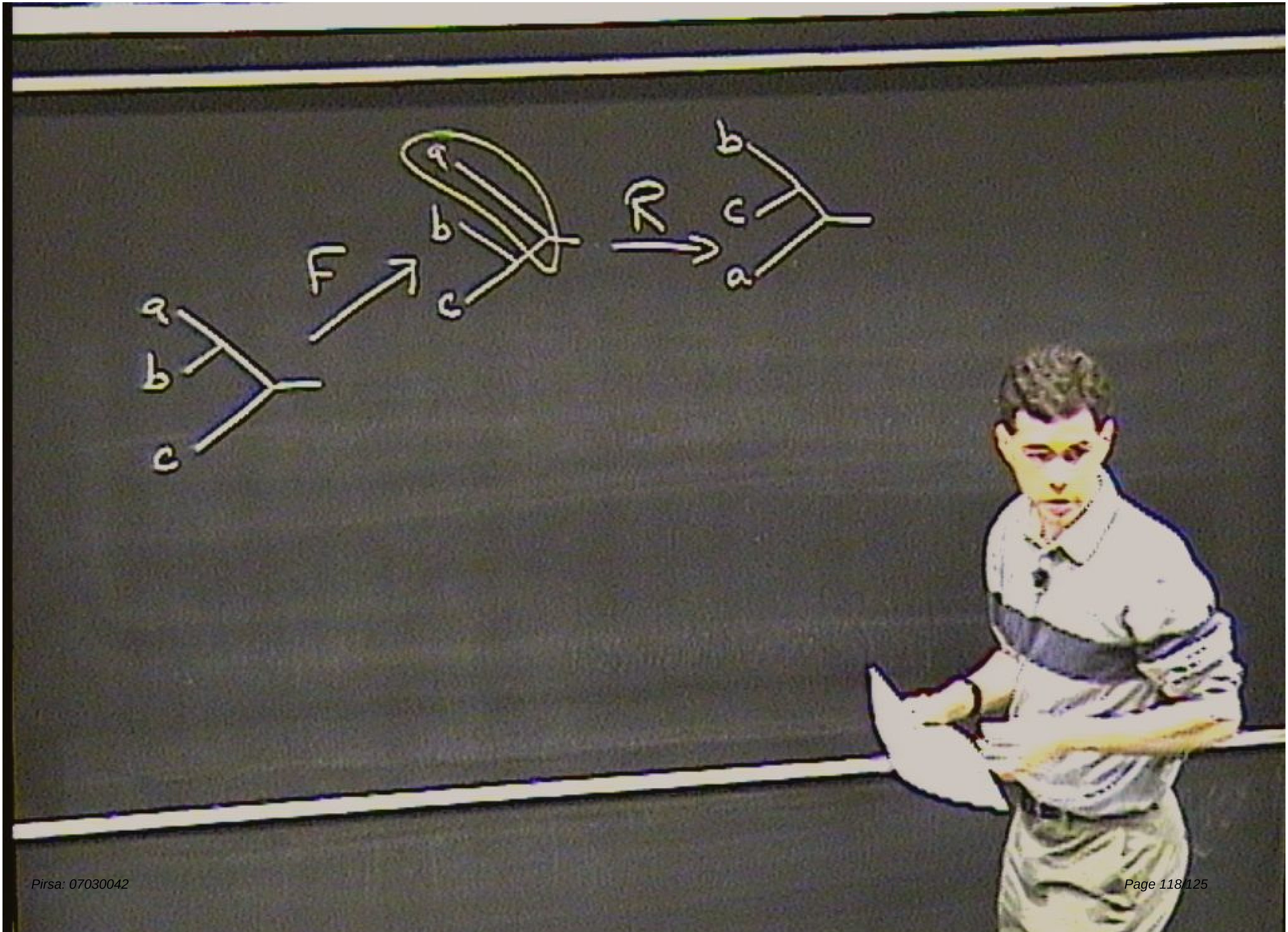
Pentagon equations

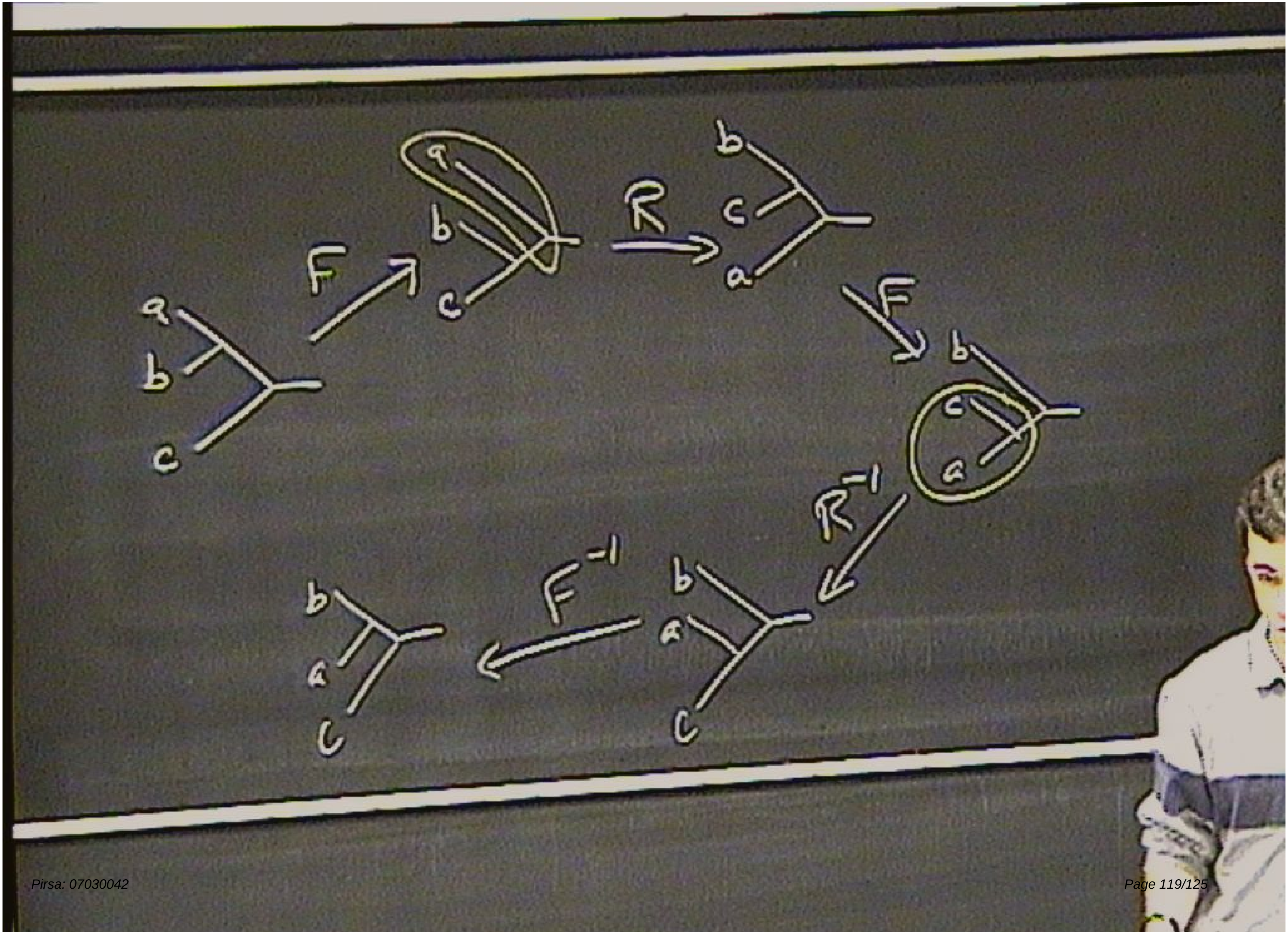
(Set constraints of F-matric


Pentagon
equations
(Set constraints
on F-matrices)
F
F
F^{-1}
F^{-1}
F^{-1}

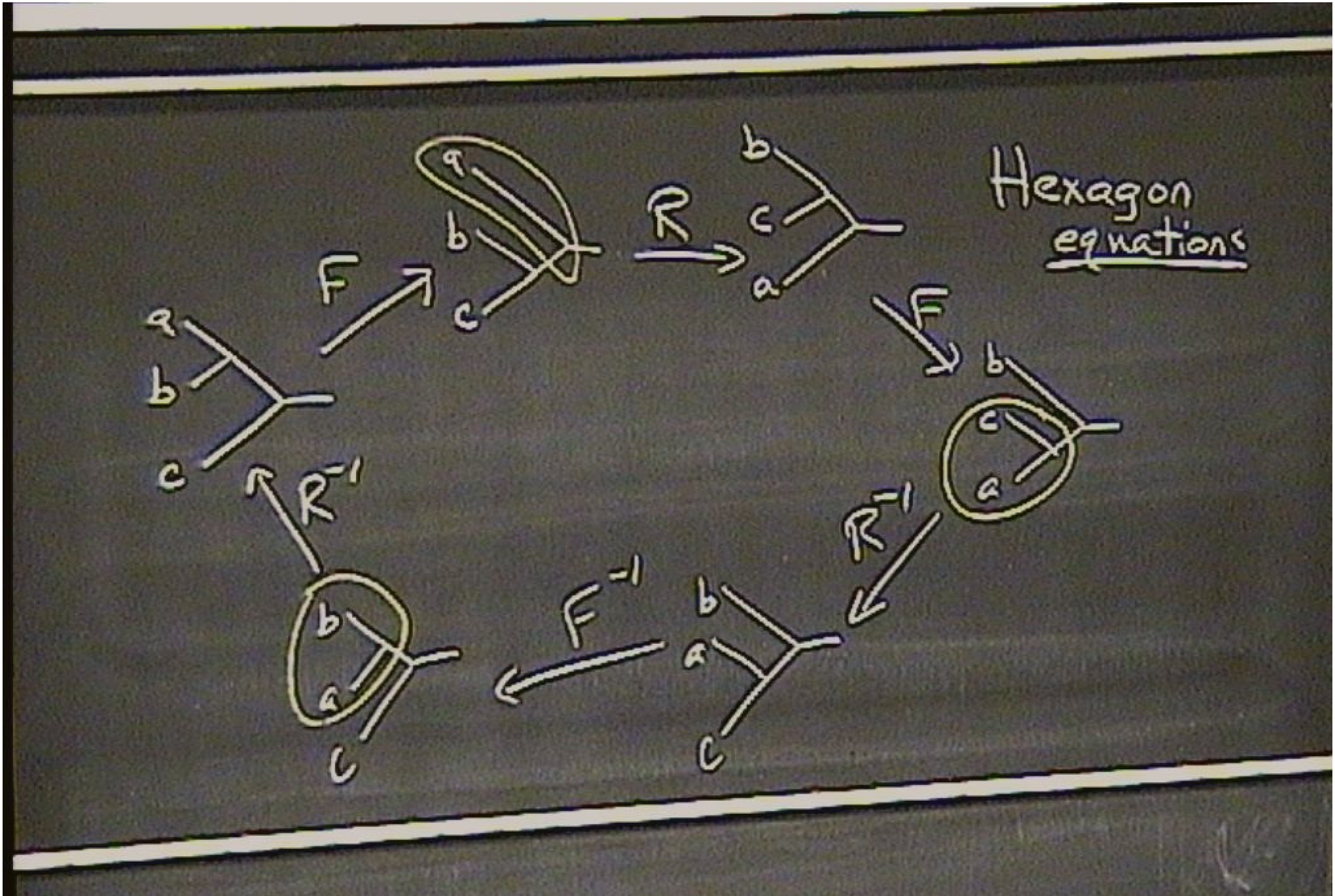

a
b
c
F
a
b
c


a
b
c
F
a
b
c

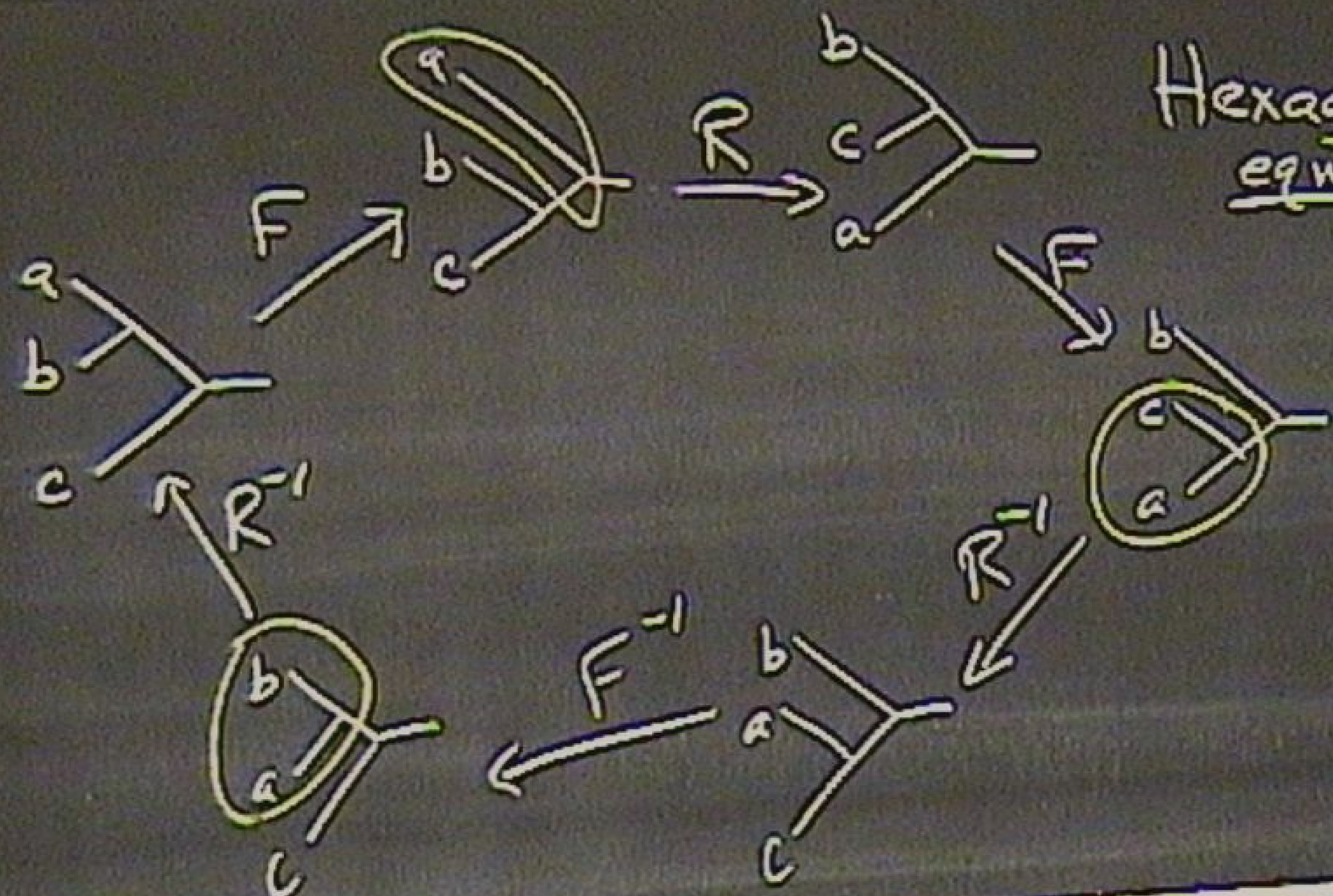

a
b
c
F
a
b
c
R
b
c
a

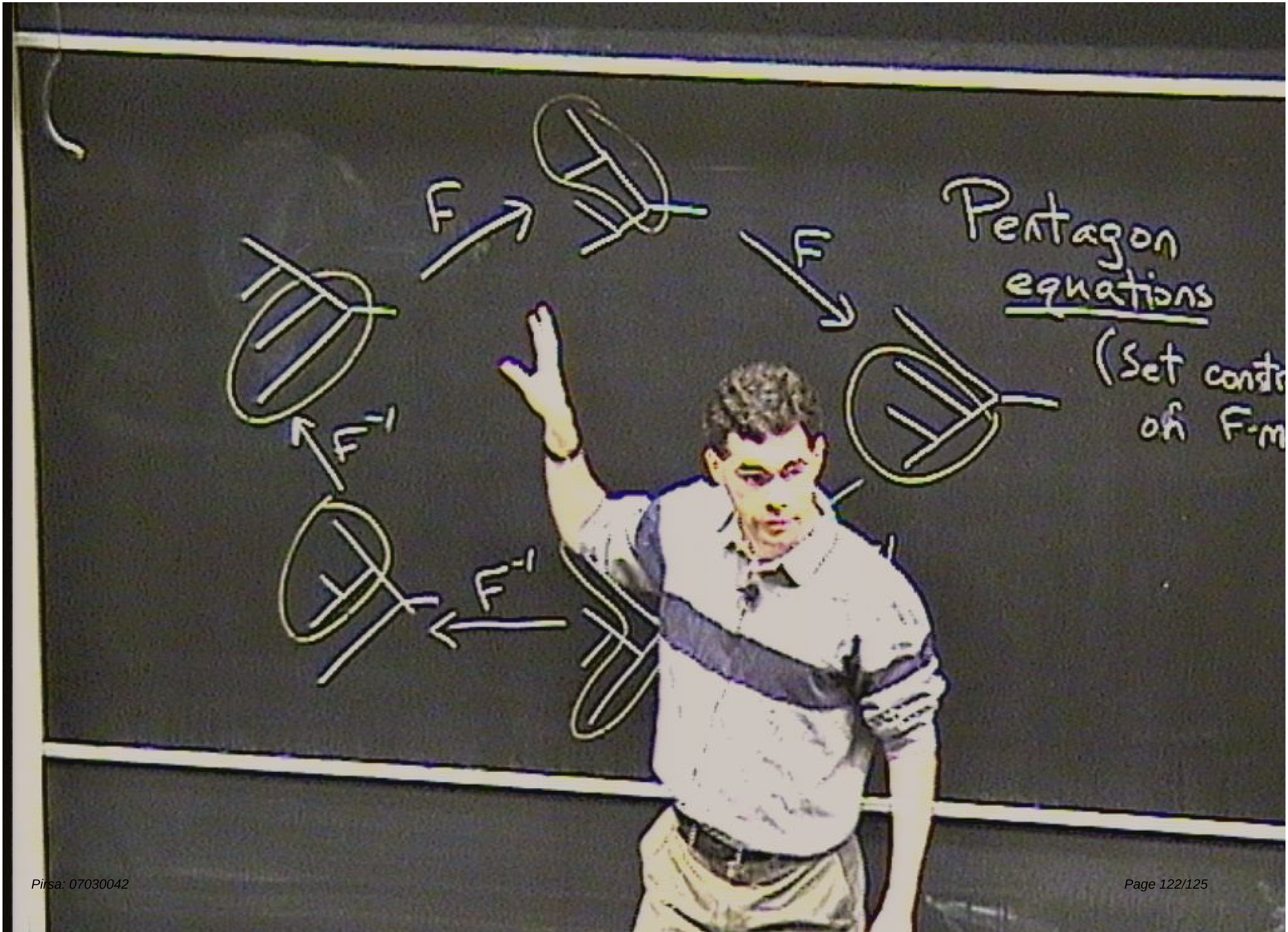


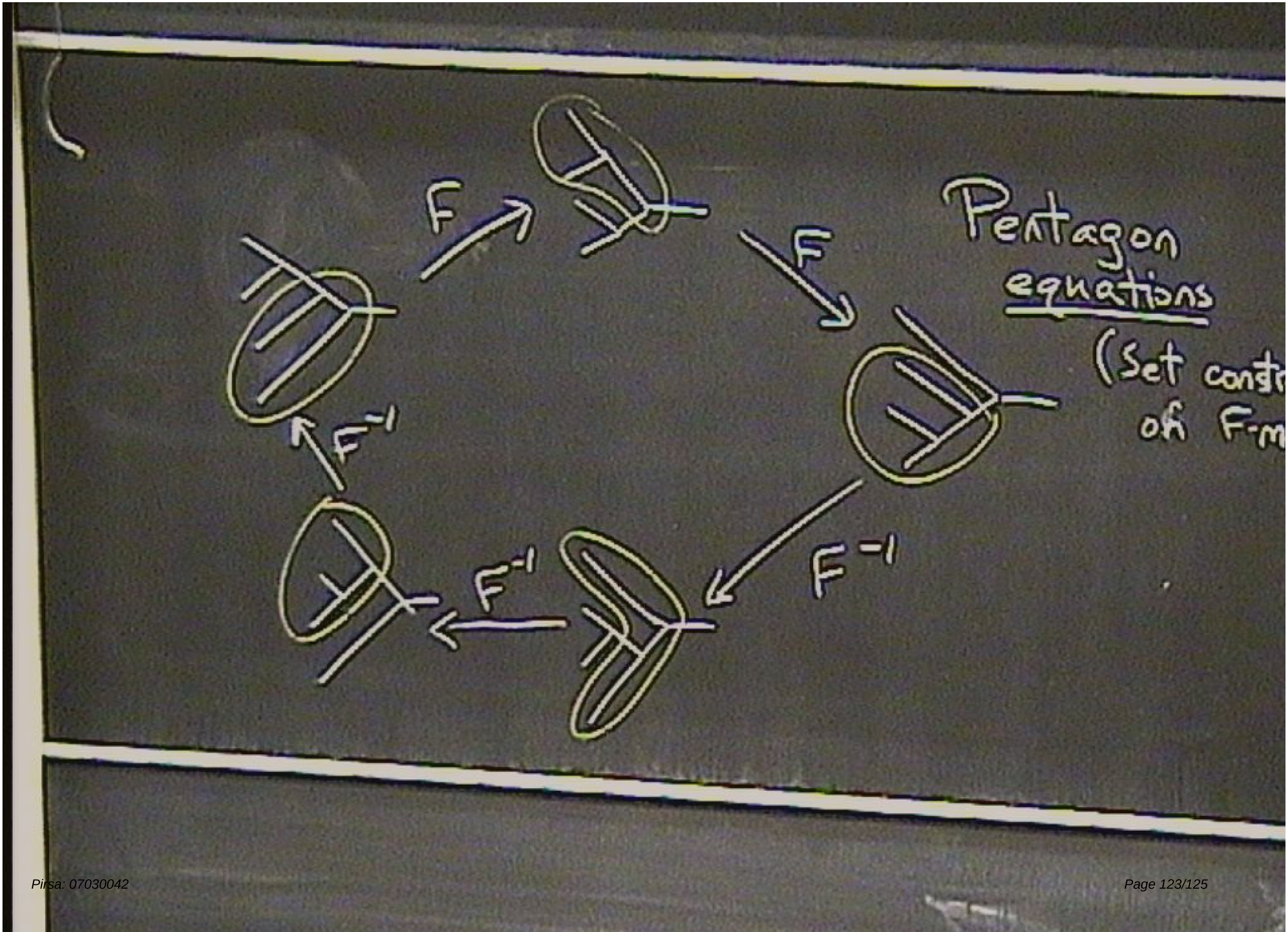
a
b
c
F
a
b
c
R
b
c
a
F
b
c
a
R^{-1}
b
a
c
F^{-1}
b
a
c

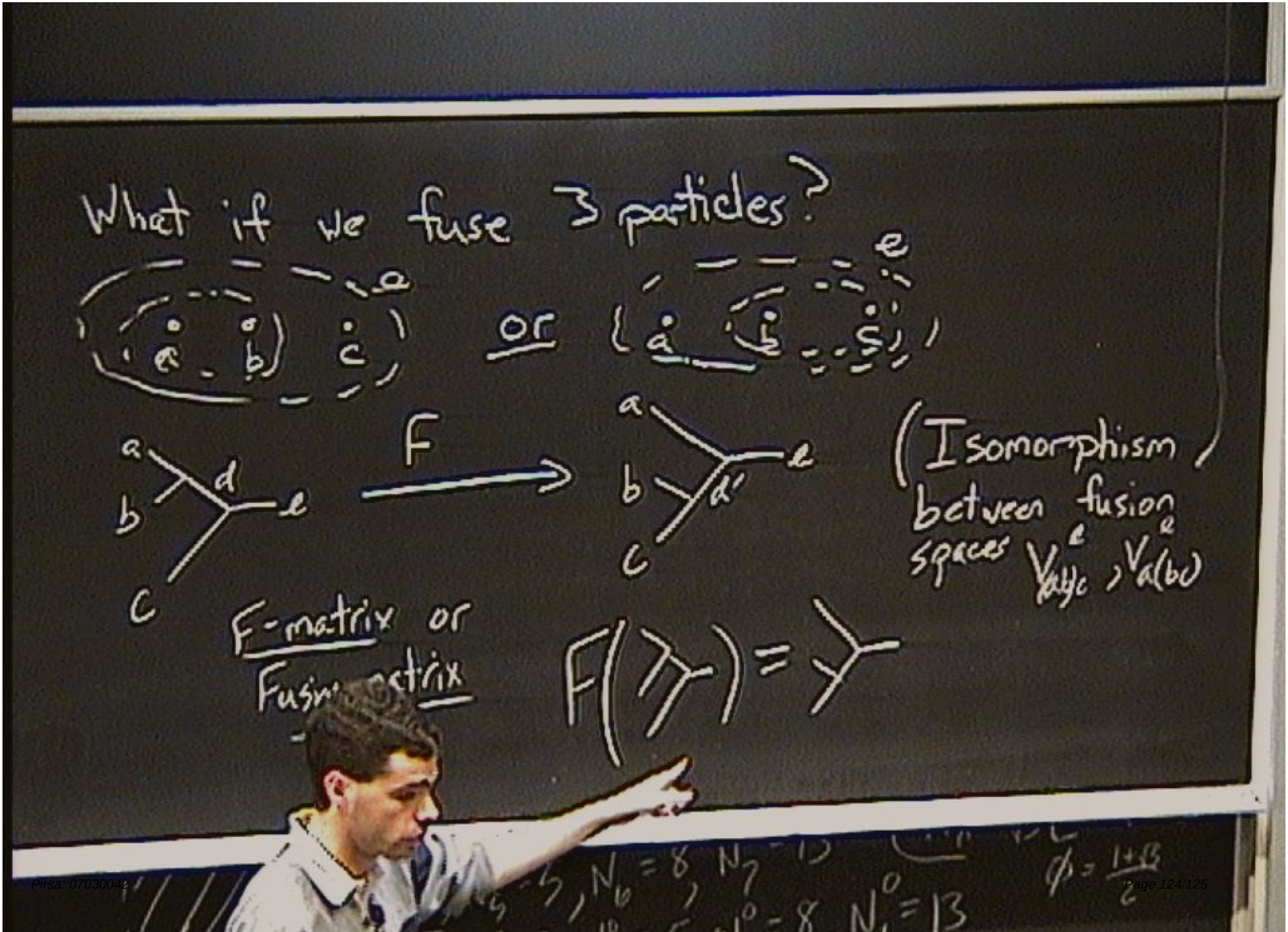

Hexagon equations
F
R
F
R⁻¹
F⁻¹
R⁻¹
a
b
c

Hexagon equations




F
F
F⁻¹
F⁻¹
Pentagon
equations
(Set


F
F
F⁻¹
F⁻¹
F⁻¹
Pentagon
equations
(Set
of

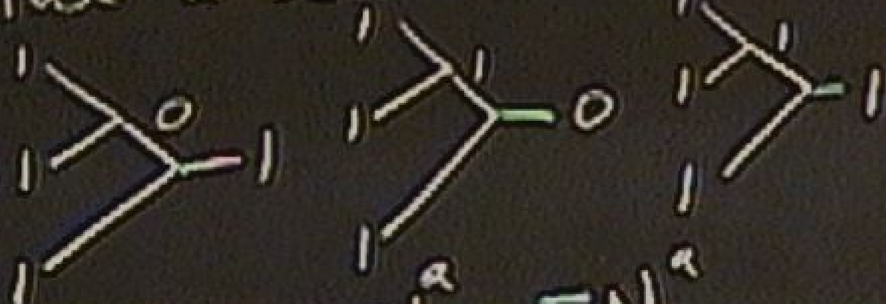

What if we fuse 3 particles?
or
F
(Isomorphism between fusion spaces $V^e_{(ab)c}$, $V^e_{a(bc)}$)
F-matrix or Fusion matrix

F-matrix or Fusion matrix

$F\Big($  $\Big) =$ 

For Fibonacci anyons:

Fuse a series of 1s



$N^0_{111} = 1$, $N^1_{111} = 2$

$N^1_2 = 1$, $N^1_3 = 2$

n 1 particles $N^a_{\underbrace{1\ldots1}_{n}} \equiv N^a_n$

$N^0_n = N^1_{n-1}$, $N^1_n = N^0_{n-1} + N^1_{n-1} = N^1_{n-1} + N^1_{n-2}$

$N^0_n = \frac{1}{\sqrt{5}}\left[\phi^n + \left(-\frac{1}{\phi}\right)^n\right]$

$\phi = \frac{1+\sqrt{5}}{2}$

$N^1_4 = 3$, $N^1_5 = 5$, $N^1_6 = 8$, $N^1_7 = 13$

$\boxed{N^0_4 = 2}$, $N^0_5 = 3$, $N^0_6 = 5$, $N^0_7 = 8$, $N^0_8 = 13$