Title: Counting BPS operators in CFTs with Sasaki-Einstein duals.

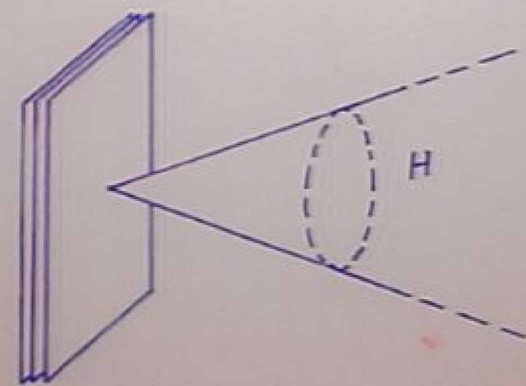
Date: Feb 27, 2007 02:00 PM

URL: http://pirsa.org/07020007

Abstract: After reviewing recent developments in the study of the AdS/CFT correspondence between the IR gauge theory living on a stack of D3-branes placed at a Calabi-Yau singularity and type IIB string theory on the near horizon geometry, we focus on the problem of counting chiral BPS operators in this class of CFTs and we match them with the dual string states in the general case of toric CY. Partition functions can be explicitly written in all sectors with fixed baryonic charge using localization over fixed points of the toric actions. These partition functions are related to the volume functions of the Sasaki-Einstein manifold and of its divisors.

THE AdS/CFT CORRESPONDENCE

•) PUT N D3-BRANES AT THE SINGULARITY OF A CALABI-YAU CONE CY_6



$\mathbb{R}^{3,1} \times CY_6$

CY_6 CONE OVER H: $\quad ds^2_{CY_6} = dr^2 + r^2 ds^2_H$

CY_6 IS CALABI YAU $\Leftrightarrow$ H IS SASAKI-EINSTEIN

•) (LOW ENERGY) AdS/CFT LIMIT:

THEORY ON D3-BRANES: $\quad$ NEAR HORIZON GEOMETRY:

$d=4$ SUPERCONFORMAL GAUGE THEORY $\Longleftrightarrow$ TYPE IIB STRING ON $AdS_5 \times H$

$$\int_H F_5 = N$$

SU(N) QUIVER

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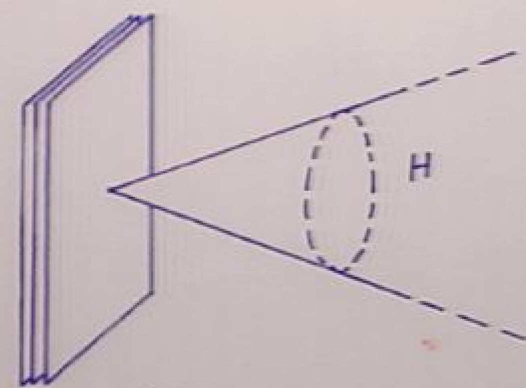

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THE SUPERCONFORMAL CASE

$\boxed{\text{4d } \mathcal{N}=1 \text{ SUPERCONFORMAL GAUGE THEORY}} \iff \boxed{\text{TYPE IIB } AdS_5 \times H}$

FORWARD PROBLEM

$\Longrightarrow$ (COMPUTE MODULI SPACE)

INVERSE PROBLEM

$\Longleftarrow$ (NON TRIVIAL!!)

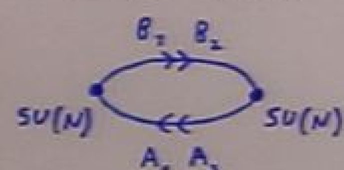
EXAMPLES:

-) $H = S^5$ AND ORBIFOLDS: $H = S^5/\Gamma$

-) $H = T^{1,1}$ $\qquad C(T^{1,1}) =$ CONIFOLD

GAUGE THEORY $\qquad\qquad$ STRING THEORY

QUIVER



$\iff$ $AdS_5 \times T^{1,1}$

$T^{1,1} = \dfrac{SU(2)\times SU(2)}{U(1)} \simeq S^2 \times S^3$

+

SUPER POT.: $W = \mathrm{Tr}\left(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right)$ $\qquad \int_{T^{1,1}} F_5 = N$

-) $Y^{p,q}$ $\quad L^{p,q,r}$

-) A SUPERCONFORMAL GAUGE THEORY HAS: $U(1)_R$ $\longleftrightarrow$ A SASAKI-EINSTEIN HAS $\xi = J\left(r \frac{\partial}{\partial r}\right)$

-) IN GENERAL THE ISOMETRIES OF H CAN BE $U(1)^r$

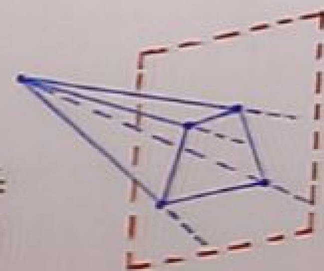
$r = 1$, $\quad r = 2$, $\quad \boxed{r = 3}$ TORIC CASE

TORIC GEOMETRY

TORIC MANIFOLD $\leftrightarrow (\mathbb{C}^*)^3$ ACTION

$U(1)^3$ ISOMETRIES

FAN C:



RATIONAL, CONVEX CONE IN $\mathbb{R}^3$

$\sim SL(3, \mathbb{Z})$

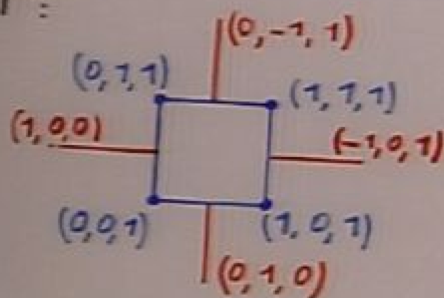
CY CONDITION:

$V_i = (x_i, y_i, 1)$

DUAL FAN C^*:

$$C^* = \{ \vec{m} \in \mathbb{R}^3 \mid (\vec{m}, \vec{V}_i) \geq 0 \quad \forall\, i = 1, 2, \dots d \}$$

ex $T^{1,1}$:



$$(0,1,0) + (0,-1,1) = (-1,0,1) + (1,0,0)$$

$$x\,y = z\,w$$

INTEGER POINTS $\vec{m} \in C^*$ $\Longleftrightarrow$ HOLOMORPHIC FUNCTIONS ON $C(H)$

DIVISORS: $V_i \leftrightarrow D_i$ $\qquad \sum_{i=1}^{d} (\vec{m}_0, \vec{V}_i)\, D_i = 0$

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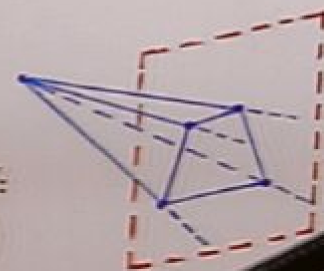

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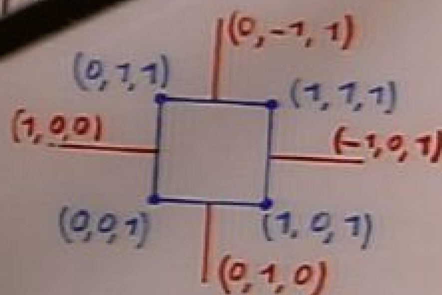
& $T^{1,1}$



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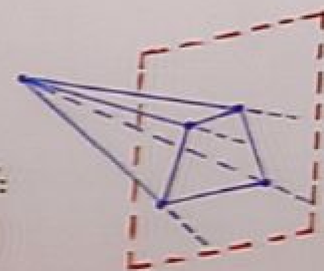

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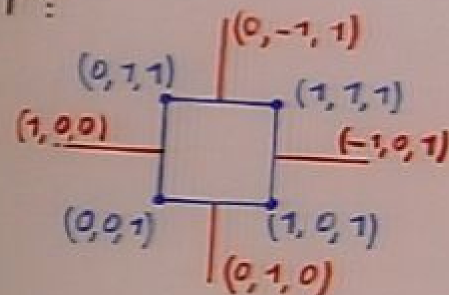

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QUIVER GAUGE THEORY IN THE TORIC CASE

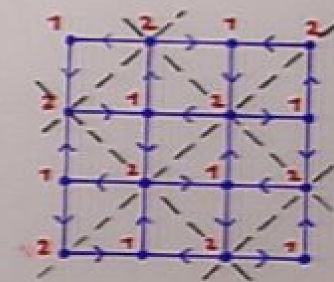
•) PERIODIC QUIVER

HANANY, KENNAWAY
FRANCO, HANANY, KENNAWAY, VEGH, WECHT

$$\left.\begin{array}{c}\text{QUIVER}\\ +\\ \text{SUPERPOTENTIAL}\end{array}\right\} \Rightarrow \text{PERIODIC QUIVER ON } T^2$$

$T^{1,1}$:

$\Longrightarrow$



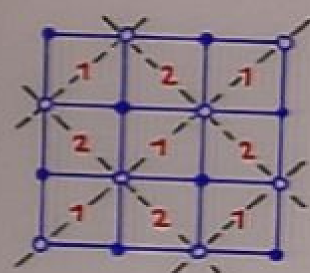
$$W = \mathrm{Tr}\left(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right)$$

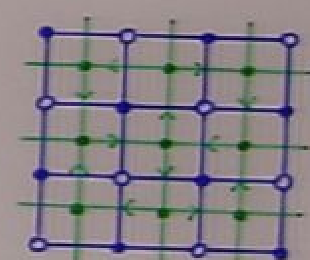
V VERTICES $\longrightarrow$ GAUGE GROUPS
E EDGES $\longrightarrow$ CHIRAL MULTIPLETS
F FACES $\longrightarrow$ SUPERPOTENTIAL TERMS

$$V - E + F = 0$$

•) DIMER

DIMER $\equiv$ DUAL GRAPH OF PERIODIC QUIVER, BIPARTITE GRAPH





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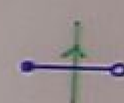

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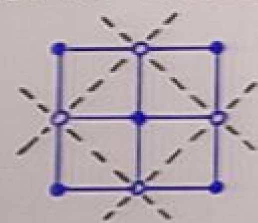
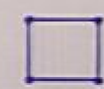
AdS/CFT IN THE TORIC CASE

GAUGE THEORY | STRING THEORY

$T^{1,1}$:



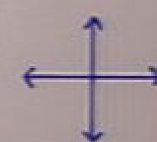
$\Longleftrightarrow$



DIMER | TORIC DIAGRAM

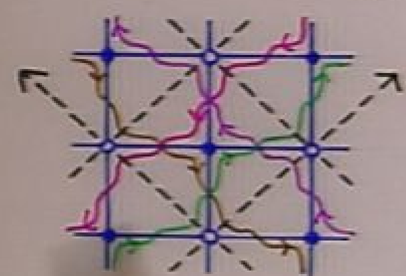
ZIG-ZAG PATH = CLOSED LOOP ON THE DIMER TURNING ALTERNATIVELY MAX. LEFT, MAX. RIGHT

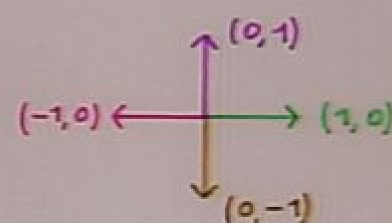
ZIG-ZAG PATHS $\Longleftrightarrow$ LEGS OF THE (p,q) WEB



HANANY, VEGH

HOMOTOPY ON T^2 $\Longleftrightarrow$ (p,q) COORDINATES



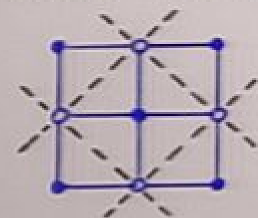
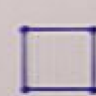


FORWARD ALGORITHM: DRAW THE ZIG-ZAG PATHS AND COUNT HOMOTOPY NUMBERS

INVERSE ALGORITHM: DRAW SUITABLE LOOPS ON A TORUS T^2 WITH

AdS/CFT IN THE TORIC CASE

GAUGE THEORY | STRING THEORY

$T^{1,1}$:  $\Longleftrightarrow$ 

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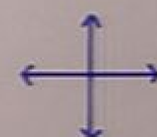

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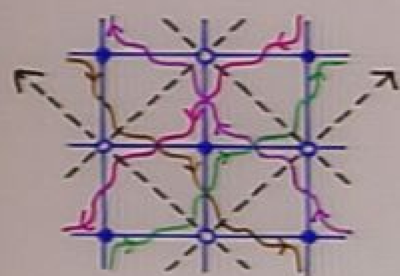

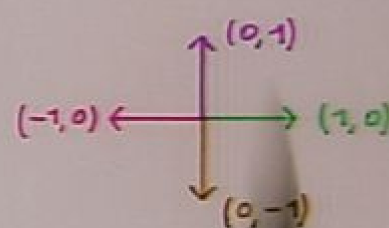

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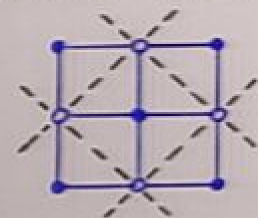
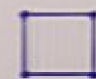
$T^{1,1}$:  $\Longleftrightarrow$ 

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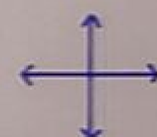
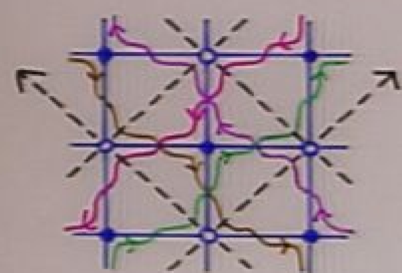
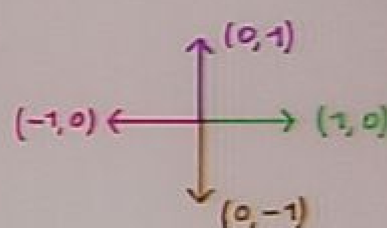


HANANY, VEGH

HOMOTOPY ON T^2 $\Longleftrightarrow$ (p,q) COORDINATES





FORWARD ALGORITHM: DRAW THE ZIG-ZAG PATHS AND COUNT HOMOTOPY NUMBERS

INVERSE ALGORITHM: DRAW SUITABLE LOOPS ON A TORUS T^2 WITH

CHECKS OF AdS/CFT (PROTECTED SECTOR)

- MODULI SPACE $\longleftrightarrow$ COMPLEX CONE $C(H)$

- CENTRAL CHARGE $\longleftrightarrow$ VOLUME OF H

$$a = \frac{\pi^3}{4\,\mathrm{Vol}(H)}$$

- BARYONS $\longleftrightarrow$ D3-BRANES WRAPPED ON $\Sigma_i \subseteq H$

  $\det \phi_i$

$$R_i = \frac{\pi\,\mathrm{Vol}(\Sigma_i)}{3\,\mathrm{Vol}(H)}$$

- MESONS $\longleftrightarrow$ SUPERGRAVITY STATES FROM g^{α}_{α}, $F^{(5)}_{\mu\nu\rho\sigma\delta}$

  $\mathrm{Tr}(\phi_1 \phi_2 \cdots \phi_K)$

$$\Delta(\Delta-4) = m^2$$

$U(1)$ SYMMETRIES

CONSERVED CURRENTS IN CFT: J^μ $\longleftrightarrow$ GAUGE POTENTIALS A_μ ON AdS_5

REDUCING ON AdS_5:

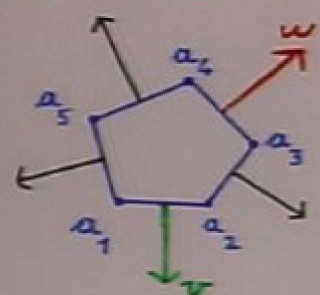
$$g_{\mu\nu} \xrightarrow[\text{ISOMETRIES}]{} U(1)_F^2 \times U(1)_R$$

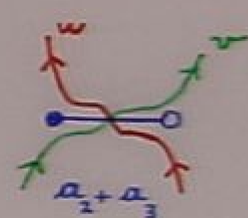
$$A^{RR}_{\mu\nu\rho\sigma} \xrightarrow[\text{3-CYCLES}]{} U(1)_B^{d-3}$$

$\Rightarrow$ IN GAUGE THEORY:

$$\underbrace{U(1)_B^{d-3} \times U(1)_F^2}_{U(1)^{d-1} \text{ GLOBAL}} \times U(1)_R \qquad \overbrace{U(1)_F^2 \times U(1)_R}^{U(1)^3 \text{ ISOMETRIES}}$$

PARAMETRIZATION OF $U(1)$ SYMMETRIES





MULTIPLICITY $\det(v, w)$

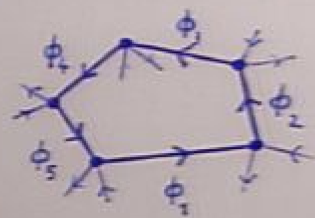
$a_1 + a_2 + \ldots + a_d = 2 \Rightarrow$ R-CHARGES

$a_1 + a_2 + \ldots + a_d = 0 \Rightarrow$ GLOBAL CHARGES

BARYONIC: $\sum_{i=1}^{d} a_i V_i = 0$

THE SPECTRUM OF MESONS

•) MESONS = CLOSED LOOPS IN THE QUIVER



$$\mathrm{Tr}\left(\phi_1\,\phi_2\,\phi_3\,\phi_4\,\phi_5\right)$$

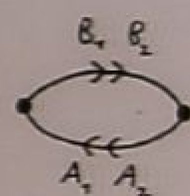
CHIRAL (PROTECTED)
Δ INDEPENDENT FROM N $\Longrightarrow$ DUAL TO SUGRA STATES FROM q^{α}_{α}, $F^{(5)}_{\mu\nu\rho\sigma\delta}$

$$\Delta(\Delta-4) = m^2$$

•) CHIRAL RING OF MESONS / F-TERMS $\Longrightarrow$ MODULI SPACE OF VACUA : $C(H)$

$(N=1)$

ex: $T^{1,1}$



$$\mathcal{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \equiv \begin{pmatrix} A_1 B_1 & A_1 B_2 \\ A_2 B_1 & A_2 B_2 \end{pmatrix}$$

$$M_{11} M_{22} - M_{12} M_{21} = 0$$

•) CHIRAL RING OF MESONS / F-TERMS $\equiv$ HOLOMORPHIC FUNCTIONS $f(z)$ ON $C(H)$

$\Box_{C(H)} f(z) = 0$ IF $f(z) = r^{\delta}\Psi$ $\Rightarrow$ $\Box_H \Psi = -\delta(\delta+4)\Psi$

$\delta = (\vec{m}, \vec{\ell}) = \Delta$ $\quad\quad$ $\partial_{\phi_k} f(z) = i\,\vec{m}_k\, f(z)$

•) RINGS:

MESONS $\equiv$ HOLOMORPHIC FUNCTIONS ON $C(H)$ $\equiv$ INTEGER POINTS IN C^*

$$C^* = \{ \vec{m} \in \mathbb{R}^3 \mid (\vec{m}, \vec{V}_i) \geq 0 \quad \forall i = 1, 2, \dots d \}$$

$\vec{m}$ = CHARGE OF $f(z)$ UNDER $U(1)^3$

•) Ψ– MAP

HANANY, HERZOG, VEGH
A.B.

COMPUTE THE TRIAL CHARGE OF A MESON M USING ZIG-ZAG PATHS

$\Rightarrow$ FOR EACH MESON M THERE EXISTS $\vec{m}$:

$$R(M) = \sum_{i=1}^{d} (\vec{m}, \vec{V}_i)\, a_i$$

$\vec{m} = (x, y, z)$ (x,y) = HOMOTOPY NUMBERS OF M ON T^2

MESONS CHARGED ONLY UNDER: $U(1)_F^2$, $U(1)_R$ AND NOT UNDER $U(1)_B^{d-3}$

•) MATCHING WITH SUGRA SPECTRUM:

$$\Delta = (\vec{m}, \vec{\ell}) \quad \Leftrightarrow \quad m^2 = \Delta(\Delta - 4)$$

$$\Delta = \frac{3}{2} R = \frac{3}{2} \sum_{i=1}^{d} (\vec{m}, \vec{V}_i)\, \bar{a}_i = (\vec{m}, \vec{\ell})$$

COUNTING MESONS → COUNT HOLOMORPHIC $f(z)$

•) CHARACTER: $\chi(q) = \mathrm{Tr}\left\{ q \mid H^0(C(H)) \right\}$

$$\chi(q) = \sum_{m} c_m q^m = \sum_{m} c_m q_1^{m_1} q_2^{m_2} q_3^{m_3}$$

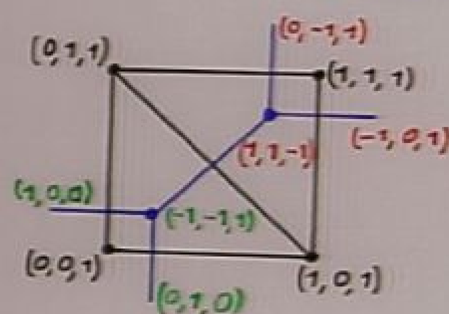
IN THE TORIC CASE: $\begin{cases} c_m = 1 & \text{IF } m \in C^* \\ c_m = 0 & \text{IF } m \notin C^* \end{cases}$

•) FROM EQUIVARIANT INDEX THEOREM:

LOCALIZATION: MARTELLI, SPARKS, YAU

$$\chi(q) = \sum_{P_I} \frac{1}{\prod_{\lambda=1}^{3} \left(1 - q^{m_I^\lambda}\right)}$$

ex $T^{1,1}$:



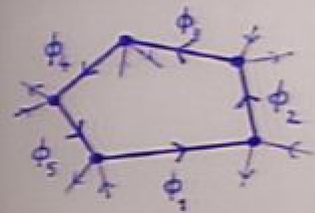
P_I	P_{II}
(1,0,0)	(-1,0,1)
(0,1,0)	(0,-1,1)
(-1,-1,1)	(1,1,-1)

$$\chi(q) = \frac{1}{(1-q_1)(1-q_2)\left(1-\frac{q_3}{q_1 q_2}\right)} + \frac{1}{\left(1-\frac{q_3}{q_1}\right)\left(1-\frac{q_3}{q_2}\right)\left(1-\frac{q_1 q_2}{q_3}\right)}$$

•) VOLUMES FROM $\chi(q)$

$$\mathrm{Vol}_H(b) = \pi^3 \lim_{t \to 0} t^3 \chi\left(e^{-tb}\right) \qquad q_i = e^{-t b_i}$$

•) MESONS = CLOSED LOOPS IN THE QUIVER



$$\mathrm{Tr}\,(\phi_1\,\phi_2\,\phi_3\,\phi_4\,\phi_5)$$

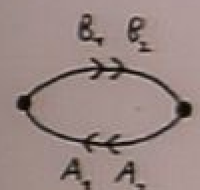
CHIRAL (PROTECTED) Δ INDEPENDENT FROM N $\Rightarrow$ DUAL TO SUGRA STATES FROM g^{α}_{α}, $F^{(5)}_{\mu\nu\rho\sigma\delta}$

$$\Delta(\Delta-4) = m^2$$

•) CHIRAL RING OF MESONS / F-TERMS $\Rightarrow$ MODULI SPACE OF VACUA : $C(H)$

$(N=1)$

ex: $T^{1,1}$



$$\mathcal{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \equiv \begin{pmatrix} A_1 B_1 & A_1 B_2 \\ A_2 B_1 & A_2 B_2 \end{pmatrix}$$

$$M_{11} M_{22} - M_{12} M_{21} = 0$$

•) CHIRAL RING OF MESONS / F-TERMS $\equiv$ HOLOMORPHIC FUNCTIONS $f(z)$ ON $C(H)$

$\Box_{C(H)} f(z) = 0$ IF $f(z) = r^{\delta}\psi$ $\Rightarrow$ $\Box_H \psi = -\delta(\delta+4)\,\psi$

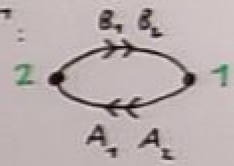
$\delta = (\vec{m}, \vec{\ell}) = \Delta$

$$\partial_{\phi_K} f(z) = i\,\vec{m}_K\, f(z)$$

$$\mathrm{Reeb} = \sum_K \vec{\ell}_K\, \partial_{\phi_K} \qquad K = 1,2,3$$

BPS BARYONIC OPERATORS

$2\times T^{1,1}$:



$SU(2)\times SU(2)\times U(1)_R \times U(1)_B$

$$U(1)_B : \begin{cases} A_i \to e^{i\alpha} A_i \\ B_i \to e^{-i\alpha} B_i \end{cases}$$

- "BASIC" BARYONIC OPERATORS:

$$\varepsilon_{p_1 \cdots p_N}\, \varepsilon^{k_1 \cdots k_N} (A_{i_1})^{p_1}_{k_1} \cdots (A_{i_N})^{p_N}_{k_N} = (\det A)_{i_1 \cdots i_N} \qquad B = N$$

$\updownarrow$ DUAL TO

STATIC D3-BRANE WRAPPED ON $\Sigma \subseteq H$

- DEFINE: $A_{I,J} = A_{i_1} B_{J_1} \cdots A_{i_m} B_{J_m} A_{i_{m+1}}$

$\Rightarrow$ GENERIC PATH FROM NODE 1 TO NODE 2

$B = 1$ NOT GAUGE INVARIANT

- GENERIC OPERATOR WITH $B = N$:

$$\varepsilon_{p_1 \cdots p_N}\, \varepsilon^{k_1 \cdots k_N} (A_{I_1, J_1})^{p_1}_{k_1} \cdots (A_{I_N, J_N})^{p_N}_{k_N}$$

$\updownarrow$ DUAL TO

MOVING D3-BRANE WRAPPED ON $\tilde{\Sigma} \subseteq H$

- BARYONIC CHARGE QUANTIZED:

$B = kN \qquad k \in \mathbb{Z}$

SUSY D3-BRANES INSIDE H

- $\mathcal{M}_d$ = CLASSICAL SUSY D3-BRANES CONFIGURATION = HOLOMORPHIC SURFACES $S \subseteq C(H)$ (MIKHAILOV)

EUCLIDEAN: $\mathbb{R}^4 \times C(H)$ D3 ON $S \in C(H)$

ADD N D3 $\mathbb{R}^4$: $\downarrow$ $\downarrow$ IF $S = \Sigma \times \gamma$ $\Sigma \subseteq H$

$\gamma^5 \times H$ D3 ON $\Sigma \times \gamma$

$\Rightarrow$ WICK ROTATE AND MAKE γ TIMELIKE

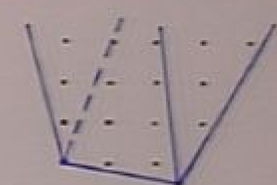
- HOLOMORPHIC SURFACE: $\chi = 0$ $\chi \in H^0(C(H), \mathcal{O}(D))$

IN TORIC GEOMETRY: $D = \sum_{i=1}^{d} c_i D_i$ $D_i \leftrightarrow V_i$

$$c_i \simeq c_i + (\vec{m}_0, \vec{V}_i)$$

$\chi_{\vec{m}}$: HOLOMORPHIC SECTIONS OF $\mathcal{O}(D)$ = INTEGER POINTS $\vec{m}$ INSIDE THE POLYTOPE P_D

$$P_D = \{ \vec{m} \in \mathbb{R}^3 \mid (\vec{m}, \vec{V}_i) + c_i \geq 0 \} \quad \forall i = 1, \dots d$$



- GEOMETRIC QUANTIZATION:

$$\mathcal{M}_d \ni \sum_{m \in P_D} h_m \chi_m = 0 \qquad h_m \sim \lambda h_m \qquad \mathcal{M}_d = CP^\infty$$

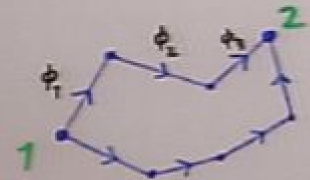
QUANTUM STATES: SECTIONS OF $\mathcal{O}(N)$ OVER CP^∞: $\int_H F_5 = N$

$$|h_{m_1} h_{m_2} \dots h_{m_N}\rangle \qquad \text{SYM. IN } m_1, \dots m_N$$

MATCHING BPS BARYONS WITH STRING STATES

•) GENERALIZING Ψ – MAP : A.B., FORCELLA, ZAFFARONI

CONSIDER IN THE QUIVER GENERIC PATHS JOINING TWO FIXED VERTICES



$$\mathcal{O} = (\phi_1\ \phi_2\ \phi_3)^{\tau}_{K}$$

NOT GAUGE INVARIANT

TRIAL R-CHARGE: $R = \sum_{i=1}^{d} \left((\vec{m}, \vec{V}_i) + c_i \right) a_i$

$(\vec{m}, \vec{V}_i) + c_i \geq 0 \Rightarrow \vec{m} \in P_{\mathcal{D}}$

INTEGER POINTS $\vec{m}$ INSIDE THE POLYTOPE $P_{\mathcal{D}}$ $=$ OPEN PATHS: $(\mathcal{O}_{\vec{m}})^{\tau}_{K}$ /F-TERMS

•) GENERALIZING BEASLEY PRESCRIPTION:

IN EACH SECTOR WITH FIXED $\mathcal{D} \leftrightarrow$ FIXED $\mathcal{B}$:

$$h_{\vec{m}} \longleftrightarrow (\mathcal{O}_{\vec{m}})^{\tau}_{K} \qquad \forall \vec{m} \in P_{\mathcal{D}}$$

$$|h_{m_1} \cdots h_{m_N}\rangle \longleftrightarrow \varepsilon_{\tau_1 \cdots \tau_N}\, \varepsilon^{K_1 \cdots K_N} (\mathcal{O}_{m_1})^{\tau_1}_{K_1} \cdots (\mathcal{O}_{m_N})^{\tau_N}_{K_N}$$

QUANTUM STATE IN STRING THEORY — GAUGE INVARIANT OPERATOR IN CFT

SYMMETRIC IN $m_1, \dots m_N$

COUNTING BPS BARYONIC OPERATORS

- $N=1$, FIXED B

$$\chi(q,D) = \mathrm{Tr}\left\{ q \,|\, H^0(C(H), \mathcal{O}(D)) \right\} = \sum_m c_m q^m$$

IN THE TORIC CASE:

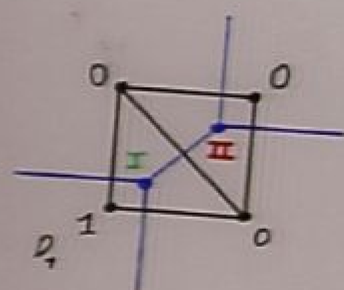
$$c_m = \begin{cases} 1 & \text{IF } m \in P_D \\ 0 & \text{IF } m \notin P_D \end{cases}$$

- FROM EQUIVARIANT INDEX THEOREM:

LOCALIZATION:

$$\chi(q,D) = \sum_{P_I} \frac{q^{m_I^0}}{\prod_{\lambda=1}^{3} \left(1 - q^{m_I^\lambda}\right)}$$

A.B., FORCELLA, ZAFFARONI



$$m_I^0 = (1,1,-1) \qquad m_{II}^0 = (0,0,0)$$

$$\begin{cases} (m_I^0, V_i) + c_i = 0 & \text{in REGION I} \\ (m_I^0, V_i) + c_i \geq 0 & \text{GENERALLY} \end{cases}$$

$$\chi(q,D_1) = \frac{\frac{q_1 q_2}{q_3}}{(1-q_1)(1-q_2)\left(1-\frac{q_3}{q_1 q_2}\right)} + \frac{1}{\left(1-\frac{q_3}{q_1}\right)\left(1-\frac{q_3}{q_2}\right)\left(1-\frac{q_1 q_2}{q_3}\right)}$$

- VOLUMES OF DIVISORS

$$\frac{\chi(e^{-t\ell}, D)}{\chi(e^{-t\ell})} = 1 + t\,\frac{\pi}{2}\,\frac{\mathrm{Vol}_D(\ell)}{\mathrm{Vol}_H(\ell)} + O(t^2)$$

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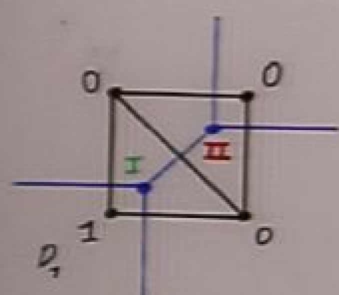

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COUNTING INTEGER $\vec{m} \in P_D$

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→ WE WANT A FUNCTION $g(q,\nu)$ THAT COUNTS SYMMETRIC PRODUCTS OF POINTS $\vec{m} \in P_D$

$$g(q,\nu) = \sum_{N=0}^{+\infty} g_N(q)\, \nu^N$$

IF $g_1(q) = \sum_{\vec{m}} c_{\vec{m}}\, q^{\vec{m}}$

$$\Rightarrow \quad g(q,\nu) = \prod_{\vec{m}} \frac{1}{(1-\nu q^{\vec{m}})^{c_{\vec{m}}}}$$

$$\log g(q,\nu) = -\sum_{\vec{m}} c_{\vec{m}} \log(1-\nu q^{\vec{m}}) = \sum_{\vec{m}} c_{\vec{m}} \sum_{k=1}^{+\infty} \frac{\nu^k q^{\vec{m}k}}{k}$$

$$= \sum_{k=1}^{+\infty} \frac{\nu^k}{k} g_1(q^k)$$

⇒ PLETHYSTIC EXPONENTIAL:

$$g(q,\nu) = P.E._\nu\left[g_1(q)\right] \equiv \exp\left(\sum_{k=1}^{+\infty} \frac{\nu^k}{k} g_1(q^k)\right)$$

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FORCELLA, HANANY, ZAFFARONI

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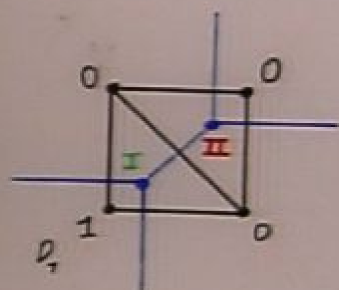

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BENVENUTI, FENG, HANANY, HE

CONCLUSIONS

FURTHER ISSUES:

→ WRITE PARTITION FUNCTIONS SUMMING OVER ALL SECTORS WITH FIXED B

FORCELLA, HANANY, ZAFFARONI

⇒ PLETHYSTIC EXPONENTIAL
PLETHYSTIC PROGRAM

⇒ UNDERSTAND MULTIPLICITIES

→ INCLUDE FERMIONS

→ $\frac{1}{4}$ BPS ??

→ RELATION WITH OTHER COUNTING PROBLEMS:
INDICES
BLACK-HOLE MICROSTATES ?
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