Title: Classical interaction cannot replace a quantum message

Date: Mar 21, 2007 04:00 PM

URL: http://pirsa.org/07030026

Abstract: We give a communication problem between two players, Alice and Bob, that can be solved by Alice sending a quantum message to Bob, for which any classical interactive protocol requires exponentially more communication.

Communication complexity: the multi-round model

Multi-Round Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

Communication complexity: the multi-round model

Multi-Round Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

Communication complexity: the multi-round model

Multi-Round Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

Communication complexity: the multi-round model

Multi-Round Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

• • •

- Bob produces an answer

Communication complexity: the multi-round model

Multi-Round Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

• • •

- Bob produces an answer

Communication complexity: the multi-round model

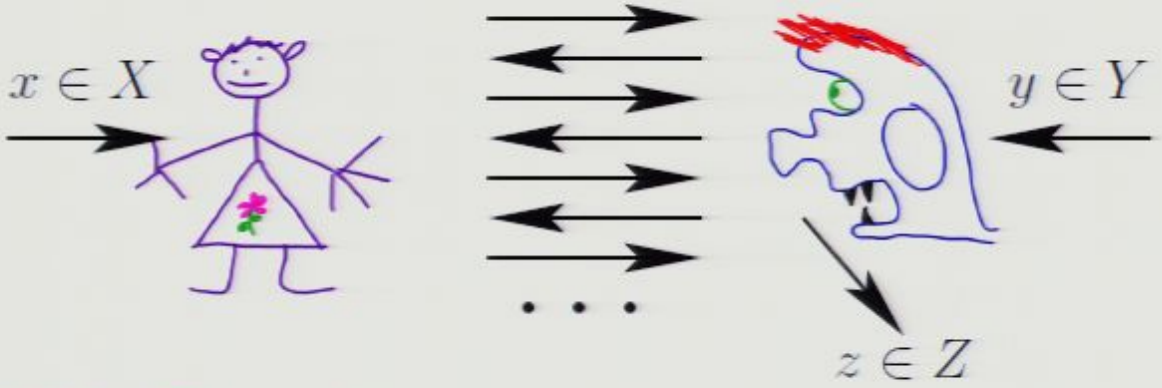
Multi-Round Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

• • •

- Bob produces an answer

Communication complexity: the multi-round model



$P \subseteq X \times Y \times Z$

Is $(x, y, z) \in P$?

Multi-Round Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

• • •

- Bob produces an answer

Communication complexity: one-way communication

$P \subseteq X \times Y \times Z$

Is $(x, y, z) \in P$?

One-Way Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

• • •

- Bob produces an answer

Communication complexity: the multi-round model

$P \subseteq X \times Y \times Z$

Is $(x, y, z) \in P$?

Multi-Round Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

• • •

- Bob produces an answer

Communication complexity: one-way communication

$P \subseteq X \times Y \times Z$

Is $(x, y, z) \in P$?

One-Way Communication:

- Alice receives x and Bob receives y
- Alice sends a message to Bob
- Bob sends a message to Alice

• • •

- Bob produces an answer

Exponential Savings from Quantum Communication

- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)

Exponential Savings from Quantum Communication

- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)
- Bounded-error protocols, Q^1 vs. R^1, for a function (Gavinsky, Kempe, Kerenidis, Raz, and de Wolf, 2007)

Exponential Savings from Quantum Communication

- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)
- Bounded-error protocols, Q^1 vs. R^1, for a function (Gavinsky, Kempe, Kerenidis, Raz, and de Wolf, 2007)
- We show a relation that can be solved with bounded error by a Q^1-protocol that is exponentially more efficient than any R-protocol.

Exponential Savings from Quantum Communication

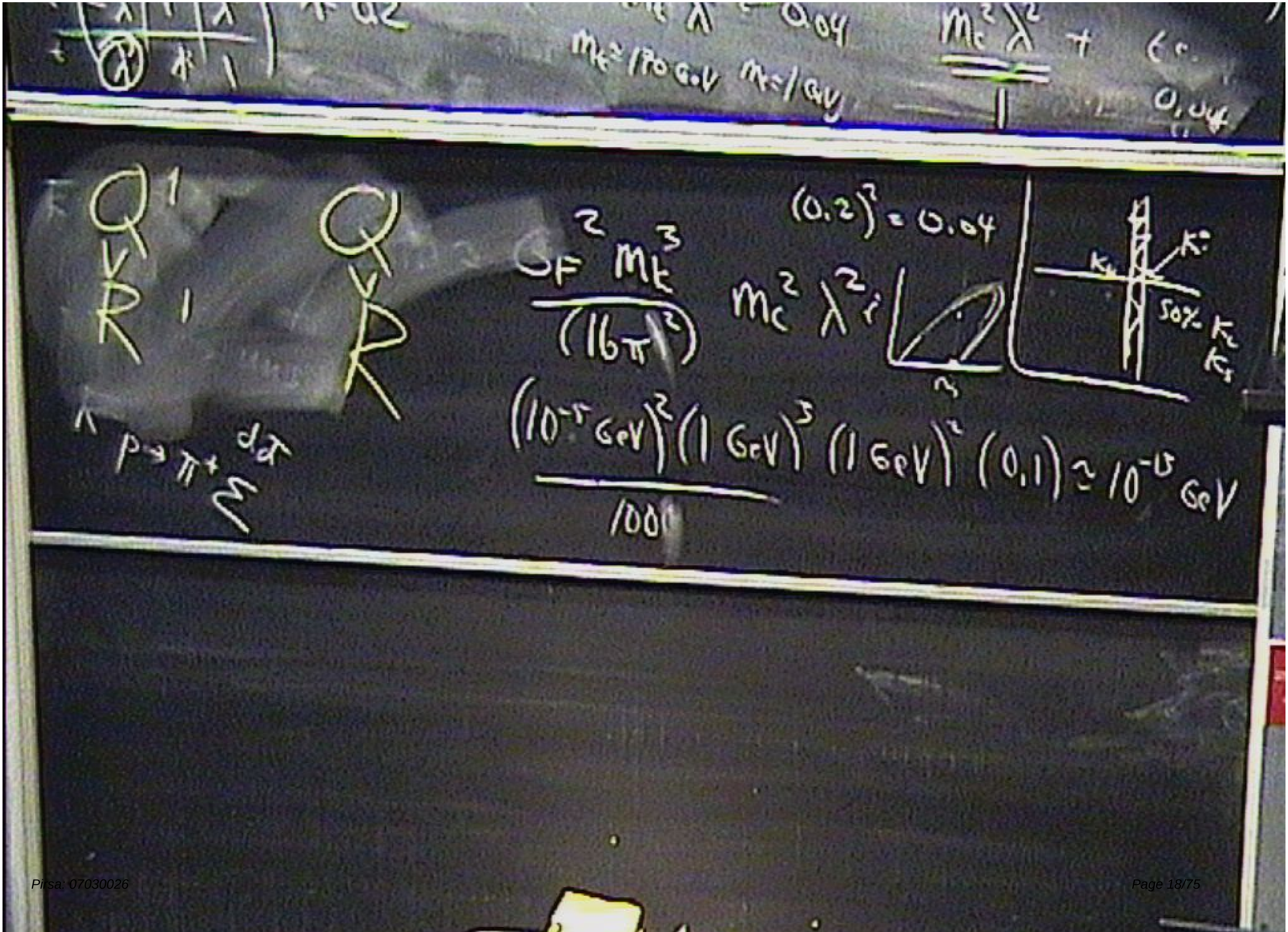
- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)
- Bounded-error protocols, Q^1 vs. R^1, for a function (Gavinsky, Kempe, Kerenidis, Raz, and de Wolf, 2007)
- We show a relation that can be solved with bounded error by a Q^1-protocol that is exponentially more efficient than any R-protocol

Exponential Savings from Quantum Communication

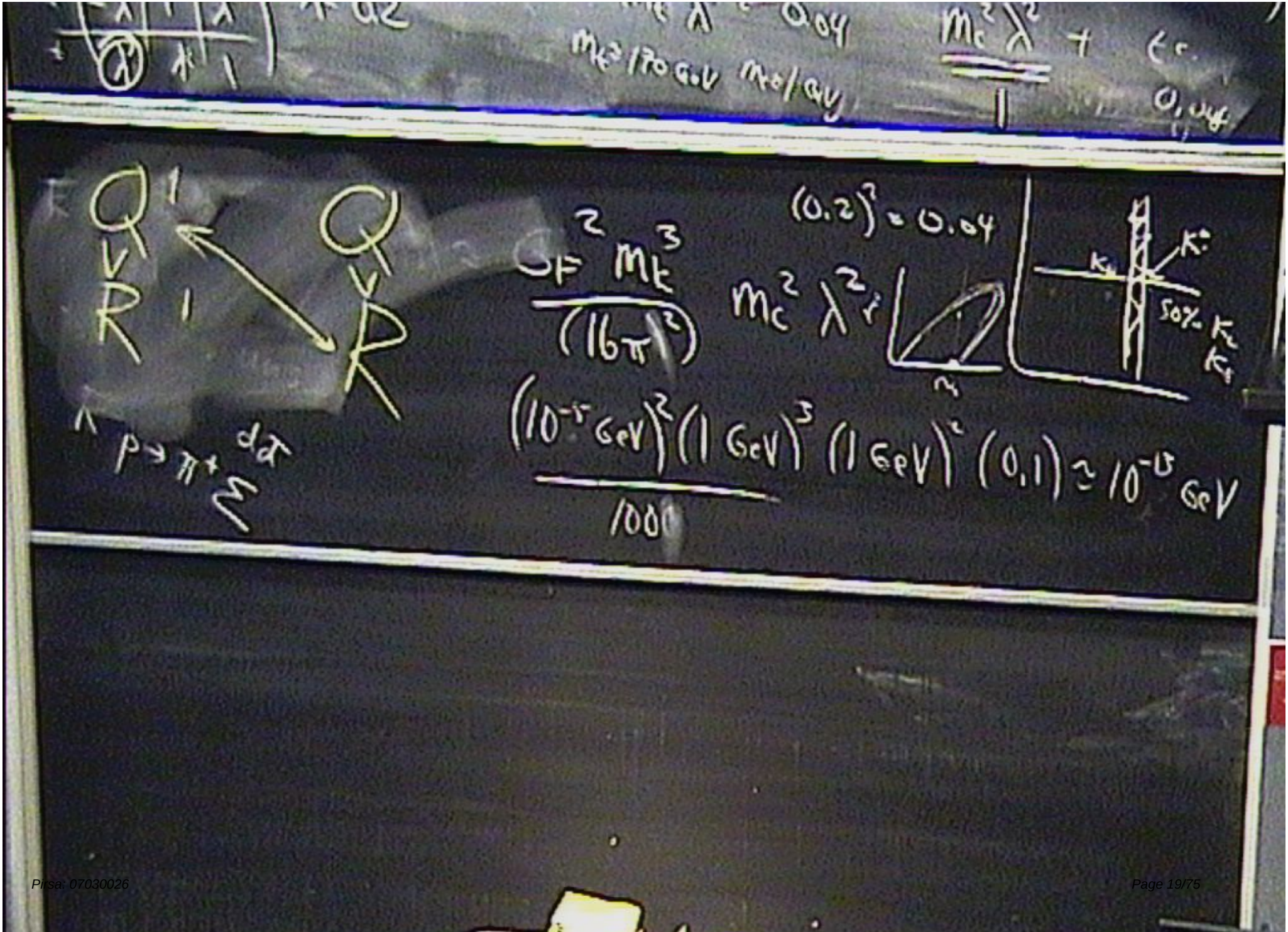
- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)
- Bounded-error protocols, Q^1 vs. R^1, for a function (Gavinsky, Kempe, Kerenidis, Raz, and de Wolf, 2007)
- We show a relation that can be solved with bounded error by a Q^1-protocol that is exponentially more efficient than any R-protocol

Exponential Savings from Quantum Communication

- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)
- Bounded-error protocols, Q^1 vs. R^1, for a function (Gavinsky, Kempe, Kerenidis, Raz, and de Wolf, 2007)
- We show a relation that can be solved with bounded error by a Q^1-protocol that is exponentially more efficient than any R-protocol



(0,2)² = 0,04
m_t = 170 GeV
m_c = 1 GeV
0,04
Q
V
R
1
G_F² m_t³ / (16π²)
m_c² λ² i
50%
K_L
(10⁻⁵ GeV)² (1 GeV)³ (1 GeV)² (0,1) / 1000 ≃ 10⁻¹³ GeV
p → π⁺


$m_c^2 \lambda^2$
$m_t^2/170\ GeV$
m_c/GeV
$(0.2)^2 = 0.04$
$m_c^2 \lambda^2 \gamma$
50%
$\Lambda \to p \pi^- \Sigma$
$\frac{G_F^2 m_t^3}{(16\pi^3)}$
$\frac{(10^{-5}\ GeV)^2 (1\ GeV)^3 (1\ GeV)^2 (0.1) \simeq 10^{-13}\ GeV}{1000}$

Exponential Savings from Quantum Communication

- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)
- Bounded-error protocols, Q^1 vs. R^1, for a function (Gavinsky, Kempe, Kerenidis, Raz, and de Wolf, 2007)
- We show a relation that can be solved with bounded error by a Q^1-protocol that is exponentially more efficient than any R-protocol

Our Communication Problem

1	*2*
1,5	3,8
4,7	2,6

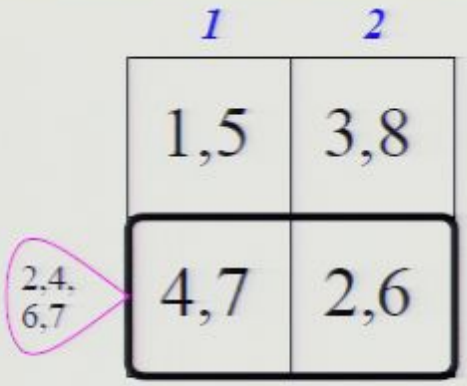
- Integers $1..2n^2$ are placed in an $n \times n$ table, two numbers in every cell; the columns are indexed $1..n$
- Alice's input consists of the elements of the last row
- Bob's input consists of the elements of each column
- Alice and Bob have to output a column index, and a number which is orthogonal to the bit-wise xor of the two elements in the corresponding cell of the last row

Our Communication Problem

1	*2*
1,5	3,8
4,7	2,6

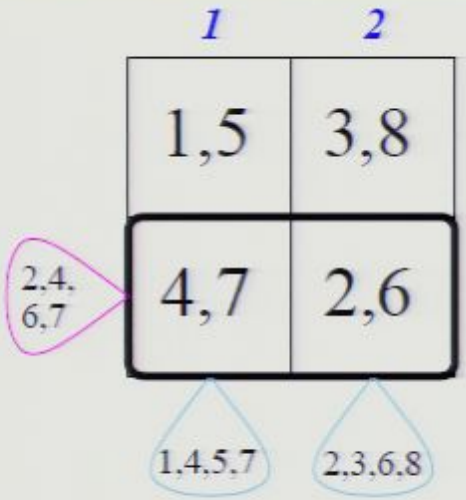
- Integers $1..2n^2$ are placed in an $n \times n$ table, two numbers in every cell; the columns are indexed $1..n$
- Alice's input consists of the elements of the last row
- Bob's input consists of the elements of each column
- Alice and Bob have to output a column index and a number which is orthogonal to the bit-wise xor of the two elements in the corresponding cell of the last row

Our Communication Problem



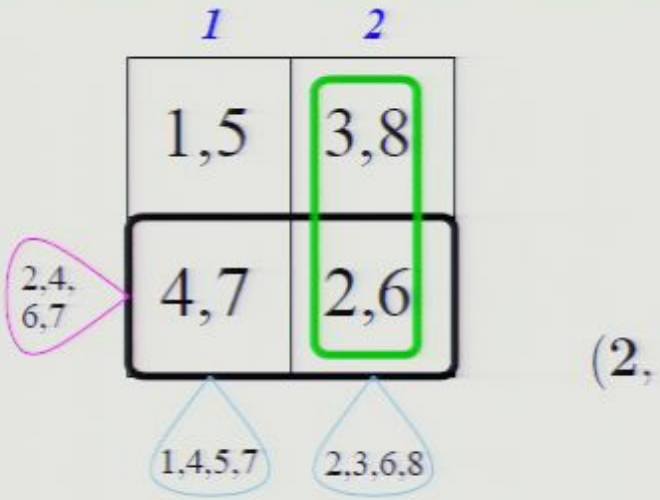
- Integers $1..2n^2$ are placed in an $n \times n$ table, two numbers in every cell; the columns are indexed $1..n$
- Alice's input consists of the elements of the last row
- Bob's input consists of the elements of each column
- Alice and Bob have to output a column index, and a number which is orthogonal to the bit-wise xor of the two elements in the corresponding cell of the last row

Our Communication Problem



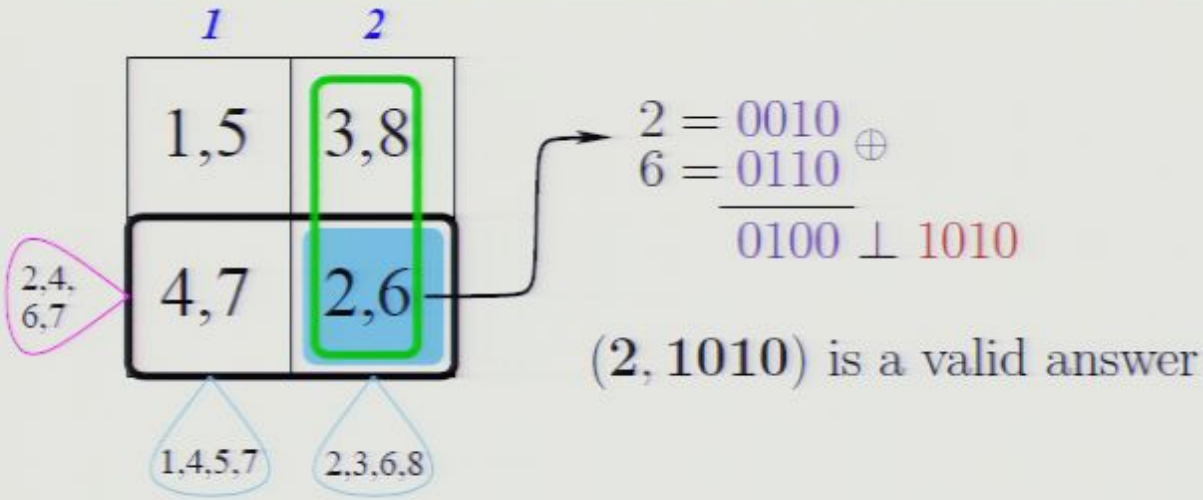
- Integers $1..2n^2$ are placed in an $n \times n$ table, two numbers in every cell; the columns are indexed $1..n$
- Alice's input consists of the elements of the last row
- Bob's input consists of the elements of each column
- Alice and Bob have to output a column index, and a number which is orthogonal to the bit-wise xor of the two elements in the corresponding cell of the last row

Our Communication Problem



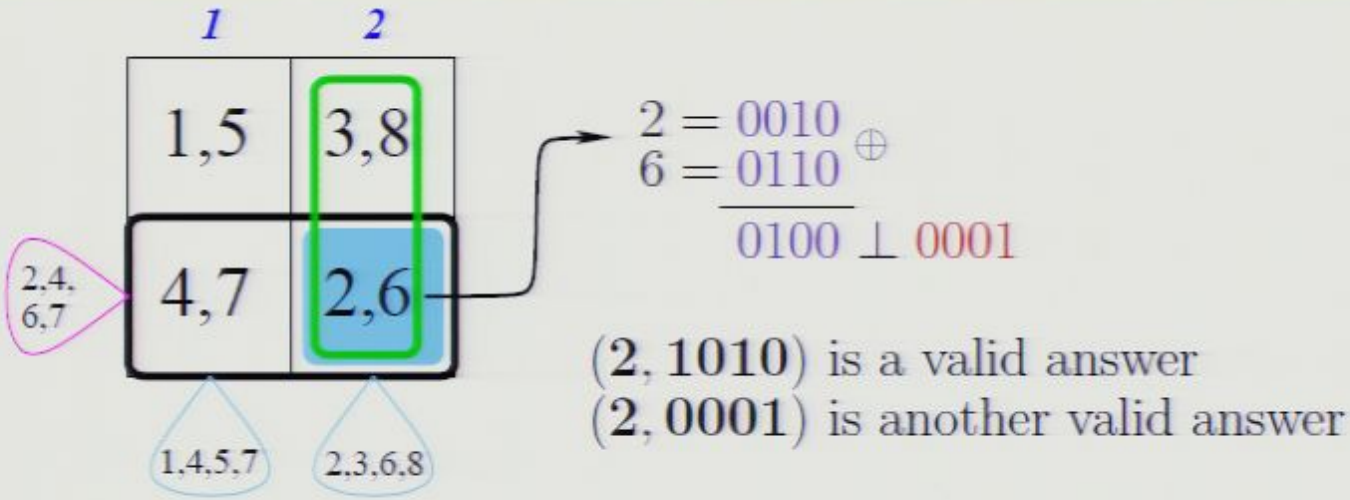
- Integers $1..2n^2$ are placed in an $n \times n$ table, two numbers in every cell; the columns are indexed $1..n$
- Alice's input consists of the elements of the last row
- Bob's input consists of the elements of each column
- Alice and Bob have to output a column index, and a number which is orthogonal to the bit-wise xor of the two elements in the corresponding cell of the last row

Our Communication Problem



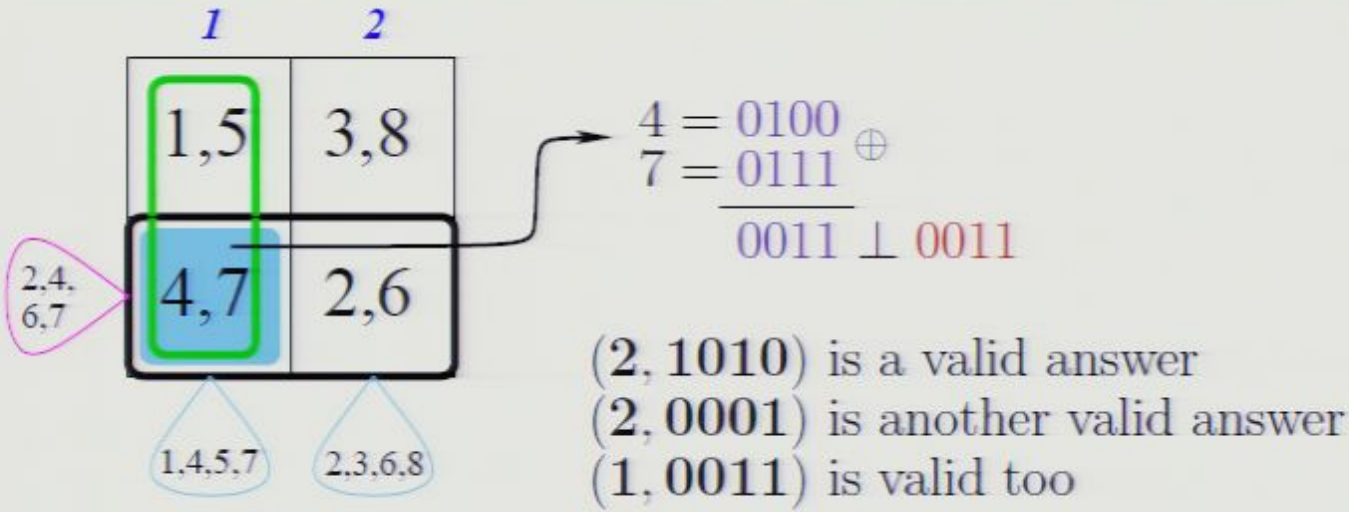
- Integers $1..2n^2$ are placed in an $n \times n$ table, two numbers in every cell; the columns are indexed $1..n$
- Alice's input consists of the elements of the last row
- Bob's input consists of the elements of each column
- Alice and Bob have to output a column index, and a number which is orthogonal to the bit-wise xor of the two elements in the corresponding cell of the last row

Our Communication Problem



- Integers $1..2n^2$ are placed in an $n \times n$ table, two numbers in every cell; the columns are indexed $1..n$
- Alice's input consists of the elements of the last row
- Bob's input consists of the elements of each column
- Alice and Bob have to output a column index, and a number which is orthogonal to the bit-wise xor of the two elements in the corresponding cell of the last row

Our Communication Problem



- Integers $1..2n^2$ are placed in an $n \times n$ table, two numbers in every cell; the columns are indexed $1..n$
- Alice's input consists of the elements of the last row
- Bob's input consists of the elements of each column
- Alice and Bob have to output a column index, and a number which is orthogonal to the bit-wise xor of the two elements in the corresponding cell of the last row

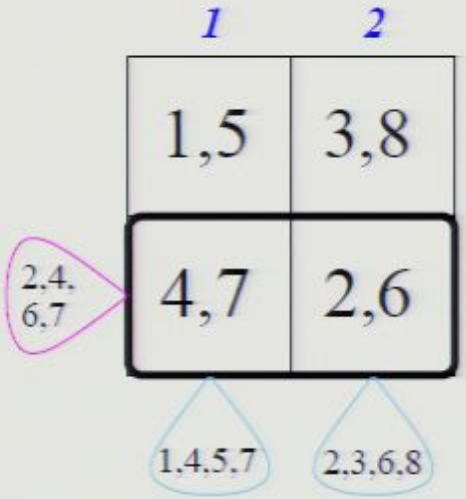
Our Statement

- Our problem is solvable by a one-way quantum protocol of cost $O(\log n)$
- The problem requires $\tilde{\Omega}(n^{1/8})$ bits of communication in the classical multi-round model
- The gap is exponential

Our Statement

- Our problem is solvable by a one-way quantum protocol of cost $O(\log n)$
- The problem requires $\tilde{\Omega}\left(n^{1/8}\right)$ bits of communication in the classical multi-round model
- The gap is exponential

Quantum One-Way Protocol

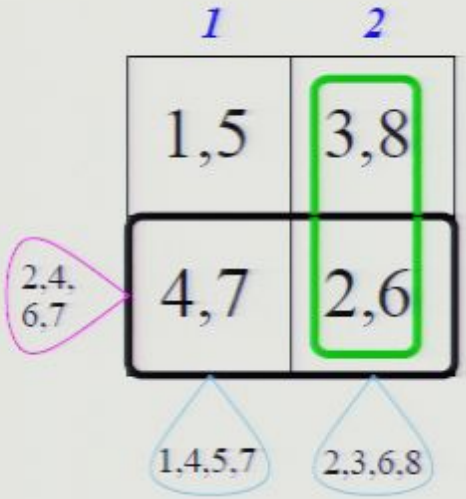


- $\frac{|2\rangle+|4\rangle+|6\rangle+|7\rangle}{2}$
- $\frac{|2\rangle+|6\rangle}{\sqrt{2}} = \frac{|0010\rangle+|0110\rangle}{\sqrt{2}}$

$\frac{\sum_{j \perp 0100} |j\rangle}{4}$

- Alice sends to Bob the superposition of the indices in the last row
- Bob projects the state to the content of one of the columns, then applies the Hadamard transform and measures in the computational basis

Quantum One-Way Protocol

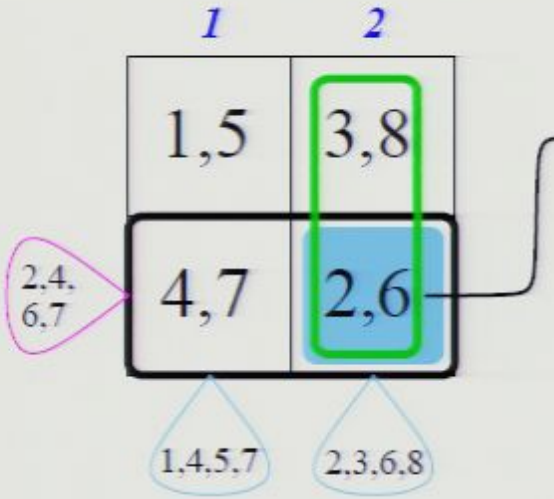


- $\frac{|2\rangle+|4\rangle+|6\rangle+|7\rangle}{2}$
- $\frac{|2\rangle+|6\rangle}{\sqrt{2}} = \frac{|0010\rangle+|0110\rangle}{\sqrt{2}}$

$\frac{\sum_{j \perp 0100}|j\rangle}{4}$

- Alice sends to Bob the superposition of the indices in the last row
- Bob projects the state to the content of one of the columns, then applies the Hadamard transform and measures in the computational basis

Quantum One-Way Protocol



$2 = 0010$
$6 = 0110$ $\oplus$
$0100 \perp 1010$

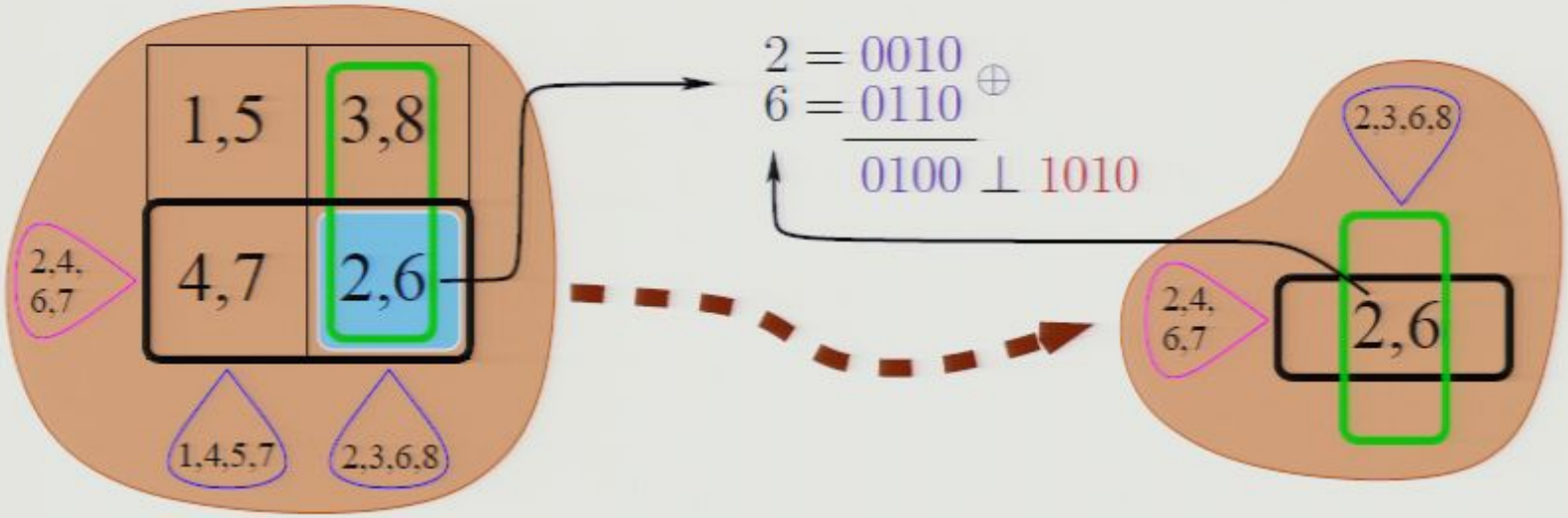
$(\mathbf{2}, \mathbf{1010})$ is valid

- $\frac{|2\rangle+|4\rangle+|6\rangle+|7\rangle}{2}$
- $\frac{|2\rangle+|6\rangle}{\sqrt{2}} = \frac{|0010\rangle+|0110\rangle}{\sqrt{2}}$

$\frac{\sum_{j \perp 0100}|j\rangle}{4}$

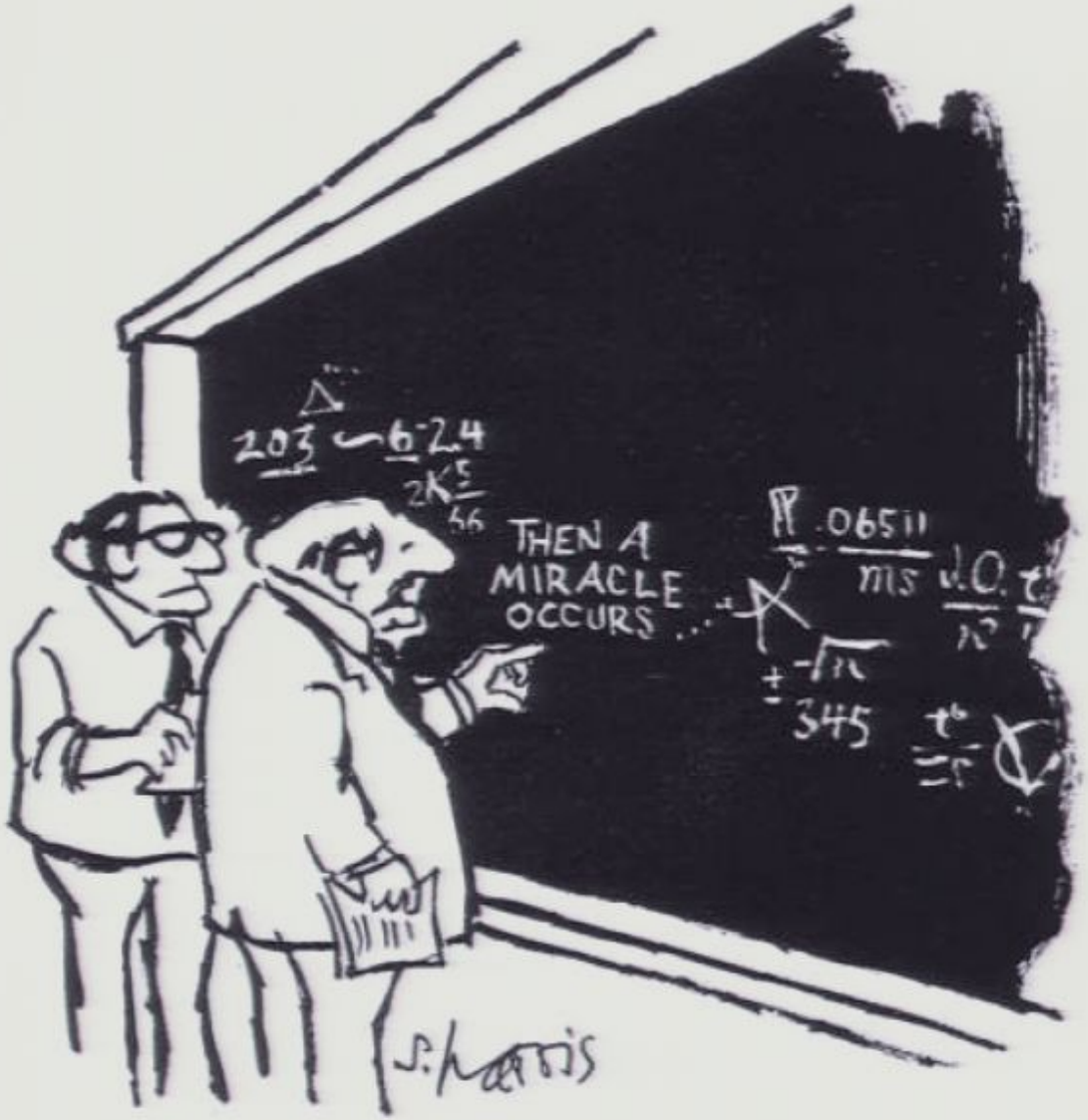
- Alice sends to Bob the superposition of the indices in the last row
- Bob projects the state to the content of one of the columns, then applies the Hadamard transform and measures in the computational basis

Classical Solution is Expensive: The First Reduction

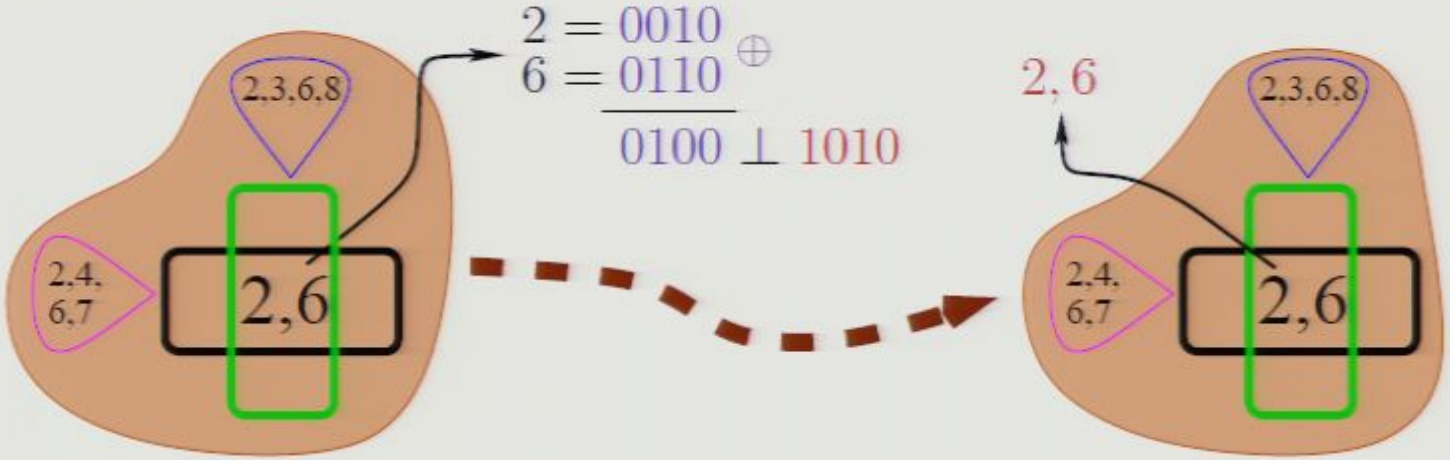


Claim

Assume that a protocol of cost k solves the original problem with small error. Then another protocol of similar cost solves the 1×1-version with probability $\frac{1}{n}$ with small error.


THEN A
MIRACLE
OCCURS

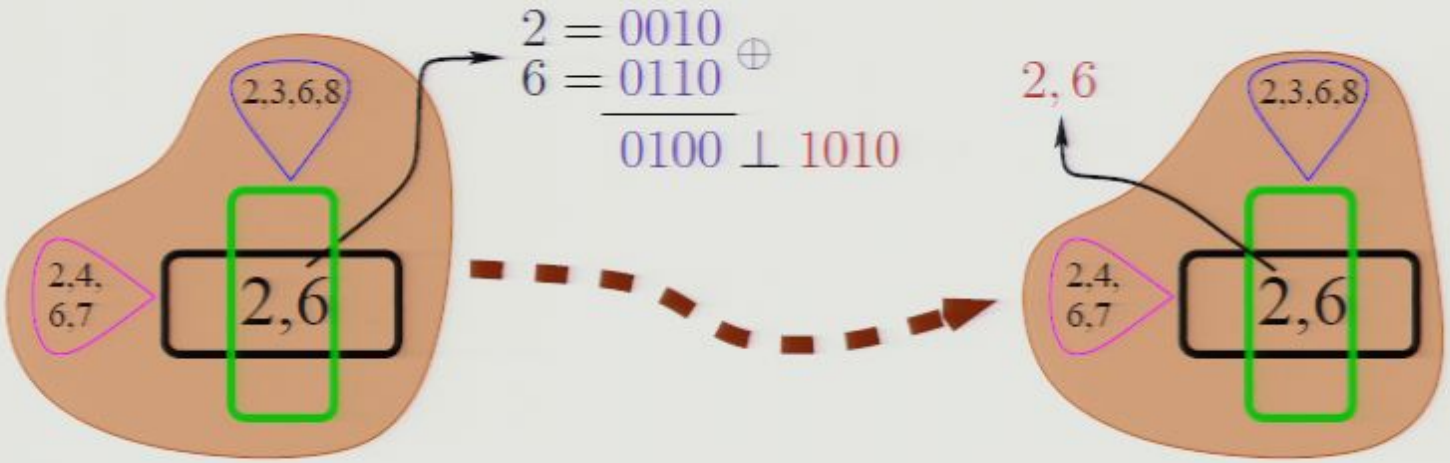
Classical Solution is Expensive: The Second Reduction



Claim

Assume that a protocol of cost k solves the 1x1-version of the problem with probability $\frac{1}{n}$ with small error. Then another protocol of similar cost solves the search 1x1-version of the problem with probability $\frac{1}{nk^2 \log^2(n)}$.

Classical Solution is Expensive: The Second Reduction

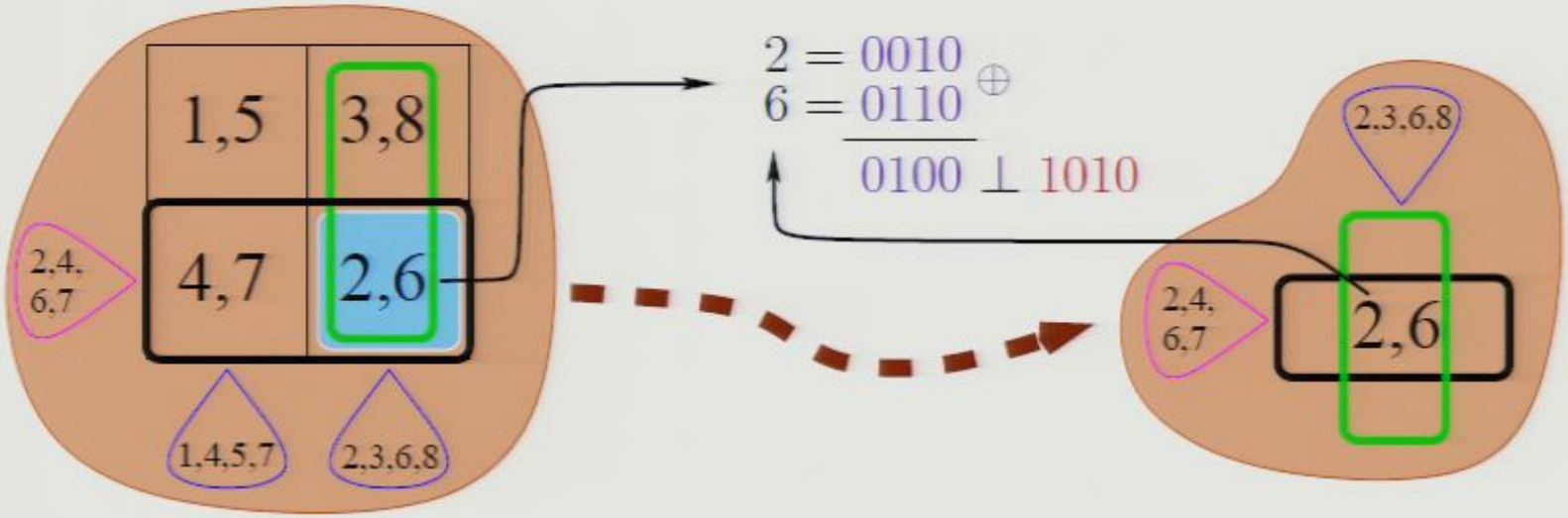


Claim

Assume that a protocol of cost k solves the 1x1-version of the problem with probability $\frac{1}{n}$ with small error. Then another protocol of similar cost solves the search 1x1-version of the problem with probability $\frac{1}{nk^2 \log^2(n)}$.

The proof is combinatorial, technical.

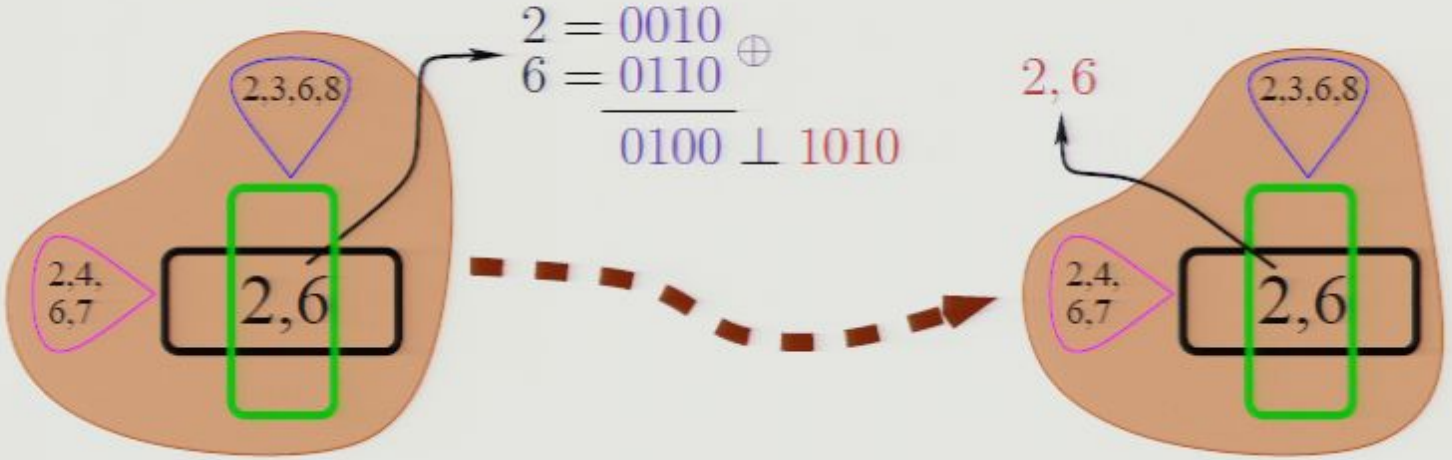
Classical Solution is Expensive: The First Reduction



Claim

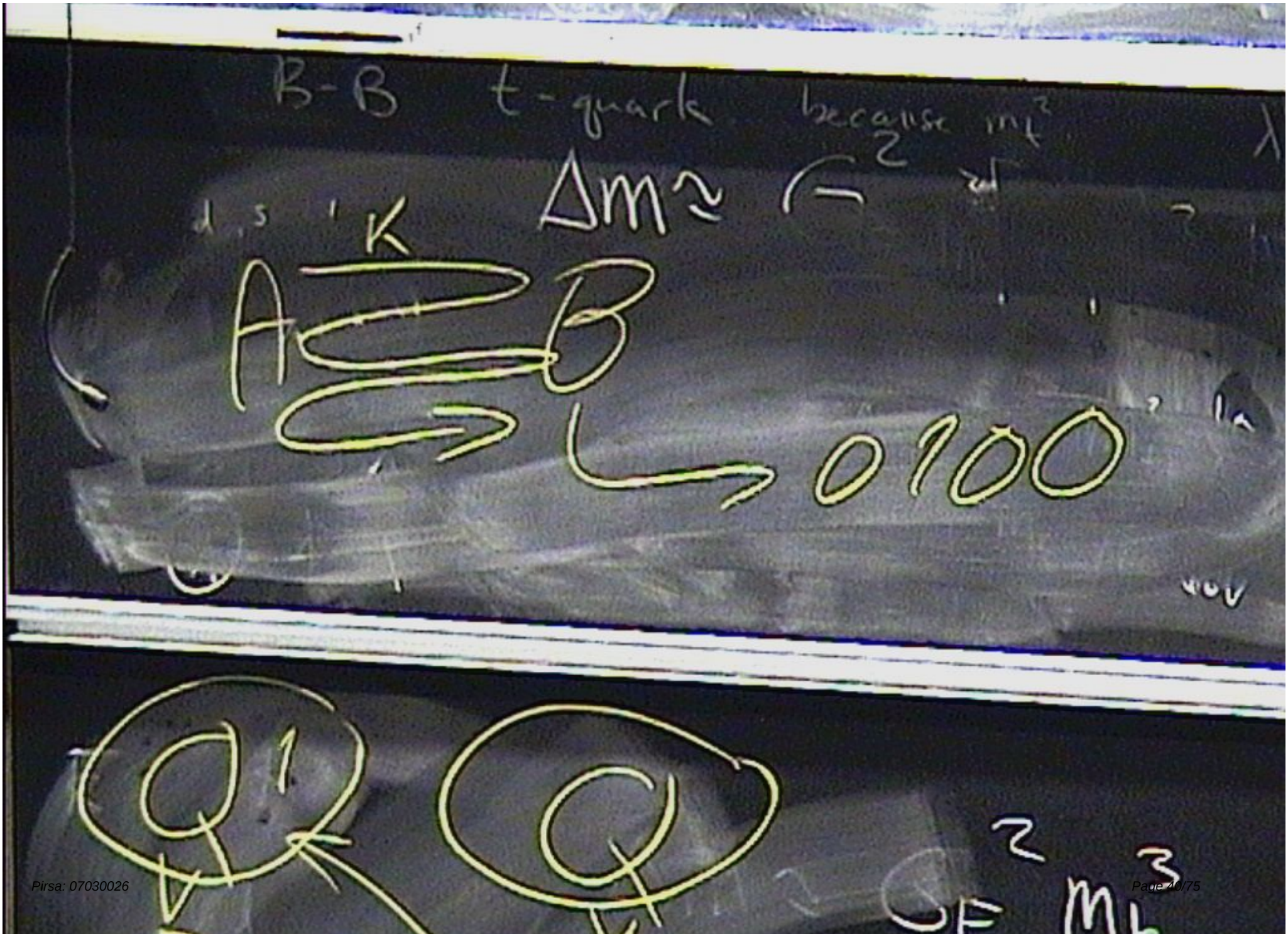
Assume that a protocol of cost k solves the original problem with small error. Then another protocol of similar cost solves the 1×1-version with probability $\frac{1}{n}$ with small error.

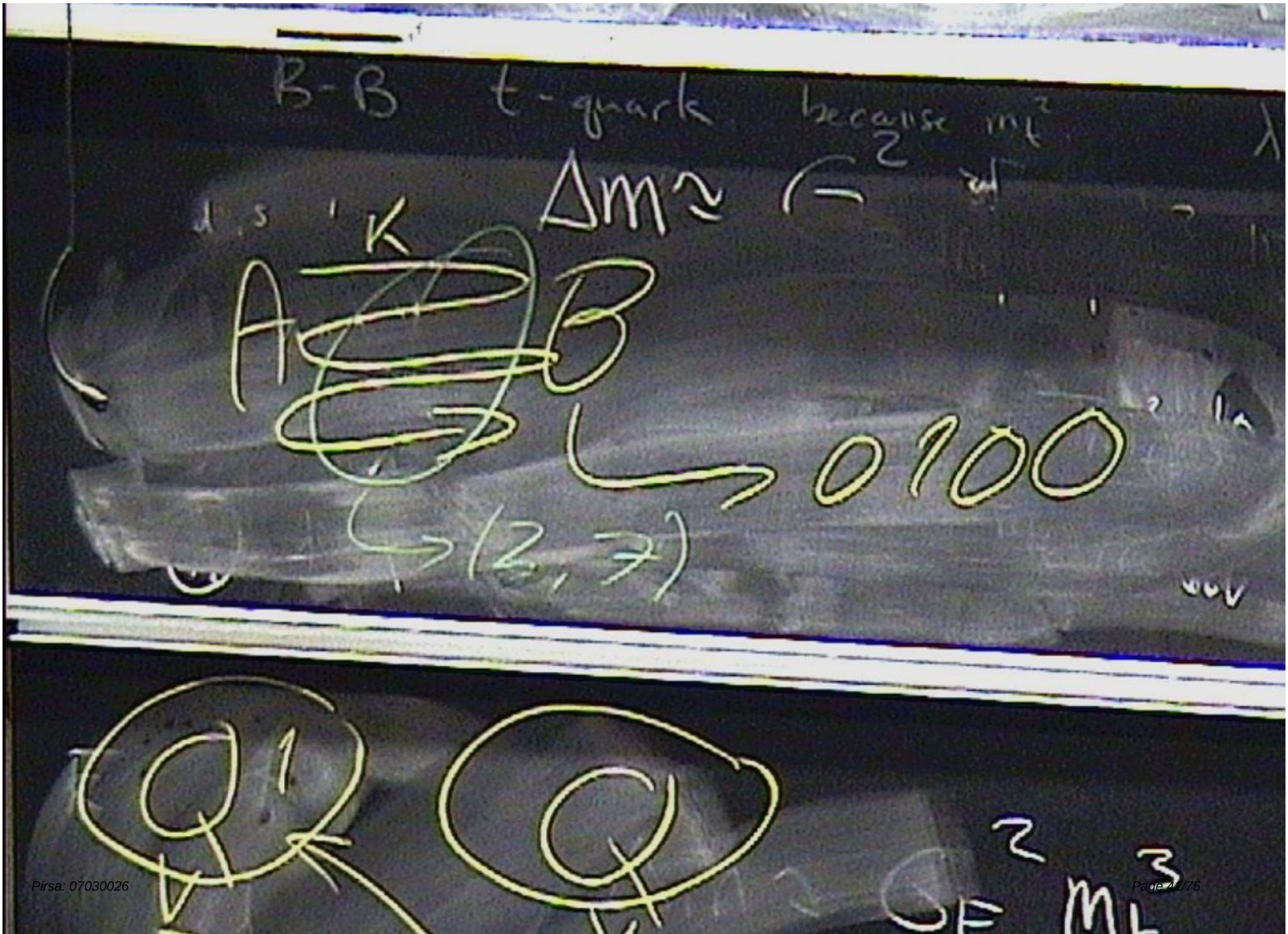
Classical Solution is Expensive: The Second Reduction



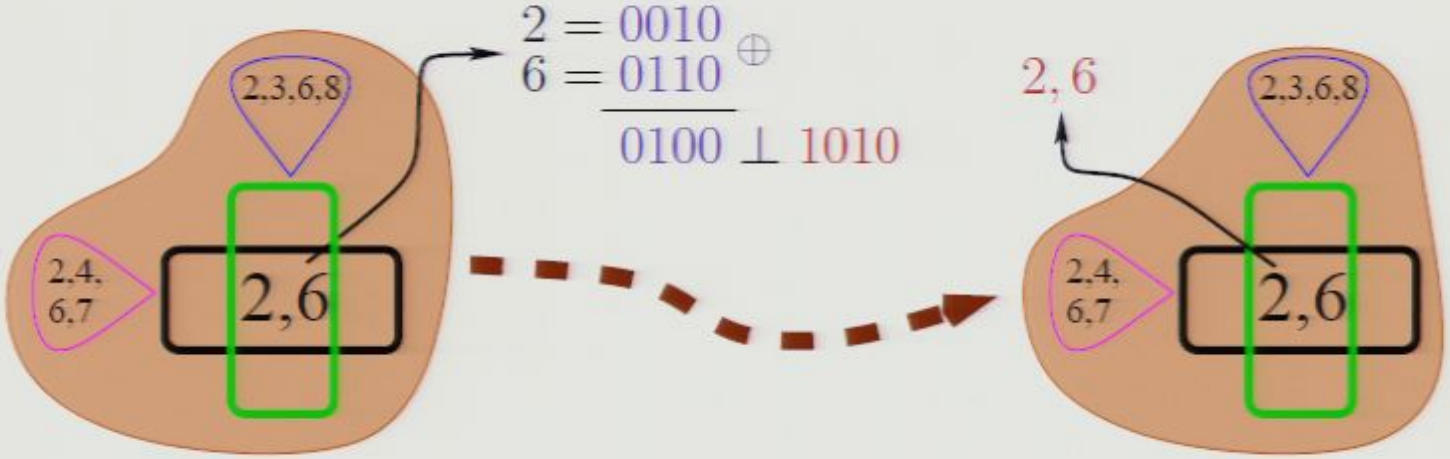
Claim

Assume that a protocol of cost k solves the 1x1-version of the problem with probability $\frac{1}{n}$ with small error. Then another protocol of similar cost solves the search 1x1-version of the problem with probability $\frac{1}{nk^2 \log^2(n)}$.


B-B t-quark because m_t^2
$\Delta m \approx$
d, s
K
A
B
0100
Q1
Q
m_b^3


B-B t-quark because m_t^2
$\Delta m \approx$
K
A B
0100
(2, 7)
Q 1
Q
m_t^3

Classical Solution is Expensive: The Second Reduction

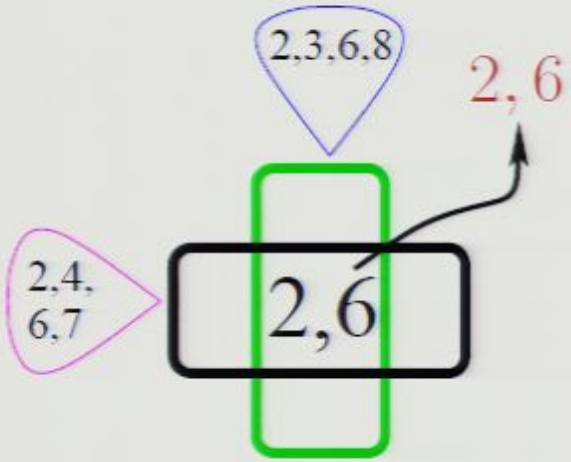


Claim

Assume that a protocol of cost k solves the 1x1-version of the problem with probability $\frac{1}{n}$ with small error. Then another protocol of similar cost solves the search 1x1-version of the problem with probability $\frac{1}{nk^2\log^2(n)}$.

The proof is combinatorial, technical.

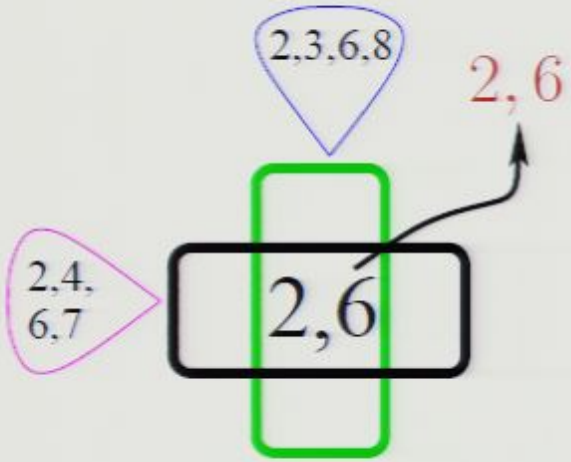
Complexity of the Search 1x1-Version



- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$

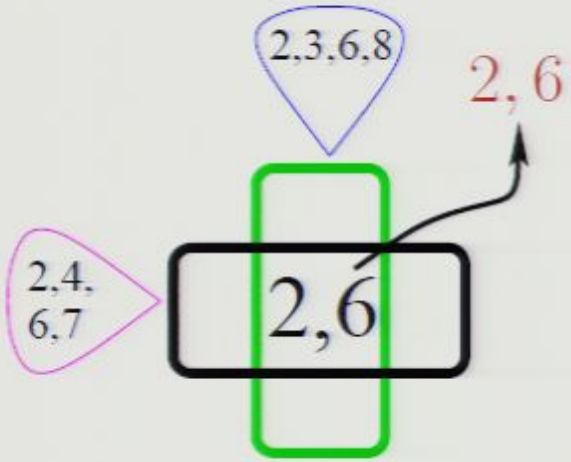
Complexity of the Search 1x1-Version



- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$

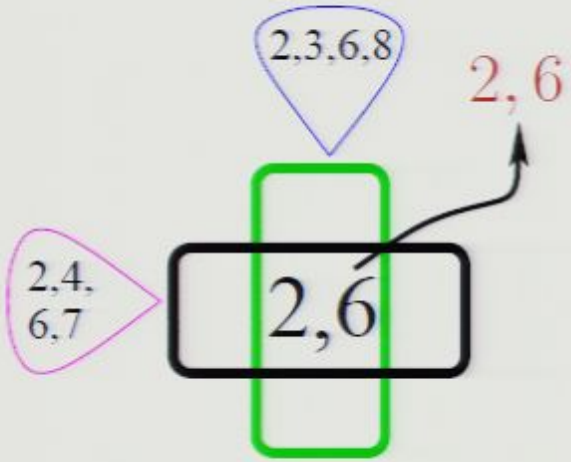
Complexity of the Search 1x1-Version



- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

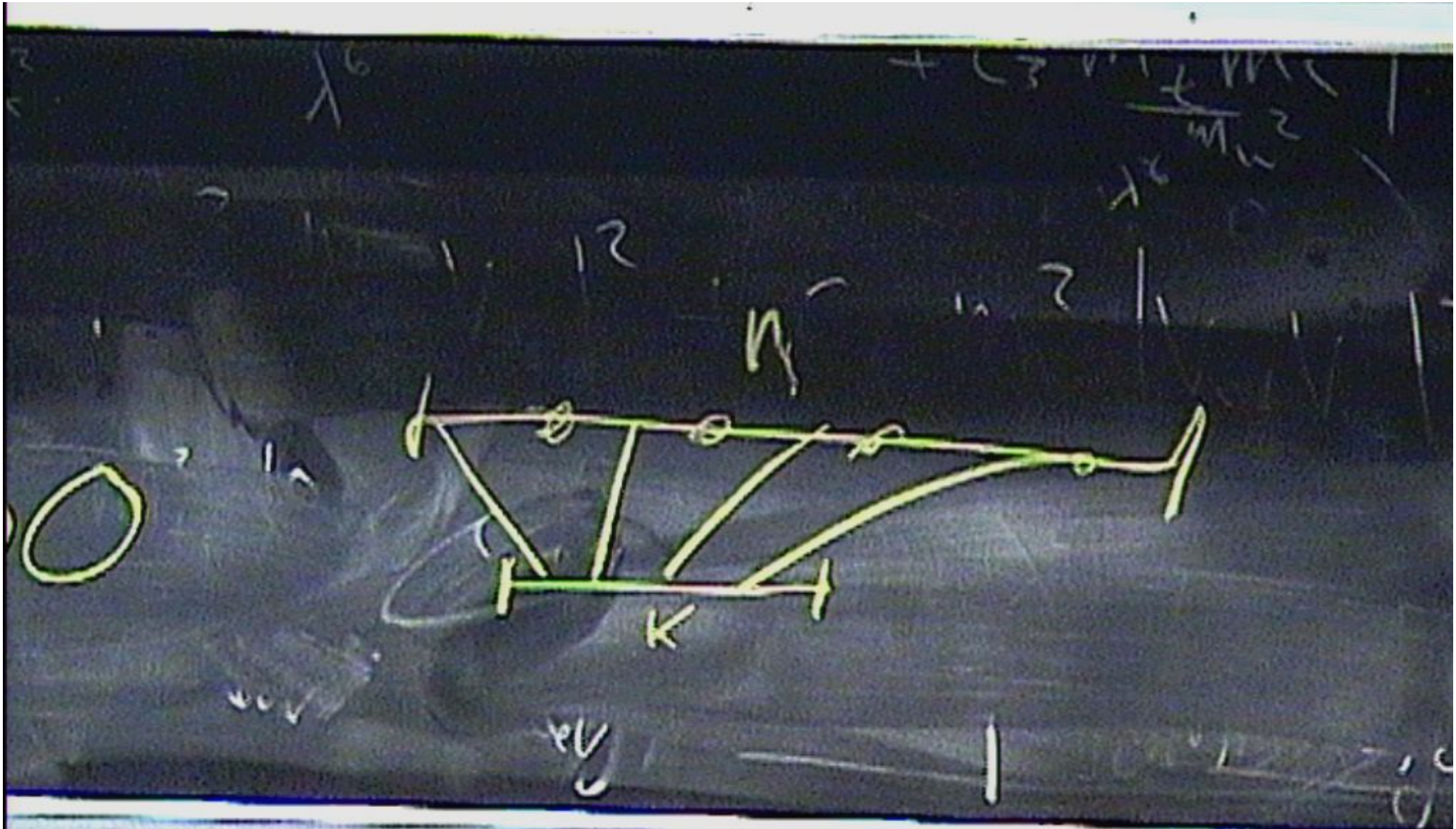
  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$

Complexity of the Search 1x1-Version

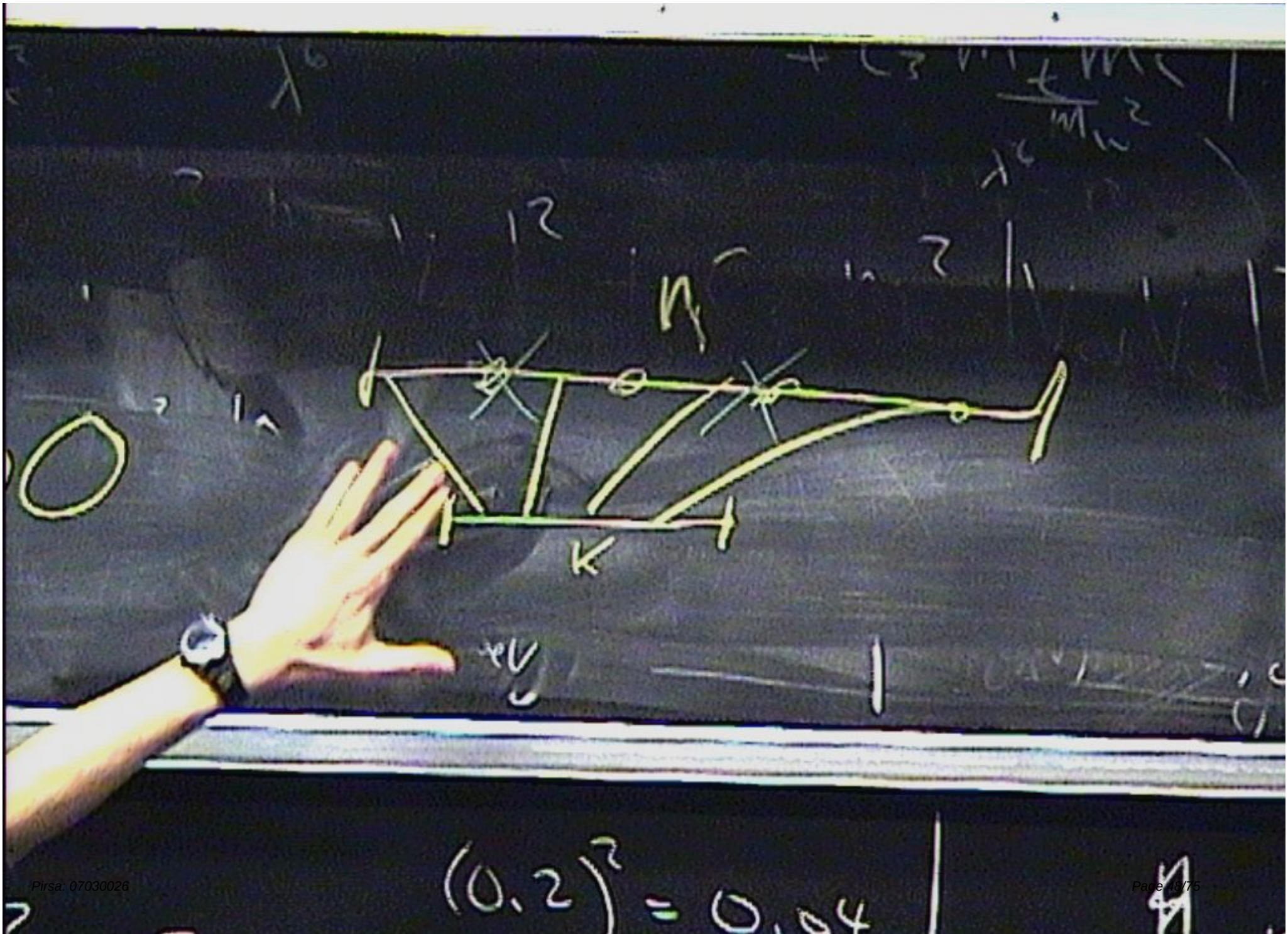


- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

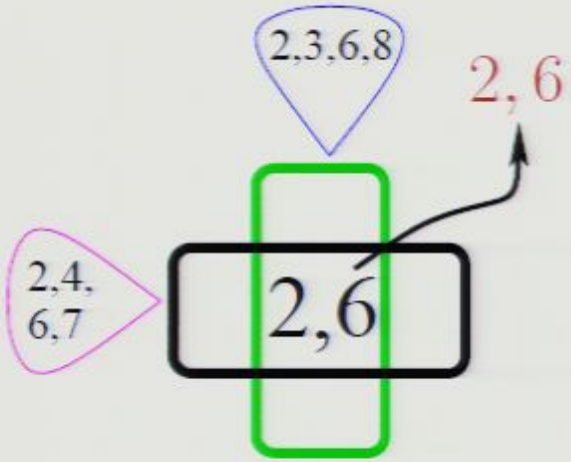
  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$



$(0,2)^2 = 0,04$


K
$(0.2)^2 = 0.04$

Complexity of the Search 1x1-Version



- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$

Another combinatorial proof, uses a lemma by Razborov (1992)

Lower Bound Summary

- If a protocol of cost k solves the original problem with small error then another protocol of similar cost solves the 1×1-version with probability $\frac{1}{n}$ with small error
- If a protocol of cost k solves the 1x1-version of the problem with probability $\frac{1}{n}$ with small error then another protocol of similar cost solves the search 1x1-version of the problem with probability $\frac{1}{nk^2 \log^2(n)}$
- The chances of a protocol of cost k to solve the search 1x1-version are $O\left(\left(\frac{k}{n}\right)^2\right)$
- This gives us the required $k \in \Omega\left(n^{1/3}\right)$

Lower Bound Summary

- If a protocol of cost k solves the original problem with small error then another protocol of similar cost solves the 1×1-version with probability $\frac{1}{n}$ with small error
- If a protocol of cost k solves the 1x1-version of the problem with probability $\frac{1}{n}$ with small error then another protocol of similar cost solves the search 1x1-version of the problem with probability $\frac{1}{nk^2 \log^2(n)}$
- The chances of a protocol of cost k to solve the search 1x1-version are $O\left(\left(\frac{k}{n}\right)^2\right)$
- This gives us the required $k \in \tilde{\Omega}\left(n^{1/3}\right)$

Lower Bound Summary

- If a protocol of cost k solves the original problem with small error then another protocol of similar cost solves the 1×1-version with probability $\frac{1}{n}$ with small error
- If a protocol of cost k solves the $1x1$-version of the problem with probability $\frac{1}{n}$ with small error then another protocol of similar cost solves the search $1x1$-version of the problem with probability $\frac{1}{nk^2 \log^2(n)}$
- The chances of a protocol of cost k to solve the search $1x1$-version are $O\left(\left(\frac{k}{n}\right)^2\right)$
- This gives us the required $k \in \tilde{\Omega}\left(n^{1/8}\right)$

Lower Bound Summary

- If a protocol of cost k solves the original problem with small error then another protocol of similar cost solves the 1×1-version with probability $\frac{1}{n}$ with small error
- If a protocol of cost k solves the $1x1$-version of the problem with probability $\frac{1}{n}$ with small error then another protocol of similar cost solves the search $1x1$-version of the problem with probability $\frac{1}{nk^2 \log^2(n)}$
- The chances of a protocol of cost k to solve the search $1x1$-version are $O\left(\left(\frac{k}{n}\right)^2\right)$
- This gives us the required $k \in \tilde{\Omega}\left(n^{1/8}\right)$

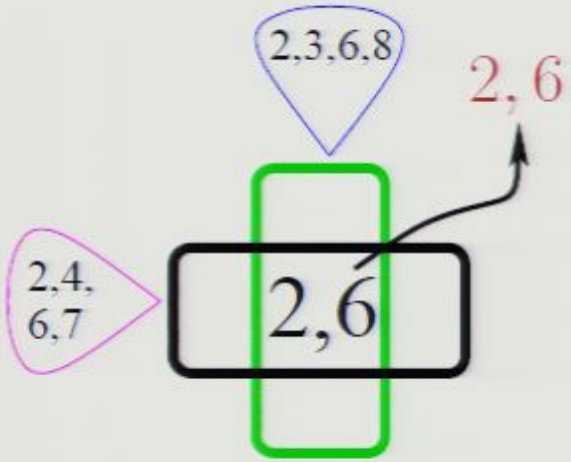
Lower Bound Summary

- If a protocol of cost k solves the original problem with small error then another protocol of similar cost solves the 1×1-version with probability $\frac{1}{n}$ with small error
- If a protocol of cost k solves the $1x1$-version of the problem with probability $\frac{1}{n}$ with small error then another protocol of similar cost solves the search $1x1$-version of the problem with probability $\frac{1}{nk^2 \log^2(n)}$
- The chances of a protocol of cost k to solve the search $1x1$-version are $O\left(\left(\frac{k}{n}\right)^2\right)$
- This gives us the required $k \in \tilde{\Omega}\left(n^{1/8}\right)$

Open problems

- Is it possible to find a functional problem that requires exponentially more expensive protocol in R than in Q^1?
- How about simultaneous protocols?
- Generally speaking, give a separation that would logically imply as many known results as possible.

Complexity of the Search 1x1-Version

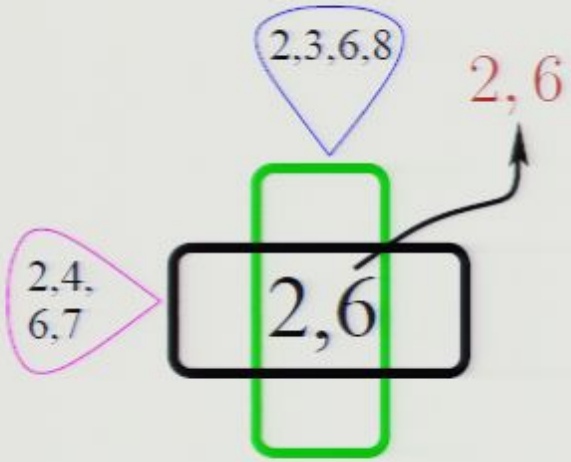


- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$

Another combinatorial proof, uses a lemma by Razborov (1992)

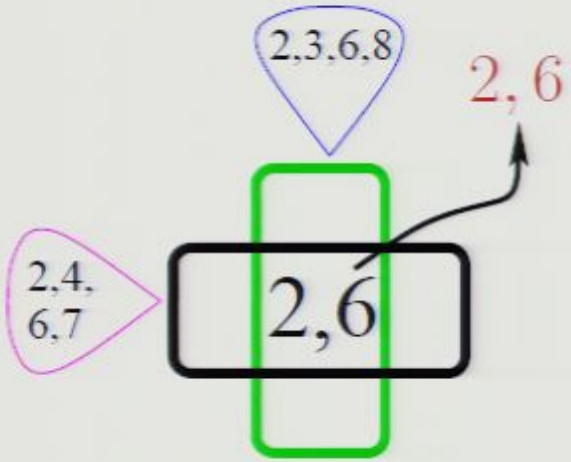
Complexity of the Search 1x1-Version



- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$

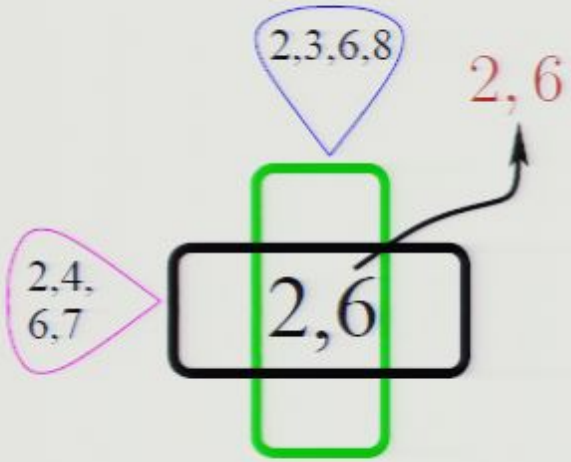
Complexity of the Search 1x1-Version



- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

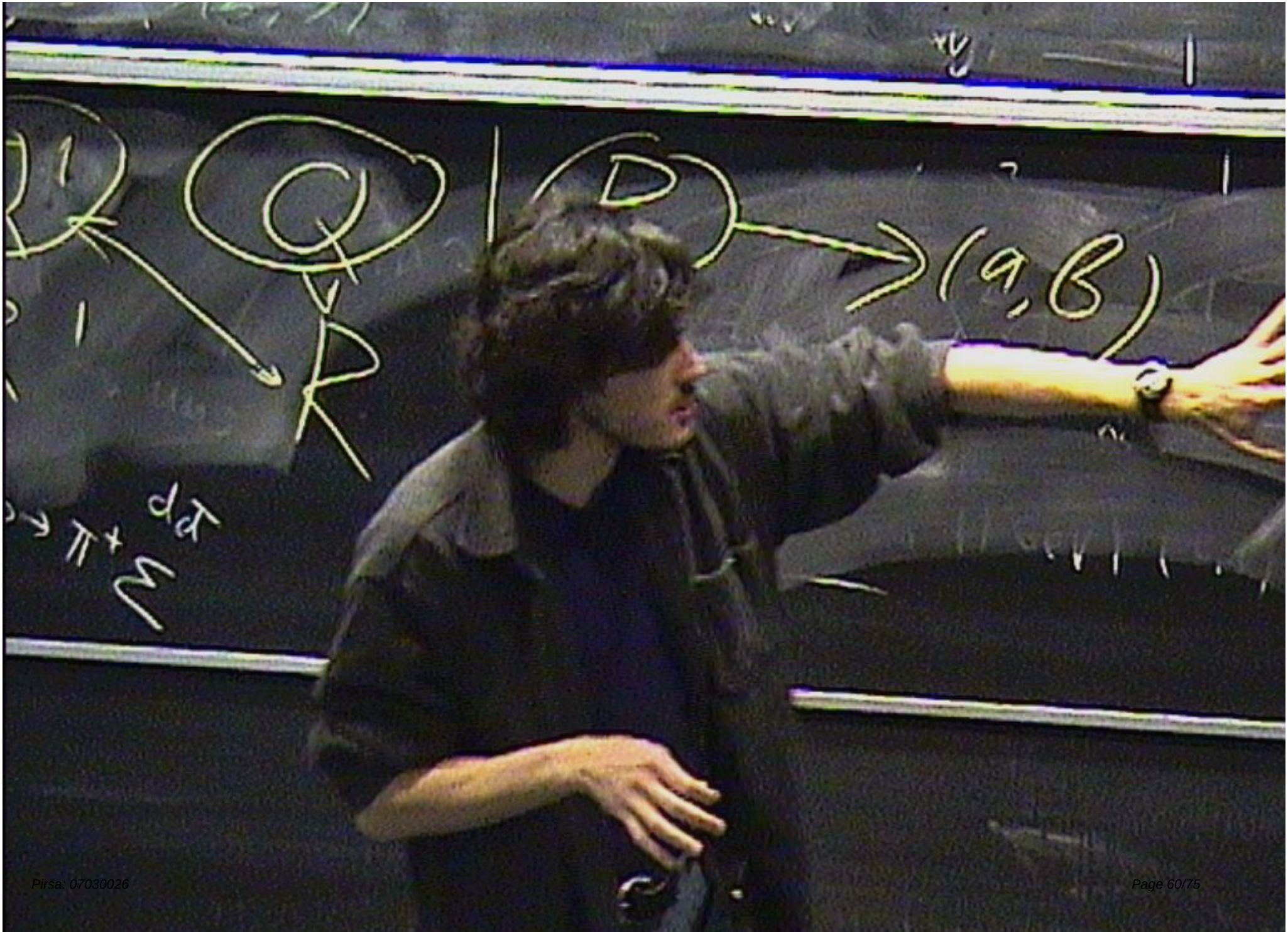
  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$

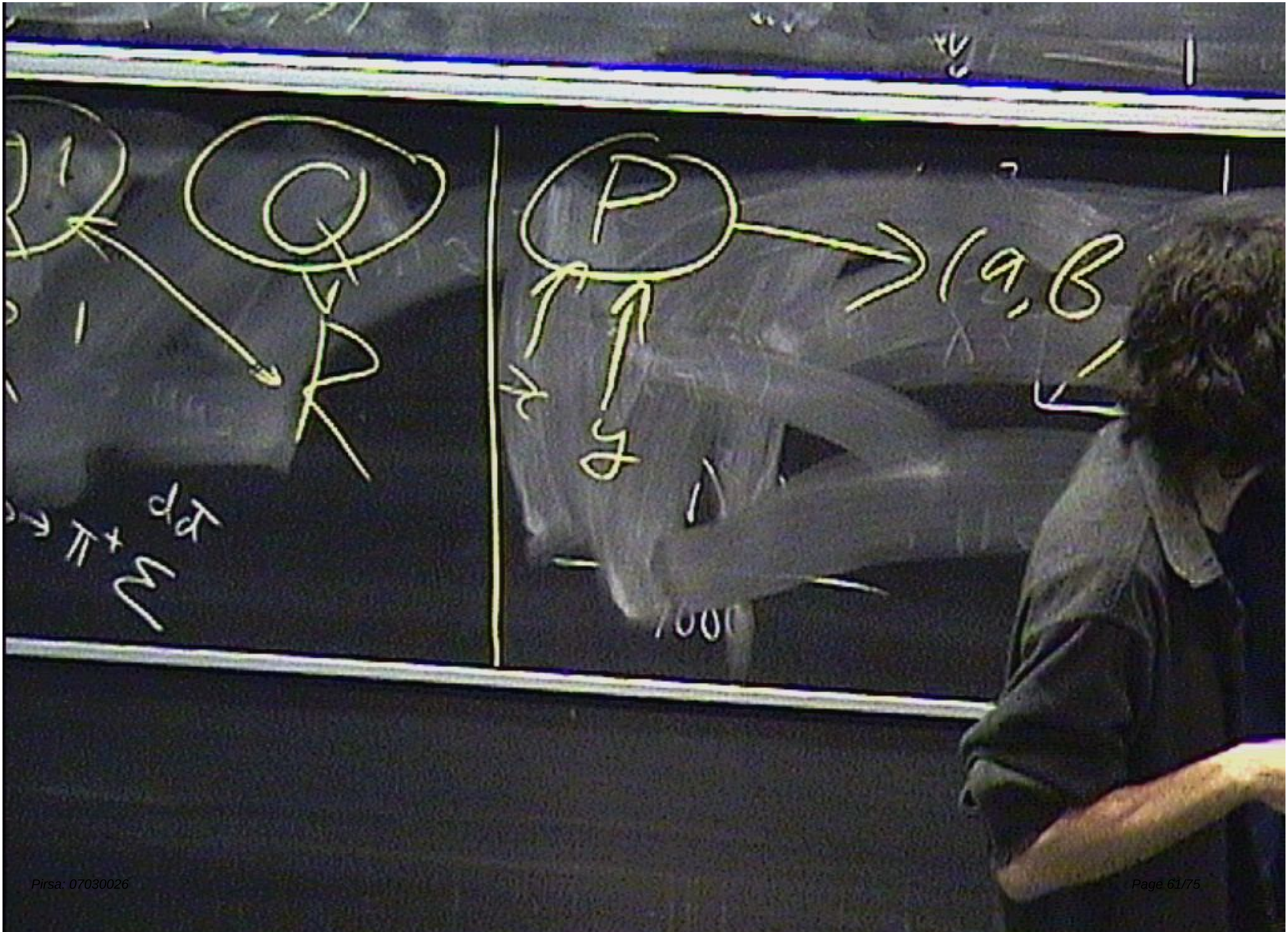
Complexity of the Search 1x1-Version

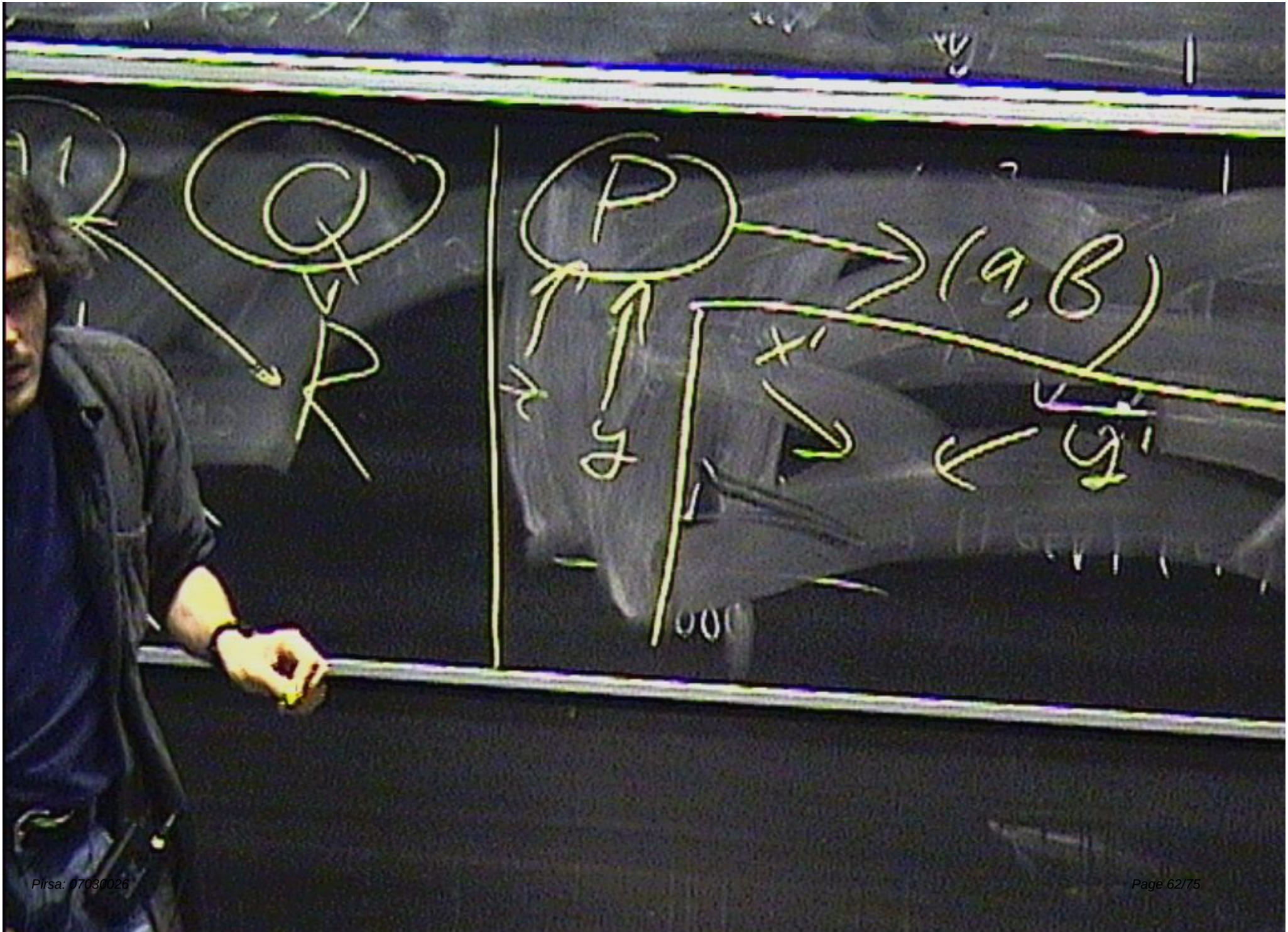


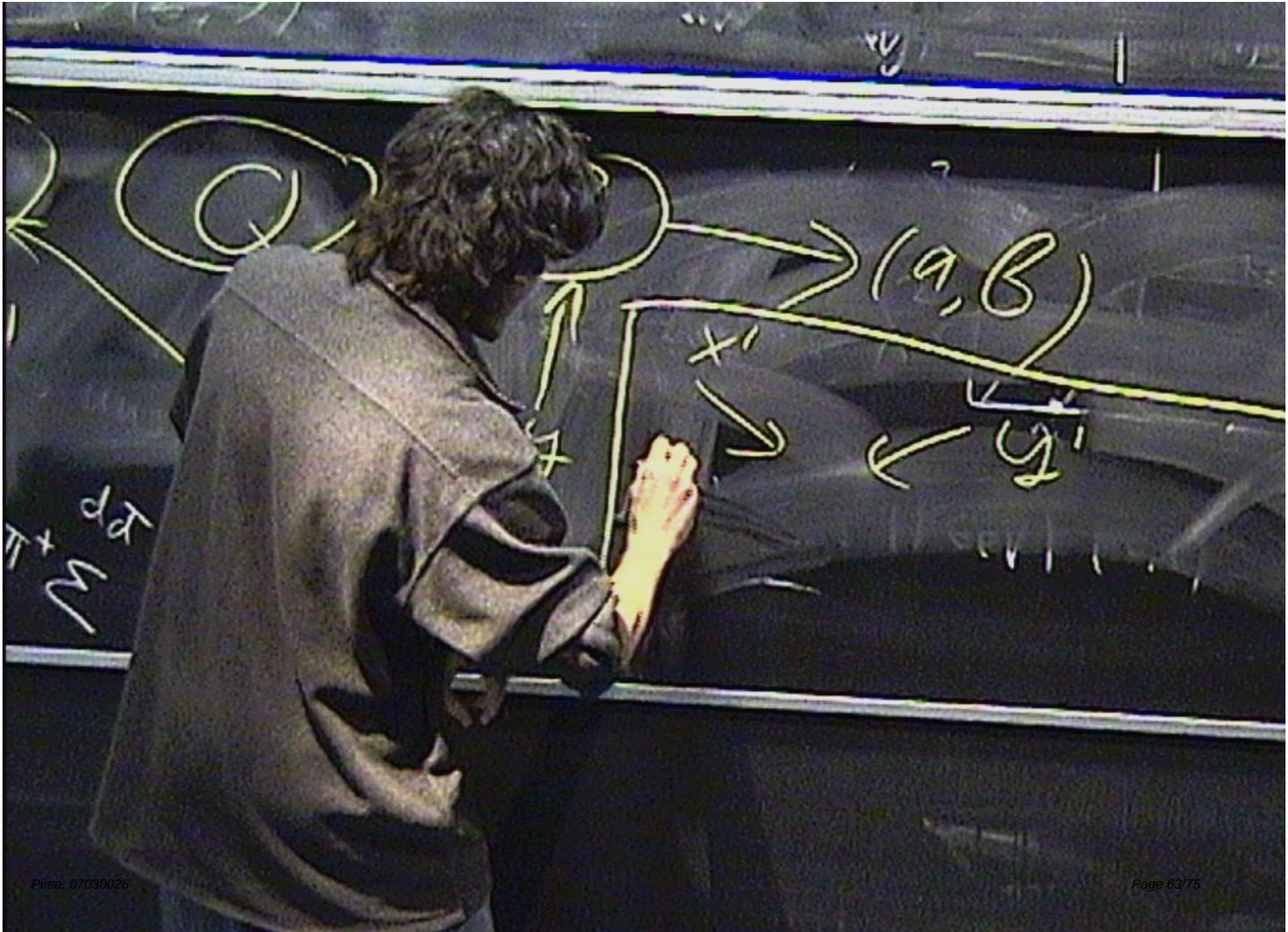
- To solve the problem with constant probability, we need $\Omega(n)$ bits of communication
- If we are allowed only k bits of communication, we can find one element of the intersection with probability $O\left(\frac{k}{n}\right)$,

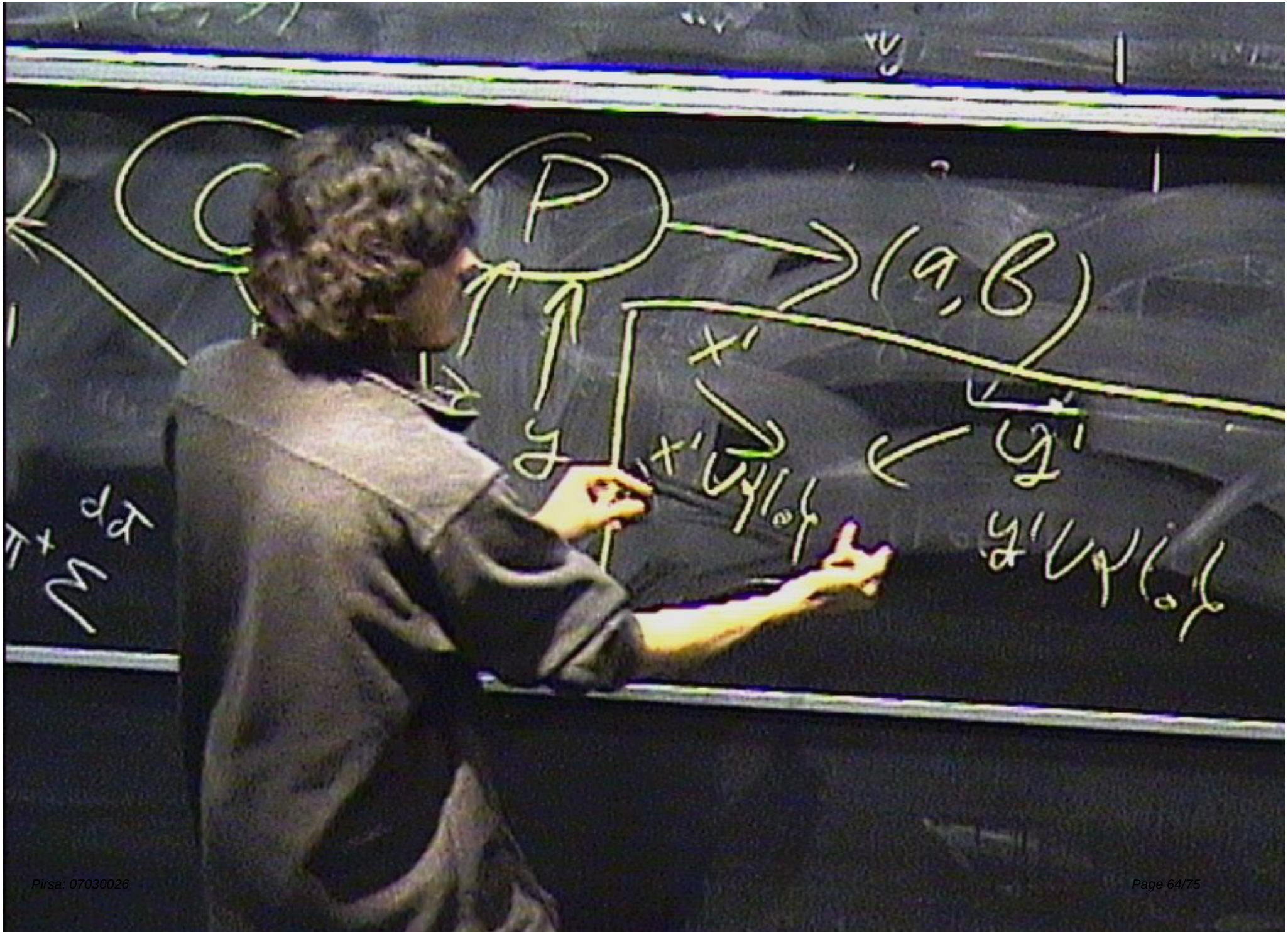
  our chances to find the both elements are $O\left(\left(\frac{k}{n}\right)^2\right)$

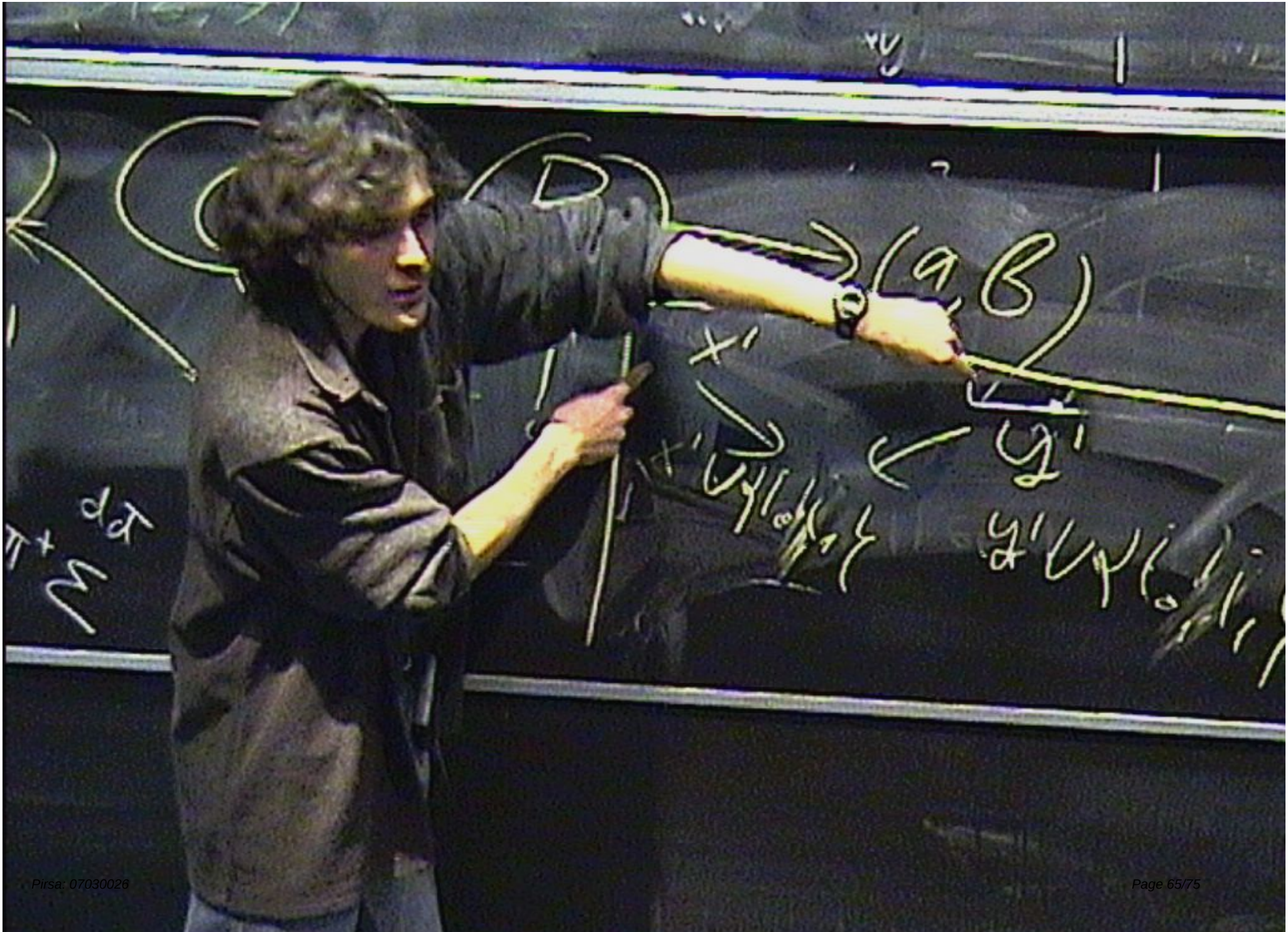

R
Q
D
(a,b)
dd
π+

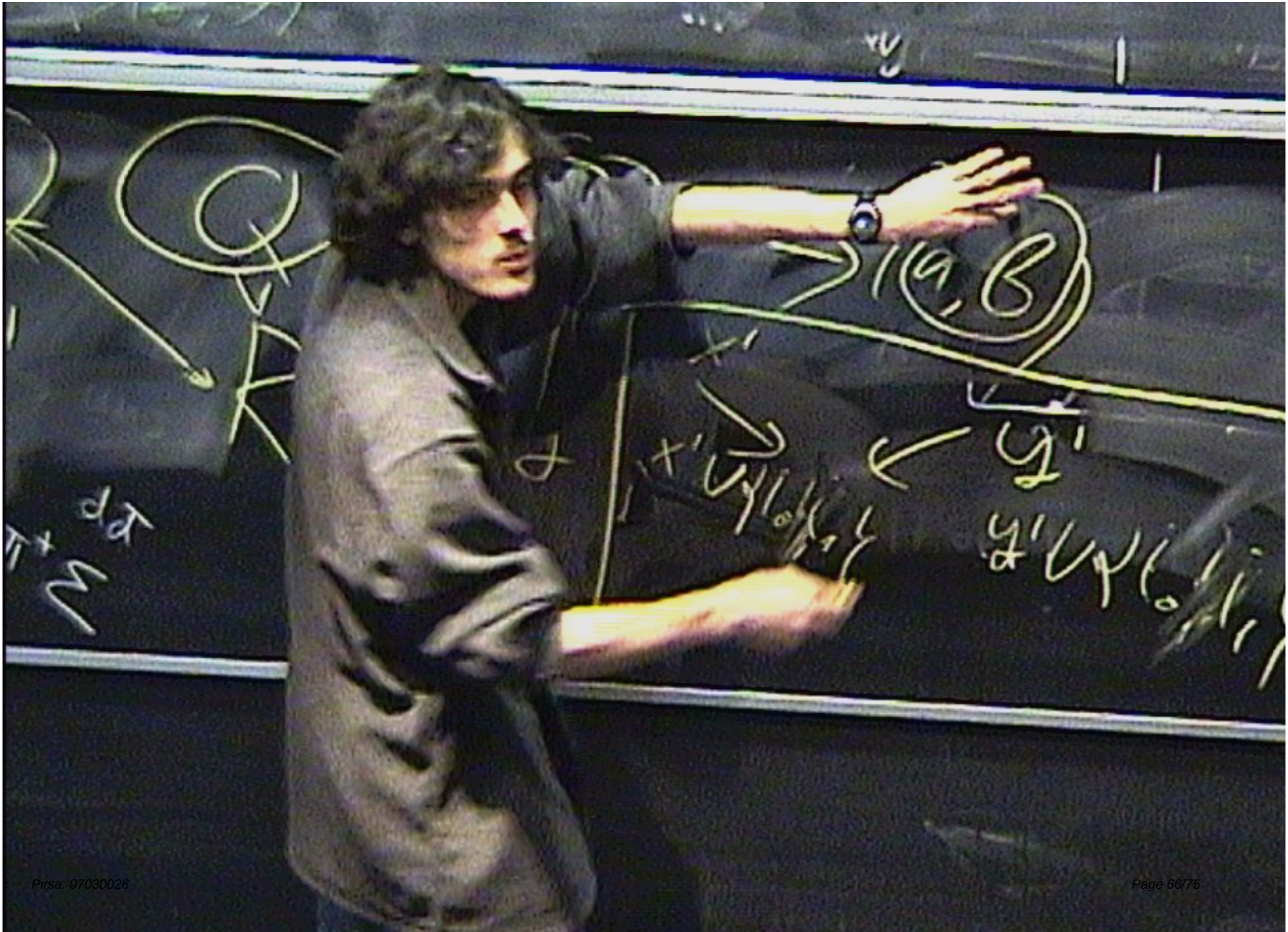

P
Q
R

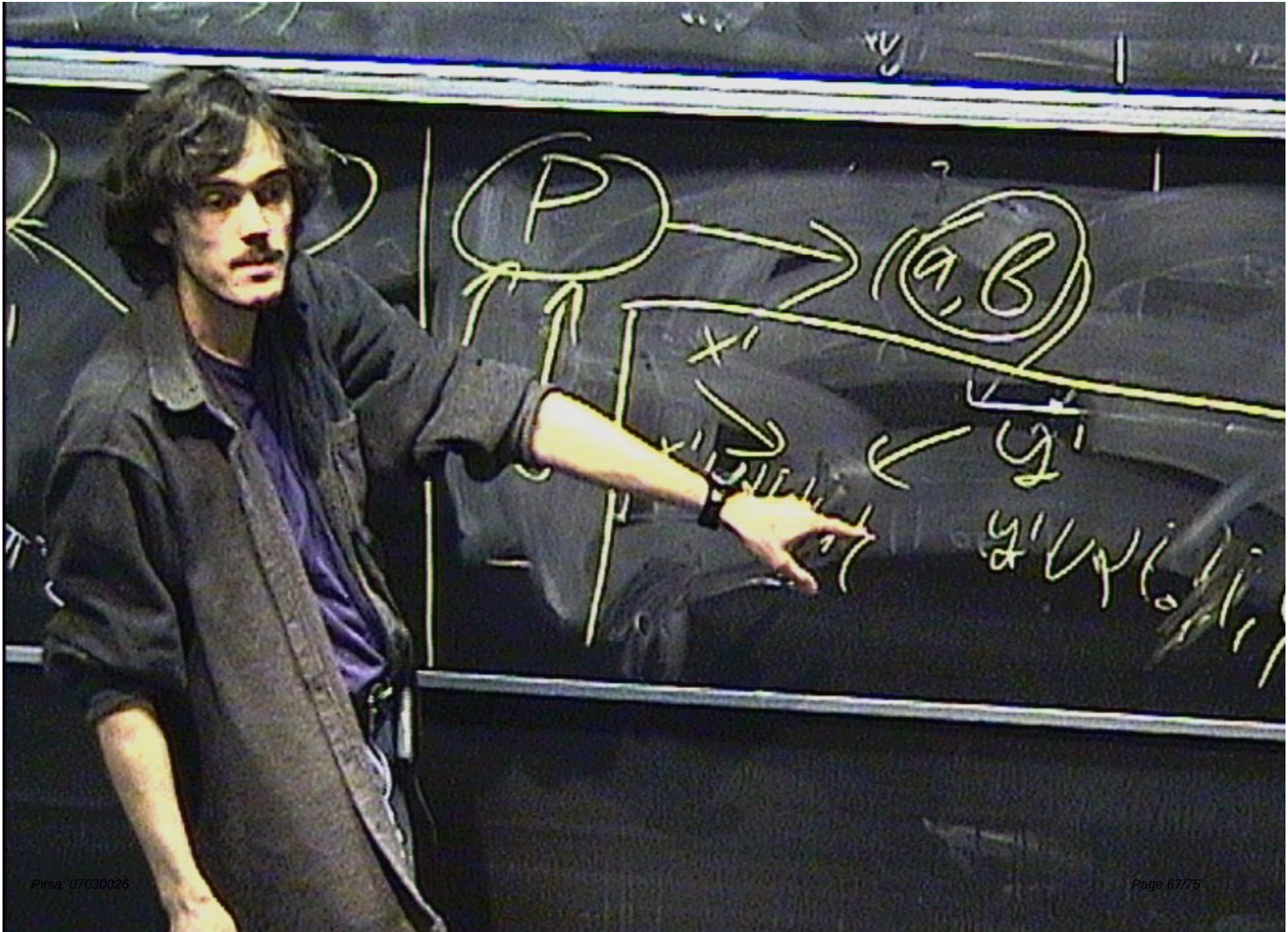

Q
R
P
(a, β)
x'
y
y'


(9,B)
x'
y'
dx


P
(q, p)


(a,b)




P
(q, β)
x'
y'

Open problems

- Is it possible to find a functional problem that requires exponentially more expensive protocol in R than in Q^1?
- How about simultaneous protocols?
- Generally speaking, give a separation that would logically imply as many known results as possible.

Open problems

- Is it possible to find a functional problem that requires exponentially more expensive protocol in R than in Q^1?

  How about simultaneous protocols?
- Generally speaking, give a separation that would logically imply as many known results as possible.

Open problems

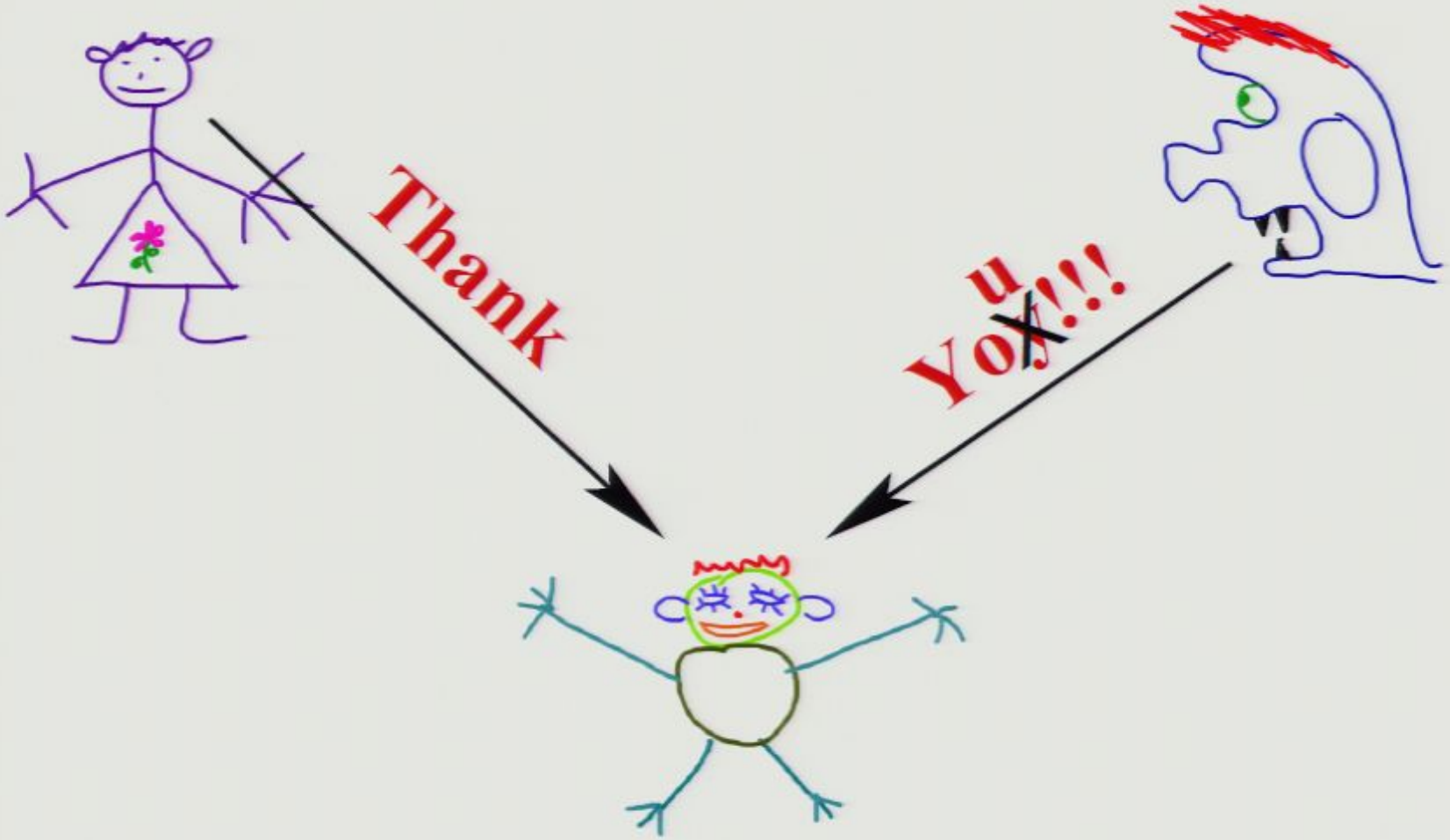
- Is it possible to find a functional problem that requires exponentially more expensive protocol in R than in Q^1?

  How about simultaneous protocols?
- Generally speaking, give a separation that would logically imply as many known results as possible.

Open problems

- Is it possible to find a functional problem that requires exponentially more expensive protocol in R than in Q^1?

  How about simultaneous protocols?
- Generally speaking, give a separation that would logically imply as many known results as possible.

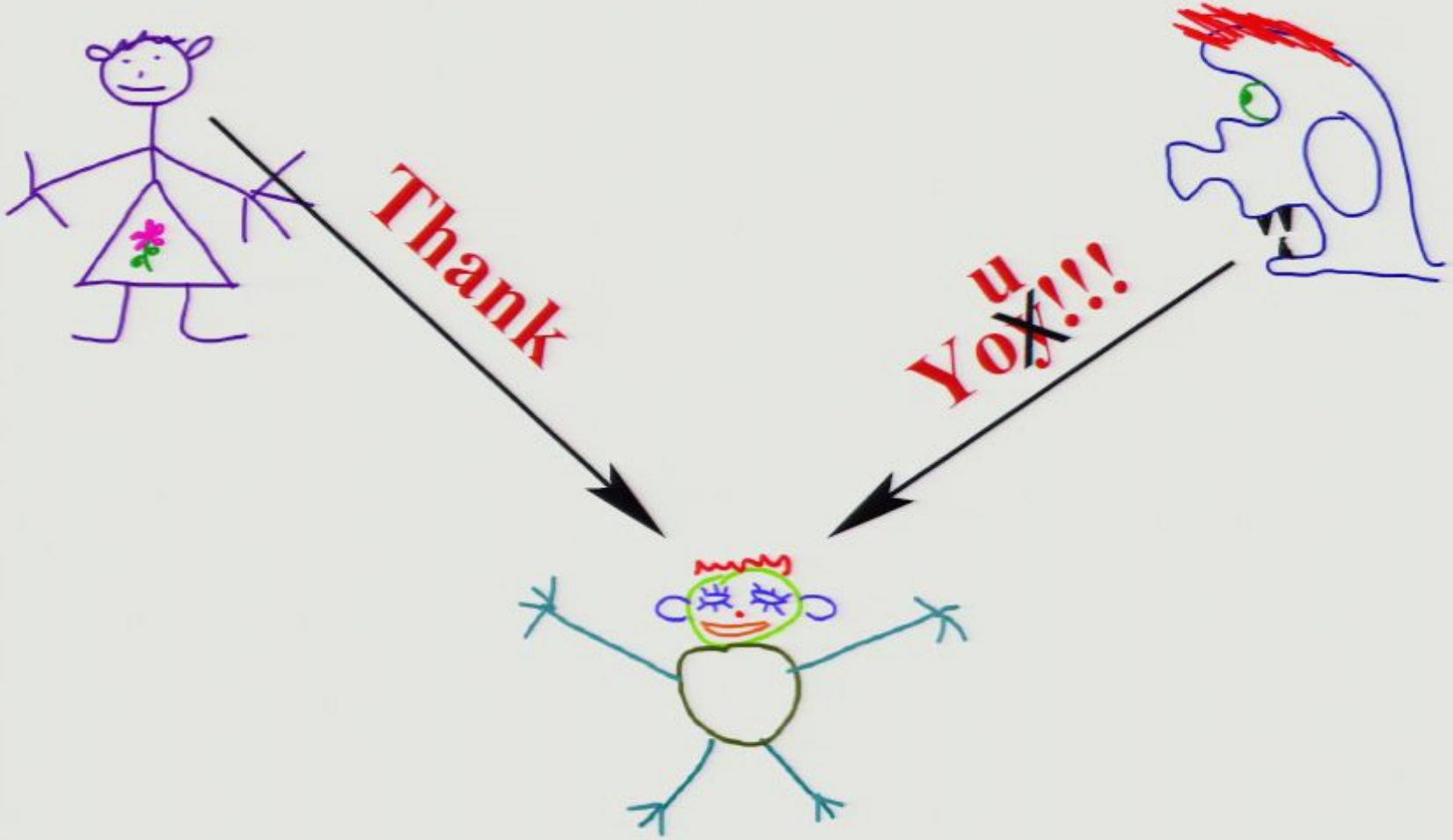

Thank
You!!!

Exponential Savings from Quantum Communication

- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)
- Bounded-error protocols, Q^1 vs. R^1, for a function (Gavinsky, Kempe, Kerenidis, Raz, and de Wolf, 2007)
- We show a relation that can be solved with bounded error by a Q^1-protocol that is exponentially more efficient than any R-protocol

Exponential Savings from Quantum Communication

- Zero-error protocols, Q vs. R and Q^1 vs. R^1 (Buhrman, Cleve, and Wigderson, 1998)
- Bounded-error protocols, Q vs. R (Raz, 1999)
- Bounded-error protocols, simultaneous protocols (Buhrman, Cleve, Watrous, and de Wolf, 2001)
- Bounded-error protocols, Q^1 vs. R^1, for a relation (Bar-Yossef, Jayram, and Kerenidis, 2004)
- Bounded-error protocols, Q^1 vs. R^1, for a function (Gavinsky, Kempe, Kerenidis, Raz, and de Wolf, 2007)
- We show a relation that can be solved with bounded error by a Q^1-protocol that is exponentially more efficient than any R-protocol


Thank
You!!!