Title: D-brane instanton effects in 4D string vacua and their phenomenological applications

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Abstract: We discuss D-brane instantons in four-dimensional string compactifications with special emphasis on Eucliden D2-branes in Type IIA orientifolds with spacetime filling D6-branes. These can induce superpotential couplings among the open string fields which are forbidden at the perturbative level since they violate some of the global U(1) symmetries generically present in string theory.
Phenomenologcially important couplings of this type include Majorana mass terms for right-handed neutrinos or mu-terms in the Higss sector of the MSSM. If realized in concrete constructions, the exponential suppression of such non-perturbative terms may 'naturally' generate the observed hierarchies characteristic of these couplings.
After discussing the general philosophy, we derive the prescription for the CFT computation of such non-perturbative superpotential couplings and exemplify the computation of Majorana mass terms in toroidal intersecting brane worlds. If time permits, we also comment on D2-instanton effetcs potentially destabilising the vacuum or modifying the D-term supersymmetry conditions.

D-brane Instanton Corrections in 4D String Vacua

and their phenomenological applications

based on:

R. Blumenhagen, M. Cvetič, T.W., hep-th/0609191

M. Cvetič, R. Richter, T.W., hep-th/0703028

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Motivation

String theory exhibits a huge number of perturbatively (meta-)stable 4D vacua
determined by zero mode approximation/
effective $\mathcal{N} = 1$ supergravity $S_{eff} \Leftrightarrow W, K, f$
SUSY vacuum given by: $DW = 0$ (+D-terms)

How do genuine quantum effects modify the landscape topography?
K loop-corrected, but
W not renormalized perturbatively
$\rightsquigarrow$ non-perturbative corrections crucial

Example:
Closed string sector in Type IIB orientifolds: dependence of W on Kähler moduli only by Euclidean D3-branes
decisive for IIB moduli fixing industry

Motivation

Non-perturbative effects in gauge sector of string vacua?

• relevant for stability of model in the first place
(e.g. Π-stability for B-type branes)

• potential to break perturbative gauge or global symmetries

• may generate perturbatively absent couplings, exponentially suppressed w.r.t. string scale
$\rightsquigarrow$ come at genuinely stringy hierarchical scale
$\leftrightarrow$ relation to peculiar scale of certain MSSM couplings?
$\rightsquigarrow$ Majorana masses for right-handed neutrinos of order $10^8 GeV < M_M < 10^{15} GeV$
$\rightsquigarrow$ hierarchically small μ-terms of order $\mathcal{O}(M_Z)$

Motivation

Various non-perturbative effects in gauge sector studied in detail in literature, e.g.

worldsheet instantons in heterotic compactifications

[Dine,Seiberg,Wenn,Witten'86],[Distler,Greene'88],[Witten'99],
[Buchbinder,Donagi,Ovrut'02],[Beasley,Witten'03,'05]

worldsheet instantons in IIA brane models

[Kachru et al.'00],[Aganacic,Vafa'00]

M2/M5-brane effects in heterotic M-theory

[Becker,Becker,Strominger'95],[Harvey, Moore'99]

D3-D(-1) system in IIB

[Green,Gutperle'97],[Billo et al.'02],[Green,Stahn'03]

Motivation

This talk:
Effects of wrapped Euclidean D-branes in Type IIA Intersecting Brane Vacua
special focus on induced superpotential terms involving charged matter fields Φ_i

$$W_{np} \simeq \prod_{i=1}^{M} \Phi_i \, e^{-S_{inst.}}$$

violating global perturbative abelian symmetries

recent related work:

[Haack,Krefl,Lust,VanProeyen,Zagermann, hep-th/0609211]

[Ibanez,Uranga, hep-th/0609213]

[Florea,Kachru,McGreevy,Saulina, hep-th/0610003]

[Buican,Malyshev,Morrison,Wijnholt,Verlinde, hep-th/0610007]

Plan of the talk

1. Motivation
2. Reminder: Intersecting Brane Worlds and anomalous U(1)
3. E2-brane instanton generated superpotentials:
   - Heuristics
   - Zero mode structure
   - CFT instanton calculus
4. Applications:
   - Majorana masses: example + CFT on $T^6/\mathbb{Z}_2 \times \mathbb{Z}'_2$
   - Outlook: Vacuum destabilisation or open string moduli fixing?
5. Conclusions

Briefing on Type IIA model building

Compactify Type IIA string theory on
$\mathcal{M}^{(10)} = \mathcal{M}^{(4)} \times CY_3$ quotiented by $\Omega\,(-1)^{F_L}\,\overline{\sigma}$

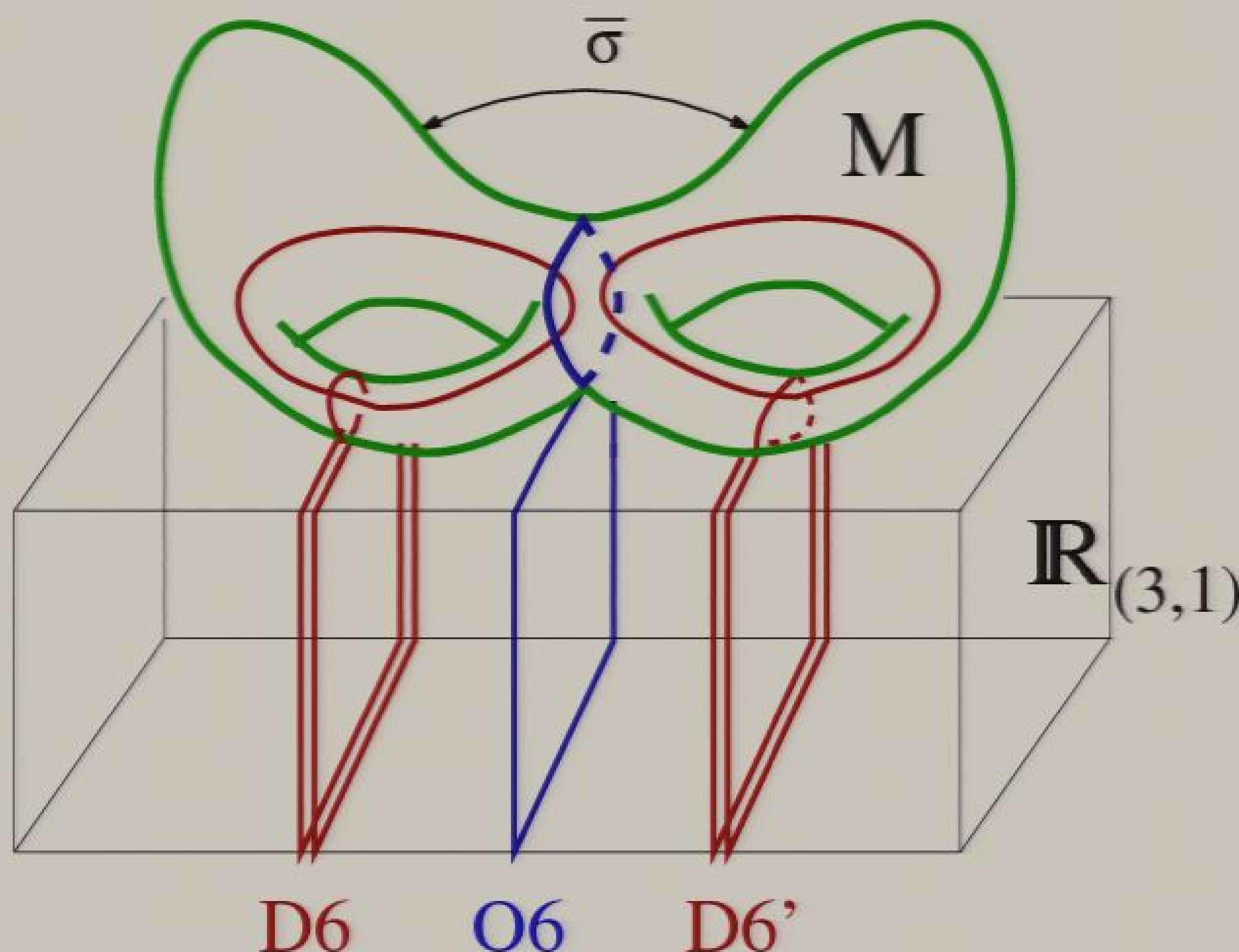
Ω: worldsheet parity, F_L: left-handed fermion number

$\overline{\sigma}$: anti-holomorphic involution on CY_3

$\rightsquigarrow$ orientifold O6-plane $\mathcal{M}^{(4)} \times \Pi_{O6}$ carries RR and NS charge
$\rightsquigarrow$ introduction of D6-branes for charge cancellation
N_a D6-branes wrap 3-cycles Π_a on CY_3 and fill $\mathcal{M}^{(4)}$
orientifold action $\rightsquigarrow$ include also image branes Π'_a

TAD: $\sum_a N_a(\Pi_a + \Pi'_a) - 4\Pi_{O6} = [0]$
$D6$-branes should wrap sLags preserving same SUSY as $O6$

chiral matter at non-trivial intersection of internal 3-cycles
chiral number of generations in $(\overline{N}_a, N_b)$: $\Pi_a \circ \Pi_b$
(top. intersection number)


σ̄
M
$\mathbb{R}_{(3,1)}$
D6
O6
D6'

Anomalous $U(1)$ and GS-mechanism

Specific signature of IBW:
gauge group $\prod_a U(N_a) = \prod_a SU(N_a) \times U(1)_a$
in general: $U(1)_a$ is anomalous
anomaly cancelled by 4D Green-Schwarz mechanism,
mediated by Chern-Simons coupling

$$S_{CS} = \sum_a N_a \, \mu_6 \int_{\mathbb{R}^{1,3} \times \Pi_a} e^{trF_a} \sum_p C_{2p+1}$$

abelian gauge potential becomes massive and anomalous $U(1)_a$ survives as a global perturbative symmetry

Only specific linear combinations of $U(1)$s are massless
$\rightsquigarrow$ in realistic models: only $U(1)_Y$ massless, but:
additional perturbative $U(1)$ forbid some desirable matter couplings e.g. right-handed neutrino masses or μ-terms

Anomalous $U(1)$ and GS-mechanism

CS-coupling induces gauging of global axionic shift symmetry:

under $U(1)_a$ gauge transformation the RR-form C_3, KK-reduced on 3-cycle $\widetilde{\Pi}$, transforms as

$$\begin{aligned} A_a &\longrightarrow A_a + d\Lambda_a \\ \int_{\widetilde{\Pi}} C^{(3)} &\longrightarrow \int_{\widetilde{\Pi}} C^{(3)} + Q_a(\widetilde{\Pi})\Lambda_a \end{aligned}$$

with $Q_a(\widetilde{\Pi}) = \frac{\ell_s^3}{2\pi} N_a \, \widetilde{\Pi} \circ (\Pi_a - \Pi_{a'})$

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Instantons-Heuristics

Strategy: Probe for non-pert. terms by computing suitable amplitudes in instanton background
Instanton background: presence of Euclidean D_p-brane wrapping internal $(p+1)$-cycle
On $X = CY_3$: $b_1(X) = 0 = b_5(X)$
$\rightsquigarrow$ relevant objects are Euclidean $D2$-branes

Rules:

- Instanton sector corresponds to local minimum of (full) string action
  $\rightsquigarrow$ $E2$-brane volume minimizing on internal sLag Ξ
- Integrate over bosonic and fermionic zero modes localized on $E2$
  $\rightsquigarrow$ All fermionic zero modes have to appear in some vertex operator

Instantons-Heuristics

Consequence:
F-terms require $E2$-sector half-BPS w.r.t. $D6$-branes/$O6$-plane,
if Ξ anti-SUSY w.r.t. $\Pi_a \rightarrow 1/2$ SUSY Q_α broken due to localisation in 4D
$\rightsquigarrow$ integrate over 2 Goldstinos θ_α
$\rightsquigarrow$ holomorphic (chiral) superpotential: instanton sector

D-terms require $E2$-sector to break all 4 supersymmetries:
Ξ on sLag non-SUSY w.r.t. background
$\rightsquigarrow$ integrate over 4 fermionic zero modes θ_α, $\overline{\theta}_{\dot{\alpha}}$

This talk:
focus on single-instanton contribs. to superpotential W

Instantons-Heuristics

$$W_{np} \propto e^{-S_{E2}} = \exp\left[\frac{2\pi}{\ell_s^3}\left(-\frac{1}{g_s}\int_\Xi \Re(\Omega_3) + i\int_\Xi C_3\right)\right]$$

exponential not gauge invariant under $U(1)_a$!

$e^{-S_{E2}} \to e^{i\,Q_a(E2)\,\Lambda_a}\; e^{-S_{E2}}$: $Q_a(E2) = N_a\; \Xi \circ (\Pi_a - \Pi'_a)$

Consequence:

If $Q_a(E2) \neq 0$ for some a, no terms $W = e^{-S_{E2}}$ possible but:

$$W = \prod_i \Phi_i\, e^{-S_{E2}} \quad \text{with} \quad \sum_i Q(\Phi_i) + Q_a(E2) = 0 \;\; \forall a$$

non-perturbative breakdown of global $U(1)$ symmetry possible

How can we understand this selection rule in terms of fermionic zero modes?

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Zero mode structure-Details

Distinguish 2 types of fermionic zero modes:

1) zero modes uncharged under $U(1)_a$:

- Goldstinos θ_α
- If cycle Ξ non-rigid:
  $b_1(\Xi)$ fermionic zero modes from open strings starting and ending on $E2 \leftrightarrow E2$-moduli
- additional zero modes also at intersection of Ξ and Ξ' counted by $\frac{1}{2}([\Xi \cap \Xi']^{\pm} + [\Xi \cap \Pi_{O6}]^{\pm})$

need to absorb latter two types: higher fermionic or closed string dependent couplings

for W_{np} dependent only on open fields of gauge sector, they have to be absent:
Ξ has to be rigid and $[\Xi \cap \Xi']^{\pm} = 0$

Zero mode structure-Details

2) zero modes charged under $U(1)_a$:
from strings between E_2 and $D6_a$
DN-boundary conditions in 4D, mixed boundary conditions along CY_3
$\rightsquigarrow$ at chiral intersection: 1 single fermionic zero mode λ_a

zero modes	Reps.	number
$\lambda_{a,I}$	$(-1_E, \Box_a)$	$I = 1, \ldots, [\Xi \cap \Pi_a]^+$
$\overline{\lambda}_{a,I}$	$(1_E, \overline{\Box}_a)$	$I = 1, \ldots, [\Xi \cap \Pi_a]^-$
$\lambda_{a',I}$	$(-1_E, \overline{\Box}_a)$	$I = 1, \ldots, [\Xi \cap \Pi'_a]^+$
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total $U(1)_a$ charge of all zero modes:
$Q_a(E2) = N_a \, \Xi \circ (\Pi_a - \Pi'_a)$
in agreement with $e^{-S_{E2}} \rightarrow e^{i\, Q_a(E2)\, \Lambda_a} \; e^{-S_{E2}}$

Instanton calculus - Outline

Wanted: (physical) matter couplings induced by
$W_{np} \simeq \prod_{i=1}^{M} \Phi_{a_i,b_i}\, e^{-S_{E2}}$

$\Phi_{a_i,b_i} = \phi_{a_i,b_i} + \theta\psi_{a_i,b_i}$: at intersection of Π_{a_i}, Π_{b_i}
suppress Chan-Paton labels for simplicity

Compute (physical) correlator in E_2-background

$$\langle \phi_{a_1,b_1} \cdot \ldots \cdot \phi_{a_{M-2},b_{M-2}} \cdot \psi_{a_{M-1},b_{M-1}} \cdot \psi_{a_M,b_M} \rangle_{E2-\text{inst}} =$$

$$\int d^4\tilde{x}_E\, d^2\tilde{\theta} \sum_{\text{conf.}} \prod_a \left(\prod_{I=1}^{[\Xi\cap\Pi_a]^+} d\tilde{\lambda}_{a,I} \right) \left(\prod_{I=1}^{[\Xi\cap\Pi_a]^-} d\tilde{\overline{\lambda}}_{a,I} \right)$$

$$\prod_k \langle \Phi^k_{a_{k_1},b_{k_1}} \cdot \ldots \cdot \Phi^k_{a_{k_r},b_{k_r}} \rangle^{g_k}_{\prod \lambda_k}$$

Instanton calculus - Outline

Which ways of splitting the $\langle \ldots \rangle^{g_k}_{\prod \lambda_k}$ are due to W?

1) Each factor has to involve at least one $E2$-boundary
2) all λ_a and the two θ_α modes have to appear precisely once
3) Holomorphy of W: only dependence on g_s via $exp(-S_{E2})$

$\rightsquigarrow$ analyse g_s-scaling of $\langle \ldots \rangle^{g_k}_{\prod \lambda_k}$:

- each disk $E2 - D6$: $(g_s)^{-1}$
  one-loop diagram (annulus/Möbius): $(g_s)^0$
- all vertex operators carry $(g_s)^0$
  ($\simeq$ frame where all tree pert. terms at $\frac{2\pi}{g_s}$)
- proper norm. of measure in terms of
  $\tilde{x}^\mu_E = \frac{x^\mu_E}{2}\sqrt{\frac{2\pi \mathcal{V}_{E2}}{g_s}}, \qquad \tilde{\theta}^\alpha = \theta^\alpha \sqrt{\frac{2\pi \mathcal{V}_{E2}}{g_s}} \qquad \tilde{\lambda} = \lambda\sqrt{\frac{2\pi}{g_s}}$ cf.
  D3-D(-1): [Billo et al.'03]

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Instanton calculus - Outline

Consequence:
from $d^4\tilde{x}_E\, d^2\tilde{\theta}\,:\ \frac{2\pi}{g_s}\,\mathcal{V}_{E2}$

no additional powers of g_s picked up iff:

- each disk carries precisely 2 λ_a vertices
- in annulus/Möbius amplitudes: no λ_a vertices appear
- and no worldsheets of genus higher than 1 are considered

Focus on physical couplings of above type, i.e. arising at lowest order in g_s

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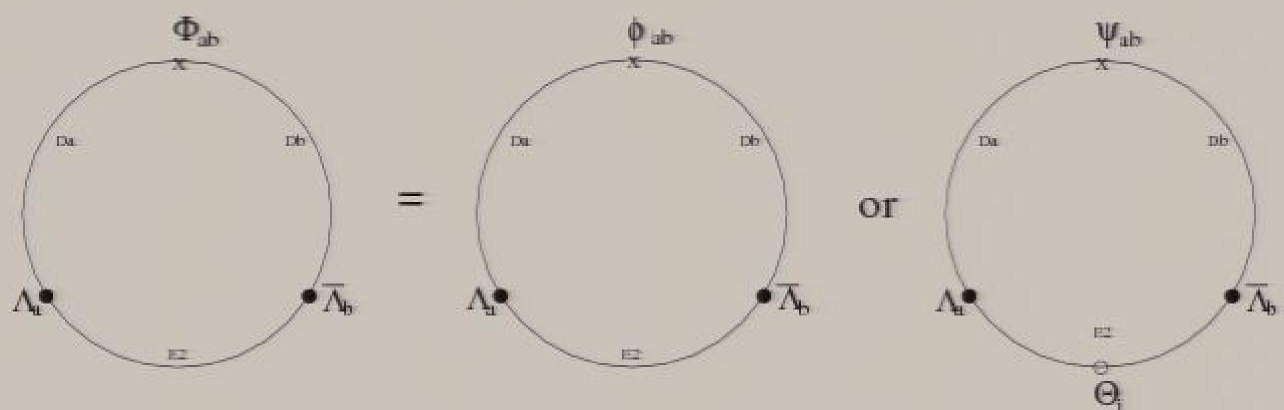
Instanton calculus - Disks

- factor off vacuum disks cf.[Polchinski'94]

$$\sum_{n=1}^{\infty} \frac{1}{n!}(\langle 1 \rangle_{disk})^n = exp(-S_{E2})$$

- appropriate insertion of θ_i vertices hand in hand with insertion of $\phi_{a_i,b_i}/\psi_{a_i,b_i}$:



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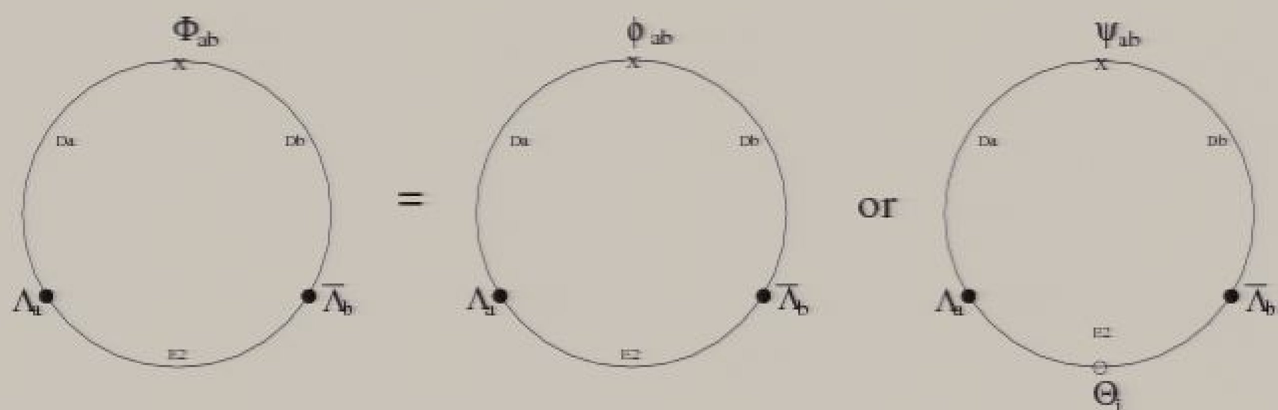
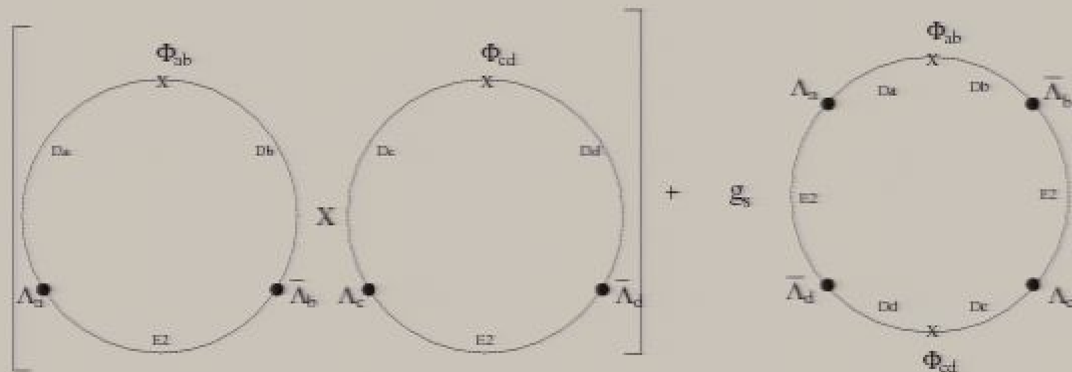


Instanton calculus - Disks

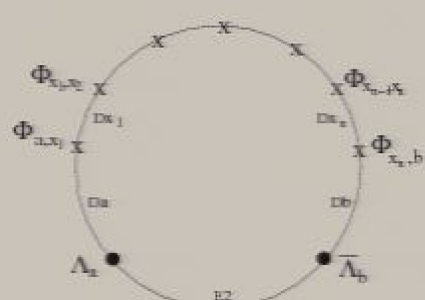
- only order g_s^0 diagrams

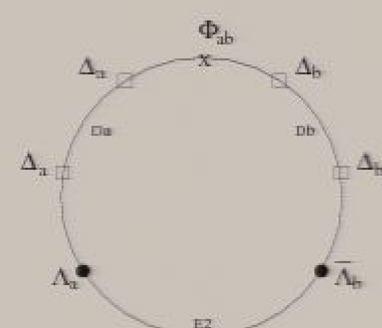


- also multi-insertion disks possible

e.g. if $D6$-brane has deformation moduli (superfields Δ_a), insertion of arbitrary number of Δ_a

$\rightsquigarrow$ overall $\exp\left(-\frac{1}{\alpha'}\mathrm{tr}(\Delta_a)\right)$-dependence on open string moduli





Instanton calculus - 1-loop amplitudes

Recall: loop-amplitudes uncharged (no λ_a-insertion)

factor off vacuum loops involving at least one $E2$ boundary and omit zero modes

$$\sum_{n=0}^{\infty} \frac{1}{n!} \left(\sum_a \left[Z'^A(E2, D6_a) + Z'^A(D6'_a, E2) \right] + Z'^M(E2, O6) \right)^n$$
$$= \exp\left(Z'_0\right),$$

$\rightsquigarrow$ regularized 1-loop determinant
related to β-function [Akerblom et al., hep-th 0612132]

Cf. worldsheet instantons in het. (0,2)-models [Witten'99]:

$$W = \frac{\mathrm{Pfaff}(\overline{\partial}_{V(-1)})}{(det' \, \overline{\partial}_{\mathcal{O}})^2 \, (det \, \overline{\partial}_{\mathcal{O}(-1)})^2} \exp(-S_{\mathrm{inst}})$$

Instanton calculus - Disks

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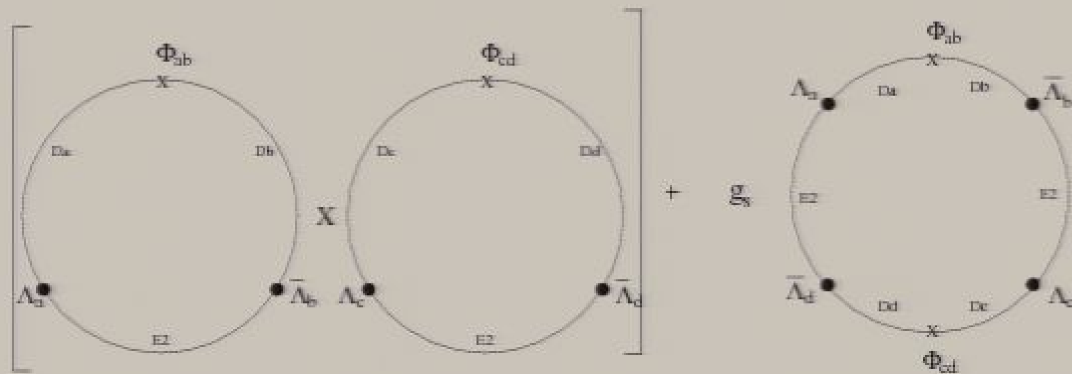

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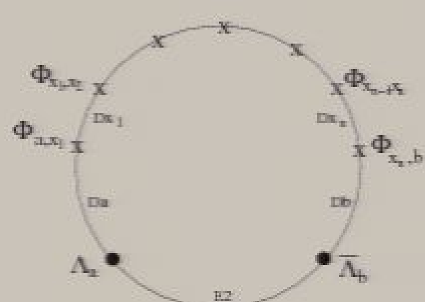

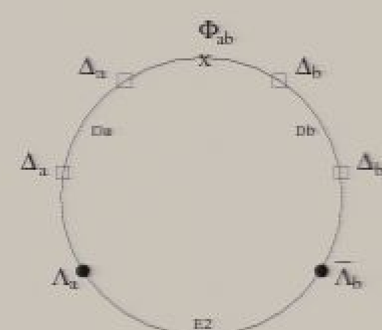

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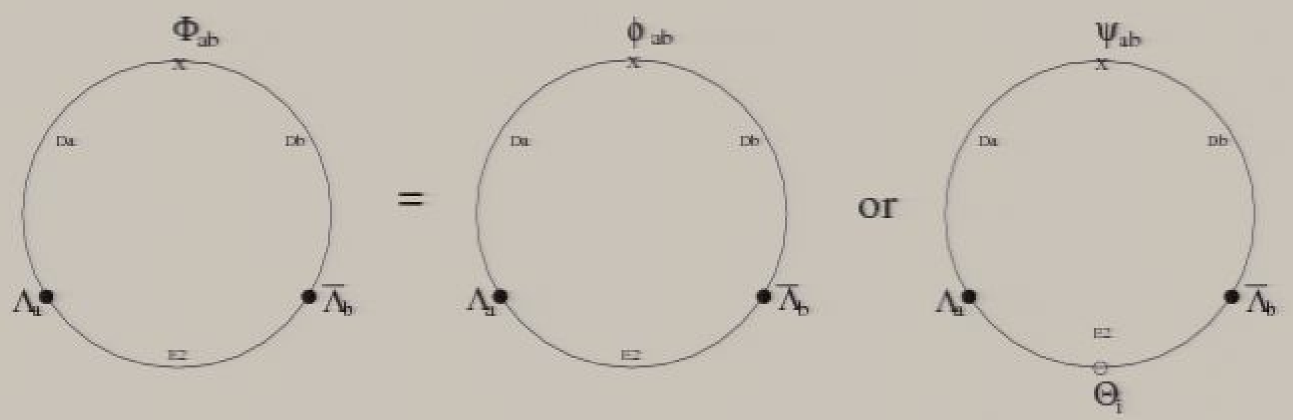

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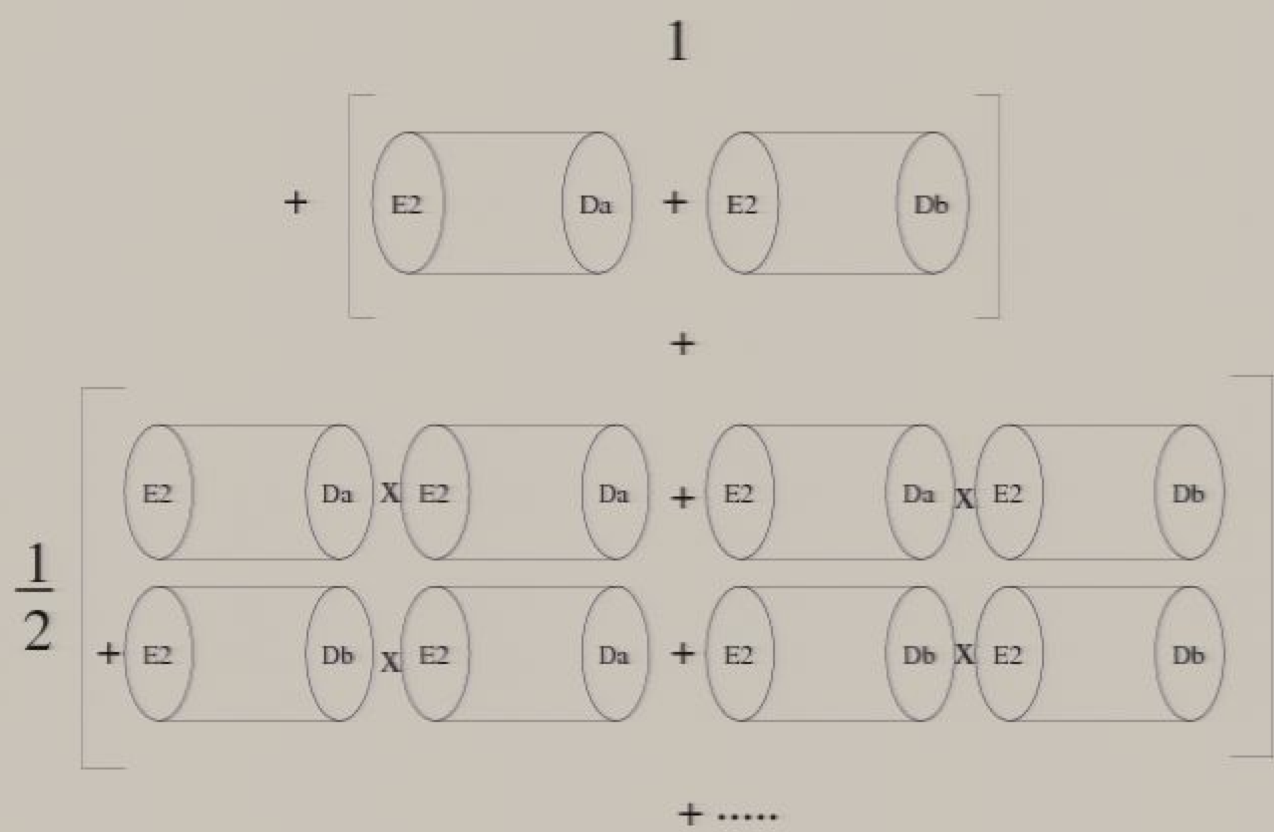
$$W = \frac{\mathrm{Pfaff}(\overline{\partial}_{V(-1)})}{(det' \, \overline{\partial}_{\mathcal{O}})^2 \, (det \, \overline{\partial}_{\mathcal{O}(-1)})^2} \exp(-S_{\text{inst}})$$

Instanton calculus - 1-loop amplitudes



Instanton calculus - 1-loop amplitudes

Recall: loop-amplitudes uncharged (no λ_a-insertion)

factor off vacuum loops involving at least one $E2$ boundary and omit zero modes

$$\sum_{n=0}^{\infty} \frac{1}{n!} \left(\sum_a \left[Z'^A(E2, D6_a) + Z'^A(D6'_a, E2) \right] + Z'^M(E2, O6) \right)^n$$
$$= \exp\left(Z'_0\right),$$

$\rightsquigarrow$ regularized 1-loop determinant
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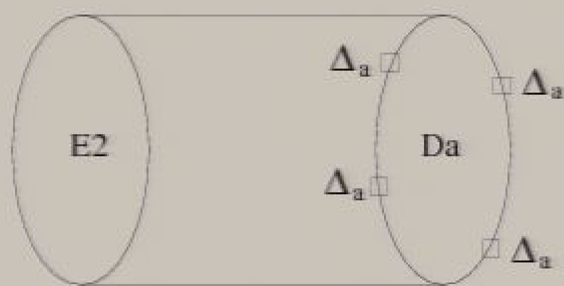
Instanton calculus - 1-loop amplitudes

also attach chains of Φ_{a_i,b_i} to 1-loop diagrams with one boundary on $E2$
In particular: moduli Δ_a

$\rightsquigarrow$ moduli dependence of 1-loop determinant



Instanton calculus - Summary

$$\langle \Phi_{a_1,b_1}(p_1)\cdot\ldots\cdot\Phi_{a_M,b_M}(p_M)\rangle_{E2-\text{inst}} =$$

$$= \frac{1}{L!}\int d^4\tilde{x}\, d^2\tilde{\theta} \sum_{\text{conf.}} \prod_a \Big(\prod_{i=1}^{[\Xi\cap\Pi_a]^+} d\widetilde{\lambda}_a^i\Big)\Big(\prod_{i=1}^{[\Xi\cap\Pi_a]^-} d\widetilde{\overline{\lambda}}_a^i\Big)$$

$$\exp(-S_{E2}) \times \exp\left(Z_0'\right)$$

$$\times \langle\widehat{\Phi}_{a_1,b_1}[\vec{x}_1]\rangle_{\lambda_{a_1},\overline{\lambda}_{b_1}}\cdot\ldots\cdot\langle\widehat{\Phi}_{a_L,b_L}[\vec{x}_L]\rangle_{\lambda_{a_L},\overline{\lambda}_{b_L}} \times$$

$$\prod_k \langle\widehat{\Phi}_{c_k,c_k}[\vec{x}_k]\rangle^{\text{loop}}_{A(E2,D6_{c_k})}$$

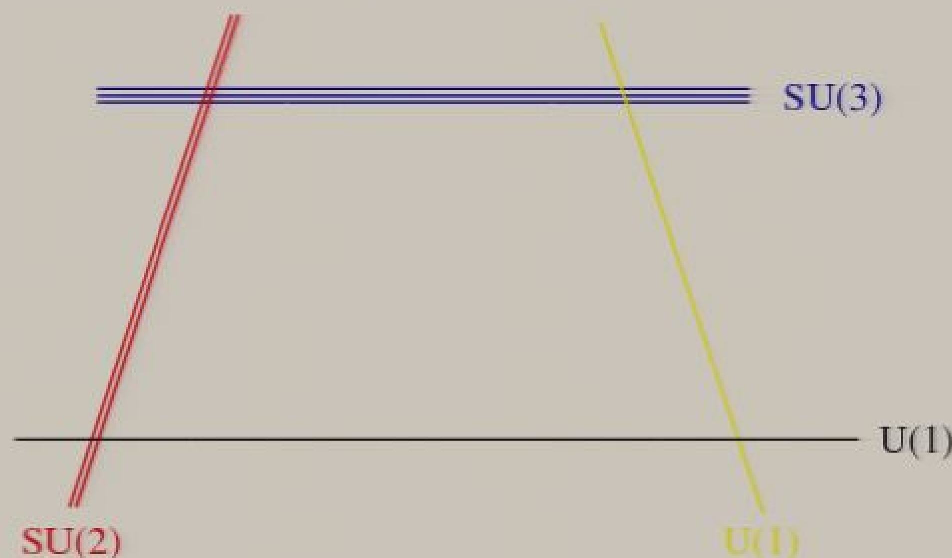
$$W = \sum_{E2} \overbrace{e^{-S_{E2}(U)}}^{\text{complex structure}} f\left(\overbrace{\exp\left(-\frac{T}{\alpha'}\right)}^{\text{WS disk instantons}}, \overbrace{\exp\left(-\frac{\text{tr}(\Delta)}{\alpha'}\right)}^{\text{D6 moduli}}, \Phi_{ab}\right)$$

Matter couplings

Generation of important perturbatively forbidden matter couplings possible

Most prominently: hierarchically large Majorana masses for right-handed neutrinos

For concreteness consider putative MSSM from IBW



Majorana Masses

Intersection	Matter	Rep.	Y
(a,b)	Q_L	$3\times(3,2)_{(1,0,0)}$	$1/3$
(a,c)	$(U_R)^c$	$3\times(\overline{3},1)_{(-1,1,0)}$	$-4/3$
(a',c)	$(D_R)^c$	$3\times(\overline{3},1)_{(-1,-1,0)}$	$2/3$
(b,d)	L_L	$3\times(1,2)_{(0,0,-1)}$	-1
(c,d)	$(E_R)^c$	$3\times(1,1)_{(0,-1,1)}$	2
(c',d)	$(N_R)^c$	$3\times(1,1)_{(0,1,1)}$	0

massive i.e. perturbative global symmetries:
baryon number $Q_B = Q_a$, lepton number $Q_L = Q_b$, Q_c, Q_d
massless hypercharge $U(1)_Y = \frac{1}{3}U(1)_a - U(1)_c + U(1)_d$
$\rightsquigarrow$ Dirac mass $W_H = H^+ L_L (N_R)^c$ present, but
Majorana mass $W_m = M_m (N_R)^c (N_R)^c$ perturbatively forbidden

Majorana Masses

Non-pert. coupling possible if CY_3 possesses rigid 3-cycle Ξ with zero mode structure

$$\Xi \cap \Pi_a = \Xi \cap \Pi'_a = \Xi \cap \Pi_b = \Xi \cap \Pi'_b = \Xi \cap \Pi_c = \Xi \cap \Pi'_d = 0$$
$$[\Xi \cap \Pi'_c]^+ = 2,\ [\Xi \cap \Pi'_c]^- = 0,\ [\Xi \cap \Pi_d]^- = 2,\ [\Xi \cap \Pi_d]^+ = 0$$

Non-pert. Majorana coupling:

$W_m = M_m\,(N_R)^c\,(N_R)^c$ with $M_m = x\,M_s\,e^{-\frac{2\pi}{\ell_s^3\,g_s}\mathrm{Vol}_{E2}}$

Use $\frac{1}{\alpha_{\rm GUT}} = \frac{1}{\ell_s^3\,g_s}\mathrm{Vol}_{D6} \rightsquigarrow M_m = x\,M_s\,e^{-\frac{2\pi}{\alpha_{\rm GUT}}\frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{D6}}}$

For seesaw mechanism need $10^8\mathrm{GeV} < M_m < 10^{15}\mathrm{GeV}$

Possible without fine tuning for $0.4 \cdot R_{D6} > R_{E2} > 0.2 \cdot R_{D6}$

(provided x of order 1)

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Concrete realisations on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$

Aim:

- Provide example of model with rigid $E2$-brane and suitable zero mode structure $\rightsquigarrow$ highly constraining
- Realize correct suppression scale for Majorana masses
- Exemplify CFT computation $\rightsquigarrow$ determine x exactly

Rigid sLags on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ with discrete torsion:

[BCMS hep-th/0502095]

rigid fractional cycles:

- Stuck at orbifold fixed points in all three twisted sectors (fractional cycles)
- homological sum of bulk cycle $S^1 \times S^1 \times S^1$ and $\sum_g \mathbb{P}^1_g \times S^1$ from each twisted sector g

Absence of $E2 - E2'$ modes requires $E2 = E2'$

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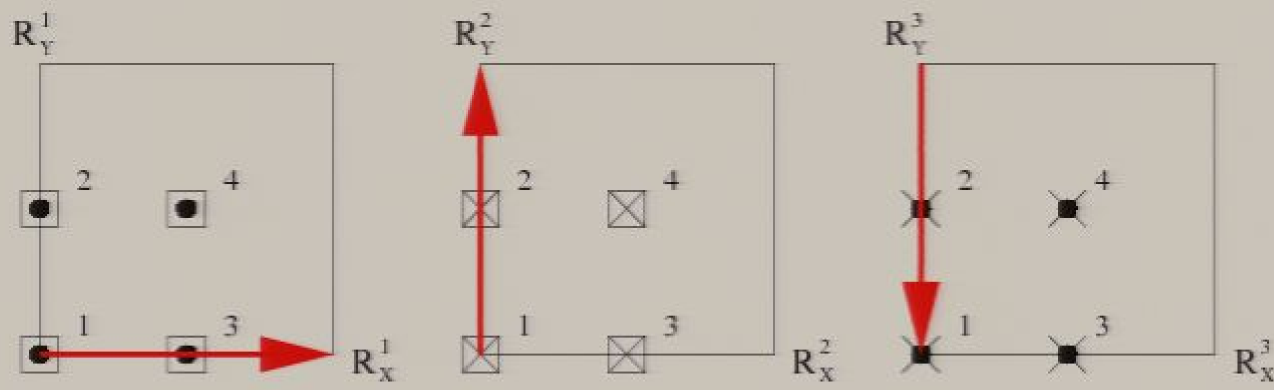

From details of orientifold action on twisted cycles:

8×8 such invariant rigid cycles

Construct supersymmetric local 3-stack GUT-like model:

sector	I_{xy}	representation	matter
(c, c')	4	Antisym	**10**
(c, a)	24	$(\overline{c}, a)$	$\overline{\mathbf{5}}$
(c, b)	-24	$(c, \overline{b})$	$\mathbf{5}_H$
(a, b)	32	$(\overline{a}, b)$	N_R^c

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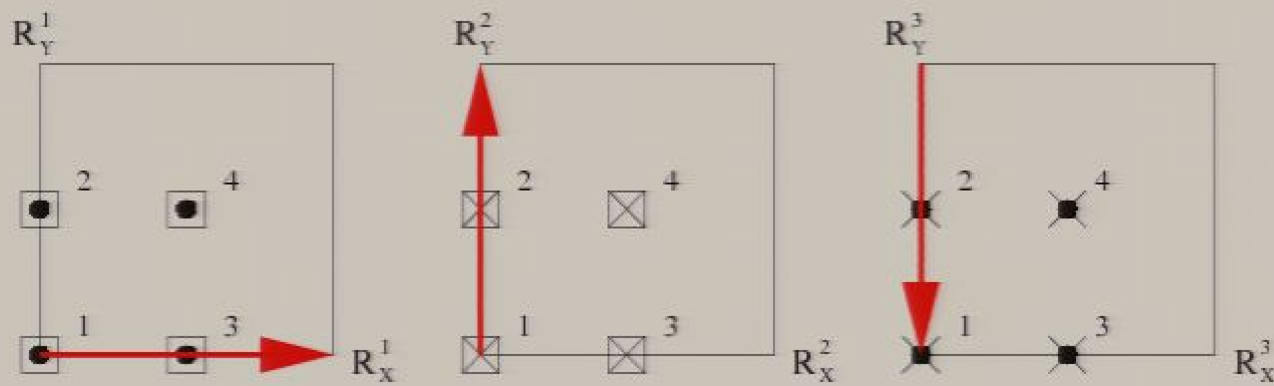

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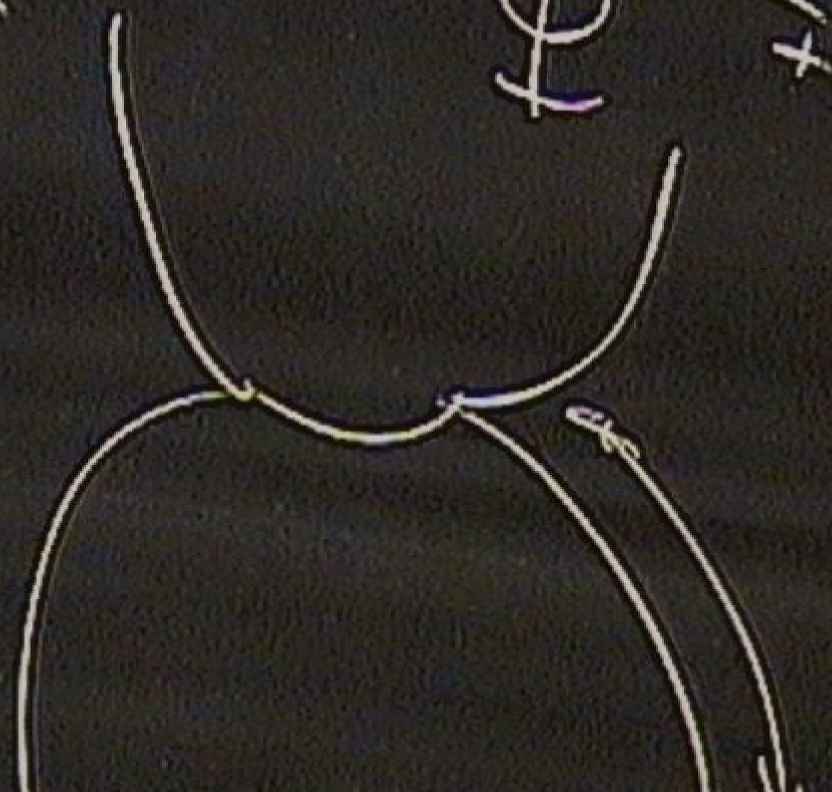
$$e^{-\frac{Vol_{E2}}{3g}} \qquad Vol_{E2} = \int \Omega_3$$

$$\Phi R = m^2 \Phi^2 + \Phi T$$



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Indeed:

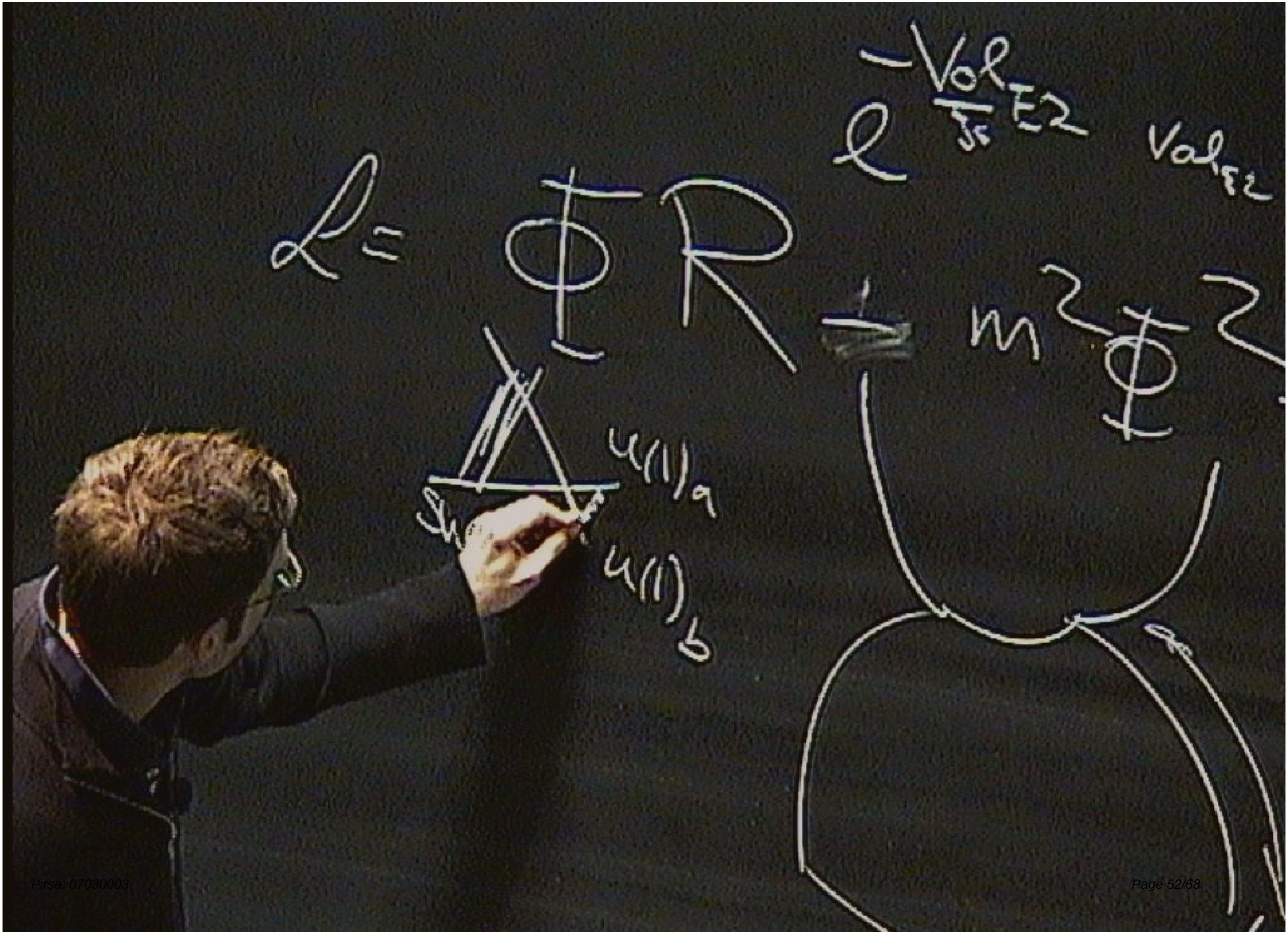
$$[\Pi_{E2} \cap \Pi_a]^+ = 2, \quad [\Pi_{E2} \cap \Pi_b]^- = 2, \quad [\Pi_{E2} \cap \Pi_c]^\pm = 0$$

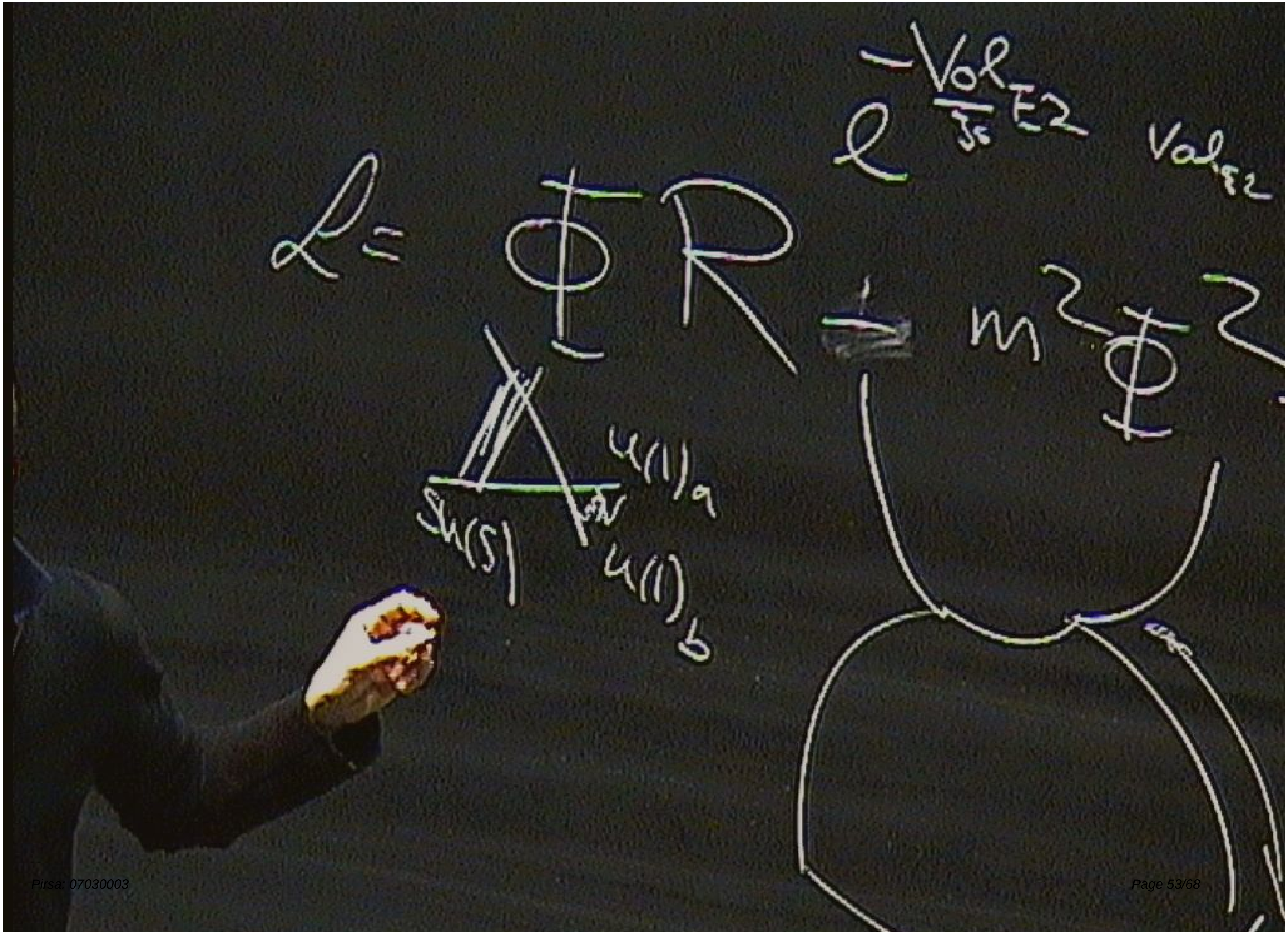
Perform detailed CFT computation of instanton induced Majorana mass by evaluation disks for each instanton

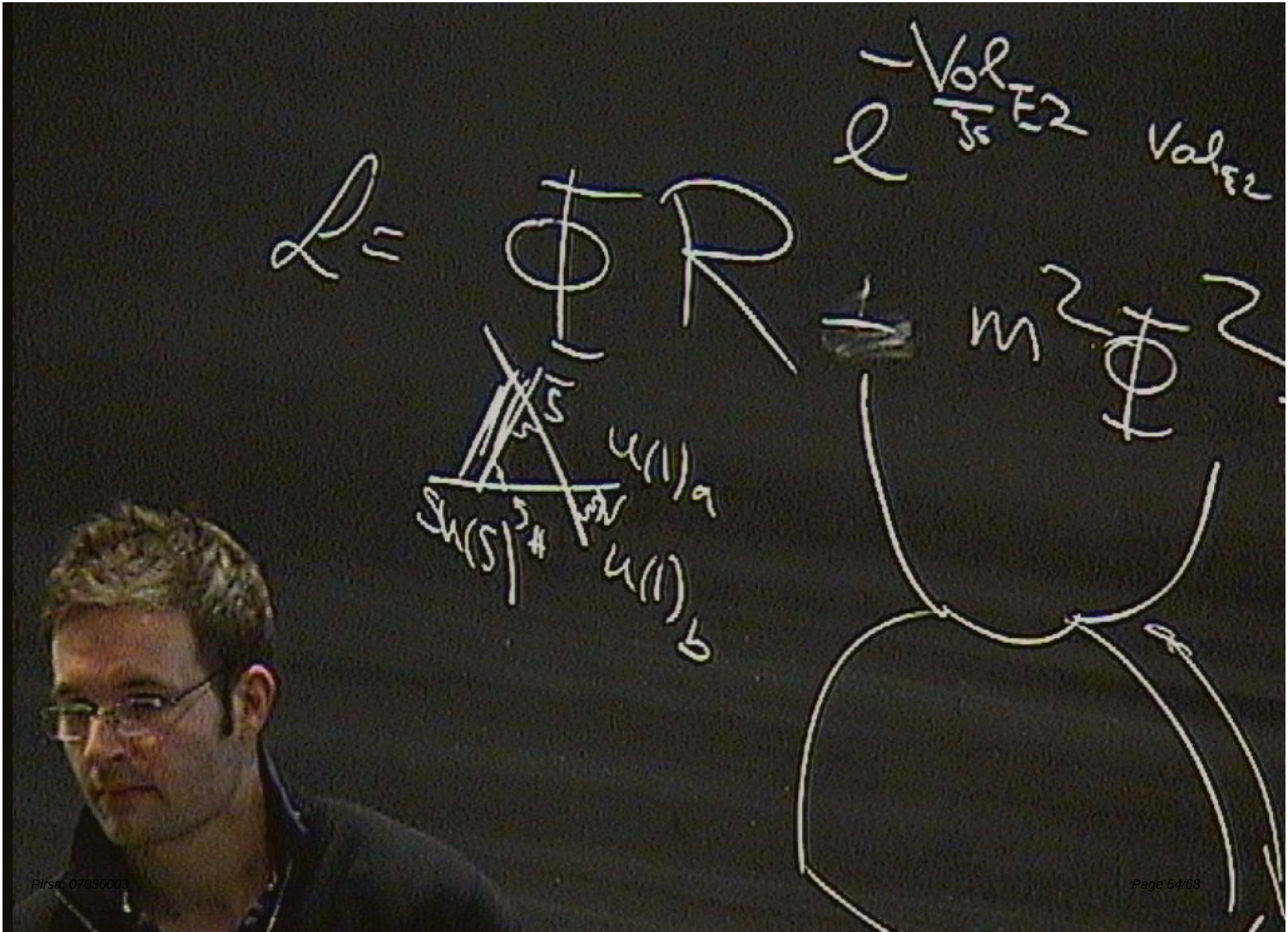
Result: $< \nu^A \nu^B >_{E2_i} = \frac{2\pi}{g_s} \mathcal{V}_{E2} \, \vec{v}^T \, \mathcal{M} \, \vec{v} \; (2\pi)^4 \delta^4(k^A + k^B)$

$$\mathcal{M} = x \, M_s \, e^{-\frac{2\pi}{\alpha_{\rm GUT}} \frac{8}{57}} \begin{pmatrix} A_i & 0 & B_i & 0 \\ 0 & C_i & 0 & D_i \\ B_i & 0 & E_i & 0 \\ 0 & D_i & 0 & F_i \end{pmatrix} ,$$

$$x = \frac{(4\pi)^{3/2}\pi^2}{16} \left[\Gamma_{1-\theta^1_{ab}, 1-\theta^1_{E2a}, 1+\theta^1_{E2b}} \prod_{I=2}^3 \Gamma_{-\theta^I_{ab}, -\theta^I_{E2a}, 1+\theta^I_{E2b}} \right]^{\frac{1}{2}} \; e^{Z'}$$


u(1)_a
u(1)_b
Vol_E2


SU(5)
U(1)a
U(1)b


u(1)a
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Concrete realisations on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$

Sum up contributions from all 64 factorizable E2-instantons, taking leading contribution (smallest triangle)
$\rightsquigarrow M_M \simeq 10^{10} GeV$
for triangles of order string scale (as required)

together with pert. Dirac masses from $\overline{\mathbf{5}}\,\mathbf{5}_H\,\mathbf{1}$ of electroweak scale
$\rightsquigarrow$ small neutrino masses of $\mathcal{O}(1eV)$ via see-saw mechanism

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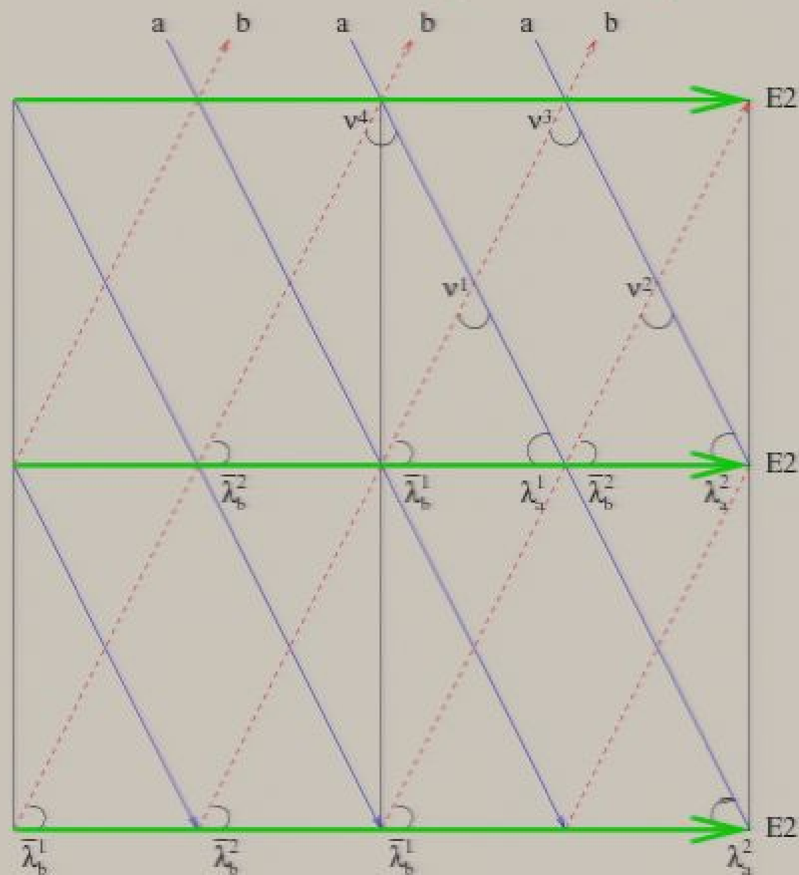
Family mixing pattern depends on relative suppression by worldsheet instantons, i.e. ratios of triangles

involves Kahler and open string (position) moduli



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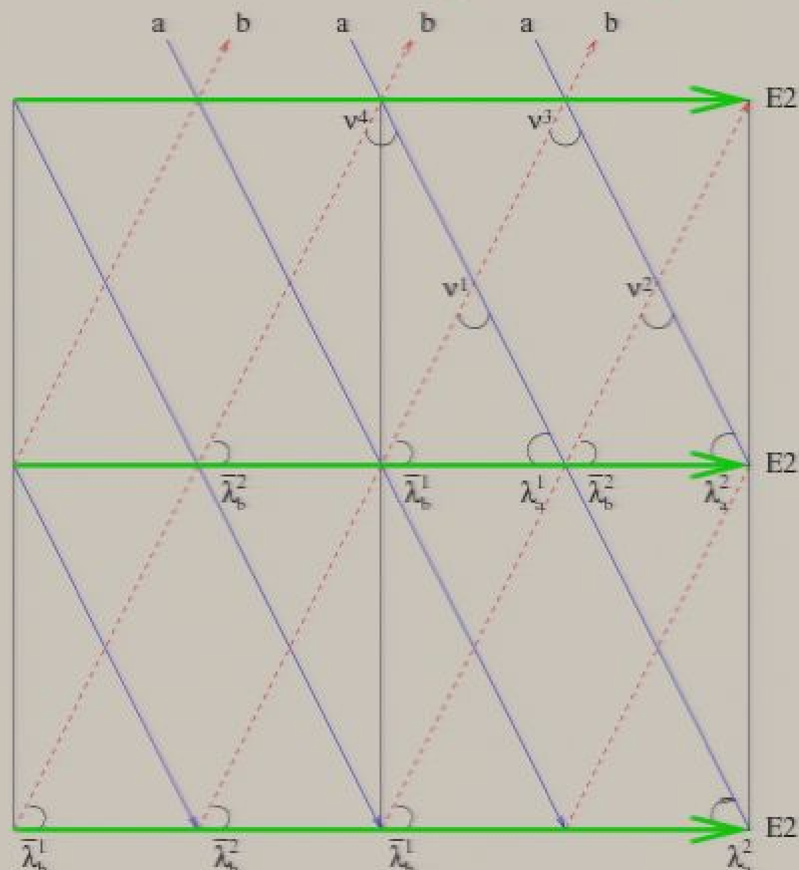

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Further applications

• in many concrete IBW constructions:
μ -term $\mu H^+ H^-$ forbidden perturbatively
$\rightsquigarrow$ can well be generated by $E2$-instantons!
$\rightsquigarrow$ appropriate volume ratio may "explain" why $\mu \simeq \mathcal{O}(M_Z)$

• IBW GUT $SU(5)$ suffer from absence of pert. trilinear Yukawas $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$, where $\mathbf{10}$ from (a,a')-intersection
Not generated by $E2$-instantons due to too many λ-modes

Proton decay?

Nice property of perturbative global $U(1)$ such as Q_B, Q_L:
Perturbative absence of dimension-four proton decay operators

$$W_4 = \lambda\,[Q_L\,(D_R)^c\,L_L] + \lambda'\,[(U_R)^c\,(D_R)^c\,(D_R)^c] + \lambda''\,[L_L\,L_L\,(E_R)^c]$$

$\rightsquigarrow$ Is proton unstable non-perturbatively?
Proton decay induced only if $\lambda\lambda' \neq 0$
careful analysis of restrictive structure of boundary combinatorics and fermionic zero modes crucial:

Find: no generation of $[Q_L\,(D_R)^c\,L_L]$ possible
$\rightsquigarrow$ no non-pert. dimension 4 proton decay operators induced in this manner

Vacuum destabilisation

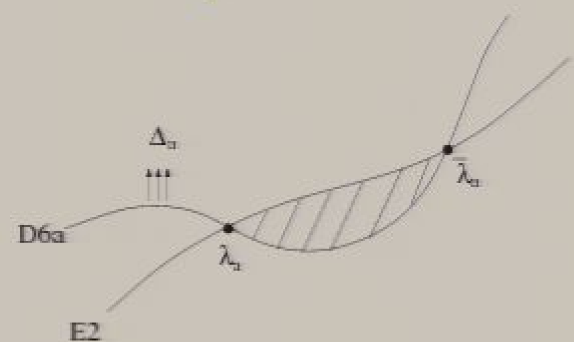
2 kinds of destabilising terms can be induced:

• In presence of precisely to fermionic zero modes λ_a, $\overline{\lambda}_b$, open string tadpole

$$W = \Phi_{a,b}\; e^{-S_{E2}}$$

is possible.

• If $E2$-brane has only non-chiral (vector-like) $\lambda - \overline{\lambda}$ modes with $D6_a$, purely moduli dependent terms induced:



Vacuum destabilisation

Does this destroy all known perturbative brane vacua?

• Both rigidity and zero mode structure rule out these processes on many backgrounds
• Interplay of several such terms can in principle also lead to fixing of open string moduli
$\rightsquigarrow$ systematic analysis in concrete examples required
• Cancellation of complete sum of these contributions likewise conceivable in principle (see Beasley/Witten)
$\rightsquigarrow$ remains to be seen!

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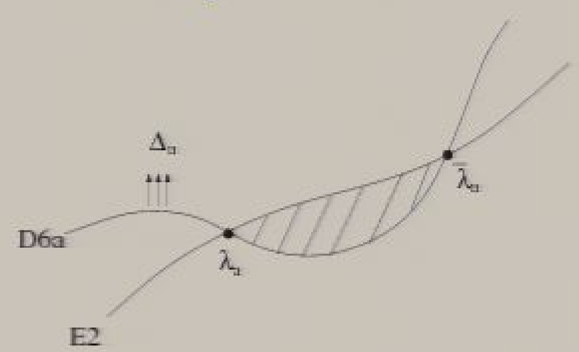

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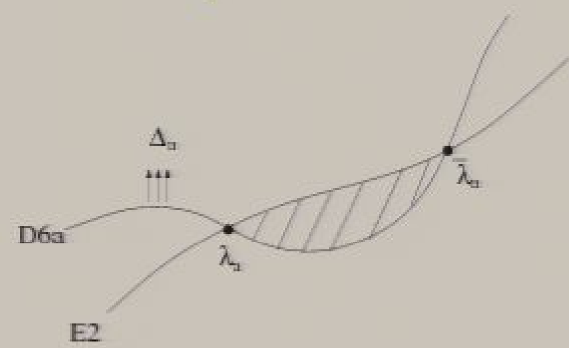

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Conclusions

studied CFT description of E2-brane instantons in Type IIA
general rules for disk and one-loop contributions

formalism applicable to vacua with exactly solvable CFT:
toroidal orientifolds, Gepner Model orientifolds

implications on phenomenology and model building:

- generation of hierarchies without fine-tuning in context of Majorana masses/ μ-terms

$\rightsquigarrow$ gave explicit local example for see-saw mechanism

Challenge: present global models with rigid cycles and realistic spectrum

- vacuum destabilisation/SUSY breaking? open string moduli fixing?

$\rightsquigarrow$ effects on open string landscape?