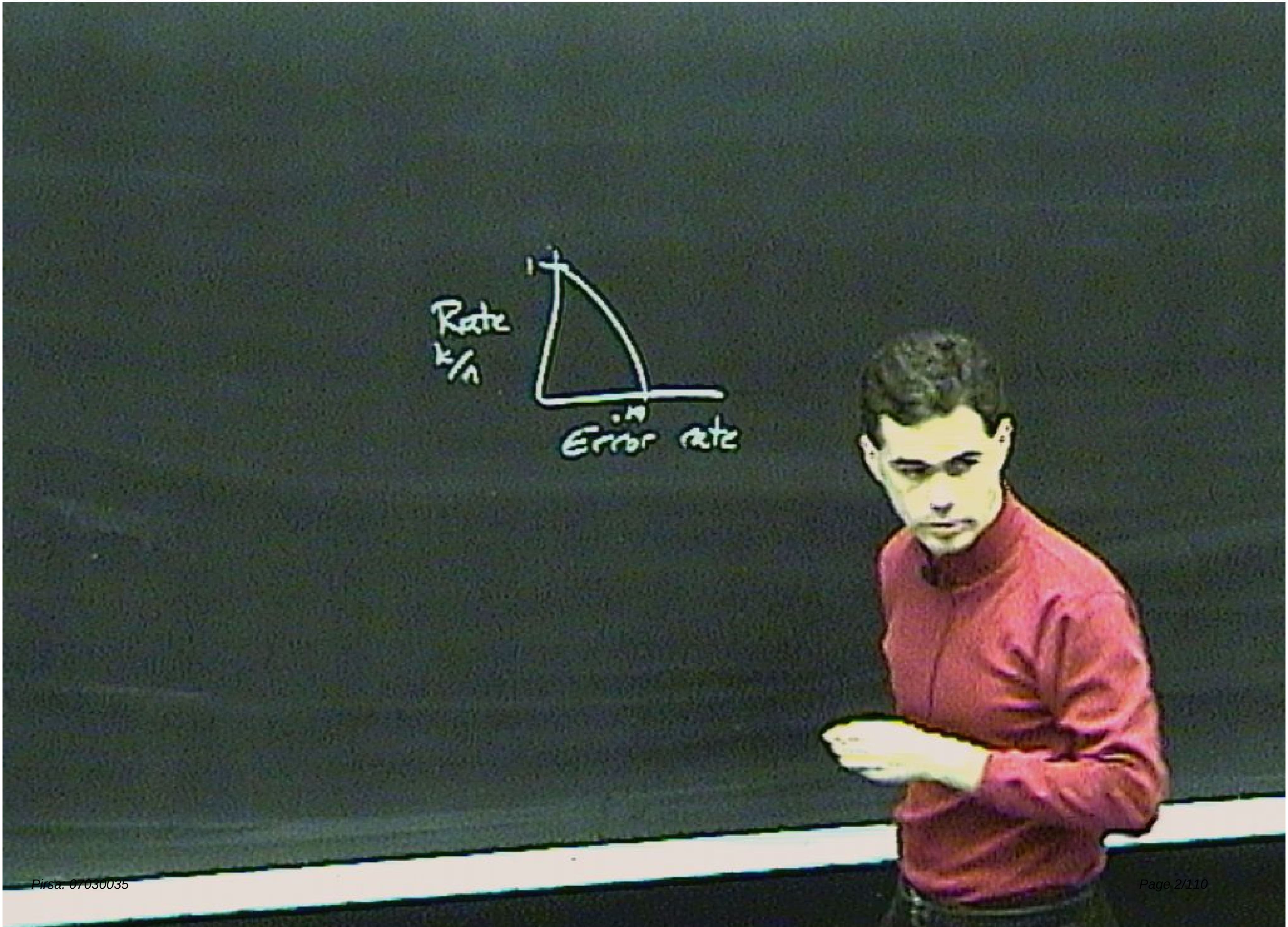
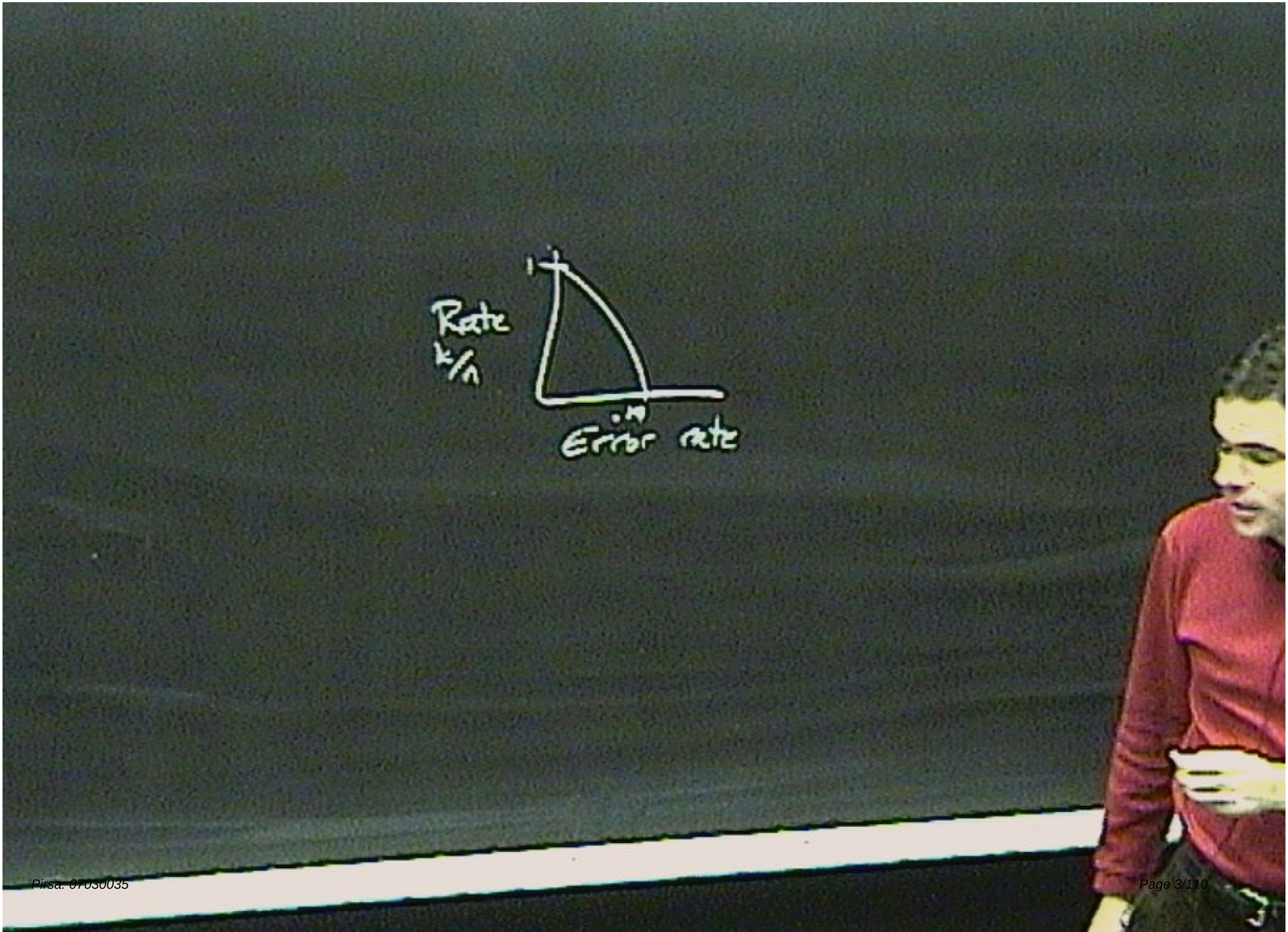
Title: Quantum Error Correction 11A

Date: Mar 20, 2007 03:30 PM

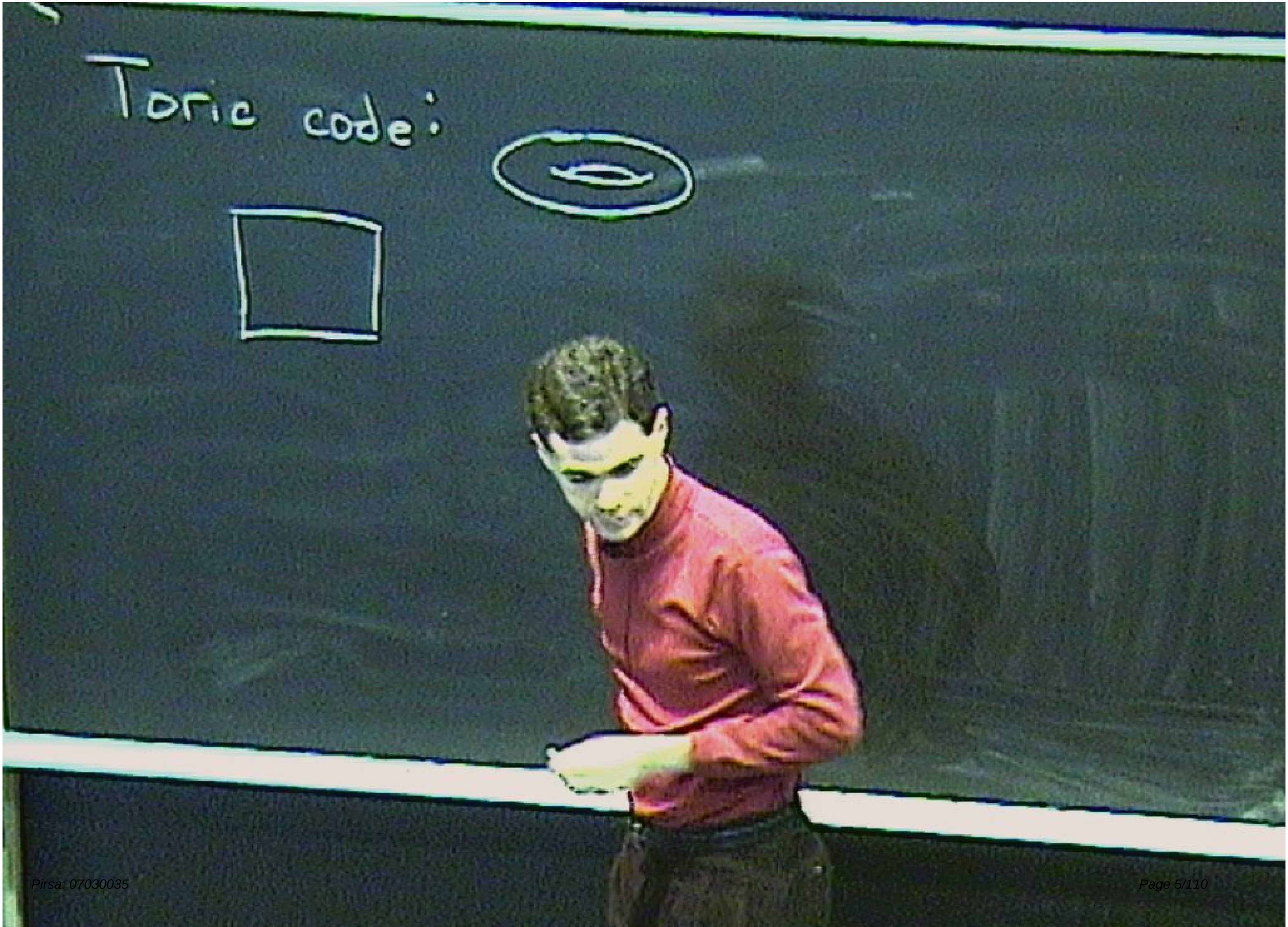
URL: http://pirsa.org/07030035

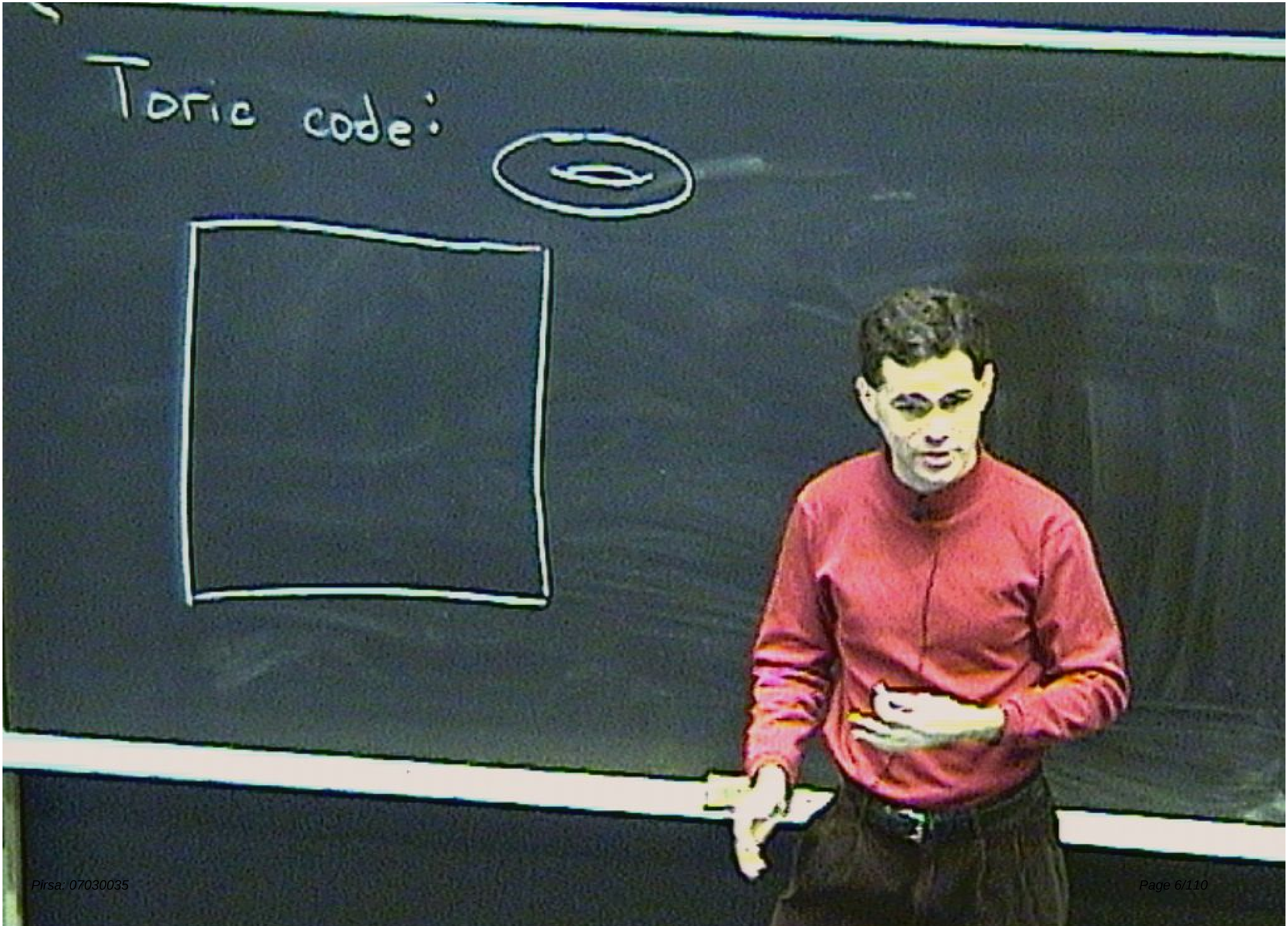
Abstract: Toric code (definition, fault-tolerance, particle model of errors), definition of qudit toric code

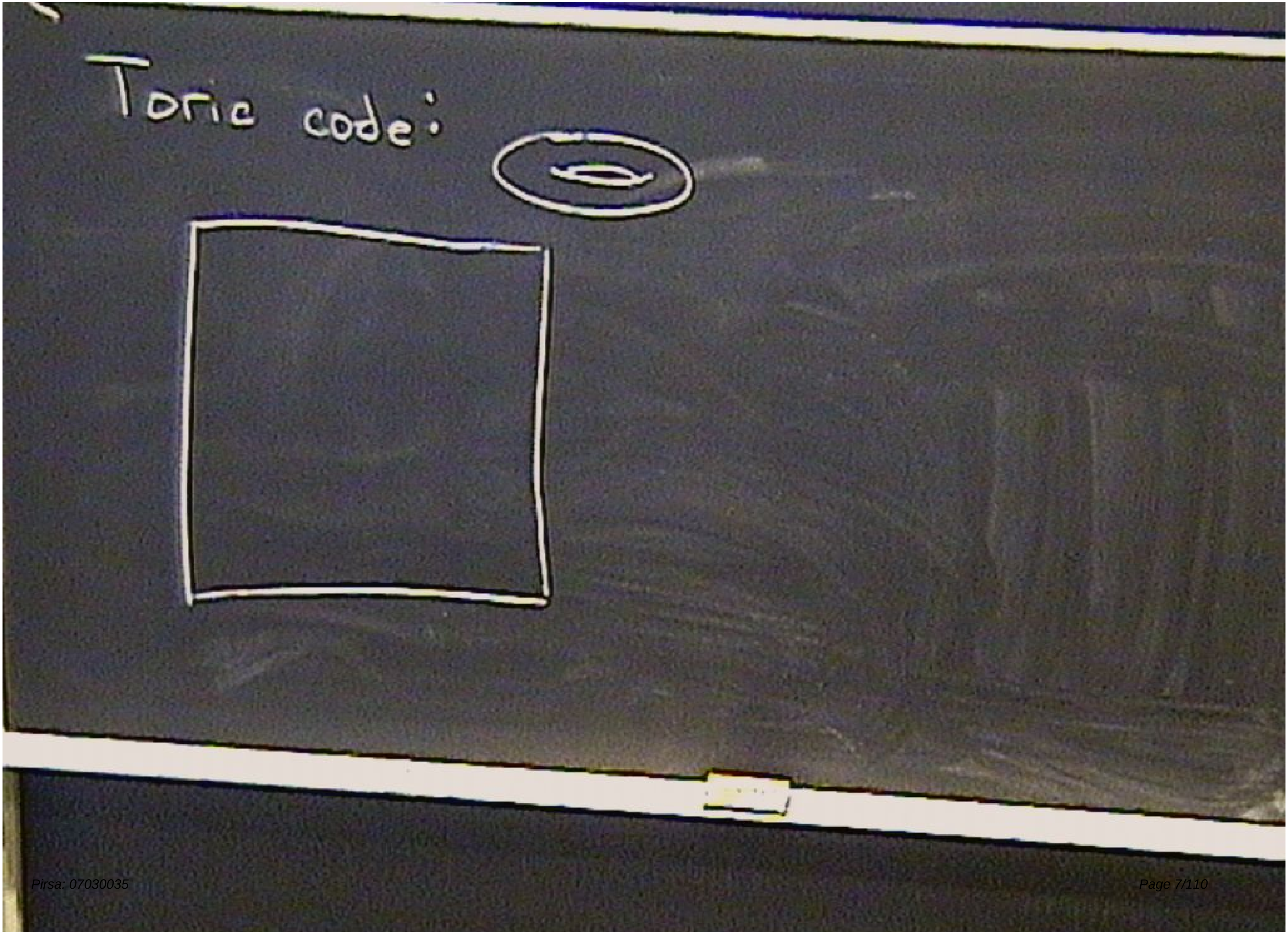

1
Rate
k/n
.11
Error rate

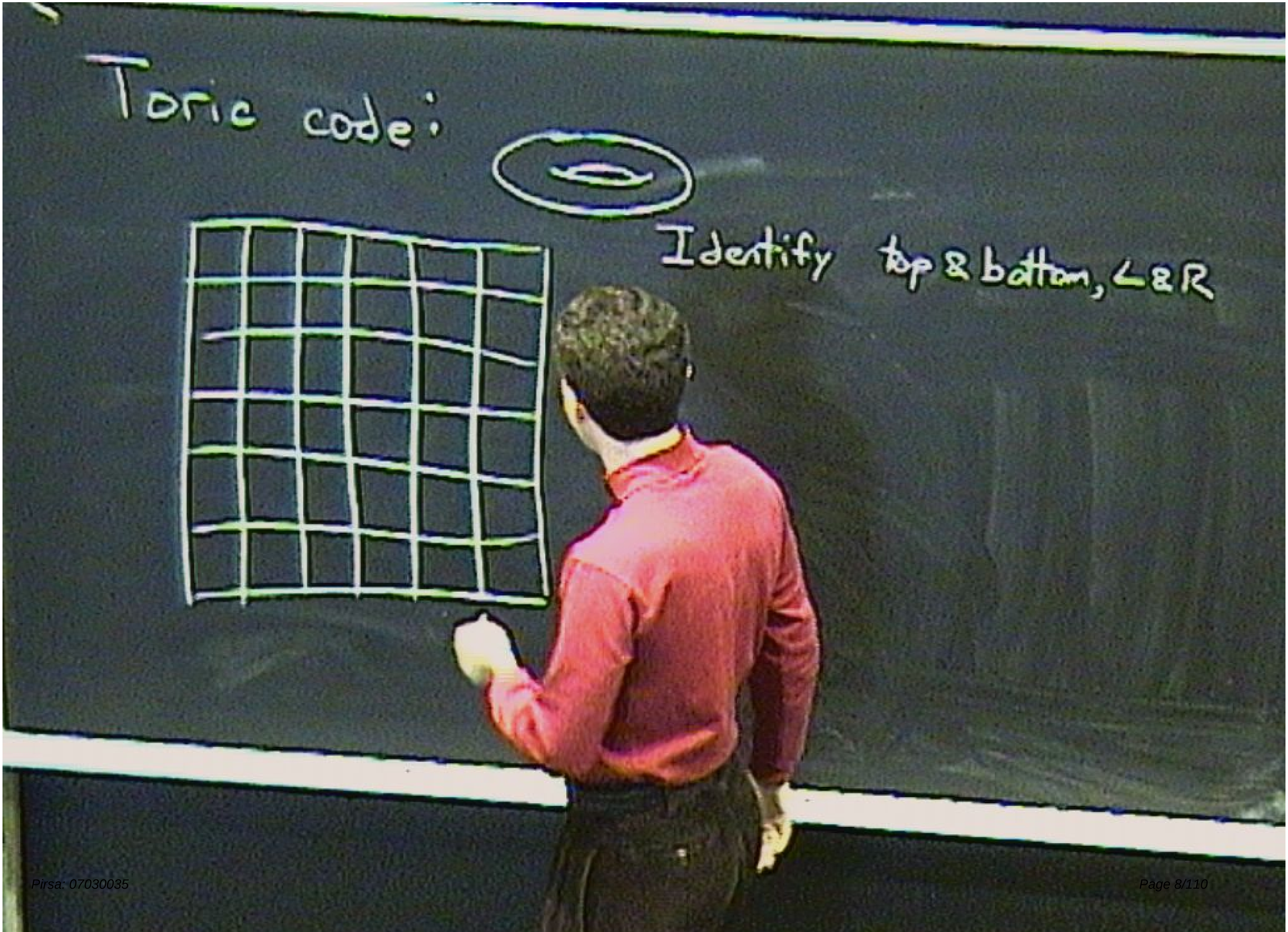

Rate
k/n
1
Error rate

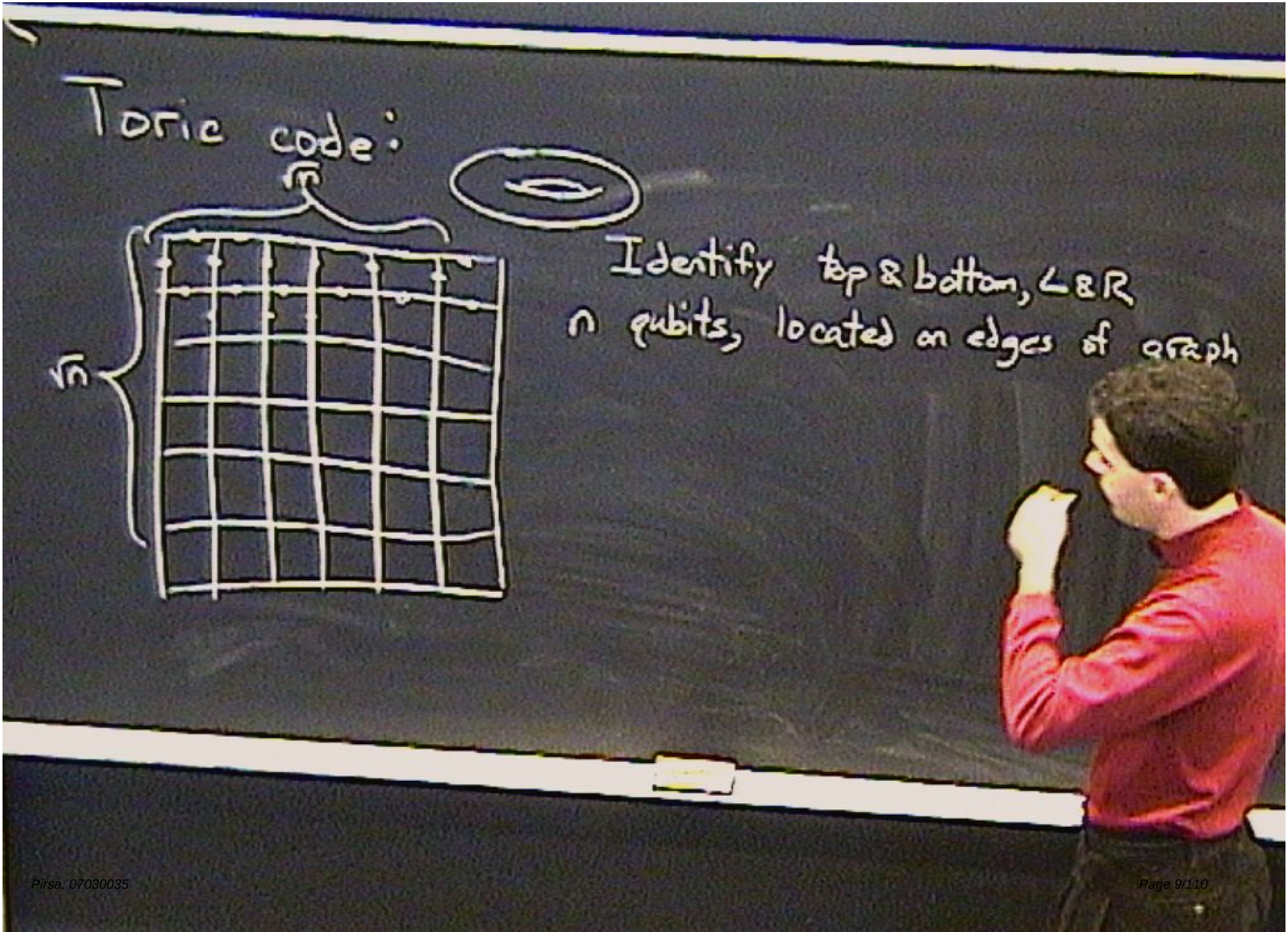
Toric code:


Toric code:

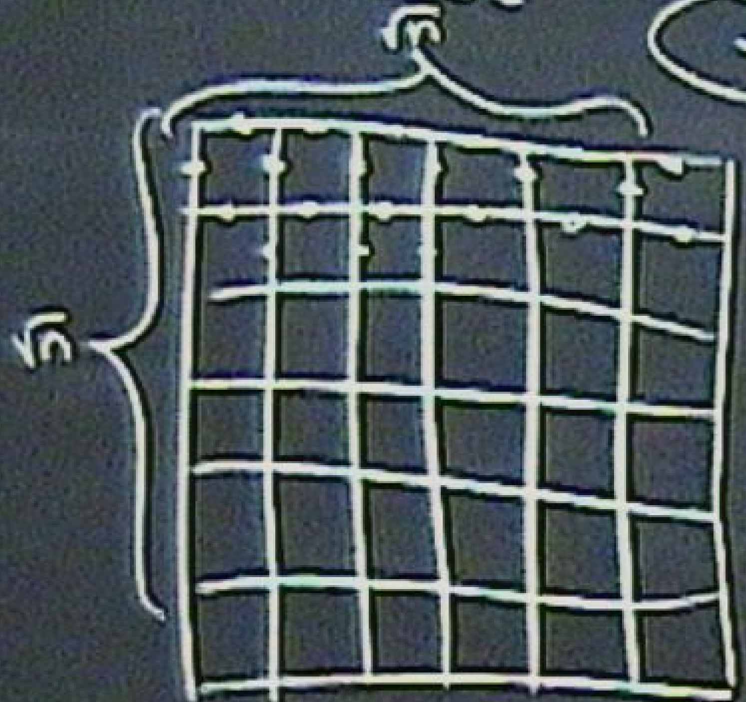

Toric code:


Toric code:


Toric code:
Identify top & bottom, L & R


Toric code:
√n
√n
Identify top & bottom, L & R
n qubits, located on edges of graph

Toric code:

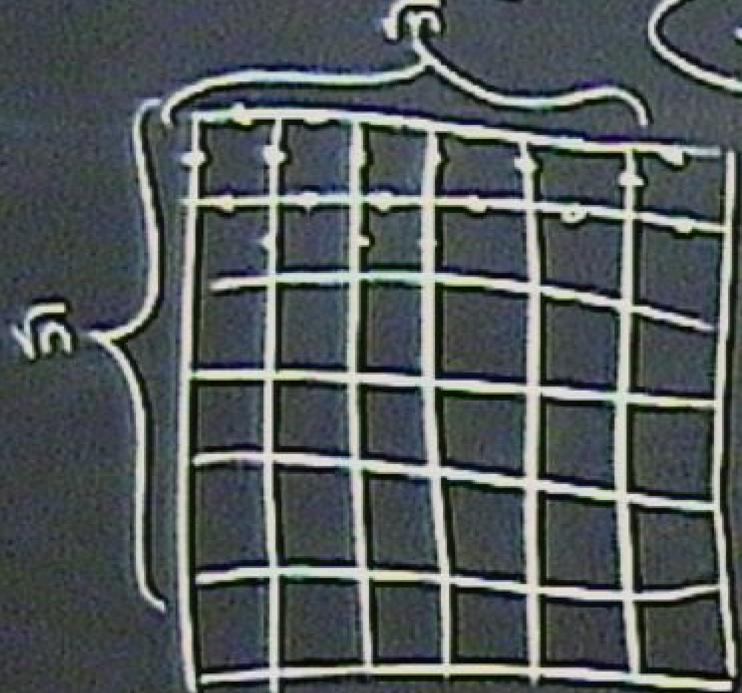


Identify top & bottom, L & R

2n qubits, located on edges of graph

Define stabilizer code:

Toric code:



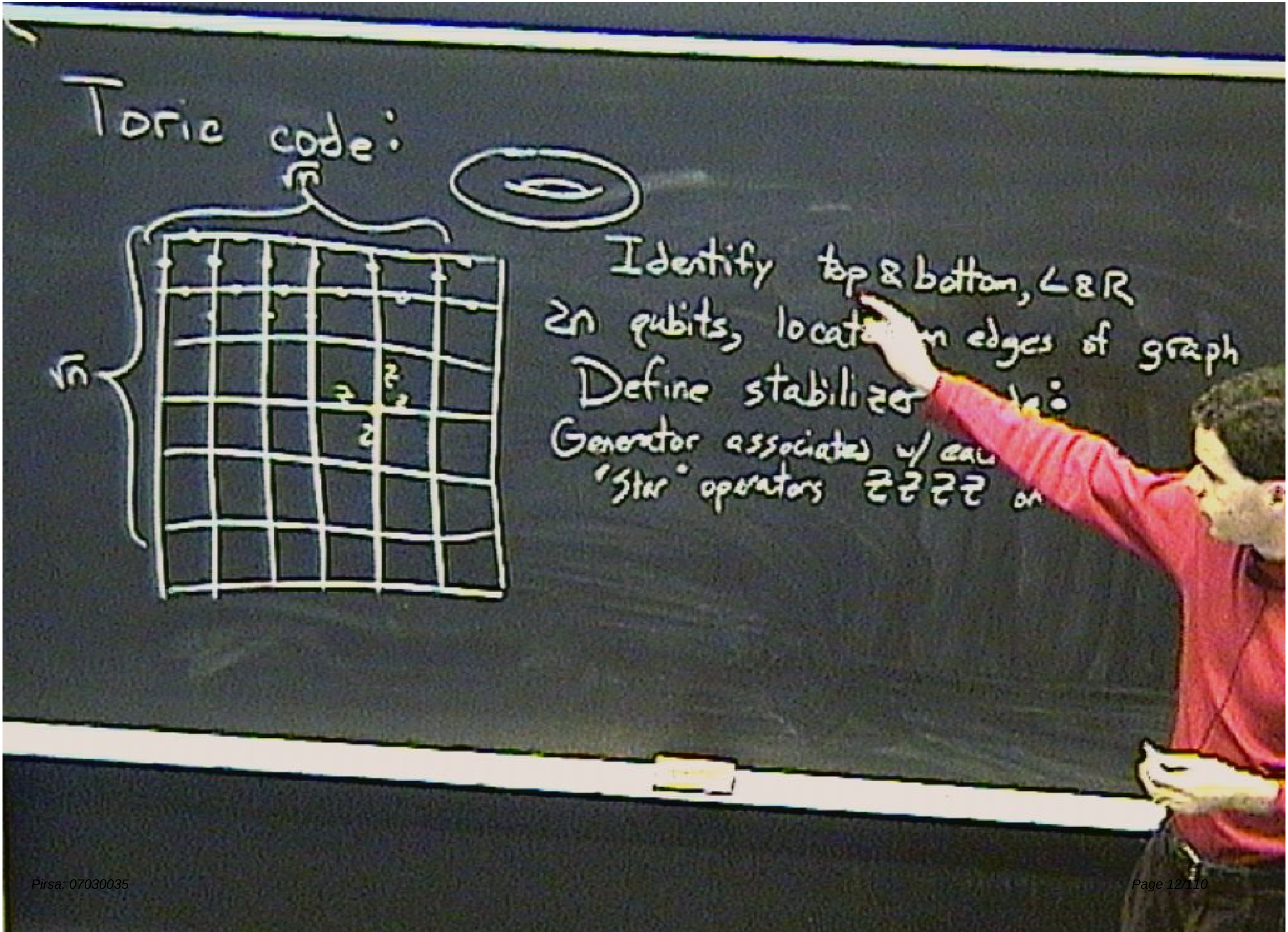
Identify top & bottom, L & R

$2n$ qubits, located on edges of graph

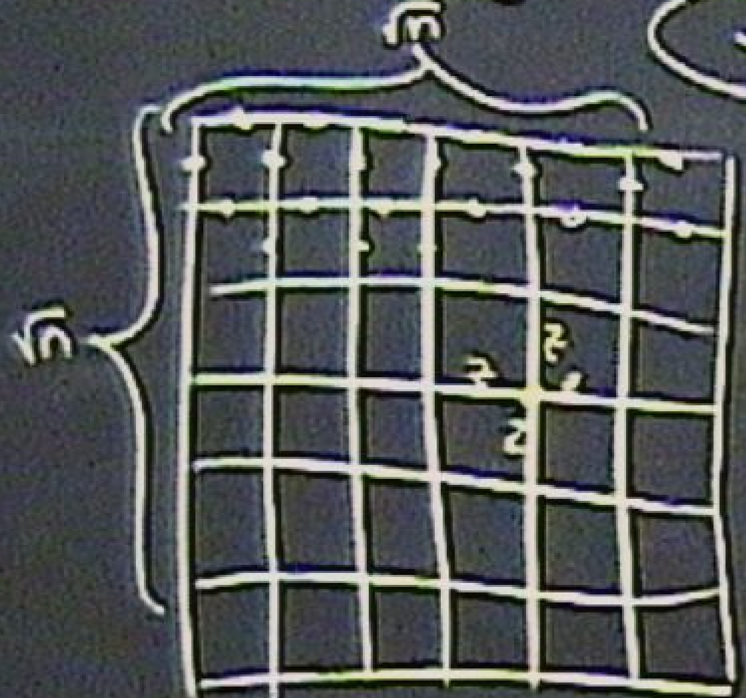
Define stabilizer code:

Generator associated w/ each vertex

"Star" operators


Toric code:
√n
√n
Identify top & bottom, L & R
2n qubits, locat n edges of graph
Define stabilize :
Generator associated w/ eac
"Star" operators ZZZZ on
Z
Z
Z
Z

Toric code:



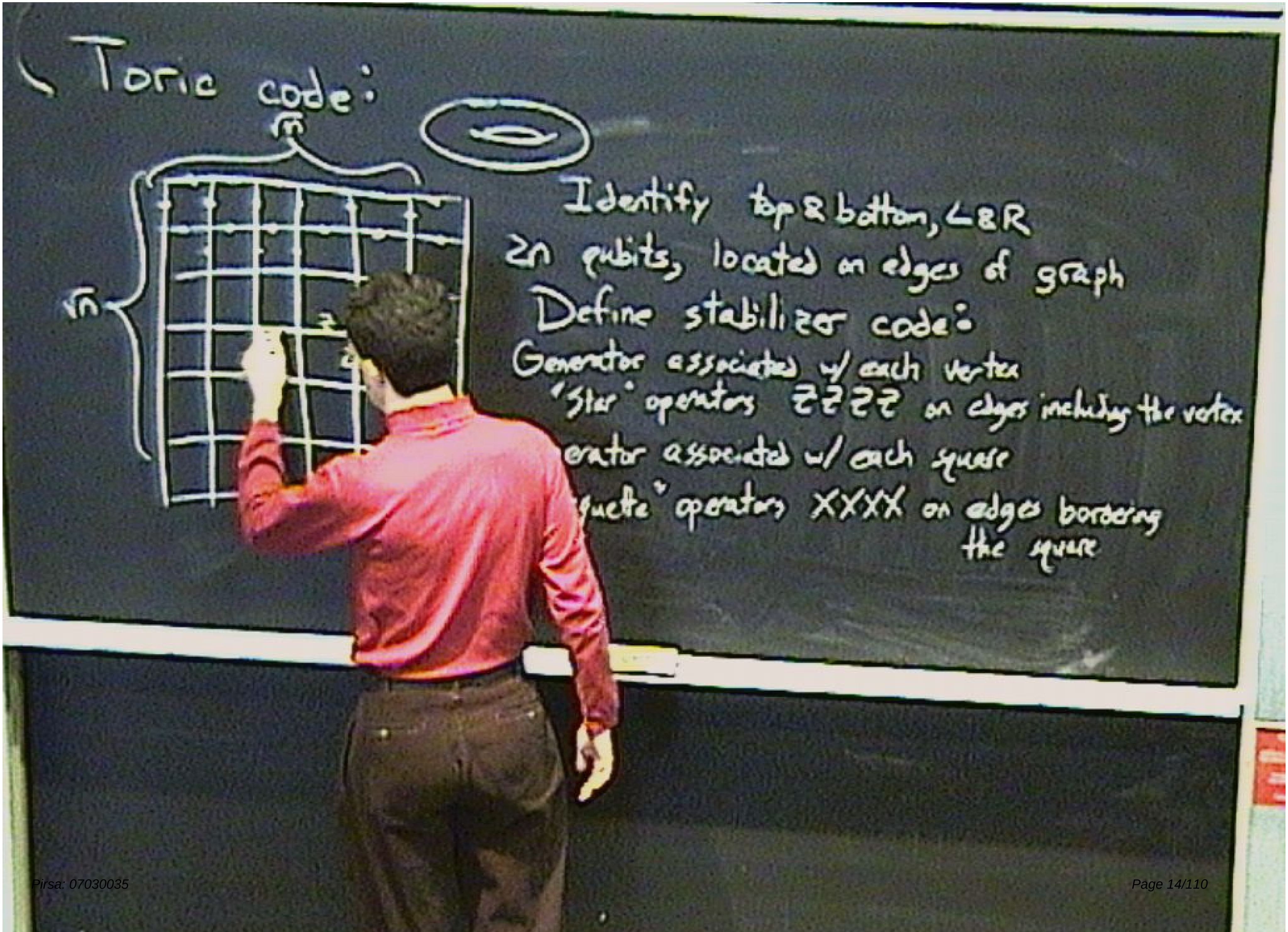
Identify top & bottom, L & R

$2n$ qubits, located on edges of graph

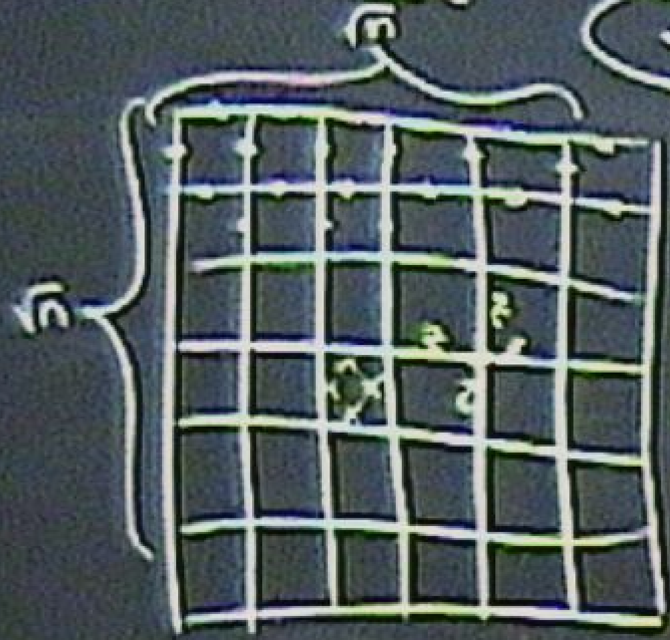
Define stabilizer code:

Generator associated w/ each vertex

"Star" operators $ZZZZ$ on edges including th


Toric code:
√n
√n
Identify top & bottom, L & R
2n qubits, located on edges of graph
Define stabilizer code:
Generator associated w/ each vertex
"Star" operators ZZZZ on edges including the vertex
erator associated w/ each square
quette" operators XXXX on edges bordering
the square

Toric code:



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$2n$ qubits, located on edges of graph

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"Star" operators $ZZZZ$ on edges including the vertex

Generator associated w/ each square

"Plaquette" operators $XXXX$ on edges bordering the square

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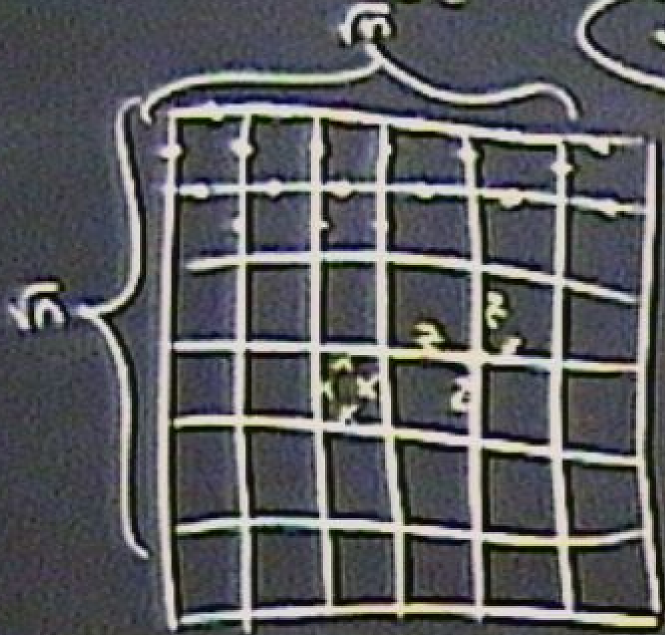

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$2n$ qubits, located on edges of graph

Define stabilizer code:

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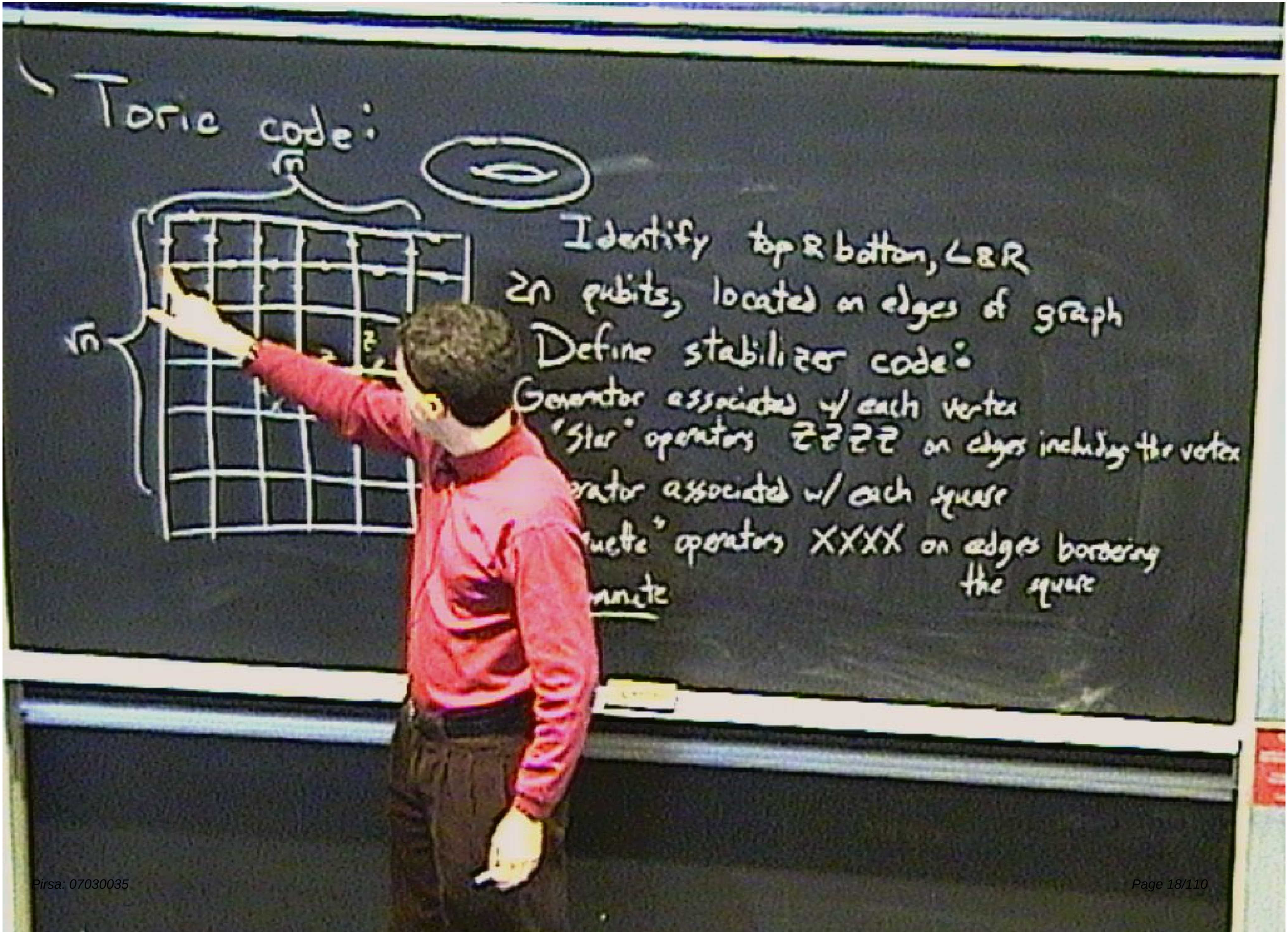
Generator associated w/ each square

"Plaquette" operators XXXX on edges bordering the square

<u>These commute</u>

Product of all stars is I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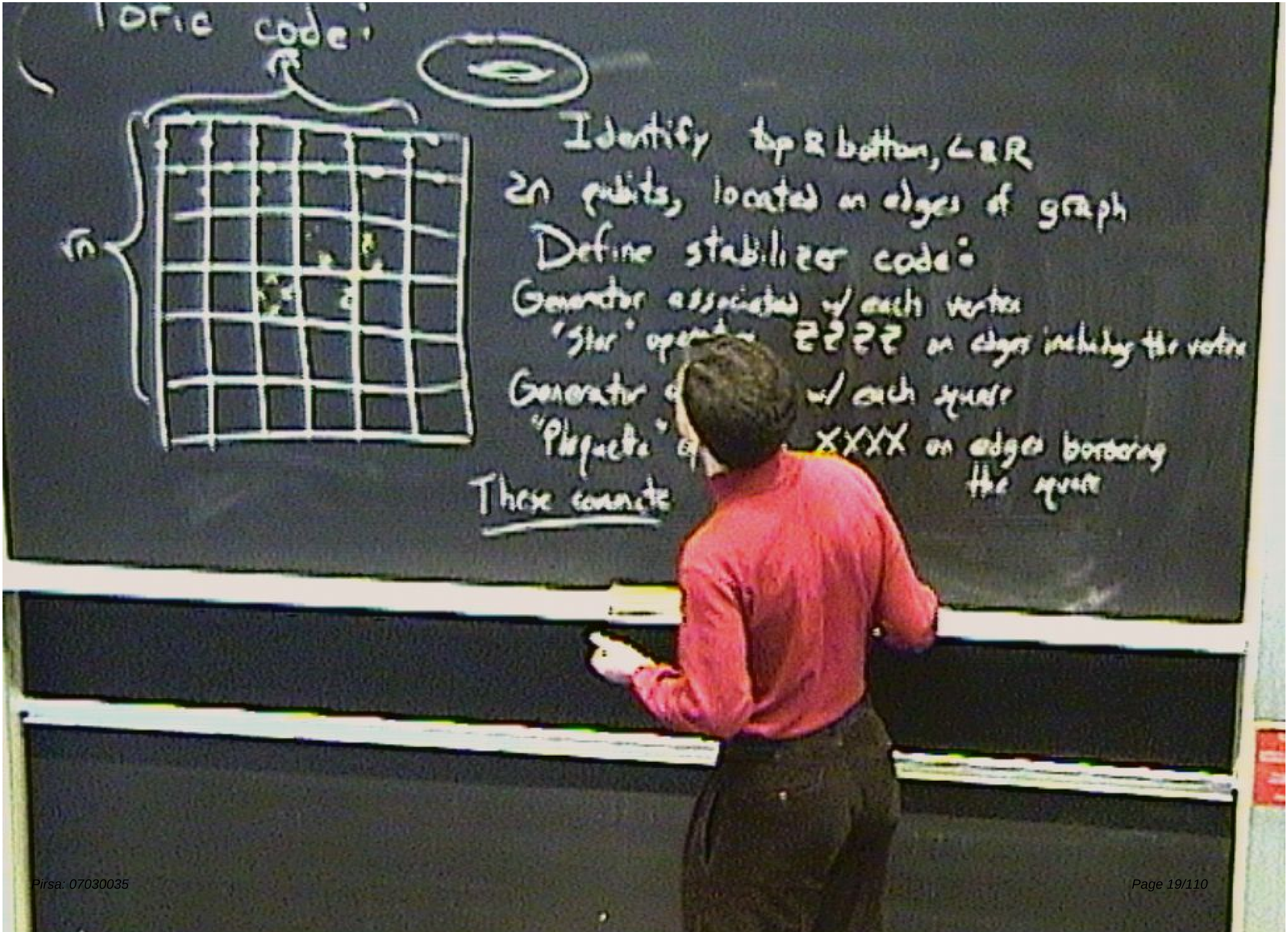
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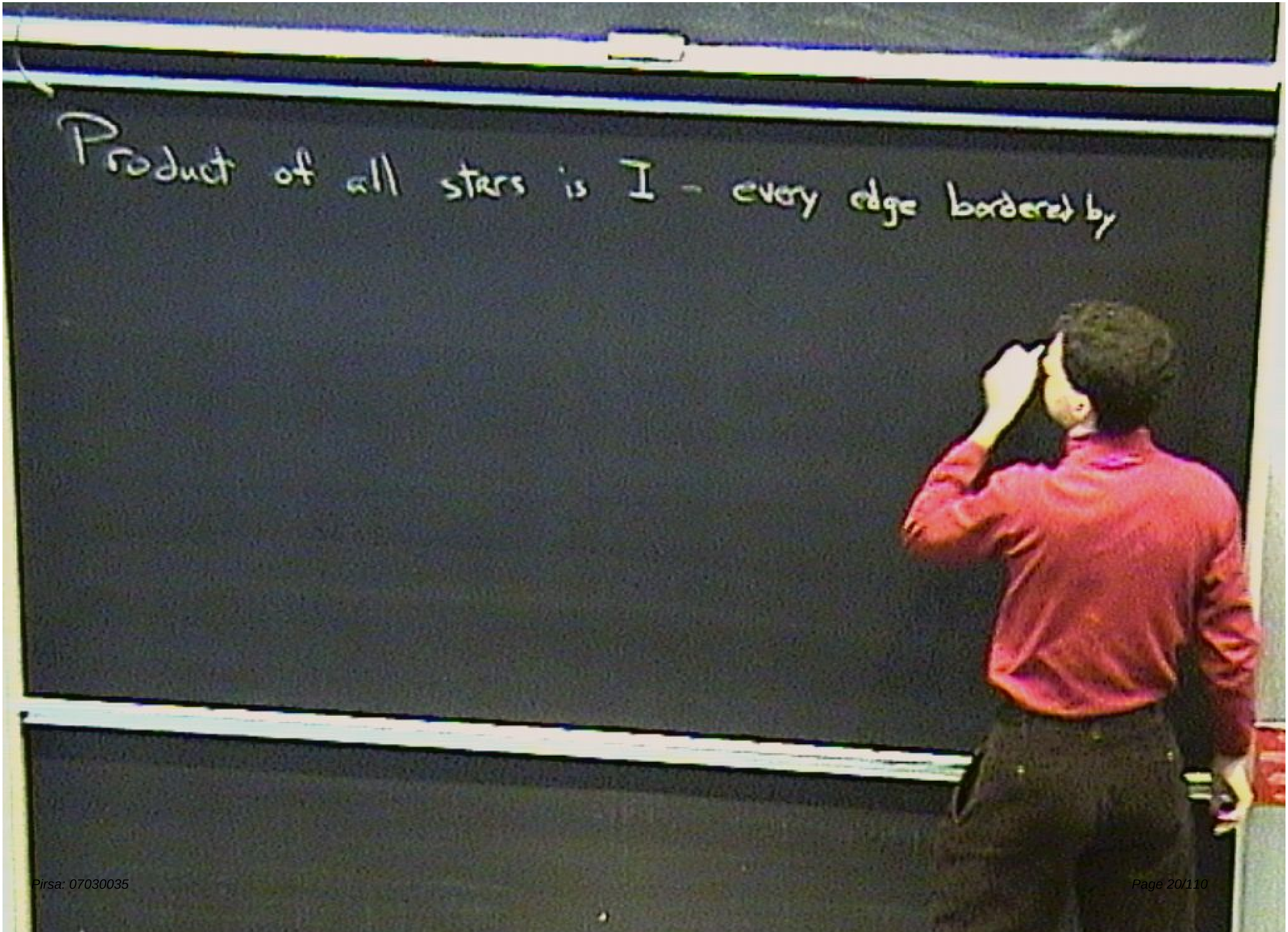
These commute


Product of all stars is I - every edge bordered by

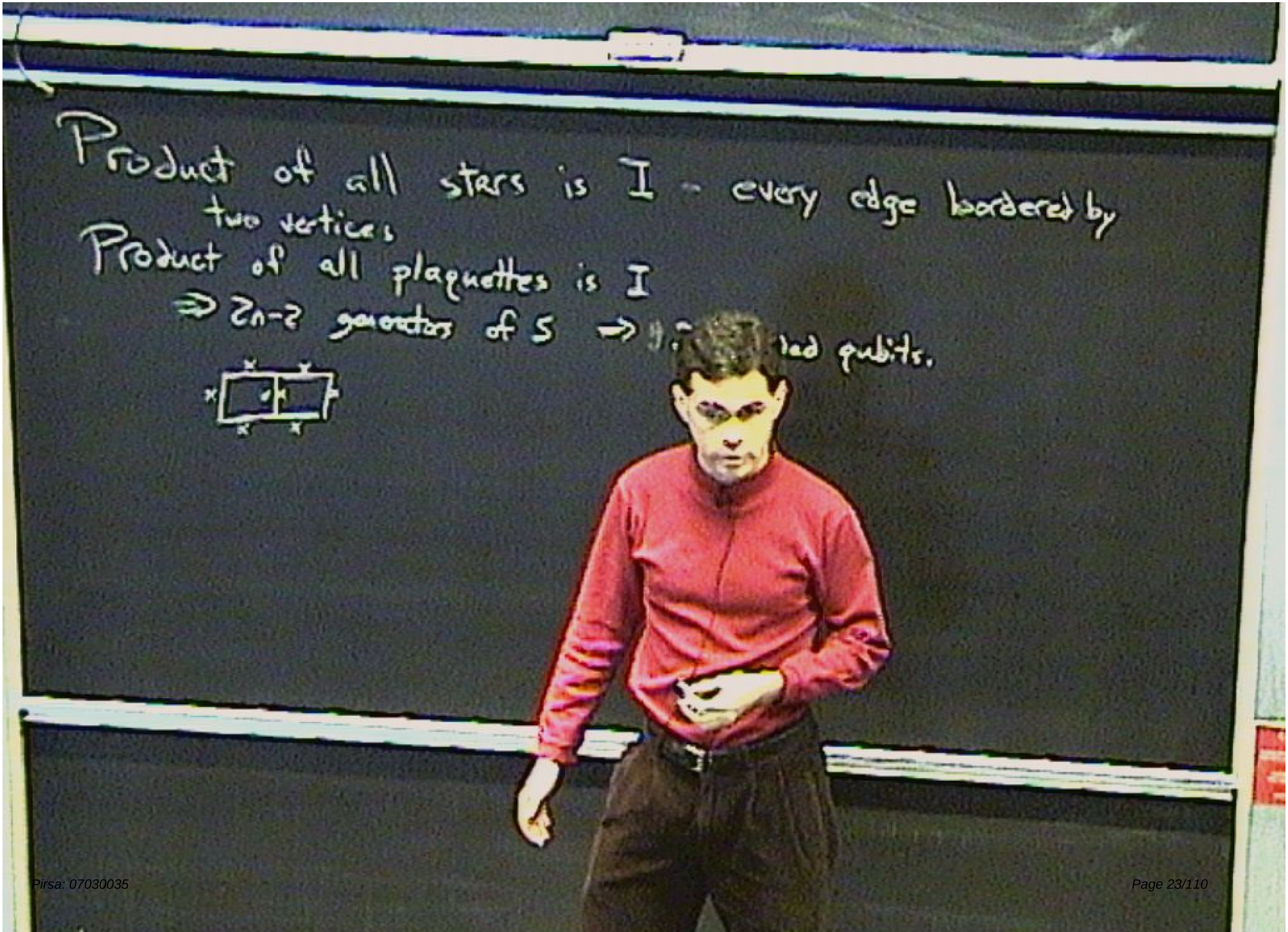
Product of all stars is I - every edge bordered by two vertices

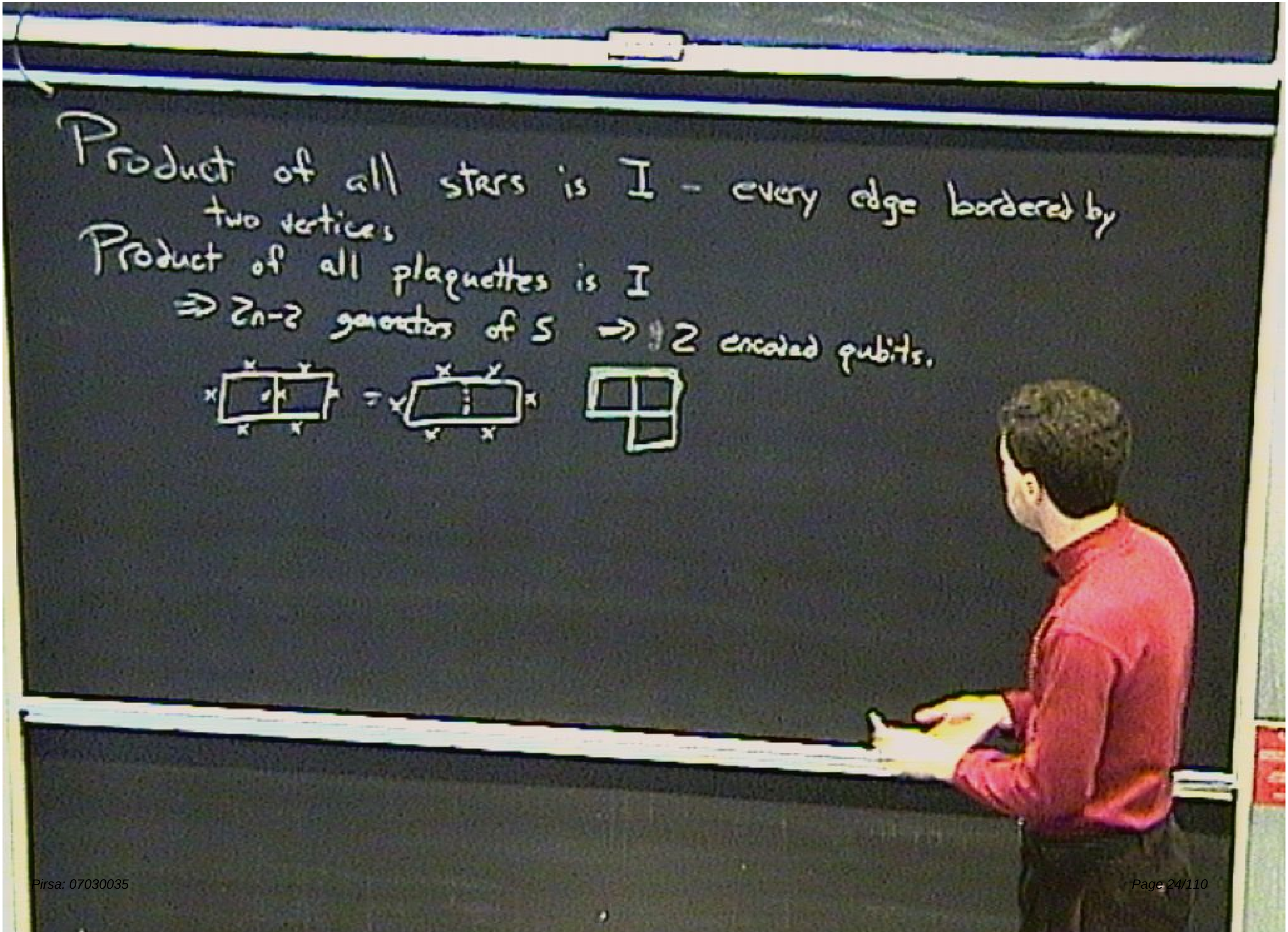
Product of all plaquettes is I

Product of all stars is I - every edge bordered by two vertices

Product of all plaquettes is I

$\Rightarrow 2n-2$ generators of S $\Rightarrow$ 2 encoded qubits.


Product of all stars is I – every edge bordered by two vertices
Product of all plaquettes is I
⇒ 2n-2 generators of S ⇒
ded qubits.


Product of all stars is I – every edge bordered by two vertices
Product of all plaquettes is I
⇒ 2n-2 generators of S ⇒ 2 encoded qubits.

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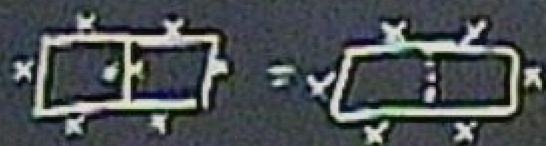
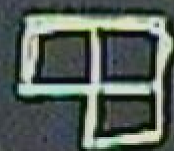
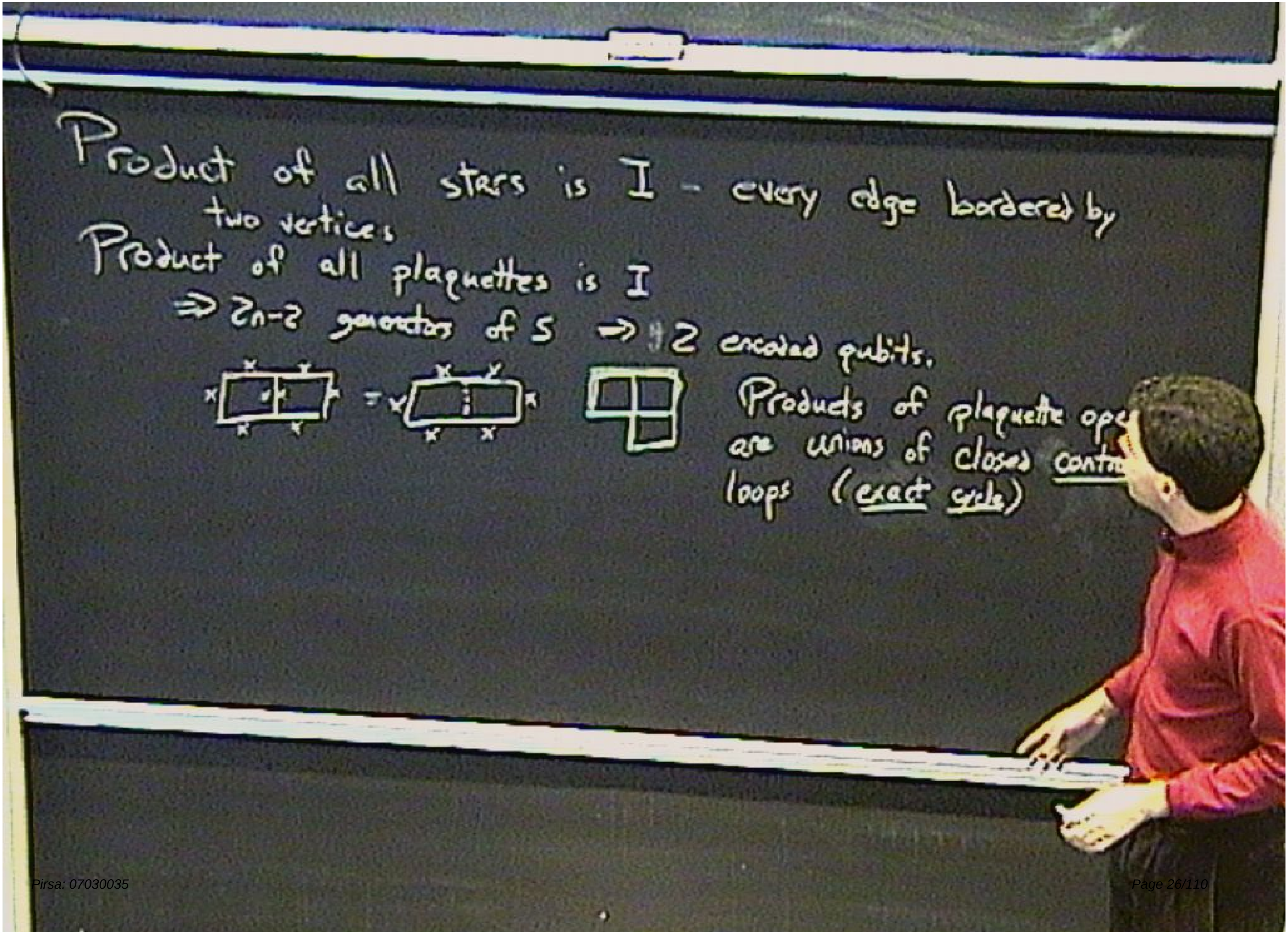




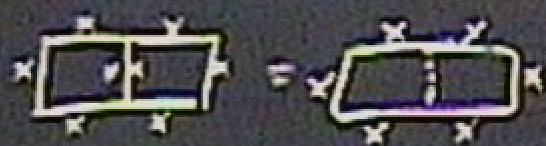
Products of plaquette operators are unions of closed loops

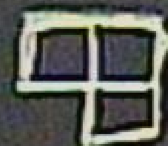

Product of all stars is I - every edge bordered by two vertices
Product of all plaquettes is I
⇒ 2n-2 generators of S ⇒ 2 encoded qubits.
Products of plaquette op
are unions of closed cont
loops (exact cycle)

Product of all stars is I – every edge bordered by two vertices.

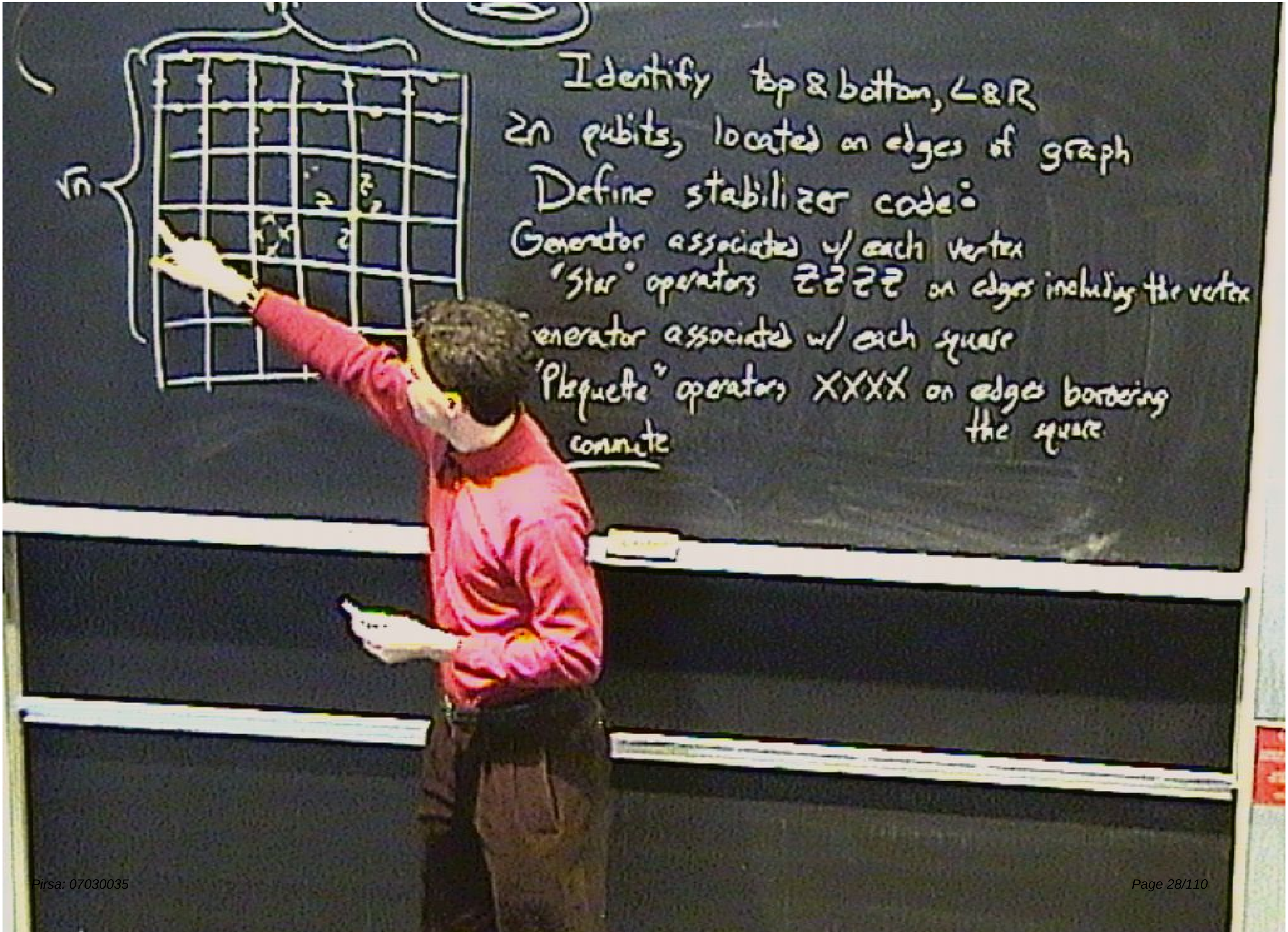
Product of all plaquettes is I

$\Rightarrow 2n-2$ generators of S $\rightarrow$ 2 encoded qubits.





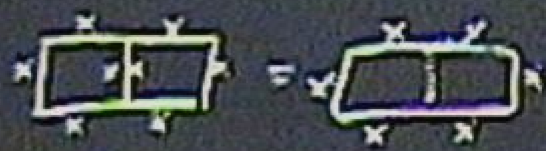
Products of plaquette operators are unions of closed contractible loops (exact cycle)

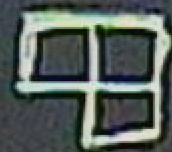

√n
Identify top & bottom, L & R
2n qubits, located on edges of graph
Define stabilizer code:
Generator associated w/ each vertex
"Star" operators ZZZZ on edges including the vertex
Generator associated w/ each square
"Plaquette" operators XXXX on edges bordering the square
commute

Product of all stars is I – every edge bordered by two vertices

Product of all plaquettes is I

$\Rightarrow$ $2n-2$ generators of S $\Rightarrow$ 2 encoded qubits.





Products of plaquette operators are unions of closed <u>contractible</u> loops (<u>exact</u> <u>cycle</u>)

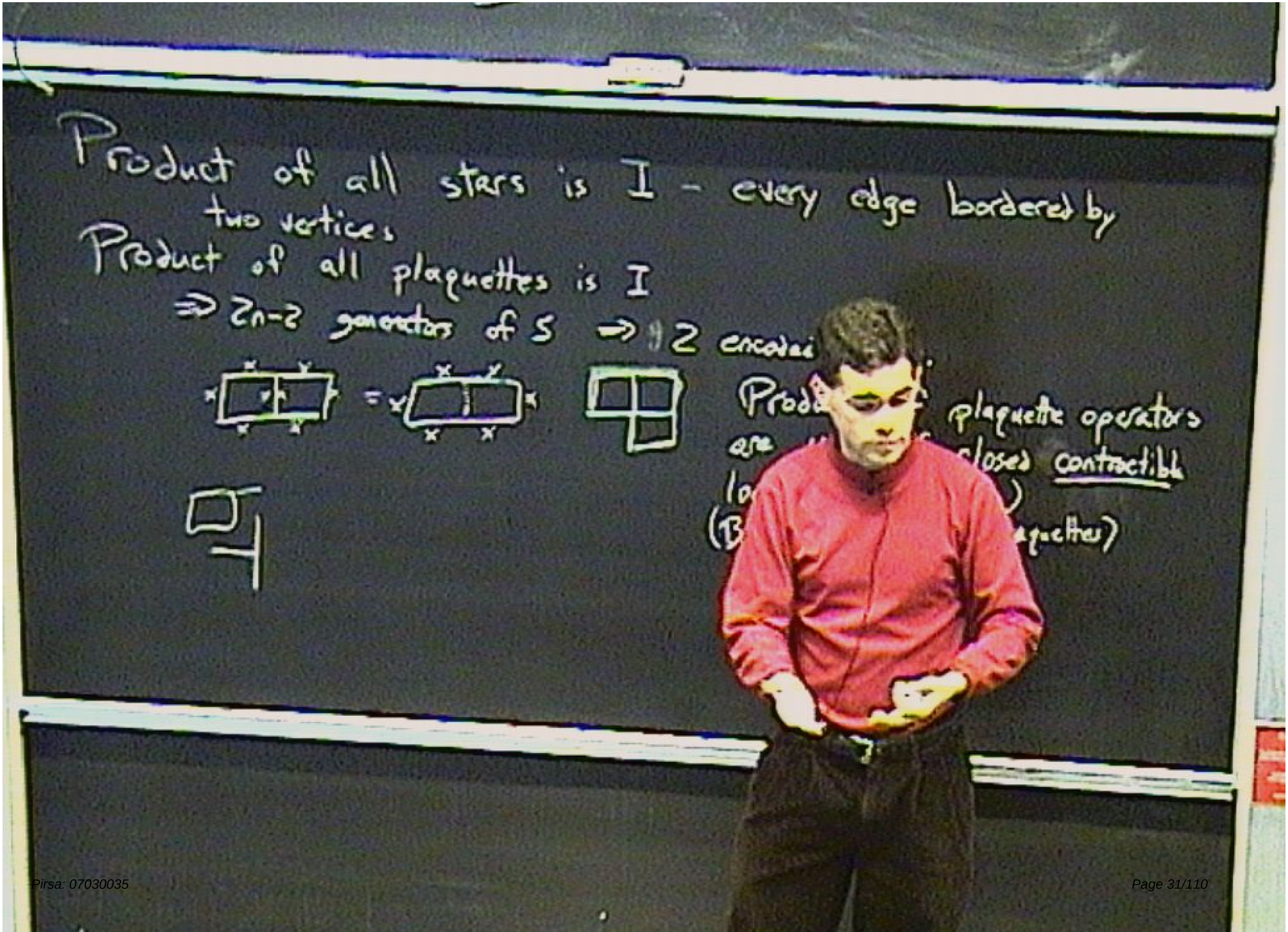
Product of all stars is I – every edge bordered by two vertices

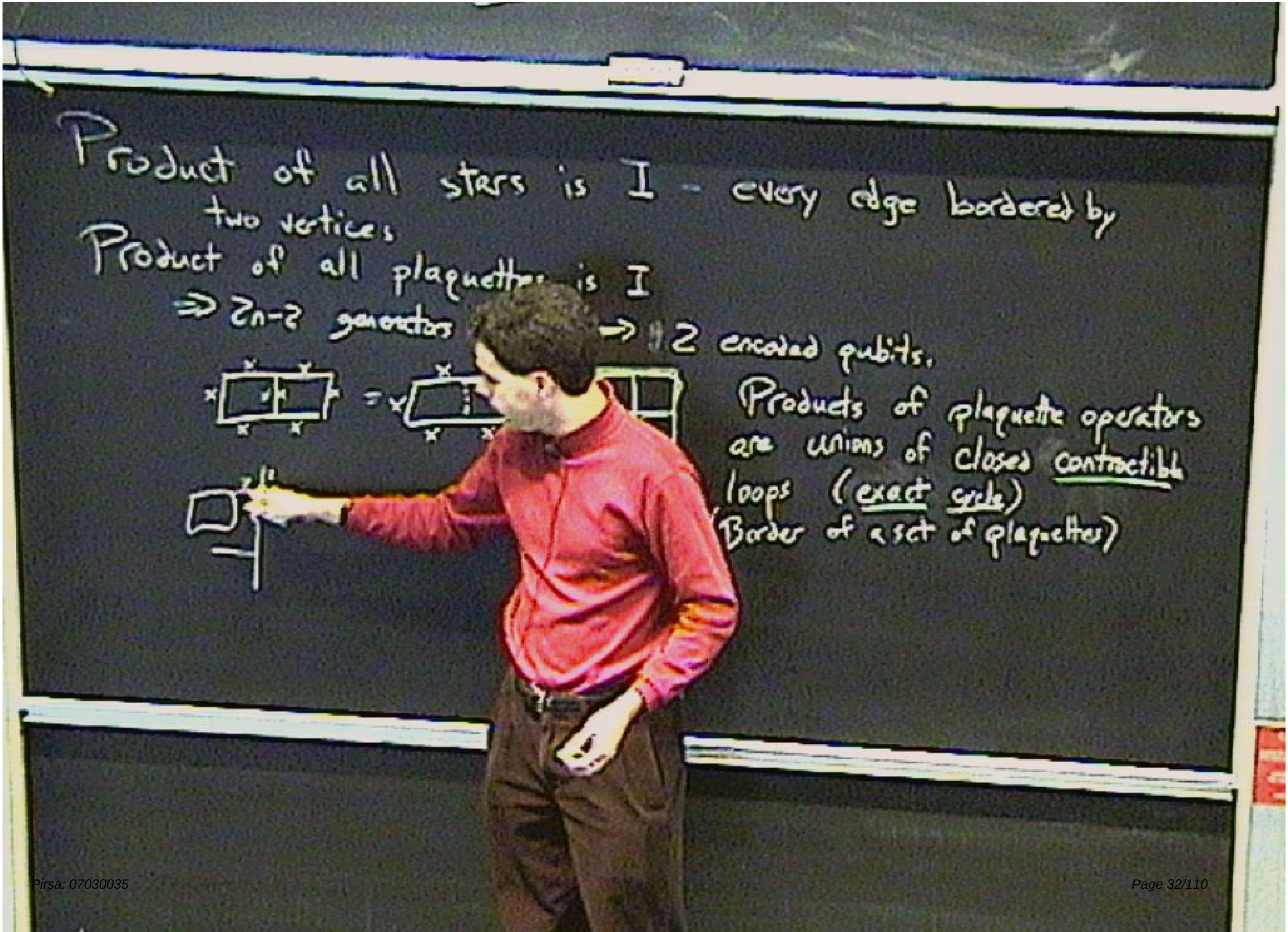
Product of all plaquettes is I

$\Rightarrow$ $2n-2$ generators of S $\Rightarrow$ 2 encoded qubits.

Products of plaquette operators are unions of closed contractible loops (exact cycle)

(Border of a set of plaquettes)


Product of all stars is I – every edge bordered by two vertices
Product of all plaquettes is I
⇒ 2n-2 generators of S → 2 encoded
plaquette operators
closed contractible


Product of all stars is I - every edge bordered by two vertices
Product of all plaquettes is I
⇒ 2n-2 generators
⇒ 2 encoded qubits.
Products of plaquette operators are unions of closed contractible loops (exact cycle)
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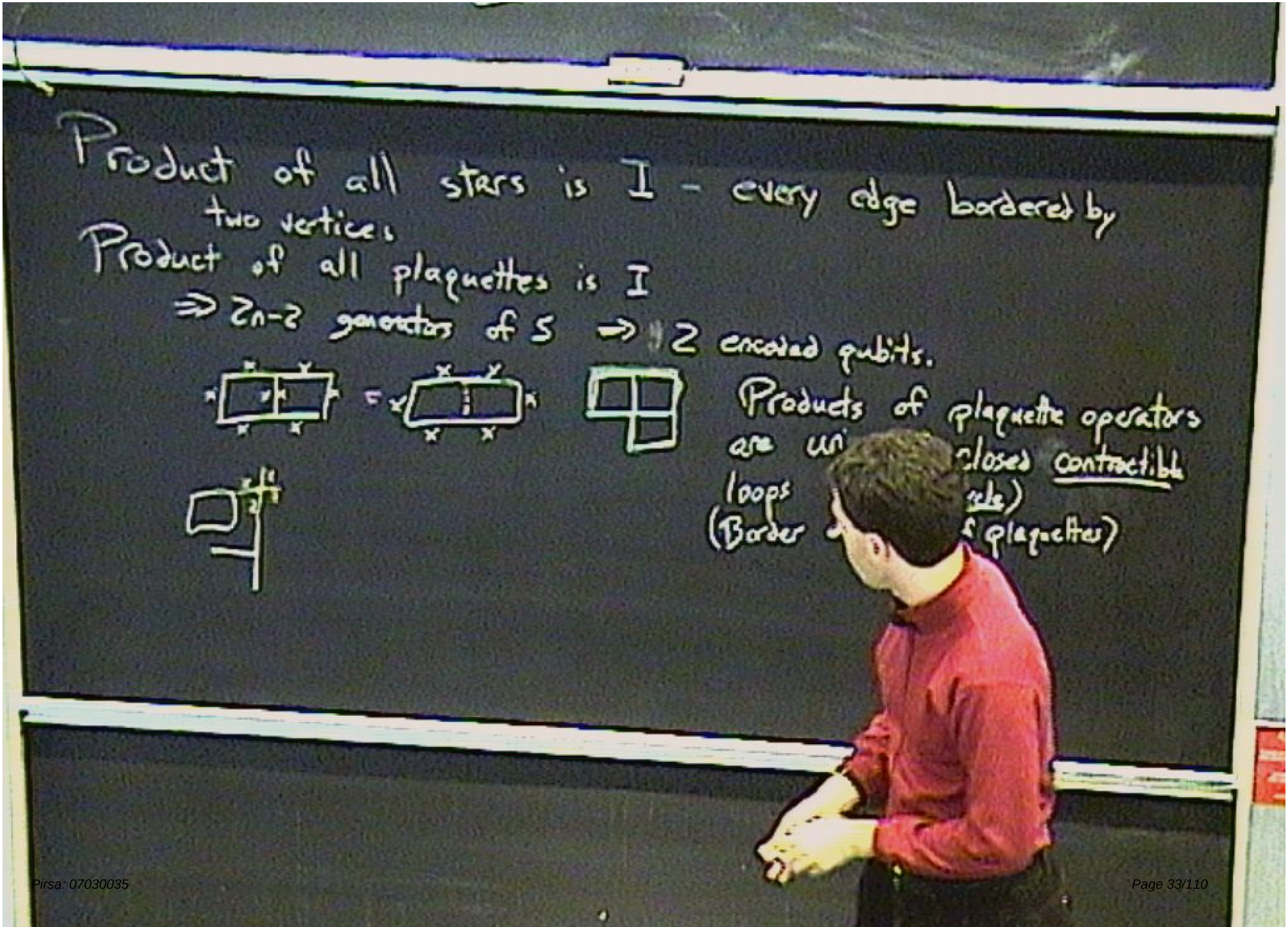
Product of all stars is I - every edge bordered by two vertices
Product of all plaquettes is I
⇒ 2n-2 generators of S ⇒ 2 encoded qubits.
Products of plaquette operators
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(Border
plaquettes)

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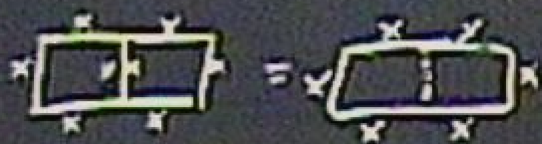
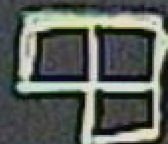
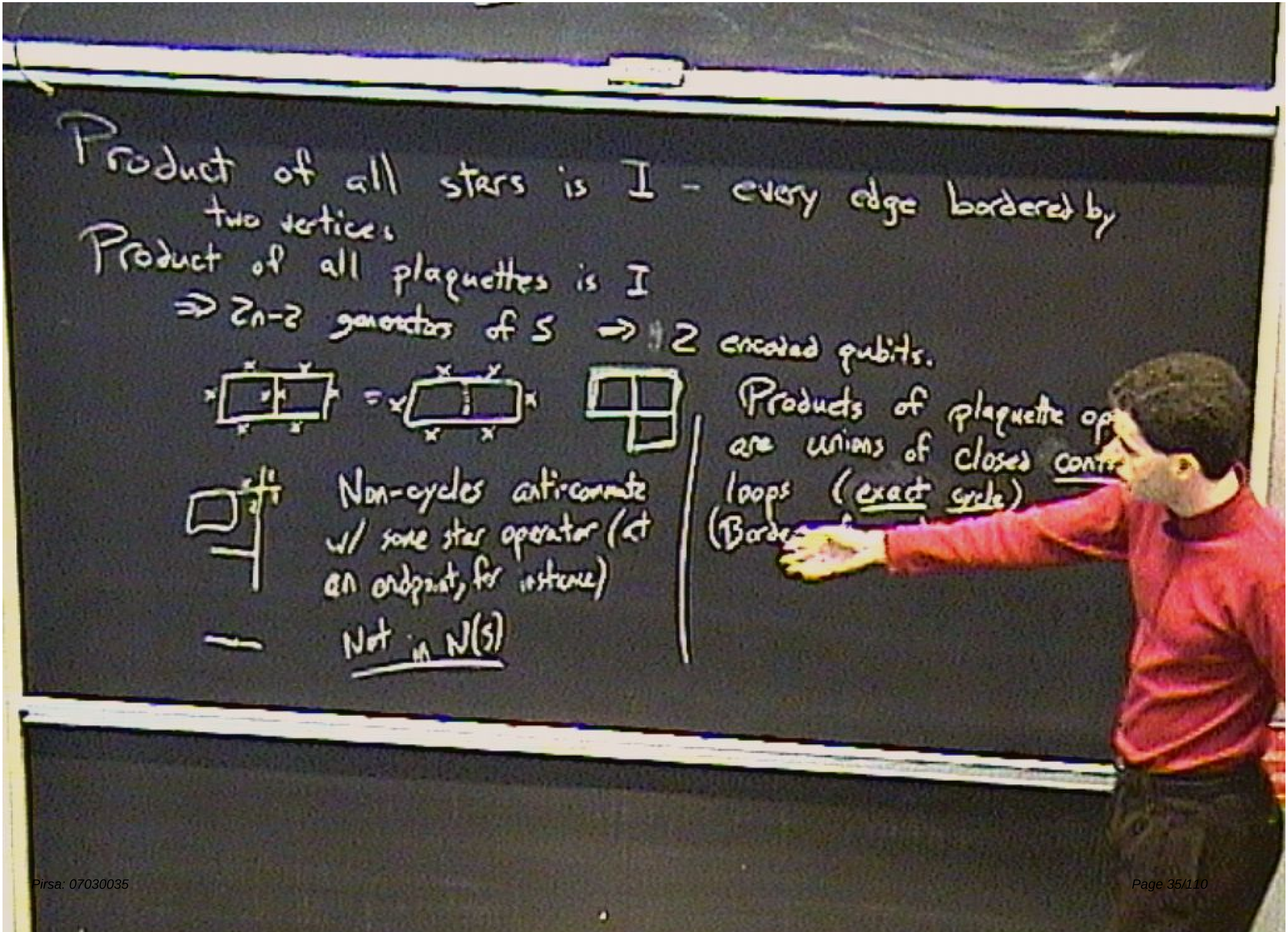
$\Rightarrow$ $2n-2$ generators of S $\Rightarrow$ 2 encoded qubits.





Non-cycles anticommute w/ some star operator (at an endpoint, for instance)

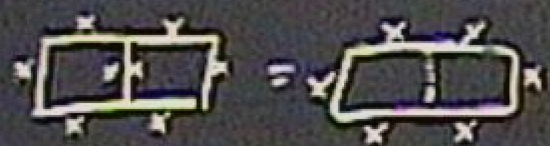
Products of plaquette operators are unions of closed contractible loops (exact cycle)
(Border of a set of plaquettes)

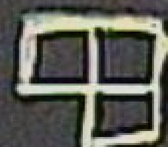

Product of all stars is I - every edge bordered by two vertices
Product of all plaquettes is I
⇒ 2n-2 generators of S → 2 encoded qubits.
Products of plaquette op
are unions of closed cont
loops (exact cycle)
Non-cycles anticommute w/ some star operator (at an endpoint, for instance)
Not in N(S)

Product of all stars is I – every edge bordered by two vertices.

Product of all plaquettes is I

$\Rightarrow$ $2n-2$ generators of S $\Rightarrow$ 2 encoded qubits.

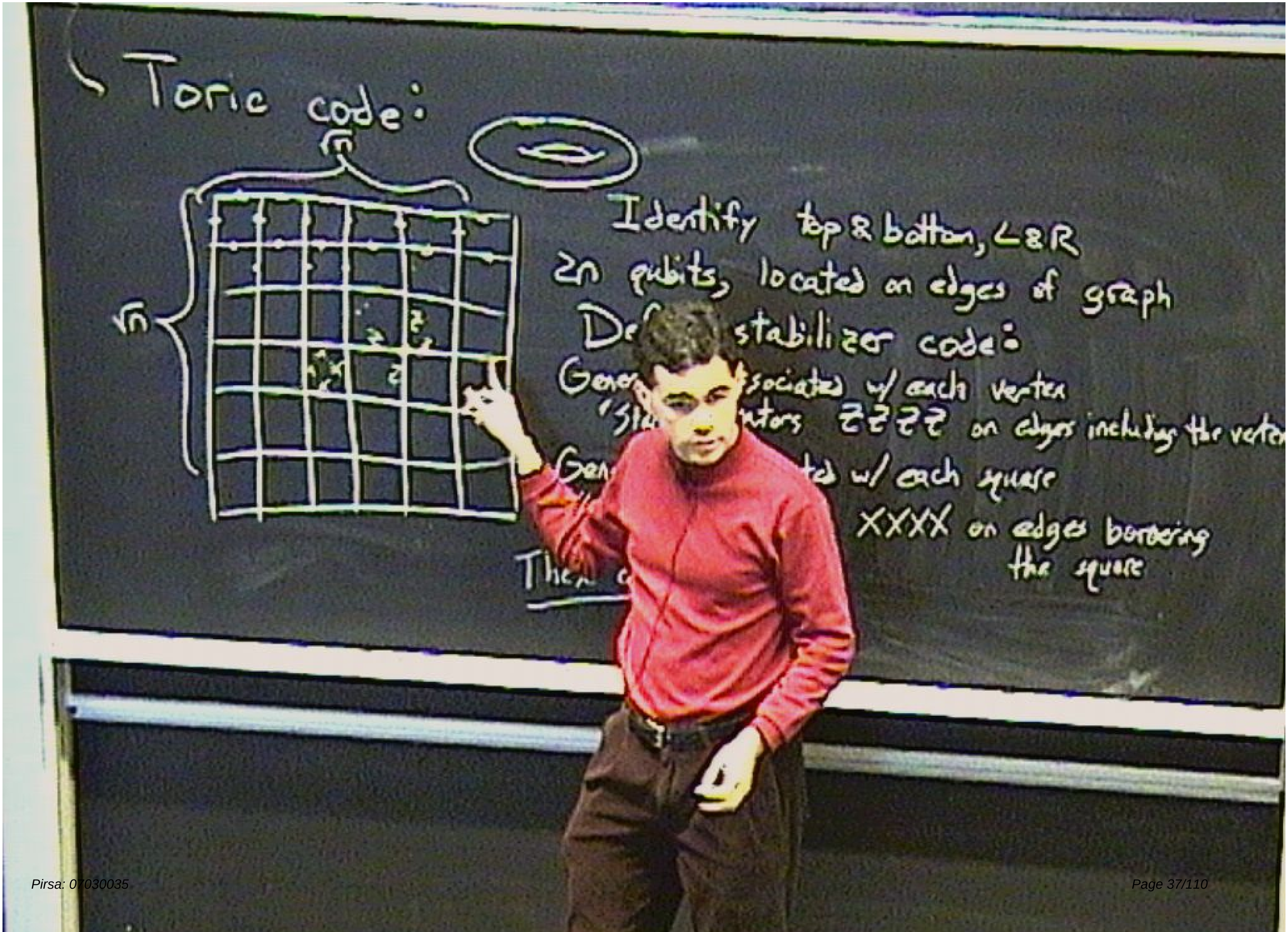


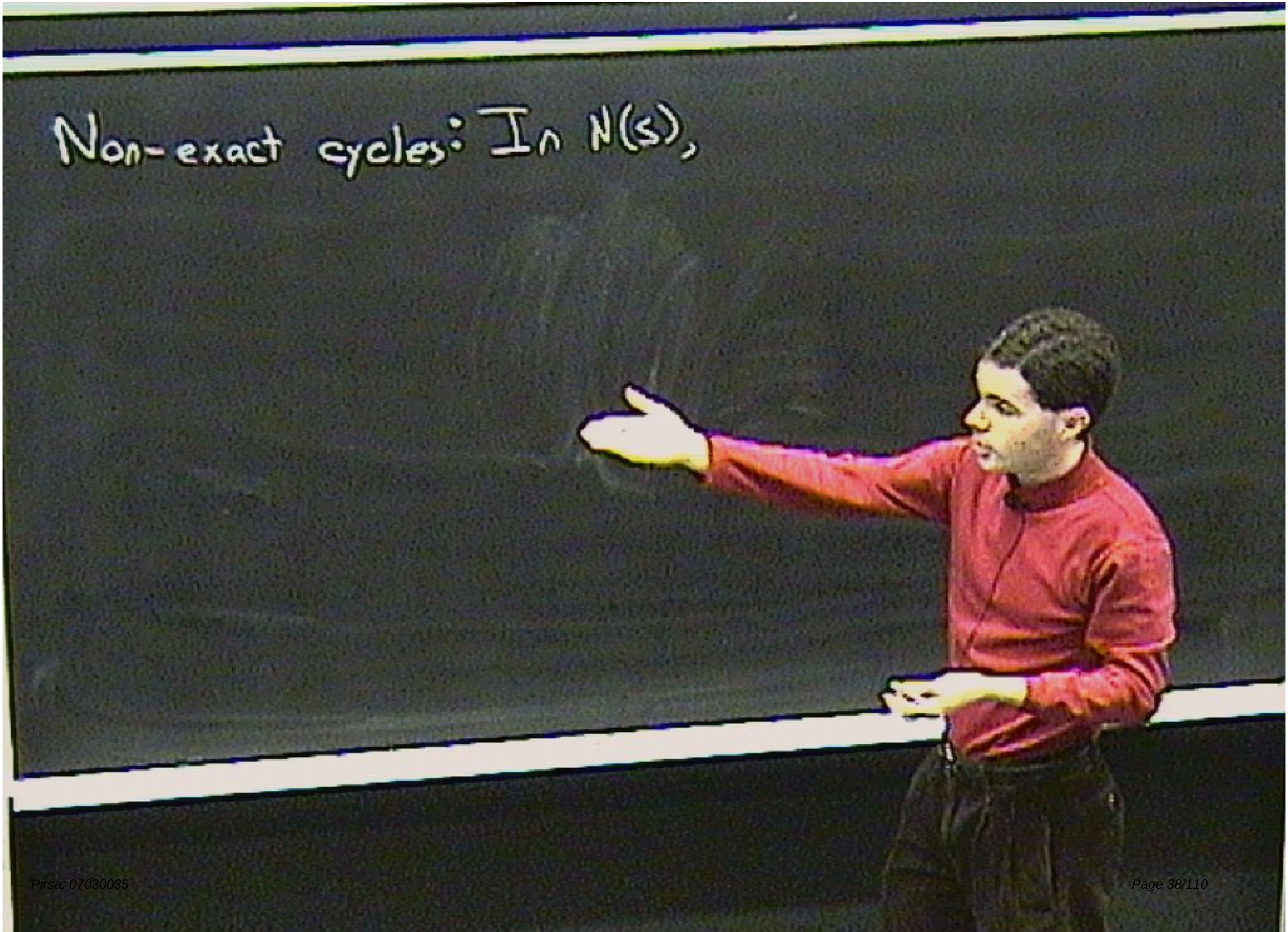


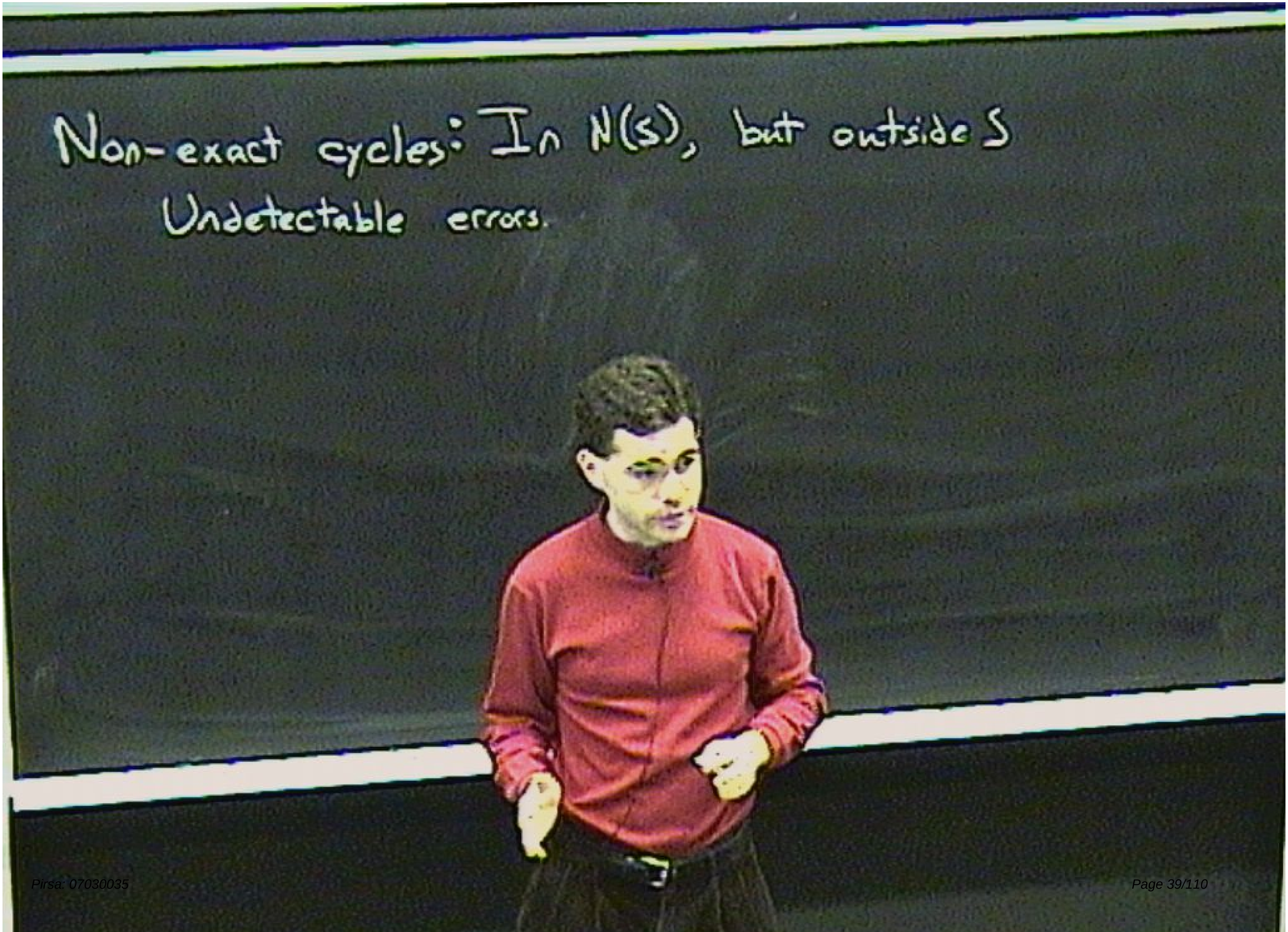
Products of plaquette operators are unions of closed <u>contractible</u> loops (<u>exact</u> <u>cycle</u>) (Border of a set of plaquettes)

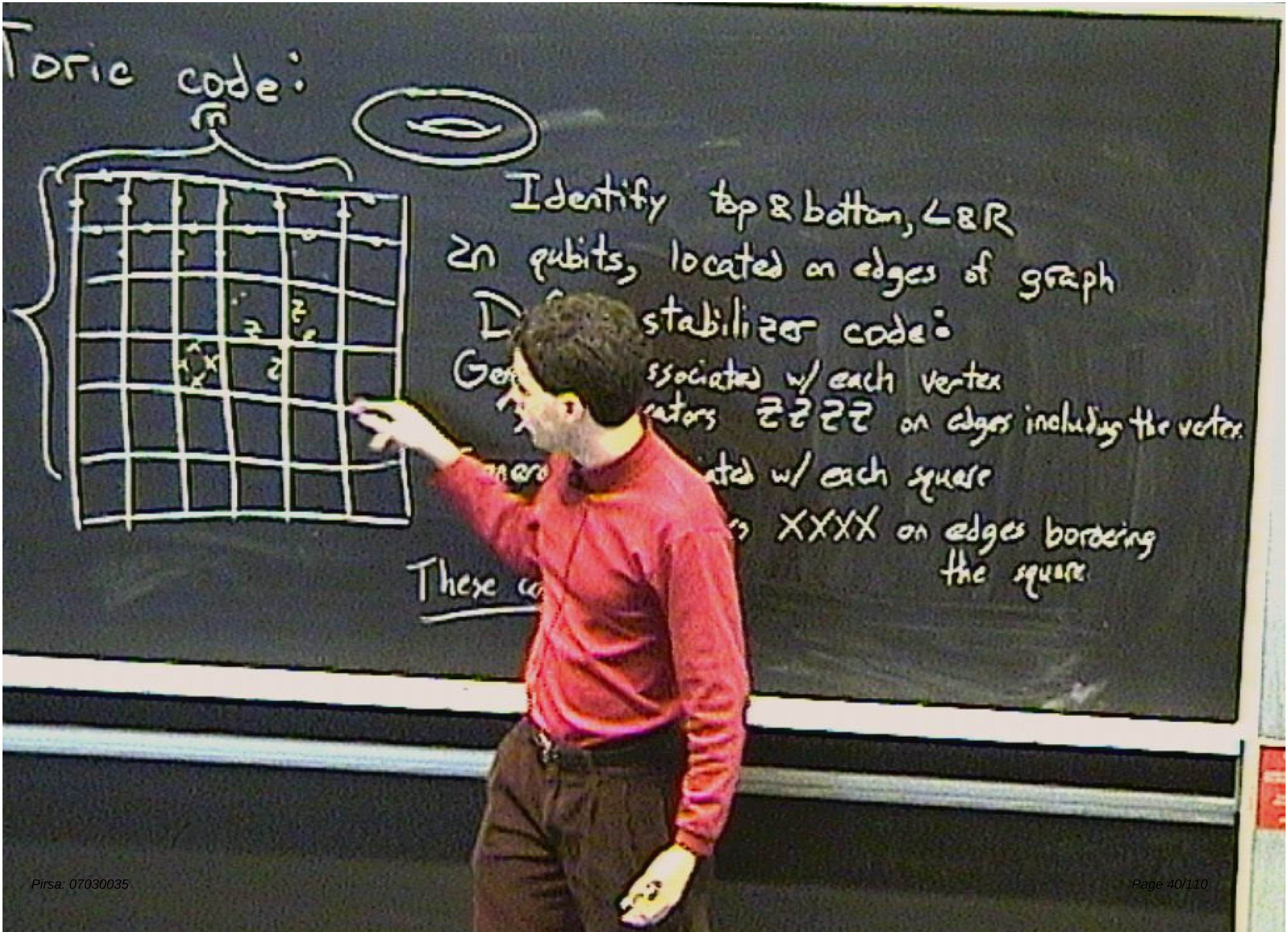
Non-cycles anticommute w/ some star operator (at an endpoint, for instance)

<u>Not in $N(S)$</u>


Toric code:
Identify top & botton, L & R
2n qubits, located on edges of graph
stabilizer code:
w/ each vertex
ZZZZ on edges including the vertex
w/ each square
XXXX on edges bordering
the square


Non-exact cycles: In N(s),


Non-exact cycles: In N(S), but outside S
Undetectable errors.



Toric code:
n
Identify top & bottom, L&R
2n qubits, located on edges of graph
stabilizer code:
w/ each vertex
ZZZZ on edges including the vertex
w/ each square
XXXX on edges bordering the square
These

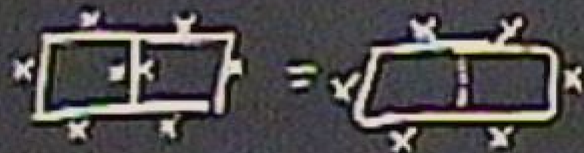
Non-exact cycles: In $N(S)$, but outside S

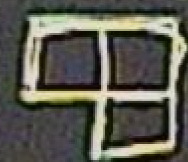
Undetectable errors. – shortest has weight $\sqrt{n}$.

Product of all stars is I – every edge bordered by two vertices

Product of all plaquettes is I

$\Rightarrow$ $2n-2$ generators of S $\Rightarrow$ # 2 encoded qubits.



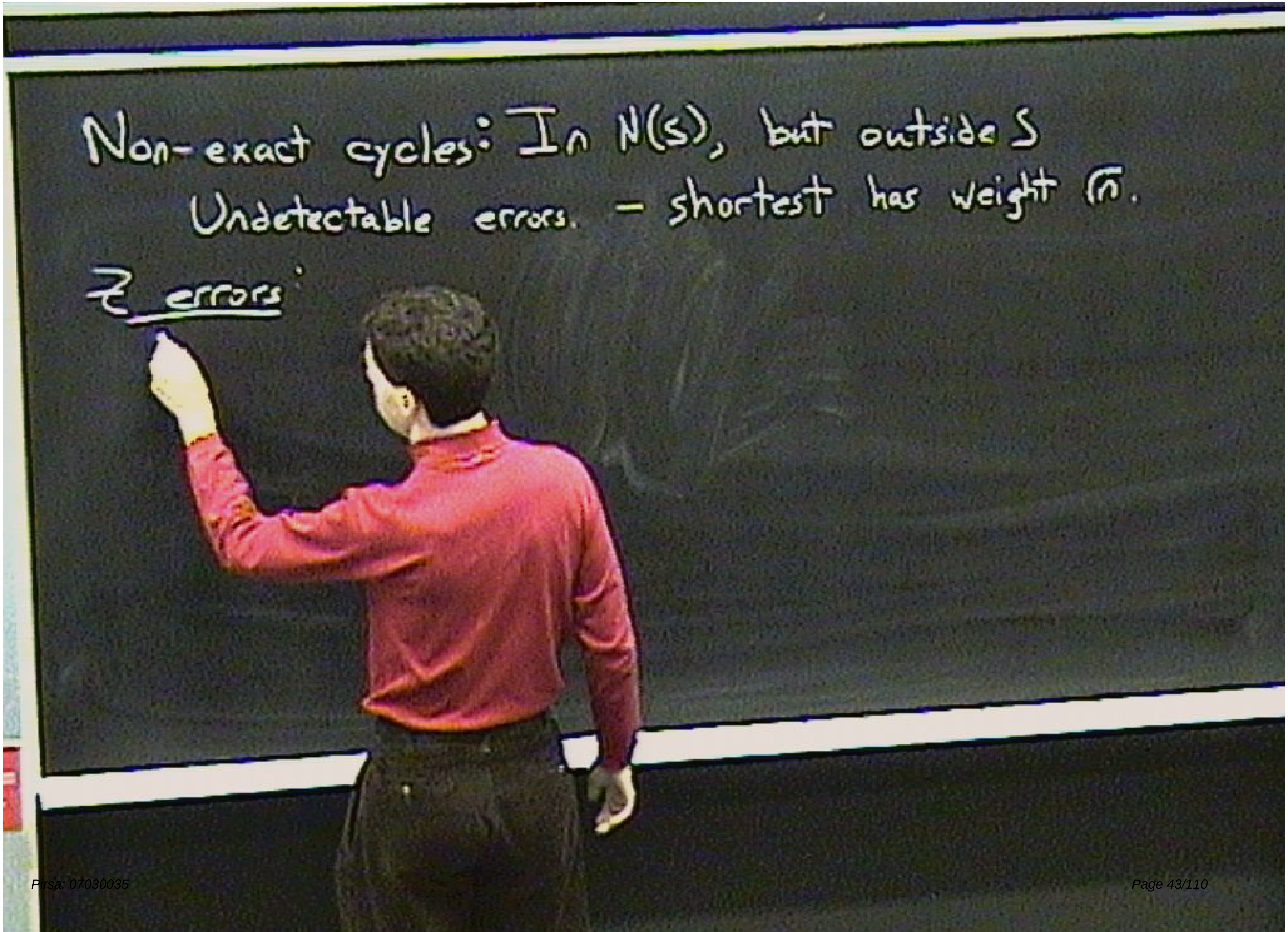


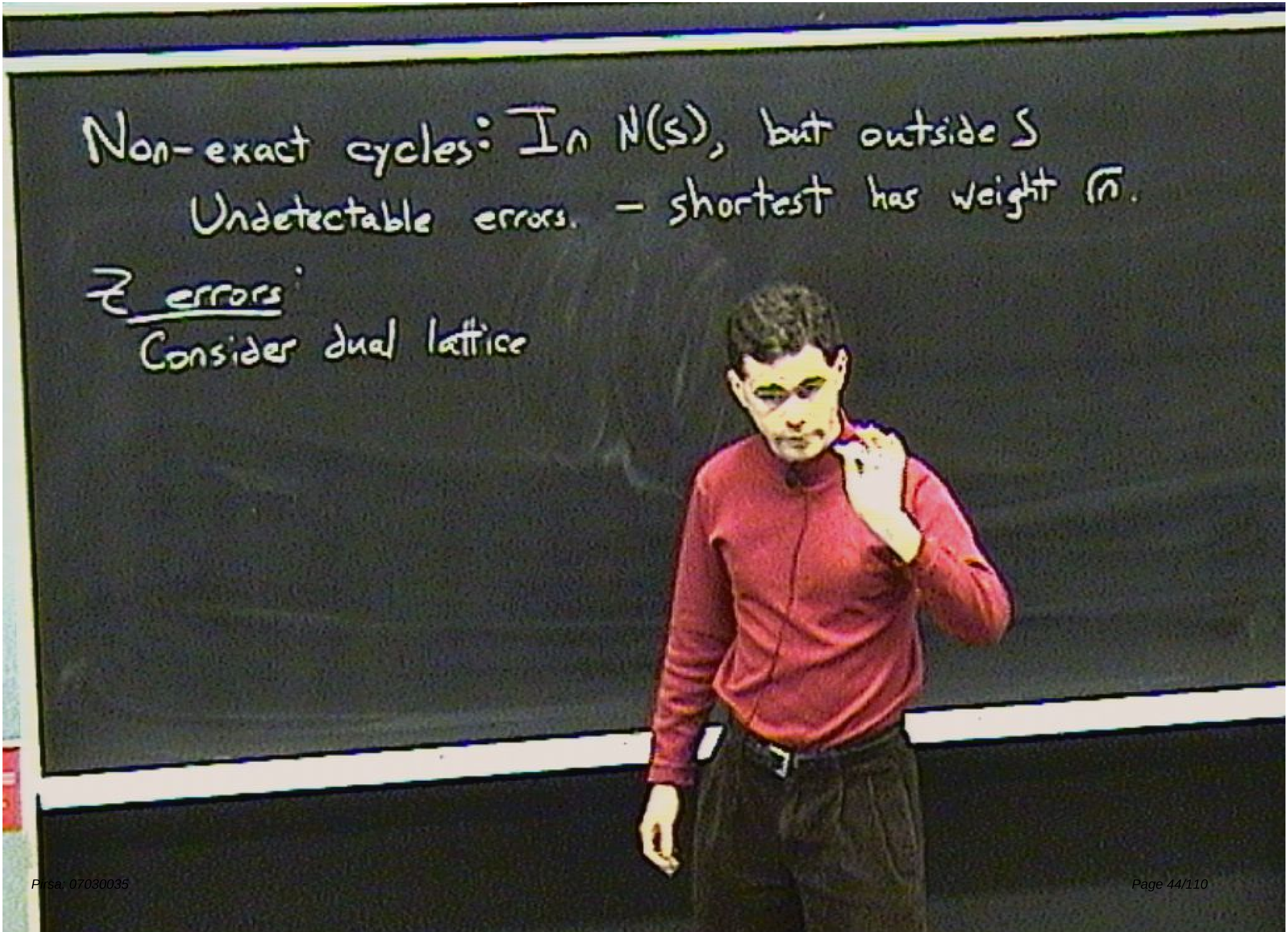
Products of plaquette operators are unions of closed <u>contractible</u> loops (<u>exact</u> <u>cycle</u>)

(Border of a set of plaquettes)

Non-cycles anti-commute w/ some star operator (at an endpoint, for instance)

<u>Not in $N(S)$</u>


Non-exact cycles: In N(S), but outside S
Undetectable errors. – shortest has weight √n.
Z errors:


Non-exact cycles: In N(S), but outside S
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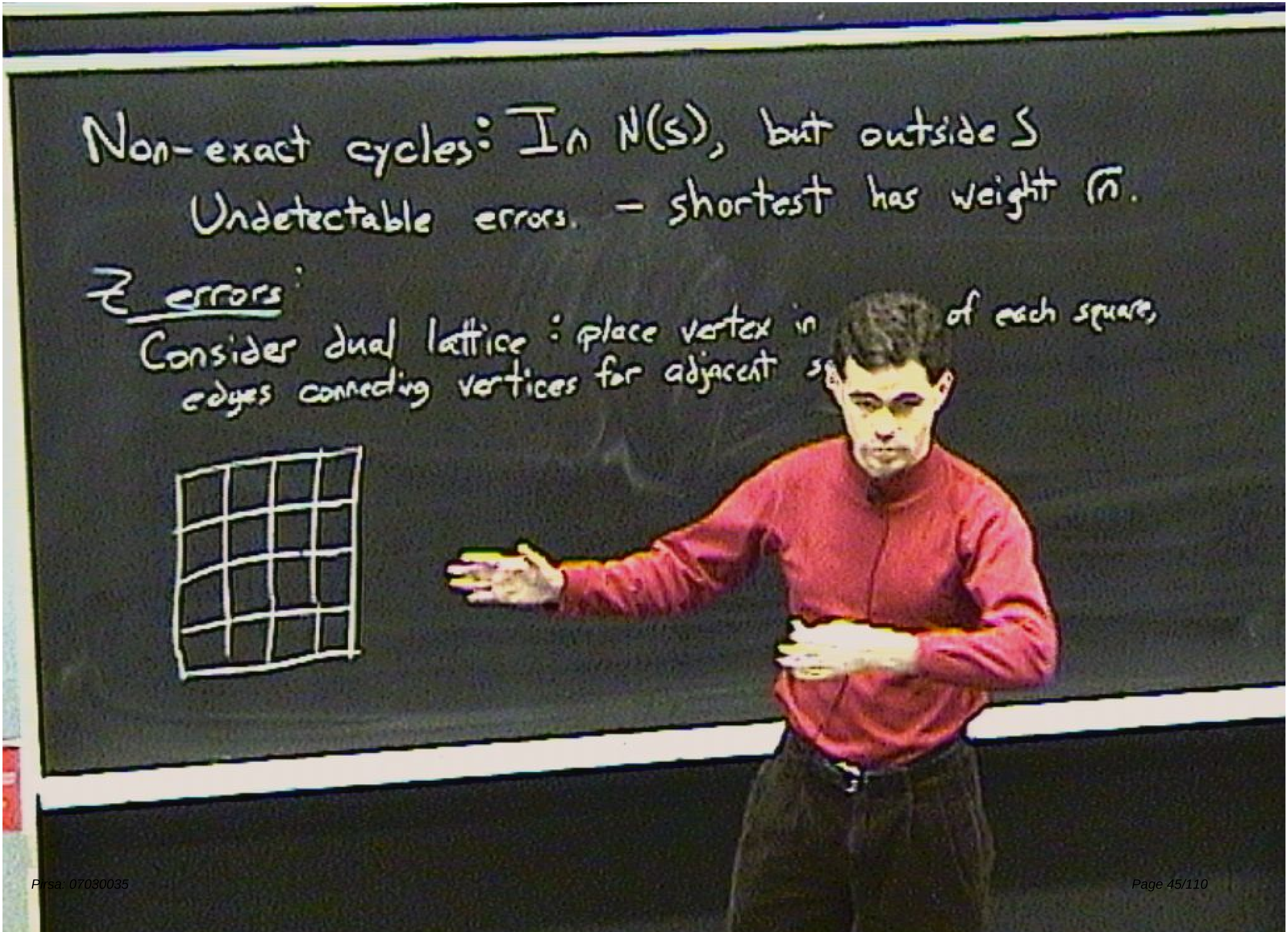
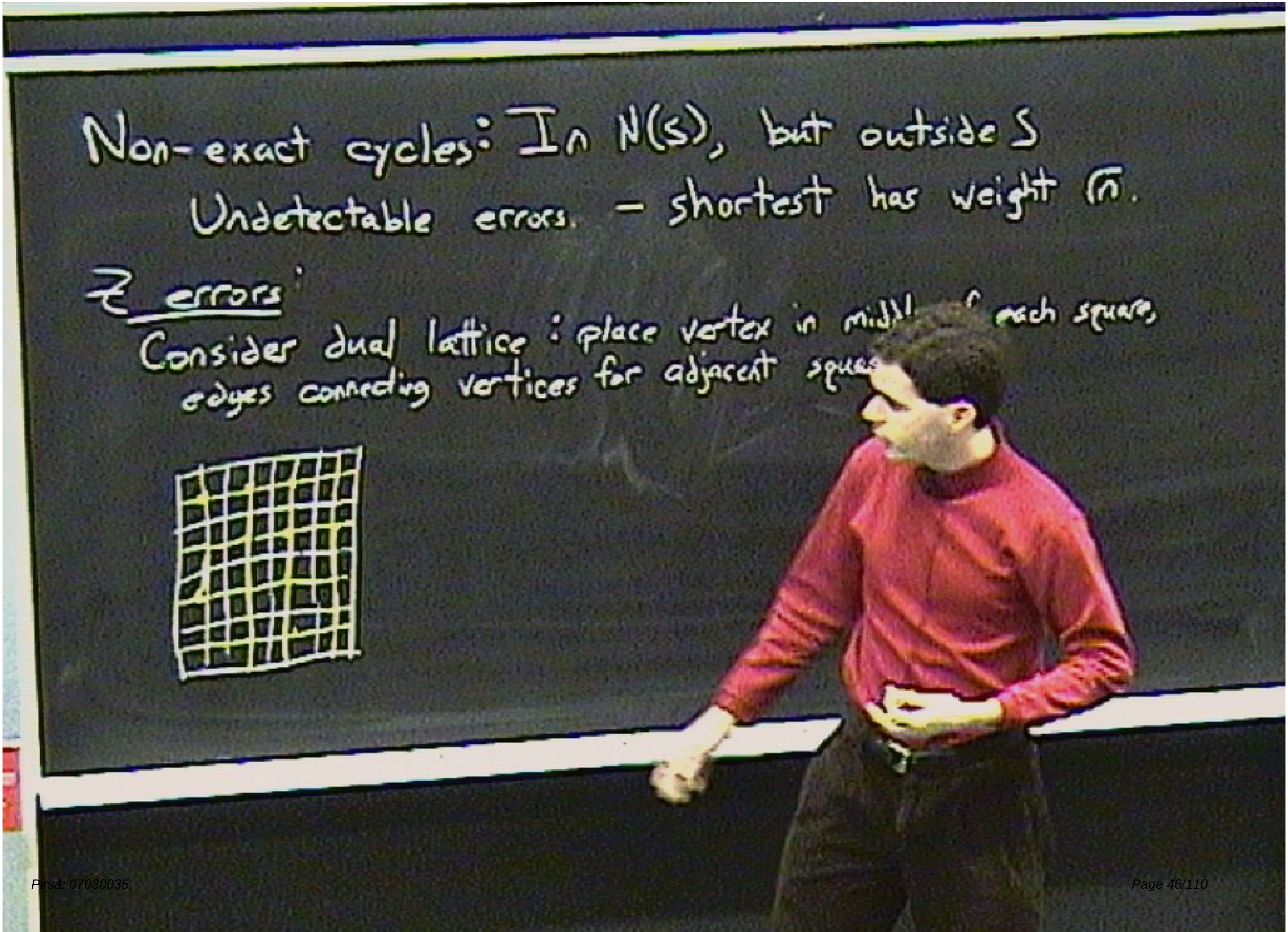
Z errors:
Consider dual lattice


Non-exact cycles: In N(S), but outside S
Undetectable errors. – shortest has weight √n.
Z errors:
Consider dual lattice: place vertex in
of each square,
edges connecting vertices for adjacent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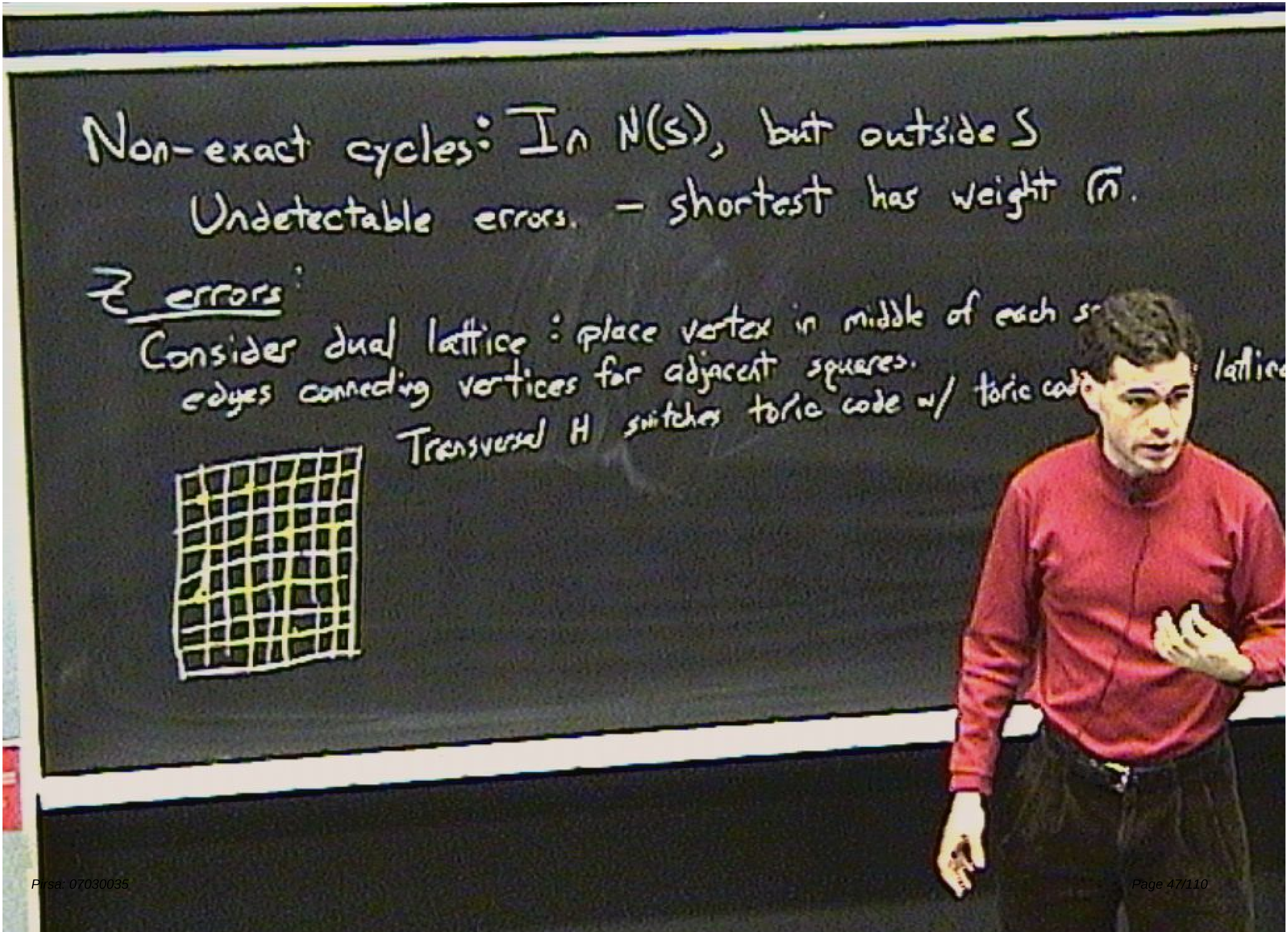
Non-exact cycles: In N(S), but outside S
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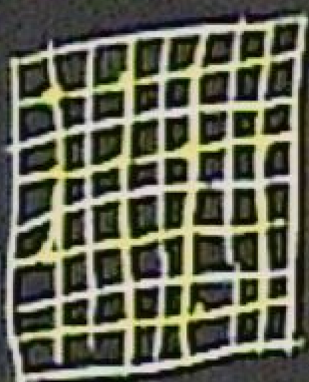
Z errors:
Consider dual lattice: place vertex in middle of each
edges connecting vertices for adjacent squares.
lattice
Transversal H switches toric code w/ toric

Non-exact cycles: In $N(S)$, but outside S

Undetectable errors. – shortest has weight $\sqrt{n}$.

Z errors:

Consider dual lattice: place vertex in middle of each square, edges connecting vertices for adjacent squares.



Transversal H switches toric code w/ toric code on dual lattice

Z stabilizer elements are exact cycles on dual lattice (cocycles)

Undetectable Z errors – non-exact cocycles.

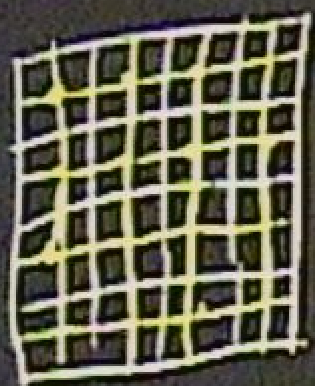
Non-exact cycles: In $N(S)$, but outside S

Undetectable errors. – shortest has weight $\sqrt{n}$.

<u>Z errors</u>:

Consider dual lattice: place vertex in middle of each square, edges connecting vertices for adjacent squares.

Transversal H switches toric code w/ toric code on dual lattice



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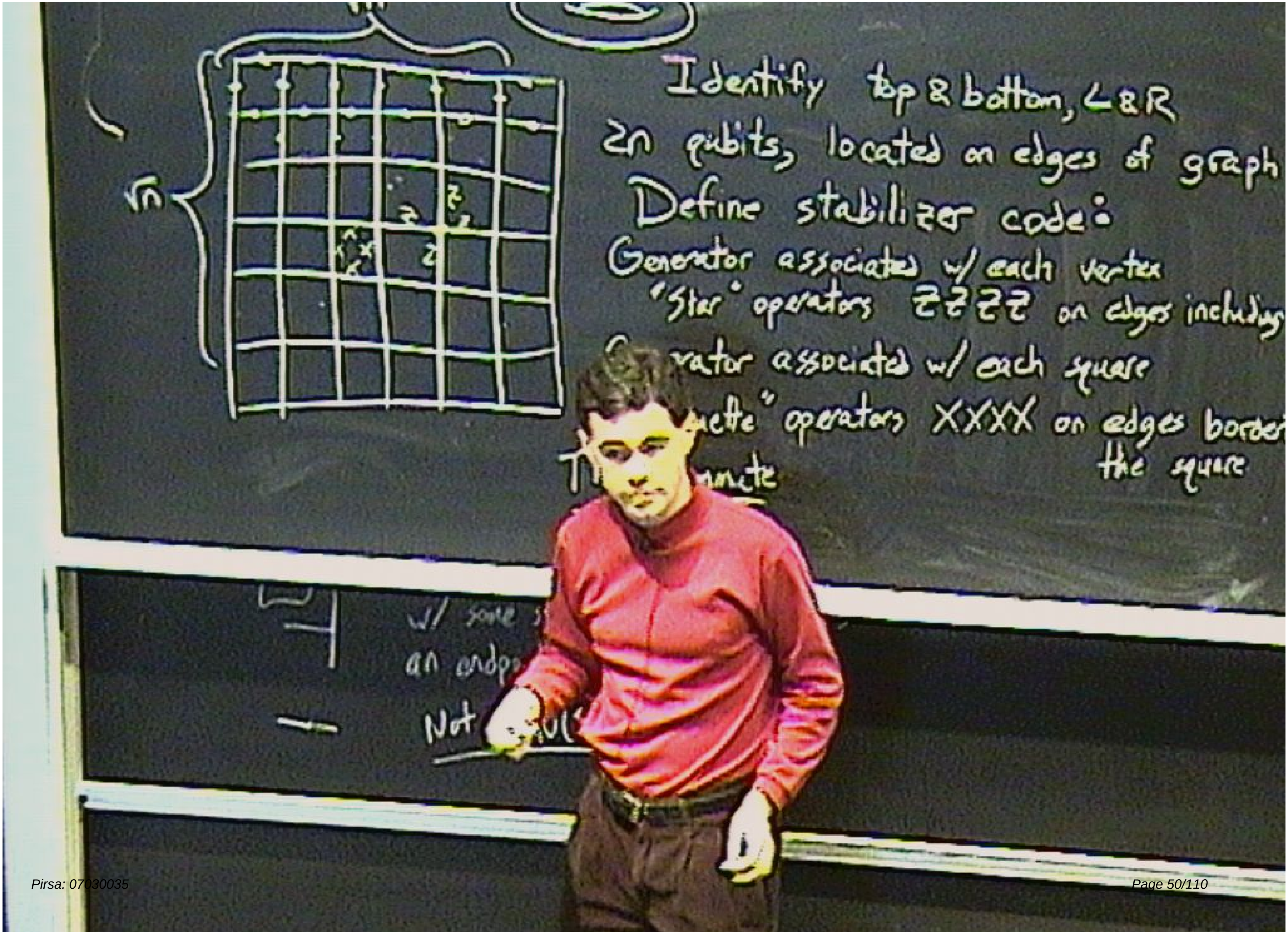
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mute
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an endp
Not

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Transversal H switches toric code w/ toric code on dual lattice

Z stabilizer elements are exact cycles on dual lattice (cocycles)

Undetectable Z errors – non-exact cocycles.

$\Rightarrow$ Code is $[[2n, 2, \sqrt{n}]]$.

Non-exact cycles: In $N(S)$, but outside S

Undetectable errors. – shortest has weight $\sqrt{n}$.

Z errors:

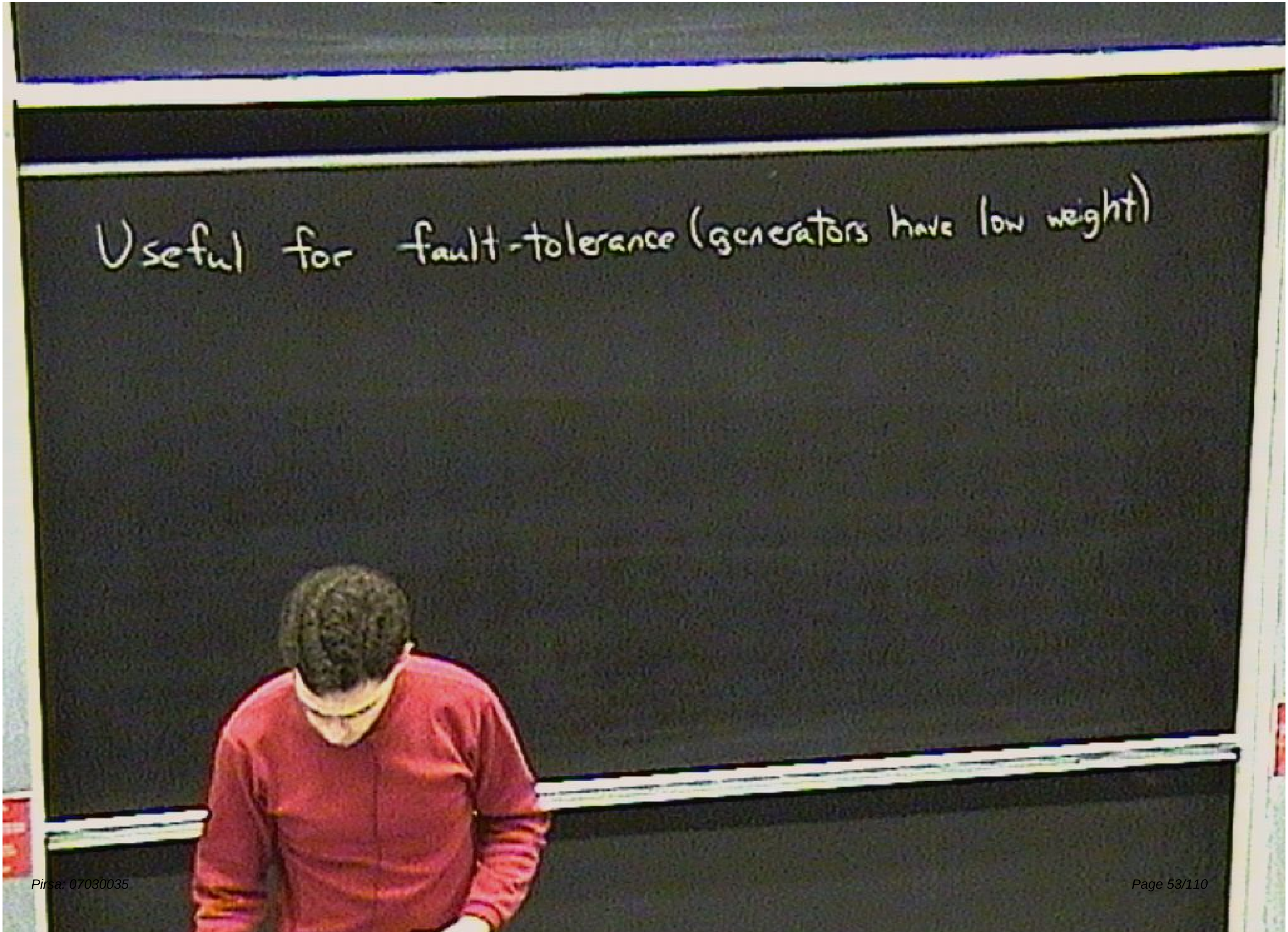
Consider dual lattice: place vertex in middle of each square, edges connecting vertices for adjacent squares.

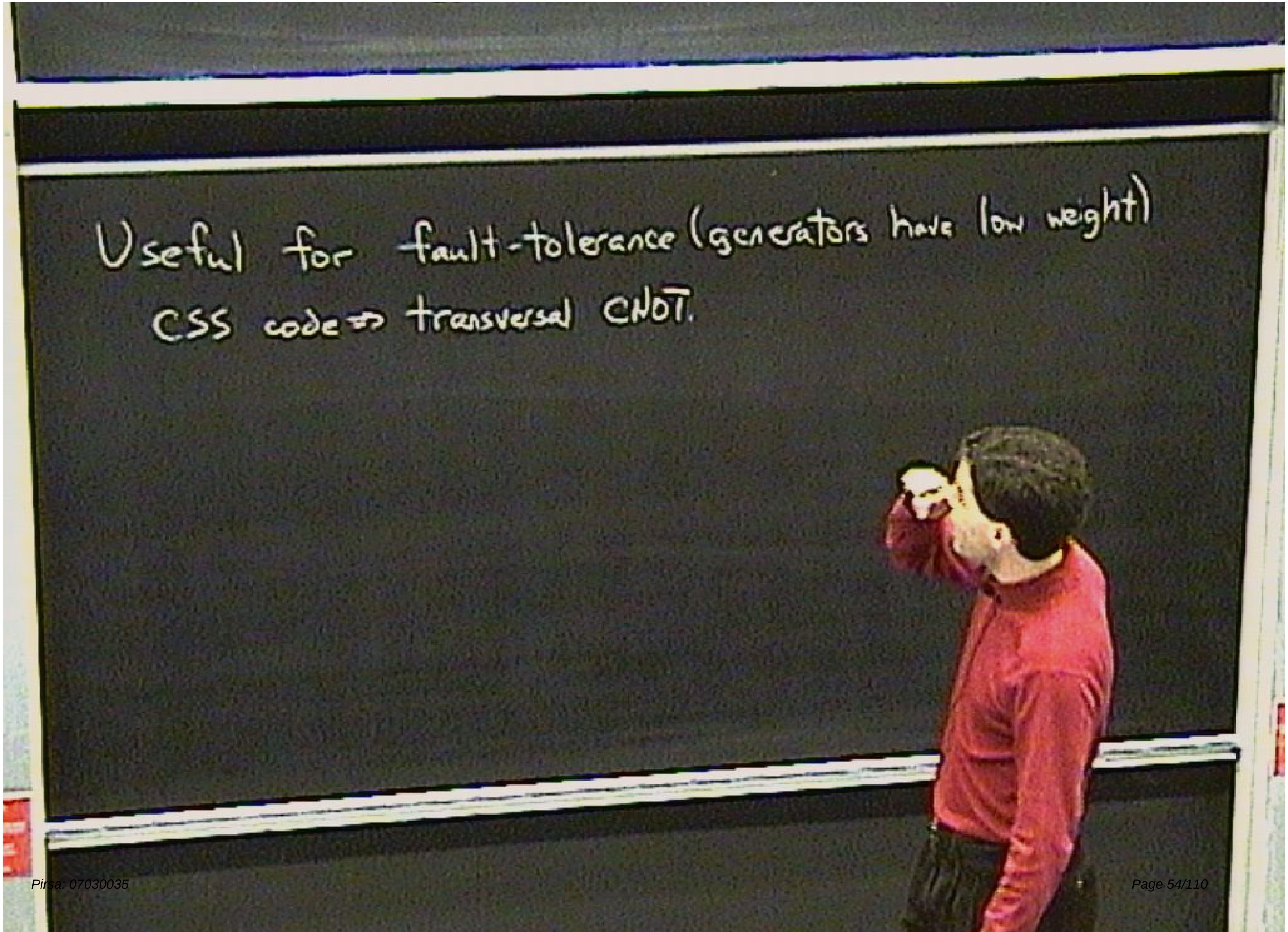
Transversal H switches toric code w/ toric code on dual lattice

Z stabilizer elements are exact cycles on dual lattice (cocycles)

Undetectable Z errors – non-exact cocycles.

$\Rightarrow$ Code is $[[2n, 2, \sqrt{n}]]$. (Codewords are associated w/ $\mathbb{Z}_2$ homology of torus.)


Useful for fault-tolerance (generators have low weight)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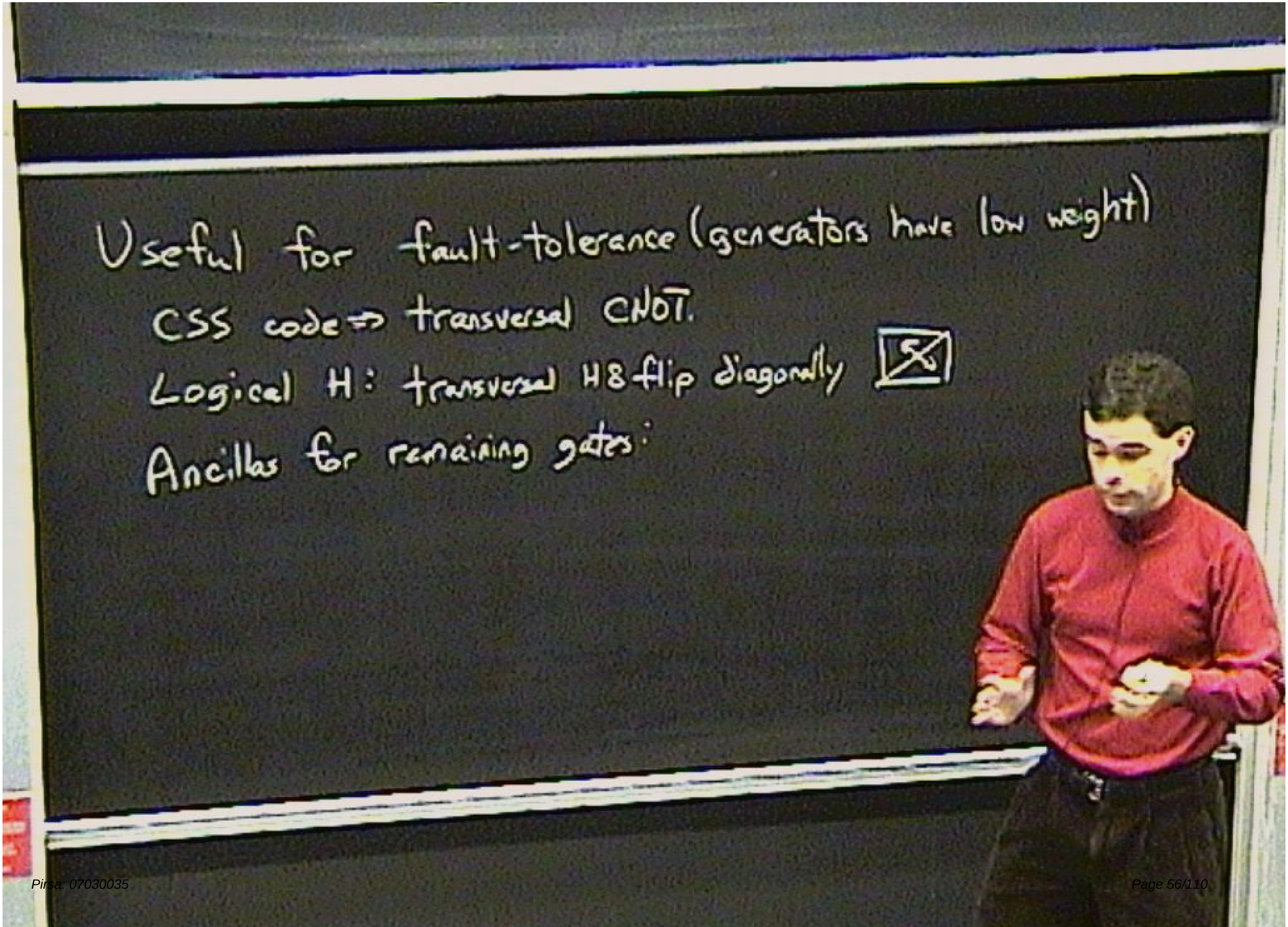
Logical H: transversal H & flip diagonally 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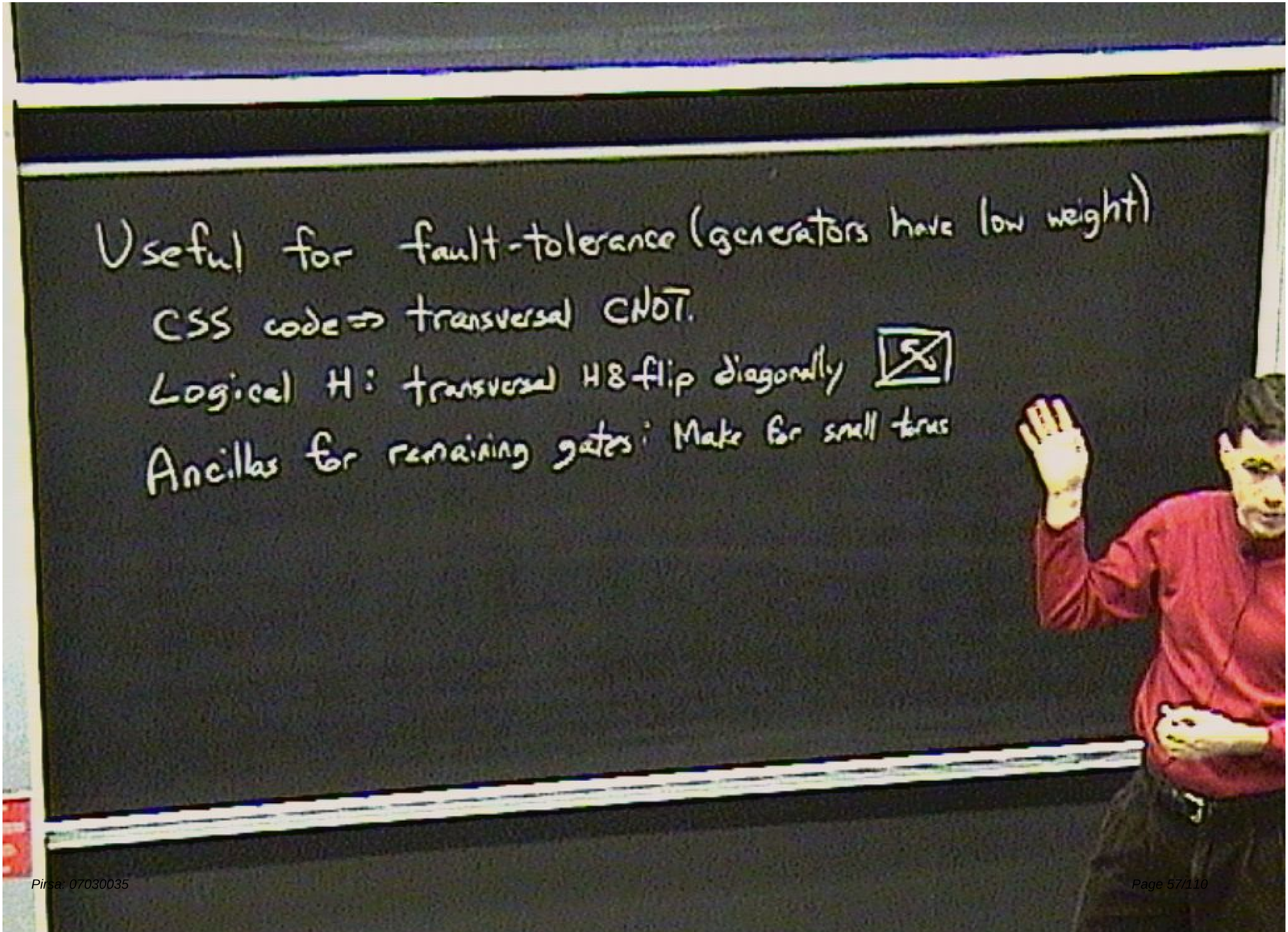
Ancillas for remaining gates: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Grow torus by adding rows & columns of qubits.

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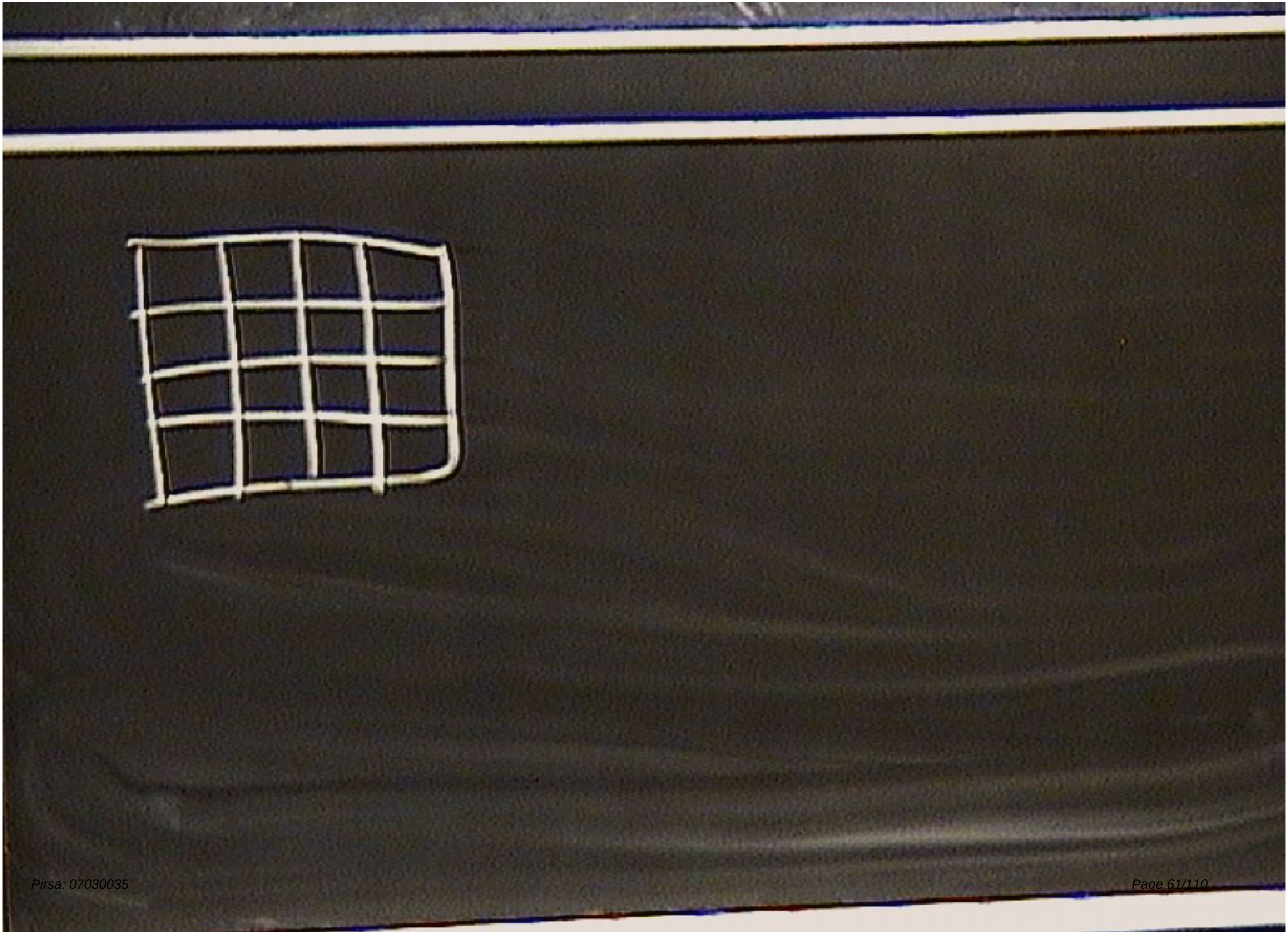
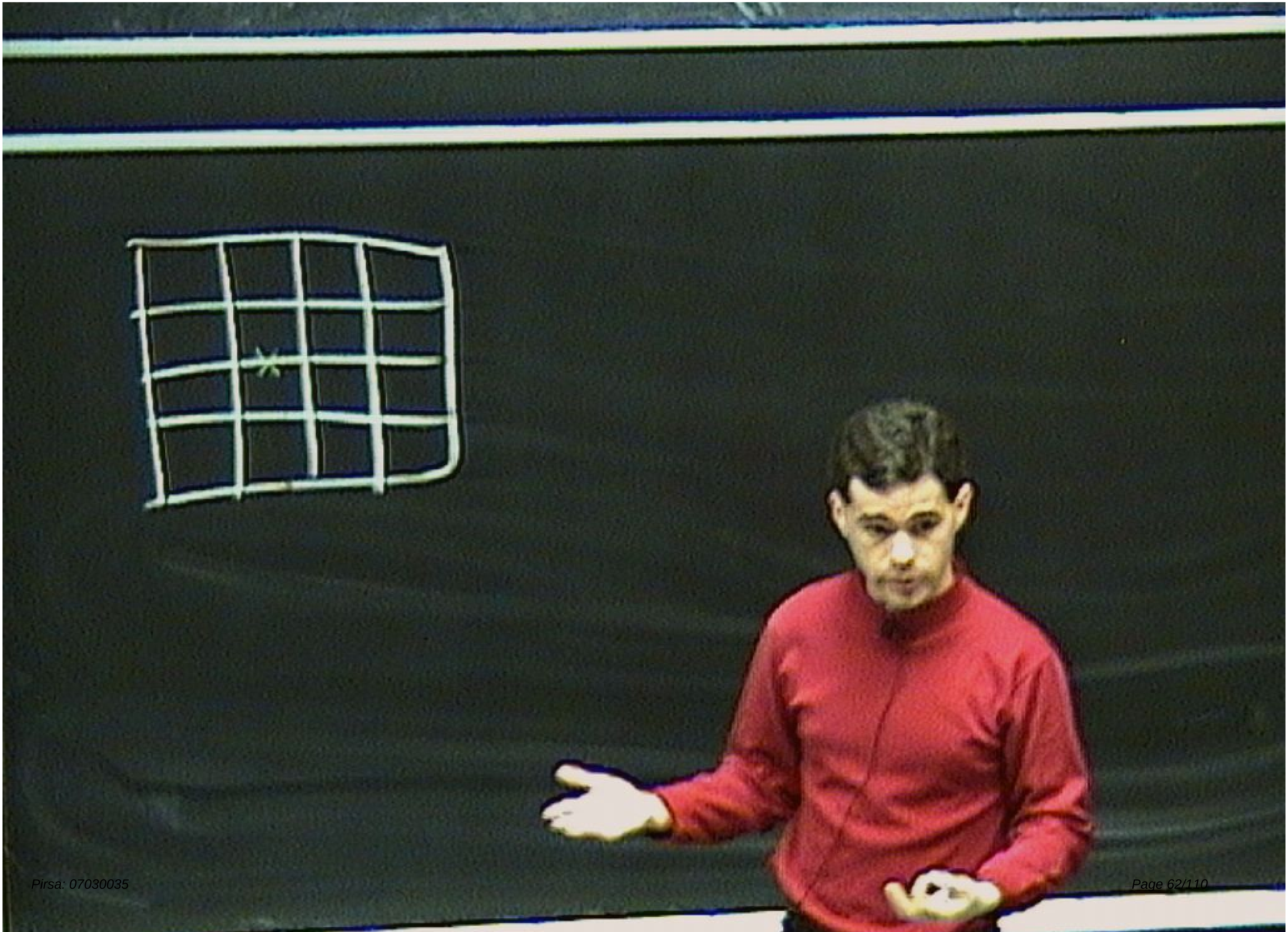
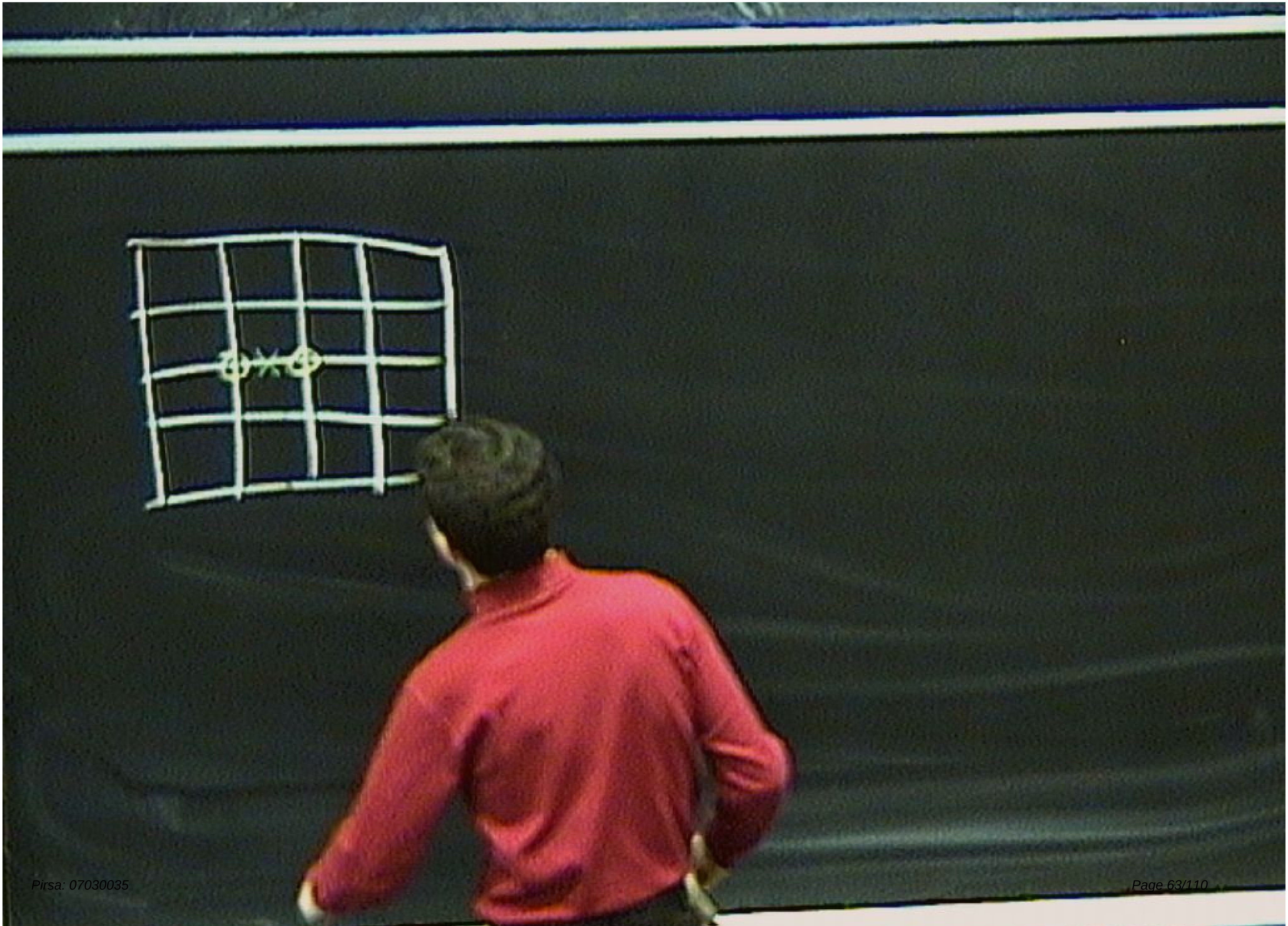
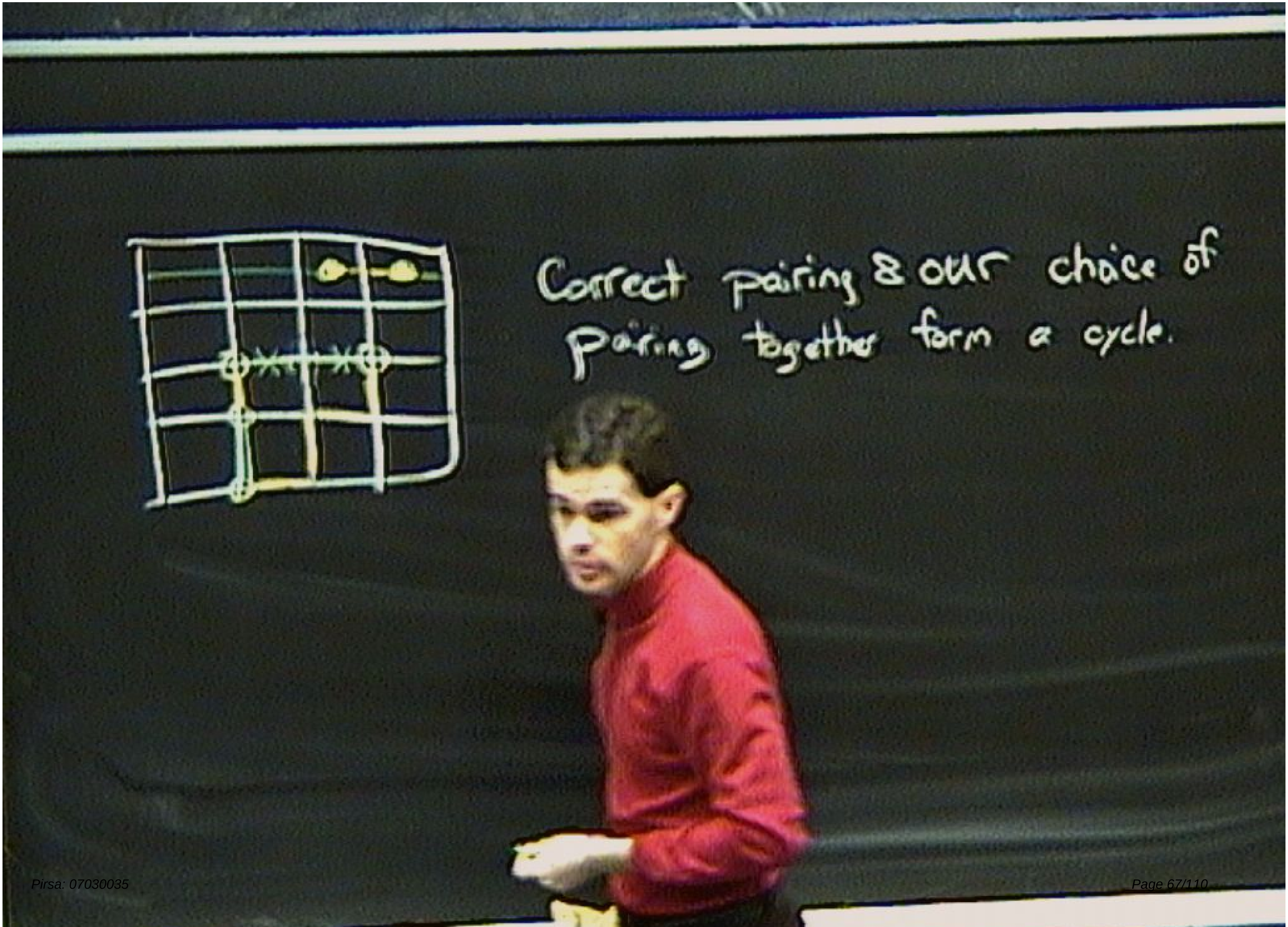
Get threshold result w/ large toric codes








Correct pairing & our choice of
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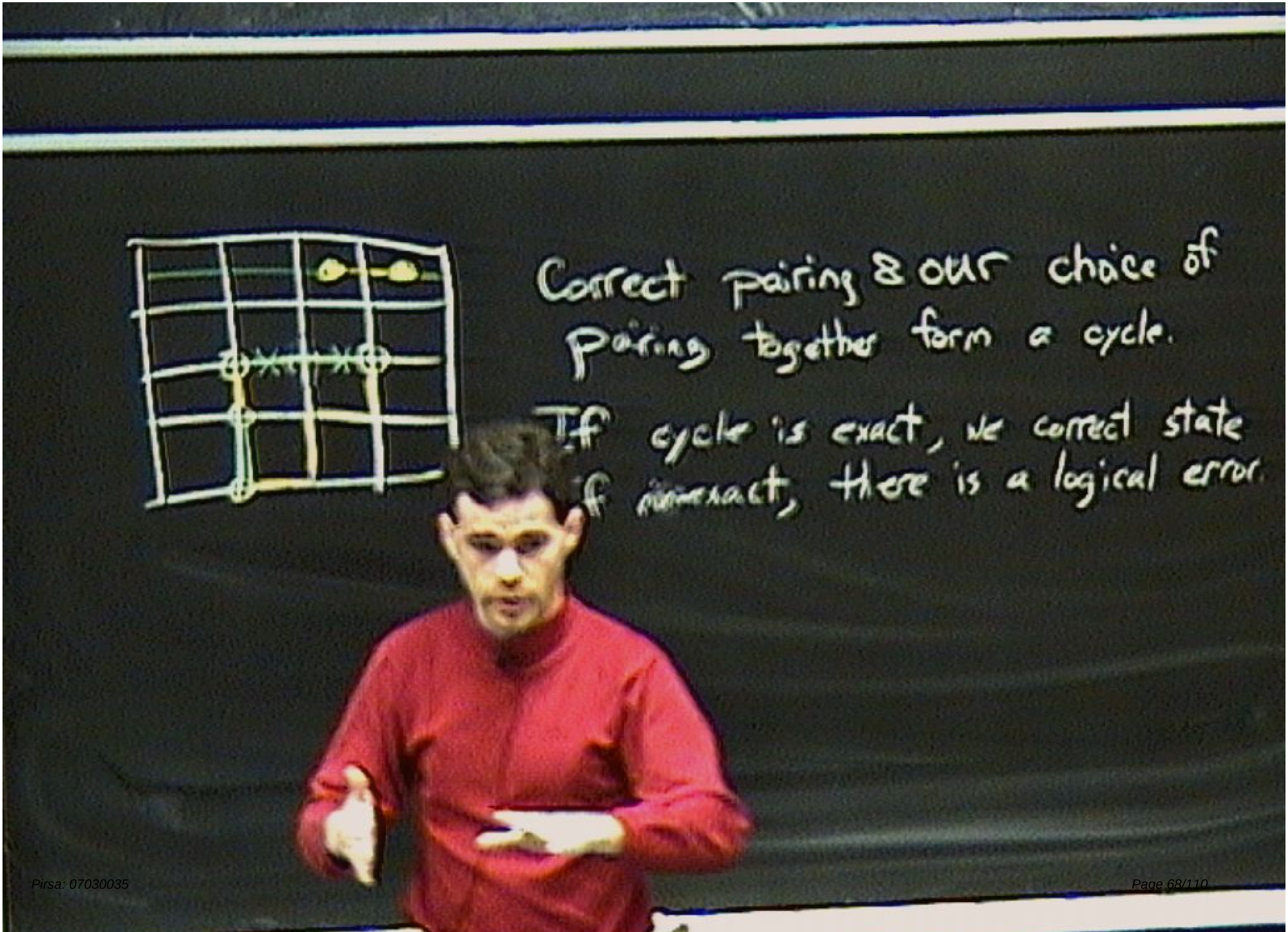
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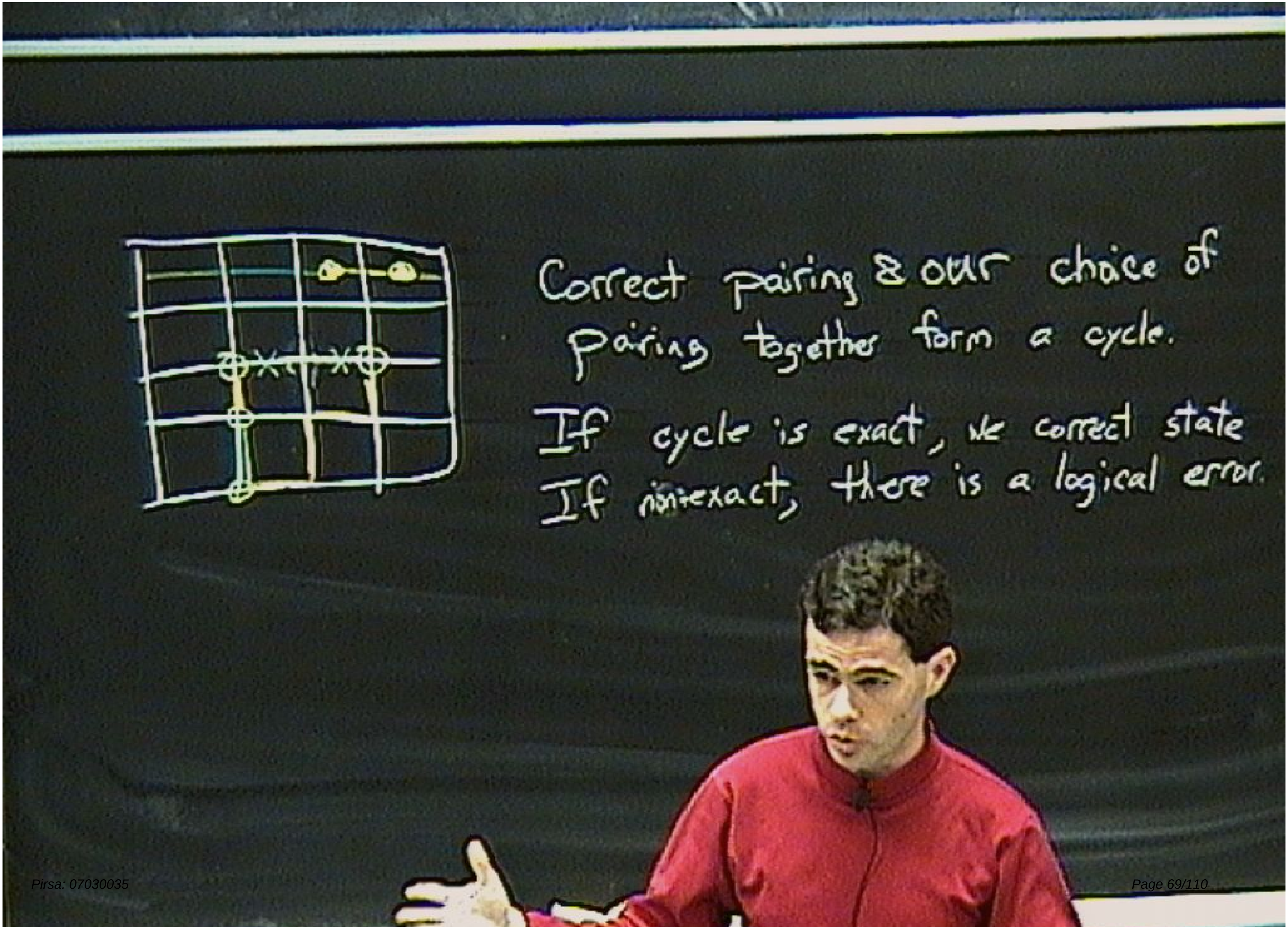
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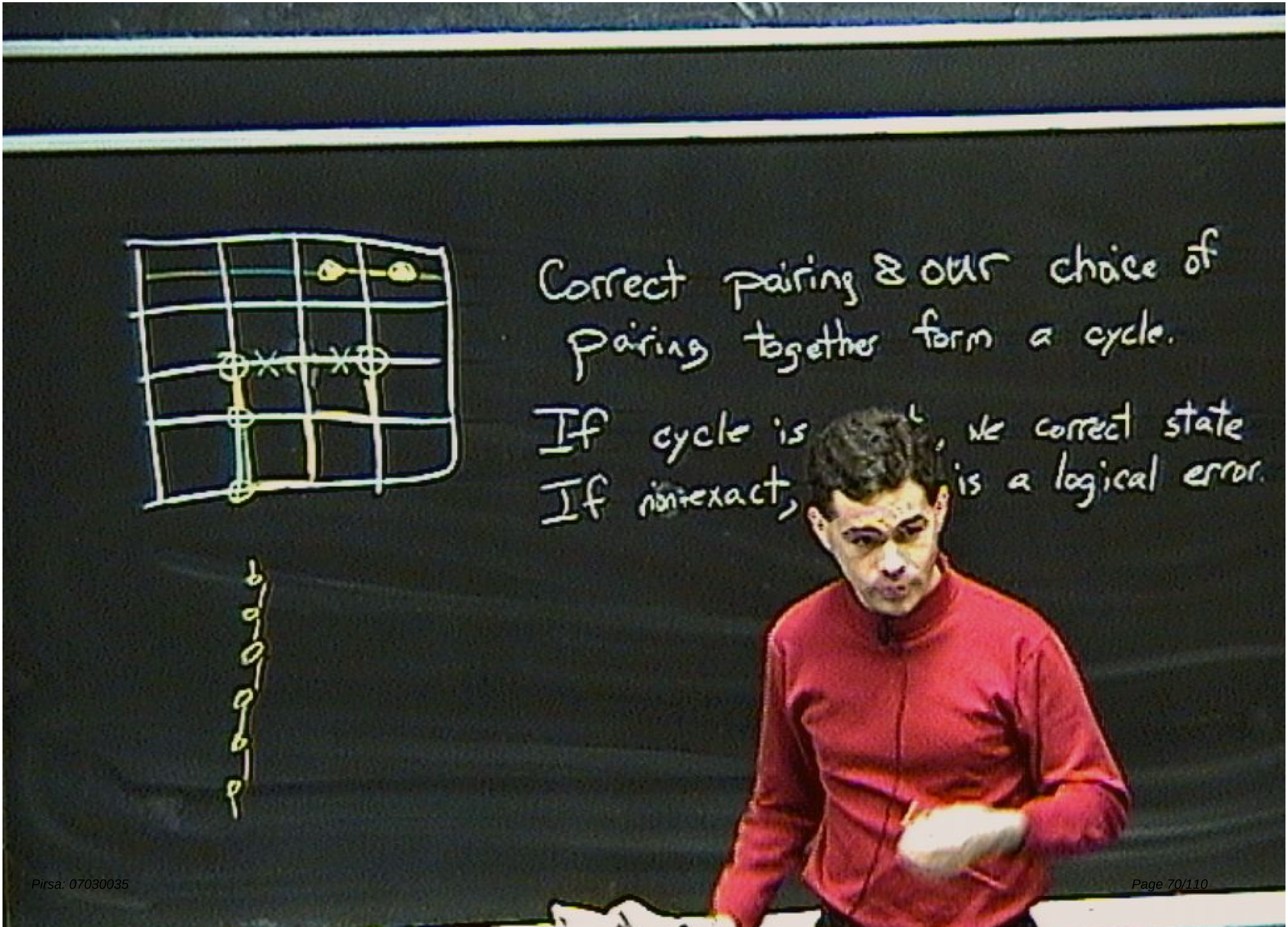
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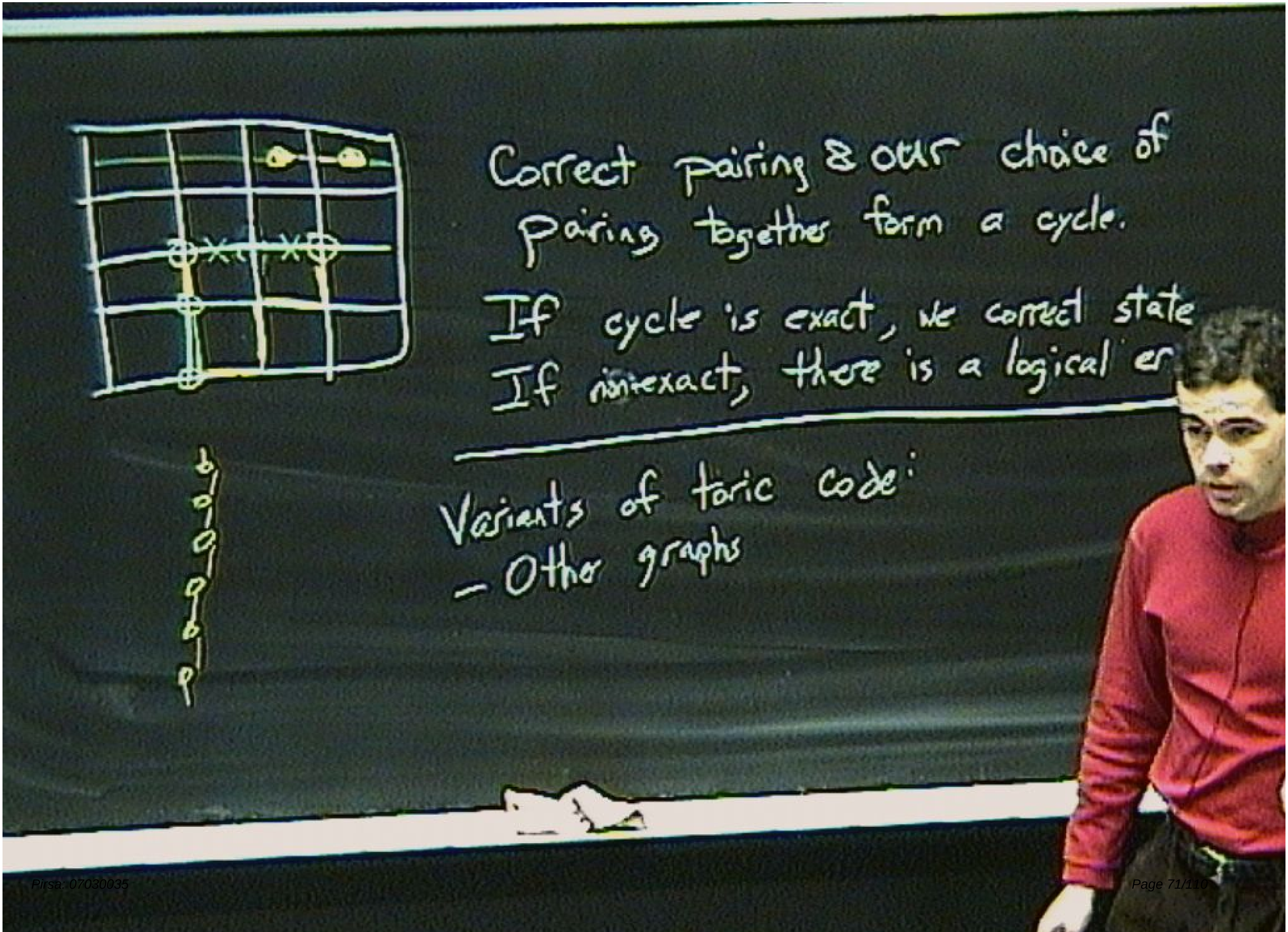
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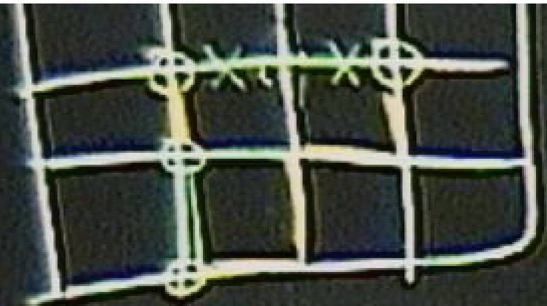
Variants of toric code:
- Other graphs



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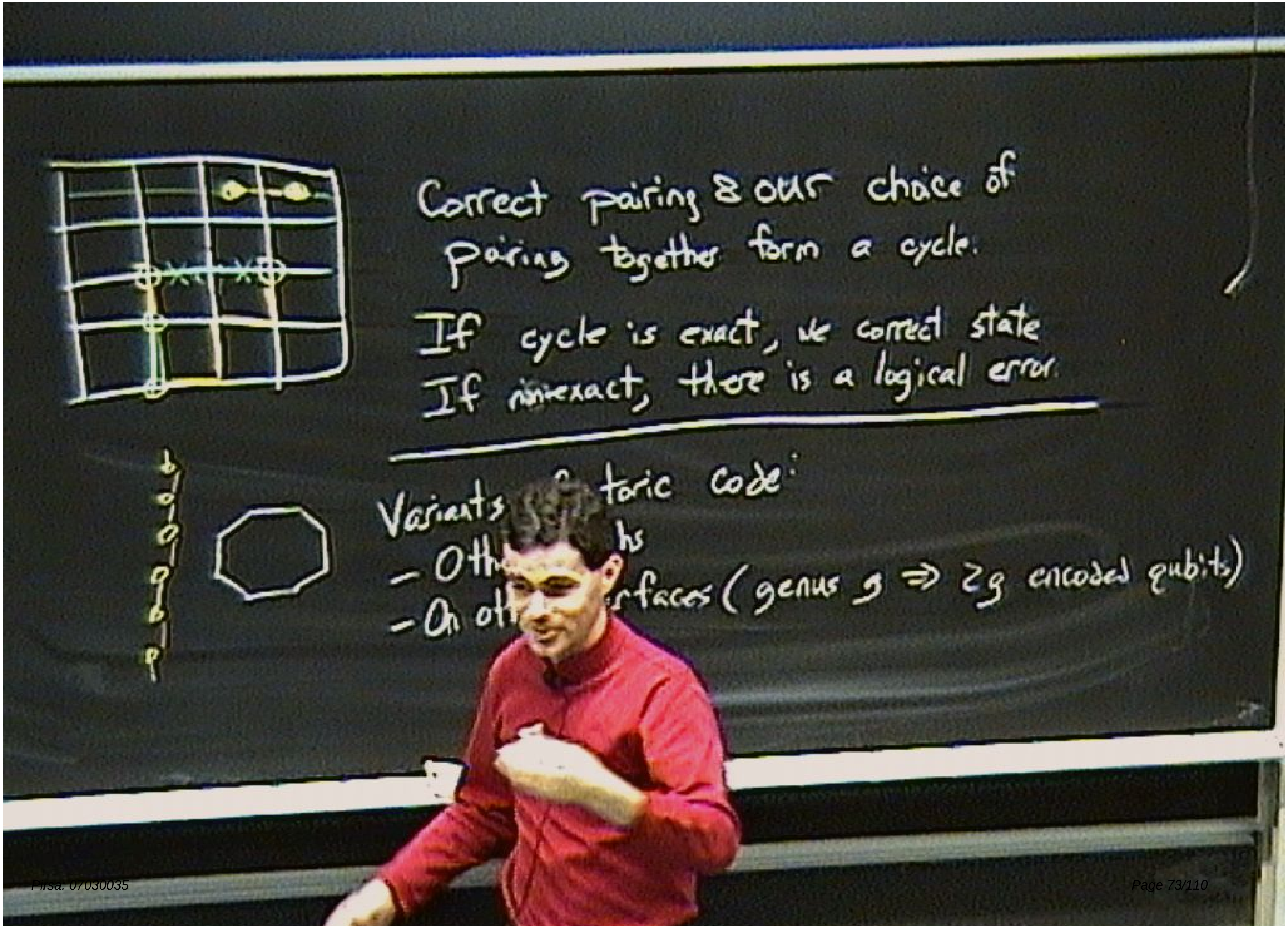
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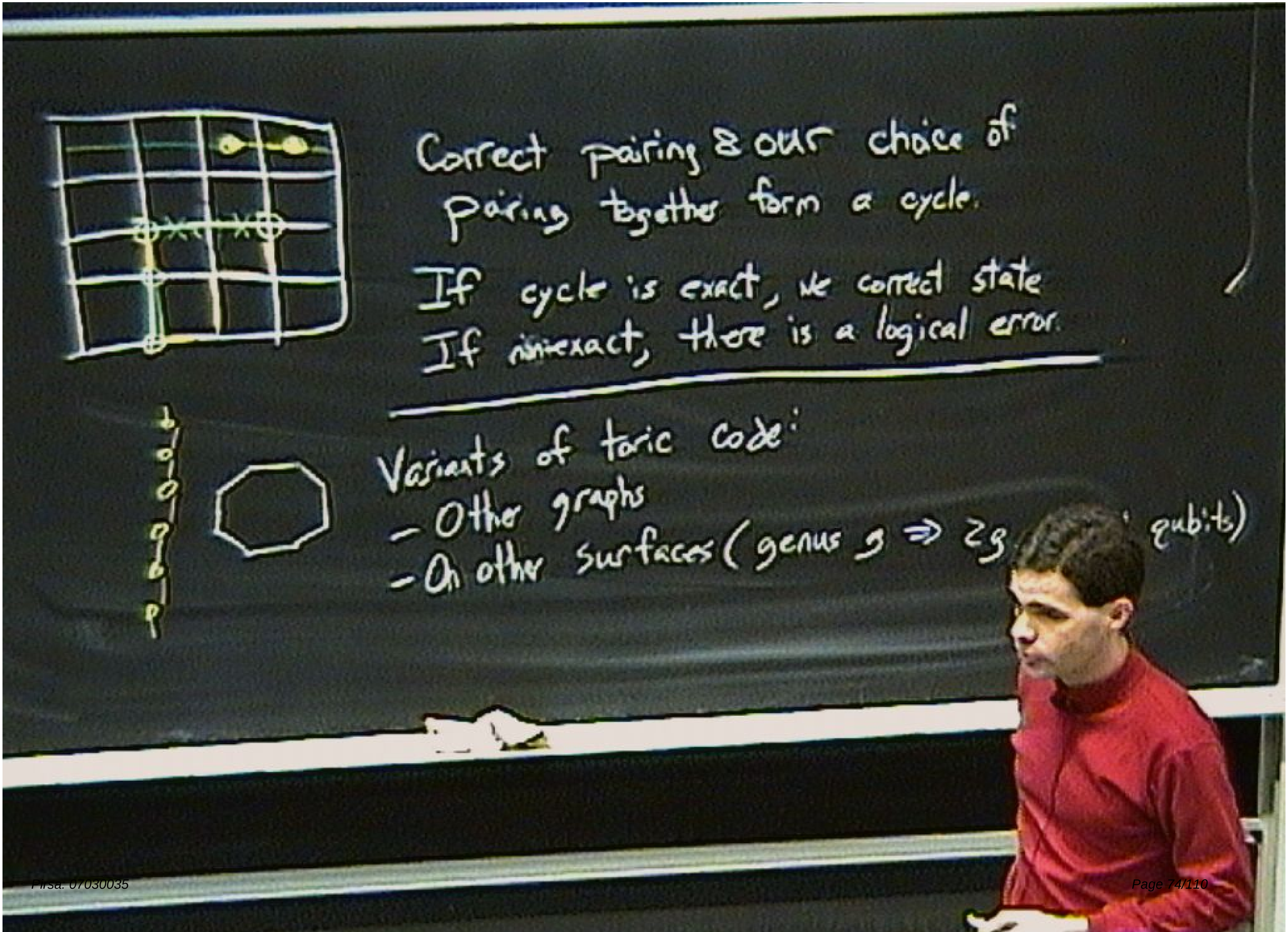
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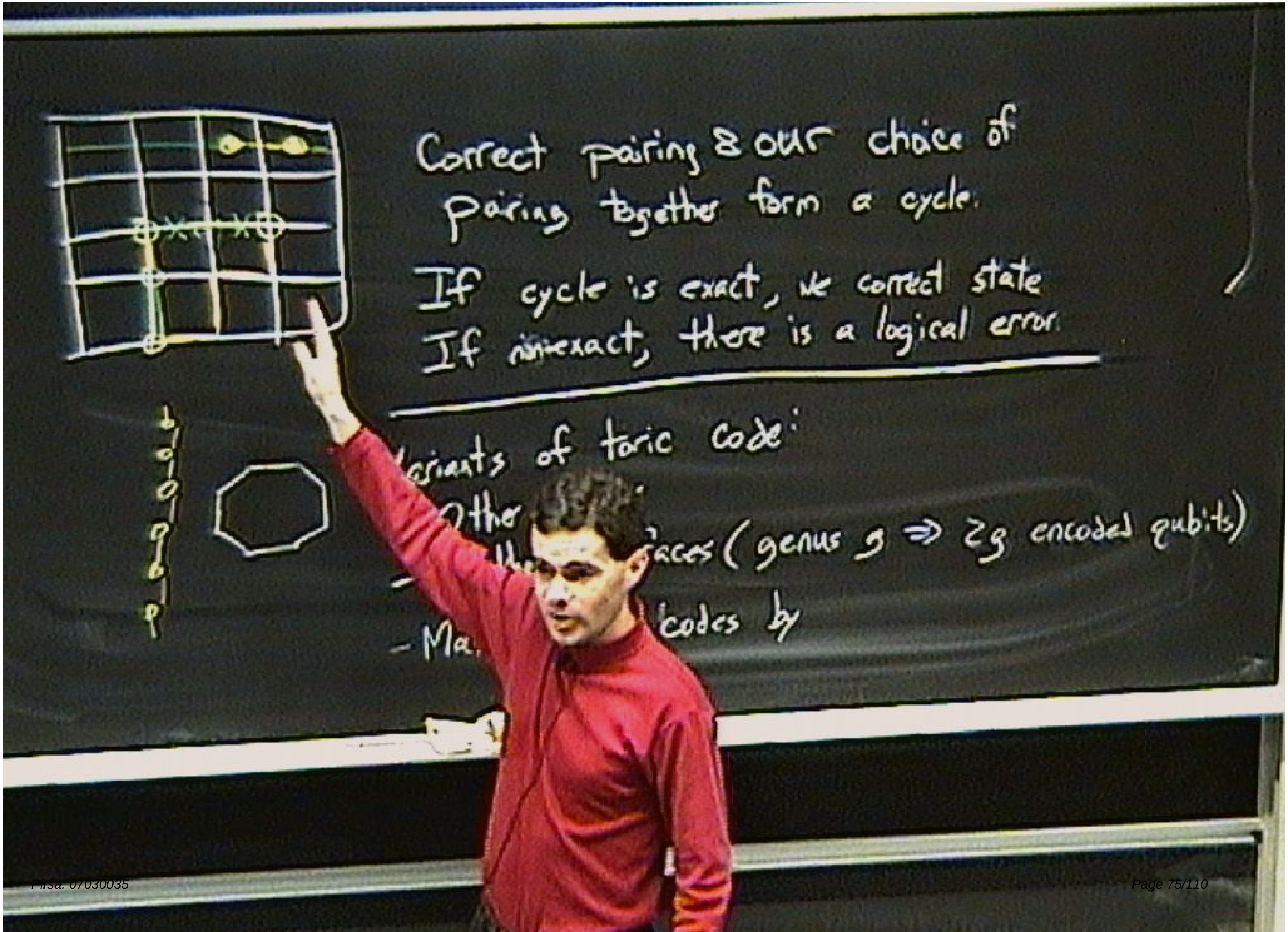
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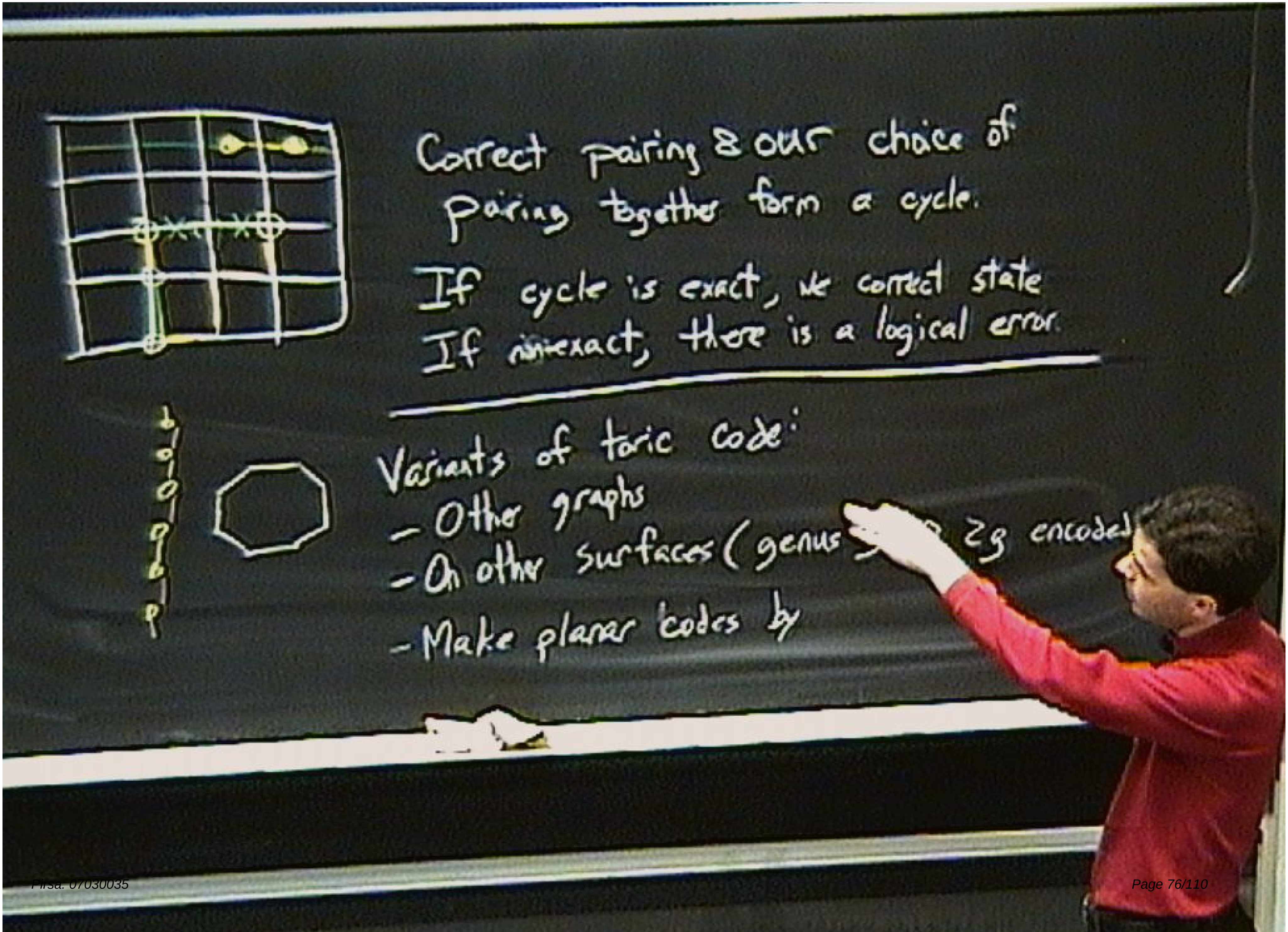
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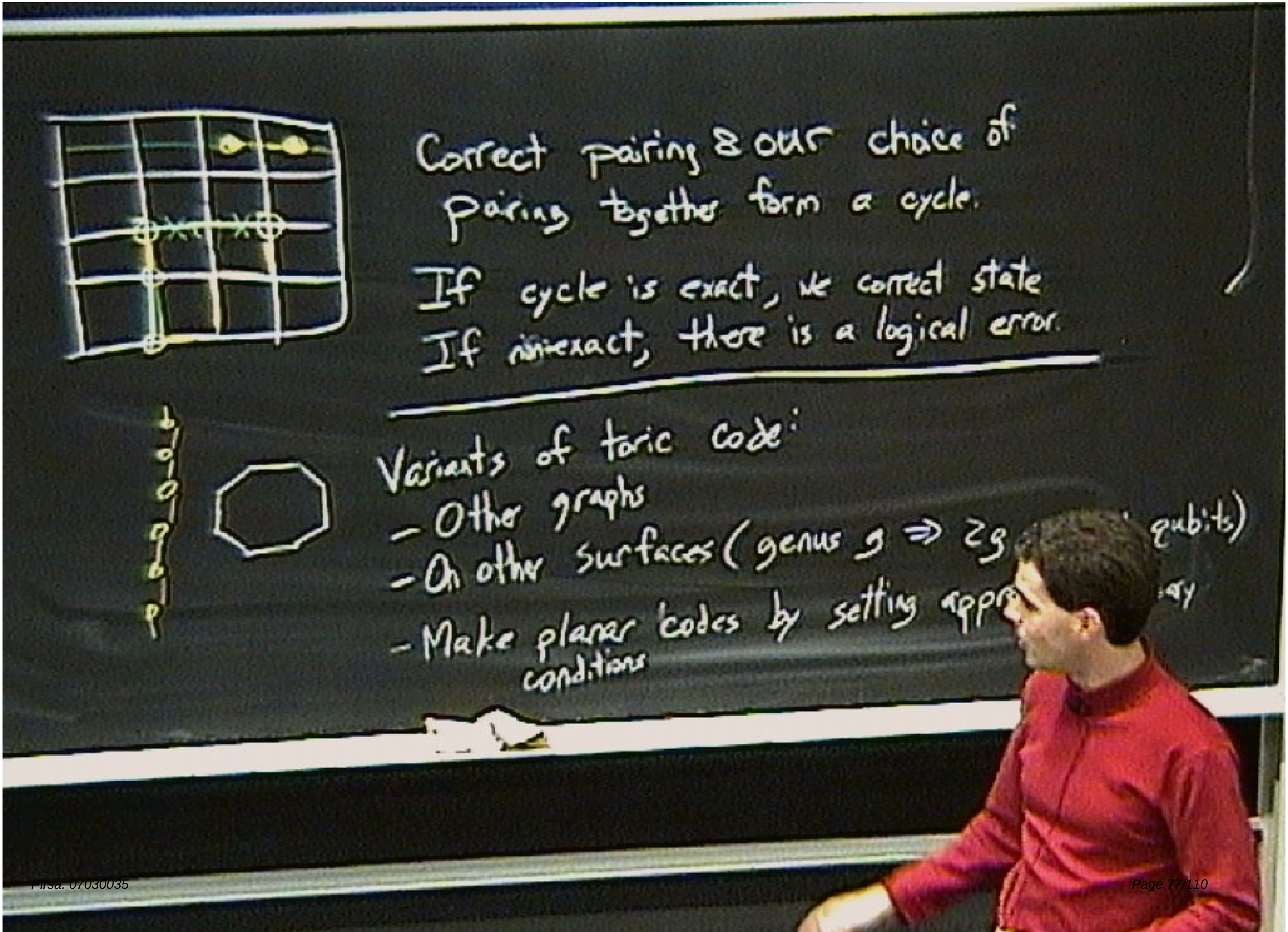
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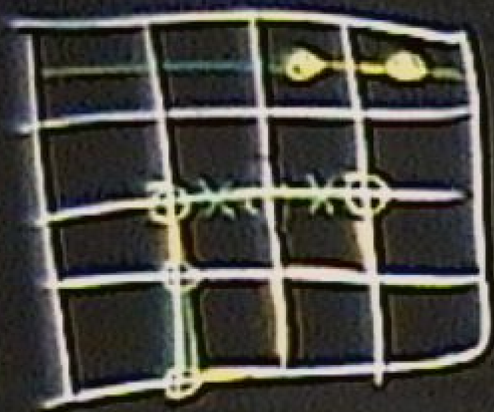

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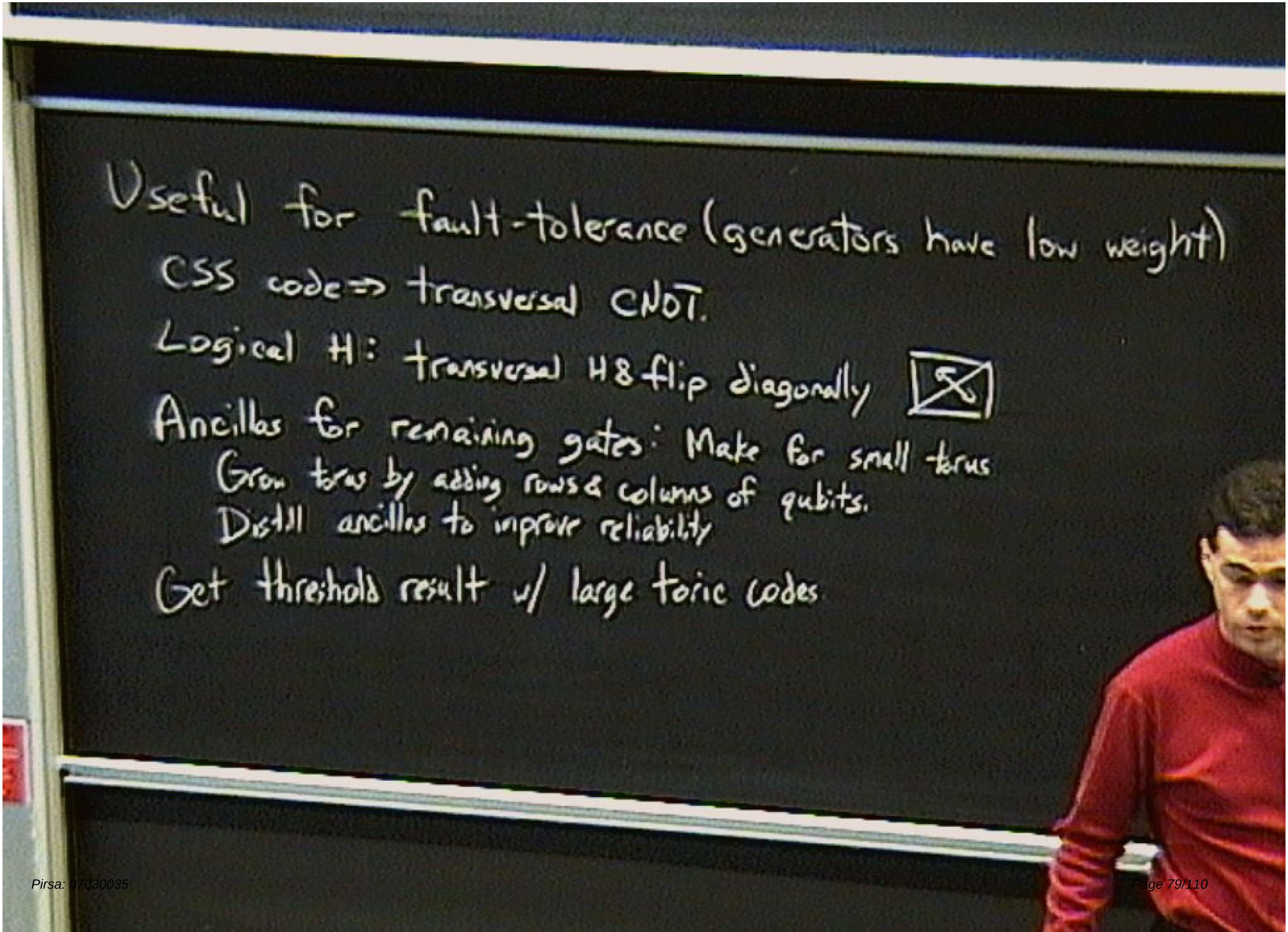
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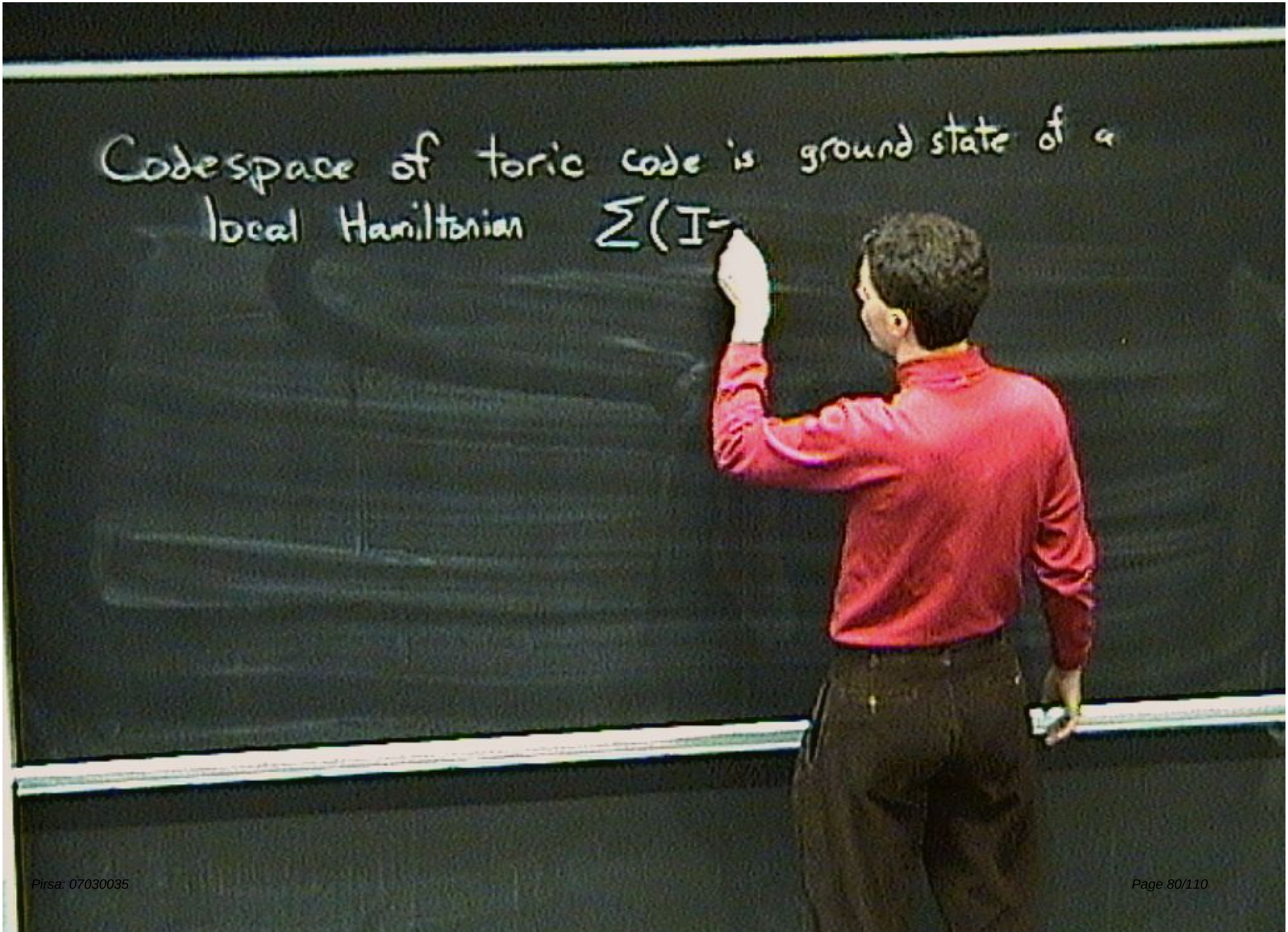
Codespace of toric code is ground state of a
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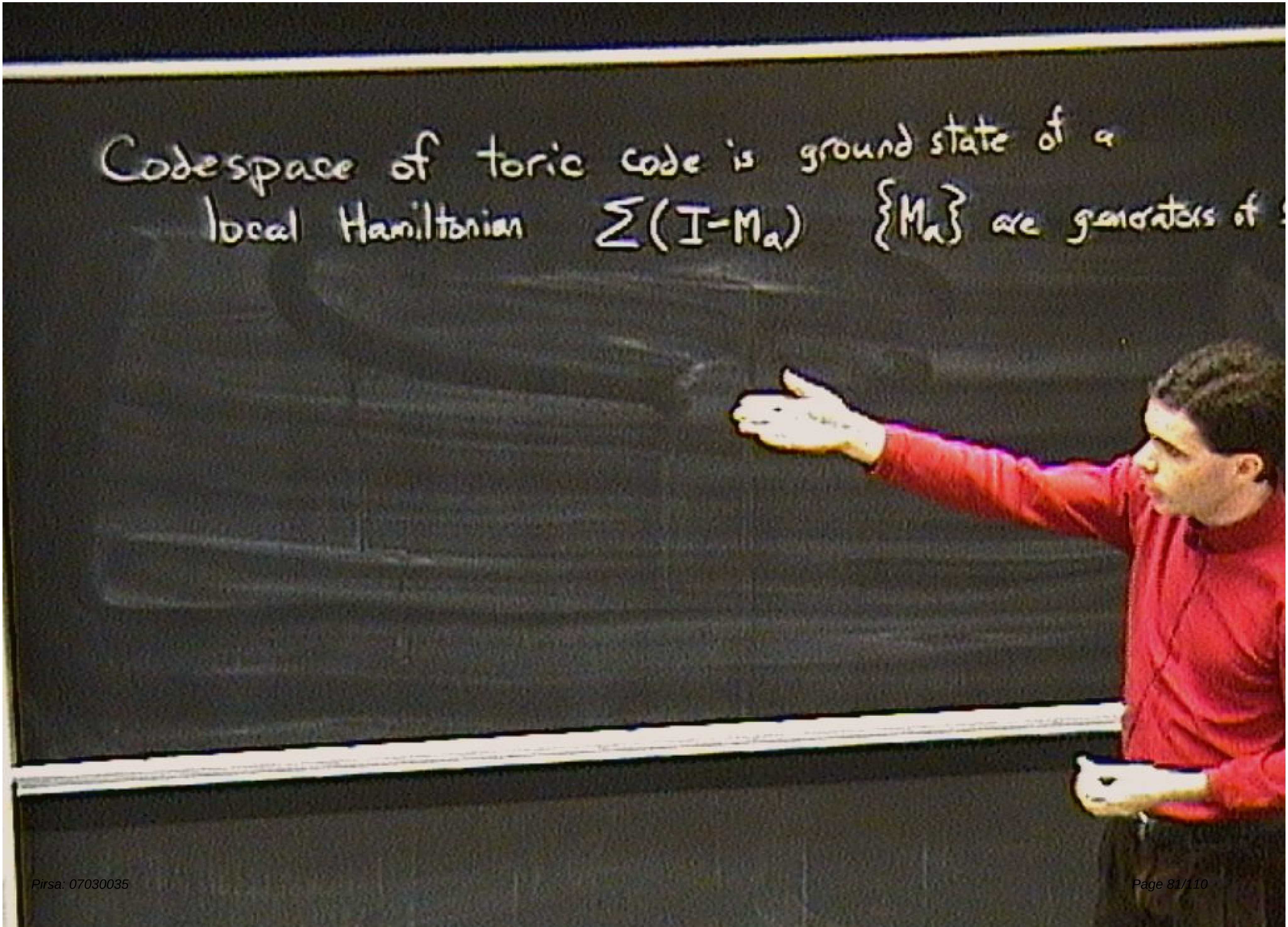
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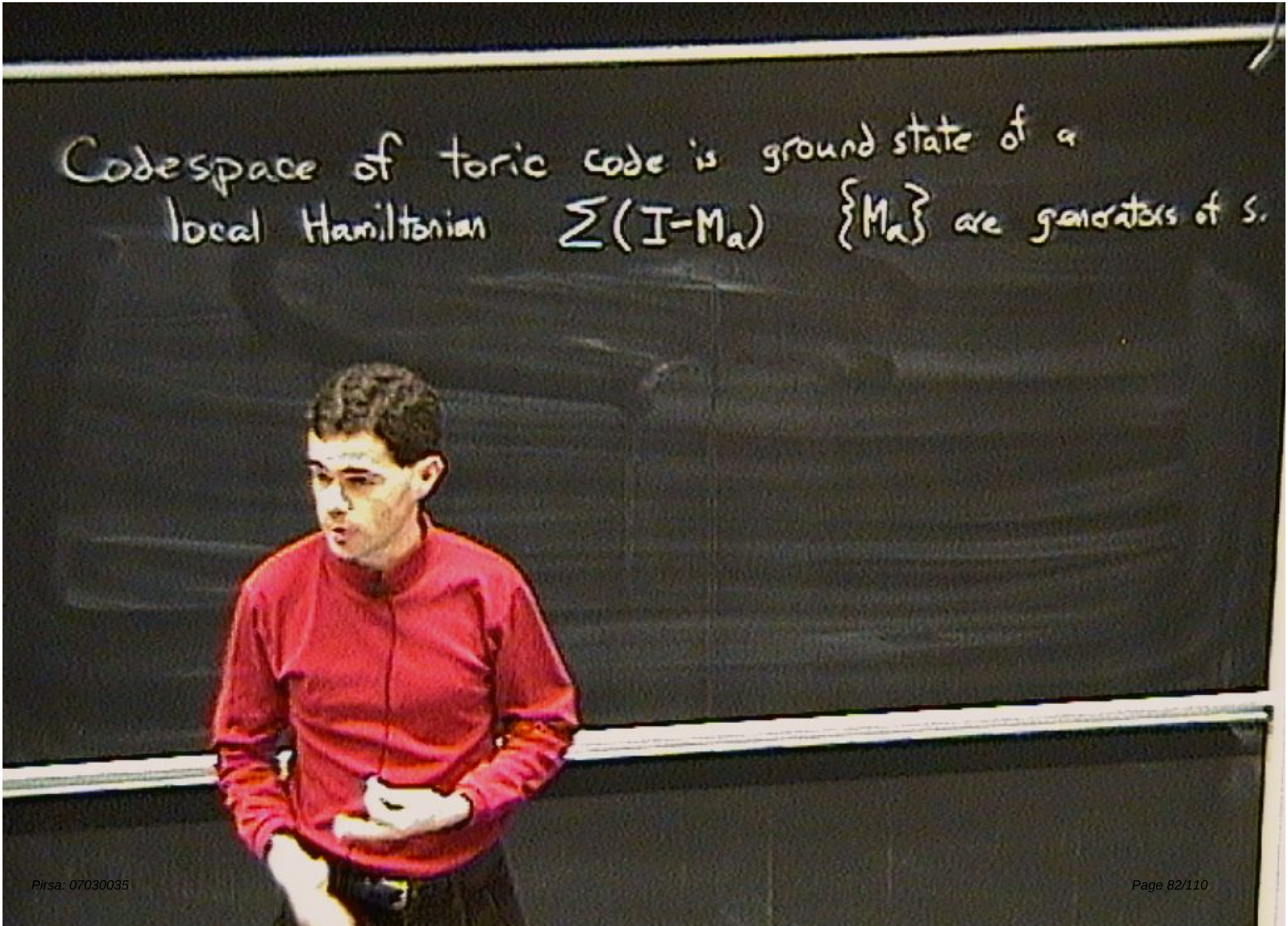
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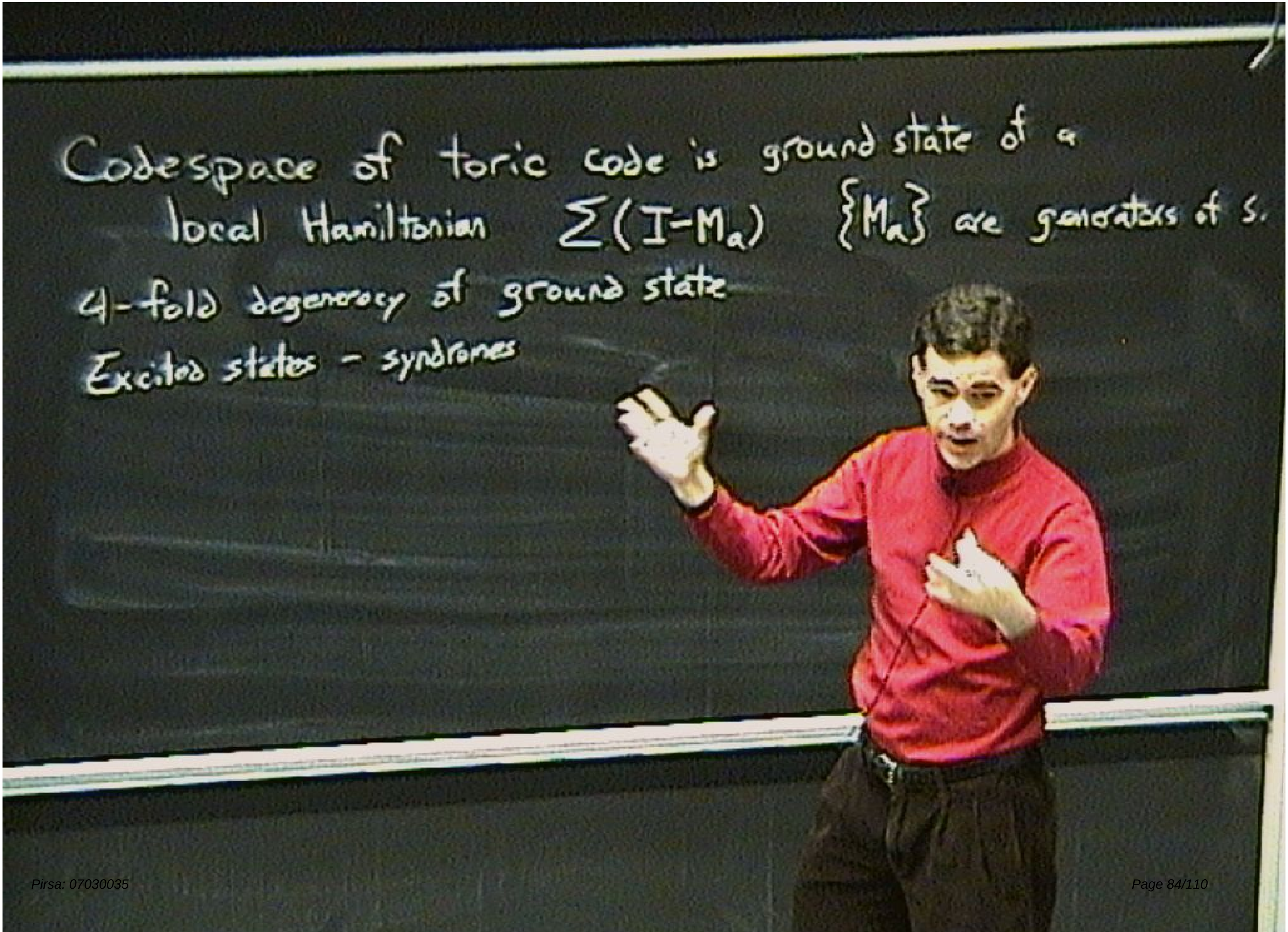
4-fold degeneracy of ground state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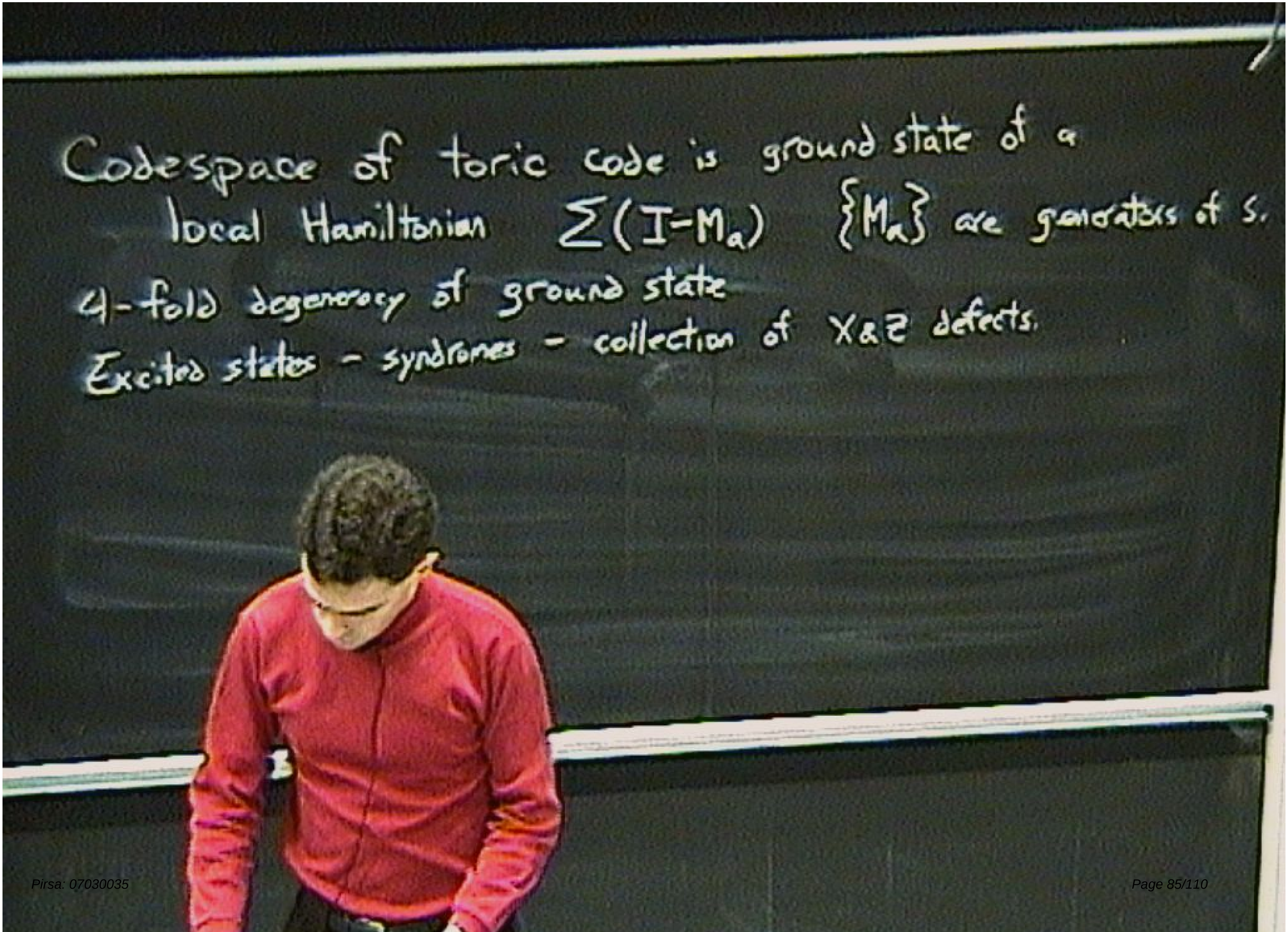
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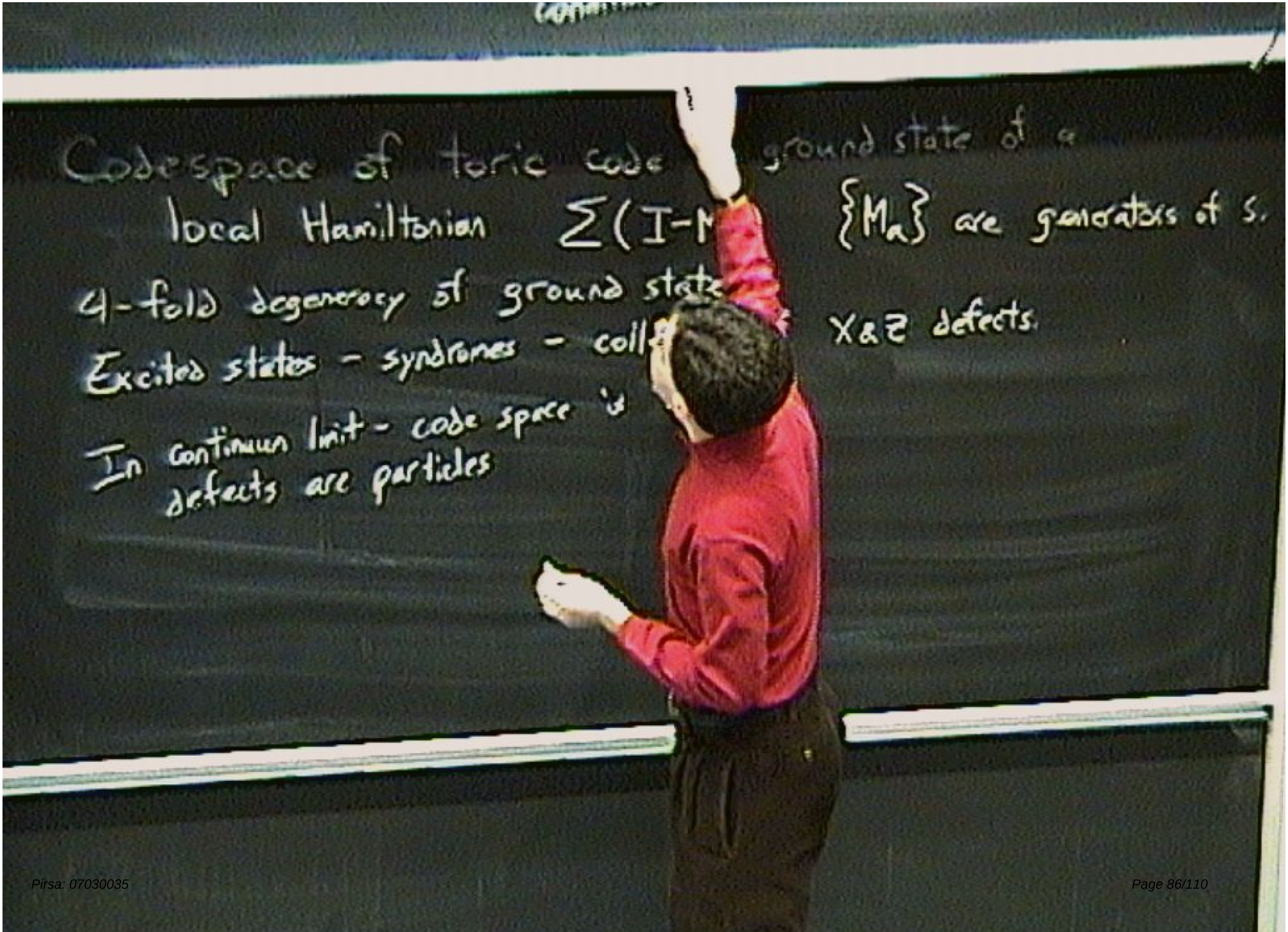
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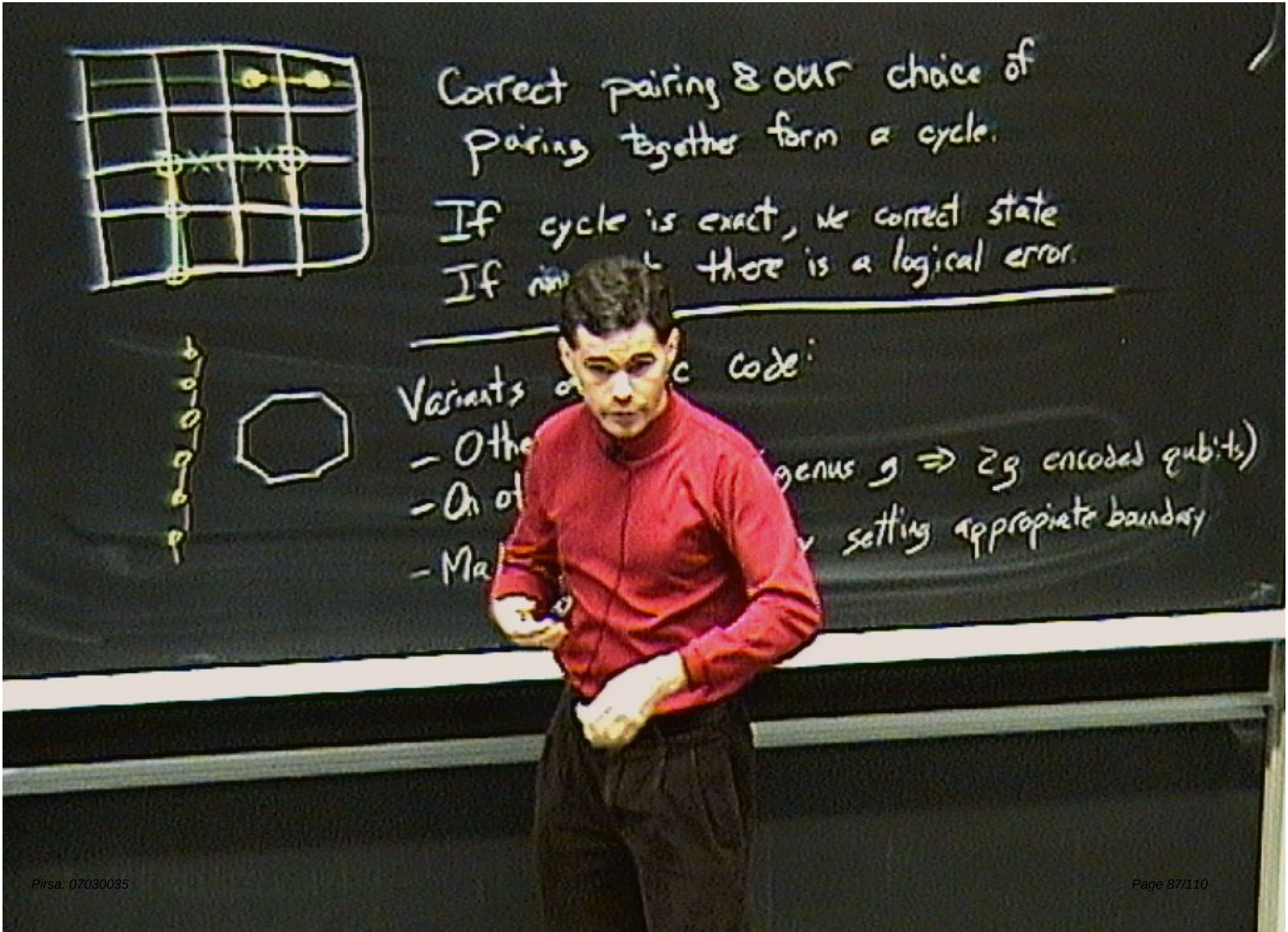
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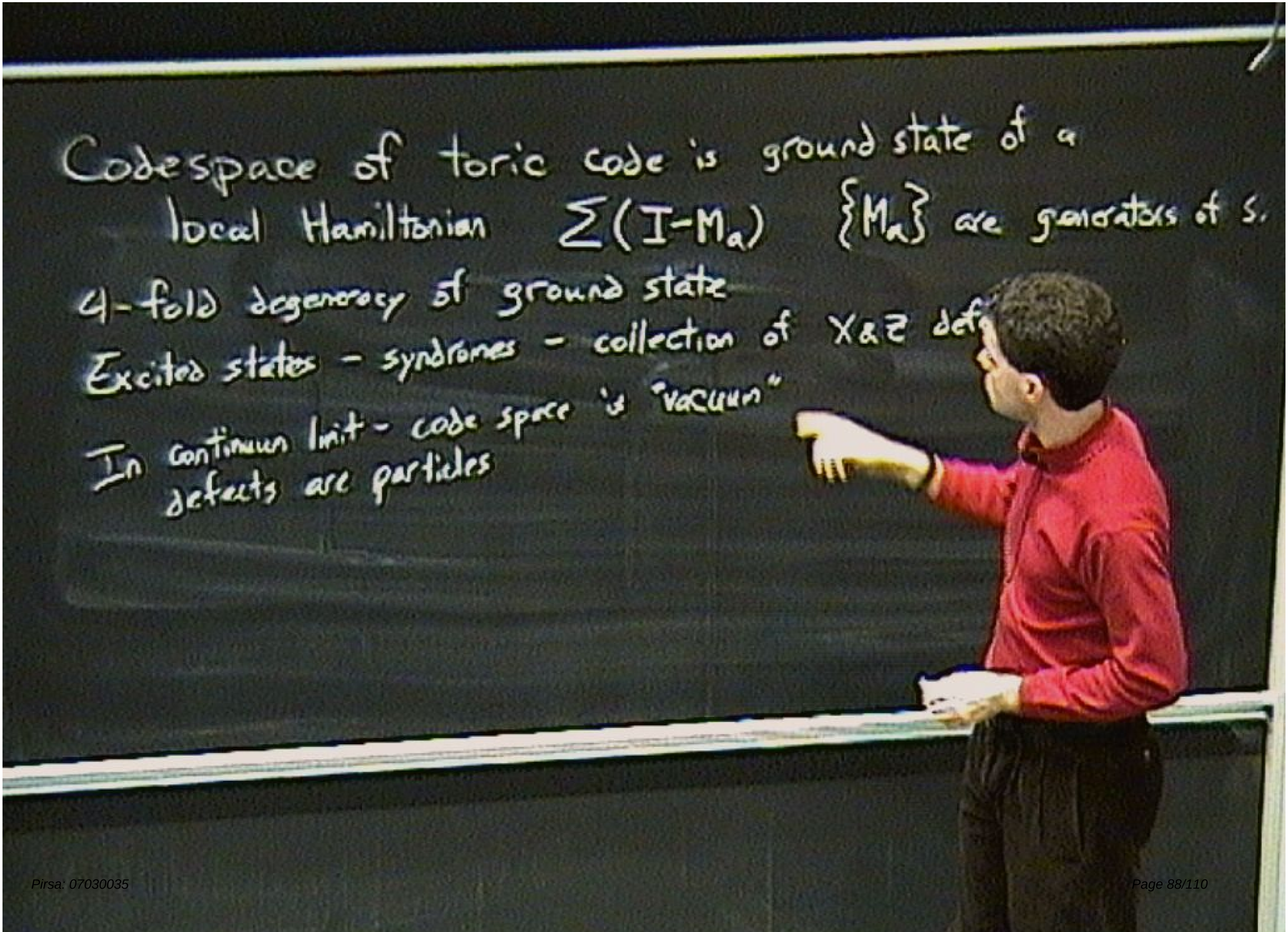
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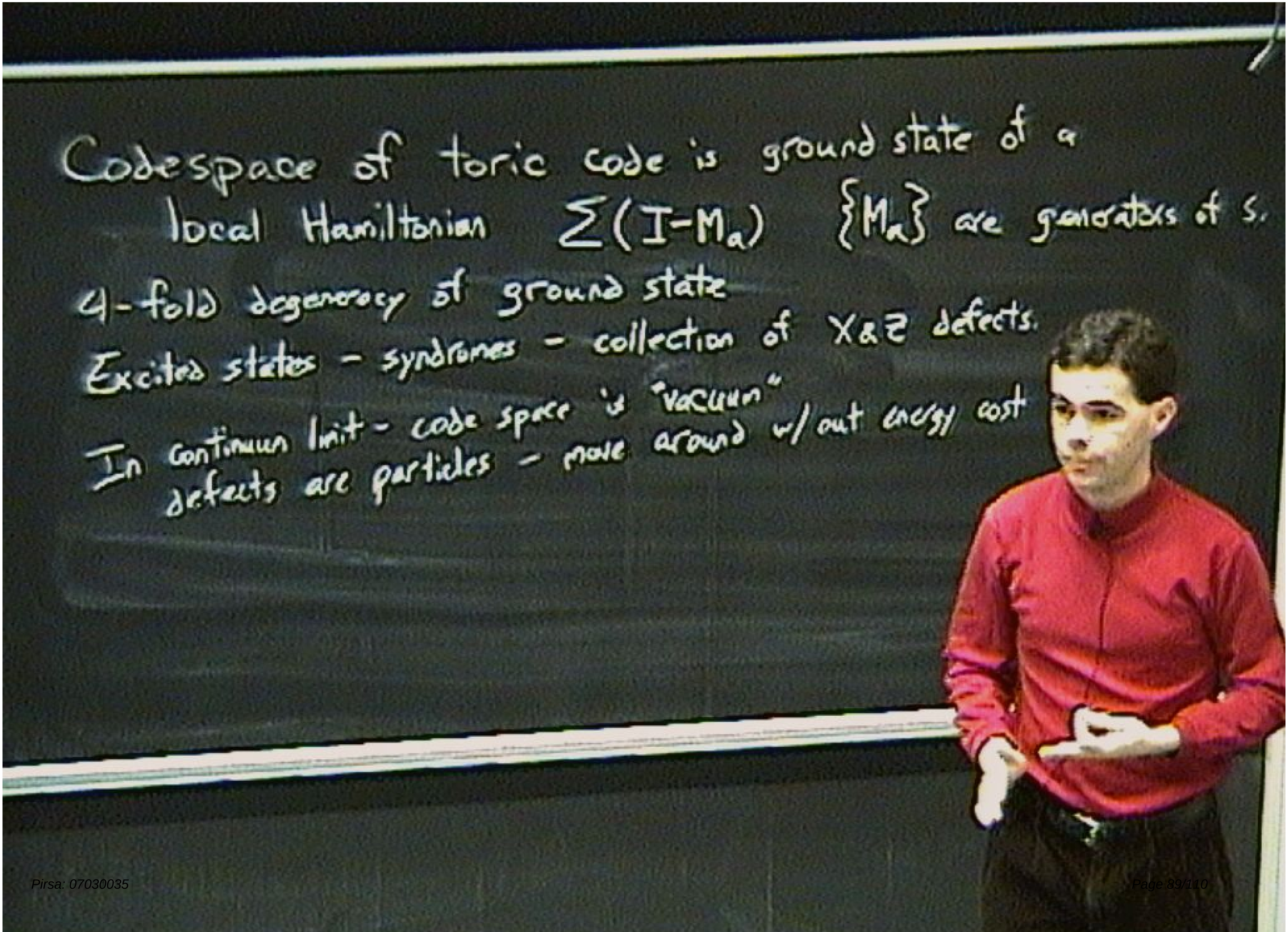
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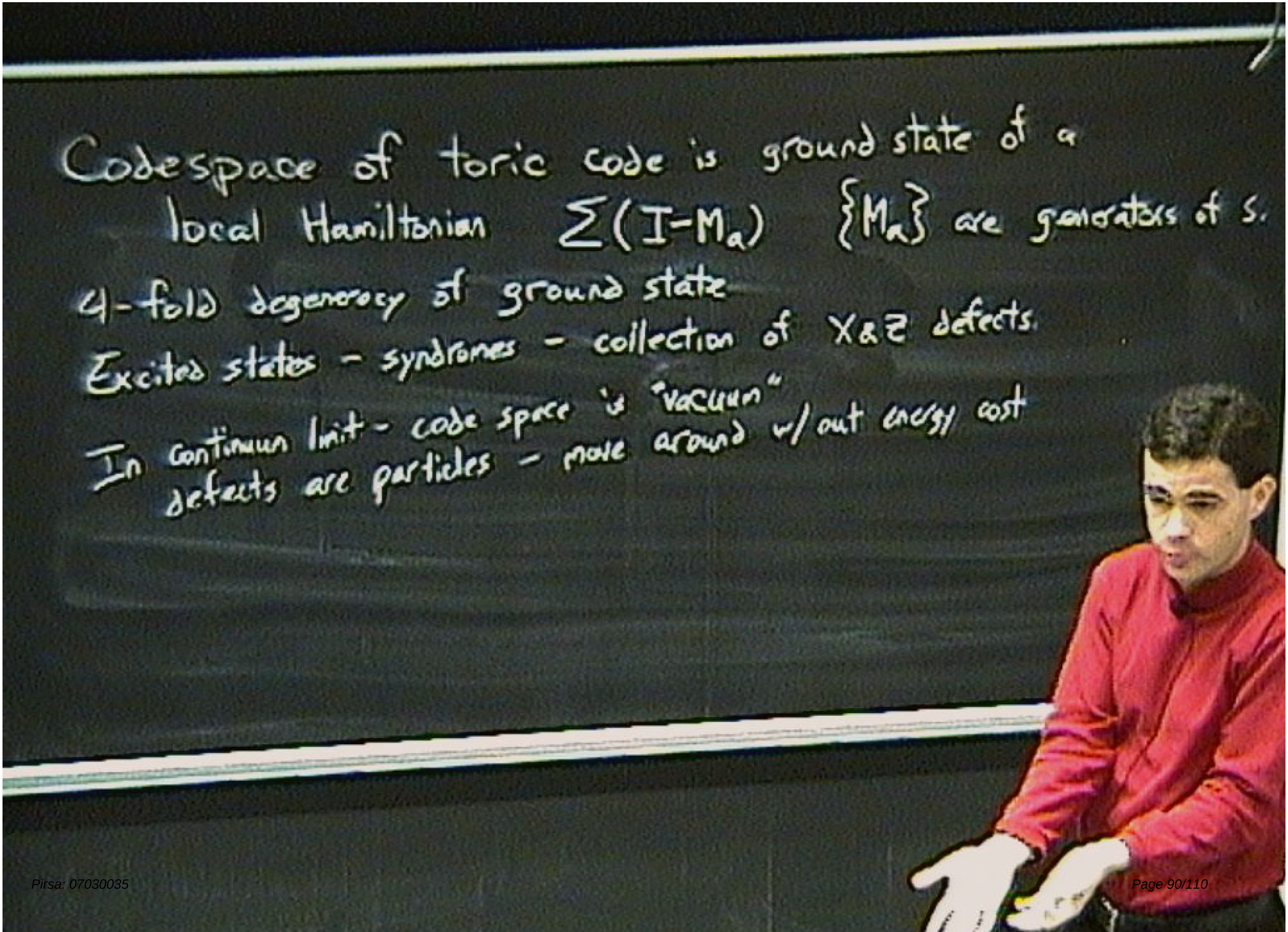
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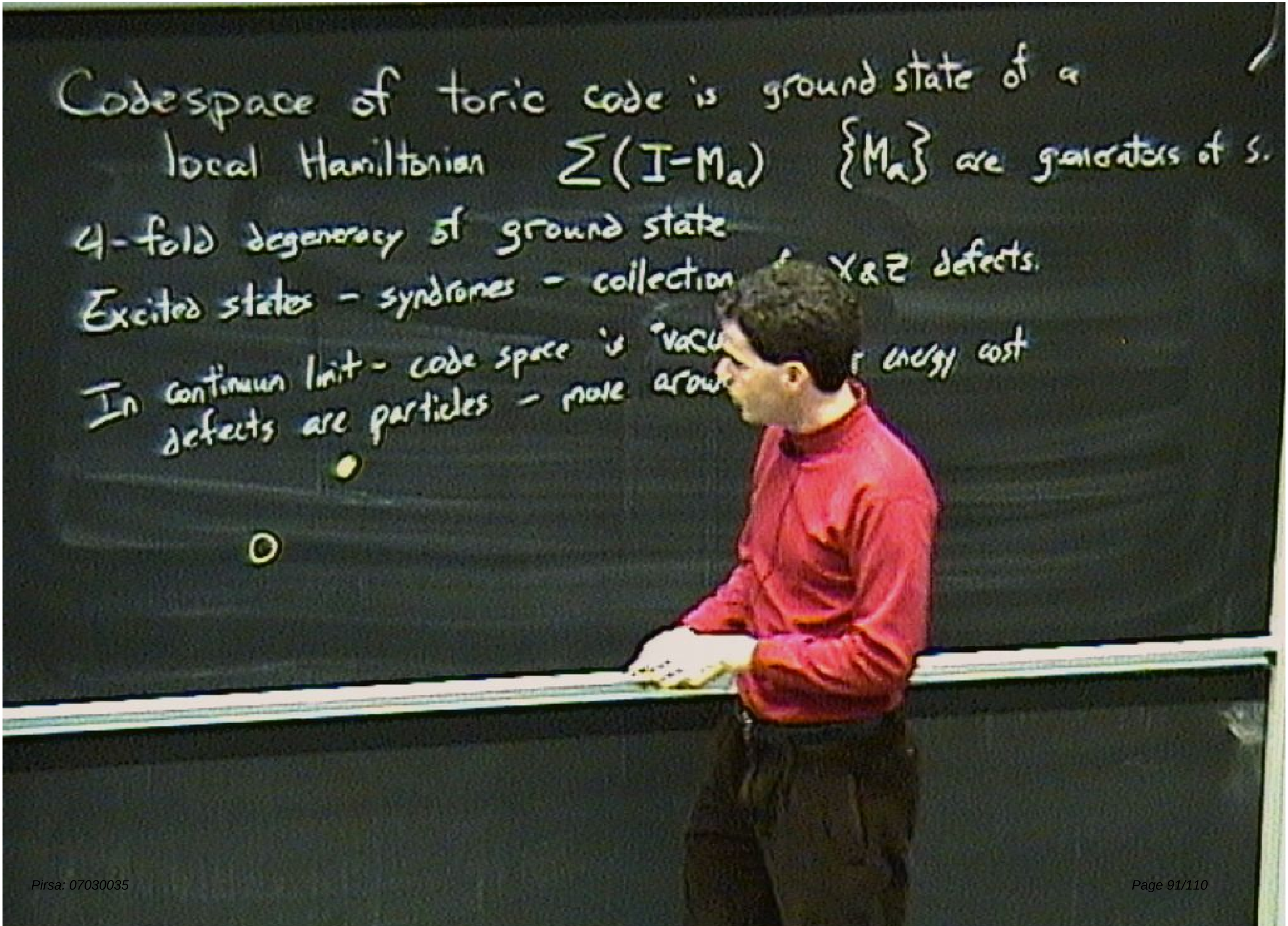
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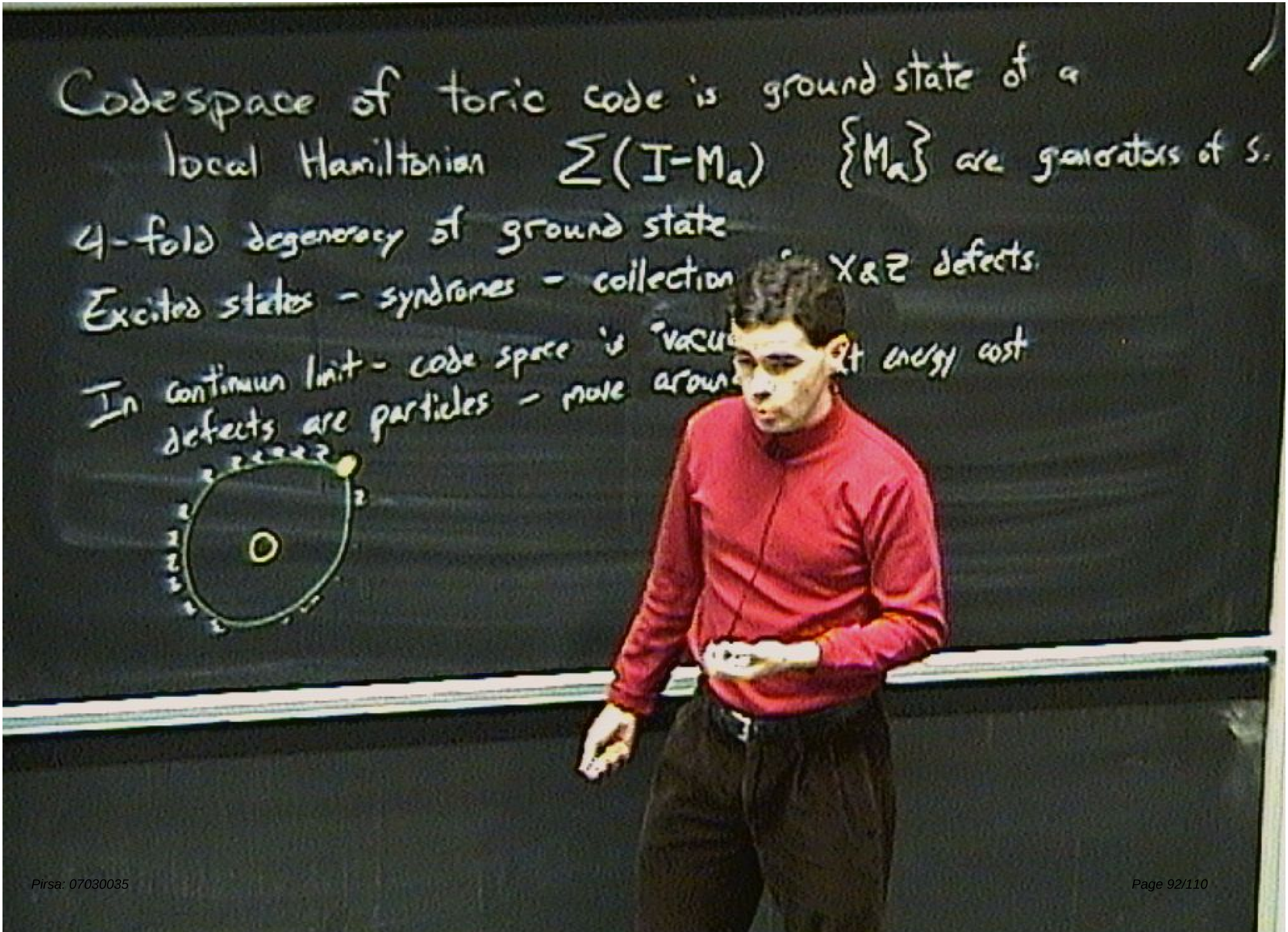
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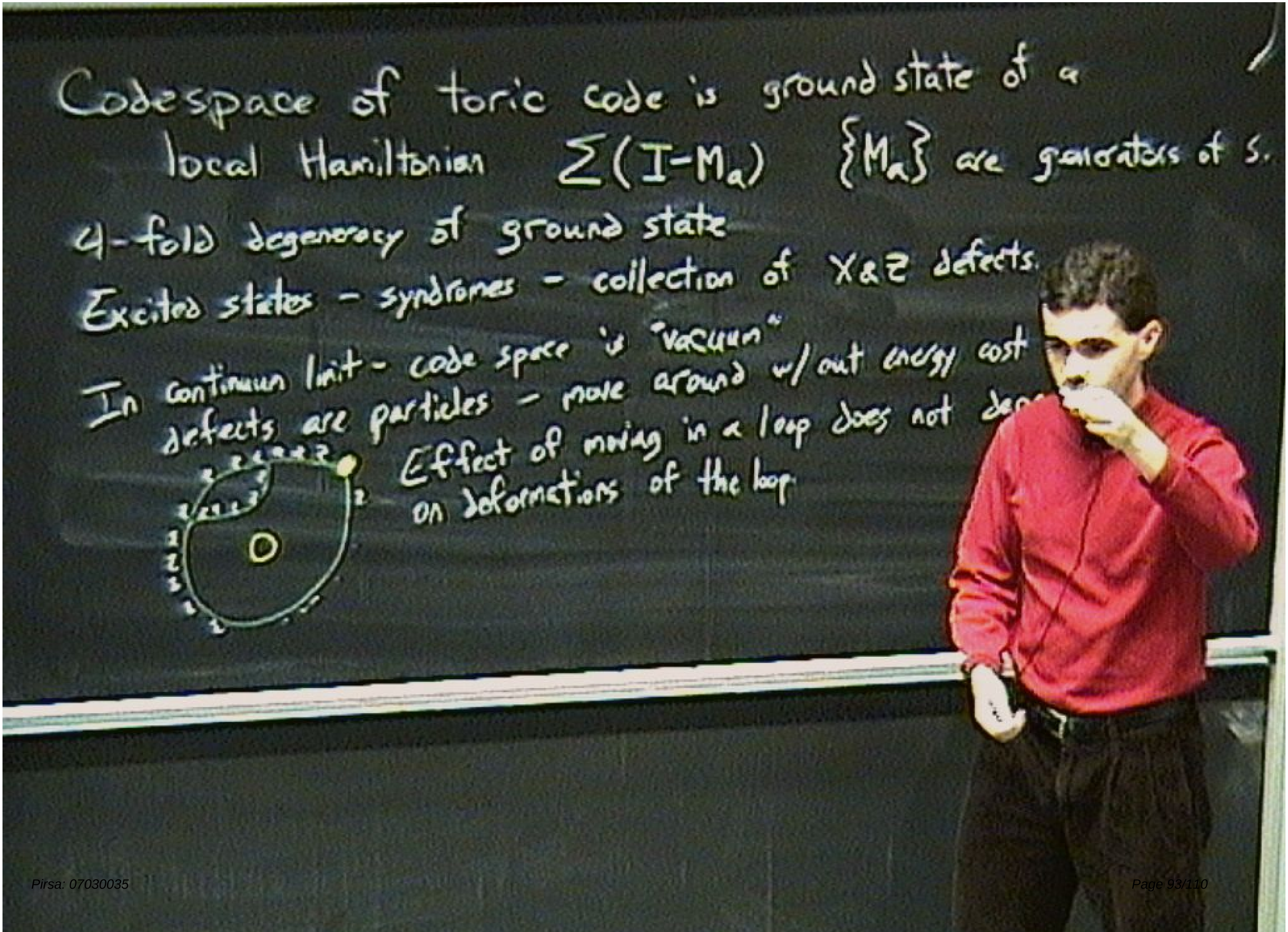
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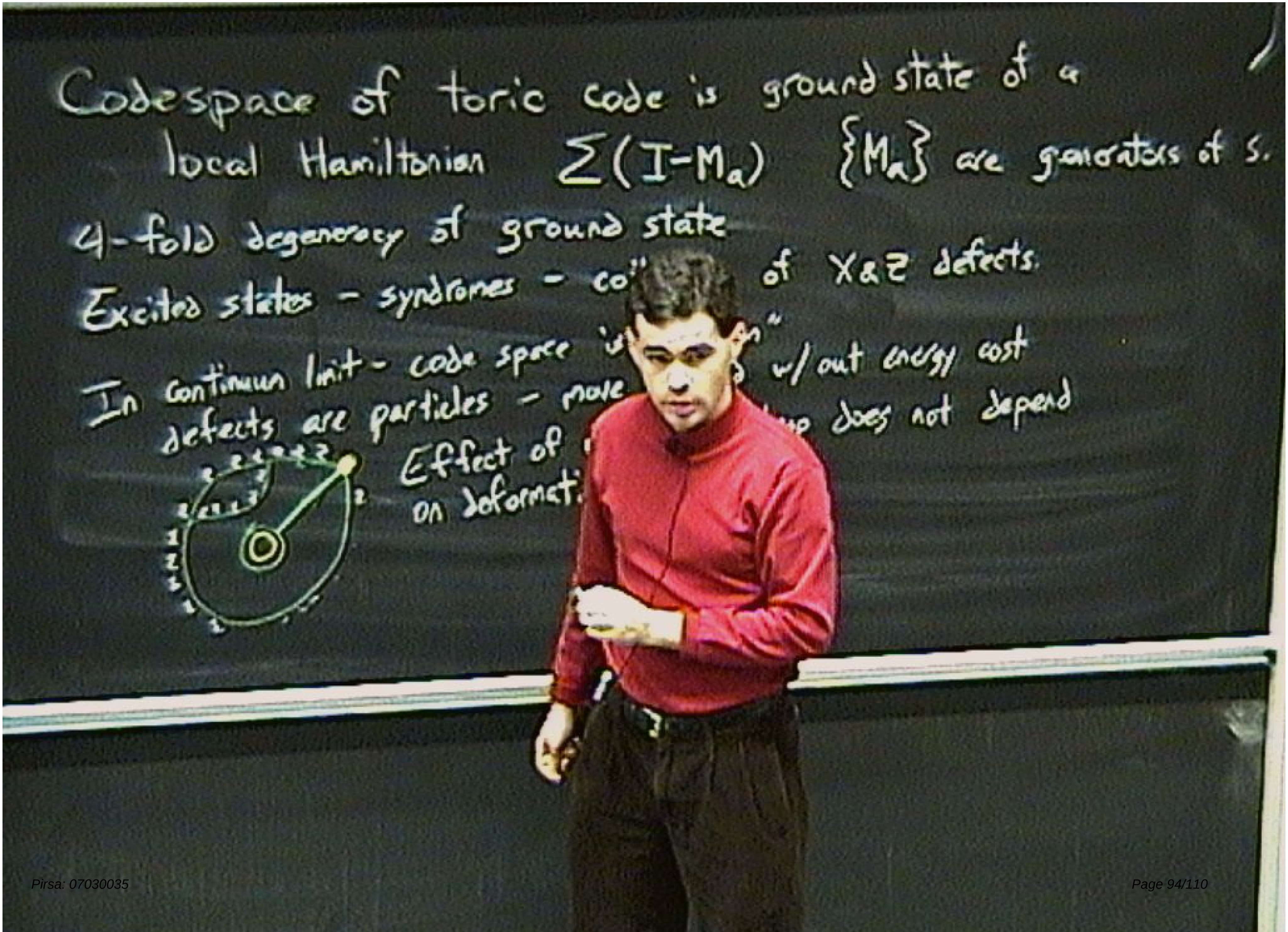

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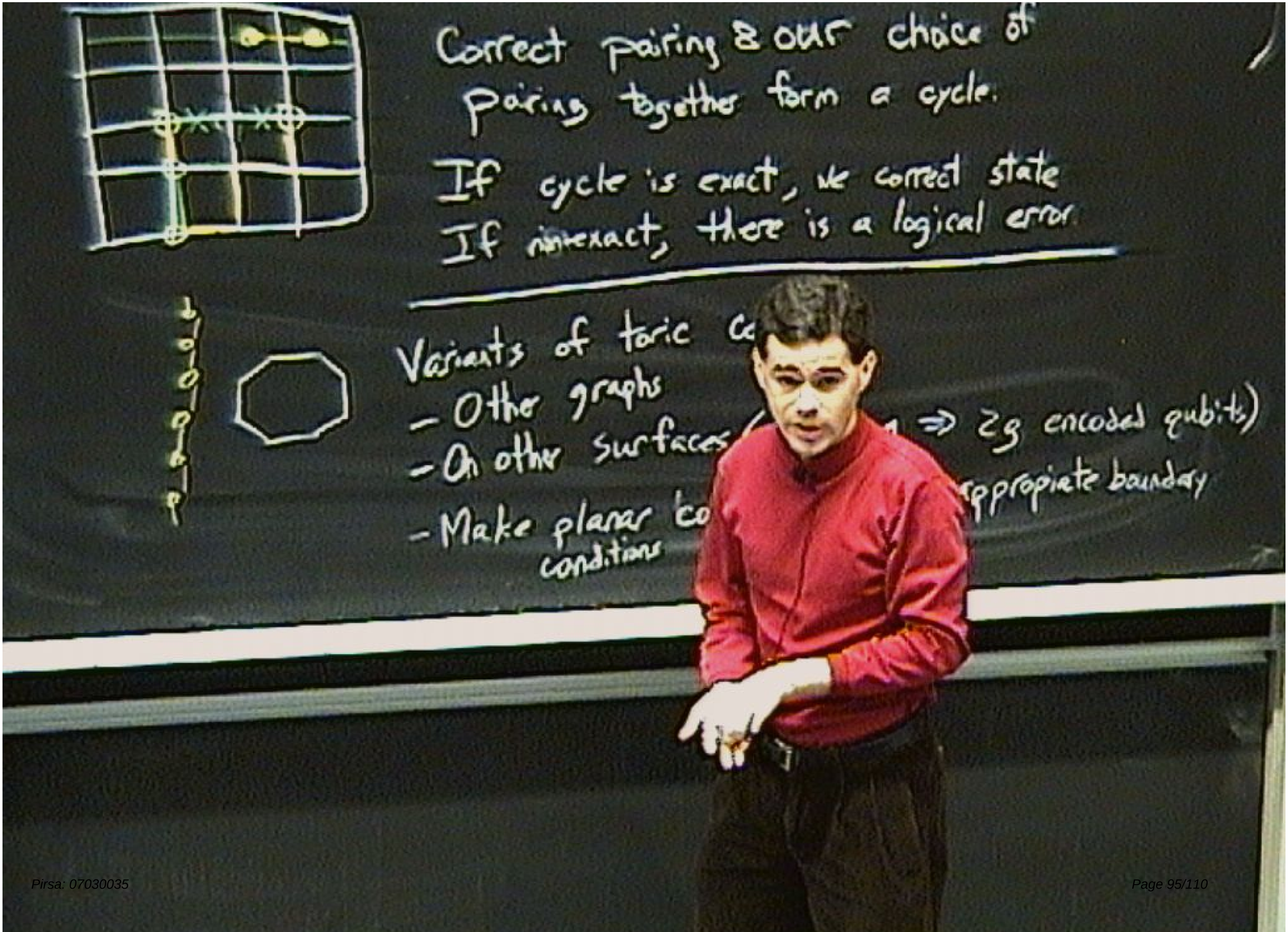

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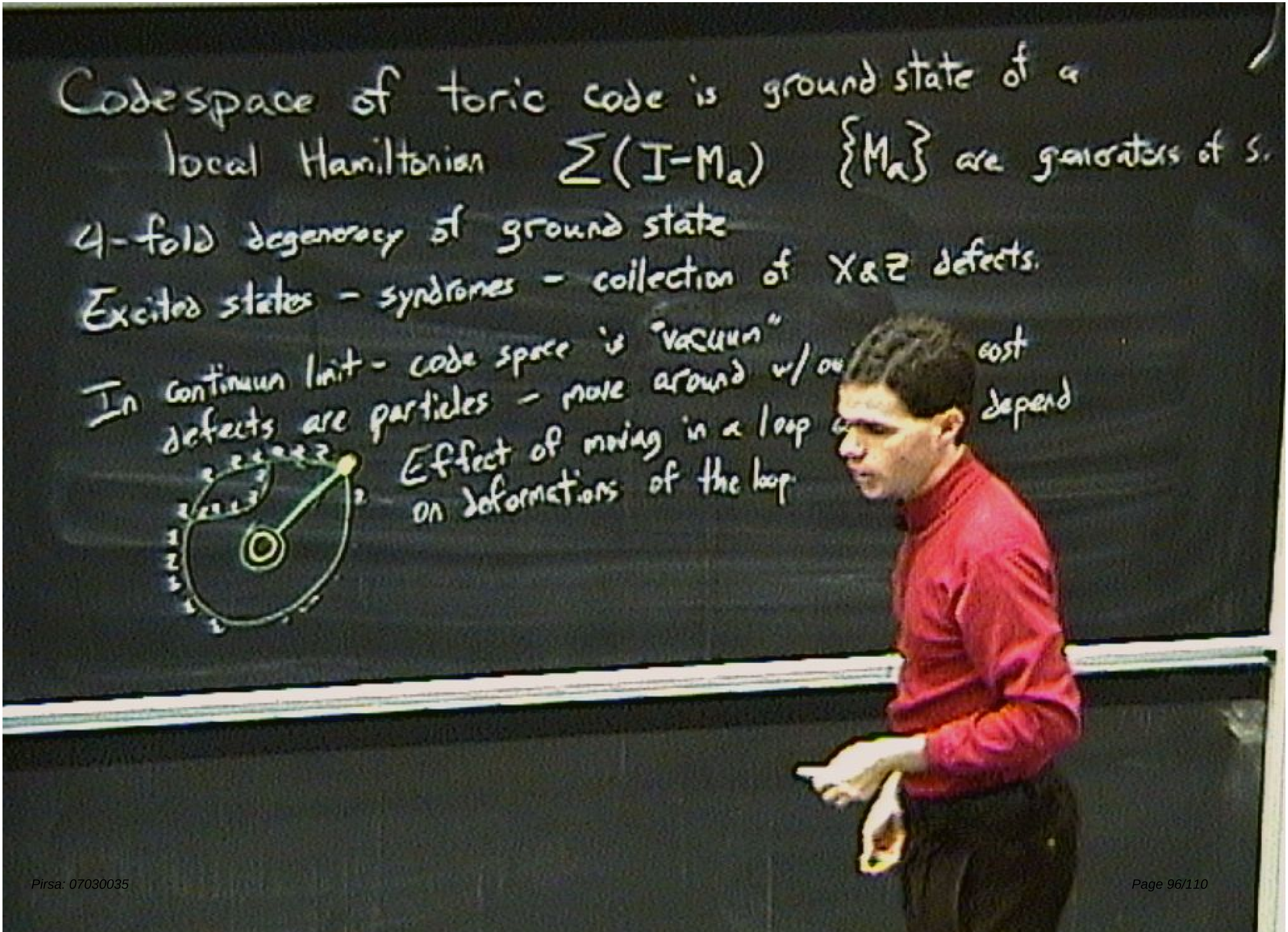
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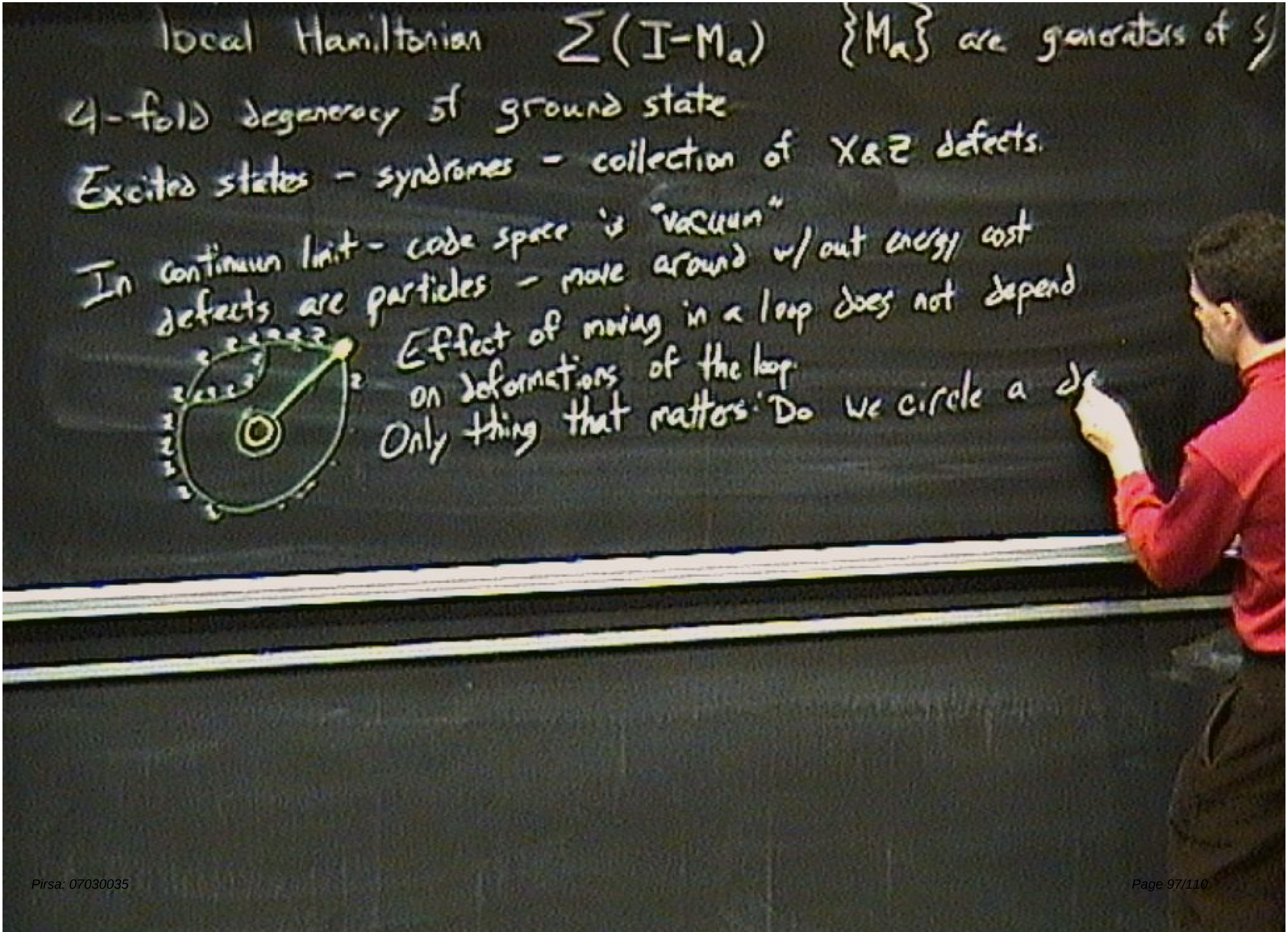
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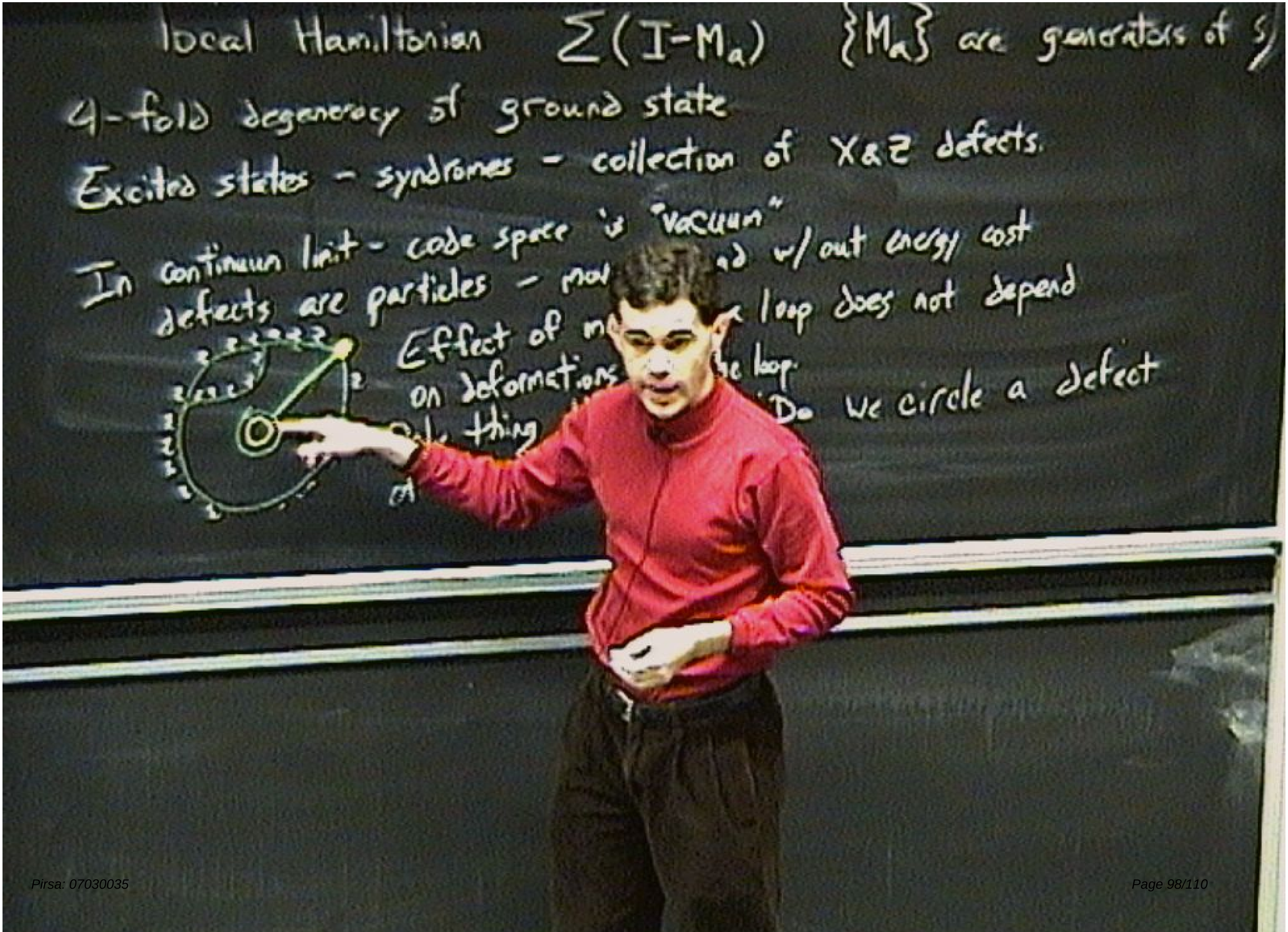
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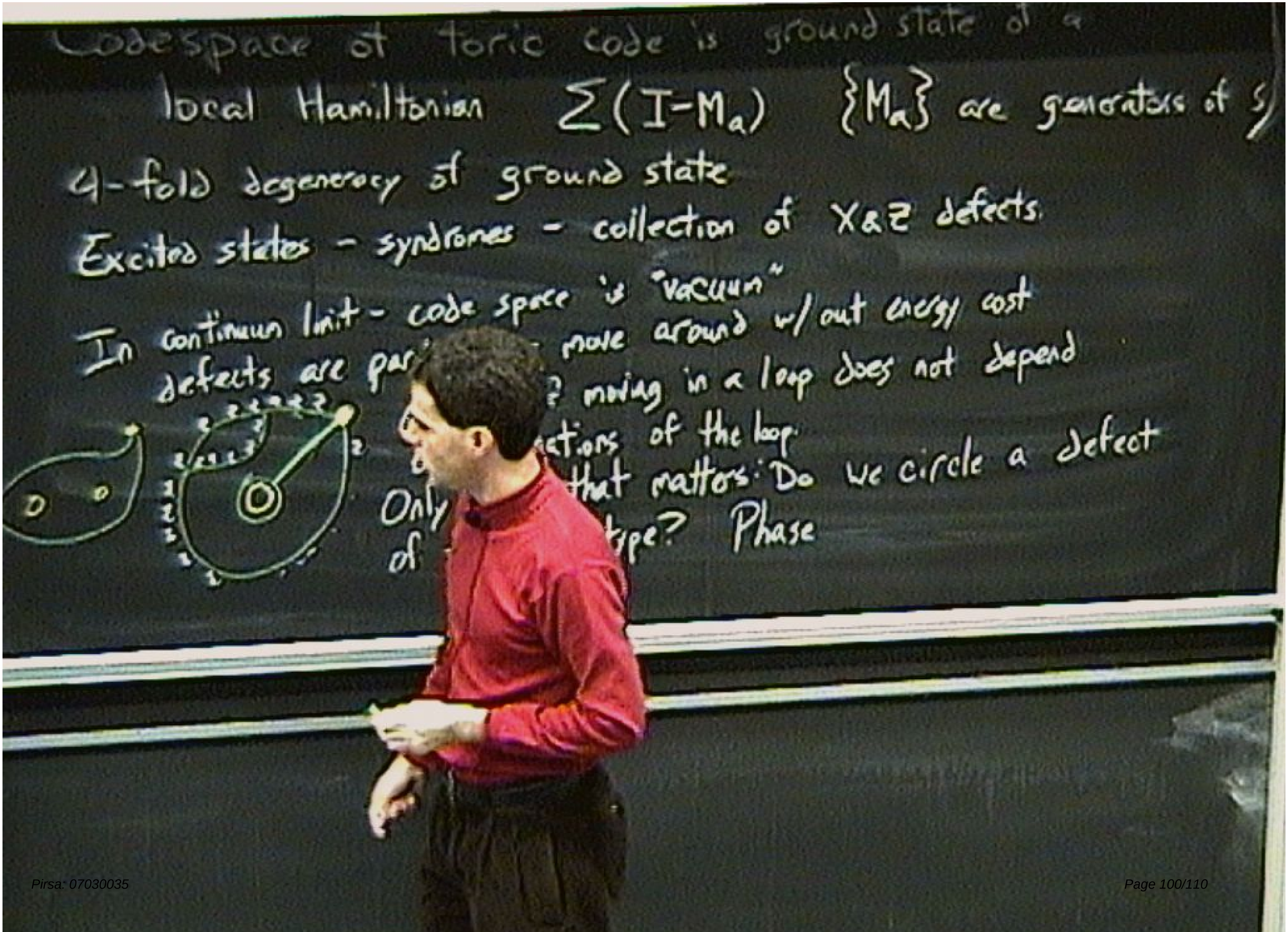

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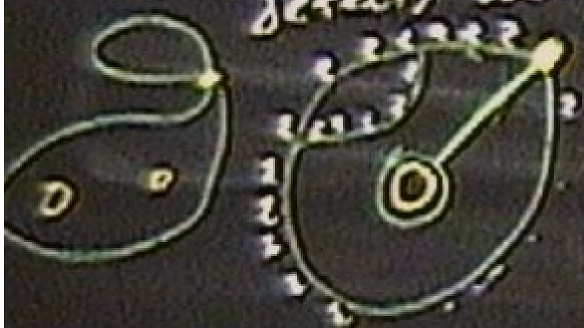

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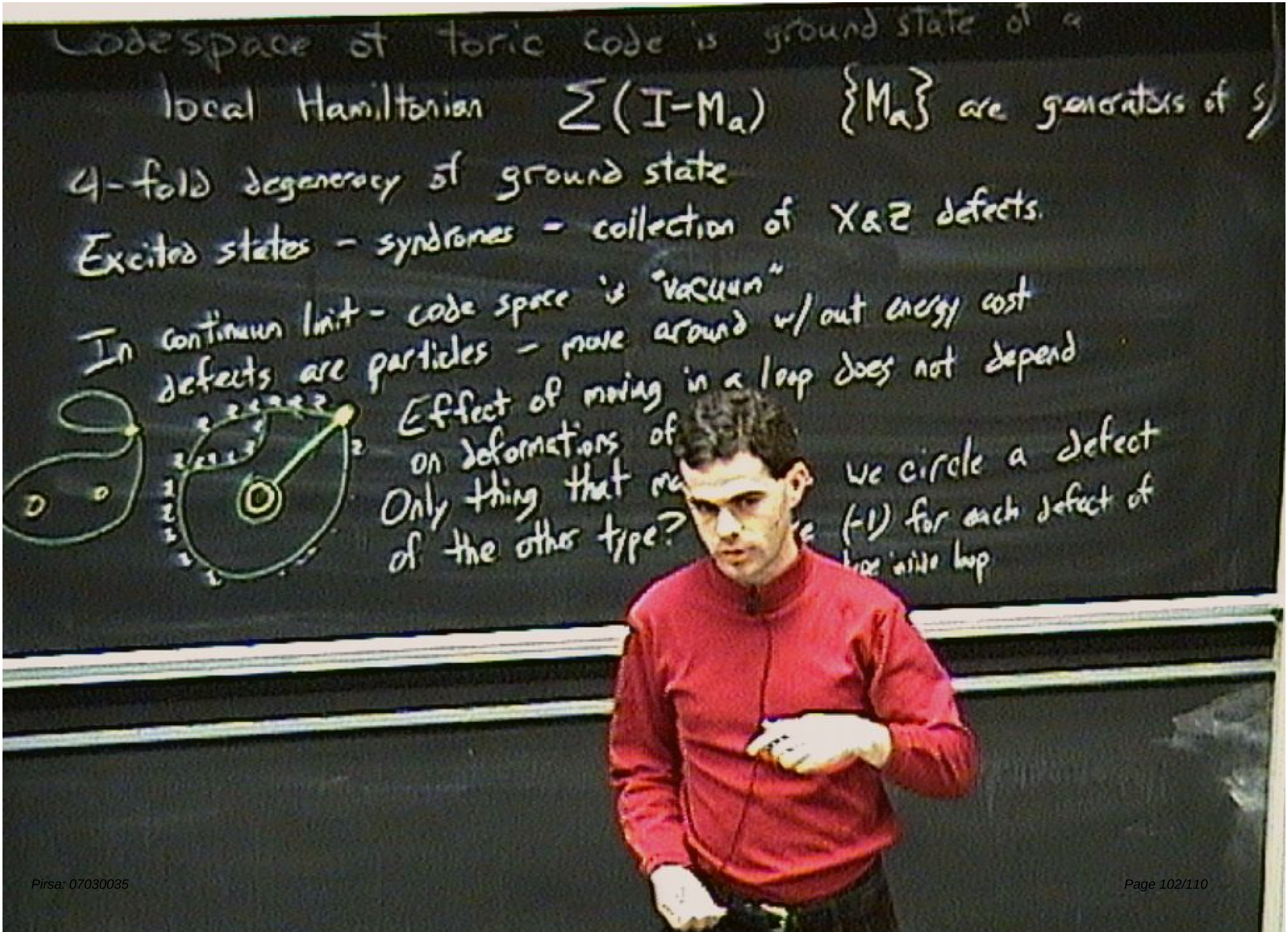
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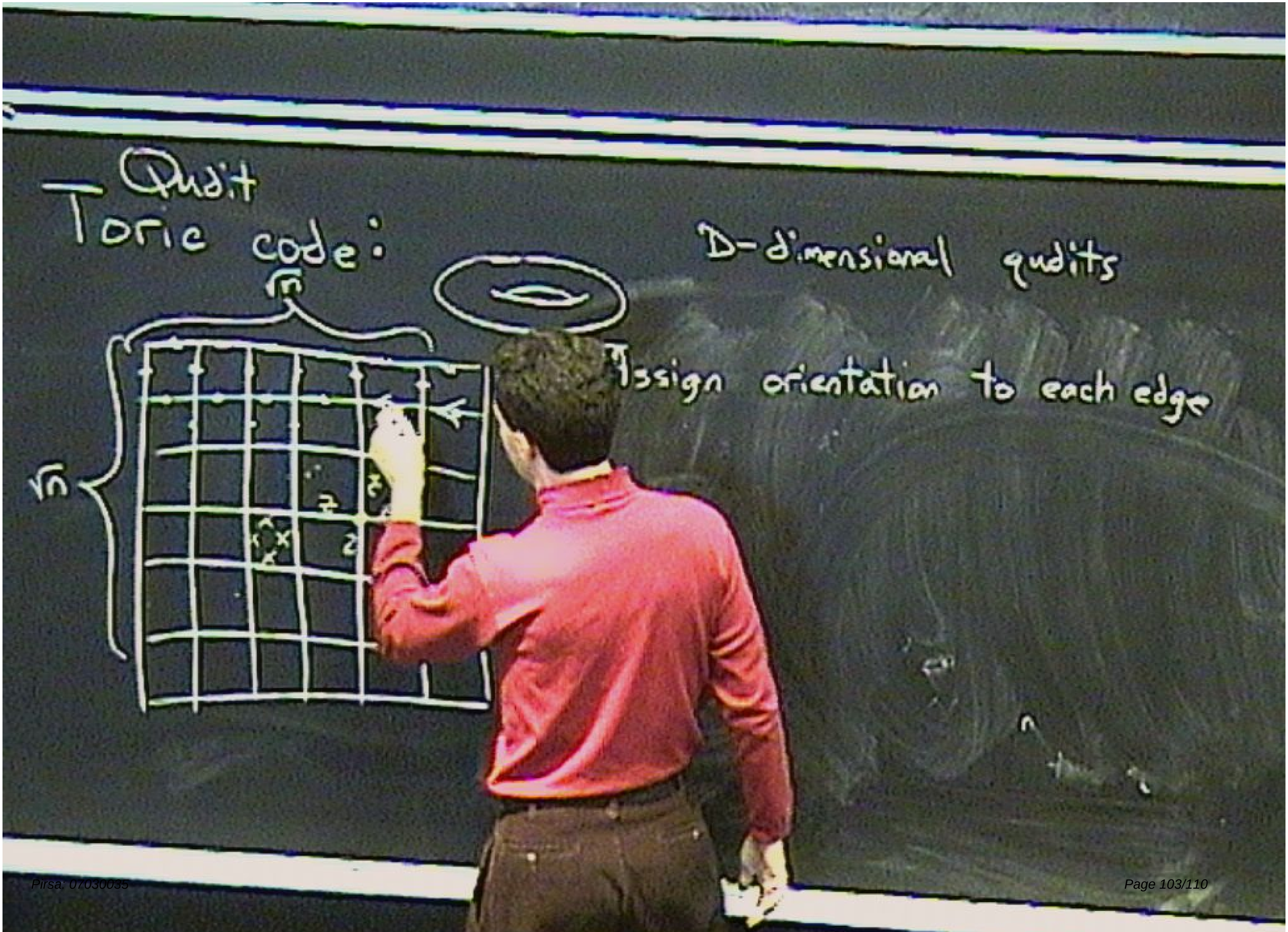
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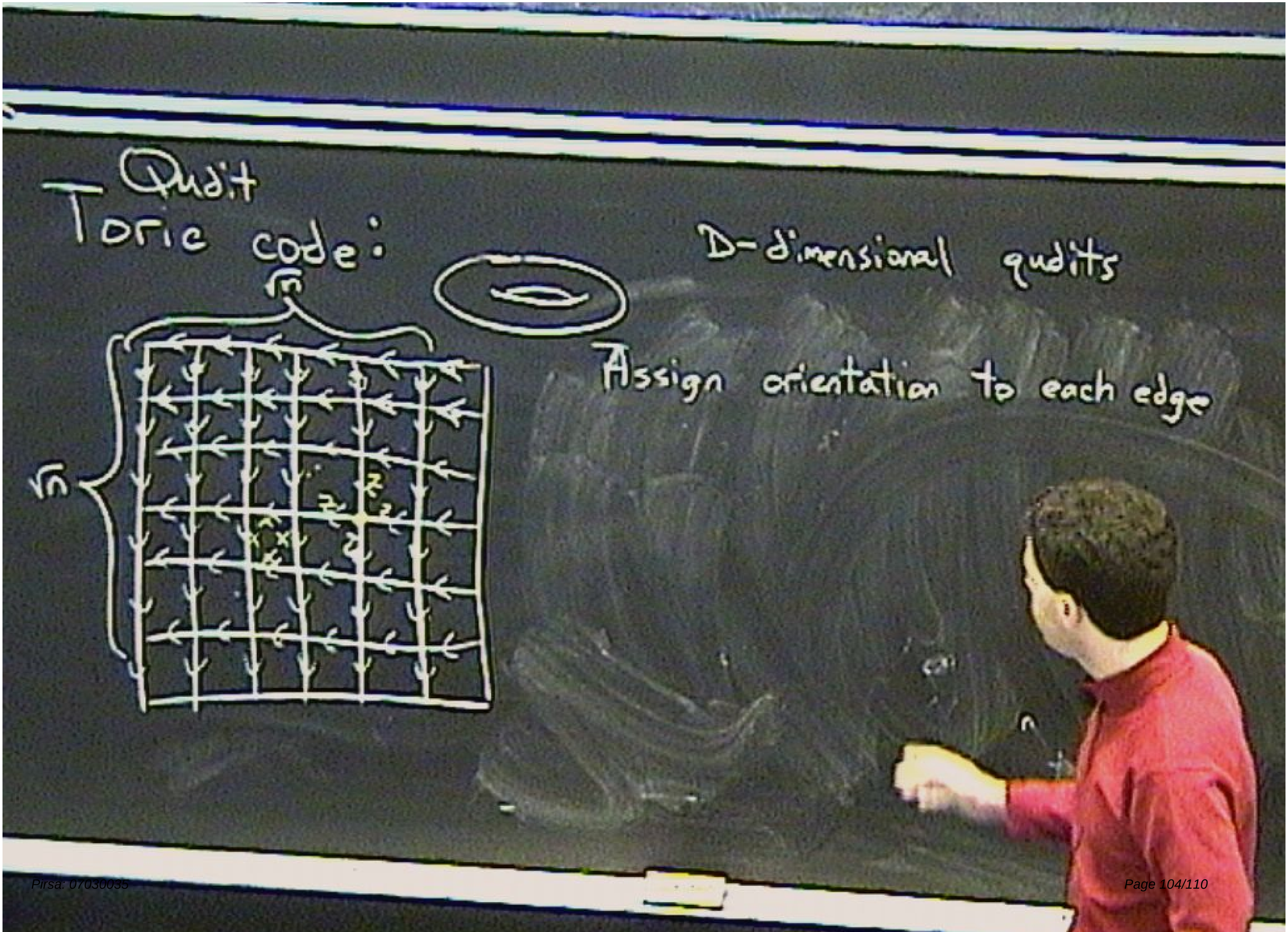
√n
D-dimensional qudits
Assign orientation to each edge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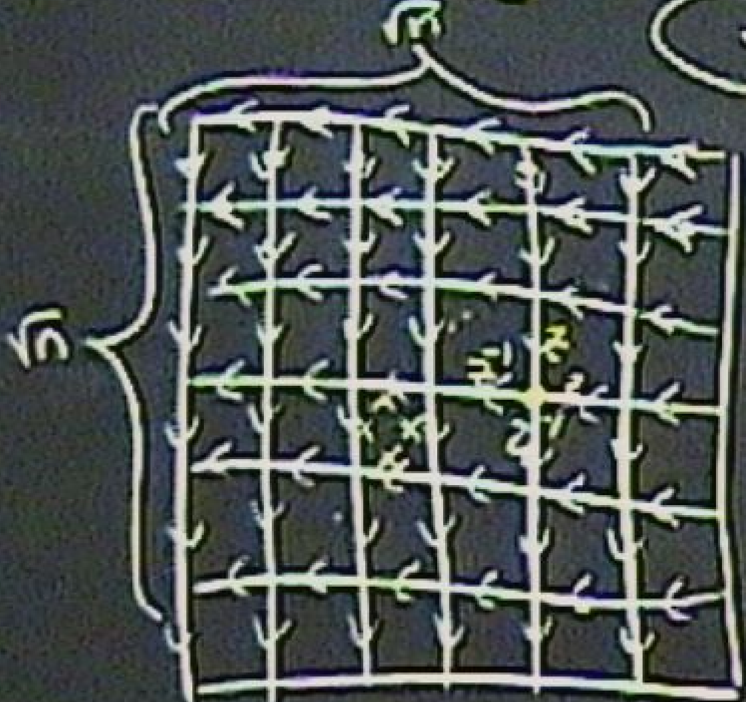

D-dimensional qudits

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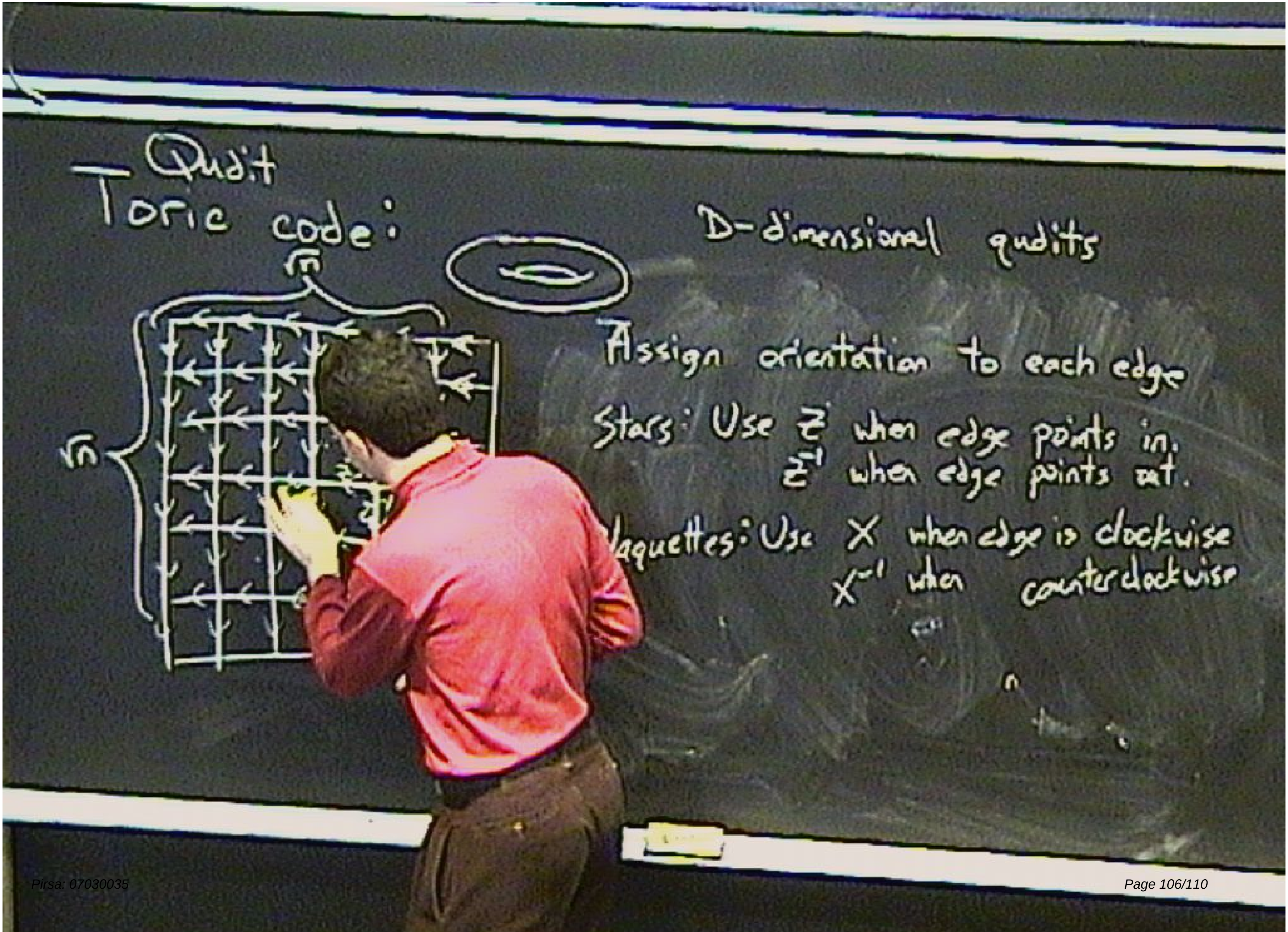
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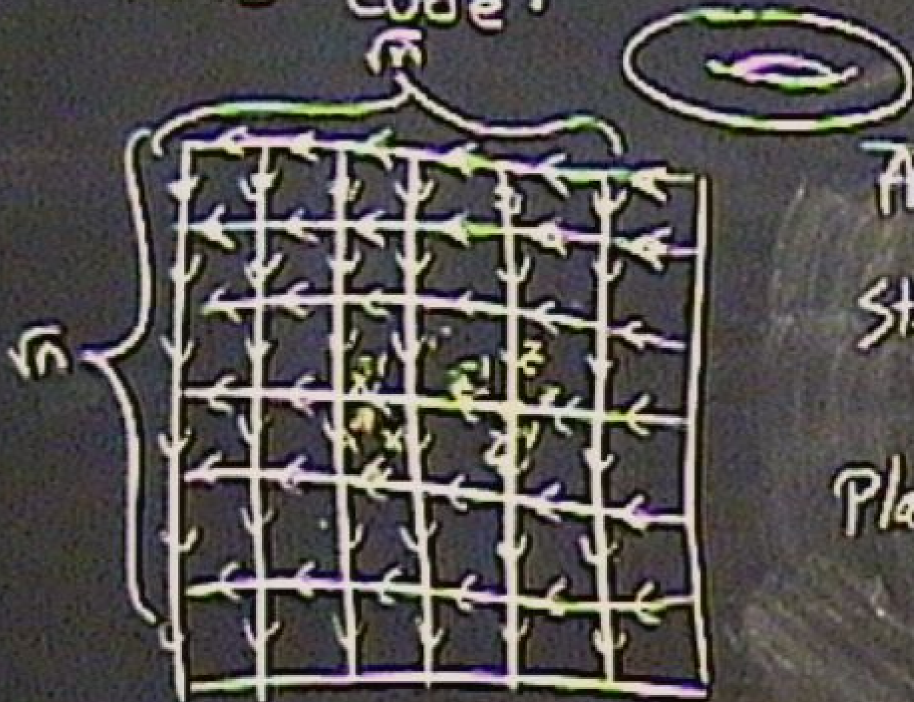
laquettes: Use X when edge is clockwise
X^-1 when counterclockwise

Qudit
Toric code:

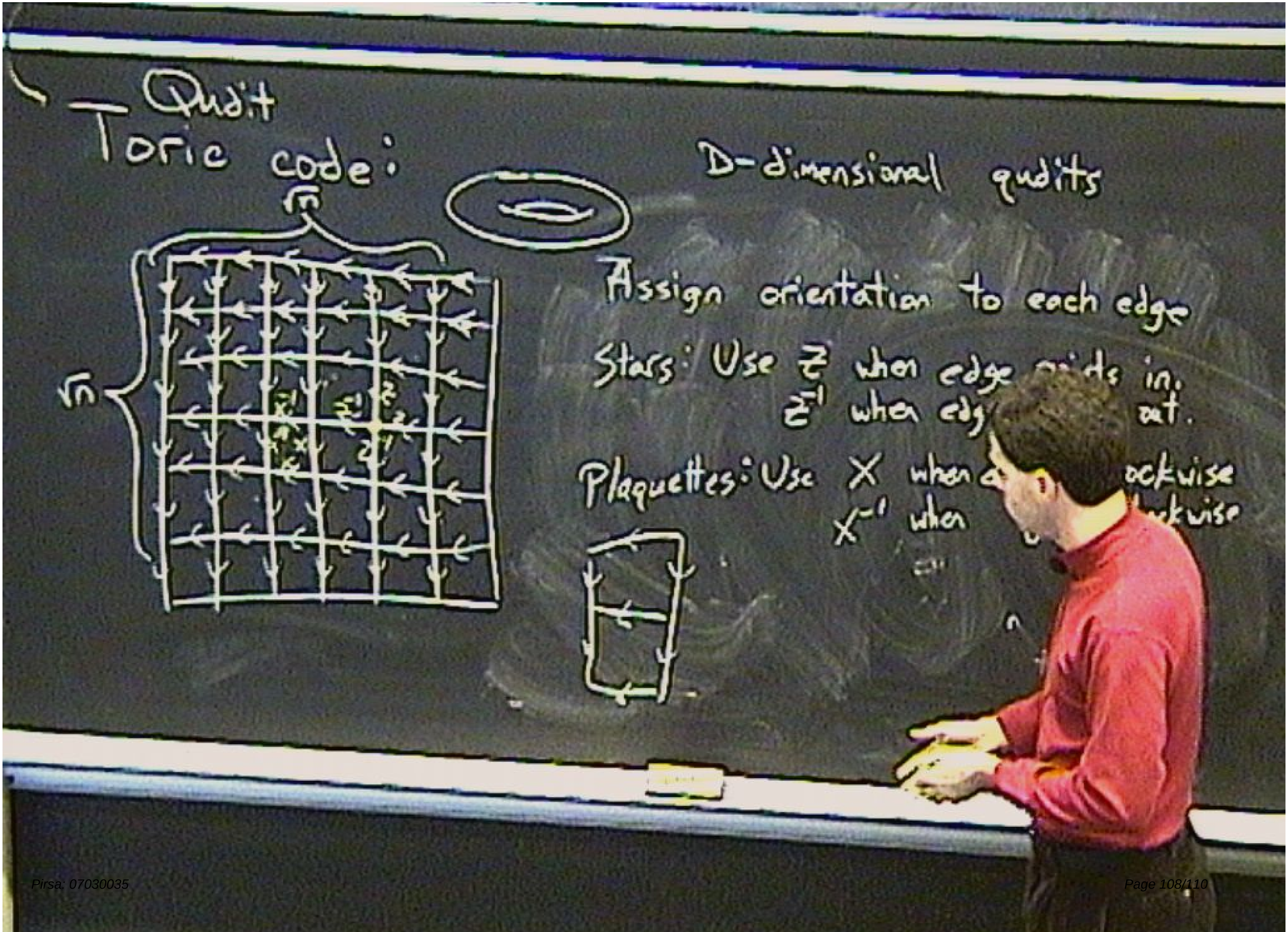


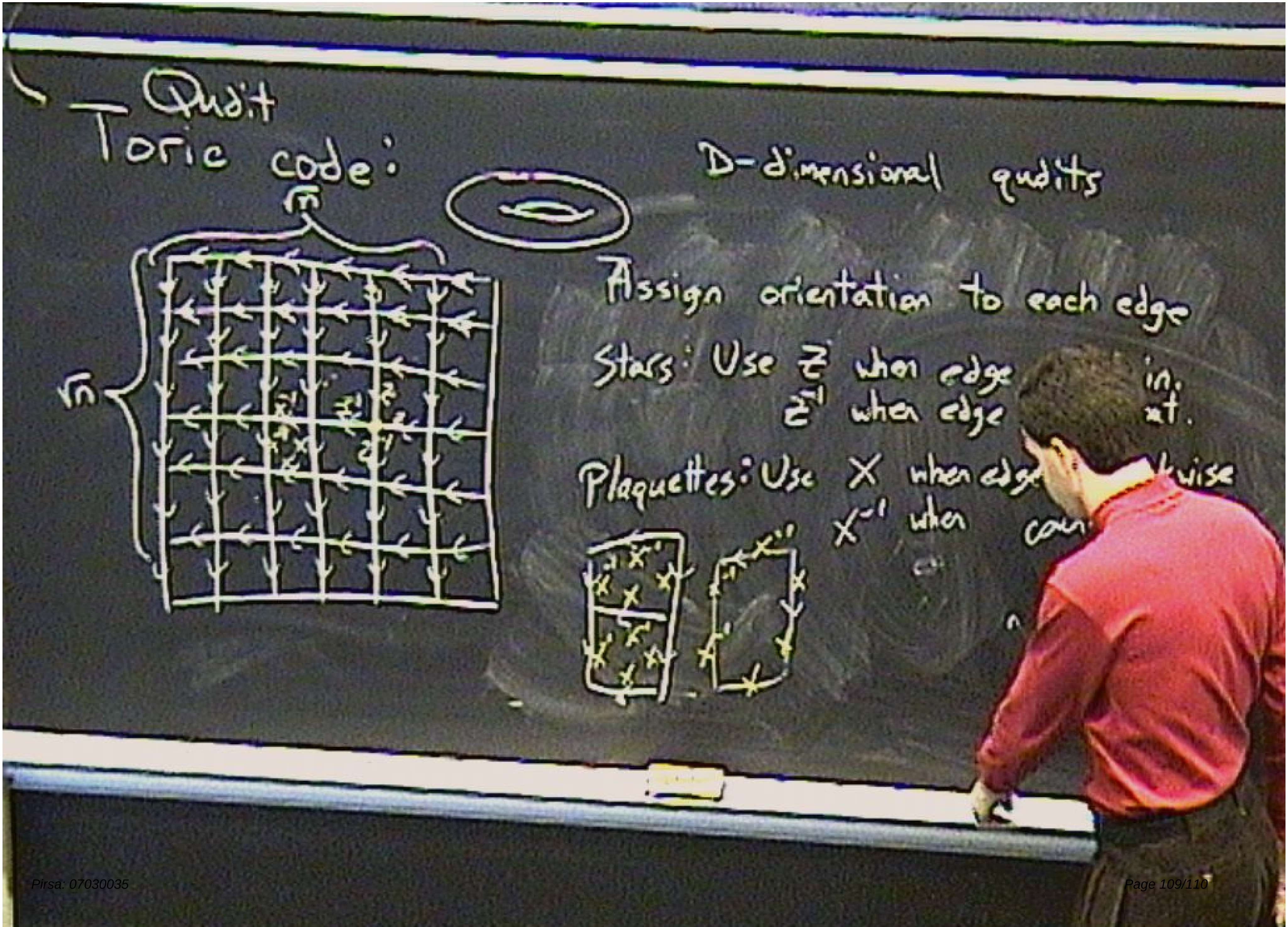
D-dimensional qudits

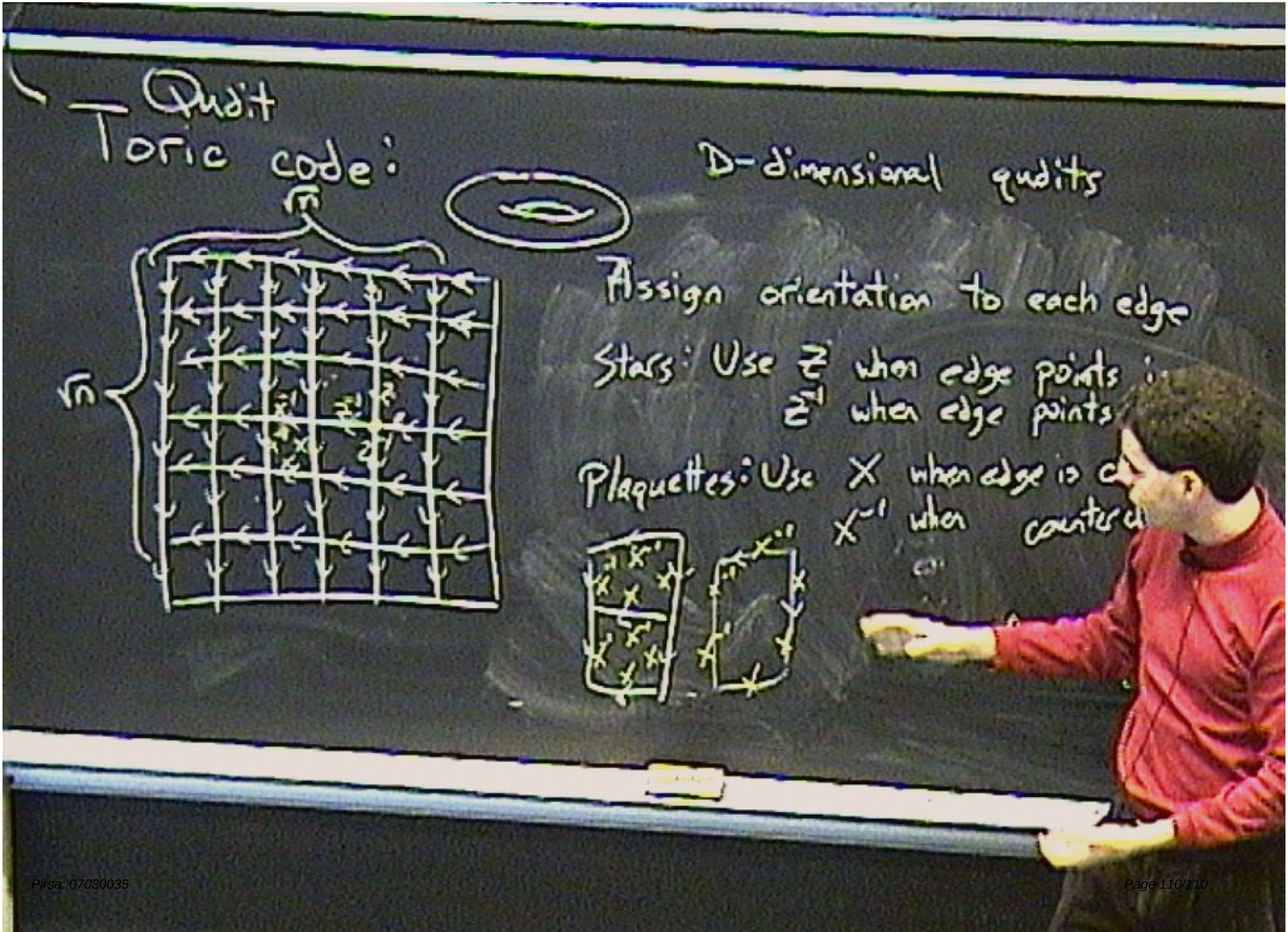
Assign orientation to each edge

Stars: Use Z when edge points in.
Z^{-1} when edge points out.

Plaquettes: Use X when edge is clockwise
X^{-1} when counterclockwise


Qudit
Toric code:
√n
√n
D-dimensional qudits
Assign orientation to each edge
Stars: Use Z when edge
in.
Z⁻¹ when edg
out.
Plaquettes: Use X when
ockwise
X⁻¹ when


Qudit
Toric code:
D-dimensional qudits
$\sqrt{n}$
$\sqrt{n}$
Assign orientation to each edge
Stars: Use Z when edge
in.
Z^{-1} when edge
ut.
Plaquettes: Use X when edge
wise
X^{-1} when
coun


Qudit
Toric code:
√n
√n
D-dimensional qudits
Assign orientation to each edge
Stars: Use Z when edge points
Z⁻¹ when edge points
Plaquettes: Use X when edge is c
X⁻¹ when counterc