Title: Quantum Error Correction 11B

Date: Mar 20, 2007 05:00 PM

URL: http://pirsa.org/07030036

Abstract: Behavior of particles in qudit toric code, braid group, basic idea of fault tolerance with non-Abelian anyons

2 types of charges: X-type & Z-type

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Can have value $0, \ldots, D-1$

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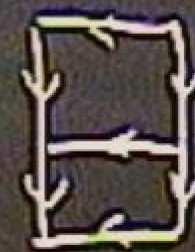

2 types of charges: X-type & Z-type
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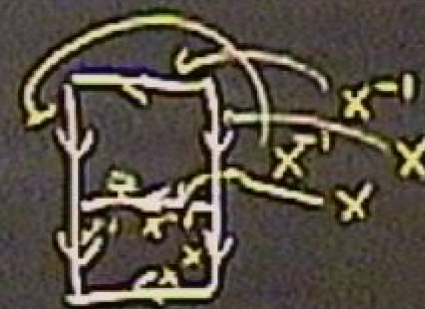
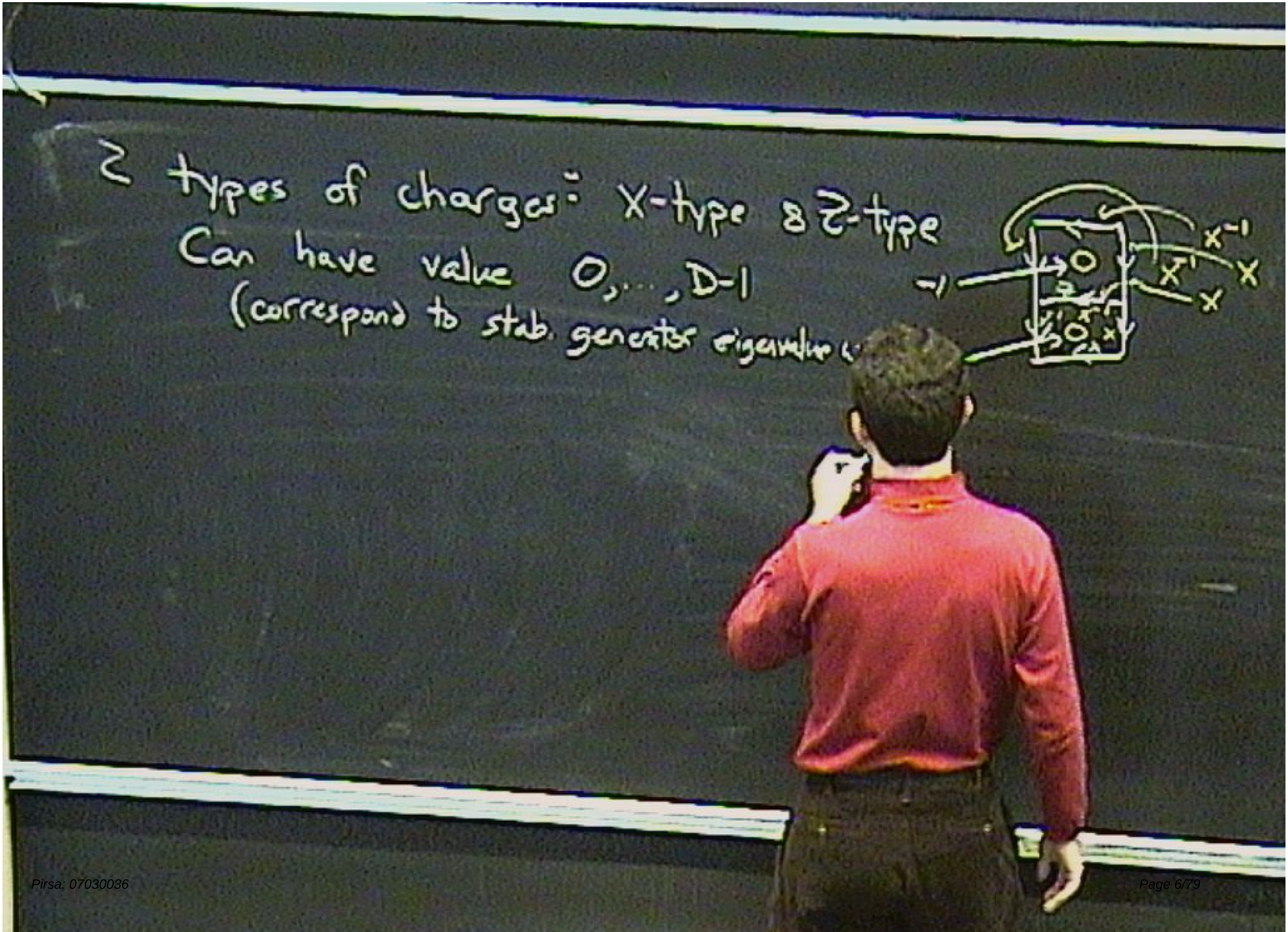
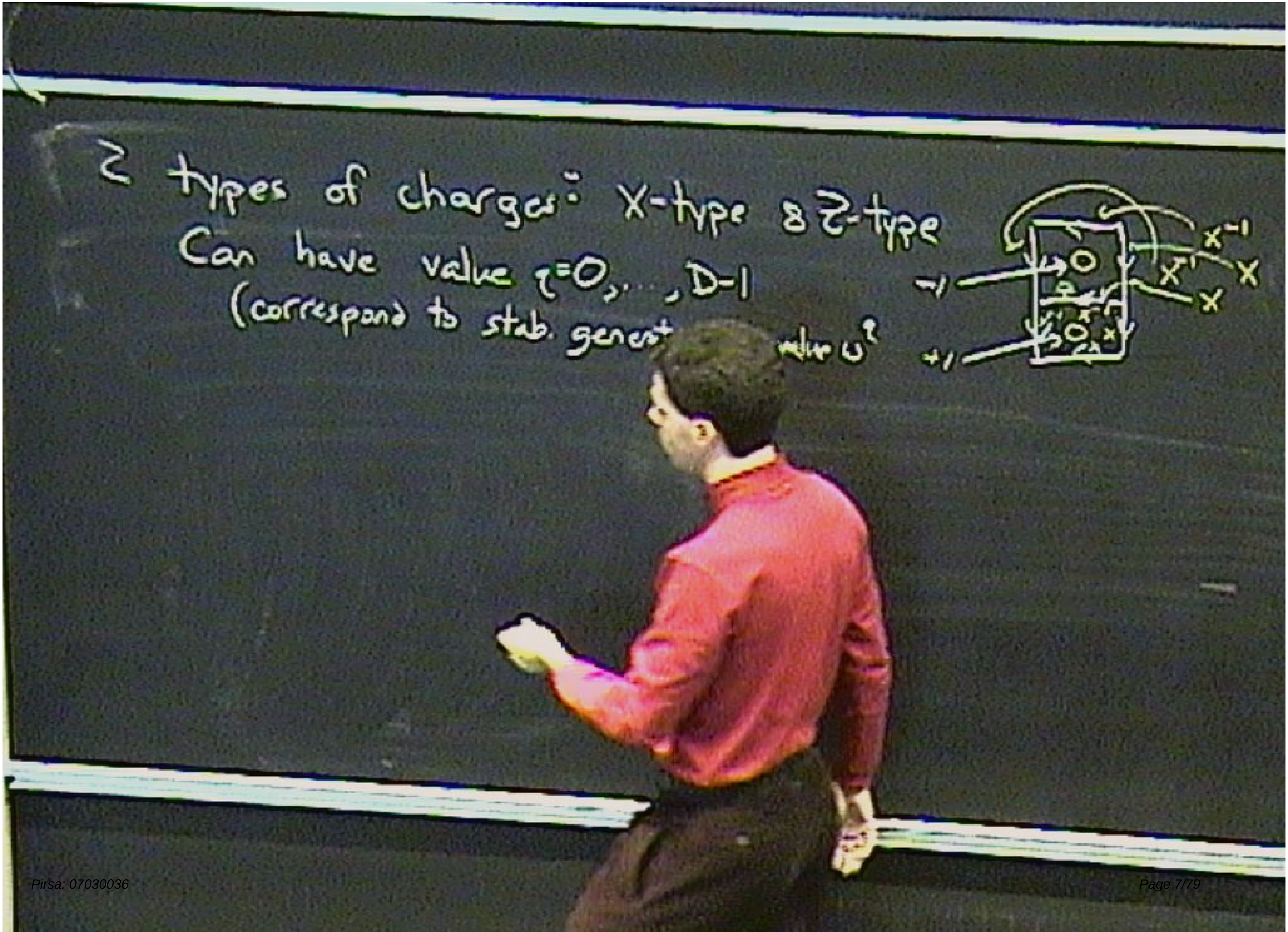



2 types of charges: X-type & Z-type
Can have value 0, ..., D-1
(correspond to stab. generator eigenvalue
-1
X
X
X

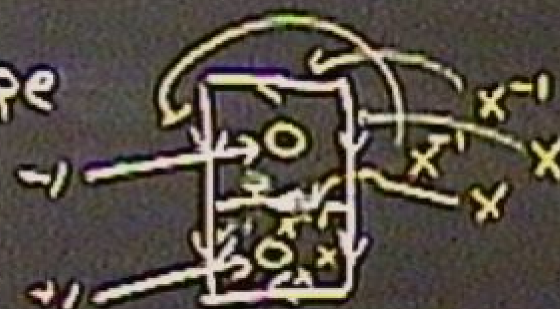

2 types of charges: X-type & Z-type
Can have value q=0, . . . , D-1
(correspond to stab. generat
value ω^q
X^-1
X
X
-1
+1

2 types of charges: X-type & Z-type

Can have value $q=0,\dots,D-1$

(correspond to stab. generator eigenvalue ω^q

$\omega = e^{2\pi i/D}$



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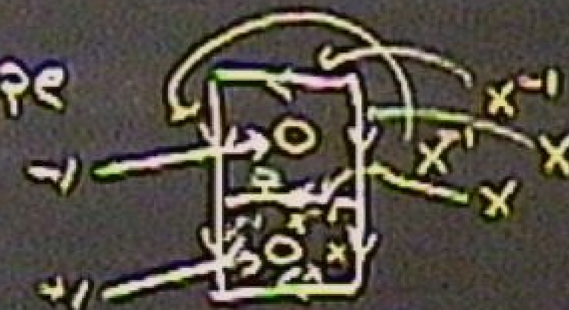
Charge conserved under movement



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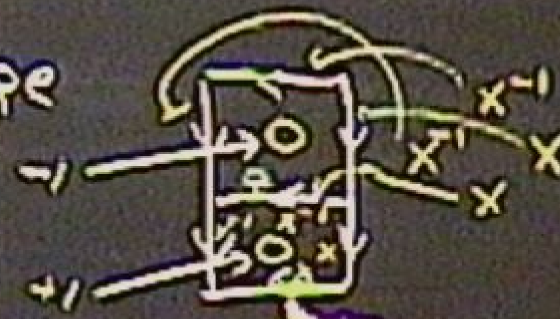

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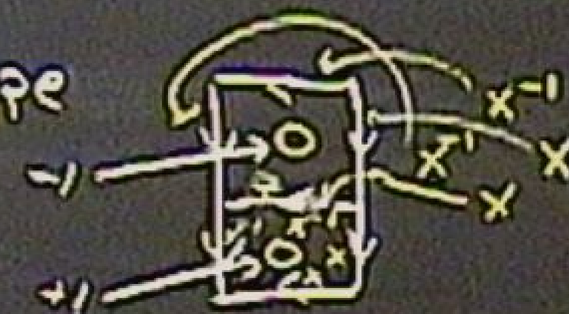
Move together particles–



2 types of charges: X-type & Z-type

Can have value $q = 0, \dots, D-1$

(correspond to stab. generator eigenvalue ω^q)

$\omega = e^{2\pi i/D}$

Charge conserved under movement

Move together particles – charges add (fusion)

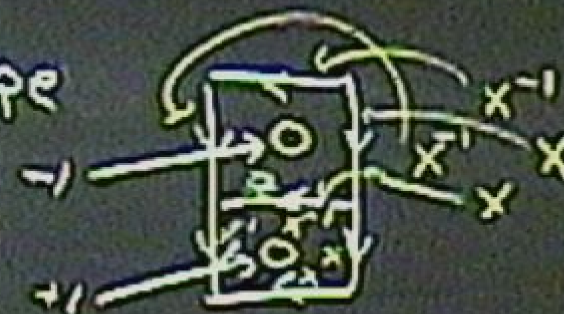
2 types of charges: X-type & Z-type

Can have value $t = 0, \dots, D-1$

(correspond to stab. generator eigenvalue ω^t

$\omega = e^{2\pi i/D}$

Charge conserved under movement



Move together particles – charges add (fusion)

Movement does not depend on distortions of path

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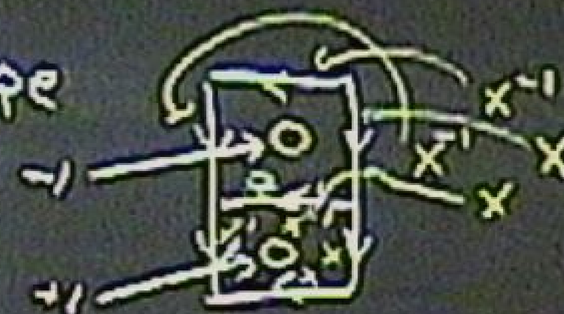

Charge conserved under movement

Move together particles – charges add (fusion)

Movement does not depend on distortions of path except when we loop around another particle.

2 types of charges: X-type & Z-type
Can have value $\tau = 0, \ldots, D-1$
(correspond to stab. generator eigenvalue ω^{τ}
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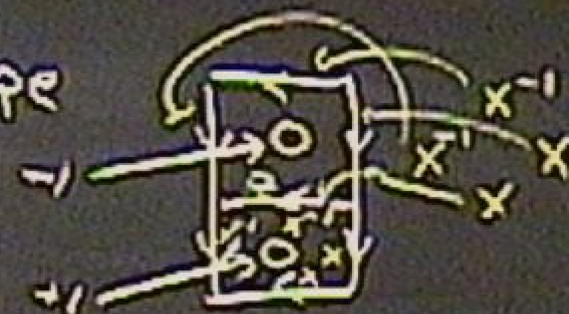

Charge conserved under movement

Move together particles – charges add (fusion)

Movement does not depend on distortions of path except when we loop around another particle. Move a charge q_1 of one type & move it in a loop containing

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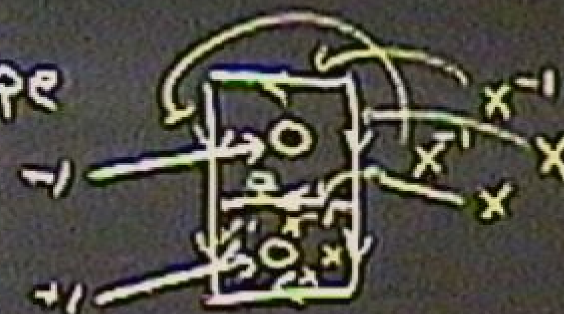
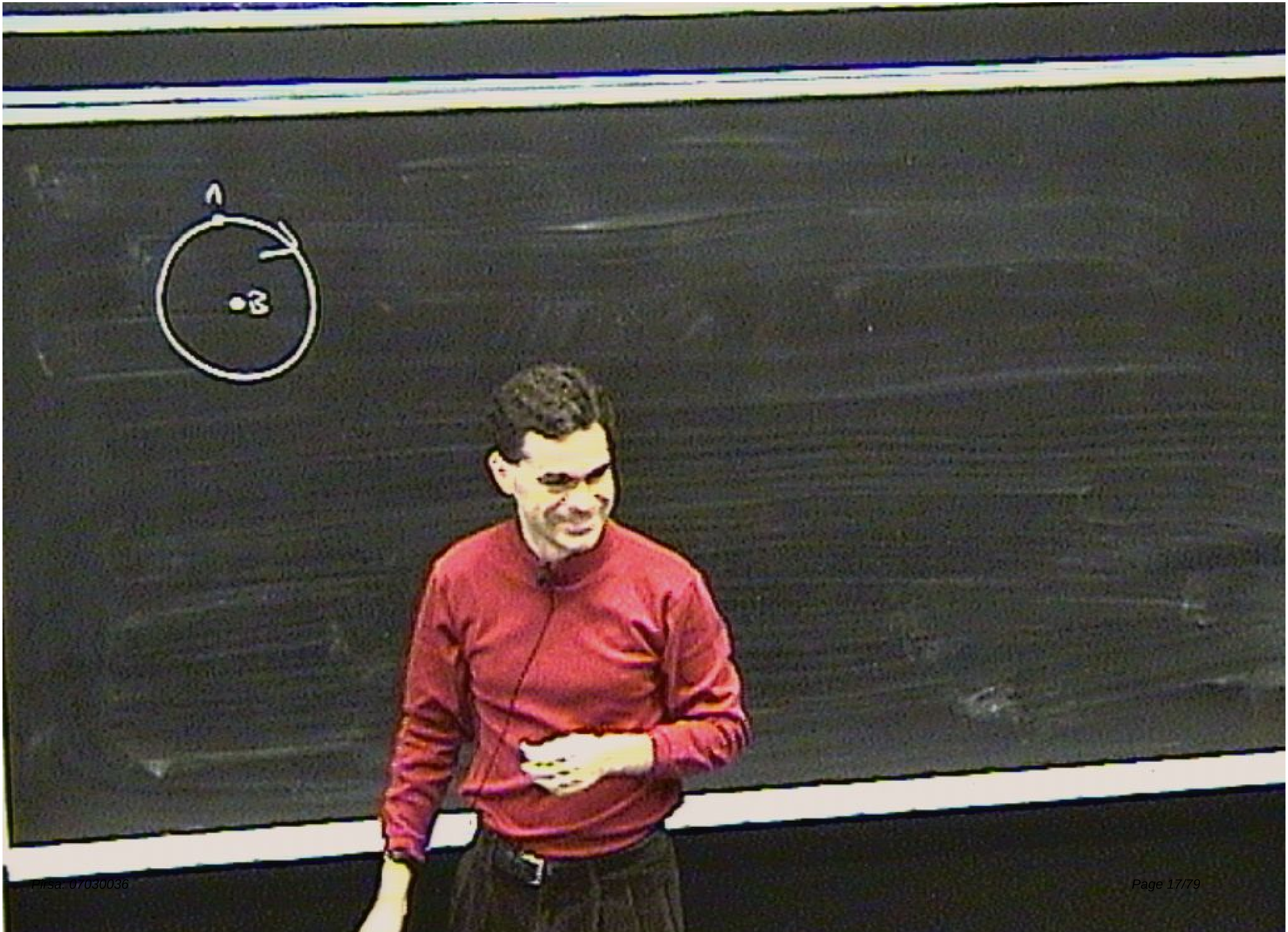
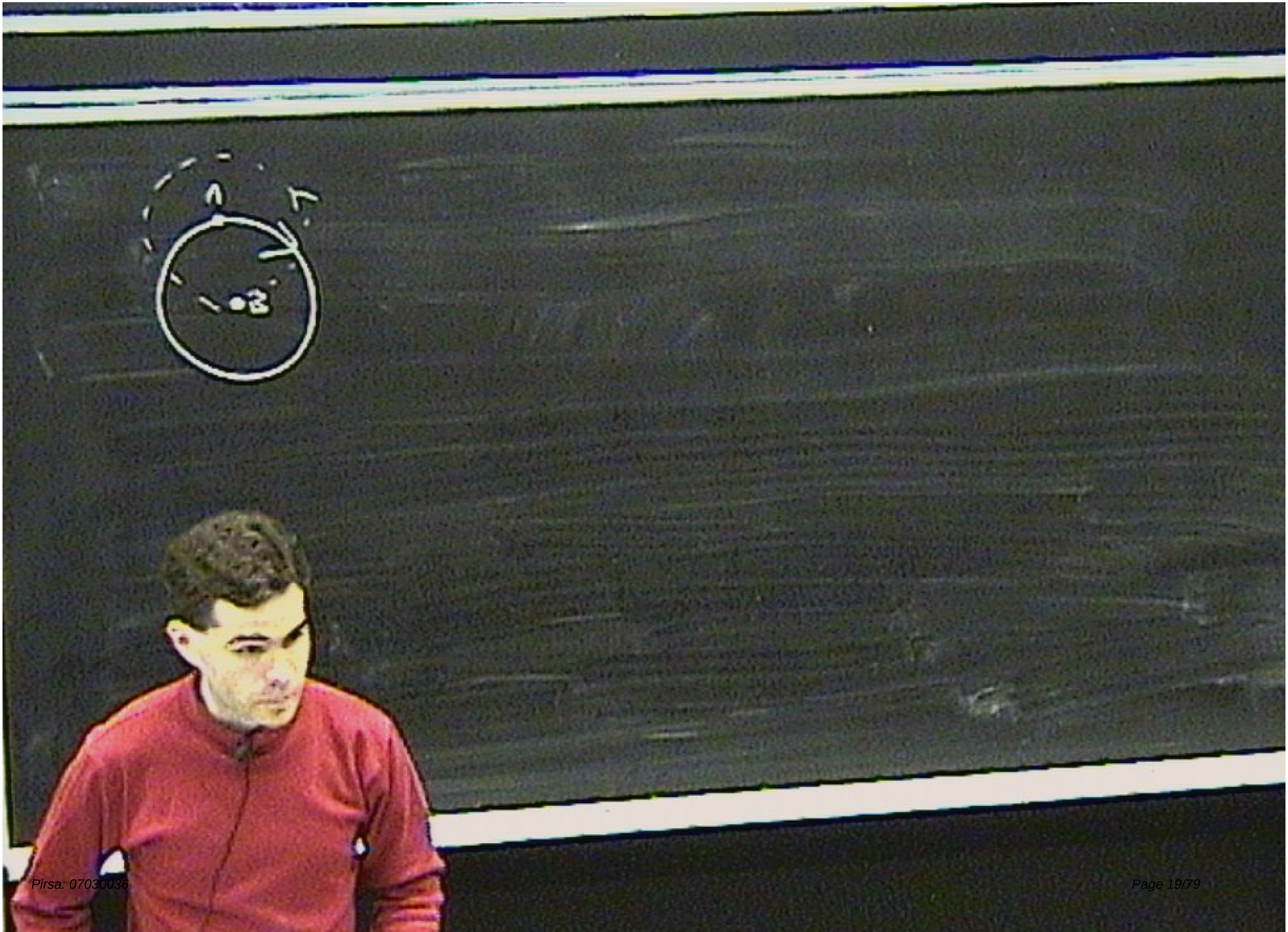
Charge <u>conserved</u> under movement



Move together particles – charges add (<u>fusion</u>)

Movement does not depend on distortions of path <u>except</u> when we loop around another particle. Move a charge q_1 of one type & move it in a loop containing total charge q_2 of other type, we get a phase $\omega^{\pm q_1 q_2}$


B



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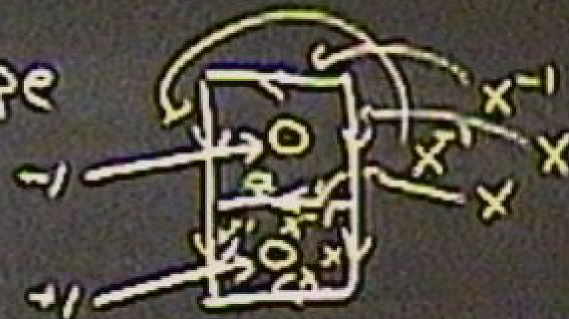

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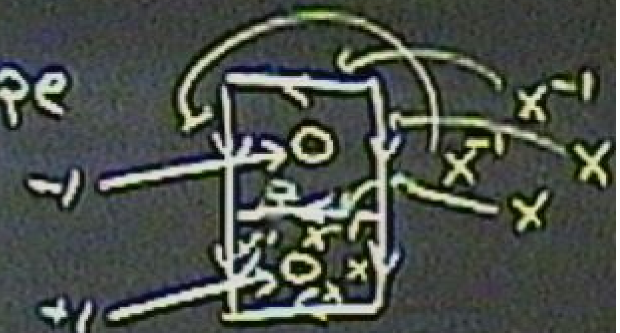

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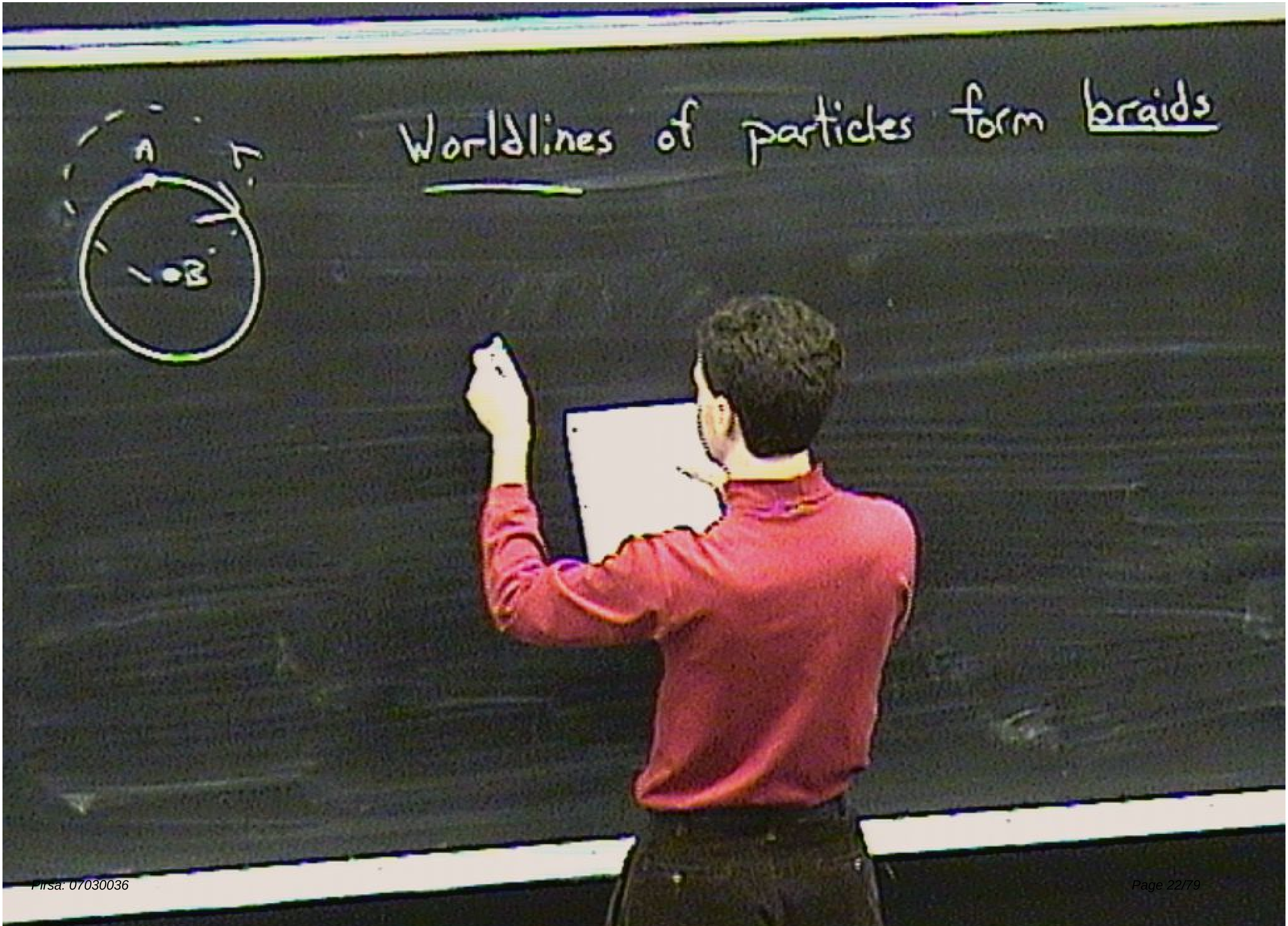
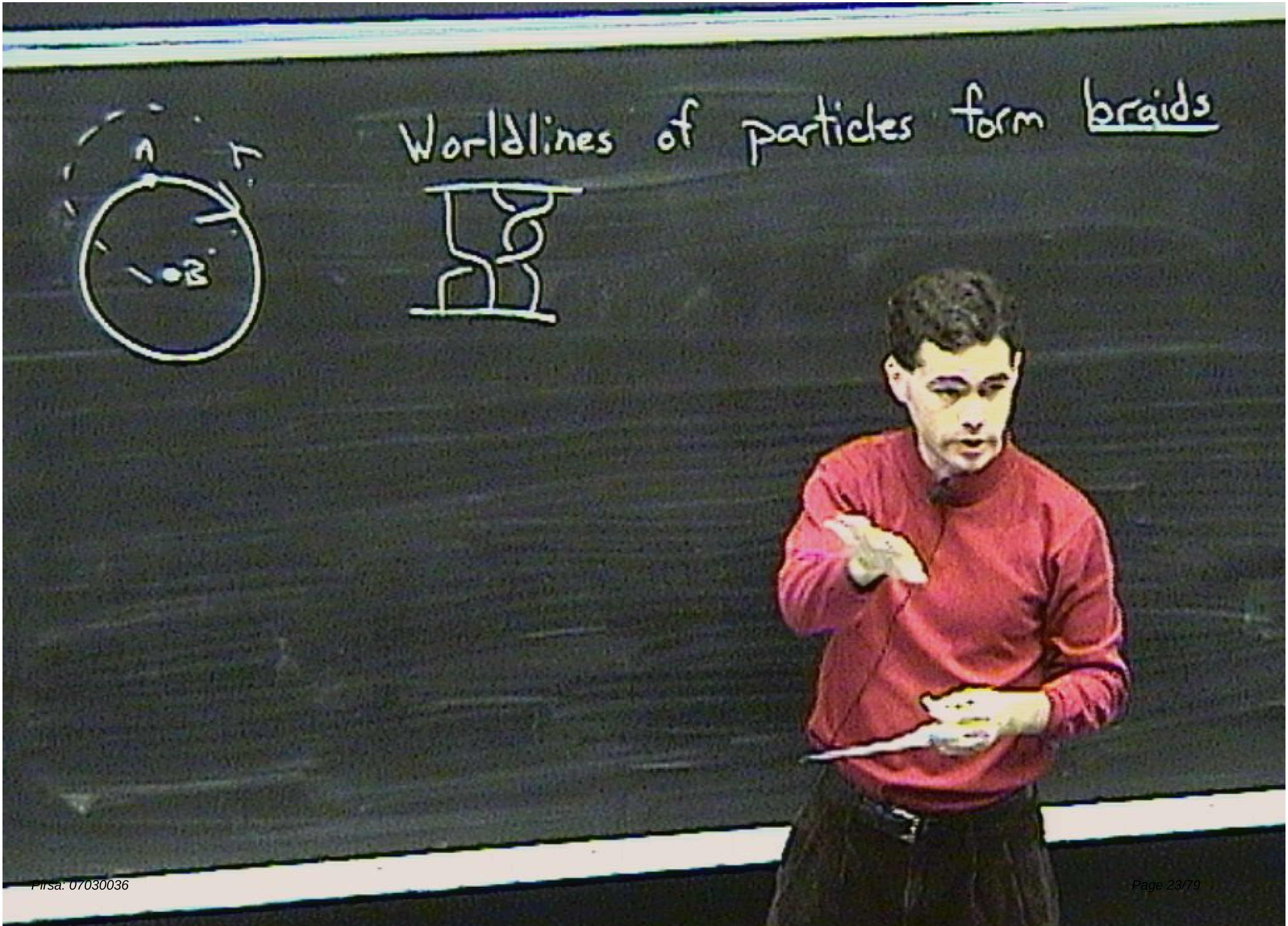
$\boxed{\text{Abelian anyons}}$


Worldlines of particles form braids
A
B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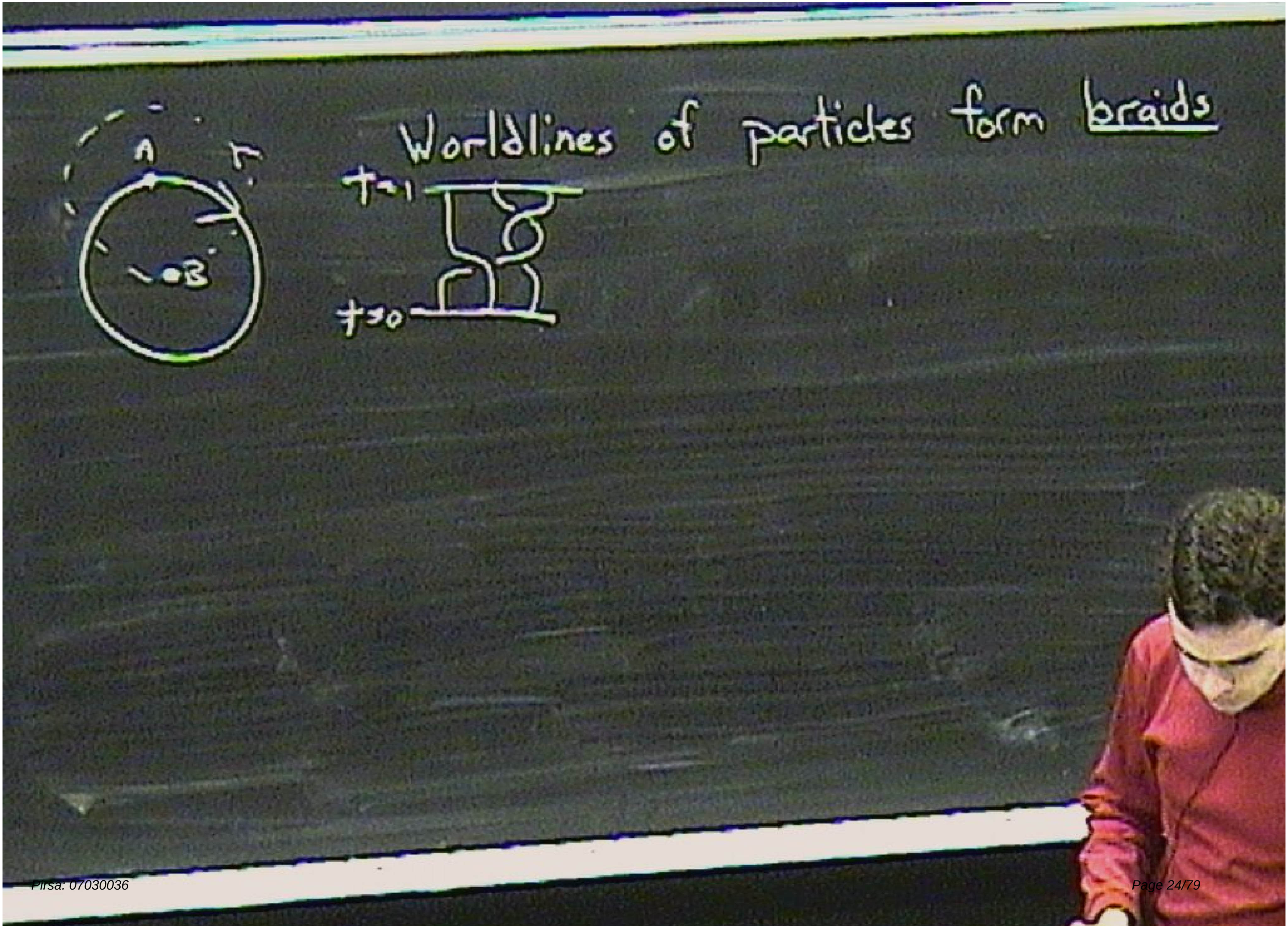
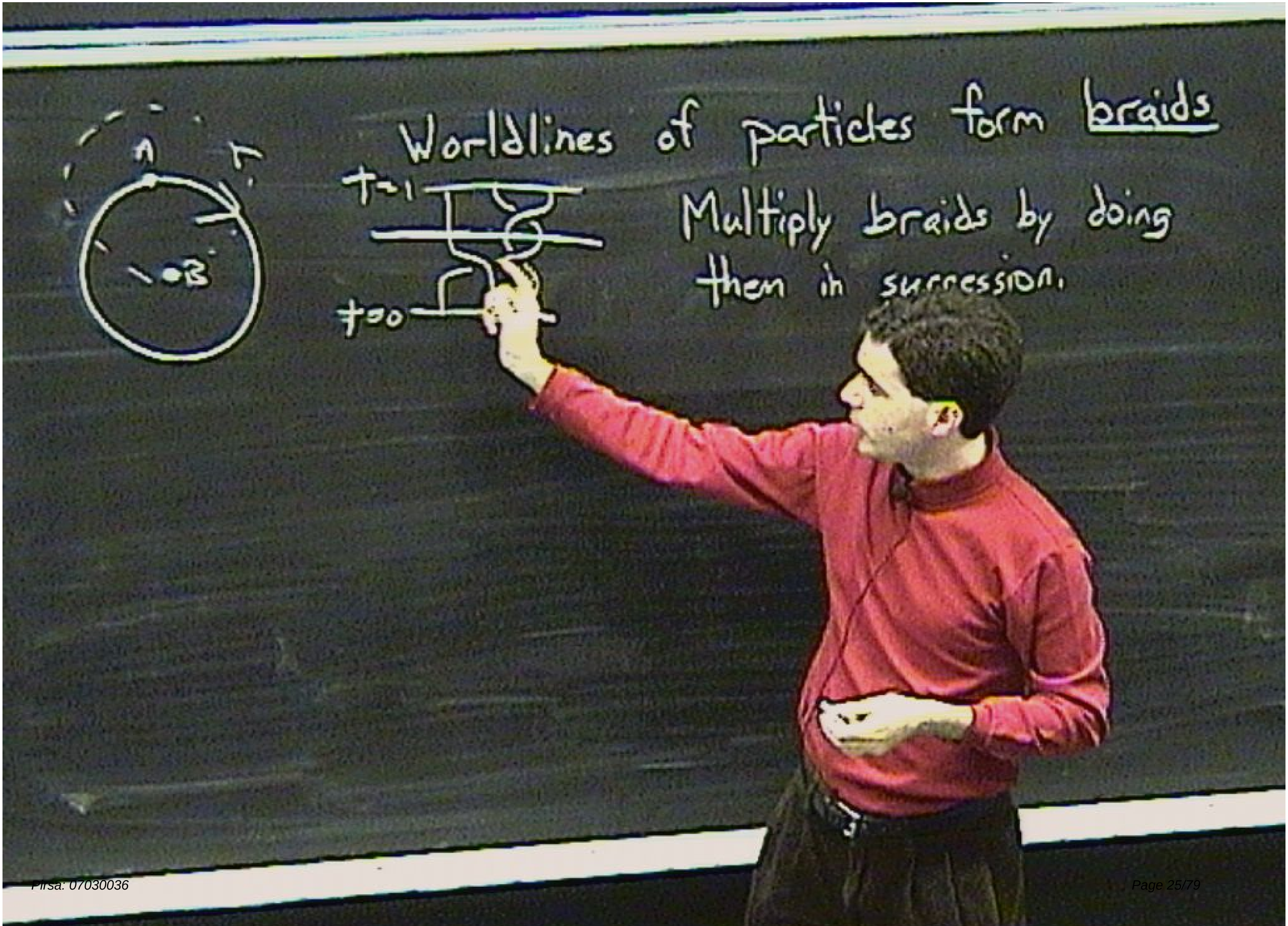
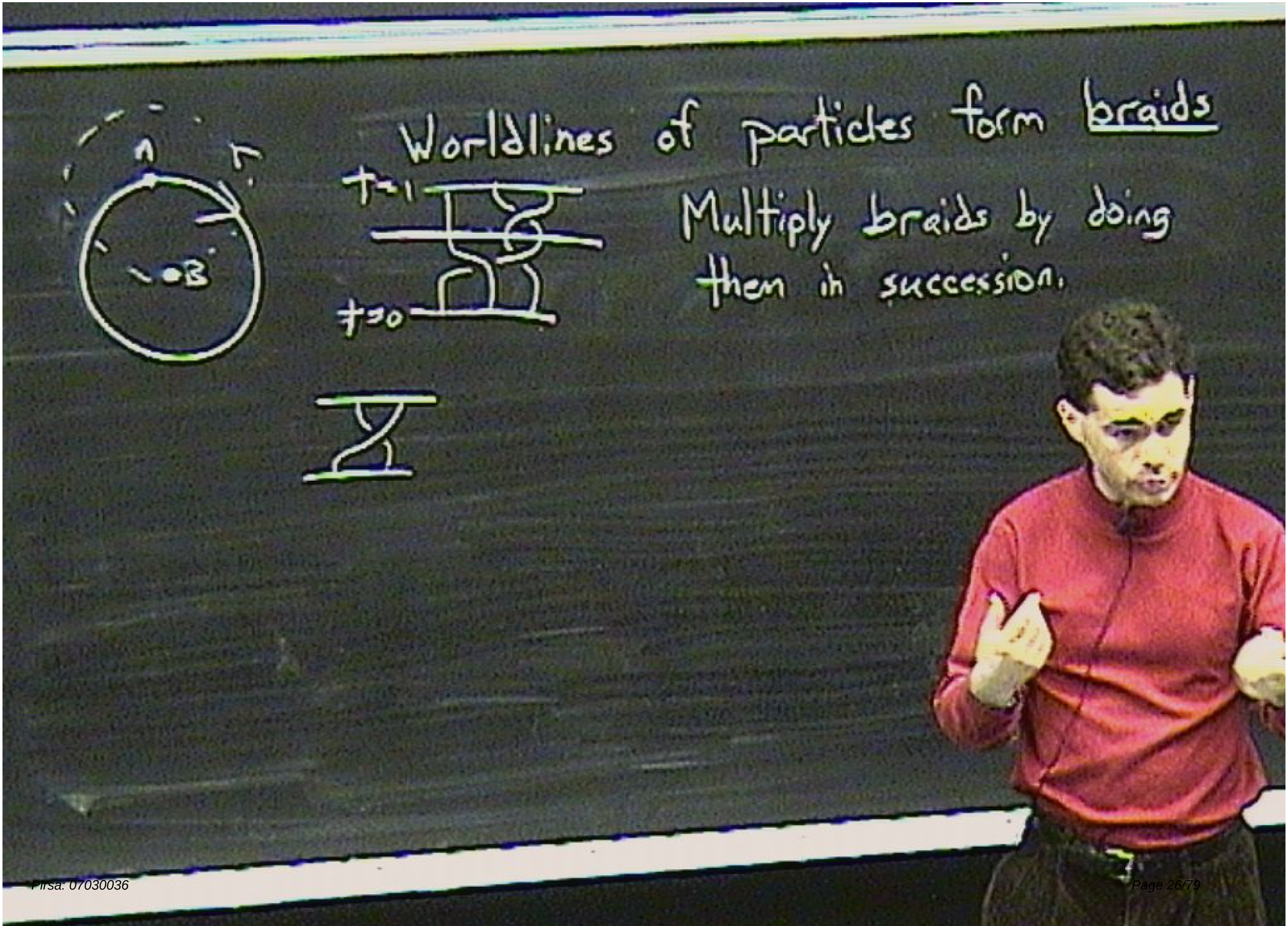
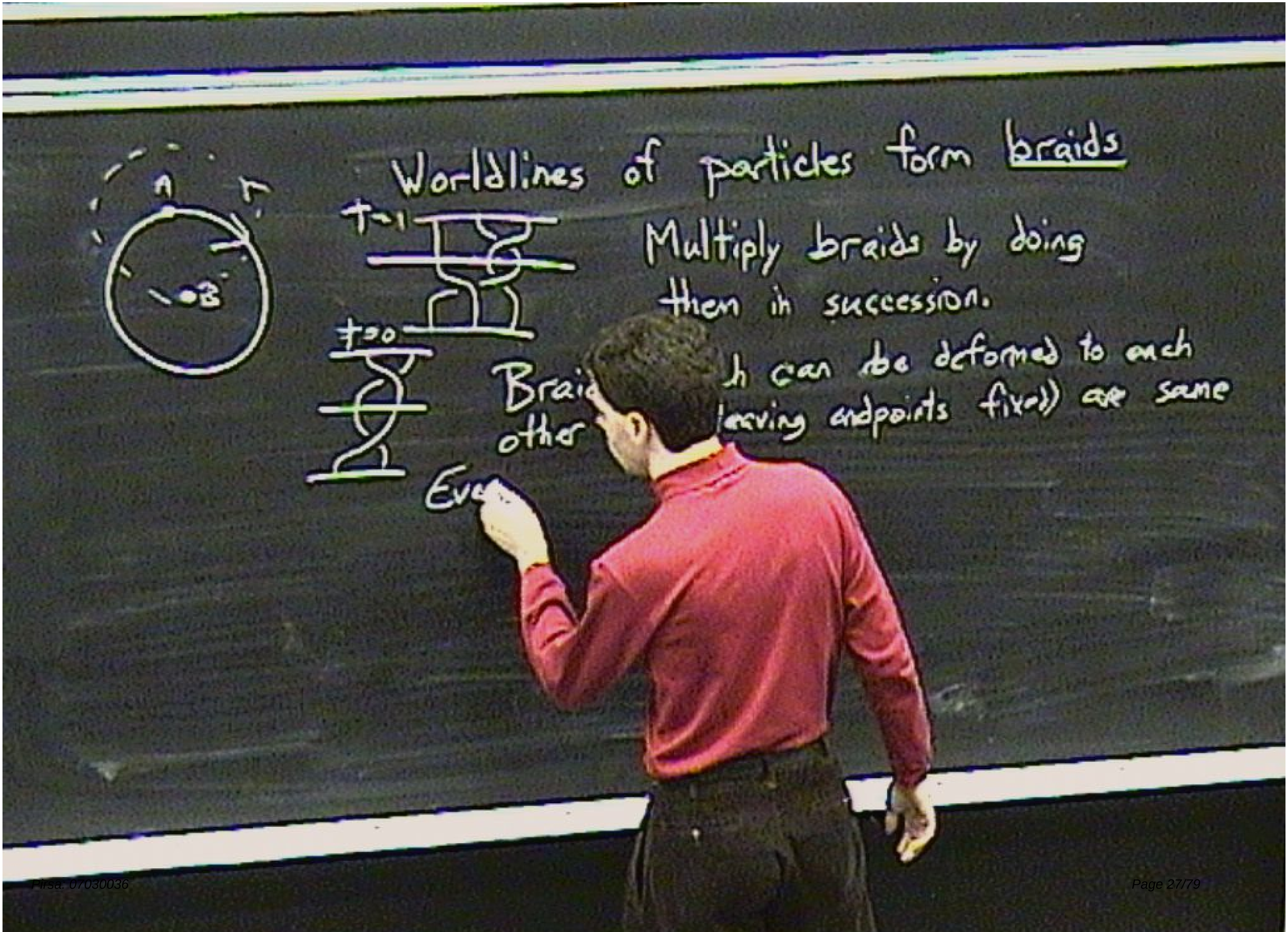
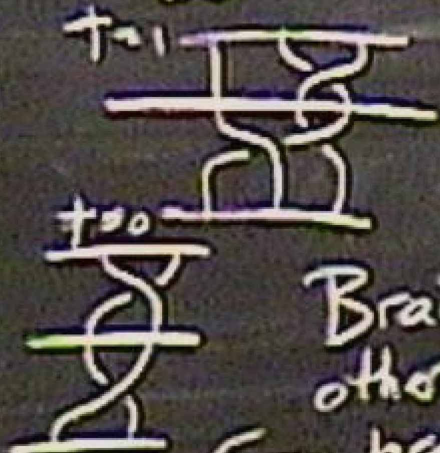

Worldlines of particles form braids
t=1
t=0
A
B


Worldlines of particles form braids
Multiply braids by doing them in succession.
t=1
t=0
A
B


Worldlines of particles form braids
Multiply braids by doing
then in succession.
t=1
t=0
B


Worldlines of particles form braids
t=1
Multiply braids by doing
then in succession.
t=0
can be deformed to each
other
leaving endpoints fixed) are same

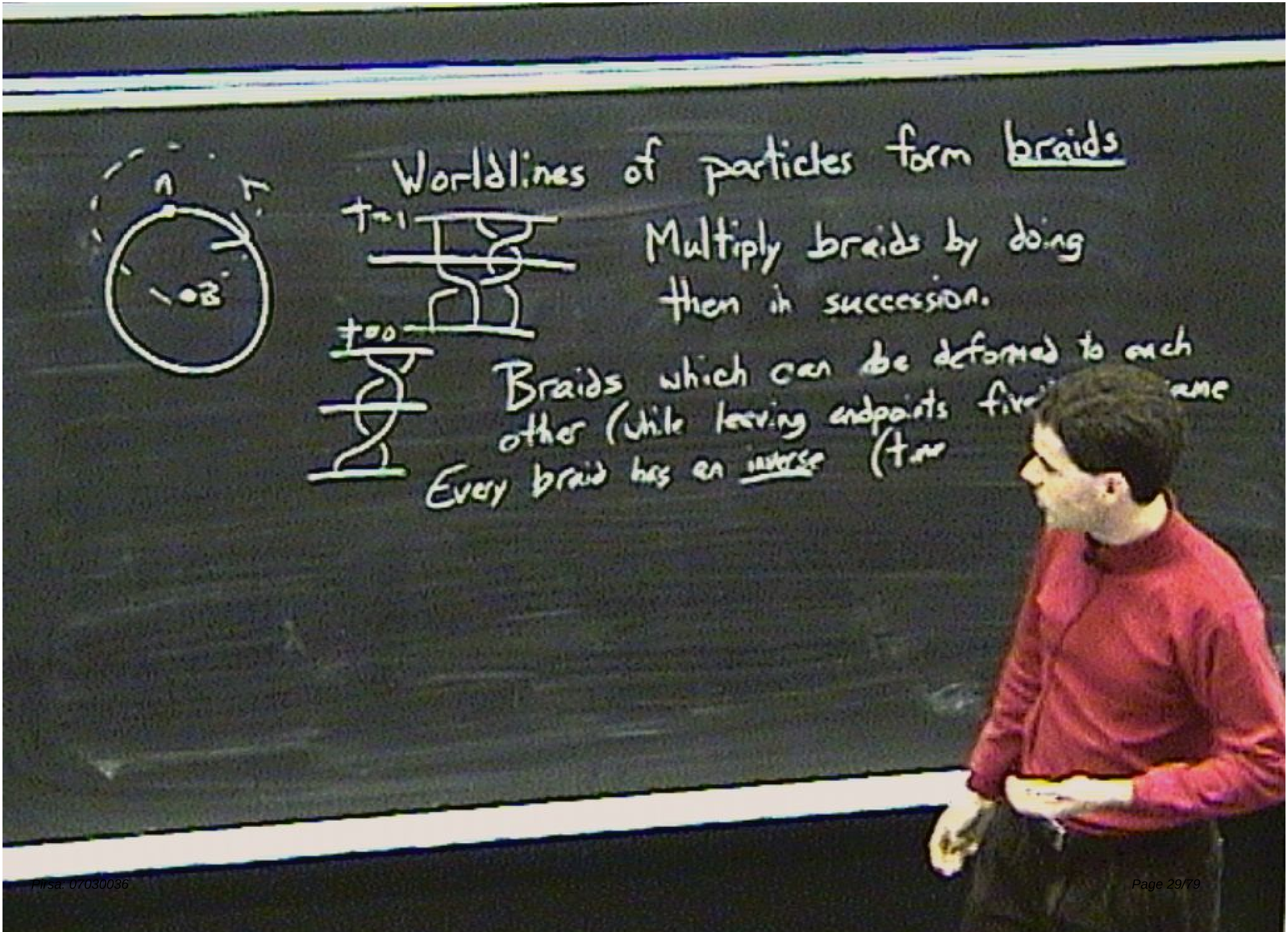
Worldlines of particles form __braids__



Multiply braids by doing them in succession.

Braids which can be deformed to each other (while leaving endpoints fixed) are same

Every braid has an __inverse__



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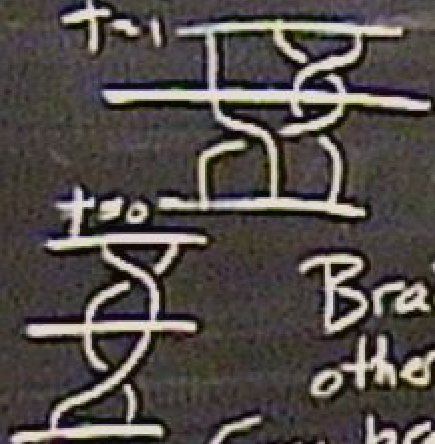
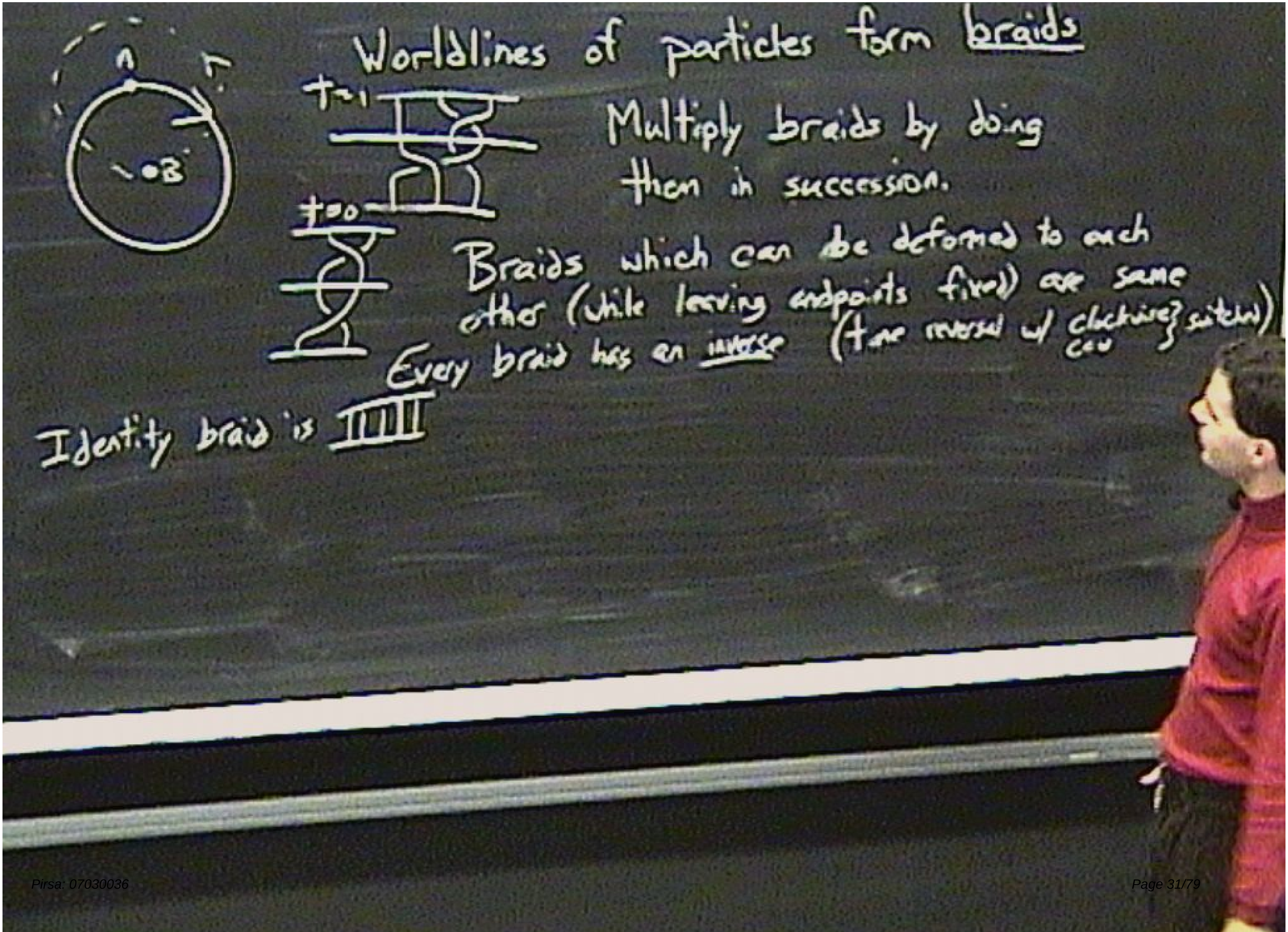
Worldlines of particles form **braids**



Multiply braids by doing them in succession.

Braids which can be deformed to each other (while leaving endpoints fixed) are same

Every braid has an inverse (time reversal w/ clockwise / ccw switched)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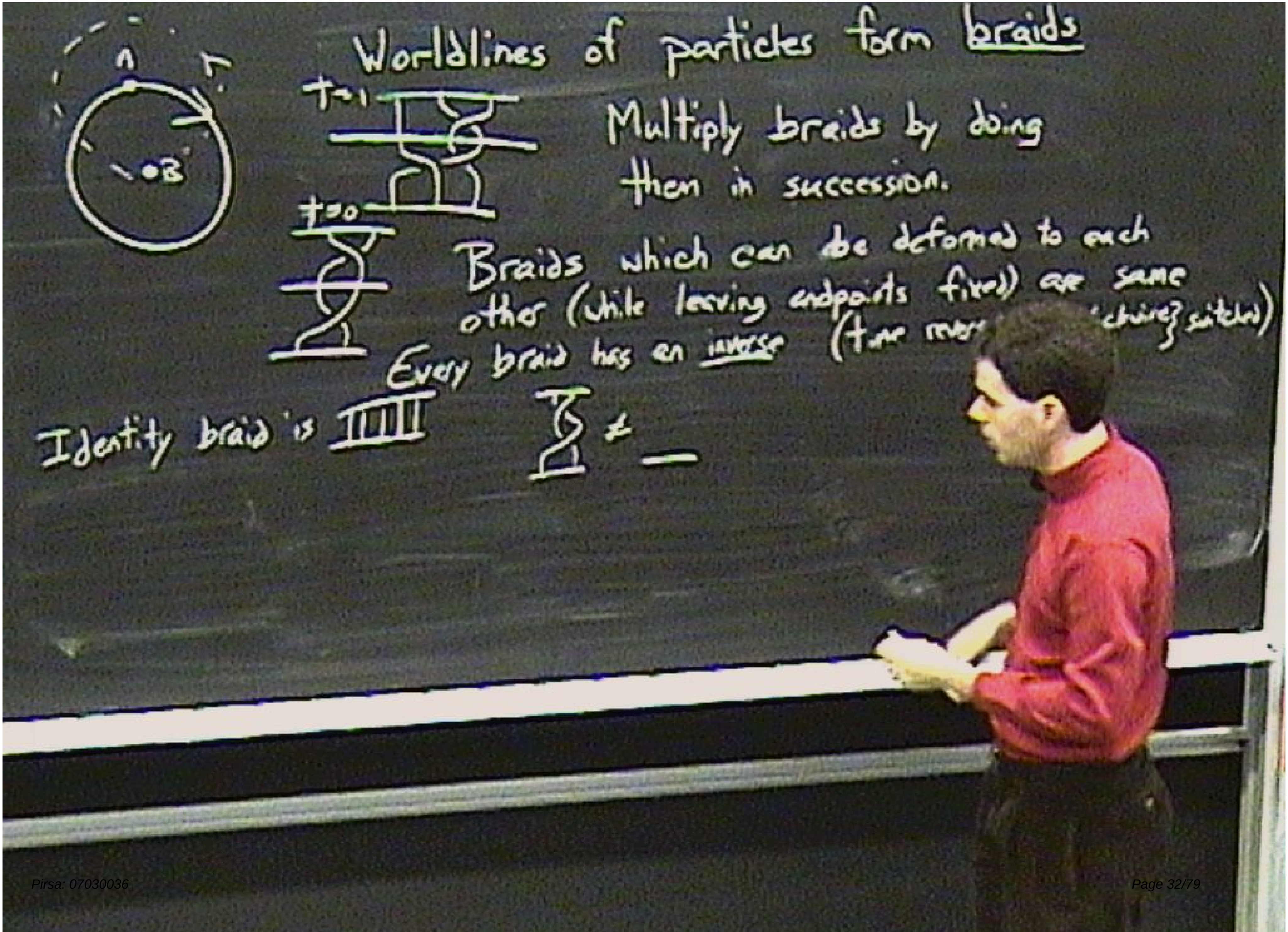
Identity braid is



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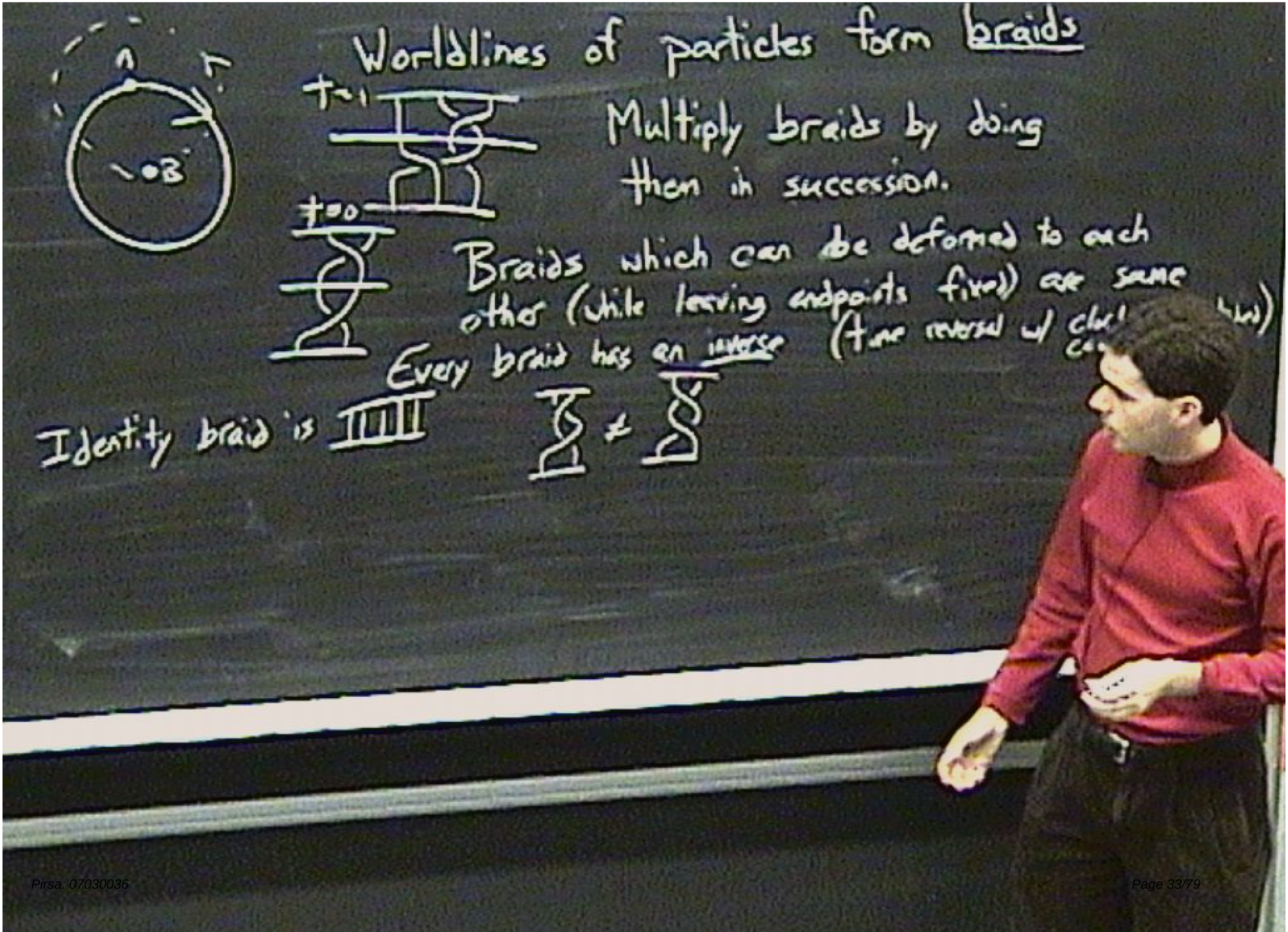
$\neq$



Worldlines of particles form braids
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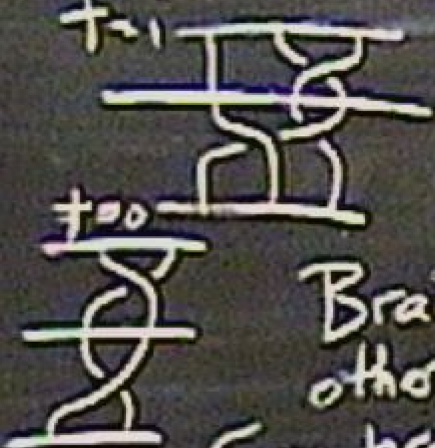

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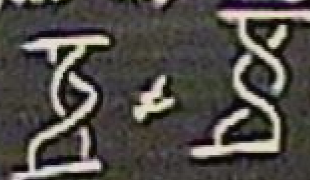

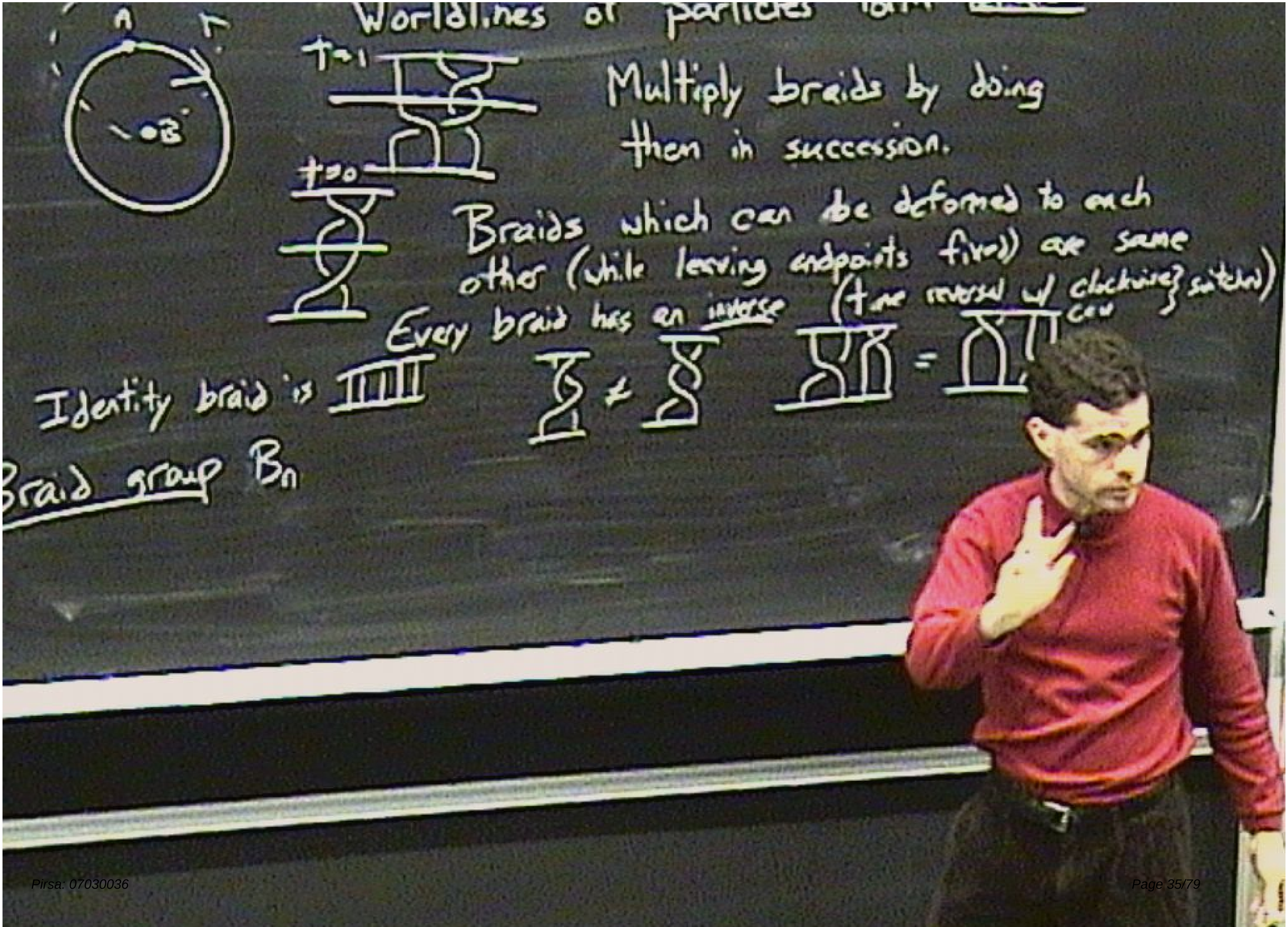
A
B
Worldlines of particles
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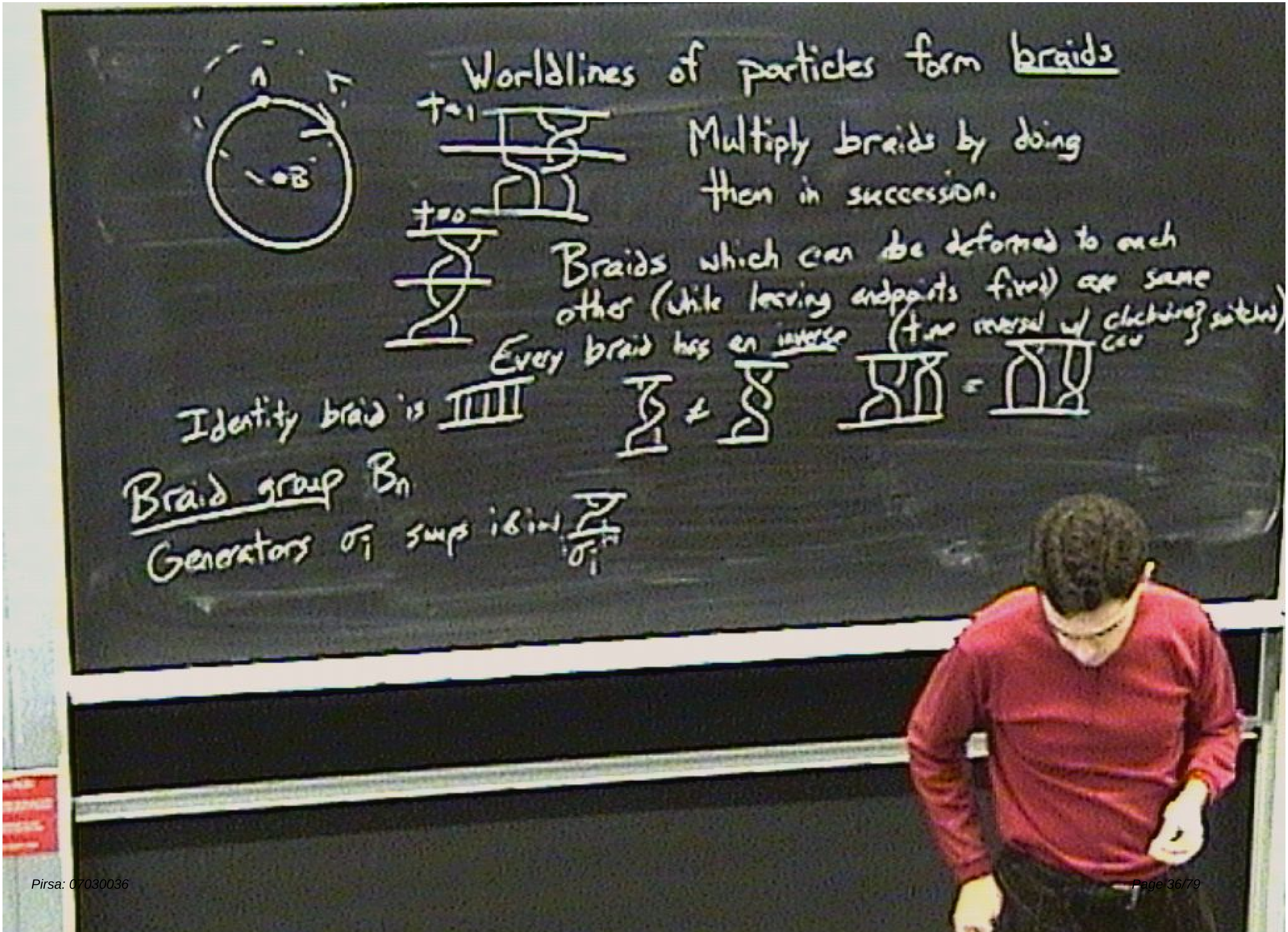
Identity braid is
Braid group B_n



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≠
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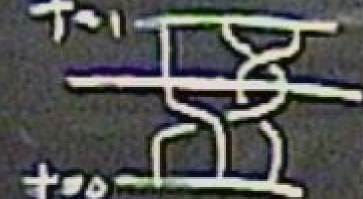
Generators σ_i swaps i & $i+1$
σ_i
σ_i^{-1}

Worldlines of particles form braids



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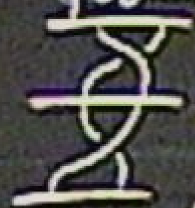

Braids which can be deformed to each other (while leaving endpoints fixed) are same

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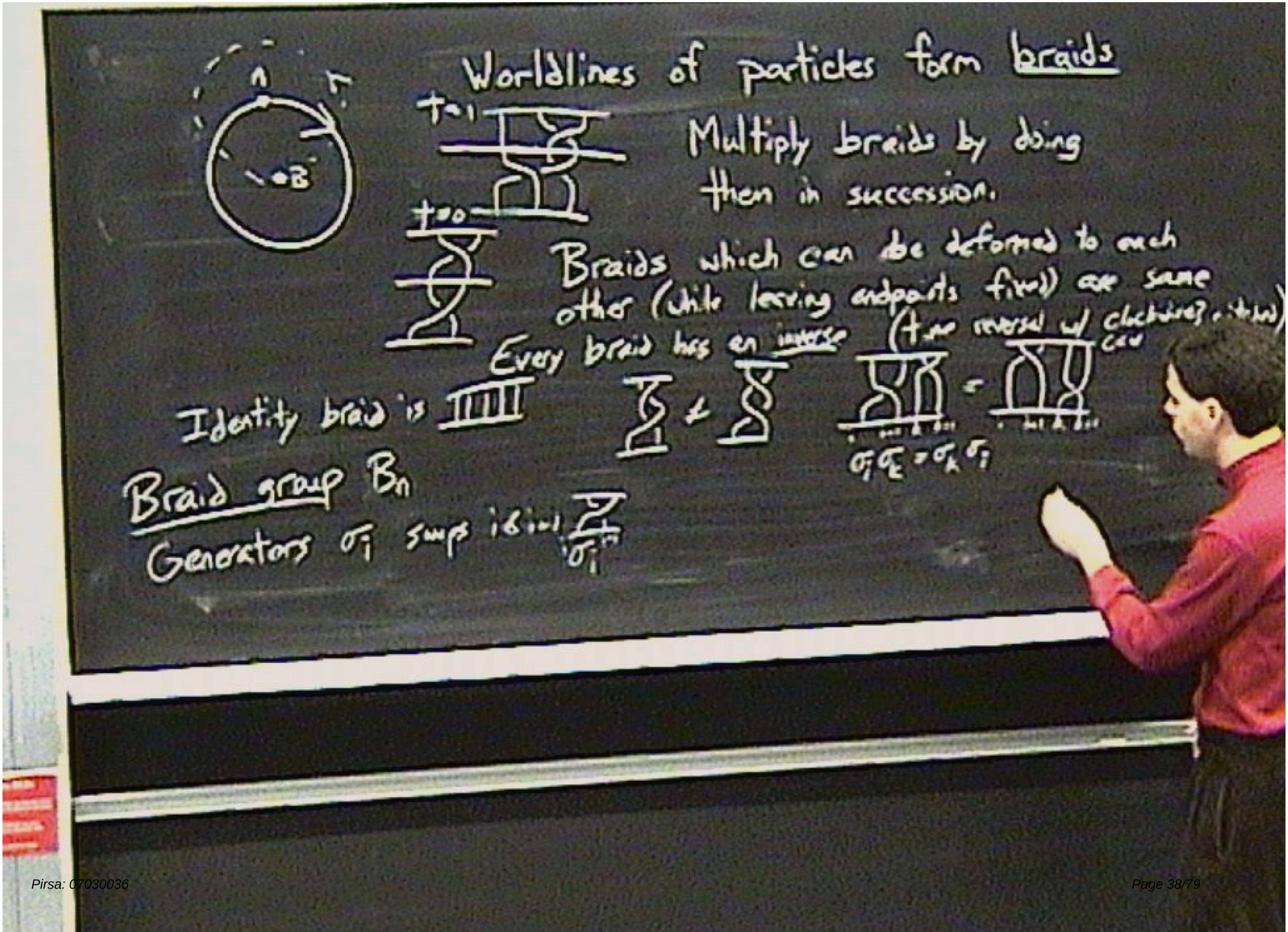
Identity braid is 

Braid group B_n

Generators σ_i swaps i & $i+1$ 



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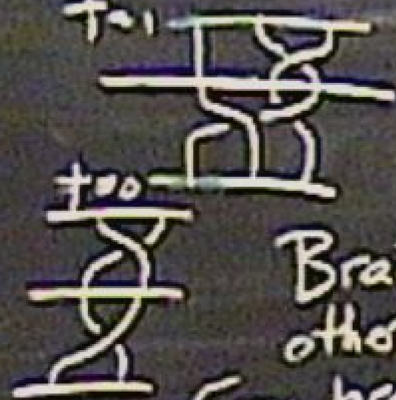
σ_i
$\sigma_i \sigma_k = \sigma_k \sigma_i$

Worldlines of particles form braids



Multiply braids by doing them in succession.

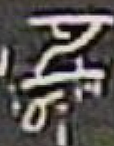
Braids which can be deformed to each other (while leaving endpoints fixed) are same

Every braid has an inverse (time reversal w/ clockwise ccw switched)

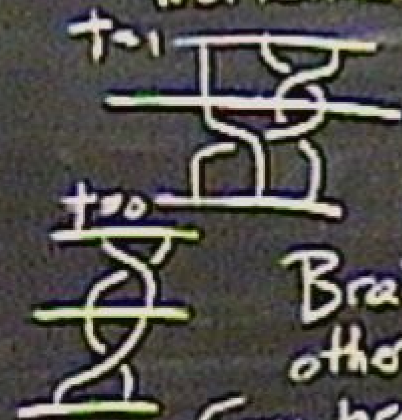
Identity braid is 

$\boxed{\sigma_i \sigma_k = \sigma_k \sigma_i \quad (|i-k| \geq 2)}$

Braid group B_n

Generators σ_i swaps i & i+1 

Worldlines of particles form braids



Multiply braids by doing them in succession.

Braids which can be deformed to each other (while leaving endpoints fixed) are same

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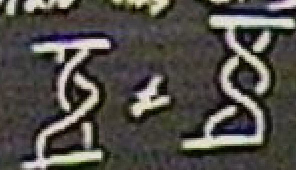


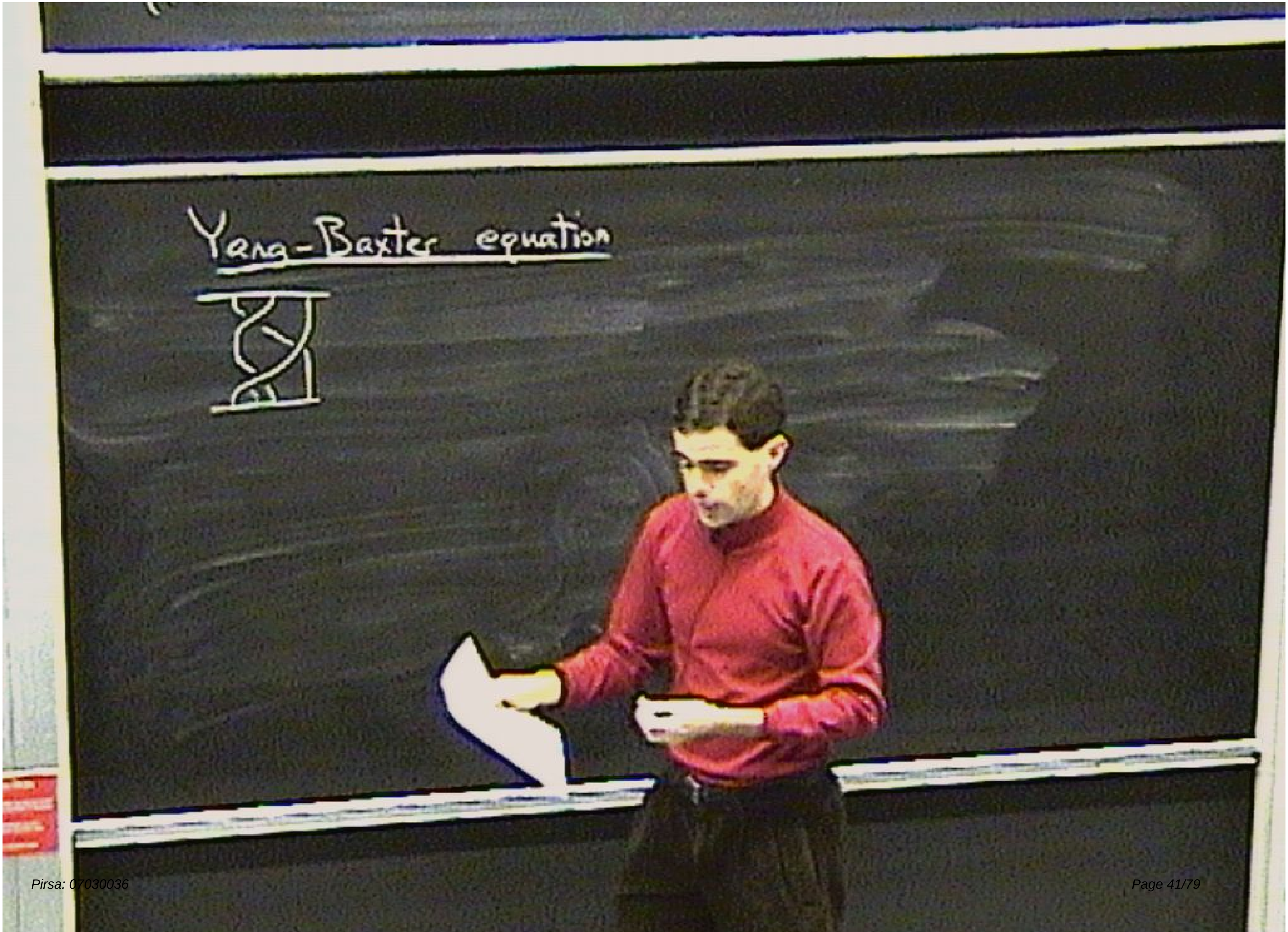
$\boxed{\sigma_i \sigma_k = \sigma_k \sigma_i \quad (|i-k| \ldots}$

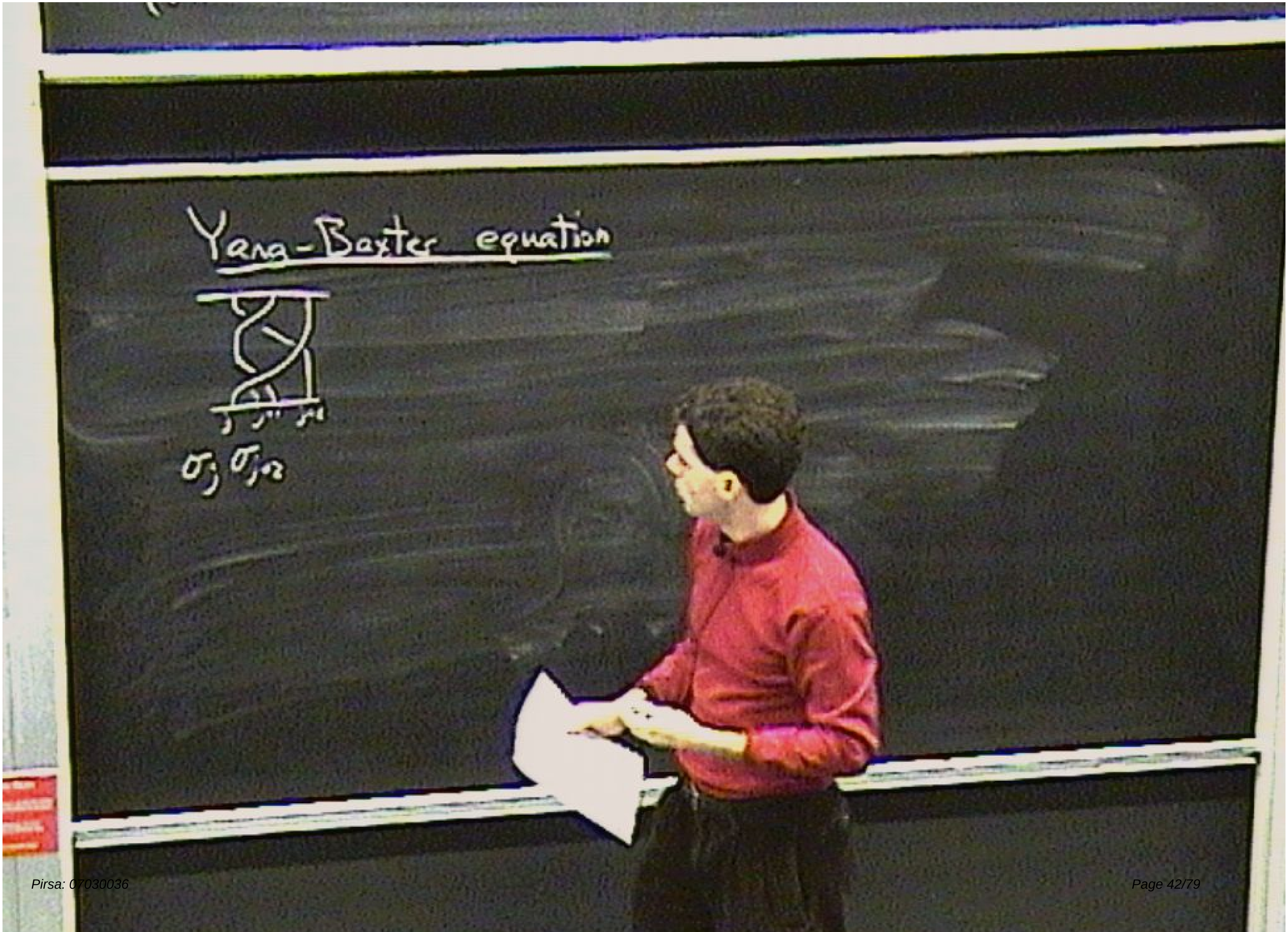
Identity braid is

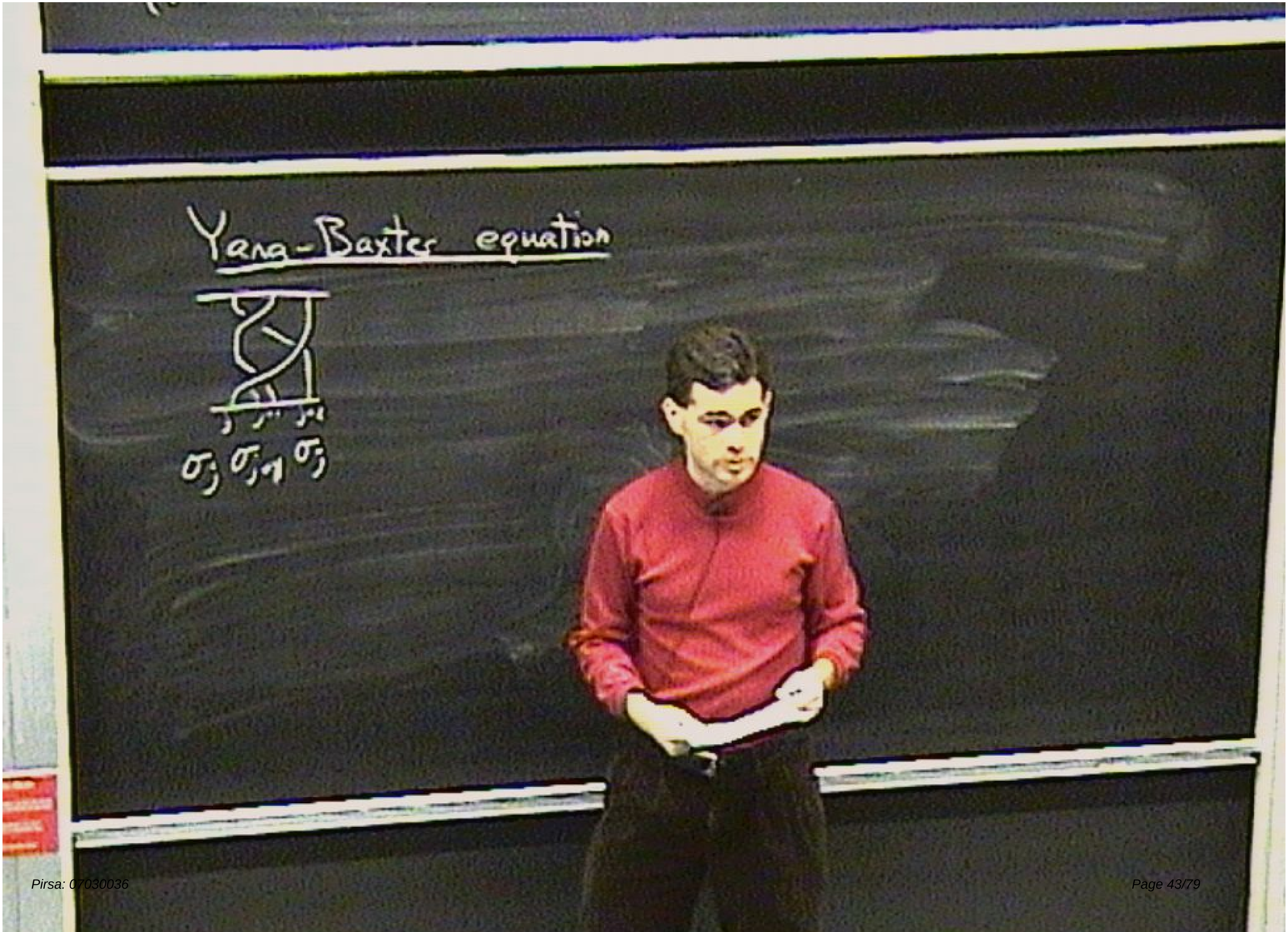
Braid group B_n

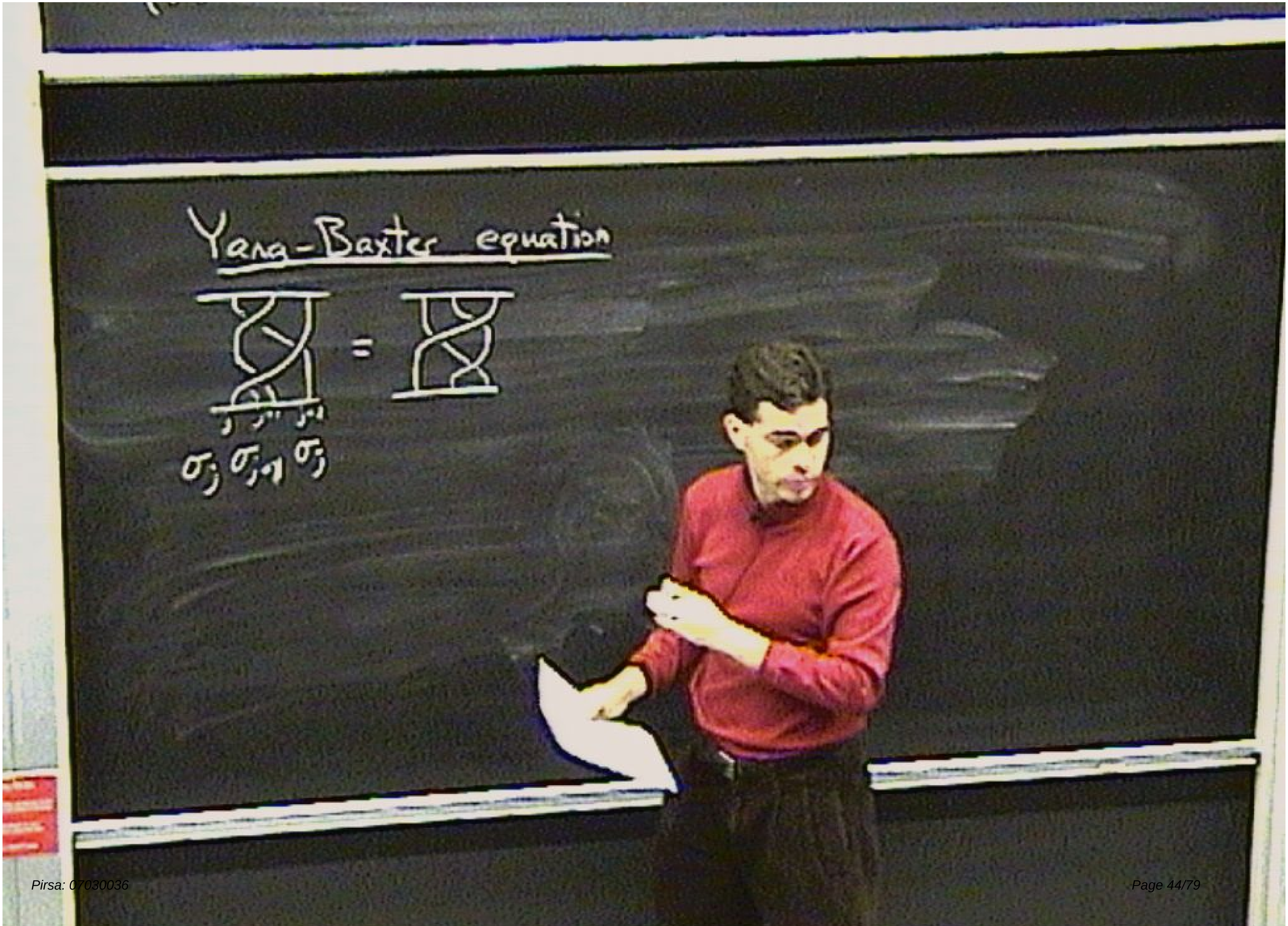
Generators σ_i swaps i & $i+1$ σ_i

(Also σ_i^{-1})


Yang-Baxter equation


Yang-Baxter equation


Yang-Baxter equation
$\sigma_j \sigma_{j+1} \sigma_j$


Yang-Baxter equation
=
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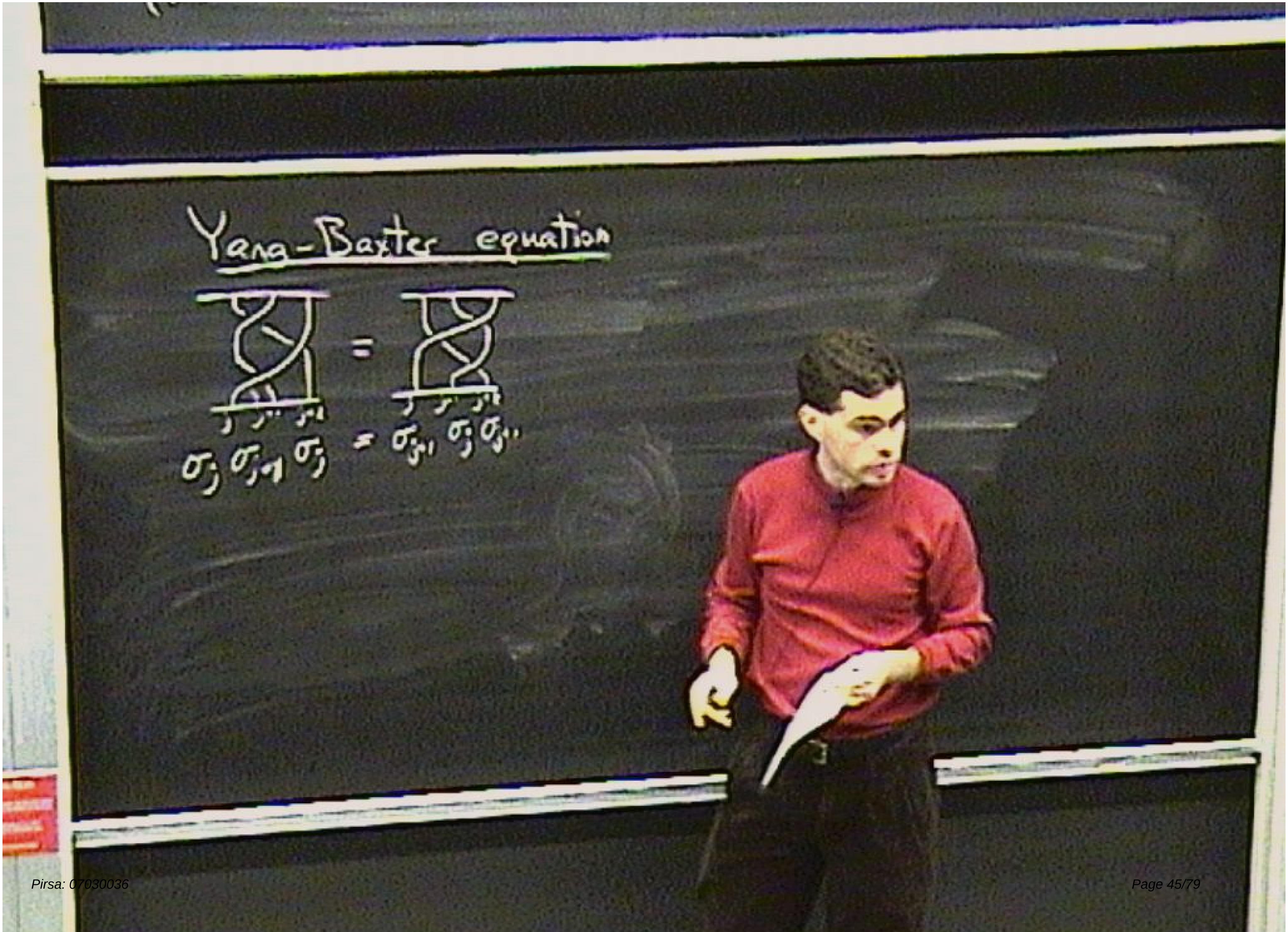
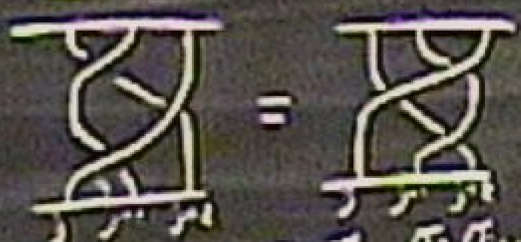
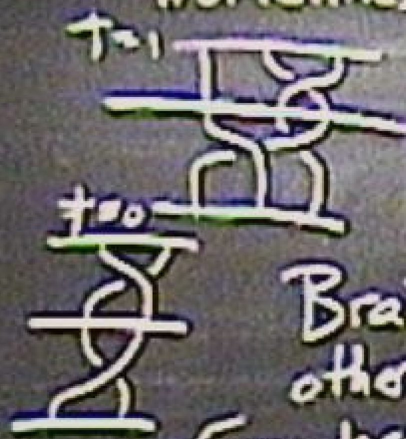

Yang-Baxter equation
$\sigma_j \sigma_{j+1} \sigma_j = \sigma_{j+1} \sigma_j \sigma_{j+1}$

Yang-Baxter equation



$$\sigma_j \sigma_{j+1} \sigma_j = \sigma_{j+1} \sigma_j \sigma_{j+1}$$

Worldlines of particles form braids



Multiply braids by doing then in succession.

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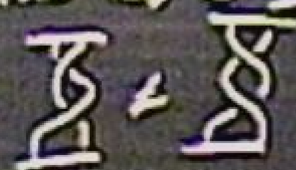


$\boxed{\sigma_i \sigma_k = \sigma_k \sigma_i \quad (|i-k| \geq 2)}$

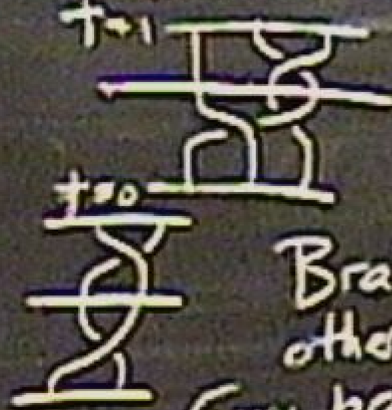
Braid group B_n

Generators σ_i swap i & i+1  "σ_i"

(Also σ_i^{-1})

Yang-Baxter equation

Worldlines of particles form braids



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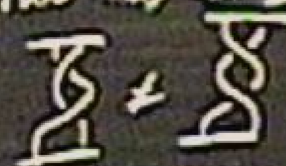


$\boxed{\sigma_i \sigma_k = \sigma_k \sigma_i \quad (|i-k| \geq 2)}$

Braid group B_n

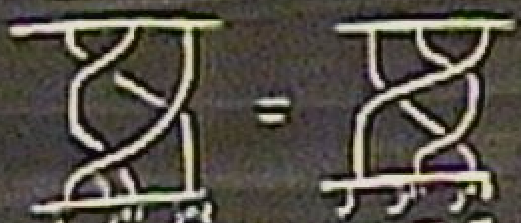
Generators σ_i swaps i & $i+1$ 

(Also σ_i^{-1})

Generators of ... σ_i

(Also σ_i^{-1})

Yang-Baxter equation



$$\sigma_j \sigma_{j+1} \sigma_j = \sigma_{j+1} \sigma_j \sigma_{j+1}$$

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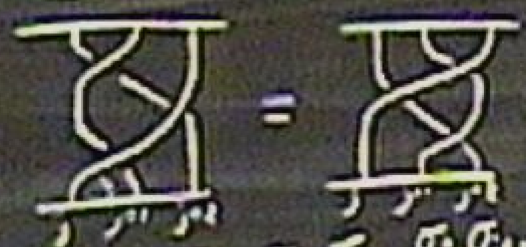

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Non-Abelian anyons will be in non-Abelian representation of the braid group

$\sigma_j \longmapsto U_j$ unitaries on qubits.

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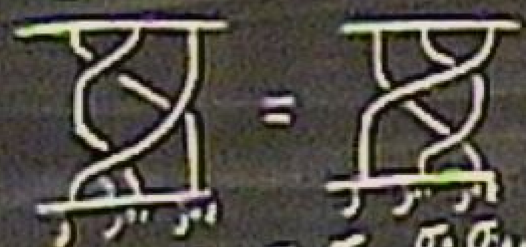

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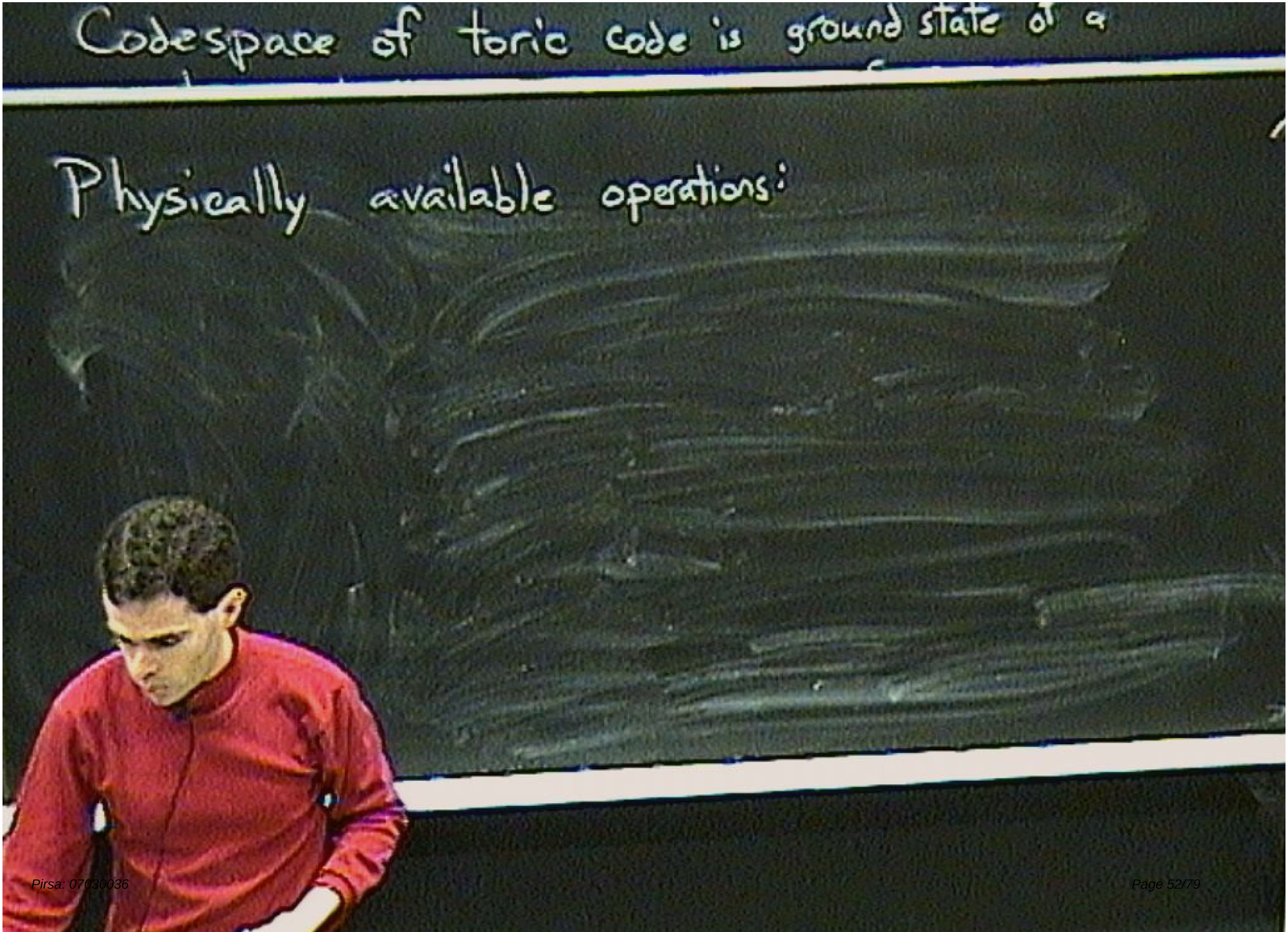
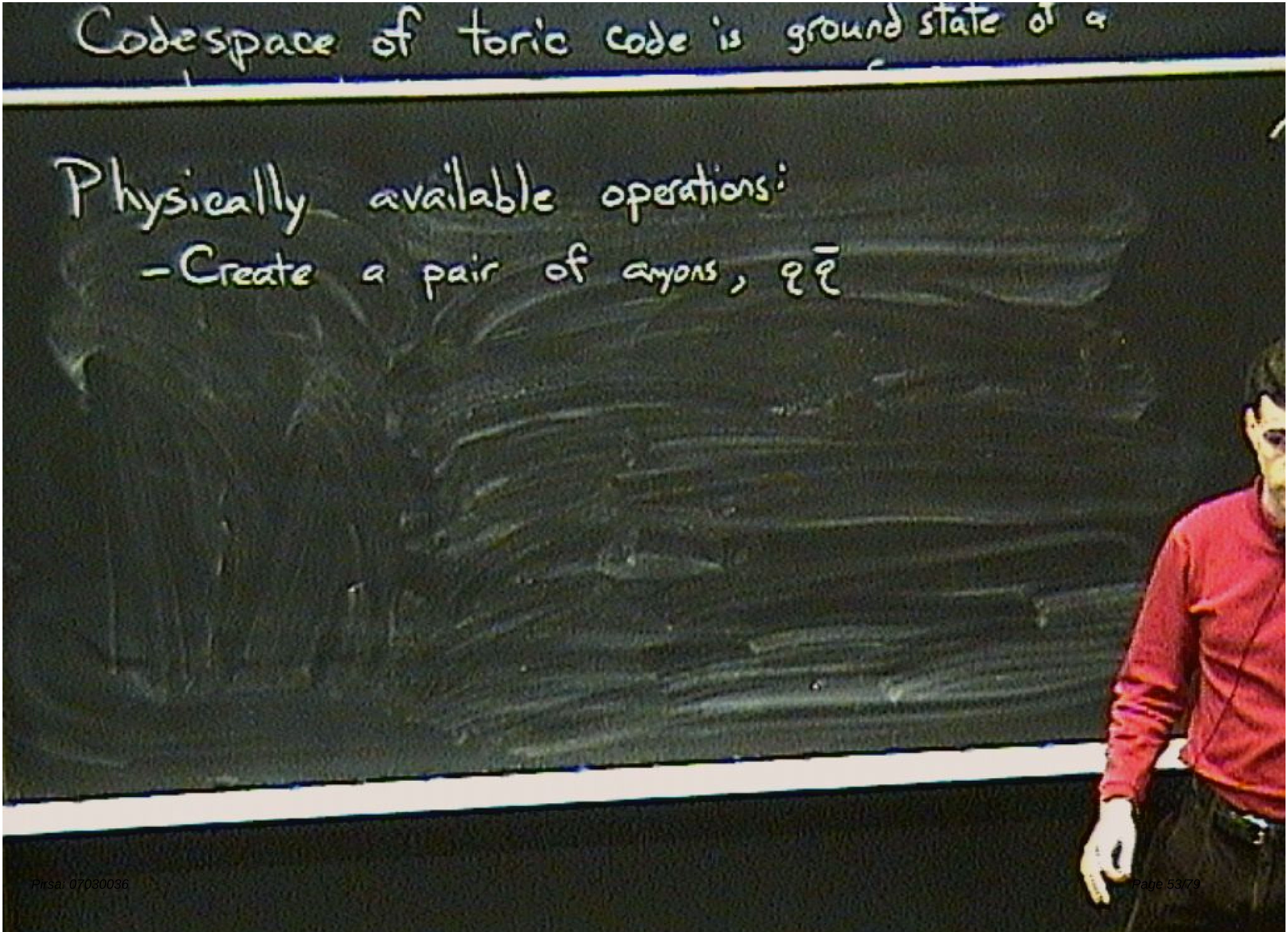

Codespace of toric code is ground state of a
Physically available operations: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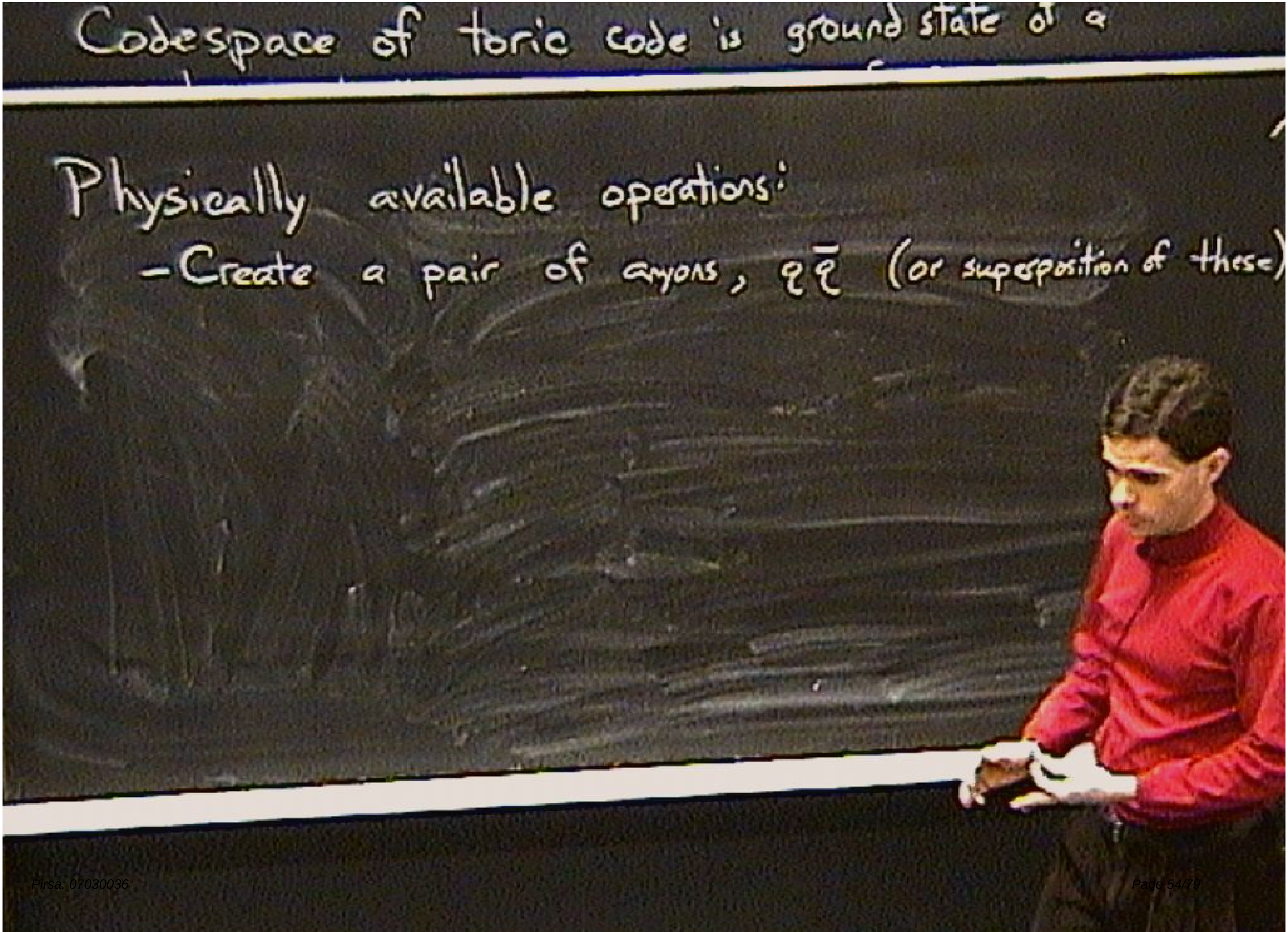
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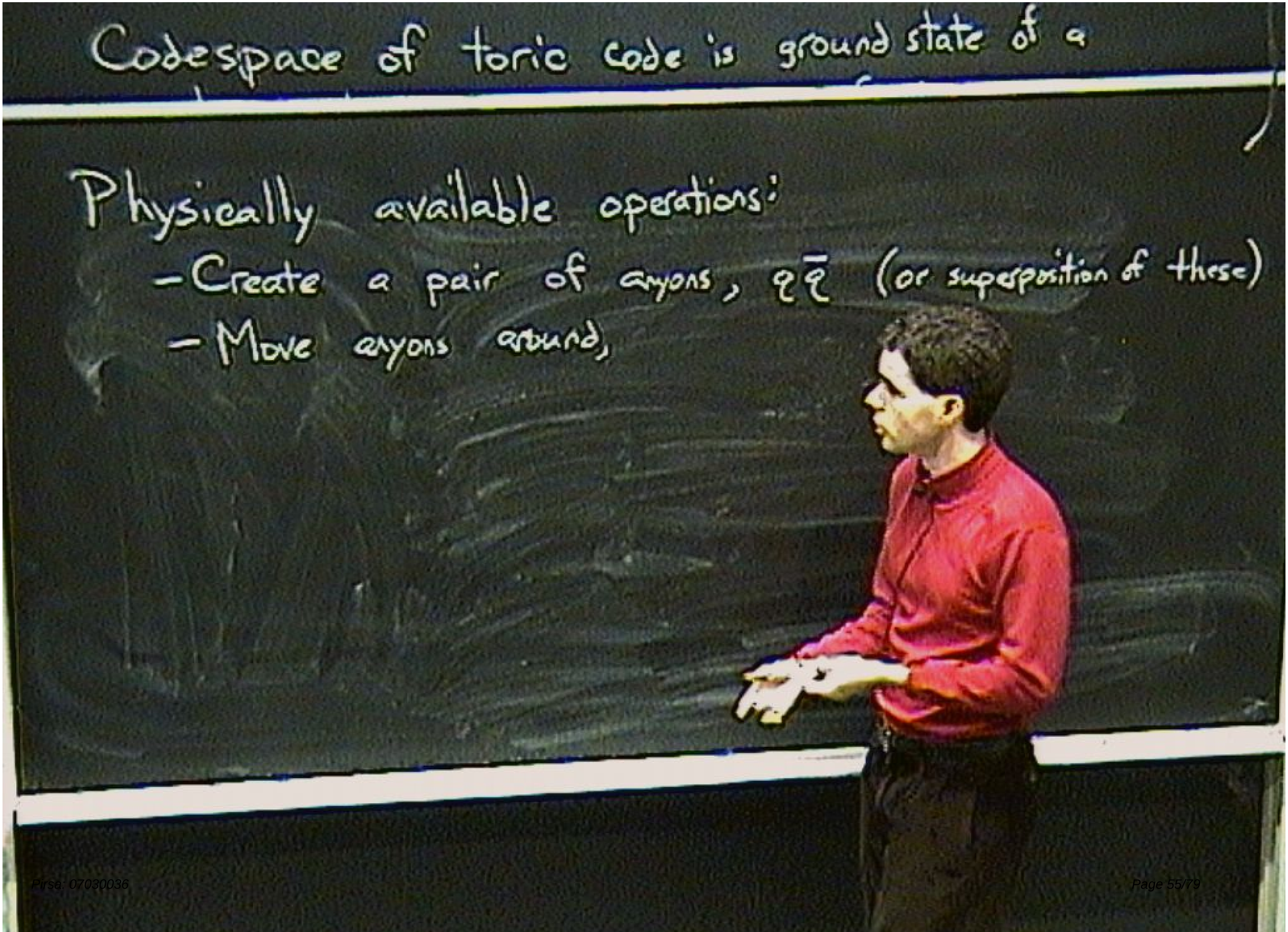
Codespace of toric code is ground state of a
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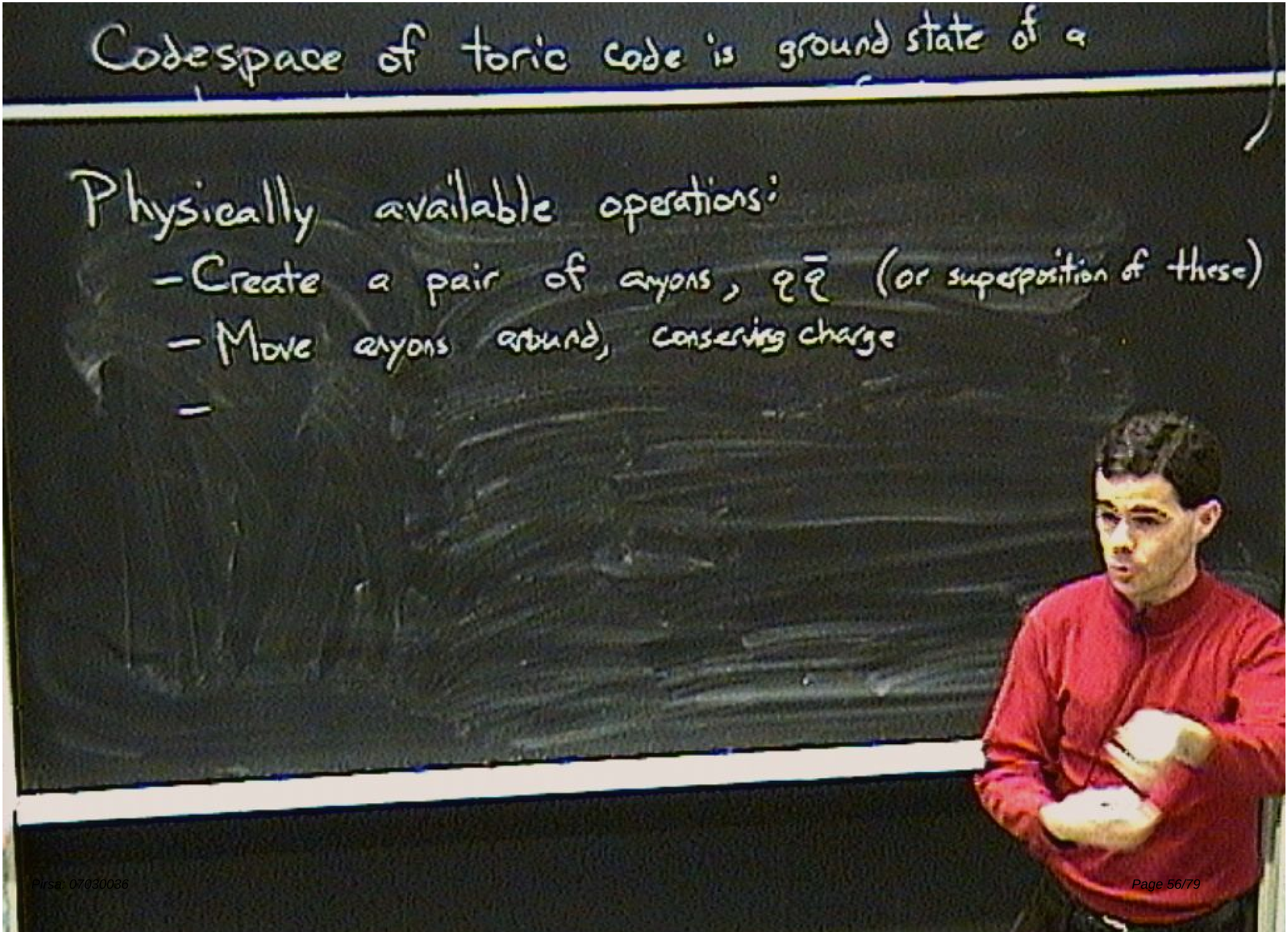
- Create a pair of anyons, $q\bar{q}$ (or superposition of these)
- Move anyons around,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- Create a pair of anyons, $q\bar{q}$ (or superposition of these)
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Codespace of toric code is ground state of a

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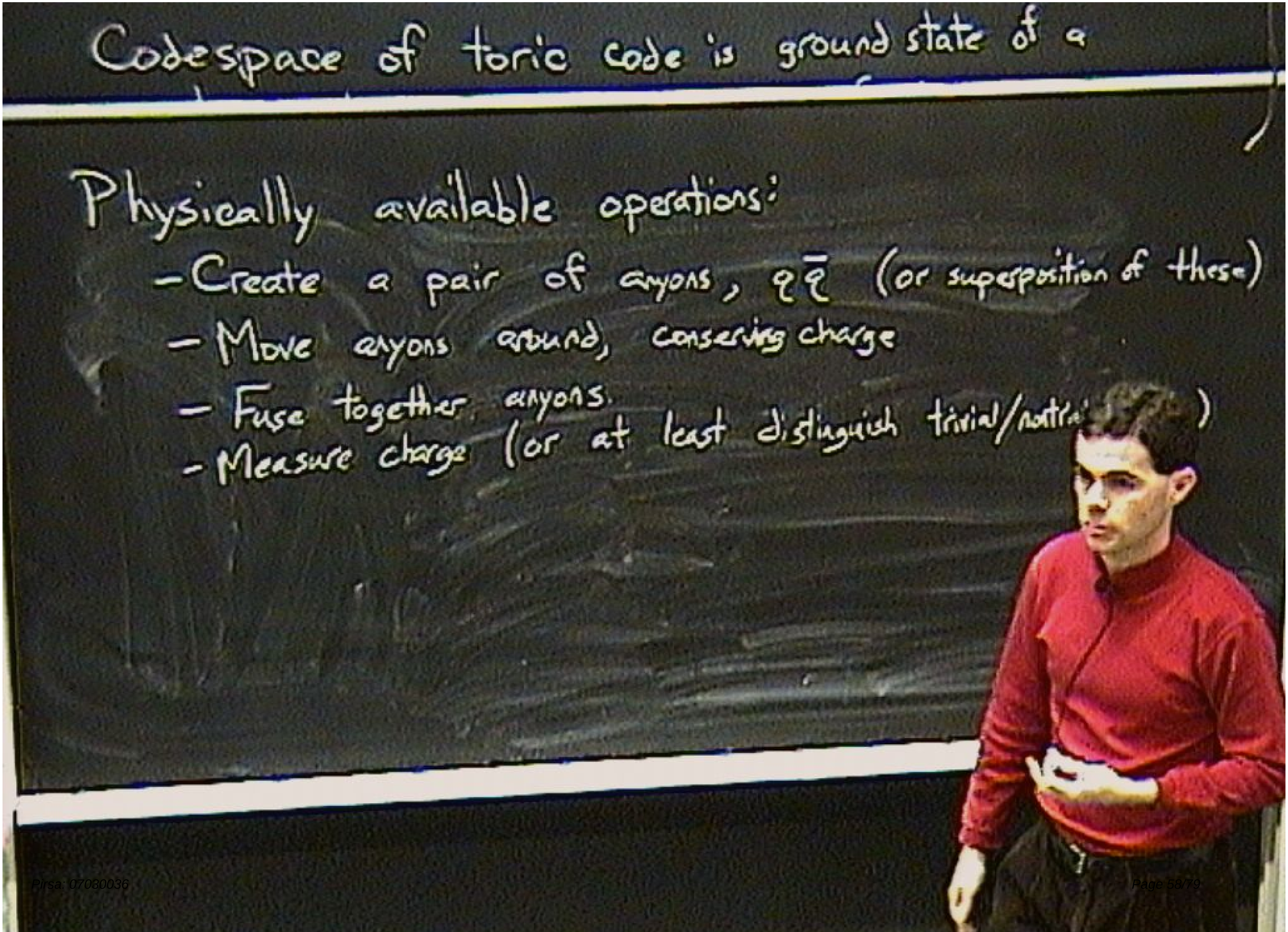
- Fuse together anyons.
- Measure charge



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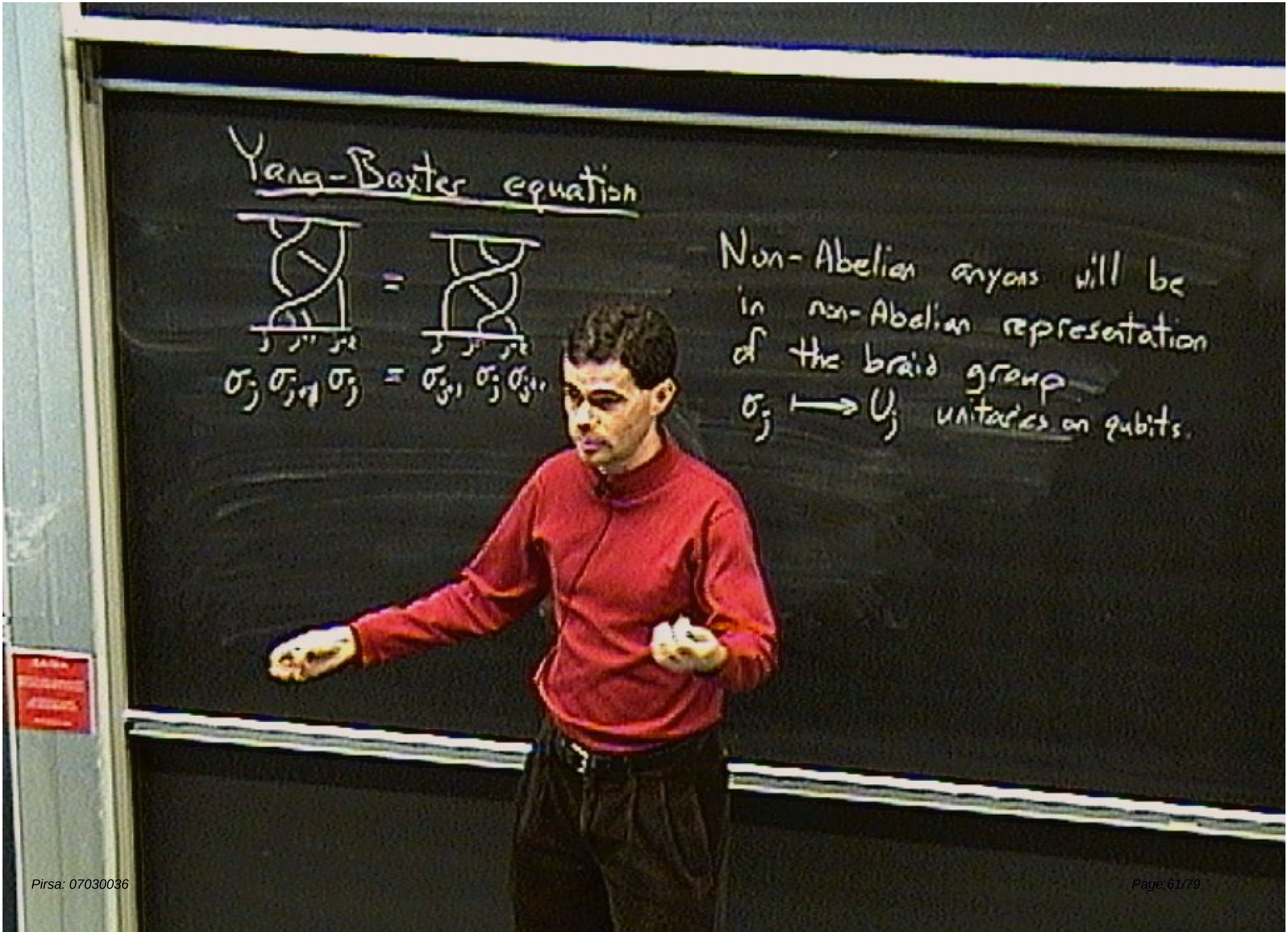
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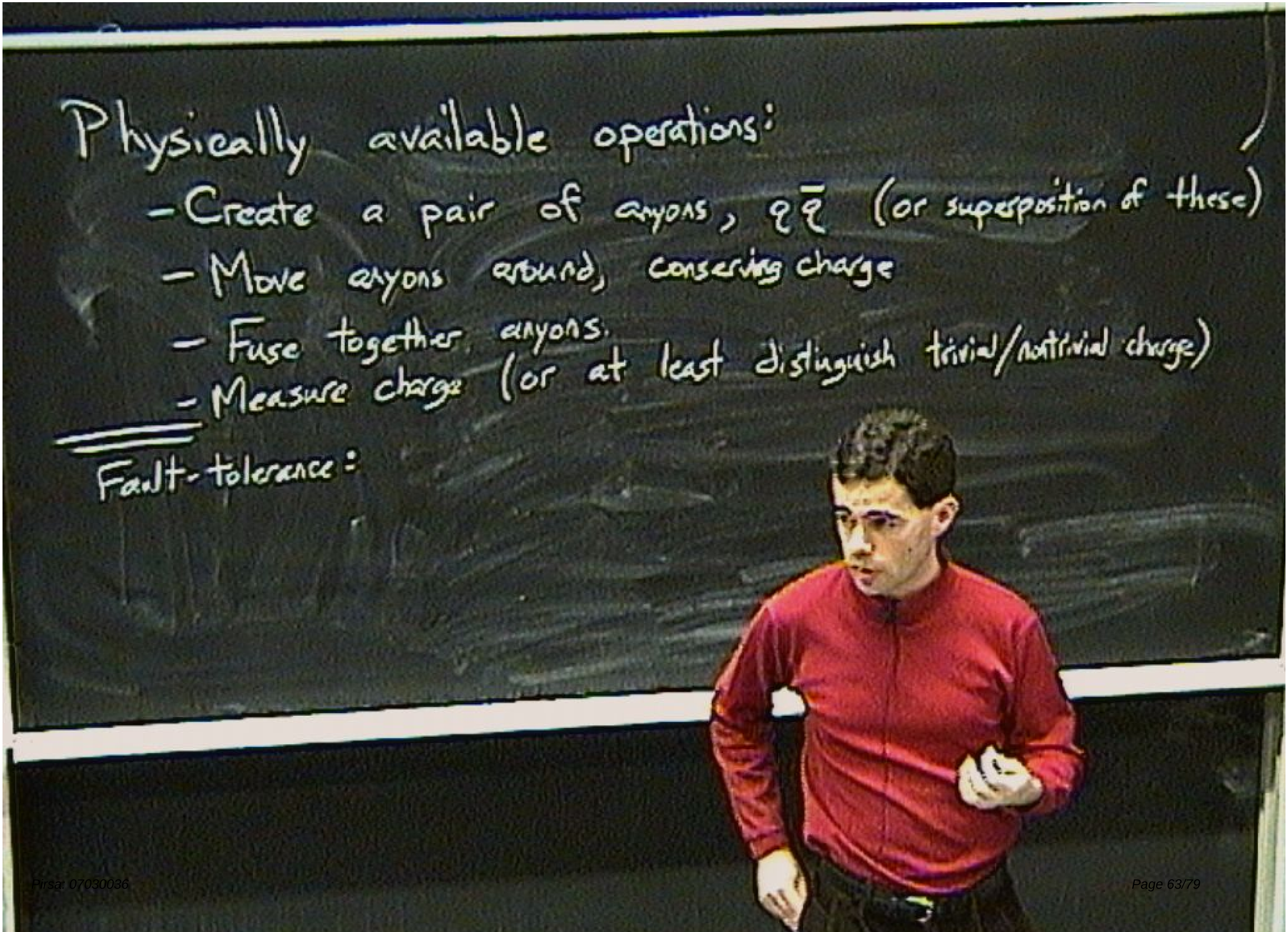
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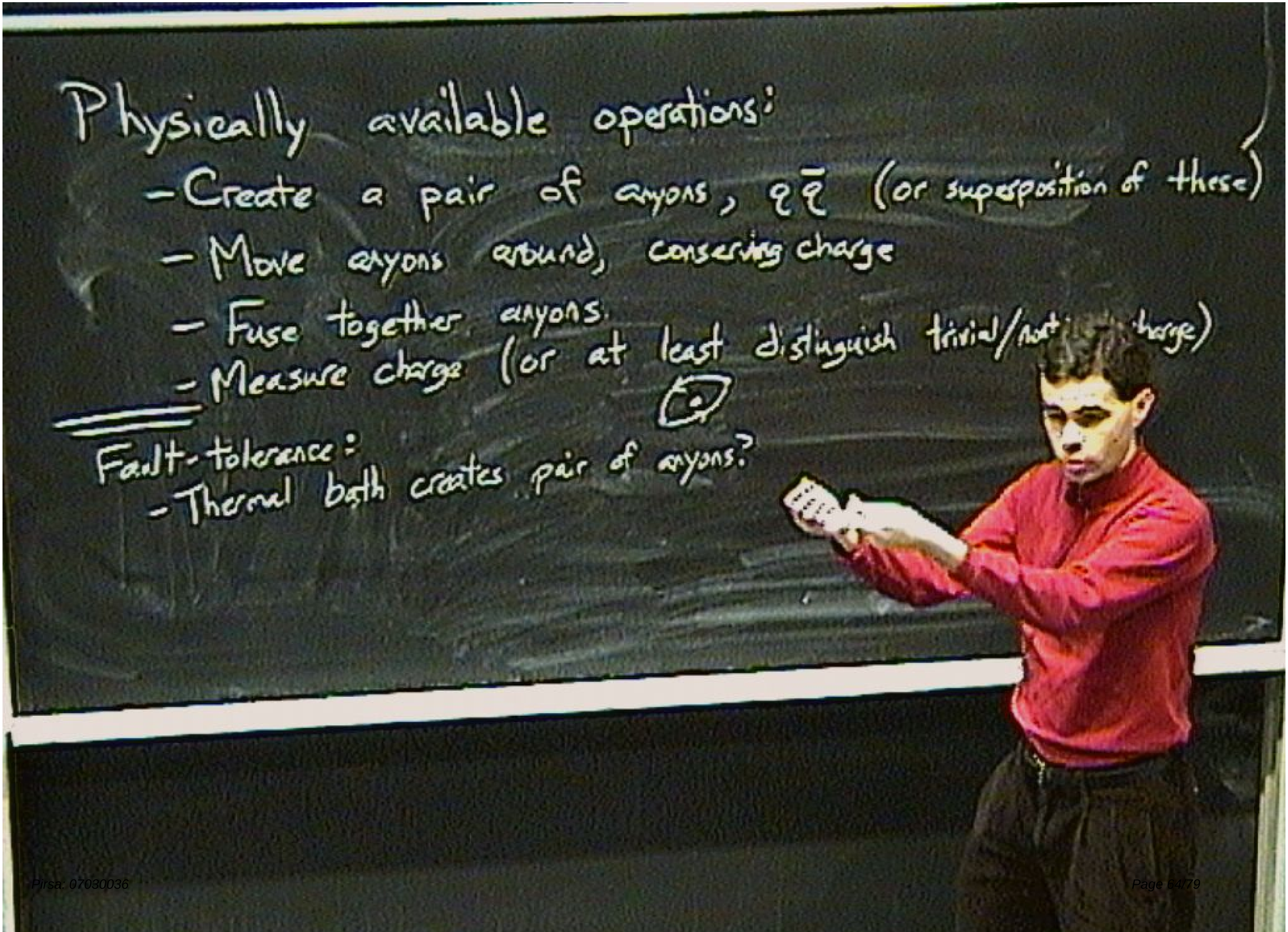
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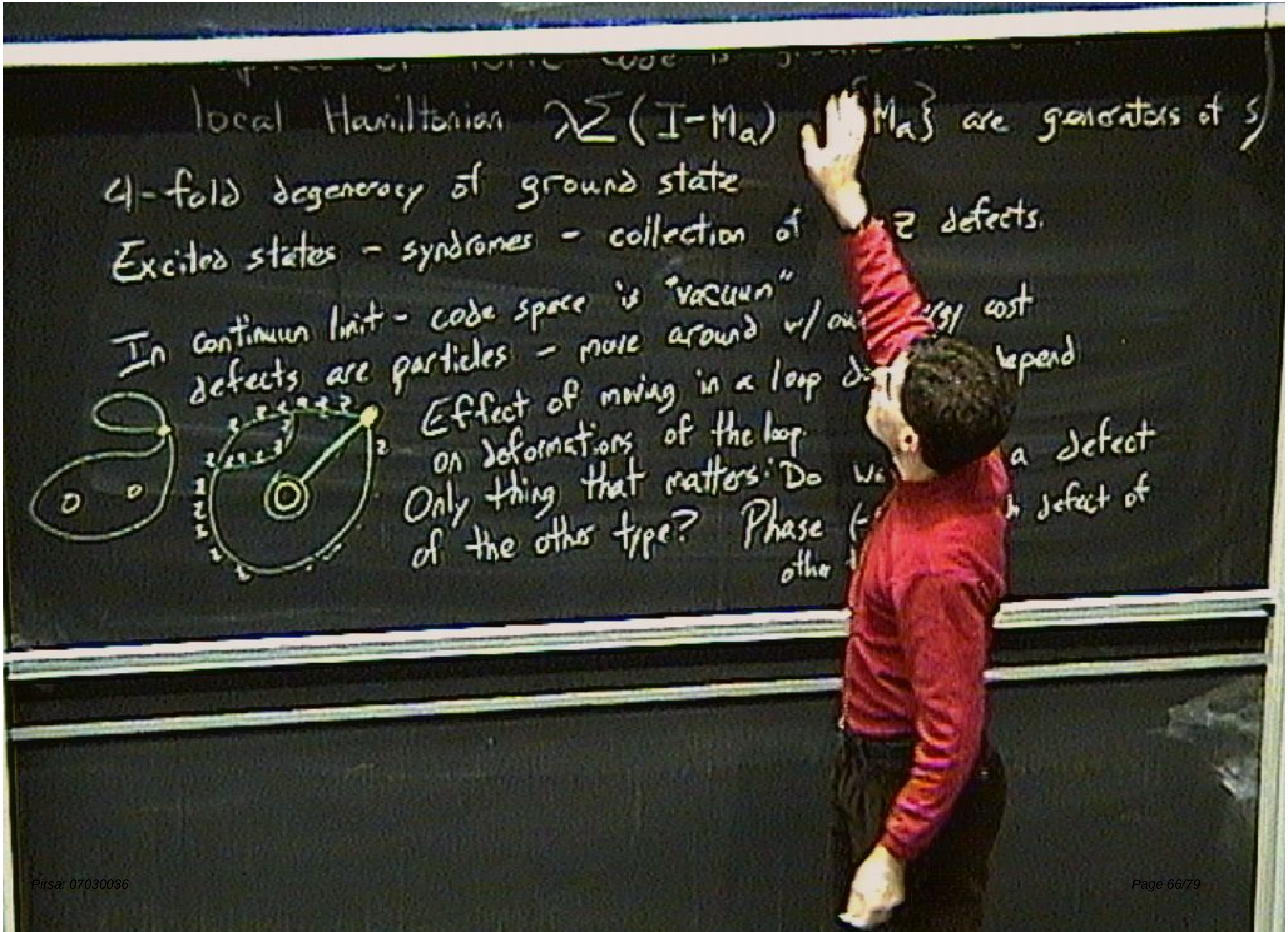
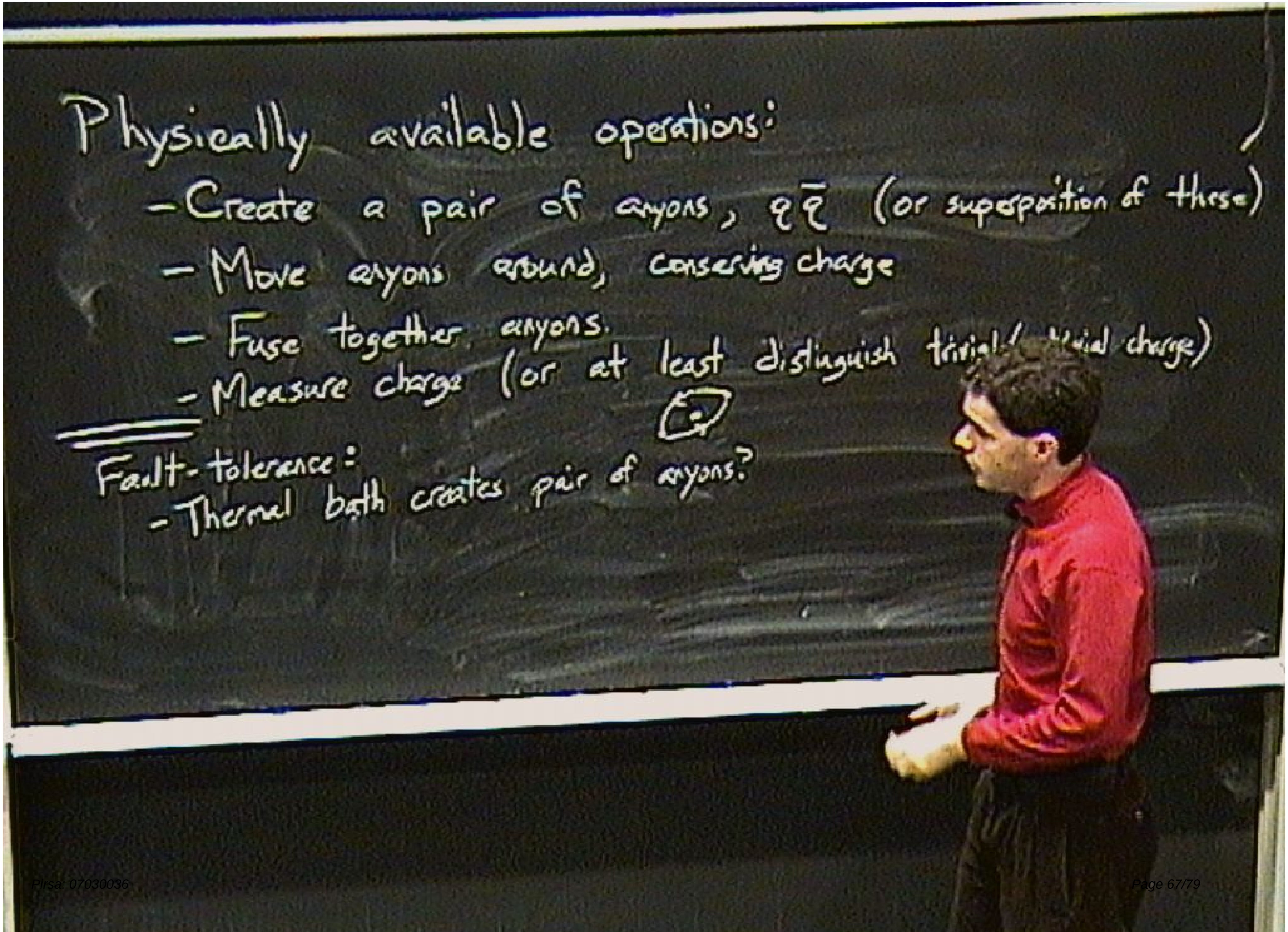

local Hamiltonian $\lambda\sum(I-M_a)$
$\{M_a\}$ are generators of S
4-fold degeneracy of ground state
Excited states - syndromes - collection of
Z defects.
In continuum limit - code space is "vacuum"
defects are particles - move around w/o
cost
Effect of moving in a loop
depend
on deformations of the loop
Only thing that matters: Do
a defect
defect of
of the other type? Phase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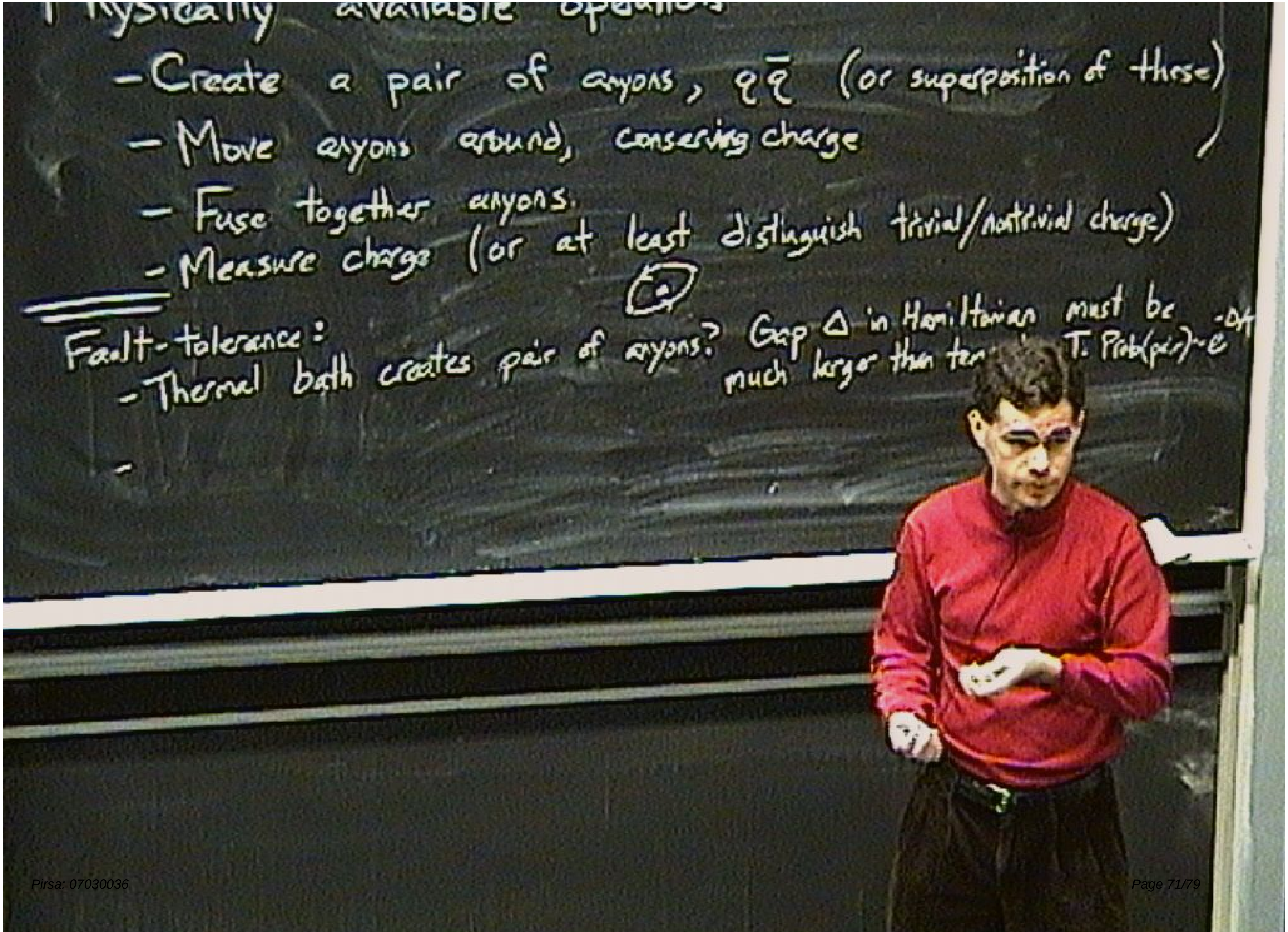
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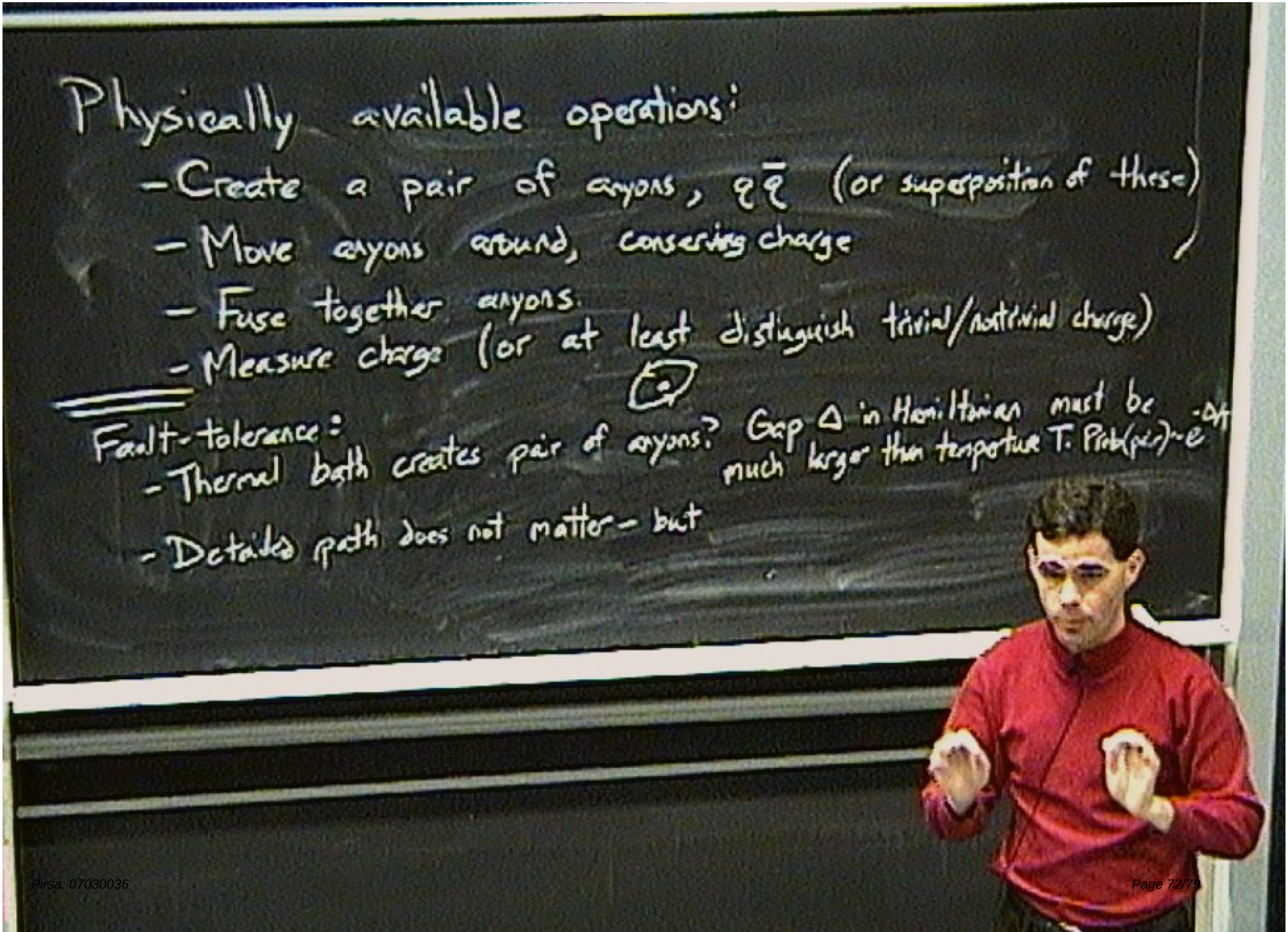
-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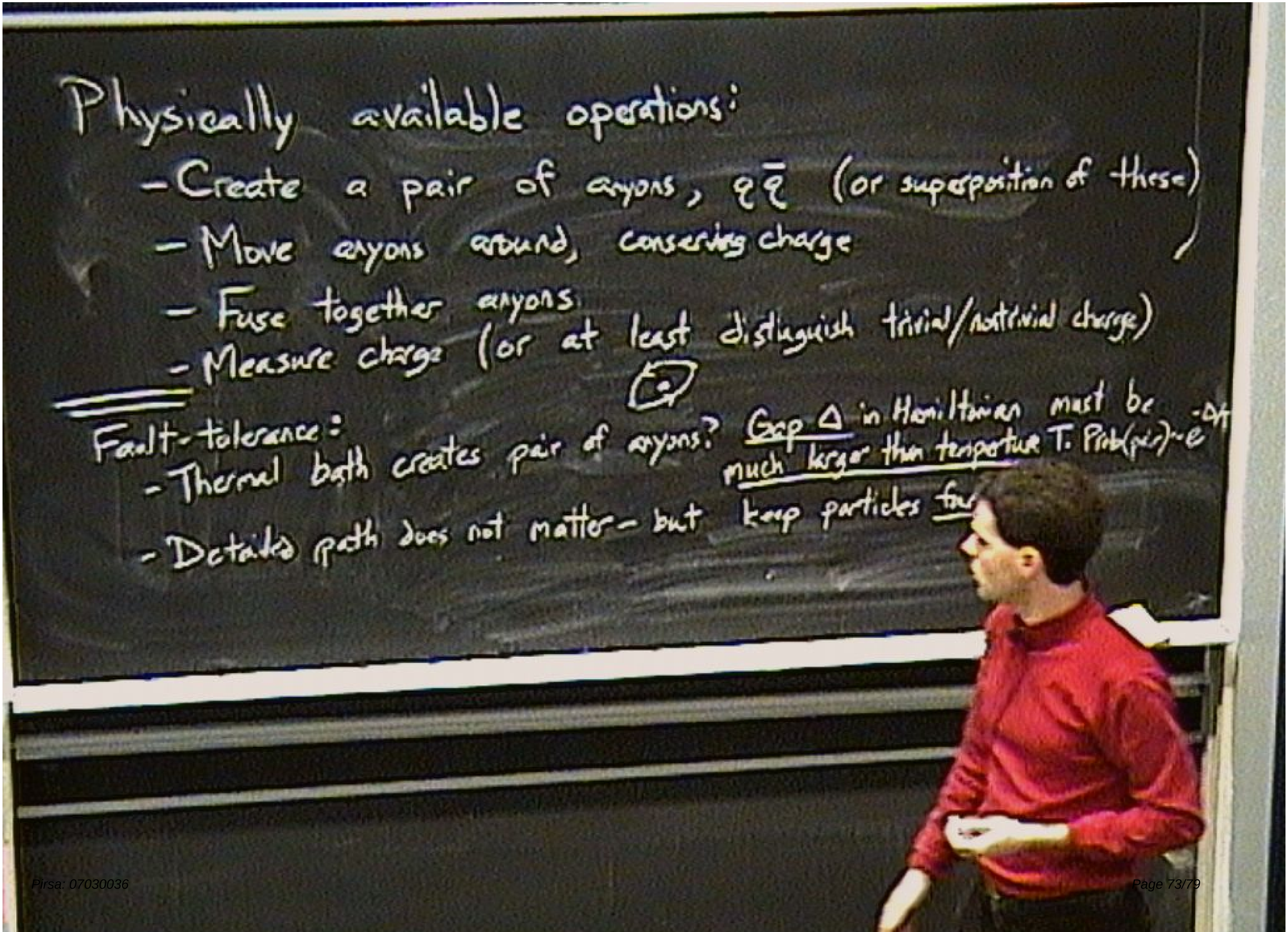

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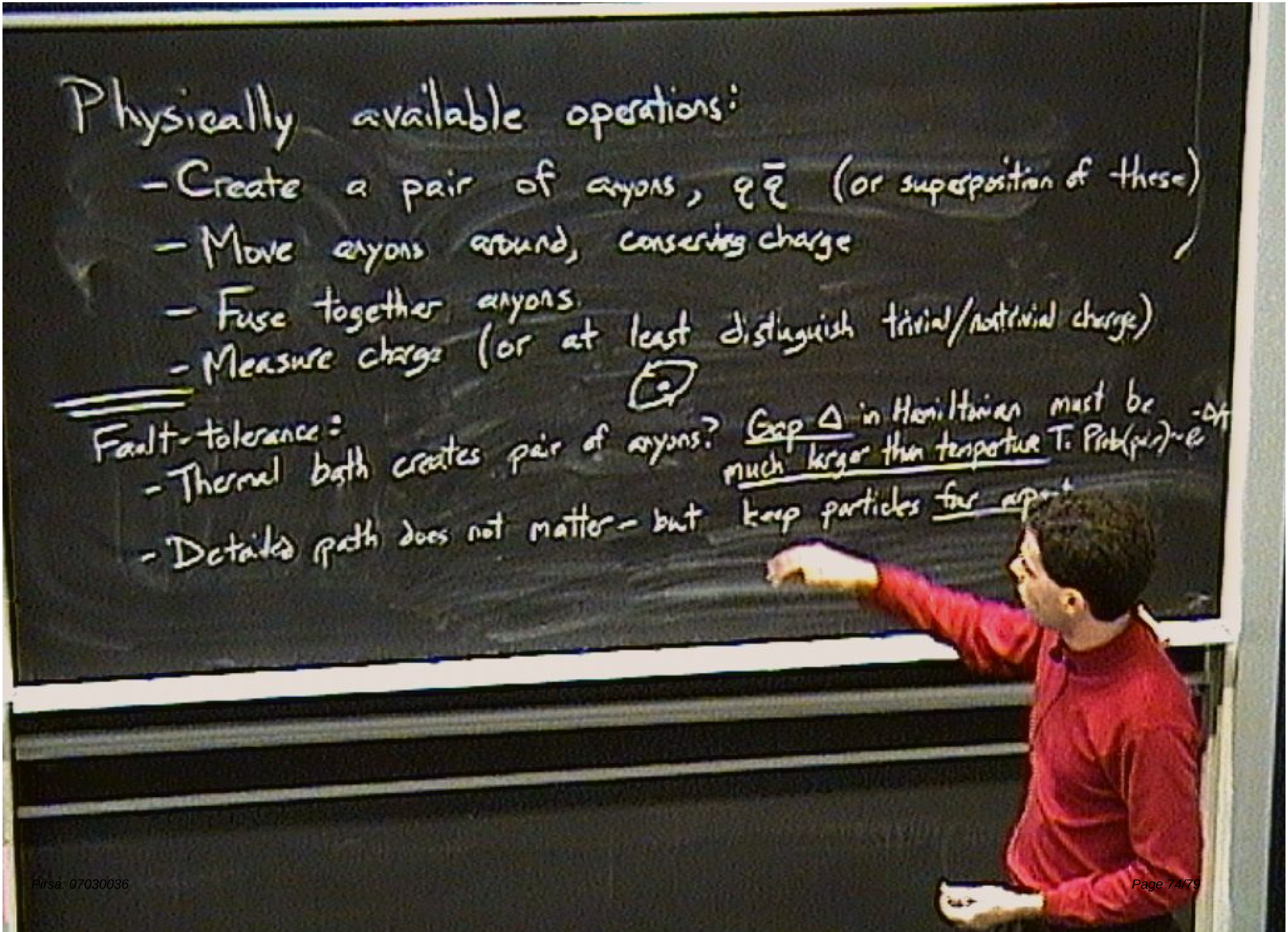
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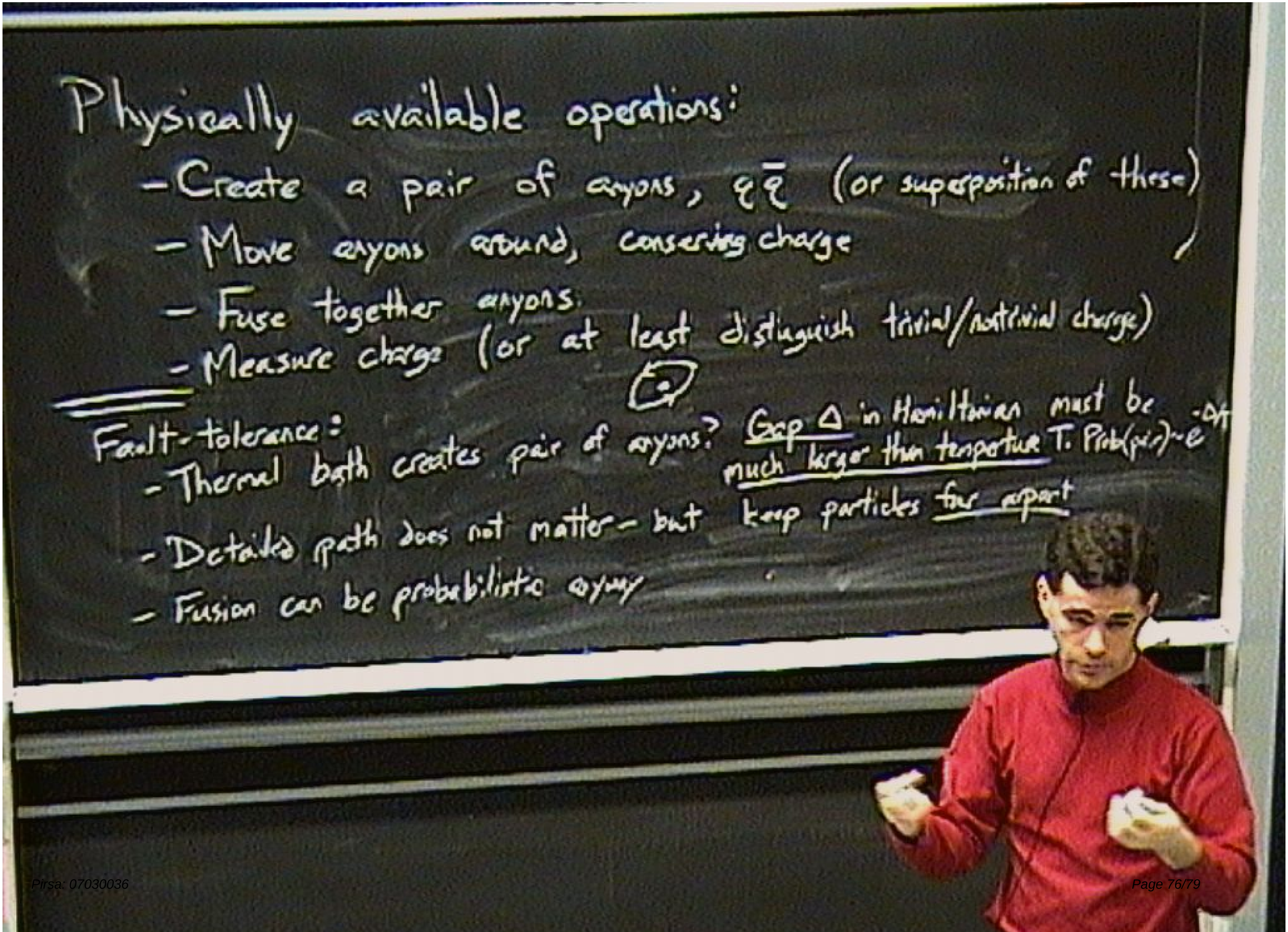
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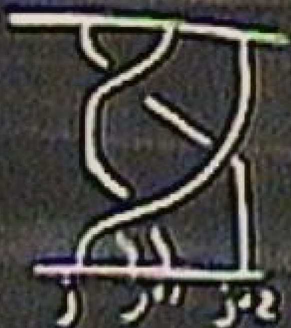

=

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Non-Abelian any

in non-Abelian

of the braid gr

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