Title: On LCSFT/MST Correspondence

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Abstract: Light-cone superstring field theory (LCSFT) and matrix string theory (MST) are closely related. Both theories at the tree level are the Green-Schwarz superstring theory in the light-cone gauge. At the interaction level, the twist fields and the spin fields in MST correspond to the string interaction vertices in LCSFT. Since the CFT fields in MST are characterized by their OPEs, we would like to realize the OPEs by the interaction vertices in LCSFT to see the correspondence. In this talk I will begin with reviews of both theories and proceed to the correspondence between them.

On LCSFT / MST Correspondence

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with I. Kishimoto & S. Teraguchi

UNANSWERED QUESTIONS OF STRING THEORY

MULTILOOP AMPLITUDE ?

STRING VACUA ?

⇨ GOOD FORMULATION

UNANSWERED QUESTIONS OF STRING THEORY

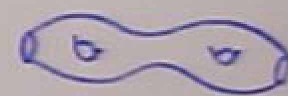
MULTILOOP AMPLITUDE ? 

STRING VACUA ? 

NONPERTURBATIVE [illegible] 

⇒ GOOD FORMULATION

Unanswered Questions of String Theory

Multiloop Amplitude ? 

String Vacua ? 

Nonperturbative DOF? 

⇨ Good Formulation

2 FORMULATIONS

FOR LIGHT-CONE QUANTIZATION

OF IIB STRING

LCSFT vs. MST

LIGHT CONE STRING FIELD THEORY — MATRIX STRING THEORY

- FREE

  IDENTICAL [GS STRING IN LC GAUGE]

- INTERACTION

  EXPLICIT VERTEX EXPRESSED IN OSCILLATORS / FORMAL OPERATORS SATISFYING OPEs

  COMPLICATED / SIMPLE

- HIGHER ORDER

  ?

RELATION ?

ONE FORMULATION COMPLEMENTS THE OTHER

MAYBE MORE ?

" STRING / GAUGE CORRESPONDENCE " ?

CAN OPEs [MST]

REALIZED BY INTERACTION VERTEX [LCSFT] ?

2 FORMULATIONS FOR LIGHT-CONE QUANTIZATION OF IIB STRING

LCSFT (LIGHT CONE STRING FIELD THEORY)	VS.	MST (MATRIX STRING THEORY)

- FREE

  IDENTICAL [GS STRING IN LC GAUGE]

- INTERACTION

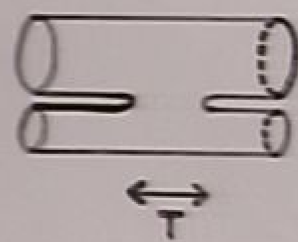
EXPLICIT VERTEX EXPRESSED IN OSCILLATORS	FORMAL OPERATORS SATISFYING OPEs
COMPLICATED	SIMPLE

- HIGHER ORDER

  ?

FIRST SIGN !!

ACCORDING TO KISHIMOTO-MATSUO-WATANABE



$\sim \frac{1}{T^6}$

USING LCSFT

MATRIX STRING INTERPRETATION:

$$[\sigma\bar{\sigma}](z,\bar{z}) \cdot [\sigma\bar{\sigma}](0,0) \sim \frac{1}{|z|^6}$$

$$6 = \left(\frac{1}{16}+\frac{1}{16}\right) \times (26-2) \times 2$$

MORE DETAILS ...

Contents

2. LCSFT

Construction

[1] GS String in LC Gauge

$$S = \int d\tau d\sigma \, [\partial X \cdot \partial X + \theta \not\partial \theta]$$

[2] Conserved Charges

$Q_0^{\dot a}$, $\tilde{Q}_0^{\dot a}$ SUSY

H_0 Hamiltonian

$Q_0^{\dot a}$, $\tilde{Q}_0^{\dot a}$, P_0^{i}, M_0^{ij}, M_0^{i-}, M_0^{+-}

[3] Algebra

$$\{ Q_0^{\dot a}, Q_0^{\dot b} \} = \{ \tilde{Q}_0^{\dot a}, \tilde{Q}_0^{\dot b} \} = 2\delta^{\dot a \dot b} H_0$$

$$\{ Q_0^{\dot a}, \tilde{Q}_0^{\dot b} \} = 0$$

...

[2′] EXPLICIT FORM OF $Q_0^{\dot a}$, $\tilde{Q}_0^{\dot a}$, H_0

$$Q_0^{\dot a} = \frac{1}{\sqrt{p^+}} \int \frac{d\sigma}{2\pi} (\gamma^i \theta)^{\dot a} (P - X')^i$$

$$\tilde{Q}_0^{\dot a} = \frac{1}{\sqrt{p^+}} \int \frac{d\sigma}{2\pi} (\gamma^i \bar{\theta})^{\dot a} (P + X')^i$$

$$H_0 = \frac{1}{p^+} \int \frac{d\sigma}{2\pi} \left[(P^i)^2 + (X^{i\prime})^2 + \theta^a \theta^{a\prime} + \bar{\theta}^a \bar{\theta}^{a\prime} \right]$$

[4] Interaction Terms, Respecting SUSY Algebra

$$Q^{\dot a} = Q_0^{\dot a} + g_s Q_1^{\dot a} + \cdots$$

$$\tilde{Q}^{\dot a} = \tilde{Q}_0^{\dot a} + g_s \tilde{Q}_1^{\dot a} + \cdots$$

$$H = H_0 + g_s H_1 + \cdots$$

$$\{Q^{\dot a}, Q^{\dot b}\} = \{\tilde{Q}^{\dot a}, \tilde{Q}^{\dot b}\} = 2\delta^{\dot a \dot b} H$$

$$\{Q^{\dot a}, \tilde{Q}^{\dot b}\} = 0$$

[5] Overlapping Condition

$|V\rangle_{12}$

[5′] EXPLICIT FORM OF $|V\rangle_{123}$

(1) PARTICLE THEORY REVISITED

COORDINATE REP.

$$V = \int d^d x\ \phi(x)^3 = \int d^d x_1\, d^d x_2\, d^d x_3\ \phi(x_1)\,\phi(x_2)\,\phi(x_3)\ \delta^d(x_1-x_2)\,\delta^d(x_2-x_3)$$

MOMENTUM REP. $\quad \phi(x) = \int d^d p\ \tilde{\phi}(p)\, e^{ipx}$

$$V = \int d^d p_1\, d^d p_2\, d^d p_3\ \tilde{\phi}(p_1)\,\tilde{\phi}(p_2)\,\tilde{\phi}(p_3)\ \delta^d(p_1+p_2+p_3)$$

OSCILLATOR REP. $\quad \tilde{\phi}(p) = \langle \phi | p \rangle$

$$|p\rangle = \exp\left(-\tfrac{1}{4}p^2 + p a^\dagger - \tfrac{1}{2} a^\dagger a^\dagger\right)|0\rangle$$

$$V = {}_1\langle\phi|\ {}_2\langle\phi|\ {}_3\langle\phi| \int \prod_{i=1}^{3} d^d p_i\ \prod_{j=1}^{3} \exp\left(-\tfrac{1}{4}p_j^2 + p_j a_j^\dagger - \tfrac{1}{2} a_j^\dagger a_j^\dagger\right)|0\rangle_{123}$$
$$\times\ \delta^d(p_1+p_2+p_3)$$

$$= {}_1\langle\phi|\ {}_2\langle\phi|\ {}_3\langle\phi|\ \exp\left(\sum_{r,s=1}^{3} \tfrac{1}{2}\, a_r^\dagger N^{r,s} a_s^\dagger\right)|0\rangle_{123}$$

$$= {}_1\langle\phi|\ {}_2\langle\phi|\ {}_3\langle\phi|\ |V\rangle_{123}$$

[5′] EXPLICIT FORM OF $|V\rangle_{123}$

(i) PARTICLE THEORY REVISITED

COORDINATE REP.

$$V = \int d^d x \; \phi(x)^3 = \int d^d x_1\, d^d x_2\, d^d x_3\; \phi(x_1)\,\phi(x_2)\,\phi(x_3)\; \delta^d(x_1 - x_3)\; \delta^d(x_2 - x_3)$$

MOMENTUM REP. $\quad \phi(x) = \int d^d p\; \tilde{\phi}(p)\; e^{ip\cdot x}$

$$V = \int d^d p_1\, d^d p_2\, d^d p_3\; \tilde{\phi}(p_1)\,\tilde{\phi}(p_2)\,\tilde{\phi}(p_3)\; \delta^d(p_1 + p_2 + p_3)$$

OSCILLATOR REP. $\quad \tilde{\phi}(p) = \langle \phi | p \rangle$

$$|p\rangle = \exp\left(-\tfrac{1}{4}p^2 + p a^+ - \tfrac{1}{2} a^+ a^+\right)|0\rangle$$

$$V = {}_1\langle\phi|\; {}_2\langle\phi|\; {}_3\langle\phi| \int \prod_{i=1}^{3} d^d p_i \; \prod_{j=1}^{3} \exp\left(-\tfrac{1}{4} p_j^2 + p_j a_j^+ - \tfrac{1}{2} a_j^+ a_j^+\right)|0\rangle_{123} \times \delta^d(p_1 + p_2 + p_3)$$

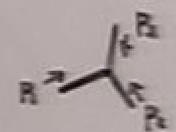
$$= {}_1\langle\phi|\; {}_2\langle\phi|\; {}_3\langle\phi| \; \exp\left(\sum_{r,s=1}^{3} \tfrac{1}{2}\, a_r^+ N^{r,s} a_s^+\right)|0\rangle_{123}$$

$$= {}_1\langle\phi|\; {}_2\langle\phi|\; {}_3\langle\phi| \; |V\rangle_{123}$$

(2) STRING THEORY

MOMENTUM REP.

$$V = \int DP_1(\sigma)\, DP_2(\sigma)\, DP_3(\sigma)\ \Phi(P_1(\sigma))\ \Phi(P_2(\sigma))\ \Phi(P_3(\sigma))$$
$$\times\ \delta(\text{"}P_1 + P_2 + P_3\text{"})$$



$$p_1 + p_2 + p_3 = 0$$

PARTICLE

STRING

$|\sigma| < \alpha_1 \qquad P_1(\sigma) + P_3(-\sigma) = 0$

$\alpha_1 < \sigma \qquad P_2(\sigma - \alpha_1) + P_3(-\sigma) = 0$

$\sigma < -\alpha_1 \qquad P_2(\sigma + \alpha_1) + P_3(-\sigma) = 0$

$$P_1(\sigma) = p_1 + \sum_{n=1}^{\infty} \sqrt{2}\left(p^{c}_{1n} \cos n\sigma + p^{s}_{1n} \sin n\sigma\right)$$

$$P_3 + \sqrt{C} A^{(1)} \frac{1}{\sqrt{C}} P_1 + \sqrt{C} A^{(2)} \frac{1}{\sqrt{C}} P_2 = 0$$

$$\left[\sqrt{C} A^{(1)} \frac{1}{\sqrt{C}}\right]_{mn} = \int_0^{\pi\alpha_1} \frac{d\sigma}{\pi\alpha_3}\ 2 \cos\frac{n\sigma}{\alpha_1} \cos\frac{m\sigma}{\alpha_3}$$

$$\left[\sqrt{C} A^{(2)} \frac{1}{\sqrt{C}}\right]_{mn} = \int_{\pi\alpha_1}^{\pi(\alpha_1+\alpha_2)} \frac{d\sigma}{\pi\alpha_3}\ 2 \cos\frac{n(\sigma - \pi\alpha_1)}{\alpha_2} \cos\frac{m\sigma}{\alpha_3}$$

$$[C]_{mn} = m\, \delta_{mn}$$

[5′] Explicit Form of $|V\rangle_{123}$

(i) Particle Theory Revisited

Coordinate Rep.

$$V = \int d^d x \; \phi(x)^3 = \int d^d x_1 \, d^d x_2 \, d^d x_3 \; \phi(x_1)\,\phi(x_2)\,\phi(x_3)\; \delta^d(x_1 - x_2)\,\delta^d(x_2 - x_3)$$

Momentum Rep. $\qquad \phi(x) = \int d^d p \; \tilde{\phi}(p)\, e^{ip\cdot x}$

$$V = \int d^d p_1 \, d^d p_2 \, d^d p_3 \; \tilde{\phi}(p_1)\,\tilde{\phi}(p_2)\,\tilde{\phi}(p_3)\; \delta^d(p_1 + p_2 + p_3)$$

Oscillator Rep. $\qquad \tilde{\phi}(p) = \langle \phi | p \rangle$

$$|p\rangle = \exp\left(-\tfrac{1}{4}p^2 + p a^\dagger - \tfrac{1}{2} a^\dagger a^\dagger\right)|0\rangle$$

$$V = {}_1\langle\phi|\,{}_2\langle\phi|\,{}_3\langle\phi| \int \prod_{i=1}^{3} d^d p_i \; \prod_{j=1}^{3} \exp\left(-\tfrac{1}{4}p_j^2 + p_j a_j^\dagger - \tfrac{1}{2} a_j^\dagger a_j^\dagger\right)|0\rangle_{123} \times \delta^d(p_1 + p_2 + p_3)$$

$$= {}_1\langle\phi|\,{}_2\langle\phi|\,{}_3\langle\phi| \; \exp\left(\sum_{r,s=1}^{3} \tfrac{1}{2} a_r^\dagger N^{r,s} a_s^\dagger\right)|0\rangle_{123}$$

$$= {}_1\langle\phi|\,{}_2\langle\phi|\,{}_3\langle\phi| \; |V\rangle_{123}$$

(2) STRING THEORY

MOMENTUM REP.

$$V = \int DP_1(\sigma)\, DP_2(\sigma)\, DP_3(\sigma)\ \Phi(P_1(\sigma))\ \Phi(P_2(\sigma))\ \Phi(P_3(\sigma))$$
$$\times\ \delta(\text{"}P_1 + P_2 + P_3\text{"})$$

$$p_1 + p_2 + p_3 = 0$$

PARTICLE

STRING

$$|\sigma| < \alpha_1 \qquad P_1(\sigma) + P_3(-\sigma) = 0$$
$$\alpha_1 < \sigma \qquad P_2(\sigma-\alpha_1) + P_3(-\sigma) = 0$$
$$\sigma < -\alpha_1 \qquad P_2(\sigma+\alpha_1) + P_3(-\sigma) = 0$$

$$P_1(\sigma) = p_1 + \sum_{n=1}^{\infty} \sqrt{2}\left(p^c_{1n}\cos n\sigma + p^s_{1n}\sin n\sigma\right)$$

$$p_3 + \sqrt{C}A^{(1)}\frac{1}{\sqrt{C}}\, p_1 + \sqrt{C}A^{(2)}\frac{1}{\sqrt{C}}\, p_2 = 0$$

$$\left[\sqrt{C}A^{(1)}\frac{1}{\sqrt{C}}\right]_{mn} = \int_0^{\pi\alpha_1}\frac{d\sigma}{\pi\alpha_3}\ 2\cos\frac{m\sigma}{\alpha_1}\cos\frac{n\sigma}{\alpha_3}$$
$$\left[\sqrt{C}A^{(2)}\frac{1}{\sqrt{C}}\right]_{mn} = \int_{\pi\alpha_1}^{\pi(\alpha_1+\alpha_2)}\frac{d\sigma}{\pi\alpha_3}\ 2\cos\frac{m(\sigma-\pi\alpha_1)}{\alpha_2}\cos\frac{n\sigma}{\alpha_3}$$

$$[C]_{mn} = m\,\delta_{mn}$$

OSCILLATOR REP.

$$V = {}_1\langle\Phi|\,{}_2\langle\Phi|\,{}_3\langle\Phi| \int \prod_{i=1}^{2} dp_i$$

$$\times \prod_{j=1}^{3} \exp\left(-\frac{1}{4} p_j^T \frac{1}{C} p_j + p_j^T \frac{1}{\sqrt{C}} a_j^\dagger - \frac{1}{2} a_j^{\dagger T} a_j^\dagger\right) |0\rangle_{123}$$

$$\times\ \delta\left(p_3 + \sqrt{C} A^{(1)} \frac{1}{\sqrt{C}} p_1 + \sqrt{C} A^{(2)} \frac{1}{\sqrt{C}} p_2\right)$$

$$= {}_1\langle\Phi|\,{}_2\langle\Phi|\,{}_3\langle\Phi|\ \exp\left(\sum_{r,s=1}^{3} \frac{1}{2} a_r^{\dagger T} N^{r,s} a_s^\dagger\right) |0\rangle_{123}$$

$$= {}_1\langle\Phi|\,{}_2\langle\Phi|\,{}_3\langle\Phi|\ |V\rangle_{123}$$

$$N^{r,s} = \delta^{r,s} - 2 A^{(r)T} \frac{1}{\Gamma} A^{(s)}$$

WITH $\quad A^{(3)} = 1\,, \quad \Gamma = \sum_{t=1}^{3} A^{(t)} A^{(t)T}$

COMBINING INCOMING / OUTGOING STRINGS,

$$|V\rangle_{123} = \exp\left(\frac{1}{2} a_3^{\dagger T} N^{3,3} a_3^\dagger + a_3^{\dagger T} N^{3,12} a_{12}^\dagger + \frac{1}{2} a_{12}^{\dagger T} N^{12,12} a_{12}^\dagger\right) |0\rangle_{123}$$

$$a_{12}^\dagger = \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix} \qquad N^{3,12} = \left(N^{3,1}\ N^{3,2}\right) \qquad N^{12,12} = \begin{pmatrix} N^{1,1} & N^{1,2} \\ N^{2,1} & N^{2,2} \end{pmatrix}$$

(3) FORMULAS

- UNITARITY   INCOMING $\leftrightarrow$ OUTGOING

$$\mathbb{P}_3 = \left(-\sqrt{c}A^{31}\frac{1}{\sqrt{c}} \quad -\sqrt{c}A^{32}\frac{1}{\sqrt{c}}\right)\begin{pmatrix}\mathbb{P}_1\\ \mathbb{P}_2\end{pmatrix}$$

UNITARY MATRIX

$\Rightarrow$ $-\frac{\omega_r}{\omega_3} A^{3rT} C A^{3s} = \delta^{r,s} C$ $(r,s=1,2)$

$\sum_{r=1}^{2} \omega_r A^{3r} \frac{1}{C} A^{3rT} = \cdots$

- $\sum_{t=1}^{2} N^{r,t} N^{t,s} = \delta^{r,s}$

$\Rightarrow$ $N^{1,1}N^{1,1} + N^{1,R}N^{R,1} = 1$

$N^{1,1}N^{1,R} + N^{1,R}N^{R,R} = 0$

[4] Interaction Terms, Respecting SUSY Algebra

$$Q^{\dot{a}} = Q_0^{\dot{a}} + g_s\, Q_1^{\dot{a}} + \cdots$$

$$\tilde{Q}^{\dot{a}} = \tilde{Q}_0^{\dot{a}} + g_s\, \tilde{Q}_1^{\dot{a}} + \cdots$$

$$H = H_0 + g_s\, H_1 + \cdots$$

$$\{Q^{\dot{a}}, Q^{\dot{b}}\} = \{\tilde{Q}^{\dot{a}}, \tilde{Q}^{\dot{b}}\} = 2\delta^{\dot{a}\dot{b}} H$$

$$\{Q^{\dot{a}}, \tilde{Q}^{\dot{b}}\} = 0$$

[5] Overlapping Condition

$|V\rangle_{123}$

[6] RESULTS

$$|Q_1^{\dot a}\rangle_{123} = \bar{Z}^i\, s^{i\dot a}(Y)\, |V\rangle_{123}$$

$$|\tilde{Q}_1^{\dot a}\rangle_{123} = Z^i\, \tilde{s}^{i\dot a}(Y)\, |V\rangle_{123}$$

$$|H_1\rangle_{123} = Z^i\, \bar{Z}^j\, v^{ji}(Y)\, |V\rangle_{123}$$

$$Z^i = N^T \alpha^{i+}, \quad \bar{Z}^i = N^T \bar{\alpha}^{i+}, \quad Y^a = N^T S^{a+}$$

COMMUTE WITH MOMENTUM CONSERVATION

[7] INTERPRETATION

Z^i HOLOMORPHIC PART OF BOSONIC MOMENTUM $(P^i + X^{i\prime})(\sigma)$

Y^a FERMIONIC MOMENTUM $\lambda^a(\sigma)$

RENORMALIZED. AT THE INTERACTION POINT

$$(P^i + X^{i\prime})(\sigma)\, |V\rangle_{123} \sim \frac{1}{\sqrt{\sigma - \sigma_I}}\, Z^i\, |V\rangle_{123}$$

$$\lambda^a(\sigma)\, |V\rangle_{123} \sim \frac{1}{\sqrt{\sigma - \sigma_I}}\, Y^a\, |V\rangle_{123}$$

[6′] Explicit Form of $S^{i\dot a}$, $\tilde S^{i\dot a}$, Z^{ij}

$$S^{i\dot a} = -\frac{i}{\sqrt{2}}\left(\eta S_1^{i\dot a} + \eta^* S_2^{i\dot a}\right)$$

$$\tilde S^{i\dot a} = \frac{i}{\sqrt{2}}\left(\eta^* S_1^{i\dot a} + \eta S_2^{i\dot a}\right)$$

$$S_1^{i\dot a} = 2\,\gamma^i_{a\dot a}\Upsilon^a + \frac{8}{6!}\,u^{i\dot a}_{abc}\,\varepsilon^{a\cdots h}\,\Upsilon^d\cdots\Upsilon^h$$

$$S_2^{i\dot a} = \frac{2}{3}\,u^{i\dot a}_{abc}\,\Upsilon^a\Upsilon^b\Upsilon^c - \frac{16}{7!}\,\gamma^i_{a\dot a}\,\varepsilon^{a\cdots h}\,\Upsilon^b\cdots\Upsilon^h$$

$$Z^{ij} = x^{ij} + y^{ij}$$

$$x^{ij} = \delta^{ij} + \frac{1}{6}\,t^{ij}_{abcd}\,\Upsilon^a\Upsilon^b\Upsilon^c\Upsilon^d + \frac{16}{8!}\,\delta^{ij}\,\varepsilon^{a\cdots h}\,\Upsilon^a\cdots\Upsilon^h$$

$$y^{ij} = \frac{2}{3}\,u^{i\dot a}_{abc}\,\Upsilon^a\Upsilon^b\Upsilon^c - \frac{16}{7!}\,\gamma^i_{a\dot a}\,\varepsilon^{a\cdots h}\,\Upsilon^b\cdots\Upsilon^h$$

WITH

$$\Gamma^i = \begin{pmatrix} & \gamma^i_{a\dot a} \\ \gamma^i_{\dot a a} & \end{pmatrix}, \quad \{\Gamma^i, \Gamma^j\} = 2\delta^{ij}$$

$$u^{i\dot a}_{abc} = -\gamma^{ij}_{[ab}\,\gamma^j_{c]\dot a}, \quad t^{ij}_{abcd} = \gamma^{ik}_{[ab}\,\gamma^{jk}_{cd]}$$

$$\eta = e^{\pi i/4}$$

DEFINE

$$[\hat{\gamma}^a]_{i\dot{a}} = [\hat{\gamma}^a]_{\dot{a}i} \equiv [\gamma^i]_{a\dot{a}}$$

$$\hat{\Gamma}^a = \begin{pmatrix} & \hat{\gamma}^a_{i\dot{a}} \\ \hat{\gamma}^a_{\dot{a}i} & \end{pmatrix} \quad , \quad \{\hat{\Gamma}^a, \hat{\Gamma}^b\} = 2\delta^{ab}$$

AND

$$\mathcal{Y} = \sqrt{2}\,\eta^{s}\, Y^a \hat{\gamma}^a$$

THEN

$$\mathcal{U}^{ij}(Y) = [\cosh \mathcal{Y}]^{ij}$$

$$\Sigma^{i\dot{a}}(Y) = [\sinh \mathcal{Y}]^{\dot{a}i}$$

$$\hat{\Sigma}^{i\dot{a}}(Y) = [\sinh \mathcal{Y}]^{i\dot{a}}$$

[6″] PROOF OF SUSY ALGEBRA

SUSY @ $O(g_s^1)$

$$\{Q_0^{\dot a}, Q_1^{\dot b}\} + \{Q_1^{\dot a}, Q_0^{\dot b}\} = 2\,\delta^{\dot a \dot b} H_1$$

$$\Updownarrow$$

$$\sum_{t=1}^{3} Q_0^{\dot a(t)} |Q_1^{\dot b}\rangle + \sum_{t=1}^{3} Q_0^{\dot b(t)} |Q_1^{\dot a}\rangle = 2\,\delta^{\dot a \dot b} |H_1\rangle$$

$$\Uparrow \qquad \begin{aligned} |Q_1^{\dot a}\rangle &= \bar{z}^i s^{i\dot a}(Y)\,|V\rangle \\ |H_1\rangle &= z^i \bar{z}^j\, v^{ij}(Y)\,|V\rangle \end{aligned}$$

$$\gamma^j_{a\dot a} D^a s^{i\dot b}(Y) + \gamma^j_{a\dot b} D^a s^{i\dot a}(Y) = 2\sqrt{2}\,\delta_{\dot a \dot b}\, v^{ij}(Y)$$

WITH $\quad D^a = \frac{1}{\gamma^2}\frac{\partial}{\partial Y^a} + \gamma^2 Y^a$

$$\Uparrow$$

$$D^a s^{i\dot a}(Y) = \sqrt{2}\,\gamma^j_{a\dot a}\, v^{ij}(Y)$$

$$\Updownarrow$$

$$D^a \, [\sinh \chi]^{\dot{a} i} = [\hat{\gamma}^a \cosh \chi]^{\dot{a} i}$$

$$\Uparrow$$

$$\frac{1}{\gamma^2} \frac{\partial}{\partial \Upsilon^a} \chi^k = k \, \hat{\gamma}^a \chi^{k-1} - k(k-1) \, \gamma^2 \Upsilon^a \chi^{k-2}$$

$$\Uparrow$$

CLIFFORD ALGEBRA $\{ \hat{\gamma}^a , \hat{\gamma}^b \} = 2 \delta^{ab}$

$$\Updownarrow$$

$$D^a\,[\sinh\chi]^{\dot{a}i} = [\hat{\gamma}^a \cosh\chi]^{\dot{a}i}$$

$$\Uparrow$$

$$\frac{1}{\gamma^2}\frac{\partial}{\partial\gamma^a}\chi^k = k\,\hat{\gamma}^a\chi^{k-1} - k(k-1)\,\gamma^c\gamma^a\chi^{k-2}$$

$$\Uparrow$$

CLIFFORD ALGEBRA $\{\hat{\gamma}^a, \hat{\gamma}^b\} = 2\,\delta^{ab}$

[1] FOURIER TRANSFORMATION

$$[\cosh X]^{ij} = \int d^2\phi \; e^{-\phi^a Y^a} \, [\cosh \phi]^{ji}$$

$$\Rightarrow \quad |\tilde{A}\rangle_{1245} = {}_{36}\langle R| \; e^{-T(\mathrm{PROP.})}$$

$$\times \; e^{-\phi^a_{123} Y^a_{123}} \, |V\rangle_{123} \; e^{-\phi^a_{456} Y^a_{456}} \, |V\rangle_{456}$$

[2] FORMULA

$$\langle 0| \exp\left(\tfrac{1}{2} \mathbb{S}^T M \mathbb{S} + \mathbb{k}^T \mathbb{S}\right) \times \exp\left(\tfrac{1}{2} \mathbb{S}^{+T} N \mathbb{S}^+ + \mathbb{y}^T \mathbb{S}^+\right) |0\rangle$$

$$= \exp\left[\tfrac{1}{2} \mathbb{k}^T N \Delta \mathbb{k} + \tfrac{1}{2} \mathbb{y}^T \Delta M \mathbb{y} + \mathbb{y}^T \Delta \mathbb{k}\right]$$

$$\Delta = (1 + MN)^{-1}$$

[3] RESULT

$$|\tilde{A}\rangle_{1245} = \int \delta(1,2,4,5) \; e^{-\phi^a_{123} Y^a_{123}} \; e^{-\phi^a_{456} Y^a_{456}}$$

$$\times \; e^{F(1,2,4,5)} \; |0\rangle_{1245}$$

WITH

[1] FOURIER TRANSFORMATION

$$[\cosh \mathcal{X}]^{ij} = \int d^a\phi \; e^{-\phi^a Y^a} \, [\cosh \phi]^{ij}$$

$$\Rightarrow \quad |\tilde{A}\rangle_{1245} = {}_{36}\langle R| \; e^{-T(\mathrm{PROP.})}$$

$$\times \; e^{-\phi^a_{123} Y^a_{123}} \, |V\rangle_{123} \; e^{-\phi^a_{456} Y^a_{456}} \, |V\rangle_{456}$$

[2] FORMULA

$$\langle 0| \exp\left(\tfrac{1}{2} \mathfrak{S}^T M \mathfrak{S} + \mathbb{k}^T \mathfrak{S}\right) \times \exp\left(\tfrac{1}{2} \mathfrak{S}^{+T} N \mathfrak{S}^{+} + \mathbb{l}^T \mathfrak{S}^{+}\right) |0\rangle$$

$$= \exp\left[\tfrac{1}{2} \mathbb{k}^T N \Delta \, \mathbb{k} + \tfrac{1}{2} \mathbb{l}^T \Delta M \, \mathbb{l} + \mathbb{l}^T \Delta \, \mathbb{k}\right]$$

$$\Delta = (1 + MN)^{-1}$$

[3] RESULT

$$|\tilde{A}\rangle_{1245} = \int \delta(1,2,4,5) \; e^{-\phi^a_{123} Y^a_{123}} \; e^{-\phi^a_{456} Y^a_{456}}$$

$$\times \; e^{F(1,2,4,5)} \, |0\rangle_{1245}$$

SURPRISE : LINEAR IN ϕ^a INSTEAD OF QUADRATIC

3. MST

$$\begin{array}{ccc} M & & S \\ \downarrow & & \circlearrowright \\ \mathrm{IIA} & \xrightarrow[T]{} & \mathrm{IIB} \end{array}$$

M(ATRIX) THEORY ⇔ LC QUANT. OF M
0+1 D SYM

⇓

MST ⇔ LC QUANT. OF IIB
1+1 D SYM

PARAMETERS $1/g_{YM}^2 = g_s^2 \alpha'$

MST vs. PERTURBATIVE STRING

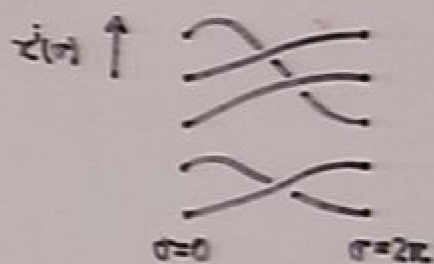
IR LIMIT ⇔ FREE STRING

LEAST IRREL. OP. ⇔ 1ST ORDER INTERACTION

FREE STRING (IR LIMIT)

$$S = \cdots + \frac{1}{g_s^2 \alpha'} [x^i(\sigma), x^j(\sigma)]^2 \quad \Rightarrow \quad [x^i(\sigma), x^j(\sigma)] = 0$$

$$\Rightarrow \quad X^i(\sigma) = U(\sigma)\, x^i(\sigma)\, U^{-1}(\sigma)$$



$$x^i(\sigma + 2\pi) = g\, x^i(\sigma)\, g^{-1} \qquad g \in S_N$$

$$H_0 = \sum_{n=1}^{N} \int_0^{2\pi} \frac{d\sigma}{2\pi} \left[(p_n^i)^2 + (x_n^{i\prime})^2 + \theta_n^a \theta_n^{a\prime} + \bar{\theta}_n^a \bar{\theta}_n^{a\prime} \right]$$

1st Order Interaction

- "Exchange"

$\sigma, \tau^i, \cdots$ Twist Fields for $x^i(z)$

$$\partial x^i(z) \cdot \sigma(0) \sim \frac{1}{\sqrt{z}} \tau^i(0)$$

$\Sigma^{\dot{a}}, \Sigma^i, \cdots$ Spin Fields for $\theta^a(z)$

$$\theta^a(z) \cdot \Sigma^{\dot{a}}(0) \sim \frac{\gamma^i_{a\dot{a}}}{\sqrt{z}} \Sigma^i(0)$$

- Dimension $(3/2, 3/2)$

$H = H_0 + g_s H_1 + \cdots$ Lorentz Scalar

$H_1 = \sqrt{\alpha'} \int d\sigma$ [Dim. 3 Op.]

$$\Rightarrow \sum_{m,n=1}^{N} \left(\tau^i \bar{\tau}^j \Sigma^i \bar{\Sigma}^j \right) \quad \text{[DVV]}$$

CONTENTS

4. Supercharges

LCSFT

$$|H_1\rangle_{123} = \mathrm{Z}^i \bar{\mathrm{Z}}^j v^{ij}(Y)\,|V\rangle_{123}$$

$$|Q_1^{\dot a}\rangle_{123} = \bar{\mathrm{Z}}^i s^{i\dot a}(Y)\,|V\rangle_{123}$$

$$|\tilde{Q}_1^{\dot a}\rangle_{123} = \mathrm{Z}^i \tilde{s}^{i\dot a}(Y)\,|V\rangle_{123}$$

$$(P^i + X^{i\prime})(\sigma)\,|V\rangle_{123} \sim \frac{1}{\sqrt{\sigma-\sigma_I}}\,\mathrm{Z}^i\,|V\rangle_{123}$$

MST

$$H_1 = \sqrt{\alpha'} \sum_{m,n} \int \frac{d\sigma}{2\pi} \left(\tau^i \bar{\tau}^j \Sigma^i \bar{\Sigma}^j\right)_{mn}$$

$$Q^{\dot a} =$$

$$\tilde{Q}^{\dot a} =$$

?

$$\partial x^i(z)\cdot\sigma(0) \sim \frac{1}{\sqrt{z}}\,\tau^i(0)$$

4. SUPERCHARGES

LCSFT

$$|H_1\rangle_{123} = Z^i \bar{Z}^j v^{ij}(Y)\, |V\rangle_{123}$$

$$|Q_1^{\dot a}\rangle_{123} = \bar{Z}^i\, s^{i\dot a}(Y)\, |V\rangle_{123}$$

$$|\tilde{Q}_1^{\dot a}\rangle_{123} = Z^i\, \tilde{s}^{i\dot a}(Y)\, |V\rangle_{123}$$

$$(P^i + X^{i\prime})(\sigma)\, |V\rangle_{123} \sim \frac{1}{\sqrt{\sigma-\sigma_I}}\, Z^i |V\rangle_{123}$$

MST

$$H_1 = \sqrt{\alpha'} \sum_{m,n} \int \frac{d\sigma}{2\pi} \left(\tau^i \bar{\tau}^j\, \Sigma^i \bar{\Sigma}^j\right)_{mn}$$

$$Q^{\dot a} = \quad ?$$

$$\tilde{Q}^{\dot a} =$$

$$\partial x^i(z) \cdot \sigma(0) \sim \frac{1}{\sqrt{z}}\, \tau^i(0)$$

$$Q_1^{\dot a} = \sqrt{\alpha'} \sum_{m,n} \int \frac{d\sigma}{2\pi} \left(\sigma \bar{\tau}^i \, \Sigma^{\dot a} \bar{\Sigma}^i \right)_{m,n}$$

$$\tilde{Q}_1^{\dot a} = \sqrt{\alpha'} \sum_{m,n} \int \frac{d\sigma}{2\pi} \left(\tau^i \bar{\sigma} \, \Sigma^i \bar{\Sigma}^{\dot a} \right)_{m,n}$$

✓ SUSY ALGEBRA AT $O(g_s^1)$

CONSISTENT WITH . . .

- PIONEER BUT PRIMITIVE SUSY ARGUMENT [DVV]

$$Q^{\dot a} = \sum_m \int \frac{d\sigma}{2\pi} \gamma^i_{a\dot a} \theta^a_m \partial x^i_m$$

$$\{ Q^{\dot a}, \sigma \Sigma^{\dot b} \} + \{ \sigma \Sigma^{\dot a}, Q^{\dot b} \} = 2 \tau^i \Sigma^i \delta^{\dot a \dot b}$$

$$[Q^{\dot a}, \tau^i \Sigma^i] = \partial (\sigma \Sigma^{\dot a})$$

- RELATION PROPOSED BY DIJKGRAAF-MOTL

$$S^{i\dot a} \leftrightarrow \Sigma^{\dot a} \bar{\Sigma}^i$$
$$\tilde{S}^{i\dot a} \leftrightarrow \Sigma^i \bar{\Sigma}^{\dot a}$$
$$V^{ji} \leftrightarrow \Sigma^i \bar{\Sigma}^j$$

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✓ SUSY ALGEBRA AT $O(g_s^2)$

CONSISTENT WITH . . .

- PIONEER BUT PRIMITIVE SUSY ARGUMENT [DVV]

$$Q^{\dot a} = \sum_m \int \frac{d\sigma}{2\pi} \gamma^i_{a\dot a} \theta^a_m \partial x^i_m$$

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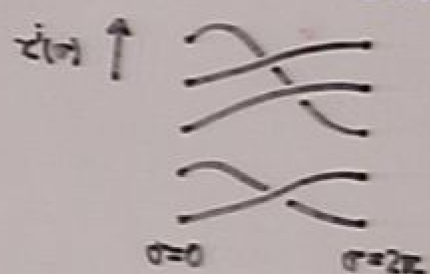
$$\Rightarrow X^i(\sigma) = U(\sigma)\, x^i(\sigma)\, U^{-1}(\sigma)$$



$$x^i(\sigma+2\pi) = g\, x^i(\sigma)\, g^{-1} \qquad g \in S_N$$

$$H_0 = \sum_{n=1}^{N} \int_0^{2\pi} \frac{d\sigma}{2\pi} \left[(p_n^i)^2 + (x_n^{i\prime})^2 + \theta_n^a \theta_n^{a\prime} + \bar{\theta}_n^a \bar{\theta}_n^{a\prime} \right]$$

5. OPEs

For Simplicity

Bosonic LCSFT vs. **"Bosonic" MST**

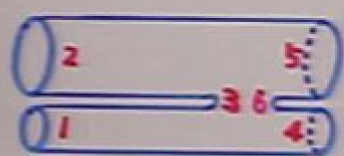
$$|H_1\rangle_{123} = |V\rangle_{123}$$

$$H_1 = \sqrt{\lambda} \sum_{m,n} \int \frac{d\sigma}{2\pi} \left(\sigma\bar{\sigma}\right)_{mn}$$

?

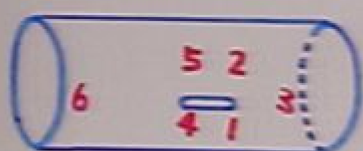
$$[\sigma\bar{\sigma}](z,\bar{z}) \cdot [\sigma\bar{\sigma}](0,0) \sim \frac{1}{|z|^{3/4}(\ln|z|)^{1/2}}$$

Tree



$$|A\rangle_{1245} = {}_{36}\langle R|\, e^{-T(\text{Prop.})} \times |V\rangle_{123}\, |V\rangle_{456}$$

1-Loop



$$|B\rangle_{36} = {}_{14}\langle R|\; {}_{25}\langle R|\, e^{-T(\text{Prop.})} \times |V\rangle_{123}\, |V\rangle_{456}$$

$\langle R|$ Reflector, Bra $\leftrightarrow$ Ket

RESULTS

$$|A\rangle_{12345} = \left[\frac{1}{T^{4}(\ln T)^{k}}\right]^{26} |R\rangle_{14} |R\rangle_{25}$$

$$|B\rangle_{36} = \left[\frac{1}{T^{4}(\ln T)^{k}}\right]^{26} |R\rangle_{36}$$

cf. AMPLITUDE

$$\int d\alpha \int dT \; {}_{36}\langle R| \, e^{-T(\mathrm{PROP.})} \, P\,|V\rangle_{123} \; P\,|V\rangle_{456}$$

$$P = \int \frac{d\theta}{2\pi} \, e^{i\theta(L_0 - \bar{L}_0)}$$ LEVEL MATCHING PROJ.

$$[\sigma\bar{\sigma}]_{nn} \leftrightarrow P\,|V\rangle$$

OR

$$\leftrightarrow |V\rangle \quad \checkmark$$

No Sum

Sample Calculations

$$a^{(34)} = \begin{pmatrix} a^{(3)} \\ a^{(4)} \end{pmatrix}$$

- $${}_{34}\langle R| = {}_{34}\langle 0|\, \exp\left\{ \frac{1}{2}\, a^{(34)T} \begin{pmatrix} & -1 \\ -1 & \end{pmatrix} a^{(34)} \right\}$$
- $$|V\rangle_{123}\, |V\rangle_{456} = \exp\left\{ \frac{1}{2}\, a^{(34)\dagger T} \begin{pmatrix} N^{3,3} & \\ & N^{3,3} \end{pmatrix} a^{(34)\dagger} \right.$$
$$+\, a^{(34)\dagger T} \begin{pmatrix} N^{3,R} & \\ & N^{3,R} \end{pmatrix} a^{(1256)\dagger}$$
$$\left. +\, \frac{1}{2}\, a^{(1256)\dagger T} \begin{pmatrix} N^{R,R} & \\ & N^{R,R} \end{pmatrix} a^{(1256)\dagger} \right\} |0\rangle_{123456}$$

$$N^{3,R} = \begin{pmatrix} N^{3,1} & N^{3,2} \end{pmatrix}$$

$$N^{R,R} = \begin{pmatrix} N^{1,1} & N^{1,2} \\ N^{2,1} & N^{2,2} \end{pmatrix}$$

▷ Formula

$$\langle 0|\, \exp\left(\frac{1}{2}\, a^{T} M a\right) \times \exp\left(\frac{1}{2}\, a^{\dagger T} N a^{\dagger} + a^{\dagger T} \ell\right) |0\rangle$$
$$= \left[\det(1 - MN)\right]^{-1/2} \exp\left(\frac{1}{2}\, \ell^{T} \frac{1}{1 - MN} M \ell\right)$$

$$\Rightarrow\ {}_{34}\langle R|\, |V\rangle_{123}\, |V\rangle_{456} = \exp\left(\frac{1}{2}\, a^{(1256)\dagger T} N' a^{(1256)\dagger}\right) |0\rangle_{1256}$$

$$N' = \begin{pmatrix} N^{R,R} & \\ & N^{R,R} \end{pmatrix} + \begin{pmatrix} N^{R,3} & \\ & N^{R,3} \end{pmatrix} \frac{1}{\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} - \begin{pmatrix} & -1 \\ -1 & \end{pmatrix} \begin{pmatrix} N^{3,3} & \\ & N^{3,3} \end{pmatrix}} \begin{pmatrix} & -1 \\ -1 & \end{pmatrix} \begin{pmatrix} N^{3,R} & \\ & N^{3,R} \end{pmatrix}$$

$$= \begin{pmatrix} & -1 \\ -1 & \end{pmatrix}$$

$$N^{3,3} N^{3,3} + N^{3,R} N^{R,3} = 1$$
$$N^{3,3} N^{3,R} + N^{3,R} N^{R,R} = 0$$

⇧

Unitarity

Super LCSFT/MST

- Various OPEs
- Prefactors

Ex.

OPE: $\Sigma^i(z)\,\bar{\Sigma}^j(\bar{z}) \cdot \Sigma^k(0)\,\bar{\Sigma}^l(0) \sim \dfrac{\delta^{ik}\delta^{jl}}{|z|^2}$

Tree

$$|A\rangle_{12456} = {}_{36}\langle R|\, e^{-T(\mathrm{Prop.})}$$

$$\times\, [\cosh \chi_{123}]^{ij}\, |V\rangle_{123}\, [\cosh \chi_{456}]^{kl}\, |V\rangle_{456}$$

[1] Fourier Transformation

$$[\cosh \mathcal{X}]^{ij} = \int d^a\phi \; e^{-\phi^a Y^a} \, [\cosh \phi]^{ji}$$

$$\Rightarrow \quad |\tilde{A}\rangle_{1245} = {}_{36}\langle R| \; e^{-T(\text{PROP})}$$

$$\times \; e^{-\phi^a_{123} Y^a_{123}} \, |V\rangle_{123} \; e^{-\phi^a_{456} Y^a_{456}} \, |V\rangle_{456}$$

[2] Formula

$$\langle 0| \exp\left(\tfrac{1}{2} \xi^T M \xi + k^T \xi\right) \times \exp\left(\tfrac{1}{2} \xi^{\dagger T} N \xi^{\dagger} + \psi^T \xi^{\dagger}\right) |0\rangle$$

$$= \exp\left[\tfrac{1}{2} k^T N \Delta k + \tfrac{1}{2} \psi^T \Delta M \psi + \psi^T \Delta k\right]$$

$$\Delta = (1 + MN)^{-1}$$

[3] Result

$$|\tilde{A}\rangle_{1245} = \int \delta(1,2,4,5) \; e^{-\phi^a_{123} Y^a_{123}} \; e^{-\phi^a_{456} Y^a_{456}}$$

$$\times \; e^{F(1,2,4,5)} \, |0\rangle_{1245}$$

Surprise: Linear in ϕ^a, Instead of Quadratic

$$\langle R| = \langle 0| \exp\left(\xi^{I T} R \, \xi^{I}\right)$$

$$|V\rangle = \exp\left(\xi^{I \dagger T} N \, \xi^{I}\right) |0\rangle$$

SUPER LCSFT/MST

- VARIOUS OPEs
- PREFACTORS

Ex.

OPE: $\Sigma_1^i(z)\,\bar{\Sigma}_1^j(\bar{z}) \cdot \Sigma_1^k(0)\,\bar{\Sigma}_1^l(0) \sim \frac{\delta^{ik}\delta^{jl}}{|z|^2}$

TREE

$$|A\rangle_{123456} = {}_{36}\langle R|\ e^{-T(\mathrm{PROP.})}$$

$$\times\ [\cosh \chi_{123}]^{\bar{ij}}\ |V\rangle_{123}\ [\cosh \chi_{456}]^{kl}\ |V\rangle_{456}$$

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$$[\cosh \not{X}]^{ij} = \int d^a\phi \; e^{-\phi^a Y^a} [\cosh \not{\phi}]^{ji}$$

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$$\times \; e^{-\phi^a_{123} Y^a_{123}} |V\rangle_{123} \; e^{-\phi^a_{456} Y^a_{456}} |V\rangle_{456}$$

[2] Formula

$$\langle 0| \exp\left(\tfrac{1}{2} S^T M S + k^T S\right) \times \exp\left(\tfrac{1}{2} S^{+T} N S^+ + \ell^T S^+\right) |0\rangle$$

$$= \exp\left[\tfrac{1}{2} k^T N \Delta k + \tfrac{1}{2} \ell^T \Delta M \ell + \ell^T \Delta k\right]$$

$$\Delta = (1 + MN)^{-1}$$

[3] Result

$$|\tilde{A}\rangle_{1245} = \int \delta(1,2,4,5) \; e^{-\phi^a_{123} Y^a_{123}} \; e^{-\phi^a_{456} Y^a_{456}}$$

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$$\langle R| = \langle 0| \exp\left(S^{I T} R S^I\right)$$

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$$Y = N^T S^I$$

[4] Inverse Fourier Transformation

$$|A\rangle_{1245} = \int \delta(1,2,4,5)\, [\cosh \Psi_{123}]^{ij}\, [\cosh \Psi_{456}]^{kl}$$

$$\times\ e^{F(1,2,4,5)}\ |0\rangle_{1245}$$

[5] Short Time Limit

$$\Psi^a_{123} \sim -\Psi^a_{456} \sim \Psi^a$$

$$[\cosh \Psi_{123}]^{ij} [\cosh \Psi_{456}]^{kl} \sim [\cosh \Psi]^{ij} [\cosh \Psi]^{kl}$$

[6] Fierz Identity

$$M_{AB}\ N_{CD} = \sum_{m=0}^{8} \frac{1}{m!}\ \hat\gamma^{a_1\cdots a_m}_{AD}\ (N\, \hat\gamma^{a_m\cdots a_1} M)_{CB}$$

$$[\cosh \Psi]^{ij} [\cosh \Psi]^{kl} = \delta_{ik}(\cdots)_{lj} + (\hat\gamma^2)_{ik}(\cdots)_{lj} + \cdots$$

Most Singular

[7] Finally

$$|A\rangle_{1245} \sim \frac{\delta_{ik}\delta_{jl}}{T^2}\ |R\rangle_{14}\ |R\rangle_{25}$$

Super LCSFT/MST

- Various OPEs
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Ex.

OPE: $\Sigma^{i}_{1}(z)\,\bar{\Sigma}^{j}_{1}(\bar{z}) \cdot \Sigma^{k}_{1}(0)\,\bar{\Sigma}^{l}_{1}(0) \sim \frac{\delta^{ik}\delta^{jl}}{|z|^2}$

$| \quad R| \; e^{-T(\mathrm{PROP})}$

$[\cosh X_{123}]^{ij}\, |V\rangle_{123}\, [\cosh X_{456}]^{kl}\, |V\rangle_{456}$

[4] Inverse Fourier Transformation

$$|A\rangle_{1245} = \int \delta(1,2,4,5)\,[\cosh \not\psi_{123}]^{ij}\,[\cosh \not\psi_{456}]^{kl}$$

$$\times\, e^{F(1,2,4,5)}\,|0\rangle_{1245}$$

[5] Short Time Limit

$$\not\psi^{a}_{123} \sim -\not\psi^{a}_{456} \sim \not\psi^{a}$$

$$[\cosh \not\psi_{123}]^{ij}\,[\cosh \not\psi_{456}]^{kl} \sim [\cosh \not\psi]^{ij}\,[\cosh \not\psi]^{kl}$$

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$\Uparrow$ Most Singular

[7] Finally

$$|A\rangle_{1245} \sim \frac{\delta_{ik}\delta_{jl}}{T^{2}}\,|R\rangle_{14}\,|R\rangle_{25}$$

Summary

LCSFT ↔ MST

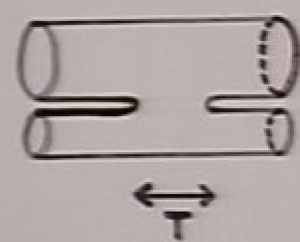
- OPEs Realized by Interaction Vertex
- LCSFT, Of/By/For People !!

Further Directions

- Higher Order Contact Terms ?
- PP-wave LCSFT

FIRST SIGN !!

ACCORDING TO KISHIMOTO-MATSUO-WATANABE



$\sim \frac{1}{T^6}$

USING LCSFT

MATRIX STRING INTERPRETATION:

$$[\sigma\bar{\sigma}](z,\bar{z}) \cdot [\sigma\bar{\sigma}](0,0) \sim \frac{1}{|z|^6}$$

$$6 = \left(\frac{1}{16}+\frac{1}{16}\right) \times (26-2) \times 2$$

MORE DETAILS ...