Title: Quantum Error Correction 6B

Date: Feb 13, 2007 05:00 PM

URL: http://pirsa.org/07020022

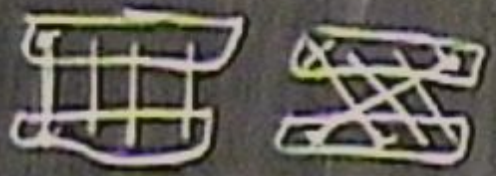
Abstract: Transversal Pauli group, transversal Clifford group for 7-qubit code, transversal gates for 5-qubit code, overview of fault-tolerant protocols.

Transversal gates:

#1 Block

#2 Code block

Interact ith qubit of one block only with ith qubit of other blocks.



1 error can only propagate to 1 error per block

(even in a sequence of transversal gates)

Terminology:

Transversal gates:

#1 Block of code

#2 Code block

Interact ith qubit of one block only with ith qubits of other blocks.

1 error can only propagate to <u>1 error per block</u>

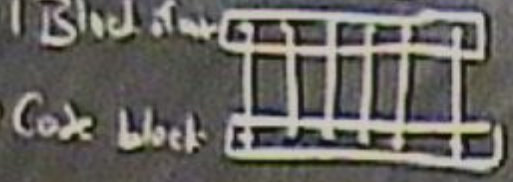
(even in a sequence of transversal gates)

Terminology: "transversal U" = $U^{\otimes n}$, physical U on each qubit

Transversal gates:

#1 Block of qubits

#2 Code block



Interact ith qubit of one block only with ith qubits of other blocks.

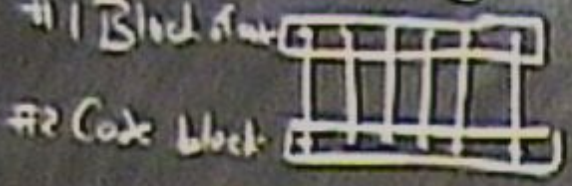
1 error can only propagate to <u>1 error per block</u>

(even in a sequence of transversal gates)

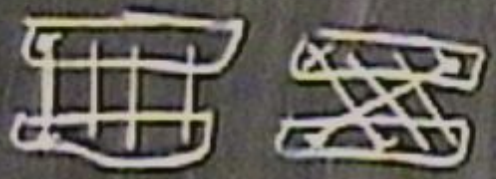
Terminology: "transversal U" = $U^{\otimes n}$, physical U on each qubit

"U can be ... transversally" = transversal gate that

Transversal gates:



Interact ith qubit of one block only with ith qubits of other blocks.



1 error can only propagate to <u>1 error per block</u>

(even in a sequence of transversal gates)

Terminology: "transversal U" = $U^{\otimes n}$, physical U on each qubit

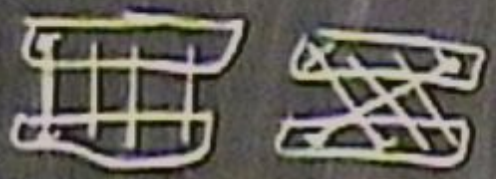
"U can be implemented transversally" = transversal gate that does a logical U

Transversal gates:

#1 Block of code

#2 Code block

Interact ith qubit of one block only with ith qubits of other blocks.



1 error can only propagate to <u>1 error per block</u>

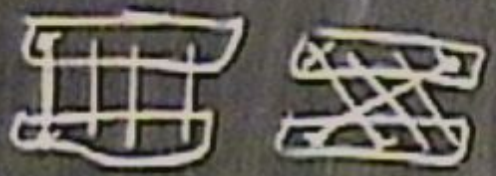
(even in a sequence of transversal gates)

Terminology: "transversal U" = $U^{\otimes n}$, physical U on each qubit

"U can be implemented transversally" = transversal gate that does a logical U (might be different physical gates on different qubits)

Transversal gates:

Interact ith qubit of one block only with ith qubits of other blocks.



1 error can only propagate to <u>1 error per block</u>

(even in a sequence of transversal gates)

Terminology: "transversal U" = $U^{\otimes n}$, physical U on each qubit

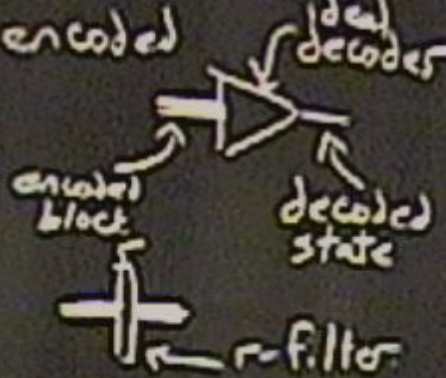
"U can be implemented transversally" = transversal gate that does a logical U (might be different physical gates on different qubits)

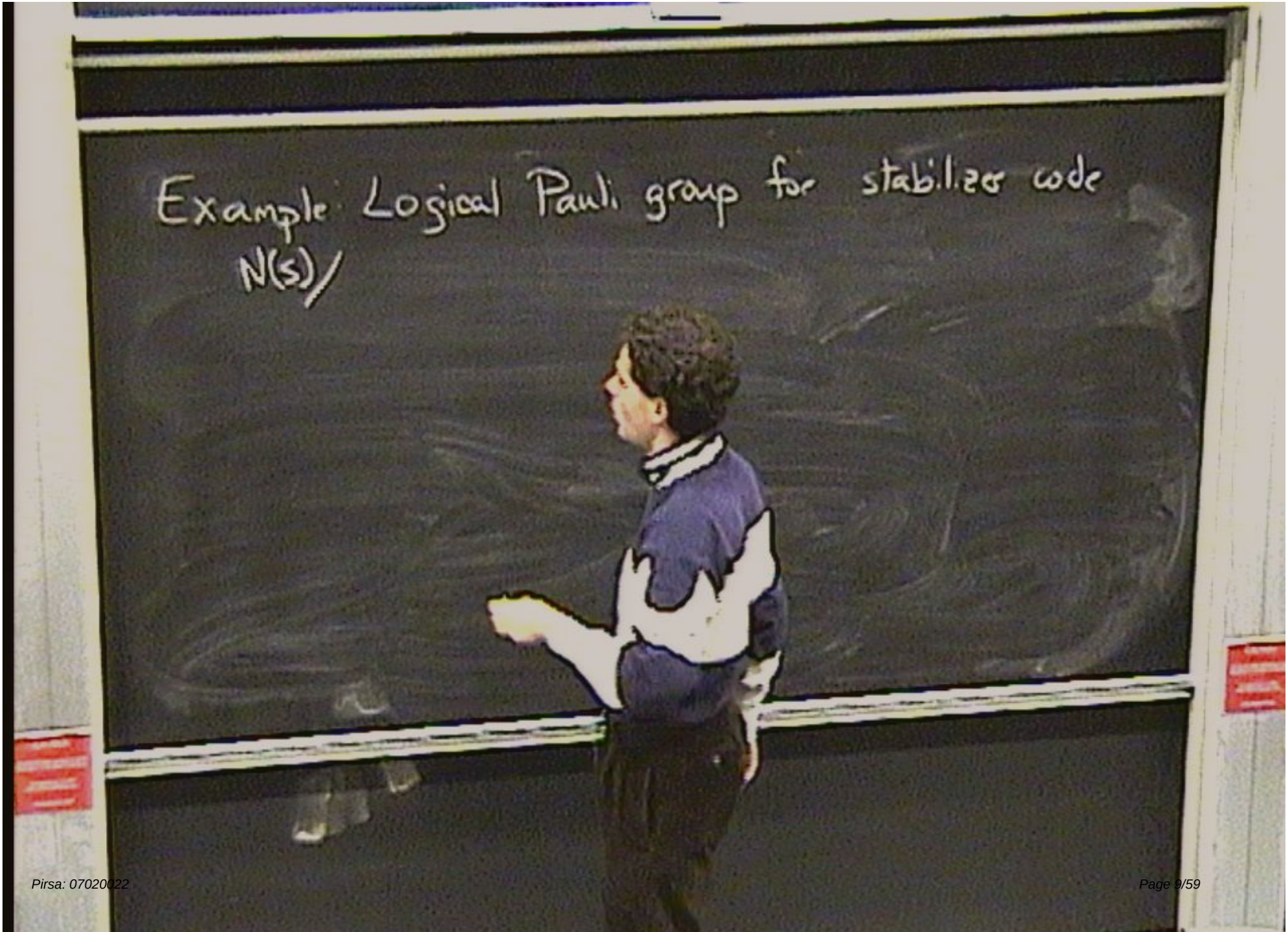
, physical U on each qubit

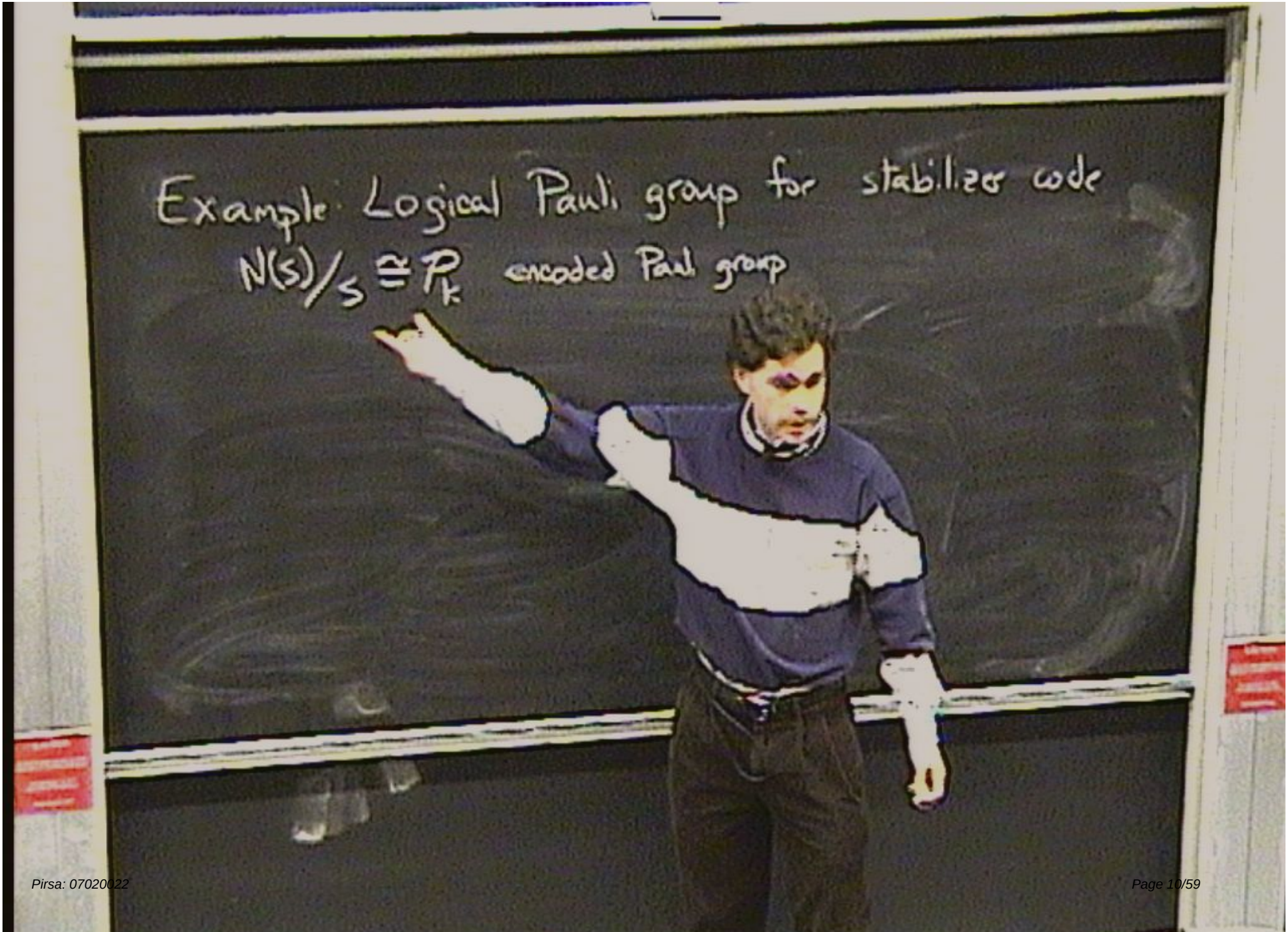
"U can be implemented transversally" = transversal gate that does a logical U (might be different physical gates on different qubits)

Def: The ideal decoder takes an encoded state, corrects any errors, & decodes.

Def: Let the r-filter be the projector on subspace spanned by codewords w/ up to r errors.



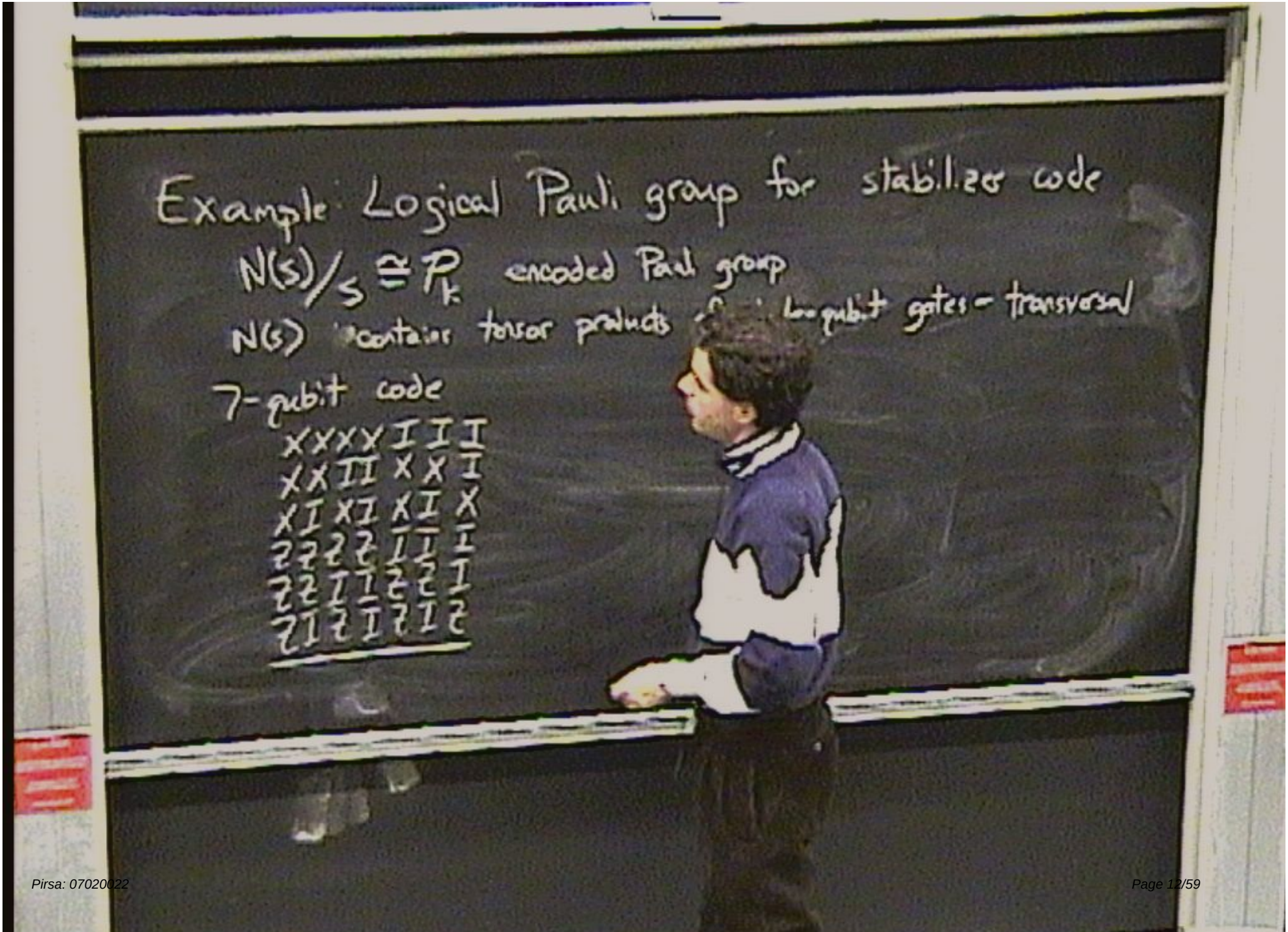

Example Logical Pauli group for stabilizer code
N(S)/

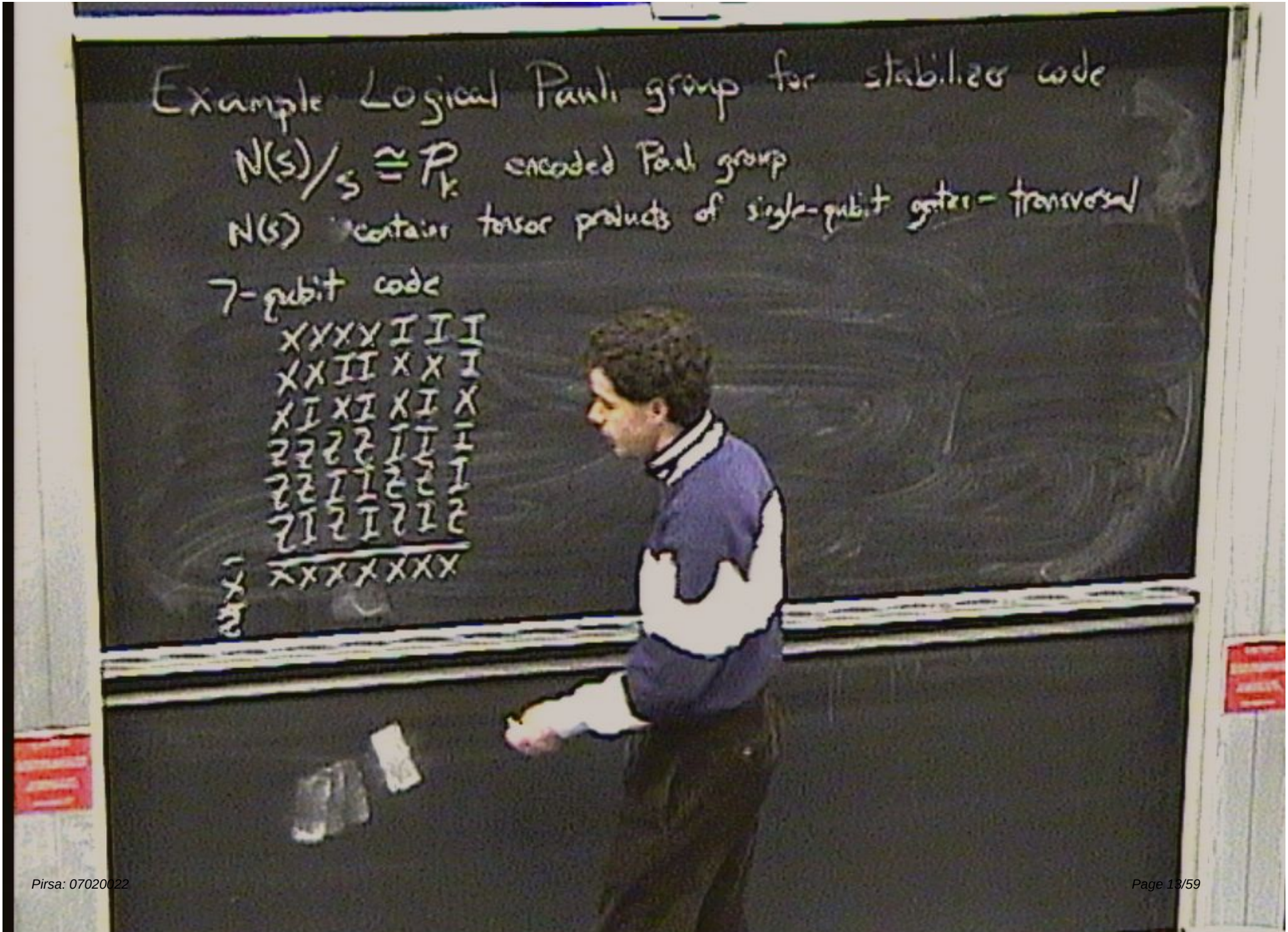

Example: Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group

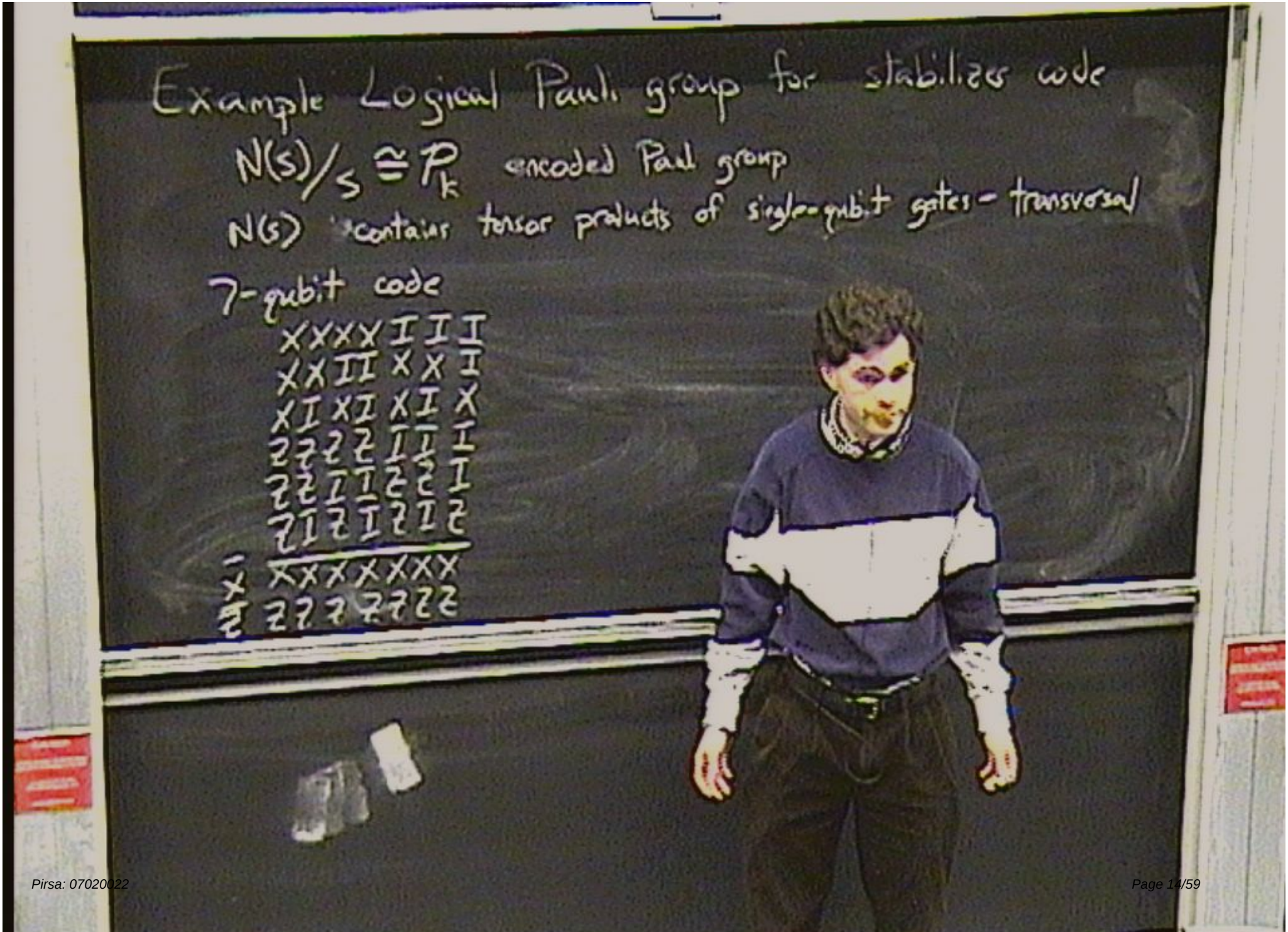
Example: Logical Pauli group for stabilizer code

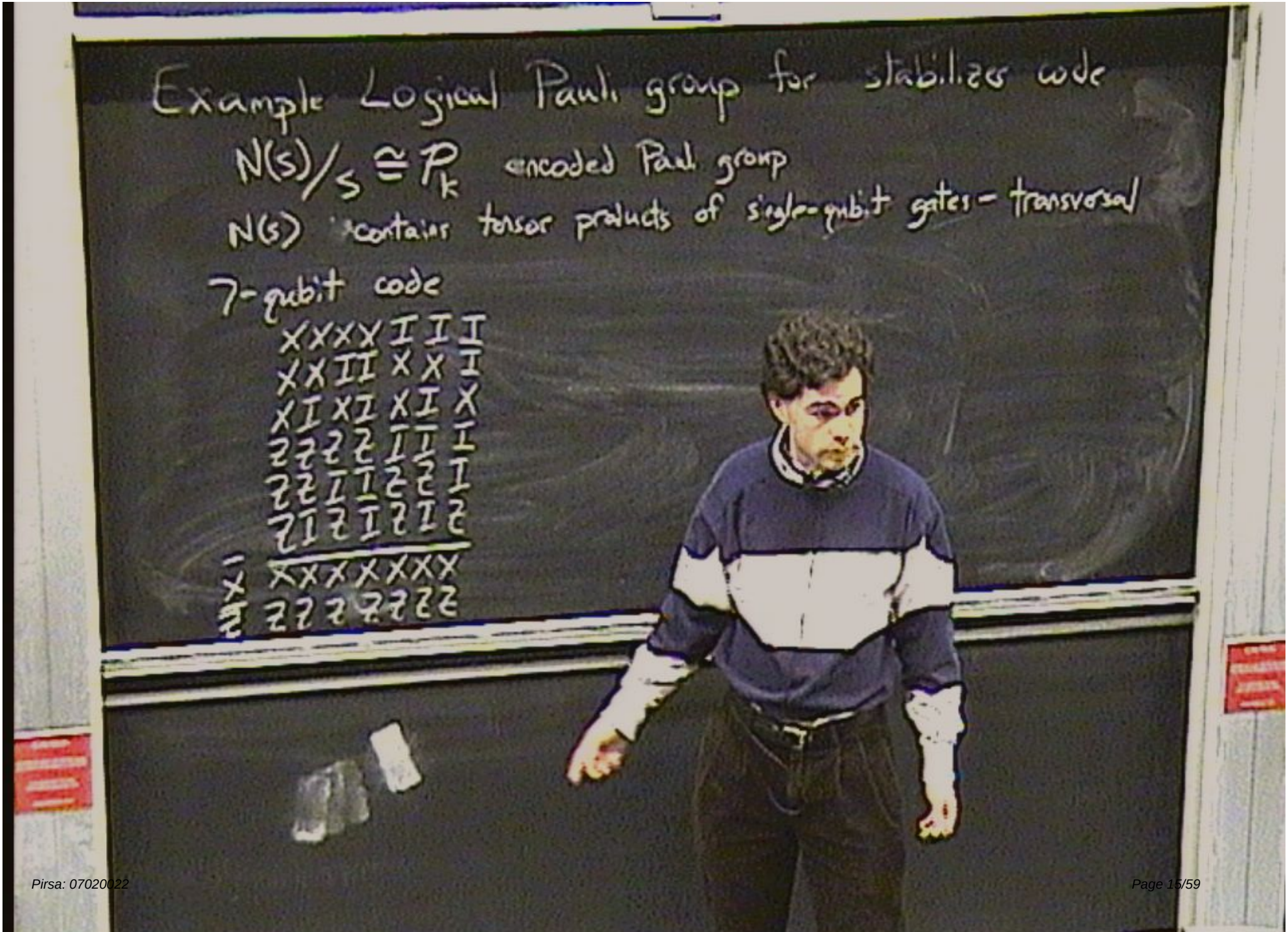
$N(S)/S \cong P_k$ encoded Pauli group

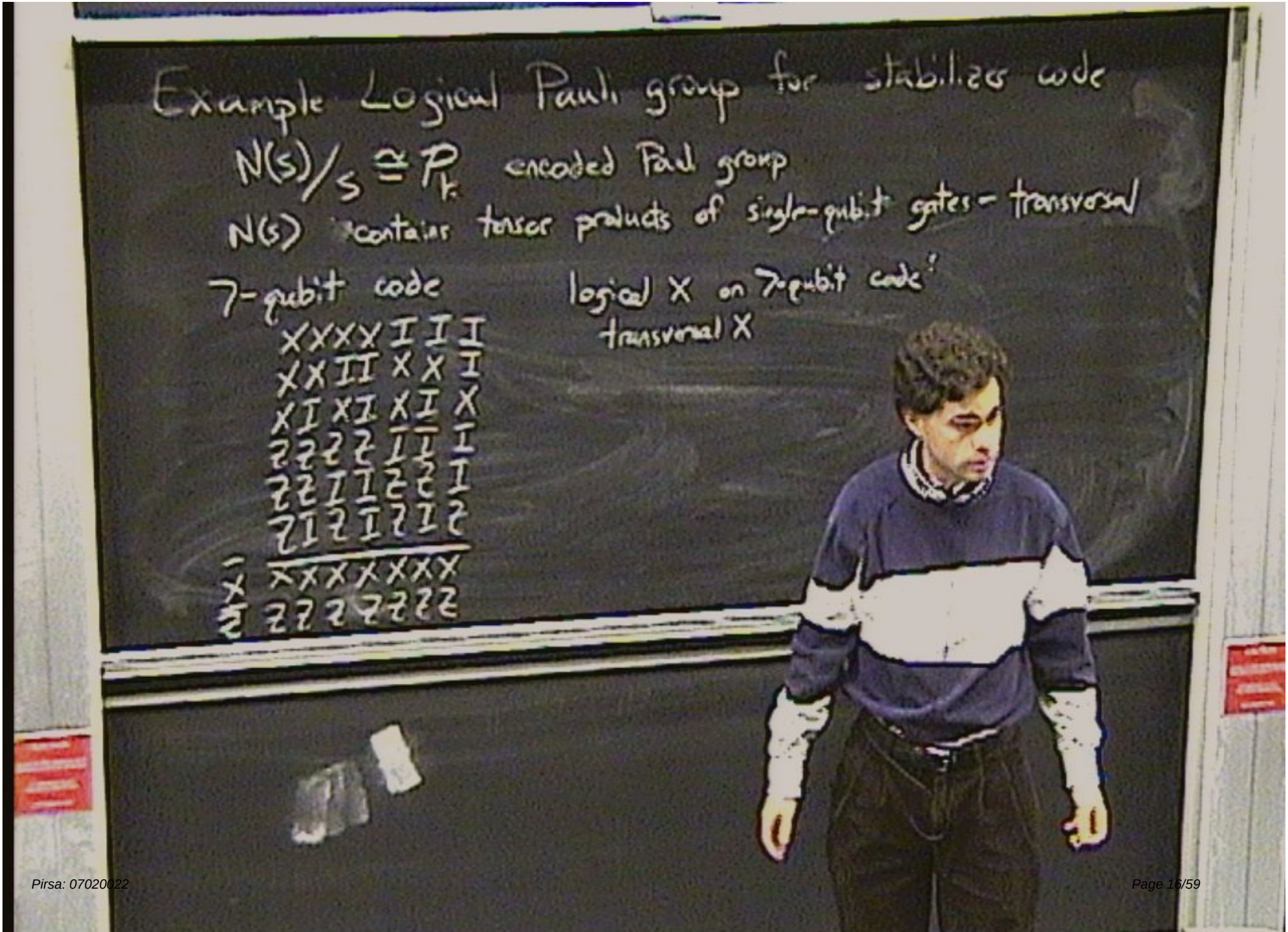
$N(S)$ contains tensor products of single-qubit gates - transversal

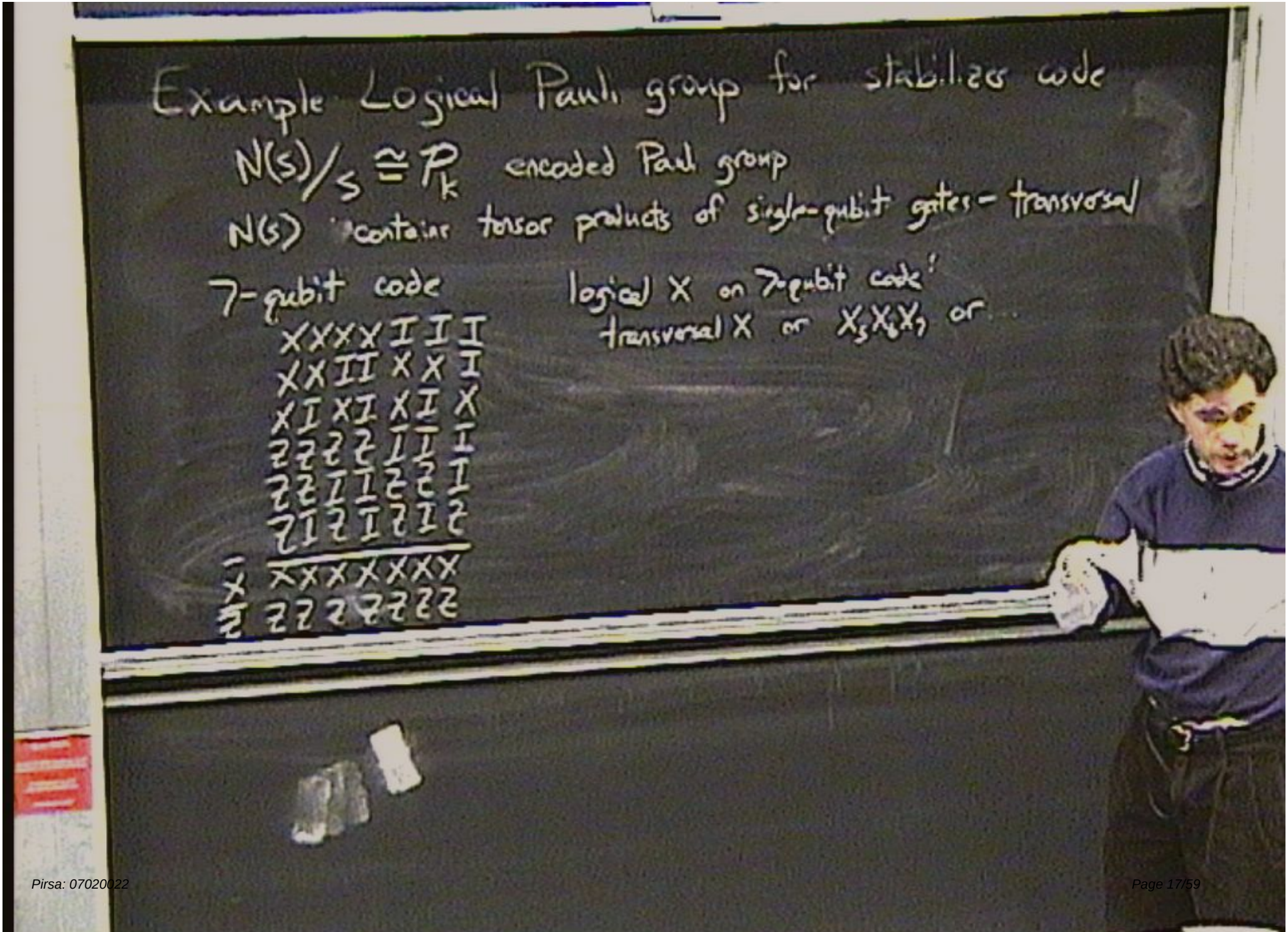

Example: Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
N(S) contains tensor products of 1-qubit gates - transversal
7-qubit code
XXXXIII
XXIIXXI
XIXIXIX
ZZZZIII
ZZIIZZI
ZIZIZIZ

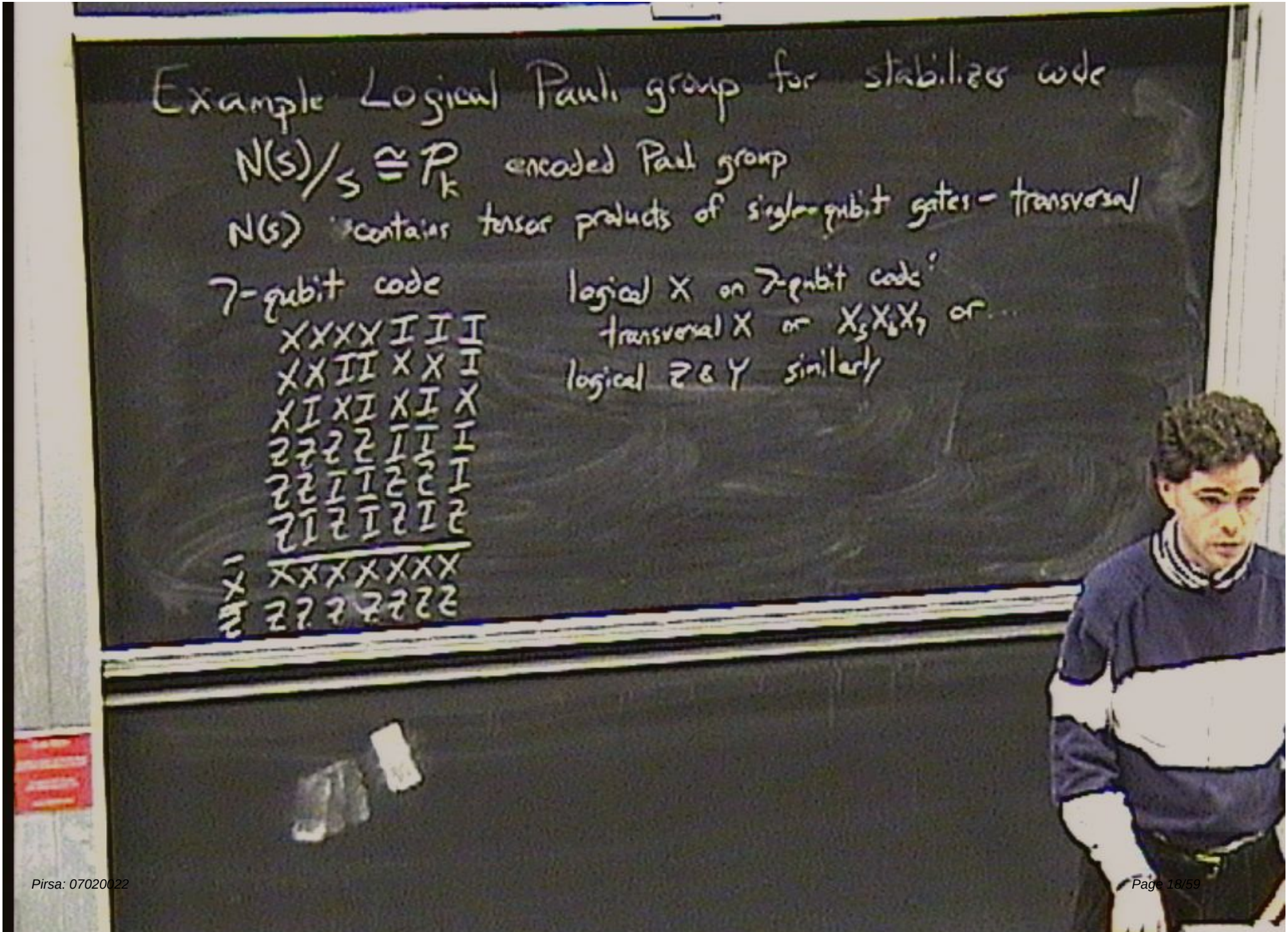

Example Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
N(S) contains tensor products of single-qubit gates - transversal
7-qubit code
XXXXIII
XXIIXXI
XIXIXIX
IIIXXXX
IIXXIIX
IXIXIXI
$\bar{X}$ XXXXXXX
$\bar{Z}$


Example Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
N(S) contains tensor products of single-qubit gates - transversal
7-qubit code
XXXXIII
XXIIXXI
XIXIXIX
ZZZZIII
ZZIIZZI
ZIZIZIZ
XXXXXXX
ZZZZZZZ


Example Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
N(S) contains tensor products of single-qubit gates - transversal
7-qubit code
XXXXIII
XXIIXXI
XIXIXIX
ZZZZIII
ZZIIZZI
ZIZIZIZ
XXXXXXX
ZZZZZZZ


Example Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
N(S) contains tensor products of single-qubit gates - transversal
7-qubit code
logical X on 7-qubit code
transversal X
$\bar{X}$
$\bar{Z}$


Example Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
N(S) contains tensor products of single-qubit gates - transversal
7-qubit code
XXXXIII
XXIIXXI
XIXIXIX
ZZZZIII
ZZIIZZI
ZIZIZIZ
$\bar{X}$ XXXXXXX
$\bar{Z}$ ZZZZZZZ
logical X on 7-qubit code
transversal X or $X_5X_6X_7$ or


Example Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
N(S) contains tensor products of single-qubit gates - transversal
7-qubit code
XXXXIII
XXIIXXI
XIXIXIX
ZZZZIII
ZZIIZZI
ZIZIZIZ
$\bar{X}$ XXXXXXX
$\bar{Z}$ ZZZZZZZ
logical X on 7-qubit code:
transversal X or $X_5X_6X_7$ or...
logical Z & Y similarly

Example Logical Pauli group for stabilizer code

$N(S)/S \cong \mathcal{P}_k$ encoded Pauli group

$N(S)$ contains tensor products of single-qubit gates – transversal

7-qubit code

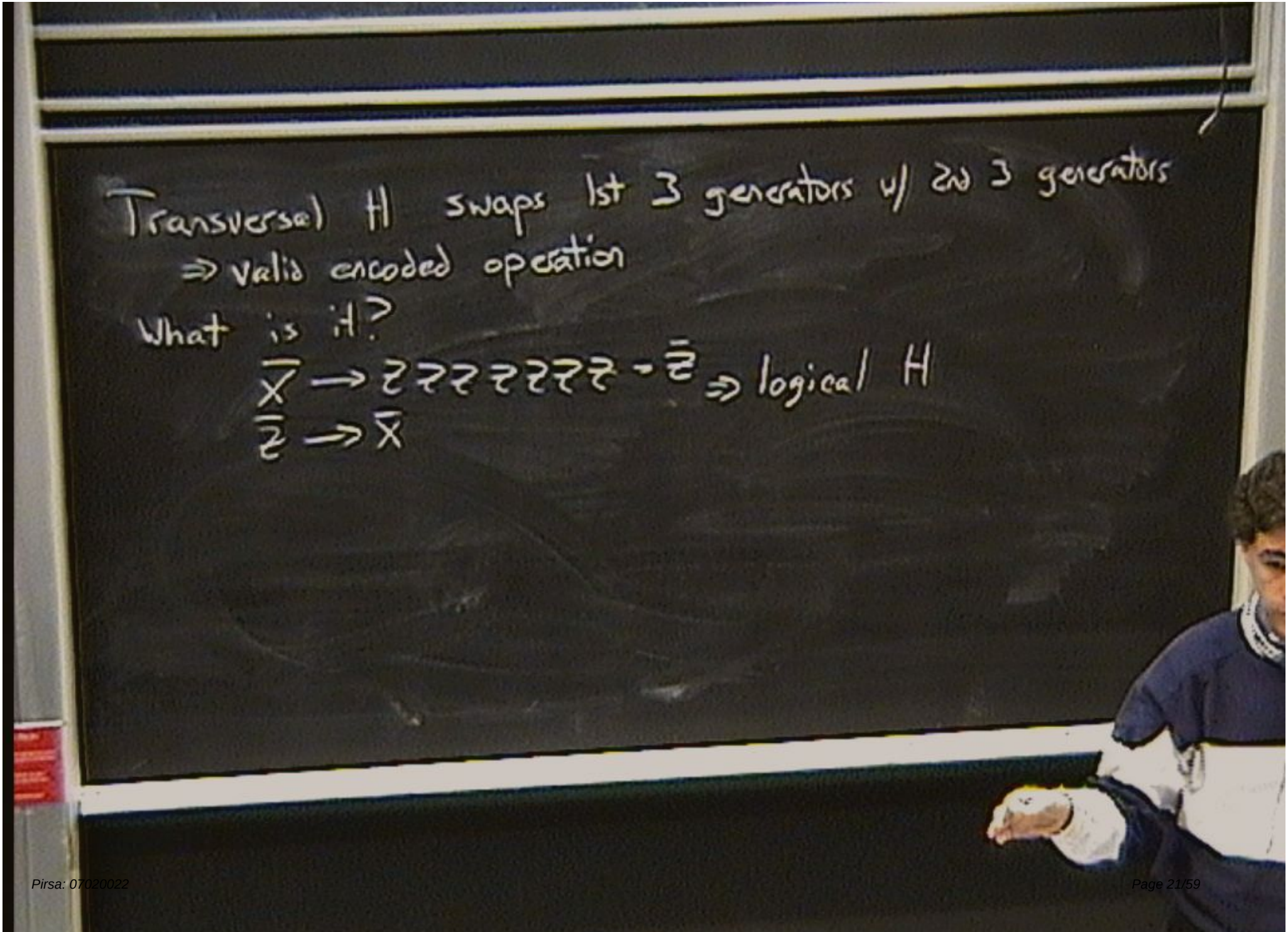
	X	X	X	X	I	I	I
	X	X	I	I	X	X	I
	X	I	X	I	X	I	X
	Z	Z	Z	Z	I	I	I
	Z	Z	I	I	Z	Z	I
	Z	I	Z	I	Z	I	Z
$\bar{X}$	X	X	X	X	X	X	X
$\bar{Z}$	Z	Z	Z	Z	Z	Z	Z

logical X on 7-qubit code:
transversal X or $X_5X_6X_7$ or...
logical Z & Y similarly

Transversal H swaps 1st 3 generators w/ 2nd 3 generators

⇒ valid encoded operation

What is it?


Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ valid encoded operation
What is H?
$\bar{X} \rightarrow \bar{Z}$
$\bar{Z} \rightarrow \bar{X}$
⇒ logical H

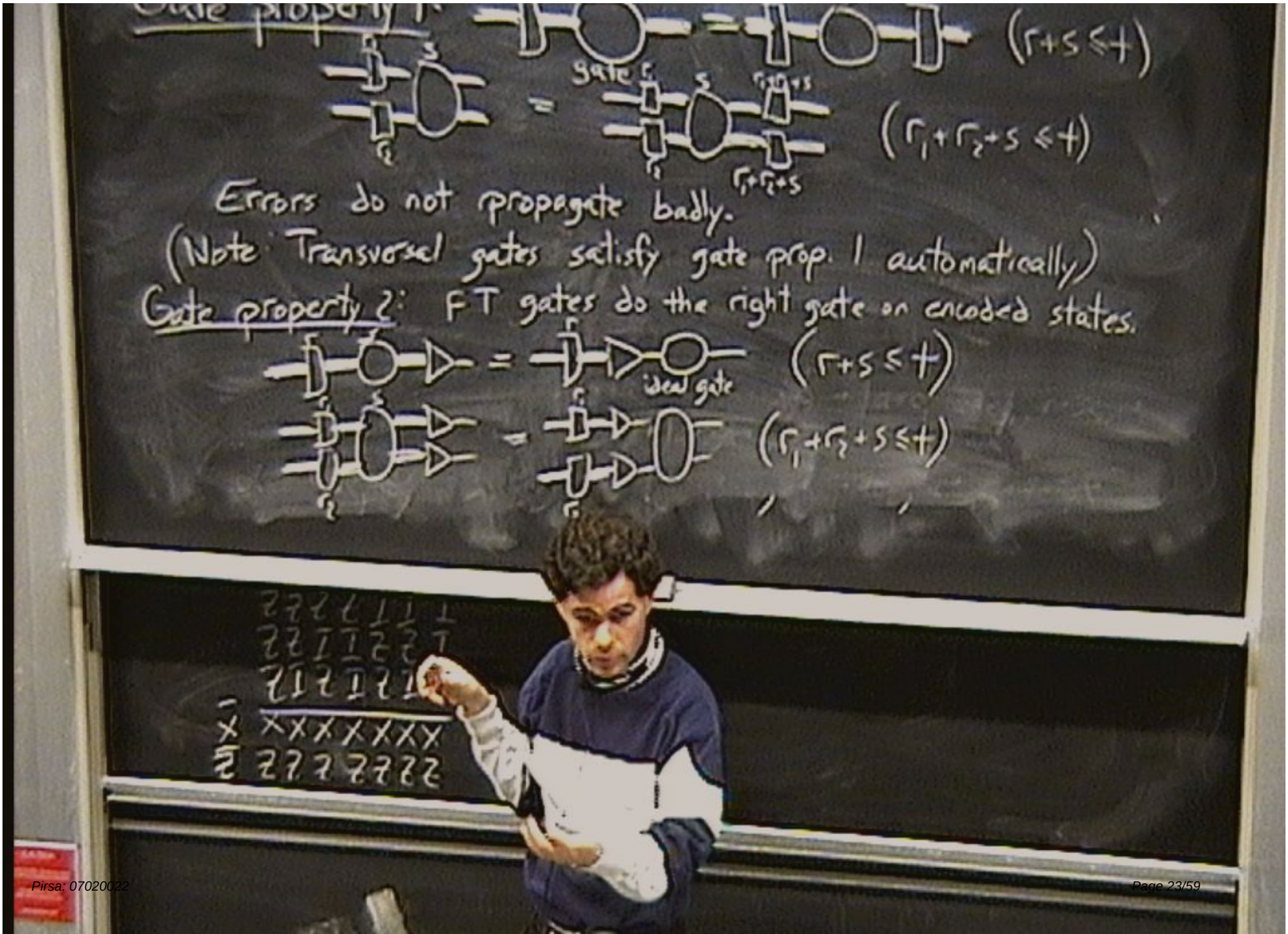
Transversal H swaps 1st 3 generators w/ 2nd 3 generators

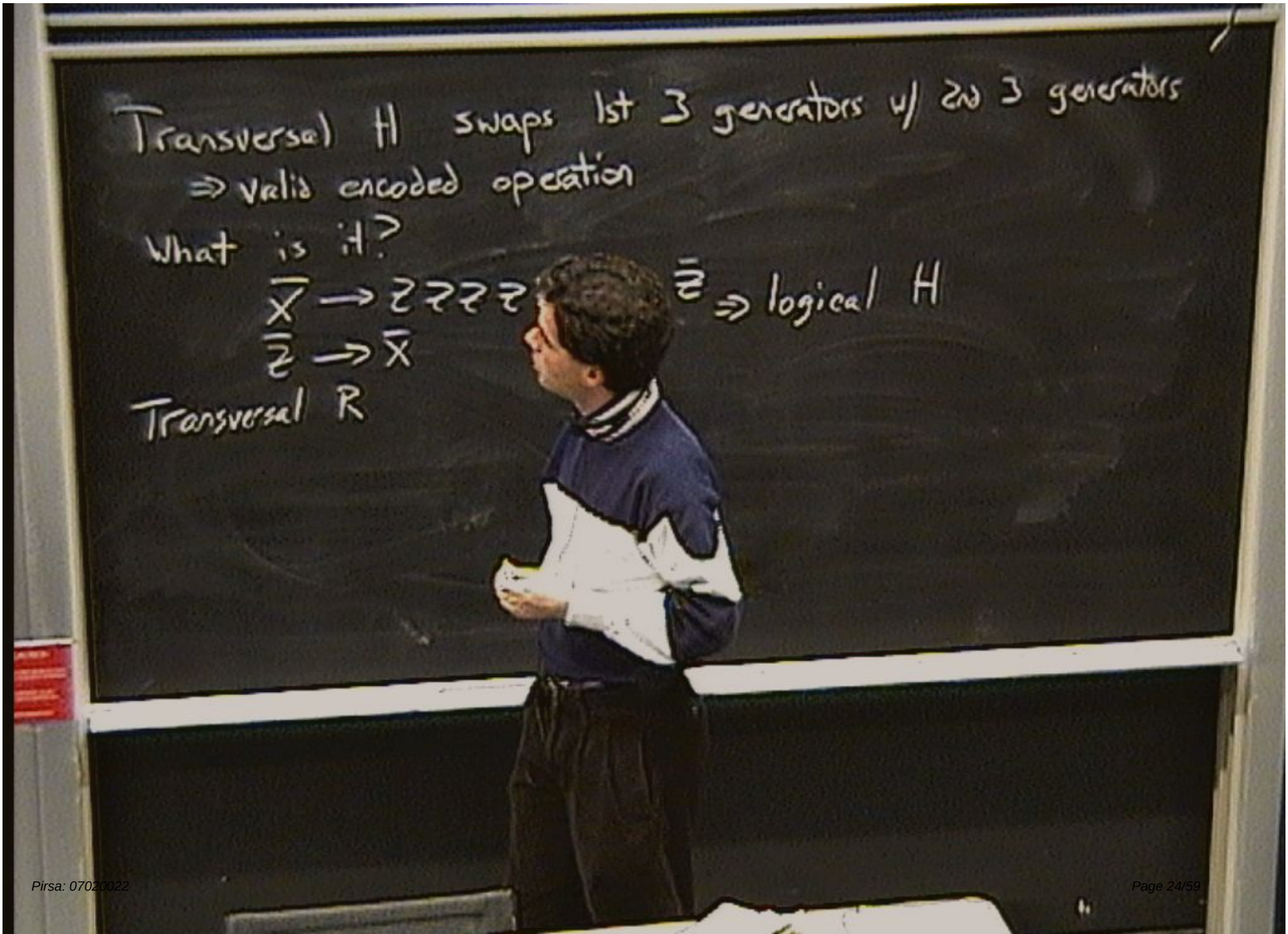
⇒ valid encoded operation

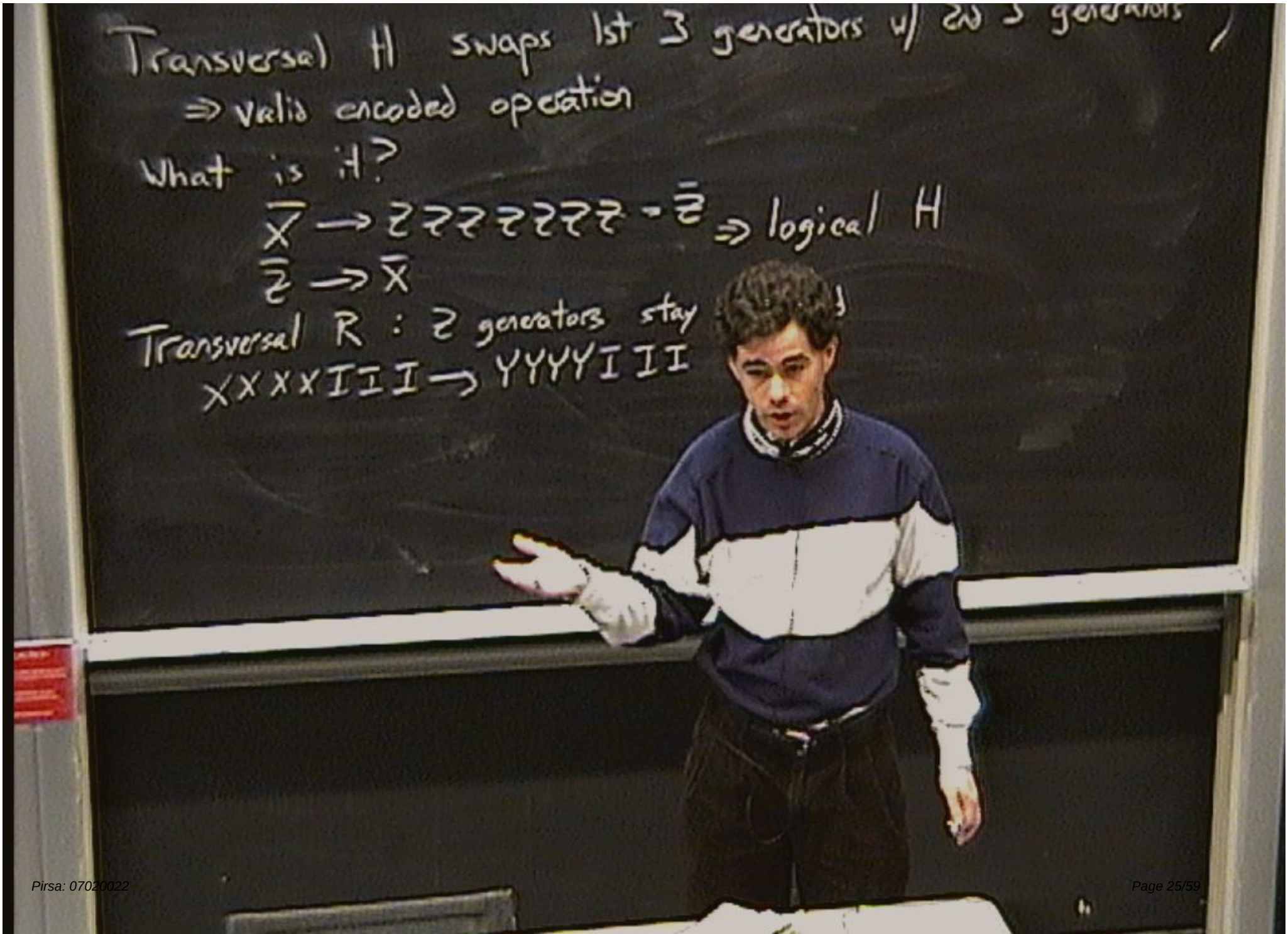
What is it?

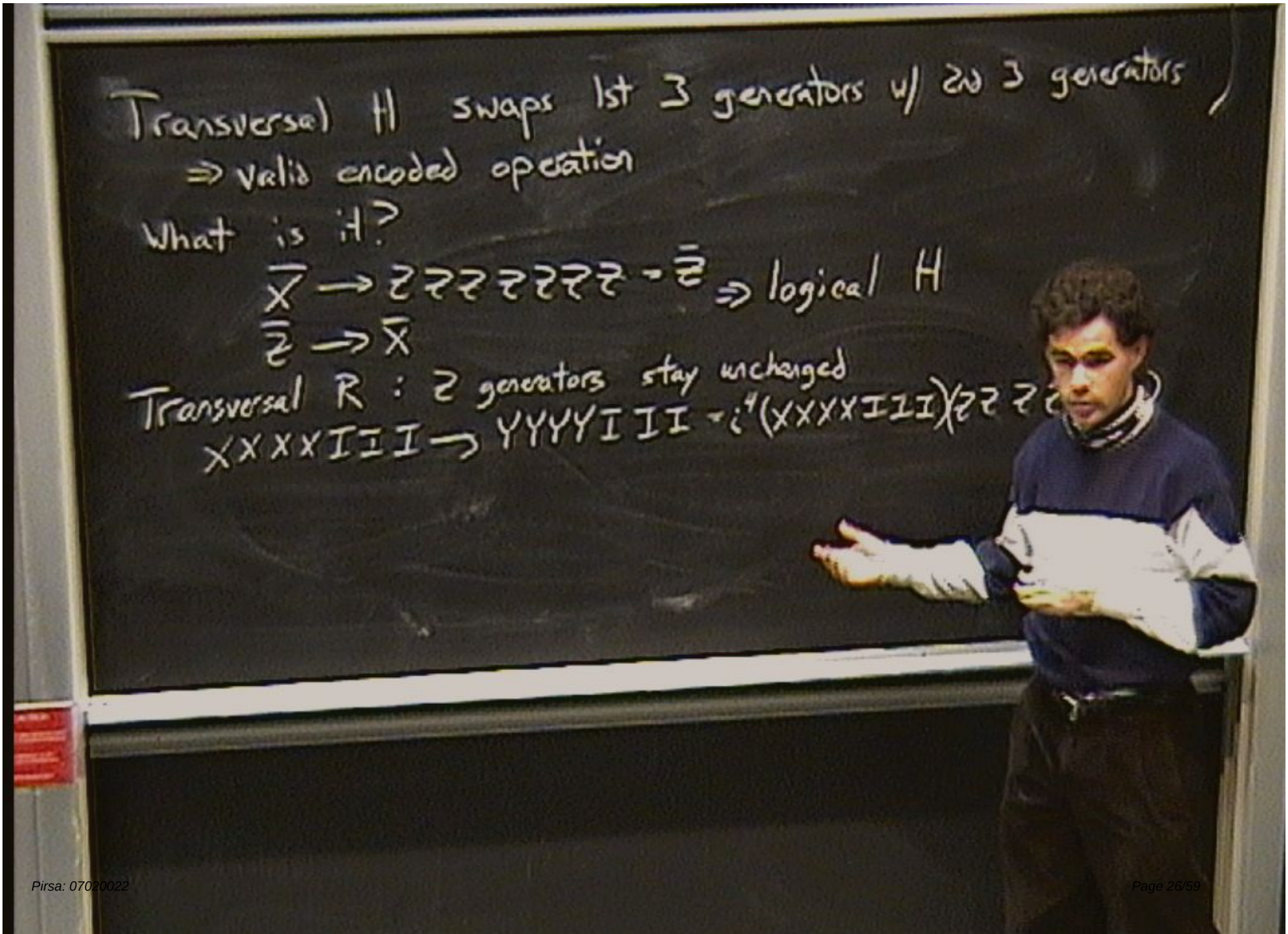
$\bar{X} \rightarrow ZZZZZZZ \cdot \bar{Z}$ ⇒ logical H

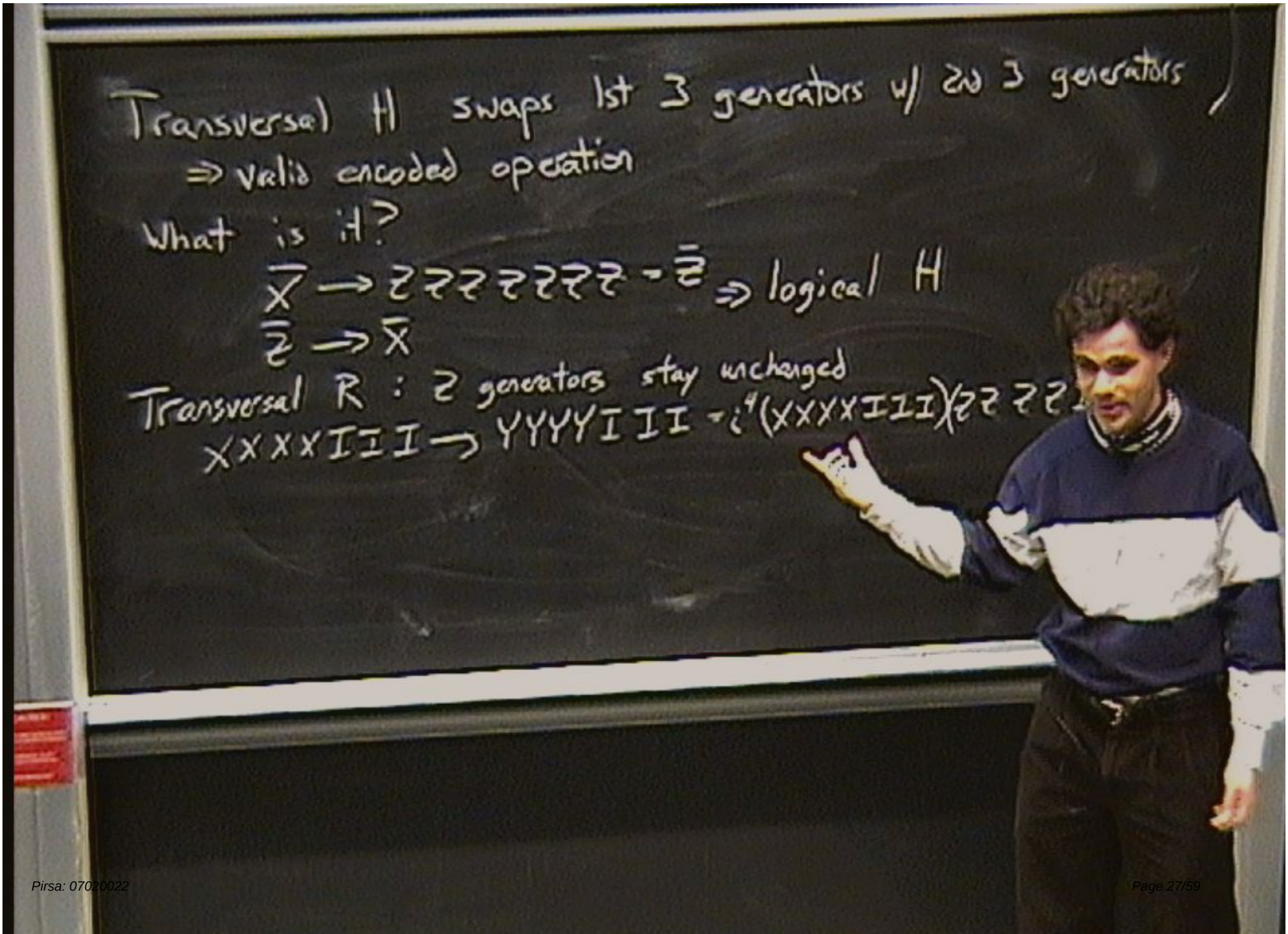
$\bar{Z} \rightarrow \bar{X}$

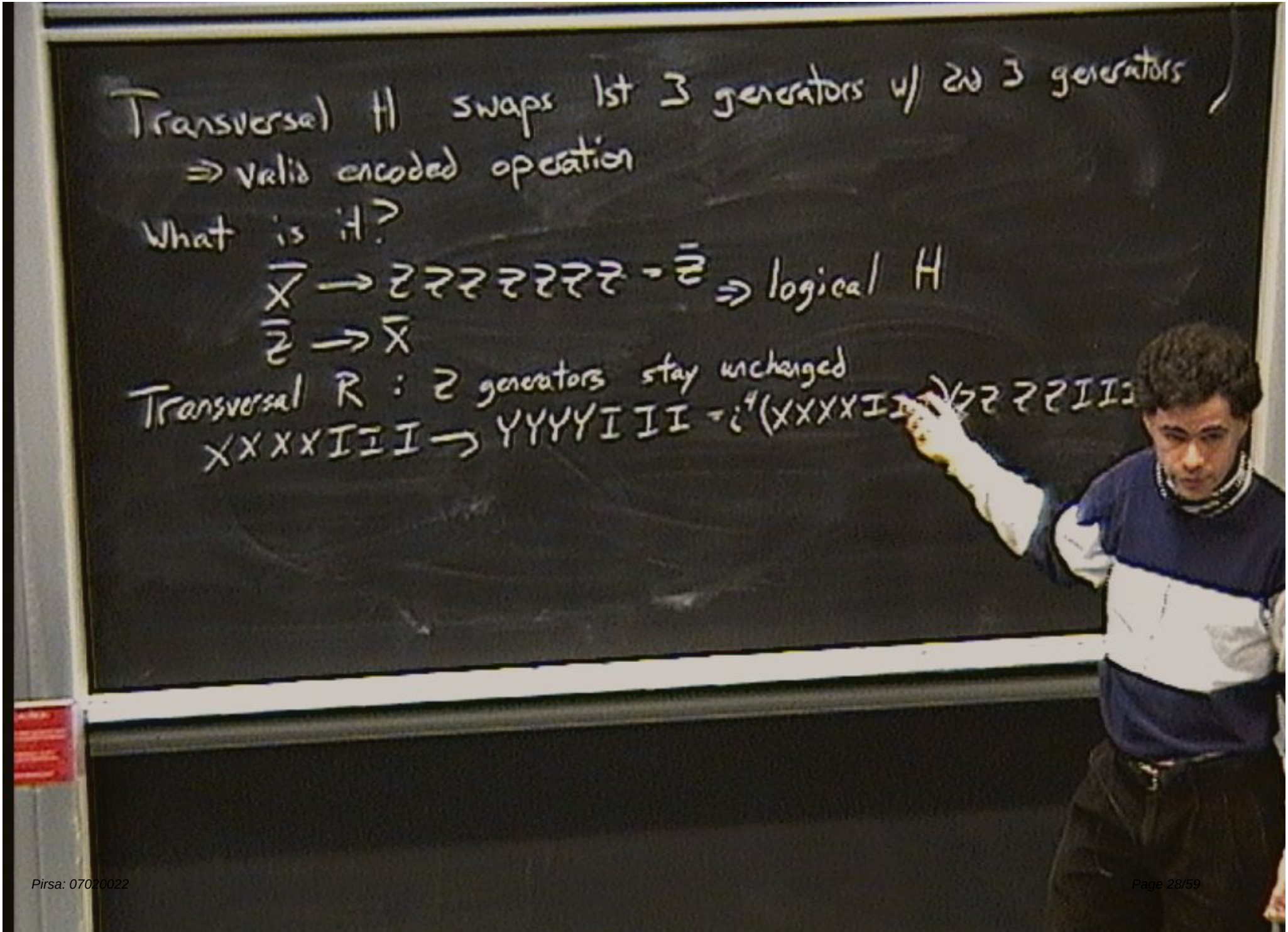

Gate property 1:
gate
$(r+s \le t)$
$(r_1+r_2+s \le t)$
Errors do not propagate badly.
(Note: Transversal gates satisfy gate prop. 1 automatically)
Gate property 2: FT gates do the right gate on encoded states.
ideal gate
$(r+s \le t)$
$(r_1+r_2+s \le t)$

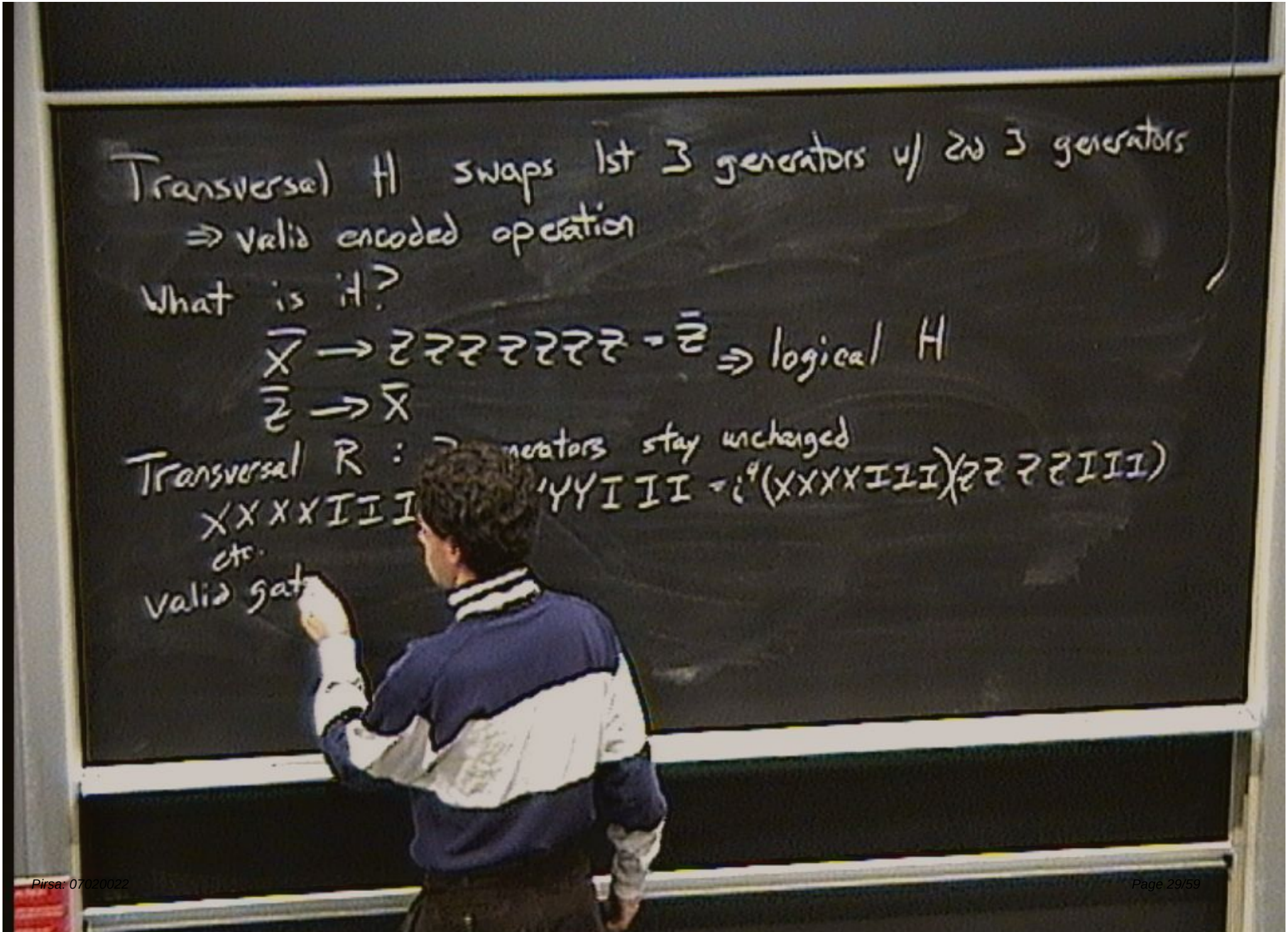

Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ valid encoded operation
What is H?
$\bar{X} \rightarrow \bar{Z}$
$\bar{Z} \rightarrow \bar{X}$
$\bar{Z} \Rightarrow$ logical H
Transversal R


Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ Valid encoded operation
What is H?
$\bar{X} \rightarrow ZZZZZZZ \cdot \bar{Z} \Rightarrow$ logical H
$\bar{Z} \rightarrow \bar{X}$
Transversal R : 2 generators stay
XXXXIII → YYYYIII


Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ valid encoded operation
What is H?
$\bar{X} \rightarrow \bar{Z}$
$\bar{Z} \rightarrow \bar{X}$
⇒ logical H
Transversal R : 2 generators stay unchanged
XXXXIIII → YYYYIIII


Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ valid encoded operation
What is H?
$\bar{X} \rightarrow ZZZZZZZ = \bar{Z}$ ⇒ logical H
$\bar{Z} \rightarrow \bar{X}$
Transversal R : 2 generators stay unchanged
XXXXIII → YYYYIII


Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ Valid encoded operation
What is H?
$\bar{X} \rightarrow ZZZZZZZ \cdot \bar{Z}$ ⇒ logical H
$\bar{Z} \rightarrow \bar{X}$
Transversal R : 2 generators stay unchanged
XXXXIII → YYYYIII


Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ valid encoded operation
What is H?
$\bar{X} \rightarrow \bar{Z}\bar{Z}\bar{Z}\bar{Z}\bar{Z}\bar{Z}\bar{Z} = -\bar{Z}$ ⇒ logical H
$\bar{Z} \rightarrow \bar{X}$
Transversal R : generators stay unchanged
XXXXIII
YYIII
$= i^4(XXXXIII)(ZZZZIII)$
etc.
Valid gat

Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ valid encoded operation

What is H?

$\bar{X} \rightarrow Z\,Z\,Z\,Z\,Z\,Z\,Z = \bar{Z}$ ⇒ logical H

$\bar{Z} \rightarrow \bar{X}$

Transversal R : 2 generators stay unchanged

$XXXXIIII \rightarrow YYYYIIII = i^4(YYYYIIII)(ZZZZIIII)$

etc.

Valid gate

$\bar{X} \rightarrow YYYYYYYY =$

$\bar{Z} \rightarrow \bar{Z}$

Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ valid encoded operation

What is H?

$\bar{X} \rightarrow ZZZZZZZ = \bar{Z}$ ⇒ logical H

$\bar{Z} \rightarrow \bar{X}$

Transversal R : 2 generators stay unchanged

$XXXXIIII \rightarrow YYYYIIII = i^4(XXXXIIII)(ZZZZIIII)$

etc.

valid gate

$\bar{X} \rightarrow YYYYYYY = i^7(XXXXXXX)(ZZZZZZZ) = -i\bar{X}\bar{Z} = -\bar{Y}$

$\bar{Z} \rightarrow \bar{Z}$

Transversal H swaps 1st 3 generators w/ 2nd 3 generators
⇒ valid encoded operation

What is $\bar{H}$?

$\bar{X} \to \bar{Z}\bar{Z}\bar{Z}\bar{Z}\bar{Z}\bar{Z}\bar{Z} = \bar{Z}$ ⇒ logical H

$\bar{Z} \to \bar{X}$

Transversal R : 2 generators stay unchanged

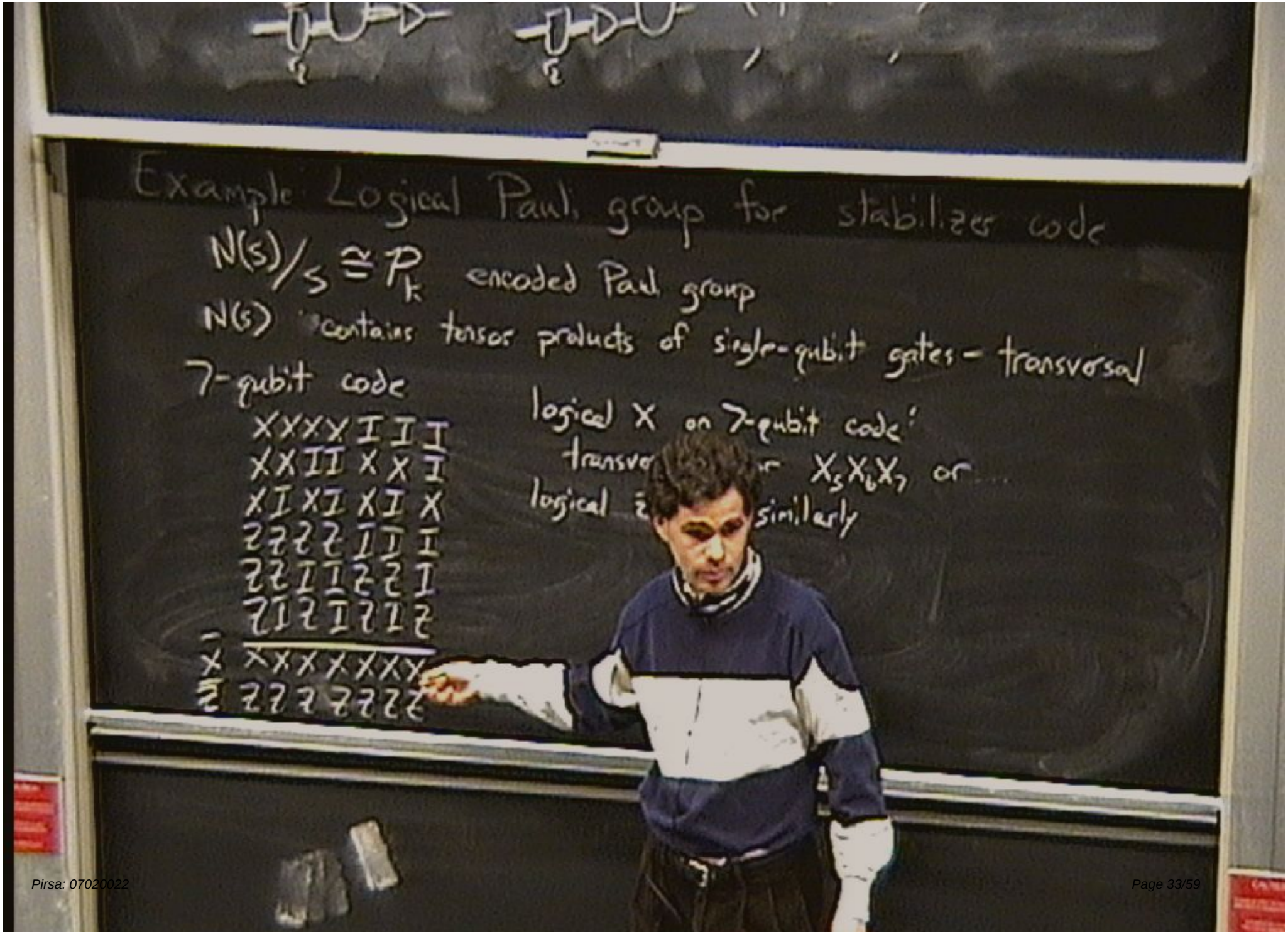
$XXXXIIII \to YYYYIIII = i^4(XXXXIIII)(ZZZZIIII)$

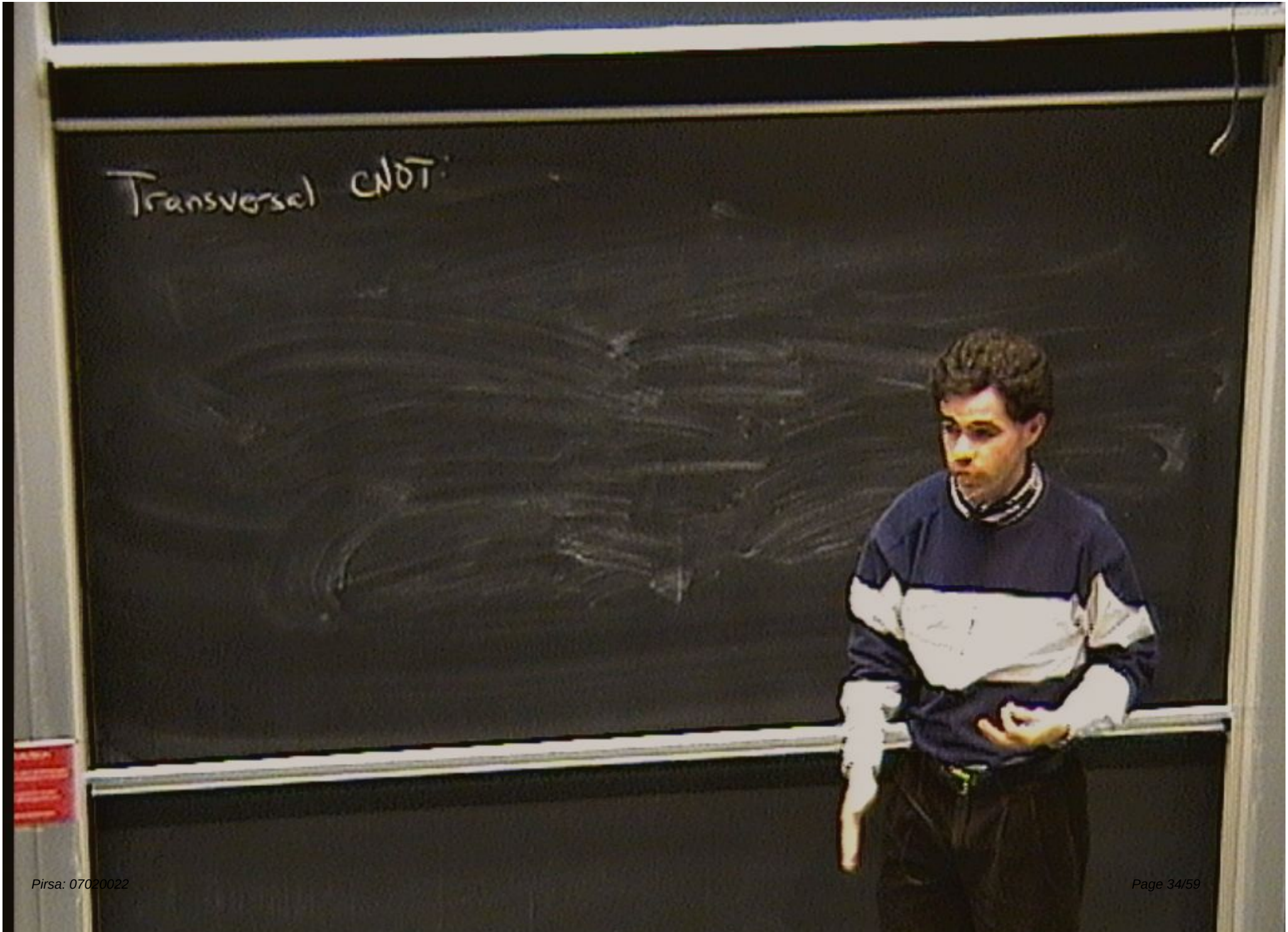
etc.

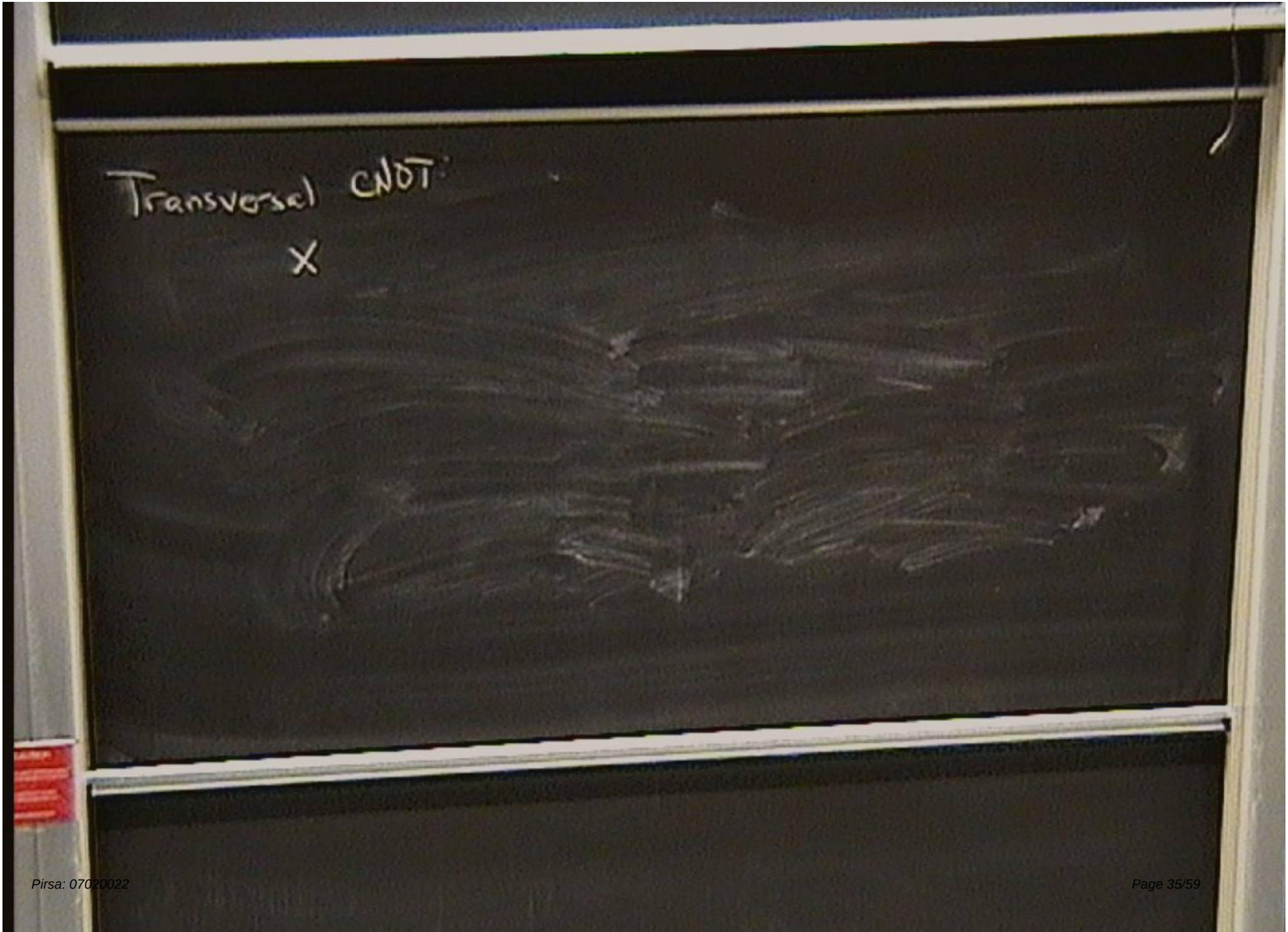
Valid gate ⇒ logical R^{-1} (To do logical R, do transversal R^{-1})

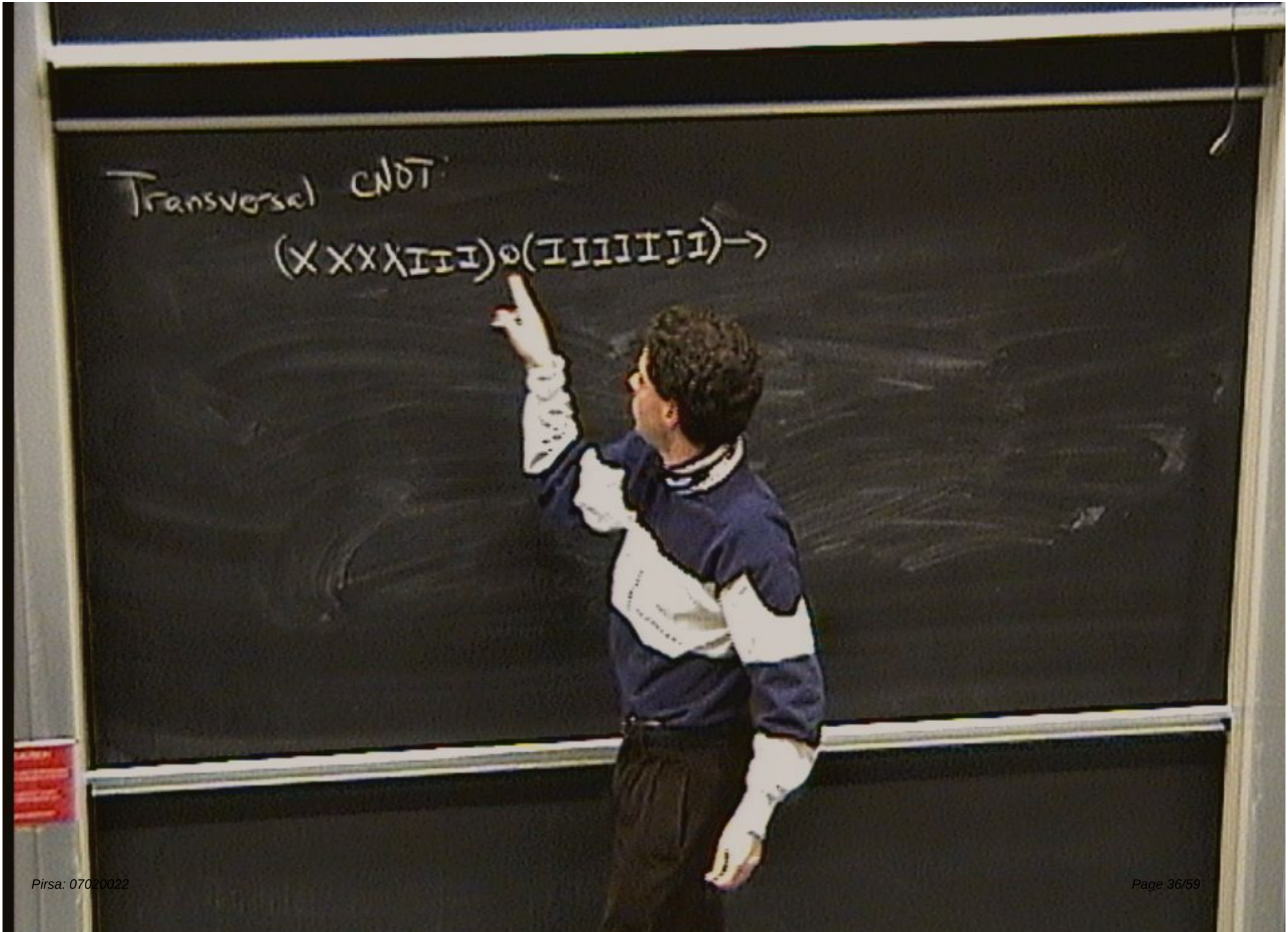
$\bar{X} \to YYYYYYY = i^7(XXXXXXX)(ZZZZZZZ) = -i\bar{X}\bar{Z} = -\bar{Y}$

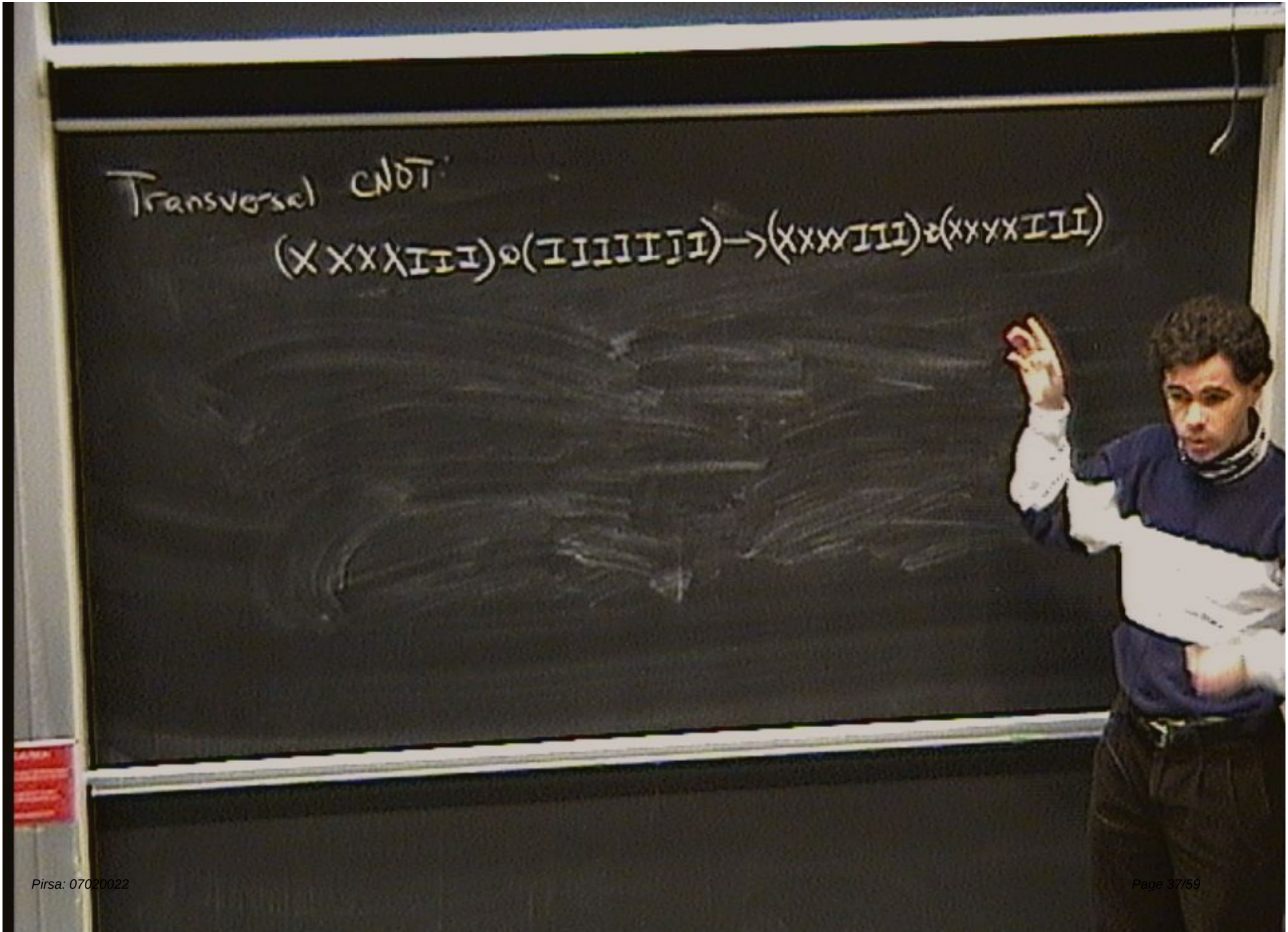
$\bar{Z} \to \bar{Z}$

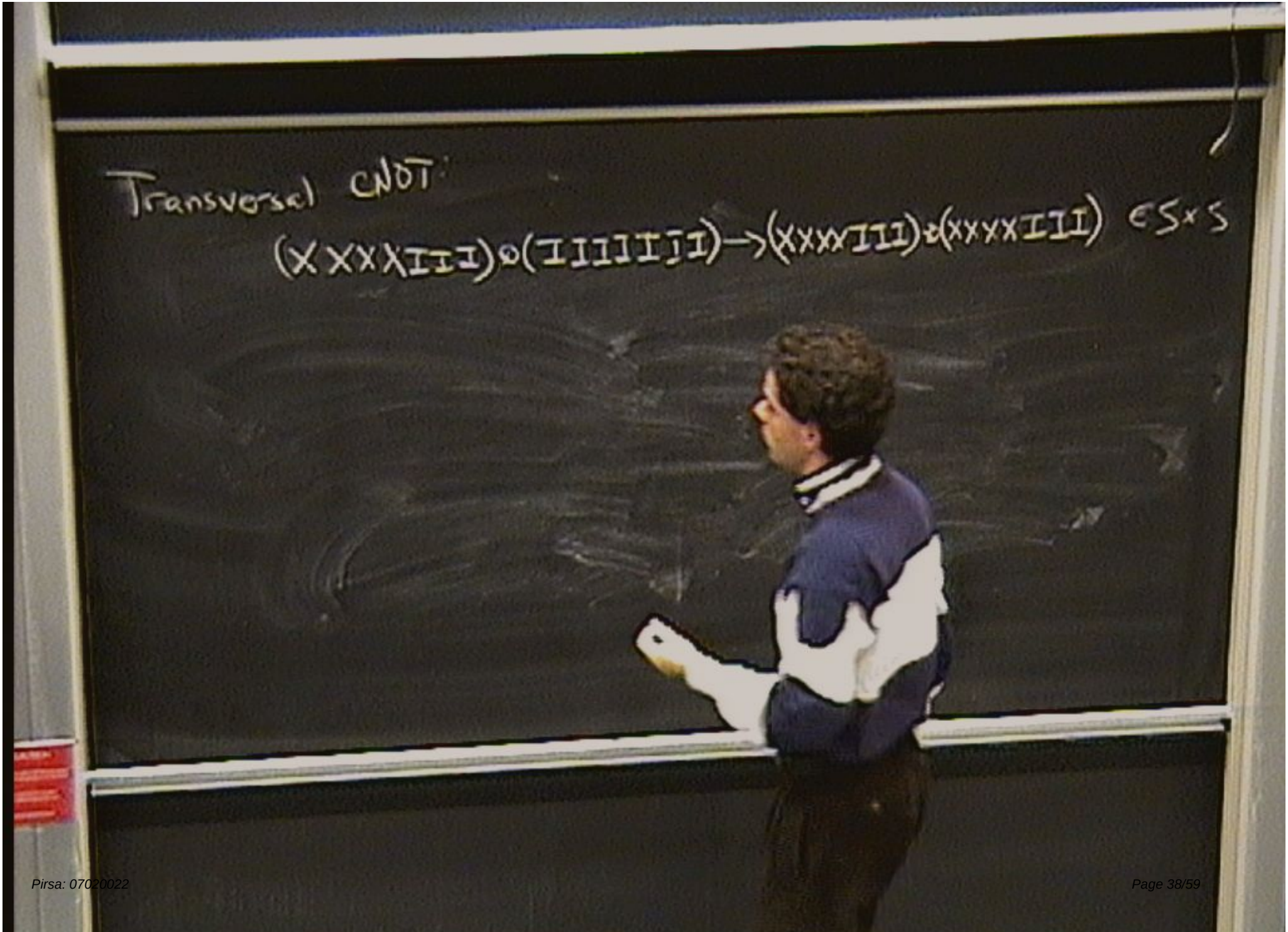

Example Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
N(S) contains tensor products of single-qubit gates - transversal
7-qubit code
XXXXIII
XXIIXXI
XIXIXIX
ZZZZIII
ZZIIZZI
ZIZIZIZ
$\bar{X}$ XXXXXXX
$\bar{Z}$ ZZZZZZZ
logical X on 7-qubit code:
transv
$X_5X_6X_7$ or
logical
similarly

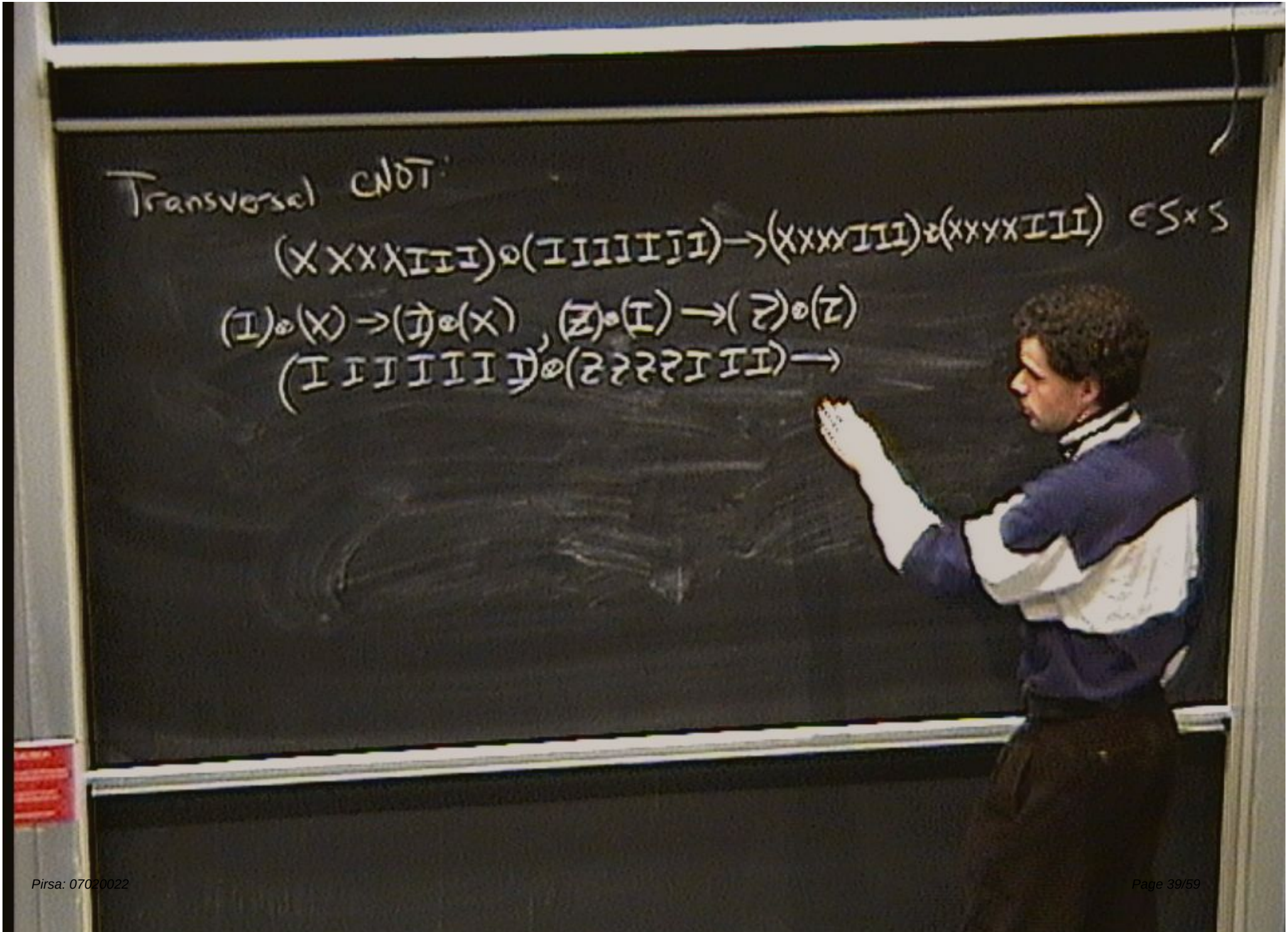

Transversal CNOT


Transversal CNOT
X


Transversal CNOT
(XXXXIII)∘(IIIIIII)→


Transversal CNOT
(XXXXIII)⊗(IIIIIII)→(XXXIII)⊗(XXXXIII)


Transversal CNOT
(XXXXIII) ⊗ (IIIIIII) → (XXXXIII) ⊗ (XXXXIII) ∈ S × S


Transversal CNOT
(X X X X I I I) ⊗ (I I I I I I I) → (X X X X I I I) ⊗ (X X X X I I I) ∈ S × S
(I) ⊗ (X) → (I) ⊗ (X) , (Z) ⊗ (I) → (Z) ⊗ (Z)
(I I I I I I I) ⊗ (Z Z Z Z I I I) →

Transversal CNOT

$(XXXXIIII) \circ (IIIIIIII) \to (XXXXIIII) \circ (XXXXIIII) \in S \times S$

$(I) \circ (X) \to (I) \circ (X)$, $(Z) \circ (I) \to (Z) \circ (Z)$

$(IIIIIIII) \circ (ZZZZIIII) \to (IIIIZZZZ)(ZZZZIIII)$

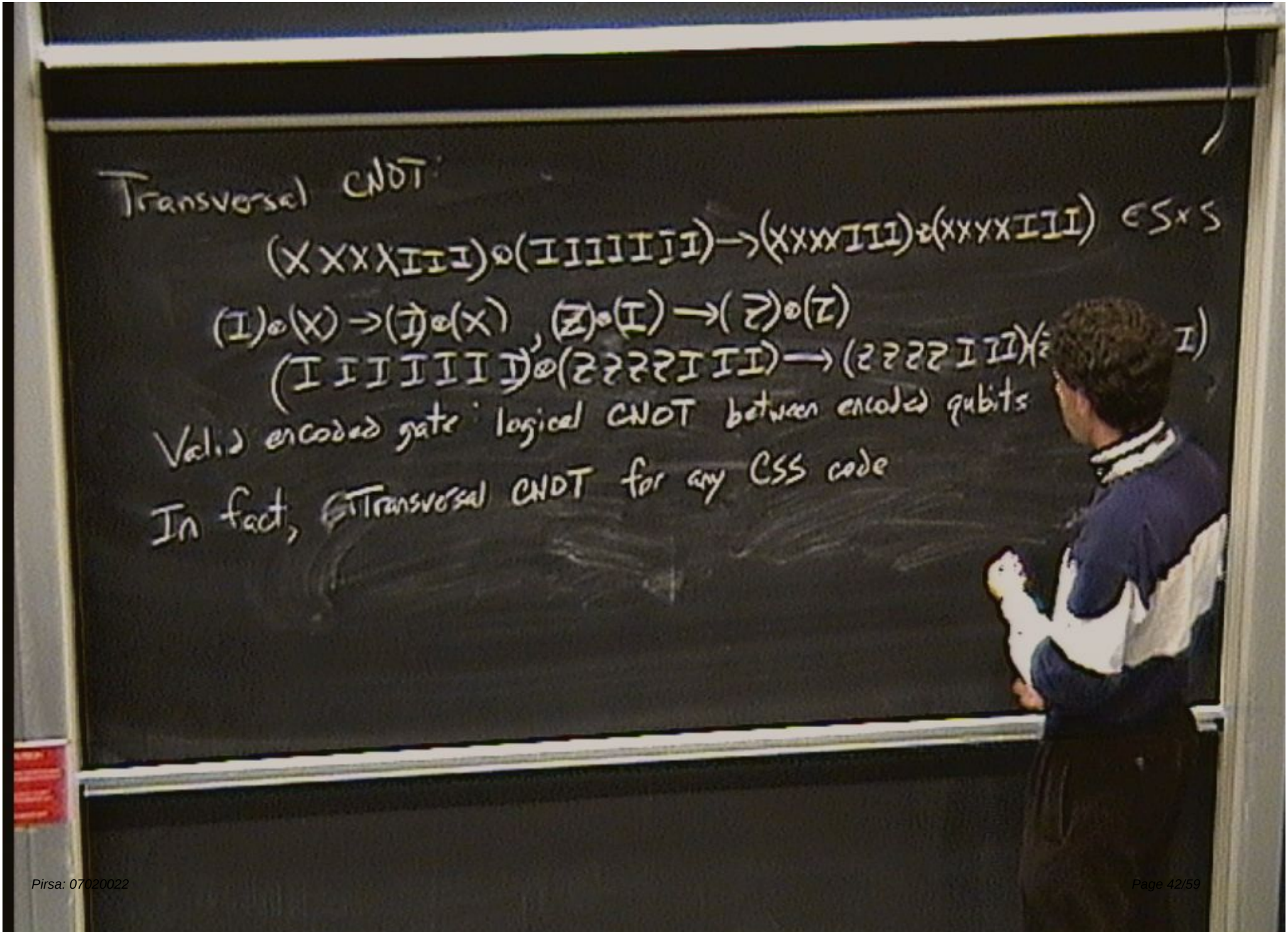
Transversal CNOT

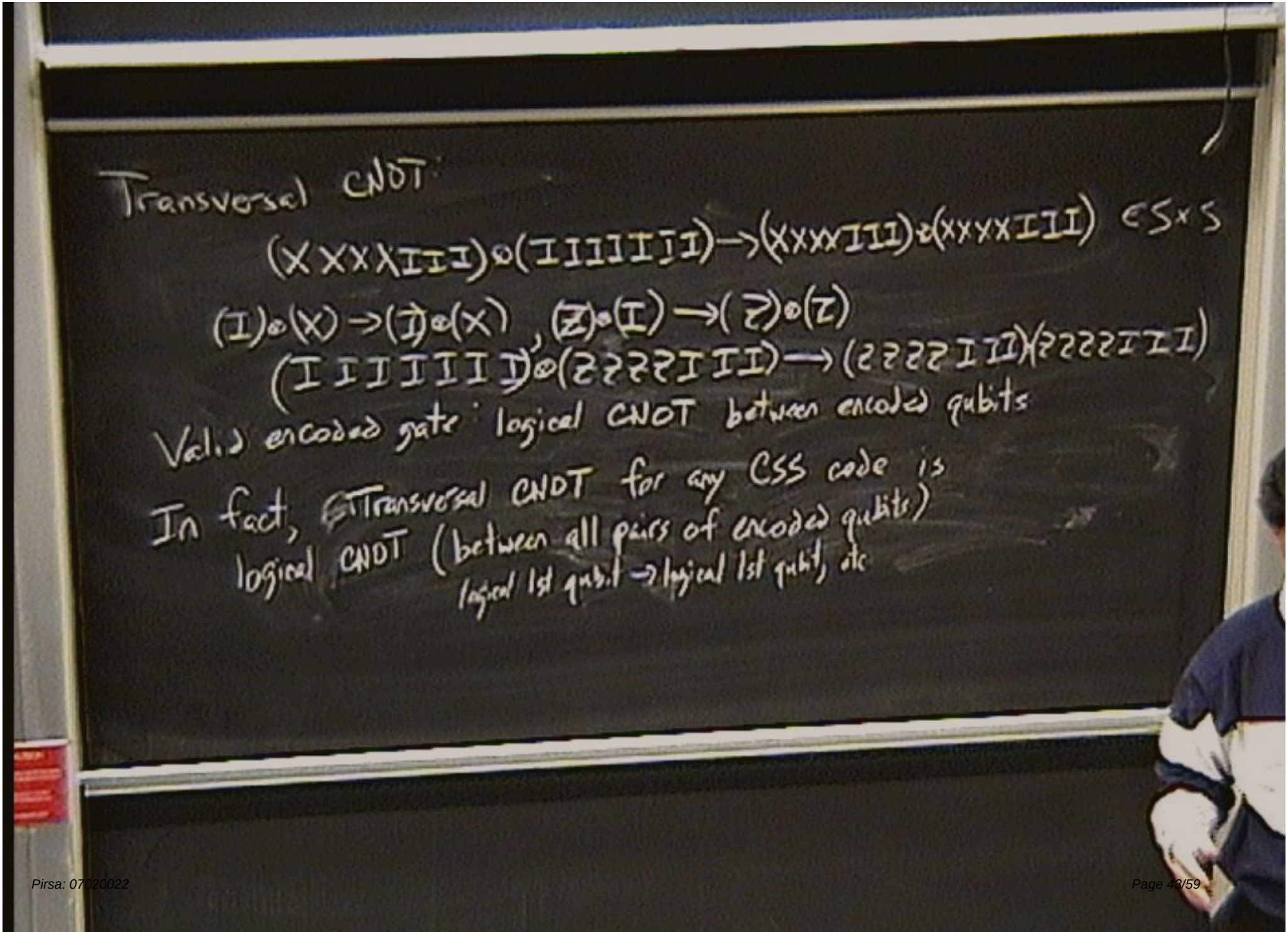
$(XXXXIIII) \otimes (IIIIIIII) \rightarrow (XXXXIIII) \otimes (XXXXIIII) \in S \times S$

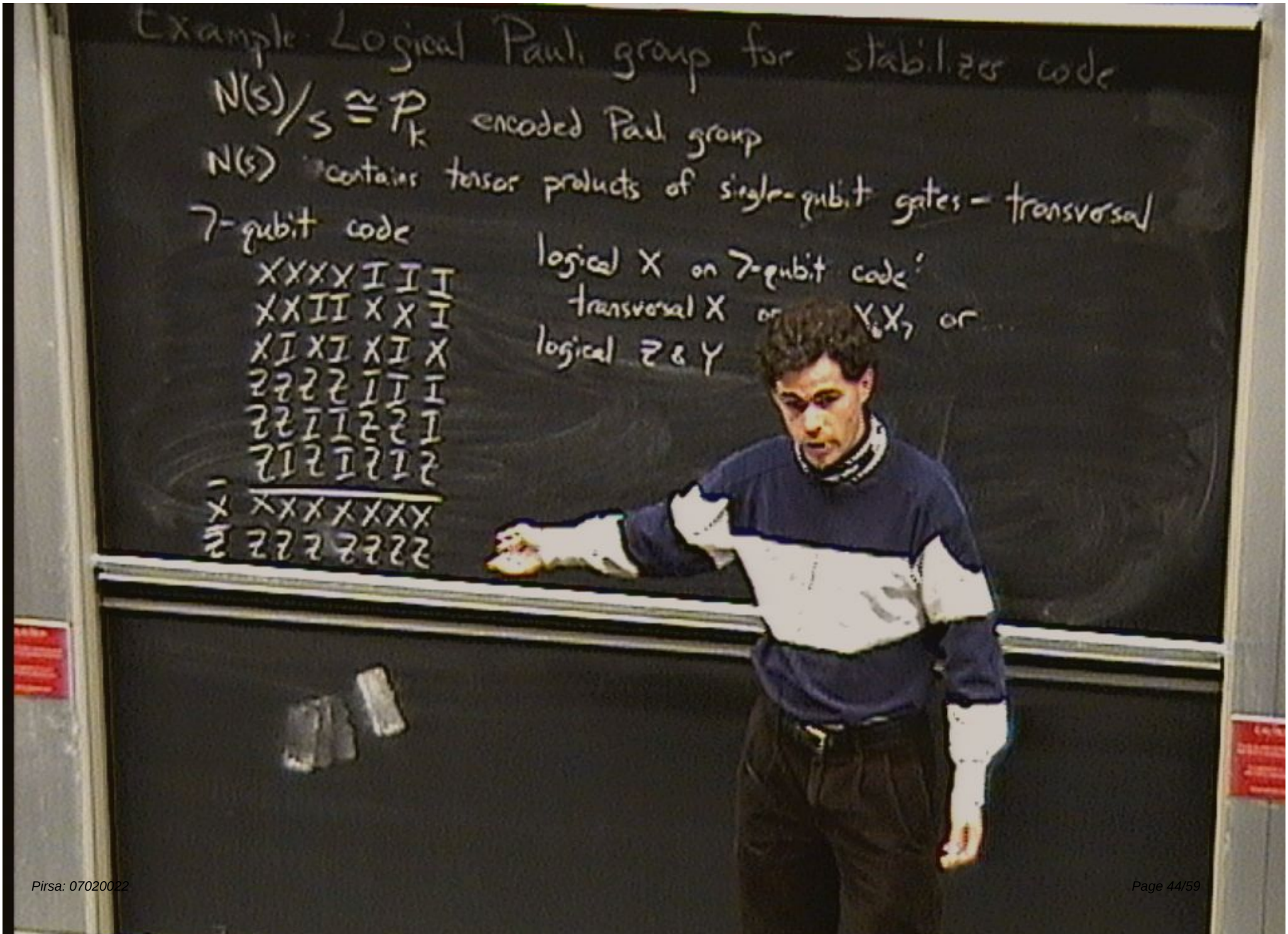
$(I) \otimes (X) \rightarrow (I) \otimes (X)$, $(Z) \otimes (I) \rightarrow (Z) \otimes (Z)$

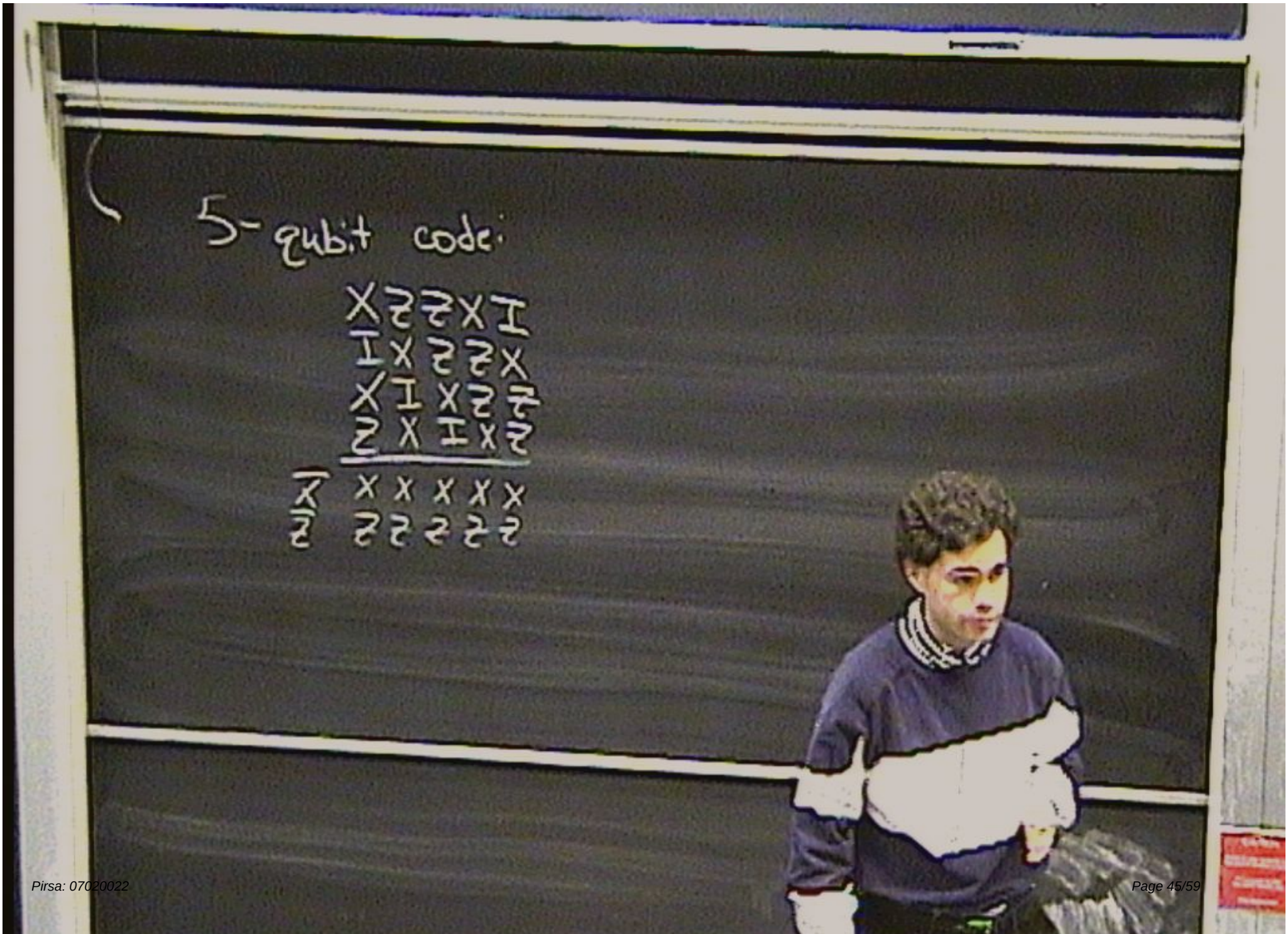
$(IIIIIIII) \otimes (ZZZZIIII) \rightarrow (ZZZZIIII)(ZZZZIIII)$

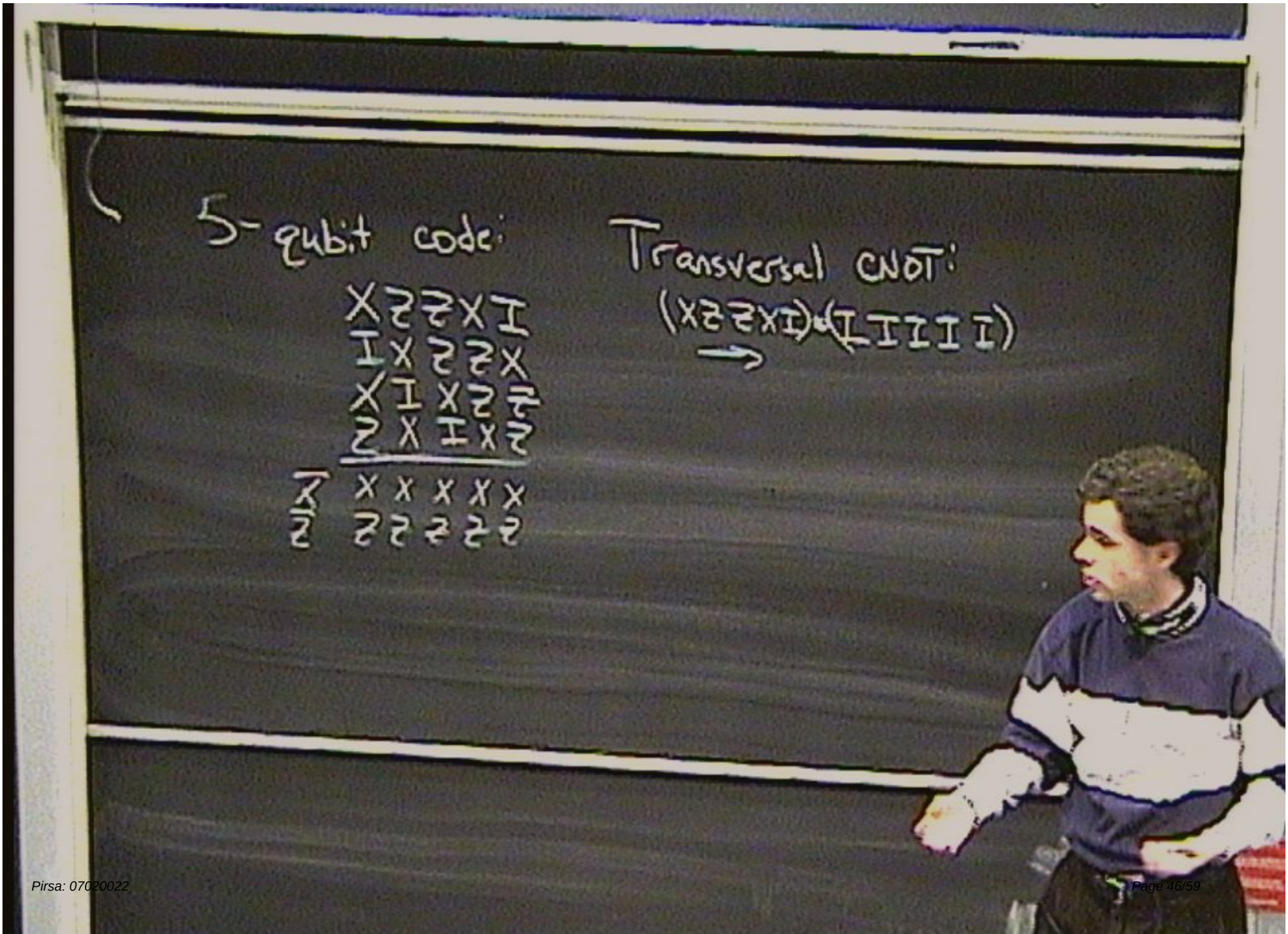
Valid encoded gate: logical CNOT between encoded qubits

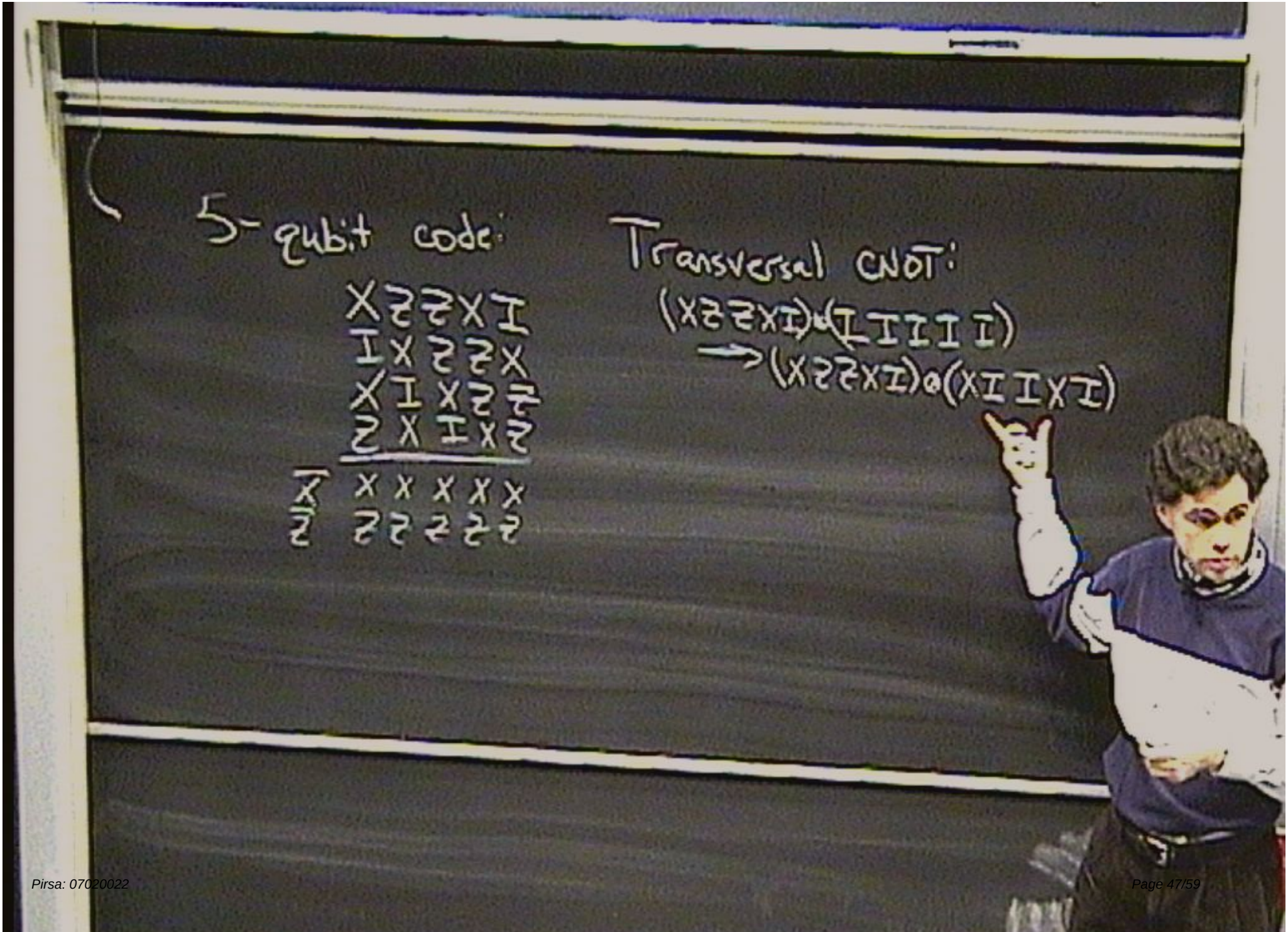

Transversal CNOT
(XXXXIII)⊗(IIIIIII) → (XXXXIII)⊗(XXXXIII) ∈ S×S
(I)⊗(X) → (I)⊗(X), (Z)⊗(I) → (Z)⊗(Z)
(IIIIIII)⊗(ZZZZIII) → (ZZZZIII)⊗(
Valid encoded gate: logical CNOT between encoded qubits
In fact, ∃ Transversal CNOT for any CSS code

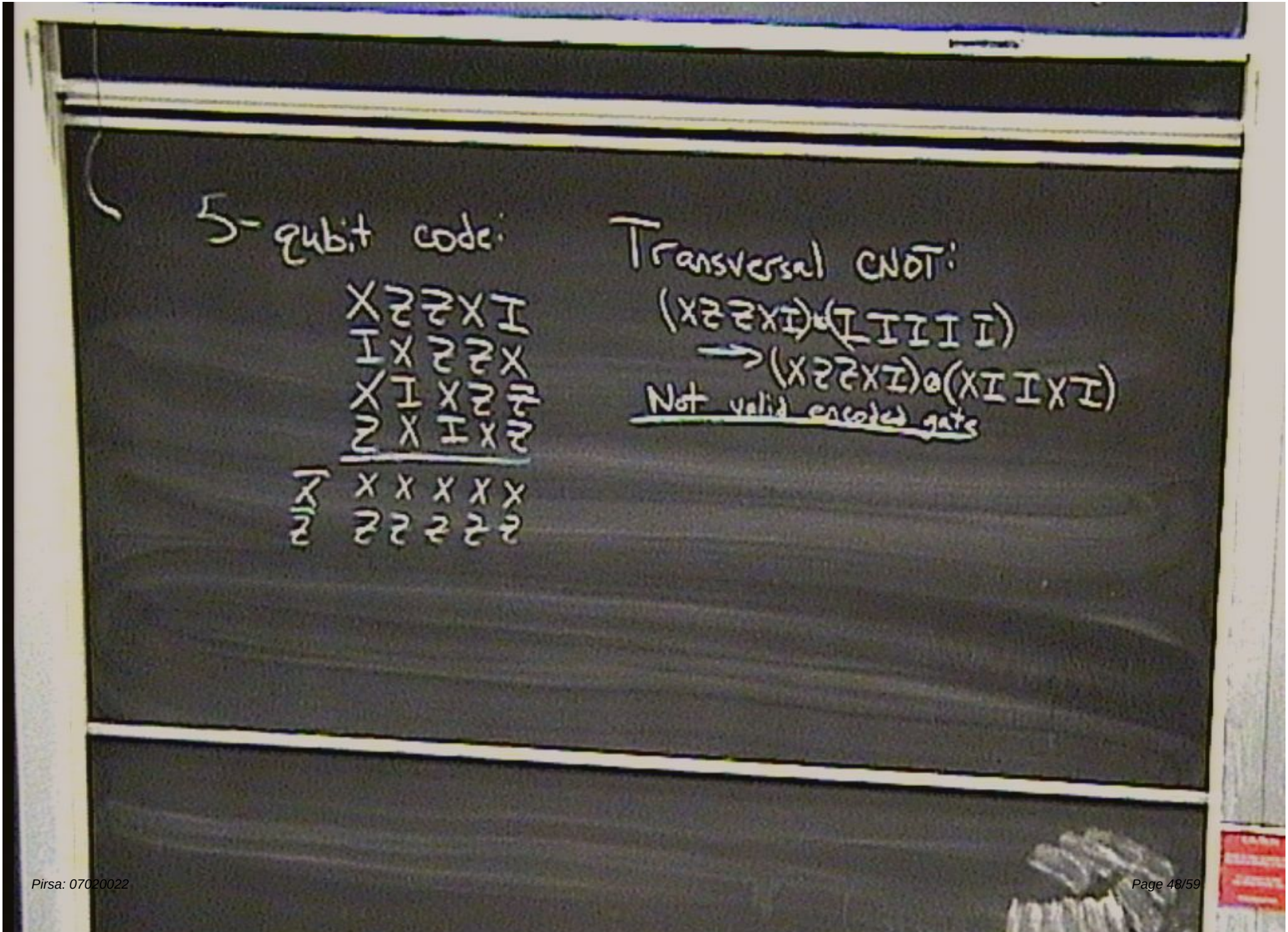

Transversal CNOT
(XXXXIIII)⊗(IIIIIIII)→(XXXXIIII)⊗(XXXXIIII) ∈S×S
(I)⊗(X)→(I)⊗(X), (Z)⊗(I)→(Z)⊗(Z)
(IIIIIIII)⊗(ZZZZIIII)→(ZZZZIIII)(ZZZZIIII)
Valid encoded gate: logical CNOT between encoded qubits
In fact, ∈T Transversal CNOT for any CSS code is
logical CNOT (between all pairs of encoded qubits)
logical 1st qubit → logical 1st qubit, etc


Example: Logical Pauli group for stabilizer code
$N(S)/S \cong P_k$ encoded Pauli group
$N(S)$ contains tensor products of single-qubit gates - transversal
7-qubit code
XXXXIII
XXIIXXI
XIXIXIX
ZZZZIII
ZZIIZZI
ZIZIZIZ
$\bar{X}$ XXXXXXX
$\bar{Z}$ ZZZZZZZ
logical X on 7-qubit code:
transversal X or Y_6X_7 or
logical Z & Y


5- qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
XXXXX
ZZZZZ


5-qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
XXXXX
ZZZZZ
Transversal CNOT:
(XZZXI)⊗(IIIII)
→


5- qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
X X X X X
Z Z Z Z Z
Transversal CNOT:
(XZZXI)⊗(IIIII)
→(XZZXI)⊗(XIIXI)


5-qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
X X X X X
Z Z Z Z Z
Transversal CNOT:
(XZZXI)⊗(IIIII)
→(XZZXI)⊗(XIIXI)
Not valid encoded gate

5-qubit code:

X	Z	Z	X	I
I	X	Z	Z	X
X	I	X	Z	Z
Z	X	I	X	Z
$\bar{X}$	X	X	X	X	X
$\bar{Z}$	Z	Z	Z	Z	Z

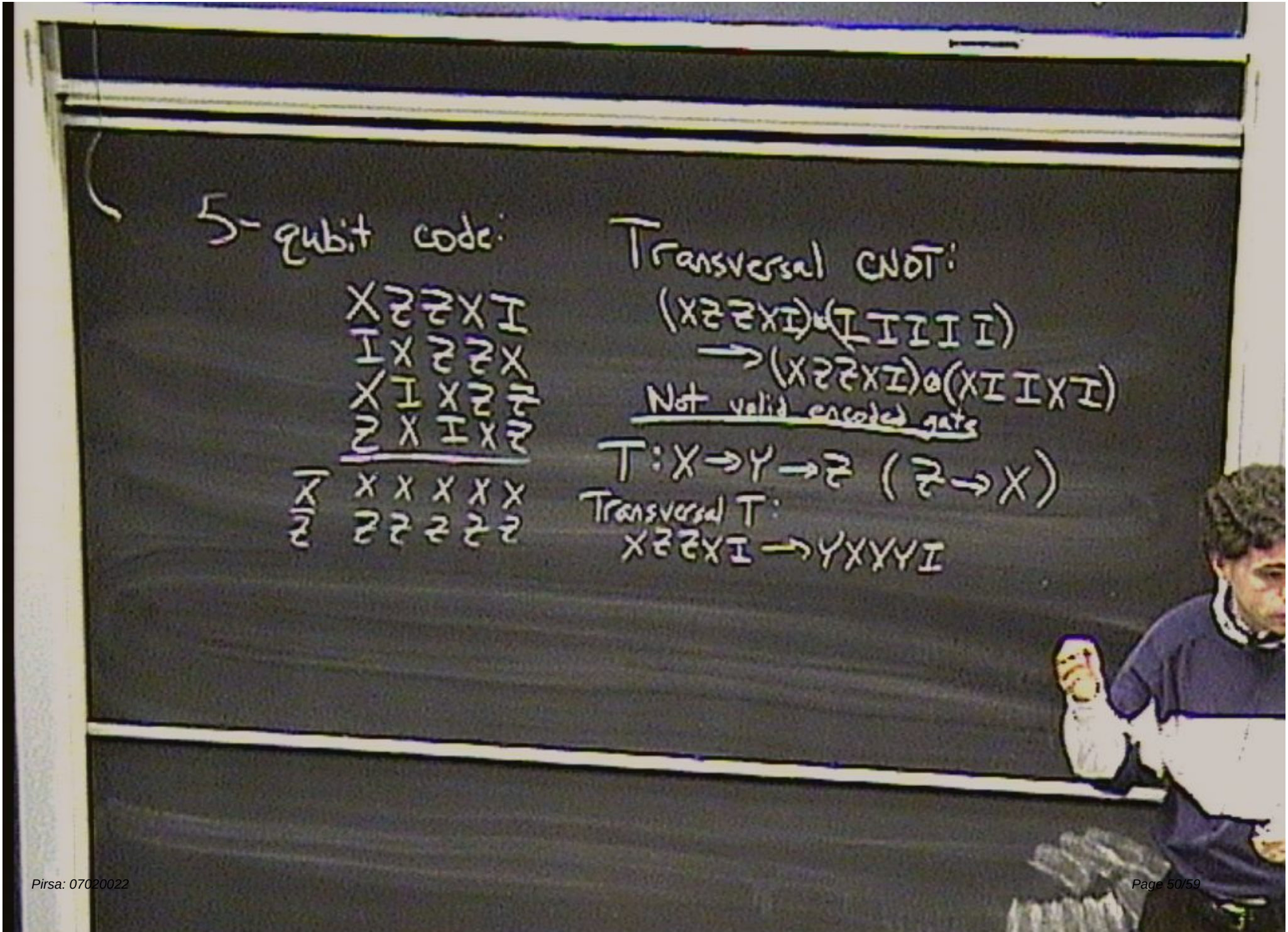
Transversal CNOT:

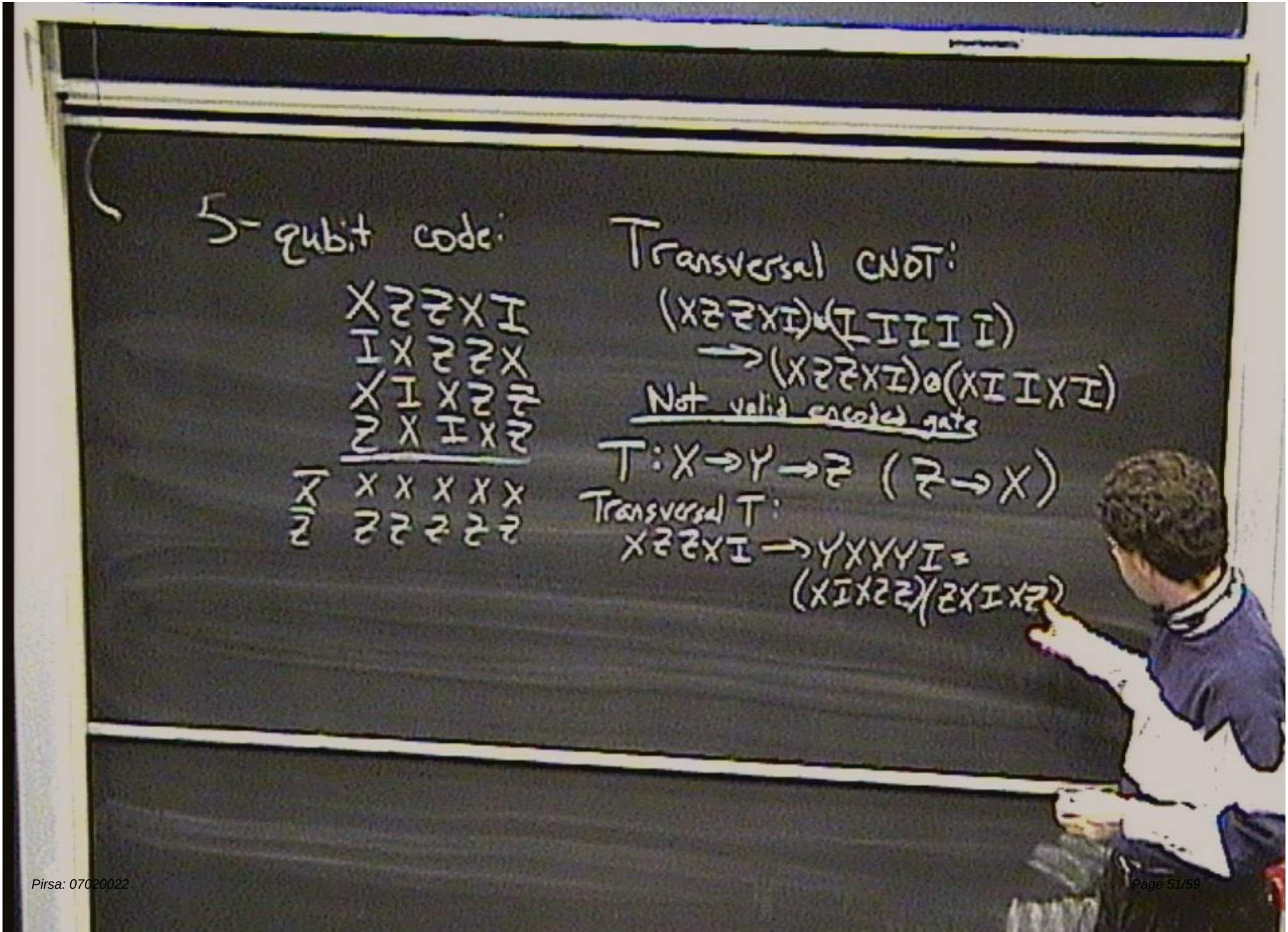
$(XZZXI) \otimes (IIIII)$

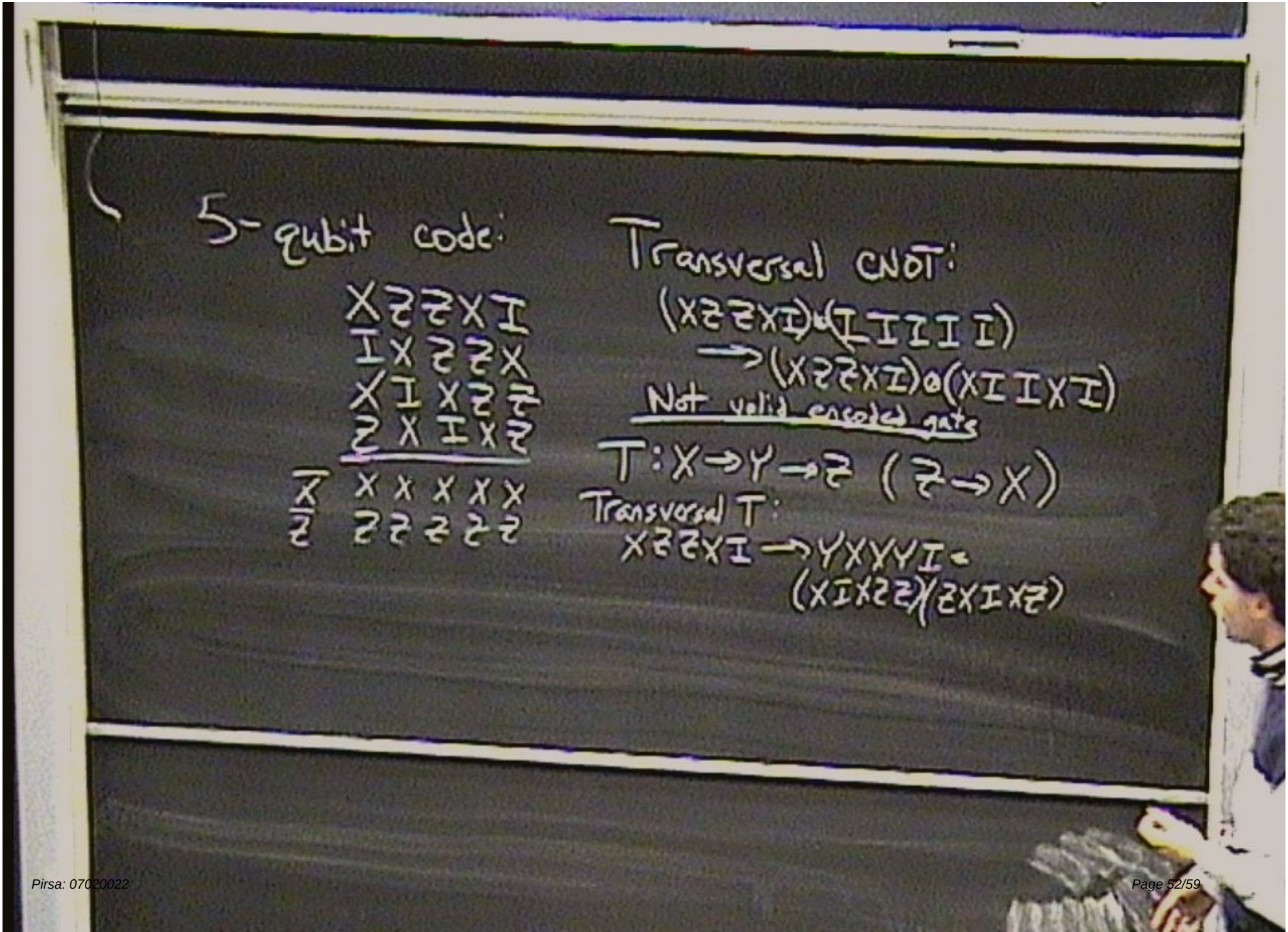
$\longrightarrow (XZZXI) \otimes (XIIXI)$

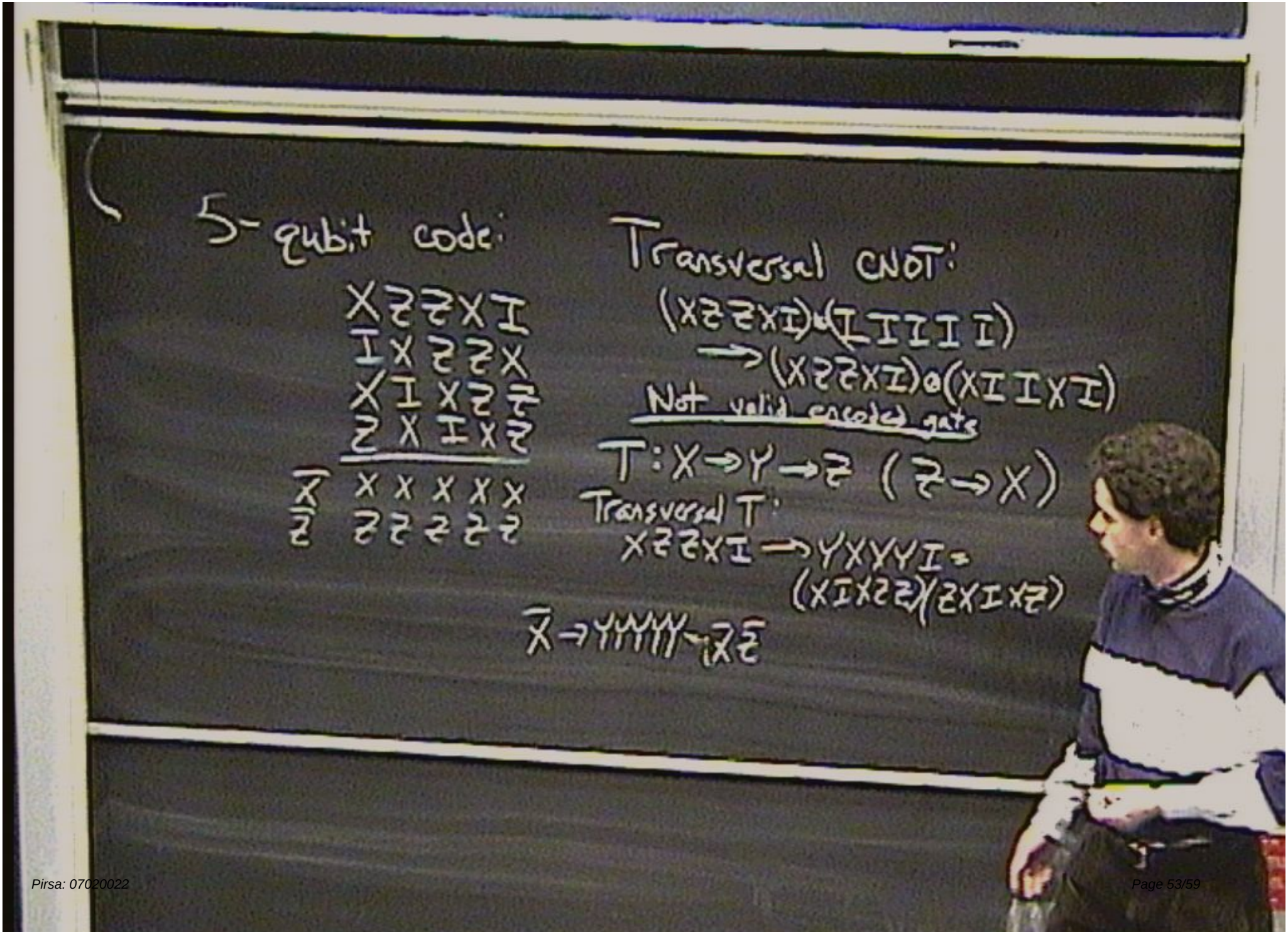
Not valid encoded gate

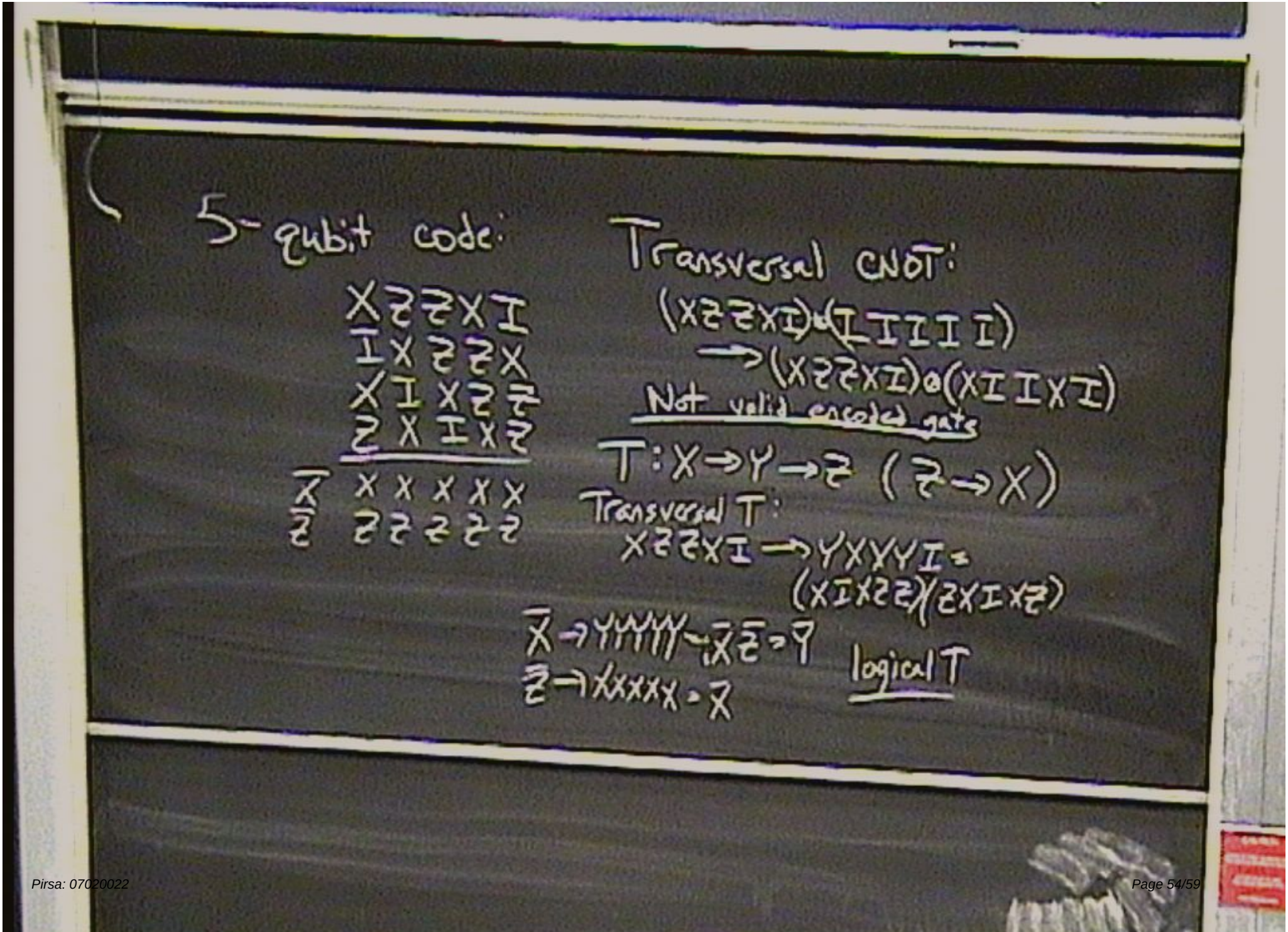
$T: X \to Y \to Z \quad (Z \to X)$

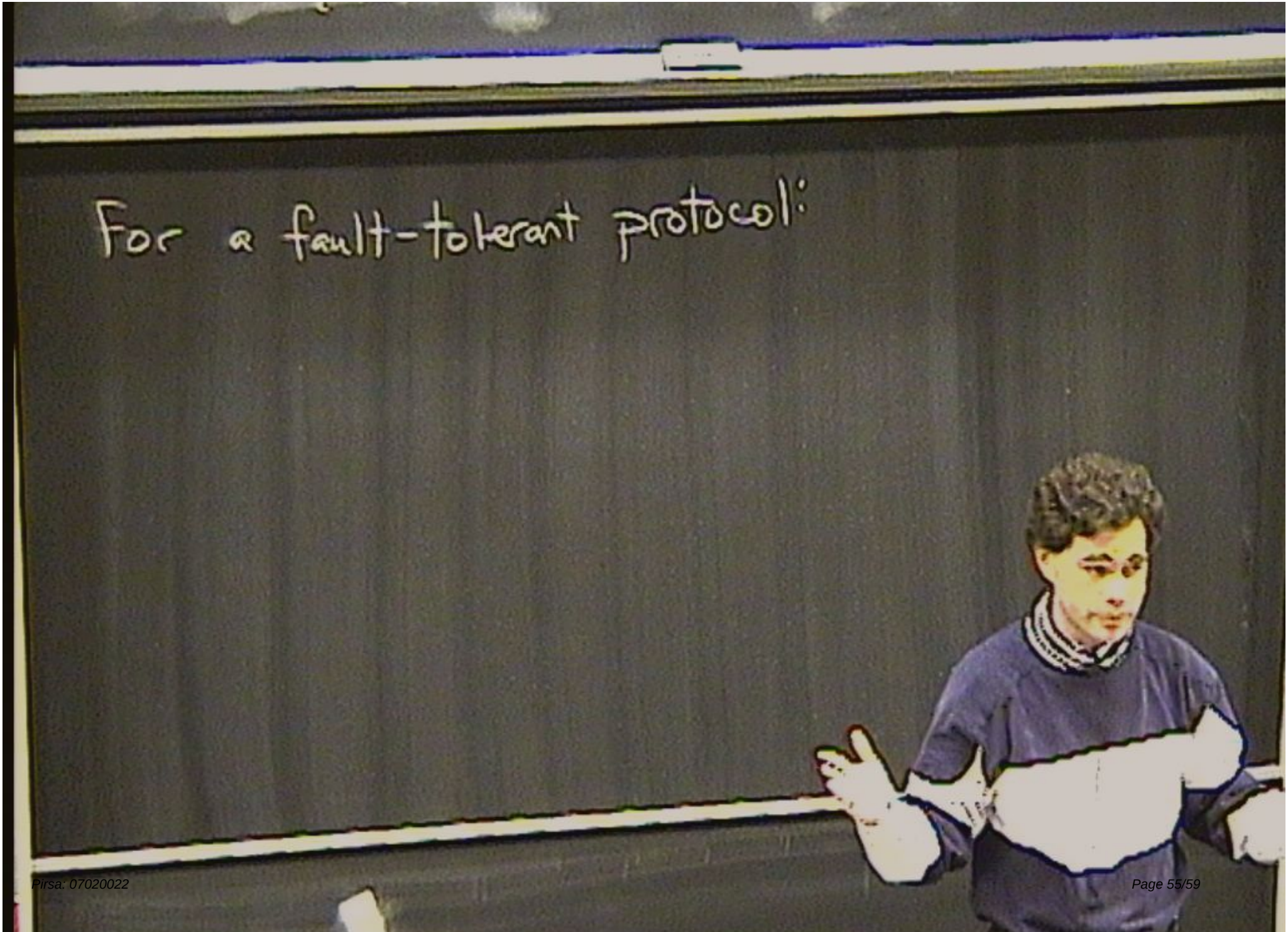

5-qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
X X X X X
Z Z Z Z Z
Transversal CNOT:
(XZZXI)⊗(IIIII)
→(XZZXI)⊗(XIIXI)
Not valid encoded gate
T: X→Y→Z (Z→X)
Transversal T:
XZZXI → YXYYI


5-qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
X̄ XXXXX
Z̄ ZZZZZ
Transversal CNOT:
(XZZXI)⊗(IIIII)
→(XZZXI)⊗(XIIXI)
Not valid encoded gate
T: X→Y→Z (Z→X)
Transversal T:
XZZXI → YXYYI =
(XIXZZ)(ZXIXZ)


5-qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
X X X X X
Z Z Z Z Z
Transversal CNOT:
(XZZXI)⊗(IIIII)
→(XZZXI)⊗(XIIXI)
Not valid encoded gate
T: X→Y→Z (Z→X)
Transversal T:
XZZXI → YXYYI
(XIXZZ)(ZXIXZ)


5-qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
X X X X X
Z Z Z Z Z
Transversal CNOT:
(XZZXI)⊗(IIIII)
→(XZZXI)⊗(XIIXI)
Not valid encoded gate
T: X→Y→Z (Z→X)
Transversal T:
XZZXI → YXYYI =
(XIXZZ)(ZXIXZ)
X̄ → YYYYY ~ ZZ


5-qubit code:
XZZXI
IXZZX
XIXZZ
ZXIXZ
X X X X X
Z Z Z Z Z
Transversal CNOT:
(XZZXI)⊗(IIIII)
→(XZZXI)⊗(XIIXI)
Not valid encoded gate
T: X→Y→Z (Z→X)
Transversal T:
XZZXI → YXYYI =
(XIXZZ)(ZXIXZ)
X̄ → YYYYY = iX̄Z̄ = Ȳ
Z̄ → XXXXX = X̄
logical T


For a fault-tolerant protocol:

For a fault-tolerant protocol:

① FT state preparation (for $|\bar{0}\rangle$)

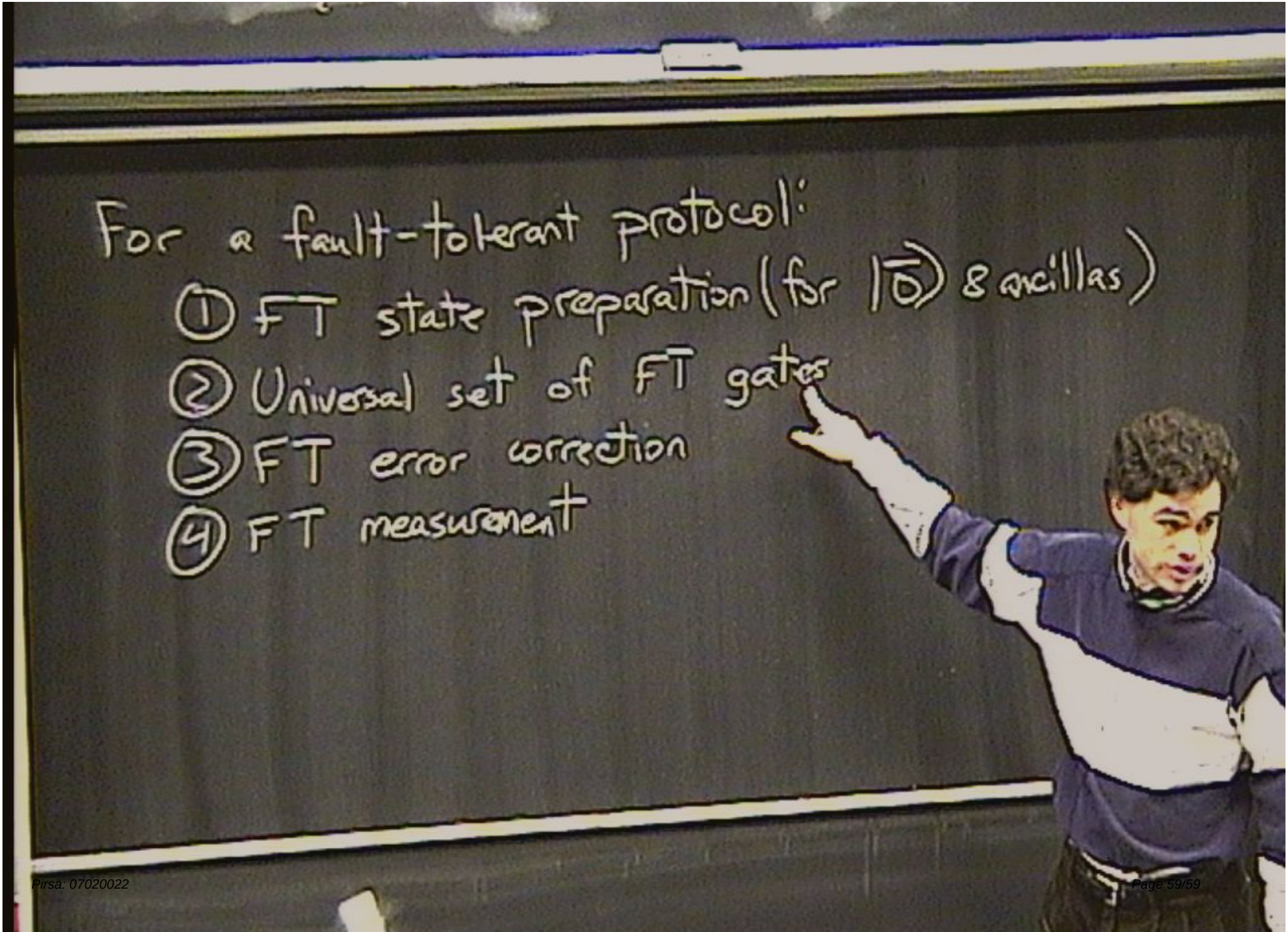
For a fault-tolerant protocol:

① FT state preparation (for $|\vec{0}\rangle$ & ancillas)

② Universal set of FT gates

For a fault-tolerant protocol:

1. FT state preparation (for $|\overline{0}\rangle$ & ancillas)
2. Universal set of FT gates
3. FT error correction


For a fault-tolerant protocol:
① FT state preparation (for $|\overline{0}\rangle$ & ancillas)
② Universal set of FT gates
③ FT error correction
④ FT measurement