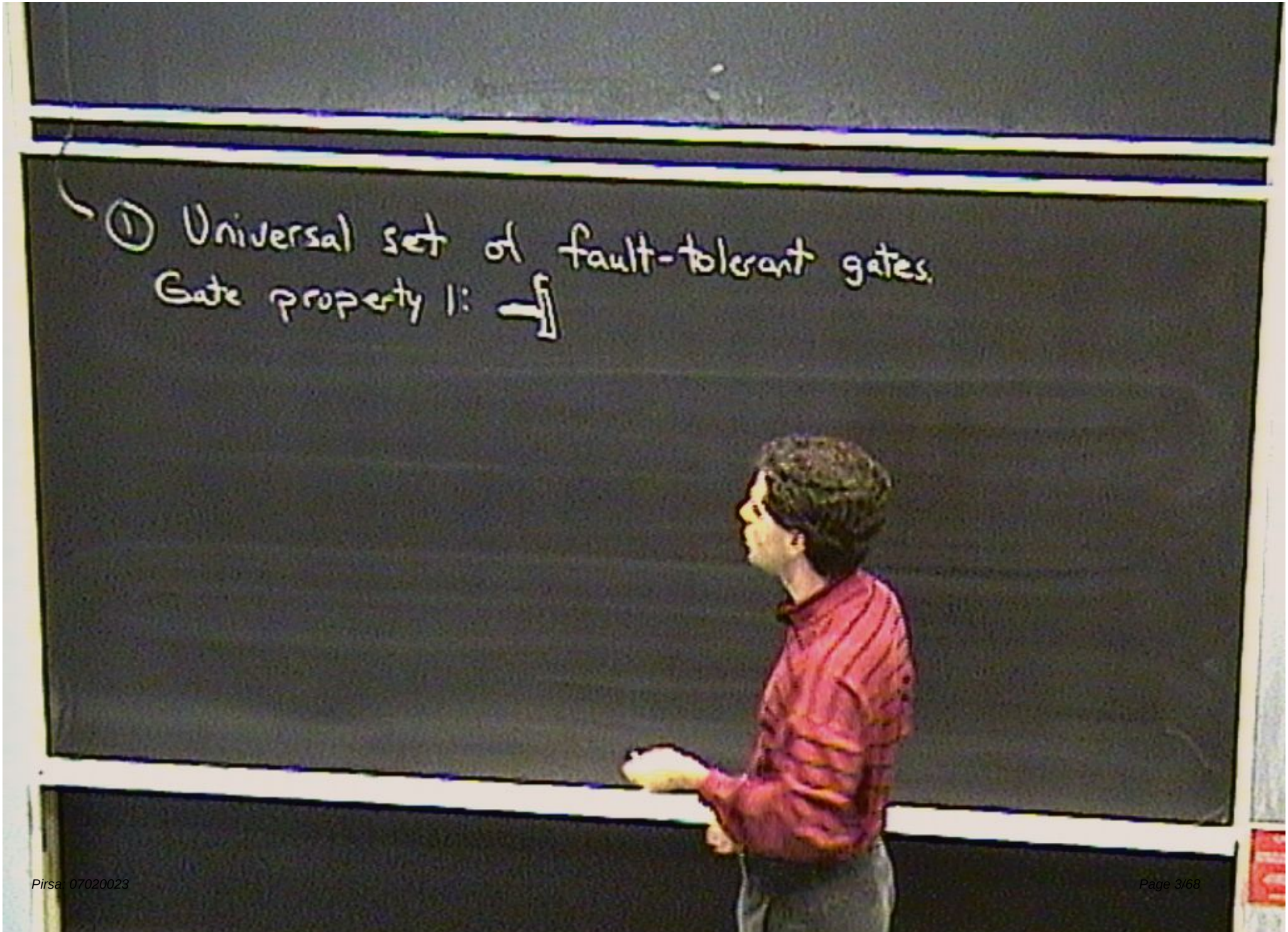
Title: Quantum Error Correction 7A

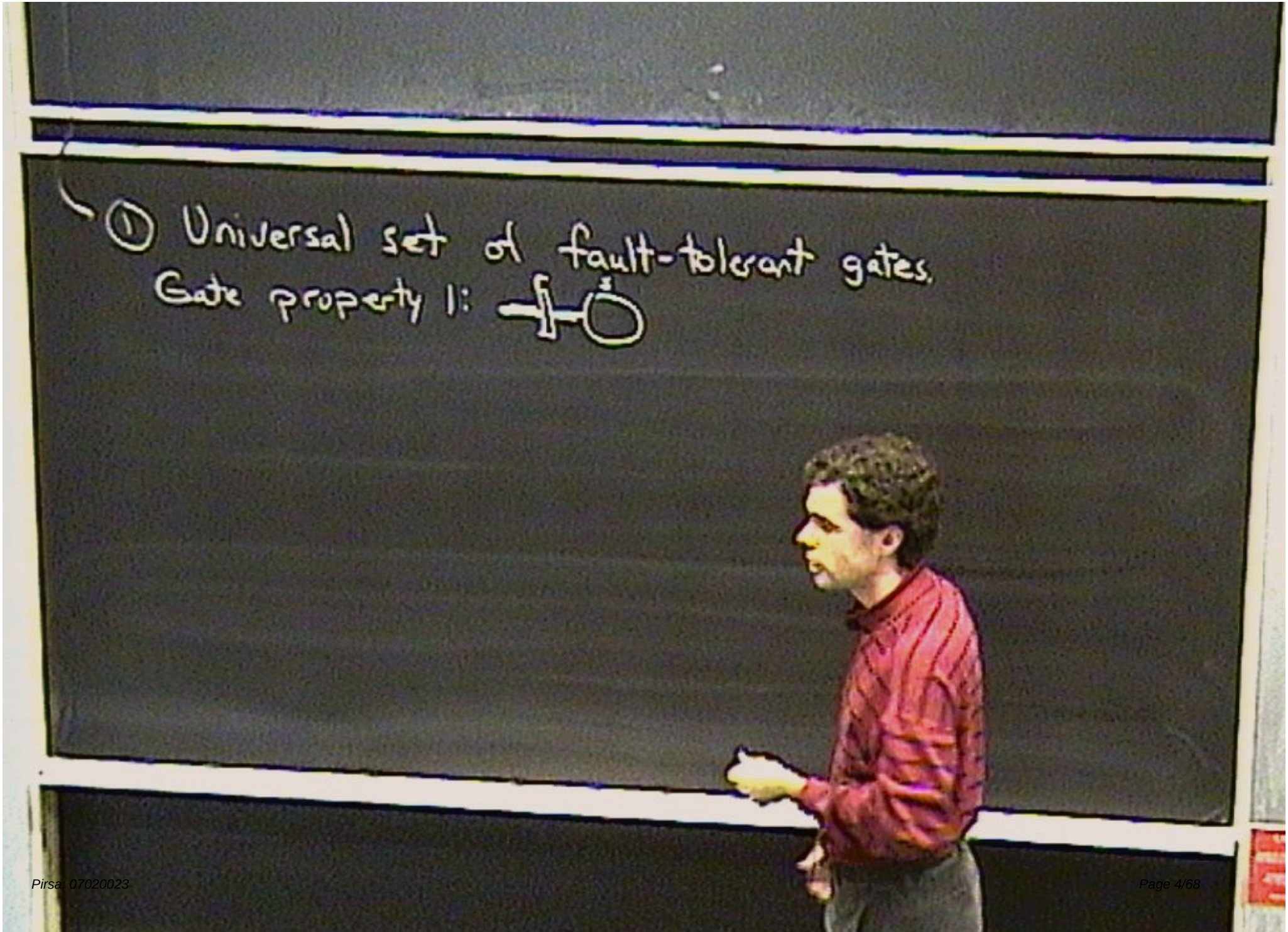
Date: Feb 20, 2007 03:30 PM

URL: http://pirsa.org/07020023

Abstract: Definition of fault tolerance, Shor error correction, fault-tolerant measurement for stabilizer and CSS codes

① Universal set of fault-tolerant gates.


① Universal set of fault-tolerant gates.
Gate property 1:


① Universal set of fault-tolerant gates.
Gate property 1:

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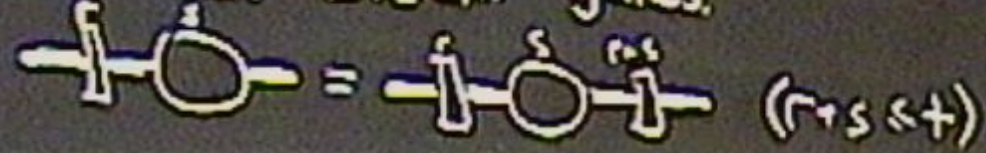
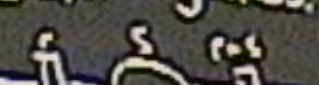
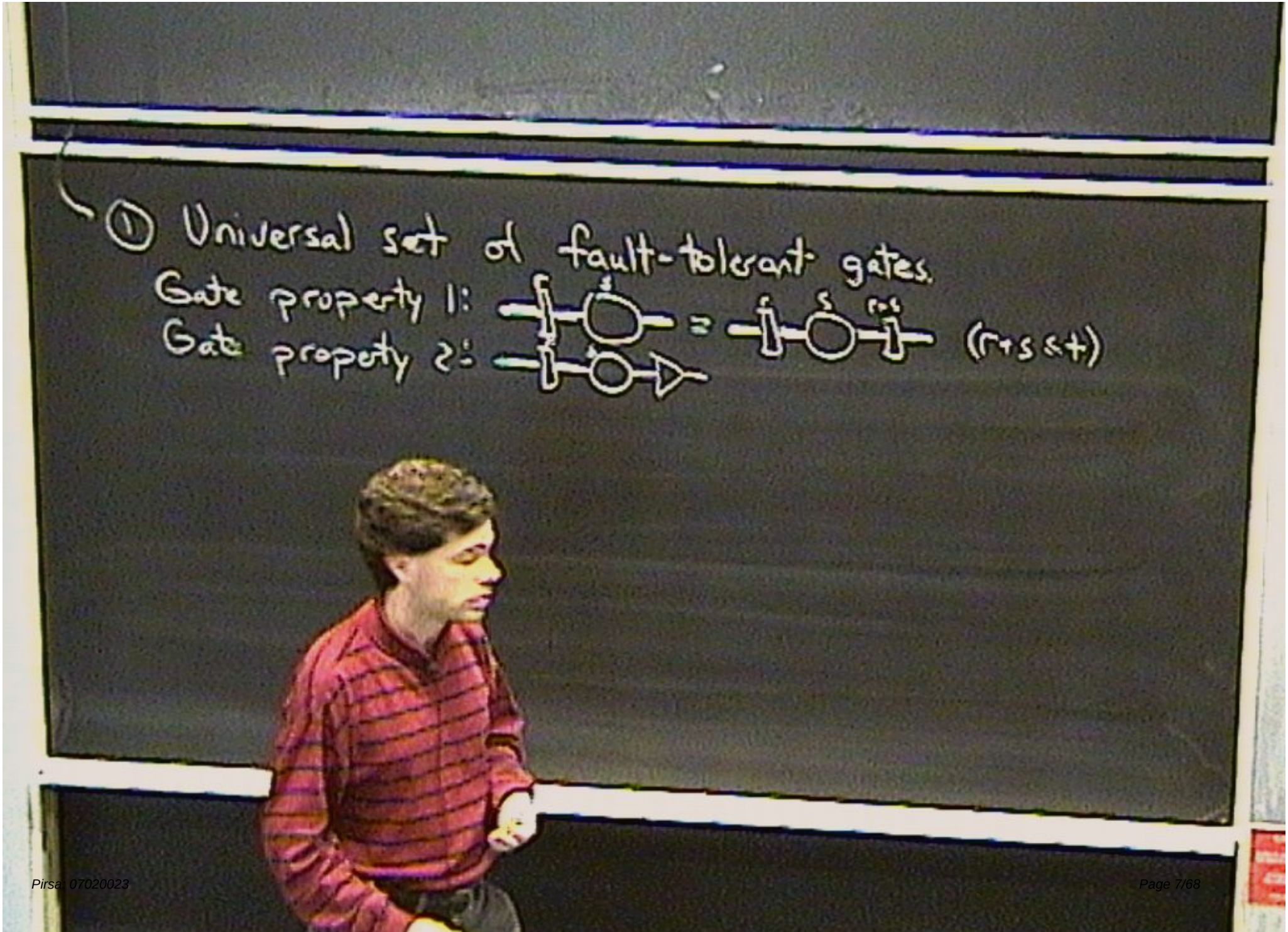
Gate property 1:  $(r+s<t)$

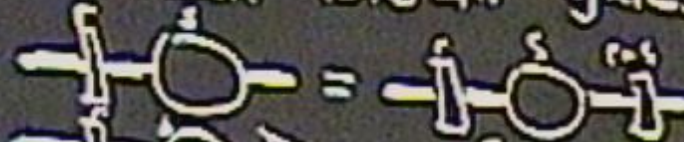
① Universal set of fault-tolerant gates.

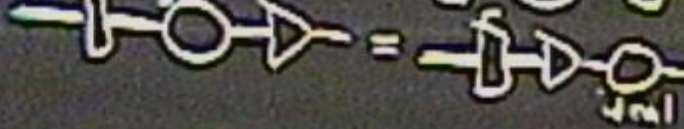
Gate property 1:

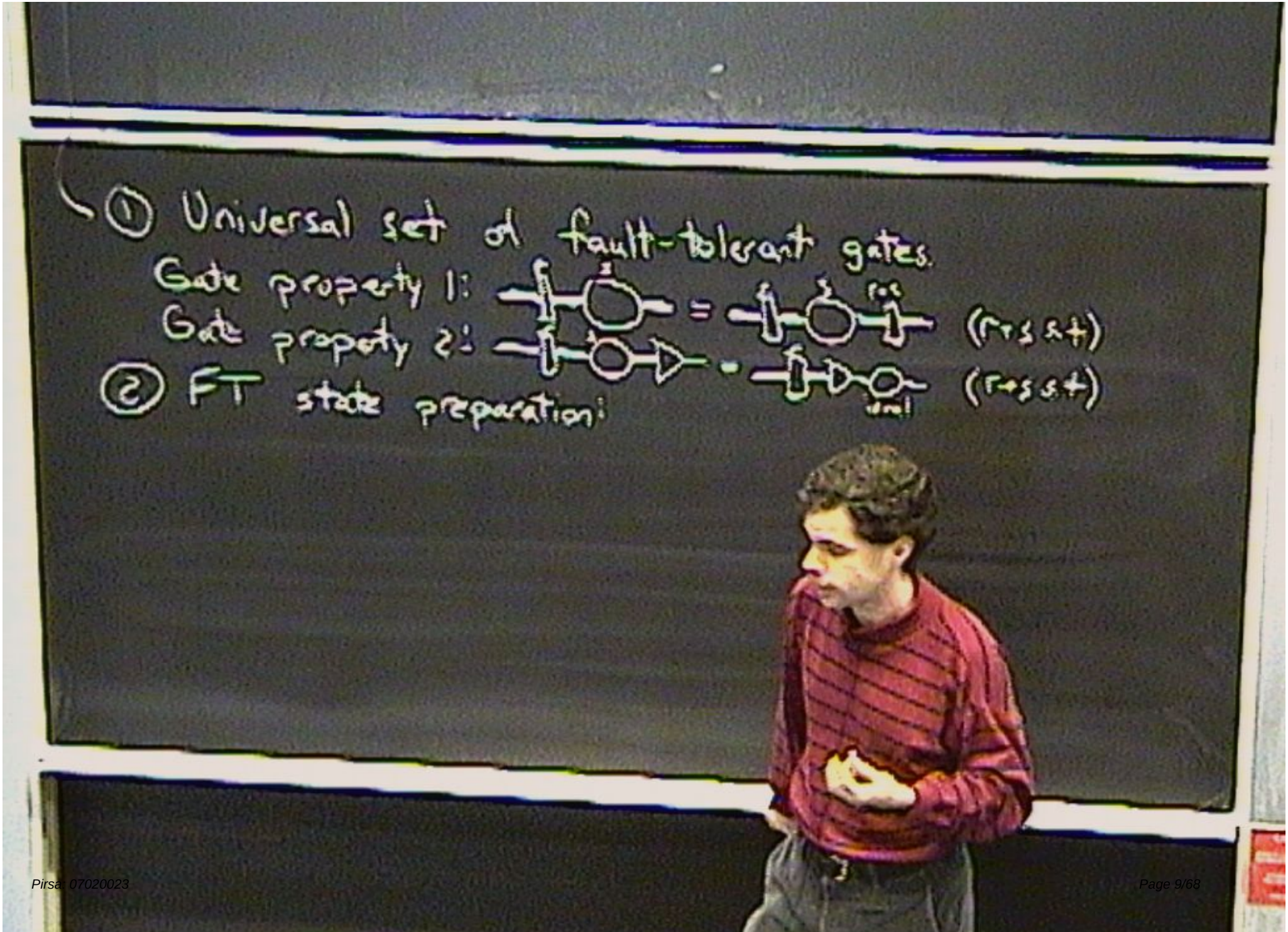
Gate property 2:  $(r+s<t)$

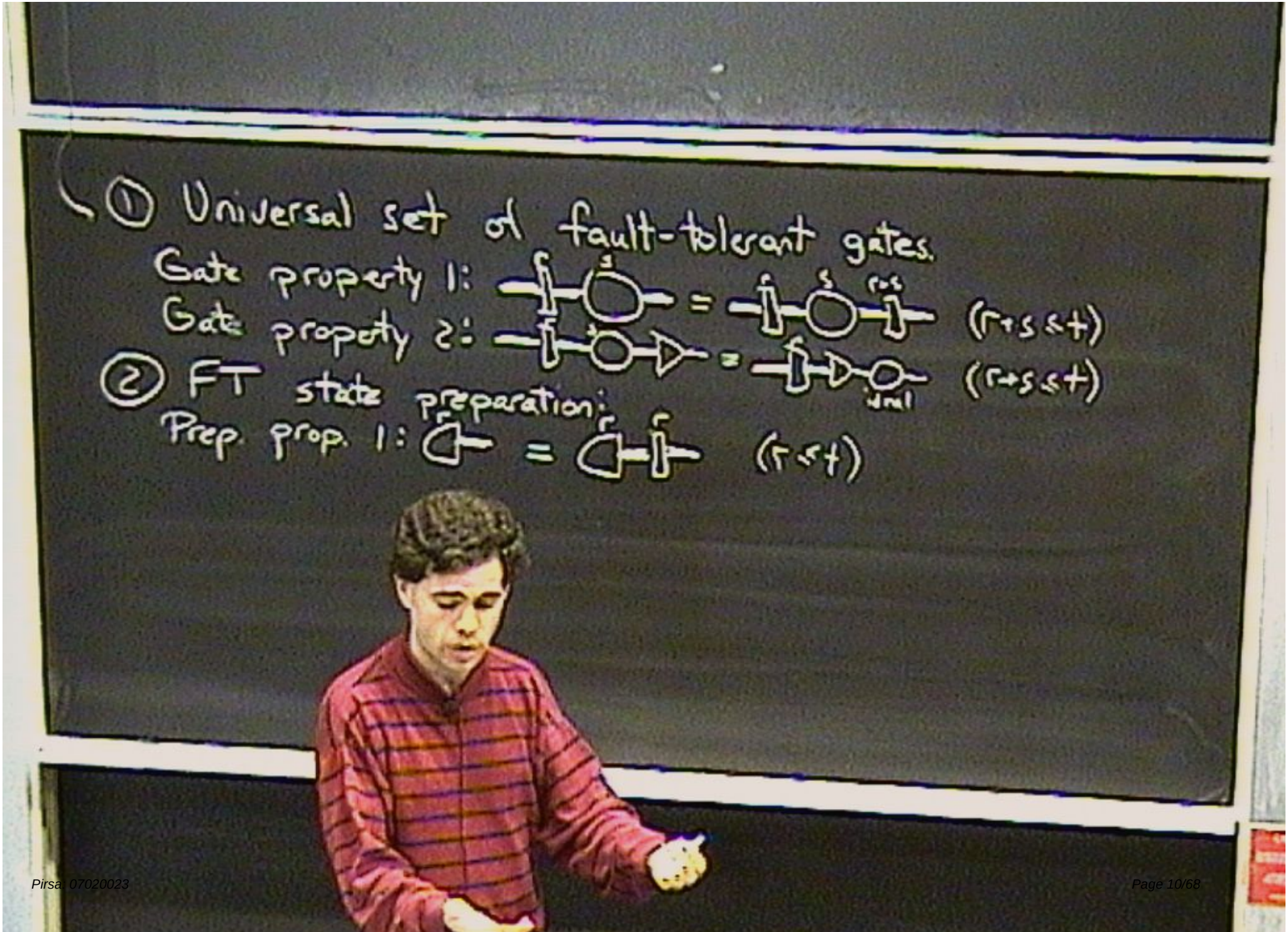

① Universal set of fault-tolerant gates.
Gate property 1:
Gate property 2:
(r+s<t)

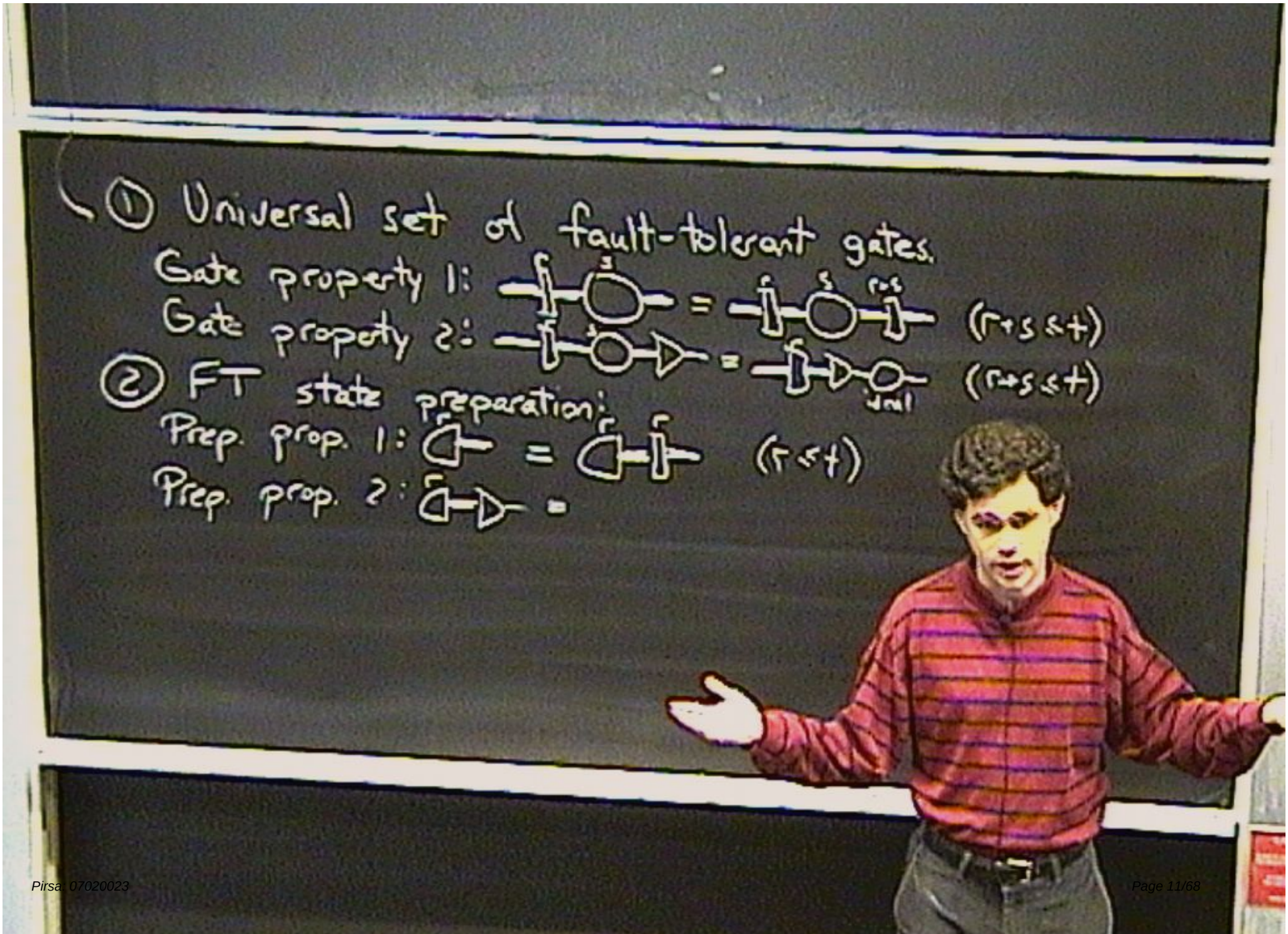
① Universal set of fault-tolerant gates.

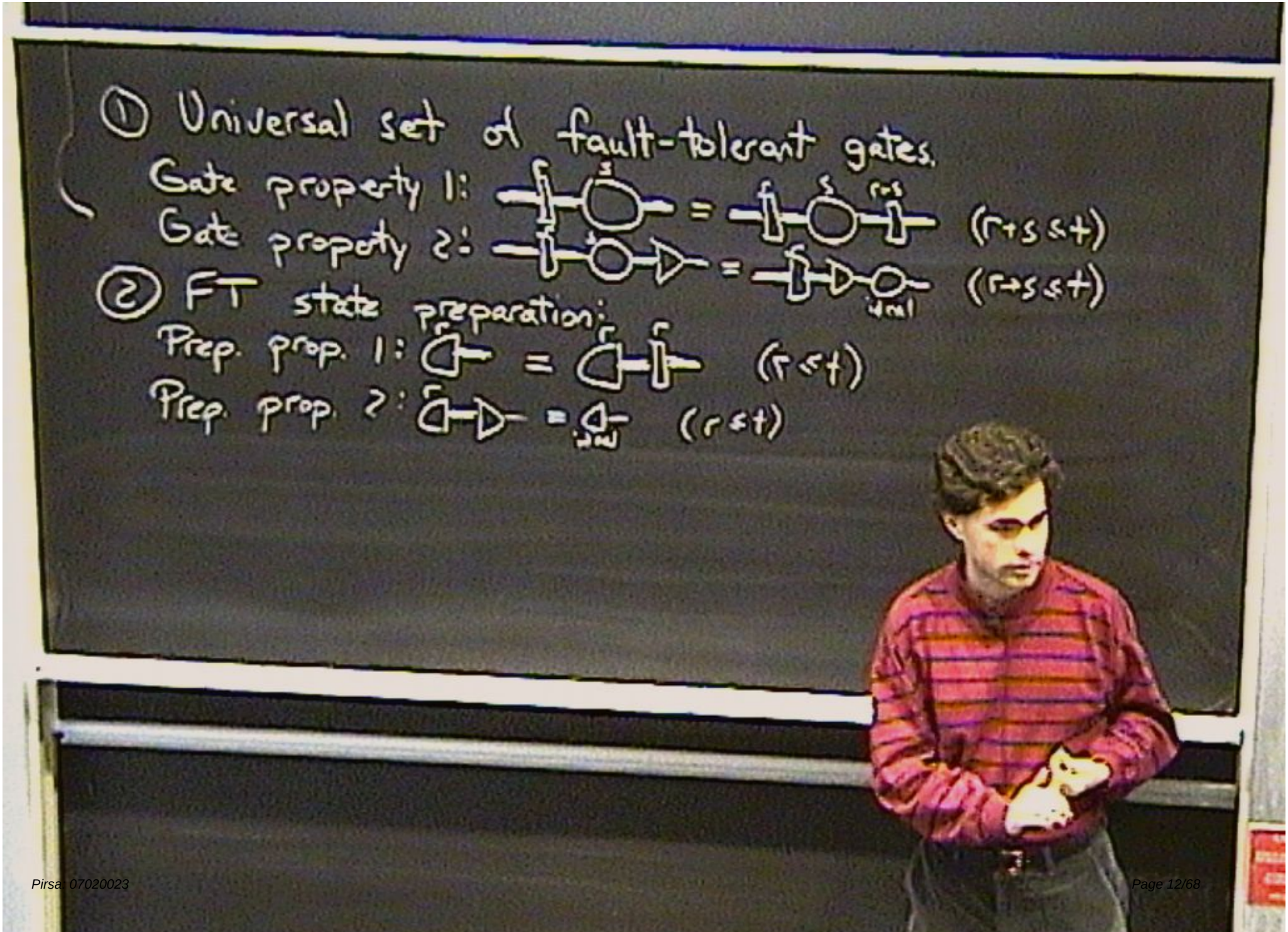
Gate property 1:  $(r+s \le t)$

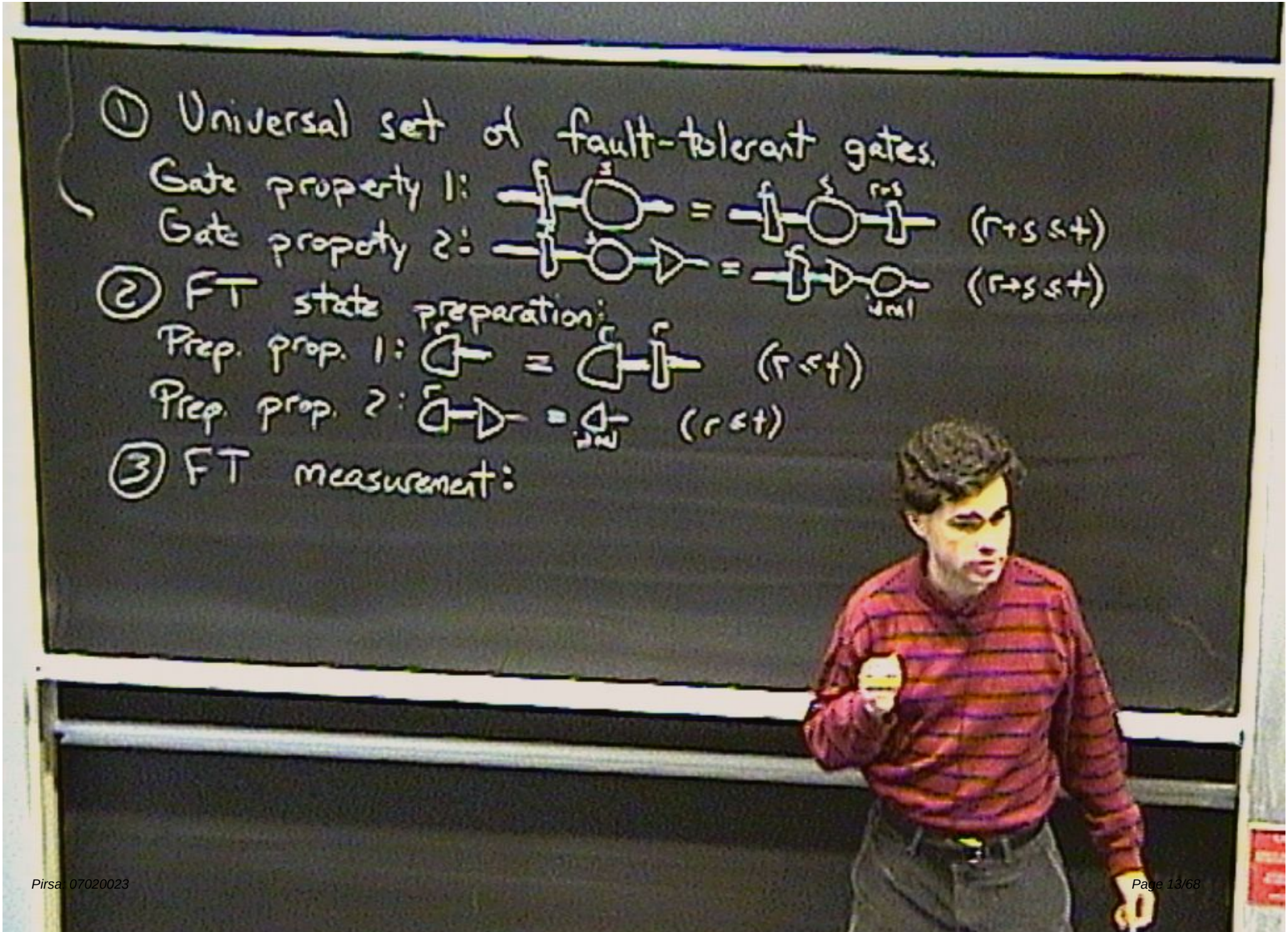
Gate property 2:  $(r+s \le t)$

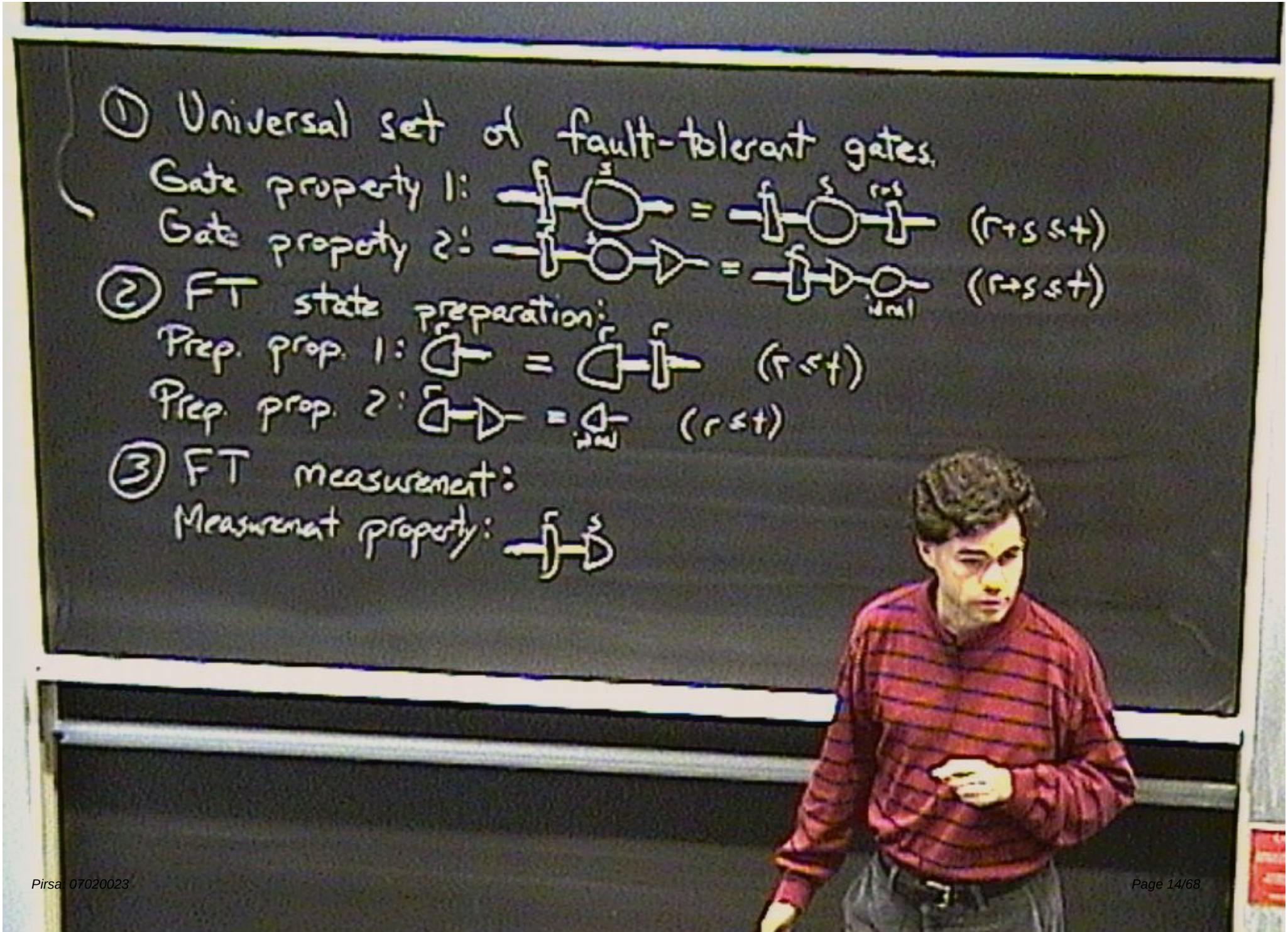

① Universal set of fault-tolerant gates.
Gate property 1:
Gate property 2:
(r+s s+t)
(r+s s+t)
② FT state preparation:

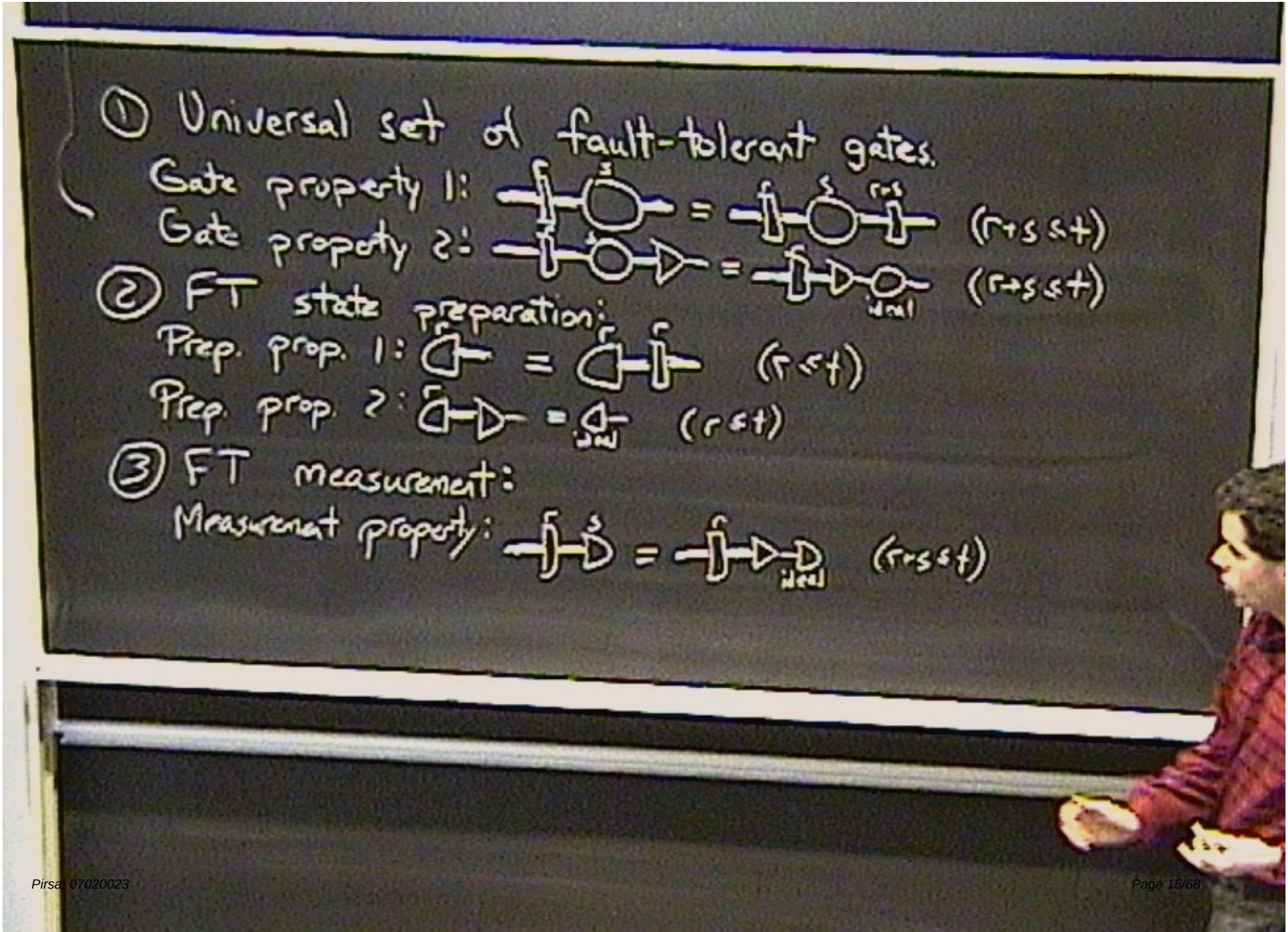

① Universal set of fault-tolerant gates.
Gate property 1:
(r+s≤t)
Gate property 2:
(r+s≤t)
② FT state preparation:
Prep. prop. 1:
(r≤t)

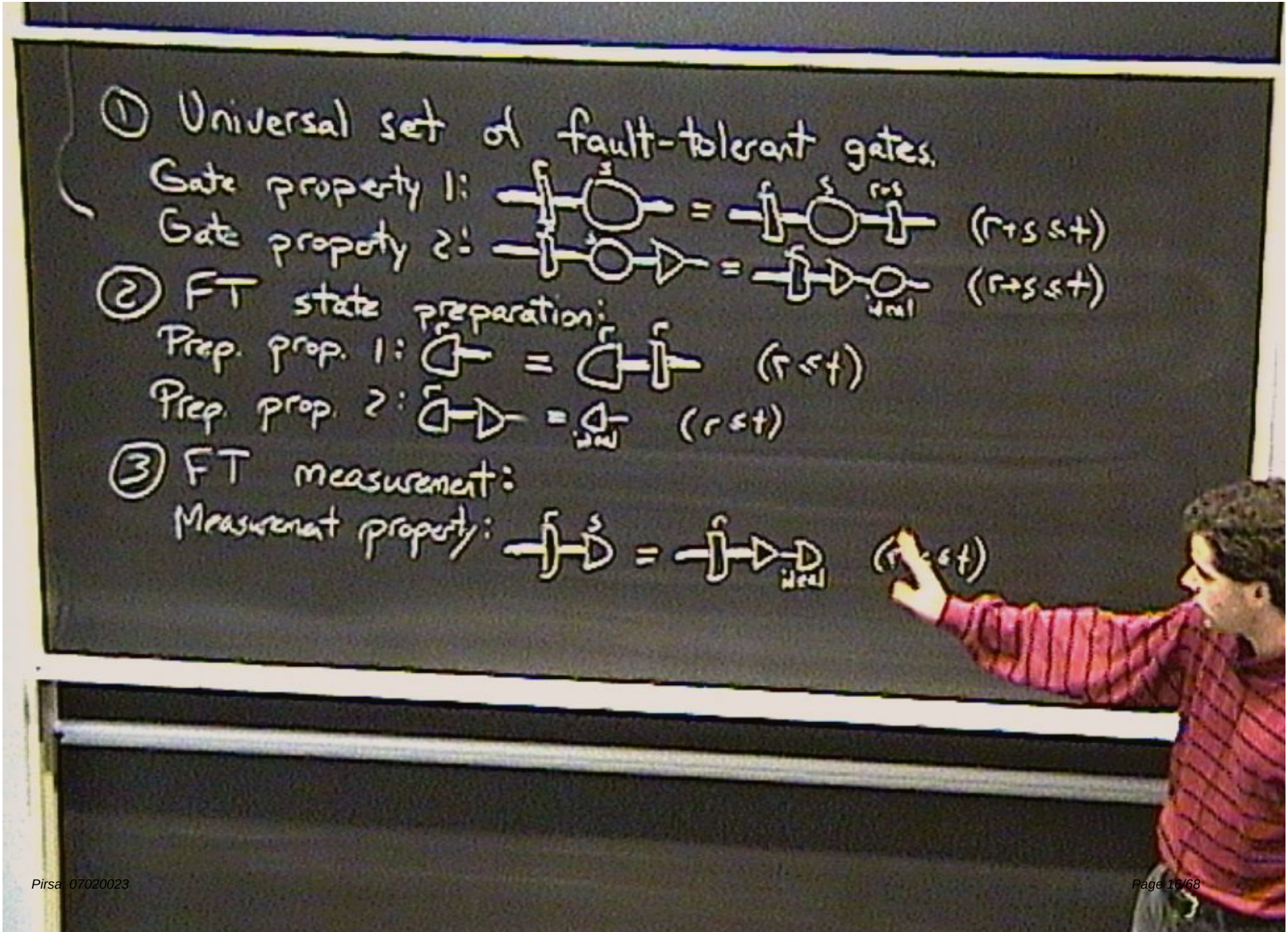

① Universal set of fault-tolerant gates.
Gate property 1:
(r+s≤t)
Gate property 2:
(r+s≤t)
② FT state preparation:
Prep. prop. 1:
(r≤t)
Prep. prop. 2:

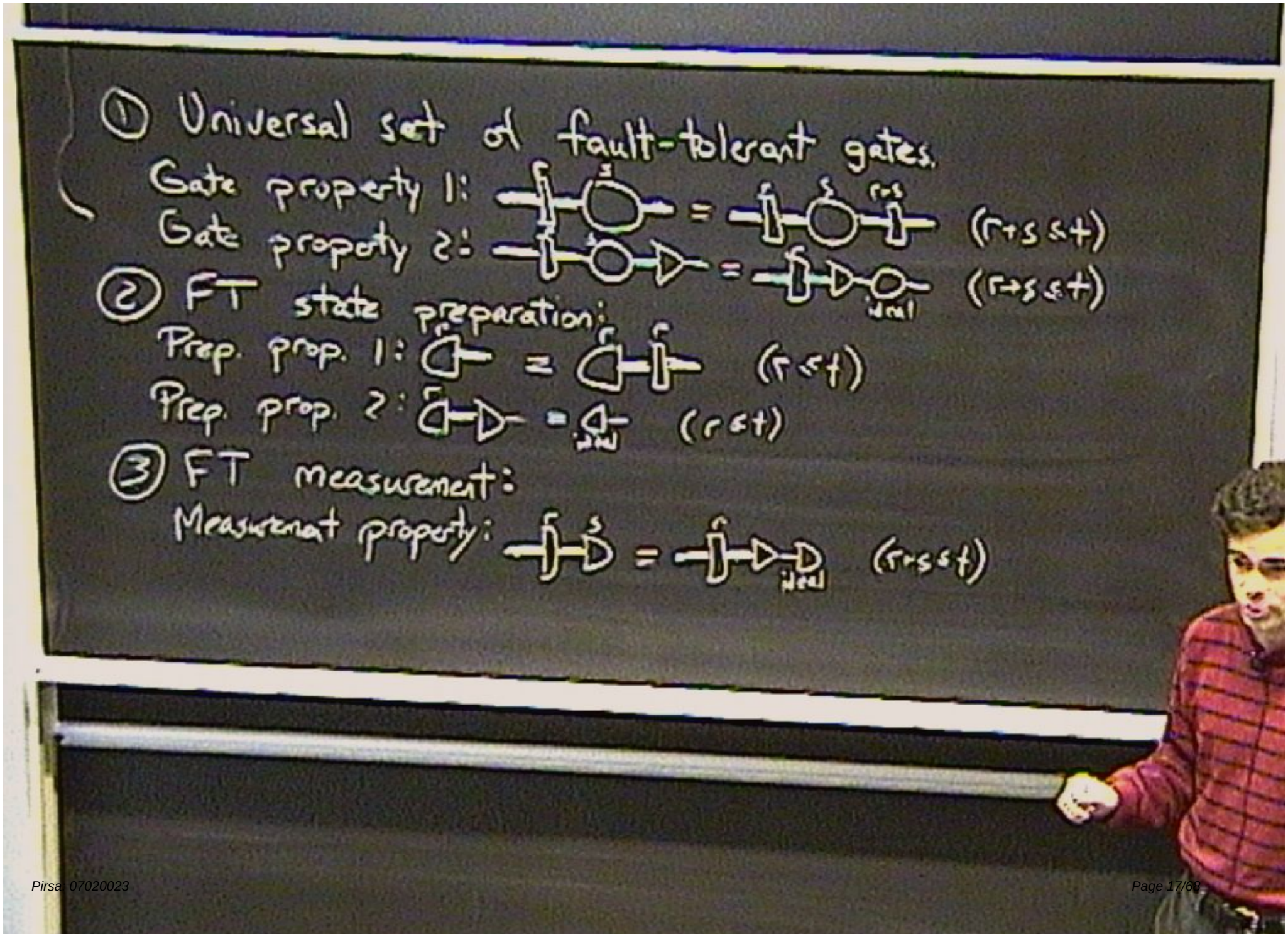

① Universal set of fault-tolerant gates.
Gate property 1:
(r+s≤t)
Gate property 2:
(r+s≤t)
② FT state preparation:
Prep. prop. 1:
(r≤t)
Prep. prop. 2:
(r≤t)


① Universal set of fault-tolerant gates.
Gate property 1:
(r+s≤t)
Gate property 2:
((r+s≤t)
② FT state preparation:
Prep. prop. 1:
(r≤t)
Prep. prop. 2:
(r≤t)
③ FT measurement:

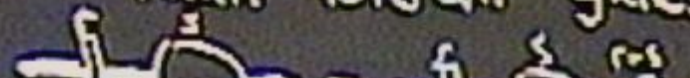

① Universal set of fault-tolerant gates.
Gate property 1:
(r+s≤t)
Gate property 2:
((r+s≤t)
② FT state preparation:
Prep. prop. 1:
(r≤t)
Prep. prop. 2:
(r≤t)
③ FT measurement:
Measurement property:


① Universal set of fault-tolerant gates.
Gate property 1:
(r+s ≤ t)
Gate property 2:
ideal
(r+s ≤ t)
② FT state preparation:
Prep. prop. 1:
(r ≤ t)
Prep. prop. 2:
ideal
(r ≤ t)
③ FT measurement:
Measurement property:
ideal
(r+s ≤ t)


① Universal set of fault-tolerant gates.
Gate property 1:
(r+s≤t)
Gate property 2:
(r+s≤t)
② FT state preparation:
Prep. prop. 1:
(r≤t)
Prep. prop. 2:
(r≤t)
③ FT measurement:
Measurement property:

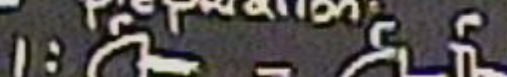

① Universal set of fault-tolerant gates.
Gate property 1:
(r+s≤t)
Gate property 2:
ideal
((r→s≤t)
② FT state preparation:
Prep. prop. 1:
(r≤t)
Prep. prop. 2:
ideal
(r≤t)
③ FT measurement:
Measurement property:
ideal
(r→s≤t)

① Universal set of fault-tolerant gates.

Gate property 1:  $(r+s \le t)$

Gate property 2: $(r+s \le t)$

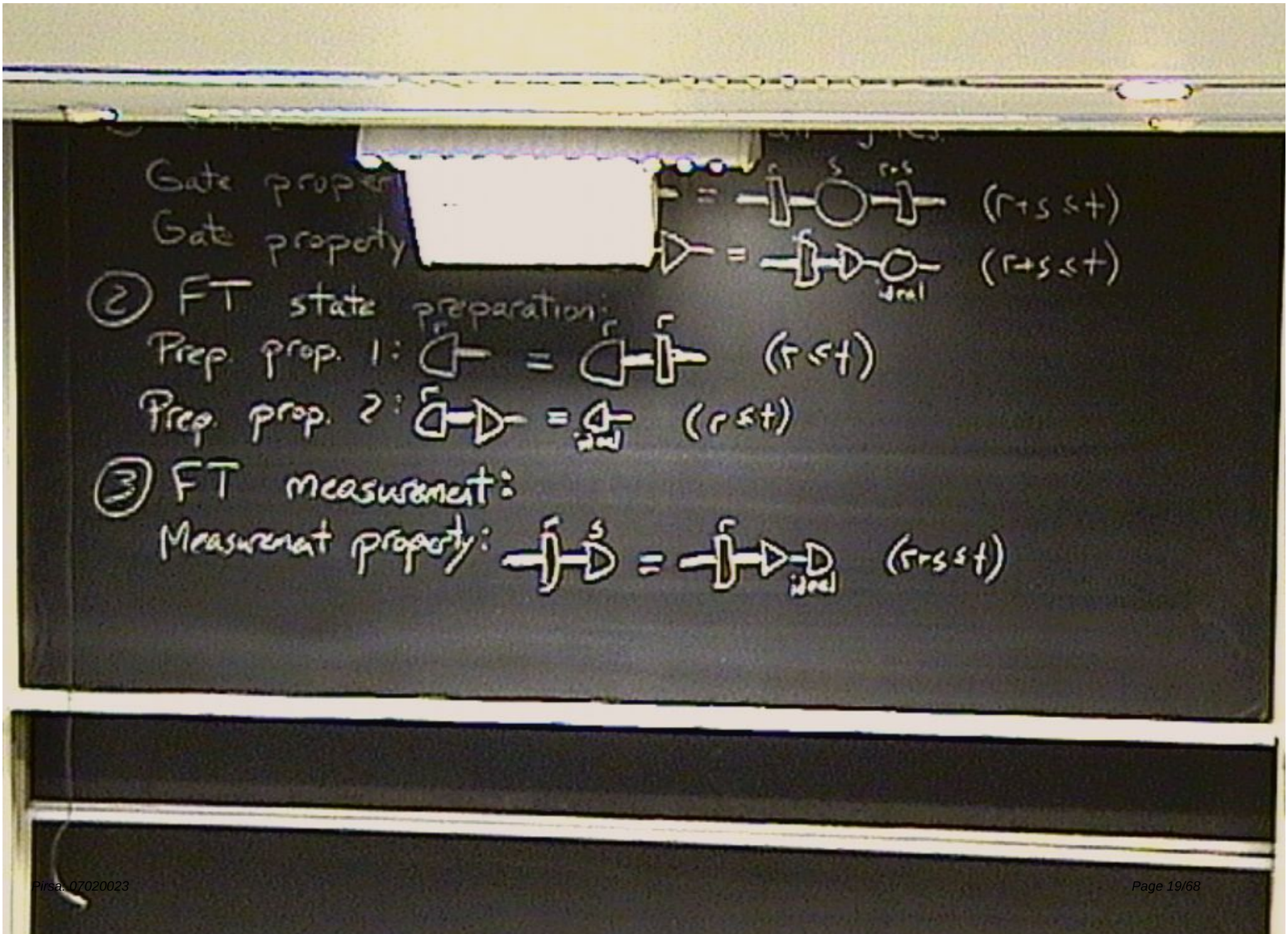
② FT state preparation:

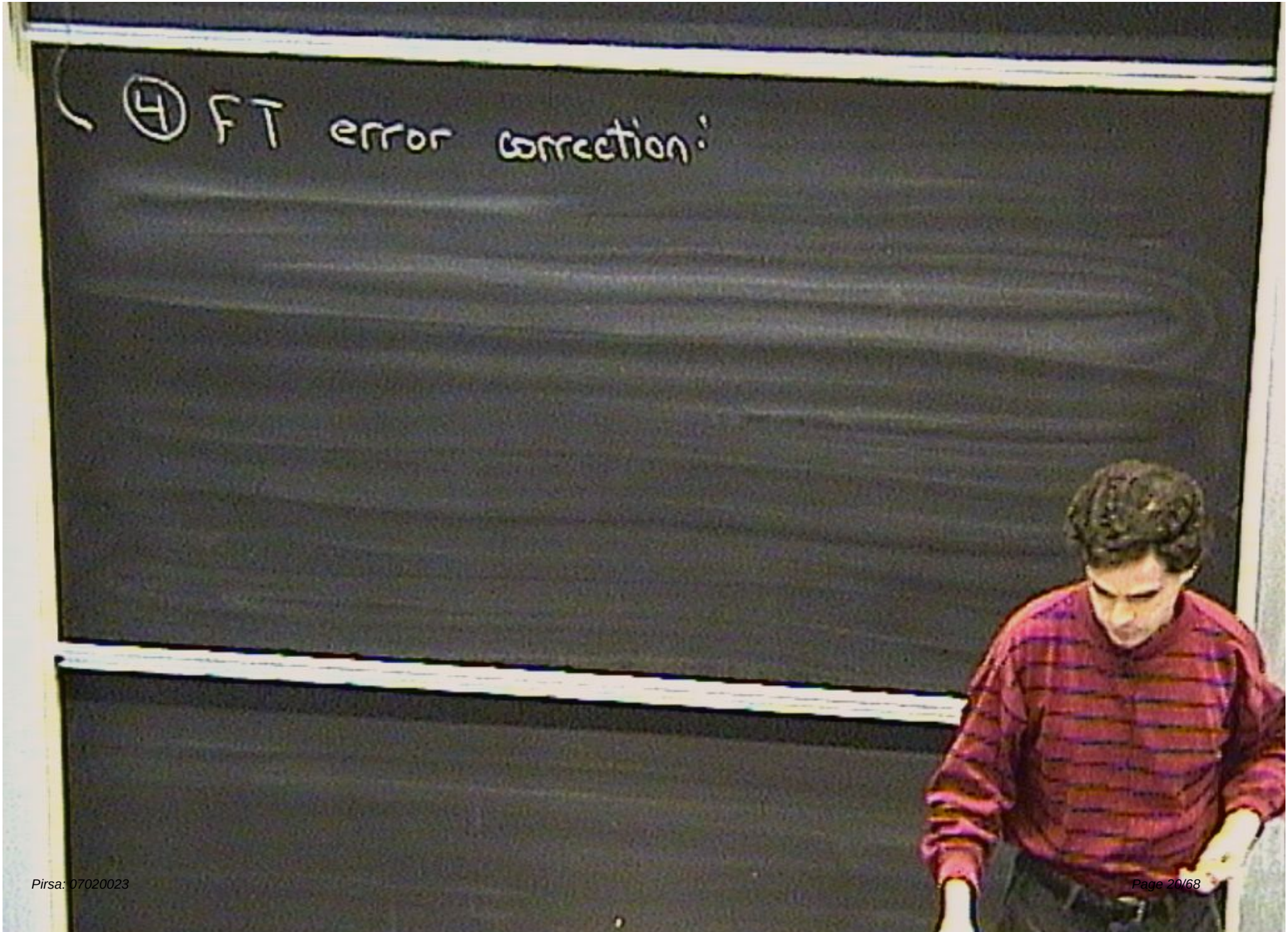
Prep. prop. 1:  $(r \le t)$

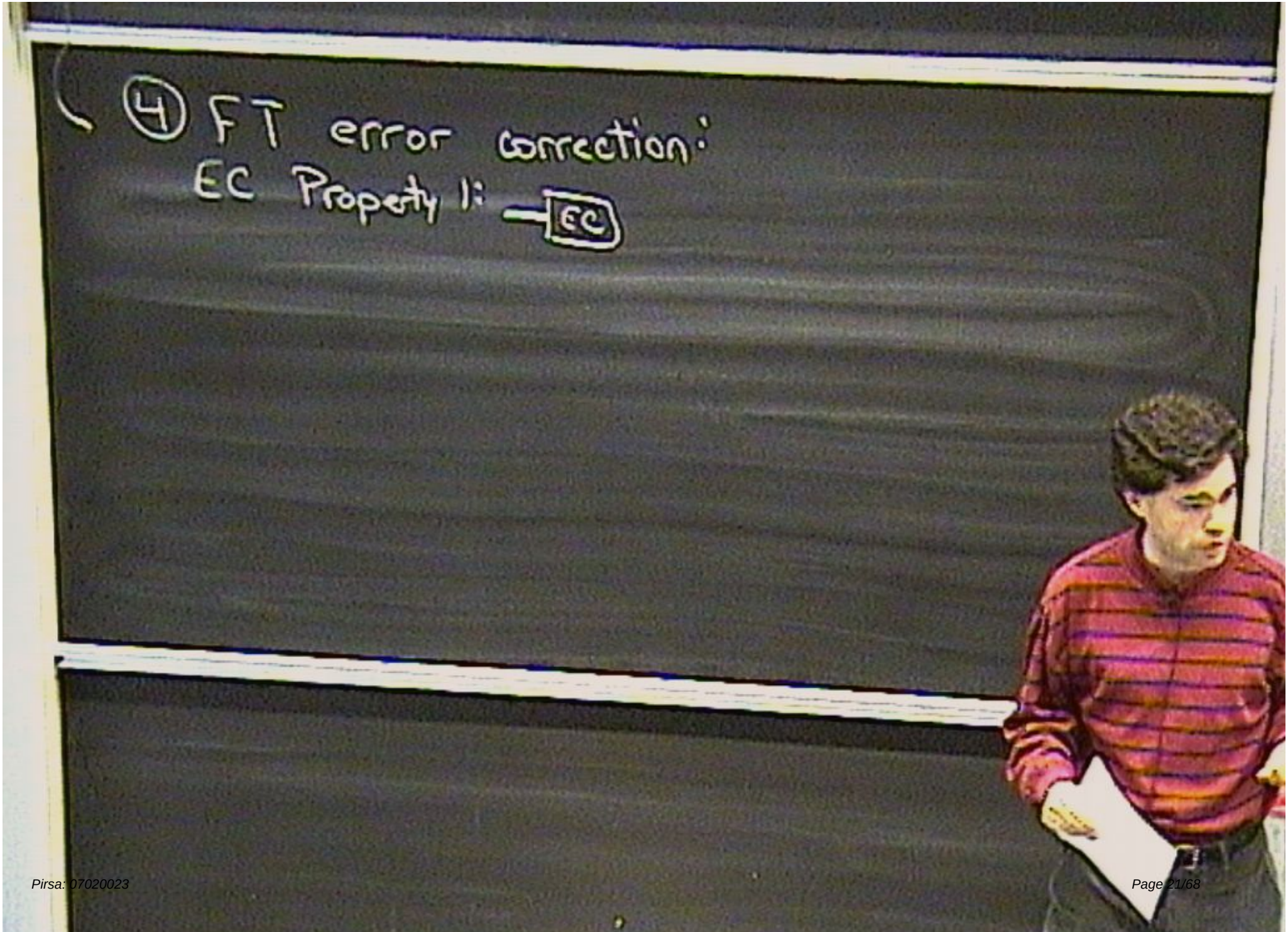
Prep. prop. 2: $(r \le t)$

③ FT measurement:

Measurement property: $(r+s \le t)$


Gate proper
Gate property
(r+s ≤ t)
(r+s ≤ t)
ideal
② FT state preparation:
Prep. prop. 1:
(r ≤ t)
Prep. prop. 2:
ideal
(r ≤ t)
③ FT measurement:
Measurement property:
ideal
(r+s ≤ t)

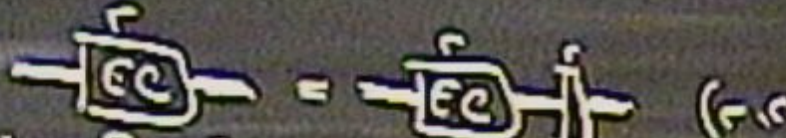

④ FT error correction!


④ FT error correction:
EC Property 1:
EC

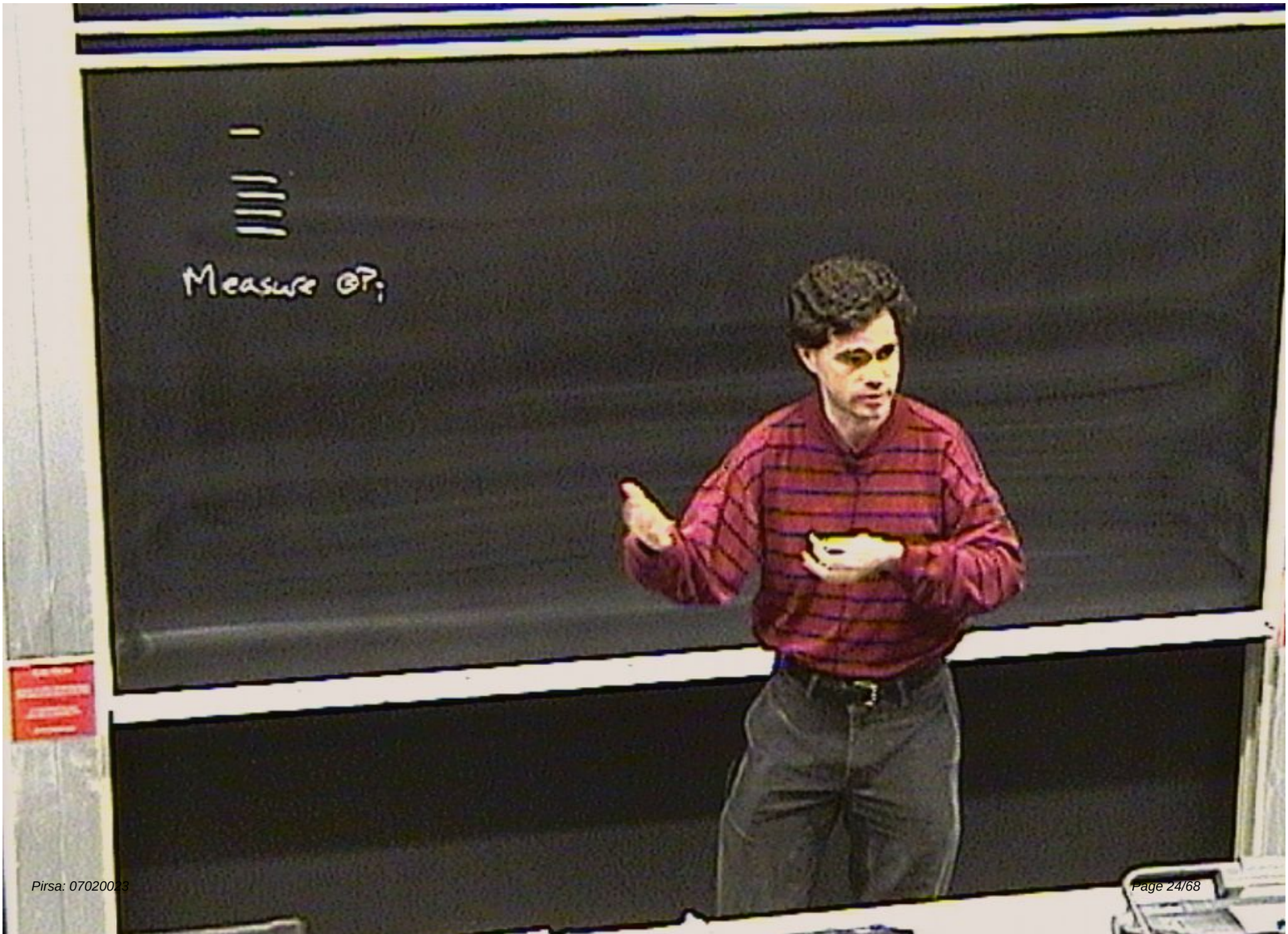
④ FT error correction:

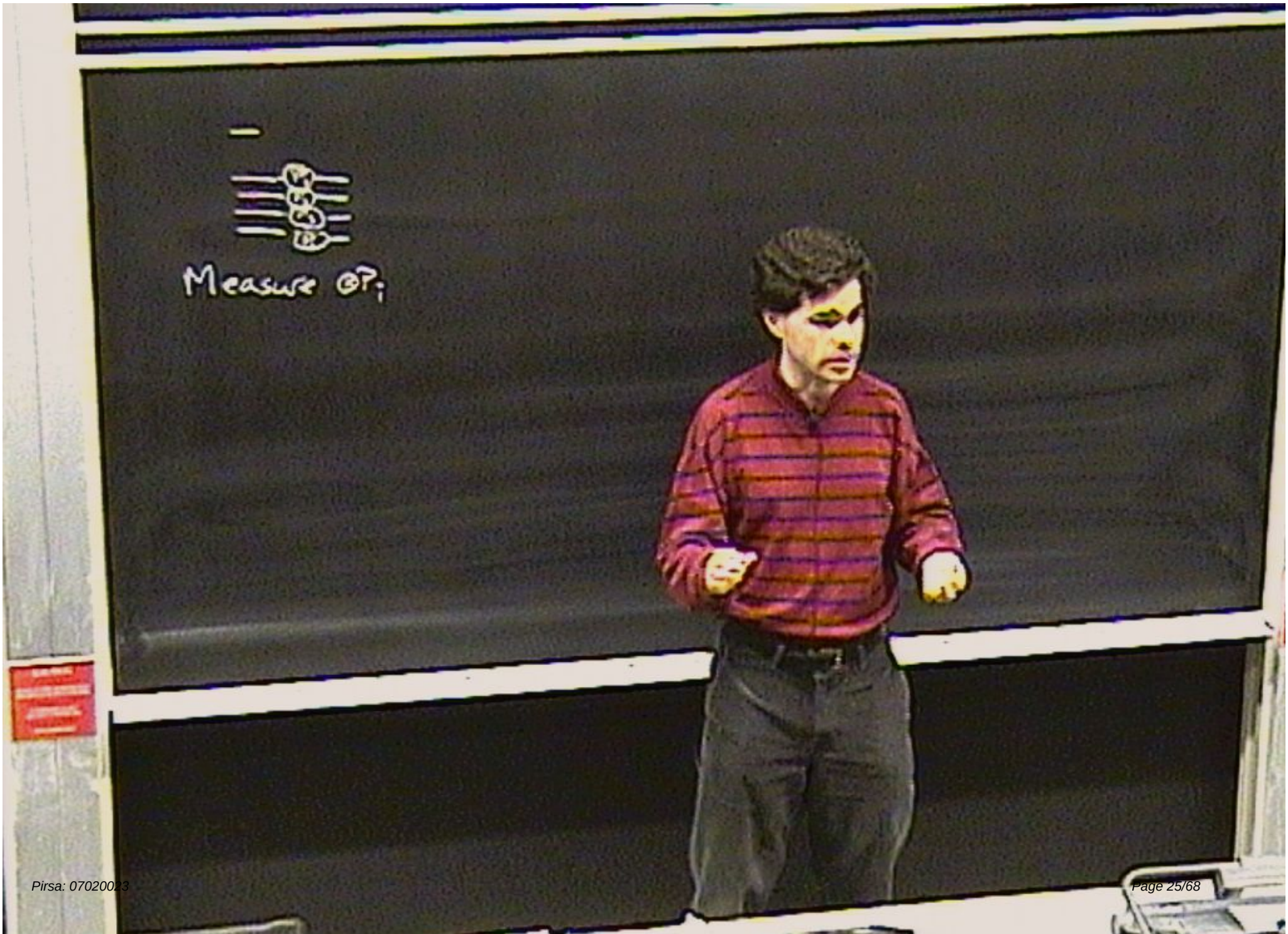
EC Property 1:

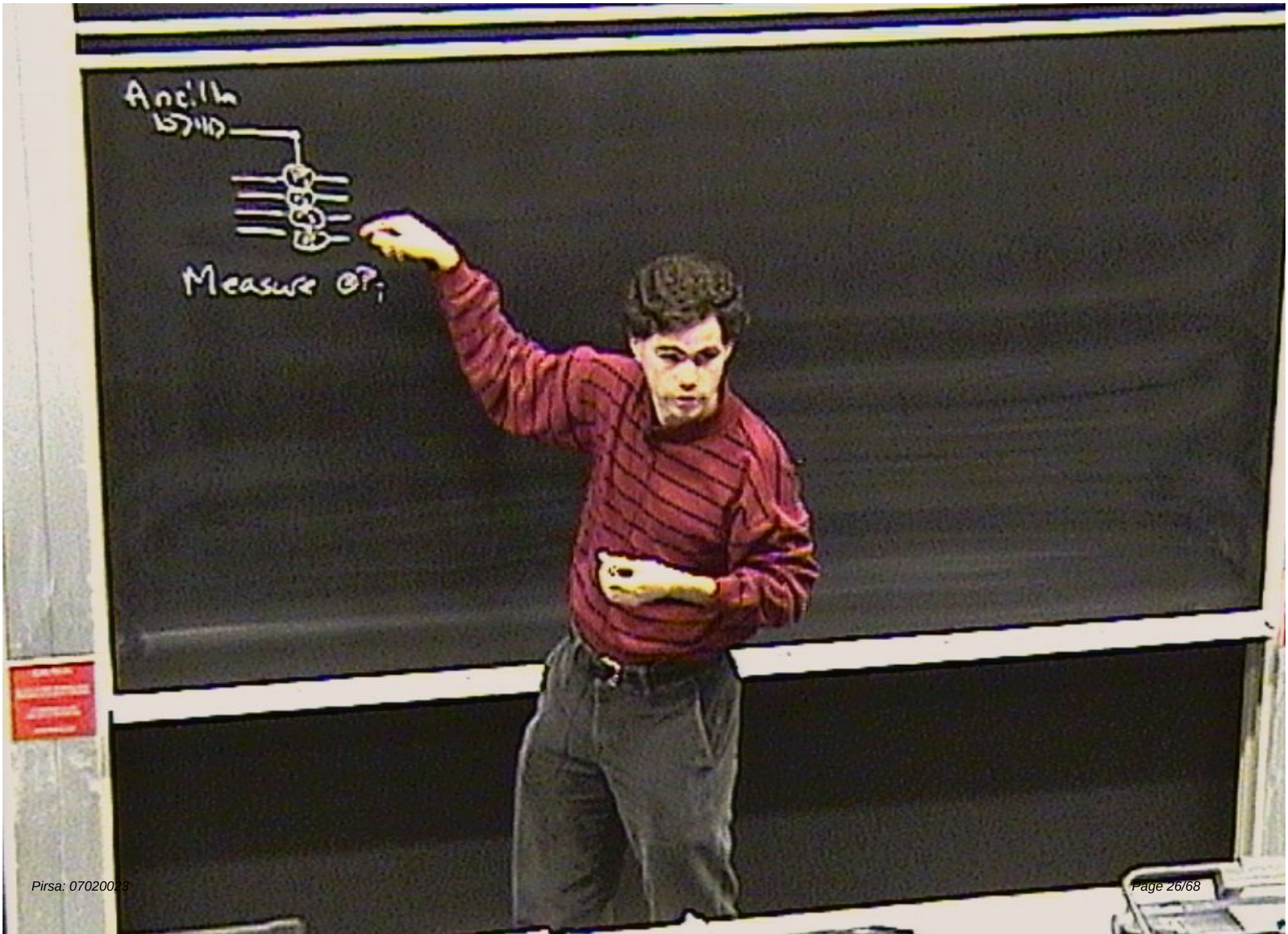
④ FT error correction:

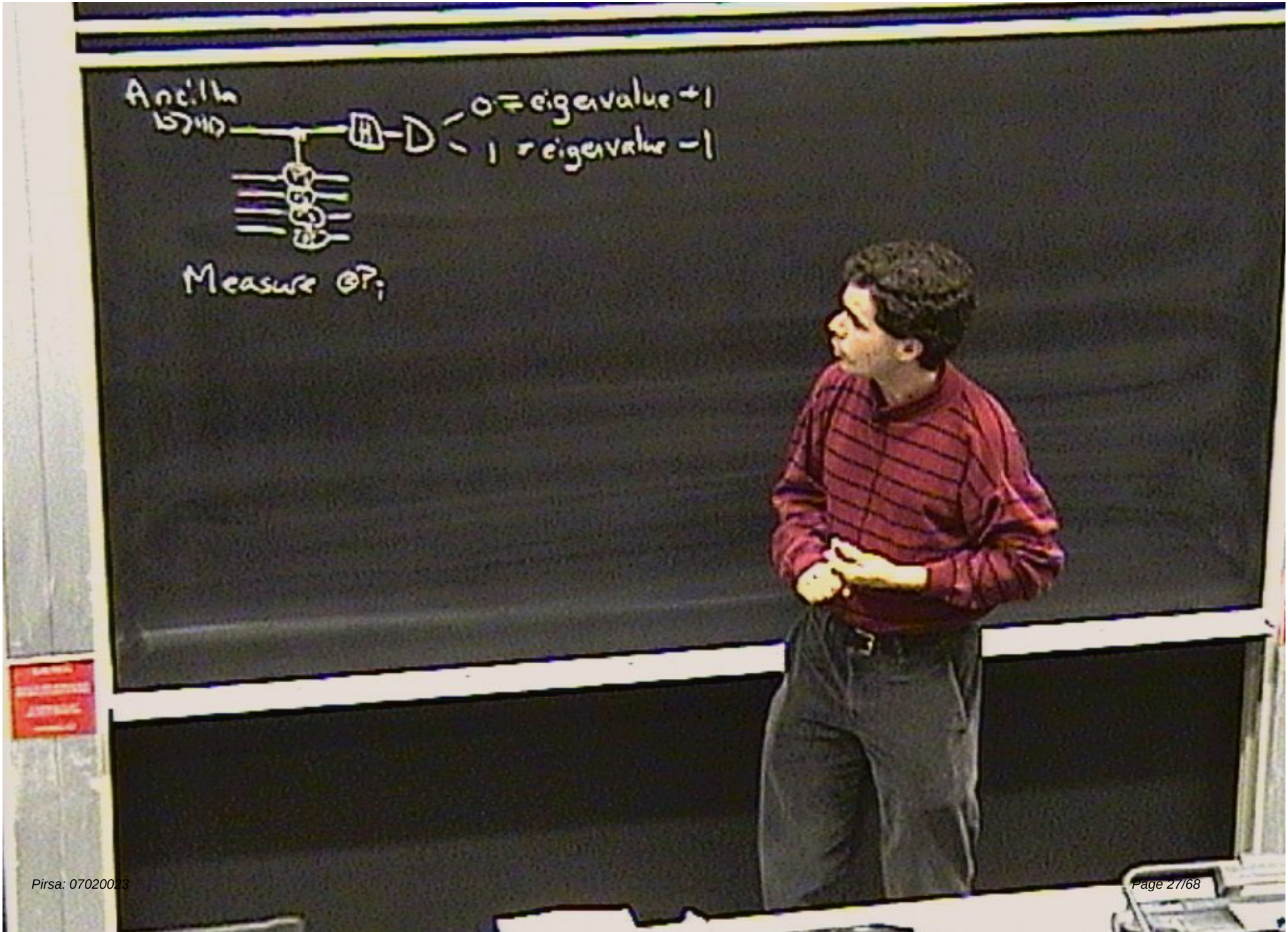
EC Property 1:  $(r<t)$

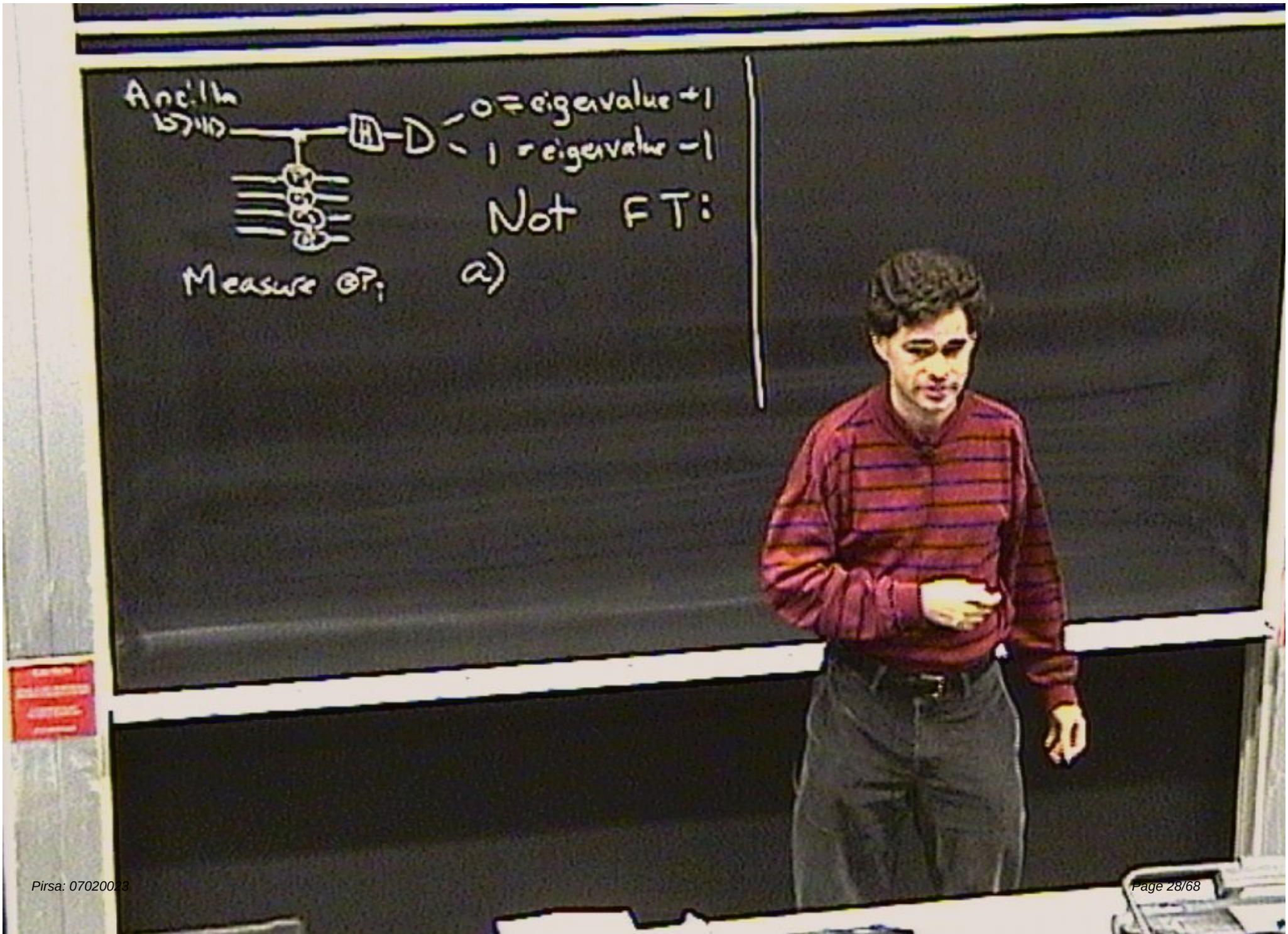
EC Property 2: $(r+s\leq t)$

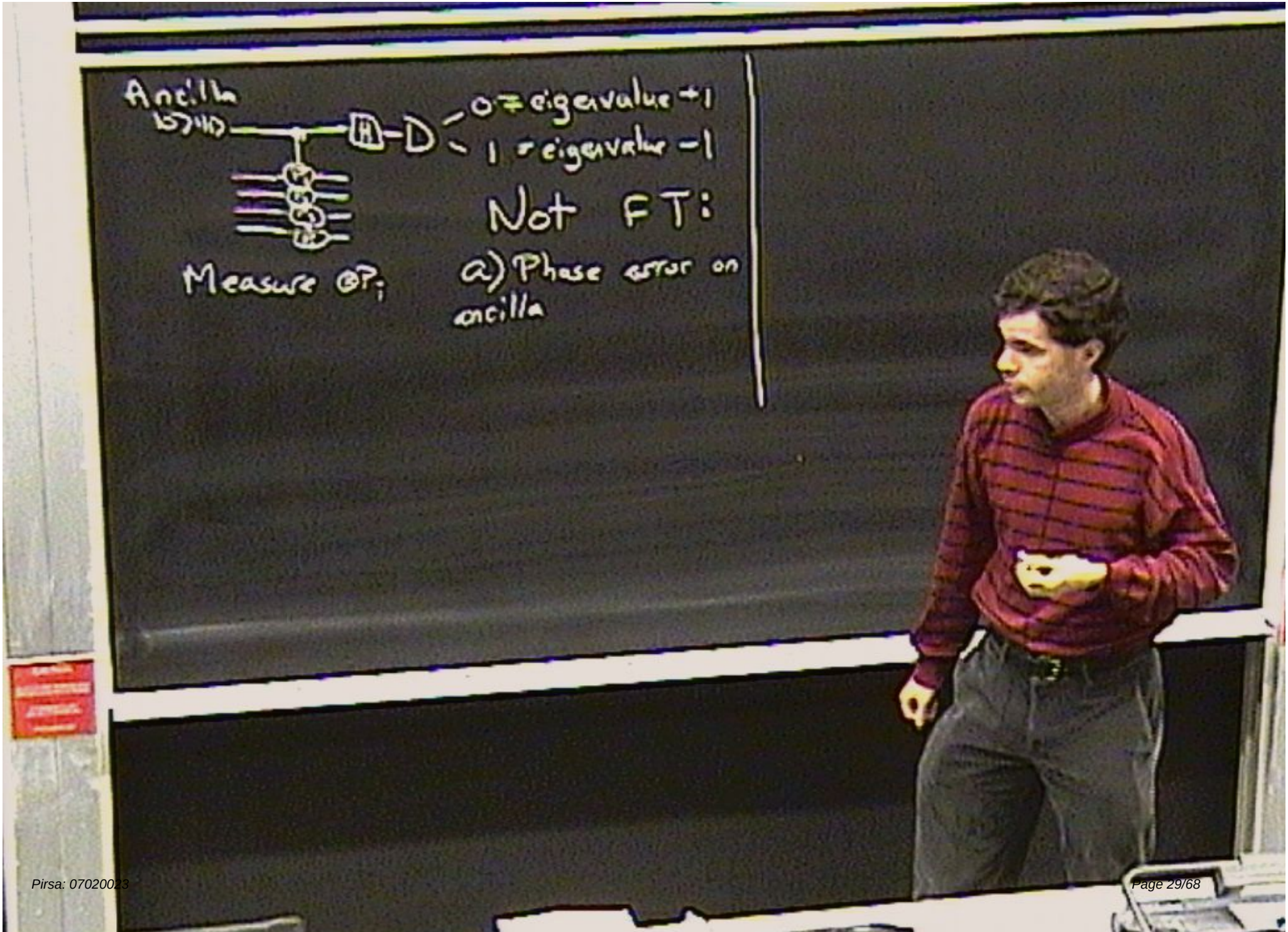

Measure $\hat{O}_P$;

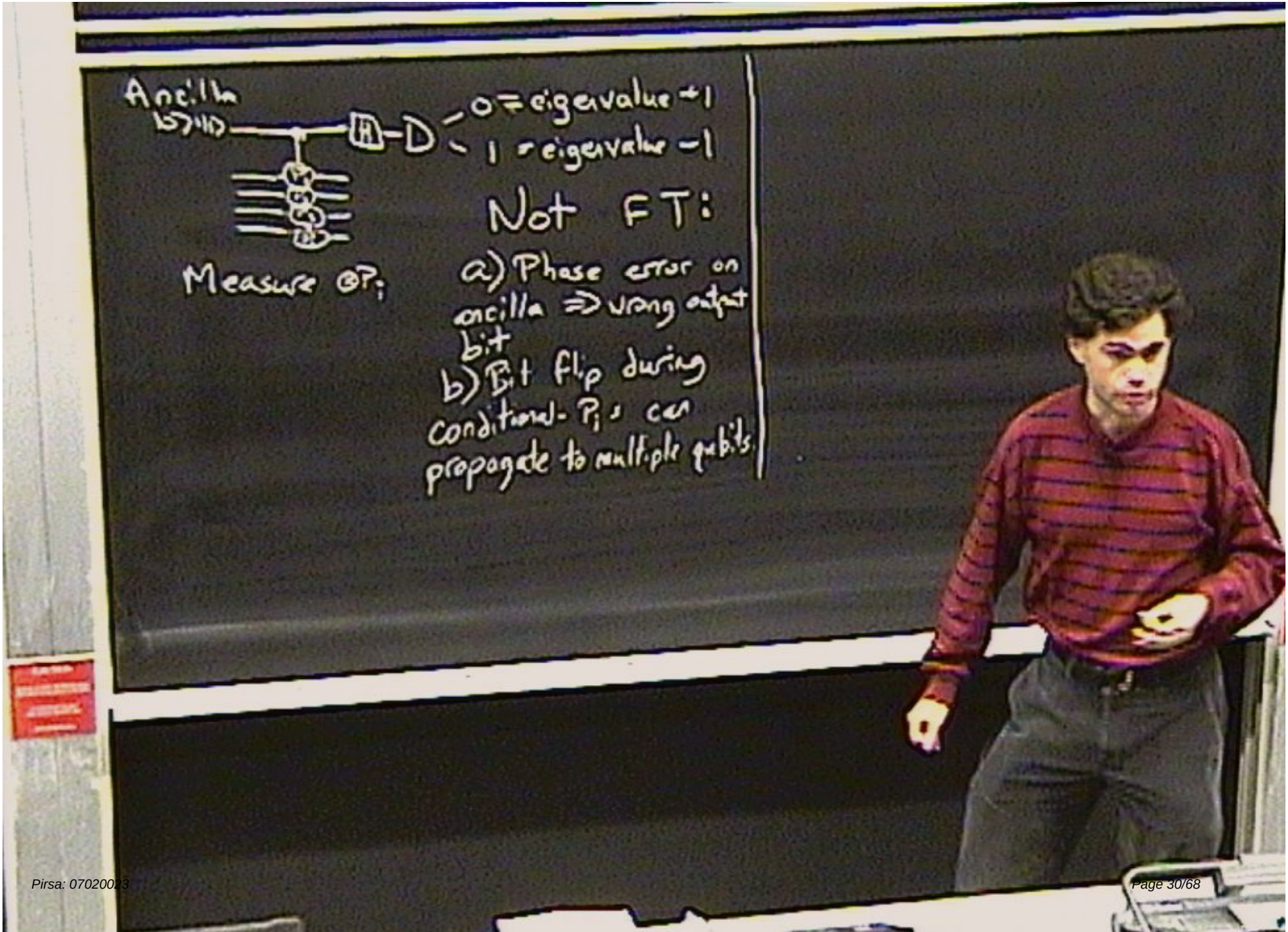

Measure of;

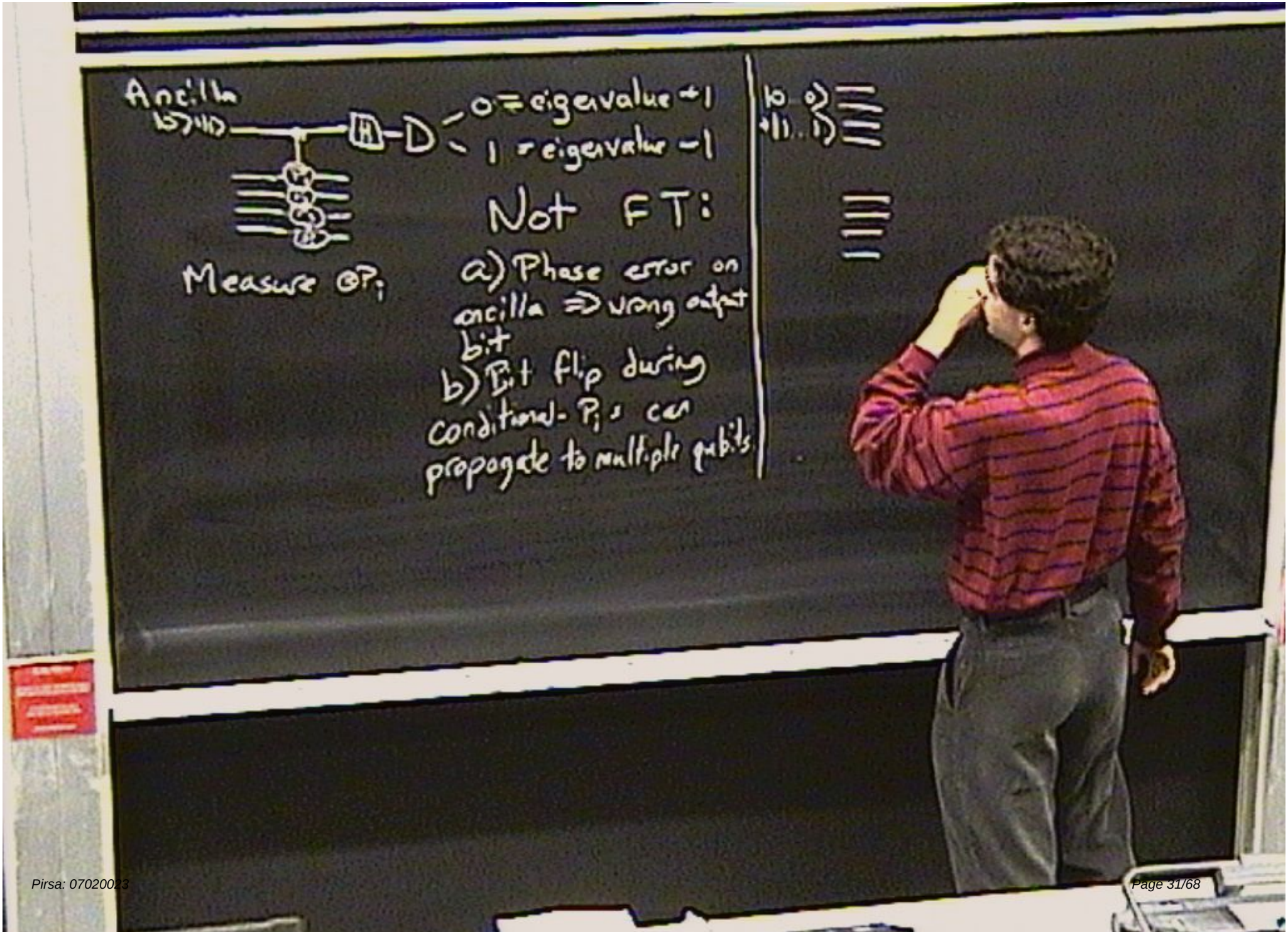

Measure

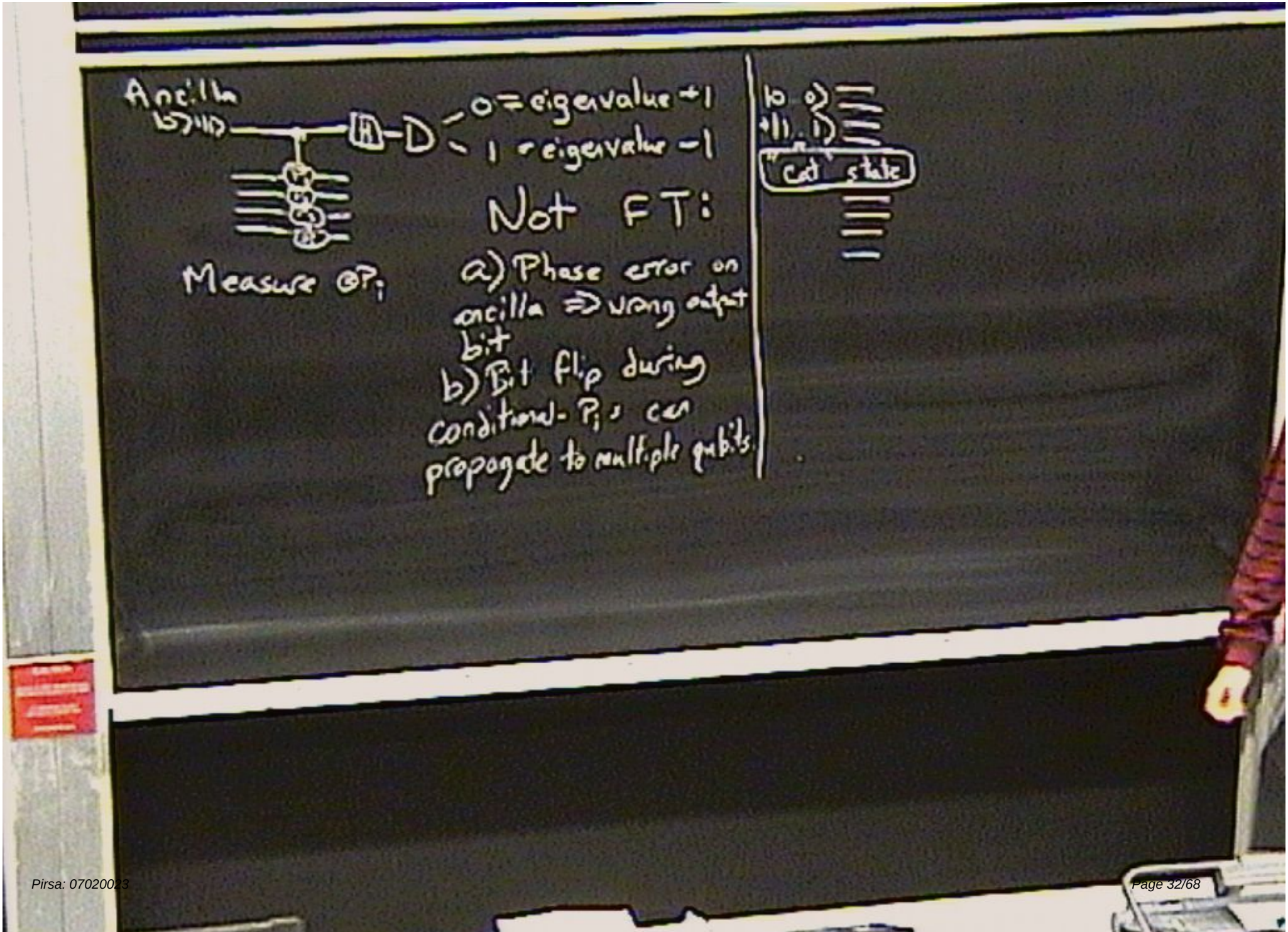

Ancilla
H
D
0 = eigenvalue +1
1 = eigenvalue -1
Measure σ_i^z

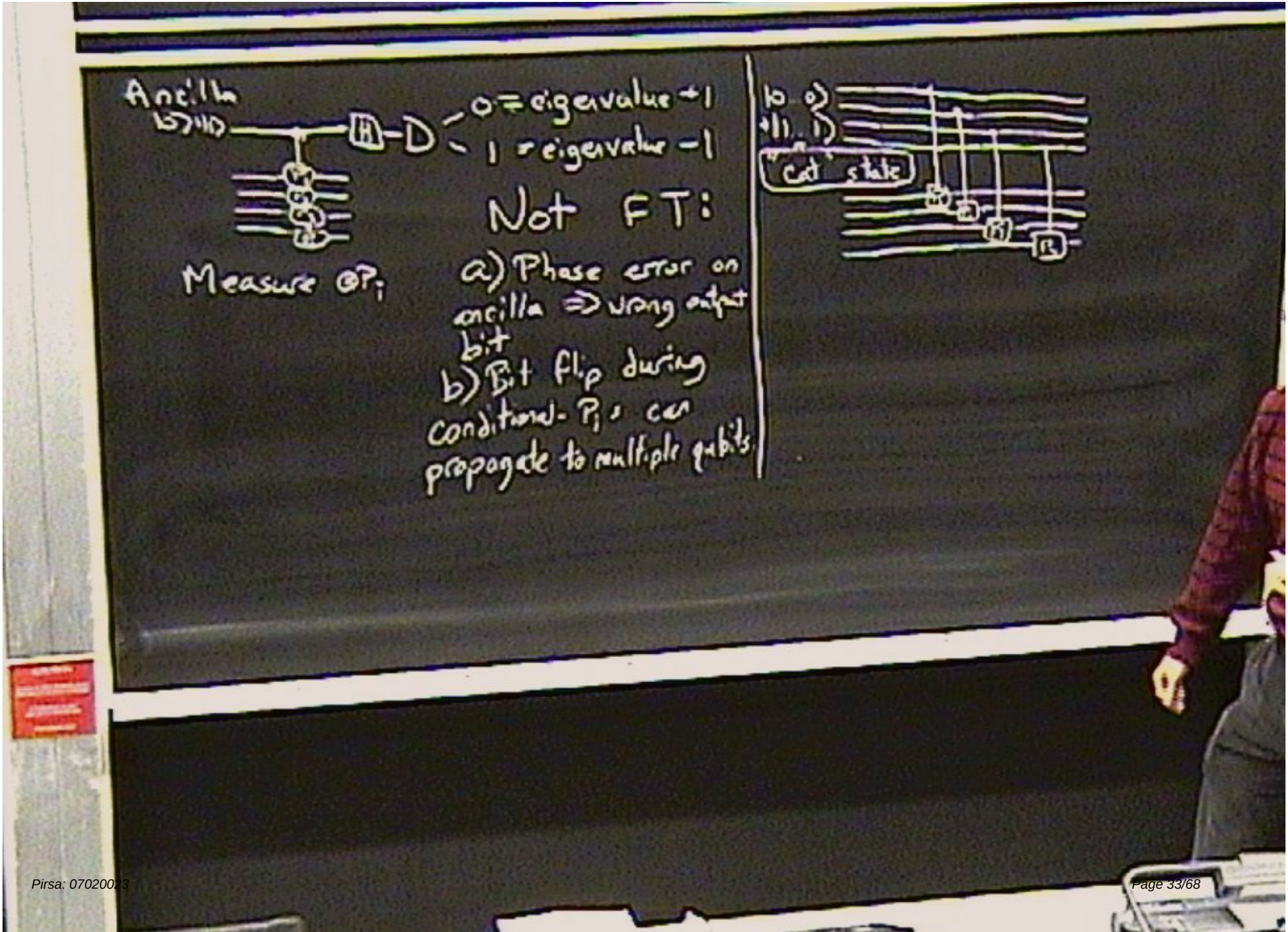

Ancilla
H
D
0 = eigenvalue +1
1 = eigenvalue -1
Not FT:
Measure OP:
a)


Ancilla
|0⟩+|1⟩
H
D
0 = eigenvalue +1
1 = eigenvalue -1
Measure $\mathcal{O}_i$
Not FT:
a) Phase error on ancilla


Ancilla
|0⟩
H
D
0 = eigenvalue +1
1 = eigenvalue -1
Measure σ_i^z
Not FT:
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional-P_i's can propagate to multiple qubits


Ancilla
|0⟩+|1⟩
H
D
0 = eigenvalue +1
1 = eigenvalue −1
Measure $\otimes P_i$
Not FT:
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional-P_i's can propagate to multiple qubits
|0 0⟩
+|1 1⟩


Ancilla
|0⟩+|1⟩
H
D
0 = eigenvalue +1
1 = eigenvalue -1
Measure $\otimes P_i$
Not FT:
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional-P_i's can propagate to multiple qubits.
|0⟩
|1⟩
Cat state


Ancilla
|0⟩+|1⟩
H
D
0 = eigenvalue +1
1 = eigenvalue -1
Measure $\otimes P_i$
Not FT:
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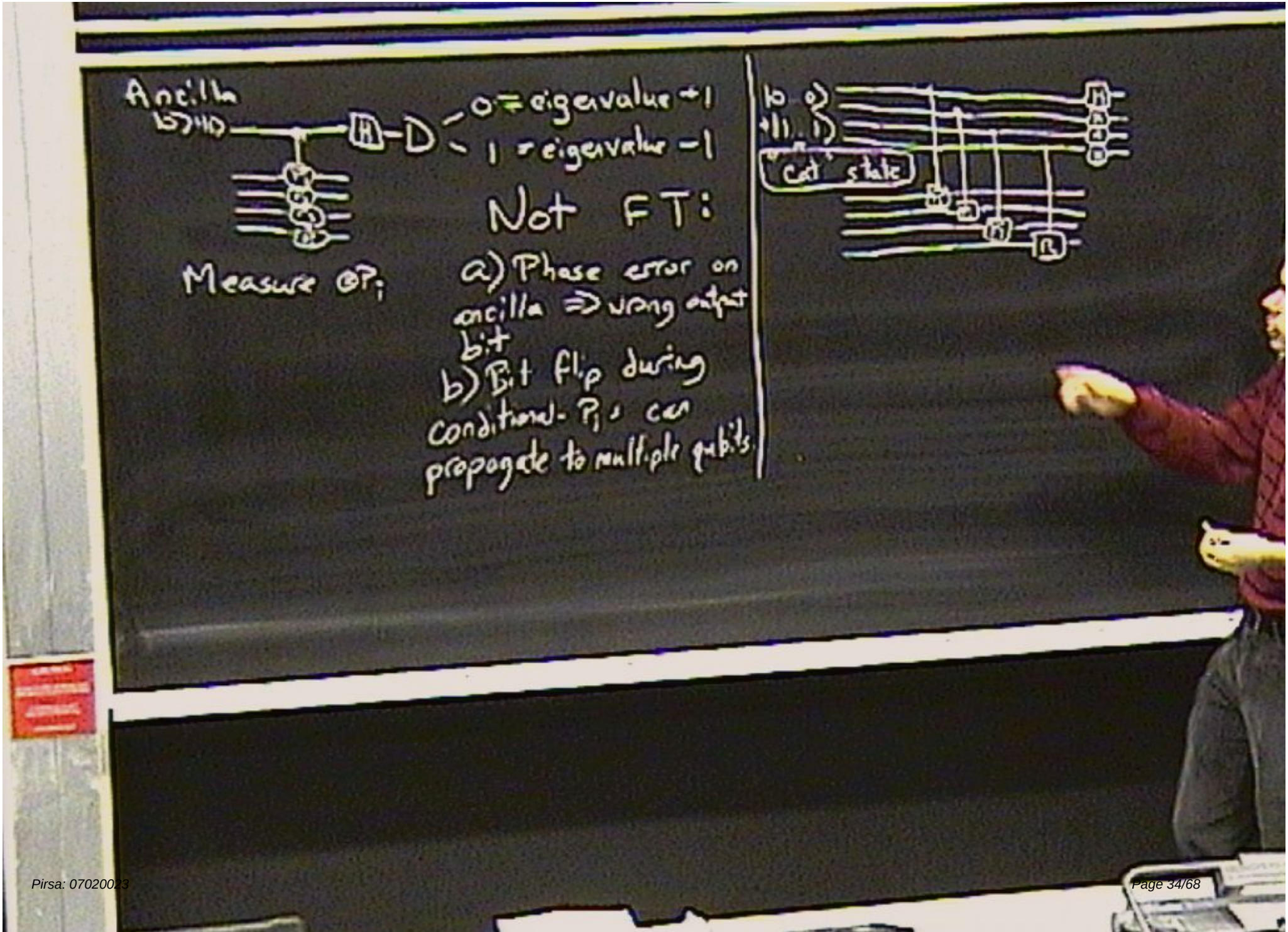
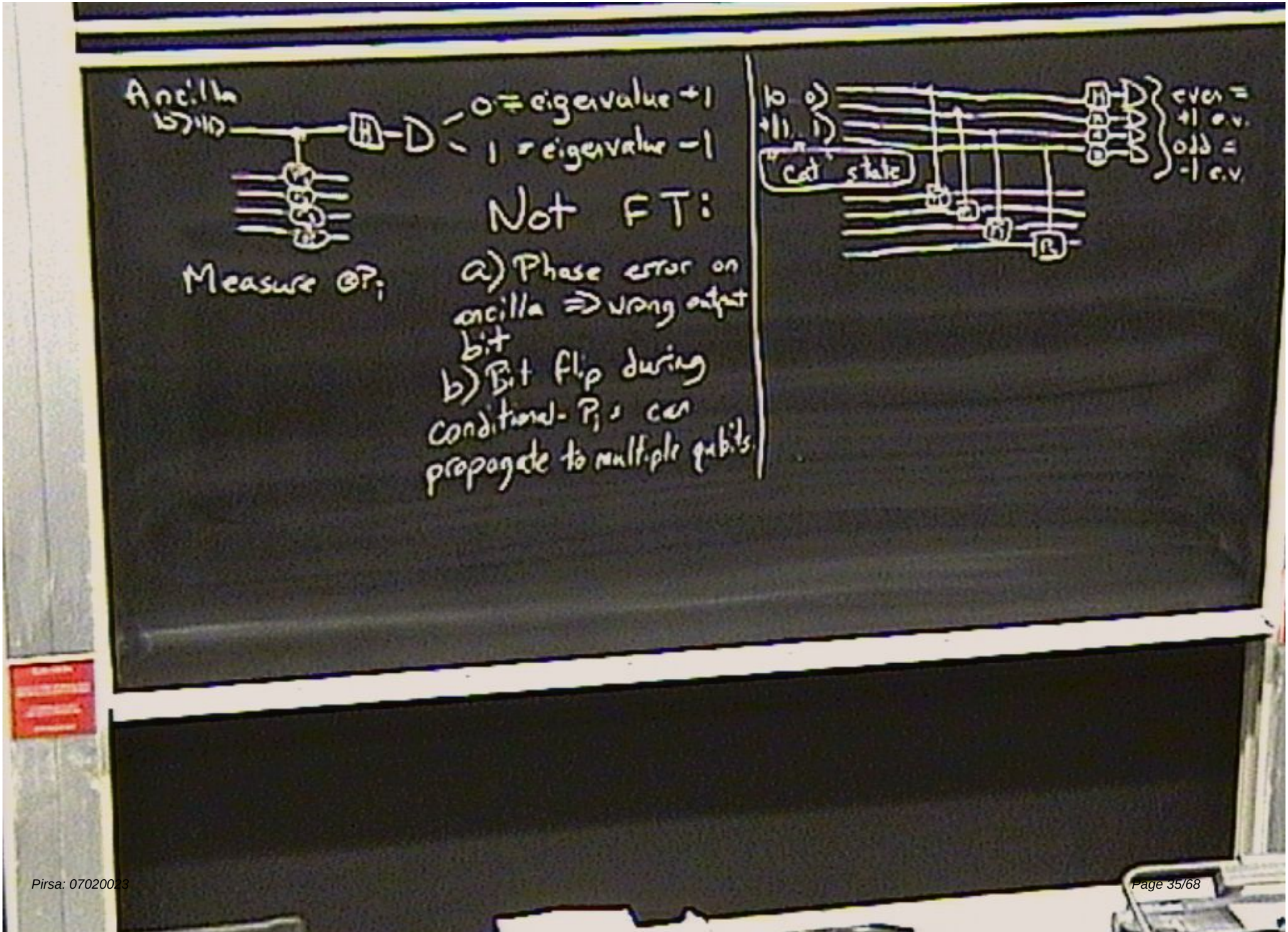
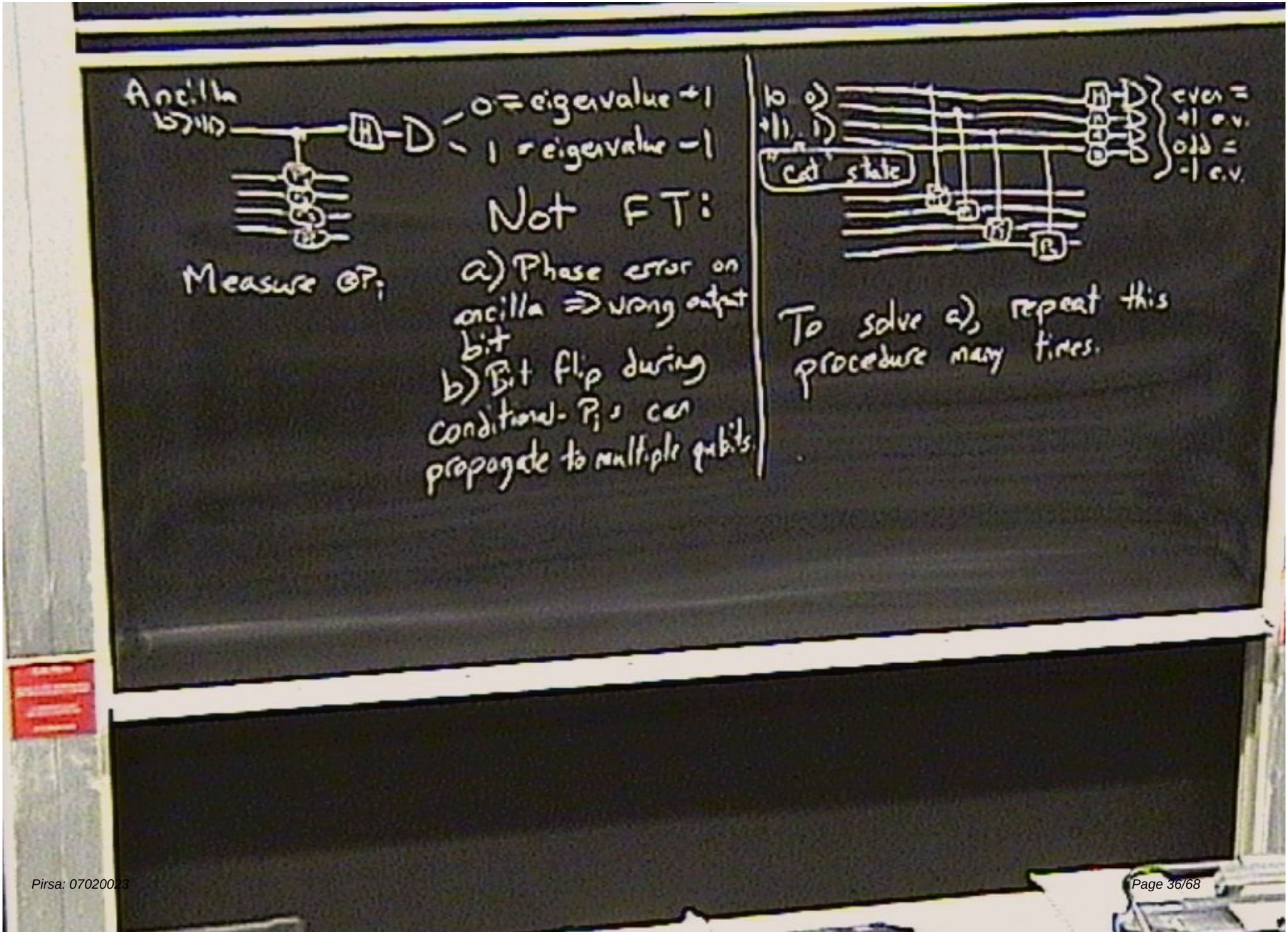
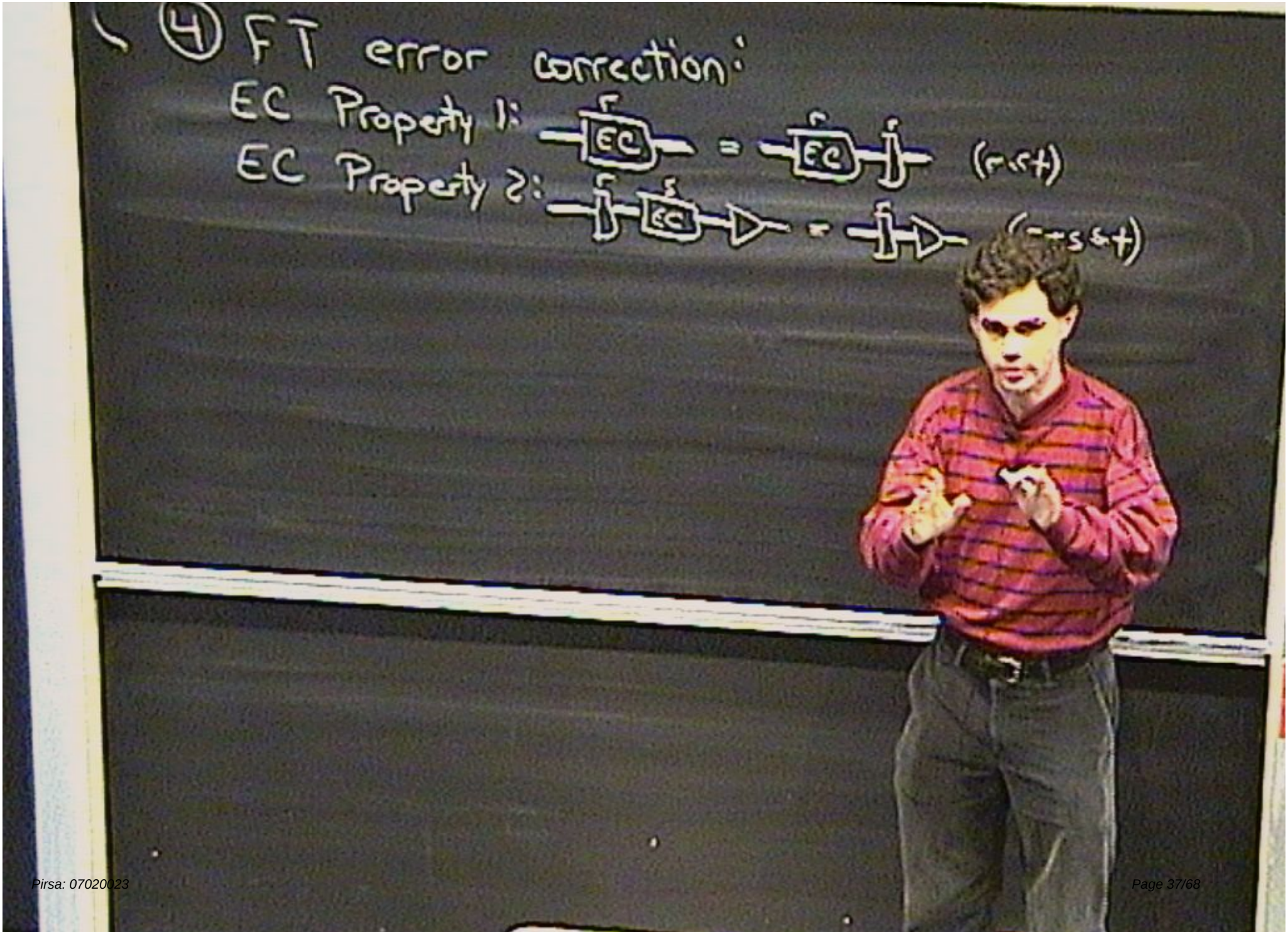
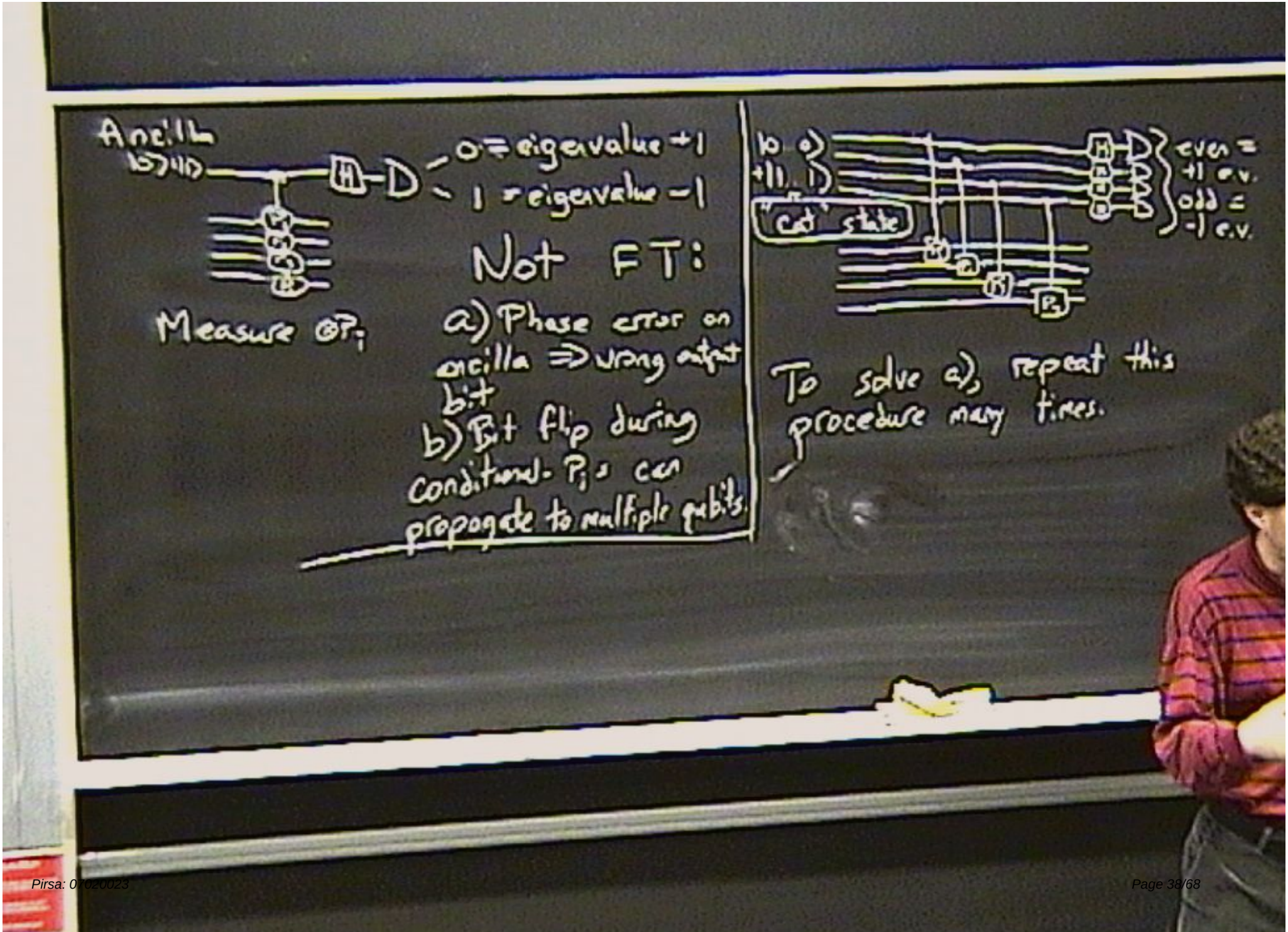
|0 0⟩
+|1 1⟩
cat state

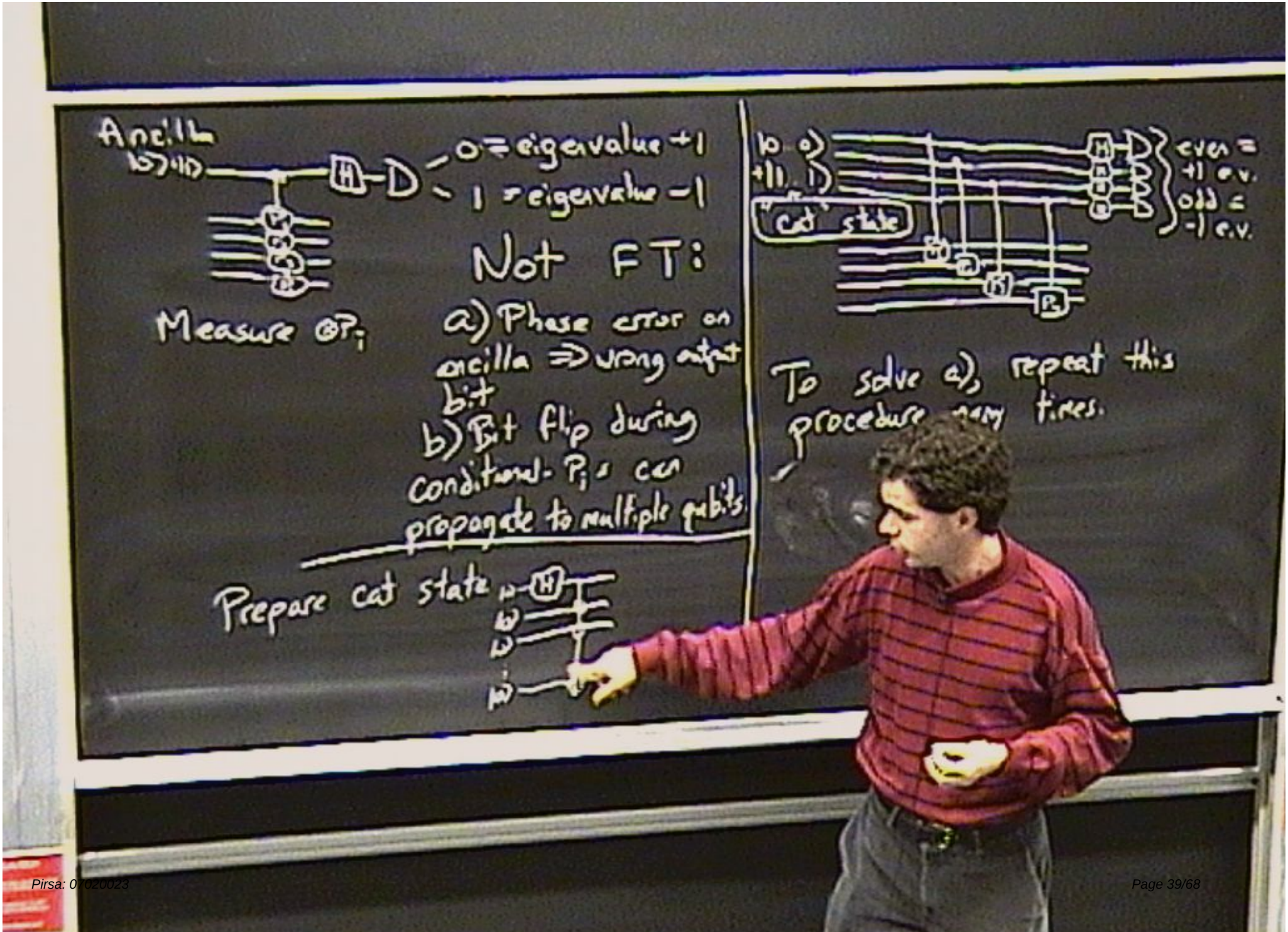

Ancilla
|0⟩+|1⟩
H
D
0 ⇒ eigenvalue +1
1 ⇒ eigenvalue −1
Measure $\mathcal{O}P_i$
Not FT:
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional-P_i's can propagate to multiple qubits
|0 0⟩
+|1 1⟩
Cat state

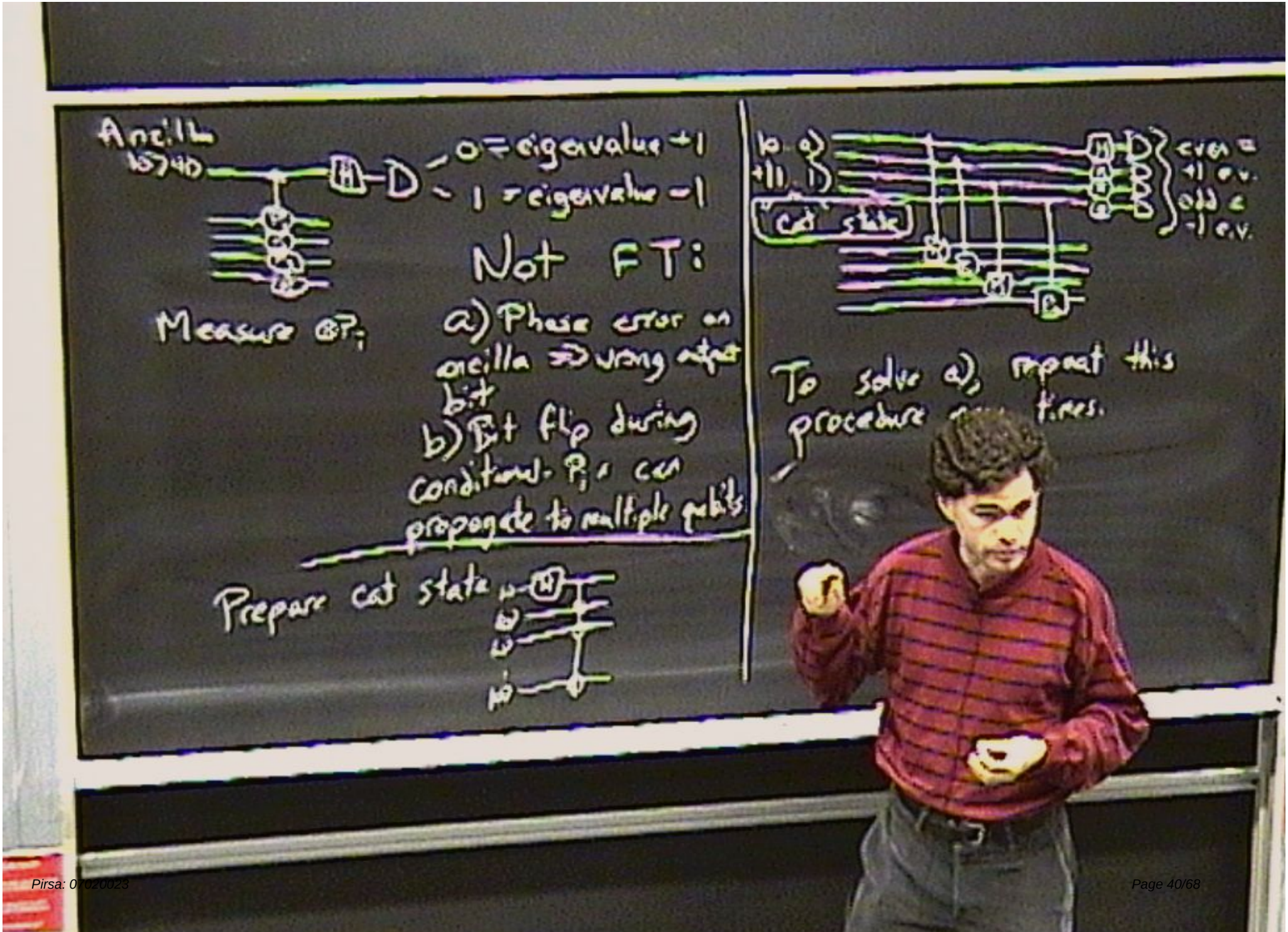

Ancilla
|0⟩
H
D
0 ≙ eigenvalue +1
1 ≙ eigenvalue −1
Measure @ P_i
Not FT:
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional-P_i's can propagate to multiple qubits
|0 0⟩
+|1 1⟩
cat state
even = +1 e.v.
odd = −1 e.v.

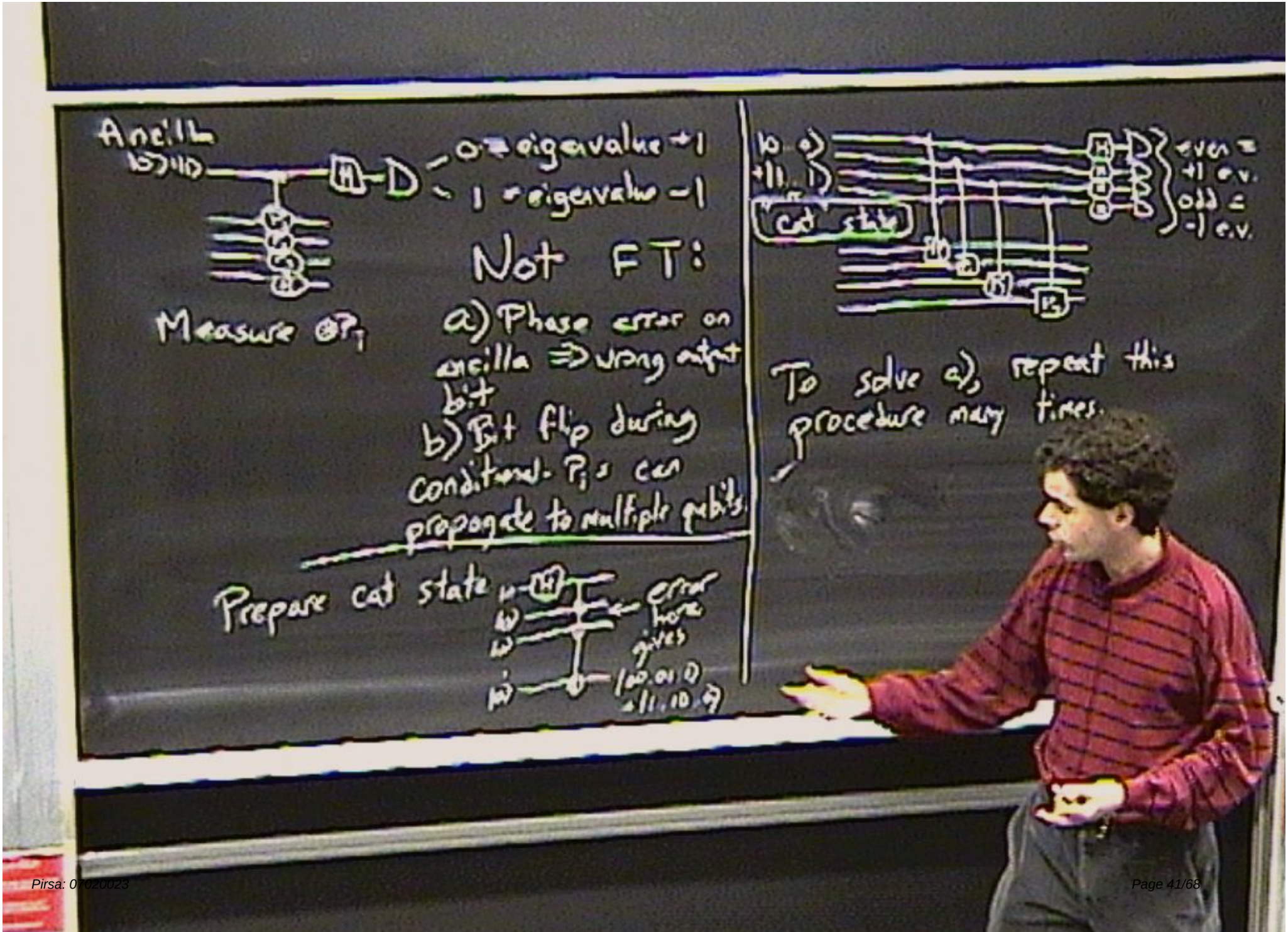

Ancilla
|0⟩+|1⟩
H
D
0 = eigenvalue +1
1 = eigenvalue −1
Measure of P_i
Not FT:
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional-P_i's can propagate to multiple qubits
|0 0⟩
+|1 1⟩
cat state
even = +1 e.v.
odd = −1 e.v.
To solve a), repeat this procedure many times.


④ FT error correction:
EC Property 1:
EC
EC
(r<t)
EC Property 2:
EC
(r≤s<t)


Ancilla
H
D
0 ⇒ eigenvalue +1
1 ⇒ eigenvalue −1
Measure P_i
Not FT:
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional. P_i's can propagate to multiple qubits.
"cat state"
even = +1 e.v.
odd = −1 e.v.
To solve a), repeat this procedure many times.


Ancilla
0 = eigenvalue +1
1 = eigenvalue −1
Not FT:
Measure $\otimes P_i$
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional-P_i's can propagate to multiple qubits.
cat state
even = +1 e.v.
odd = −1 e.v.
To solve a), repeat this procedure many times.
Prepare cat state


Ancilla
0 = eigenvalue +1
1 = eigenvalue −1
Not FT:
Measure $\otimes P_i$
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional P_i can propagate to multiple qubits
Prepare cat state
cat state
even = +1 e.v.
odd = −1 e.v.
To solve a), repeat this procedure
times.


Ancilla
0 = eigenvalue +1
1 = eigenvalue −1
Not FT:
Measure ⊗P_i
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional-P_i's can propagate to multiple qubits
Prepare cat state
error here gives
cat state
even = +1 e.v.
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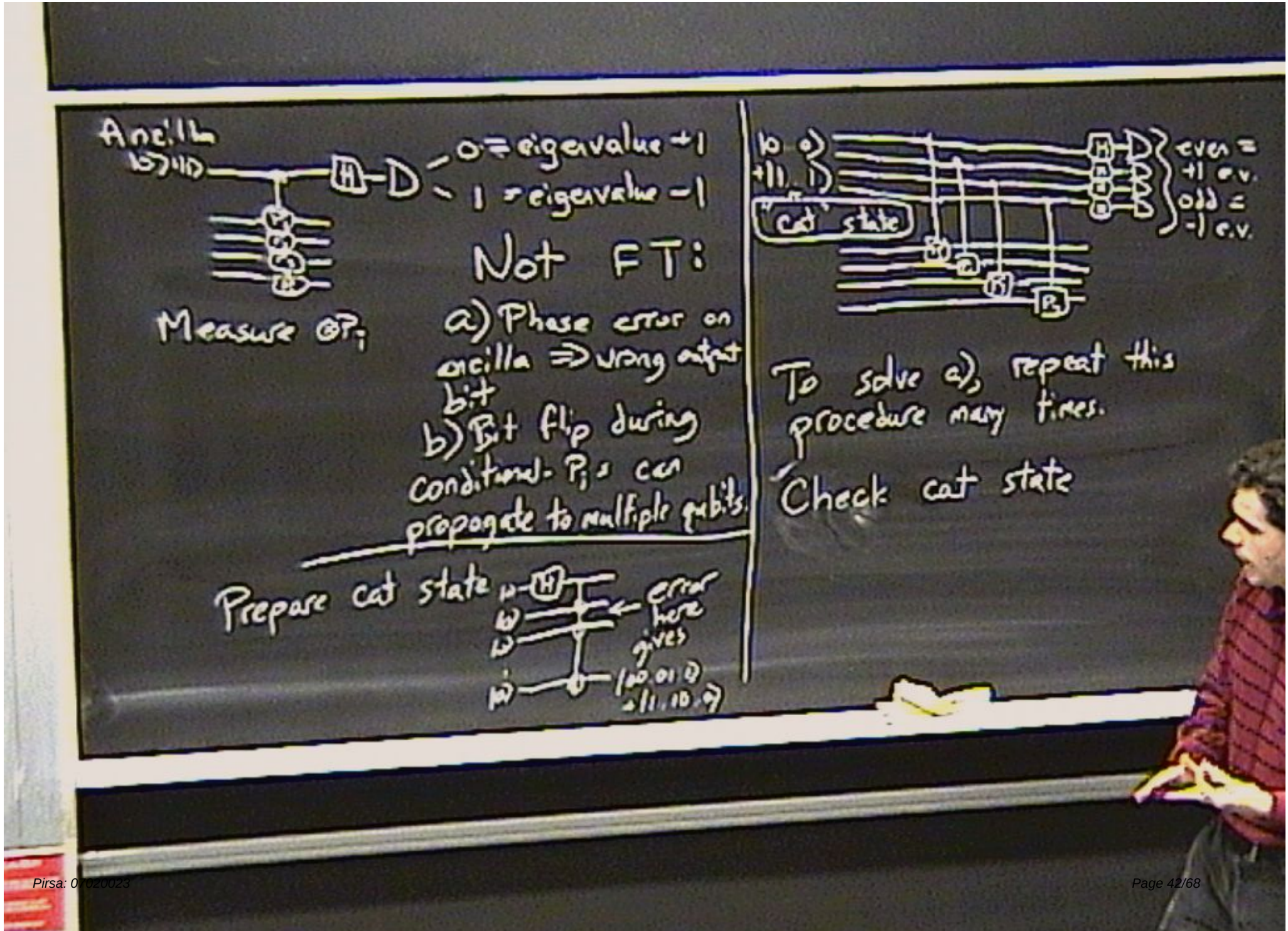
To solve a), repeat this procedure many times.


Ancilla
H
D
0 = eigenvalue +1
1 = eigenvalue -1
Measure $\otimes P_i$
Not FT:
a) Phase error on ancilla ⇒ wrong output bit
b) Bit flip during conditional. P_i's can propagate to multiple qubits.
cat state
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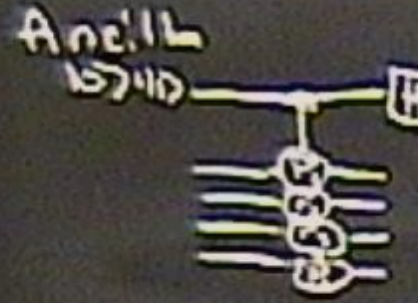
Check cat state
Prepare cat state
error here gives
|00..01⟩ + |11..10⟩

Ancilla



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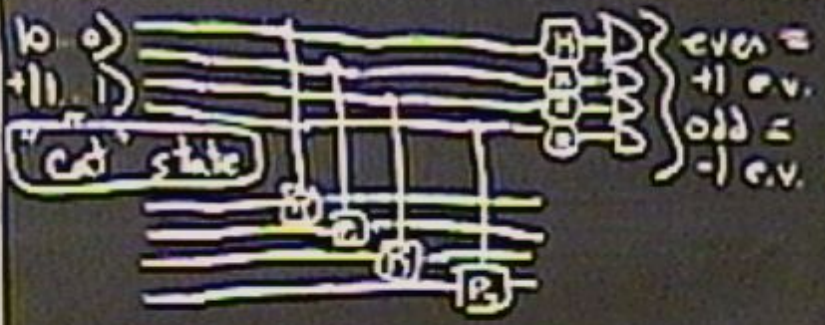
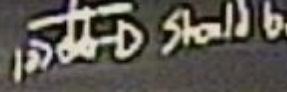


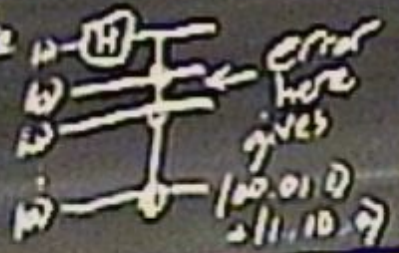
To solve a), repeat this procedure many times.

Check cat state:

E.g. compare random pairs of bits



Prepare cat state



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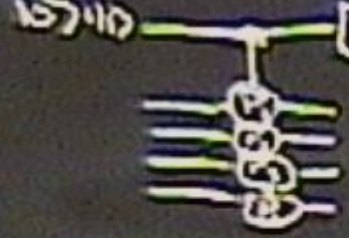

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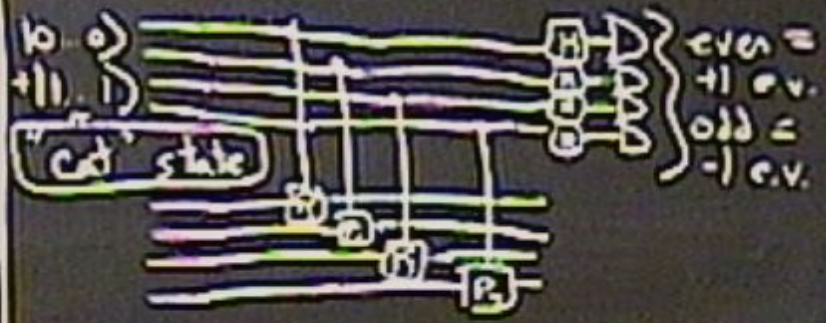

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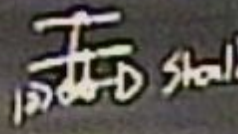

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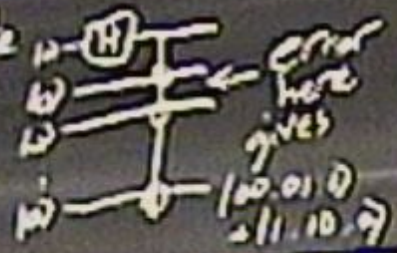
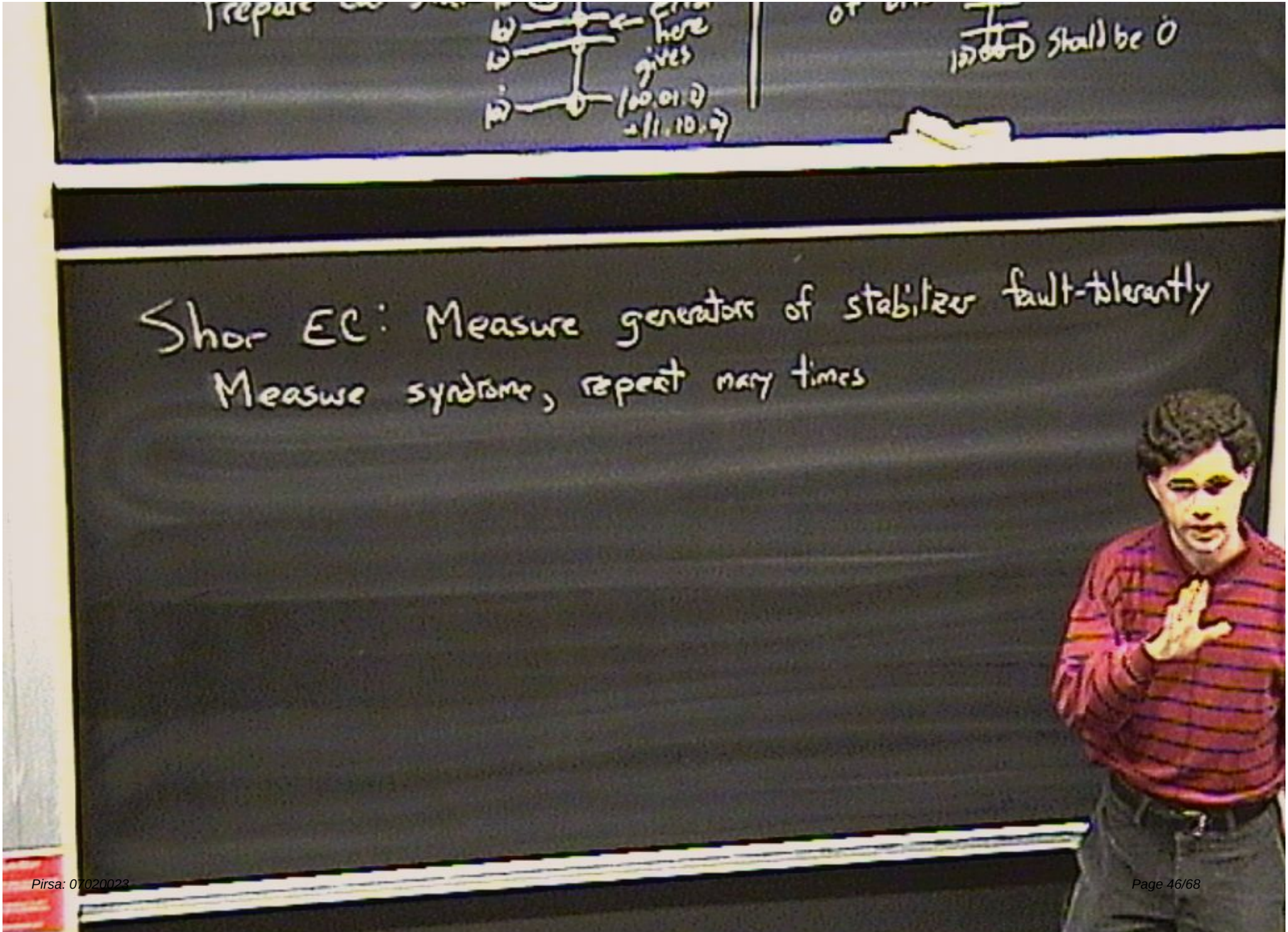


shall be 0

Shor EC: Measure generators of stabilizer fault-tolerantly


here
gives
Shall be 0
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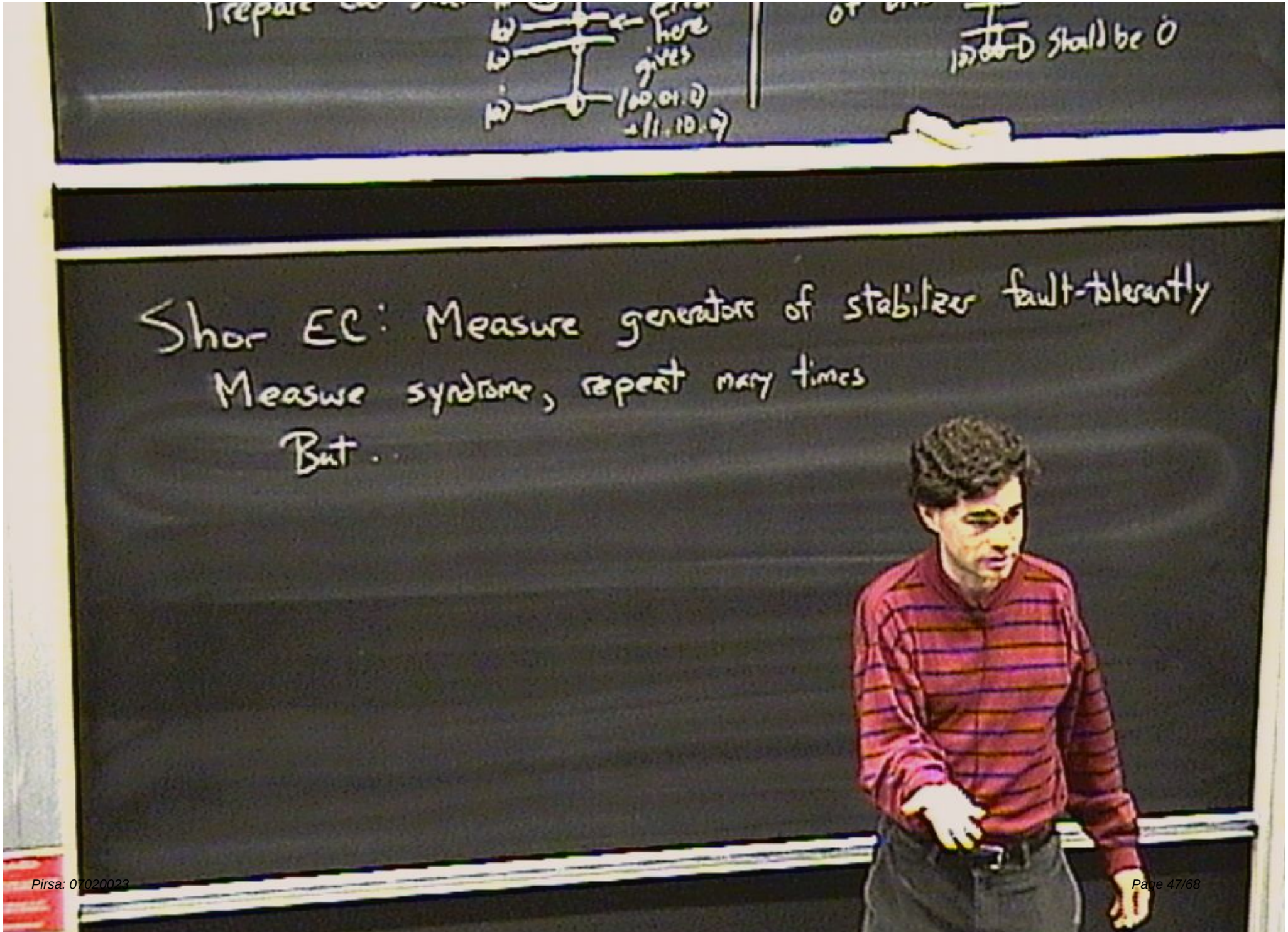
Measure syndrome, repeat many times


Shor EC: Measure generators of stabilizer fault-tolerantly
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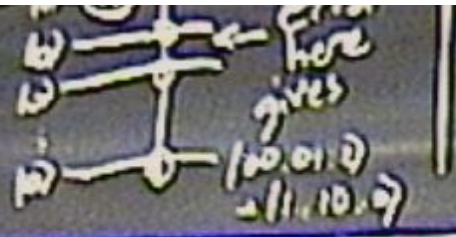
But...



shall be 0

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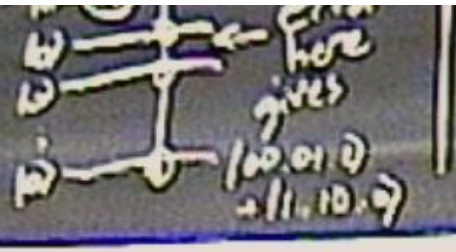
But... an error in data block during syndrome measurement will change the syndrome.



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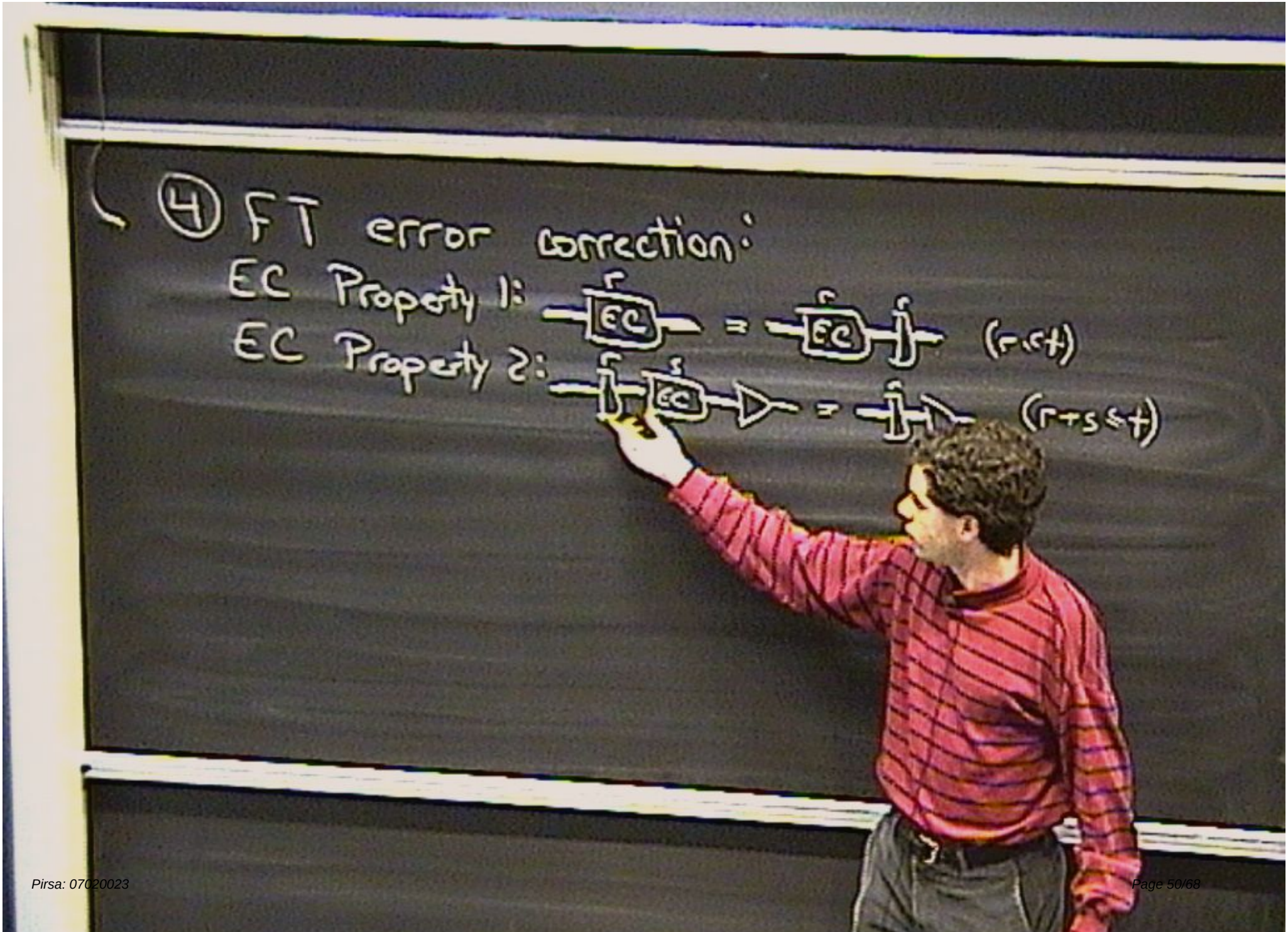
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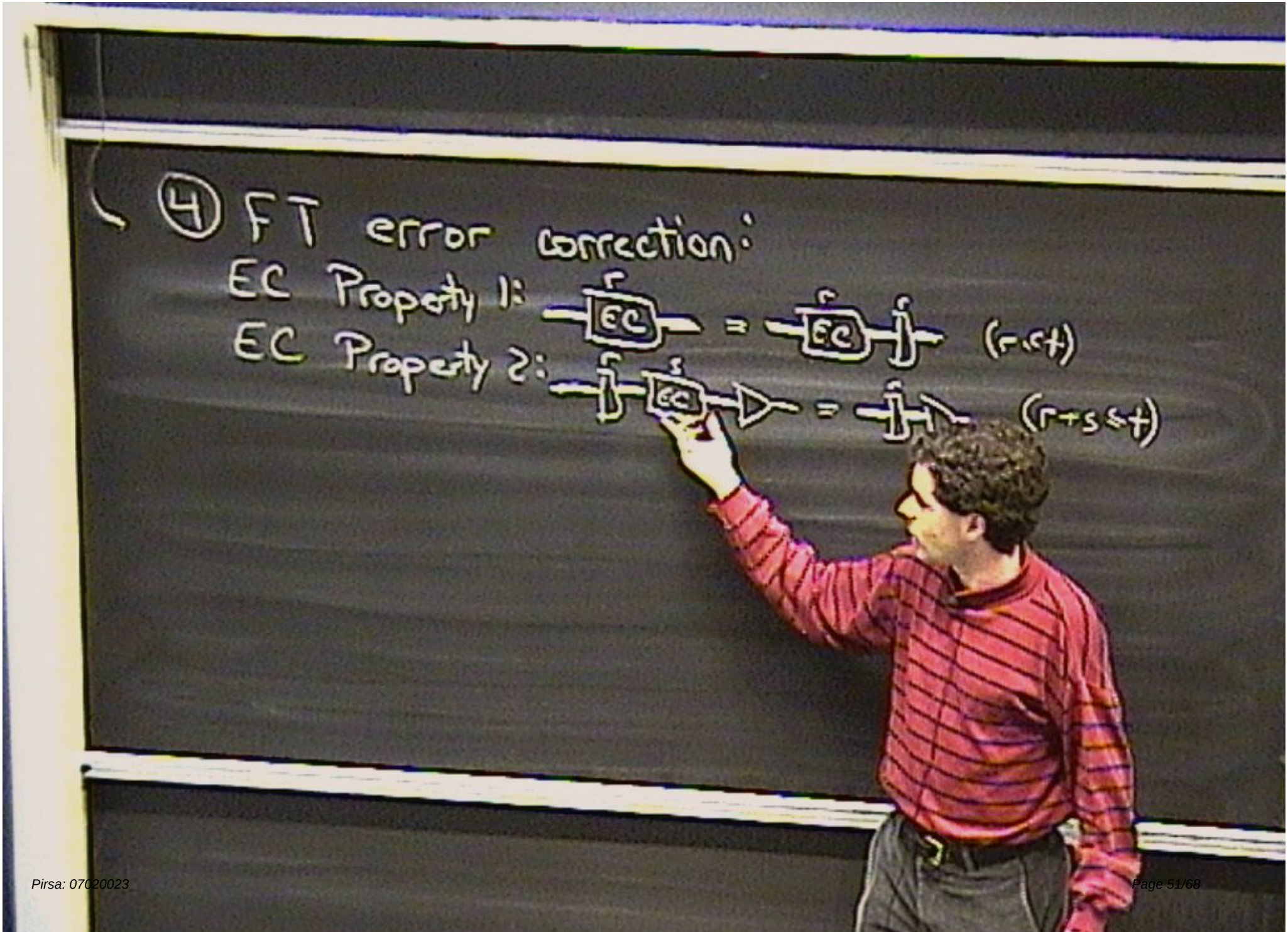
EC Property 1:
EC
EC
(r<t)
EC Property 2:
EC
(r+s≤t)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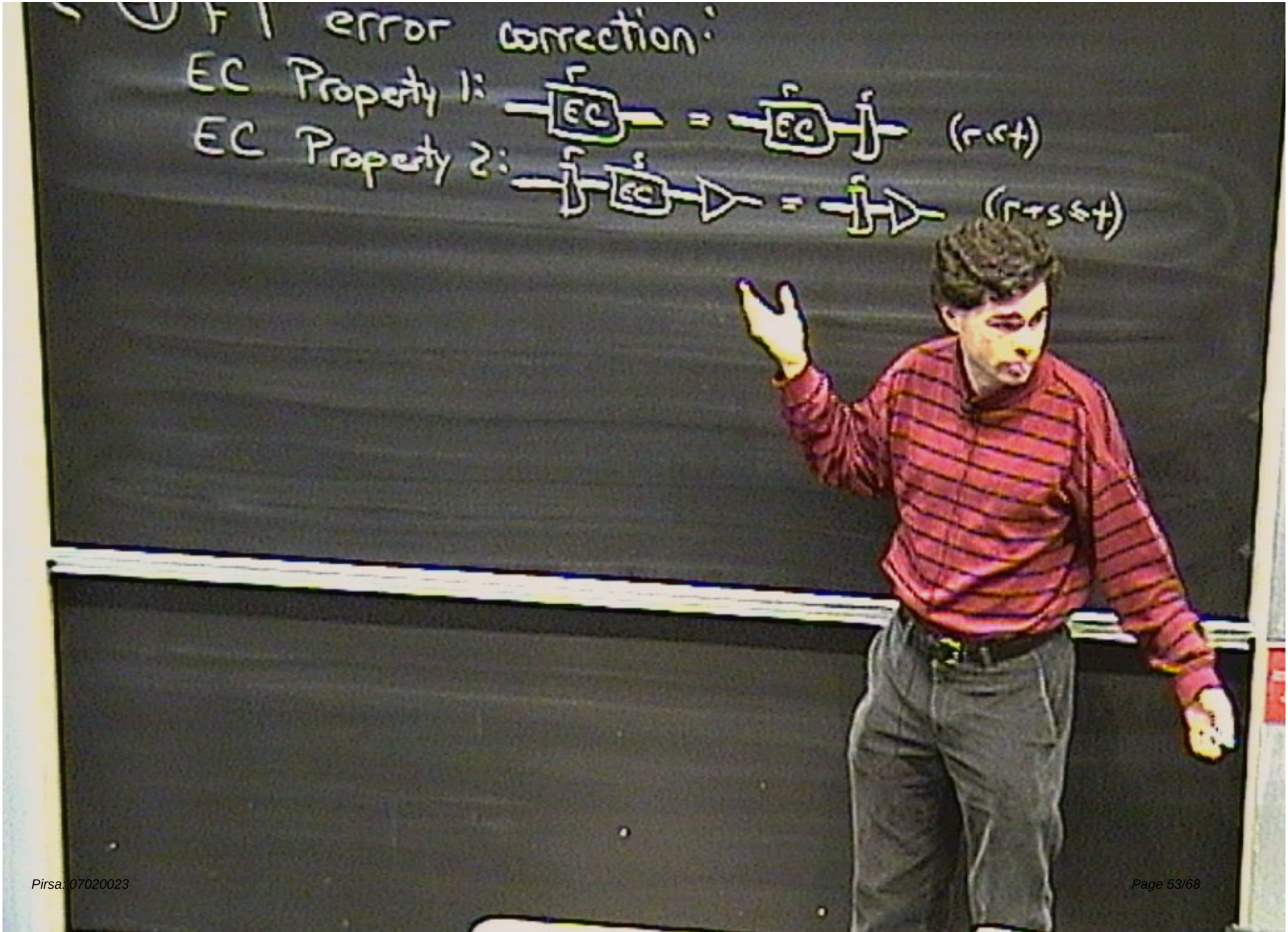
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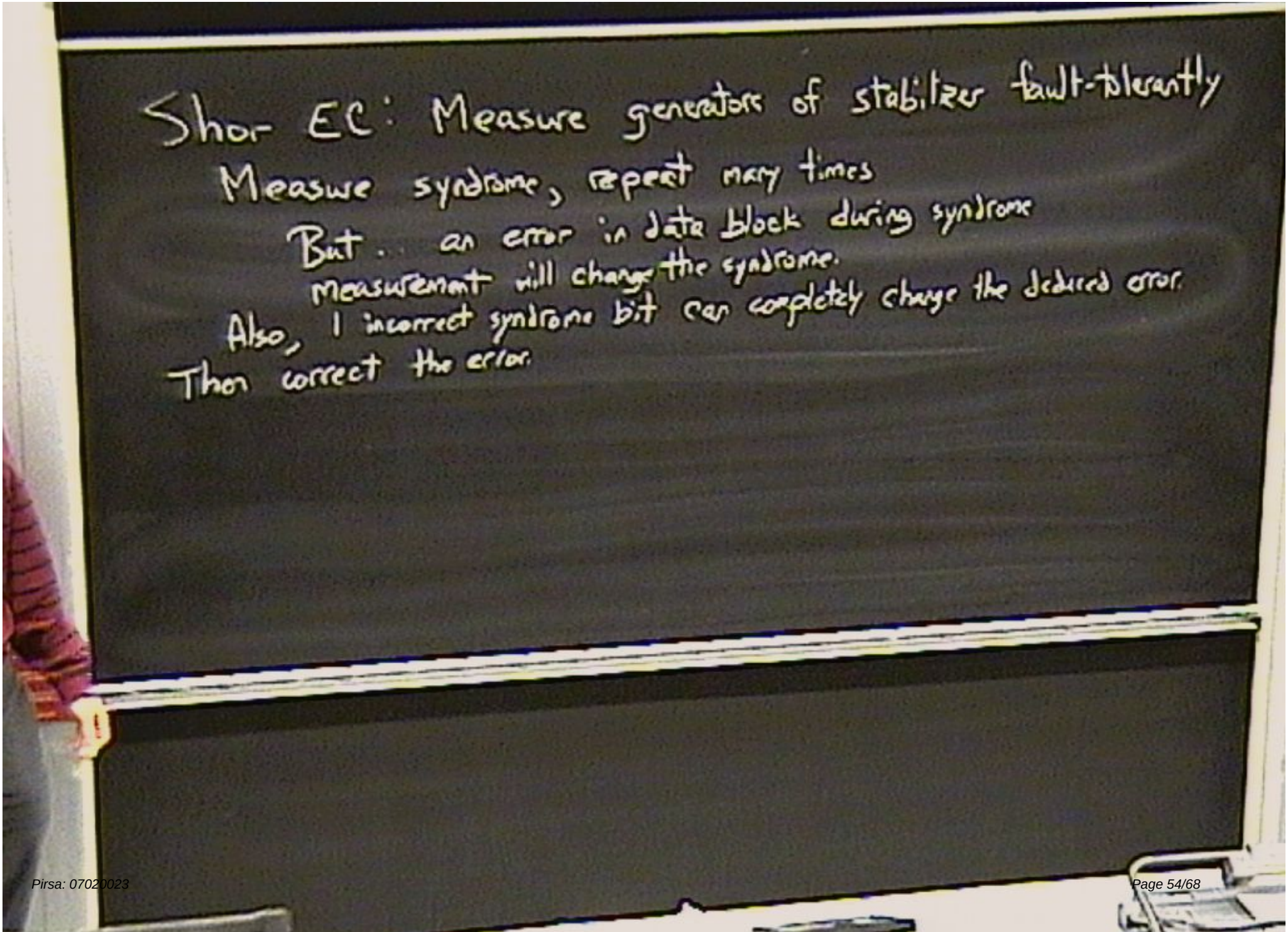
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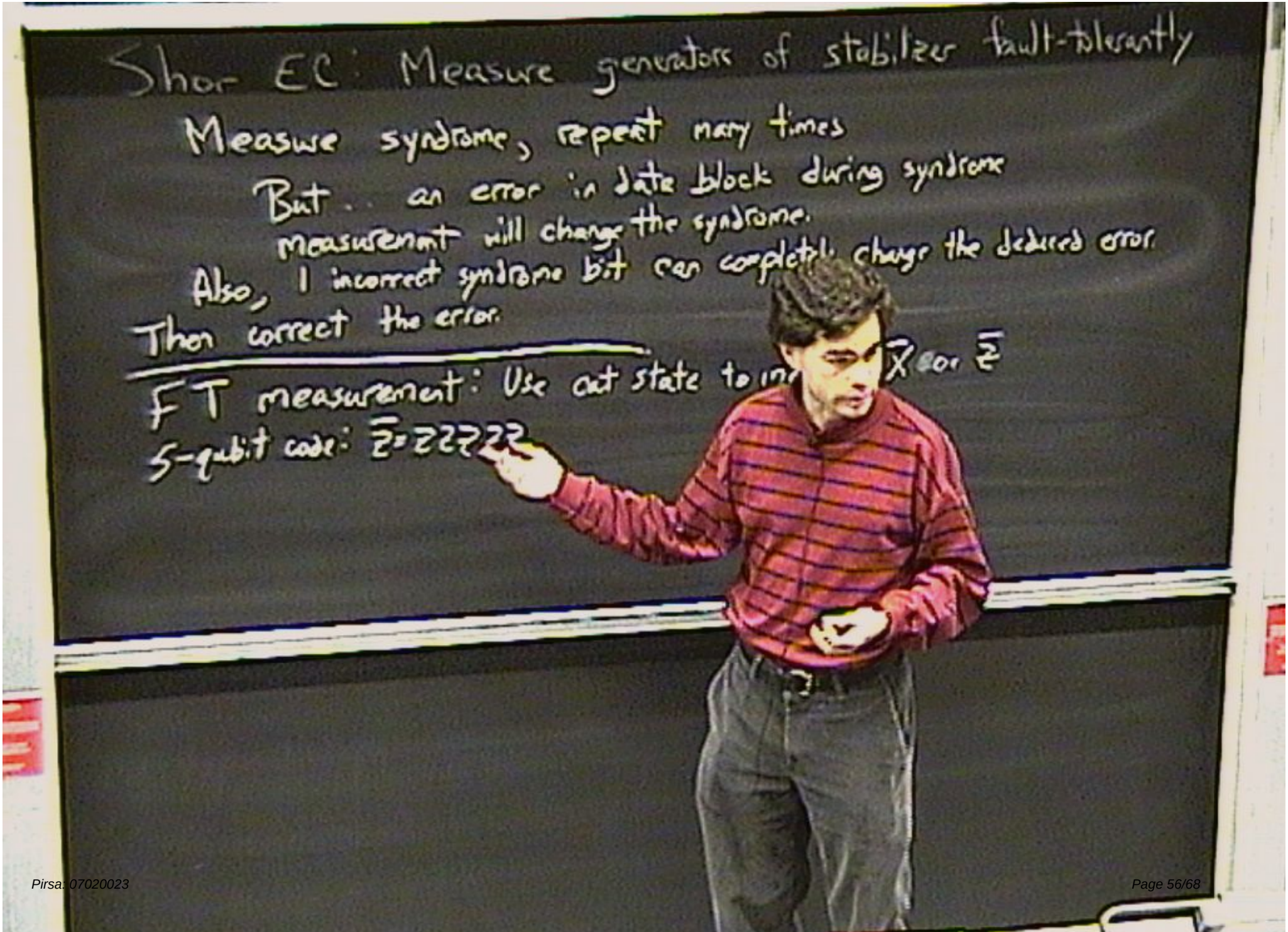
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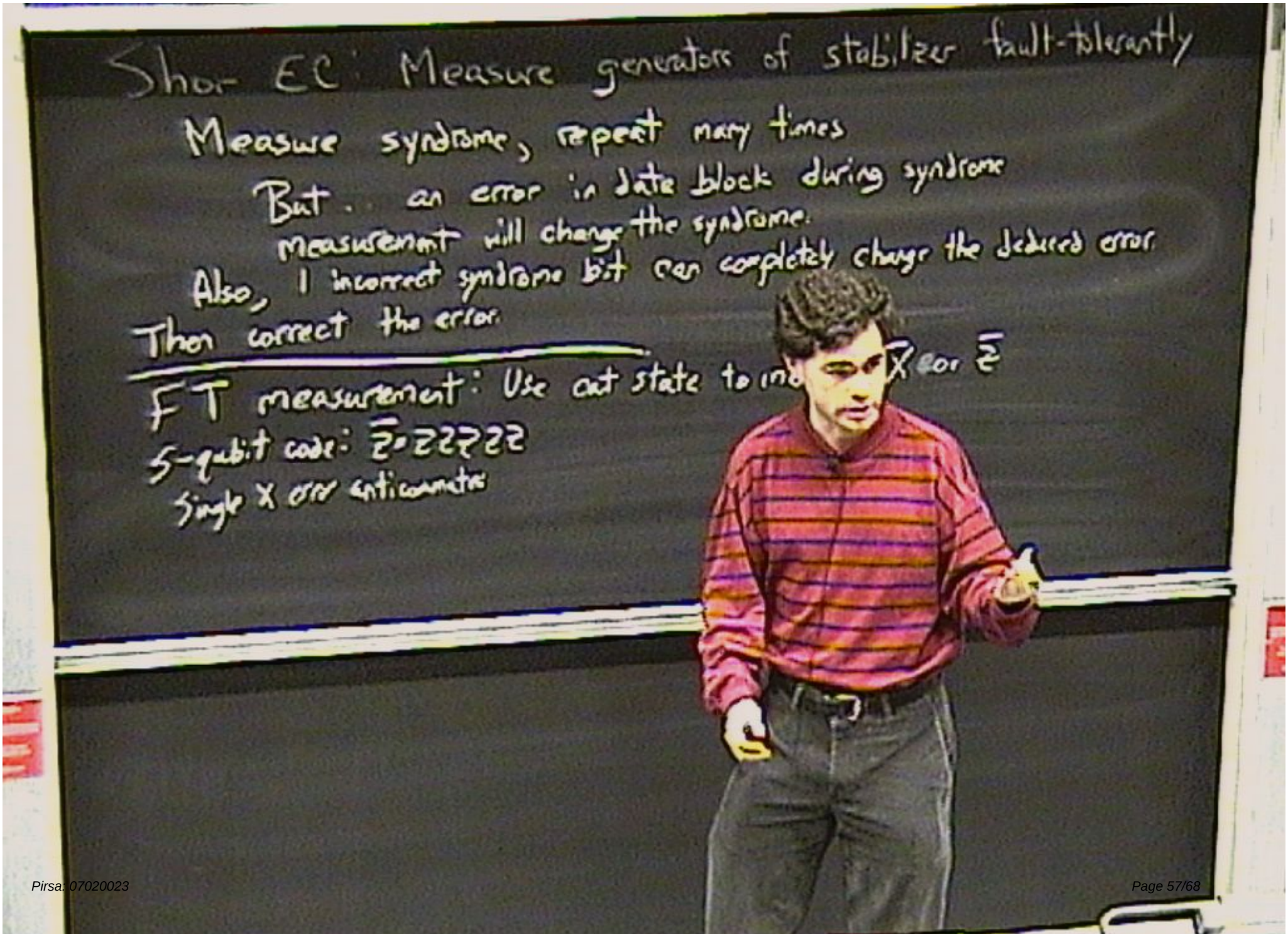
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FT measurement: Use cat state to ind
X or Z̄
5-qubit code: Z̄=ZZZZZ
Single X error anticommutes

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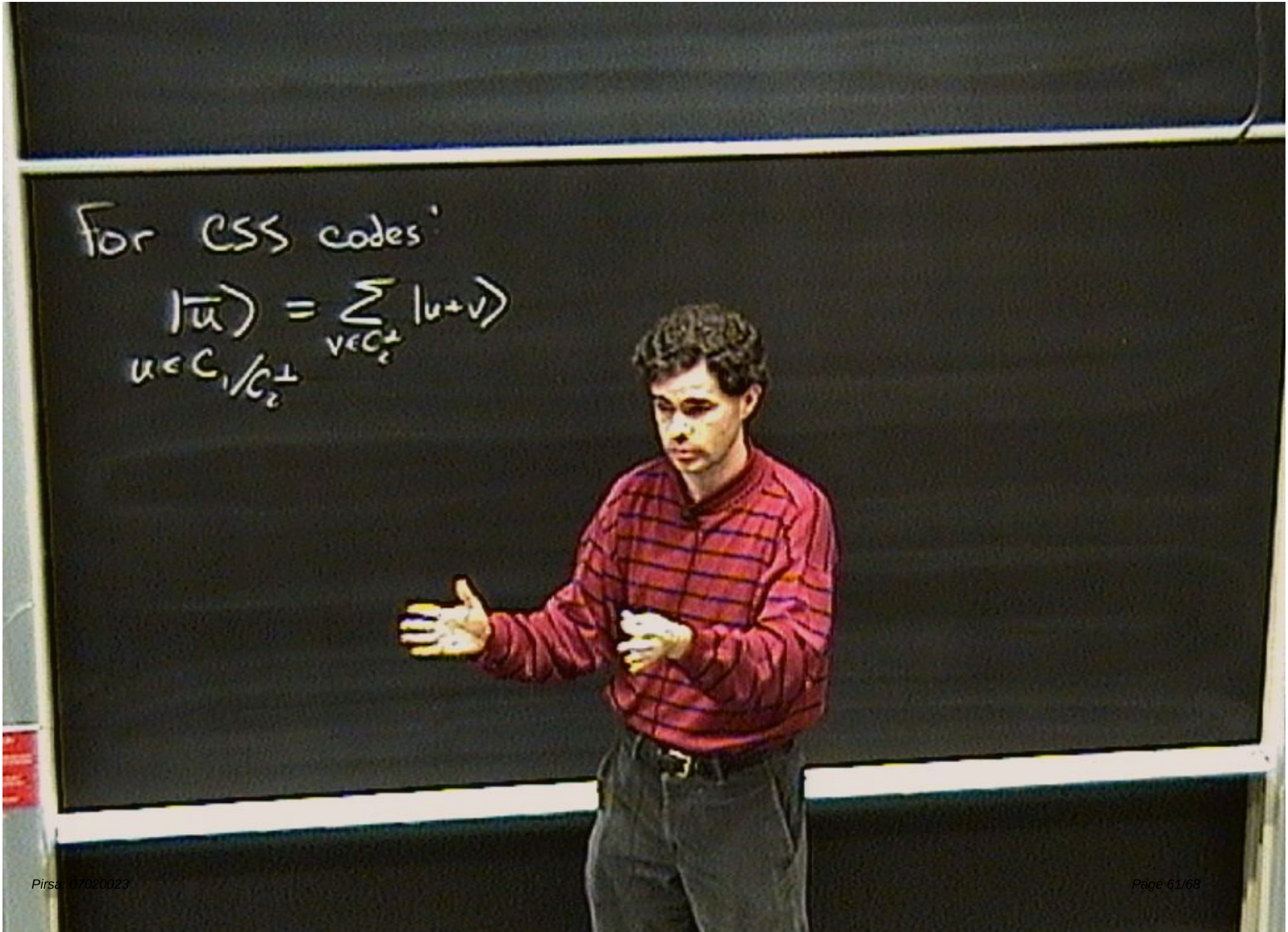
For CSS codes
$|\overline{u}\rangle = \sum_{v \in C_2^\perp} |u+v\rangle$
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$$|\overline{u}\rangle = \sum_{v \in C_2^\perp} |u+v\rangle \qquad u \in C_1/C_2^\perp$$

$\Rightarrow$ Measure transversally. When no errors, get random codewords from $u+C_2^\perp$.
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Bit flip errors $e \rightarrow \sum_{v \in C_2^\perp} |u+v+e\rangle$

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Codewords of C_1 w/ error e ⇒ classical error correction corrects

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Deduce u as before

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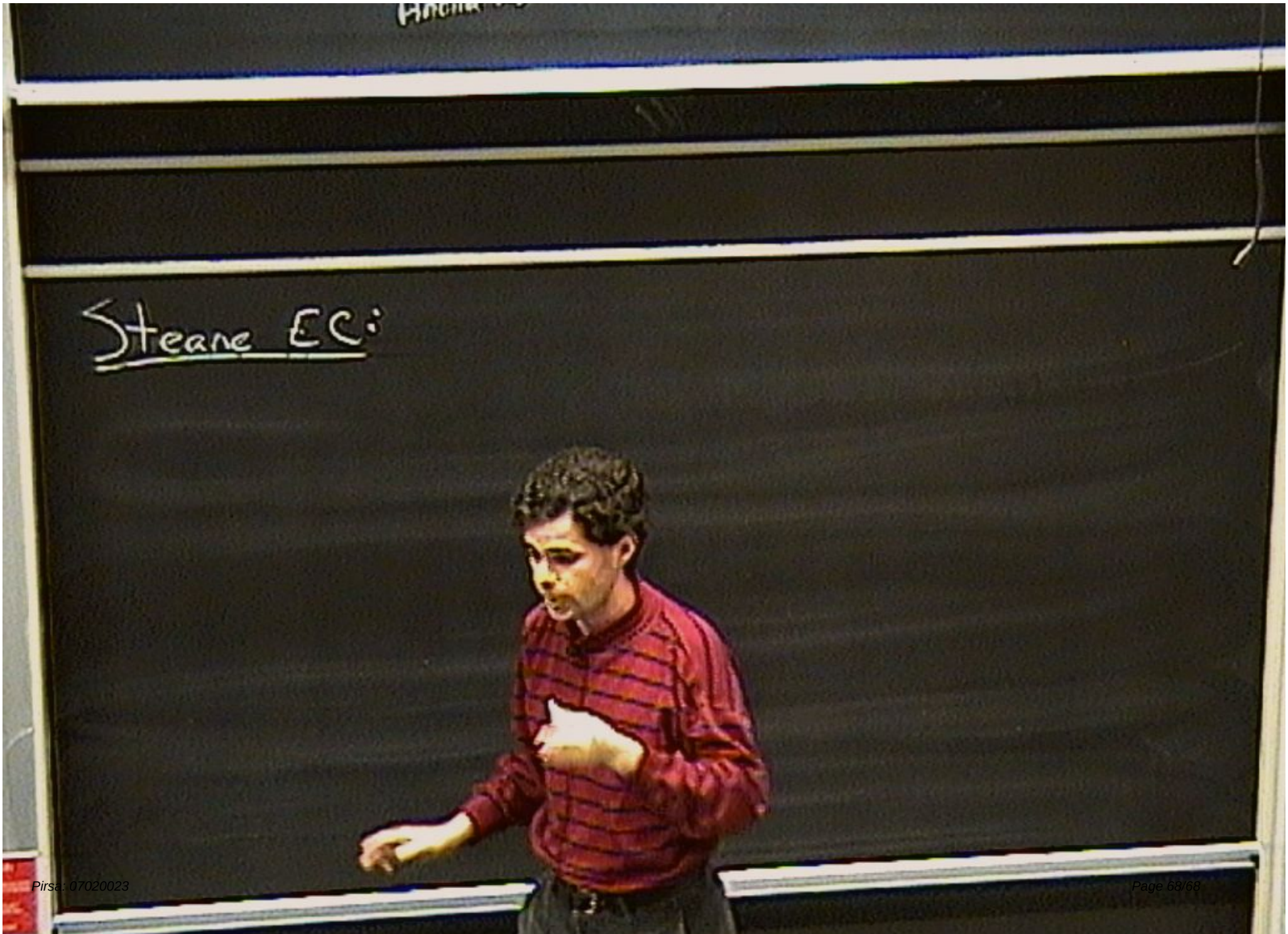
Steane EC: