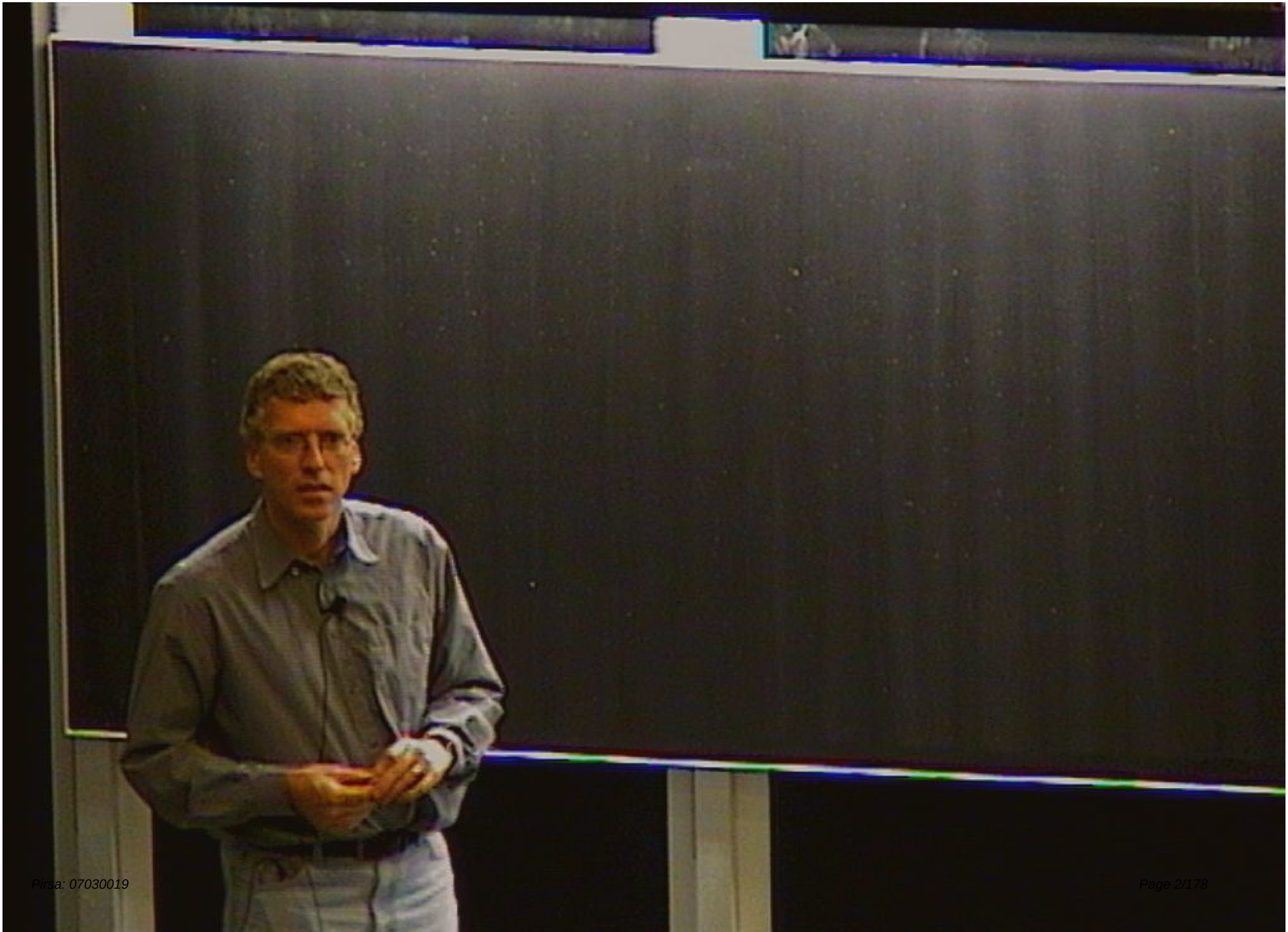
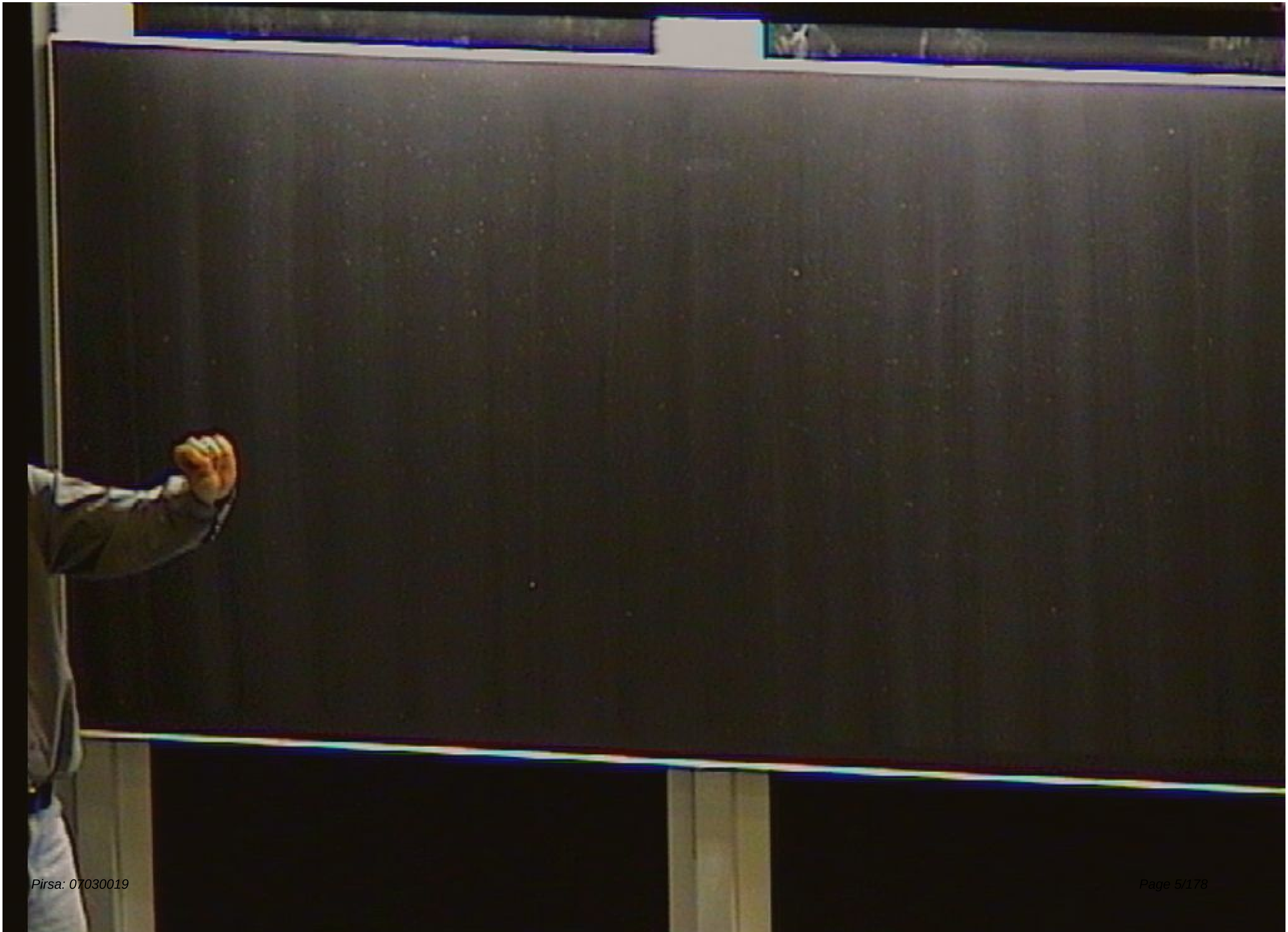
Title: Graduate Course on Standard Model & Quantum Field Theory - 14B

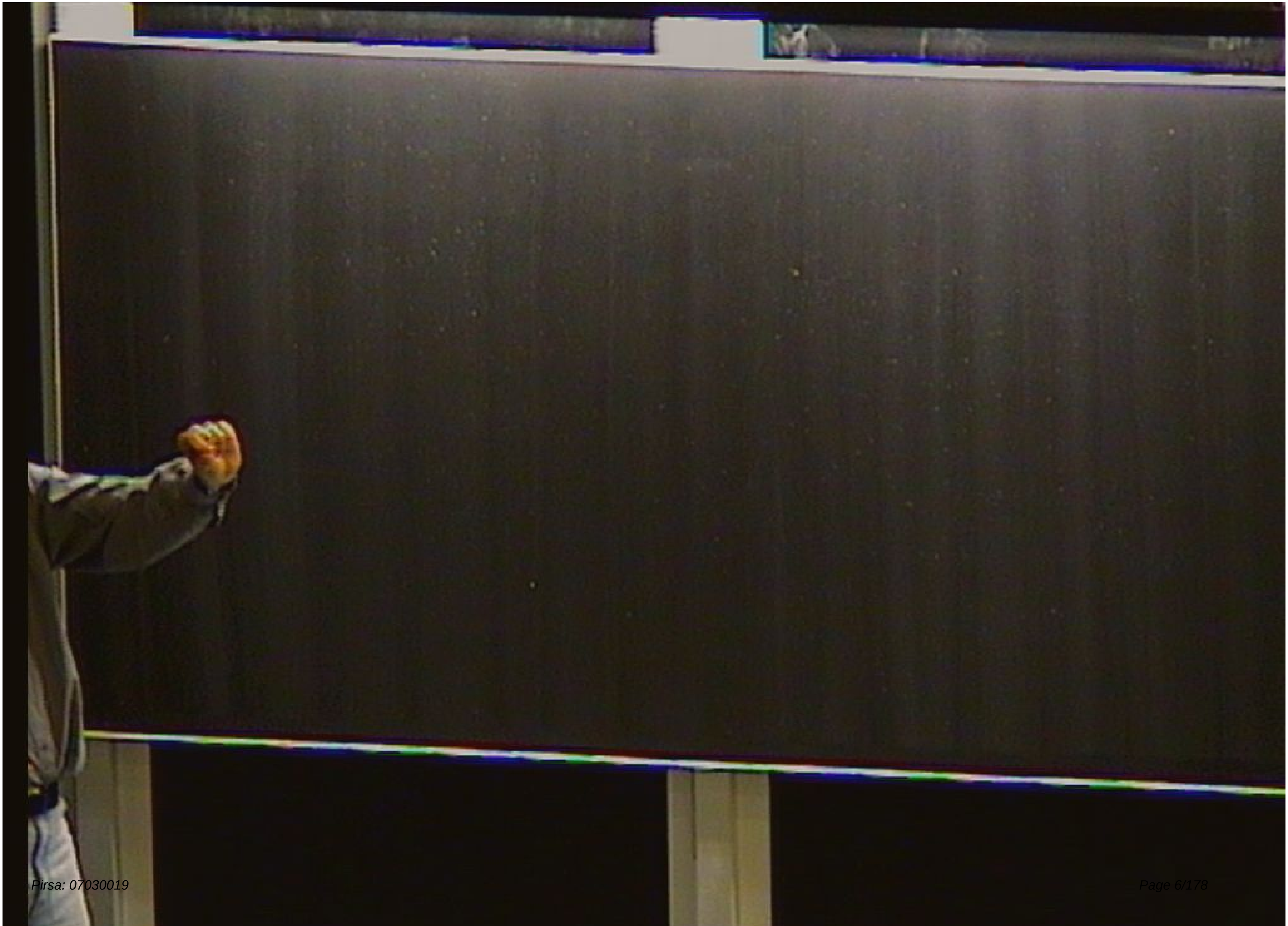
Date: Mar 07, 2007 03:30 PM

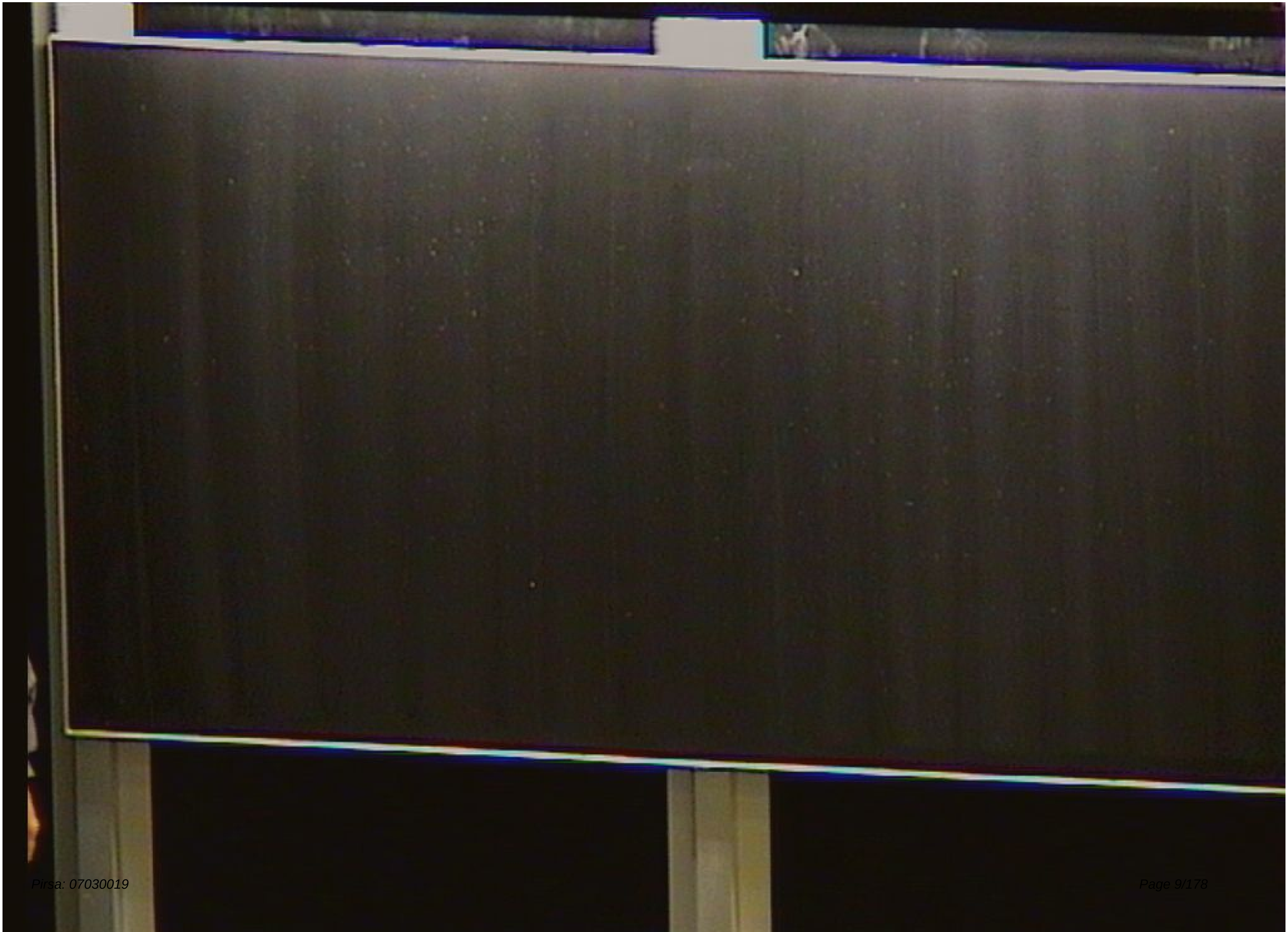
URL: http://pirsa.org/07030019

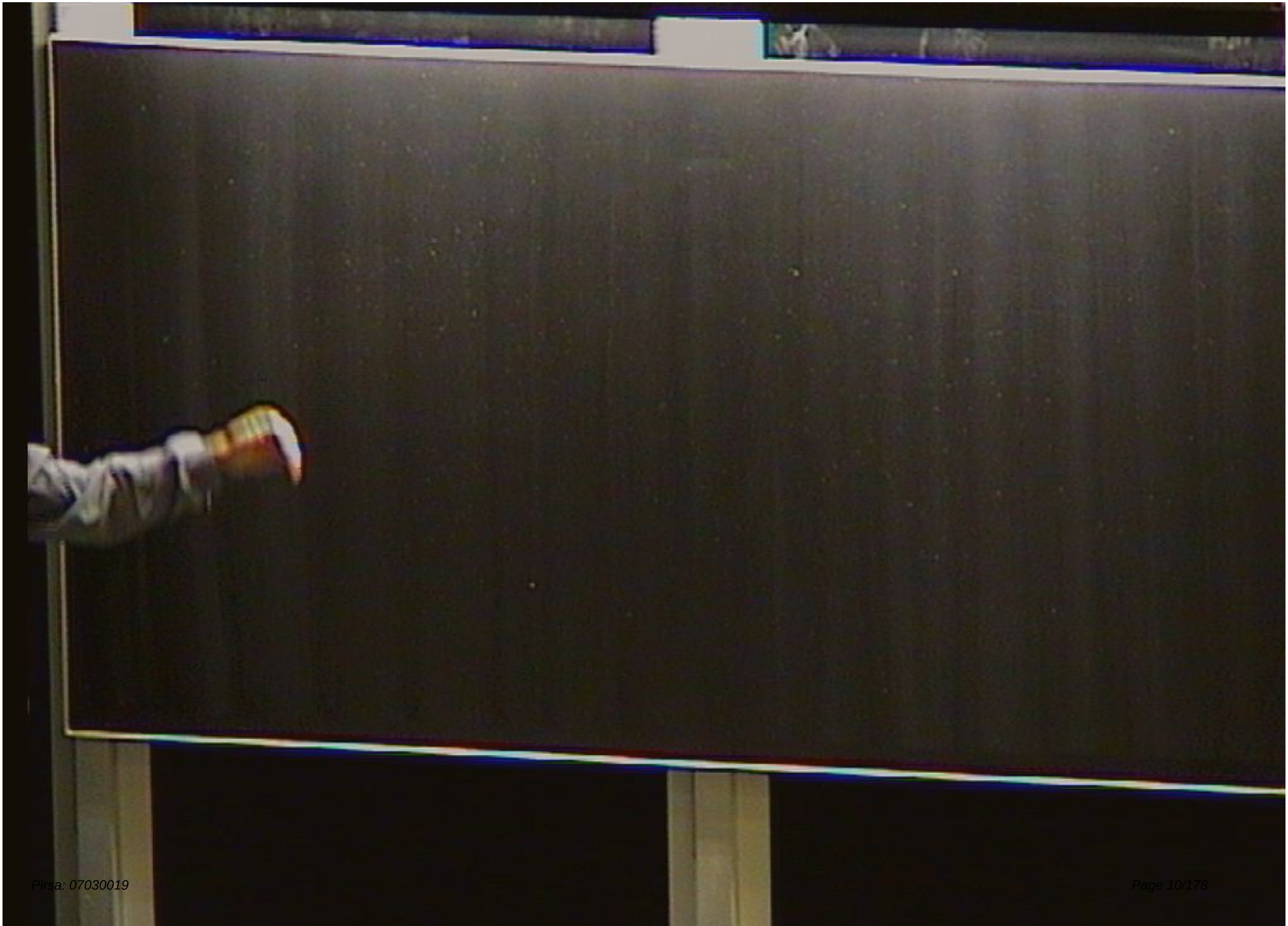
Abstract: Graduate Course on Standard Model & Quantum Field Theory

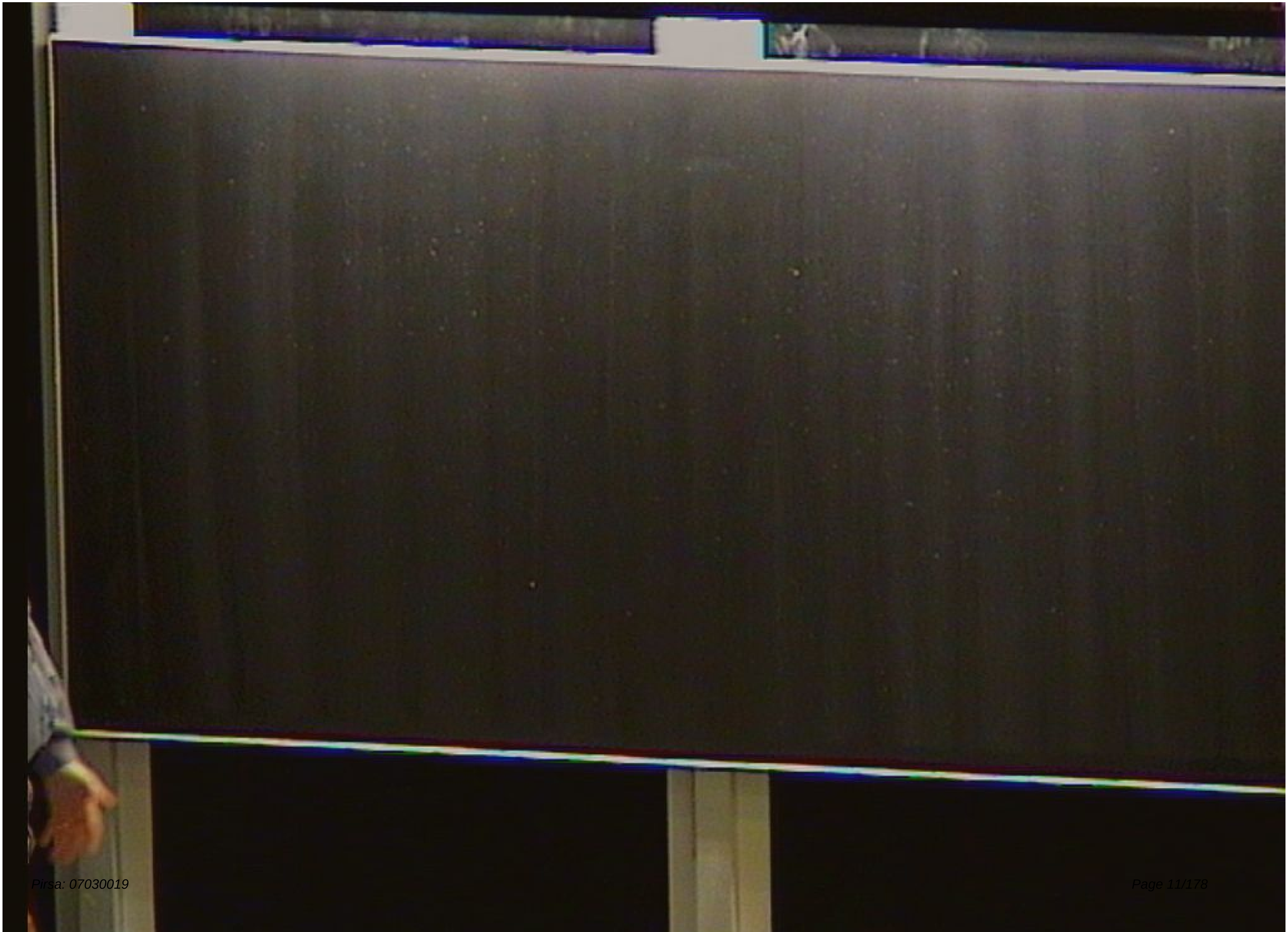


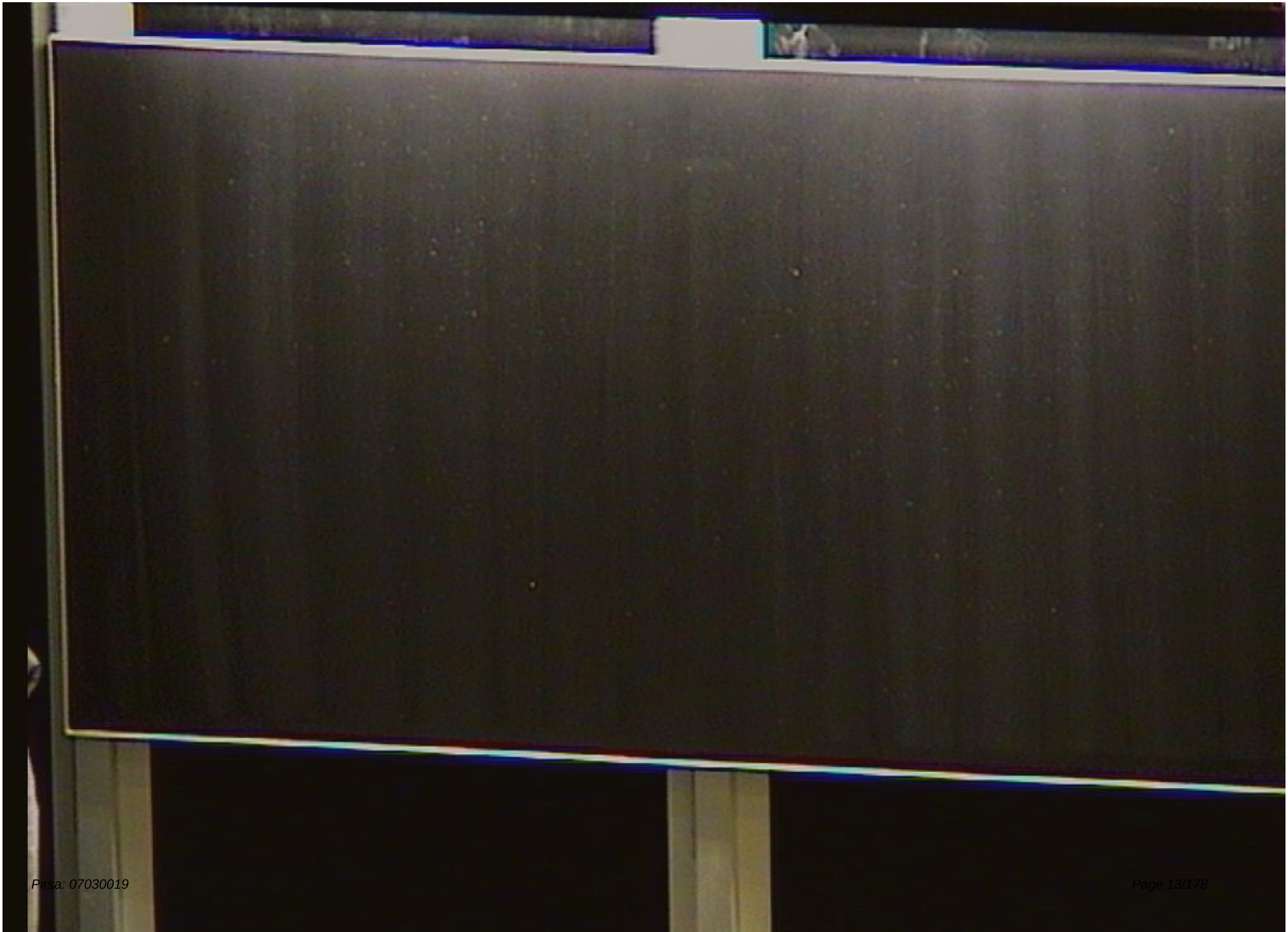


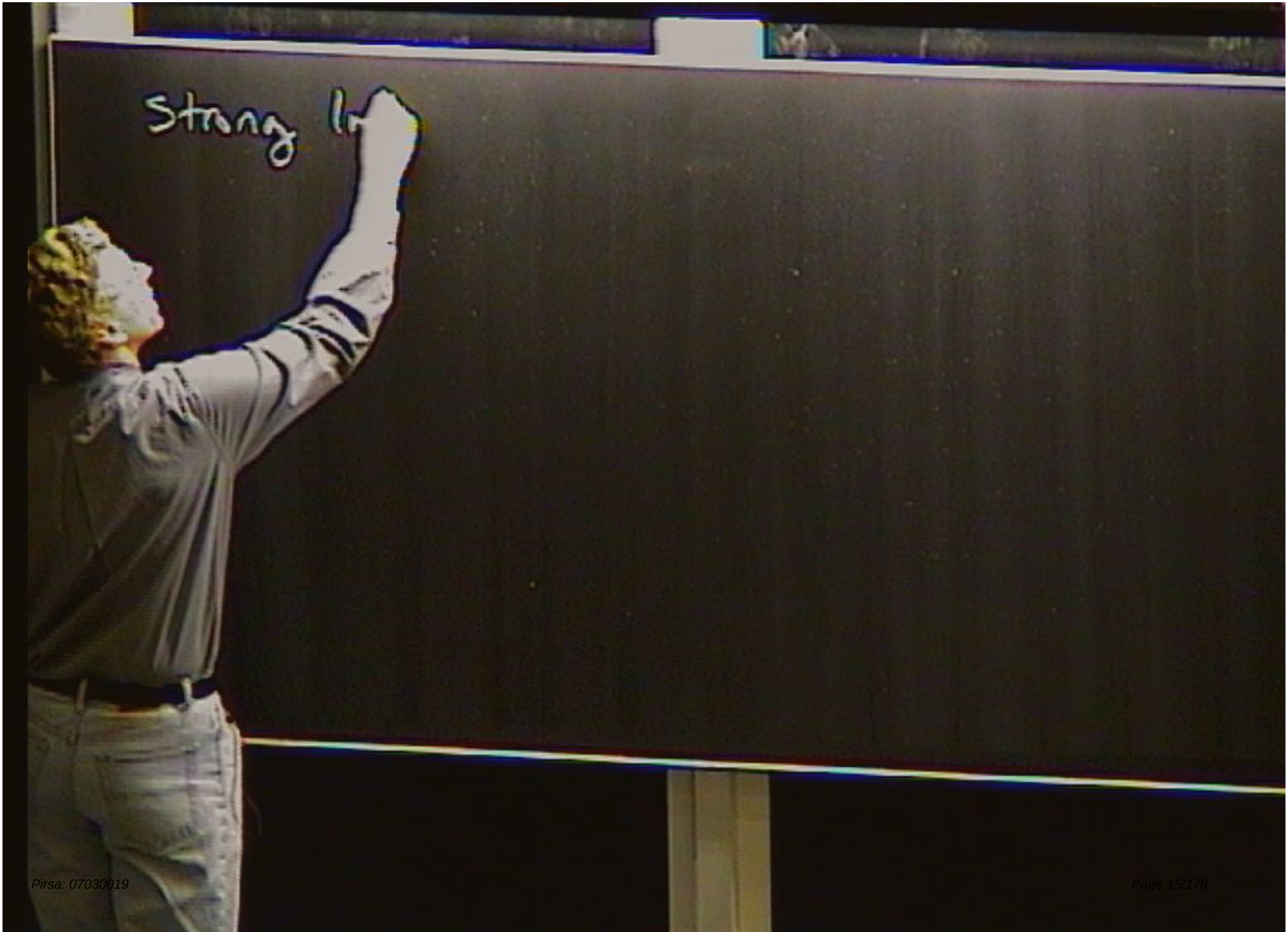


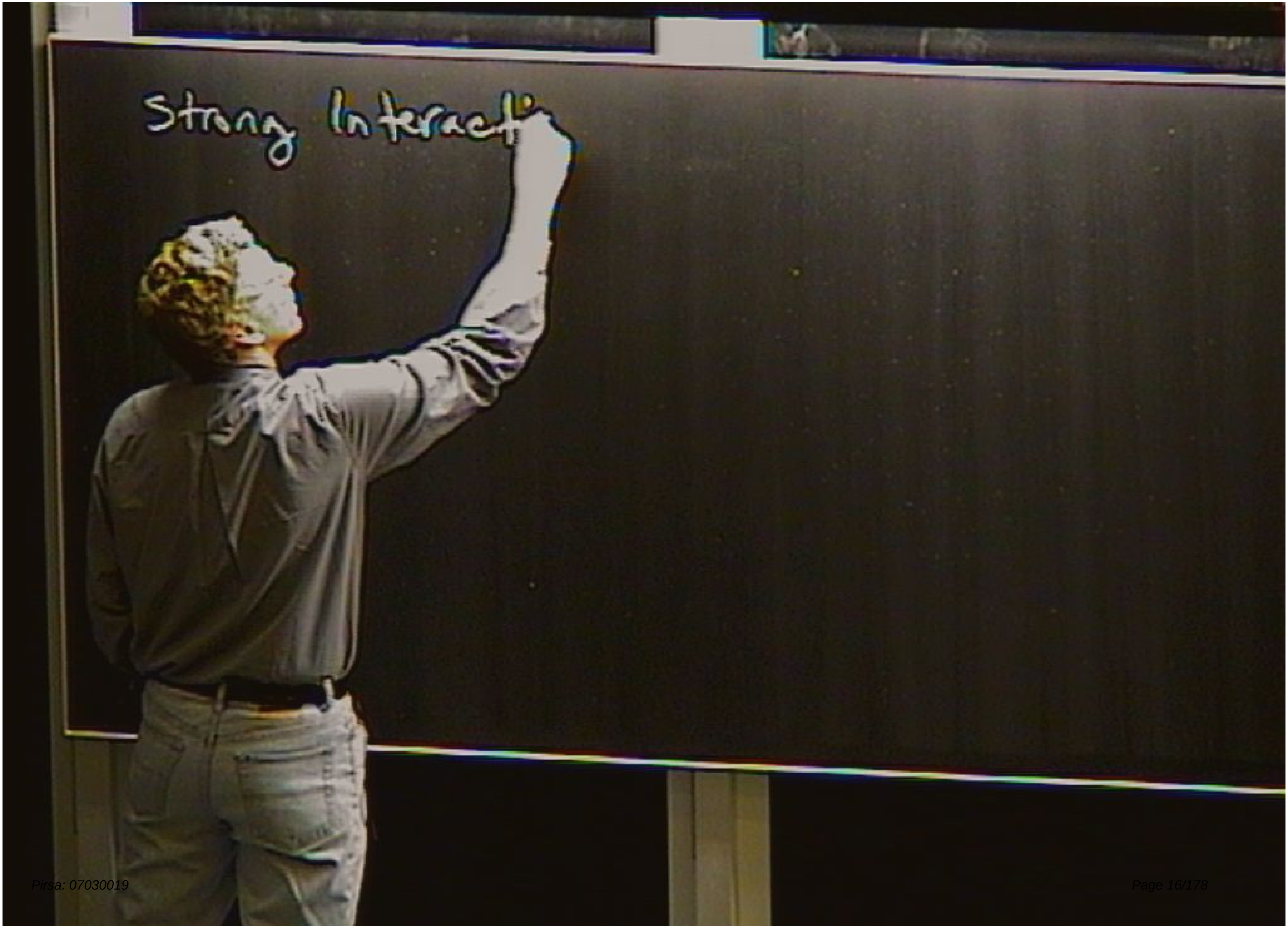









Strong

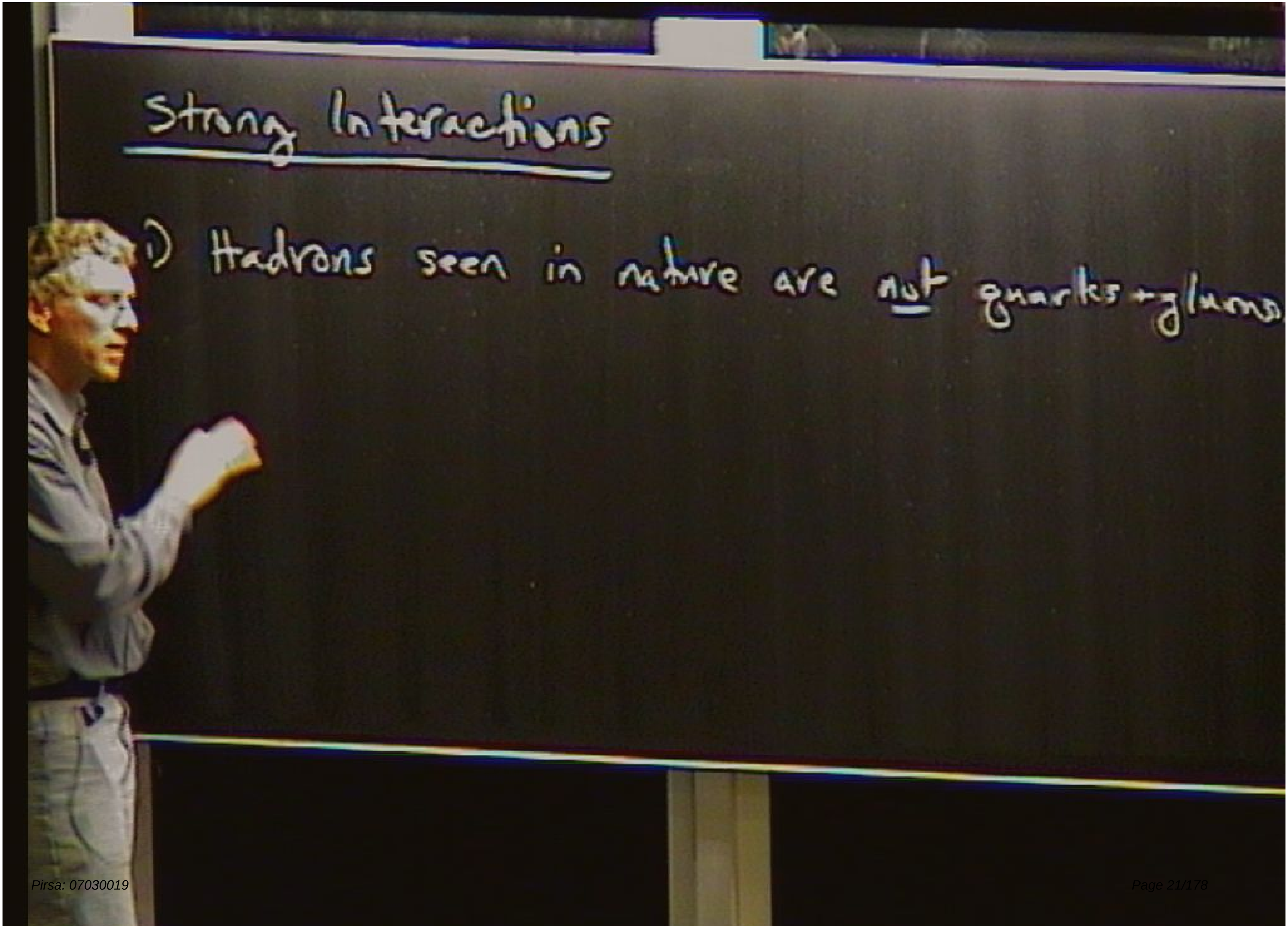

Strong Interacti

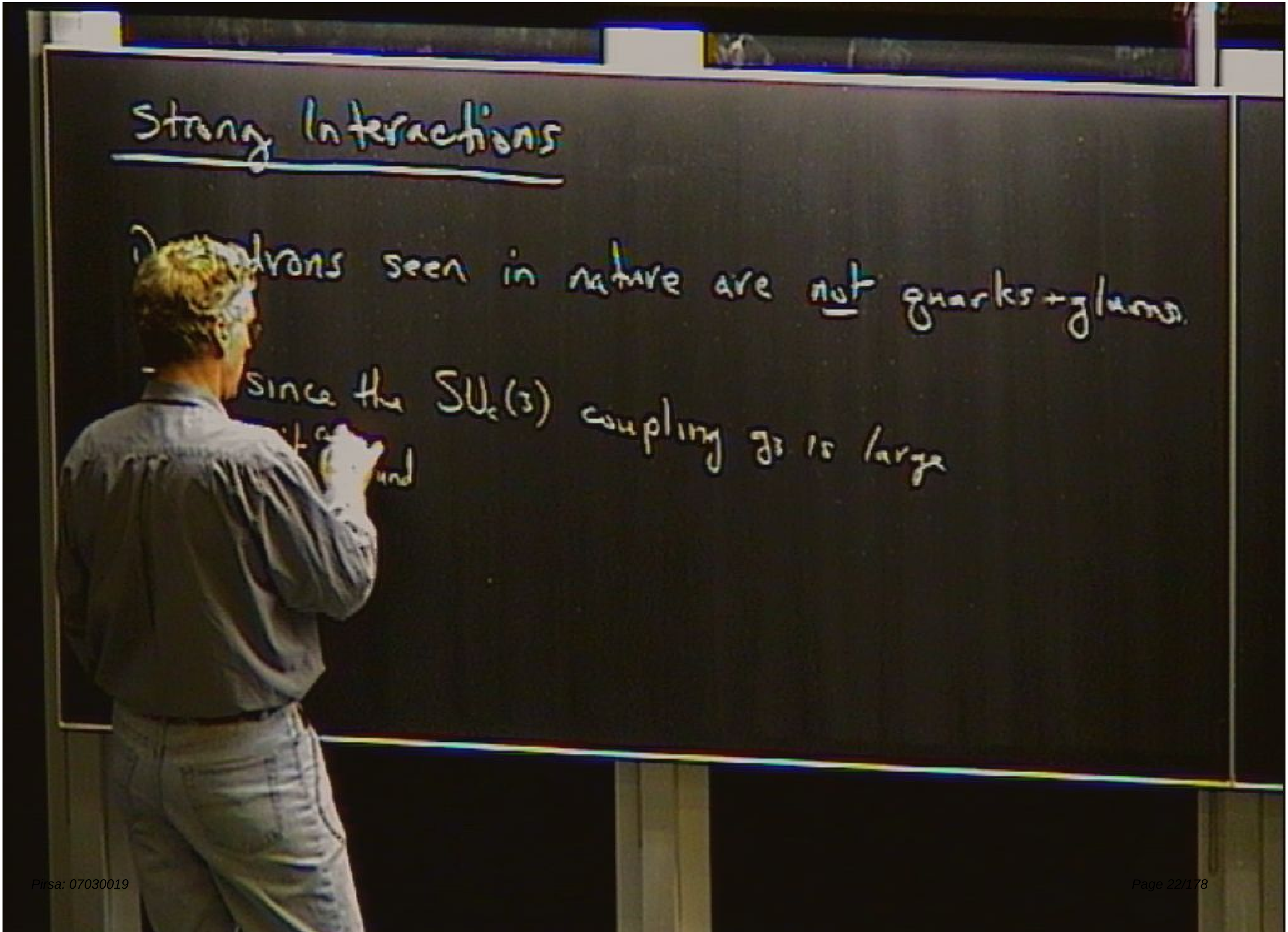
Strong Interactions

Strong Interactions

Strong Interactions

Strong Interactions

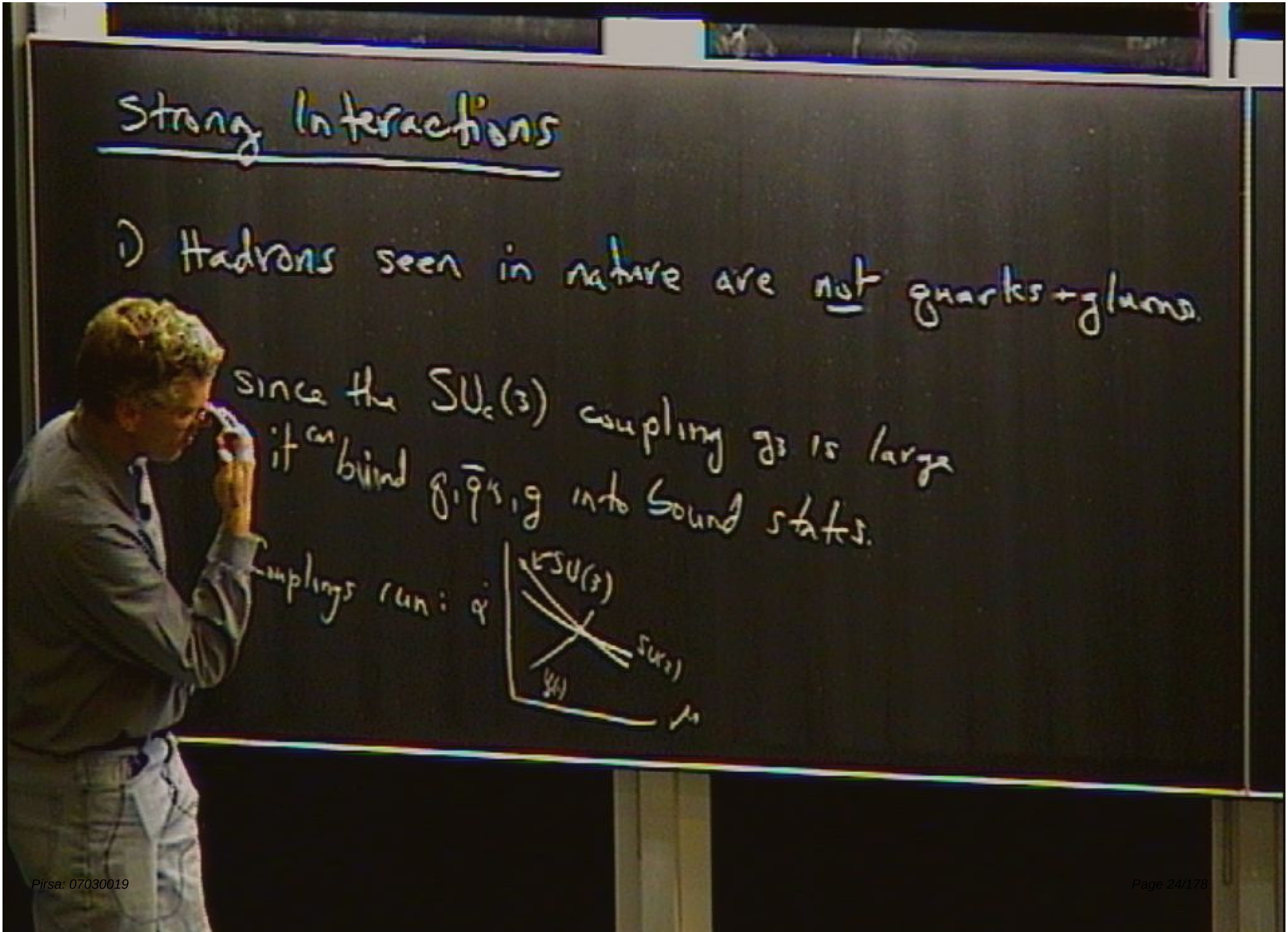

Strong Interactions
1) Hadrons seen in nature are not quarks + gluons


Strong Interactions
seen in nature are not quarks + gluons
since the SU(3) coupling is large

Strong Interactions

1) Hadrons seen in nature are <u>not</u> quarks + gluons.

— since the $SU_c(3)$ coupling g_3 is large it can bind $q, \bar{q}$'s, g into bound states.

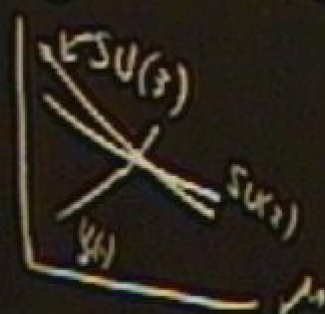

Strong Interactions
1) Hadrons seen in nature are not quarks + gluons
since the SU_c(3) coupling g3 is large
it can bind q, q̄'s, g into bound states.
Couplings run: α^i
SU(3)
SU(2)
U(1)
μ

Strong Interactions

1) Hadrons seen in nature are not quarks + gluons.

— since the $SU_c(3)$ coupling g_3 is large it can bind $q, \bar{q}$'s, g into bound states.

Couplings run: α



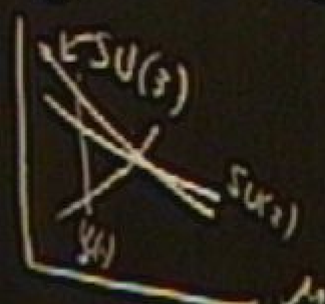
eg $\alpha_3(M_Z) =$

Strong Interactions

1) Hadrons seen in nature are <u>not</u> quarks + gluons.

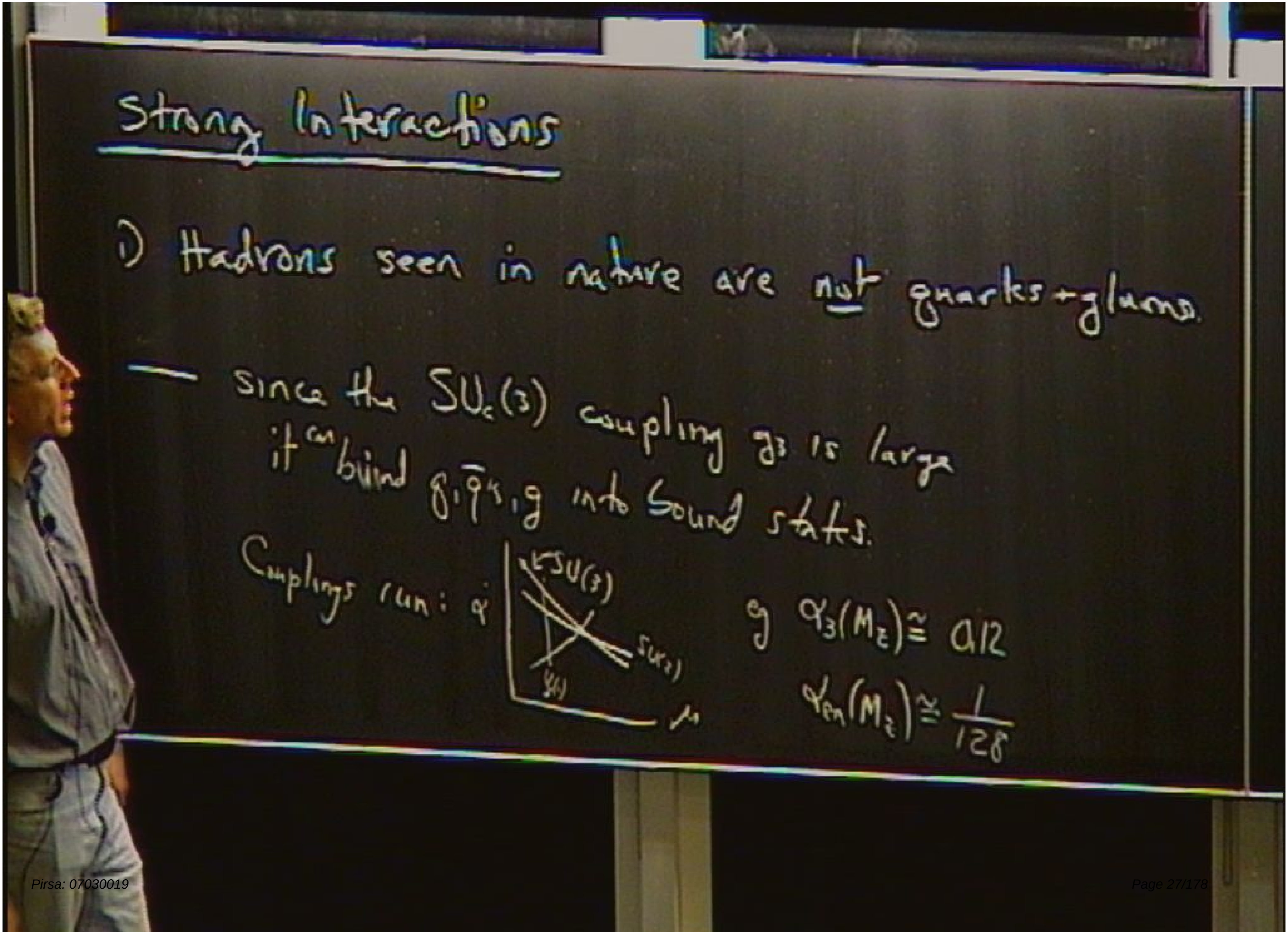
Since the $SU_c(3)$ coupling g_3 is large it can bind $q, \bar{q}$'s, g into bound states.

...s run: α^i



e.g. $\alpha_3(M_Z) \cong 0.12$

$\alpha_{em}(M_Z) \cong \frac{1}{128}$

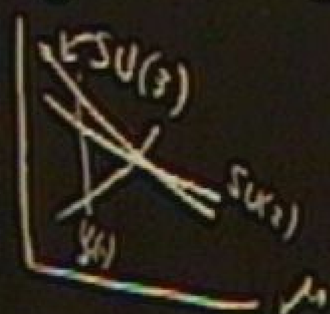

Strong Interactions
1) Hadrons seen in nature are not quarks + gluons.
— since the SU_c(3) coupling g3 is large
it can bind q, q̄'s, g into bound states.
Couplings run: α
SU(3)
SU(2)
U(1)
μ
eg α3(Mz) ≅ 0.12
αem(Mz) ≅ 1/128

Strong Interactions

1) Hadrons seen in nature are not quarks + gluons.

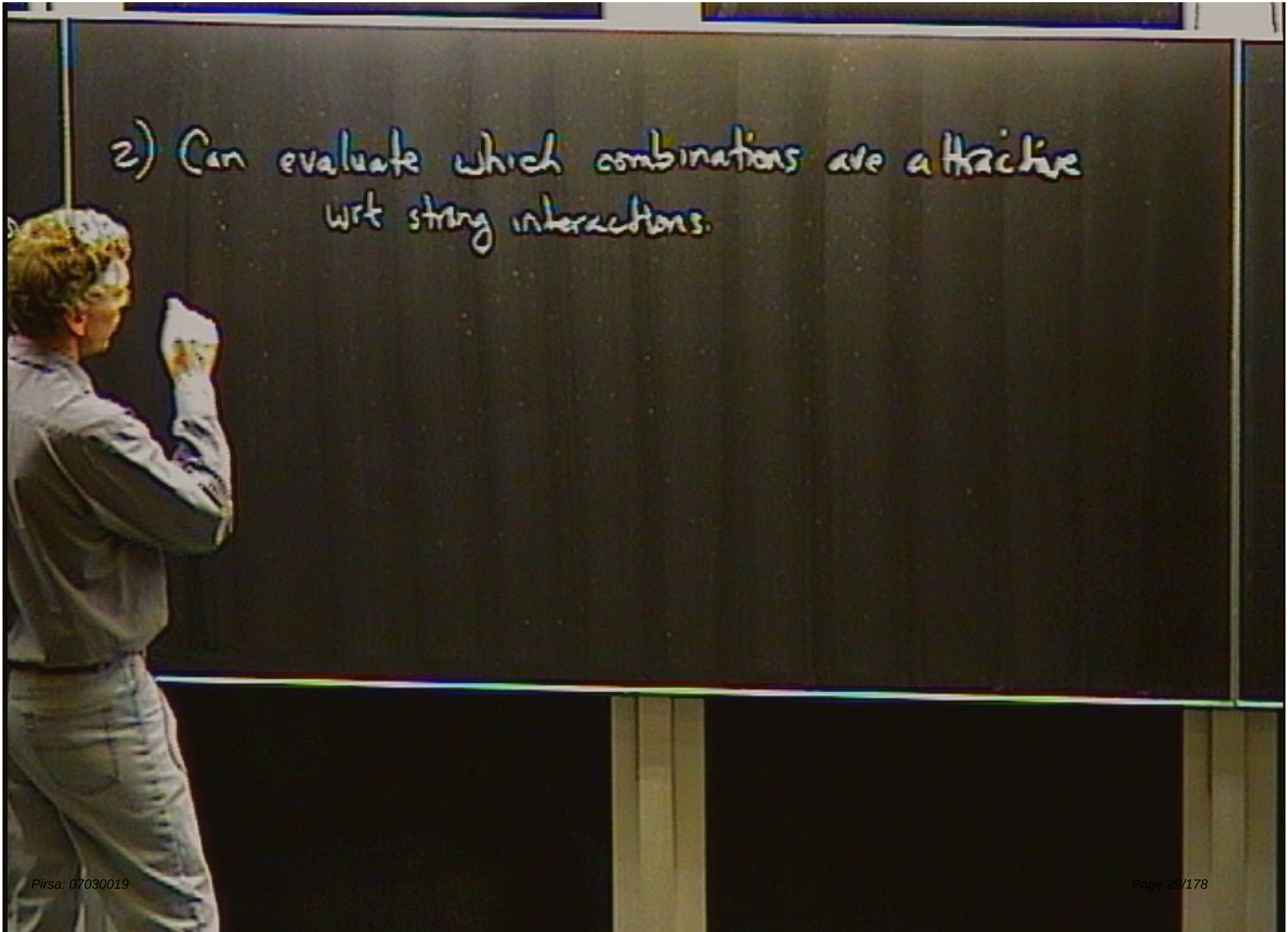
— since the $SU_c(3)$ coupling g_3 is large it can bind $q, \bar{q}$'s, g into bound states.

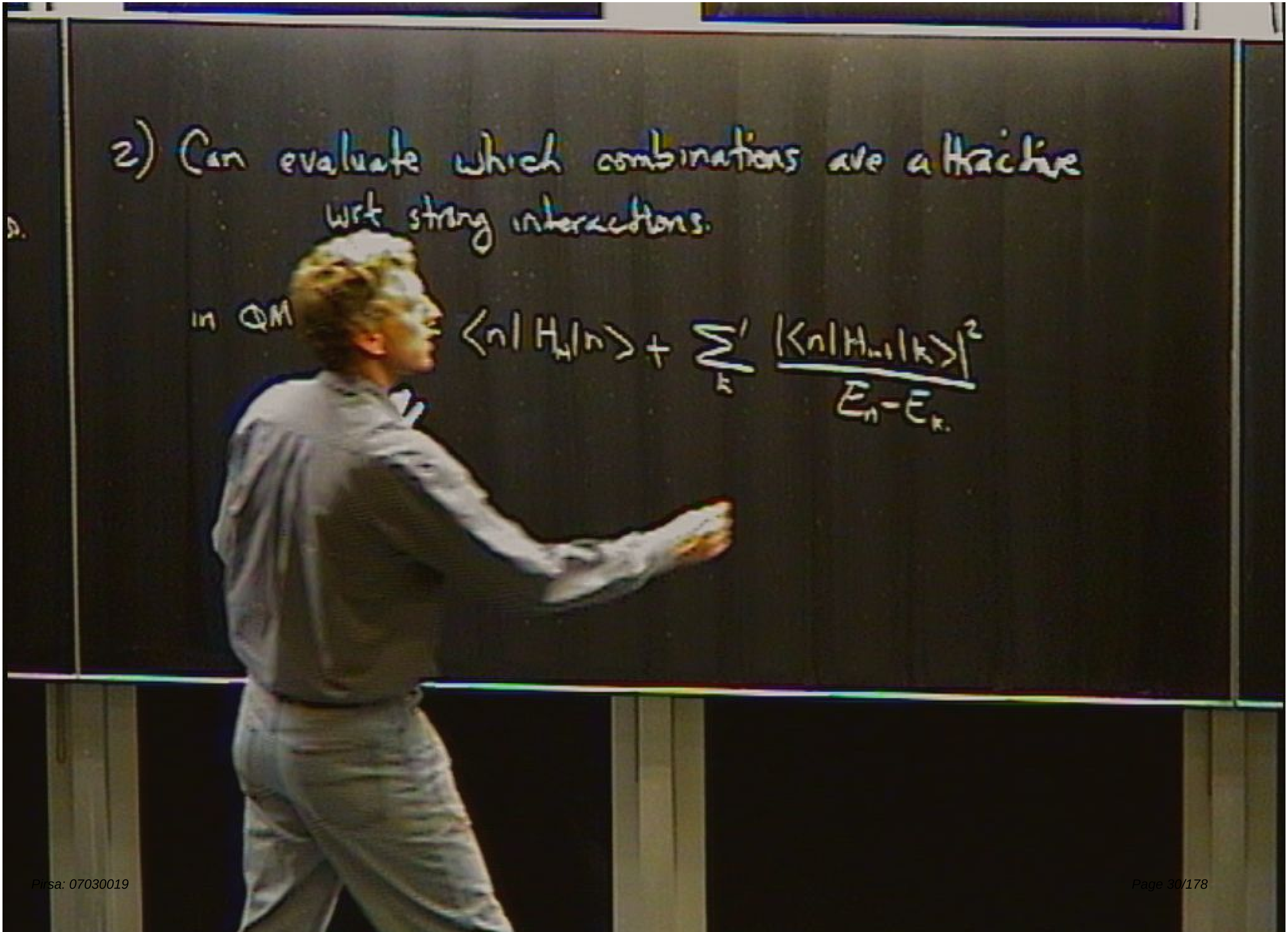
Couplings run: α



e.g. $\alpha_3(M_Z) \simeq 0.12$

$\alpha_{em}(M_Z) \simeq \frac{1}{128}$


2) Can evaluate which combinations are attractive
wrt string interactions.


2) Can evaluate which combinations are attractive
wrt strong interactions.
in QM
$\langle n | H_{int} | n \rangle + \sum_k' \frac{|\langle n | H_{int} | k \rangle|^2}{E_n - E_k}$

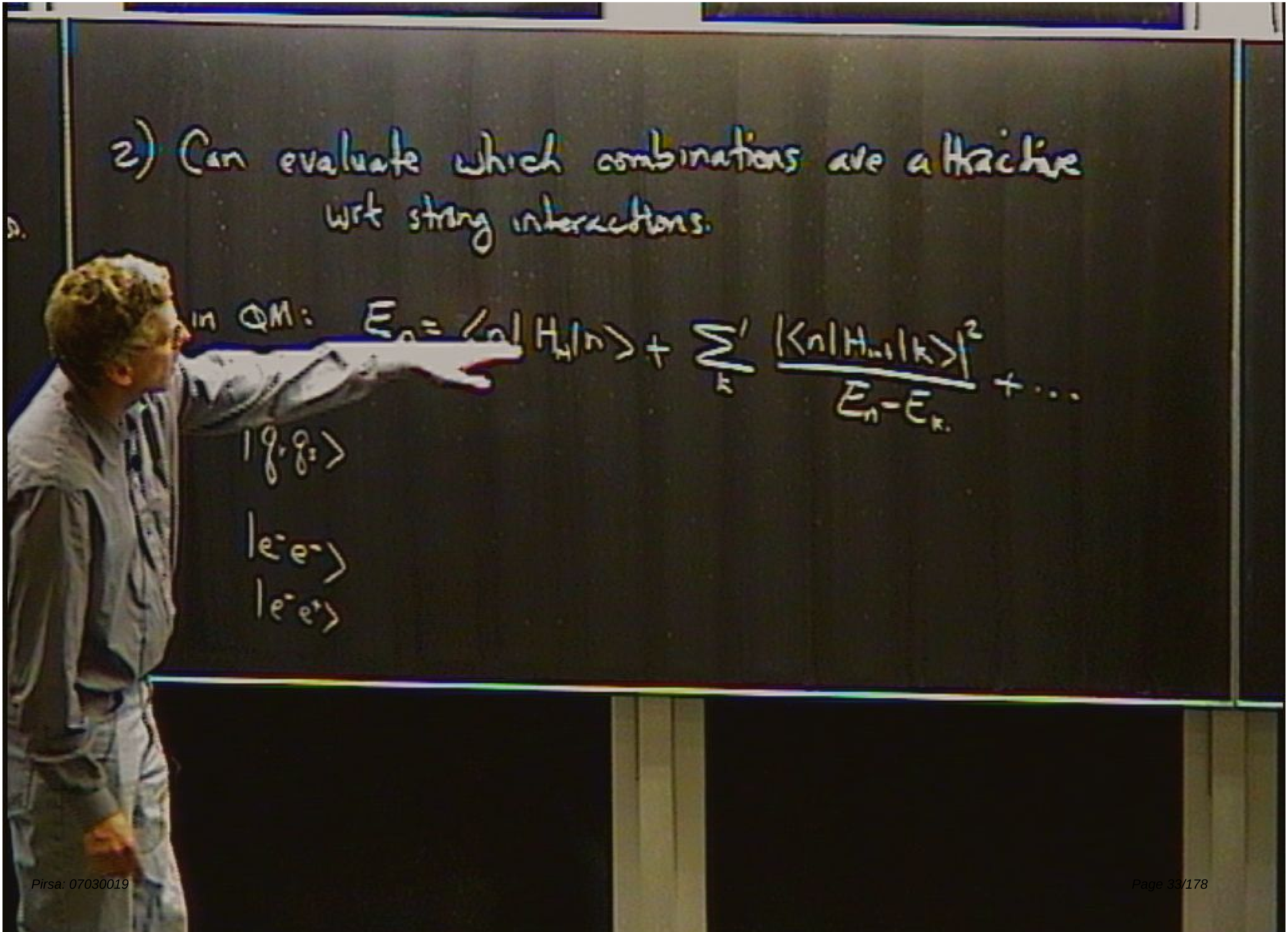
2) Can evaluate which combinations are attractive
wrt strong interactions.

in QM: $E_n = \langle n | H_{int} | n \rangle + \sum_k{}' \frac{|\langle n | H_{int} | k \rangle|^2}{E_n - E_k} + \dots$

2) Can evaluate which combinations are attractive wrt strong interactions.

in QM: $E_n = \langle n | H_{int} | n \rangle + \sum_k{}' \frac{|\langle n | H_{int} | k \rangle|^2}{E_n - E_k} + \dots$

$|q_1 q_2\rangle$


2) Can evaluate which combinations are attractive
wrt strong interactions.
in QM: $E_n = \langle n | H_I | n \rangle + \sum_k' \frac{|\langle n | H_{int} | k \rangle|^2}{E_n - E_k} + \ldots$
$|q_1 q_2\rangle$
$|e^- e^-\rangle$
$|e^- e^+\rangle$

2) Can evaluate which combinations are attractive wrt strong interactions.

in QM: $E_n = \langle n|H_{int}|n\rangle + \sum_k{}' \frac{|\langle n|H_{int}|k\rangle|^2}{E_n - E_k} + \dots$

$|q_1 q_2\rangle$ $\quad \mathcal{L}_{int} = i g_s (\bar{q} \gamma^\mu \lambda_a q) G^a_\mu$

$|e^- e^-\rangle$

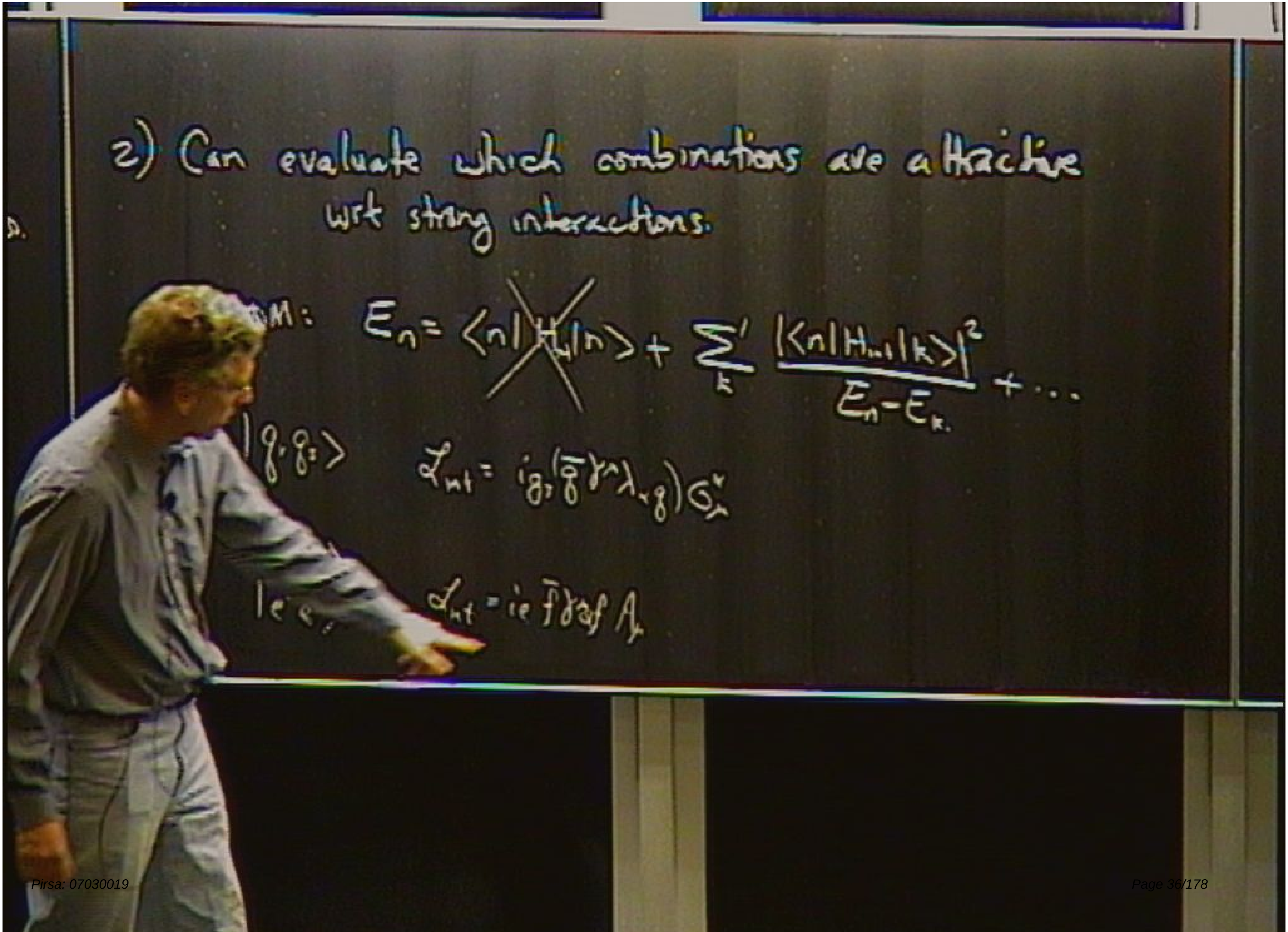
$|e^- e^+\rangle$ $\quad \mathcal{L}_{int} = ie \bar{f} \gamma^\mu f A_\mu$

2) Can evaluate which combinations are attractive wrt strong interactions.

in QM: $E_n = \cancel{\langle n|H_{int}|n\rangle} + \sum_k{}' \frac{|\langle n|H_{int}|k\rangle|^2}{E_n - E_k} + \dots$

$|q_i q_j\rangle$ $\qquad \mathcal{L}_{int} = i g_s (\bar{q} \gamma^\mu \lambda_a q) G^a_\mu$

$|e^-\rangle$ $|e^+\rangle$ $\qquad \mathcal{L}_{int} = ie \bar{f} \gamma^\mu f A_\mu$


2) Can evaluate which combinations are attractive
wrt strong interactions.
$E_n = \cancel{\langle n | H_{int} | n \rangle} + \sum_k' \frac{|\langle n | H_{int} | k \rangle|^2}{E_n - E_k} + \dots$
$\mathcal{L}_{int} = i g_s (\bar{q} \gamma^\mu \lambda_a q) G^a_\mu$
$\mathcal{L}_{int} = ie \bar{f} \gamma^\mu f A_\mu$

2) Can evaluate which combinations are attractive wrt strong interactions.

in QM: $E_n = \cancel{\langle n|H_{int}|n\rangle} + \sum_k{}' \frac{|\langle n|H_{int}|k\rangle|^2}{E_n - E_k} + \dots$

$|q_1 q_2\rangle$ $\quad \mathcal{L}_{int} = i g_s (\bar{q} \gamma^\mu \lambda_a q) G^a_\mu$

$|e^- e^-\rangle$

$|e^- e^+\rangle$ $\quad \mathcal{L}_{int} = ie \bar{f} \gamma^\mu f A_\mu$

2) Can evaluate which combinations are attractive wrt strong interactions.

in QM: $\cancel{\langle n | H_{int} | n \rangle} + \sum_k{}' \frac{|\langle n | H_{int} | k \rangle|^2}{E_n - E_k} + \dots$

$\mathcal{L}_{int} = i g_s (\bar{q} \gamma^\mu \lambda_a q) G^a_\mu$

$\mathcal{L}_{int} = ie \bar{f} \gamma f A_\mu$ $\delta E_k = \varepsilon$

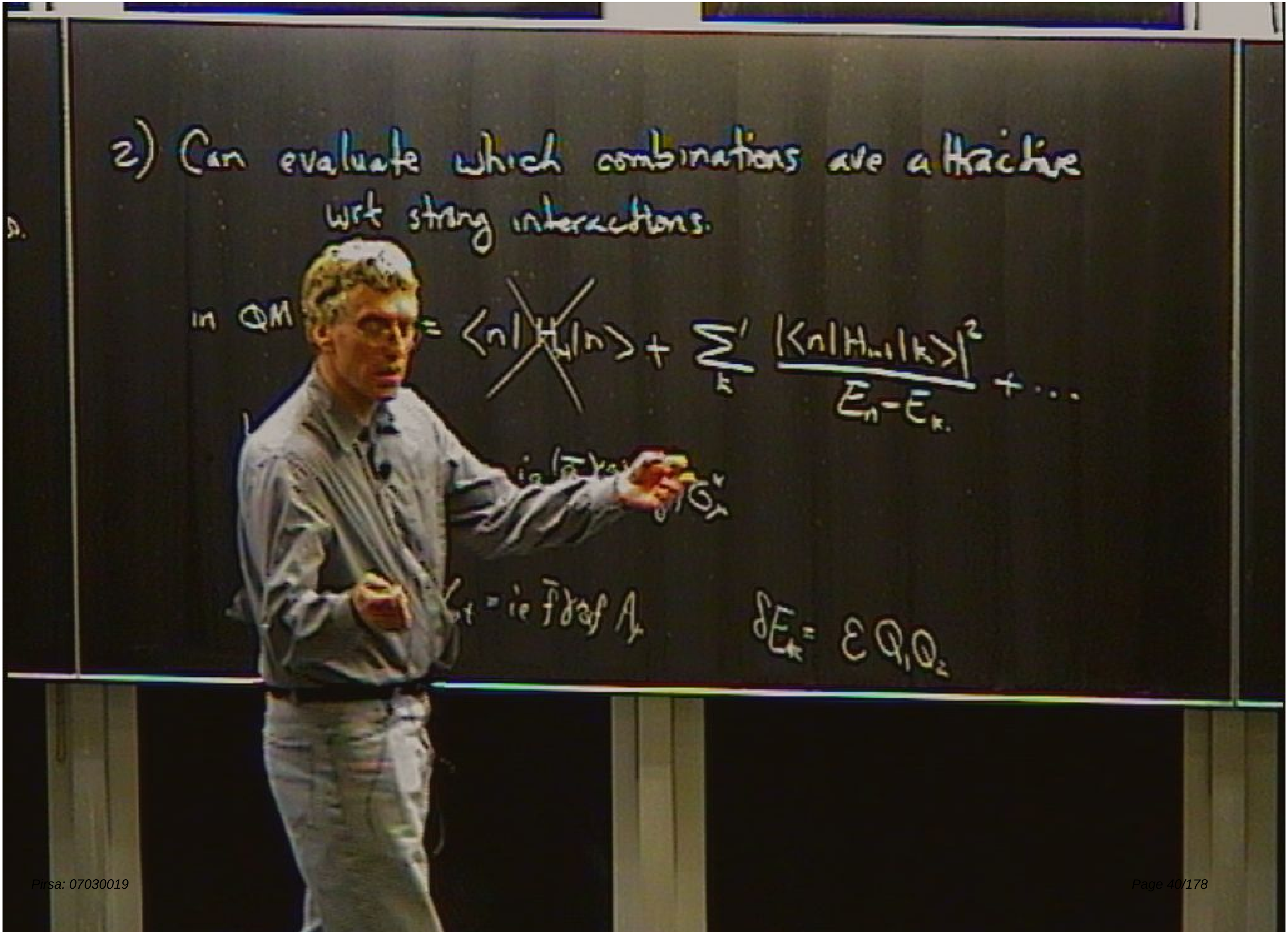
2) Can evaluate which combinations are attractive wrt strong interactions.

in QM: $\delta E_n = \cancel{\langle n| H_{int} |n\rangle} + \sum_k{}' \frac{|\langle n|H_{int}|k\rangle|^2}{E_n - E_k} + \dots$

$|q_1 q_2\rangle$ $\quad \mathcal{L}_{int} = i g_s (\bar{q} \gamma^\mu \lambda_a q) G_\mu^a$

$|e^- e^-\rangle$

$|e^- e^+\rangle$ $\quad \mathcal{L}_{int} = ie \bar{f} \gamma^\mu f A_\mu$ $\quad \delta E_k = \varepsilon$


2) Can evaluate which combinations are attractive
wrt strong interactions.
in QM
= ⟨n| H_int |n⟩ + Σ'_k |⟨n|H_int|k⟩|^2 / (E_n − E_k) + ...
δE_k = ε Q_1 Q_2

2) Can evaluate which combinations are attractive wrt strong interactions.

in QM: $\delta E_n = \cancel{\langle n|H_{int}|n\rangle} + \sum_k{}' \frac{|\langle n|H_{int}|k\rangle|^2}{E_n - E_k} + \dots$

$|q_1 q_2\rangle$ $\qquad \mathcal{L}_{int} = i g_s (\bar{q} \gamma^\mu \lambda_a q) G^a_\mu$

$|\bar{q}_1 \bar{q}_2\rangle$

$\mathcal{L}_{int} = ie \bar{f} \gamma^\mu f A_\mu \qquad \delta E_k = \varepsilon Q_1 Q_2 \qquad \varepsilon > 0$ (like charges repel)

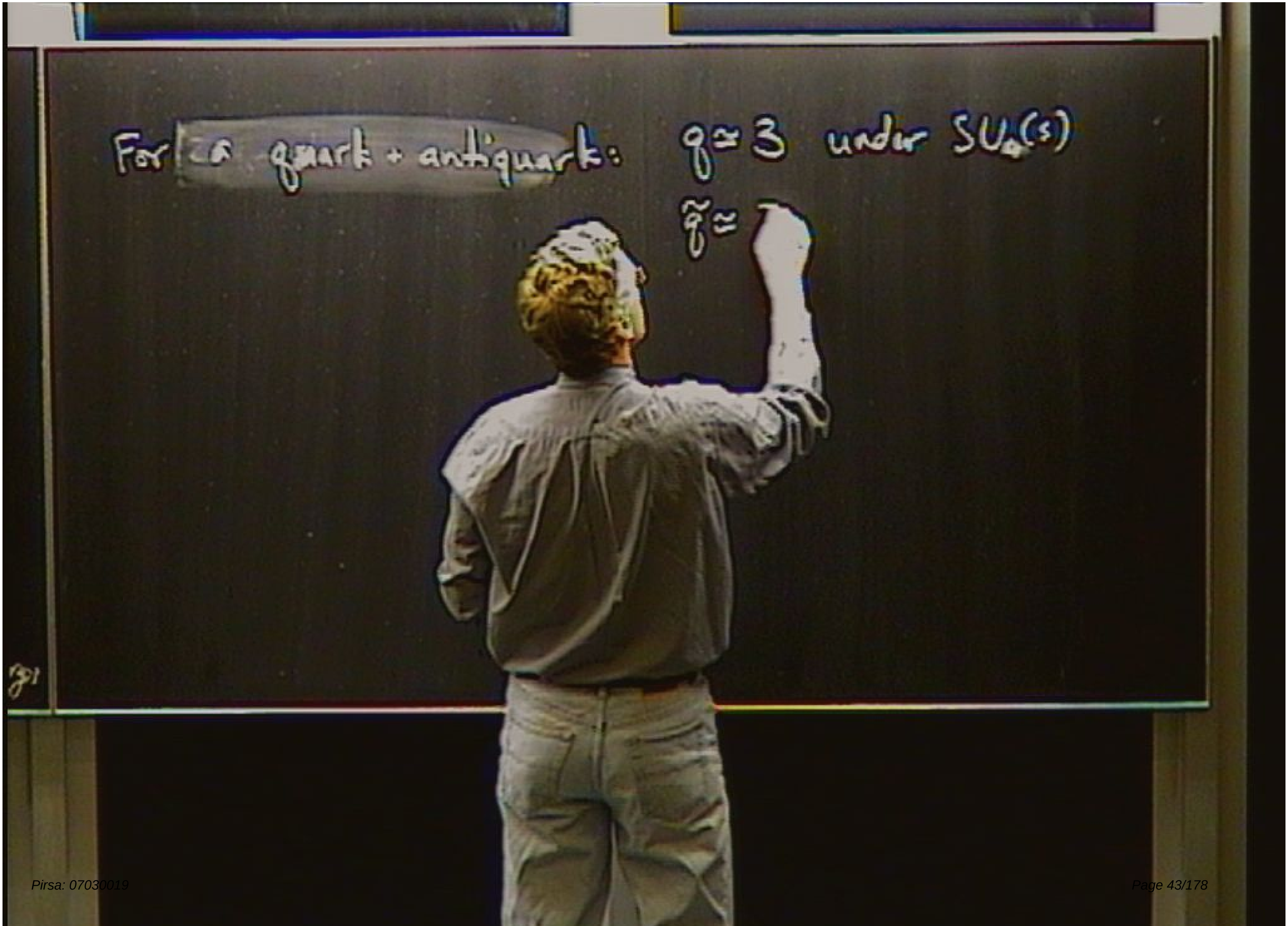
2) Can evaluate which combinations are attractive wrt strong interactions.

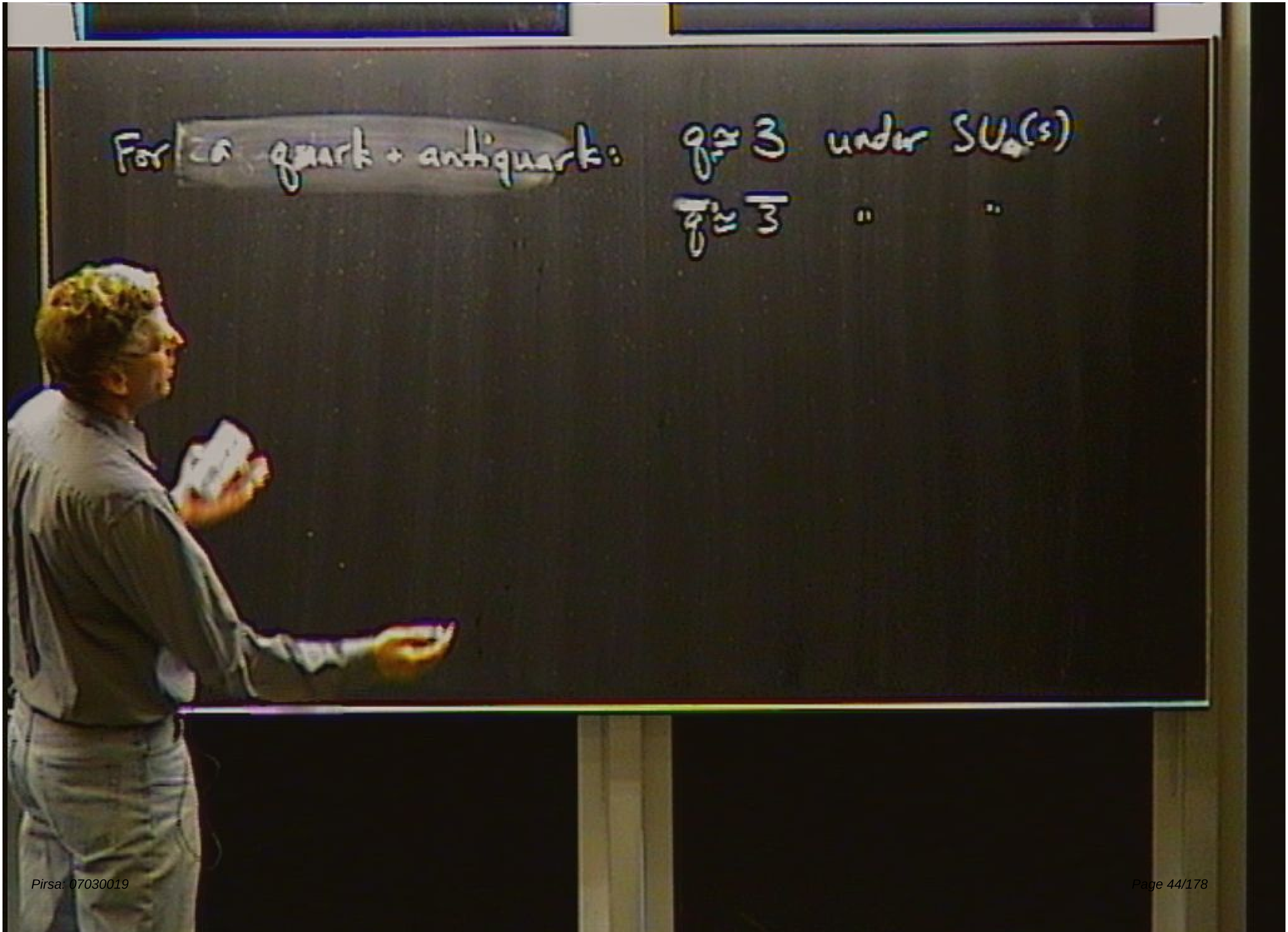
in QM: $\delta E_n = \cancel{\langle n | H_{int} | n \rangle} + \sum_k{}' \frac{|\langle n | H_{int} | k \rangle|^2}{E_n - E_k} + \dots$

$|q_1 q_2\rangle$ $\quad \mathcal{L}_{int} = i g_s (\bar{q} \gamma^\mu \lambda_a q) G^a_\mu$

$|\bar{q}_1 \bar{q}_2\rangle$

$\mathcal{L}_{int} = ie \bar{f} \gamma^\mu f A_\mu$ $\qquad \delta E_k = \varepsilon Q_1 Q_2$ $\qquad \varepsilon > 0$ (like charges repel)

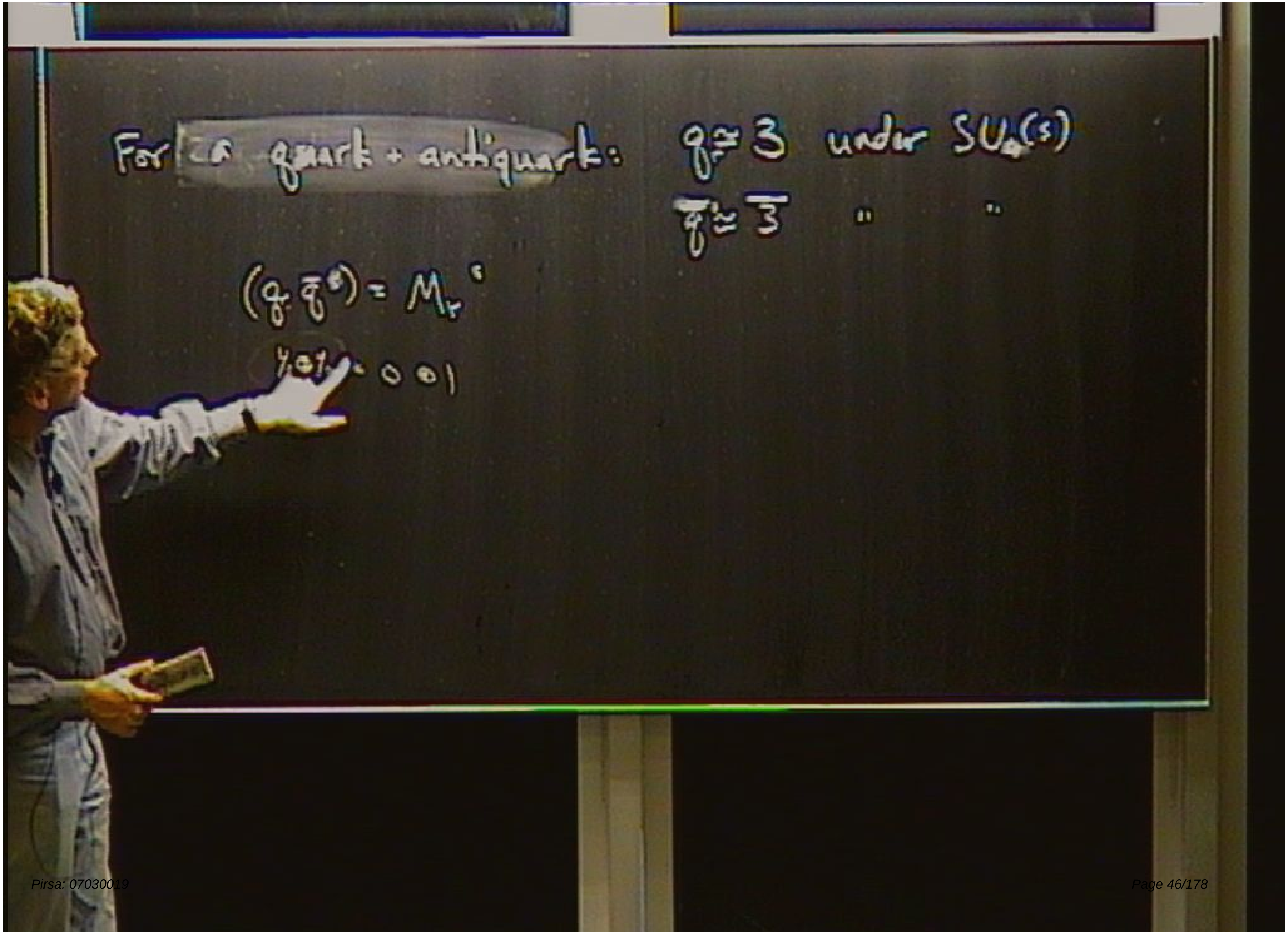

For a quark + antiquark: q ≃ 3 under SU_c(3)
q̃ ≃


For a quark + antiquark: $q \simeq 3$ under $SU_c(3)$
$\bar{q} \simeq \bar{3}$ " "

For a quark + antiquark: $q \simeq 3$ under $SU_c(3)$

$\bar{q} \simeq \bar{3}$ " "

$(q_i \bar{q}^j) = M_i^{\ j}$

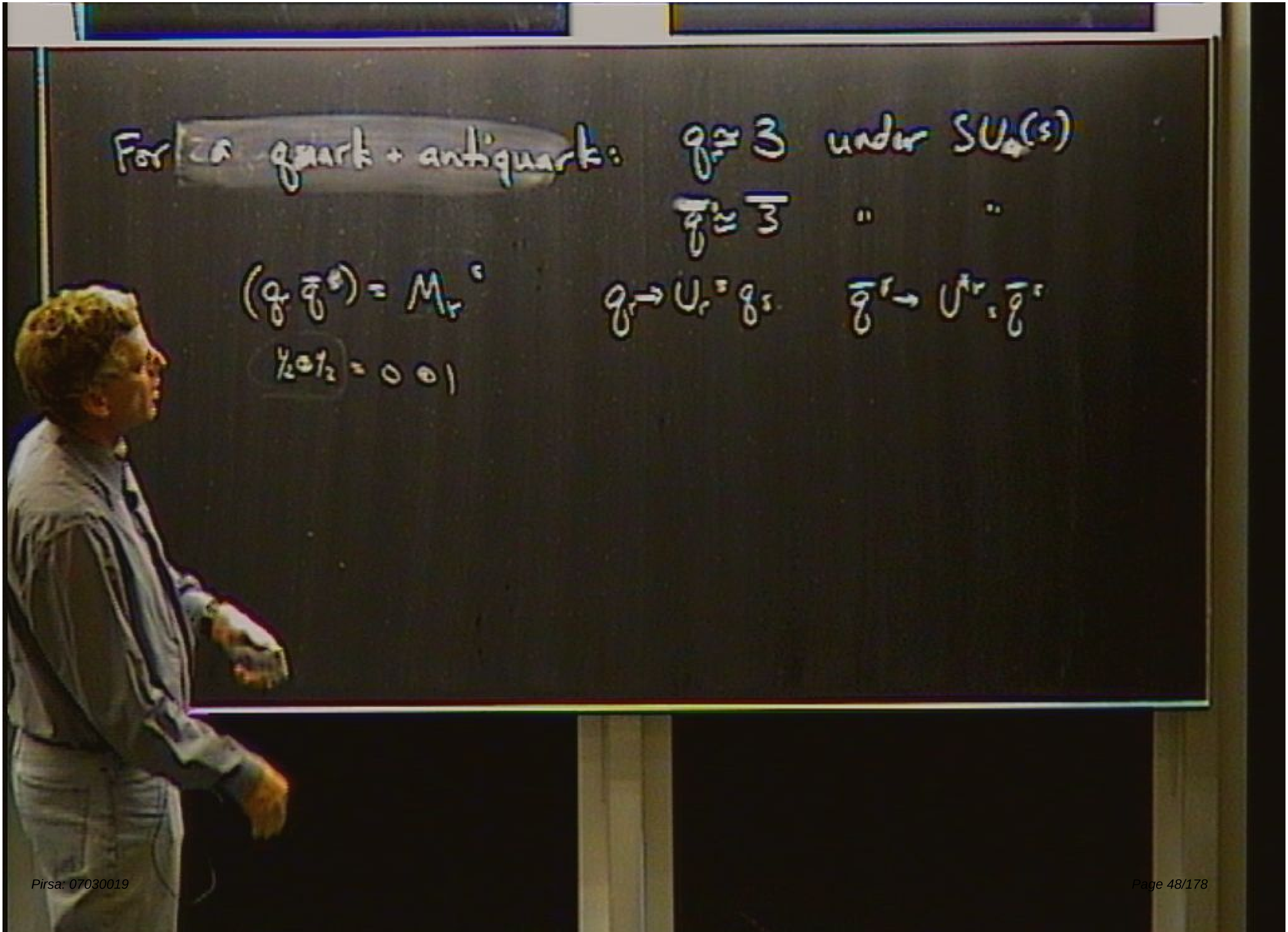

For a quark + antiquark:
q ≃ 3 under SU(3)
q̄ ≃ 3̄ " "
0 ⊕ 1

For quark + antiquark: $q \simeq 3$ under $SU_c(3)$

$\bar{q} \simeq \bar{3}$ " "

$(q\,\bar{q}^{s}) = M_r{}^{s}$

$\oplus 1$


For a quark + antiquark: $q \simeq 3$ under $SU_c(3)$
$\bar{q} \simeq \bar{3}$ " "
$(q_r \bar{q}^s) = M_r{}^s$
$q_r \to U_r{}^s q_s$
$\bar{q}^r \to U^{*r}{}_s \bar{q}^s$
$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

For a quark + antiquark: $q_r \simeq 3$ under $SU_c(3)$

$\bar{q}^r \simeq \bar{3}$ " "

$(q_r \bar{q}^s) = M_r{}^s$ $\qquad$ $q_r \to U_r{}^s q_s$ $\qquad$ $\bar{q}^r \to U^{*r}{}_s \bar{q}^s$

$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

$M \to U M U^\dagger$

For a quark + antiquark: $q \simeq 3$ under $SU_c(3)$

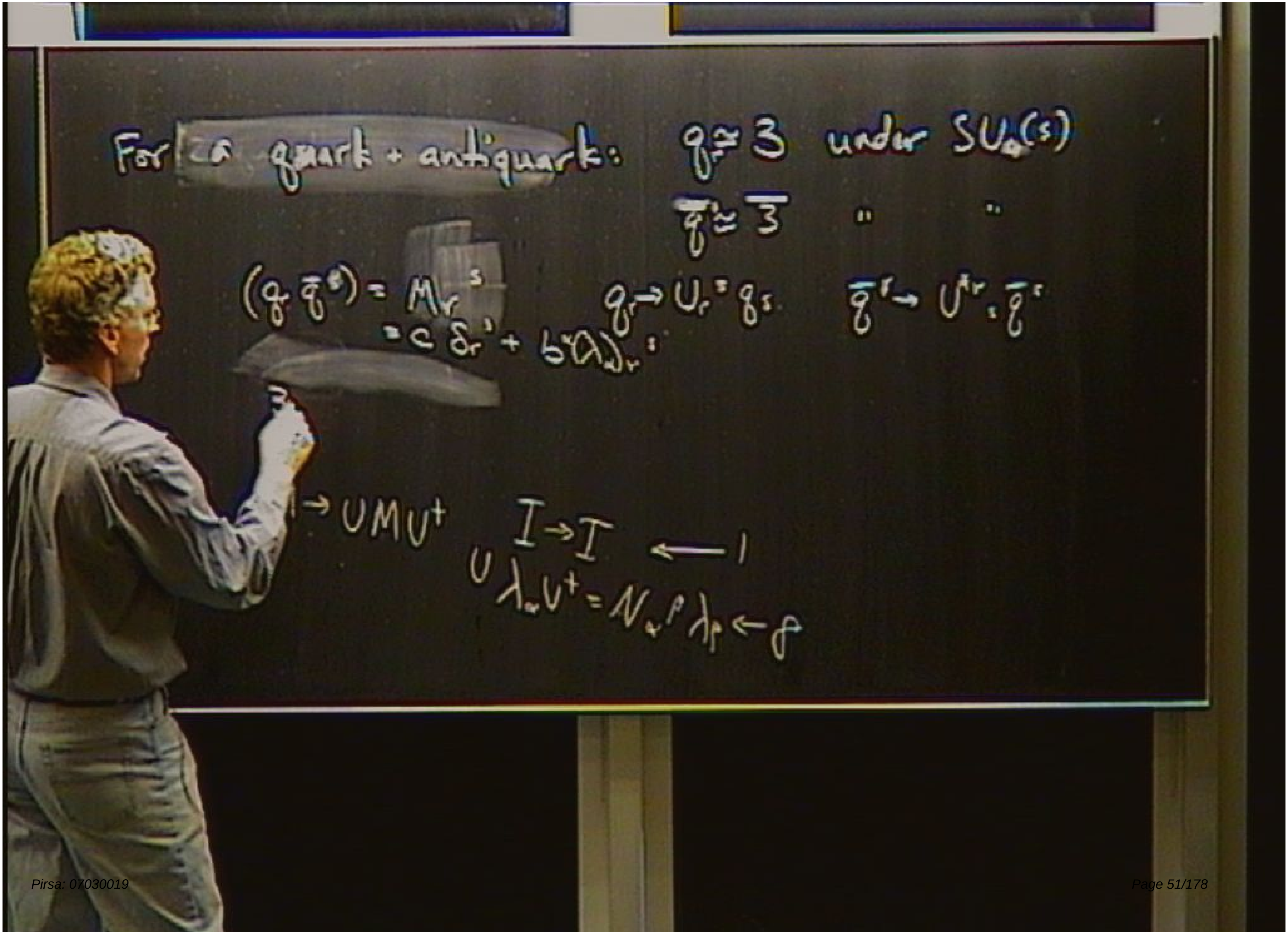
$\bar{q} \simeq \bar{3}$ " "

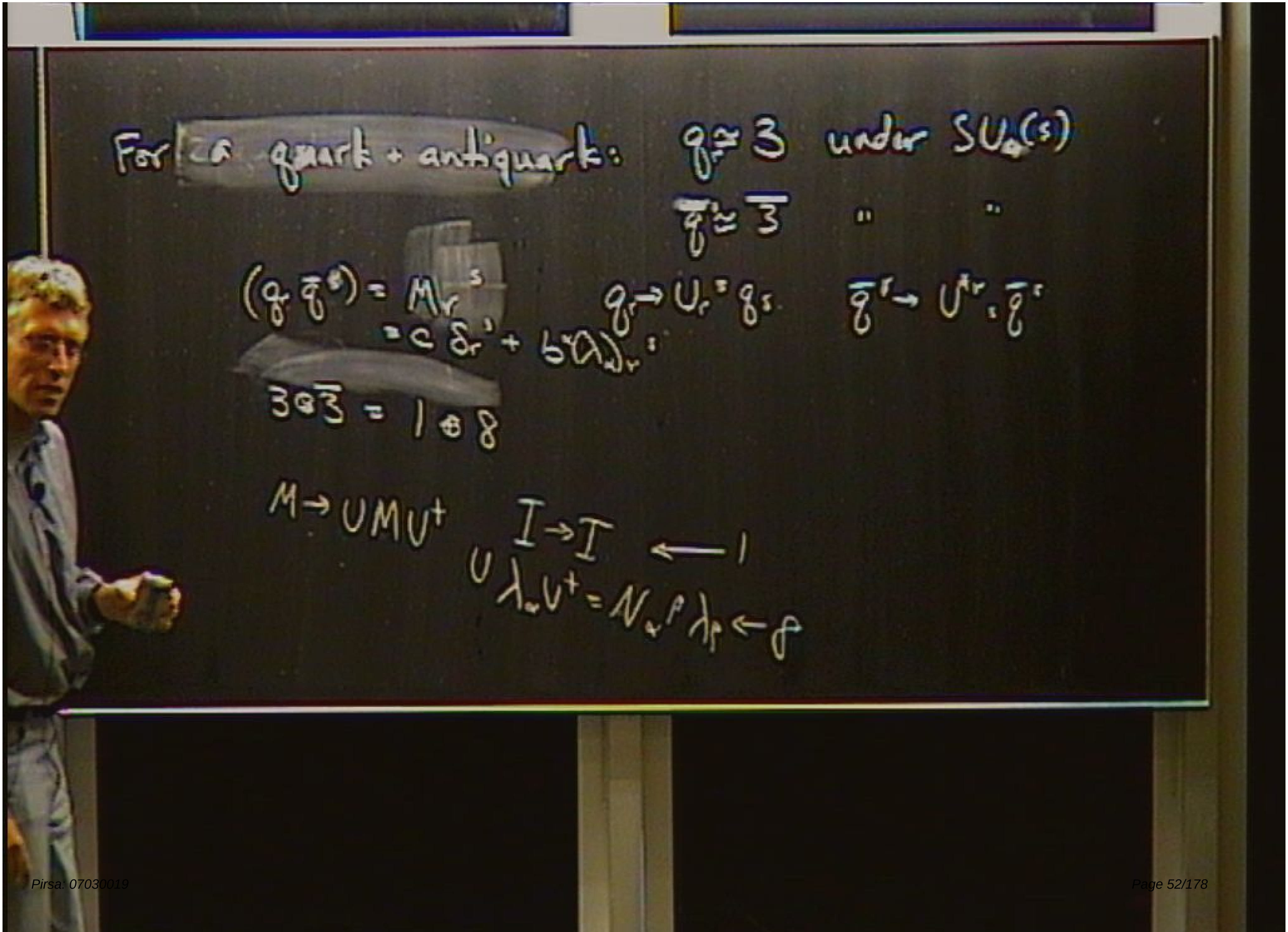
$(q_r \bar{q}^s) = M_r{}^s$ $q_r \to U_r{}^s q_s$ $\bar{q}^r \to U^{\dagger r}{}_s \bar{q}^s$

$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

$M \to U M U^\dagger$ $I \to I$

$U \lambda_\alpha U^\dagger = N_\alpha{}^\beta \lambda_\beta$


For a quark + antiquark: $q_r \simeq 3$ under $SU_c(3)$
$\bar{q}^r \simeq \bar{3}$ " "
$(q_r \bar{q}^s) = M_r{}^s$
$= c\,\delta_r{}^s + b^\alpha (\lambda_\alpha)_r{}^s$
$q_r \to U_r{}^s q_s$
$\bar{q}^r \to U^{*r}{}_s \bar{q}^s$
$\to UMU^\dagger$
$I \to I \longleftarrow 1$
$U \lambda_\alpha U^\dagger = N_\alpha{}^\beta \lambda_\beta \leftarrow 8$


For a quark + antiquark: q ≅ 3 under SU_c(3)
q̄ ≅ 3̄ " "
(q_r q̄^s) = M_r^s
= c δ_r^s + b^α (λ_α)_r^s
q_r → U_r^s q_s
q̄^r → U^{*r}_s q̄^s
3 ⊗ 3̄ = 1 ⊕ 8
M → UMU†
I → I ⟵ 1
U λ_α U† = N_α^β λ_β ← 8

For a quark + antiquark: $q \simeq 3$ under $SU_c(3)$

$\bar{q} \simeq \bar{3}$ " "

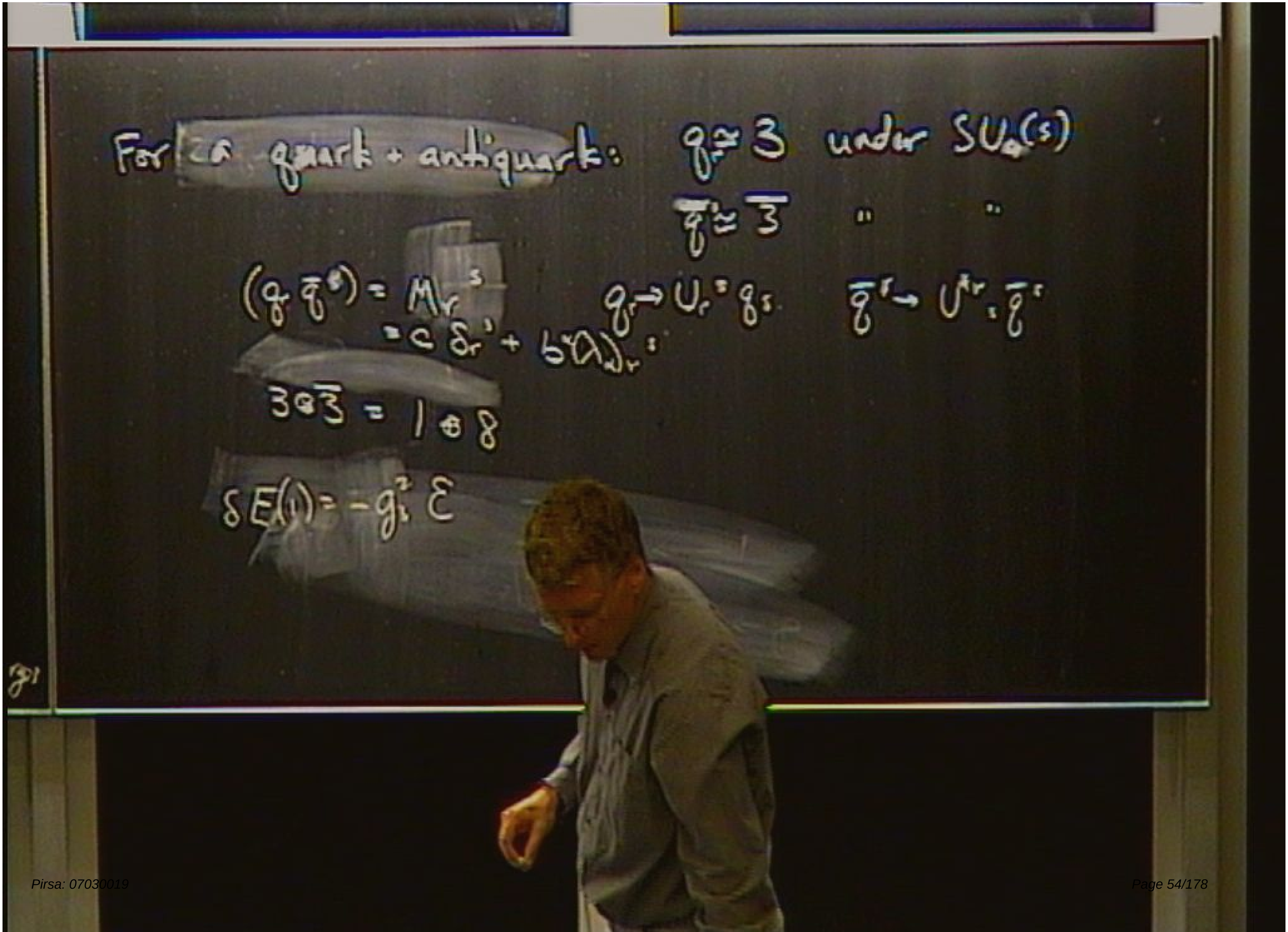
$(q_r \bar{q}^s) = M_r{}^s$ $\quad q_r \to U_r{}^s q_s \qquad \bar{q}^r \to U^{*r}{}_s \bar{q}^s$

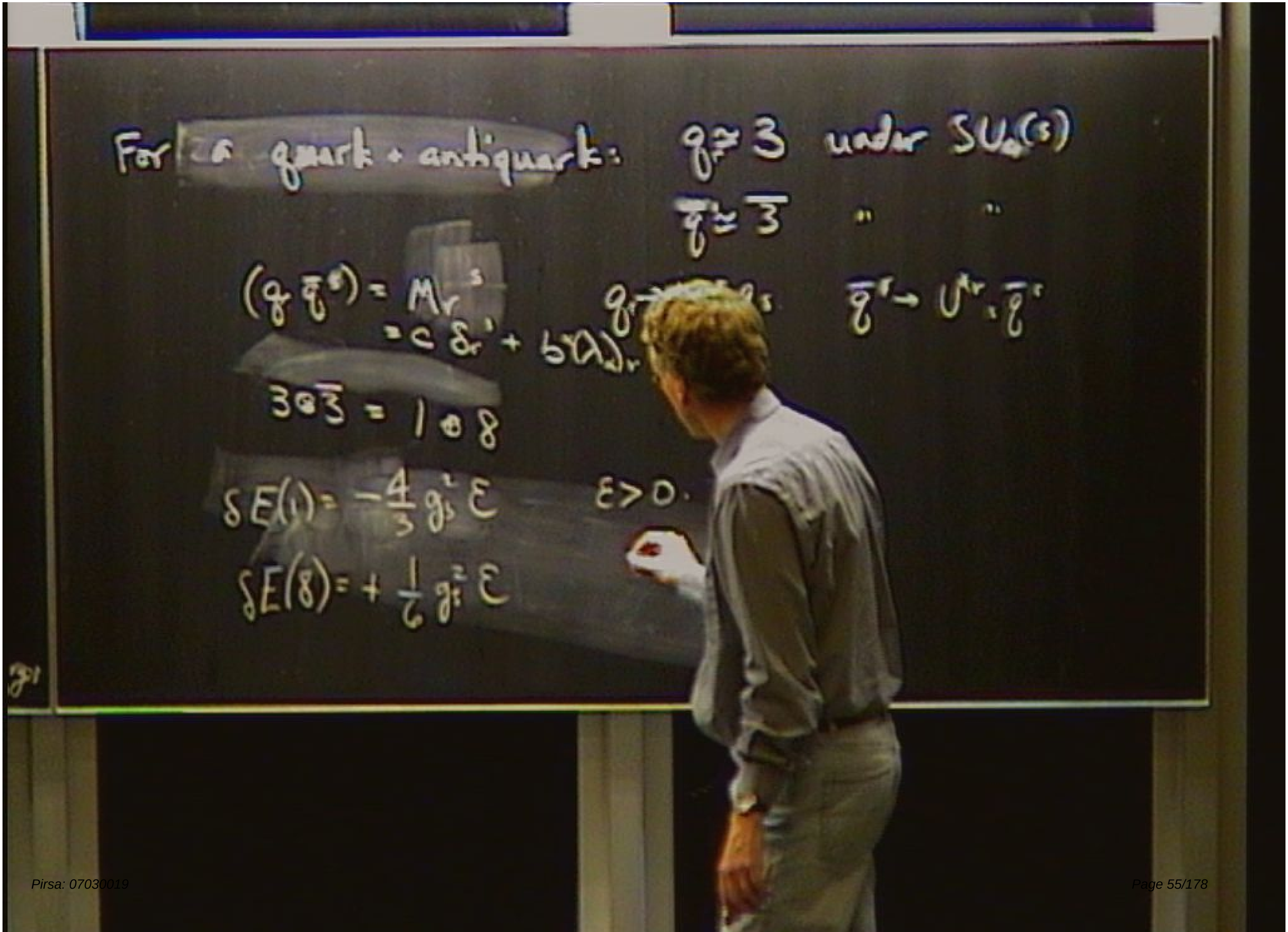
$= c\, \delta_r{}^s + b^\alpha (\lambda_\alpha)_r{}^s$

$3 \otimes \bar{3} = 1 \oplus 8$

$M \to U M U^\dagger \qquad I \to I \longleftarrow 1$

$U \lambda_\alpha U^\dagger = N_\alpha{}^\beta \lambda_\beta \leftarrow 8$


For a quark + antiquark: q ≃ 3 under SU_c(3)
q̄ ≃ 3̄ " "
(q q̄^s) = M_r^s
= c δ_r^s + b (λ)_r^s
q_r → U_r^s q_s
q̄^r → U*^r_s q̄^s
3 ⊗ 3̄ = 1 ⊕ 8
δE(1) = −g_s^2 ε


For a quark + antiquark: $q \cong 3$ under $SU_c(3)$
$\bar{q} \cong \bar{3}$ " "
$(q_r \bar{q}^s) = M_r{}^s$
$= c\,\delta_r{}^s + b^a (\lambda_a)_r{}^s$
$\bar{q}^r \to U^{*r}{}_s \bar{q}^s$
$3 \otimes \bar{3} = 1 \oplus 8$
$\delta E(1) = -\frac{4}{3} g_s^2 \varepsilon$
$\varepsilon > 0.$
$\delta E(8) = +\frac{1}{6} g_s^2 \varepsilon$

For a quark + antiquark: $q_r \simeq 3$ under $SU_c(3)$

$\bar{q}^r \simeq \bar{3}$ " "

$(q_r \bar{q}^s) = M_r{}^s$ $\quad q_r \to U_r{}^s q_s \quad \bar{q}^r \to U^{*r}{}_s \bar{q}^s$

$= c\, \delta_r{}^s + b^a (\lambda_a)_r{}^s$

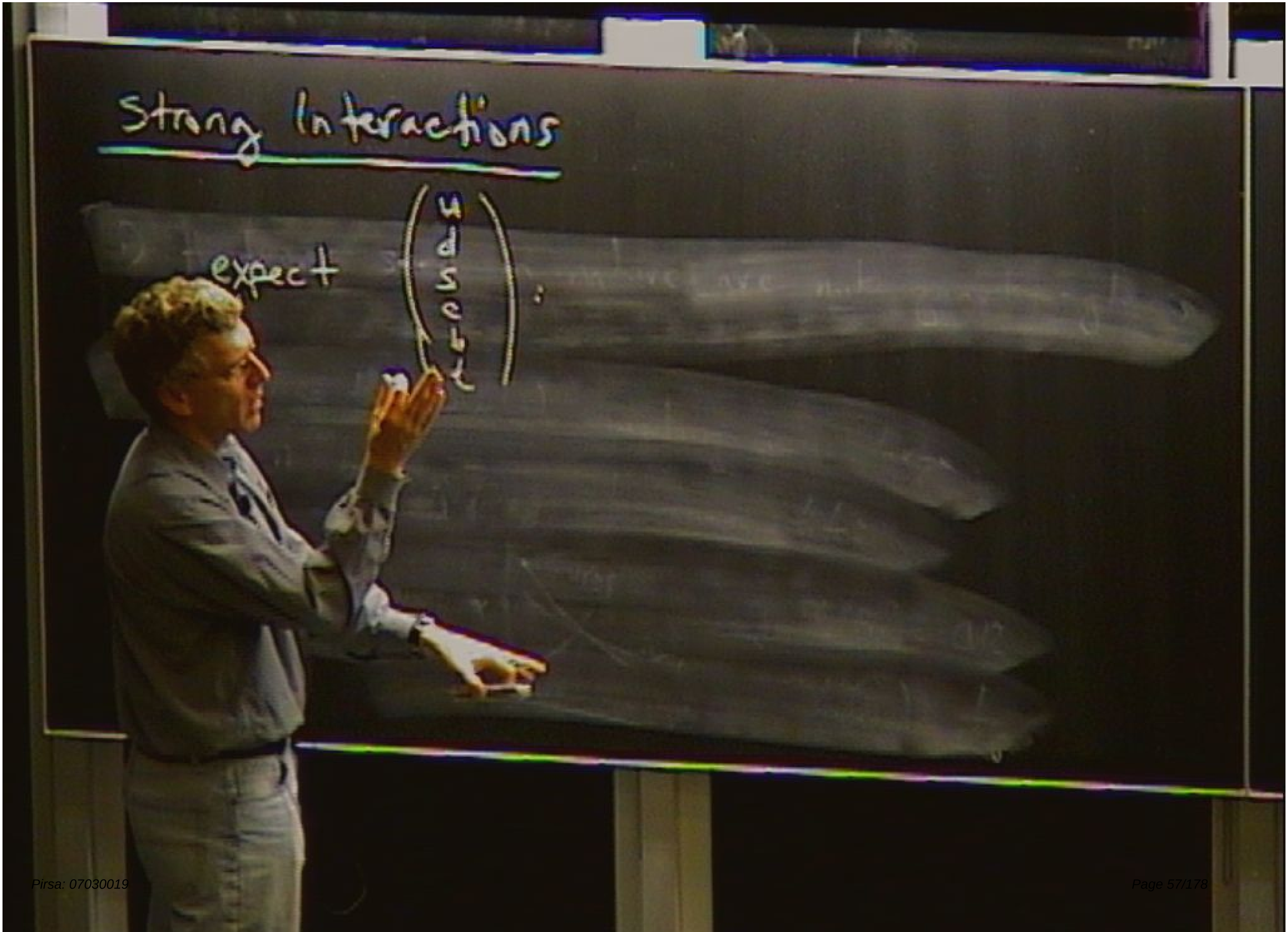
$3 \otimes \bar{3} = 1 \oplus 8$

$\delta E(1) = -\frac{4}{3} g_s^2 \mathcal{E} \qquad \mathcal{E} > 0.$

$\delta E(8) = +\frac{1}{6} g_s^2 \mathcal{E}$

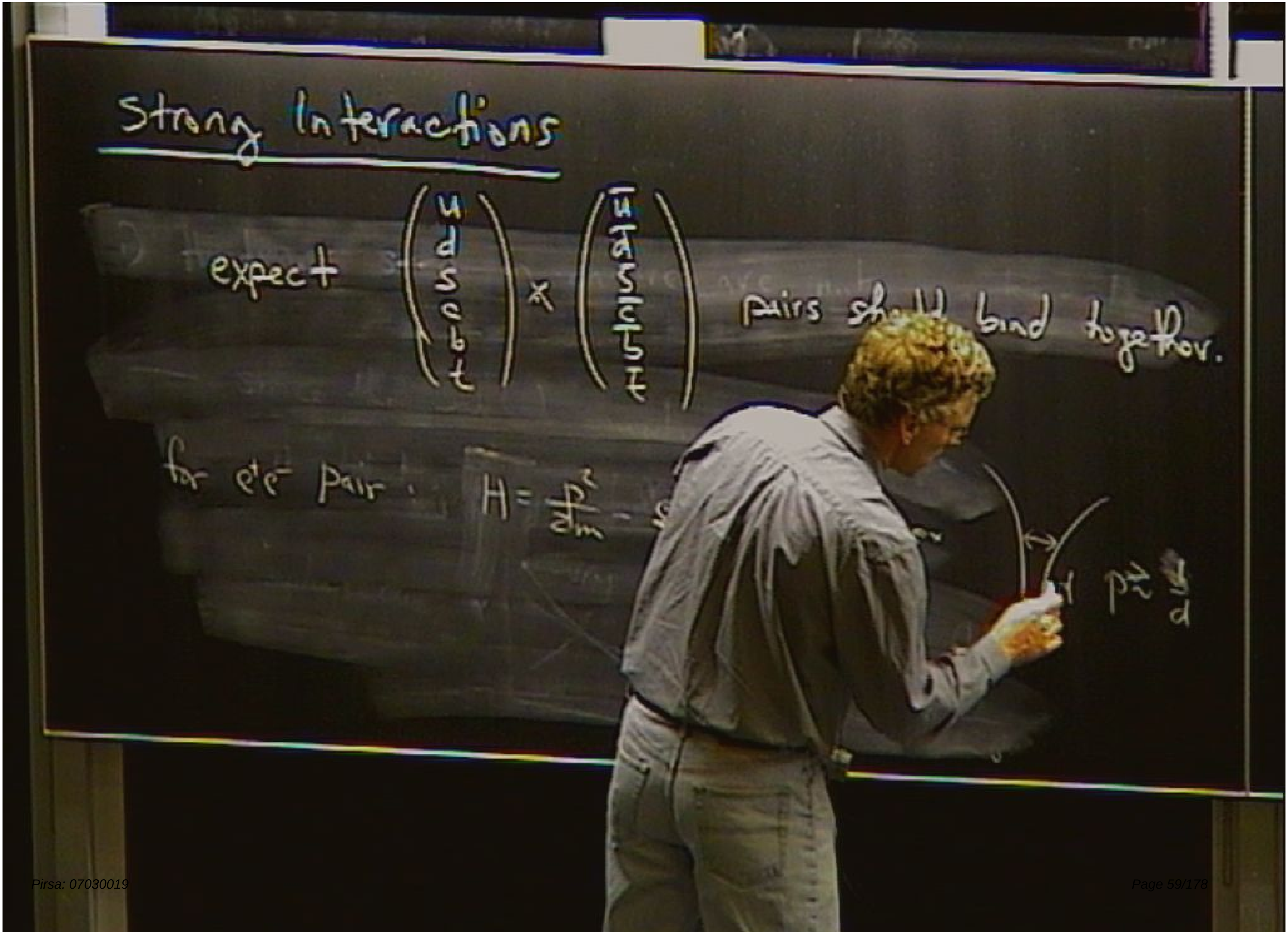
singlet channel is attractive

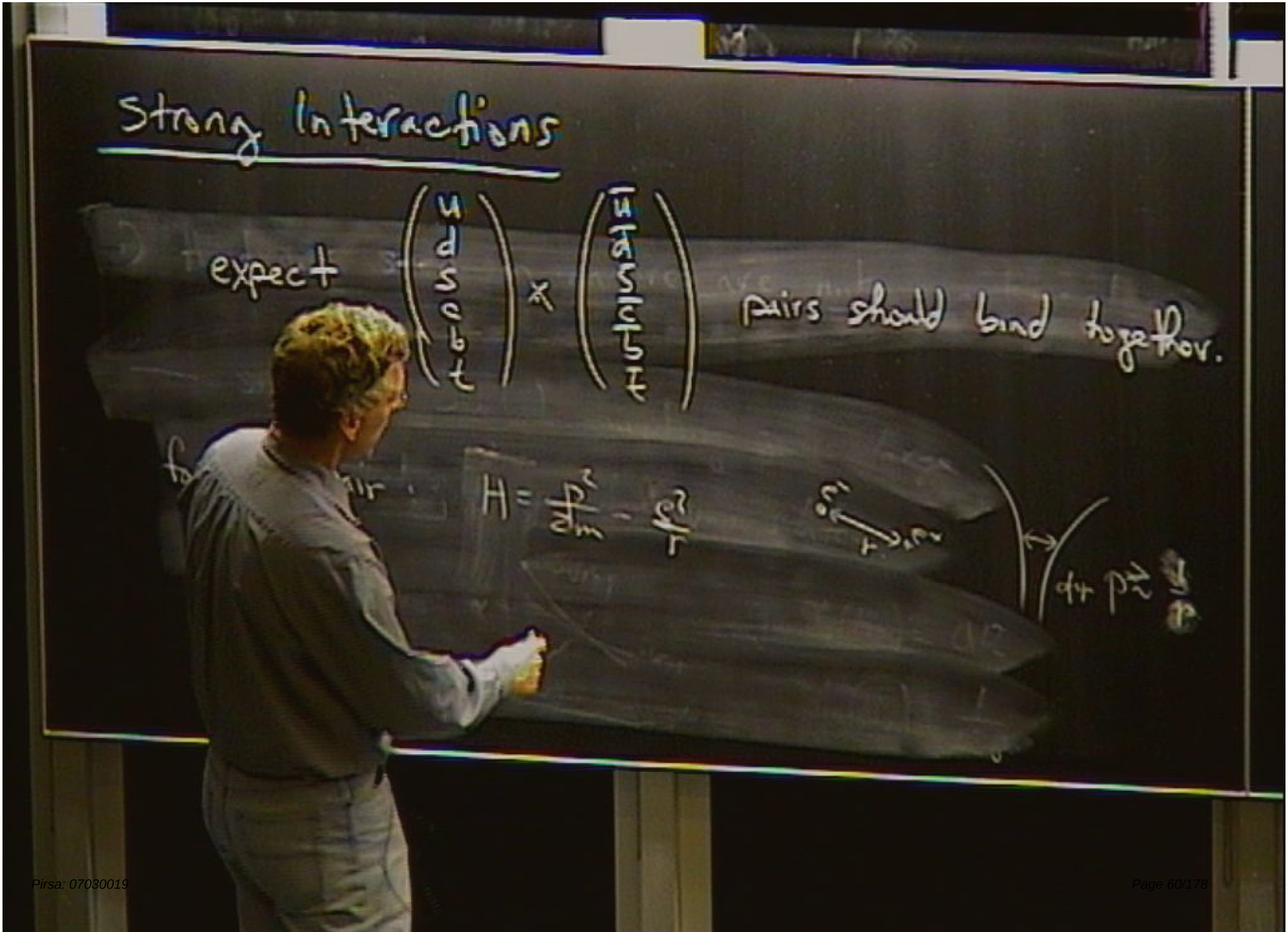
octet is repulsive (weak coupling)

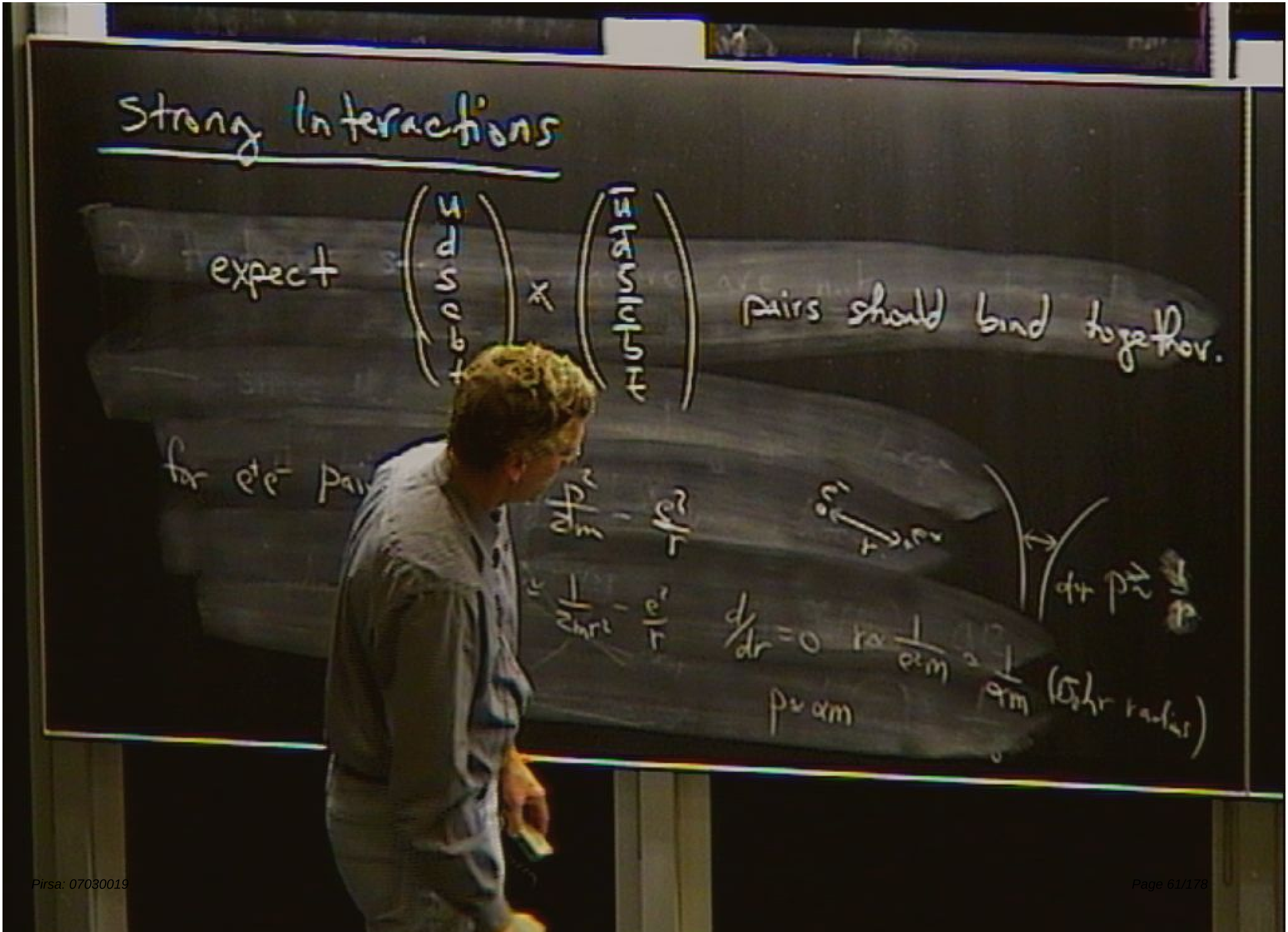

Strong Interactions
expect
u
d
s
c
b
t

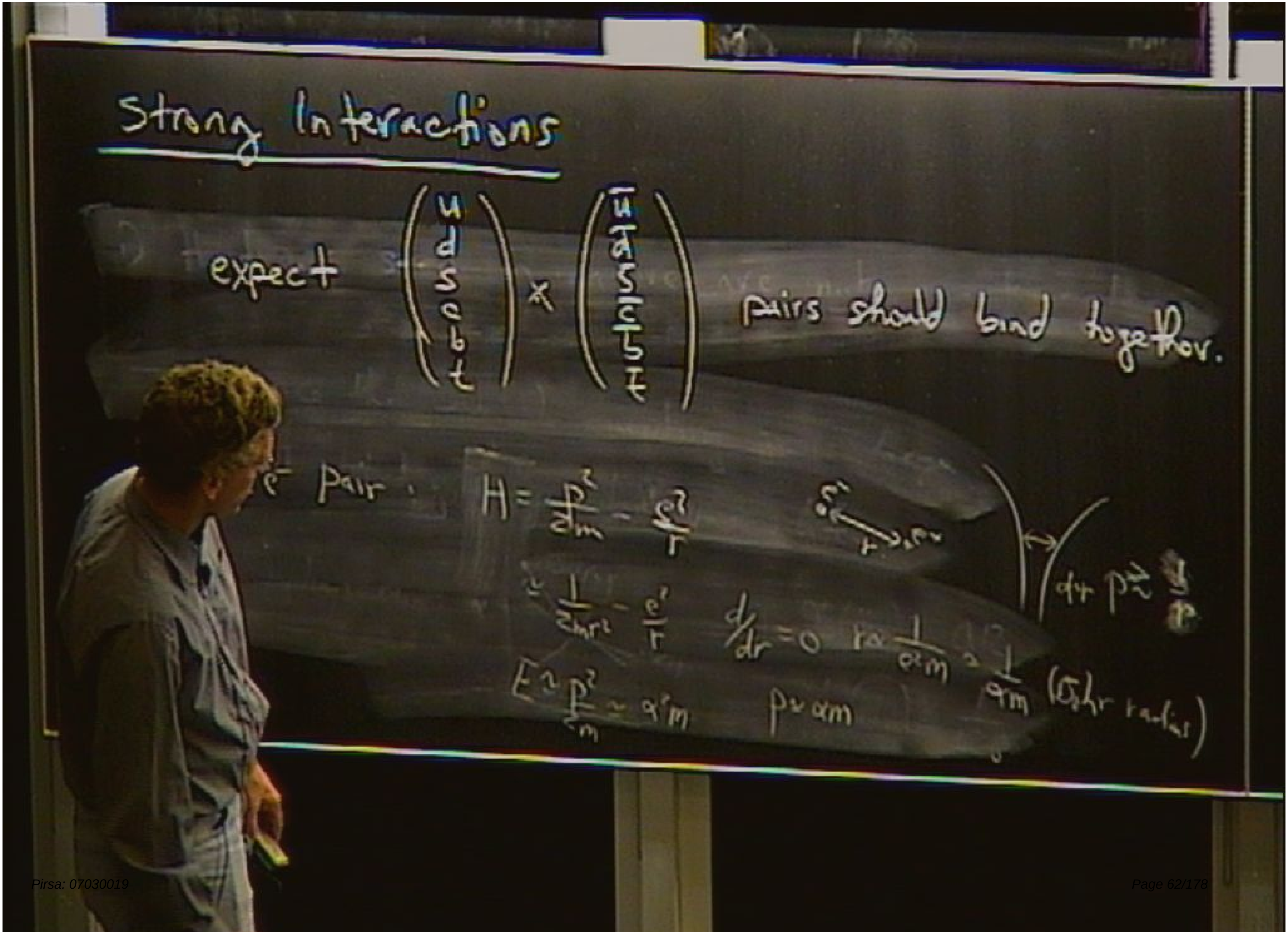
Strong Interactions

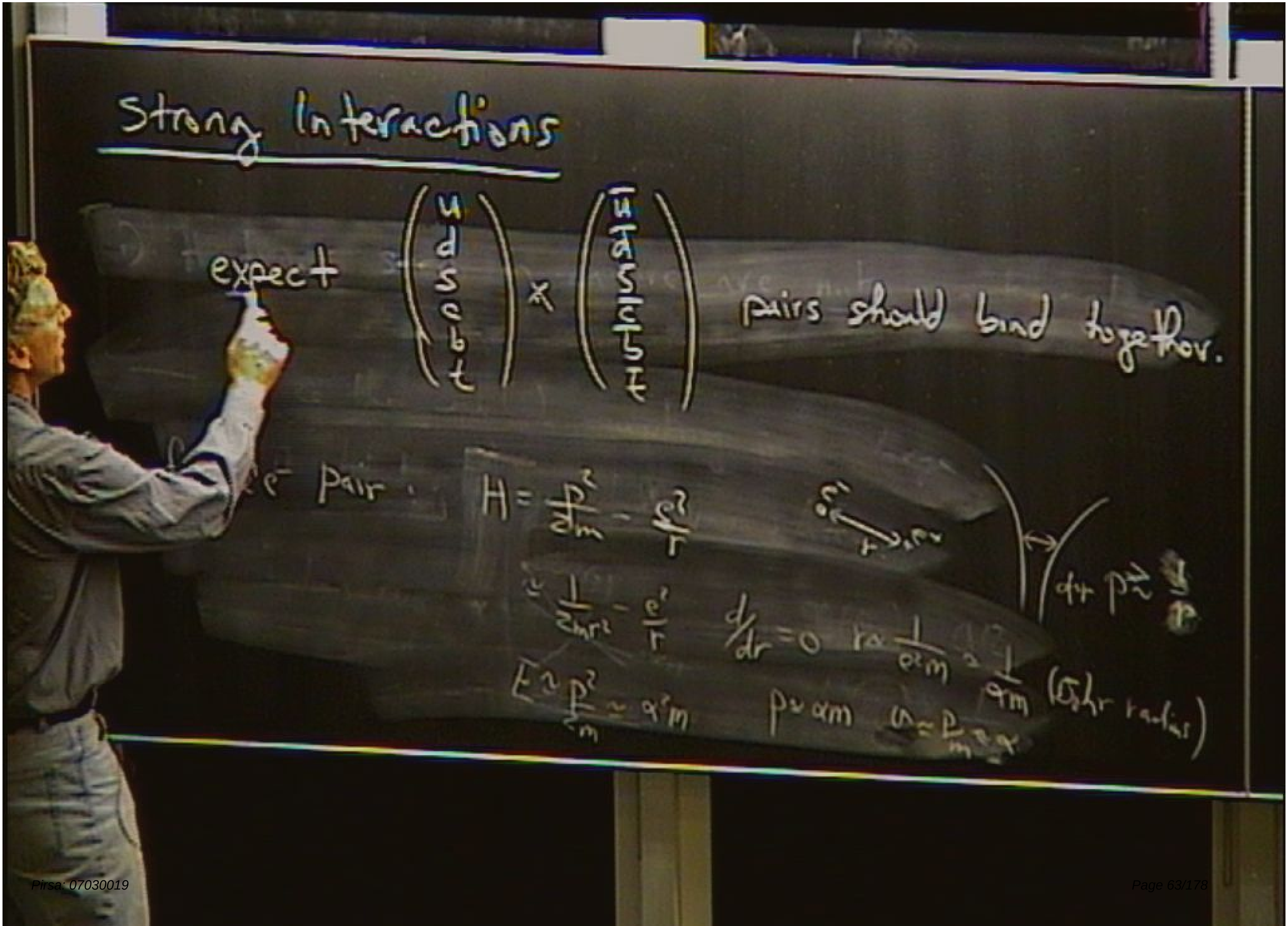
expect $\begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix}$ pairs should bind together.

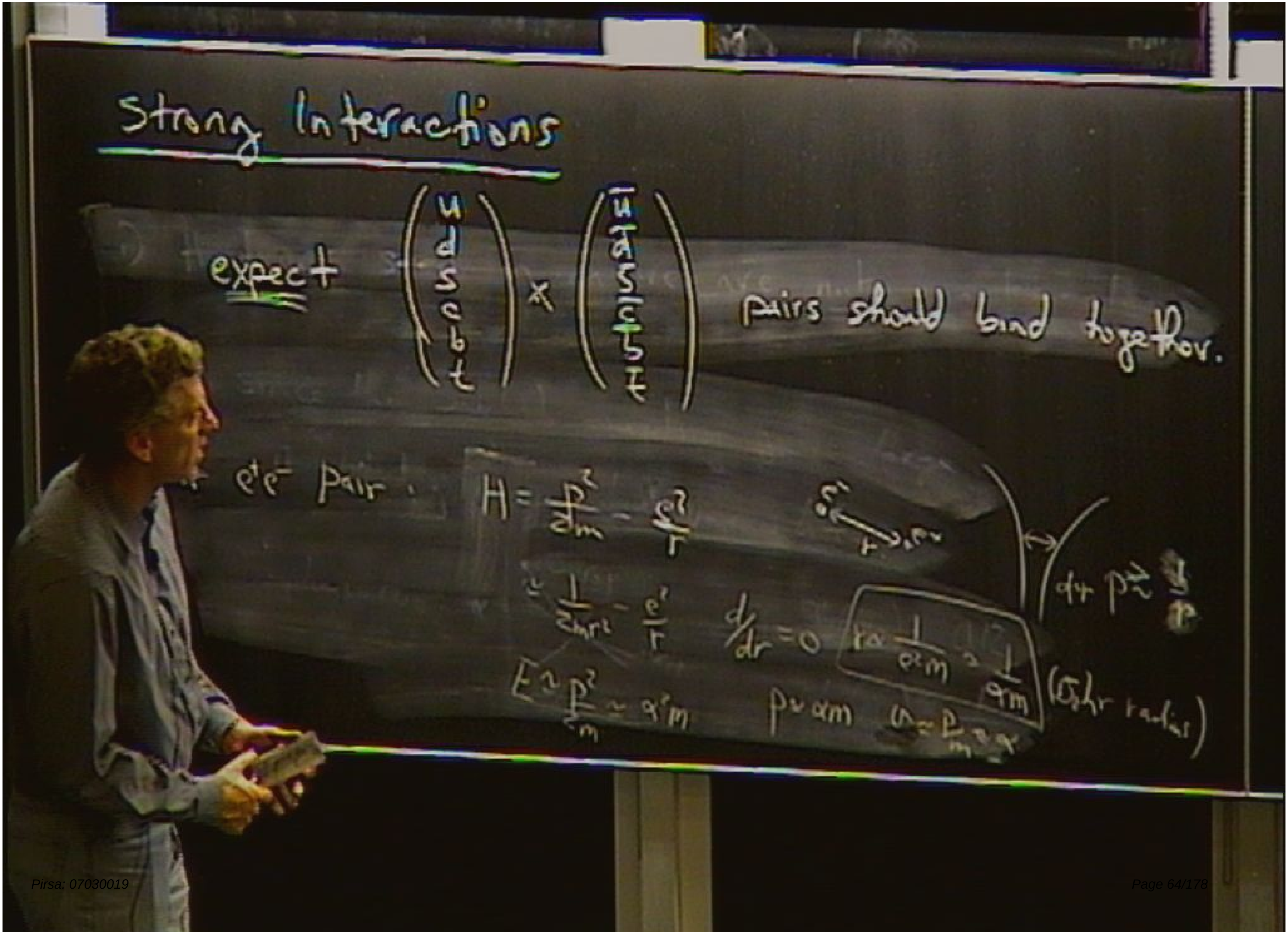

Strong Interactions
expect
u d s c b t
x
pairs sh... bind together.
for e⁺e⁻ pair:
H = p²/2m

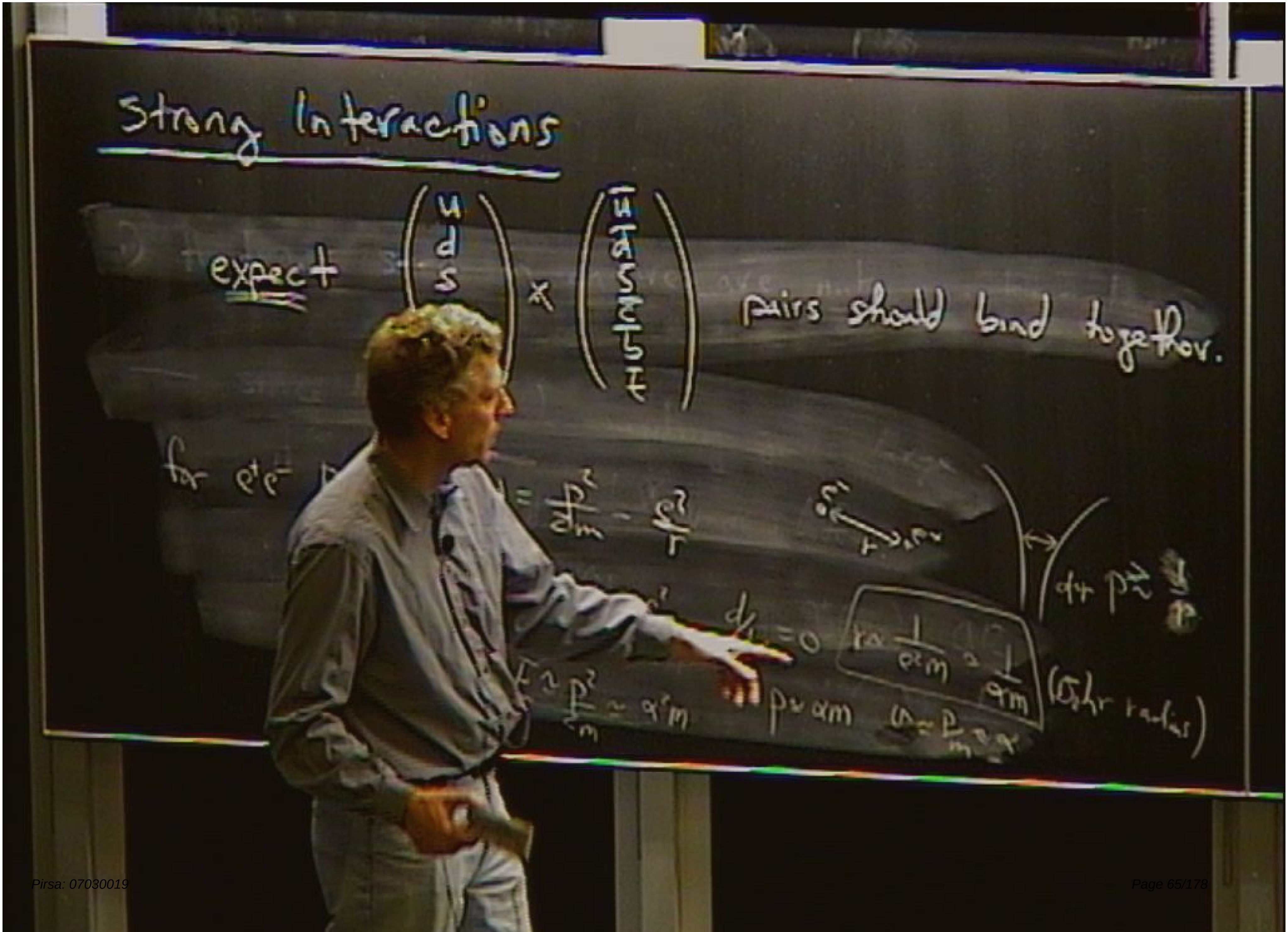

Strong Interactions
expect
u d s c b t
pairs should bind together.
H =

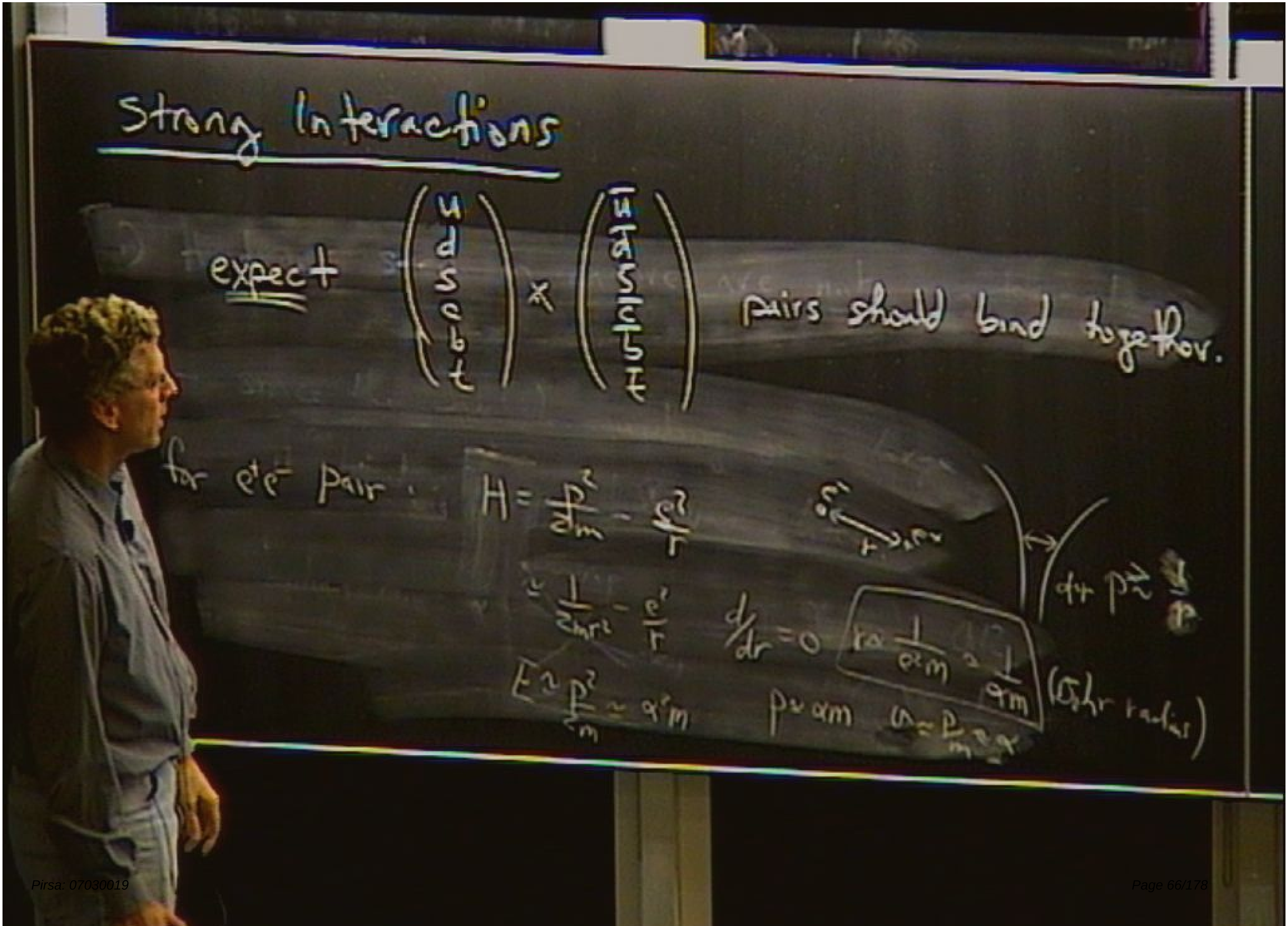

Strong Interactions
expect
pairs should bind together.
p ~ αm
(Bohr radius)

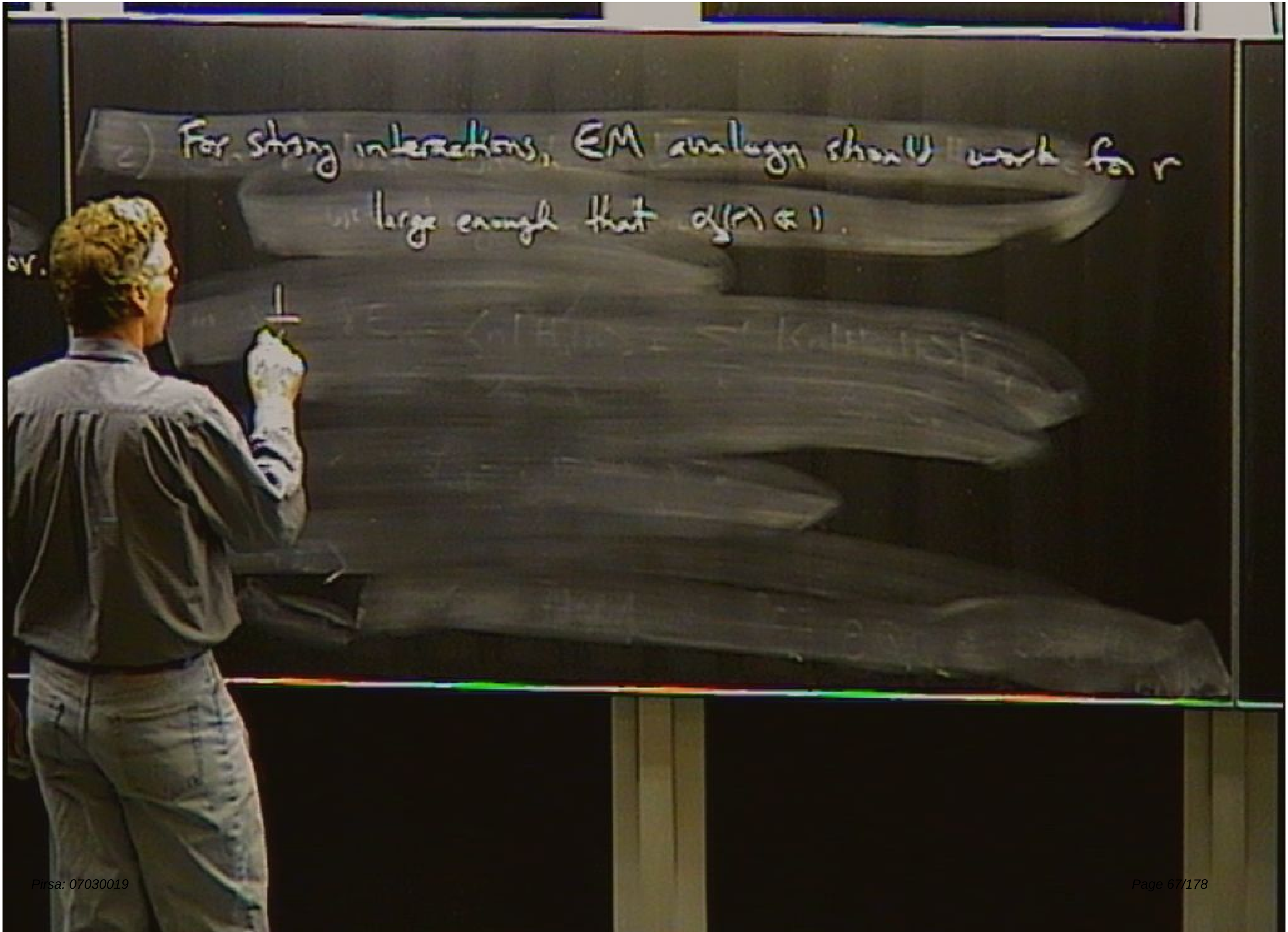

Strong Interactions
expect
pairs should bind together.
pair
$H = \frac{p^2}{2m} - \frac{e^2}{r}$
$\frac{d}{dr} = 0$
(Bohr radius)
$p \sim \alpha m$


Strong Interactions
expect
u d s c b t
x
ū d̄ s̄ c̄ b̄ t̄
pairs should bind together.
e⁻ pair
H = p²/2m − e²/r
≈ 1/2mr² − e²/r
d/dr = 0
r ≈ 1/e²m ≈ 1/αm
(Bohr radius)
E ≈ p²/2m ≈ α²m
p ≈ αm
v ≈ p/m ≈ α


Strong Interactions
expect
pairs should bind together.
e+e- pair.
H = p²/2m - e²/r
d/dr = 0
(Bohr radius)


Strong Interactions
expect
pairs should bind together.
for e+e−
(Bohr radius)


Strong Interactions
expect
u d s c b t
x
ū d̄ s̄ c̄ b̄ t̄
pairs should bind together.
for e+e- pair:
H = p²/2m - e²/r
≈ 1/2mr² - e²/r
d/dr = 0
E ≈ p²/2m ≈ α²m
p ≈ αm
(Bohr radius)


(2) For strong interactions, EM analogy should work for r
large enough that $\alpha_s(r) \ll 1$.
ov.

For strong interactions, EM analogy should work for r large enough that $\alpha_s(r) \ll 1$.

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\mu_0)} + b\alpha_s^2 \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

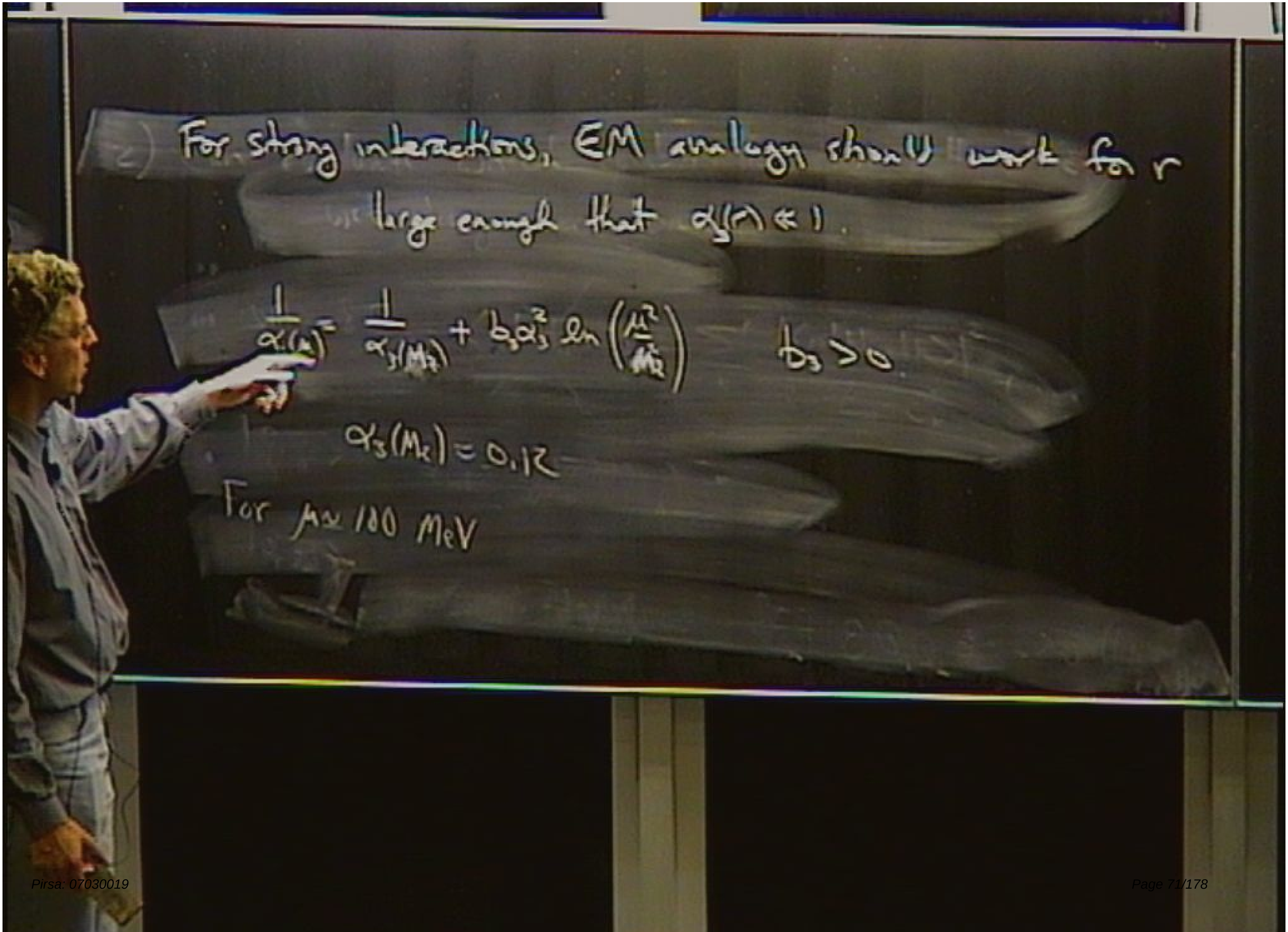
For strong interactions, EM analogy should work for r large enough that $\alpha_s(r) \ll 1$.

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\mu_0)} + b_3 \alpha_s^2 \ln\left(\frac{\mu^2}{\mu_0^2}\right) \qquad b_3 > 0$$

For strong interactions, EM analogy should work for r large enough that $\alpha_3(r) \ll 1$.

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_3(M_Z)} + b_3 \alpha_3^2 \ln\left(\frac{\mu^2}{M_Z^2}\right) \qquad b_3 > 0$$

$$\alpha_3(M_Z) = 0.12$$


For strong interactions, EM analogy should work for r
large enough that α_s(r) ≪ 1.
1/α_s(μ) = 1/α_3(M_Z) + b_3 α_3^2 ln(μ^2/M_Z^2)
b_3 > 0
α_3(M_Z) = 0.12
For μ ≈ 100 MeV

For strong interactions, EM analogy should work for r large enough that $\alpha_3(r) \ll 1$.

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_3(M_Z)} + b_3 \alpha_3^2 \ln\left(\frac{\mu^2}{M_Z^2}\right) \qquad b_3 > 0$$

$\alpha \approx 4\pi$

$$\alpha_3(M_Z) = 0.12$$

For $\mu \approx 100$ MeV

2) For strong interactions, EM analogy should work for r large enough that $\alpha_3(r) \ll 1$.

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_3(M_Z)} + b_3 \alpha_3^2 \ln\left(\frac{\mu^2}{M_Z^2}\right) \qquad b_3 > 0$$

$\alpha \simeq 4\pi$

$\alpha_3(M_Z) = 0.12$

For $\mu \simeq 100$ MeV $\quad \alpha_3 \gtrsim O(1)$

$\mu \simeq 1$ GeV

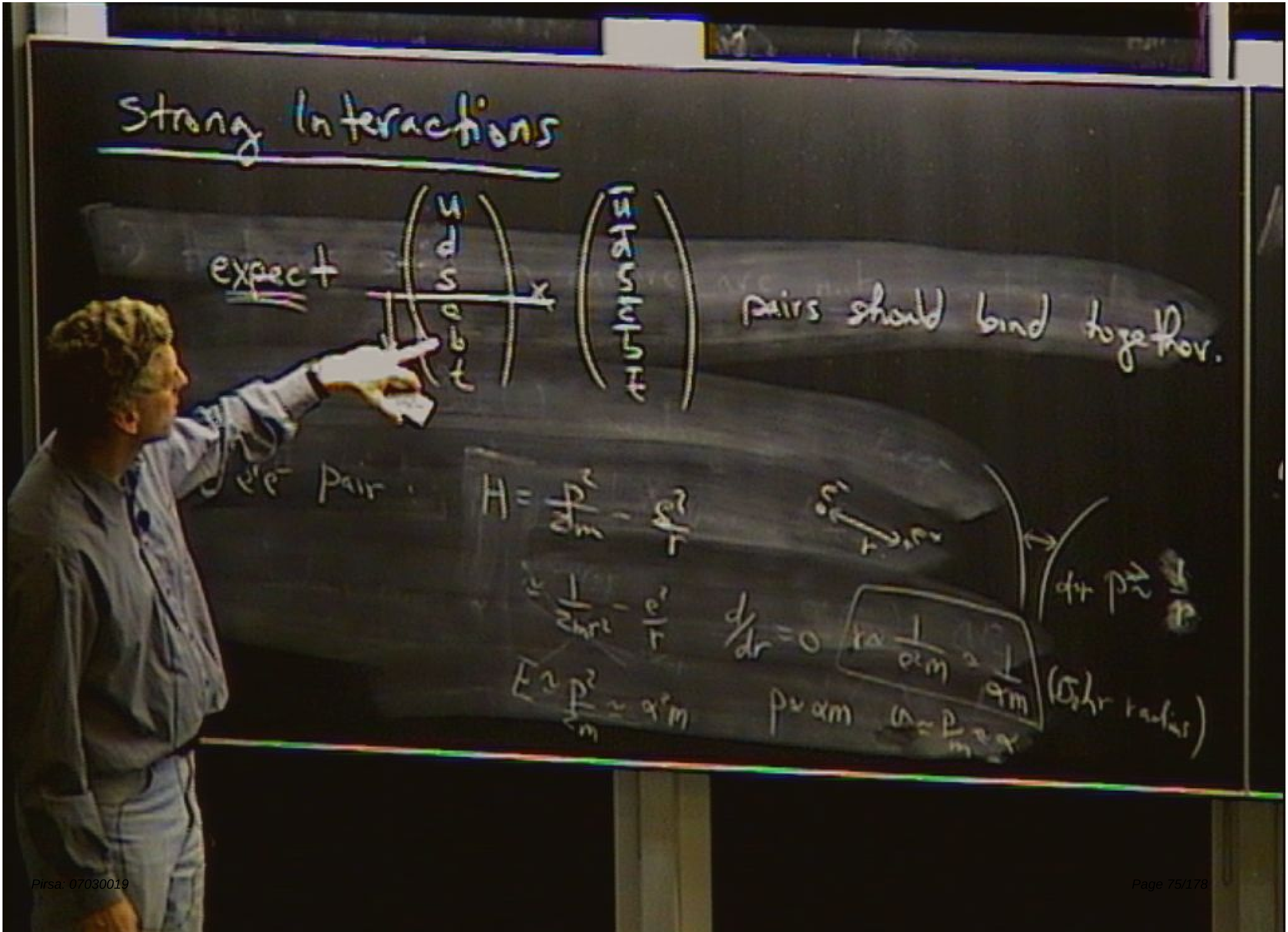
$\Lambda_{QCD} \simeq 200$ MeV

QCD scale

expect QED-like bound states.

with $\delta E \ll m_q$ for $q\bar{q}$

if $m_q \gg \Lambda_{QCD}$.


Strong Interactions
expect
pairs should bind together.
pair
(Bohr radius)

Strong Interactions

expect $\begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix}$ pairs should bind together.

for e^+e^- pair: $H = \frac{p^2}{2m} - \frac{e^2}{r}$

$\sim \frac{1}{2mr^2} - \frac{e^2}{r}$ $\quad d/dr = 0$ $\quad \boxed{r \sim \frac{1}{e^2 m} \sim \frac{1}{\alpha m}}$ (Bohr radius)

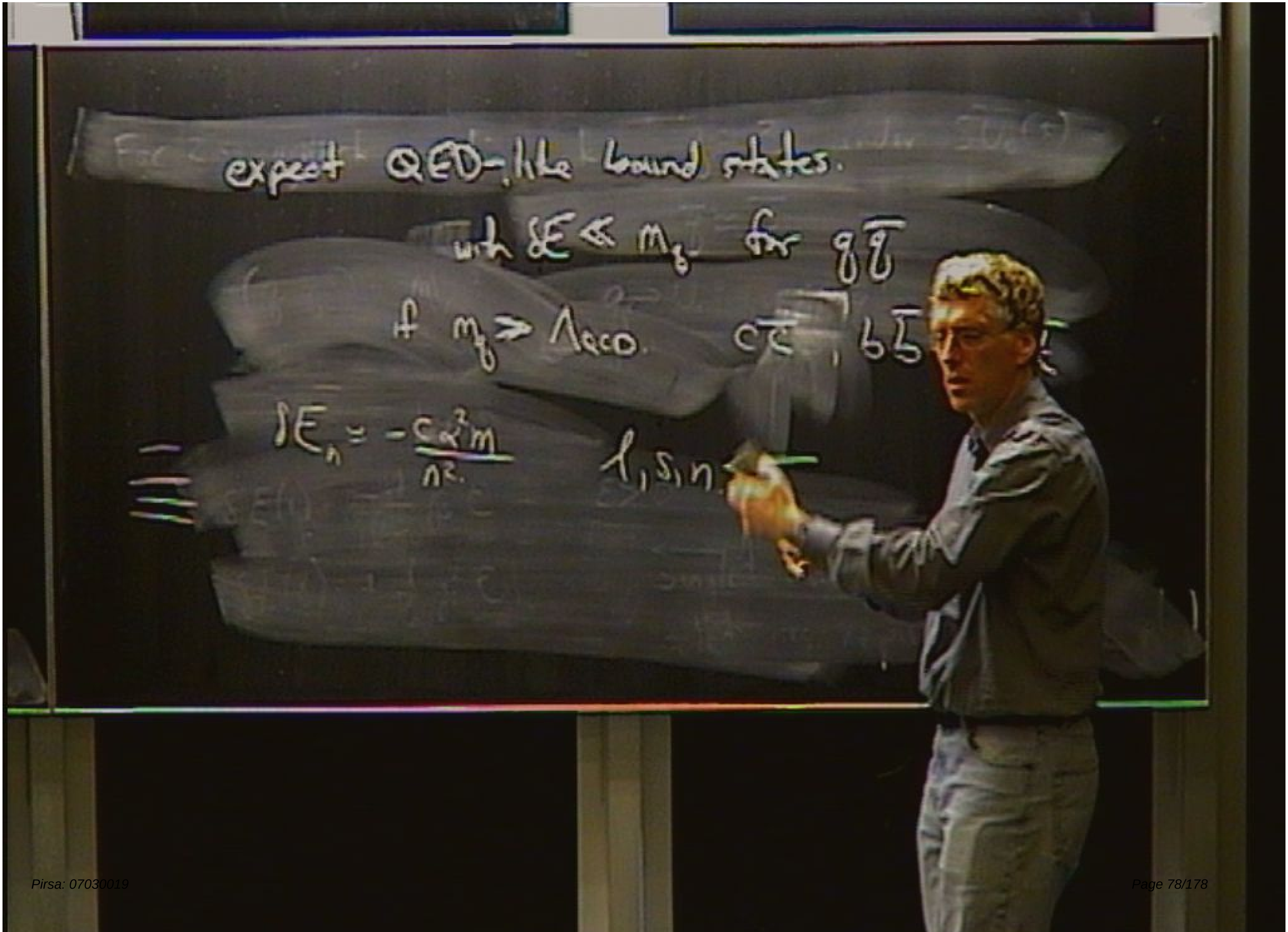
$\Delta r \, p \gtrsim \hbar$

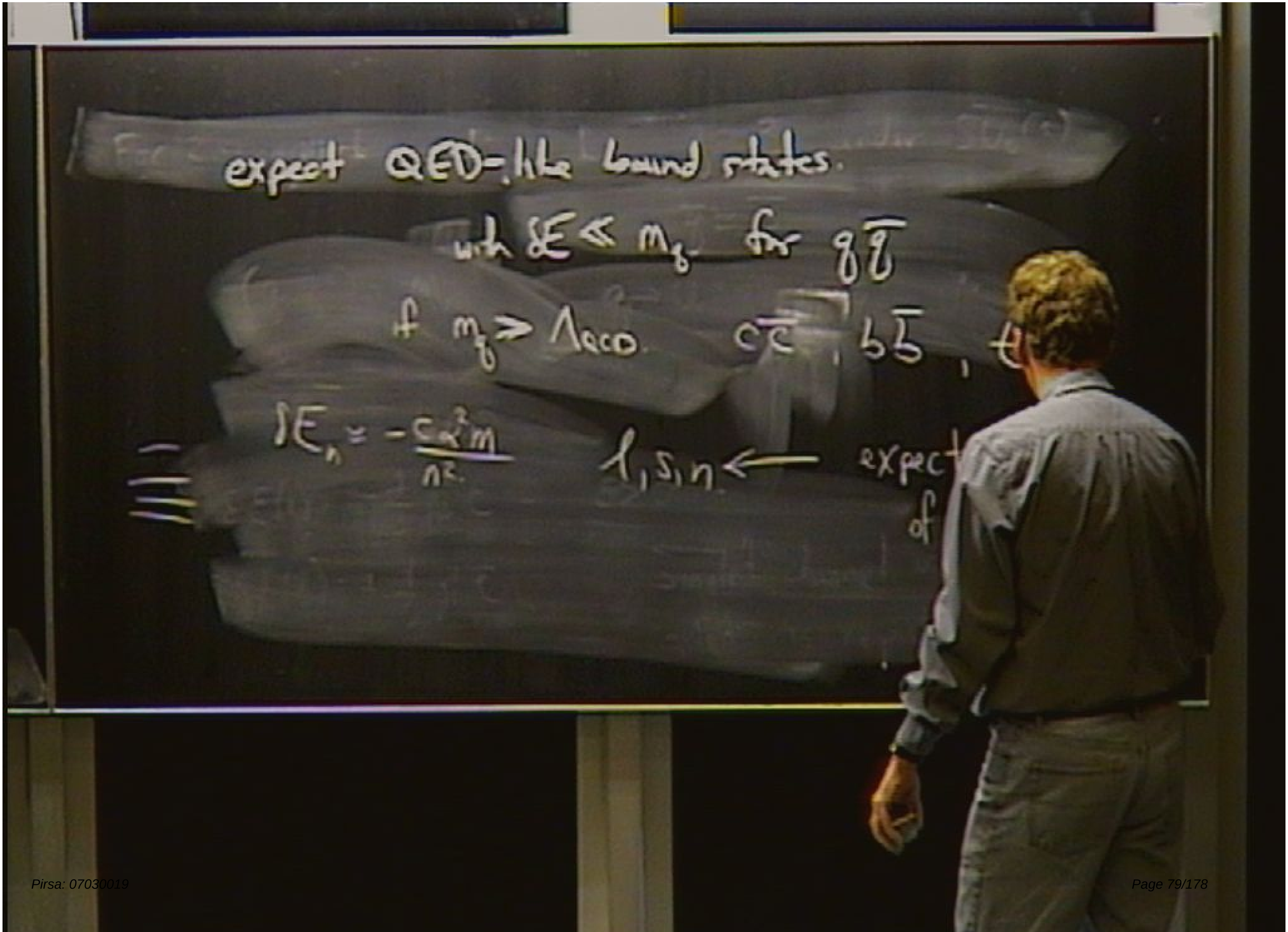
$E \sim \frac{p^2}{m} \sim \alpha^2 m \qquad p \sim \alpha m \qquad v \sim \frac{p}{m} \sim \alpha$

expect QED-like bound states.

with $\delta E \ll m_q$ for $q\bar{q}$

if $m_q \gg \Lambda_{QCD}$ $c\bar{c}$, $b\bar{b}$, $t\bar{t}$


expect QED-like bound states.
with δE ≪ m_q for q q̄
if m_q ≫ Λ_QCD c c̄, b b̄
δE_n = −cα²m / n²
l, s, n


expect QED-like bound states.
with δE ≪ m_q for q q̄
if m_q ≫ Λ_QCD c c̄, b b̄, t t̄
δE_n = -c α² m / n²
l, s, n ← expect
of

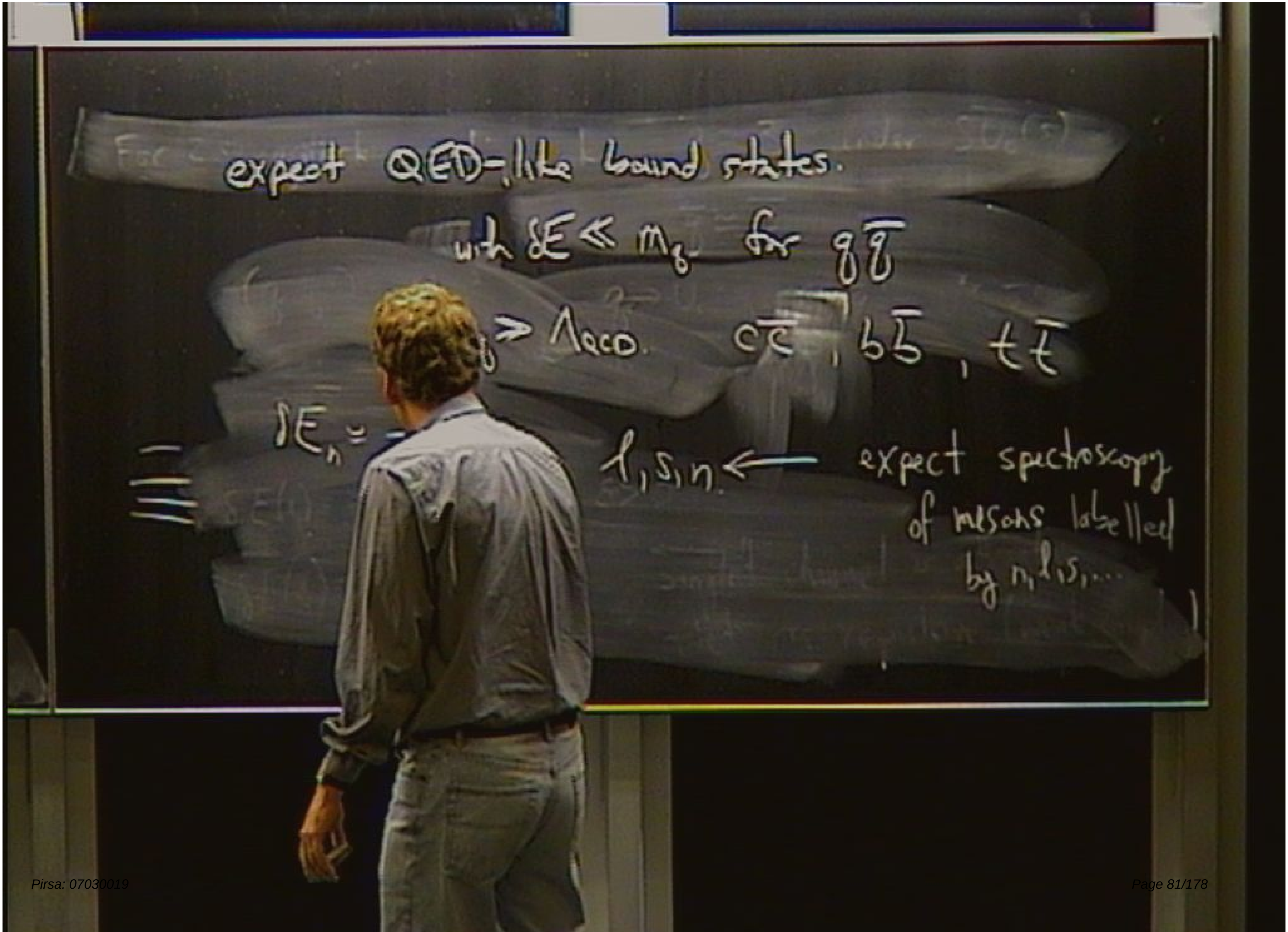
expect QED-like bound states.

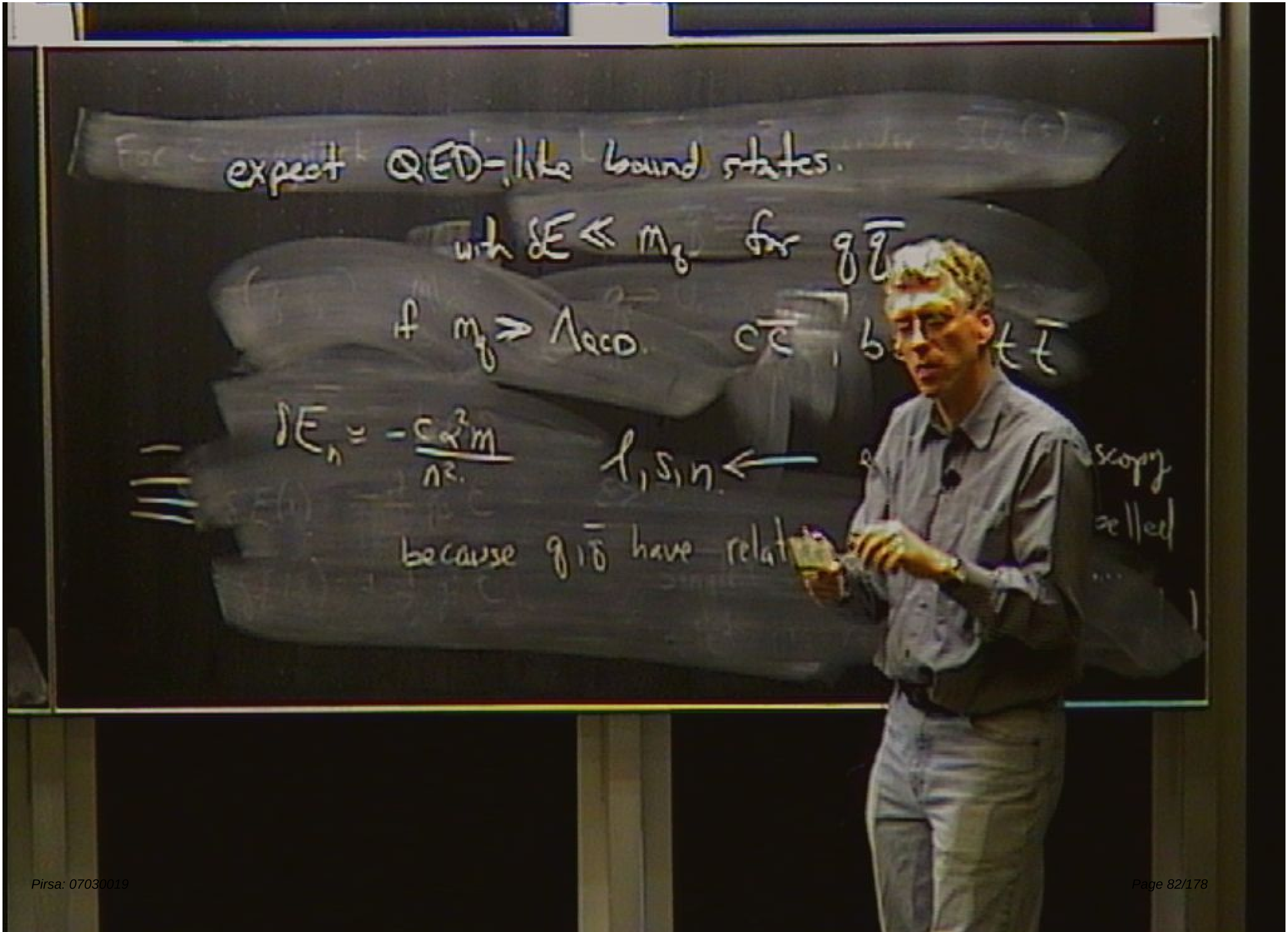
with $\delta E \ll m_q$ for $q\bar{q}$

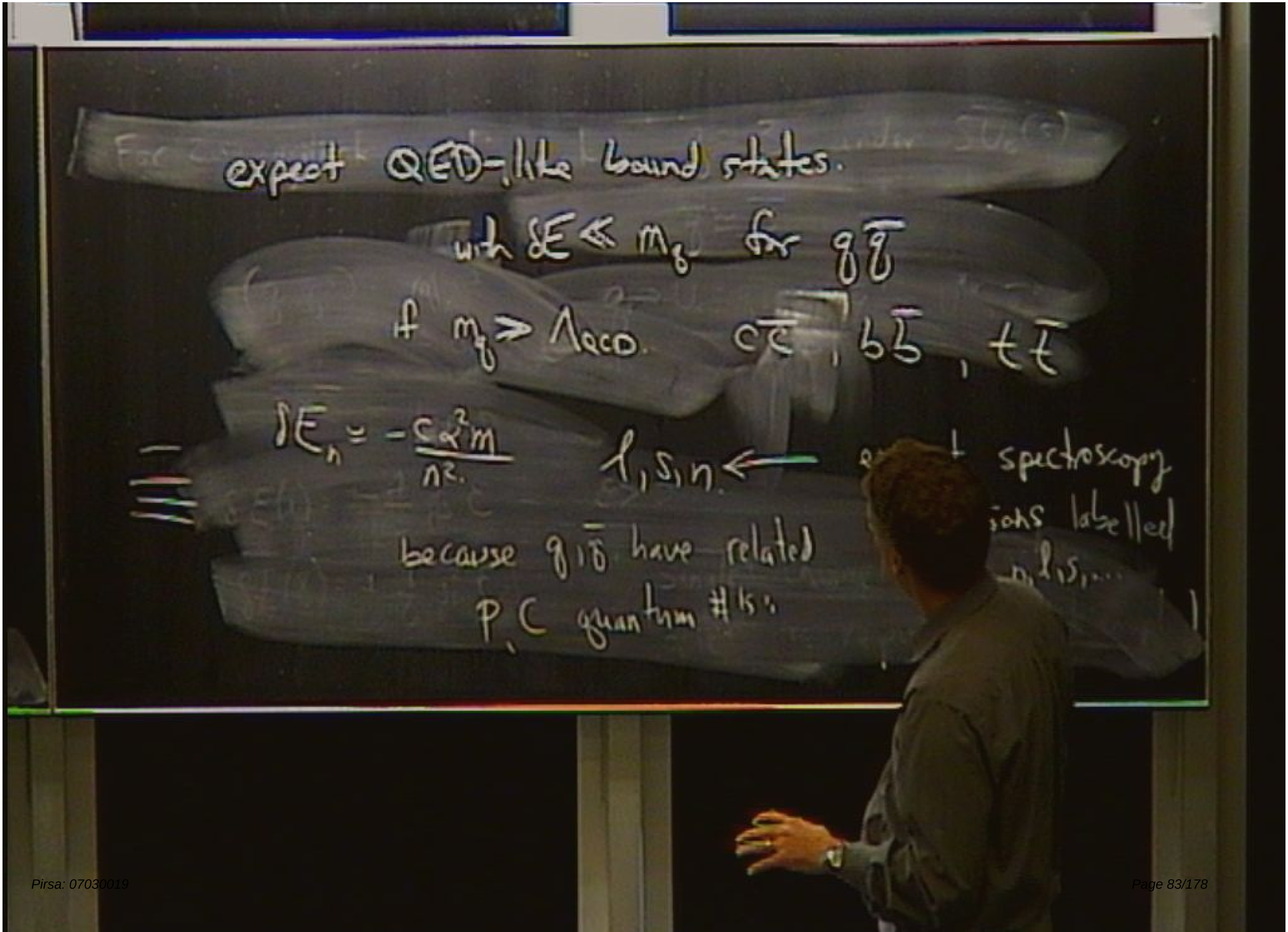
if $m_q \gg \Lambda_{QCD}$. $c\bar{c}$, $b\bar{b}$, t

$$\delta E_n \simeq -\frac{c\alpha^2 m}{n^2}$$

$l, s, n \leftarrow$ expect spect of mesons by n, l


expect QED-like bound states.
with δE ≪ m_q for q q̄
> Λ_QCD. c c̄, b b̄, t t̄
δE_n ≈
l, s, n ← expect spectroscopy
of mesons labelled
by n, l, s, ...


expect QED-like bound states.
with δE ≪ m_q for q q̄
if m_q ≫ Λ_QCD c c̄, b b̄, t t̄
δE_n = -cα²m / n²
l, s, n


expect QED-like bound states.
with δE ≪ m_q for q q̄
if m_q ≫ Λ_QCD. c c̄, b b̄, t t̄
δE_n ≈ -cα²m/n²
l, s, n
spectroscopy
ions labelled
because q, q̄ have related
P, C quantum #'s.

expect QED-like bound states.

with $\delta E \ll m_q$ for $q\bar{q}$

if $m_q \gg \Lambda_{QCD}$. $c\bar{c}$, $b\bar{b}$, $t\bar{t}$

$$\delta E_n \simeq -\frac{c\,\alpha^2 m}{n^2}.$$

$l, s, n \leftarrow$ expect spectroscopy of mesons labelled by $n, l, s, \ldots$

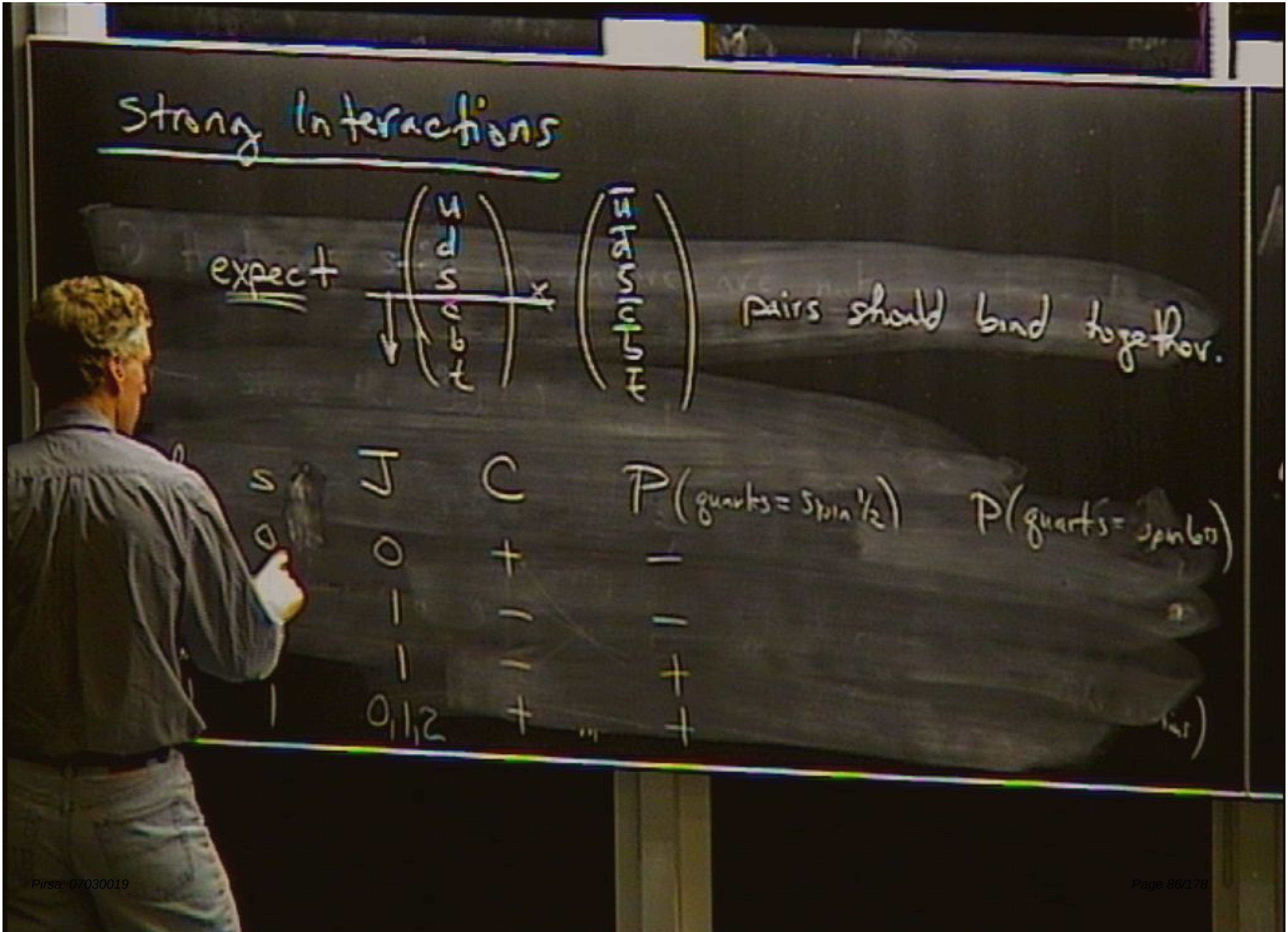
because $q, \bar{q}$ have related P, C quantum #'s:

Strong Interactions

$$\text{expect} \quad \begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix} \quad \text{pairs should bind together.}$$

$l \quad s \quad J \quad C \quad P(\text{quarks} = \text{spin } 1/2) \quad P(\text{quarks} = \text{spinless})$

0



Strong Interactions
expect
u d s c b t
x
ū d̄ s̄ c̄ b̄ t̄
pairs should bind together.
s J C P(quarks = spin ½) P(quarks = spin 0)
0 0 + −
1 − −
1 − +
1 0,1,2 + ... +

Strong Interactions

expect $\begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix}$ pairs should bind together.

l	s	J	C	P(quarks = spin ½)	P(quarks = spin 0)
0	0	0	+	−	
0	1	1	−	−	+
1	0	1	−	+	
1	1	0,1,2	+	+	−

Strong Interactions

expect $\begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix}$ pairs should bind together.

ℓ	s	J	C	P(quarks = spin ½)	P(quarks = spin 0)
0	0	0	+	−	
0	1	1	−	−	+
1	0	1	−	+	
1	1	0,1,2	+	+	−

expect QED-like bound states.

with $\delta E \ll m_q$ for $q\bar{q}$

if $m_q \gg \Lambda_{QCD}$. $c\bar{c}$ $t\bar{t}$

J/ψ

$\delta E_n \simeq -\frac{c\alpha^2 m}{n^2}$ $l, s, n \leftarrow$

because $q, \bar{q}$ have related

P, C quantum #'s:

expect QED-like bound states.

with $\delta E \ll m_q$ for $q\bar{q}$

if $m_q \gg \Lambda_{QCD}$. $c\bar{c}$, $b\bar{b}$, $t\bar{t}$

J/ψ

$\delta E_n \simeq -\frac{c\,\alpha^2 m}{n^2}$

$l, s, n \leftarrow$ expect spectroscopy of mesons labelled by $n, l, s, \ldots$

because $q, \bar{q}$ have related P, C quantum #'s:

expect QED-like bound states.

with $\delta E \ll m_q$ for $q\bar{q}$

if $m_q \gg \Lambda_{QCD}$ $c\bar{c}$, $b\bar{b}$

J/ψ Υ

$$\delta E_n = -\frac{c\alpha^2 m}{n^2}$$

$l, s, n \leftarrow$ expect

of m

because $q, \bar{q}$ have related

P, C quantum #'s:

expect QED-like bound states.

with $\delta E \ll m_q$ for $q\bar{q}$

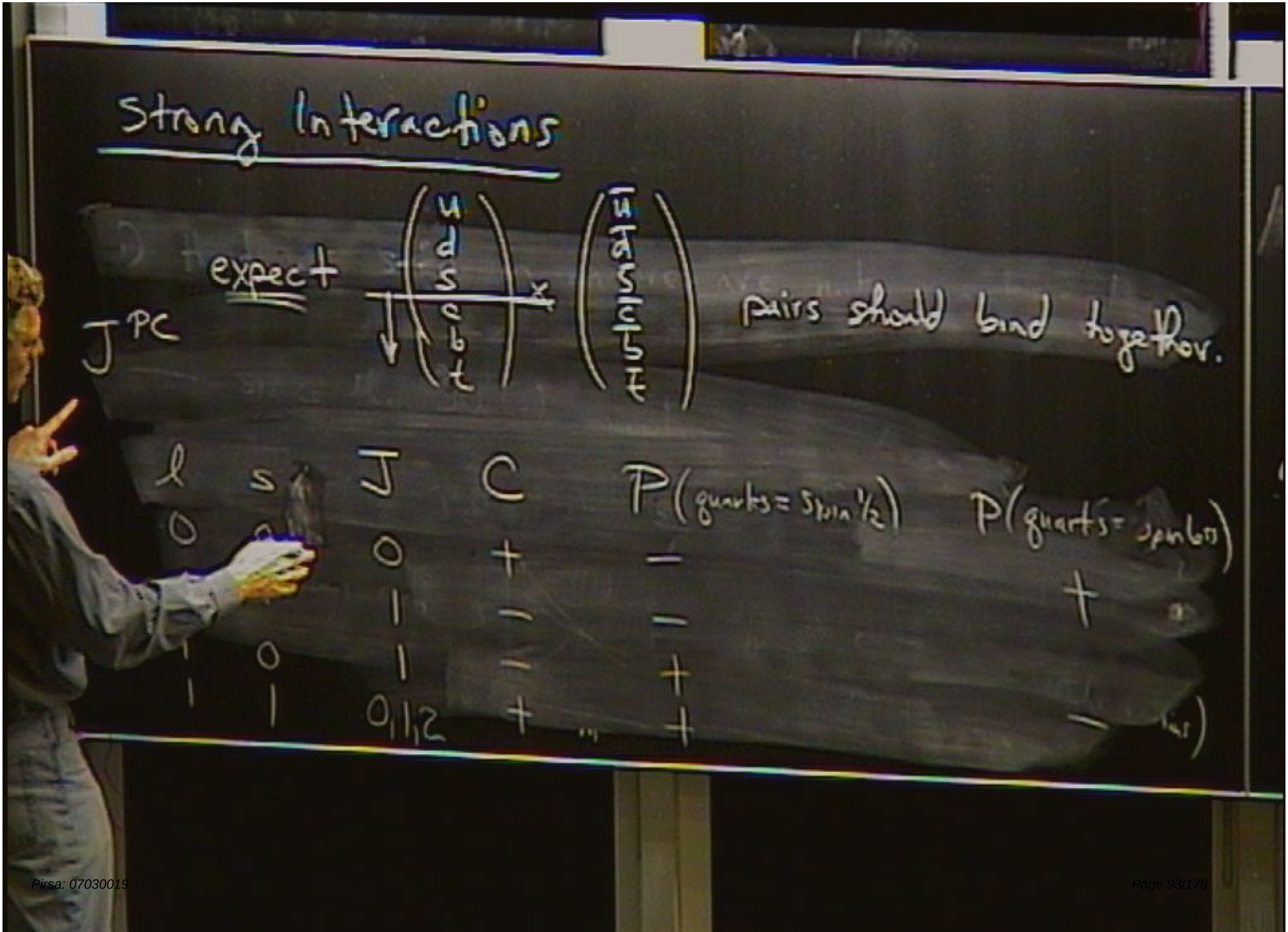
if $m_q \gg \Lambda_{QCD}$. $c\bar{c}$, $b\bar{b}$, $t\bar{t}$

J/ψ Υ

$$\delta E_n \simeq -\frac{c\,\alpha^2 m}{n^2}$$

$l, s, n \leftarrow$ expect spectroscopy of mesons labelled by $n, l, s, \ldots$

because $q, \bar{q}$ have related P, C quantum #'s:


Strong Interactions
expect
J PC
u d s c b t
x
ū d̄ s̄ c̄ b̄ t̄
pairs should bind together.
l s J C P(quarks = spin ½) P(quarks = spin 0)
0 0 0 + −
1 − − +
1 0 1 − +
1 1 0,1,2 + " + −

Strong Interactions

expect

J^{PC} $\quad 0^{-+}$

$$\begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix}$$ pairs should bind together.

ℓ	s	J	C	P(quarks = spin 1/2)	P(quarks = spin 0)
0	0	0	+	−	
0	1	1	−	−	+
1	0	1	−	+	
1	1	0,1,2	+	+	−

Strong Interactions

expect $\begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix}$ pairs should bind together.

J^{PC}

0^{-+}

1^{--}

ℓ	s	J	C	P(quarks = spin ½)	P(quarks = spin 0)
0	0	0	+	−	
0	1	1	−	−	+
1	0	1	−	+	
1	1	0,1,2	+	+	−

Strong Interactions

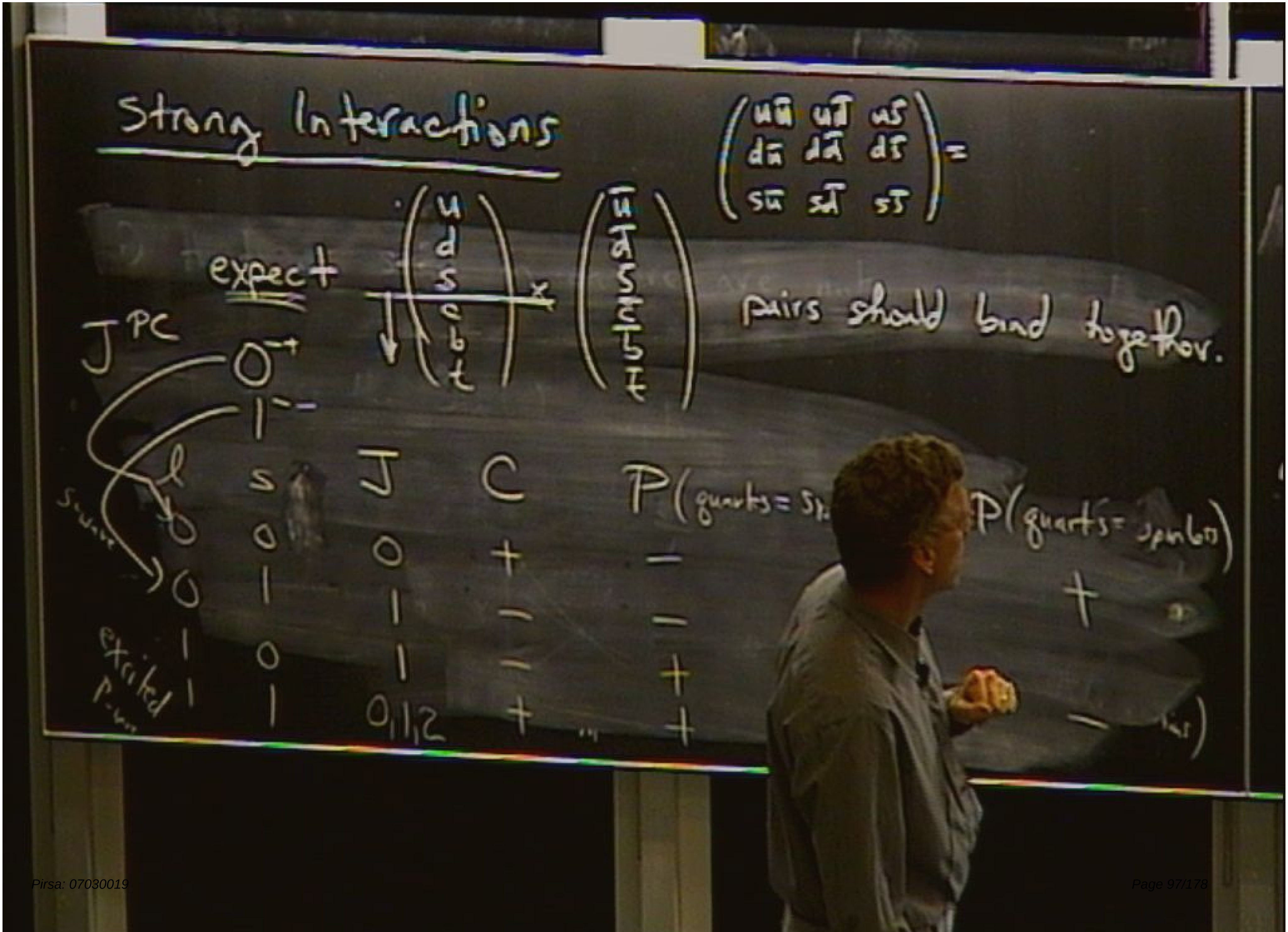
expect $\begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix}$ pairs should bind together.

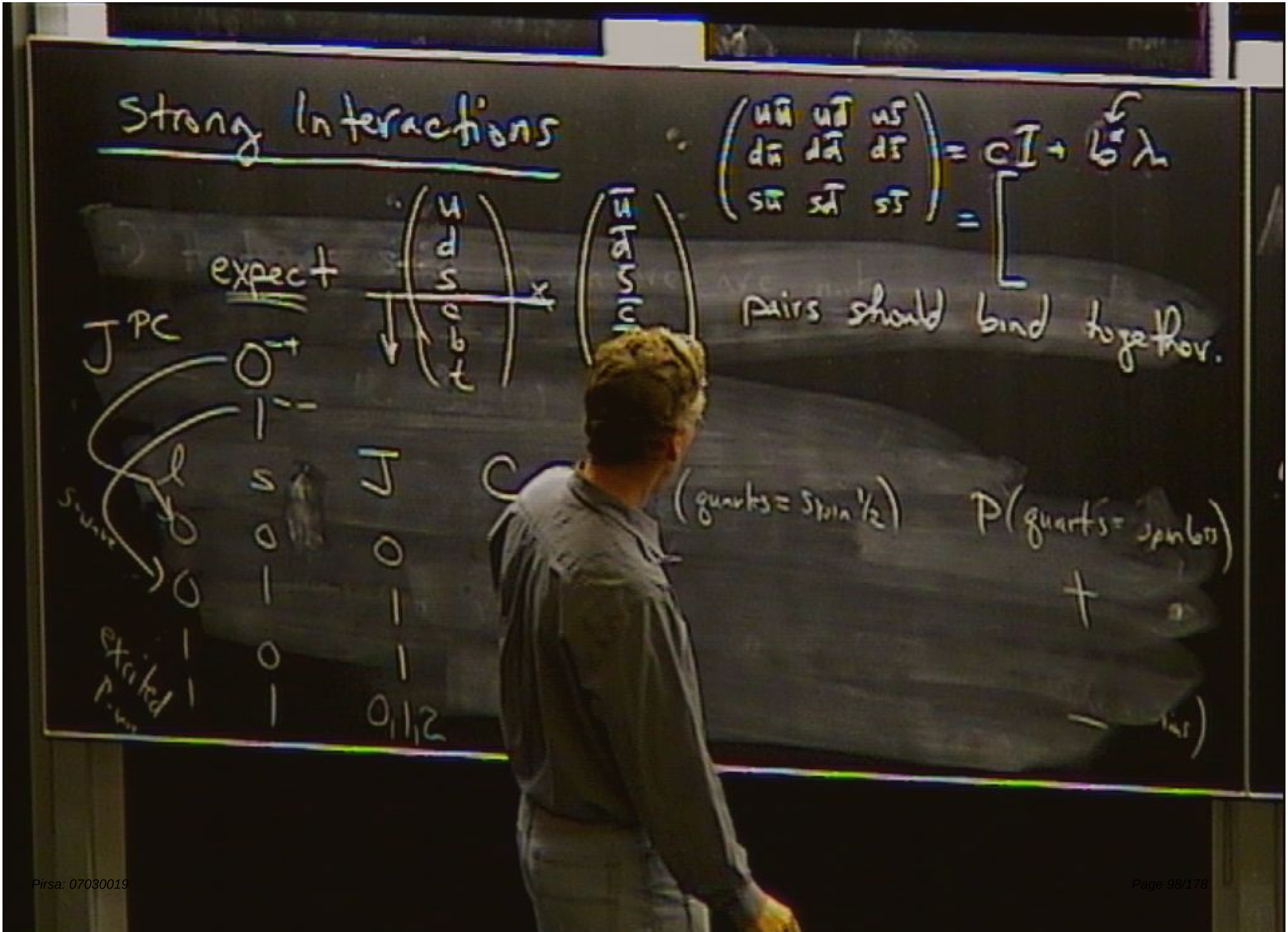
J^{PC}

0^{-+}

1^{--}

	l	s	J	C	P(quarks = spin ½)	P(quarks = spin 0)
S-wave	0	0	0	+	−	
	0	1	1	−	−	+
excited P-wave	1	0	1	−	+	
	1	1	0,1,2	+	+	−


Strong Interactions
expect
u d s c b t
ū d̄ s̄ c̄ b̄ t̄
uū ud̄ us̄
dū dd̄ ds̄
sū sd̄ ss̄
pairs should bind together.
J PC
0 −+
1 −−
l s J C P(quarks= spinless) P(quarks= spin 1/2)
0 0 0 + − −
0 1 1 − − +
1 0 1 − +
1 1 0,1,2 + + −
S-wave
excited
P-wave


Strong Interactions
expect
J^{PC}
0^{-+}
1^{--}
pairs should bind together.
(quarks = spin ½)
P(quarks = spinless)
excited

Strong Interactions

$$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = cI + b^a \lambda_a$$

$$= [\quad]$$

expect

$$\begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \\ \bar{c} \\ \bar{b} \\ \bar{t} \end{pmatrix}$$

pairs should bind together.

J^{PC}

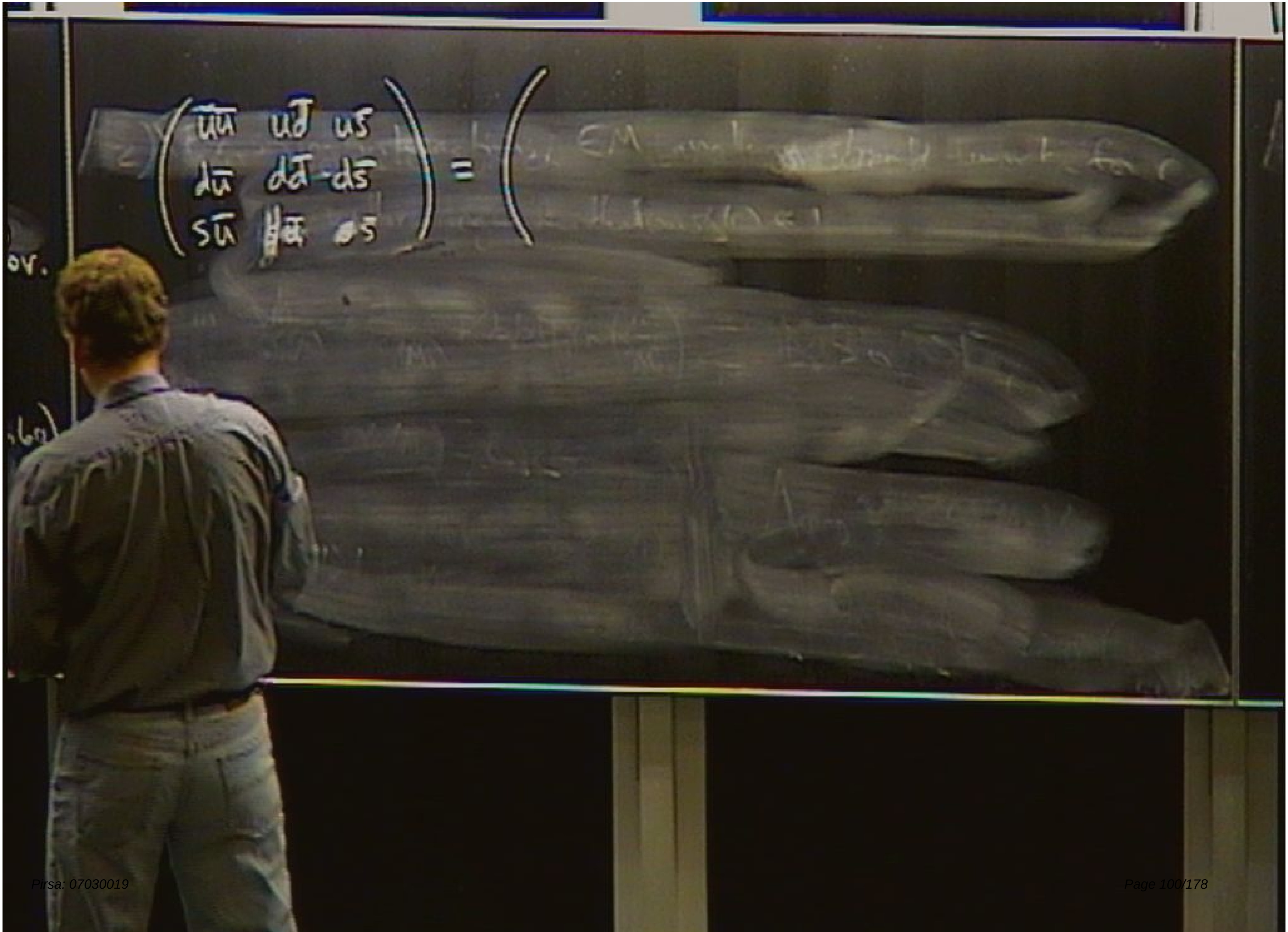
0^{-+}

1^{--}

ℓ	s	J	C	P(quarks = spin 1/2)	P(quarks = spin 0)
0	0	0	+	−	
0	1	1	−	−	+
1	0	1	−	+	
1	1	0,1,2	+	+	−

excited P-wave

S-wave

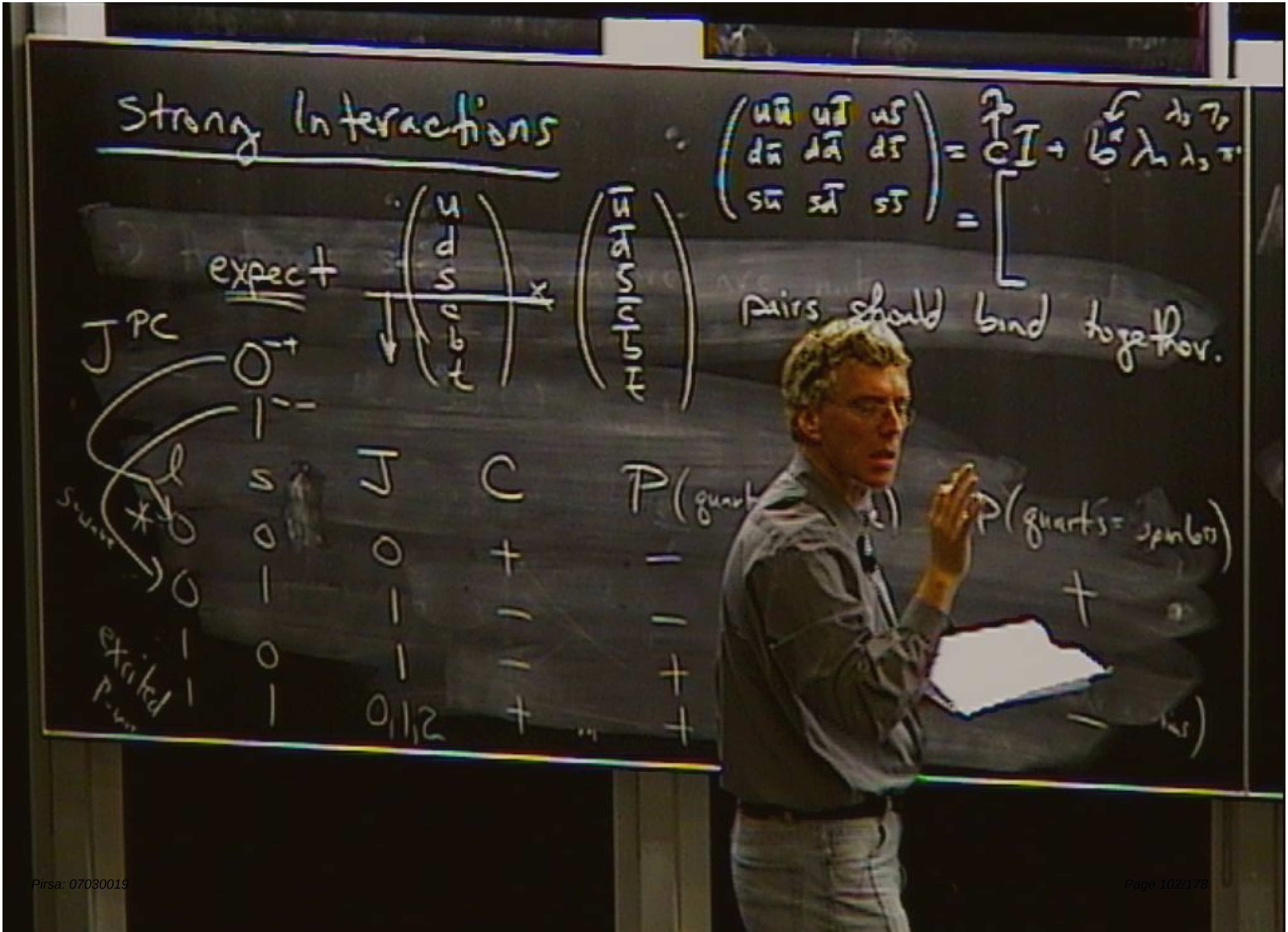

uū ud̄ us̄
dū dd̄-ds̄

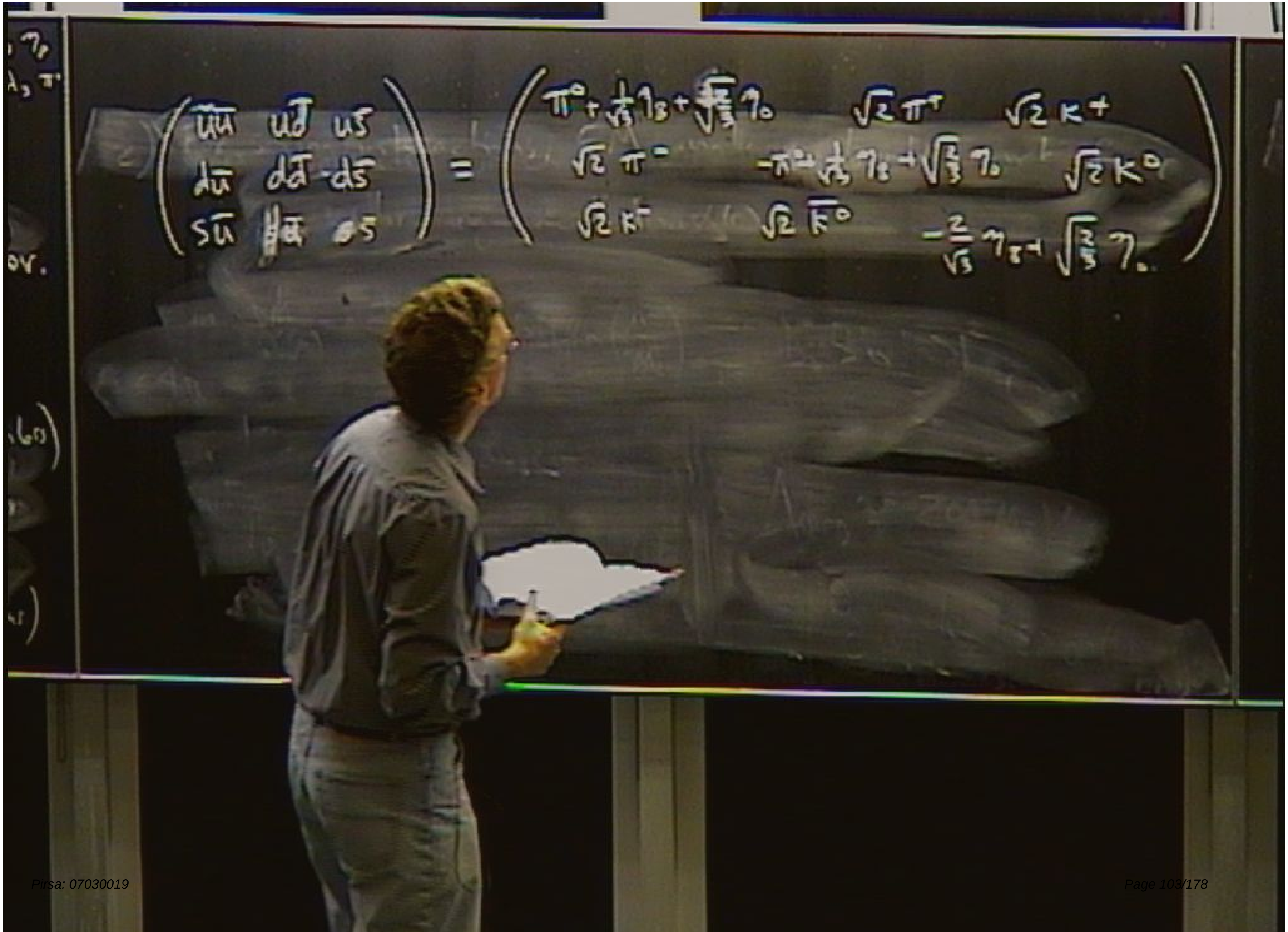
ov.

,60)

r)

$$\begin{pmatrix} \bar{u}u & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} - d\bar{s} & \\ s\bar{u} & [illegible] & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\tfrac{2}{3}}\eta_0 & & \\ & & \\ & & \end{pmatrix}$$


Strong Interactions
expect
J^PC
0^-+
1^--
l s J C P(quarks) P(quarks = spinless)
0 0 0 + −
0 1 1 − − +
1 0 1 − +
1 1 0,1,2 + +
excited
pairs should bind together.

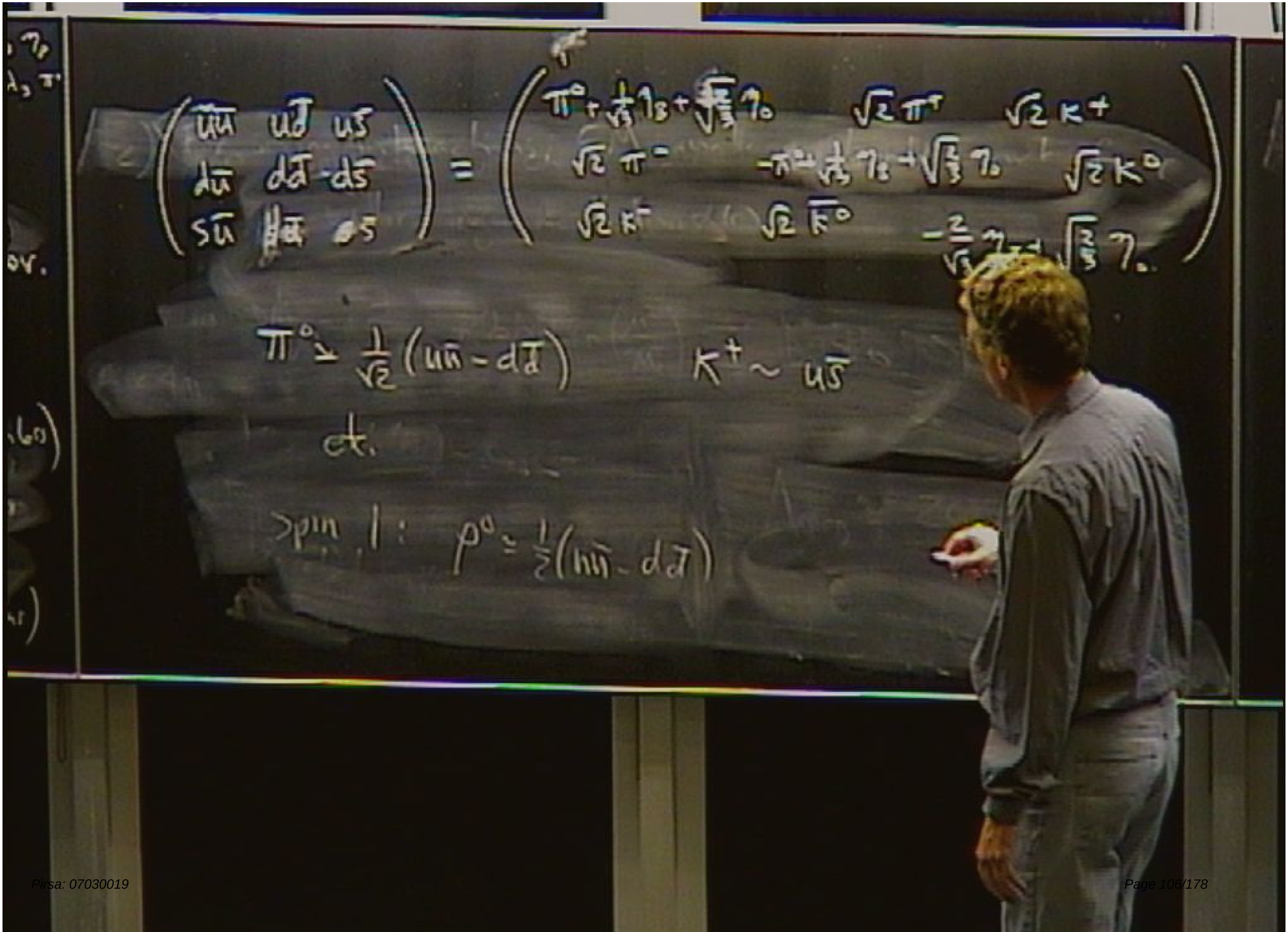


$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$

$$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

$$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\,\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\,\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\,\eta_0 \end{pmatrix}$$

$$\pi^0 \sim \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$


$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$
$\pi^0 \simeq \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$K^+ \sim u\bar{s}$
etc.
Spin 1: $\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

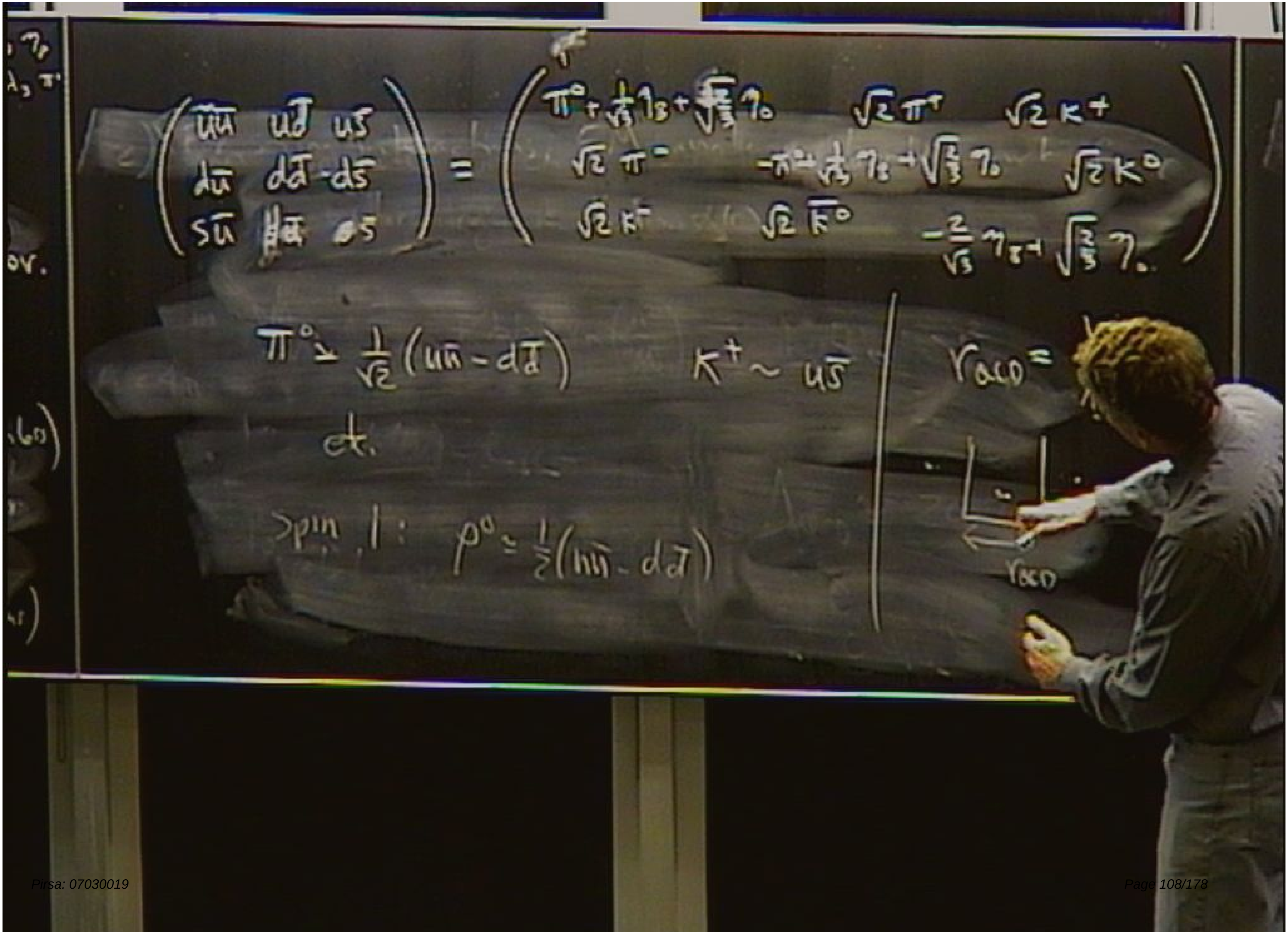
$$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

$\pi^0 \simeq \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ $\quad K^+ \sim u\bar{s}$

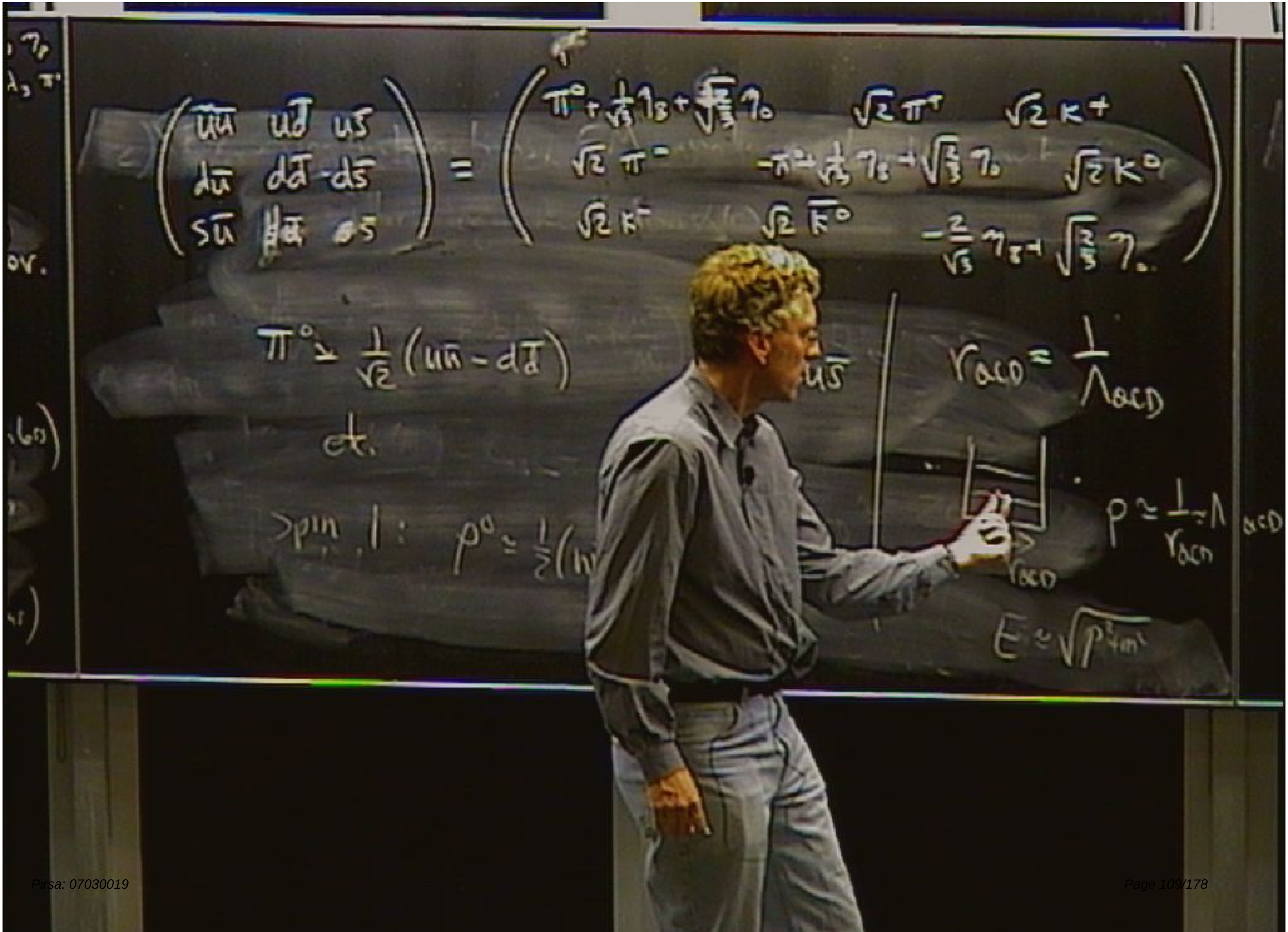
etc.

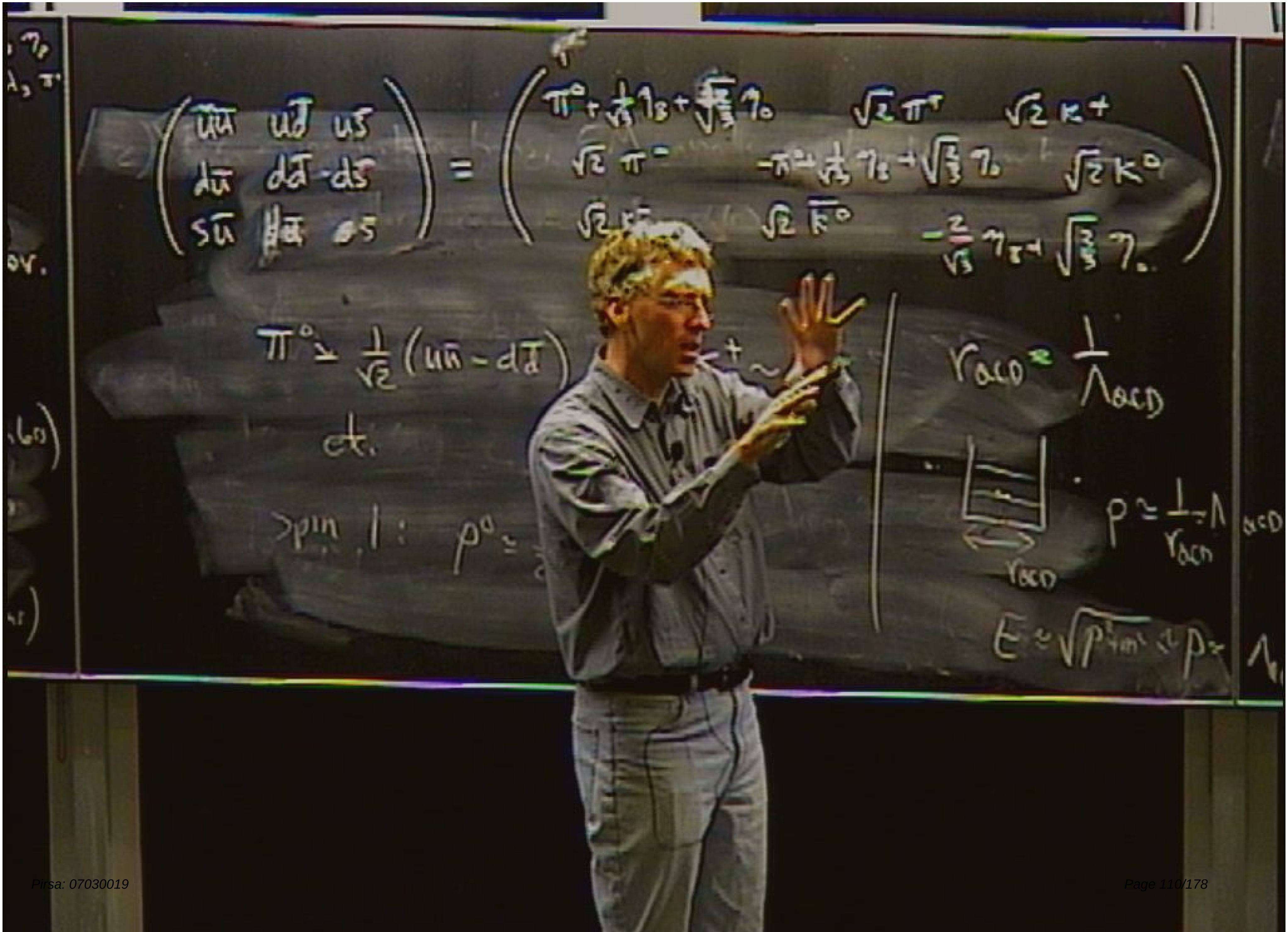
Spin 1: $\rho^0 \simeq \frac{1}{2}(u\bar{u} - d\bar{d})$

$r_{QCD} = \frac{1}{\Lambda_{QCD}}$



$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$
$\pi^0 \simeq \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$K^+ \sim u\bar{s}$
etc.
Spin 1: $\rho^0 \simeq \frac{1}{2}(u\bar{u} - d\bar{d})$
$r_{QCD} =$
r_{QCD}


π° ≈ 1/√2 (uū − dd̄)
etc.
Spin 1:
r_QCD = 1/Λ_QCD
ρ ≈ 1/r_QCD ~ Λ_QCD
E ≈ √(p²+m²)


ūu ud̄ us̄
dū dd̄ ds̄
sū
π⁰ ≃ 1/√2 (uū − dd̄)
etc.
Spin 1: ρ⁰ ≃
r_QCD ≈ 1/Λ_QCD
r_QCD
p ≃ 1/r_QCD Λ_QCD
E ≃ √(p²+m²)

$$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

$\pi^0 \simeq \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ $\qquad K^+ \sim u\bar{s}$

etc.

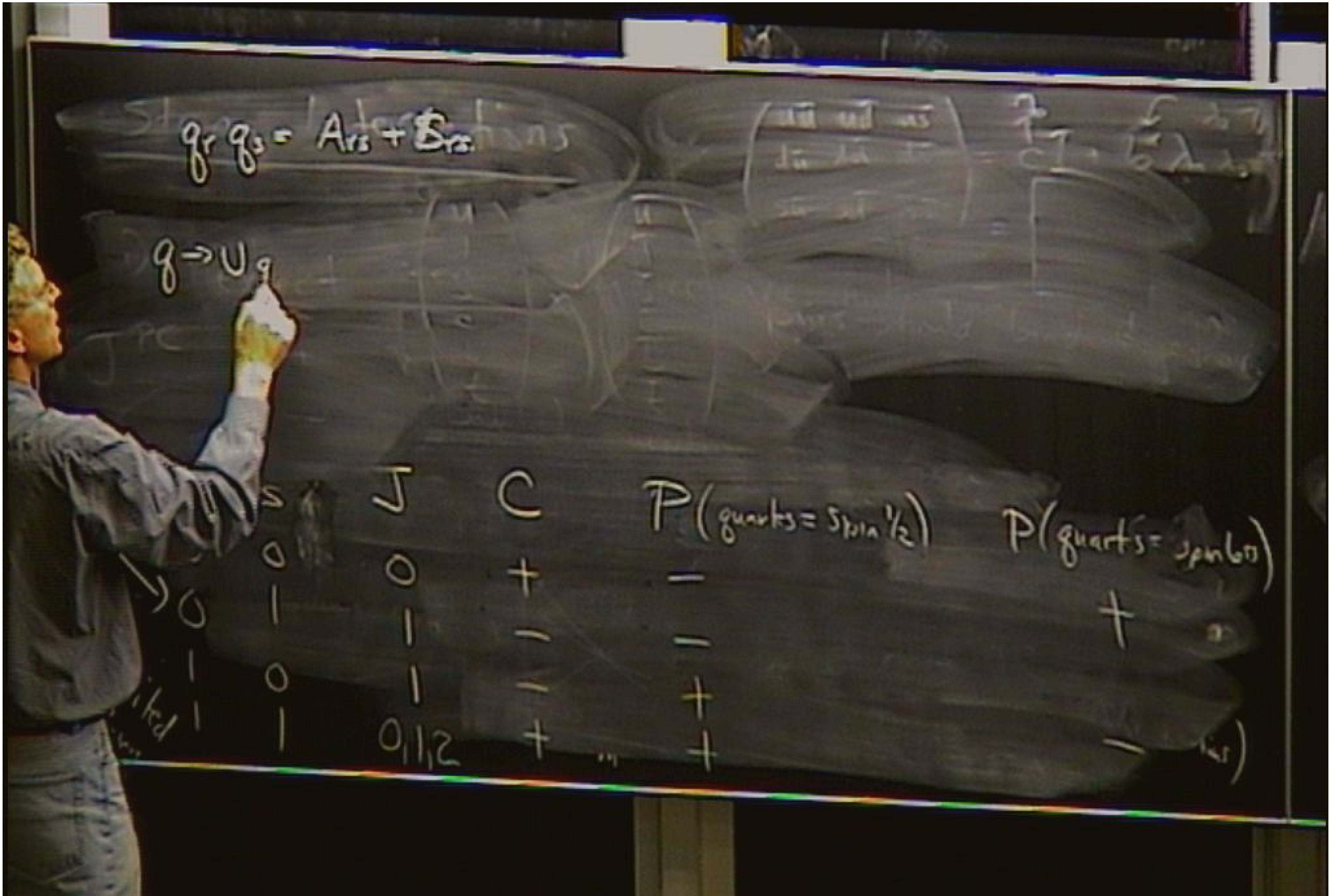
Spin 1: $\rho^0 \simeq \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

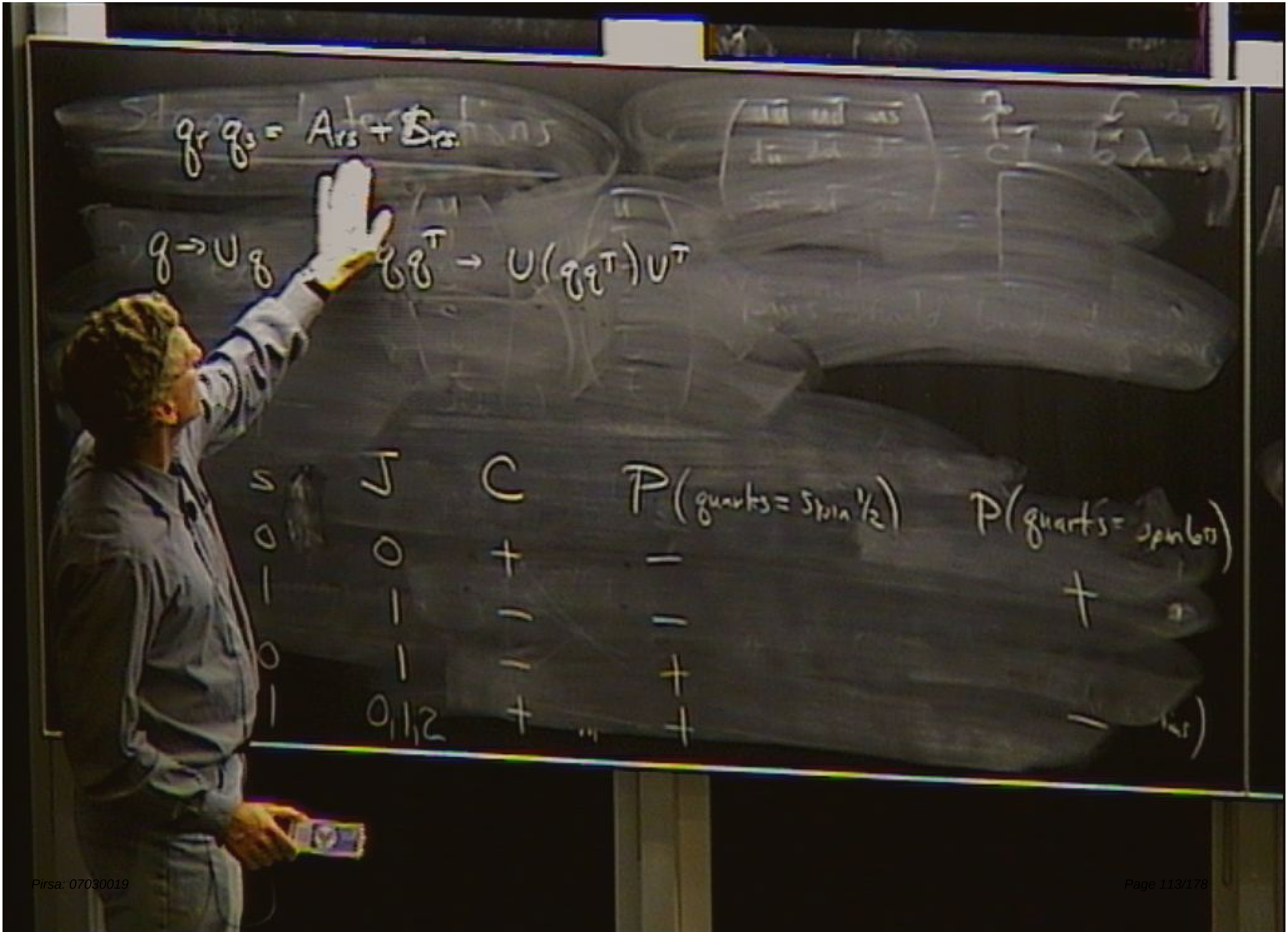
$r_{QCD} = \frac{1}{\Lambda_{QCD}}$

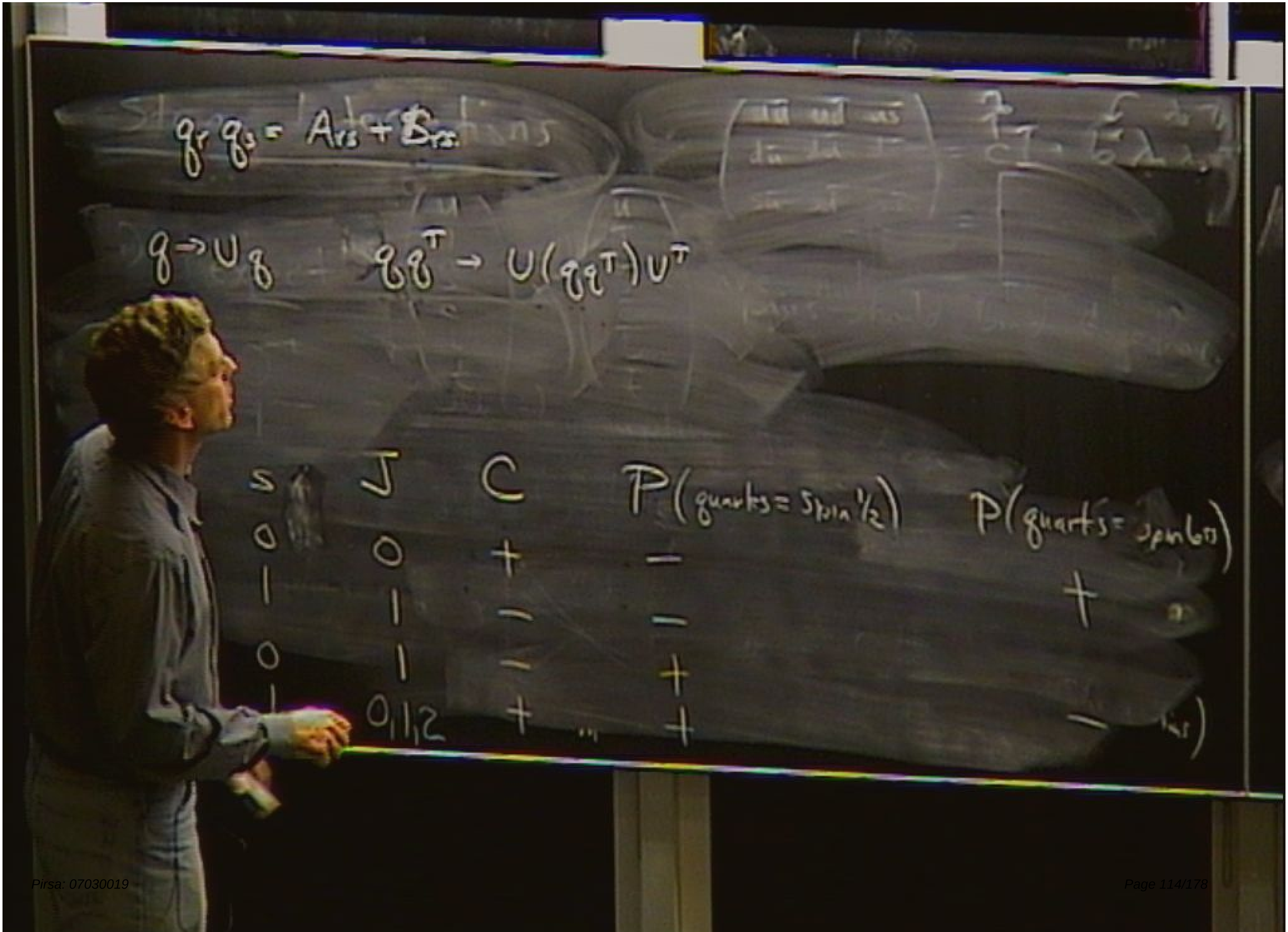
r_{QCD}

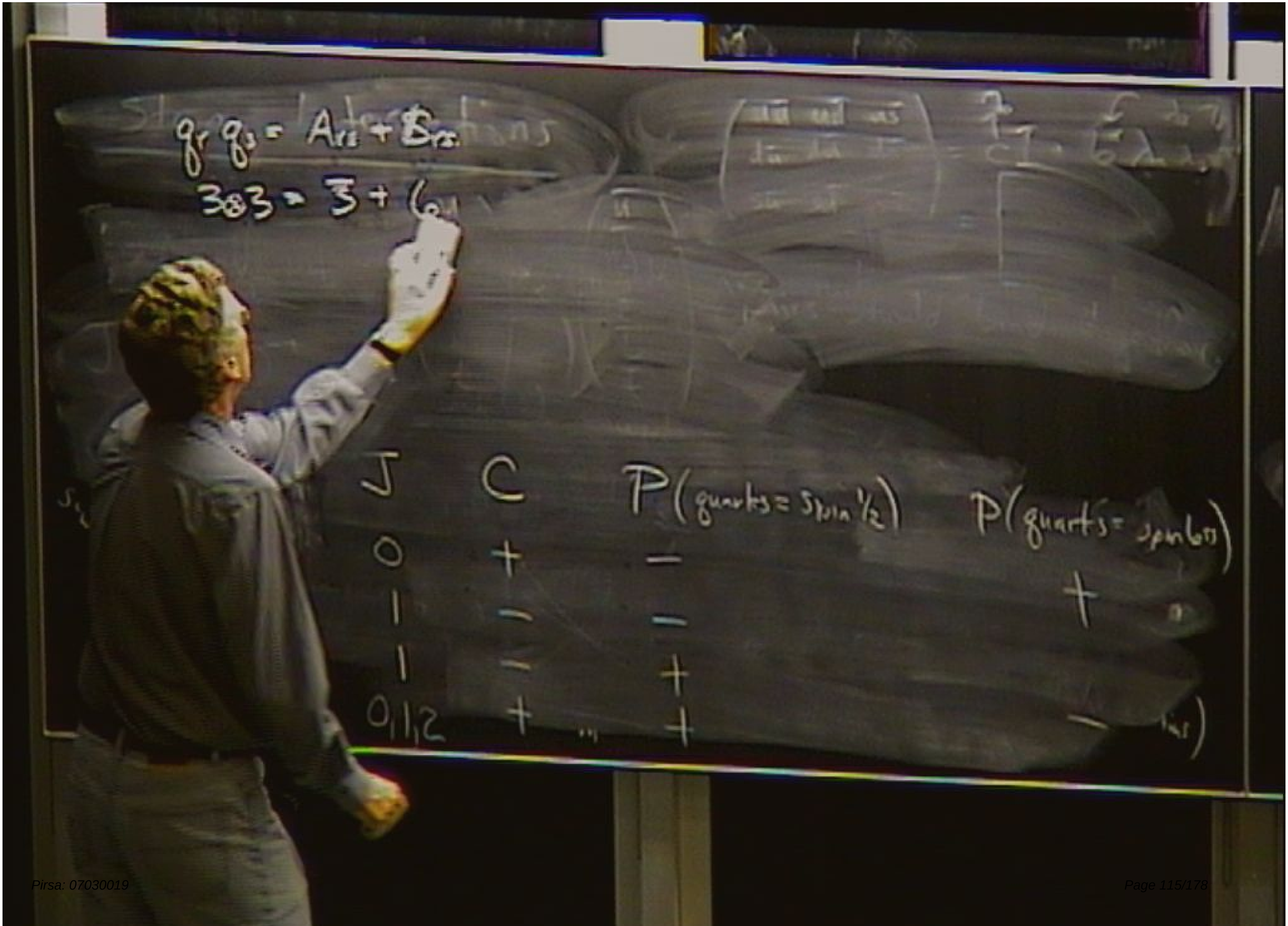
$p \simeq \frac{1}{r_{QCD}} \simeq \Lambda_{QCD}$

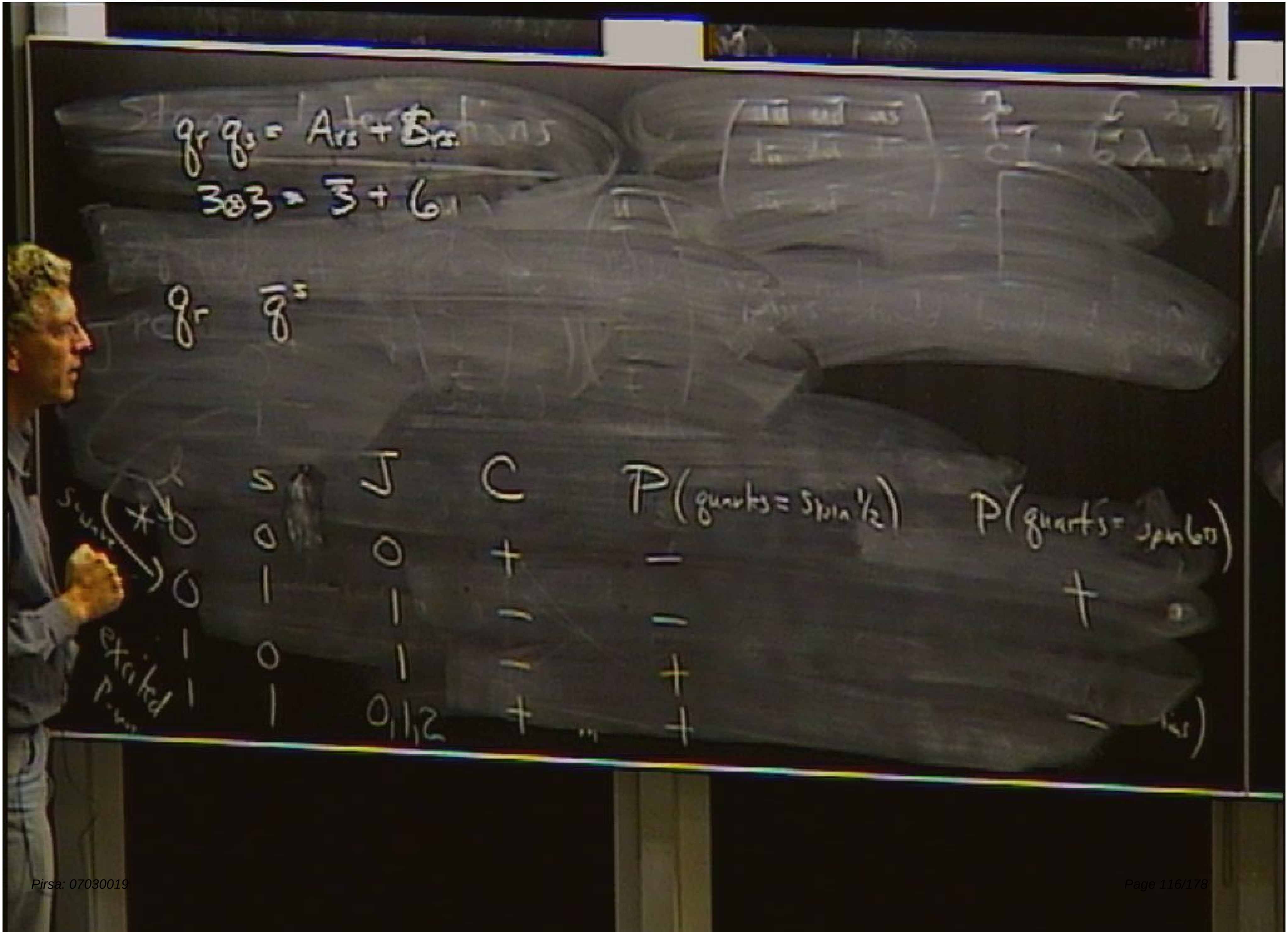
$E \simeq \sqrt{p^2+m^2} \simeq p \simeq \Lambda$

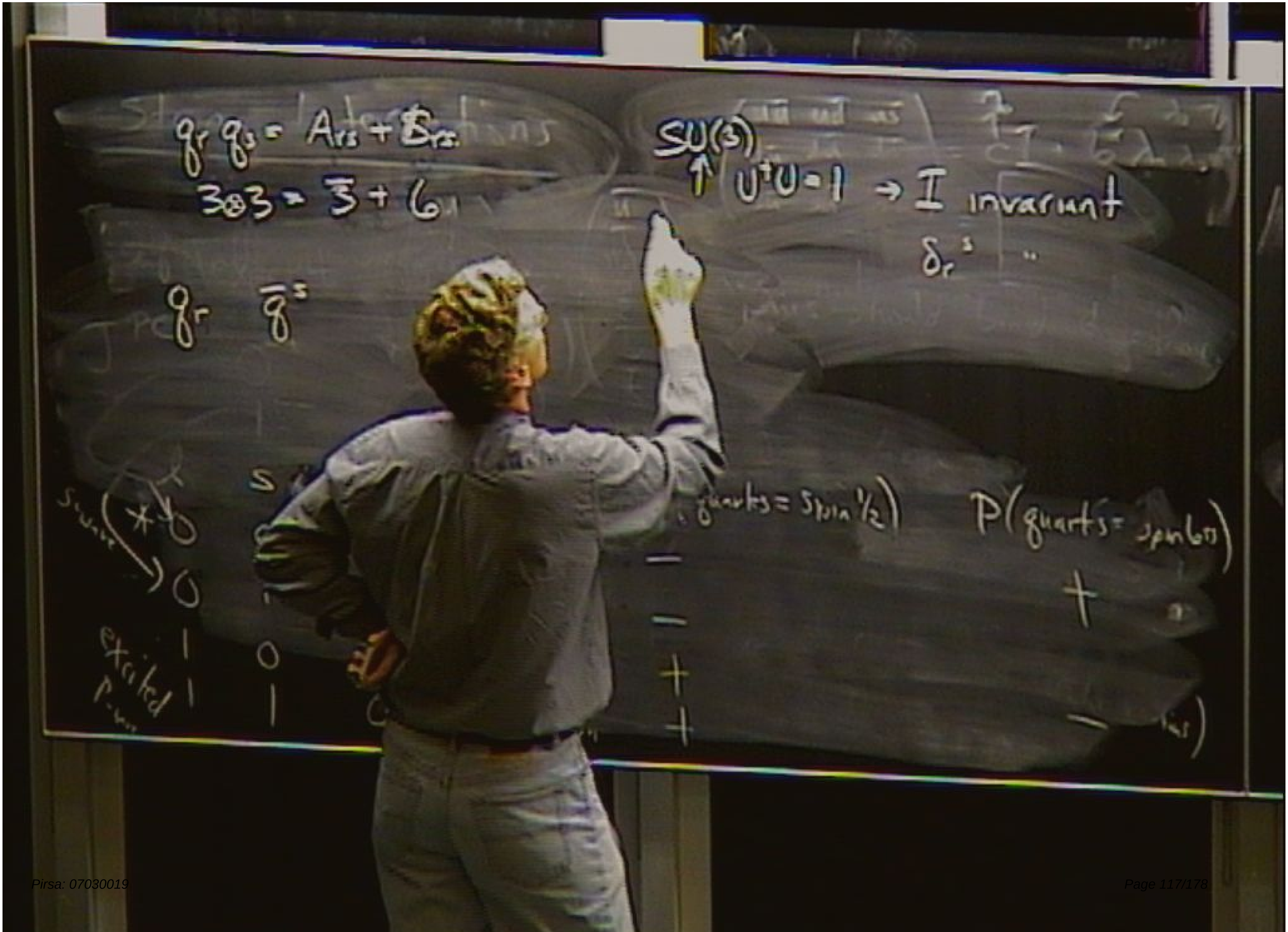

qr qs = Ars + Brs
q → U q
S J C P(quarks = spin ½) P(quarks = spin 0)
0 0 + −
0 1 1 − − +
1 0 1 − +
1 1 0,1,2 + + −

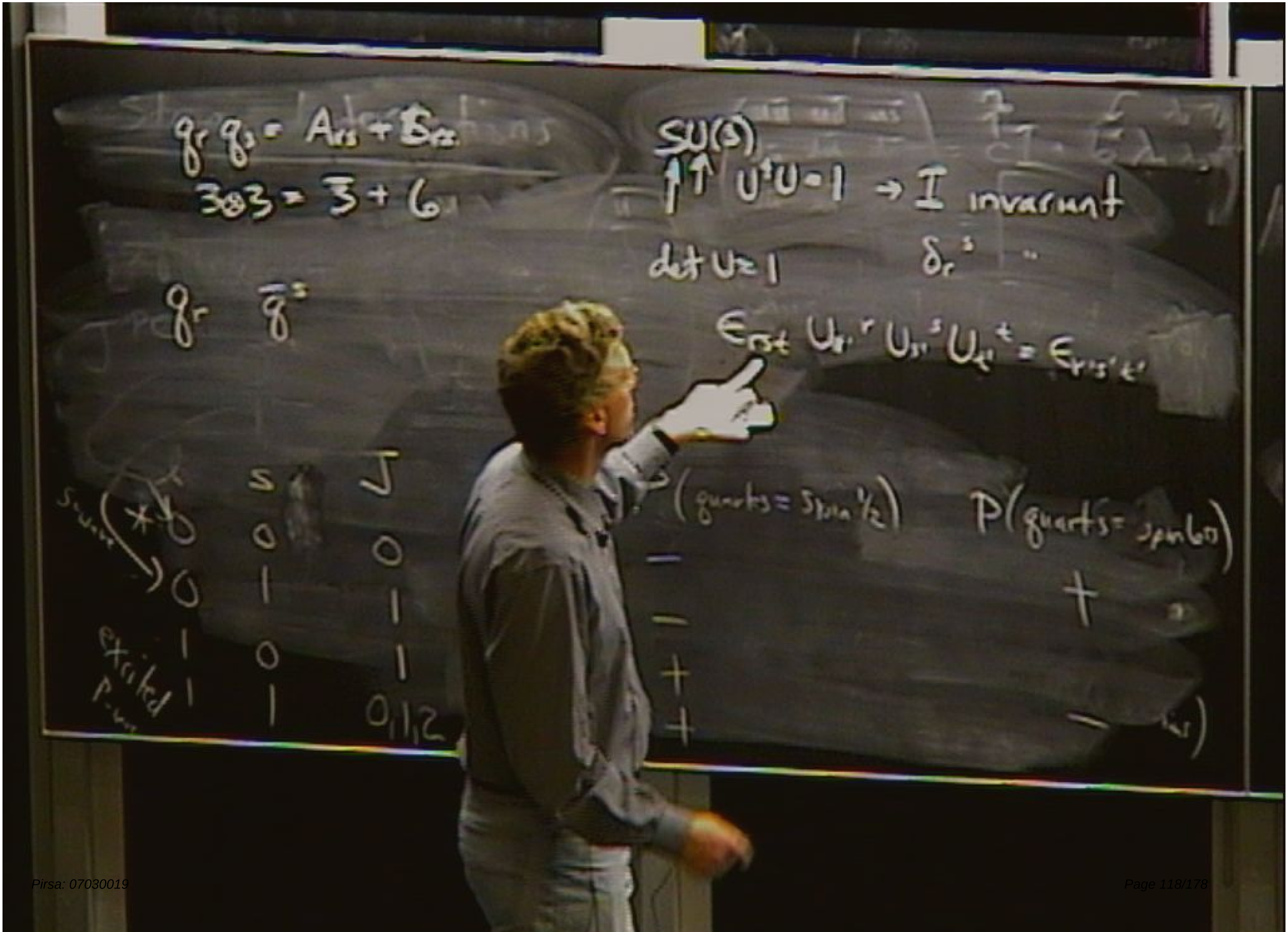

q_r q_s = A_rs + B_rs
q → Uq
qq^T → U(qq^T)U^T
S J C P(quarks = spin 1/2) P(quarks = spinless)
0 0 + −
1 1 − − +
0 1 − +
1 0,1,2 + + −

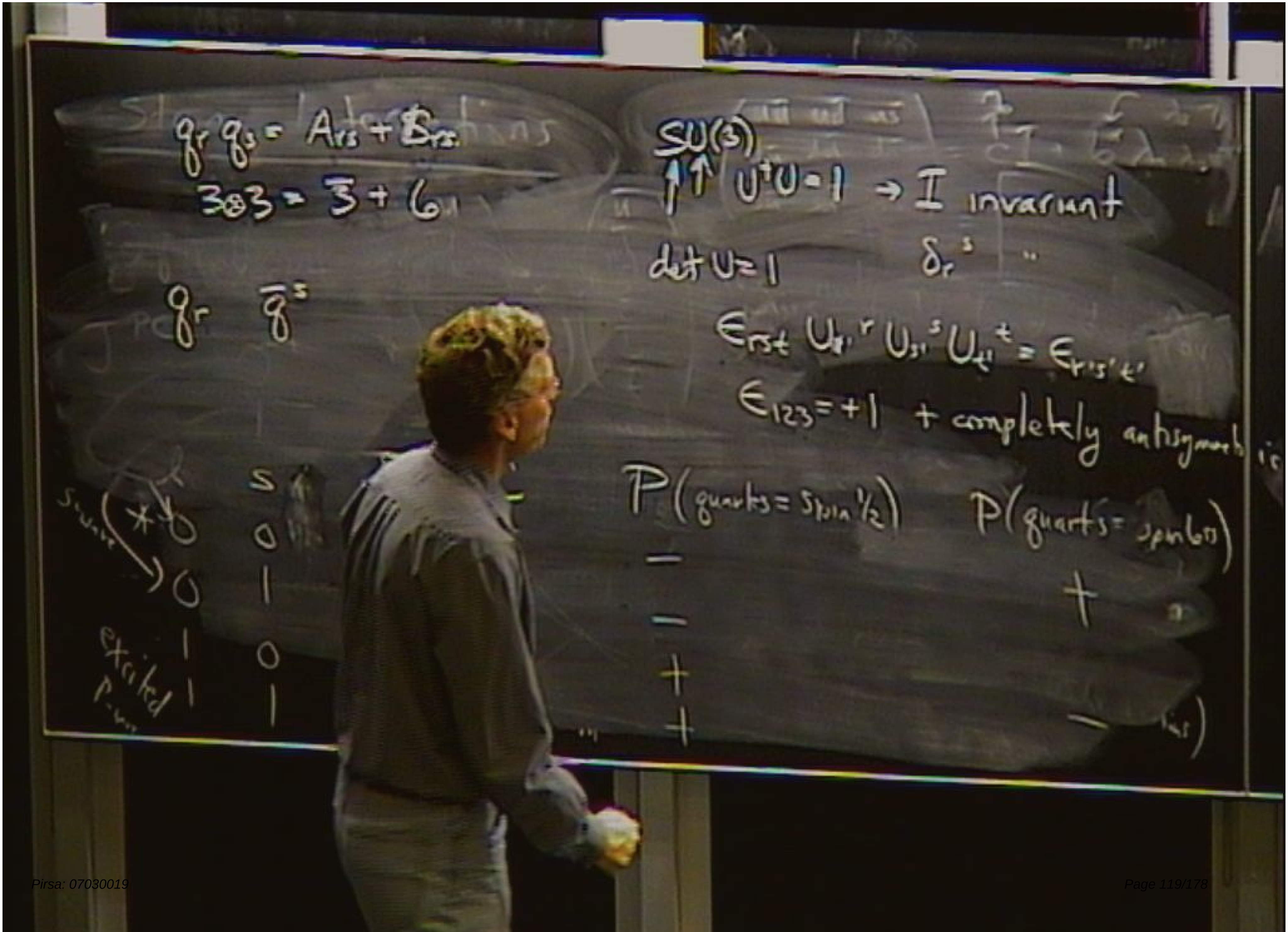

q_r q_s = A_rs + B_rs
q → U q
q q^T → U(q q^T) U^T
S J C P(quarks = spin 1/2) P(quarks = spin 0)
0 0 + −
1 1 − − +
0 1 − +
1 0,1,2 + + −


q_r q_s = A_rs + B_rs
3⊗3 = 3̄ + 6
J C P(quarks = spin ½) P(quarks = spinless)
0 + −
1 − − +
1 − +
0,1,2 + + −

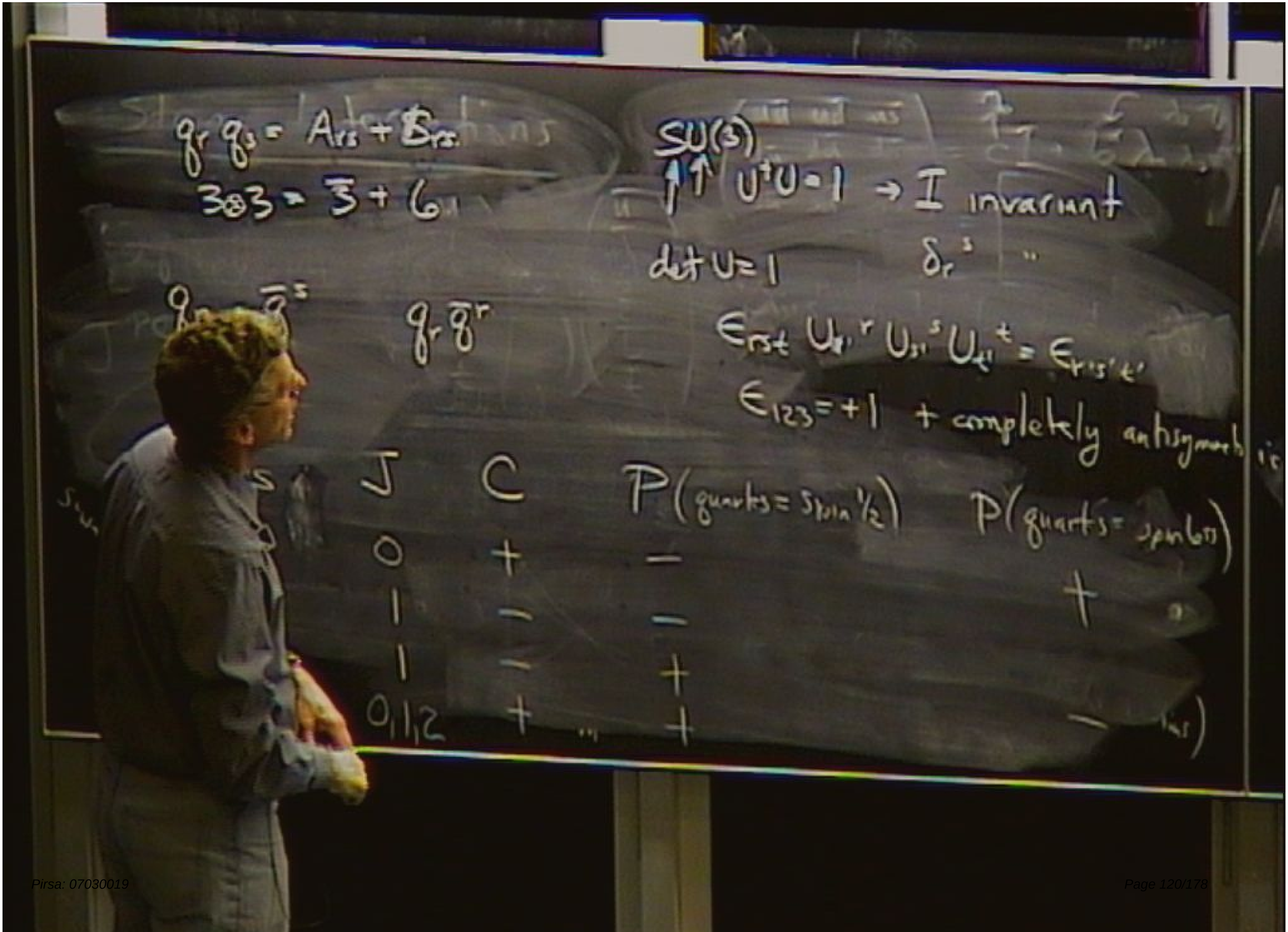

q_r q_s = A_rs + B_rs
3⊗3 = 3̄ + 6
q_r q̄^s
S J C P(quarks = spin ½) P(quarks = spin 0)
S-wave
0 0 0 + −
0 1 1 − − +
excited P-wave
1 0 1 − +
1 1 0,1,2 + + −


q_r q_s = A_rs + B_rs
3⊗3 = 3̄ + 6
SU(3)
U†U = 1 → I invariant
δ_r^s
q_r q̄^s
quarks = spin 1/2)
excited


q_r q_s = A_rs + S_rs
3⊗3 = 3̄ + 6
SU(3)
U†U = 1 → I invariant
det U = 1
δ_r^s
q_r q̄^s
ε_rst U_r'^r U_s'^s U_t'^t = ε_r's't'
S J
S-wave
excited P-wave
0 0 0
0 1 1
1 0 1
1 1 0,1,2
P(quarks = spin 1/2)
P(quarks = spin 0)
—
—
+
+
+
—



q_r q_s = A_rs + B_rs
3⊗3 = 3̄ + 6
q_r q̄^s
SU(3)
U†U = 1 → I invariant
det U = 1
δ_r^s
ε_rst U_r'^r U_s'^s U_t'^t = ε_r's't'
ε_123 = +1 + completely antisymmetric
P(quarks = spin 1/2)
excited


$q_r q_s = A_{rs} + B_{rs}$
$3 \otimes 3 = \bar{3} + 6$
SU(3)
$U^\dagger U = 1 \rightarrow I$ invariant
$\det U = 1$
δ_r^s "
$\epsilon_{rst} U_{r'}{}^r U_{s'}{}^s U_{t'}{}^t = \epsilon_{r's't'}$
$\epsilon_{123} = +1$ + completely antisymmetric
$q_r \bar{q}^s$
$q_r \bar{q}^r$
J
C
P(quarks = spin 1/2)
P(quarks = spin 0)
0 + −
1 − − +
1 − +
0,1,2 + " + −

$q_r q_s = A_{rs} + B_{rs}$

$3 \otimes 3 = \bar{3} + 6$

$SU(3)$

$U^\dagger U = 1 \rightarrow I$ invariant

$\det U = 1$ $\quad \delta_r^{\ s}$ "

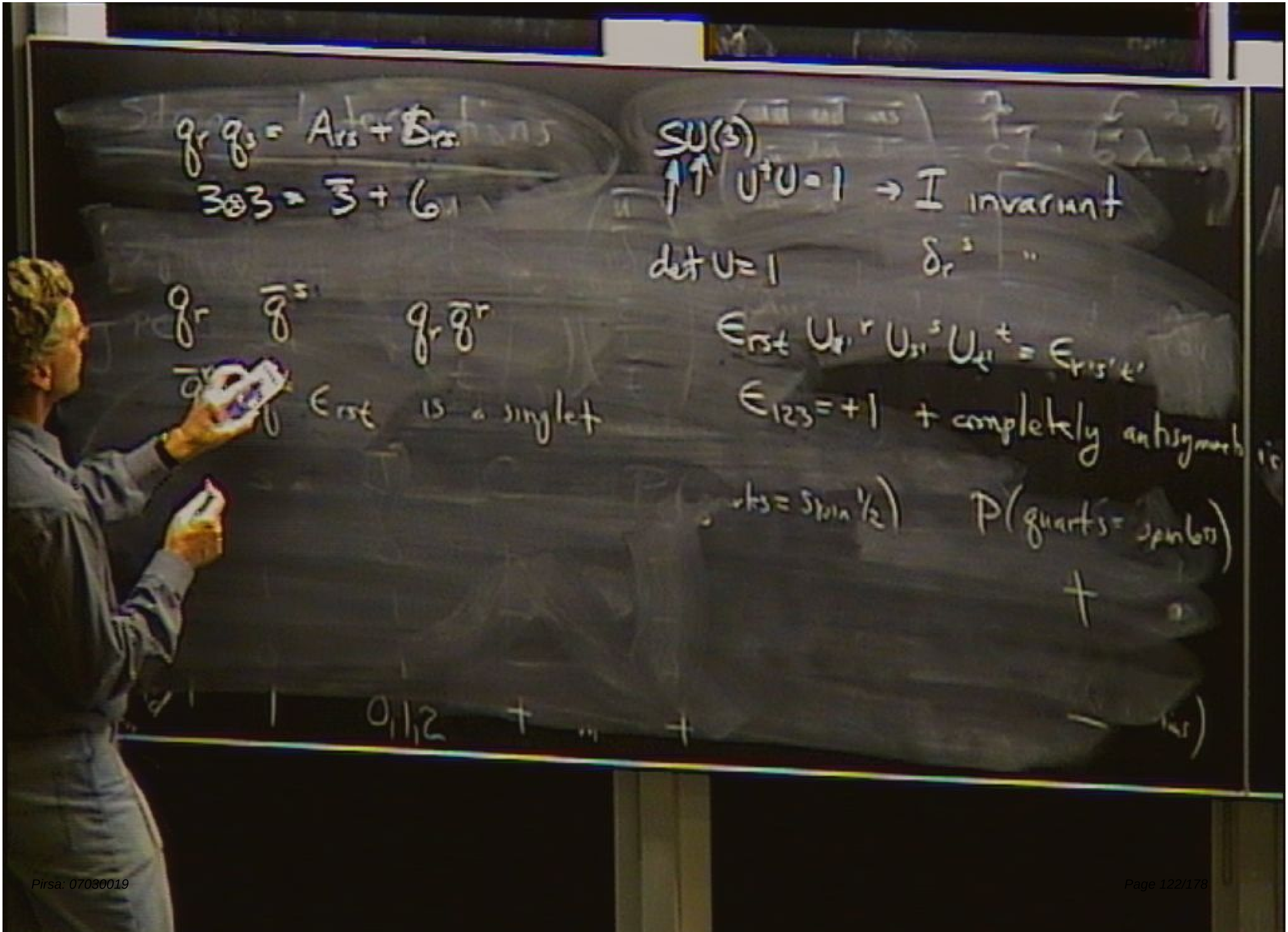
$q_r \quad \bar{q}^s \quad q_r \bar{q}^r$

$\epsilon_{rst} U_{r'}^{\ r} U_{s'}^{\ s} U_{t'}^{\ t} = \epsilon_{r's't'}$

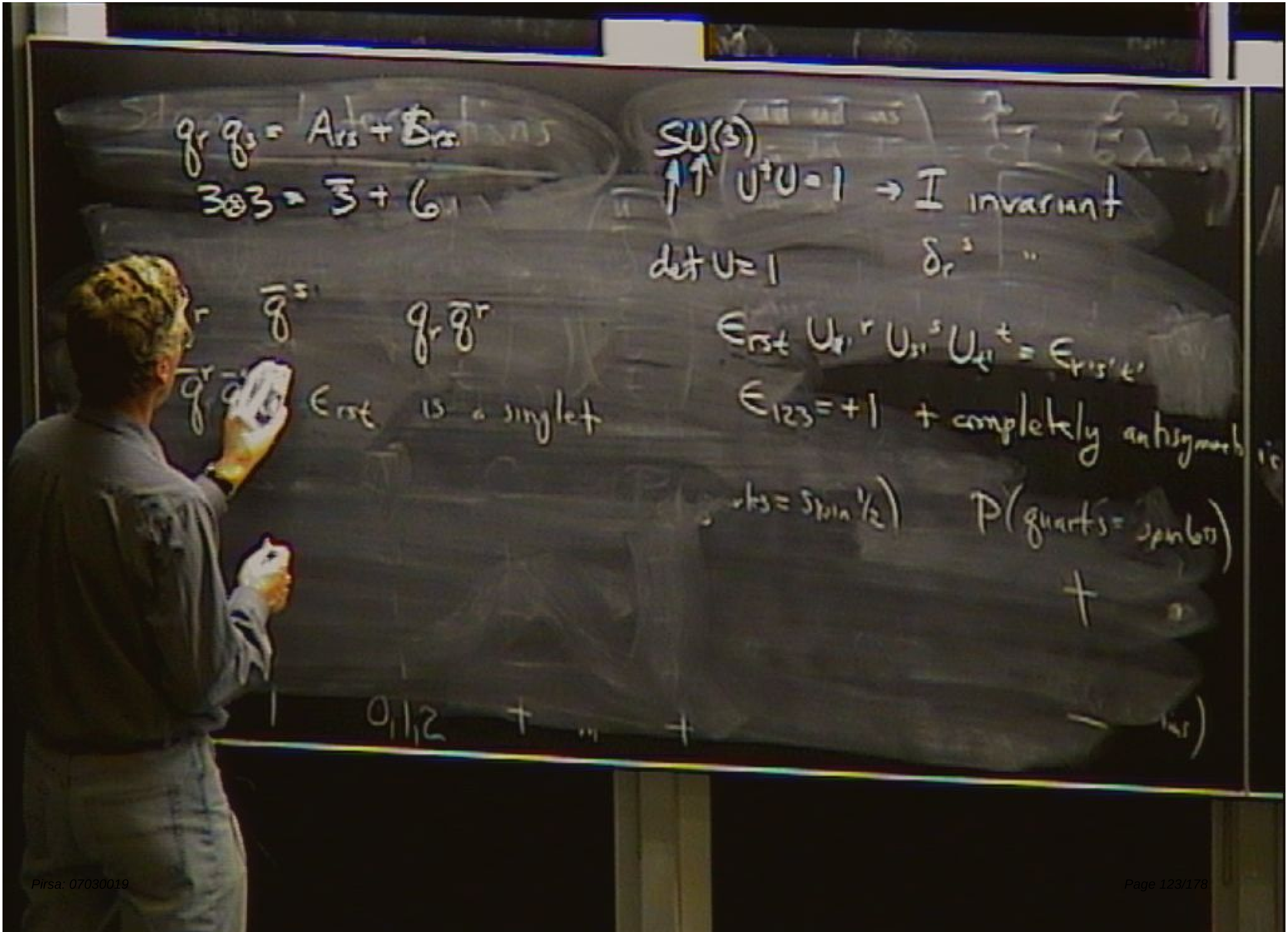
$\bar{q}^r \bar{q}^s \bar{q}^t \epsilon_{rst}$

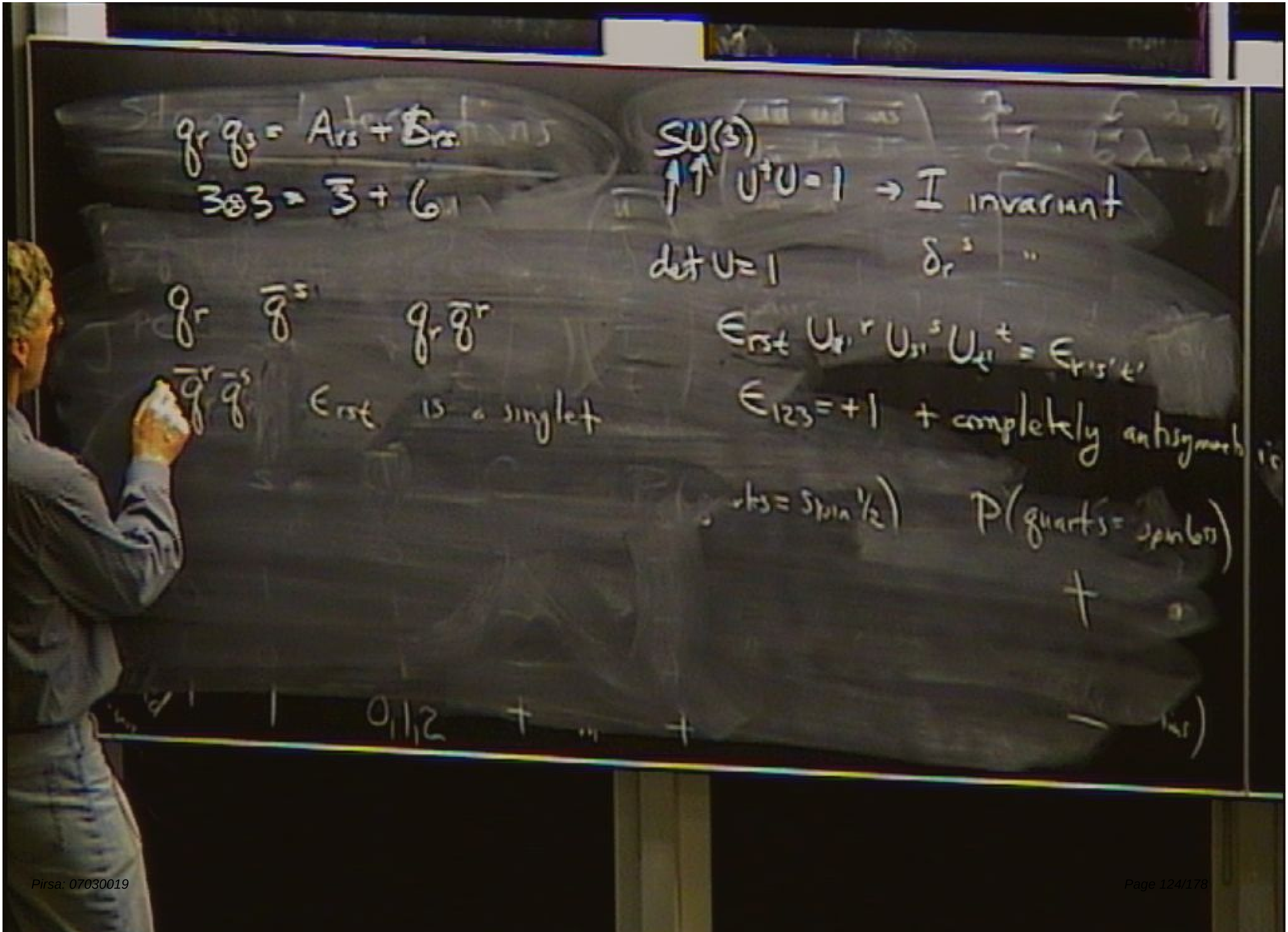
$\epsilon_{123} = +1$ + completely antisymmetric

	S	J	C	P(quarks = spin ½)	P(quarks = spin 0)
0	0	0	+	−	
0	1	1	−	−	+
	0	1	−	+	
1	1	0,1,2	+ "	+	−



$q_r q_s = A_{rs} + B_{rs}$
$3 \otimes 3 = \bar{3} + 6$
SU(3)
$U^\dagger U = 1 \rightarrow I$ invariant
$\det U = 1$
δ_r^s "
$q_r \quad \bar{q}^s \qquad q_r \bar{q}^r$
ϵ_{rst} is a singlet
$\epsilon_{rst} U_{r'}{}^r U_{s'}{}^s U_{t'}{}^t = \epsilon_{r's't'}$
$\epsilon_{123} = +1$ + completely antisymmetric
rks = spin 1/2)
P(quarks = spin 0)
0,1,2


q_r q_s = A_rs + B_rs
3⊗3 = 3̄ + 6
SU(3)
U†U = 1 → I invariant
det U = 1
δ_r^s
ε_rst U_r'^r U_s'^s U_t'^t = ε_r's't'
ε_123 = +1 + completely antisymmetric
ε_rst is a singlet
q̄^s
q_r q̄^r
P(quarks = spin 1/2)
0,1,2


qr qs = Ars + Brs
3⊗3 = 3̄ + 6
SU(3)
U†U = 1 → I invariant
det U = 1
δr s "
qr q̄s qr q̄r
q̄r q̄s
Erst is a singlet
Erst Ur'r Us's Ut't = Er's't'
E123 = +1 + completely antisymmetric
P(quarks = spin 0)
0,1,2 +

$q_r q_s = A_{rs} + B_{rs}$

$3 \otimes 3 = \bar{3} + 6$

SU(3)

$U^\dagger U = 1 \rightarrow I$ invariant

$\det U = 1$ $\qquad \delta_r^{\ s}$ "

$q_r \quad \bar{q}^s \qquad q_r \bar{q}^r$

$\epsilon_{rst} U_{r'}^{\ r} U_{s'}^{\ s} U_{t'}^{\ t} = \epsilon_{r's't'}$

$= \bar{q}^r \bar{q}^s \quad \epsilon_{rst}$ is a singlet

$\epsilon_{123} = +1$ + completely antisymmetric

...rks = spin ½) $\qquad$ P(quarks = spin 0?)

0, 1, 2

$q_r q_s = A_{rs} + B_{rs}$

$3 \otimes 3 = \bar{3} + 6$

SU(3)

$U^\dagger U = 1 \to I$ invariant

$\det U = 1$ $\qquad \delta_r^{\ s}$ "

$q_r \qquad \bar{q}^s \qquad q_r \bar{q}^r$

$\epsilon_{rst} U_{r'}^{\ r} U_{s'}^{\ s} U_{t'}^{\ t} = \epsilon_{r's't'}$

$X_t = \bar{q}^r \bar{q}^s \epsilon_{rst}$ is a singlet

$\epsilon_{123} = +1$ + completely antisymmetric

...rks = spin 1/2) $\quad$ P(quarks = spin 0)

+

0,1,2 + ,, +

$q_r q_s = A_{rs} + B_{rs}$

$3 \otimes 3 = \bar{3} + 6$

$q_r \quad \bar{q}^s \qquad q_r \bar{q}^r$

$X_t = \bar{q}^r \bar{q}^s \quad \epsilon_{rst}$ is a singlet

SU(3)

$U^\dagger U = 1 \rightarrow I$ invariant

$\det U = 1 \qquad \delta_r^{\ s}$ "

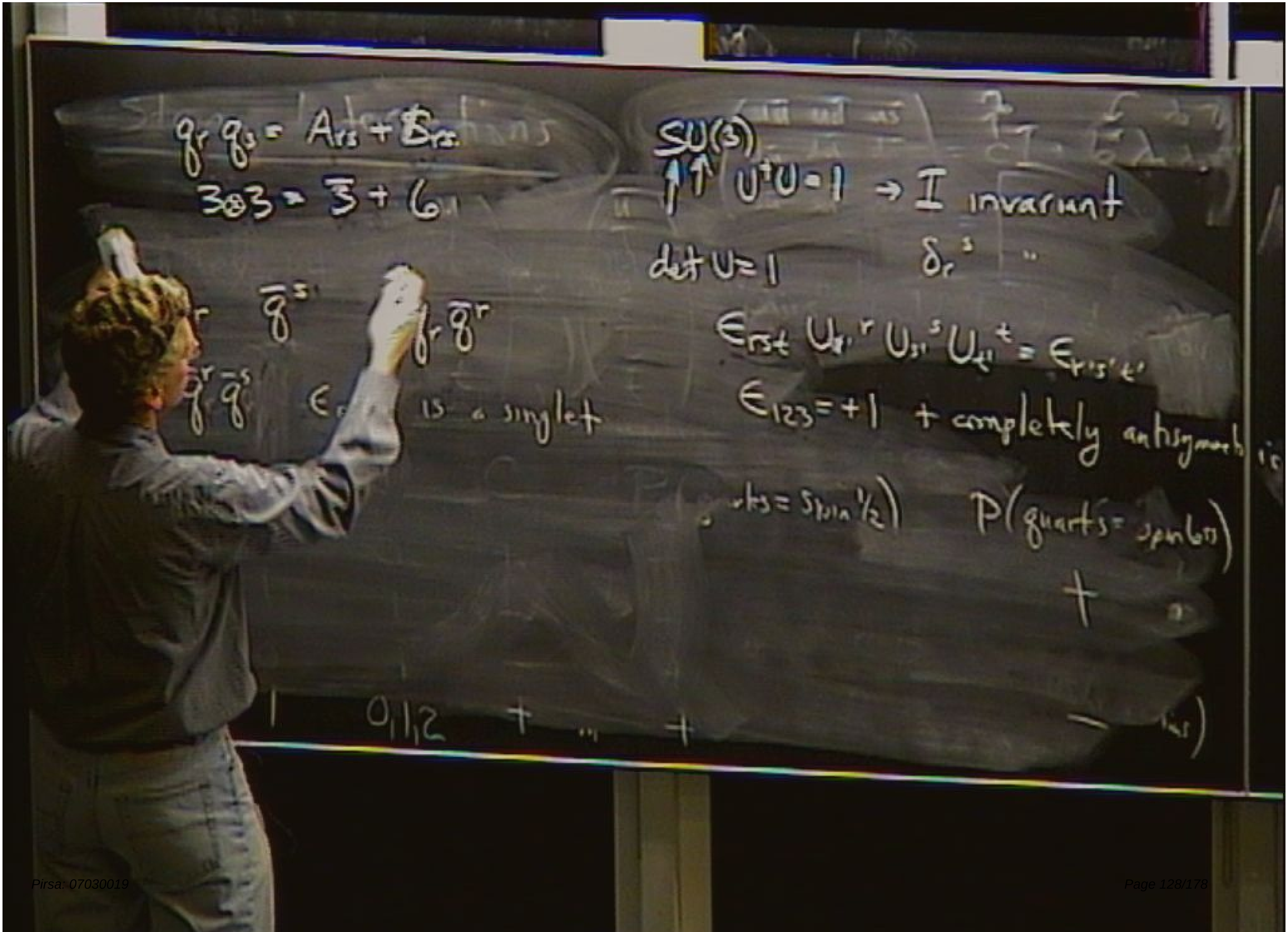
$\epsilon_{rst}\, U_{r'}^{\ r} U_{s'}^{\ s} U_{t'}^{\ t} = \epsilon_{r's't'}$

$\epsilon_{123} = +1$ + completely antisymmetric

...rks = spin 1/2) P(quarks = spin 0)

+

0, 1, 2 + " + — ...)


q_r q_s = A_rs + B_rs
3⊗3 = 3̄ + 6
SU(3)
U†U = 1 → I invariant
det U = 1 δ_r^s "
ε_rst U_r'^r U_s'^s U_t'^t = ε_r's't'
ε_123 = +1 + completely antisymmetric
is a singlet
P(quarks = spin 1/2)
0,1,2

$q_r q_s = A_{rs} + B_{rs}$

$3 \otimes 3 = \bar{3} + 6$

$SU(3)$

$U^\dagger U = 1 \rightarrow I$ invariant

$\det U = 1 \qquad \delta_r^s$ "

$q_r \quad \bar{q}^s \qquad q_r \bar{q}^r$

$\epsilon_{rst} U_{r'}{}^r U_{s'}{}^s U_{t'}{}^t = \epsilon_{r's't'}$

$\bar{q}_t = \bar{q}^r \bar{q}^s$ $\quad \epsilon_{rst}$ is a singlet

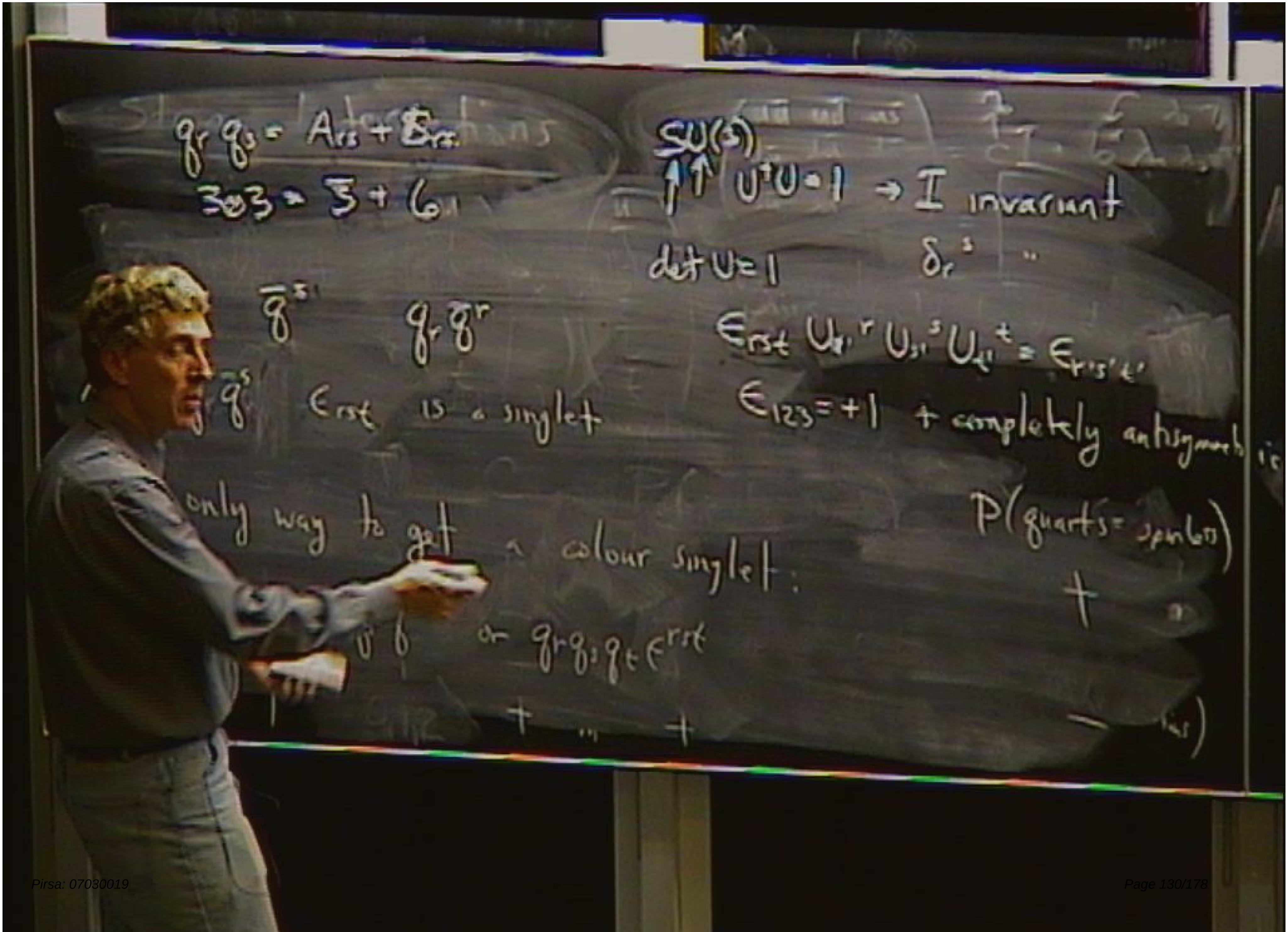
$\epsilon_{123} = +1$ + completely antisymmetric

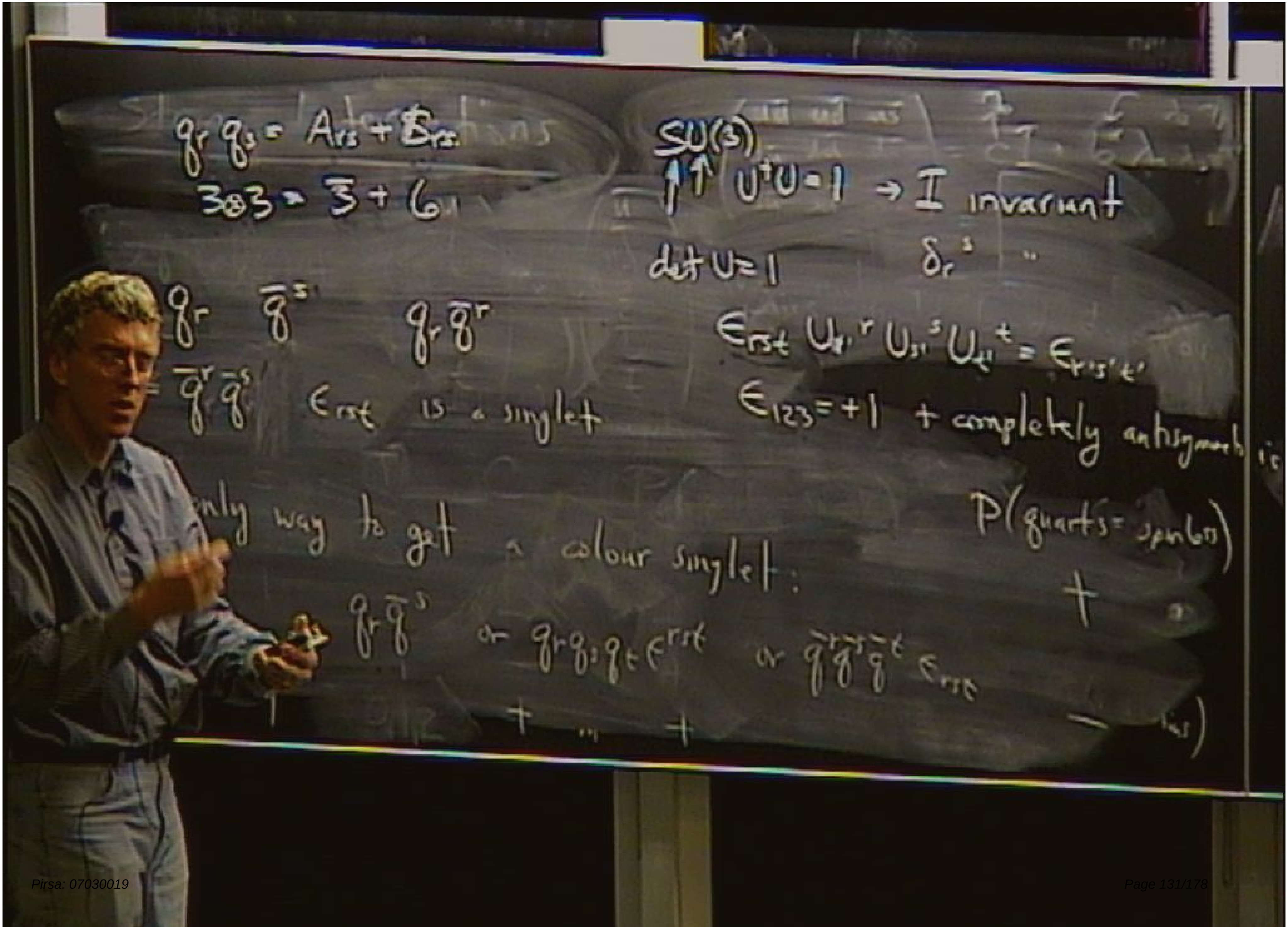
$\epsilon^{rst} q_r q_s = y^t$

...rks = spin ½) $\qquad$ P(quarks = spin 0)

+

0, 1, 2 $\quad$ + $\quad$ " $\quad$ +


$q_r q_s = A_{rs} + B_{rs}$
$3 \otimes 3 = \bar{3} + 6$
SU(3)
$U^\dagger U = 1 \rightarrow I$ invariant
det U = 1
δ_r^s "
$\bar{q}^s$
$q_r \bar{q}^r$
$\epsilon_{rst} U_{r'}{}^r U_{s'}{}^s U_{t'}{}^t = \epsilon_{r's't'}$
ϵ_{rst} is a singlet
$\epsilon_{123} = +1$ + completely antisymmetric
only way to get a colour singlet:
P(quarks = spin 1/2)
or $q_r q_s q_t \epsilon^{rst}$


$q_r q_s = A_{rs} + B_{rs}$
$3 \otimes 3 = \bar{3} + 6$
SU(3)
$U^\dagger U = 1 \rightarrow I$ invariant
$\det U = 1$
δ_r^s "
$q_r \quad \bar{q}^s \quad q_r \bar{q}^r$
$\epsilon_{rst} U_{r'}{}^r U_{s'}{}^s U_{t'}{}^t = \epsilon_{r's't'}$
$\bar{q}^r \bar{q}^s$ ϵ_{rst} is a singlet
$\epsilon_{123} = +1$ + completely antisymmetric
only way to get a colour singlet:
$q_r \bar{q}^s$ or $q_r q_s q_t \epsilon^{rst}$ or $\bar{q}^r \bar{q}^s \bar{q}^t \epsilon_{rst}$

$q_r q_s = A_{rs} + B_{rs}$

$3 \otimes 3 = \bar{3} + 6$

SU(3)

$U^\dagger U = 1 \to I$ invariant

$\det U = 1$ $\qquad \delta_r{}^s$ "

$q_r \qquad \bar{q}^s \qquad\qquad q_r \bar{q}^r$

$\epsilon_{rst}\, U_{r'}{}^r\, U_{s'}{}^s\, U_{t'}{}^t = \epsilon_{r's't'}$

$= \bar{q}^r \bar{q}^s$ $\quad \epsilon_{rst}$ is a singlet

$\epsilon_{123} = +1$ + completely antisymmetric

The only way to get a colour singlet:

P(quarks = spin...)

$q_r \bar{q}^s$ or $q_r q_s q_t \epsilon^{rst}$ or $\bar{q}^r \bar{q}^s \bar{q}^t \epsilon_{rst}$

mesons $\qquad$ baryons

$q_r q_s = A_{rs} + B_{rs}$

$3 \otimes 3 = \bar{3} + 6$

SU(3)

$U^\dagger U = 1 \rightarrow I$ invariant

$\det U = 1$ $\qquad \delta_r^{\ s}$ "

$q_r \qquad \bar{q}^s \qquad q_r \bar{q}^r$

$\epsilon_{rst} U_{r'}^{\ r} U_{s'}^{\ s} U_{t'}^{\ t} = \epsilon_{r's't'}$

$X_t = \bar{q}^r \bar{q}^s \epsilon_{rst}$ is a singlet

$\epsilon_{123} = +1$ + completely antisymmetric

The only way to get a colour singlet:

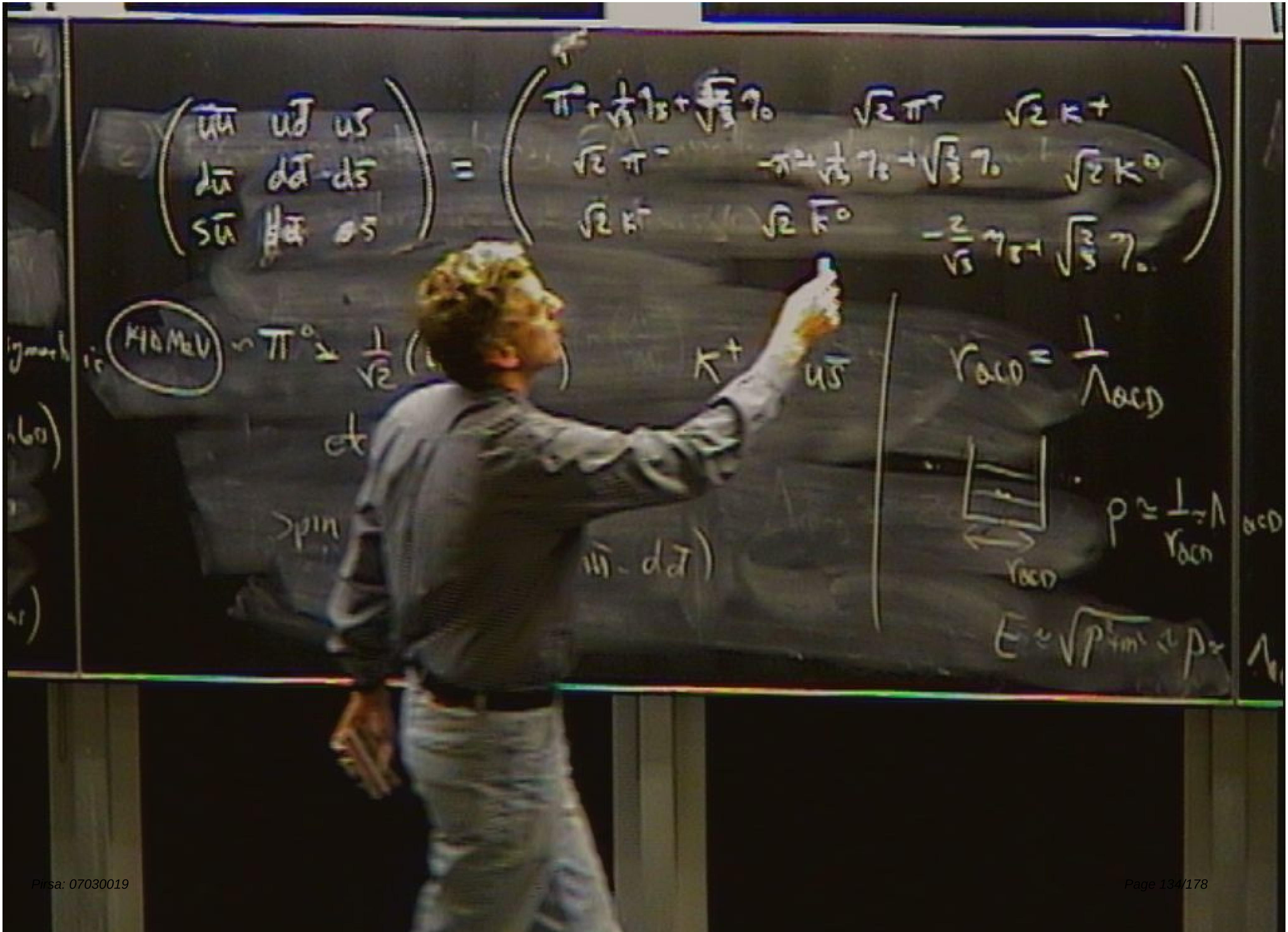
P(quarks = spin 1/2)

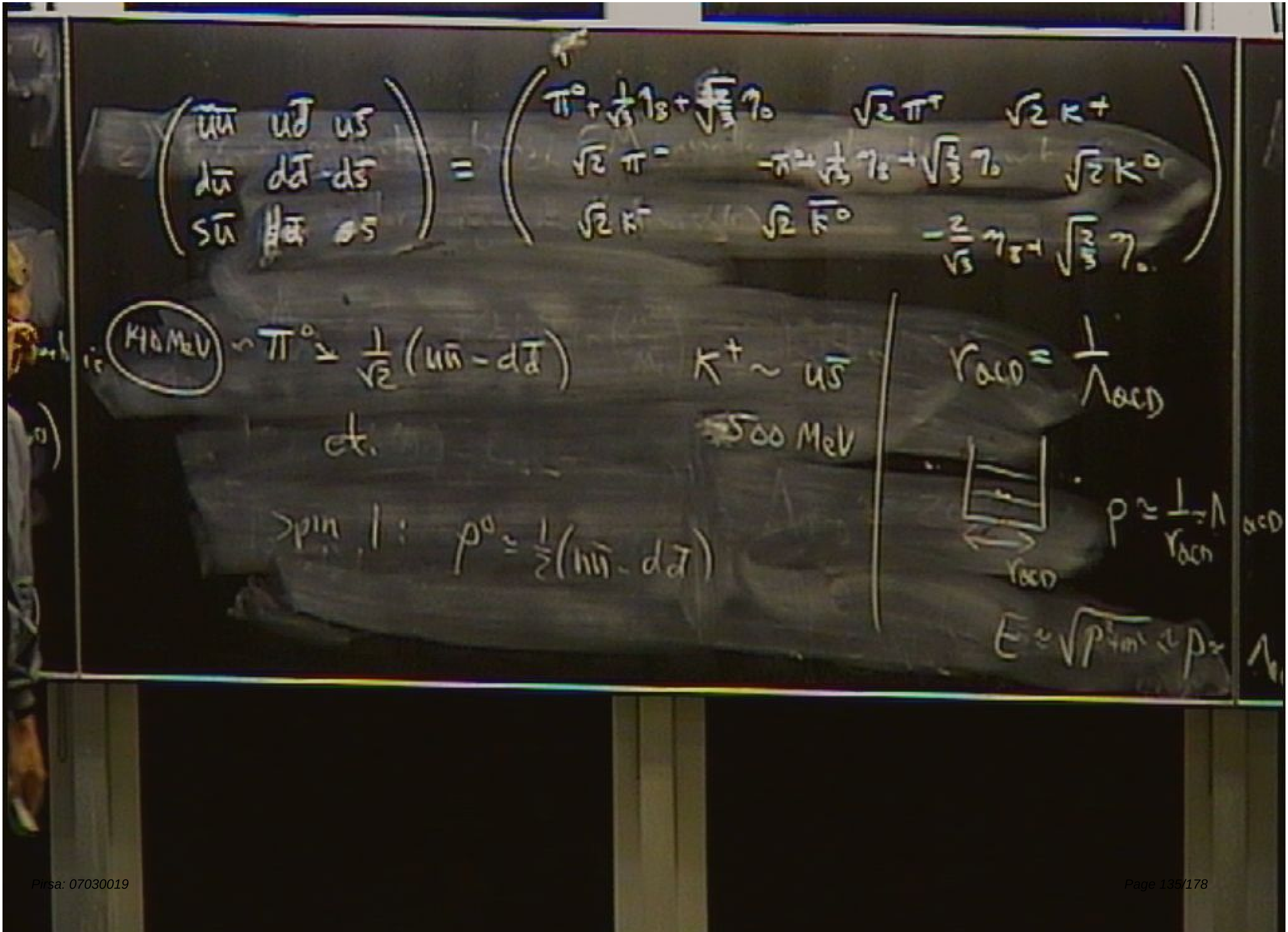
$\underline{q_r \bar{q}^s}$ or $\underline{q_r q_s q_t \epsilon^{rst}}$ or $\bar{q}^r \bar{q}^s \bar{q}^t \epsilon_{rst}$

mesons. $\qquad$ baryons

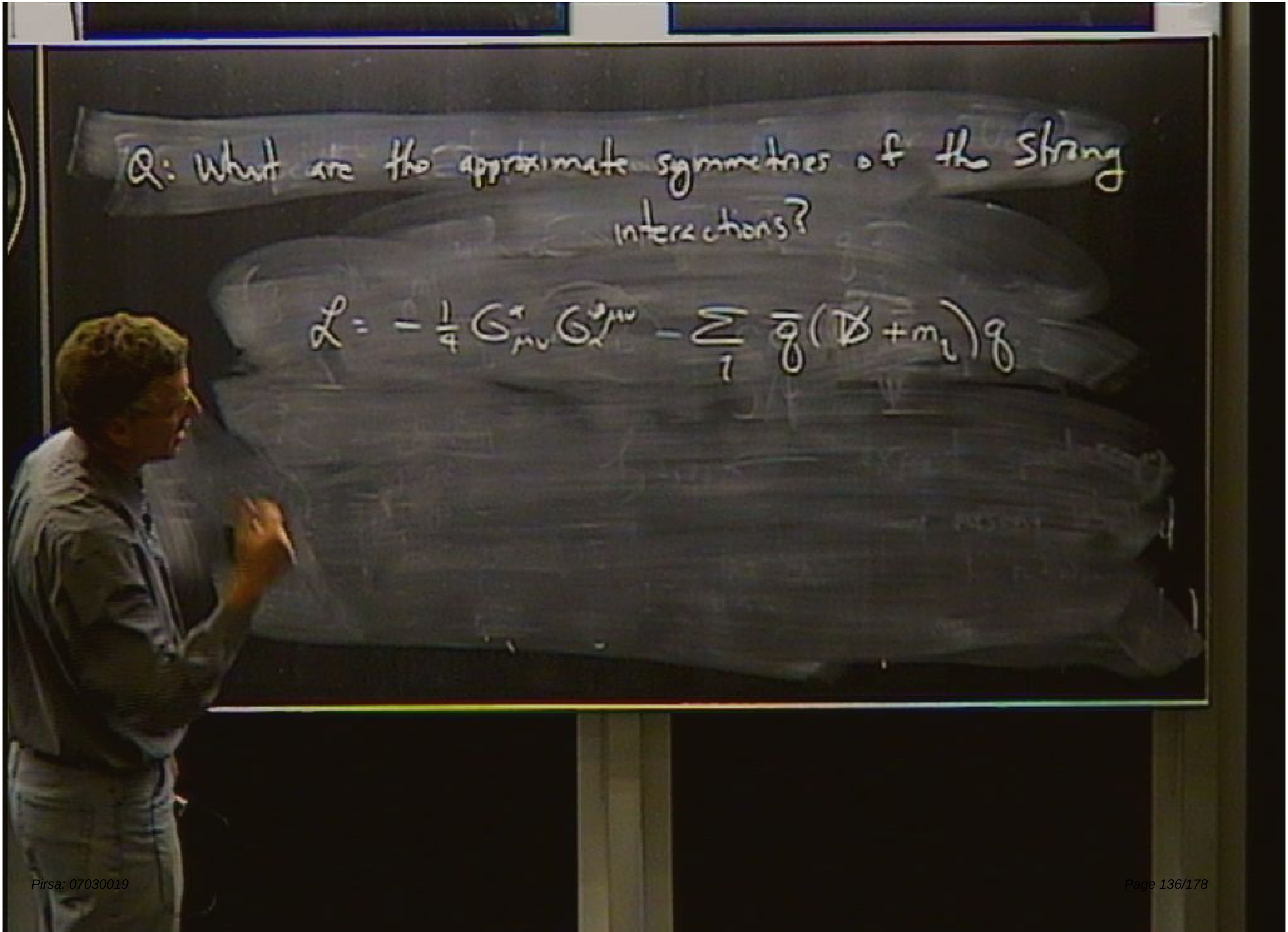
$+$

$-$ ...)





$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$
140 MeV $\sim \pi^0 \simeq \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
$K^+ \sim u\bar{s}$
etc.
500 MeV
Spin 1: $\rho^0 \simeq \frac{1}{2}(u\bar{u} - d\bar{d})$
$r_{QCD} = \frac{1}{\Lambda_{QCD}}$
r_{QCD}
$p \simeq \frac{1}{r_{QCD}} \sim \Lambda_{QCD}$
$E \simeq \sqrt{p^2 + m^2} \simeq p \simeq \Lambda$

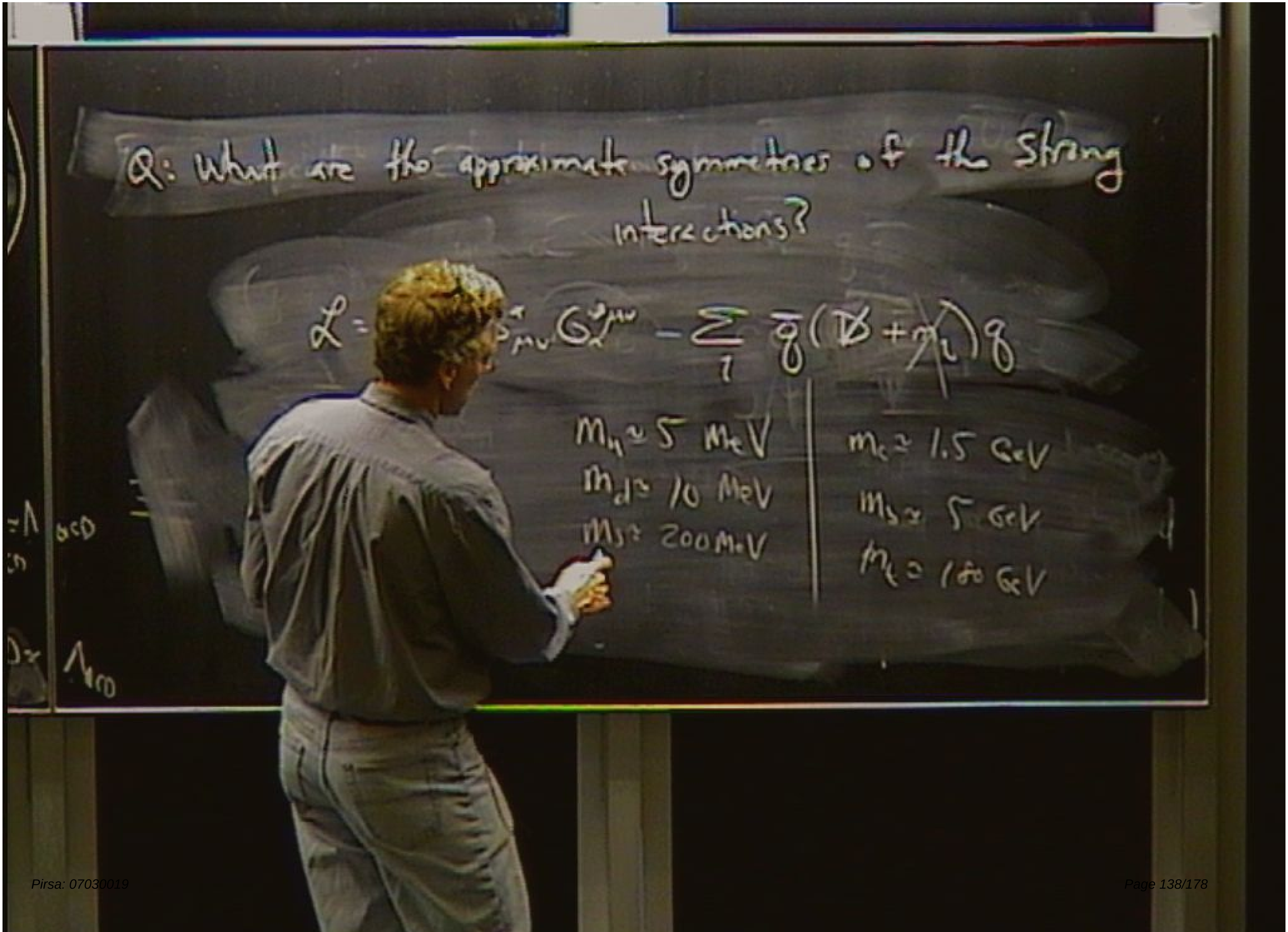

Q: What are the approximate symmetries of the strong interactions?
$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} - \sum_i \bar{q}_i (\not{D} + m_i) q_i$

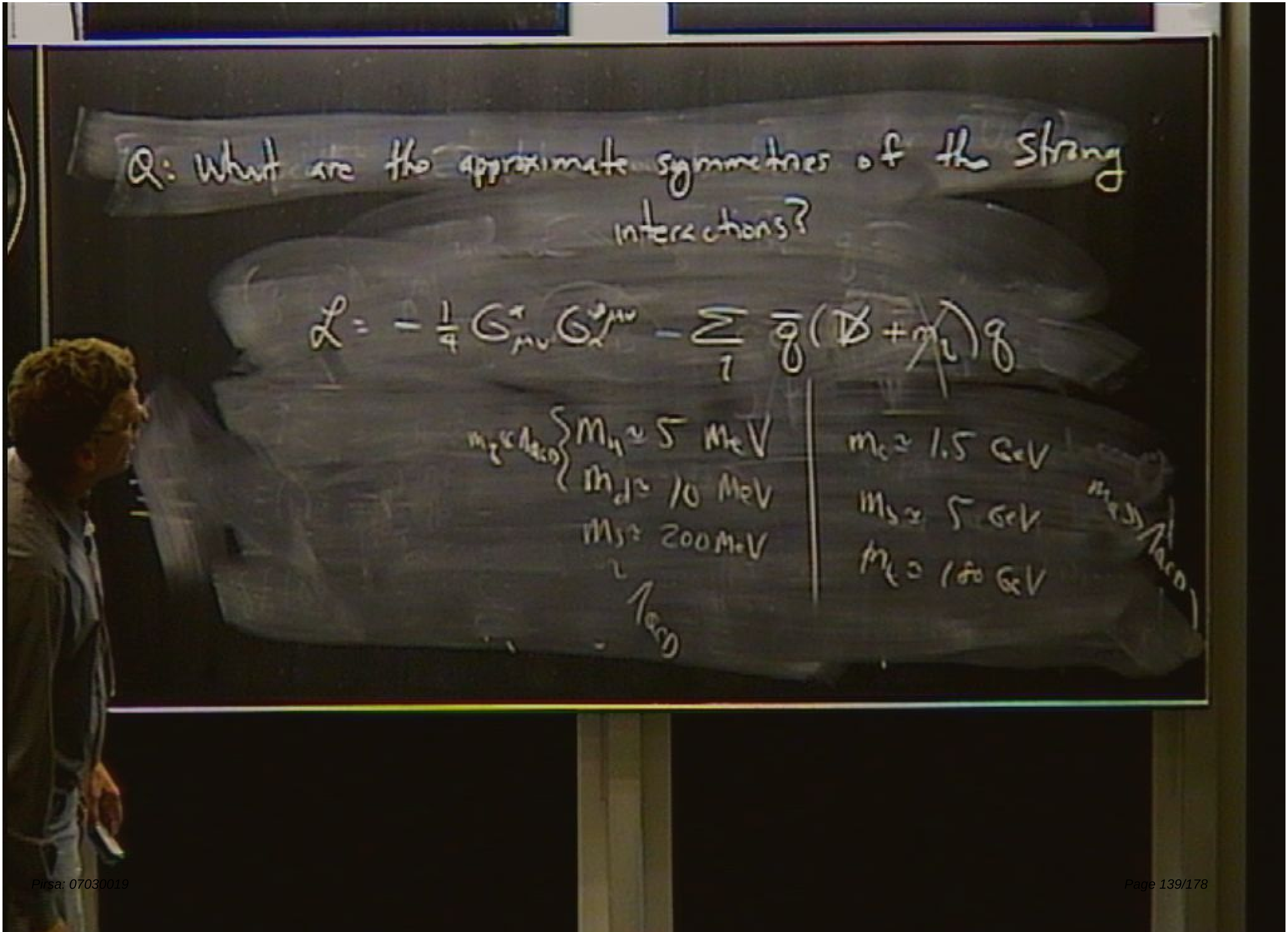
Q: What are the approximate symmetries of the strong interactions?

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \sum_i \bar{q}_i (\not{D} + m_i) q_i$$

$\approx \Lambda_{QCD}$

Λ_{QCD}


Q: What are the approximate symmetries of the Strong interactions?
m_u ≃ 5 MeV
m_d ≃ 10 MeV
m_s ≃ 200 MeV
m_c ≃ 1.5 GeV
m_b ≃ 5 GeV
m_t ≃ 180 GeV


Q: What are the approximate symmetries of the strong interactions?
$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} - \sum_i \bar{q} (\not{D} + m_i) q$
$m_i \ll \Lambda_{QCD}$ $\{ m_u \simeq 5$ MeV, $m_d \simeq 10$ MeV
$m_s \simeq 200$ MeV $\sim \Lambda_{QCD}$
$m_c \simeq 1.5$ GeV
$m_b \simeq 5$ GeV
$m_t \simeq 180$ GeV
$m_{c,b,t} \gg \Lambda_{QCD}$

Q: What are the approximate symmetries of the strong interactions?

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} - \sum_q \bar{q}(\not{D} + m_q) q$$

for u,d, (possibly for s) $m_q \simeq 0$ should be a good approx

$m_q \ll \Lambda_{QCD}$ $\begin{cases} m_u \simeq 5 \text{ MeV} \\ m_d \simeq 10 \text{ MeV} \end{cases}$

$m_s \simeq 200$ MeV

$\sim \Lambda_{QCD}$

$m_c \simeq 1.5$ GeV

$m_b \simeq 5$ GeV

$m_t \simeq 180$ GeV

$m_q \gg \Lambda_{QCD}$

Q: What are the approximate symmetries of the strong interactions?

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \sum_i \bar{q}\,(\not{D} + m_i)\,q$$

for u,d, (possibly for s) $m_i \approx 0$

should be a good approx

$m_i \ll \Lambda_{QCD}$ $\begin{cases} m_u \approx 5 \text{ MeV} \\ m_d \approx 10 \text{ MeV} \\ m_s \approx 200 \text{ MeV} \end{cases}$

$\frac{m_i}{\Lambda_{QCD}}$

$m_c \approx 1.5$ GeV	
$m_b \approx 5$ GeV	$m_{c,b,t} \gg \Lambda_{QCD}$
$m_t \approx 180$ GeV	

Q: What are the approximate symmetries of the strong interactions?

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \sum_q \bar{q}(\not{D} + m_q) q$$

for u,d, (possibly for s) $m_q \approx 0$ should be a good approx

$m_q \ll \Lambda_{QCD}$ $\begin{cases} m_u \simeq 5\ \text{MeV} \\ m_d \simeq 10\ \text{MeV} \end{cases}$

$m_s \simeq 200\ \text{MeV}$

$\sim \Lambda_{QCD}$

$m_c \simeq 1.5\ \text{GeV}$

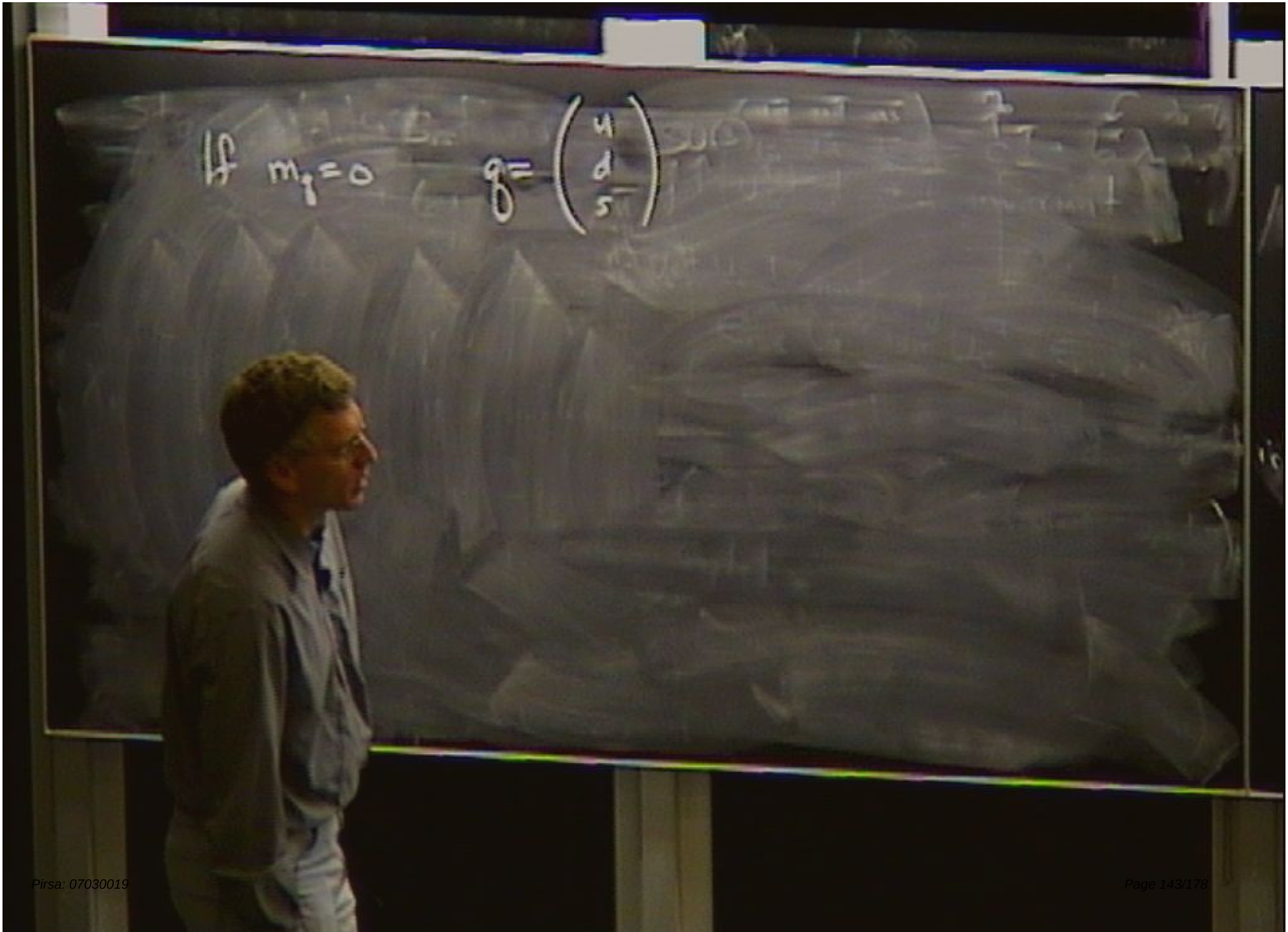
$m_b \simeq 5\ \text{GeV}$

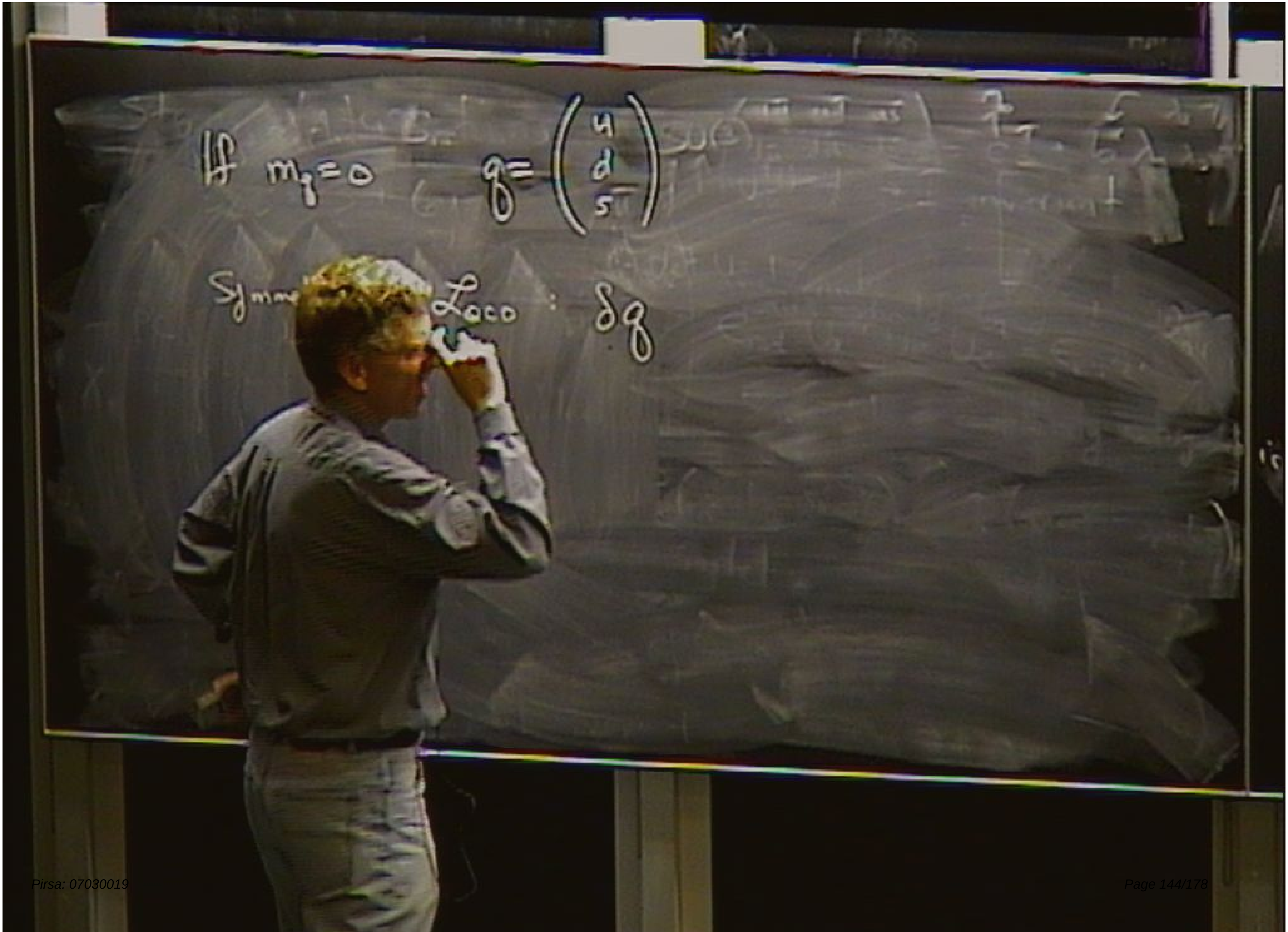
$m_t \simeq 180\ \text{GeV}$

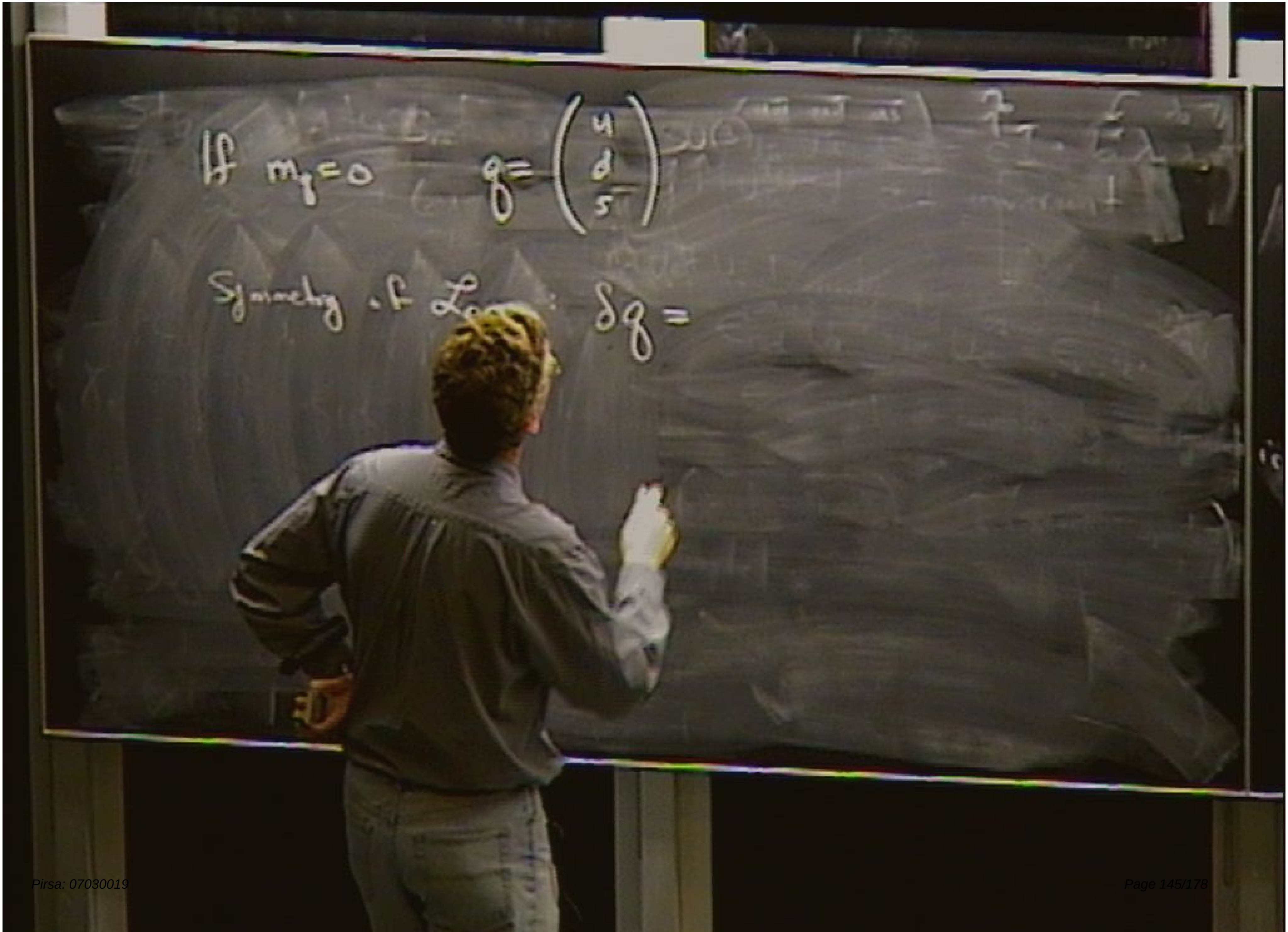
$m_{c,b,t} \gg \Lambda_{QCD}$

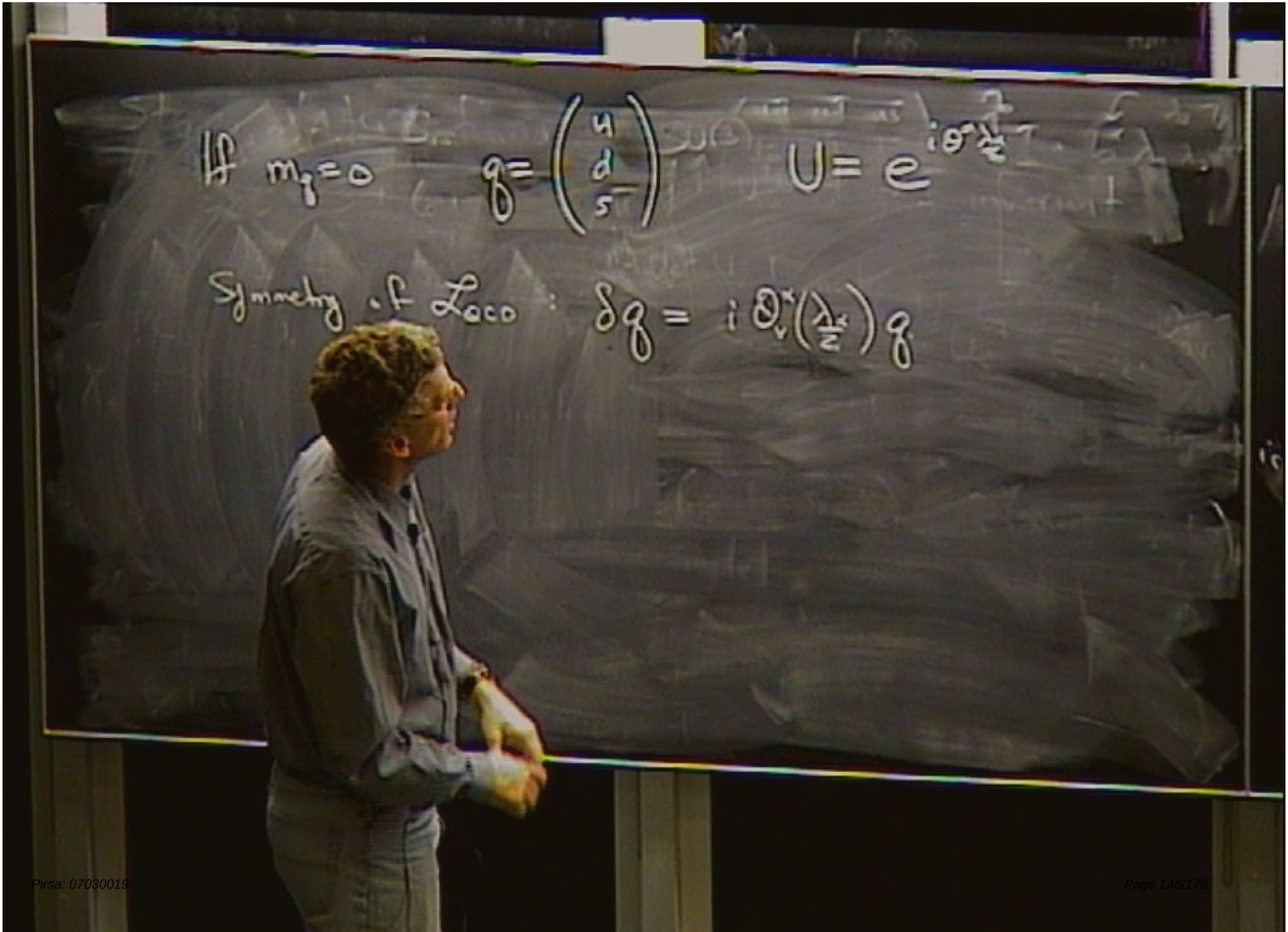
$\frac{m_q}{\Lambda_{QCD}}$ chiral perturbation th.

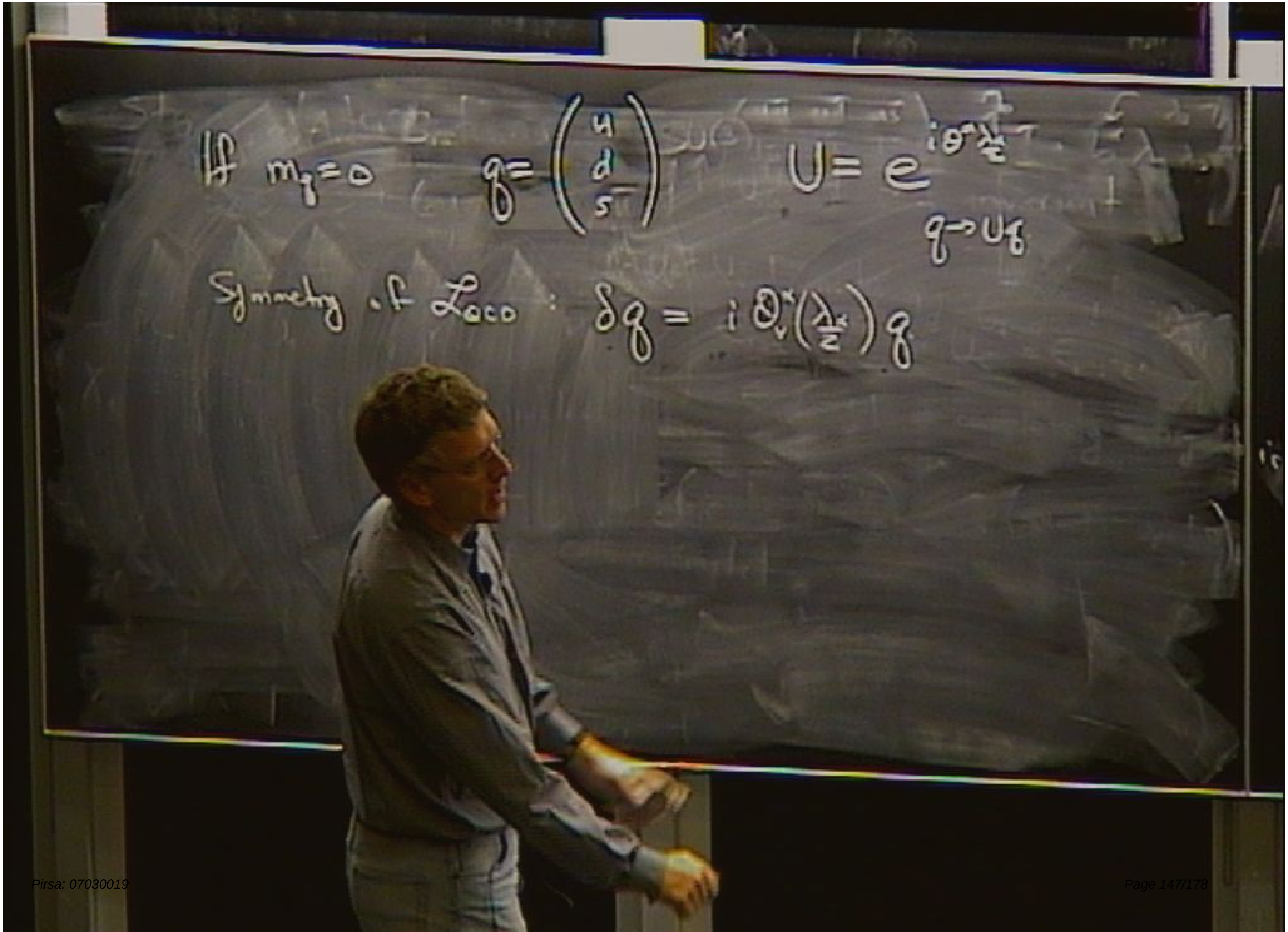
Λ_{QCD}

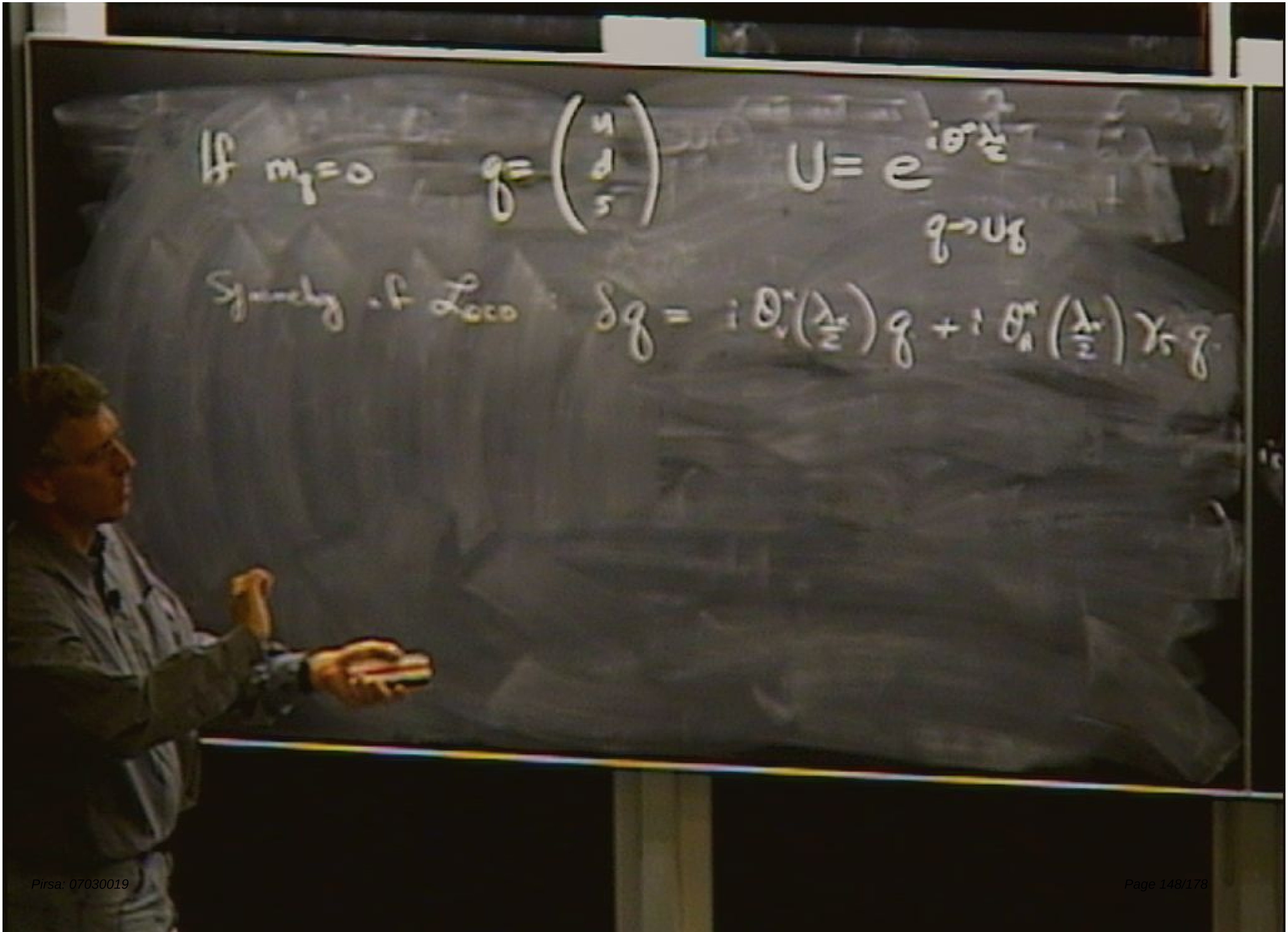

If $m_q = 0$
$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$

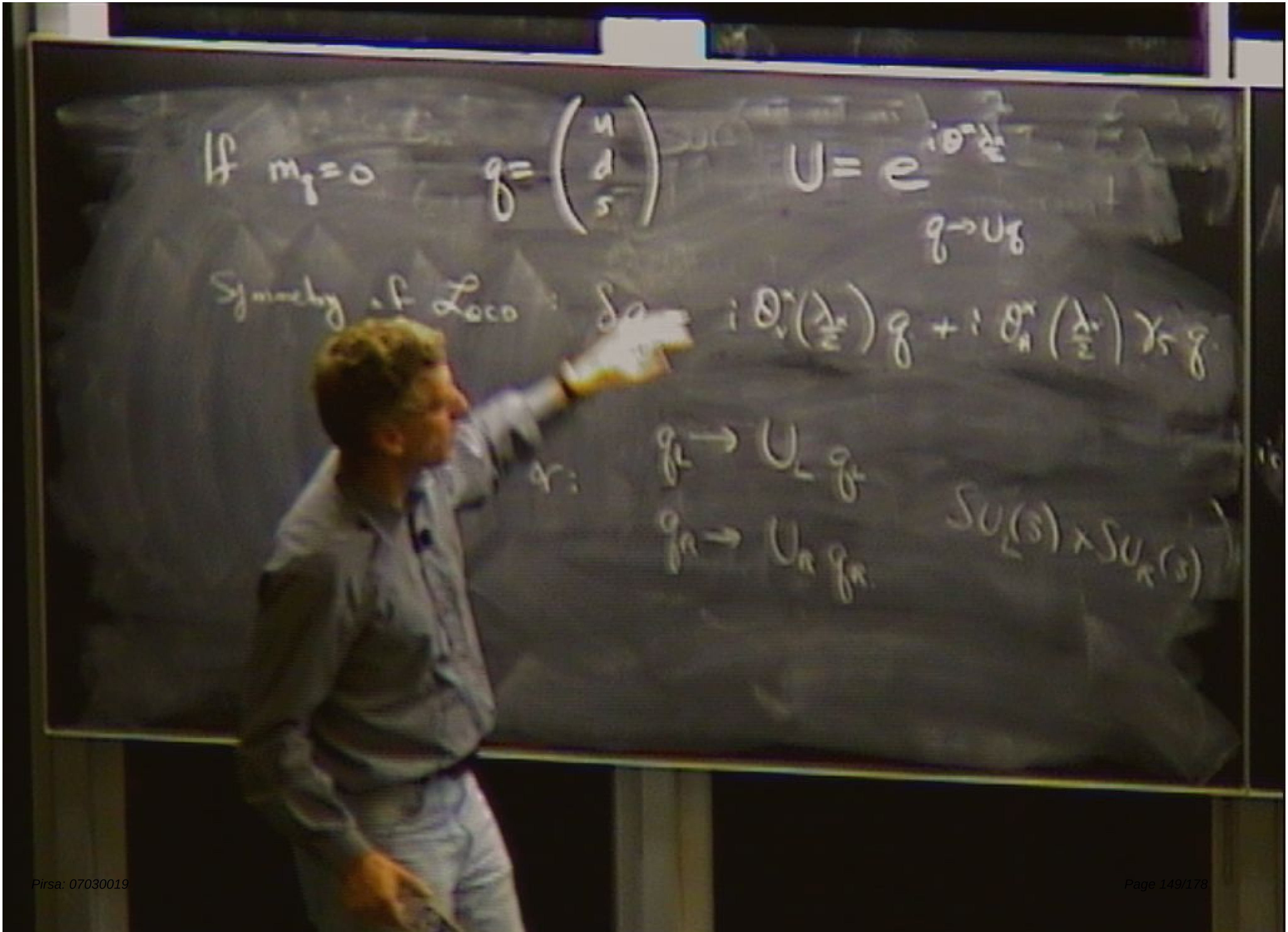

If m_q = 0
q = (u d s)
L_QCD :
δq

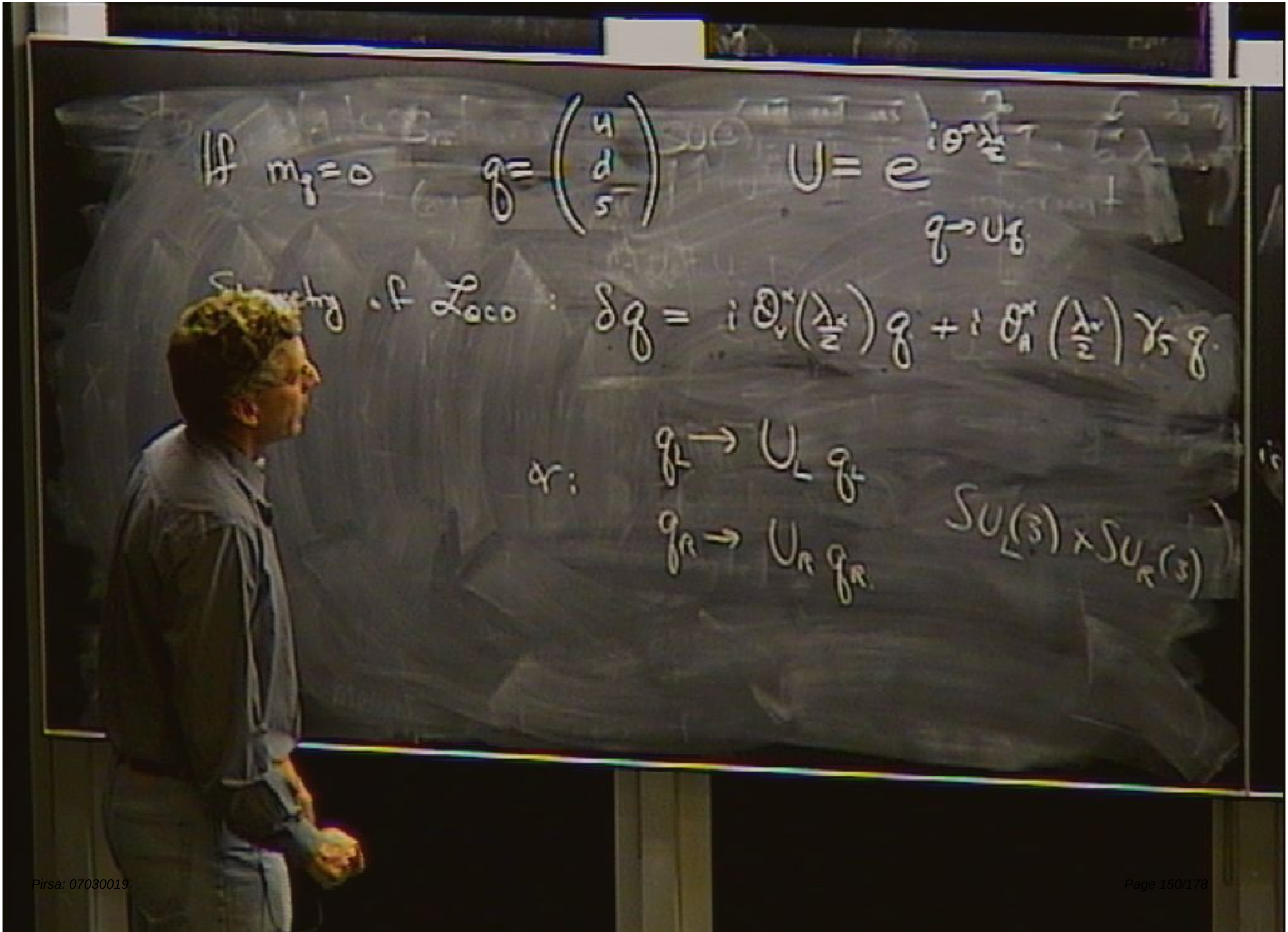

If $m_q = 0$ $\quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$
Symmetry of $\mathcal{L}$: $\delta q =$

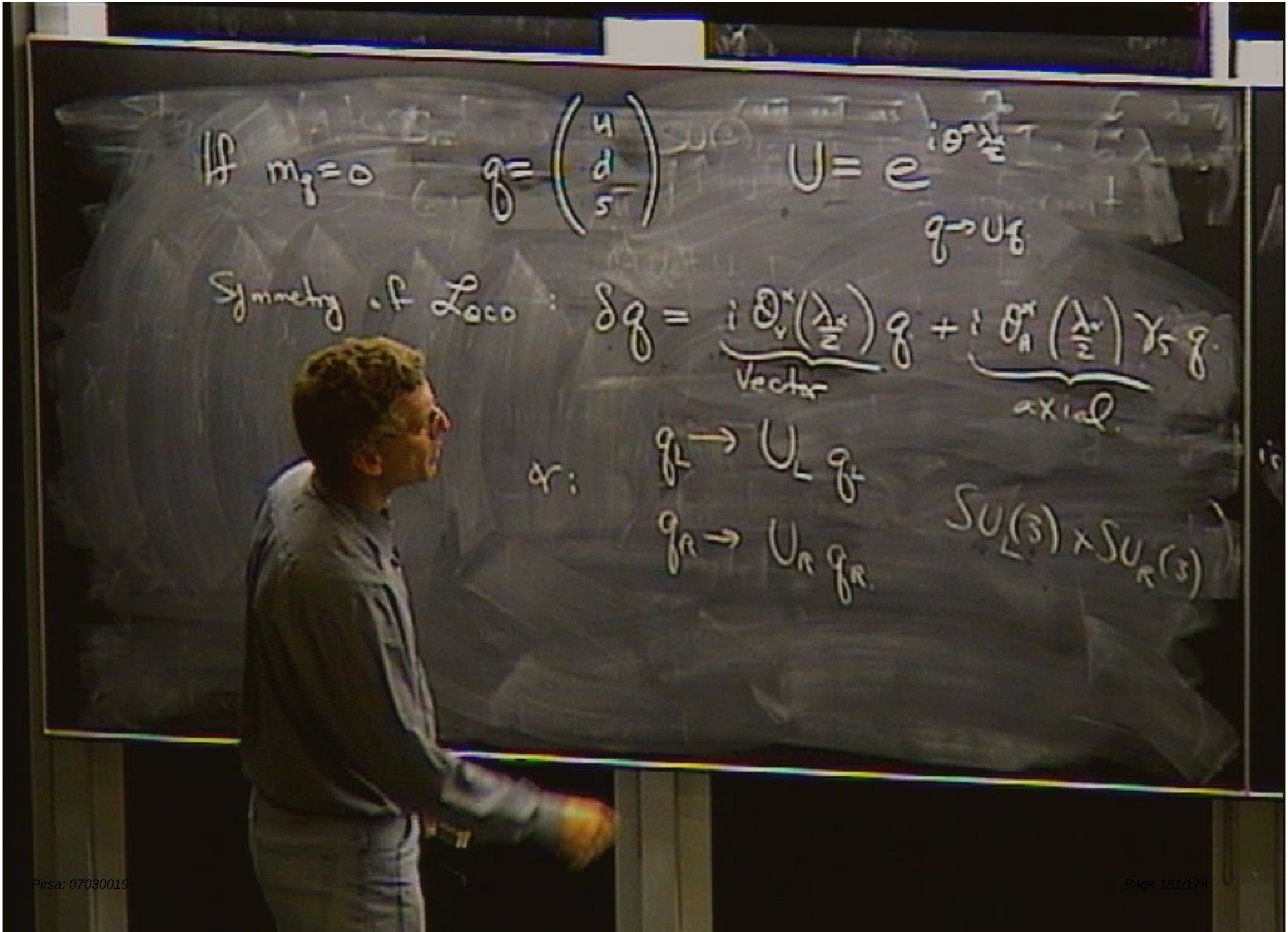

If $m_q=0$ $q=\begin{pmatrix}u\\d\\s\end{pmatrix}$ $U=e^{i\theta^\alpha\frac{\lambda^\alpha}{2}}$
Symmetry of $\mathcal{L}_{QCD}$: $\delta q = i\,\theta_V^\alpha\left(\frac{\lambda^\alpha}{2}\right)q$

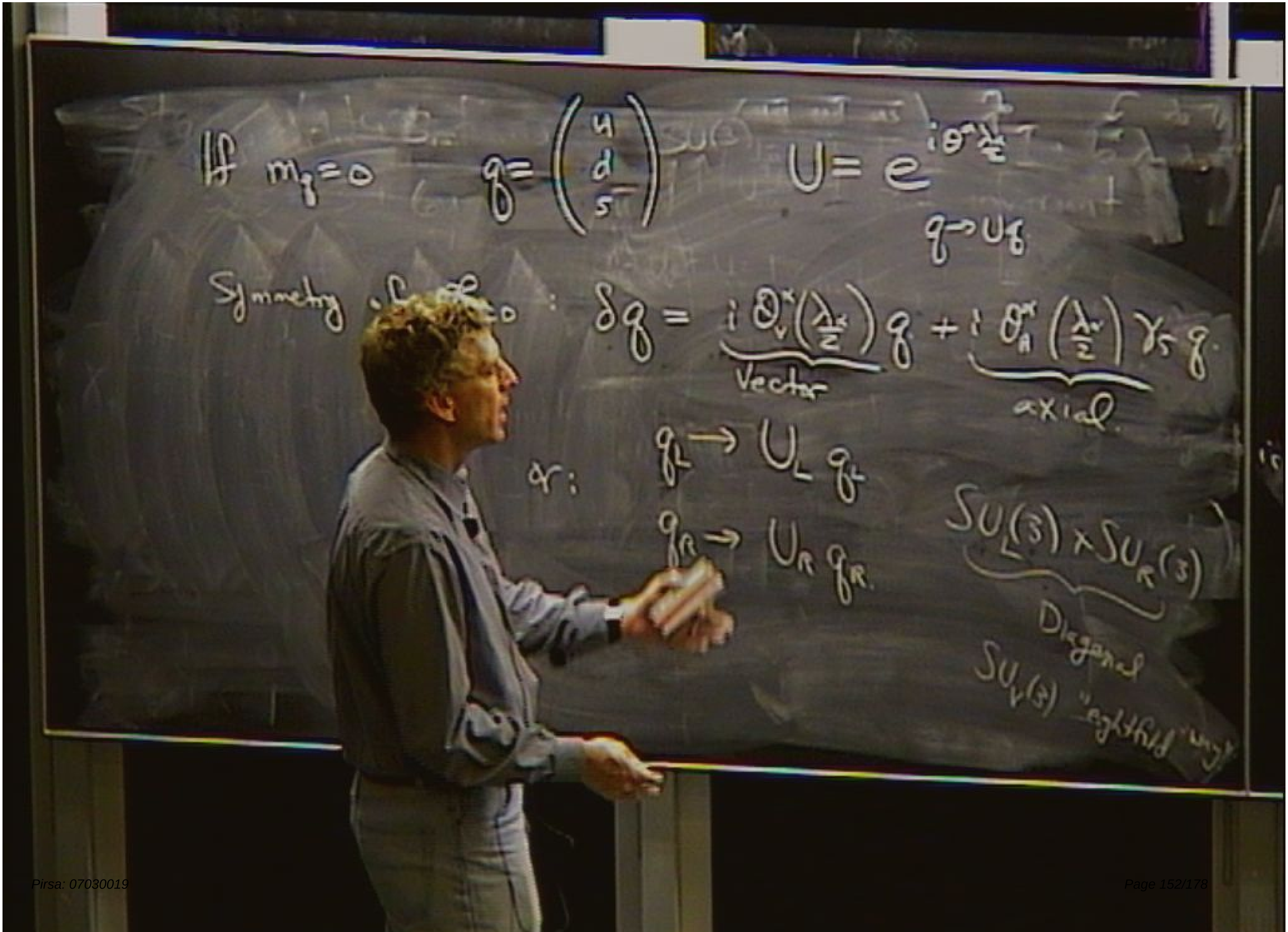

If $m_q=0$ $q=\begin{pmatrix}u\\d\\s\end{pmatrix}$ $U=e^{i\theta^\alpha\frac{\lambda^\alpha}{2}}$
$q\to Uq$
Symmetry of $\mathcal{L}_{QCD}$: $\delta q = i\theta_V^\alpha\left(\frac{\lambda^\alpha}{2}\right)q$

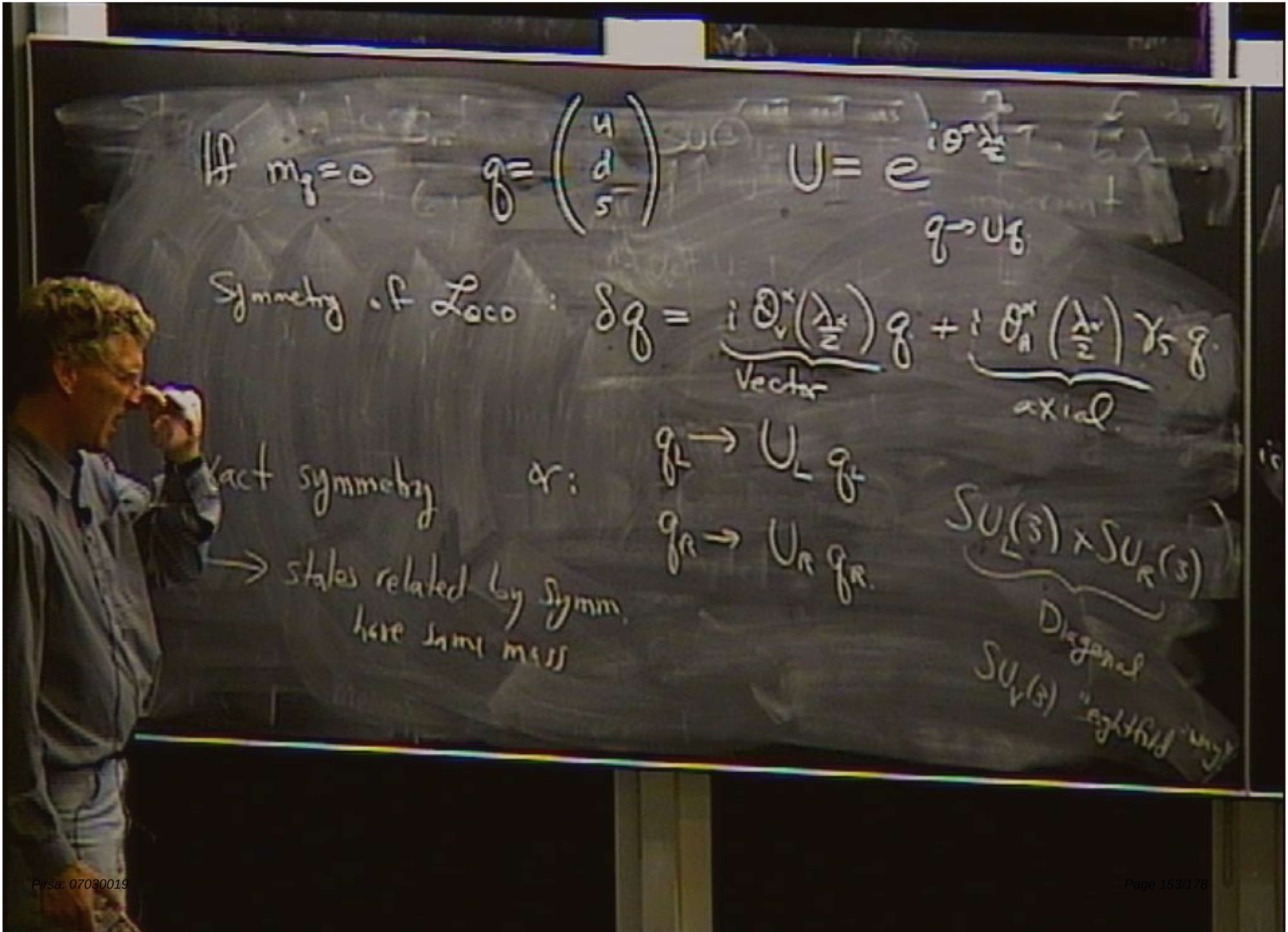

If $m_q = 0$
$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$
$U = e^{i\theta^a \frac{\lambda^a}{2}}$
$q \to Uq$
Symmetry of $\mathcal{L}_{QCD}$: $\delta q = i\theta_V^a\left(\frac{\lambda^a}{2}\right)q + i\theta_A^a\left(\frac{\lambda^a}{2}\right)\gamma_5 q$

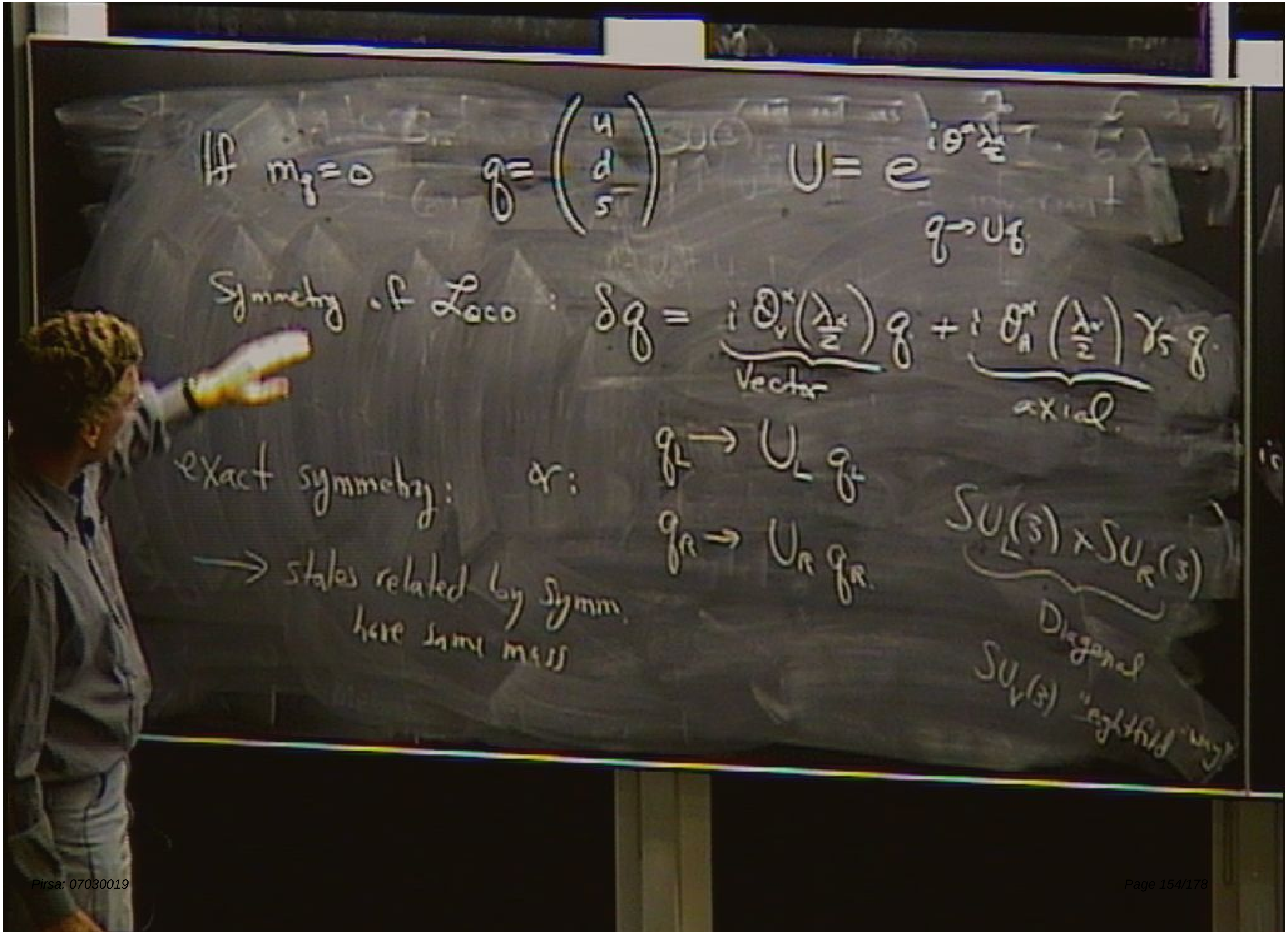

If $m_q = 0$
$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$
$U = e^{i\theta^a \frac{\lambda^a}{2}}$
$q \to Uq$
Symmetry of $\mathcal{L}_{QCD}$: $\delta q = i\theta_V^a \left(\frac{\lambda^a}{2}\right) q + i\theta_A^a \left(\frac{\lambda^a}{2}\right) \gamma_5 q$
$q_L \to U_L q_L$
$q_R \to U_R q_R$
$SU_L(3) \times SU_R(3)$

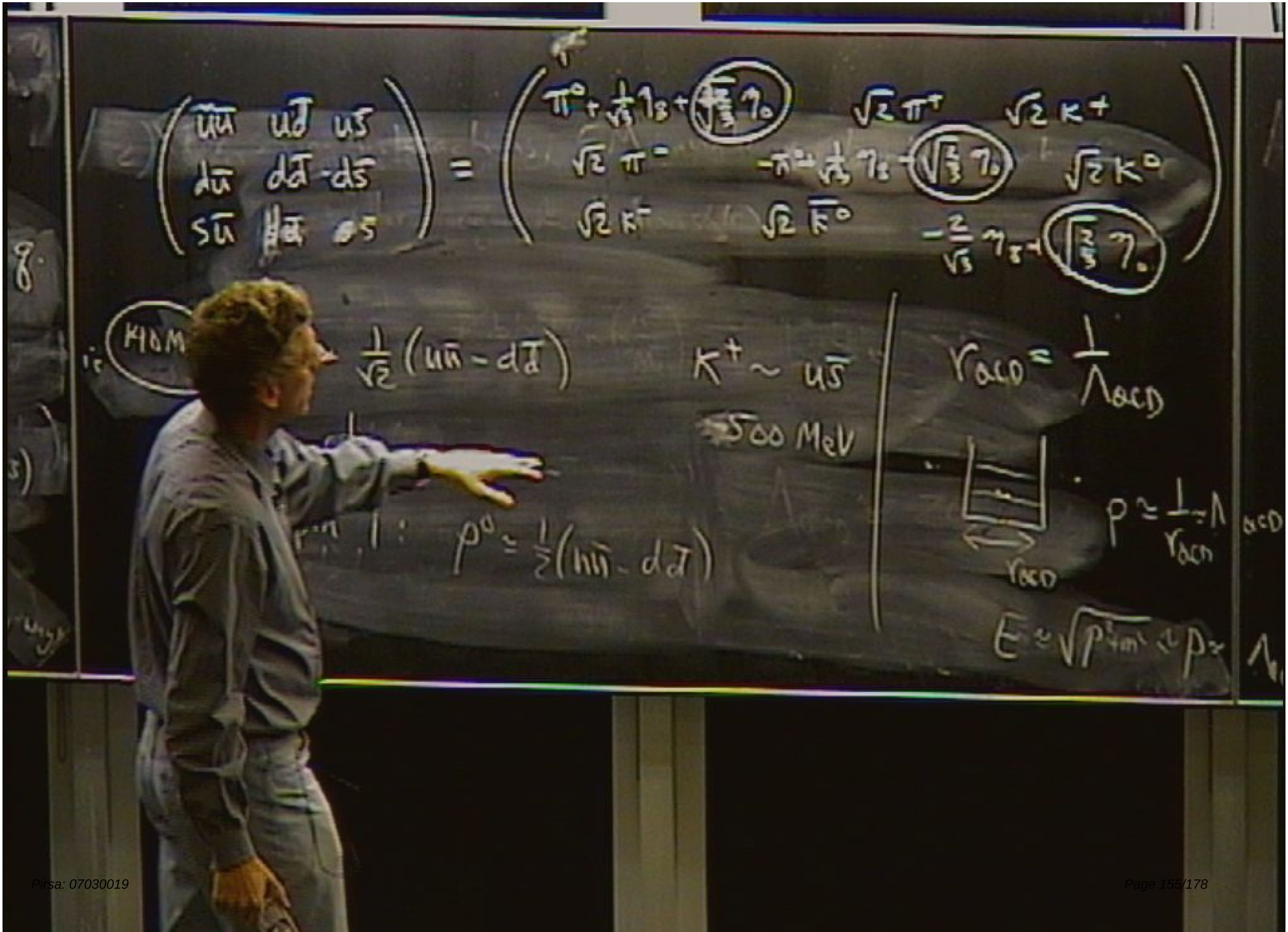

If $m_q = 0$ $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ $U = e^{i\theta^\alpha \frac{\lambda^\alpha}{2}}$
$q \to Uq$
Symmetry of $\mathcal{L}_{QCD}$: $\delta q = i\,\theta_V^\alpha \left(\frac{\lambda^\alpha}{2}\right) q + i\,\theta_A^\alpha \left(\frac{\lambda^\alpha}{2}\right) \gamma_5 q$
or: $q_L \to U_L\, q_L$
$q_R \to U_R\, q_R$
$SU_L(3) \times SU_R(3)$

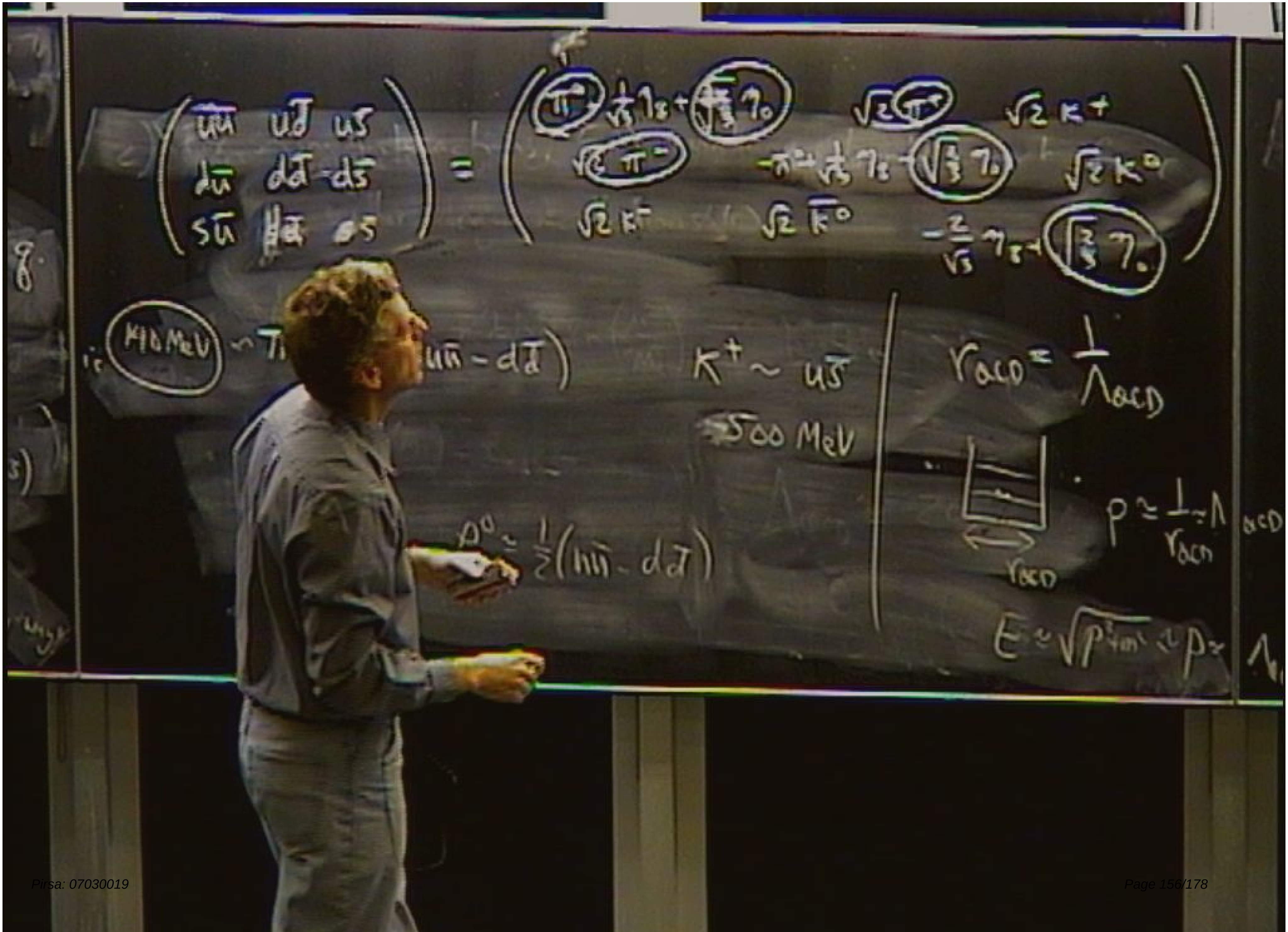

If $m_q = 0$
$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$
$U = e^{i\theta^\alpha \frac{\lambda^\alpha}{2}}$
$q \to Uq$
Symmetry of $\mathcal{L}_{QCD}$: $\delta q = \underbrace{i\,\theta_V^\alpha\left(\frac{\lambda^\alpha}{2}\right)}_{\text{Vector}} q + \underbrace{i\,\theta_A^\alpha\left(\frac{\lambda^\alpha}{2}\right)\gamma_5}_{\text{axial}} q$
or: $q_L \to U_L\, q_L$
$q_R \to U_R\, q_R$
$SU_L(3) \times SU_R(3)$

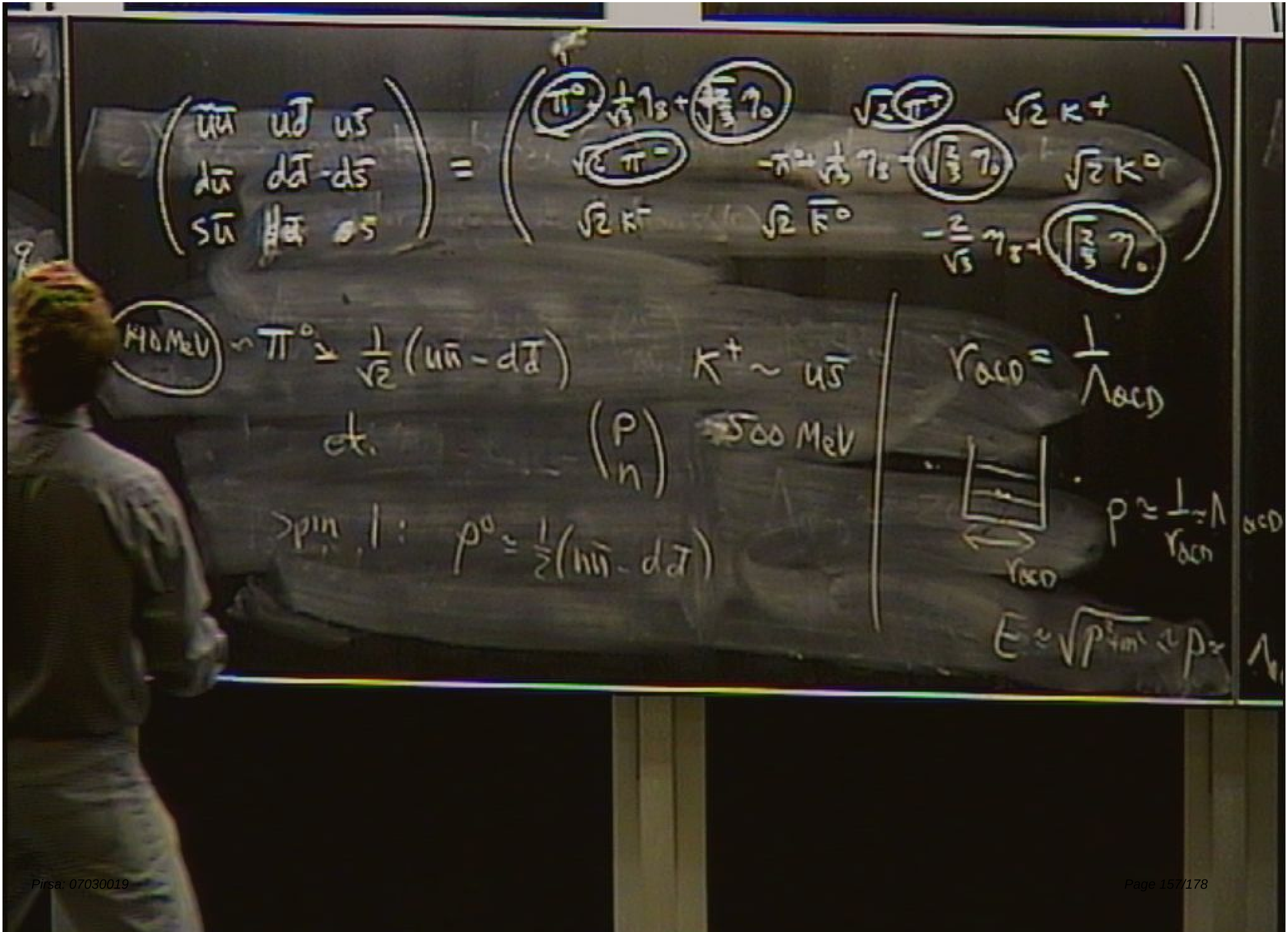

If $m_q = 0$
$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$
$U = e^{i\theta^\alpha \frac{\lambda^\alpha}{2}}$
$q \to Uq$
Symmetry of $\mathcal{L}_0$: $\delta q = \underbrace{i\theta_V^\alpha \left(\frac{\lambda^\alpha}{2}\right) q}_{\text{Vector}} + \underbrace{i\theta_A^\alpha \left(\frac{\lambda^\alpha}{2}\right) \gamma_5 q}_{\text{axial}}$
or: $q_L \to U_L q_L$
$q_R \to U_R q_R$
$\underbrace{SU_L(3) \times SU_R(3)}_{\text{Diagonal}}$
$SU_V(3)$ "eightfold way"

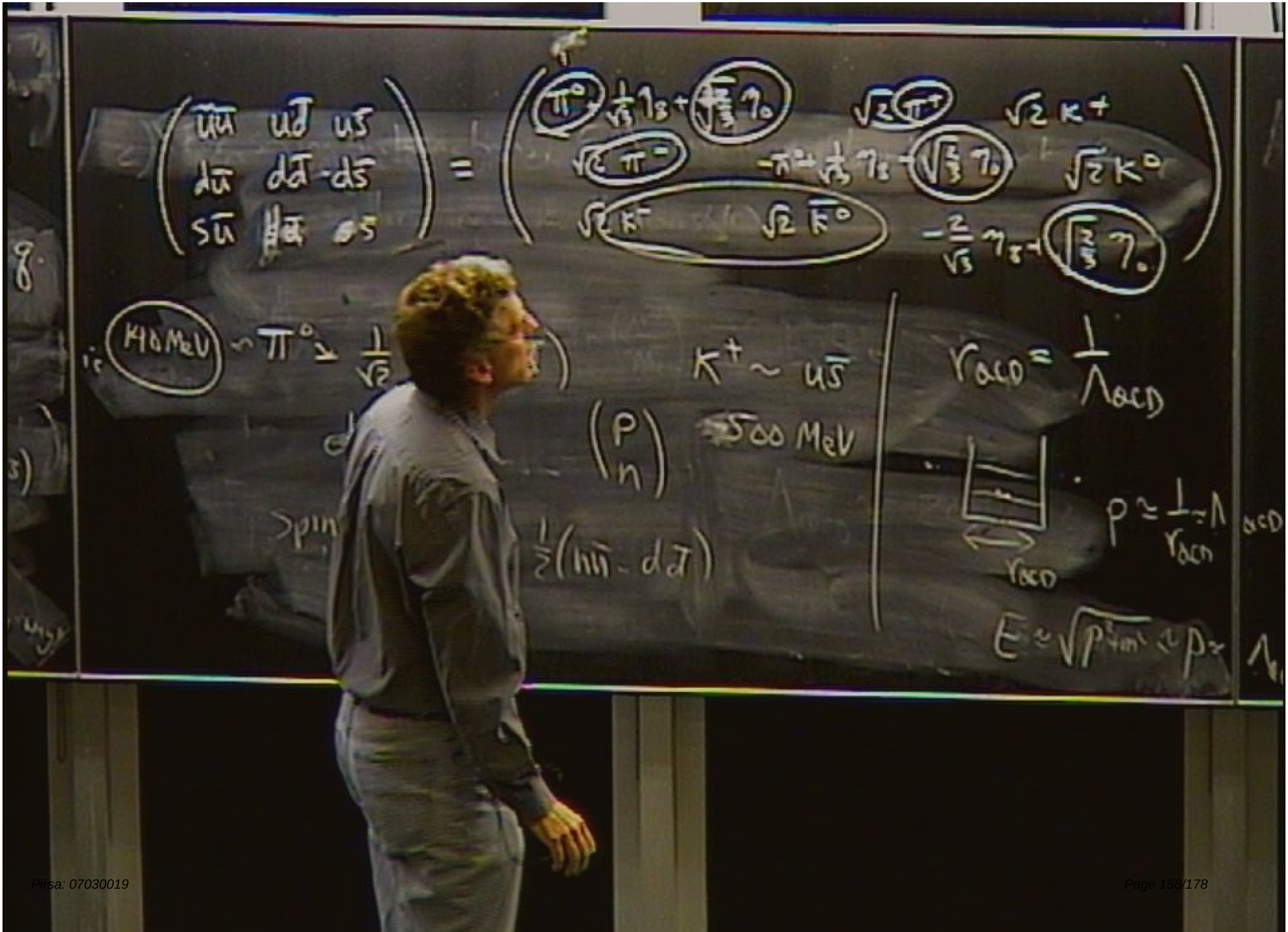

If m_q=0
q = (u d s)
U = e^{iθ^α λ_α/2}
q → Uq
Symmetry of L_QCD :
δq = i θ_V^α (λ_α/2) q + i θ_A^α (λ_α/2) γ_5 q
Vector
axial
Exact symmetry ⇒:
q_L → U_L q_L
q_R → U_R q_R
→ states related by symm. have same mass
SU_L(3) × SU_R(3)
Diagonal
SU_V(3) "eightfold way"


If $m_q = 0$ $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ $U = e^{i\theta^\alpha \frac{\lambda_\alpha}{2}}$
$q \to Uq$
Symmetry of $\mathcal{L}_{QCD}$: $\delta q = \underbrace{i\,\theta_V^\alpha \left(\frac{\lambda_\alpha}{2}\right)}_{\text{Vector}} q + \underbrace{i\,\theta_A^\alpha \left(\frac{\lambda_\alpha}{2}\right) \gamma_5}_{\text{axial}} q$
exact symmetry: or: $q_L \to U_L\, q_L$
$q_R \to U_R\, q_R$
$\underbrace{SU_L(3) \times SU_R(3)}$
Diagonal
$SU_V(3)$ "eightfold way"
$\longrightarrow$ states related by symm.
here same mass


K⁺ ~ us̄
500 MeV
r_QCD = 1/Λ_QCD
r_QCD
p ≃ 1/r_QCD ≃ Λ_QCD
E ≃ √(p²+m²)


K⁺ ~ us̄
500 MeV
r_QCD ≈ 1/Λ_QCD
p ≈ 1/r_QCD ~ Λ_QCD
r_QCD
E ≈ √(p²+m²) ≈ p ≈ Λ


140 MeV ~ π⁰ ≃ 1/√2 (uū − dd̄)
K⁺ ~ us̄
etc.
(p n) ~ 500 MeV
spin 1 : ρ⁰ ≃ 1/√2 (uū − dd̄)
r_QCD = 1/Λ_QCD
r_QCD
p ≃ 1/r_QCD ~ Λ_QCD


√2 K+
√2 K0
140 MeV
K+ ~ us̄
500 MeV
r_QCD = 1/Λ_QCD
r_QCD
p ≃ 1/r_QCD ~ Λ_QCD

If $m_q = 0$ $\quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ $\quad U = e^{i\theta^\alpha \frac{\lambda^\alpha}{2}}$

$q \to Uq$

Symmetry of $\mathcal{L}_{QCD}$: $\delta q = \underbrace{i\,\theta_V^\alpha \left(\frac{\lambda^\alpha}{2}\right)}_{\text{Vector}} q + \underbrace{i\,\theta_A^\alpha \left(\frac{\lambda^\alpha}{2}\right) \gamma_5}_{\text{axial}} q$

[illegible]act symmetry: $\quad$ or: $\quad q_L \to U_L\, q_L$

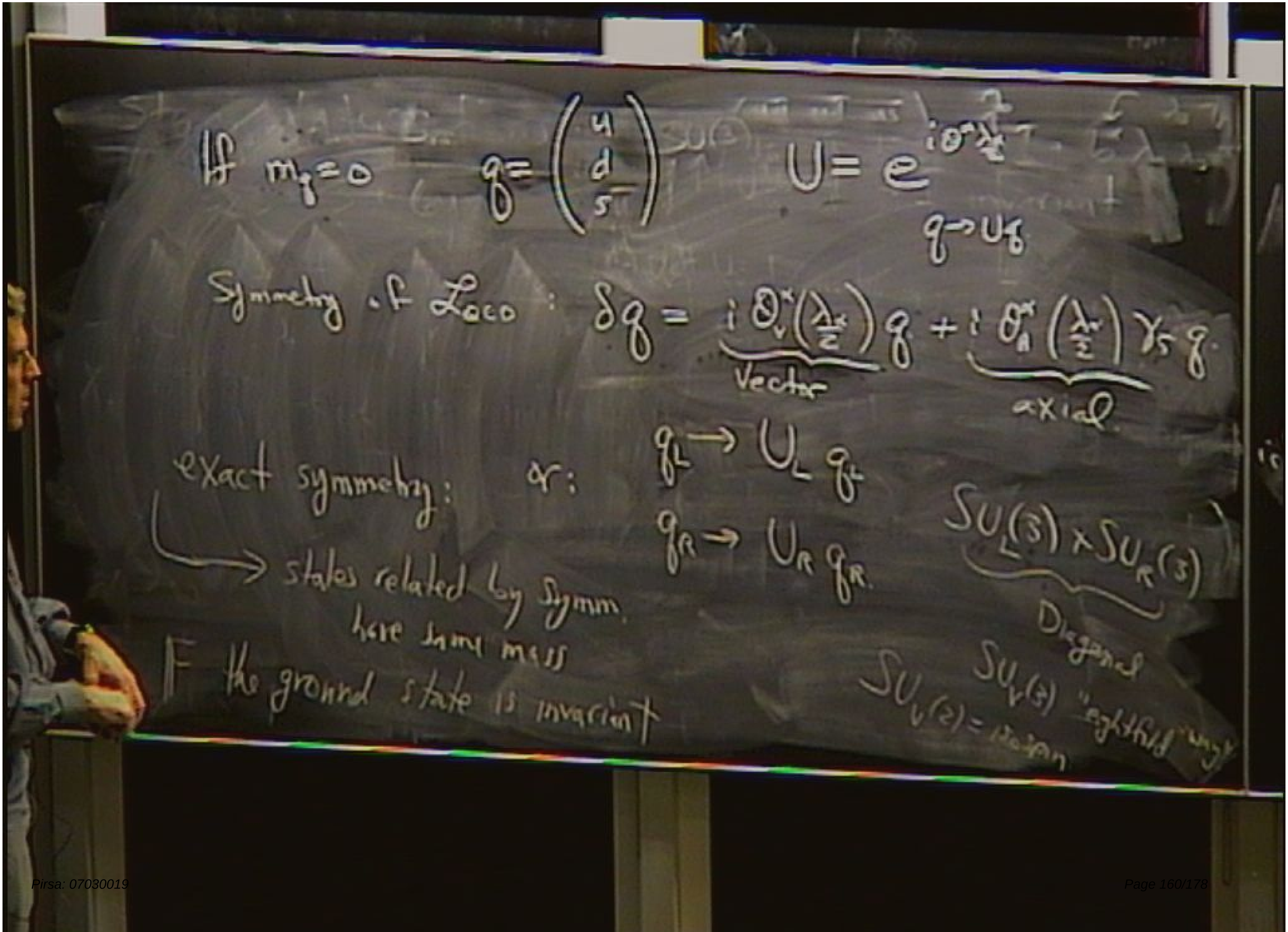
$q_R \to U_R\, q_R$

→ states related by symm.
here same mass

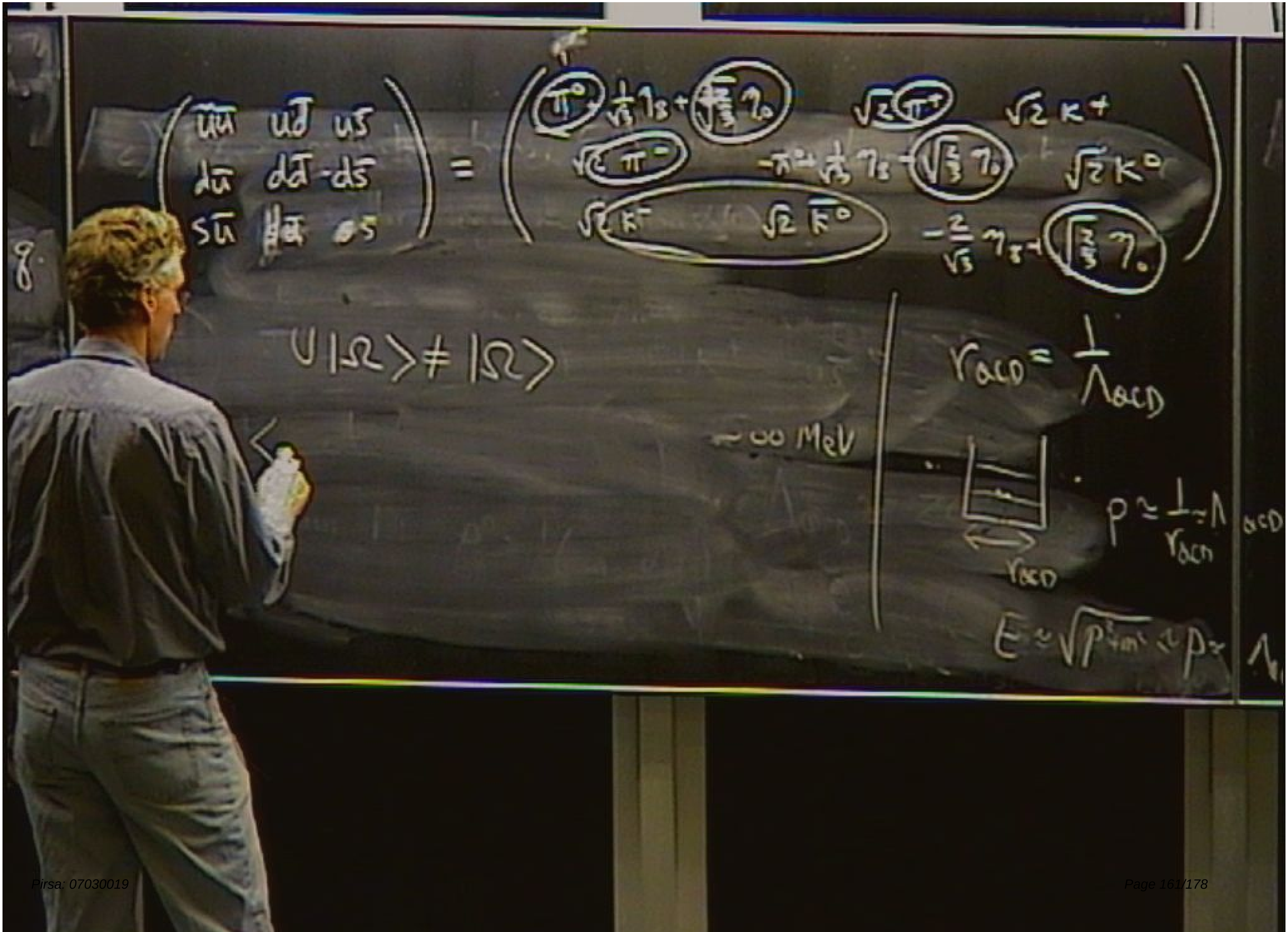
$\underbrace{SU_L(3) \times SU_R(3)}_{\text{Diagonal}}$

$SU_V(3)$ "eightfold way"

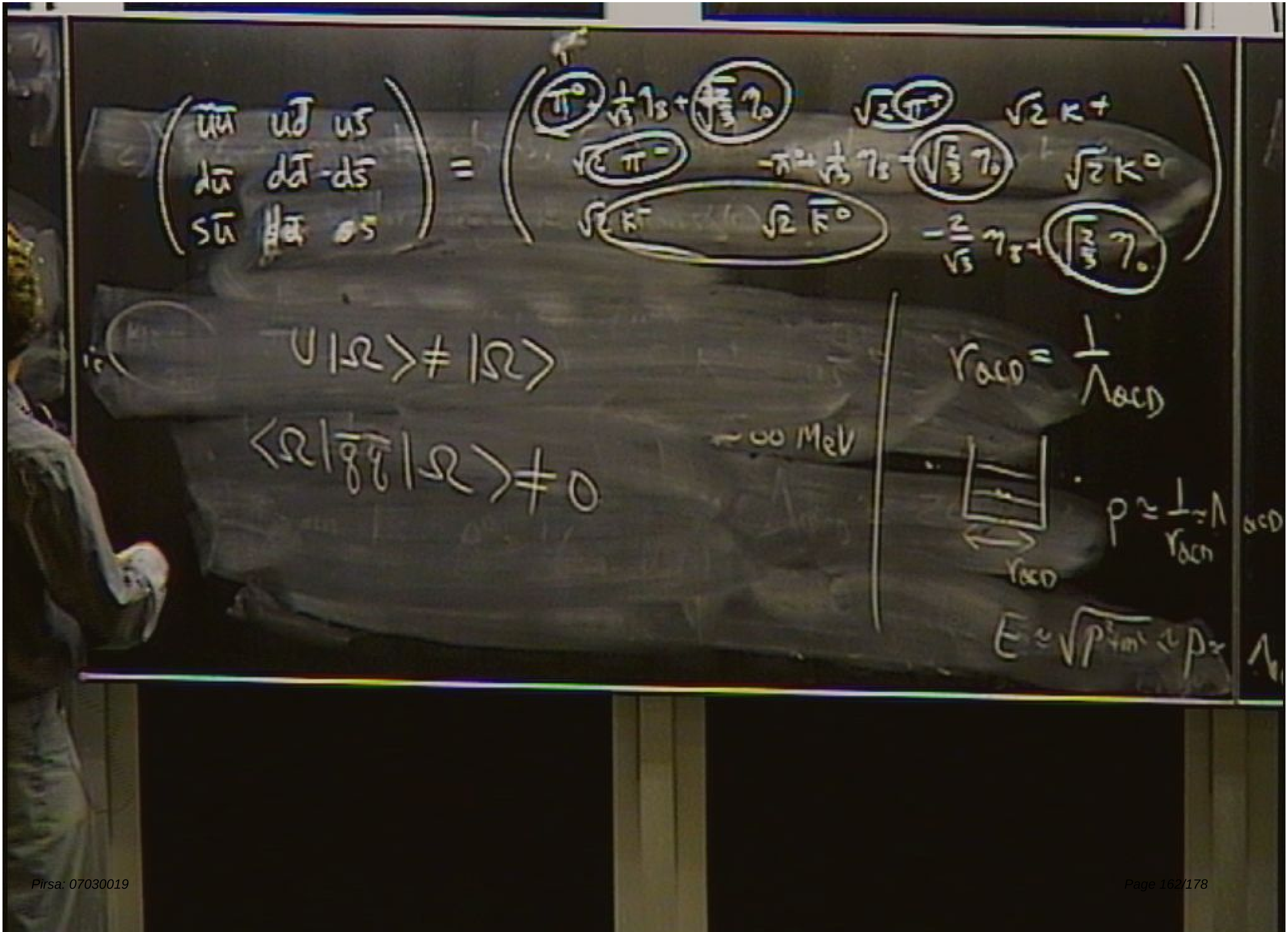
$SU_V(2)$ = isospin



If $m_q = 0$
$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$
$U = e^{i\theta^a \frac{\lambda_a}{2}}$
$q \to Uq$
Symmetry of $\mathcal{L}_{QCD}$: $\delta q = \underbrace{i\,\theta_V^a \left(\frac{\lambda_a}{2}\right)}_{\text{Vector}} q + \underbrace{i\,\theta_A^a \left(\frac{\lambda_a}{2}\right)}_{\text{axial}} \gamma_5 q$
exact symmetry:
or:
$q_L \to U_L q_L$
$q_R \to U_R q_R$
$\underbrace{SU_L(3) \times SU_R(3)}_{\text{Diagonal}}$
states related by symm. have same mass
$SU_V(3)$ "eightfold way"
$SU_V(2)$ = isospin
If the ground state is invariant



$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$
$U|\Omega\rangle \neq |\Omega\rangle$
MeV
$r_{QCD} = \frac{1}{\Lambda_{QCD}}$
r_{QCD}
$p \simeq \frac{1}{r_{QCD}} \sim \Lambda_{QCD}$
$E \simeq \sqrt{p^2 + m^2} \simeq p \simeq \Lambda$



$$\begin{pmatrix} u\bar u & u\bar d & u\bar s \\ d\bar u & d\bar d & d\bar s \\ s\bar u & s\bar d & s\bar s \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt3}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt2\,\pi^+ & \sqrt2\,K^+ \\ \sqrt2\,\pi^- & -\pi^0 + \frac{1}{\sqrt3}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt2\,K^0 \\ \sqrt2\,K^- & \sqrt2\,\bar K^0 & -\frac{2}{\sqrt3}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$
$U|\Omega\rangle \neq |\Omega\rangle$
$\langle\Omega|\bar q q|\Omega\rangle \neq 0$
MeV
$r_{QCD} = \frac{1}{\Lambda_{QCD}}$
r_{QCD}
$p \simeq \frac{1}{r_{QCD}} \sim \Lambda_{QCD}$
$E \simeq \sqrt{p^2 + m^2} \simeq p \simeq \Lambda$

$$\begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

evidence $U|\Omega\rangle \neq |\Omega\rangle$

is: $\rightarrow \langle\Omega|\bar{q}q|\Omega\rangle \neq 0$

~ 00 MeV

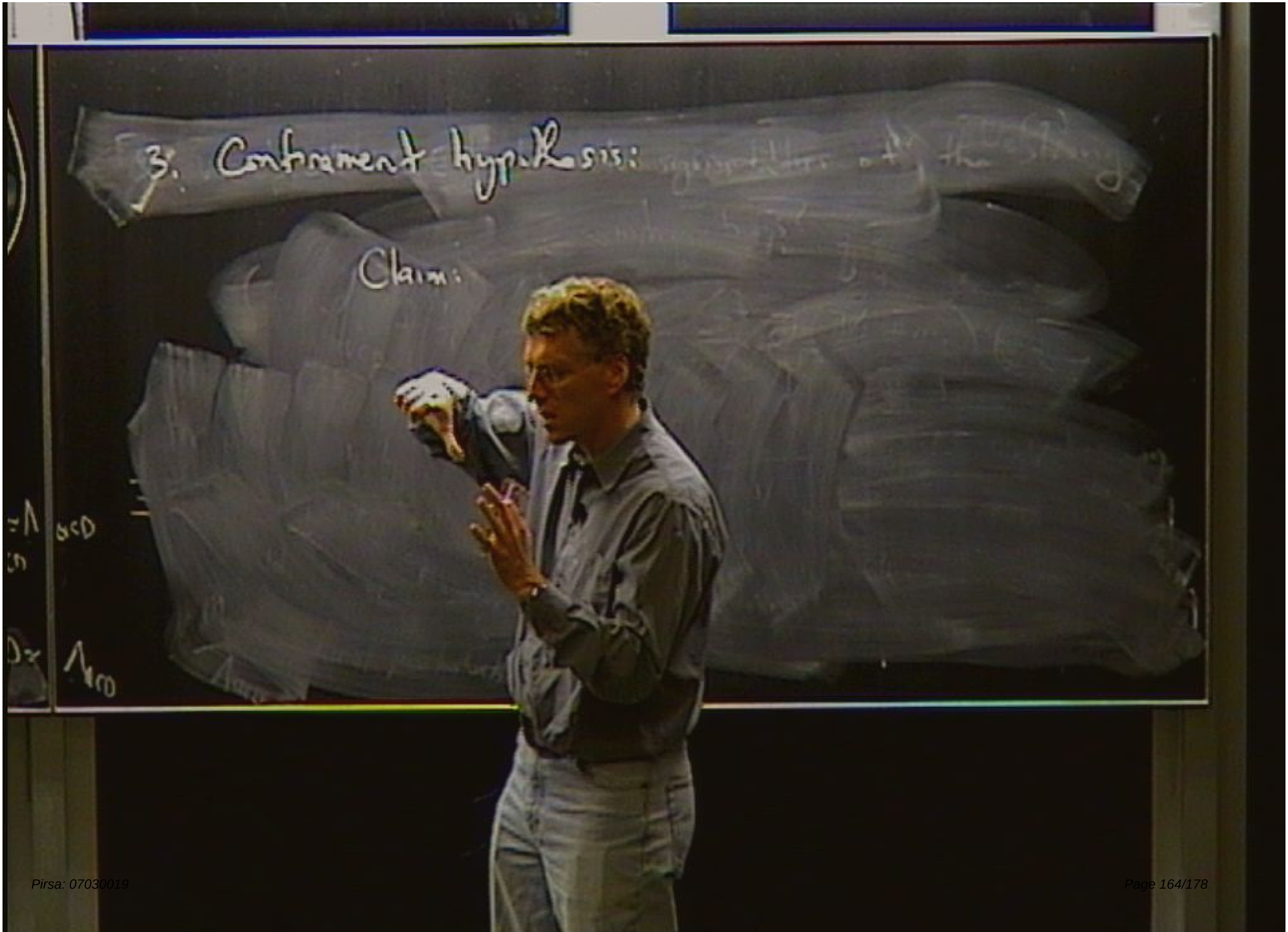
so $SU_A(3)$ is spontaneously broken.

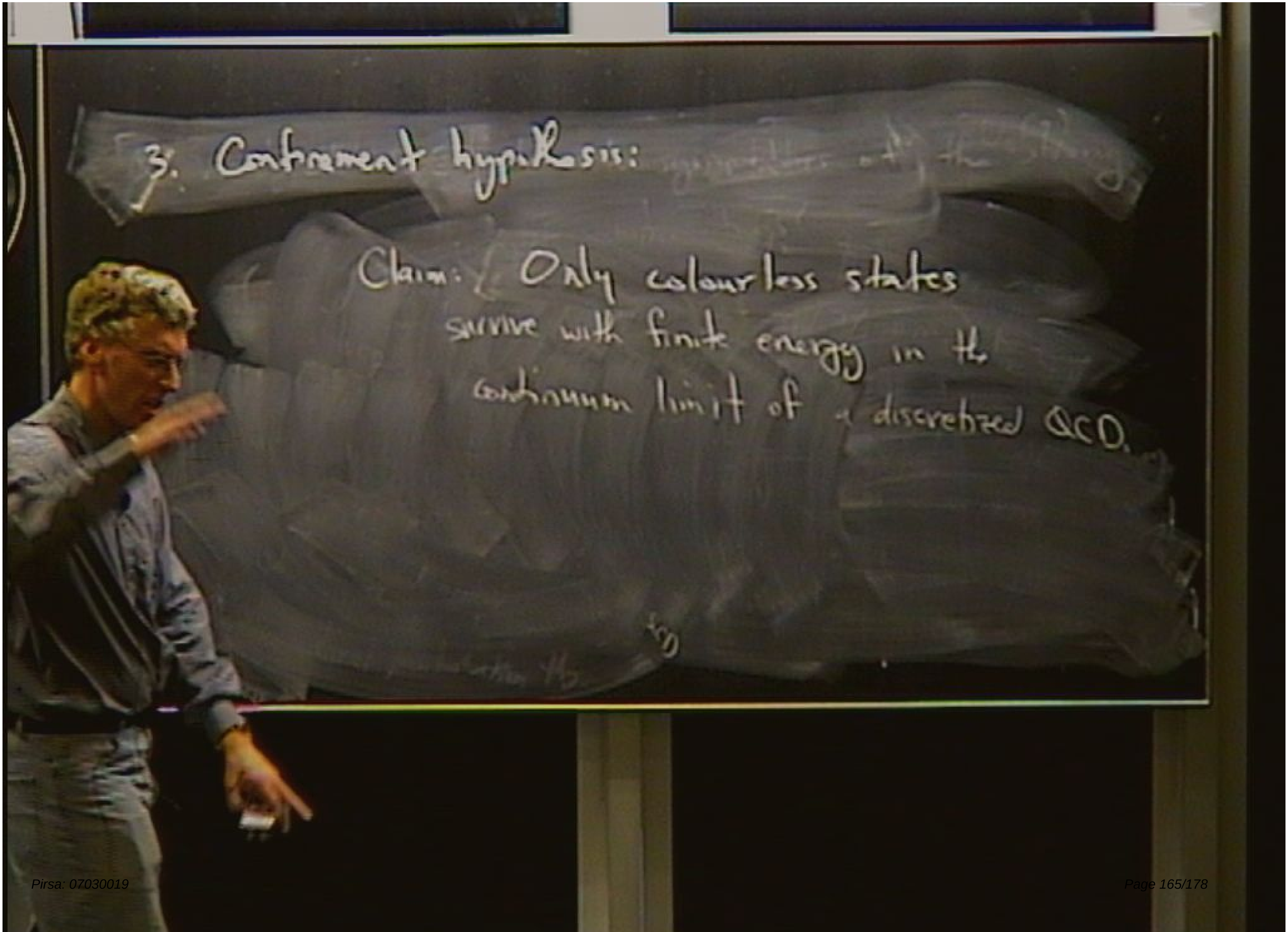
$$r_{QCD} = \frac{1}{\Lambda_{QCD}}$$

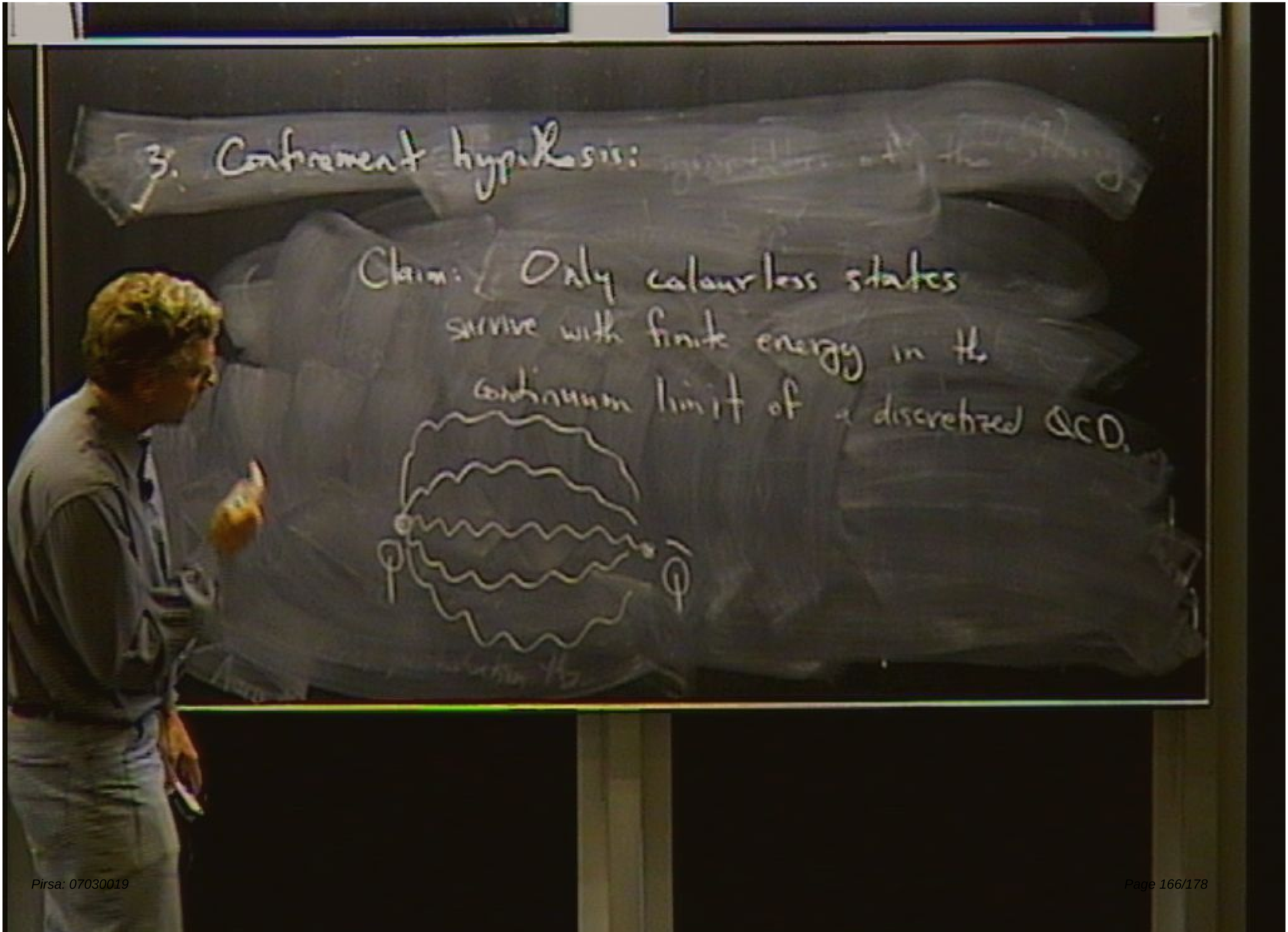
r_{QCD}

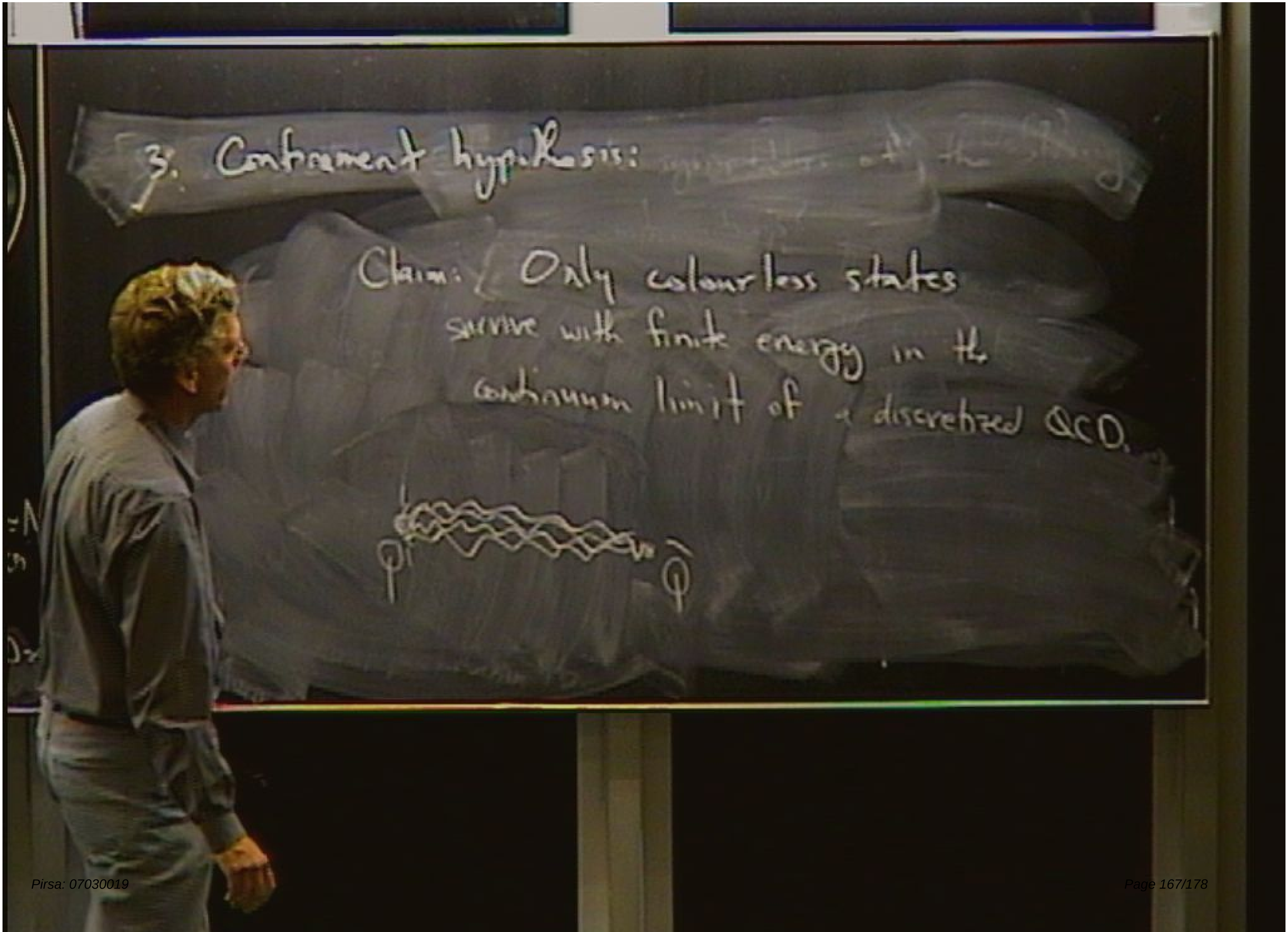
$$p \simeq \frac{1}{r_{QCD}} \sim \Lambda_{QCD}$$

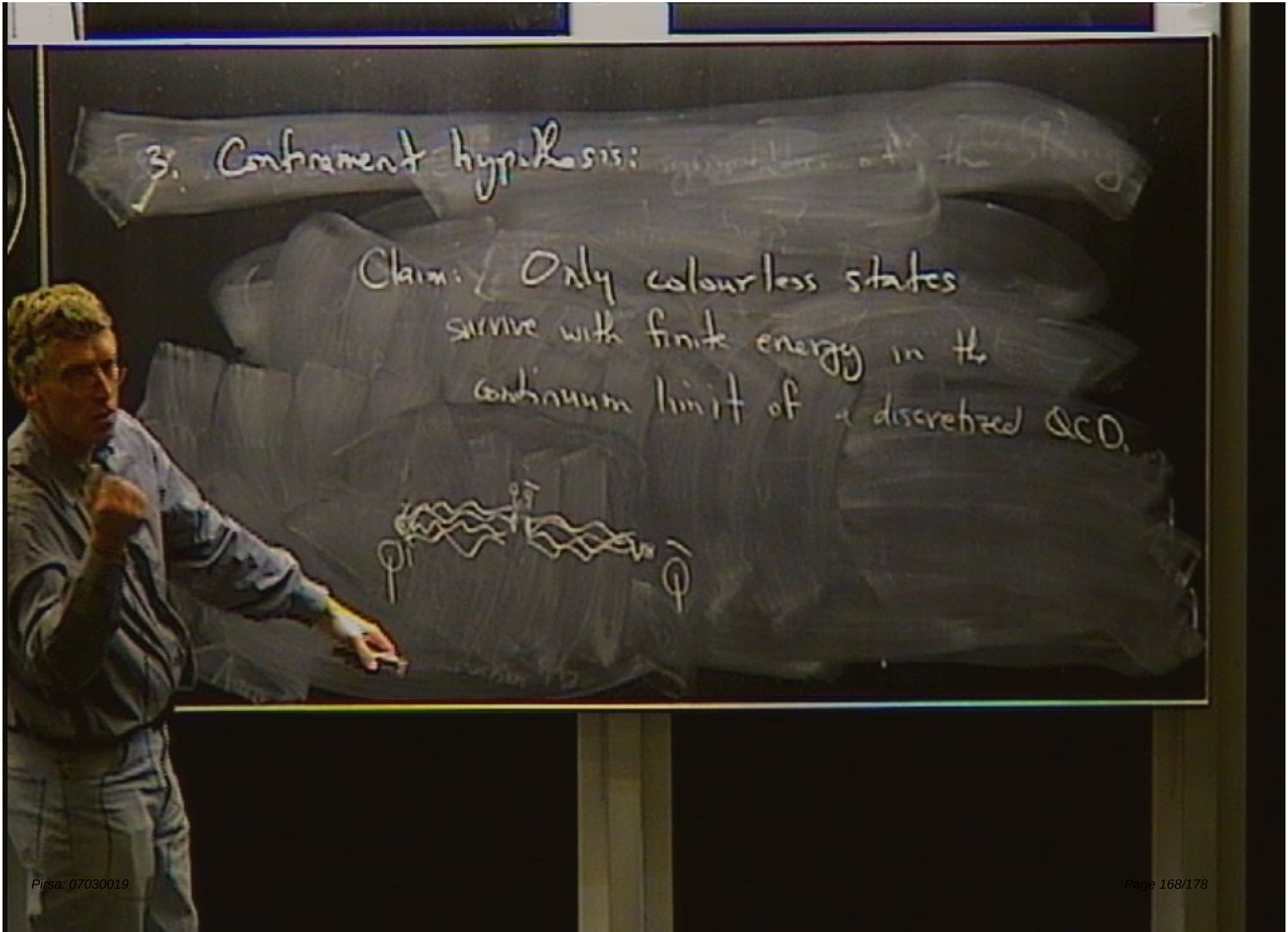
$$E \simeq \sqrt{p^2 + m^2} \simeq p \simeq \Lambda$$

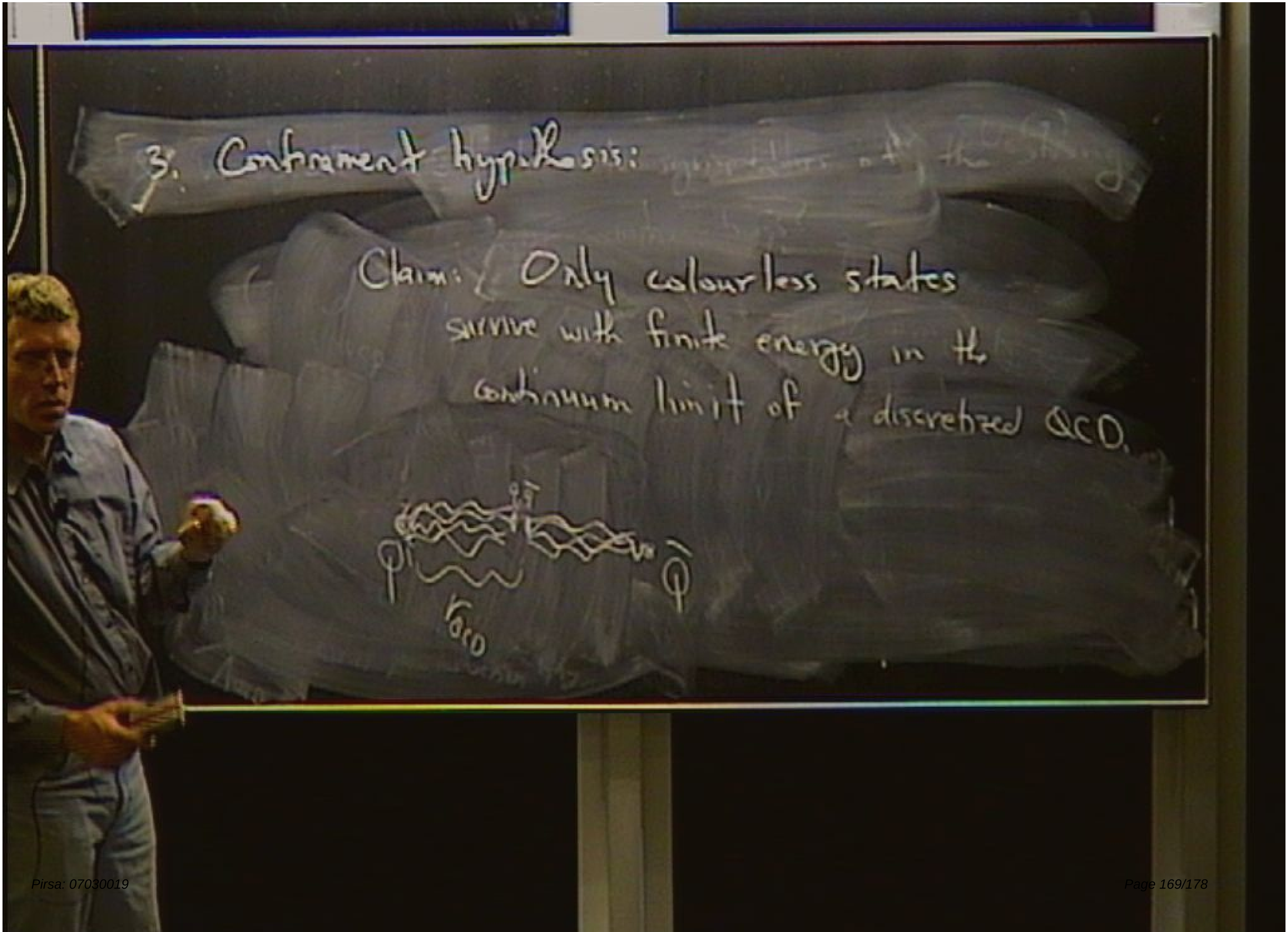

3. Confinement hypothesis:
Claim:

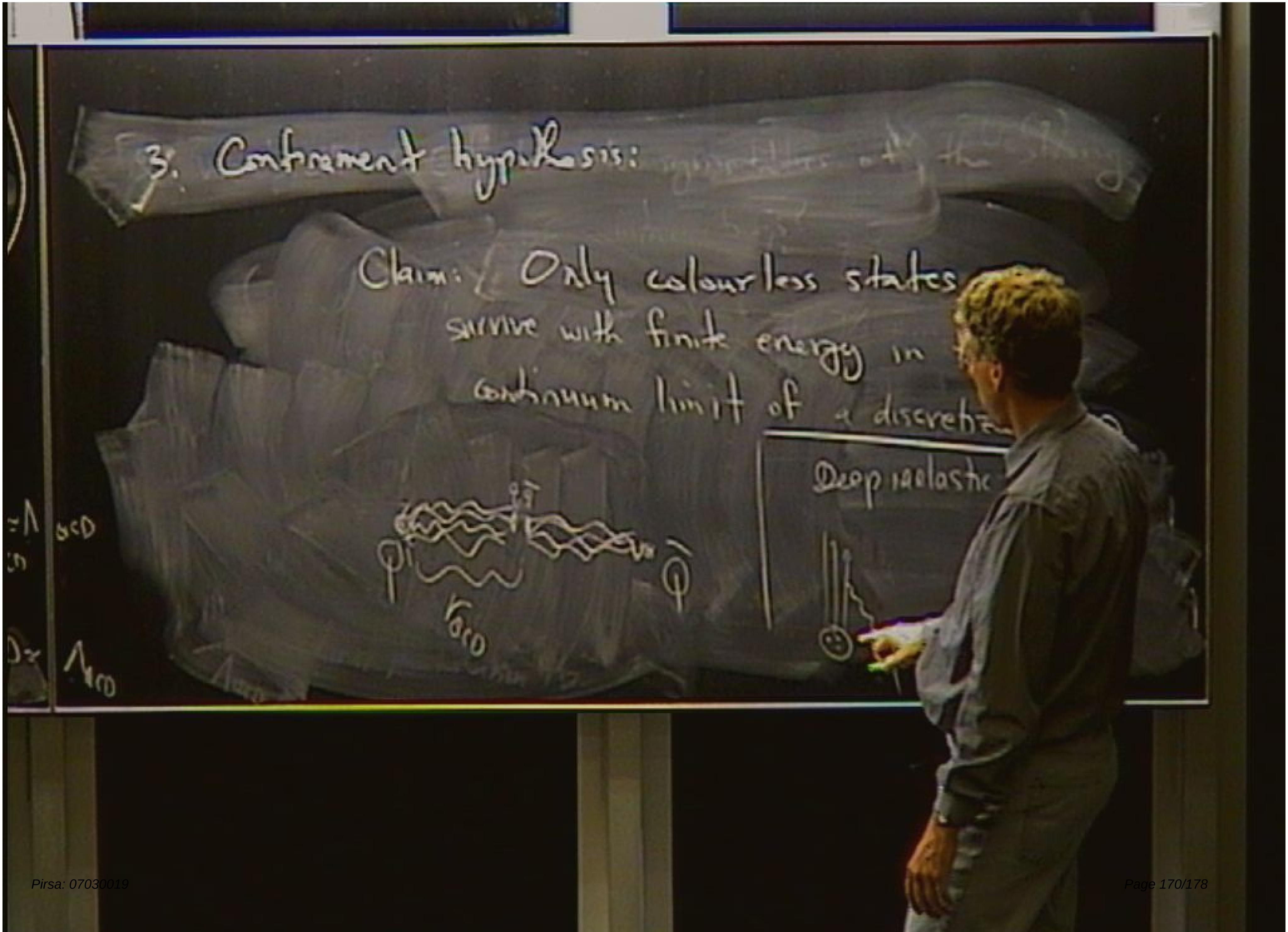

3. Confinement hypothesis:
Claim: Only colourless states
survive with finite energy in the
continuum limit of a discretized QCD.

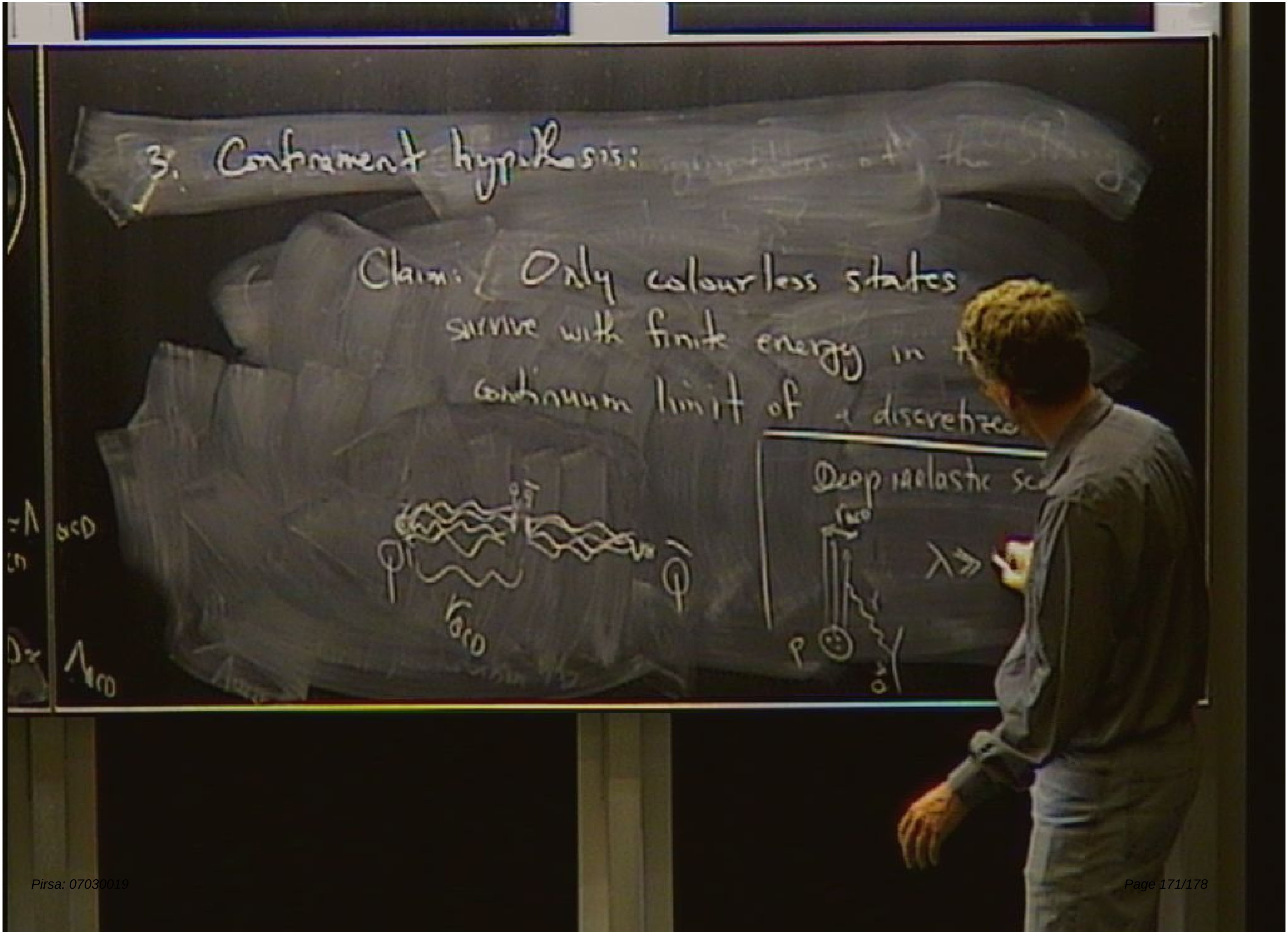

3. Confinement hypothesis:
Claim: Only colourless states
survive with finite energy in the
continuum limit of a discretized QCD.

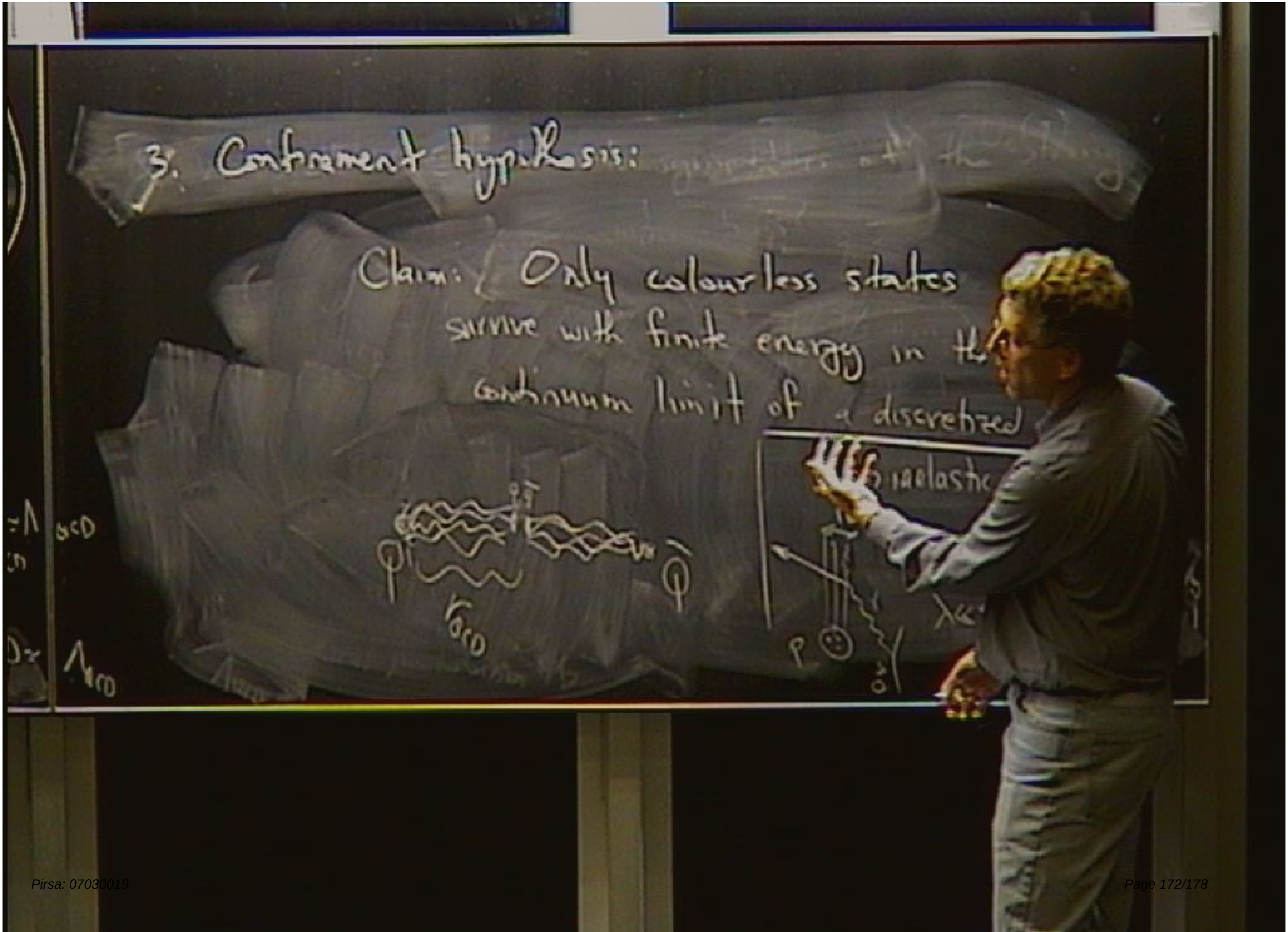

3. Confinement hypothesis:
Claim: Only colourless states survive with finite energy in the continuum limit of a discretized QCD.
q
q̄

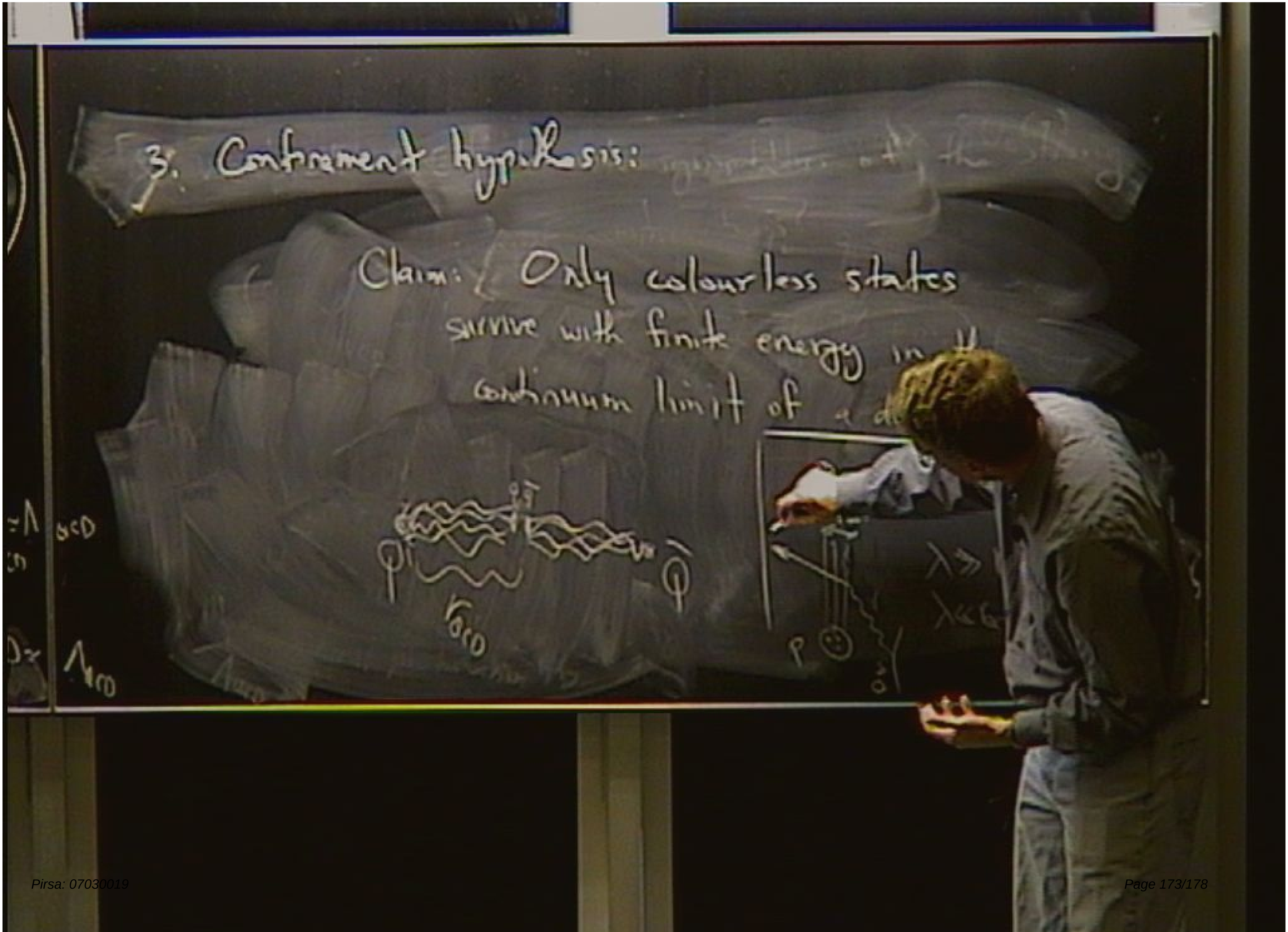

3. Confinement hypothesis:
Claim: Only colourless states survive with finite energy in the continuum limit of a discretized QCD.

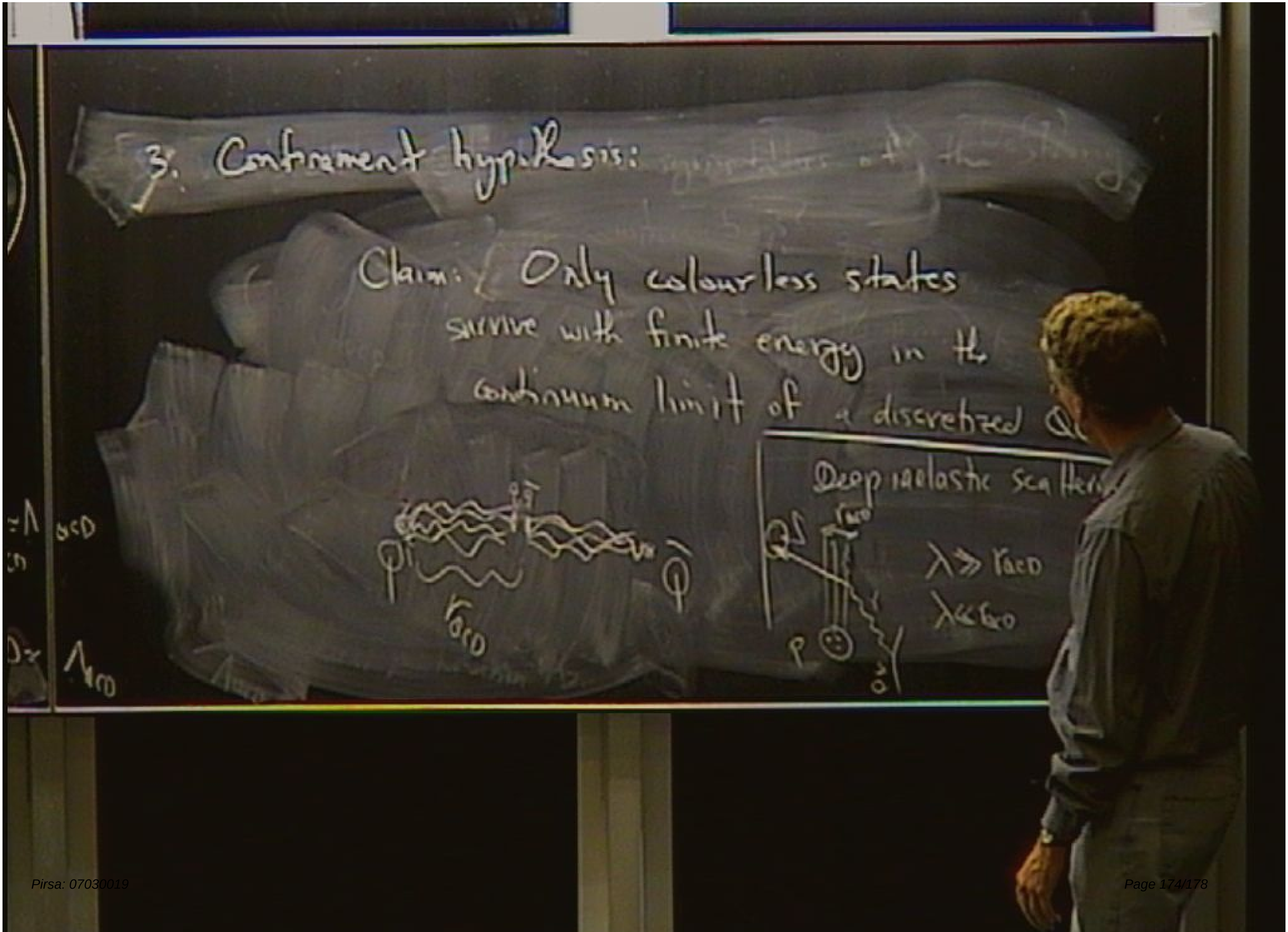

3. Confinement hypothesis:
Claim: Only colourless states survive with finite energy in the continuum limit of a discretized QCD.

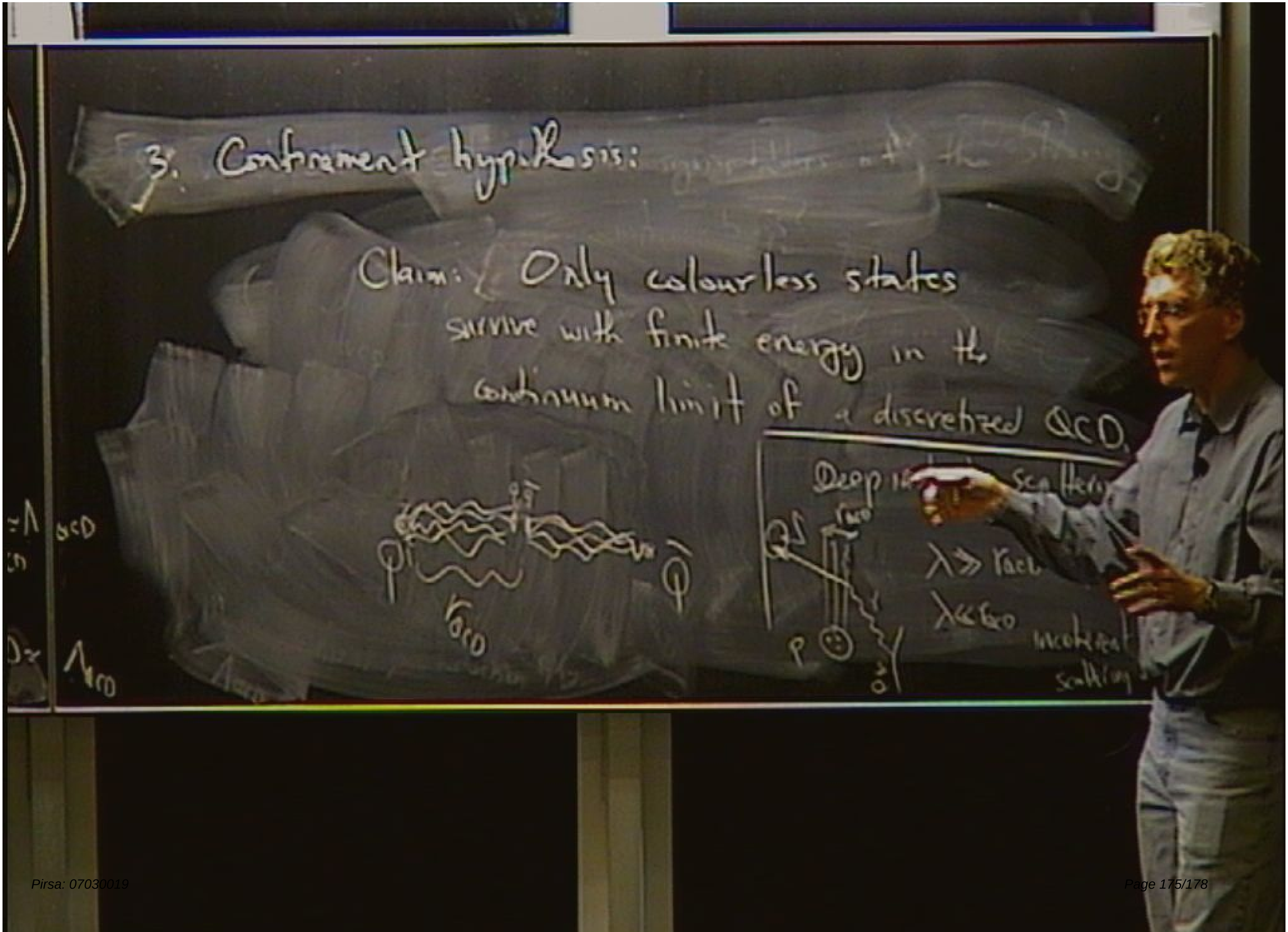

3. Confinement hypothesis:
Claim: Only colourless states
survive with finite energy in
continuum limit of a
Deep inelastic

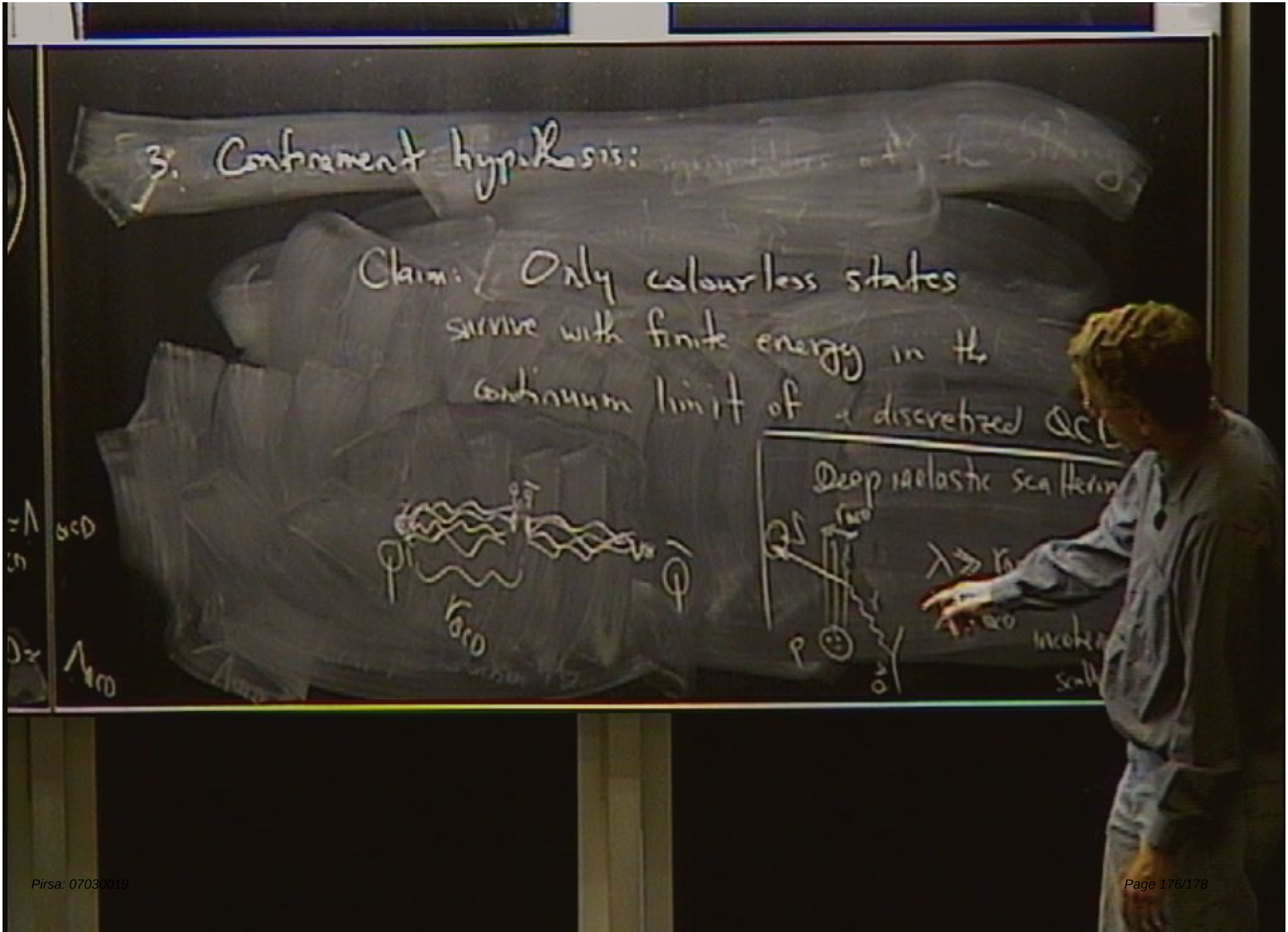

3. Confinement hypothesis:
Claim: Only colourless states
survive with finite energy in
continuum limit of a discretized
Deep inelastic sc

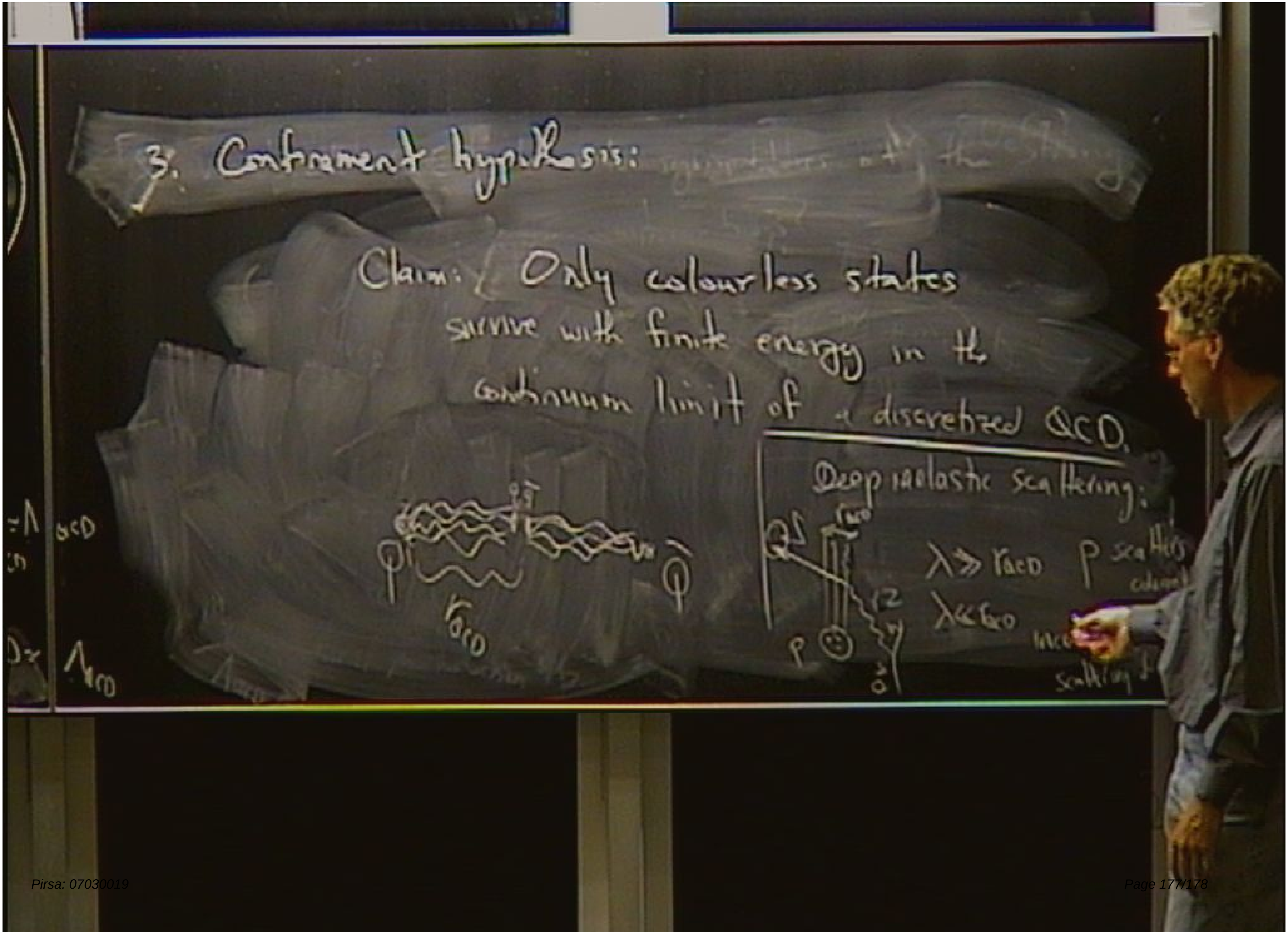

3. Confinement hypothesis:
Claim: Only colourless states
survive with finite energy in the
continuum limit of a discretized
inelastic


3. Confinement hypothesis:
Claim: Only colourless states
survive with finite energy in
continuum limit of


3. Confinement hypothesis:
Claim: Only colourless states
survive with finite energy in the
continuum limit of a discretized QC
Deep inelastic scattering
λ ≫ r_QCD
λ ≪ r_QCD
r_QCD


3. Confinement hypothesis:
Claim: Only colourless states survive with finite energy in the continuum limit of a discretized QCD.
Deep
scattering
incoherent
scattering

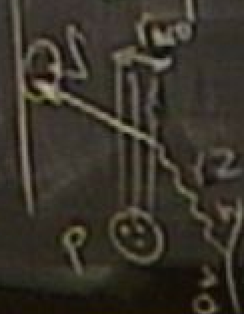

3. Confinement hypothesis:
Claim: Only colourless states survive with finite energy in the continuum limit of a discretized QC
Deep inelastic scatterin


3. Confinement hypothesis:
Claim: Only colourless states survive with finite energy in the continuum limit of a discretized QCD.
Deep inelastic scattering:
λ ≫ rQCD
λ ≪ rQCD
rQCD

3. Confinement hypothesis:

Claim: Only colourless states survive with finite energy in the continuum limit of a discretized QCD.

Deep inelastic scattering:



$\lambda \gg r_{QCD}$ p scatters coherently

$\lambda \ll r_{QCD}$ incoherent scattering of quarks