Title: Holographic QCD and Pion Mass

Date: Mar 20, 2007 02:00 PM

URL: http://pirsa.org/07030007

Abstract: To realize massive pions, I propose a variation of the holographic model of massless QCD using the D4/D8/D8bar-brane configuration proposed by Sakai and Sugimoto. The deformation breaks the chiral symmetry explicitly and I compute the mass of the pions and vector mesons. The observed value of the pion mass can be obtained. I also argue a chiral perturbation corresponding to the deformation.

March 2007
Talk at MIT, Brown, and Perimeter

Holographic QCD and Pion Mass

Koji Hashimoto
(Univ. of Tokyo, Komaba)

hep-th/0703024
Work in collaboration with T. Hirayama and A. Miwa

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Plan of this talk

1. More realistic model of holographic QCD?
2. Idea for pion mass in Sakai-Sugimoto
3. Computation of the pion mass
4. Corresponding chiral perturbation
5. Summary and discussions

1. More realistic model of holographic QCD?

"Need of pion mass in SS"

Holographic QCD

Apply AdS/CFT correspondence to QCD!

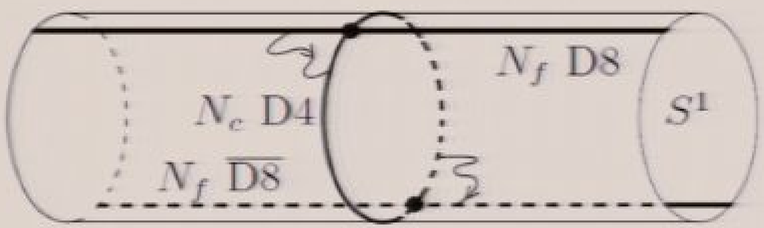
Although it is in large N, it might work for analysing strong coupling regime of QCD‥‥

- Meson and baryon spectra
- Chiral lagrangians, interactions
- Thermal phase transitions, QCD phase diagram
- Jet quenching, viscosity

Not just reproducing observed phenomena, but being predictive with interesting dual gravity descriptions

Sakai-Sugimoto model

A good holographic model of massless QCD.



	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D4	○	○	○	○					
D8	○	○	○		○	○	○	○	○
$\overline{\text{D8}}$	○	○	○		○	○	○	○	○

- Nc D4-branes wrapping S^1 with anti-periodic b.c. for gauginos $\rightarrow$ pure YM at low energy Witten
- Nf D8 and Nf D8bar intersecting with the D4 $\rightarrow$ left- and right- handed quarks

Exact matter content of massless QCD!

Near horizon limit

Replacing the D4s by their geometry :

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2) \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$e^\phi = g_s \left(\frac{U}{R}\right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{V_4}\epsilon_4, \quad f(U) = 1 - \frac{U_{KK}^3}{U^3}$$

AdS/CFT

Strong coupling of massless large Nc QCD

- Probe D8 dynamics ($N_c \gg N_f$) $\longrightarrow$ meson sector
- Bulk gravity dynamics $\longrightarrow$ glueballs etc.

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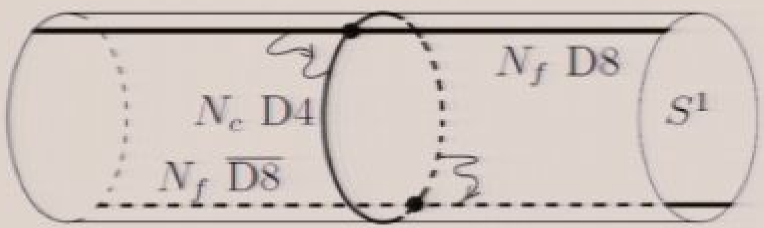

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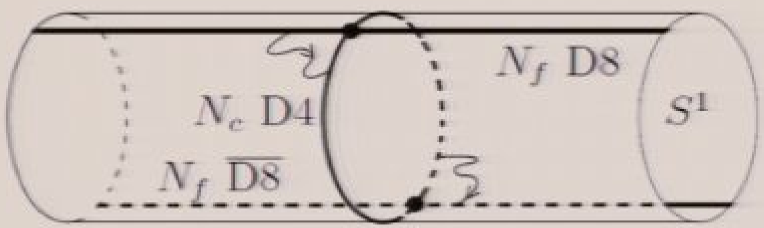

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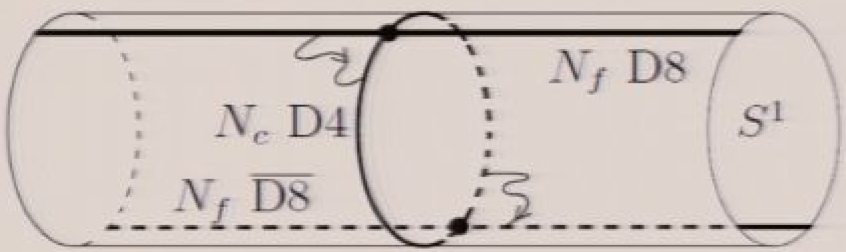
Spontaneous chiral S.B. in SS



Massless QCD
at weak coupling

Chiral sym. : D8: U(N_f)$_L$
D8bar:U(N_f)$_R$

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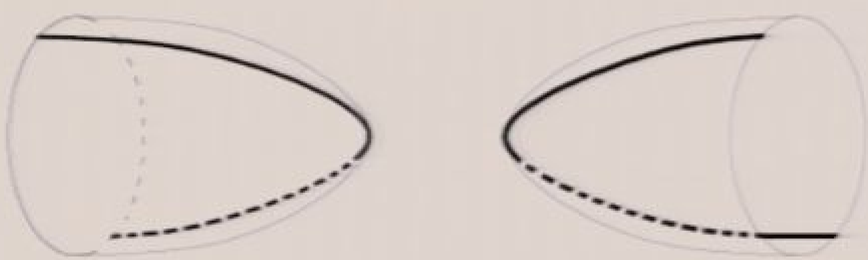

Strong coupling,
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D8 and D8bar are connected: U(N_f)$_V$

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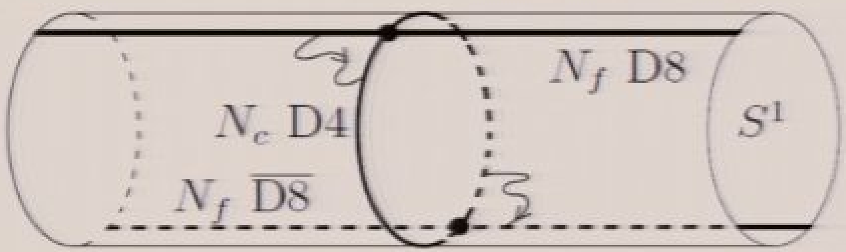

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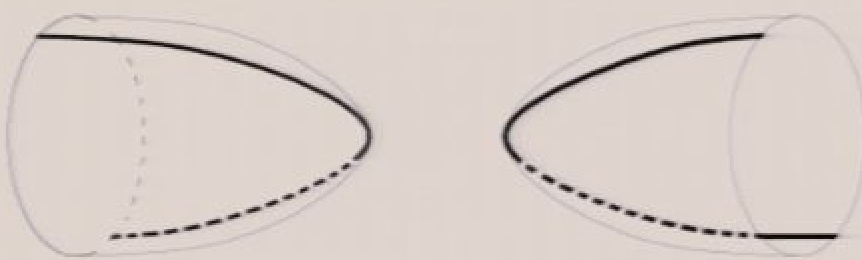

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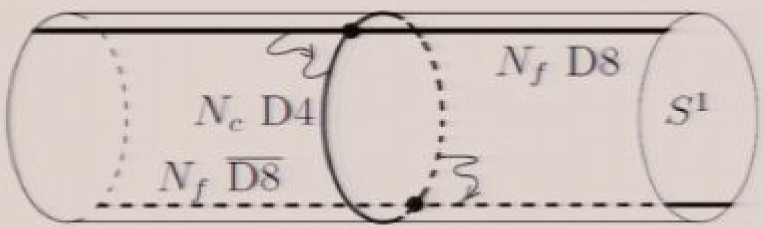

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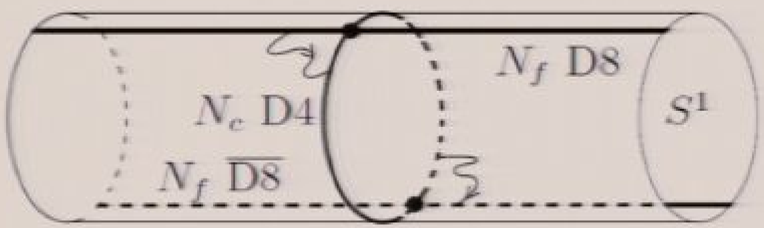

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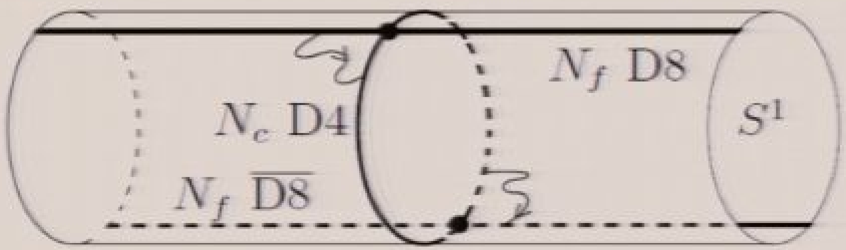

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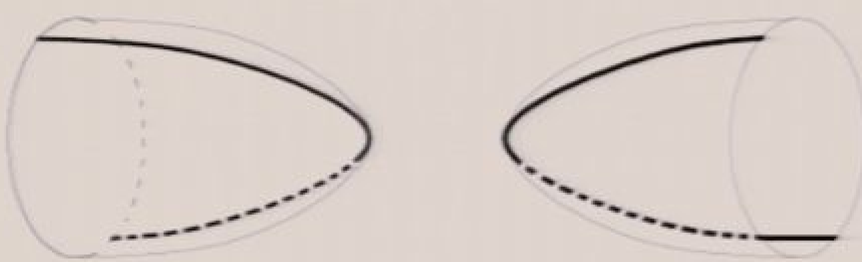

Strong coupling,
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D8 and D8bar are connected: $U(N_f)_V$

Upshots of the SS model

- Vector/scalar mesons = KK modes of gauge fields on the connected probe D8.
- Chiral langrangian = the probe D8-brane action.
- And many more: baryon spectrum, realization of hidden local symmetry, etc..

Results are quite consistent with observed experimental values.

Comparison to other holographic models:

- Spontaneous chiral sym. br. is seen in geometry
- Clear understanding of weak coupling definition

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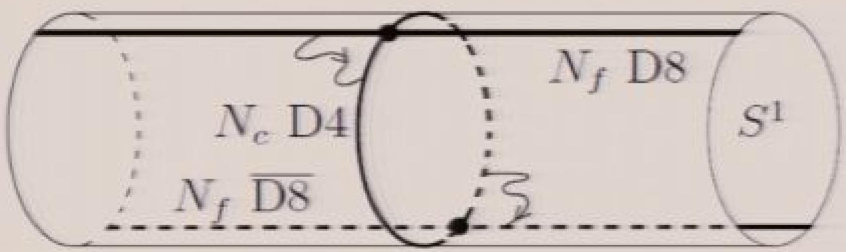

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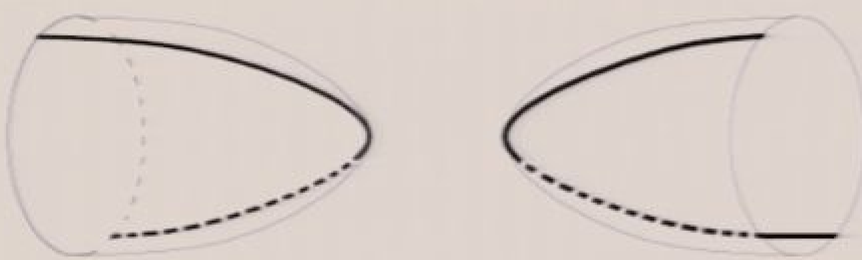

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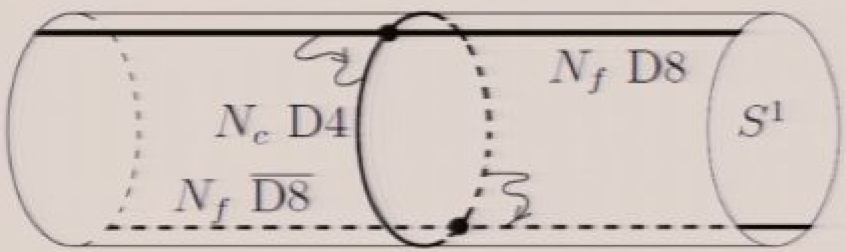

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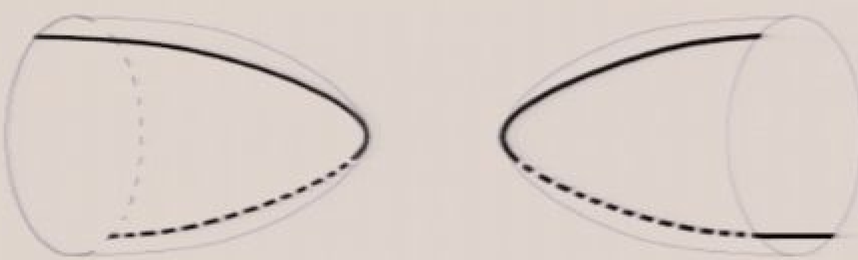

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Numerical results of SS

Vector meson spectrum

not established → (a_1')

	ρ	a_1	ρ'	(a_1')	ρ''
exp.(MeV)	776	1230	1465	(1640)	1720
SS model	[776]	1189	1607	2023	2435
ratio	[1]	1.03	0.911	(0.811)	0.706

Chiral lagrangian

$$\frac{1}{32e_S^2}\mathrm{Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 = L_1 P_1 + L_2 P_2 + L_3 P_3$$

	L_1	L_2	L_3
exp.($\times 10^{-3}$)	0.4 ± 0.3	1.4 ± 0.3	-3.5 ± 1.1
SS model	0.584	1.17	-3.51

(Tables taken from Sugimoto's presentation)

Problems! in the Sakai-Sugimoto

D-brane configuration severely constrains the model.

→ Problems in the SS model :

1) Pion is massless.

- No explicit breaking of the chiral symmetry.

cf) D4/D6 model Kruczenski-Mateos-Myers-Winters

2) Non-decoupling of higher dimensional DoFs.

- Single unique scale is the KK scale.

We give a resolution of the problem 1)

Why are pions massless in SS?

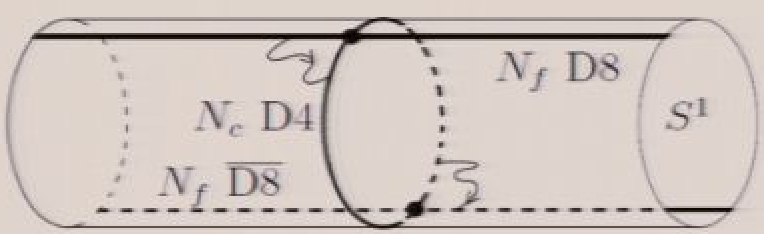
Answer : Quarks are massless in the D4D8.

→ No explicit chiral symmetry breaking

→ Pions are exactly NG bosons

Then, why quarks are massless in SS?

Because: It's difficult to separate D8 from D4



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Co-dimension = 6 → Chiral fermions

Importance of massive pion

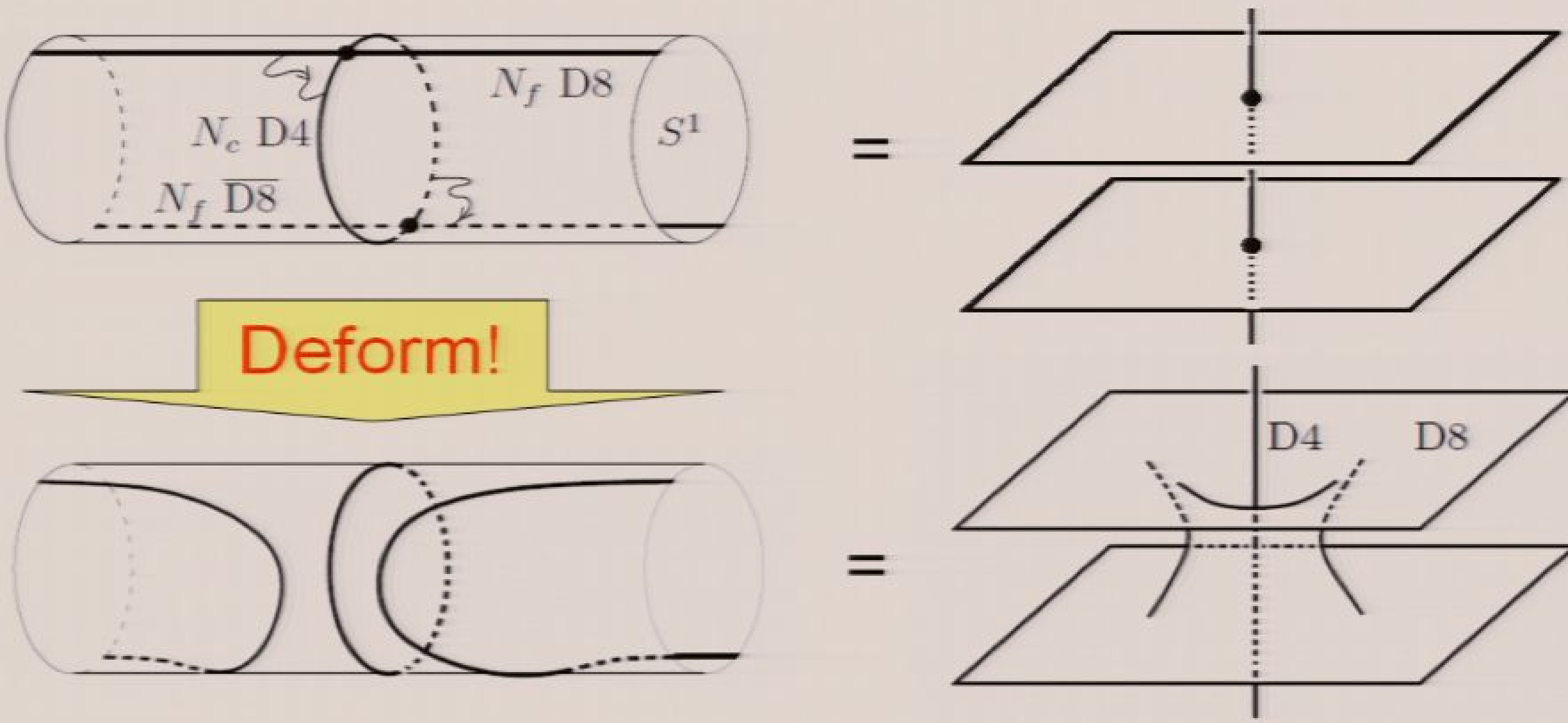
1) For constructing more realistic models
2) To understand properties of the lightest hadron
   (a) At low energy, pion mass is an indispensable ingredient in low energy effective lagrangian.
   (b) Spontaneous vs. explicit breaking of the chiral symmetry.
3) In SS, we need to consider only energy lower than the KK scale (technical reason)

We introduce pion mass in SS,
by deforming the shape of probe D8-brane

2. Our idea for pion mass in Sakai-Sugimoto

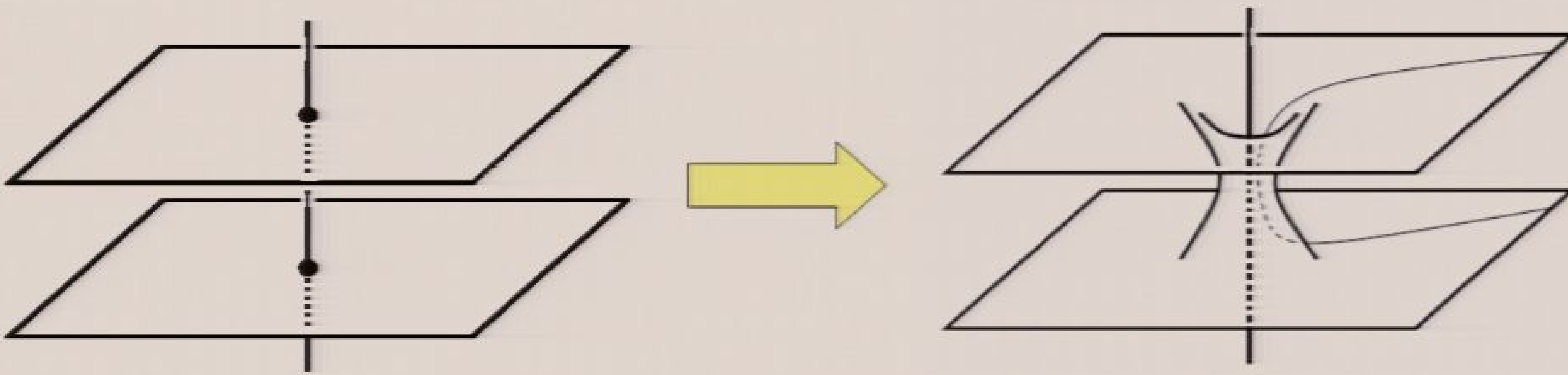
"Additional D4 gives pion mass"

Explicit chiral sym. breaking?



- D8s are connected not by the background geometry
- Quarks are expected to be massive

How can we connect D8 and D8bar in flat spacetime (weak coupling regime)?



Introduction of different D-brane charge on D8

Charge conservation forces D8 and D8bar to connect

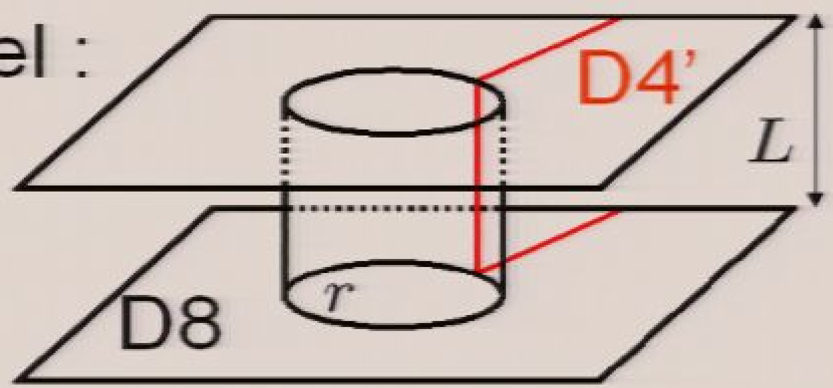
→ To achieve this, we introduce a D4'-brane bound in the D8

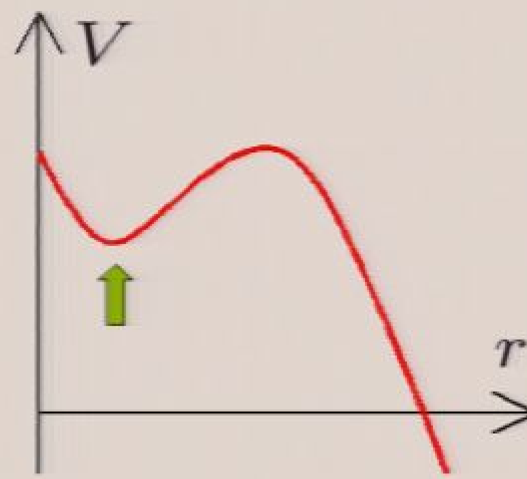
Throat D8 configuration

There is a "throat" D8-D4' bound state configuration with throat radius l_s^2/L : "BIon"

Callan-Maldacena
Constable-Myers-Tafjord

Toy model :



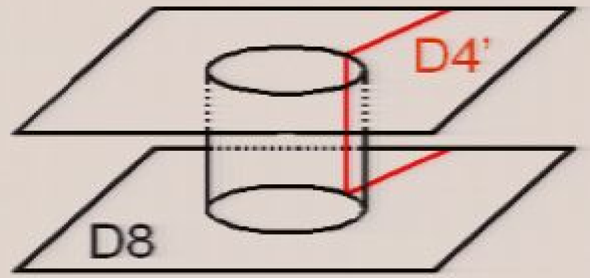


$$V = T_{\mathrm{D8}}(-r^5 + Lr^4) + T_{\mathrm{D4}}(-r + L)$$

Quark mass ~ [throat radius] × [string tension] ~ $1/L$

Bound D4 as YM instanton on D8

a D4' bound in D8
= YM instanton on D8
Witten, Douglas



The direction of the D4' worldvolume is along radial direction → We introduce YM instanton in angular 4-sphere of probe D8 worldvolume

- Tunable parameter : instanton size μ^{-1}
  [size → 0] → D4' can be separated from D8
  → Reduction to the SS
  (0 size instanton → radius 0 → massless quark)

Summary of our idea

1. To connect D8 and D8bar in flat space, we introduce another D4' charge on D8 D8bar
2. The D4' is a YM instanton in 4-sphere.

We re-analyze SS :
fluctuation analysis of the connected D8
in the D4 near horizon geometry,
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→ Hadron spectra/interaction with massive pion

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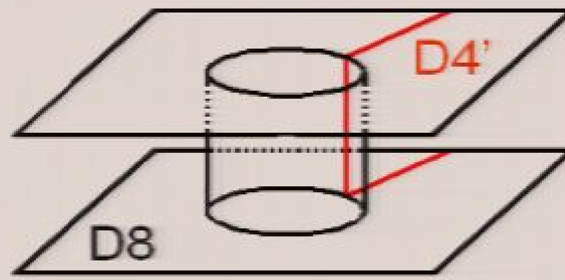

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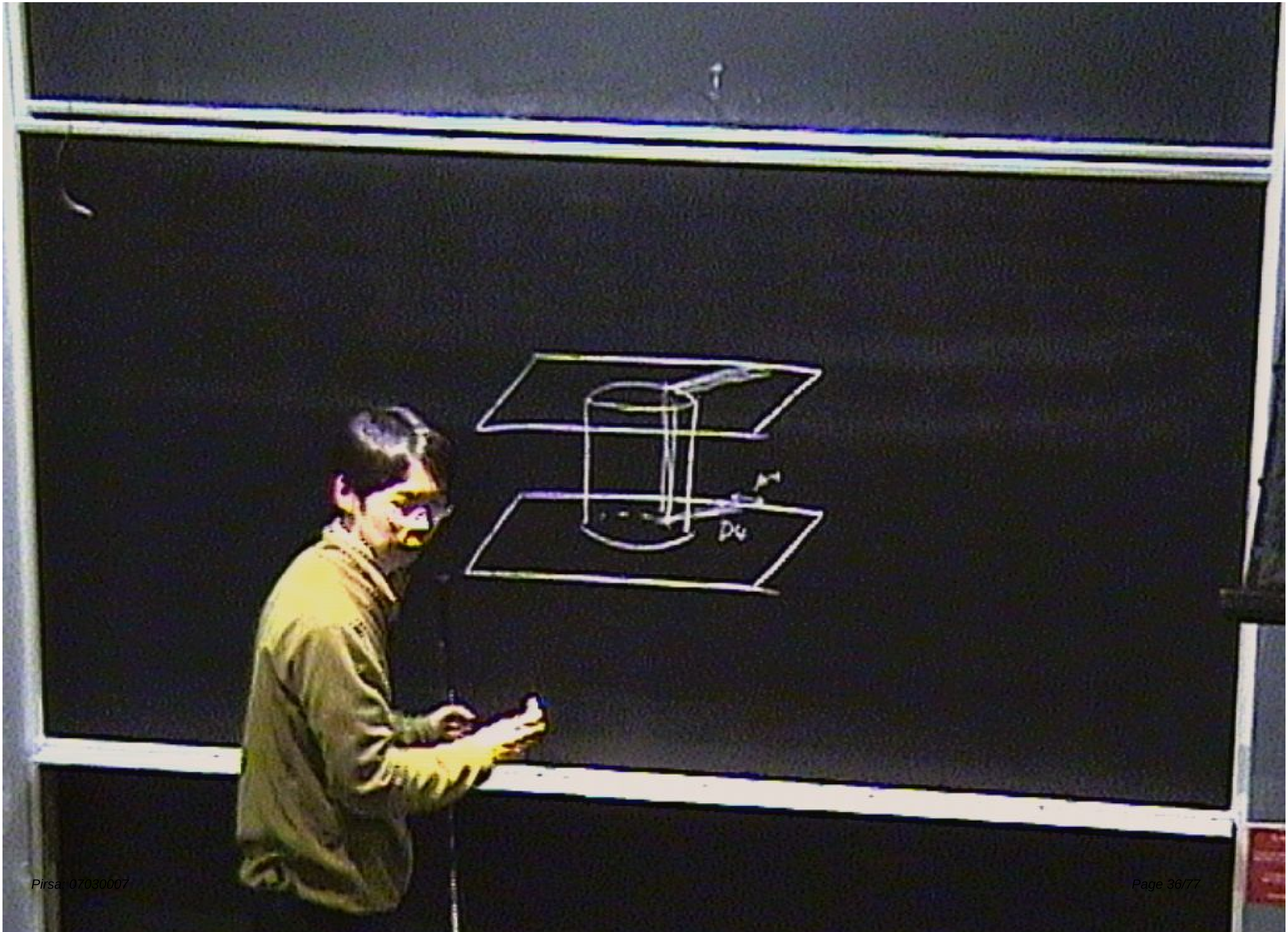
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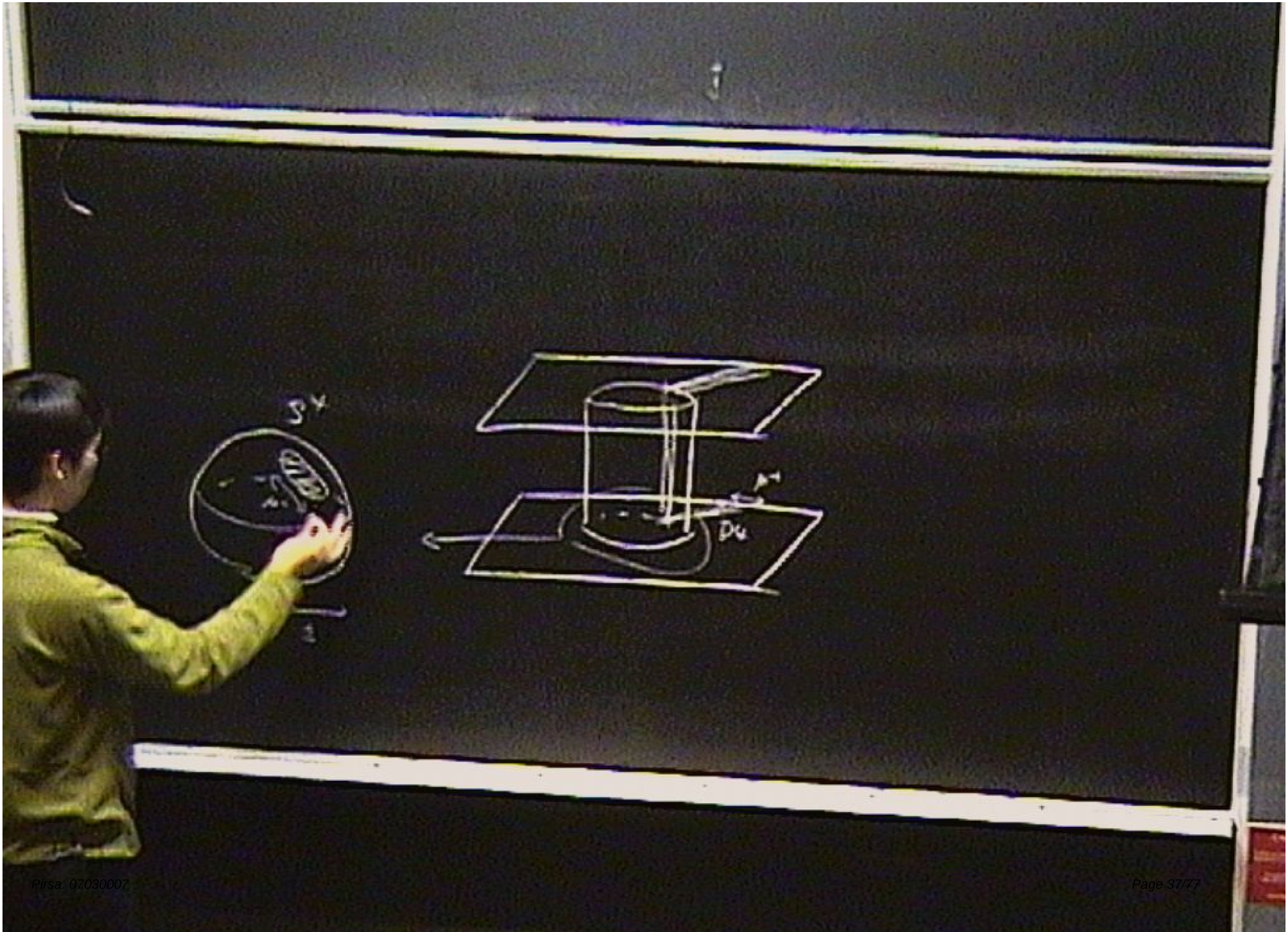
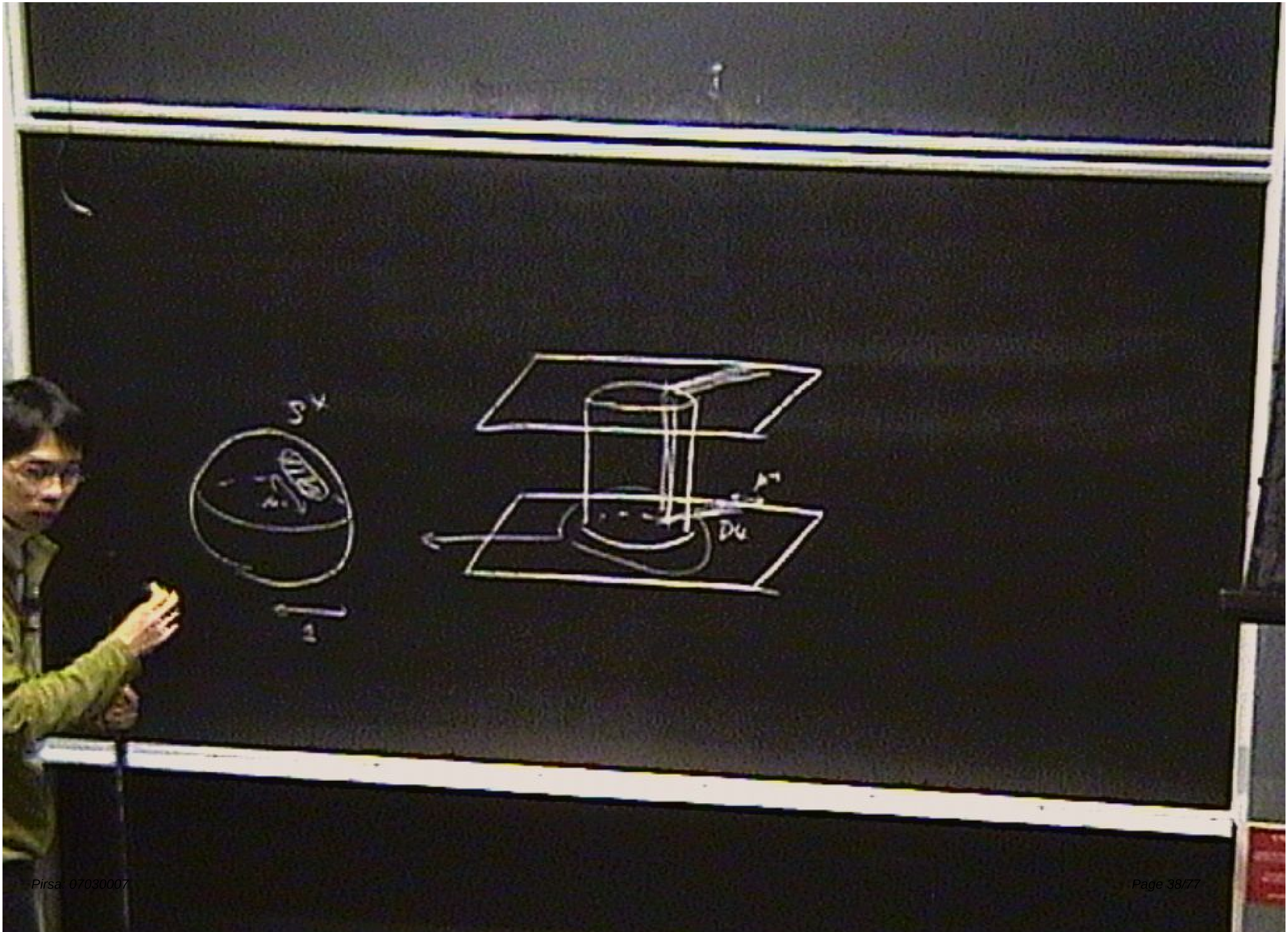
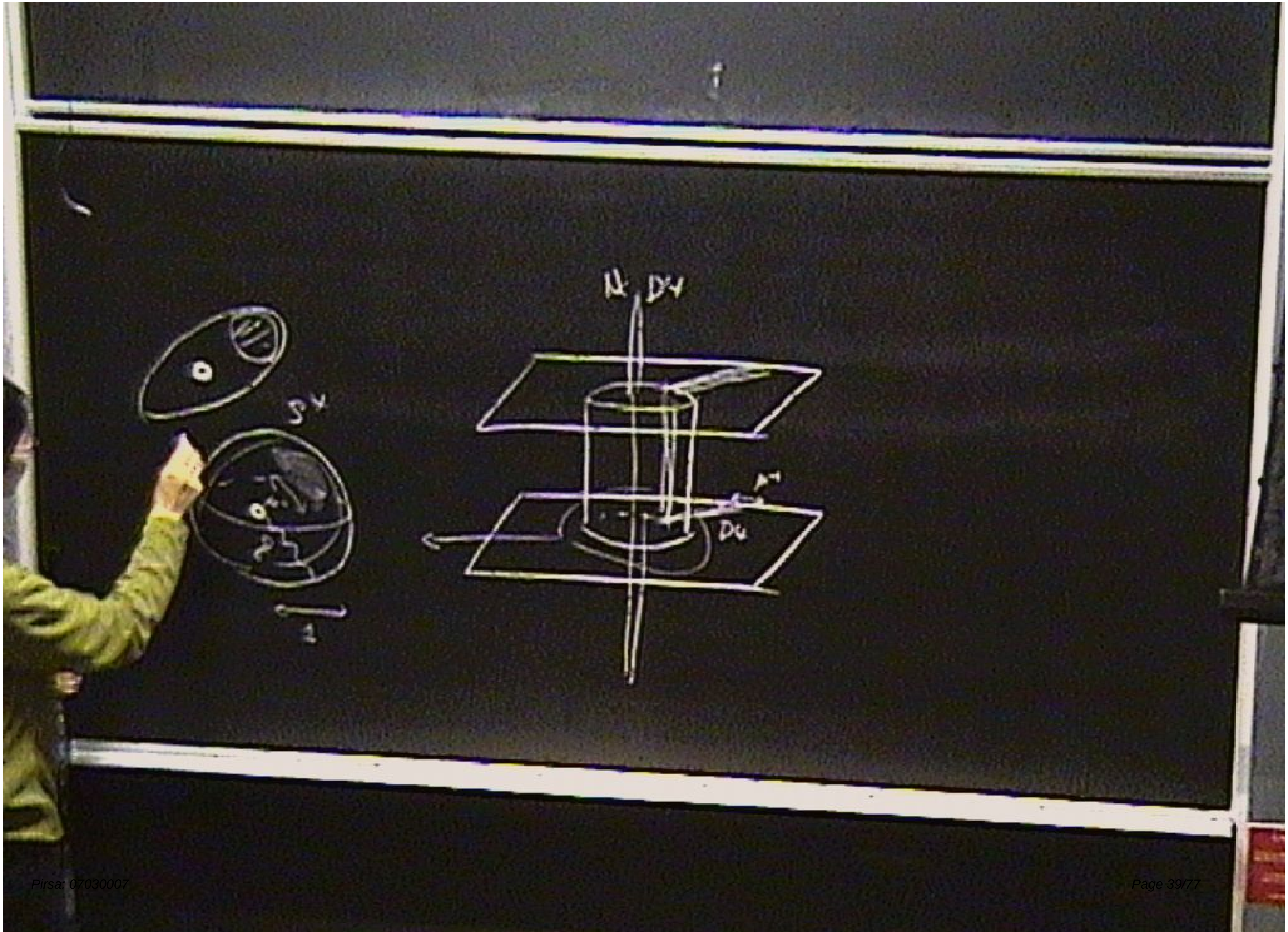
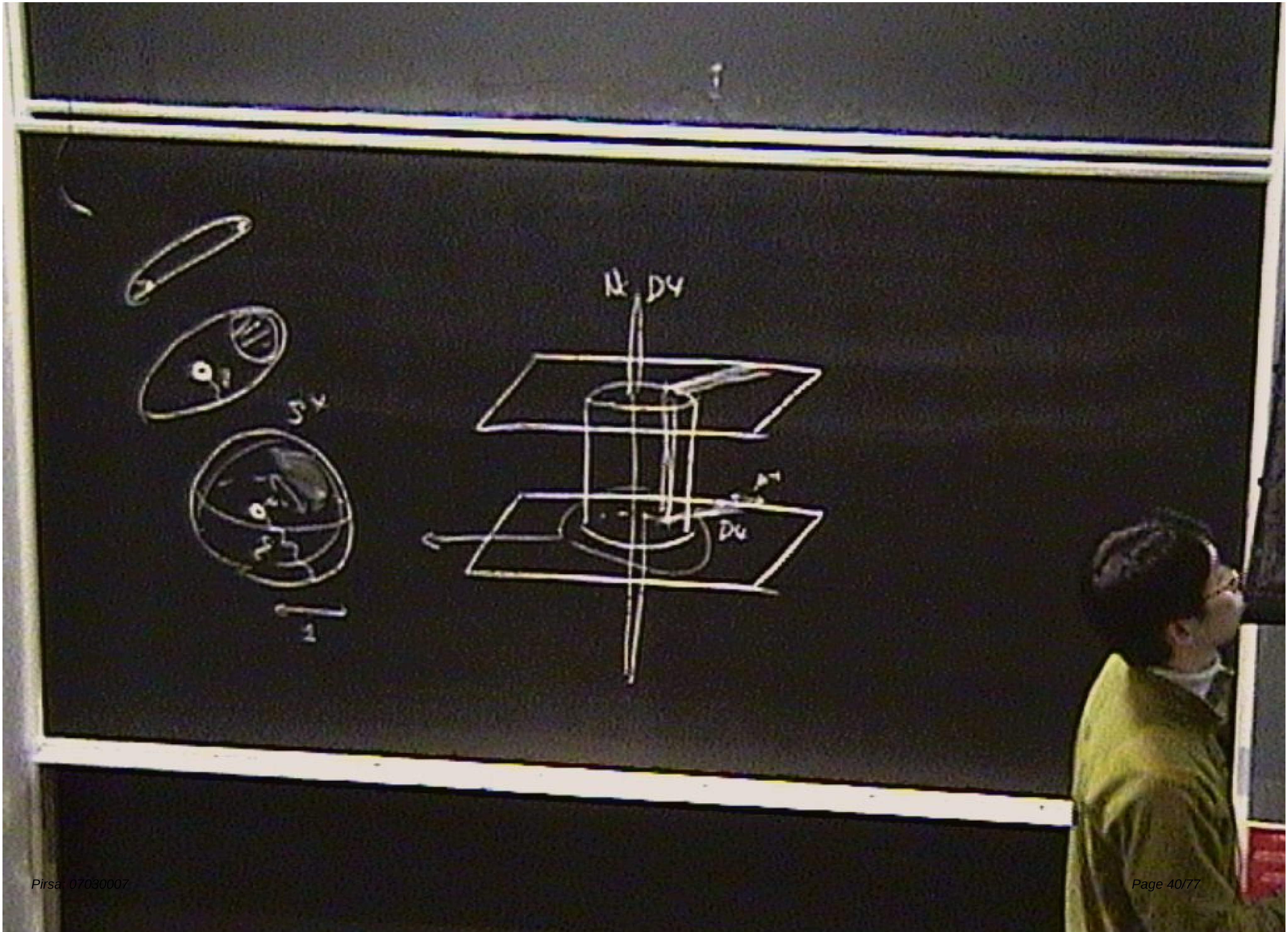

S^4






S4
1
N DY
D4

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3. Computation of pion mass

"Observed value of pion mass reproduced"

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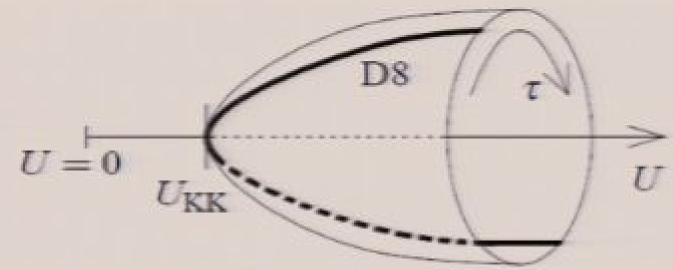
Instanton background

We work with probe D8 action:

$$T_{D8}(2\pi\alpha')^2 \int d^9\sigma e^{-\phi}\sqrt{-\det g}\frac{1}{2}\mathrm{Tr}F_{MN}F^{MN}$$



Induced metric from D4 geometry:

$$ds^2_{D8} = g_{MN}d\sigma^M d\sigma^N = \left(\frac{U_z}{R}\right)^{3/2} dx_4^2 + \frac{4}{9}\left(\frac{R}{U_z}\right)^{3/2}\frac{U_{\mathrm{KK}}}{U_z}dz^2 + \left(\frac{R}{U_z}\right)^{3/2} U_z^2 d\Omega_4^2.$$

Instanton is a classical solution in angular 4-sphere:

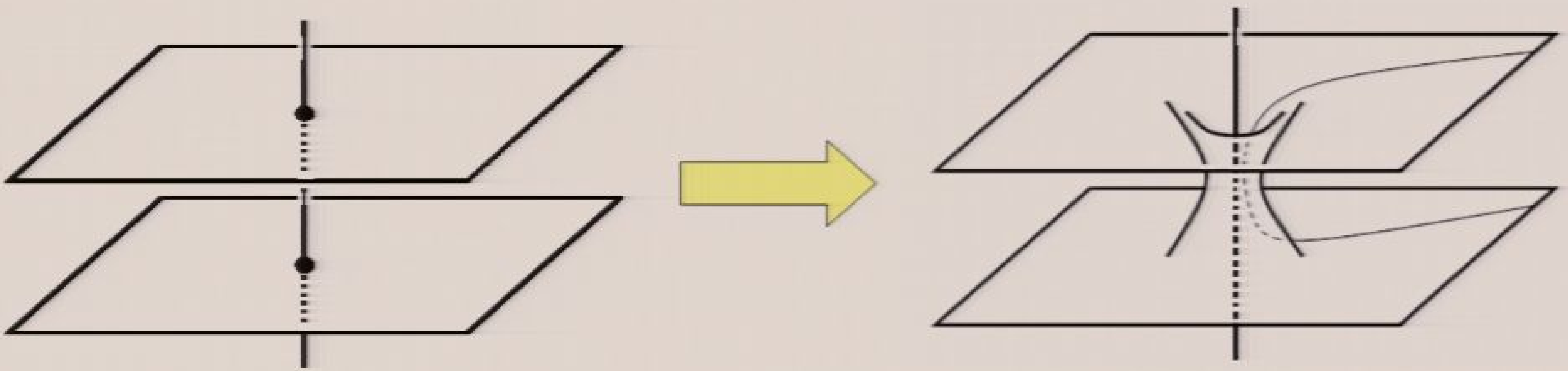
$$A_\mu = 0, \quad A_z = 0, \quad A_i = A_i(\theta^j)$$

SS:

KK modes from A_μ	$\longrightarrow$ vector mesons
KK modes from A_z	$\longrightarrow$ pion

We do the fluctuation analysis in the b.g. instanton

How can we connect D8 and D8bar in flat spacetime (weak coupling regime)?

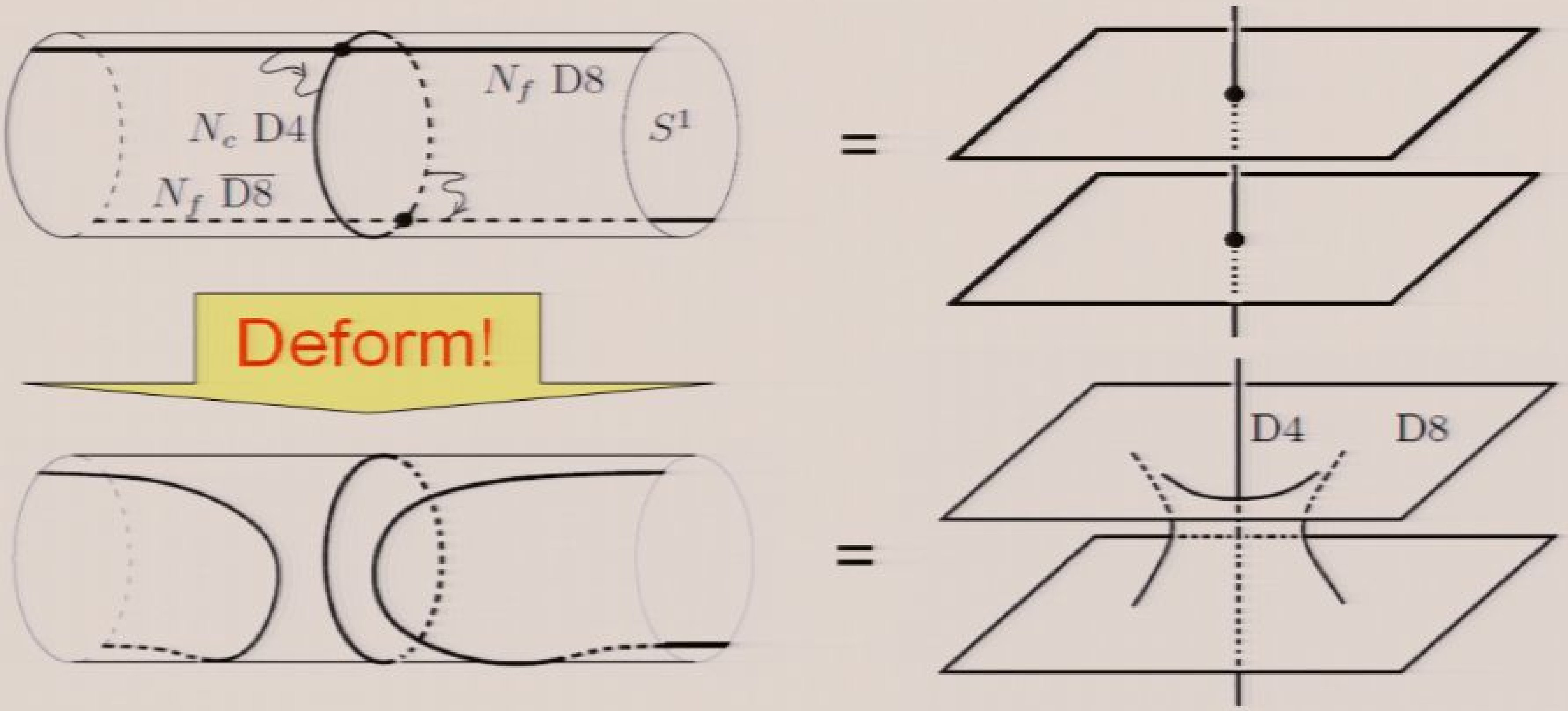


Introduction of different D-brane charge on D8

Charge conservation forces D8 and D8bar to connect

→ To achieve this, we introduce a D4'-brane bound in the D8

Explicit chiral sym. breaking?



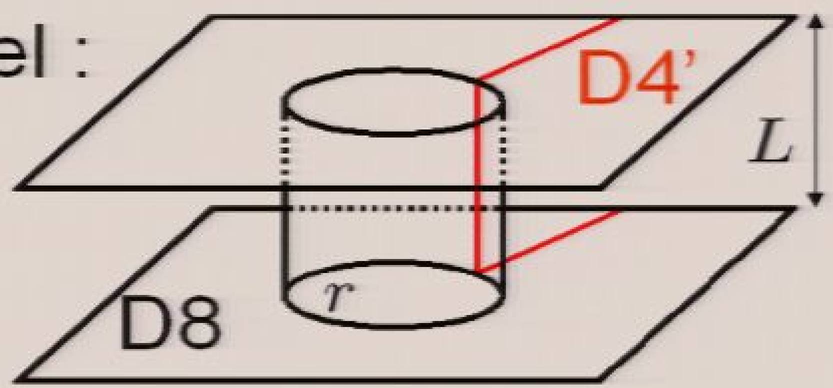
- D8s are connected not by the background geometry
- Quarks are expected to be massive

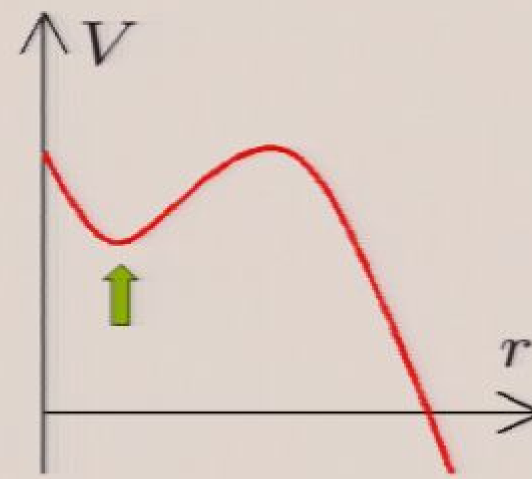
Throat D8 configuration

There is a "throat" D8-D4' bound state configuration with throat radius l_s^2/L : "BIon"

Callan-Maldacena
Constable-Myers-Tafjord

Toy model :





$$V = T_{D8}(-r^5 + Lr^4) + T_{D4}(-r + L)$$

Quark mass ~ [throat radius] × [string tension] ~ $1/L$

Summary of our idea

1. To connect D8 and D8bar in flat space, we introduce another D4' charge on D8 D8bar
2. The D4' is a YM instanton in 4-sphere.

We re-analyze SS :
fluctuation analysis of the connected D8
in the D4 near horizon geometry,
with the instanton background

→ Hadron spectra/interaction with massive pion

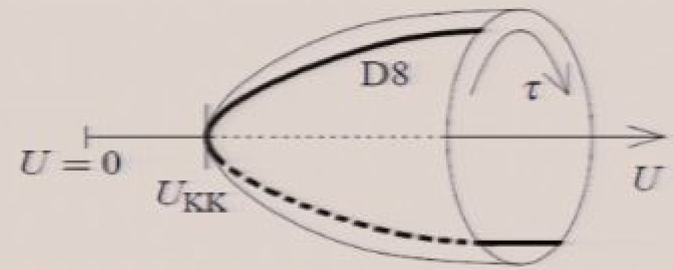
3. Computation of pion mass

"Observed value of pion mass reproduced"

Instanton background

We work with probe D8 action:

$$T_{D8}(2\pi\alpha')^2 \int d^9\sigma e^{-\phi}\sqrt{-\det g}\frac{1}{2}\mathrm{Tr}F_{MN}F^{MN}$$



Induced metric from D4 geometry:

$$ds^2_{D8} = g_{MN}d\sigma^M d\sigma^N = \left(\frac{U_z}{R}\right)^{3/2} dx_4^2 + \frac{4}{9}\left(\frac{R}{U_z}\right)^{3/2}\frac{U_{KK}}{U_z}dz^2 + \left(\frac{R}{U_z}\right)^{3/2} U_z^2 d\Omega_4^2.$$

Instanton is a classical solution in angular 4-sphere:

$$A_\mu = 0, \quad A_z = 0, \quad A_i = A_i(\theta^j)$$

SS:

KK modes from A_μ $\longrightarrow$ vector mesons
KK modes from A_z $\longrightarrow$ pion

We do the fluctuation analysis in the b.g. instanton

S^4 KK reduction

Action in components : $S_{D8} = \tilde{T}(2\pi\alpha')^2 \int d^4x \mathscr{L}$

$$\mathscr{L} = \int dz \frac{d\Omega_4}{V_4} 2\mathrm{Tr}\left\{ \frac{R^3}{4U_z} \eta^{\mu\nu}\eta^{\rho\sigma} F_{\mu\rho}F_{\nu\sigma} + \frac{9}{8}\frac{U_z^3}{U_{\mathrm{KK}}} \eta^{\mu\nu} F_{\mu z}F_{\nu z} \right.$$

$$\left. + \frac{1}{2}\eta^{\mu\nu} h^{ij} D_i A_\mu D_j A_\nu + \frac{9}{8}\frac{U_z^4}{R^3 U_{\mathrm{KK}}} h^{ij} D_i A_z D_j A_z + (\text{ terms with } A_i) \right\}$$

Pion in A_z obtains mass from $[A_i^{\mathrm{inst}}, A_z]^2$

Decompose: $A_\mu(x,z,X) = \tilde{A}_\mu(x,z)\zeta(\rho), \quad A_z(x,z,X) = \tilde{A}_z(x,z)\zeta(\rho)$

Eigen eq: $-\partial_\rho(\rho^3 4(\rho^2+1)^{-2}\partial_\rho\zeta) + 8L\frac{\rho^5}{(\mu^2+\rho^2)^2}\zeta = \rho^3 L^2 \varepsilon^2 \zeta$

Then we obtain mass terms,

$$\int dz \left\{ \frac{R^3}{4U_z}\eta^{\mu\nu}\eta^{\rho\sigma}\tilde{F}^a_{\mu\rho}\tilde{F}^a_{\nu\sigma} + \frac{9}{8}\frac{U_z^3}{U_{\mathrm{KK}}}\eta^{\mu\nu}\tilde{F}^a_{\mu z}\tilde{F}^a_{\nu z} + \frac{1}{2}\varepsilon^2\eta^{\mu\nu}\tilde{A}^a_\mu\tilde{A}^a_\nu + \frac{9}{8}\frac{U_z^4}{R^3 U_{\mathrm{KK}}}\varepsilon^2\tilde{A}^a_z\tilde{A}^a_z \right.$$

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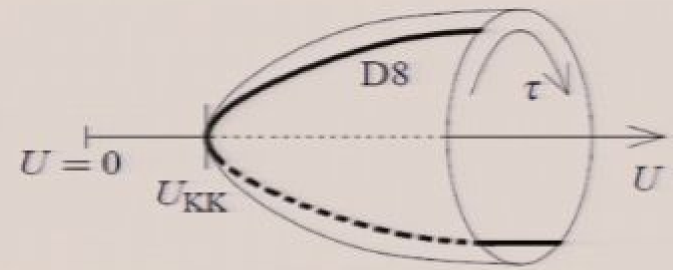

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Radial KK reduction

Vector mesons come from decomposition

$$\tilde{A}_\mu(x,z) = \sum_{m\geq 1} \tilde{A}_\mu^{(m)}(x)\,\psi_m(z)$$

for which mass eigen equation is

$$-\partial_z(K\partial_z\psi_m) + \frac{4}{9}\varepsilon^2 U_{\mathrm{KK}}^{-2}\psi_m = \lambda_m U_{\mathrm{KK}}^{-2} K^{-1/3}\psi_m \quad \left(K \equiv \left(\frac{U_z}{U_{\mathrm{KK}}}\right)^3\right)$$

Pion $\tilde{A}_z^{(0)} \equiv \varphi$ is included in the decomposition

$$\tilde{A}_z(x,z) = \sum_{m\geq 0} \tilde{A}_z^{(m)}(x)\,\phi_m(z) \qquad \phi_m \equiv \partial_z\psi_m \quad (m\geq 1), \qquad \phi_0 \propto \frac{1}{K}$$

We need a field redefinition to absorb cross terms

$$\tilde{B}_\mu^{a,(m)} \equiv \tilde{A}_\mu^{a,(m)} - \frac{U_{\mathrm{KK}}^2}{\lambda_m}\sum_{n\geq 1} K_{mn}\partial_\mu \tilde{A}_z^{a,(n)} \qquad K_{mn} \equiv \int dz K\phi_m\phi_n$$

Pion/vector meson spectrum

Final lagrangian: $K'_{mn} = K_{mn} - \sum_{p\geq 1} K_{mp}\frac{U_{KK}^2}{\lambda_p}K_{pn}$ $\quad M_{mn} \equiv \int dz K^{4/3}\phi_m\phi_n$

$$\mathscr{L} = -\frac{9U_{KK}^2}{8}\left[K_{00}(\partial_\mu\varphi^a)^2 + \frac{4}{9}\varepsilon^2 M_{KK}^2 M_{00}(\varphi^a)^2\right] - \frac{R^3}{U_{KK}}\sum_{m\geq 1}\left[\frac{1}{4}(\tilde{F}_{\mu\nu}^{a,(m)})^2 + \frac{1}{2}\lambda_m M_{KK}^2(\tilde{B}_\mu^{a,(m)})^2\right]$$
$$-\frac{9U_{KK}^2}{8}\sum_{m,n\geq 1}\left[K'_{mn}\partial_\mu\tilde{A}_z^{a,(m)}\partial^\mu\tilde{A}_z^{a,(n)} + \frac{4}{9}\varepsilon^2 M_{KK}^2 M_{mn}\tilde{A}_z^{a,(m)}\tilde{A}_z^{a,(n)}\right] - \varepsilon^2 U_{KK}^2 M_{KK}^2\sum_{m\geq 1} M_{0n}\varphi^a\tilde{A}_z^{a,(n)}$$

μ^{-1}		0	0.02	0.05	1/13.0	0.1	0.2	1.0
ε		0	0.0488	0.120	0.180	0.230	0.423	1.41
$m_{\pi^\pm,\pi^0}$	(140,135)	0	36.4	88.7	132	167	285	624
m_ρ	(776)	(776)	(776)	(776)	(776)	(776)	(776)	(776)
m_{a_1}	(1230)	1189	1188	1186	1183	1179	1162	1046
$m_{\rho'}$	(1465)	1607	1607	1603	1596	1589	1550	1308

Meson spectrum of SS is not much modified.

4. Corresponding chiral perturbation

"Four Fermi? Higher dimensional?"

Ingredients of chiral perturbation

Chiral symmetry is asymptotic gauge symmetries:

$$g_\pm \equiv \lim_{z\to\pm\infty} g(x,z,\theta) \qquad (g_+, g_-) \in SU(N_f)_L \times SU(N_f)_R$$

Building blocks of chiral lagrangian:

$$U(x,\theta) \equiv \mathrm{P}\exp\left\{ -\int_{-\infty}^{\infty} dz A_z(x,z,\theta) \right\} \qquad U \to g_+ U g_-^{-1}$$

$$A_i^\pm(\theta) \equiv A_i^{\mathrm{inst}}(x, z=\pm\infty, \theta) = A_i^{\mathrm{inst}}(\theta) \qquad \begin{array}{l} A_i^+ \to g_+ A_i^+ g_+^{-1} \\ A_i^- \to g_- A_i^- g_-^{-1} \end{array}$$

SS: D8 action in terms of U = Chiral lagrangian

Instanton = External source in chiral perturbation

"U" and 9 dim. YM fields

We move to $A_z = 0$ gauge.

For $z \to \pm\infty$,

$$A_\mu(x,z,\theta) \to \xi_\pm(x,\theta)\partial_\mu \xi_\pm^{-1}(x,\theta)$$

$$A_i(x,z,\theta) \to \xi_\pm(x,\theta)(A_i^\pm(\theta) + \partial_i)\xi_\pm^{-1}(x,\theta)$$

$$\xi_\pm^{-1}(x,\theta) \equiv \mathrm{P}\exp\left\{-\int_0^{\pm\infty} dz' A_z(x,z',\theta)\right\}$$

$$\xi_+ \to h(x,\theta)\xi_+ g_+^{-1}$$

$$\xi_- \to h(x,\theta)\xi_- g_-^{-1}$$

We further use residual gauge sym $h(x,\theta) \equiv g(x, z=0, \theta)$ to put $\xi_- = 1$ and thus $U = \xi_+\xi_-^{-1} \to \xi_+$

So the expansion of the gauge fields is

$$A_\mu(x,z,\theta) = U^{-1}(x,\theta)\partial_\mu U(x,\theta)\psi_+(z) + \text{higher modes},$$

$$A_i(x,z,\theta) = U^{-1}(x,\theta)(A_i^+(\theta) + \partial_i)U(x,\theta)\,\widetilde{\psi}_+(z) + A_i^-(\theta)\,\widetilde{\psi}_-(z) + \text{higher modes}$$

$$\psi_\pm(z) = \frac{1}{2}\left(1 \pm \frac{C_{-1}(z)}{C_{-1}(\infty)}\right), \quad \widetilde{\psi}_\pm(z) = \frac{1}{2}\left(1 \pm \frac{C_{-4/3}(z)}{C_{-4/3}(\infty)}\right), \quad C_n(z) = \int_0^z dz K^n$$

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Chiral Lagrangian

Substituting the expression gives 9-dim. action

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Expanding $U(x,\theta) = \exp\left(2i\pi(x)/f_\pi + \text{higher } S^4 \text{ KK modes}\right)$
and defining $U(x) = \exp(2i\pi(x)/f_\pi)$, we obtain 4-d. action

$$\mathscr{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(U^{-1}\partial_\mu U)^2 + C\int \frac{d\Omega_4}{V_4}\mathrm{Tr}\left(U^{-1}A_i^+ U A_i^-\right) + \mathscr{O}(\mu^{-4})$$

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Possible perturbation in QCD?

Usual pion mass term in chiral lagrangian is
$\mathrm{tr}(UM^\dagger + MU^\dagger)$ while ours is $\mathrm{tr}(U^{-1}A_i^+ U A_i^-)$

From the chiral charge of the external fields

$$A_i^+ \to g_+ A_i^+ g_+^{-1} \qquad A_i^- \to g_- A_i^- g_-^{-1}$$

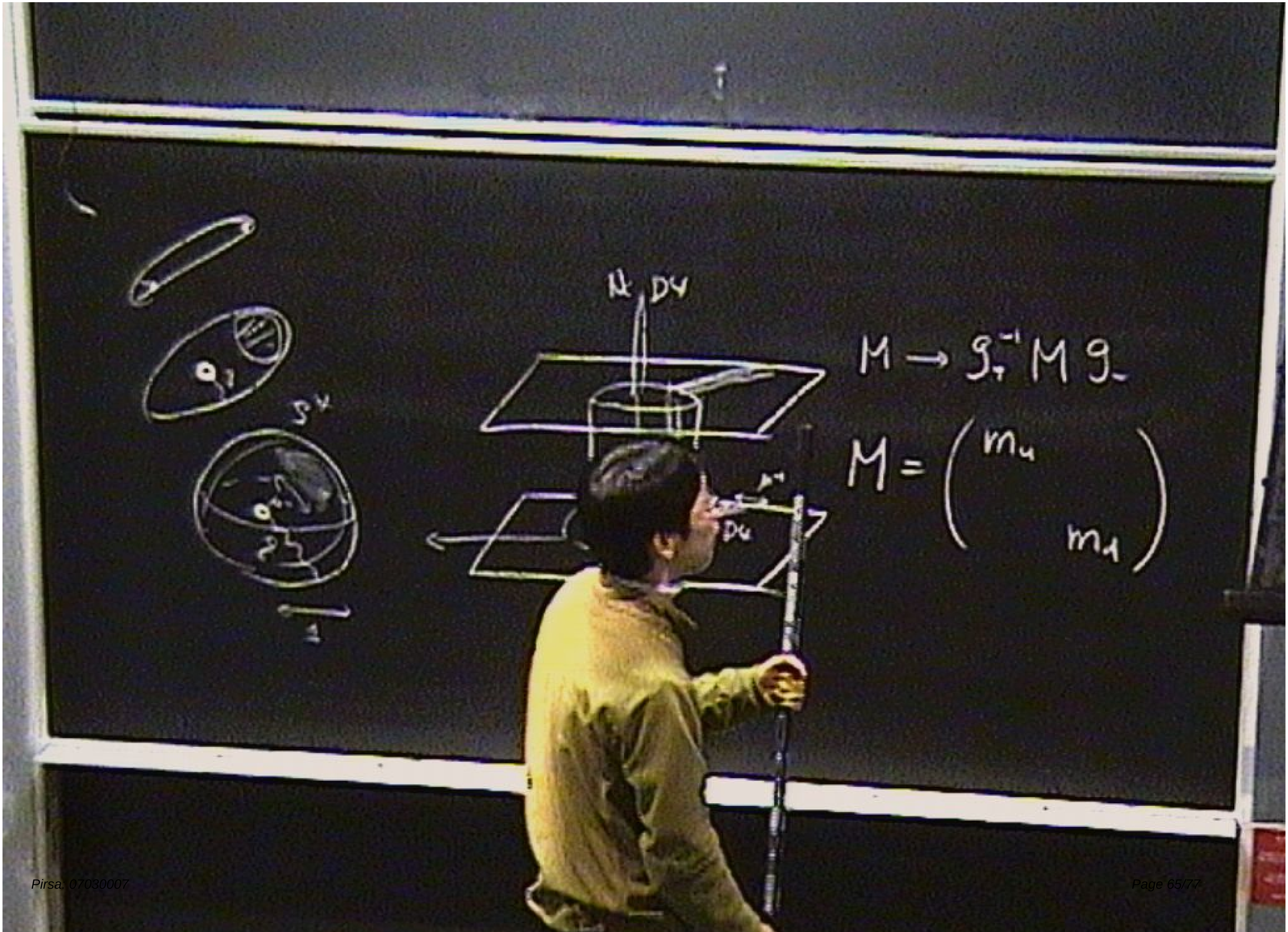
possible perturbations are :

(i) Four-Fermi term?

$$\mathscr{L} = \mathscr{L}_{QCD} + \boxed{G_{bq}^{ap} \overline{q}_{La} q_R{}^q \overline{q}_{Rp} q_L{}^b} + h.c.$$

(ii) Higher orders in quark mass term?

$A_i^+ \sim MM^\dagger, \quad A_i^- \sim M^\dagger M$ cf) Stern phase


M → g₊⁻¹ M g₋
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5. Summary and discussions

"There can be no question, my dear Watson"

Summary

- Sakai-Sugimoto + additional D4' charge
  → Local deformation of the D8 shape
  = explicit chiral symmetry breaking
  → massive pion
- Arbitrarily small pion mass : as a perturbation
- Meson spectrum not altered much
- Throat picture suggests quark mass term
- Chiral lagrangian shows four-Fermi term or higher order in quark mass

Discussions

- Pion mass for the axial U(1) part?
  $\rightarrow$ Non-commutative instanton on 4 sphere
- μ -dependence of the throat radius :
  One can see that for large μ the string length in the throat is shorter $\rightarrow$ consistent
- Quark mass from Tachyons on D8 D8bar?
- θ dependence $\rightarrow$ ∞ # of external source
- Derivation of nuclear force?

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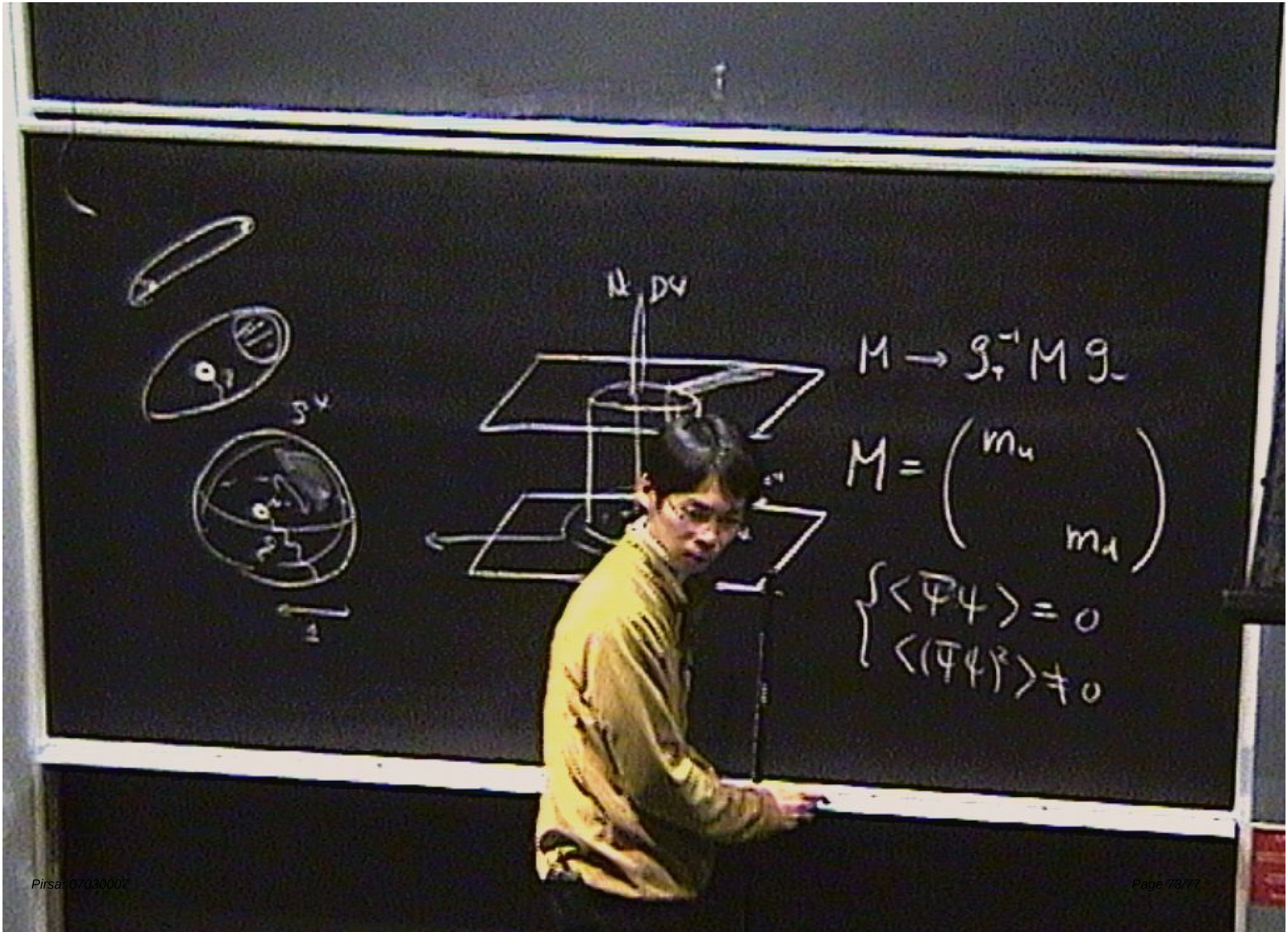
possible perturbations are :

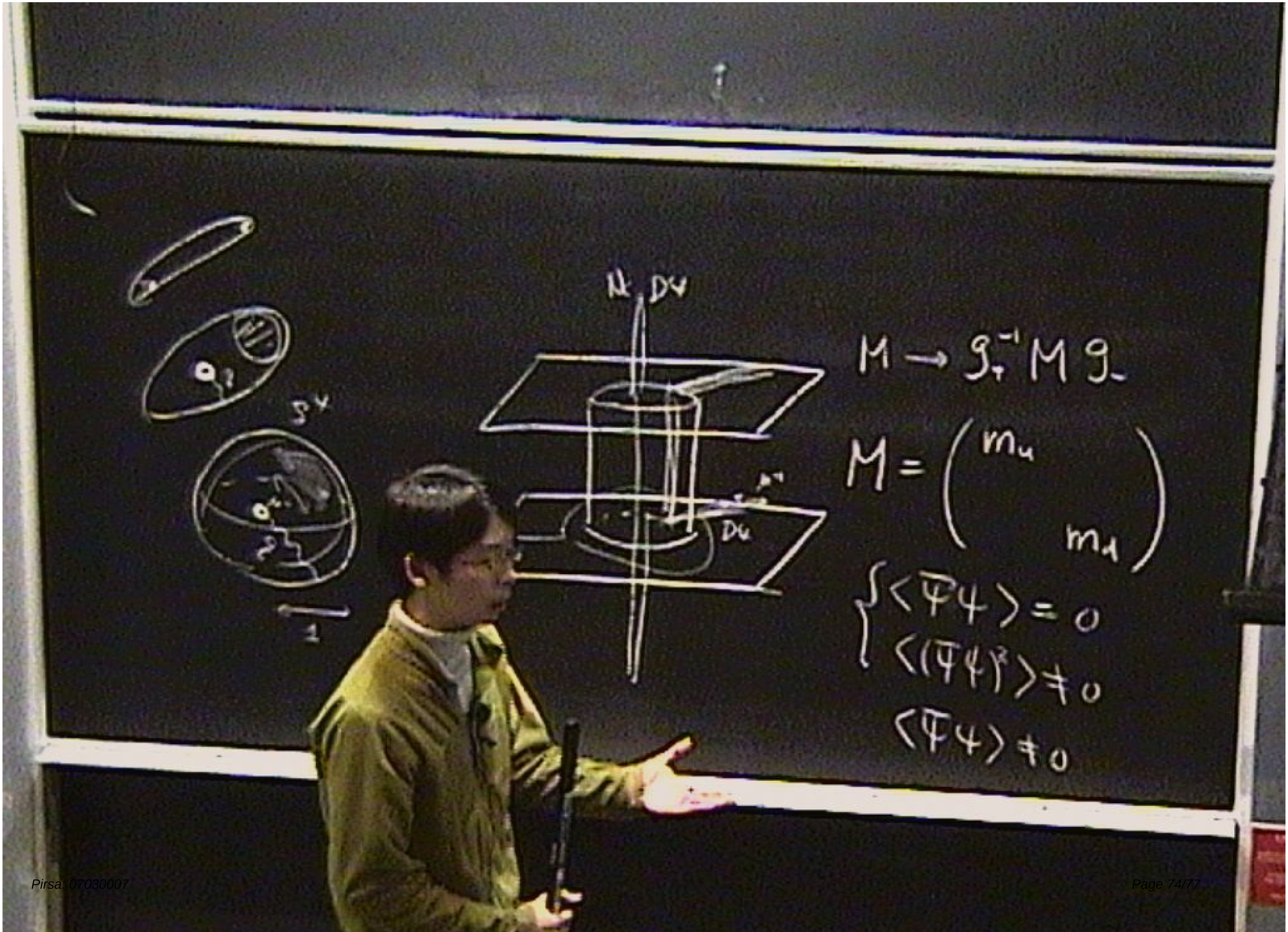
(i) Four-Fermi term?

$$\mathscr{L} = \mathscr{L}_{QCD} + \boxed{G_{bq}^{ap} \bar{q}_{La} q_R{}^q \bar{q}_{Rp} q_L{}^b} + h.c.$$

(ii) Higher orders in quark mass term?

$$A_i^+ \sim MM^\dagger, \quad A_i^- \sim M^\dagger M \qquad \text{cf) Stern phase}$$


S4
M → g+^-1 M g-
M = (mu md)
< Ψ̄ψ > = 0
< (Ψ̄ψ)^2 > ≠ 0



N DW
S⁴
DW
M → g₊⁻¹ M g₋
M = (mᵤ mₐ)
⟨Ψ̄ψ⟩ = 0
⟨(Ψ̄ψ)²⟩ ≠ 0
⟨Ψ̄ψ⟩ ≠ 0

Discussions

- Pion mass for the axial U(1) part?
  $\rightarrow$ Non-commutative instanton on 4 sphere
- μ -dependence of the throat radius :
  One can see that for large μ the string length in the throat is shorter $\rightarrow$ consistent
- Quark mass from Tachyons on D8 D8bar?
- θ dependence $\rightarrow \infty$ # of external source
- Derivation of nuclear force?

"U" and 9 dim. YM fields

We move to $A_z = 0$ gauge.

For $z \to \pm\infty$,
$$A_\mu(x,z,\theta) \to \xi_\pm(x,\theta)\partial_\mu \xi_\pm^{-1}(x,\theta)$$
$$A_i(x,z,\theta) \to \xi_\pm(x,\theta)(A_i^\pm(\theta) + \partial_i)\xi_\pm^{-1}(x,\theta)$$

$$\xi_\pm^{-1}(x,\theta) \equiv \mathrm{P}\exp\left\{-\int_0^{\pm\infty} dz' A_z(x,z',\theta)\right\}$$

$$\xi_+ \to h(x,\theta)\xi_+ g_+^{-1}$$
$$\xi_- \to h(x,\theta)\xi_- g_-^{-1}$$

We further use residual gauge sym $h(x,\theta) \equiv g(x,z=0,\theta)$ to put $\xi_- = 1$ and thus $U = \xi_+\xi_-^{-1} \to \xi_+$

So the expansion of the gauge fields is

$$A_\mu(x,z,\theta) = U^{-1}(x,\theta)\partial_\mu U(x,\theta)\psi_+(z) + \text{higher modes},$$
$$A_i(x,z,\theta) = U^{-1}(x,\theta)(A_i^+(\theta) + \partial_i)U(x,\theta)\widetilde{\psi}_+(z) + A_i^-(\theta)\widetilde{\psi}_-(z) + \text{higher modes}$$
$$\psi_\pm(z) = \frac{1}{2}\left(1 \pm \frac{C_{-1}(z)}{C_{-1}(\infty)}\right), \quad \widetilde{\psi}_\pm(z) = \frac{1}{2}\left(1 \pm \frac{C_{-4/3}(z)}{C_{-4/3}(\infty)}\right), \quad C_n(z) = \int_0^z dz K^n$$

Why are pions massless in SS?

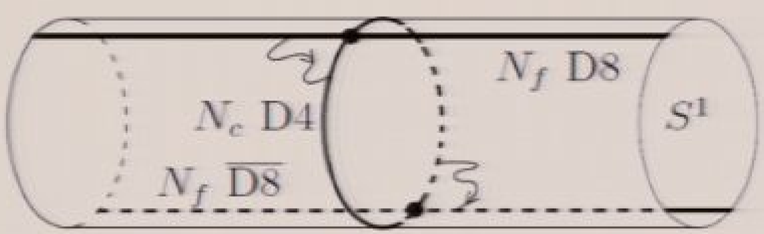
Answer : Quarks are massless in the D4D8.

➡ No explicit chiral symmetry breaking

➡ Pions are exactly NG bosons

Then, why quarks are massless in SS?

Because: It's difficult to separate D8 from D4



	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D4	○	○	○	○					
D8	○	○	○		○	○	○	○	○
$\overline{\text{D8}}$	○	○	○		○	○	○	○	○

Co-dimension = 6 $\rightarrow$ Chiral fermions