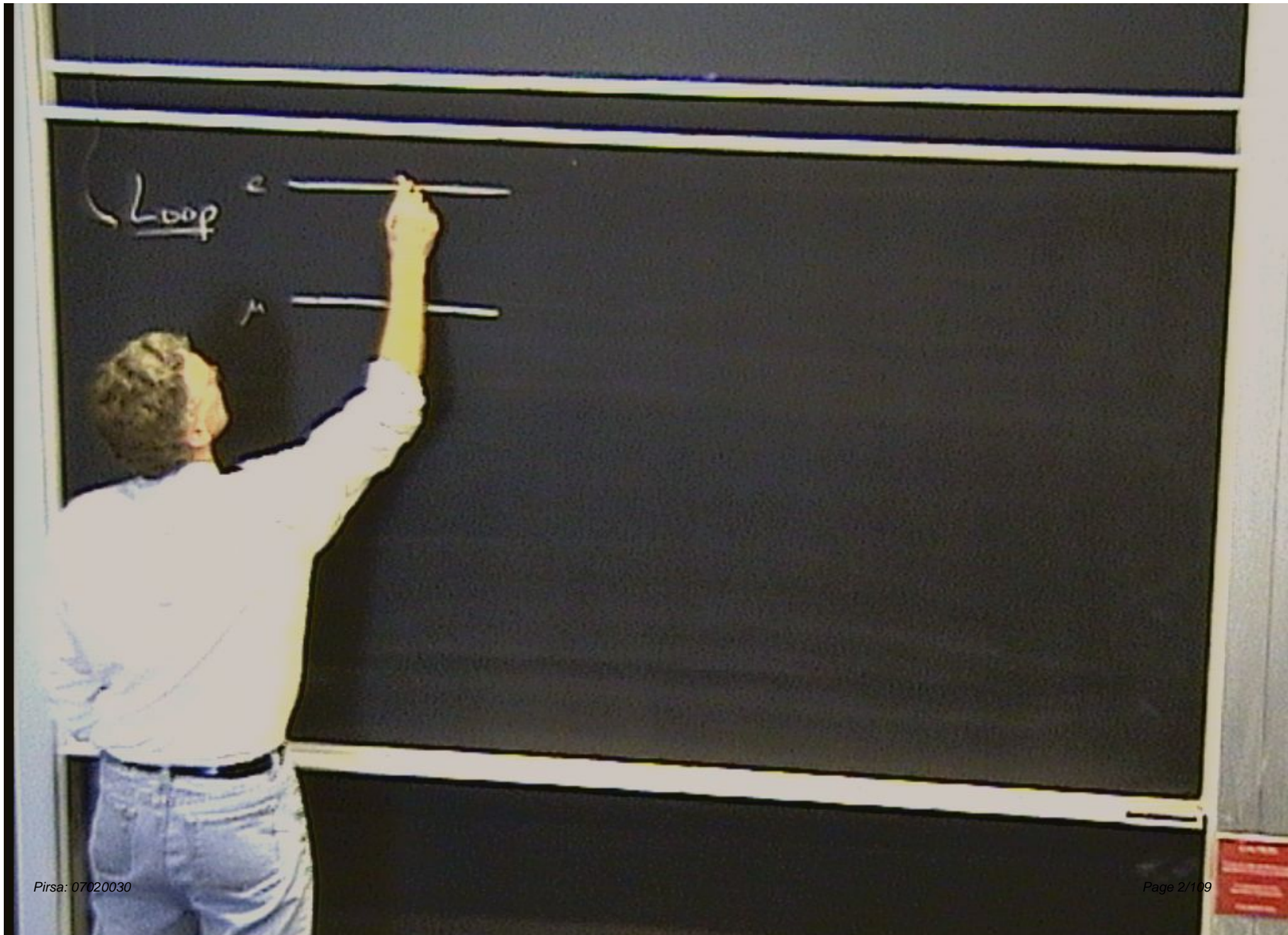


Title: Graduate Course on Standard Model & Quantum Field Theory - 13A

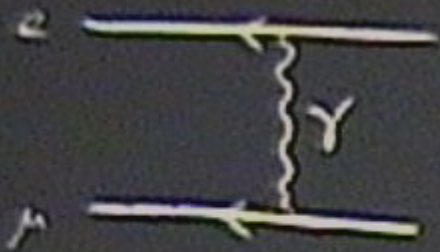
Date: Feb 28, 2007 11:00 AM

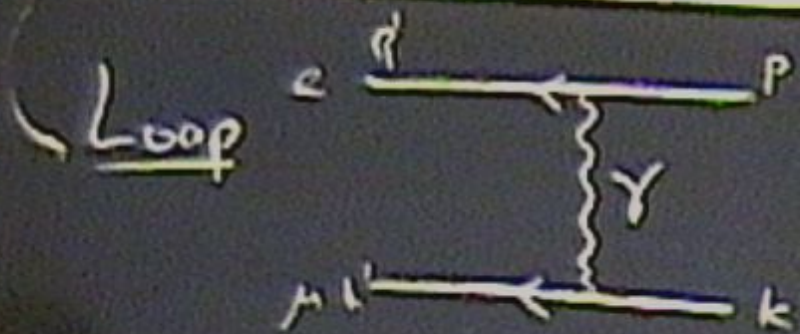
URL: <http://pirsa.org/07020030>

Abstract: Graduate Course on Standard Model & Quantum Field Theory

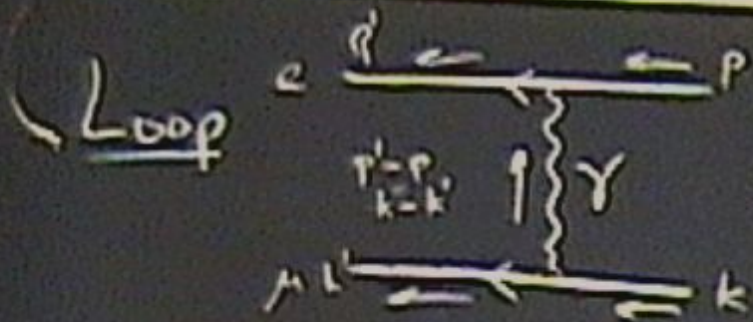


Loop





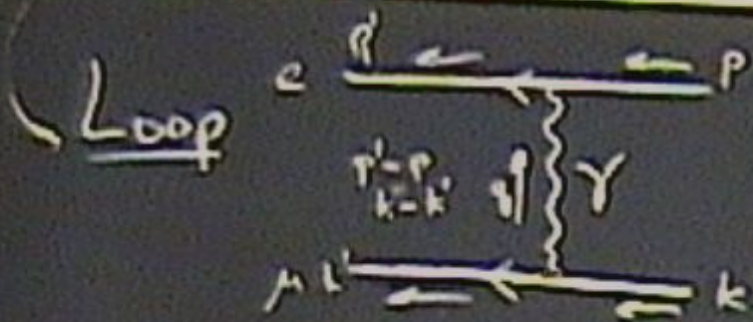
$$\langle e(p)\mu(k) | S | e(p)\mu(k) \rangle = \frac{1}{2!} (-i)^2$$



$$\mathcal{L}_{em} = ie \bar{\psi} \gamma^\mu \psi A_\mu$$

$$\langle e(p') \mu(k') | S | e(p) \mu(k) \rangle = \frac{1}{2!} (-i)^2 [\bar{u}(k) \gamma^\mu u(k)] [\bar{u}(p') \gamma^\nu u(p)]$$

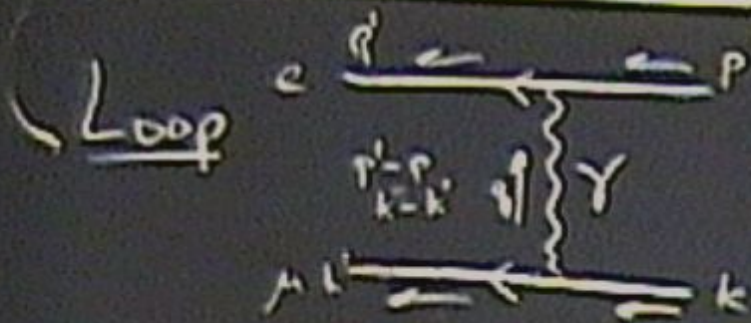
$$\times \frac{\eta_{\mu\nu}}{(p' - p)^2 - i\epsilon}$$



$$\mathcal{L}_{em} = ie \bar{\psi} \gamma^\mu \psi A_\mu$$

$$\langle e(p) \mu(k) | S | e(p) \mu(k) \rangle = \int d^4x \frac{1}{2!} (ie)^2 [\bar{u}(k) \gamma^\mu u(k)] [\bar{u}(p) \gamma^\nu u(p)]$$

$$\times \frac{1}{(2\pi)^4} \int d^4q \frac{-i \eta_{\mu\nu}}{q^2 - i\epsilon} (2\pi)^4 \delta^4(q - p' + p) (2\pi)^4 \delta^4(q - k' + k)$$



$$\mathcal{L}_{em} = ie \bar{\psi} \gamma^\mu \psi A_\mu$$

$$\langle e(p) \mu(k) | S | e(p) \mu(k) \rangle = \frac{1}{2} \epsilon^{\mu\nu} (i) [\bar{u}(k) \gamma^\mu u(k)] [\bar{u}(p) \gamma^\nu u(p)]$$

$$\times \frac{\eta_{\mu\nu}}{(q^2 - i\epsilon)}$$

$$= \frac{ie^2}{2} (2\pi)^4 \delta^4(p+k-p'-k') [\bar{u} \gamma^\mu u] [\bar{u} \gamma^\nu u] \frac{\eta_{\mu\nu}}{(p-k)^2 - i\epsilon}$$

$(2\pi)^4 \delta^4(p+k-p'-k')$

$$\langle e(p)_{\mu}(k') | S | e(p)_{\mu}(k) \rangle = \frac{1}{2} \epsilon^{(i)} [\bar{u}(k) \gamma^{\mu} u(k)] [\bar{u}(p) \gamma^{\nu} u(p)]$$



$$\frac{1}{(p-k)^2 - i\epsilon}$$

$$(2\pi)^4 \delta^4(p-k-p')$$

$$= \frac{1}{2} \frac{1}{(2\pi)^4} \delta^4(p+k-p'-k') [\bar{u} \gamma^{\mu} u] [\bar{u} \gamma^{\nu} u] \frac{1}{(p-k)^2 - i\epsilon}$$

$$\langle e(p)_{\mu}(k') | S | e(p)_{\mu}(k) \rangle =$$



$$\langle e(p)\mu(k) | S | e(p)\mu(k) \rangle = \frac{1}{2} \epsilon^{ij} [\bar{u}(k) \gamma^i u(k)] [\bar{u}(p) \gamma^j u(p)]$$



$$\frac{1}{(p-k)^2 - i\epsilon}$$

$$(2\pi)^4 \delta^4(p-k)$$

$$= \frac{1}{2} (2\pi)^4 \delta^4(p+k-p-k) [\bar{u} \gamma^i u] [\bar{u} \gamma^j u] \frac{1}{(p-k)^2 - i\epsilon}$$

$$\langle e(p)\mu(k) | S | e(p)\mu(k) \rangle = \frac{1}{4} \epsilon^{ij}$$

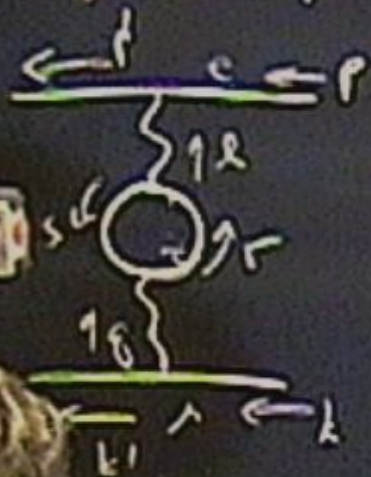


$$= (2\pi)^4 \delta^4(p+k-p'-k') [\bar{u} \gamma^\mu u] [\bar{u} \gamma_\mu u] \frac{1}{(p-p')^2 - i\epsilon}$$

$$\langle e(p) \mu(k) | S | e(p) \mu(k) \rangle = \frac{e^4}{4i}$$



$$\langle e(p')_{\mu}(k') | S | e(p)_{\mu}(k) \rangle = \frac{e^4}{4!} \mathcal{Z}$$

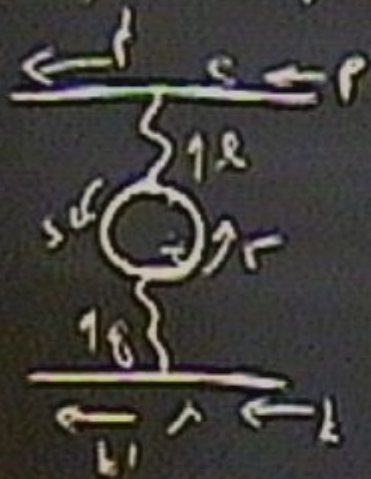


$$e^{iL} - e^{-iL}$$

$$L = (\vec{E} \cdot \vec{A} - \vec{A} \cdot \vec{E} - \vec{E} \cdot \vec{A}) e$$

(17)

$$\langle e(p') \mu(k') | S | e(p) \mu(k) \rangle = \frac{i^4 e^4}{4!} \mathcal{Z}$$

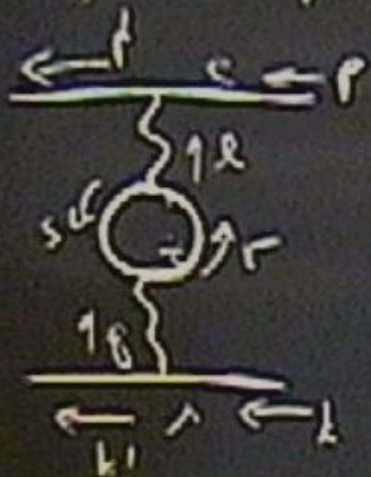


$$e^{iL} \sim e^{-iH}$$

$$L = (E_A + \vec{p} \cdot \vec{A} + \vec{p} \cdot \vec{A}) e$$

$$i \int (i \vec{A} \cdot \vec{p} - \vec{A} \cdot \vec{p}) e$$

$$\langle e(p') \mu(k') | S | e(p) \mu(k) \rangle = \frac{i^4 e^4}{4!} \mathcal{Z}$$



$$(x+y)^2 = x^2 + 2xy + y^2$$

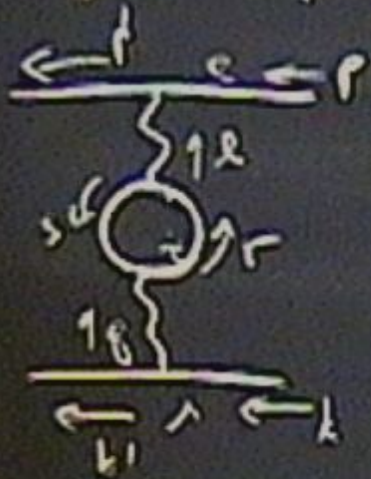
$$e^{iL} = e^{-iH}$$

$$L = (E_A + \vec{p} \cdot \vec{A} + \vec{p} \cdot \vec{A})_e$$

$$i \int (\vec{\pi} \cdot \vec{A} + \vec{p} \cdot \vec{A})_e$$

$$= \frac{i^4}{4!} \left[4! e^4 (\vec{\pi} \cdot \vec{A})^2 \dots \right]$$

$$\langle e(p) \mu(k) | S | e(p) \mu(k) \rangle = \frac{i^4 e^4}{4!} \mathcal{Z}$$



$$e^{iL} = e^{-iH} \quad (x+y)^2 = x^2 + 2xy + y^2$$

$$L = (eA + \bar{\psi}A + \psi A) c$$

$$i^4 \frac{(\int \bar{\psi} A + \psi A) c}{4!} = \frac{i^4}{4!} \left[4! e c A (\int \bar{\psi} A) (\int \psi A) \right]$$



$$\mathcal{L}_{em} = ie \bar{\psi} \gamma^\mu \psi A_\mu$$

$$\langle e(p) \mu(k) | S | e(p) \mu(k) \rangle = 2 \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} [\bar{u}(k) \gamma^\mu u(k)] [\bar{u}(p) \gamma^\nu u(p)]$$

$$\frac{1}{(p-p')^2 - i\epsilon}$$

$$(2\pi)^4 \delta^4(p - k + k')$$

$$= \frac{ie^2}{2} (2\pi)^4 \delta^4(p + k - p' - k') [\bar{u} \gamma^\mu u] [\bar{u} \gamma^\nu u] \frac{1}{(p-p')^2 - i\epsilon}$$



$$\langle \psi(r)|\psi(r) \rangle = \frac{i e^4}{4!} 2 \cdot 4 \cdot 3$$



$$\langle e(p)_{\mu}(k) | S | e(p)_{\mu}(k) \rangle = \frac{e^4}{4!} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^{\mu} u(k') \bar{u}(p) \gamma^{\nu} u(p')$$



$$\langle e(p)_{\mu} | S | e(p)_{\nu} | \rangle = \frac{e^4}{4!} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^{\mu} u(k') \bar{u}(p) \gamma^{\nu} u(p')$$



$$\int d^4x d^4y d^4z d^4q \frac{1}{[k_0^2]_+}$$

$$\langle e^{i(p_1)_\mu} \psi(x) e^{i(p_2)_\mu} \psi(x) \rangle = \frac{e^4}{4!} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4q d^4r d^4s d^4q \frac{1}{[2\pi i]^4} [i(2\pi i)]^4$$

$$\langle S(e(p), p(t)) | S(e(p), p(t)) \rangle = \frac{e^4}{4!} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4x d^4y d^4s d^4g \frac{1}{[2\pi i]^4} [i(2\pi)^4]^4$$

$$\delta^4(p' - p - k) \delta^4(p + s - r) \delta^4(g + s - r) \delta^4(g + k' - k)$$

$$\langle \psi(p) \psi(k) | S | \psi(p) \psi(k) \rangle = \frac{e^4}{4!} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4x d^4y d^4s d^4q \frac{1}{[2\pi i]^4} [i(2\pi)^4]^4$$

$$\delta^4(p' - p - q) \delta^4(p + s - r) \delta^4(q + s - r) \delta^4(q + k' - k)$$

$$\frac{1}{k^2 - i\epsilon} \frac{1}{q^2 - i\epsilon}$$

$$\langle \psi(r) \psi^\dagger(s) \psi(p) \psi^\dagger(k) \rangle = -\frac{e^4}{4!} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4r d^4s d^4q \frac{1}{[2\pi i]^4} [i(2\pi)^4]^4$$

$$\delta^4(p' - p - k) \delta^4(p + s - r) \delta^4(q + s - r) \delta^4(q + k' - k)$$

$$\frac{1}{k^2 - i\epsilon} \frac{1}{q^2 - i\epsilon} \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r} + m}{r^2 + m^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{s} + m}{s^2 + m^2 - i\epsilon} \right) \right]$$

$$\langle \psi(p) | \psi(p) \rangle = -\frac{e^4}{4\pi} \cancel{2 \cdot 4 \cdot 3} \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4x d^4y d^4z d^4q \frac{1}{[2\pi i]^4} [i(2\pi i)]^4$$

$$\delta^4(p'-p-q) \delta^4(p+q-r) \delta^4(q+s-r) \delta^4(q+k'-k)$$

$$\frac{1}{k^2-ic} \frac{1}{q^2-ic} \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r}+m}{r^2+m^2-ic} \right) \gamma^\nu \left(\frac{-i\not{s}+m}{s^2+m^2-ic} \right) \right]$$

$$= -e^4 [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \delta^4(k'-p'-k-p) \int d^4r \frac{1}{r^2}$$

$$\langle \psi(p) \psi(k) | S | \psi(p) \psi(k) \rangle = -\frac{e^4}{4\pi} \cancel{2 \cdot 4 \cdot 3} \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4x d^4y d^4s d^4q \frac{1}{(2\pi)^4} [i(2\pi)^4]^4$$

$$\delta^4(p' - p - q) \delta^4(p + s - r) \delta^4(q + s - r) \delta^4(q + k' - k)$$

$$\frac{1}{k' - ic} \frac{1}{q' - ic} \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r} + m}{r^2 + m^2 - ic} \right) \gamma^\nu \left(\frac{-i\not{s} + m}{s^2 + m^2 - ic} \right) \right]$$

$$= -e^4 [\bar{u}(k') u(k)] [\bar{u}(p') u(p)] \delta^4(k' - p' - k - p) \int d^4r \frac{1}{(r - p')^2 - ic} \frac{1}{(k - k')^2 - ic}$$

$$\times \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r} + m}{r^2 + m^2 - ic} \right) \gamma^\nu (-i) \right]$$

$$\langle e(r) p(k) | S | e(p) p(k) \rangle = -\frac{e^4}{4\pi} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4x d^4y d^4s d^4q \frac{1}{(2\pi)^4} [i(2\pi)^4]^4$$

$$\delta^4(p'-p-q) \delta^4(p+s-r) \delta^4(q+s-r) \delta^4(q+k'-k)$$

$$\frac{1}{k^2 - i\epsilon} \frac{1}{q^2 - i\epsilon} \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r} + m}{r^2 + m^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{s} + m}{s^2 + m^2 - i\epsilon} \right) \right]$$

$$= -e^4 [\bar{u}(k') u(k)] [\bar{u}(p') u(p)] \delta^4(k'-p'-k-p) \int d^4r \frac{1}{r^2 - i\epsilon} \frac{1}{(k-k')^2 - i\epsilon} \times \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r} + m}{r^2 + m^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{r} + m}{r^2 + m^2 - i\epsilon} \right) \right]$$

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-r) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\mu u]$$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma_\mu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

x

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(k) \right\}$$

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon} \\ \times \left\{ \gamma_{\mu\nu} + \pi_{\mu\nu}(p'-p) \right\}$$

where $\pi_{\mu\nu}(q) =$

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \gamma_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $\Pi_{\mu\nu}(q) = ie^2 \int \frac{d^4r}{(2\pi)^4} \text{Tr} \left\{ \gamma_\mu \right\}$

$$= -ie^2 (2\pi)^4 \delta^4(k'+p'-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \gamma_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $\Pi_{\mu\nu}(q) = ie^2 \int \frac{d^4r}{(2\pi)^4} \text{Tr} \left\{ \gamma_\mu \left[\frac{-i(\not{r} + m)}{r^2 + m^2 - i\epsilon} \right] \gamma_\nu \left[\frac{-i(\not{q} + m)}{q^2 + m^2 - i\epsilon} \right] \right\}$

$$\langle e(p) \mu(t) | S | e(p) \mu(t) \rangle = -\frac{e^4}{4\pi} \cancel{2 \cdot 4 \cdot 3} \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4x d^4y d^4s d^4q \cancel{\left[\frac{1}{(2\pi)^4} \right]^4} \left[(2\pi)^4 \right]^4$$

$$\delta^4(p' - p - q) \delta^4(p + s - r) \delta^4(q + s - r) \delta^4(q + k' - k)$$

$$s = r - q$$

$$r = p + q$$

$$\frac{1}{l^2 - i\epsilon} \frac{1}{q^2 - i\epsilon} \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{l} + m}{l^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{q} + m}{q^2 - i\epsilon} \right) \right]$$

$$= -e^4 [\bar{u}(k') \gamma^\mu u(k)] [\bar{u}(p) \gamma^\nu u(p')] \delta^4(k' - p' - k - p) \int d^4r \frac{1}{(r^2 - i\epsilon)} \frac{1}{(p' - r)^2 - i\epsilon}$$

$$\times \text{Tr} \left[\gamma^\rho \left(\frac{-i\not{r} + m}{r^2 - i\epsilon} \right) \gamma^\sigma \left(\frac{-i\not{p}' + m}{(p' - r)^2 - i\epsilon} \right) \right]$$

$$\langle e(r) p(t) | S | e(p) p(t) \rangle = -\frac{e^4}{4\pi} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4r d^4s d^4q \frac{1}{[2\pi i]^4} [i(2\pi i)]^4$$

$$\delta^4(p' - p - q) \delta^4(p + s - r) \delta^4(q + s - r) \delta^4(q + k' - k)$$

$$s = r - q$$

$$r = (p' - p)$$

$$\frac{1}{r^2 - i\epsilon} \frac{1}{q^2 - i\epsilon} \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r} + m}{r^2 + m^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{q} + m}{q^2 + m^2 - i\epsilon} \right) \right]$$

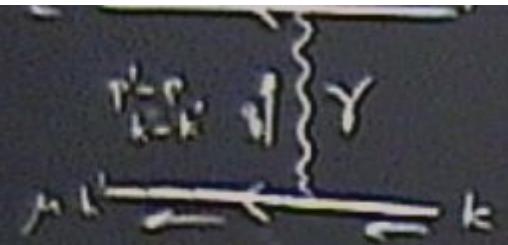
$$= -e^4 [\bar{u}(k') u(k)] [\bar{u}(p') u(p)] \delta^4(k' - p' - k - p) \int d^4r \frac{1}{r^2 - i\epsilon} \frac{1}{(p - p')^2 - i\epsilon}$$

$$\times \text{Tr} \left[\gamma^\rho \left(\frac{-i\not{r} + m}{r^2 + m^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{p} + m}{(p - p')^2 + m^2 - i\epsilon} \right) \right]$$

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p+p')^2 - i\epsilon} \\ \times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(p-p') \right\}$$

where $\Pi_{\mu\nu}(q) = ie^2 \int \frac{d^4r}{(2\pi)^4} \text{Tr} \left\{ \gamma_\mu \left[\frac{-i(\not{r} + m)}{r^2 + m^2 - i\epsilon} \right] \gamma_\nu \left[\frac{-i(\not{r} - \not{q} + m)}{(r-q)^2 + m^2 - i\epsilon} \right] \right\}$

Loop



$$\mathcal{L}_{em} = ie \bar{\psi} \gamma^\mu \psi A_\mu$$

$$\langle e(p) \mu(k) | S | e(p) \mu(k) \rangle = 2 \frac{1}{2} \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} [\bar{u}(p) \gamma^\mu u(k) \int \frac{d^4 q}{(2\pi)^4} \gamma^\nu u(p)]$$

$$\frac{\gamma_{\mu\nu}}{(p-q)^2 - i\epsilon}$$

$$= \frac{ie^2}{(2\pi)^4} \delta^4(p+k-q)$$

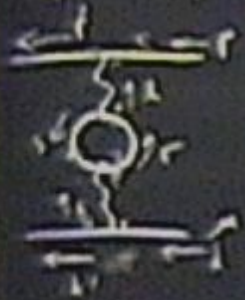
$$\int \frac{d^4 q}{(p-q)^2 - i\epsilon}$$

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\not{q} + m}{i\not{p} - i\epsilon} \right) \right]$$

$$\int \frac{d^4 q}{(p-q)^2 - i\epsilon}$$

$$= -\frac{ie^2}{4\pi} (2\pi)^4 \delta^4(p+k-p'-k') [\bar{u} \gamma^\mu u] [\bar{u} \gamma^\nu u] \frac{1}{(p-p')^2 - i\epsilon}$$

$$\langle e(p) \mu(k) | S | e(p) \mu(k) \rangle = -\frac{ie^2}{4\pi} 2 \cdot 4 \cdot 3 \bar{u}(k) \gamma^\mu u(k') \bar{u}(p) \gamma^\nu u(p')$$



$$\int d^4x d^4y d^4s d^4q \frac{1}{(2\pi)^4} [i(2\pi)^4]^4$$

$$\delta^4(p'-p-q) \delta^4(p+s-r) \delta^4(q+s-r) \delta^4(q+k'-k)$$

$$\frac{1}{l^2 - i\epsilon} \frac{1}{q^2 - i\epsilon} \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{l} + m}{l^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{s} + m}{s^2 - i\epsilon} \right) \right]$$

$$= -e^2 [\bar{u} \gamma^\mu u] [\bar{u} \gamma^\nu u] \delta^4(k+p'-k-r) \int d^4r \frac{1}{(p-p')^2 - i\epsilon} \frac{1}{(p-p')^2 - i\epsilon}$$

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \gamma_{\mu\nu} + \Pi_{\mu\nu}(p'-p) \right\}$$

where $\Pi_{\mu\nu}(q) = -ie^2 \int \frac{d^4r}{(2\pi)^4} \text{Tr} \left\{ \gamma_\mu \left[\frac{-i(\not{r} + \not{q})}{r^2 + m^2 - i\epsilon} \right] \gamma_\nu \left[\frac{-i(\not{r} - \not{q})}{(r-q)^2 + m^2 - i\epsilon} \right] \right\}$

polarization $= -ie^2 \int \frac{d^4r}{(2\pi)^4} \frac{N_{\mu\nu}}{D}$

$$D = (r^2 + m^2 - i\epsilon)((r-q)^2 + m^2 - i\epsilon) \quad N_{\mu\nu} = \text{Tr} \left\{ \gamma_\mu (-i\not{r} + m) \gamma_\nu [-i(\not{r}-\not{q}) + m] \right\}$$

Goal: Make D even under $\gamma^M \rightarrow -\gamma^M$

Goal: Make D even under $\gamma^m \rightarrow -\gamma^m$

Feynman trick: $\frac{1}{AB} =$

Goal: Make D even under $\gamma^m \rightarrow -\gamma^m$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{\underbrace{[(A-B)x + B]}_u} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2}$$

$$u = (A-B)x$$

Goal: Make D even under $\sqrt{x} \rightarrow -\sqrt{x}$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{\underbrace{[(A-B)x + B]}_u} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B} \left[-\frac{1}{u} \right]_B^A = \frac{1}{A-B} \left[\frac{1}{B} - \frac{1}{A} \right]$$

$$du = (A-B)dx$$

Goal: Make D even under $\gamma^A \rightarrow -\gamma^A$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{\underbrace{[(A-B)x + B]}_u} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B} \left[-\frac{1}{u} \right]_B^A = \frac{1}{A-B} \left[\frac{1}{B} - \frac{1}{A} \right]$$

$$du = (A-B)dx$$

$$\frac{1}{D} = \frac{1}{(r^2 + m^2 - i\epsilon)[(r-p)^2 + m^2 - i\epsilon]} = \int_0^1 dx \frac{1}{\underbrace{[(r^2 + m^2 - i\epsilon)x + ((r-p)^2 + m^2 - i\epsilon)(1-x)]^2}_{r^2 + 2r \cdot p(1-x) + p^2(1-x)}}$$

Goal: Make D even under $r \rightarrow -r$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{\underbrace{[(A-B)x + B]}_u} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B} \left[-\frac{1}{u} \right]_B^A = \frac{1}{A-B} \left[\frac{1}{B} - \frac{1}{A} \right]$$

$du = (A-B)dx$

$$\frac{1}{D} = \frac{1}{(r^2 + m^2 - \epsilon)^2} = \int_0^1 dx \frac{1}{\underbrace{[(r^2 + m^2 - \epsilon)x + (r - q)^2 + m^2 - \epsilon]}_{r^2 + 2r(q-A) + q^2 + m^2 - \epsilon}}^2$$

Goal: Make D even under $r \rightarrow -r$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{[(A-B)x + B]^2} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B} \left[-\frac{1}{u} \right]_B^A = \frac{1}{A-B} \left[\frac{1}{B} - \frac{1}{A} \right]$$

$du = (A-B) dx$

$$\frac{1}{D} = \int_0^1 dx \frac{1}{\left[\frac{(r^2 - m^2 - i\epsilon)(1-x) + (r-p)^2 + m^2 - i\epsilon}{r^2 + 2r \cdot p \cdot x + p^2 + m^2 - i\epsilon} \right]^2}$$

Goal: Make D even under $r \rightarrow -r$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{\underbrace{[(A-B)x + B]}_u^2} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B} \left[-\frac{1}{u} \right]_B^A = \frac{1}{A-B} \left[\frac{1}{B} - \frac{1}{A} \right]$$

$$du = (A-B)dx$$

$$\frac{1}{D} = \frac{1}{(r^2 + m^2 - i\epsilon)[(r-p)^2 + m^2 - i\epsilon]} = \int_0^1 dx \frac{1}{\underbrace{[(r^2 + m^2 - i\epsilon) + (r-p)^2 + m^2 - i\epsilon]}_{r^2 + 2r \cdot p + (p^2 + m^2 - i\epsilon)}}^2$$

$$r^2 - 2r \cdot q x = (r - x q)^2 - x^2 q^2$$

$$[] = (r - x q)^2 + q^2 x(1-x) + m^2 - i \epsilon$$

$$r - x q = \hat{r}$$

$$r^2 - 2r \cdot q x = (r - xq)^2 - x^2 q^2$$

$$L = (r - xq)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$$r - xq = \hat{r} \quad dqr = d\hat{r}$$

$$r^2 - 2r \cdot q x = (r - x q)^2 - x^2 q^2$$

$$[] = (r - x q)^2 + q^2 x(1-x) + m^2 - i \epsilon$$

$$r - x q = \hat{r} \quad d^4 r = d^4 \hat{r} \quad r = \hat{r} + x q \text{ in } N_{\text{pr.}}$$

$$r^2 - 2r \cdot q x = (r - x q)^2 - x^2 q^2$$

$$[] = (r - x q)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$$r - x q = \hat{r} \quad dq r = d\hat{r} \quad r = \hat{r} + x q \text{ in } N_{\mu}$$

$$\Pi_{\mu\nu} = -i e^2 \int \frac{d^4 \hat{r}}{(2\pi)^4} \int_0^1 dx \frac{1}{[\hat{r}^2 + q^2 x(1-x) + m^2 - i\epsilon]}$$

$$r^2 - 2r \cdot q x = (r - x q)^2 - x^2 q^2$$

$$[] = (r^2 - x q)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$$r - x q = \hat{r} \quad dq r = d\hat{r} \quad r = \hat{r} + x q \text{ in } N_{\text{pr}}$$

$$\Pi_{\mu\nu} = -i e^2 \int \frac{d^4 \hat{r}}{(2\pi)^4} \int_0^1 dx \frac{\text{Tr} [\gamma_\mu [-i(\hat{r} + x q) + m] \gamma_\nu [-i(\hat{r} + (1-x)q) + m]]}{[i^2 + q^2 x(1-x) + x^2 - i\epsilon]^2}$$

$$r^2 - 2r \cdot q x = (r - x q)^2 - x^2 q^2$$

$$[] = (r - x q)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$$r - x q = \hat{r} \quad dq r = d\hat{r} \quad r = \hat{r} + x q \text{ in } N_{\text{pr.}}$$

$$\Pi_{\mu\nu} = -i e^2 \int \frac{d^4 \hat{r}}{(2\pi)^4} \int_0^1 dx \frac{\text{Tr} [\gamma_\mu [-i(\hat{r} + x q) + m] \gamma_\nu [-i(\hat{r} + (1-x)q) + m]]}{[i^2 + q^2 x(1-x) + m^2 - i\epsilon]}$$

Can drop cross terms of the form $\int \frac{d^4 r}{D} r_\mu = 0$

$$r^2 - 2r \cdot q x = (r - x q)^2 - x^2 q^2$$

$$[] = (r - x q)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$$r - x q = \hat{r} \quad d^4 r = d^4 \hat{r} \quad r = \hat{r} + x q \text{ in } N_{\text{pr.}}$$

$$\Pi_{\mu\nu} = -i e^2 \int \frac{d^4 \hat{r}}{(2\pi)^4} \int_0^1 dx \frac{\text{Tr} [\gamma_\mu [-i(\hat{r} + x q) + m] \gamma_\nu [-i(\hat{r} + (1-x)q) + m]]}{[\hat{r}^2 + q^2 x(1-x) + m^2 - i\epsilon]}$$

Can drop cross terms of the form $\int \frac{d^4 \hat{r}}{D} \hat{r}^\mu q^\nu = 0$

$$r^2 - 2r \cdot qv = (r - xq)^2 - x^2 q^2$$

$$[] = (r^2 - xq)^2 + q^2 x(1-x) + m^2 - i\epsilon$$

$$r - xq = \hat{r}^1 \quad d^4r = d^4\hat{r} \quad r = \hat{r} + xq \text{ in } N_{\mu\nu}$$

$$\Pi_{\mu\nu} = -ie^2 \int \frac{d^4\hat{r}}{(2\pi)^4} \int_0^1 dx \frac{\text{Tr} \left[\gamma_\mu \left[-i(\hat{r}^2 + xq) + m \right] \gamma_\nu \left[-i(\hat{r}^2 + (1-x)q) + m \right] \right]}{\left[\hat{r}^2 + q^2 x(1-x) + m^2 - i\epsilon \right]}$$

Can drop cross terms of the form $\int \frac{d^4\hat{r}}{D} \hat{r}^\mu q^\nu = 0$
 Drop all terms in the trace.

$$\Pi_{\mu\nu} = -ie^2$$

Drop my front x 10 vol.

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 q}{(2\pi)^4} \frac{\text{Tr} [m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{x} \gamma_\mu \not{q} - \gamma_\nu \not{x} \not{q} \gamma_\mu]}{[q^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$



Drop my front x 10 1000

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\text{Tr} [m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{x} \gamma_\mu \not{x} - \gamma_\nu \not{x} \not{r} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)}$$

$$I_2 = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu$$



Drop my term in the denominator

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4r}{(2\pi)^4} \frac{\text{Tr}[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{x} \gamma_\mu \not{x} - \gamma_\nu \not{x} \not{r} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4r}{D(r)}$$

$$I_2(q) = \int \frac{d^4r}{D(r)} r_\mu r_\nu \propto \eta_{\mu\nu}$$



$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\text{Tr}[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{x} \gamma_\mu \not{r} - \gamma_\nu \not{x} \not{r} \gamma_\mu]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r)} \quad I_2 = \int \frac{d^4 r}{D(r)} r_\mu r_\nu = \frac{1}{4} g_{\mu\nu} \int \frac{d^4 r}{D(r)} r^2$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\text{Tr}[m^2 \gamma_\mu \gamma_\nu - \gamma_\nu \not{x} \gamma_\mu \not{r} - \gamma_\nu \not{x} \not{r} \gamma_\mu]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$\underline{I_1} = \int \frac{d^4 r}{D(r)} \quad \underline{I_A(H)} = \int \frac{d^4 r}{D(r)} \gamma_\mu \gamma_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4 r}{D(r)} r^2$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\text{Tr}[m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{x} \gamma_\nu \not{x} - \gamma_\nu \not{x} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$\underline{I_1} = \int \frac{d^4 r}{D(r)} \quad \underline{I_2(\eta)} = \int \frac{d^4 r}{D(r)} \gamma_\mu \gamma_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4 r}{D(r)} r^2$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4r}{(2\pi)^4} \frac{\text{Tr} [m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{x} \gamma_\nu \not{x} - \gamma_\nu \not{x} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4r}{D(r^2)} \quad I_2(\not{q}) = \int \frac{d^4r}{D(r^2)} \not{r} \quad \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4r}{D(r^2)} r^2$$

$$D = (r^2 + M^2 - i\epsilon)^2 \quad M^2 = m^2 + x(1-x)q^2$$

$$\frac{1}{D} = \frac{1}{[r^2 + \tilde{r}^2 + M^2 - i\epsilon]^2}$$

$$r^2 = \tilde{r}^2 + M^2 - i\epsilon$$

$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\text{Tr} [m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{x} \gamma_\nu \not{x} - \gamma_\nu \not{x} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \quad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$

$$D = (r^2 + M^2 - i\epsilon)^2 \quad M^2 = m^2 + x(1-x)q^2$$

$$\frac{1}{D} = \frac{1}{[r^2 + M^2 - i\epsilon]^2}$$

$$r_0^2 = \vec{r}^2 + M^2 - i\epsilon$$



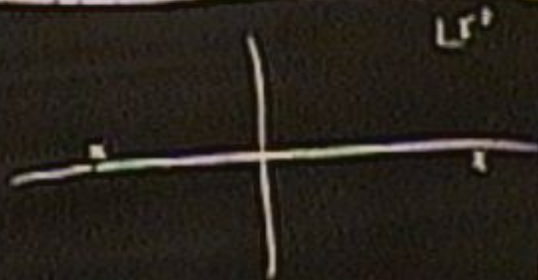
$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\text{Tr} [m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{x} \gamma_\nu \not{x} - \gamma_\nu \not{x} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \quad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} g_{\mu\nu} \int \frac{d^4 r}{D(r^2)}$$

$$D = (r^2 + M^2 - i\epsilon)^2 \quad M^2 = m^2 + x(1-x)q^2$$

$$\frac{1}{D} = \frac{1}{[-r_0^2 + \vec{r}^2 + M^2 - i\epsilon]^2}$$

$$r_0^2 = \vec{r}^2 + M^2 - i\epsilon$$

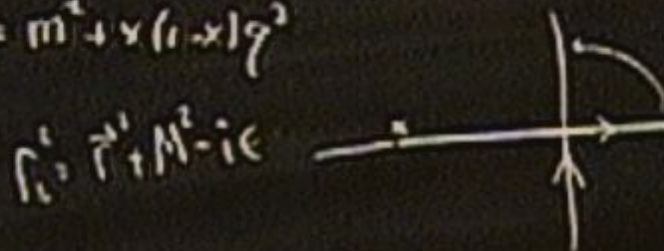


$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\text{Tr} [m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{x} \gamma_\nu \not{x} - \gamma_\nu \not{x} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \quad I_{\mu\nu}(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} g_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$

$$D = (r^2 + M^2 - i\epsilon)^2 \quad M^2 = m^2 + x(1-x)q^2$$

$$\frac{1}{D} = \frac{1}{[-r_0^2 + \vec{r}^2 + M^2 - i\epsilon]^2}$$

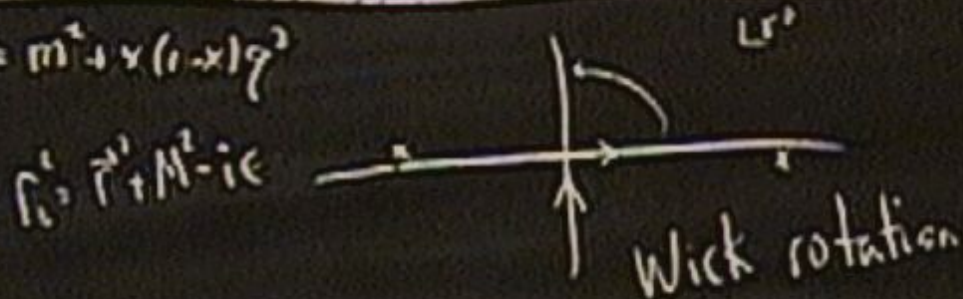


$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4 r}{(2\pi)^4} \frac{\text{Tr} [m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{x} \gamma_\nu \not{x} - \gamma_\nu \not{x} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4 r}{D(r^2)} \quad I_2(q) = \int \frac{d^4 r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4 r}{D(r^2)} r^2$$

$$D = (r^2 + M^2 - i\epsilon)^2 \quad M^2 = m^2 + x(1-x)q^2$$

$$\frac{1}{D} = \frac{1}{[-\tilde{r}^2 + \tilde{r}^2 + M^2 - i\epsilon]^2}$$



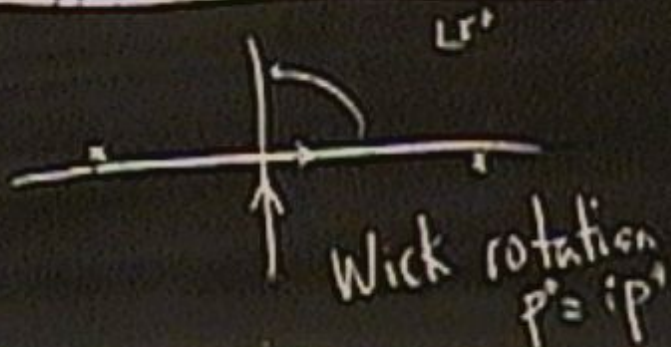
$$\Pi_{\mu\nu} = -ie^2 \int_0^1 dx \int \frac{d^4r}{(2\pi)^4} \frac{\text{Tr} [m^2 \gamma_\mu \gamma_\nu - \gamma_\mu \not{x} \gamma_\nu \not{x} - \gamma_\nu \not{x} \gamma_\mu \not{x}]}{[r^2 + x(1-x)q^2 + m^2 - i\epsilon]^2}$$

$$I_1 = \int \frac{d^4r}{D(r^2)} \quad I_2(\not{q}) = \int \frac{d^4r}{D(r^2)} r_\mu r_\nu = \frac{1}{4} \eta_{\mu\nu} \int \frac{d^4r}{D(r^2)} r^2$$

$$D = (r^2 + M^2 - i\epsilon)^2 \quad M^2 = m^2 + x(1-x)q^2$$

$$\frac{1}{D} = \frac{1}{[-r_0^2 + \vec{r}^2 + M^2 - i\epsilon]^2}$$

$$r_0^2 = \vec{r}^2 + M^2 - i\epsilon$$



$$I = \int \frac{d^4r}{D(r^2)} r^{2n} = \int dr^0 d^3r \frac{r^{2n}}{D(r^2)}$$



$$I = \int \frac{d^4 r}{D(r^2)} r^{2n} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \quad r^L = -p_0^2 + \vec{p}^2$$
$$= i \int d^4 r_E \frac{(r_E^{2n})}{D(r_E^2)} \quad r_E^2 = p_0^2 + \vec{p}^2$$

$$I = \int \frac{d^4 r}{D(r^2)} r^{2n} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \quad r^L = -p^0^2 + \vec{p}^2$$

$$= i \int d^4 r_E \frac{(r_E^2)^n}{D(r_E^2)} \quad r_E^2 = p^0^2 + \vec{p}^2$$

$$= i \int \frac{d^4 r_E (r_E^2)^n}{[r^2 + M^2]^2}$$

$$I = \int \frac{d^4 r}{D(r^2)} r^{2n} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \quad r^2 = -p_0^2 + \vec{p}^2$$

$$= i \int d^4 r_E \frac{(r_E^2)^n}{D(r_E^2)} \quad r_E^2 = +$$

$$= i \int \frac{d^4 r_E (r_E^2)^n}{[r_E^2 + M^2]^2} = i \int r^3 dr \frac{r^{2n}}{(r^2 + M^2)^2} \int d^4 \Omega$$

$$I = \int \frac{d^4 r}{D(r^2)} r^{2n} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \quad r^L = -p^0^2 + \vec{p}^2$$

$$= i \int d^4 r_E \frac{(r_E^2)^n}{D(r_E^2)} \quad r_E^2 = p^0^2 + \vec{p}^2$$

$$= i \int \frac{d^4 r_E (r_E^2)^n}{[r_E^2 + M^2]^2} = i \int r^3 dr \frac{r^{2n}}{(r^2 + M^2)^2} \int_{S_3} d^3 \Omega$$

Goal: Make D even under $r \rightarrow -r$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[A + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{\underbrace{[(A-B)x + B]}_u} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B} \left[-\frac{1}{u} \right]_B^A = \frac{1}{A-B} \left[\frac{1}{B} - \frac{1}{A} \right]$$

$du = (A-B)dx$

$$\frac{1}{D} = \frac{1}{(r^2 + m^2 - i\epsilon)(r-p)^2 + m^2 - i\epsilon} = \int_0^1 dx \frac{1}{\underbrace{(r^2 + m^2 - i\epsilon)x + [(r-p)^2 + m^2 - i\epsilon](1-x)}_{(r^2 + p^2 - 2rp + m^2 - i\epsilon)}}$$

$\int_0^1 dx$

Goal: Matrix D even under $\gamma^A \rightarrow -\gamma^A$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{[A \cdot x + B(1-x)]^2} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B} \left[-\frac{1}{u} \right]_B^A = \frac{1}{A-B} \left[\frac{1}{B} - \frac{1}{A} \right]$$

$du = (A-B)dx$

$$\frac{1}{D} = \frac{1}{(r^2 + m^2 - i\epsilon)(r-p)^2 + m^2 - i\epsilon} = \int_0^1 dx \frac{1}{\underbrace{(r^2 + m^2 - i\epsilon)A + (r-p)^2 + m^2 - i\epsilon} \cdot B}$$

$\int_0^1 dx e^{-i\epsilon x} = \left[\frac{e^{-i\epsilon x}}{-i\epsilon} \right]_0^1$

Goal: Matrix D even under $\gamma^A \rightarrow -\gamma^A$

Feynman trick: $\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + B(1-x)]^2}$

$$\int_0^1 dx \frac{1}{\underbrace{[Ax + B(1-x)]}_u} = \int_B^A \frac{du}{(A-B)} \frac{1}{u^2} = \frac{1}{A-B} \left[-\frac{1}{u} \right]_B^A = \frac{1}{A-B} \left[\frac{1}{B} - \frac{1}{A} \right]$$

$du = (A-B)dx$

$$\frac{1}{D} = \frac{1}{(r^2 + m^2 - i\epsilon)[(r-p)^2 + m^2 - i\epsilon]} = \int_0^1 dx \frac{1}{\underbrace{(r^2 + m^2 - i\epsilon) + [(r-p)^2 + m^2 - i\epsilon]x}_{(r+2r_1)^2 + m^2 - i\epsilon}}$$

$\int_0^1 dx e^{-i\epsilon x} = \left[\frac{e^{-i\epsilon x}}{-i\epsilon} \right]_0^1$

$$p = ip$$

$$I = \int \frac{d^4 r}{D(r^2)} r^{2n} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \quad r^2 = -p_0^2 + \vec{p}^2$$

$$= i \int d^4 r_E \frac{(r_E^2)^n}{D(r_E^2)} \quad r_E^2 = p_0^2 + \vec{p}^2$$

$$= i \int \frac{d^4 r_E (r_E^2)^n}{[r_E^2 + M^2]^2} = i \int r^3 dr \frac{r^{2n}}{(r^2 + M^2)^2} \underbrace{\int_{S_3} d\Omega}_{2\pi^2}$$

$$I = \int \frac{d^4 r}{D(r^2)} r^{2n} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \quad r^2 = -p_0^2 + \vec{p}^2$$

$$= i \int d^4 r_E \frac{(r_E^2)^n}{D(r_E^2)} \quad r_E^2 = p_0^2 + \vec{p}^2$$

$$= i \int \frac{d^4 r_E (r_E^2)^n}{[r_E^2 + M^2]^2} = i \int r^3 dr \frac{r^{2n}}{(r^2 + M^2)^2} \underbrace{\int_{S_3} d^3 \Omega}$$

Volume of an n -sphere:
$$\Omega_k = \frac{2 \pi^{(k+1)/2}}{\Gamma(\frac{k+1}{2})}$$

$\Gamma(z)$ is Euler's gamma fn.

$$\Gamma(n+1) = n!$$

Γ

Volume of an n -sphere: $\Omega_k = \frac{2\pi^{(k+1)/2}}{\Gamma(\frac{k+1}{2})}$

$\Gamma(z)$ is Euler's gamma fn.

$$\Gamma(n+1) = n!$$

$$\Gamma(z) = \int_0^{\infty} dt e^{-t} t^{z-1}$$

Volume of an n -sphere: $\Omega_k = \frac{2\pi^{(k+1)/2}}{\Gamma(\frac{k+1}{2})}$

$\Gamma(z)$ is Euler's gamma fn.

$$\Gamma(n+1) = n!$$

$$\Gamma(1) = 0! = 1$$

$$\Gamma(2) = 1! = 1$$

$$\Gamma(z) = \int_0^{\infty} dt e^{-t} t^{z-1}$$

circle: $k=1$ $\Omega_1 = \frac{2\pi}{\Gamma(1)} = 2\pi$

Volume of an k -sphere: $\Omega_k = \frac{2\pi^{(k+1)/2}}{\Gamma(\frac{k+1}{2})}$

$\Gamma(z)$ is Euler's gamma fn.

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(z+1) = z\Gamma(z)$$

$$\Gamma(1) = 1$$

$$1! = 1$$

$$\Gamma(z) = \int_0^{\infty} dt e^{-t} t^z$$

circle: $k=1$ $\Omega_1 = \frac{2\pi}{\Gamma(1)}$

plane: $k=2$ $\frac{2\pi^{3/2}}{\Gamma(3/2)}$

Volume of an n -sphere: $\Omega_k = \frac{2\pi^{(k+1)/2}}{\Gamma(\frac{k+1}{2})}$

$\Gamma(z)$ is Euler's gamma fn.
 $\Gamma(\frac{1}{2}) = \sqrt{\pi}$
 $\Gamma(n+1) = n!$
 $\Gamma(1) = 0! = 1$
 $\Gamma(2) = 1! = 1$
 $\Gamma(z+1) = z\Gamma(z)$

$$\Gamma(z) = \int_0^{\infty} dt e^{-t} t^{z-1}$$

circle: $k=1$ $\Omega_1 = \frac{2\pi}{\Gamma(1)} = 2\pi$

2-sphere: $k=2$: $\frac{2\pi^{3/2}}{\Gamma(3/2)} = \frac{2\pi^{3/2}}{\frac{1}{2}\sqrt{\pi}} = 4\pi$

3-sphere: $k=3$: $\frac{2\pi^2}{\Gamma(2)} = 2\pi^2$

$$I = \int \frac{d^4 r}{D(r^2)} r^{2n} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \quad r^2 = -p_0^2 + \vec{p}^2$$

$$= i \int d^4 r_E \frac{(r_E^2)^n}{D(r_E^2)} \quad r_E^2 = p_0^2 + \vec{p}^2$$

$$= i \int \frac{d^4 r_E (r_E^2)^n}{[r_E^2 + M^2]^2} = i \int_0^\infty r^3 dr \frac{r^{2n}}{(r^2 + M^2)^2} \int_{S_3} d^3 \Omega$$

ultraviolet divergent
 $2\pi^2$

$p = ip$

$$I = \int \frac{d^4 r}{D(r^2)} r^{2n} = \int dr^0 d^3 r \frac{r^{2n}}{D(r^2)} \quad r^2 = -p_0^2 + \vec{p}^2$$

$$= i \int d^4 r_e \frac{(r_e^2)^n}{D(r_e^2)} \quad r_e^2 = p_0^2 + \vec{p}^2$$

$$= i \int \frac{d^4 r_e (r_e^2)^n}{[r_e^2 + M^2]^2} = i \int_0^\infty r_e^3 dr_e \frac{r_e^{2n}}{(r_e^2 + M^2)^2} \int_{S_3} d\Omega$$

ultraviolet divergent
 $2\pi^2$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1$$

$$\times \int_{-\infty}^{\infty} \delta(x) \left(\frac{1}{1+im^2} \right) \delta(x) \left(\frac{-1+im^2}{1+im^2} \right) dx$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1$$

If you cut this integral off : things go bad ...



$$x \int_{-\infty}^{\infty} \left[\delta^{\mu} \left(\frac{1}{r^2} \right) \delta^{\nu} \left(\frac{-1}{r^2} \right) \right] \dots$$

If you cut this integral off: things go bad...

$$1) \int_0^{\infty} dr^2 (\dots) = ?$$

$$2 \text{Tr}[\gamma^0 (\not{p} + \not{k} - \not{p}' - \not{k}') \not{u} \not{u}] \frac{1}{(p-p)_{\mu}^2}$$

$\langle \psi | \gamma^0 (\not{p} + \not{k} - \not{p}' - \not{k}') | \psi \rangle = \frac{1}{4} \text{Tr}[\gamma^0 (\not{p} + \not{k} - \not{p}' - \not{k}') \not{u} \not{u}]$



$$\int d^4x d^4y d^4z d^4w \frac{1}{(2\pi)^4} [i(2\pi)^4]^4$$

$$\delta^4(p-q-l) \delta^4(p-s-r) \delta^4(q-s-r) \delta^4(y-k'-k)$$

$$\frac{1}{k^2 - i\epsilon} \frac{1}{q^2 - i\epsilon} \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r} + m}{r^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{s} + m}{s^2 - i\epsilon} \right) \right]$$

$$= -e^4 [\bar{u}(p) \not{u} \gamma^\mu u] \delta^4(k-p'-k-p) \int d^4r \frac{1}{r^2 - i\epsilon} \frac{1}{(k-p)^2 - i\epsilon} \times \text{Tr} \left[\gamma^\mu \left(\frac{-i\not{r} + m}{r^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{s} + m}{s^2 - i\epsilon} \right) \right]$$



$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\not{V} + m}{r^2 m^2 - i\epsilon} \right) \gamma^\alpha \left(\frac{-i\not{V} + m}{(r^2 m^2 - i\epsilon)} \right) \right] \frac{1}{(p' - p)^2 - i\epsilon}$$

If you cut this integral off: things go bad...

$$1) \int_0^{r^2} dr^2 (\dots) = ?$$

Answer depends on which
integral limit you
choose to keep as the
integration variable.

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\not{V} + m}{r^2 m^2 - i\epsilon} \right) \gamma^\alpha \left(\frac{-i\not{V} + m}{r^2 m^2 - i\epsilon} \right) \right] \frac{1}{(p-p')^2 - i\epsilon} \frac{1}{(p-p')^2 - i\epsilon}$$

If you cut this integral off: things go bad ...

1) $\int_0^{\Lambda} dr^2 (\dots) = ?$

Answer depends on which
integral line you
choose to keep as the
integration variable.

2) cut things off by modifying α :

eg $\alpha = -i(\not{\alpha})^2 - m^2 + \epsilon$

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\not{V} + m}{r^2 m^2 - i\epsilon} \right) \gamma^\alpha \left(\frac{-i\not{V} + m}{r^2 m^2 - i\epsilon} \right) \right] \frac{1}{(p' - p)^2 - i\epsilon}$$

If you cut this integral off: things go bad ...

1) $\int_0^{\Lambda^2} dr^2 (\dots) = ?$

Answer depends on which
interval line you
choose to keep as the
integration variable.

2) cut things off by modifying α :

eg $\alpha = i((\not{x})^2 - m^2)^2 - \epsilon(\not{x}, \not{x}) = \not{x}(\dots)\not{x} = \not{x}(p^2 + m^2)\not{x}$

$\rightarrow \rightarrow \frac{1}{(\dots)} = \frac{1}{p^2 + m^2}$

$$\times \text{Tr} \left[\gamma^{\beta} \left(\frac{-i\nu + m}{r^2 + m^2 - i\epsilon} \right) \gamma^{\nu} \left(\frac{-i\nu + m}{(r-p)^2 - i\epsilon} \right) \right] \frac{10^p}{(p-p)^2 - i\epsilon}$$

If you cut this integral off: things go bad...

1) $\int_0^{\Lambda^2} dr^2 (\dots) = ?$

Answer depends on which
integral line you
choose to keep as the
integration variable.

2) cut things off by modifying α :

eg $\alpha = -i(\nu^2)^2 - m^2 + i\epsilon = -\frac{1}{2}(\nu^2 + m^2) = \phi(\dots)\phi = \phi(p^2 + m^2)\phi$

$\rightarrow \frac{1}{(\dots)} = \frac{1}{p^2 + m^2 + i\epsilon} = \frac{1}{p^2 - A_1} + \frac{1}{p^2 - A_2}$ no. of ϵ_1, ϵ_2 negative.

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\not{V} + m}{r^2 + m^2 - i\epsilon} \right) \gamma^\alpha \left(\frac{-i\not{V} + m}{r^2 + m^2 - i\epsilon} \right) \right] \frac{1}{(p' - p)^2 - i\epsilon} \frac{1}{(p' - p)^2 - i\epsilon}$$

If you cut this integral off: things go bad ...

1) $\int_0^{r^2} dr^2 (\dots) = ?$

Answer depends on which
interval line you
choose to keep as the
integration variable.

2)

ff by modifying ϵ :

$\langle \text{OT}(\dots) \rangle$

$$\frac{1}{(\dots)} = \frac{1}{r^2 + m^2 - i\epsilon} = \frac{1}{r^2 - A_1} + \frac{1}{r^2 - A_2} \quad \text{no. of } \epsilon_1, \epsilon_2 \text{ is negative.}$$

$$\psi(\dots) \psi = \psi(p^2 + m^2) \psi$$

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\not{p} + m}{p^2 + m^2 - i\epsilon} \right) \gamma^\alpha \left(\frac{-i\not{p}' + m}{(p')^2 + m^2 - i\epsilon} \right) \right] \frac{10^p}{(p-p')^2 - i\epsilon}$$

If you cut this integral off: things go bad ...

1) $\int_0^{\infty} dr^2 (\dots) = ?$

Answer depends on which
integral line you
choose to keep as the
integration variable.

2) cut things off by modifying α :

\downarrow
 $\text{Tr}(\dots)$

eg $\alpha = i(\not{a}\not{x})^2 - m^2 + i\epsilon = \frac{1}{2}(\not{a}\not{x} + \not{x}\not{a})^2 = \not{x}(\dots)\not{x} = \not{x}(p^2 + m^2)\not{x}$

$\rightarrow \frac{1}{(\dots)} = \frac{1}{p^2 + m^2 + i\epsilon} = \frac{1}{p^2 - A_1} + \frac{1}{p^2 - A_2}$ no. of ϵ_1, ϵ_2 is negative.

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\nu + m}{r^2 + m^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\nu + m}{r^2 + m^2 - i\epsilon} \right) \right] \frac{1}{(p-p')^2 - i\epsilon}$$

If you cut this integral off: things go bad...

1) $\int_0^{\infty} dr^2 (\dots) = ?$

Answer depends on which interval line you choose to keep as the integration variable.

$\not\partial - iA_4$

2) cut things off by modifying $\not\partial$:

$\langle \text{Dir}(\not\partial) | 0 \rangle$

eg $\not\partial = i(\not{x})^2 - m^2 \not{x}^2 = \not{x}(\dots)\not{x} = \not{x}(p^2 + m^2)\not{x}$

$\rightarrow \frac{1}{(\dots)} = \frac{1}{p^2 + m^2 + i\epsilon} = \frac{1}{p^2 - \mu_1^2} + \frac{1}{p^2 - \mu_2^2}$ no. of μ_i is negative.

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\gamma + m}{r^2 + m^2 - i\epsilon} \right) \gamma^\alpha \left(\frac{-i\gamma + m}{r^2 + m^2 - i\epsilon} \right) \right] \frac{1}{(p' - p)^2 - i\epsilon}$$

If you cut this integral off: things go bad...

1) $\int_0^{r^2} dr^2 (\dots) = ?$

Answer depends on which interval here you choose to keep as the integration variable.

$\partial\psi = iA_2\psi$

2) cut things off by modifying α :

$\langle \text{OT}(\psi, \psi) |_0 \rangle$

eg $\alpha_0 = i((\partial x)^2 - m^2)\psi^2 - \frac{1}{2}(\partial_0 \psi)^2 = \psi(\dots)\psi = \psi(p^2 + m^2)\psi$

$\rightarrow \frac{1}{(\dots)} = \frac{1}{p^2 + m^2 + i\epsilon} = \frac{1}{p^2 - A_1} + \frac{1}{p^2 - A_2}$ no. of ϵ_1, ϵ_2 is negative.

Dimensional Regularization:

$$\int_0^\infty d^4 r_e \frac{(r_e^2)^A}{(r_e^2 + M^2)^B} =$$

$$\frac{d^4 r_e (r_e^2)^A}{(r_e^2 + M^2)^B}$$

Dimensional Regularization:

$$\int_0^\infty d^n r \frac{(r^2)^A}{(r^2 + M^2)^B} = \int_0^\infty d^n r \frac{(r^2)^A}{(r^2 + M^2)^B}$$
$$= \Omega_{n-1} \int_0^\infty dr \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

Dimensional Regularization:

$$\int_{\mathbb{R}^n} d^n r \frac{(r^2)^A}{(r^2 + M^2)^B} = \int_0^\infty d^n r \frac{(r^2)^A}{(r^2 + M^2)^B} = \Omega_{n-1} \int_0^\infty dr \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

Regard this as an analytic fn of n where n is $4 - 2\epsilon$

Dimensional Regularization:

the integral
for any fixed A, B
converges if n is
sufficiently negative.

Define $I(n)$

$$I = \int_0^\infty d^4 r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B} = \int_0^\infty d^n r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$
$$= \Omega_{n-1} \int_0^\infty dr \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

Regard this as an analytic fn of n where n
is complex

Dimensional Regularization:

the integral
for any fixed A, B
converges if n is
sufficiently negative.

$$I = \int_{\mathbb{R}^n} d^n r \frac{(r^2)^A}{(r^2 + M^2)^B} = \int_0^\infty d^n r \frac{(r^2)^A}{(r^2 + M^2)^B}$$

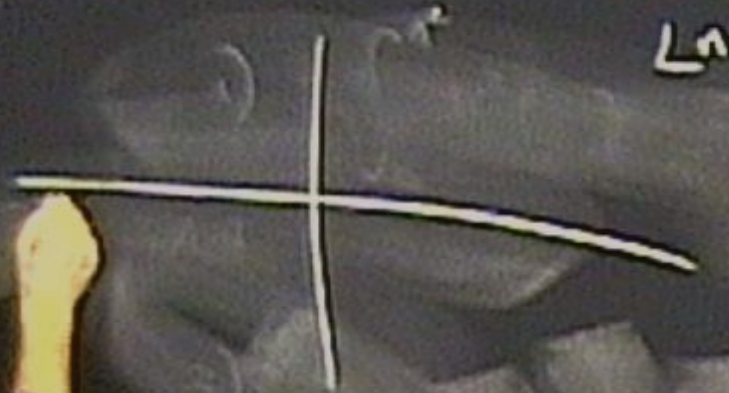
$$= \Omega_{n-1} \int_0^\infty dr \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

Define $I(n)$ by analytically
continuing the first result at
 $\lim_{n \rightarrow -\infty}$

Regard this as an analytic fn of n where n
is complex

$$\times \text{Tr} \left[\gamma^{\mu} \left(\frac{-i\nu + m}{r^2 + m^2 - i\epsilon} \right) \gamma^{\nu} \left(\frac{-i\nu + m}{r^2 + m^2 - i\epsilon} \right) \right] \frac{1}{(p-p')^2 - i\epsilon} \frac{1}{(p-p')^2 - i\epsilon}$$

If you cut this integral off: things go bad ...



in which
integral here you
can't keep the
integration variable.

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + m^2} dx$$

$$\text{eg } \int_{-\infty}^{\infty} \frac{1}{x^2 + m^2} dx = \int_{-\infty}^{\infty} \frac{1}{(x - im)(x + im)} dx = \int_{-\infty}^{\infty} \frac{1}{x^2 + m^2} dx$$

$$\rightarrow \frac{1}{x^2 + m^2} = \frac{1}{(x - im)(x + im)} = \frac{A}{x - im} + \frac{B}{x + im}$$

no. of poles is negative.

$$\times \text{Tr} \left[\gamma^{\mu} \left(\frac{-i\not{p} + m}{p^2 - m^2 - i\epsilon} \right) \gamma^{\nu} \left(\frac{-i\not{p} + m}{p^2 - m^2 - i\epsilon} \right) \right]$$

If you cut this integral off: things go bad ...



internal loop you
 can't map the
 integration variable.

eg $\frac{1}{p^2 - m^2 - i\epsilon} = \frac{1}{(p^2 - m^2) - i\epsilon} = \frac{1}{(p^2 - m^2) - i\epsilon} = \frac{1}{(p^2 - m^2) - i\epsilon}$

$$\rightarrow \frac{1}{(p^2 - m^2) - i\epsilon} = \frac{1}{p^2 - m^2 - i\epsilon} = \frac{1}{p^2 - m^2} + \frac{1}{p^2 - m^2}$$

no. of ϵ 's is negative.

$$\times \text{Tr} \left[\gamma^{\mu} \left(\frac{-i\not{p} + m}{p^2 - m^2 - i\epsilon} \right) \gamma^{\nu} \left(\frac{-i\not{p}' + m}{(p')^2 - m^2 - i\epsilon} \right) \right]$$

If you cut this integral off: things go bad ...



... which
integral here you
cut to remove the
integration variable.

\downarrow
 $\langle \text{dir}(\not{x}) \rangle_0$

$$\frac{1}{p^2 - m^2 - i\epsilon} = \frac{1}{p^2 - m^2} + \frac{C_1}{p^2 - m^2 - i\epsilon} + \frac{C_2}{p^2 - m^2 + i\epsilon}$$

no. of C_1, C_2 is negative.

$$\times \text{Tr} \left[\gamma^{\mu} \left(\frac{-i\not{v} + m}{r^2 + m^2 - i\epsilon} \right) \gamma^{\nu} \left(\frac{-i\not{v} + m}{r^2 + m^2 - i\epsilon} \right) \right] \frac{1}{(k-p)^2 - i\epsilon}$$

If you cut this integral off: things go bad ...



internal line you
 close to complete the
 integration variable.

of:

$$\text{eg } \frac{1}{x^2 - m^2 + i\epsilon} = \frac{1}{(x - m + i\epsilon/2)(x + m - i\epsilon/2)} = \frac{1}{2m} \left(\frac{1}{x - m + i\epsilon/2} - \frac{1}{x + m - i\epsilon/2} \right)$$

$$\rightarrow \frac{1}{(\dots)} = \frac{1}{r^2 - A} = \frac{1}{r^2 - A_1} + \frac{1}{r^2 - A_2}$$

no. of ϵ_1, ϵ_2 is negative.

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\nu + m}{r^2 m^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\nu + m}{r^2 m^2 - i\epsilon} \right) \right] \frac{1}{(p' - p)^2 - i\epsilon} \frac{1}{(p' - p)^2 - i\epsilon}$$

If you cut this integral off: things go bad ...



... which internal line you choose to integrate the integration variable.

\downarrow
 $\langle \text{OT}(\dots) \rangle_0$

eg $\mathcal{D}_0 = -i(\partial x)^2 - m^2 + i\epsilon = \frac{1}{2}(\partial_0 x)^2 = \phi(\dots)\phi = \phi(p^2 + m^2)\phi$

$\rightarrow \frac{1}{(\dots)} = \frac{1}{p^2 + m^2 + i\epsilon} = \frac{1}{p^2 - A_1} + \frac{1}{p^2 - A_2}$ no. of f_1, f_2 is negative.

Dimensional Regularization:

the integral
for any fixed A, B
converges if n is
sufficiently negative.

$$I = \int_{D^4} d^4 r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B} = \int_{D^n} d^n r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

$$= \Omega_{n-1} \int_0^\infty dr \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

Define $I(n)$ by analytically
continuing the finite result at
 $\lim_{n \rightarrow -\infty}$

Regard this as an analytic fn of n where n
is complex

Dimensional Regularization:

the integral
for any fixed A, B
converges if n is
sufficiently negative.

$$I = \int_{D^4} d^4 r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B} = \int_{D^n} d^n r_E \frac{(r_E^2)^A}{(r_E^2 + M^2)^B}$$

$$= \Omega_{n-1} \int_0^\infty dr \frac{r^{n-1+2A}}{(r^2 + M^2)^B}$$

Define $I(n)$ by analytically
continuing the finite result at
 $\lim_{n \rightarrow -\infty}$

Regard this as an analytic fn of n where n
is complex

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \Pi_{\mu\nu}(q) \right\}$$

where $\Pi_{\mu\nu}(q) = -ie^2 \int \frac{d^4r}{(2\pi)^4} \text{Tr} \left\{ \gamma_\mu \left[\frac{-i(\not{V} + m)}{r^2 + m^2 - i\epsilon} \right] \gamma_\nu \left[\frac{-i(\not{r} - \not{q} + m)}{(r-q)^2 + m^2 - i\epsilon} \right] \right\}$

vacuum polarization $= -ie^2 \int \frac{d^4r}{(2\pi)^4} \frac{N_{\mu\nu}}{D}$

$$D = (r-q)^2 + m^2 - i\epsilon \quad N_{\mu\nu} = \text{Tr} \left\{ \gamma_\mu (-i\not{r} + m) \gamma_\nu [-i(\not{r}-\not{q}) + m] \right\}$$

$$= 2\pi \quad \text{3-sphere } k=3: \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$$

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \frac{\Pi_{\mu\nu}(p'-p)}{(p'-p)^2} \right\}$$

where $\Pi_{\mu\nu}(q) = -ie^2 \int \frac{d^4r}{(2\pi)^4} \text{Tr} \left\{ \gamma_\mu \left[\frac{-i(\not{r} + m)}{r^2 + m^2 - i\epsilon} \right] \gamma_\nu \left[\frac{-i(\not{r}-q) + m}{(r-q)^2 + m^2 - i\epsilon} \right] \right\}$

vacuum polarization $= -ie^2 \int \frac{d^4r}{(2\pi)^4} \frac{N_{\mu\nu}}{D}$

$$= (r^2 + m^2 - i\epsilon)((r-q)^2 + m^2 - i\epsilon) \quad N_{\mu\nu} = \text{Tr} \left\{ \gamma_\mu (-i\not{r} + m) \gamma_\nu [-i(\not{r}-q) + m] \right\}$$

$$\frac{2\pi}{\Gamma(1)} = 2\pi \quad \text{3-sphere } k=3: \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$$

$$= -ie^2 (2\pi)^4 \delta^4(k+p-k-p) [\bar{u}\gamma^\mu u] [\bar{u}\gamma^\nu u] \frac{1}{(p'-p)^2 - i\epsilon}$$

$$\times \left\{ \eta_{\mu\nu} + \frac{\Pi_{\mu\nu}(q)}{(p'-p)^2 - i\epsilon} \right\}$$

where $\Pi_{\mu\nu}(q) = -ie^2 \int \frac{d^4r}{(2\pi)^4} \text{Tr} \left\{ \gamma_\mu \left[\frac{-i(\not{r} + m)}{r^2 + m^2 - i\epsilon} \right] \gamma_\nu \left[\frac{-i(\not{r} - q + m)}{(r-q)^2 + m^2 - i\epsilon} \right] \right\}$

vacuum polarization $= -ie^2 \int \frac{d^4r}{(2\pi)^4} \frac{N_{\mu\nu}}{D}$

$$D = (r^2 + m^2 - i\epsilon)((r-q)^2 + m^2 - i\epsilon) \quad N_{\mu\nu} = \text{Tr} \left\{ \gamma_\mu (-i\not{r} + m) \gamma_\nu (-i\not{r} - q + m) \right\}$$

circle: $k=1$ $\Omega_1 = \frac{2\pi}{\Gamma(1)} = 2\pi$

3-sphere $k=3$: $\frac{2\pi^2}{\Gamma(2)} = 2\pi^2$

$$\langle e(p) \mu(k) | S | e(p) \mu(k) \rangle = 2 \frac{1}{2!} \frac{1}{2!} (i) [\bar{u}(k) \gamma^\mu u(k)] [\bar{u}(p) \gamma^\nu u(p)]$$



$$\times \frac{1}{(p-k)^2 - i\epsilon}$$

$$(2\pi)^4 \delta^4(p-k+k')$$

$$= \frac{i e^2}{2!} (2\pi)^4 \delta^4(p+k-p'-k') [\bar{u} \gamma^\mu u] [\bar{u} \gamma^\nu u] \frac{1}{(p-p')^2 - i\epsilon}$$

$$\times \text{Tr} \left[\gamma^\beta \left(\frac{-i\not{p} + m}{p^2 - i\epsilon} \right) \gamma^\nu \left(\frac{-i\not{p}' + m}{p'^2 - i\epsilon} \right) \right]$$

If you cut this integral off: things go bad...



depends on which interval you choose to keep the