Title: Standard 4-D gravity on a brane in six dimensional flux compactifications

Date: Mar 06, 2007 11:00 AM

URL: http://pirsa.org/07030009

Abstract: We consider a six-dimensional space-time, in which two of the dimensions are compactified by a flux. Matter can be localized on a codimension one brane coupled to the bulk gauge field and wrapped around an axis of symmetry of the internal space. By studying the linear perturbations around this background, we show that the gravitational interaction between sources on the brane is described by Einstein 4d gravity at large distances. Our model provides a consistent setup for the study of gravity in a football compactification, without having to deal with the complications of a deltaÂ–like, codimension two brane. Moreover, it allows us to identify the origin of the problems that emerge when one takes the limit of a codimension-two brane.

Standard 4D gravity on a brane in 6D flux compactification

Lorenzo Sorbo

UMass Amherst

M. Peloso, LS, G. Tasinato, hep-th/0603026, PRD

Standard 4D gravity on a brane in 6D flux compactification

Lorenzo Sorbo

UMass Amherst

M. Peloso, LS, G. Tasinato, hep-th/0603026, PRD

Brane models and large extra dimensions

ADD, RS

Motivated by particle physics (hierarchy problem)...

...very interesting for gravity

(e.g. brane cosmology)

Gravity of codimension-1 brane models studied in great detail

But brane models can have more than just one extra dimension!

ADD with 2 extra dims, an especially interesting case...

...since codimension-1 is so well studied, the next step is codimension-2!

let us start considering just a codimension-2 0-brane: a point in 2+1 spacetime:

Gravity in 2+1 dimensions is nondynamical

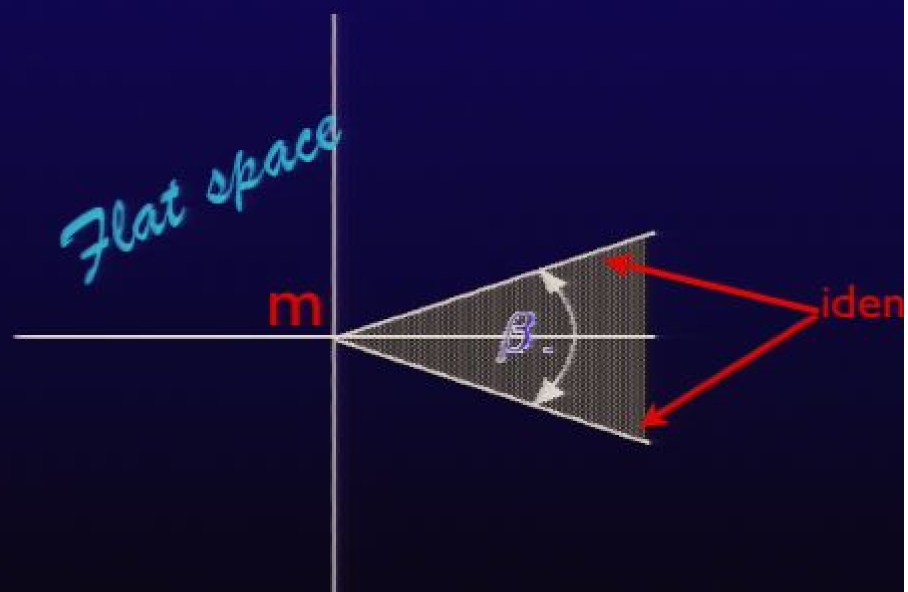
$$\boxed{R^{\mu\nu}_{\alpha\beta} = \epsilon^{\mu\nu\sigma}\epsilon_{\alpha\beta\lambda}G^{\lambda}_{\sigma}} \longrightarrow$$ away from sources, Riemann=0

For a pointlike mass m

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\theta^2$$

$$\boxed{0 \leq \theta \leq 2\pi\,(1 - 4Gm)}$$

deficit angle ß



In analogous way, a 3-brane in a 6d bulk
does not curve the bulk, but generates just a deficit angle

The curvature singularity at the tip of the cone
matches the singular stress energy tensor
associated to the brane tension

⬇

It is possible to find solutions with a flat brane
for whatever value of the brane tension!

A realistic setting: compactification on a 2-sphere

Freund and Rubin, 8

Matter content:

- Bulk cosmological constant Λ
- Bulk U(1) gauge field F_{MN}

$$S = \int d^6x \sqrt{-g_6} \left[M^4 R - \Lambda - \frac{1}{4} F_{AB} F^{AB} \right]$$

Gives Minkowski x S^2 if

$$F_{ij} = B \sqrt{g_2}\, \epsilon_{ij} \quad , \quad \{i,\, j\} = \{\theta,\, \phi\}$$

with the constraint $\boxed{\Lambda = \dfrac{B^2}{2}}$

equivalent to fine-tuning the cosmological constant to zero

- radius of the sphere $R = \dfrac{M^2}{B}$

Could this be used to help with the cosmological constant problem

Yes, in the spirit of *self tuning*:
the theory may contain some fine-tunings,
but they do not involve the brane tension.

If matter lives only on the brane,
matter loops will contribute only to the brane tension

Can always find flat solutions
for any matter content of the brane

e.g. with broken susy on the bran

...but we need a consistent theory with 4d gravity on the brane!

(a flat infinite bulk cannot localize gravity, of course..

A realistic setting: compactification on a 2-sphere

Freund and Rubin, 8

Matter content:

- Bulk cosmological constant Λ
- Bulk U(1) gauge field F_{MN}

$$S = \int d^6x \sqrt{-g_6} \left[M^4 R - \Lambda - \frac{1}{4} F_{AB} F^{AB} \right]$$

Gives Minkowski x S² if

$$F_{ij} = B \sqrt{g_2}\, \epsilon_{ij} \quad , \quad \{i, j\} = \{\theta, \phi\}$$

with the constraint $\boxed{\Lambda = \frac{B^2}{2}}$

equivalent to fine-tuning the cosmological constant to zero

- radius of the sphere $R = \frac{M^2}{B}$

Could this be used to help with the cosmological constant problem

Yes, in the spirit of *self tuning*:
the theory may contain some fine-tunings,
but they do not involve the brane tension.

If matter lives only on the brane,
matter loops will contribute only to the brane tension

Can always find flat solutions
for any matter content of the brane

e.g. with broken susy on the bran

...but we need a consistent theory with 4d gravity on the brane!

(a flat infinite bulk cannot localize gravity of course..

A realistic setting: compactification on a 2-sphere

Freund and Rubin, 8

Matter content:

- Bulk cosmological constant Λ
- Bulk U(1) gauge field F_{MN}

$$S = \int d^6x \sqrt{-g_6} \left[M^4 R - \Lambda - \frac{1}{4} F_{AB} F^{AB} \right]$$

Gives Minkowski x S^2 if

$$F_{ij} = B \sqrt{g_2}\, \epsilon_{ij} \quad , \quad \{i, j\} = \{\theta, \phi\}$$

with the constraint $\boxed{\Lambda = \frac{B^2}{2}}$

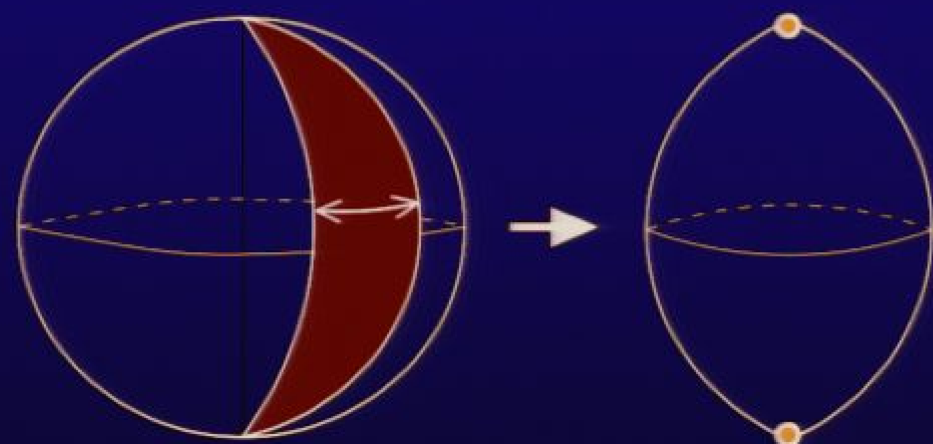
equivalent to fine-tuning the cosmological constant to zero

- radius of the sphere $R = \dfrac{M^2}{B}$

...then add two branes at the poles:

Sundrum, 98
Carroll and Guica, 03

(Z_2 symmetry across the equator + azymuthal symmetry)



Football compactification

a deficit angle β is generated at the poles

$$1-\beta=\frac{T}{2\pi M^4}$$

where T is the brane tension

changing the brane tension has the only effect
of changing the bulk deficit angle

can get flat 4D solutions
for any value of the brane tension

is the Cosmological Constant problem
alleviated this way?

t should not... : at distances larger than R the four dimensional effective theory i

just gravity + a (finite) bunch of matter fields.

Weinberg's no-go theorem applies. So something must go wrong.

changing the brane tension has the only effect
of changing the bulk deficit angle

can get flat 4D solutions
for any value of the brane tension

is the Cosmological Constant problem
alleviated this way?

t should not... : at distances larger than R the four dimensional effective theory i
just gravity + a (finite) bunch of matter fields.

Weinberg's no-go theorem applies. So something must go wrong.

t should not... : at distances larger than R the four dimensional effective theory i
just gravity + a (finite) bunch of matter fields.

Weinberg's no-go theorem applies. So something must go wrong.

...and indeed, there is a first problem...

Aghababaie et al 03, Navarro 0

if there are fields charged under the bulk U(1) group, then the flux is quantized

the deficit angle is quantized

$$\beta = \frac{N}{2eM^2R}$$

N=0,1,2,...

the tension T can only take a discrete set of values

...and even if there is no matter that is charged under the U(1), the flux must be *conserved:*

Garriga and Porrati, 04

the product of magnetic field times internal volume cannot change

- suppose a phase transition on the brane changes the tension T
- the deficit angle should change
- the internal volume should change
- if the magnetic field does not change, then the magnetic flux must change

...and even if there is no matter that is charged under the U(1), the flux must be *conserved:*

Garriga and Porrati, 04

the product of magnetic field times internal volume cannot change

- suppose a phase transition on the brane changes the tension T
- the deficit angle should change
- the internal volume should change
- if the magnetic field does not change, then the magnetic flux must change

but this is forbidden by flux conservation!

so the only option is that the deficit angle stays constant and the brane "curves" and starts expanding

...and even if there is no matter that is charged under the U(1), the flux must be *conserved:*

Garriga and Porrati, 04

the product of magnetic field times internal volume cannot change

- suppose a phase transition on the brane changes the tension T
- the deficit angle should change
- the internal volume should change
- if the magnetic field does not change, then the magnetic flux must change

but this is forbidden by flux conservation!

so the only option is that the deficit angle stays constant and the brane "curves" and starts expanding

...*unless* the phase transition is accompanied by emission of objects (branes) charged under the magnetic field, that change the magnetic field

but this is just Brown-Teitelboim with two extra dimensions!

...or a simplified version of Bousso-Polchinsk

...so the football is probably not good for self-tuning...

...still it is an interesting example of:

- 6D ADD scenario where the stabilization mechanism is consistently taken into account

- Brown-Teitelboim/Bousso-Polchinski brane model

One more reason to study codimension-2 braneworlds:

gravity of codimension-2 objects is special

(independently of phenomenological applications

eroch and Traschen, 88

The metric associated to a codimension-2 singular object does not give well defined Einstein equations

(Riemann tensor not well defined even in a distributional sense

. delta-like, codimension-2 brane can give different external metrics depending on the way the delta-like limit is taken.

(different from codimension-1 case)

...even worse than that:

There are contraints on the forms of matter that can be localized on a delta-like codimension-2 brane!

Cline et al., Bostock et al. 0

Let us study the Einstein equations locally, close to the brane:

$$ds^2 = g_{\mu\nu}(x,\, r)\, dx^\mu dx^\nu - L^2(x,\, r)\, d\theta^2 - dr^2$$

$$\widehat{g}_{\mu\nu}(x) + \mathcal{O}(r^2) \qquad \beta\, r + \mathcal{O}(r^2)$$

$$T_{MN} = \begin{pmatrix} \widehat{T}_{\mu\nu}(x)\, \frac{\delta(r)}{2\pi L} & 0 \\ 0 & 0 \end{pmatrix}$$

...then we match the singular parts of Einstein equations:

use:

$$\frac{L''}{L} = -(1-\beta)\frac{\delta(r)}{L} + \text{regular}$$

to obtain

$$\boxed{2\pi(1-\beta)M^4\,\widehat{g}_{\mu\nu} = \widehat{T}_{\mu\nu}}$$

Only tension can be localized on the delta-like defect!

One possible way out

Bostock et al. 03

Add higher curvature terms (Gauss-Bonnet) in the bulk

[consistent with String Theo

$$\delta S = M^4 \int d^6x \sqrt{-g}\, \alpha \left(R^2 - 4\, R_{AB}\, R^{AB} + R_{ABCD}\, R^{ABCD}\right)$$

In 6D, only higher curvature term in the lagrangian that still leads to 2nd order differential equations
Lovelock 71

New equation for singular parts:

$$2\pi\,(1-\beta)\, M^4 \left[\widehat{g}_{\mu\nu} + 4\alpha\widehat{G}_{\mu\nu} + \alpha\frac{\beta}{1-\beta}\widehat{W}_{\mu\nu}\right] = \widehat{T}_{\mu\nu}$$

Weyl term from the bulk $W \sim g^{-1}\,\partial_r g\,\partial_r g$

This is certainly a possibility, but...

- What does the bulk look like in this case?

[The higher curvature terms will deform the bulk solution, need to re-study the global properties of the system]

- Does it mean that we *have* to introduce a Gauss-Bonnet term in our theory?

Seems to be too strong a conclusion...

Our perspective

Peloso, LS, Tasinato 06

- Keep Einstein gravity in the bulk

- In the spirit of Geroch and Traschen, consider a thick defect

  [We do not consider this as a regularization, but as a physical structure of the defect

- In order to be able to use our knowledge of codimension-1 branes, use a 4-brane wrapped around an axis of the system

What are we looking for:

- Possibility to put matter with arbitrary equation of state on the brane

- Behavior of gravity in football compactification

- Physical understanding of the origin of the constraint on the equation of state for matter localized on a delta-like brane

Our system

"Inside" sphere

$$ds_2^2 = R_i^2\left(d\theta^2 + \cos^2\theta d\phi^2\right)$$

$$F_{\theta\phi} = M^2 R_i \cos\theta$$

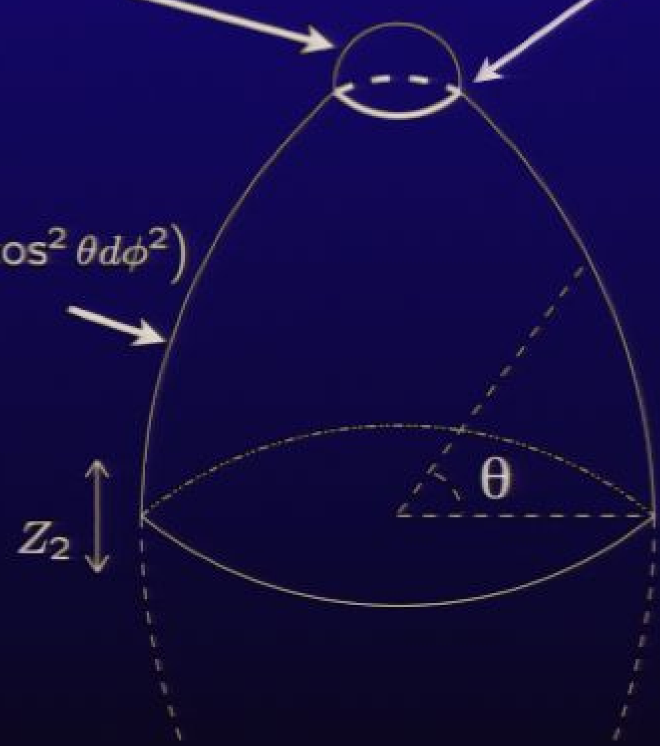
4-brane located at $\theta = \bar{\theta}$

"Outside" ball

$$ds_2^2 = R_o^2\left(d\theta^2 + \beta^2\cos^2\theta d\phi^2\right)$$

$$F_{\theta\phi} = M^2 \beta R_o \cos\theta$$

$$R_i = \beta\, R_0$$



Let us put some numbers, to fix ideas...

This is a good model for a 5D Standard Model in a 6D bulk (ADD)

$\beta = O(1)$, R~mm

so that the fundamental Planck scale is M~TeV

The radius of the brane is

$$r_{\rm brane} = R\,\cos\bar{\theta}$$

and we can set it to be ~ TeV^{-1}

Before adding matter...

what do we put on the brane?

Cannot put pure tension!

1- Junction conditions want $T_{\mu\nu} \propto g_{\mu\nu}$ while $T_{\phi\phi} = 0$

Chacko, Nelson 99

2- The brane must carry magnetic charge

Aghababaie et al. 03

Need to include brane matter charged under the bulk U(1)

The brane action

Charged field $|\phi| = v$

(explicitly) breaks U (1)

$$S = -\int d^5x\sqrt{-\gamma}\left[\lambda_s + \frac{v^2}{2}\left(\partial_M\sigma - eA_M\right)\left(\partial^M\sigma - eA^M\right)\right]$$

Phase of the field ϕ

- Maxwell equations determine e^2v^2 in terms bulk quantities
- Brane position determined by $\tan^2\bar{\theta} = \dfrac{2\lambda_s}{M^4e^2v^2}$
- After setting the brane fields to their *vev*, the brane stress energy tensor is $T_{\mu\nu} = T_4\, g_{\mu\nu}\ ,\quad T_{\phi\phi} = 0\ ,$ as required by the junction conditions
- Deficit angle $1-\beta = \dfrac{T_4}{2M^4\pi\sin\bar{\theta}}$ gives the correct limit as $\bar{\theta}\to\pi/2$

(limit of delta-like brane)

And now let us add matter to the brane

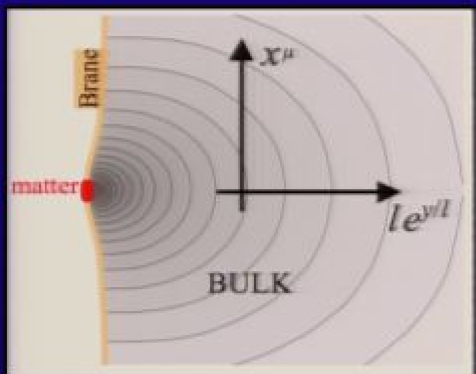
in the form of some (small) stress energy tensor $T_{\mu\nu}$

Allowed deformations of the system (scalar, vector, tensor) :

$$
\boxed{
\begin{aligned}
ds^2 &= (1+2\,\Phi)\,d\vartheta^2 + 2\,A\,d\vartheta\,d\varphi + (1+2\,C)\,\cos^2\theta\,d\varphi^2 \\
&+ 2\,(T_\mu + \partial_\mu T)\,d\vartheta dx^\mu + 2\,(V_\mu + \partial_\mu V)\,d\varphi\,dx^\mu \\
&+ \left\{\eta_{\mu\nu}\,(1+2\,\Psi) + 2\,E_{,\mu\nu} + E_{(\mu,\nu)} + h_{\mu\nu}\right\}\,dx^\mu\,dx^\nu \\
\delta A_M &= \{a_\vartheta,\ a_\varphi\,,\ \partial_\mu a + a_\mu\} \quad , \quad \delta\sigma \quad , \quad \theta_{\text{brane}} = \bar{\theta} + \zeta\,(x)
\end{aligned}
}
$$

- Fix gauge
- Solve Einstein/Maxwell equations in the bulk
- Require regularity at the pole
- Require Z_2 symmetry at the equator
- Continuity+jumps at the brane (Israel junction conditions)

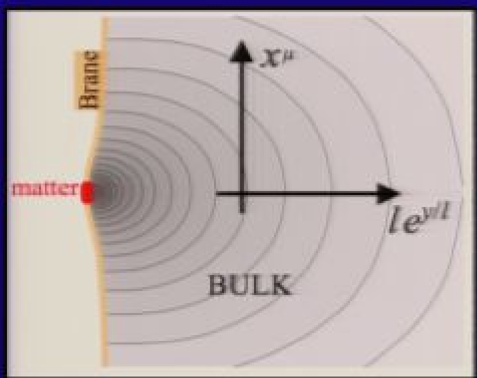
A crucial mode: the brane bending



In RS, crucial to get 4D gravity: couples to the trace of the matter stress-energy

Garriga and Tanaka 99,
Giddings, Katz and Randall 00

A crucial mode: the brane bending



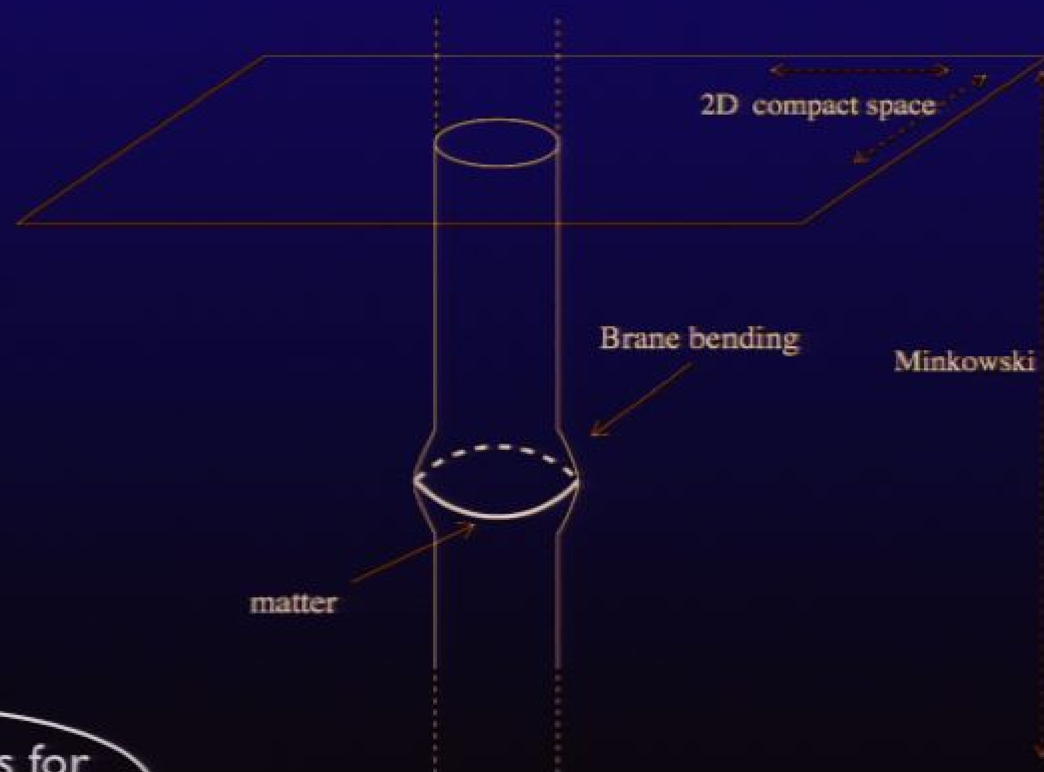
In RS, crucial to get 4D gravity: couples to the trace of the matter stress-energy

Garriga and Tanaka 99,
Giddings, Katz and Randall 00

In our case, this is the mode that allows to put matter with arbitrary equation of state on the brane:

$$\left[\partial^2\zeta\right]_J = \frac{T}{3M^4} \quad , \quad T \equiv T^\mu_\mu$$

ump across the brane)



No scalar modes for the delta like brane

Graesser, Kile and Wang 04

Assumptions

We assume that the brane radius is hierarchically smaller than the bulk radius

Modes are constant in the ϕ direction, nontrivial dependence on θ

Analysis limited only to zero modes (enough for long-range gravity)

Getting 4D gravity

Result is analogous to Garriga and Tanaka:
perturbative equations of motion get contributions from tensor modes and from bending mode

Israel junction conditions give:

$$\partial_\vartheta\left(\cos\theta\,\partial_\vartheta h_{\mu\nu}\right)+\cos\theta\,\partial^2 h_{\mu\nu}=-2\delta\left(\theta-\bar{\theta}\right)\frac{\cos\bar{\theta}}{M^4}\left(T_{\mu\nu}-\frac{T}{3}\eta_{\mu\nu}+M^4\left[\zeta_{,\mu\nu}\right]_J\right)$$

Getting 4D gravity

Result is analogous to Garriga and Tanaka:
perturbative equations of motion get contributions
from tensor modes and from bending mode

Israel junction conditions give:

$$\partial_\vartheta \left(\cos\theta \, \partial_\vartheta h_{\mu\nu}\right) + \cos\theta \, \partial^2 h_{\mu\nu} = -2\delta\left(\theta - \bar{\theta}\right) \frac{\cos\bar{\theta}}{M^4} \left(T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu} + M^4 \left[\zeta_{,\mu\nu}\right]_J\right)$$

Getting 4D gravity

Result is analogous to Garriga and Tanaka: perturbative equations of motion get contributions from tensor modes and from bending mode

Israel junction conditions give:

$$\partial_\vartheta\left(\cos\theta\,\partial_\vartheta h_{\mu\nu}\right)+\cos\theta\,\partial^2 h_{\mu\nu}=-2\delta\left(\theta-\bar\theta\right)\frac{\cos\bar\theta}{M^4}\left(T_{\mu\nu}-\frac{T}{3}\gamma_{\mu\nu}+M^4\left[\zeta_{,\mu\nu}\right]_J\right)$$

We are interested only in the induced 4D metric $g^{(4)}_{\mu\nu}$ and we define an effective 4D stress energy tensor as the integral along ϕ of the brane stress energy tensor:

$$T^{(4)}_{\mu\nu}=\int_0^{2\pi R\beta}d\phi\sqrt{\gamma^{(0)}_{\phi\phi}}\,T_{\mu\nu}=2\pi R\beta\cos\bar\theta\,T_{\mu\nu}$$

Einstein equations on the brane

$$\Upsilon = \left(-2\,\Psi + \frac{2\pi\, R\beta}{V_2} \cos\bar{\theta}\; [\zeta]_J \right)\bigg|_{\text{brane}}$$

$$R^{(4)}_{\mu\nu} = \frac{1}{M^4 V_2}\left[T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{2}\eta_{\mu\nu} \right] + \left(\frac{1}{2}\partial^2 \Upsilon\, \eta_{\mu\nu} + \Upsilon_{,\mu\nu} \right)$$

ADD M_p^2

Einstein equations on the brane

$$\Upsilon = \left(-2\,\Psi + \frac{2\pi\, R\beta}{V_2}\,\cos\bar{\theta}\;[\zeta]_J \right)\Bigg|_{\text{brane}}$$

$$R^{(4)}_{\mu\nu} = \frac{1}{M^4 V_2}\left[T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{2}\eta_{\mu\nu} \right] + \left(\frac{1}{2}\partial^2 \Upsilon\, \eta_{\mu\nu} + \Upsilon_{,\mu\nu} \right)$$

ADD M_p^2

Standard 4D gravity

Einstein equations on the brane

$$\Upsilon = \left(-2\,\Psi + \frac{2\pi\, R\beta}{V_2}\cos\bar\theta\;[\zeta]_J\right)\Big|_{\text{brane}}$$

$$R^{(4)}_{\mu\nu} = \frac{1}{M^4 V_2}\left[T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{2}\eta_{\mu\nu}\right] + \left(\frac{1}{2}\partial^2\Upsilon\,\eta_{\mu\nu} + \Upsilon_{,\mu\nu}\right)$$

ADD M_p^2

Standard 4D gravity

Brans-Dicke scalar

Einstein equations on the brane

$$\Upsilon = \left(-2\,\Psi + \frac{2\pi\,R\beta}{V_2}\cos\bar{\theta}\;[\zeta]_J\right)\Big|_{\text{brane}}$$

$$R^{(4)}_{\mu\nu} = \frac{1}{M^4 V_2}\left[T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{2}\eta_{\mu\nu}\right] + \left(\frac{1}{2}\partial^2\Upsilon\,\eta_{\mu\nu} + \Upsilon_{,\mu\nu}\right)$$

ADD M_p^2

Standard 4D gravity

Brans-Dicke scalar

Scalar-tensor theory

(Not surprisingly, $\Delta V_2 \propto \Upsilon$)

...but the Brans-Dicke scalar is heavy!

Indeed, the relation between Υ and T can be found explicitly

By replacing it into Einstein's equations we get

$$R^{(4)}_{\mu\nu} = \frac{1}{M_p^2}\left[T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{2}\eta_{\mu\nu}\right] + \boxed{\mathcal{F}\left(\beta, \bar{\theta}\right)\frac{R^2}{M_p^2}\left(\frac{1}{2}\eta_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu\right)\left(\frac{T^{(4)}}{3} - T^{(4)\,\phi}_{\phi}\right)}$$

regular and O (1)

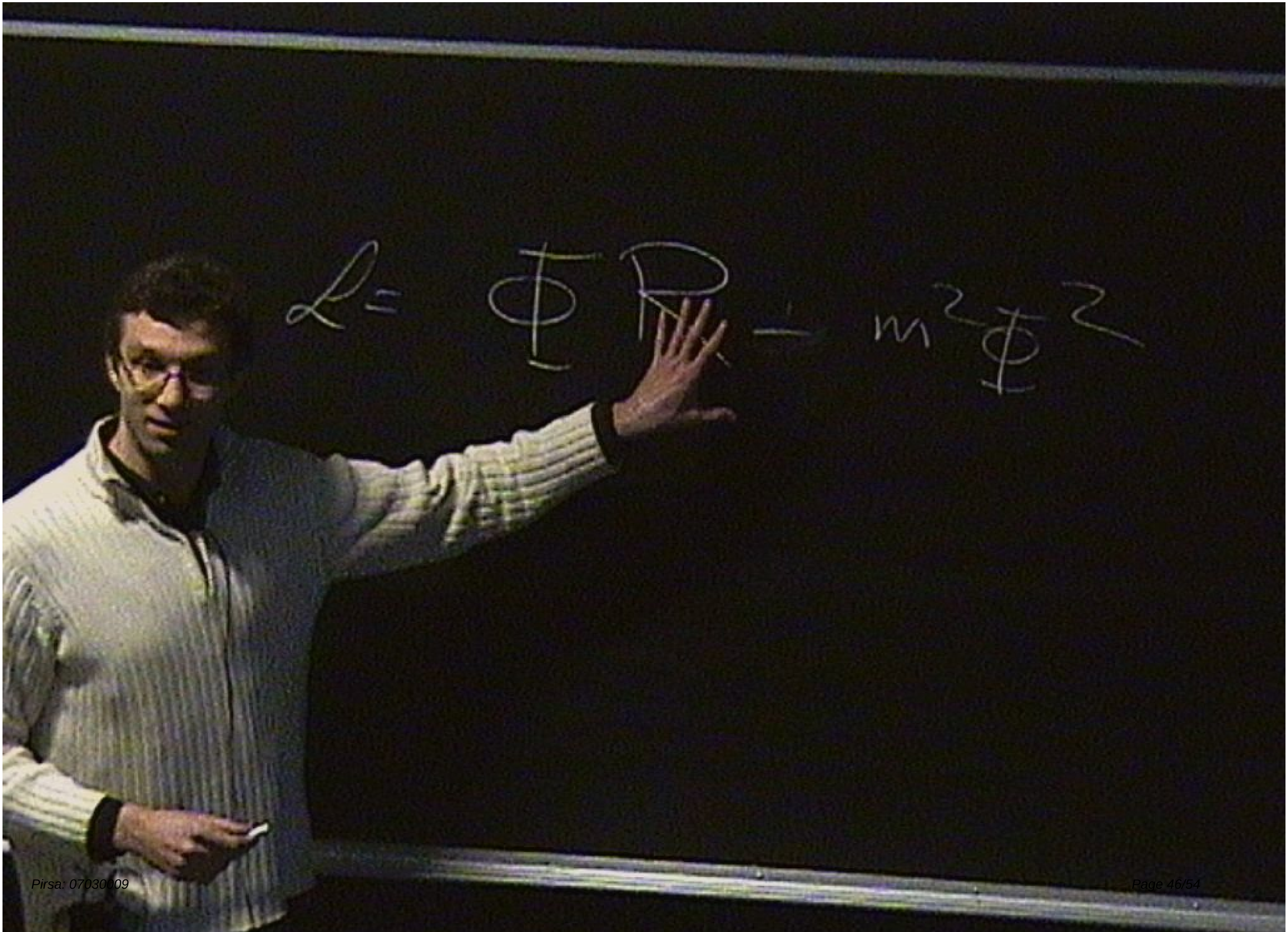
extra term is negligible for L>>R, where L is th
scale of variation of matter on the brane

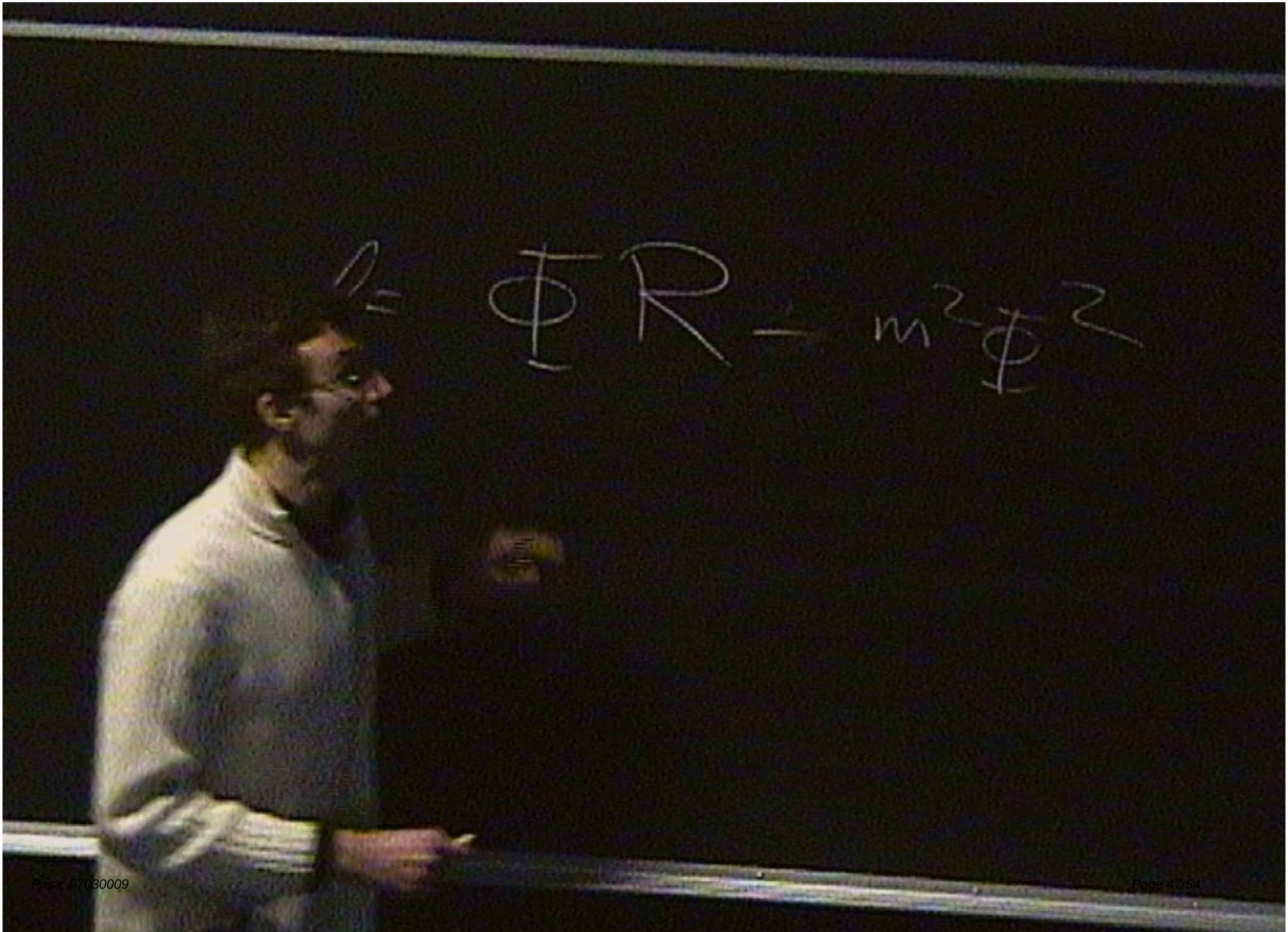
Standard 4D gravity is recovered

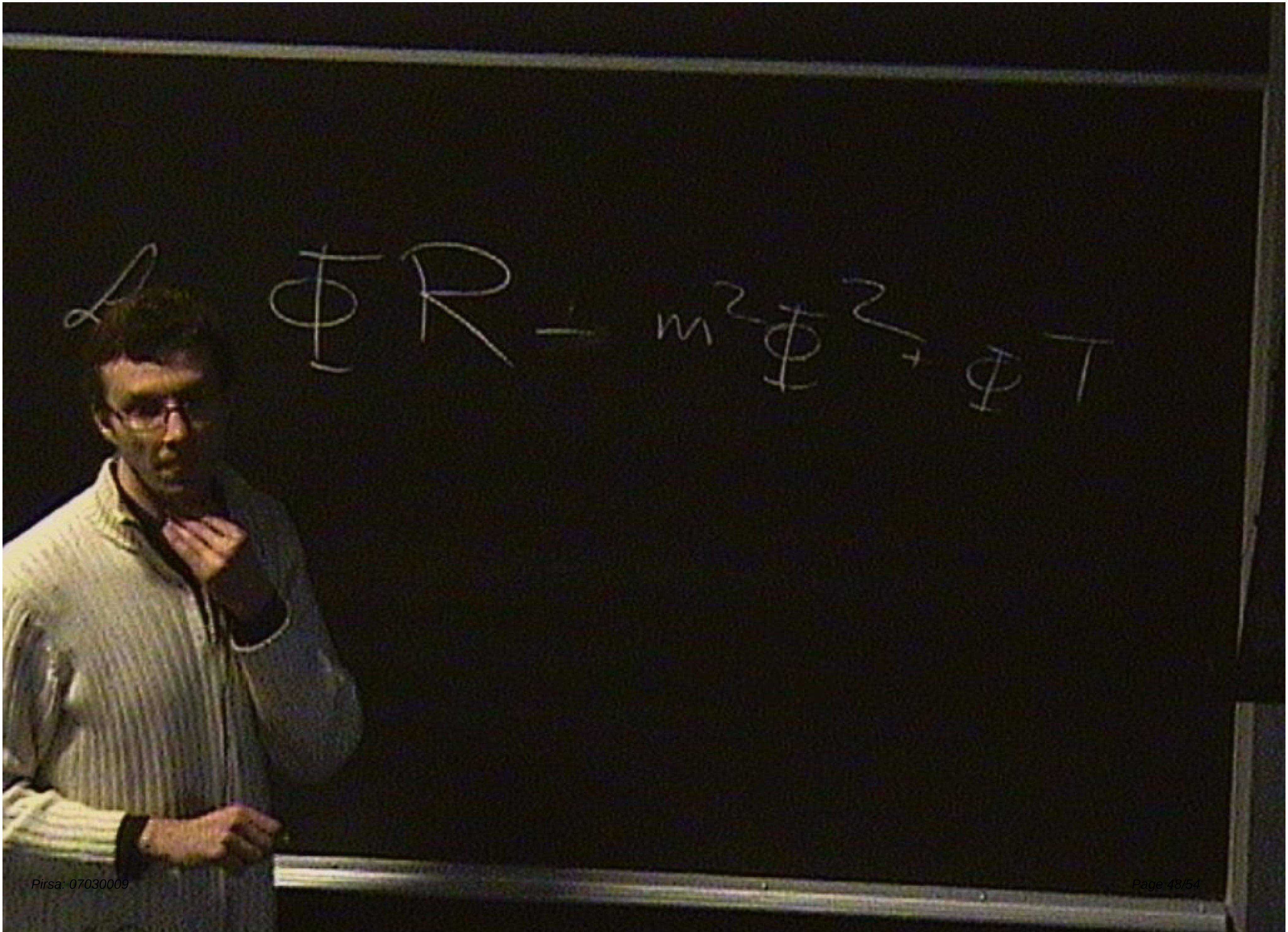
Stabilization fails at scales compatible with compactification radius

Can be understood as an effect of causality in the bulk:

The stabilization mechanism takes a time ~ R to act


$\mathcal{L} = \Phi R - m^2 \Phi^2$


$\Phi R - m^2 \Phi^2$



$$\mathcal{L} = \Phi R - m^2 \Phi^2 + \Phi T$$

What happens in the limit of a delta-like defect?

This limit corresponds to $\bar{\theta} \to \pi/2$

The brane stress-energy tensor obeys

$$T^{(4)}_{\mu\nu} = 2\pi \, R\beta \cos\bar{\theta} \, T_{\mu\nu}$$

where we want to keep $T^{(4)}_{\mu\nu}$ constant

$T_{\mu\nu}$ has to diverge

From $\left[\partial^2\zeta\right]_J = \dfrac{T}{3M^4}$, bending mode diverges

unless $T^{(4)}=0$. *Strong coupling problem*

What happens in the limit of a delta-like defect?

This limit corresponds to $\bar{\theta} \to \pi/2$

The brane stress-energy tensor obeys

$$T^{(4)}_{\mu\nu} = 2\pi \, R\beta \cos\bar{\theta} \, T_{\mu\nu}$$

where we want to keep $T^{(4)}_{\mu\nu}$ constant

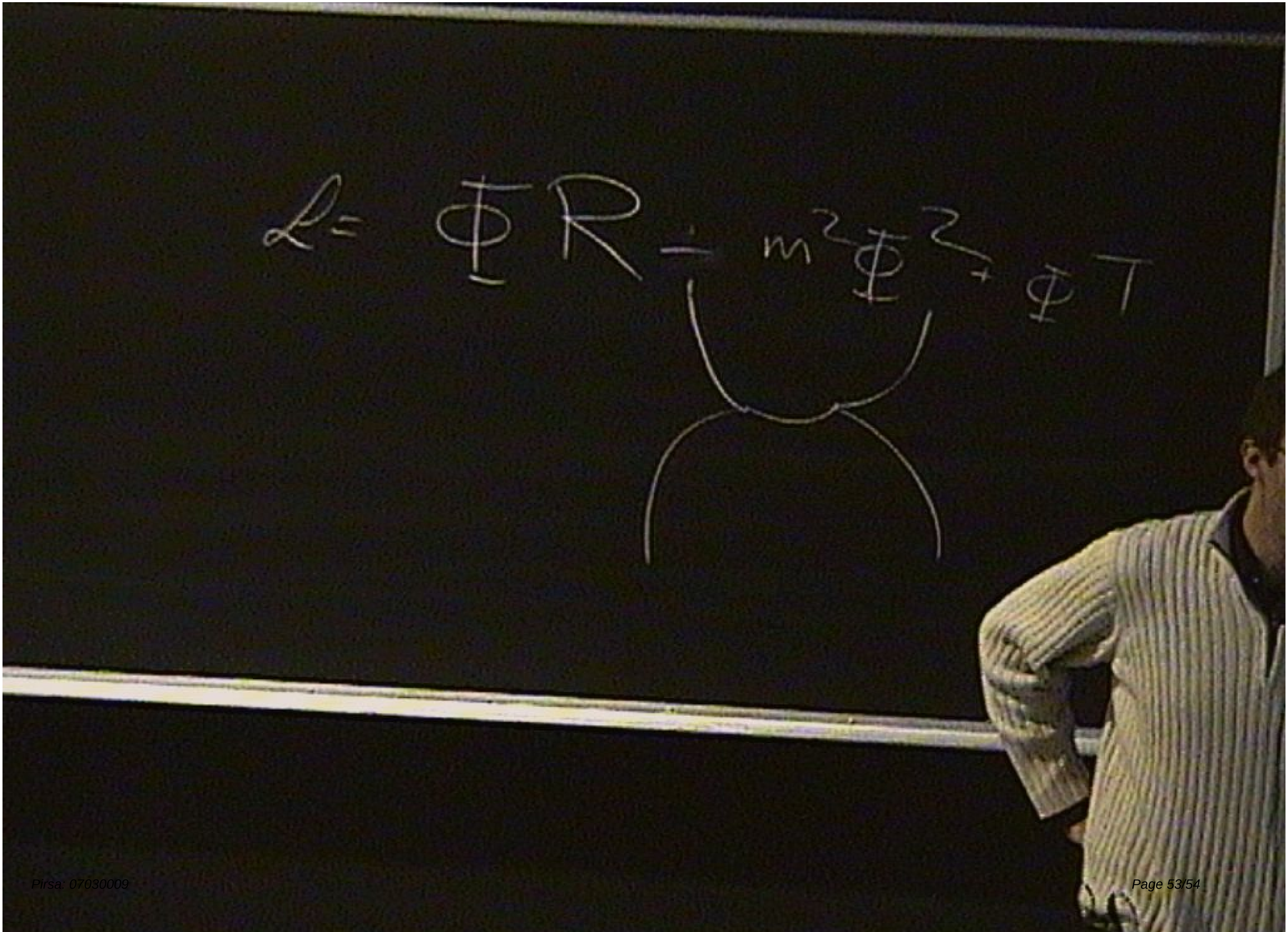
$T_{\mu\nu}$ has to diverge

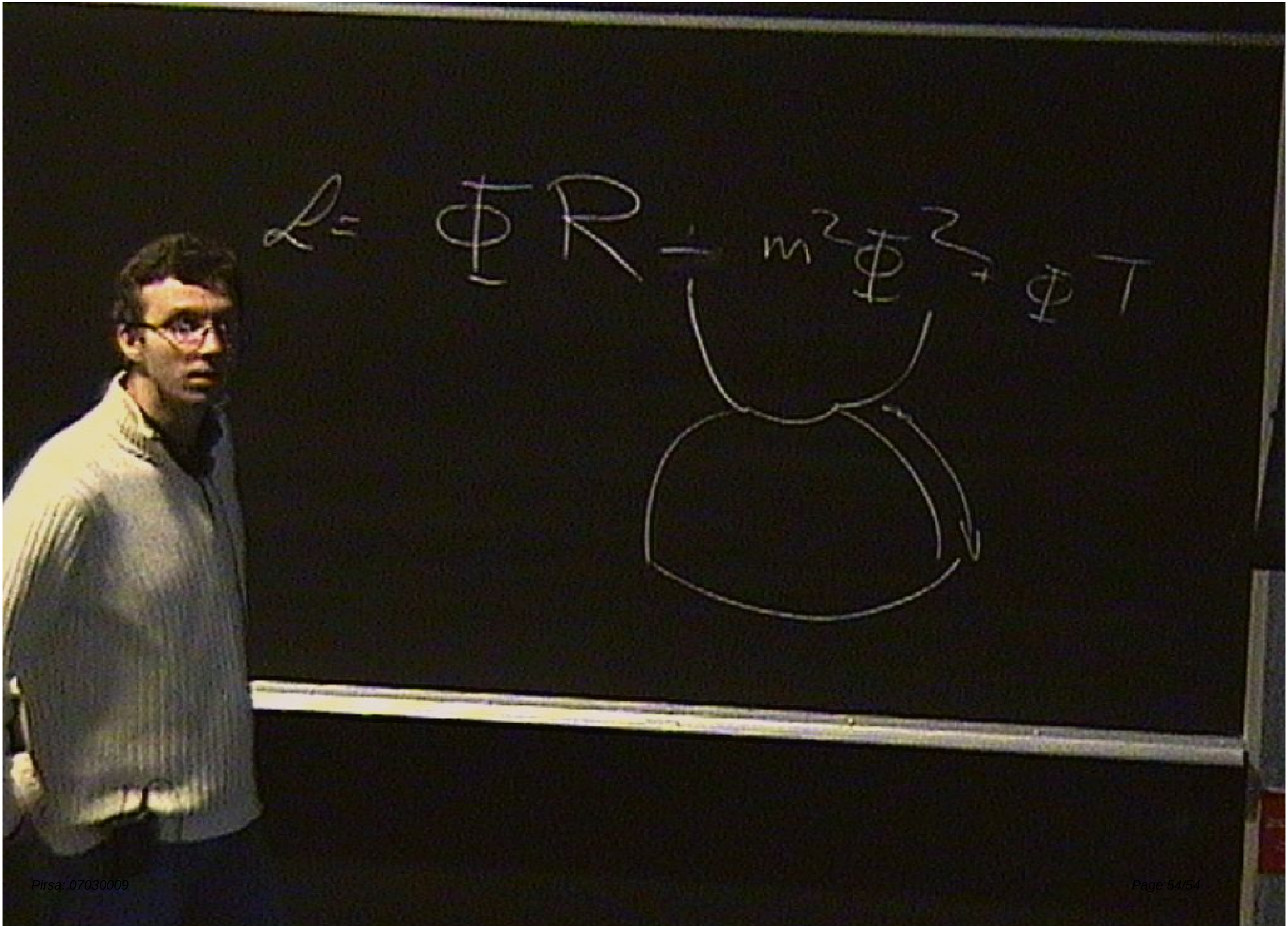
From $\left[\partial^2\zeta\right]_J = \dfrac{T}{3M^4}$, bending mode diverges

unless $T^{(4)}$=0. *Strong coupling problem*

Conclusions

- Built a regular version of the football (brane Universe in flux compactification)
- 4D gravity is recovered at distances >> R
- Pathologies emerge in the delta-like codimension-2 limit
- First complete analysis (to our knowledge) of weak gravity in a stabilized 6D brane-worlds


$\mathcal{L} = \Phi R - m^2 \Phi^2 + \Phi T$


$\mathcal{L} = \Phi R - m^2 \Phi^2 + \Phi T$