

Title: The power of forgetting

Date: Mar 26, 2007 11:00 AM

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Abstract: Thermodynamics places surprisingly few fundamental constraints on information processing. In fact, most people would argue that it imposes only one, known as Landauer's Principle: a process erasing one bit of information must release an amount  $kT \ln 2$  of heat. It is this simple observation that finally led to the exorcism of Maxwell's Demon from statistical mechanics, more than a century after he first appeared.

Ignoring the lesson implicit in this early advance, however, quantum information theorists have been surprisingly slow to embrace erasure as a fundamental primitive. Over the past couple of years, however, it has become clear that a detailed understanding of how difficult it is to erase correlations leads to a nearly complete synthesis and simplification of the known results of asymptotic quantum information theory. As it turns out, surprisingly many of the tasks of interest, from distilling high-quality entanglement to sending quantum data through a noisy medium to many receivers, can be understood as variants of erasure. I'll sketch the main ideas behind these discoveries and end with some speculations on what lessons the new picture might have for understanding information loss in real physical systems.

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# The power of forgetting

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Patrick Hayden (McGill University)

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# Landauer's Principle

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State 1

# Landauer's Principle

**Erasure:** A process that, regardless of the input state, results in output state 0

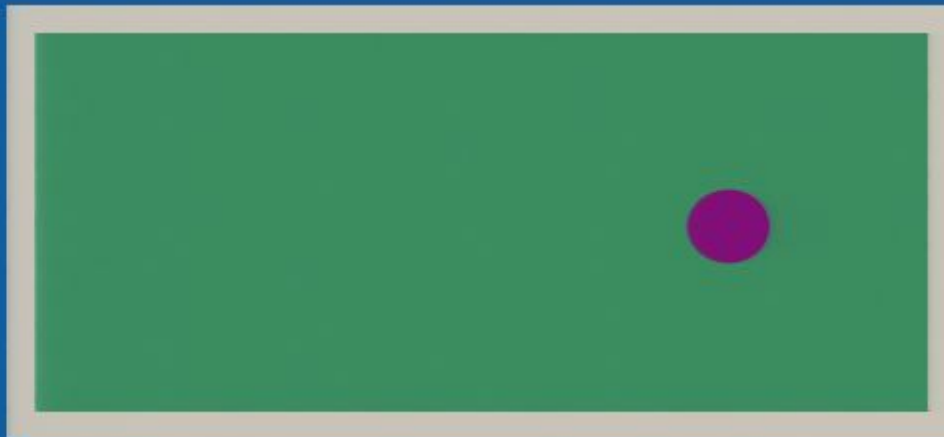


State 1



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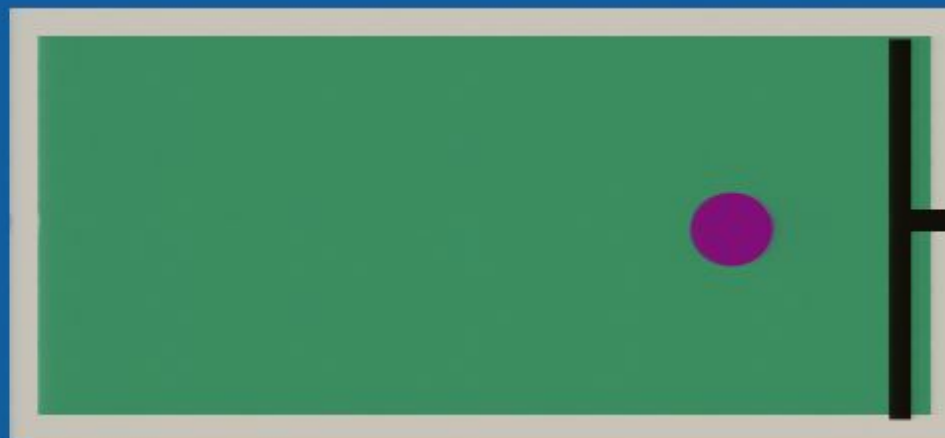
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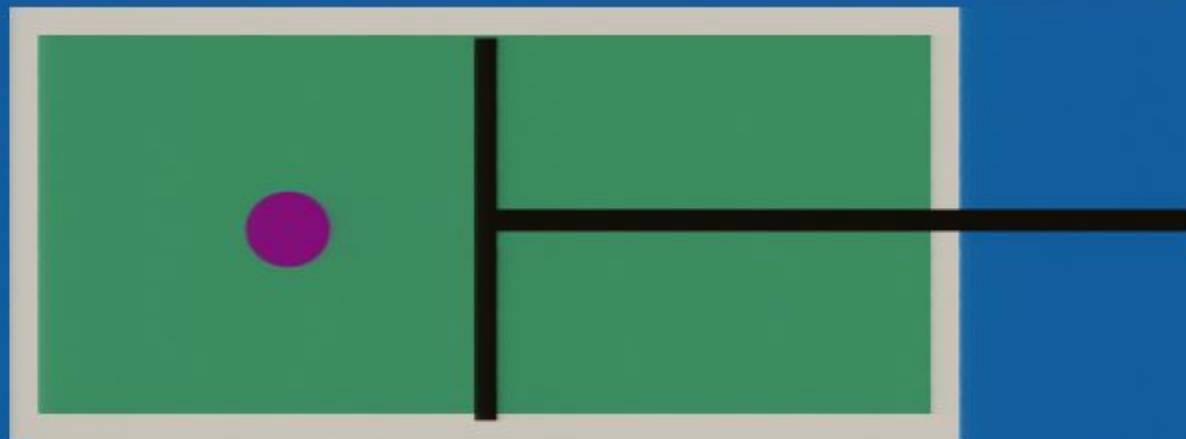
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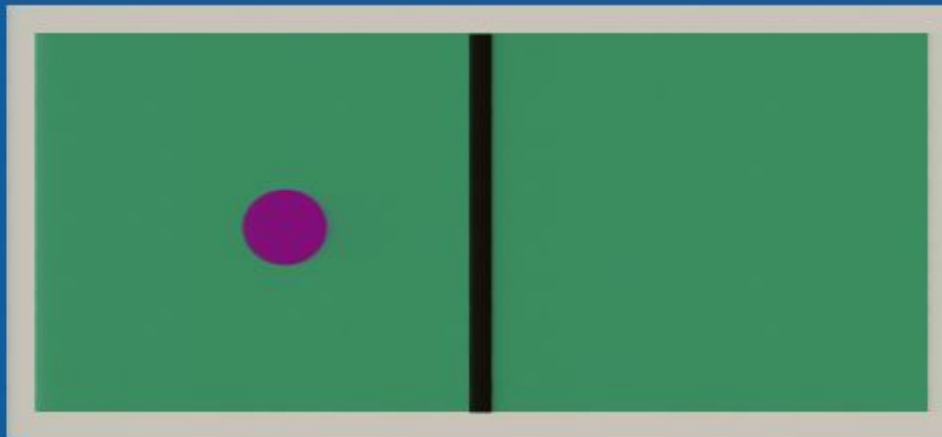
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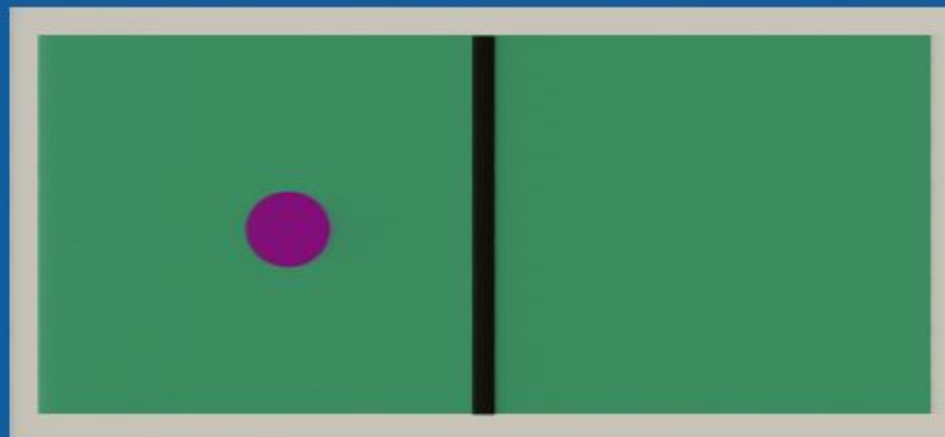
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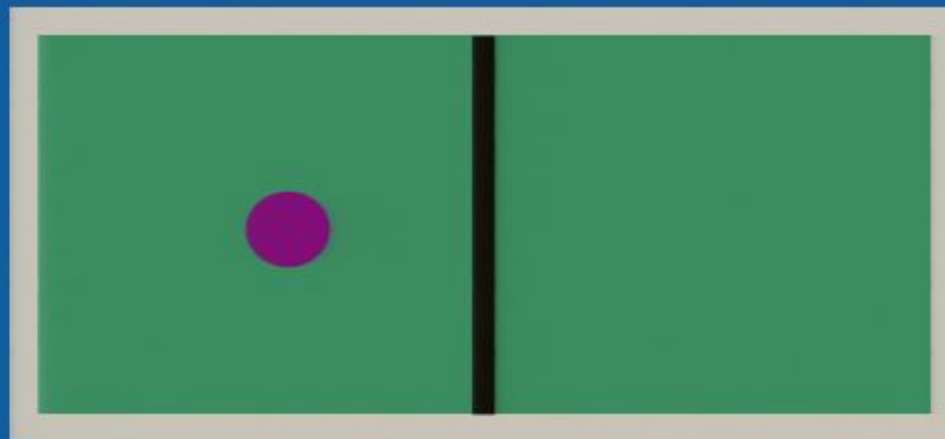


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Erasing information requires a process that can reduce in uncertainty

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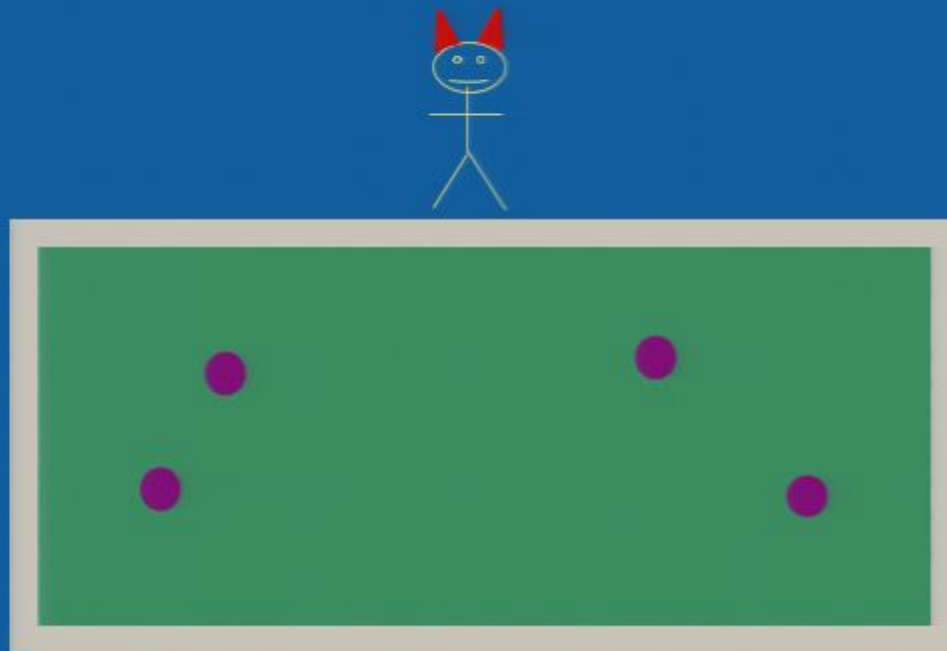
Erasing information requires a process that can reduce in uncertainty

Thermodynamic entropy of the gas is reduced by  $k \ln 2$  so amount  $kT \ln 2$  of heat released to the environment.



# Maxwell's Demon

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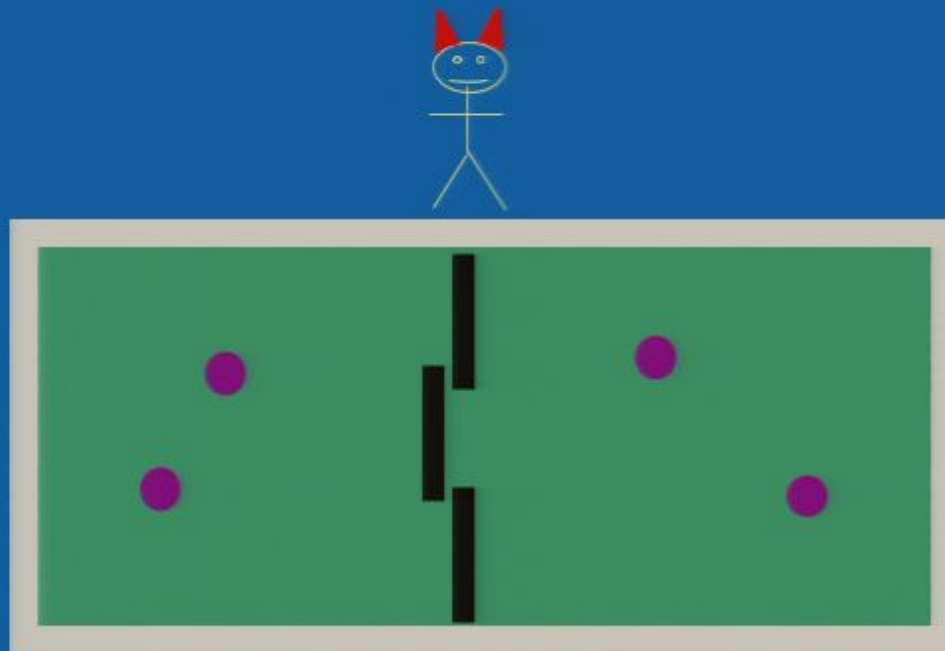


Gas originally at equilibrium. Demon inserts partition.



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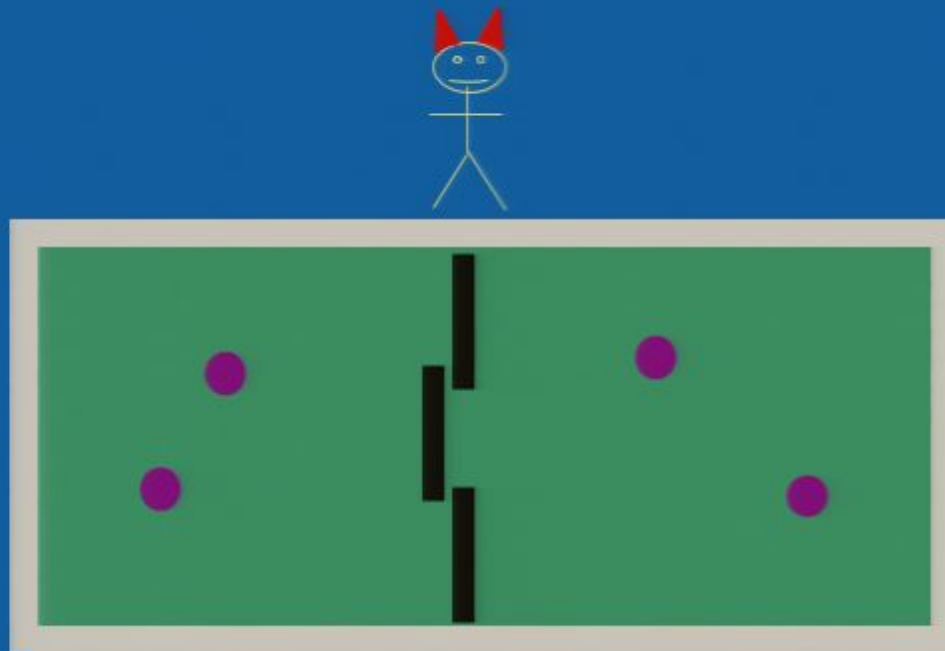


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Demon uses door to allow particles to move left to right, but not right to left.

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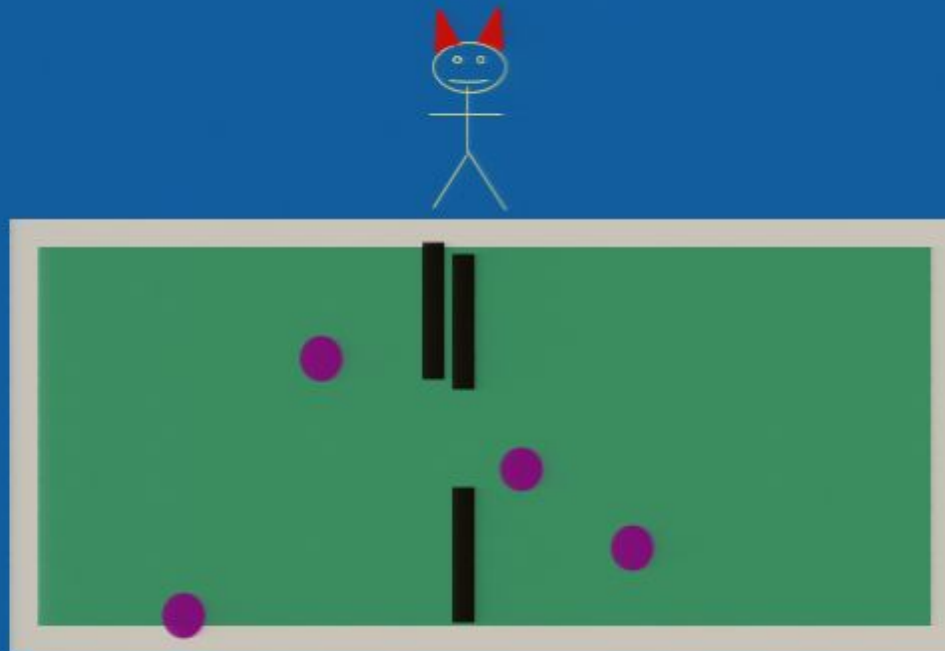
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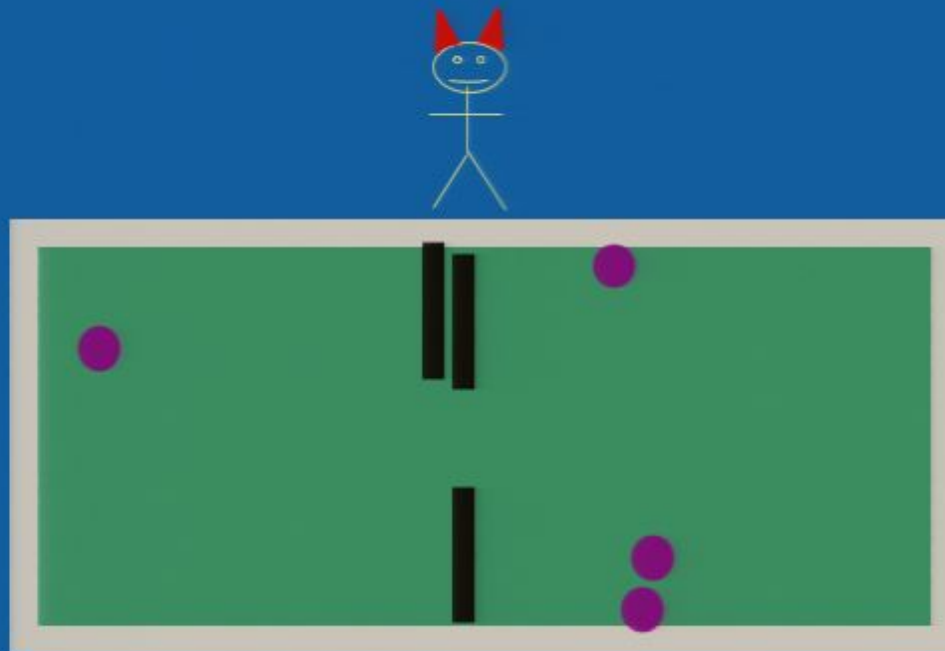


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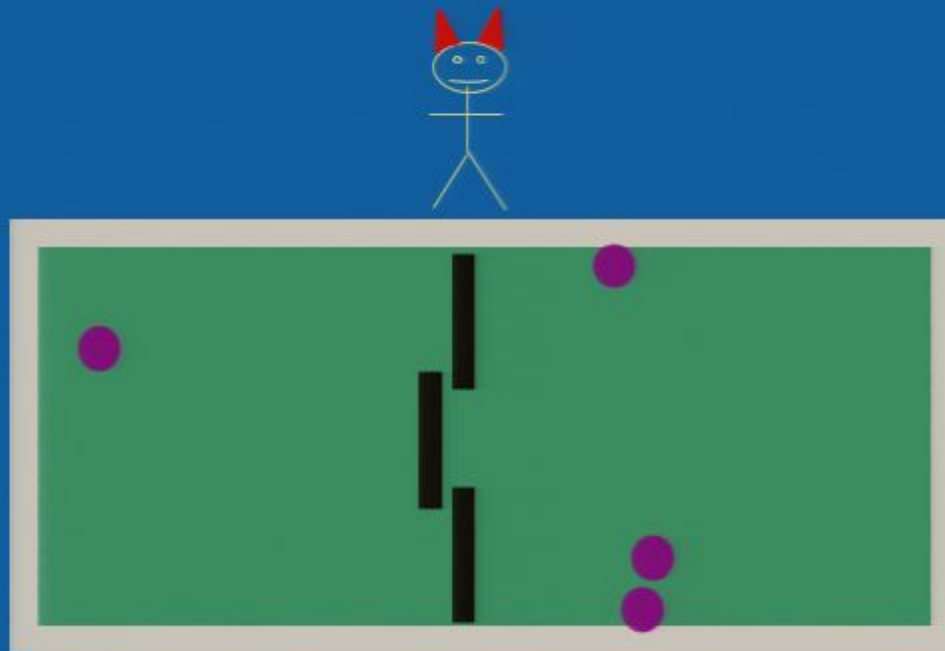


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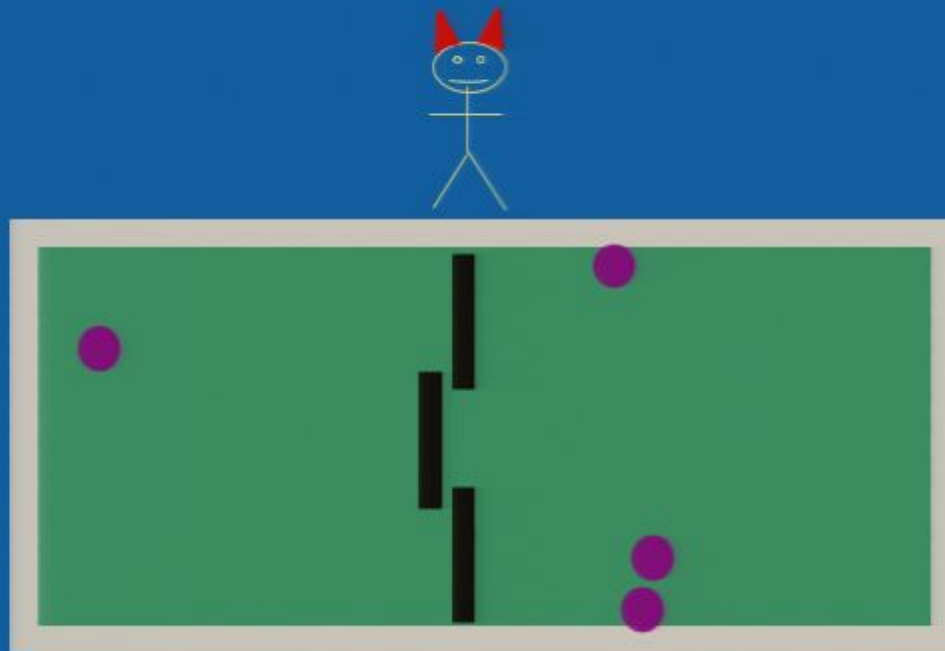
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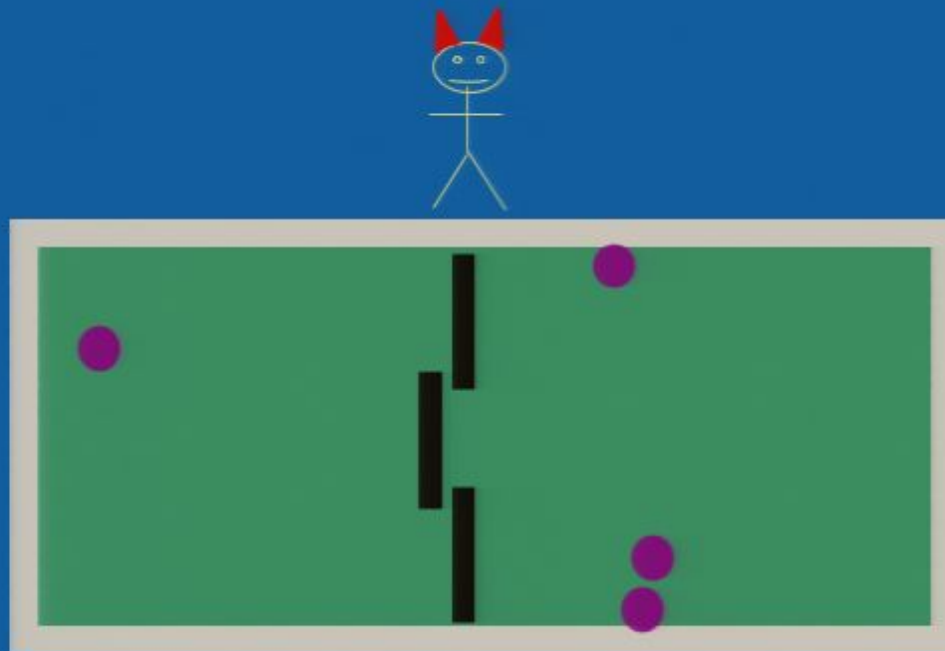


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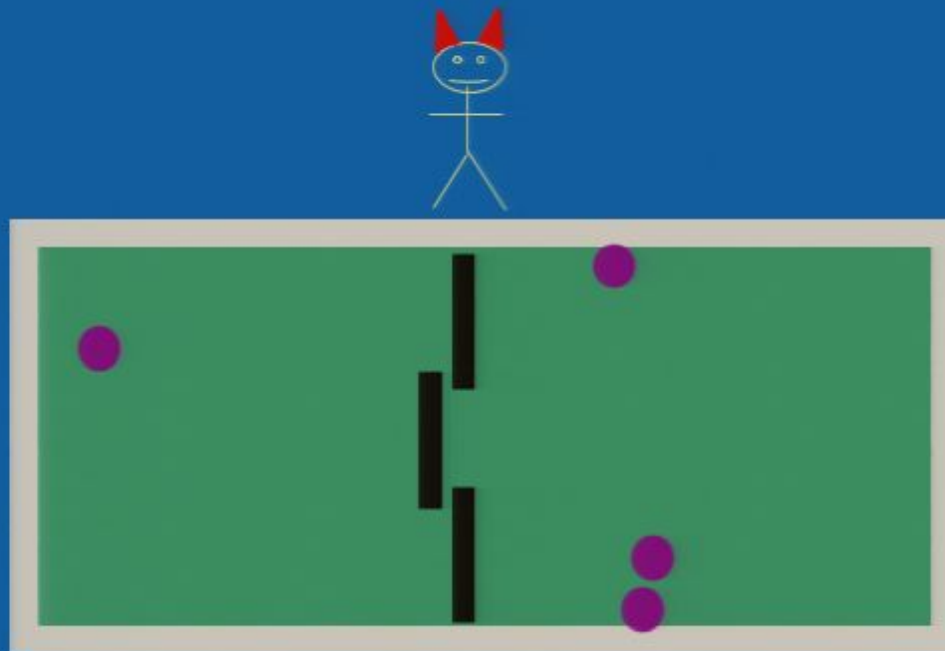


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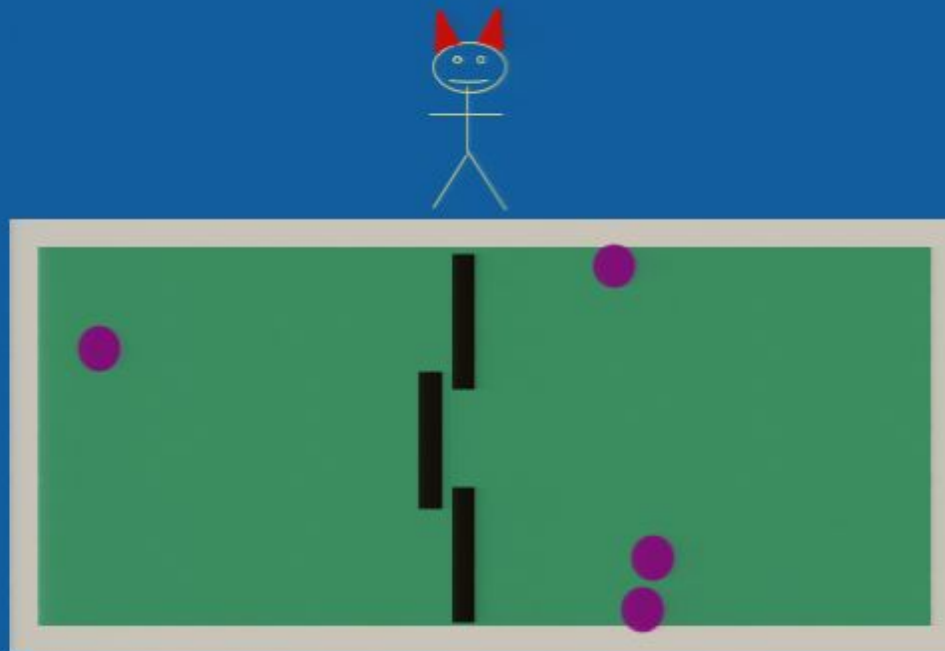
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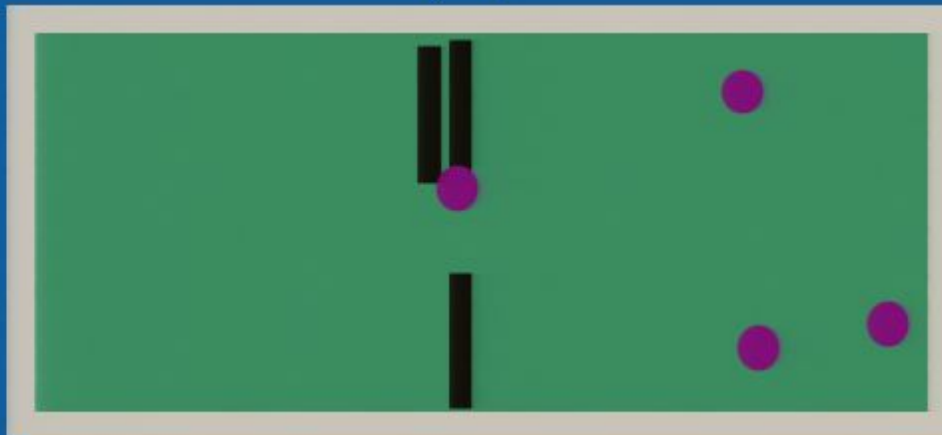


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Particle coming!



Another one!



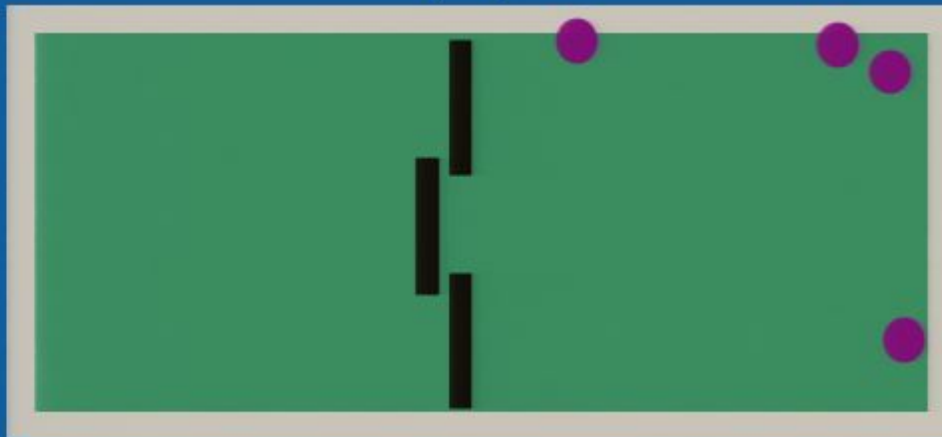
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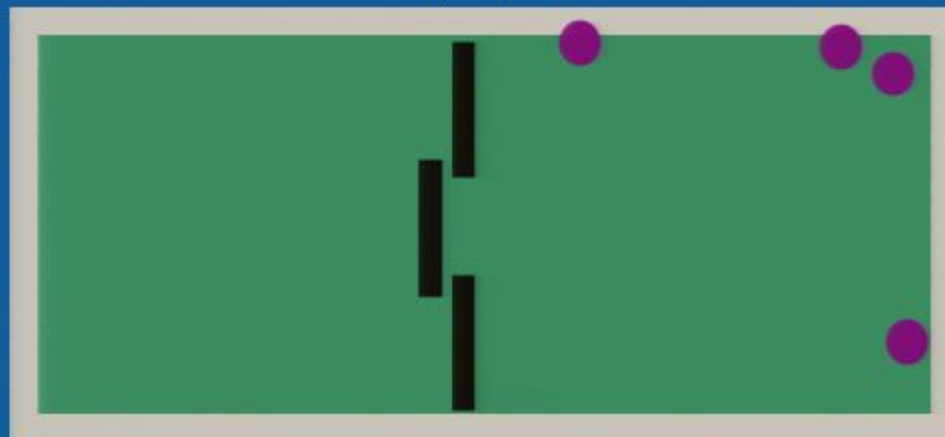
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**Finite memory:** he must eventually start *forgetting*.

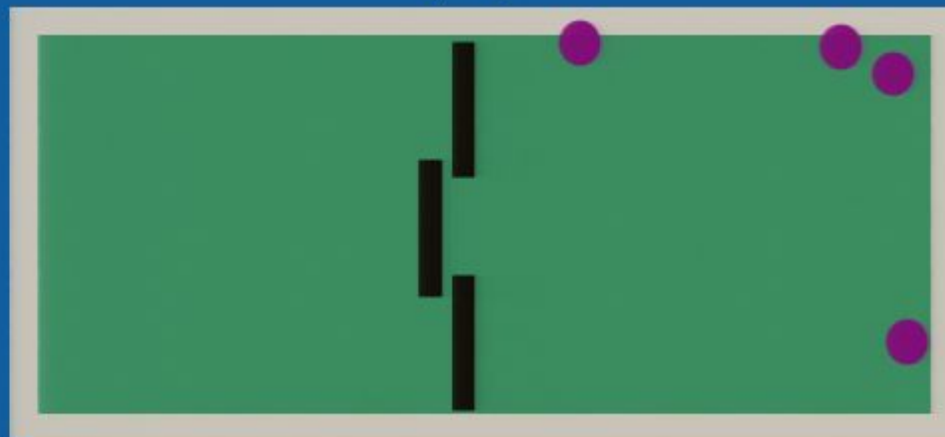


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*Landauer's principle* restores global increase of entropy.

# Church of the Larger Hilbert Space





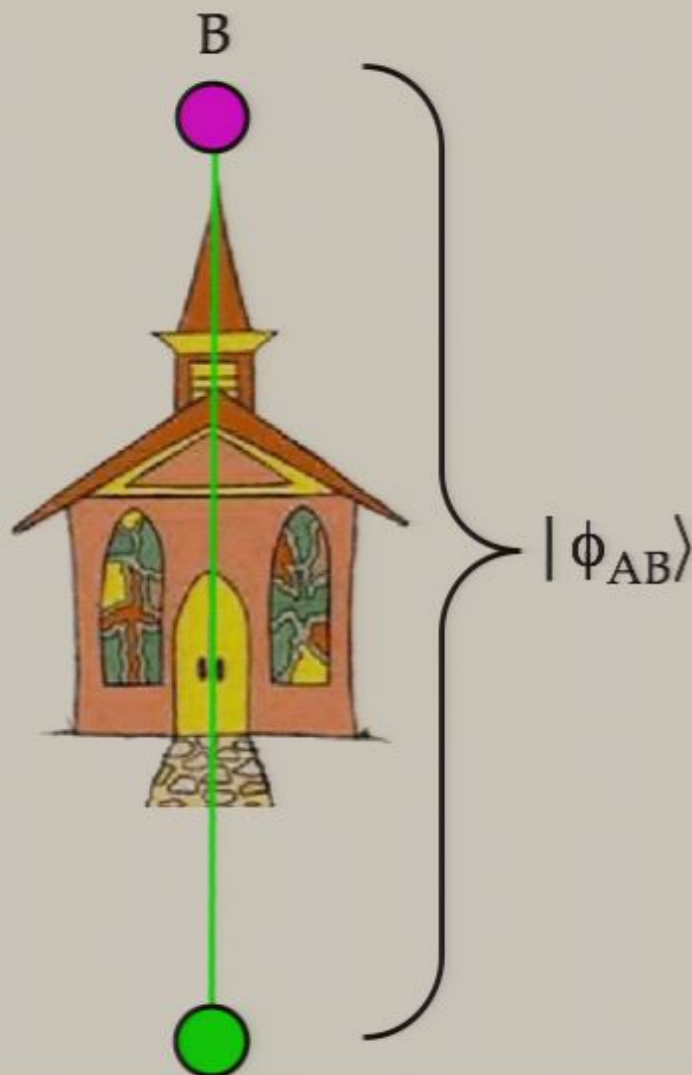
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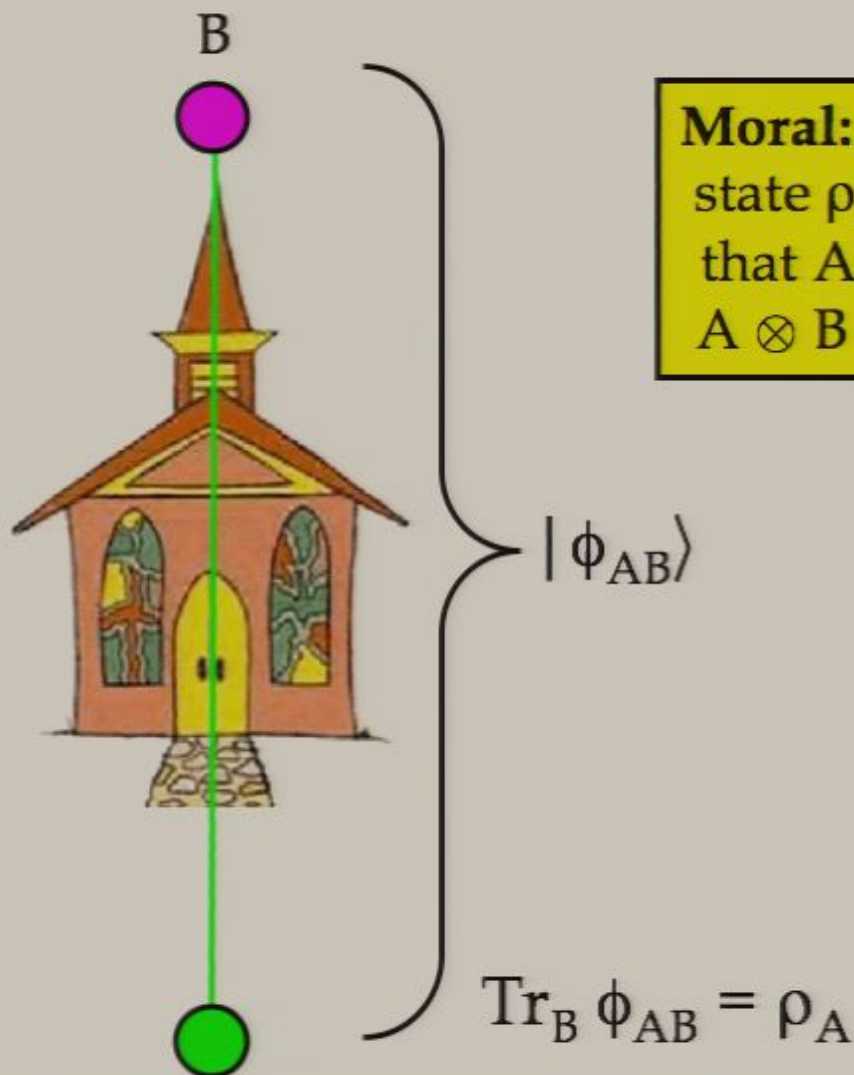
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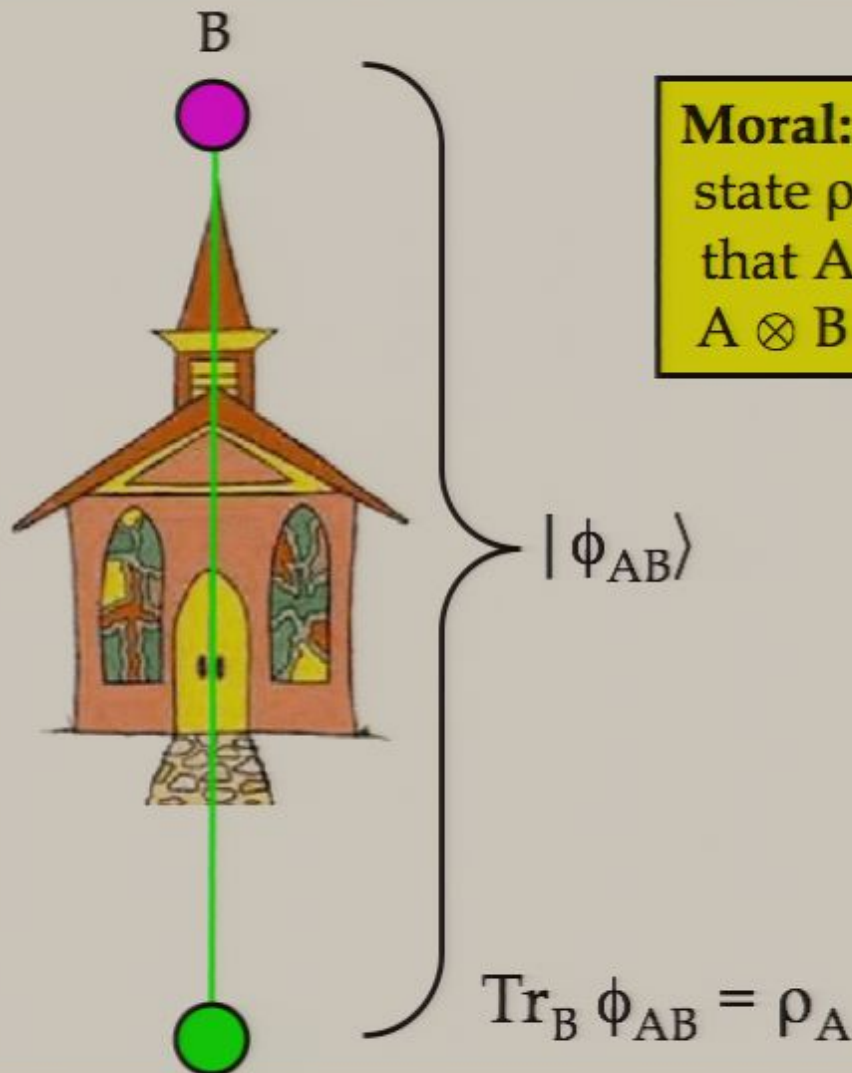
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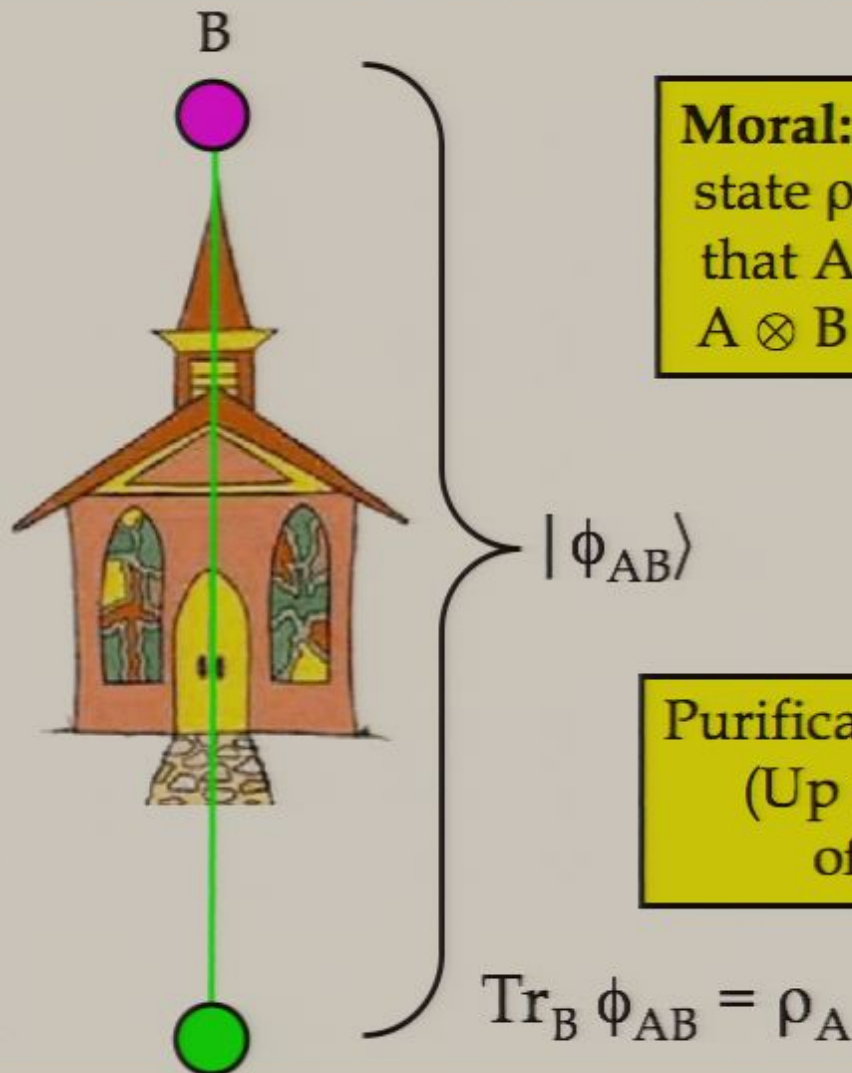


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(Up to local transformations  
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# Purification and correlation



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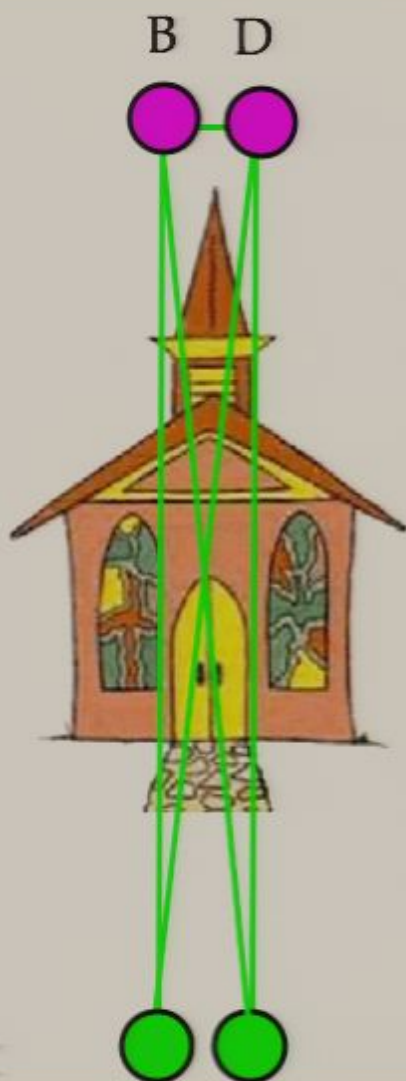
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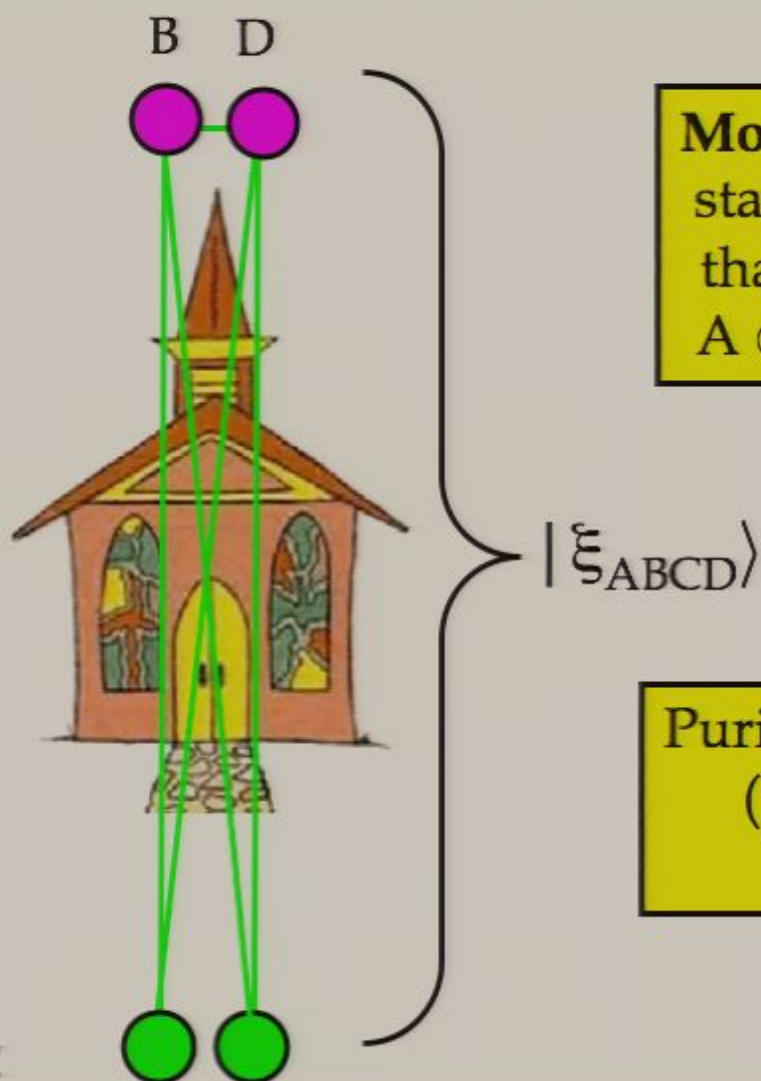
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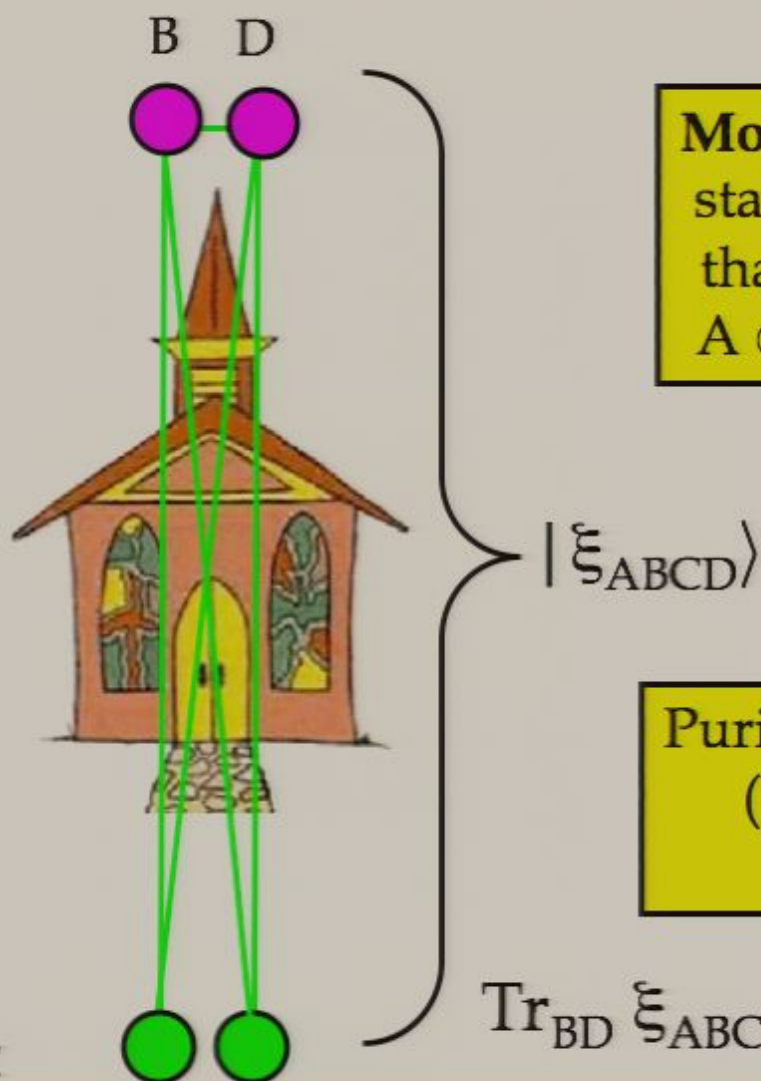


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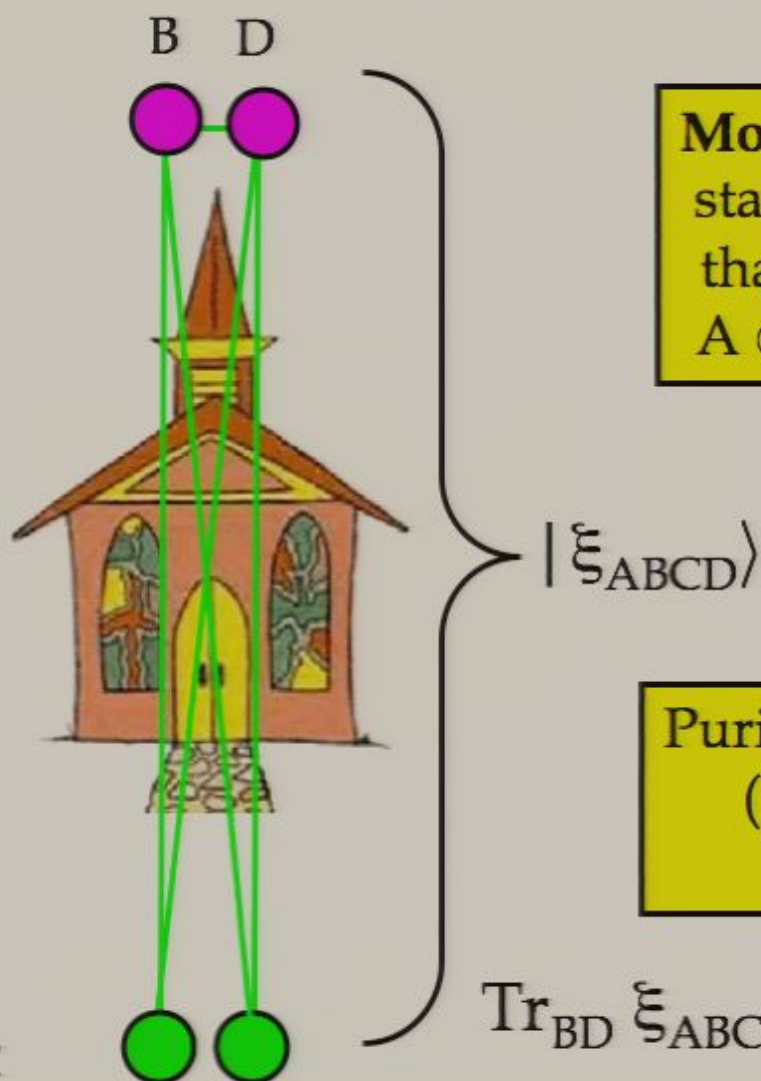


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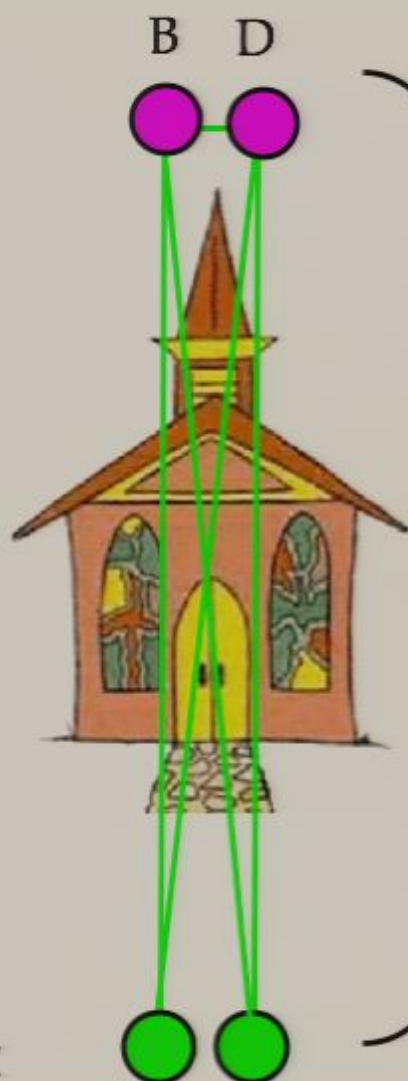
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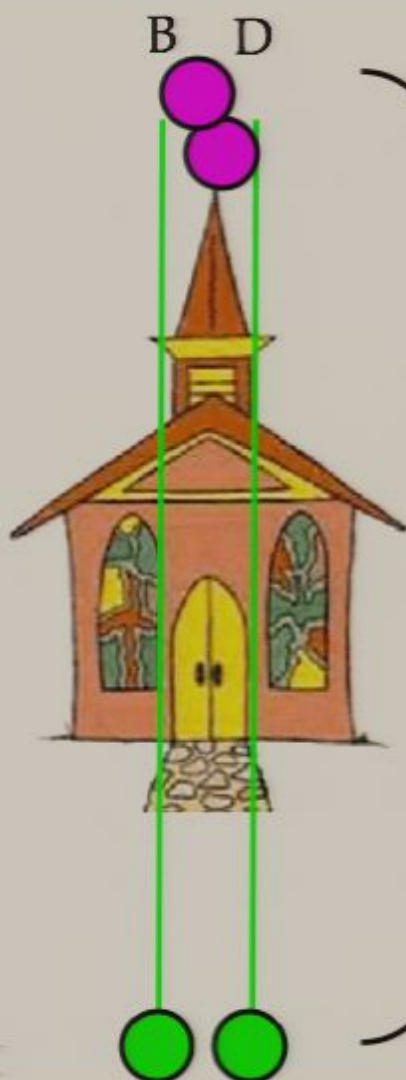
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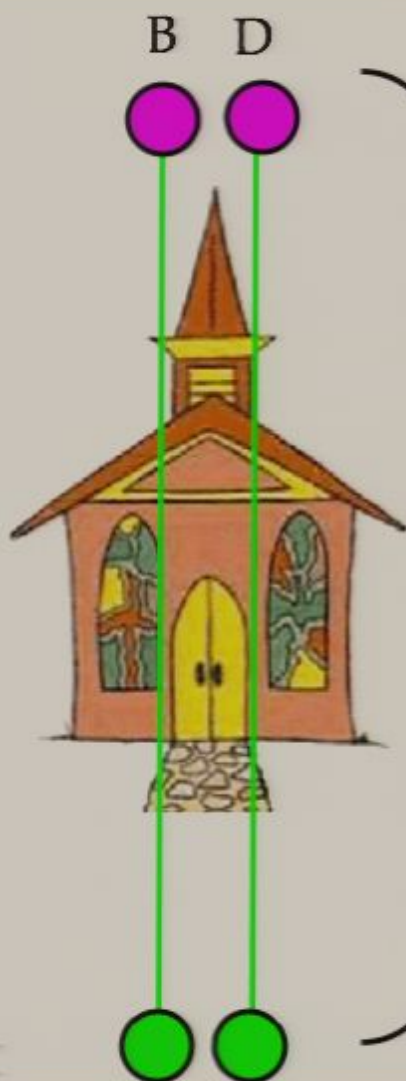
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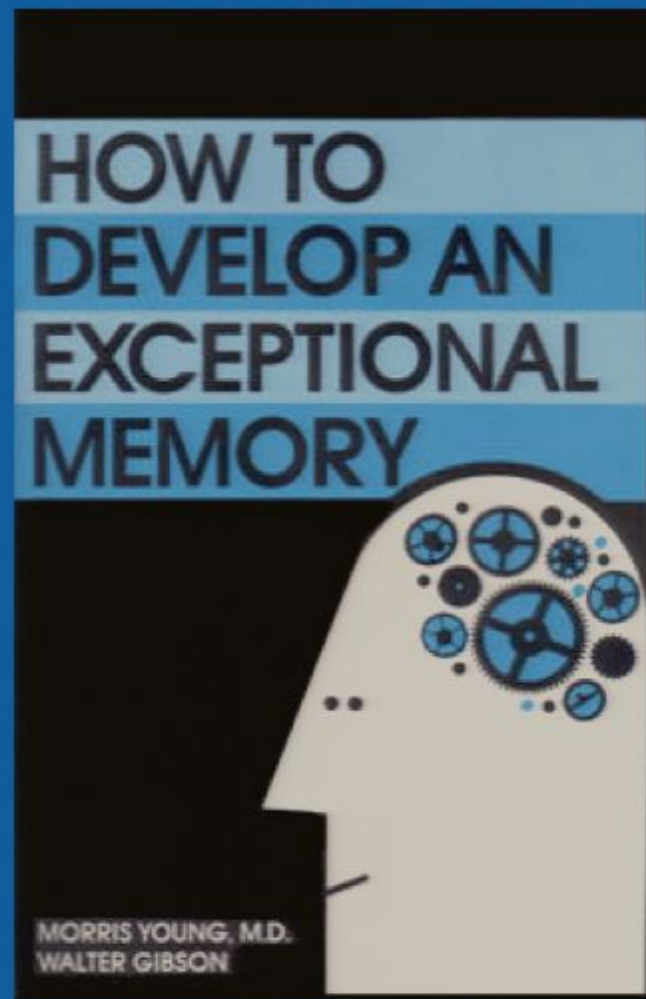
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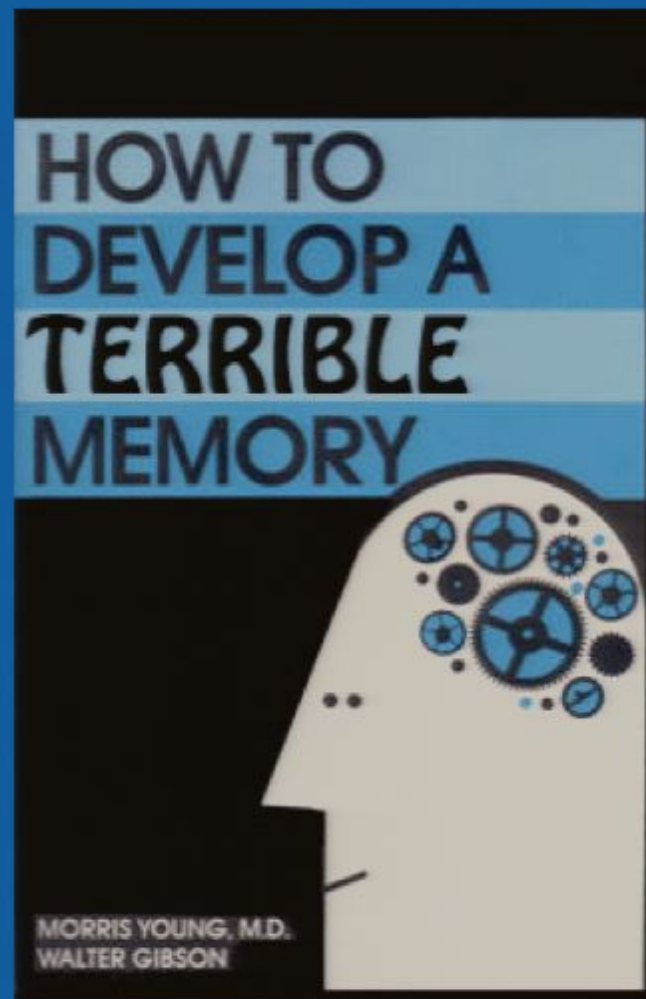
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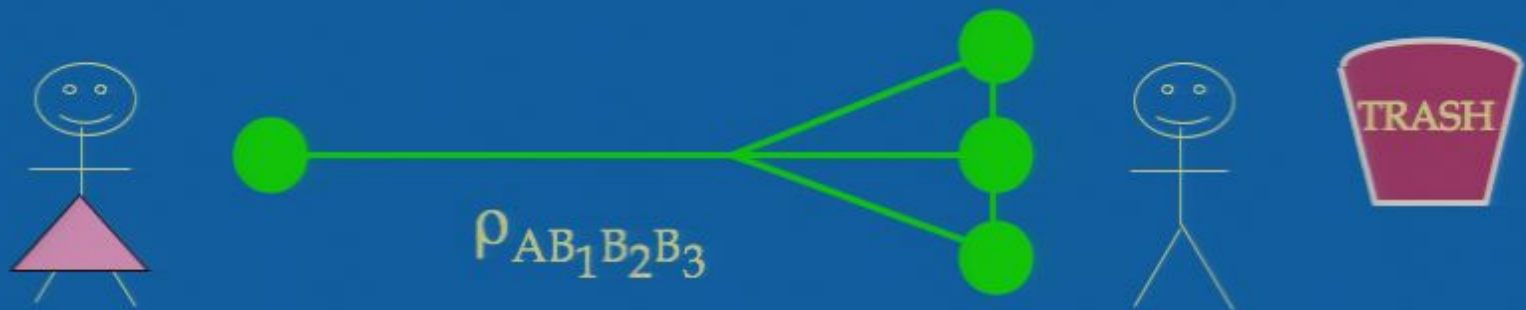


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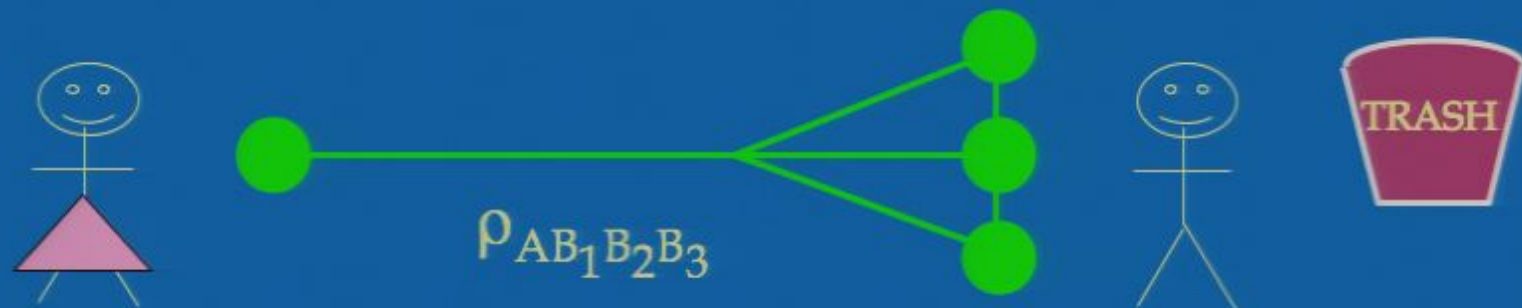


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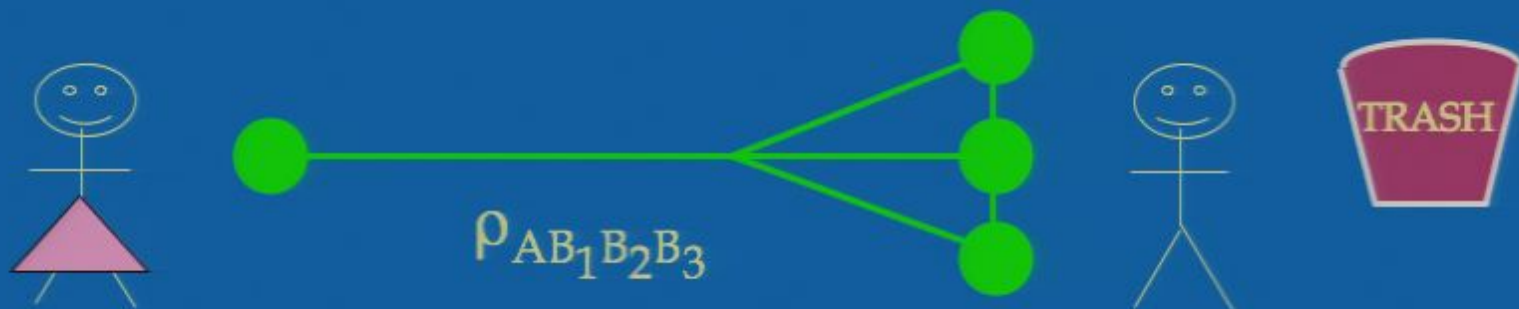


# The art of forgetting



How can Bob unilaterally destroy his correlation with Alice?

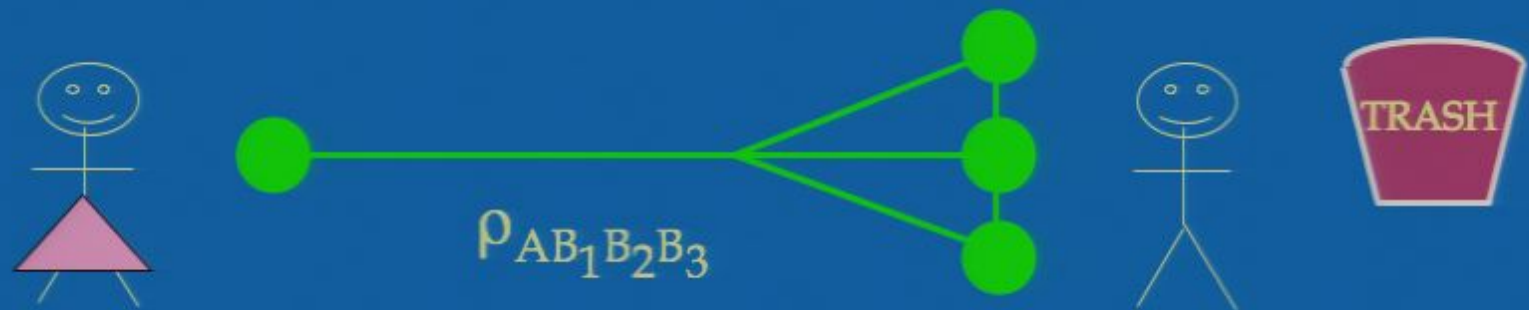
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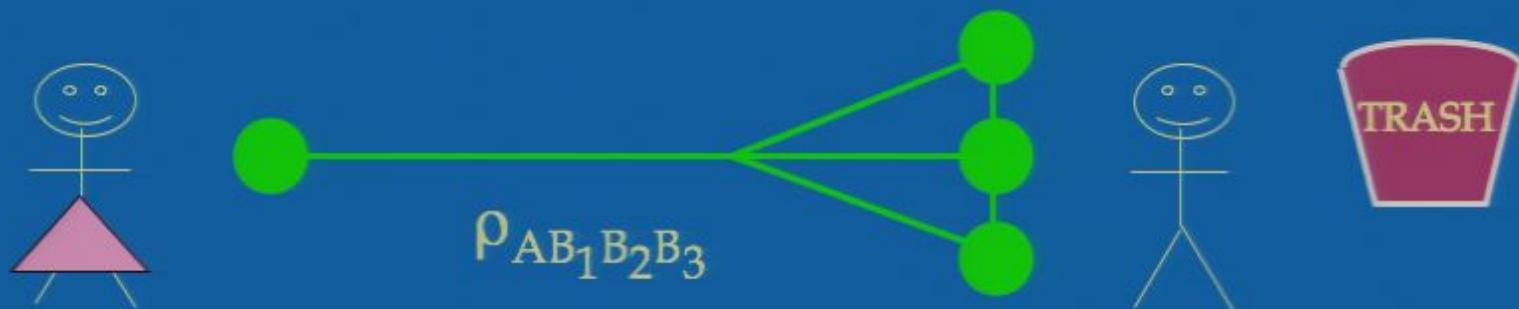
What is the minimal number of particles he must discard before the remaining state is uncorrelated?

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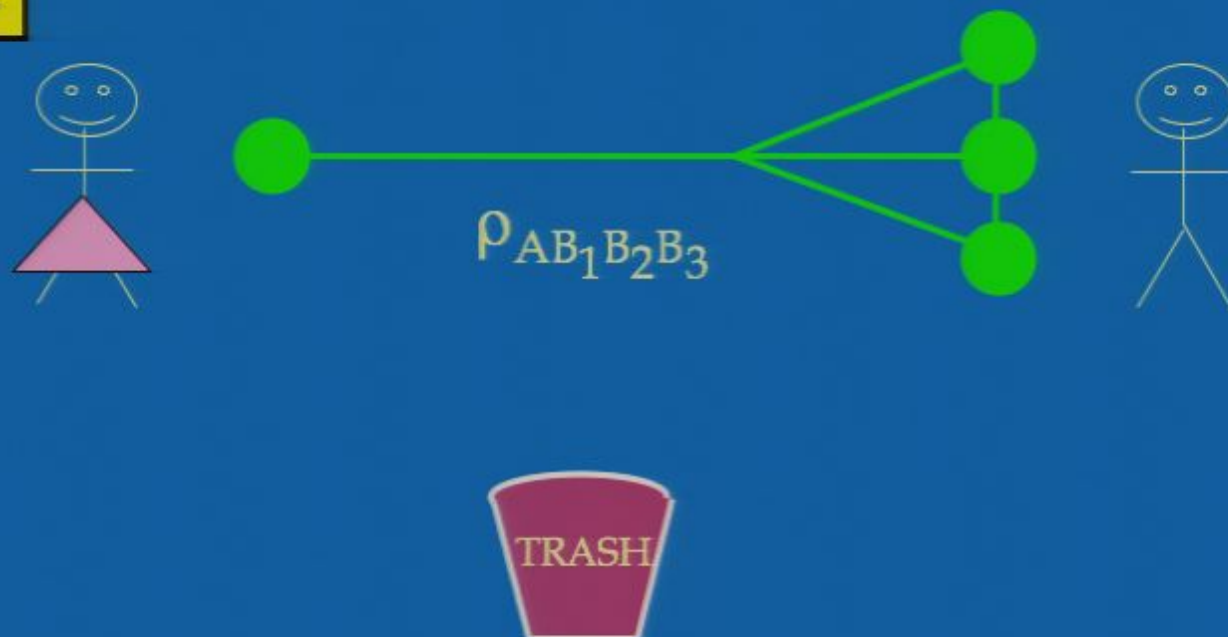
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What is the minimal number of particles he must discard before the remaining state is uncorrelated?

In this case, by discarding 2 particles, Bob succeeded in eliminating all correlations with Alice's particle

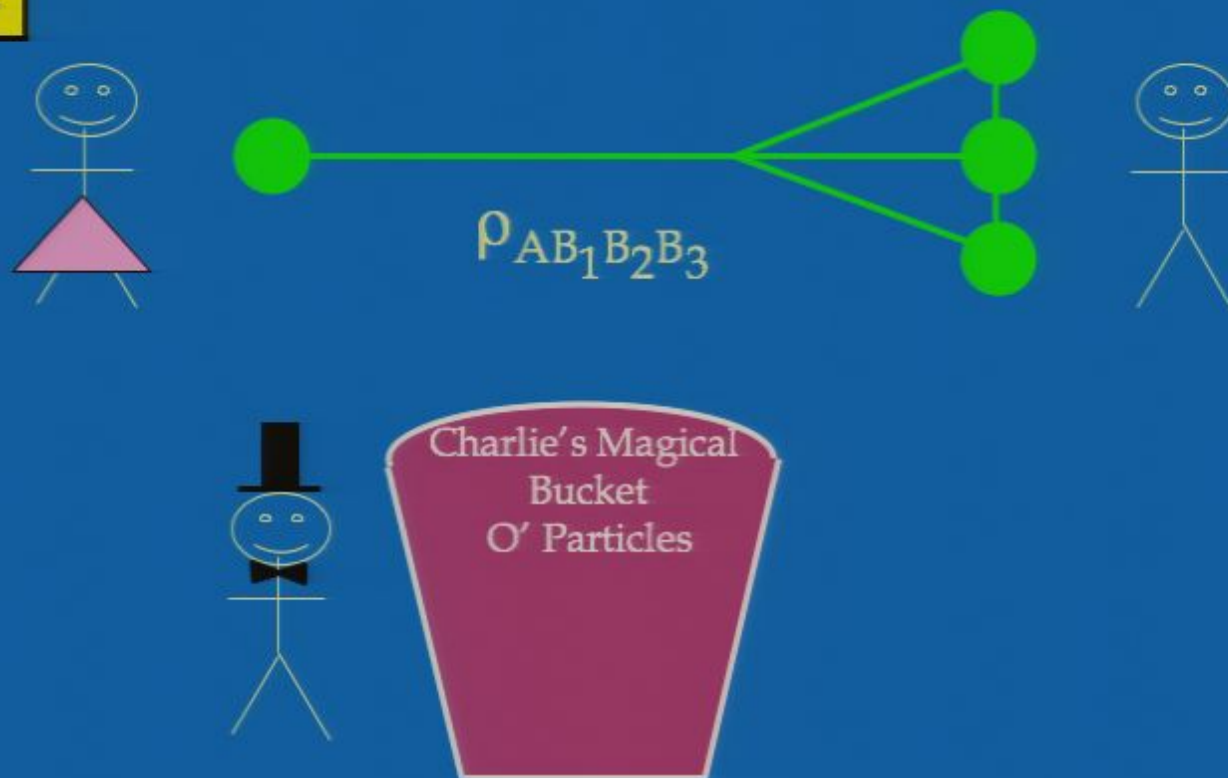
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Watch again:



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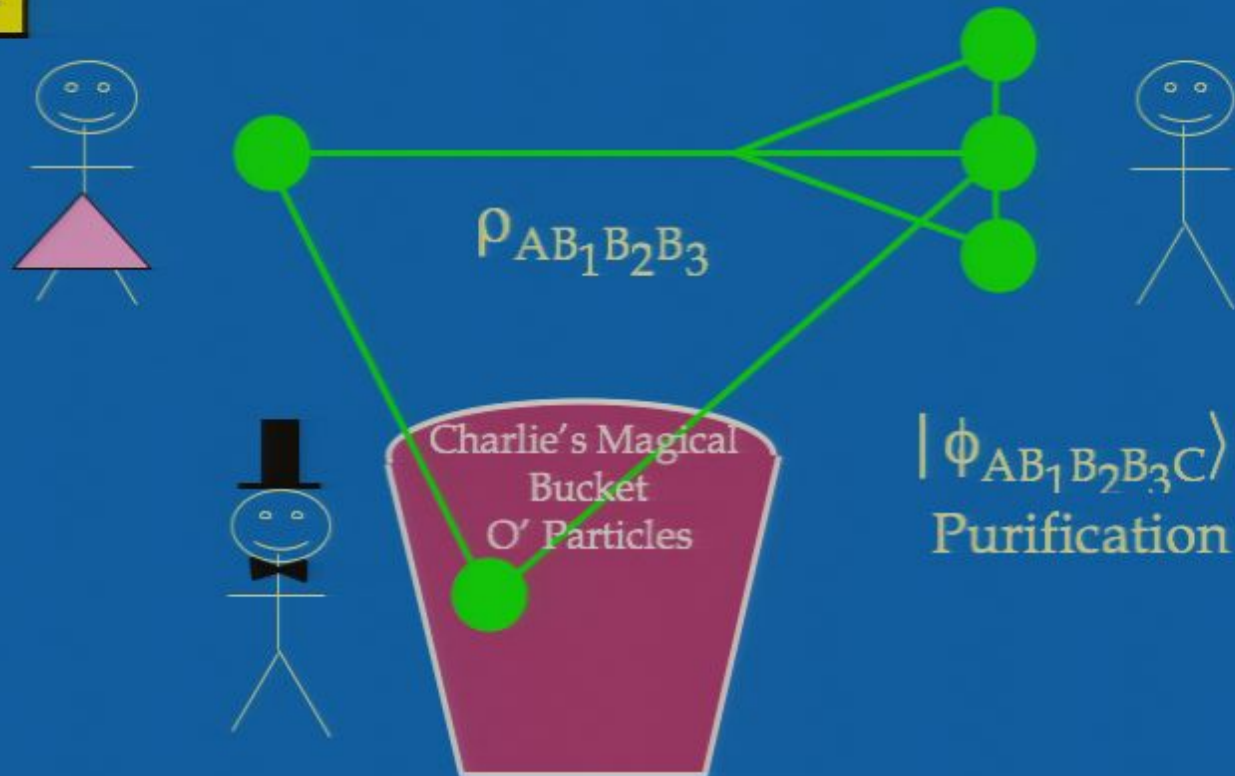
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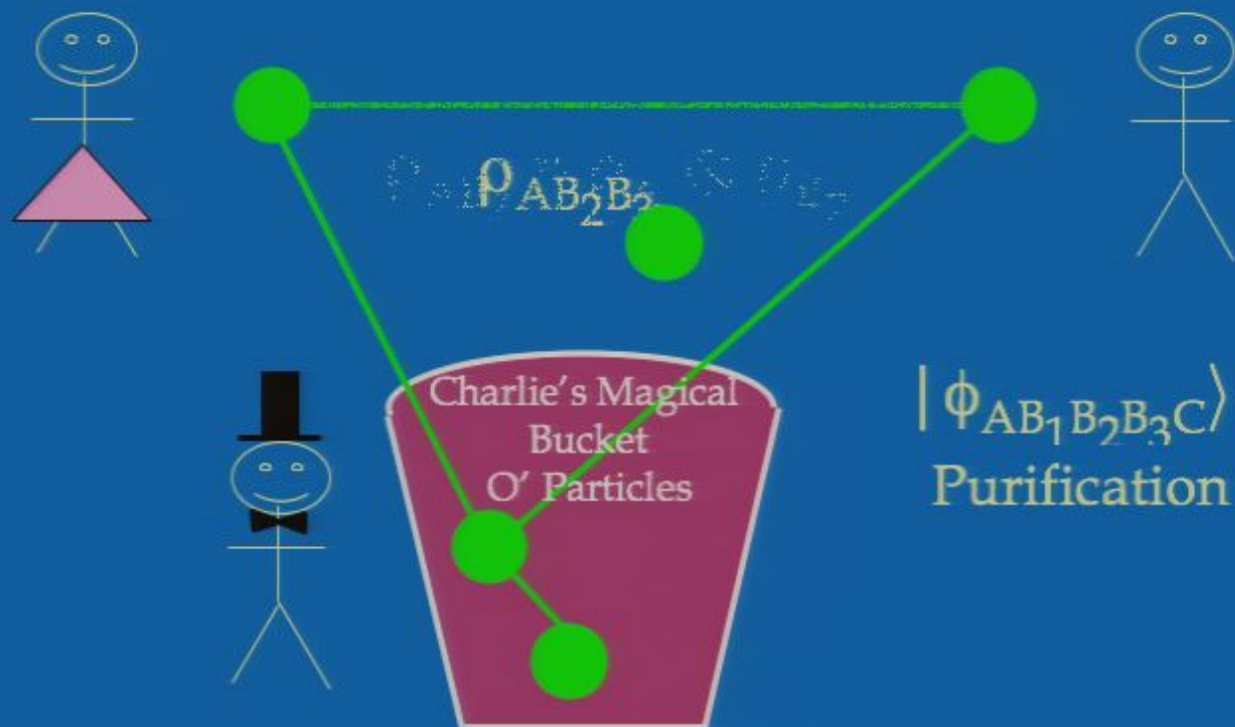
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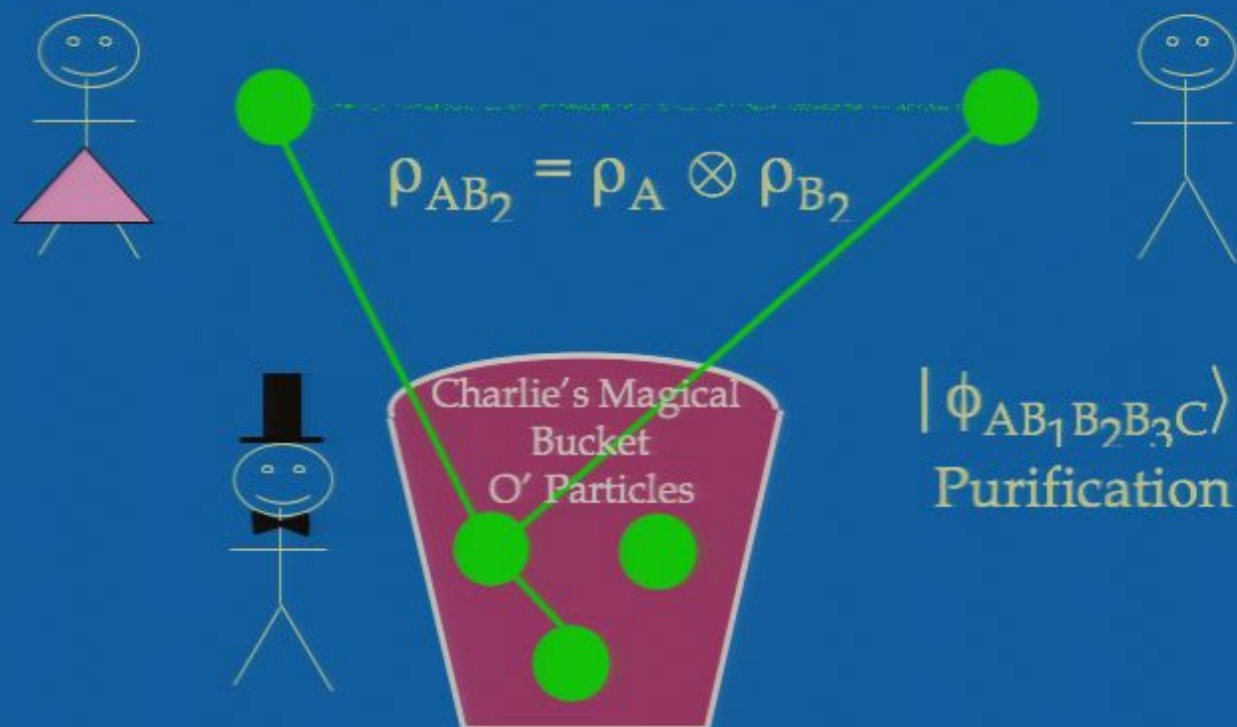
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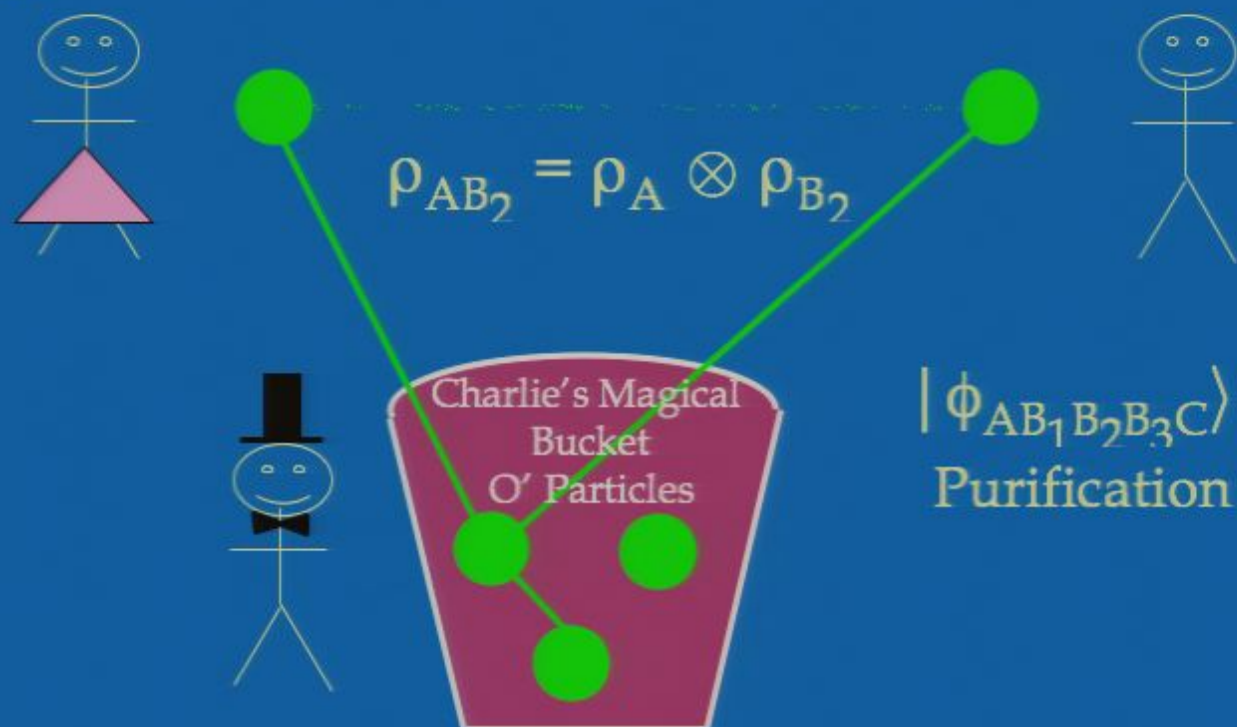
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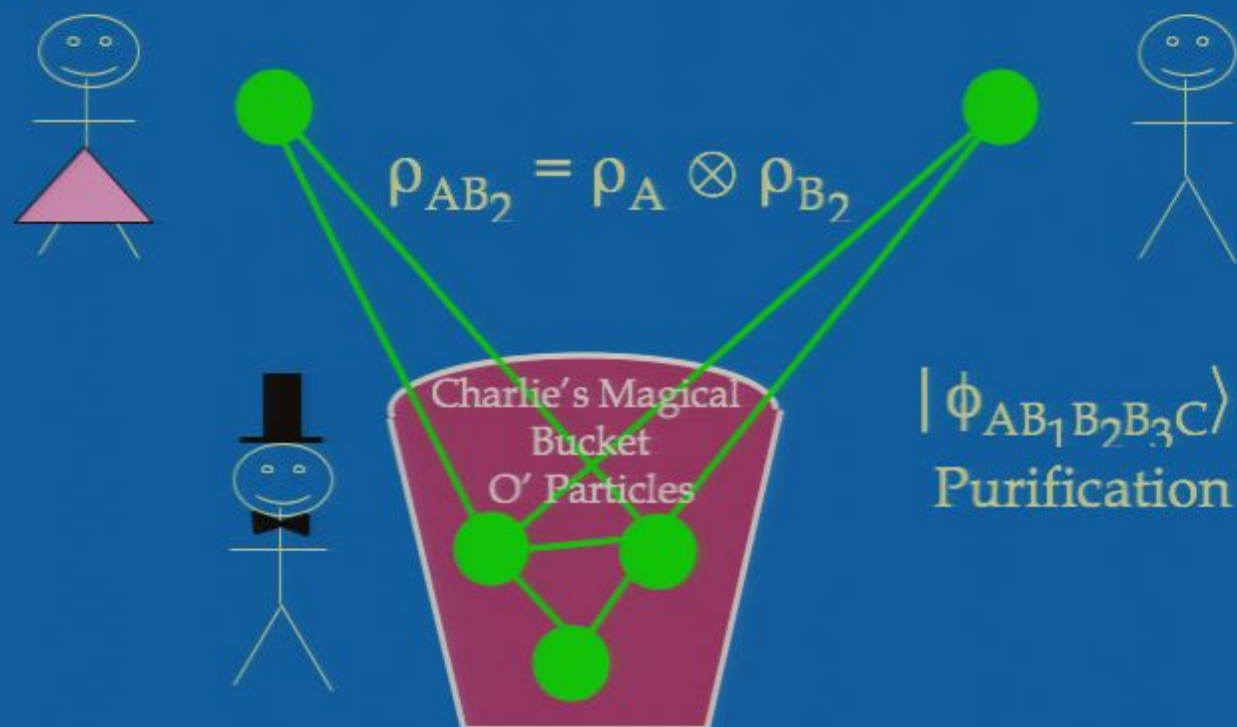
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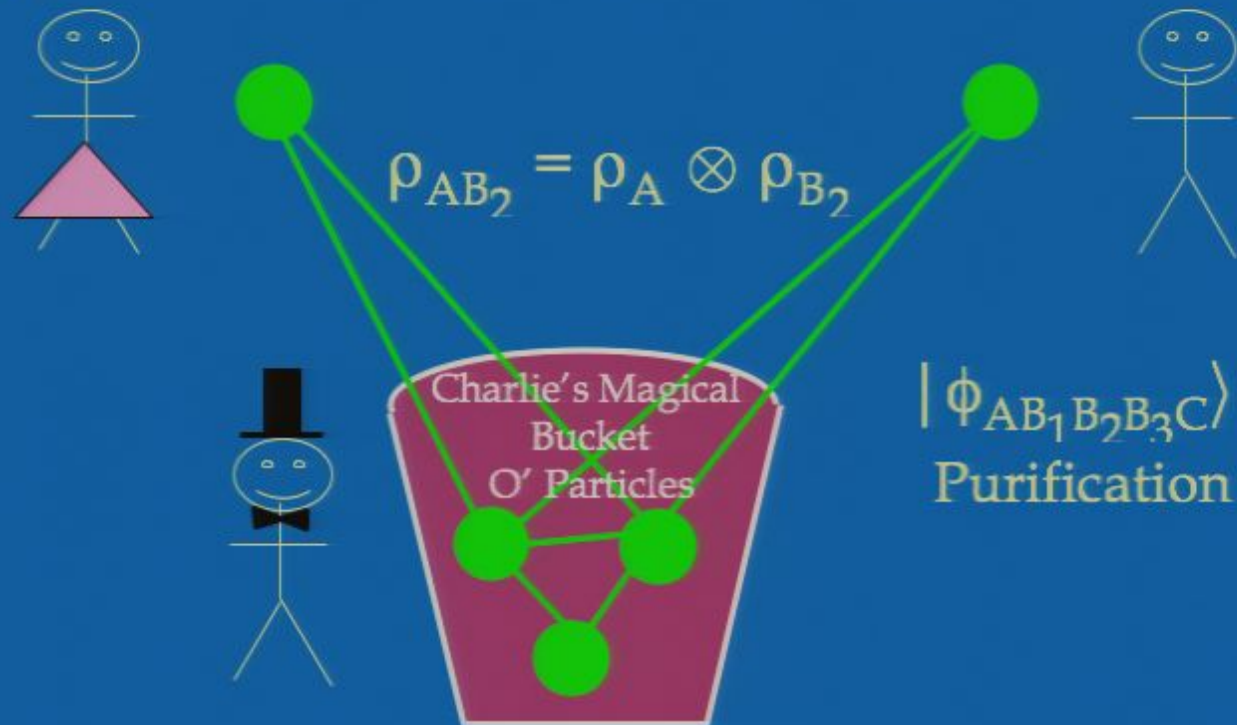
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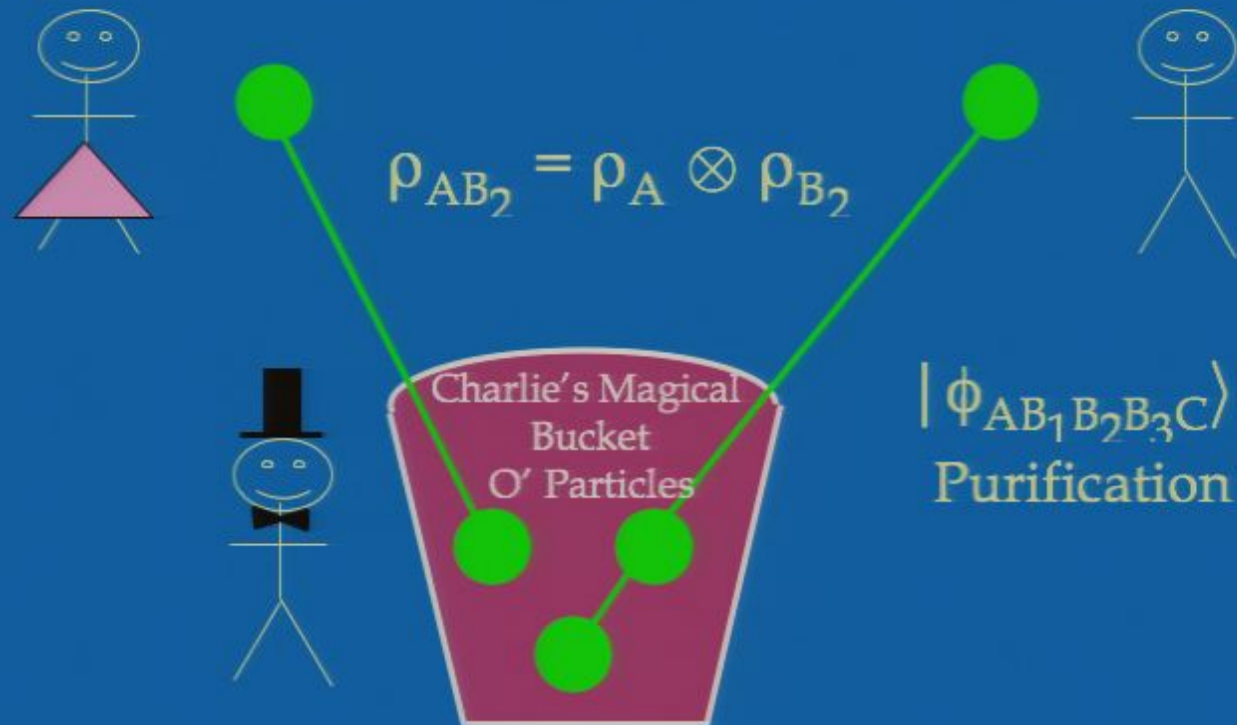
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All purifications equivalent up to a local transformation in Charlie's lab.

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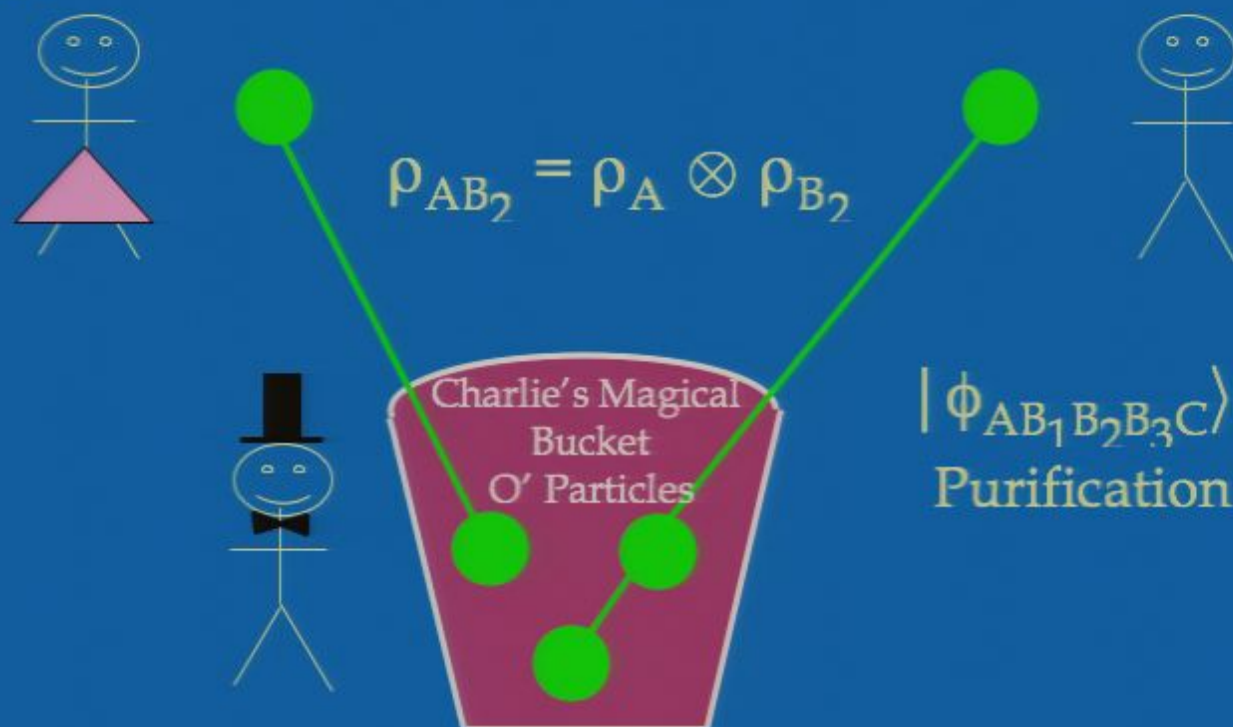
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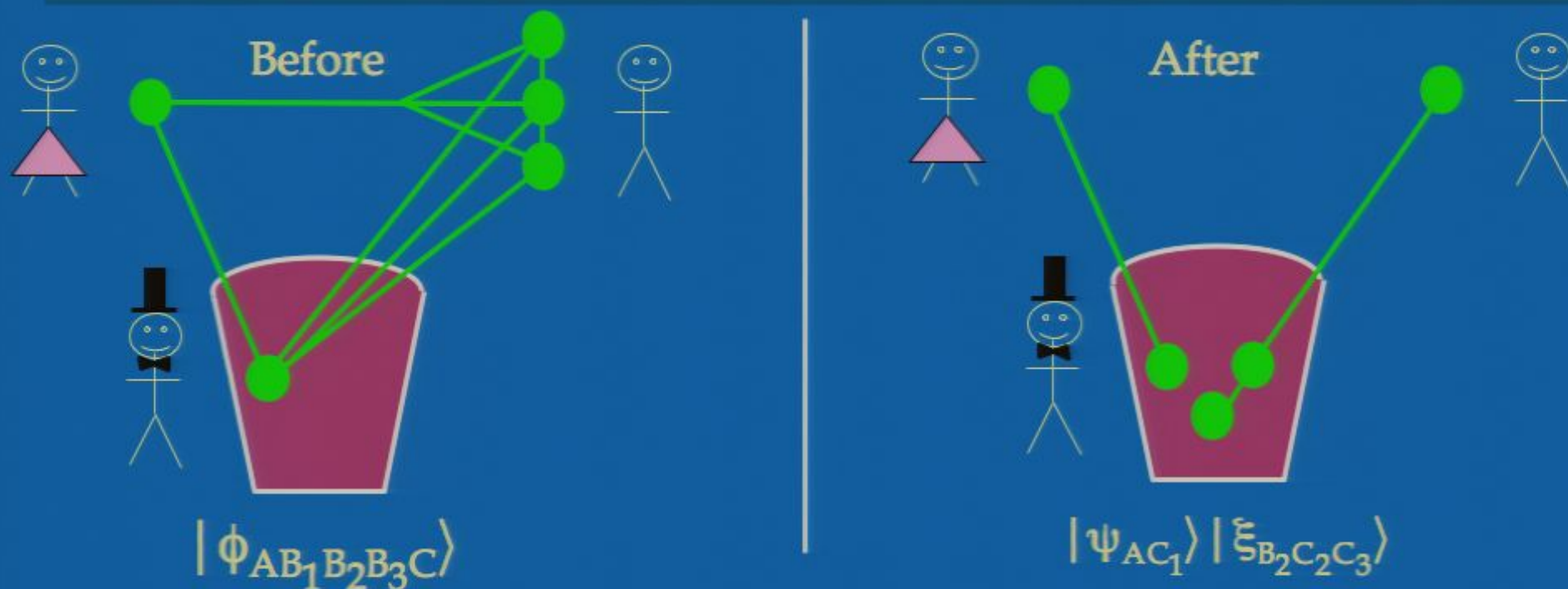


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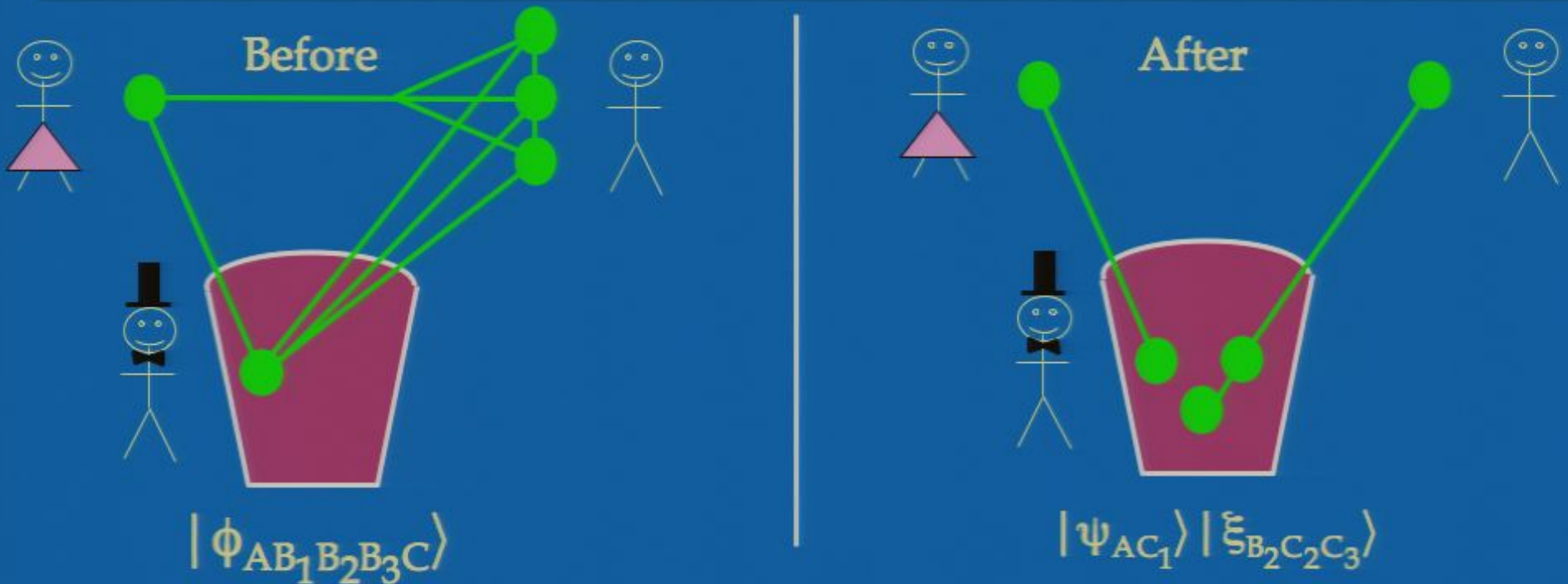
Charlie holds **uncorrelated** purifications of **both** Alice's particle and Bob's remaining particles.



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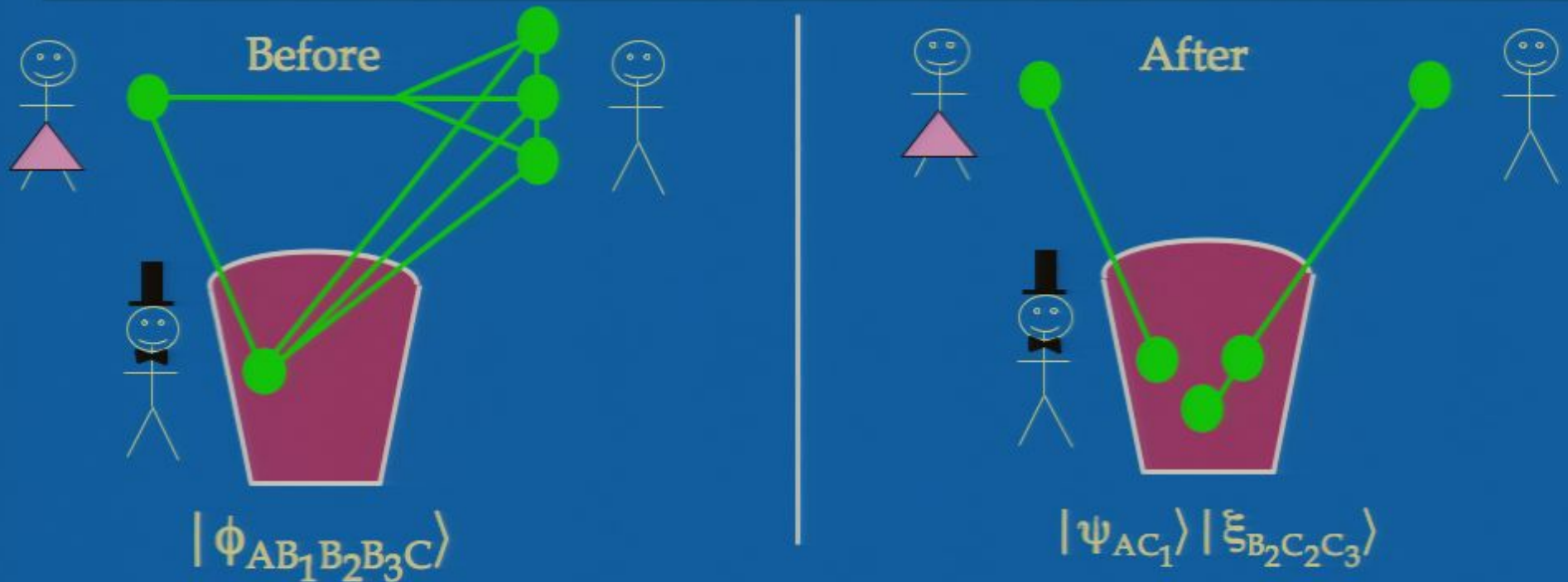
# The benefits of forgetting: Applied theology



Alice never did anything  $\Rightarrow$  Her marginal state  $\phi_A = \psi_A$  is unchanged



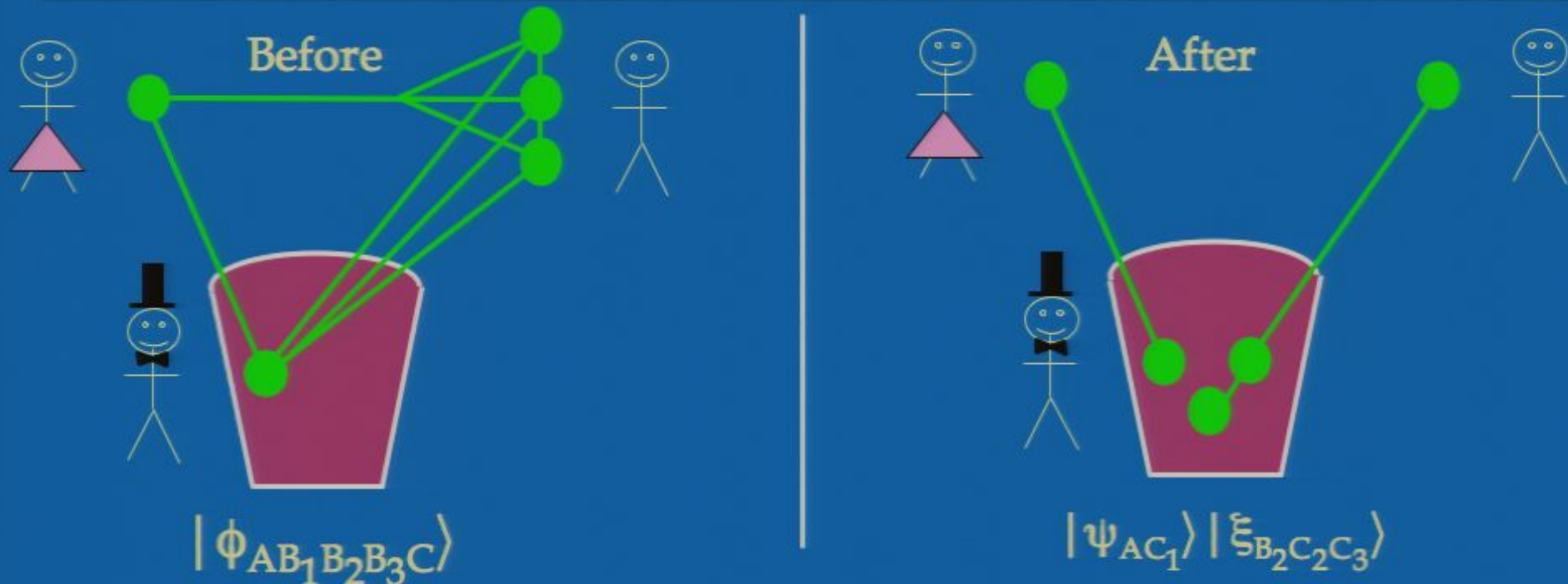
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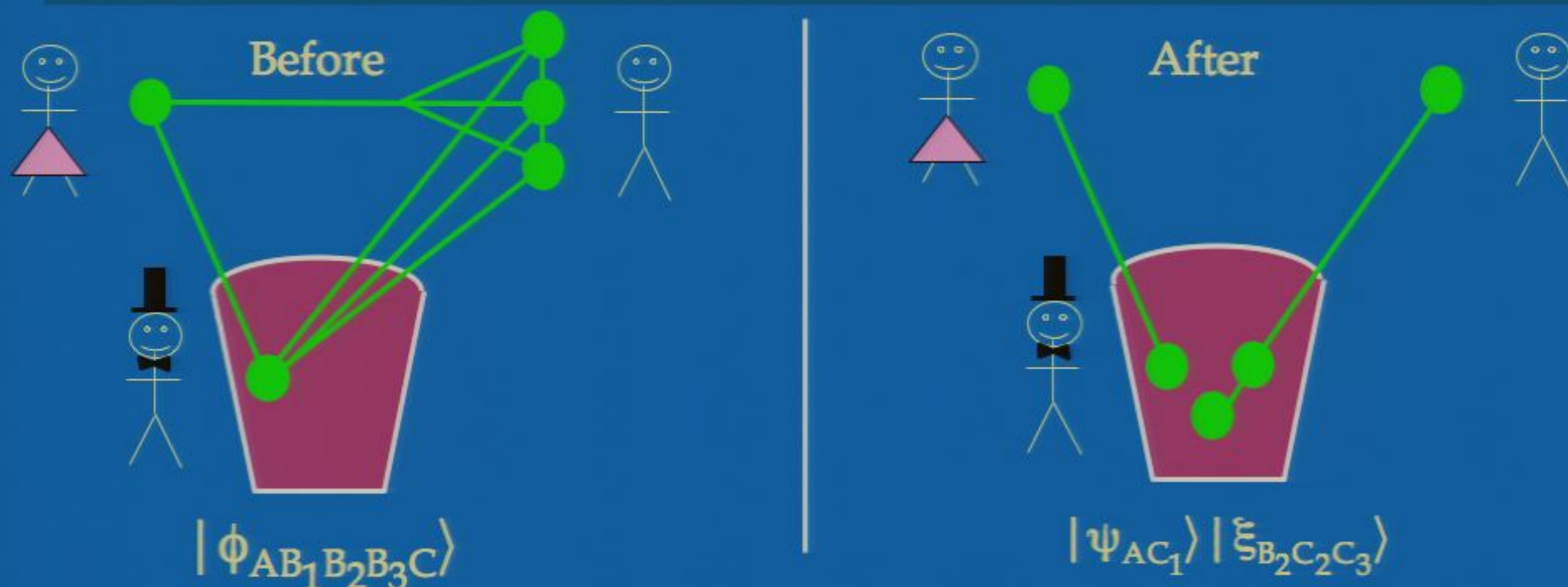
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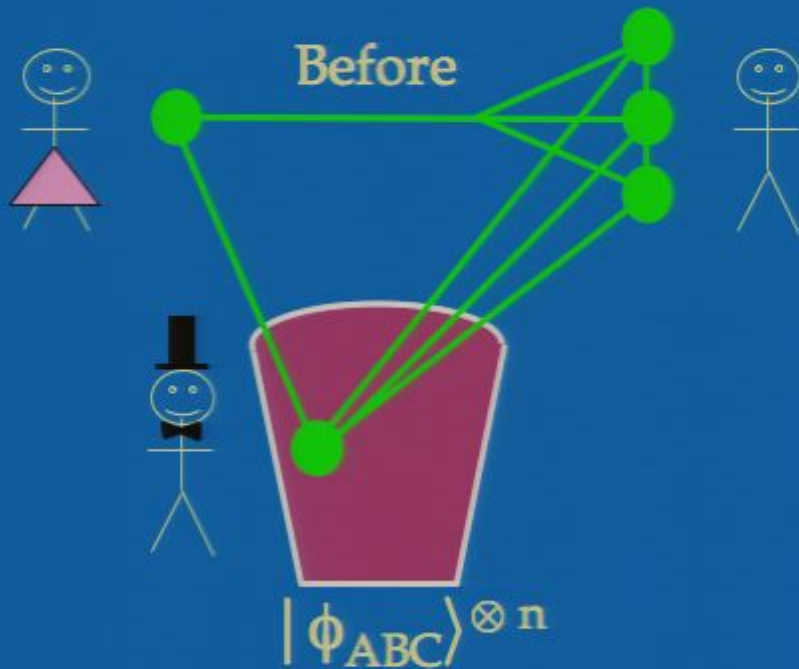
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**Bob transferred** his Alice entanglement to Charlie  
and **distilled** entanglement with Charlie, just by discarding particles!



# Time for some formulas:

## How much does Bob need to send?



Uncertainty: von Neumann entropy

$$H(A)_\rho = H(\rho_A) = -\text{tr}[\rho_A \log \rho_A]$$

Correlation: mutual information

$$I(A;B)_\rho = H(A)_\rho + H(B)_\rho - H(AB)_\rho$$

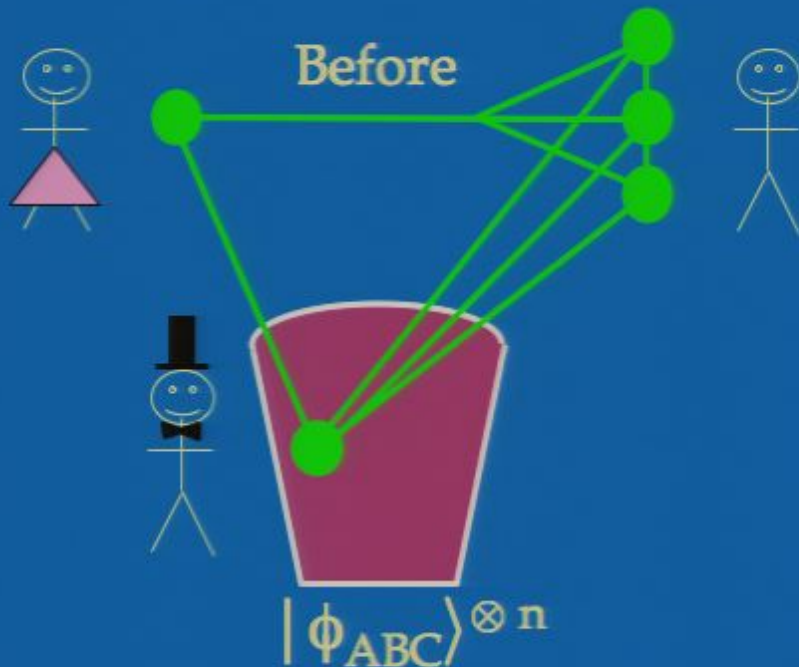
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$$\frac{1}{\sqrt{2}} (\lvert 0 \rangle^A \lvert 0 \rangle^B + \lvert 1 \rangle^A \lvert 1 \rangle^B)$$



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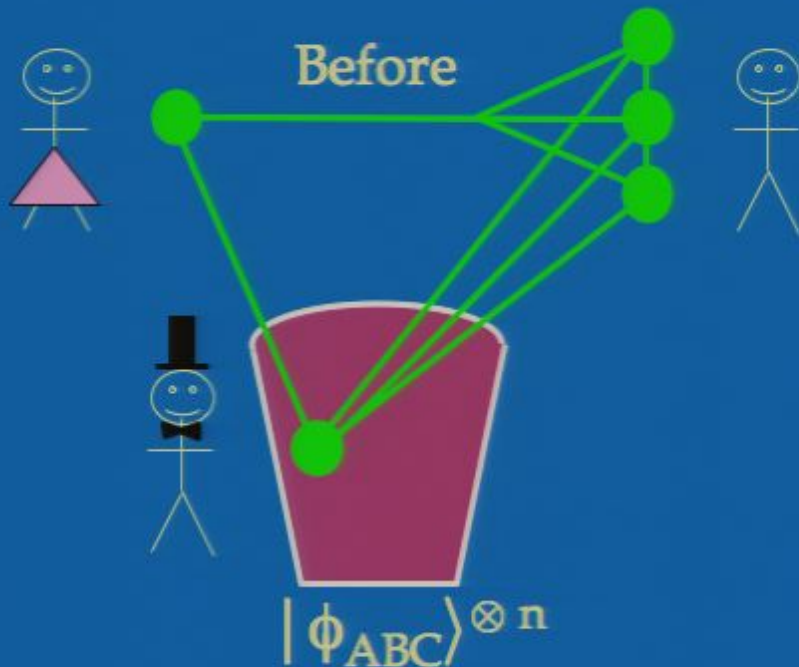
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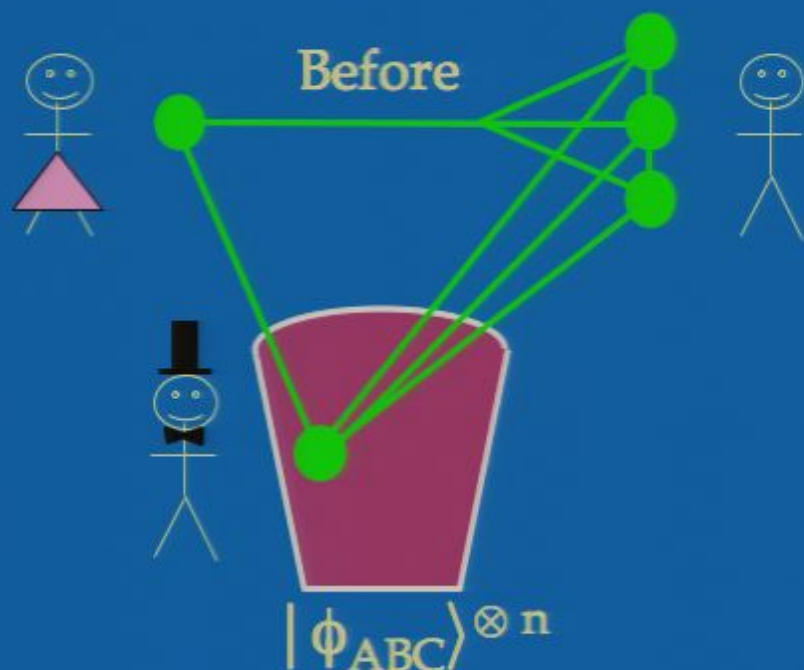
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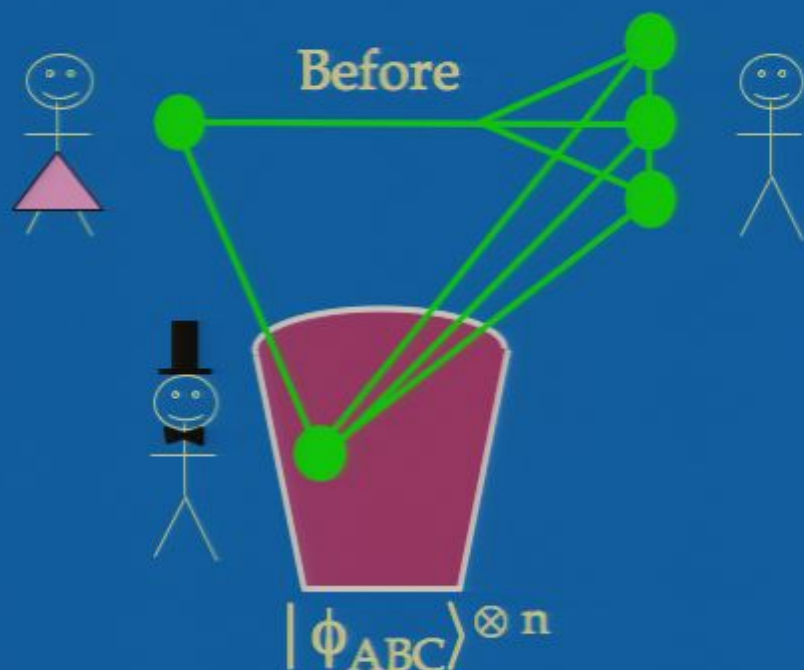
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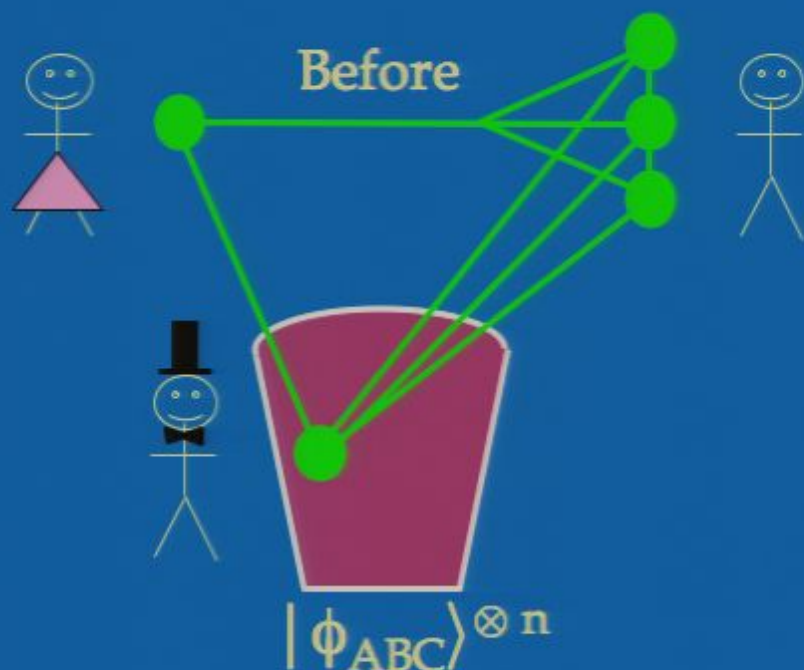
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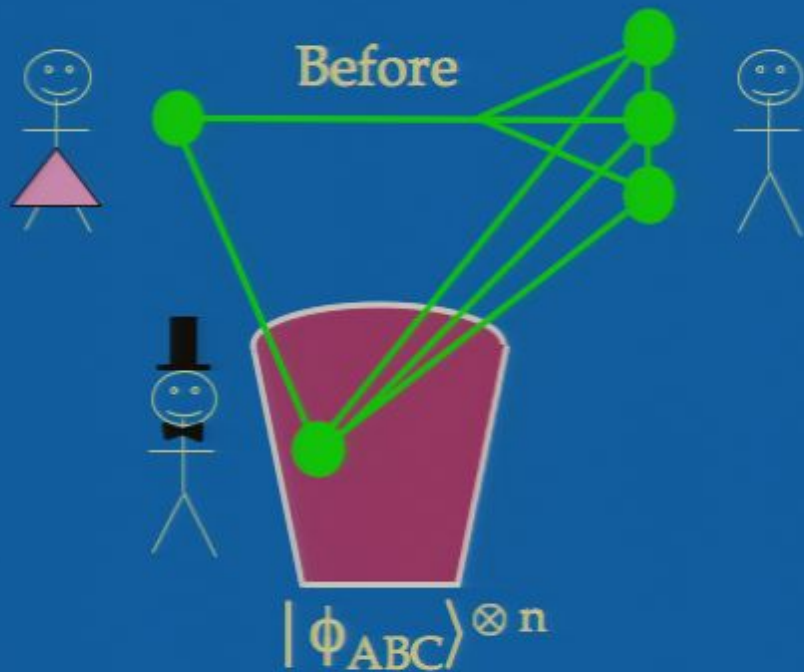
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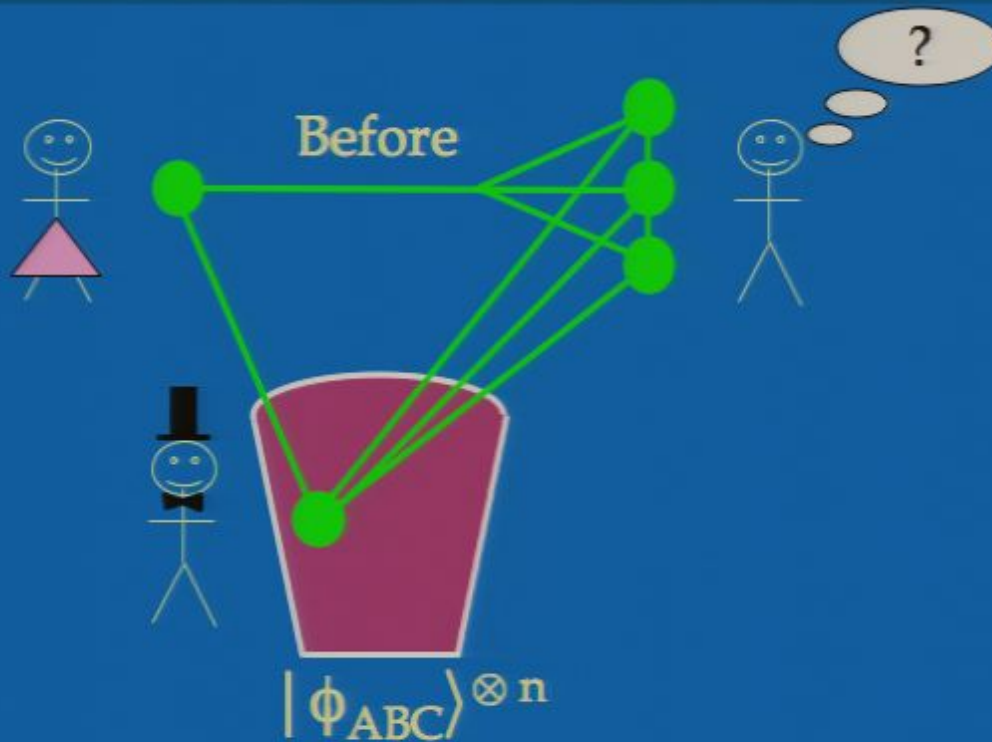
Bob should (ideally) send around  $nI(A;B)/2$  qubits to Charlie



# How does Bob choose *which* qubits?



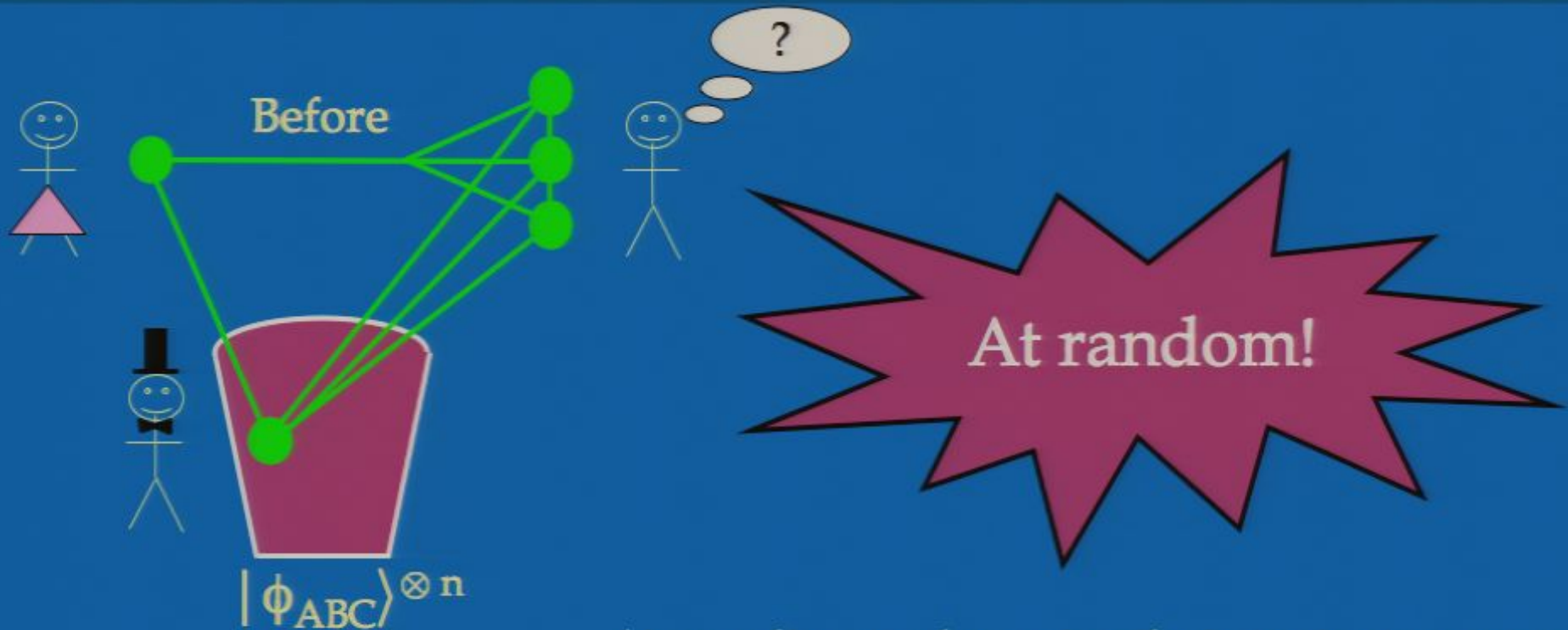
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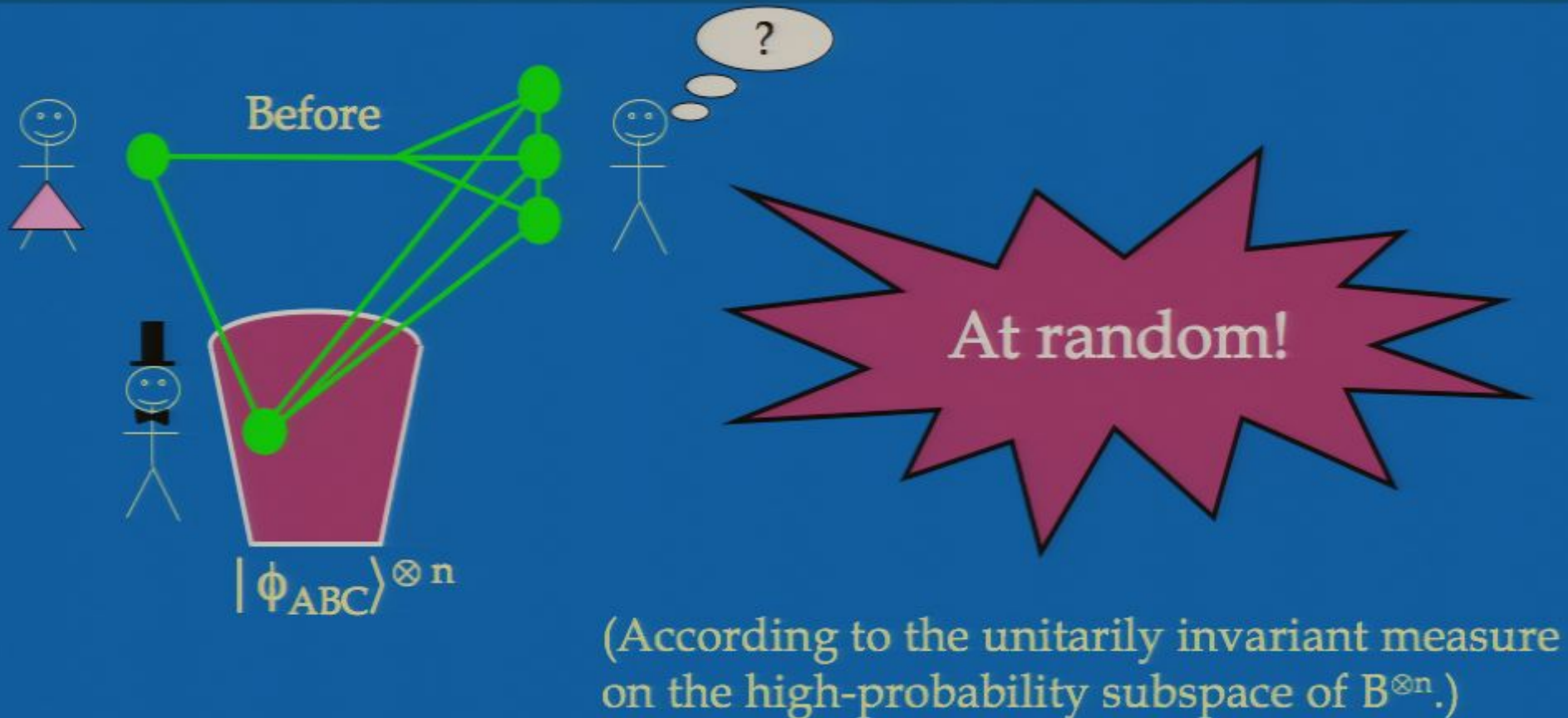
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(According to the unitarily invariant measure on the high-probability subspace of  $B^{\otimes n}$ .)



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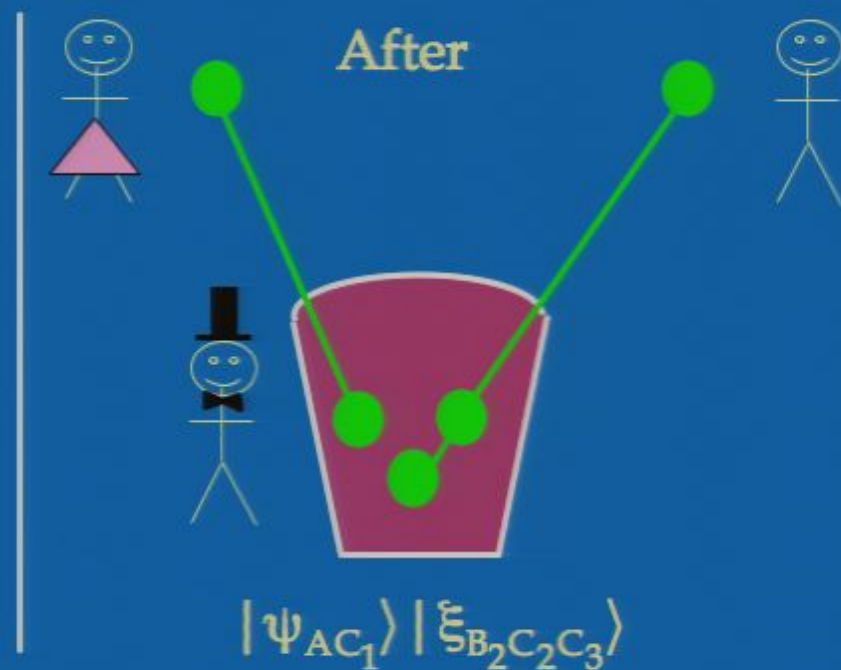


Bob can ignore the correlation structure of his state!

# Final accounting

## Investment:

Bob sends Charlie  $\sim n[I(A;B)_\phi]/2$  qubits



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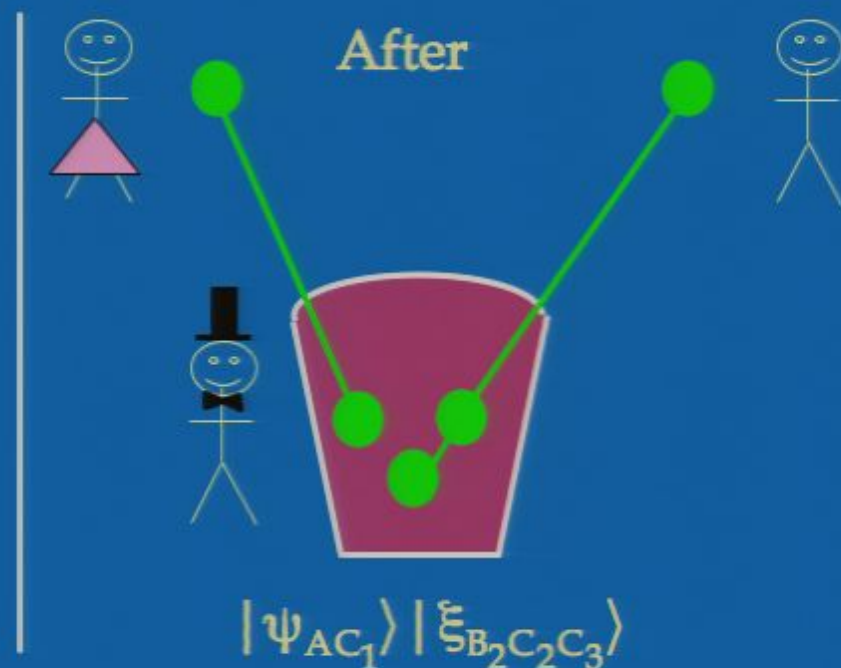
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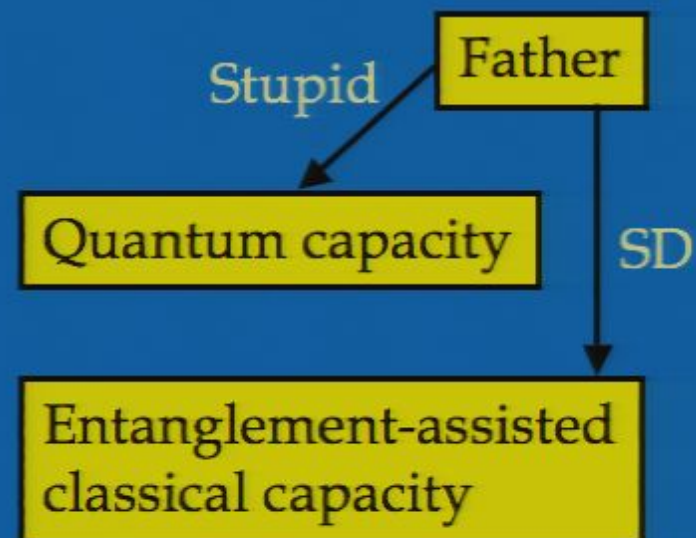
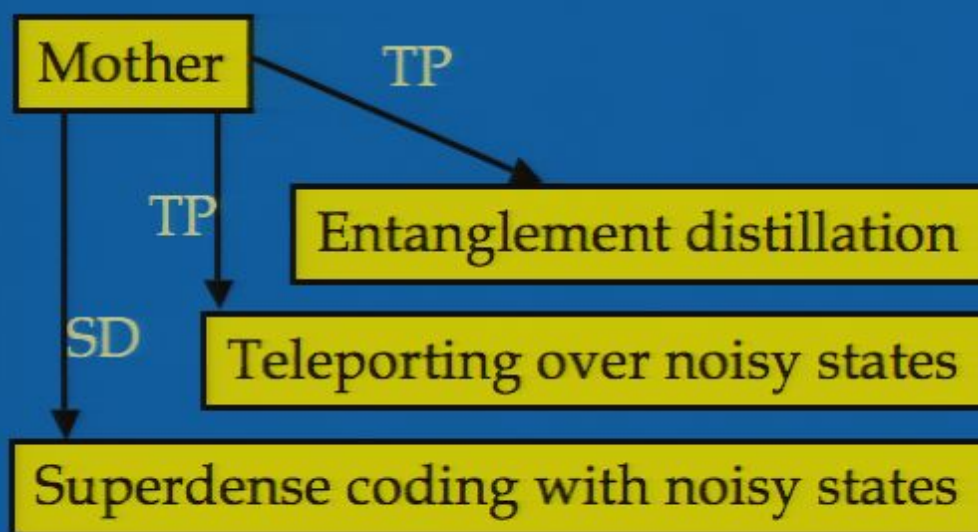
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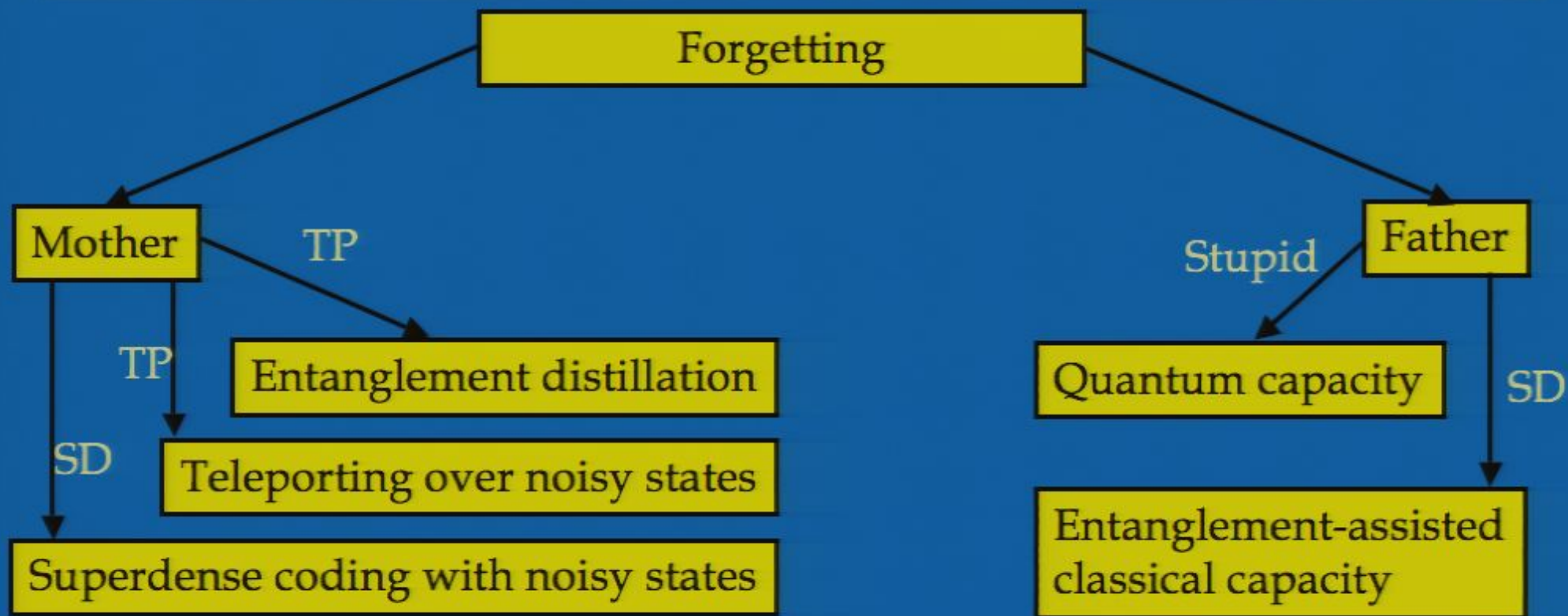
OK – but what good is it?



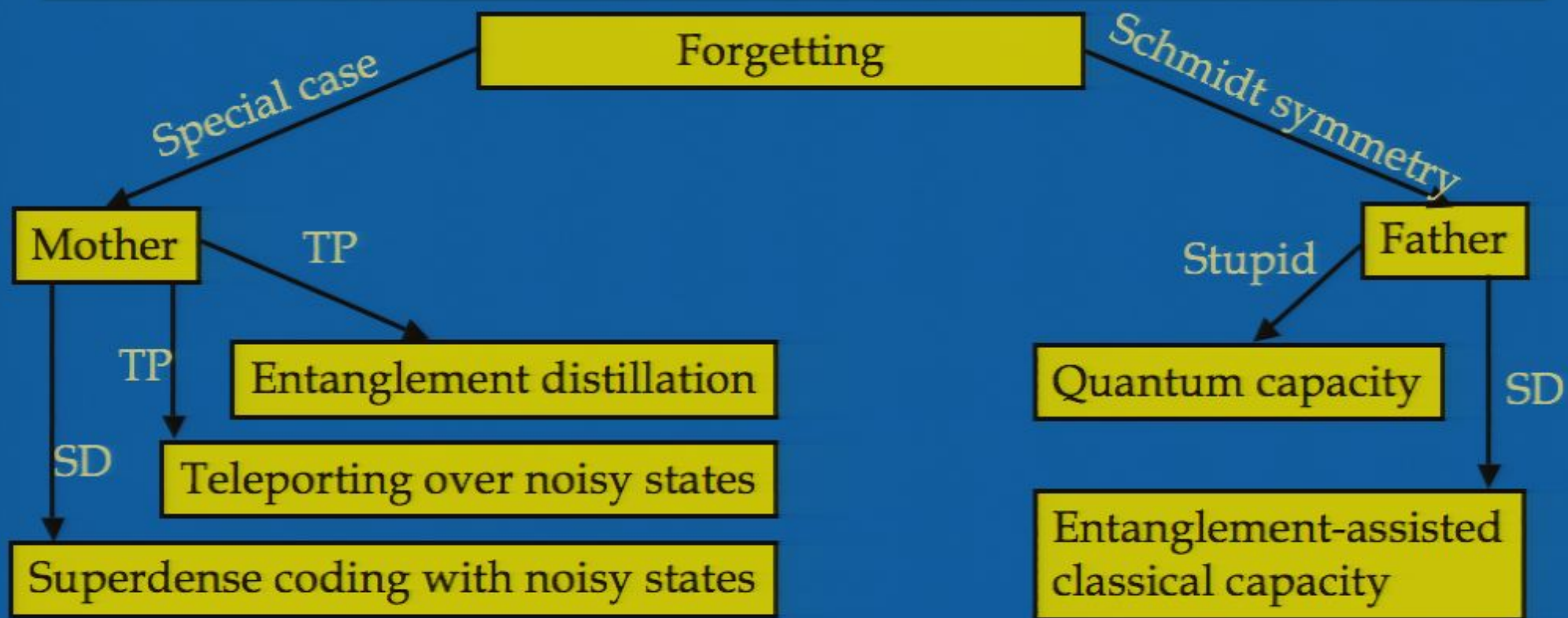
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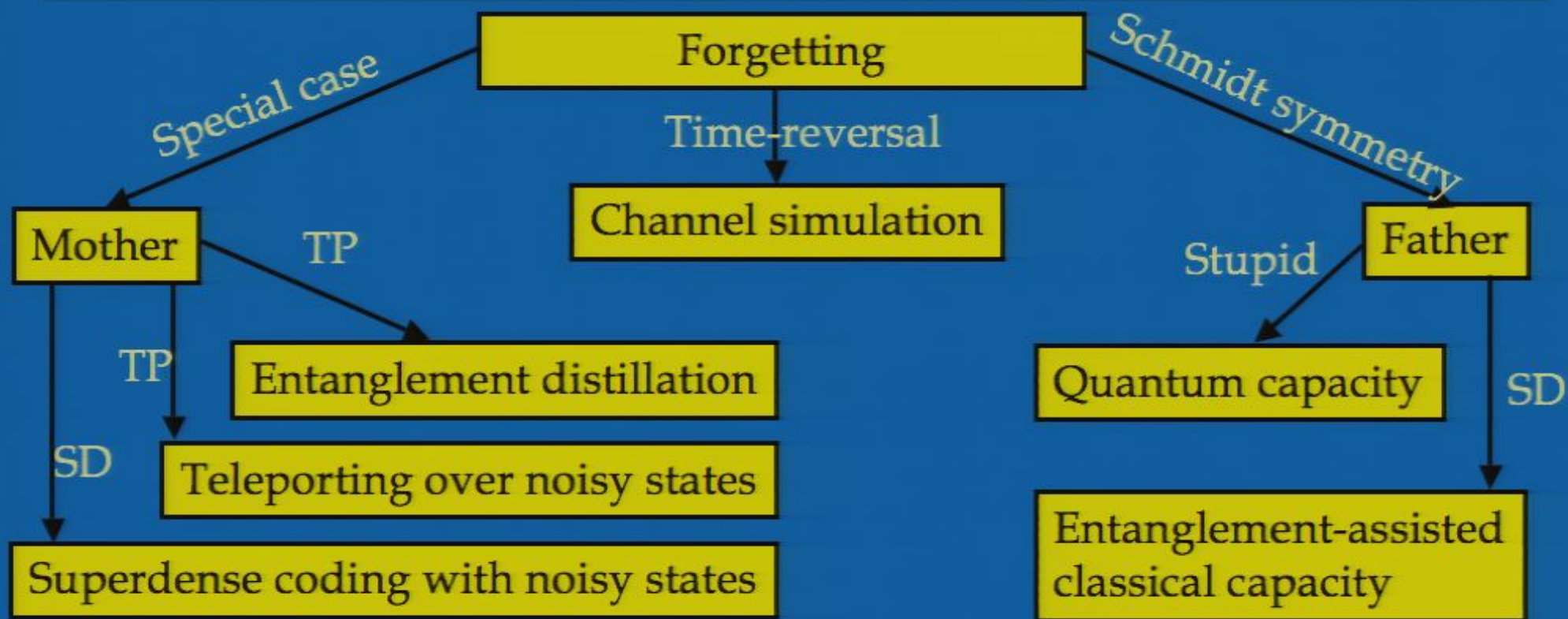
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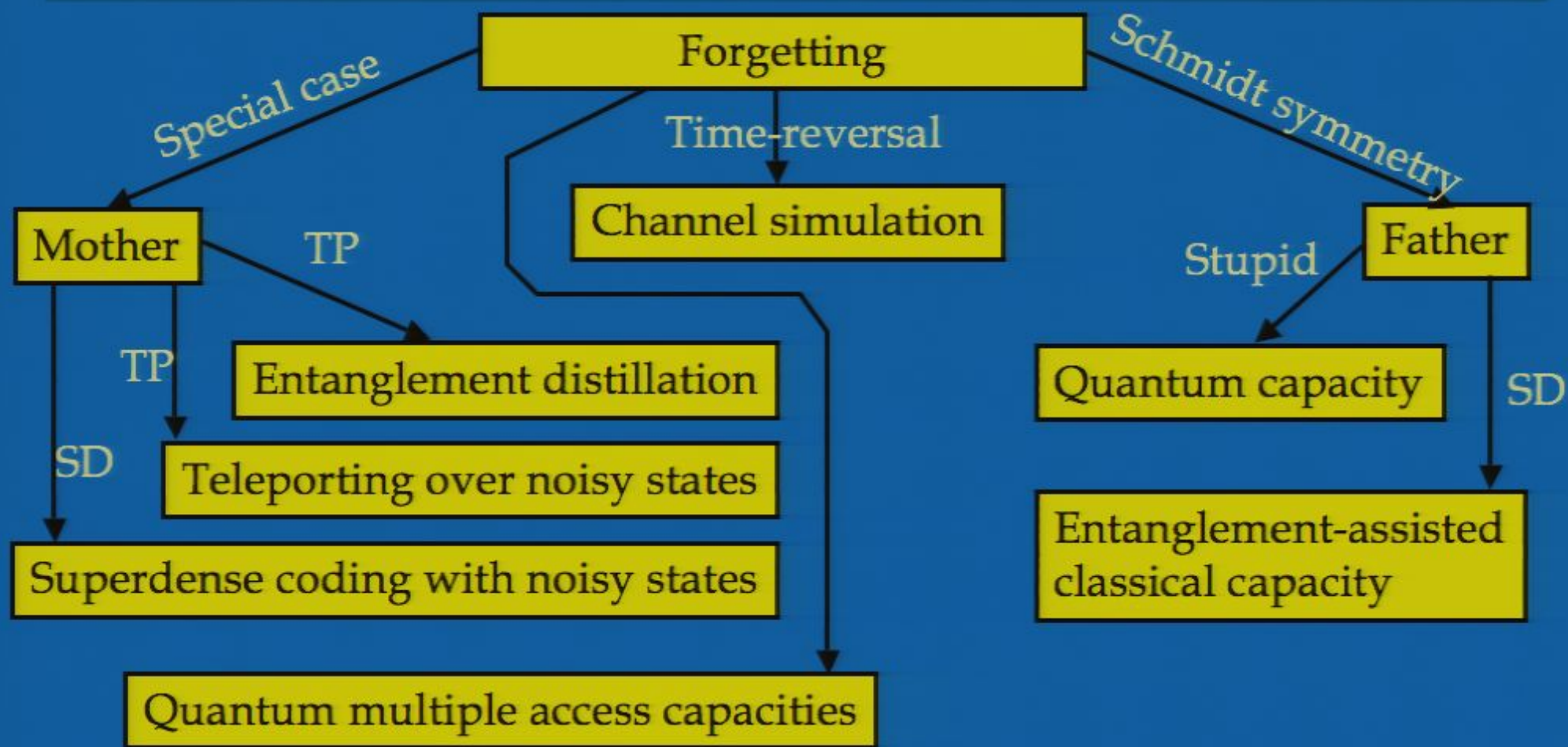


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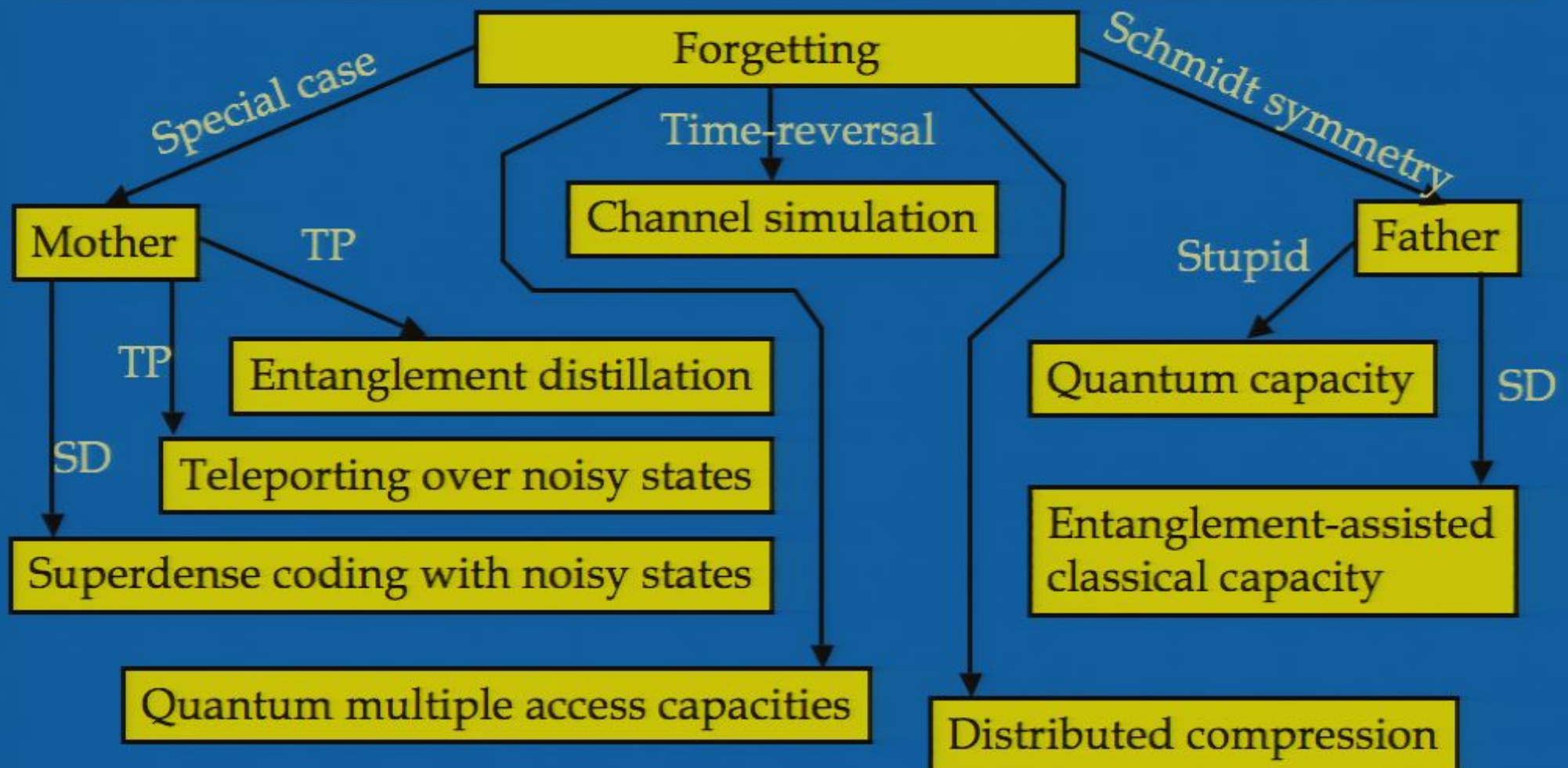




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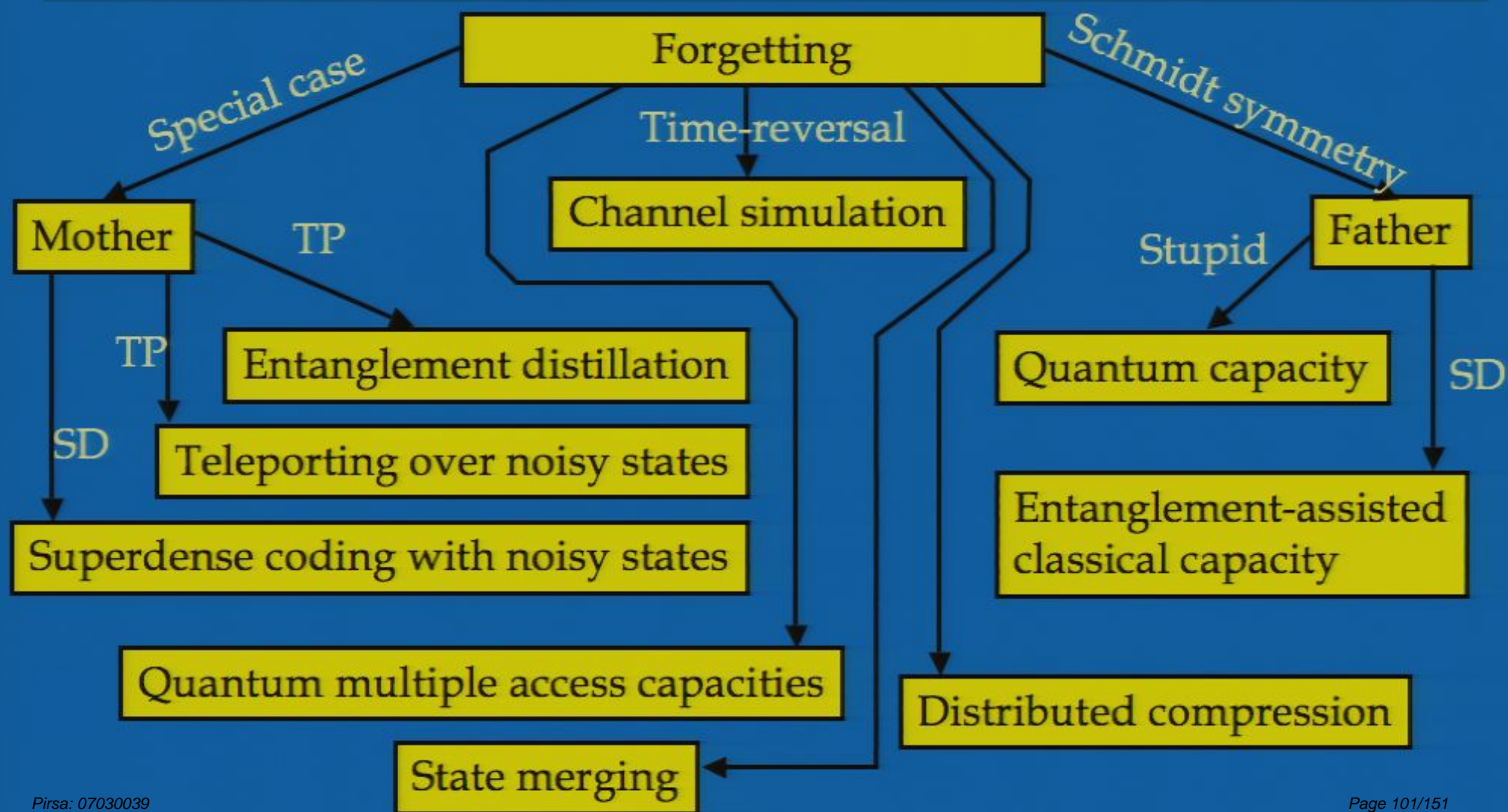


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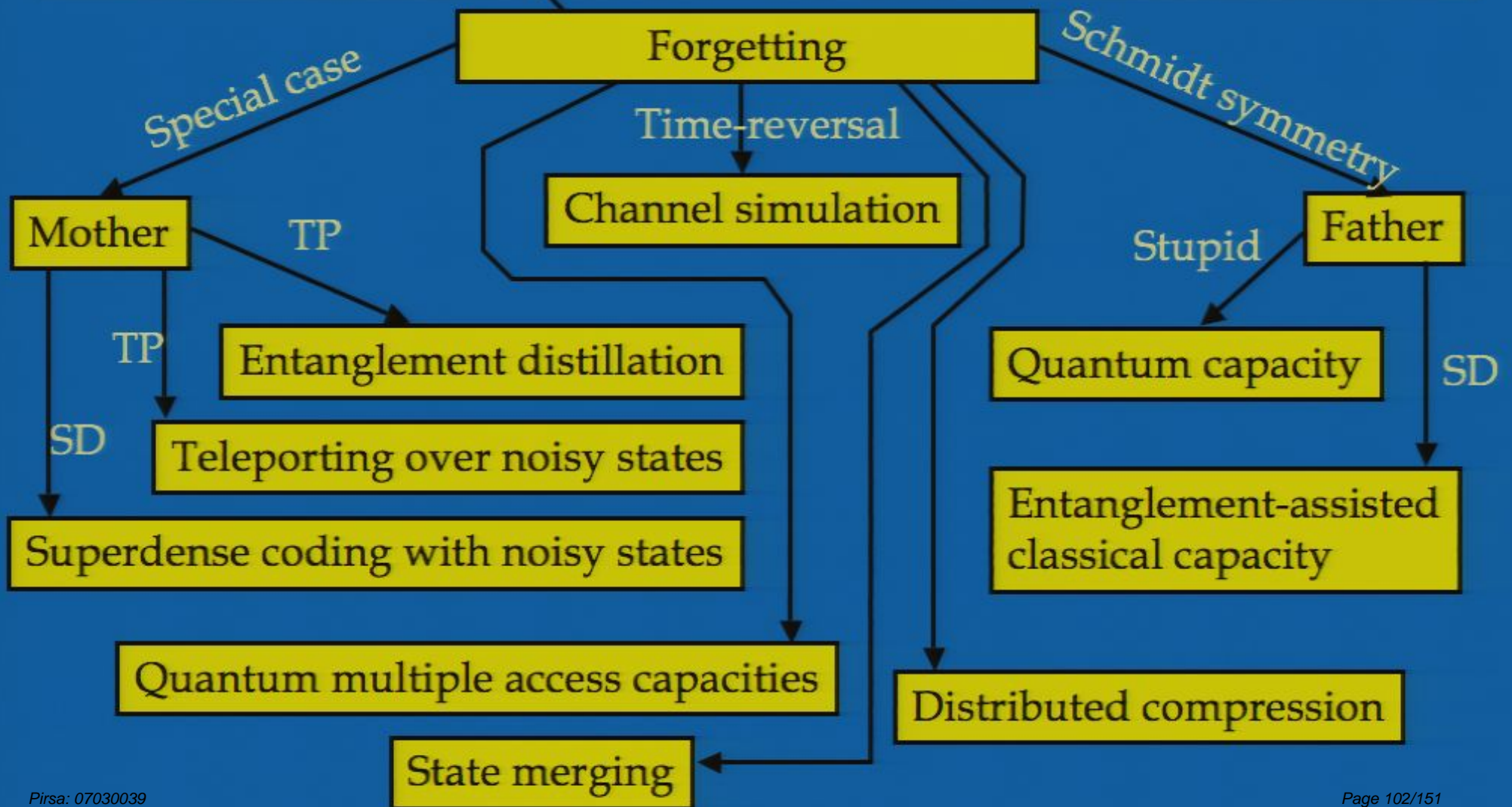


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## Capacities of quantum broadcast channels

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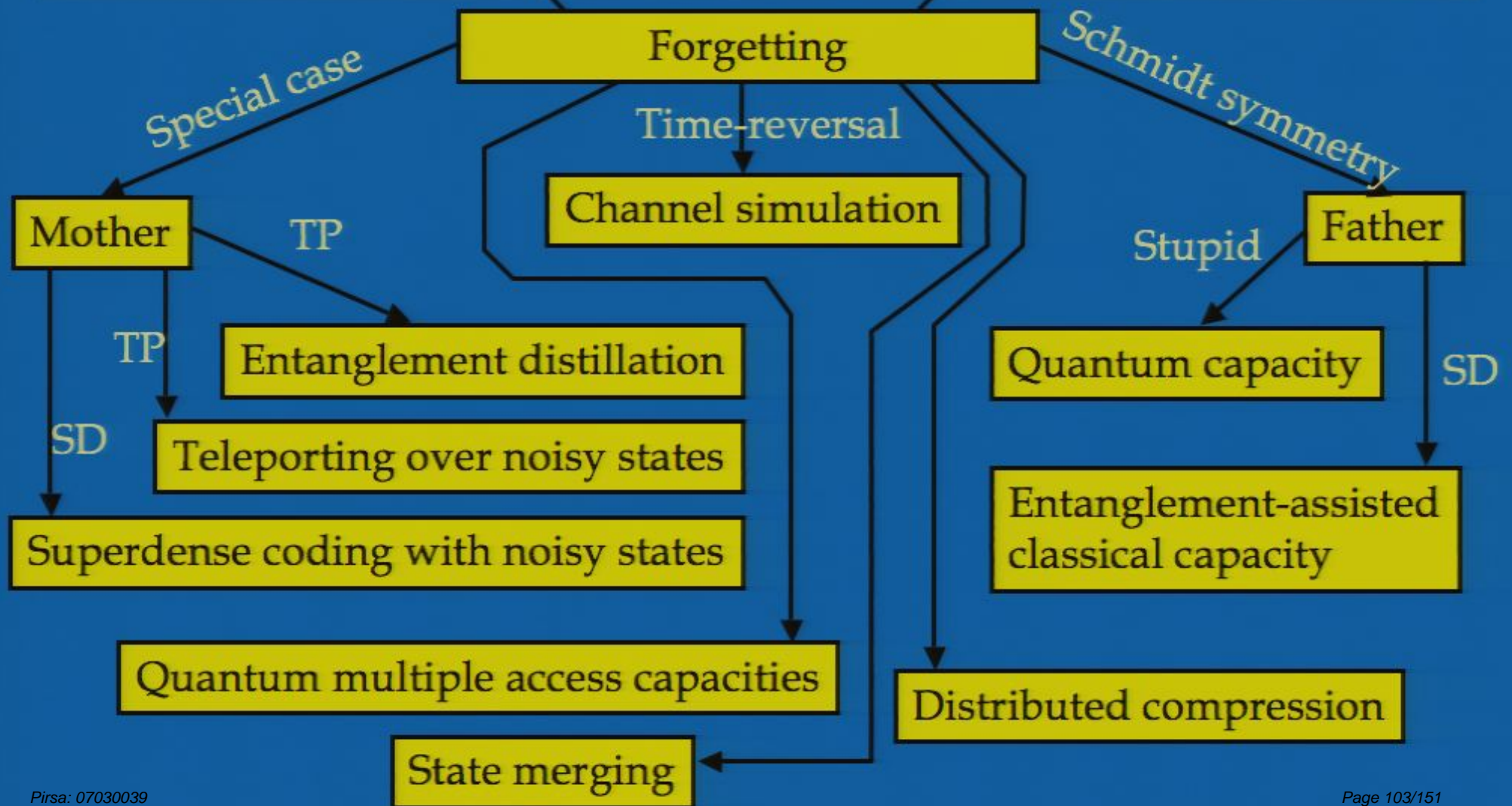




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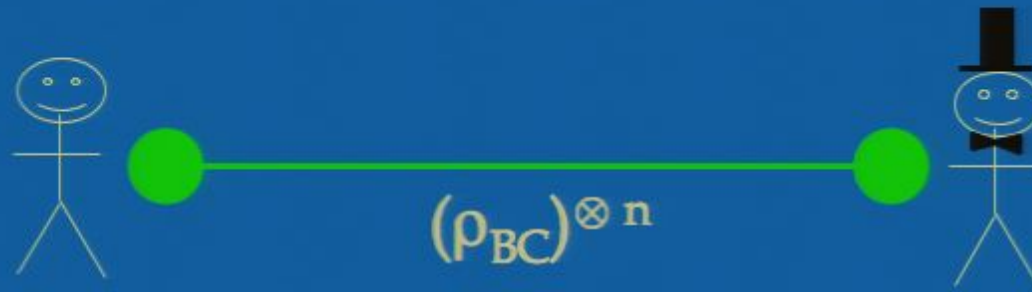
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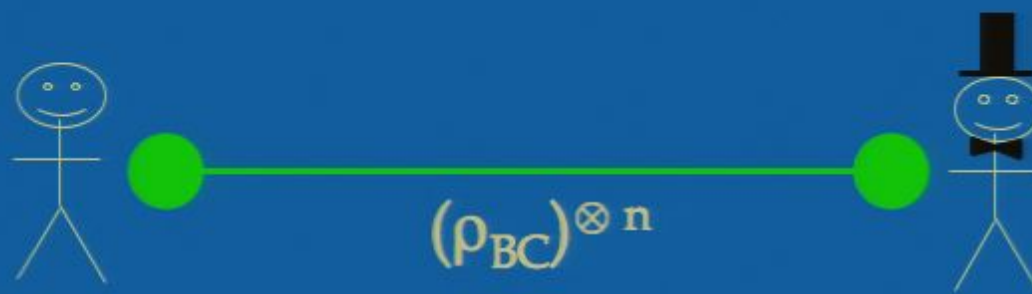


# Entanglement distillation

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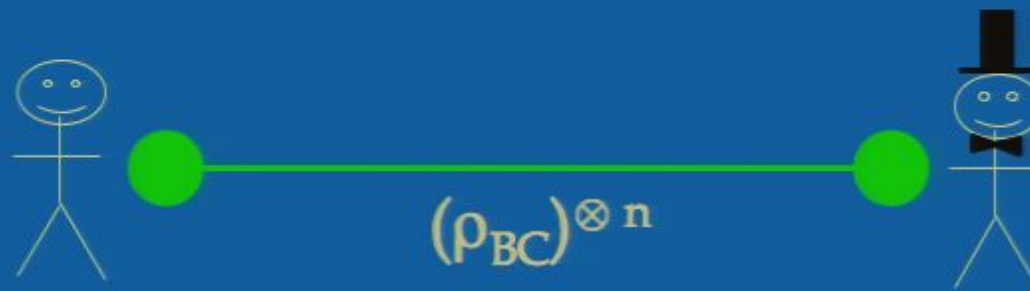
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Bob and Charlie share many copies of a noisy entangled state and would like to convert it to ebits.

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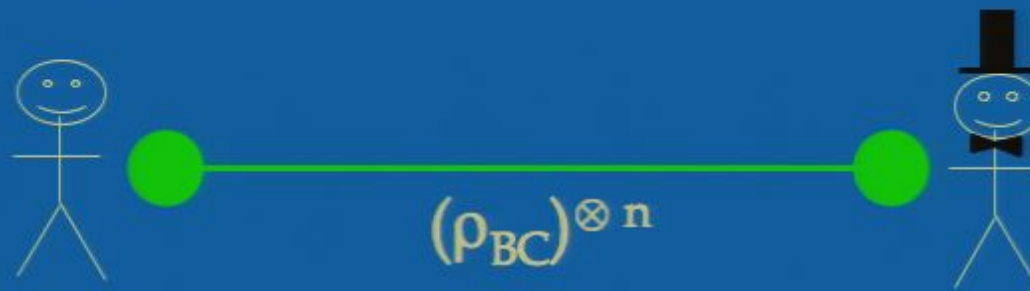


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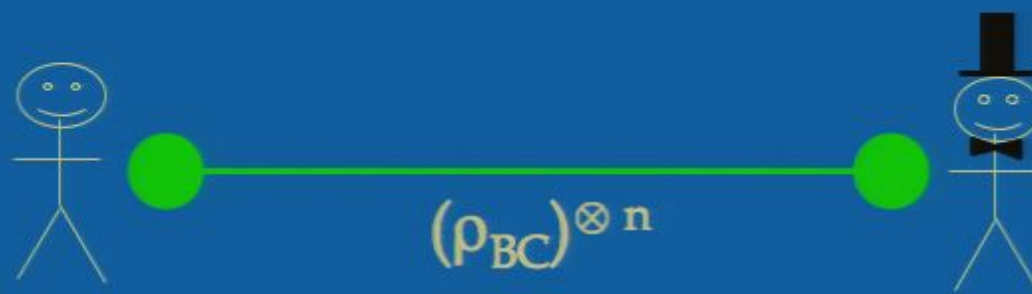


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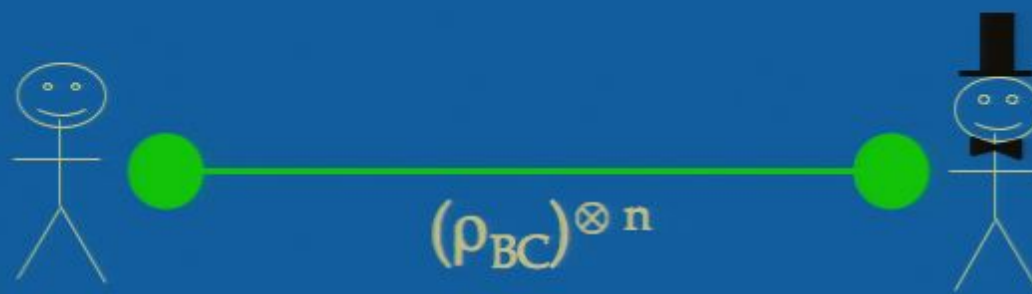
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But while the breakthrough may make sense to experts in quantum physics, it will leave the man in the street completely baffled.

Dr Winter, a lecturer at the university's department of mathematics, has been scratching his head to come up with an explanation of his theory.

And his example will still leave most people wondering why he bothered.

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Dr Winter worked with two other experts, Jonathan Oppenheim from Cambridge University and Michal Horodecki from the Institute for Theoretical Physics, in Gdansk, Poland, to

**by Ian Turner**

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develop the theory. Their work has now been published in the leading science journal *Nature*.

Quantum physics tries to explain the behaviour of particles, such as electrons, protons, and neutrons, which are smaller than an atom.

Scientists claim that in the quantum world, there are things we just cannot know, no matter how clever we are.

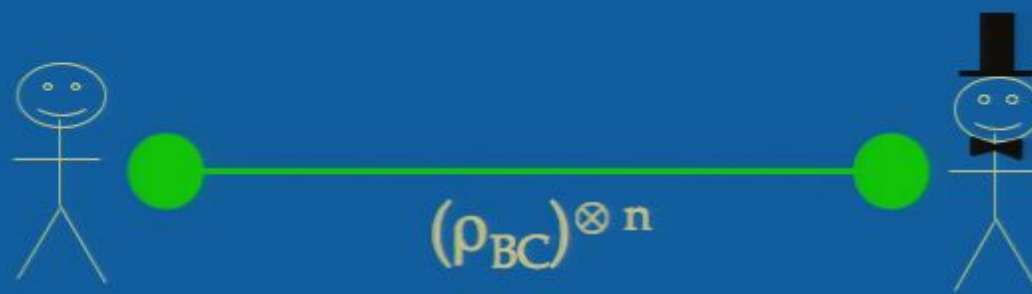
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He said: "We can quantify information in terms of how much stuff I need to send you before you get to know something. In the case of negative quantum information, you can get to know something without me sending you any quantum particles. Page 110/151  
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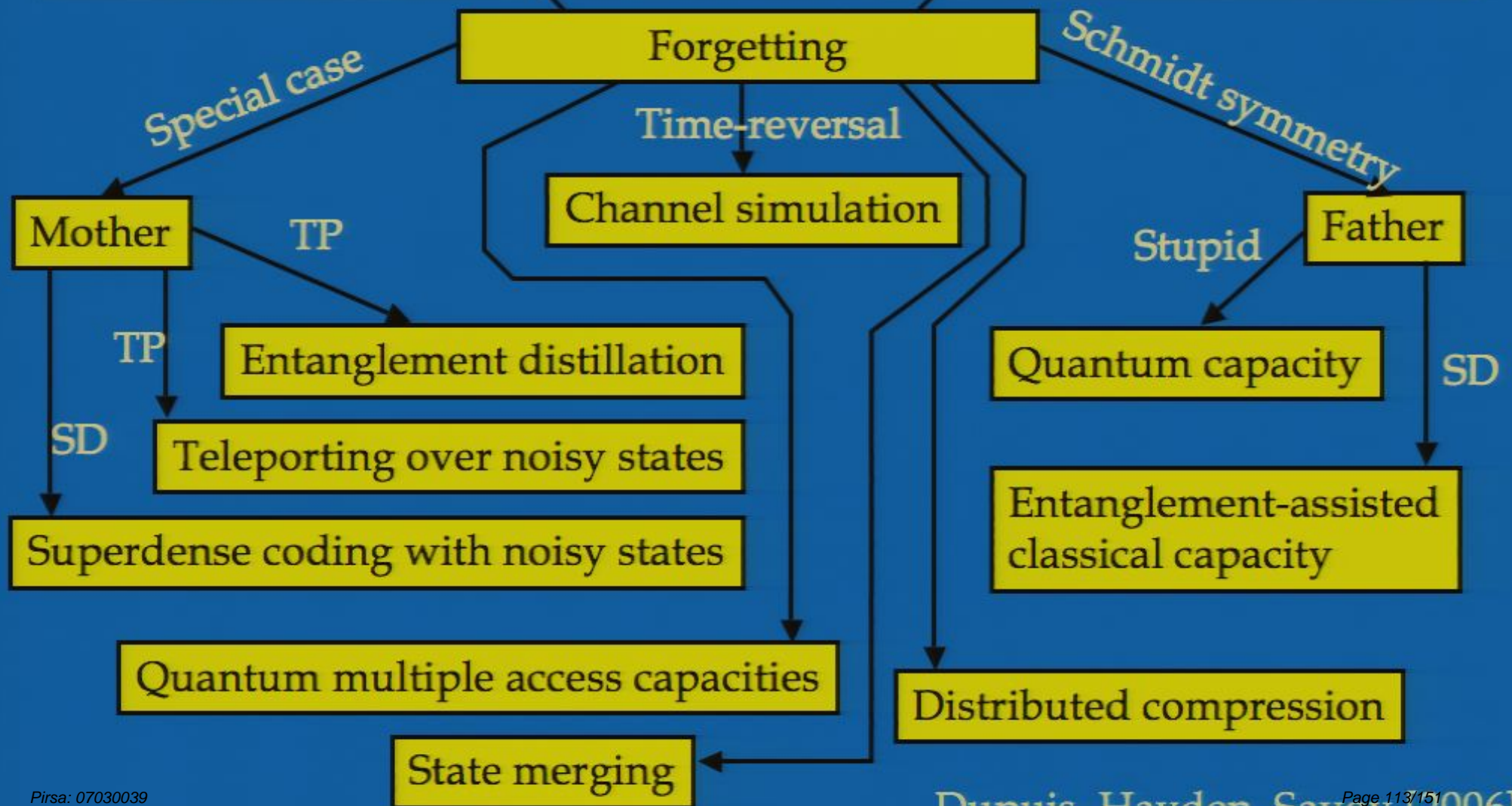
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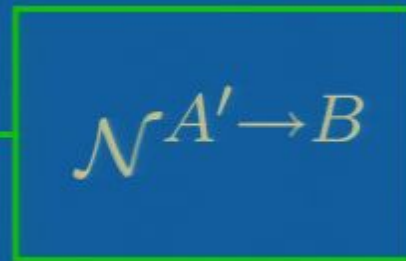
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# Mother of all protocols



# Entanglement-assisted communication

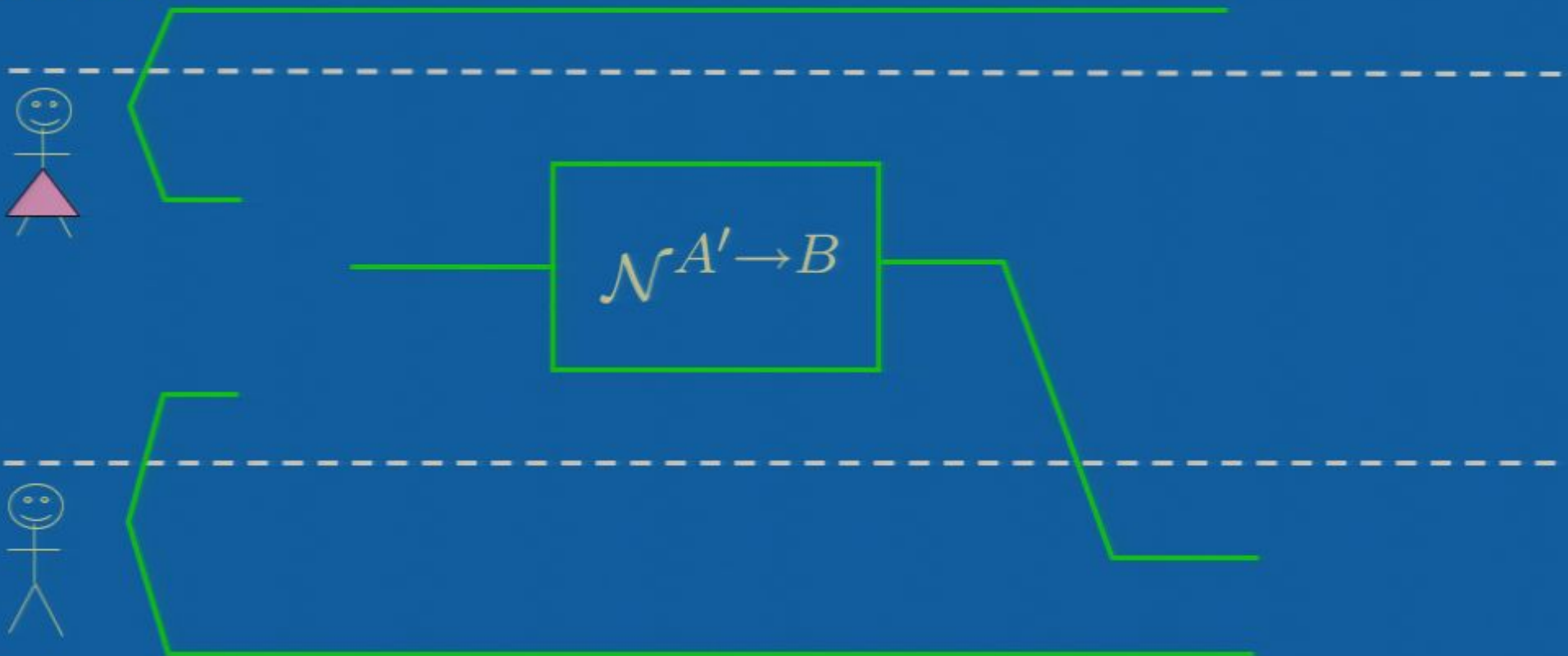
Reference





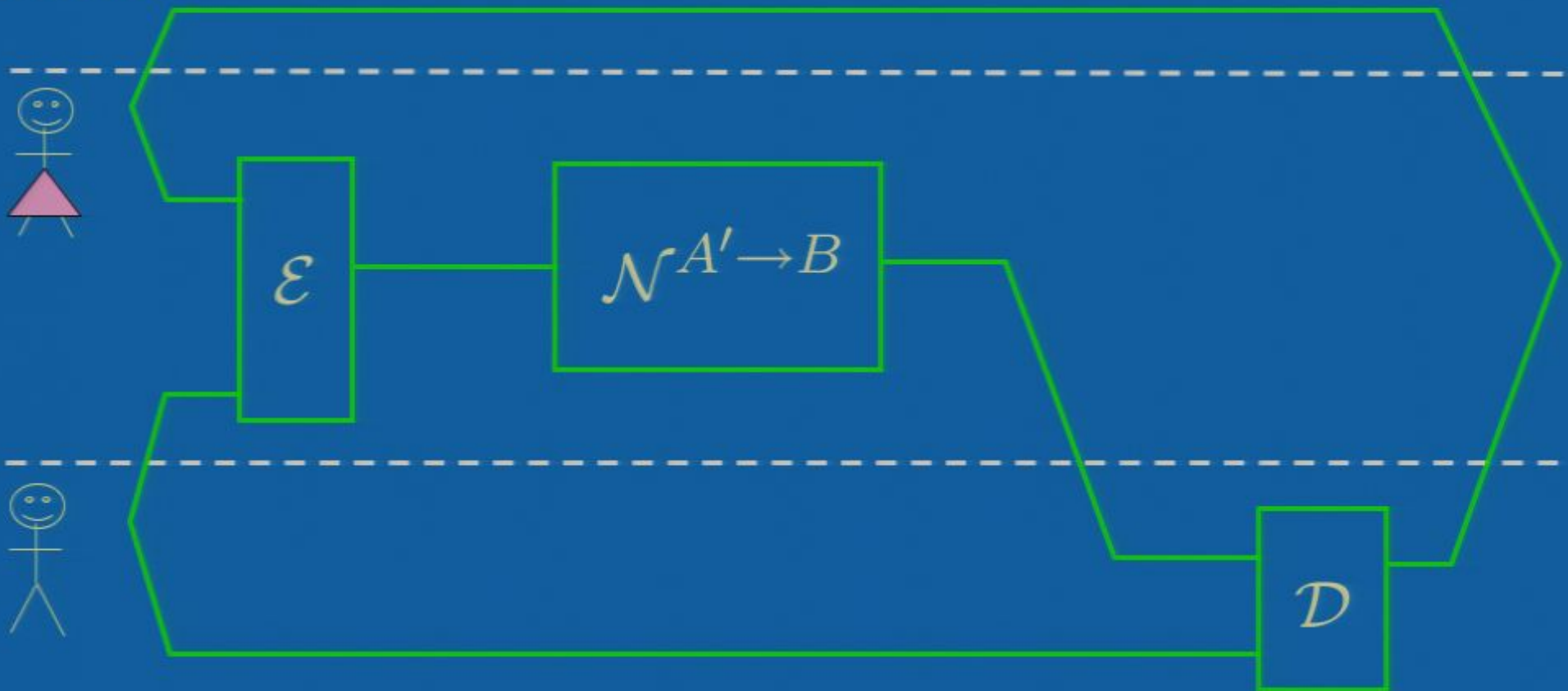
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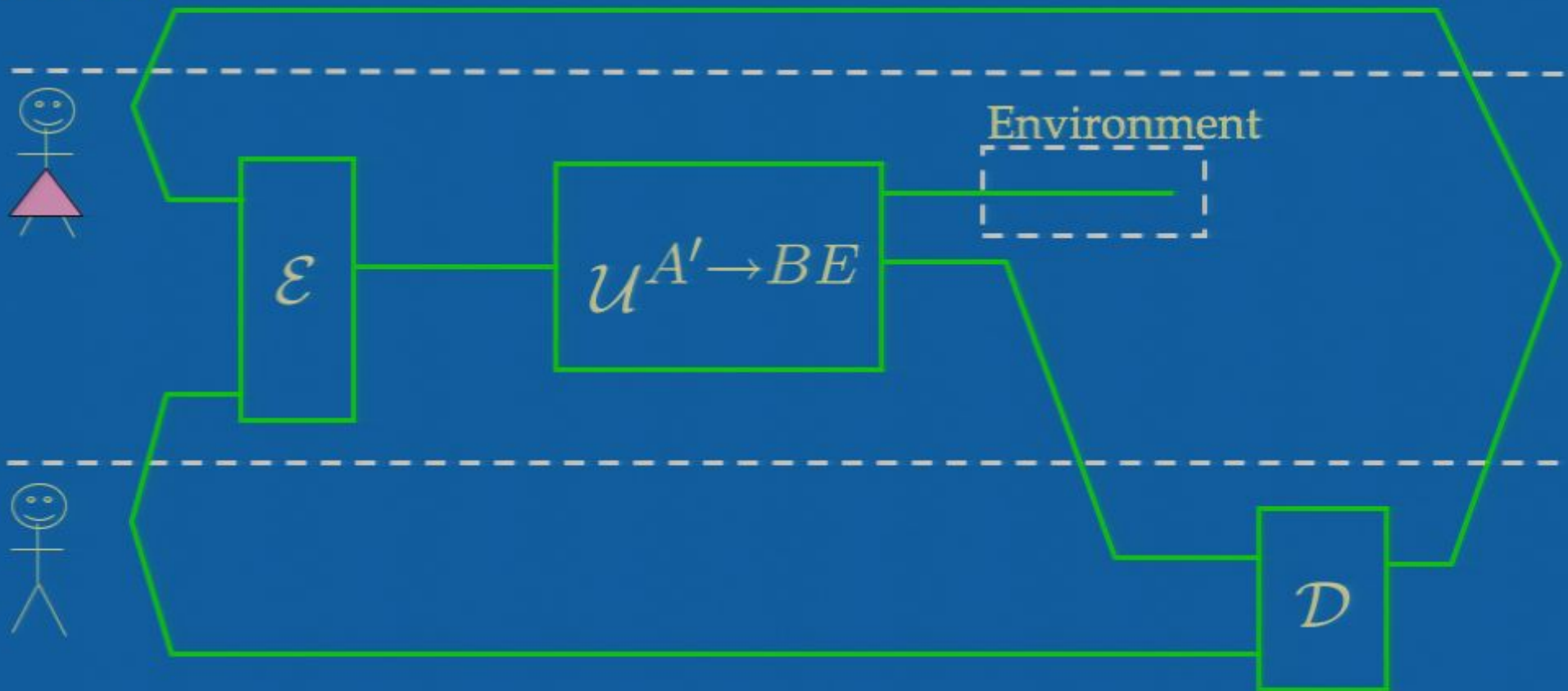
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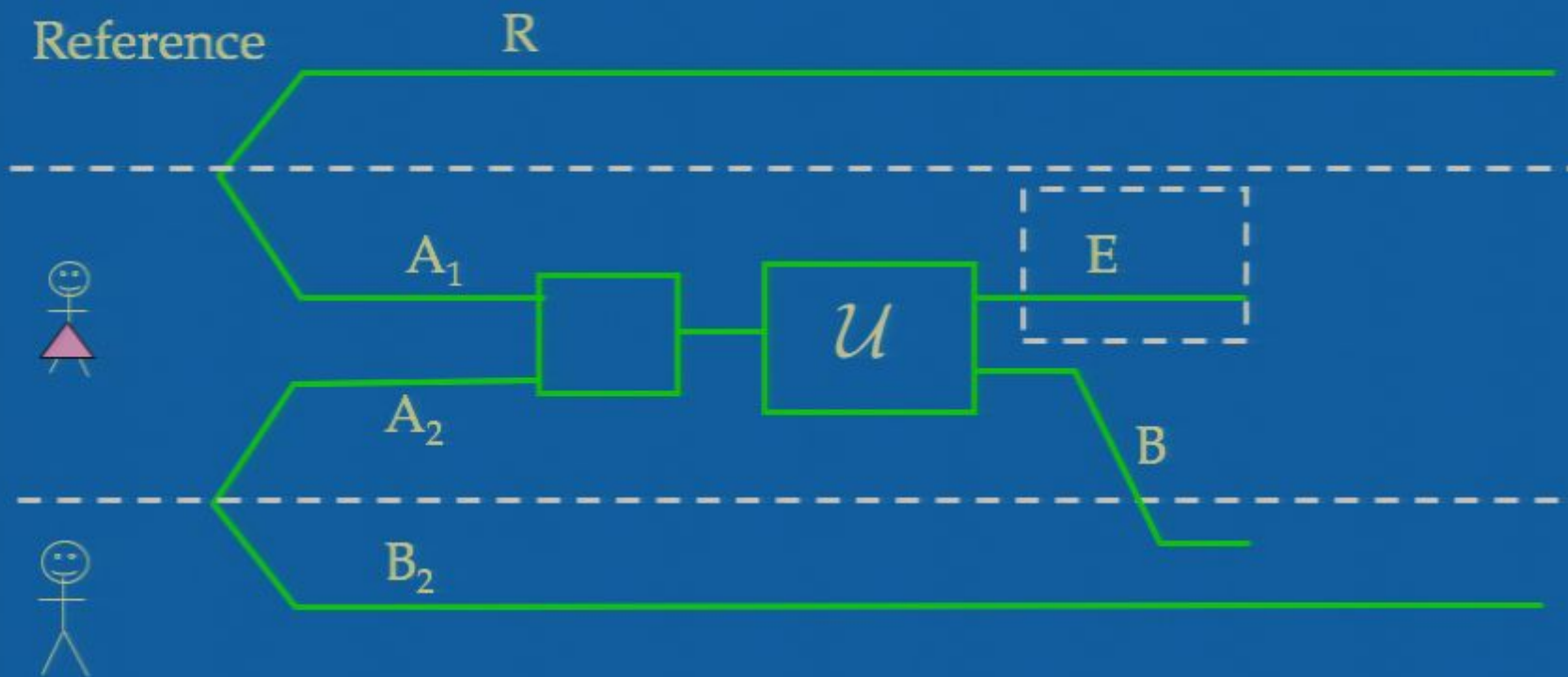


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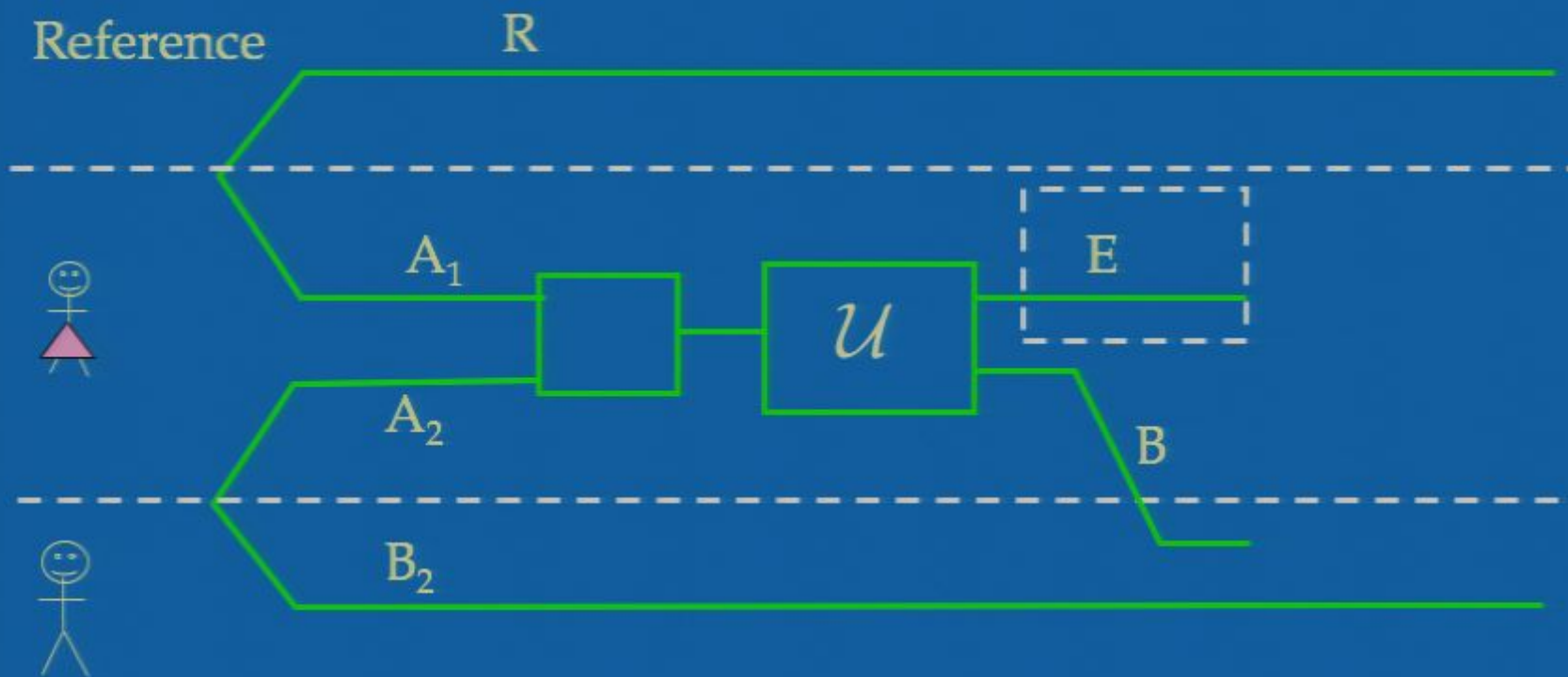


# Reduction to forgetting





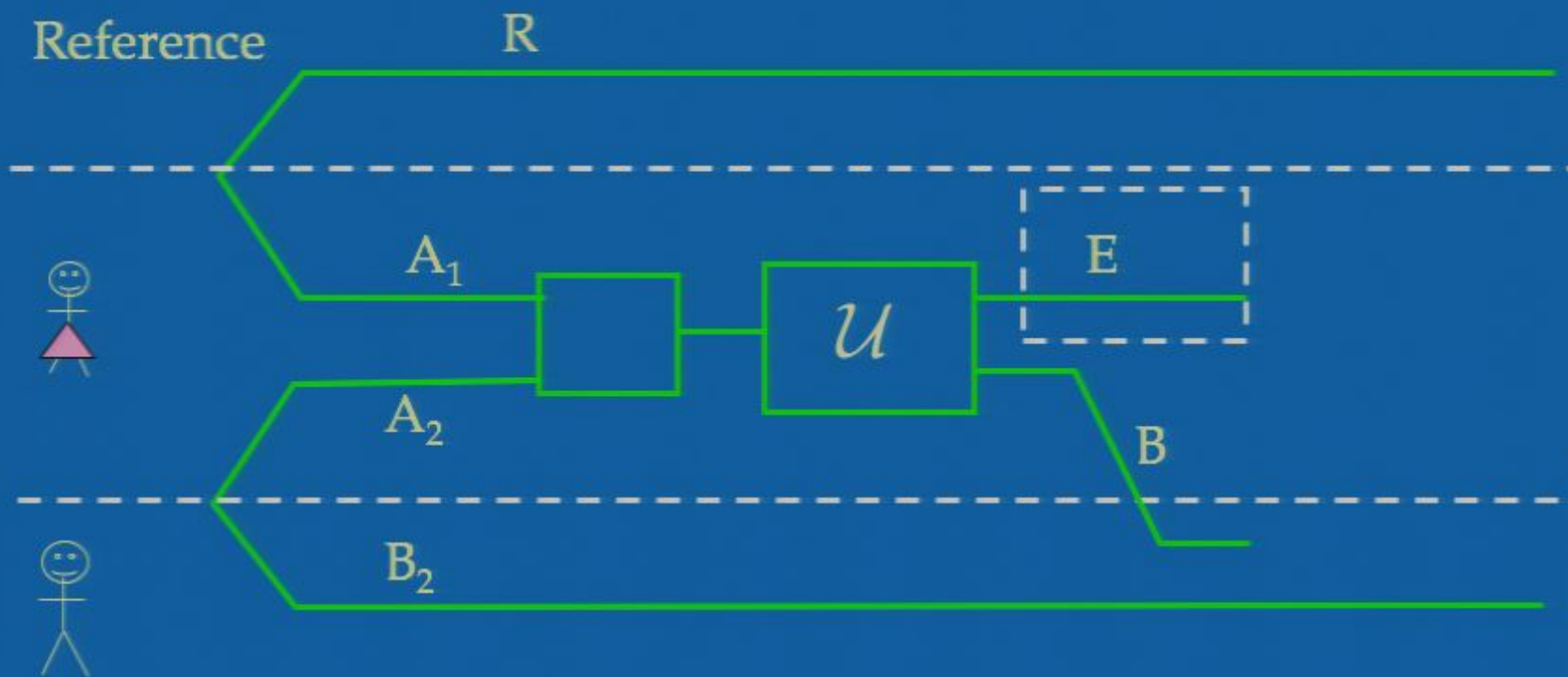
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*Who needs to do the forgetting?*

It is sufficient to ensure that there is a product state on  $R \otimes E$

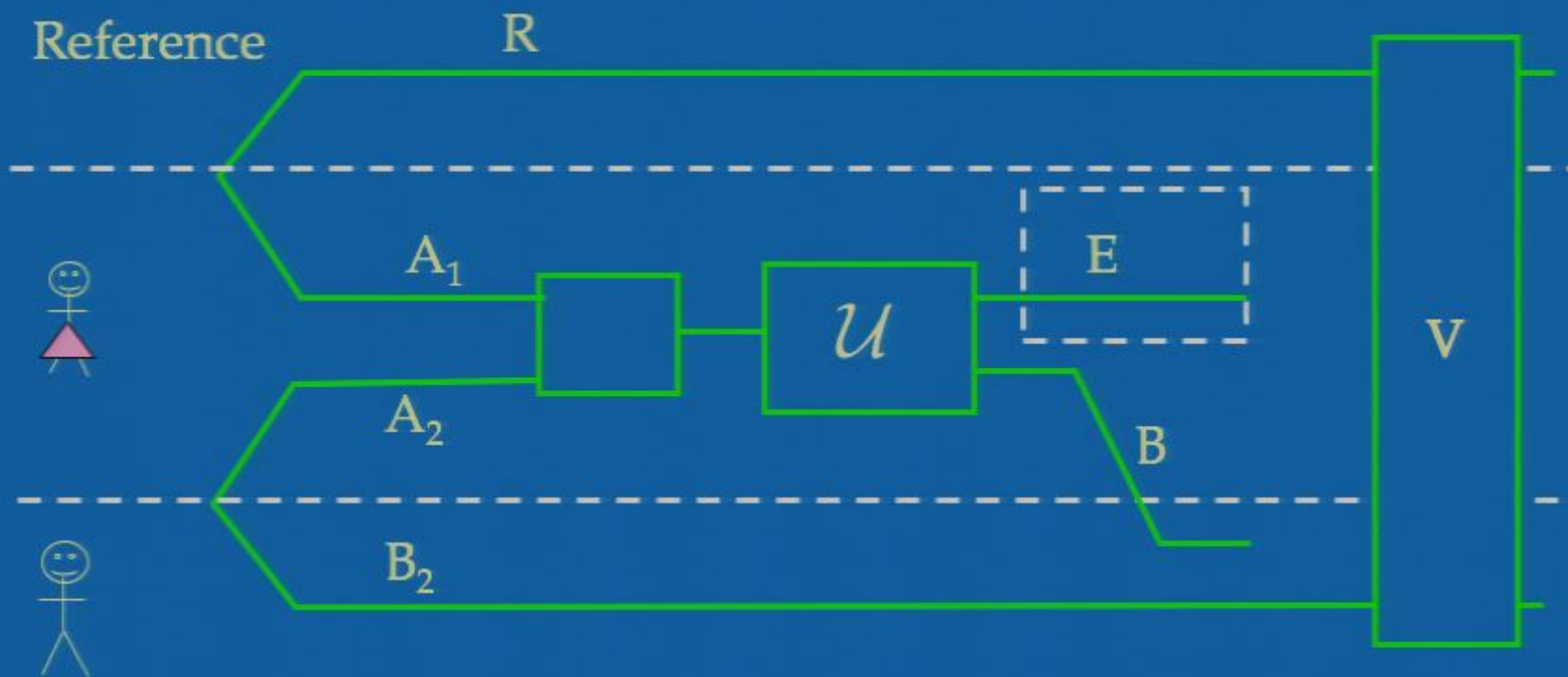
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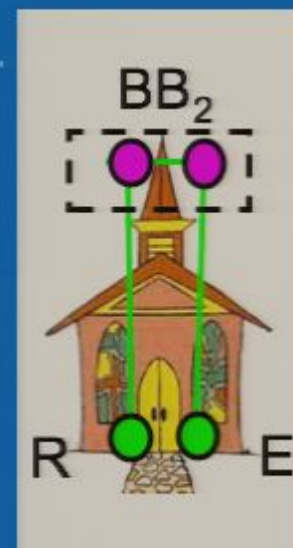


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*Imagine:* apply a random unitary  $V$  to  $RB_2$ .

*Result:* For sufficiently large  $B_2$ , product state on  $R \otimes E$  !



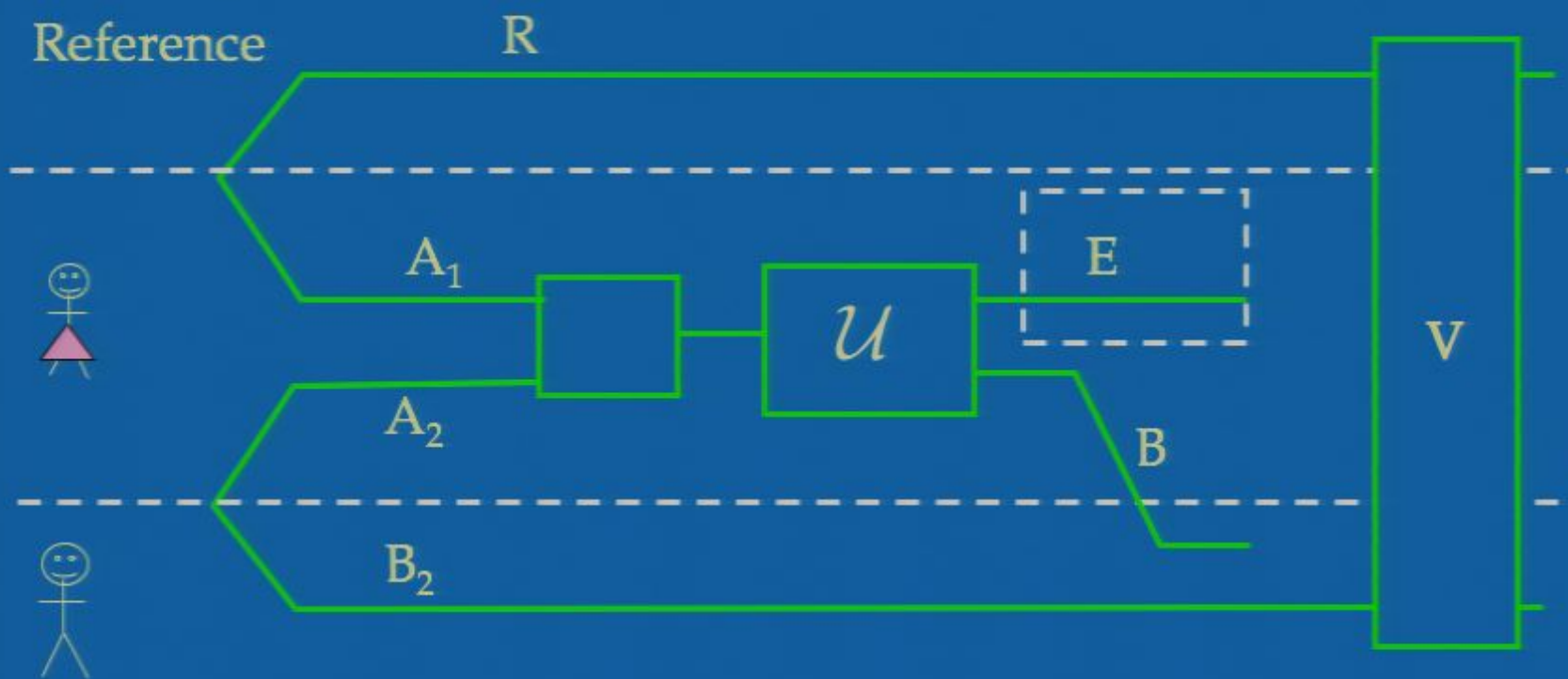


$$|\Phi\rangle = \frac{1}{\sqrt{2}} (|0\rangle^A |0\rangle^B + |1\rangle^A |1\rangle^B)$$

$$(A \otimes I) |\Phi\rangle = (I \otimes A^\dagger) |\Phi\rangle$$



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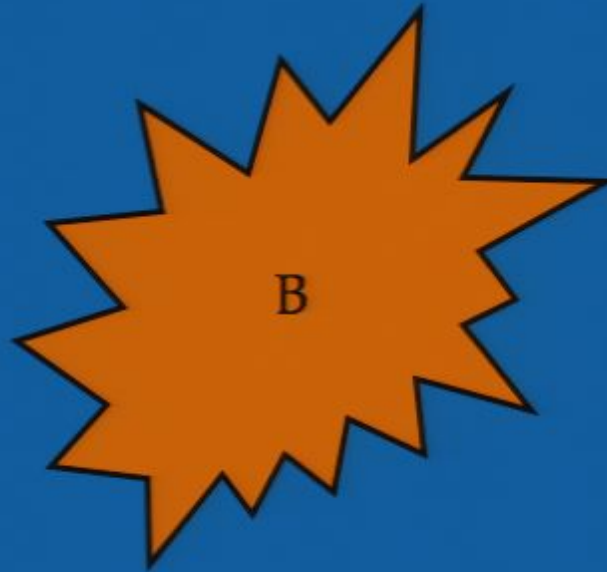
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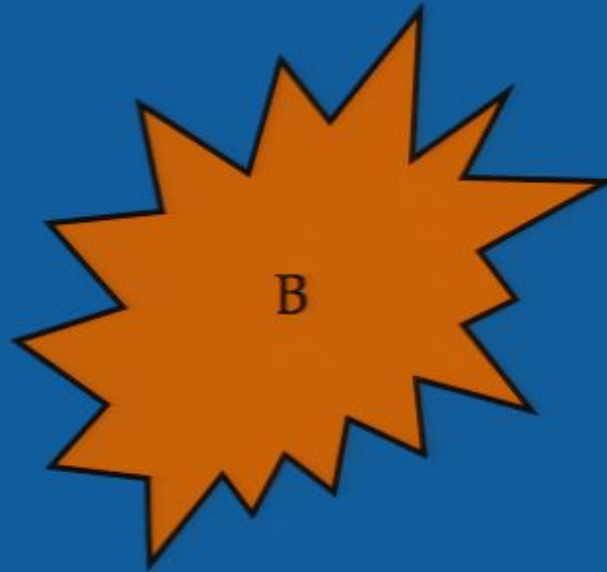
# Random dynamics and information leakage

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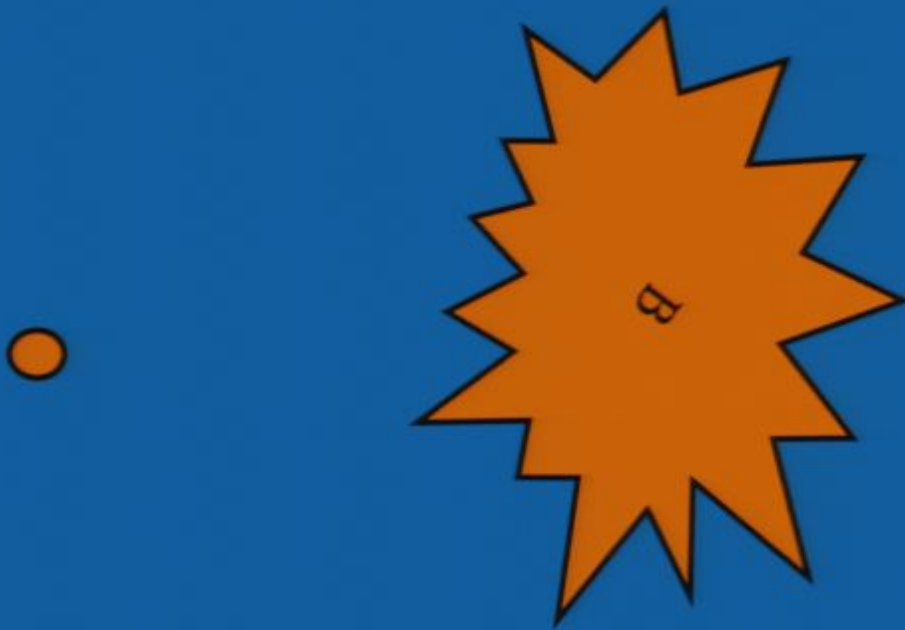
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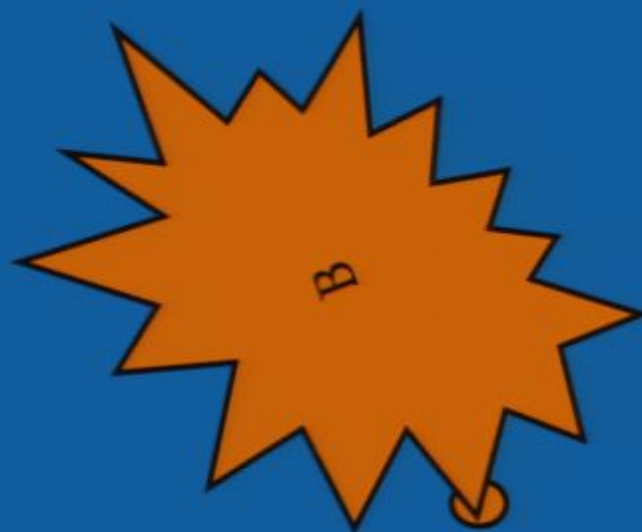
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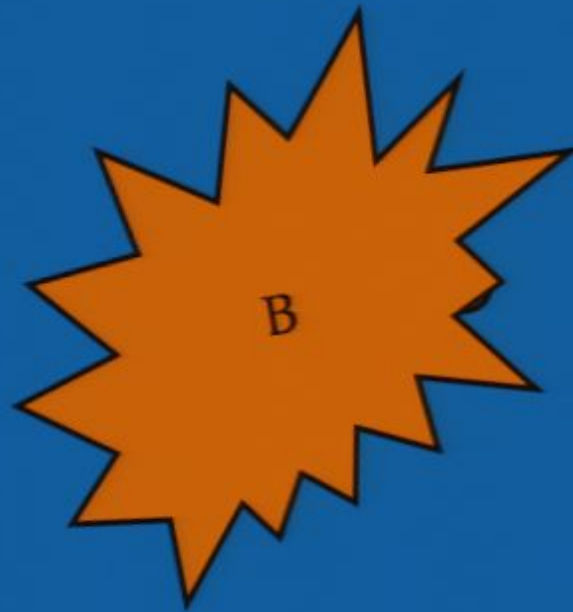
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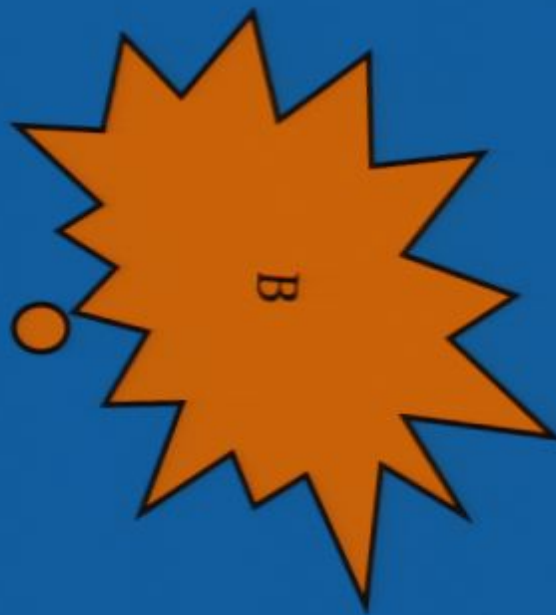
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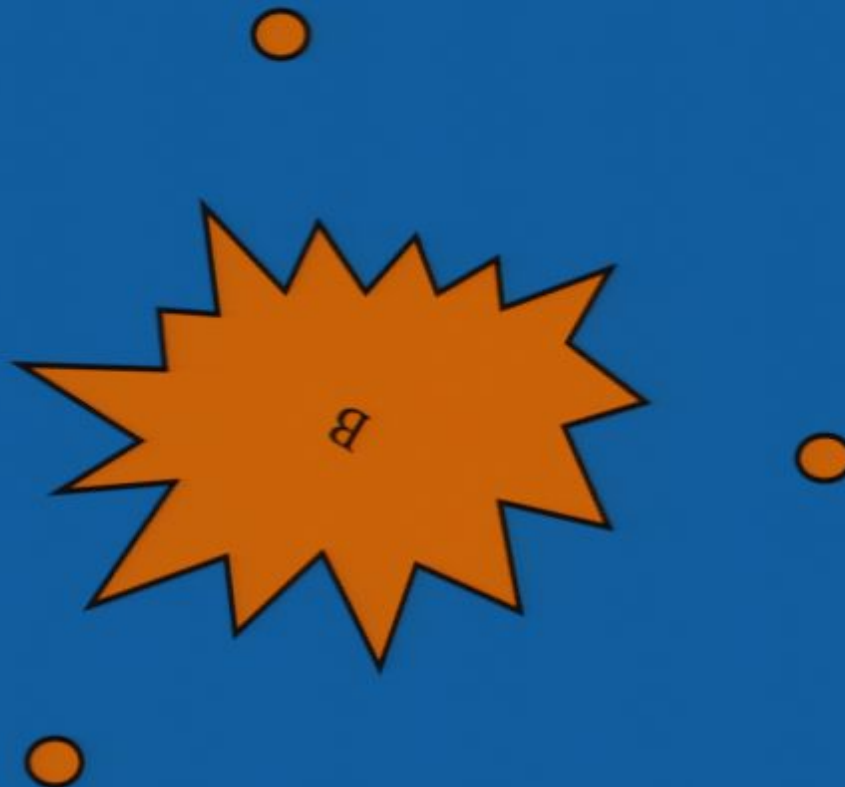
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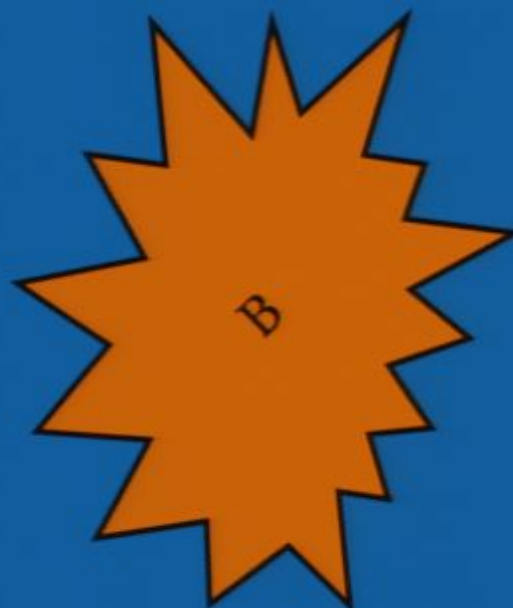
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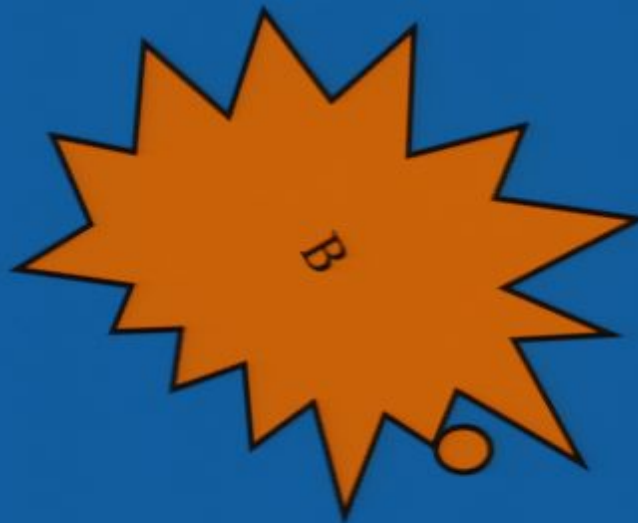
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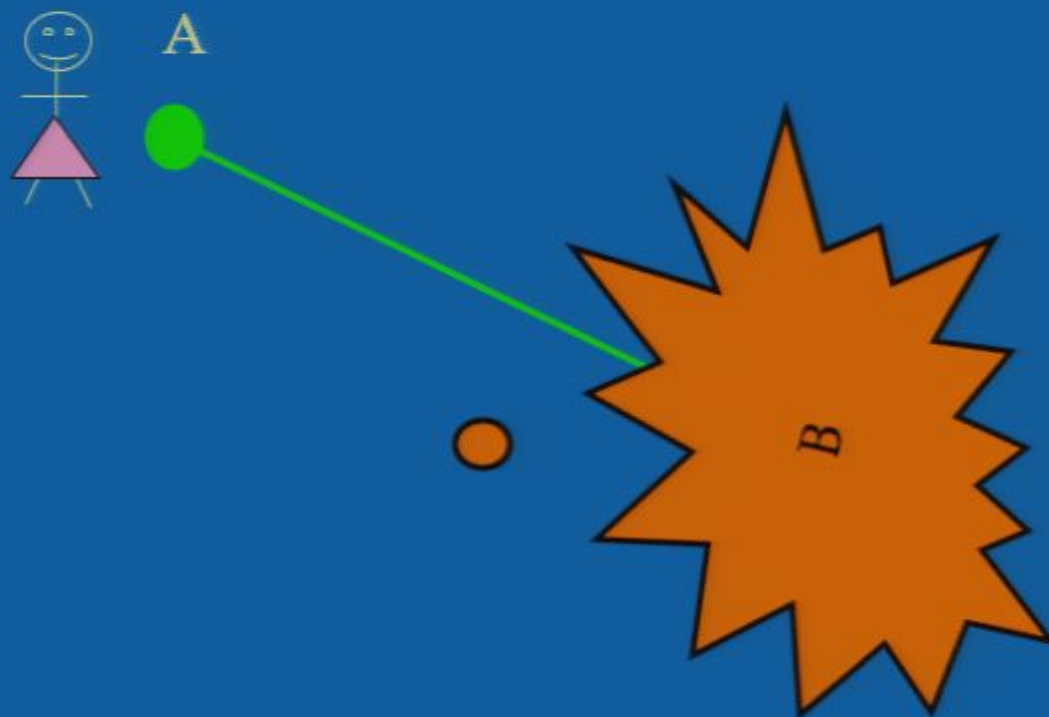
# Random dynamics and information leakage

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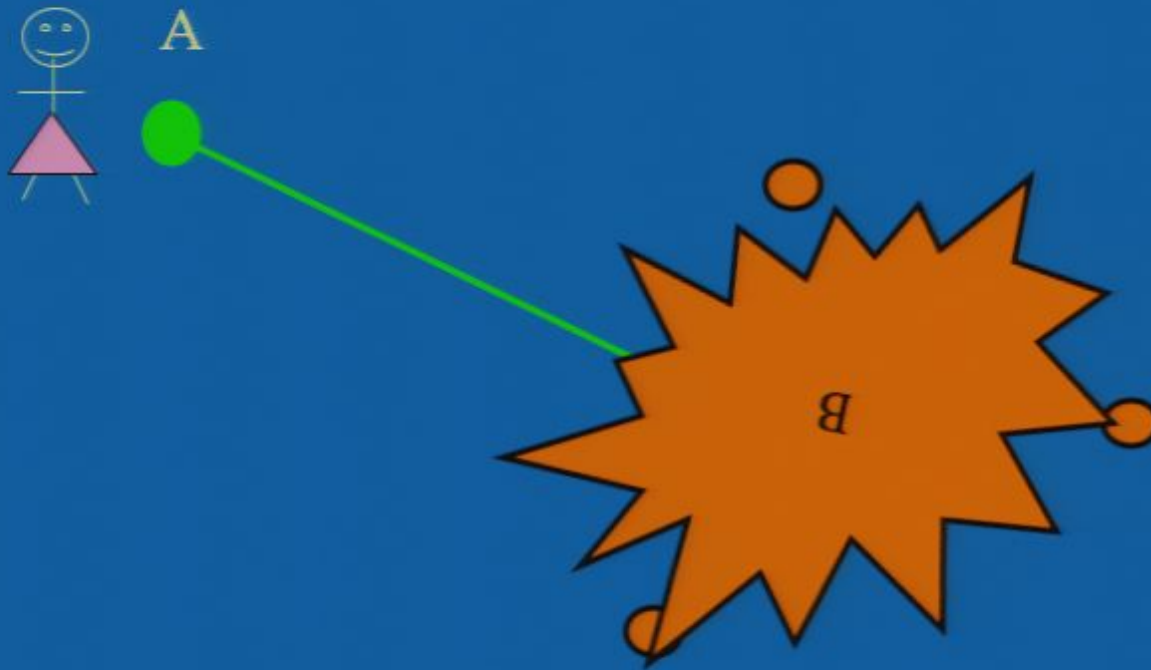


# Random dynamics and information leakage

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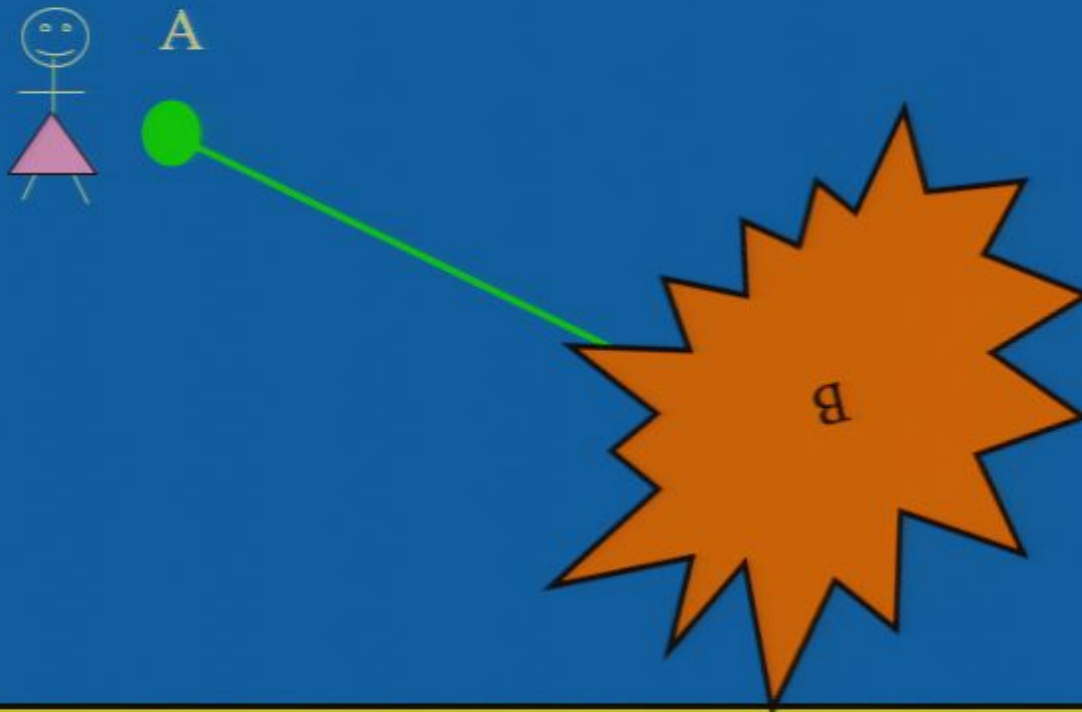
# Random dynamics and information leakage



How long must Alice wait until the “information about A” gets radiated?



# Random dynamics and information leakage



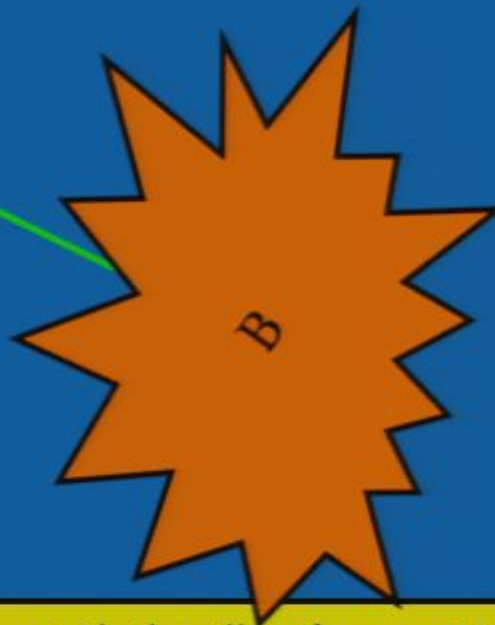
How long must Alice wait until the “information about A” gets radiated?

Equivalently, how long until the orange blob has forgotten about A?

# Random dynamics and information leakage



A



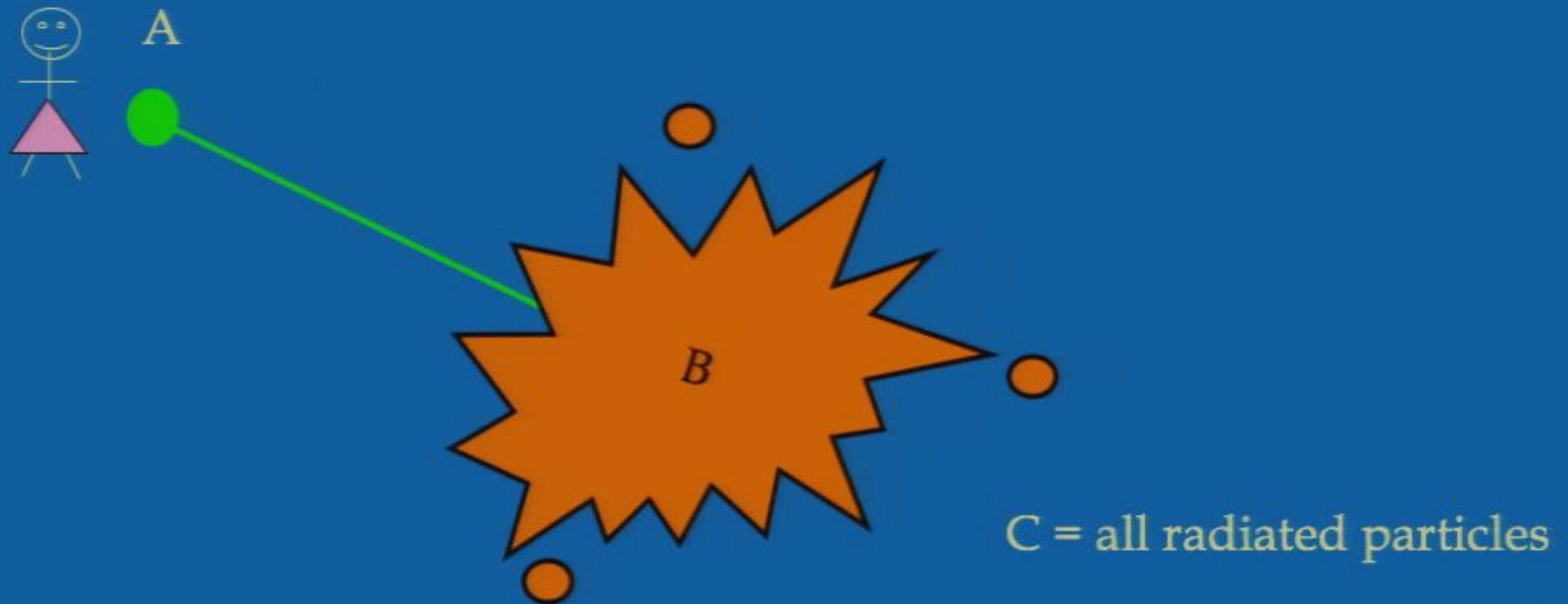
B

C = all radiated particles

How long must Alice wait until the “information about A” gets radiated?

Equivalently, how long until the orange blob has forgotten about A?

# Random dynamics and information leakage



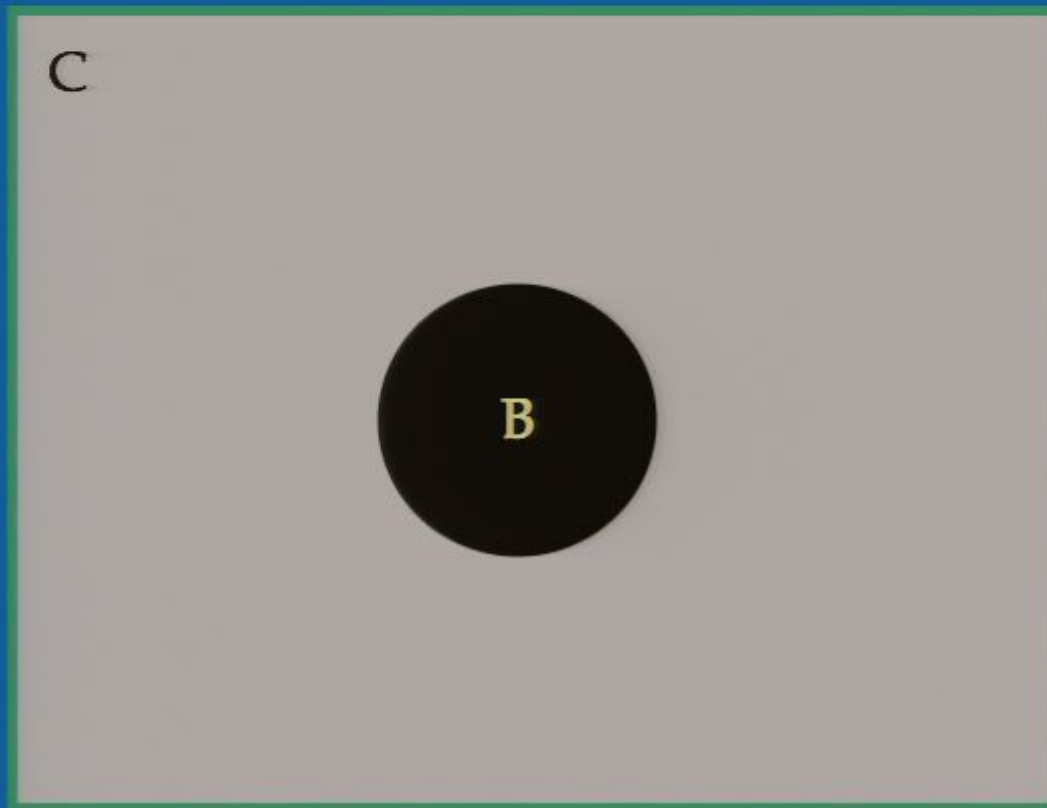
How long must Alice wait until the “information about A” gets radiated?

Equivalently, how long until the orange blob has forgotten about A?

For sufficiently mixing dynamics,  
information about A is released *almost immediately*

# Lessons for black hole information loss?

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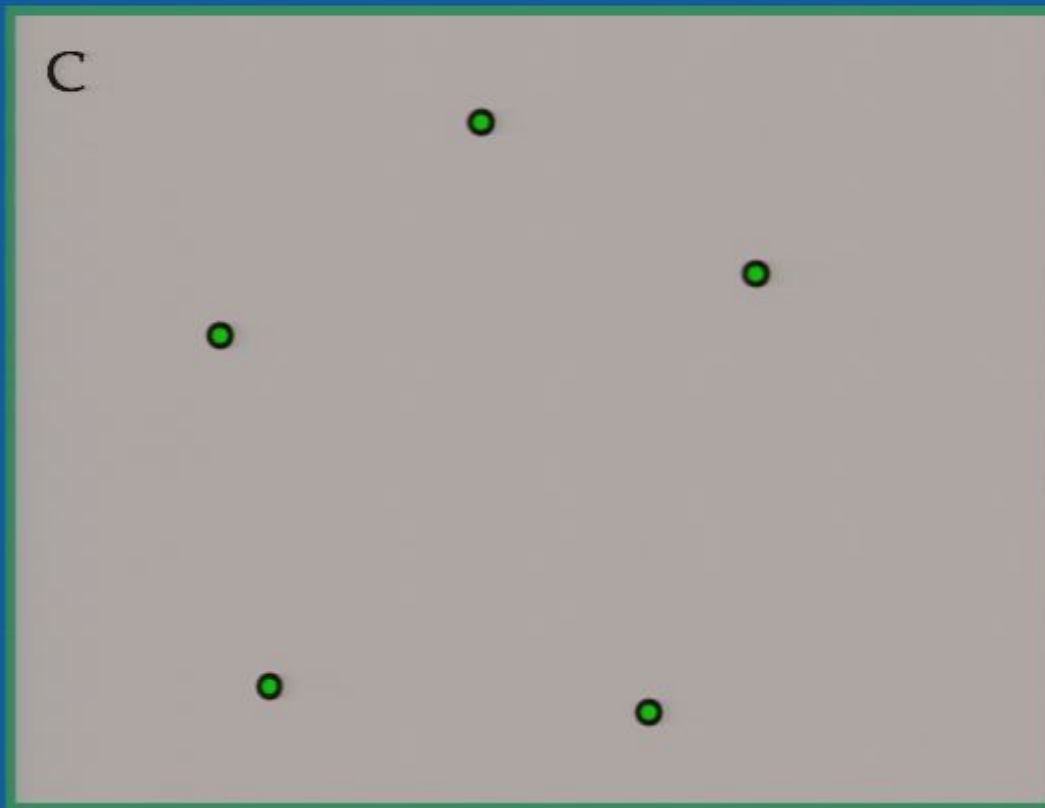
$t_0$ : Pure state:

B: Black hole

C: Rest of universe



# Lessons for black hole information loss?



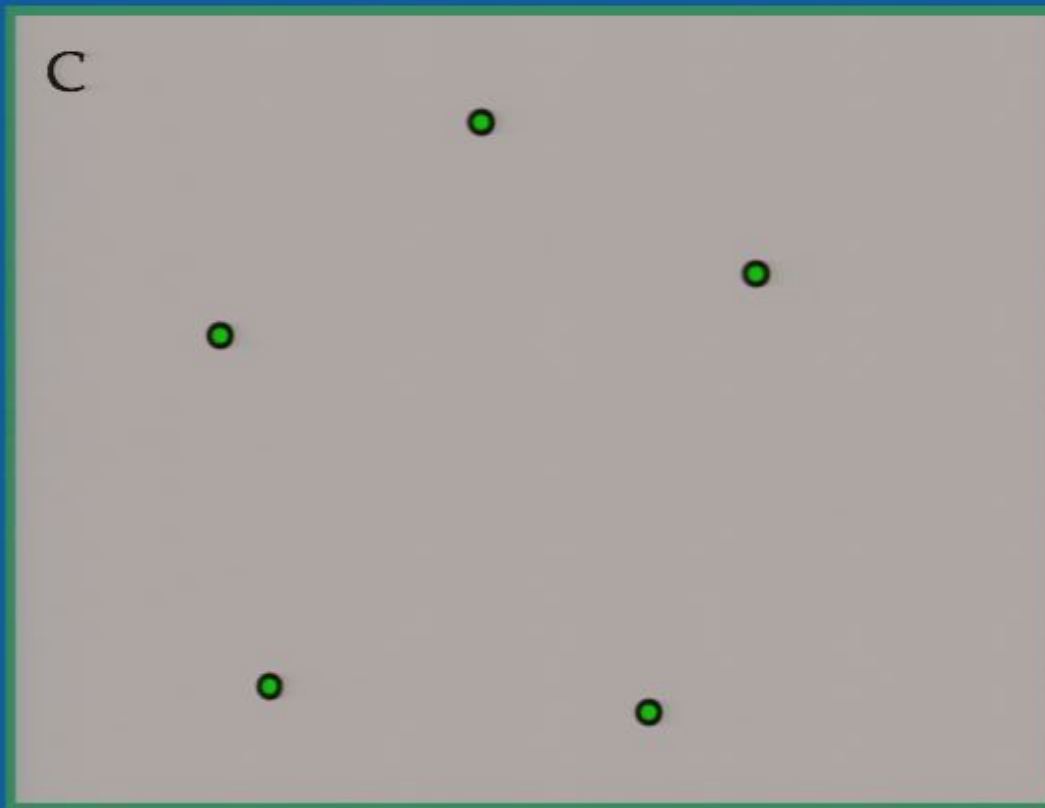
$t_0$ : Pure state:

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$t_1$ : Thermal Hawking radiation

# Lessons for black hole information loss?



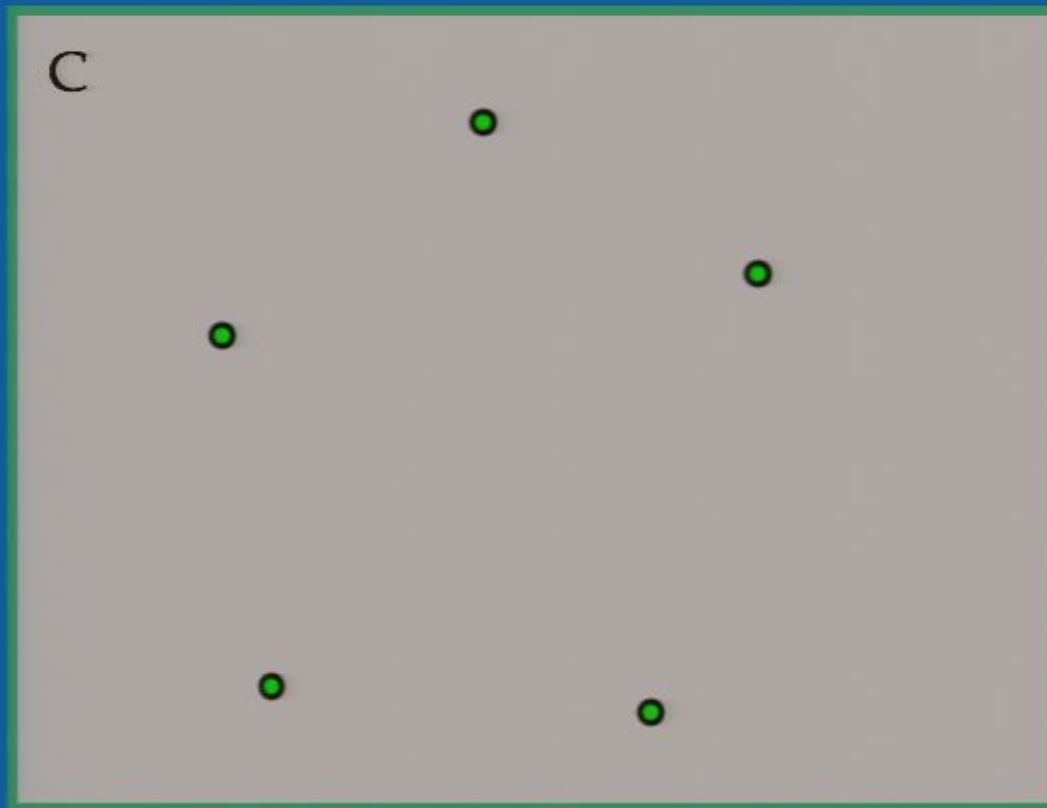
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# Lessons for black hole information loss?



$t_0$ : Pure state:

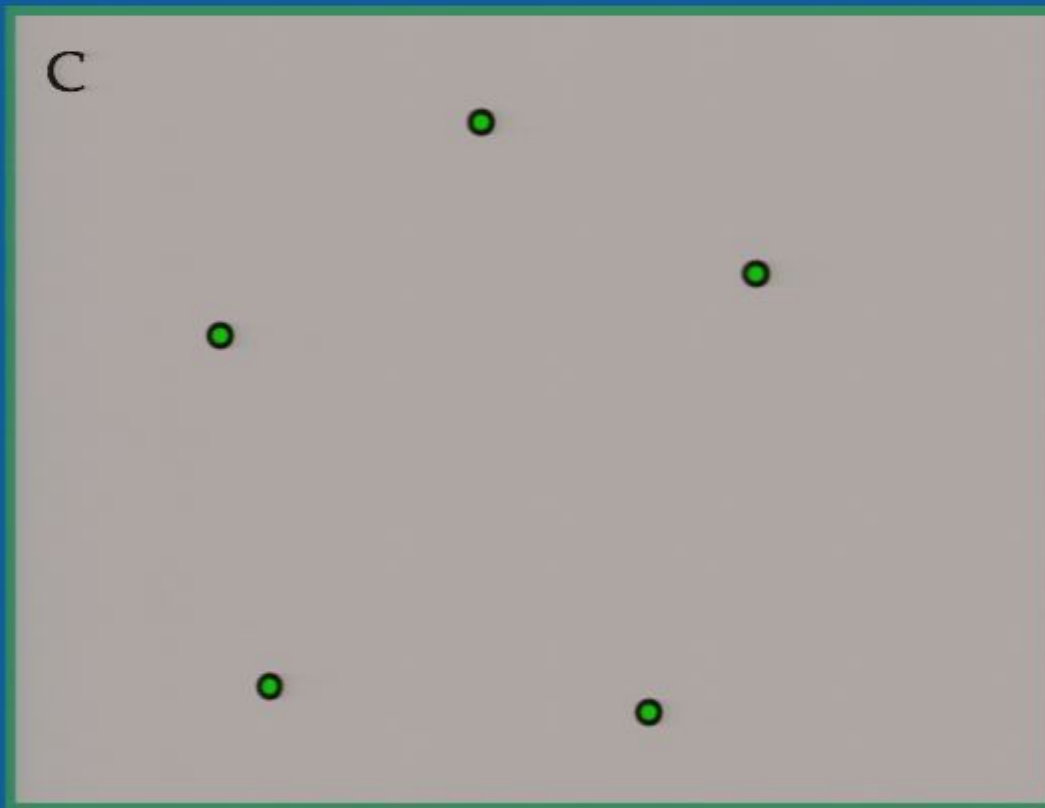
B: Black hole

C: Rest of universe

$t_1$ : Thermal Hawking radiation

$t_2$ : Radiation but no black hole

# Lessons for black hole information loss?



$t_0$ : Pure state:

B: Black hole

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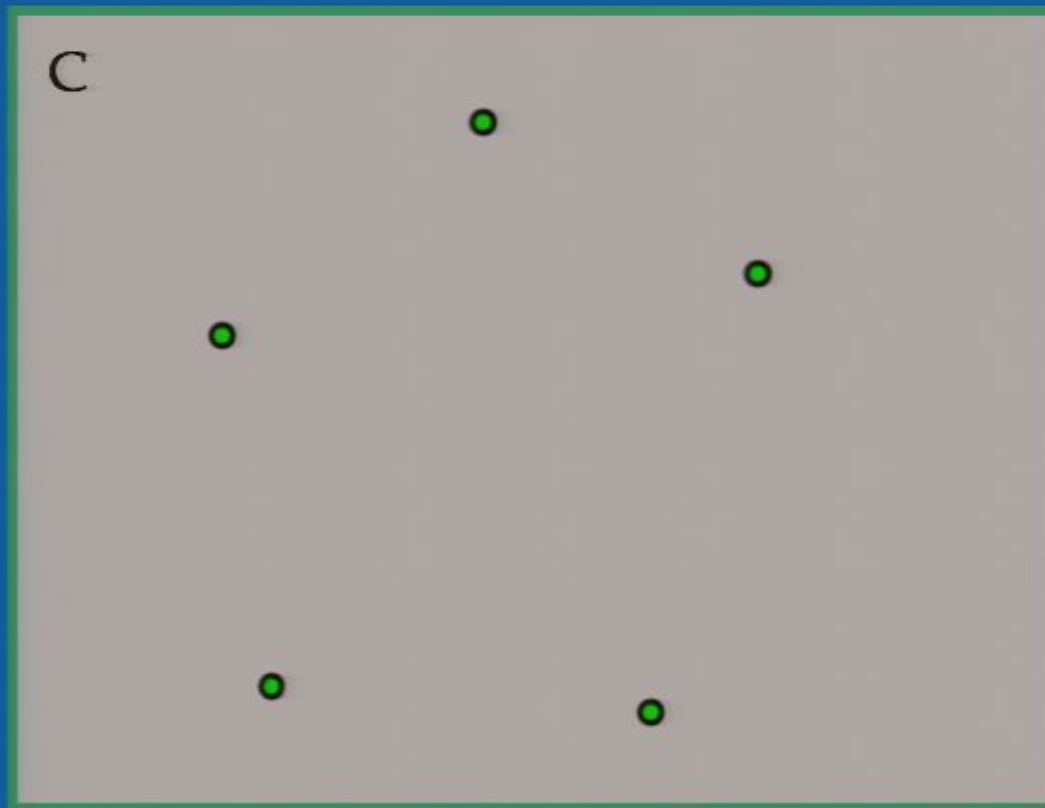
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**Standard question:** Is final state mixed or pure?



# Lessons for black hole information loss?



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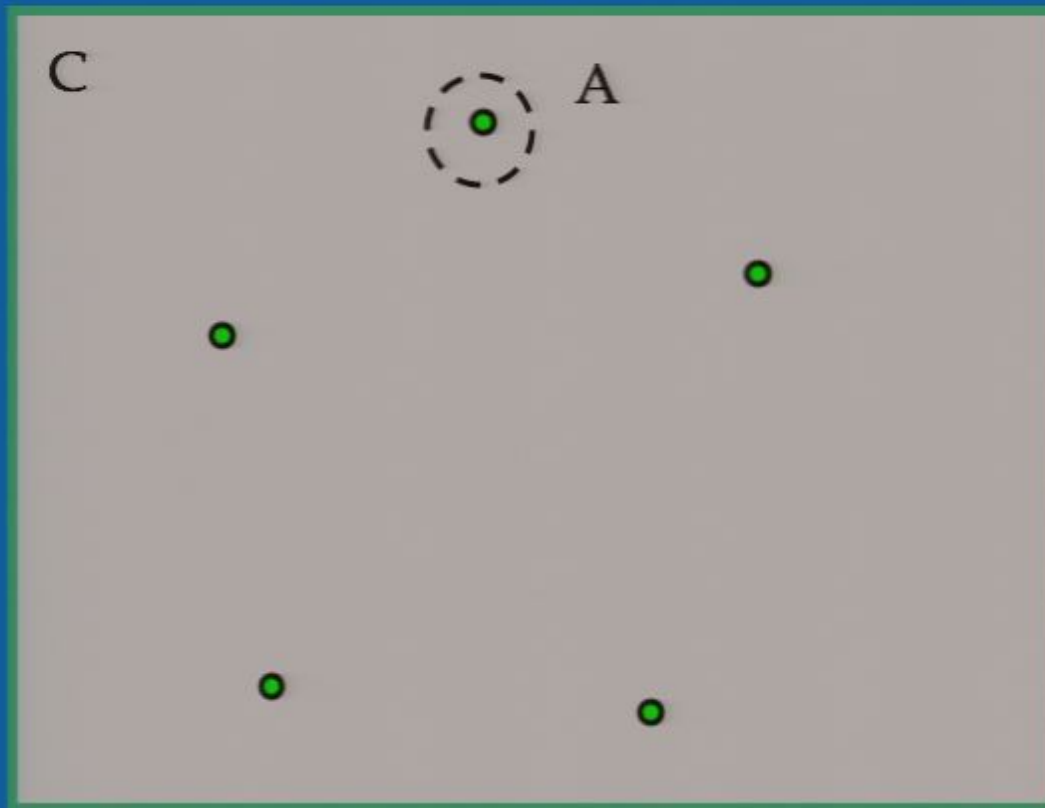
$t_1$ : Thermal Hawking radiation

$t_2$ : Radiation but no black hole

**Standard question:** Is final state mixed or pure?

**New question:** Is final state of *some* radiation purified by rest of universe?

# Lessons for black hole information loss?



$t_0$ : Pure state:

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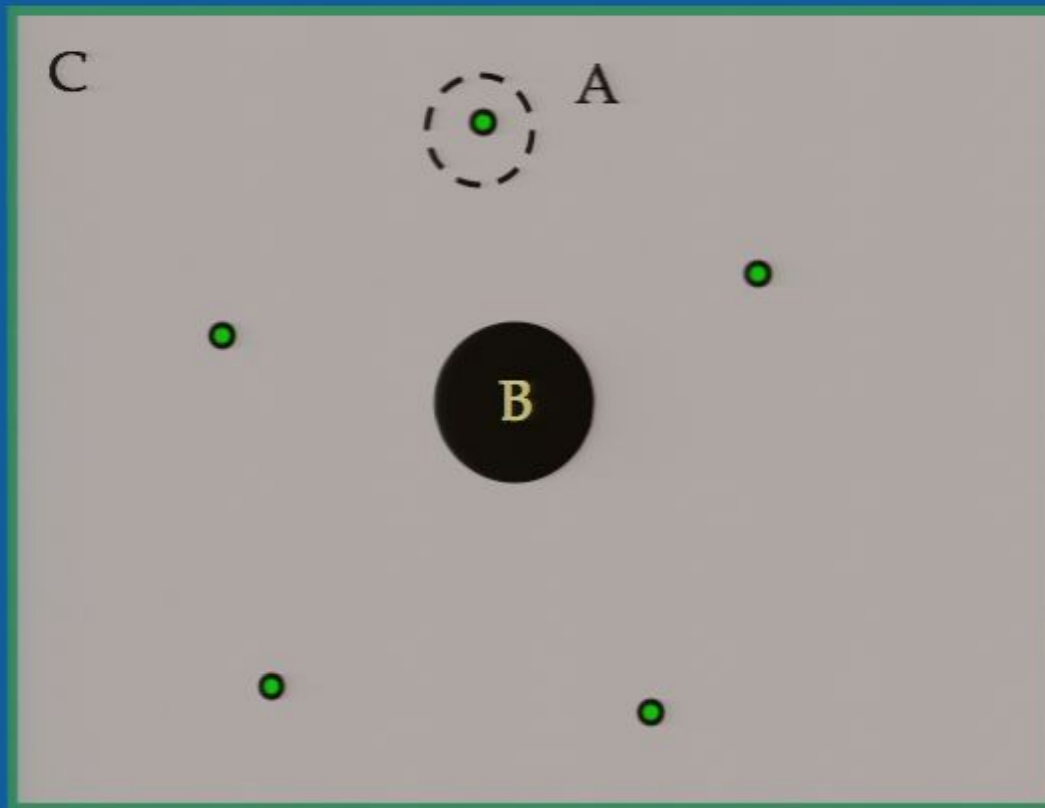
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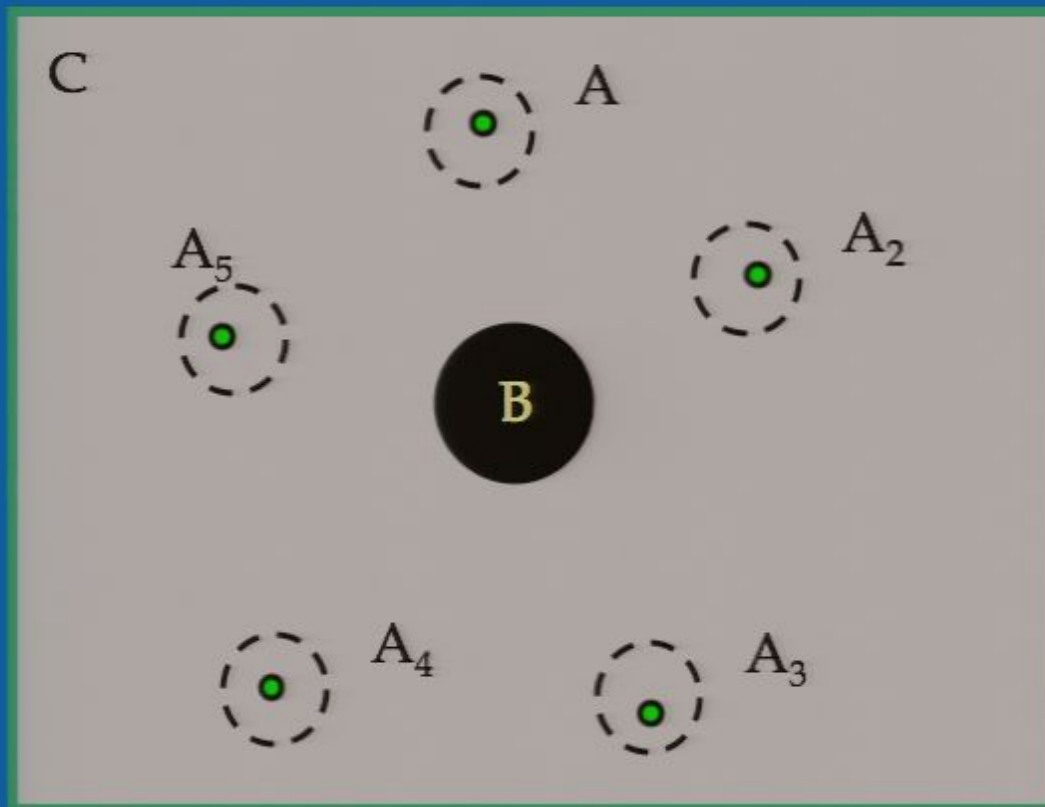
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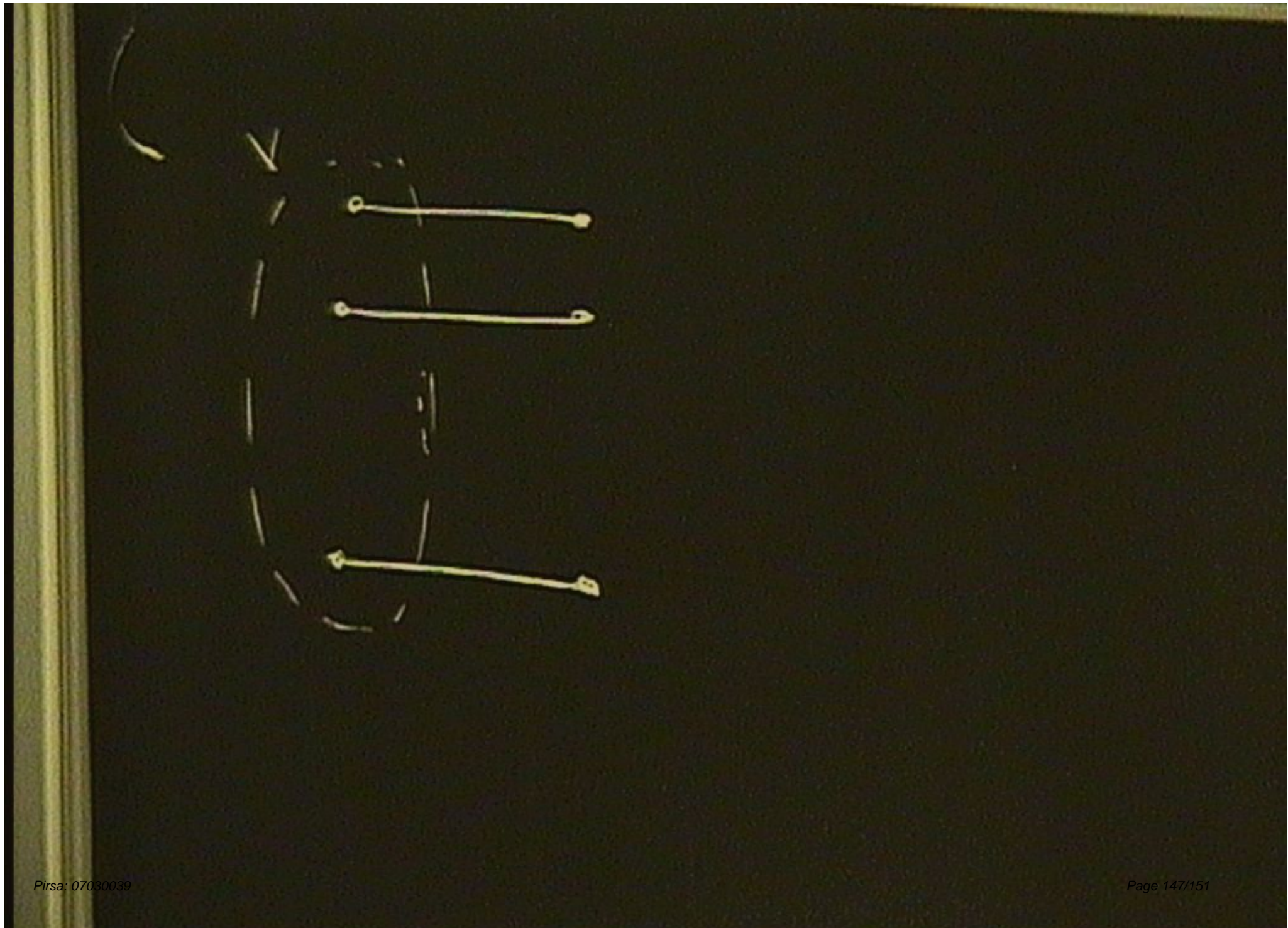
$t_1$ : Thermal Hawking radiation

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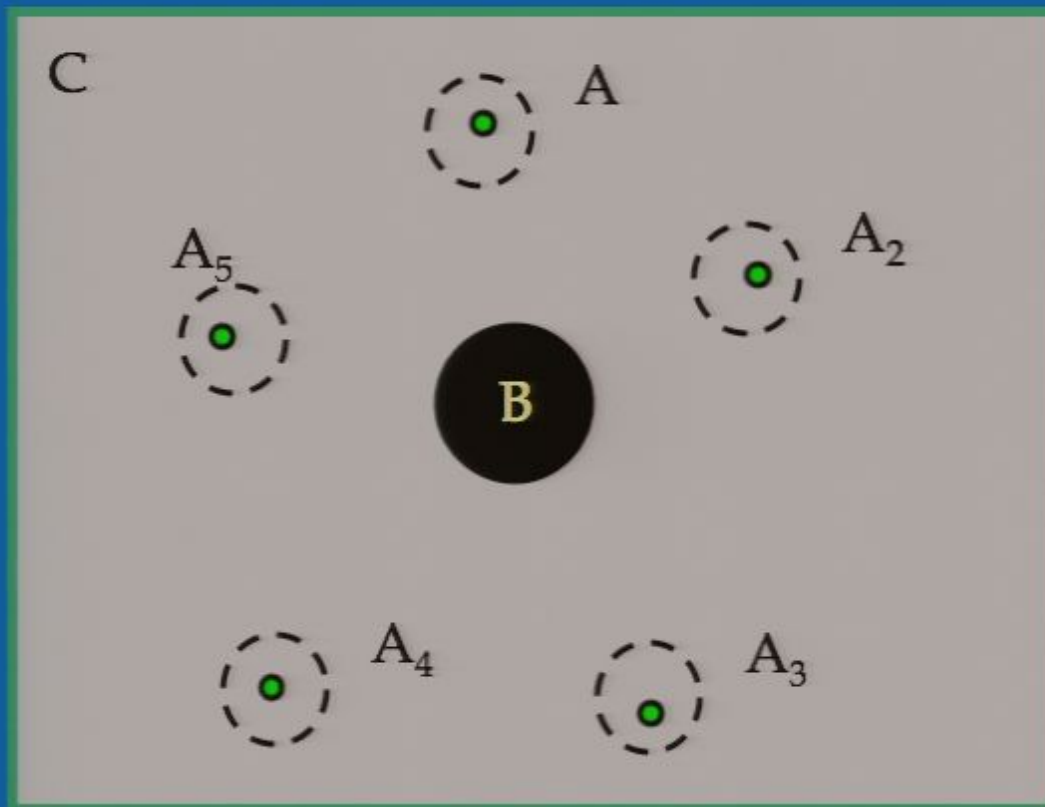
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# Lessons for black hole information loss?



$t_0$ : Pure state:

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$t_1$ : Thermal Hawking radiation

$t_2$ : Radiation but no black hole

**Standard question:** Is final state mixed or pure?

**New question:** Is final state of *some* radiation purified by rest of universe?



# Summary

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- Forgetting is a basic primitive for quantum information theory
- Detailed understanding of how to do it most efficiently
- These methods are generated by generic unitary transformations: could be useful for understanding real physics

<http://arxiv.org/abs/quant-ph/0606225>

