Title: Dark Matter, Dark Energy, or Worse?

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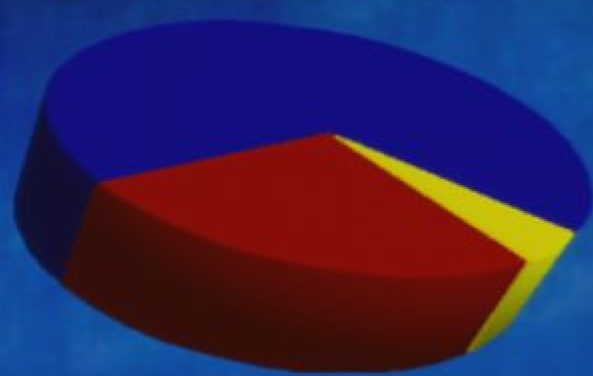
Abstract: tba

Beyond Dark Matter and Dark Energy

Sean Carroll
Caltech



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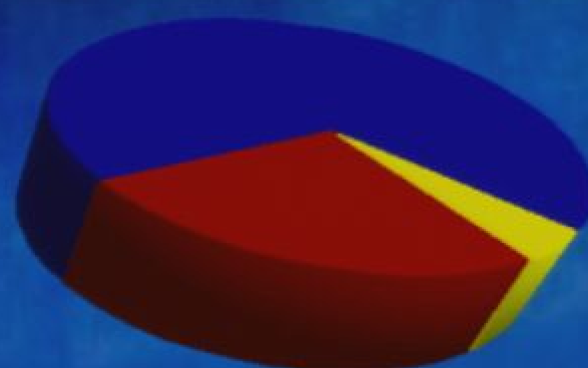
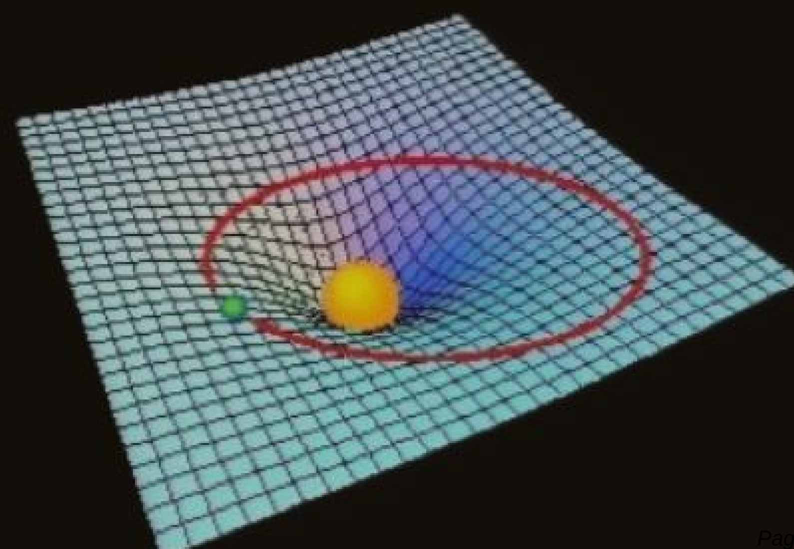


General relativity:
gravity is the curvature of spacetime



Spacetime geometry is described by the metric $g_{\mu\nu}$. The curvature scalar $R[g_{\mu\nu}]$ is the most basic scalar quantity characterizing the curvature of spacetime at each point. The simplest action possible is thus

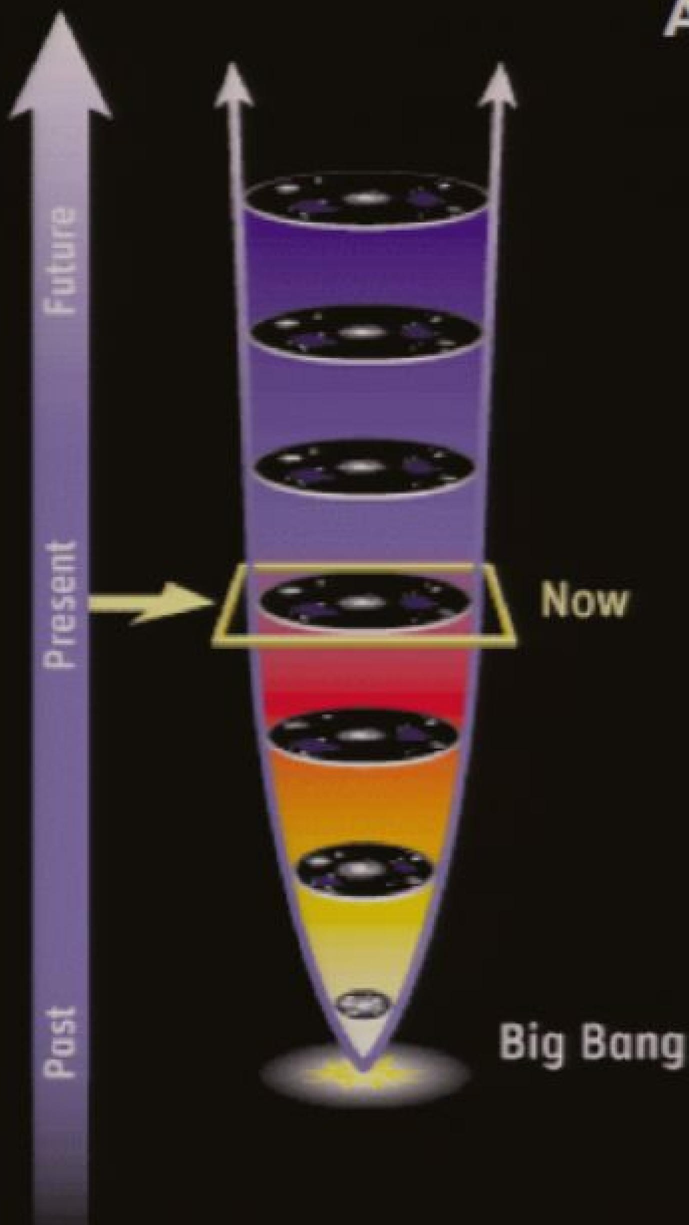
$$S = \frac{1}{16\pi G} \int R \, d^4x + S_{(\mathrm{matter})}$$

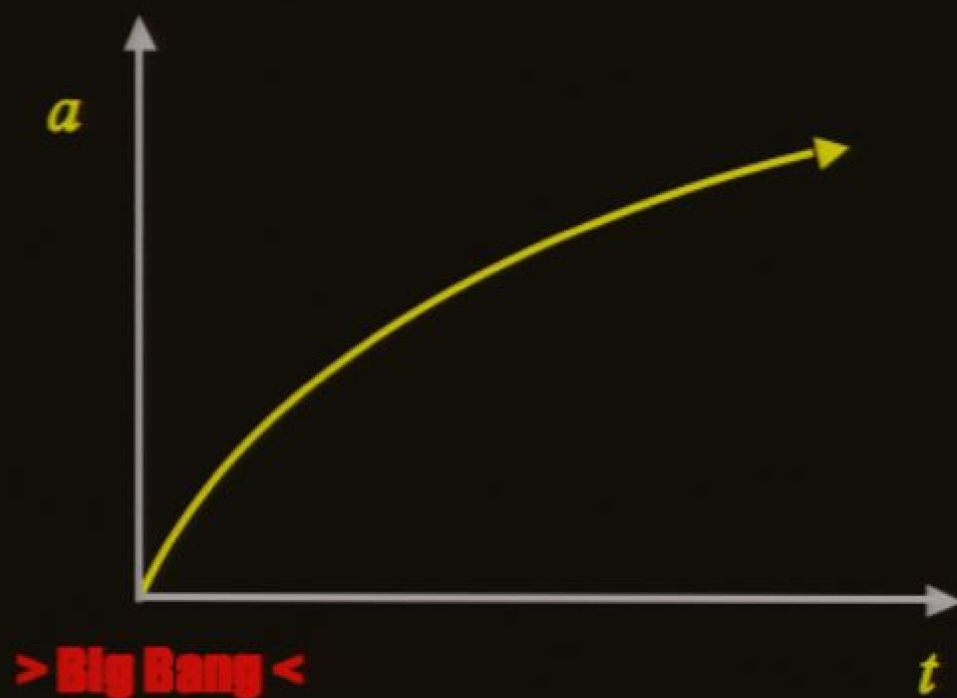
Varying with respect to $g_{\mu\nu}$ gives Einstein's equation:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}^{(\mathrm{matter})}$$

$G_{\mu\nu}$ is the Einstein tensor, characterizing curvature, and $T_{\mu\nu}$ is the energy-momentum tensor of matter.

Apply GR to the whole universe:
uniform (homogeneous and isotropic) space expanding as a function of time.

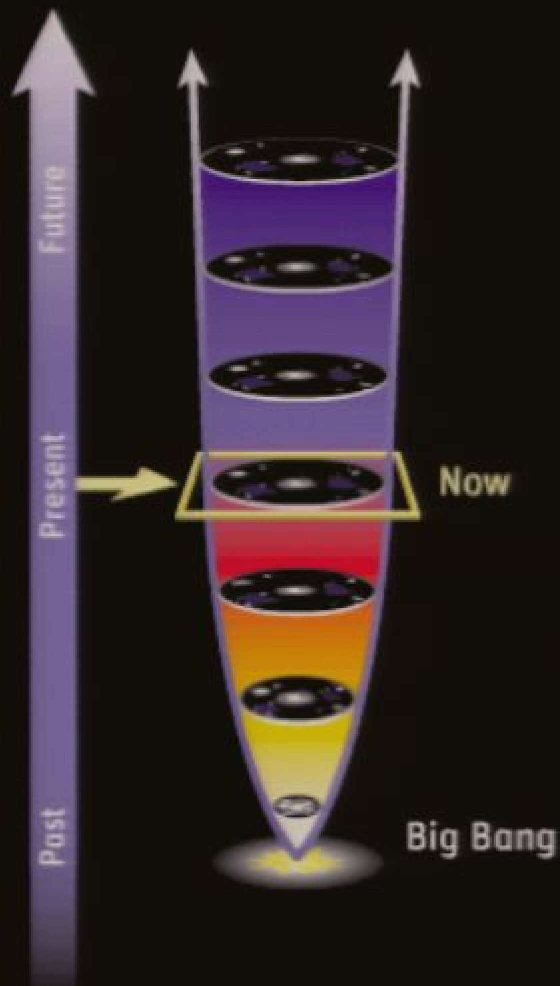


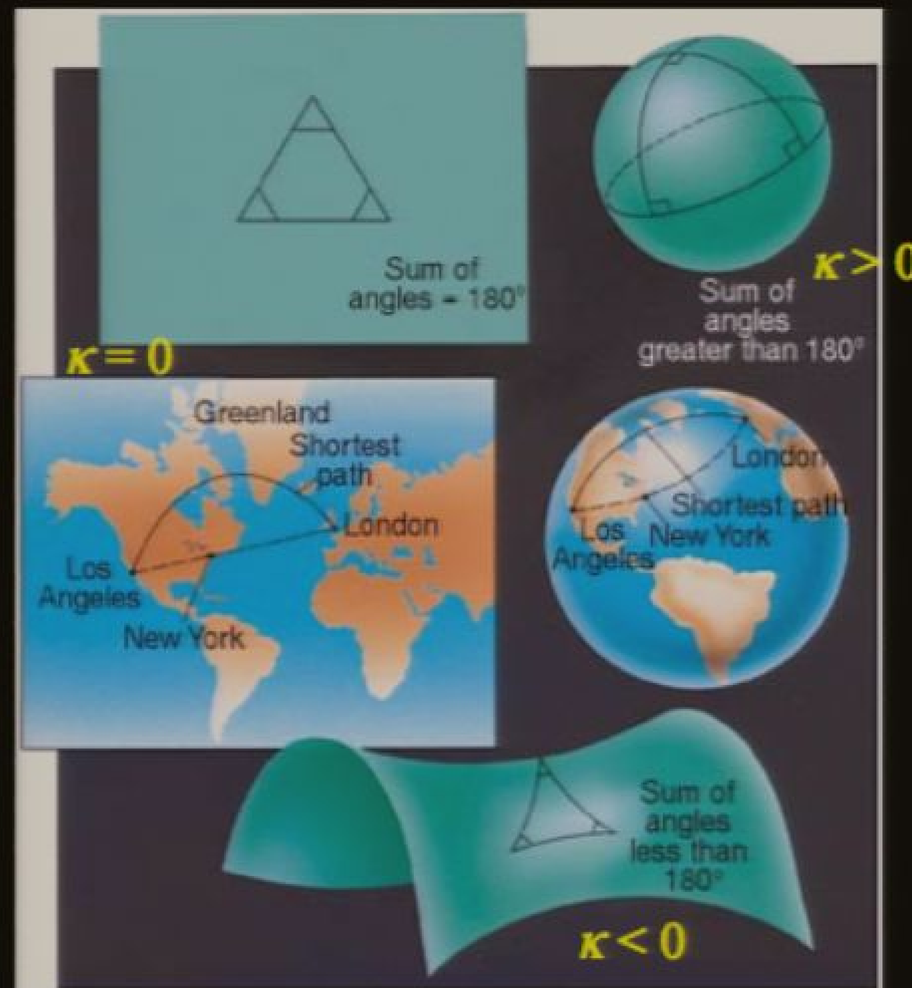


Relative size at different times is measured by the scale factor $a(t)$.

[Sky & Telescope]

Applied to a smooth, expanding universe, the curvature of spacetime gets two contributions.





Expansion rate $H=\dot{a}/a$

Spatial curvature κ

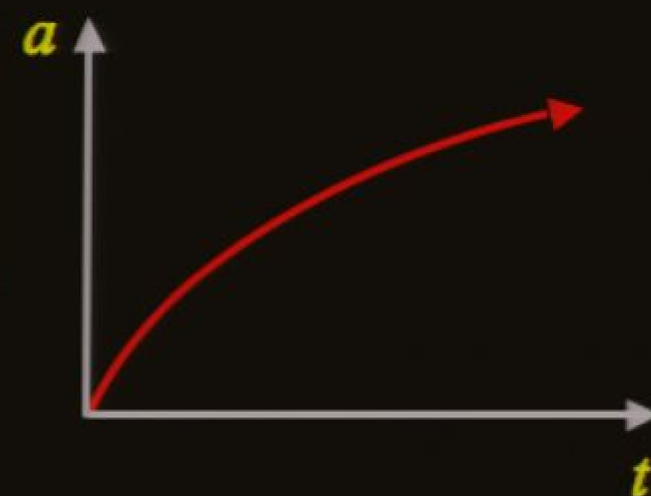
We can use Einstein's equation to relate the expansion of the universe to spatial curvature and the energy density.

spacetime curvature $$G_{\mu\nu} = 8\pi G T_{\mu\nu}^{(\mathrm{matter})}$$ *energy and momentum*

Applied to cosmology, this gives the Friedmann equation:

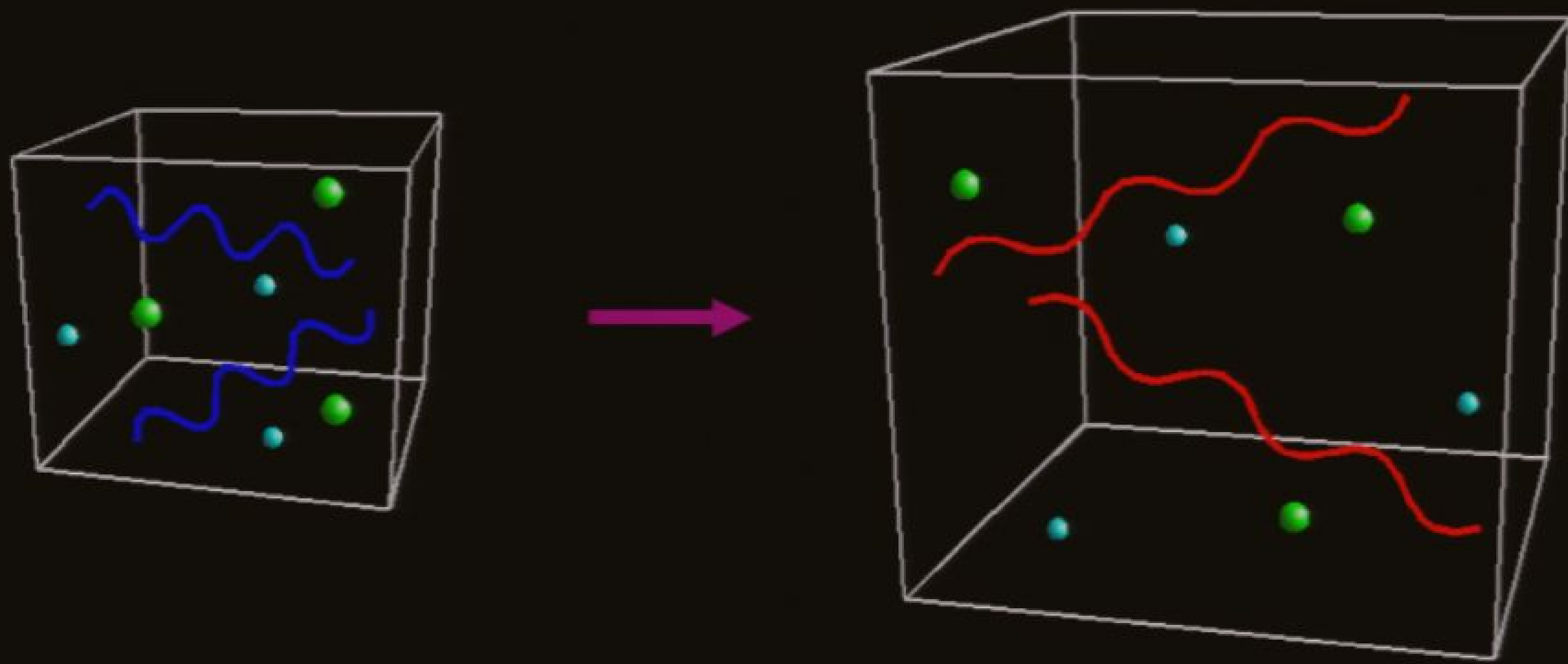
$$\boxed{H^2 + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\rho(a)}$$

expansion rate *curvature of space* *energy density*



If we know κ, and ρ as a function of a, we can solve for the expansion history $a(t)$.

Expansion dilutes matter (cold particles) and redshifts radiation.



So the energy density in matter simply goes down inversely with the increase in volume: $\rho_{\rm M} \propto a^{-3}$

And the energy density in radiation diminishes more quickly as each photon loses energy: $\rho_{\rm R} \propto a^{-4}$

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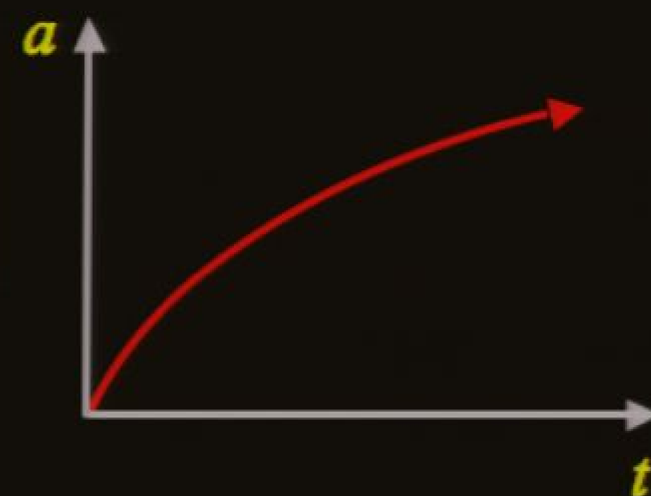

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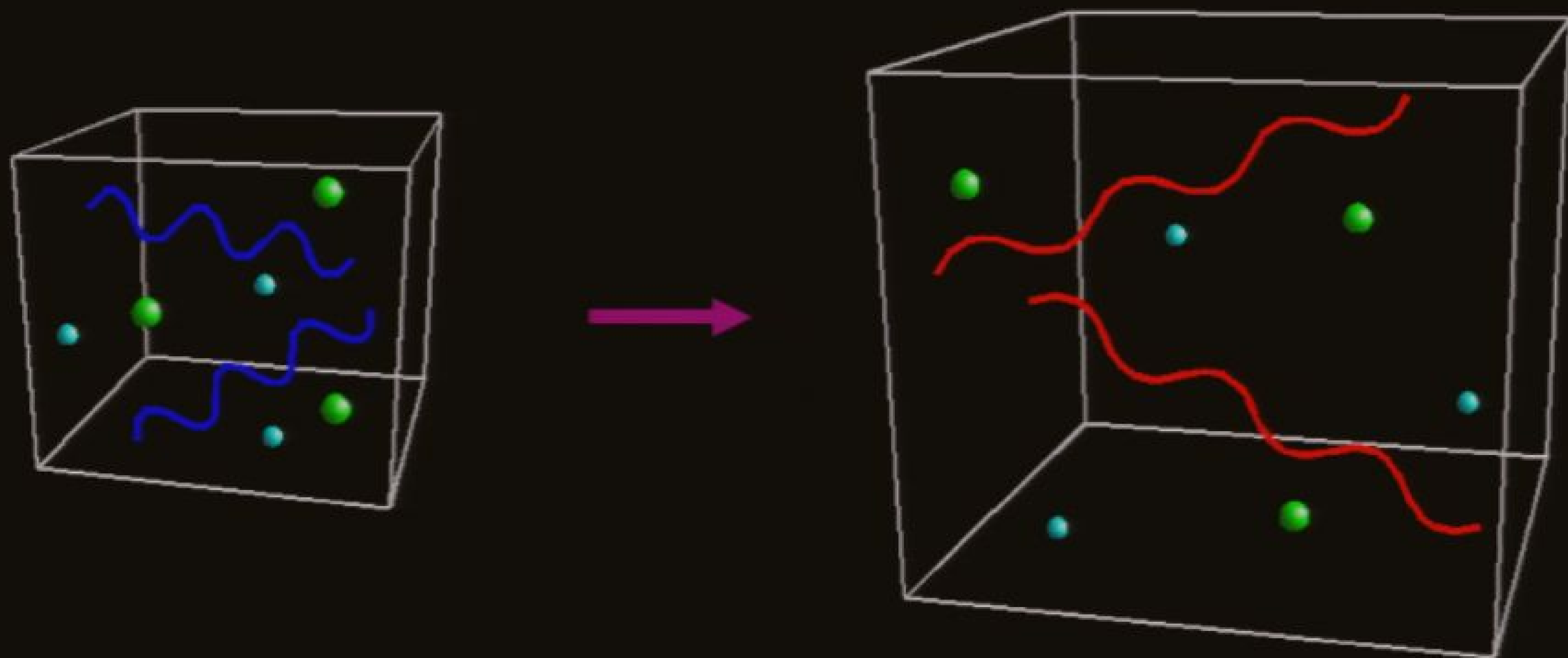


So the energy density in matter simply goes down inversely with the increase in volume: $\rho_{\mathrm{M}} \propto a^{-3}$

And the energy density in radiation diminishes more quickly as each photon loses energy: $\rho_{\mathrm{R}} \propto a^{-4}$

Some matter is “ordinary” -- protons, neutrons, electrons, for that matter any of the particles of the Standard Model. But much of it is dark.

We can detect dark matter through its gravitational field – e.g. through gravitational lensing of background galaxies by clusters.

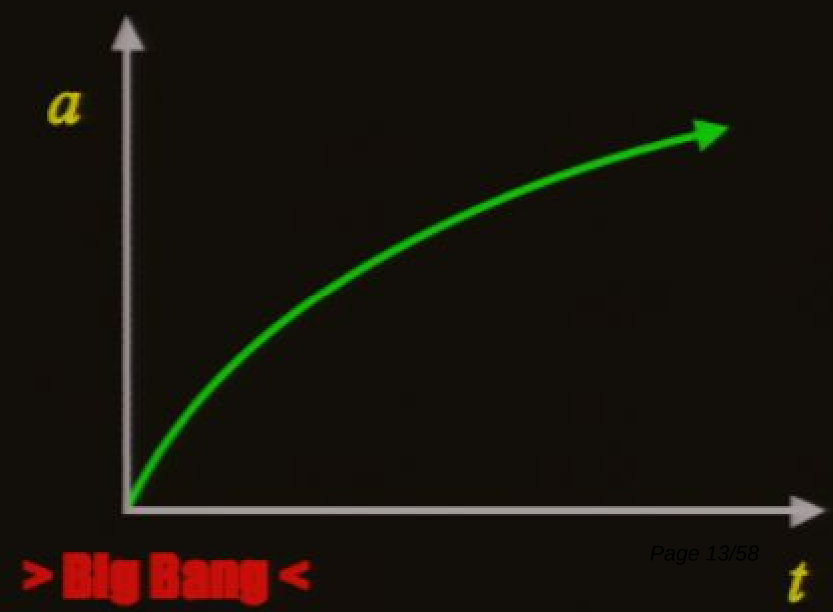
Whatever the dark matter is, it's not a particle we've discovered – it's something new.

The Friedmann equation with matter and radiation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{\rho_{\mathrm{M0}}}{a^3} + \frac{\rho_{\mathrm{R0}}}{a^4}\right) - \frac{\kappa}{a^2}$$

Multiply by a^2 to get: $\dot{a}^2 \propto \frac{\rho_{\mathrm{M0}}}{a} + \frac{\rho_{\mathrm{R0}}}{a^2} + \mathrm{const}$

If a is *increasing*, each term on the right is *decreasing*; we therefore predict the universe should be decelerating ($\dot{a}$ decreasing).

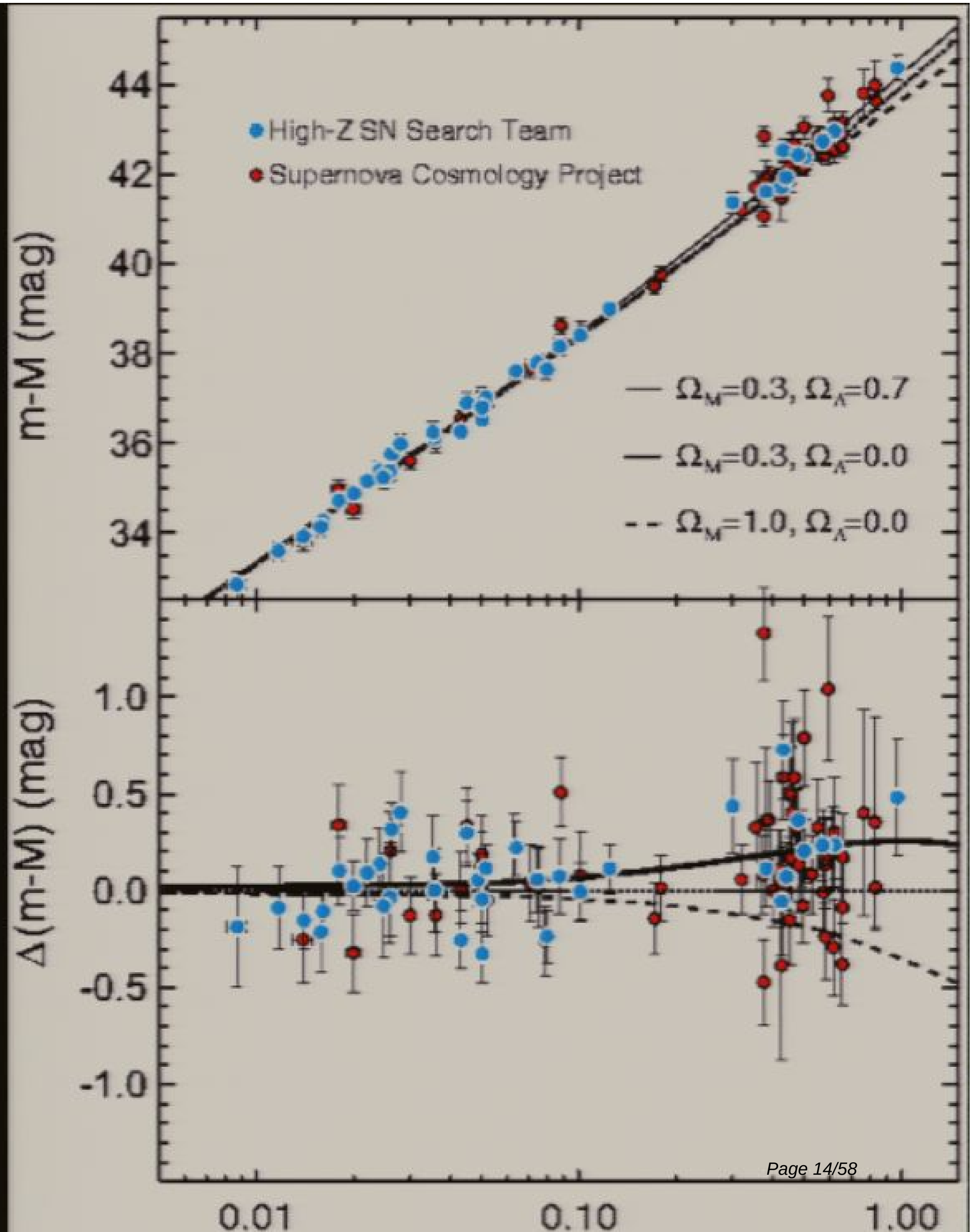


But it isn't.

Type Ia supernovae are standardizable candles; observations of many at high redshift test the time evolution of the expansion rate.

Result: the universe is <u>accelerating</u>!

There seems to be a sort of energy density which <u>doesn't</u> decay away: "dark energy."



[Riess et al. 1998; Perlmutter et al. 1998]

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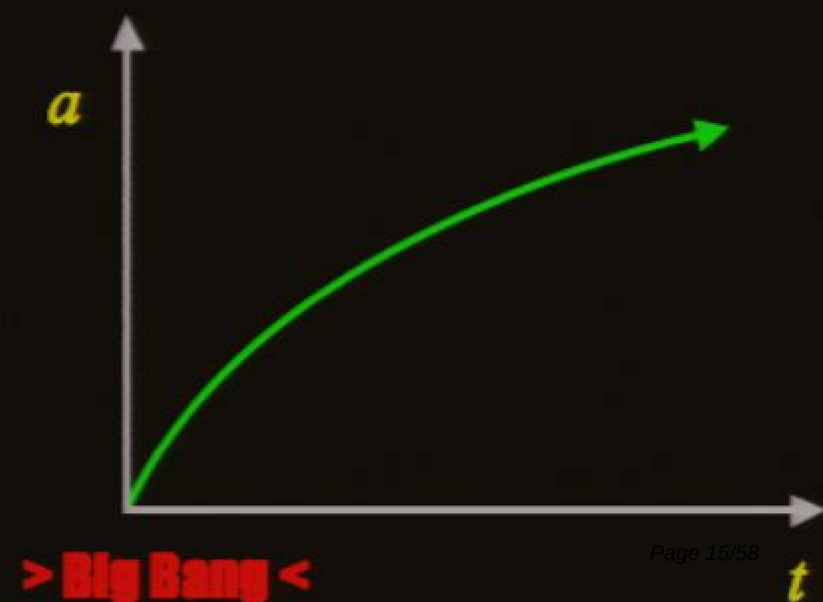

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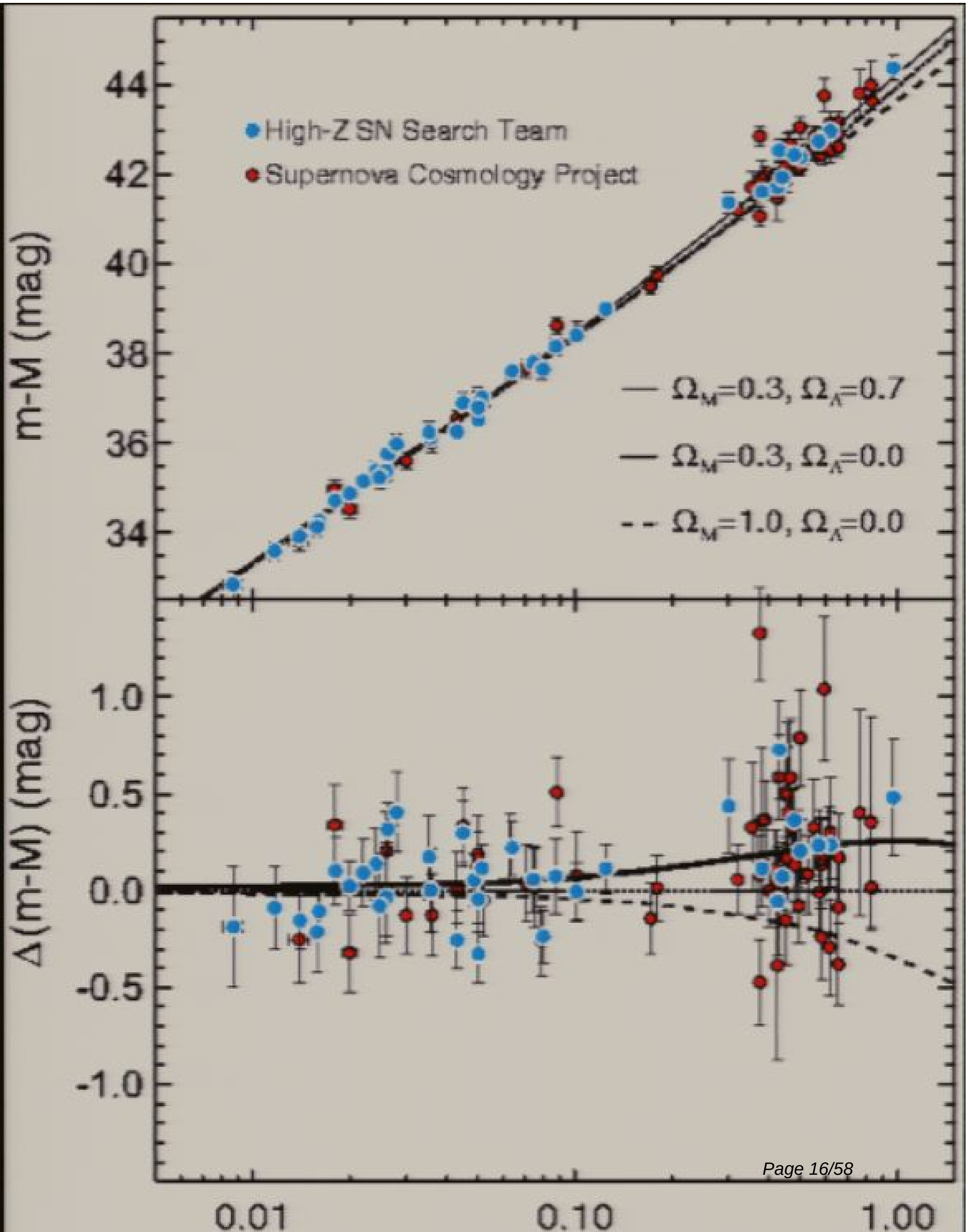
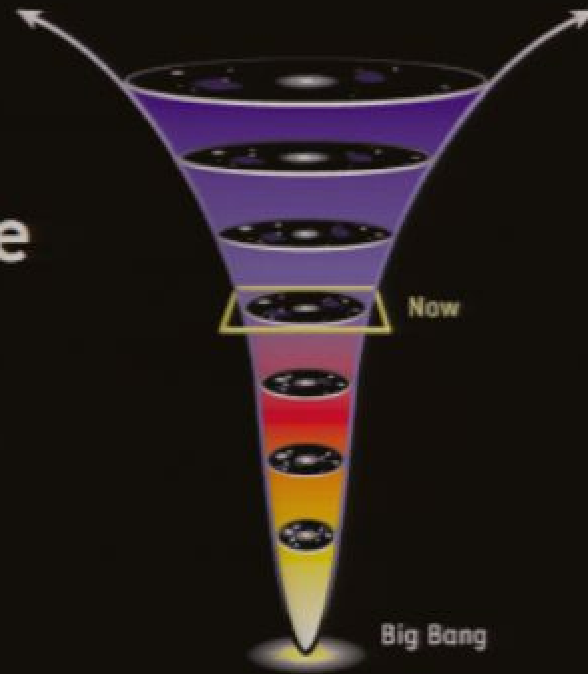


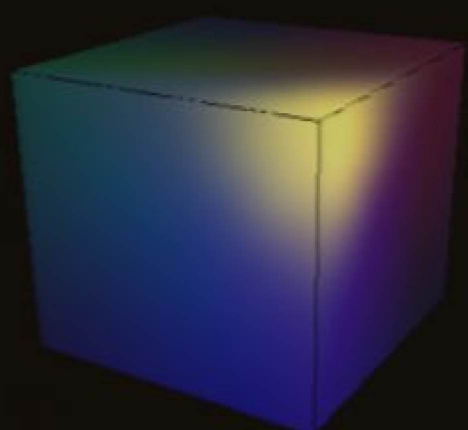
[Riess et al. 1998; Perlmutter et al. 1998]

Dark Energy is characterized by:

- smoothly distributed through space
- negative pressure, $w = p/\rho \approx -1$.
- varies slowly (if at all) with time

$$\rho \sim a^{-3(1+w)} \approx \text{constant}$$



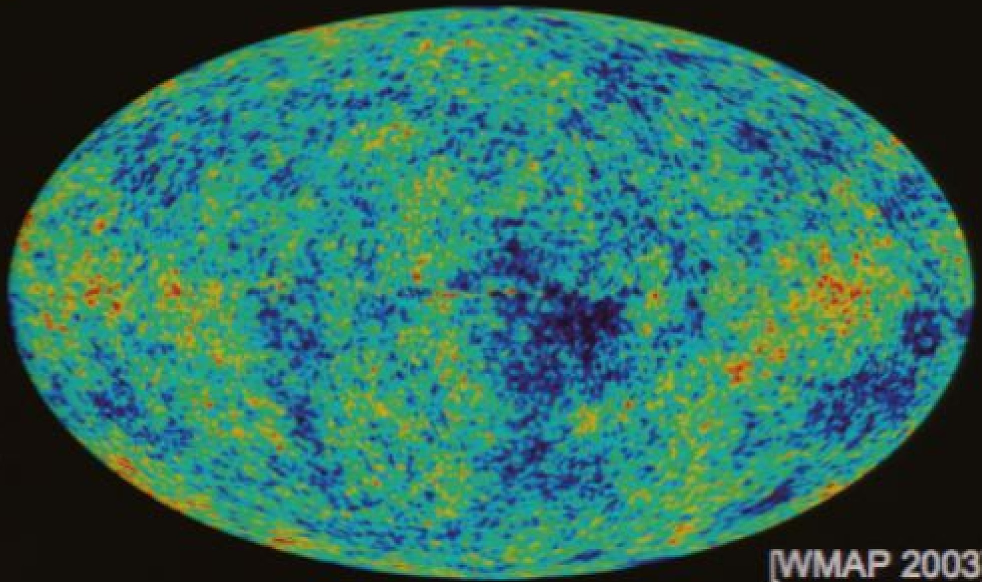


(artist's impression of vacuum energy)

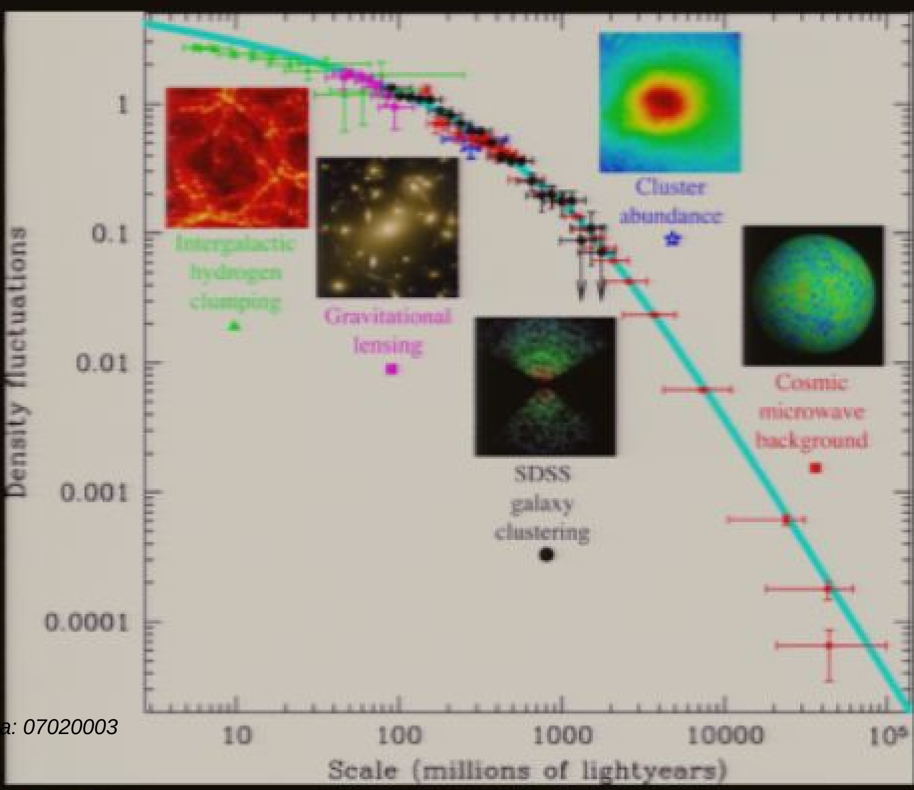
Dark energy could be exactly constant through space and time: vacuum energy (i.e. the cosmological constant Λ). Or it could be dynamical (quintessence, etc.).

Consistency Checks

Fluctuations in the Cosmic Microwave Background peak at a characteristic length scale of 370,000 light years; observing the corresponding angular scale measures the geometry of space.

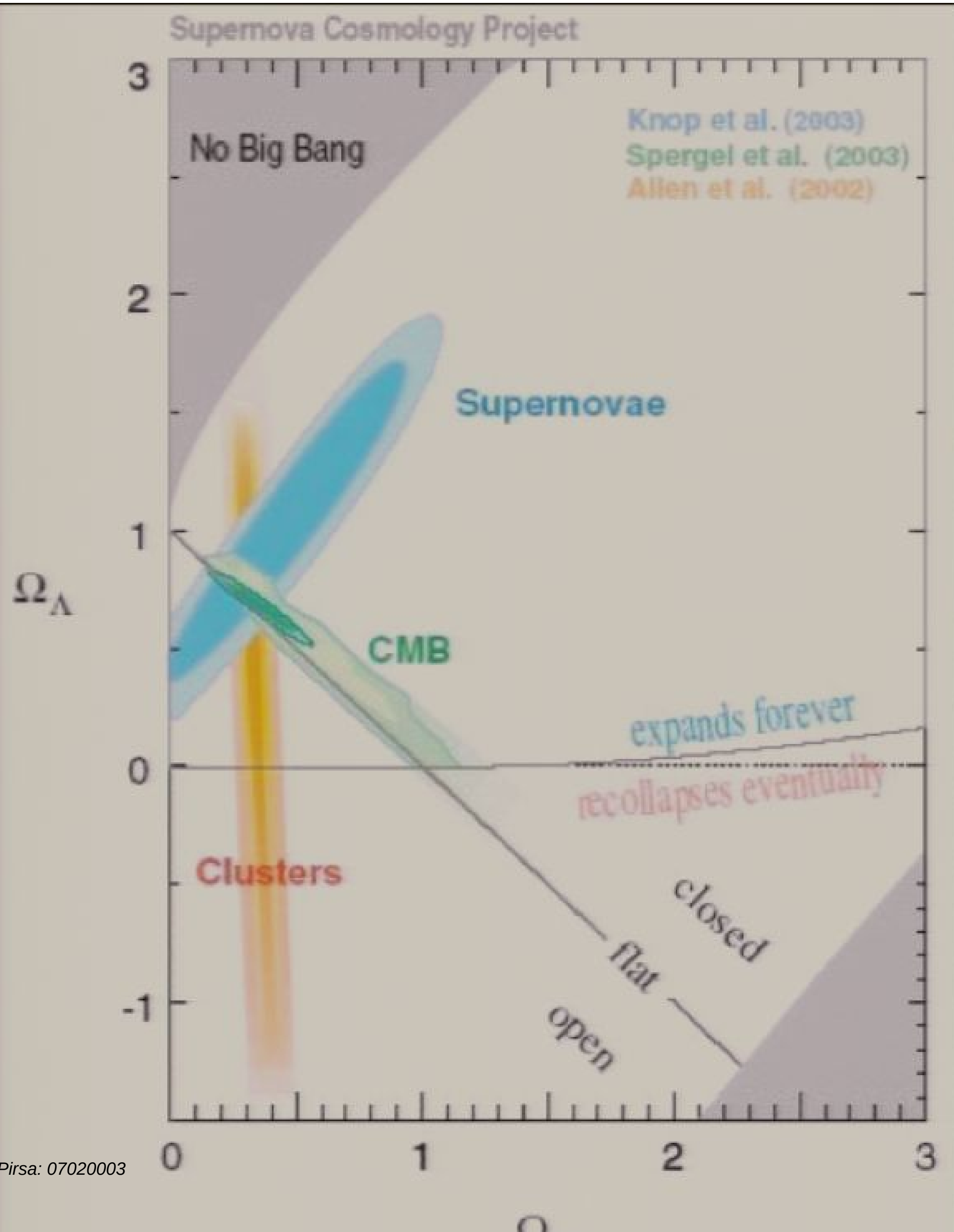


[WMAP 2003]



Evolution of large-scale structure from small early perturbations to today depends on expansion history of the universe.

Results: need for dark energy confirmed.

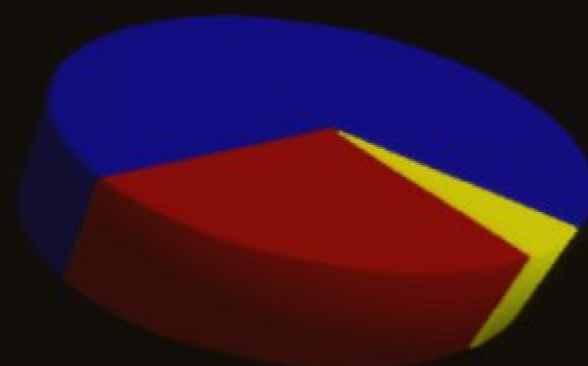


Concordance:

5% Ordinary Matter

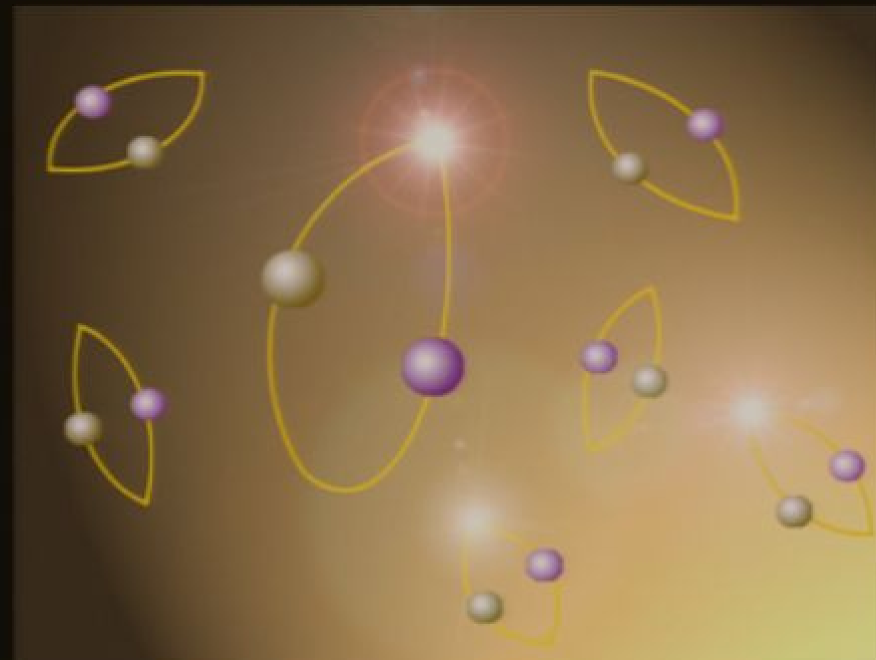
25% Dark Matter

70% Dark Energy



But:
this universe has issues.

One issue: why is the vacuum energy so small?



We know that virtual particles couple to photons (e.g. Lamb shift); why not to gravity?

Naively: $\rho_{\mathrm{vac}} = \infty$, or at least $\rho_{\mathrm{vac}} = E_{\mathrm{Pl}}/L_{\mathrm{Pl}}^{3} = 10^{120}\,\rho_{\mathrm{vac}}^{(\mathrm{obs})}$.

Could gravity be the culprit?

We infer the existence of dark matter and dark energy. Could it be a problem with general relativity? (Sure.)

Field theories (like GR) are characterized by :

- ✓ Degrees of Freedom (vibrational modes) -- number, spin.
- ✓ Propagation (massive/Yukawa, massless/Coulomb, etc).
- ✓ Interactions (coupling to other fields & themselves).

Inventing a new theory means specifying these things.

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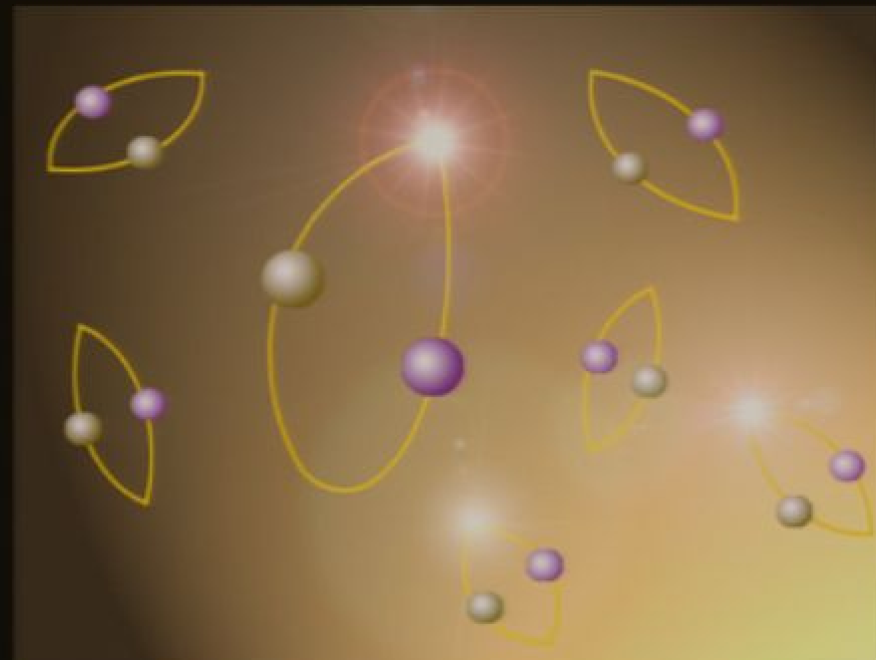

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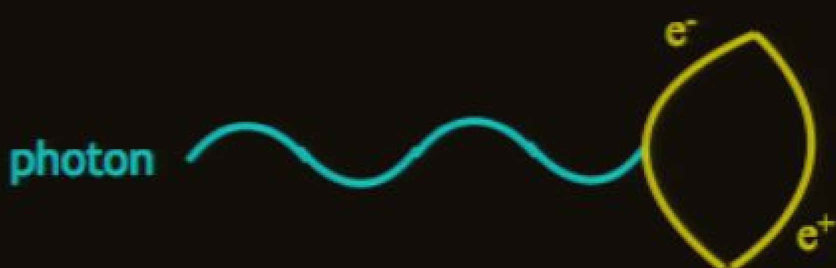

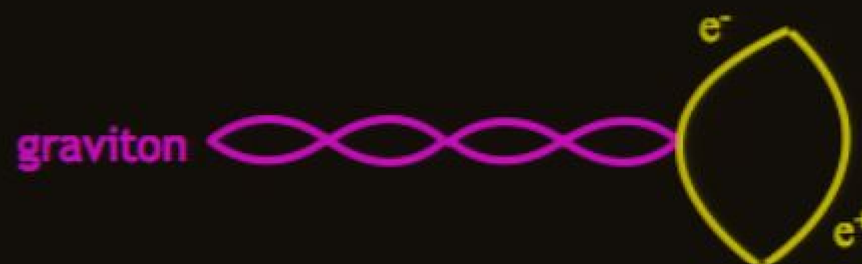

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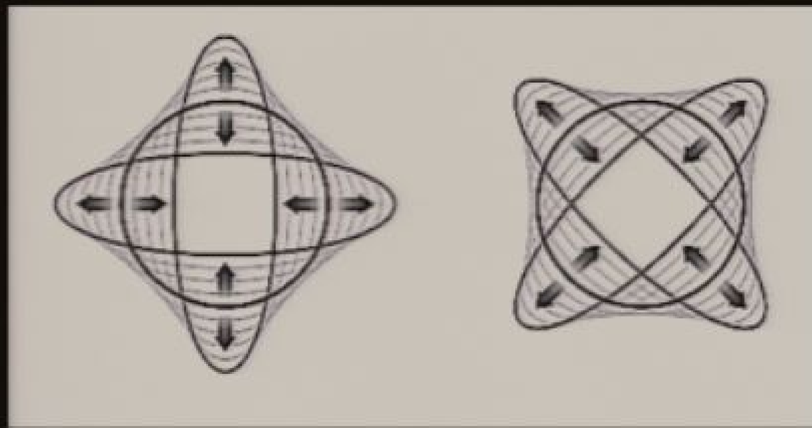
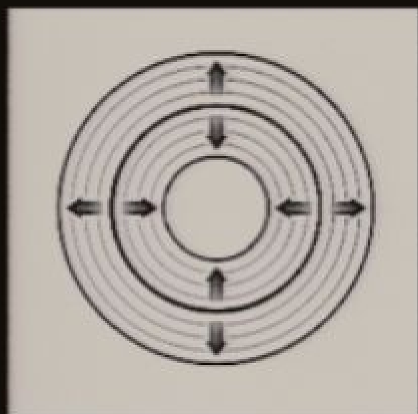
For example, in GR we have the graviton, which is:

- spin-2
- massless
- coupled to $T_{\mu\nu}$



A scalar (spin-0) graviton would look like this:



Scalar-Tensor Gravity

Introduce a **scalar field $\phi(x)$** that determines the strength of gravity. Einstein's equation

$$G_{\mu\nu} = 8\pi G T^{(\mathrm{m})}_{\mu\nu}$$

is replaced by

$$G_{\mu\nu} = f(\phi)\left[T^{(\mathrm{m})}_{\mu\nu} + T^{(\phi)}_{\mu\nu}\right]$$

variable "Newton's constant" extra energy-momentum from ϕ

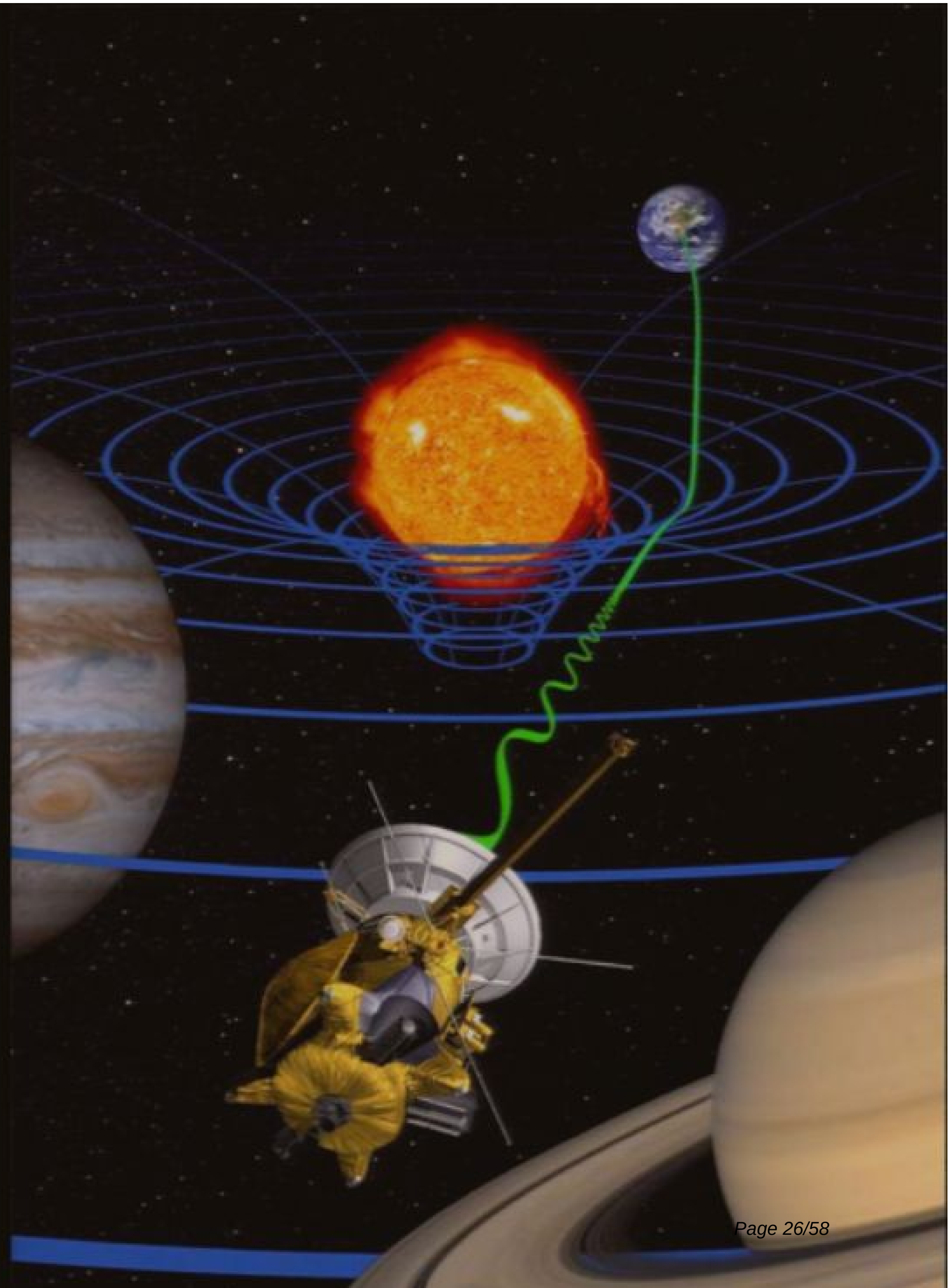
The new field $\phi(x)$ is an extra degree of freedom; an independently-propagating scalar particle.

The new scalar ϕ is sourced by planets and the Sun, distorting the metric away from Schwarzschild. It can be tested many ways, e.g. from the time delay of signals from the Cassini mission.

Experiments constrain the "Brans-Dicke parameter" ω to be

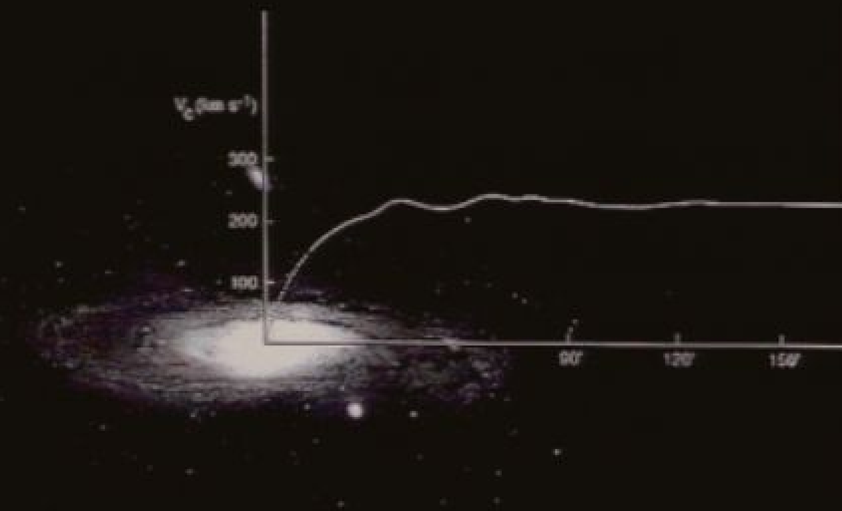
$$\omega > 40{,}000 \ ,$$

where $\omega = \infty$ is GR.



Modified Newtonian Dynamics -- MOND

Milgrom (1984) noticed a remarkable fact: dark matter is only needed in galaxies once the acceleration due to gravity dips below $a_0 = 10^{-8}\ \mathrm{cm/s^2} \sim cH_0$.



He proposed a phenomenological force law, MOND, in which gravity falls off more slowly when it's weaker:

$$F \propto \begin{cases} 1/r^2, & a > a_0, \\ 1/r, & a < a_0. \end{cases}$$

Bekenstein (2004) introduced *TeVeS*, a relativistic version of MOND featuring the metric, a fixed-norm vector U_μ, scalar field ϕ, and Lagrange multipliers η and λ:

$$S = \frac{1}{16\pi G}\int d^4x\,(R + \mathcal{L}_U + \mathcal{L}_\phi)$$

where

$$\mathcal{L}_U = -\frac{1}{2}KF^{\mu\nu}F_{\mu\nu} + \lambda(g^{\mu\nu}U_\mu U_\nu + 1)$$

$$\mathcal{L}_\phi = -\mu_0\eta(g^{\mu\nu} - U^\mu U^\nu)\partial_\mu\phi\partial_\nu\phi - V(\eta)$$

$$V(\eta) = \frac{3\mu_0}{128\pi l_B^2}\left[\eta(4 + 2\eta - 4\eta^2 + \eta^3) + 2\ln^2(\eta - 1)\right]$$

Not something you'd stumble upon by accident.

Bullet Cluster

[Clowe et al.]

Bullet Cluster

Bullet Cluster

Bullet Cluster

Bullet Cluster

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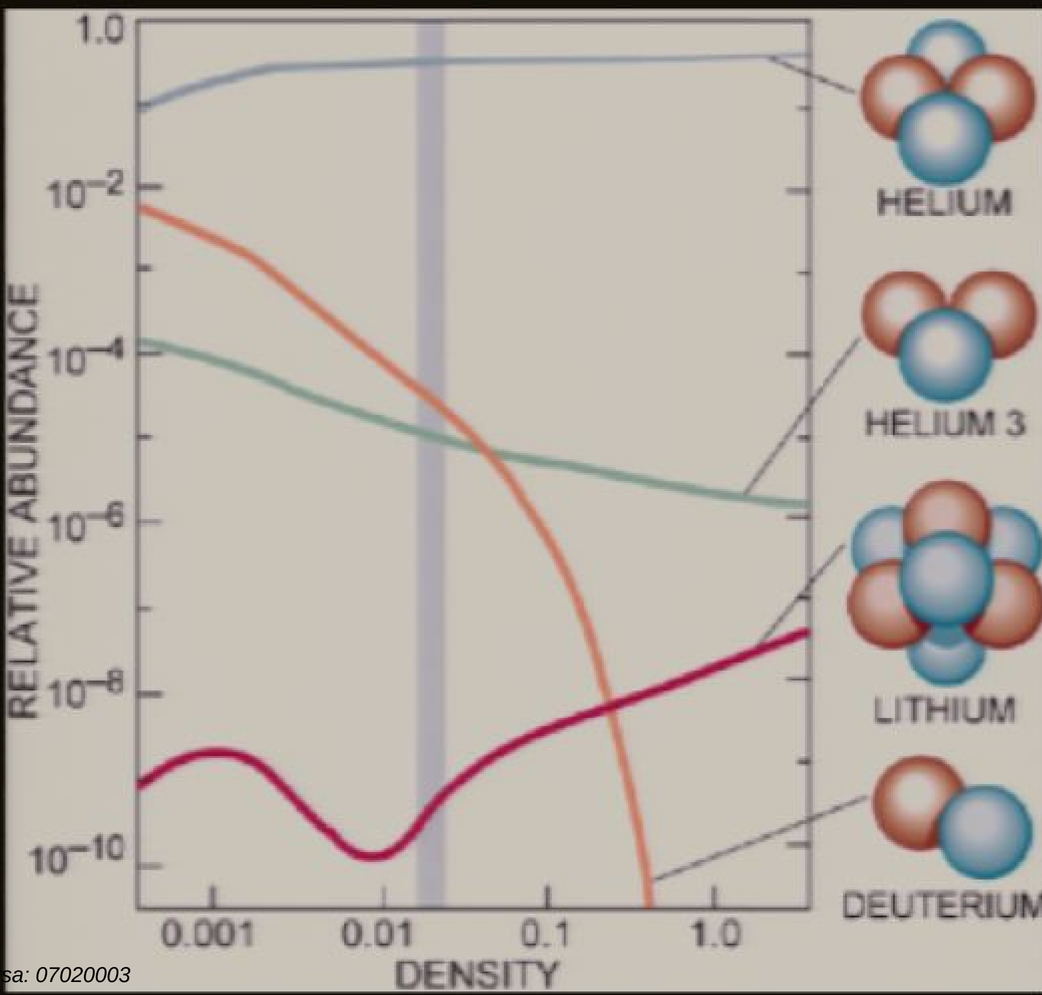
Bullet Cluster

Moral: Dark Matter is Real.

What about the expansion/acceleration of the universe?

Big Bang Nucleosynthesis occurred when the universe was about one minute old, 10^{-9} its current size.



Relic abundances depend on the expansion rate at that time, so provide an excellent test of the validity of the Friedmann equation,

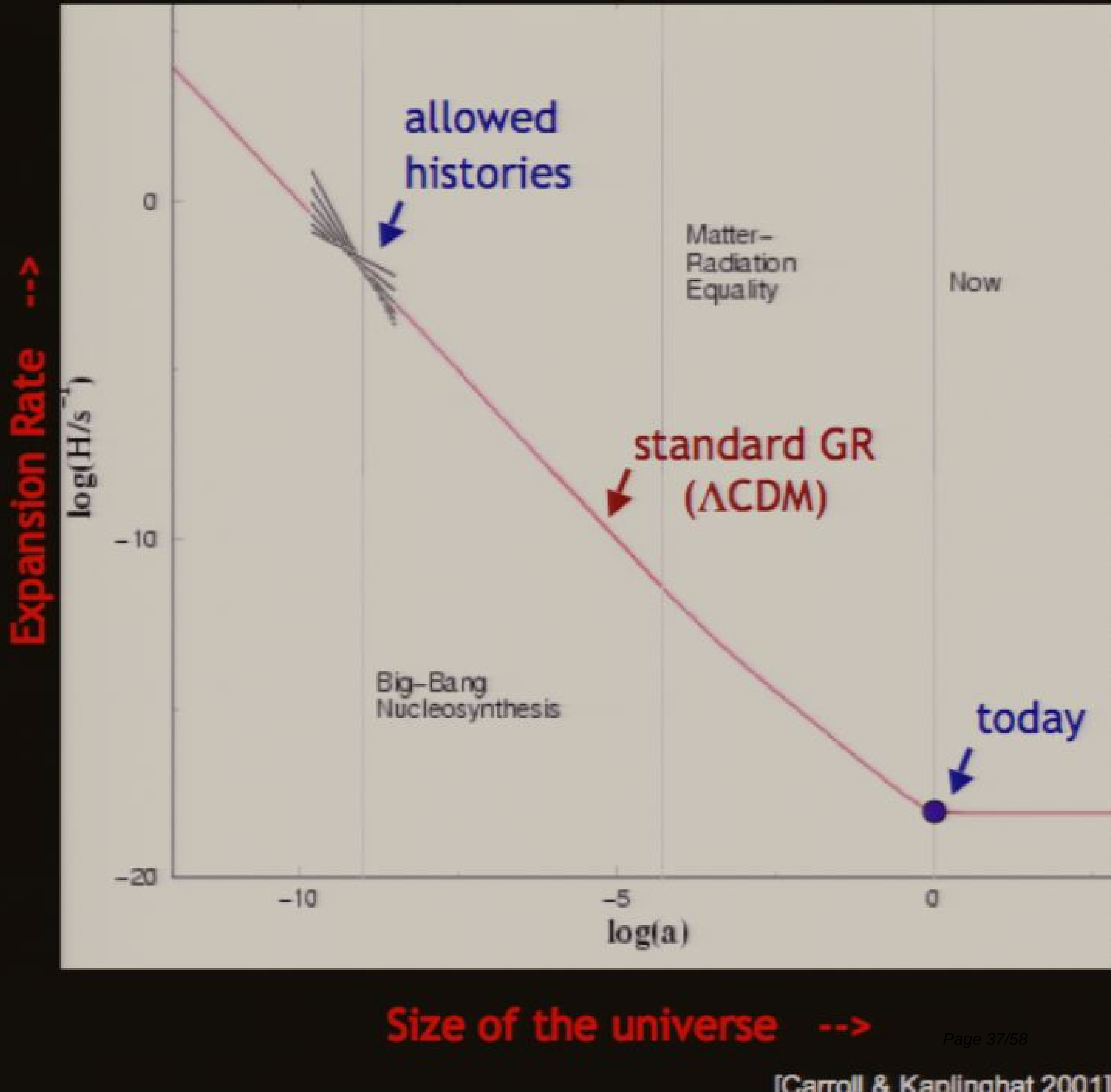
$$H^2 = \frac{8\pi G}{3}\rho$$

not to mention the value of G.

Result:

Different expansion rates during BBN are allowed, but they must be very similar overall to the GR prediction.

Deviations from GR must only turn on rather late.



Can we modify gravity purely in four dimensions, with an ordinary field theory, to make the universe accelerate at late times? Simplest possibility: replace

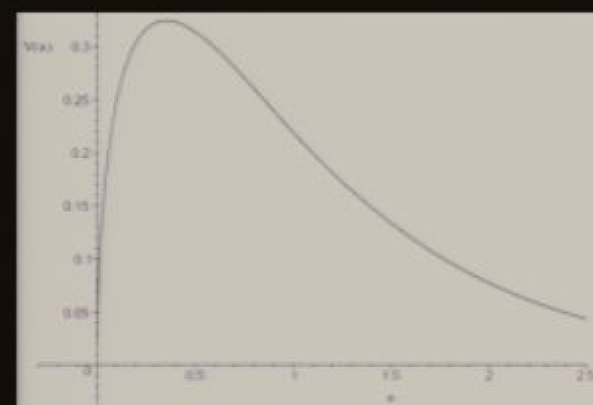
$$S = \int R \, d^4x$$

with

$$S = \int \left(R - \frac{1}{R} \right) d^4x$$

[Carroll, Duvvuri, Trodden & Turner 2003]

The vacuum in this theory is not flat space, but an accelerating universe!
But: the modified action brings a new tachyonic scalar degree of freedom to life.



This is secretly a scalar-tensor theory, dramatically ruled out by Solar-System tests of GR.

[Chiba 2003; Erickcek, Smith & Kamionkowski 2006]

This is a generic problem.

- Weak-field GR is a theory of massless spin-2 gravitons. Their dynamics is essentially unique; it's hard to modify that behavior without new degrees of freedom.

- Loophole 1: somehow hide the scalar by giving it a location-dependent mass, either from matter effects ("chameleons") or other invariants ($R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$).

  [Khoury & Weltman 2003]

  [Carroll, DeFelice, Duvvuri, Easson, Trodden & Turner 2006; Navarro & Van Acoleyen 2005; Mena, Santiago & Weller 2005]

- Loophole 2: the Friedmann equation, $H^2 = (8\pi G/3)\rho$, has nothing to do with gravitons; it's a constraint. We could change Einstein's equation from $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ to $G_{\mu\nu} = 8\pi G\, f_{\mu\nu}$, where $f_{\mu\nu}$ is some function of $T_{\mu\nu}$.

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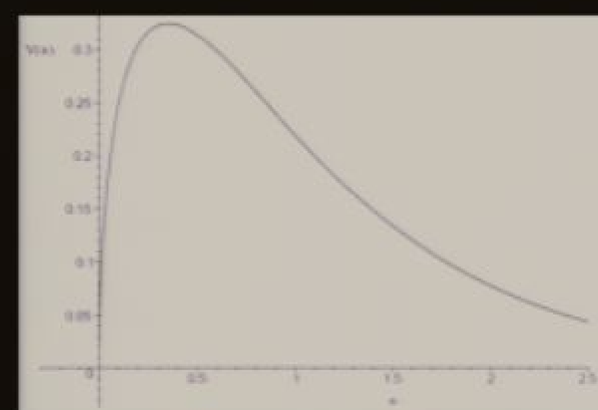

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Yes we can: "Modified-Source Gravity."

We specify a new function $\psi(T)$ that depends on the trace of the energy-momentum tensor, $T = -\rho + 3p$, where ρ is the energy density and p is the pressure.

The new field equations take the form

$$G_{\mu\nu} = 8\pi G \left[e^{-2\psi} T^{(\mathrm{m})}_{\mu\nu} + T^{(\psi)}_{\mu\nu} \right]$$

density-dependent rescaling of Newton's constant

"ψ energy-momentum tensor"; determined in terms of $T^{(matter)}$.

Exactly like scalar-tensor theory, but with the scalar determined by the ordinary matter fields.

In the modified-source-gravity equation of motion

$$G_{\mu\nu} = 8\pi G \left[e^{-2\psi} T^{(\mathrm{m})}_{\mu\nu} + T^{(\psi)}_{\mu\nu} \right]$$

the energy-momentum tensor for ψ looks like

$$\begin{aligned} T^{(\psi)}_{\mu\nu} = & \left[(\nabla\psi)^2 + 2\nabla^2\psi - e^{-2\psi} U(\psi) \right] g_{\mu\nu} \\ & -2\nabla_\mu\psi\nabla_\nu\psi + 2\nabla_\mu\nabla_\nu\psi \end{aligned}$$

$U(\psi)$ is a "potential" that defines $\psi(T)$ via

$$\frac{dU}{d\psi} - 4U(\psi) = -T$$

So the metric ultimately depends only on the matter energy-momentum – no new degrees of freedom.

[Flanagan 2005; Carroll, Sawicki, Silvestri & Trodden 2006]

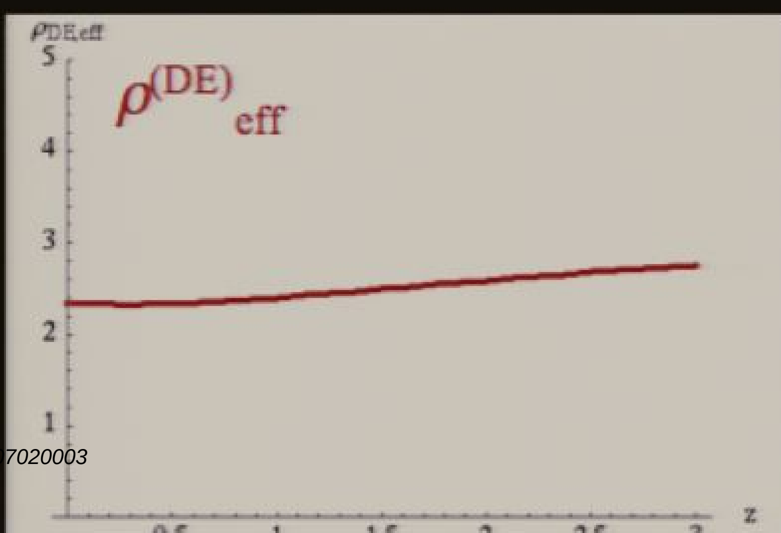
Cosmology in modified-source gravity

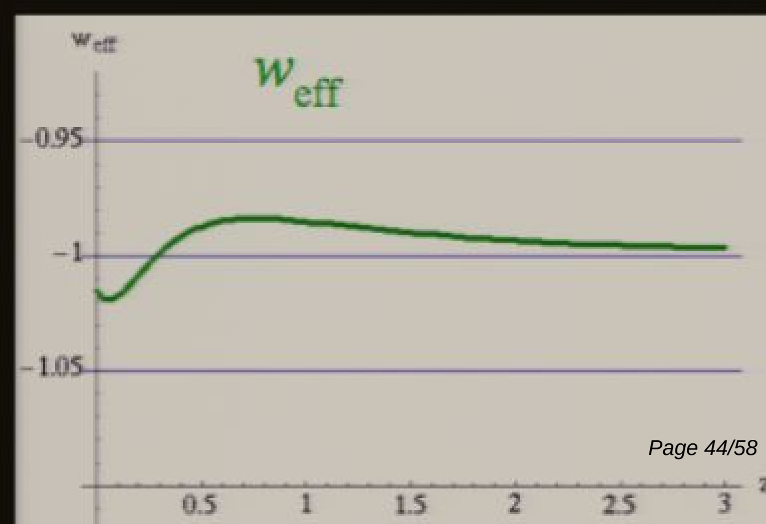
The effective Friedmann equation is

$$H^2 = \frac{8\pi G}{3} e^{-2\psi} \underbrace{\left[1 - 3\rho\left(\frac{d\psi}{d\rho}\right)\right]^2}_{\text{density-dependent correction to Newton's constant}} [\rho + U(\psi)]$$

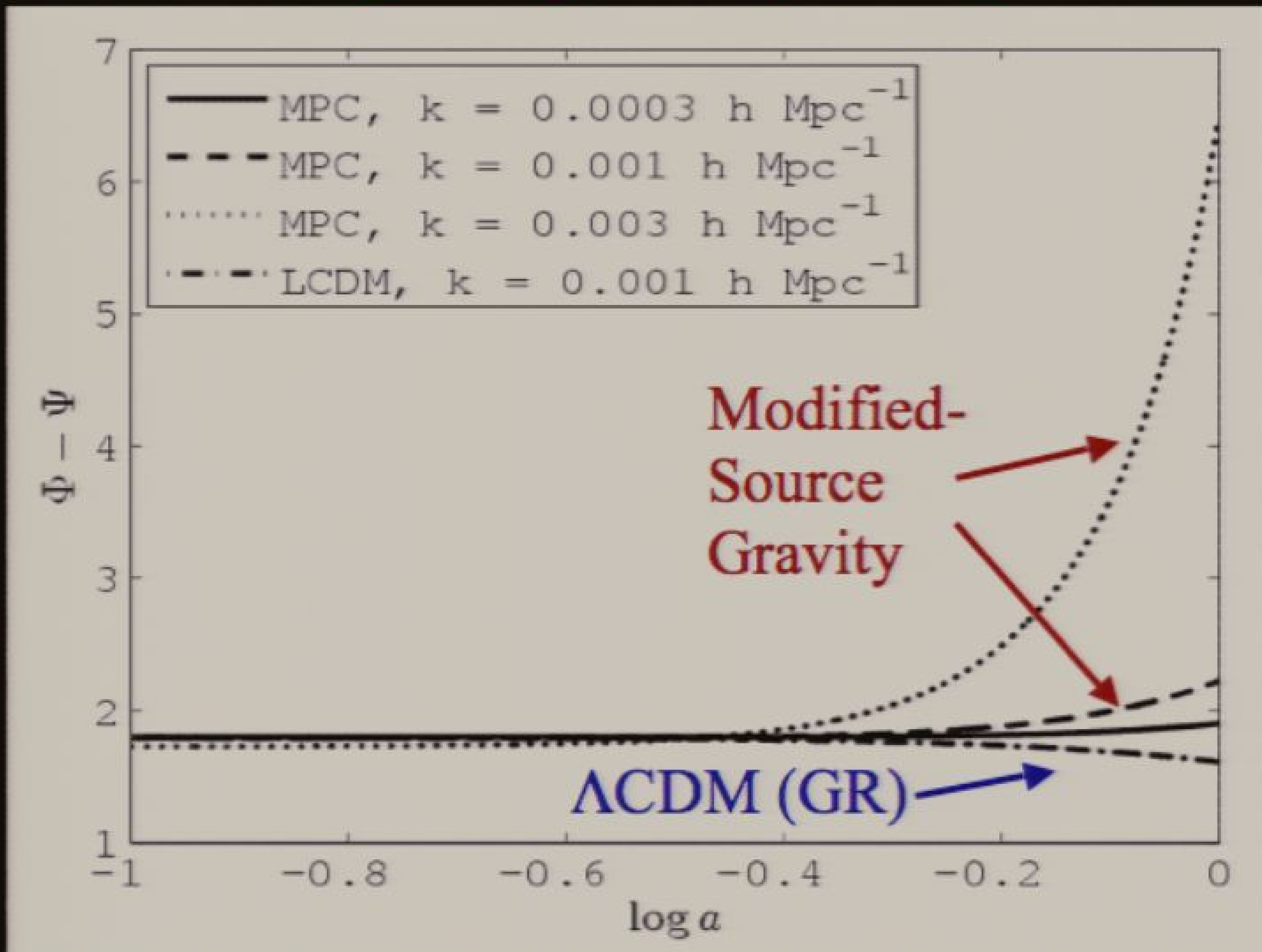
ordinary matter energy density

density-dependent vacuum energy





MSG changes late-time evolution of perturbations: small scales begin to grow explosively.



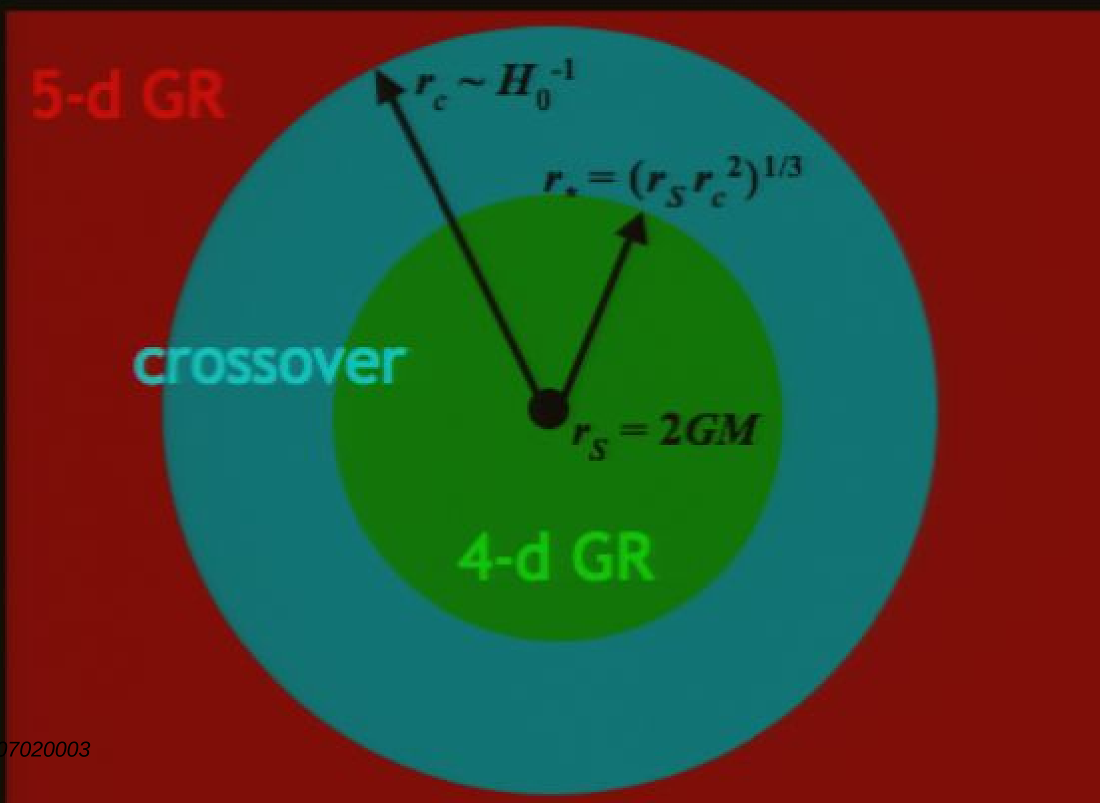
Not especially promising! But nonlinearities make it difficult to say anything definitive.

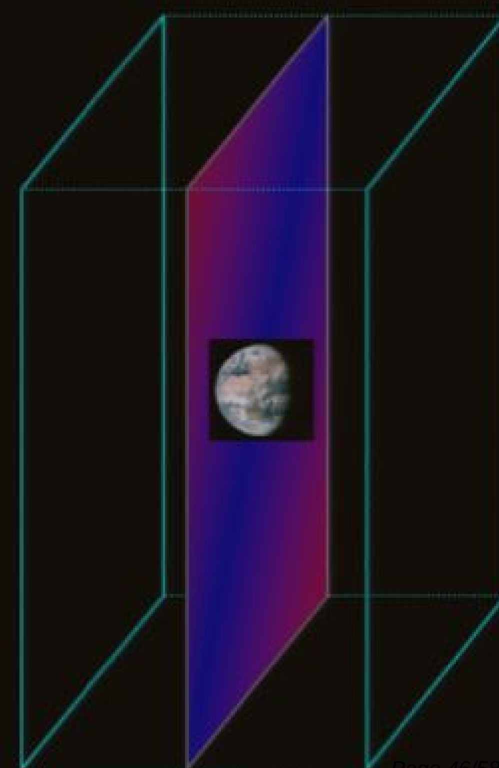
Dvali, Gabadadze, & Porrati (DGP) gravity: an infinite extra dimension, with <u>gravity stronger in the bulk</u>; 5-d kicks in at large distances.

$$S = \frac{M^2}{2}\int R_4\, d^4x + \frac{M^2}{2r_c}\int R_5\, d^5x$$

4-d gravity

5-d gravity enhanced by $r_c \sim H_0^{-1}$





[Dvali, Gabadadze & Porrati 2000]

Self-acceleration in DGP cosmology

Imagine that somehow the cosmological constant is set to zero in both brane and bulk. The DGP version of the Friedmann equation is then

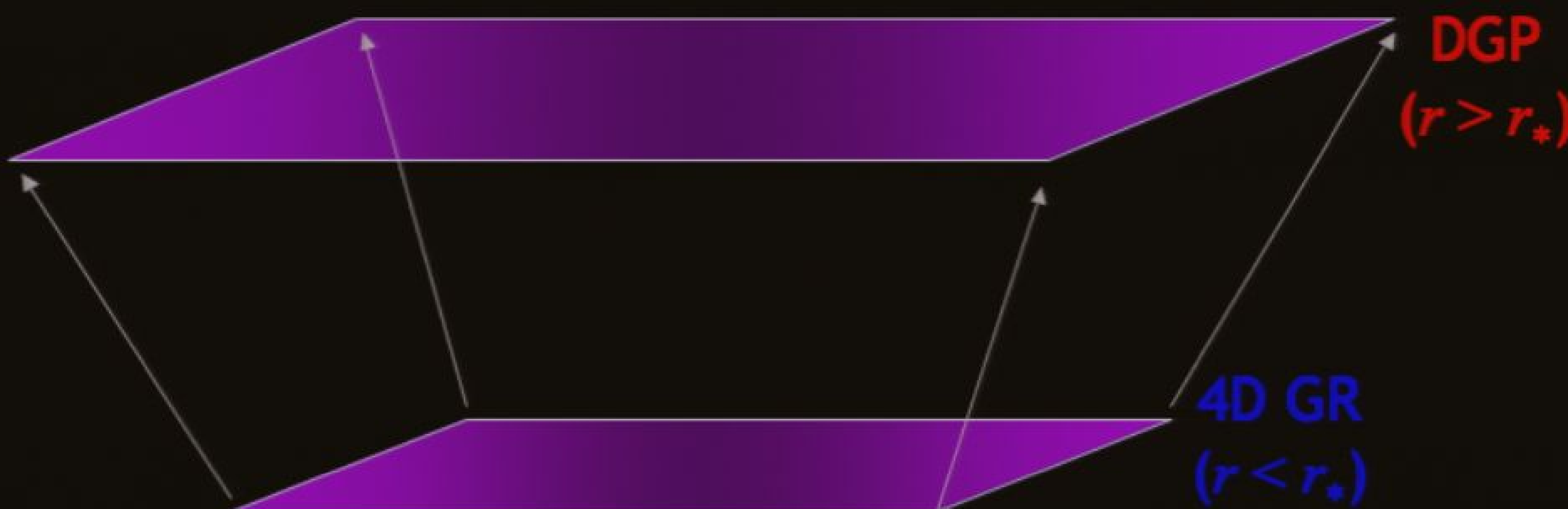
$$H^2 - \frac{H}{r_c} = \frac{8\pi G}{3}\rho$$

This exhibits self-acceleration: for $\rho = 0$, there is a de Sitter solution with $H = 1/r_c$ = constant. [Deffayet 2001]

The acceleration is somewhat mild; equivalent to an equation-of-state parameter $w_{eff} \sim -0.7$ – on the verge of being inconsistent with present data. (Also: strong coupling, ghosts, swampland, etc.)

Perturbation evolution

As the universe expands, modes get stretched, and evolve from the 4-d GR regime into the scalar-tensor crossover ("DGP") regime.



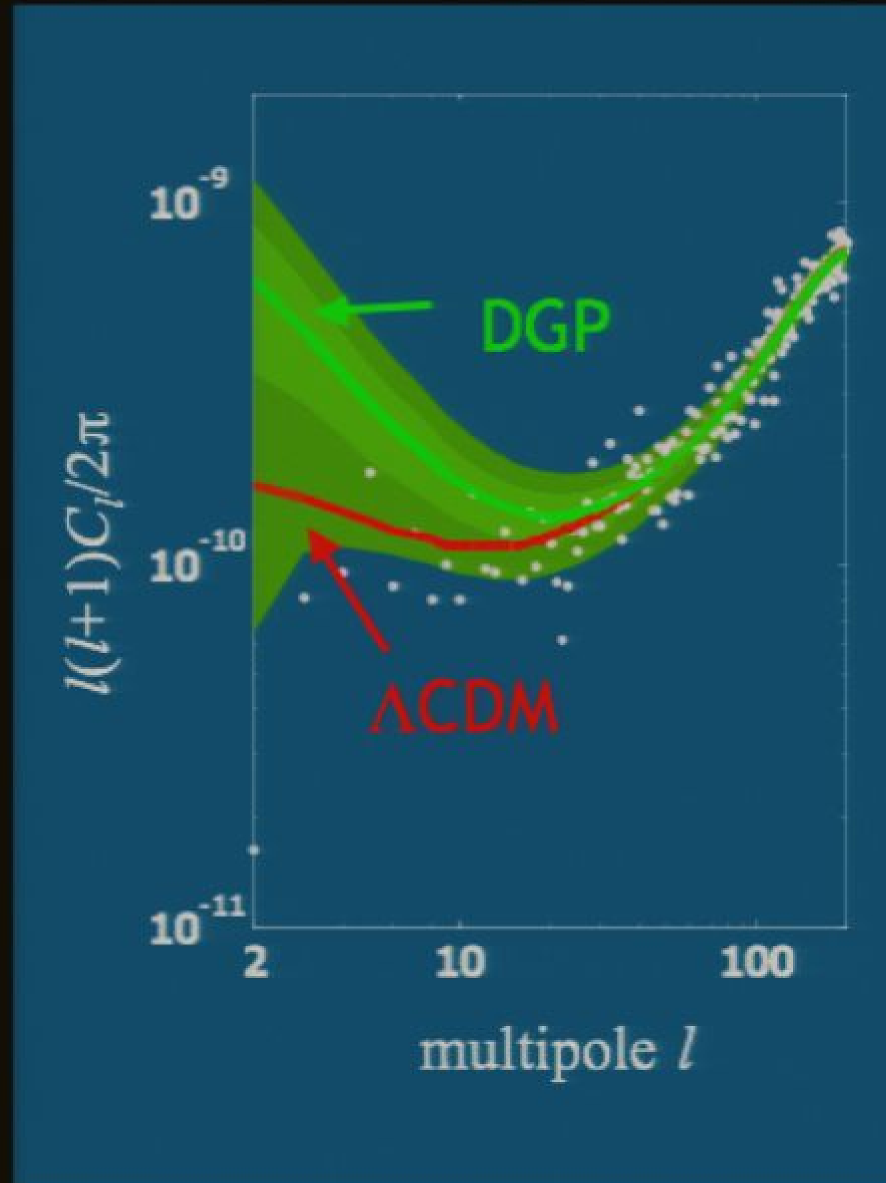
Scalar-tensor effects become important for long-wavelength modes at late times. Bulk effects important!

[Deffayet 2001; Lue, Scoccimarro & Starkman 2004; Koyama & Maartens 2006]

Large-scale CMB anisotropies in DGP vs. ΛCDM:

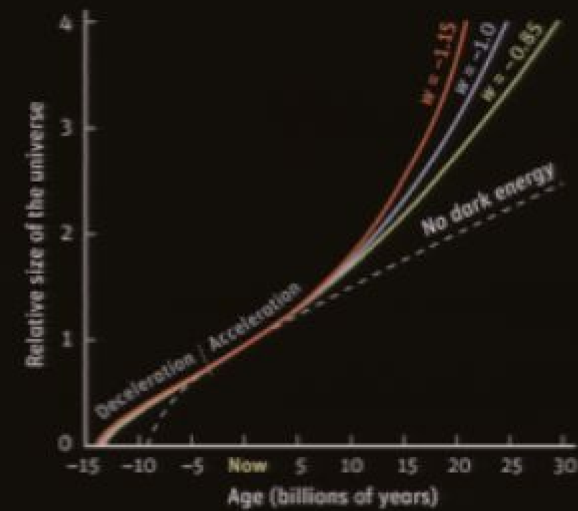
The DGP evolution equations imply an effective "stress" that causes the scalar gravitational potentials Φ and Ψ to diverge. This enhances the Integrated Sachs-Wolfe effect, caused by photons moving through time-dependent potentials.

Upshot: DGP has larger large-scale anisotropy than GR (not what the data want).



[Sawicki & Carroll 2005; Song, Sawicki & Hu 2006]

The lesson: we can **test GR on cosmological scales,** by comparing kinematic probes of DE to dynamical ones, and looking for consistency.



Kinematic probes [only sensitive to $a(t)$]:

- Standard candles (distance vs. redshift)
- Baryon oscillations (angular distances)

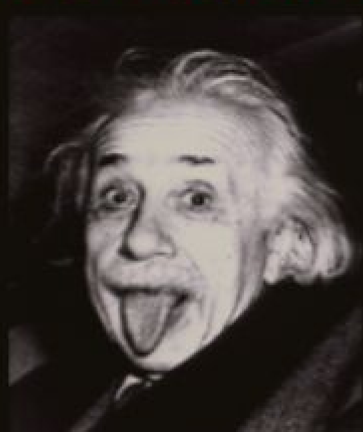
Dynamical probes [sensitive to $a(t)$ and growth factor]:

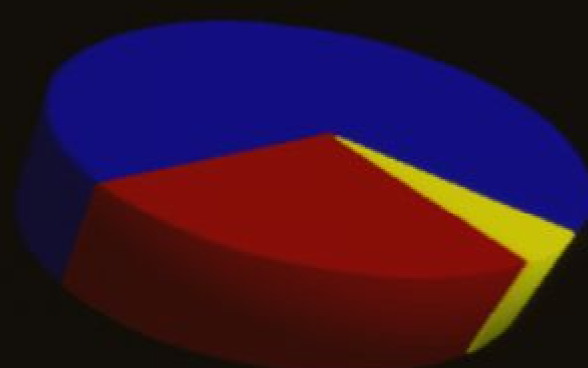
- Weak lensing
- Cluster counts (SZ effect)

[cf. Lue & Starkman; Ishak, Upadhye & Spergel; Linder; Albrecht et al., Dark Energy Task Force Report]

Outlook

- Observational evidence is conclusive that something is happening – dark stuff, or worse.
- Dark matter definitely exists; we detect gravity where the ordinary matter is not.
- Dark energy is less well understood; the data demand something, and modified-gravity models are not yet very promising.
- 95% of the universe is dark -- let's keep an open mind.





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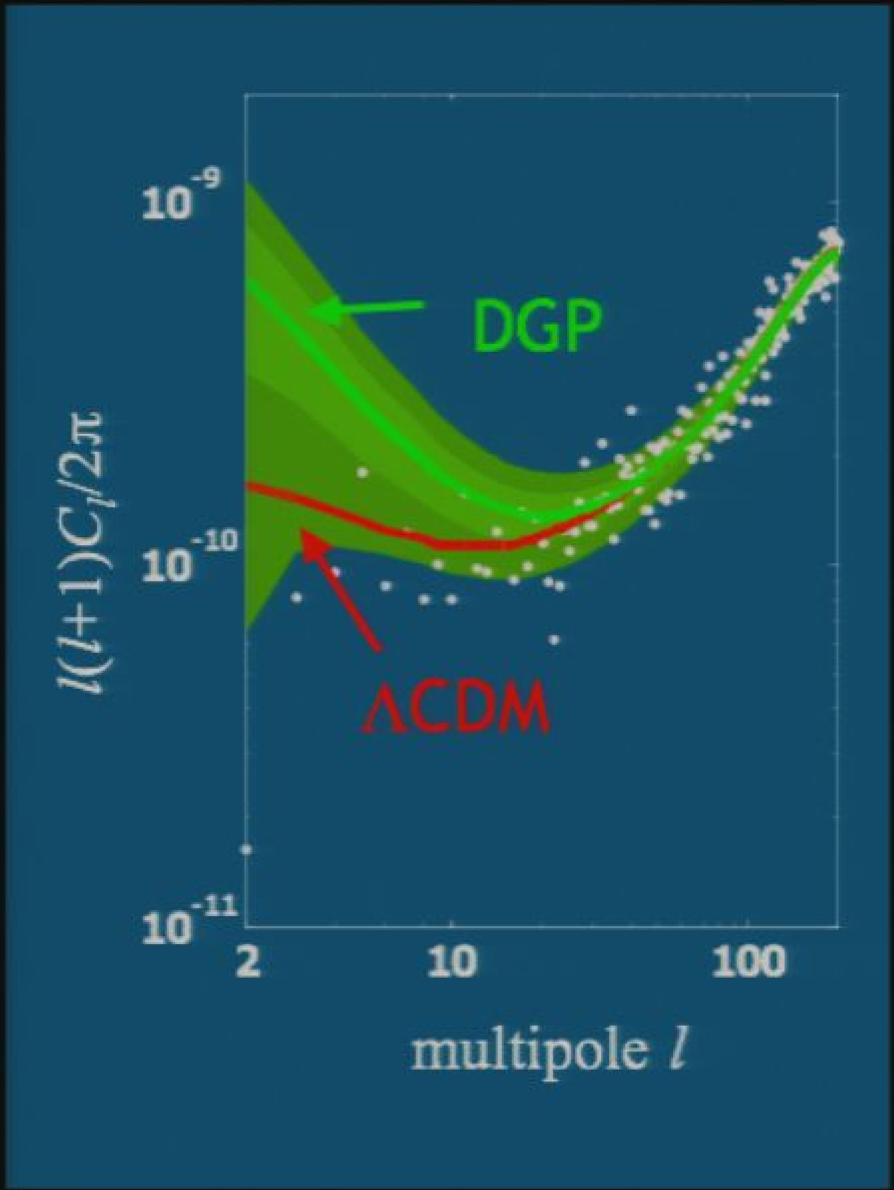

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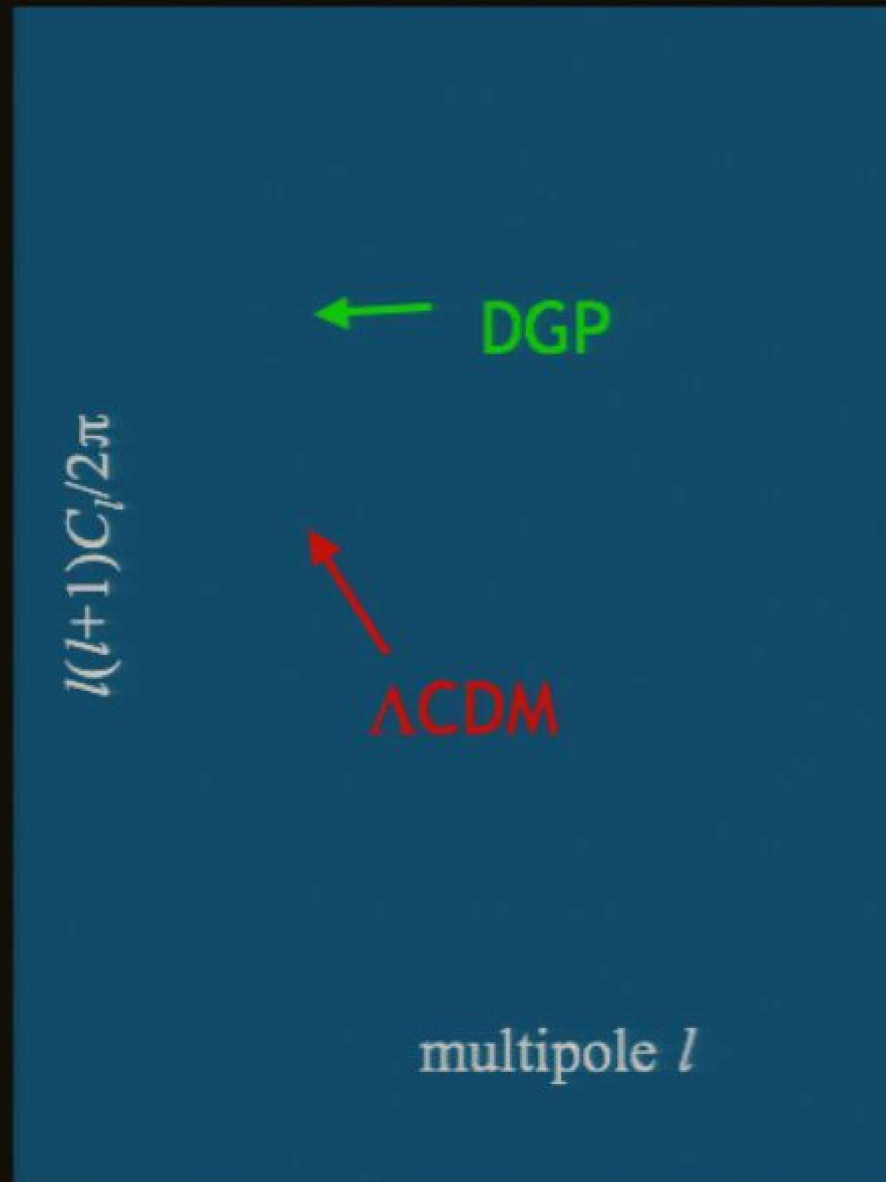

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The acceleration is somewhat mild; equivalent to an equation-of-state parameter $w_{eff} \sim -0.7$ – on the verge of being inconsistent with present data. (Also: strong coupling, ghosts, swampland, etc.)

Can we modify gravity purely in four dimensions, with an ordinary field theory, to make the universe accelerate at late times? Simplest possibility: replace

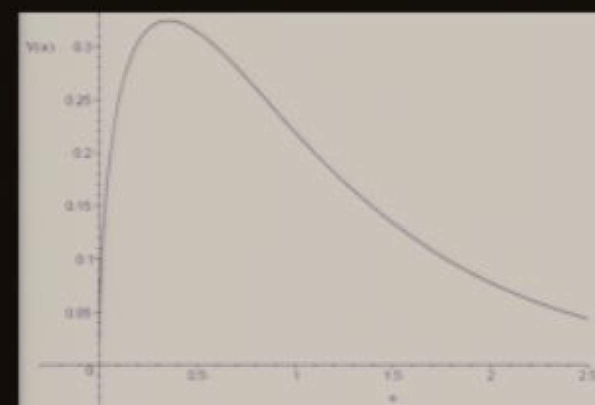
$$S = \int R \, d^4x$$

with

$$S = \int \left(R - \frac{1}{R} \right) d^4x$$

[Carroll, Duvvuri, Trodden & Turner 2003]

The vacuum in this theory is not flat space, but an accelerating universe!



But: the modified action brings a new tachyonic scalar degree of freedom to life.

This is secretly a scalar-tensor theory, dramatically ruled out by Solar-System tests of GR.

[Chiba 2003; Erickcek, Smith & Kamionkowski 2006]

Bekenstein (2004) introduced *TeVeS*, a relativistic version of MOND featuring the metric, a fixed-norm vector U_μ, scalar field ϕ, and Lagrange multipliers η and λ:

$$S = \frac{1}{16\pi G}\int d^4x\,(R + \mathcal{L}_U + \mathcal{L}_\phi)$$

where

$$\mathcal{L}_U = -\frac{1}{2}KF^{\mu\nu}F_{\mu\nu} + \lambda(g^{\mu\nu}U_\mu U_\nu + 1)$$

$$\mathcal{L}_\phi = -\mu_0\eta(g^{\mu\nu} - U^\mu U^\nu)\partial_\mu\phi\partial_\nu\phi - V(\eta)$$

$$V(\eta) = \frac{3\mu_0}{128\pi l_B^2}\left[\eta(4 + 2\eta - 4\eta^2 + \eta^3) + 2\ln^2(\eta - 1)\right]$$

Not something you'd stumble upon by accident.

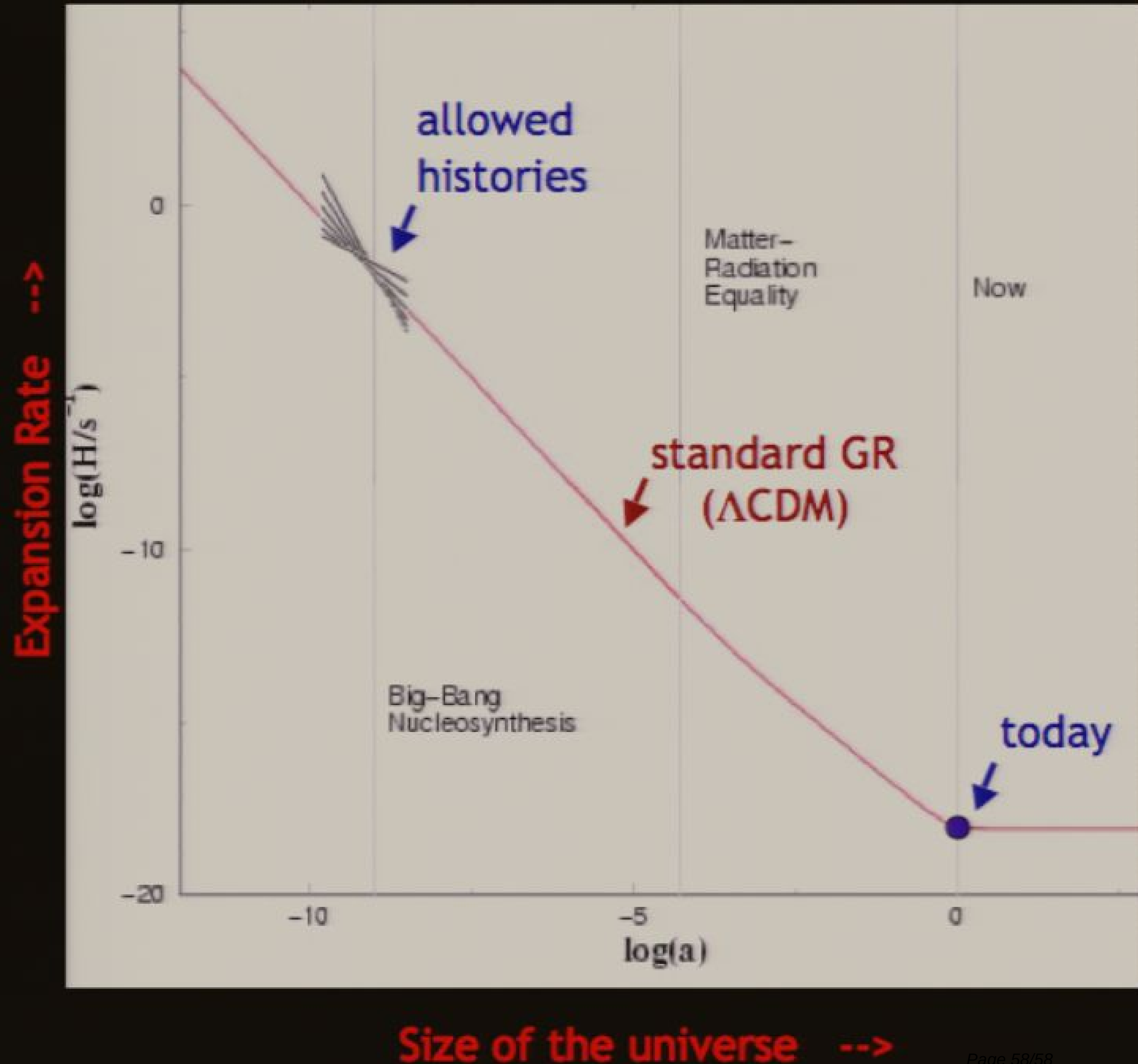
Bullet Cluster

Moral: Dark Matter is Real.

Result:

Different expansion rates during BBN are allowed, but they must be very similar overall to the GR prediction.

Deviations from GR must only turn on rather late.



[Carroll & Kaplinghat 2001]