Title: Graduate Course on Standard Model & Quantum Field Theory - 13A

Date: Feb 28, 2007 11:00 AM

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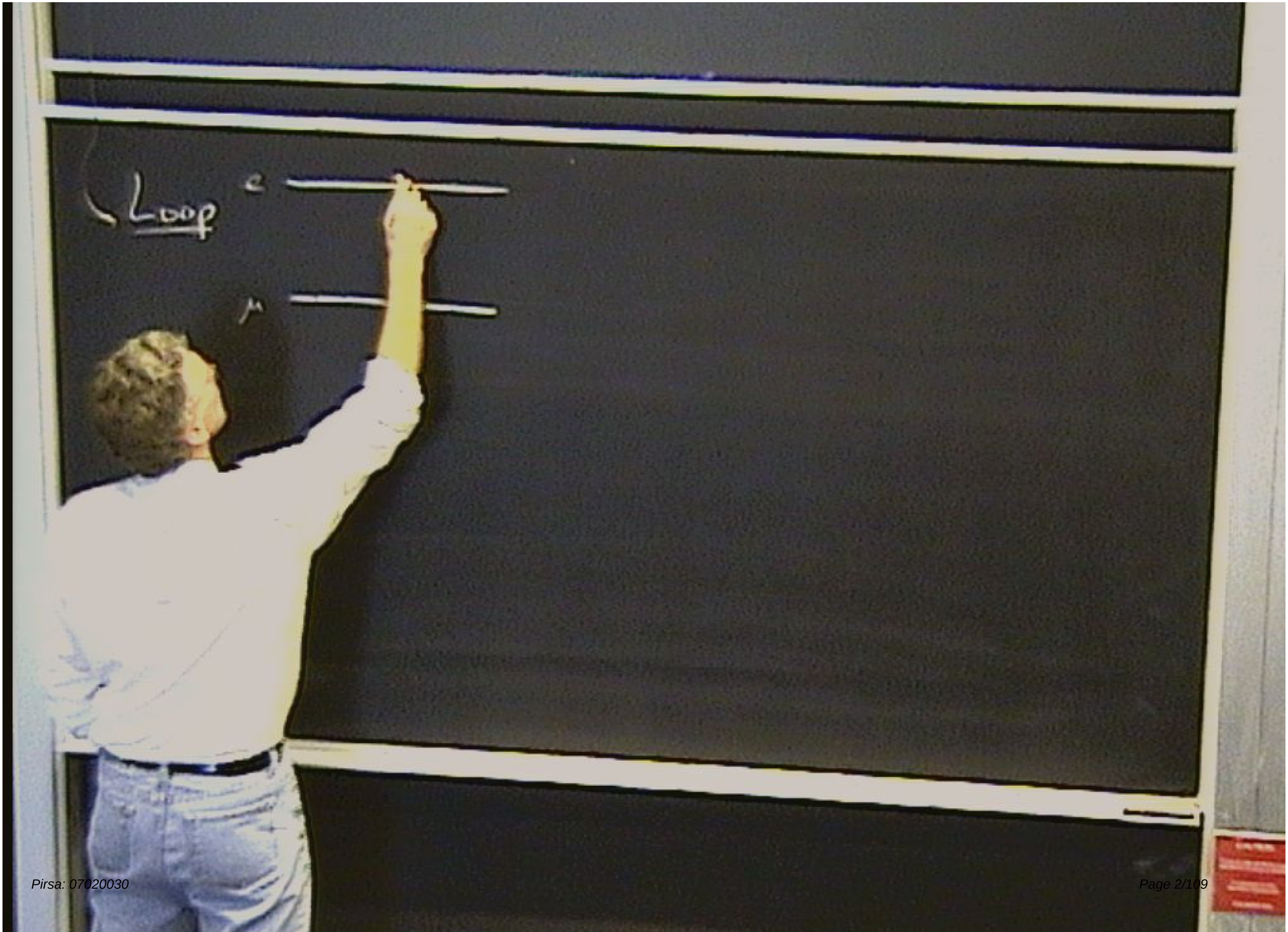
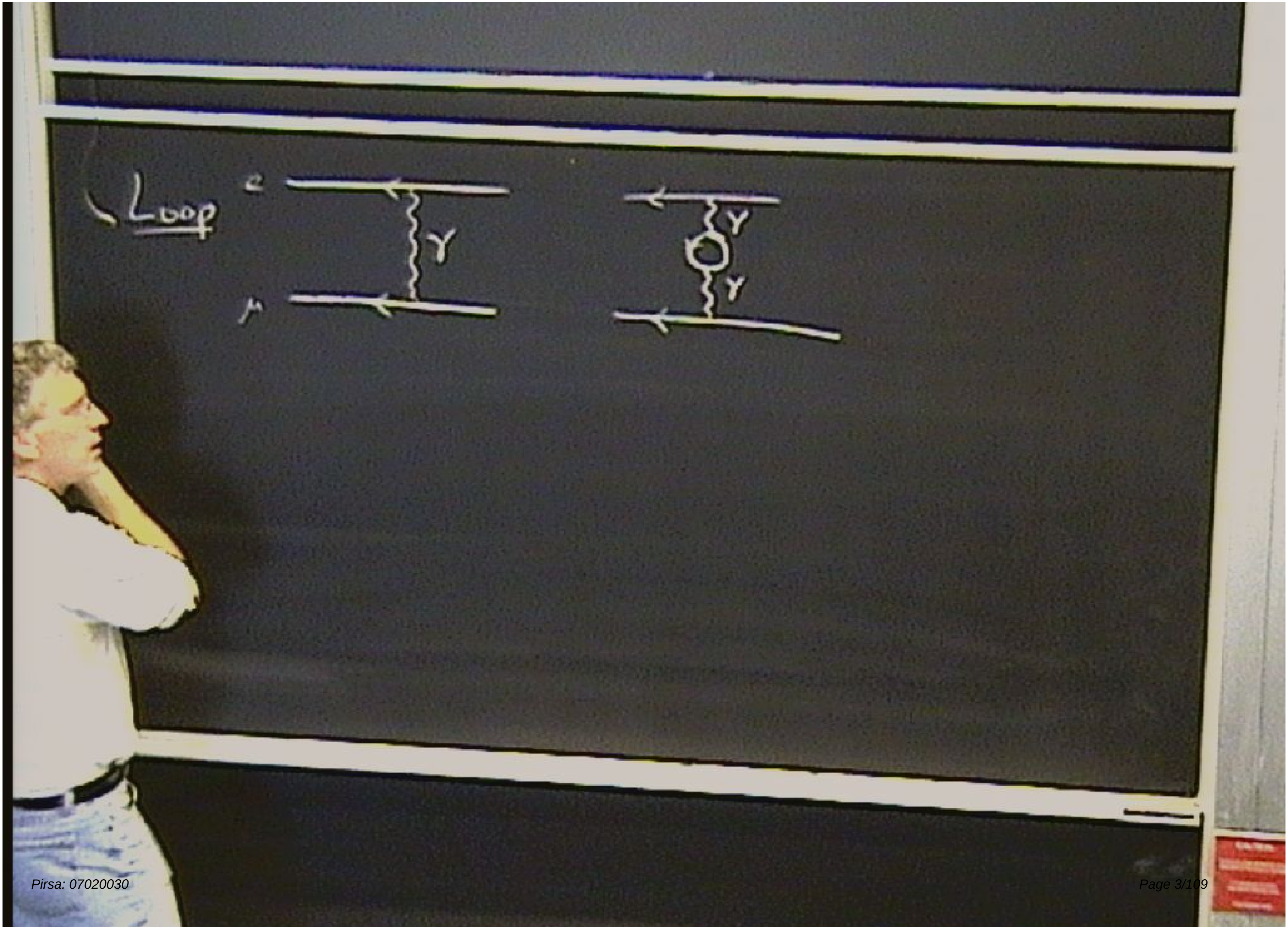
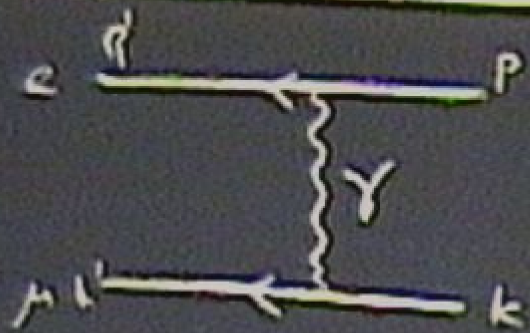
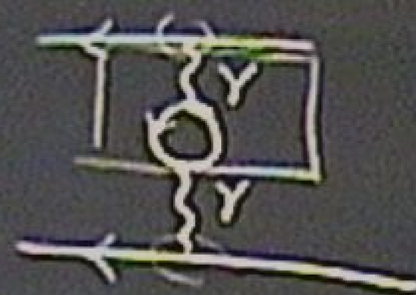
Abstract: Graduate Course on Standard Model & Quantum Field Theory


Loop
e
μ

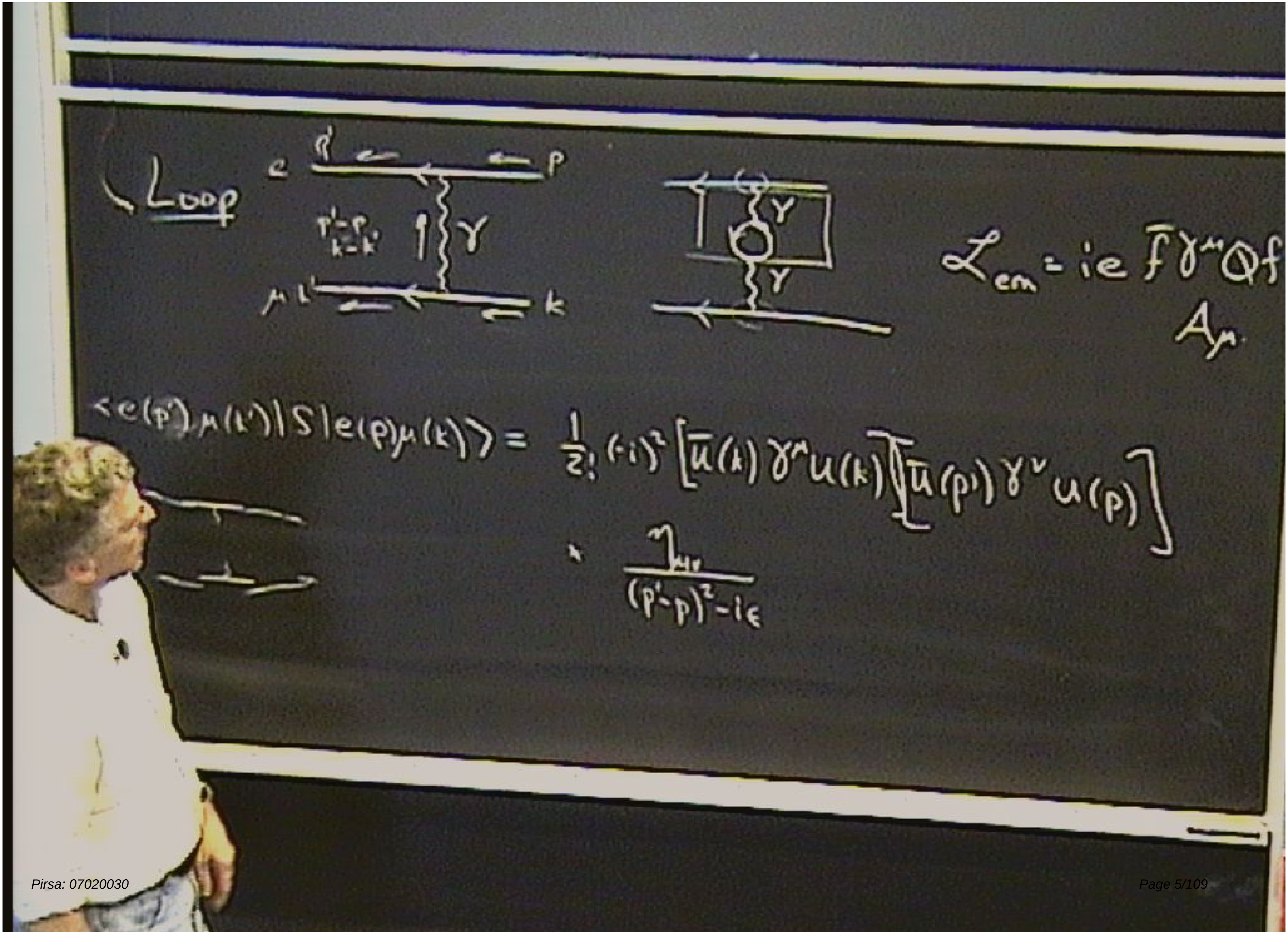

Loop
e
μ
γ
γ
γ

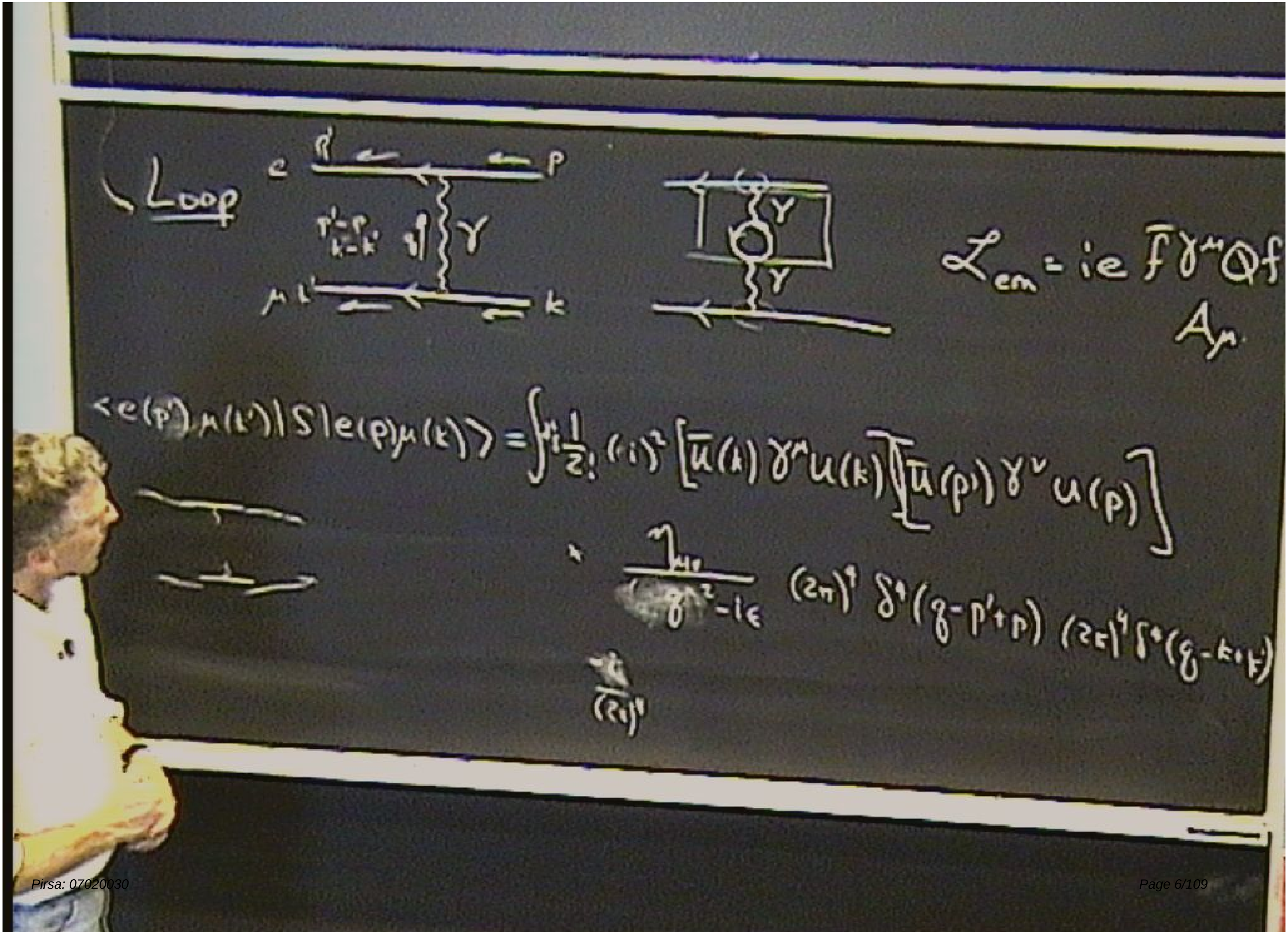
Loop

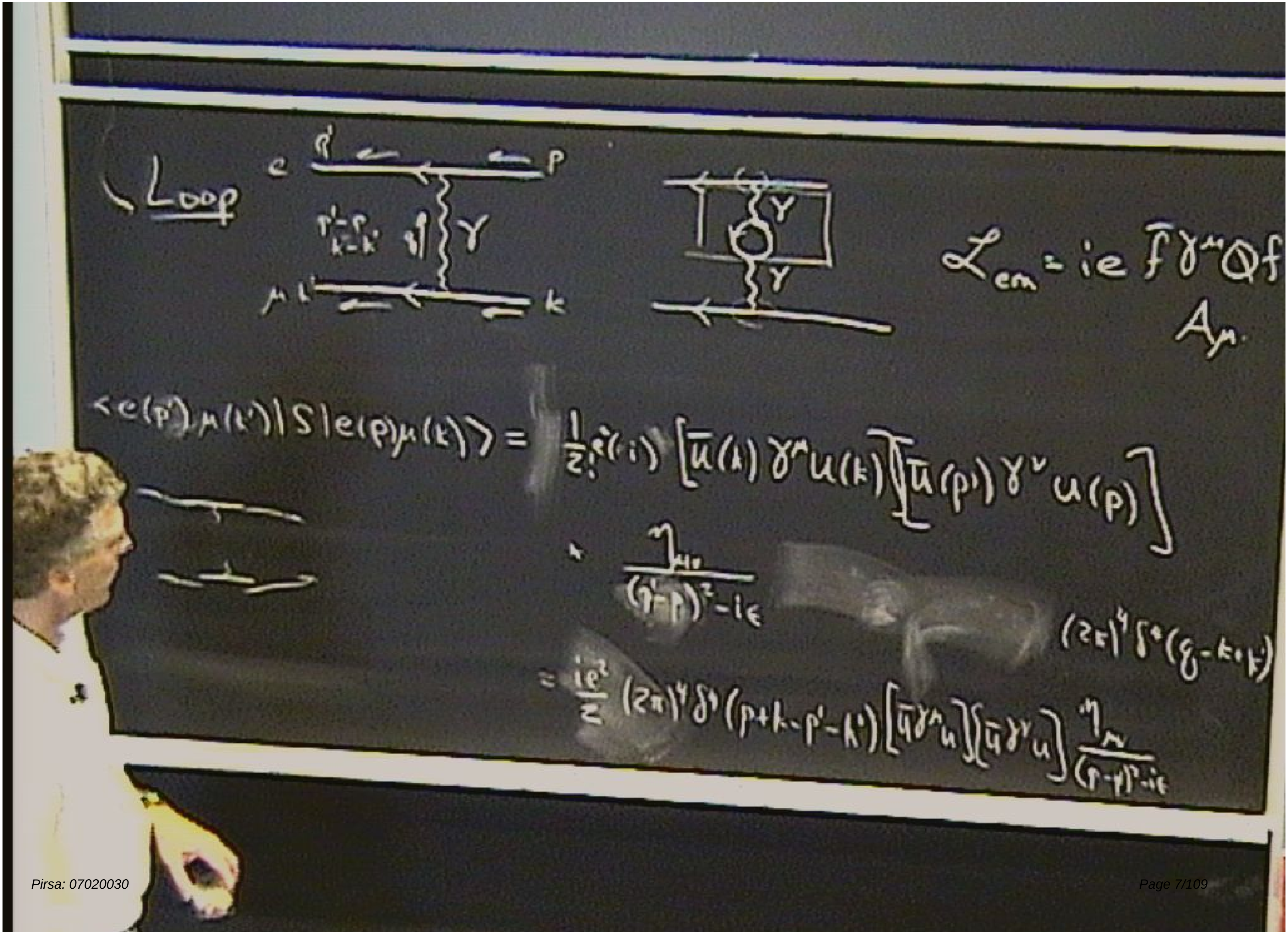




$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{1}{2!}(-i)^2$$


Loop
e p' p
p'-p, k-k' γ
μ k' k
γ
γ
$\mathcal{L}_{em} = ie \bar{f} \gamma^\mu Q f A_\mu$
$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{1}{2!}(-i)^2 [\bar{u}(k')\gamma^\mu u(k)][\bar{u}(p')\gamma^\nu u(p)] \times \frac{\eta_{\mu\nu}}{(p'-p)^2 - i\epsilon}$


Loop
e
p
γ
μ
k
$\mathcal{L}_{em} = ie\bar{f}\gamma^\mu Q f A_\mu$
$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \int \frac{1}{2!}(i)^2 [\bar{u}(k')\gamma^\mu u(k)][\bar{u}(p')\gamma^\nu u(p)]$
$\frac{\eta_{\mu\nu}}{q^2 - i\epsilon}(2\pi)^4\delta^4(q-p'+p)(2\pi)^4\delta^4(q-k+k')$
$\frac{d^4q}{(2\pi)^4}$


Loop
$\mathcal{L}_{em} = ie\bar{f}\gamma^\mu Q f A_\mu$
$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{1}{2!}e^2(i)^2[\bar{u}(k')\gamma^\mu u(k)][\bar{u}(p')\gamma^\nu u(p)]$
$\frac{\eta_{\mu\nu}}{(p'-p)^2-i\epsilon}$
$(2\pi)^4\delta^4(q-k+k')$
$= \frac{ie^2}{2}(2\pi)^4\delta^4(p+k-p'-k')[\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u]\frac{\eta_{\mu\nu}}{(p-p')^2-i\epsilon}$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{1}{2!}e^2(i)\,\left[\bar u(k')\gamma^\mu u(k)\right]\left[\bar u(p')\gamma^\nu u(p)\right]$$

$$\times\ \frac{\eta_{\mu\nu}}{(\ldots)^2-i\epsilon}\qquad (2\pi)^4\delta^4(q-$$

$$= \frac{ie^2}{2}(2\pi)^4\delta^4(p+k-p'-k')\left[\bar u\gamma^\mu u\right]\left[\bar u\gamma^\nu u\right]\frac{\eta_{\mu\nu}}{(p-p')^2-i\epsilon}$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle =$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{1}{2!}e^2(i)\,\left[\bar{u}(k')\gamma^\mu u(k)\right]\left[\bar{u}(p')\gamma^\nu u(p)\right]$$

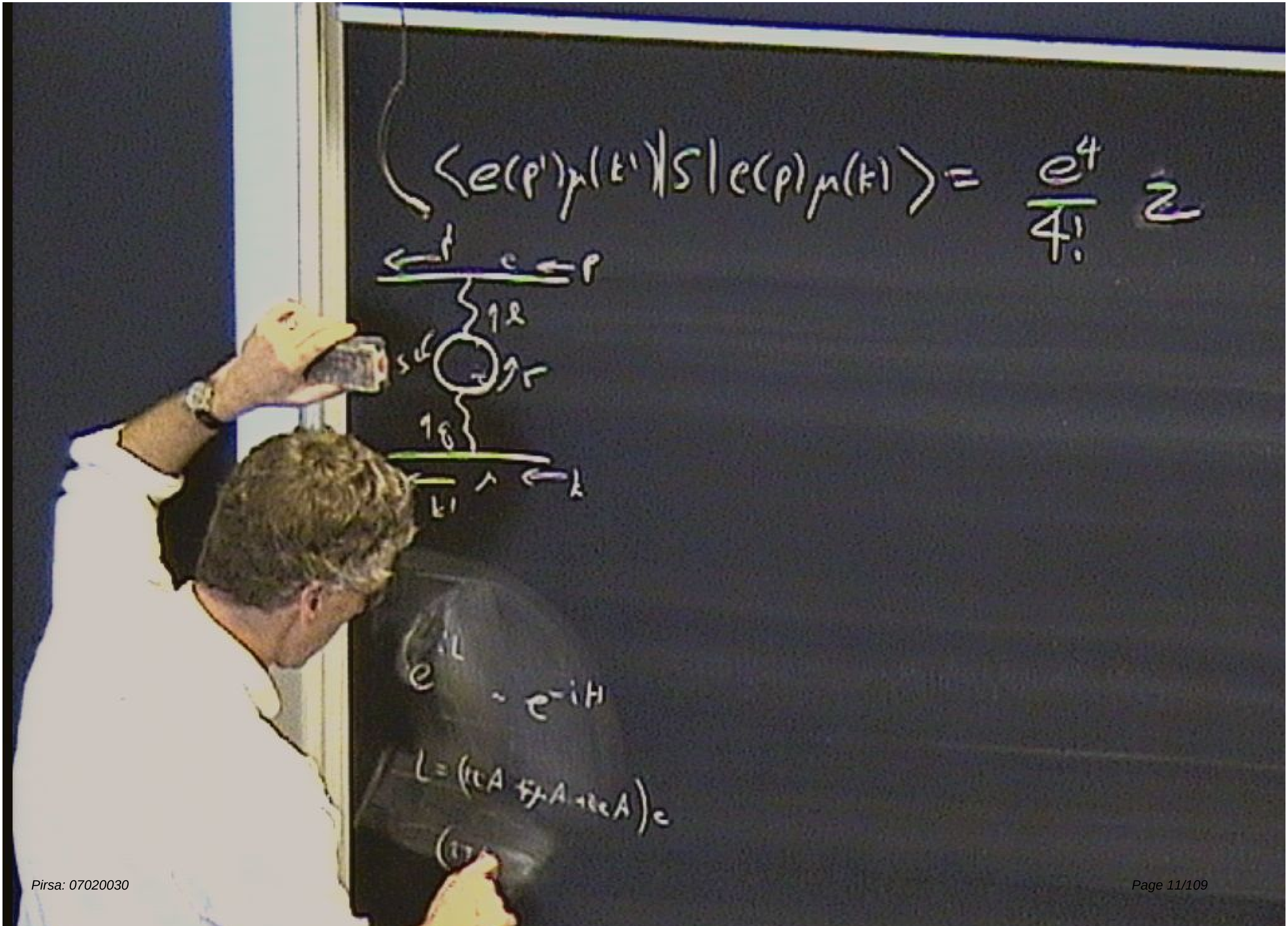
$$\times\ \frac{\eta_{\mu\nu}}{(p'-p)^2-i\epsilon}\ (2\pi)^4\delta^4(q-$$

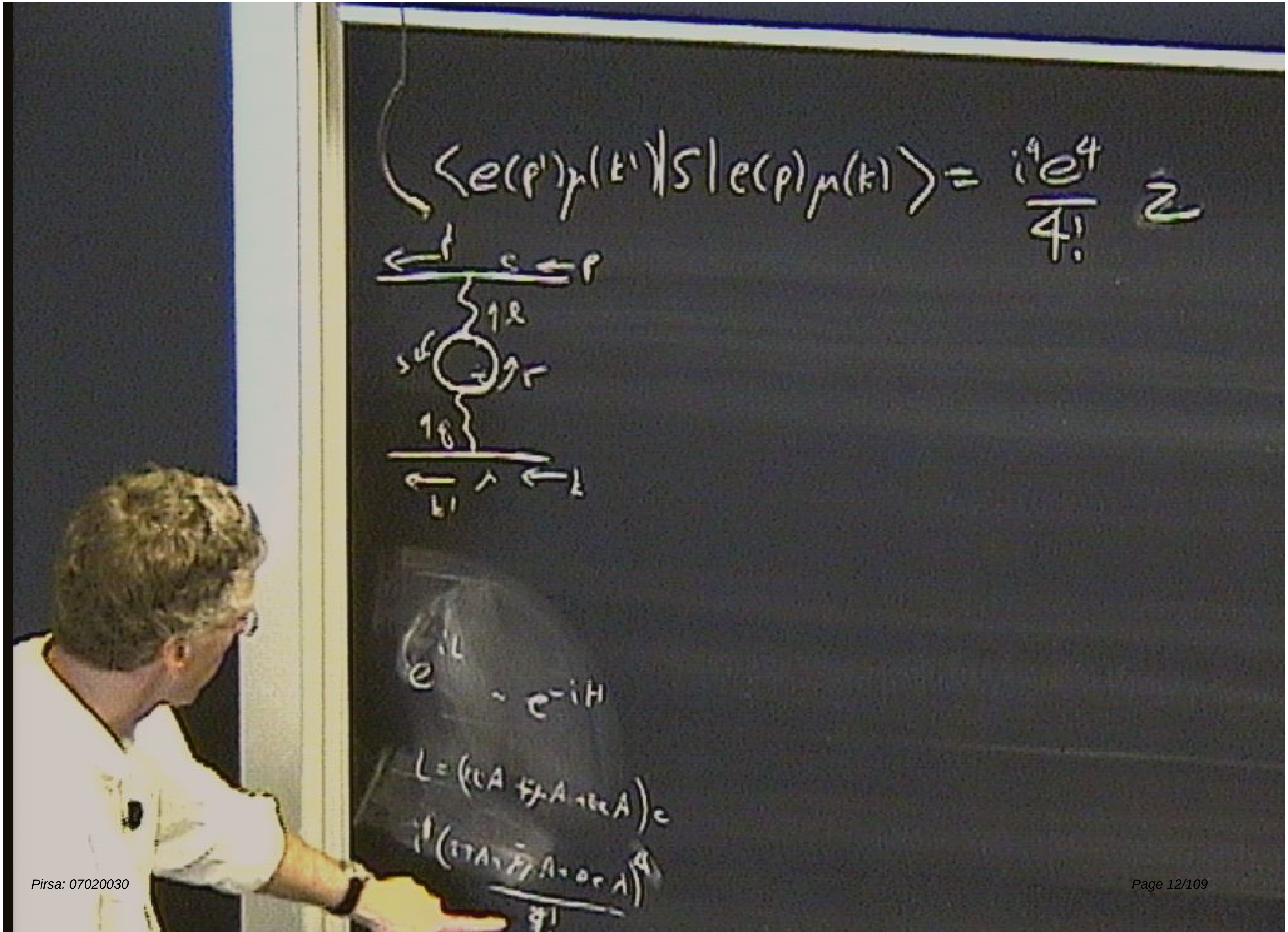
$$= \frac{ie^2}{2}(2\pi)^4\delta^4(p+k-p'-k')\left[\bar{u}\gamma^\mu u\right]\left[\bar{u}\gamma^\nu u\right]\frac{\eta_{\mu\nu}}{(p-p')^2-i\epsilon}$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{e^4}{4!}$$

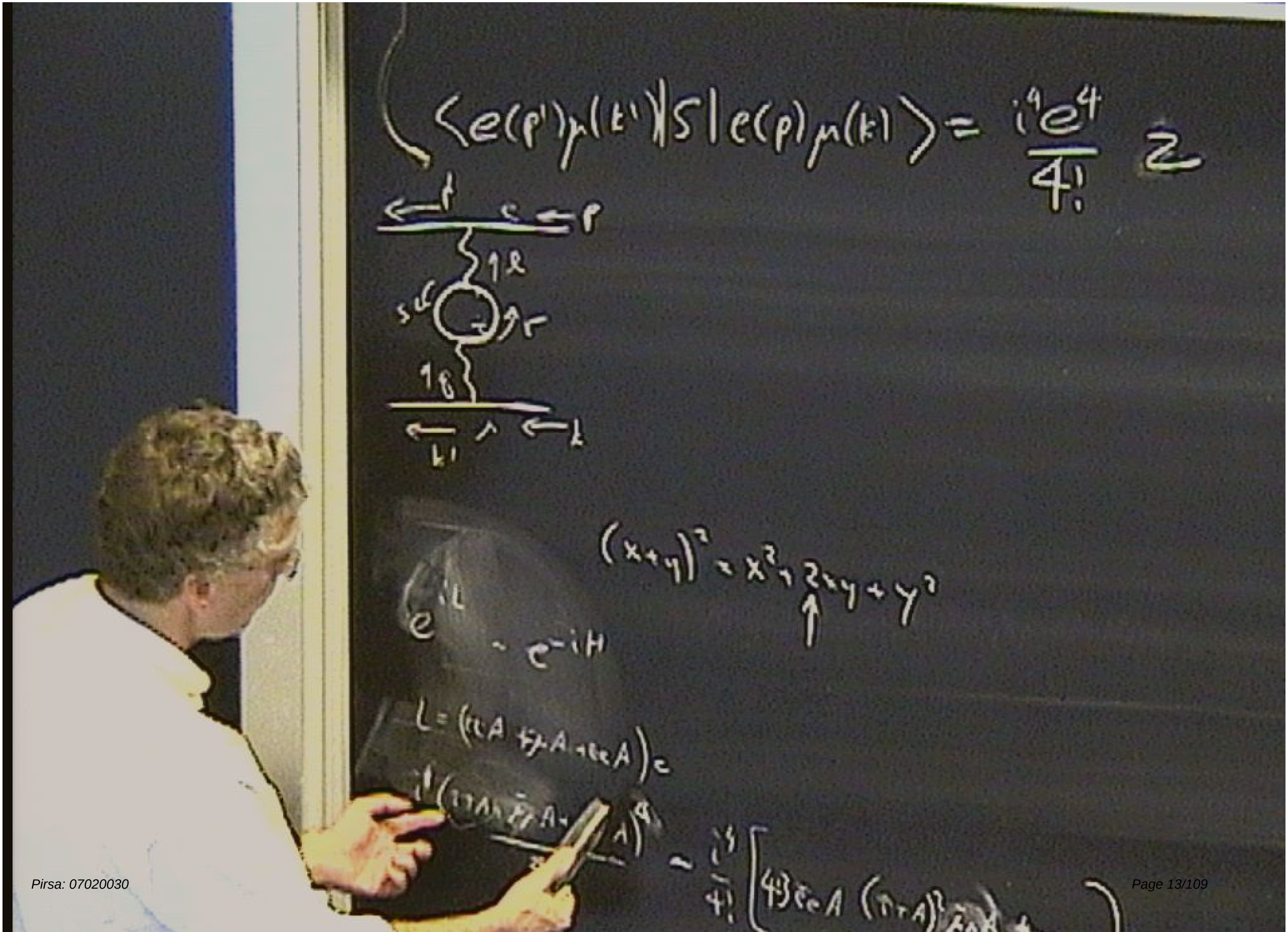
$$\ldots \delta^4(p+k-p'-k')\left[\bar{u}\gamma^\mu u\right]\left[\bar{u}\gamma_\mu u\right]\frac{1}{(p-p')^2-i\epsilon}$$

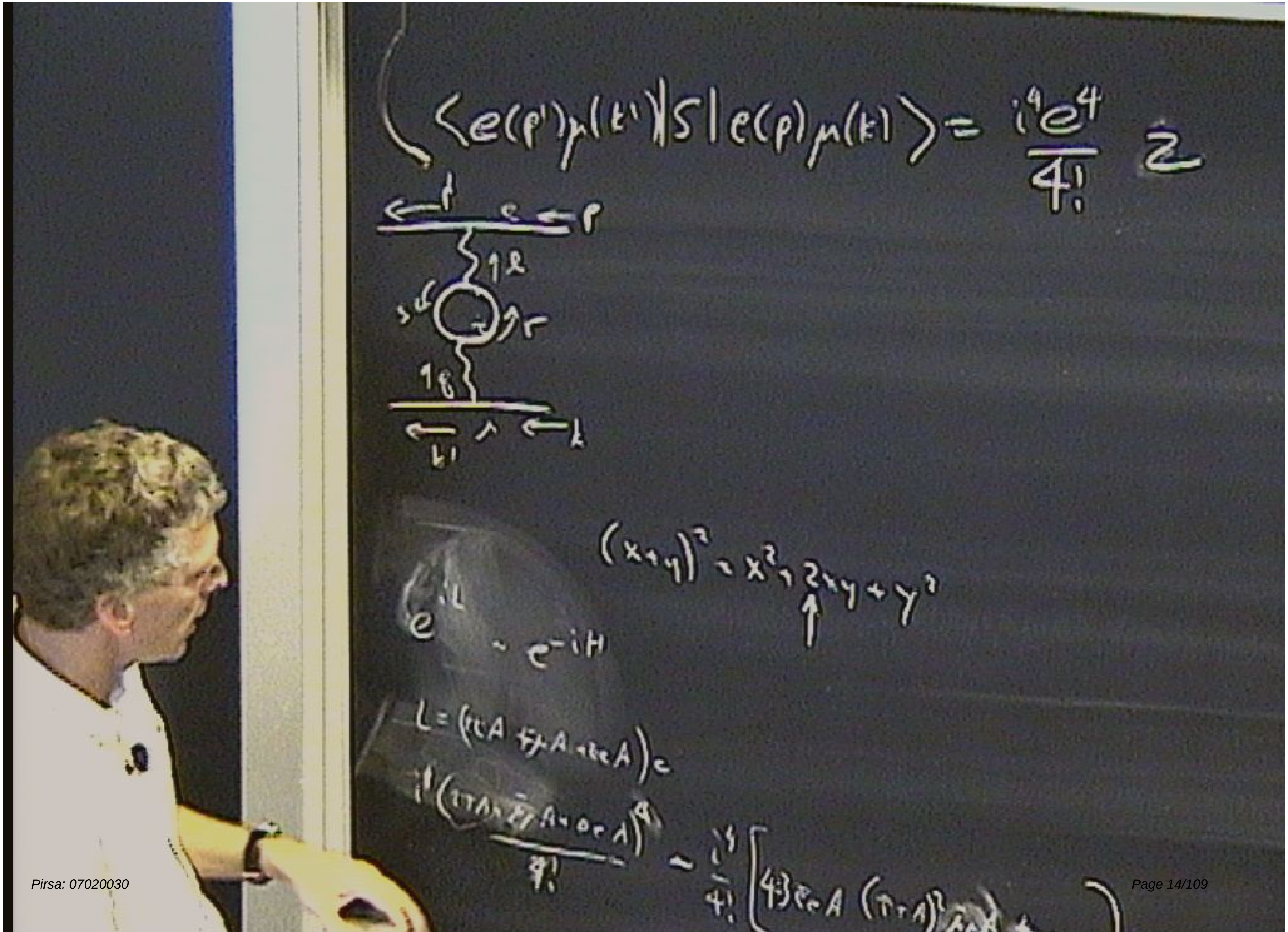
$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{e^4}{4!}$$

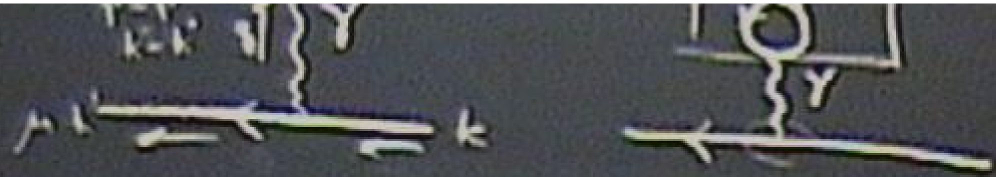

⟨e(p')μ(k')|S|e(p)μ(k)⟩ = e^4/4! Z
e^{iL} ~ e^{-iH}



⟨e(p')μ(k')|S|e(p)μ(k)⟩ = i^4 e^4 / 4! Σ
e^{iL} ~ e^{-iH}


$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{i^4 e^4}{4!} Z$
$(x+y)^2 = x^2 + 2xy + y^2$
$e^{iL} \sim e^{-iH}$


⟨e(p')μ(k')|S|e(p)μ(k)⟩ = i^4 e^4 / 4! Z
(x+y)^2 = x^2 + 2xy + y^2
e^{iL} ~ e^{-iH}

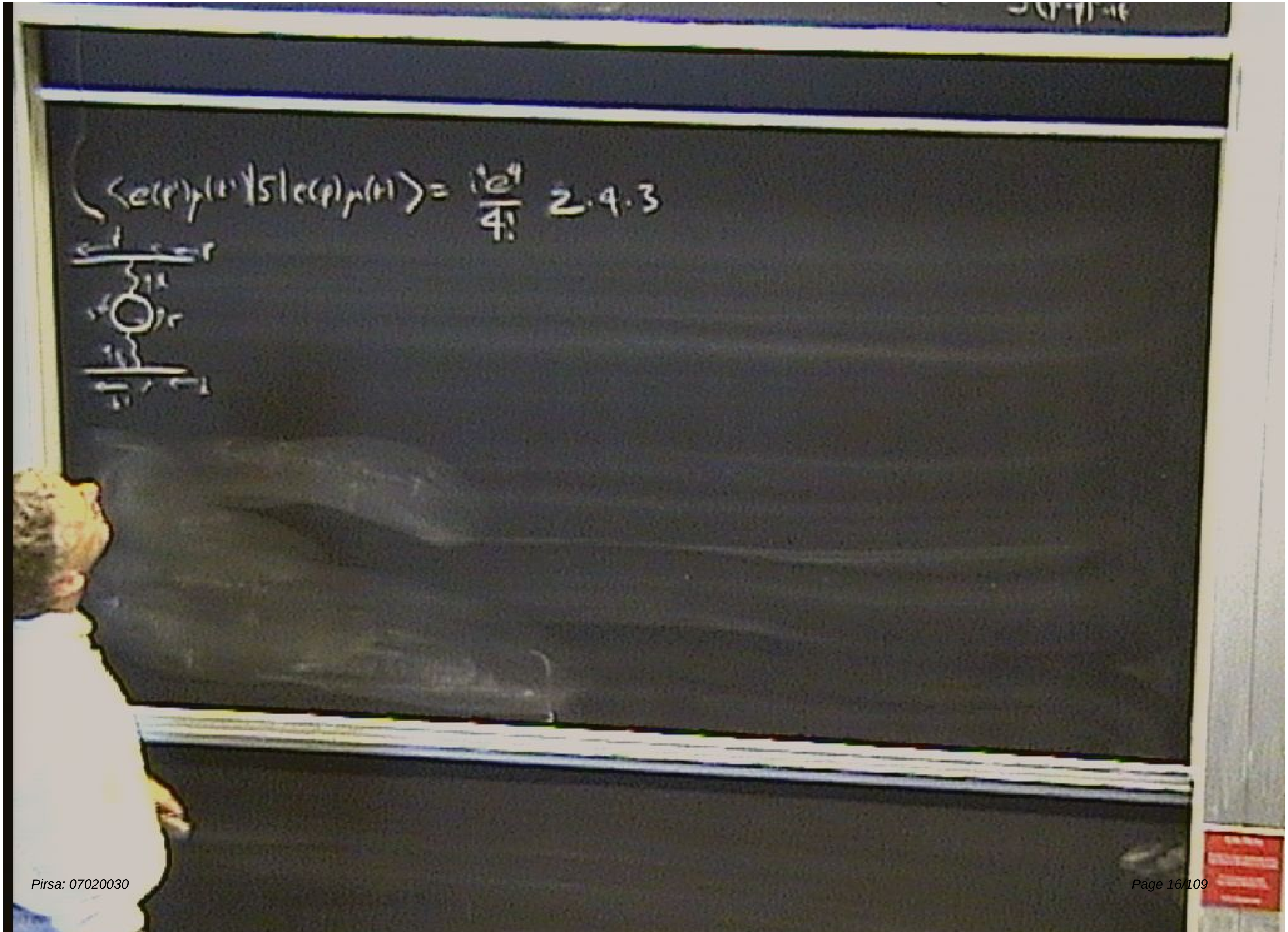


$$\mathcal{L}_{em} = ie\,\bar f \gamma^\mu Q f\, A_\mu$$

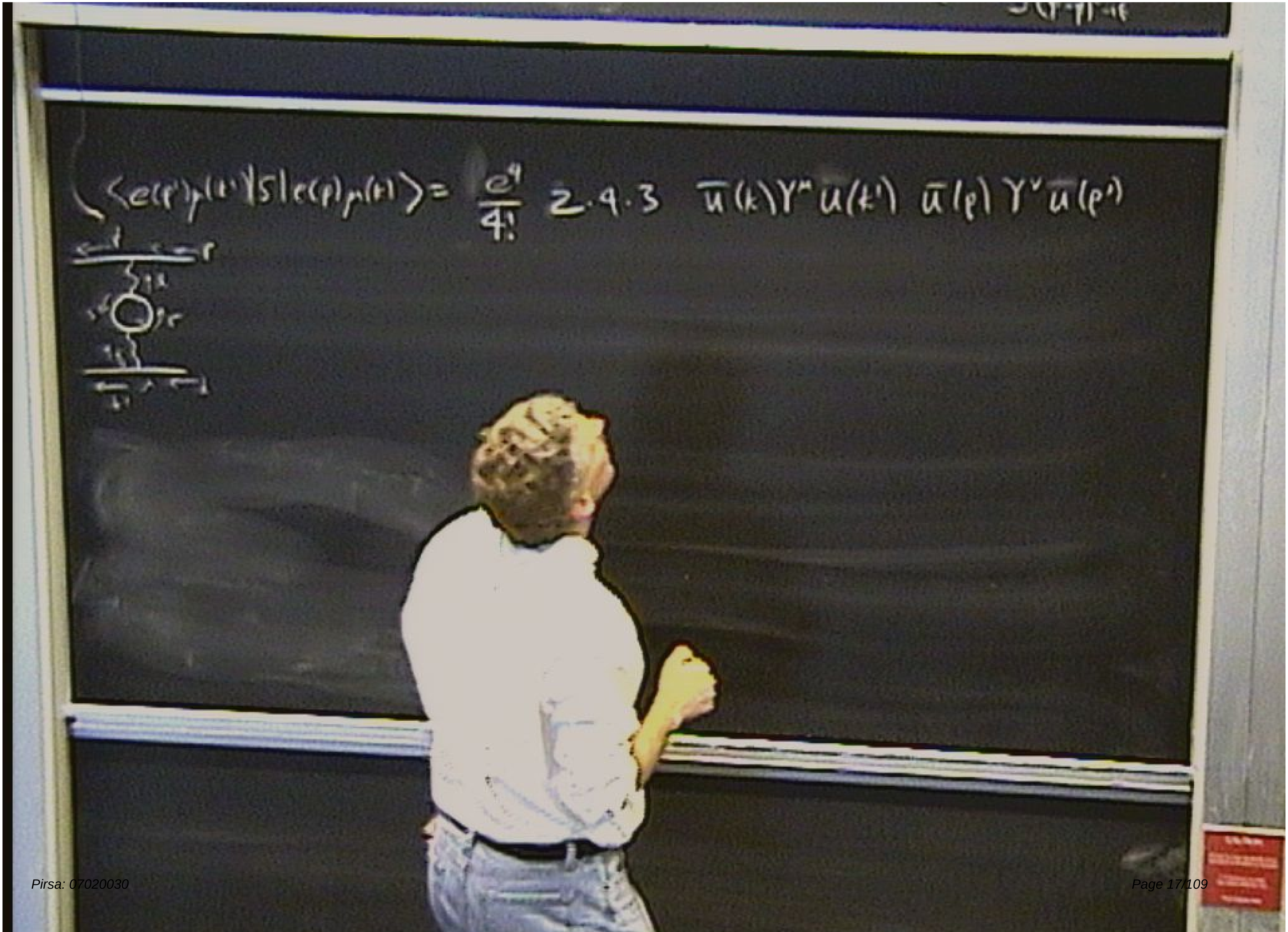
$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = 2\,\frac{1}{2!}e^2(-i)\left[\bar u(k')\gamma^\mu u(k)\right]\left[\bar u(p')\gamma^\nu u(p)\right]$$

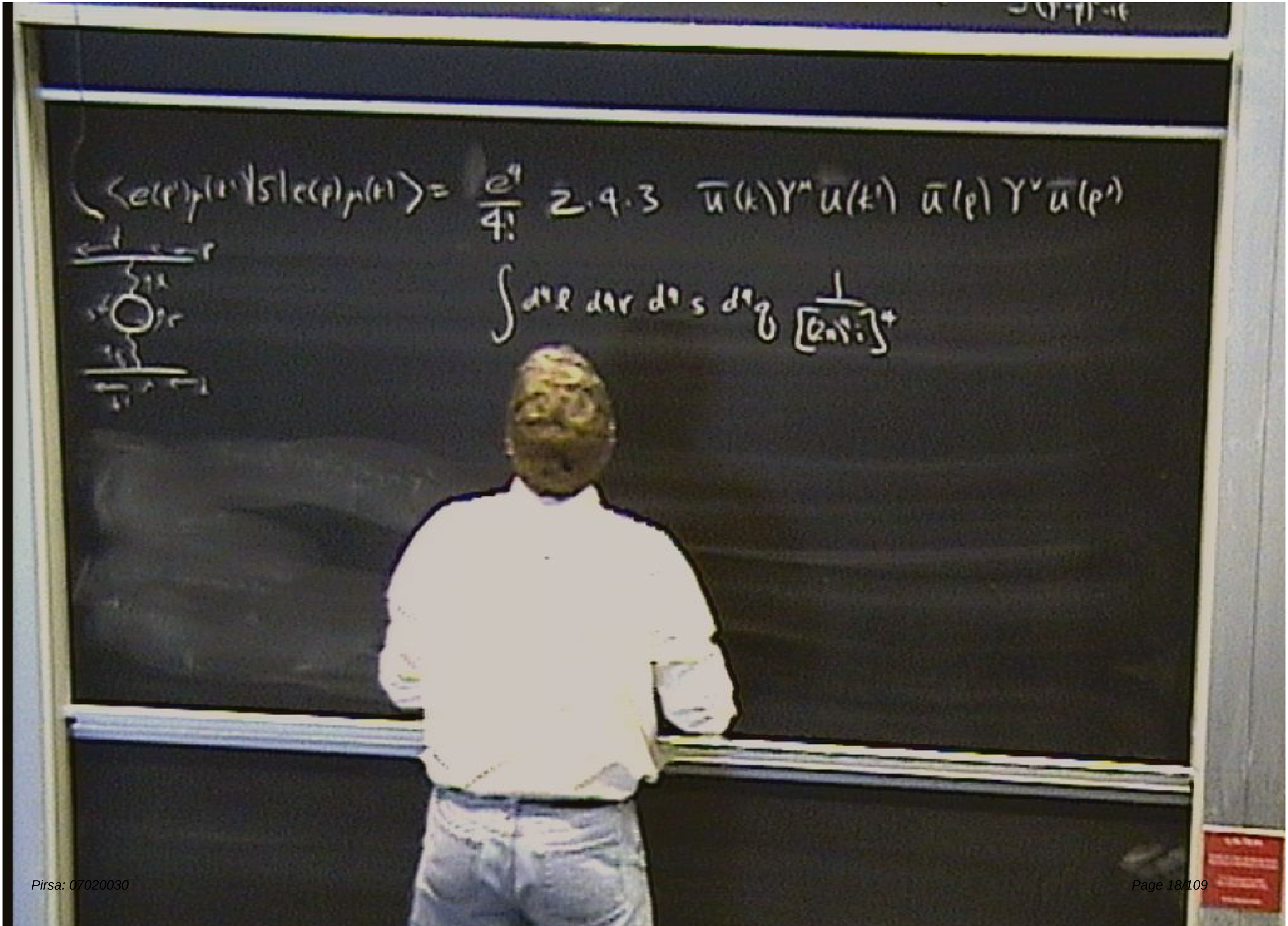
$$\cdot\ \frac{\eta_{\mu\nu}}{(p'-p)^2-i\epsilon}\qquad (2\pi)^4\delta^4(q-k+k')$$

$$= \frac{ie^2}{2}(2\pi)^4\delta^4(p+k-p'-k')\left[\bar u\gamma^\mu u\right]\left[\bar u\gamma^\nu u\right]\frac{\eta_{\mu\nu}}{(p-p')^2-i\epsilon}$$

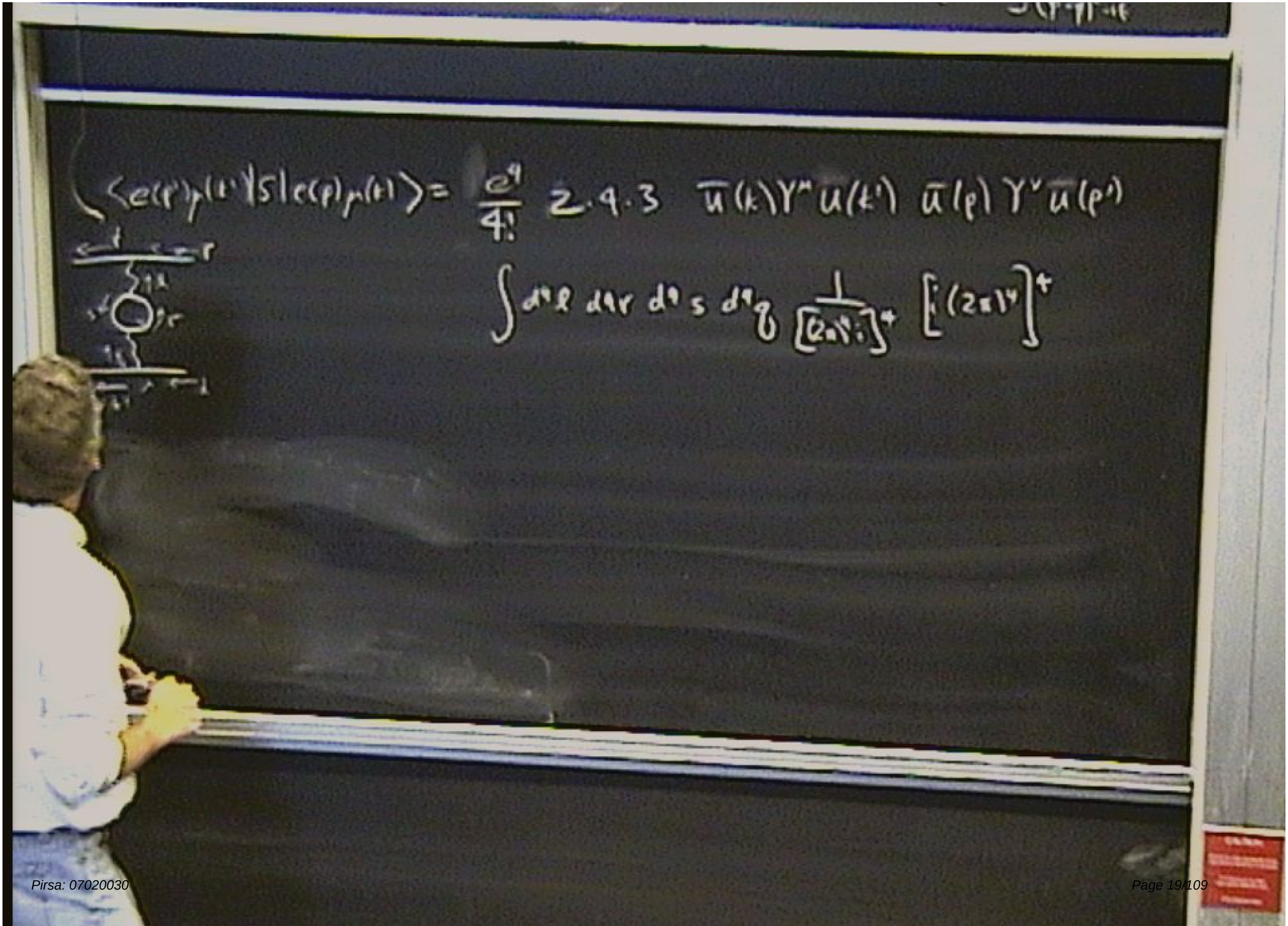


$\langle e(p')\mu(r')|S|e(p)\mu(r)\rangle = \frac{ie^4}{4!}\, 2\cdot 4\cdot 3$


⟨e(p')μ(k')|S|e(p)μ(k)⟩ = e^4/4! 2·4·3 ū(k)γ^μ u(k') ū(p)γ^ν u(p')



⟨e(p′)μ(k′)|S|e(p)μ(k)⟩ = e⁴/4! 2·4·3 ū(k)γ^μ u(k′) ū(p)γ^ν u(p′)
∫ d⁴l d⁴r d⁴s d⁴q


⟨e(k')μ(p')|S|e(p)μ(k)⟩ = e⁴/4! 2·4·3 ū(k)γ^μ u(k') ū(p)γ^ν u(p')
∫ d⁴ℓ d⁴r d⁴s d⁴q 1/[(2π)⁴ i]⁴ [i(2π)⁴]⁴

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{e^4}{4!}\, 2\cdot 4\cdot 3\ \bar{u}(k)\gamma^\mu u(k')\ \bar{u}(p)\gamma^\nu u(p')$$

$$\int d^4\ell\, d^4r\, d^4s\, d^4q\ \frac{1}{[(2\pi)^4 i]^4}\left[i(2\pi)^4\right]^4$$

$$\delta^4(p'-p-\ell)\,\delta^4(\ell+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{e^4}{4!}\, 2\cdot 4\cdot 3\ \bar{u}(k)\gamma^\mu u(k')\ \bar{u}(p)\gamma^\nu u(p')$$

$$\int d^4l\, d^4r\, d^4s\, d^4q\ \frac{1}{[(2\pi)^4 i]^4}\left[i(2\pi)^4\right]^4$$

$$\delta^4(p'-p-l)\,\delta^4(p+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$$\frac{\eta_{\mu\sigma}}{l^2-i\epsilon}\ \frac{\eta_{\nu\rho}}{q^2-i\epsilon}$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = -\frac{e^4}{4!}\, 2\cdot 4\cdot 3 \;\; \bar{u}(k)\gamma^\mu u(k') \;\; \bar{u}(p)\gamma^\nu u(p')$$

$$\int d^4\ell\, d^4r\, d^4s\, d^4q\, \frac{1}{[(2\pi)^4 i]^4}\left[i(2\pi)^4\right]^4$$

$$\delta^4(p'-p-\ell)\,\delta^4(\ell+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$$\frac{\eta_{\mu\sigma}}{\ell^2-i\epsilon}\,\frac{\eta_{\nu\rho}}{q^2-i\epsilon}\,\mathrm{Tr}\left[\gamma^\rho\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\sigma\left(\frac{-i\not{s}+m}{s^2+m^2-i\epsilon}\right)\right]$$

$$\langle e(k')\mu(p')|S|e(k)\mu(p)\rangle = \frac{-e^4}{4!}\,\cancel{2\cdot 4}\cdot 3\ \ \bar u(k)\gamma^\mu u(k')\ \ \bar u(p)\gamma^\nu u(p')$$

$$\int d^4\ell\, d^4r\, d^4s\, d^4q\ \ \cancel{\frac{1}{[(2\pi)^4]^4}}\ \ \cancel{\left[i(2\pi)^4\right]^4}$$

$$\delta^4(p'-p-\ell)\,\delta^4(p+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$$\frac{\eta_{\mu\alpha}}{\ell^2-i\epsilon}\ \frac{\eta_{\nu\beta}}{q^2-i\epsilon}\ \mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not r+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i\not s+m}{s^2+m^2-i\epsilon}\right)\right]$$

$$= -e^4\,[\bar u\gamma^\mu u][\bar u\gamma^\nu u]\,\delta^4(k'+p'-k-p)\int d^4r\ \frac{\eta_{\mu\alpha}}{\ }$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = -\frac{e^4}{4!}\, 2\cdot\cancel{4}\cdot 3\;\; \bar{u}(k)\gamma^\mu u(k')\;\; \bar{u}(p)\gamma^\nu u(p')$$

$$\int d^4\ell\, d^4r\, d^4s\, d^4q\;\; \cancel{\frac{1}{[(2\pi)^4]^4}}\;\; \cancel{\left[i(2\pi)^4\right]^4}$$

$$\delta^4(p'-p-\ell)\,\delta^4(\ell+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$$\frac{\eta_{\mu\alpha}}{\ell^2-i\epsilon}\;\frac{\eta_{\nu\beta}}{q^2-i\epsilon}\;\mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i\not{s}+m}{s^2+m^2-i\epsilon}\right)\right]$$

$$= -e^4\,[\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u]\,\delta^4(k'+p'-k-p)\int d^4r\;\frac{\eta_{\mu\alpha}}{(p'-p)^2-i\epsilon}\;\frac{\eta_{\nu\beta}}{(k-k')^2-i\epsilon}$$

$$\times\;\mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(-i(\right.\right.$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = -\frac{e^4}{4!}\,\cancel{2\cdot 4}\cdot 3\;\bar{u}(k)\gamma^\mu u(k')\;\bar{u}(p)\gamma^\nu u(p')$$

$$\int d^4\ell\, d^4r\, d^4s\, d^4q\; \cancel{\frac{1}{[(2\pi)^4]^4}}\;\cancel{\left[i(2\pi)^4\right]^4}$$

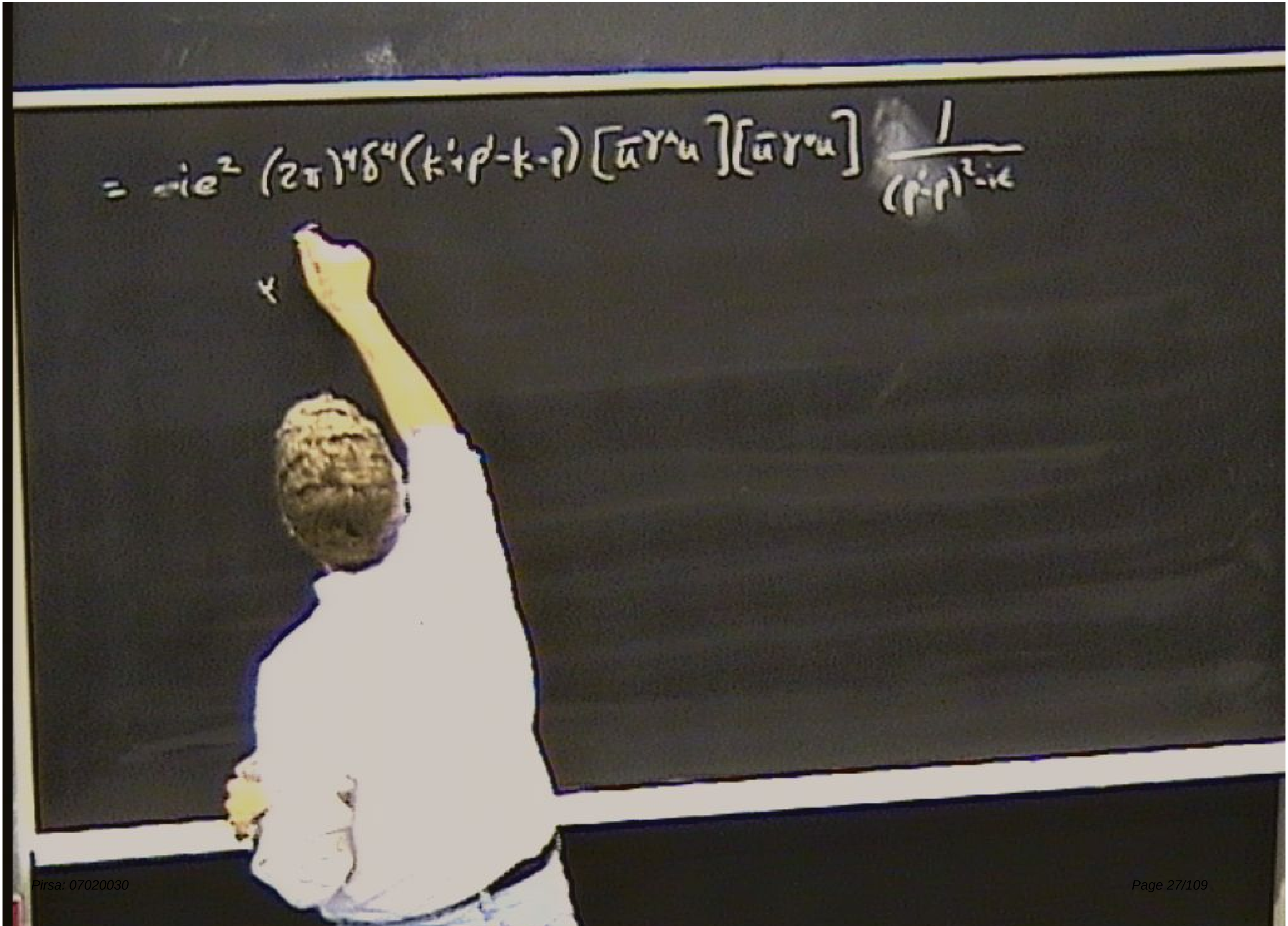
$$\delta^4(p'-p-\ell)\,\delta^4(\ell+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$$\frac{\eta_{\mu\alpha}}{\ell^2-i\epsilon}\,\frac{\eta_{\nu\beta}}{q^2-i\epsilon}\,\mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i\not{s}+m}{s^2+m^2-i\epsilon}\right)\right]$$

$$= -e^4\,[\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u]\,\delta^4(k'+p'-k-p)\int d^4r\,\frac{\eta_{\mu\alpha}}{(p'-p)^2-i\epsilon}\,\frac{\eta_{\nu\beta}}{(k-k')^2-i\epsilon}$$

$$\times\,\mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i(\not{r}-\not{\ell})+m}{(r-\ell)^2+m^2-i\epsilon}\right)\right]$$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u][\bar{u}\gamma_\mu u]$$


$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$
x

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(\right\}$$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $\Pi_{\mu\nu}(q) =$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $\Pi_{\mu\nu}(q) = ie^2 \int \frac{d^4 r}{(2\pi)^4} \mathrm{Tr}\left\{ \gamma_\mu \right.$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $\Pi_{\mu\nu}(q) = ie^2 \int \frac{d^4r}{(2\pi)^4} \mathrm{Tr}\left\{ \gamma_\mu \left[\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right] \gamma_\nu \left[\frac{-i\not{q}+m}{q^2+m^2-i\epsilon}\right.\right.$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = -\frac{e^4}{4!}\,\cancel{2\cdot 4\cdot 3}\;\bar{u}(k)\gamma^\mu u(k')\;\bar{u}(p)\gamma^\nu u(p')$$

$$\int d^4\ell\, d^4r\, d^4s\, d^4q\; \cancel{\frac{1}{[(2\pi)^4]^4}}\;\cancel{\left[i(2\pi)^4\right]^4}$$

$$\delta^4(p'-p-\ell)\,\delta^4(\ell+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$s = r-\ell$

$= r-(p-p')$

$$\frac{\eta_{\mu\alpha}}{\ell^2-i\epsilon}\;\frac{\eta_{\nu\beta}}{q^2-i\epsilon}\;\mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i\not{s}+m}{s^2+m^2-i\epsilon}\right)\right]$$

$$= -e^4\,[\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u]\,\delta^4(k'+p'-k-p)\int d^4r\;\frac{\eta_{\mu\alpha}}{(p'-p)^2-i\epsilon}\;\frac{\eta_{\nu\beta}}{(p'-p)^2-i\epsilon}$$

$$\times\;\mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\nu\left(\frac{-i(\not{r}-\not{p}+\not{p}')+m}{(r-p+p')^2+m^2-i\epsilon}\right)\right]$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = -\frac{e^4}{4!}\,\cancel{2\cdot 4}\cdot 3\ \bar{u}(k)\gamma^\mu u(k')\ \bar{u}(p)\gamma^\nu u(p')$$

$$\int d^4\ell\, d^4r\, d^4s\, d^4q\ \cancel{\frac{1}{[(2\pi)^4]^4}}\ \cancel{[i(2\pi)^4]^4}$$

$$\delta^4(p'-p-\ell)\,\delta^4(\ell+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$s = r-\ell$

$= r-(p'-p)$

$$\frac{\eta_{\mu\alpha}}{\ell^2-i\epsilon}\ \frac{\eta_{\nu\beta}}{q^2-i\epsilon}\ \mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i\not{s}+m}{s^2+m^2-i\epsilon}\right)\right]$$

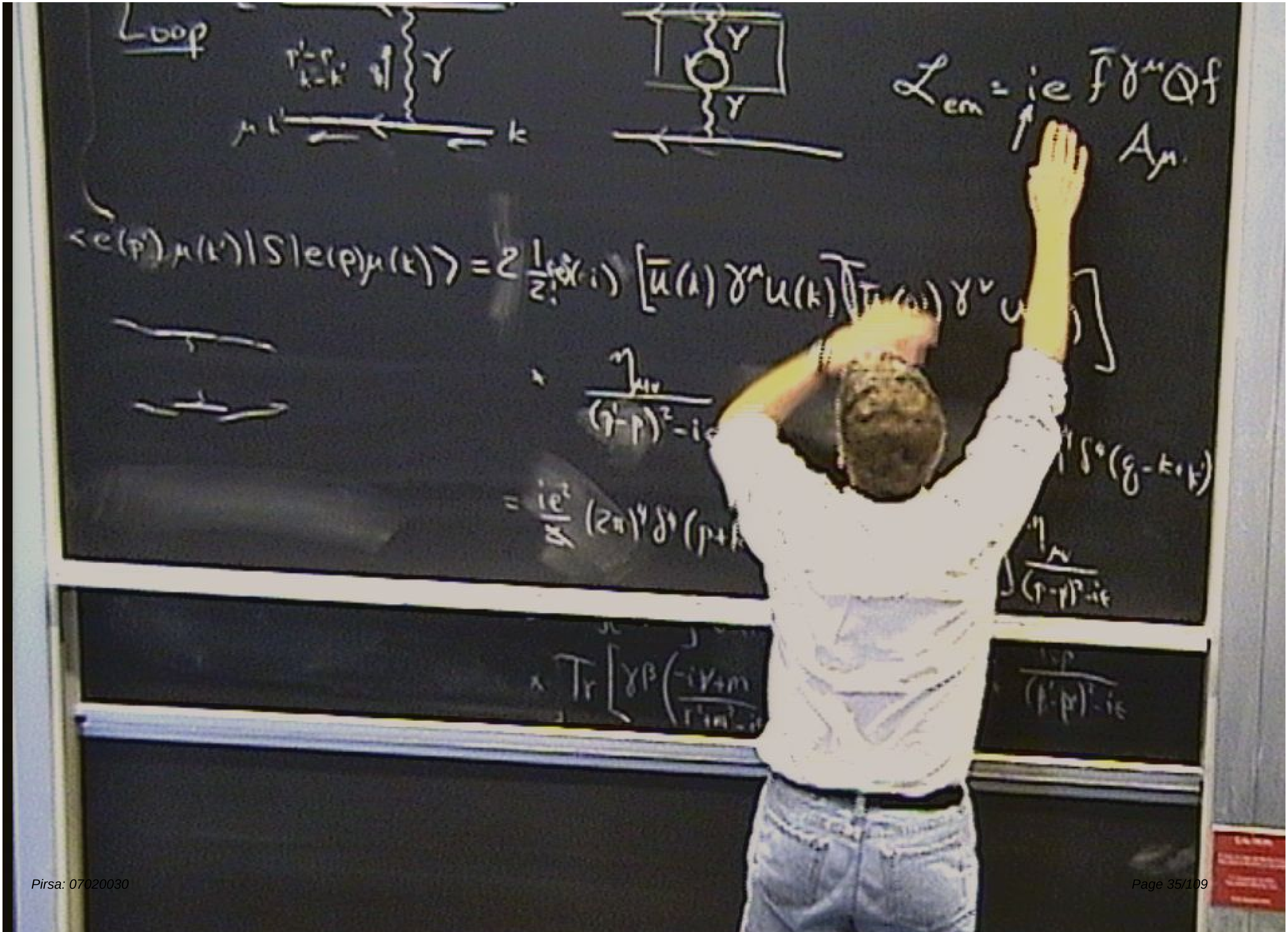
$$= -e^4\,[\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u]\,\delta^4(k'+p'-k-p)\int d^4r\ \frac{\eta_{\mu\alpha}}{(p'-p)^2-i\epsilon}\ \frac{\eta_{\nu\beta}}{(k'-k)^2-i\epsilon}$$

$$\times\ \mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i(\ldots)+m}{(r-\ldots)^2+m^2-i\epsilon}\right)\right]$$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $$\Pi_{\mu\nu}(q) = ie^2 \int \frac{d^4 r}{(2\pi)^4} \mathrm{Tr} \left\{ \gamma_\mu \left[\frac{-i\not{r} + m}{r^2 + m^2 - i\epsilon} \right] \gamma_\nu \left[\frac{-i(\not{r} - \not{q}) + m}{(r-q)^2 + m^2 - i\epsilon} \right] \right\}$$


Loop
γ
k
L_em = ie f̄ γ^μ Q f A_μ
<e(p′) μ(k′) | S | e(p) μ(k)> = 2 1/2!

$$(2\pi)^4\delta^4(q-k+k')$$

$$= \frac{-ie^2}{\cdots}(2\pi)^4\delta^4(p+k-p'-k')\left[\bar u\gamma^\mu u\right]\left[\bar u\gamma^\nu u\right]\frac{\eta_{\mu\nu}}{(p-p')^2-i\epsilon}$$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = \frac{-ie^4}{4!}\,\cancel{2\cdot 4}\cdot 3\;\;\bar u(k)\gamma^\mu u(k')\;\;\bar u(p)\gamma^\nu u(p')$$

$$\int d^4\ell\, d^4r\, d^4s\, d^4q\;\cancel{\frac{1}{[(2\pi)^4]^4}}\;\cancel{\left[i(2\pi)^4\right]^4}$$

$$\delta^4(p'-p-\ell)\,\delta^4(\ell+s-r)\,\delta^4(q+s-r)\,\delta^4(q+k'-k)$$

$s = r-\ell$

$= r-(p'-p)$

$$\frac{\eta_{\mu\alpha}}{\ell^2-i\epsilon}\,\frac{\eta_{\nu\beta}}{q^2-i\epsilon}\,\mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not r+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i\not s+m}{s^2+m^2-i\epsilon}\right)\right]$$

$$= -e^4\left[\bar u\gamma^\mu u\right]\left[\bar u\gamma^\nu u\right]\delta^4(k'+p'-k-p)\int d^4r\,\frac{\eta_{\mu\alpha}}{(p'-p)^2-i\epsilon}\,\frac{\eta_{\nu\beta}}{(p'-p)^2-i\epsilon}$$

$$\times\,\mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not r+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i(\not r-\not p'+\not p)+m}{(r-p'+p)^2+m^2-i\epsilon}\right)\right]$$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $\Pi_{\mu\nu}(q) = -ie^2 \int \frac{d^4r}{(2\pi)^4} \mathrm{Tr}\left\{ \gamma_\mu \left[\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right] \gamma_\nu \left[\frac{-i(\not{r}-\not{q})+m}{(r-q)^2+m^2-i\epsilon}\right]\right\}$

olarization $\quad = -ie^2 \int \frac{d^4r}{(2\pi)^4} \frac{N_{\mu\nu}}{D}$

$$D = (r^2+m^2-i\epsilon)((r-q)^2+m^2-i\epsilon) \qquad N_{\lambda\nu} = \mathrm{Tr}\left\{\gamma_\nu(-i\not{r}+m)\gamma_\lambda[-i(\not{r}-\not{q})+m]\right\}$$

Goal: Make D even under $\gamma^\mu \to -\gamma^\mu$

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Feynman trick: $\frac{1}{AB} =$

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Feynman trick: $$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{[Ax + B(1-x)]^2}$$

$$\int_0^1 dx \, \frac{1}{[\underbrace{(A-B)x + B}_{u}]^2} = \int_B \frac{du}{(A-B)} \frac{1}{u^2}$$

$du = (A-B)dx$

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Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{[\underbrace{(A-B)x + B}_{u}]^2} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B}\left[-\frac{1}{u}\right]_B^A = \frac{1}{A-B}\left[\frac{1}{B} - \frac{1}{A}\right]$$

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Goal: Make D even under $r^\mu \to -r^\mu$

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$du = (A-B)dx$

$$\frac{1}{D} = \frac{1}{(r^2+m^2-i\epsilon)[(r-q)^2+m^2-i\epsilon]} = \int_0^1 dx \frac{1}{[\underbrace{(r^2+m^2-i\epsilon)x + [(r-q)^2+m^2-i\epsilon](1-x)}_{r^2-2r\cdot q(1-x)+q^2(1-x)}]^2}$$

Goal: Make D even under $r^\mu \to -r^\mu$

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$du = (A-B)dx$

$$\frac{1}{D} = \frac{1}{(r^2+m^2 \cdots]} = \int_0^1 dx \, \frac{1}{[\underbrace{(r^2+m^2-i\epsilon)(1-x)+[(r-q)^2+m^2-i\epsilon](x)}_{r^2-2r\cdot q(x)+q^2x+m^2-i\epsilon}]^2}$$

Goal: Make D even under $r^\mu \to -r^\mu$

Feynman trick: $$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{[Ax + B(1-x)]^2}$$

$$\int_0^1 dx \, \frac{1}{[\underbrace{(A-B)x + B}_{u}]^2} = \int_B^A \frac{du}{(A-B)} \, \frac{1}{u^2} = \frac{1}{A-B}\left[-\frac{1}{u}\right]_B^A = \frac{1}{A-B}\left[\frac{1}{B} - \frac{1}{A}\right]$$

$du = (A-B)$ ...

$$\frac{1}{D} = \ldots [(r-q)^2 + m^2 - i\epsilon]} = \int_0^1 dx \, \frac{1}{[\underbrace{(r^2+m^2-i\epsilon)(1-x) + [(r-q)^2+m^2-i\epsilon](x)}_{r^2 - 2r\cdot q(x) + q^2 x + m^2 - i\epsilon}]^2}$$

Goal: Make D even under $r^\mu \to -r^\mu$

Feynman trick: $$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{[Ax + B(1-x)]^2}$$

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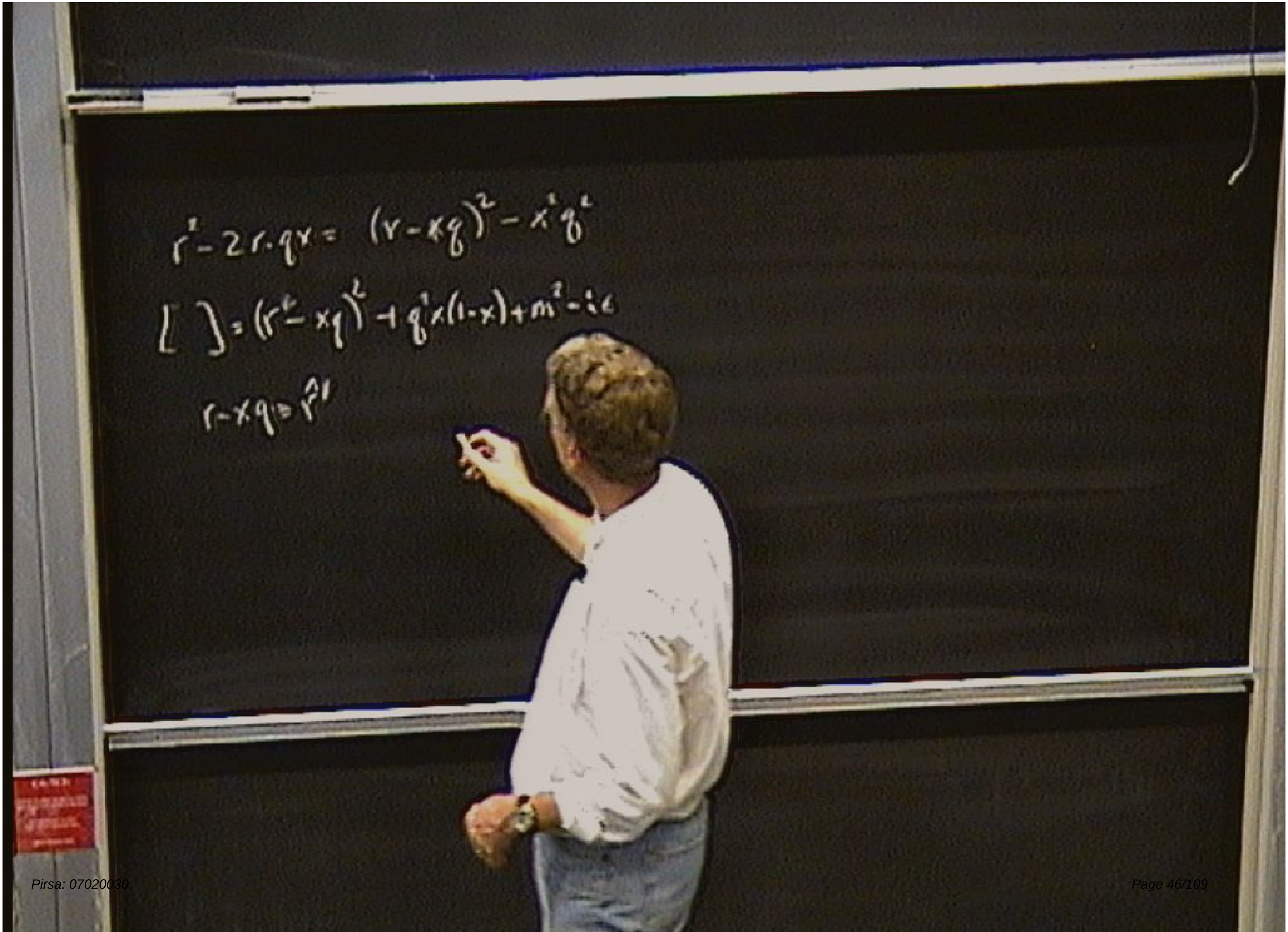
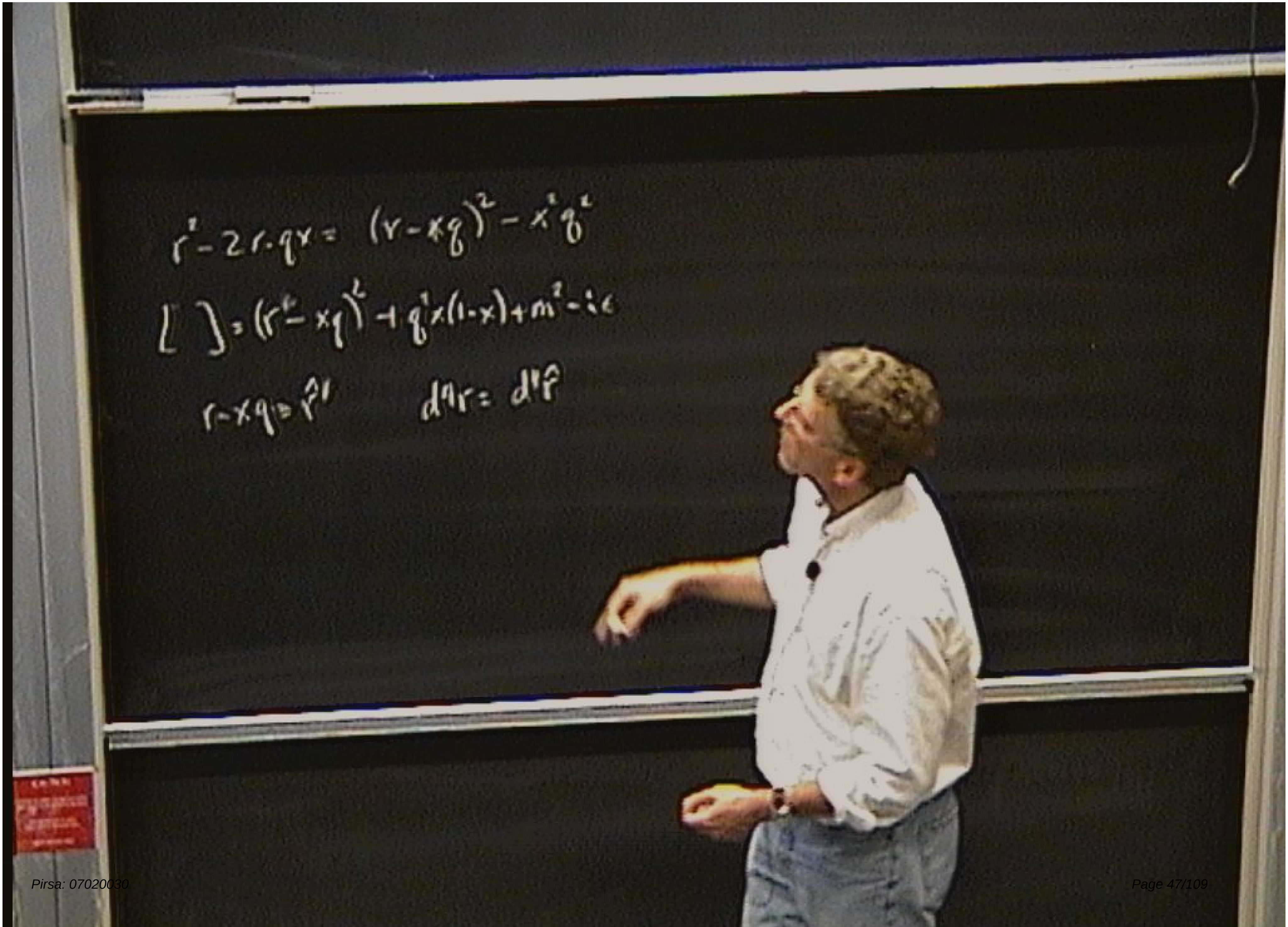
$du = (A-B)dx$

$$\frac{1}{D} = \frac{1}{(r^2+m^2-i\epsilon)[(r-q)^2+m^2-i\epsilon]} = \int_0^1 dx \, \frac{1}{[\underbrace{(r^2+m^2-i\epsilon)x + [(r-q)^2+m^2-i\epsilon](1-x)}_{r^2 - 2r\cdot q(1-x) + q^2(1-x) + m^2 - i\epsilon}]^2}$$


$r^2 - 2r\cdot q x = (r - xq)^2 - x^2 q^2$
$[\ ] = (r - xq)^2 + q^2 x(1-x) + m^2 - i\epsilon$
$r - xq \to r'$


r² - 2r·qx = (r - xq)² - x²q²
[ ] = (r - xq)² + q²x(1-x) + m² - iε
r - xq = r'
d⁴r = d⁴r'

$r^2 - 2r\cdot q x = (r - xq)^2 - x^2 q^2$

$[\ ] = (r - xq)^2 + q^2 x(1-x) + m^2 - i\epsilon$

$r - xq \Rightarrow \hat{r}'$ $\quad d^4 r = d^4 \hat{r}$ $\quad r = \hat{r} + xq$ in $N_{\mu\nu}$.

$$r^2 - 2r\cdot q x = (r - xq)^2 - x^2 q^2$$

$$[\ ] = (r - xq)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

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$$\Pi_{\mu\nu} = -ie^2 \int \frac{d^4\hat{r}}{(2\pi)^4} \int_0^1 dx \frac{}{[\hat{r}^2 + q^2 x(1-x) + m^2 - i\epsilon]^2}$$

$$r^2 - 2r\cdot q x = (r - xq)^2 - x^2 q^2$$

$$[\ \ ] = (r - xq)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$$r - xq \Rightarrow \hat{r} \qquad d^4 r = d^4 \hat{r} \qquad r = \hat{r} + xq \text{ in } N_{\mu\nu}$$

$$\Pi_{\mu\nu} = -ie^2 \int \frac{d^4\hat{r}}{(2\pi)^4} \int_0^1 dx \frac{\mathrm{Tr}\left[\gamma_\mu \left[-i(\hat{\not r} + x\not q) + m\right] \gamma_\nu \left[-i(\hat{\not r} + (1-x)\not q) + m\right]\right]}{\left[\hat{r}^2 + q^2 x(1-x) + m^2 - i\epsilon\right]^2}$$

$$r^2 - 2r\cdot q x = (r - xq)^2 - x^2 q^2$$

$$[\ ] = (r - xq)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$r - xq = \tilde{r}$ $\quad d^4r = d^4\tilde{r}$ $\quad r = \tilde{r} + xq$ in $N_{\mu\nu}$.

$$\Pi_{\mu\nu} = -ie^2 \int \frac{d^4\tilde{r}}{(2\pi)^4} \int_0^1 dx \frac{\mathrm{Tr}\left[\gamma_\mu\left[-i(\tilde{r} + x q) + m\right]\gamma_\nu\left[-i(\tilde{r} + (x-1)q) + m\right]\right]}{\left[\tilde{r}^2 + q^2 x(1-x) + m^2 - i\epsilon\right]^2}$$

Can drop cross terms of the form $\int \frac{d^4 r}{D}\, r\cdot q = 0$

$$r^2 - 2r\cdot q x = (r - xq)^2 - x^2 q^2$$

$$[\ ] = (r - xq)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$$r - xq = \hat{r} \qquad d^4 r = d^4 \hat{r} \qquad r = \hat{r} + xq \text{ in } N_{\mu\nu}.$$

$$\Pi_{\mu\nu} = -ie^2 \int \frac{d^4\hat{r}}{(2\pi)^4} \int_0^1 dx \frac{\mathrm{Tr}\left[\gamma_\mu\left[-i(\hat{\not r} + x\not q) + m\right]\gamma_\nu\left[-i(\hat{\not r} + (x-1)\not q) + m\right]\right]}{\left[\hat{r}^2 + q^2 x(1-x) + m^2 - i\epsilon\right]^2}$$

Can drop cross terms of the form $\int \frac{d^4\hat{r}}{D} \hat{r}^\mu q^\nu = 0$

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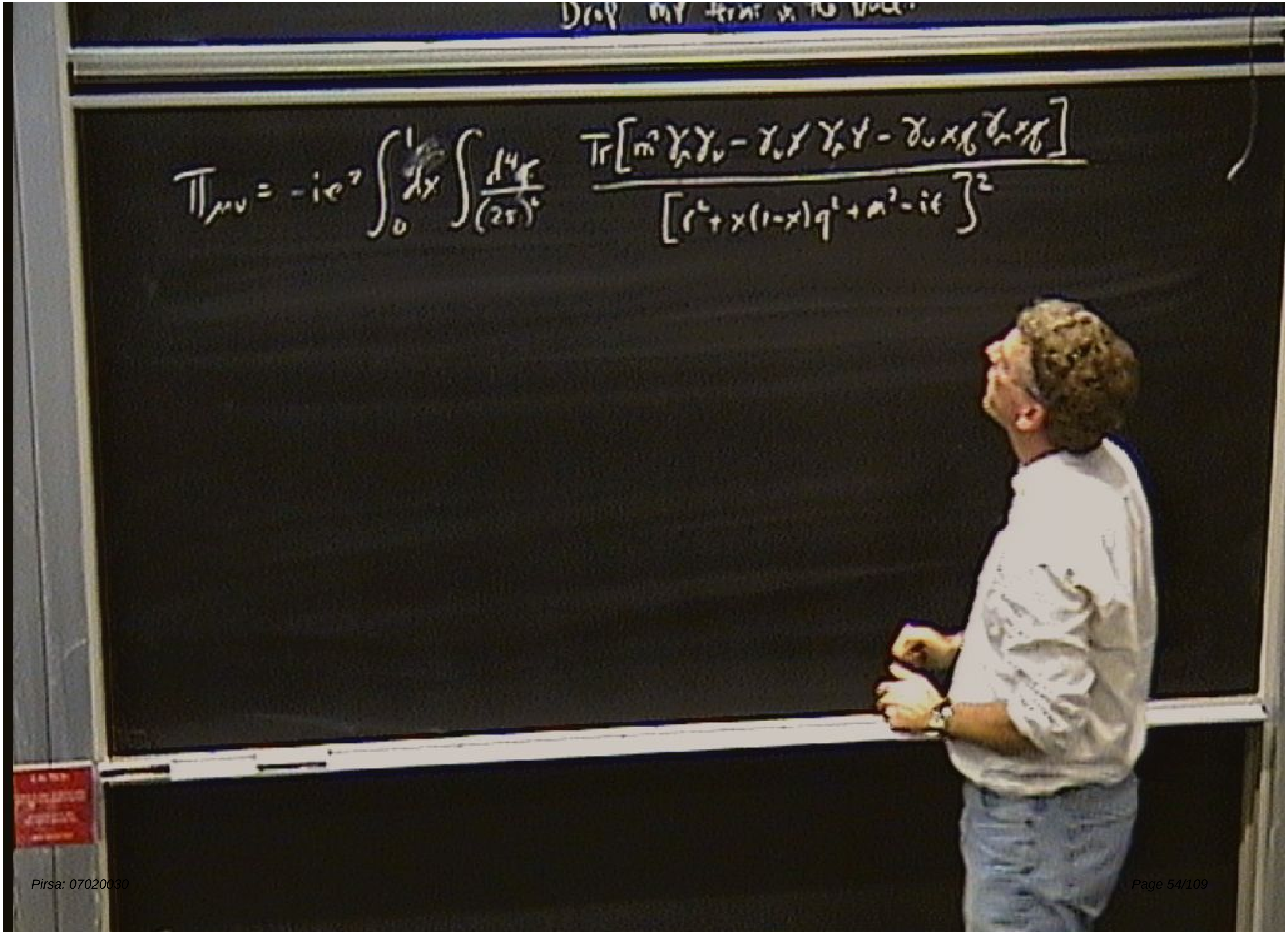
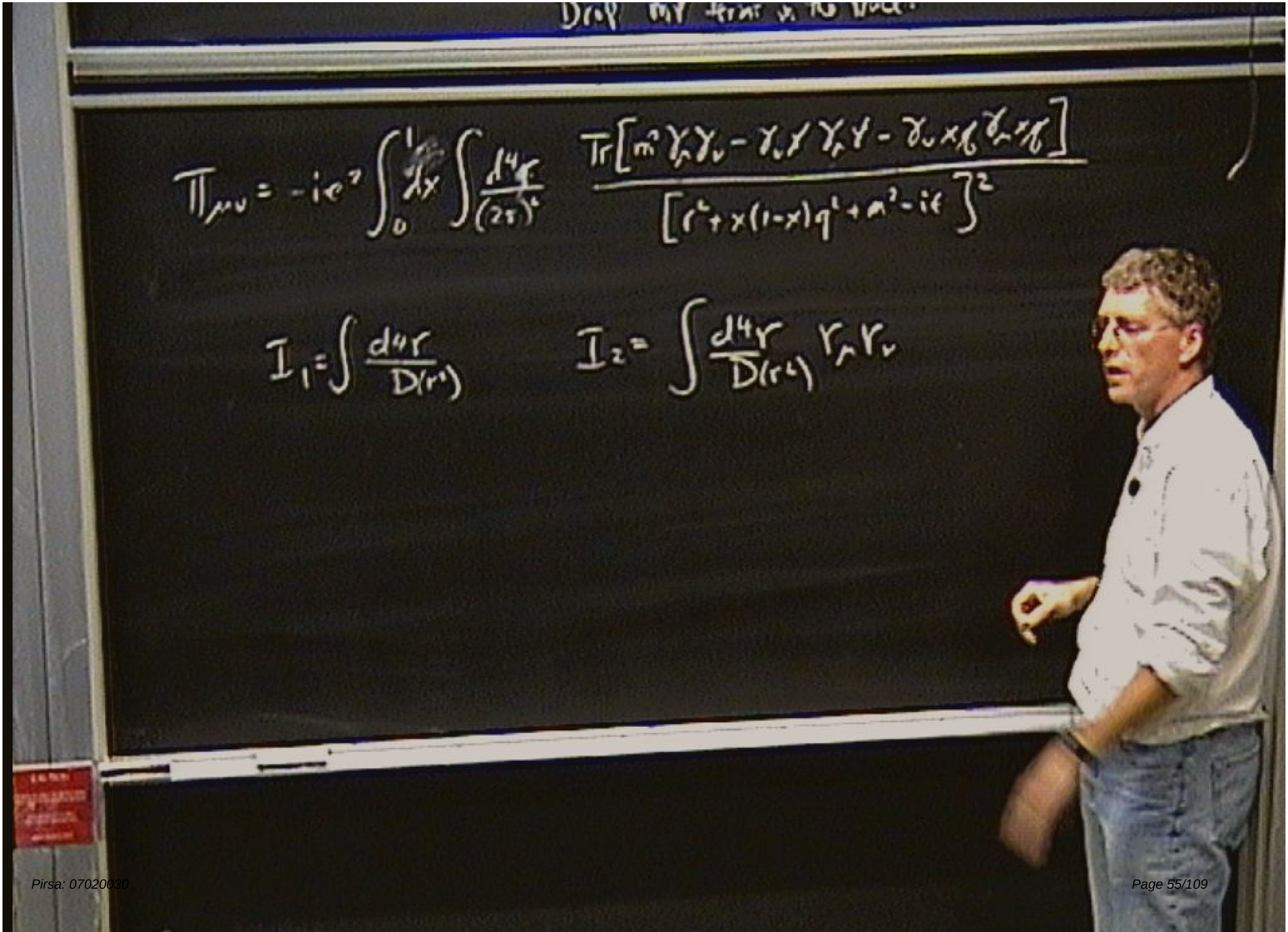
Can drop cross terms of the form $\int \frac{d^4\hat{r}}{D} \hat{r}^\mu q^\nu = 0$

Drop odd terms in the trace.

$$\Pi_{\mu\nu} = -ie^2$$


$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4\ell}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not\ell \gamma_\mu \not\ell - \gamma_\nu x \not q \gamma_\mu x \not q\right]}{\left[\ell^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$


$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\mathrm{Tr}[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$
$I_1 = \int \frac{d^4 r}{D(r^2)}$
$I_2 = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu$

Drop my term in the back.

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4r}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}\right]}{\left[r^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$I_1 = \int \frac{d^4r}{D(r^2)} \qquad I_2(q) = \int \frac{d^4r}{D(r^2)} r_\mu r_\nu \propto \eta_{\mu\nu}$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}\right]}{\left[r^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \qquad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$

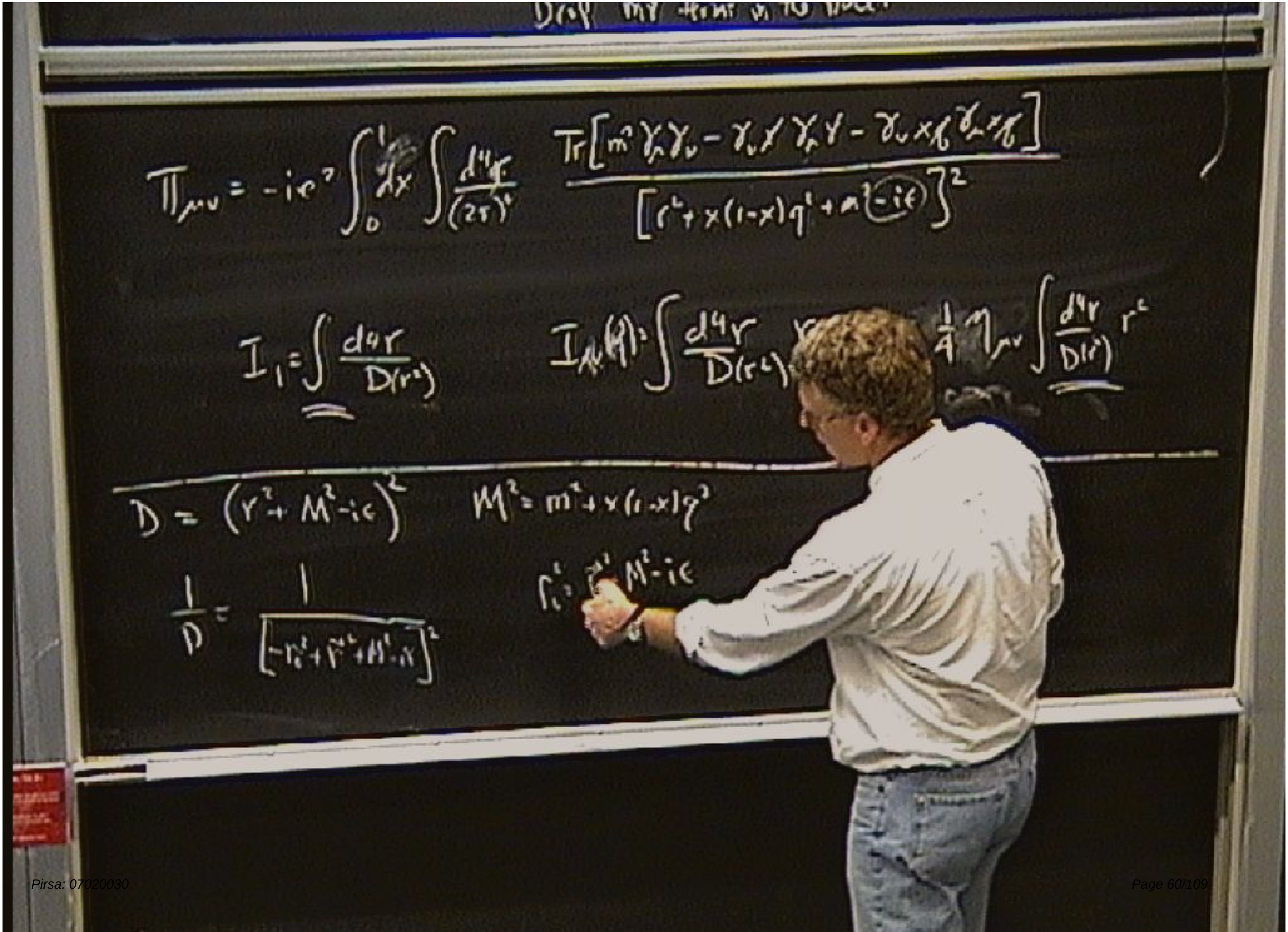
Drop my terms in to back.

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}\right]}{\left[r^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \qquad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2\gamma_\mu\gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}\right]}{\left[r^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \qquad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4}\eta_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$


$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4q}{(2\pi)^4} \frac{\mathrm{Tr}[m^2\gamma_\mu\gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$
$I_1 = \int \frac{d^4r}{D(r^2)}$
$I_{\mu\nu}(q) = \int \frac{d^4r}{D(r^2)}$
$\frac{1}{4}\eta_{\mu\nu}\int \frac{d^4r}{D(r^2)} r^2$
$D = (r^2 + M^2 - i\epsilon)^2$
$M^2 = m^2 + x(1-x)q^2$
$\frac{1}{D} = \frac{1}{[-r_0^2 + \vec{r}^2 + M^2 - i\epsilon]^2}$
$r_1^2 = \vec{r}^2 + M^2 - i\epsilon$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 q}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}\right]}{\left[r^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \qquad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$

$$D = \left(r^2 + M^2 - i\epsilon\right)^2 \qquad M^2 = m^2 + x(1-x)q^2$$

$$r_0^2 = \vec{r}^2 + M^2 - i\epsilon$$

$$\frac{1}{D} = \frac{1}{\left[-r_0^2 + \vec{r}^2 + M^2 - i\epsilon\right]^2}$$

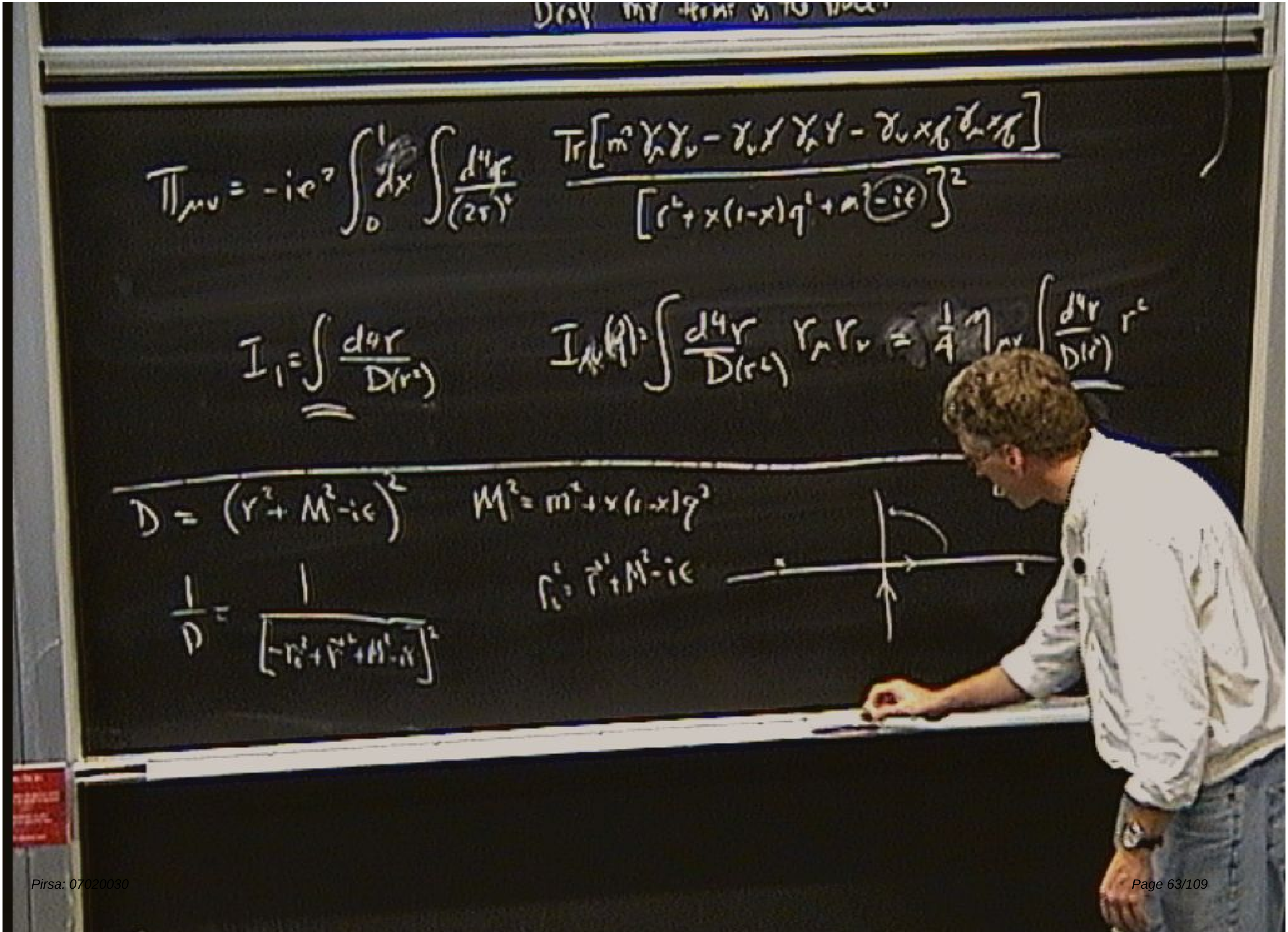
$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 q}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}\right]}{\left[r^2 + x(1-x) q^2 + m^2 - i\epsilon\right]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \qquad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} \eta_{\mu\nu} \int$$

$$D = (r^2 + M^2 - i\epsilon)^2 \qquad M^2 = m^2 + x(1-x) q^2$$

$$r_0^2 = \vec{r}^2 + M^2 - i\epsilon$$

$$\frac{1}{D} = \frac{1}{\left[-r_0^2 + \vec{r}^2 + M^2 - i\epsilon\right]^2}$$

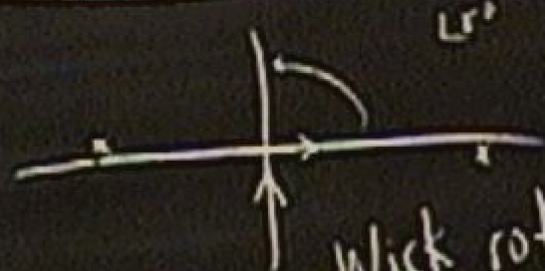

$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\mathrm{Tr}[m^2\gamma_\mu\gamma_\nu - \gamma_\nu \not r \gamma_\mu \not r - \gamma_\nu x \not q \gamma_\mu x \not q]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$
$I_1 = \int \frac{d^4 r}{D(r^2)}$
$I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4}\eta_{\mu\nu}\int \frac{d^4 r}{D(r^2)} r^2$
$D = (r^2 + M^2 - i\epsilon)^2$
$M^2 = m^2 + x(1-x)q^2$
$r_0^2 = \vec{r}^2 + M^2 - i\epsilon$
$\frac{1}{D} = \frac{1}{[-r_0^2 + \vec{r}^2 + M^2 - i\epsilon]^2}$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 q}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r}\, \gamma_\mu \not{r} - \gamma_\nu x \not{q}\, \gamma_\mu x \not{q}\right]}{\left[r^2 + x(1-x)q^2 + m^2 - i\epsilon\right]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \qquad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4}\eta_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$

$$D = \left(r^2 + M^2 - i\epsilon\right)^2 \qquad M^2 = m^2 + x(1-x)q^2$$

$$\frac{1}{D} = \frac{1}{\left[-r_0^2 + \vec{r}^{\,2} + M^2 - i\epsilon\right]^2} \qquad r_0^2 = \vec{r}^{\,2} + M^2 - i\epsilon$$



Wick rotation

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 q}{(2\pi)^4} \frac{\mathrm{Tr}\left[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{r} \gamma_\mu \not{r} - \gamma_\nu x \not{q} \gamma_\mu x \not{q}\right]}{\left[r^2 + x(1-x) q^2 + m^2 - i\epsilon\right]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \qquad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$

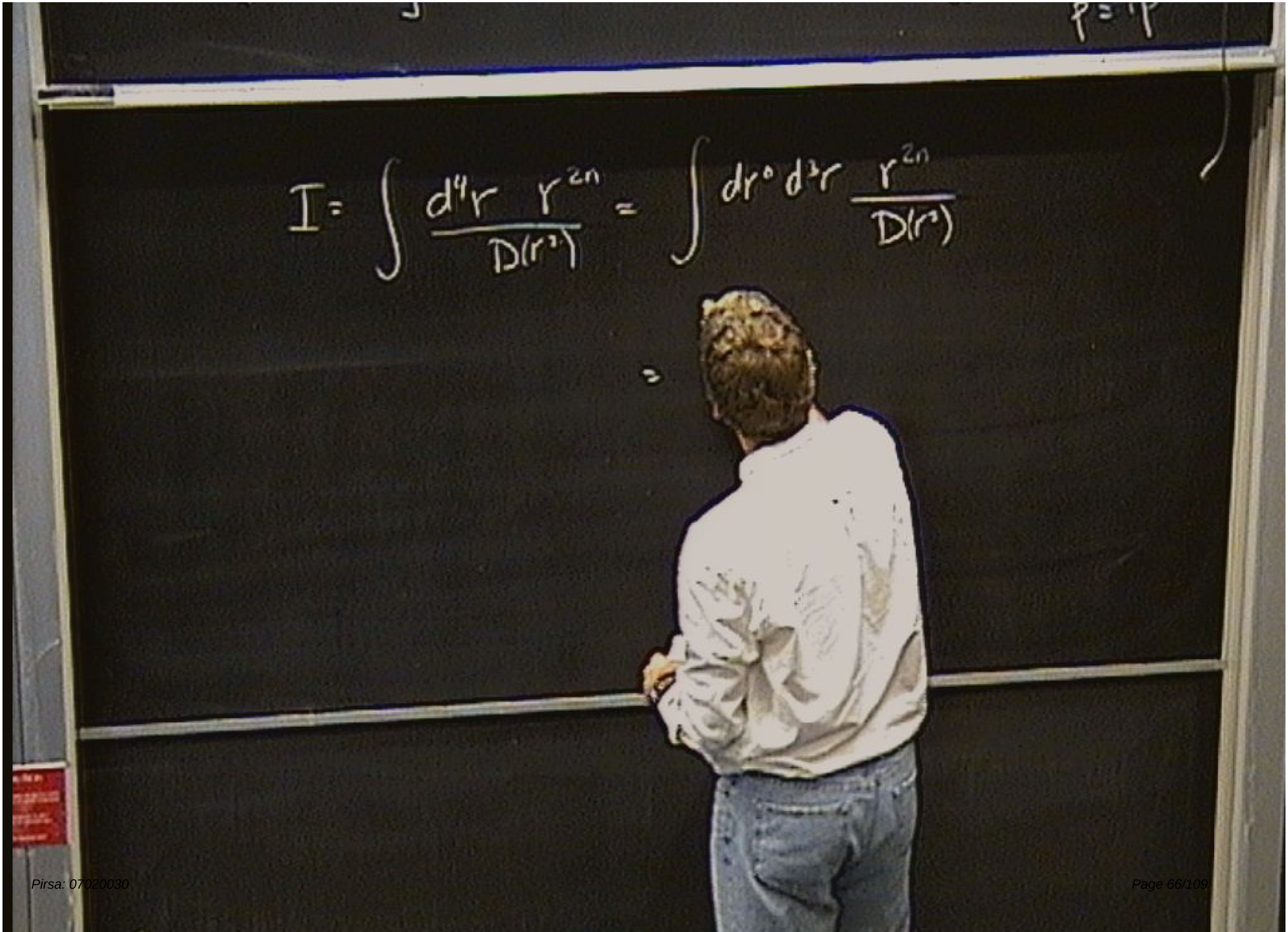
$$D = \left(r^2 + M^2 - i\epsilon\right)^2 \qquad M^2 = m^2 + x(1-x) q^2$$

$$\frac{1}{D} = \frac{1}{\left[-r_0^2 + \vec{r}^{\,2} + M^2 - i\epsilon\right]^2} \qquad r_0^2 = \vec{r}^{\,2} + M^2 - i\epsilon$$

r^0

Wick rotation

$p^0 = ip^4$


$I = \int \frac{d^4r \; r^{2n}}{D(r^2)} = \int dr^0 \, d^3r \, \frac{r^{2n}}{D(r^2)}$
=

$$I = \int \frac{d^4r \; r^{2n}}{D(r^2)} = \int dr^0 d^3r \frac{r^{2n}}{D(r^2)} \qquad r^2 = -p_0^2 + \vec{p}^2$$

$$= i \int d^4r_E \frac{((r_E^2))^n}{D(r_E^2)} \qquad r_E^2 = +p_4^2 + \vec{p}^2$$

$$I = \int \frac{d^4r \; r^{2n}}{D(r^2)} = \int dr^0 \, d^3r \, \frac{r^{2n}}{D(r^2)}$$

$r^2 = -p_0^2 + \vec{p}^2$

$$= i \int d^4 r_E \, \frac{\left((r_E^2)\right)^n}{D(r_E^2)}$$

$r_E^2 = +p_4^2 + \vec{p}^2$

$$= i \int \frac{d^4 r_E \, (r_E^2)^n}{\left[r^2 + M^2\right]^2}$$

$$I = \int \frac{d^4r \; r^{2n}}{D(r^2)} = \int dr^0 d^3r \; \frac{r^{2n}}{D(r^2)} \qquad r^2 = -p_0^2 + p^2$$

$$= i \int d^4 r_E \; \frac{((r_E^2))^n}{D(r_E^2)} \qquad r_E^2 = +$$

$$= i \int \frac{d^4 r_E \, (r_E^2)^n}{\left[r_E^2 + M^2\right]^2} = i \int r^3 dr \; \frac{r^{2n}}{(r^2 + M^2)^2} \int d^3\Omega$$

$$I = \int \frac{d^4 r \; r^{2n}}{D(r^2)} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \qquad r^2 = -p_0^2 + \vec{p}^2$$

$$= i \int d^4 r_E \frac{\left((r_E^2)\right)^n}{D(r_E^2)} \qquad r_E^2 = +p_4^2 + \vec{p}^2$$

$$= i \int \frac{d^4 r_E \,(r_E^2)^n}{\left[r_E^2 + M^2\right]^2} = i \int r^3 dr \frac{r^{2n}}{(r^2 + M^2)^2} \int_{S_3} d^3\Omega$$

Goal: Make D even under $r^\mu \to -r^\mu$

Feynman trick: $$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{[Ax + B(1-x)]^2}$$

$$\int_0^1 dx \, \frac{1}{[\underbrace{(A-B)x+B}_{u}]^2} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B}\left[-\frac{1}{u}\right]_B^A = \frac{1}{A-B}\left[\frac{1}{B} - \frac{1}{A}\right]$$

$du = (A-B)dx$

$$\frac{1}{D} = \frac{1}{(r^2+m^2-i\epsilon)[(r-q)^2+m^2-i\epsilon]} = \int_0^1 dx \, \frac{1}{[\underbrace{(r^2+m^2-i\epsilon)(1-x)+[(r-q)^2+m^2-i\epsilon](x)}_{r^2-2r\cdot q(x)+q^2 x+m^2-i\epsilon}]^2}$$

$\int e^{-x^2/\ell} \, d^n x$

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$$\int e^{-r^2/\ell} d^n r = \left[\int_{-\infty}^{\infty} e^{-x^2/\ell} dx\right]^n$$

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$$\int e^{-r_E^2/\Lambda} d^n r_E = \left[\int_{-\infty}^{\infty} e^{-x^2/\Lambda} dx\right]^n$$

$p^0 = i p^4$

$$I = \int \frac{d^4 r \; r^{2n}}{D(r^2)} = \int dr^0 \, d^3 r \, \frac{r^{2n}}{D(r^2)} \qquad r^2 = -p_0^2 + \vec{p}^2$$

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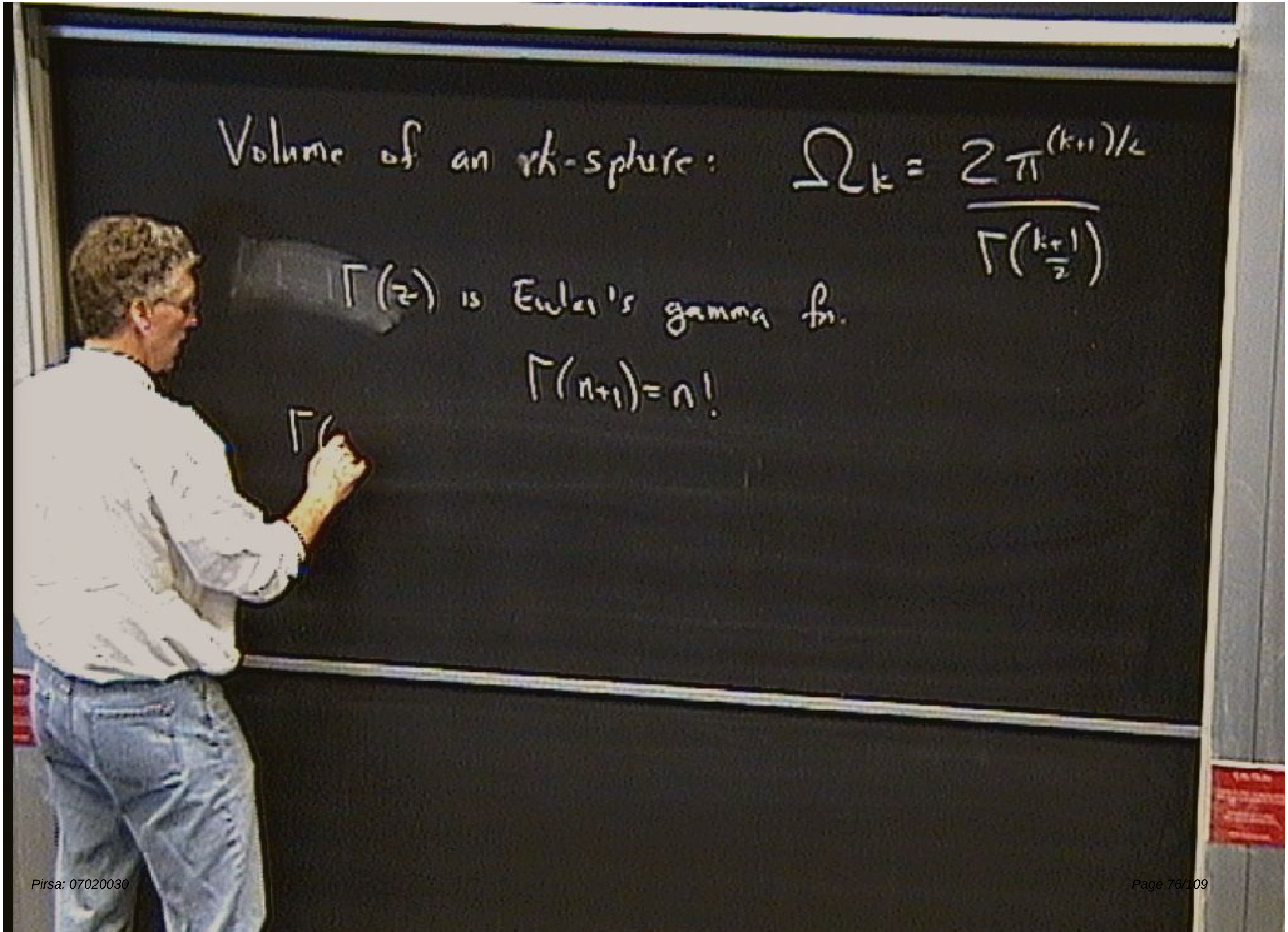
Volume of an rk-sphere: $\Omega_k = \frac{2\pi^{(k+1)/2}}{\Gamma(\frac{k+1}{2})}$
$\Gamma(z)$ is Euler's gamma fn.
$\Gamma(n+1) = n!$
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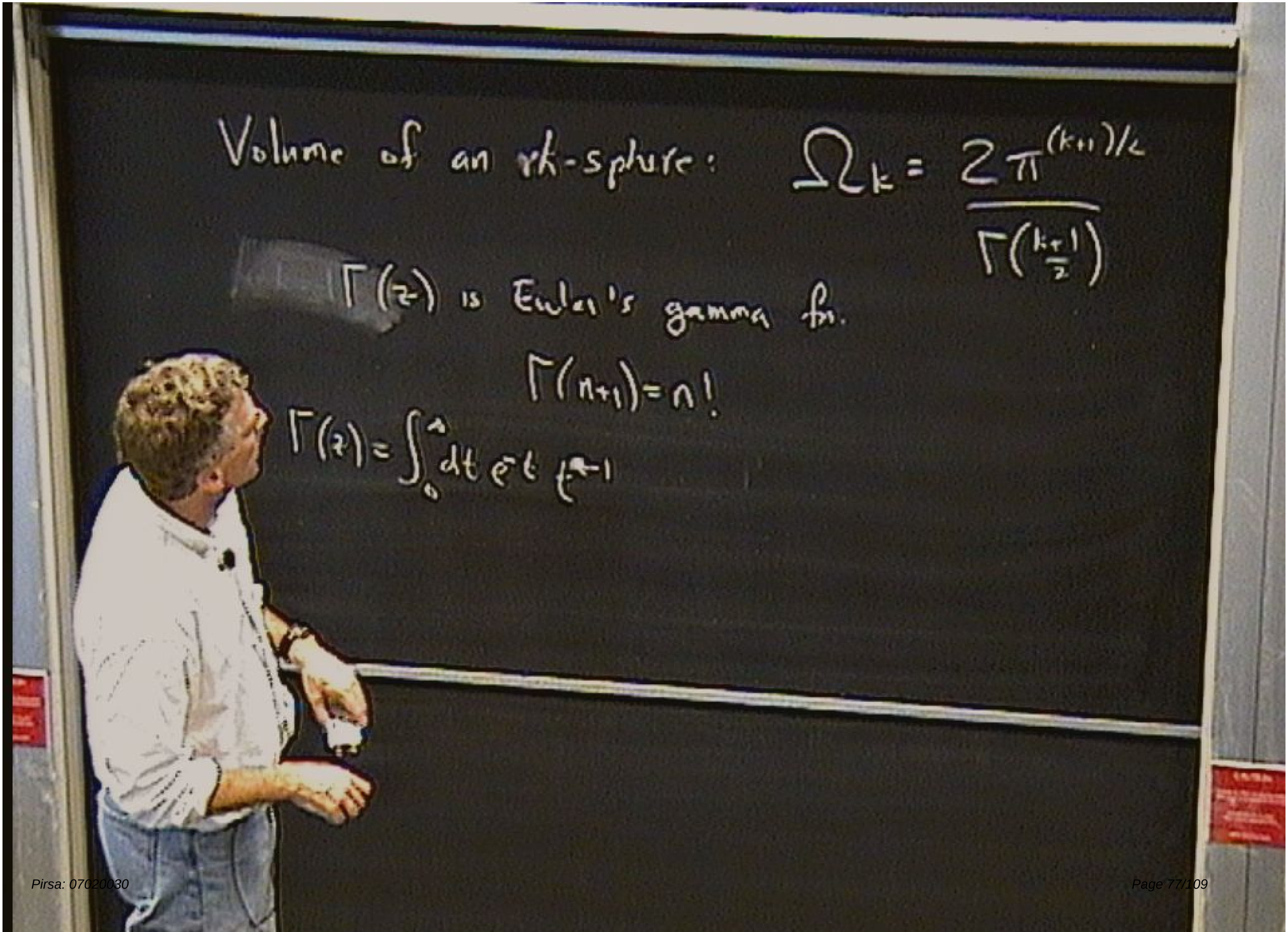
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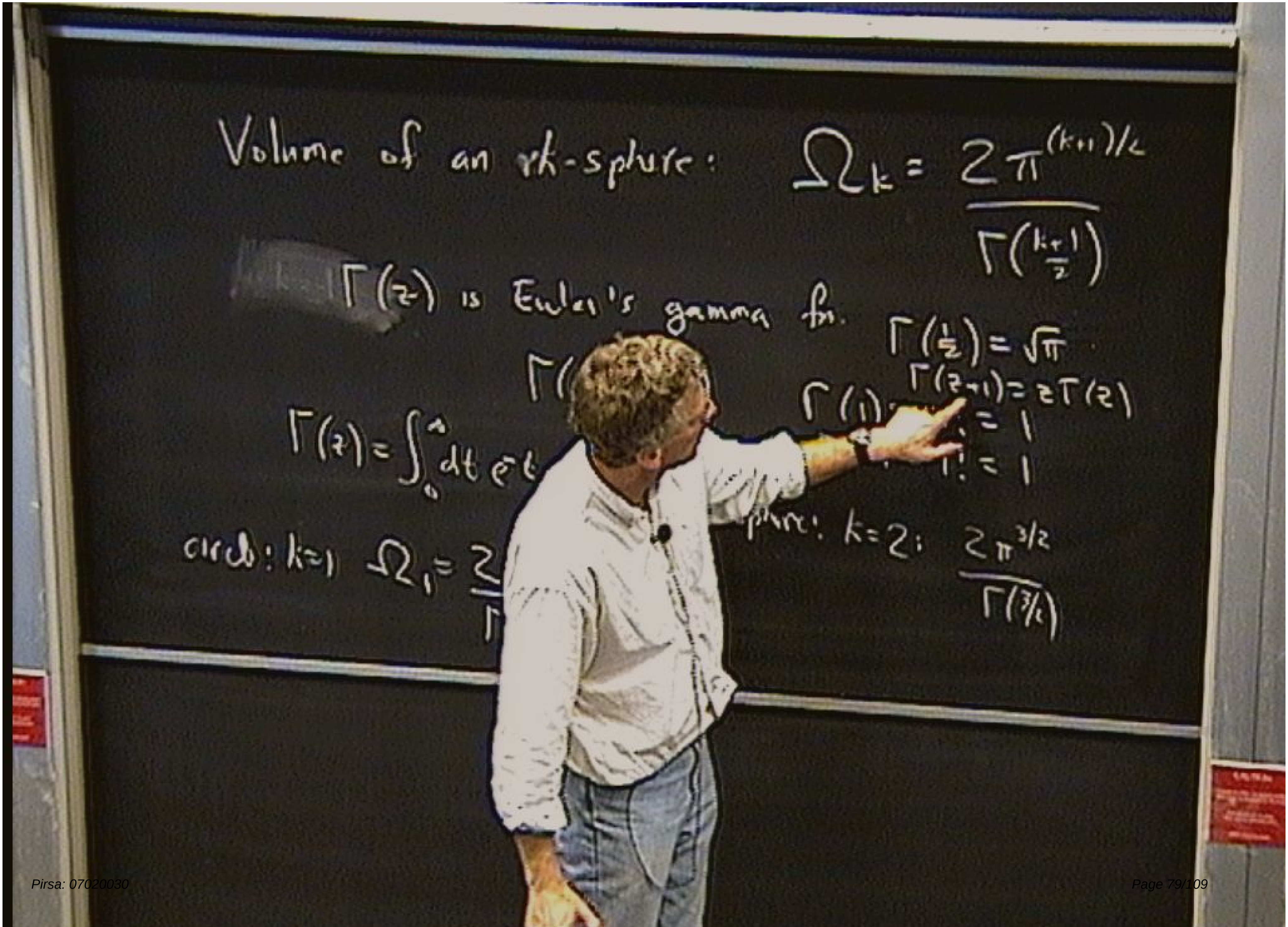
$\Gamma(z) = \int_0^\infty dt\, e^{-t}\, t^{z-1}$ $\qquad \Gamma(2) = 1! = 1$

circle: $k=1$ $\quad \Omega_1 = \dfrac{2\pi}{\Gamma(1)} = 2\pi$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2-sphere: $k=2$: $\dfrac{2\pi^{3/2}}{\Gamma(3/2)} = \dfrac{2\pi^{3/2}}{\frac{1}{2}\sqrt{\pi}} = 4\pi$

circle: $k=1$ $\Omega_1 = \dfrac{2\pi}{\Gamma(1)} = 2\pi$

3-sphere $k=3$: $\dfrac{2\pi^2}{\Gamma(2)} = 2\pi^2$

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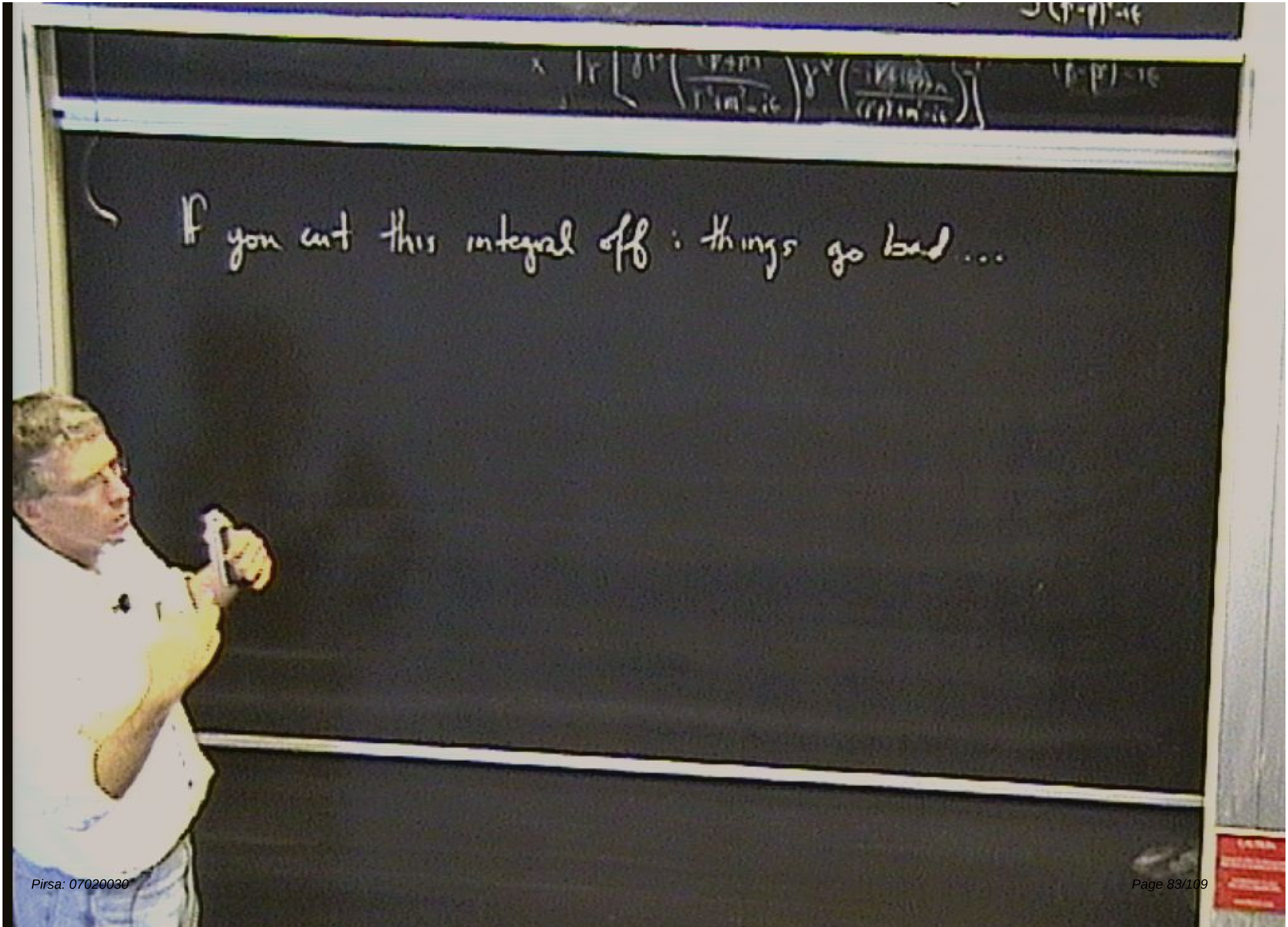
If you cut this integral off : things go bad ...

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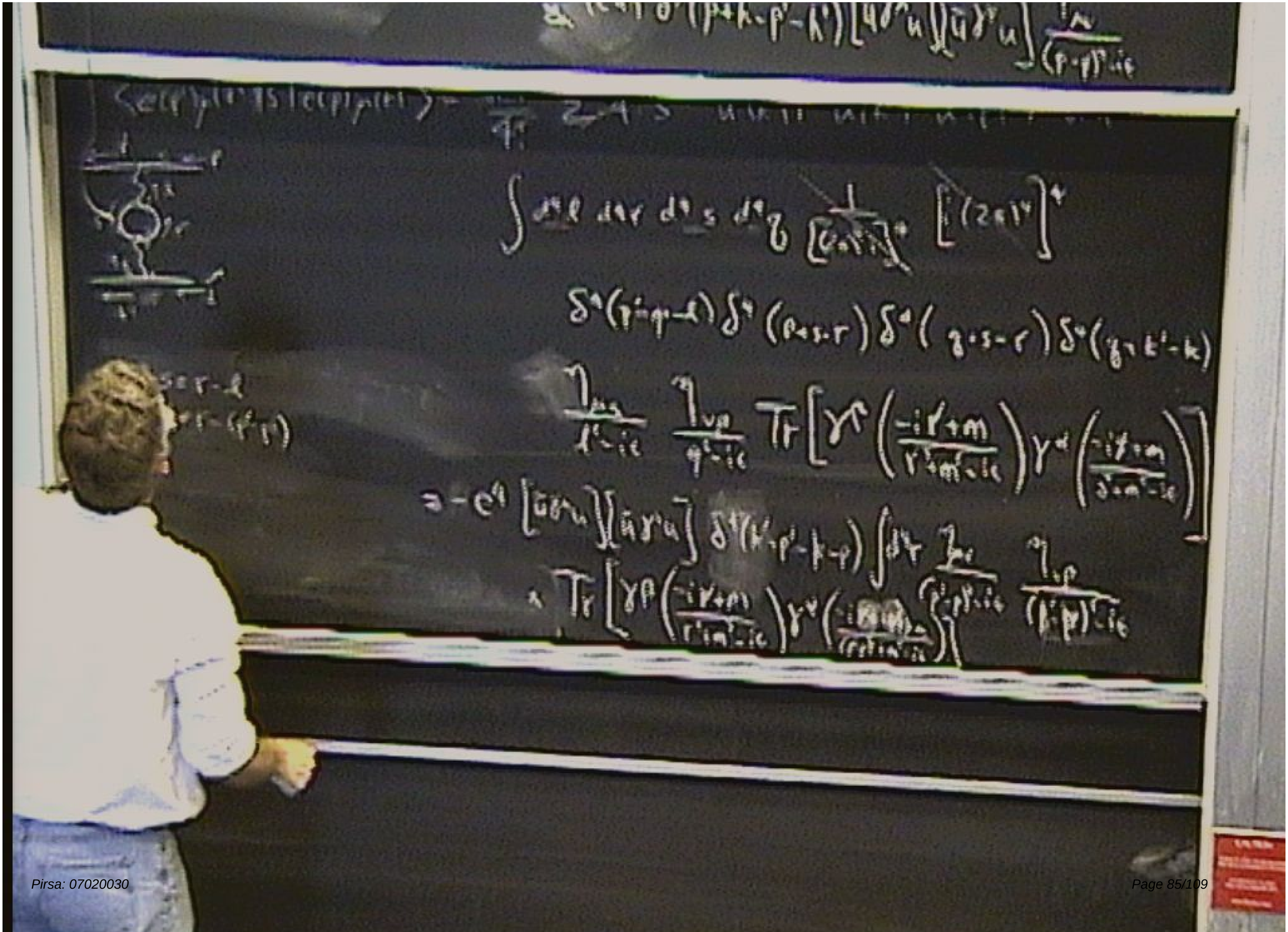
1) $\int_0^{\Lambda^2} dr^2 (\cdots) = ?$



$$\times \mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{\ell}+m}{\ell^2+m^2-i\epsilon}\right)\gamma^\nu\left(\frac{-i(\not{\ell}+\not{p})+m}{(\ell+p)^2+m^2-i\epsilon}\right)\right] \quad \frac{\cdots}{(p'-p)^2-i\epsilon}$$

If you cut this integral off : things go bad ...

i) $\int_0^{\Lambda^2} dr^2 (\ldots) = ?$ Answer depends on which internal line you chose to keep as the integration variable.

$$\times \, \mathrm{Tr}\left[\gamma^\beta \left(\frac{-i\not{p}+m}{p^2+m^2-i\epsilon}\right)\gamma^\nu\left(\frac{-i(\not{p}+\not{q})+m}{(p+q)^2+m^2-i\epsilon}\right)\right] \quad \frac{1}{(p'-p)^2-i\epsilon}$$

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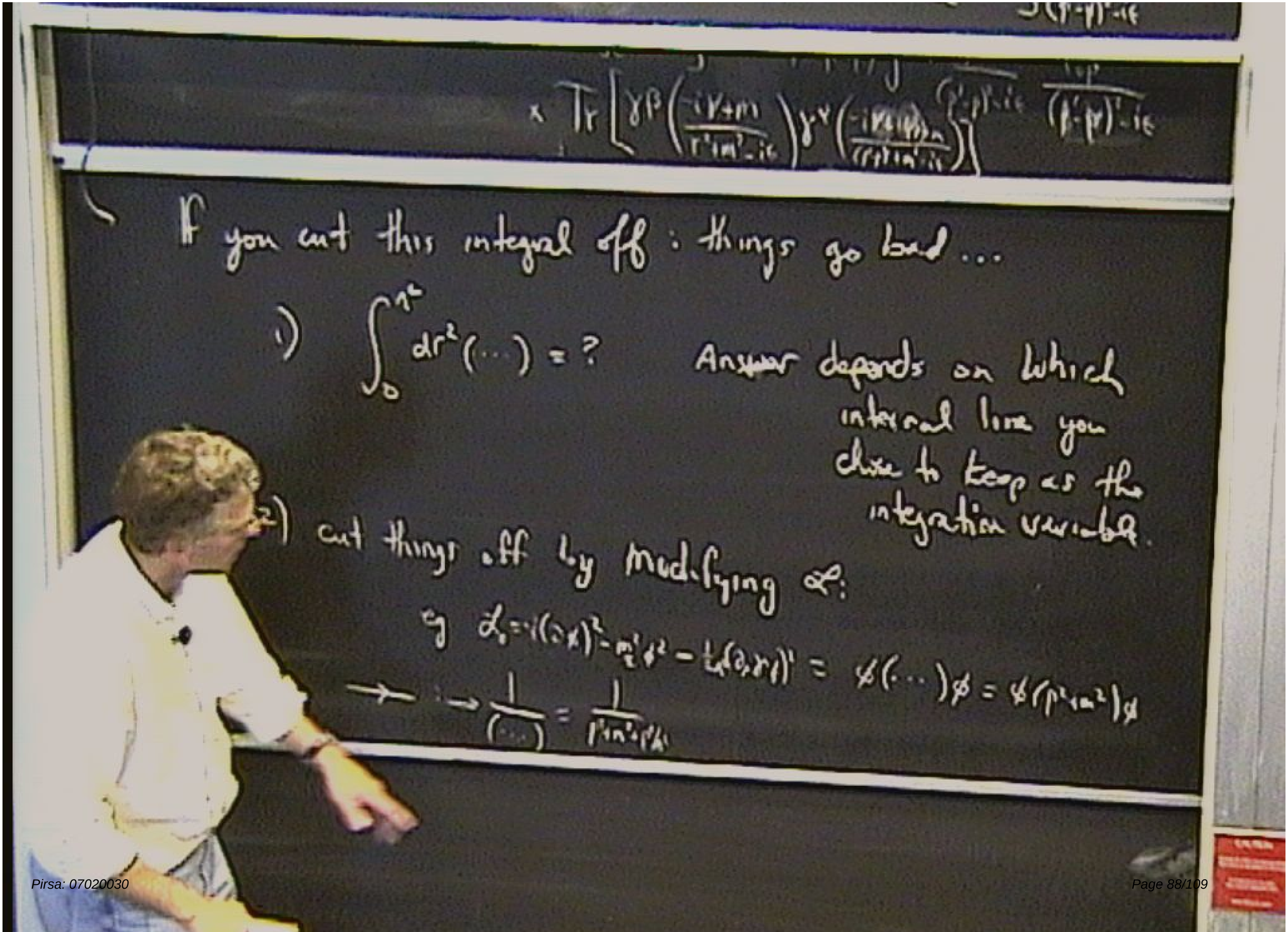
2) cut things off by modifying $\mathcal{L}$:

eg $\mathcal{L} = -\frac{1}{2}(\partial\chi)^2 - m_\chi^2\chi^2$


× Tr[γ^β((-ip̸+m)/(p²+m²-iε))γ^ν((-i(p̸+q̸)+m)/((p+q)²+m²-iε))]
If you cut this integral off : things go bad ...
1) ∫_0^{Λ²} dr²(...) = ?
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eg ℒ_0 = -½(∂φ)² - m²/2 φ² - 1/(2Λ²)(∂²φ)² = φ(...)φ = φ(p²+m²)φ
→ i → 1/(...) = 1/(p²+m²+p⁴/Λ²)

$$\times \, \mathrm{Tr}\left[\gamma^\mu \left(\frac{-i\not{p}+m}{p^2+m^2-i\epsilon}\right)\gamma^\nu\left(\frac{-i(\not{p}+\not{q})+m}{(p+q)^2+m^2-i\epsilon}\right)\right] \quad \frac{1}{(p'-p)^2-i\epsilon}$$

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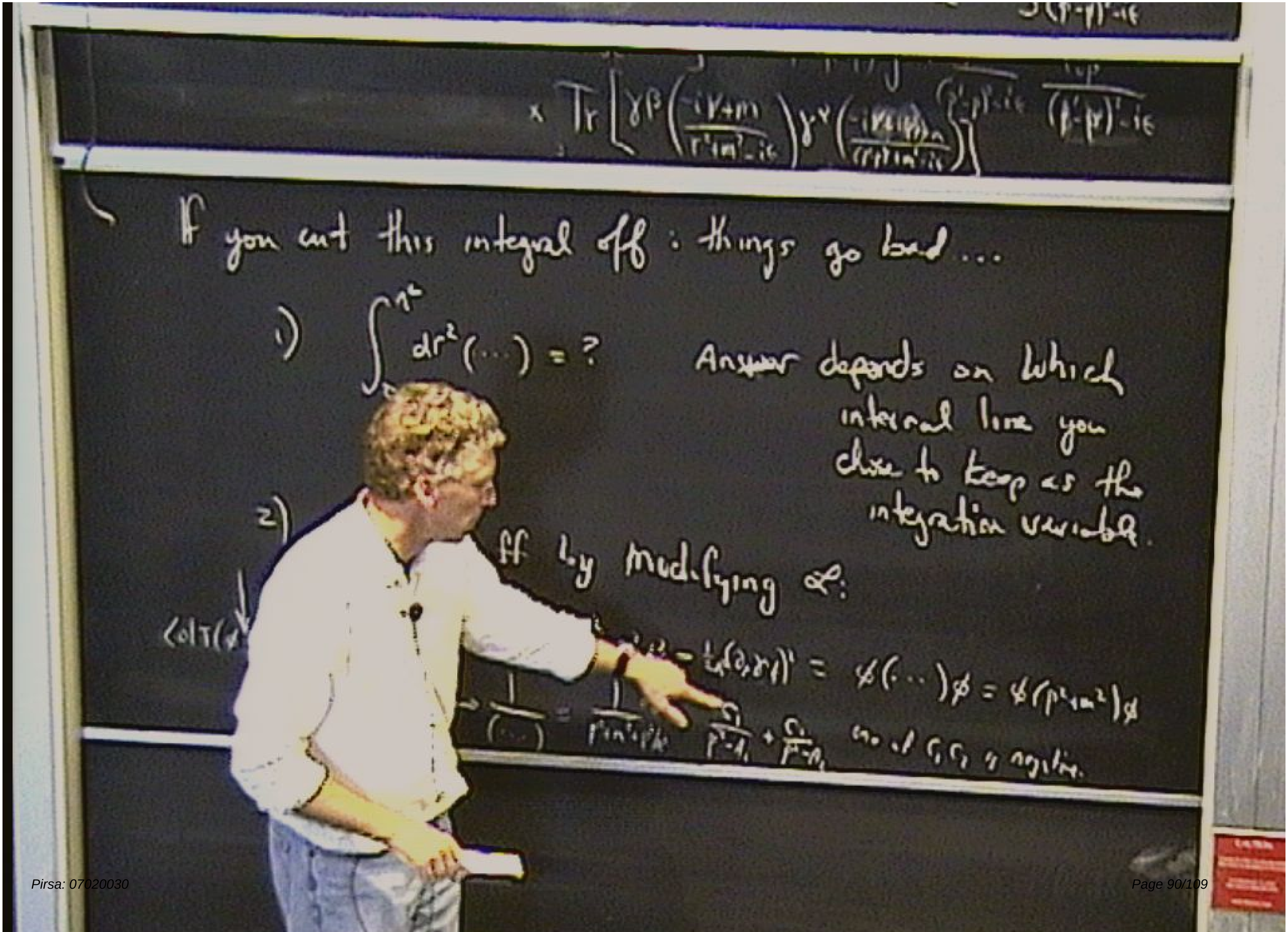
$\rightarrow \; : \rightarrow \frac{1}{(\cdots)} = \frac{1}{p^2+m^2+p^4/\Lambda^2} = \frac{c_1}{p^2-A_1} + \frac{c_2}{p^2-A_2}$ one of c_1, c_2 is negative.



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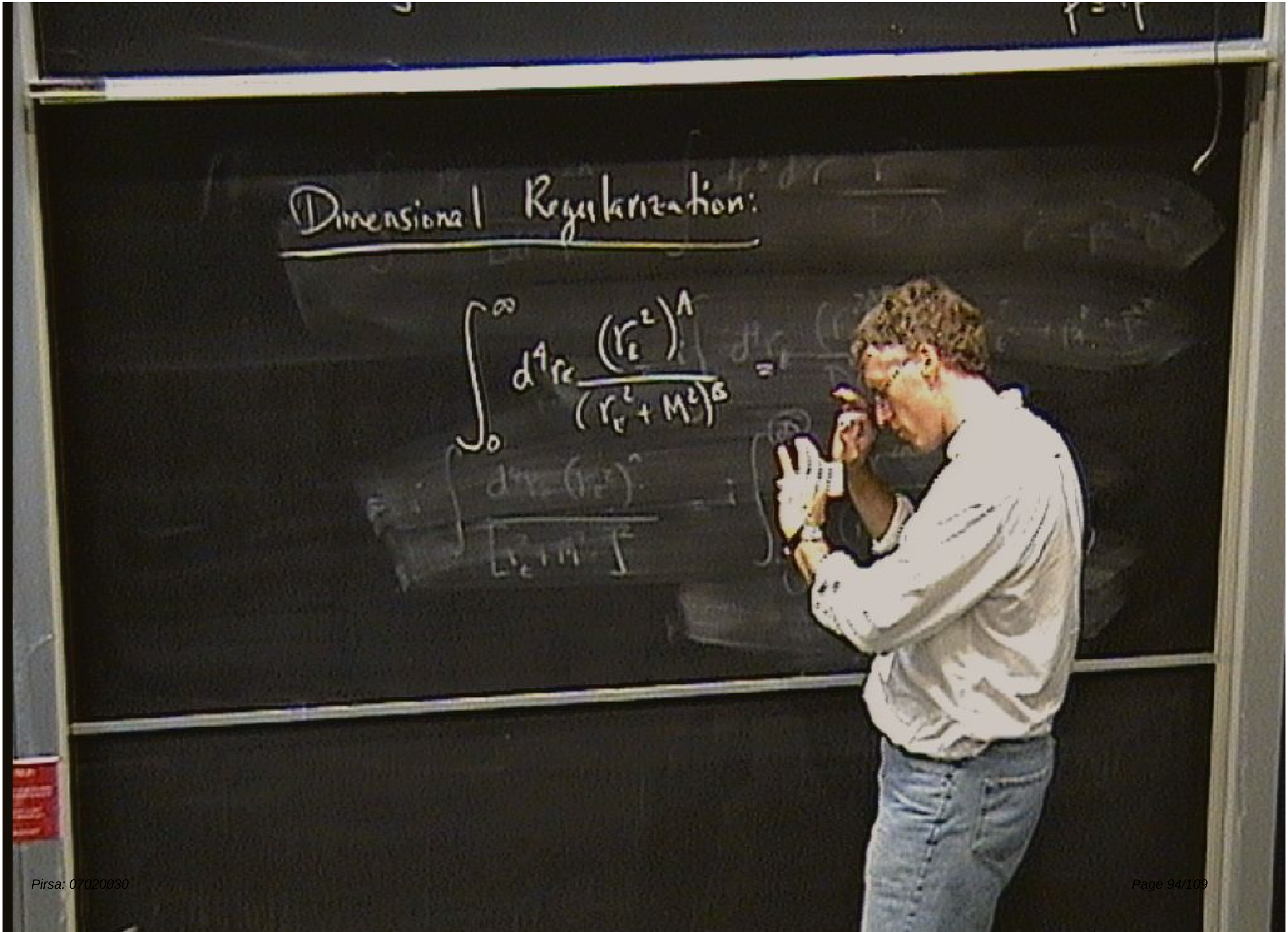
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Regard this as an analytic fn of n where n is complex

Dimensional Regularization:

the integral for any fixed A, B converges if n is sufficiently negative.

Define $I(n)$

$$I = \int_0^\infty d^4 r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B} = \int_0^\infty d^n r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

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$$\times \operatorname{Tr}\left[\gamma^{\mu}\left(\frac{-i\not{p}+m}{p^2+m^2-i\epsilon}\right)\gamma^{\nu}\left(\frac{-i(\not{p}+\not{k})+m}{(p+k)^2+m^2-i\epsilon}\right)\right] \frac{1}{(p'-p)^2-i\epsilon}$$

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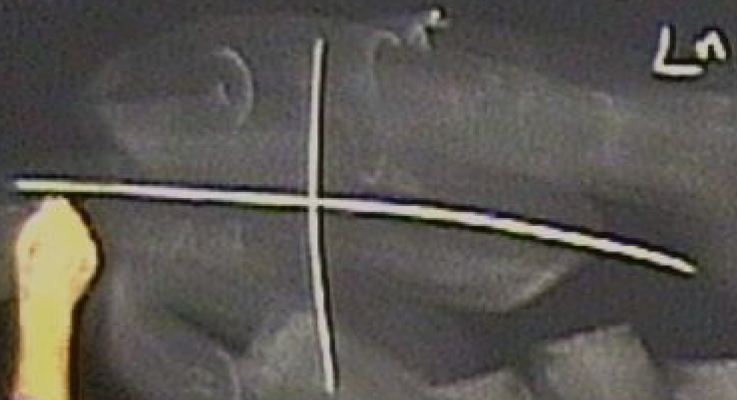

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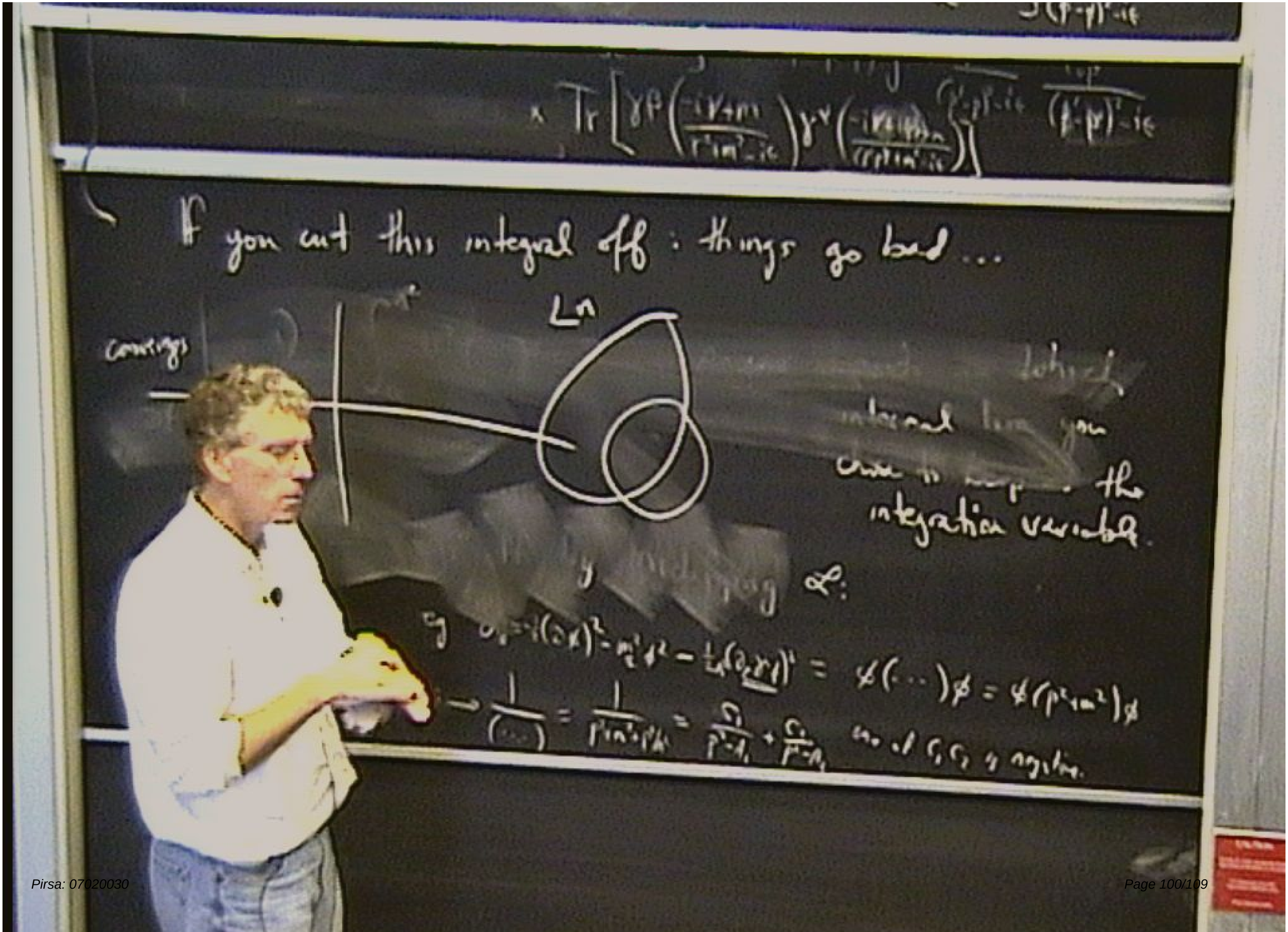
If you cut this integral off : things go bad ...
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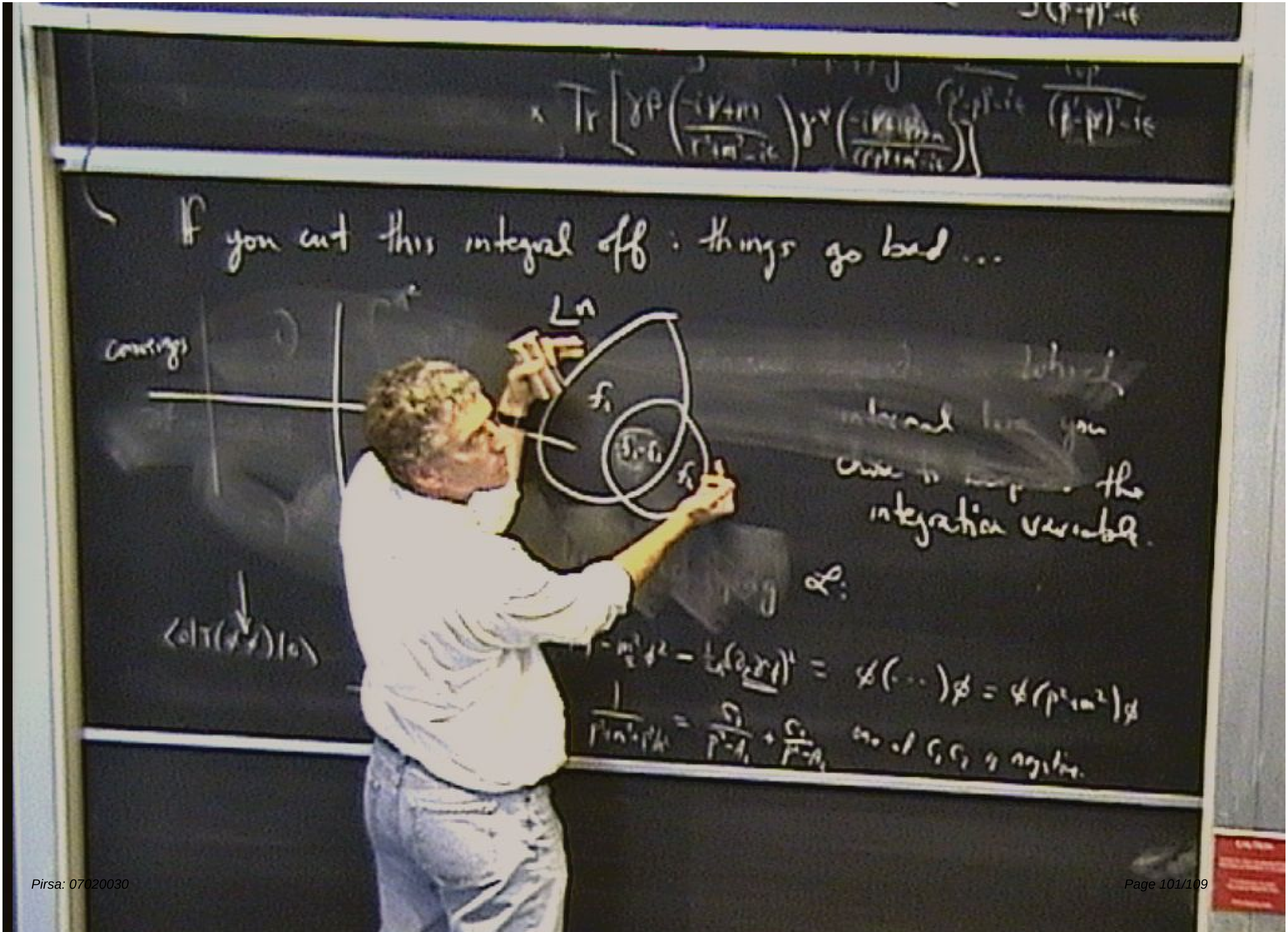
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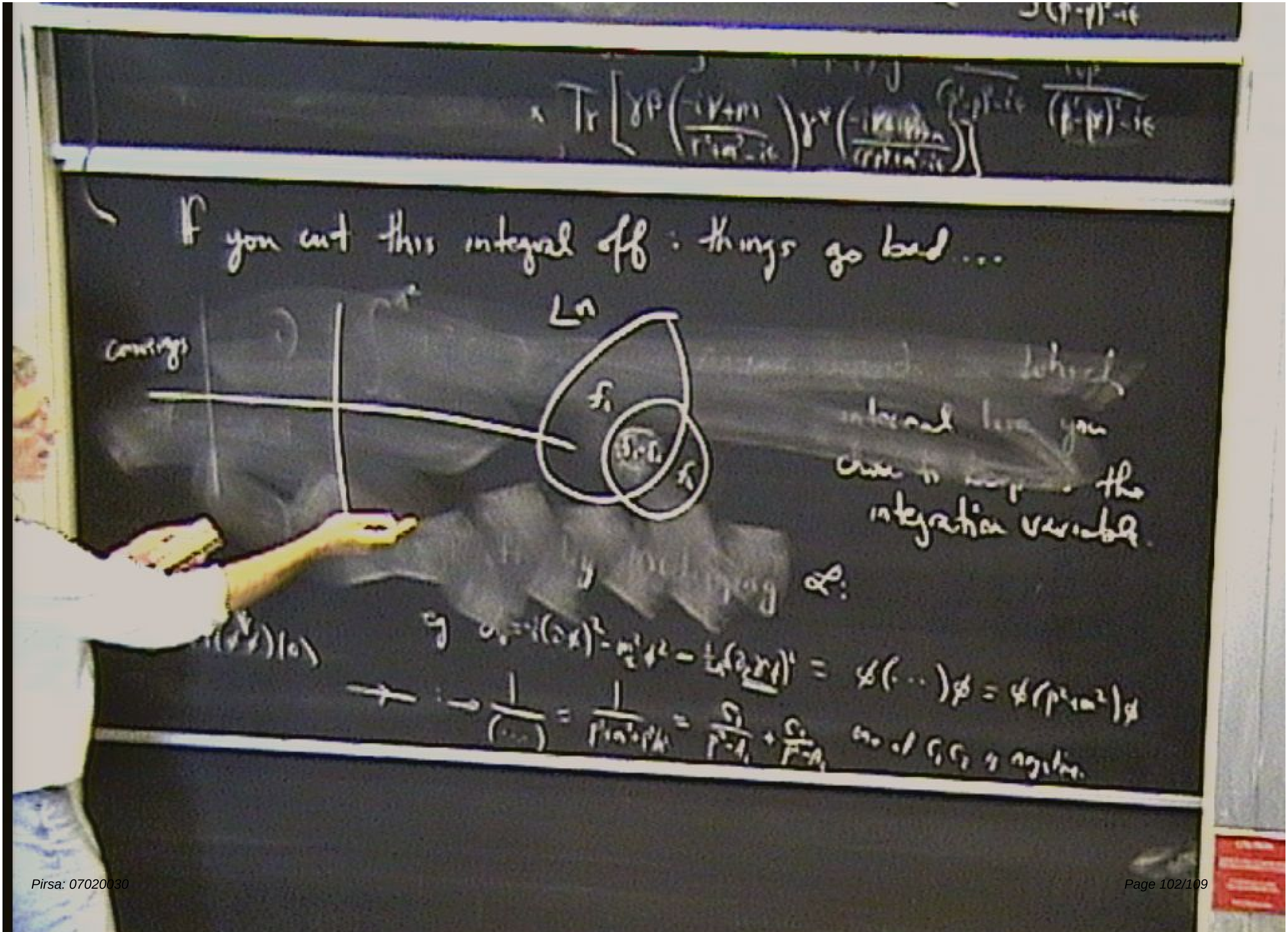
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Regard this as an analytic fn of n where n is complex

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Define $I(n)$ by analytically continuing the finite result at $n \to 4$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p)\,[\bar{u}\gamma^\mu u][\bar{u}\gamma^\nu u]\,\frac{1}{(p'-p)^2-i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $\Pi_{\mu\nu}(q) = -ie^2 \int \frac{d^4r}{(2\pi)^4}\, \mathrm{Tr}\left\{ \gamma_\mu \left[\frac{-i(\not{r}+m)}{r^2+m^2-i\epsilon}\right] \gamma_\nu \left[\frac{-i(\not{r}-\not{q})+m}{(r-q)^2+m^2-i\epsilon}\right]\right\}$

vacuum polarization $= -ie^2 \int \frac{d^4r}{(2\pi)^4}\, \frac{N_{\mu\nu}}{D}$

$D = (\ldots-i\epsilon)((r-q)^2+m^2-i\epsilon)$ $\quad N_{\mu\nu} = \mathrm{Tr}\left\{\gamma_\mu(-i\not{r}+m)\gamma_\nu[-i(\not{r}-\not{q})+m]\right\}$

$\Gamma(3/2)$ $\quad \frac{1}{2}\sqrt{\pi}$

$= 2\pi$ $\quad$ 3-sphere $k=3$: $\frac{2\pi^2}{\Gamma(2)} = 2\pi^2$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) \left[\bar{u}\gamma^\mu u\right]\left[\bar{u}\gamma^\nu u\right] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \frac{\Pi_{\mu\nu}(p'-p)}{(p'-p)^2} \right\}$$

where $\Pi_{\mu\nu}(q) = -ie^2 \int \frac{d^4r}{(2\pi)^4} \, \mathrm{Tr}\left\{ \gamma_\mu \left[\frac{-i(\not{r}+m)}{r^2+m^2-i\epsilon}\right] \gamma_\nu \left[\frac{-i(\not{r}-\not{q})+m}{(r-q)^2+m^2-i\epsilon}\right] \right\}$

vacuum polarization $= -ie^2 \int \frac{d^4r}{(2\pi)^4} \frac{N_{\mu\nu}}{D}$

$= (r^2+m^2-i\epsilon)((r-q)^2+m^2-i\epsilon)$ $\quad N_{\mu\nu} = \mathrm{Tr}\left\{\gamma_\nu(-i\not{r}+m)\gamma_\mu\left[-i(\not{r}-\not{q})+m\right]\right\}$

$\frac{\quad}{\Gamma(3/2)} = \frac{\quad}{\frac{1}{2}\sqrt{\pi}} = 4\pi$

$= \frac{2\pi}{\Gamma(1)} = 2\pi$ $\quad$ 3-sphere $k=3$: $\frac{2\pi^2}{\Gamma(2)} = 2\pi^2$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p)\,[\bar u \gamma^\mu u][\bar u \gamma^\nu u]\,\frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \frac{\Pi_{\mu\nu}(p'-p)}{(p'-p)^2 - i\epsilon} \right\}$$

where $\Pi_{\mu\nu}(q) = -ie^2 \int \frac{d^4r}{(2\pi)^4}\, \mathrm{Tr}\left\{\gamma_\mu \left[\frac{-i(\not r + m)}{r^2+m^2-i\epsilon}\right]\gamma_\nu \left[\frac{-i(\not r - \not q)+m}{(r-q)^2+m^2-i\epsilon}\right]\right\}$

vacuum polarization $= -ie^2\int \frac{d^4r}{(2\pi)^4}\,\frac{N_{\mu\nu}}{D}$

$D = (r^2+m^2-i\epsilon)((r-q)^2+m^2-i\epsilon)$ $\quad N_{\lambda\nu} = \mathrm{Tr}\{\gamma_\nu(-i\not r+m)\gamma_\lambda[-i(\not r-\not q)+m]\}$

$\frac{\;}{\Gamma(3/2)} = \frac{\;}{\frac{1}{2}\sqrt{\pi}} = 4\pi$

circle: $k=1$ $\quad \Omega_1 = \frac{2\pi}{\Gamma(1)} = 2\pi$ $\qquad$ 3-sphere $k=3$: $\frac{2\pi^2}{\Gamma(2)} = 2\pi^2$

$$\langle e(p')\mu(k')|S|e(p)\mu(k)\rangle = 2\,\frac{1}{2!}(-ie)^2(i)\left[\bar{u}(k)\gamma^\mu u(k)\right]\left[\bar{u}(p')\gamma^\nu u(p)\right]$$

$$\times\ \frac{\eta_{\mu\nu}}{(p'-p)^2-i\epsilon}\ (2\pi)^4\delta^4(q-k+k')$$

$$= -\frac{ie^2}{\ldots}(2\pi)^4\delta^4(p+k-p'-k')\left[\bar{u}\gamma^\mu u\right]\left[\bar{u}\gamma^\nu u\right]\frac{\eta_{\mu\nu}}{(p-p')^2-i\epsilon}$$

$$\times\ \mathrm{Tr}\left[\gamma^\beta\left(\frac{-i\not{r}+m}{r^2+m^2-i\epsilon}\right)\gamma^\alpha\left(\frac{-i\not{r}+(\not{p})+m}{(r+p)^2+m^2-i\epsilon}\right)\right]\ \frac{\ldots}{(p'-p)^2-i\epsilon}\ \frac{\ldots}{(p'-p)^2-i\epsilon}$$

If you cut this integral off: things go bad...

converges

Λ

$\mathcal{H}_1$

$\mathcal{H}_2$

$\mathcal{H}_2 \cap \mathcal{H}_1$

... which integral line you chose to keep ... the