Title: Transport properties of strongly coupled plasmas from black hole physics

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Abstract: Shear viscosity is a transport coefficient in the hydrodynamic description of liquids, gases and plasmas. The ratio of the shear viscosity and the volume density of the entropy has the dimension of the ratio of two fundamental constants - the Planck constant and the Boltzmann constant - and characterizes how close a given fluid is to a perfect fluid. Transport coefficients are notoriously difficult to compute from first principles.
Recent progress in string theory, in particular the development of the gauge-gravity duality, has enabled one to approach this problem from a totally unexpected perspective.
In my talk, I will describe the connection between the dynamics of black hole horizons (encoded in their quasinormal spectra) and the hydrodynamics of certain strongly coupled plasmas. I will comment on the relevance of this approach for the interpretation of data obtained in experiments on heavy ion collisions.

Transport properties of strongly coupled plasmas from black hole physics

Andrei Starinets

Perimeter Institute for Theoretical Physics

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Prologue

- We certainly have an intuitive notion of what viscosity, thermal conductivity and other transport coefficients mean

- As physical quantities, however, they are hard to compute from first principles, especially for liquids

- Here we discuss how transport coefficients in a large class of MODELS can be computed from higher-dimensional gravity

Holographic principle ('t Hooft, Susskind) relates a d+1 dimensional theory with gravitational d.o.f. to a non-gravitational theory in dimension d

Hydrodynamic properties of strongly interacting hot plasmas in 4 dimensions

can be related (for certain models!)

to fluctuations and dynamics of 5-dimensional black holes

Experimental and theoretical motivation

- Heavy ion collision program at RHIC, LHC (2000-2008-2020 ??)

- Studies of hot and dense nuclear matter

$T \sim 10^{12}\, K \sim 150,000\, T_{\odot}$ $\quad \epsilon \sim 5\,\mathrm{Gev/fm^3} \sim 30\,\epsilon_{\mathrm{nucleus}}$

- Abundance of experimental results, poor theoretical understanding:

- the collision apparently creates a fireball of "quark-gluon fluid"

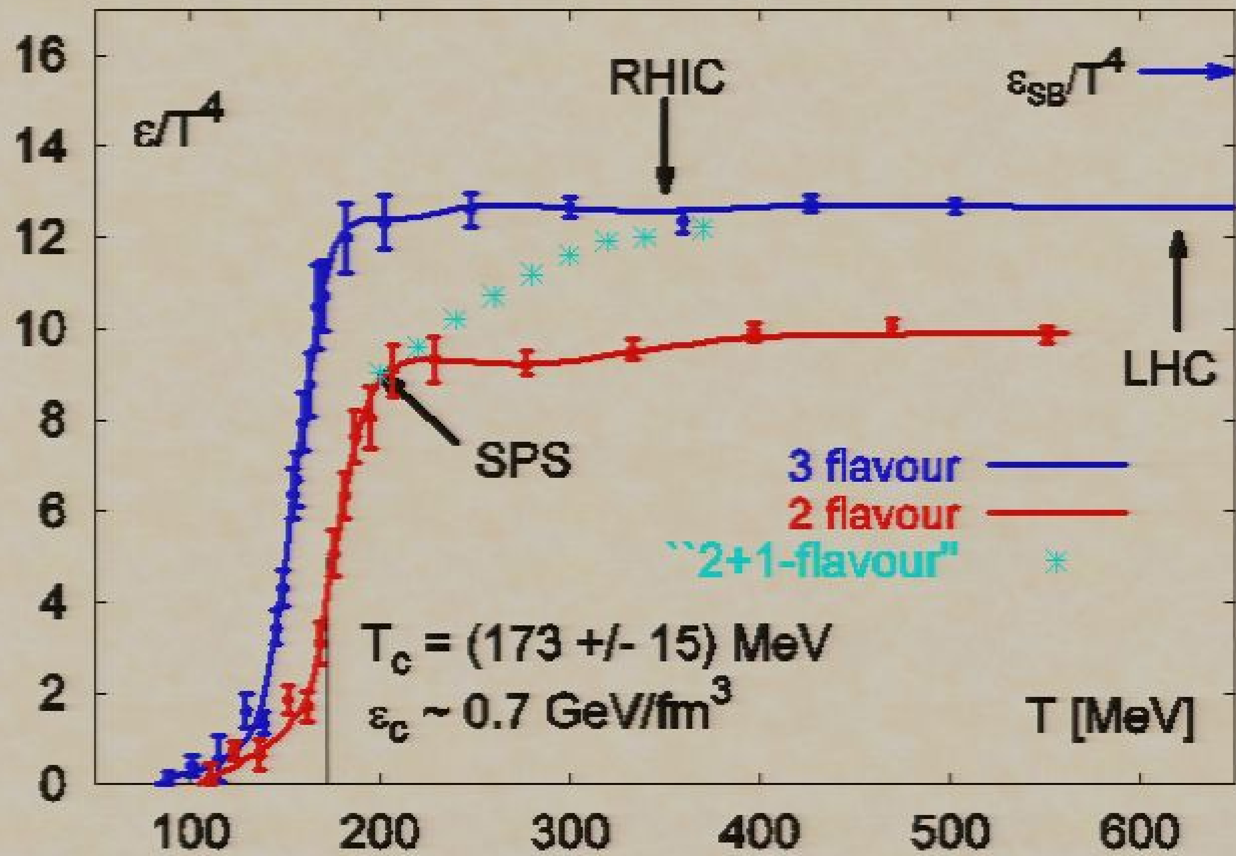
- need to understand both thermodynamics and kinetics

-in particular, need theoretical predictions for parameters entering equations of relativistic hydrodynamics – viscosity etc – computed from the underlying microscopic theory (thermal QCD)

-this is difficult since the fireball is a strongly interacting nuclear fluid, not a dilute gas

The challenge of RHIC

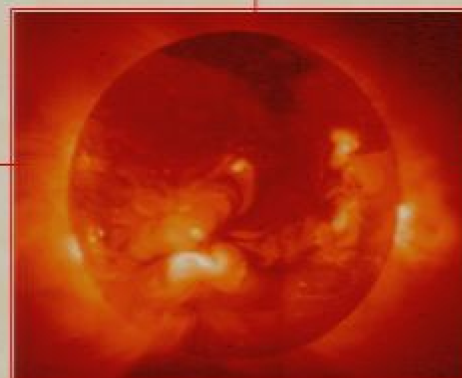
Energy density
vs
temperature



QCD deconfinement transition (lattice data)

$\alpha_s(T_{\mathrm{RHIC}}) \sim O(1)$

Quantum field theories at finite temperature/density



Equilibrium

entropy
equation of state
....

Near-equilibrium

transport coefficients
emission rates
....

perturbative — pQCD

non-perturbative — Lattice

perturbative — kinetic theory

non-perturbative — ????

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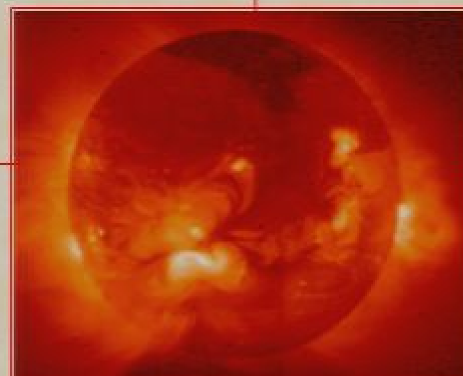

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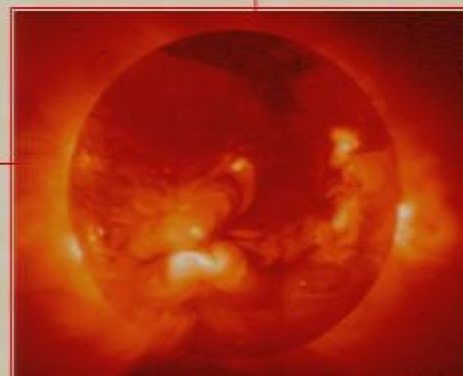

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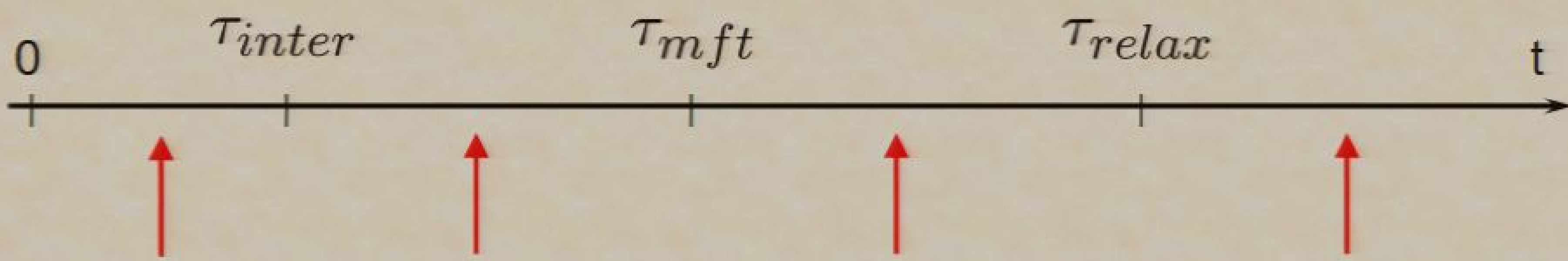
The hydrodynamic regime

Hierarchy of times (example)



Mechanical description	Kinetic theory	Hydrodynamic approximation	Equilibrium thermodynamics
Coordinates, momenta of individual particles	Coordinate- and time-dependent distribution functions	Local densities of conserved charges $\epsilon(x,t)$	Globally conserved charges E

Hydro regime: $\tau_{\text{micro}} \ll \tau \ll t_{\text{global}}$ $l_{\text{micro}} \ll l \ll L_{\text{global}}$

Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

Example: charge diffusion

Conservation law

$$\partial_t j^0 + \partial_i j^i = 0$$

Constitutive relation

[Fick's law (1855)]

$$j_i = -D\,\partial_i j^0 + O[(\nabla j^0)^2, \nabla^2 j^0]$$

Diffusion equation

$$\partial_t j^0 = D \nabla^2 j^0$$

Dispersion relation

$$\omega = -i\,D\,q^2 + \cdots$$

Expansion parameters: $\omega \ll T, \quad q \ll T$

Similarly, one can analyze another conserved quantity – energy-momentum tensor:

$$\partial_\mu T^{\mu\nu} = 0$$

This is equivalent to analyzing fluctuations of energy and pressure

$$\langle T^{00} \rangle = \epsilon \qquad \langle T^{ij} \rangle = P\,\delta^{ij}$$

We obtain a dispersion relation for the sound wave:

$$\omega = \pm v_s\, q - \frac{i}{2(\epsilon + P)} \left(\frac{4}{3}\eta + \zeta \right) q^2$$

In quantum field theory, the dispersion relations such as

$$\omega = \pm v_s\, q - \frac{i}{2(\epsilon + P)} \left(\frac{4}{3}\eta + \zeta\right) q^2$$

appear as poles of the retarded correlation functions, e.g.

$$\langle T_{00}(k)\, T_{00}(-k)\rangle \sim \frac{q^2\, T^4}{\omega^2 - q^2/3 + i\omega q^2/3\pi T}$$

- in the hydro approximation - $\omega/T \ll 1\,, \quad q/T \ll 1$

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to compute transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In a certain regime, the correlators can be computed from black hole physics
using the gauge-gravity duality in string theory

- Transport coefficients and the speed of sound can be computed from string theory for SOME thermal gauge theories in nonperturbative regime

- This calculation is based on the approach known as "gauge/gravity duality" or "AdS/CFT correspondence"

AdS = Anti de Sitter space
CFT = Conformal Field Theory

Maldacena, 1997; Gubser, Klebanov, Polyakov; Witten, 1998

In quantum field theory, the dispersion relations such as

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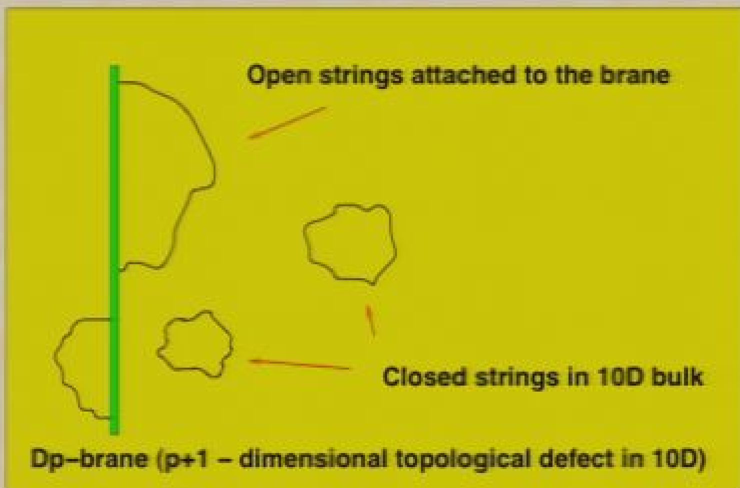
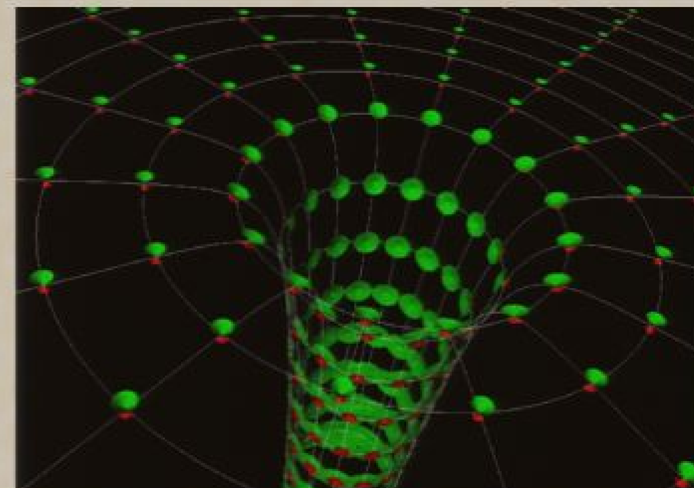
Gauge-gravity duality in string theory





Perturbative string theory: open and closed strings
(at low energy, gauge fields and gravity, correspondingly)

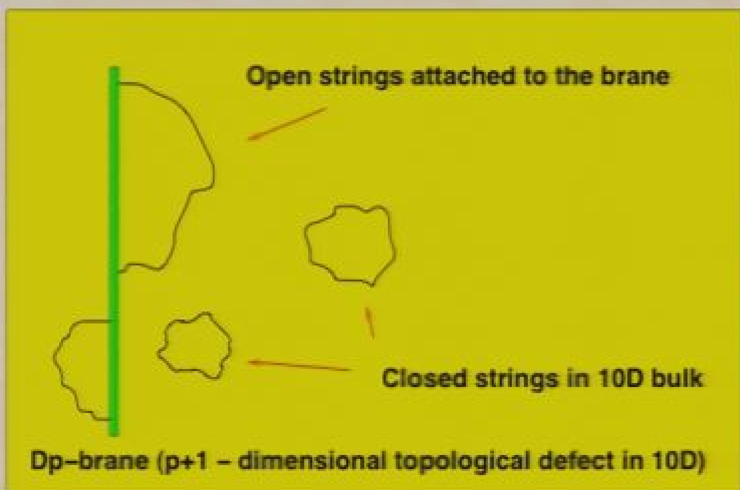
Nonperturbative theory: D-branes ("topological defects" in 10d)

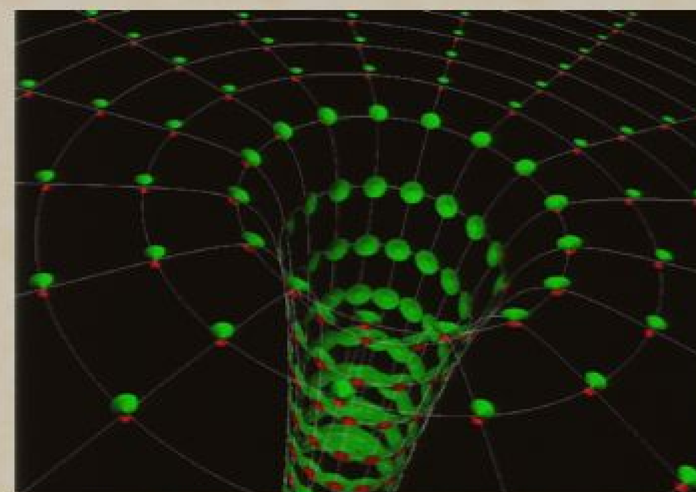
Complementary description of D-branes by open (closed) strings:

$g_{YM}^2 N_c \ll 1$ perturbative gauge theory description OK

$g_{YM}^2 N_c \gg 1\,, \;\; N_c \gg 1$ perturbative gravity description OK

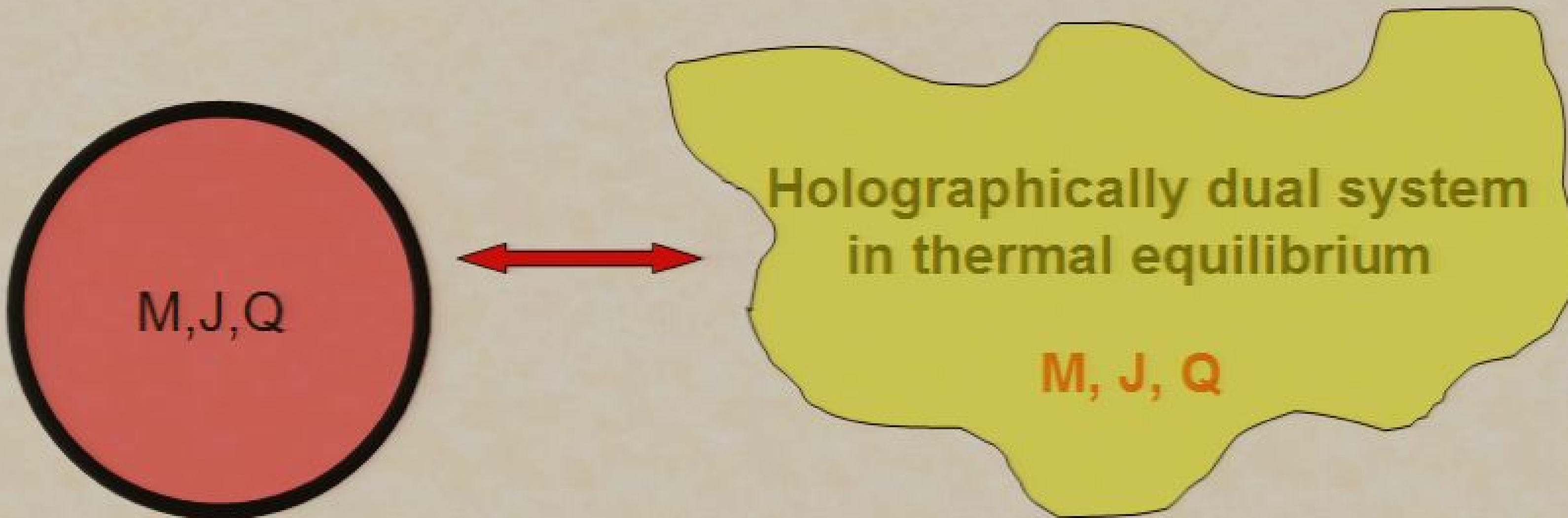
Gauge-gravity duality





- Two descriptions of a D-brane (topolog. defect)
- 4-dim gauge theory at strong coupling = 5-dim gravity at weak coupling
- Build a dictionary between the two descriptions

$$\langle e^{\int d^4x\, \phi_0\, \mathcal{O}} \rangle_{\text{field theory}} = e^{-S_{\text{grav}}[\phi]}$$



T_{Hawking} $S_{\text{Bekenstein-Hawking}} = A/4$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $\Longleftrightarrow$????

$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box"\ h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$

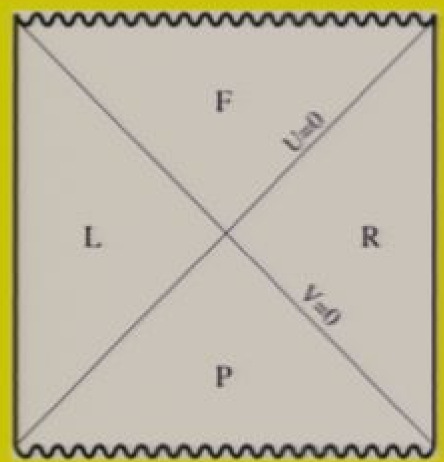
$$\partial_t j^0 + \partial_i j^i = 0$$

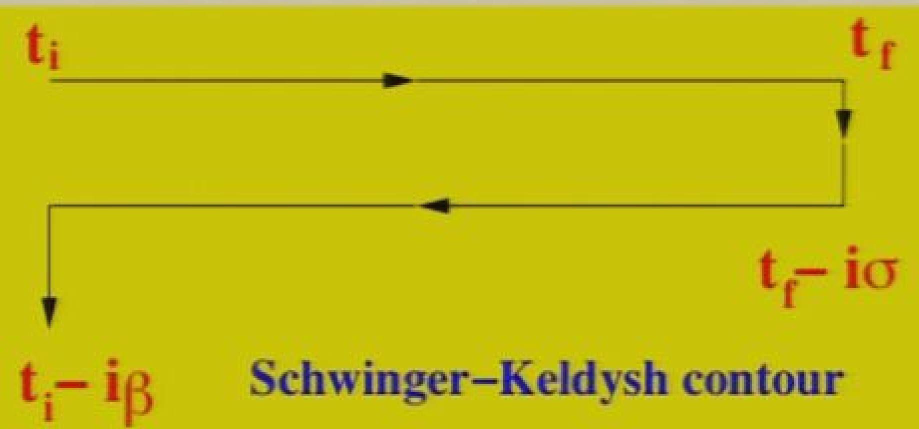
$$\partial_t j^0 = D\nabla^2 j^0$$

Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$

Computing finite-temperature correlation functions from gravity

- Need to solve 5d e.o.m. of the dual fields propagating in asymptotically AdS space
- Can compute Minkowski-space 4d correlators
- Gravity maps into real-time finite-temperature formalism (Son and A.S., 2001; Herzog and Son, 2002)





Predictions of hydrodynamics

Hydrodynamics predicts that the retarded correlator

$$\langle T_{00}(k)\, T_{00}(-k)\rangle$$

has a "sound wave" pole at

$$\omega = v_s\, q - \frac{i}{2(\epsilon + P)}\left(\frac{4}{3}\eta + \zeta\right) q^2$$

Moreover, in conformal theory $\epsilon = 3P \implies v_s^2 = \frac{\partial P}{\partial \epsilon} = 1/3$

Now look at the correlators obtained from gravity

$$\langle T_{00}(k) T_{00}(-k) \rangle = \frac{3N^2\pi^2 T^4 q^2}{2(\omega^2 - q^2/3 + i\omega q^2/3\pi T)} + \cdots$$

The correlator has poles at $\omega = \pm \frac{q}{\sqrt{3}} - i \frac{q^2}{6\pi T} + \cdots$

The speed of sound coincides with the hydro prediction!

$$\left.\begin{aligned} \eta &= \frac{\pi N^2 T^3}{8} \\ s &= \frac{\pi^2}{2} N^2 T^3 \end{aligned}\right\} \quad \frac{\eta}{s} = \frac{1}{4\pi}$$

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Transport (kinetic) coefficients

- Shear viscosity η
- Bulk viscosity ζ
- Charge diffusion constant D_Q
- Thermal conductivity κ_T
- Electrical conductivity σ

Transport coefficients in N=4 SYM

- Shear viscosity $\eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2} \right) \right]$
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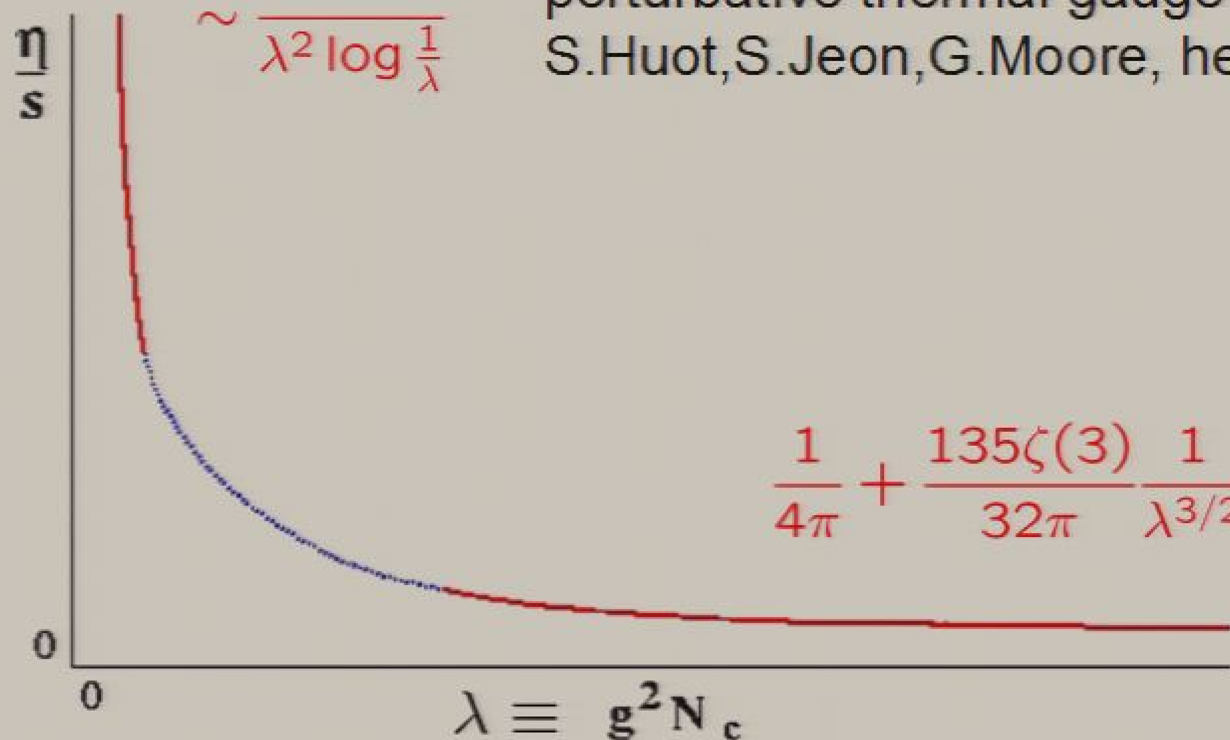
- Electrical conductivity $\sigma = e^2 \frac{N_c^2 T}{16 \pi} + \cdots$

Shear viscosity in $\mathcal{N} = 4$ SYM

$\sim \frac{1}{\lambda^2 \log \frac{1}{\lambda}}$ perturbative thermal gauge theory

S.Huot,S.Jeon,G.Moore, hep-ph/0608062



$$\frac{1}{4\pi} + \frac{135\zeta(3)}{32\pi}\frac{1}{\lambda^{3/2}} + \cdots$$

Correction to $1/4\pi$: A.Buchel, J.Liu, A.S., hep-th/0406264

Universality of η/s

Theorem:

For a thermal gauge theory, the ratio of shear viscosity to entropy density is equal to $1/4\pi$
in the regime described by a dual gravity theory

(e.g. at $g^2_{YM}N_c = \infty\,, N_c = \infty$ in $\mathcal{N} = 4$ SYM)

Remarks:

- Extended to non-zero chemical potential:

  Benincasa, Buchel, Naryshkin, hep-th/0610145

- Extended to models with fundamental fermions in the limit $N_f/N_c \ll 1$

  Mateos, Myers, Thomson, hep-th/0610184

- *String/Gravity dual to QCD is currently unknown*

Shear viscosity in $\mathcal{N}=4$ SYM

perturbative thermal gauge theory
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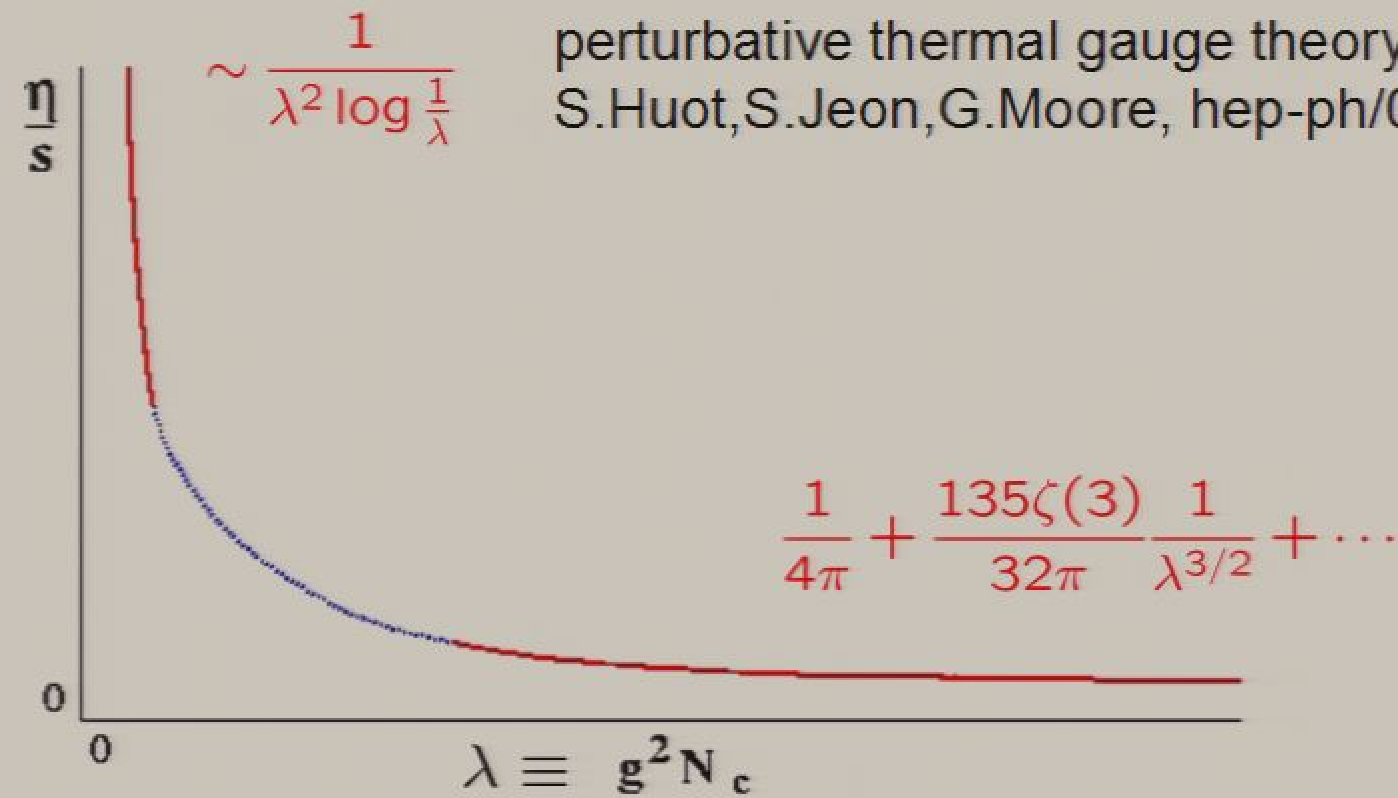

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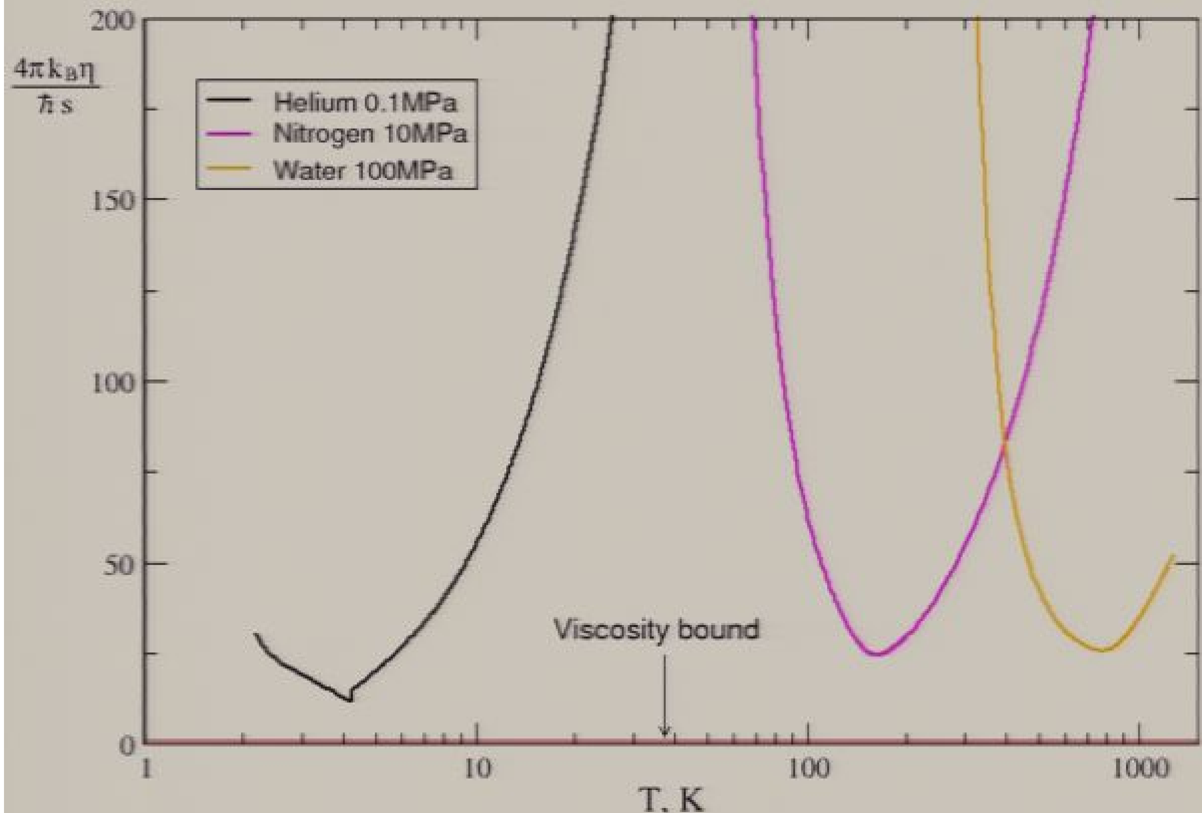
A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13}\, K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

A hand-waving argument

$$\eta \sim \rho v l \sim \rho v^2 \tau \sim n m v^2 \tau \sim n \epsilon \tau$$

$$s \sim n$$

Thus $$\frac{\eta}{s} \sim \epsilon\tau \geq \hbar$$

Gravity duals fix the coefficient: $$\frac{\eta}{s} \geq \hbar/4\pi$$

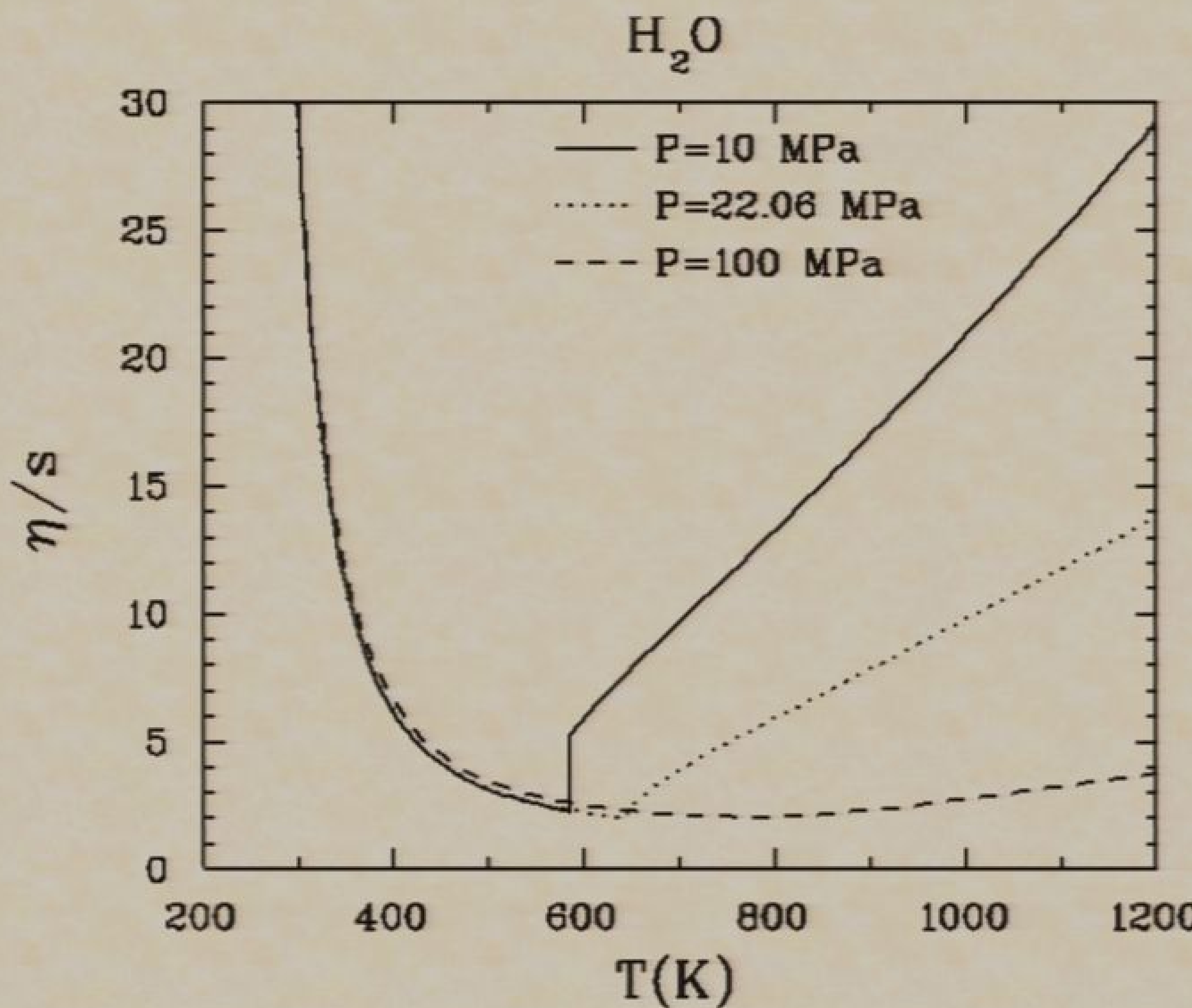
Can we test

$$\eta/s \geq 1/4\pi$$

experimentally?

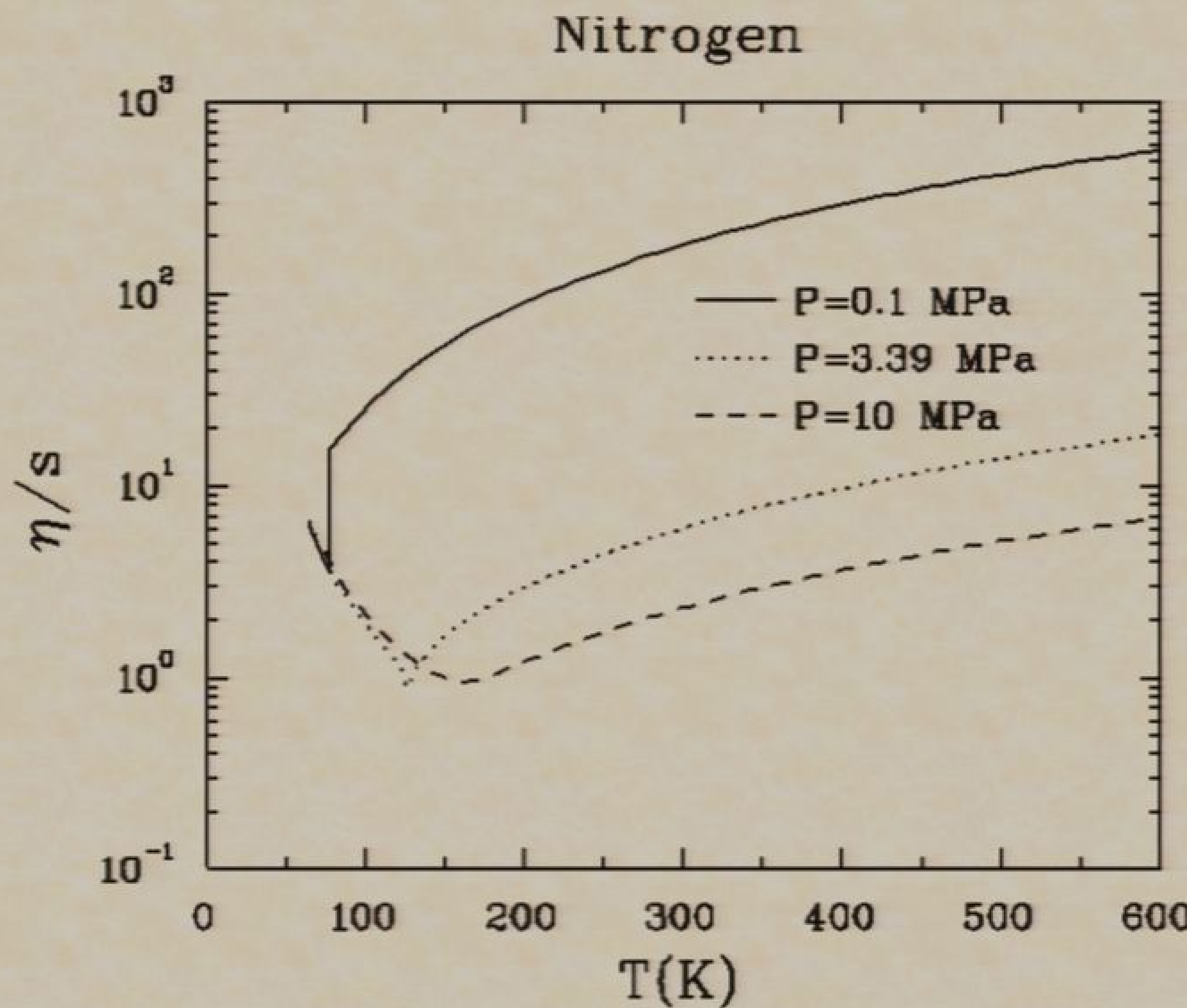
A characteristic feature of systems saturating the bound: strong interactions

- Heavy ion collisions - experiments at RHIC
- (Indirect) lattice QCD simulations
- Trapped atoms – strongly interacting Fermi systems



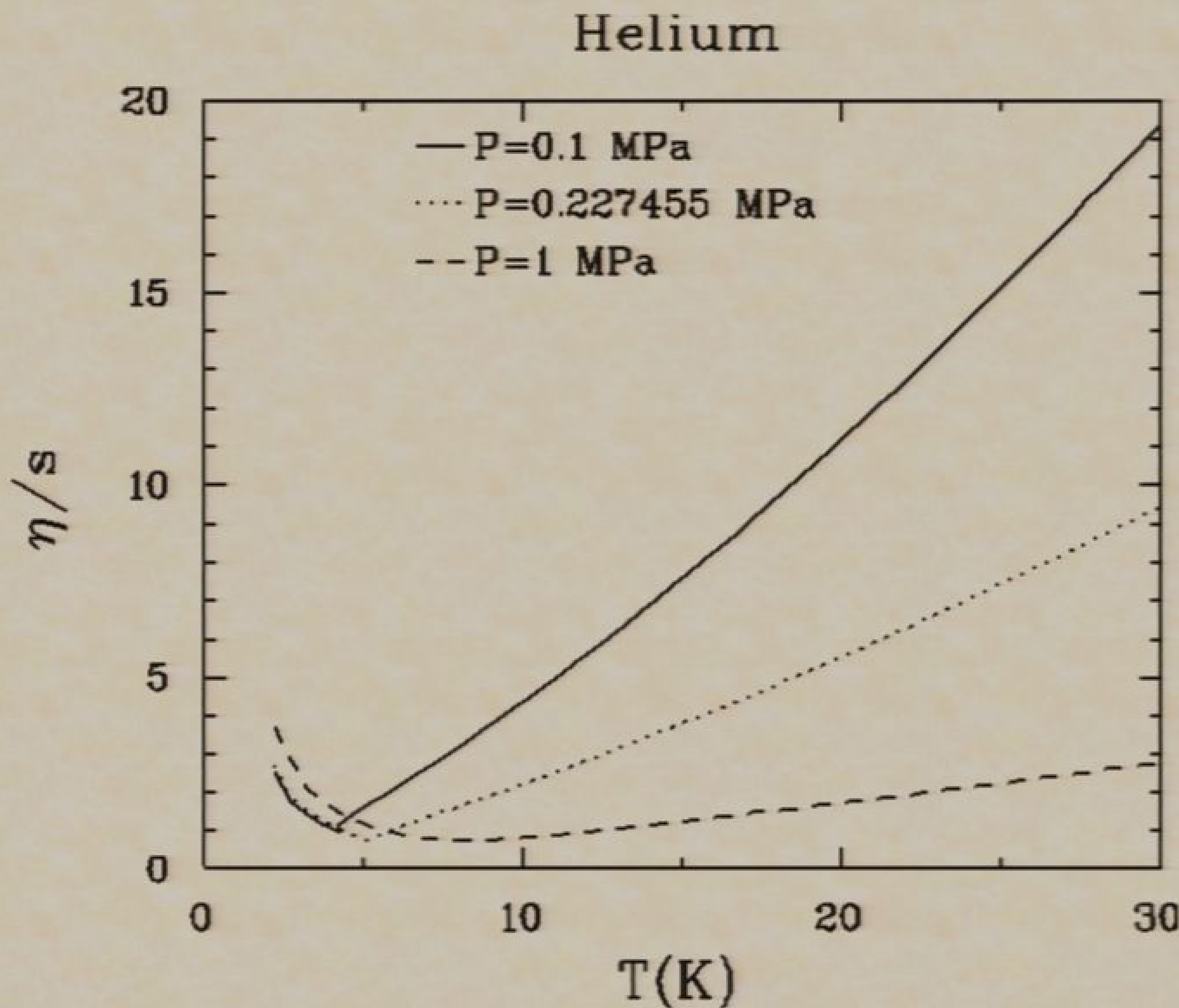
$(\eta/s)_{\rm min} \sim 25$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032



$(\eta/s)_{\mathrm{min}} \sim 23$ in units of $\frac{\hbar}{4\pi k_B}$

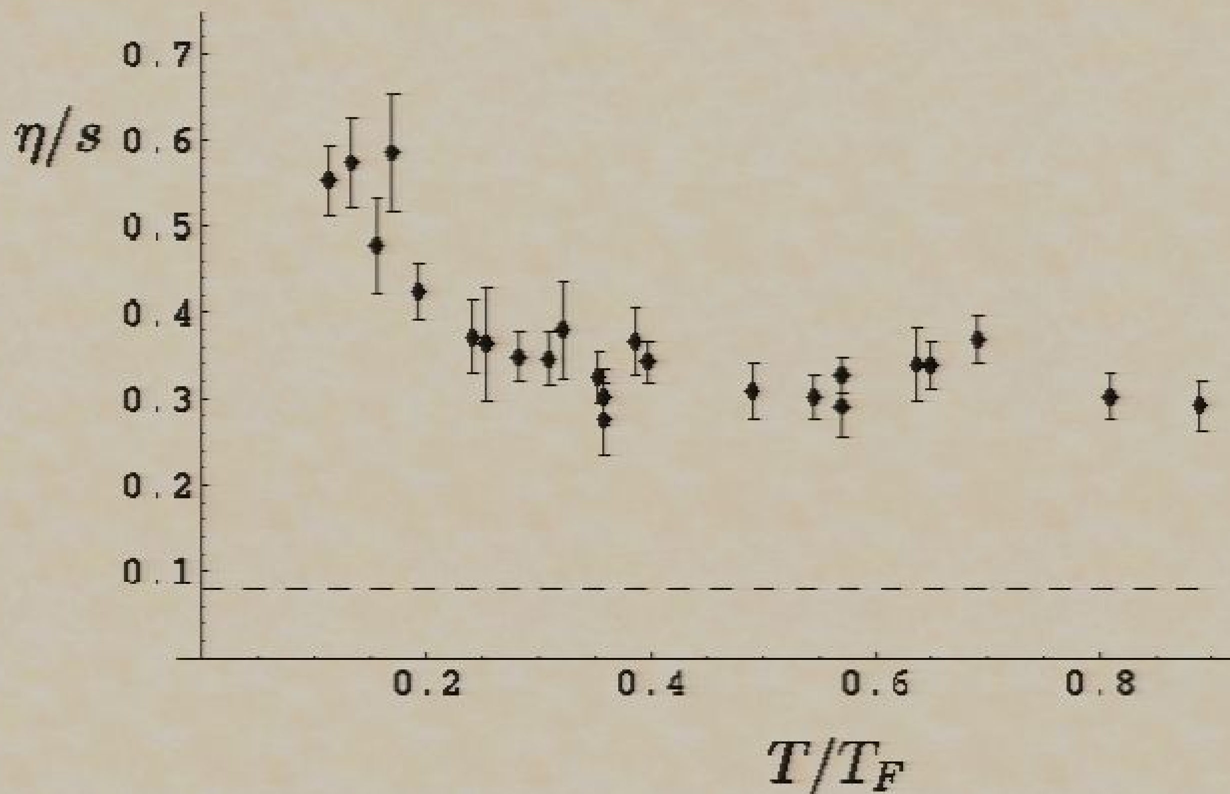
Chernai, Kapusta, McLerran, nucl-th/0604032



$(\eta/s)_{\rm min} \sim 8.8$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

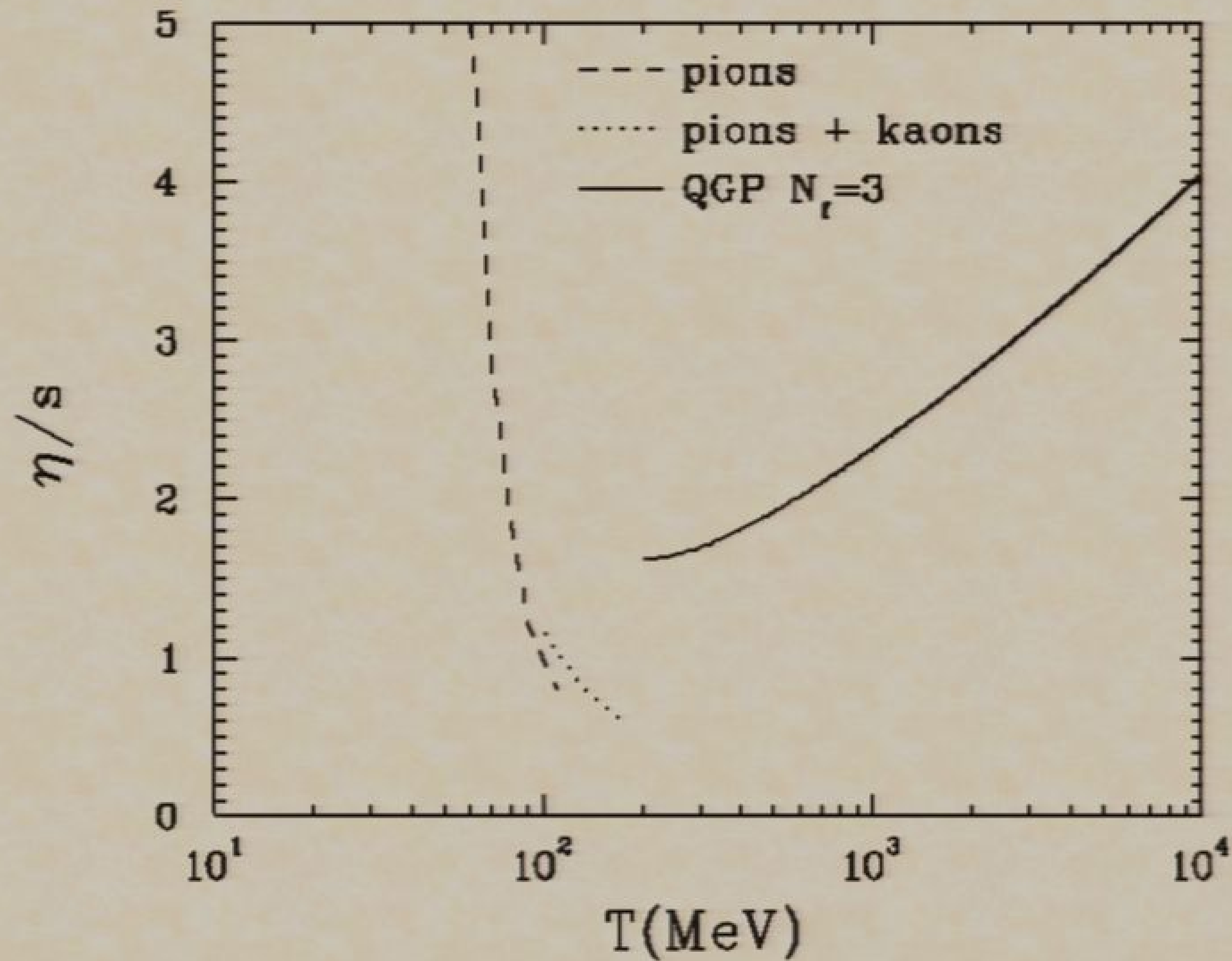
Viscosity-entropy ratio of a trapped Fermi gas



$\eta/s \sim 4.2$ in units of $\frac{\hbar}{4\pi k_B}$ T.Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

QCD



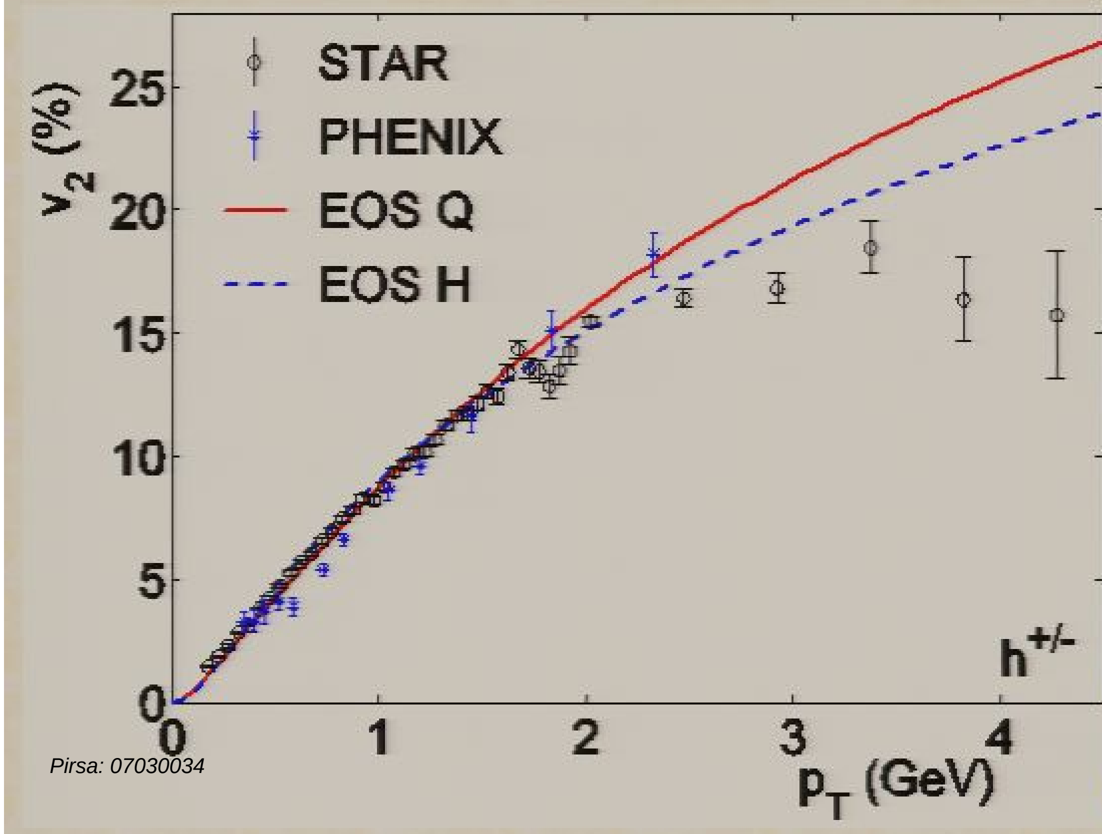
Chernai, Kapusta, McLerran, nucl-th/0604032

Viscosity "measurements" at RHIC

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$$0 < \eta/s \leq 3.8 \times \frac{\hbar}{4\pi k_B}$$

e.g. Baier, Romatschke, nucl-th/0610108

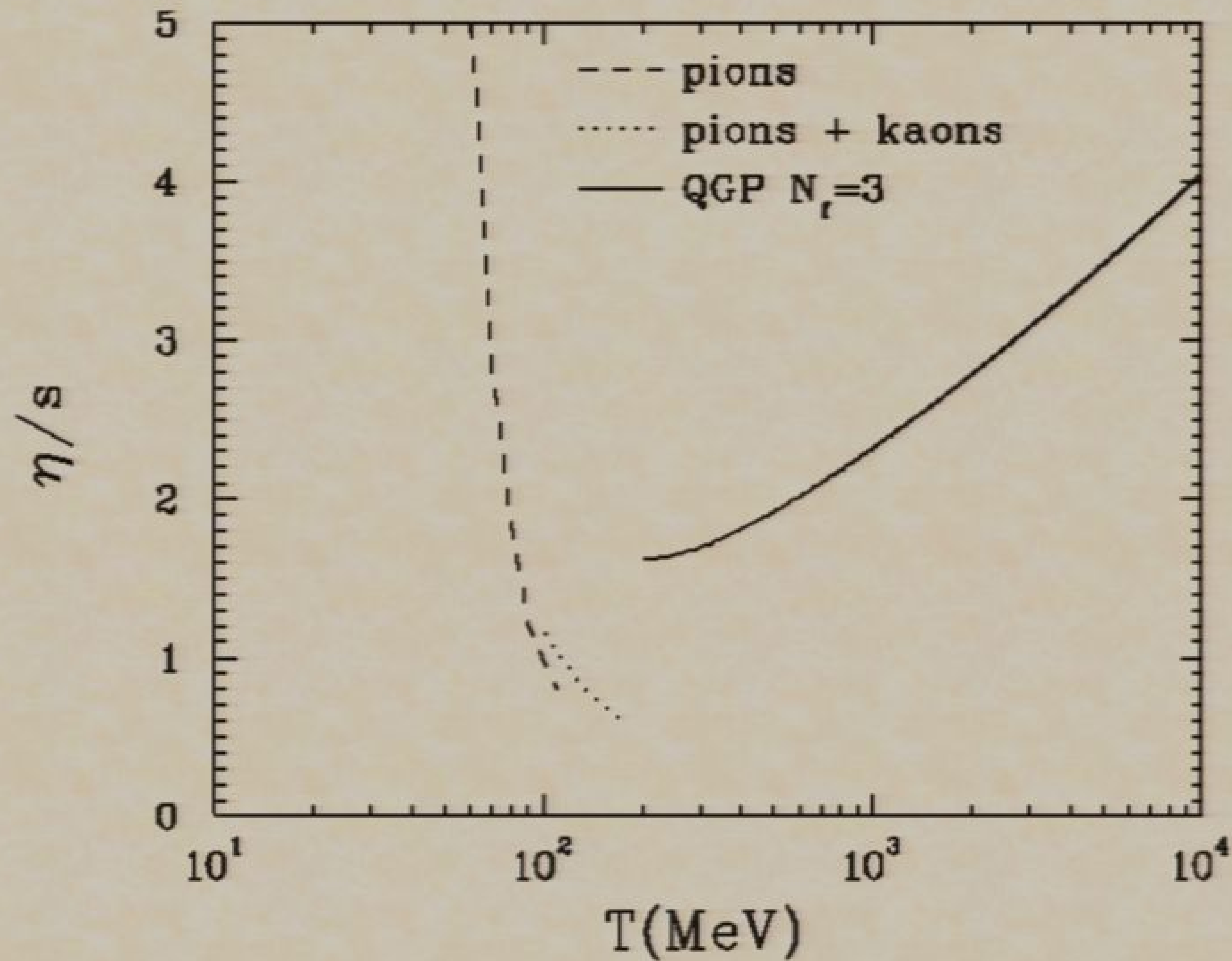
Perturbative QCD:

$$\eta/s\,(T_{\rm RHIC}) \approx (20 \sim 23) \times \frac{\hbar}{4\pi k_B}$$

Chernai, Kapusta, McLerran, nucl-th/0604032

SYM: $\eta/s \approx (1.1 \sim 3.5) \times \frac{\hbar}{4\pi k_B}$

QCD



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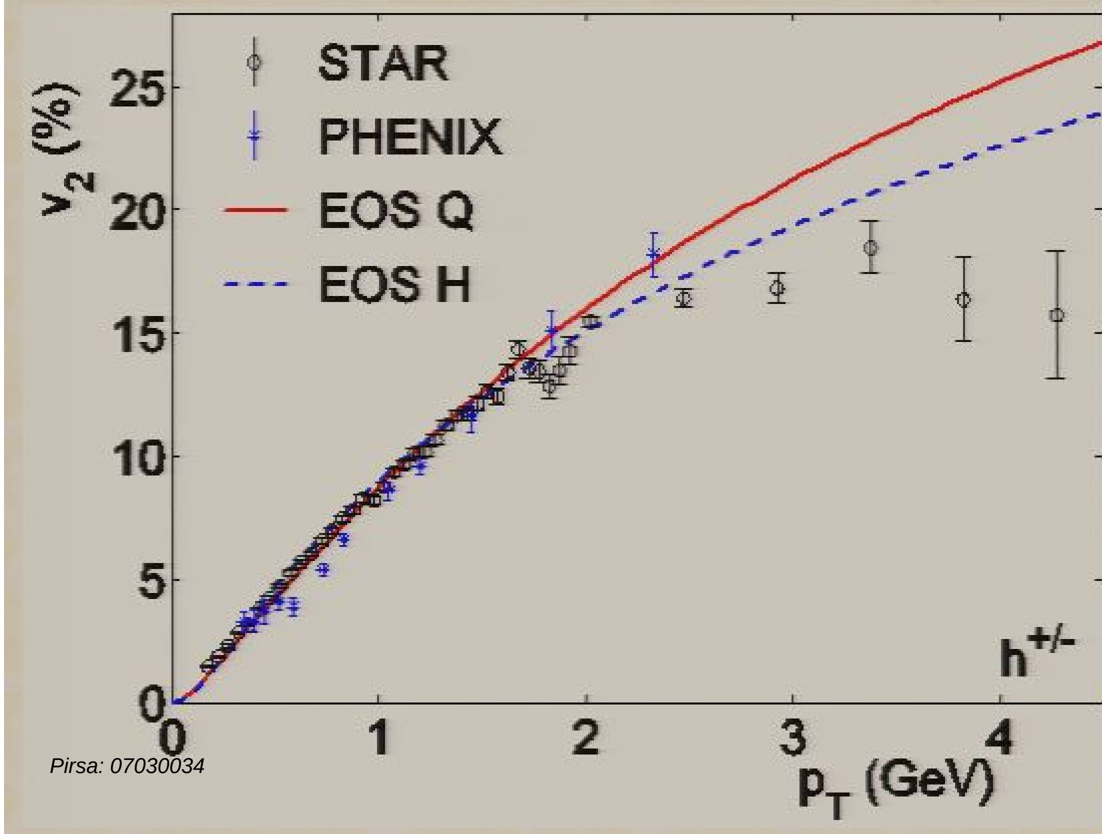

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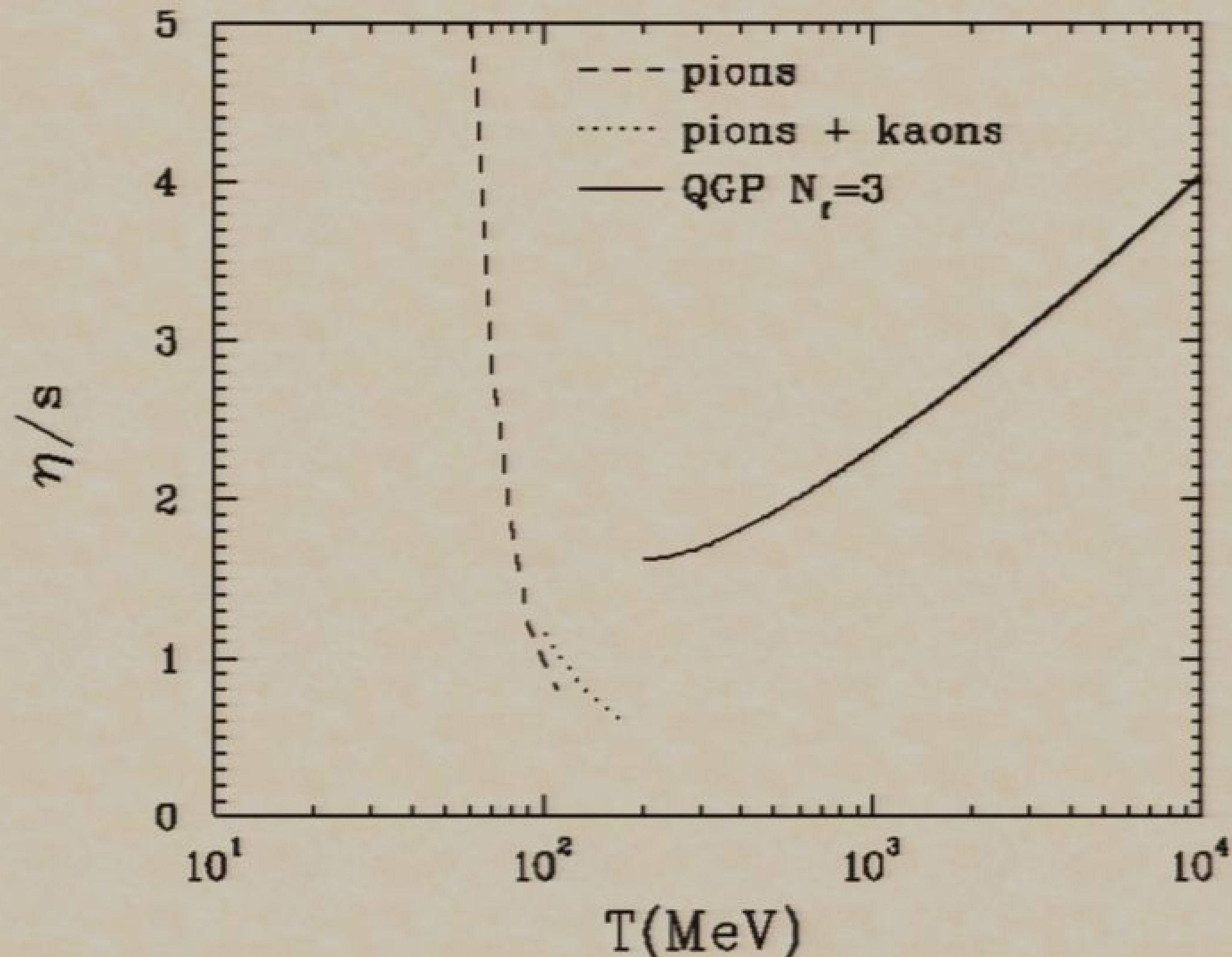
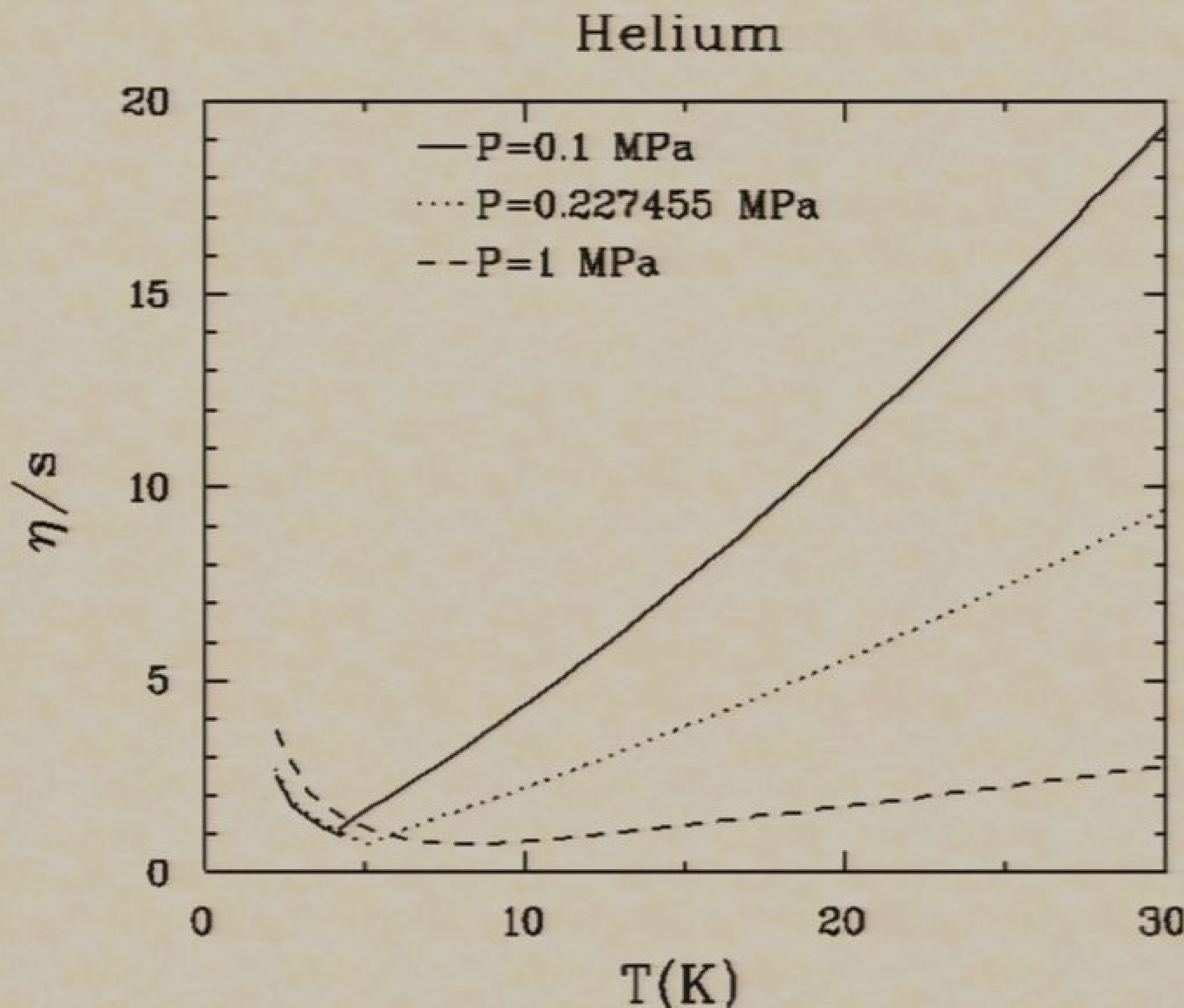
SYM: $\eta/s \approx (1.1 \sim 3.5) \times \frac{\hbar}{4\pi k_B}$

QCD

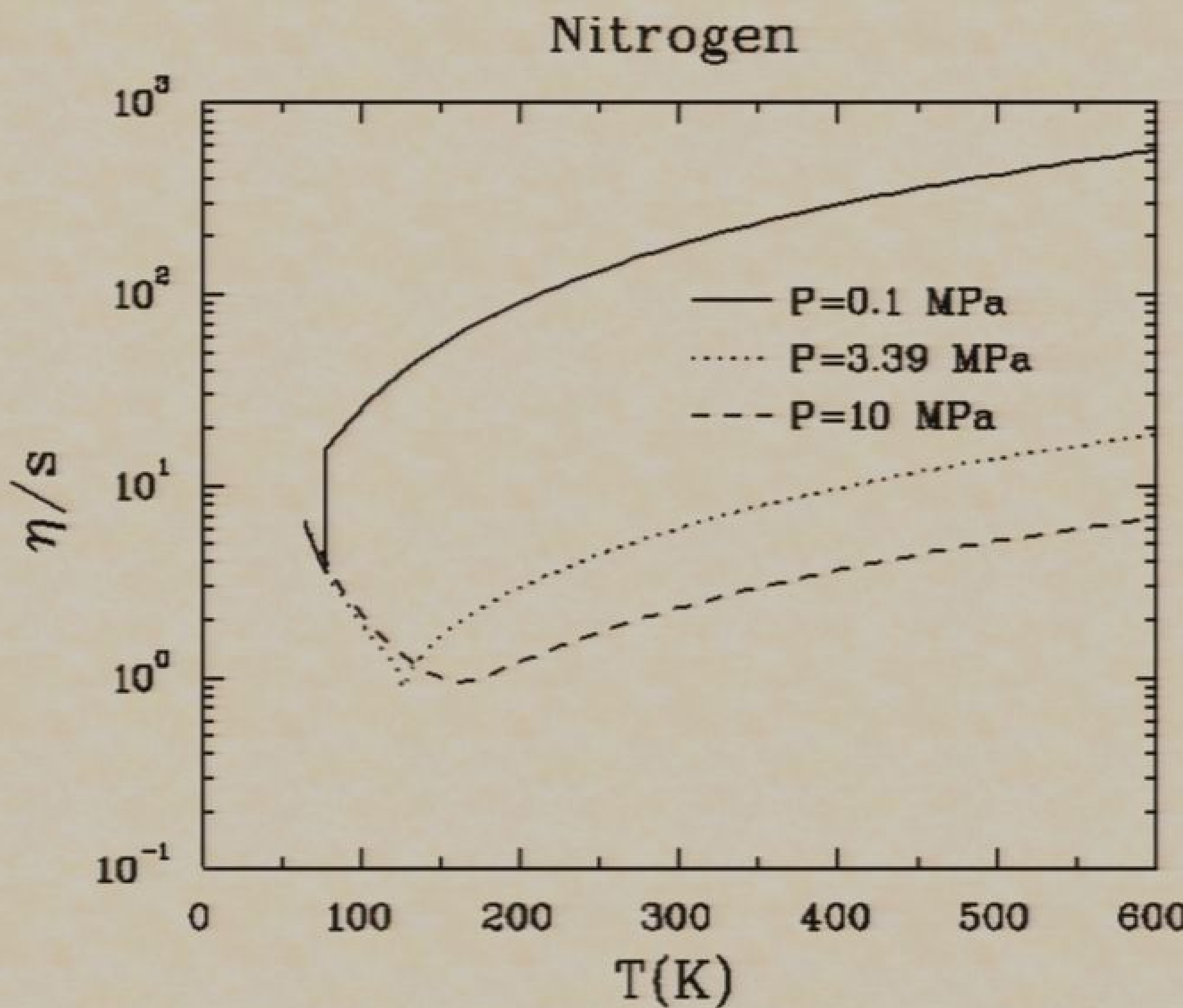


Chernai, Kapusta, McLerran, nucl-th/0604032



$$(\eta/s)_{\text{min}} \sim 8.8 \quad \text{in units of } \frac{\hbar}{4\pi k_B}$$

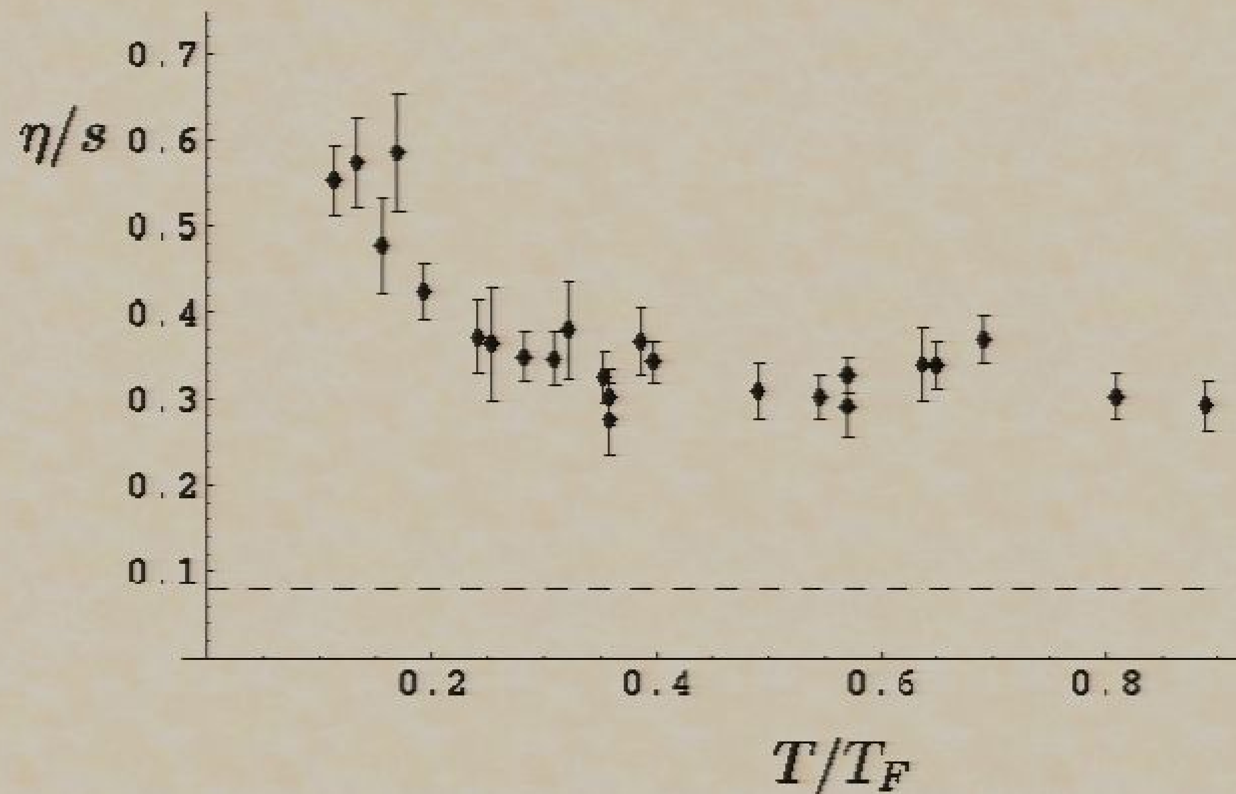
Chernai, Kapusta, McLerran, nucl-th/0604032



$(\eta/s)_{\text{min}} \sim 23$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

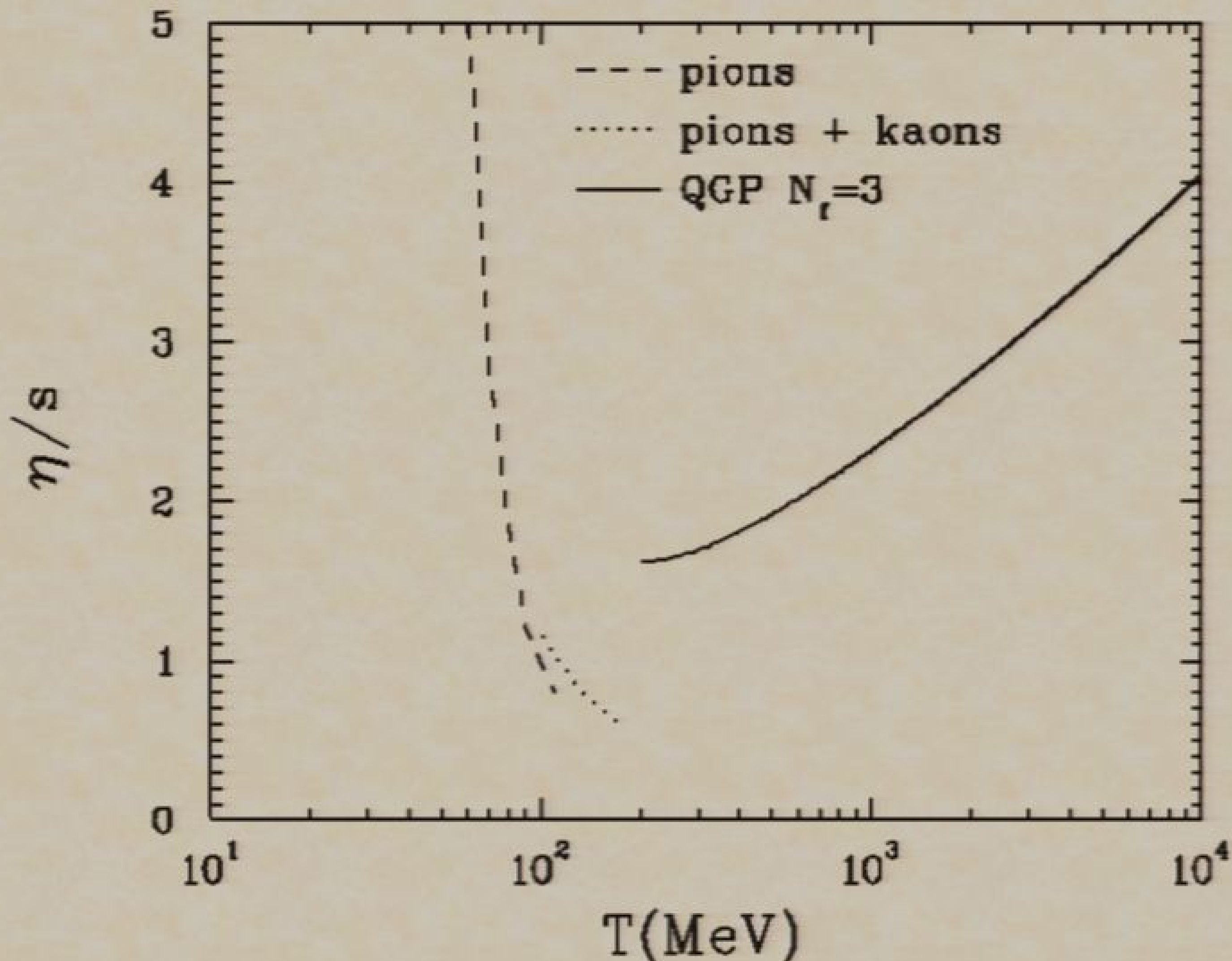
Viscosity-entropy ratio of a trapped Fermi gas



$\eta/s \sim 4.2$ in units of $\frac{\hbar}{4\pi k_B}$ T.Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

QCD



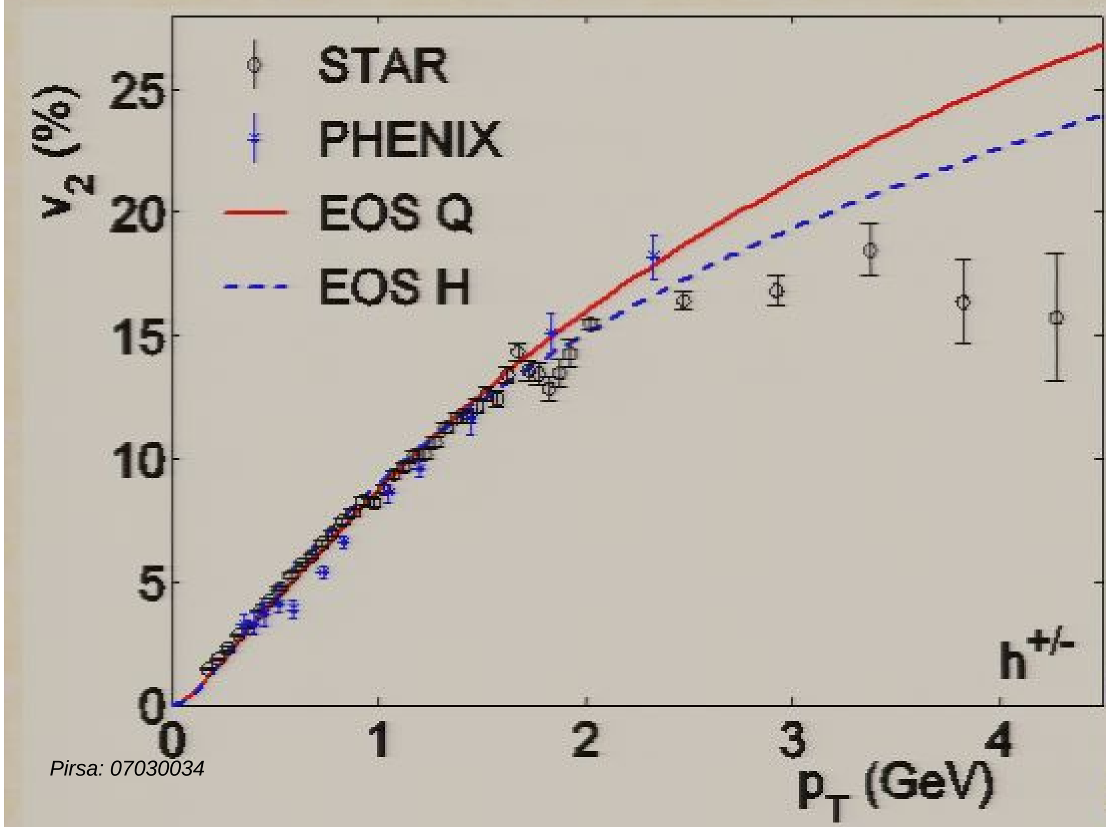
Chernai, Kapusta, McLerran, nucl-th/0604032

Viscosity "measurements" at RHIC

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$$0 < \eta/s \leq 3.8 \times \frac{\hbar}{4\pi k_B}$$

e.g. Baier, Romatschke, nucl-th/0610108

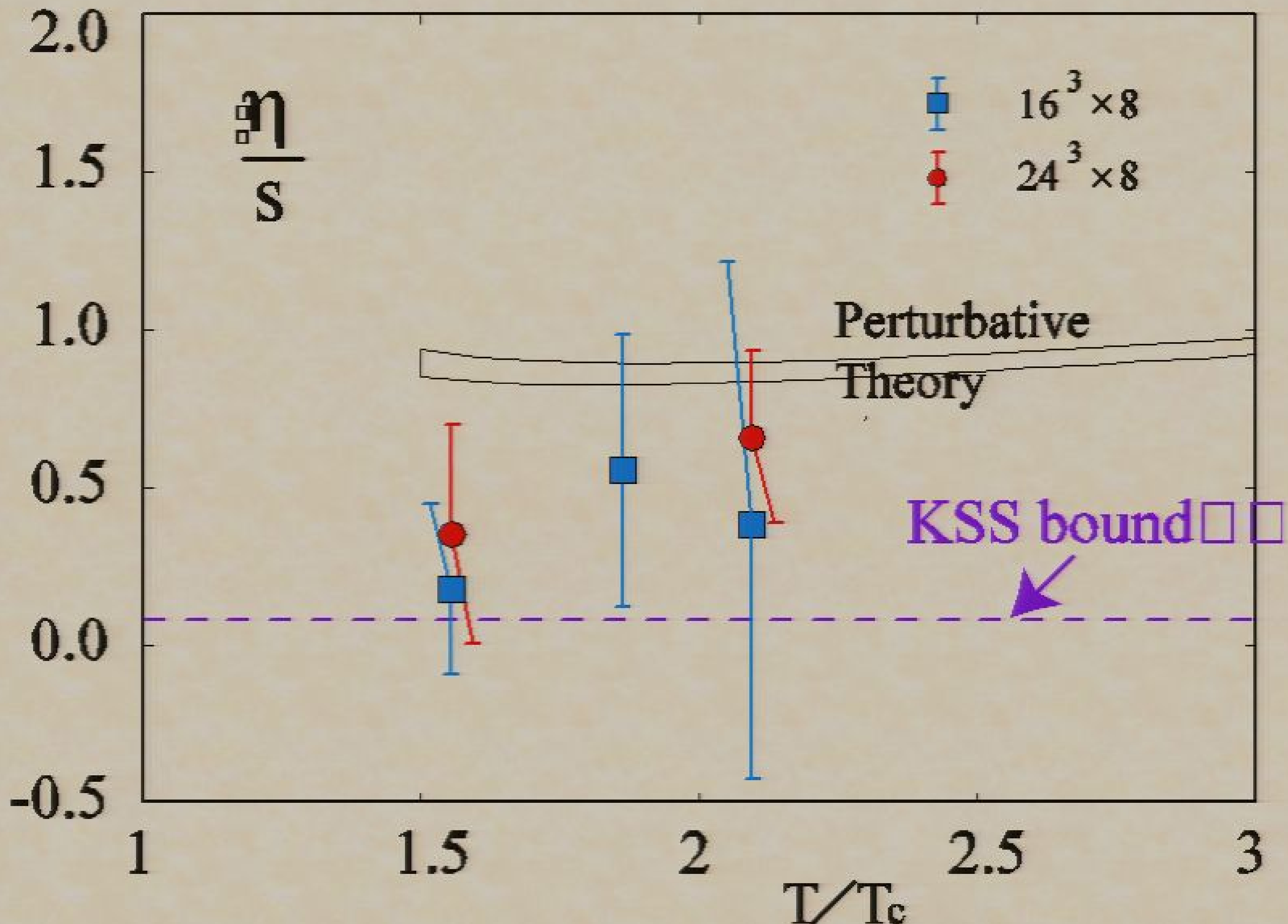
Perturbative QCD:

$$\eta/s\,(T_{\rm RHIC}) \approx (20 \sim 23) \times \frac{\hbar}{4\pi k_B}$$

Chernai, Kapusta, McLerran, nucl-th/0604032

SYM: $\eta/s \approx (1.1 \sim 3.5) \times \frac{\hbar}{4\pi k_B}$

Lattice test of the viscosity/entropy bound $\eta/s \geq 1/4\pi$?



A.Nakamura, S.Sakai, hep-lat/0510039

The “species problem”

Classical dilute gas with a LARGE number of components

has a large Gibbs mixing entropy

$$s = n \ln \left[\frac{1}{n} \left(\frac{m k_B T}{2\pi\hbar^2} \right)^{3/2} \right] + \frac{5}{2} n + n \ln N_f$$

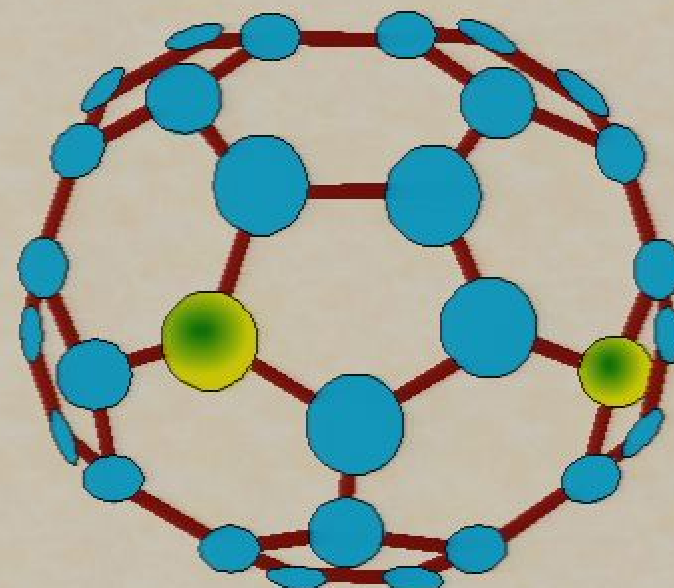
$$\eta \sim \frac{\sqrt{m k_B T}}{d^2}$$

To have $\frac{\eta}{s} < \frac{\hbar}{4\pi k_B}$ with C_{60}

need $N_f > 10^{4000}$ (Dam Son, 2007)

To have $\frac{\eta}{s} \sim 8.8 \frac{\hbar}{4\pi k_B}$ need

$N_f \sim 10^{450}$ species



Buckminsterfullerene C_{60}

a.k.a.”buckyball”

A.Dobado, F.Llanes-Estrada, hep-th/0703132

Epilogue

- On the level of theoretical models, there exists a connection between near-equilibrium regime of certain strongly coupled thermal field theories and fluctuations of black holes
- This connection allows us to compute transport coefficients for these theories
- At the moment, this method is the only theoretical tool available to study the near-equilibrium regime of strongly coupled thermal field theories
- The result for the shear viscosity turns out to be universal for all such theories in the limit of infinitely strong coupling
- Stimulating for experimental/theoretical research in other fields

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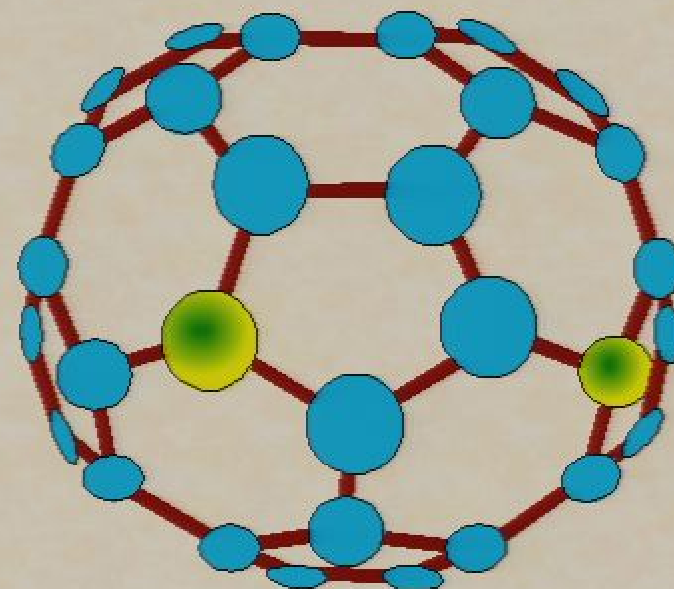

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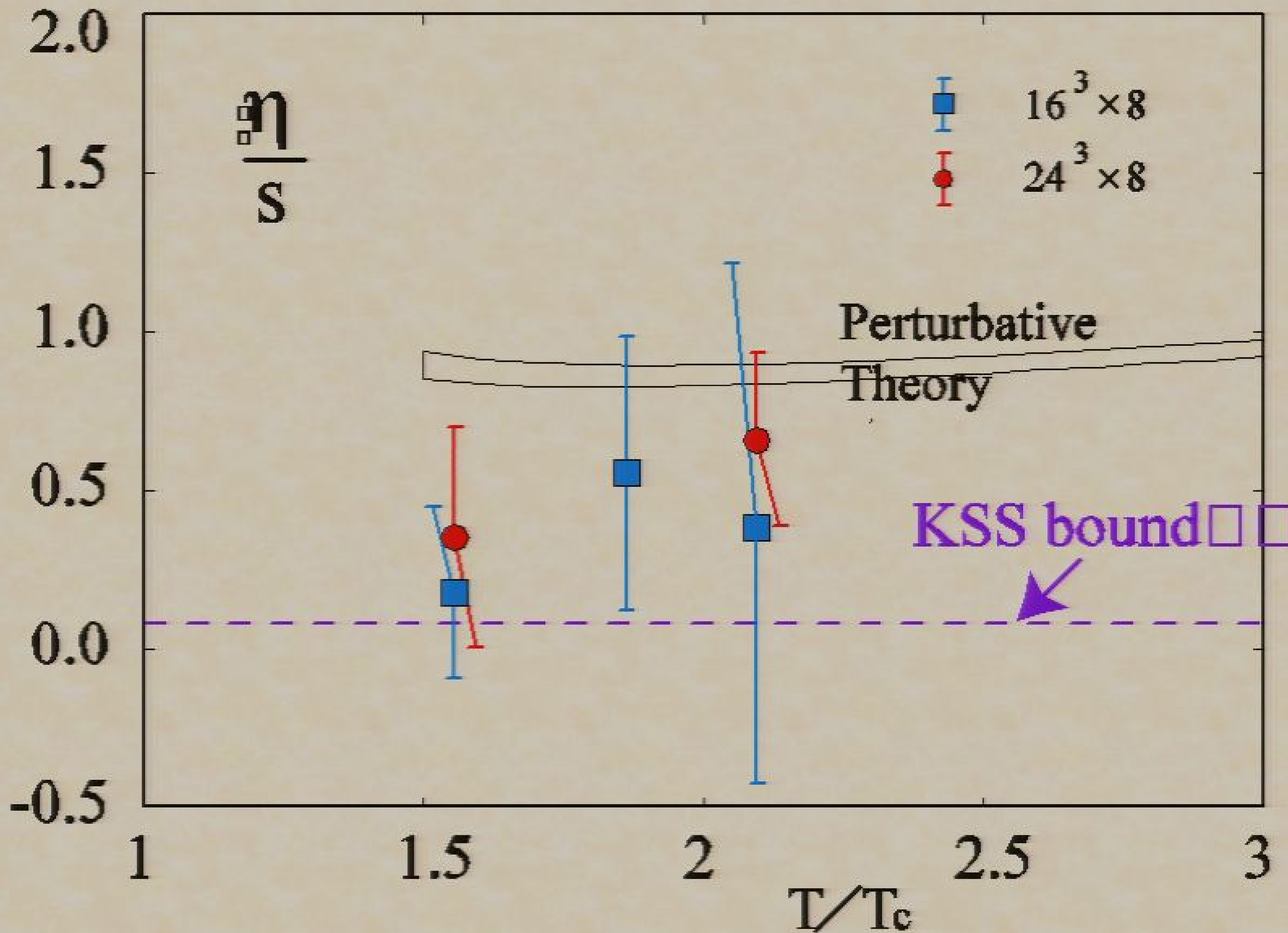

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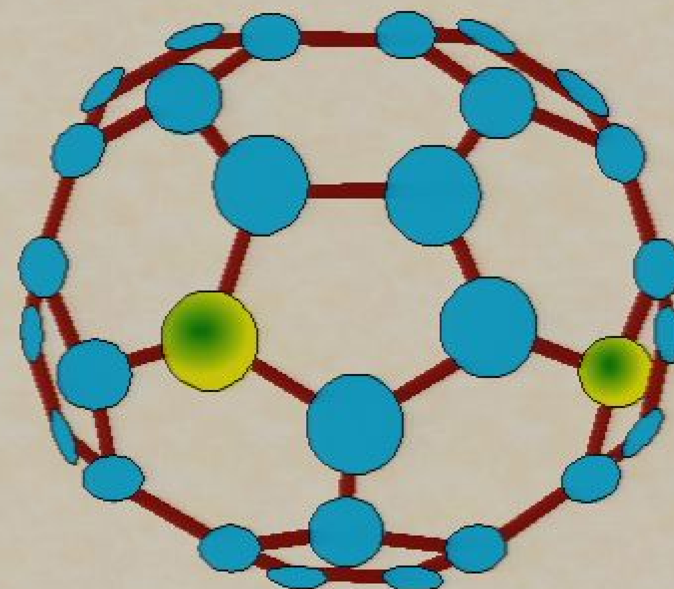

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A.Dobado, F.Llanes-Estrada, hep-th/0703132

T.Cohen, hep-th/0702136