Title: Black Holes and Holography: Mini-Course - Lecture 3

Date: Mar 22, 2007 10:30 AM

URL: http://pirsa.org/07030030

Abstract: <div id="Cleaner">Distinguished theoretical physicist Leonard Susskind, Professor of Physics at Stanford, will give a series of lectures on Black Holes and Holography at Perimeter Institute in Waterloo. This mini-course is open to all university professors and students.

t = sch time

t = Sch time

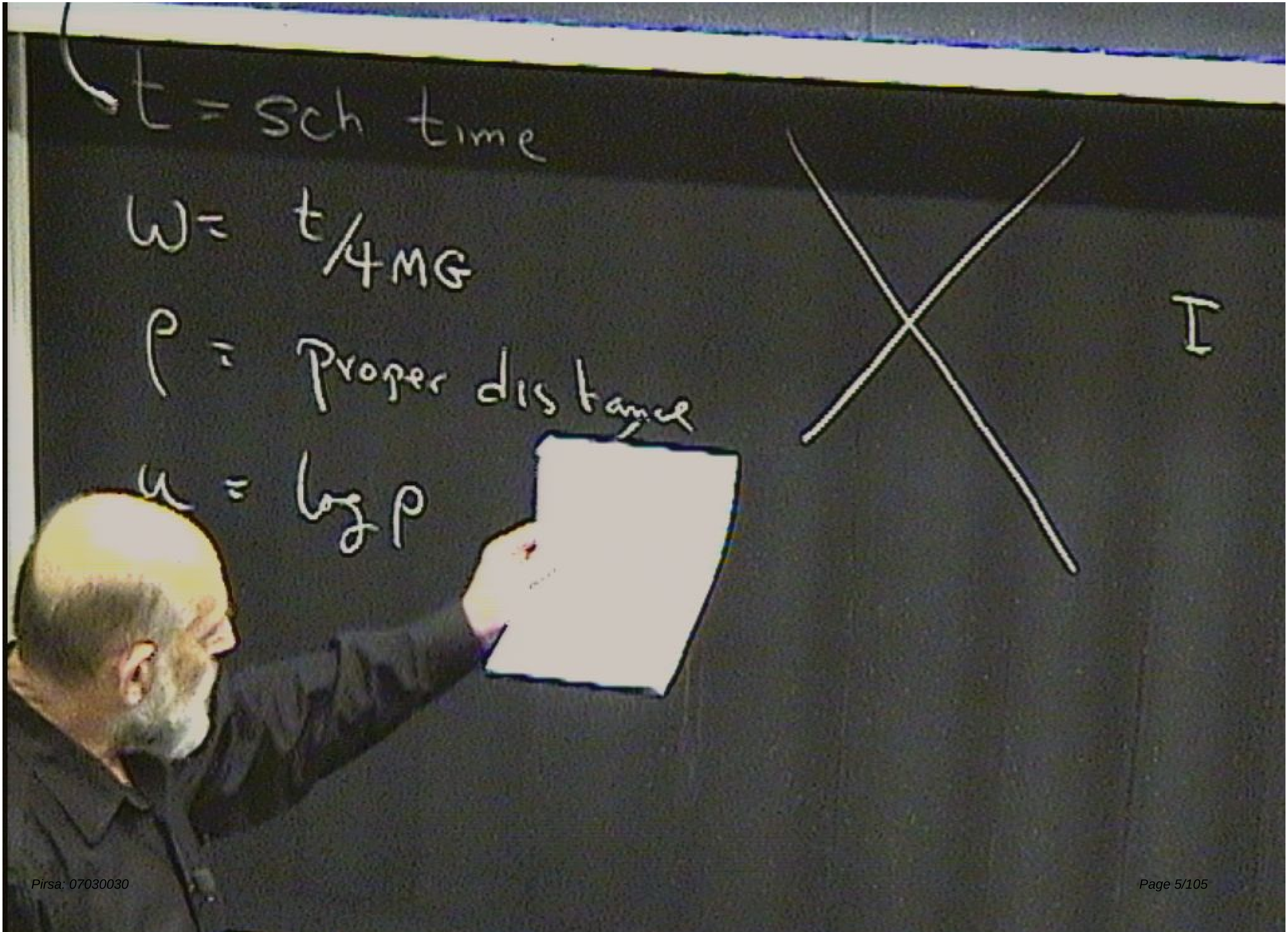
$w = t/4MG$

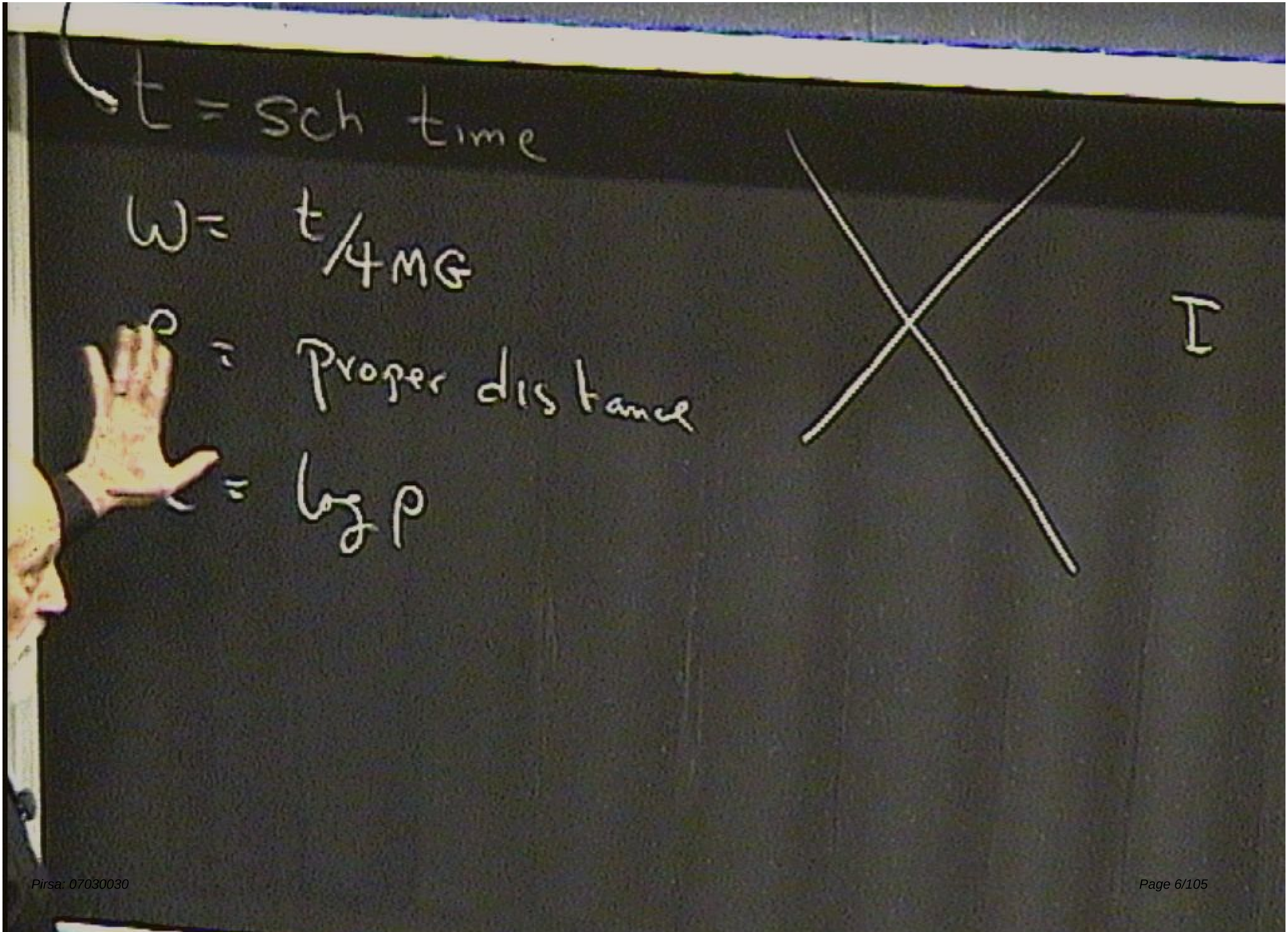
t = Sch time

$\omega = t/4MG$

ρ = Proper distance

$u = \log \rho$


t = sch time
w = t/4MG
ρ = proper distance
u = log ρ
I


t = sch time
w = t/4MG
ρ = proper distance
= log ρ
I

t = sch time

$w = t/4MG$

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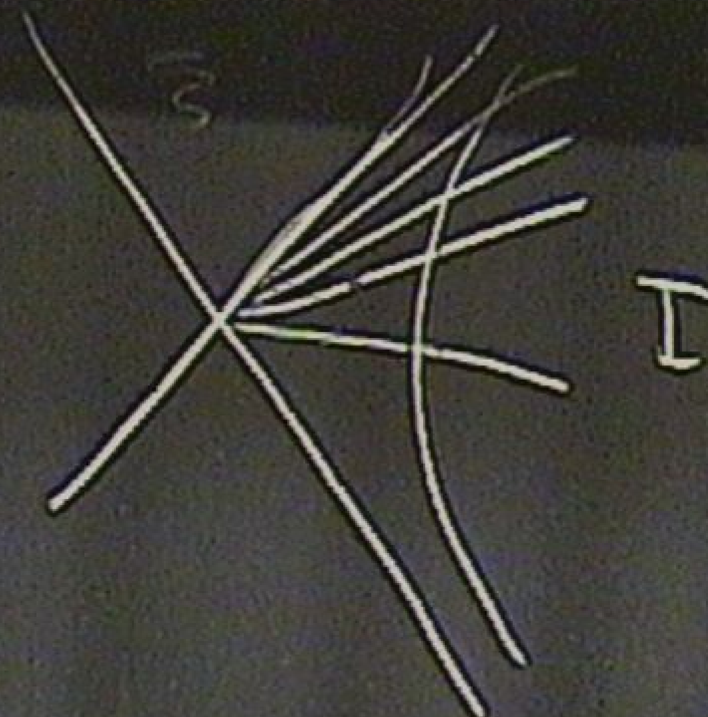
$u = \log \rho$

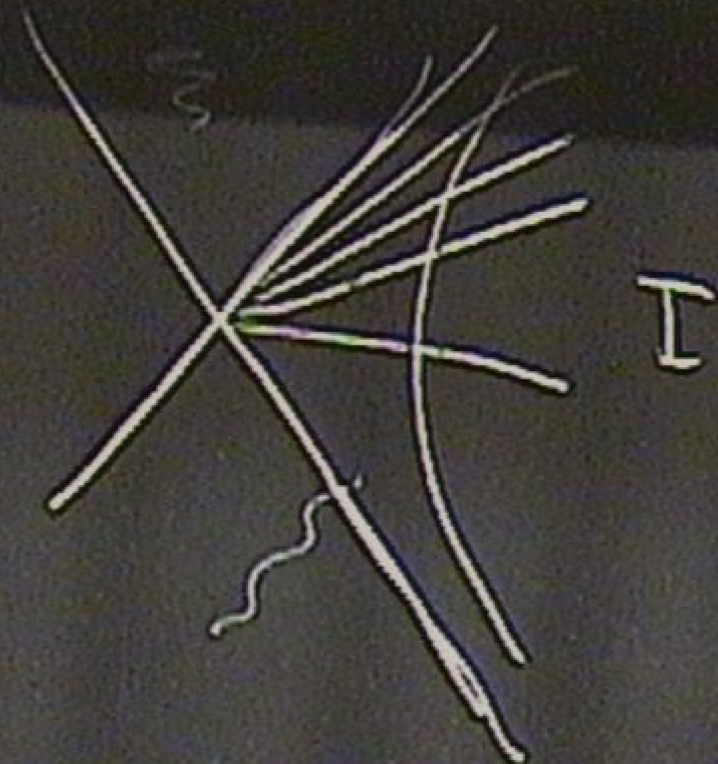


t = Sch time

$\omega = t/4MG$

ρ = proper distance

$u = \log \rho$



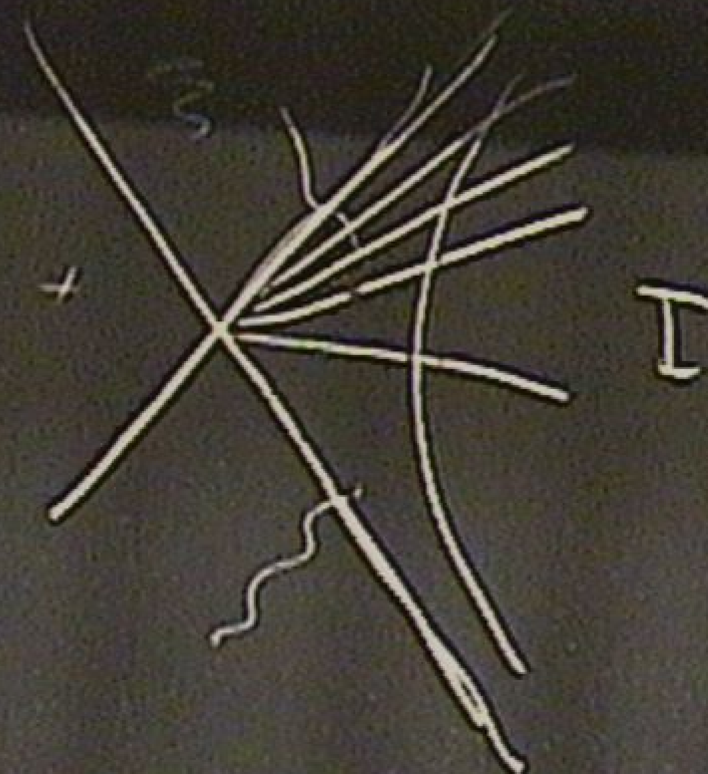
t = Sch time

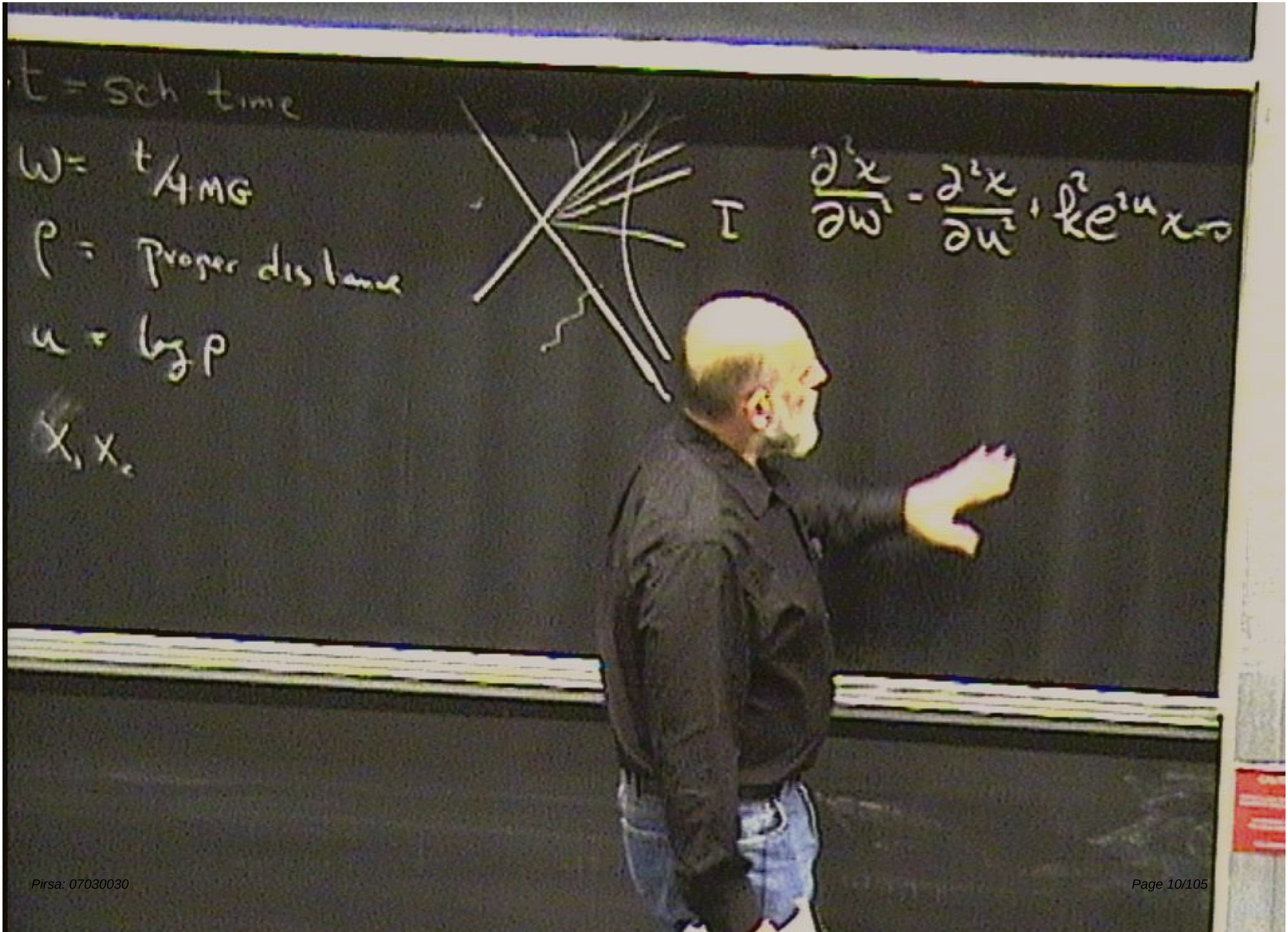
$W = t/4MG$

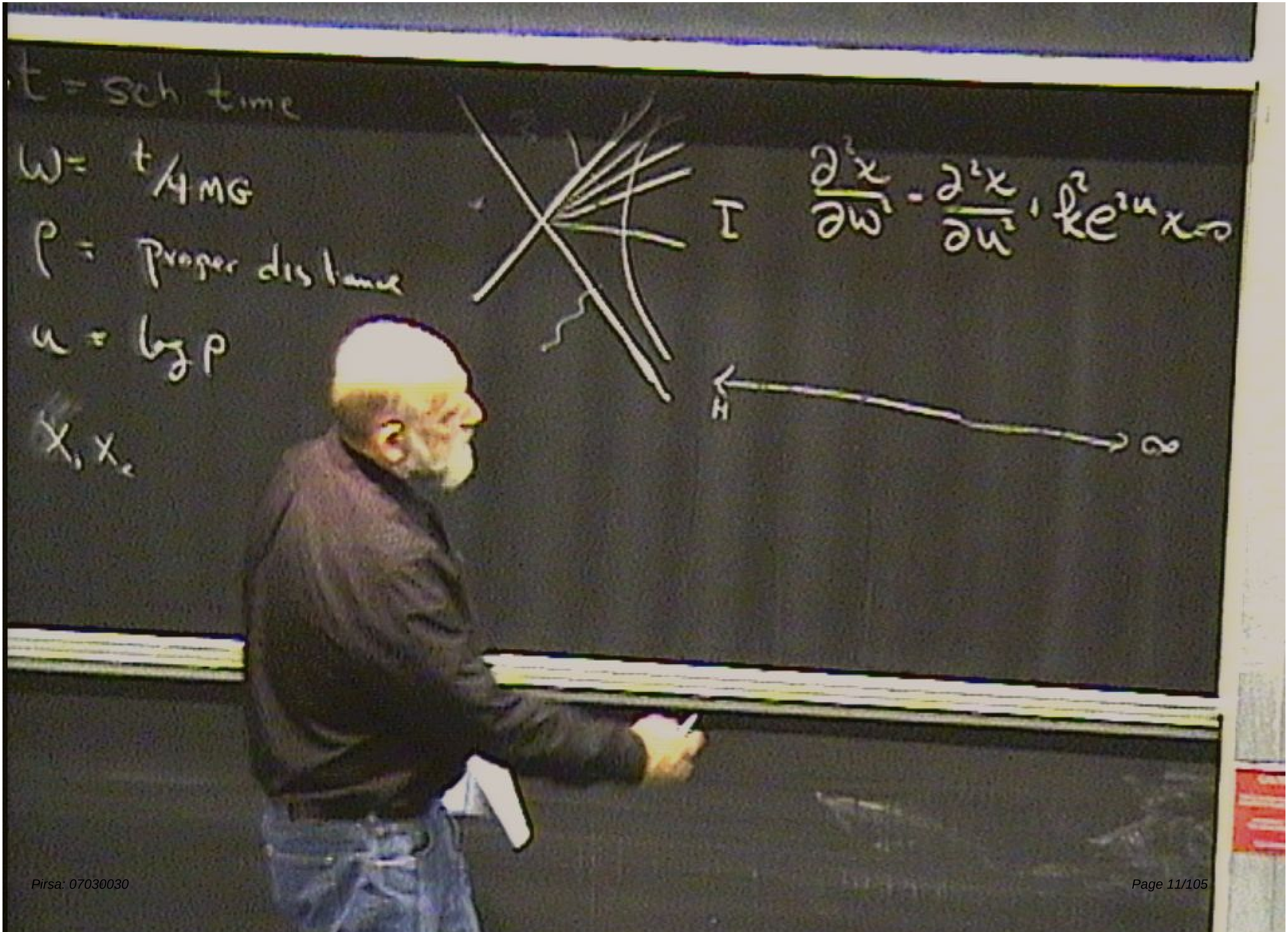
ρ = Proper distance

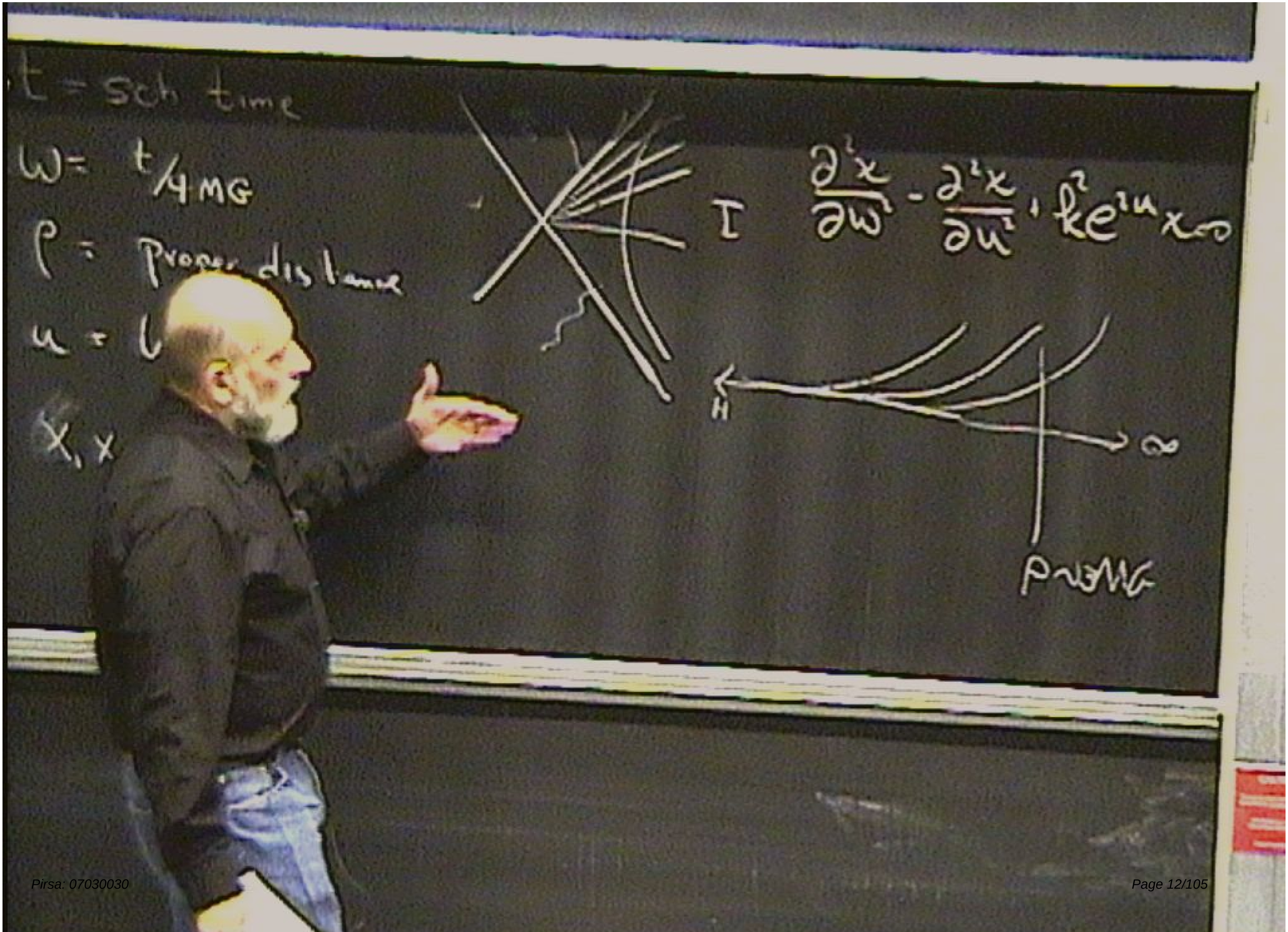
$u = \log \rho$

$X_1 \; X_2$




t = sch time
$\omega = t/4MG$
ρ = proper distance
$u = \log \rho$
$X_1 X_2$
I
$\frac{\partial^2 x}{\partial \omega^2} - \frac{\partial^2 x}{\partial u^2} + k^2 e^{2u} x = 0$


t = sch time
ω = t/4MG
ρ = proper distance
u = log ρ
X₁, X₂
I
∂²X/∂ω² − ∂²X/∂u² + k² e^{2u} X = 0
H
∞



t = sch time
ω = t/4MG
ρ = proper distance
u =
X, x
I
$\frac{\partial^2 x}{\partial \omega^2} - \frac{\partial^2 x}{\partial u^2} + k^2 e^{2u} x = 0$
H
∞

t = sch time

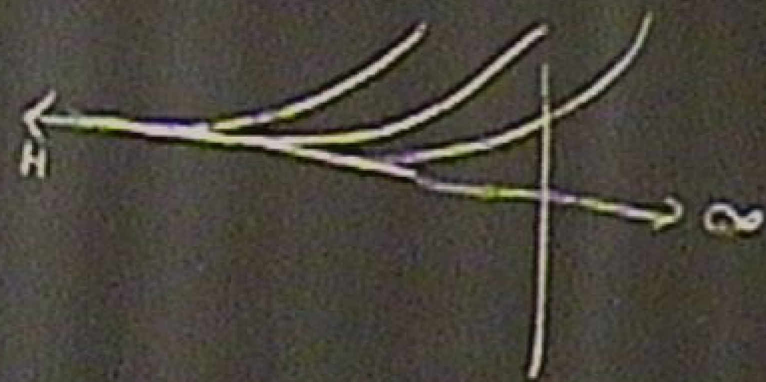
$\omega = t/4MG$

ρ = proper distance

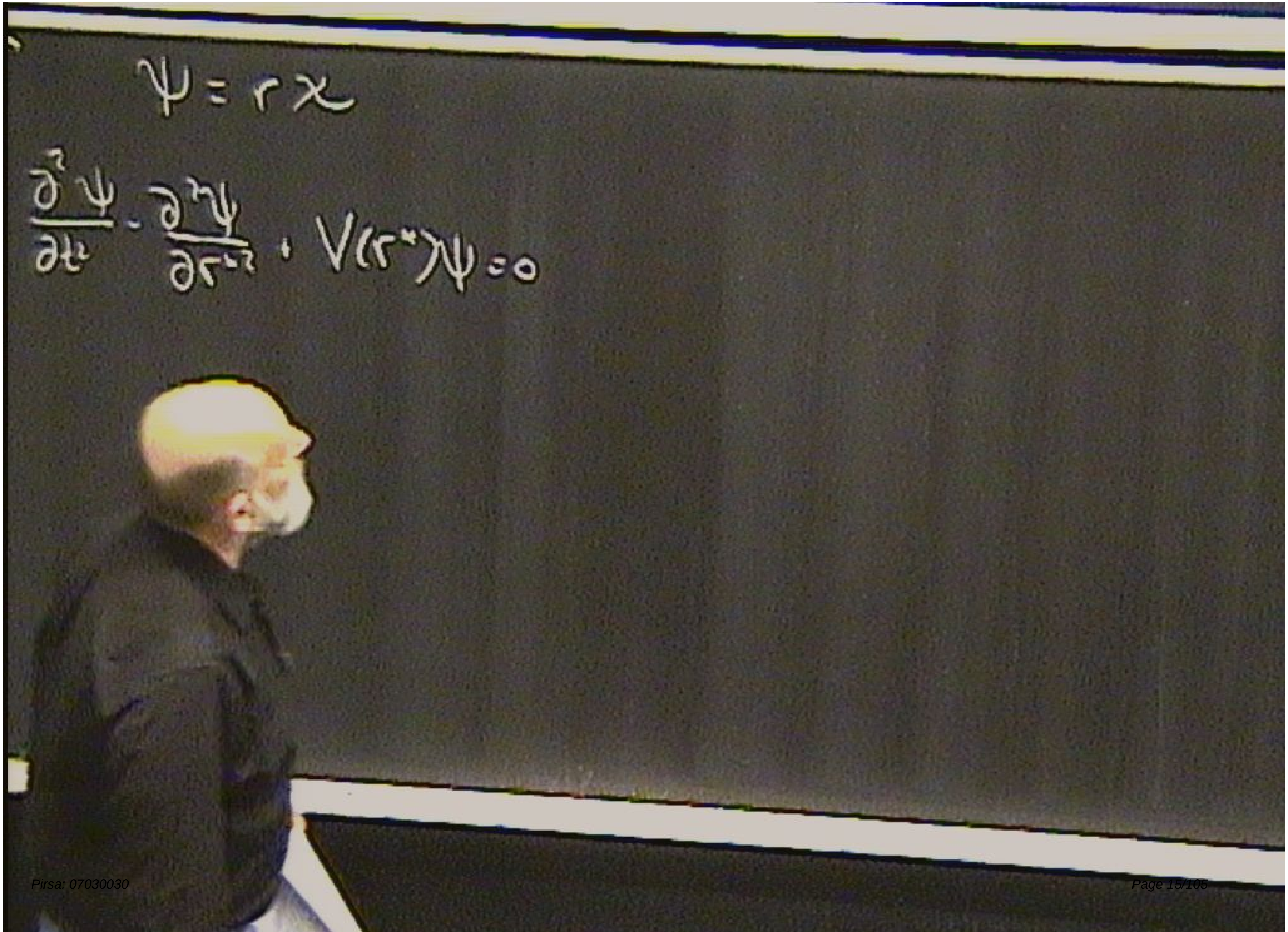
$u = \log \rho$

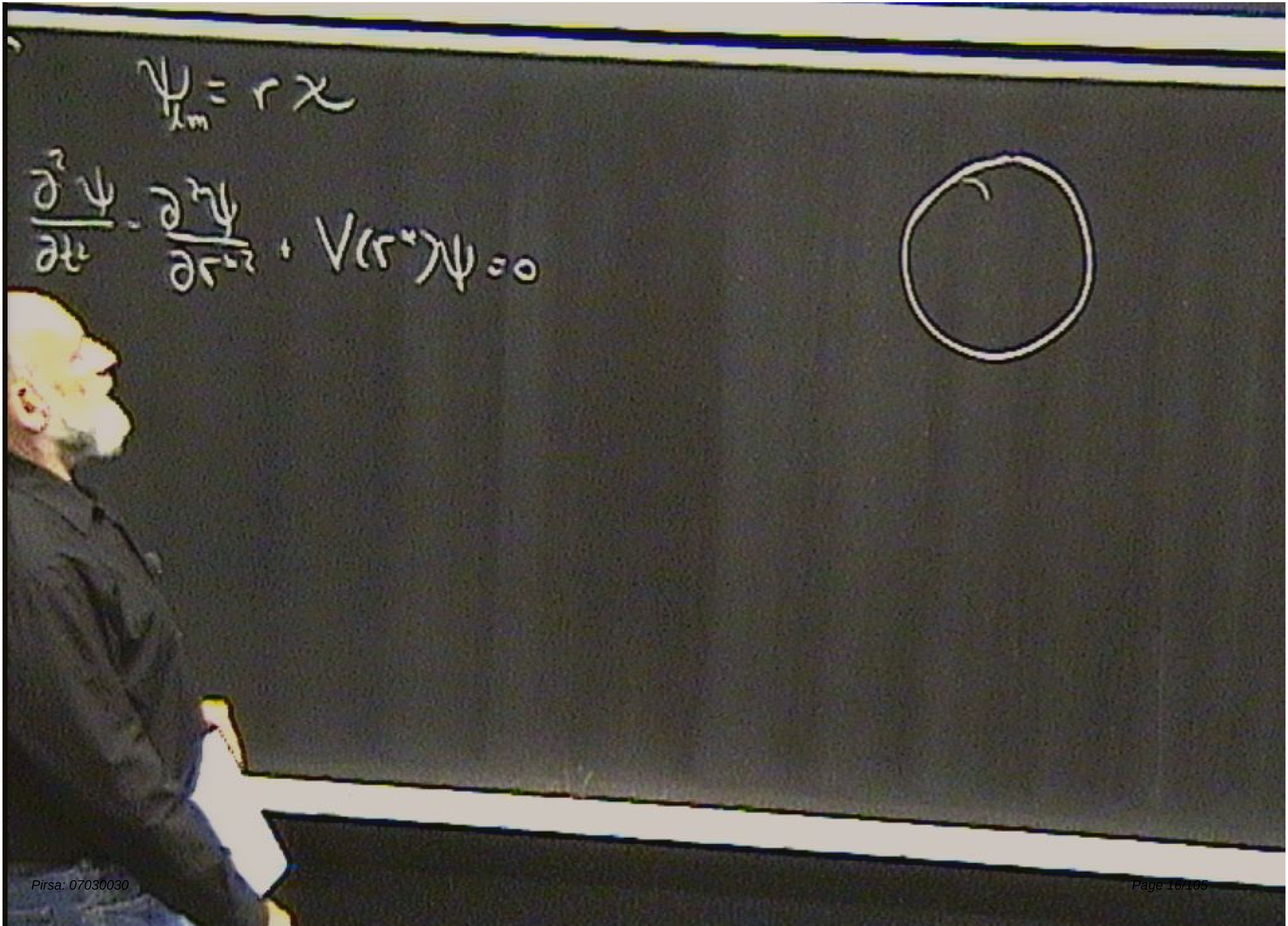
$X_1\ X_2$

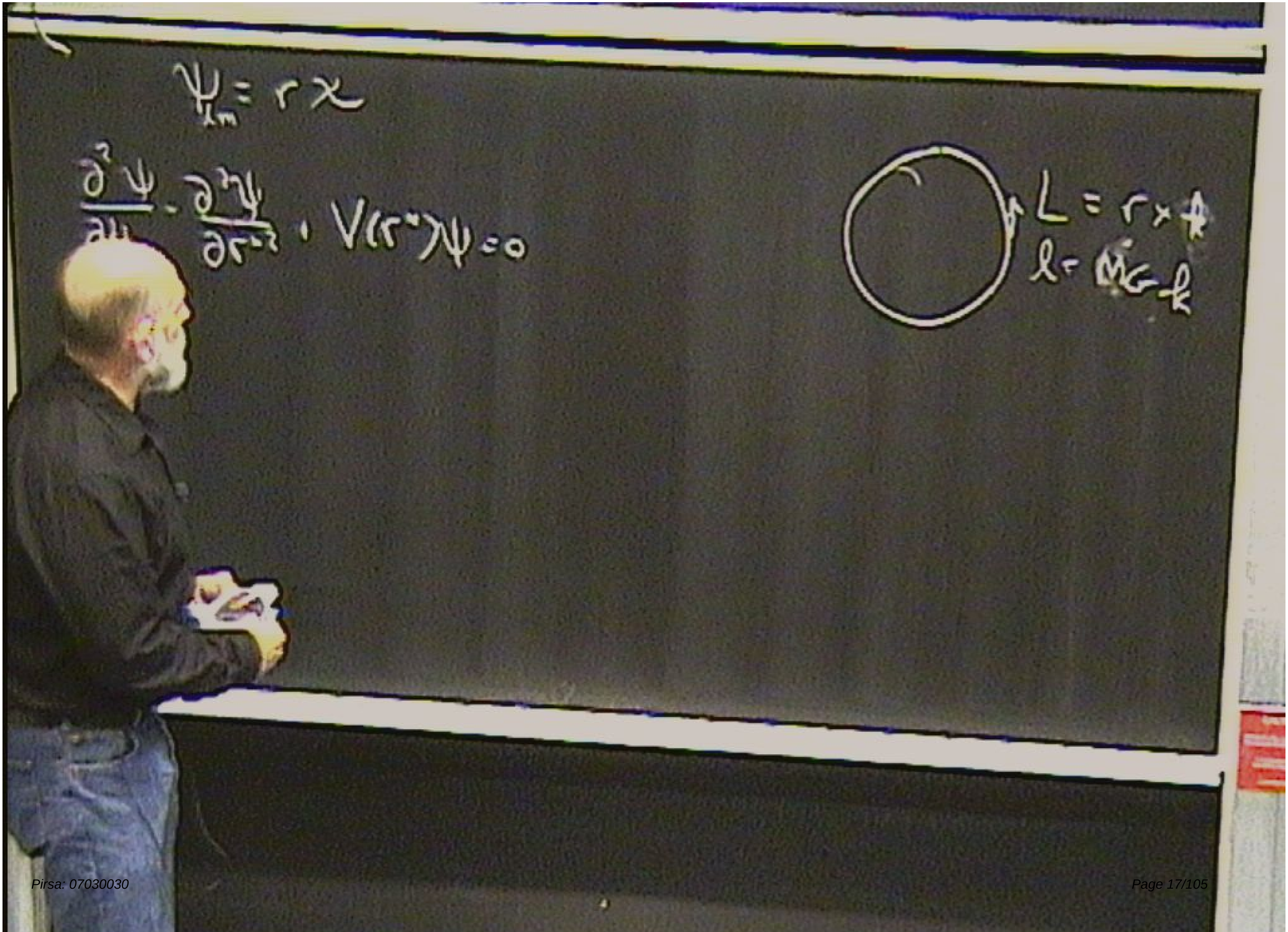
$$\frac{\partial^2 X}{\partial \omega^2} - \frac{\partial^2 X}{\partial u^2} + k^2 e^{2u} X = 0$$



$\psi = r x$


$\psi = r\chi$
$\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial r^{*2}} + V(r^*)\psi = 0$

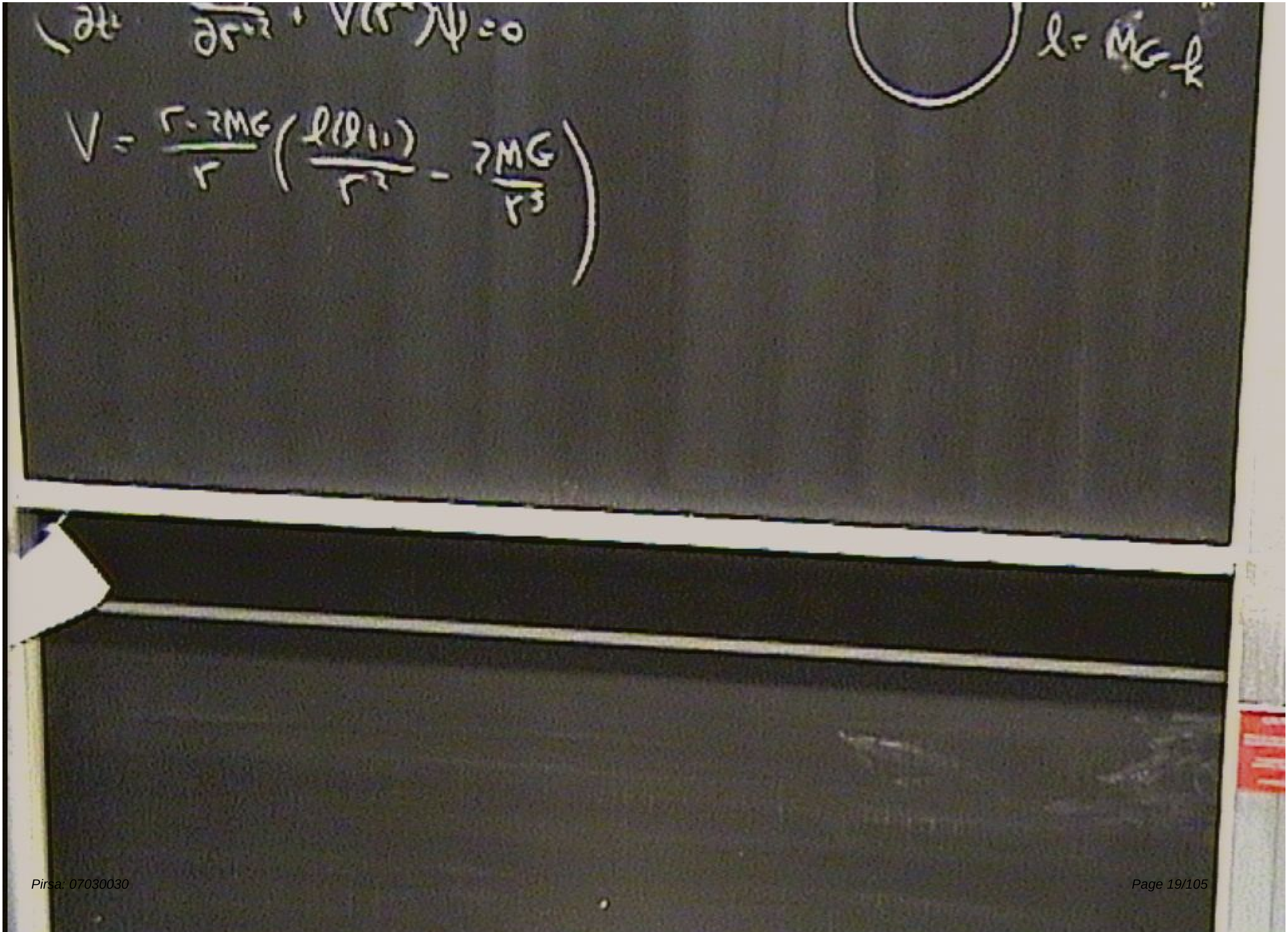

$\psi_{lm} = r\chi$
$\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial r^{*2}} + V(r^*)\psi = 0$


ψ_lm = r χ
∂²ψ/∂t² − ∂²ψ/∂r*² + V(r*)ψ = 0
L = r × p

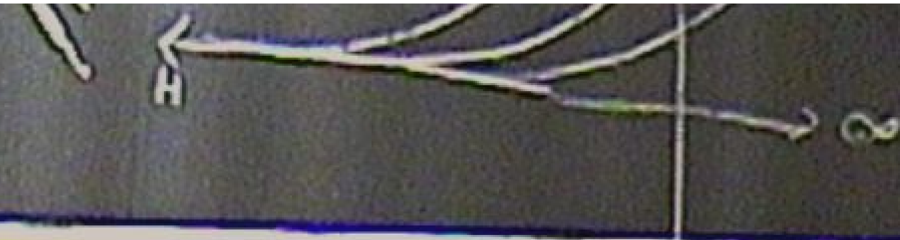
$$\left(\partial_t^2 - \partial_{r^*}^2 + V(r^*)\right)\psi = 0$$

$$\ell = MG\,k$$

$$V = \frac{r-2MG}{r}\left(\frac{\ell(\ell+1)}{r^2} - \frac{2MG}{r^3}\right)$$



$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^{*2}} + V(r^*)\right)\psi = 0$
$l = MG\,k$
$V = \frac{r-2MG}{r}\left(\frac{l(l+1)}{r^2} - \frac{2MG}{r^3}\right)$

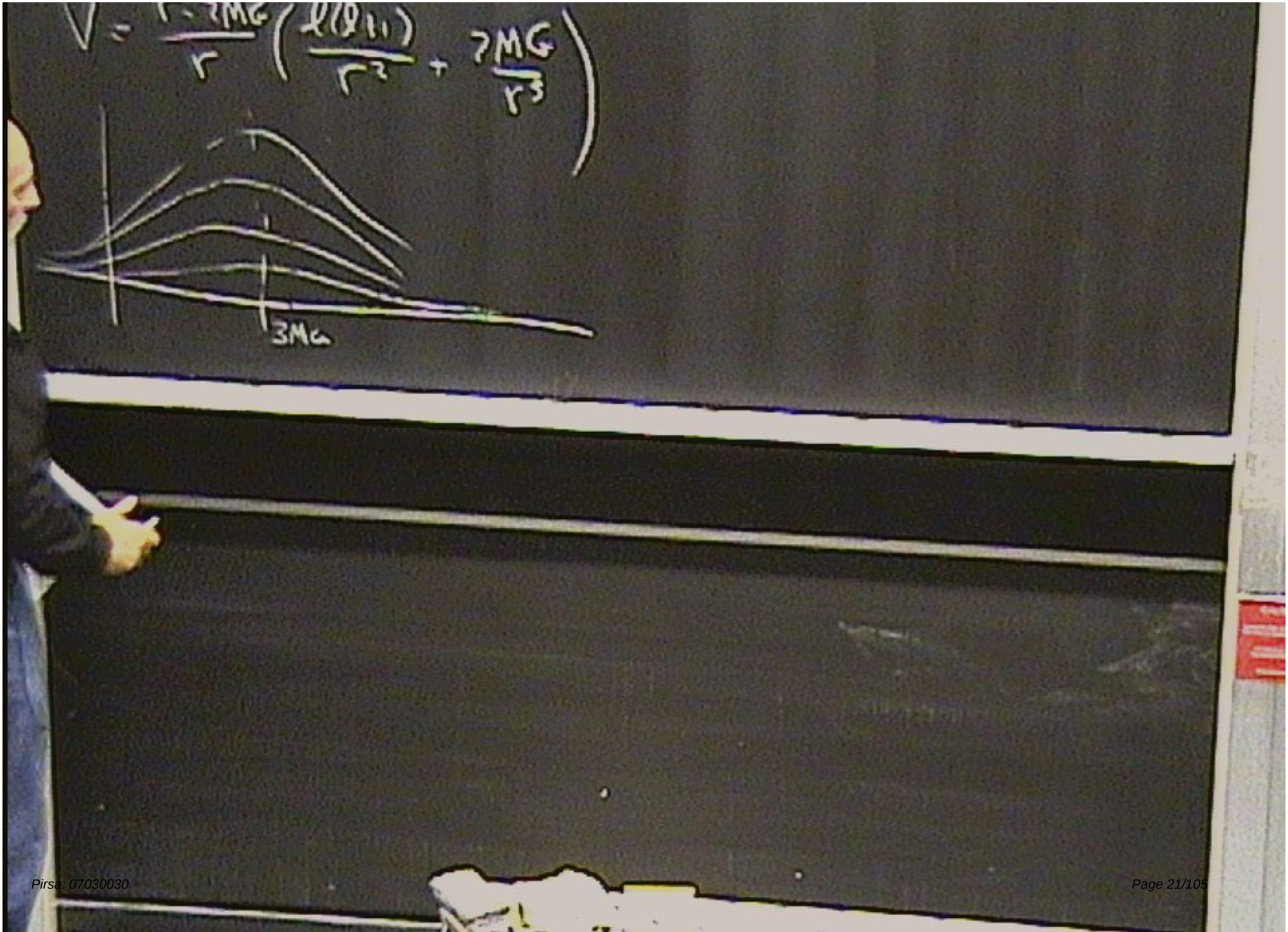


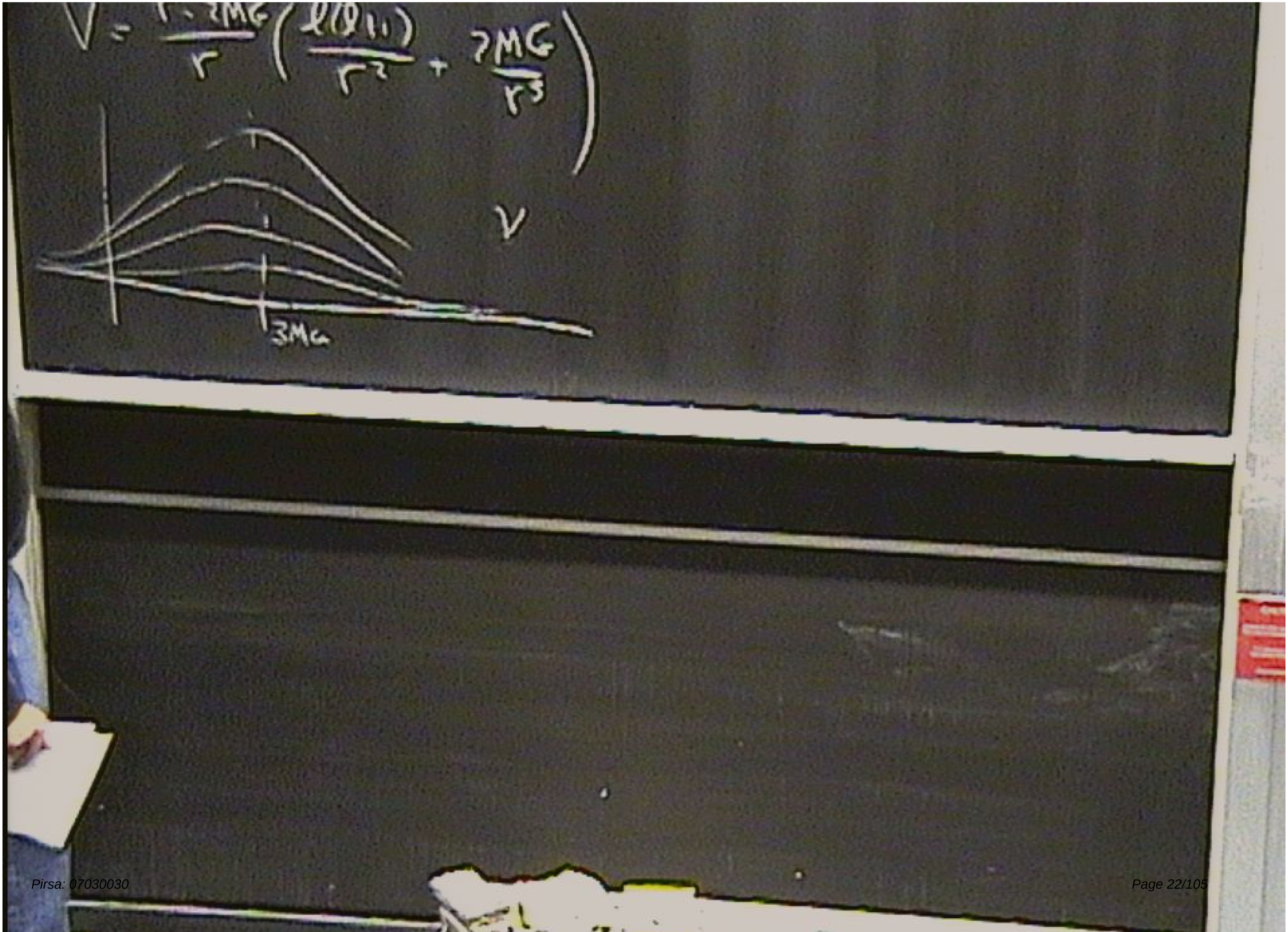
$$\psi_{\ell m} = r\chi$$

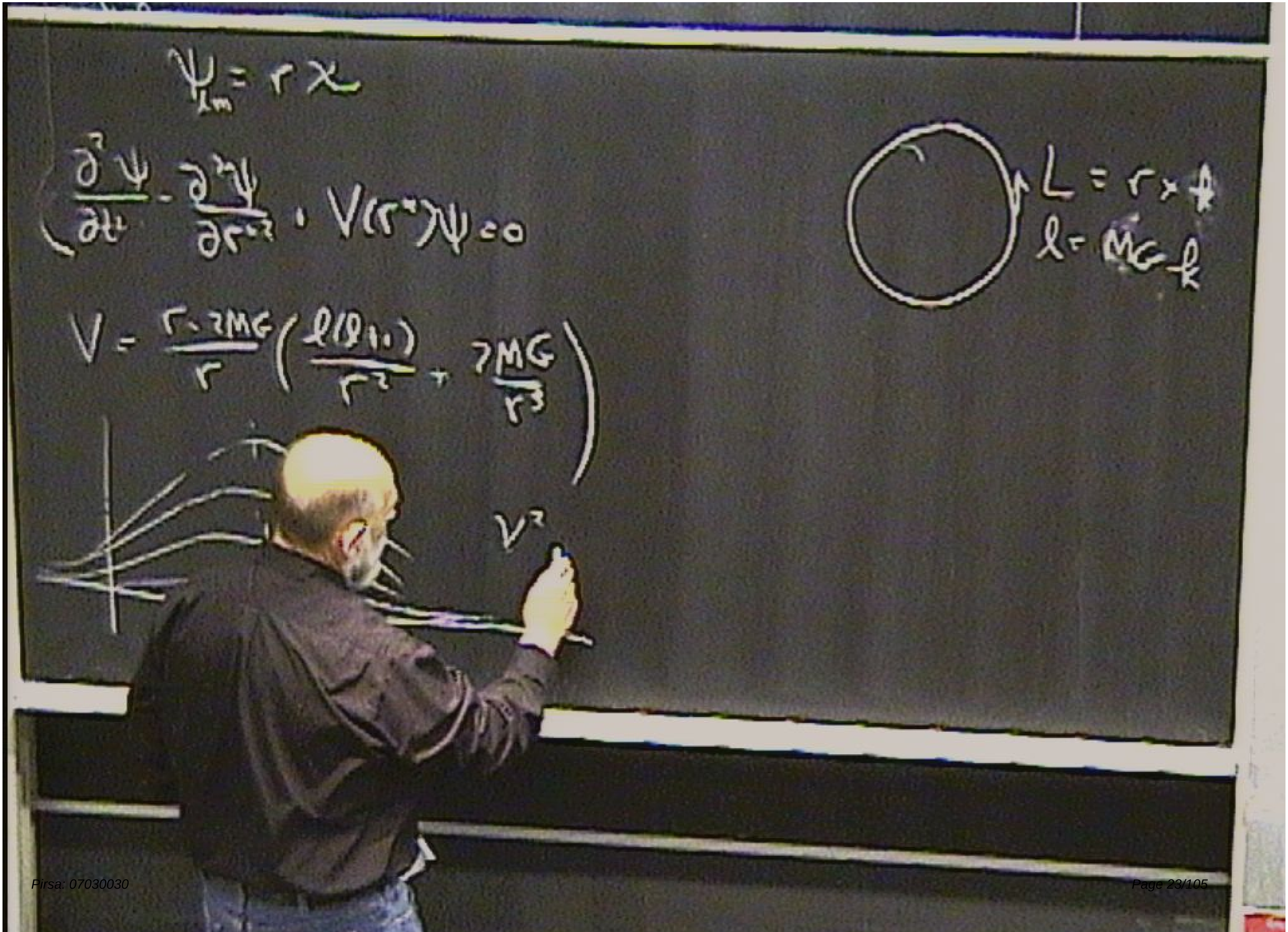
$$\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial r^{*2}} + V(r^*)\psi = 0$$

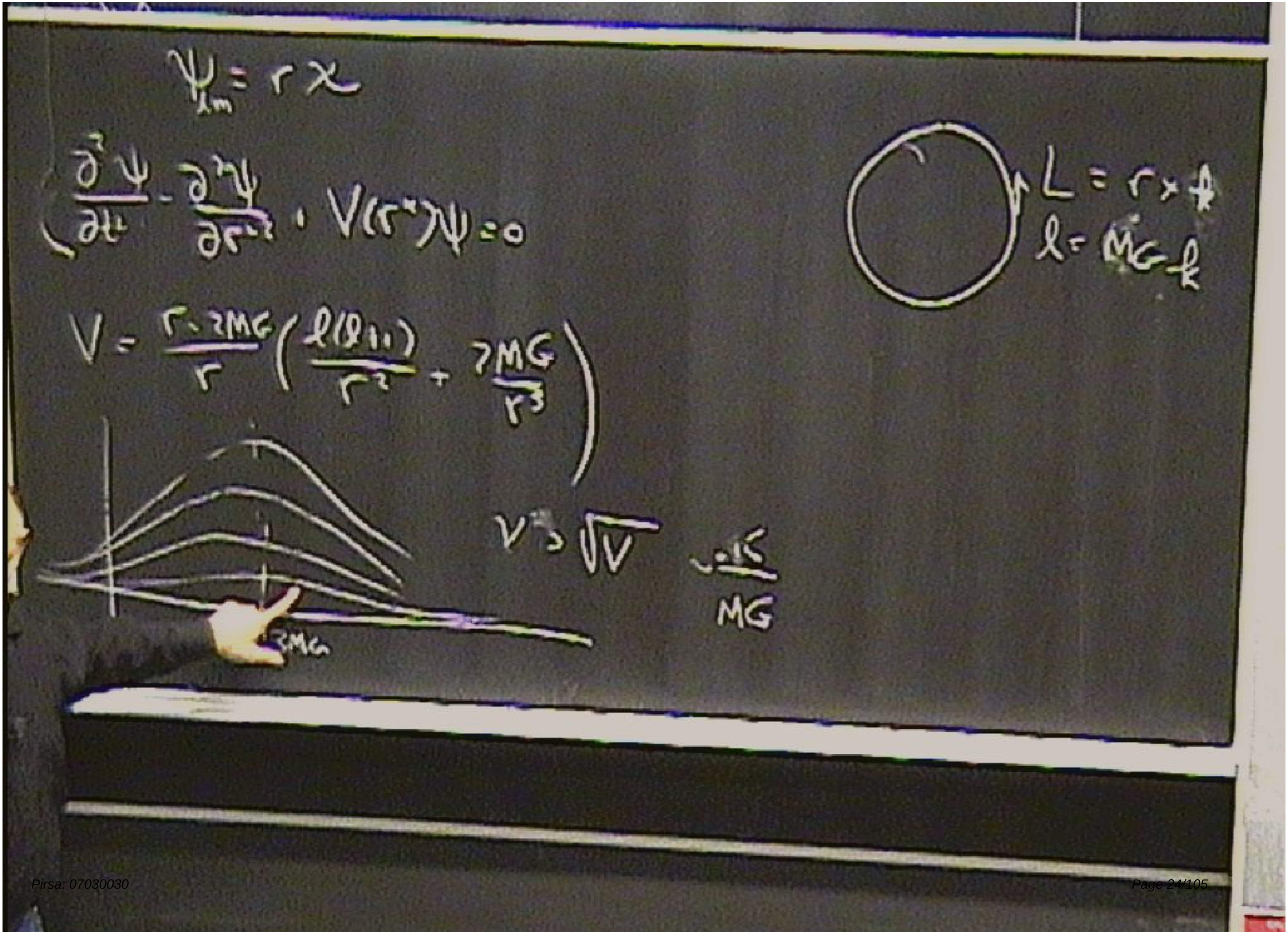
$$V = \frac{r - 2MG}{r}\left(\frac{\ell(\ell+1)}{r^2} - \frac{2MG}{r^3}\right)$$

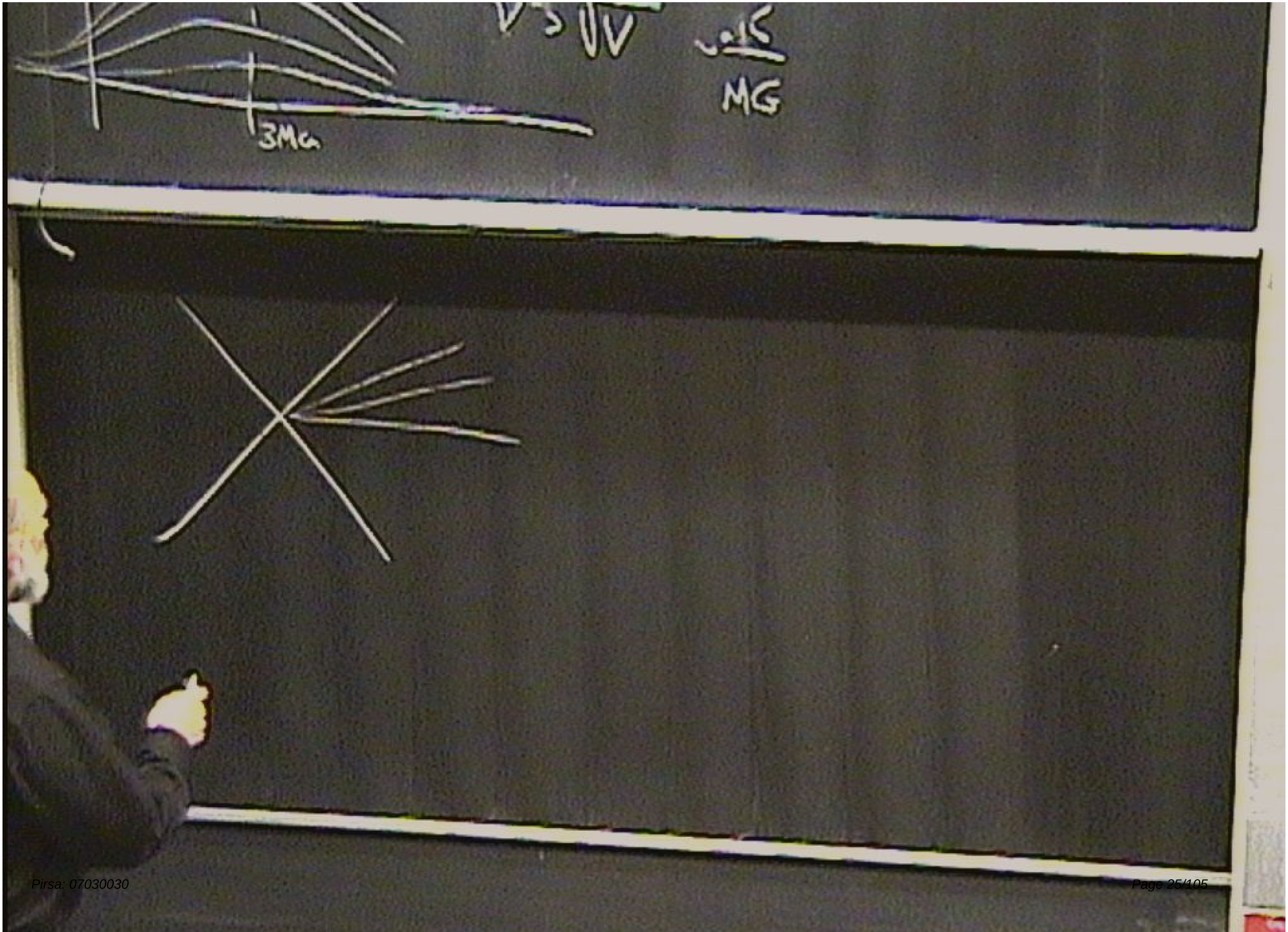
$$L = r \times k$$
$$\ell = MG\,k$$

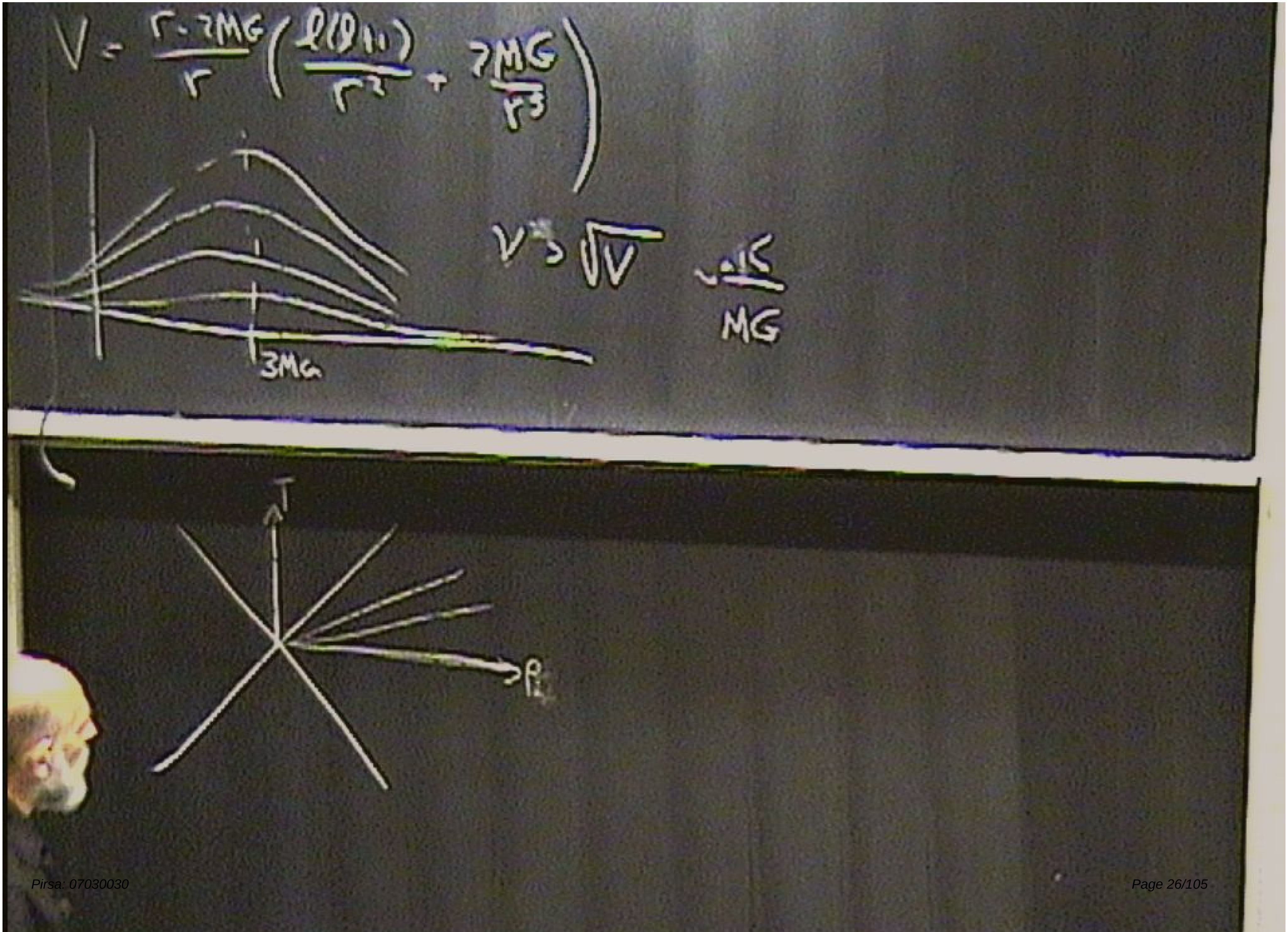

$V = \frac{r-2MG}{r}\left(\frac{\ell(\ell+1)}{r^2} + \frac{2MG}{r^3}\right)$
3MG

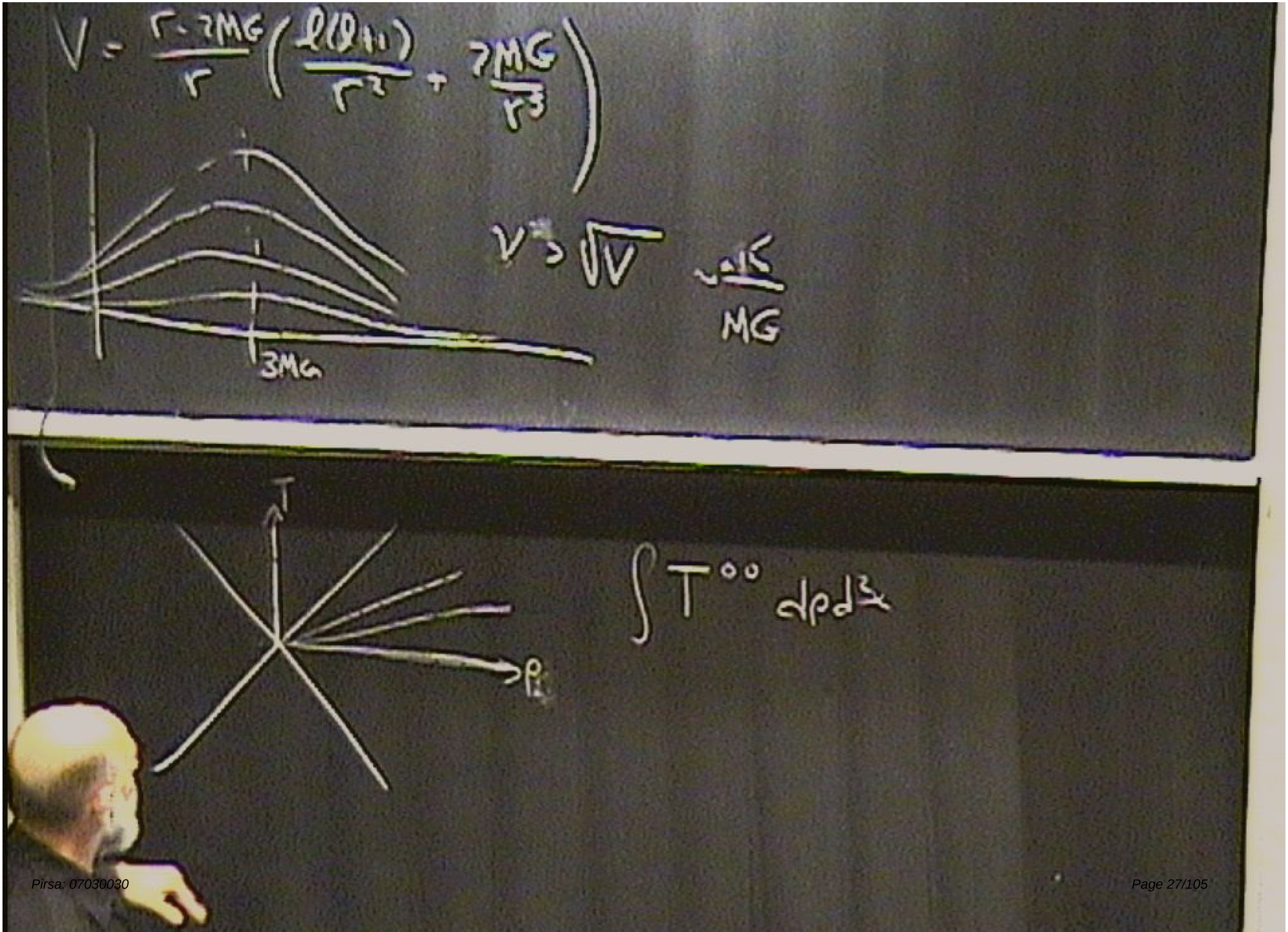

V
3MG

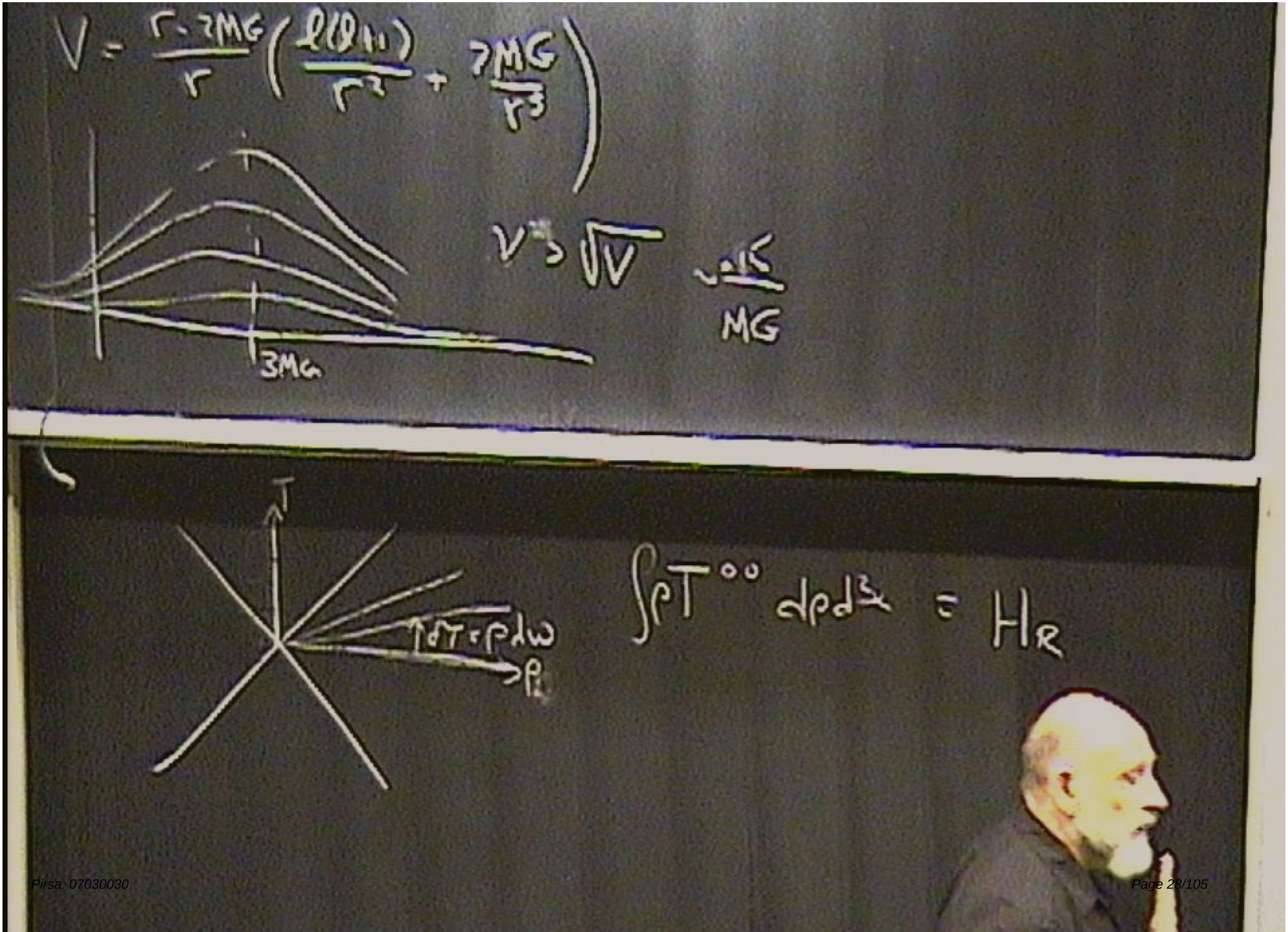

ψ_lm = r χ
∂²ψ/∂t² − ∂²ψ/∂r*² + V(r*)ψ = 0
V = (r − 2MG)/r (l(l+1)/r² + 2MG/r³)
L = r × k
l = MG k
V²

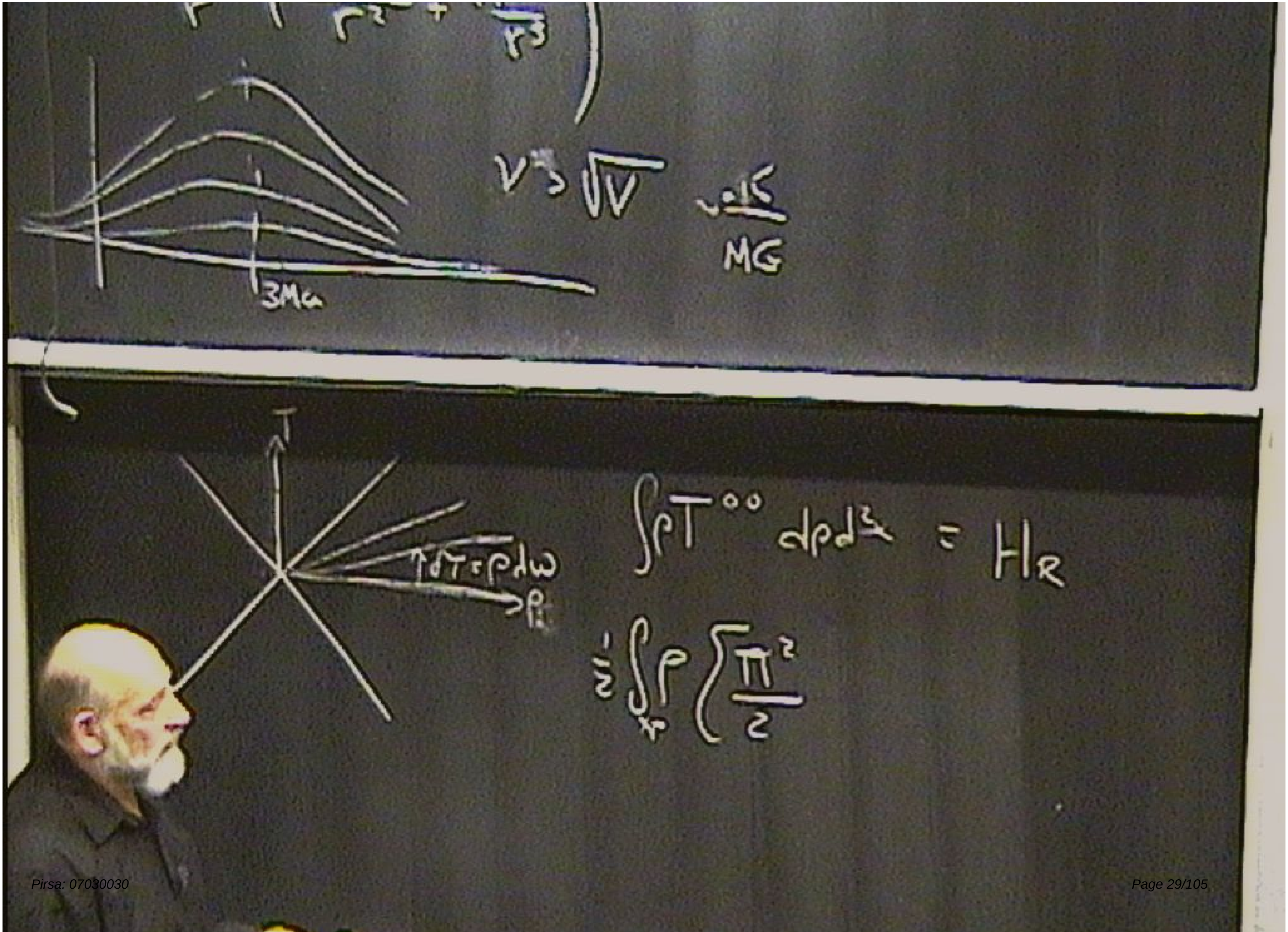

$\psi_{lm} = r\chi$
$\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial r^{*2}} + V(r^*)\psi = 0$
$V = \frac{r-2MG}{r}\left(\frac{l(l+1)}{r^2} + \frac{2MG}{r^3}\right)$
$L = r \times k$
$l = MGk$
$\sqrt{V}$
$\frac{.15}{MG}$
$2MG$

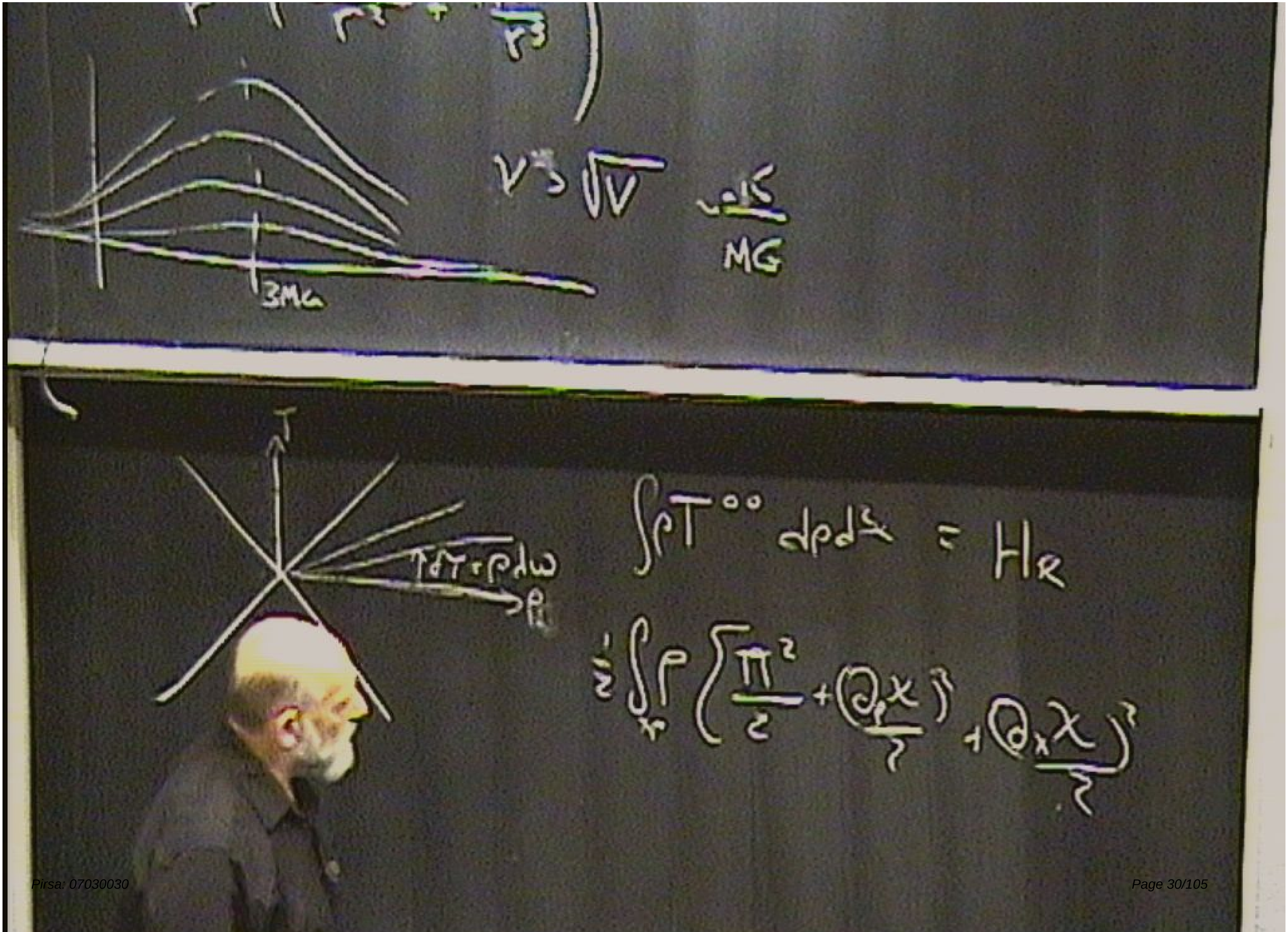

3MG
MG

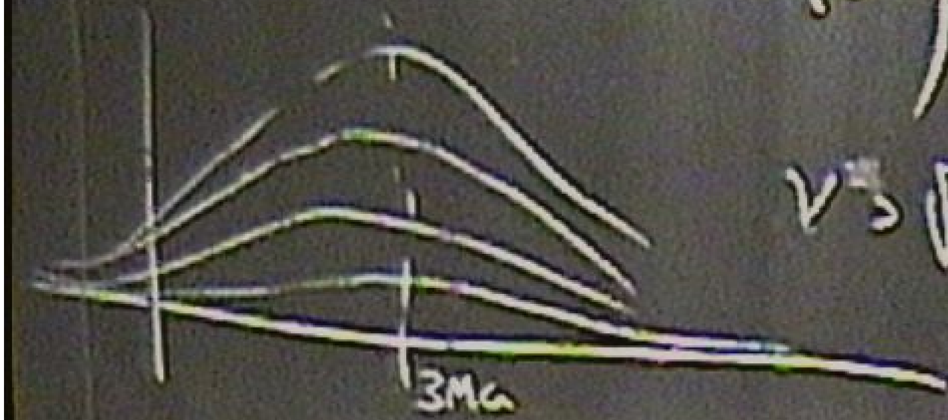

V = (r - 2MG)/r (l(l+1)/r^2 + 2MG/r^3)
3MG
v > √V
MG
T


V = (r-2MG)/r (l(l+1)/r² + 2MG/r³)
3MG
MG
∫T⁰⁰ dp d³x
T

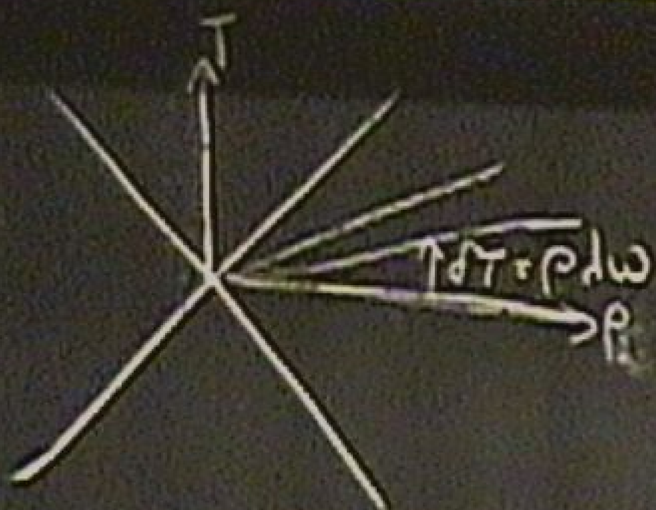

V = (r − 2MG)/r (l(l+1)/r² + 2MG/r³)
3MG
MG
∫ρT⁰⁰ dρ d³x = H_R


3MG
MG
∫ρT⁰⁰ dρd³x = H_R


3MG
MG

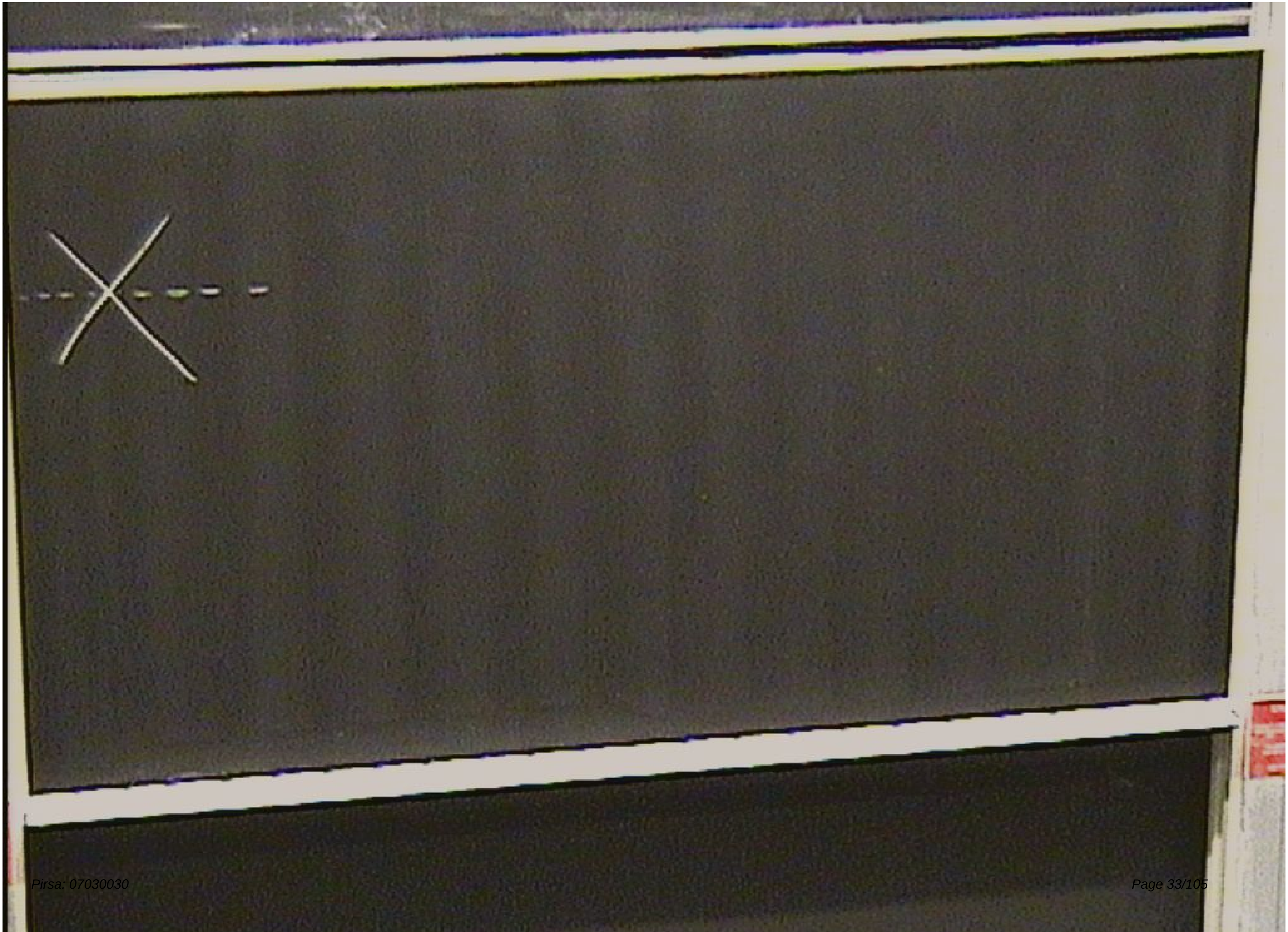


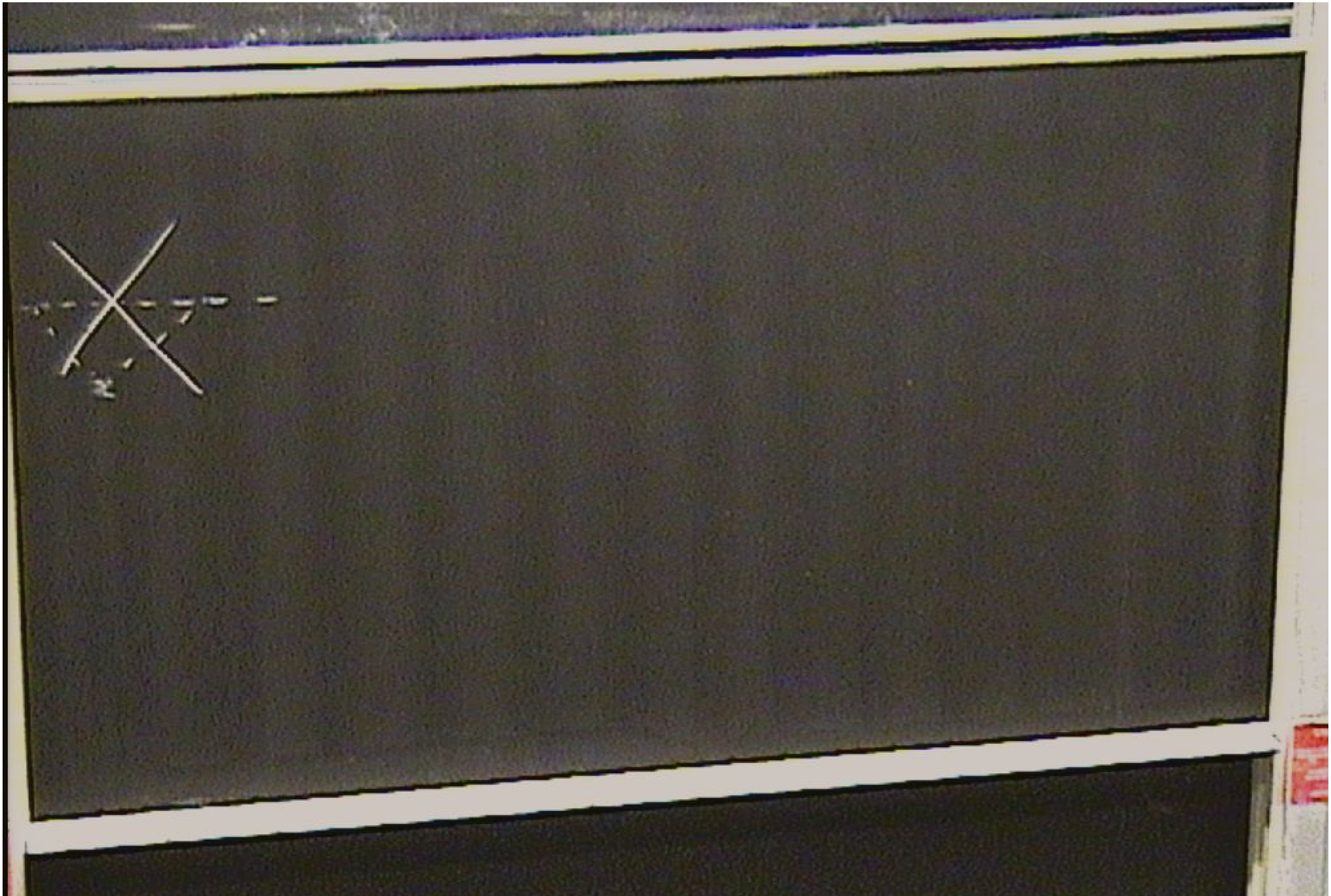
$V \geq \sqrt{V}$ $\frac{.015}{MG}$

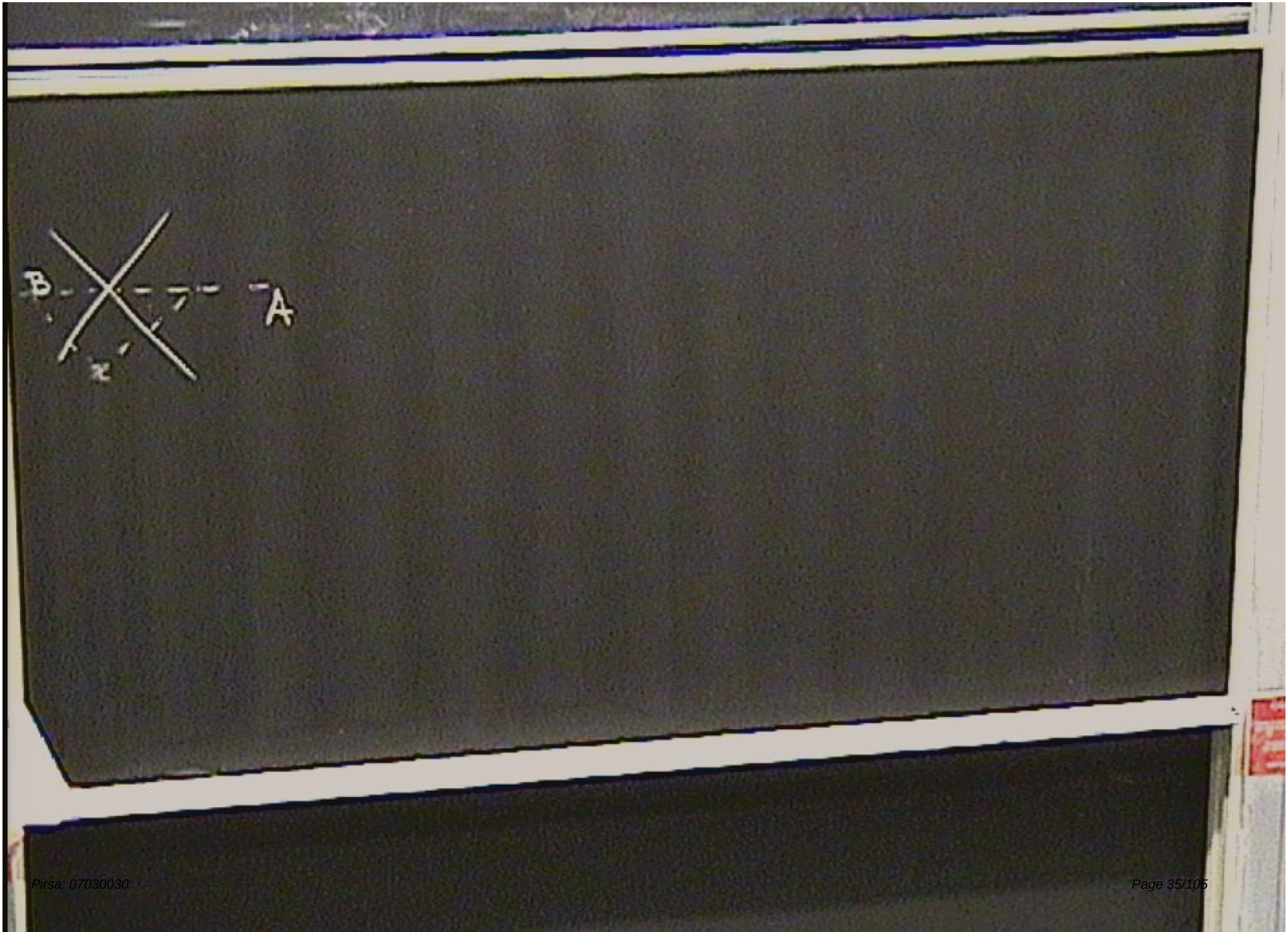


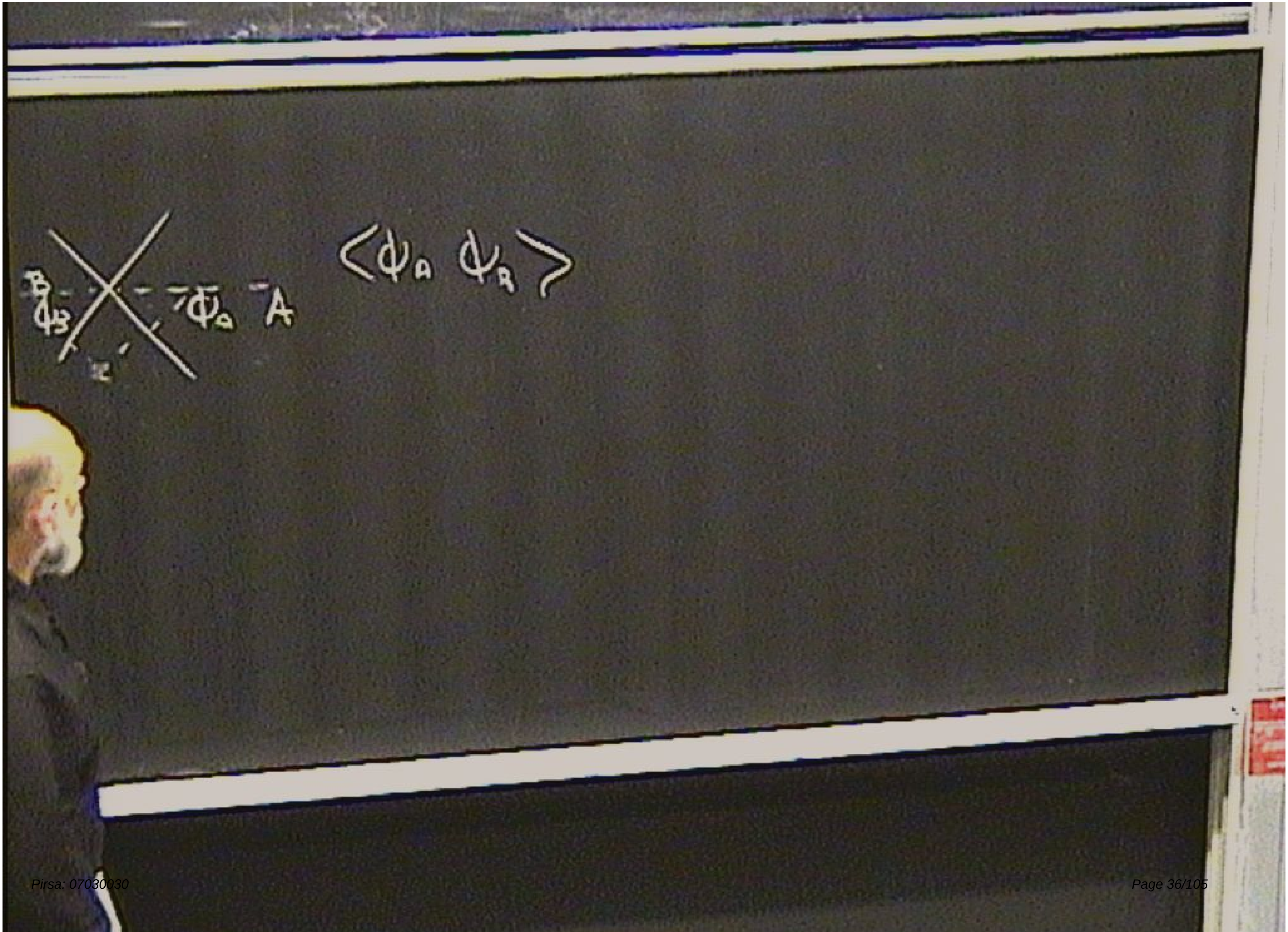
$$\boxed{\int \rho\, T^{00}\, d\rho\, d^3x = H_R}$$

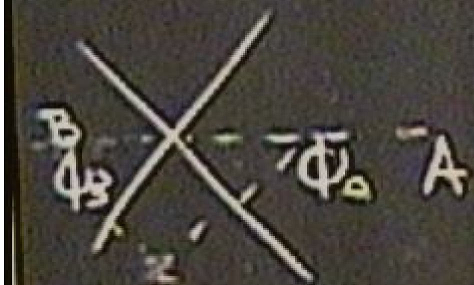
$$\frac{1}{2}\int_x \rho \cancel{\left(\frac{\Pi^2}{2} \quad \frac{(\partial_\rho \chi)^2}{2}\right)} + \frac{(\partial_x \chi)^2}{2}$$



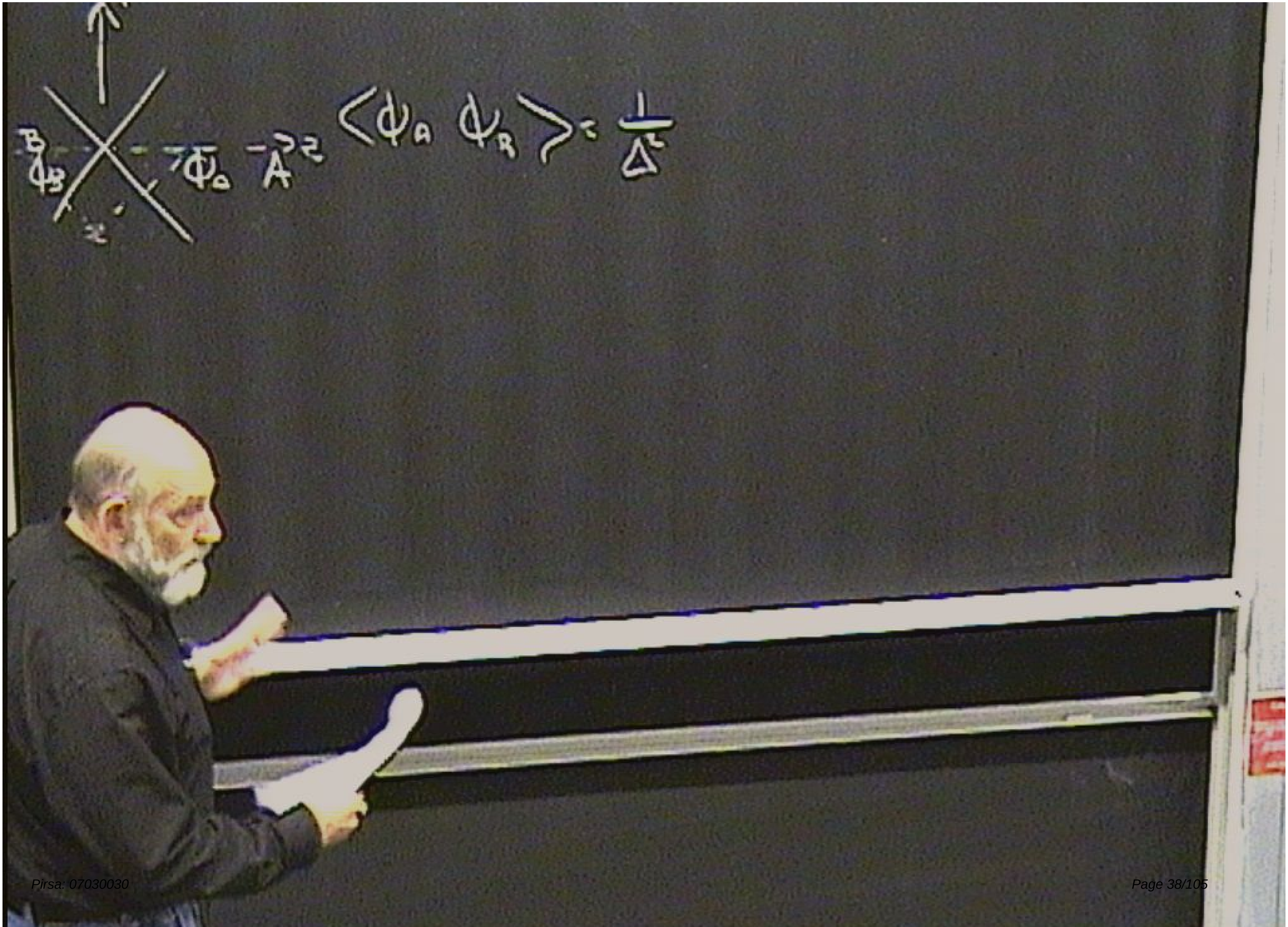


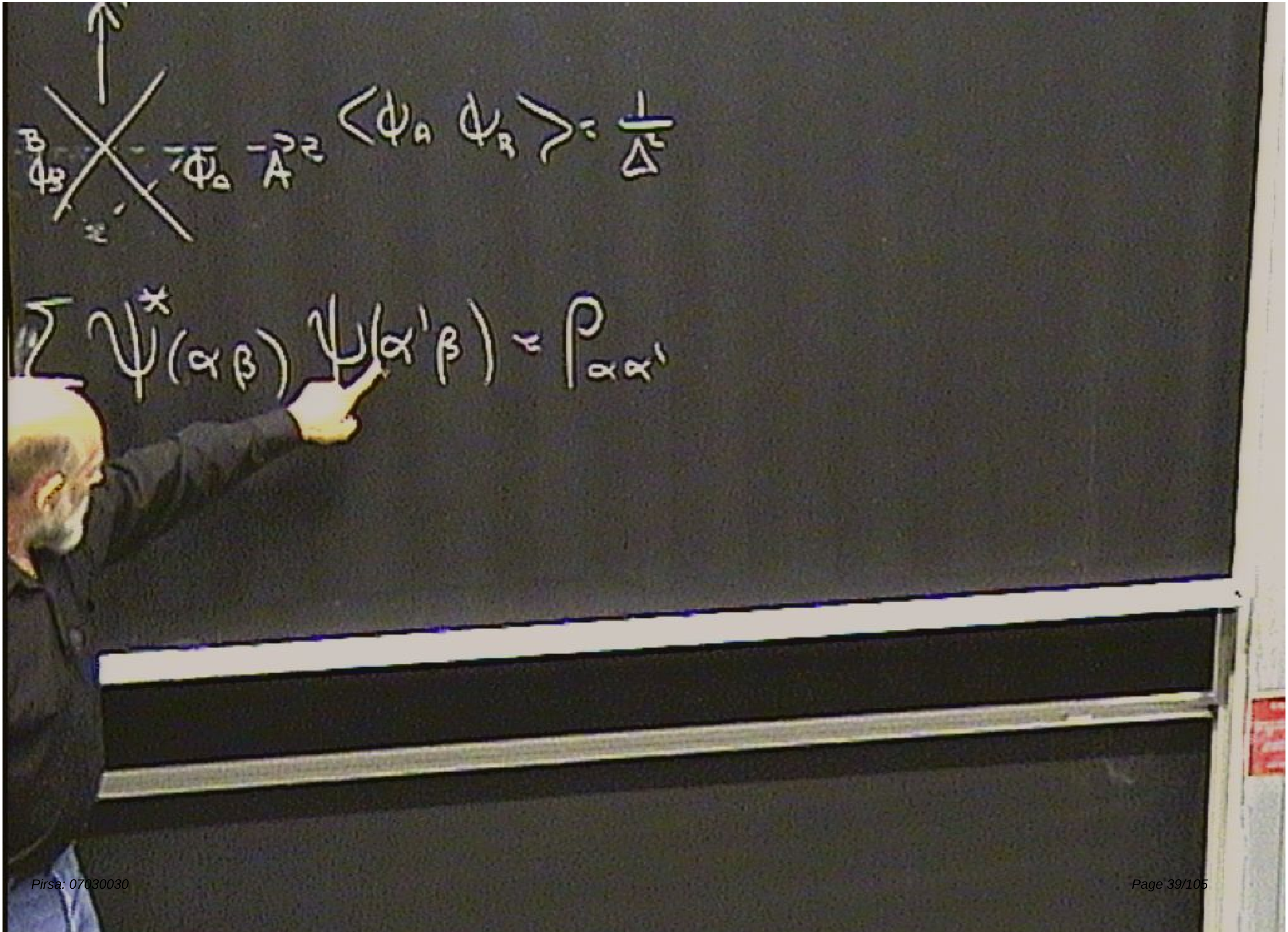

B
A
x

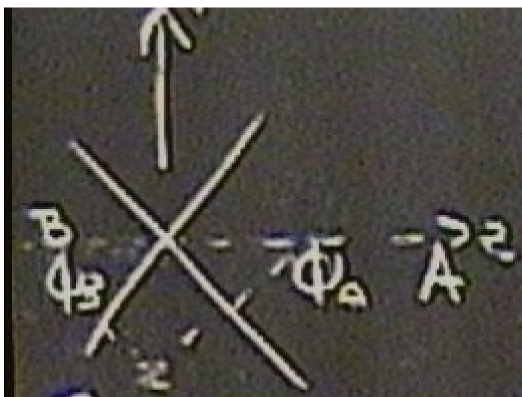

$\langle \phi_A \, \phi_B \rangle$
ϕ_B
ϕ_a
A



$$\langle \phi_A \, \phi_B \rangle = \frac{1}{\Delta^2}$$


$\langle \phi_A \phi_B \rangle = \frac{1}{\Delta^2}$

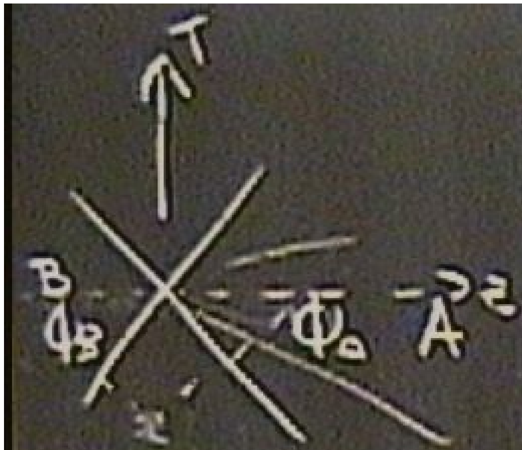

$\langle \phi_A \phi_B \rangle = \frac{1}{\Delta^2}$
$\sum \psi^*(\alpha\beta)\,\psi(\alpha'\beta) = \rho_{\alpha\alpha'}$



$$\langle \phi_A \, \phi_B \rangle = \frac{1}{\Delta^2}$$

$$\psi^*(\alpha\beta)\,\psi(\alpha'\beta) = \rho_{\alpha\alpha'}$$

$$\langle A \rangle = \mathrm{Tr}\,\rho A$$
$$= \mathrm{Tr}\,A\rho$$

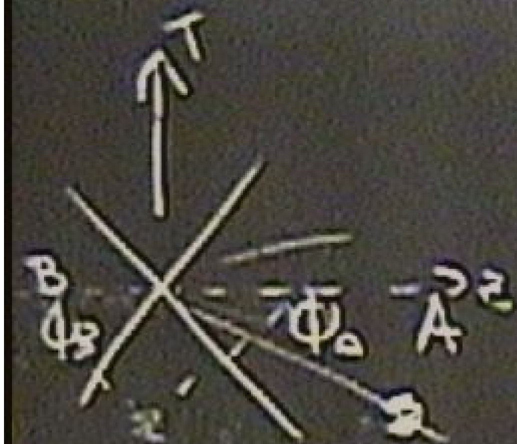


$$\langle \phi_A \, \phi_B \rangle = \frac{1}{\Delta^2}$$

$$\sum_\beta \psi^*(\alpha\beta)\,\psi(\alpha'\beta) = \rho_{\alpha\alpha'}$$

$$\langle A \rangle = \mathrm{Tr}\,\rho A$$
$$= \mathrm{Tr}\,A\rho$$

$$\frac{d\rho_A}{dt} = i\left[\rho_A \; H_R\right]$$

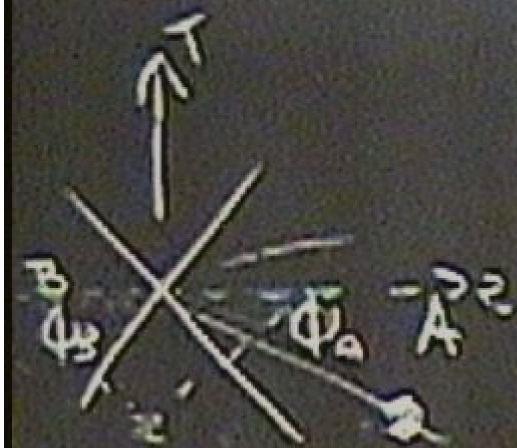


$$\langle \phi_A \; \phi_B \rangle = \frac{1}{\Delta^2}$$

$$\sum \psi^*(\alpha\beta)\, \psi(\alpha'\beta) = \rho_{\alpha\alpha'}$$

$$\langle A \rangle = \mathrm{Tr}\,\rho A$$
$$= \mathrm{Tr}\,A\rho$$

$$\frac{d\rho_A}{dt} = i\,[\rho_A \;\; H_R]$$

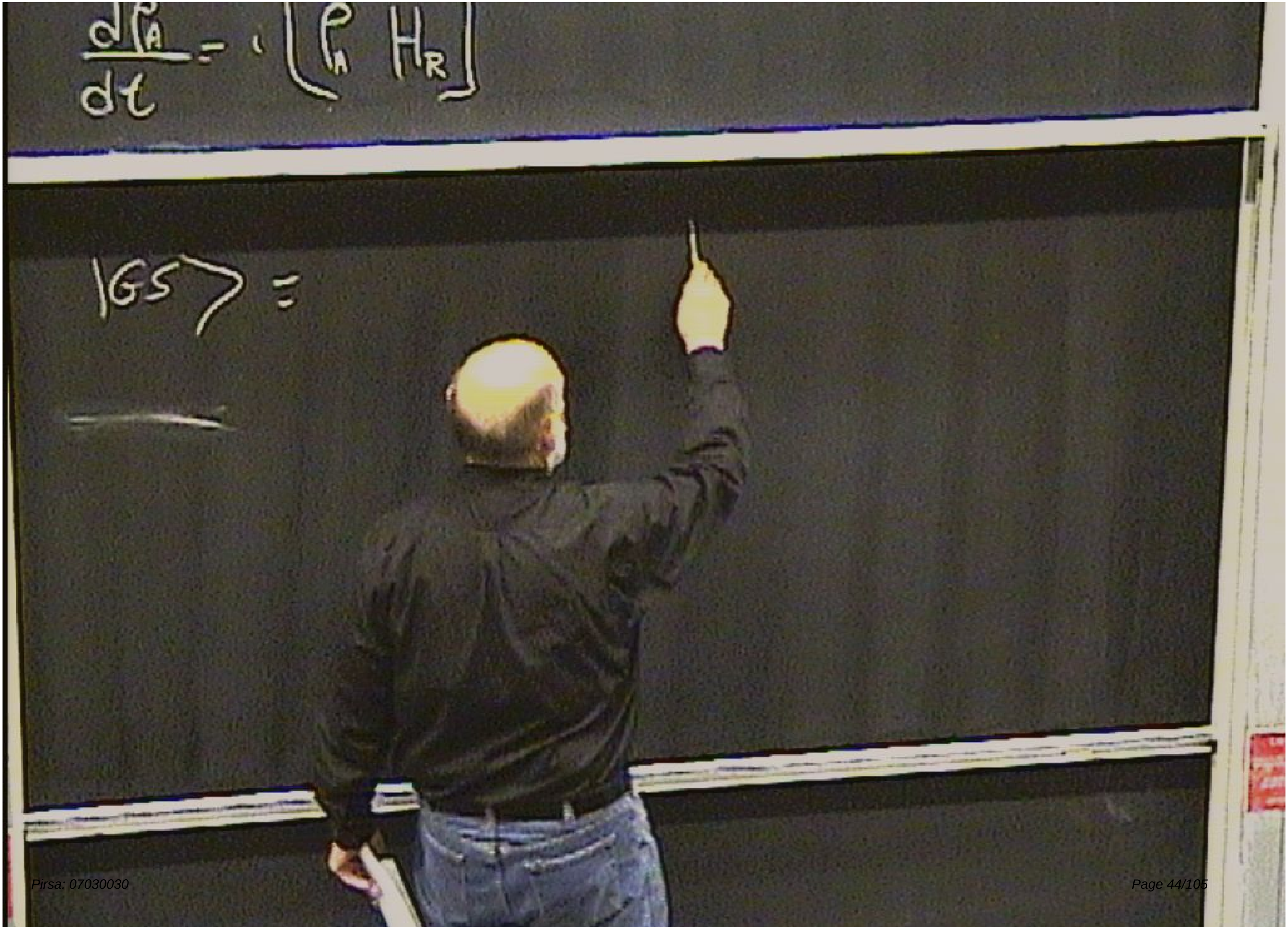


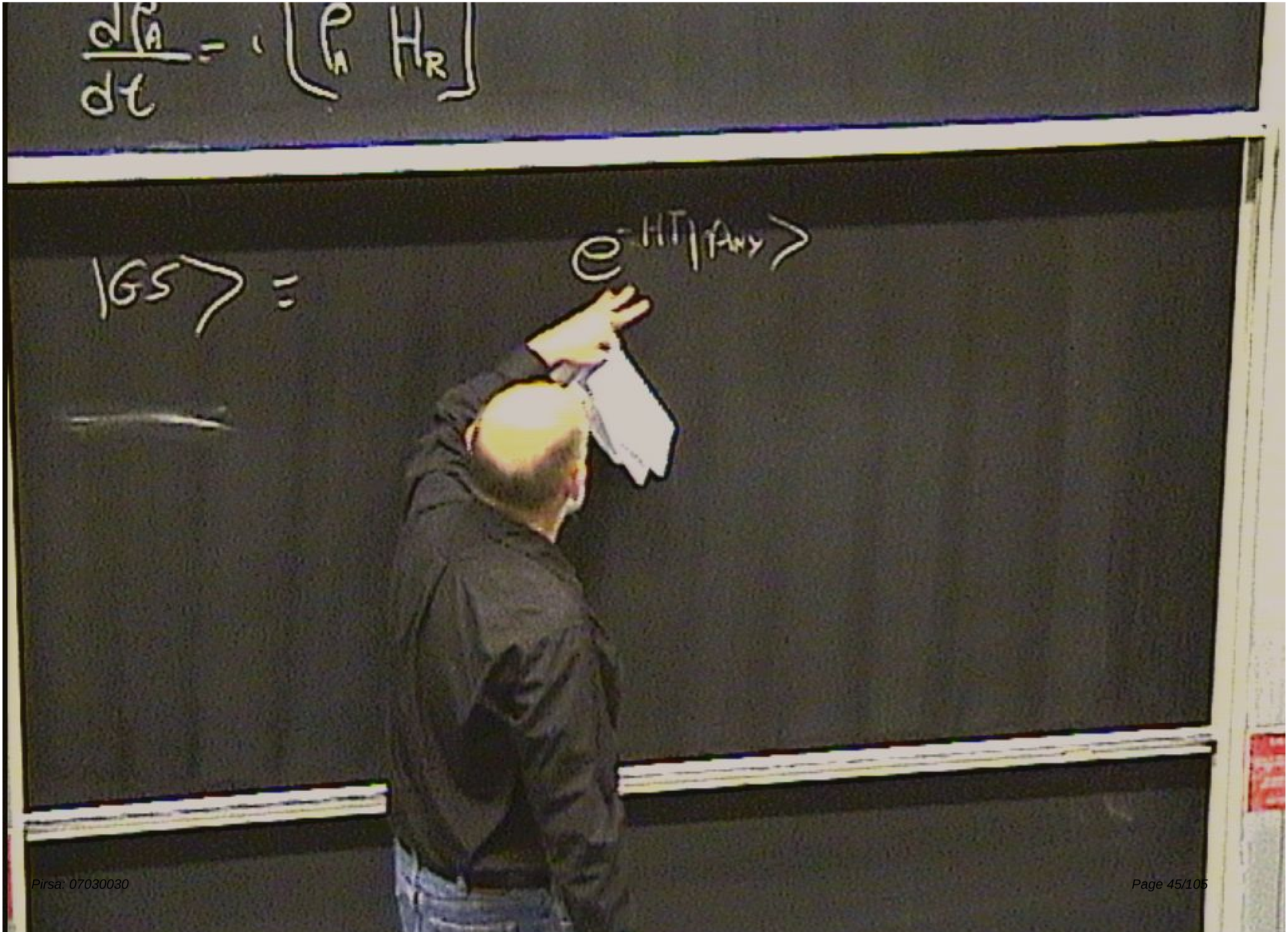
$$\langle \phi_A \, \phi_B \rangle = \frac{1}{\Delta^2}$$

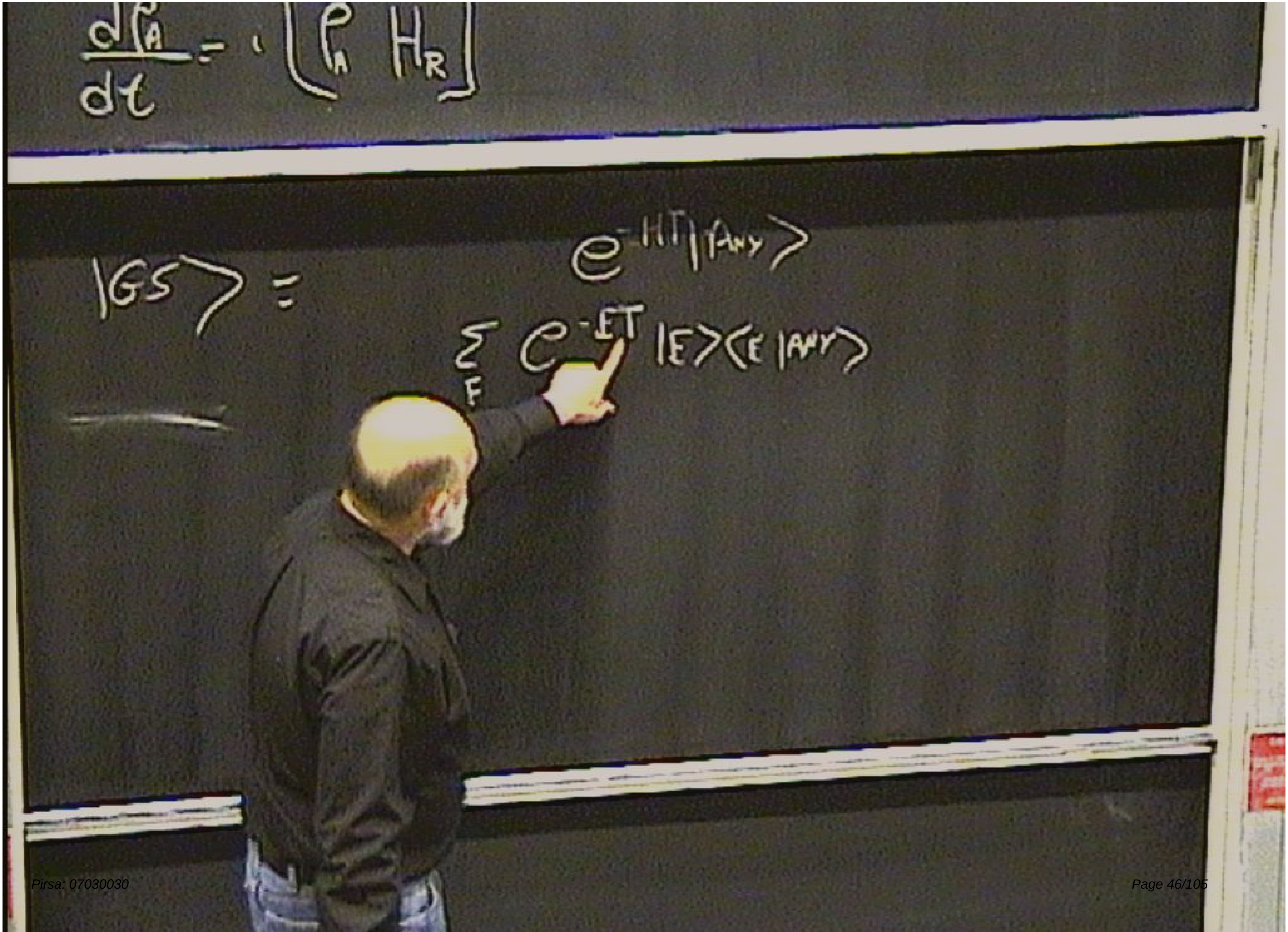
$$\sum_\beta \psi^*(\alpha\beta)\,\psi(\alpha'\beta) = \rho_{\alpha\alpha'}$$

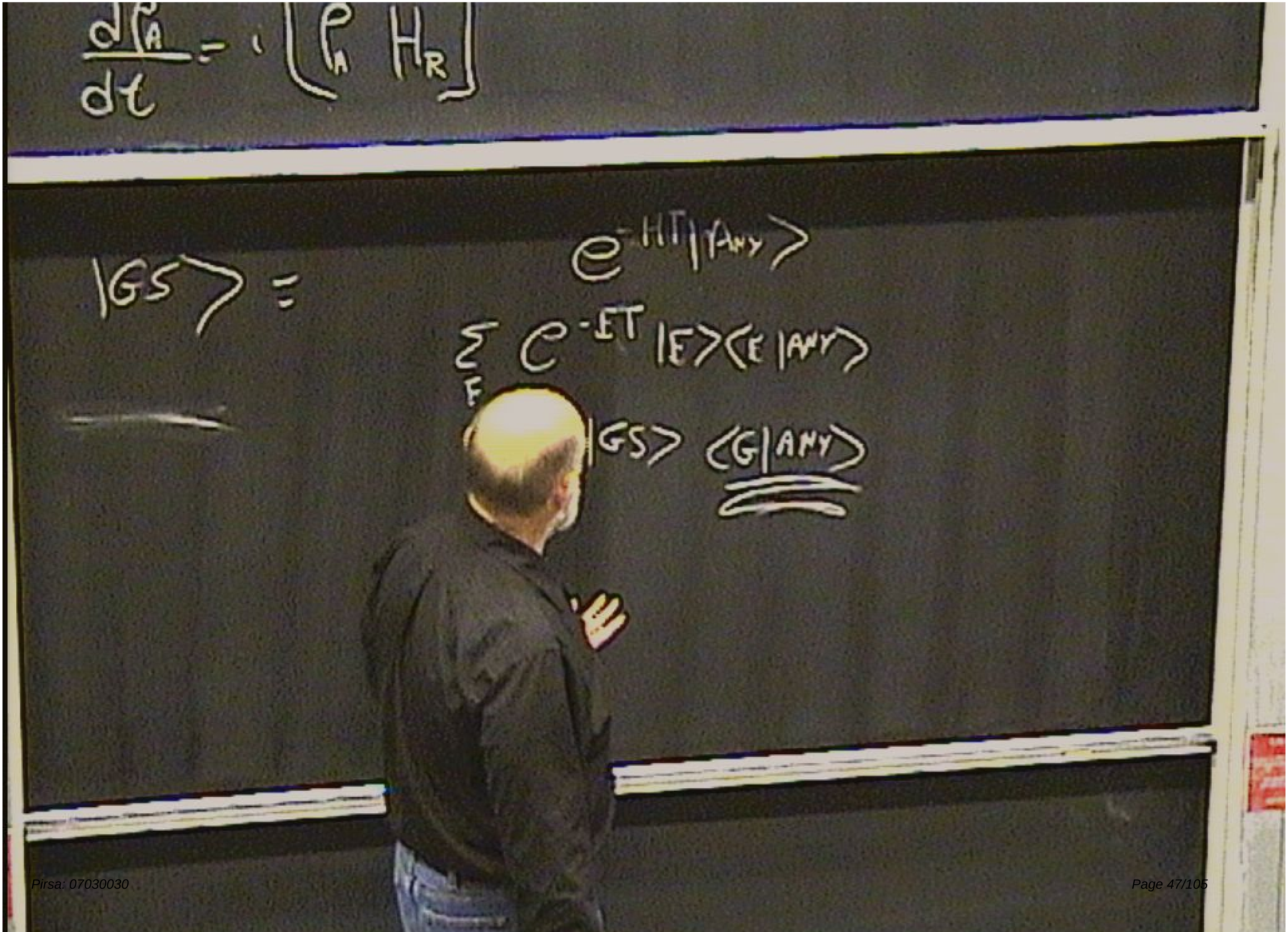
$$\langle A \rangle = \mathrm{Tr}\,\rho A = \mathrm{Tr}\,A\rho$$

$$\frac{d\rho_A}{dt} = i\left[\rho_A \;\; H_R\right]$$

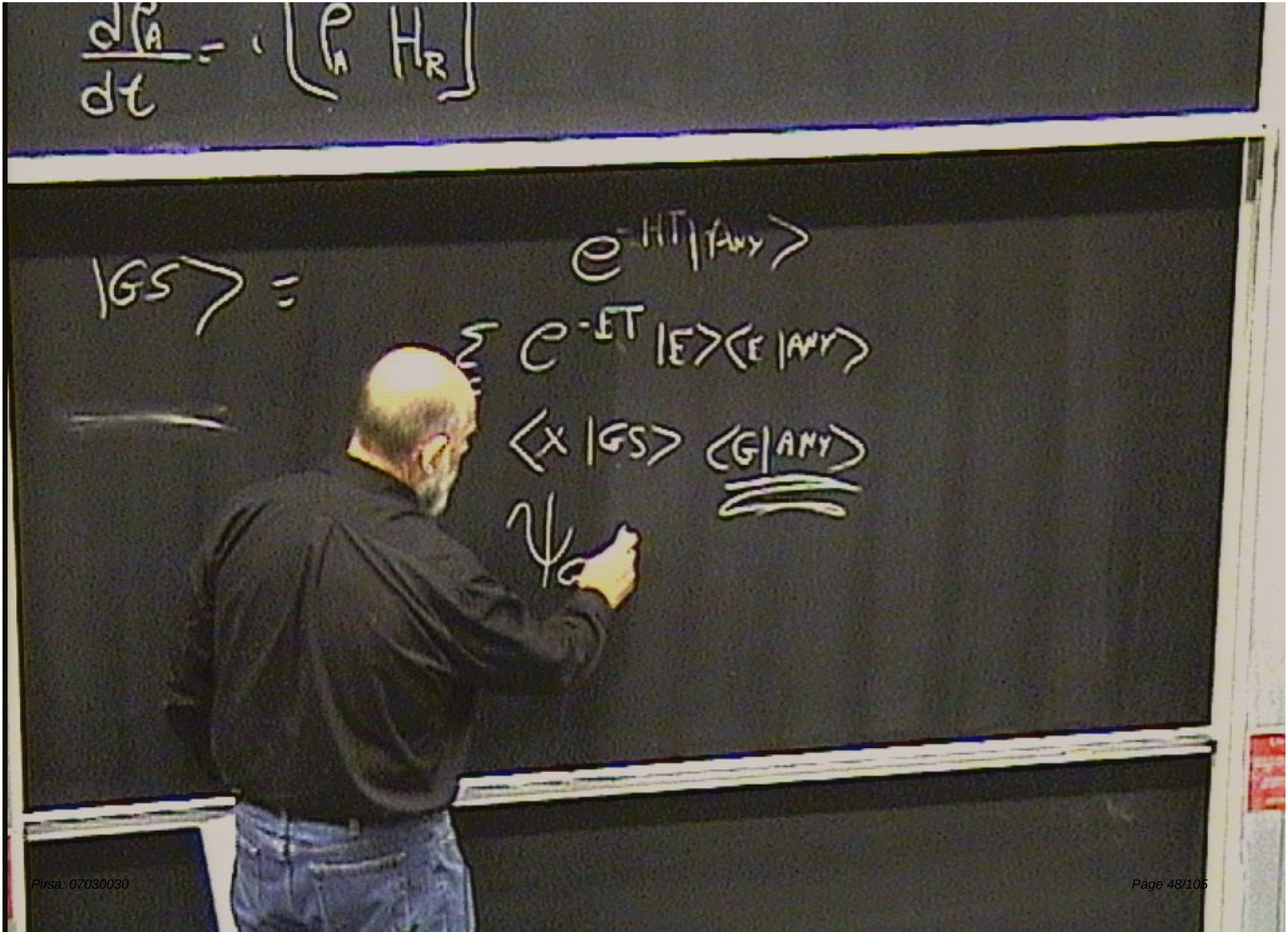

$\frac{d\rho_A}{dt} = i[\rho_A \ H_R]$
$|GS\rangle =$


$\frac{d\rho_A}{dt} = i\,[\rho_A\ H_R]$
$|GS\rangle =$
$e^{-HT}|\psi_{Any}\rangle$

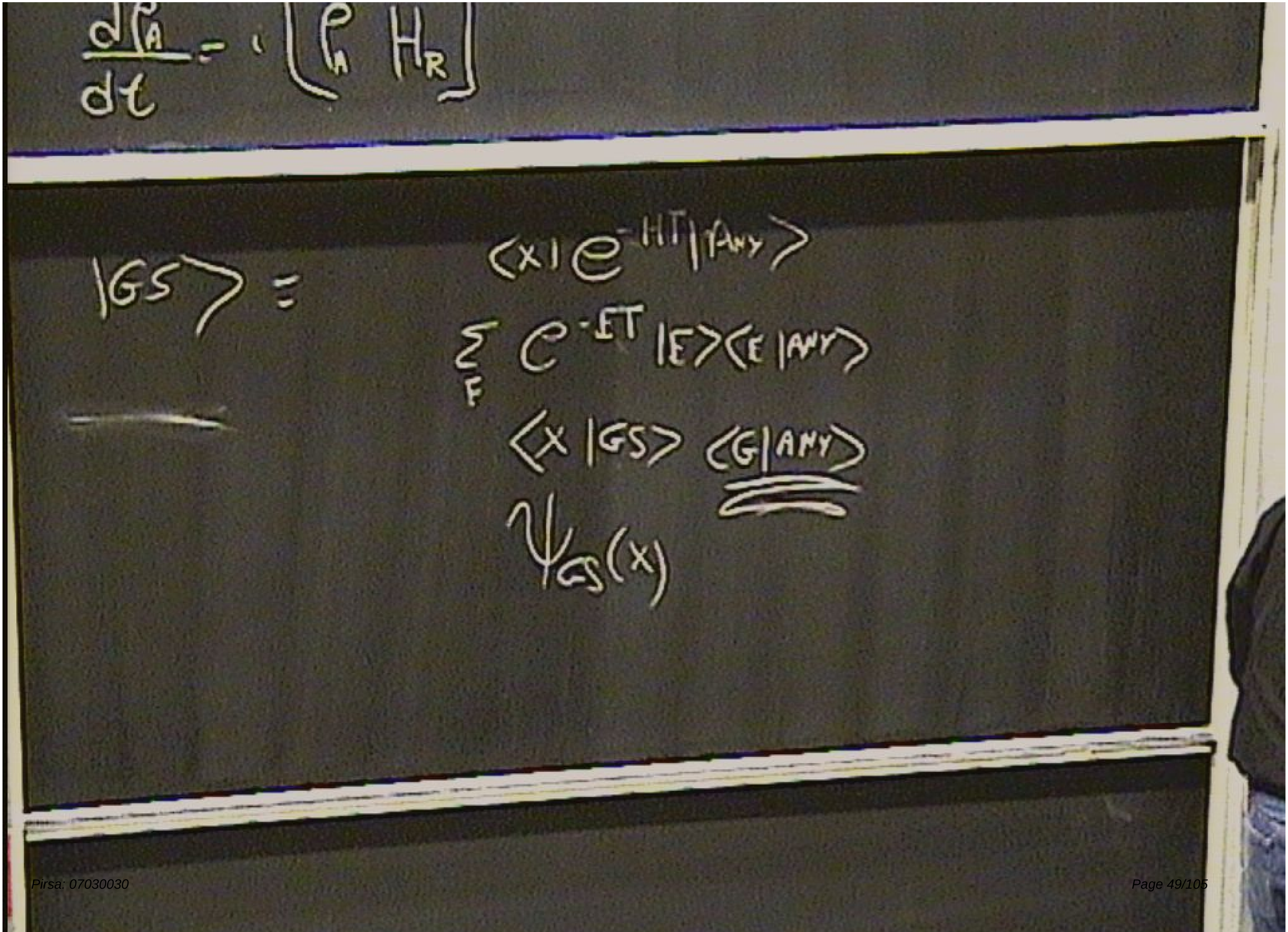

$\frac{d\rho_A}{dt} = i\,[\rho_A\ H_R]$
$|GS\rangle =$
$e^{-HT}\,|\,|ANY\rangle$
$\sum_E e^{-ET}\,|E\rangle\langle E\,|ANY\rangle$

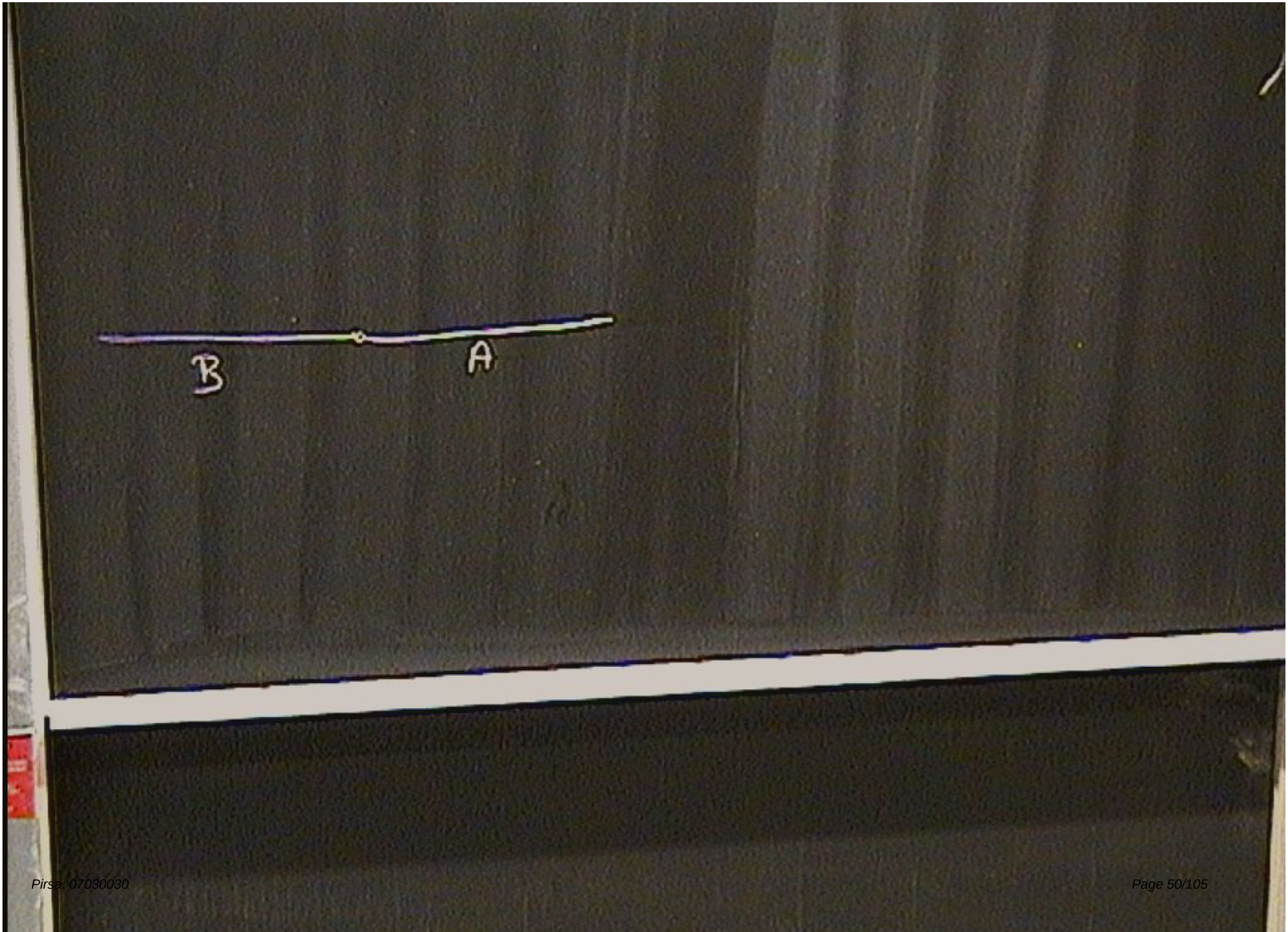


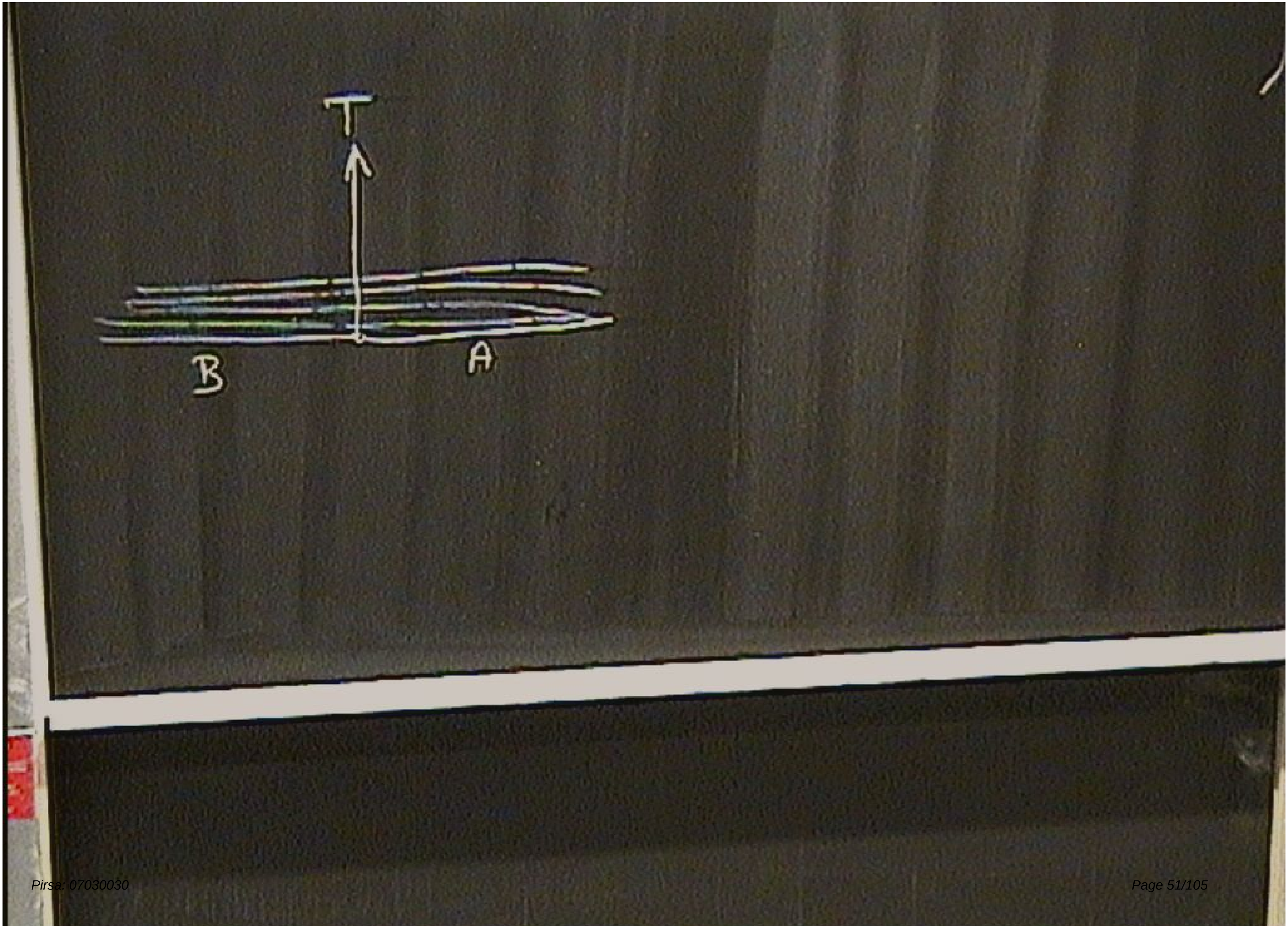
$\frac{d\rho_A}{dt} = i[\rho_A \; H_R]$
$|GS\rangle =$
$e^{-HT}|Any\rangle$
$\sum_E e^{-ET}|E\rangle\langle E|Any\rangle$
$|GS\rangle \langle G|Any\rangle$

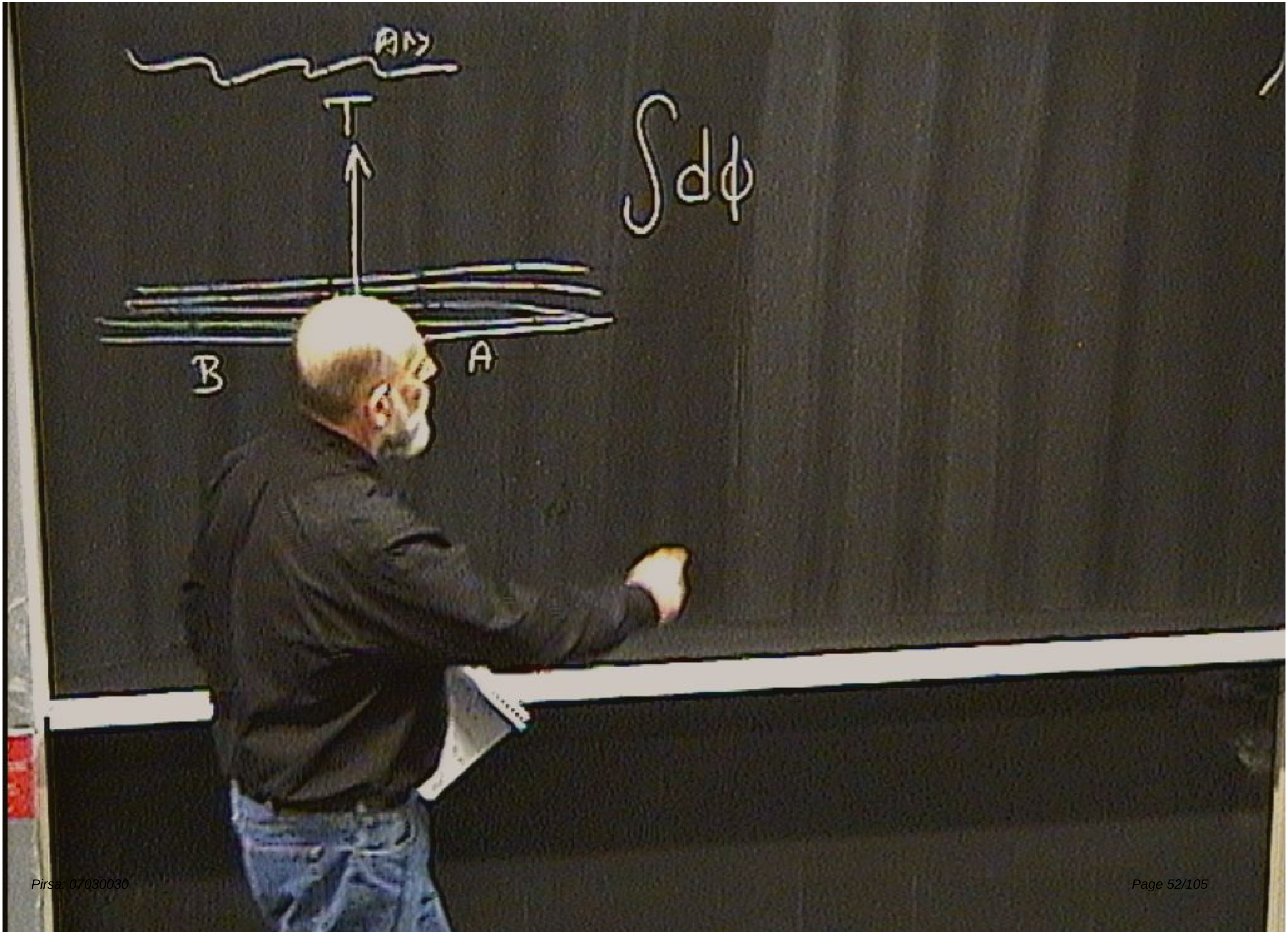


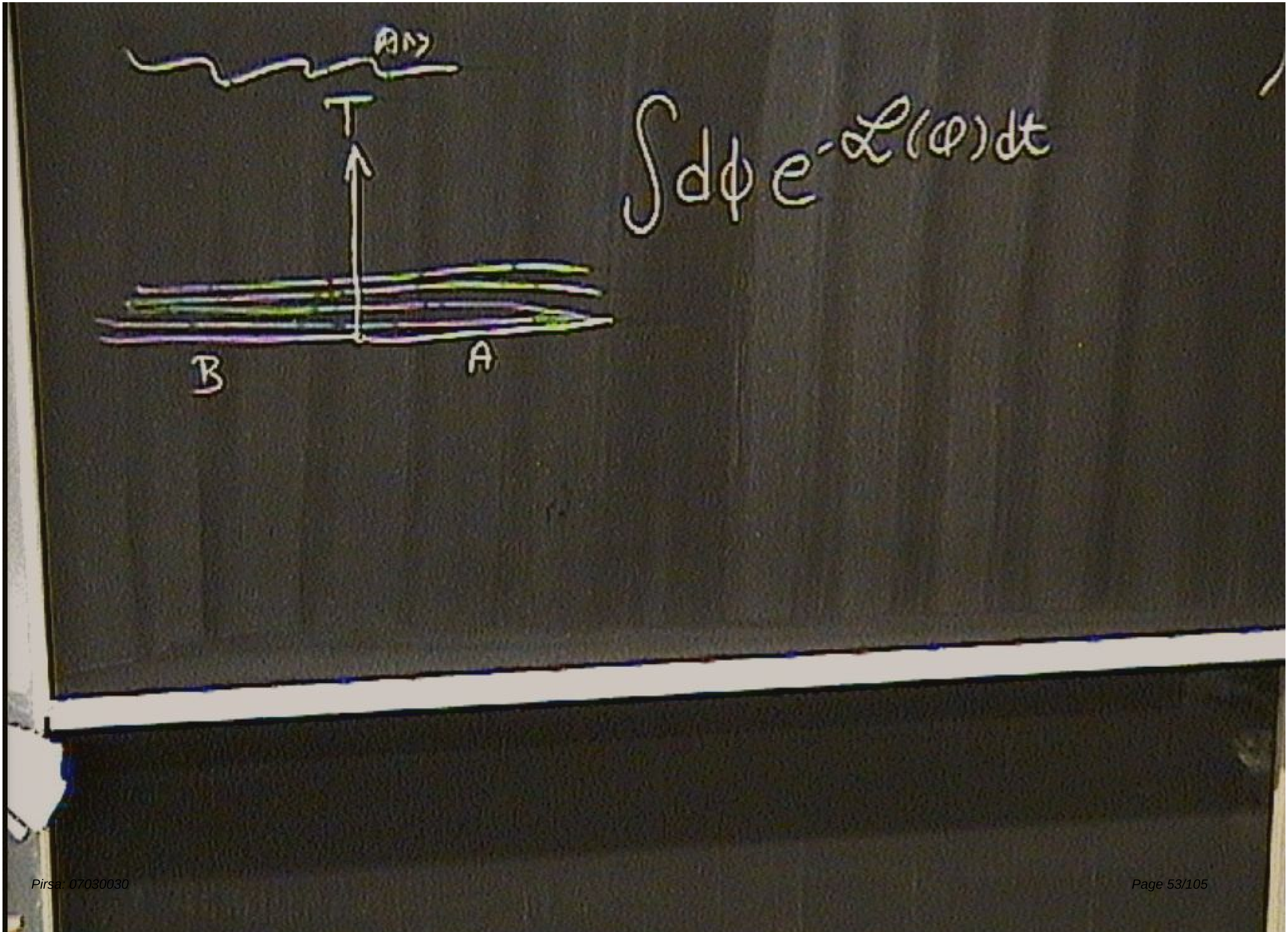
$\frac{d\rho_A}{dt} = i\,[\rho_A \; H_R]$
$|GS\rangle =$
$e^{-HT}|ANY\rangle$
$\sum e^{-ET}|E\rangle\langle E|ANY\rangle$
$\langle x|GS\rangle \; \langle G|ANY\rangle$
ψ_G

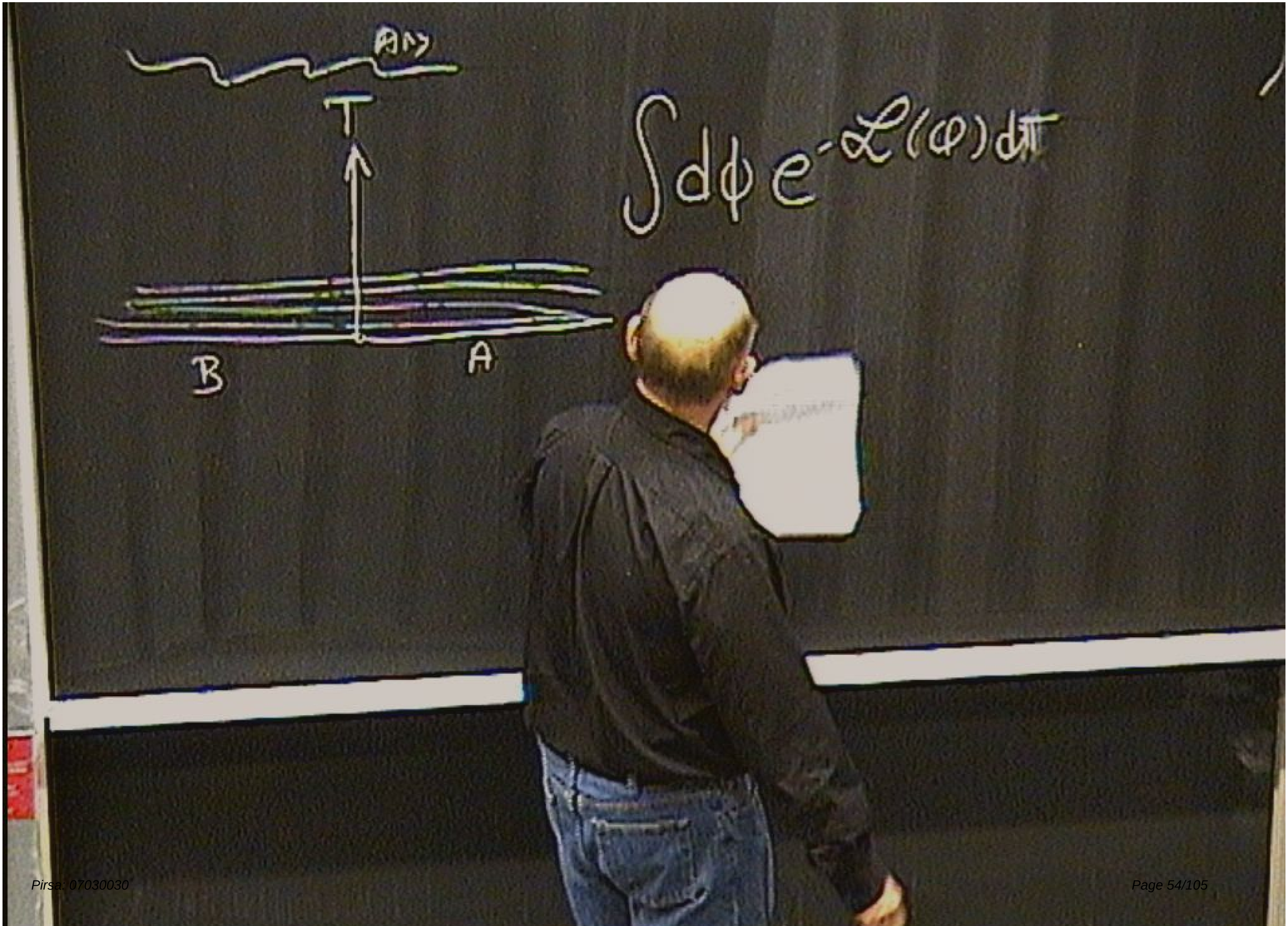

$\frac{d\rho_A}{dt} = i[\rho_A \; H_R]$
$|GS\rangle =$
$\langle x | e^{-HT} | \psi_{ANY} \rangle$
$\sum_E e^{-ET} |E\rangle\langle E | ANY\rangle$
$\langle x | GS\rangle \; \langle G | ANY\rangle$
$\psi_{GS}(x)$

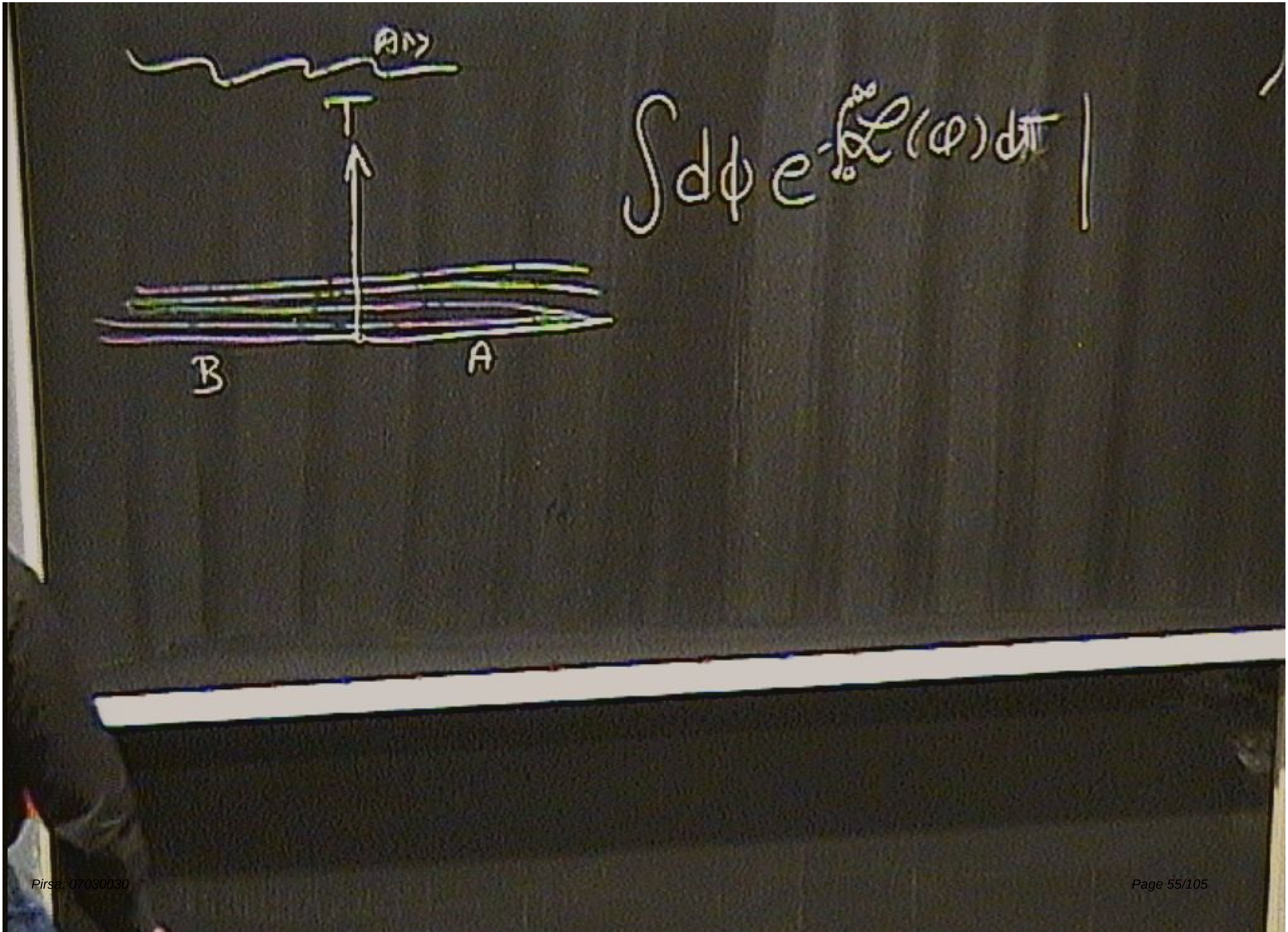

B
A

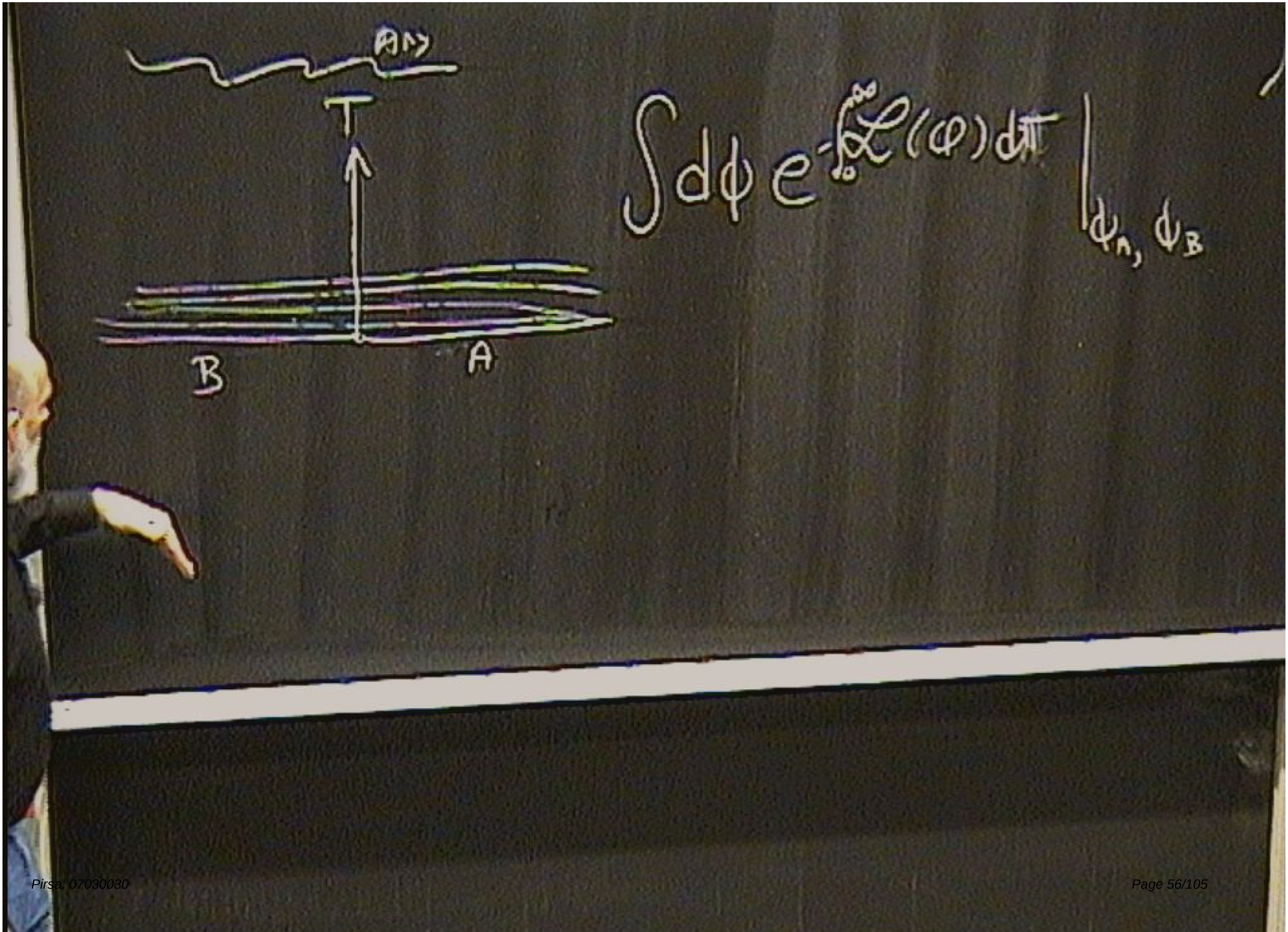

T
B
A

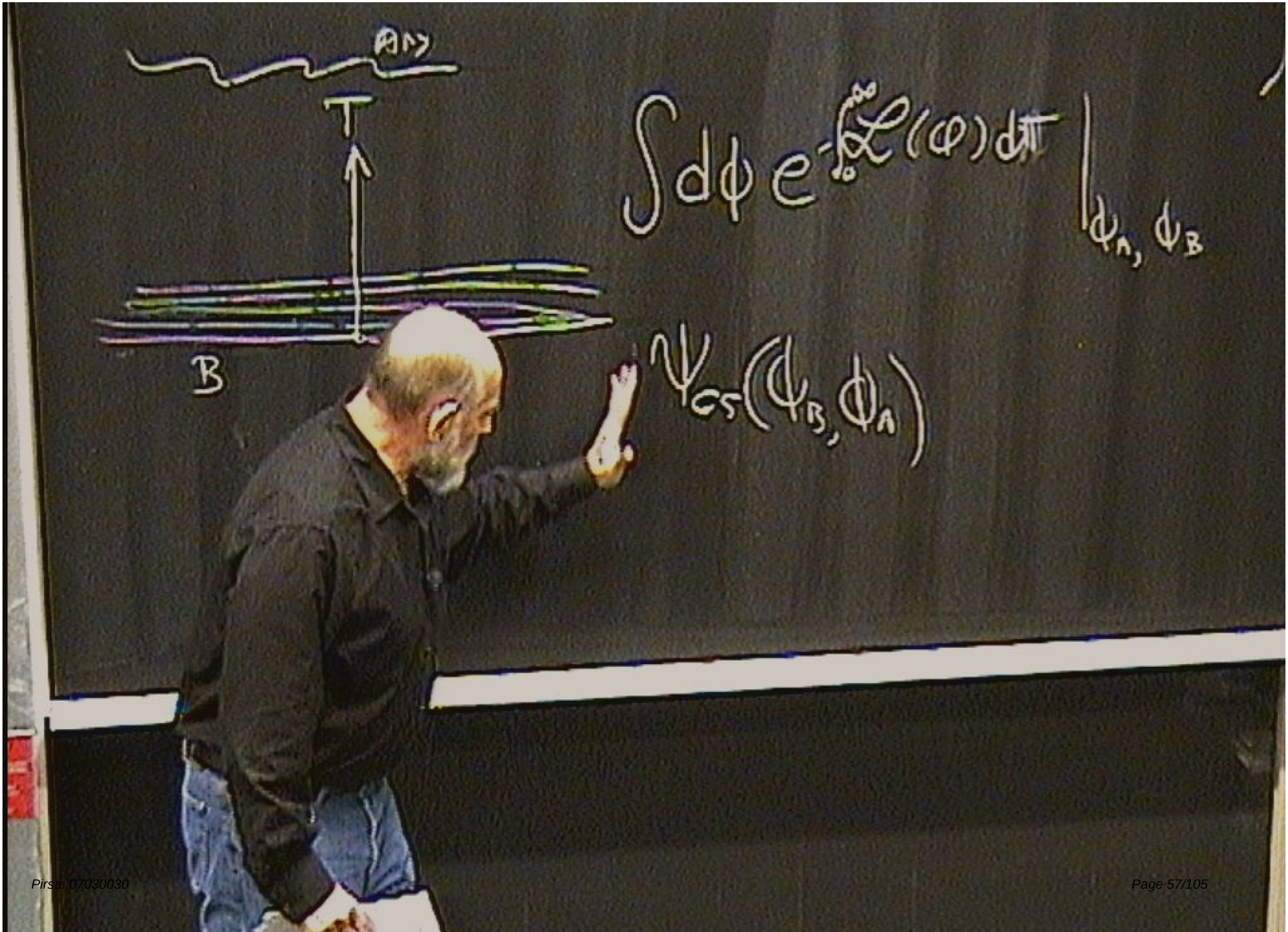

T
∫dφ
B
A

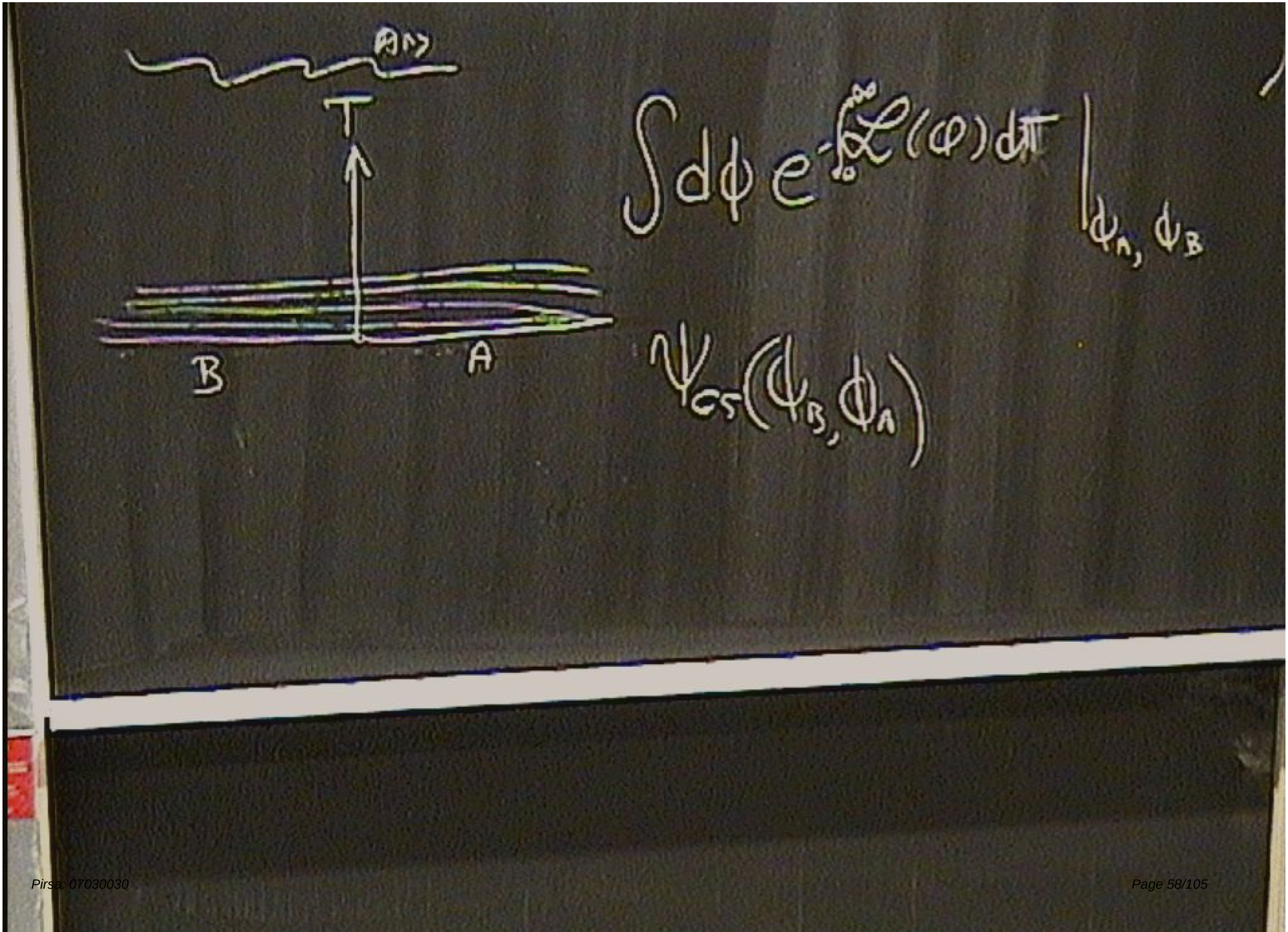

T
B
A
$\int d\phi\, e^{-\mathcal{L}(\varphi)dt}$

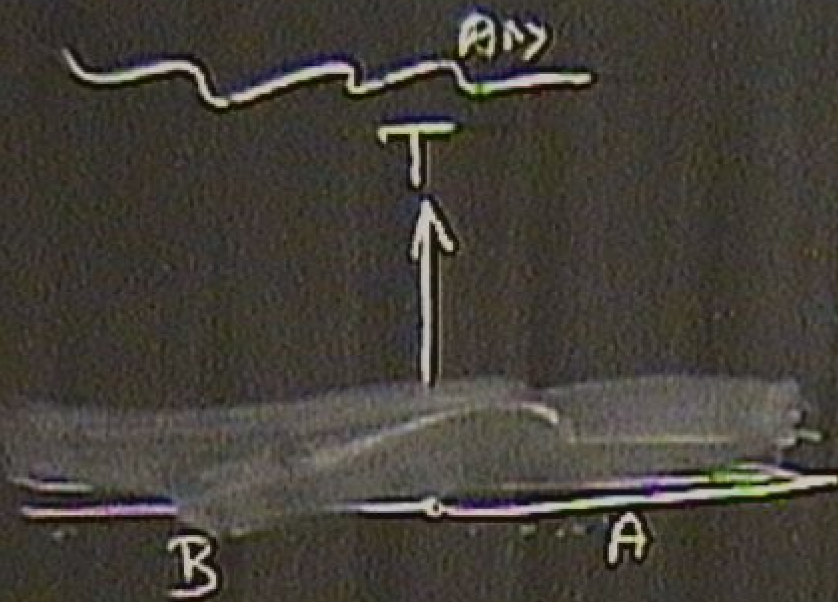

T
B
A
$\int d\phi\, e^{-\mathcal{L}(\phi)\, dT}$


T
B
A
$\int d\phi\, e^{-\int \mathcal{L}(\phi)\, d\tau}$


T
B
A
∫dφ e^{-∫L(φ)dτ} |φ_A, φ_B

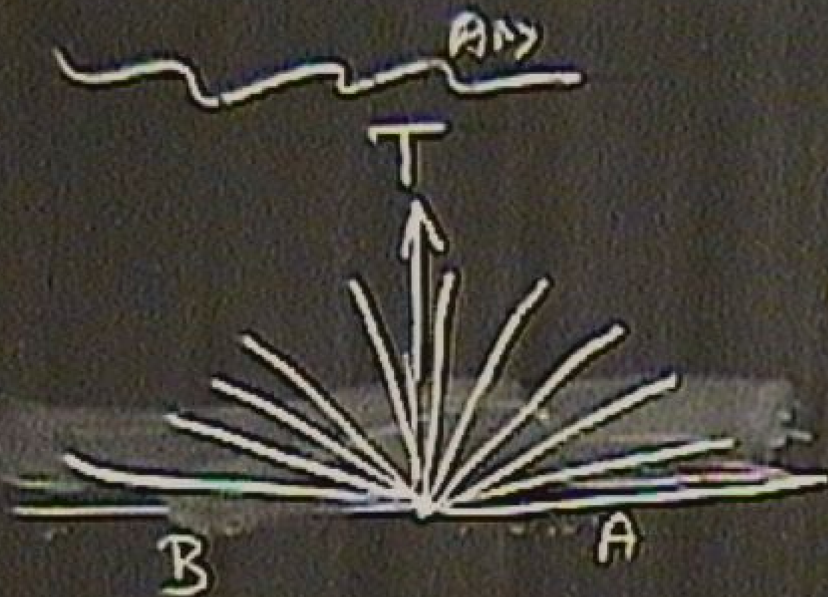

T
B


T
B
A
$\int d\phi \, e^{-\int \mathcal{L}(\phi) d\tau} \Big|_{\phi_A, \phi_B}$
$\Psi_{GS}(\phi_B, \phi_A)$



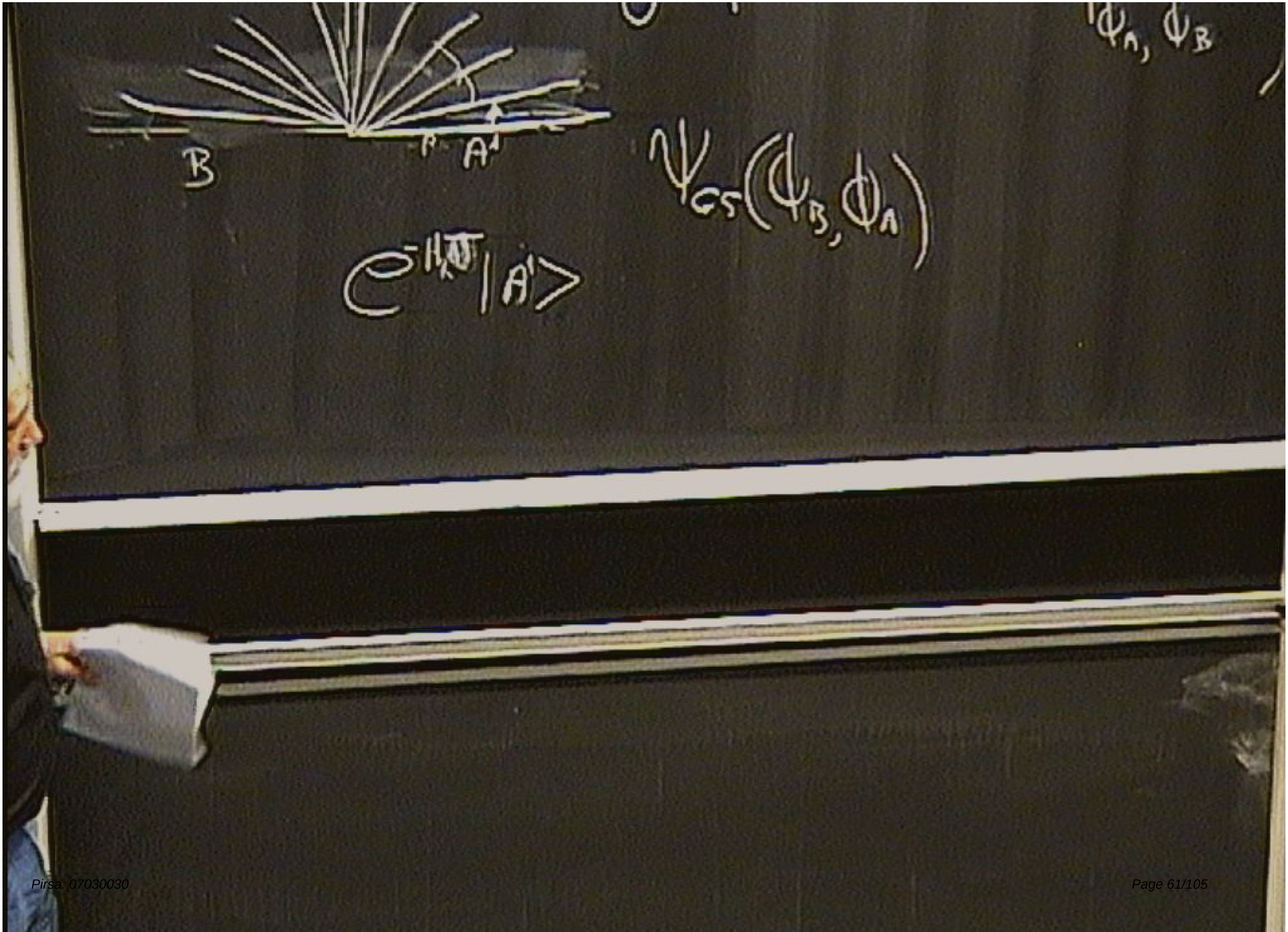
$$\int d\phi \, e^{-\int_0^\infty \mathcal{L}(\phi)\, d\tau} \Big|_{\phi_A,\, \phi_B}$$

$$\Psi_{GS}(\phi_B, \phi_A)$$

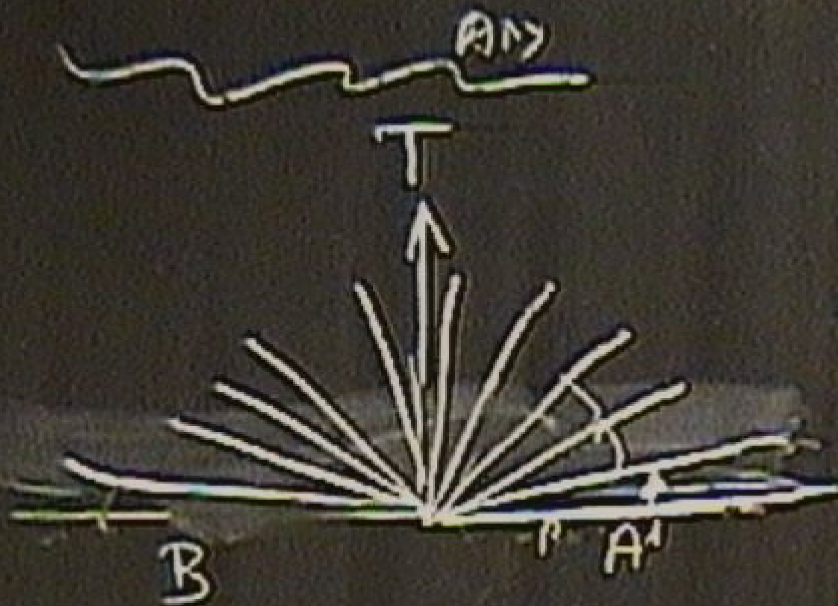


$$\int d\phi \, e^{-\int_0^\infty \mathcal{L}(\phi)\, d\tau} \Big|_{\phi_A,\, \phi_B}$$

$$\Psi_{GS}(\phi_B, \phi_A)$$



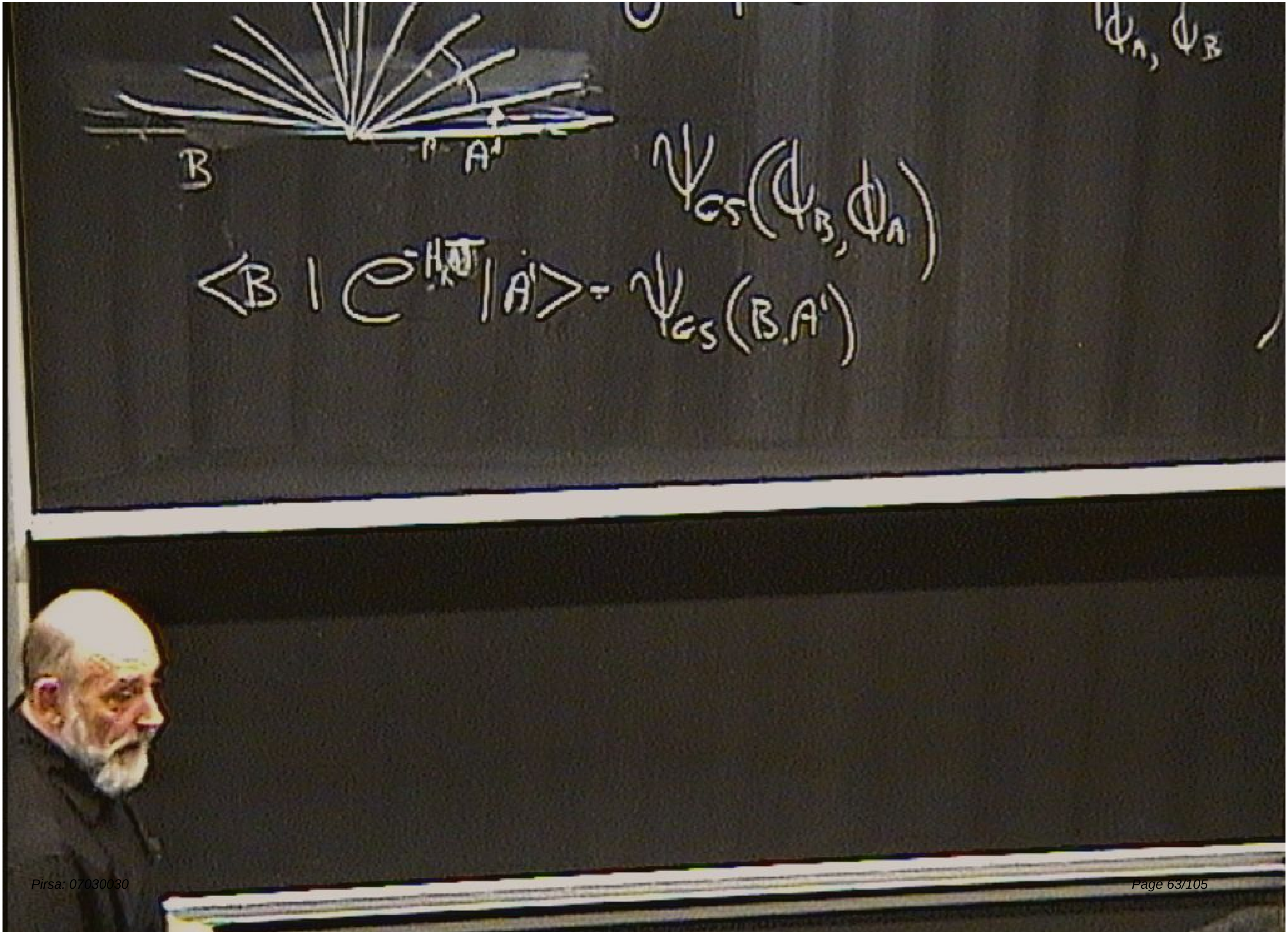
B
A'
φA, φB
ΨGS(φB, φA)
e^{-H_R π}|A'⟩



$$\int d\phi\, e^{-\int_0^{\infty} \mathcal{L}(\phi)\, d\tau} \Big|_{\phi_A,\, \phi_B}$$

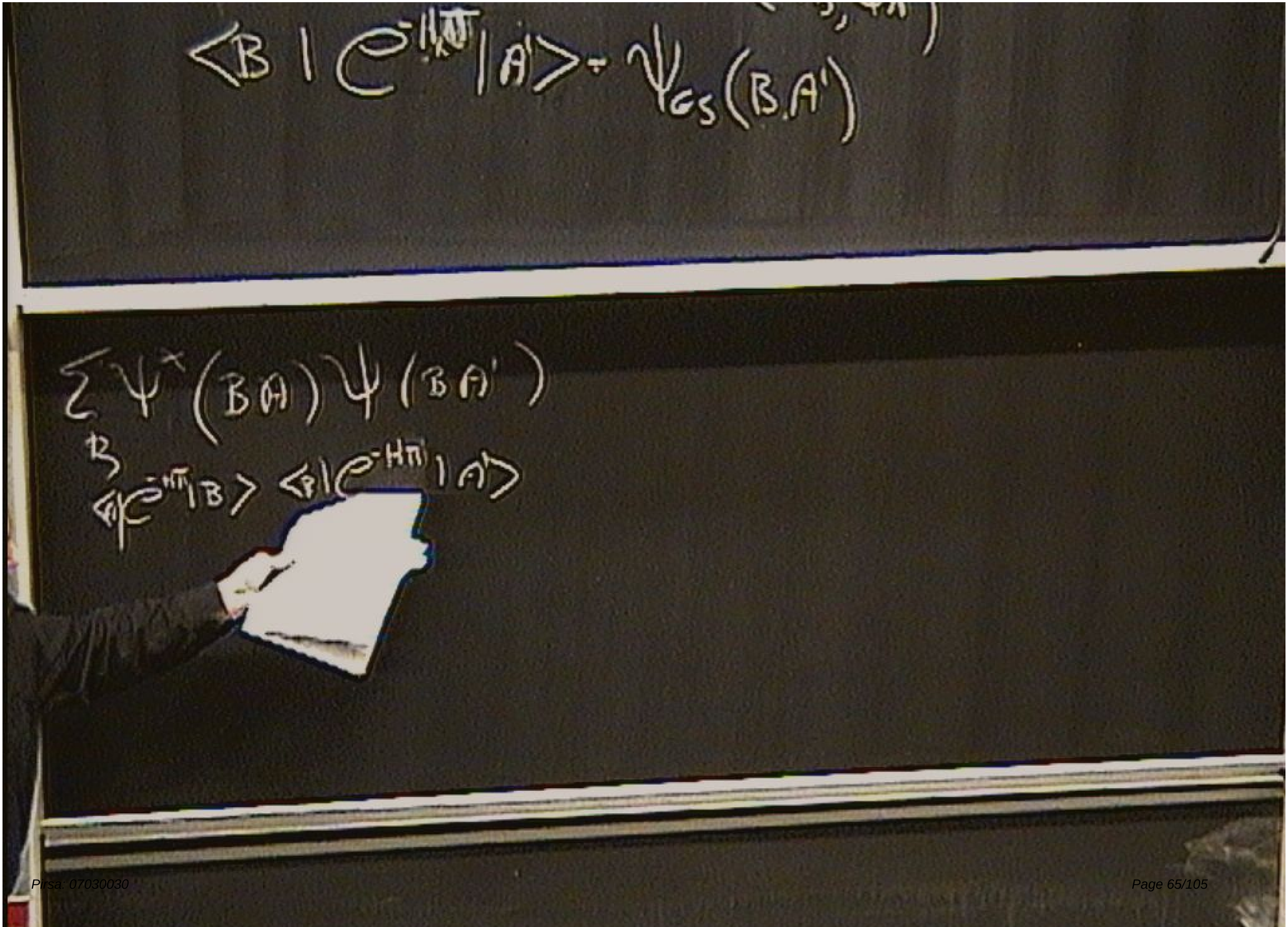
$$\Psi_{GS}(\phi_B, \phi_A)$$

$$\langle B \,|\, e^{-H T} \,|\, A' \rangle$$


φA, φB
B
A A'
ΨGS(φB, φA)
<B | e^-HRτ | A'> = ΨGS(B, A')

$$\langle B | e^{-iHT} | A' \rangle = \psi_{GS}(B, A')$$

$$\sum_{B} \psi^{*}(B A)\, \psi(B A')$$



$\langle B | e^{-H\tau/\hbar} | A' \rangle = \psi_{GS}(B, A')$
$\sum_B \psi^*(B A)\, \psi(B A')$
$\langle A | e^{-H\tau} | B \rangle \langle B | e^{-H\tau} | A' \rangle$

$$\langle B | e^{-H_R \pi} | A' \rangle = \Psi_{GS}(B, A')$$

$$\sum_B \Psi^*(B, A)\, \Psi(B, A')$$

$$\langle A | e^{-H\pi} | B \rangle \langle B | e^{-H\pi} | A' \rangle$$

$$\langle A | e^{-2\pi H_R} | A' \rangle$$

$$\langle B | e^{-H_R \pi} | A' \rangle = \psi_{GS}(B, A')$$

$$\sum_B \psi^*(B, A)\, \psi(B, A')$$

$$\langle A | e^{-H\pi} | B \rangle \langle B | e^{-H\pi} | A' \rangle$$

$$\langle A | e^{-2\pi H_R} | A' \rangle \qquad \rho_A = e^{-2\pi H_R}$$

$$\langle B | e^{-H_R \pi} | A' \rangle = \Psi_{GS}(B, A')$$

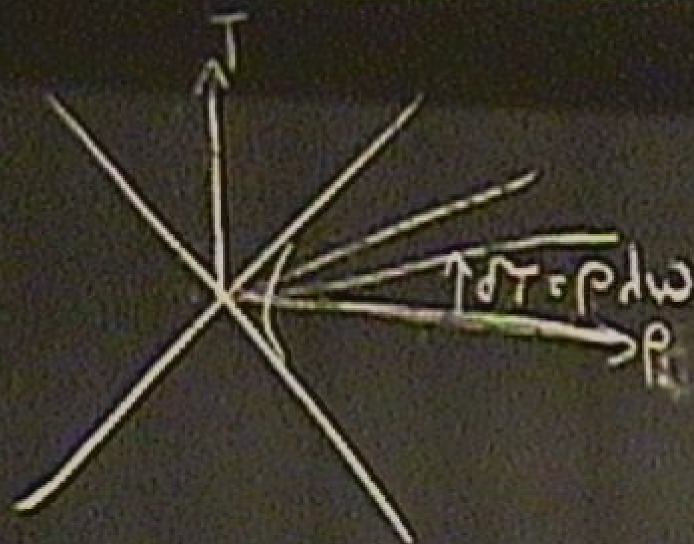
$$\sum_B \Psi^*(B, A)\, \Psi(B, A')$$

$$\langle A | e^{-H\pi} | B \rangle \langle B | e^{-H\pi} | A' \rangle$$

$$\langle A | e^{-2\pi H_R} | A' \rangle \qquad \rho_A = e^{-2\pi H_R}$$

$$\boxed{T_R = \frac{1}{2\pi}}$$

$3Mc$



$$\boxed{\int \rho T^{00}\, d\rho\, d^3x = H_R}$$

$$\frac{1}{2}\int_{\rho} \rho \left(\cancel{\frac{\pi^2}{\rho^2} + \frac{(\partial_\rho \chi)^2}{\rho^2}} + \frac{(\partial_x \chi)^2}{\rho^2} \right)$$

$$\sum_B \Psi^*(B\,A)\,\Psi(B\,A')$$

$$\langle A|e^{-\pi H}|B\rangle\,\langle B|e^{-H\pi}|A'\rangle$$

$$\langle A|e^{-2\pi H_R}|A'\rangle \qquad \rho_A = e^{-2\pi H_R}$$

$$\boxed{T_R = \frac{1}{2\pi}}$$

$$d\tau = \sqrt{g_{00}}\, d\omega$$

$$\sqrt{g_{\omega\omega}} = \rho$$

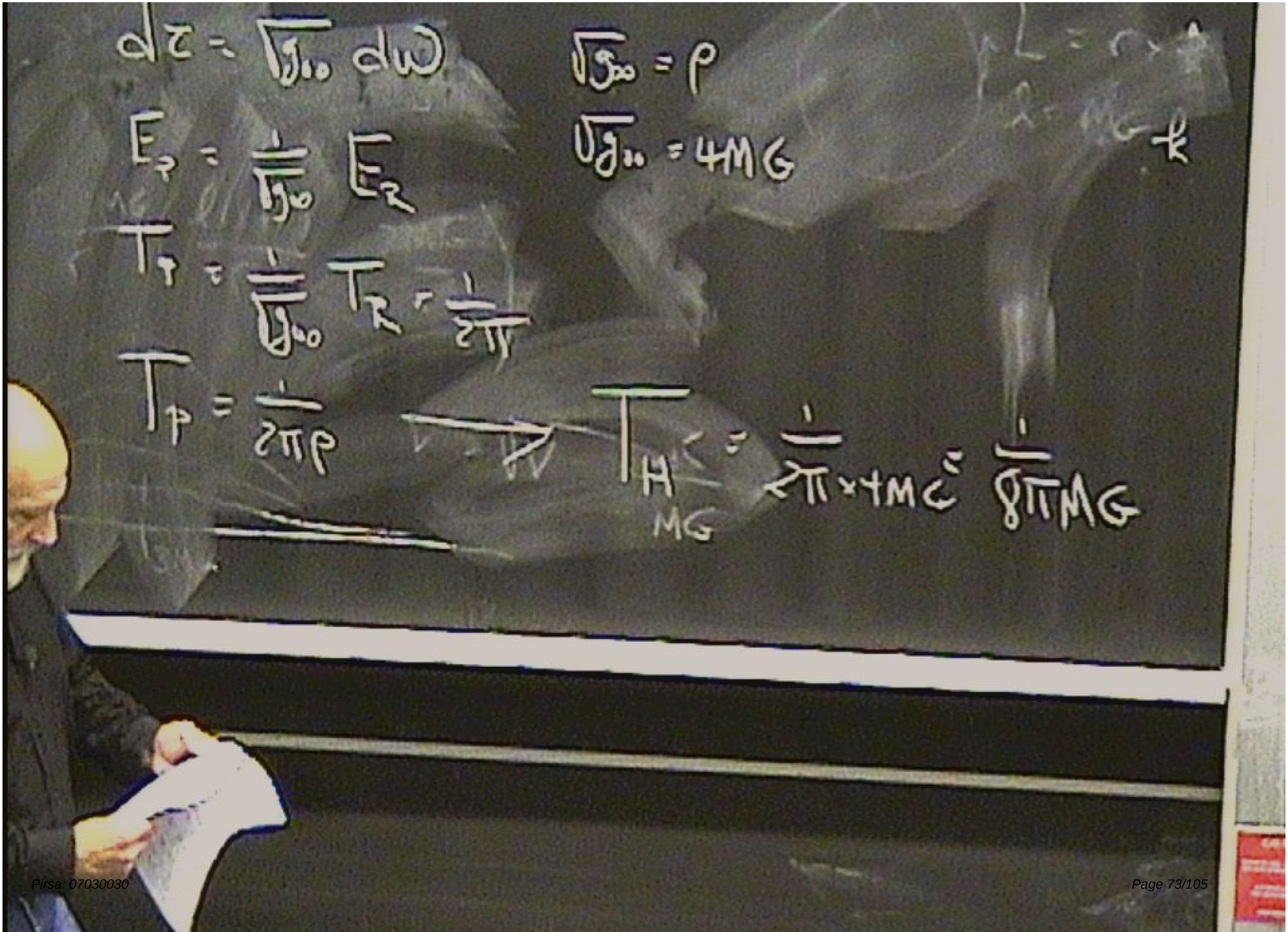
$$\sqrt{g_{11}} = 4MG$$

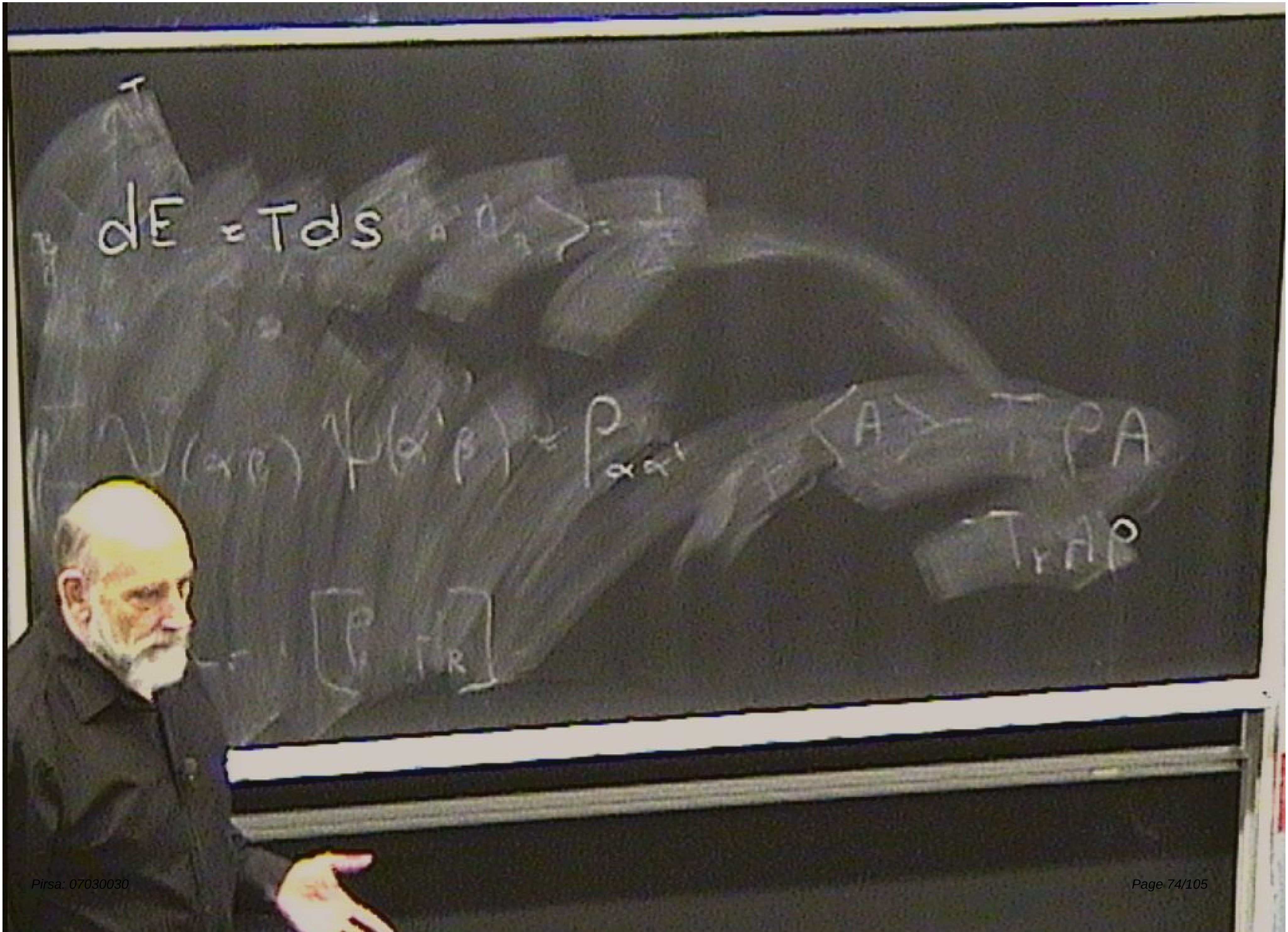
$$d\tau = \sqrt{g_{00}}\, d\omega \qquad \sqrt{g_{00}} = \rho$$

$$E_P = \frac{1}{\sqrt{g_{00}}} E_R \qquad \sqrt{g_{00}} = 4MG$$

$$T_P = \frac{1}{\sqrt{g_{00}}} T_R = \frac{1}{2\pi}$$

$$T_P = \frac{1}{2\pi\rho} \quad \Longrightarrow \quad T_{\underset{MG}{H}} = \frac{1}{2\pi \times 4MG} = \frac{1}{8\pi MG}$$


$d\tau = \sqrt{g_{00}}\, d\omega$
$\sqrt{g_{00}} = \rho$
$E_? = \frac{1}{\sqrt{g_{00}}} E_R$
$\sqrt{g_{00}} = 4MG$
$T_? = \frac{1}{\sqrt{g_{00}}} T_R = \frac{1}{2\pi}$
$T_p = \frac{1}{2\pi\rho} \Longrightarrow T_{H_{MG}} = \frac{1}{2\pi \times 4MG} = \frac{1}{8\pi MG}$


dE = Tds
⟨A⟩ = Tr ρA
Tr Aρ

T

$dE = T\,dS$

$M = \frac{1}{8\pi MG}\,dS$

$8 \ldots (\alpha\beta)\,\psi(\alpha'\beta) = \rho_{\alpha\alpha'}$

$\langle A \rangle = \mathrm{Tr}\,\rho A$

$= \mathrm{Tr}\,A\rho$

$dE = T\,dS$

$M = \frac{1}{8\pi M G} dS$

$\frac{8\pi G}{2} dM^2 = dD$

$S = 4\pi M^2 G$

$\langle A \rangle = \mathrm{Tr}\,\rho A$

$= \mathrm{Tr}\,A\rho$

$$dE = T\,dS$$

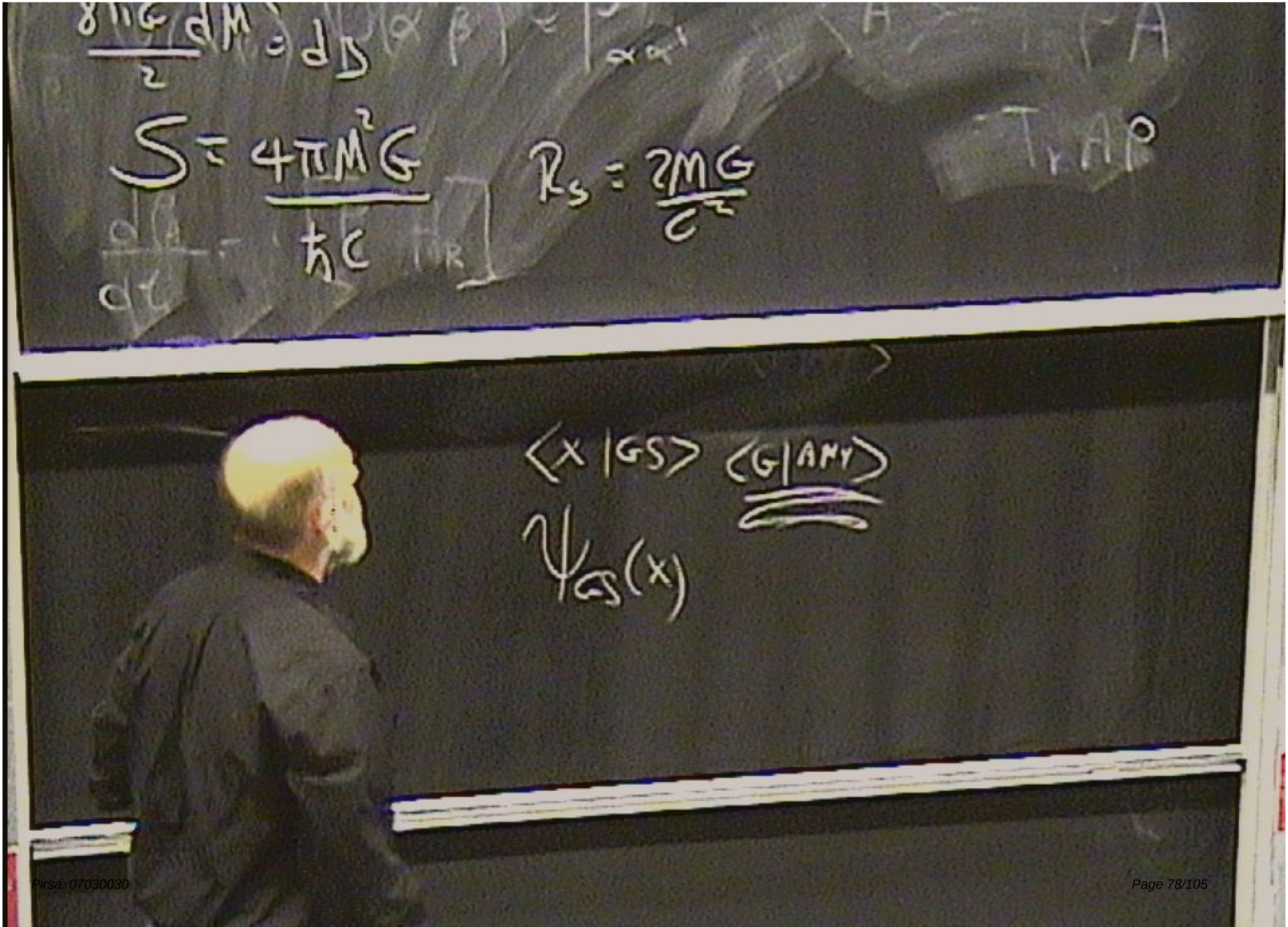
$$M = \frac{1}{8\pi M G}\,dS$$

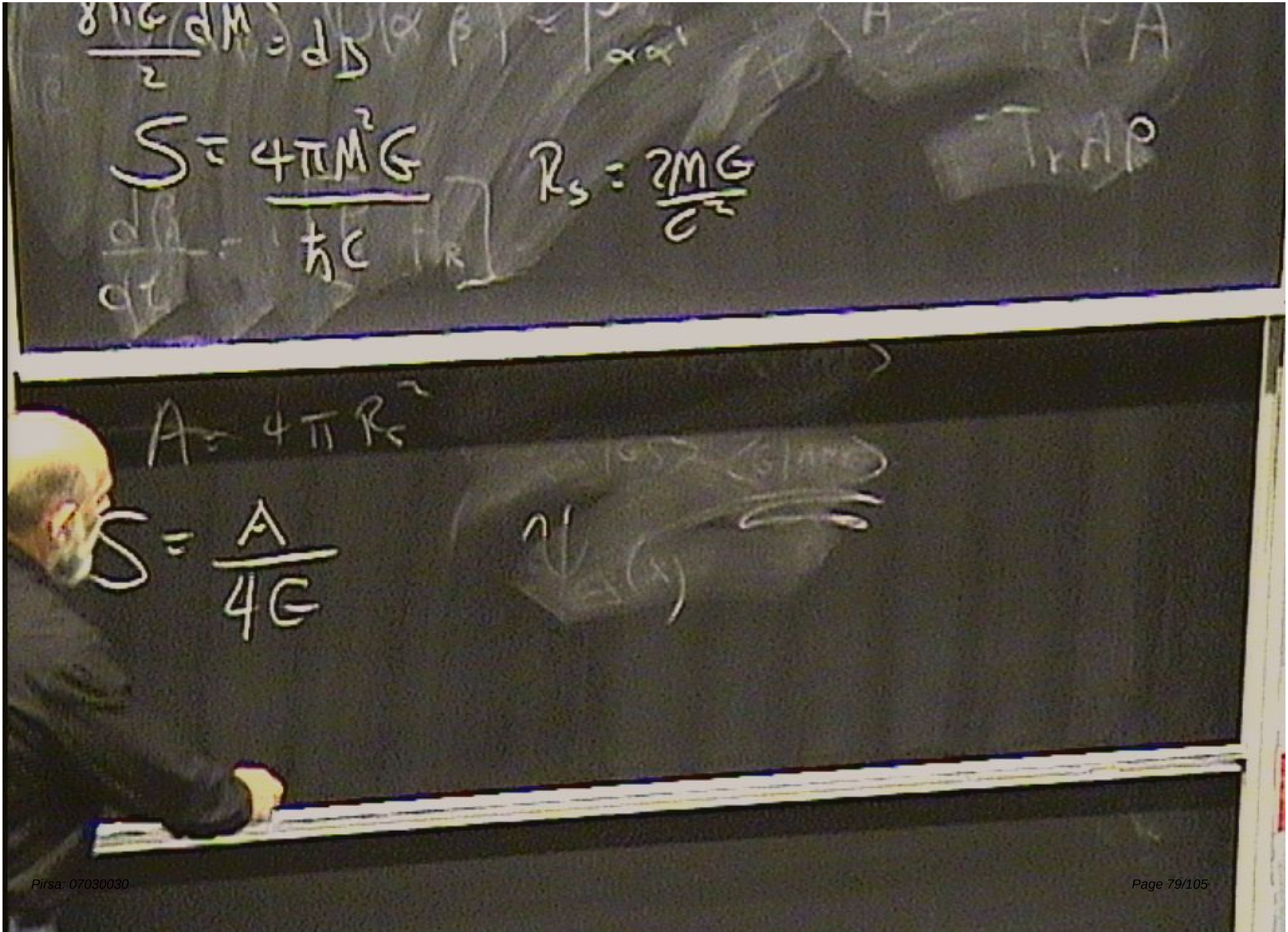
$$\frac{8\pi G}{2}\,dM^2 = dD$$

$$\langle A \rangle = \mathrm{Tr}\,\rho A$$

$$= \mathrm{Tr}\,A\rho$$

$$S = \frac{4\pi M^2 G}{\hbar c}$$


S = 4πM²G / ħc
R_s = 2MG / c²
Tr A ρ
⟨x|GS⟩ ⟨G|ANY⟩
ψ_GS(x)

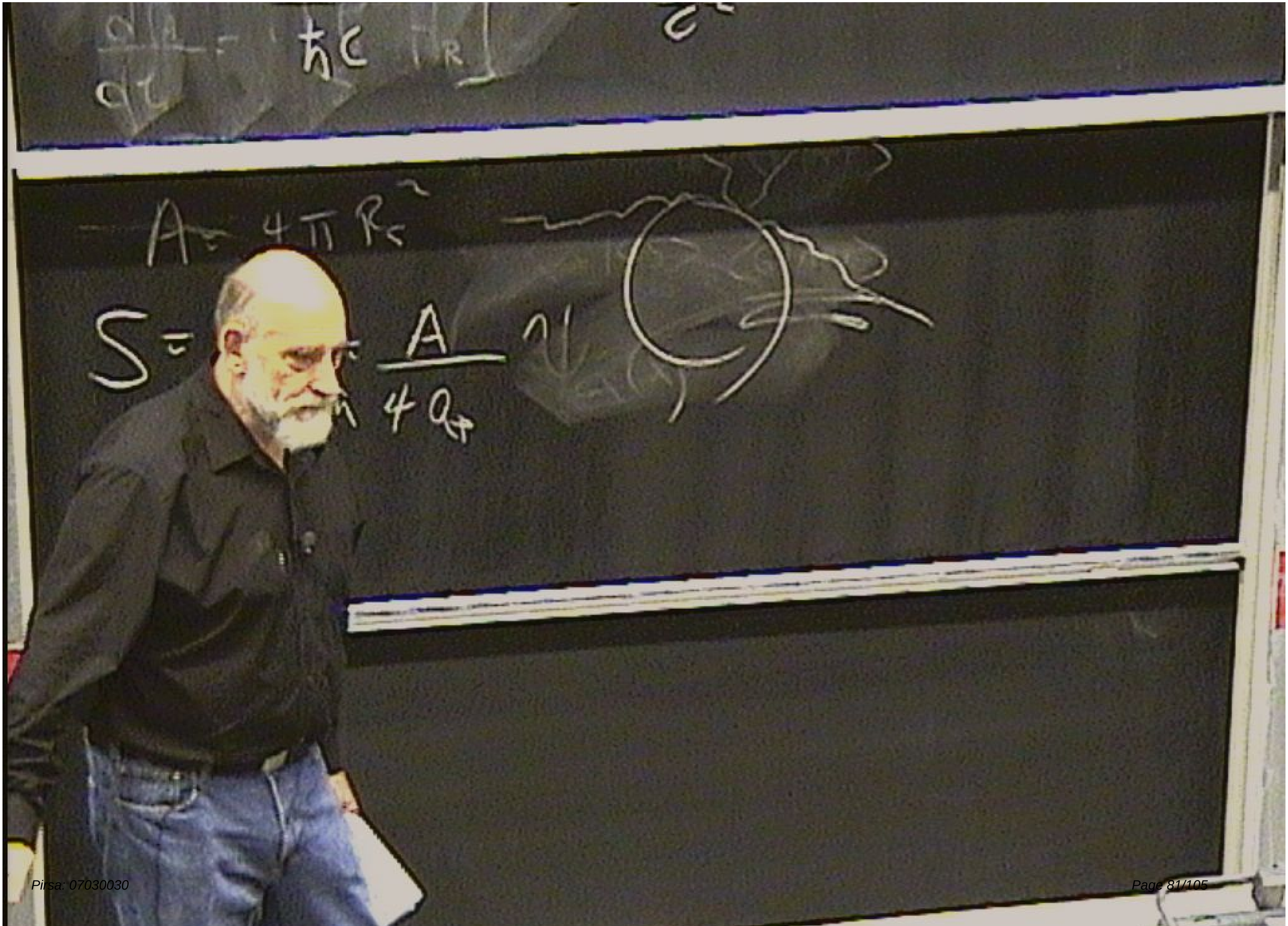

S = 4πM²G / ħc
R_s = 2MG / c²
A = 4π R_s²
S = A / 4G

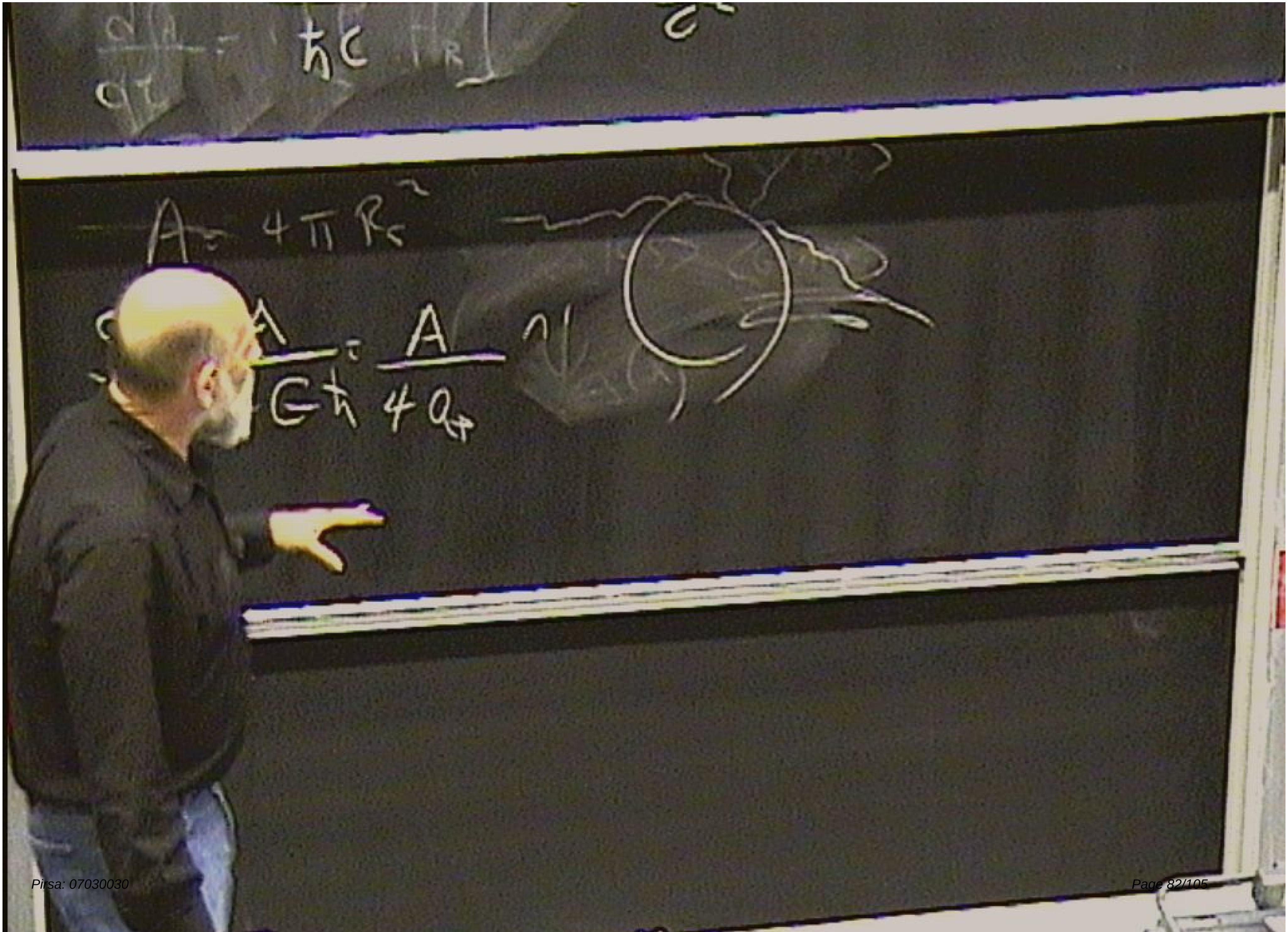
$$\frac{8\pi G\, dM}{2} = dB$$

$$S = \frac{4\pi M^2 G}{\hbar c} \qquad R_s = \frac{2MG}{c^2}$$

$$A = 4\pi R_s^2$$

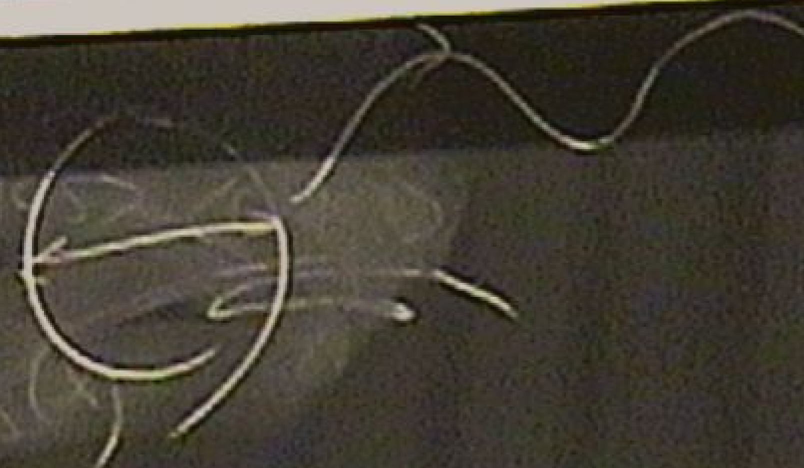
$$S = \frac{A}{4G\hbar} = \frac{A}{4a_P}$$

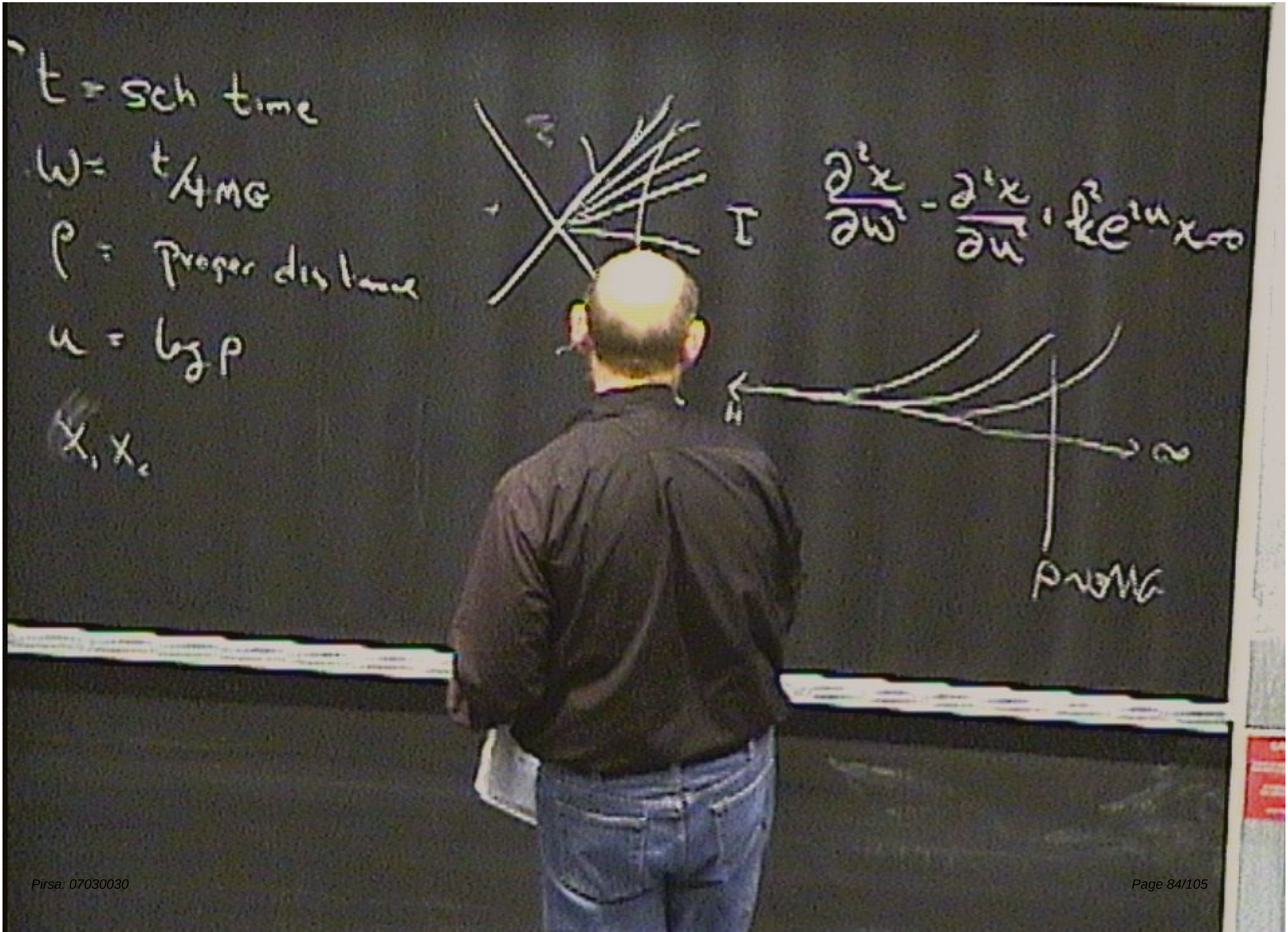


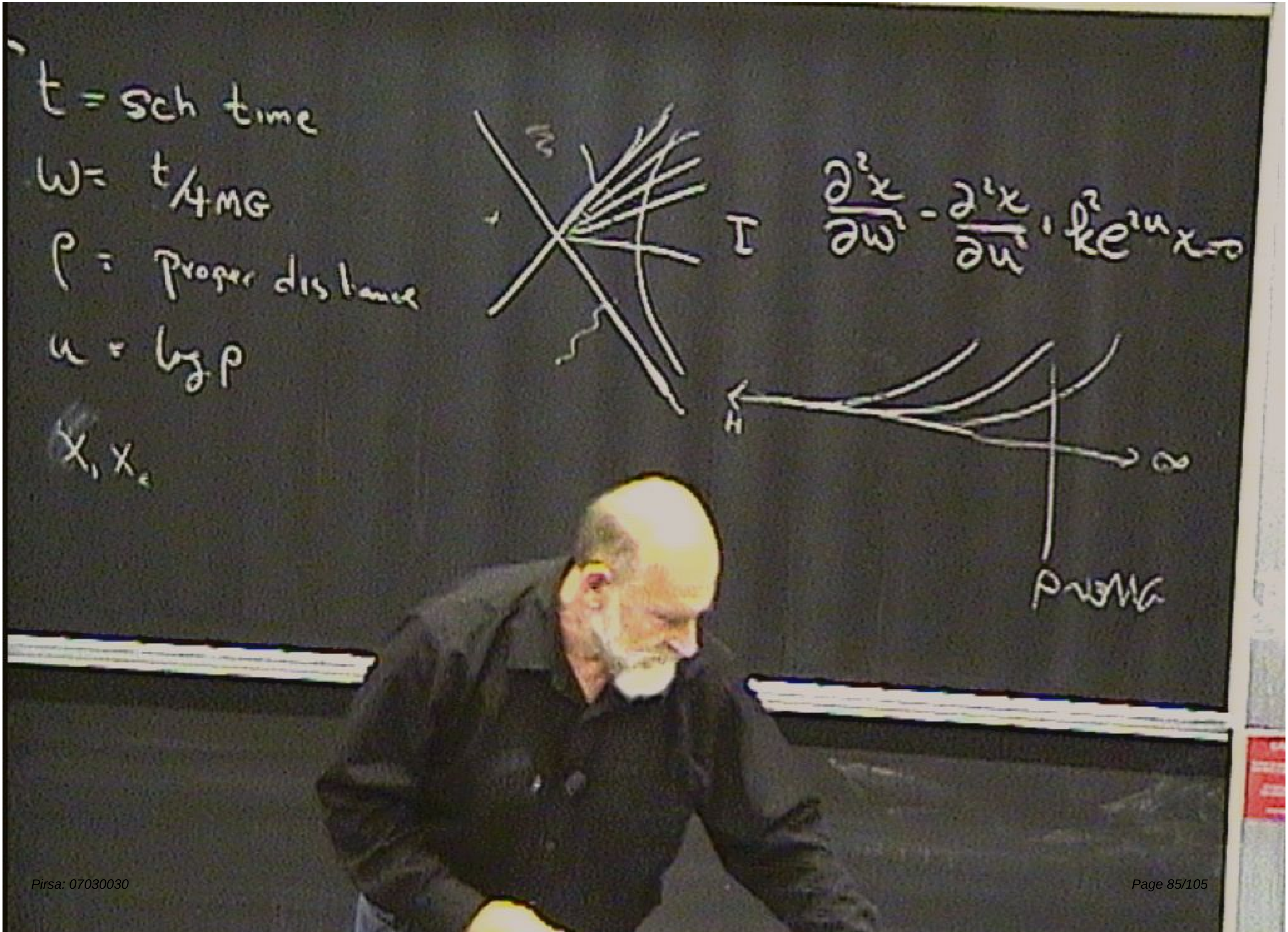

A = 4π R_s^2
A/4Għ = A/4ℓ_P

$A = 4\pi R_s^2$

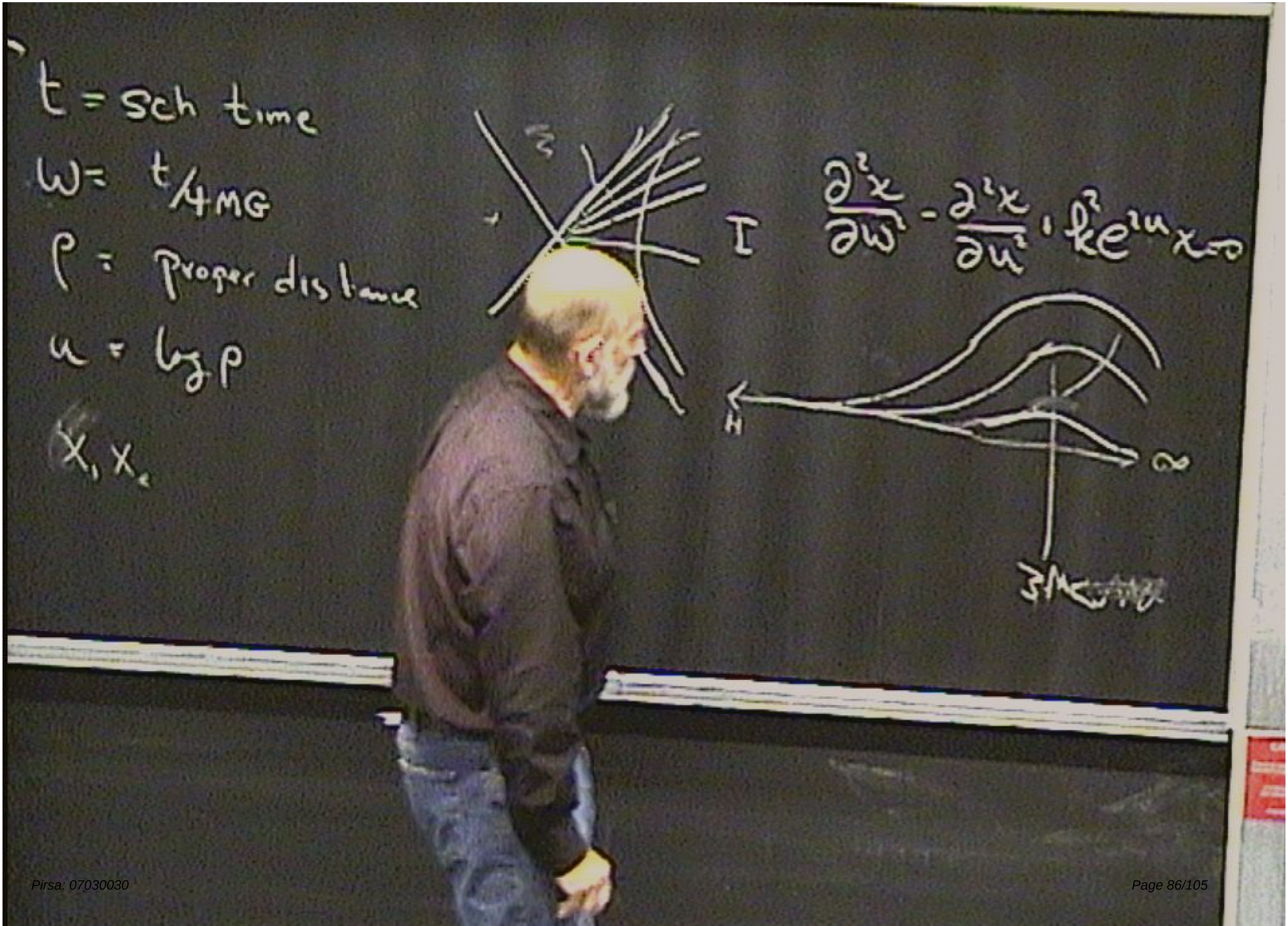
$$S = \frac{A}{4G\hbar} = \frac{A}{4\,a_P}$$

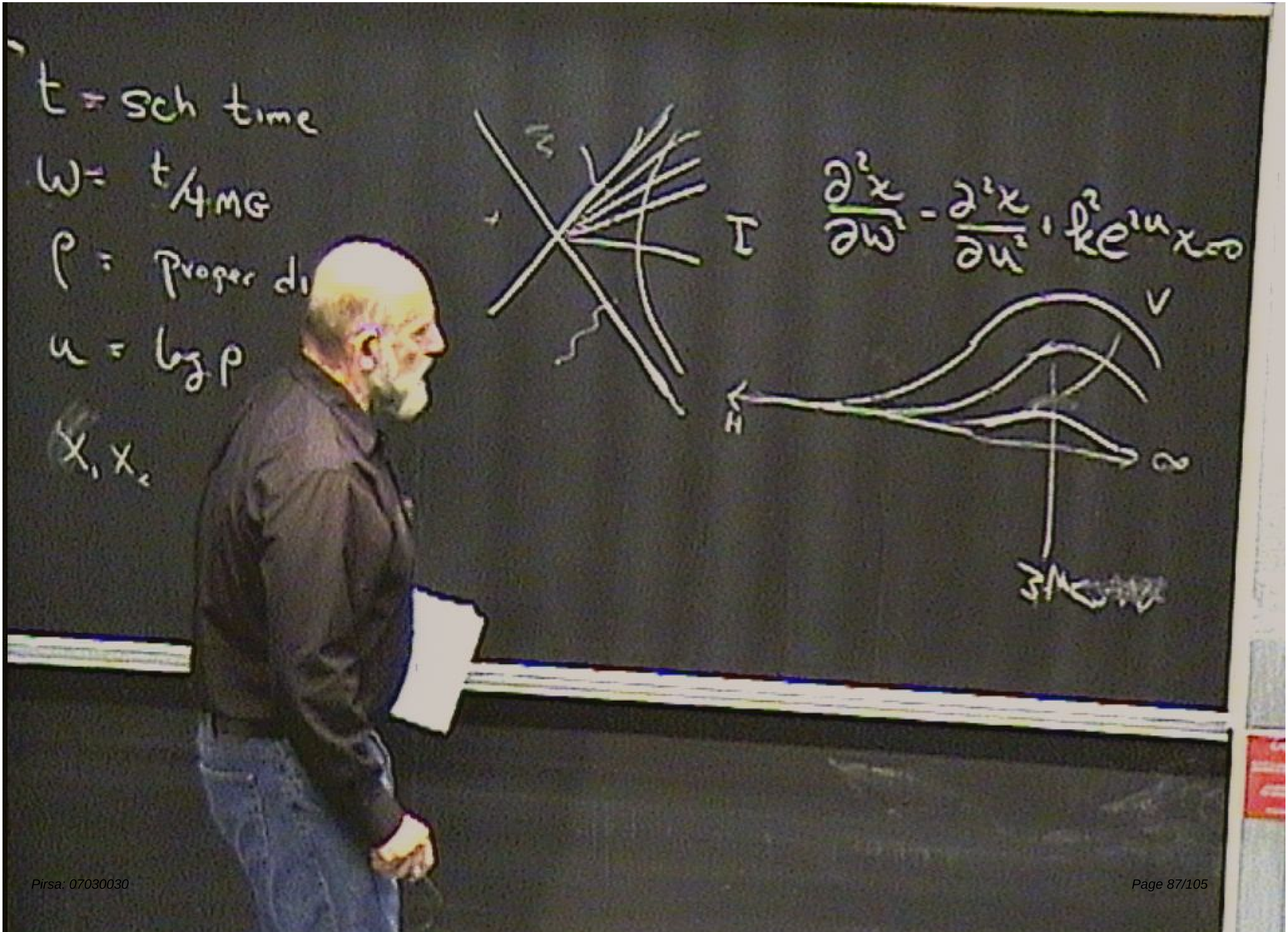


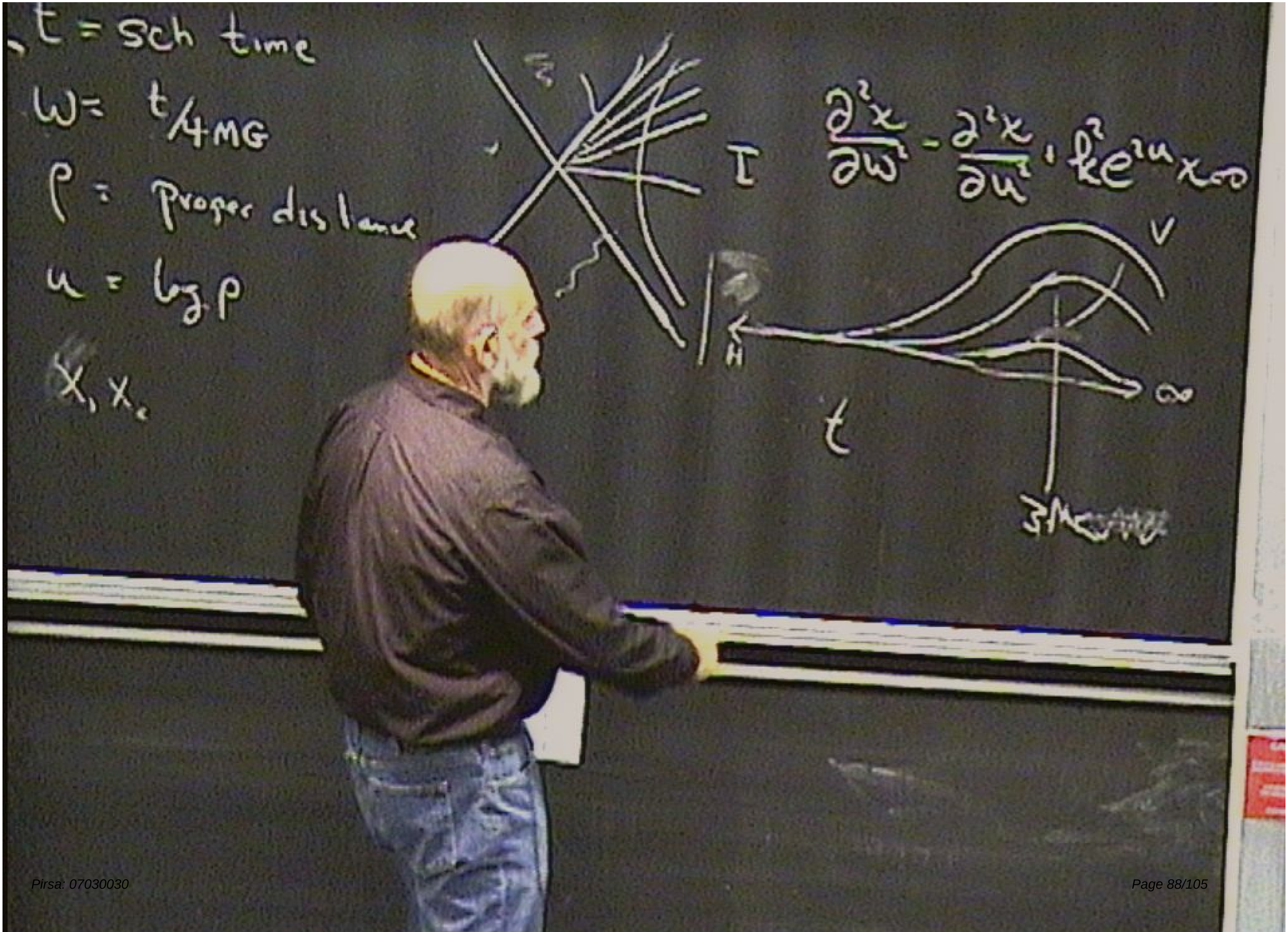

t = sch time
w = t/4MG
ρ = proper distance
u = log ρ
I
ρ ~ 0.4MG
∞

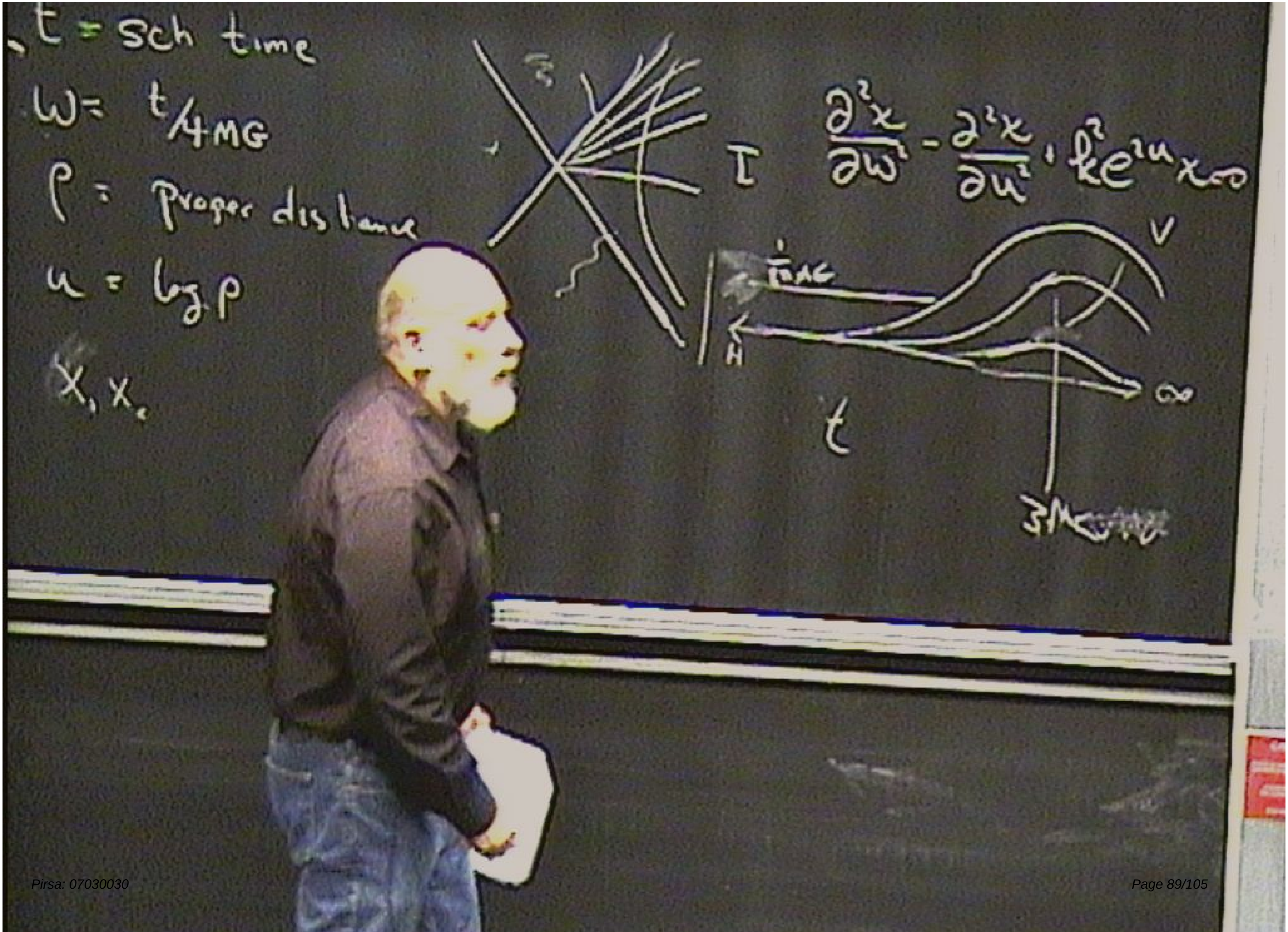


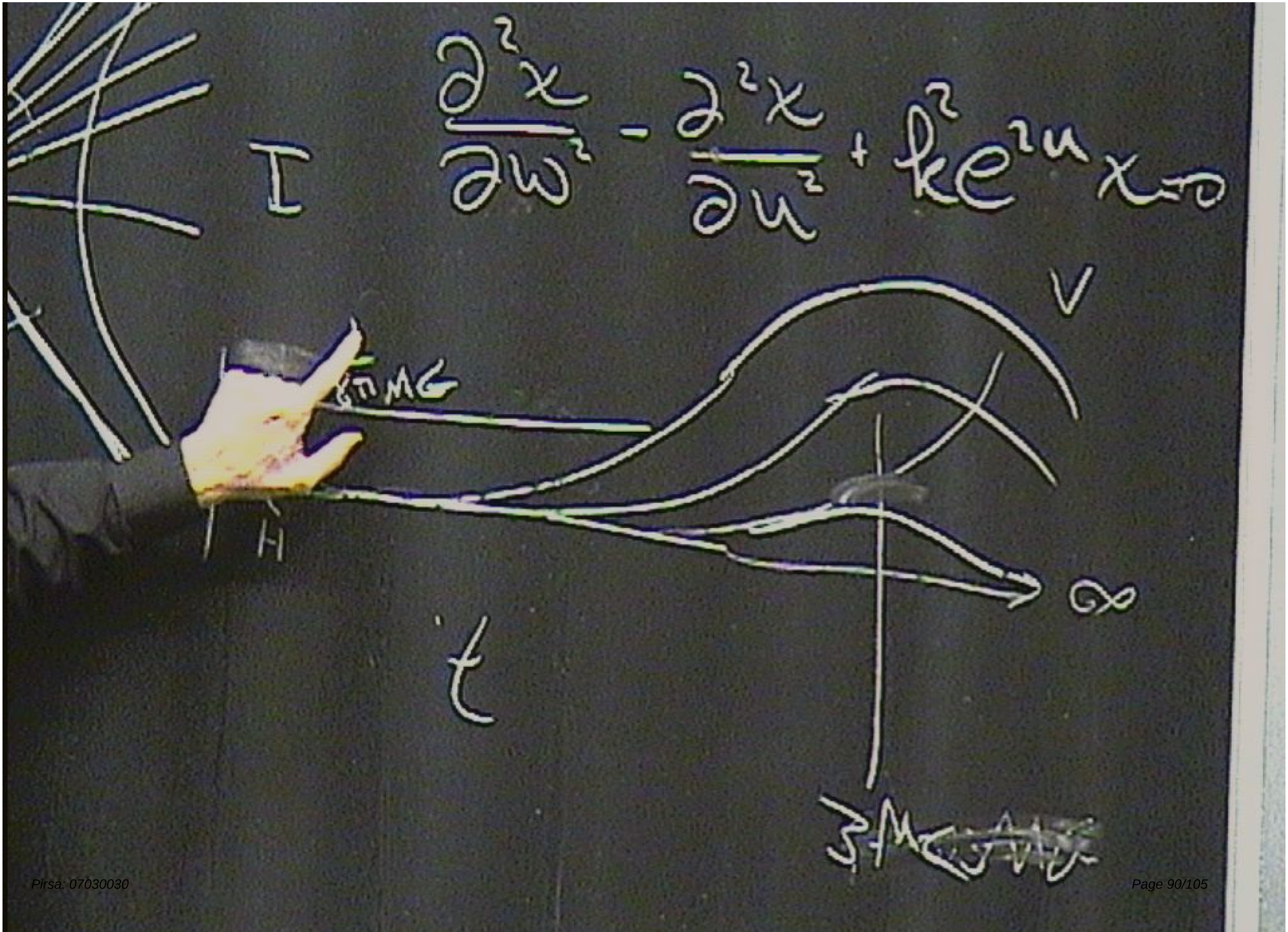
t = sch time
w = t/4MG
ρ = proper distance
u = log ρ
X_1, X_2
I
$\frac{\partial^2 x}{\partial w^2} - \frac{\partial^2 x}{\partial u^2} + k^2 e^{2u} x = 0$
H
∞

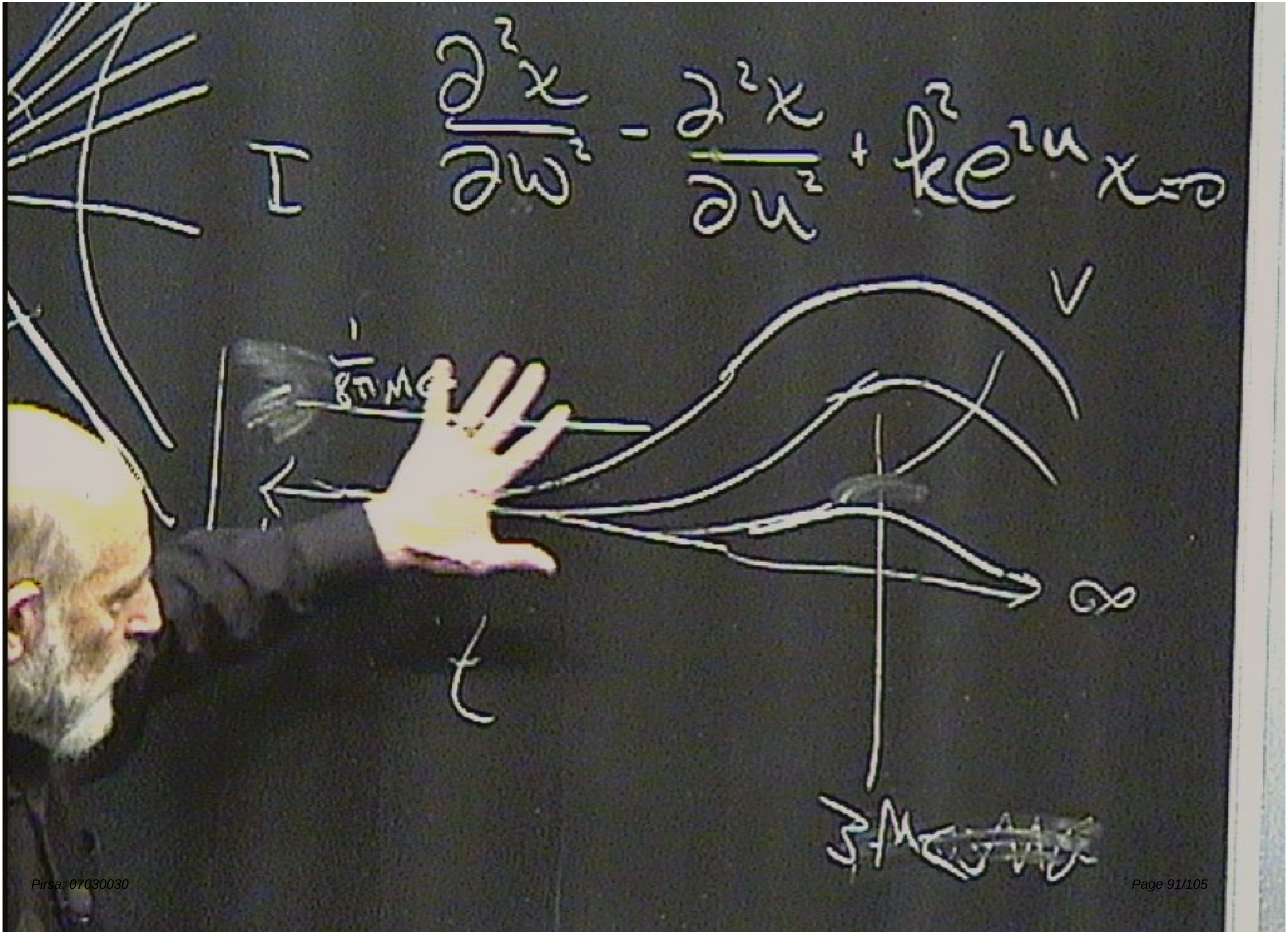

t = sch time
w = t/4MG
ρ = proper distance
u = log ρ
X1, X2
I
$\frac{\partial^2 x}{\partial w^2} - \frac{\partial^2 x}{\partial u^2} + k^2 e^{2u} x = 0$
H
∞

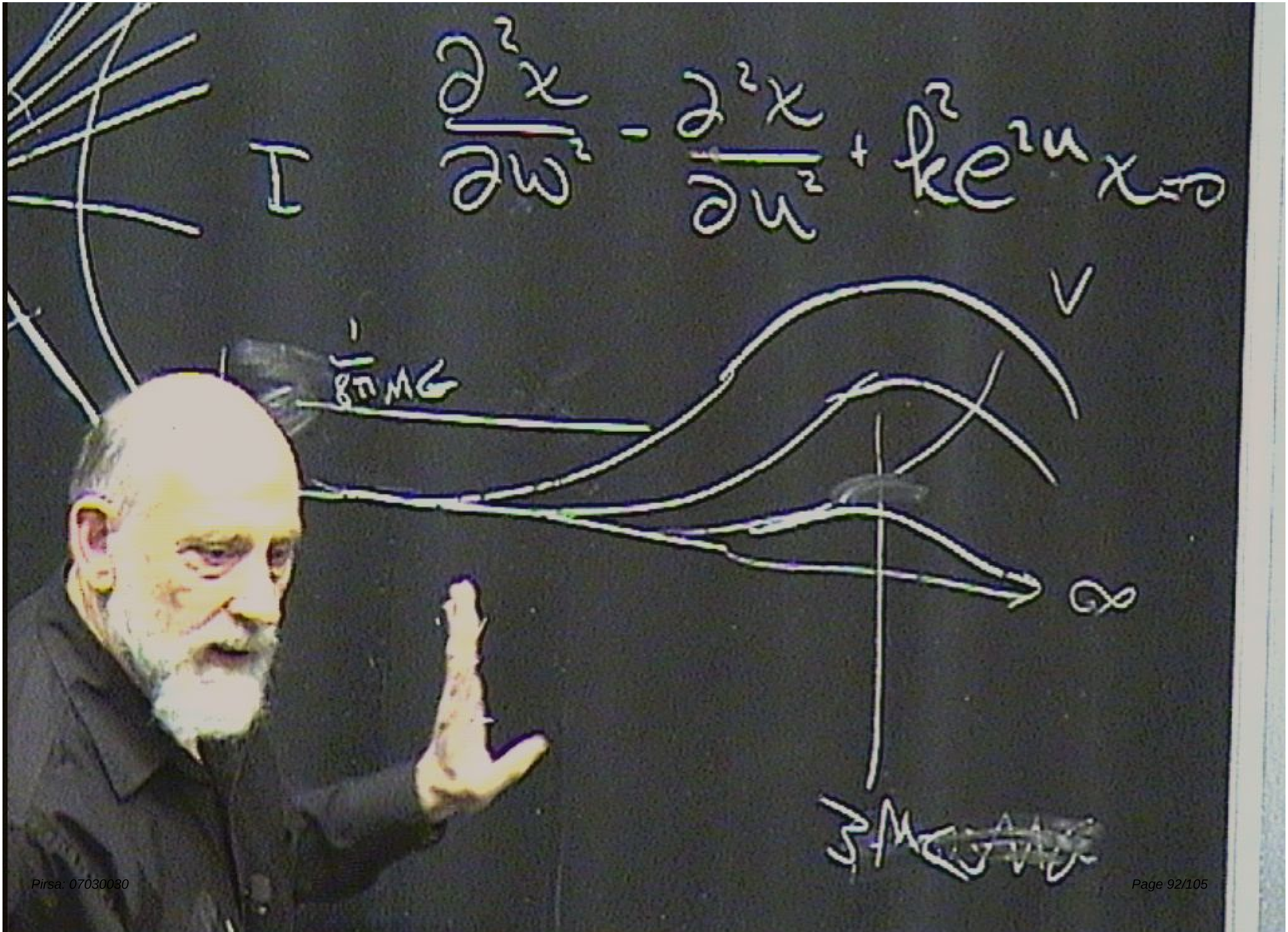

t = sch time
ω = t/4MG
ρ = proper di
u = log ρ
X₁, X₂
I
∂²x/∂ω² − ∂²x/∂u² + k²e^{2u}x = 0
V
H
∞


t = Sch time
ω = t/4MG
ρ = proper distance
u = log ρ
X, X
I
V
∞
t
H

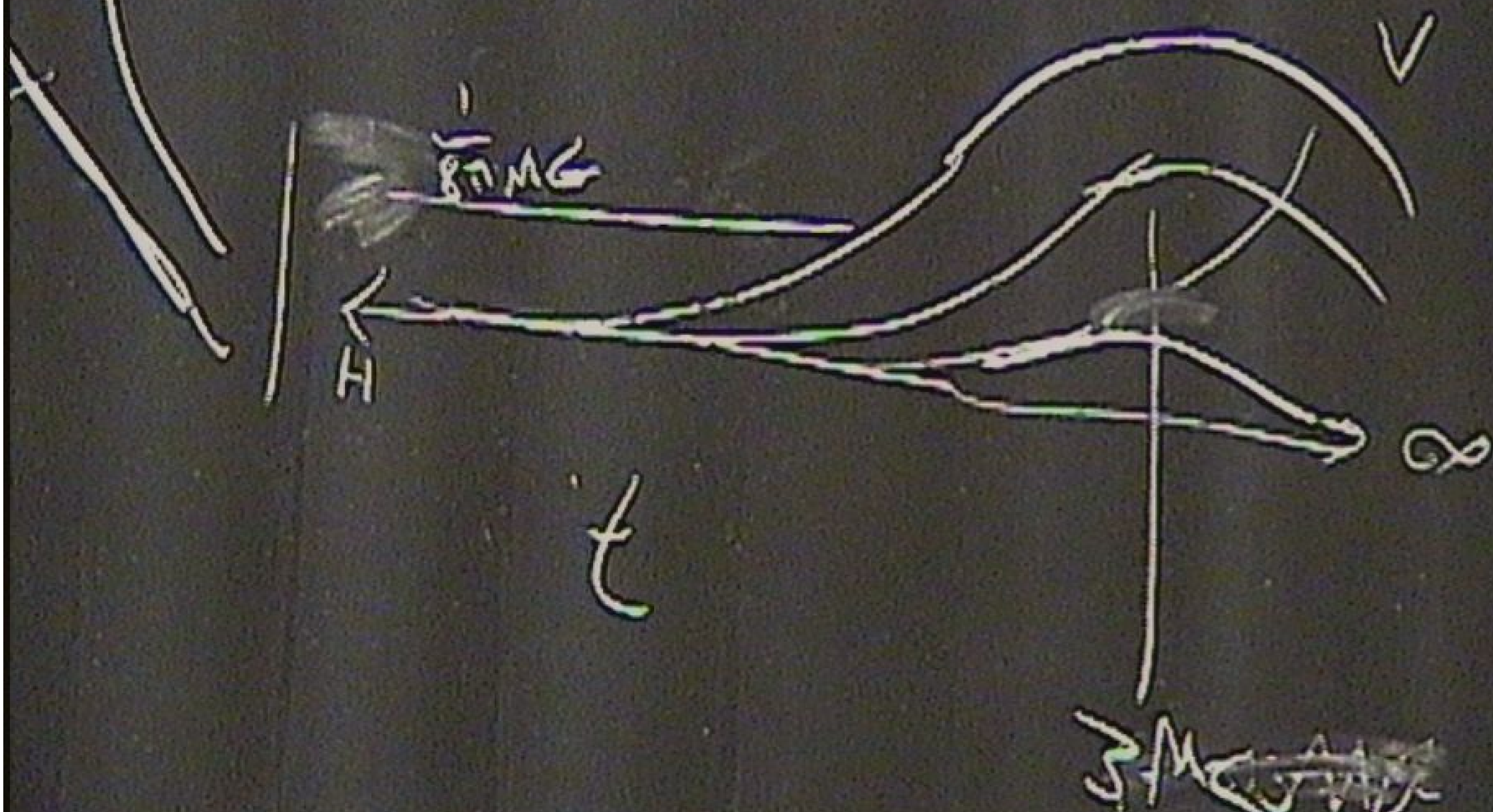

t = Sch time
ω = t/4MG
ρ = proper distance
u = log ρ
X, X
I
V
H
∞
t

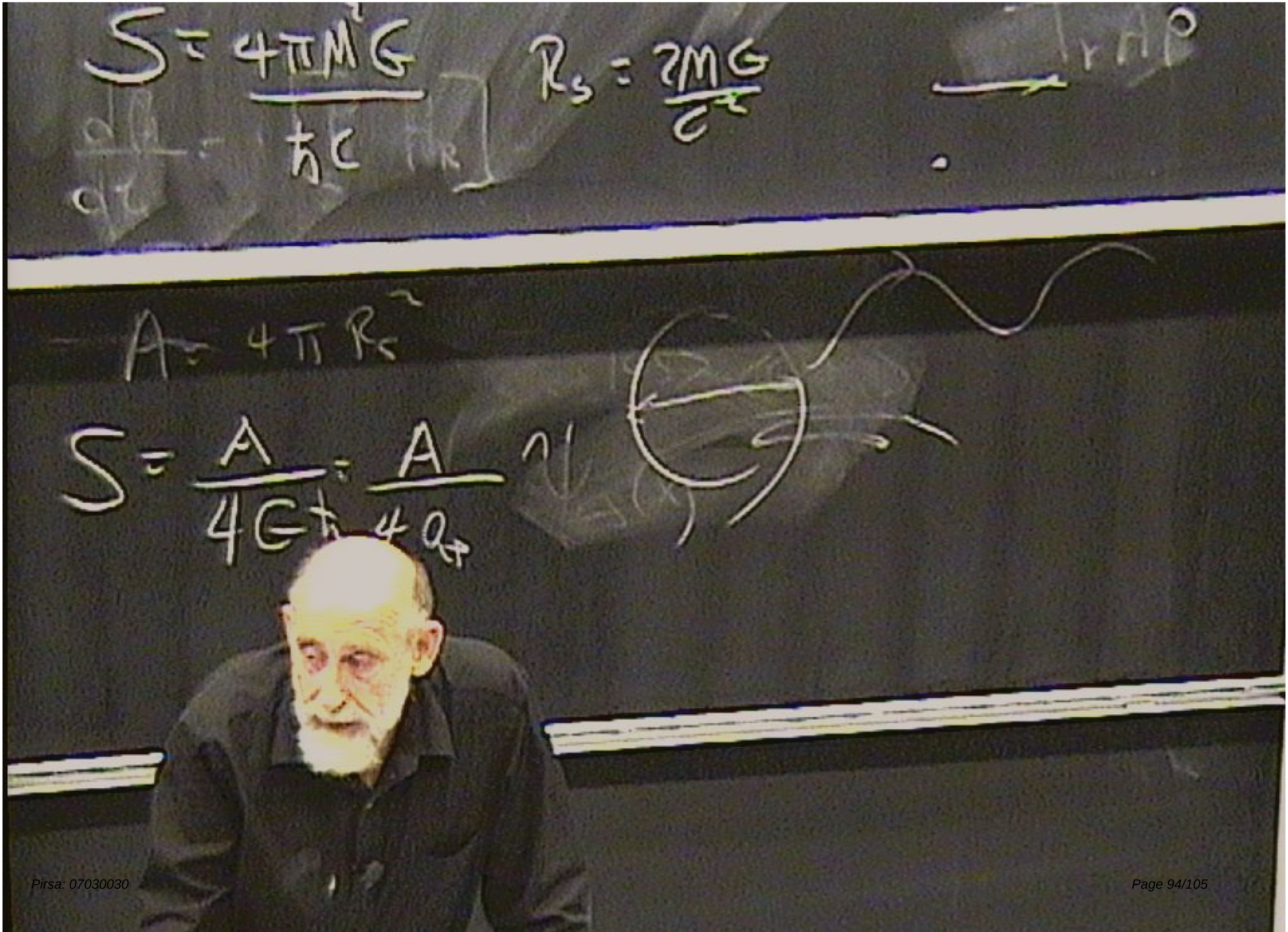

I
$\frac{\partial^2 x}{\partial w^2} - \frac{\partial^2 x}{\partial u^2} + k^2 e^{2u} x = 0$
V
H
∞
t

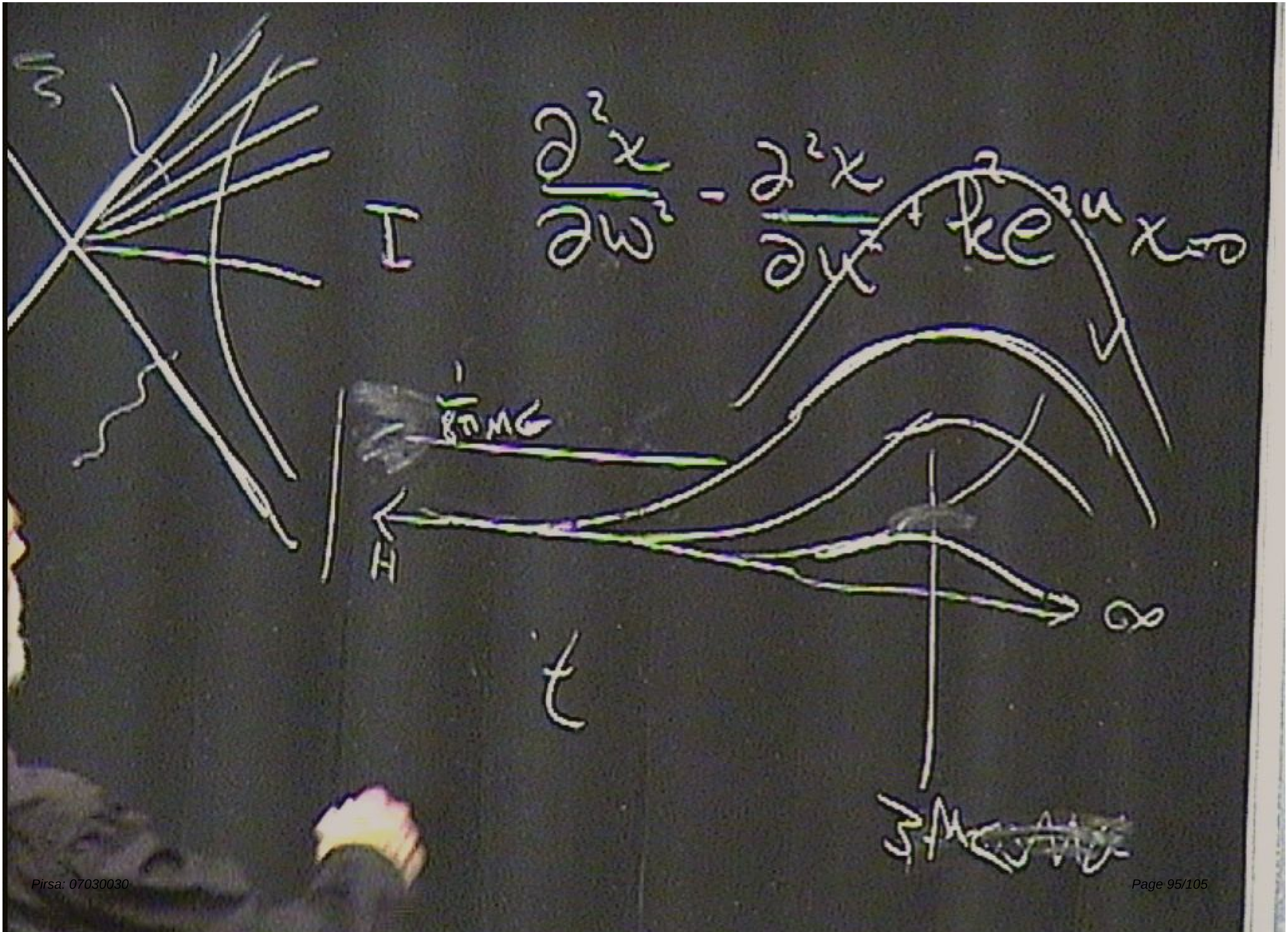

I
$\frac{\partial^2 x}{\partial w^2} - \frac{\partial^2 x}{\partial u^2} + k^2 e^{2u} x = 0$
V
$\frac{1}{8\pi M}$
∞
t

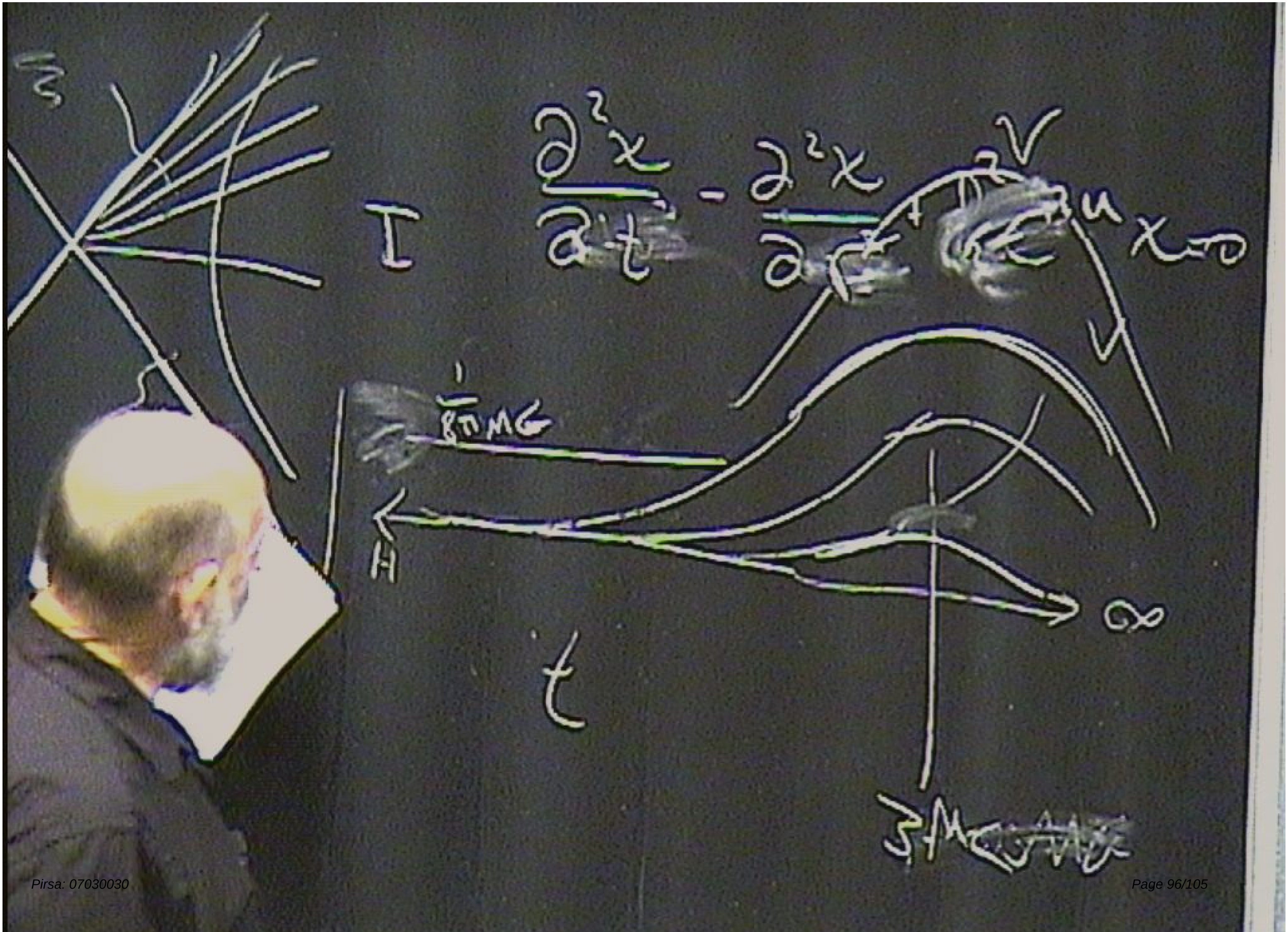

I
$\frac{\partial^2 x}{\partial w^2} - \frac{\partial^2 x}{\partial u^2} + k^2 e^{2u} x = 0$
$\frac{1}{8\pi MG}$
V
∞

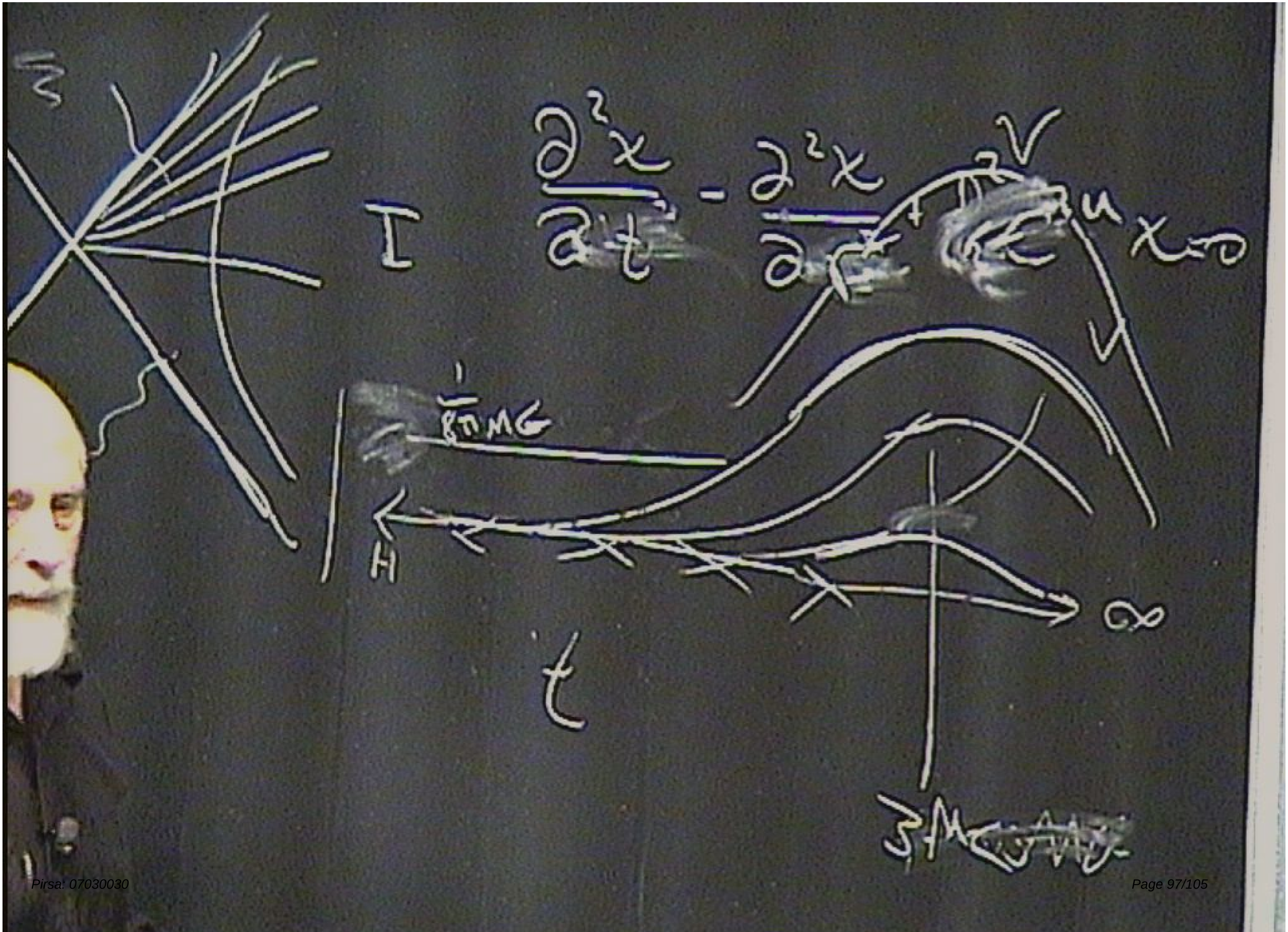
I $\quad \dfrac{\partial^2 x}{\partial w^2} - \dfrac{\partial^2 x}{\partial u^2} + k^2 e^{2u} x = 0$

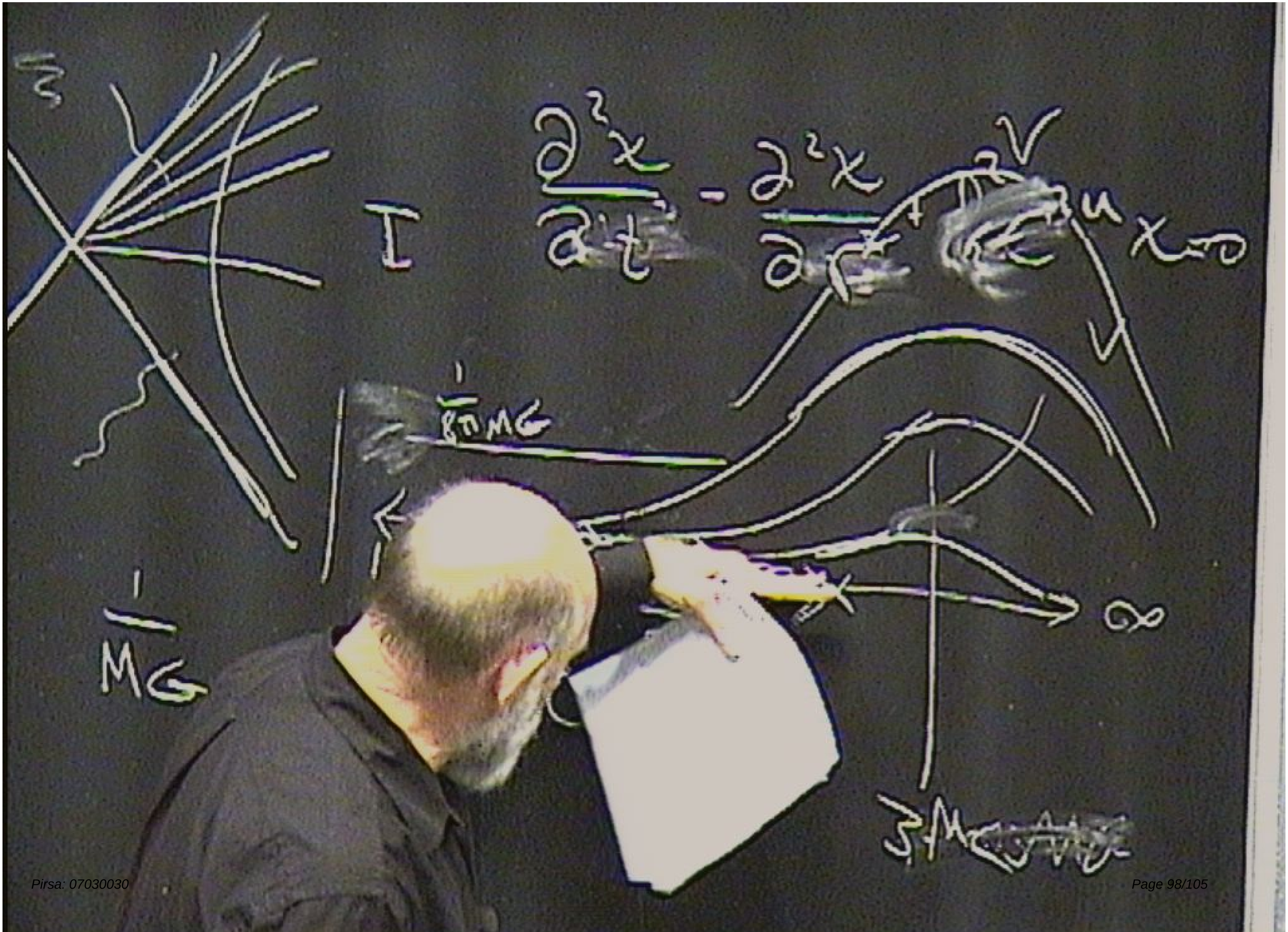



S = 4πM²G / ħc
Rs = 2MG / c²
A = 4π Rs²
S = A / 4Għ = A / 4 ℓp

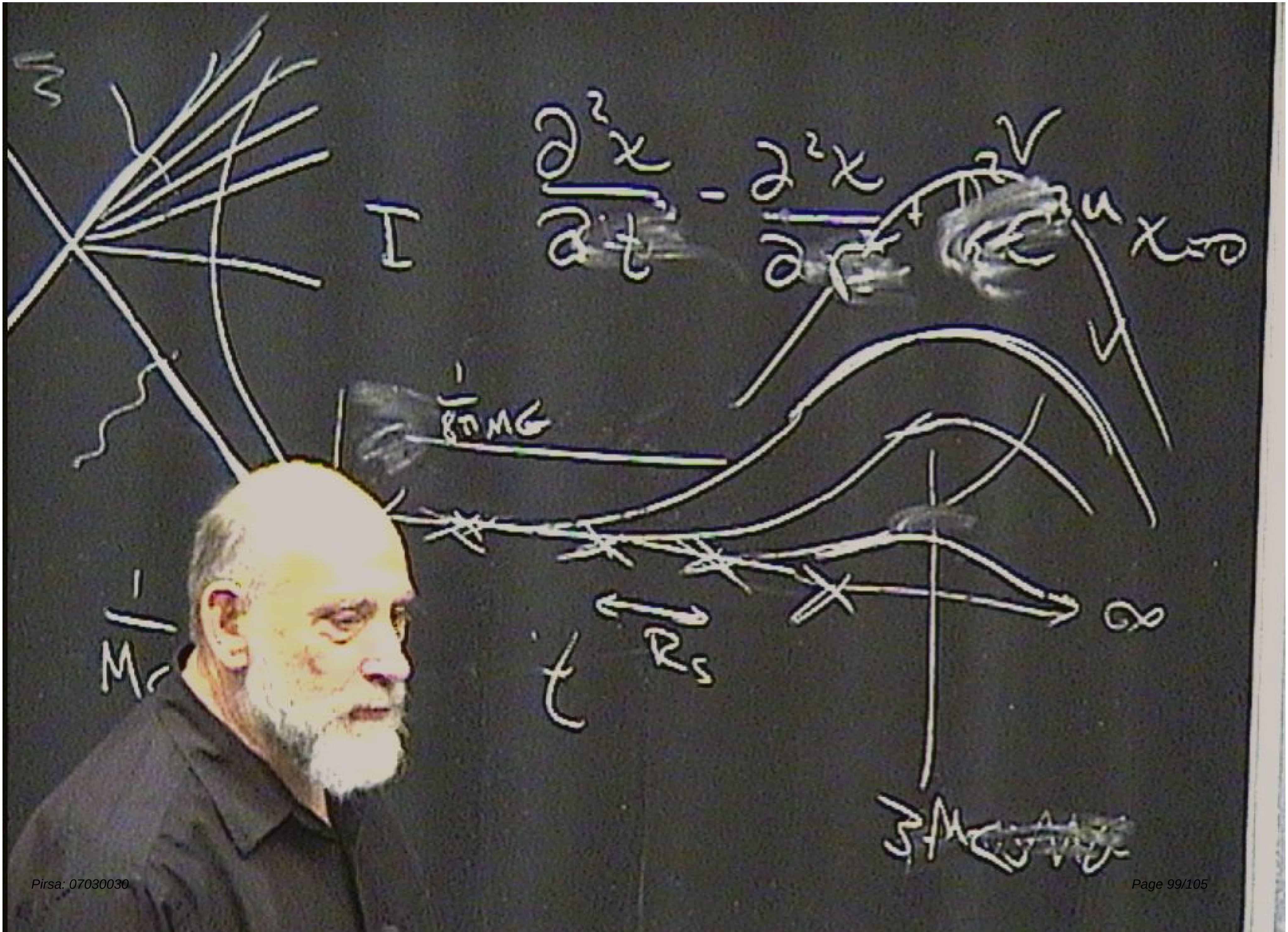

I
$\frac{\partial^2 x}{\partial w^2} - \frac{\partial^2 x}{\partial y^2} + k e^{2u} x = 0$
$\frac{1}{8\pi MG}$
H
∞
t

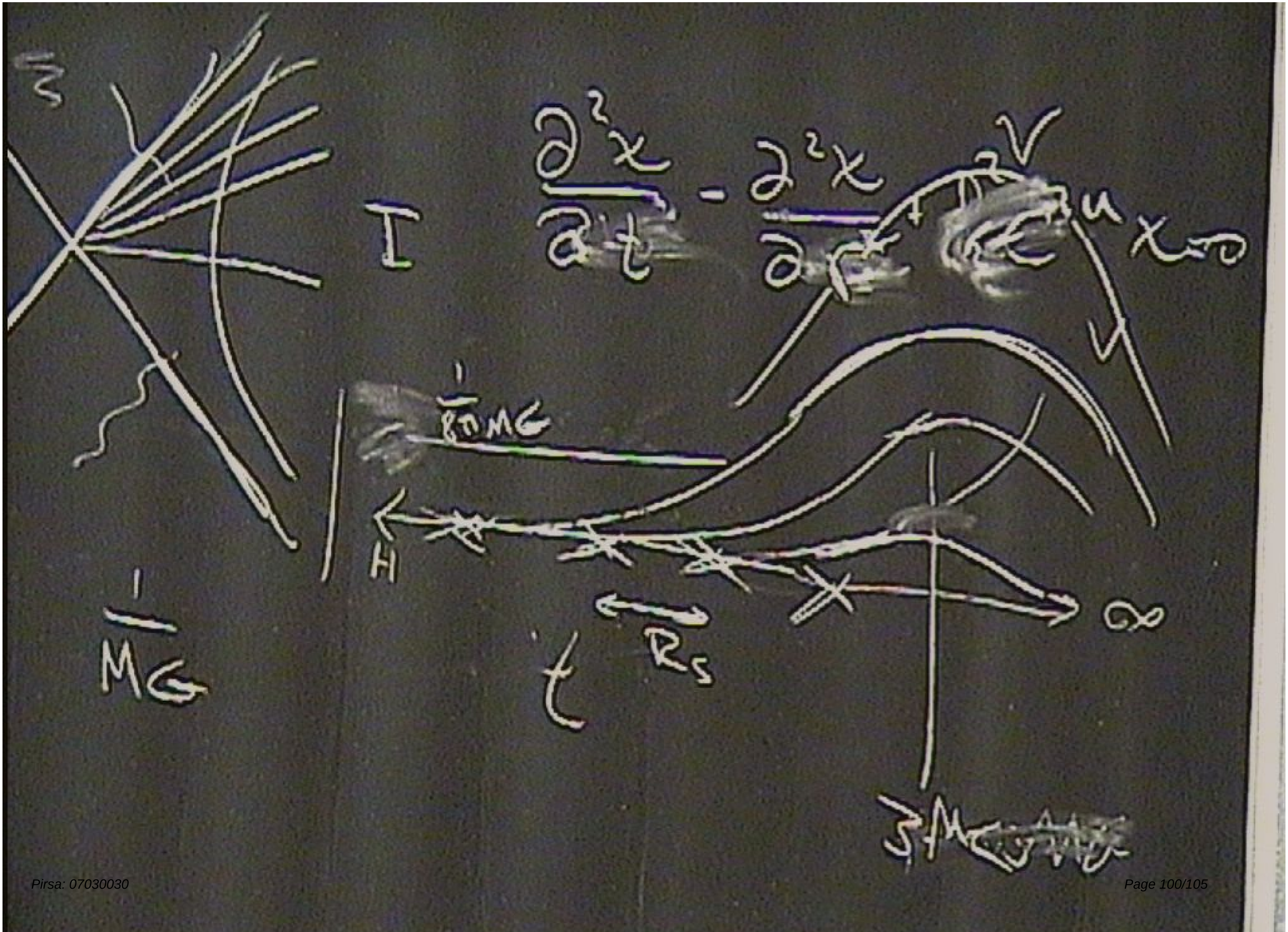

I
$\frac{\partial^2 x}{\partial t^2} - \frac{\partial^2 x}{\partial r^2}$
$\frac{1}{8\pi MG}$
H
∞
t


I
8πMG
H
∞
t



I
1/8πMG
1/MG
∞


I
8πMG
t
Rs
∞


I
$\frac{\partial^2 x}{\partial t^2} - \frac{\partial^2 x}{\partial r^2}$
V
$\frac{1}{8\pi MG}$
H
$\frac{1}{MG}$
t
R_s
∞
3MG

$$dE = T\,dS$$

$$M = \frac{1}{8\pi M G}\,dS$$

$$L = \frac{1}{(MG)^2}$$

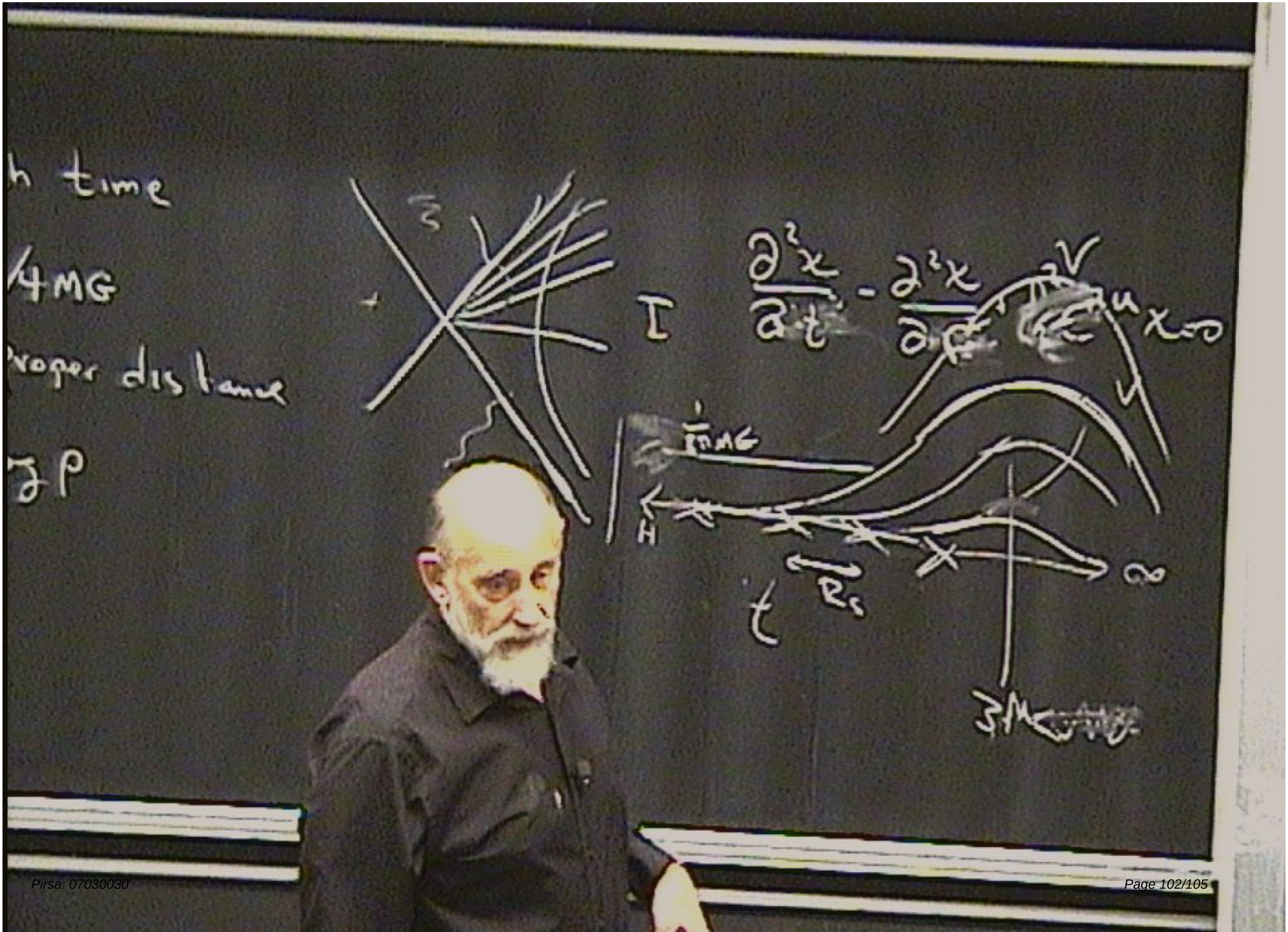
$$\frac{8\pi G}{2}\,dM^3 = d_D$$

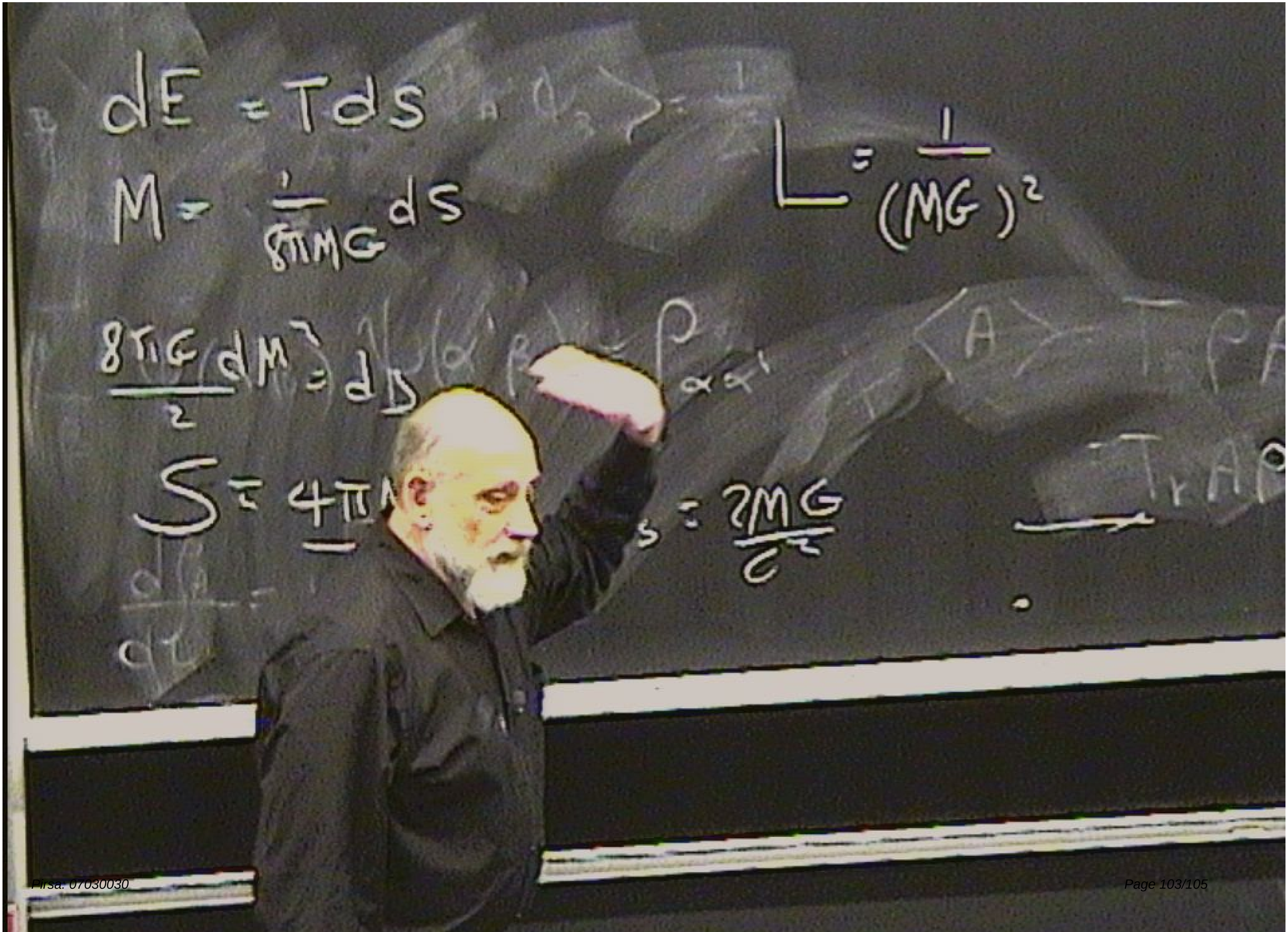
$$\langle A \rangle = \mathrm{Tr}\,\rho A$$

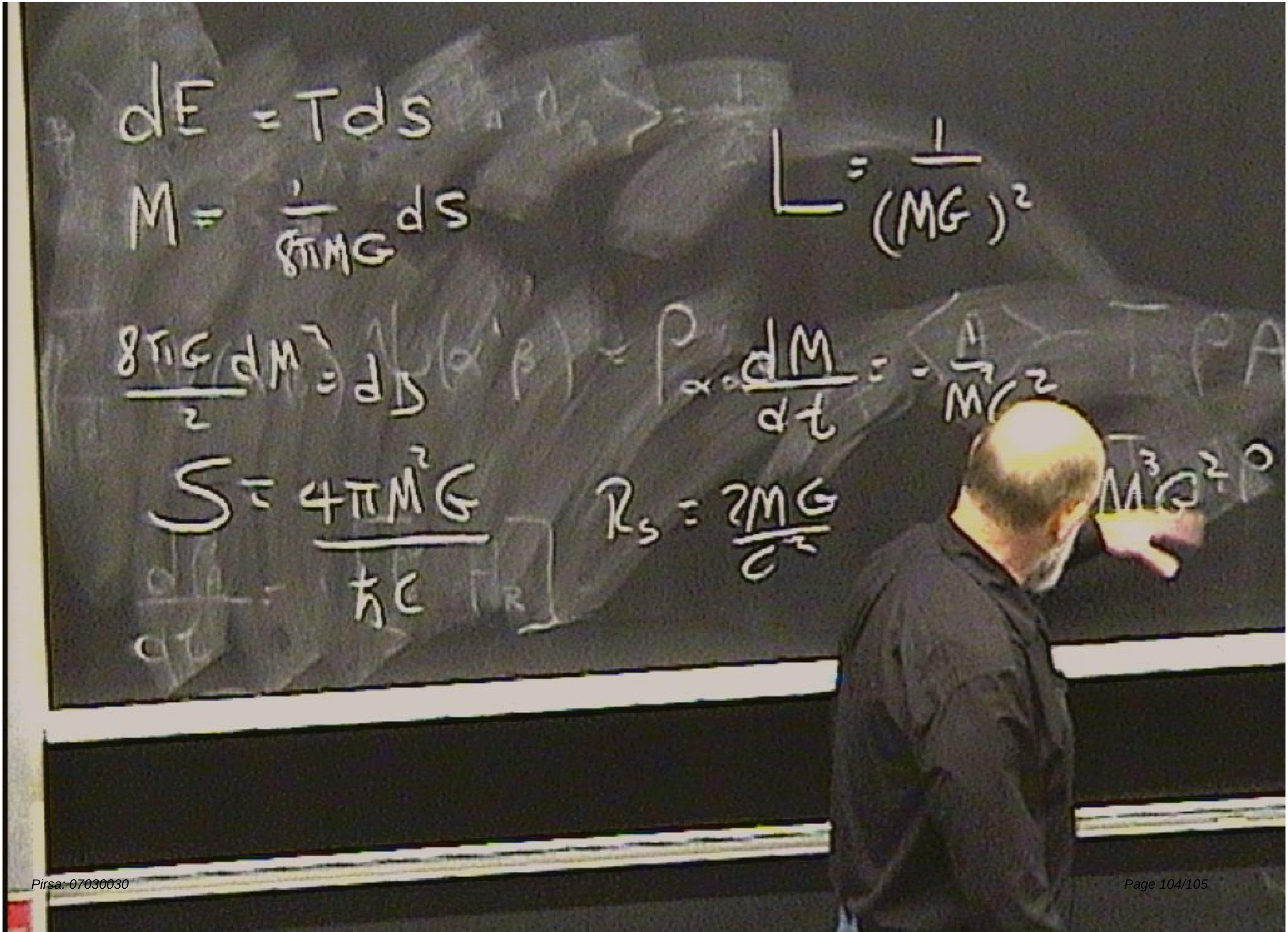
$$S = \frac{4\pi M^2 G}{\hbar c}$$

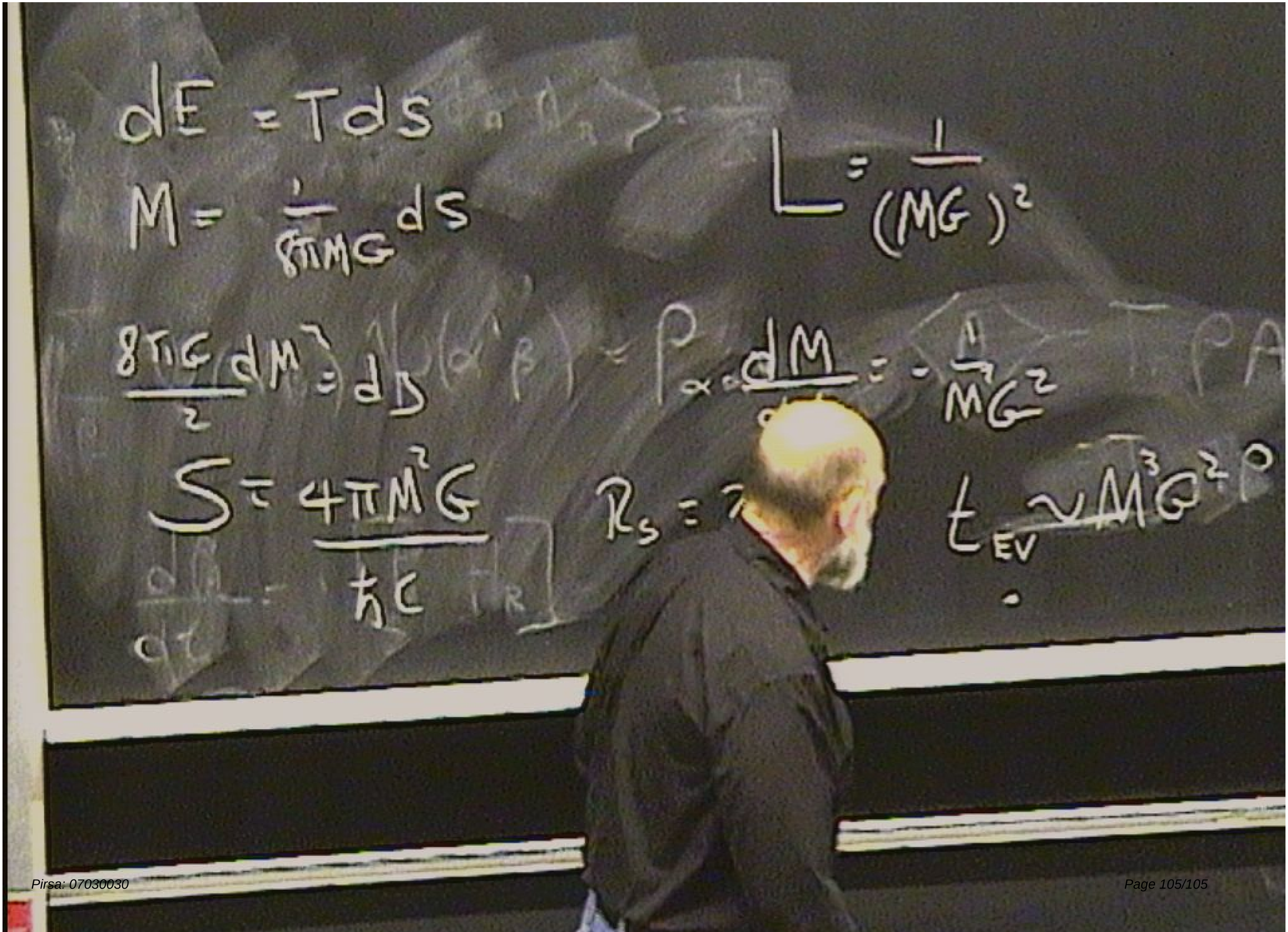
$$R_s = \frac{2MG}{c^2}$$

$$\mathrm{Tr}\,A\rho$$


time
4MG
distance
V
H
∞


dE = Tds
M = 1/8πMG ds
L = 1/(MG)²
S = 4π
= 2MG/c²


dE = Tds
M = 1/8πMG dS
L = 1/(MG)²
dM/dt = -A/M²
S = 4πM²G/ħc
Rₛ = 2MG/c²


dE = TdS
M = 1/8πMG dS
L = 1/(MG)²
dM = - 1/MG²
S = 4πM²G/ħc
R_S =
t_EV ~ M³G²