

Title: Quantum Error Correction 10B

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URL: <http://pirsa.org/07030023>

Abstract: One- and two-way quantum capacities for the erasure channel, upper and lower bounds on the quantum capacities for the depolarizing channel, coherent information

Alice wishes to send k qubits to Bob with n physical qubits transmitted. Channel is $\mathcal{N}^{\otimes n}$. She wants fidelity $\rightarrow 1$ as $n \rightarrow \infty$.

Def. The one-way quantum channel capacity $Q_1(\mathcal{N}) = \sup k/n$, with sup taken over QECs, with fidelity $\rightarrow 1$ as $n \rightarrow \infty$.

The two-way quantum capacity $Q_2(\mathcal{N}) = \sup k/n$, with sup over all procedures involving n quantum transmissions, followed by A&B doing local gates & measurements and classical communications. (A encodes, before transmission)

Distillable entanglement $D_d(\rho)$ is $\sup k/n$, over 2-EDPs that take $\rho^{\otimes n}$ to $| \bar{\Phi} \rangle^{\otimes n}$ with fidelity $\rightarrow 1$ as $n \rightarrow \infty$

Thm: Forward classical communication does not help Q_1 .

Proof

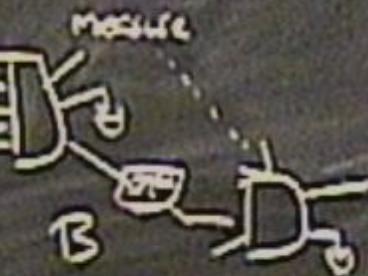
Ikm: Forward classical communication does not help Q_1 .

Proof sketch: $A - \overbrace{B}^{\text{measure}} \rightarrow C$

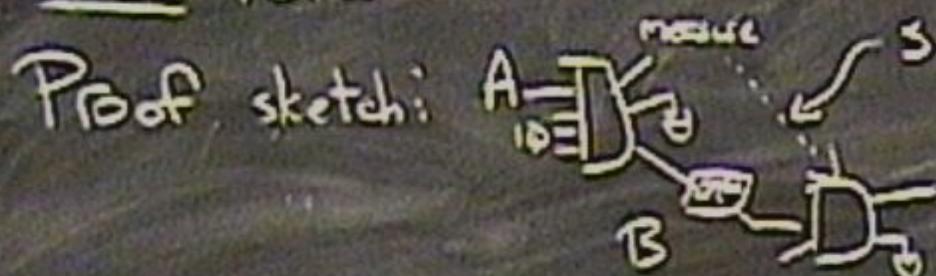


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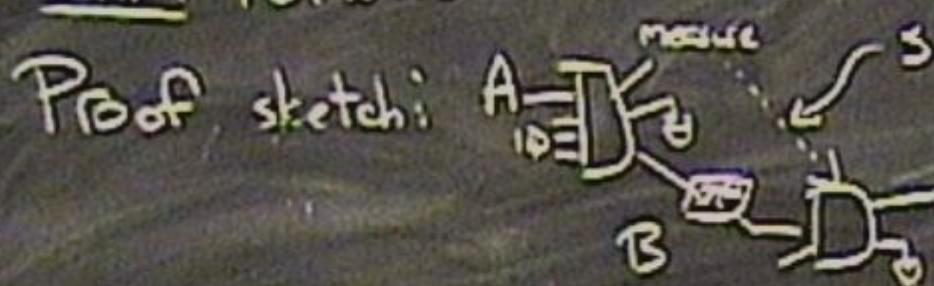
Proof sketch:



Idea: Forward classical communication does not help Q_1 .
 $S = \text{Alice's measurement result}$



Idea: Forward classical communication does not help Q_1 .



$s = \text{Alice's measurement result}$

\Rightarrow Given outcome s , Alice's input ρ

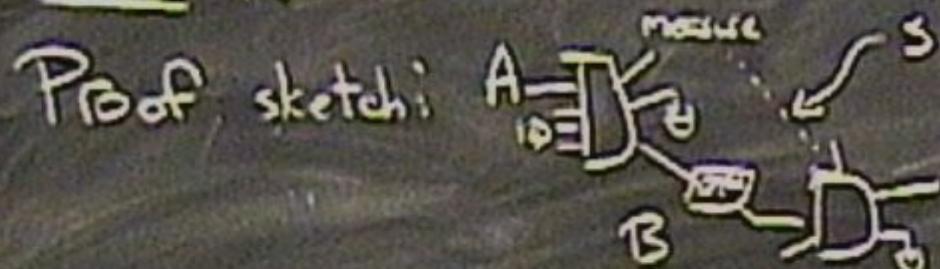
$\rightarrow O_s(\rho)$ (CP map)

Ikm: Forward classical communication does not help Q_1 .

Proof sketch:
 $s = \text{Alice's measurement result}$
 $\Rightarrow \text{Given outcome } s, A\& B \text{ input } \rho$
 $\rightarrow O_s(\rho) \text{ - map}$

Overall operation has high fidelity

Thm: Forward classical communication does not help Q₁.



S = Alice's measurement result

\Rightarrow Given outcome s, A's input ρ

$\rightarrow O_s(\rho)$ (CP map)

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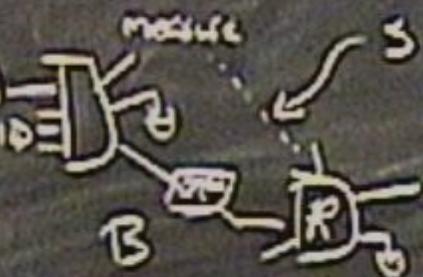
Proof sketch: $A \xrightarrow{\text{measure}} S$

$S = \text{Alice's measurement result}$

$\Rightarrow \text{Given outcome } s, A\text{'s input } \rho$
 $\rightarrow O_s(\rho) \quad (\text{C})$

Overall operation has high fidelity, averaged over $s \rightarrow$
 s' , call s' , for which the fidelity $R(O_s(\rho))$

Idea: Forward classical communication does not help Q_1 .

Proof sketch:  $s = \text{Alice's measurement result}$
 $\Rightarrow \text{Given outcome } s, \text{ Alice's input } \rho$
 $\rightarrow O_s(\rho) \text{ (CP map)}$

Overall operation has high fidelity averaged over $s \Rightarrow$ must be some s , call s' , for which the fidelity $F(R(O_{s'}(\rho)), \rho)$ is also near 1.

I think: Forward classical communication does not help Q_1 .

Proof sketch: $A \xrightarrow{\text{measure}} S \xrightarrow{\text{Alice's measurement result}} O_S(\rho) \xrightarrow{\text{(CP map)}} R \xrightarrow{\text{Bob's input}} \rho$

Overall operation has high fidelity, averaged over $S \Rightarrow$ must be some s , call s' , for which the fidelity $F(R(O_{s'}(\rho)), \rho)$ is also near 1.

Thm: Forward classical communication does not help Q_1 .

Proof sketch: A quantum circuit diagram showing a sequence of operations. It starts with a box labeled 'A' containing a unitary matrix, followed by a measurement operation labeled 'measure'. The outcome of this measurement is labeled 's'. After the measurement, there is a wire labeled 'B' containing a unitary matrix, followed by another unitary matrix labeled 'R'. The entire process is labeled $O_s(\rho)$ (CP map).

$s = \text{Alice's measurement result}$

\Rightarrow Given outcome s , A's input ρ

$\rightarrow O_s(\rho)$ (CP map)

Overall operation has high fidelity averaged over $s \Rightarrow$ must be some s , call s' , for which the fidelity $F(R(O_s(\rho)), \rho)$ is also near 1.

If fidelity = 1, O_s is unitary encoding

Ilm: Forward classical communication does not help Q_1 .

Proof sketch: $A \xrightarrow{\text{ID}} \boxed{K} \xleftarrow{\text{measure}} S$

$S = \text{Alice's measurement result}$

$\Rightarrow \text{Given outcome } s, A\text{'s input } \rho$

$\rightarrow O_s(\rho) \text{ (CP map)}$

Overall operation has high fidelity, averaged over $S \Rightarrow$ must be some s , call s' , for which the fidelity $F(R(O_{s'}(\rho)), \rho)$ is also near 1.

If fidelity = 1, $O_{s'}$ is unitary encryp. If fidelity is near 1,
 $O_{s'}$ is close to a un

Thm: Forward classical communication does not help Q_1 .

Proof sketch: $A \xrightarrow{\text{DE}} \xrightarrow{\text{measure}} S$

$S = \text{Alice's measurement result}$
 $\Rightarrow \text{Given outcome } s, A\text{'s input } \rho$
 $\rightarrow O_s(\rho) \text{ (CP map)}$

Overall operation has high fidelity, averaged over $s \in S$.
If fidelity = 1, O_s is unitary encoding. If fidelity is near 1,
there is O' st. O' is unitary encoding.

Thm: Forward classical communication does not help Q_1 .

Proof sketch: $A \xrightarrow{\text{unitary}} B \xrightarrow{\text{measure}} S$

$S = \text{Alice's measurement result}$

$\Rightarrow \text{Given outcome } s, A\text{'s input } \rho$
 $\rightarrow O_s(\rho) \text{ (CP map)}$

Overall operation has high fidelity, averaged over $S \Rightarrow$ must be some s , call s' , for which the fidelity $F(R(O_s(\rho)), \rho)$ is also near 1.

If fidelity = 1, O_s is unitary encoding. If fidelity is near 1,
there is O' st. O' is unitary encoding, $F(R(O'(\rho)), \rho)$ is close to 1.

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$$Q_2(\rho)$$

Alice wishes to send k qubits to Bob with a physical channel that transmits $n^{\otimes n}$. She wants fidelity $\rightarrow 1$ as $n \rightarrow \infty$.

Def: The one-way quantum channel capacity $Q_{\text{fw}}(\mathcal{N}) = \sup \frac{k}{n}$, with sup taken over QECs, with fidelity $\rightarrow 1$ as $n \rightarrow \infty$.

The two-way quantum capacity $Q_2(\mathcal{N}) = \sup \frac{k}{n}$, with sup over all procedures involving n quantum transmissions, followed by A&B doing local gates & measurements and classical communications. (A creates before transmission)

Distillable entanglement $D_d(\rho)$ is $\sup \frac{k}{n}$, over C-EDPs that take $\rho^{\otimes n}$ to $|E\rangle\langle E|$ with fidelity $\rightarrow 1$ as $n \rightarrow \infty$

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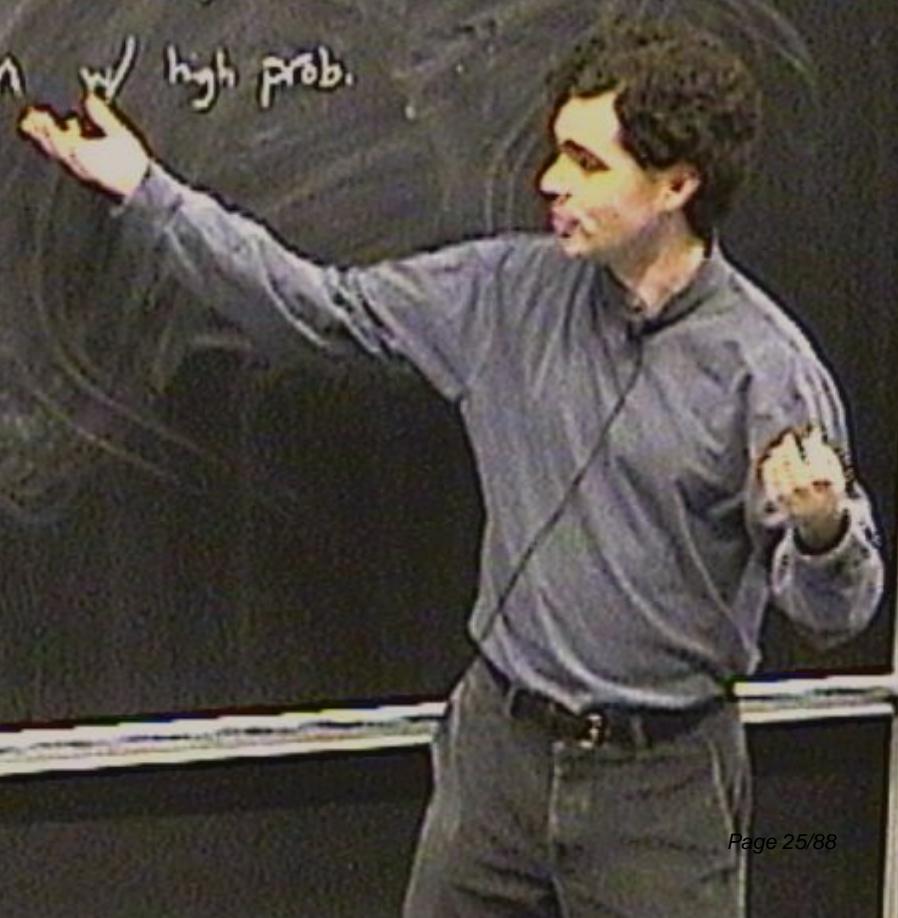
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$$Q_2(p)$$

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$$Q_2(p) \geq 1-p$$



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$Q_2(p) \geq 1-p$ (B tells A which are erased & they discard them)

$$Q_1(p) \geq$$

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Random stabilizer code w/ r generators.

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Random stabilizer code w/ r generators.

Any two errors EGF. What is the probability EGF have same syndrome?

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Random stabilizer code w/ r generators.

Any two errors $E \in F$. What is the probability $E \in F$ have same syndrome? = $\text{Prob}(E^T F \in N(s))$

(Bob learns which qubits are erased, Alice does not)

qubits erased is near p_n w/ high prob.

$Q_2(p) \geq 1-p$ (B tells A which are erased & they discard them)

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Random stabilizer code w/ r generators.

Any two errors EGF what is the probability EGF have same syndrome? = $\text{Prob.}(E^T F \in N(S)) = |N(S)| / |P_0| - 1$

Erasure channel: Qubit is erased w/ prob. p , left alone w/ prob. $1-p$.
(Bob learns which qubits are erased, Alice does not.)

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$$Q_1(p) \geq$$

Random stabilizer code w/ r generators.

Any two ^{distinct} errors EGF. What is the probability EGF have same syndrome?

$$\text{Prob}(E^i F^j | N(S)) = |N(S)| / (p_n^{r-1}) \approx 2^{n-r} / 2^n = 2^{-r}$$

Erasure channel: Qubit is erased w/ prob p , left alone w/ prob. $1-p$.

(Bob learns which qubits are erased, Alice does not)

qubits erased is near pN w/ high prob.

$Q_2(p) \geq 1-p$ (B talk A which are erased they discard them)

$Q_1(p) \geq$

Random stabilizer code w/ r gen.

Any two ^{distinct} errors $E \in F$. What is the prob $E \in F$ have same syndrome?

$$= \text{Prob}(E^T F \in N(S)) = 1 - \frac{\text{Prob}(E^T F \in N(S))}{2^{2n}} = 1 - \frac{2^{n-r}}{2^n} = 2^{-r}$$

Actual error E

Erasure channel: Qubit is erased w/ prob p , left alone w/ prob $1-p$.
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Random stabilizer code w/ r generators.

Any two ^{distinct} errors $E \in F$. What is the probability $E \in F$ have same syndrome?
 $= \text{Prob}(E^T F \cap N(S)) = |N(S)| / (p^r 1 - 1) \approx 2^{n-r} / 2^n = 2^{-r}$

Actual error E - What is prob that there is another $F \neq S$ same syndrome?

Erasure channel: Qubit is erased w/ prob p , left alone w/ prob $1-p$.

(Bob learns which qubits are erased, Alice does not)

qubits erased is near pN w/ high prob.

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Random stabilizer code w/ r generators.

Any two errors $E \in F$. What is the probability $E \in F$ have same syndrome?
 $= \text{Prob}(E^T F \cap N(S)) = |N(S)| / |P_n| - 1 = 2^{n-r} / 2^n = 2^{-r}$

Actual error E - What is prob that there is another $F \in \mathcal{F}$ w/ same syndrome? $4^{n-r} / 2^n < 1$
if $r > \log n$

(Bob learns which qubits are erased, Alice does not)

qubits erased is near p_n w/ high prob.

$$Q_2(p) \geq 1-p \quad (\text{B tells A which are erased so they discard them})$$

$$Q_1(p) \geq 1-2p$$

Random stabilizer code w/ r generators.

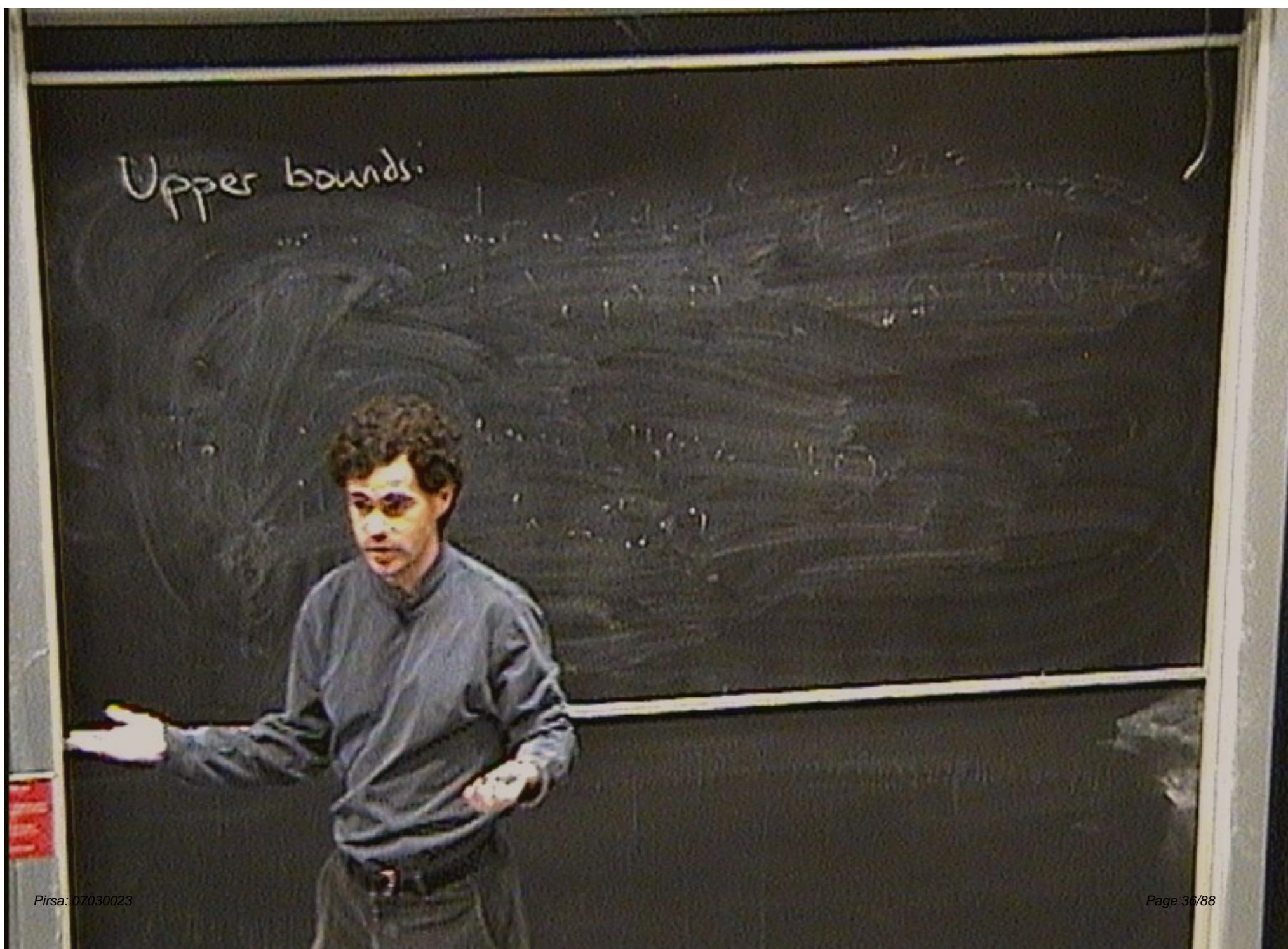
Any two errors $E \in F$. What is the probability $E \in F$ have same syndrome? $= \text{Prob}(E^T F \cap N(S)) = |N(S)| / |P_n| - 1 \approx 2^{n-r} / 2^n = 2^{-r}$

Actual error E - What is prob that there is another $F \cup S$ syndrome?

$$4^{m/r} < 1 \quad \text{if } r > 2m$$

$$\text{Choose } r = 2pn \Rightarrow k = n - 2pn \Rightarrow \frac{k}{n} = 1 - 2p$$

Upper bounds:



Upper bounds:

Mix. I channel w/ π : M_{π}^{-1}



Upper bounds:

$$\text{Mix. I channel w/ } \pi : M_1 - \varrho M + (1-\varrho)I$$

$$\text{Claim: } Q(n) \leq \varrho Q(n) + (1-\varrho)Q(I) \quad \text{for } Q_1/Q_2$$

Upper bounds:

$$\text{Mix. I channel w } \pi_1 : M_1 = q\pi + (1-q)I$$

Claim: $Q(m) \leq qQ(n) + (1-q)Q(I)$ for Q_1/Q_2

Proof: Suppose we want to send k qubits through π_1 . First send
 $(1-q)k$ randomly chosen qubit for π_1

Upper bounds:

$$\text{Mix. I channel } \pi_{\lambda} : M_{\lambda} = q\lambda + (1-q)\mathbb{I}$$

$$\text{Claim } Q(n) \leq qQ(n) + (1-q)Q(\mathbb{I}) \quad \text{for } Q_1/Q_2$$

Proof: Suppose we want to send k qubits through π_{λ} . First encode $(1-q)k$ randomly chosen qubit for π_{λ} , using $(1-q)k/Q(n)$

Upper bounds:

$$\text{Mix. I channel w/ } \pi_1 : M_1 - q\pi_1 + (1-q)I$$

Claim: $Q(m) \leq qQ(n) + (1-q)Q(I)$ for Q_1/Q_2

Proof: Suppose we want to send k qubits through π_1 . First encode $(1-q)k$ randomly chosen qubit for π_1 , using $(1-q)k/Q(n)$ physical qubits

Upper bounds:

$$\text{Mix. I channel w/ } \pi : M_q - qM + (1-q)I$$

Claim: $Q(m) \leq qQ(n) + (1-q)Q(I)$ for Q_1/Q_2

Proof: Suppose we want to encode m qubits through π . First encode $(1-q)k$ randomly chosen qubits using $(1-q)k/Q(n)$ physical qubits. Then encode qk additional qubits giving $\frac{(1-q)k}{Q(n)}$ qubits, which we encode.

for $M \Rightarrow$

Upper bounds:

$$\text{Mix. I channel w/ } \pi_1 : M_q - q\pi_1 + (1-q)\mathbb{I}$$

Claim: $Q(m) \leq qQ(n) + (1-q)Q(\mathbb{I})$ for Q_1/Q_2

Proof: Suppose we want to send k qubits through π_1 . First encode

$(1-q)k$ randomly chosen qubit for π_1 using $(1-q)k/Q(n)$ physical qubits

- qk additional qubits gives $\left(qk + \frac{(1-q)k}{Q(n)}\right)$ qubits, which we encode

for $M_q \rightarrow \left(\frac{qk + \frac{(1-q)k}{Q(n)}}{Q(n)}\right)$ physical qubits!

Upper bounds:

$$\text{Mix. I channel w/ } \pi_1 : M_1 - qM + (1-q)I$$

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$(1-q)k$ randomly chosen qubit for π_1 using $(1-q)k/Q(n)$ qubits

- qk additional qubits gives $\left(qk + \frac{(1-q)k}{Q(n)}\right)$ qubits, which we can

for $M_1 \geq \frac{\left(qk + \frac{(1-q)k}{Q(n)}\right)}{Q(n)}$ physical qubits

Upper bounds:

$$\text{Mix. I channel } \rightarrow \mathcal{N} : M_1 - q\mathcal{N} + (1-q)\mathbb{I}$$

Claim: $Q(n) \leq qQ(n) + (1-q)Q(\mathbb{I})$ for Q_1/Q_2

Proof: Suppose we want to send k qubits through \mathcal{N} .

First encode $\mathcal{N} \rightarrow M_1 - \frac{k}{Q(n)}$ physical qubits. Then a fraction $1-f$ of qubits are encoded for $\mathcal{N} \rightarrow \left[\frac{1-f}{Q(n)} + q\right] \frac{k}{Q(n)}$ physical qubits.

Decode outer layer $\rightarrow (1-f)\left(\frac{k}{Q(n)}\right)$ qubits experienced \mathbb{I} , $(\frac{k}{Q(n)})$ unencoded.

Upper bounds:

$$\text{Mix. I channel } \nu \pi : M_2 - \epsilon M + (1-\epsilon)I$$

Claim: $Q(m) \leq \epsilon Q(n) + (1-\epsilon)Q(I)$ for Q_1/Q_2

Proof: Suppose we want to send k qubits through π .

First encode $m = \frac{k}{Q(n)}$ physical qubits. Then $\frac{k}{Q(n)}$ qubits are encoded for $\pi \rightarrow \left[\frac{1-\epsilon}{Q(n)} + \epsilon \right] \frac{k}{Q(n)}$.
ie. $\frac{k}{n}$ qubits are experienced π .

Decode outer layer $\Rightarrow (1-\epsilon)\left(\frac{k}{Q(n)}\right)$ qubits experienced I ,

i.e. all experienced $m \Rightarrow k$ qubits.

$$\frac{k}{n} \leq$$

Upper bounds:

$$\text{Mix. I channel } \pi : M_q = q\mathcal{N} + (1-q)\mathbb{I}$$

Claim: $Q(m) \leq qQ(n) + (1-q)Q(\mathbb{I})$ for Q_1/Q_2

Proof: Suppose we want to send k qubits through π .

First encode off $m \cdot \frac{k}{Q(m)}$ physical qubits. Then a fraction $1-e$ of qubits are encoded for $\pi \rightarrow \left[\frac{1-e}{Q(n)} + q\right] \frac{k}{Q(m)}$ physical qubits.

Decode outer layer $\Rightarrow (1-e)\left(\frac{k}{Q(m)}\right)$ qubits experienced \mathbb{I}_2 , $\frac{k}{Q(m)}$ experienced \mathbb{I}_1

i.e. all experienced $m \Rightarrow k$ qubits using $n = Q(n)$

$$\frac{k}{n} \leq Q(n) \Leftrightarrow \frac{Q(m)}{(1-e) + q\left(\frac{k}{Q(m)}\right)} \leq 1$$

Upper bounds:

$$\text{Mix. I channel } \nu \pi : M_2 - q\pi + (1-q)\mathbb{I}$$

Claim: $Q(m) \leq qQ(n) + (1-q)Q(\mathbb{I})$ for Q_1/Q_2

Proof: Suppose we want to send k qubits through π .

First encode after $m = \frac{k}{(1-q)}$ physical qubits. Then a fraction $1-q$ of qubits are encoded for $n \rightarrow \left[\frac{1-q}{Q(n)} + q \right] \frac{k}{Q(m)}$ physical qubits.

Decode outer layer $\Rightarrow (1-q)(\frac{k}{Q(m)})$ qubits experienced \mathbb{I} , $\frac{k}{Q(m)}$ experie-

i.e. all experienced $m \Rightarrow k$ qubits using $n = \frac{(1-q)(k)}{Q(n) Q(m)}$

$$\frac{k}{n} \leq Q(n) \Leftrightarrow \frac{Q(m)}{(1-q) \cdot q(Q(m))} \leq 1$$

Upper bounds:

$$\text{Mix. I channel w/ } \pi : M_2 - qM + (1-q)I$$

Claim: $Q(n) \leq qQ(n) + (1-q)Q(I)$ for Q_1/Q_2

Proof: Suppose we want to send k qubits through π .
First encode off $M - \frac{k}{Q(n)}$ physical qubits. Then a fraction $1-\epsilon$ of qubits are encoded for $\pi \rightarrow \left[\frac{1-\epsilon}{Q(n)} + q\right] \frac{k}{Q(n)}$ physical qubits experienced π .

Decode outer layer $\rightarrow (1-\epsilon)\left(\frac{k}{Q(n)}\right)$ qubits experienced I , $\frac{k}{Q(n)}$ experienced π .
ie. all experienced $\pi \rightarrow k$ qubits using $n = \frac{(1-\epsilon)k}{Q(n)(1-\epsilon)}$

$$\frac{k}{n} \leq Q(n) \Leftrightarrow \frac{Q(n)}{(1-\epsilon) + q\left(\frac{k}{Q(n)}\right)} \leq 1$$

Upper bounds:

$$\text{Mix. I channel } \nu \mathcal{N} : M_q - q\mathcal{N} + (1-q)\mathbb{I}$$

Claim: $Q(m) \leq qQ(n) + (1-q)Q(\mathbb{I})$ for Q_1/Q_2 (if $Q(n) > 0$)

Proof: Suppose we want to send k qubits through \mathcal{N} .

First encode $m \rightarrow \frac{k}{Q(n)}$ physical qubits. Then a fraction $1-\epsilon$ of qubits are encoded for $n \rightarrow \left[\frac{1-\epsilon}{Q(n)} + \epsilon\right] \frac{k}{Q(n)}$ physical qubits.

Decode outer layer $\rightarrow (1-\epsilon)\left(\frac{k}{Q(n)}\right)$ qubits experienced \mathbb{I} , $\frac{k}{Q(n)}$ experienced \mathcal{N} .

i.e. all experienced $n \rightarrow k$ qubits using $n = \frac{(1-\epsilon)\frac{k}{Q(n)}}{Q(n) - Q(\mathbb{I})}$

$$\frac{k}{n} \leq Q(n) \Leftrightarrow \frac{Q(n)}{(1-\epsilon) + \epsilon\left(\frac{k}{Q(n)}\right)} \leq 1$$

Erasure channel: Qubit is erased w/ prob. p , left alone w/prob. $1-p$.
(Bob learns which qubits are erased, Alice does not.)

qubits erased is near pN w/ high prob.

$$Q_2(p) \approx 1-p \quad (\text{B tells A which are erased \& they discard them})$$

$$Q_1(p) \geq 1-2p$$

Random stabilizer code w/ r generators.

Any two errors $E \& F$ what is the probability $E \& F$ syndrome?

$$= \text{Prob}(E^T F^T N(S)) = |N(S)| / |P_n| - 1 \approx 2^{2n-r}$$

Actual error E - What is prob that there is another F w/ same syndrome?

$$\text{Choose } r = 2pn \Rightarrow k = n - 2pn \Rightarrow \frac{k}{n} = 1 - 2p$$

$$\begin{aligned} <1 \\ > 2pn \end{aligned}$$

Erasure channel: Qubit is erased w/ prob. p , left alone w/ prob. $1-p$.
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qubits erased is near pN w/ high prob.

$Q_2(p) \approx 1-p$ (B tells A which are erased & they discard them)

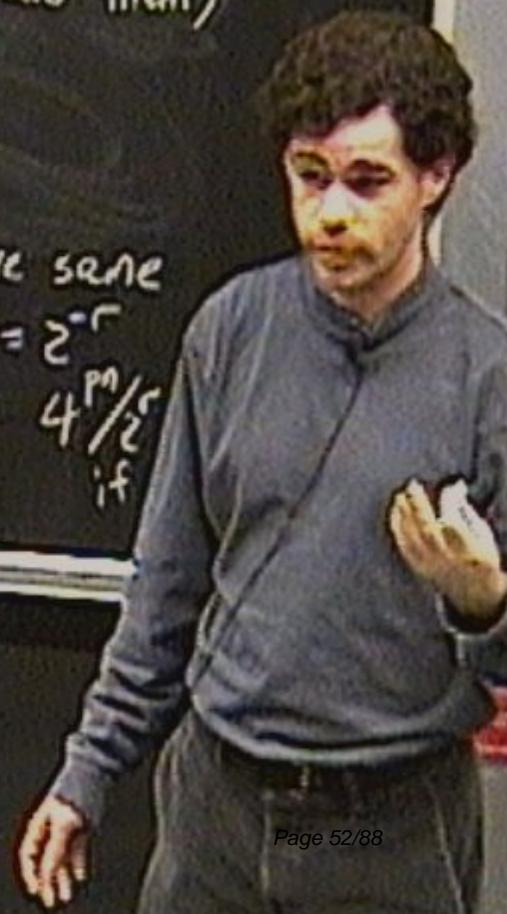
$Q_1(p) \geq 1-2p$ (Upper bound)

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Any two errors E&F. What is the probability E&F have same syndrome?
 $= \text{Prob}(E^T F \in N(S)) = |N(S)| / (|P_n| - 1) \approx 2^{2n-r} / 2^n = 2^{-r}$

Actual error E - What is prob that there is another F w/ same syndrome?

Choose $r=2pn \Rightarrow k=1-2pn \Rightarrow \frac{k}{n} = 1-2p$



Erasure channel: Qubit is erased w/ prob. p , left alone w/ prob. $1-p$.
(Bob learns which qubits are erased, Alice does not.)

qubits erased is near pN w/ high prob.

$$Q_2(p) \approx 1-p \quad (\text{Bob tells Alice which are erased \& they discard them})$$

$$Q_1(p) \geq 1-2p \quad (\text{Upper bound. No cloning} \Rightarrow Q_1(\frac{1}{2})=0)$$

Random stabilizer code w/ r generators.

Any two errors E & F. What is the probability E & F have same syndrome?
 $= \text{Prob}(E^T F \in N(S)) = |N(S)| / (|P_n| - 1) \approx 2^{2n-r} / 2^n = 2^{-r}$

Actual error E - What is prob that there is another F w/ same syndrome?

$$\text{Choose } r=2pn \Rightarrow k=1-2pn \Rightarrow \frac{k}{n} = 1-2p$$

Depolarizing channel:



- depends on precise measurement
Example: 4 patient EPP pairs

Measure $\frac{X}{Z} \cdot \frac{Y}{Z}$ or $\frac{1}{S} \cdot \frac{1}{S}$

) "correct" to get 1 good pair
) Detect an error - keep 4th pair
Don't detect error - keep remaining decimal
pair from lit 3.

Depolarizing channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

Depends on previous measurement outcome
Example: 4 patient EPP pairs, 1st > 2nd
Measure XXX or $1st > 2nd$
 ZZZ

correct to get 1 good pair
reject as error - keep 4th pair
select error - keep remaining desired
pair from 1st 3.

Depolarizing channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$
⇒ entangled state $(1-p)|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|X^+\rangle\langle X^+| + \frac{p}{3}|Y^+\rangle\langle Y^+| + \frac{p}{3}|Z^+\rangle\langle Z^+|$

depends on project measurement order
Example: 4 qubit EPR pair, 1 error
Measure $XXX \text{ or } 1st > ZZZ \text{ or } 1st >$

correct to get 1 good pair
+ an error - keep 4th pair
Don't select error - keep remaining needed
pair for fit 3.

Depolarizing channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

⇒ entangled state $(1-p)|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^-| + \frac{p}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^-|$

$|\Psi^\pm\rangle = |00\rangle \pm |11\rangle$, $|\Psi^\mp\rangle = |01\rangle \pm |10\rangle$ "Werner state"

depends on previous measurement outcome

Example: 4 qubit EPR pairs, 1 error, A & B want to "correct" to get 1 good pair
 Measure XX or YY on 1st 2 qubits (on each side)

$\left. \begin{array}{l} \text{Detect an error - keep 4th pair} \\ \text{Don't detect error - keep remaining pair from it} \end{array} \right\}$

Depolarizing channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$
⇒ entangled state $(1-p)|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^-| + \frac{p}{3}|\Psi^+\rangle\langle\Psi^-| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^+|$
 $|\Psi^\pm\rangle = |00\rangle \pm |11\rangle, |\Psi^\mp\rangle = |01\rangle \pm |10\rangle$ "Werner state"

Q₁: Use random stabilizer code, with r generators.

depends on precise measurement
Example: 4 qubit EPR pairs,
Measure XXXX or 1st > 2nd (or vice versa)

+ to get 1 good pair
error - keep 4th pair
error - keep remaining needed
pair from 1st 3.

Depolarizing channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$
 \Rightarrow entangled state $(1-p)|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|X\Psi^+\rangle\langle X\Psi^+| + \frac{p}{3}|Y\Psi^+\rangle\langle Y\Psi^+| + \frac{p}{3}|Z\Psi^+\rangle\langle Z\Psi^+|$
 $|\Psi^+\rangle = |00\rangle \pm |11\rangle, |\Psi^-\rangle = |01\rangle \pm |10\rangle$ "Werner state"

Q1: Use random stabilizer code, with r generators.
 $\text{Prob}(\text{another error has same syndrome as } E) \leq n_{\text{typ}}/2^r - n_{\text{typ}}$

depends on previous measurement outcome
 Example: 4 qubit EPR pairs, 1 error, A & B want it to "correct" to get 1 good pair
 Measure $XXXZ$ or $1st> qubits (\text{on each side}) \left. \begin{array}{l} \text{Detect an error - keep 4th pair} \\ \text{Don't detect error - keep remaining pair from 1st} \end{array} \right\}$

Depolarizing channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$
 \Leftrightarrow entangled state $(1-p)|\Phi^+\rangle\langle\Phi^+| + \frac{p}{3}|\Phi^-\rangle\langle\Phi^-| + \frac{p}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^-|$
 $|\Phi^\pm\rangle = |00\rangle \pm |11\rangle, |\Psi^\pm\rangle = |01\rangle \pm |10\rangle$ "Werner state"

Q1: Use random stabilizer code, with r generators.
 Prob (another error has same syndrome as E) $\leq \mathcal{N}_{typ}/2^r$ \mathcal{N}_{typ} = # errors in the typical set

depends on genuine measurement outcome
 Example: 4 qubit EPR pairs, 1 error, A & B want to "correct" to get 1 good pair
 Measure XXX or $1st >$ 2 bits (in each side) } Detect an error - keep 4th pair
 Measure ZZZ or $1st >$ 2 bits (in each side) } Don't detect error - keep remaining 3rd pair from 1st 3.

Depolarizing channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

\Rightarrow entangled state $(1-p)|\Phi^+\rangle\langle\Phi^+| + \frac{p}{3}|\Phi^-\rangle\langle\Phi^-| + \frac{p}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^-|$

$|\Phi^\pm\rangle = |00\rangle \pm |11\rangle, |\Psi^\pm\rangle = |01\rangle \pm |10\rangle$ "Werner state"

Q₁: Use random stabilizer codes with r generators.
 Prob (another error has same syndrome as E) $\leq \mathcal{N}_{typ}/2^r$ \mathcal{N}_{typ} = # errors in the typical set

$\frac{1}{n} \log |\mathcal{N}_{typ}| = h(p) + p \log 3, h(p) = -p \log p - (1-p) \log(1-p)$

1-EDP detects E iff QECC detects $E \Leftrightarrow E \in N(s) \setminus S$

Stabilizer 2-EDP: Adaptive stabilizer code whose error depends on previous measurement outcomes.

Example: 4 qubit EPP pass, 1 error, ABB went to "correct" to measure XXX or 1st > qubits (on each side)

{ Detect an error
Don't select error -

Depolarizing channel: $N(p) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$
 \Rightarrow entangled state $(1-p)|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^-| + \frac{p}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^-|$
 $|\Psi^+\rangle = |00\rangle \pm |11\rangle, |\Psi^-\rangle = |01\rangle \pm |10\rangle$ "Werner state"

Q: Use random stabilizer code, with r generators.
 Prob (another error has same syndrome as E) $\in \mathcal{N}_{typ}/2^r$ \mathcal{N}_{typ} = # errors in the typical set
 $\frac{1}{n} \log |\mathcal{N}_{typ}| = h(p) + p \log 3, h(p) = -p \log p - (1-p) \log (1-p)$
 $\frac{1}{n} \log |\mathcal{N}_{typ}| \sim n[h(p) + p \log 3], \mathcal{N}_{typ} < 2^n$ when $r > n[h(p) + p \log 3]$
 $\Rightarrow \mathcal{N}_{typ} \sim 2^{n[h(p) + p \log 3]}, \Rightarrow \frac{k}{n} \leq 1 - h(p) - p \log 3$

1-EDP detects E iff QECC detects $E \Leftrightarrow E \in N(S) \setminus S$

Stabilizer 2-EDP: Adaptive stabilizer code whose choice of generator depends on previous measurement outcomes

Example: 4 qubit EPR pair, 1 error, A&B want to "correct" to get measure XXX or 1st > qubits (on each side)

{ Detect an error - keep from
Don't detect error - keep from pair from

Depolarizing channel: $\mathcal{N}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

\Rightarrow entangled state $(1-p)|\Psi^+\rangle\langle\Psi^+| + \frac{p}{3}|X^+\rangle\langle X^+| + \frac{p}{3}|\Psi^-\rangle\langle\Psi^-| + \frac{p}{3}|Y^+\rangle\langle Y^+|$

$|\Psi^\pm\rangle = |00\rangle \pm |11\rangle, |\Psi^\mp\rangle = |01\rangle \pm |10\rangle$ "Werner state"

Q₁: Use random stabilizer code, with r generators.

Prob (another error has same syndrome as E) $\leq n_{typ}/2^r$ n_{typ} = # errors in the typical set

$\frac{1}{n} \log |n_{typ}| = h(p) + p \log 3, h(p) = -p \log p - (1-p) \log (1-p)$

$\frac{1}{n} \log |n_{typ}| \sim 2^{n(h(p) + p \log 3)}$, $n_{typ} < 2^r$ when $r > n[h(p) + p \log 3]$

$\Rightarrow n_{typ} \sim 2^{n(h(p) + p \log 3)}, \frac{k}{n} \leq 1 - h(p) - p \log 3 \leq Q_1(p)$

1-EDP detects E iff QECC detects E $\Leftrightarrow E \in N(S) \setminus S$

Stabilizer 2-EDP: Adaptive stabilizer code where choice of generators depends on previous measurement outcomes

Example: 4 qubit EPR pairs, 1 error, A&B want to "correct" to get 4 pure XXXX or 1st > qubits (on each side)

{ Detect an error - keep pair from it }
Don't select error - keep pair from it

Depolarizing channel:

$$\frac{1}{3}X\rho X + \frac{1}{3}Y\rho Y + \frac{1}{3}Z\rho Z$$

\Leftrightarrow entangled state $(1-p)|\Phi^+\rangle\langle\Phi^+| + p|\Psi^+\rangle\langle\Psi^+| + p|\Psi^-\rangle\langle\Psi^-|$

$$|\Phi^\pm\rangle = |00\rangle \pm |11\rangle, |\Psi^\pm\rangle = |01\rangle \pm |10\rangle \quad \text{"Werner state"}$$

Q_1 : Use random stabilizer code, with r generators.

Prob (another error has same syndrome as E) $\leq \mathcal{N}_{typ}/2^r$ $\mathcal{N}_{typ} = \# \text{errors in the typical set}$

$$\frac{1}{n} \log |\mathcal{N}_{typ}| = h(p) + p \log 3, \quad h(p) = -p \log p - (1-p) \log (1-p)$$

$$\Rightarrow \mathcal{N}_{typ} \sim 2^{n[h(p) + p \log 3]}, \quad \mathcal{N}_{typ} < 2^r \text{ when } r > n[h(p) + p \log 3]$$

$$\frac{k}{n} \leq 1 - h(p) - p \log 3 \leq Q_1(p)$$
$$\rightarrow 0 \text{ at } p = 0.18929$$

Upper bounds.

$$\text{Mix. I channel w/ } \mathcal{N}: M_1 = q \mathcal{N} \cdot (1 -$$

Upper bound: Environment steals qubits w/ prob. $\frac{1}{2}$,



Upper bound: Environment steals qubits w/ prob. $\frac{1}{2}$,
replaces w/ random qubit. $\Rightarrow 0$ capacity by no-cloning.

$$1-P = \frac{1}{2} - \frac{1}{4}$$

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replaces w/ random qubit. \rightarrow 0 capacity by no-cloning.

$$1-p = \frac{1}{2} - \frac{1}{2}p$$

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$$1 - p = \frac{1}{2} - \frac{1}{4}p$$

Upper bound: Environment steals qubits w/ prob. $\frac{1}{2}$,
replaces w/ random qubit $\Rightarrow 0$ capacity by no-cloning.

$$1-P = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} \Rightarrow Q_1\left(\frac{3}{8}\right) = 0.$$

Upper bound: Environment steals qubits w/ prob. $\frac{1}{2}$,
replaces w/ random qubit. $\Rightarrow 0$ capacity by no-cloning.

$$1-P = \frac{1}{2} - \frac{1}{4} \cdot \frac{1}{2} = \frac{5}{8} \Rightarrow Q_1\left(\frac{3}{2}\right) = 0.$$

Optimal symmetric (imperfect) cloner:

$$|0\rangle \rightarrow \sqrt{\frac{2}{3}}|00\rangle|A\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|B\rangle$$

$$|1\rangle \rightarrow \sqrt{\frac{2}{3}}|11\rangle|B\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|A\rangle$$

Upper bound: Environment steals qubits w/ prob. $\frac{1}{2}$,
replaces w/ random qubit. $\Rightarrow 0$ capacity by no-cloning.

$$1-p = \frac{1}{2} - \frac{1}{4} \cdot \frac{1}{2} = \frac{5}{8} \Rightarrow Q_1\left(\frac{3}{2}\right) = 0.$$

Optimal symmetric (imperfect) cloner:

$$|D\rangle \rightarrow \sqrt{\frac{2}{3}}|100\rangle|A\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|B\rangle \sim p = \frac{1}{4} \Rightarrow Q_1\left(\frac{1}{4}\right) = 0$$

$$|I\rangle \rightarrow \sqrt{\frac{2}{3}}|111\rangle|B\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|A\rangle$$

$\rightarrow 0$ at $P=0.10161$

Upper bound: Environment steals qubits w/ prob. $\frac{1}{2}$,
replaces w/ random qubit $\Rightarrow 0$ capacity by no-cloning.

$$1-P = \frac{1}{2} - \frac{1}{4}\frac{1}{2} = \frac{5}{8} \Rightarrow Q_1\left(\frac{3}{8}\right) = 0.$$

Optimal symmetric (imperfect) cloner:

$$|D\rangle \rightarrow \sqrt{\frac{1}{3}}|00\rangle|A\rangle + \sqrt{\frac{1}{3}}|\Psi^+\rangle|B\rangle \sim P = \frac{1}{4} \Rightarrow Q_1\left(\frac{1}{4}\right) = 0$$

$$|I\rangle \rightarrow \sqrt{\frac{2}{3}}|11\rangle|B\rangle + \sqrt{\frac{1}{3}}|\Psi^+\rangle|A\rangle$$

Can do better than random code for Q_1 !

$\rightarrow 0$ at $P=0.10161$

Upper bound: Environment steals qubits w/ prob. $\frac{1}{2}$,
replaces w/ random qubit $\Rightarrow 0$ capacity by no-cloning.

$$1-P = \frac{1}{2} - \frac{1}{4} \cdot \frac{1}{2} = \frac{5}{8} \Rightarrow Q_1\left(\frac{3}{8}\right) = 0.$$

Optimal symmetric (imperfect) cloner:

$$|D\rangle \rightarrow \sqrt{\frac{2}{3}}|100\rangle|A\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|B\rangle \sim P = \frac{1}{9} \Rightarrow Q_1\left(\frac{1}{9}\right) = 0$$

$$|D\rangle \rightarrow \sqrt{\frac{2}{3}}|111\rangle|B\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|A\rangle$$

Can do better than random code for Q_1 ! Use degeneracy
Concatenate random w/ repetition code

$\rightarrow 0$ at $p=0.19086$

Upper bound. Environment steals qubits w/ prob. $\frac{1}{2}$,
replaces w/ random qubit. $\Rightarrow 0$ capacity by no-cloning.

$$1-p = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{8} \Rightarrow Q_1\left(\frac{3}{8}\right) = 0.$$

Optimal symmetric (imperfect) cloner:

$$|D\rangle \mapsto \sqrt{\frac{2}{3}}|00\rangle|A\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|B\rangle \sim p = \frac{1}{9} \Rightarrow Q_1\left(\frac{1}{9}\right) = 0$$

$$|I\rangle \mapsto \sqrt{\frac{2}{3}}|11\rangle|B\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|A\rangle$$

Can do better than random code for Q_1 . Use degeneracy

Concatenate random or repetition code - can get $Q_1(p) > 0$ for
 $p \neq 0.19086$.

$\rightarrow 0$ at $P=0.19088$

Upper bound: Environment steals qubits w/ prob. $\frac{1}{2}$,
replaces w/ random qubit $\Rightarrow 0$ capacity by no-cloning.

$$1-P = \frac{1}{2} - \frac{1}{4} \cdot \frac{1}{2} = \frac{5}{8} \Rightarrow Q_1\left(\frac{3}{8}\right) = 0.$$

Optimal symmetric (imperfect) cloner:

$$|D\rangle \rightarrow \sqrt{\frac{2}{3}}|00\rangle|A\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|B\rangle \sim P = \frac{1}{9} \Rightarrow Q_1\left(\frac{1}{9}\right) = 0$$

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Can do better than random code for Q_1 ! Use degeneracy

Concatenate random w/ repetition code - can get $Q_1(P) > 0$ for
 $P \neq 0.19088$.

Q_2 for depolarizing channel:

$$Q_2(t) = 0 \text{ because}$$

Q_2 for depolarizing channel:

$Q_2(\frac{1}{2}) = 0$ because $p = \frac{1}{2}$ Werner state is separable.

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$Q_2(p)>0$ for $p<\frac{1}{2}$

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Def

Q_2 for depolarizing channel:

$Q_2(\frac{1}{2})=0$ because $\rho=\frac{1}{2}$ Werner state is separable.

$\underline{\underline{Q_2(\rho) > 0}}$ for $\rho < \frac{1}{2}$.

Def. Purify π to unitary on channel & environment E

The coherent information is $I(\rho, \pi) = S(\pi(\rho)) - S(E)$,

where $S(E)$ is the entropy of E after the channel is used.

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Thm:

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Thm: $Q_1(\pi) = \lim_{n \rightarrow \infty} \max_{\rho} \frac{1}{n} I(\rho^n, \pi^n)$

Upper bound. Entanglement does not help C_1
replaces w/ random qubit $\Rightarrow C$ capacity by no-cloning

$$1-P = \frac{1}{2} + \frac{1}{2} - \frac{\epsilon}{2} \Rightarrow Q_1\left(\frac{2}{3}\right) = 0.$$

Optimal symmetric (\sim perfect) clone:

$$|D\rangle \rightarrow \sqrt{\frac{1}{3}}|100\rangle|A\rangle + \sqrt{\frac{2}{3}}|1\Sigma\rangle|B\rangle \sim P = \frac{1}{3} \Rightarrow Q_1\left(\frac{1}{3}\right) = 0$$

$$|D\rangle \rightarrow \sqrt{\frac{1}{3}}|110\rangle|B\rangle + \sqrt{\frac{2}{3}}|1\Sigma\rangle|A\rangle$$

Can be better than random code for Q_1 due to degeneracy
Concentric random or repetition code - can get $Q_1 > 0$ for $P \neq 0.142857$.

Upper bound. Environment steals qubits w prob. $\frac{1}{2}$,
replaces w random qubit $\Rightarrow 0$ capacity by no-cloning.

$$1-P = \frac{1}{2} - \frac{1}{4}P^2 = \frac{5}{8} \Rightarrow Q_1\left(\frac{3}{8}\right) = 0.$$

Optimal symmetric (imperfect) cloner:

$$|D\rangle \rightarrow \sqrt{\frac{2}{3}}|100\rangle|A\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|B\rangle \sim P = \frac{1}{2} \Rightarrow Q_1\left(\frac{1}{2}\right) = 0$$

$$|D\rangle \rightarrow \sqrt{\frac{2}{3}}|11\rangle|B\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|A\rangle$$

$$|D\rangle \rightarrow \sqrt{\frac{2}{3}}|11\rangle|B\rangle + \sqrt{\frac{1}{3}}|\Psi'\rangle|A\rangle$$

Can do better than random code for Q_1 ! Use degeneracy
Concatenate random or repetition code - can get $Q_1(P) > 0$ for
 $P \neq 0.19088$.

Q_2 for depolarizing channel:

$Q_2(\frac{1}{2}) = 0$ because $P = \frac{1}{2}$ Werner state is separable.

$Q_2(P) > 0$ for $P < \frac{1}{2}$.

Def. Purify π to unitary on channels at E .

The coherent information is $I(\rho, \pi) = S(\pi) - S(E)$,

where $S(E)$ is the entropy of E after the

Theorem: π is exactly correctable $\Leftrightarrow I(\rho, \pi) = 0$

$$\text{Thm: } Q_1(\pi) = \lim_{n \rightarrow \infty} \max_{\rho} \frac{1}{n} I(\rho^n, \pi^n)$$