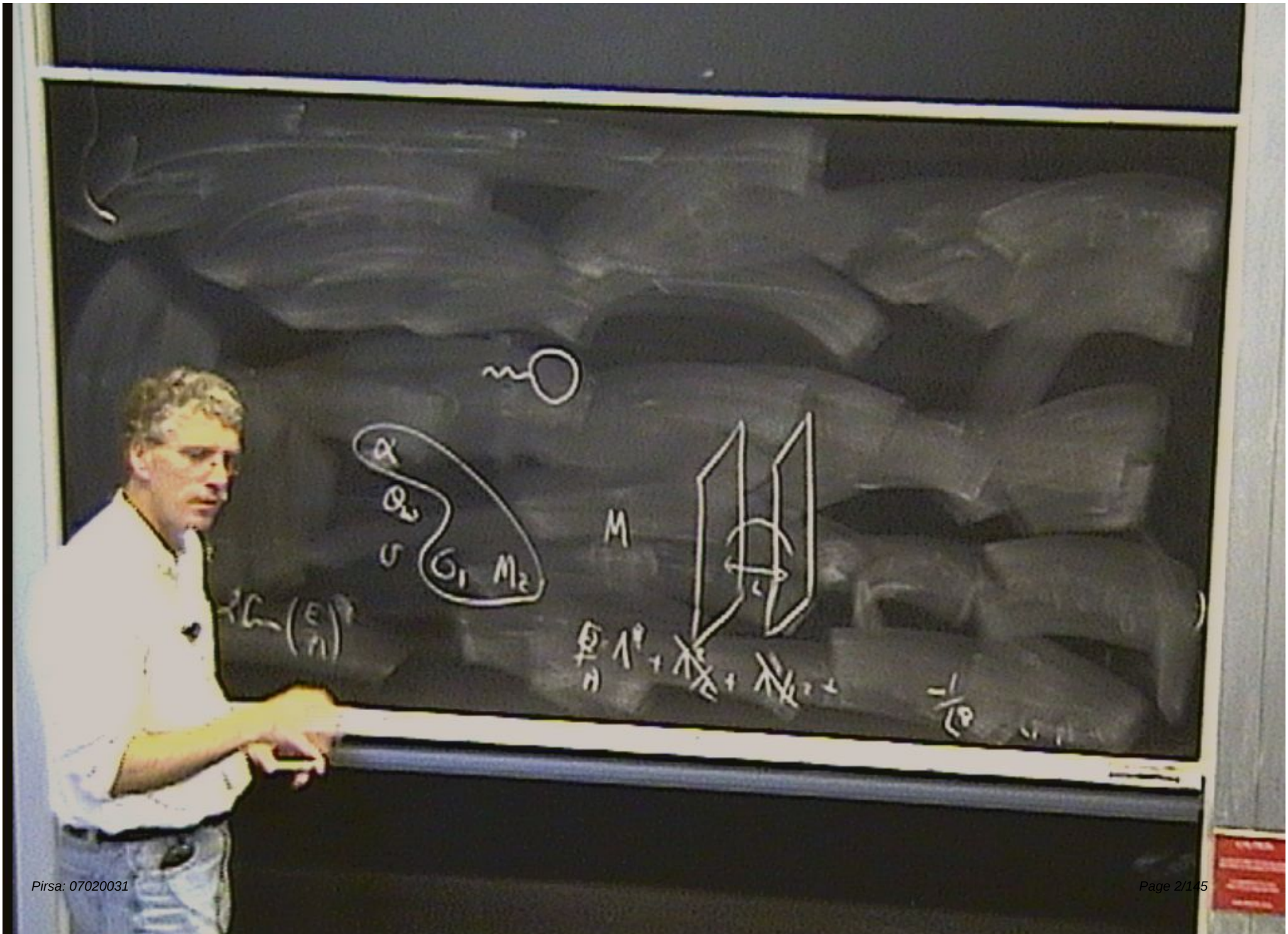
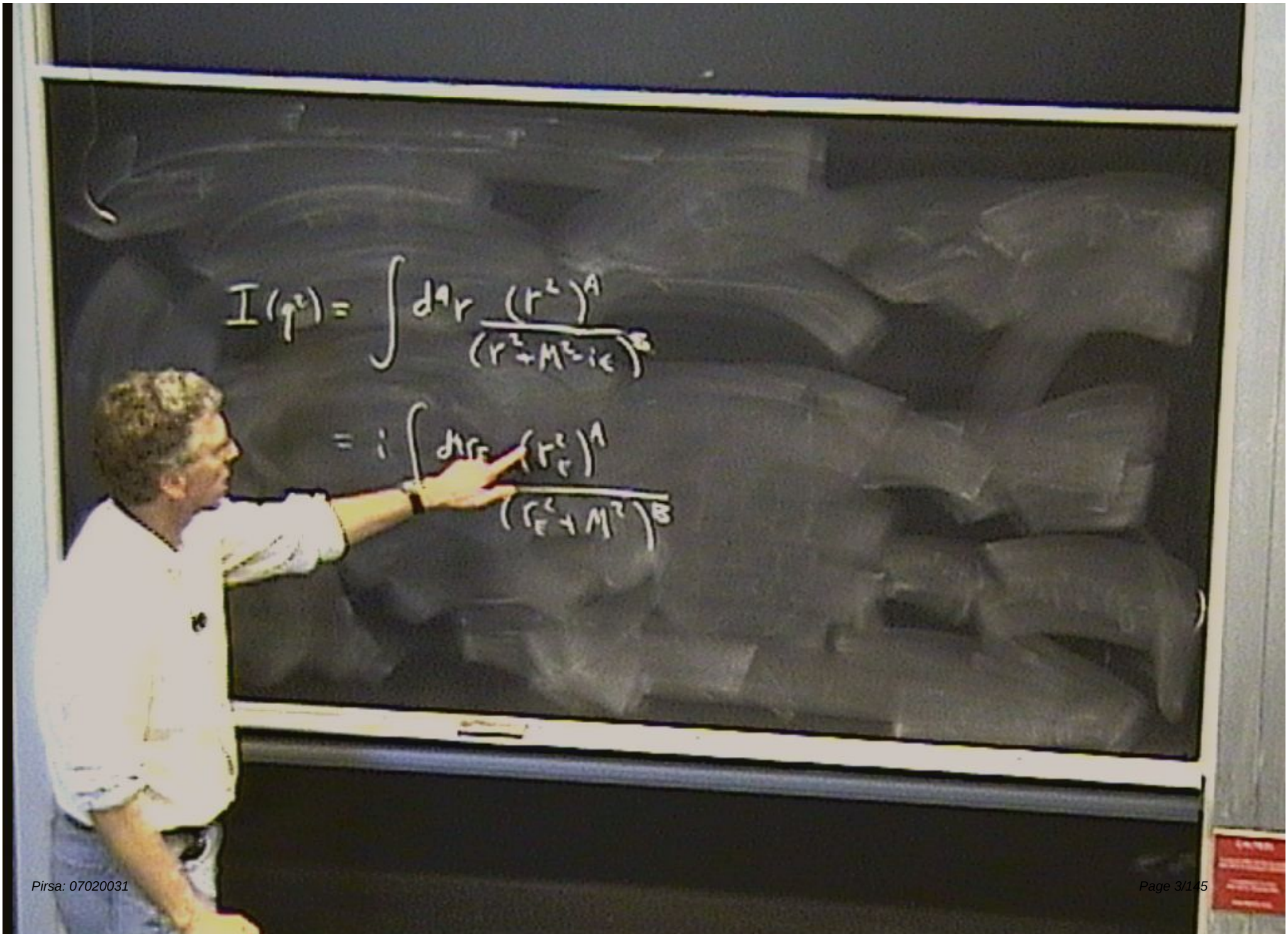
Title: Graduate Course on Standard Model & Quantum Field Theory - 13B

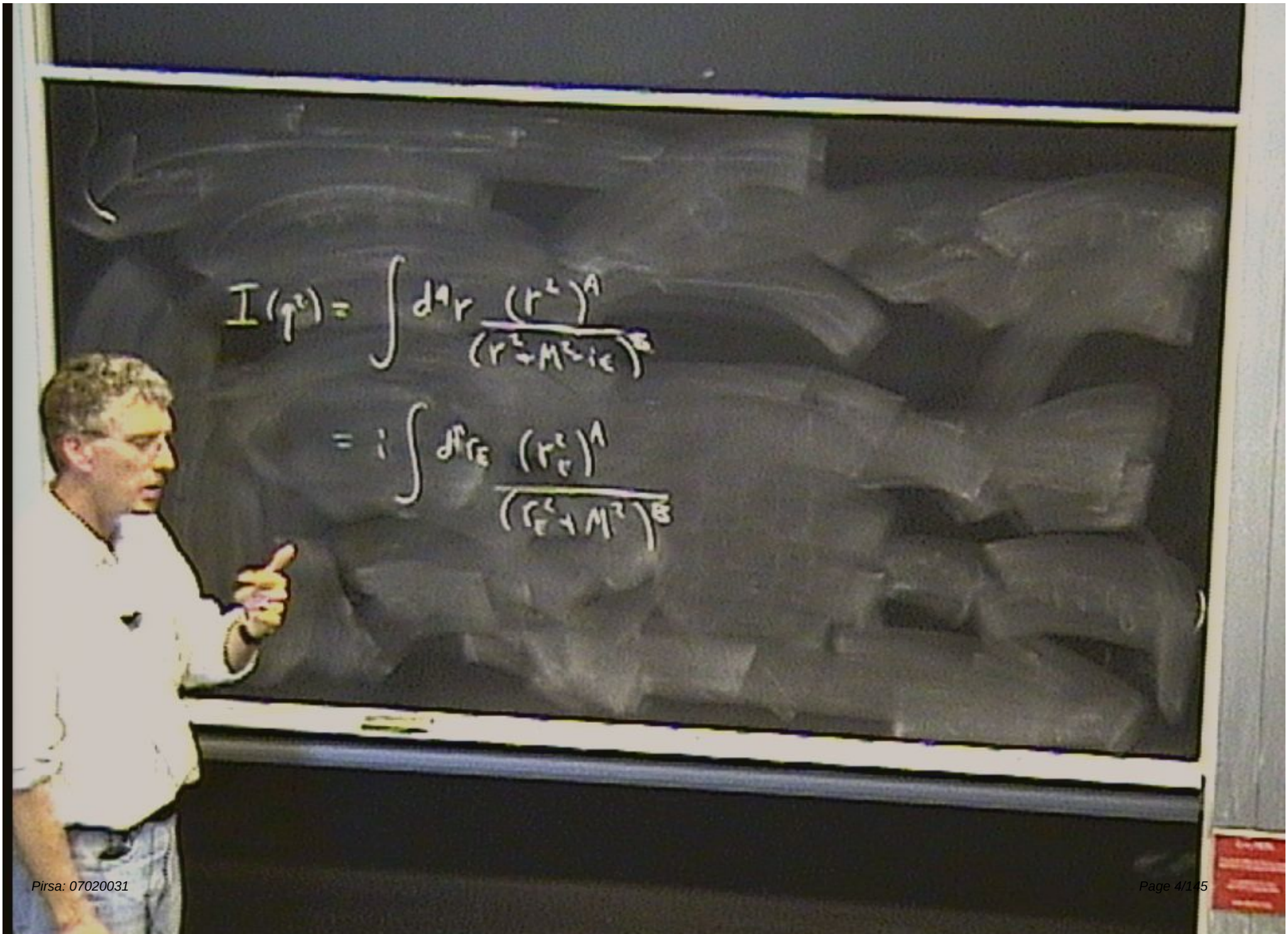
Date: Feb 28, 2007 03:30 PM

URL: http://pirsa.org/07020031

Abstract: Graduate Course on Standard Model & Quantum Field Theory


M


$I(q^2) = \int d^4r \frac{(r^2)^A}{(r^2+M^2-i\epsilon)^B}$
$= i \int d^4r_E \frac{(r_E^2)^A}{(r_E^2+M^2)^B}$


$I(p^2) = \int d^4r \frac{(r^2)^A}{(r^2 - M^2 + i\epsilon)^B}$
$= i \int d^4r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$

$$I(\ldots) = \int d^4r \, \frac{(r^2)^A}{(r^2 + M^2 - i\epsilon)^B}$$

$$= i \int d^4 r_E \, \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

$$\frac{\partial^c I}{\partial (M^2)^c} = \text{finite}$$

$$I(\ldots) = \int d^4r \, \frac{(r^2)^A}{(r^2 + M^2 - i\epsilon)^B}$$

$$= i \int d^4 r_E \, \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

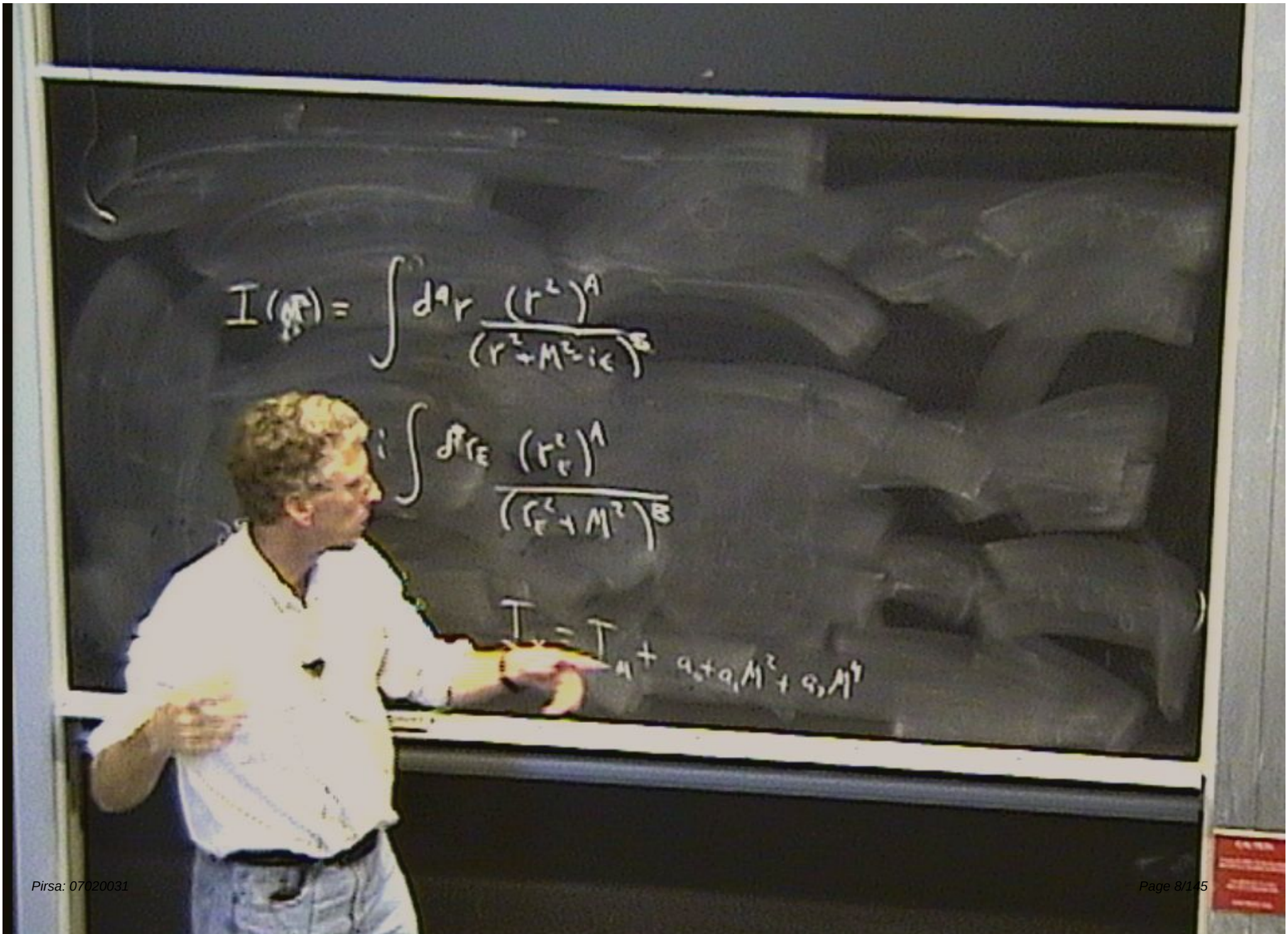
$$\frac{\partial I}{\partial M^2} = \text{finite}$$

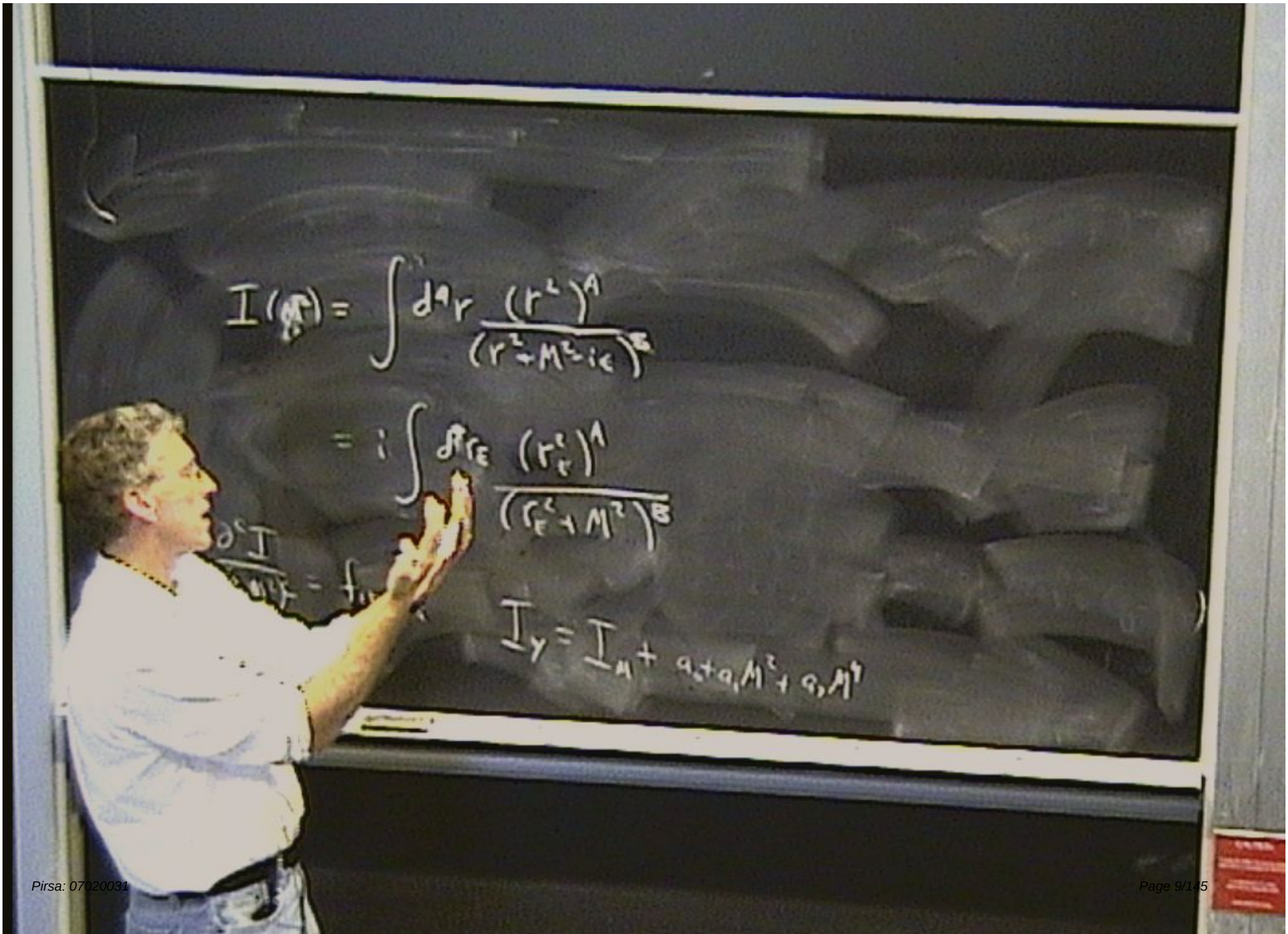
$$I_y = I_M +$$

$$I(\ldots) = \int d^4 r \, \frac{(r^2)^A}{(r^2 + M^2 - i\epsilon)^B}$$

$$= i \int d^4 r_E \, \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

$$\partial \frac{I}{M^2} = \text{finite} \qquad I_y = I_M +$$


$I(M^2) = \int d^4r \frac{(r^2)^A}{(r^2+M^2-i\epsilon)^B}$
$i\int d^4r_E \frac{(r_E^2)^A}{(r_E^2+M^2)^B}$
$I_\epsilon = I_M + a_0 + a_1 M^2 + a_2 M^4$


$I(M^2) = \int d^4r \frac{(r^2)^A}{(r^2+M^2-i\epsilon)^B}$
$= i \int d^4r_E \frac{(r_E^2)^A}{(r_E^2+M^2)^B}$
$I_Y = I_M + a_0 + a_1 M^2 + a_2 M^4$

$$I(M^2) = \int d^4r \, \frac{(r^2)^A}{(r^2 + M^2 - i\epsilon)^B}$$

$$= i \int d^4r_E \, \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

$$\frac{\partial^C I}{\partial (M^2)^C} = \text{finite}$$

$$I_Y = I_M + a_0 + a_1 M^2 + a_2 M^4$$

$$I(\ldots) = \int d^n r \, \frac{(r^2)^A}{(r^2 + M^2 - i\epsilon)^B}$$

$$= i \int d^n r_E \, \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

$$= i \Omega_{n-1} \int_0^\infty dr \, \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

$$I(A,B) = \int d^n r \, \frac{(r^2)^A}{(r^2 + M^2 - i\epsilon)^B}$$

$$= i \int d^n r_E \, \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

$$= i \Omega_{n-1} \int_0^\infty dr \, \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$= \int_0^1 dt \, \frac{t^{x-1}}{(1+t)^{x+y}}$$

$$I(M^2) = \int \frac{d^n r}{(r^2 + M^2 - i\epsilon)^B}$$

$$= i \int d^n r_E \, \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

$$= i\Omega_{n-1} \int_0^\infty dr \, \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

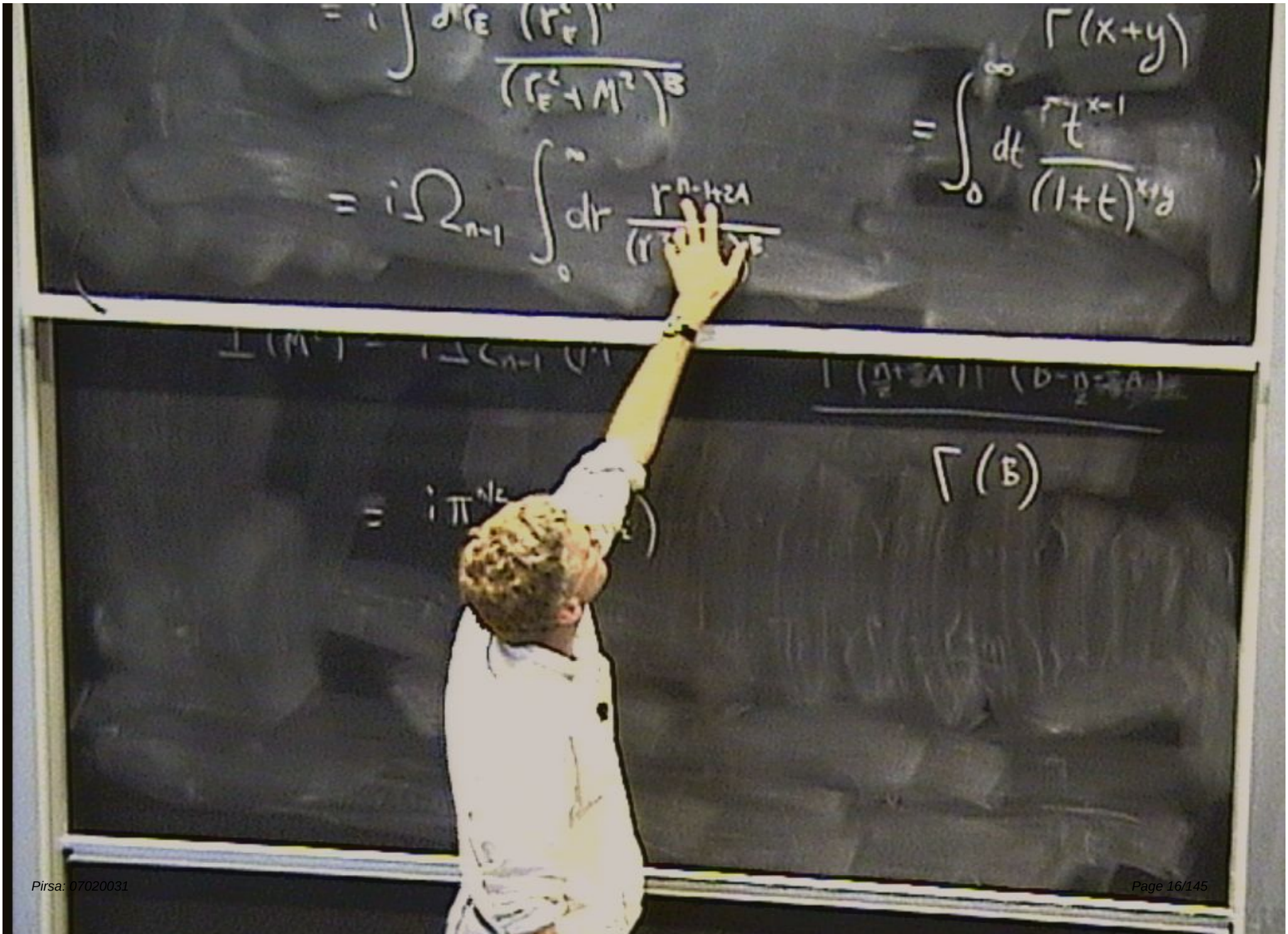
$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

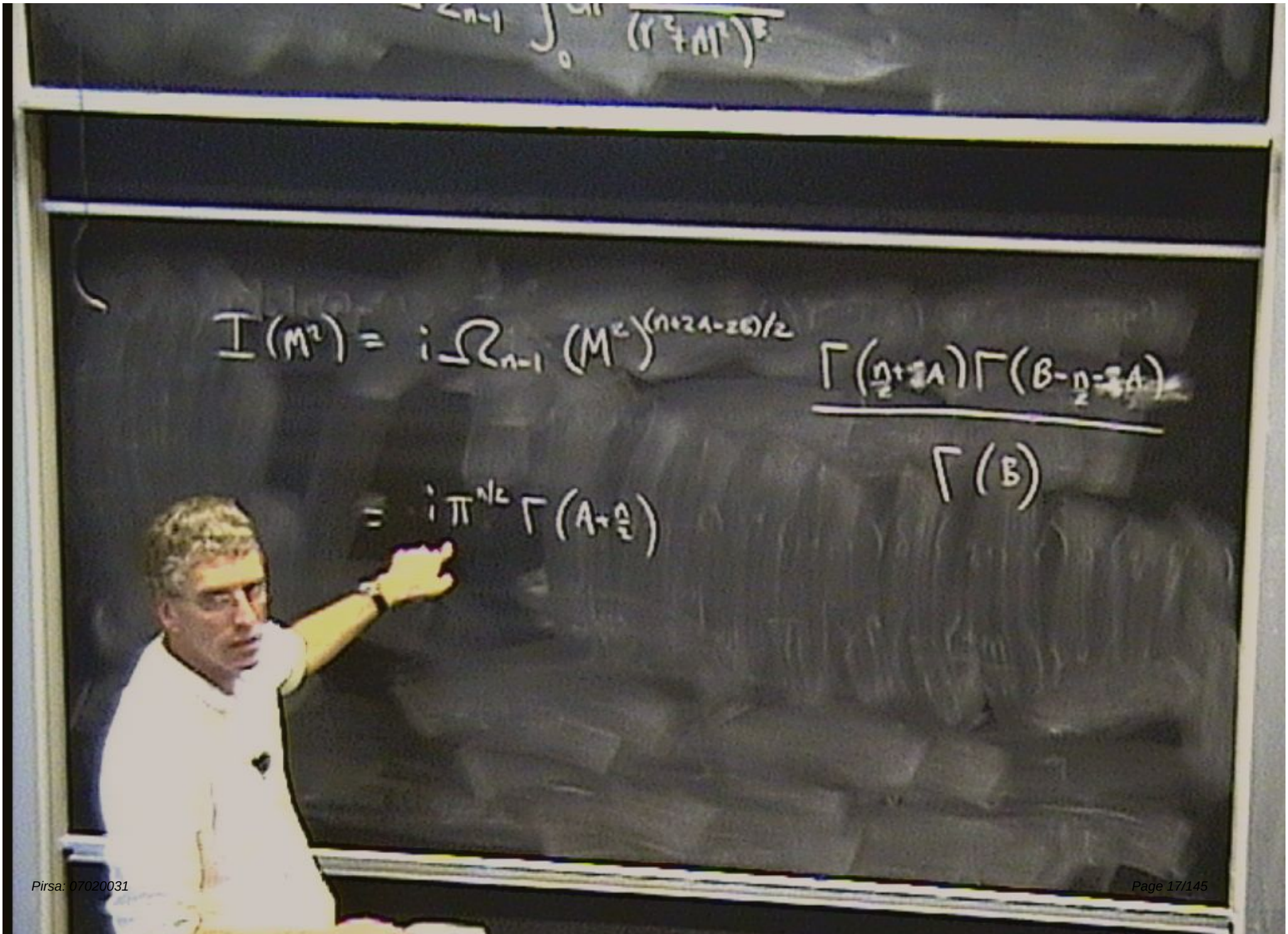
$$= \int_0^\infty dt \, \frac{t^{x-1}}{(1+t)^{x+y}}$$

$$I(M^2) = i\Omega_{n-1}$$

$$I(M^2) = \Omega_{n-1} (M^2)^{(n+2A-2B)/2} \frac{\Gamma(n+2A)\Gamma(B-n-2A)}{\Gamma(B)}$$

$$I(M^2) = i\,\Omega_{n-1}\,(M^2)^{(n+2A-2B)/2}\,\frac{\Gamma(n+2A)\,\Gamma(B-n-2A)}{\Gamma(B)}$$

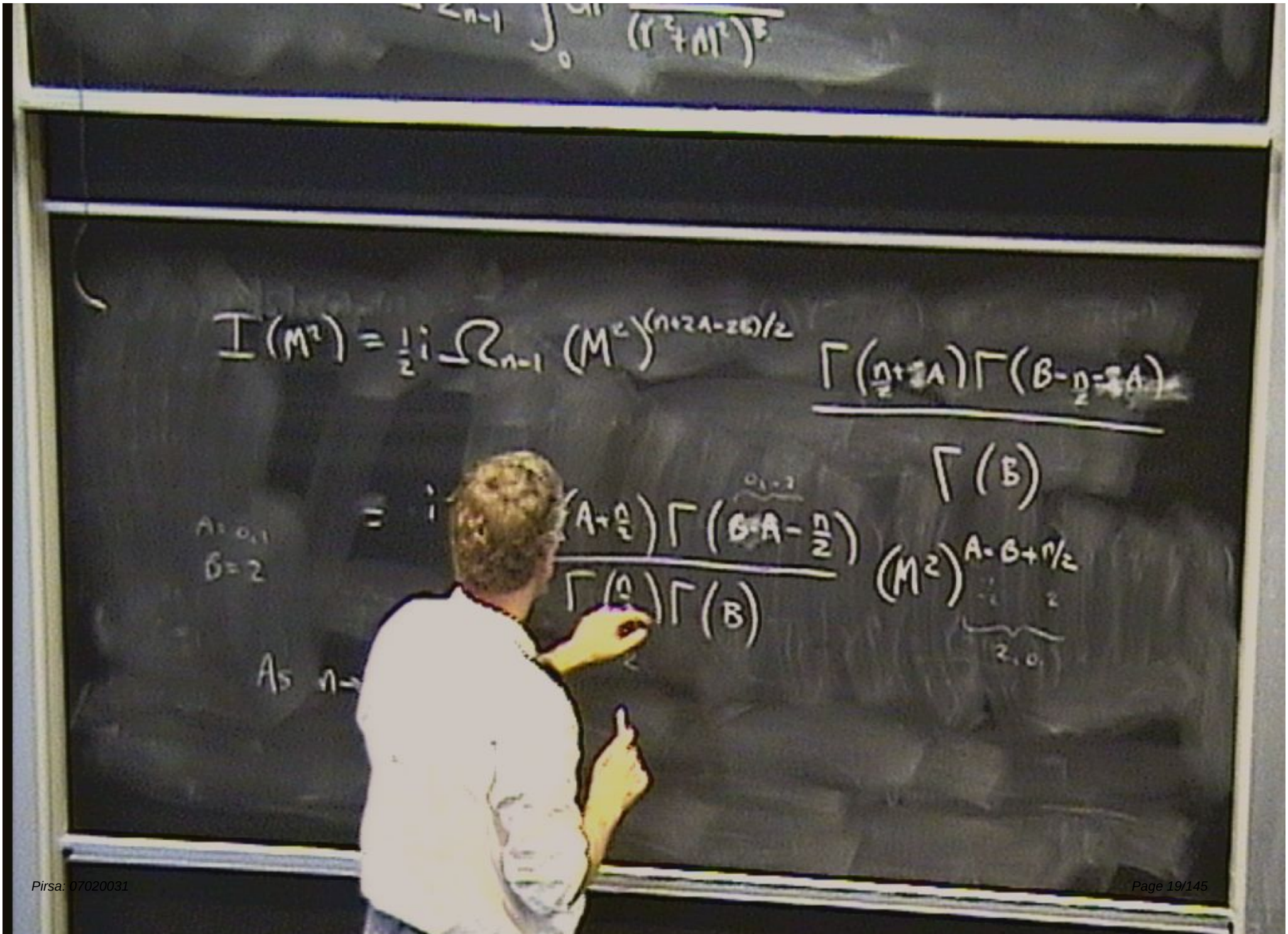

$= i\int d^4 r_E \frac{(r_E^2)^A}{(r_E^2+M^2)^B}$
$= i\Omega_{n-1}\int_0^\infty dr \frac{r^{n-1+2A}}{(r^2+M^2)^B}$
$\Gamma(x+y)$
$= \int_0^\infty dt \frac{t^{x-1}}{(1+t)^{x+y}}$
$\Gamma(B)$
$= i\pi^{n/2}$


$I(M^2) = i\Omega_{n-1}(M^2)^{(n+2A-2B)/2} \dfrac{\Gamma(\frac{n}{2}+A)\Gamma(B-\frac{n}{2}-A)}{\Gamma(B)}$
$= i\pi^{n/2}\Gamma(A+\frac{n}{2})$

$$\Omega_{n-1} \int_0^\infty \frac{dr}{(r^2+M^2)^B}$$

$$I(M^2) = \frac{1}{2} i \Omega_{n-1} (M^2)^{(n+2A-2B)/2} \frac{\Gamma\left(\frac{n}{2}+A\right)\Gamma\left(B-\frac{n}{2}-A\right)}{\Gamma(B)}$$

$$= i\pi^{n/2} \frac{\Gamma\left(A+\frac{n}{2}\right)\Gamma\left(B-A-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma(B)} (M^2)^{A-B+n/2}$$


$I(M^2) = \frac{1}{2} i \Omega_{n-1} (M^2)^{(n+2A-2B)/2} \frac{\Gamma(\frac{n}{2}+A)\Gamma(B-\frac{n}{2}-A)}{\Gamma(B)}$
$= i \frac{\Gamma(A+\frac{n}{2})\Gamma(B-A-\frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(B)} (M^2)^{A-B+n/2}$
A = 0,1
B = 2
As n→

$$\Omega_{n-1} \int_0^\infty \frac{dr}{(r^2+M^2)^B}$$

$$I(M^2) = \frac{1}{2} i \Omega_{n-1} (M^2)^{(n+2A-2B)/2} \frac{\Gamma\left(\frac{n}{2}+A\right)\Gamma\left(B-\frac{n}{2}-A\right)}{\Gamma(B)}$$

$$= i\pi^{n/2} \frac{\Gamma\left(A+\frac{n}{2}\right)\Gamma\left(\overbrace{B-A-\frac{n}{2}}^{0,-2}\right)}{\Gamma\left(\underbrace{\frac{n}{2}}_{2}\right)\Gamma(B)} (M^2)^{\underbrace{A-B+n/2}_{2,0,}}$$

$0,1$

2

As $n \to 4$

$\Gamma(z)$ has simple poles at the non-positive integers. $z = 0, -1, -2, -3, \ldots$

$$I(M^2) = \frac{1}{2} i \Omega_{n-1} (M^2)^{(n+2A-2B)/2} \frac{\Gamma\left(\frac{n}{2}+2A\right)\Gamma\left(B-\frac{n}{2}-2A\right)}{\Gamma(B)}$$

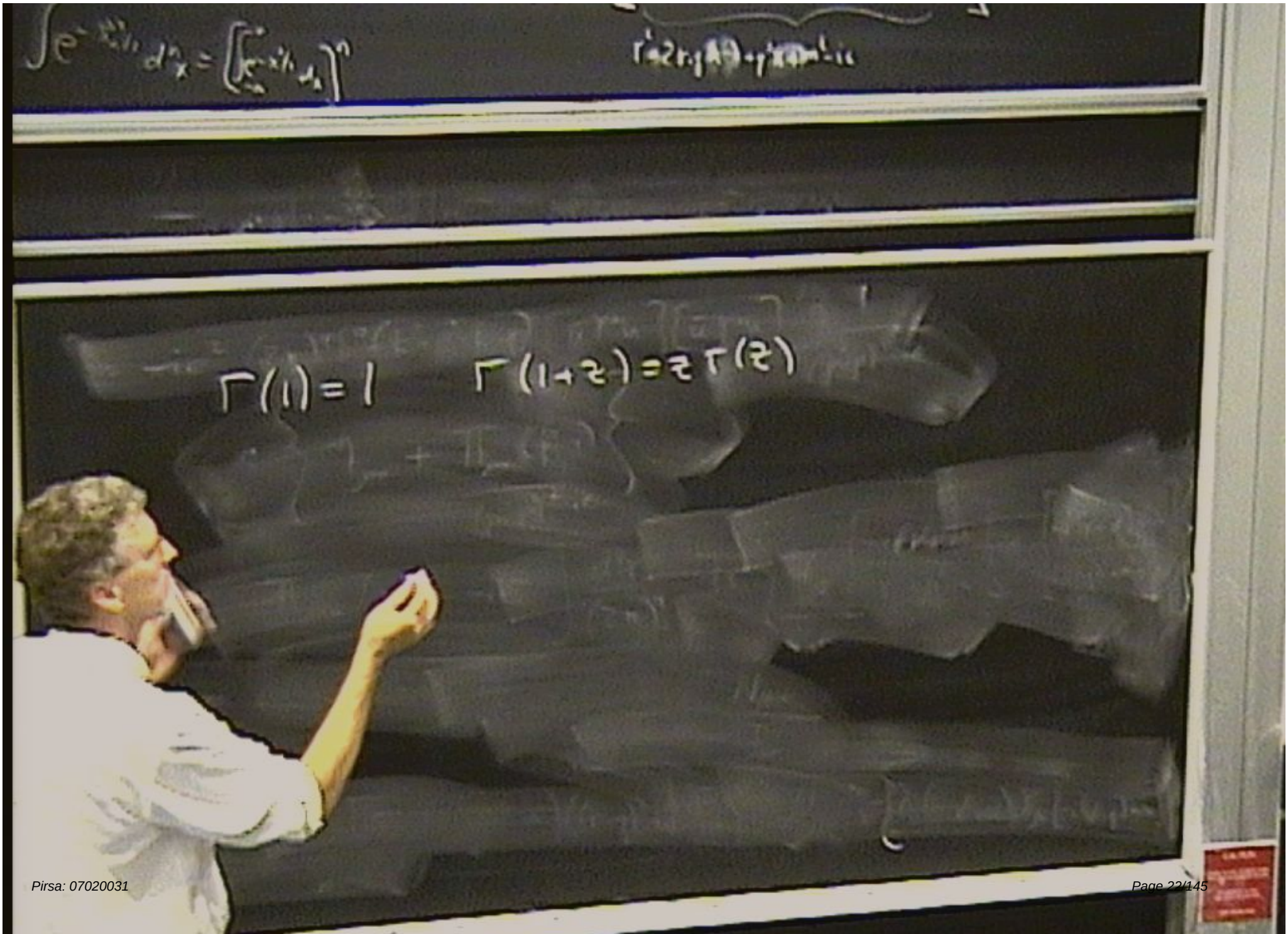
$A = 0, 1$

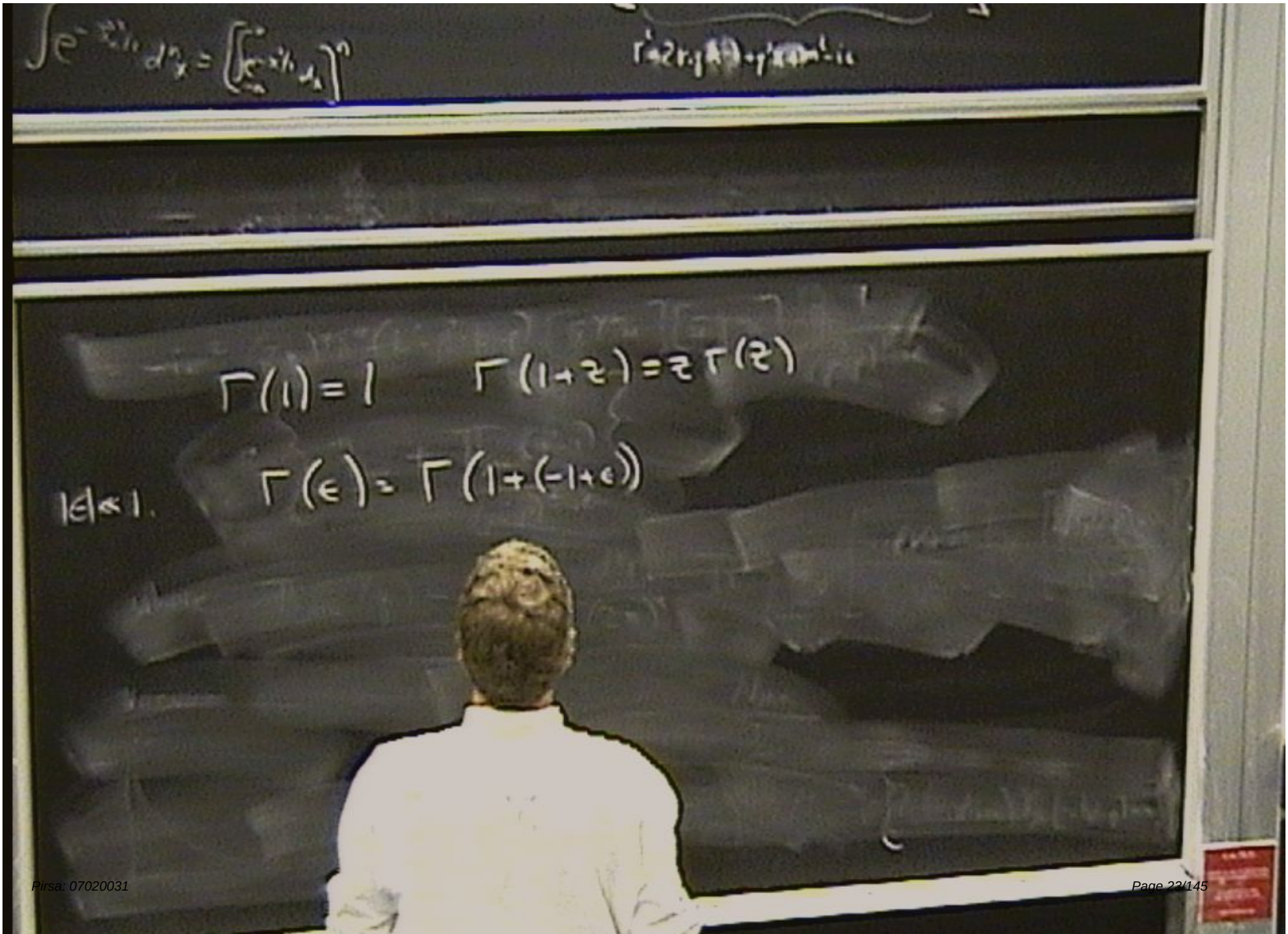
$B = 2$

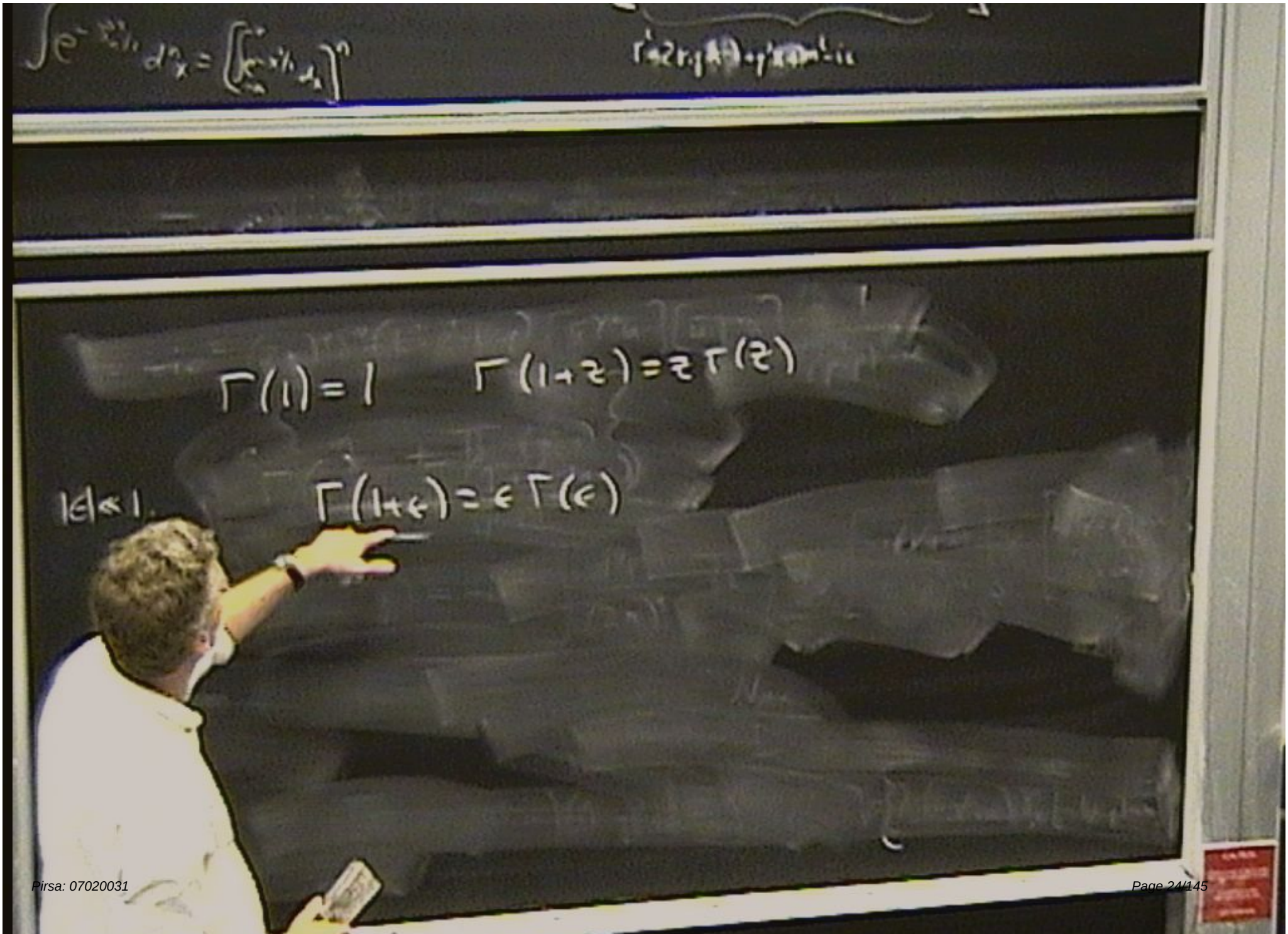
$$= i\pi^{n/2} \frac{\Gamma\left(A+\frac{n}{2}\right)\Gamma\left(\overbrace{B-A}^{0,-1}-\frac{n}{2}\right)}{\Gamma\left(\underbrace{\frac{n}{2}}_{2}\right)\Gamma(B)} (M^2)^{\underbrace{A-B+n/2}_{2,0}}$$

As $n \to 4$

$\Gamma(z)$ has simple poles at the non-positive integers $z = 0, -1, -2, -3, \ldots$


$\Gamma(1) = 1$
$\Gamma(1+z) = z\,\Gamma(z)$


$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$
$|\epsilon| \ll 1. \qquad \Gamma(\epsilon) = \Gamma(1+(-1+\epsilon))$


$\Gamma(1) = 1$
$\Gamma(1+z) = z\,\Gamma(z)$
$|\epsilon| \ll 1.$
$\Gamma(1+\epsilon) = \epsilon\,\Gamma(\epsilon)$

$$\int e^{-x^2/2}\, d^n x = \left[\int_{-\infty}^{\infty} e^{-x^2/2}\, dx\right]^n$$

$$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$$

$|\epsilon| \ll 1.$ $\qquad \underline{\Gamma(1+\epsilon)} = \epsilon\,\Gamma(\epsilon)$

$$\Gamma(1+\epsilon) = 1 + \left.\frac{d\Gamma}{dz}\right|_{z=1} \epsilon + \cdots$$

$$= 1 +$$

$$\int e^{-x^2/2} d^n x = \left[\int_{-\infty}^{\infty} e^{-x^2/2} dx\right]^n$$

$$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$$

$|\epsilon| \ll 1.$

$$\underline{\Gamma(1+\epsilon)} = \epsilon\,\Gamma(\epsilon)$$

$$\Gamma(1+\epsilon) = 1 + \left.\frac{d\Gamma}{dz}\right|_{z=1} \epsilon + \cdots$$

$$= 1 - \gamma\epsilon + \ldots$$

γ > Euler Mascheroni = 0.5

$$\int e^{-\frac{x^2}{2\sigma^2}} d^n x = \left[\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} dx\right]^n$$

$$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$$

$|\epsilon| \ll 1.$

$$\underline{\Gamma(1+\epsilon)} = \epsilon\,\Gamma(\epsilon) \qquad \Gamma(\epsilon) = \frac{1}{\epsilon}\Gamma(1+\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon)$$

$$\Gamma(-1+\epsilon)(-1+\epsilon) = \Gamma(\epsilon)$$

$$\Gamma(1+\epsilon) = 1 + \left.\frac{d\Gamma}{dz}\right|_{z=1} \epsilon + \cdots$$

$$= 1 - \gamma\epsilon + \ldots$$

γ = Euler Mascheroni = 0.5772

$$\int e^{-x^2/2}\, d^n x = \left[\int_{-\infty}^{\infty} e^{-x^2/2}\, dx\right]^n$$

$$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$$

$|\epsilon| \ll 1.$

$$\underline{\Gamma(1+\epsilon)} = \epsilon\,\Gamma(\epsilon) \qquad \Gamma(\epsilon) = \frac{1}{\epsilon}\Gamma(1+\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon)$$

$$\Gamma(-1+\epsilon)(-1+\epsilon) = \Gamma(\epsilon)$$

$$\Gamma(1+\epsilon) = 1 + \left.\frac{d\Gamma}{dz}\right|_{z=1} \epsilon + \cdots \qquad \Gamma(-1+\epsilon) = \frac{1}{-1+\epsilon}\,\frac{1}{\epsilon}\,\Gamma(1+\epsilon)$$

$$= 1 - \gamma\epsilon + \ldots$$

γ > Euler Mascheroni = 0.5772157...

$$\int e^{-\frac{x^2}{2\sigma^2}} d^n x = \left[\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} dx\right]^n$$

$$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$$

$|\epsilon| \ll 1.$

$$\underline{\Gamma(1+\epsilon)} = \epsilon\,\Gamma(\epsilon) \qquad \Gamma(\epsilon) = \frac{1}{\epsilon}\Gamma(1+\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon)$$

$$\Gamma(-1+\epsilon)(-1+\epsilon) = \Gamma(\epsilon)$$

$$\Gamma(1+\epsilon) = 1 + \left.\frac{d\Gamma}{dz}\right|_{z=1} \epsilon + \cdots \qquad \Gamma(-1+\epsilon) = \frac{1}{-1+\epsilon}\,\frac{1}{\epsilon}\,\Gamma(1+\epsilon)$$

$$= 1 - \gamma\epsilon + \ldots$$

γ > Euler Mascheroni

$$\int e^{-x^2/2} d^n x = \left[\int_{-\infty}^{\infty} e^{-x^2/2} dx\right]^n$$

$$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$$

$|\epsilon| \ll 1.$

$$\underline{\Gamma(1+\epsilon)} = \epsilon\,\Gamma(\epsilon) \qquad \Gamma(\epsilon) = \frac{1}{\epsilon}\Gamma(1+\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon)$$

$$\Gamma(-1+\epsilon)(-1+\epsilon) = \Gamma(\epsilon)$$

$$\Gamma(1+\epsilon) = 1 + \left.\frac{d\Gamma}{dz}\right|_{z=1} \epsilon + \cdots \qquad \Gamma(-1+\epsilon) = \frac{1}{-1+\epsilon}\,\frac{1}{\epsilon}\,\Gamma(1+\epsilon)$$

$$= 1 - \gamma\epsilon + \ldots$$

γ = Euler Mascheroni = 0.5772157...

$$\int e^{-x^2/2}\, d^n x = \left[\int_{-\infty}^{\infty} e^{-x^2/2}\, dx\right]^n$$

$$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$$

$|\epsilon| \ll 1.$

$$\Gamma(1+\epsilon) = \epsilon\,\Gamma(\epsilon) \qquad \Gamma(\epsilon) = \frac{1}{\epsilon}\Gamma(1+\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon)$$

$$\Gamma(-1+\epsilon)(-1+\epsilon) = \Gamma(\epsilon)$$

$$\Gamma(1+\epsilon) = 1 + \left.\frac{d\Gamma}{dz}\right|_{z=1} \epsilon + \cdots \qquad \Gamma(-1+\epsilon) = \frac{1}{-1+\epsilon}\,\frac{1}{\epsilon}\,\Gamma(1+\epsilon)$$

$$= 1 - \gamma\epsilon + \ldots \qquad \Gamma(-k+\epsilon) = \frac{(-)^k}{\epsilon} - C_k + O(\epsilon)$$

$$\int e^{-x^2/2}\, d^n x = \left[\int_{-\infty}^{\infty} e^{-x^2/2}\, dx\right]^n$$

$$\Gamma(1) = 1 \qquad \Gamma(1+z) = z\,\Gamma(z)$$

$|\epsilon| \ll 1.$

$$\underline{\Gamma(1+\epsilon)} = \epsilon\,\Gamma(\epsilon) \qquad \Gamma(\epsilon) = \frac{1}{\epsilon}\Gamma(1+\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon)$$

$$\Gamma(-1+\epsilon)(-1+\epsilon) = \Gamma(\epsilon)$$

$$\Gamma(1+\epsilon) = 1 + \left.\frac{d\Gamma}{dz}\right|_{z=1} \epsilon + \cdots \qquad \Gamma(-1+\epsilon) = \frac{1}{-1+\epsilon}\,\frac{1}{\epsilon}\,\Gamma(1+\epsilon)$$

$$= 1 - \gamma\epsilon + \ldots \qquad \Gamma(-k+\epsilon) = \frac{(-)^k}{\epsilon} - C_k + O(\epsilon)$$

$$I(M^2) = \frac{1}{2} i \, \Omega_{n-1} \, (M^2)^{(n+2A-2B)/2} \, \frac{\Gamma\left(\frac{n}{2}+A\right)\Gamma\left(B-\frac{n}{2}-A\right)}{\Gamma(B)}$$

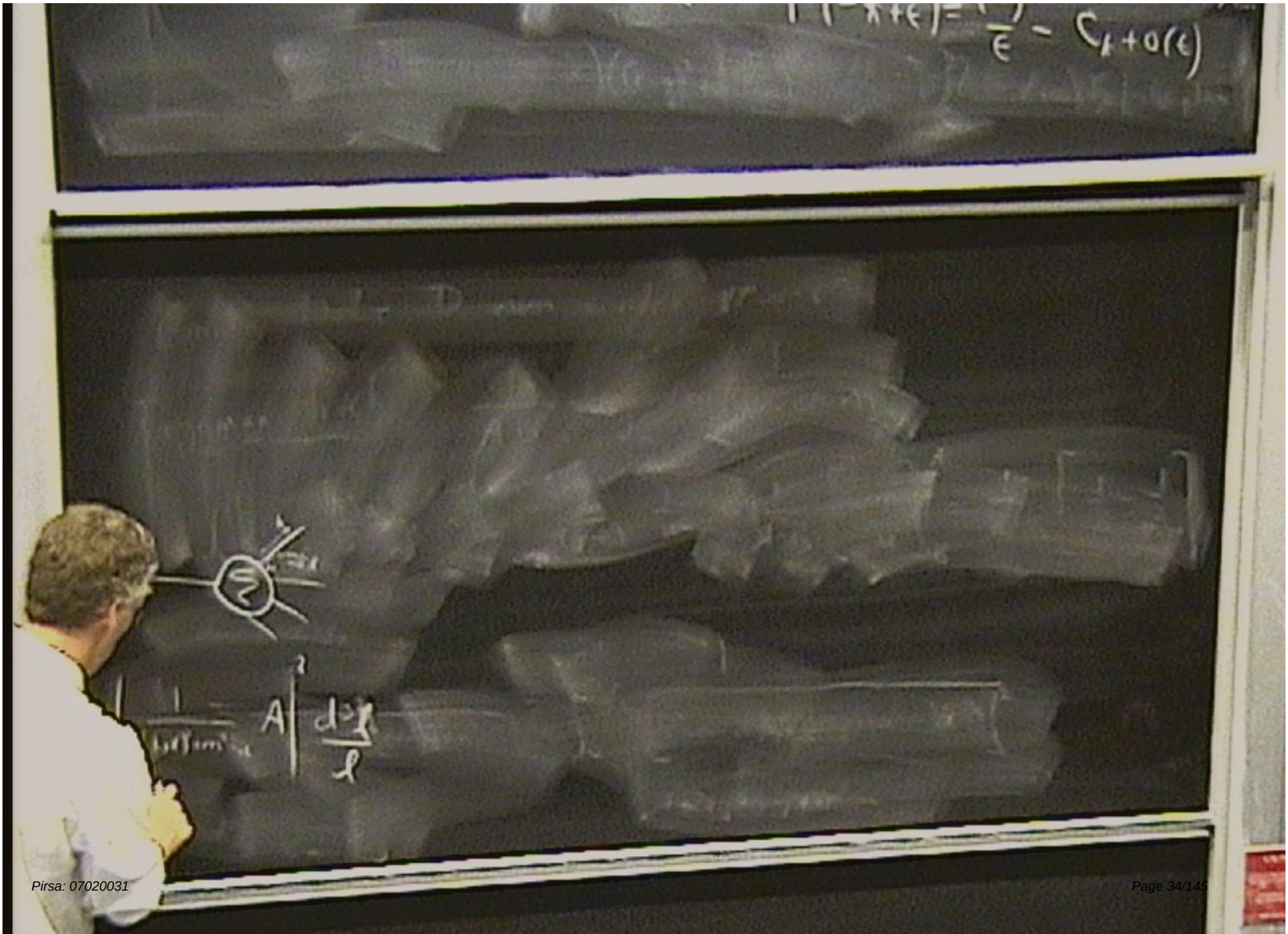
$$= i \pi^{n/2} \frac{\Gamma\left(A+\frac{n}{2}\right)\Gamma\left(\overbrace{B-A-\frac{n}{2}}^{0,-2}\right)}{\Gamma\left(\underbrace{\frac{n}{2}}_{2}\right)\Gamma(B)} (M^2)^{\underbrace{A-B+n/2}_{2,\,0,}}$$

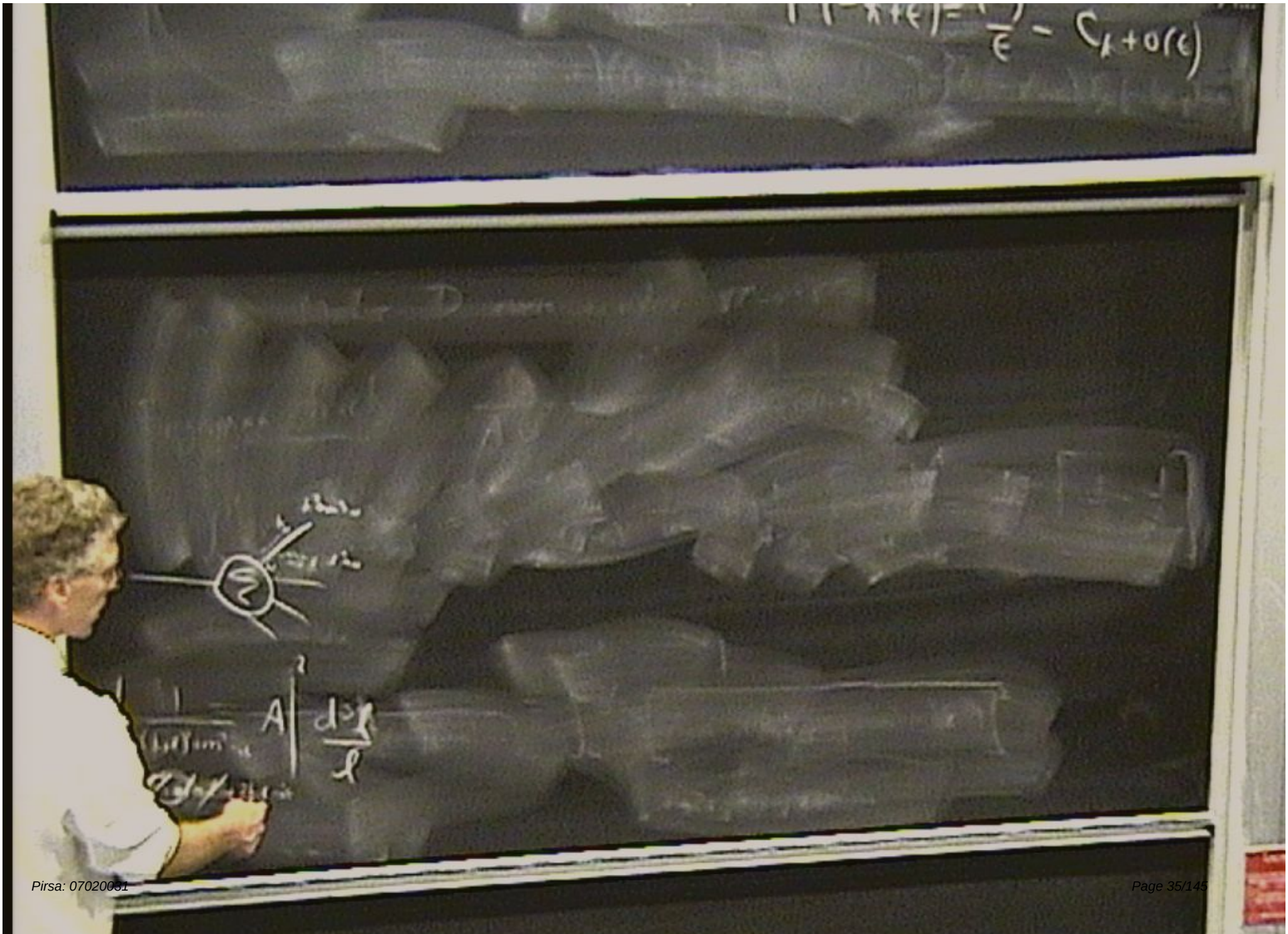
$A = 0, 1$

$B = 2$

As $n \to 4$

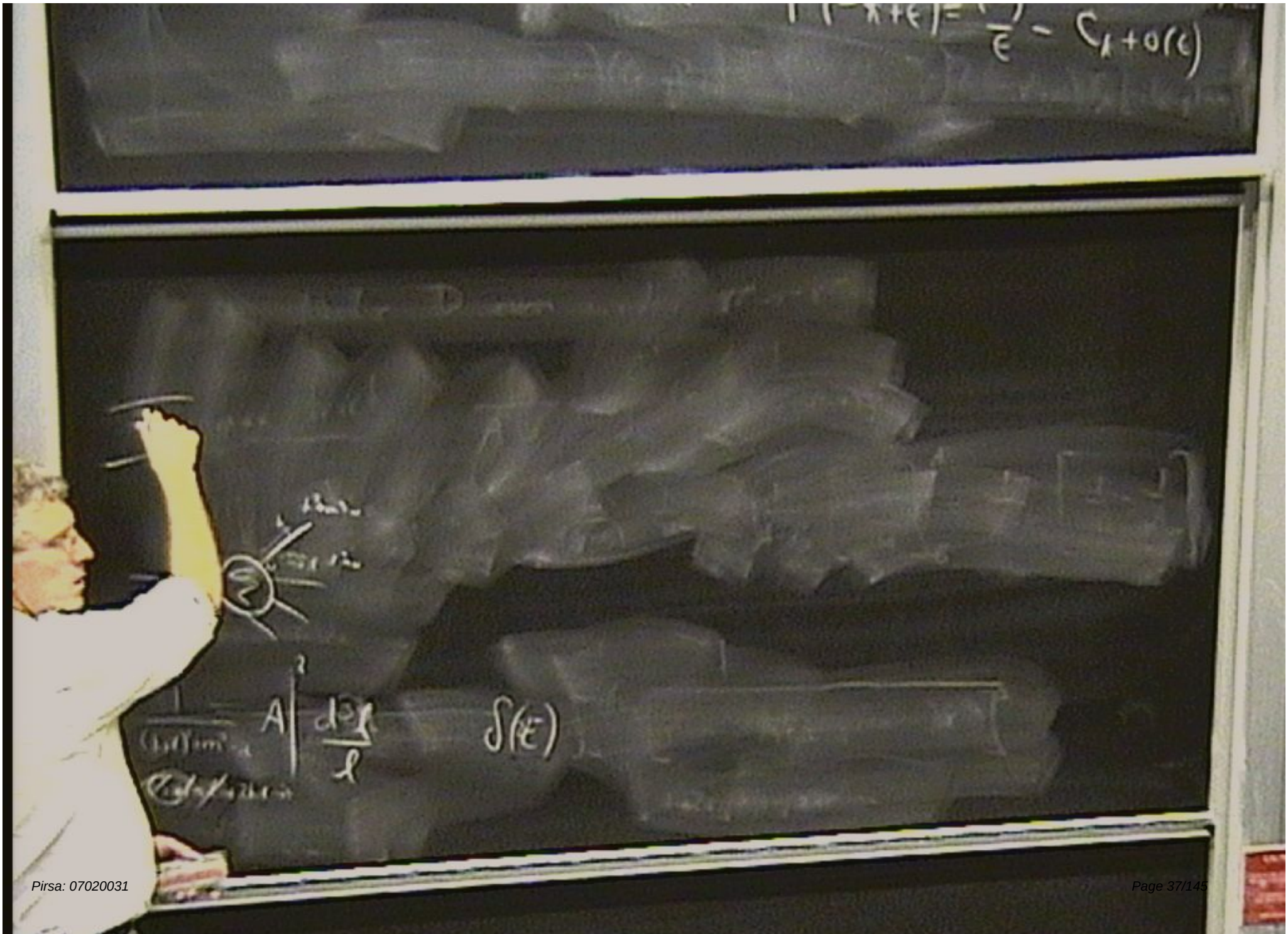
$\Gamma(z)$ has simple poles at the non-positive integers $z = 0, -1, -2, -3, \ldots$


$\frac{1}{\epsilon} - C_k + O(\epsilon)$
$A \int \frac{d^2 x}{\ell}$


$\frac{1}{\epsilon} - C_\lambda + O(\epsilon)$
A

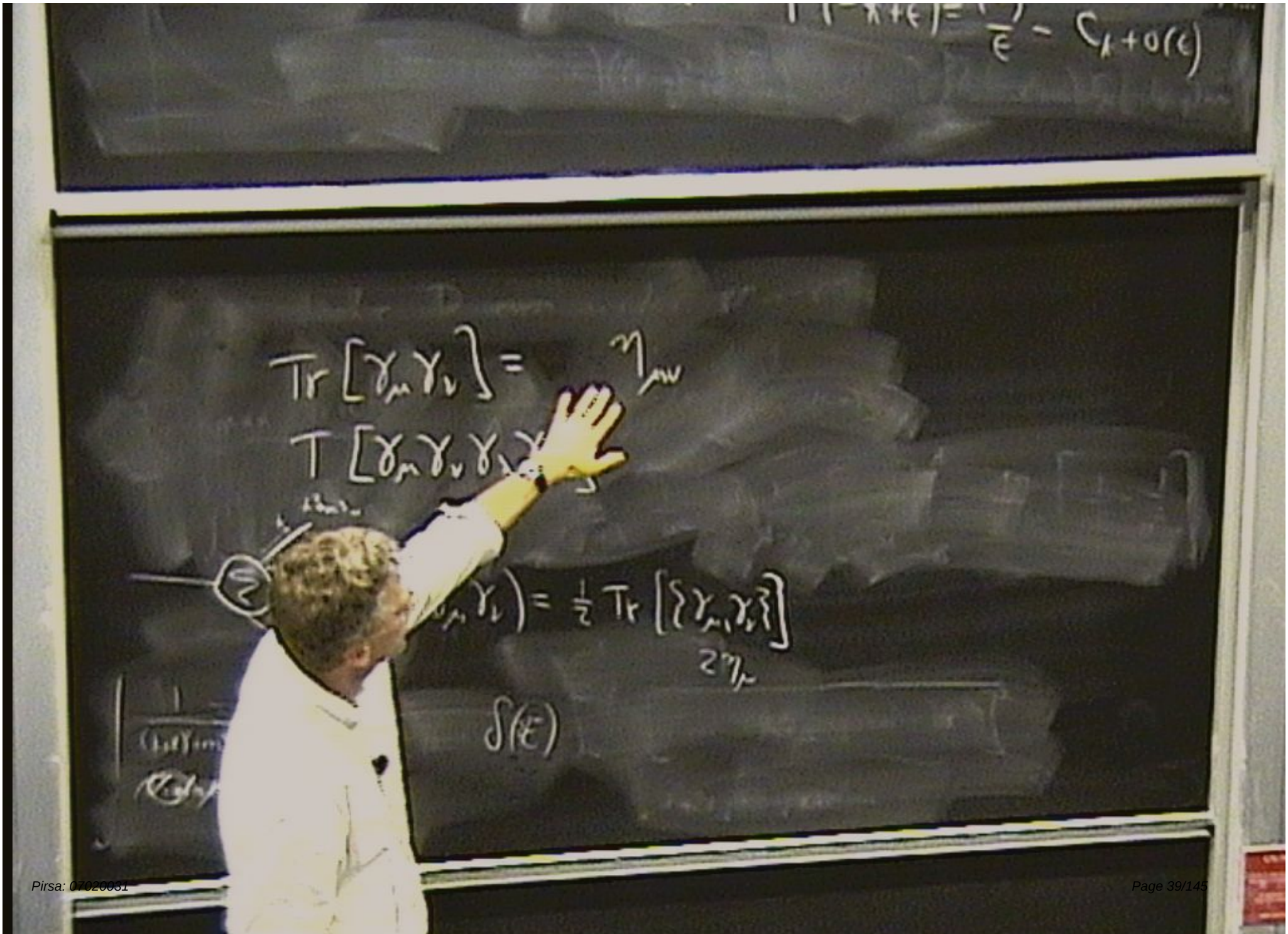
$$\Gamma(-\lambda+\epsilon) = \frac{1}{\epsilon} - C_\lambda + O(\epsilon)$$

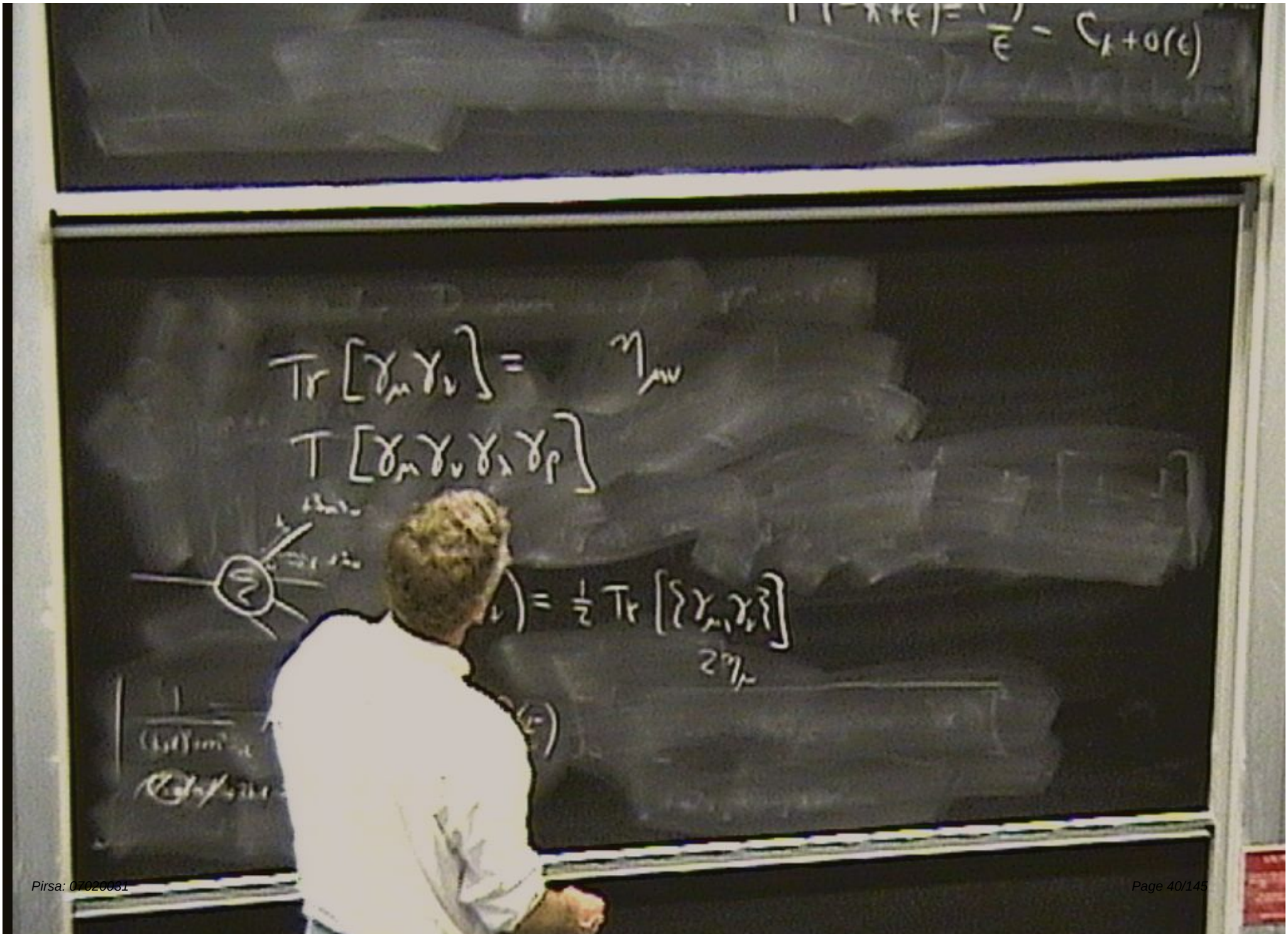
$$\left| \frac{1}{(4\pi)^{D/2}\,\Gamma(\lambda)} A \right|^2 \frac{d^D p}{\ell}$$

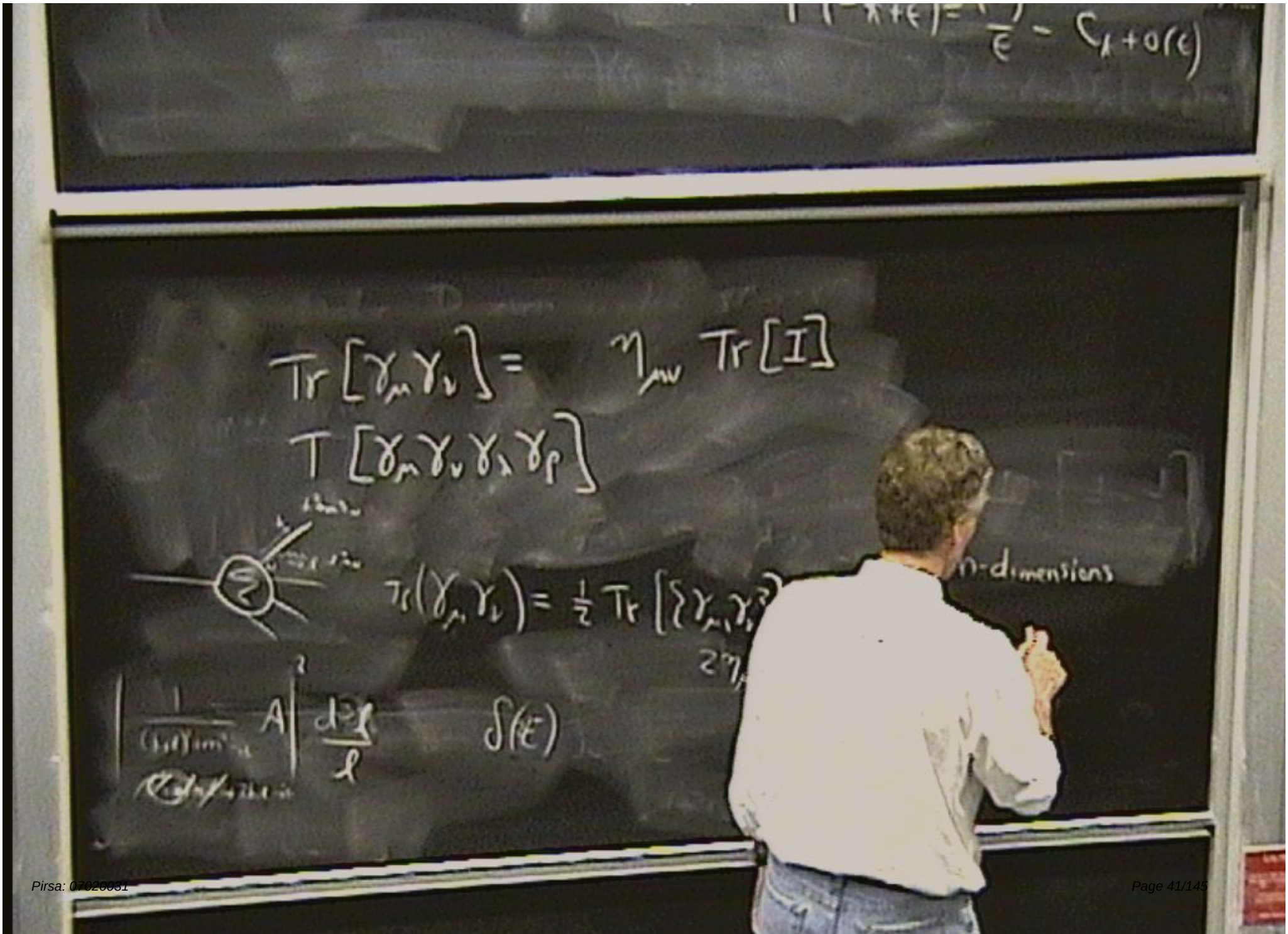

$\Gamma(-x+\epsilon) = \frac{1}{\epsilon} - C_x + O(\epsilon)$
$A \int \frac{d^2 l}{l}$
$\delta(\epsilon)$

$$\Gamma(1-x+\epsilon) = \frac{1}{\epsilon} - C_x + O(\epsilon)$$

$$\left| \frac{1}{(\ldots)^{\ldots}} A \right|^2 \frac{d^{D}\ell}{\ell} \qquad \delta(\epsilon)$$


Tr [γμ γν] = ημν
T [γμ γν γλ
γμ γν) = 1/2 Tr [{γμ, γν}]
2ημν
δ(ε)


Tr [γμ γν] = ημν
T [γμ γν γλ γρ]
= 1/2 Tr [{γμ, γν}]
2ημν


Tr[γμ γν] = ημν Tr[I]
T[γμ γν γλ γρ]
Tr(γμ γν) = ½ Tr[{γμ γν
n-dimensions
δ(ε)

$$\Gamma(-x+\epsilon) = \frac{(\ )}{\epsilon} - C_x + O(\epsilon)$$

$$\mathrm{Tr}[\gamma_\mu \gamma_\nu] = \eta_{\mu\nu}\, \mathrm{Tr}[I]$$

$$\mathrm{T}[\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho]$$

$$\mathrm{Tr}(\gamma_\mu \gamma_\nu) = \frac{1}{2}\mathrm{Tr}[\underbrace{\{\gamma_\mu, \gamma_\nu\}}_{2\eta_{\mu\nu}}]$$

In n-dimensions

a spinor $2^{n/2}$

(eg n=4

$$\frac{1}{(\ )^{\ }} A \int \frac{d^2 l}{l} \qquad \delta(\epsilon)$$

$$\Gamma(-x+\epsilon) = \frac{1}{\epsilon} - C_x + O(\epsilon)$$

$$\mathrm{Tr}[\gamma_\mu \gamma_\nu] = \eta_{\mu\nu}\, \mathrm{Tr}[I]$$

$$\mathrm{T}[\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho]$$

$$\mathrm{Tr}(\gamma \ldots \mathrm{Tr}[\{\gamma_\mu, \gamma_\nu\}]$$

$$2\eta_\mu$$

$$\left| \frac{1}{(\ldots)^{\ldots}} A \right|^2 \frac{d^D}{\ldots} \ldots (\epsilon)$$

In n-dimensions

a spinor $2^{n/2} = D$

$(g, n=4 \quad D=4)$

$$\Gamma(-x+\epsilon) = \frac{1}{\epsilon} - C_x + O(\epsilon)$$

$$\mathrm{Tr}[\gamma_\mu \gamma_\nu] = \eta_{\mu\nu}\,\mathrm{Tr}[I] = 2^{n/2}\,\eta_{\mu\nu}$$

$$\mathrm{T}[\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho] = (\mathrm{Tr}\, I)(\eta_{\mu\nu}\eta_{\lambda\rho} - \eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda})$$

$$\mathrm{Tr}(\gamma \ldots = \mathrm{Tr}[\{\gamma_\mu, \gamma_\nu\}]$$

$$2\eta_{\mu\nu}$$

$$\left| \frac{1}{(\ldots)^{\ldots}} A \right|^2 \frac{d^2 l}{l}$$

In n-dimensions

a spinor $2^{n/2} = D$

(e.g. $n=4$ $D=4$)

$$\Gamma(-k+\epsilon) = \frac{(\ldots)}{\epsilon} = C_k + O(\epsilon)$$

$$\mathrm{Tr}[\gamma_\mu \gamma_\nu] = \eta_{\mu\nu}\, \mathrm{Tr}[I] = 2^{n/2}\, \eta_{\mu\nu}$$

$$\mathrm{T}[\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho] = (\eta_{\mu\nu}\eta_{\lambda\rho} - \eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda})$$

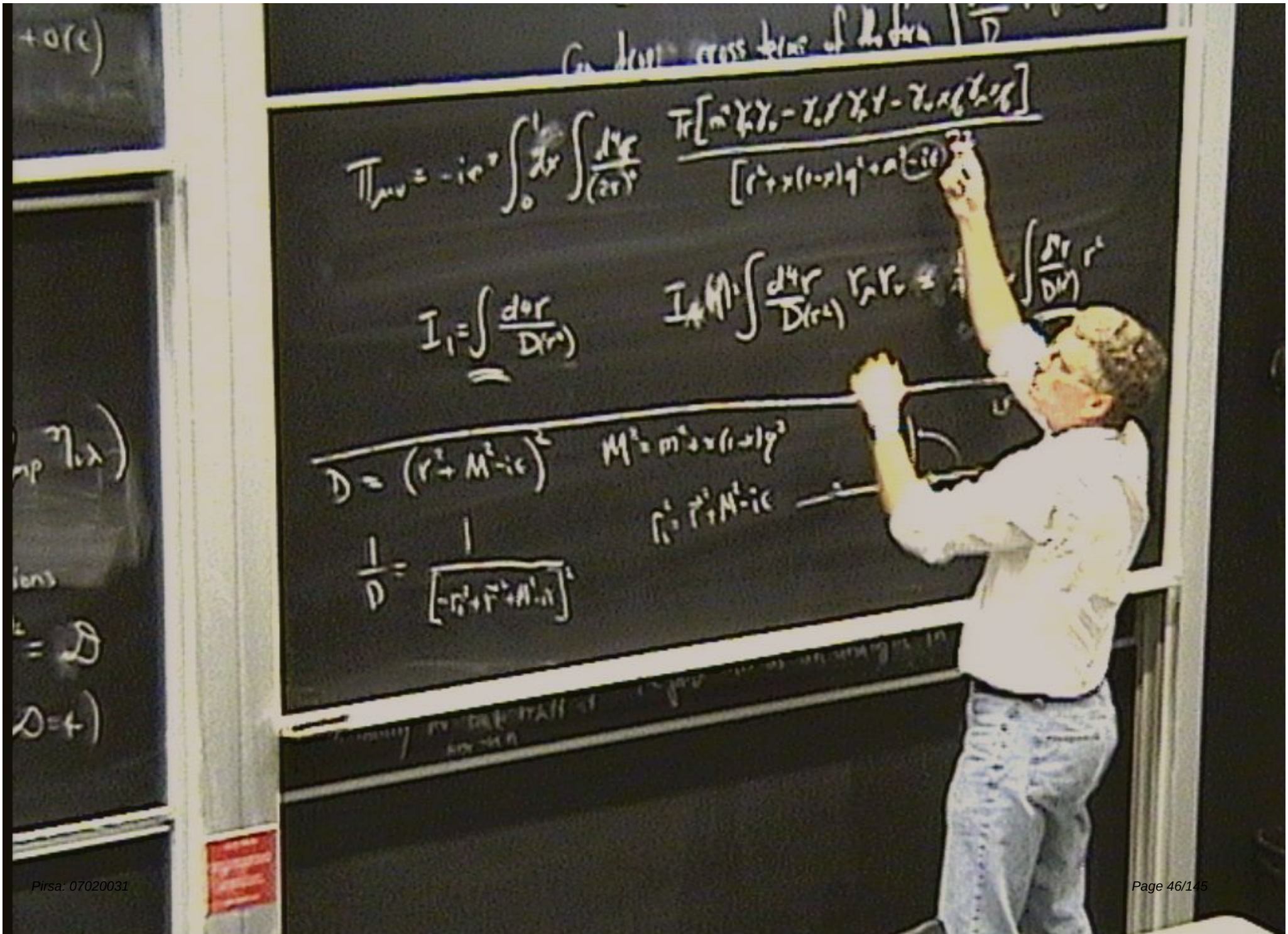
$$\mathrm{Tr}(\gamma_\mu \gamma_\nu) = \frac{1}{2}\mathrm{Tr}[\underbrace{\{\gamma_\mu, \gamma_\nu\}}_{2\eta_{\mu\nu}}]$$

$$\left| \frac{1}{(\ldots)^{\ldots}} A \right|^2 \frac{d^{2-\epsilon}\ell}{\ell} \qquad \delta(\epsilon)$$

In n-dimensions

a spinor $2^{n/2} = D$

(eg $n=4$ $D=4$)


$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4r}{(2\pi)^4} \frac{\mathrm{Tr}[\ldots]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$
$I_1 = \int \frac{d^4r}{D(r)}$
$D = (r^2 + M^2 - i\epsilon)^2$
$M^2 = m^2 + x(1-x)q^2$
$\frac{1}{D} = \frac{1}{[-r_0^2 + \vec{r}^2 + M^2 - i\epsilon]^2}$

$$\mathrm{Tr}[\gamma_\mu \gamma_\nu] = \eta_{\mu\nu}\,\mathrm{Tr}[I] = 2^{n/2}\,\eta_{\mu\nu}$$

$$\mathrm{T}[\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho] = (\eta_{\mu\nu}\eta_{\lambda\rho} - \eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda})$$

$$\mathrm{Tr}(\gamma_\mu \gamma_\nu) = \frac{1}{2}\mathrm{Tr}\left[\underbrace{\{\gamma_\mu, \gamma_\nu\}}_{2\eta_\mu}\right]$$

$$\left|\frac{1}{(\ldots)}\,A\right|^2 \frac{d^3 p}{p} \qquad \delta(E)$$

In n-dimensions

a spinor $2^{n/2} = D$

(eg $n=4$ $D=4$)

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{k} \gamma_\mu \not{k} - \gamma_\nu \not{k}\not{q} \gamma_\mu \not{k}\not{q}\right]}{\left[k^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{k} \gamma_\nu \not{k} - \gamma_\mu \not{k} \gamma_\nu \not{k}\right]}{\left[k^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= -ie^2$$

Can drop cross terms of the form

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 q}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{q} \gamma_\mu \not{q} - \gamma_\nu \not{q} \gamma_\mu \not{q}\right]}{\left[q^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= ie^2 \int_0^1 dx \int \frac{d^n p}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(p^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) p^2 \eta_{\mu\nu} + x(1-x)\left(2 q_\mu q_\nu - \right. \right.$$

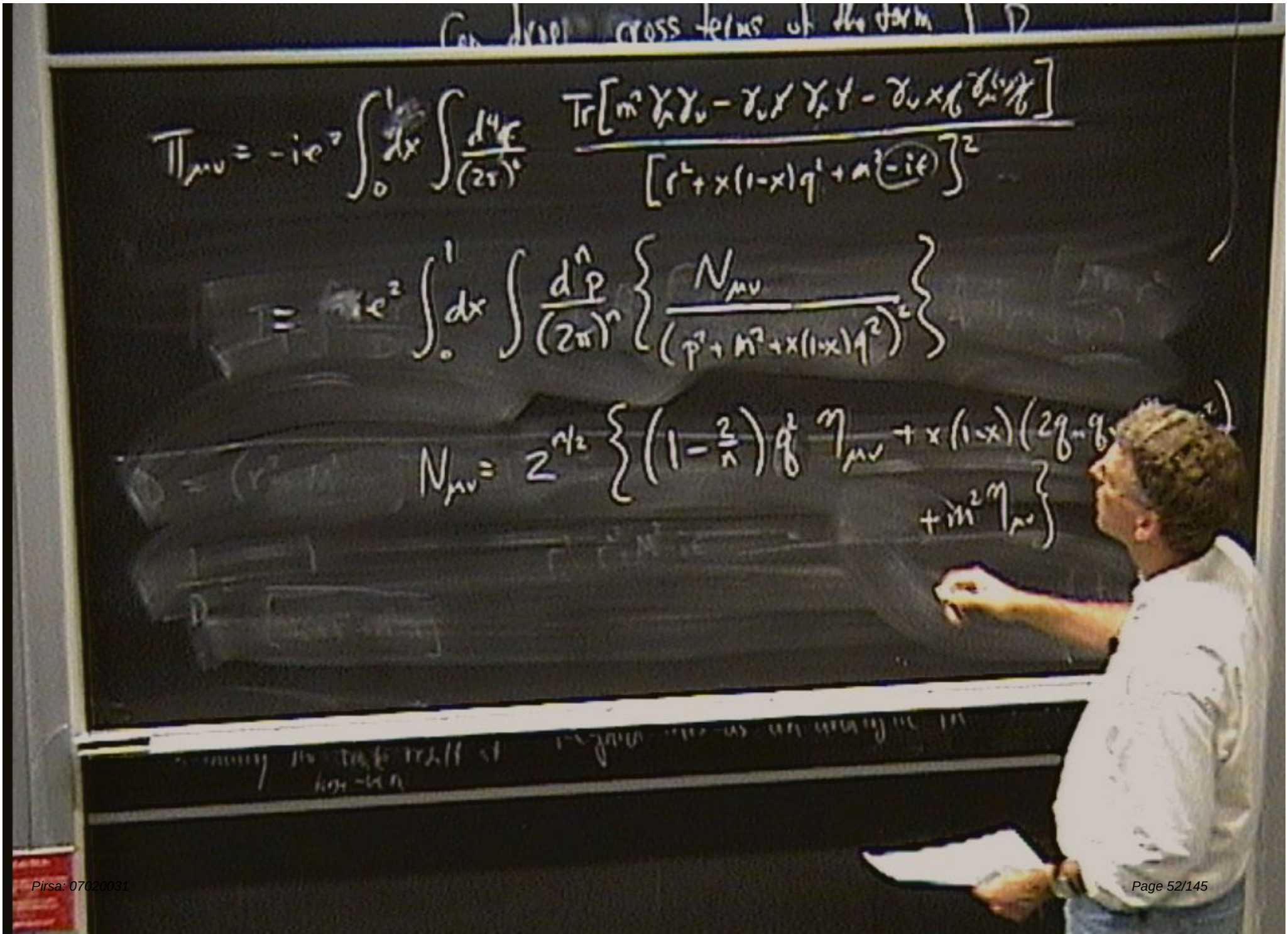
$$\left. + m^2 \eta_{\mu\nu} \right\}$$

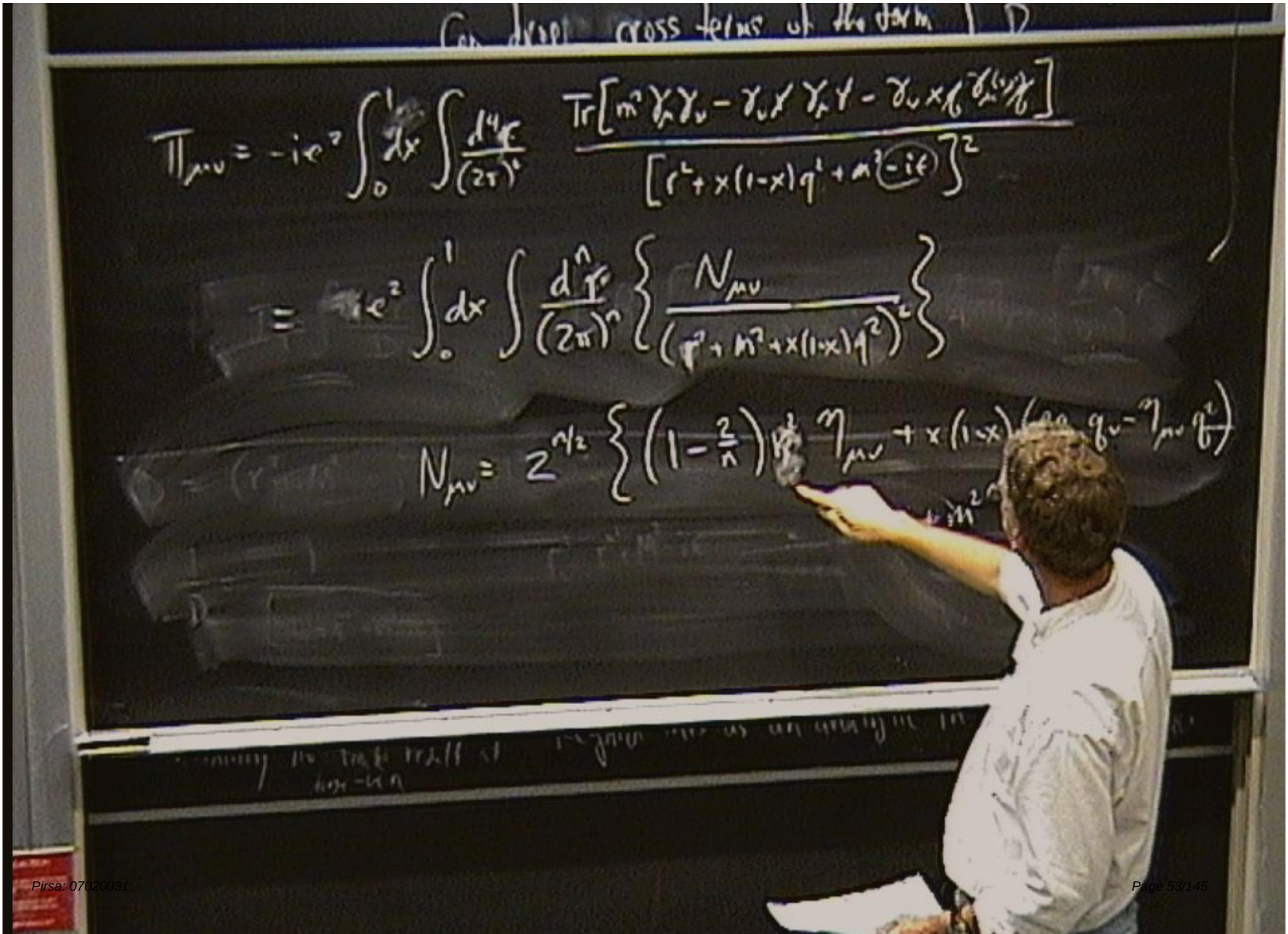
Can drop cross terms of the form

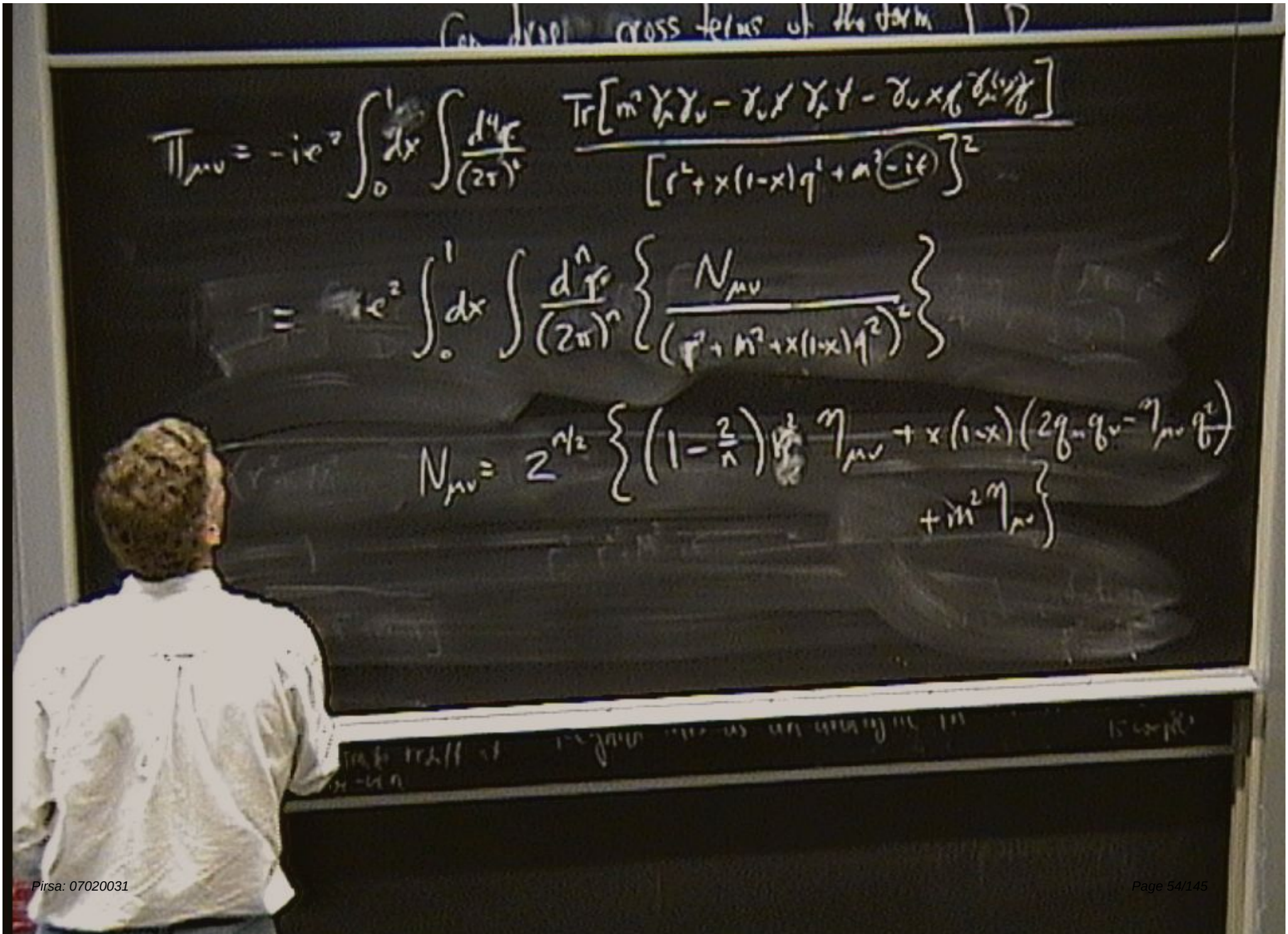
$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 p}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{q} \gamma_\mu \not{q} - \gamma_\nu \not{p} \gamma_\mu \not{p}\right]}{\left[p^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= ie^2 \int_0^1 dx \int \frac{d^n p}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(p^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) q^2 \eta_{\mu\nu} + x(1-x)\left(2 q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + m^2 \eta_{\mu\nu} \right\}$$


$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[m^2\gamma_\mu\gamma_\nu - \gamma_\nu \not{k}\gamma_\mu \not{k} - \gamma_\nu \not{k}\gamma_\mu \not{q}]}{[k^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$
$= ie^2 \int_0^1 dx \int \frac{d^n p}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{(p^2 + m^2 + x(1-x)q^2)^2} \right\}$
$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) p^2 \eta_{\mu\nu} + x(1-x)(2q_\mu q_\nu \ldots) + m^2 \eta_{\mu\nu} \right\}$


$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[m^2\gamma_\mu\gamma_\nu - \gamma_\nu \not{k}\gamma_\mu \not{k} - \gamma_\nu \not{k}\gamma_\mu \not{q}]}{[k^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$
$= ie^2 \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{(k^2 + m^2 + x(1-x)q^2)^2} \right\}$
$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)(q_\mu q_\nu - \eta_{\mu\nu} q^2) \right.$


$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[m^2\gamma_\mu\gamma_\nu - \gamma_\nu \not{k}\gamma_\mu \not{k} - \gamma_\nu \not{k}\gamma_\mu \not{q}]}{[k^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$
$= ie^2 \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{(k^2 + m^2 + x(1-x)q^2)^2} \right\}$
$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)(2q_\mu q_\nu - \eta_{\mu\nu} q^2) + m^2 \eta_{\mu\nu} \right\}$

Can drop cross terms of the form

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{k} \gamma_\mu \not{k} - \gamma_\nu \not{k} \gamma_\mu \not{q}\right]}{\left[k^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= ie^2 \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(k^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + m^2 \eta_{\mu\nu} \right\}$$

$$= \mu^{4-n} \int_0^1 dx \int \frac{d^n q}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{(q^2 + M^2 + x(1-x)q^2)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + M^2 \eta_{\mu\nu} \right\}$$

converges

suff

Define

$$J = \Omega_{n-1} \int_0^\infty dr \frac{r^{n-1-2A}}{(r^2 + M^2)^B}$$

Regard this as an analytic fn of n where n is complex

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 q}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{q} \gamma_\nu \not{q} - \gamma_\mu \not{q} \gamma_\nu \not{q}\right]}{\left[q^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= e^2 \int_0^1 dx \int \frac{d^n q}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(q^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

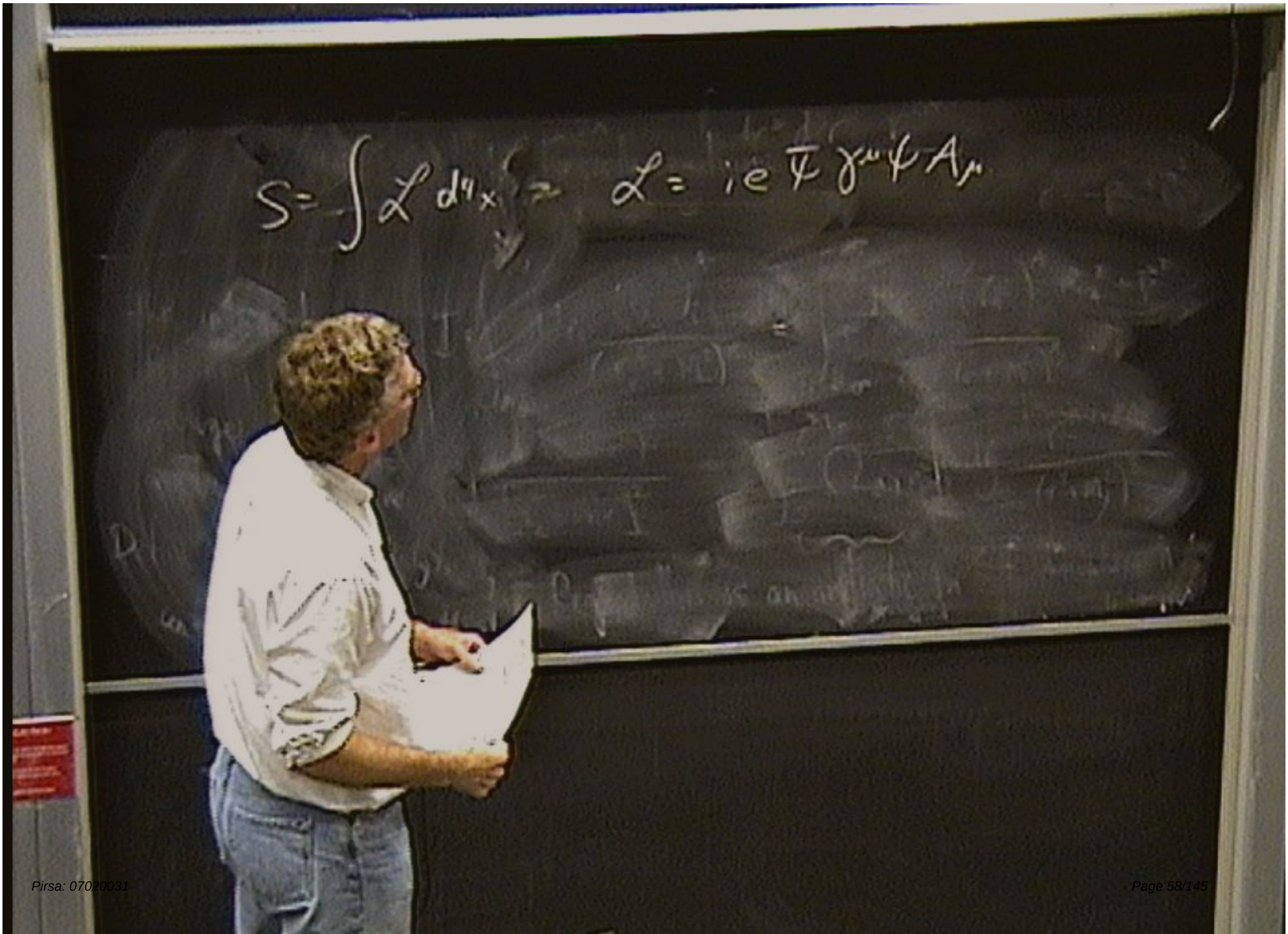
$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + m^2 \eta_{\mu\nu} \right\}$$

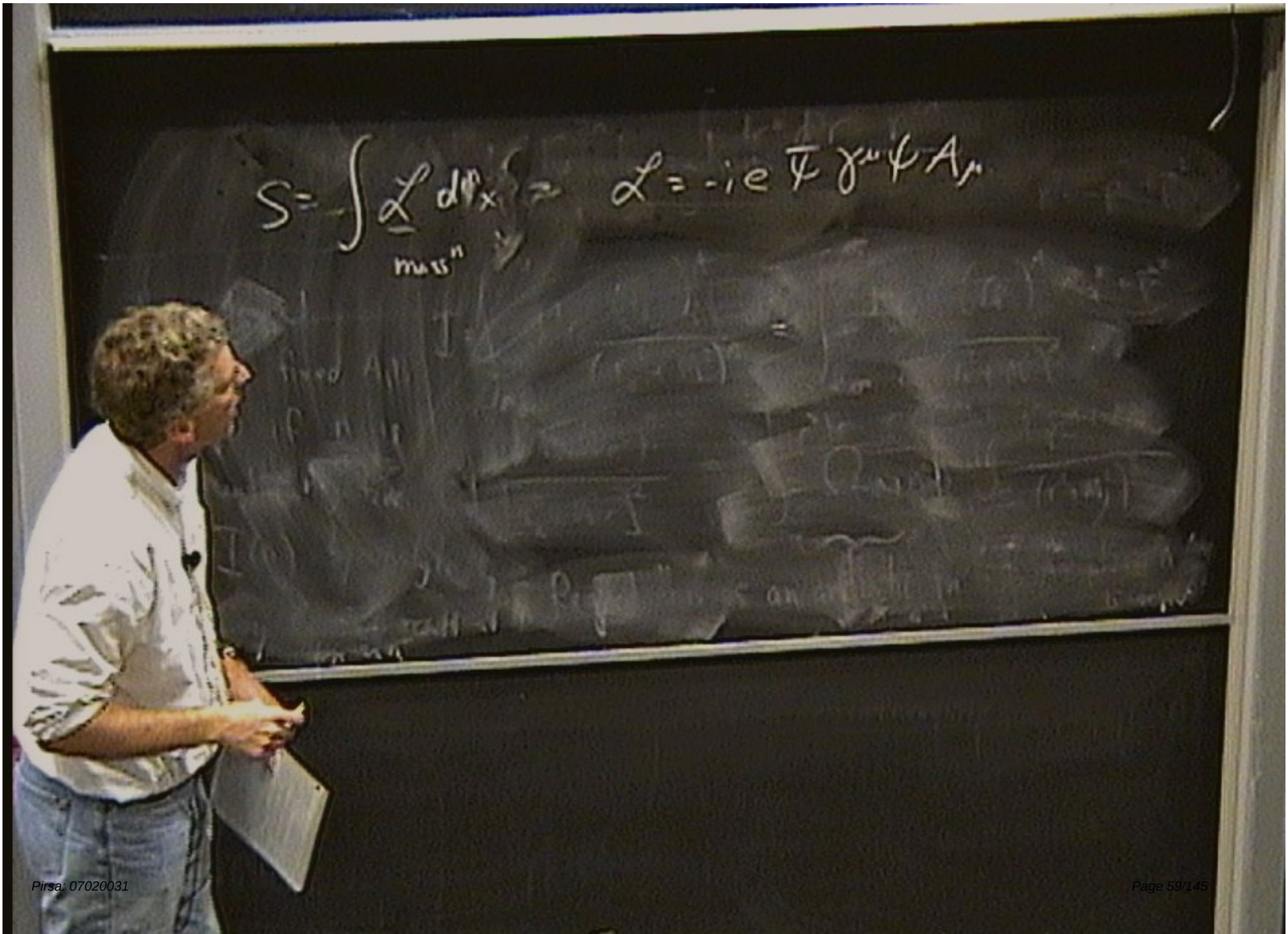
$$I = \Omega_{n-1} \int_0^\infty dr \frac{r^{n-1+2\Delta}}{(r^2+M^2)^b}$$

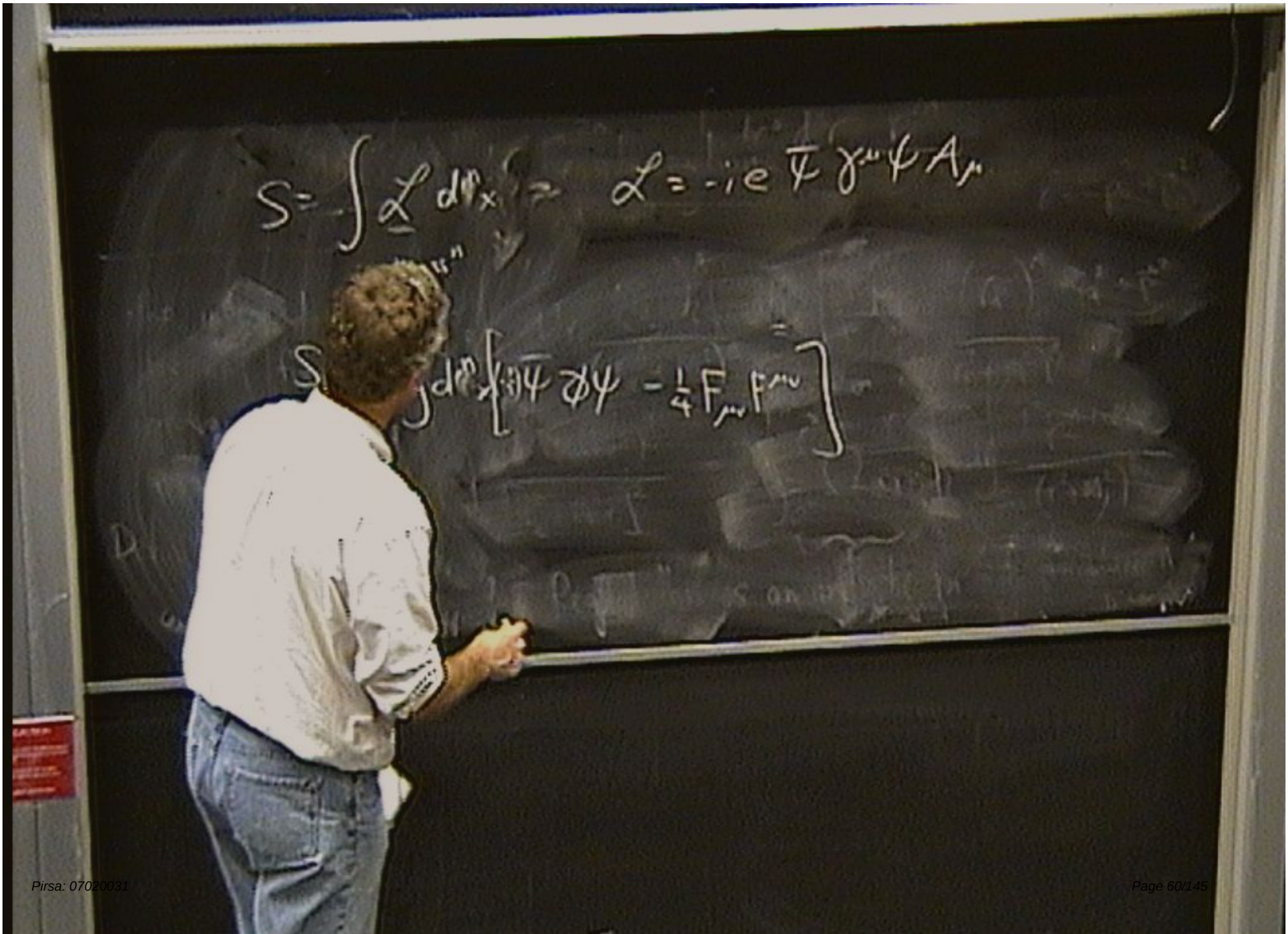
Def

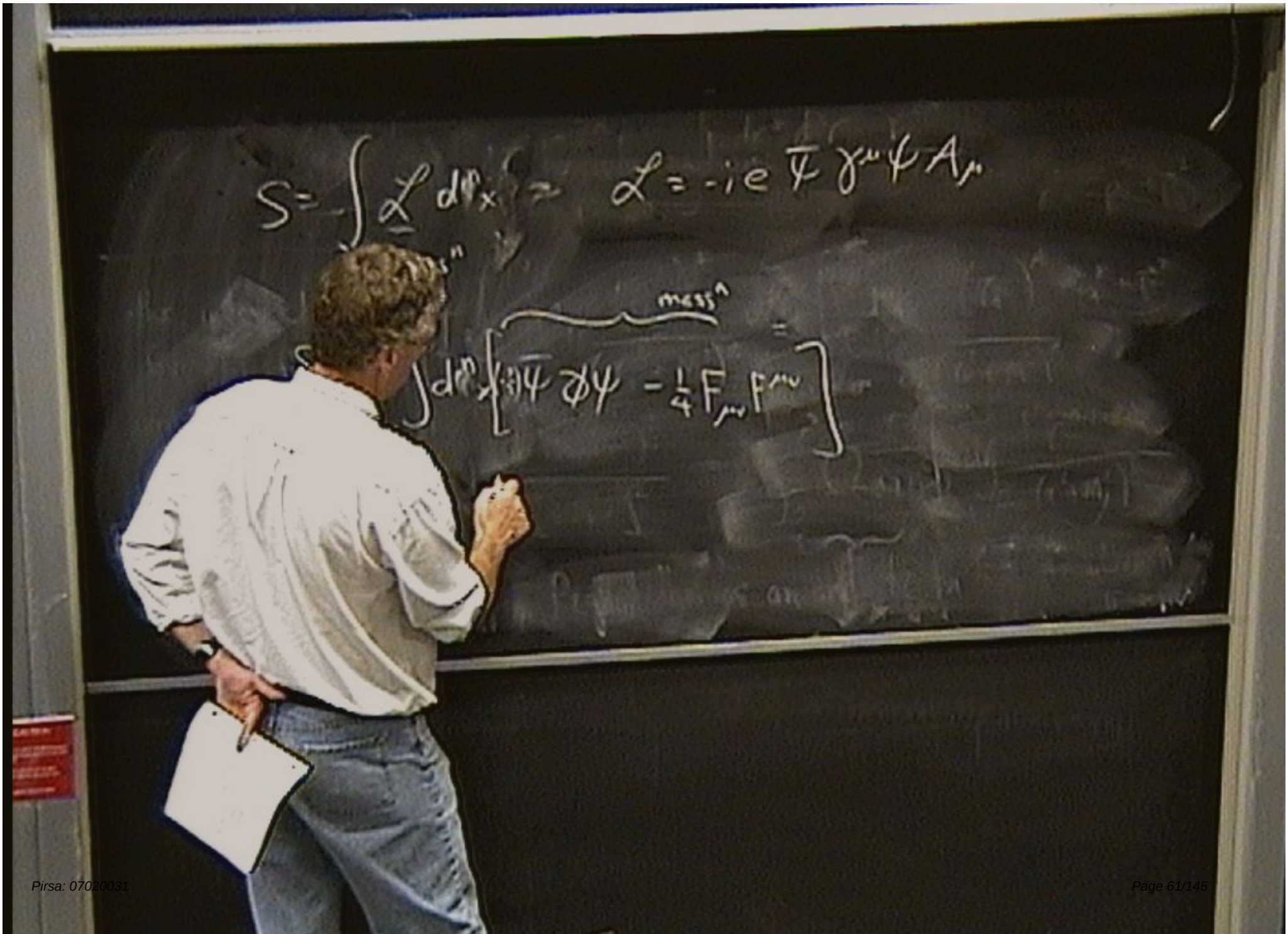
ytically

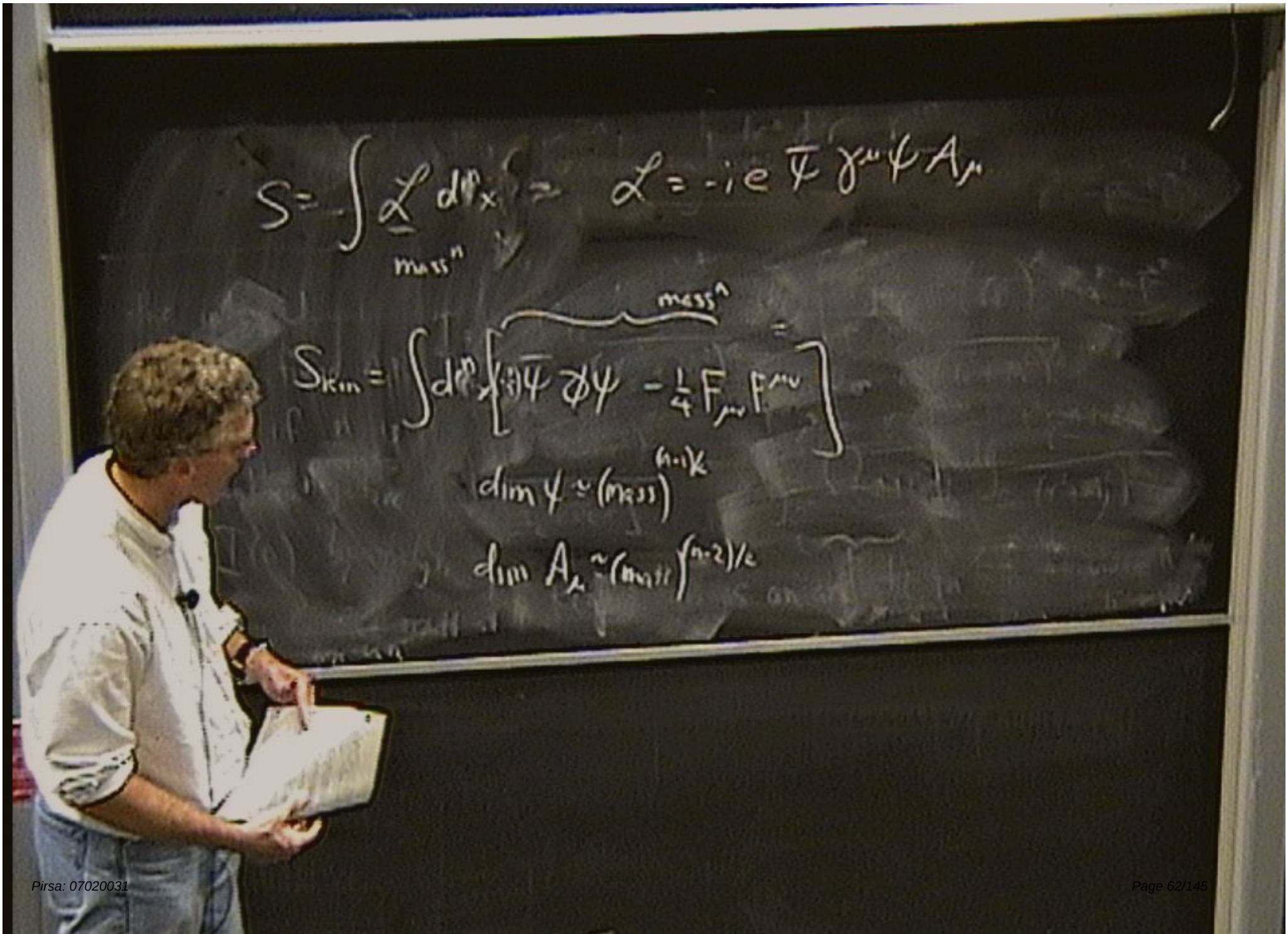
Regard this as an analytic fn of n where n is some


$S = \int \mathcal{L}\, d^4x$
$\mathcal{L} = ie\bar{\psi}\gamma^\mu\psi A_\mu$


$S = \int \mathcal{L}\, d^4x$
mass^n
$\mathcal{L} = -ie\bar{\psi}\gamma^\mu\psi A_\mu$


$S = \int \mathcal{L}\, d^4x \quad \mathcal{L} = -ie\bar{\psi}\gamma^\mu\psi A_\mu$
$S = \int d^4x \left[i\bar{\psi}\not{\partial}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right]$


$S = \int \mathcal{L}\, d^n x \Rightarrow \mathcal{L} = -ie\bar{\psi}\gamma^\mu\psi A_\mu$
mass^n
$\int d^n x \left[\bar{\psi}\not{\partial}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right]$


$S = \int \mathcal{L}\, d^n x \Rightarrow \mathcal{L} = -ie\bar{\psi}\gamma^\mu\psi A_\mu$
mass^n
mass^n
$S_{kin} = \int d^n x \left[(i)\bar{\psi}\not{\partial}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right]$
$\dim\psi \sim (\text{mass})^{(n-1)/2}$
$\dim A_\mu \sim (\text{mass})^{(n-2)/2}$

$$S = \int \underbrace{\mathcal{L}}_{\text{mass}^n} d^n x \quad \mathcal{L} = -ie\bar{\psi}\gamma^\mu\psi A_\mu$$

$$S_{kin} = \int d^n x \overbrace{\left[i\bar{\psi}\not{\partial}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right]}^{\text{mass}^n}$$

$$\left.\begin{aligned} \dim\psi &\sim (\text{mass})^{(n-1)/2} \\ \dim A_\mu &\sim (\text{mass})^{(n-2)/2} \end{aligned}\right\} \bar{\psi}\gamma^\mu\psi A_\mu \quad (\text{mass})^{n-1+(n-2)} \sim (\text{mass})$$

$$S = \int \underbrace{\mathcal{L}}_{\text{mass}^n} d^n x \qquad \mathcal{L} = -ie\,\bar{\psi}\gamma^\mu\psi A_\mu$$

$$S_{kin} = \int d^n x \Big[\overbrace{(i\partial\!\!\!/)\bar{\psi}\,\partial\!\!\!/\psi}^{\text{mass}^n} - \tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} \Big]$$

$$\dim \psi \simeq (\text{mass})^{(n-1)/2}$$

$$\dim A_\mu \simeq (\text{mass})^{(n-2)/2}$$

$$\bar{\psi}\gamma^\mu\psi A_\mu$$

$$(\text{mass})^{n-1+(n-1)} = (\text{mass}$$

$$S = \int \underbrace{\mathcal{L}}_{\text{mass}^n} d^n x \quad \Rightarrow \quad \mathcal{L} = -ie\,\bar{\psi}\gamma^\mu\psi A_\mu$$

$$S_{kin} = \int d^n x \Big[\overbrace{(i\partial\!\!\!/)\bar{\psi}\,\partial\!\!\!/\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}^{\text{mass}^n} \Big]$$

$$\dim \psi \simeq (\text{mass})^{(n-1)/2}$$

$$\dim A_\mu \simeq (\text{mass})^{(n-2)/2}$$

$$e\bar{\psi}\gamma^\mu\psi A_\mu \qquad (\text{mass})^{n-1+(n-1)}$$

$$\dim e = (\text{ma}$$

$$S = \int \underbrace{\mathcal{L}}_{\text{mass}^n} d^n x \quad \Rightarrow \quad \mathcal{L} = -ie\,\bar{\psi}\gamma^\mu\psi A_\mu$$

$$S_{kin} = \int d^n x \Big[\overbrace{(i)\bar{\psi}\,\partial\!\!\!/\,\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}^{\text{mass}^n} \Big]$$

$$\dim e = (\text{mass})^{n-(\frac{n}{2}-1)} = (\text{mass})$$

$$\dim \psi \simeq (\text{mass})^{(n-1)/2}$$

$$\dim A_\mu \simeq (\text{mass})^{(n-2)/2}$$

$$e\bar{\psi}\gamma^\mu\psi A_\mu \qquad (\text{mass})^{n-1+(\frac{n}{2}-1)} \simeq (\text{mass})^{3-\frac{n}{2}=2}$$

$$S = \int \underbrace{\mathcal{L}}_{\text{mass}^n} d^n x \quad \Rightarrow \quad \mathcal{L} = -ie\,\bar{\psi}\gamma^\mu\psi A_\mu$$

$$S_{kin} = \int d^n x \Big[\overbrace{(i)\bar{\psi}\,\partial\!\!\!/\,\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}^{\text{mass}^n} \Big]$$

$$\dim e = (\text{mass})^{n-(\frac{1}{2}n-1)} = (\text{mass})^{n/2+2} = (\text{mass})^{(n+4)/2}$$

$$\dim \psi \simeq (\text{mass})^{(n-1)/2}$$

$$\dim A_\mu \simeq (\text{mass})^{(n-2)/2}$$

$$e\bar{\psi}\gamma^\mu\psi A_\mu \qquad (\text{mass})^{n-1+(\frac{1}{2}n-1)} = (\text{mass})^{3n/2-2}$$

$$= e^2 \int_0^1 dx \int \frac{d^n q}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(q^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + m^2 \eta_{\mu\nu} \right\}$$

$$S = \int \underbrace{\mathcal{L}}_{\text{mass}^n} d^n x \quad \mathcal{L} = -ie\bar{\psi}\gamma^\mu\psi A_\mu$$

$$S = \int d^n x \left[\overbrace{\quad}^{\text{mass}^n} \right] \qquad \dim e = (\text{mass})^{n-\left(\frac{n}{2}-1\right)}$$

$$S = \int \underbrace{\mathcal{L}}_{\text{mass}^n} d^n x \quad \mathcal{L} = -ie\bar{\psi}\gamma^\mu\psi A_\mu$$

$$S_{kin} = \int d^n x \Big[\overbrace{(i)\bar{\psi}\,\partial\!\!\!/\,\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}^{\text{mass}^n} \Big]$$

$$\dim e = (\text{mass})^{n-(\ldots)}$$
$$= (\text{mass})^{-n/2+2}$$
$$= (\text{mass})^{(4-n)/2}$$

$$\dim \psi \sim (\text{mass})^{(n-1)/2}$$

$$\dim A_\mu \sim (\text{mass})^{(n-2)/2}$$

$$e\bar{\psi}\gamma^\mu\psi A_\mu$$
$$(\text{mass})^{n-1+(n-1)} = (\text{mass})^{3n/2-2}$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2\gamma_\mu\gamma_\nu - \gamma_\nu \not{k}\gamma_\mu \not{k} - \gamma_\nu \not{k}\gamma_\mu \not{q}\right]}{\left[k^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= \mu^{4-n} e_0^2 \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(k^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + m^2 \eta_{\mu\nu} \right\}$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4\ell}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{\ell} \gamma_\mu \not{\ell} - \gamma_\nu \not{\ell} \not{q} \gamma_\mu \not{q}\right]}{\left[\ell^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= \mu^{4-n} e_0^2 \int_0^1 dx \int \frac{d^n\ell}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(\ell^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \ell^2 \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + m^2 \eta_{\mu\nu} \right\}$$

$$\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \ldots$$

mass^n

$$e = e_0 \mu^{(4-n)/2}$$

$$n = \left(\tfrac{4}{4-\epsilon}\right)$$

$$\dim e = (\text{mass})^{n/2+2}$$

$$S = \int d^n x \left[\overbrace{\sqrt{-g}\, R}^{\text{mass}^n} + \ldots\right]$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4\ell}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{\ell} \gamma_\mu \not{\ell} - \gamma_\nu \not{q} \gamma_\mu \not{\ell}\right]}{\left[\ell^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= \mu^{4-n} e_0^2 \int_0^1 dx \int \frac{d^n\ell}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(\ell^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \ell^2 \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + m^2 \eta_{\mu\nu} \right\}$$

$$e = e_0 \mu^{(4-n)/2}$$

$$\dim e = (\text{mass})^{n/2}$$

$$r^2 - 2r\cdot qx = (r - xq)^2 - x^2 q^2$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu \not{r} \not{q} \gamma_\mu \not{q}\right]}{\left[r^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$= \mu^{4-n} e_0^2 \int_0^1 dx \int \frac{d^n r}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{\left(r^2 + m^2 + x(1-x)q^2\right)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) r^2 \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - \eta_{\mu\nu} q^2\right) + m^2 \eta_{\mu\nu} \right\}$$

$\frac{1}{e^2}(F_\mu, F^\mu \ldots)$

mass^n

$e = e_0 \mu^{(4-n)/2}$

$n = \left(\frac{4}{4-\epsilon}\right)$

$$S = \int d^n x \left[\overbrace{\ldots}^{\text{mass}^n} \right]$$

$\dim e = (\text{mass})^{n/2-2}$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{q-2} f_{\mu\nu}$$

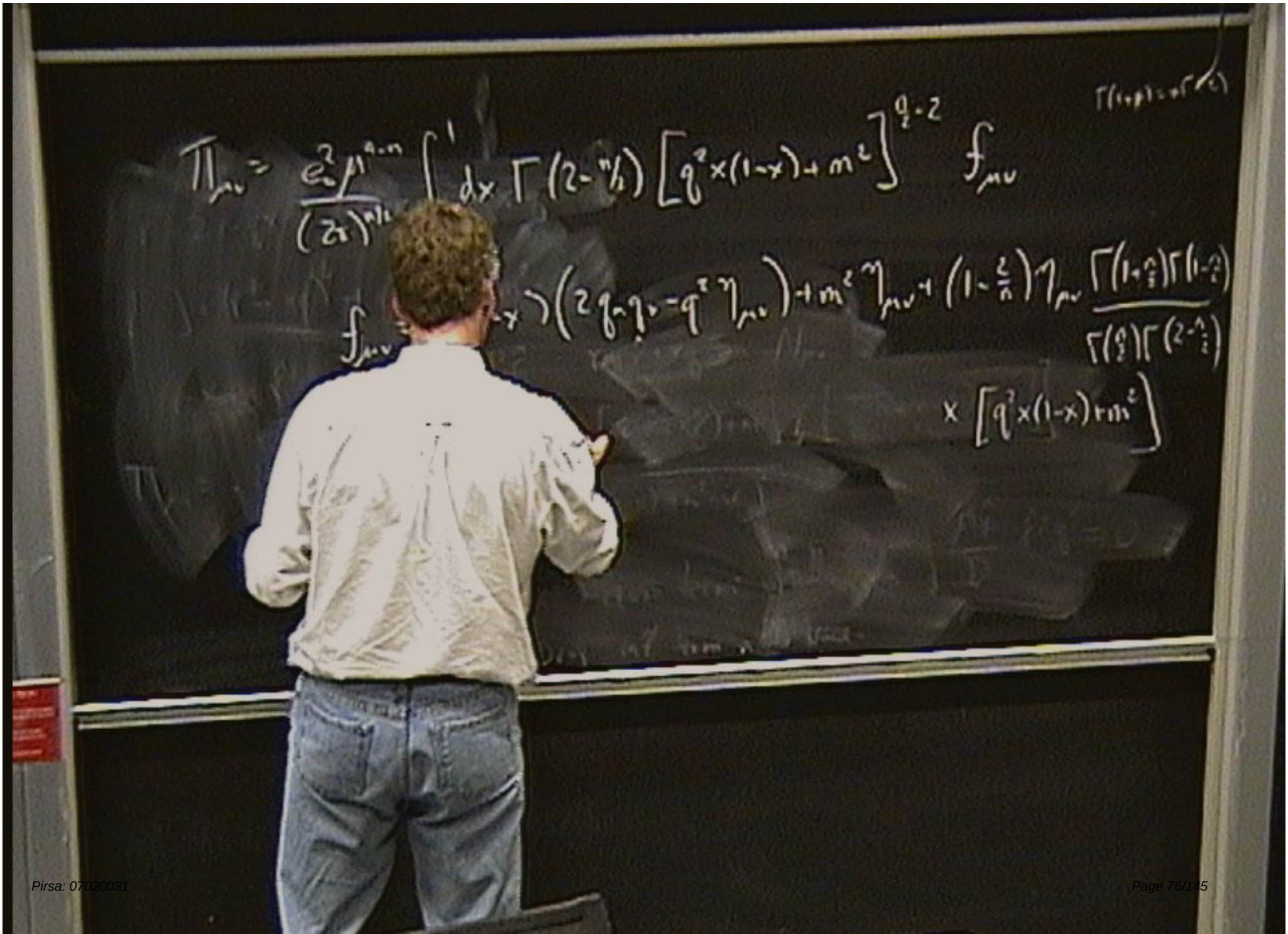
$$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1 - \frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma(1+\frac{n}{2})\Gamma(\ldots}{\Gamma(\frac{n}{2})\Gamma(\ldots}$$

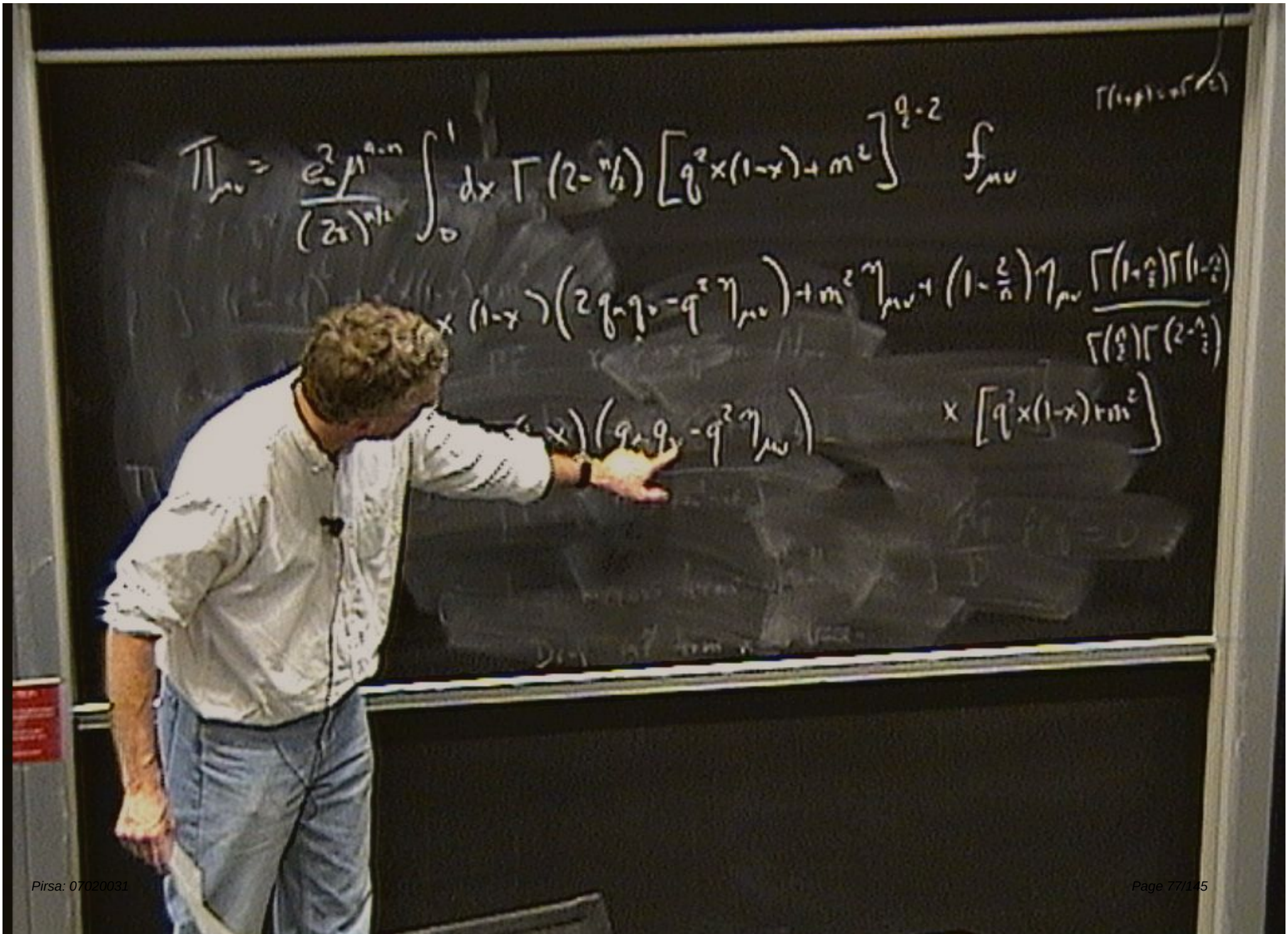
$$\times \left[q^2 x(1-x) + m^2\right]$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1 - \frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma(1+\frac{n}{2})\Gamma(1-n)}{\Gamma(\frac{n}{2})\Gamma(\ldots)}$$

$$\times \left[q^2 x(1-x) + m^2\right]$$


$\Gamma(1+p) = p\Gamma(p)$
$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$
$f_{\mu\nu} = \ldots x)(2q_\mu q_\nu - q^2 \eta_{\mu\nu}) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma(1+\frac{n}{2})\Gamma(1-\frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(2-\frac{n}{2})}$
$\times \left[q^2 x(1-x) + m^2\right]$


$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{q-2} f_{\mu\nu}$
$\times (1-x)(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}) + m^2 \eta_{\mu\nu} + (1-\frac{2}{n}) \eta_{\mu\nu} \frac{\Gamma(1+\frac{n}{2})\Gamma(1-\frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(2-\frac{n}{2})}$
$\times (1-x)(q_\mu q_\nu - q^2 \eta_{\mu\nu})$
$\times \left[q^2 x(1-x) + m^2\right]$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$\Gamma(1+p) = p\Gamma(p)$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma(2)\Gamma\left(2-\frac{n}{2}\right)}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\Gamma(1+p) = p\,\Gamma(p)$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0$$

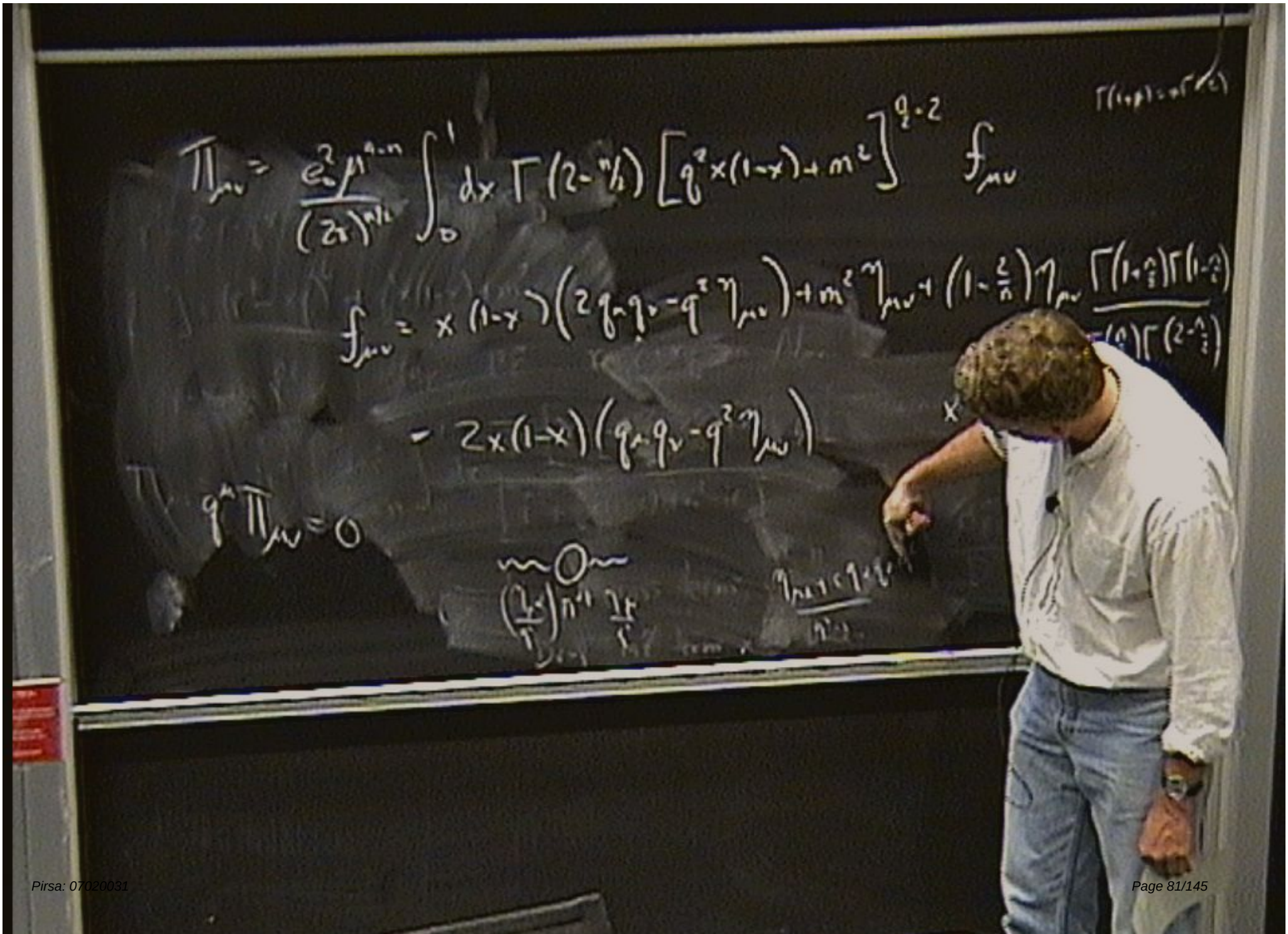
$$\Gamma(1+p) = p\,\Gamma(p)$$

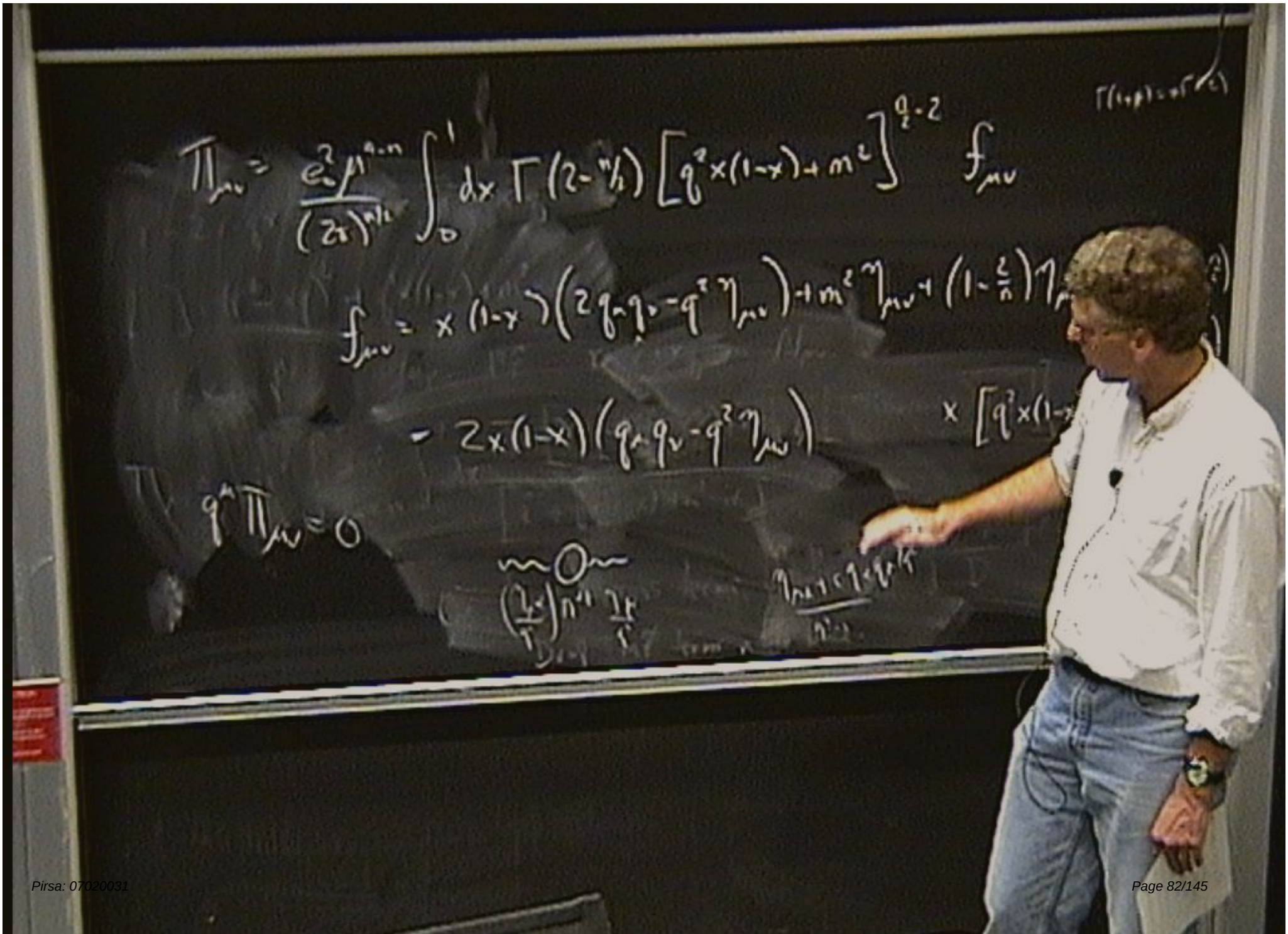
$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{q-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1 - \frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0$$


$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$
$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu}$
$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$
$q^\mu \Pi_{\mu\nu} = 0$


$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{4}{2}-2} f_{\mu\nu}$
$f_{\mu\nu} = x(1-x)(2q_\mu q_\nu - q^2 \eta_{\mu\nu}) + m^2 \eta_{\mu\nu} + (1-\frac{2}{n})\eta$
$- 2x(1-x)(q_\mu q_\nu - q^2 \eta_{\mu\nu})$
$q^\mu \Pi_{\mu\nu} = 0$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1 - \frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma(1+\frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(2-\frac{n}{2})} \times \left[q^2 x(1-x) + m^2\right]$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\left(\frac{2x}{2}\right)^{n+1} \frac{2x}{2}$$

$$\frac{\eta_{n+1} + q_\mu q_\nu / k^2}{n^2 - 1}$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

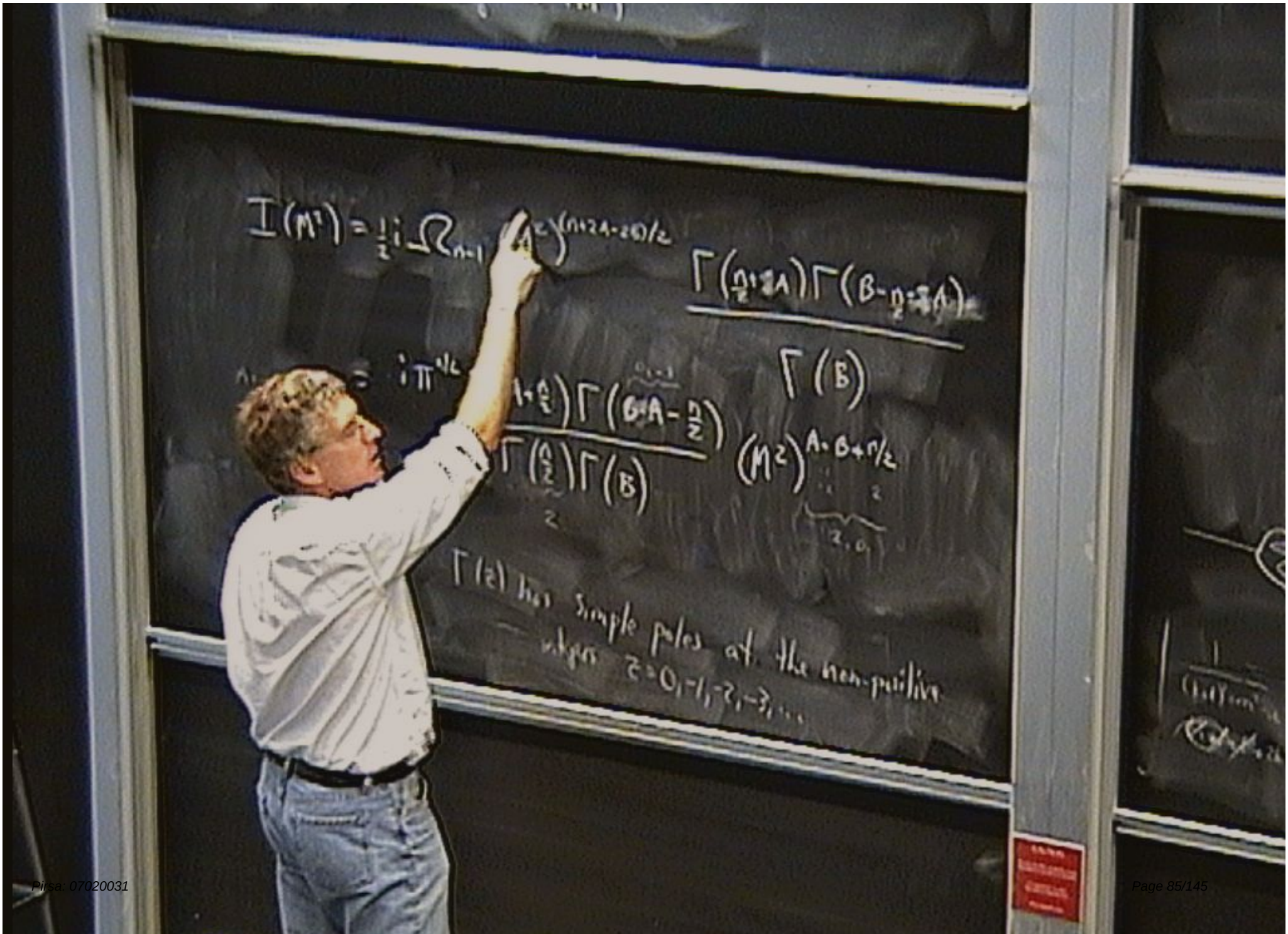
$$\Gamma(1+p) = p\,\Gamma(p)$$

$$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1 - \frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)}$$

$$- 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\left(\frac{2x}{n}\right) n^{n+1} \frac{2x}{n} \qquad \frac{\eta_{n+1} = q + q_0/k}{n!}$$


Γ(z) has simple poles at the non-positive integers z = 0, -1, -2, -3, ...

$$I(M^2) = \int \frac{\quad}{(r^2+M^2-i\epsilon)^B}$$

$$= i\int d^n k_E \frac{(k_E^2)^A}{((k_E^2+M^2)^B}$$

$$= i\Omega_{n-1}\int_0^\infty dr \frac{r^{n-1+2A}}{(r^2+M^2)^B}$$

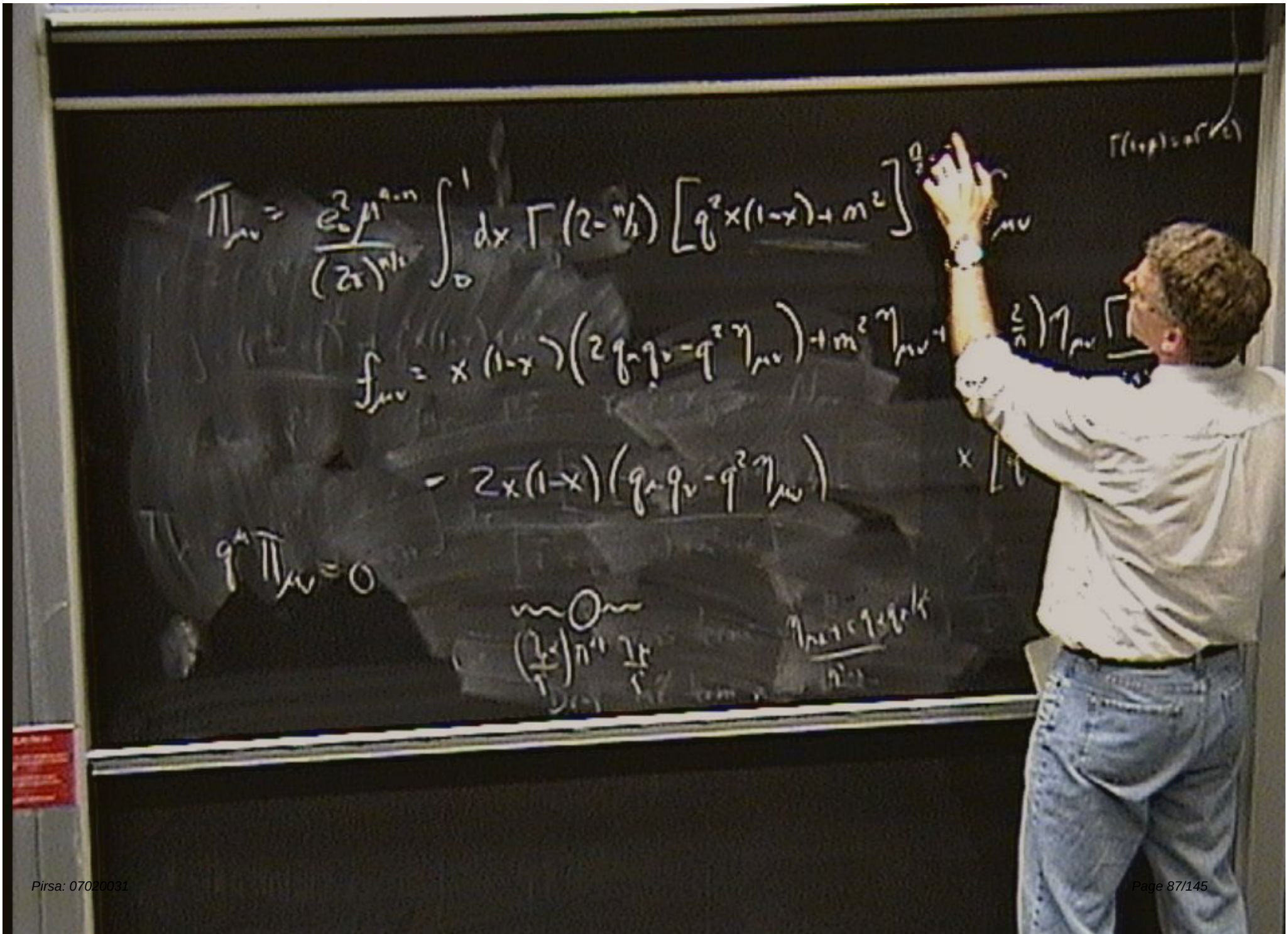
$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$= \int_0^\infty dt \frac{t^{x-1}}{(1+t)^{x+y}}$$

$|\epsilon| \ll 1.$

$$I(M^2) = \frac{1}{2} i\,\Omega_{n-1}\,(M^2)^{(n+2A-2B)/2}\,\frac{\Gamma(\ldots A)\Gamma(B-\frac{n}{2}-A)}{\Gamma(B)}$$

$$= i\pi^{d/2}\Gamma(A+\ldots)\Gamma\ldots$$


$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2}$
$f_{\mu\nu} = x(1-x)(2q_\mu q_\nu - q^2 \eta_{\mu\nu}) + m^2 \eta_{\mu\nu}$
$= 2x(1-x)(q_\mu q_\nu - q^2 \eta_{\mu\nu})$
$q^\mu \Pi_{\mu\nu} = 0$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$\Gamma(1+p) = p\Gamma(p)$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma(1+\frac{n}{2})\Gamma(1-\frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(2-\frac{n}{2})}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + \right.$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\left(\frac{q^\mu}{q}\right)^{n-1} \frac{q^\nu}{q}$$

$$\frac{\eta_{\mu\nu} + q_\mu q_\nu/q^2}{n-2}$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{q-2} f_{\mu\nu}$$

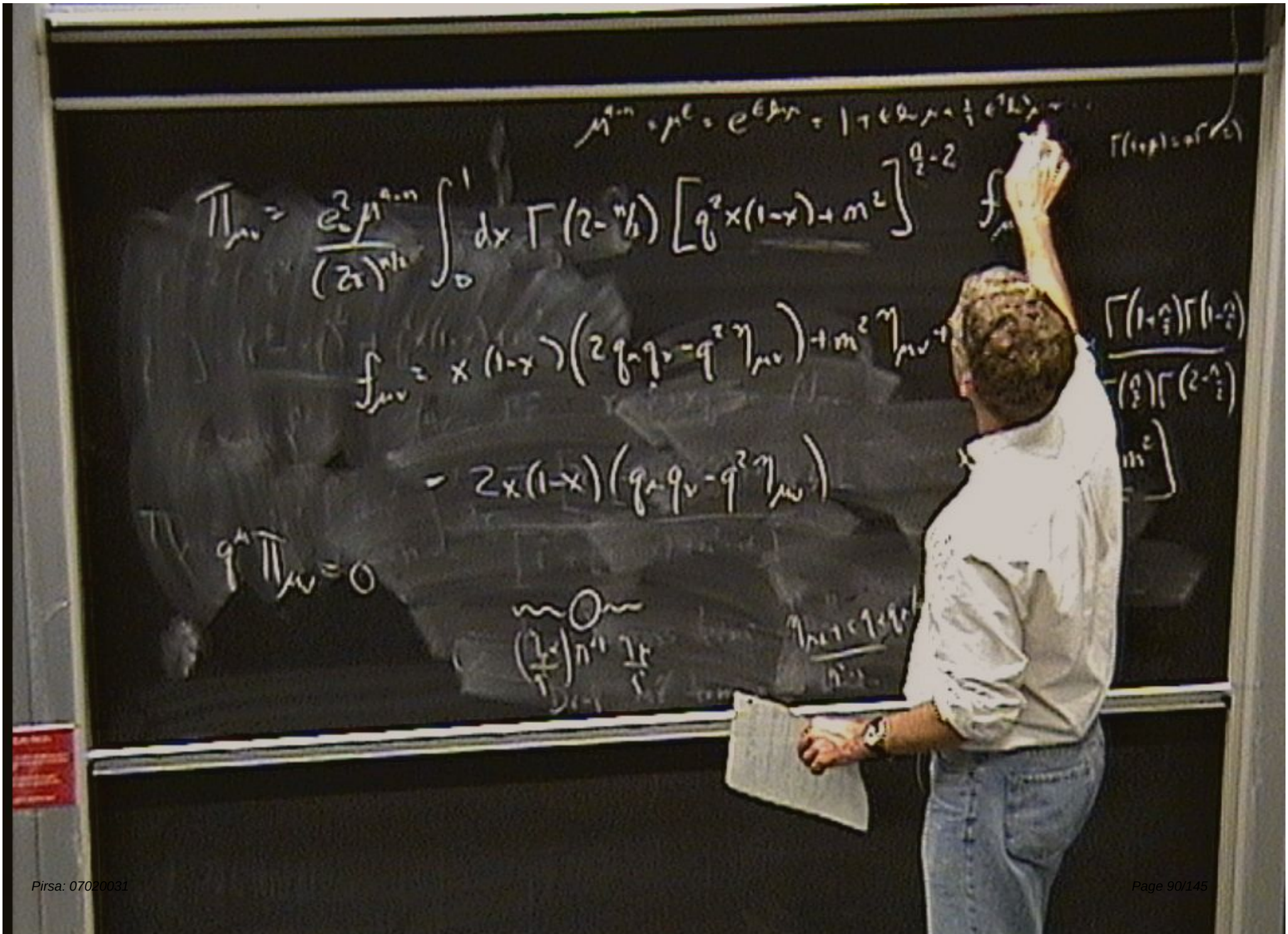
$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma(1-\frac{n}{2})\Gamma(1}{\Gamma(\frac{n}{2})\Gamma(}$$

$$- 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\left(\frac{2\pi}{q}\right)^{n-1} \frac{2\pi}{q}$$

$$\frac{\eta_{\mu\nu} + q_\mu q_\nu / k^2}{k^2}$$


$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$
$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu}$
$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$
$q^\mu \Pi_{\mu\nu} = 0$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon)$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma(1-\frac{n}{2})\Gamma(1-\frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(2-\frac{n}{2})}$$

$$\times \left[q^2 x(1-x) + m^2\right]$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\left(\frac{q_\mu}{q^2}\right) n^{\mu\nu} \frac{q_\nu}{q^2}$$

$$\frac{\eta_{\mu\nu} + c\, q_\mu q_\nu / k^2}{q^2 + \cdots}$$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon)$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\left(\frac{2x}{q}\right)^{n/2} \frac{2x}{q^2} \qquad \frac{\eta_{\mu\nu} + q_\mu q_\nu/k^2}{n/2}$$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon)$$

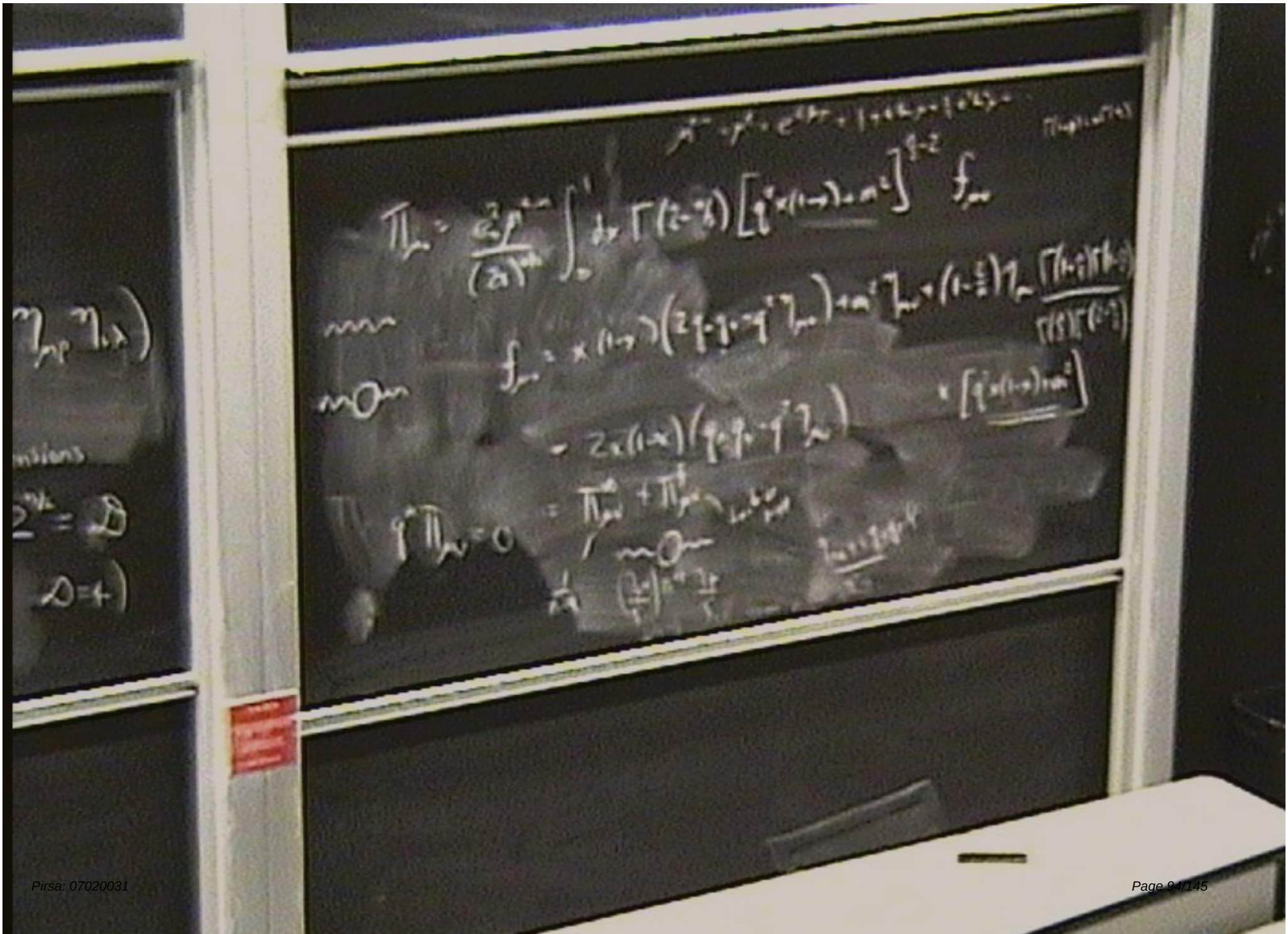
$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)}$$

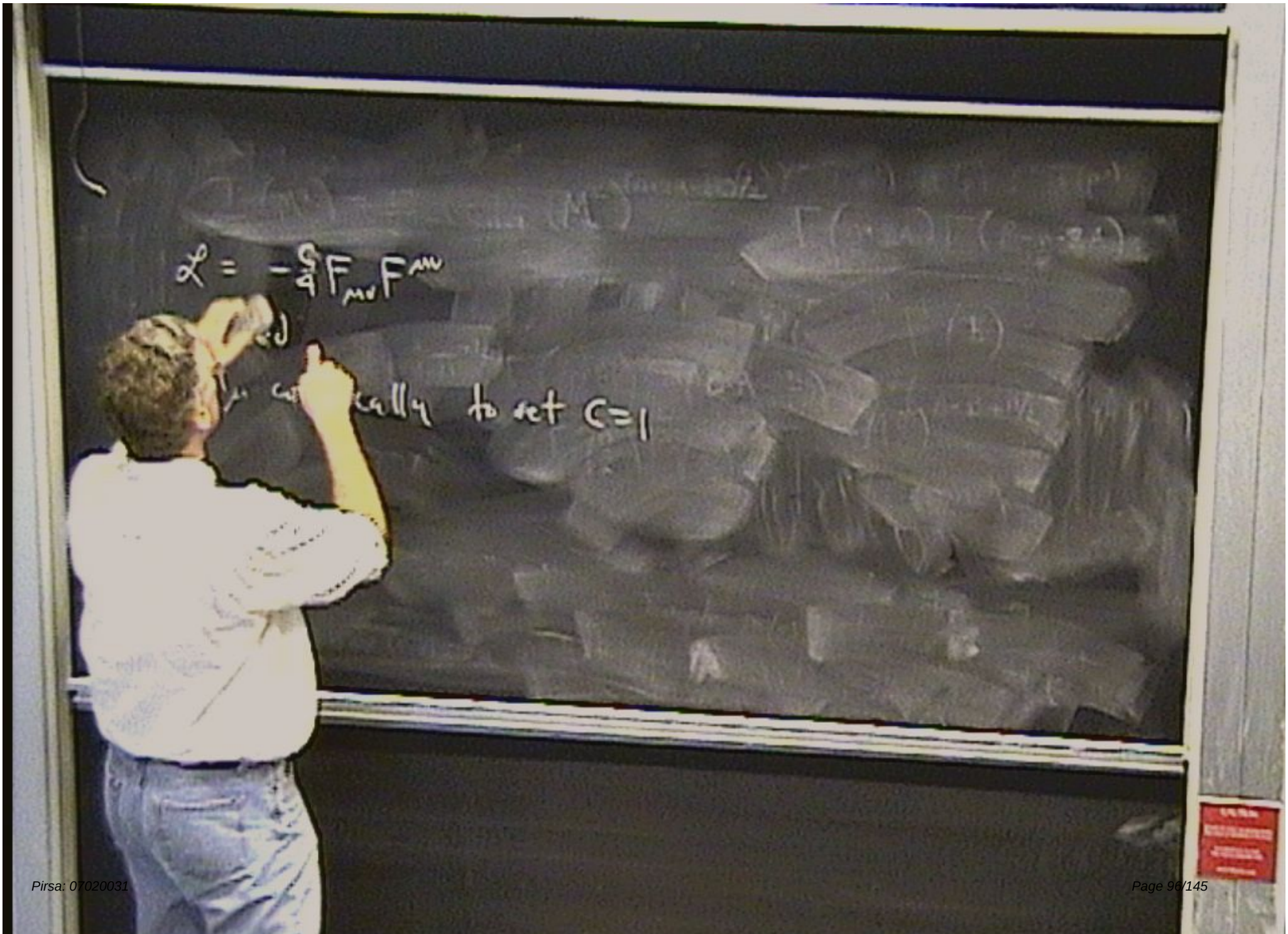
$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0 \qquad = \Pi^{\infty}_{\mu\nu} + \Pi^{f}_{\mu\nu}$$

$$\frac{1}{n-4} \qquad \left(\frac{q_\mu}{q}\right)^{n-4} \frac{q_\mu}{q} \qquad \frac{\eta_{\mu\alpha} + q_\mu q_\alpha/q^2}{n-4}$$



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

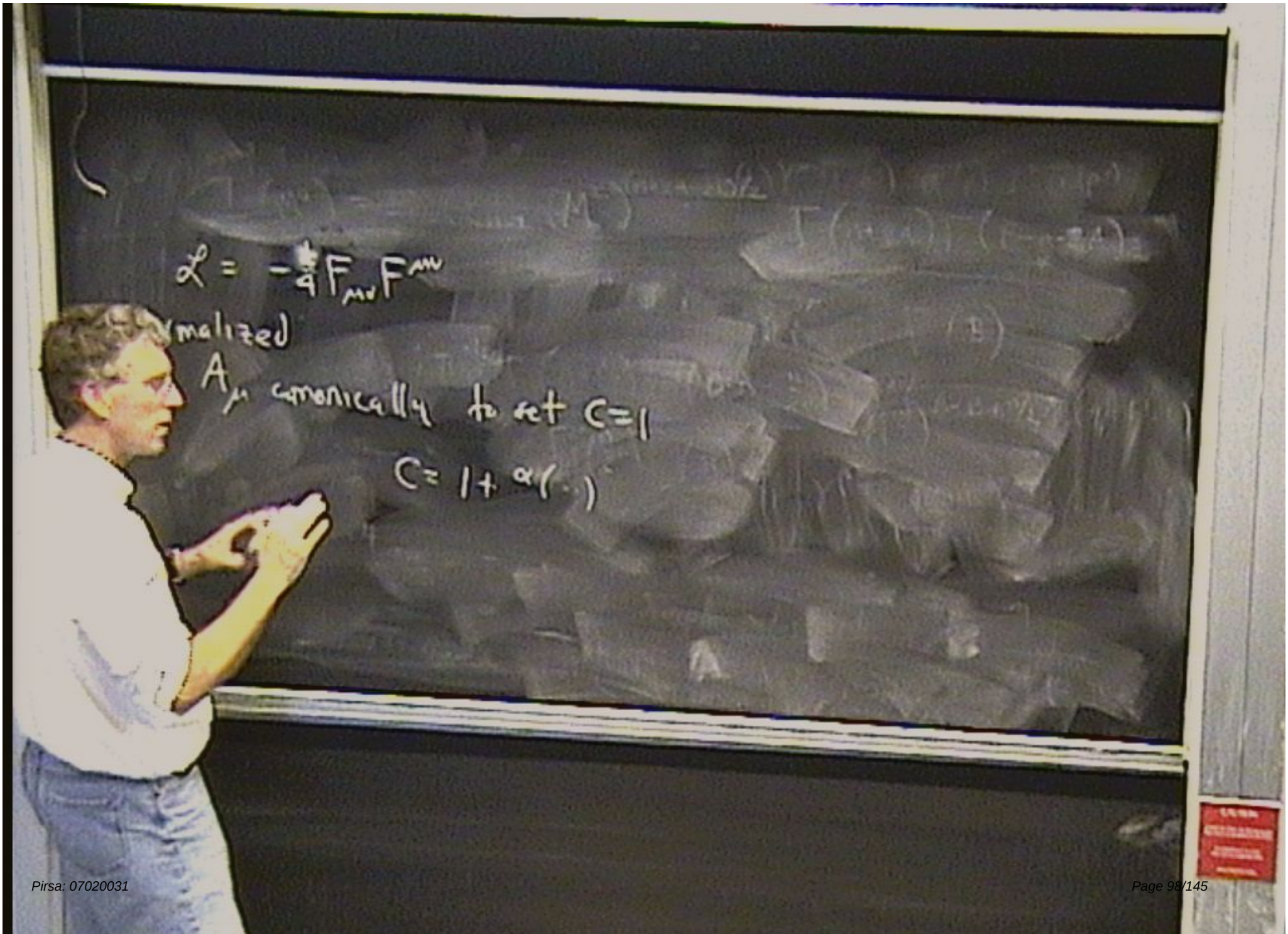

$\mathcal{L} = -\frac{c}{4} F_{\mu\nu} F^{\mu\nu}$
to set $c=1$

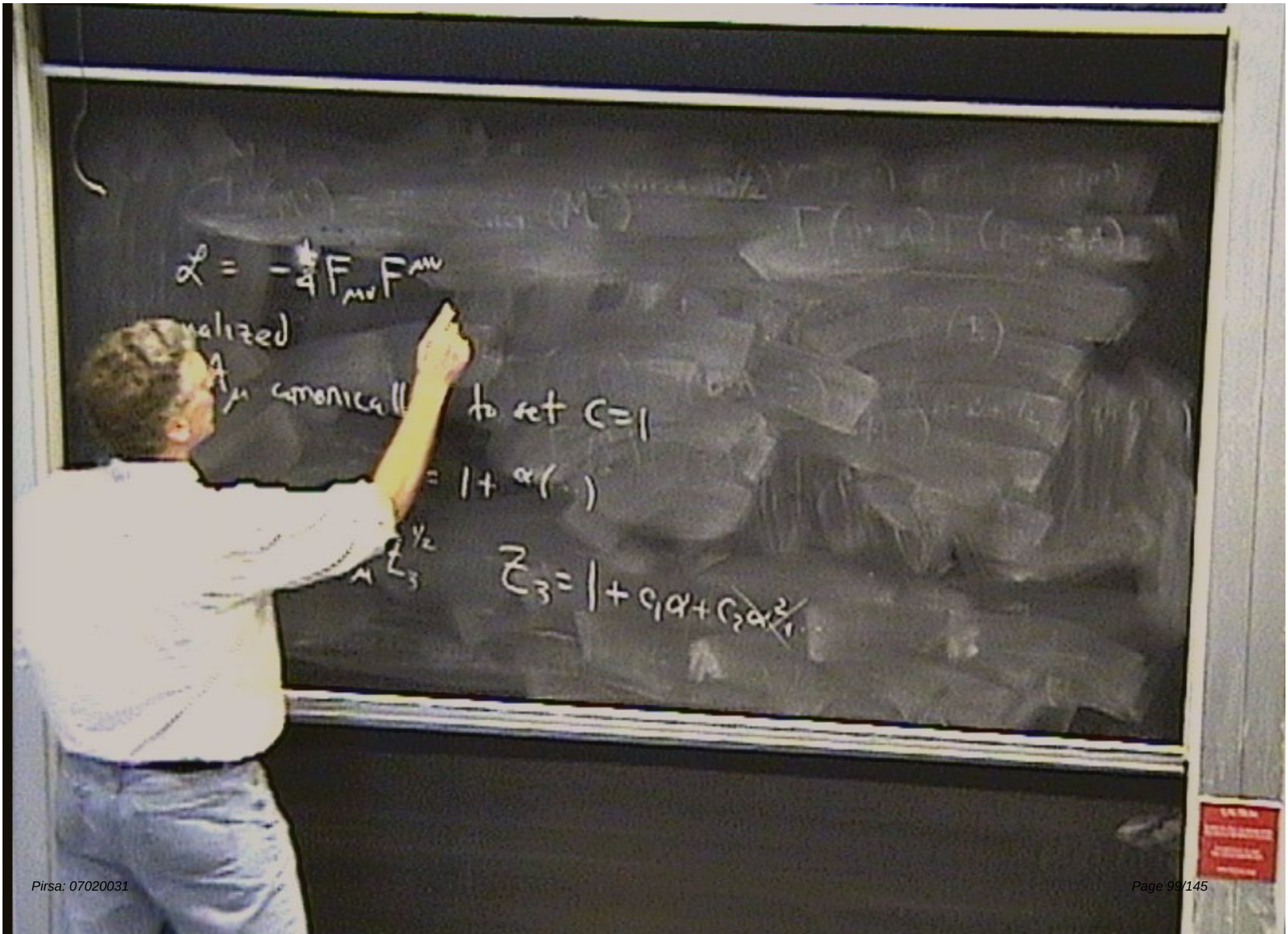
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

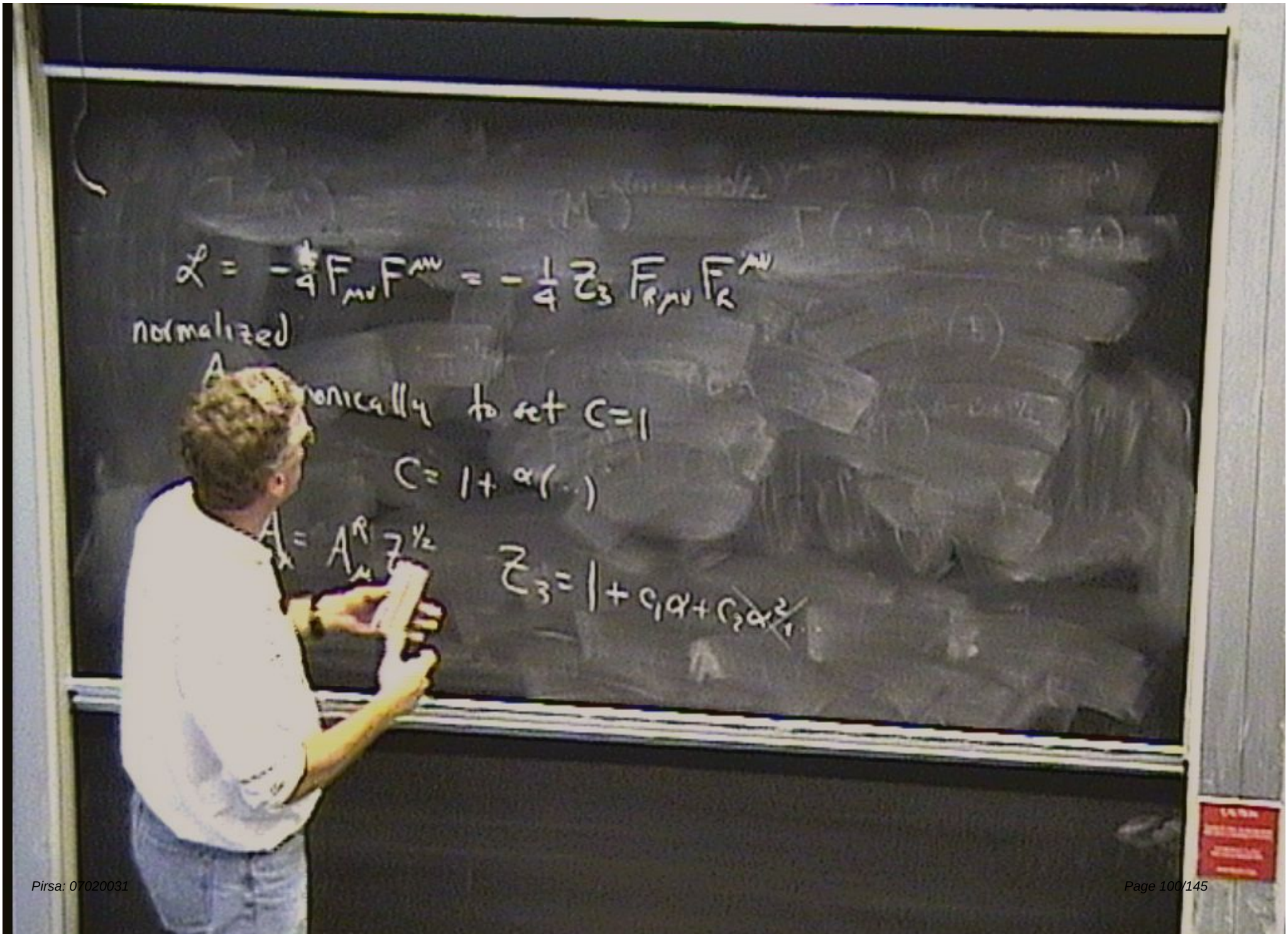
normalized

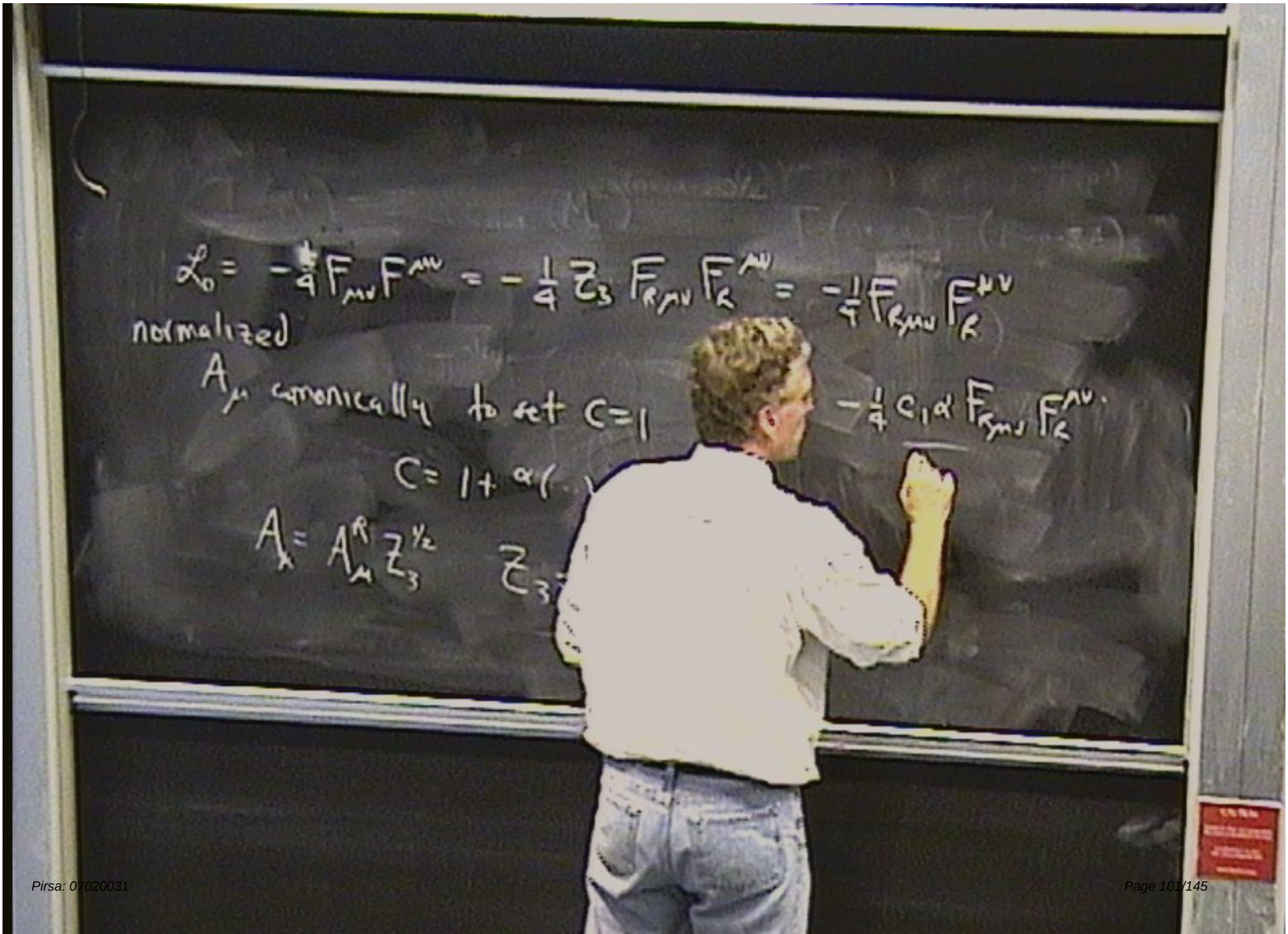
A_μ canonically to set $C=1$

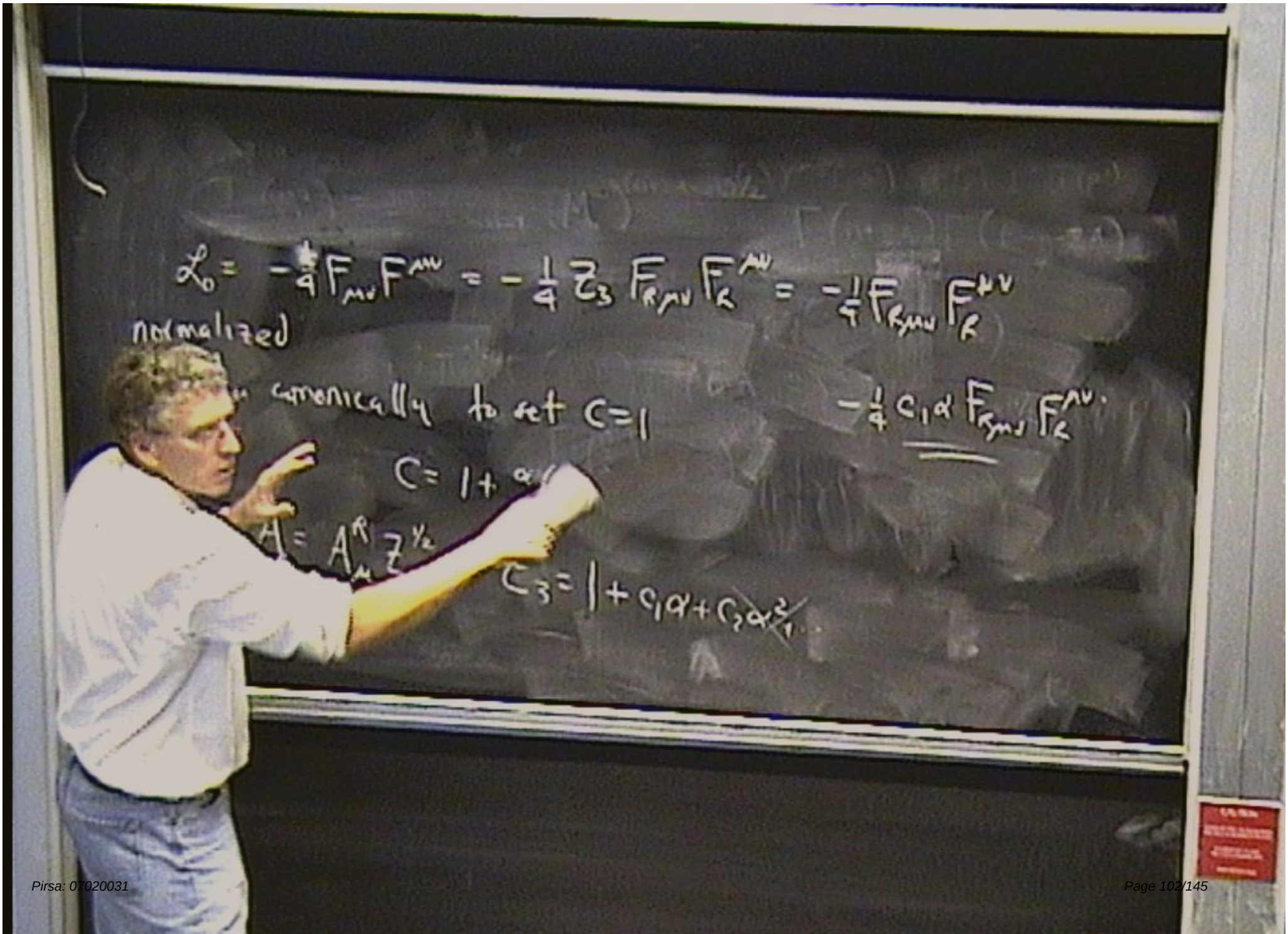
$$C = 1 + \alpha(\ldots)$$


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
malized
A_μ canonically to set $c=1$
$c = 1 + \alpha(\ldots)$


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
to set $c=1$
$= 1 + \alpha(\ldots)$
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu}$
normalized
A
onically to set $C=1$
$C = 1 + \alpha(\ldots)$
$A = A_\mu^R Z^{1/2}$
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$


$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
normalized
A_μ canonically to set $c=1$
$-\frac{1}{4} c_1 \alpha' F_{R\mu\nu} F_R^{\mu\nu}$
$c = 1 + \alpha($
$A_\mu = A_\mu^R Z_3^{1/2}$
Z_3


$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
normalized
canonically to set $C=1$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
$C = 1 + \alpha$
$A = A_\mu^R Z^{1/2}$
$C_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$$

normalized

A_μ canonically to set $C=1$

$$C = 1 + \alpha(\ldots)$$

$$A_\mu = A_\mu^R Z_3^{1/2} \qquad Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$$

$$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$$

stick this into $\mathcal{L}_{int}$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma\left(2 - \frac{n}{2}\right) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + \left(1 - \frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma(\cdots)} \left[q^2 x(1-x) + m^2\right]$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

$$= \Pi^{\infty}_{\mu\nu} + \Pi^{f}_{\mu\nu}$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\frac{1}{n-4} \quad \left(\frac{\cdots}{\epsilon}\right)$$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma\left(2 - \tfrac{n}{2}\right) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2 q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) +$$

$$= \Pi^{\infty}_{\mu\nu} + \Pi^{f}_{\mu\nu}$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\frac{1}{n-4} \qquad \left(\frac{q^2}{\mu^2}\right)^{n-4} \frac{q_\mu}{q^2}$$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

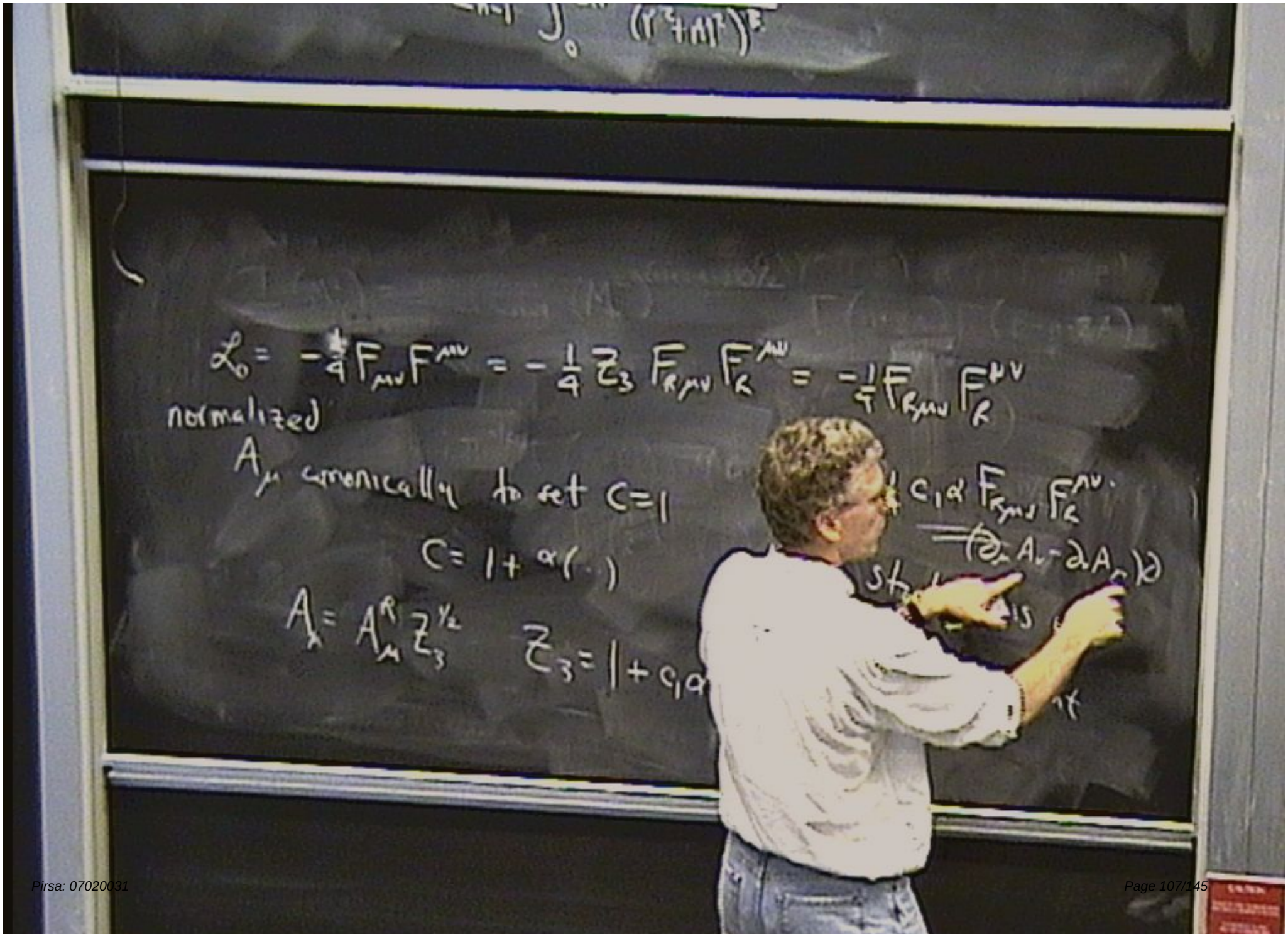
$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + c\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

$$\times \left[q^2 x(1-x)\right.$$

$$q^\mu \Pi_{\mu\nu} = 0 \qquad = \Pi^{(0)}_{\mu\nu} + \Pi^{f}_{\mu\nu}$$

$$\frac{1}{n-4} \qquad \left(\frac{k^2}{q^2}\right)^{n+1} \frac{2k}{q^2}$$


$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
normalized
A_μ canonically to set $c=1$
$c = 1 + \alpha(\ldots)$
$A_\mu = A_\mu^R Z_3^{1/2}$
$Z_3 = 1 + c_1 \alpha$

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$$

normalized

A_μ canonically to set $c=1$

$$c = 1 + \alpha(\ldots)$$

$$A_\mu = A_\mu^R Z_3^{1/2} \qquad Z_3 = 1 + c_1\alpha + \cancel{c_2\alpha^2} \ldots$$

$$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$$

$$= (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

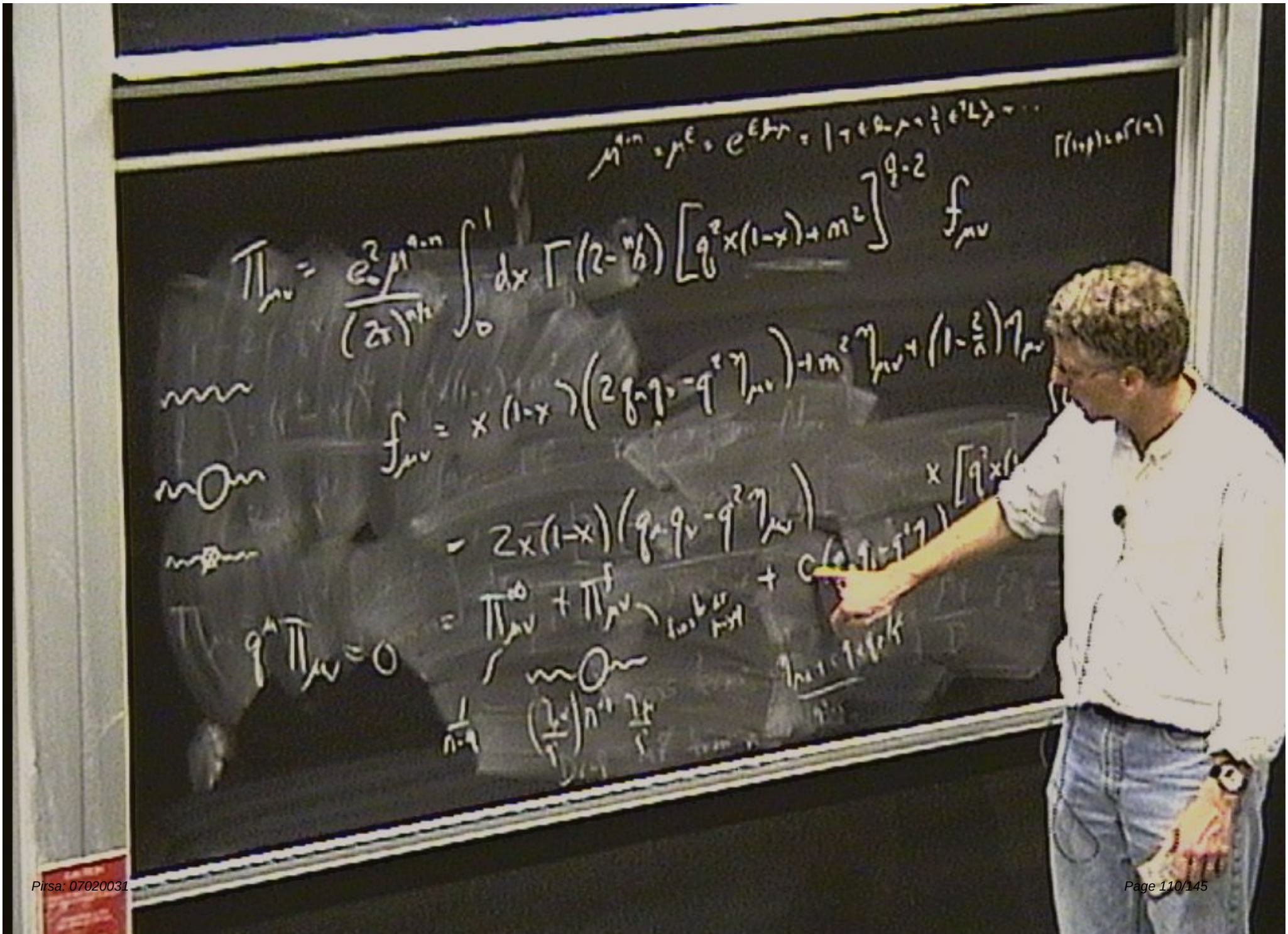
stick this into $\mathcal{L}_{int}$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma\left(2-\frac{n}{2}\right) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma(1)\Gamma\left(2-\frac{n}{2}\right)} \times \left[q^2 x(1-x) + m^2\right]$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + c\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

$$q^\mu \Pi_{\mu\nu} = 0 \qquad = \Pi^{(0)}_{\mu\nu} + \Pi^{f}_{\mu\nu}$$


$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{q-2} f_{\mu\nu}$
$f_{\mu\nu} = x(1-x)(2q_\mu q_\nu - q^2 \eta_{\mu\nu}) + m^2 \eta_{\mu\nu} + (1-\frac{2}{n})\eta_{\mu\nu}$
$= 2x(1-x)(q_\mu q_\nu - q^2 \eta_{\mu\nu})$
$q^\mu \Pi_{\mu\nu} = 0$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon)$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma(1+\frac{n}{2})\Gamma(1\ldots}{\Gamma(\frac{n}{2})\Gamma(2-\frac{n}{2})}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \times \left[q^2 x(1-x) + m^2\right]$$

$$= \Pi^{(0)}_{\mu\nu} + \Pi^{f}_{\mu\nu} + O\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

$$q^\mu \Pi_{\mu\nu} = 0$$

$$\frac{1}{n-4} \quad \left(\frac{2\pi}{q^2}\right)^{n/2} \frac{2\pi}{q^2}$$

$$\frac{\eta_{\mu\nu} + q_\mu q_\nu/k^2}{k^2 - \ldots} \quad \int \langle 0|T A_\mu^{in}(x) A_\nu^{in}(y)|0\rangle e^{ik(x-y)} = \frac{1}{\ldots}$$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon)$$

$$\Pi_{\mu\nu} = \frac{e^2 \mu^{4-n}}{(2\pi)^{n+1}} \int_0^1 dx\, \Gamma\left(2-\frac{n}{2}\right)\left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma(1)\Gamma\left(2-\frac{n}{2}\right)} \times \left[q^2 x(1-x) + m^2\right]$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + O\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

$$q^\mu \Pi_{\mu\nu} = 0 \qquad = \Pi^{\infty}_{\mu\nu} + \Pi^{f}_{\mu\nu}$$

$$\frac{1}{n-4} \qquad \left(\frac{2\pi}{q^2}\right)^{n/2} \frac{2\pi}{q^2}$$

$$\frac{q_\mu q_\nu - q^2 \eta_{\mu\nu}}{q^2} \qquad \int \langle 0 | T A^\mu_\alpha(x) A^\nu_\beta(0) | 0 \rangle e^{iqx} d^4x \qquad \frac{1}{q^2}$$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots \qquad \Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon)$$

$$\Pi_{\mu\nu} = \frac{e^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma\left(2-\frac{n}{2}\right) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + O\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0 \qquad = \Pi^{\infty}_{\mu\nu} + \Pi^{f}_{\mu\nu}$$

$$\frac{1}{n-4} \qquad \left(\frac{2\pi}{\mu}\right)^{n-4} \frac{2\pi}{\epsilon}$$

$$\frac{\eta_{\mu\nu} + q_\mu q_\nu / k^2}{k^2} \qquad \int \langle 0 | T A^\mu(x) A^\nu(y) | 0 \rangle e^{ik(x-y)} d^4x$$

$$= \left(p^2 \eta_{\mu\nu} - p_\mu p_\nu\right) \Pi(p^2)$$

$$\mu^{4-n} = \mu^{\epsilon} = e^{\epsilon \ln \mu} = 1 + \epsilon \ln \mu + \frac{1}{2}\epsilon^2 \ln^2 \mu + \cdots$$

$$\Gamma(1+\epsilon) = \epsilon\Gamma(\epsilon)$$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma\left(2-\frac{n}{2}\right)\left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{\epsilon}{2}\right)\Gamma\left(1-\frac{\epsilon}{2}\right)}{\Gamma(\epsilon)\Gamma\left(2-\frac{n}{2}\right)}$$

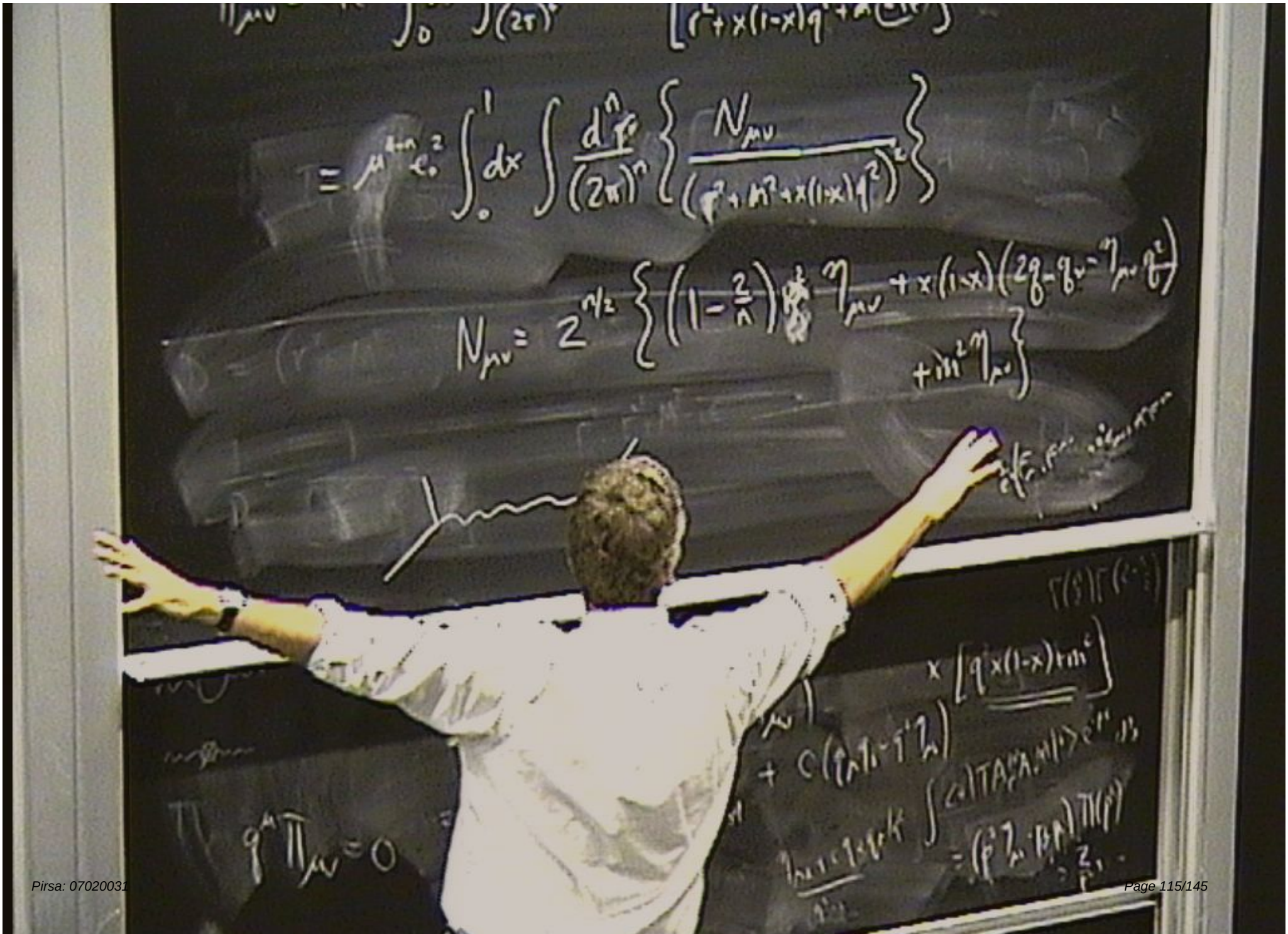
$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + O\left(\eta_{\mu\nu} q^2 - q^2 \eta_{\mu\nu}\right) \times \left[q^2 x(1-x) + m^2\right]$$

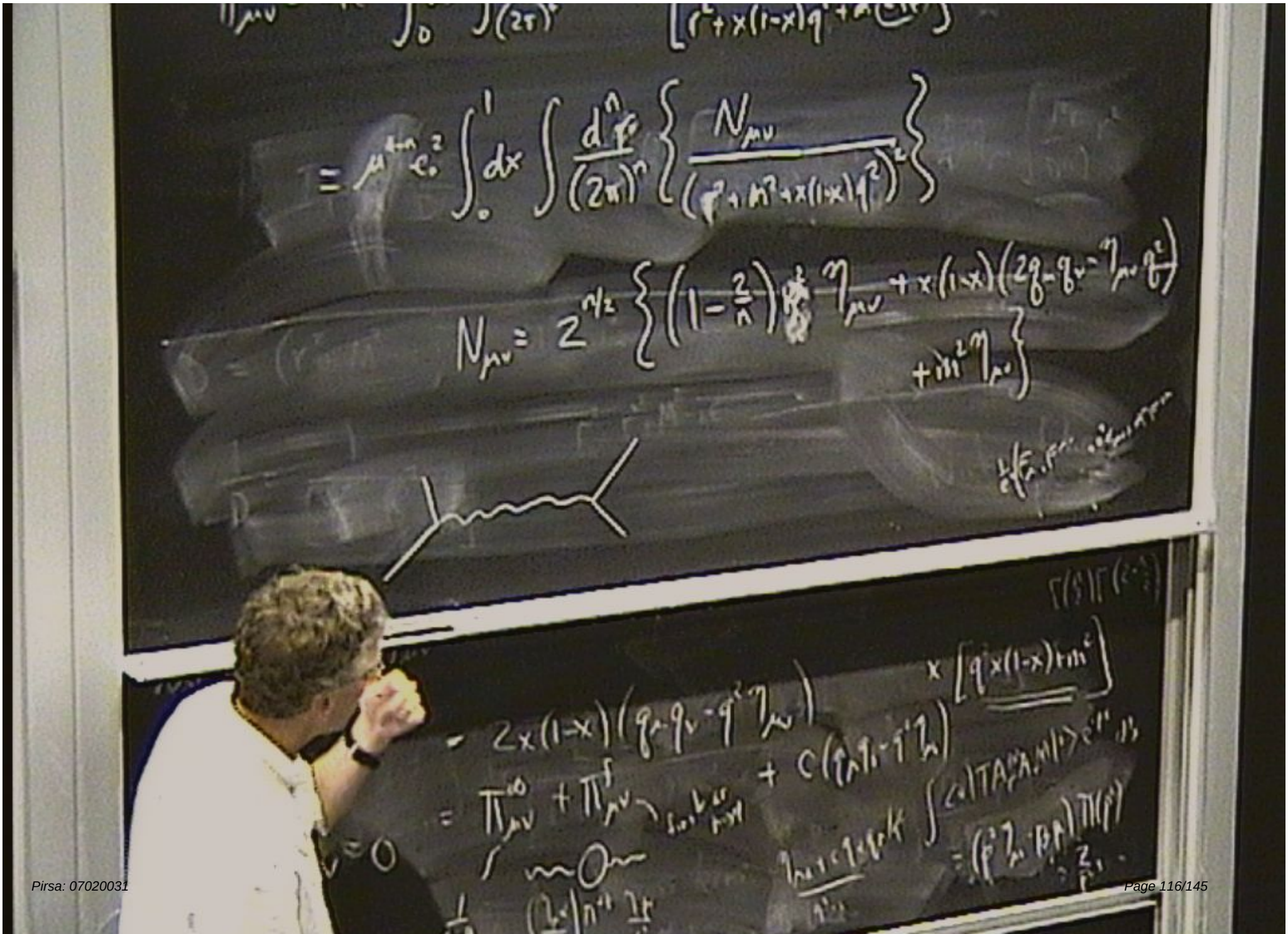
$$= \Pi^{\infty}_{\mu\nu} + \Pi^{f}_{\mu\nu}$$

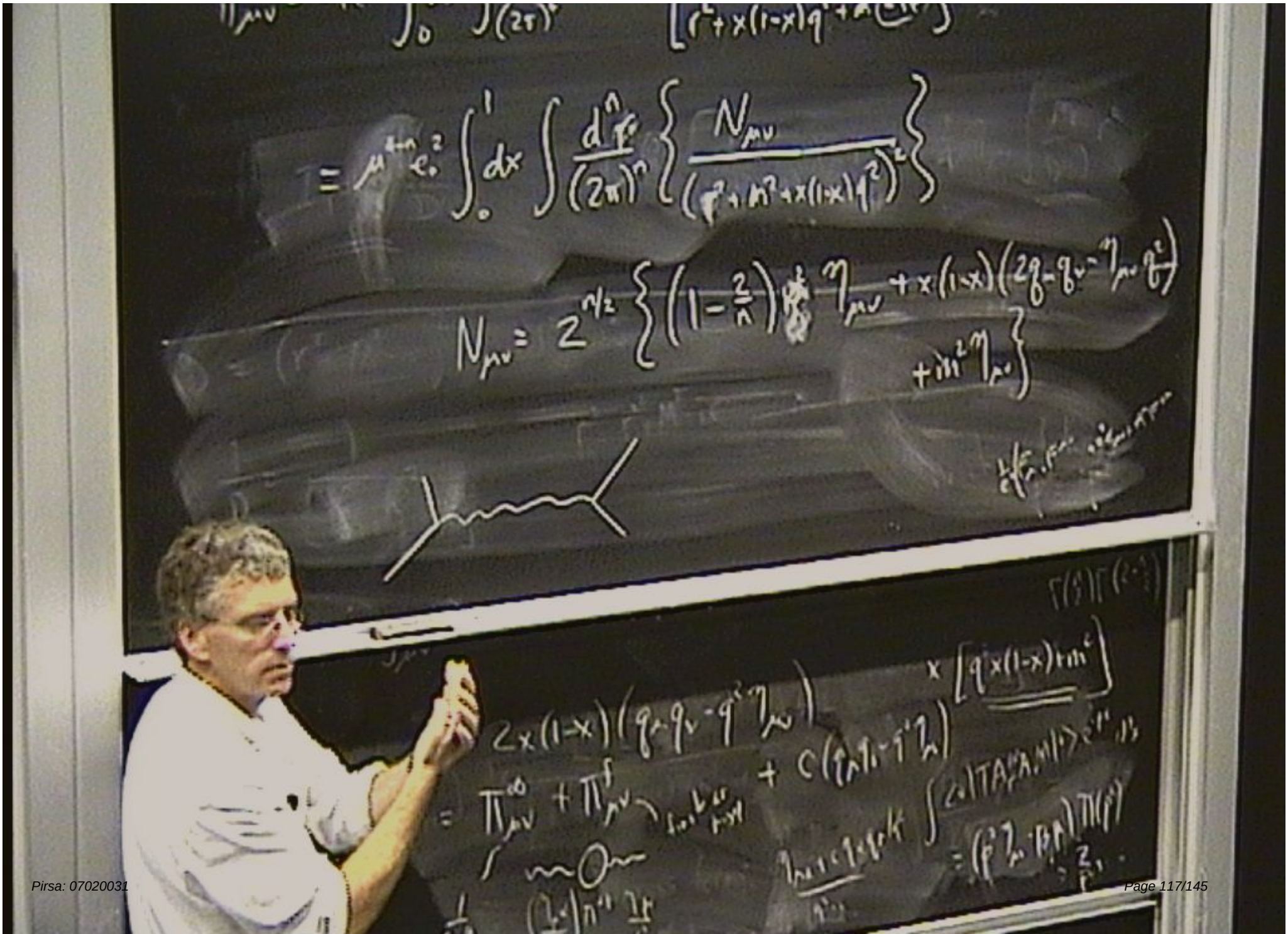
$$q^\mu \Pi_{\mu\nu} = 0$$

$$\frac{1}{n-4} \quad \left(\frac{2\pi}{q^2}\right)^{n/2} \frac{2\pi}{q^2}$$

$$\frac{\eta_{\mu\nu} + c\, q_\mu q_\nu}{n-1} \int \langle 0|T A_\mu(x) A_\nu(0)|0\rangle e^{ipx} d^4x = \left(p^2 \eta_{\mu\nu} - p_\mu p_\nu\right)\Pi(p^2) + \frac{Z_3}{p^2} \cdots$$


$= \mu^{4-n} e_0^2 \int_0^1 dx \int \frac{d^n p}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{(p^2 + m^2 + x(1-x)q^2)^2} \right\}$
$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)(2q - q_\nu - \eta_{\mu\nu} q^2) + m^2 \eta_{\mu\nu} \right\}$
$q^\mu \Pi_{\mu\nu} = 0$


$= \mu^{4-n} e_0^2 \int_0^1 dx \int \frac{d^n p}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{(p^2 + m^2 + x(1-x)q^2)^2} \right\}$
$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)(2q_\mu q_\nu - \eta_{\mu\nu} q^2) + m^2 \eta_{\mu\nu} \right\}$
$= 2x(1-x)(q_\mu q_\nu - q^2 \eta_{\mu\nu})$
$= \Pi^{(0)}_{\mu\nu} + \Pi^{f}_{\mu\nu}$


$= \mu^{4-n} e_0^2 \int_0^1 dx \int \frac{d^n p}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{(p^2 + m^2 + x(1-x)q^2)^2} \right\}$
$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) \eta_{\mu\nu} + x(1-x)(2q_\mu q_\nu - \eta_{\mu\nu} q^2) + m^2 \eta_{\mu\nu} \right\}$

$$= \mu^{4-n} e_0^2 \int_0^1 dx \int \frac{d^n p}{(2\pi)^n} \left\{ \frac{N_{\mu\nu}}{(p^2 + m^2 + x(1-x)q^2)^2} \right\}$$

$$N_{\mu\nu} = 2^{n/2} \left\{ \left(1 - \frac{2}{n}\right) p^2 \eta_{\mu\nu} + x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} \right\}$$

$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \times \left[q^2 x(1-x) + m^2\right]$$

$$q^\mu \Pi_{\mu\nu} = 0 \qquad = \Pi^{\infty}_{\mu\nu} + \Pi^{f}_{\mu\nu} + c\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$$

normalized

A_μ canonically to set $c=1$

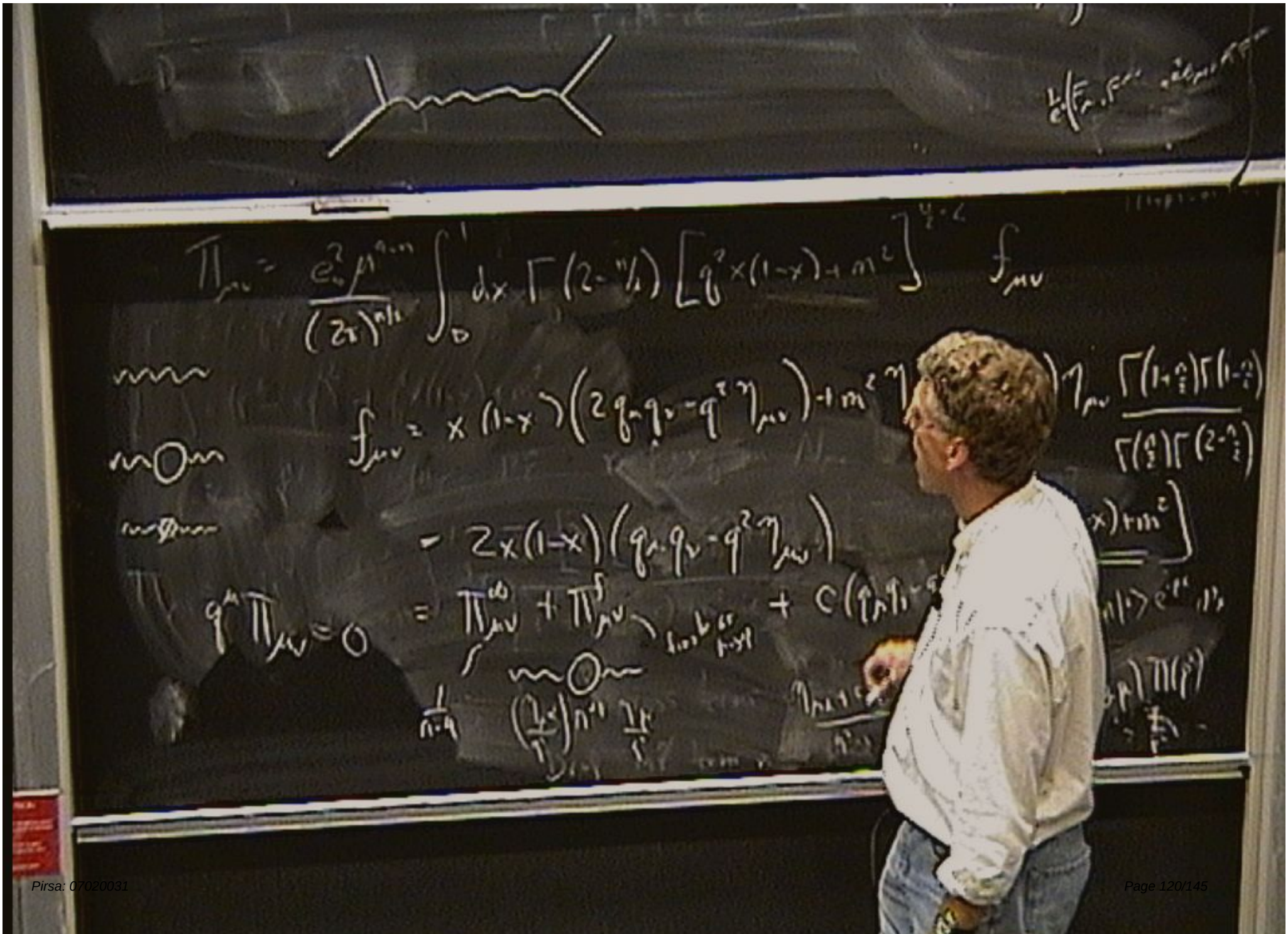
$$c = 1 + \alpha(\ldots)$$

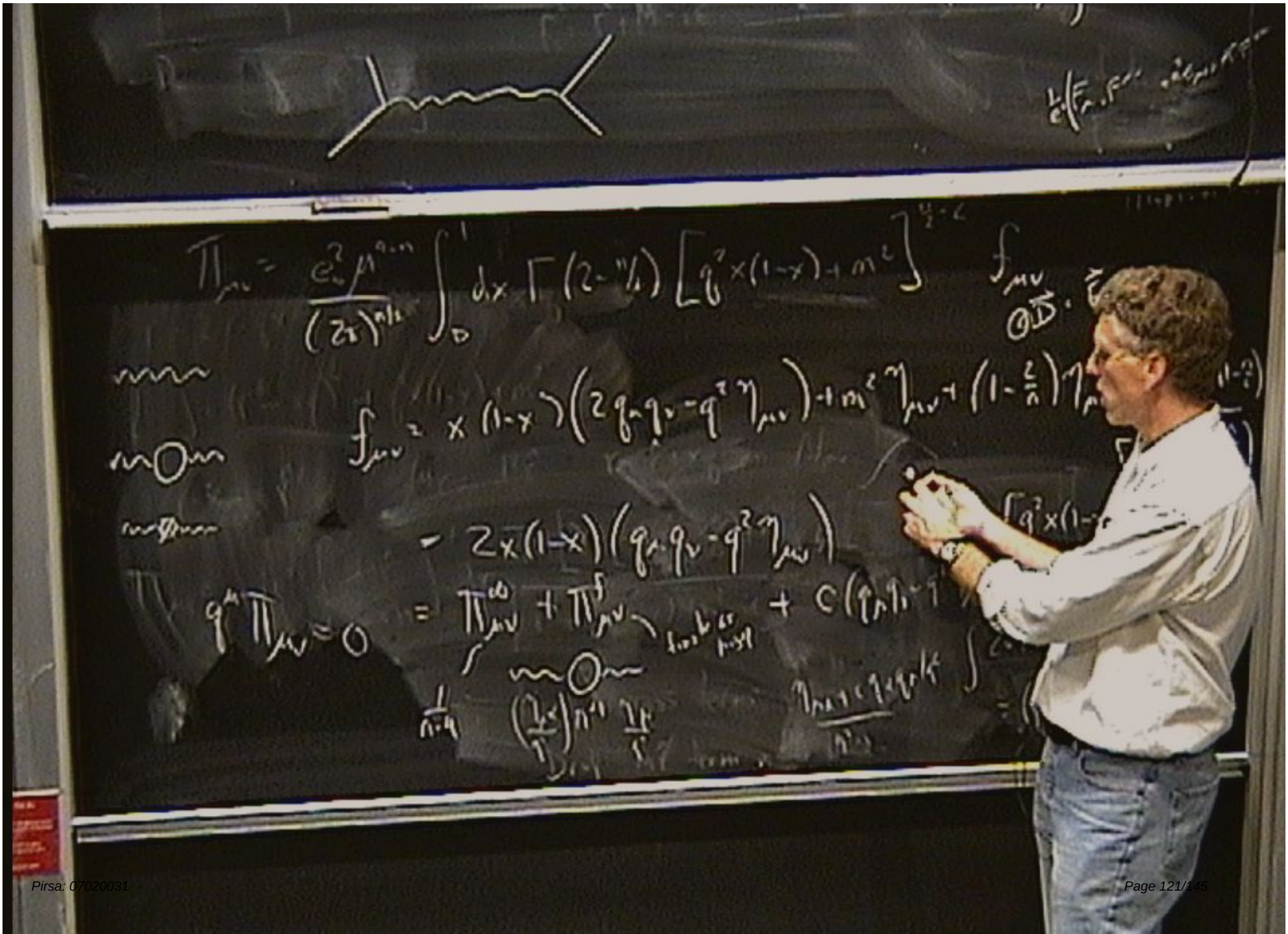
$$A_\mu = A_\mu^R Z_3^{1/2} \qquad Z_3 = 1 + c_1\alpha + \cancel{c_2\alpha^2} + \ldots$$

$$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$$

$$= (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

stick this into $\mathcal{L}_{int}$


$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{n/2-2} f_{\mu\nu}$
$f_{\mu\nu} = x(1-x)(2q_\mu q_\nu - q^2 \eta_{\mu\nu}) + m^2 \eta_{\mu\nu} \frac{\Gamma(1+\frac{n}{2})\Gamma(1-\frac{n}{2})}{\Gamma(\frac{n}{2})\Gamma(2-\frac{n}{2})}$
$= 2x(1-x)(q_\mu q_\nu - q^2 \eta_{\mu\nu})$
$q^\mu \Pi_{\mu\nu} = 0$
$= \Pi^{(0)}_{\mu\nu} + \Pi^{f}_{\mu\nu}$


$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma(2-n/2) \left[q^2 x(1-x) + m^2\right]^{n/2-2}$
$f_{\mu\nu} = x(1-x)(2q_\mu q_\nu - q^2 \eta_{\mu\nu}) + m^2 \eta_{\mu\nu} + (1-\frac{2}{n})\eta_{\mu\nu}$
$= 2x(1-x)(q_\mu q_\nu - q^2 \eta_{\mu\nu})$
$q^\mu \Pi_{\mu\nu} = 0$
$= \Pi^{(0)}_{\mu\nu} + \Pi^{f}_{\mu\nu}$

$$\Pi_{\mu\nu} = \frac{e_0^2 \mu^{4-n}}{(2\pi)^{n/2}} \int_0^1 dx\, \Gamma\left(2-\tfrac{n}{2}\right) \left[q^2 x(1-x) + m^2\right]^{\frac{n}{2}-2} f_{\mu\nu}$$

$\vec{B} \cdot \vec{E}$

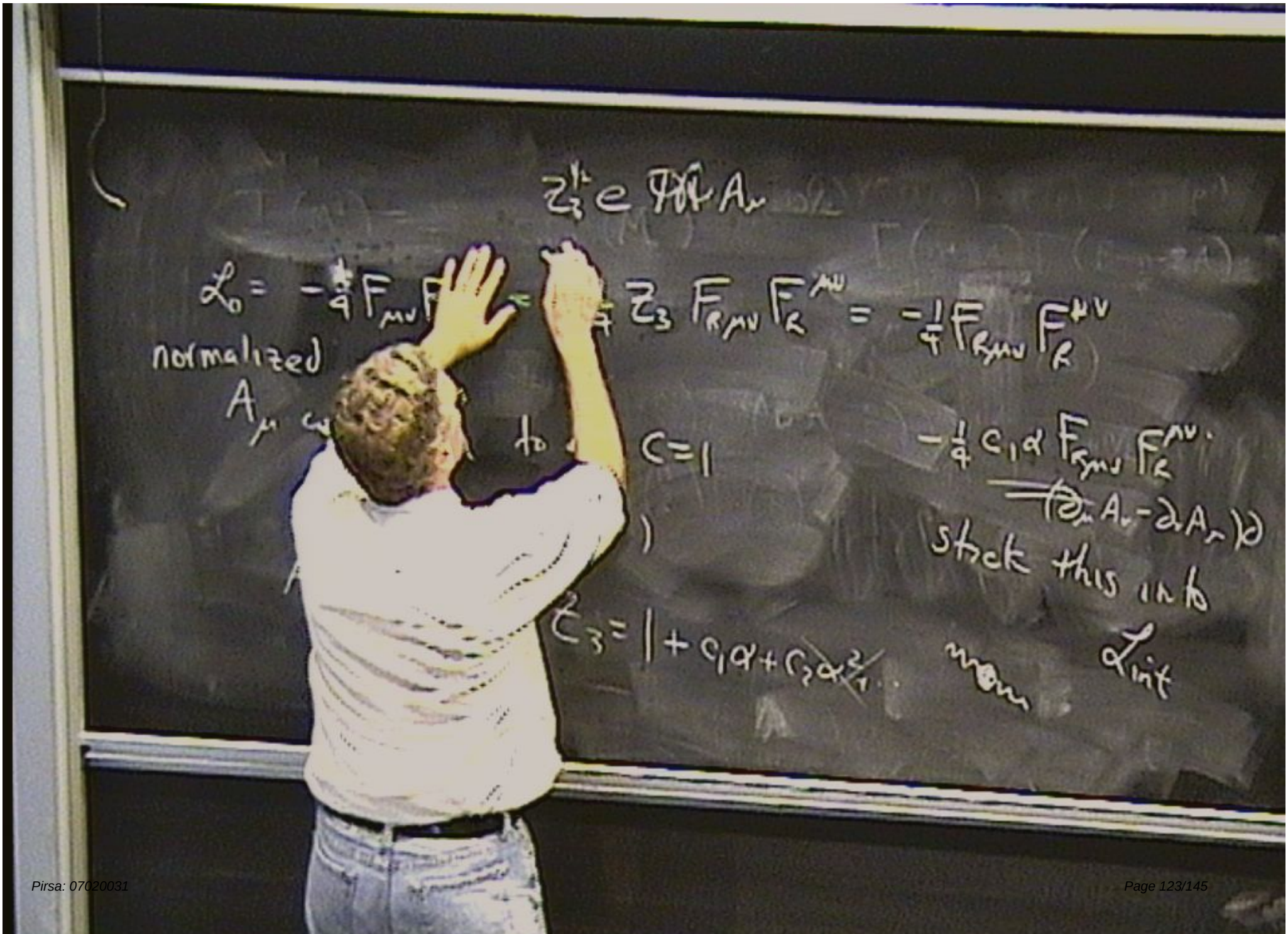
$$f_{\mu\nu} = x(1-x)\left(2q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) + m^2 \eta_{\mu\nu} + \left(1-\frac{2}{n}\right)\eta_{\mu\nu} \frac{\Gamma\left(1+\frac{n}{2}\right)\Gamma\left(1-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)}$$

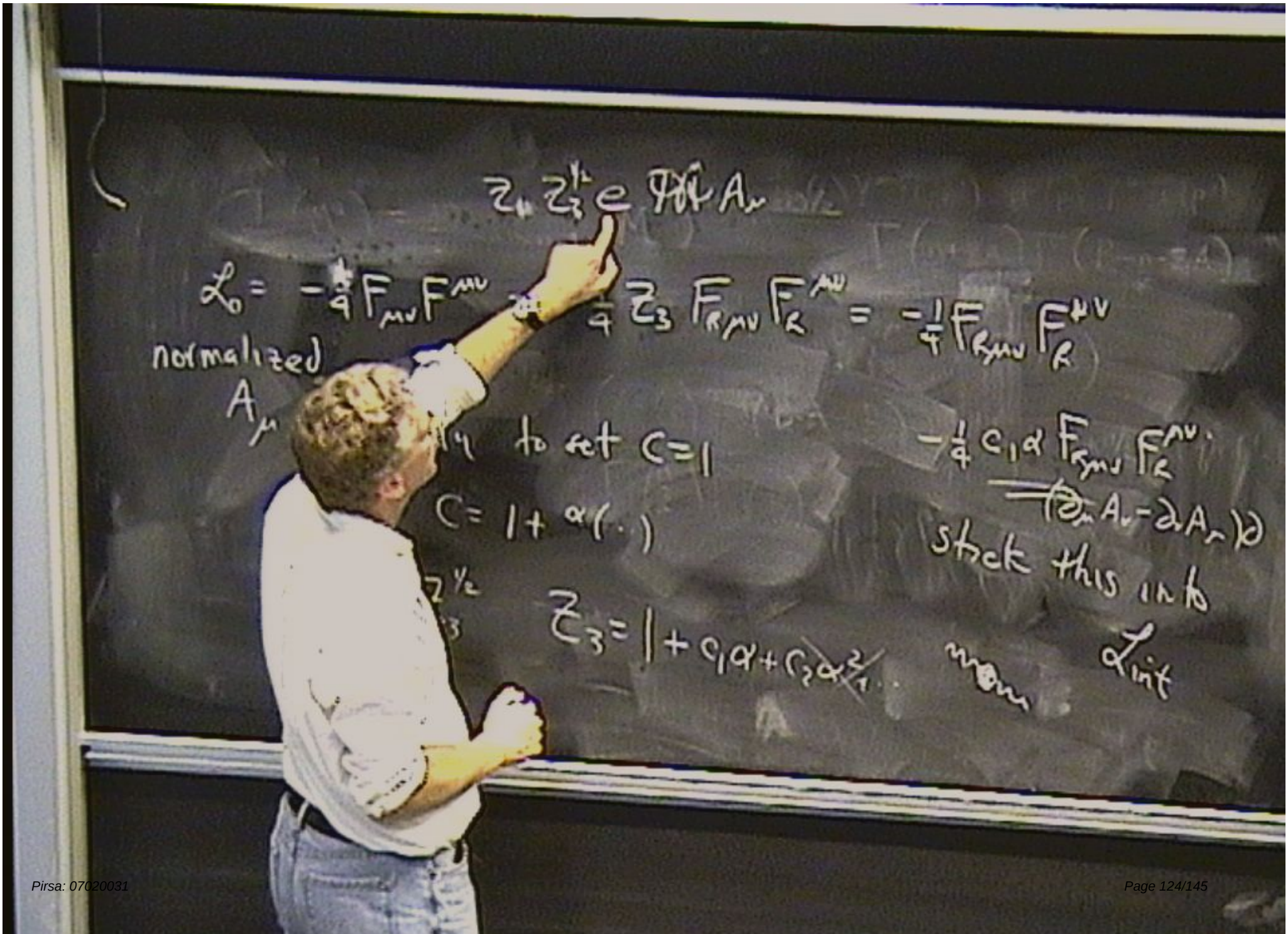
$$= 2x(1-x)\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right) \qquad \times \left[q^2 x(1-x) + m^2\right]$$

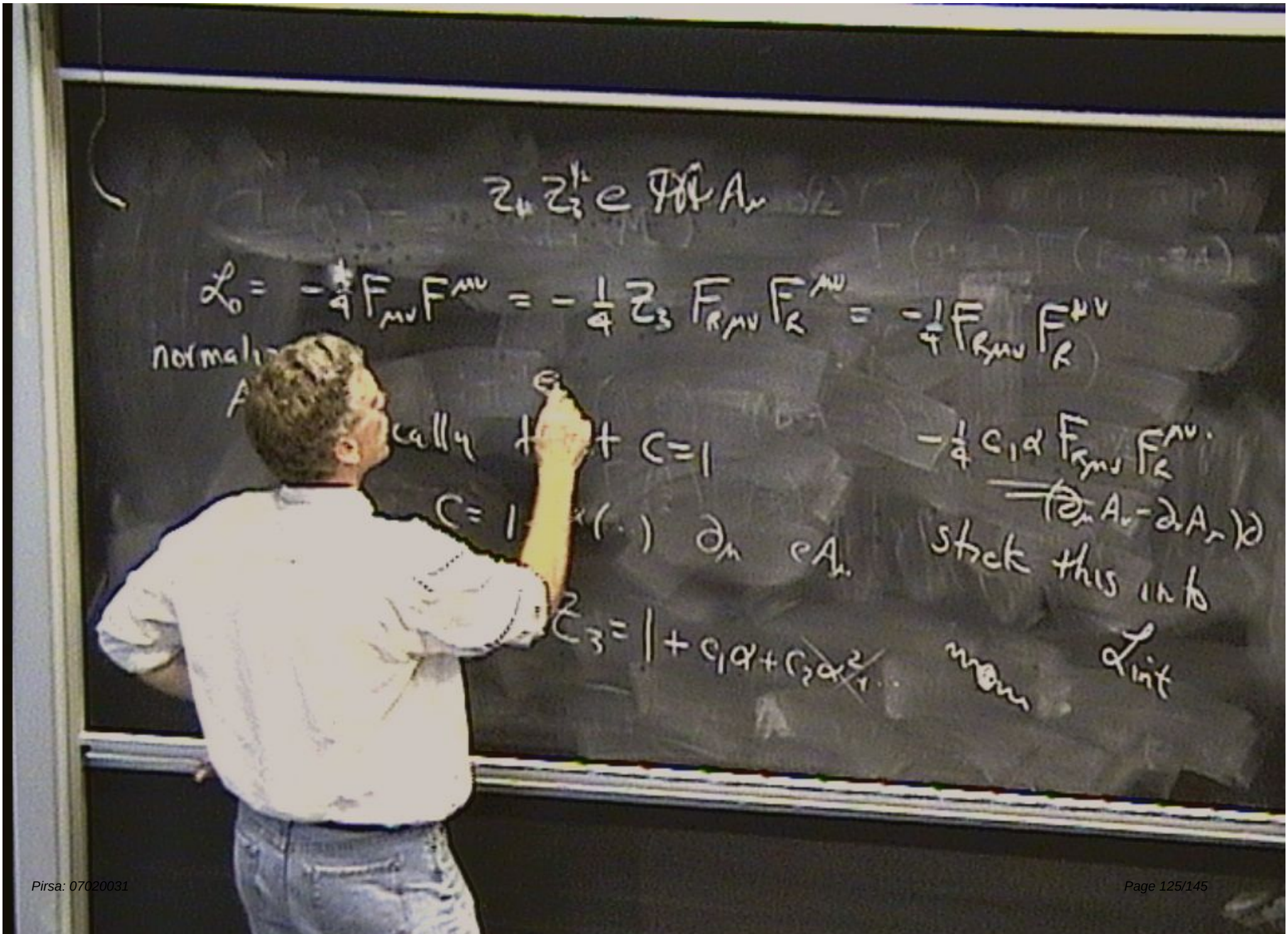
$$q^\mu \Pi_{\mu\nu} = 0 \qquad = \Pi^{\infty}_{\mu\nu} + \Pi^{f}_{\mu\nu} + c\left(q_\mu q_\nu - q^2 \eta_{\mu\nu}\right)$$

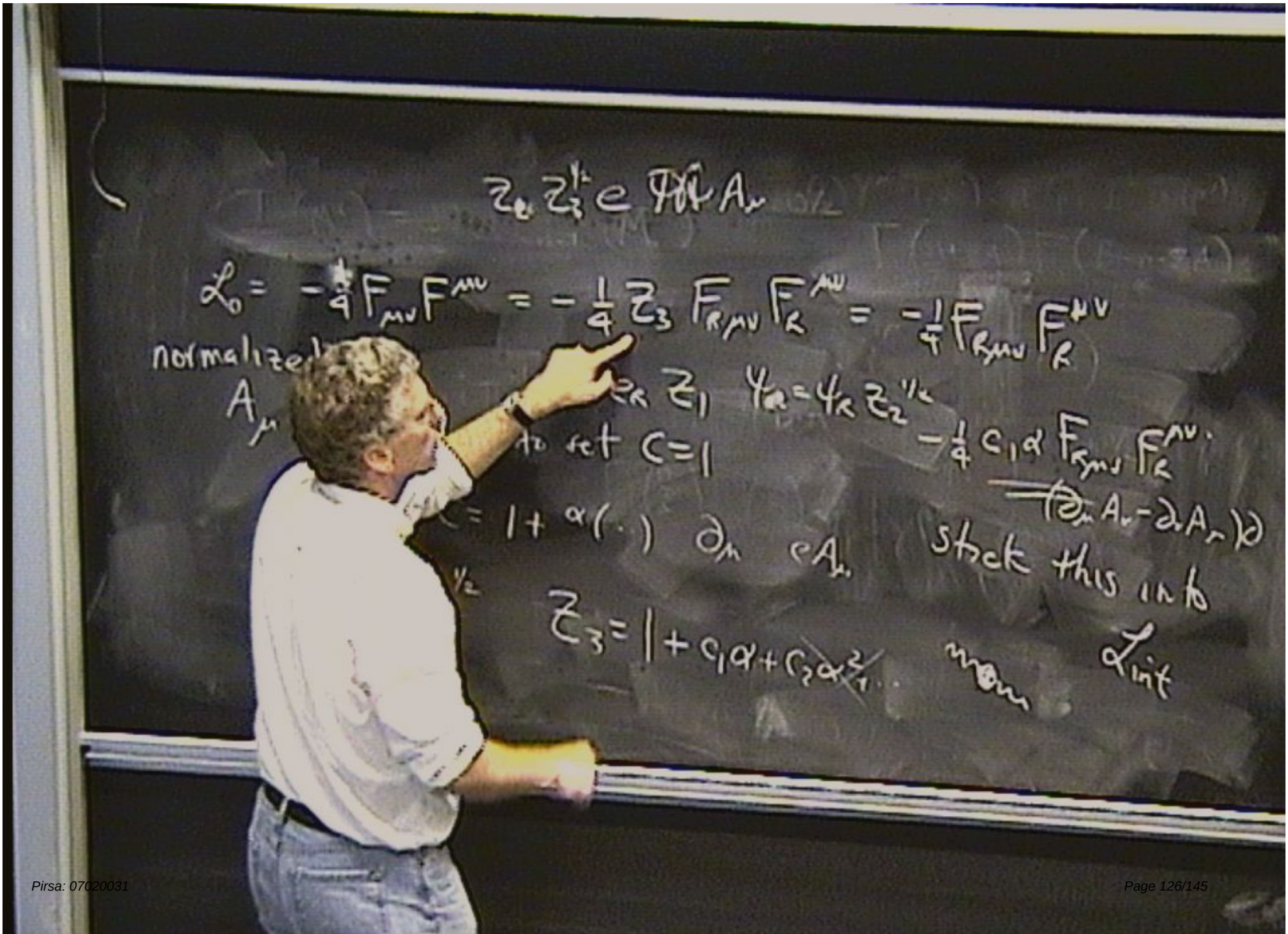
$$\frac{1}{n-4} \left(\frac{q_\mu q}{q^2}\right)^{n-4} \frac{q_\mu q}{q^2} \qquad \frac{\eta_{\mu\nu} + c\, q_\mu q_\nu}{n-1} \int \langle 0 | T A^\mu(x) A^\nu(0) | 0 \rangle e^{iqx} d^4x$$

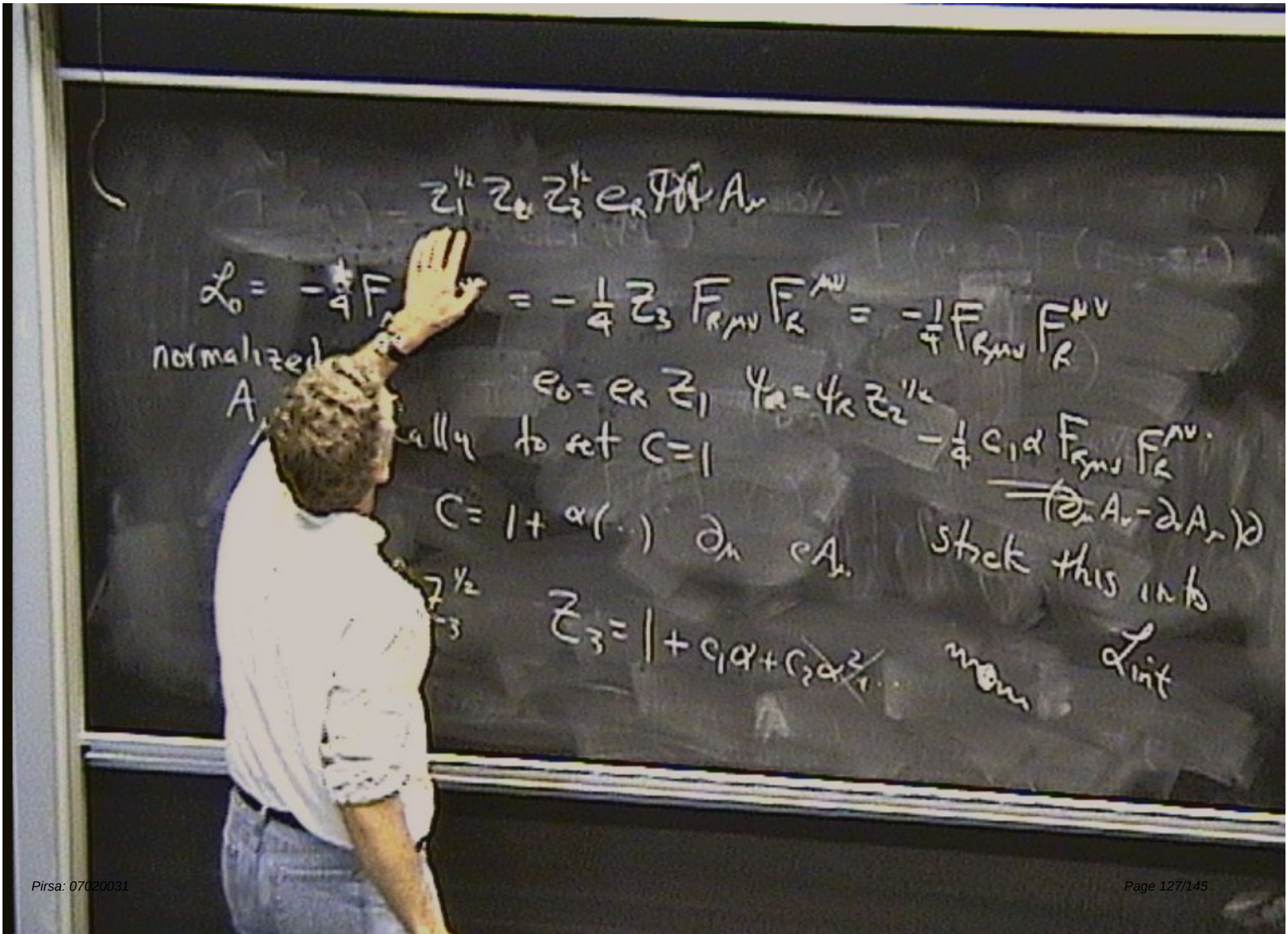
$$= \left(p^2 \eta_{\mu\nu} - p_\mu p_\nu\right) \Pi(p^2)$$

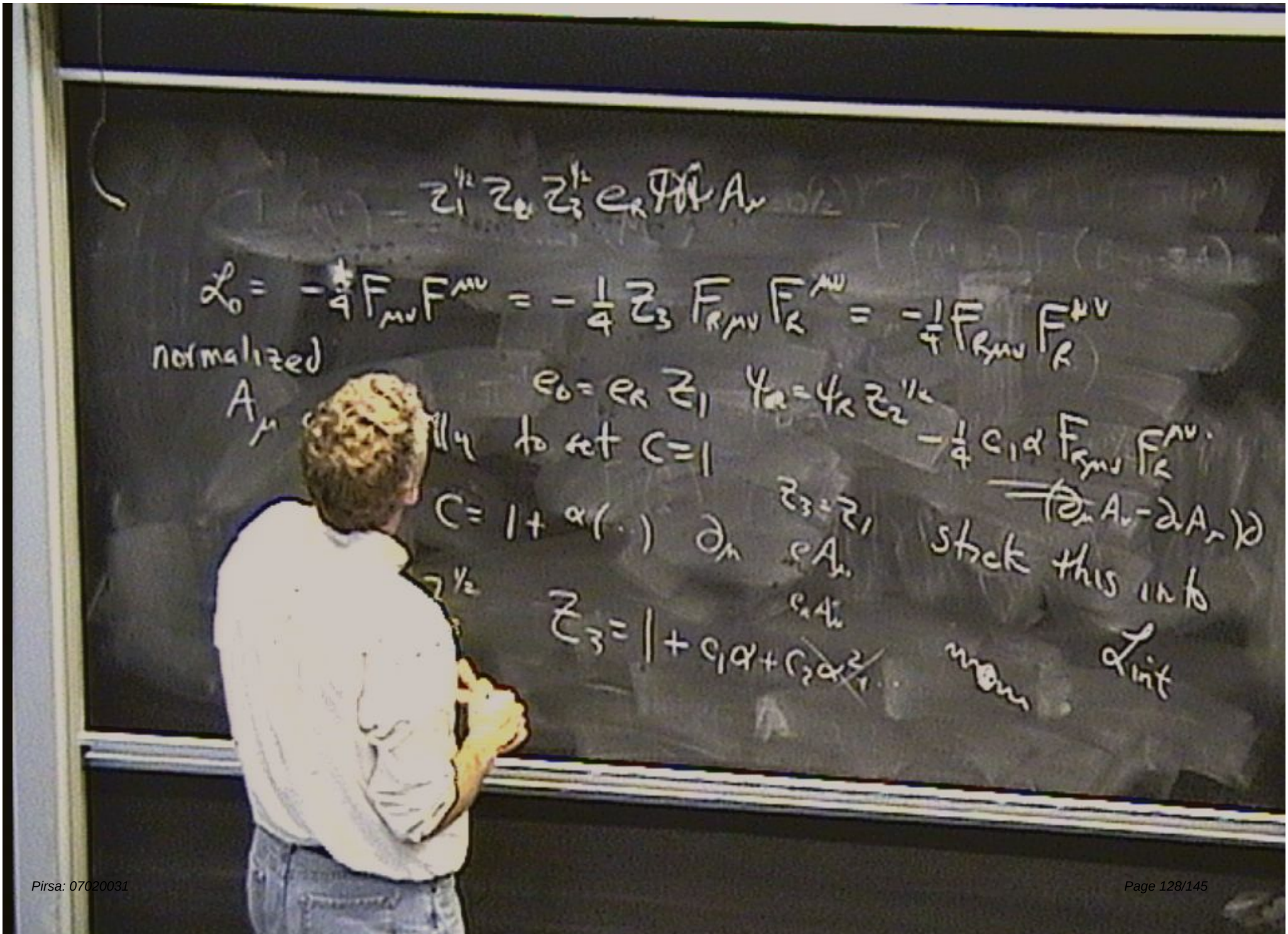

normalized
stick this into
$\mathcal{L}_{int}$


$Z_1 Z_3^{1/2} e$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$-\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
normalized
A_μ
to set $C=1$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
$C = 1 + \alpha(\cdot)$
stick this into
$\mathcal{L}_{int}$
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$


$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
stick this into
$\mathcal{L}_{int}$


$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}Z_3 F_{R\mu\nu}F_R^{\mu\nu} = -\frac{1}{4}F_{R\mu\nu}F_R^{\mu\nu}$
normalized
A_μ
to set $c=1$
$-\frac{1}{4}c_1\alpha F_{R\mu\nu}F_R^{\mu\nu}$
$(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$
stick this into
$\mathcal{L}_{int}$
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$


$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}_R \gamma^\mu \psi_R A_\mu$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
normalized
$e_0 = e_R Z_1 \quad \psi_R = \psi_R Z_2^{1/2}$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
$(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$
$C = 1 + \alpha(\cdot\cdot) \quad \partial_\mu \quad eA_\mu$
stick this into
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$
$\mathcal{L}_{int}$


$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
normalized
A_μ
$e_0 = e_R Z_1$
to set $C=1$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
$C = 1 + \alpha(\cdot)$
$Z_3 = Z_1$
stick this into
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$
$\mathcal{L}_{int}$

$$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}_R \gamma^\mu A_\mu$$

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$$

normalized

$$e_0 = e_R Z_1 \qquad \psi_0 = \psi_R Z_2^{1/2} \qquad -\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$$

A_μ canonically to set $C=1$

$$C = 1 + \alpha(\cdot\cdot) \qquad Z_3 = Z_1 \qquad \partial_\mu A_\nu - \partial A_\nu)^2$$

$$A_\mu = A_\mu^R Z_3^{1/2} \qquad Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \cdots$$

$c A_\mu$

$c_R A_\mu$

stick this into $\mathcal{L}_{int}$

mom

$$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}_R \gamma^\mu \psi_R A_\mu$$

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$$

normalized

$$e_0 = e_R Z_1 \qquad \psi_0 = \psi_R Z_2^{1/2}$$

A_μ canonically to set $C=1$

$$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$$

$$C = 1 + \alpha(\cdot) \qquad Z_3 = Z_1 \qquad \partial_\mu \quad c_1 A_\mu \qquad (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

stick this into

$$A_\mu = A_\mu^R Z_3^{1/2} \qquad Z_3 = 1 + c_1\alpha + \cancel{c_2\alpha^2} \ldots \qquad c_2 A_\mu^4 \qquad \mathcal{L}_{int}$$

mom

$$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}_R \gamma^\mu \psi_R A_\mu$$

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$$

normalized

$$e_0 = e_R Z_1 \qquad \psi_0 = \psi_R Z_2^{1/2} \qquad -\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$$

A_μ canonically to get $C=1$

$$C = 1 + \alpha(\cdot) \qquad Z_3 = Z_1 \qquad (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

$$\partial_\mu \quad e A_\mu \qquad \text{stick this into}$$

$$A_\mu = A_\mu^R Z_3^{1/2} \qquad Z_3 = 1 + c_1\alpha + \cancel{c_2\alpha^2} \qquad e_R A_\mu^R \qquad \mathcal{L}_{int}$$

$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}\psi A_\nu$

$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$

normalized

$e_0 = e_R Z_1 \quad \psi_0 = \psi_R Z_2^{1/2}$

A_μ canonically to get $C=1$

$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$

$\longrightarrow (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$

$C = 1 + \alpha(\ldots) \quad \partial_\mu \quad Z_3 = Z_1$

$e A_\mu$

stick this in to

$A_\mu = A_\mu^R Z_3^{1/2} \quad Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$

$e_R A_\mu$

$\mathcal{L}_{int}$

$D = 4 - L$

$$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}_R \gamma^\mu \psi_R A_\mu$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$$

normalized

$$e_0 = e_R Z_1 \qquad \psi_0 = \psi_R Z_2^{1/2}$$

canonically to set $C=1$

$$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$$

$$C = 1 + \alpha(\cdot) \qquad Z_3 = Z_1$$

$$\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

$$\partial_\mu \quad e A_\mu$$

stick this into

$$A_\mu = A_\mu^R Z_3^{1/2} \qquad e_R A_\mu^R$$

$$Z_3 = 1 + c_1 \alpha + c_2 \alpha^2 + \ldots$$

$$\mathcal{L}_{int}$$

$D = 4 - \epsilon$

$Z_1^{1/2} Z_0 Z_3^{1/2} e_R \bar{\psi}_R \not{A} \psi_R$

$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$

normalized

$e_0 = e_R Z_1 \quad \psi_0 = \psi_R Z_2^{1/2}$

A_μ canonically to set $C=1$

$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$

$C = 1 + \alpha(\cdot) \quad \partial_\mu \quad Z_3 = Z_1$

$(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$

$e A_\mu$

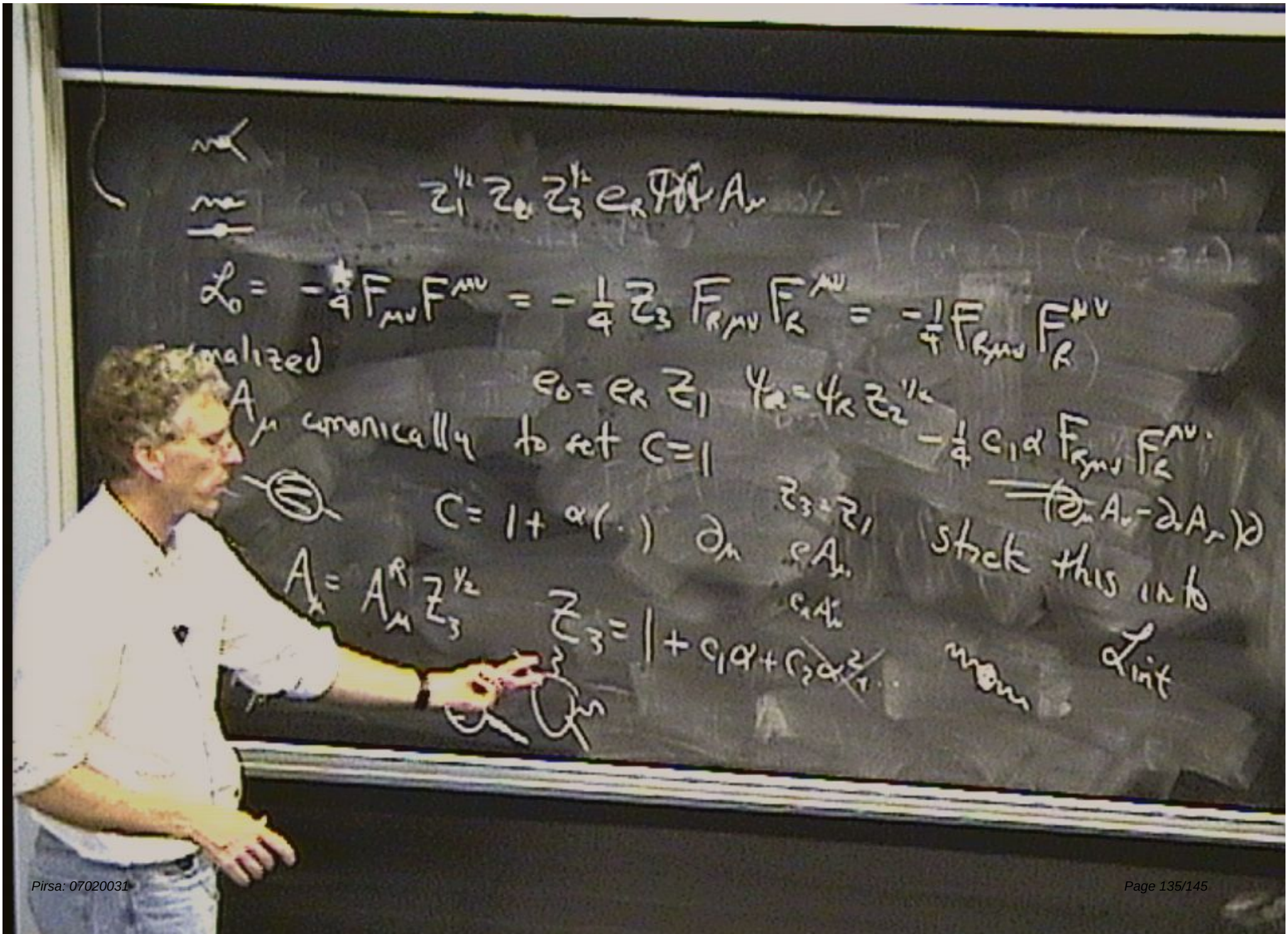
stick this into

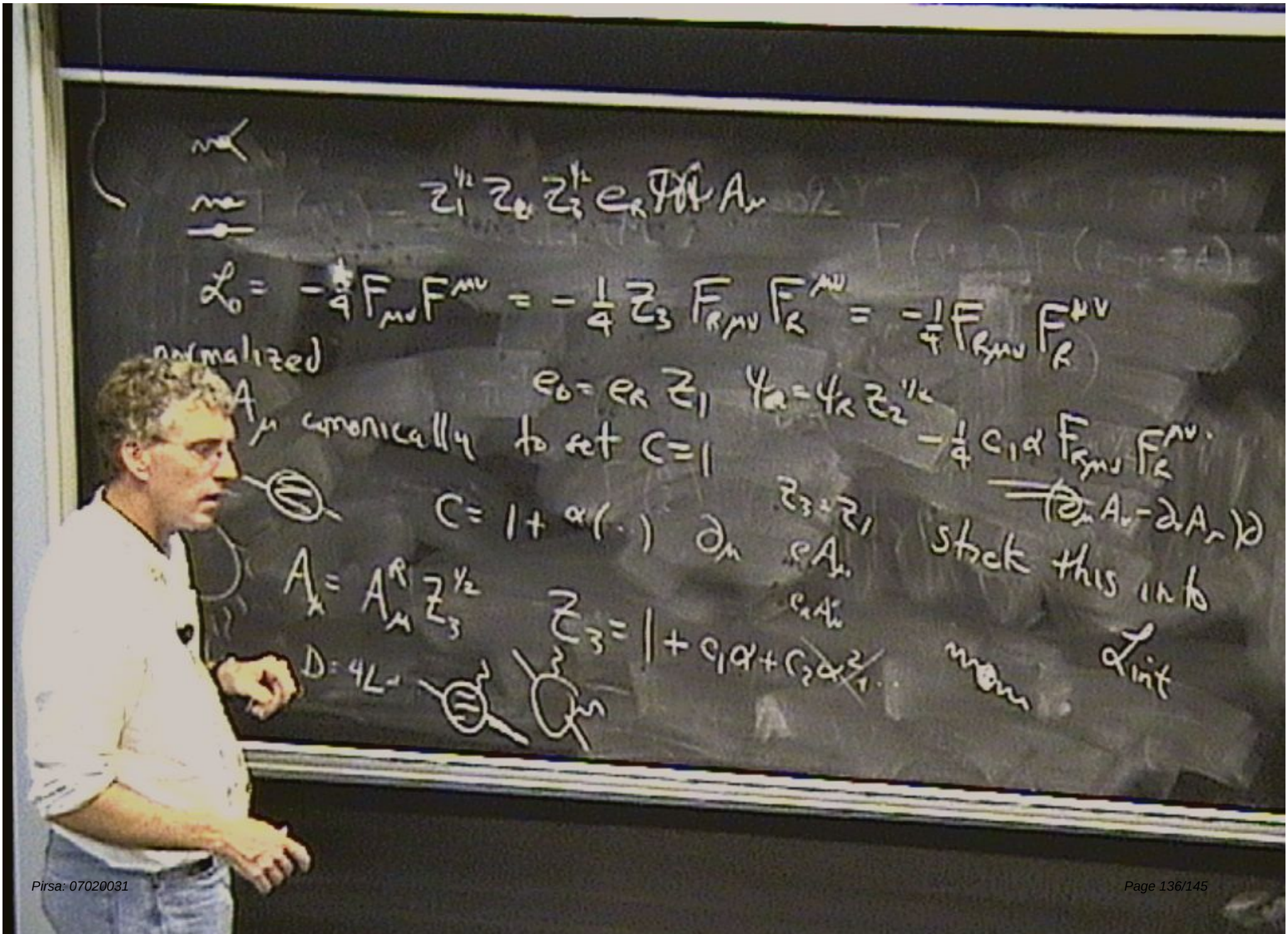
$A_\mu = A_\mu^R Z_3^{1/2} \quad Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$

$e_R A_\mu$

$\mathcal{L}_{int}$

$D = 4 - \epsilon$


$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \psi_R A_\nu$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
$e_0 = e_R Z_1 \quad \psi_0 = \psi_R Z_2^{1/2}$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
A_μ canonically to get $C=1$
$C = 1 + \alpha(\cdot)$
$Z_3 = Z_1$
$(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$
stick this into
$A_\mu = A_\mu^R Z_3^{1/2}$
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$
$\mathcal{L}_{int}$


$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \psi_R A_\nu$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
normalized
$e_0 = e_R Z_1 \quad \psi_0 = \psi_R Z_2^{1/2}$
A_μ canonically to set $C=1$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
$C = 1 + \alpha(\cdot)$
$Z_3 = Z_1$
$\partial_\mu (\partial_\mu A_\nu - \partial_\nu A_\mu)$
$A_\mu = A_\mu^R Z_3^{1/2}$
$e_R A_\mu$
stick this into
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$
$e_R A_\mu$
$\mathcal{L}_{int}$
D=4

$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \psi_R A_\nu$

$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$

lized

$e_0 = e_R Z_1 \quad \psi_0 = \psi_R Z_2^{1/2}$

A_μ canonically to get $C=1$

$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$

$(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$

$C = 1 + \alpha(\cdot\cdot) \quad \partial_\mu \quad Z_3 = Z_1$

$e A_\mu$ stick this into

$A_\mu = A_\mu^R Z_3^{1/2} \quad Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$

$e_R A_\mu$

$\mathcal{L}_{int}$

$D = 4 - L$

$Z_1^{1/2} Z_0 Z_3^{1/2} e_R \bar{\psi}\psi A_\nu$

$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$

normalized

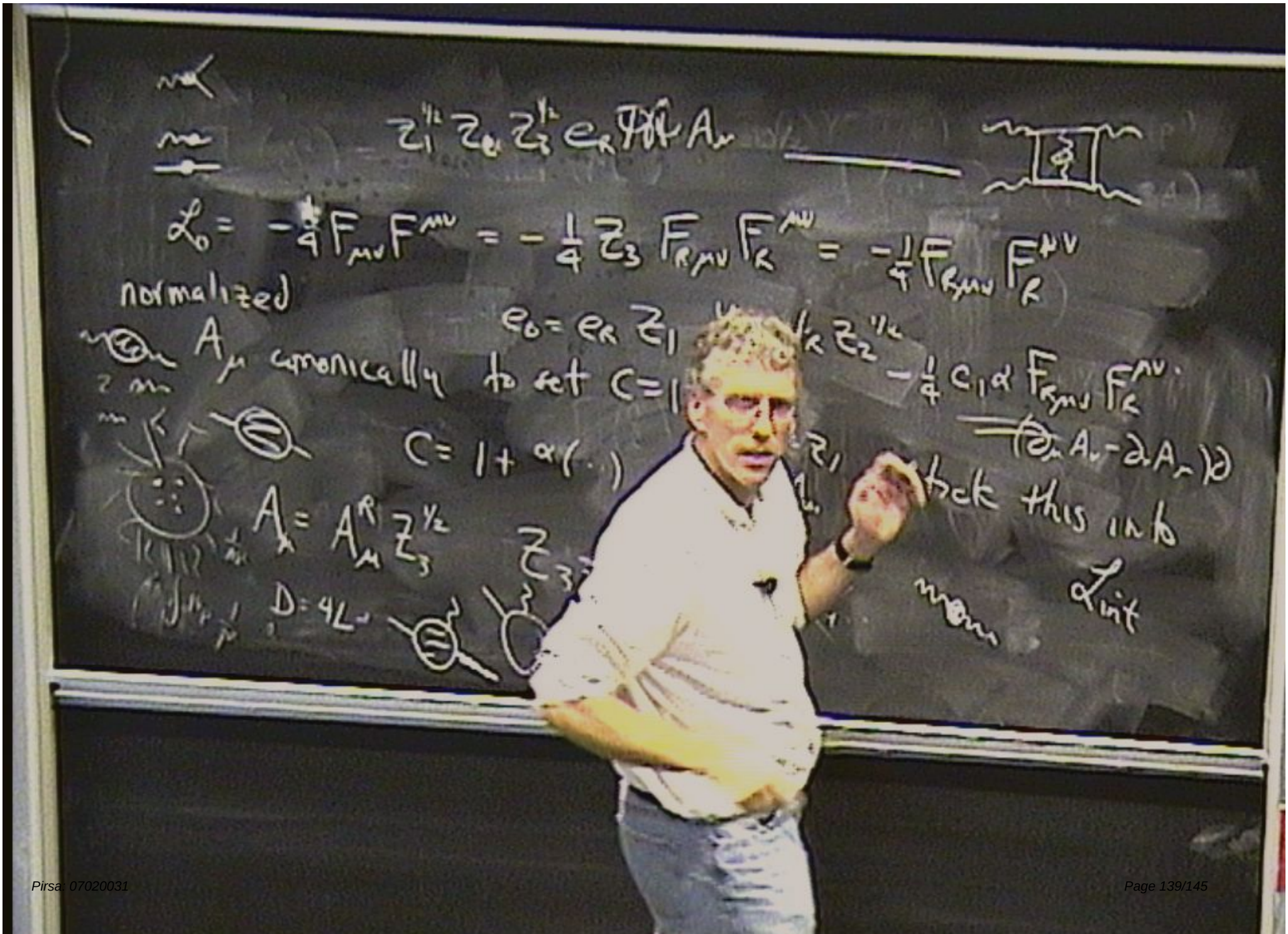
$e_0 = e_R Z_1 \quad \psi_0 = \psi_R Z_2^{1/2} \quad -\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$

A_μ canonically to set $C=1$

$C = 1 + \alpha(\cdot) \quad \partial_\mu \quad Z_3 = Z_1 \quad (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$

$A_\mu = A_\mu^R Z_3^{1/2} \quad Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$ stick this into $\mathcal{L}_{int}$

$D = 4L$


$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}\gamma^\mu A_\mu$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
normalized
$e_0 = e_R Z_1$
A_μ canonically to set $C=1$
$-\frac{1}{4} C_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
$C = 1 + \alpha(\ldots)$
$(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial$
$A_\mu = A_\mu^R Z_3^{1/2}$
Z_3
$D = 4L$
$\mathcal{L}_{int}$

$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}_R \not{A} \psi_R$

$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$

normali

$e_0 = e_R Z_1 \quad \psi_0 = \psi_R Z_2^{1/2}$

canonically to set $C=1$

$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$

$(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial$

$C = 1 + \alpha(\cdot\cdot) \quad \partial_\mu \quad Z_3 = Z_1 \quad eA_\mu$

stick this into

$A_\mu = A_\mu^R Z_3^{1/2} \quad Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$

$e_R A_\mu^R$

$\mathcal{L}_{int}$

$D = 4 - \epsilon$

$Z_1^{1/2} Z_2 Z_3^{1/2} e_R \bar{\psi}_R \not{A} \psi_R$

$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$

normalized

$e_0 = e_R Z_1 \quad \psi_0 = \psi_R Z_2^{1/2} \quad -\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$

A_μ canonically to set $c=1$

$(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial$

$c = 1 + \alpha(\ldots) \quad \partial_\mu \quad Z_3 = Z_1 \quad e A_\mu$

stick this into

$A_\mu = A_\mu^R Z_3^{1/2} \quad Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots \quad e_R A_\mu$

α_{int}

$D = 4 - \epsilon$

Pirsa: 07020031

Page 141/145

$$\mathrm{Tr}[\gamma_\mu \gamma_\nu] = \eta_{\mu\nu}\,\mathrm{Tr}[I] = 2^{n/2}\,\eta_{\mu\nu}$$

$$\mathrm{T}[\gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho] = 2^{n/2}(\eta_{\mu\nu}\eta_{\lambda\rho} - \eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda})$$

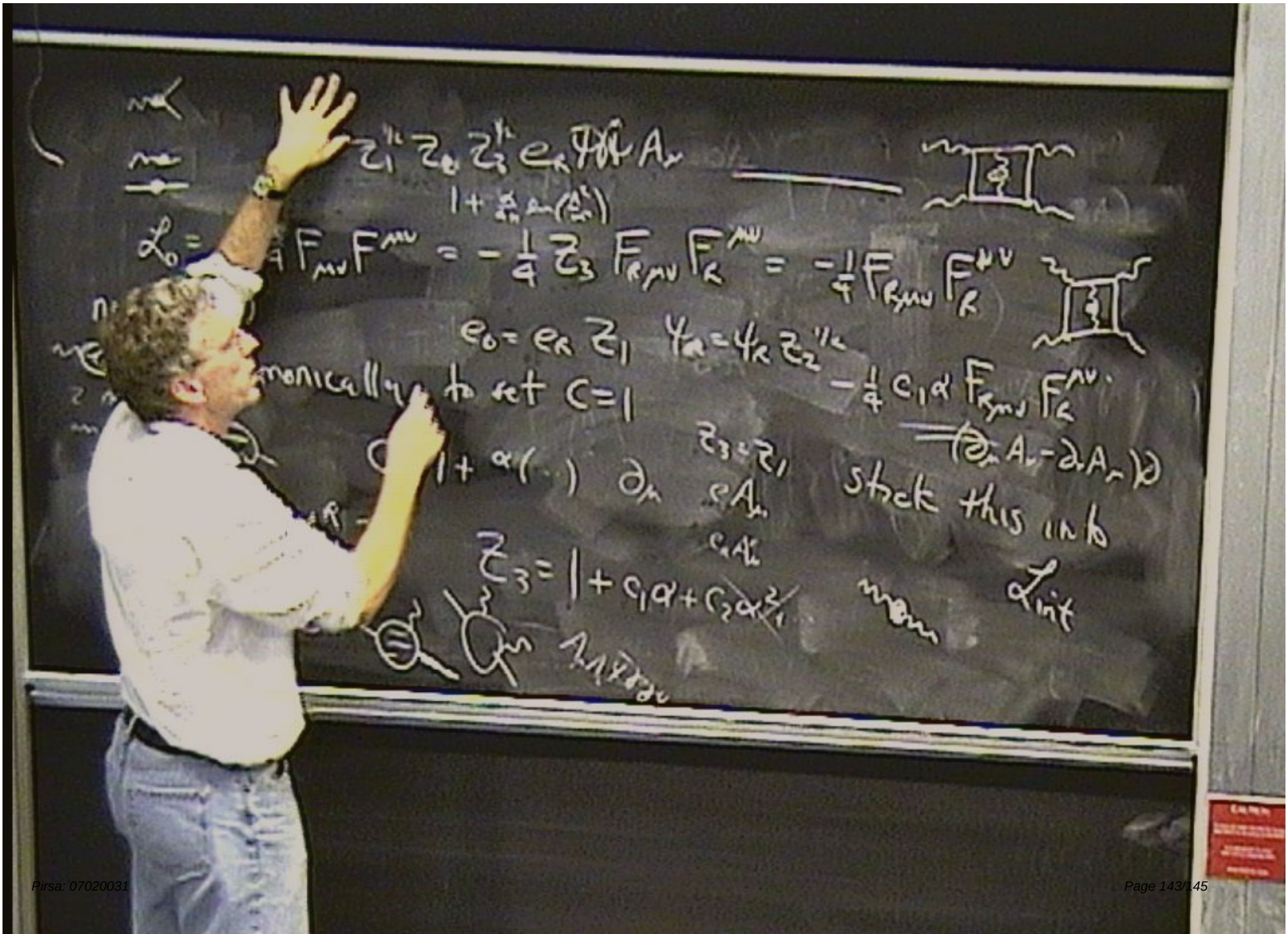
$$\mathrm{Tr}(\gamma_\mu \gamma_\nu) = \frac{1}{2}\mathrm{Tr}\underbrace{[\{\gamma_\mu, \gamma_\nu\}]}_{2\eta_\mu}$$

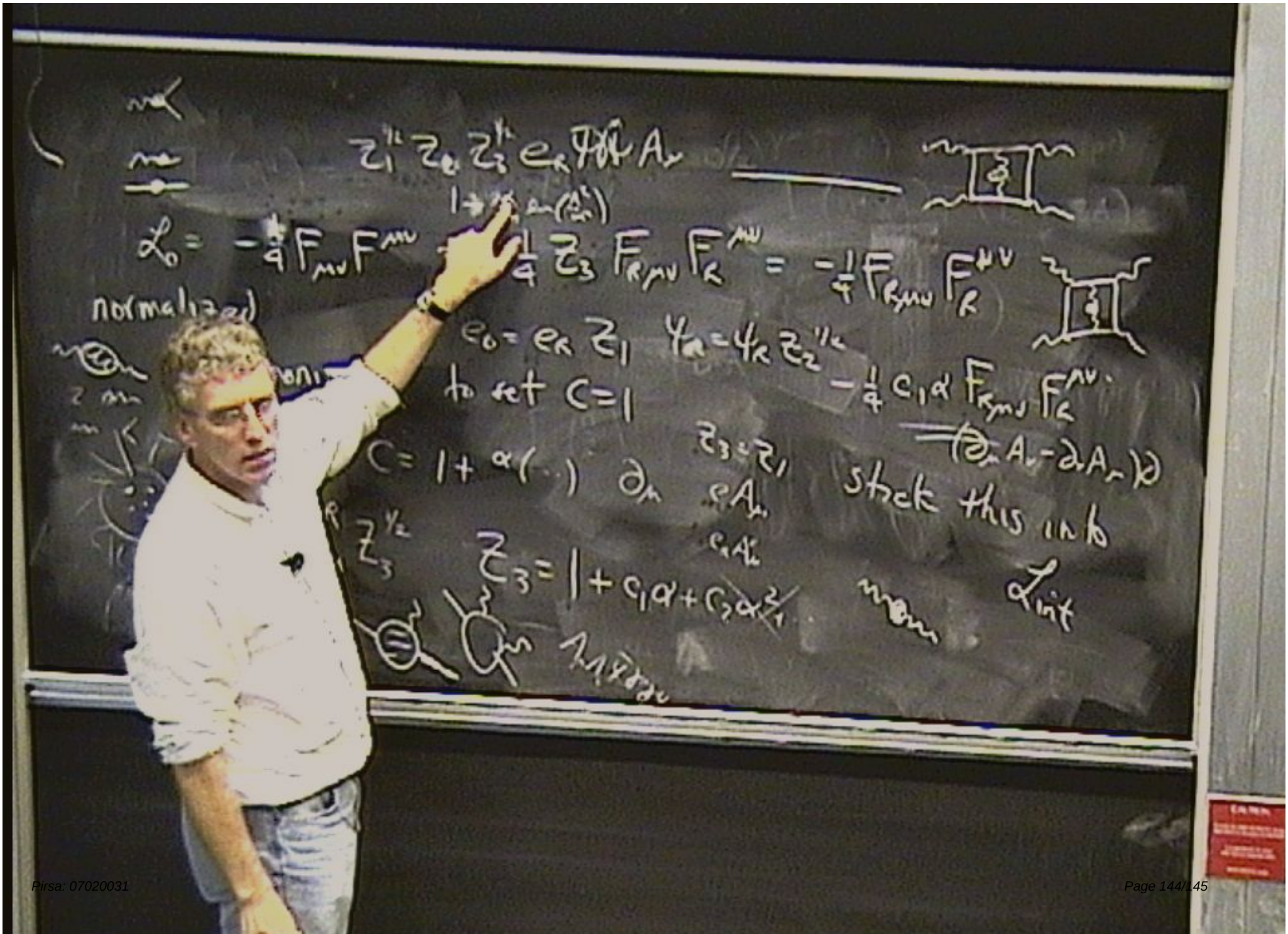
$$A \int \frac{d^D p}{\ell} \qquad \delta(\not{t})$$

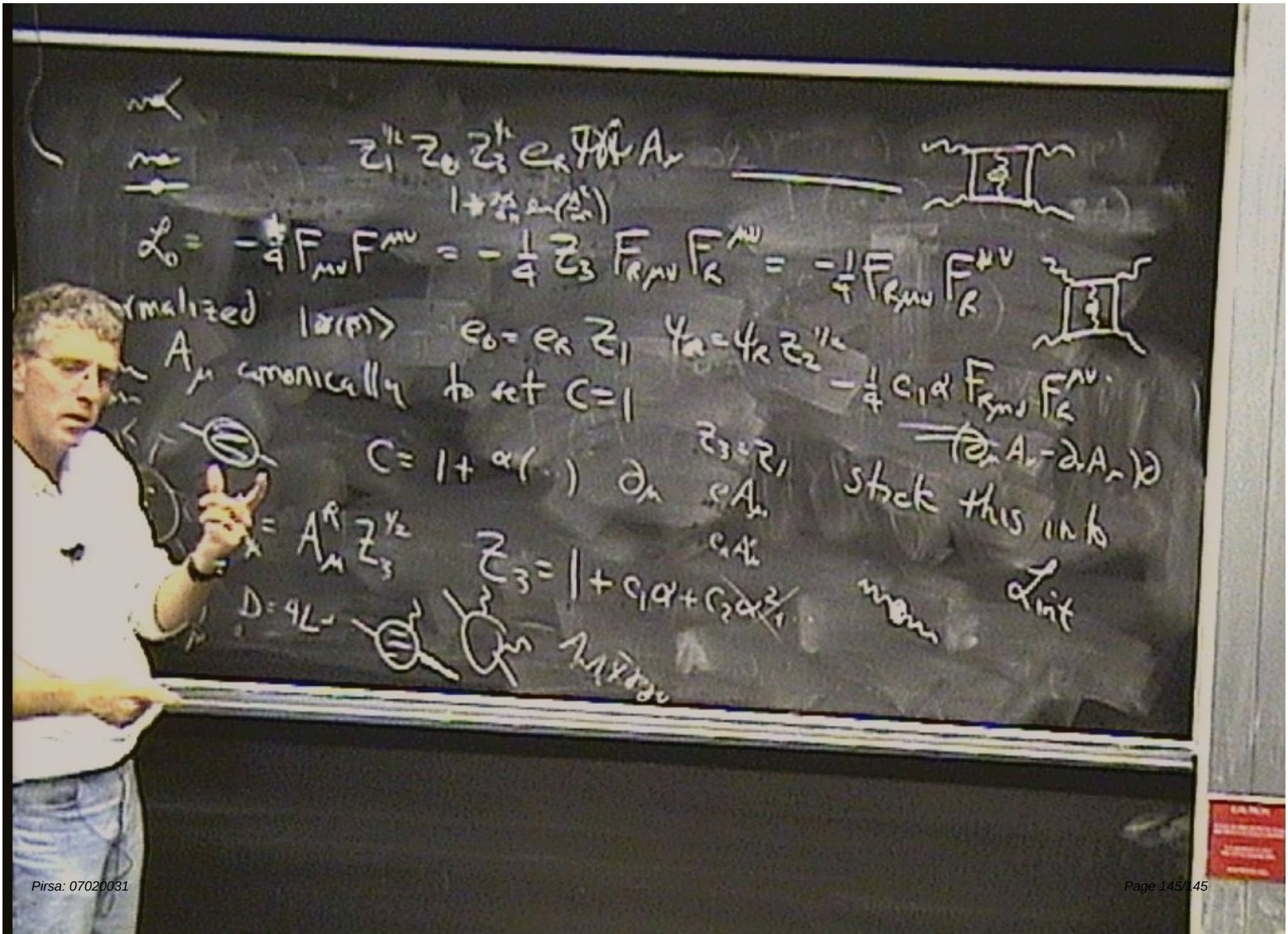
In n-dimensions

a spinor $2^{n/2} = D$

(e.g. $n=4$ $D=4$)


$e_0 = e_R Z_1$
$\psi_0 = \psi_R Z_2^{1/2}$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
monically to set $c=1$
$Z_3 = Z_1$
stick this in to
$\mathcal{L}_{int}$
$Z_3 = 1 + c_1\alpha + c_2\alpha^2$


normalize
to set C=1
stick this into
$\mathcal{L}_{int}$


$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} Z_3 F_{R\mu\nu} F_R^{\mu\nu} = -\frac{1}{4} F_{R\mu\nu} F_R^{\mu\nu}$
$e_0 = e_R Z_1$
$-\frac{1}{4} c_1 \alpha F_{R\mu\nu} F_R^{\mu\nu}$
A_μ canonically to set $C=1$
$C = 1 + \alpha(\ldots)$
$Z_3 = Z_1$
stick this into
$\mathcal{L}_{int}$
$Z_3 = 1 + c_1\alpha + c_2\alpha^2 + \ldots$