Title: Cosmological tests of general relativity

Date: Feb 20, 2007 11:00 AM

URL: http://pirsa.org/07020010

Abstract: The general relativity has been tested from mm scales to solar system scales. The discovery of cosmic acceleration motivates the study of infrared modification of gravity at horizon scales.
The cosmic expansion can be accelerated by dark energy without any correction to GR, but alternatively it can be explained by the modified gravity at large scales without introducing the unknown exotic energy. We introduce the linear structure formation theory of DGP and f(R) gravity, and present what it the strategy to test general relativity at cosmological scales.

Cosmological tests on General Relativity

Yong-Seon Song

Enrico Fermi Institute & KICP

University of Chicago

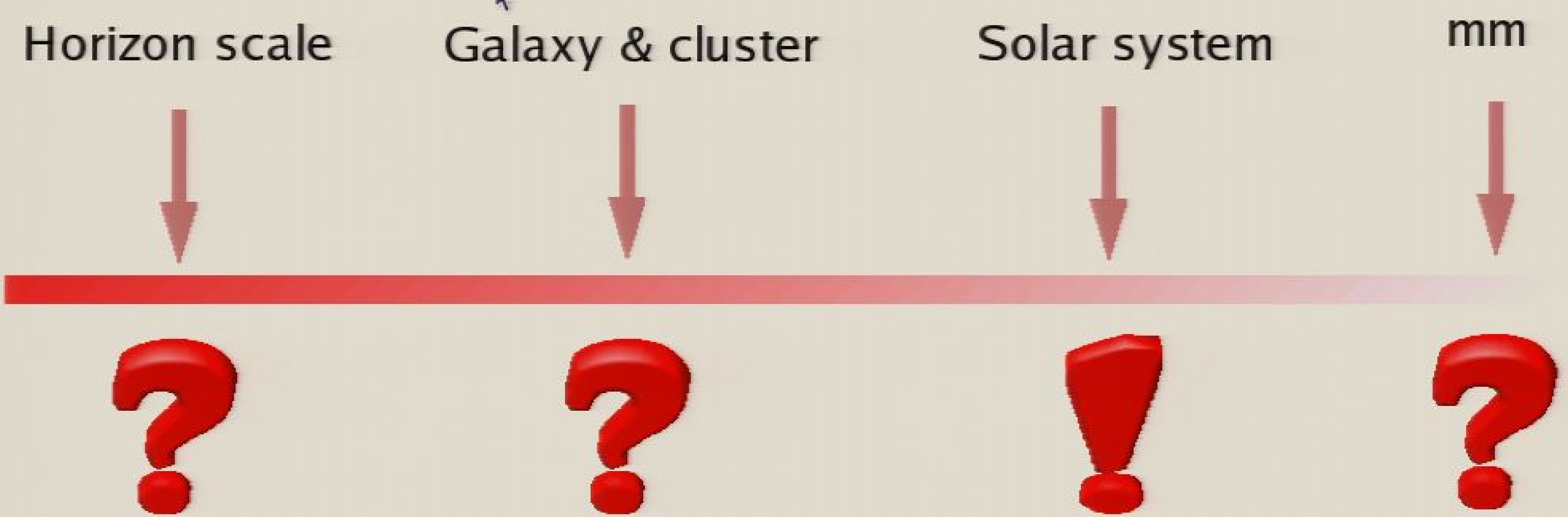
Contents

- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

Contents

- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

Scales of probing general relativity

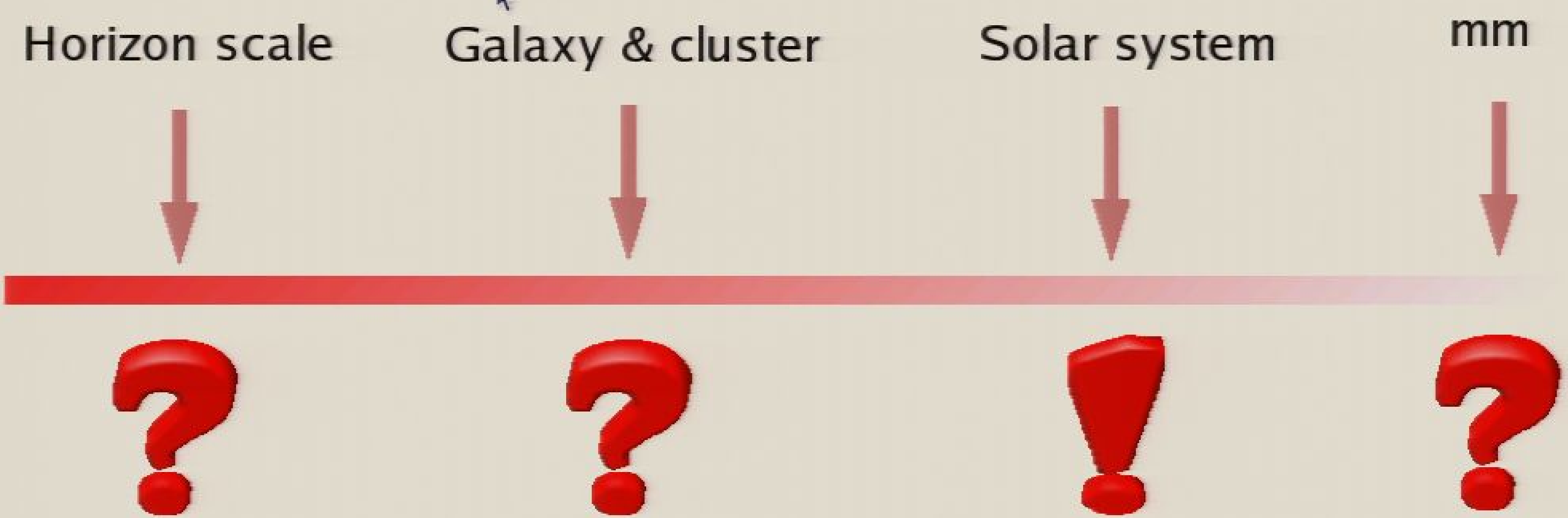


Nature of the cosmic acceleration

Dark energy:	○
General relativity:	Unbroken

Dark energy:	×
General relativity:	Broken

Scales of probing general relativity



Nature of the cosmic acceleration

Dark energy: ❍

General relativity: Unbroken

Dark energy: ×

General relativity: Broken

Nature of the cosmic acceleration

Dark energy:	○
General relativity:	Unbroken

Dark energy:	×
General relativity:	Broken

Modified gravity models to explain the cosmic acceleration

- Brane world model (DGP type): *Dvali. Gabadadze and Poratti (2000)*

  4D brane where we live is embeded on the extra dimension. The gravity leaks into the extra dimension and get weakened. In the self-accelerated branch, the cosmic expansion of the 4D brane is accelerated without any dark energy.

- f(R) theory: *Carroll, Duvvuri, Trodden and Turner (2003)*

  The cosmic acceleration can be generated by introducing nonlinear corrections to the Einstein-Hilbert Lagrangian which become dominant only at late time (low curvatures).

Modified gravity models to explain the cosmic acceleration

- Brane world model (DGP type): *Dvali. Gabadadze and Poratti (2000)*

  4D brane where we live is embeded on the extra dimension. The gravity leaks into the extra dimension and get weakened. In the self-accelerated branch, the cosmic expansion of the 4D brane is accelerated without any dark energy.

- f(R) theory: *Carroll, Duvvuri, Trodden and Turner (2003)*

  The cosmic acceleration can be generated by introducing nonlinear corrections to the Einstein-Hilbert Lagrangian which become dominant only at late time (low curvatures).

Contents

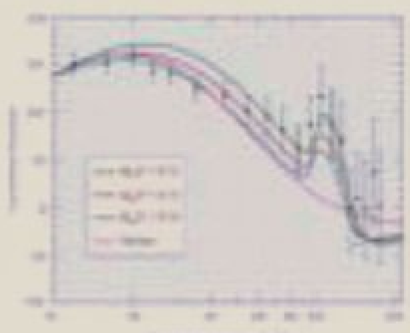
- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

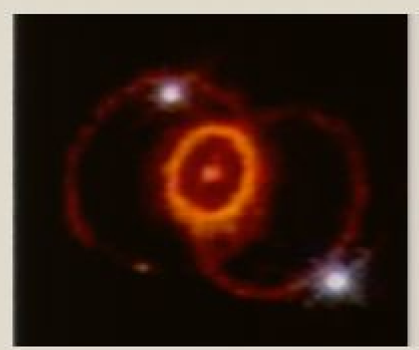
Triumph on cosmological constant

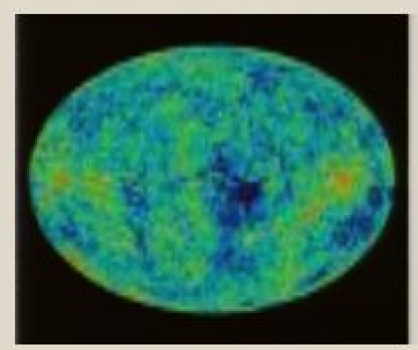
The best fit model to current observation is a cosomological constant with cold dark matter, which requests flatness of curvature.

Row redshift

High redshift





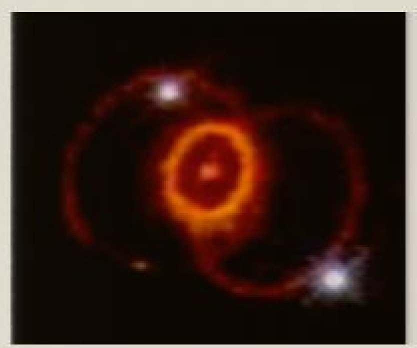


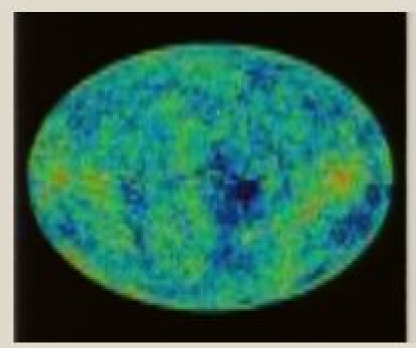
Triumph on cosmological constant

The best fit model to current observation is a cosomological constant with cold dark matter, which requests flatness of curvature.

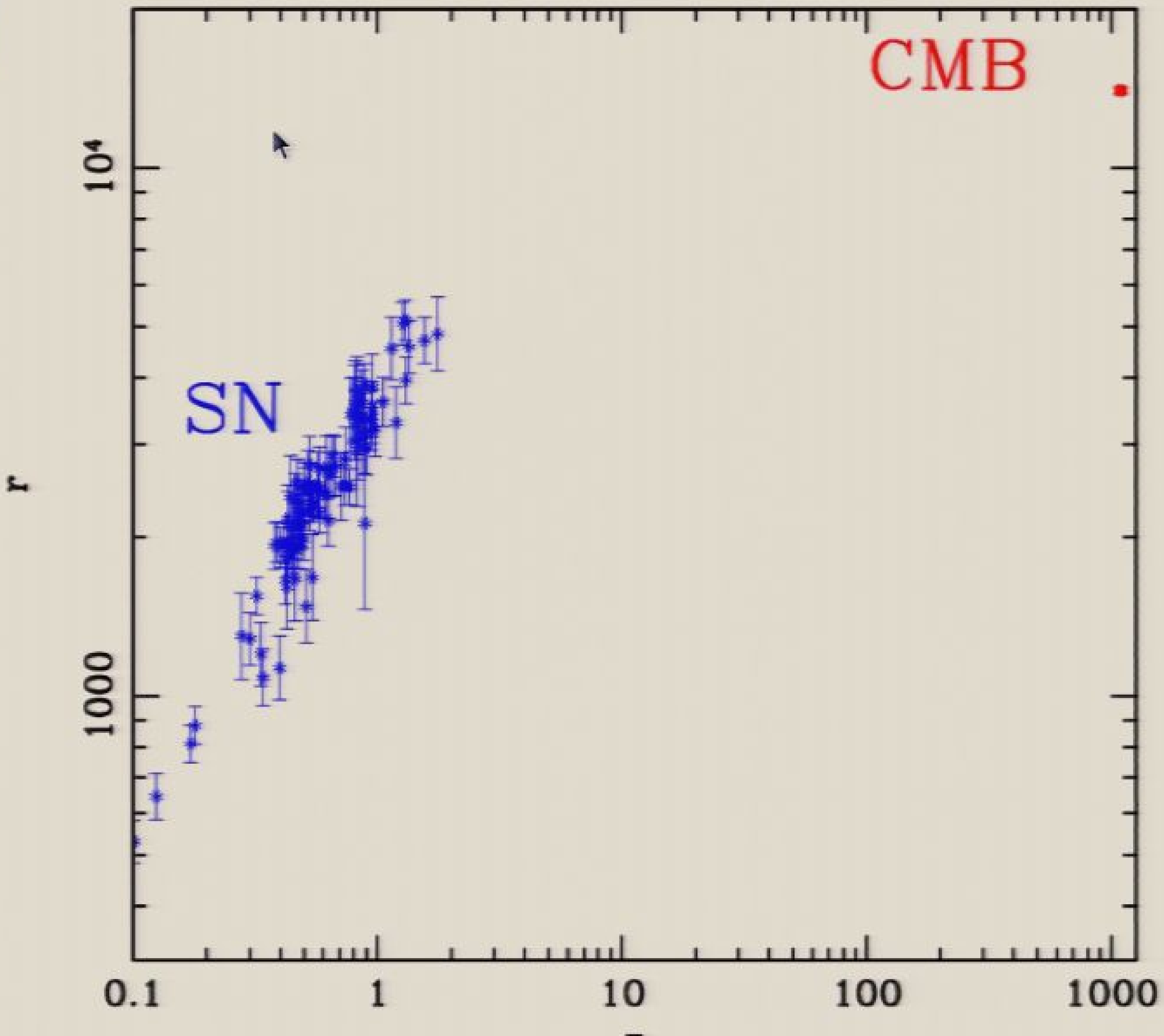
Row redshift

High redshift

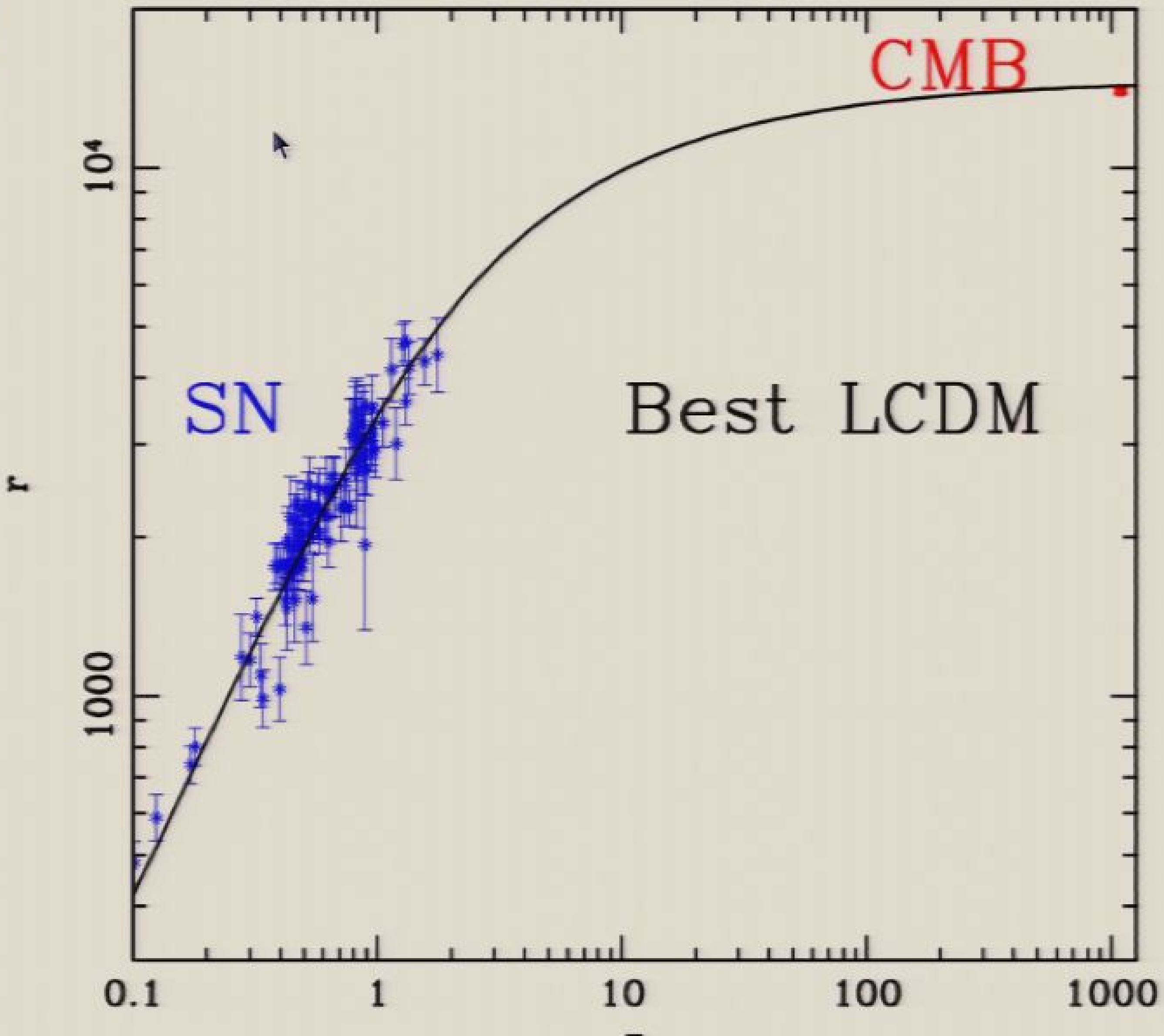




Leverage arm in geometrical test



Leverage arm in geometrical test



Contents

- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

Expansion rate of DGP self-acceleration branch

The expansion rate H(z) of DGP self acceleration branch is similar to that of dark energy model with $w_{eff}(z) > -1$.

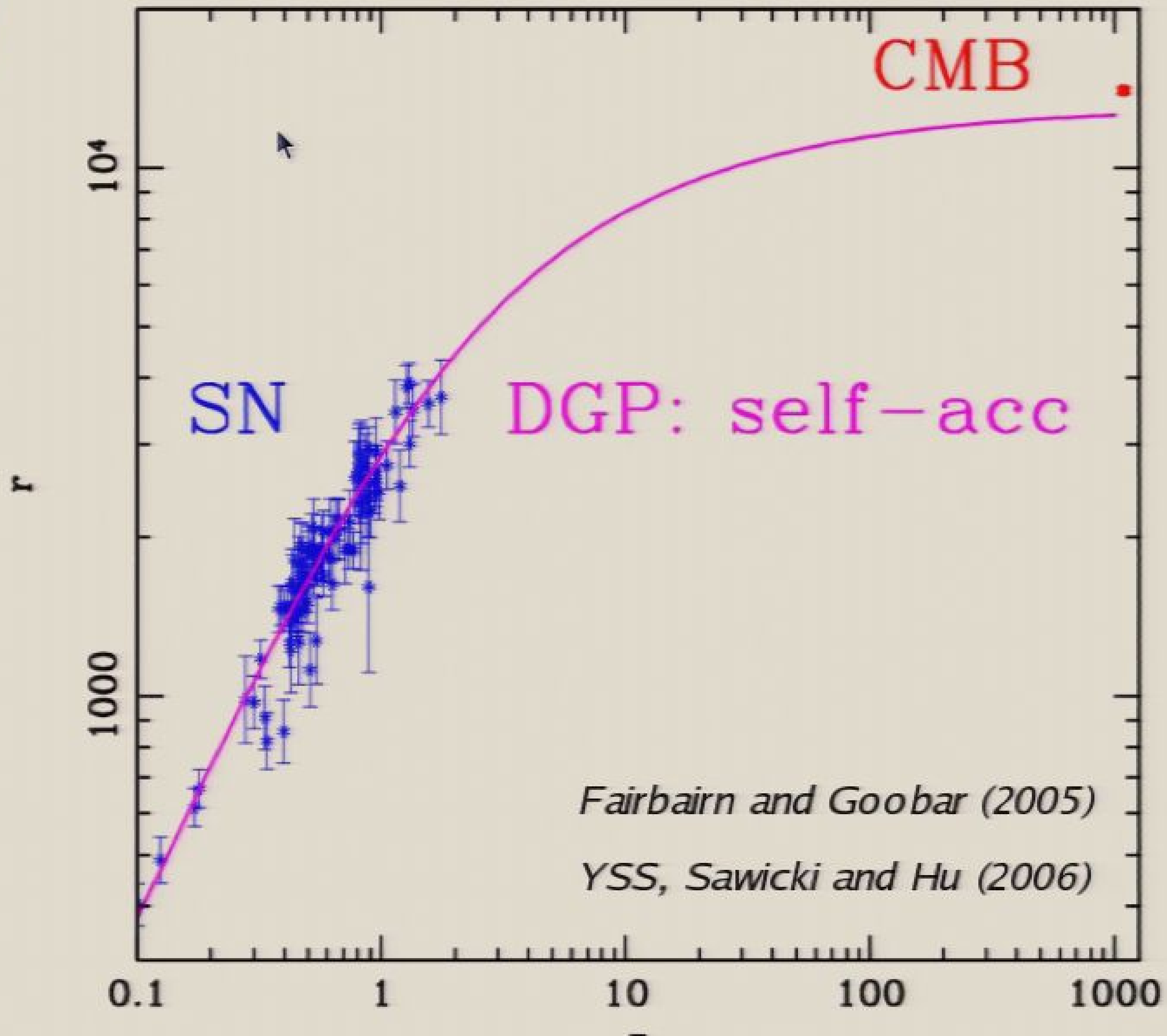
In DE of $w_{eff}(z) > -1$, with fixed $\omega_m = \Omega_m h^2$ by CMB prior

$$w_{eff}(z) > -1 \longrightarrow \Omega_{DE} \uparrow \longrightarrow H_0 \uparrow$$

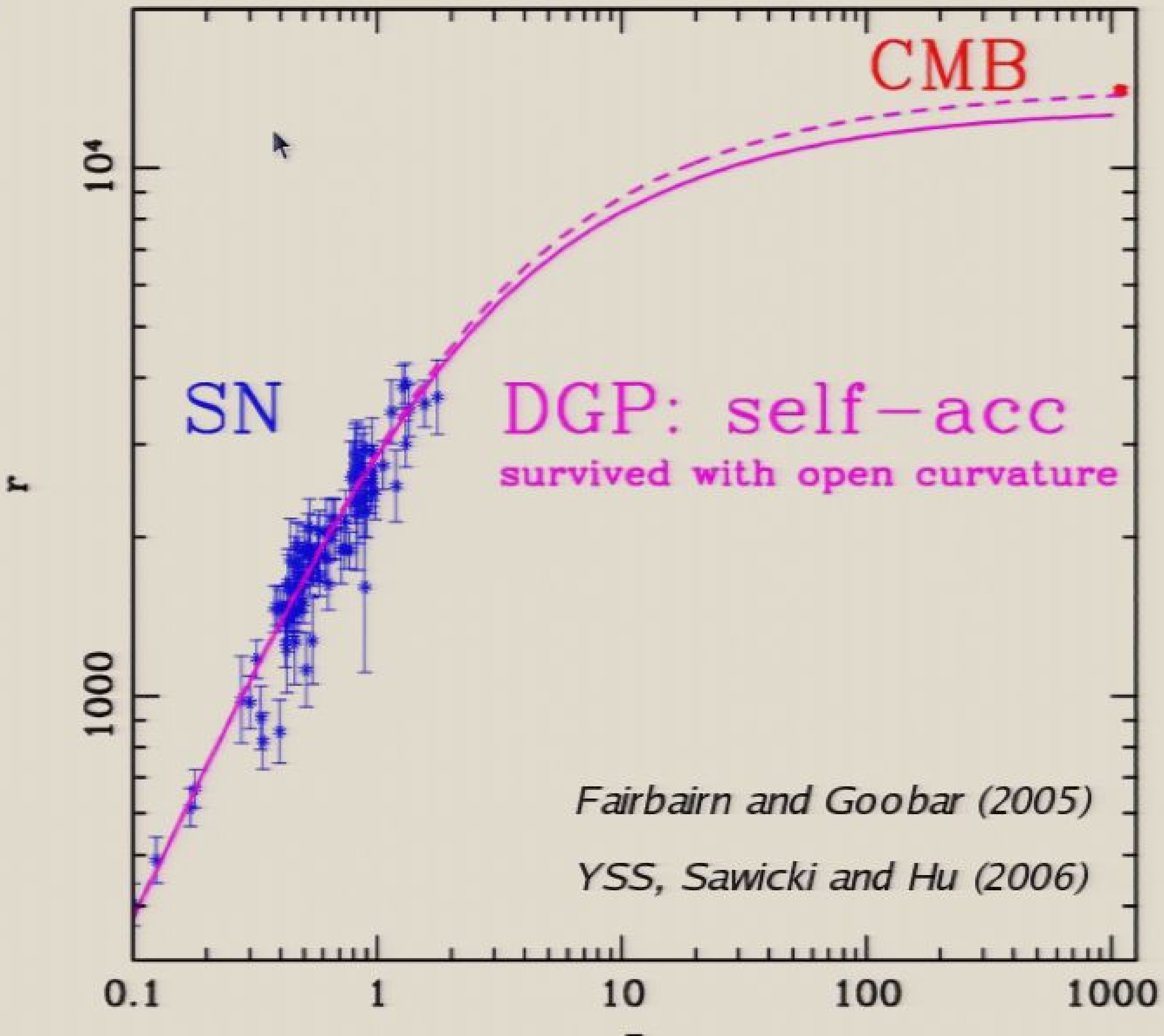
DGP self-acc branch also has high H_0, which leads

- Failure to fit $D(z_{lss}=1089)$ by CMB – cured by open curvature
- Failure to fit H0 measurement at row redshift

Open curvature save DGP self-acc from failure to fit CMB



Open curvature save DGP self-acc from failure to fit CMB



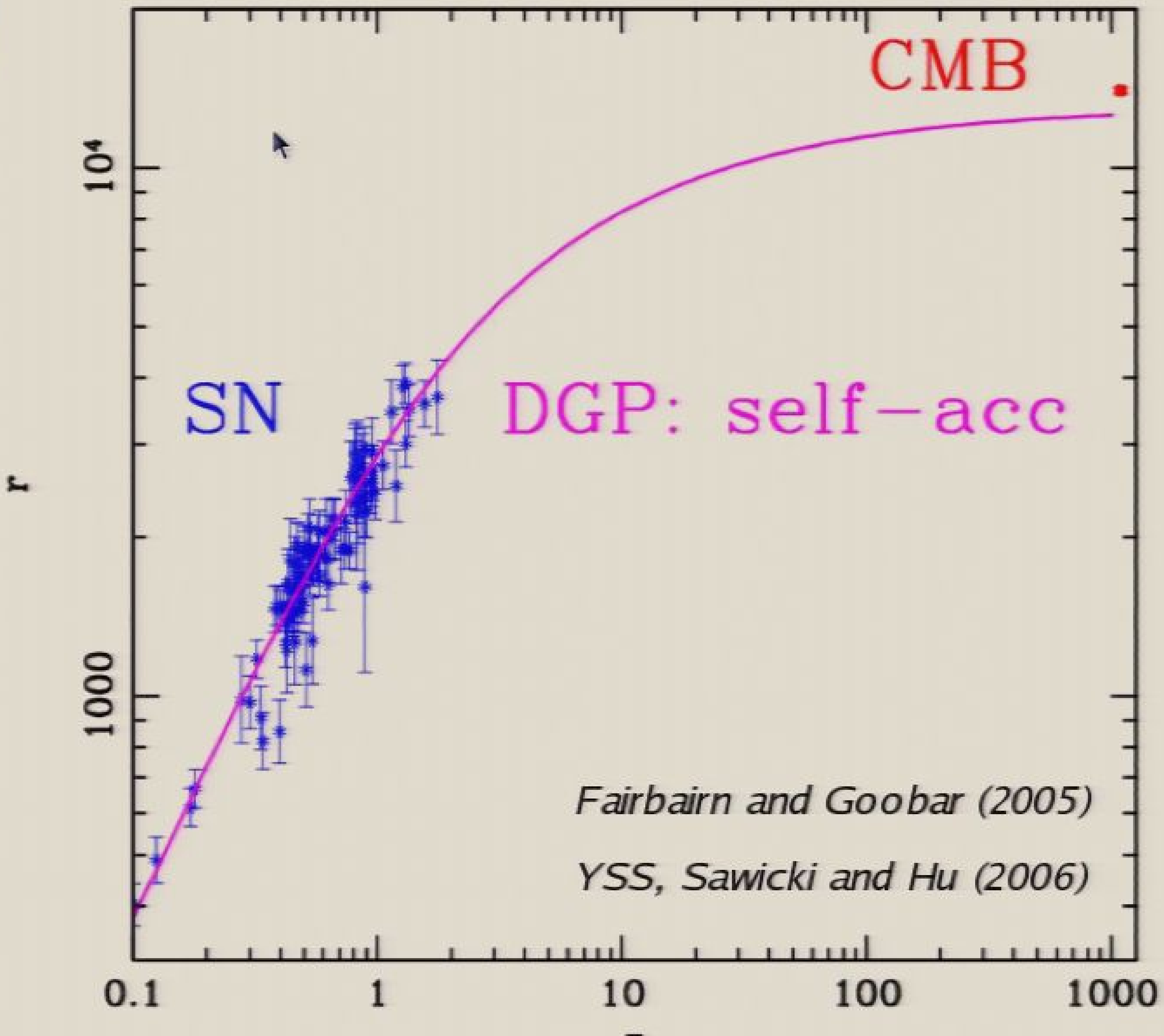
But still distinguishable by different H_0

YSS, Sawicki and Hu (2006)

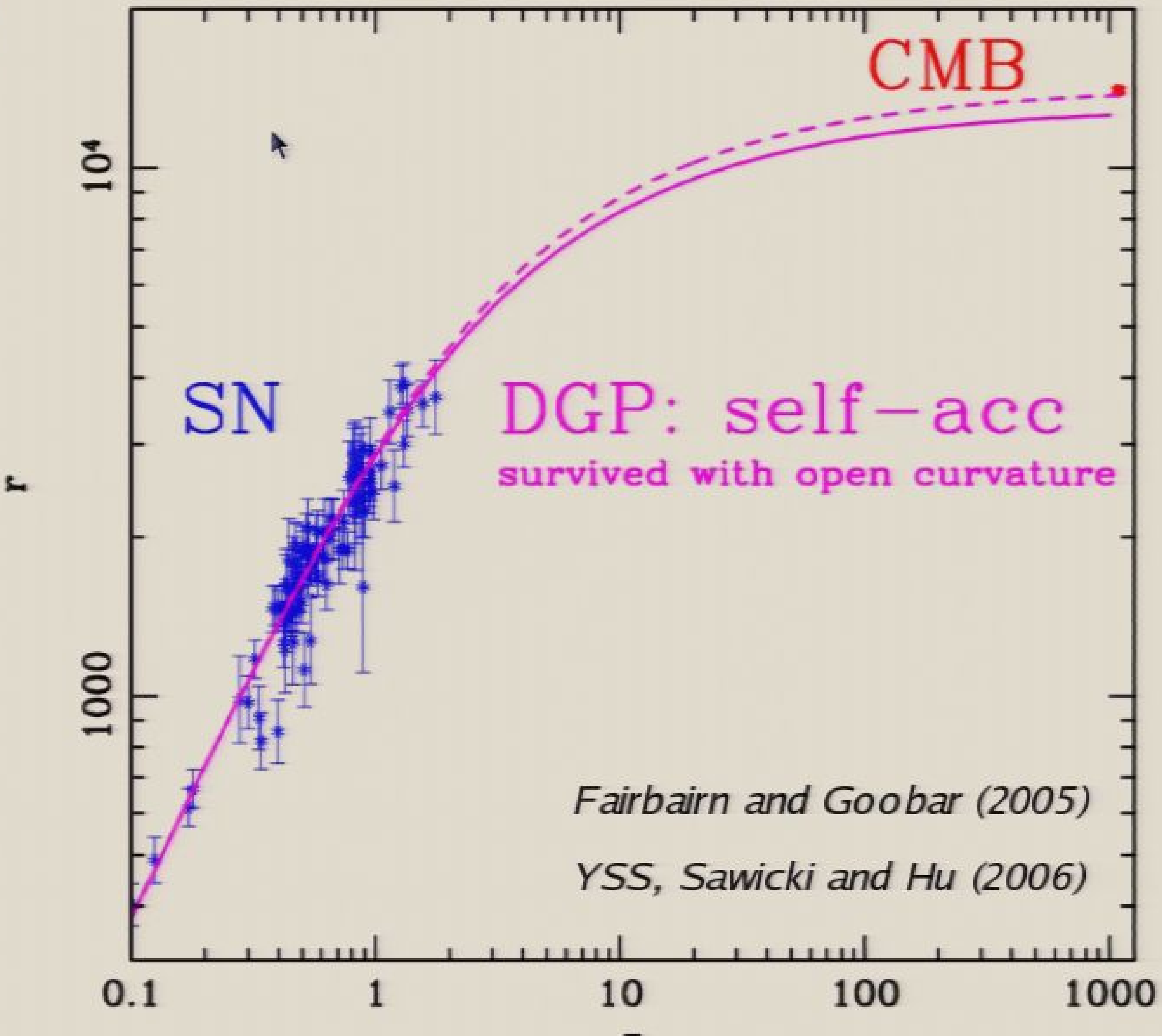
The measurement of H0 by Hubble key project gives already tension to DGP self-acceleration branch

BAO at z=0.35 which can be another test on H0, clearly rules out DGP self-acceleration branch.

Open curvature save DGP self-acc from failure to fit CMB



Open curvature save DGP self-acc from failure to fit CMB



But still distinguishable by different H_0

YSS, Sawicki and Hu (2006)

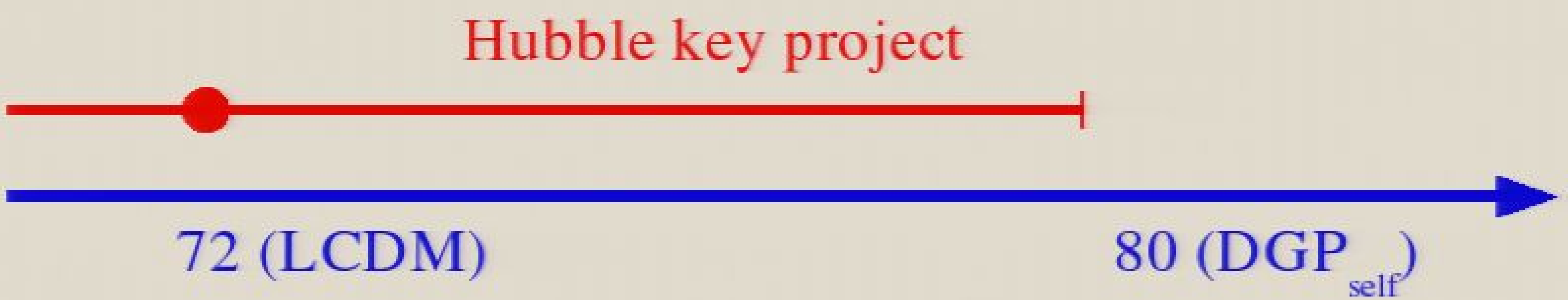
The measurement of H0 by Hubble key project gives already tension to DGP self-acceleration branch

BAO at z=0.35 which can be another test on H0, clearly rules out DGP self-acceleration branch.

But still distinguishable by different H_0

YSS, Sawicki and Hu (2006)

The measurement of H0 by Hubble key project gives already tension to DGP self-acceleration branch



But still distinguishable by different H_0

YSS, Sawicki and Hu (2006)

The measurement of H0 by Hubble key project gives already tension to DGP self-acceleration branch

We are not able to use BAO test yet for Modified gravity models of which we do not have any information on non-linear scales

Expansion rate of DGP normal branch

The expansion rate H(z) of DGP self acceleration branch is similar to that of dark energy model with $w_{eff}(z) < -1$.

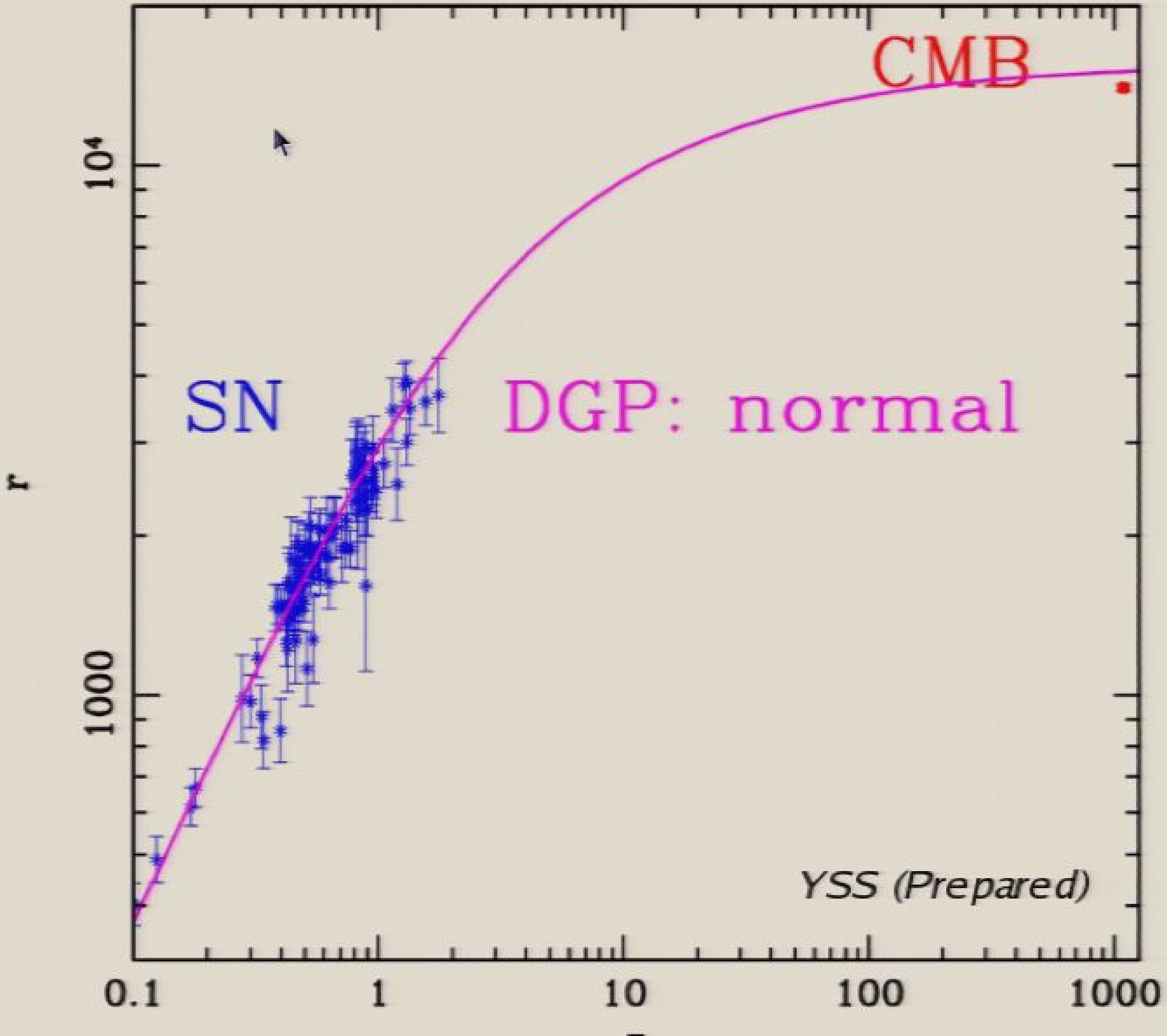
In DE of $w_{eff}(z) < -1$, with fixed $\omega_m = \Omega_m h^2$ by CMB prior

$$w_{eff}(z) < -1 \longrightarrow \Omega_{DE} \downarrow \longrightarrow H_0 \downarrow$$

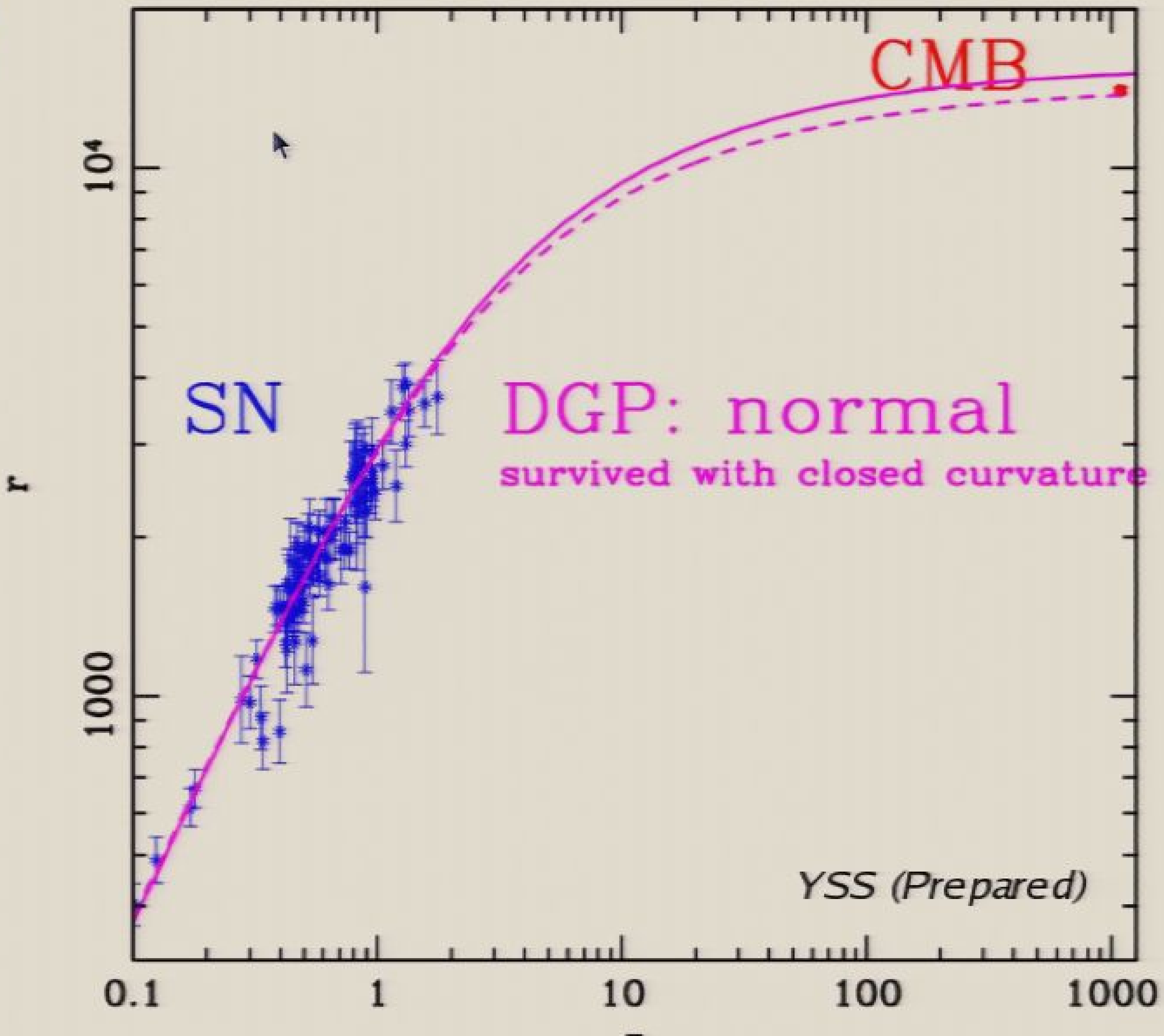
DGP normal branch also has low H_0, which leads

- Failure to fit $D(z_{lss}=1089)$ by CMB – cured by closed curvature
- Failure to fit H0 measurement at row redshift

Closed curvature save DGP normal from failure to fit CMB



Closed curvature save DGP normal from failure to fit CMB



Contents

- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

Background expansion of f(R) gravity

There are two branches which show different behaviors with the given f(R) gravity Einstein Hilbert action in Jordan frame

$$S=\int d^4x\sqrt{-g}\left[R_J+f(R_J)+L_m\right]$$

$$B=\frac{f_{RR}}{1+f_R}R_J'\frac{H}{H'}$$

- B<0 (f_{RR}<0): Inconsistent to GR at high curvature limit, accordingly perturbation is broken down.

- B>0 (f_{RR}>0): Consistent to GR at high curvature limit, and perturbation is stabilized.

YSS, Hu and Sawicki (2006)

Critiques and resolutions

- YSS, Hu and Sawicki:
  - B<0 branch: if it has GR limit at high curvature, then perturbations are blown up.
  - B>0 branch: it has GR limit at high curvature, and perturbations are stabilised.
- Faraoni: f(R) models does not have matter dominated era (i.e. no good GR limit).
- Bean et.al: B<0 branch perturbations are stabilised,

Critiques and resolutions

- YSS, Hu and Sawicki:
  - B<0 branch: if it has GR limit at high curvature, then perturbations are blown up.
  - B>0 branch: it has GR limit at high curvature, and perturbations are stabilised.
- Faraoni: B<0 branch f(R) models does not have matter dominated era (i.e. no good GR limit).
- Bean et.al: B<0 branch perturbations are stabilised, with background expansion which missed matter dominated era.

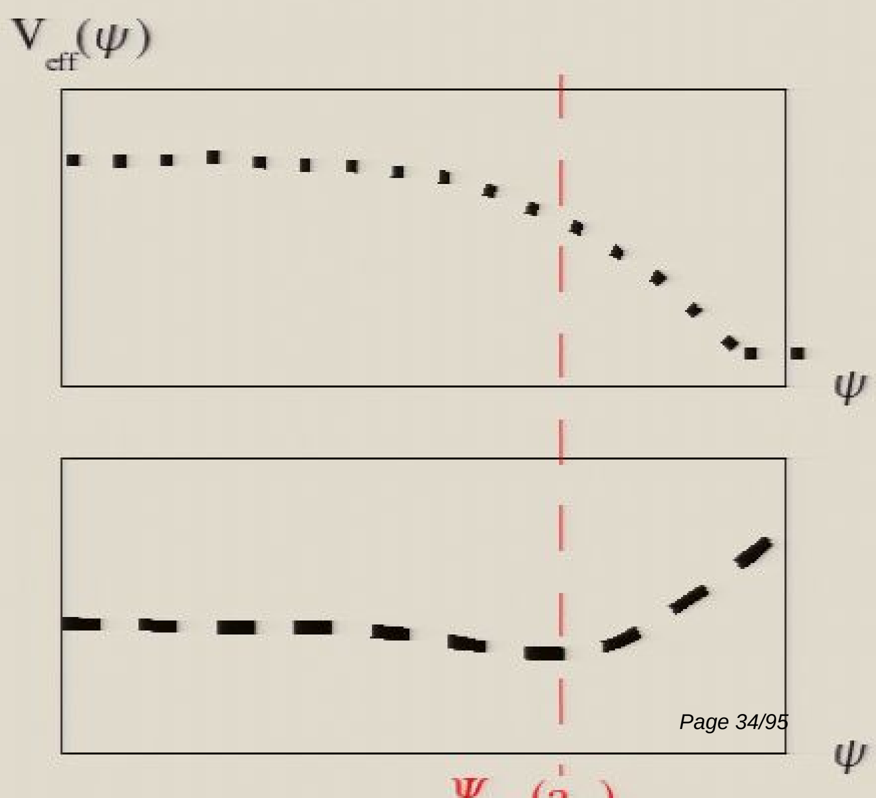
View from Einstein frame

Faraoni (2006)
YSS, Hu and Sawicki (2006)
Bean et.al. (2007)
Sawicki and Hu (2007)

It was shown that $f=-1/R_J$ model does not have matter domination epoch. It is studied extensively in both branches

- Specific models in B<0 branch

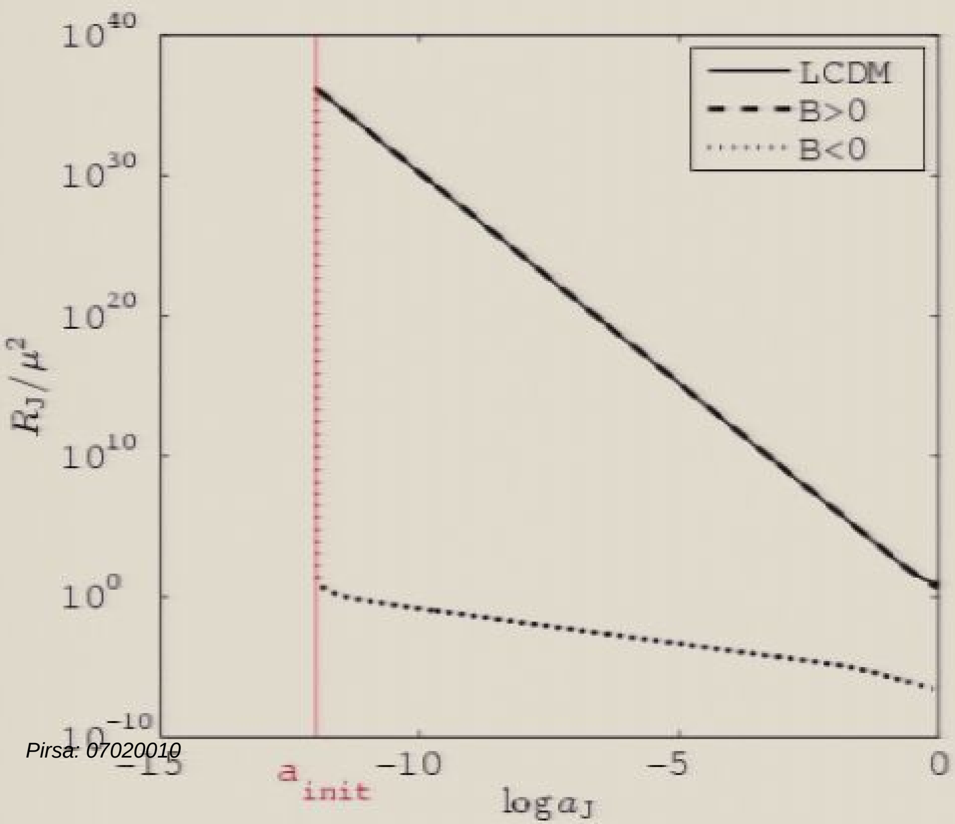
- Specific models in B>0 branch

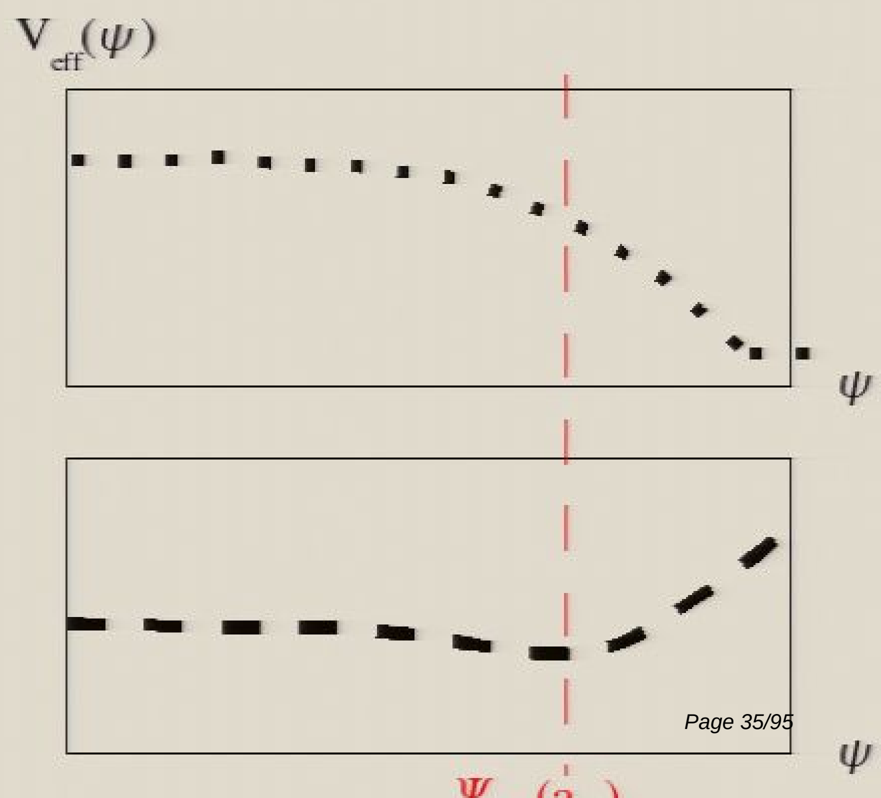


View from Einstein frame

Faraoni (2006)
YSS, Hu and Sawicki (2006)
Bean et.al. (2007)
Sawicki and Hu (2007)

It was shown that $f=-1/R_J$ model does not have matter domination epoch. It is studied extensively in both branches

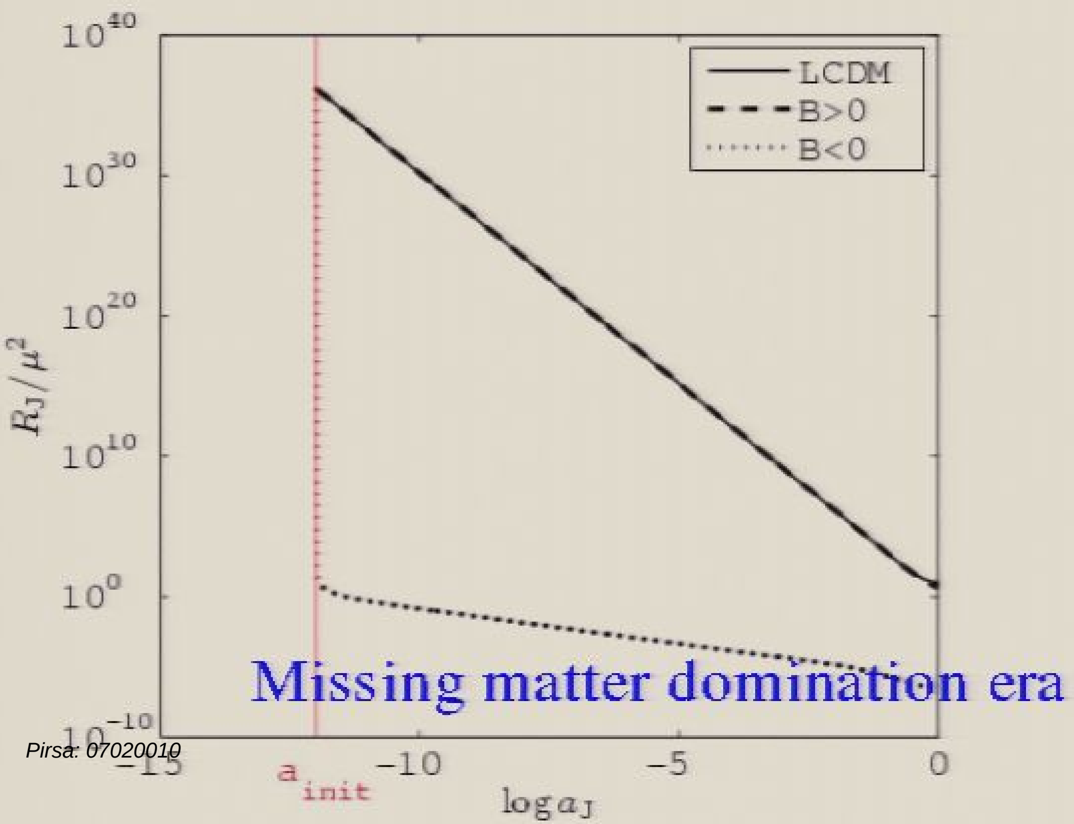


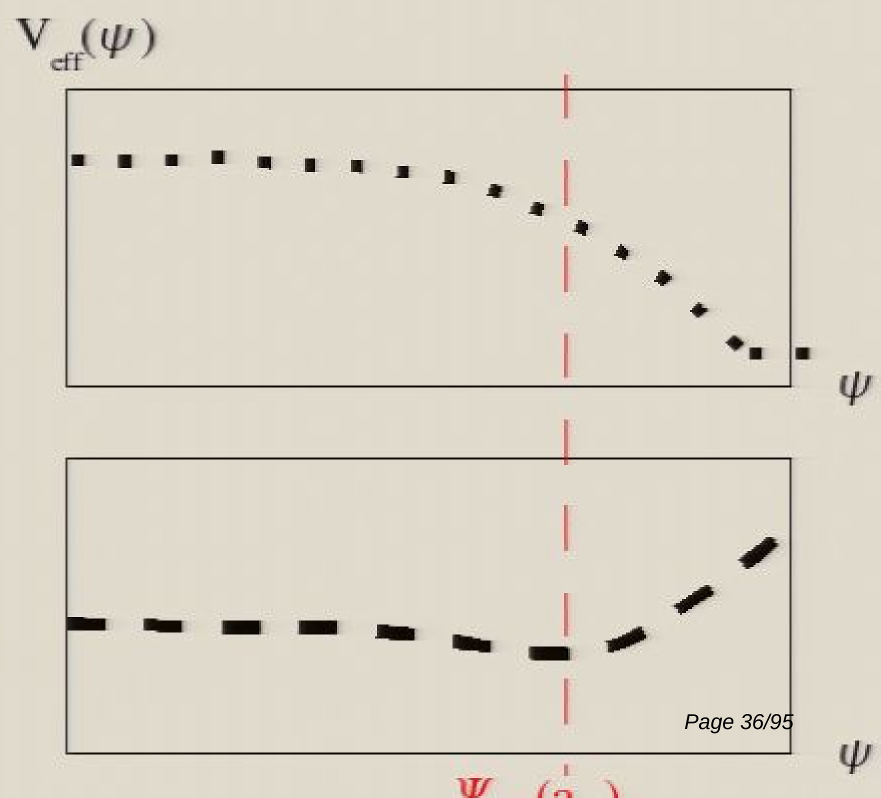


View from Einstein frame

Faraoni (2006)
YSS, Hu and Sawicki (2006)
Bean et.al. (2007)
Sawicki and Hu (2007)

It was shown that $f=-1/R_J$ model does not have matter domination epoch. It is studied extensively in both branches

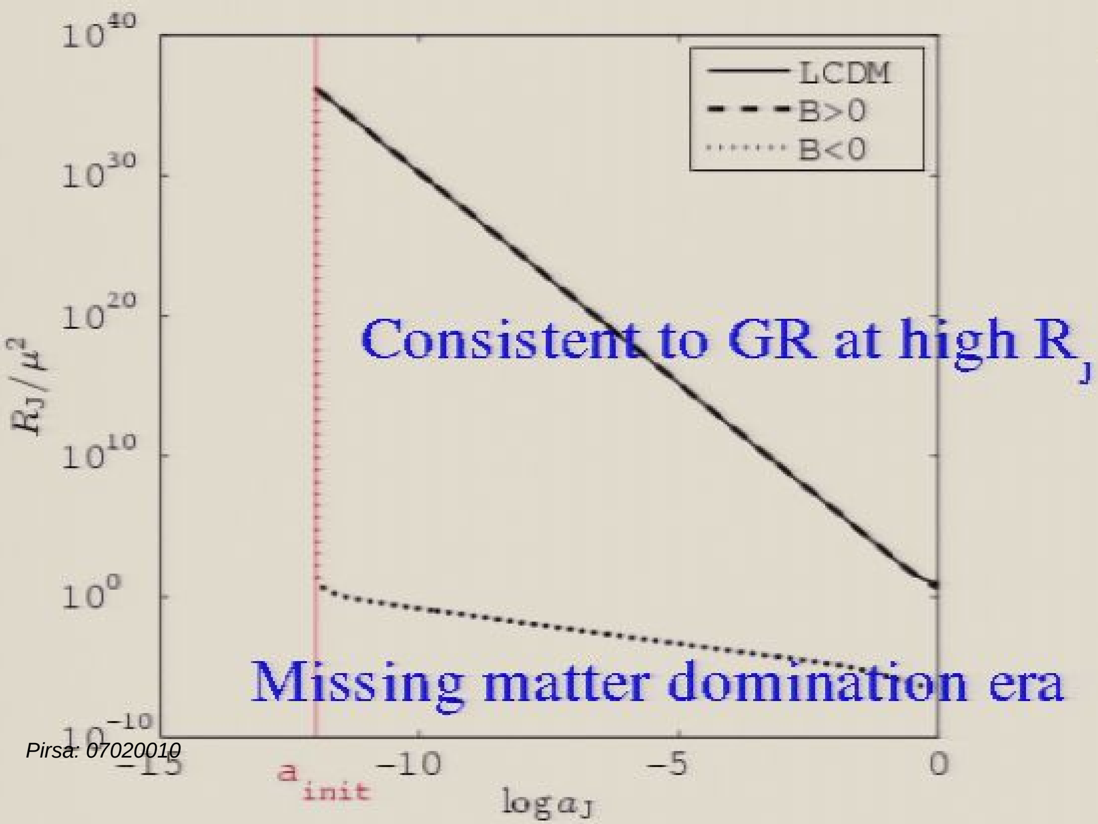


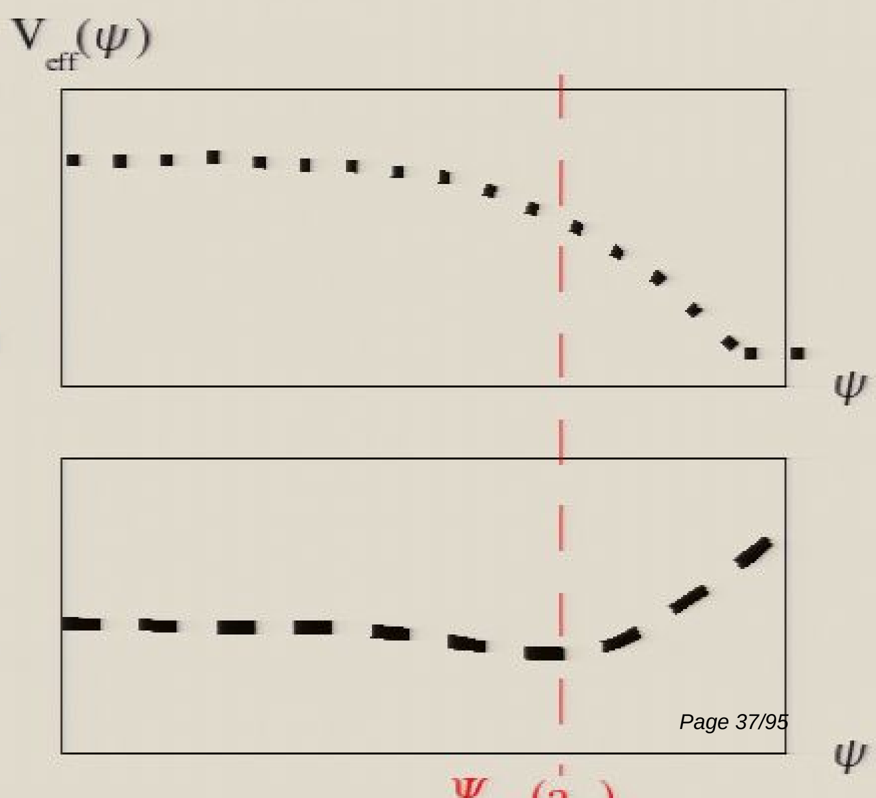


View from Einstein frame

Faraoni (2006)
YSS, Hu and Sawicki (2006)
Bean et.al. (2007)
Sawicki and Hu (2007)

It was shown that $f=-1/R_J$ model does not have matter domination epoch. It is studied extensively in both branches





View from Jordan frame: anagram

Tom Marvolo Riddle

$$H^2 - f_R(HH' + H^2) + \frac{1}{6} f + H^2 f_{RR} R' = \frac{8\pi G_N}{3} \rho_m$$

View from Jordan frame: anagram

$$H^2 - f_R(HH' + H^2) + \frac{1}{6} f + H^2 f_{RR} R' = \frac{8\pi G_N}{3} \rho_m$$

View from Jordan frame: anagram

an Lo d V de ort

$$H^2 - f_R(HH' + H^2) + \frac{1}{6} f + H^2 f_{RR} R' = \frac{8\pi G_N}{3} \rho_m$$

View from Jordan frame: anagram

I am Lord Voldemort

$$H^2 - f_R(HH' + H^2) + \frac{1}{6}f + H^2 f_{RR} R' = \frac{8\pi G_N}{3}\rho_m$$

View from Jordan frame: anagram

$$H^2 - f_R(HH' + H^2) + \frac{1}{6}f + H^2 f_{RR} R' = \frac{8\pi G_N}{3}\rho_m$$

View from Jordan frame: anagram

View from Jordan frame: anagram

$$^2 f_{RR} \quad f_R(R \quad H^2) = \frac{8\pi}{} \quad \rho_m -$$

View from Jordan frame: anagram

$$H^2 f_{RR} R' - f_R (HH' + H^2) + \frac{1}{6} f = \frac{8\pi G_N}{3} \rho_m - H^2$$

Background solution: inhomogeneous+homogeneous

We are able not only to get the function of f(R) which gives the identical H(z) with given any dark energy model, but also to get many different f(R) models by using homogeous solutions.

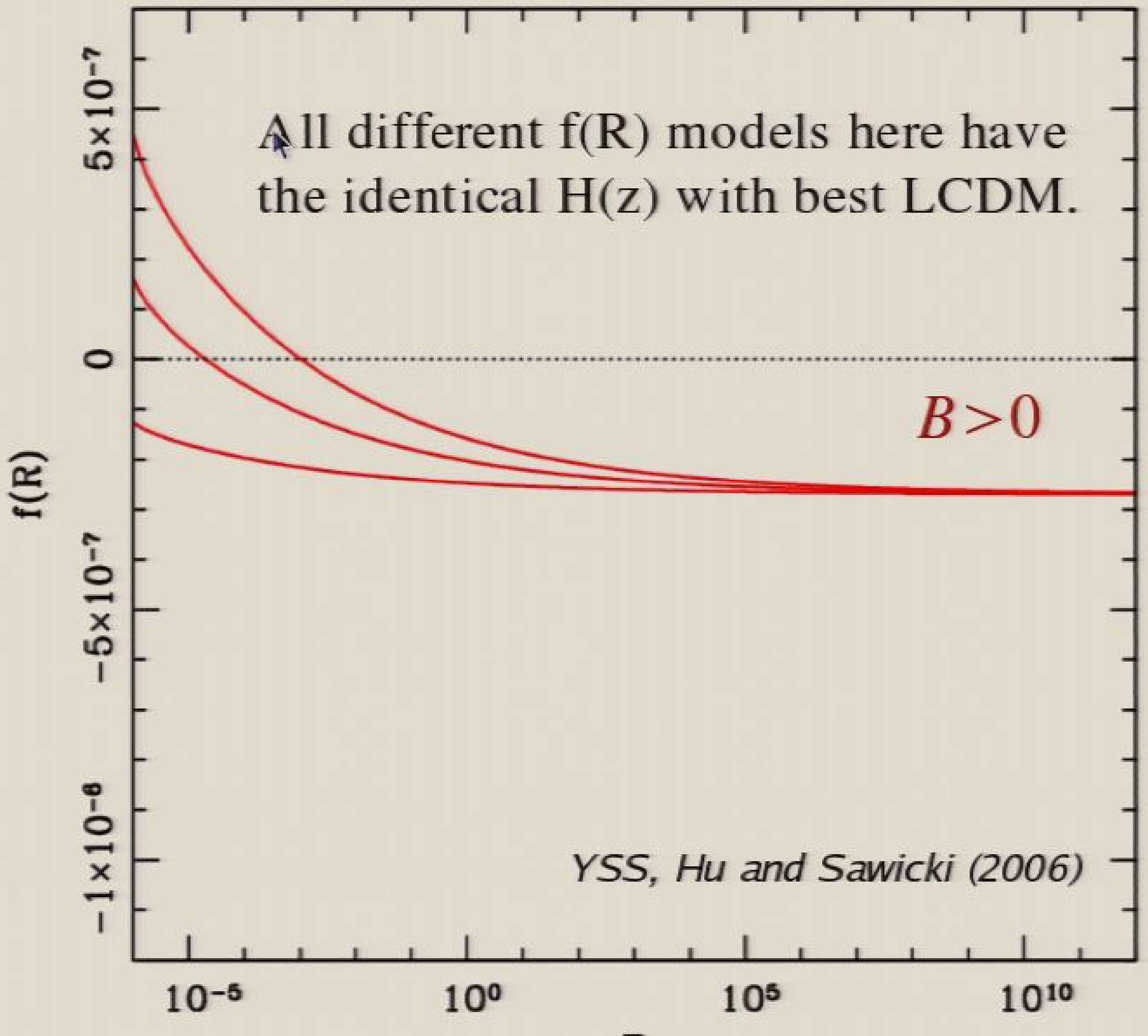
- Inhomogenneous solution: fixing H(z) with given dark energy model.

$$H^2 f_{RR} R' - f_R (HH' + H^2) + \frac{1}{6} f = \frac{8\pi G_N}{3} \rho_{DE}$$

- Homogeous solution: giving diverse f(R) even with identical H(z).

$$H^2 f_{RR} R' - f_R (HH' + H^2) + \frac{1}{6} f = 0$$

Non-uniqueness of f(R) models with identical H(z)



Summary of geometrical test on modified gravity models

DGP models are under pressure with geometrical test alone. It is expected that highly accurate measurement of H_0 possibly rules out DGP. But it survivies at this moment, it waits for the alternative cosmological test.

Some f(R) models are already ruled out by inconsistent expansion history to the current observation. But there are other types of f(R) models which have healthy background and even can provide identical H(z) of DE. We also need alternative cosmological tools to discriminate those models.

Summary of geometrical test on modified gravity models

DGP models are under pressure with geometrical test alone. It is expected that highly accurate measurement of H_0 possibly rules out DGP. But it survivies at this moment, it waits for the alternative cosmological test.

Some f(R) models are already ruled out by inconsistent expansion history to the current observation. But there are other types of f(R) models which have healthy background and even can provide identical H(z) of DE. We also need alternative cosmological tools to discriminate those models.

Contents

- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

Summary of geometrical test on modified gravity models

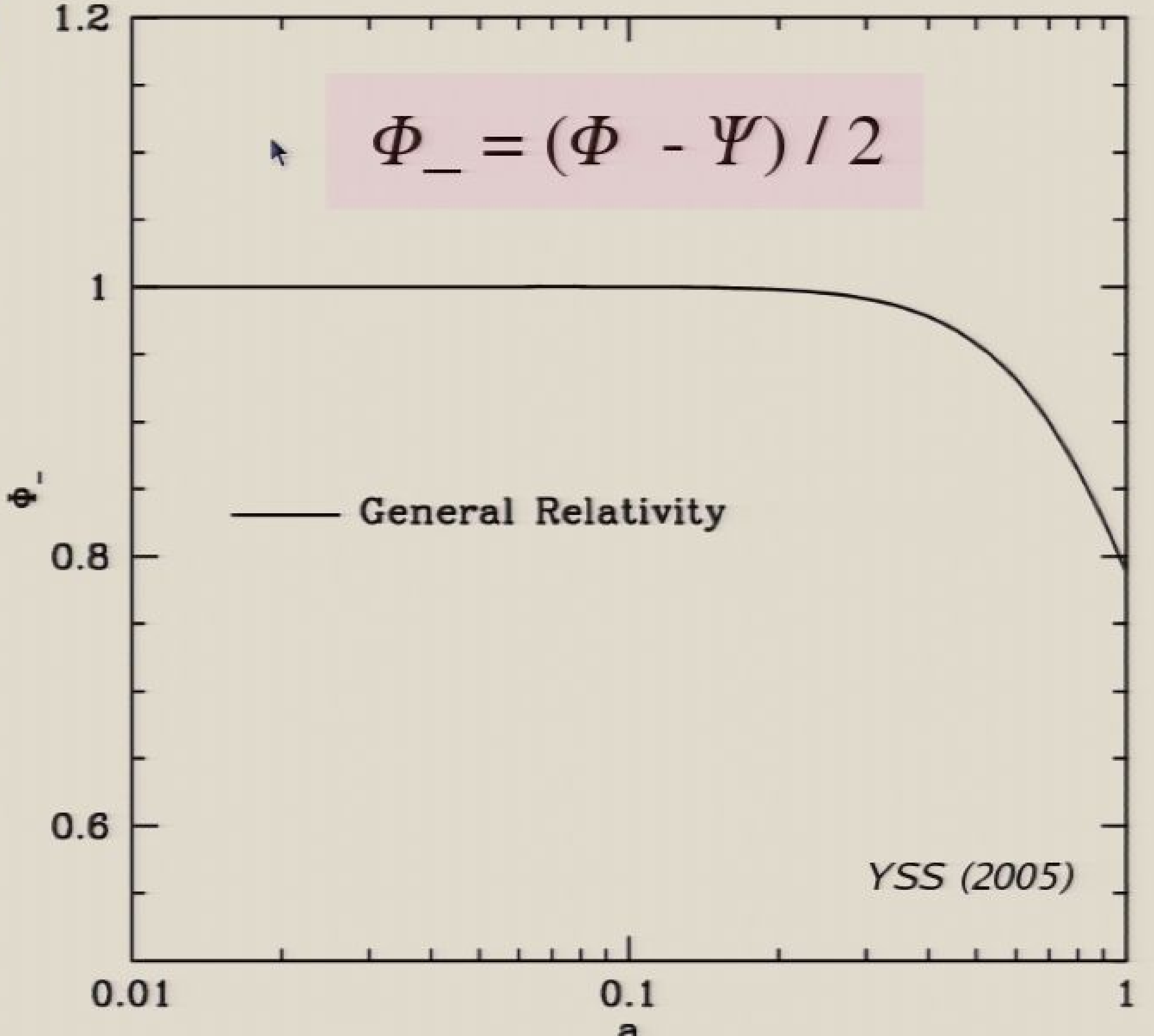
DGP models are under pressure with geometrical test alone. It is expected that highly accurate measurement of H_0 possibly rules out DGP. But it survivies at this moment, it waits for the alternative cosmological test.

Some f(R) models are already ruled out by inconsistent expansion history to the current observation. But there are other types of f(R) models which have healthy background and even can provide identical H(z) of DE. We also need alternative cosmological tools to discriminate those models.

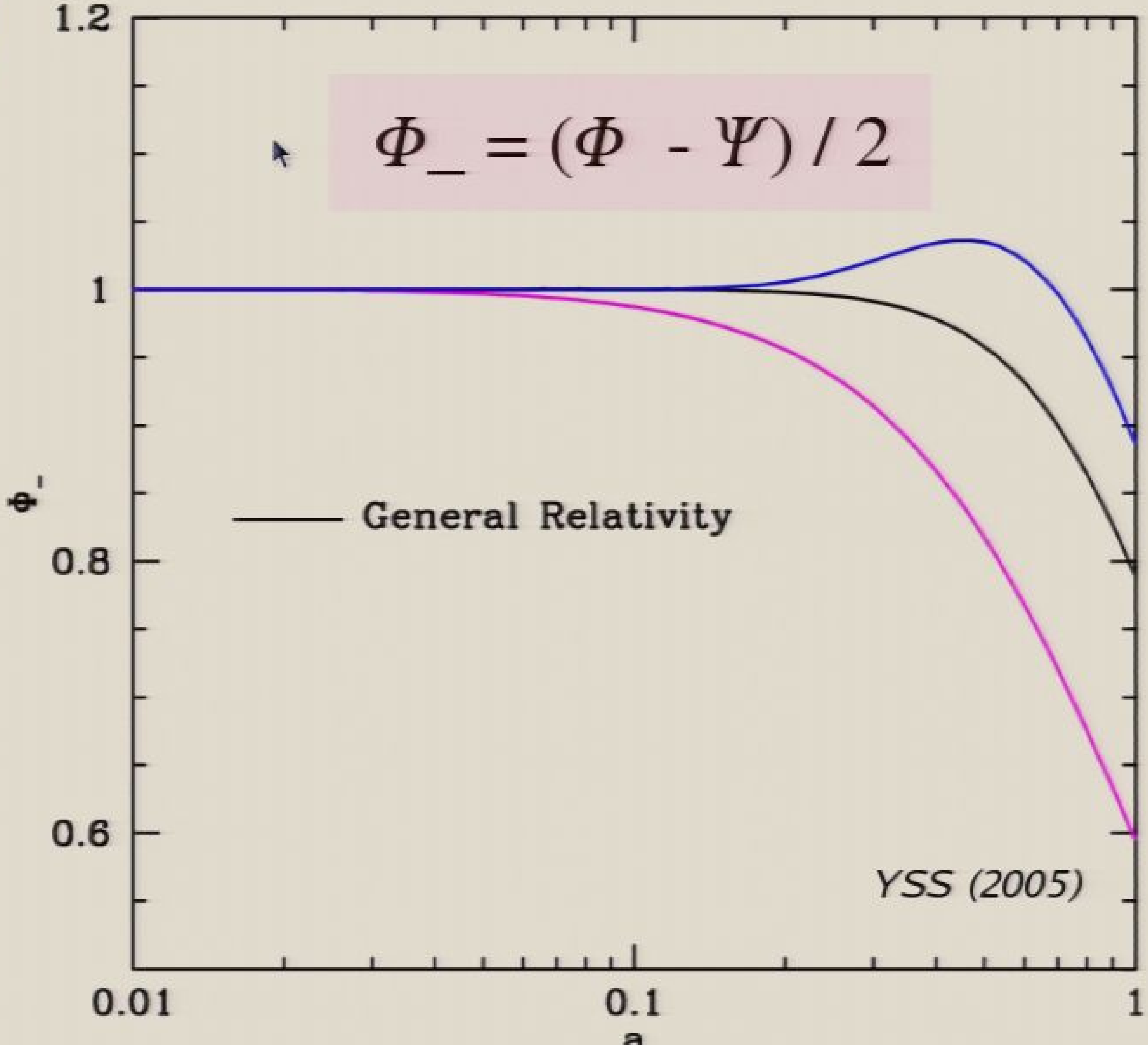
Contents

- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

An interesting quantity of linear perturbations



An interesting quantity of linear perturbations



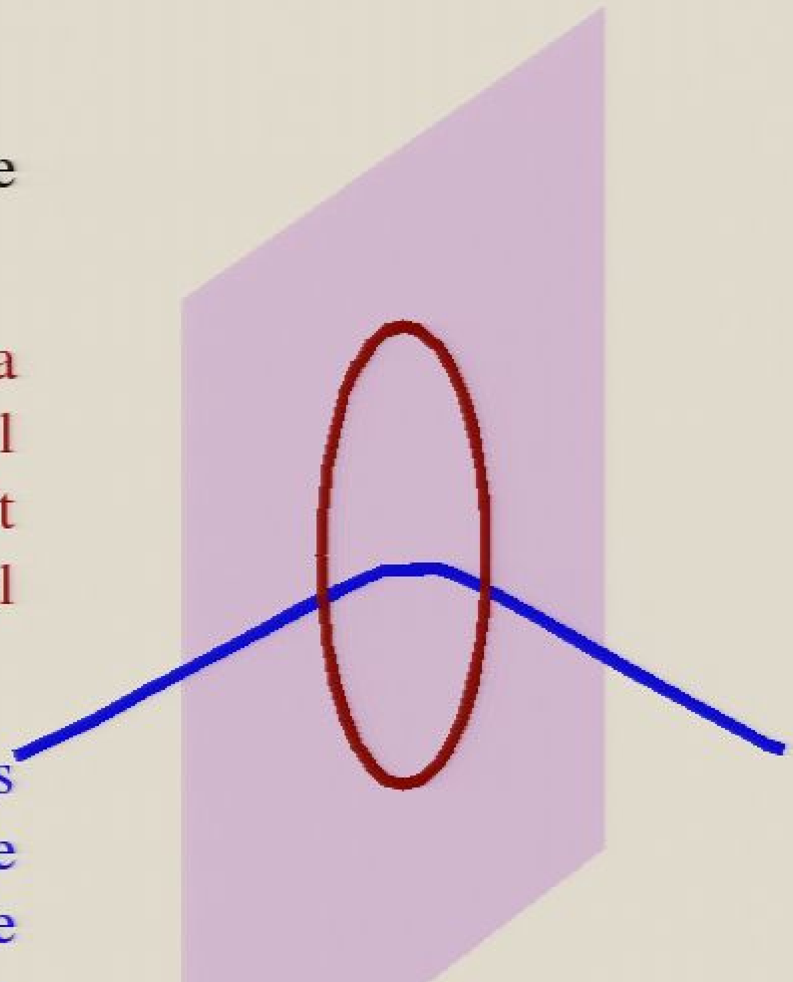
Contents

- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

Dynamic equations of linear perturbations on brane

Dynamics equations of perturbations are given by maximal symmetry.

- Bianchi identity on 4D brane: it has a closed form. All 4D quantities are well defined including anisotropy tensor. It has an additional 5^{th} dimensional dependence in first order.
- The propagation of perturbations through bulk appears as a wave smearing into the bulk. It gives the information of 5^{th} dimensional gradient perturbations at brane, and close the 4D Bianchi identity on brane.



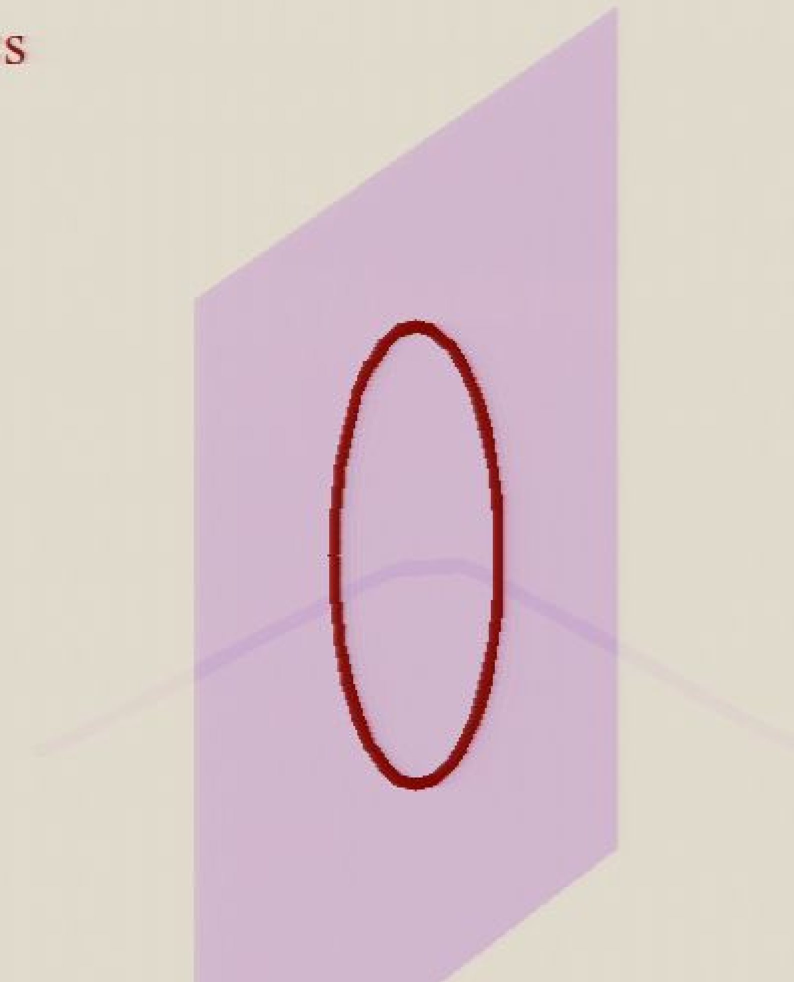
Mukohyama (2000)

Deffayet (2002)

Sawicki, YSS and Hu (2006)

Large scale linear perturbations: well-inside horizon

The 5^{th}-D effect is negligible. Dynamics is closed on 4D Bianchi identity.



- Smaller 5D Planck mass boosts $\Phi\_^{\mathrm{GR}}$.
- Weyl fluid density contributes decrement of Φ^{GR} .
- Anisotropy relations between Φ and Ψ contributes decrement of $\Phi\_^{\mathrm{GR}}$.

↓

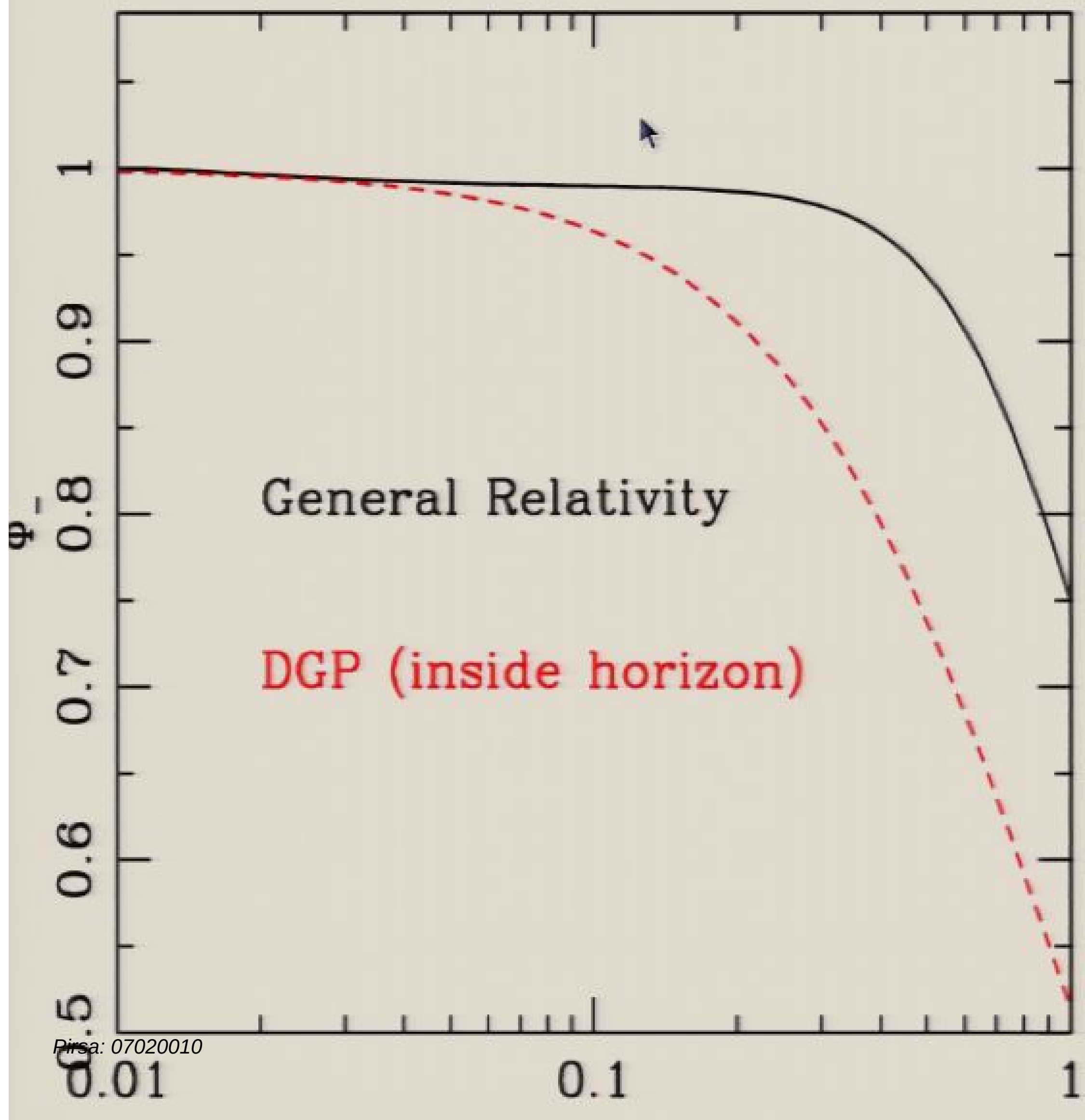
- New decrement of $\Phi\_^{\mathrm{DGP}}$ against $\Phi\_^{\mathrm{GR}}$.

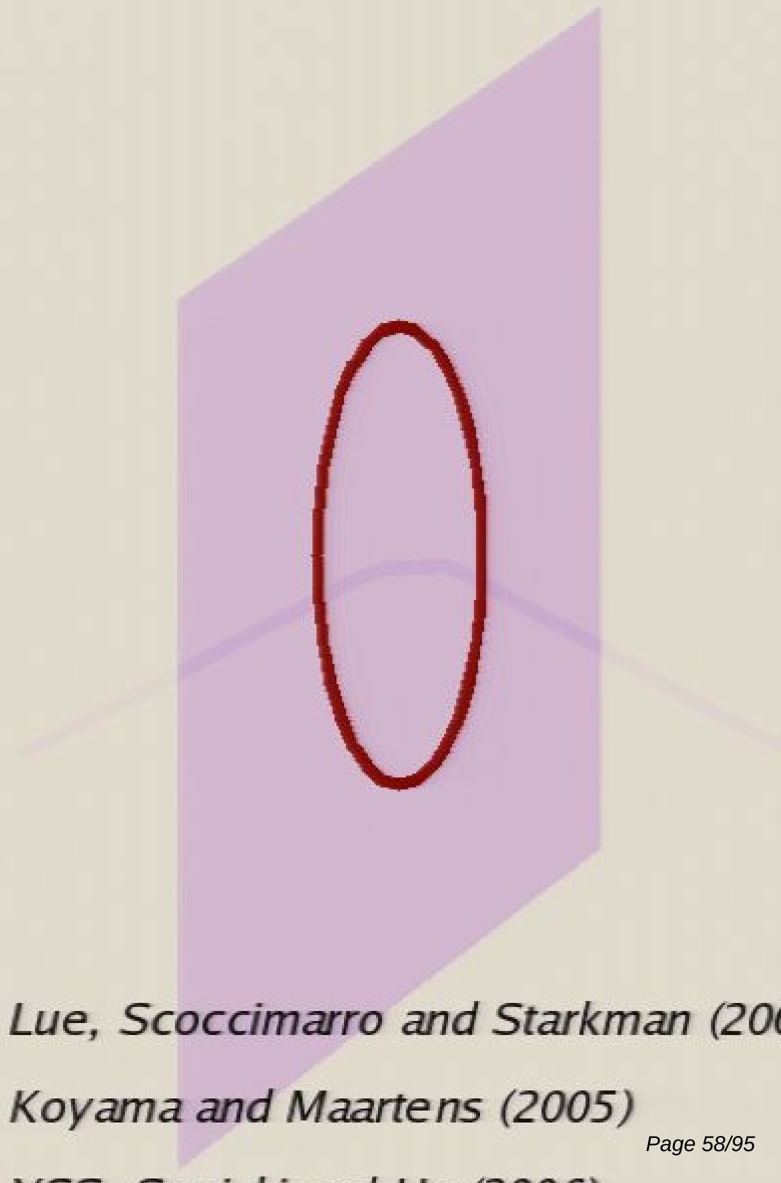
Lue, Scoccimarro and Starkman (2004)

Koyama and Maartens (2005)

YSS, Sawicki and Hu (2006)

Large scale linear perturbations: well-inside horizon





Lue, Scoccimarro and Starkman (2004)

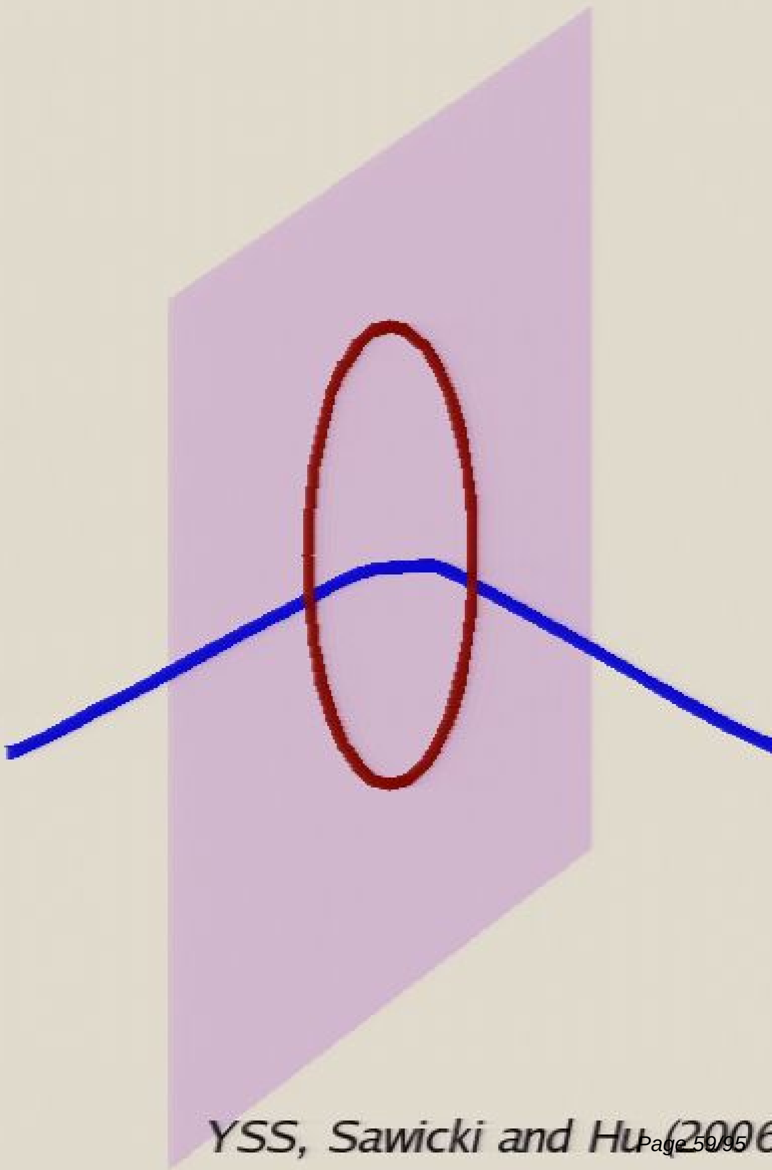
Koyama and Maartens (2005)

YSS, Sawicki and Hu (2006)

Large scale linear perturbations: near horizon

The 5^{th}-D effect is important at near horizon.

- First, we solve 4D Bianchi without considering 5^{th}-D effect.
- Second, we solve Master equation with boundary condition at causal horizon and correct 5^{th}-D effect on 4D Bianchi.
- Third, we iterate the above process until the convergence.

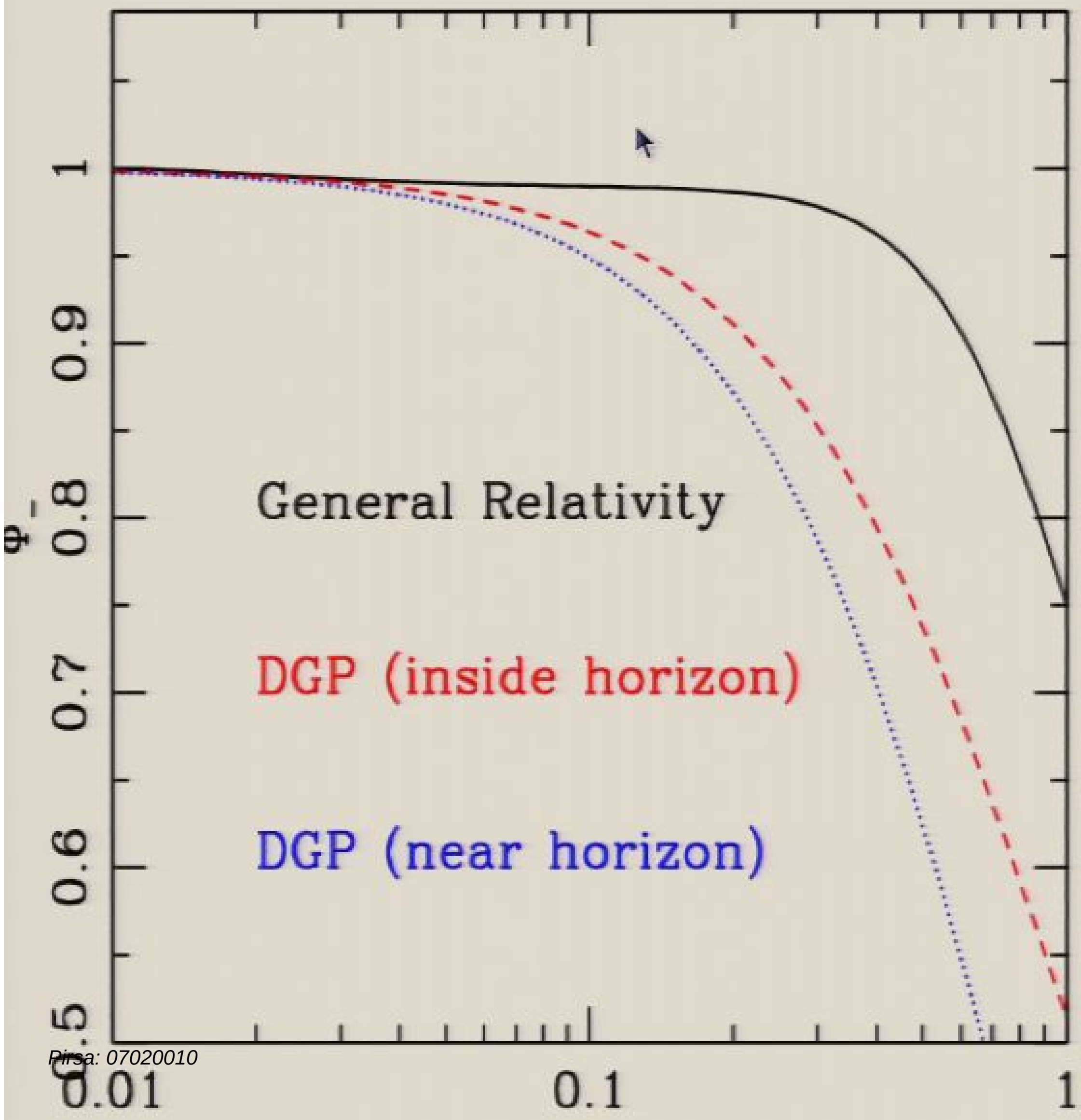


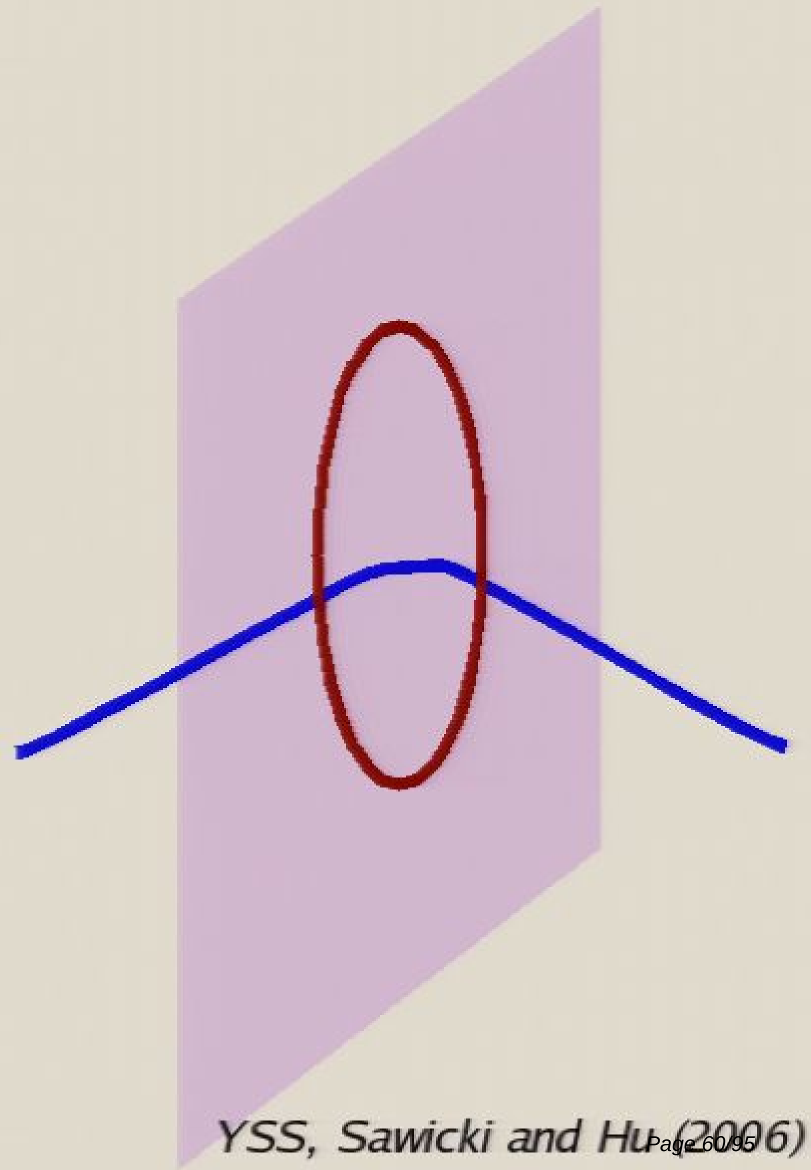
↓

- We found the stable solution with the extra decrement $\Phi\_$.

YSS, Sawicki and Hu (2006)

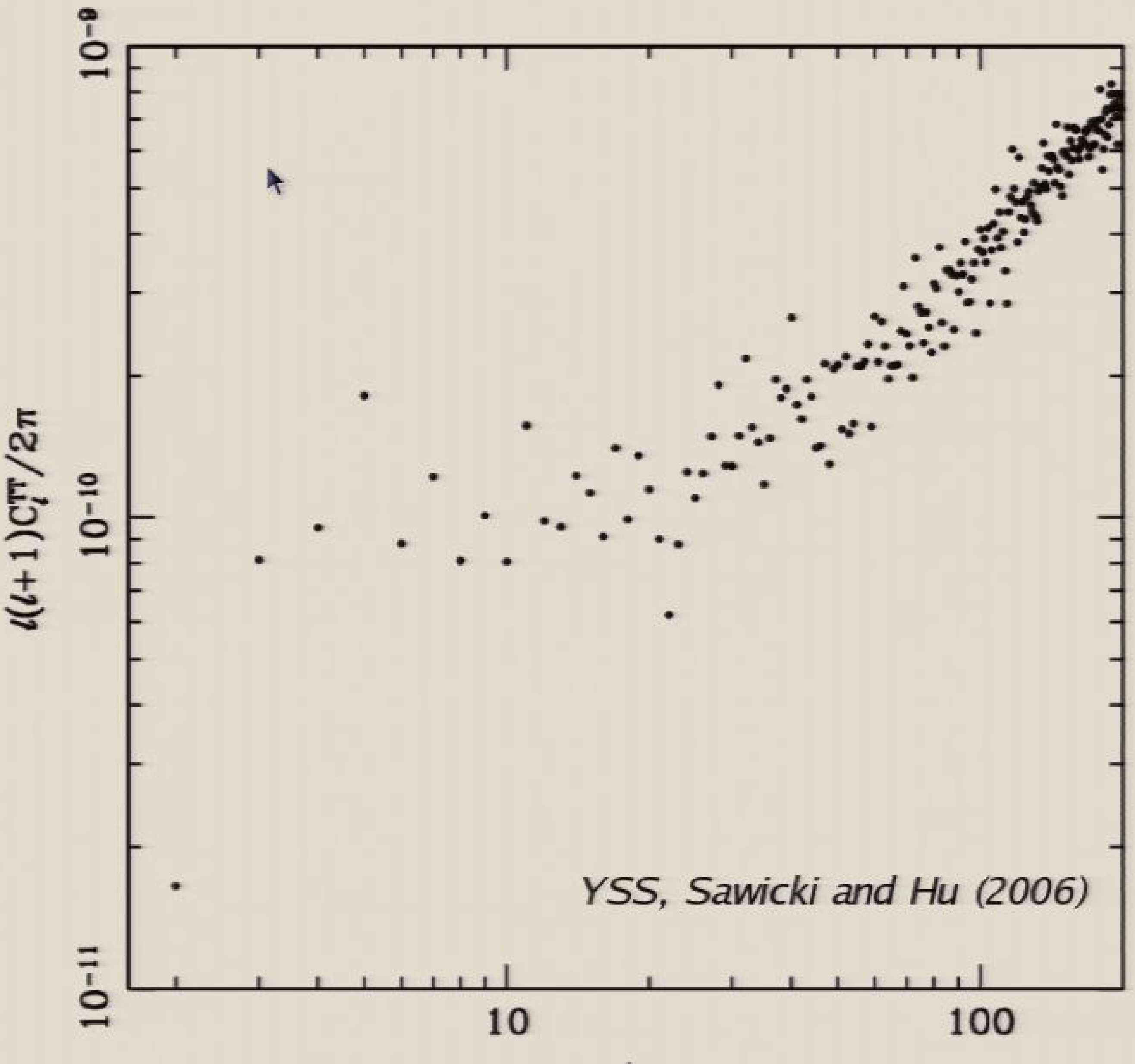
Large scale linear perturbations: near horizon



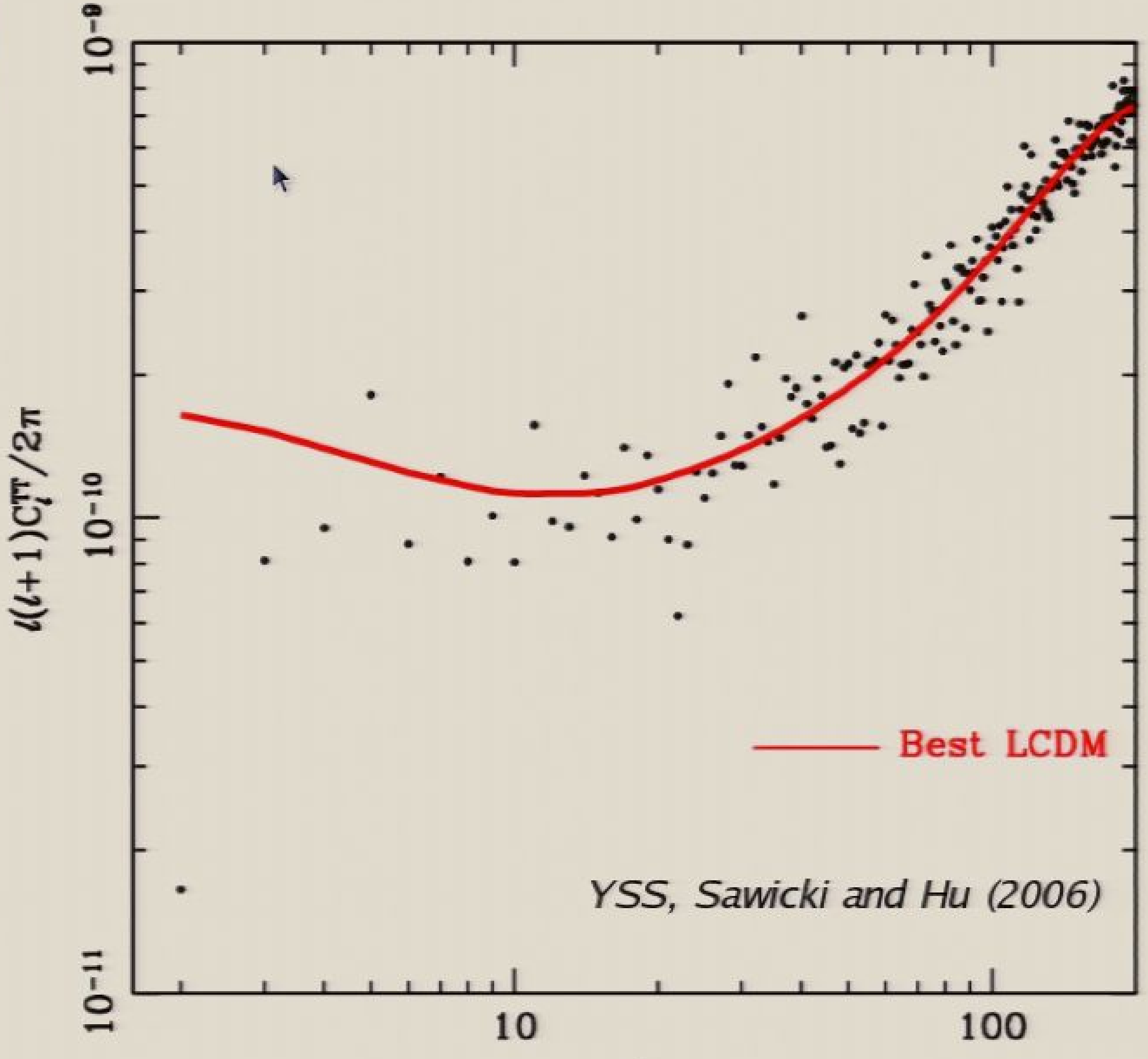


YSS, Sawicki and Hu (2006)

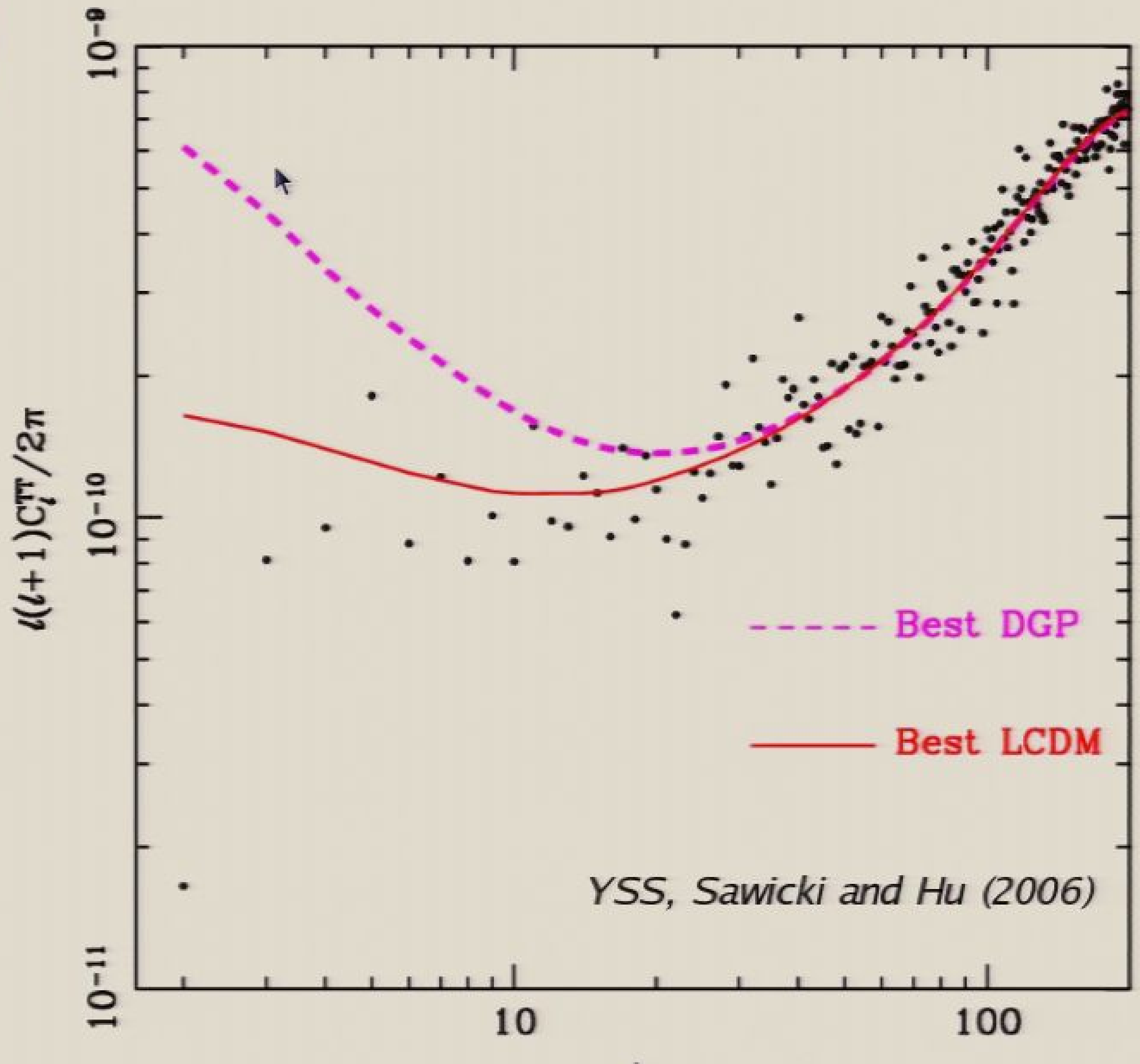
DGP fit to WMAP data



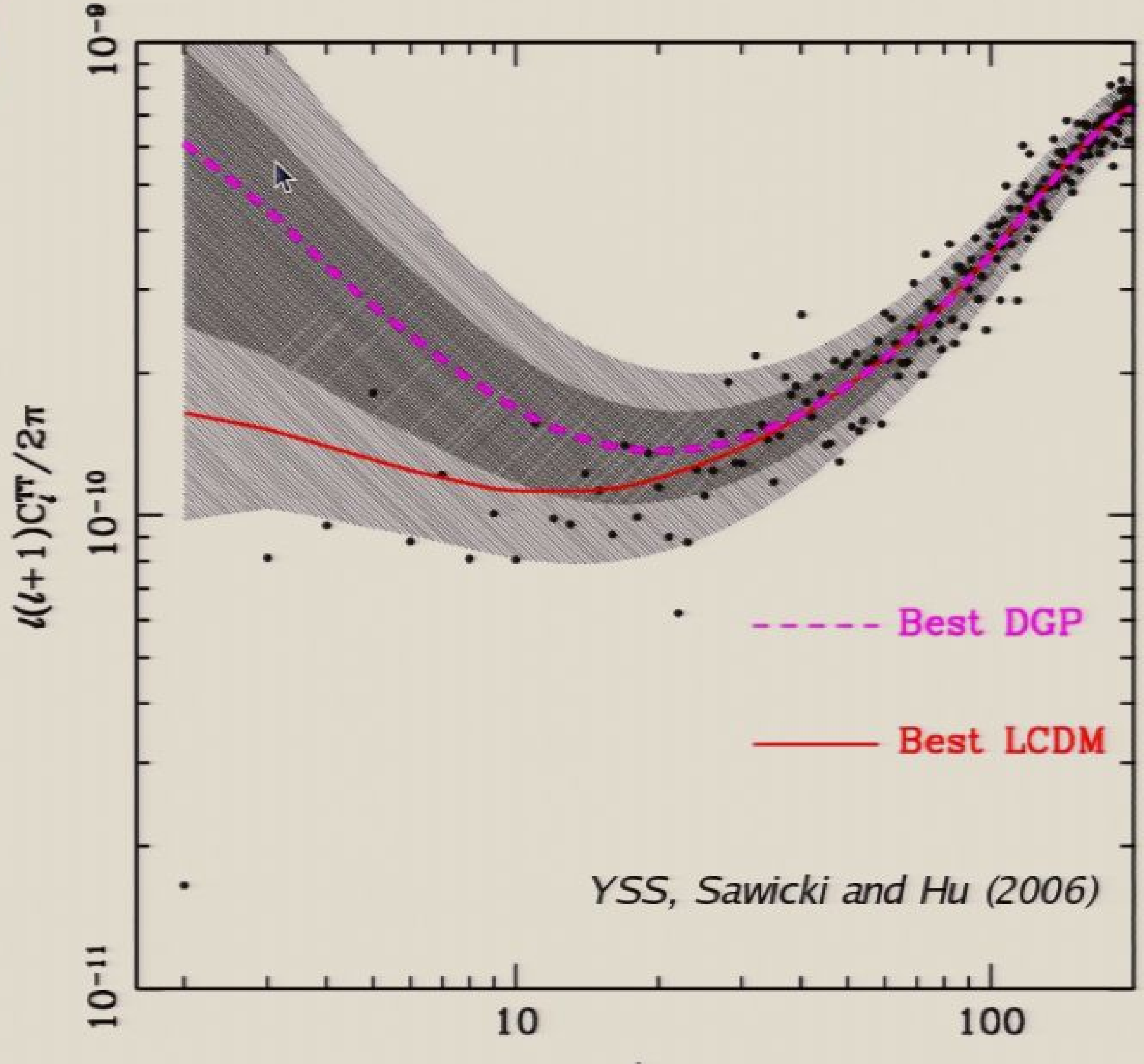
DGP fit to WMAP data



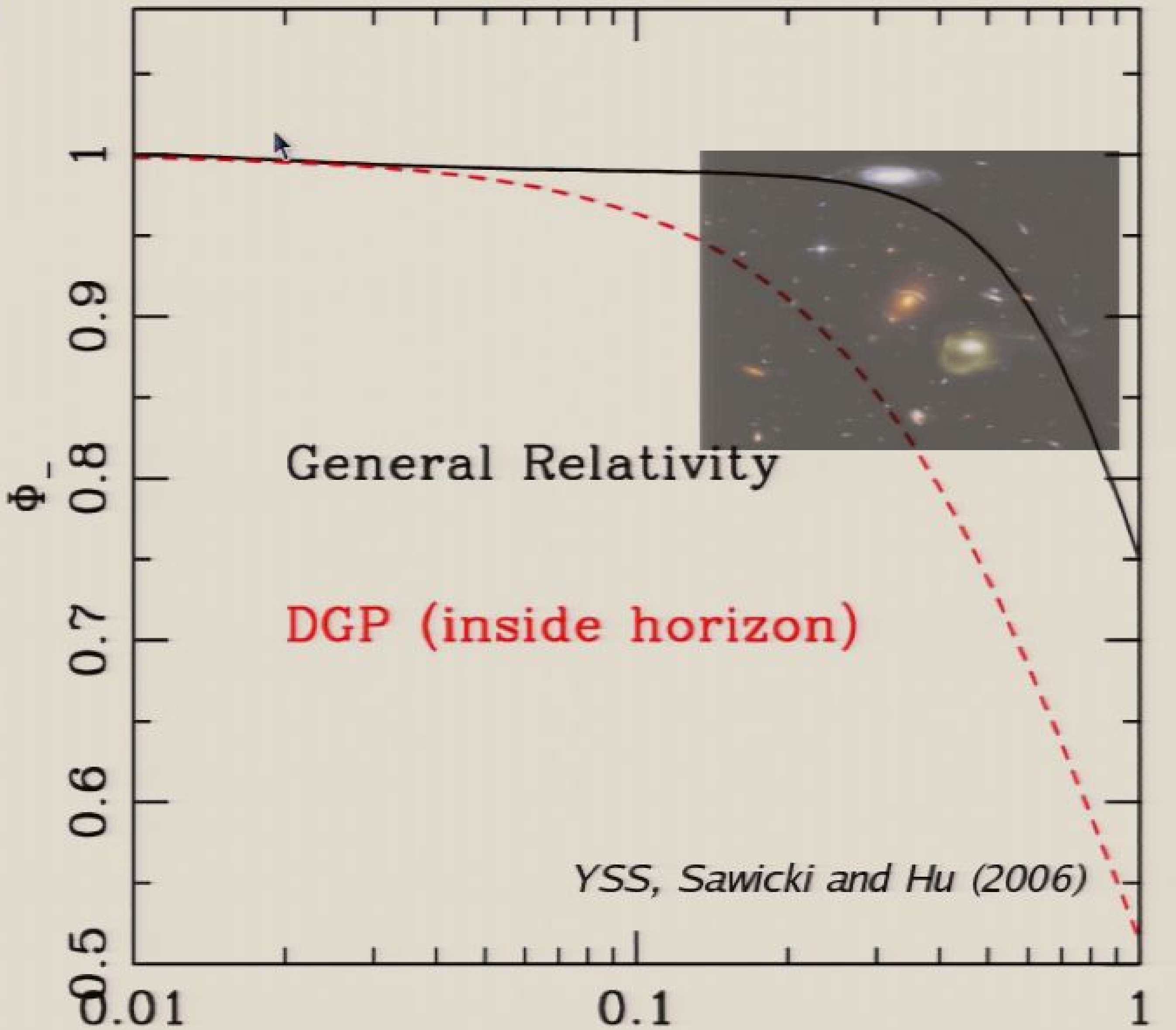
DGP fit to WMAP data



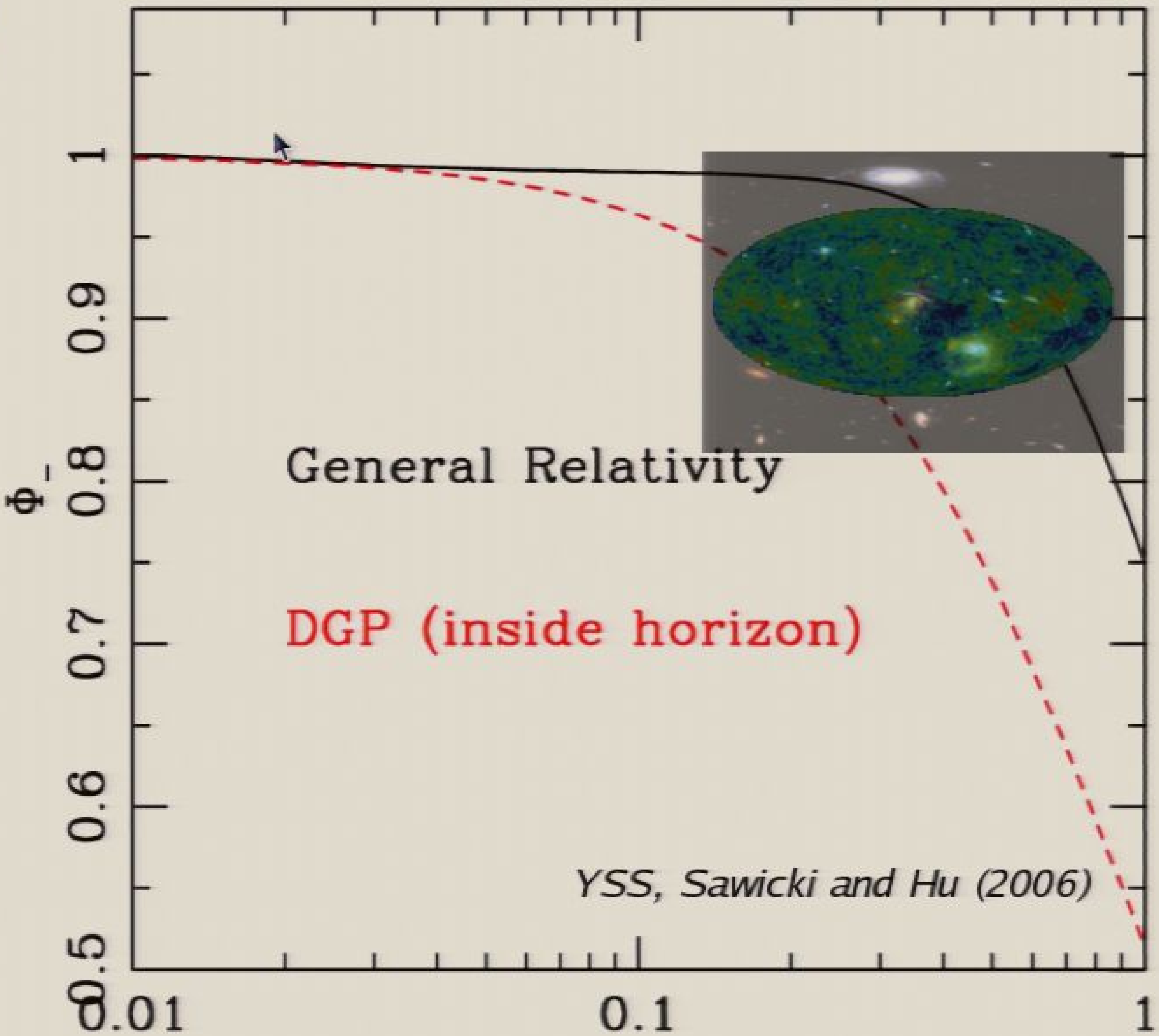
DGP fit to WMAP data



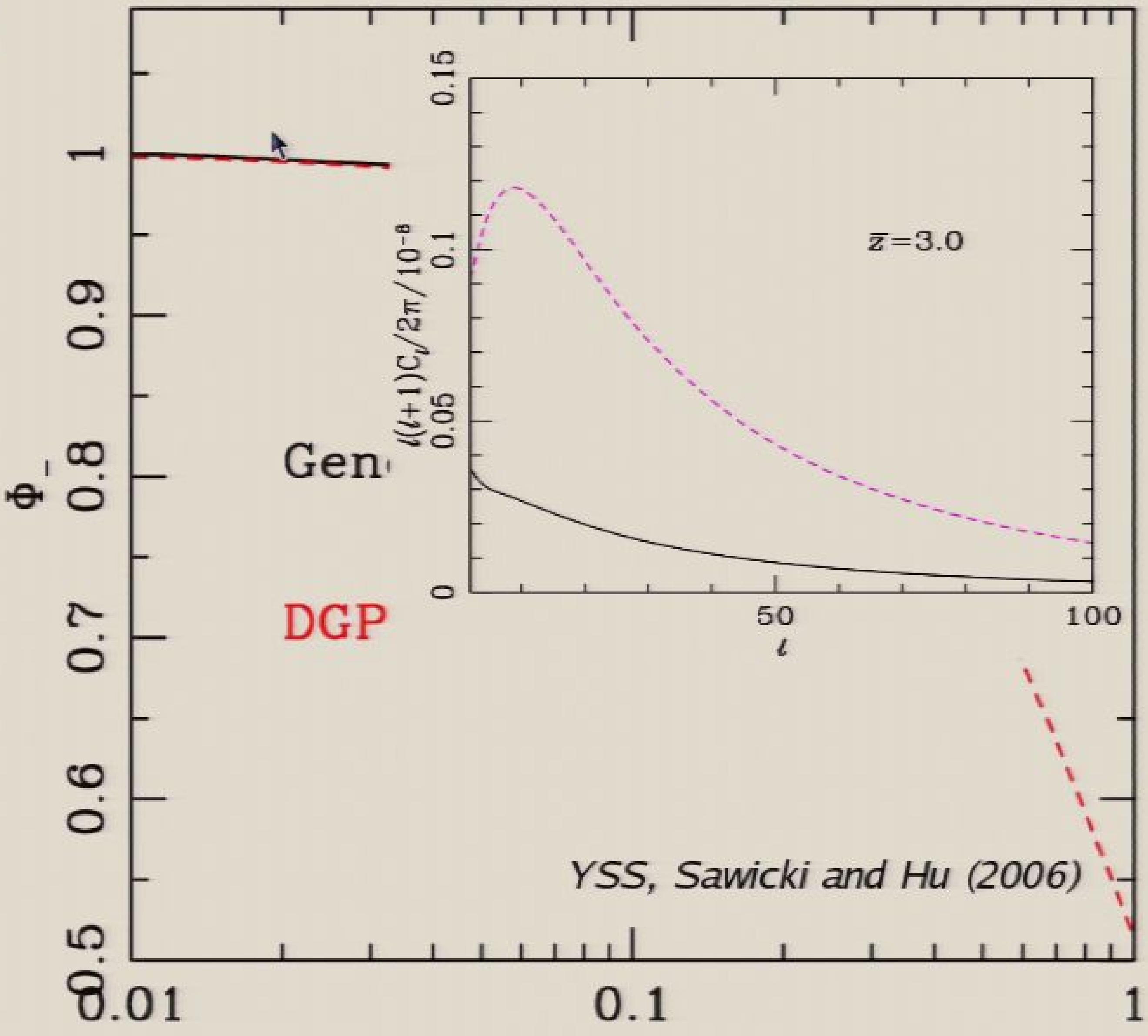
Test DGP with galaxy-ISW cross correlations



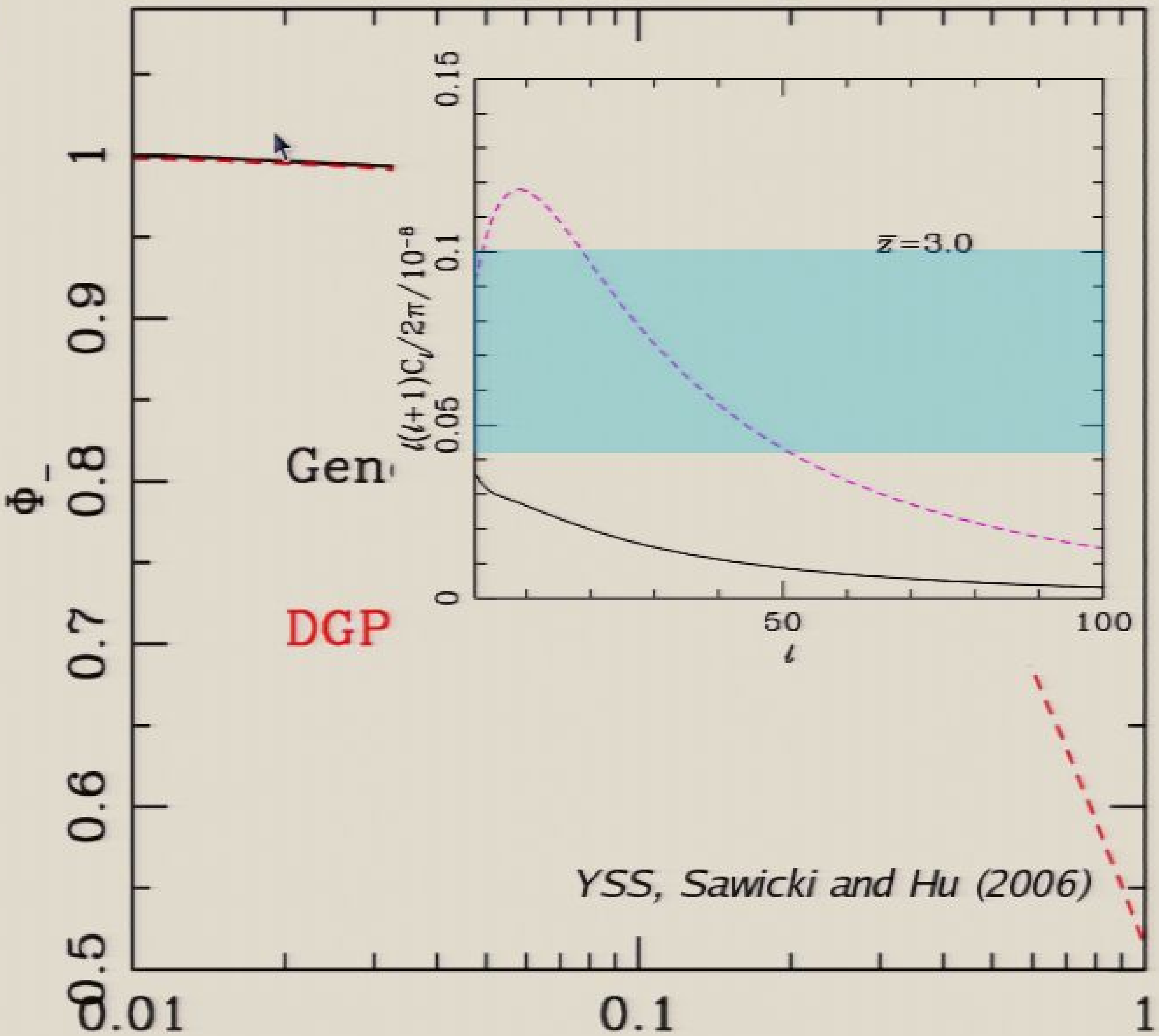
Test DGP with galaxy-ISW cross correlations



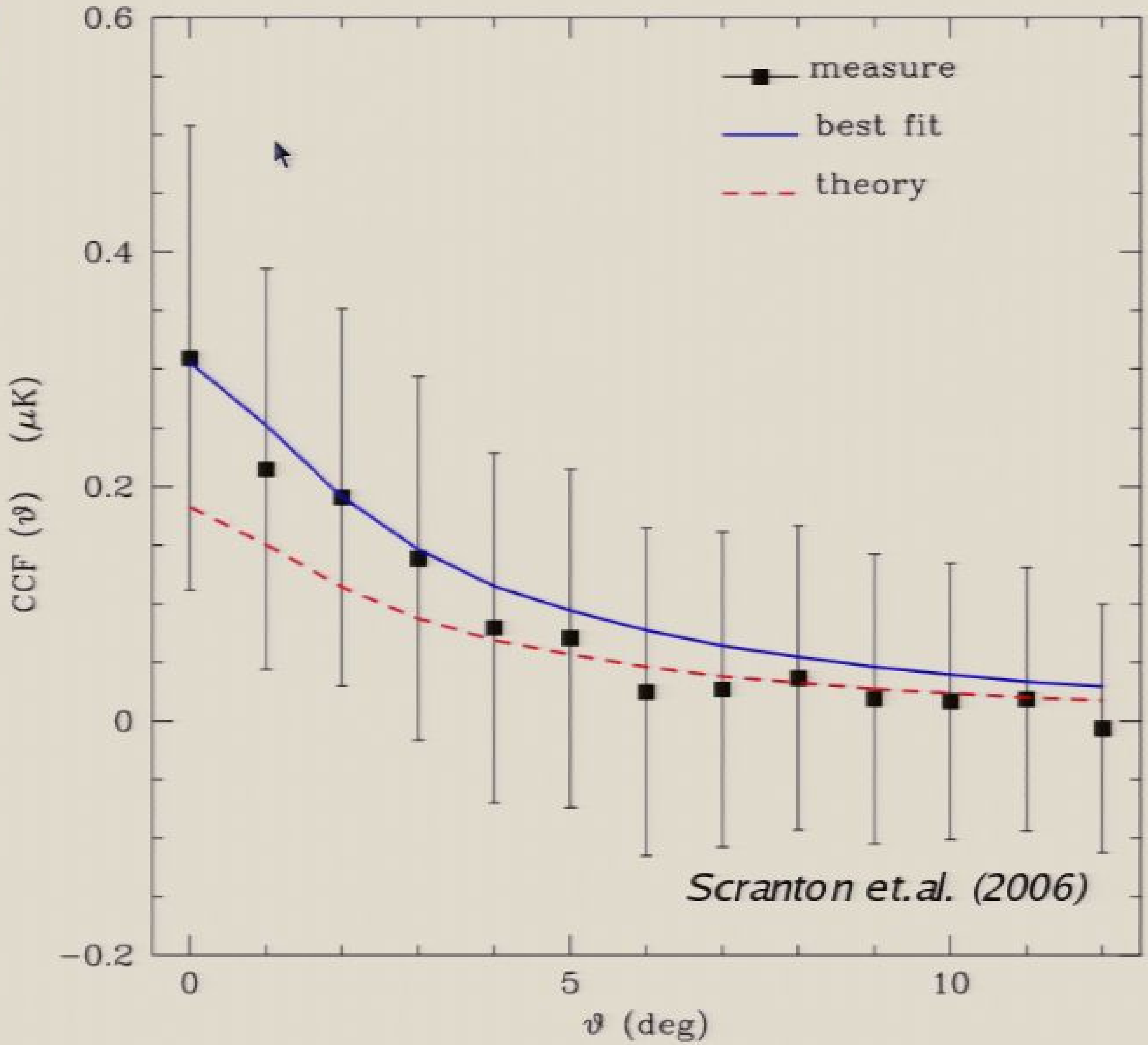
Test DGP with galaxy-ISW cross correlations



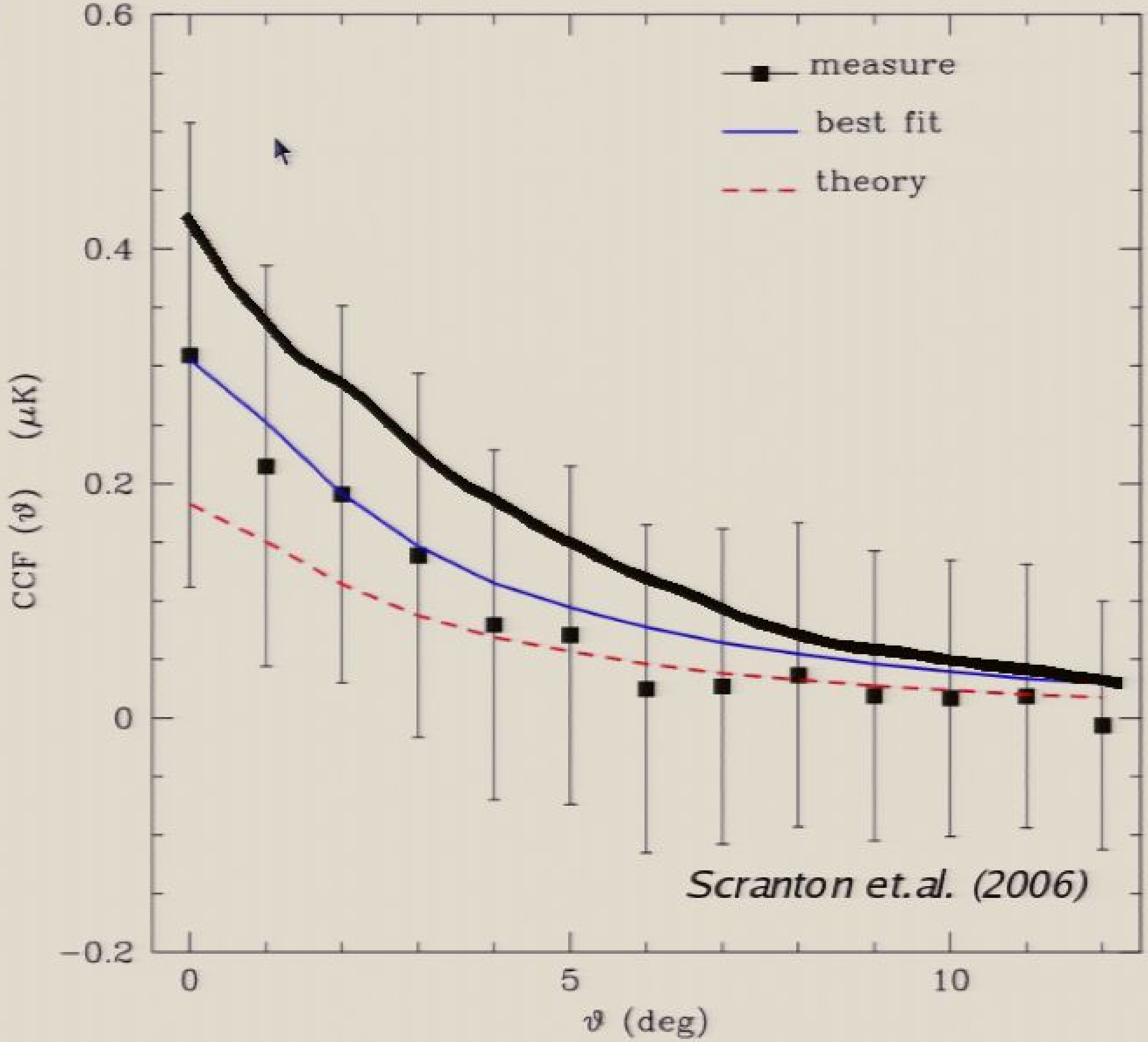
Test DGP with galaxy-ISW cross correlations



Recent release on galaxy-ISW at high redshift z~1.5



Recent release on galaxy-ISW at high redshift z~1.5



Contents

- Motivation for test GR at cosmological scales
- Geometrical test on modified gravity models
  - DGP models
  - f(R) gravity
- Structure formation test on modified gravity models
  - DGP models
  - f(R) gravity
- Conclusion

Large scale structure at super horizon scales

- The perturbed metric is; $d^2s = -(1+2\Psi)d^2t + a^2(1+2\Phi)d^2x$

- The evolution of metric and density perturbations of a background Robertson-Walker universe can be determined by
  - The Friedman equation
  - Local energy-momentum conservation

- Once the relation between Φ and Ψ is given, then the following eq determined kinematically is solvable,

$$\Phi'' - \Psi' - \frac{H''}{H'}(\Phi' - \Psi) - \frac{H'}{H}\Psi = 0$$

Bertschinger (2006); Wands, Malik, Lyth and Liddle (2000)

Test of modified gravity dynamics at superhorizon scales

- DGP: dynamic eqs directly give Bertschinger's + source ϵ_{DGP} ($\epsilon_{DGP} \rightarrow 0$ at $k/aH \rightarrow 0$).

$$\Phi'' - \Psi' - \frac{H''}{H'}(\Phi' - \Psi) - \frac{H'}{H}\Psi = \epsilon_{DGP}$$

Sawicki, YSS and Hu (2006)

- f(R): a combination of dynamic eqs at $k/aH \rightarrow 0$ limit is consistent with Berschinger's.

$$\Phi'' - \Psi' - \frac{H''}{H'}(\Phi' - \Psi) - \frac{H'}{H}\Psi = 0$$

YSS, Hu and Sawicki (2006)

Large scale structure of f(R) inside horizon

- DGP : dynamic eqs directly give Bertschinger's + source ϵ_{DGP} ($\epsilon_{DGP} \to 0$ at $k \to 0$).

$$\Phi'' - \Psi' - \frac{H''}{H'}(\Phi' - \Psi) - \frac{H'}{H}\Psi = \epsilon_{DGP}$$

- f(R): a combination of dynamic eqs at $k \to 0$ limit is consistent with Berschinger's.

$$\Phi'' - \Psi' - \frac{H''}{H'}(\Phi' - \Psi) - \frac{H'}{H}\Psi = 0$$

Large scale structure of f(R) inside horizon

- DGP : dynamic eqs directly give Bertschinger's + source ϵ_{DGP} ($\epsilon_{DGP} \to 0$ at $k \to 0$).

$$\Phi'' - \Psi' - \frac{H''}{H'}(\Phi' - \Psi) - \frac{H'}{H}\Psi = \epsilon_{DGP}$$

$$f_{RR}\epsilon_{f(R)}'' + C_1 f_{RR}\epsilon_{f(R)}' + C_2\epsilon_{f(R)} = 0$$

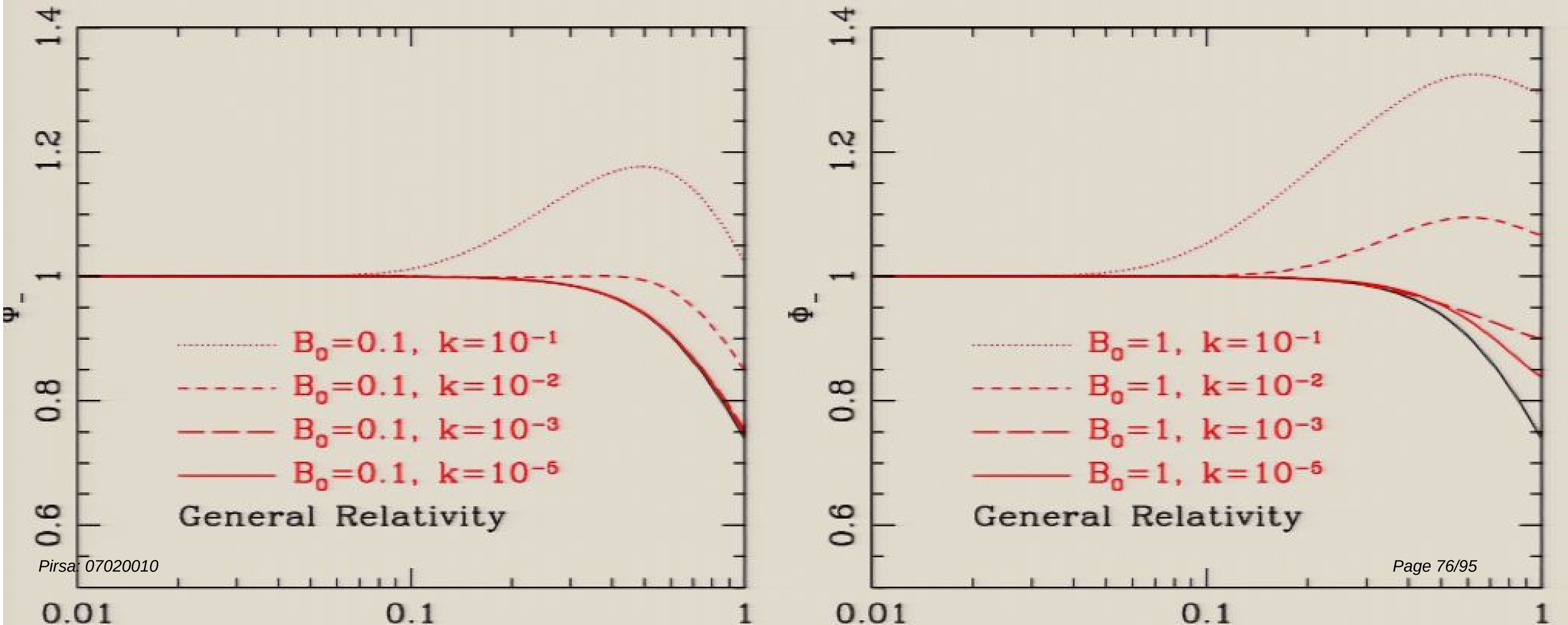
- f(R): a combination of dynamic eqs at k→0 limit is consistent with Berschinger's.

$$\Phi'' - \Psi' - \frac{H''}{H'}(\Phi' - \Psi) - \frac{H'}{H}\Psi = \epsilon_{f(R)}$$

LSS perturbations of B>0 branch f(R) gravity

YSS, Hu and Sawicki (2006)

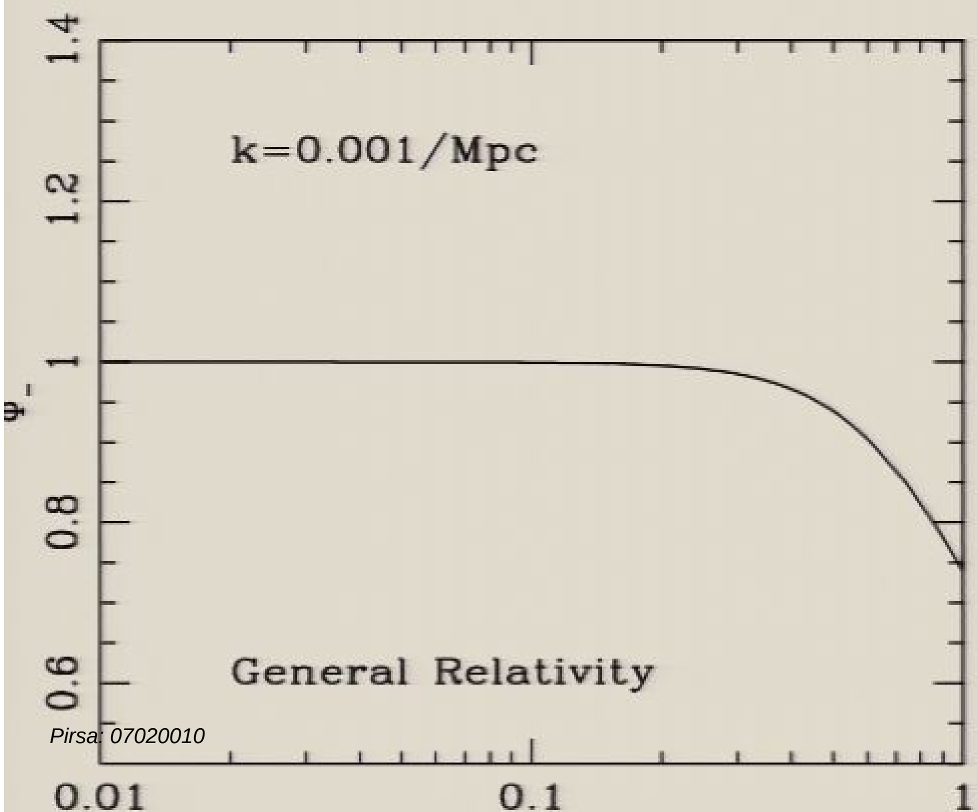
The Einstein and conservation equations depends on $\epsilon_{f(R)}$. We get a lss formation of B>0 branch f(R) gravity model by simultaneously solving the above equations and $\epsilon_{f(R)}$ equation.

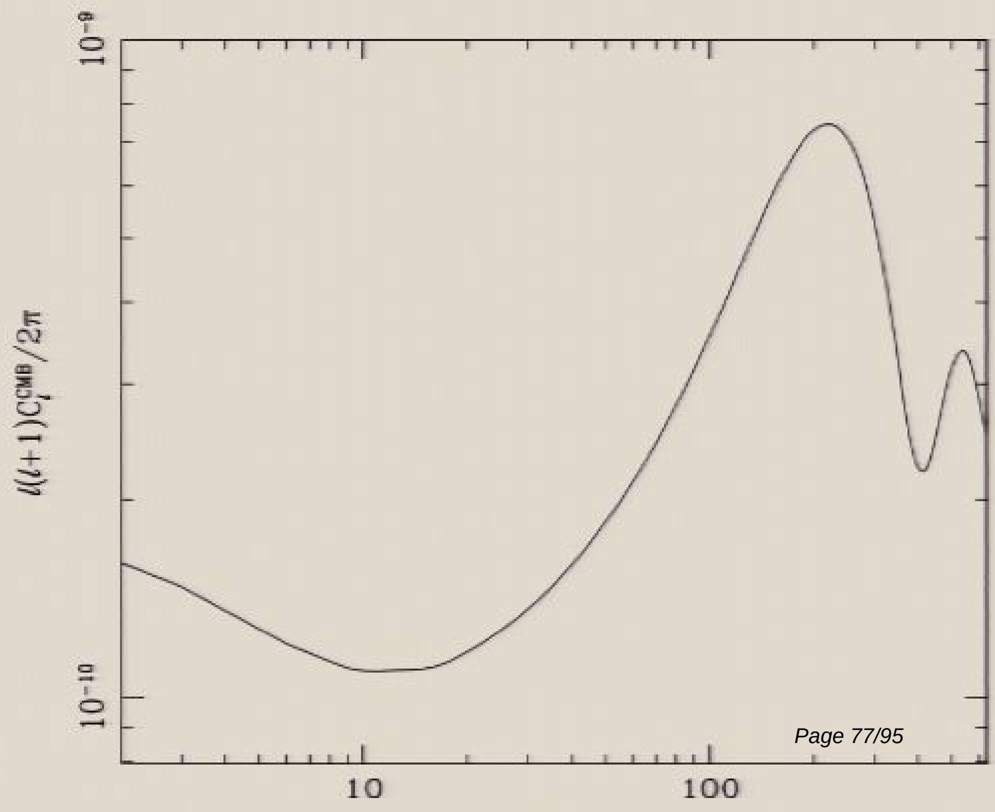


Test with CMB ISW effects

YSS, Hu and Sawicki (2006)

CMB ISW effects are mostly contributed by scales around k=0.001/Mpc. The total amount of decay will boosts $\Phi\_$ large scale CMB anisotropy structure differently with B_0.

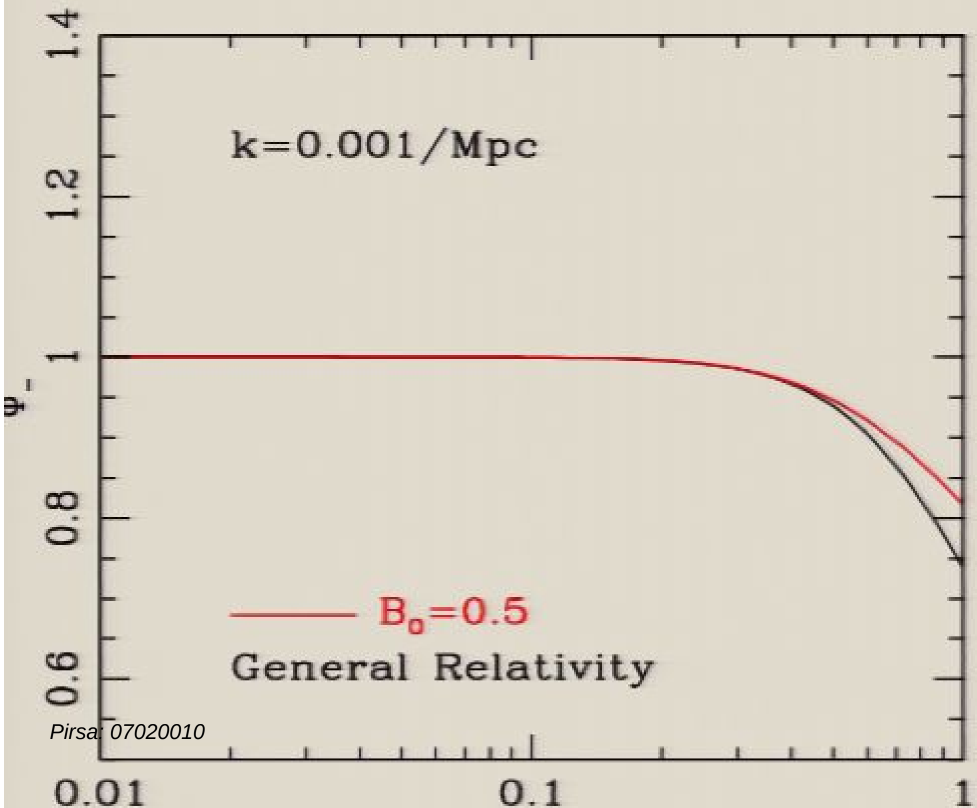


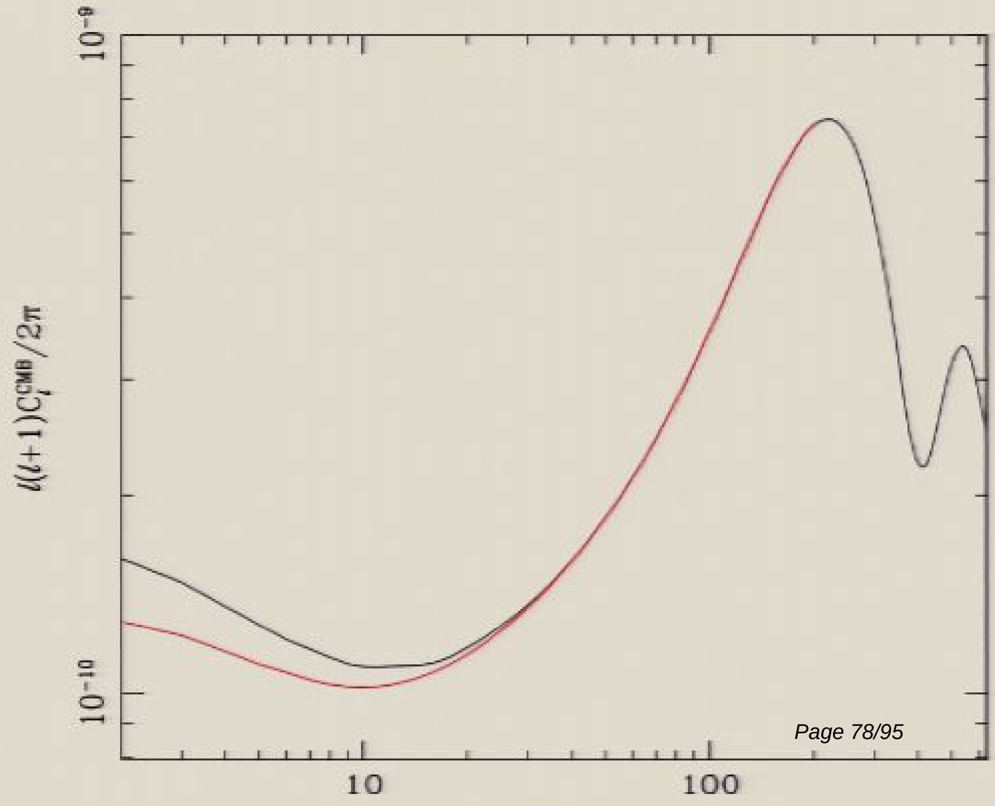


Test with CMB ISW effects

YSS, Hu and Sawicki (2006)

CMB ISW effects are mostly contributed by scales around k=0.001/Mpc. The total amount of decay will boosts $\Phi\_$ large scale CMB anisotropy structure differently with B_0.

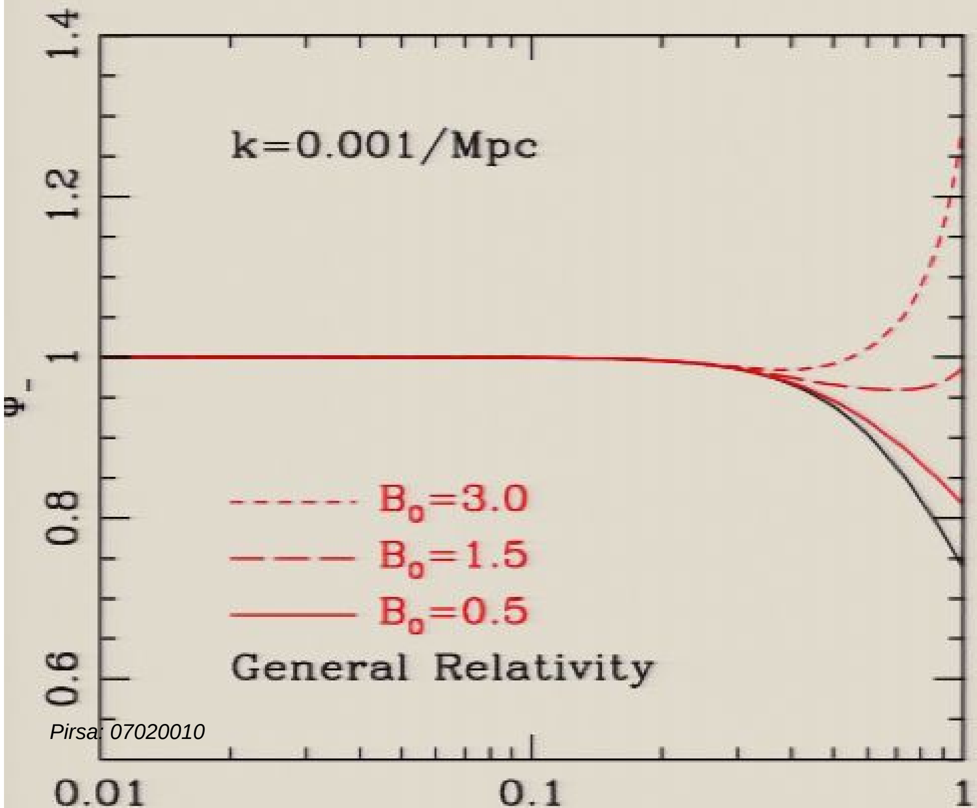


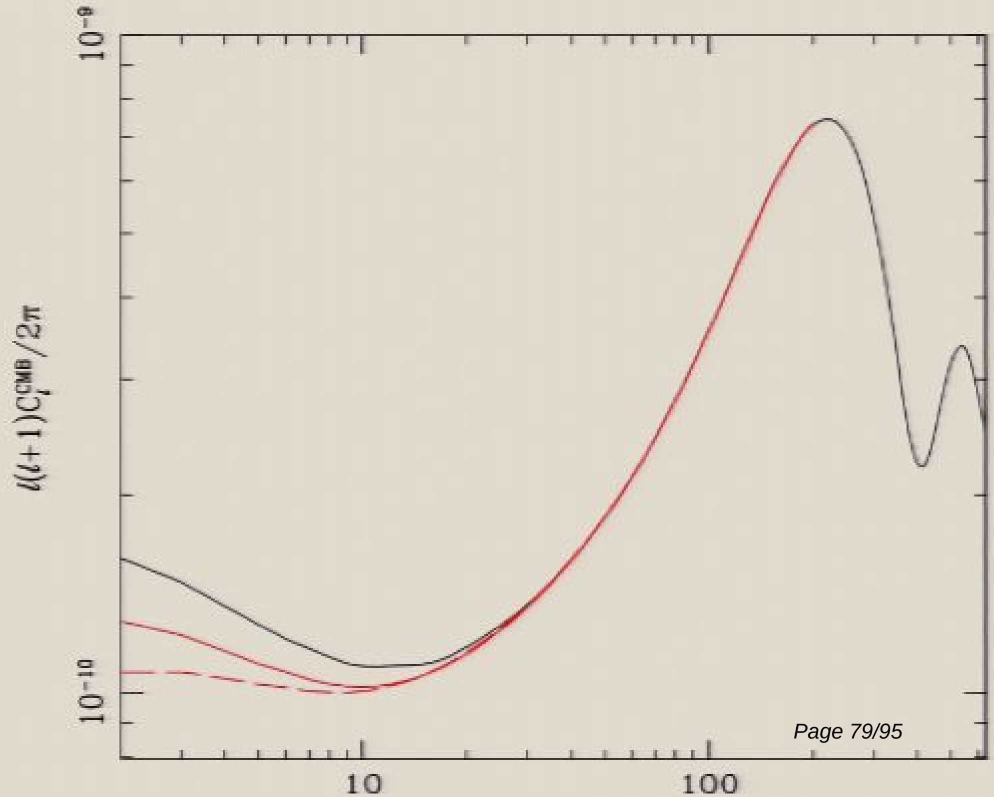


Test with CMB ISW effects

YSS, Hu and Sawicki (2006)

CMB ISW effects are mostly contributed by scales around k=0.001/Mpc. The total amount of decay will boosts Φ_- large scale CMB anisotropy structure differently with B_0.

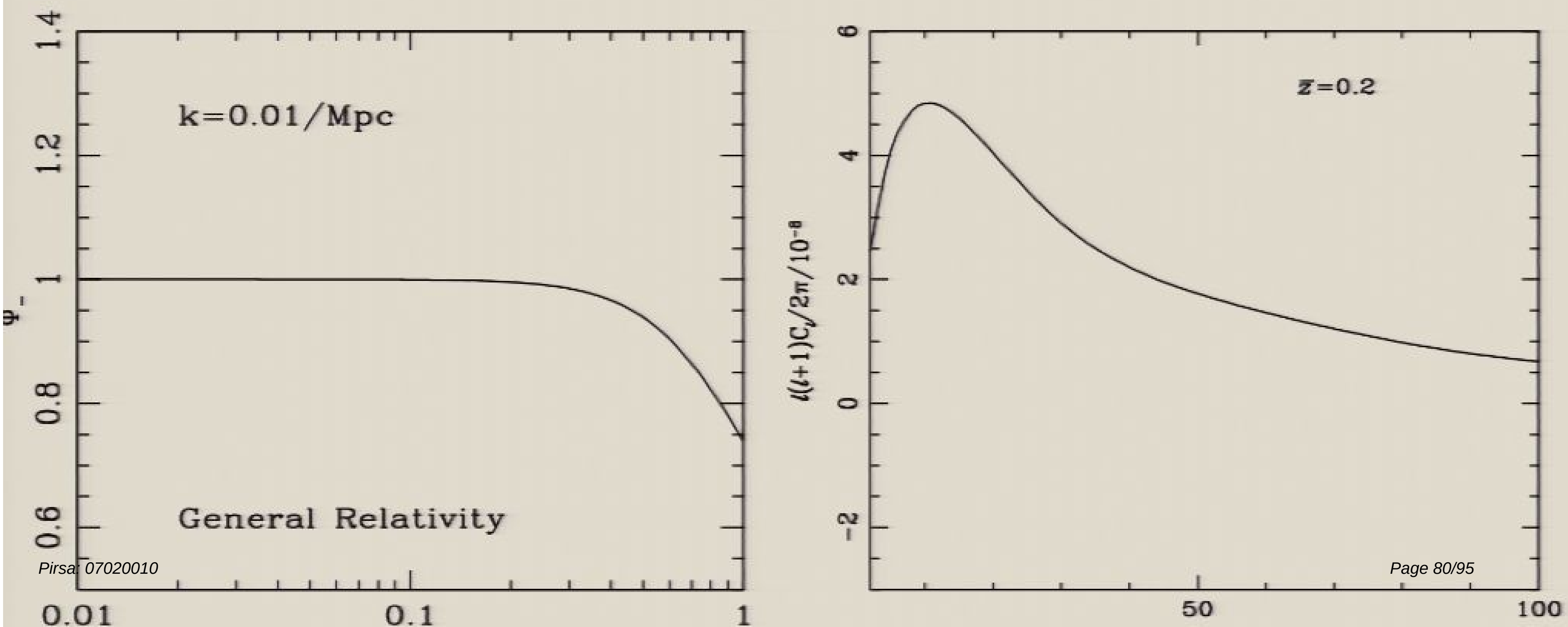




Test with Gal-ISW cross-correlations

YSS, Hu and Sawicki (2006)

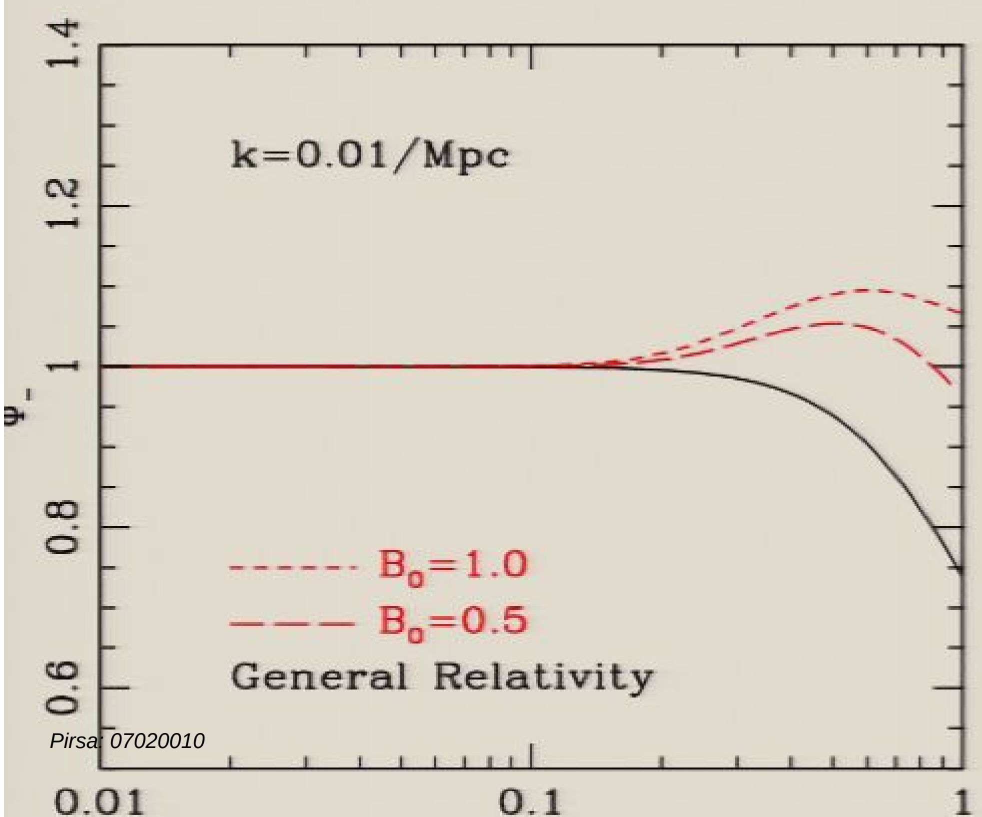
Galaxy-ISW effects are mostly contributed by scales around k=0.01/Mpc. This cross correlation can isolate the decrement in specific time, and probe the sign of variation $\Phi\_$.

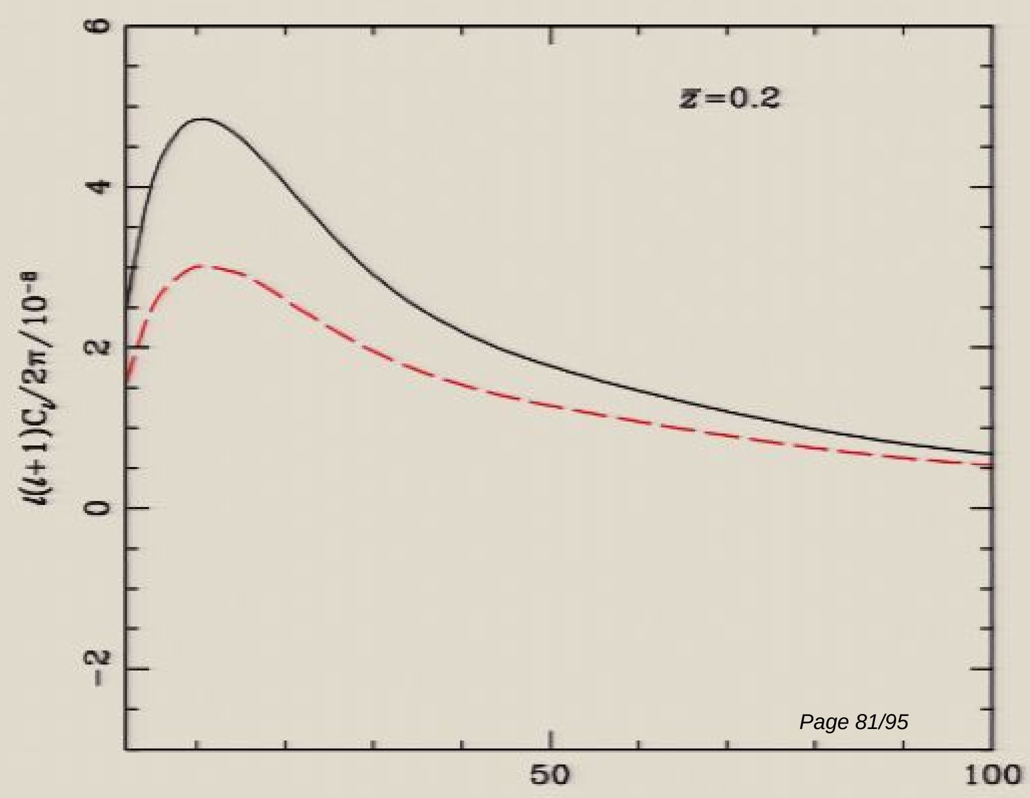


Test with Gal-ISW cross-correlations

YSS, Hu and Sawicki (2006)

Galaxy-ISW effects are mostly contributed by scales around k=0.01/Mpc. This cross correlation can isolate the decrement in specific time, and probe the sign of variation Φ_-.

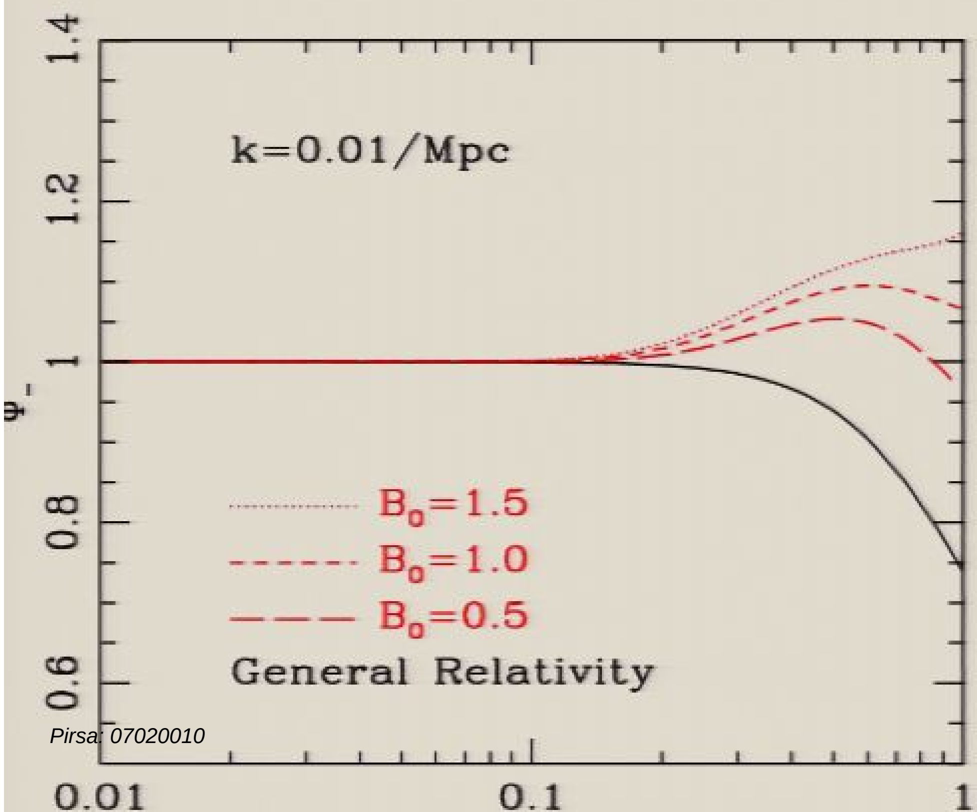


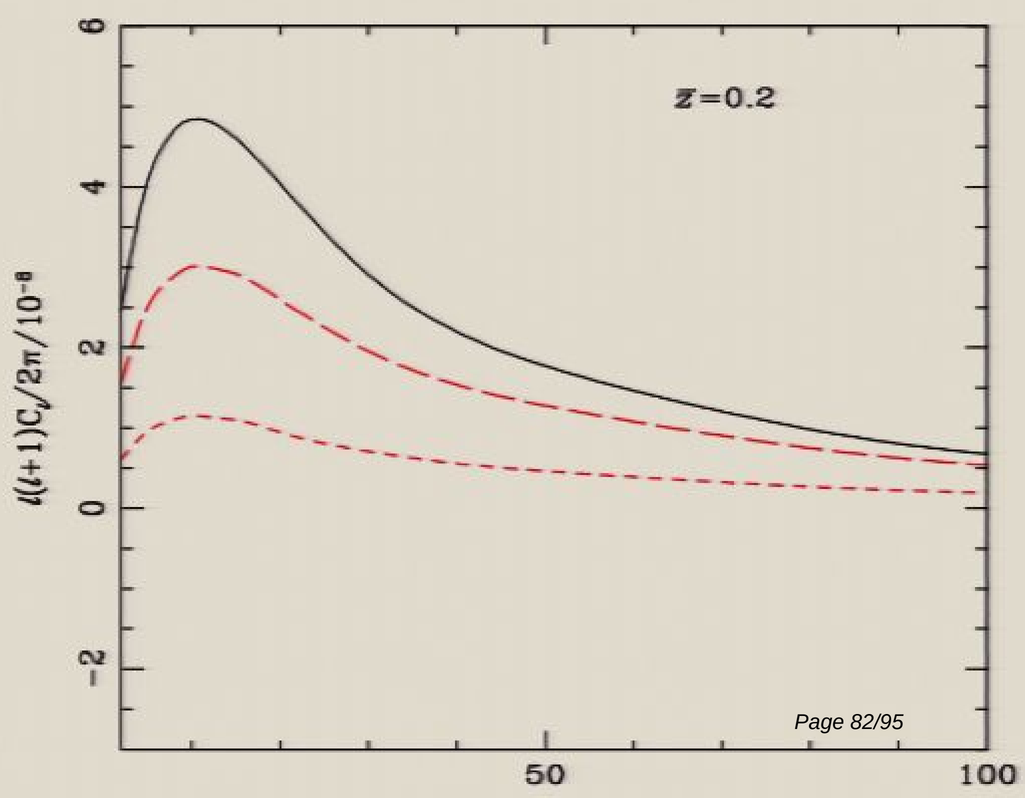


Test with Gal-ISW cross-correlations

YSS, Hu and Sawicki (2006)

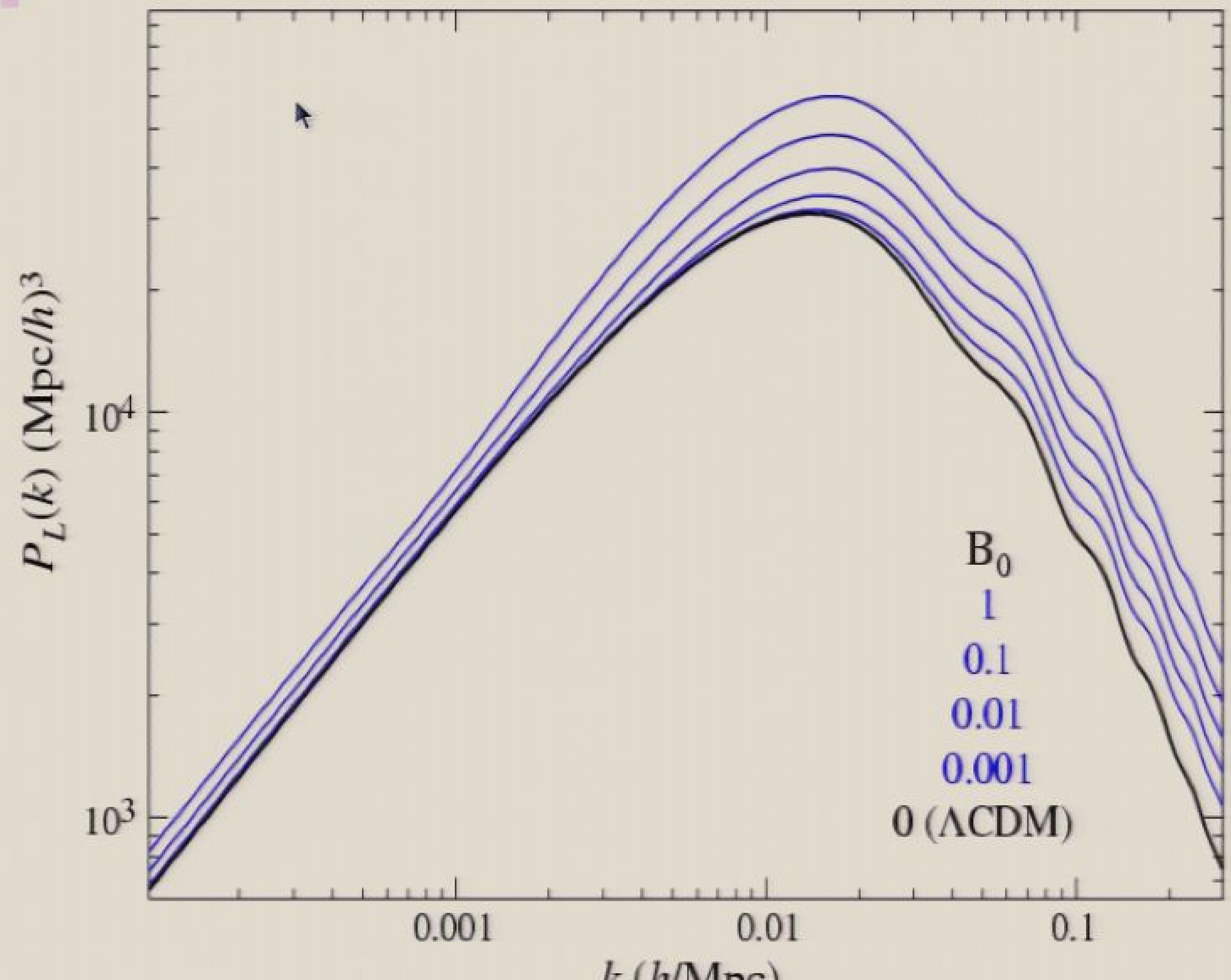
Galaxy-ISW effects are mostly contributed by scales around k=0.01/Mpc. This cross correlation can isolate the decrement in specific time, and probe the sign of variation $\Phi\_$.





Phenomenolgical effects: Power spectra

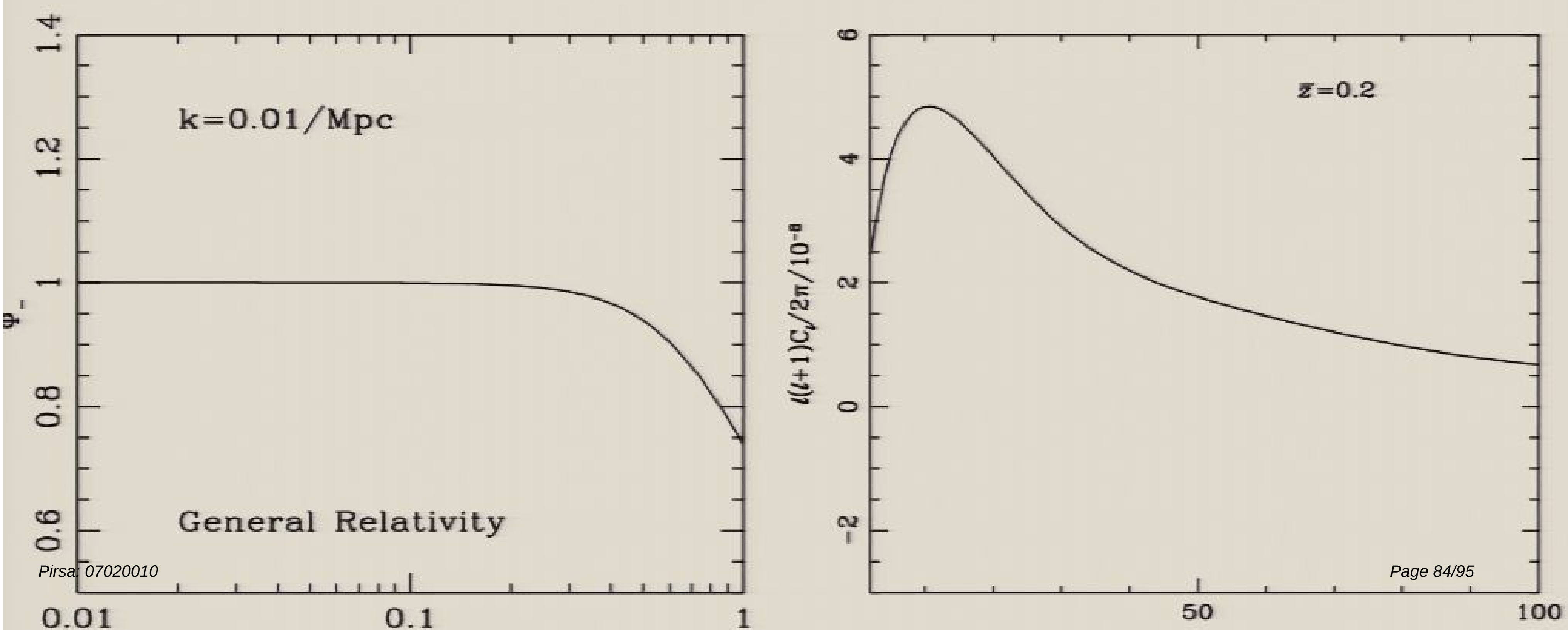
YSS, Hu and Sawicki (2006)



Test with Gal-ISW cross-correlations

YSS, Hu and Sawicki (2006)

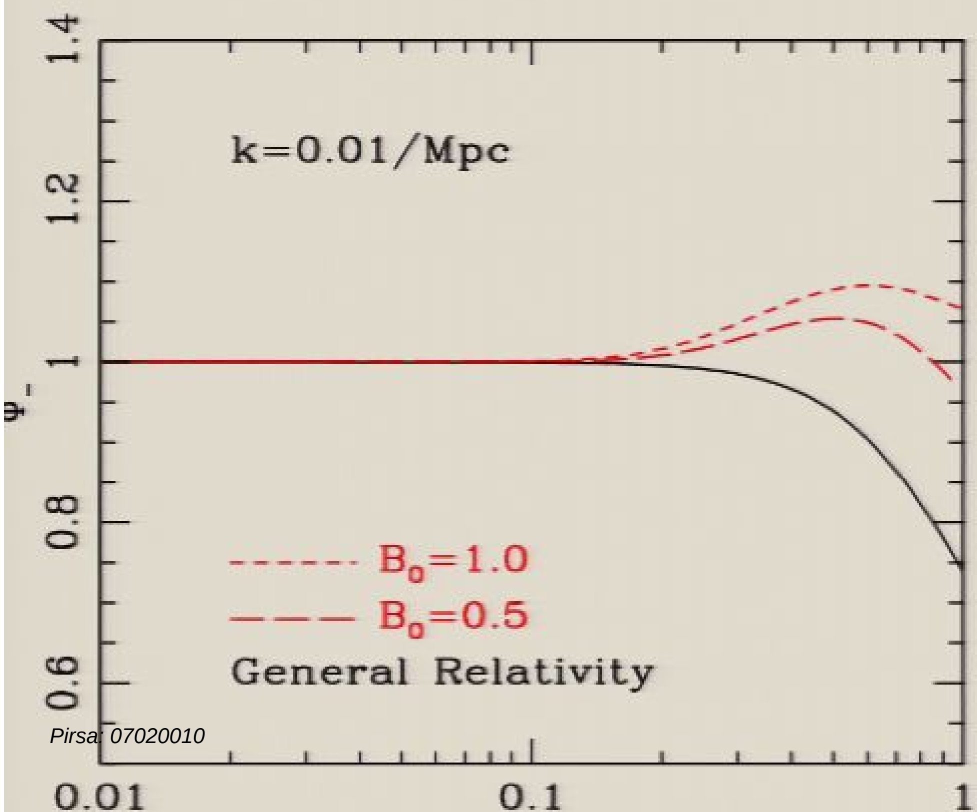
Galaxy-ISW effects are mostly contributed by scales around k=0.01/Mpc. This cross correlation can isolate the decrement in specific time, and probe the sign of variation Φ_-.

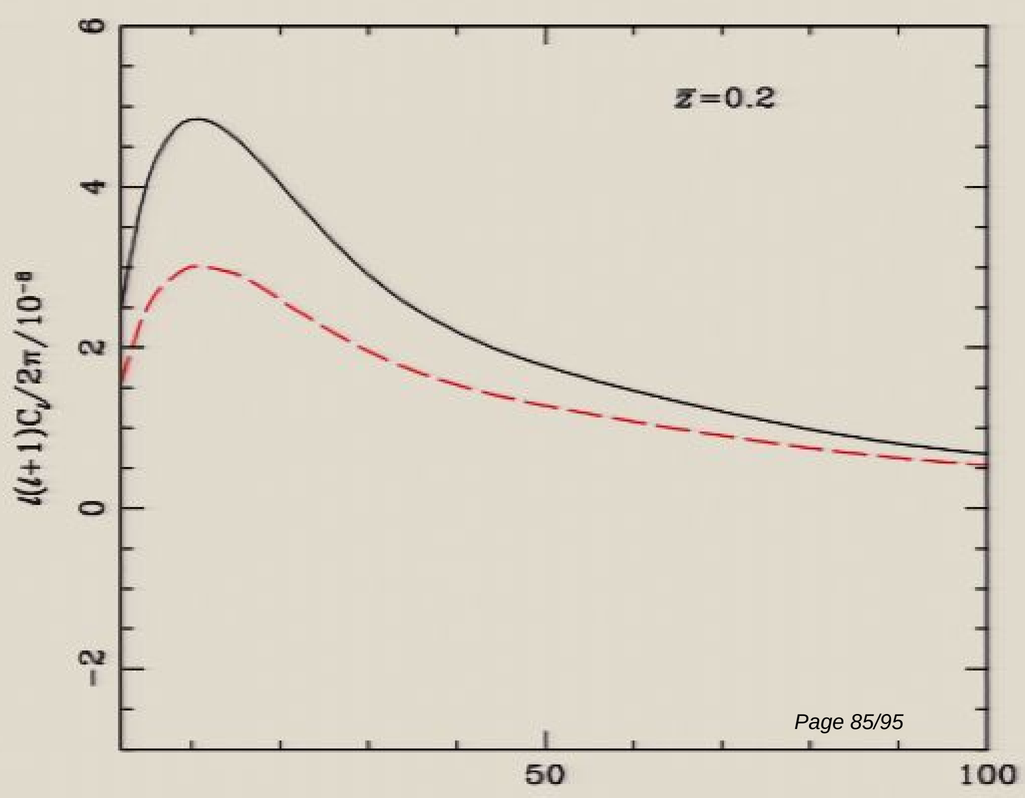


Test with Gal-ISW cross-correlations

YSS, Hu and Sawicki (2006)

Galaxy-ISW effects are mostly contributed by scales around k=0.01/Mpc. This cross correlation can isolate the decrement in specific time, and probe the sign of variation Φ_-.

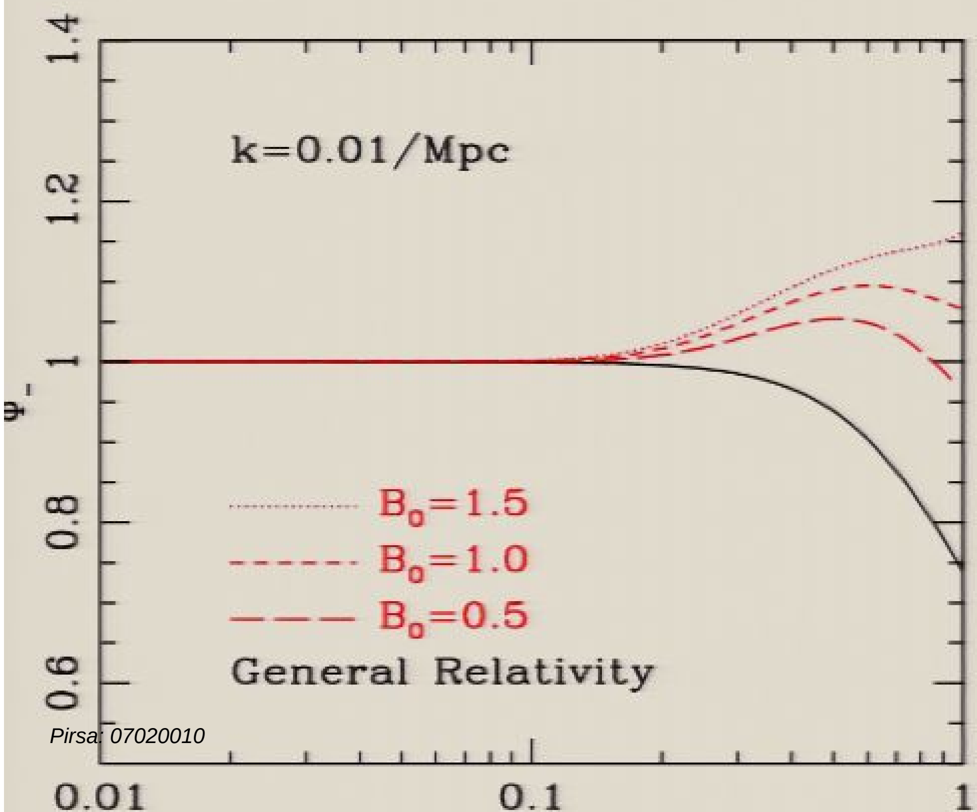


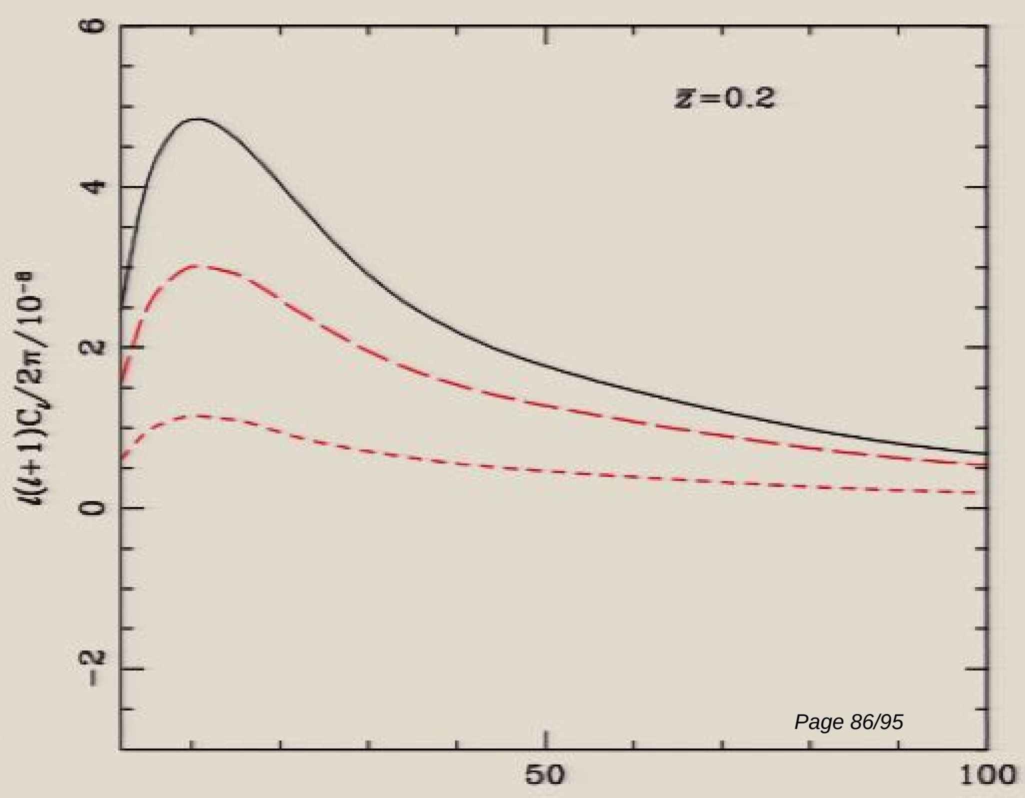


Test with Gal-ISW cross-correlations

YSS, Hu and Sawicki (2006)

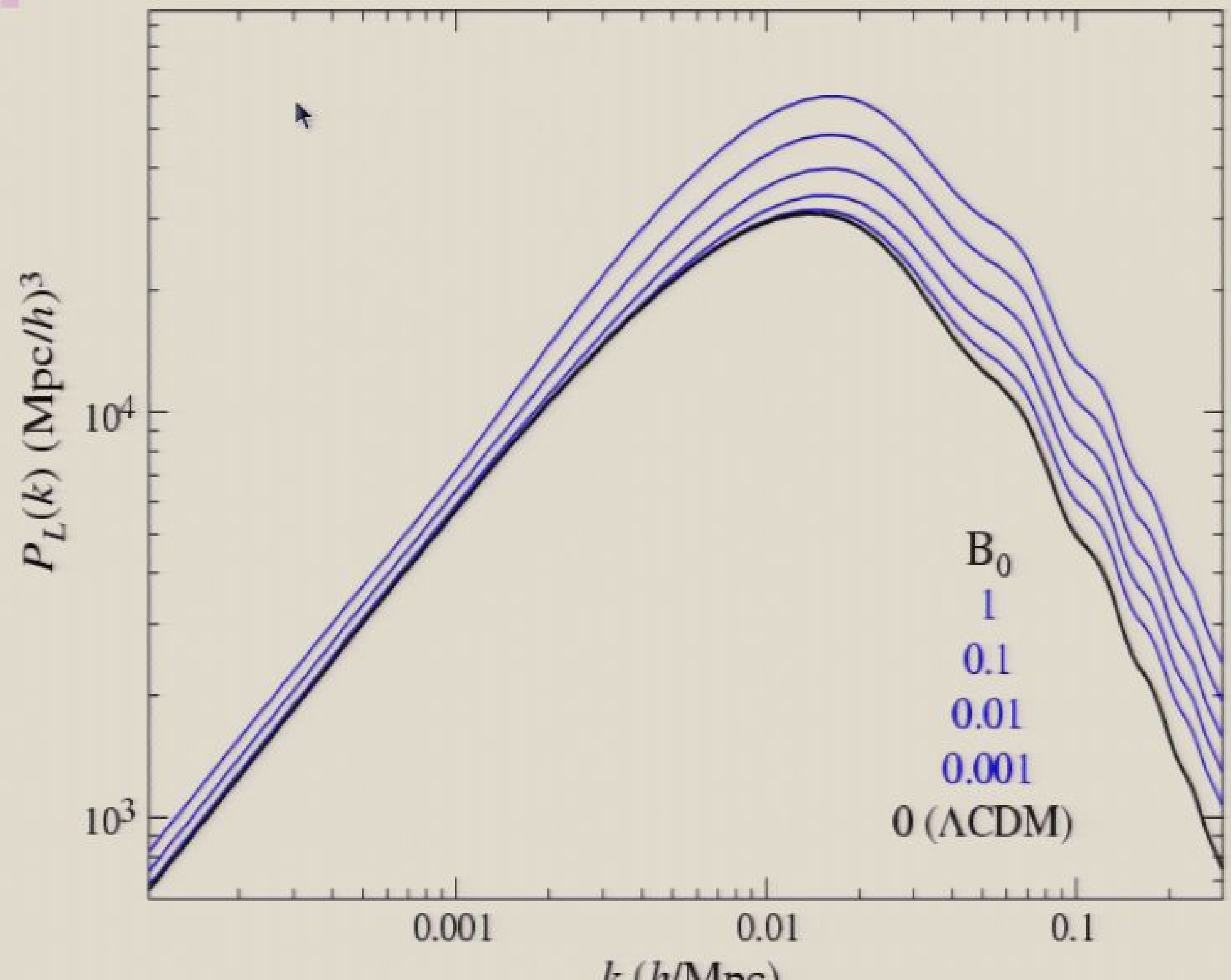
Galaxy-ISW effects are mostly contributed by scales around k=0.01/Mpc. This cross correlation can isolate the decrement in specific time, and probe the sign of variation Φ_{-}.





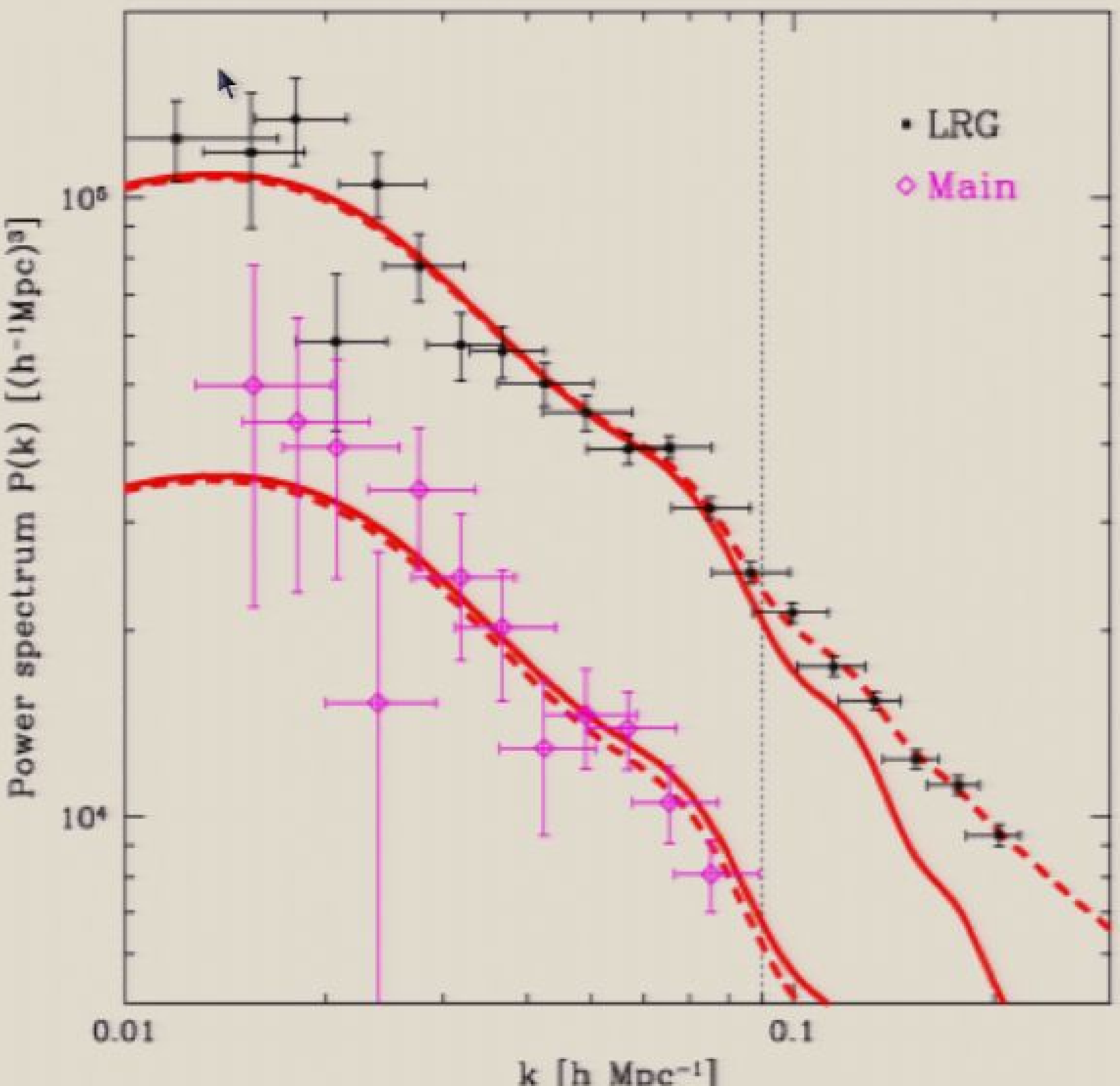
Phenomenolgical effects: Power spectra

YSS, Hu and Sawicki (2006)



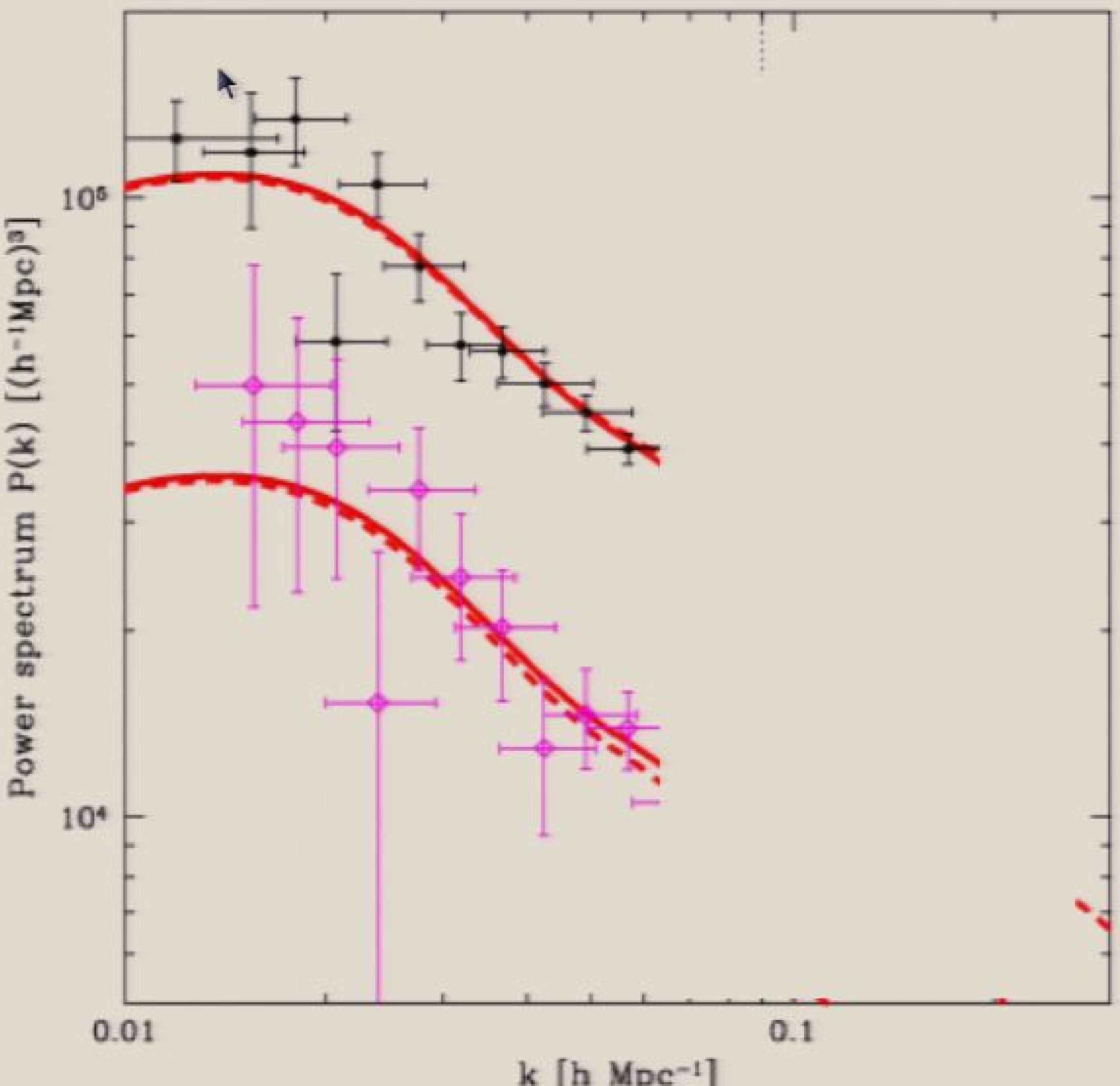
SDSS fit to f(R) gravity

Tegmark et.al. (2006)

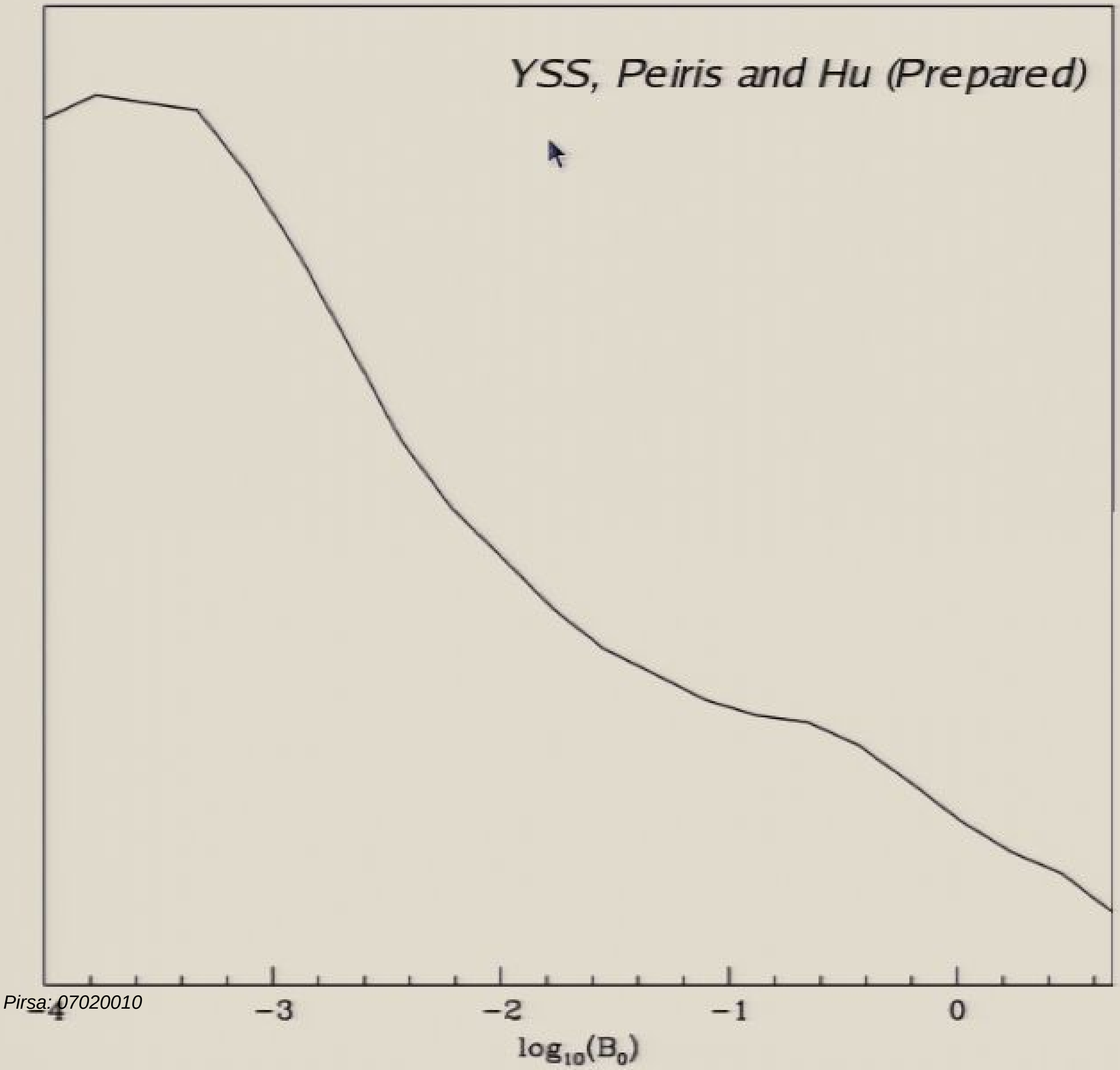


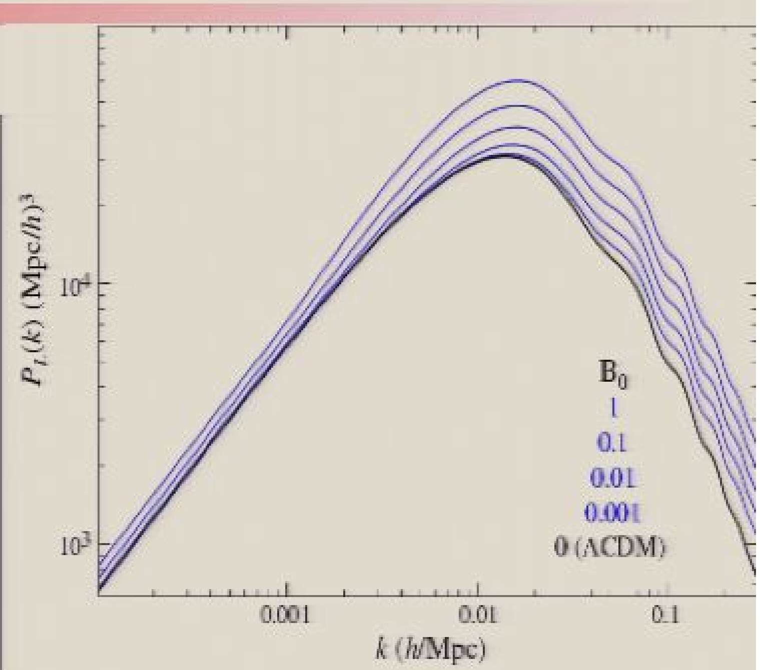
SDSS fit to f(R) gravity

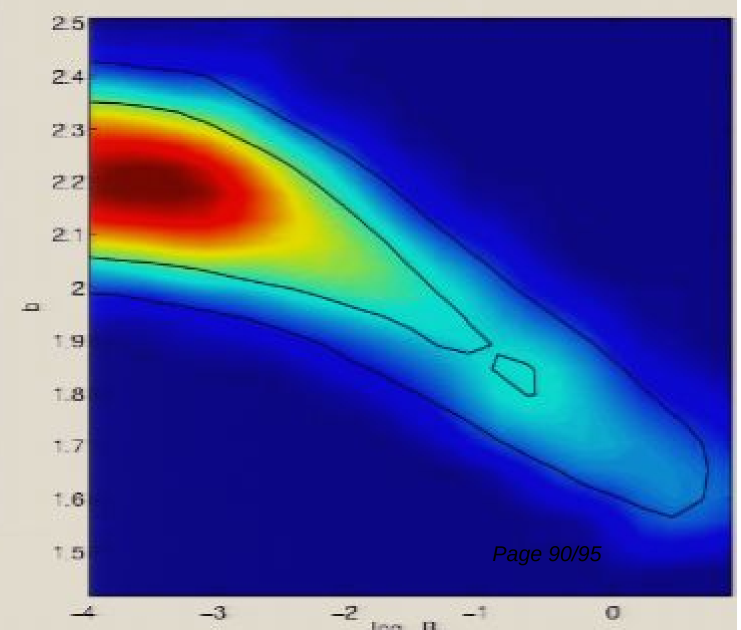
Tegmark et.al. (2006)



New upper bound of B by WMAP+SNLS+SDSS



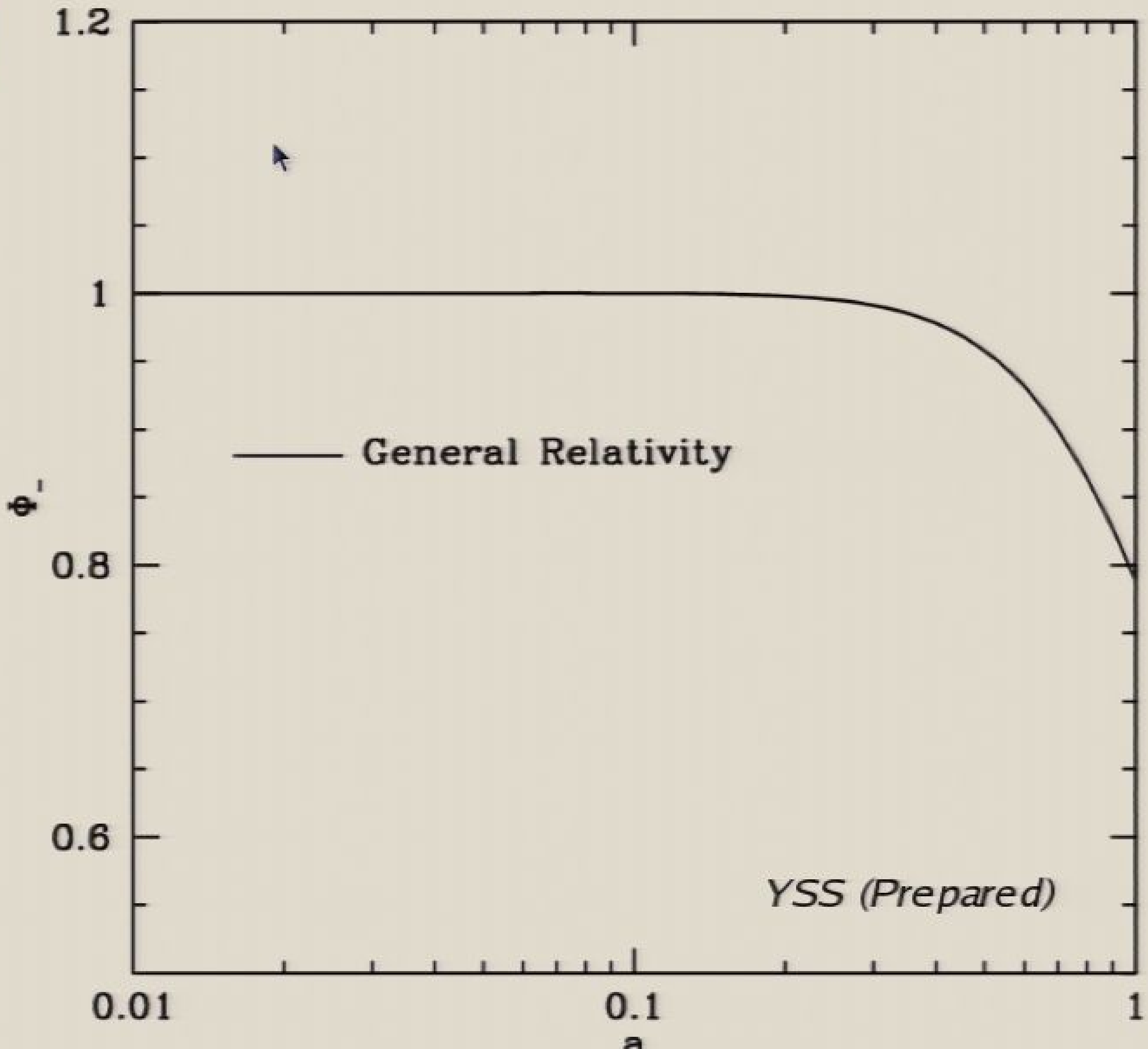




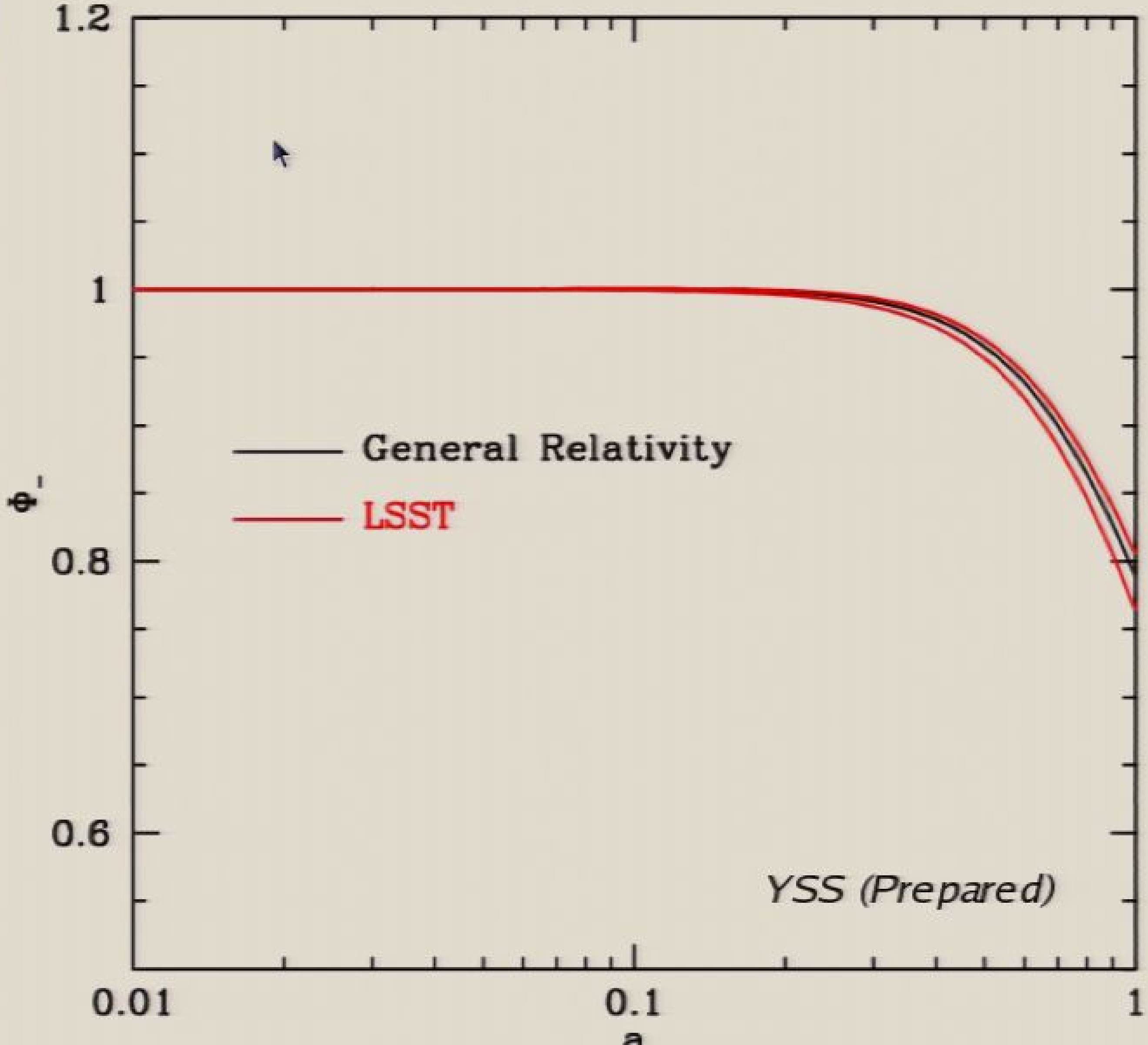
Conclusion

- A geometrical test on modified models puts a guide line for each modified gravity model
- We derive the linear perturbations at large scales, and propose phenomenological tools to test each modified gravity models.
- The future interest in this test is not just only matter of ruling or taking modified gravity models, but to determine the limit of validity of GR at largest scales in the universe.
- For accurate test on GR, we need to know the non-linear structure formation of modified gravity models.

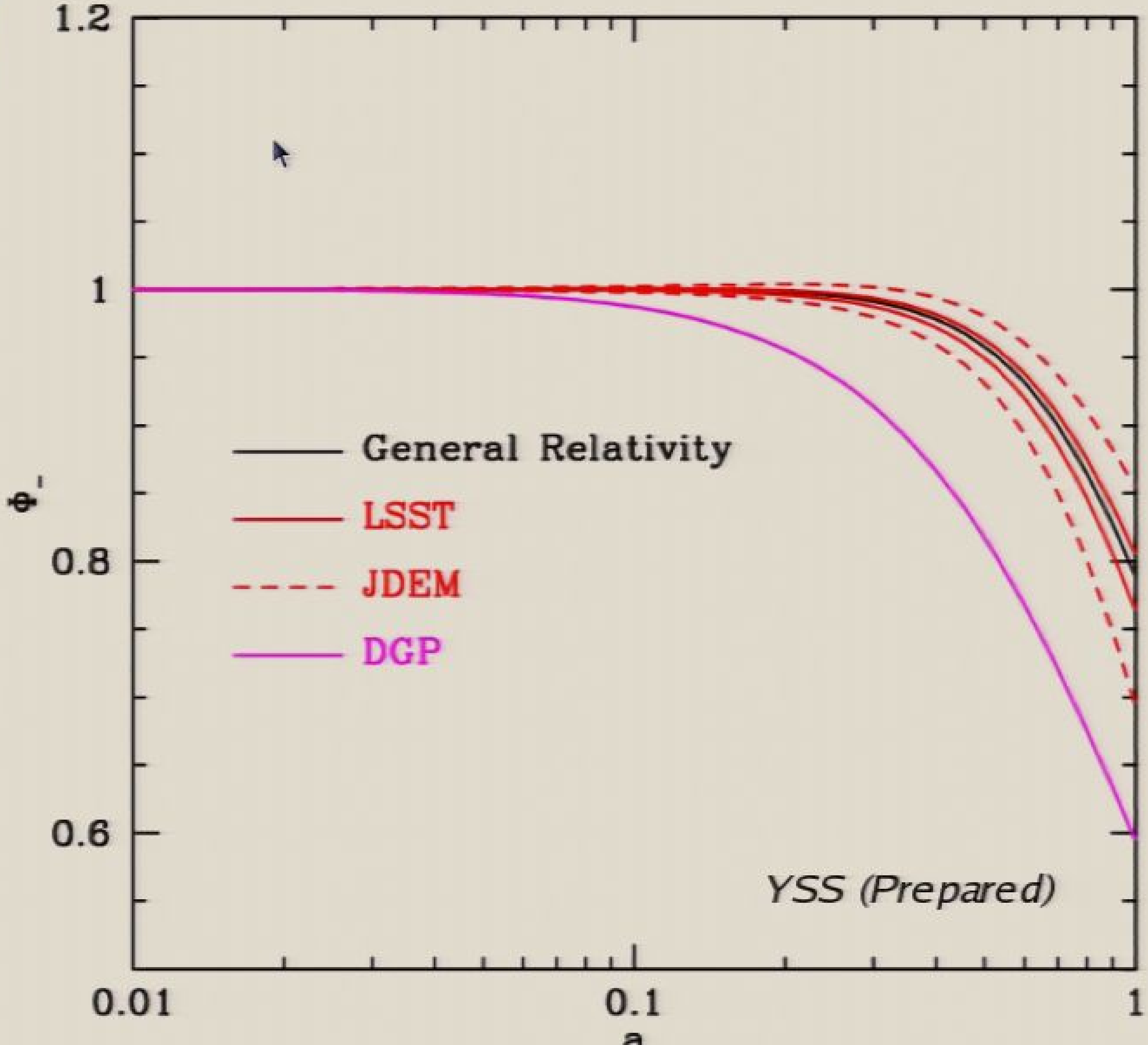
Future test



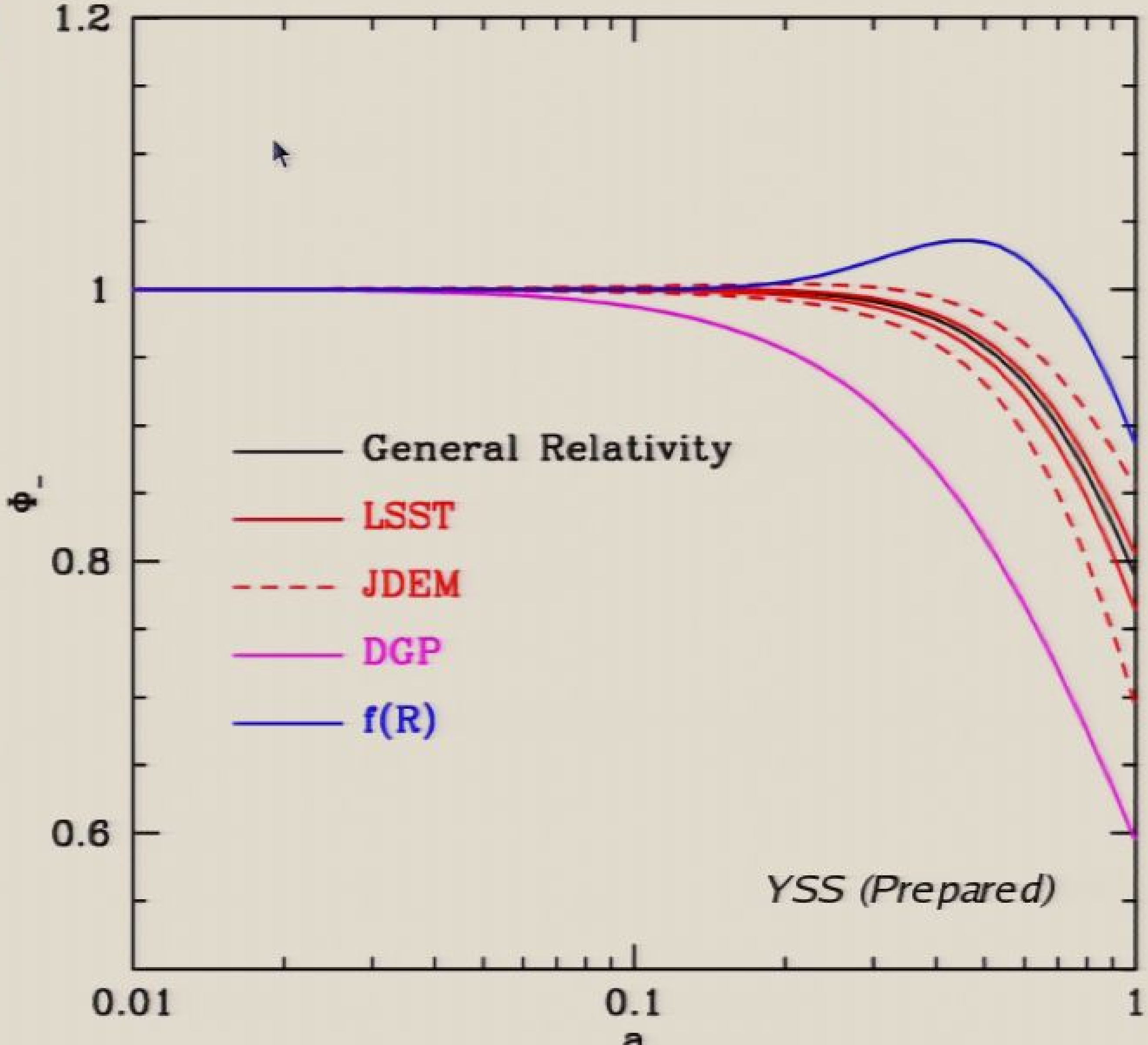
Future test



Future test


1.2
1
0.8
0.6
0.01
0.1
1
a
φ
General Relativity
LSST
JDEM
DGP
YSS (Prepared)

Future test



1.2
1
0.8
0.6
0.01
0.1
1
a
General Relativity
LSST
JDEM
DGP
f(R)
YSS (Prepared)