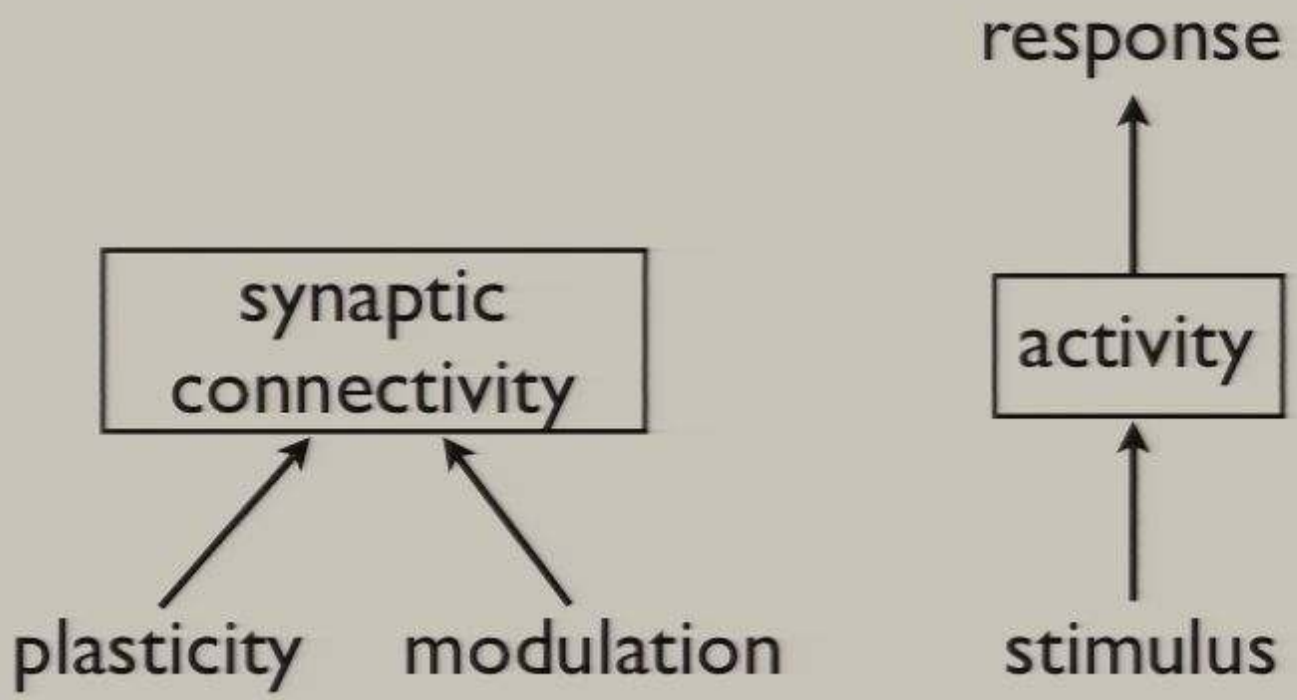
Title: The Dynamics of Neural Networks

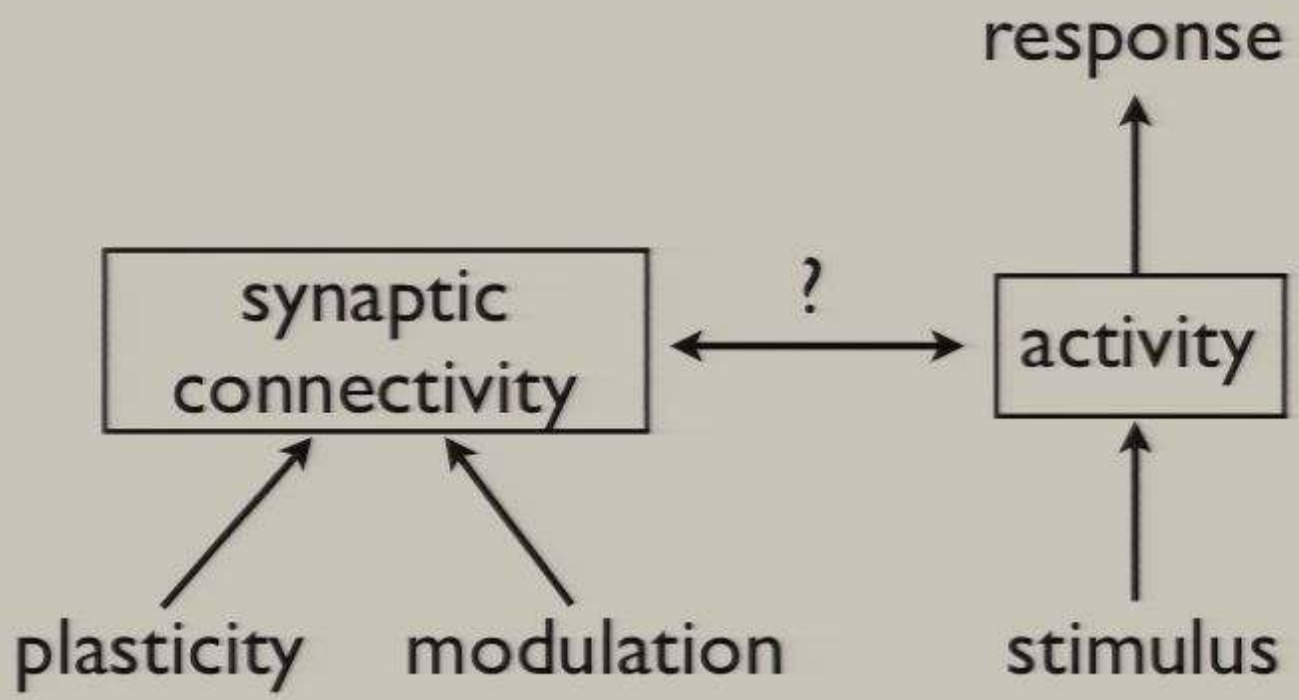
Date: Feb 21, 2007 02:00 PM

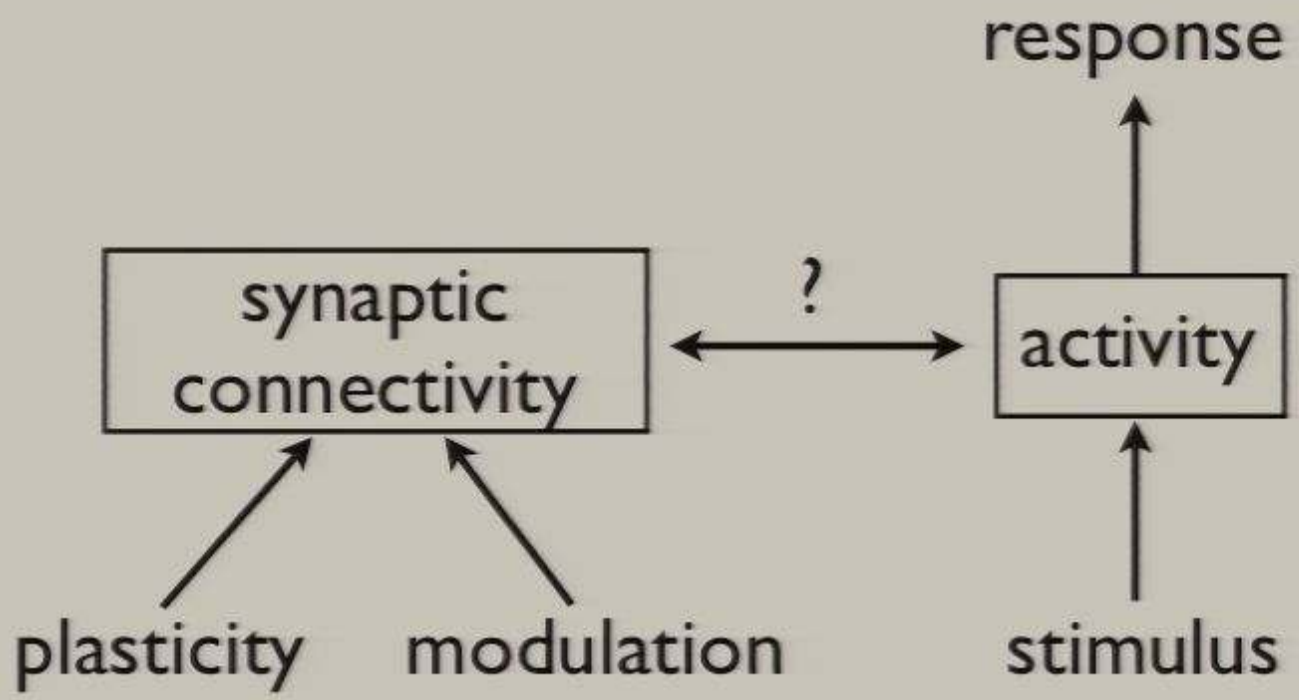
URL: http://pirsa.org/07020002

Abstract: Neural circuits exhibit complex patterns of spontaneous activity. I will discuss how neural network models can reproduce this activity and how it interacts with responses evoked by sensory stimuli. This work involves analytic, mean-field and computer analyses of large systems of nonlinear differential equations with random parameters.

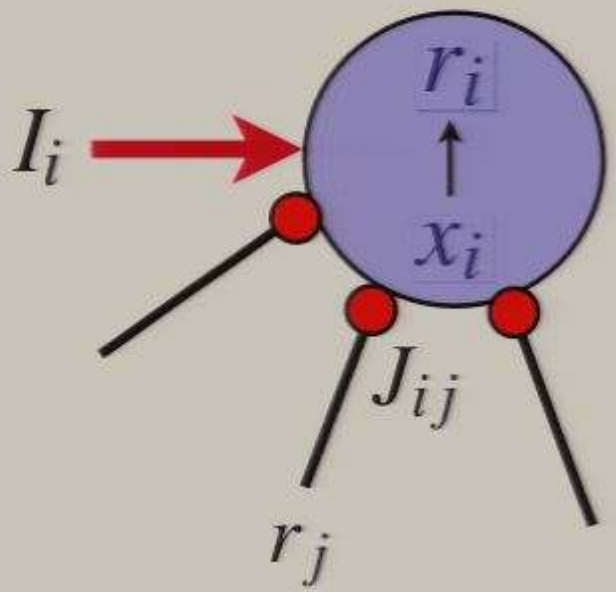
response
activity
stimulus


response
synaptic connectivity
activity
plasticity
modulation
stimulus


response
synaptic
connectivity
?
activity
plasticity
modulation
stimulus

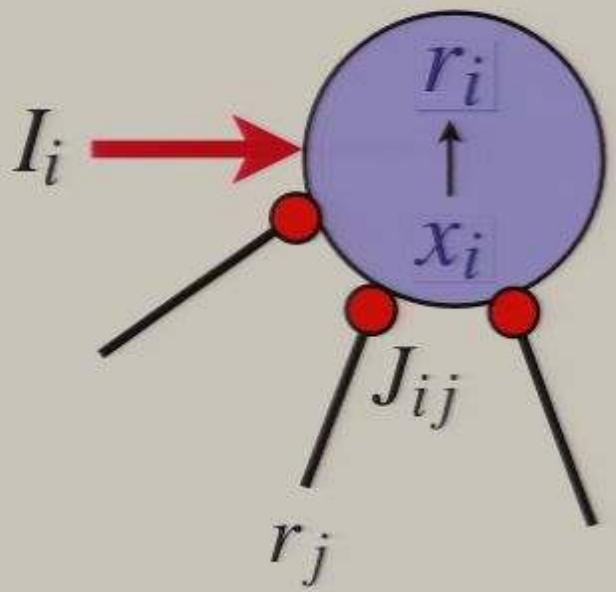

response
synaptic
connectivity
?
activity
plasticity
modulation
stimulus

$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} r_j + I_i$$



synaptic weights

$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} r_j + I_i$$



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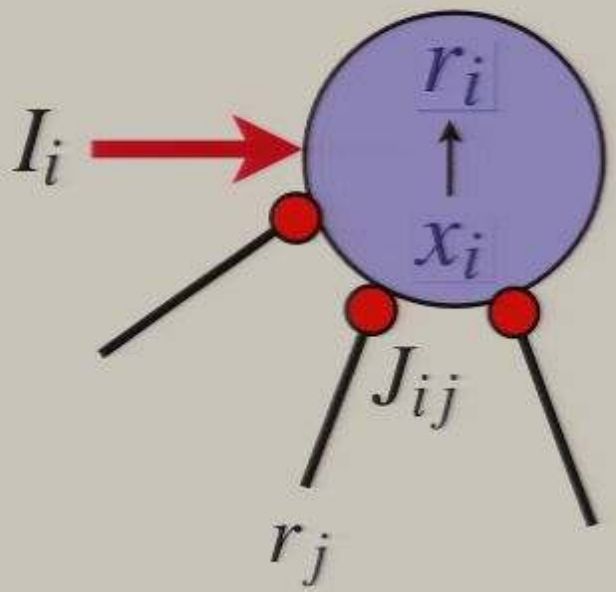

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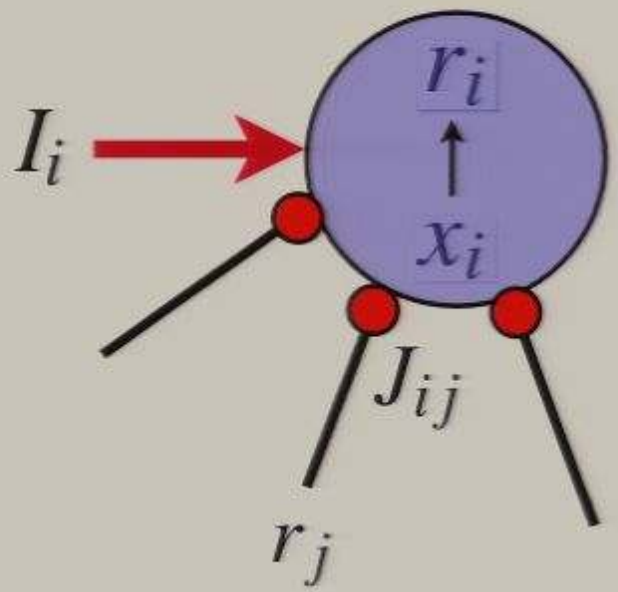

synaptic weights

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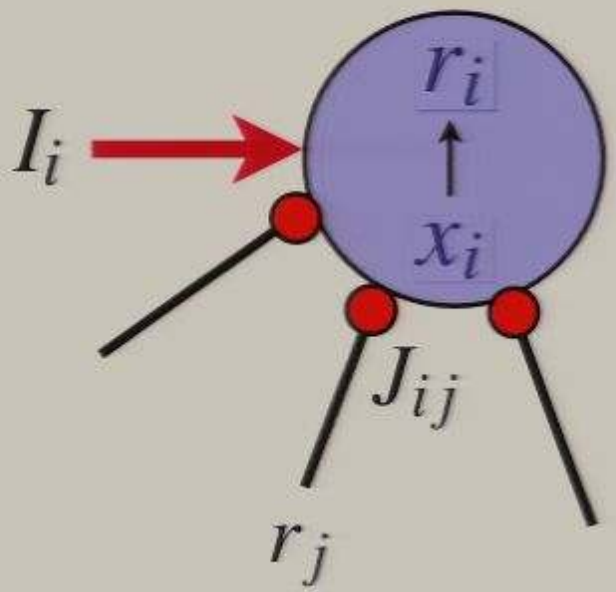
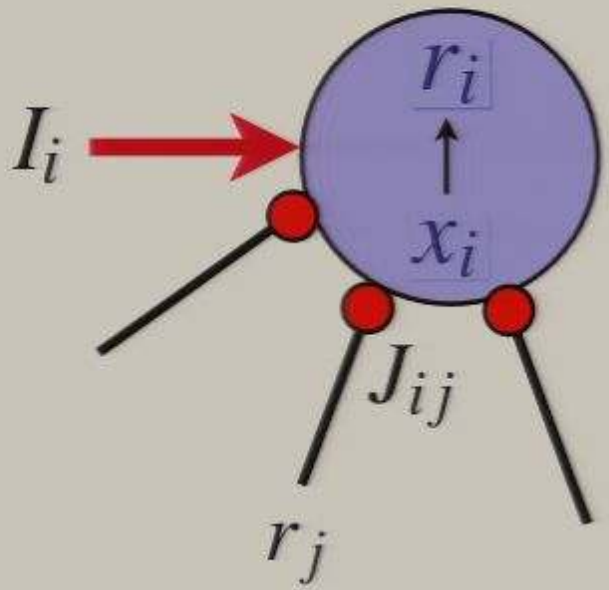
"stimulus" external input



synaptic weights

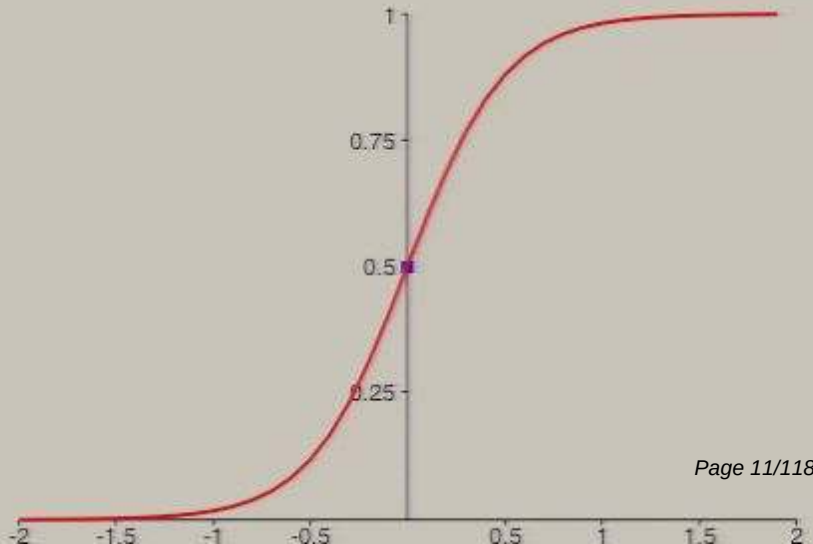
$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} r_j + I_i$$

"stimulus"
external input



"firing rate"

$$r_j = f(x_j)$$



synaptic weights

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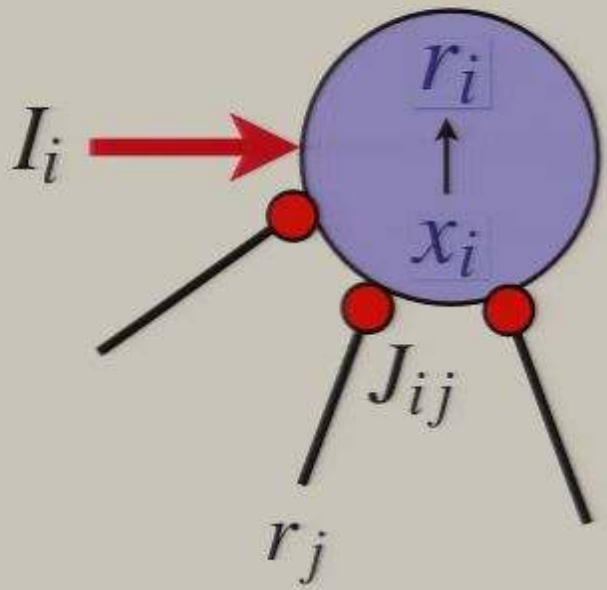
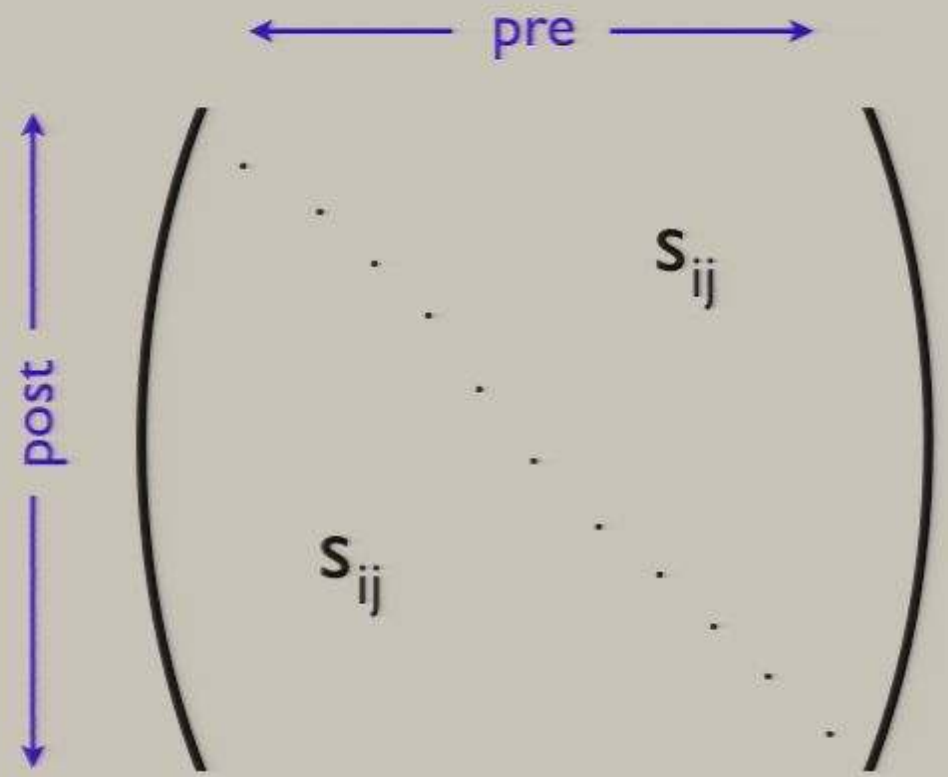
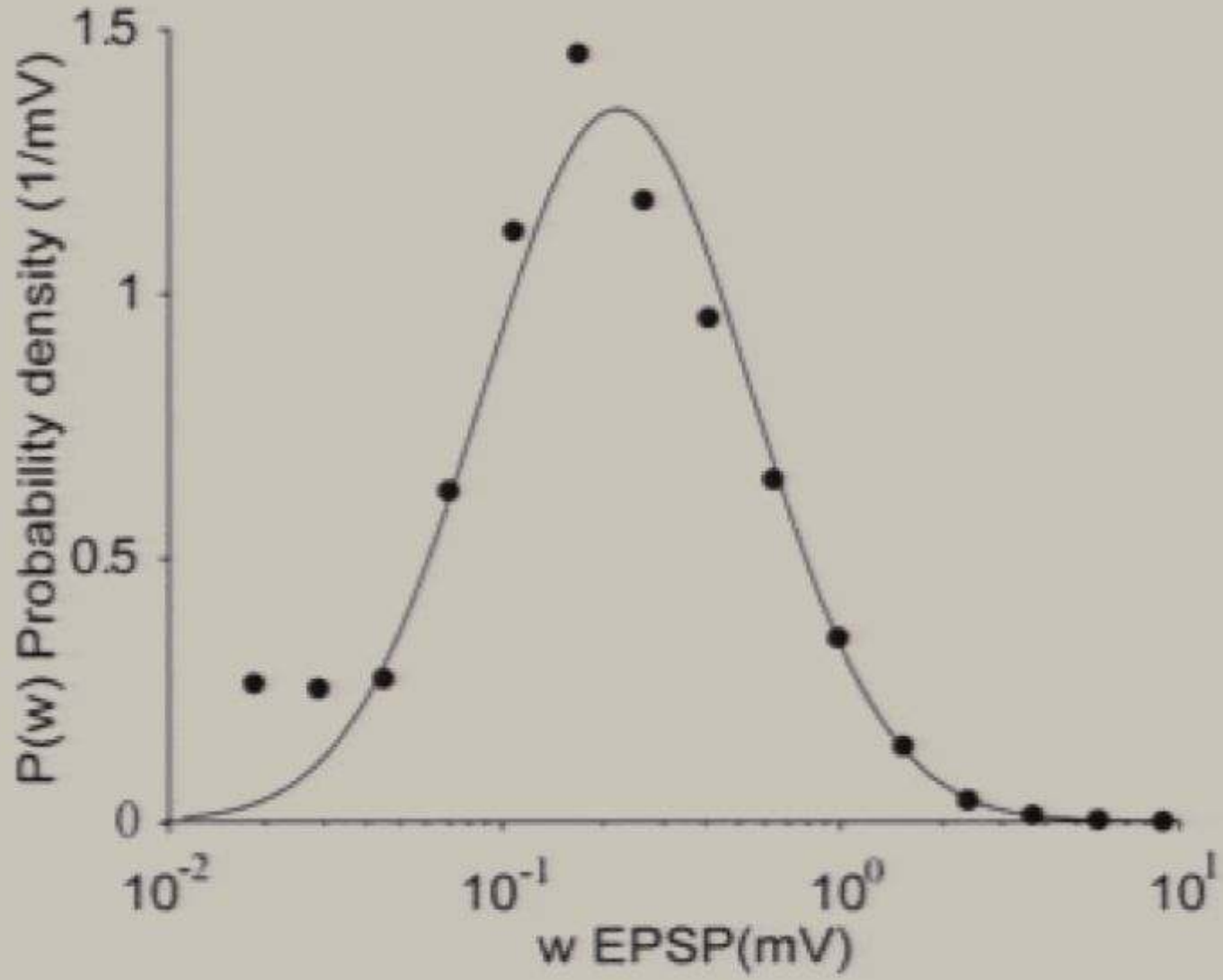


synaptic connectivity



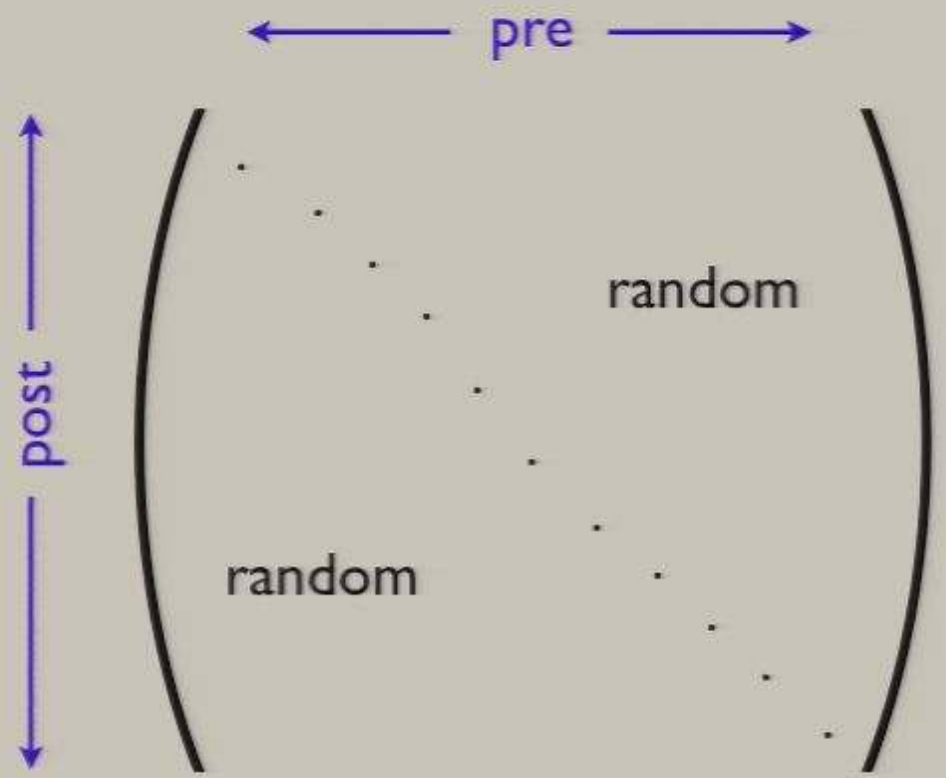
$$i = 1, 2, 3, \ldots, N$$
$$j = 1, 2, 3, \ldots, N$$

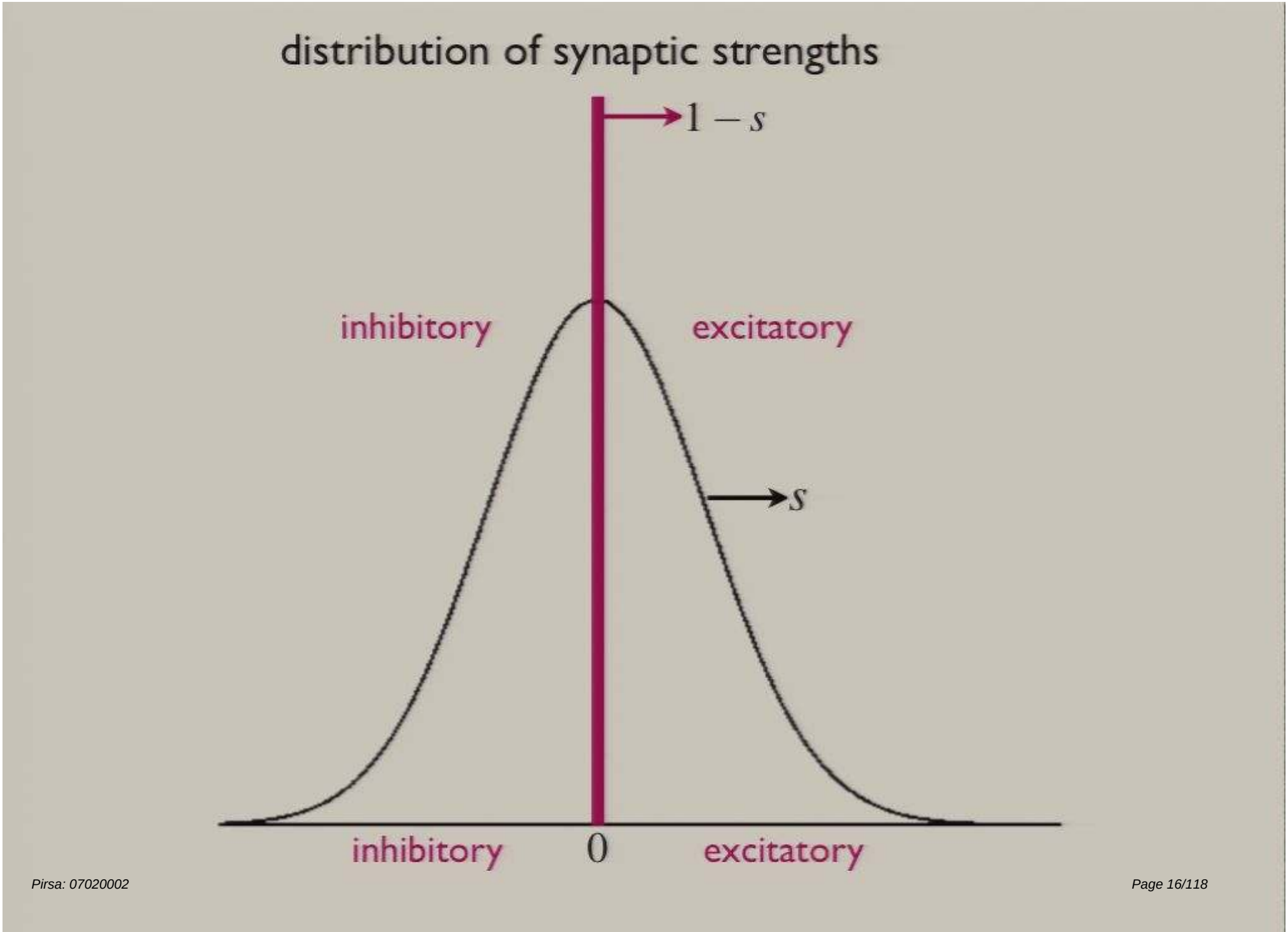
$$p[w] = 0.426\exp[-(\ln[w] + 0.702)^2/(2 \times 0.9355)^2]/w$$



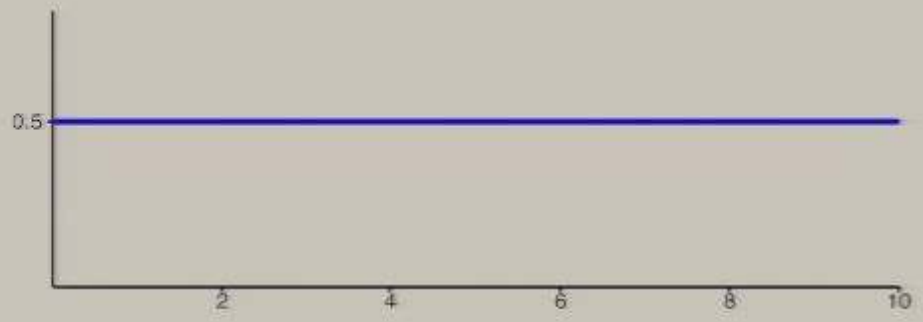
Song, Sjostrom, Reigl, Nelson, Chklovskii, 2005

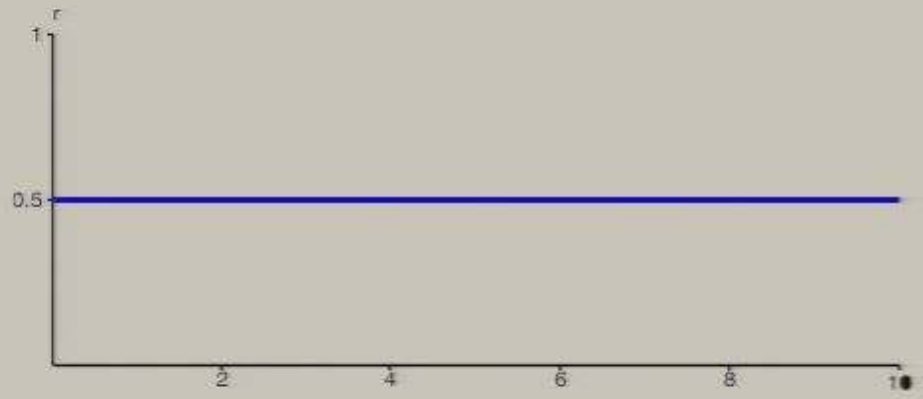
synaptic connectivity

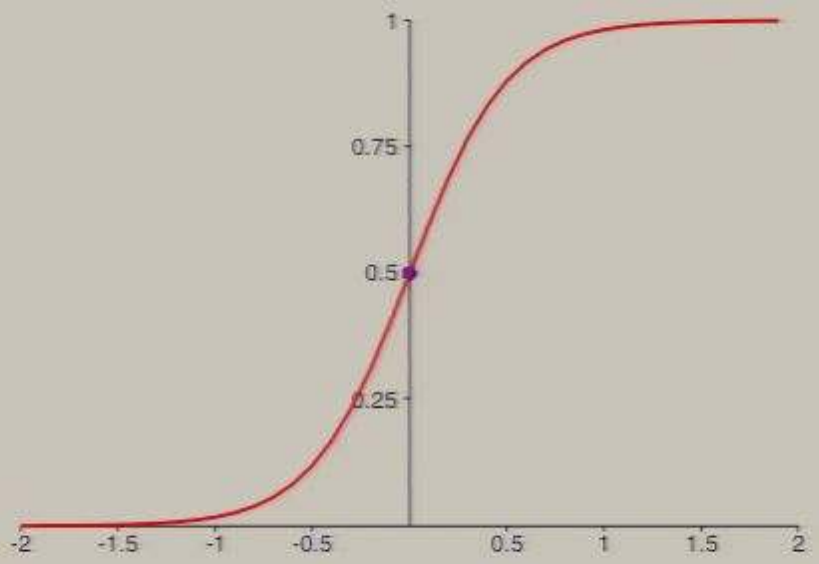


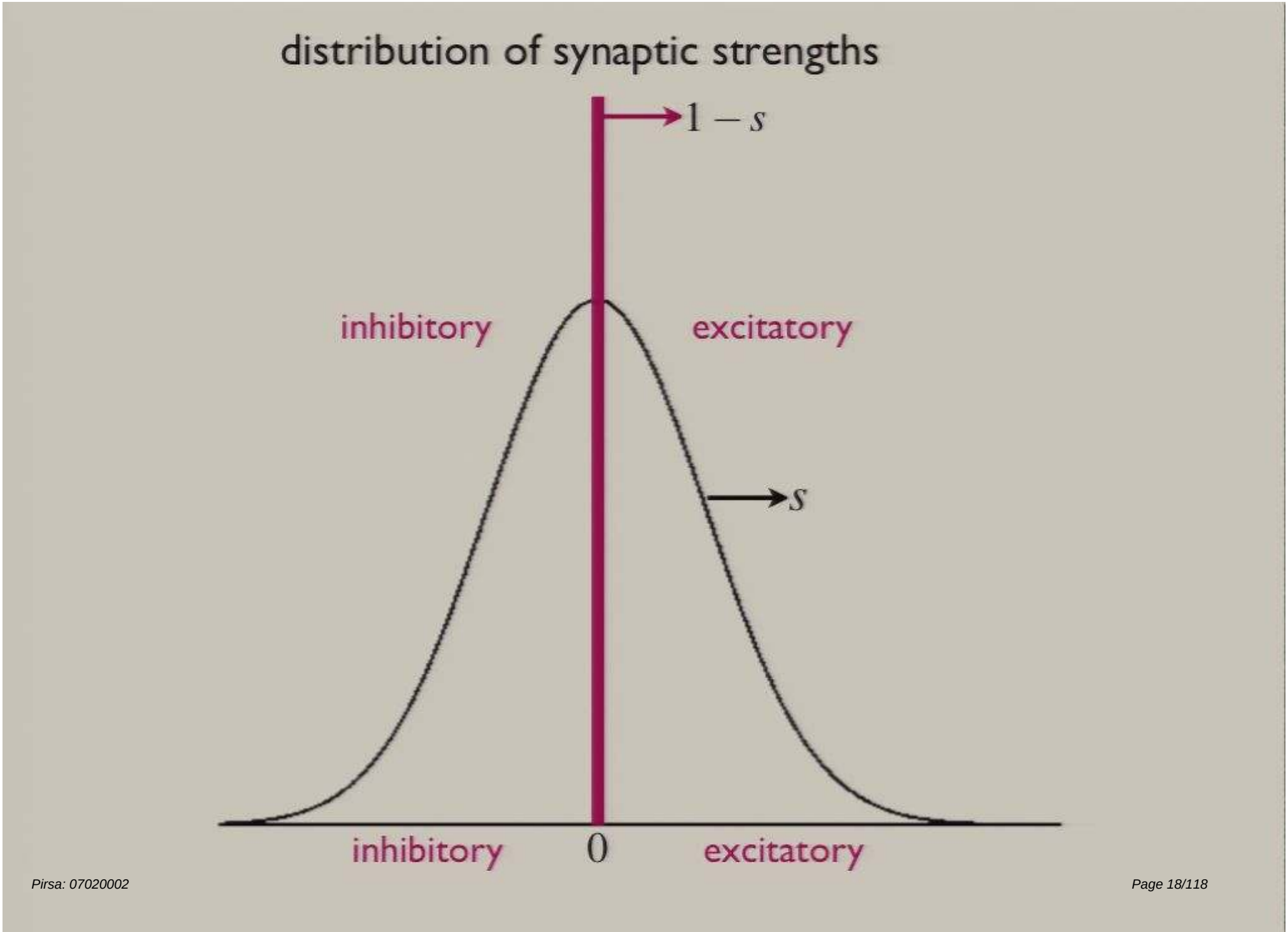

distribution of synaptic strengths
1 − s
inhibitory
excitatory
s
inhibitory
0
excitatory

trivial fixed point





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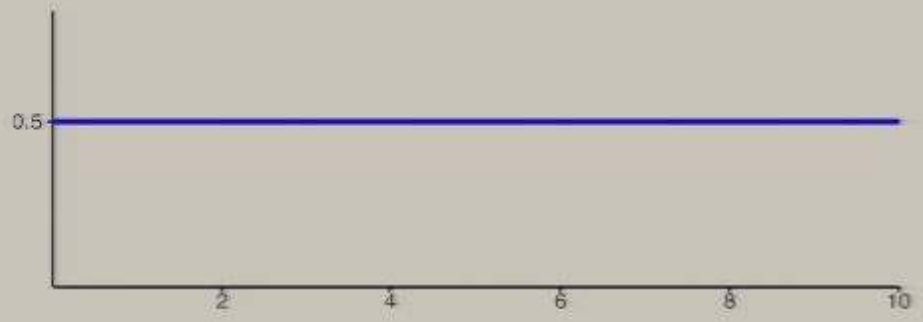
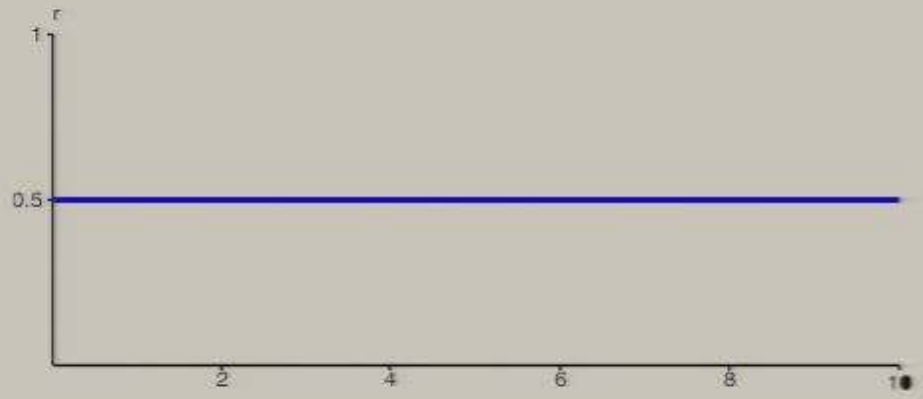
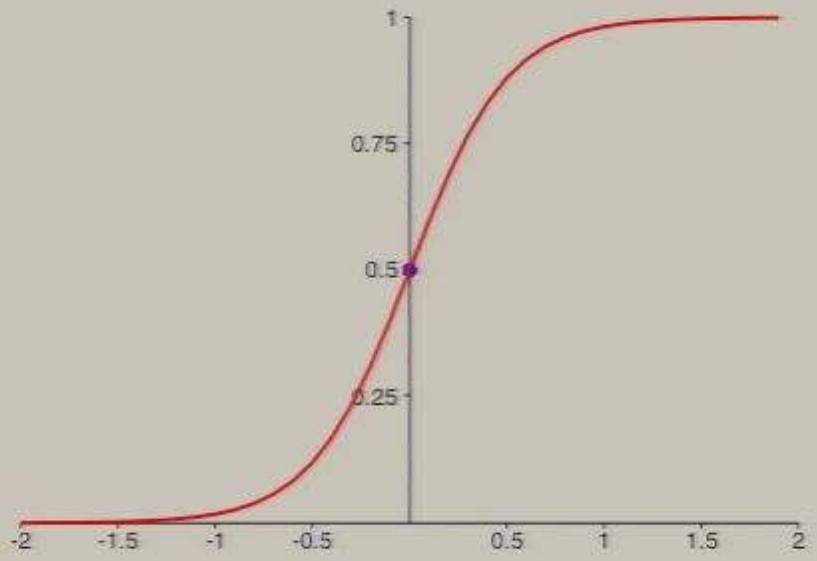
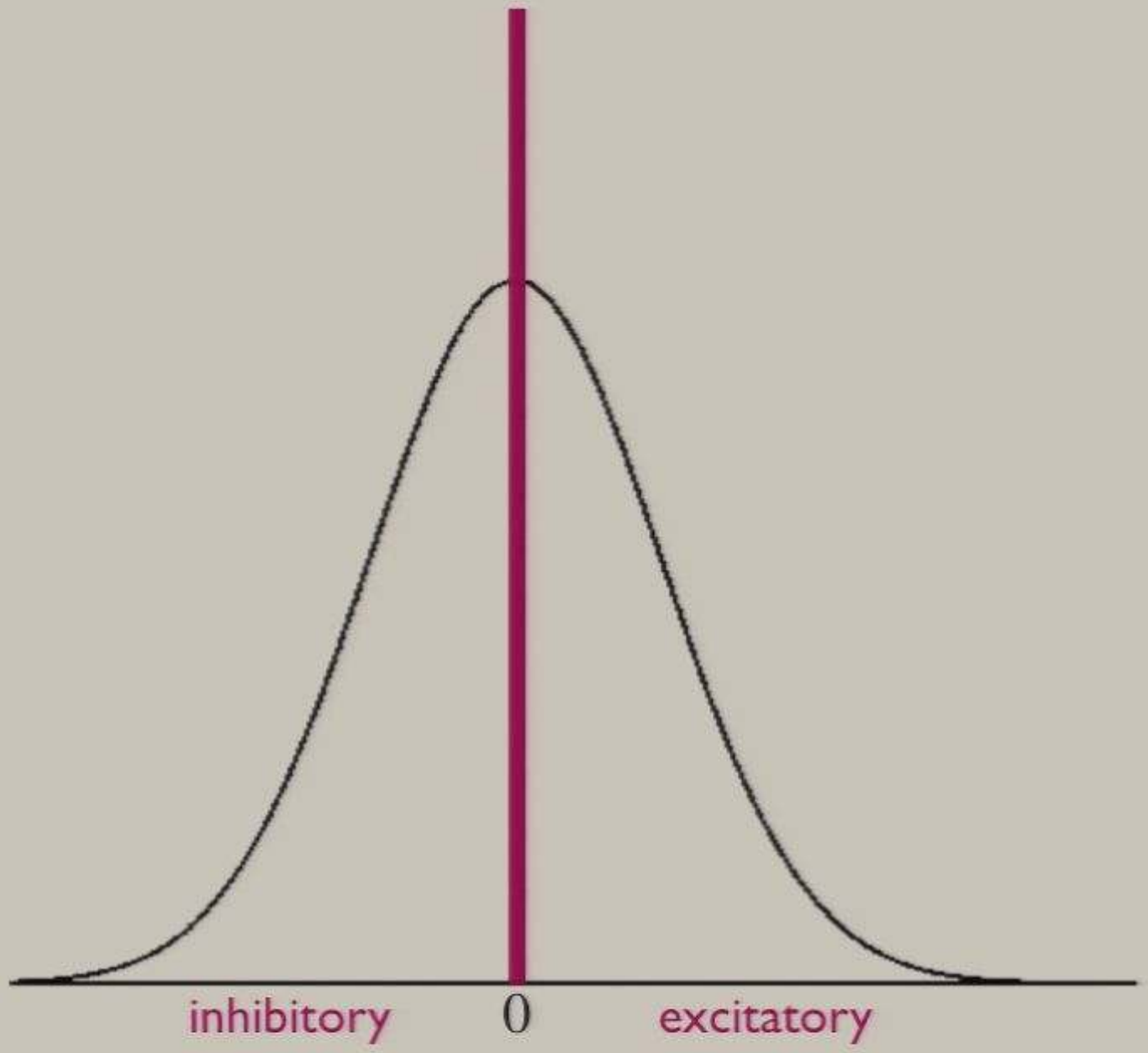
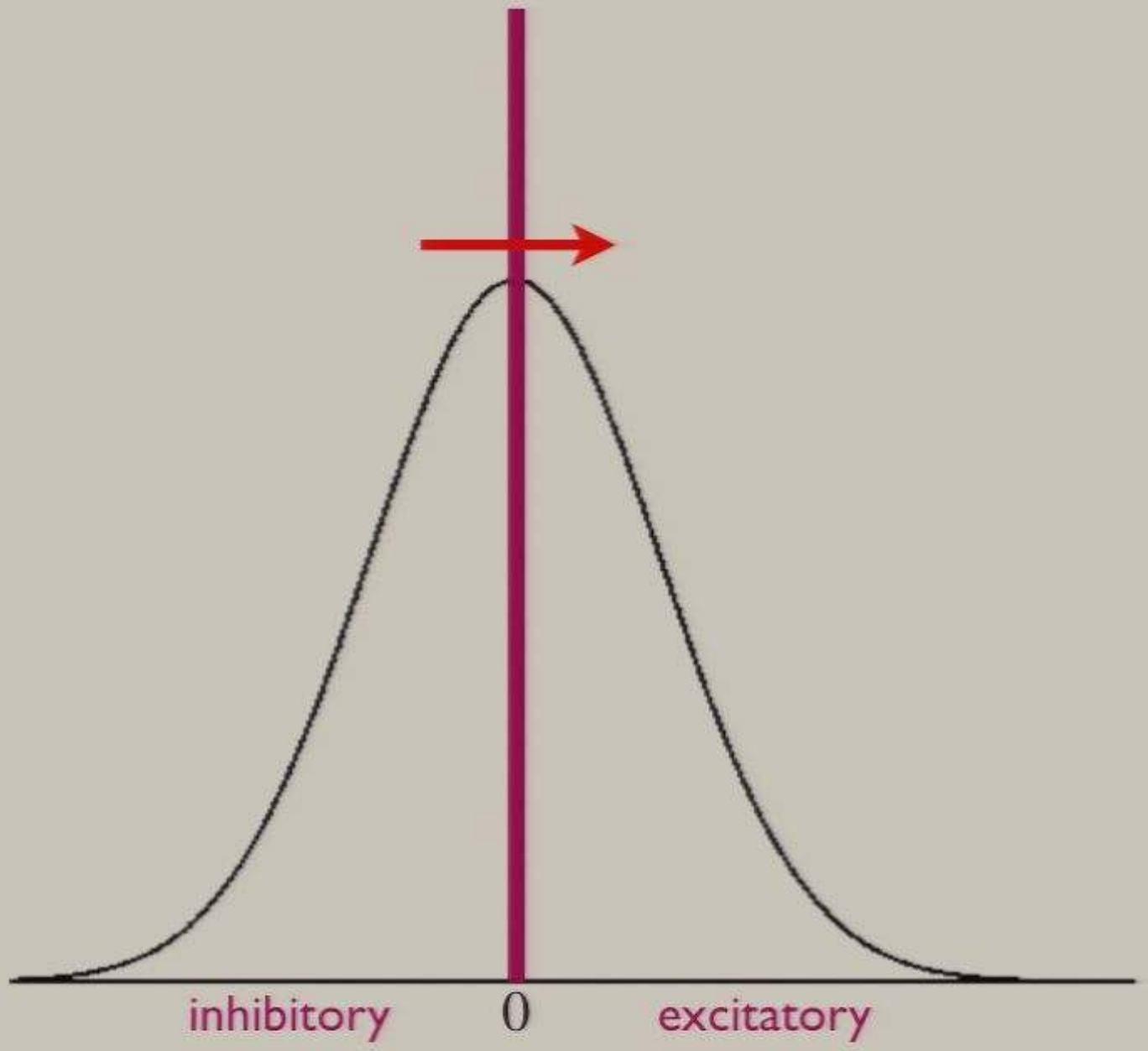


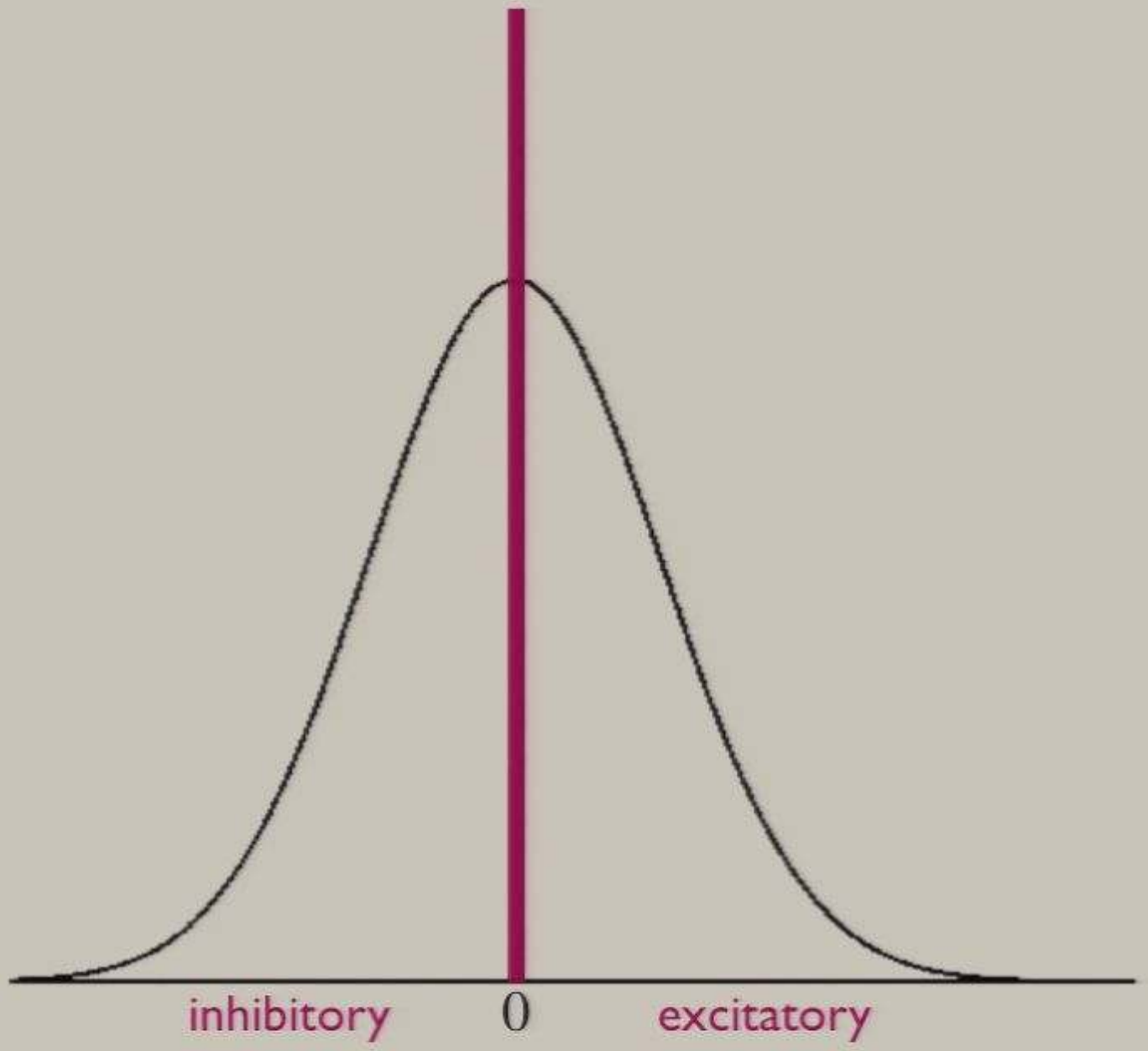


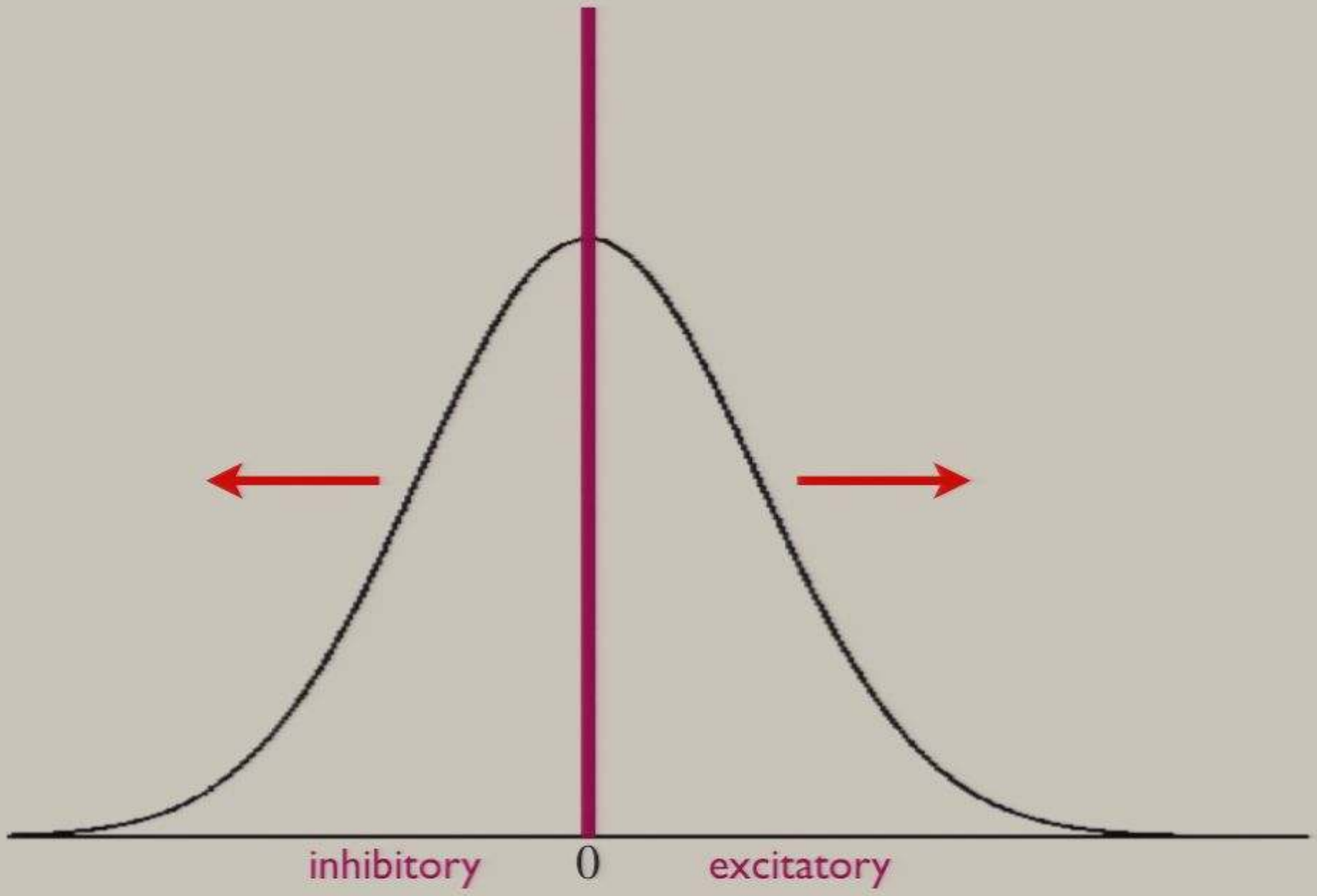


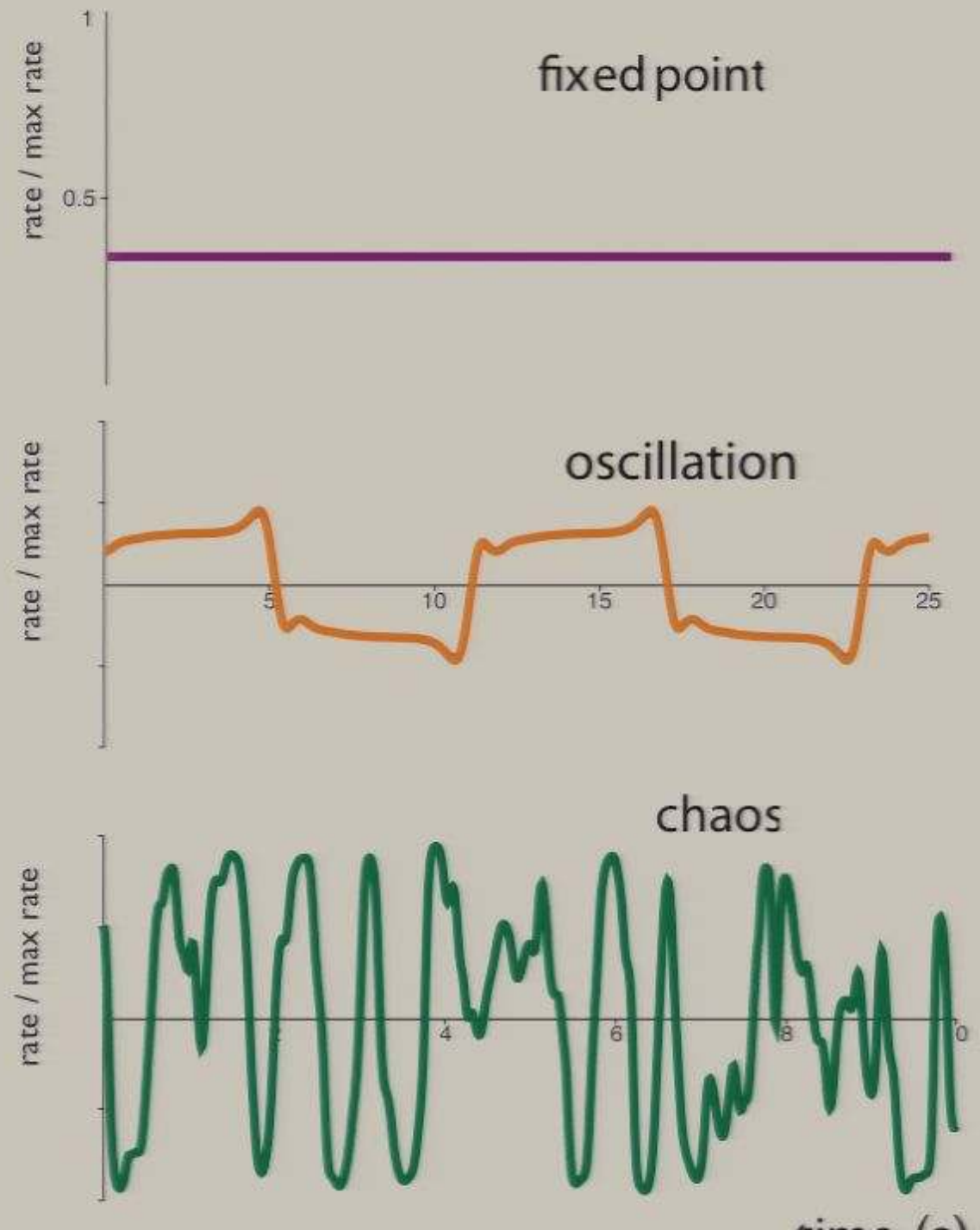

inhibitory
0
excitatory

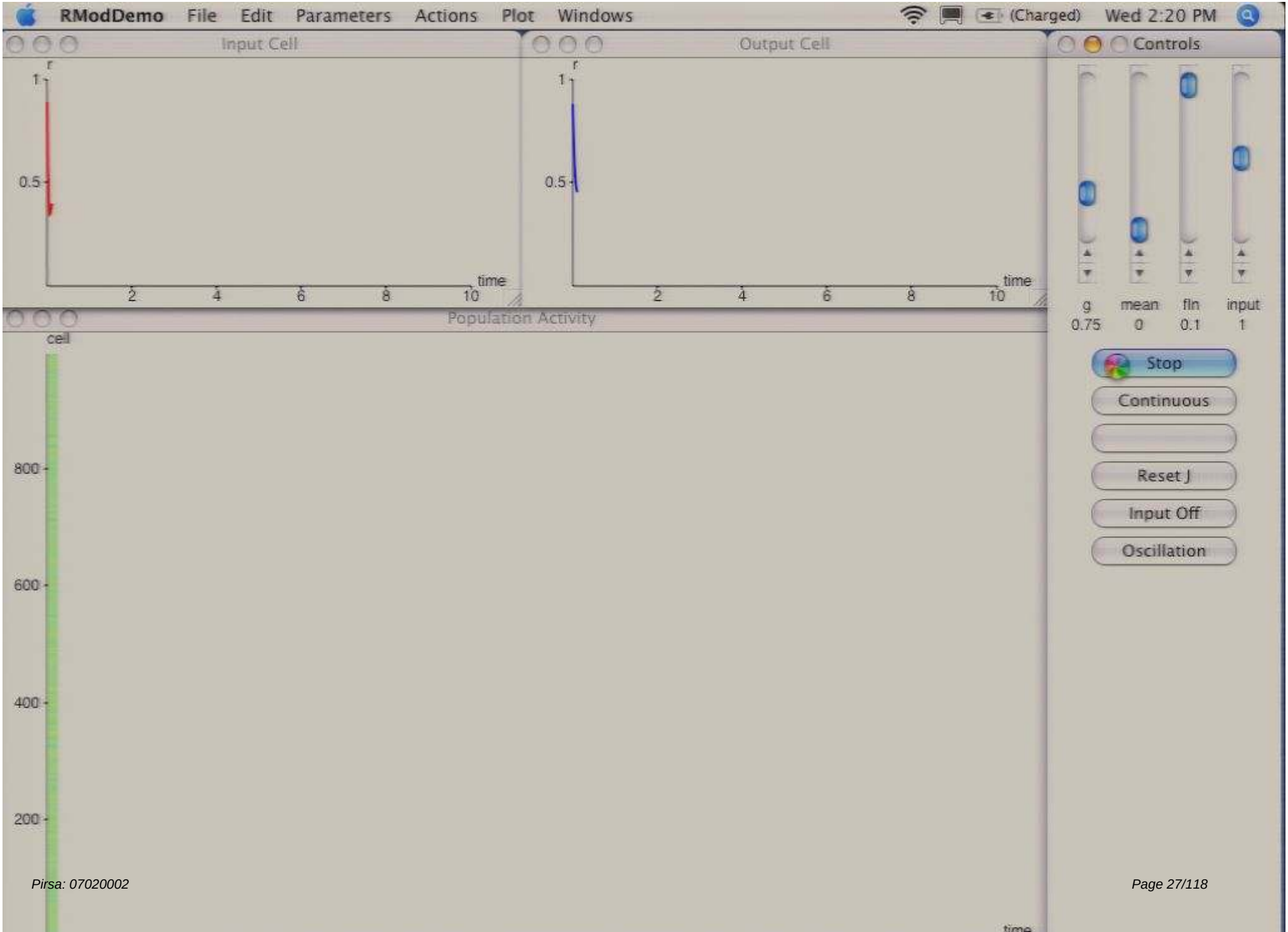

inhibitory
0
excitatory

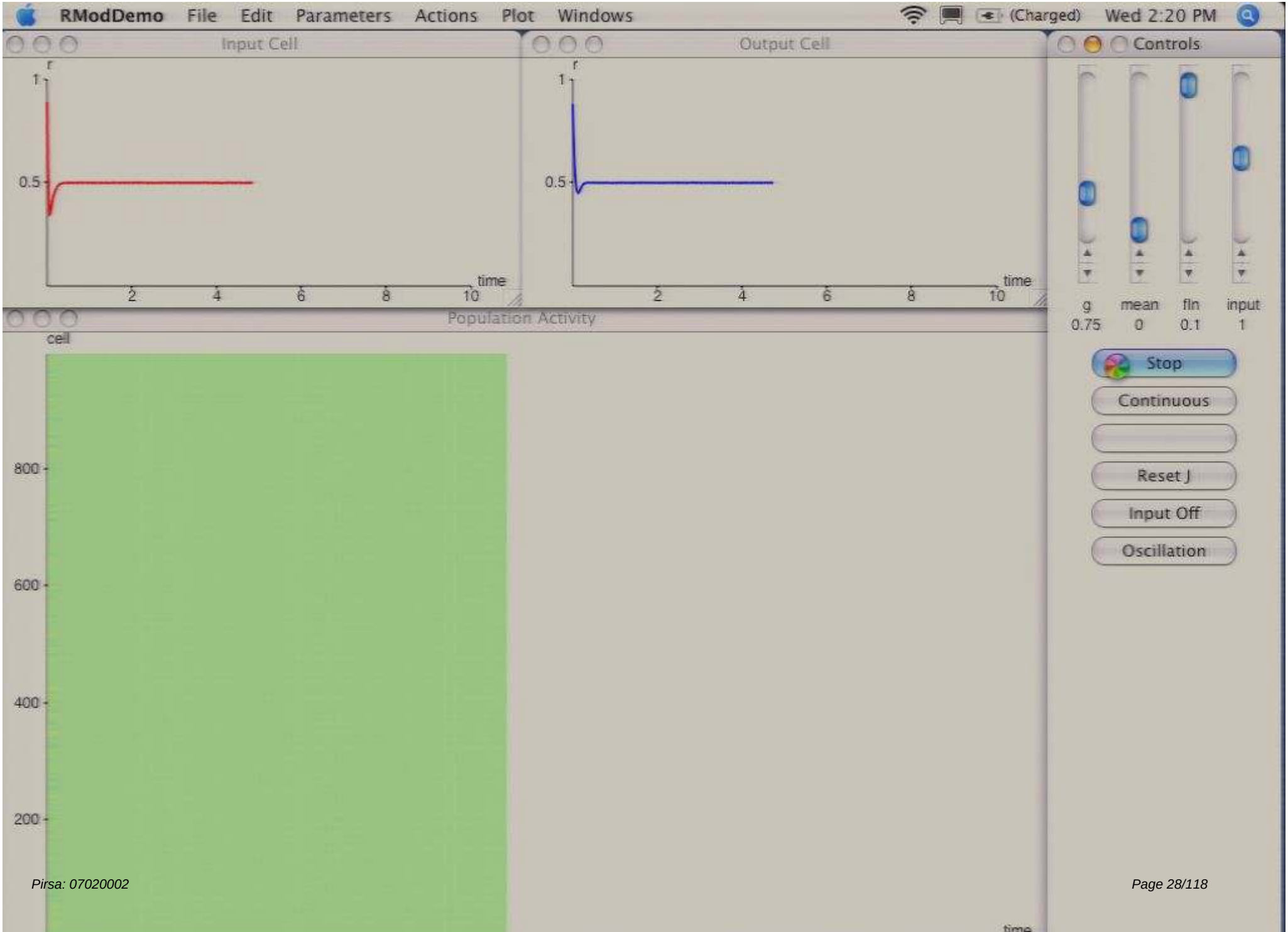
fixed point

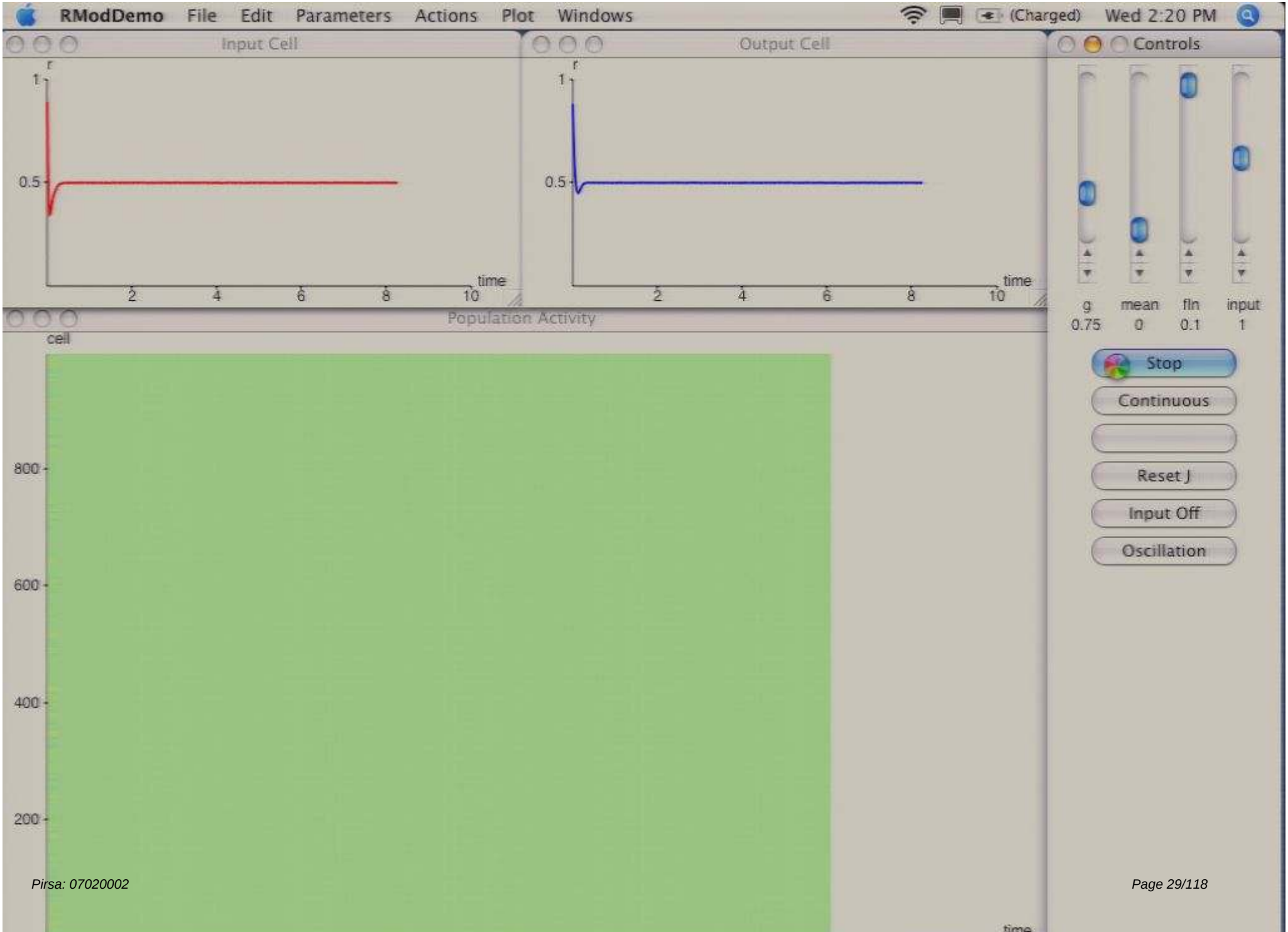

inhibitory
0
excitatory


inhibitory
0
excitatory


fixed point
rate / max rate
1
0.5
oscillation
rate / max rate
5
10
15
20
25
chaos
rate / max rate
4
6
8
0


RModDemo File Edit Parameters Actions Plot Windows (Charged) Wed 2:20 PM
Input Cell
r
1
0.5
time
2 4 6 8 10
Output Cell
r
1
0.5
time
2 4 6 8 10
Controls
g 0.75
mean 0
fln 0.1
input 1
Stop
Continuous
Reset J
Input Off
Oscillation
Population Activity
cell
800
600
400
200
time


RModDemo File Edit Parameters Actions Plot Windows (Charged) Wed 2:20 PM
Input Cell
r
1
0.5
time
2 4 6 8 10
Output Cell
r
1
0.5
time
2 4 6 8 10
Controls
g 0.75
mean 0
fln 0.1
input 1
Stop
Continuous
Reset J
Input Off
Oscillation
Population Activity
cell
800
600
400
200
time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Input Cell
r
1
0.5
2 4 6 8 10
time
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Controls
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mean 0
fln 0.1
input 1
Stop
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Oscillation
Population Activity
cell
800
600
400
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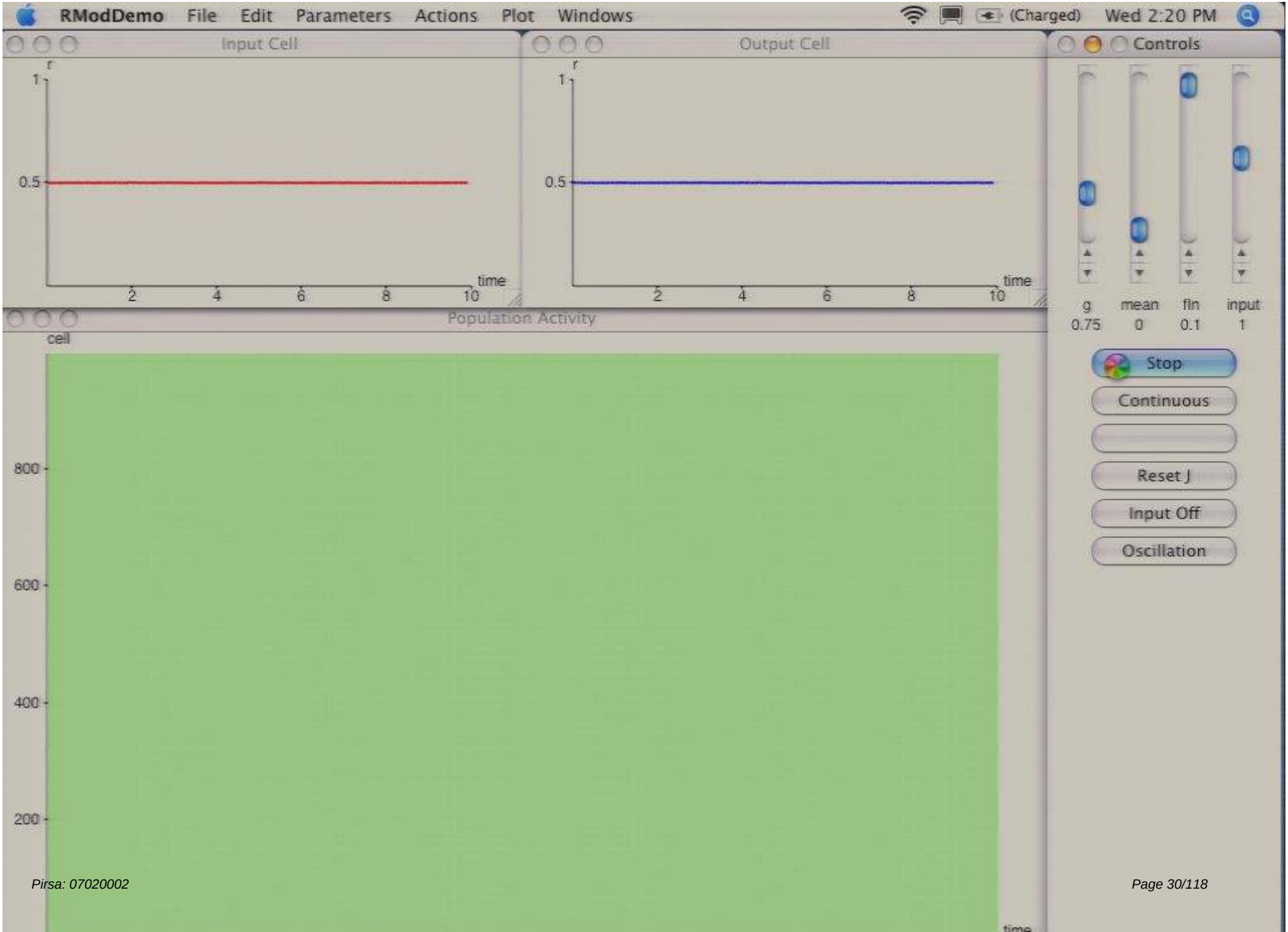
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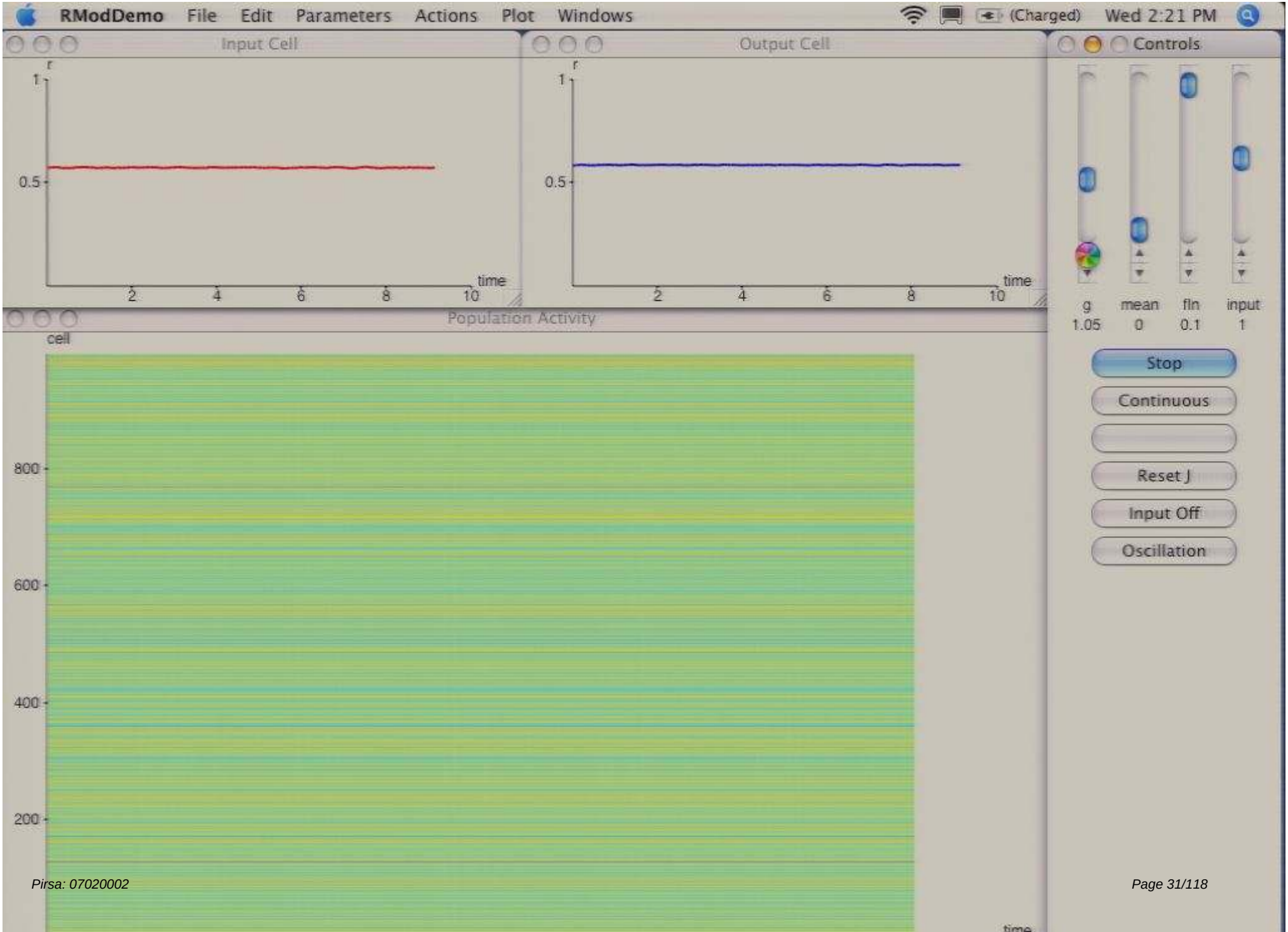
Input Cell
r
1
0.5
time
2 4 6 8 10
Output Cell
r
1
0.5
time
2 4 6 8 10
Controls
g 0.75
mean 0
fln 0.1
input 1
Stop
Continuous
Reset J
Input Off
Oscillation
Population Activity
cell
800
600
400
200
time


RModDemo File Edit Parameters Actions Plot Windows (Charged) Wed 2:21 PM
Input Cell
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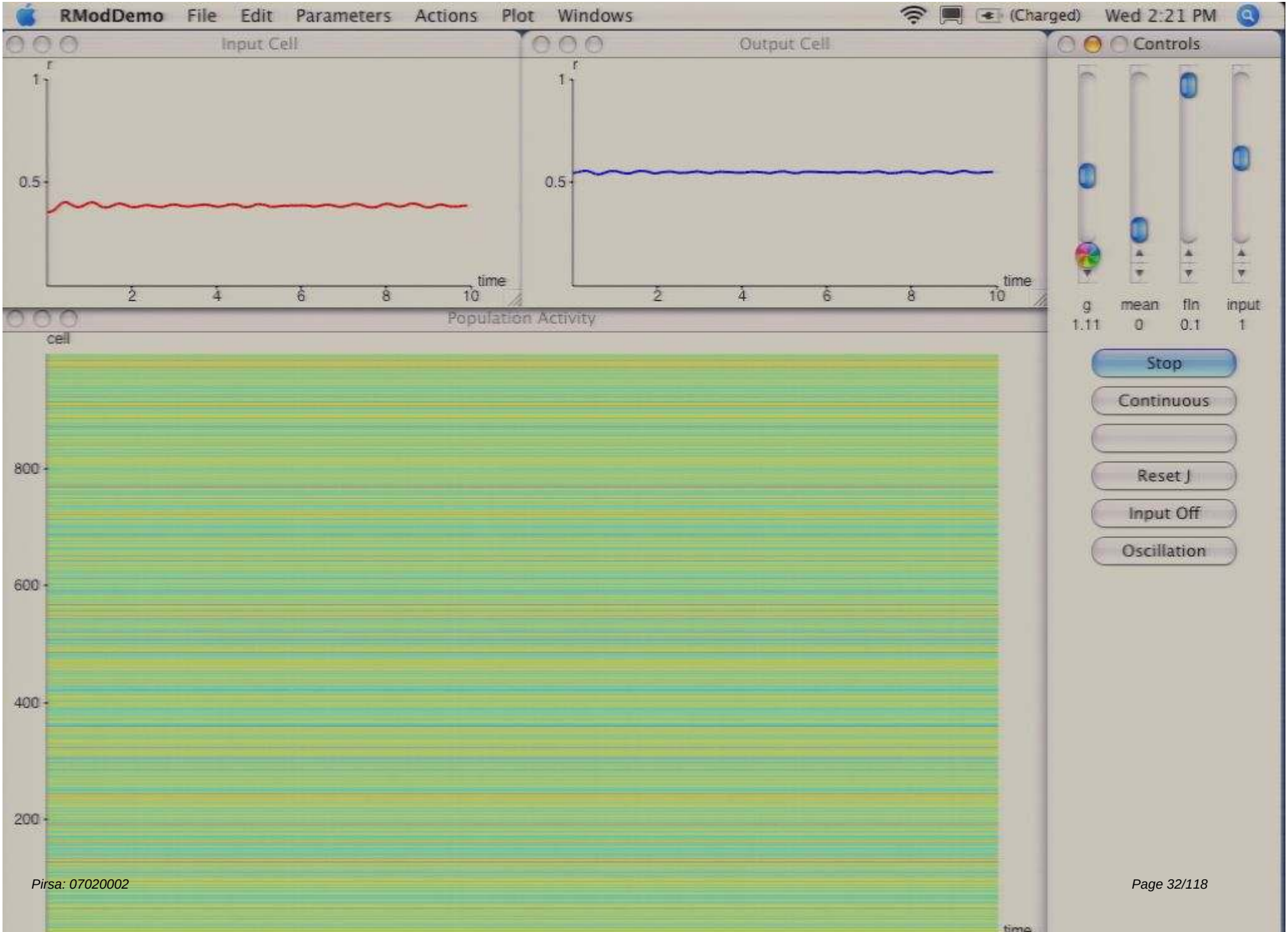
1
0.5
time
2 4 6 8 10
Output Cell
r
1
0.5
time
2 4 6 8 10
Controls
g 1.05
mean 0
fln 0.1
input 1
Stop
Continuous
Reset J
Input Off
Oscillation
Population Activity
cell
800
600
400
200
time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Input Cell
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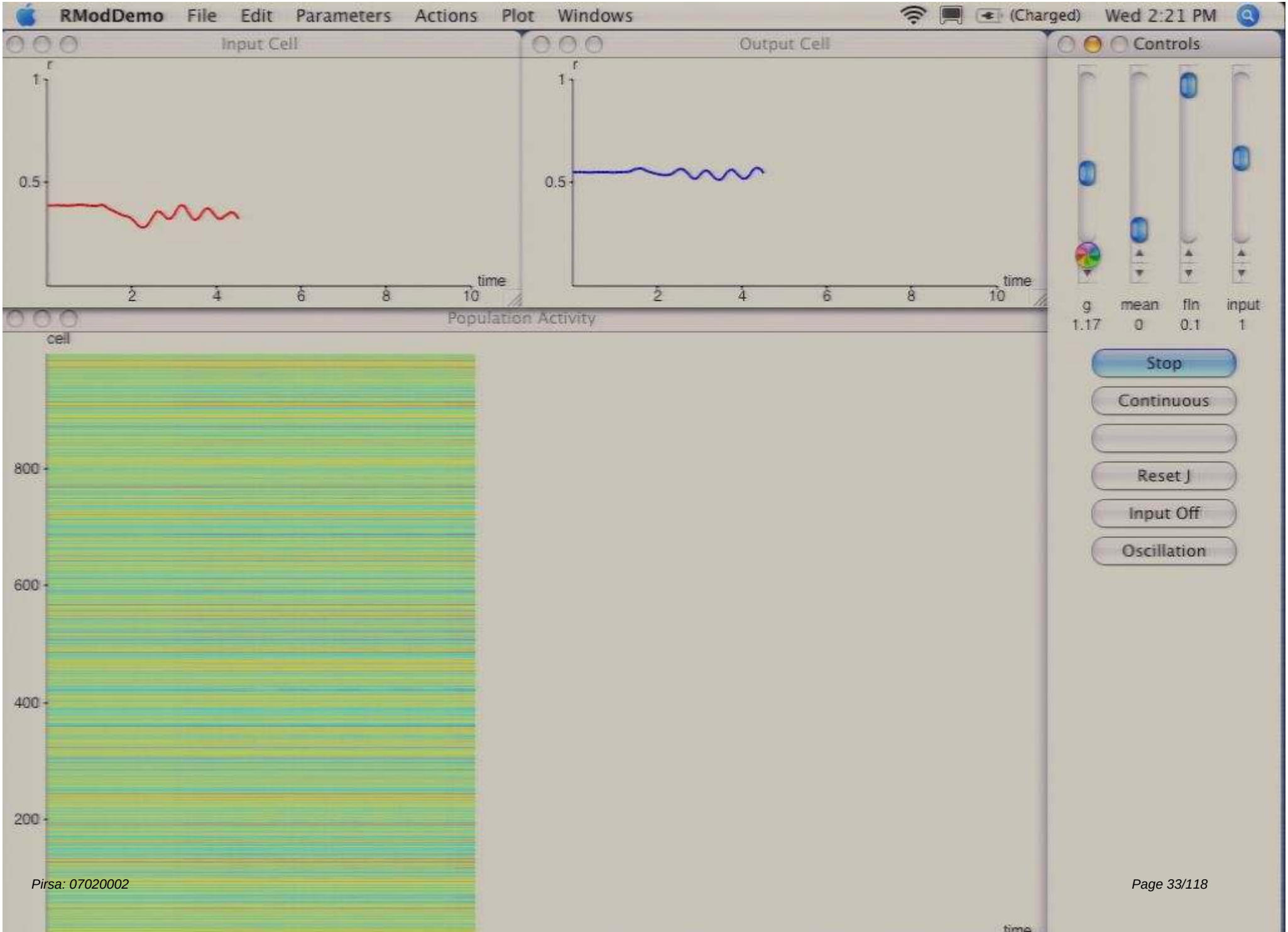
0.5
time
2 4 6 8 10
Output Cell
r
1
0.5
time
2 4 6 8 10
Controls
g 1.11
mean 0
fln 0.1
input 1
Stop
Continuous
Reset J
Input Off
Oscillation
Population Activity
cell
800
600
400
200
time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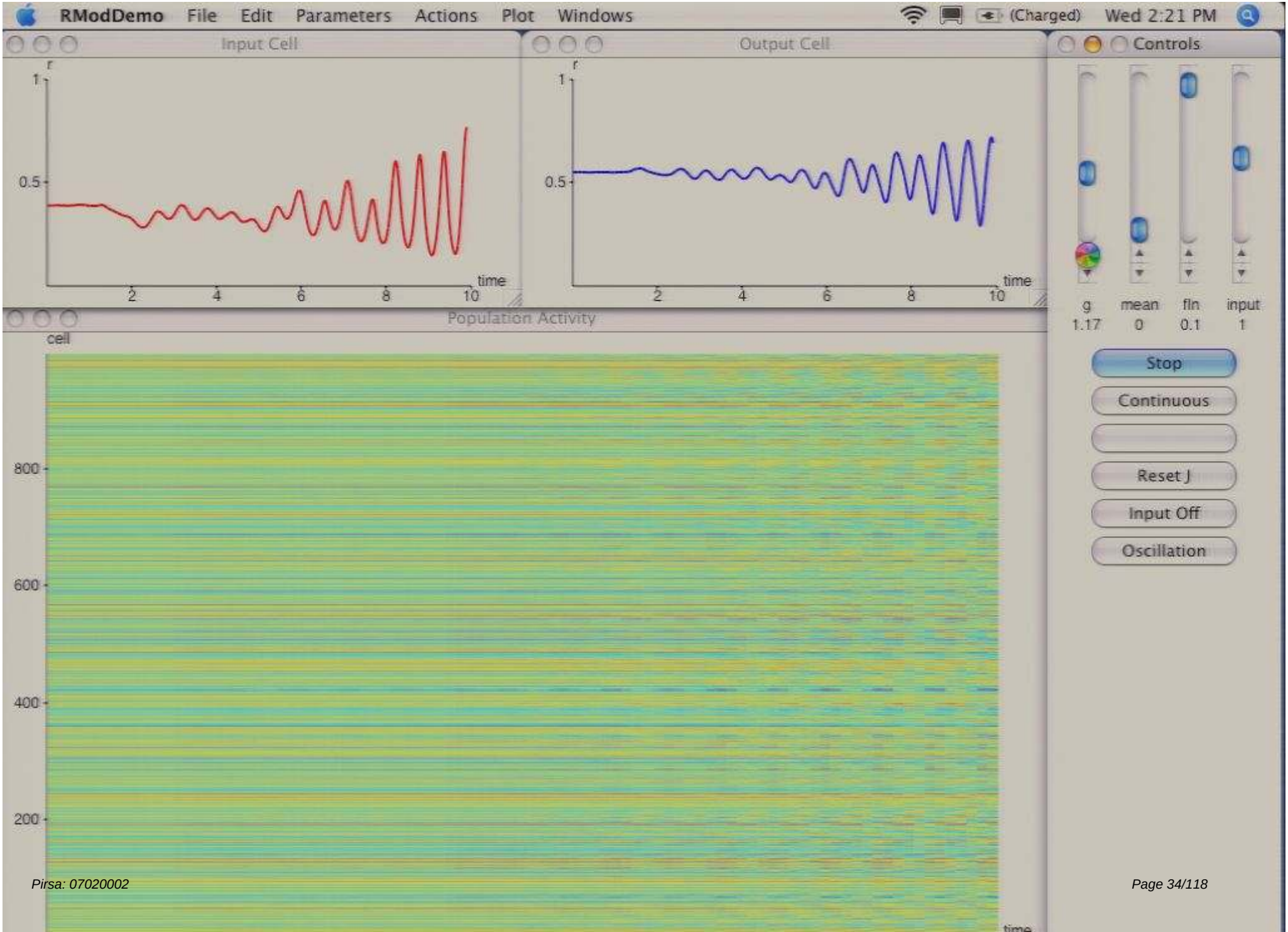
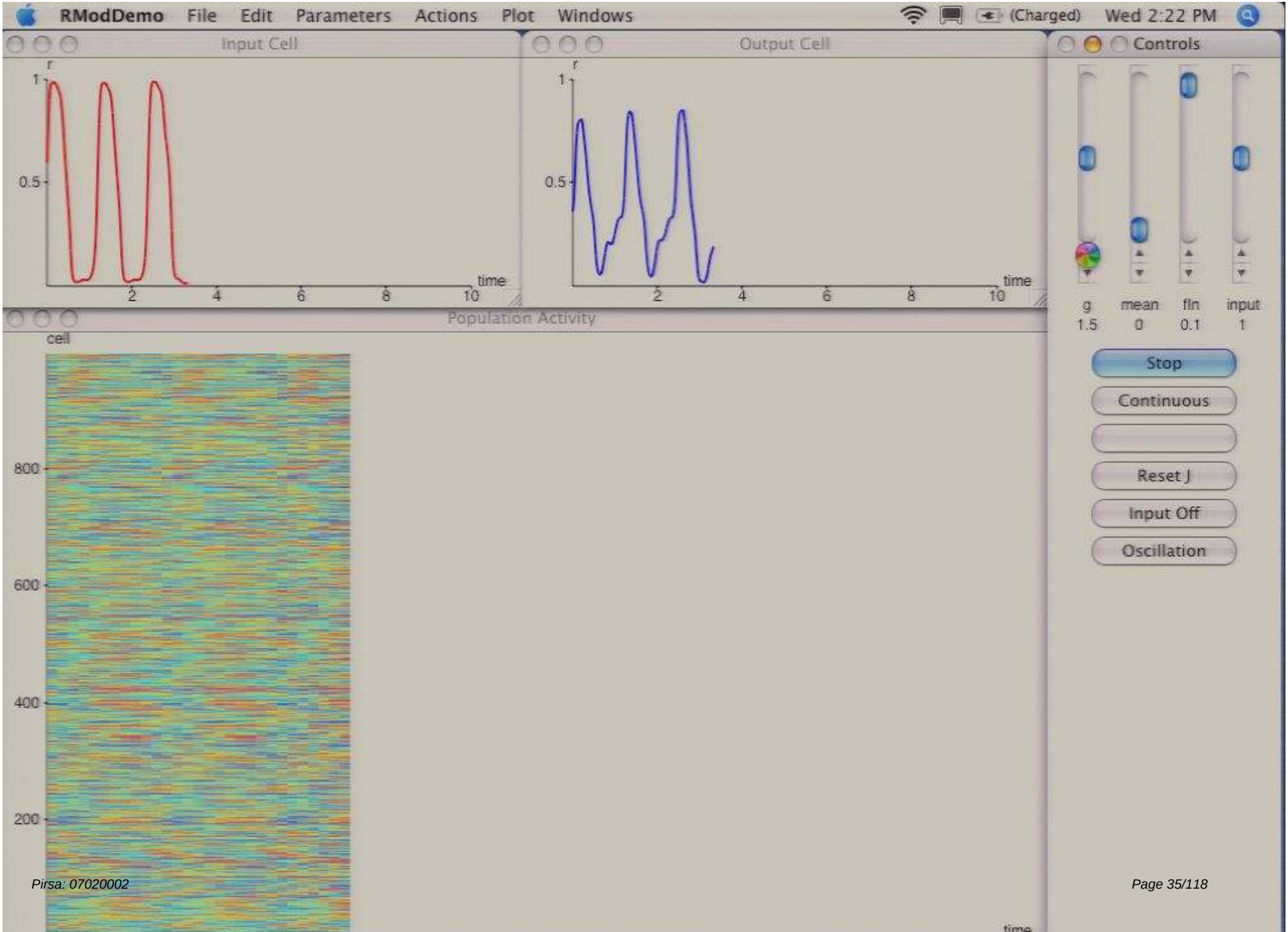
Input Cell
r
1
0.5
time
2 4 6 8 10
Output Cell
r
1
0.5
time
2 4 6 8 10
Controls
g 1.17
mean 0
fln 0.1
input 1
Stop
Continuous
Reset J
Input Off
Oscillation
Population Activity
cell
800
600
400
200
time


RModDemo File Edit Parameters Actions Plot Windows (Charged) Wed 2:21 PM
Input Cell
Output Cell
Controls
r
1
0.5
time
2 4 6 8 10
g 1.17
mean 0
fln 0.1
input 1
Stop
Continuous
Reset J
Input Off
Oscillation
Population Activity
cell
800
600
400
200
time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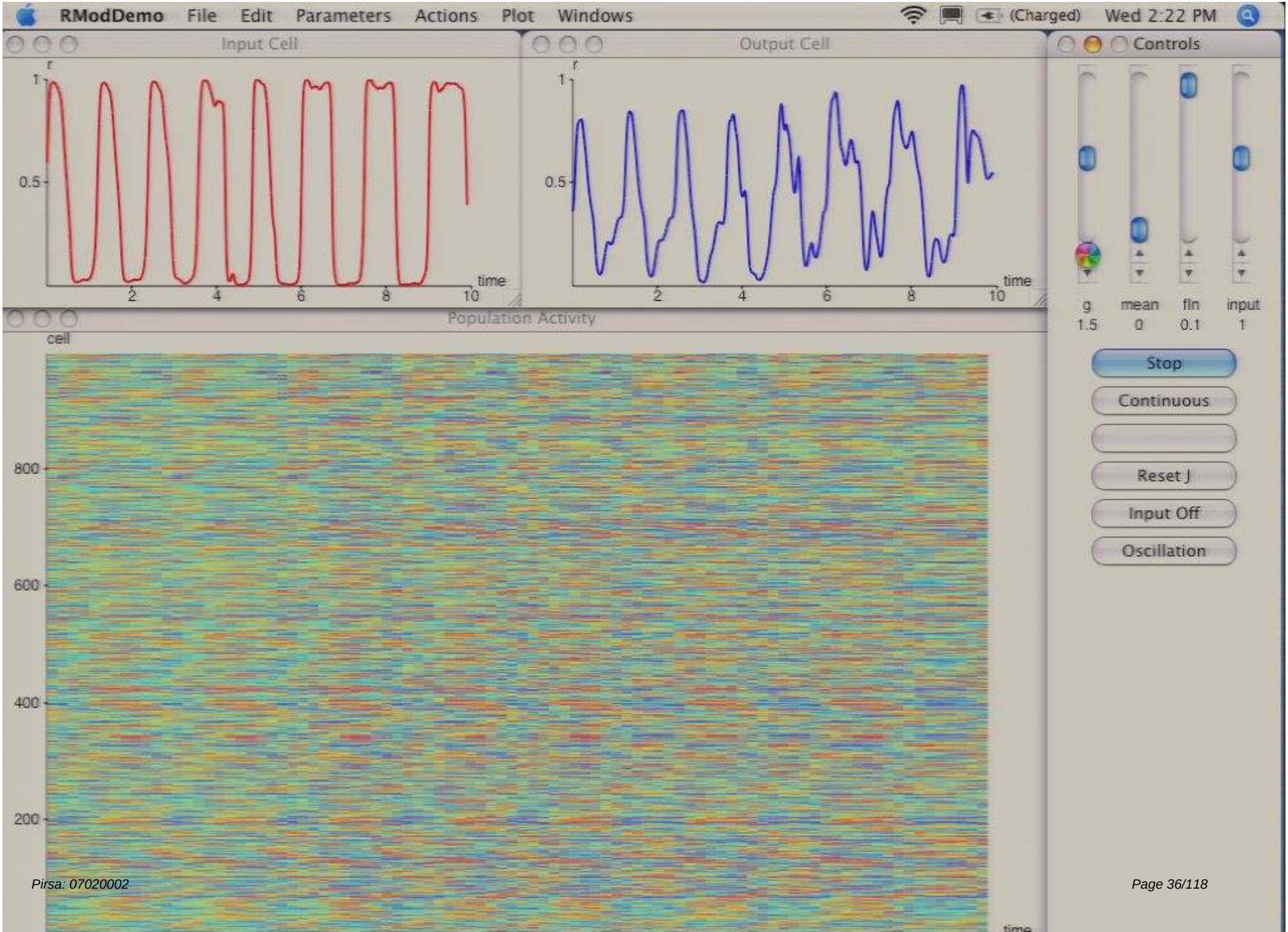
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(Charged) Wed 2:22 PM
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input 1
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Continuous
Reset J
Input Off
Oscillation
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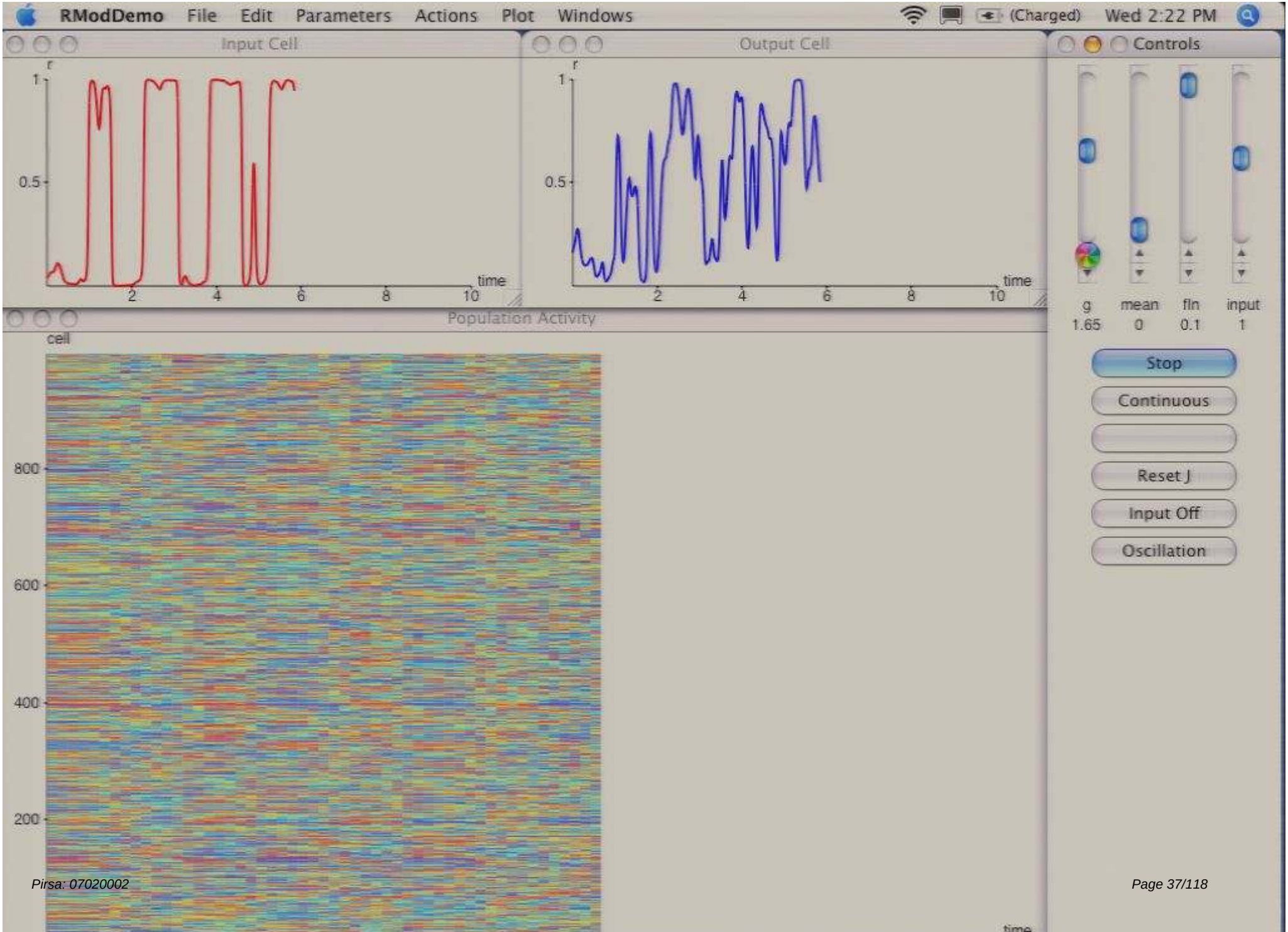
cell
800
600
400
200
time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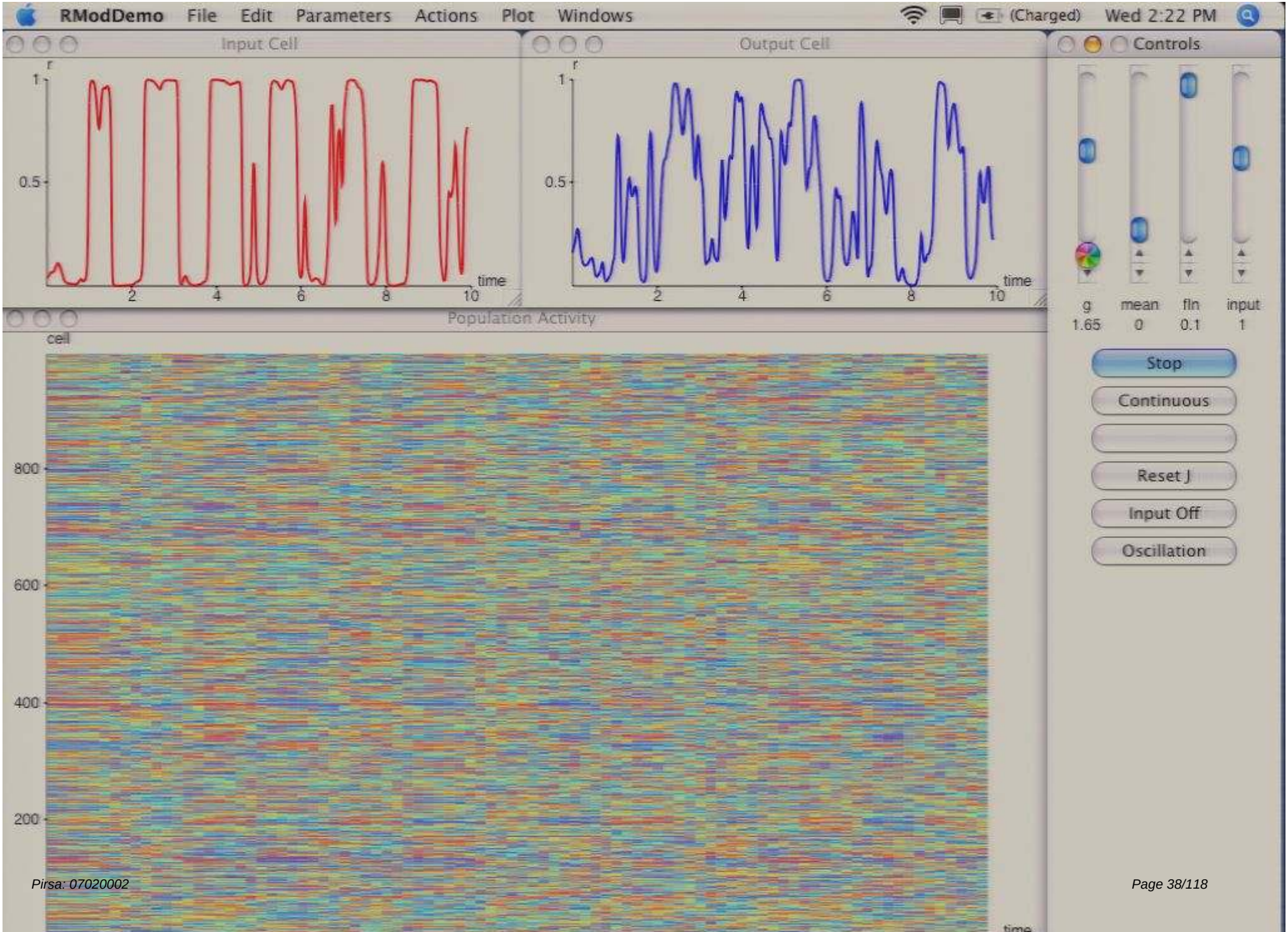
Input Cell
Output Cell
Controls
r
time
Population Activity
cell
g 1.65
mean 0
fln 0.1
input 1
Stop
Continuous
Reset J
Input Off
Oscillation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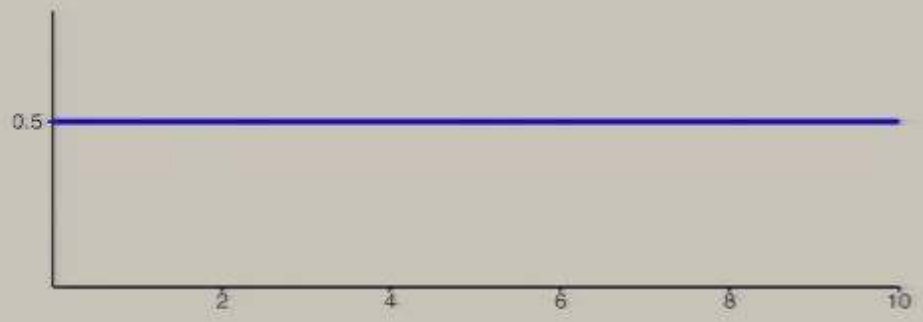
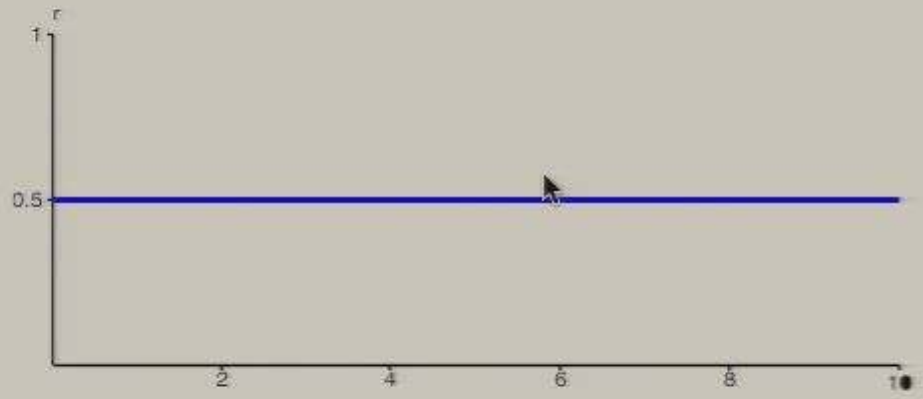
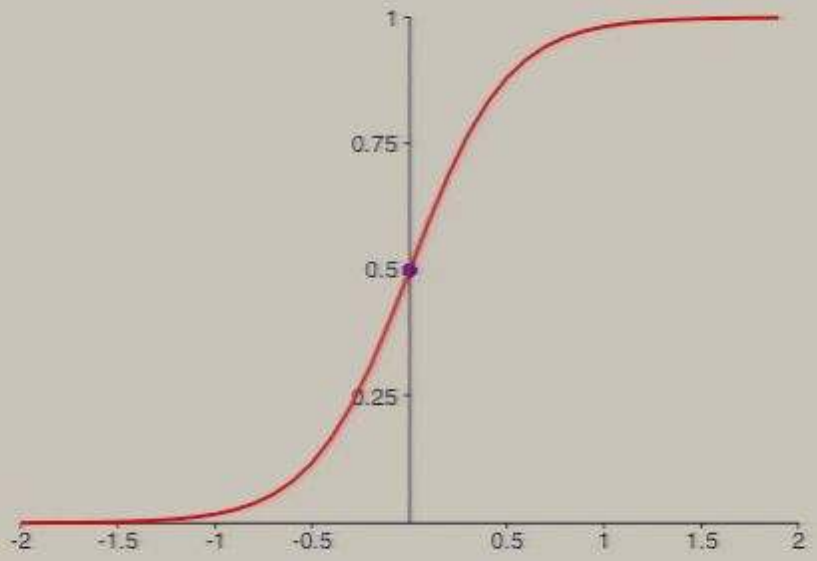
Input Cell
r
1
0.5
time
2 4 6 8 10
Output Cell
r
1
0.5
time
2 4 6 8 10
Controls
g mean fln input
1.65 0 0.1 1
Stop
Continuous
Reset J
Input Off
Oscillation
Population Activity
cell
800
600
400
200
time

trivial fixed point







$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} f(x_j)$$

$$x_i = \bar{x}_i$$

$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} f(x_j)$$

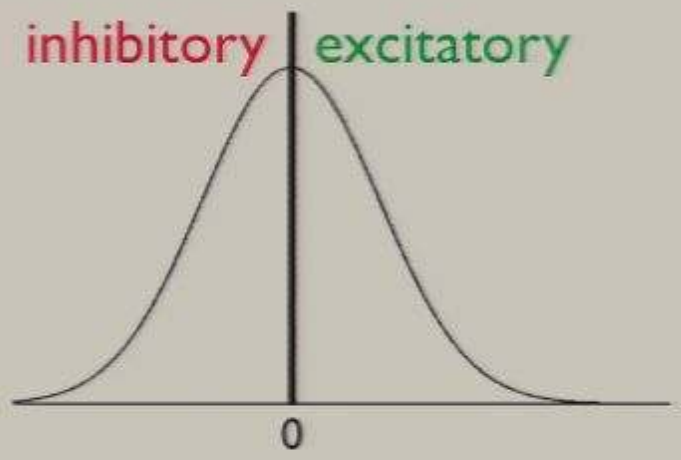
$$x_i = \bar{x}_i$$

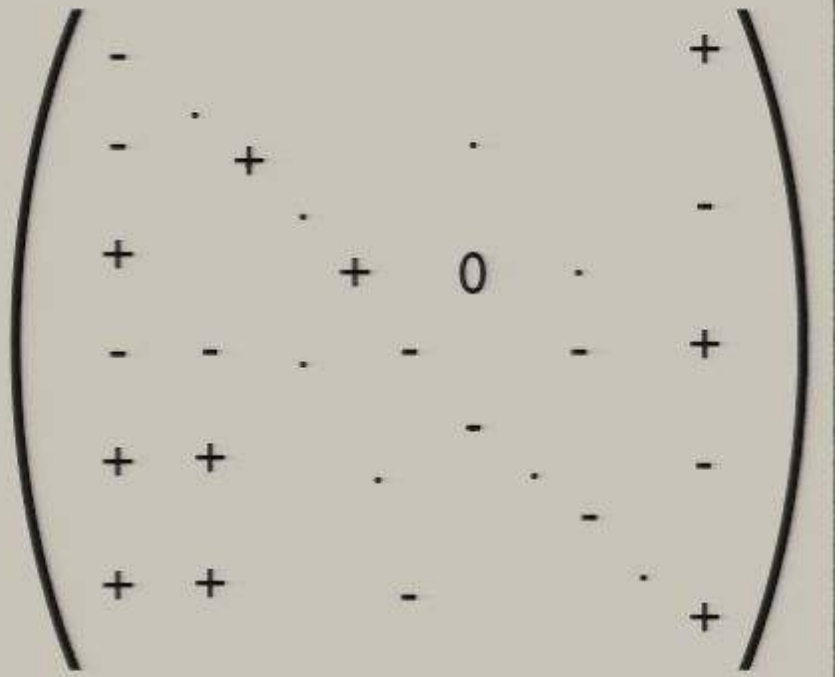
$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} f(x_j)$$

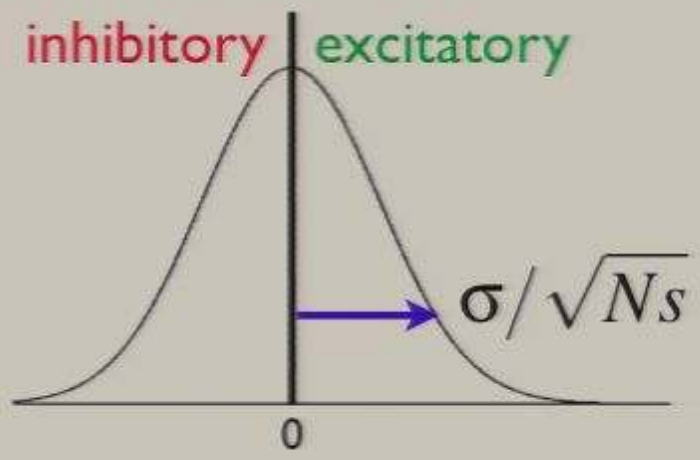
$$x_i = \bar{x}_i$$

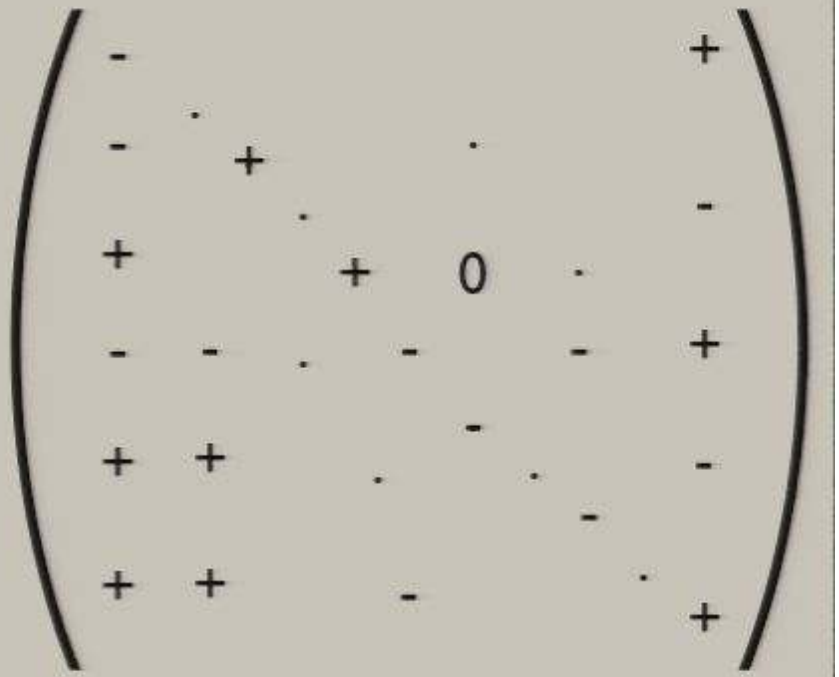
stability matrix

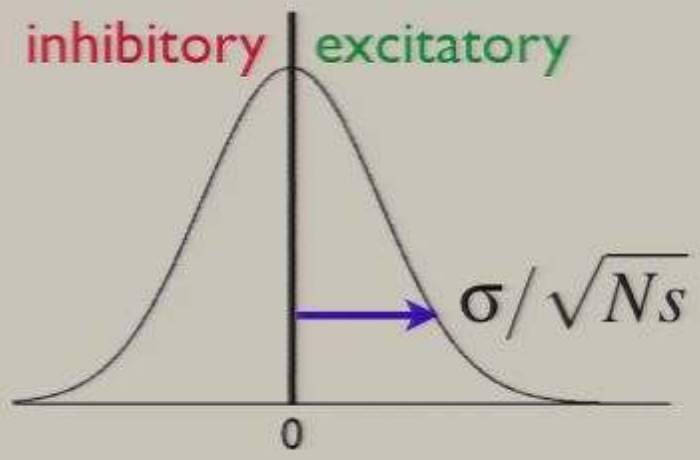
$$M_{ij} = J_{ij} f'(\bar{x}_j)$$

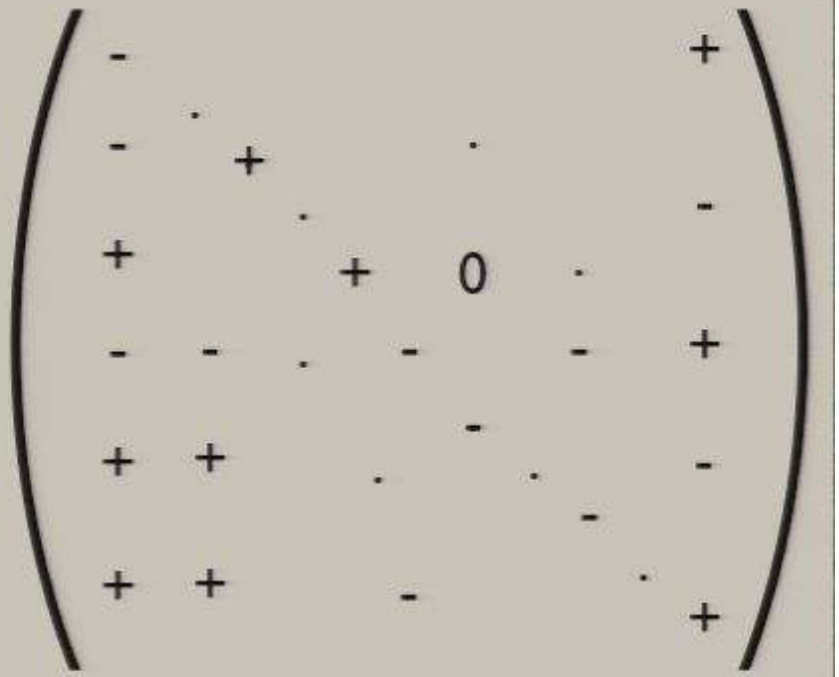

inhibitory
excitatory
0

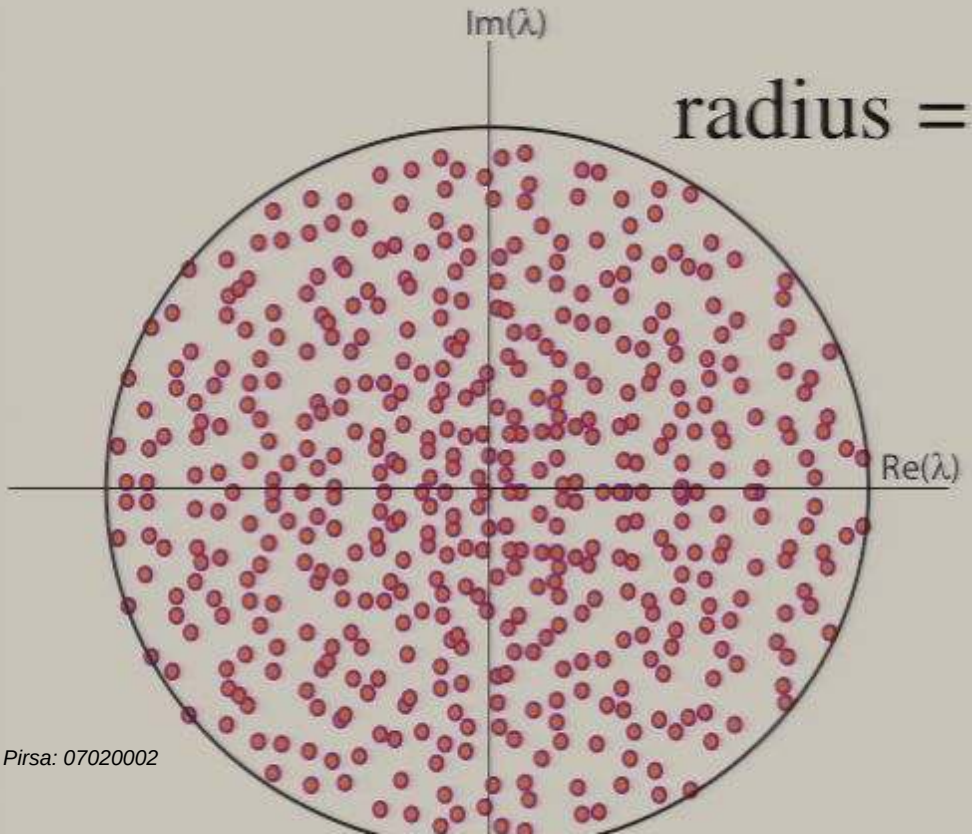



inhibitory
excitatory
$\sigma/\sqrt{Ns}$
0

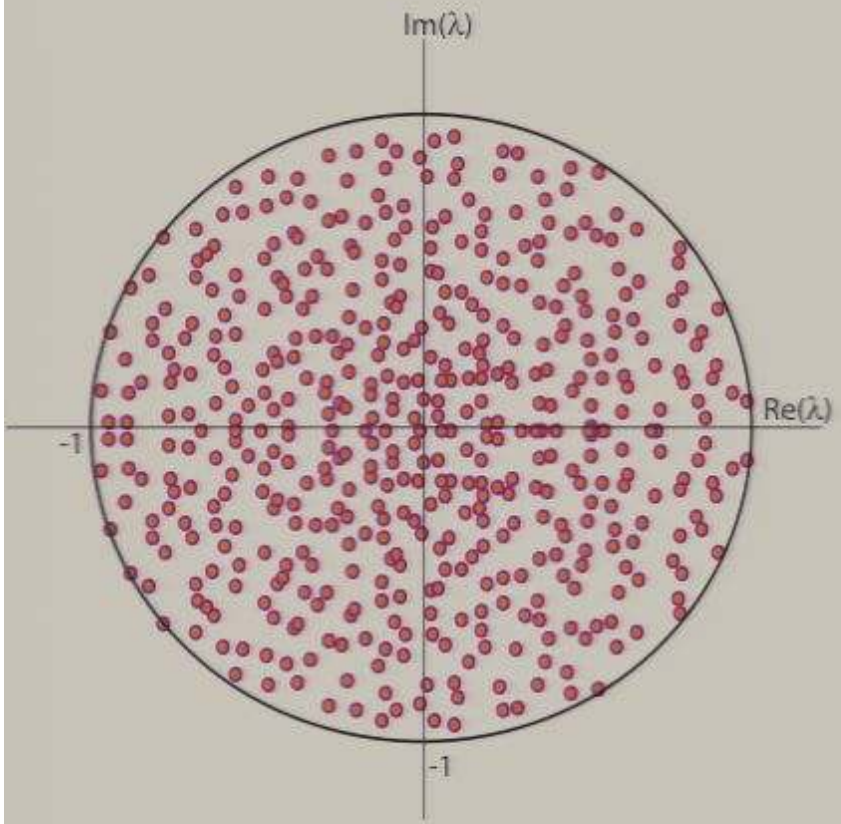


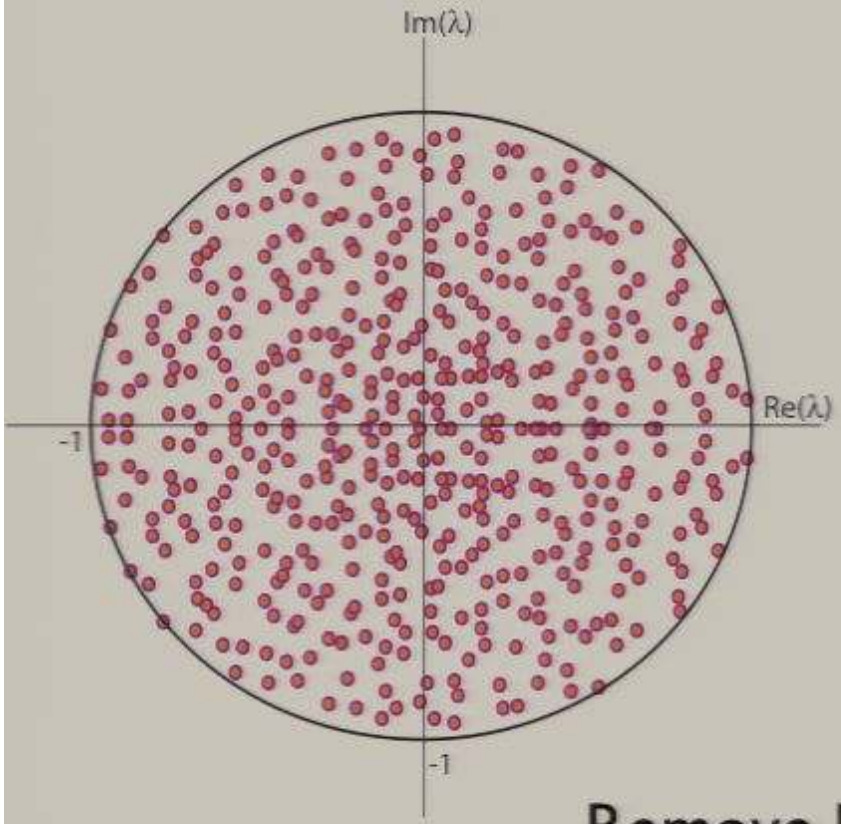




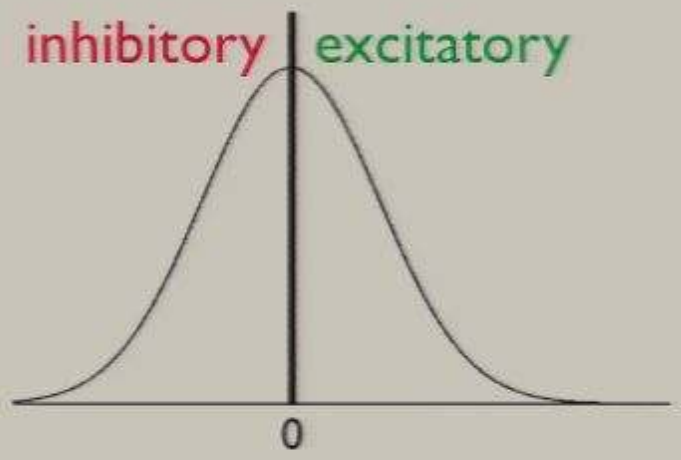


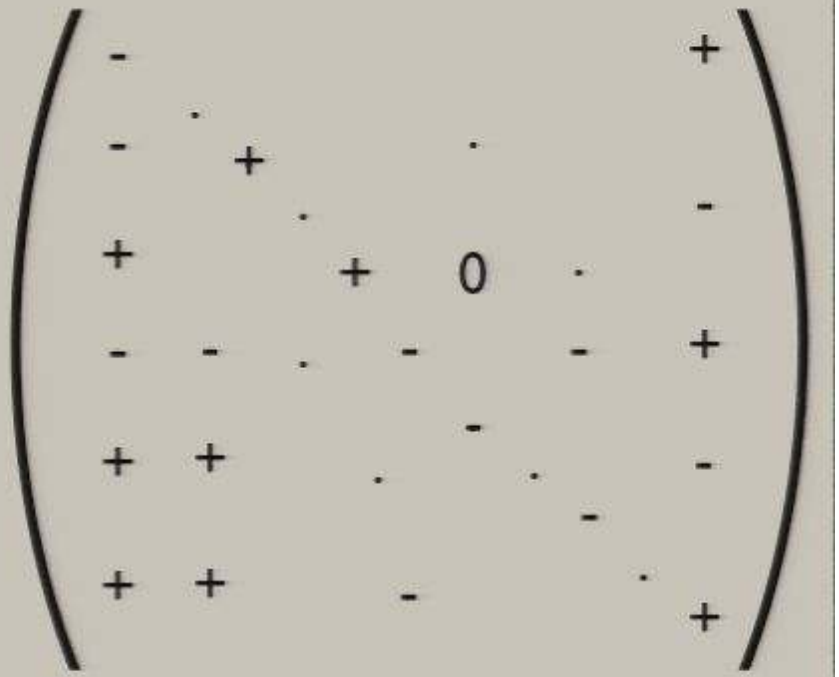
Girko, 1985.

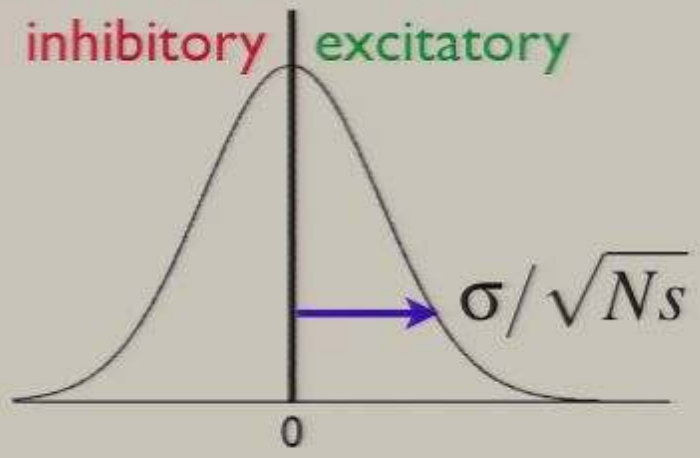

Im(λ)
Re(λ)
-1
-1

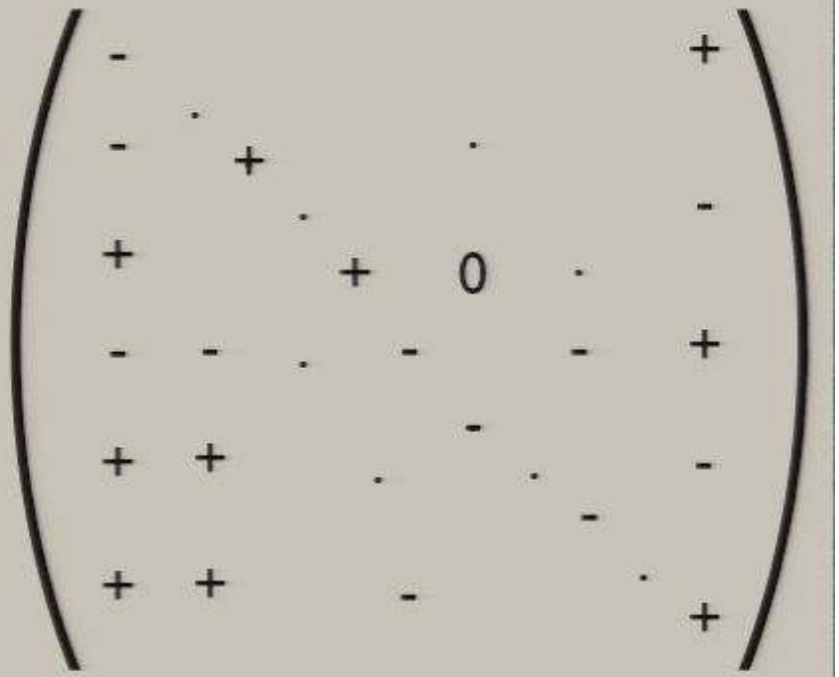


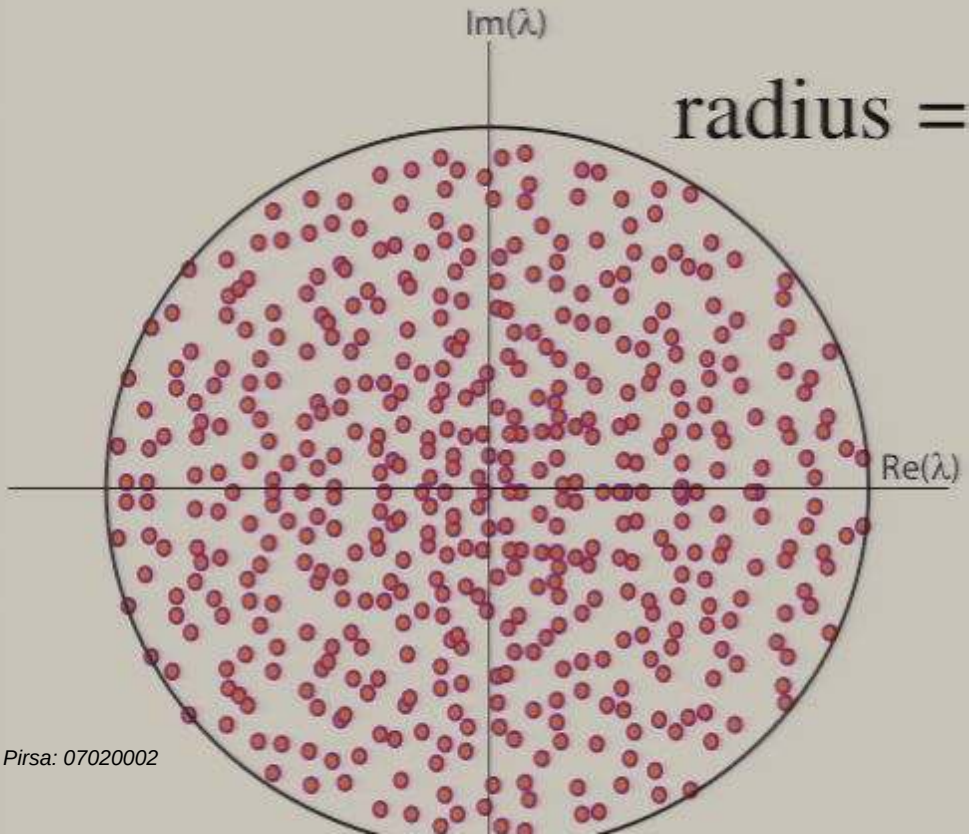
Remove lower 1/2
Flip
Add 1
*1/time constant


inhibitory
excitatory
0



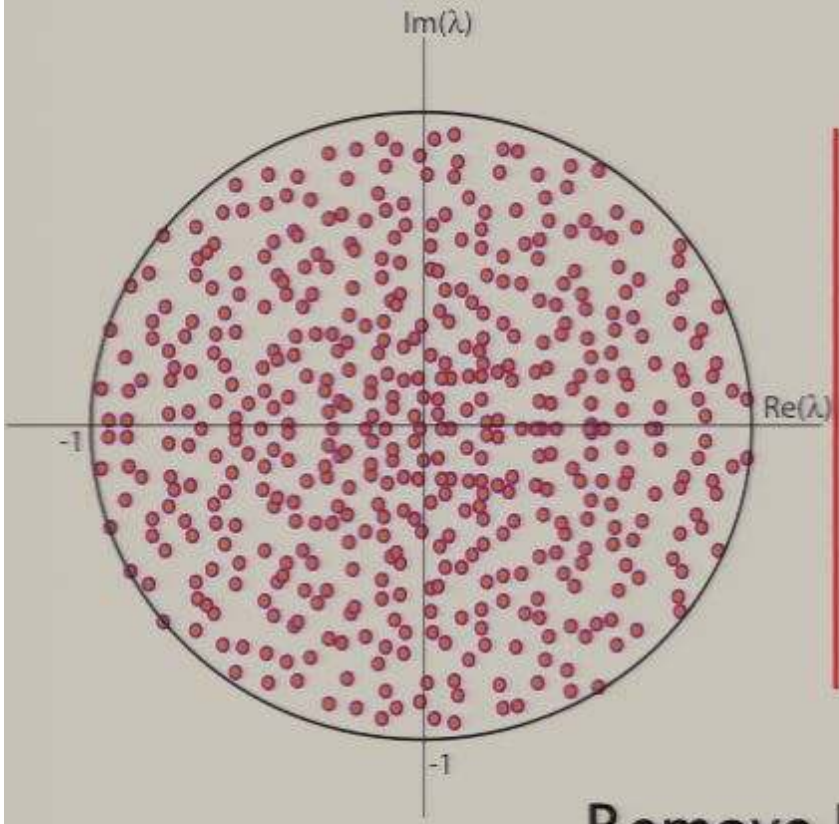


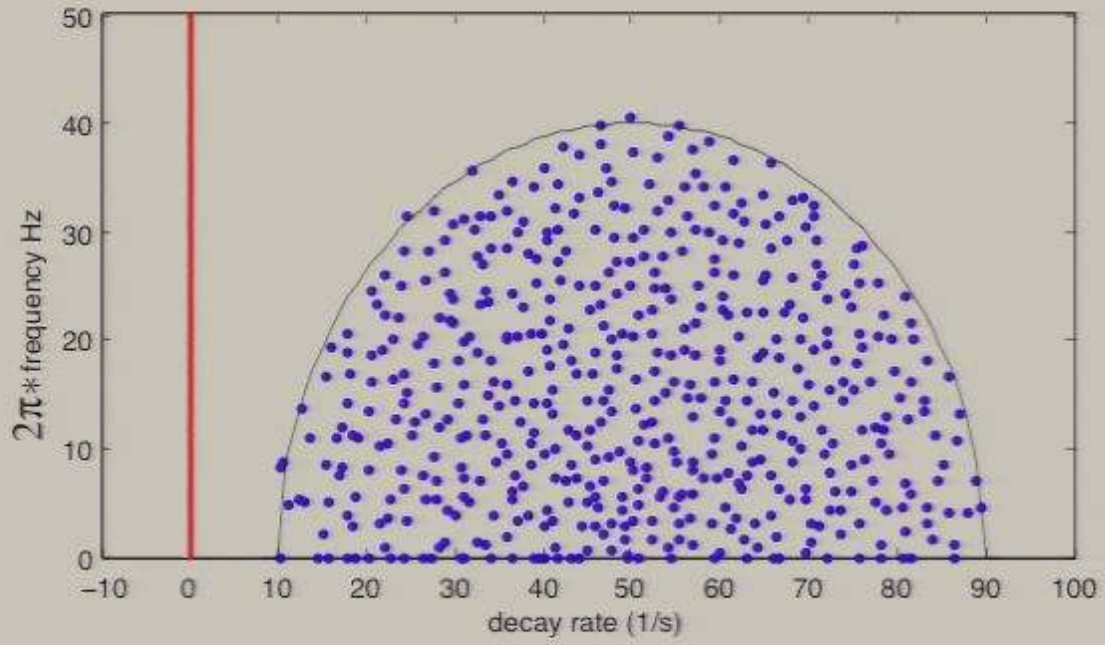




radius = σ

Girko, 1985.





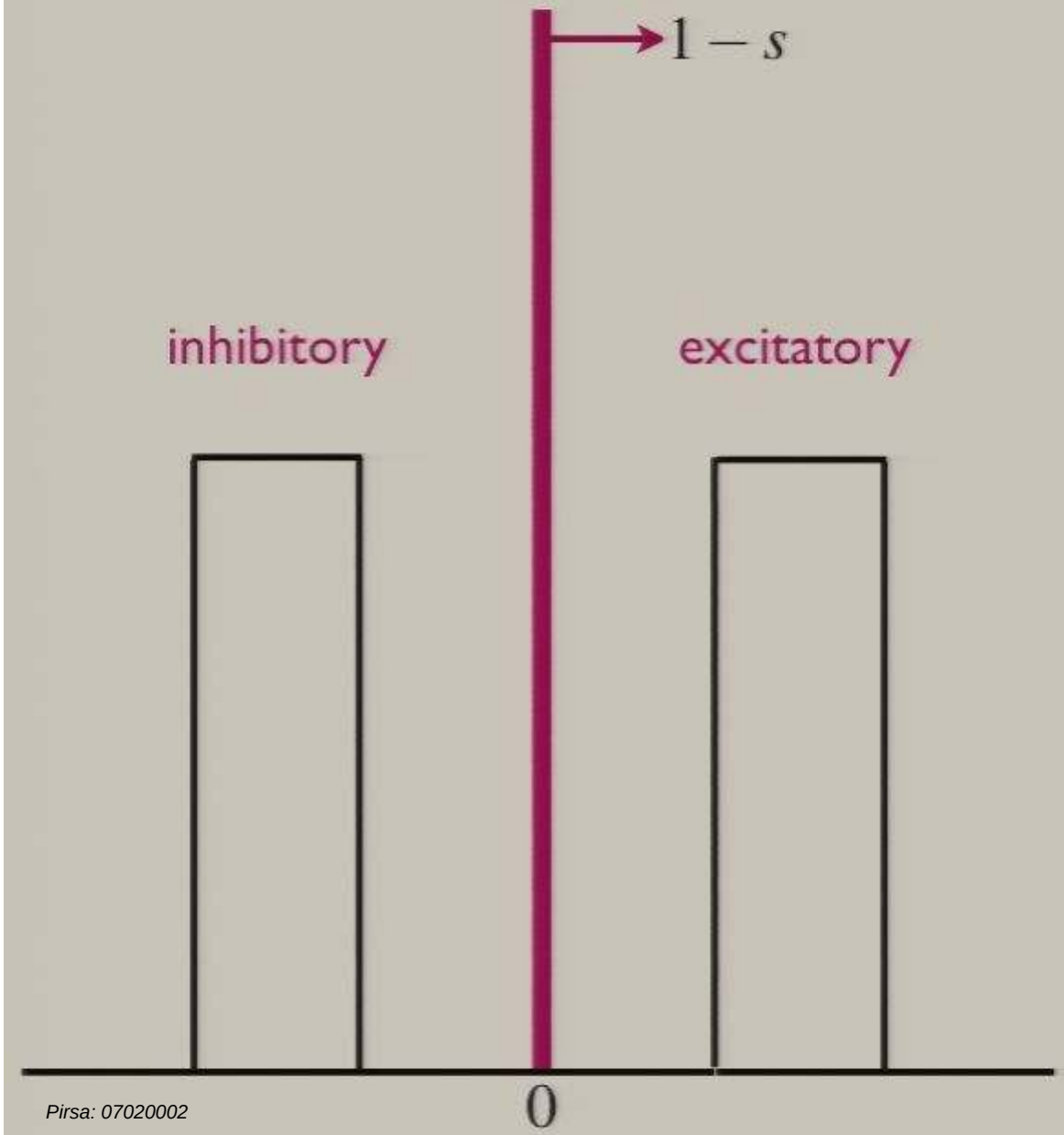
Remove lower 1/2

Flip

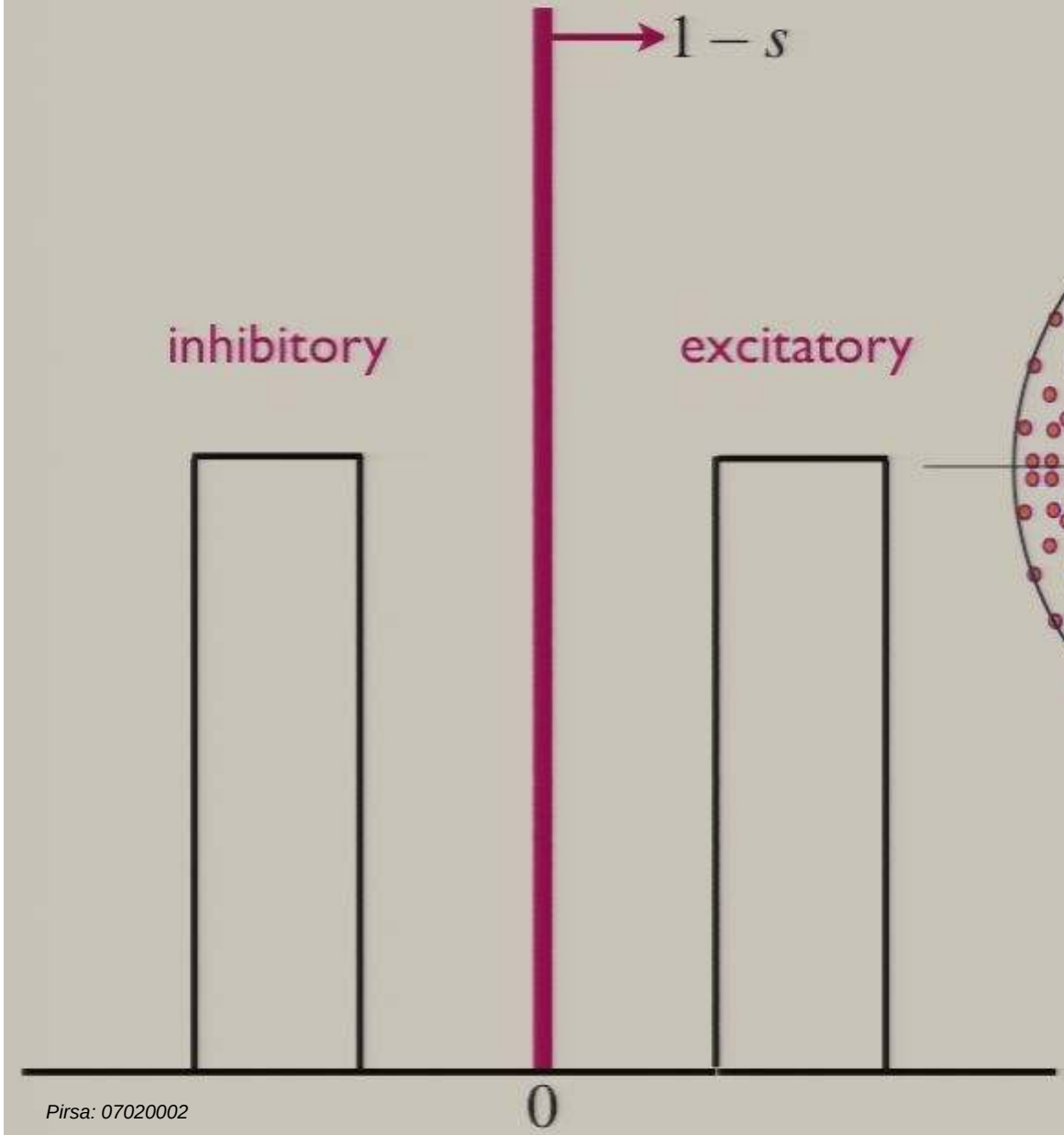
Add 1

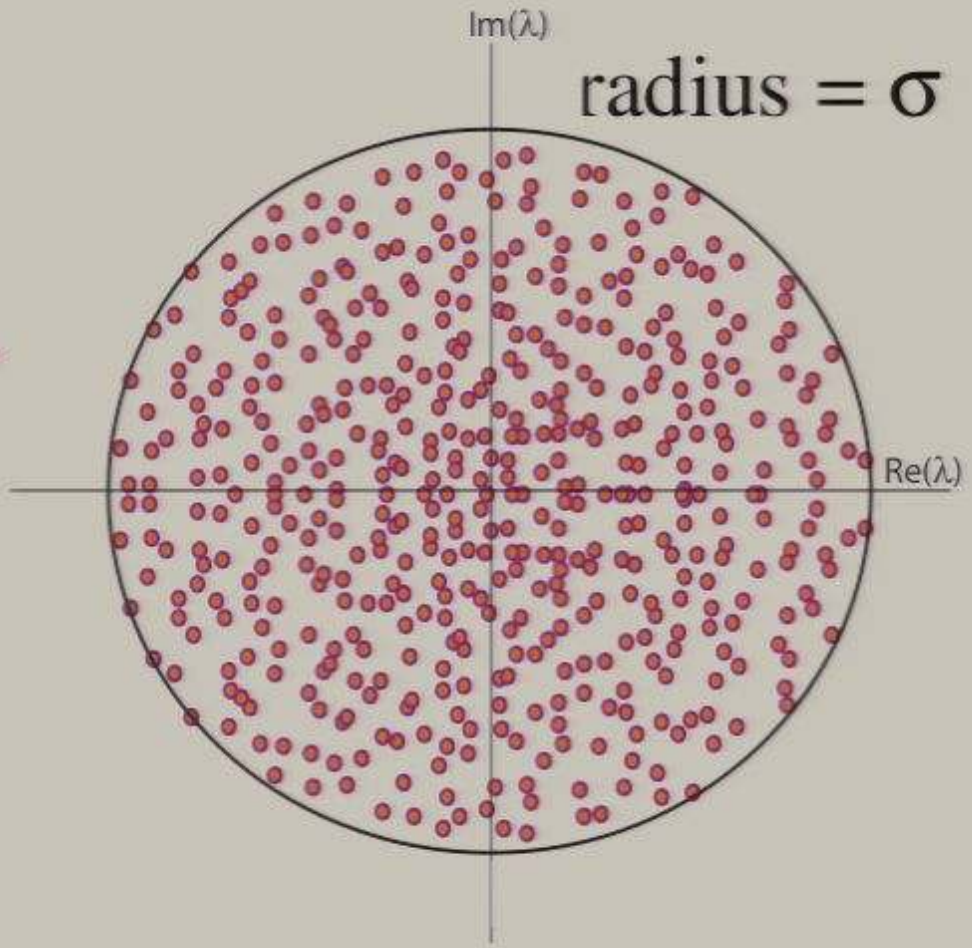
*1/time constant

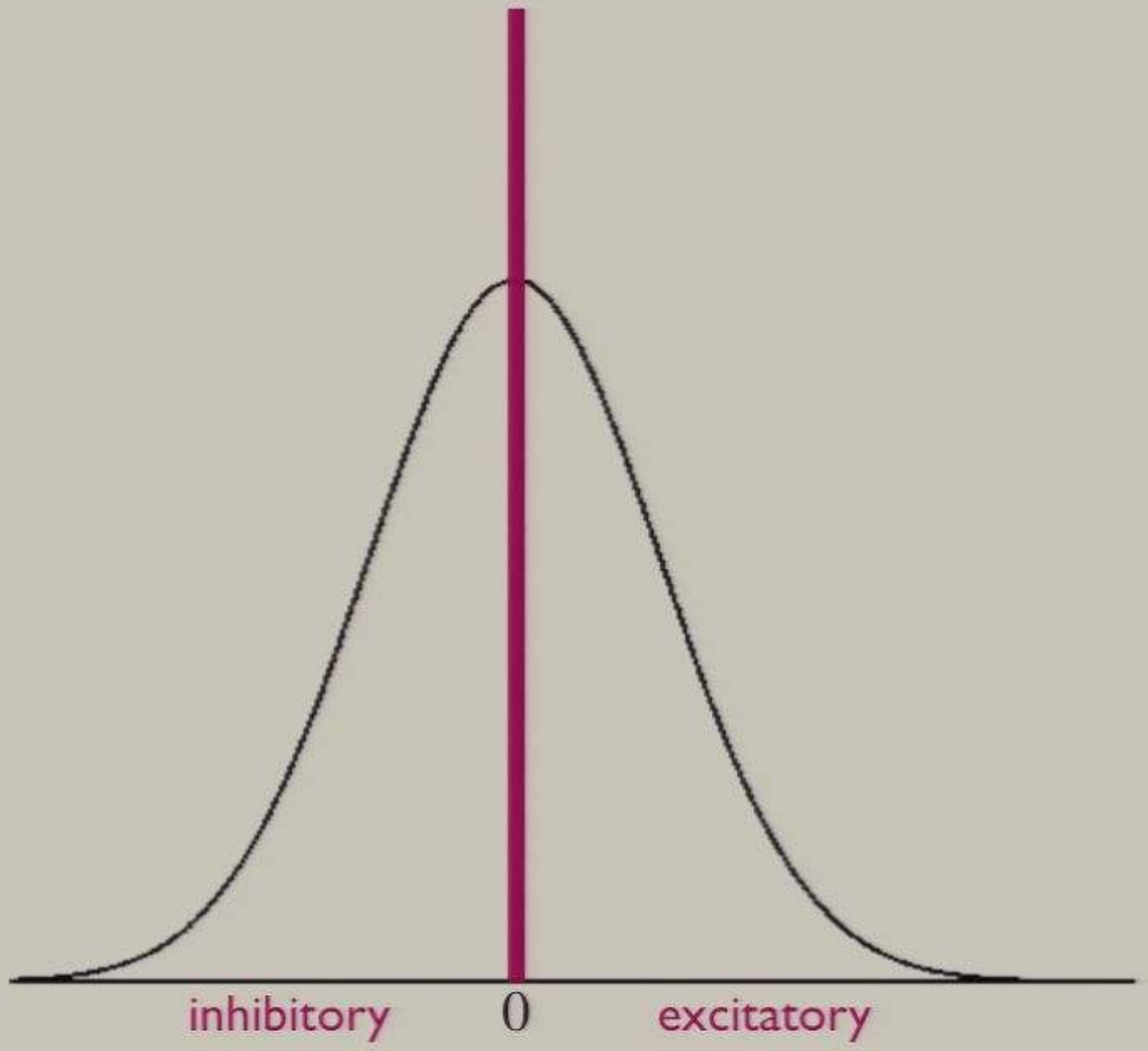
distribution of synaptic strengths

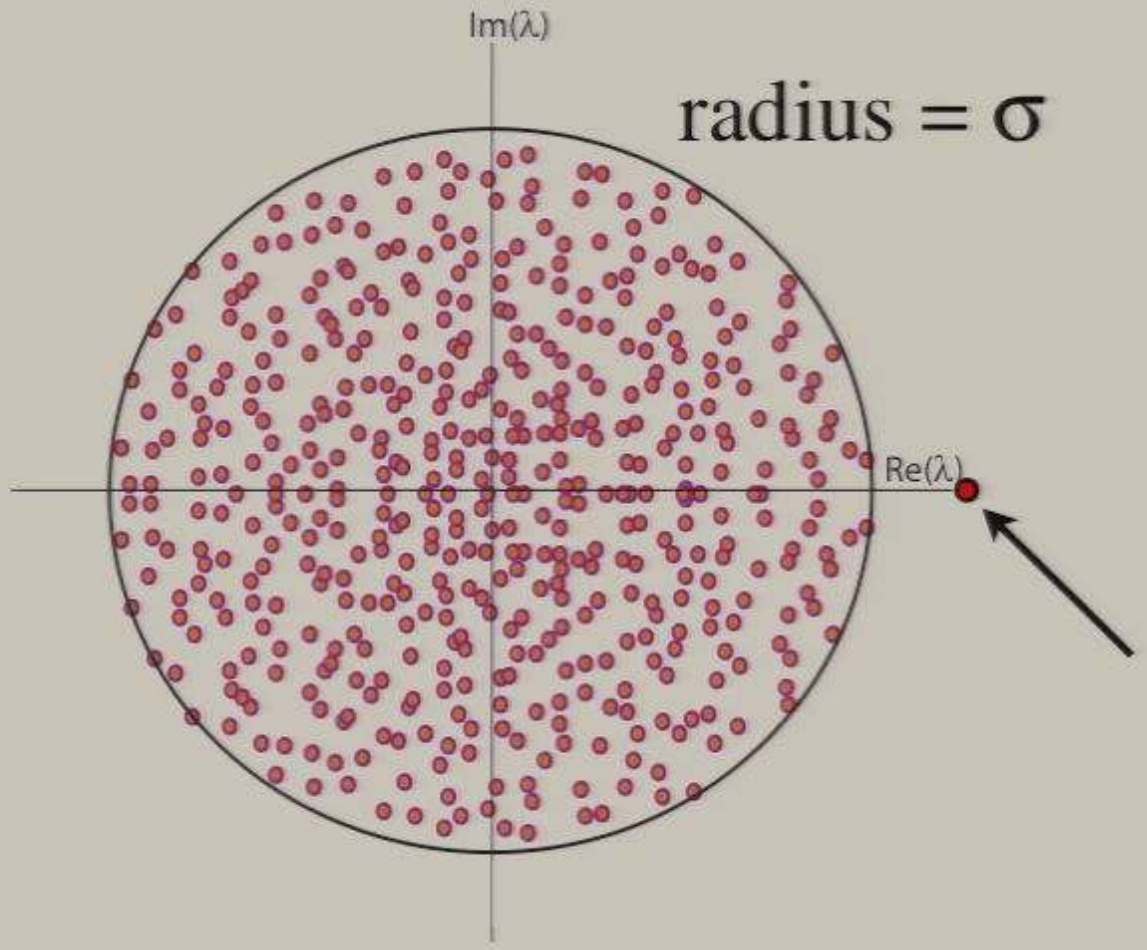


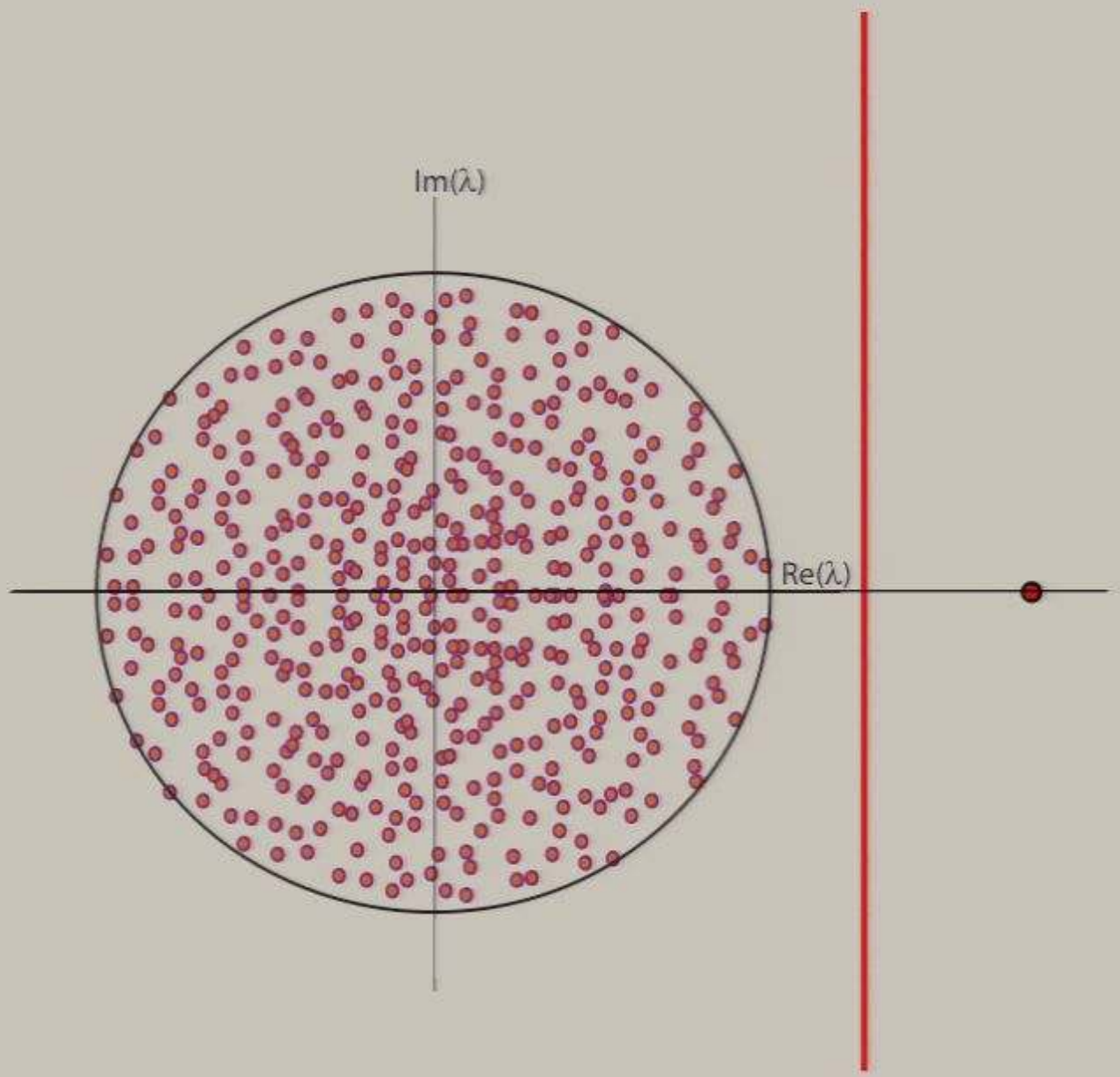
distribution of synaptic strengths

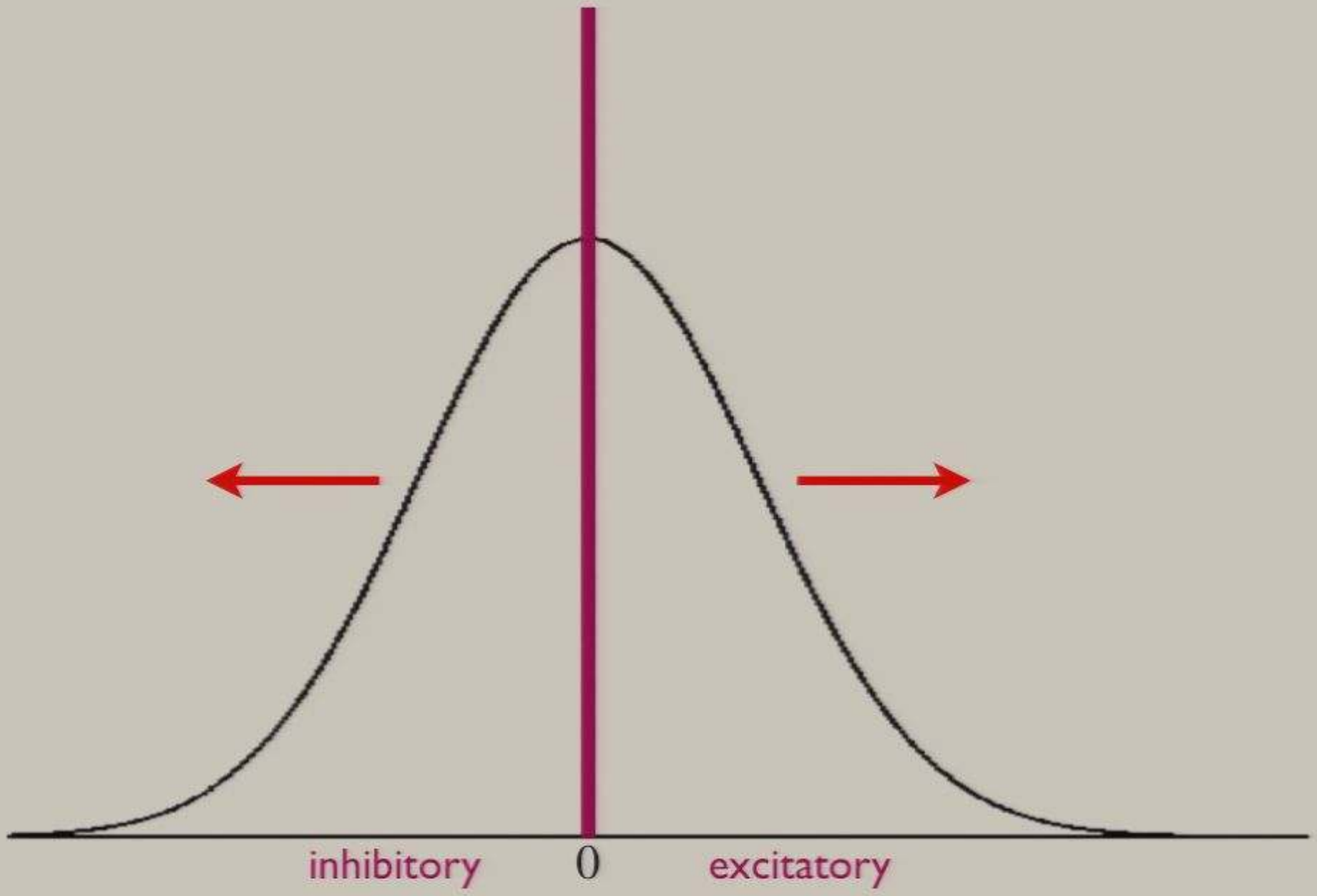


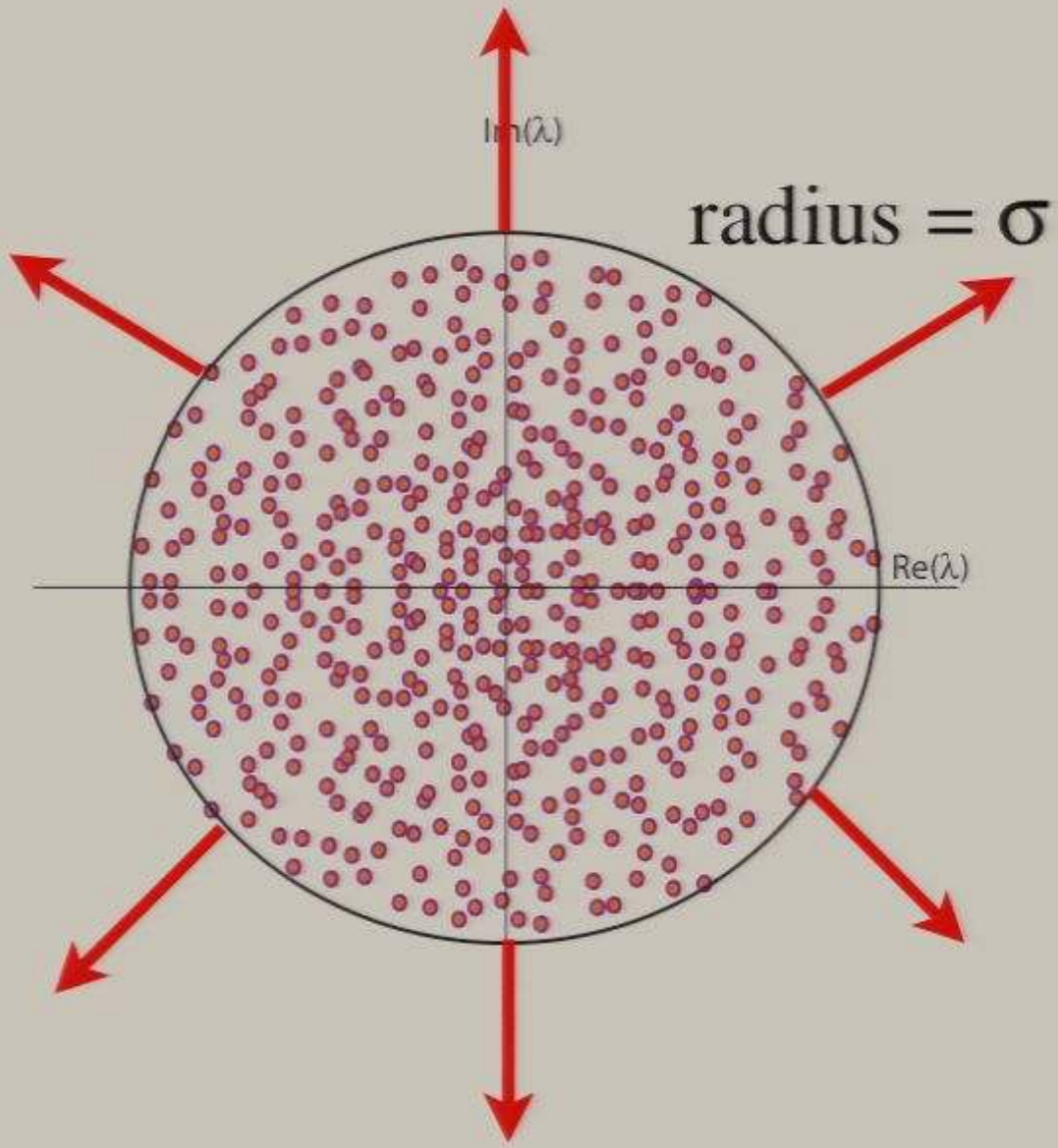


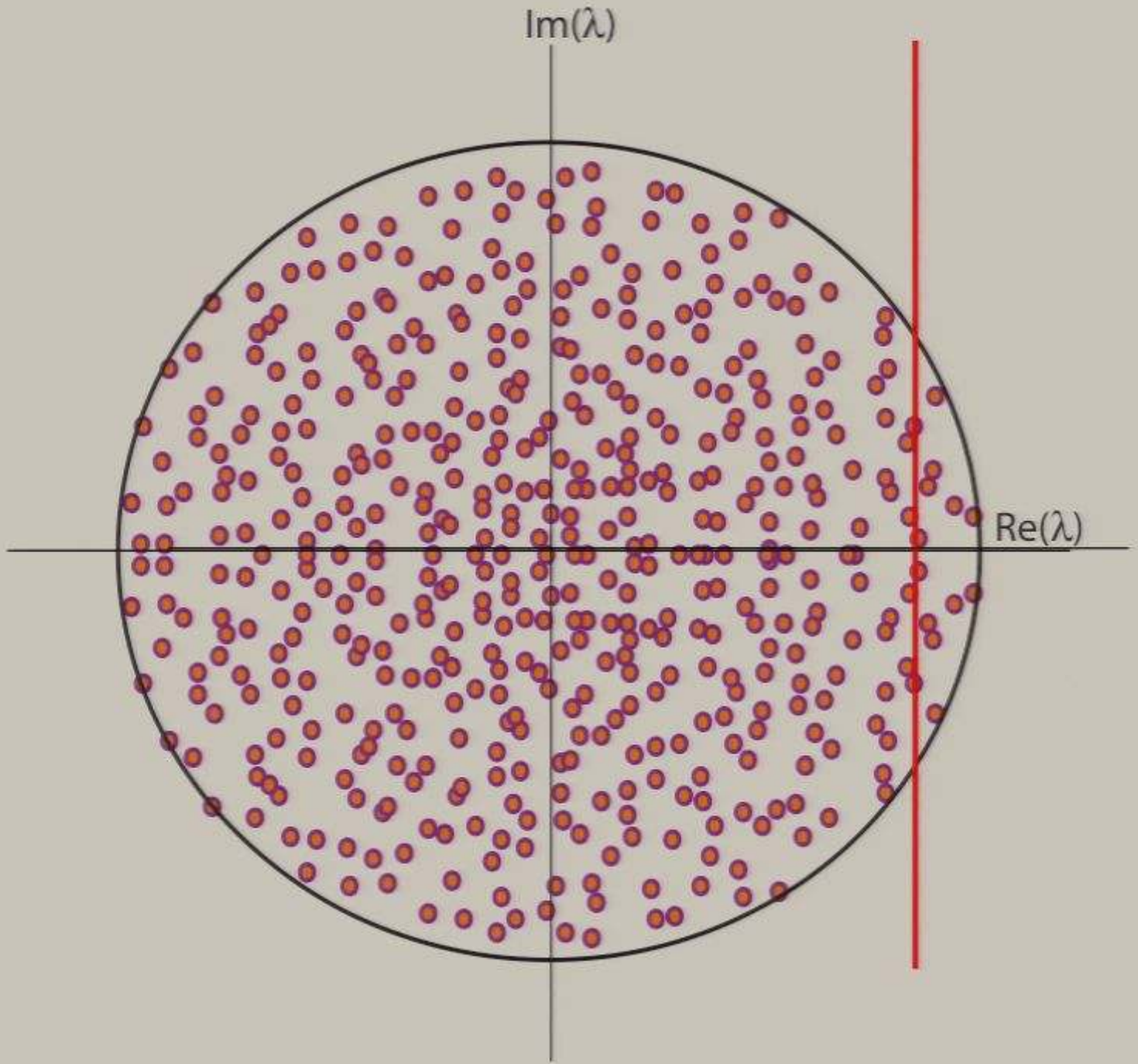

inhibitory
0
excitatory


Im(λ)
radius = σ
Re(λ)

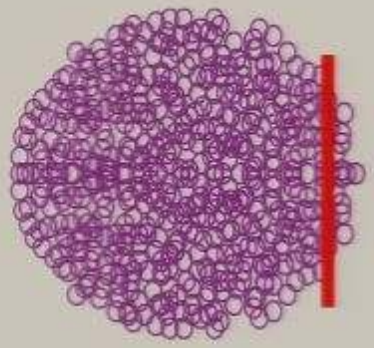

Im(λ)
Re(λ)


inhibitory
0
excitatory

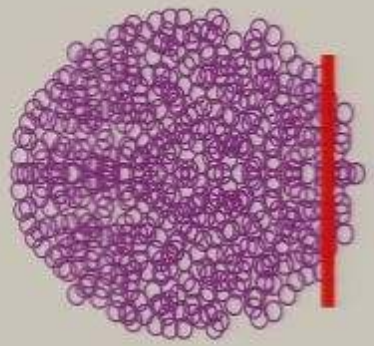

Im(λ)
radius = σ
Re(λ)


Im(λ)
Re(λ)

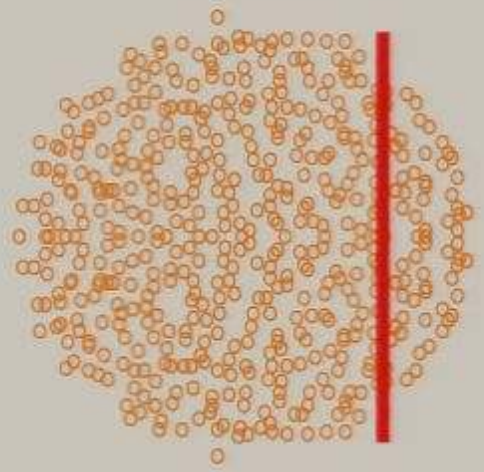
fixed point

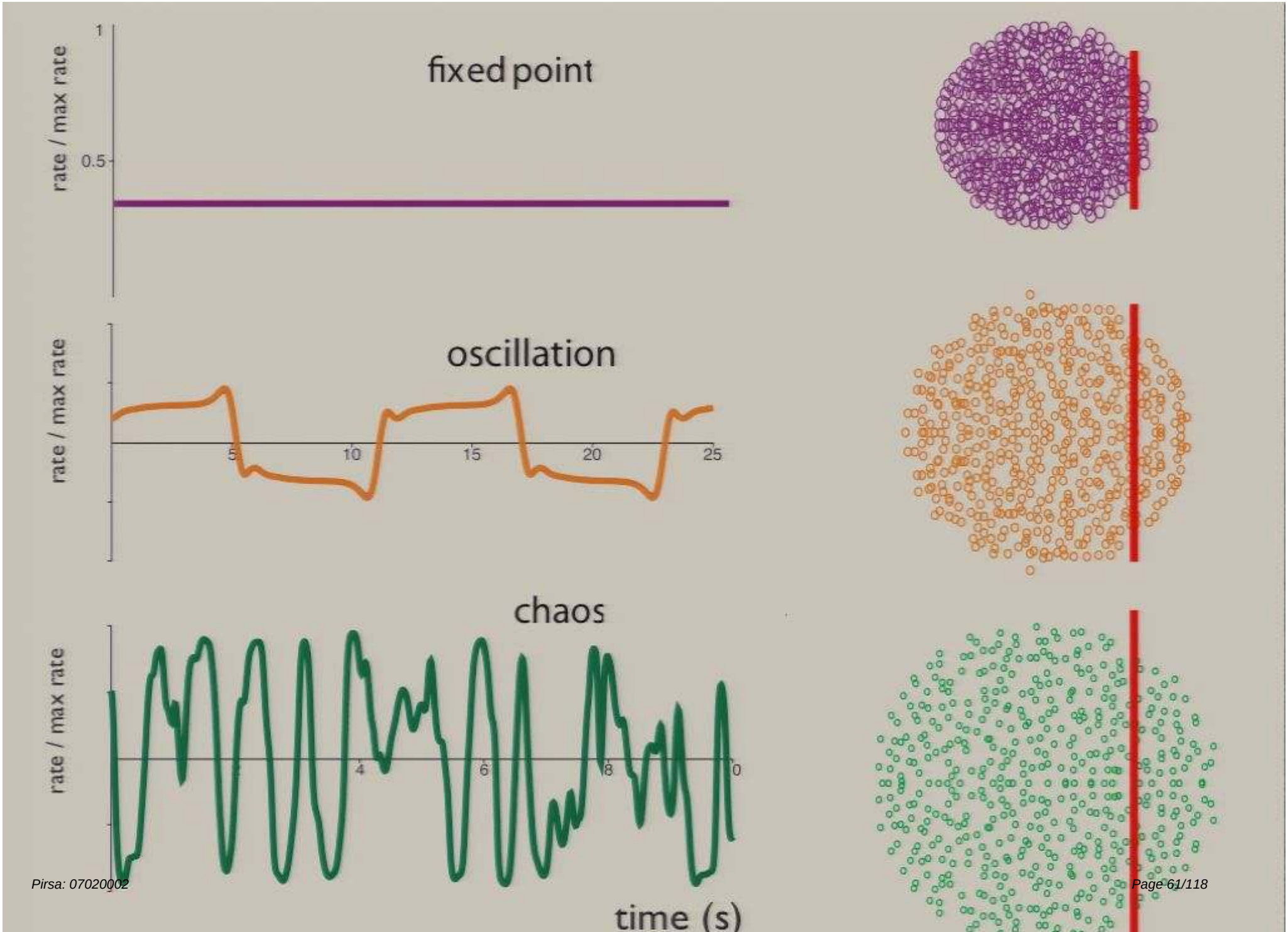


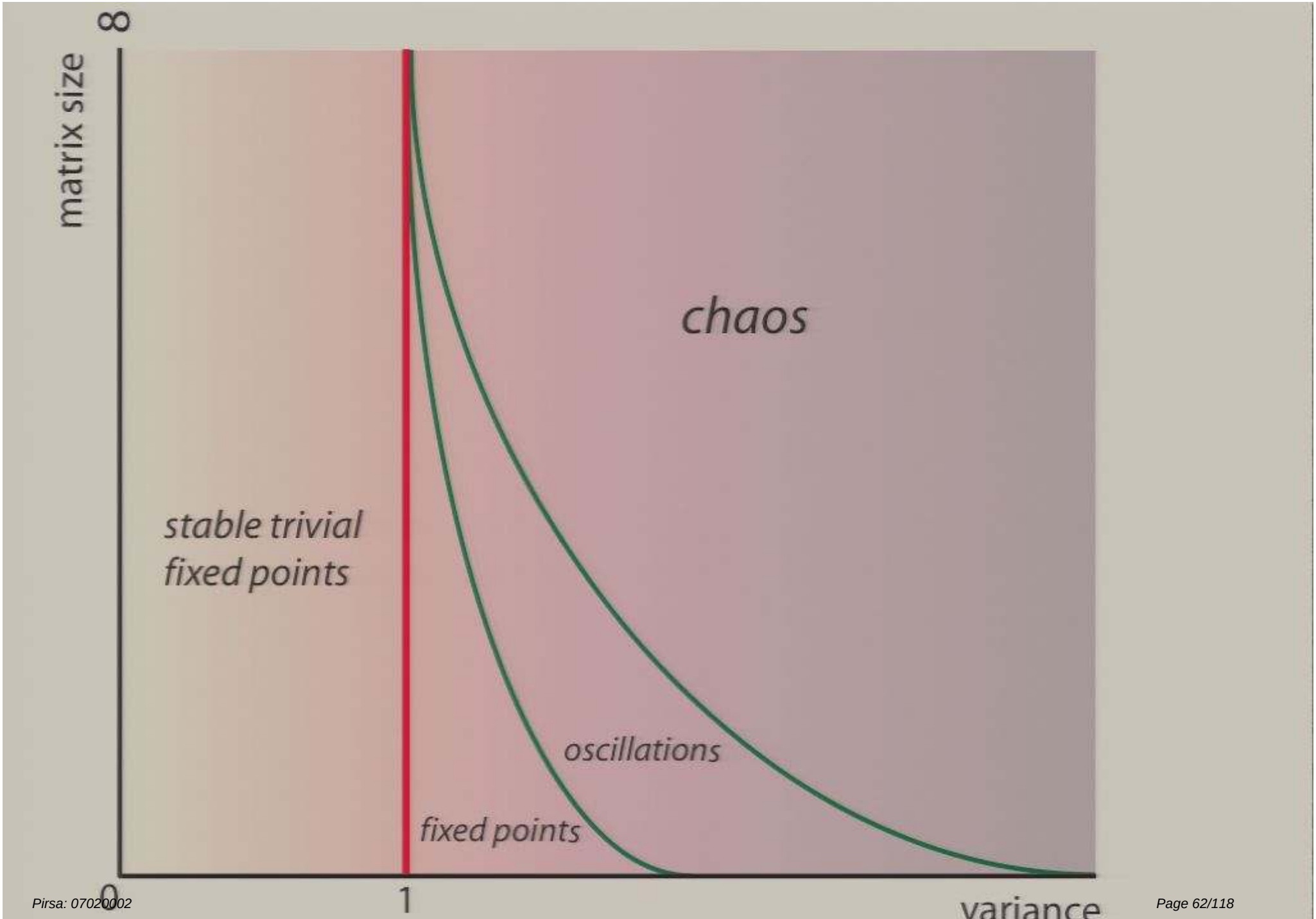
fixed point
rate / max rate
1
0.5



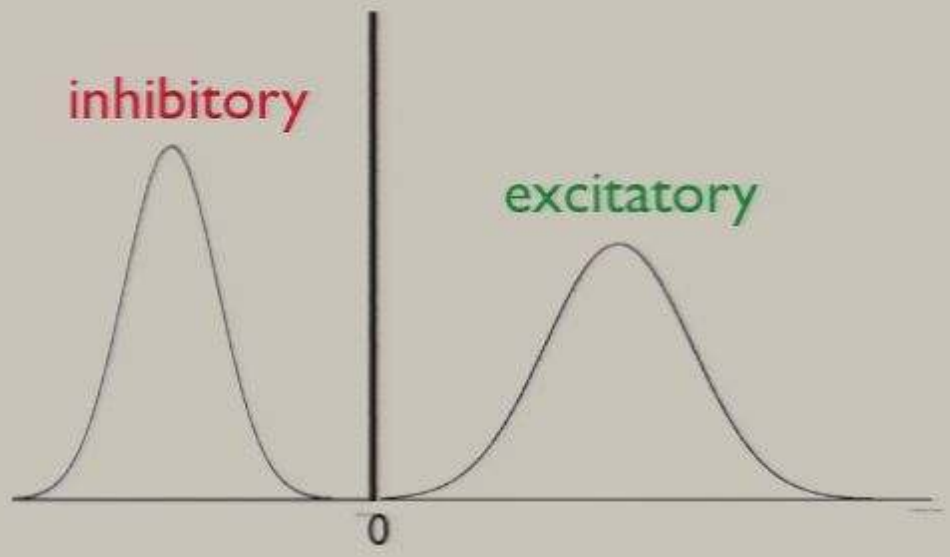
oscillation
rate / max rate
5
10
15
20
25

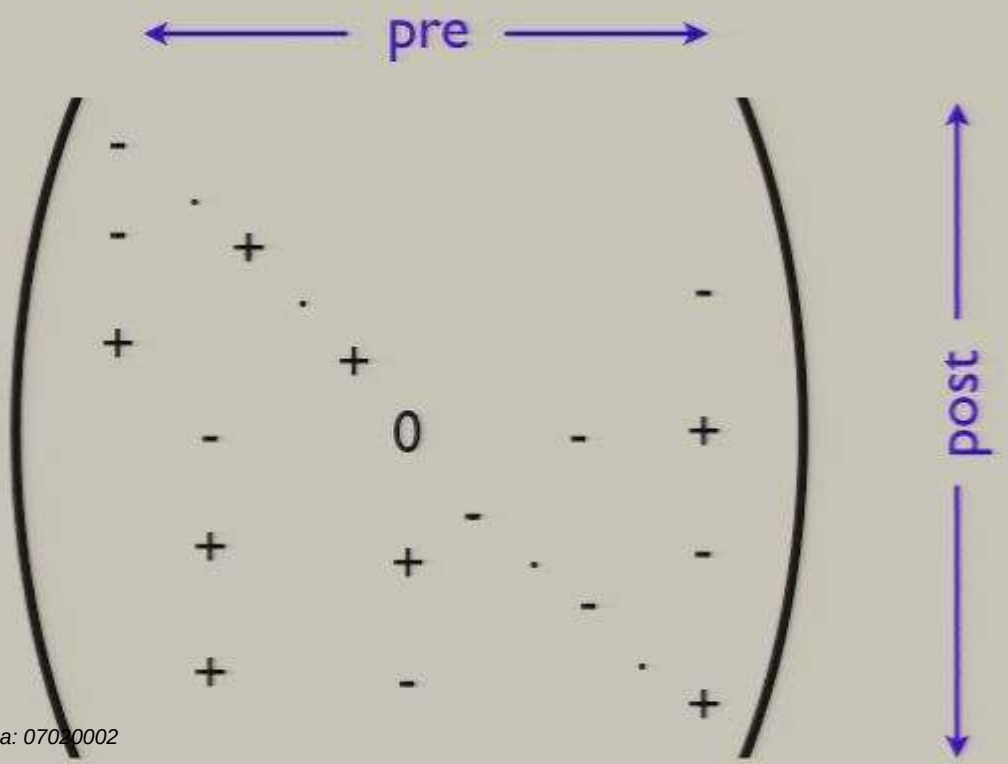


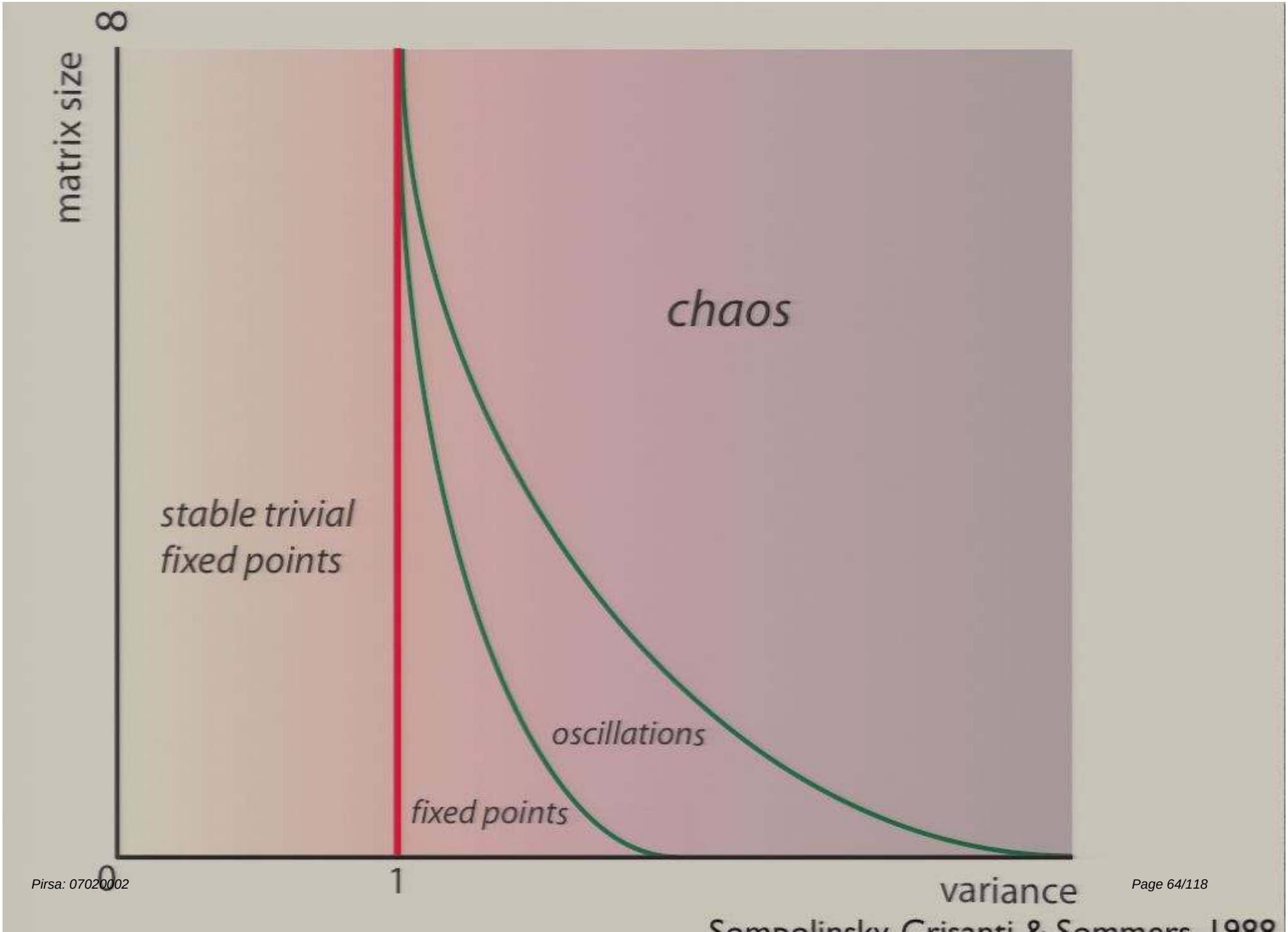

fixed point
rate / max rate
1
0.5
oscillation
rate / max rate
5
10
15
20
25
chaos
rate / max rate
4
6
8
time (s)

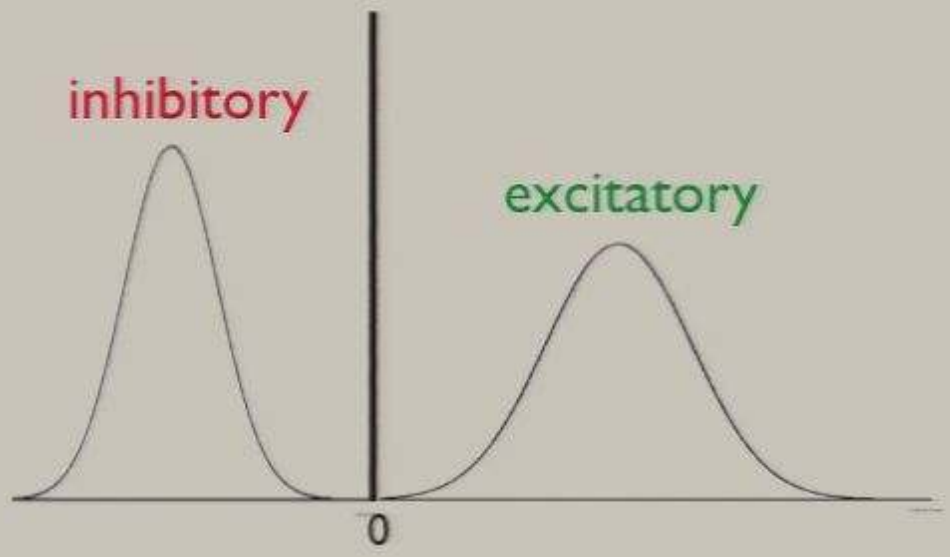


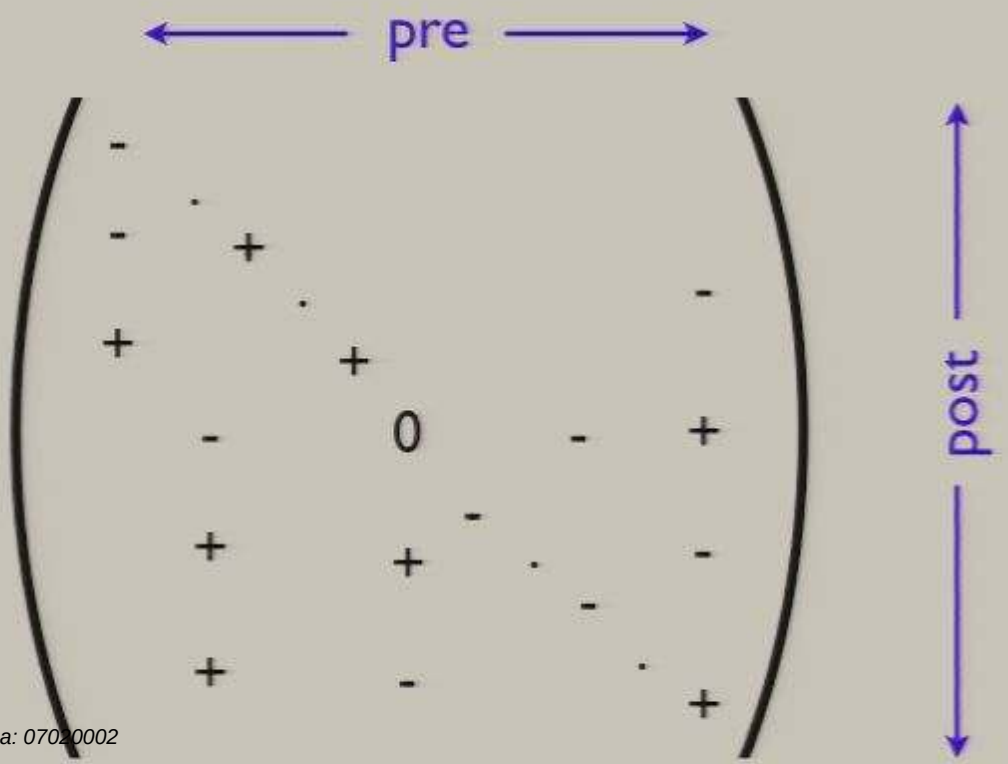
Sompolinsky, Crisanti & Sommers, 1988

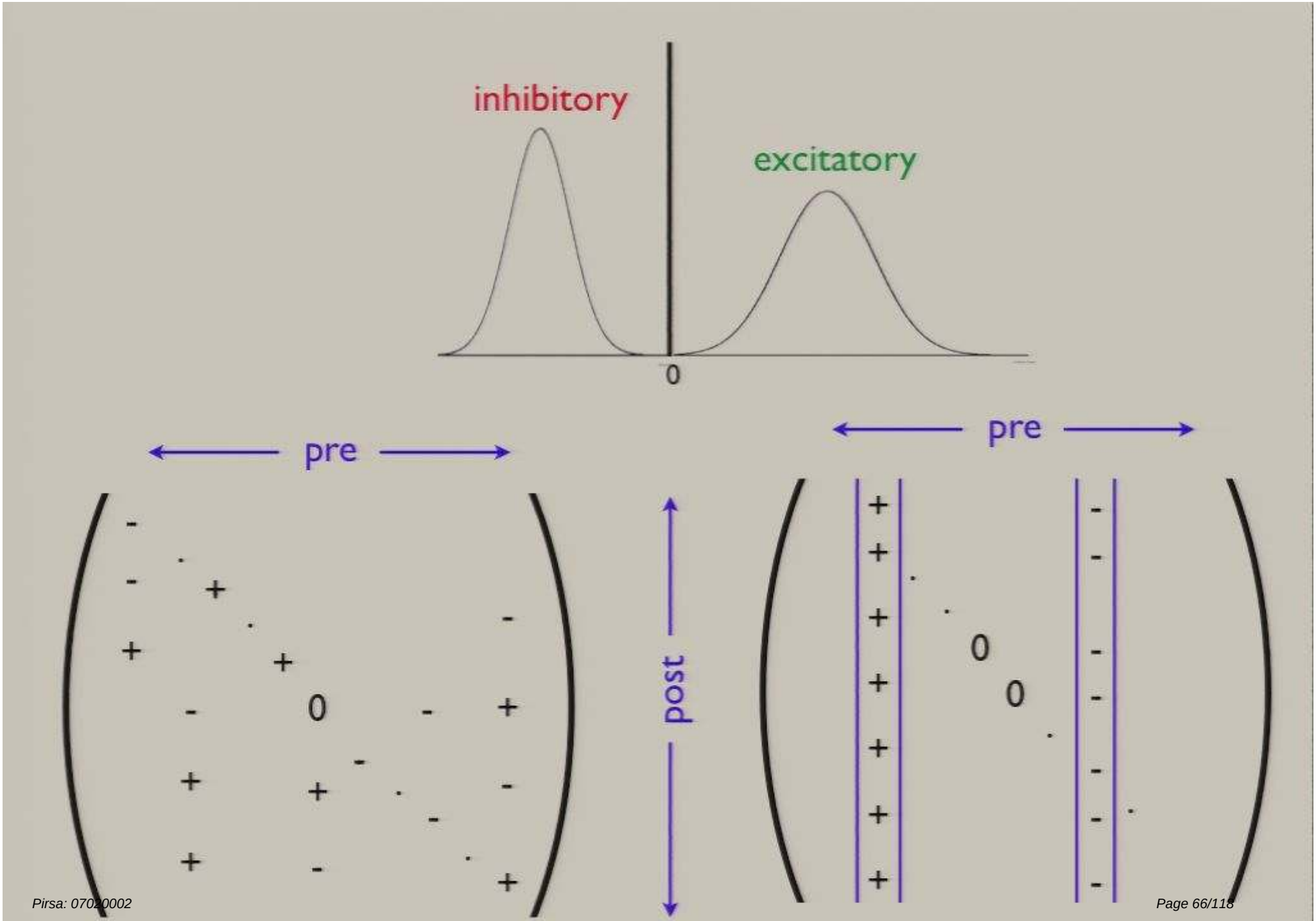

inhibitory
excitatory
0


pre
post
0


matrix size
∞
chaos
stable trivial
fixed points
oscillations
fixed points
0
1
variance

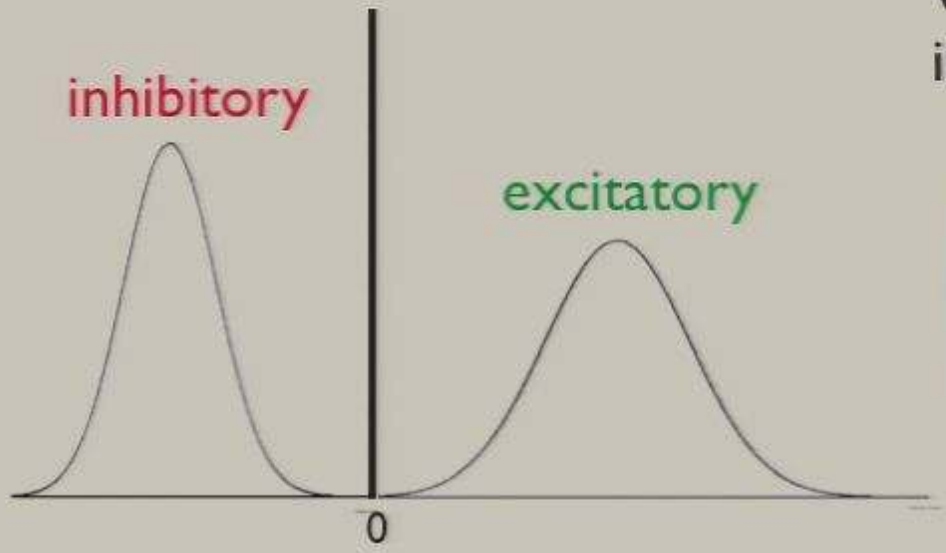

inhibitory
excitatory
0


pre
post
0


inhibitory
excitatory
0
pre
pre
post
0
0

with excitatory or inhibitory synapses

$$\begin{pmatrix} + & - & + \\ - & \cdot & \cdot \\ + & \cdot & + \\ - & & \cdot \end{pmatrix}$$

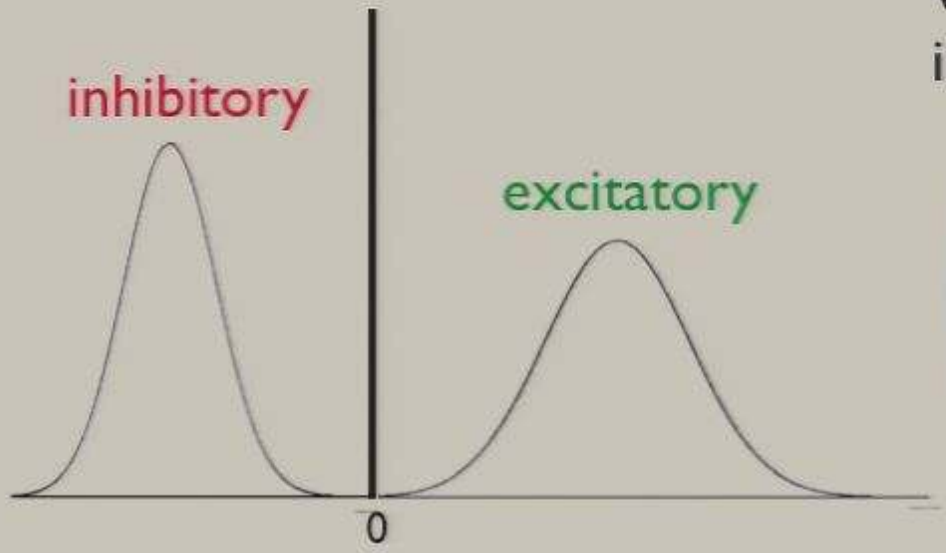


with excitatory or inhibitory neurons

$$\begin{pmatrix} + & \cdot & - \\ + & \cdot & - \\ + & \cdot & - \\ + & & - \end{pmatrix}$$

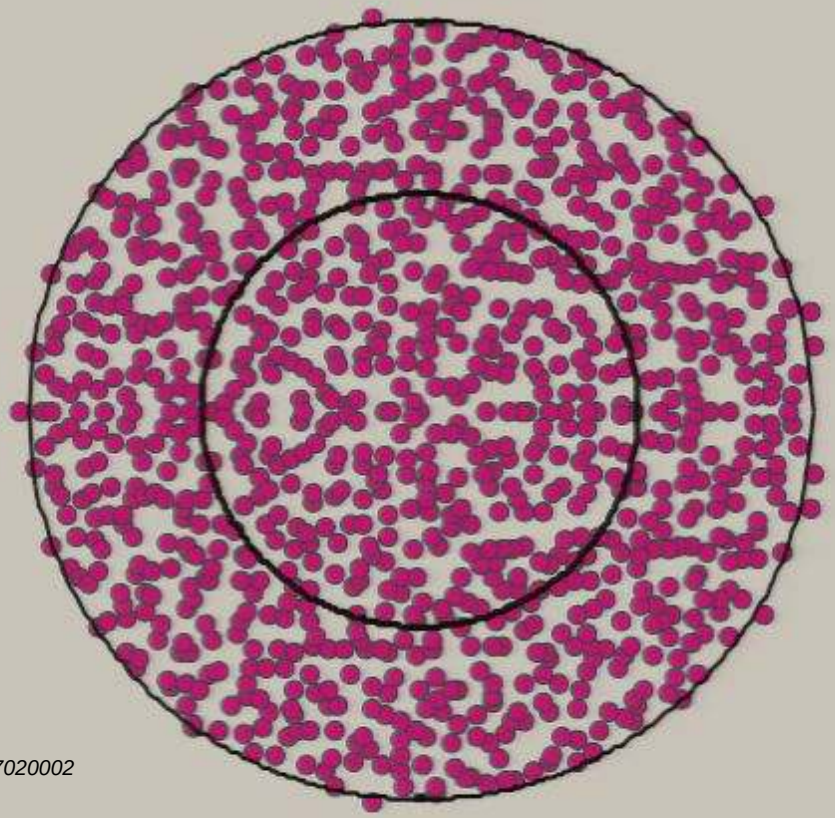
with excitatory or inhibitory synapses

$$\begin{pmatrix} + & - & + \\ - & \cdot & \cdot \\ + & \cdot & + \\ - & & \cdot \end{pmatrix}$$

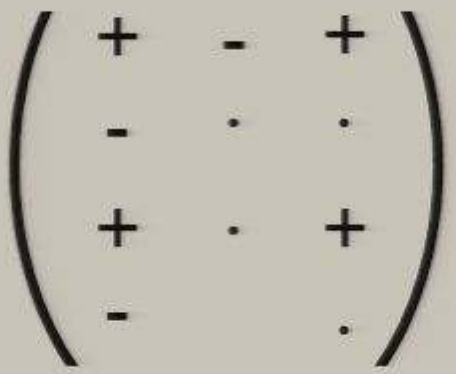


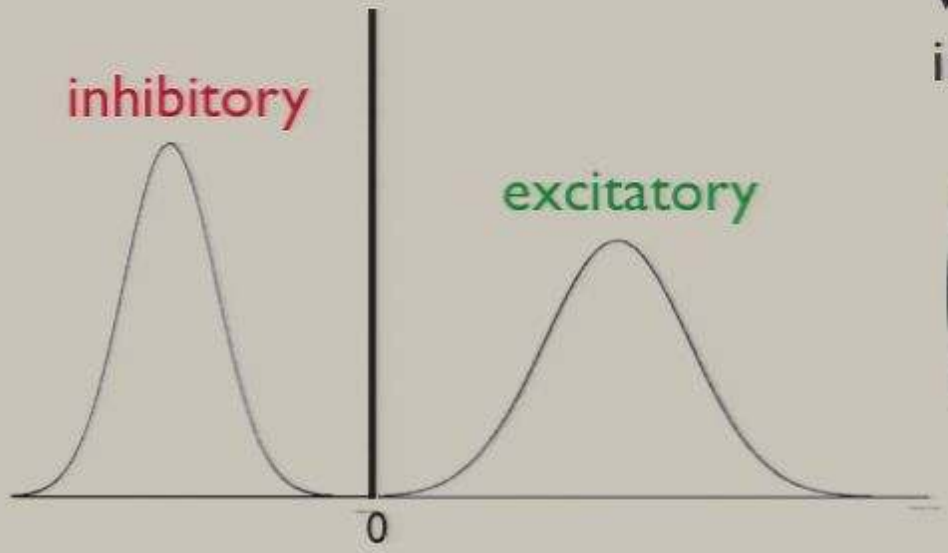
with excitatory or inhibitory neurons

$$\begin{pmatrix} + & \cdot & - \\ + & \cdot & - \\ + & \cdot & - \\ + & & - \end{pmatrix}$$

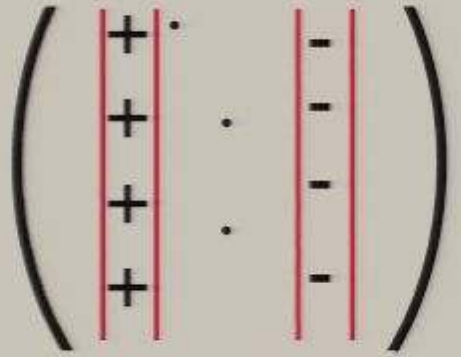


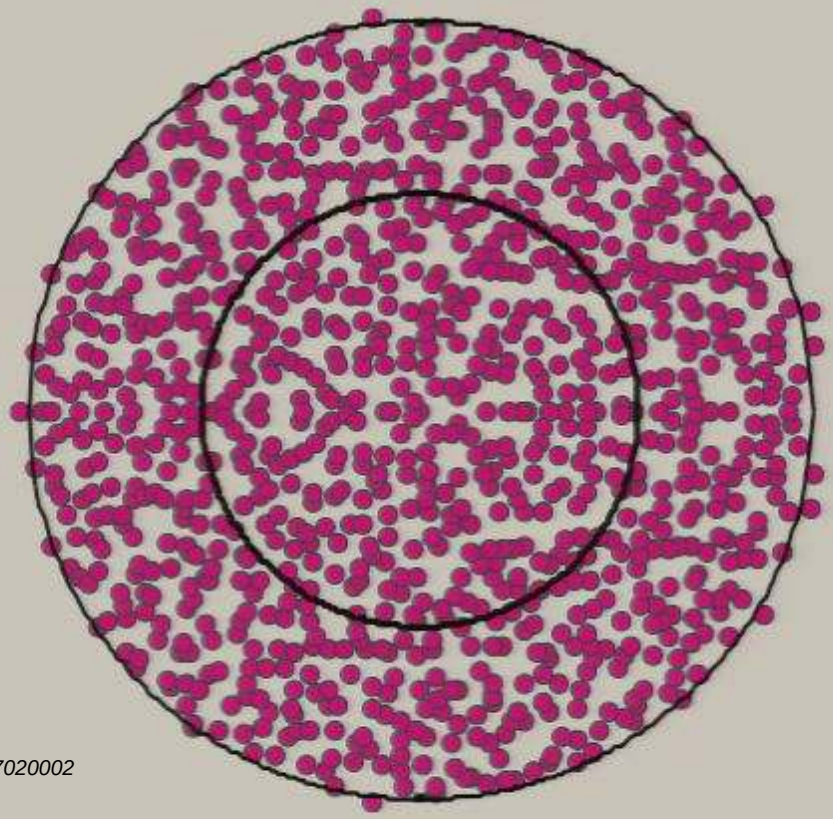
with excitatory or inhibitory synapses

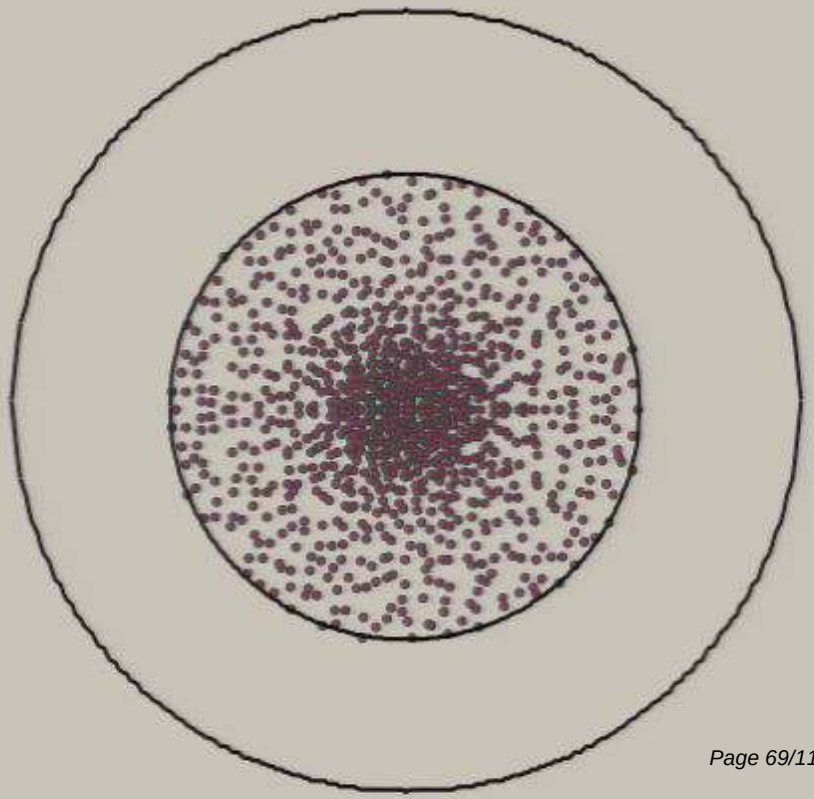




with excitatory or inhibitory neurons







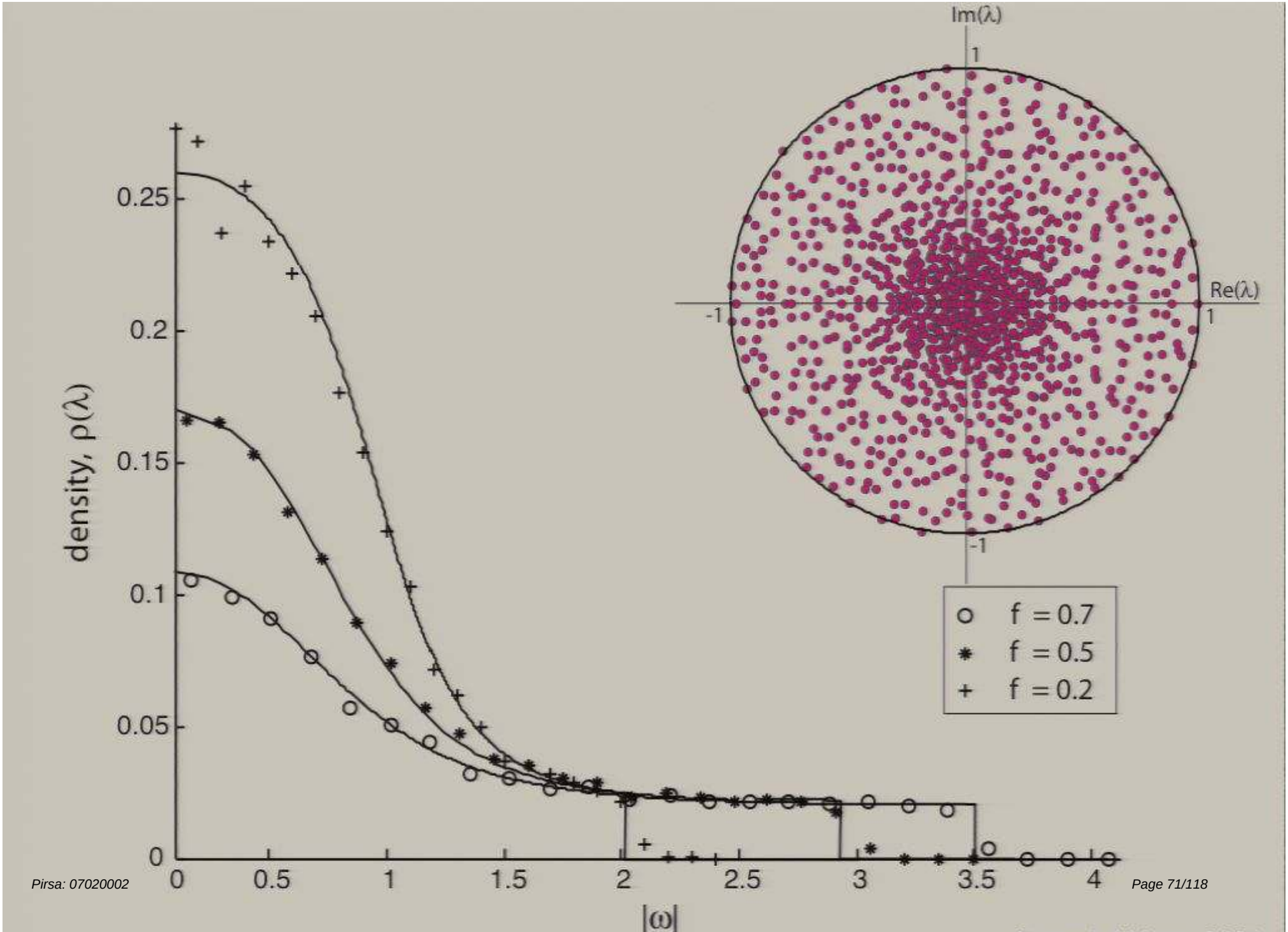
Calculation of Eigenvalue Density

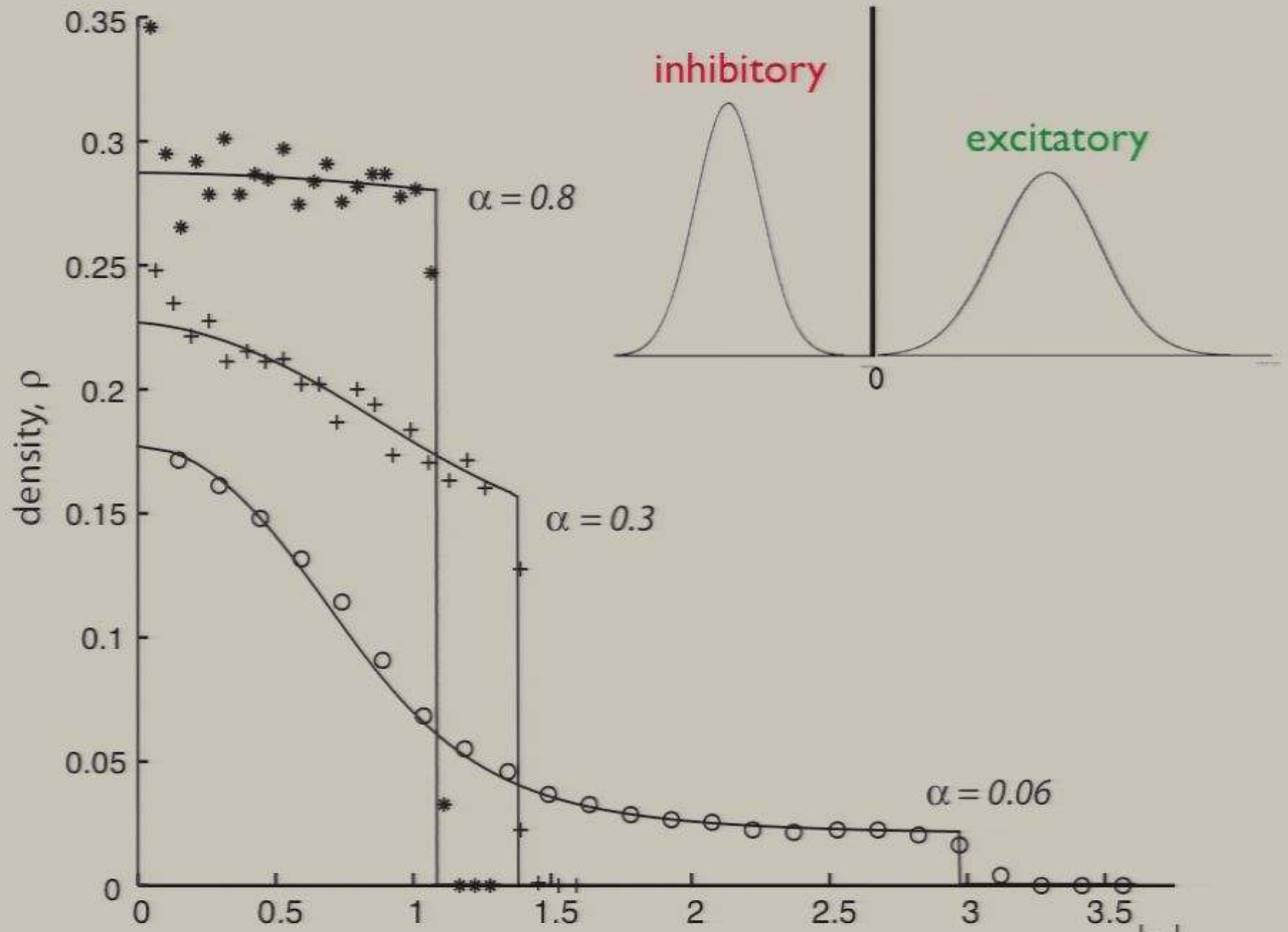
$$\rho = \frac{1}{\pi}\left(|\omega|^2\phi'' + \phi'\right)$$

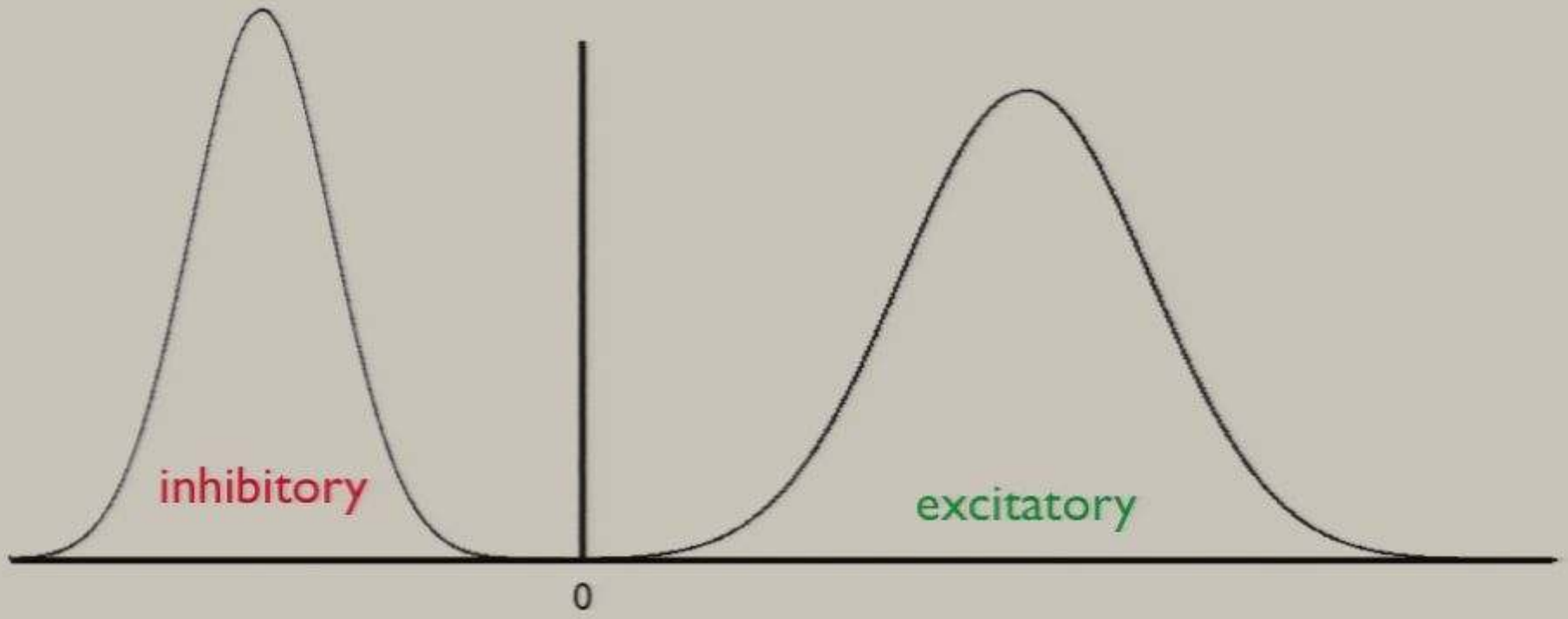
$$\phi = -\frac{1}{N}\ln\int\prod_{i=1}^{N}\frac{dz_i^* dz_i}{\pi}\prod_{i,j=1}^{N}\sqrt{\frac{N}{2\pi}}dJ_{ij}\exp(-NQ)$$

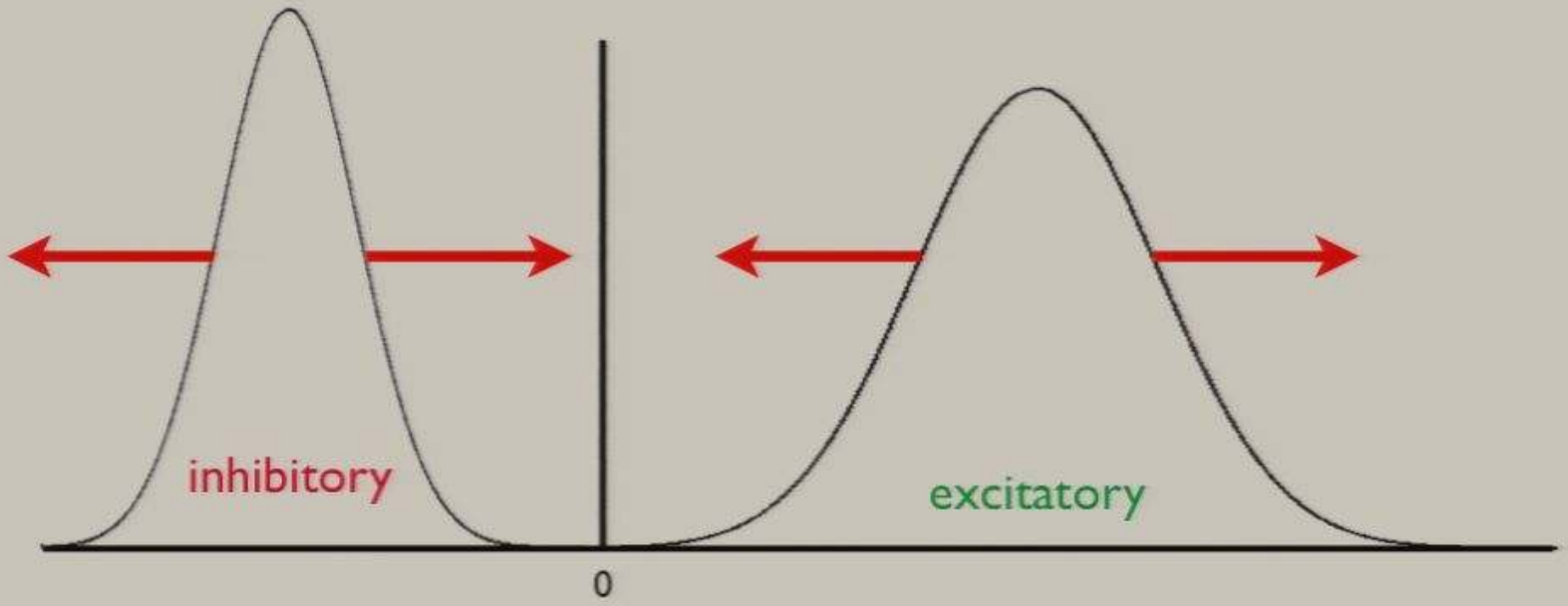
$$Q = \frac{1}{N}\sum_{i,j,k} z_i^*(\omega^*\delta_{ik} - J_{ki}\sigma_i)(\omega\delta_{kj} - J_{kj}\sigma_j)z_j + \frac{1}{2}\sum_{i,j}J_{ij}^2 + \frac{\varepsilon}{N}\sum_i z_i^* z_i.$$

Sommers, Crisanti, Sompolinsky & Stein, 1988.

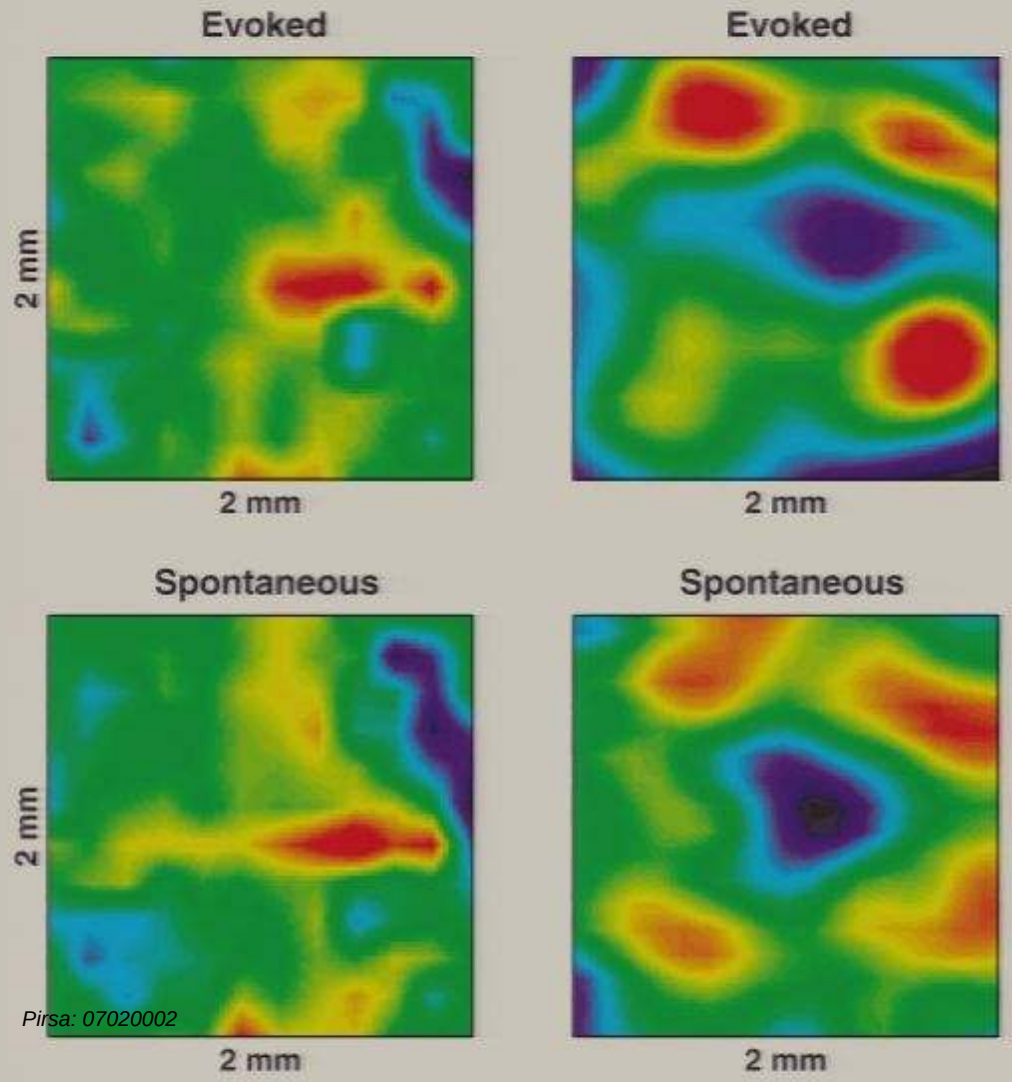

Im(λ)
1
-1
1
Re(λ)
-1
density, ρ(λ)
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5
3
3.5
4
|ω|
f = 0.7
f = 0.5
f = 0.2


inhibitory
excitatory
0
density, ρ
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5
3
3.5
α = 0.8
α = 0.3
α = 0.06


inhibitory
excitatory
0

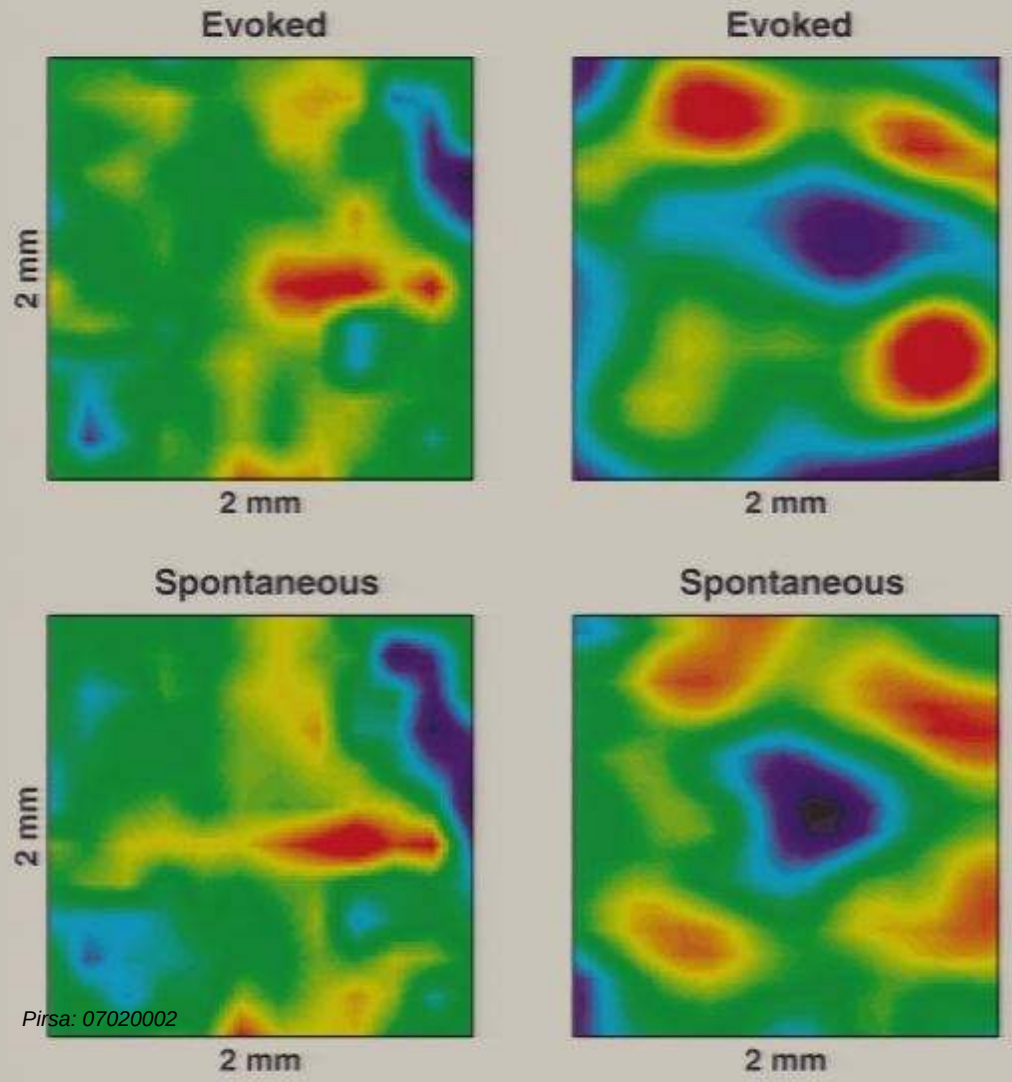

inhibitory
excitatory
0

spontaneous activity ⟷ evoked activity



Anderson, Lammpl, Gillespie & Ferster, 2000.
Beierlein, Fall, Rinzel & Yuste, 2002.
Kenet, Bibitchkov, Tsodyks, Grinvald & Arieli, 2003.
Petersen, Grinvald & Sakmann, 2003.
Fiser, Chiu & Weliky, 2004.
MacLean, Watson, Aaron & Yuste, 2006.

spontaneous activity ⟷ evoked activity



Anderson, Lammpl, Gillespie & Ferster, 2000.
Beierlein, Fall, Rinzel & Yuste, 2002.
Kenet, Bibitchkov, Tsodyks, Grinvald & Arieli, 2003.
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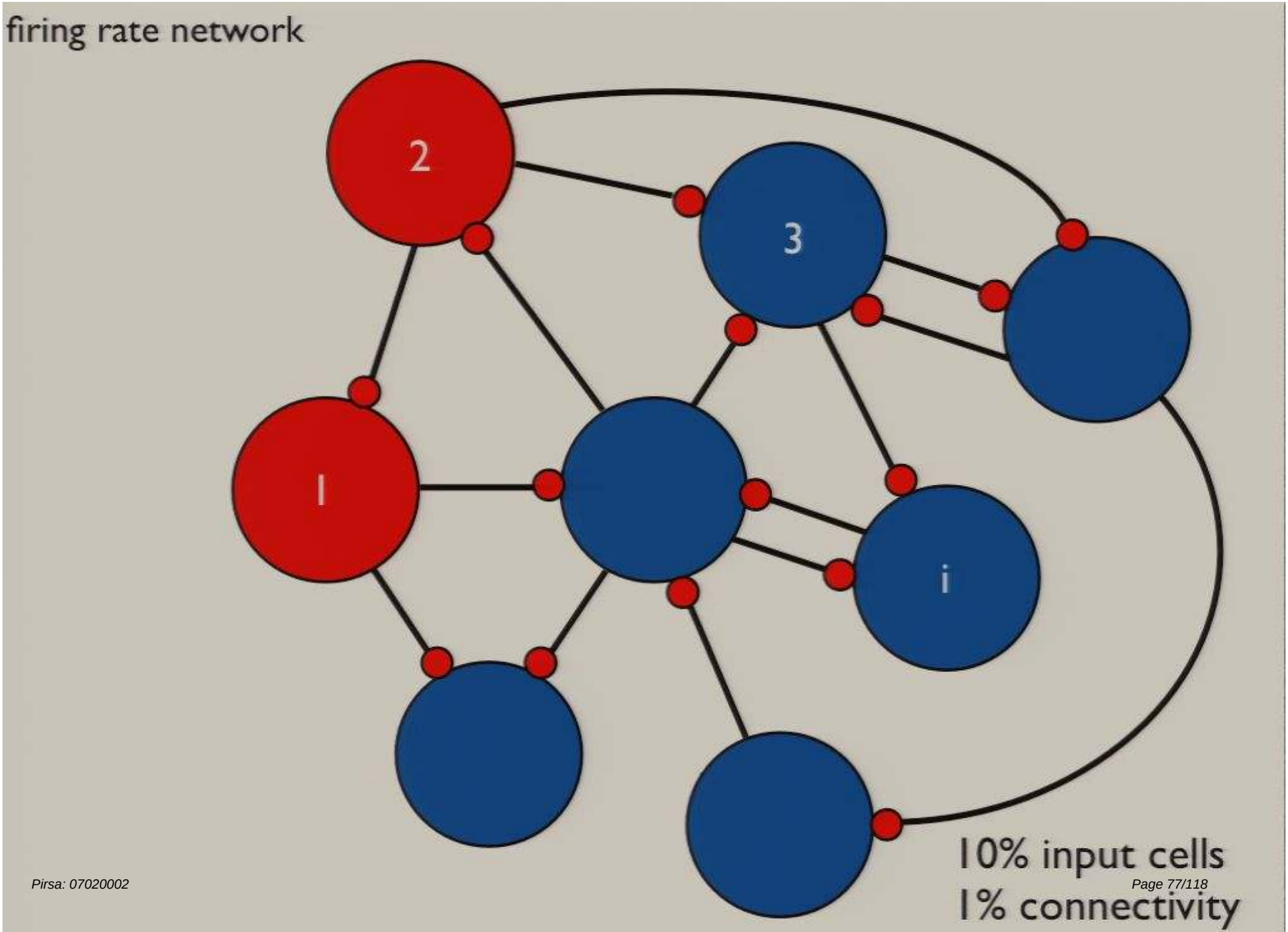
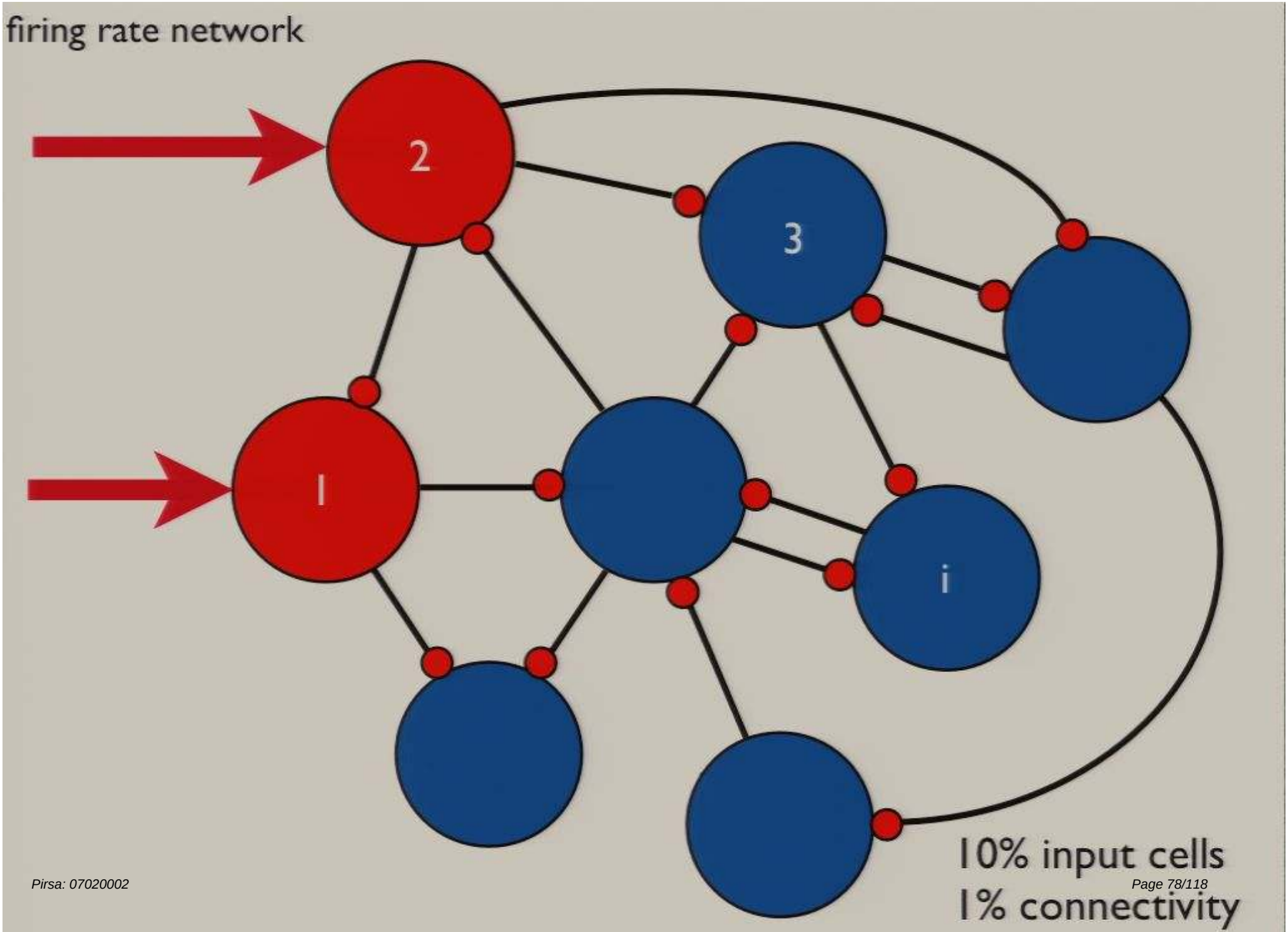
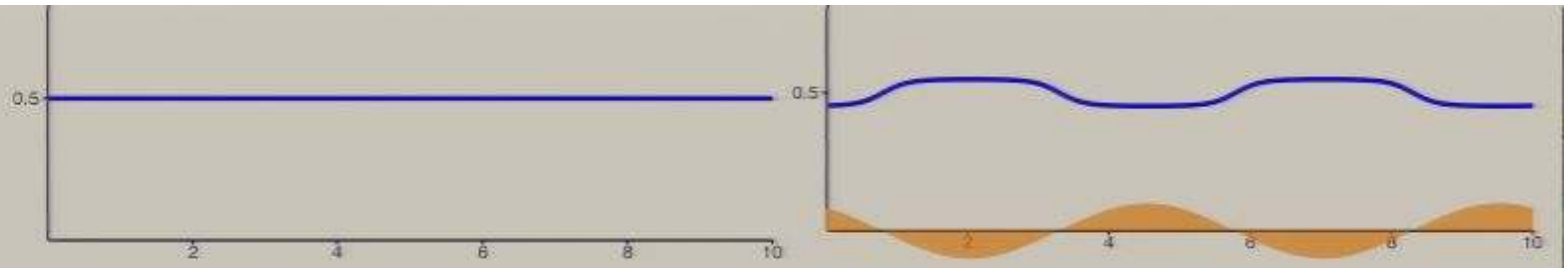
Fiser, Chiu & Weliky, 2004.
MacLean, Watson, Aaron & Yuste, 2006.

Kanaka Rajan

Haim Sompolinsky
Evan Schaffer


firing rate network
2
3
1
i
10% input cells
1% connectivity


firing rate network
2
3
1
i
10% input cells
1% connectivity



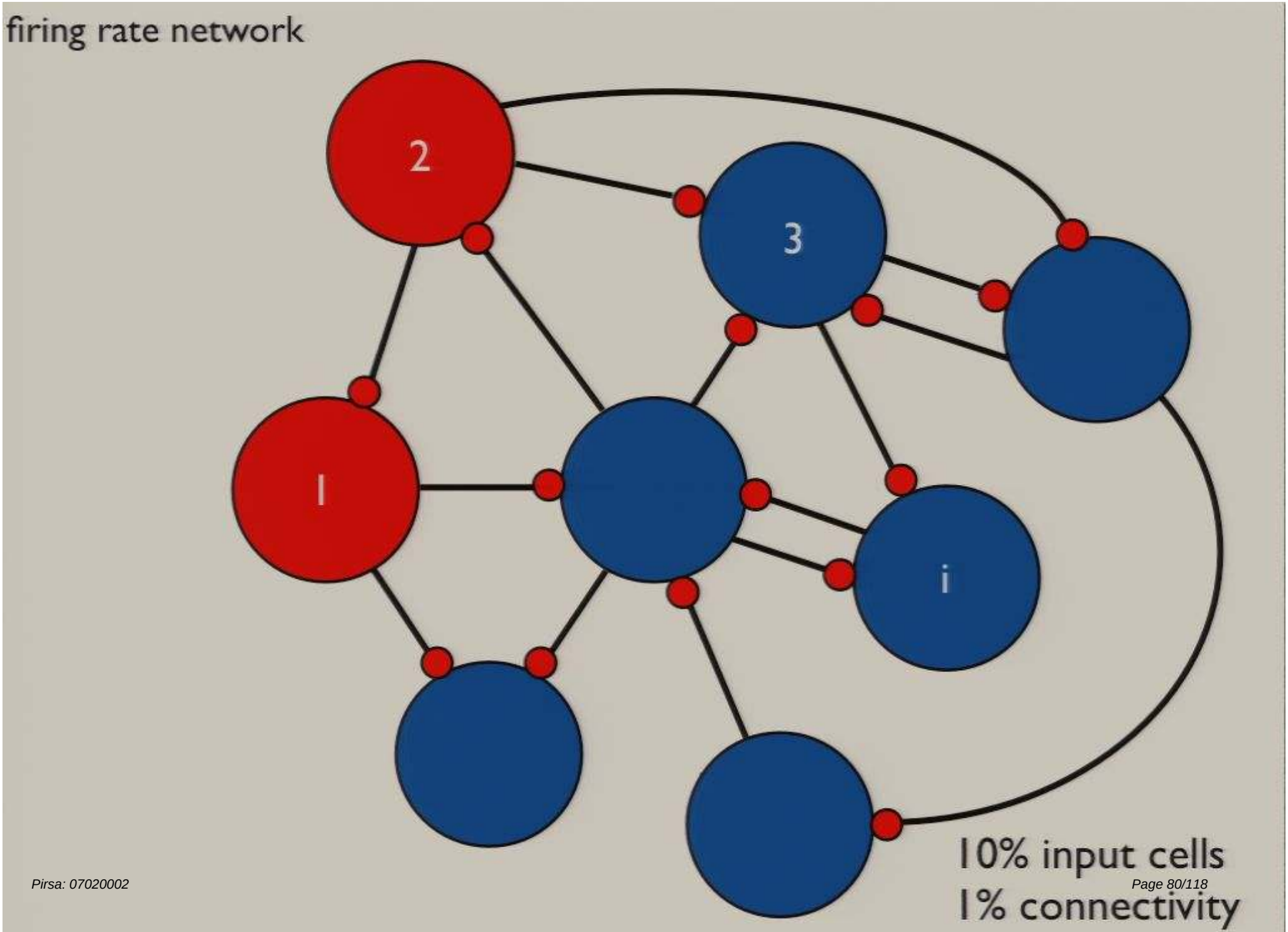
$\sigma = 0.5$

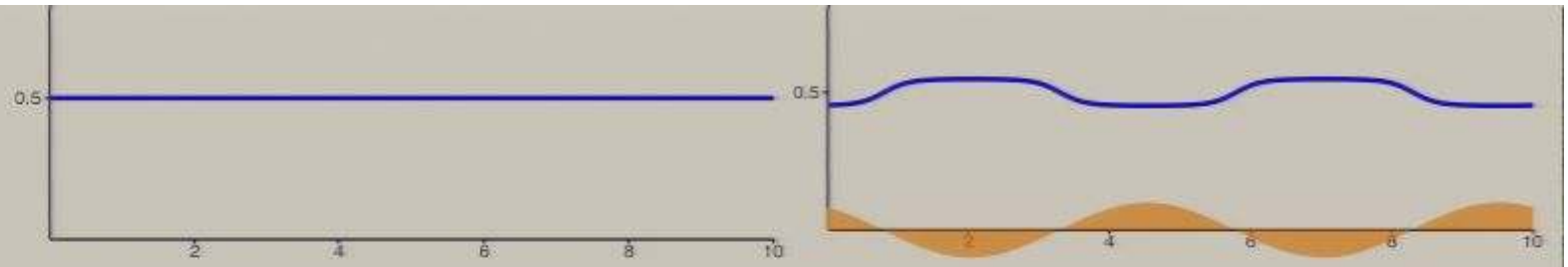
$\sigma = 1.0$

$\sigma = 1.5$

$\sigma = 2.0$

increasing variance


firing rate network
2
3
1
i
10% input cells
1% connectivity



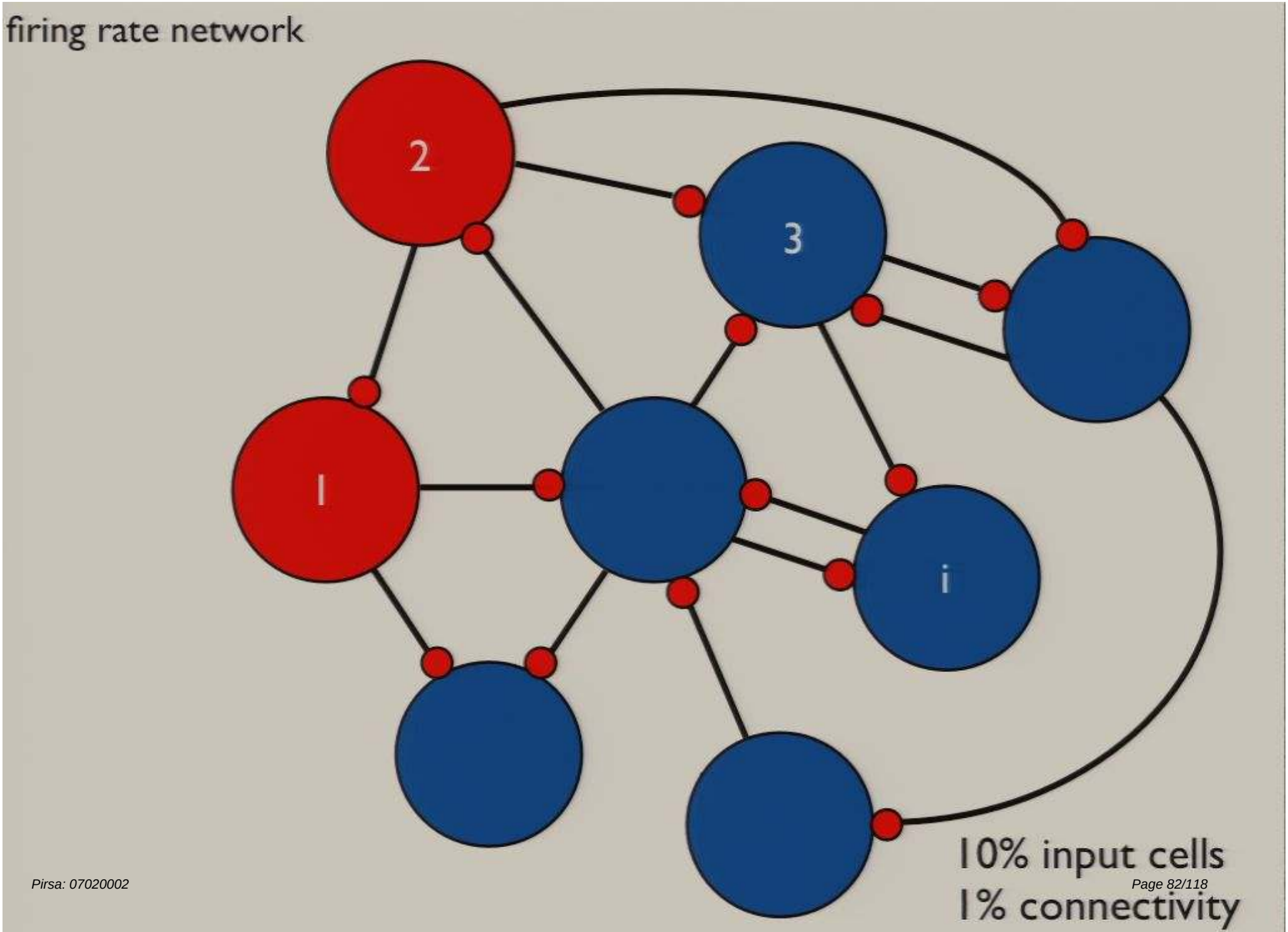
$\sigma = 0.5$

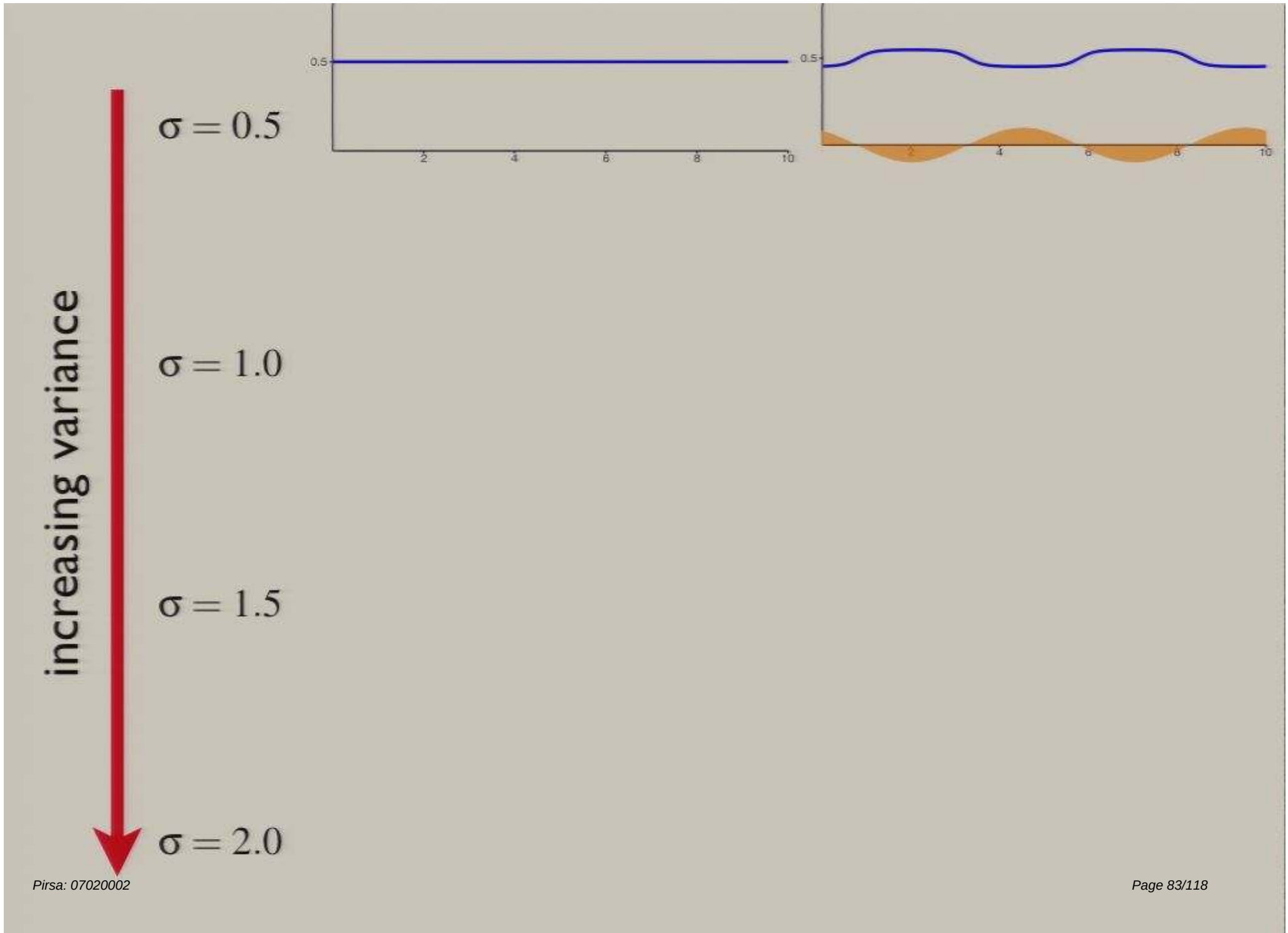
$\sigma = 1.0$

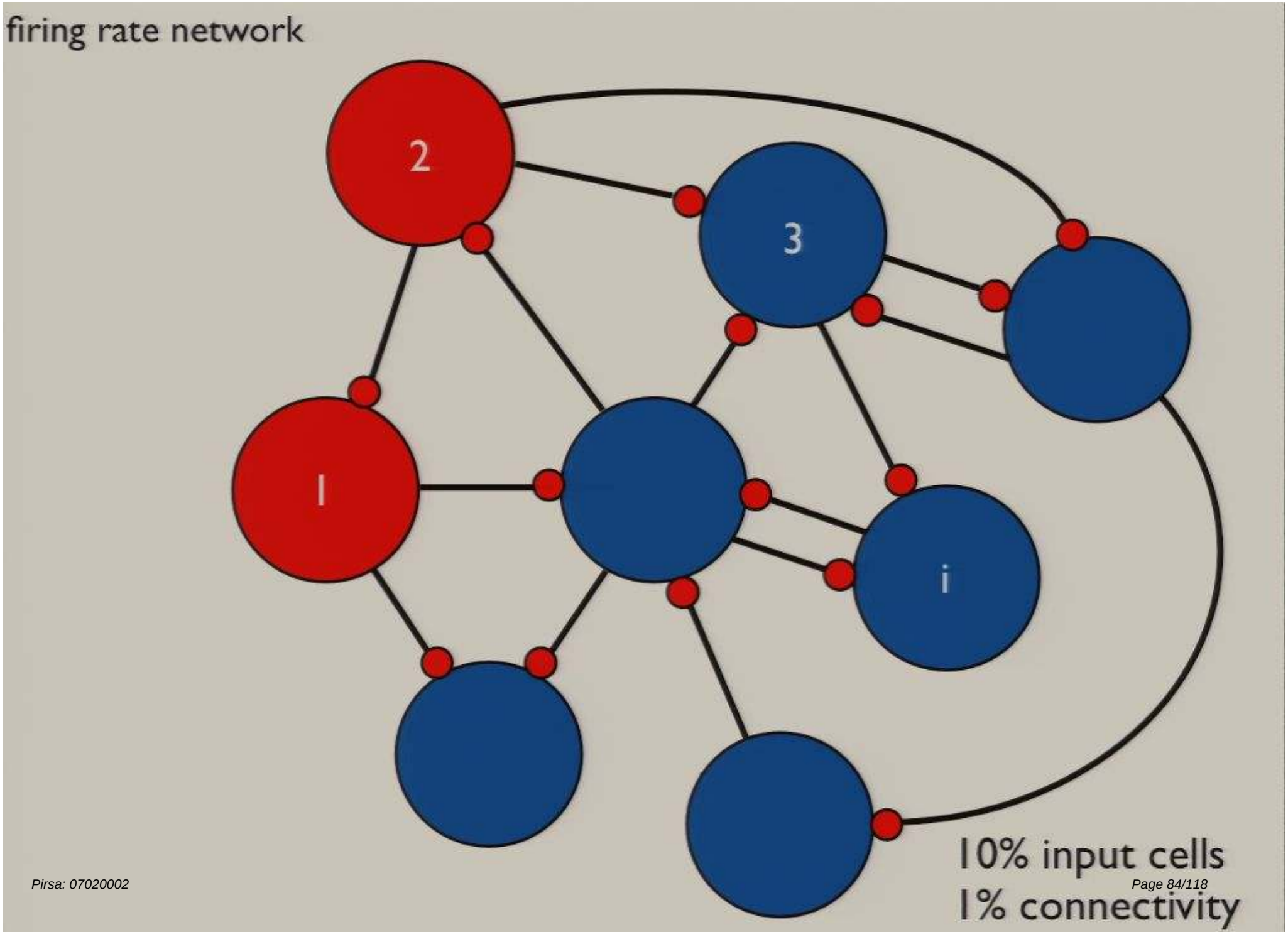
$\sigma = 1.5$

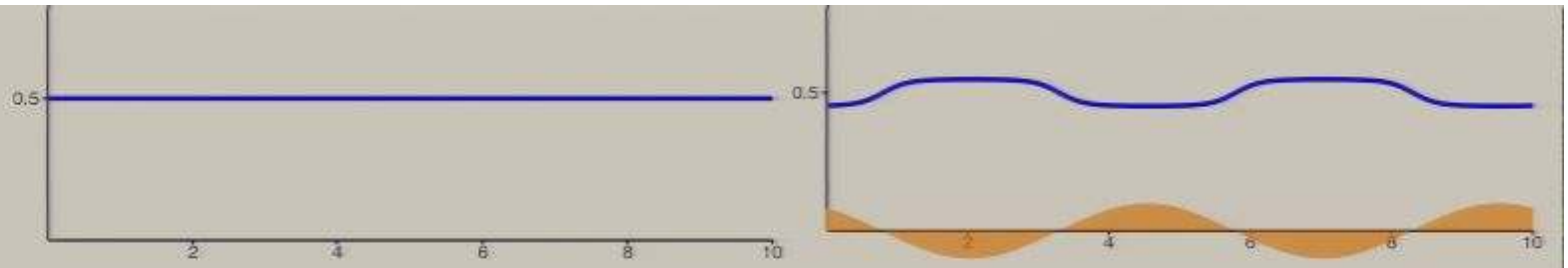
$\sigma = 2.0$

increasing variance


firing rate network
2
3
1
i
10% input cells
1% connectivity


increasing variance
σ = 0.5
σ = 1.0
σ = 1.5
σ = 2.0
0.5
2
4
6
8
10
0.5
2
4
6
8
10


firing rate network
2
3
1
i
10% input cells
1% connectivity



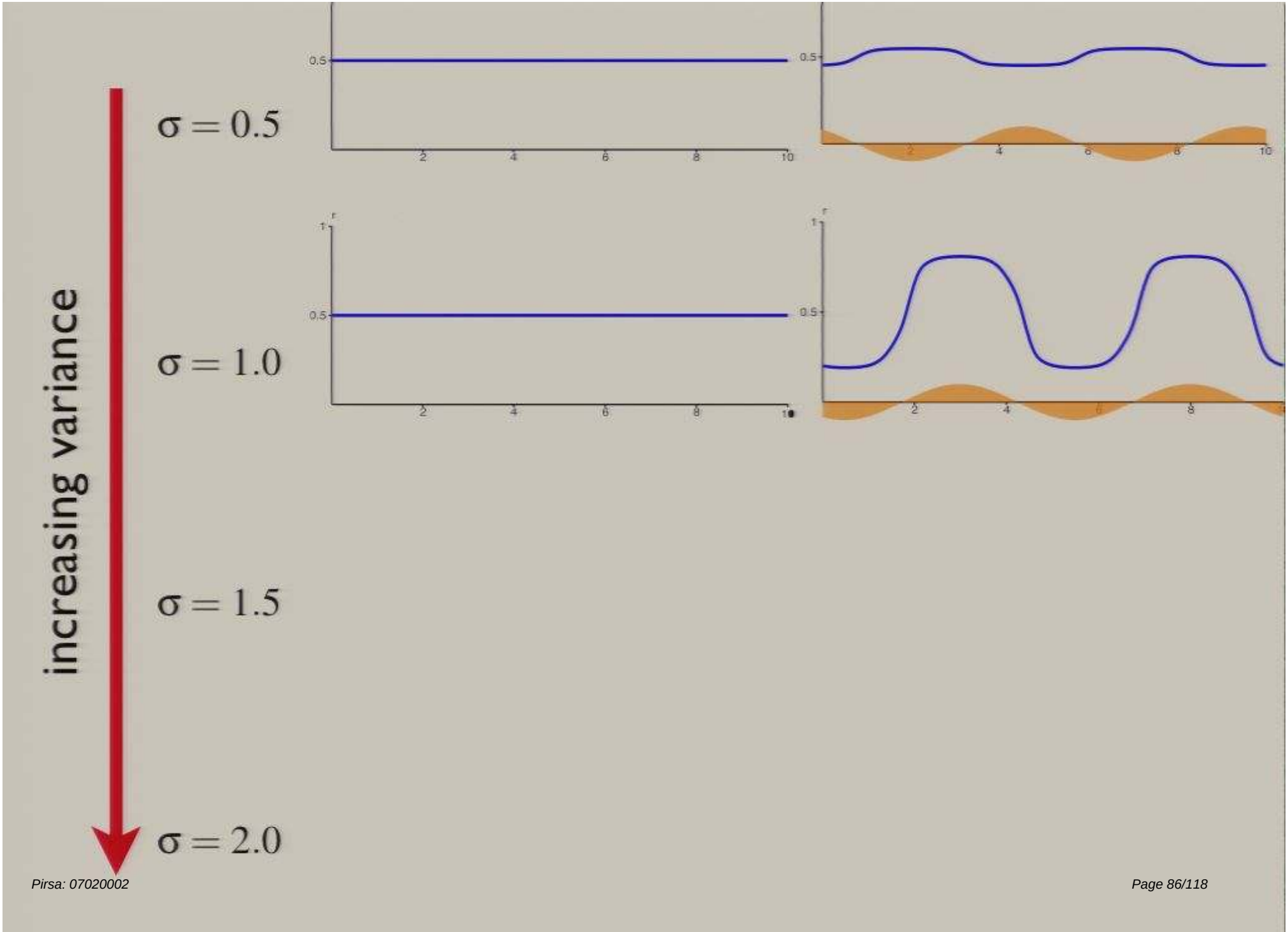
$\sigma = 0.5$

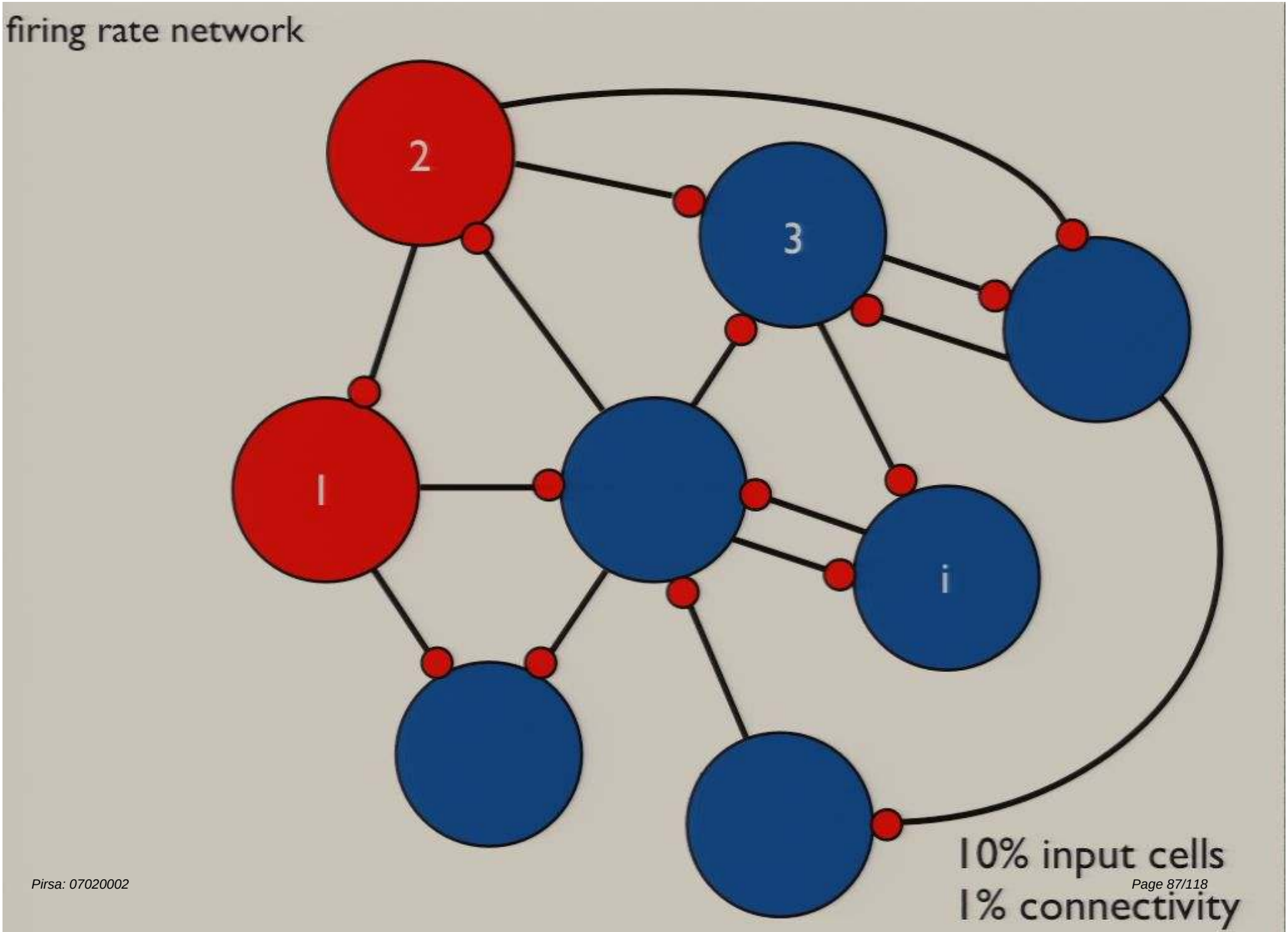
$\sigma = 1.0$

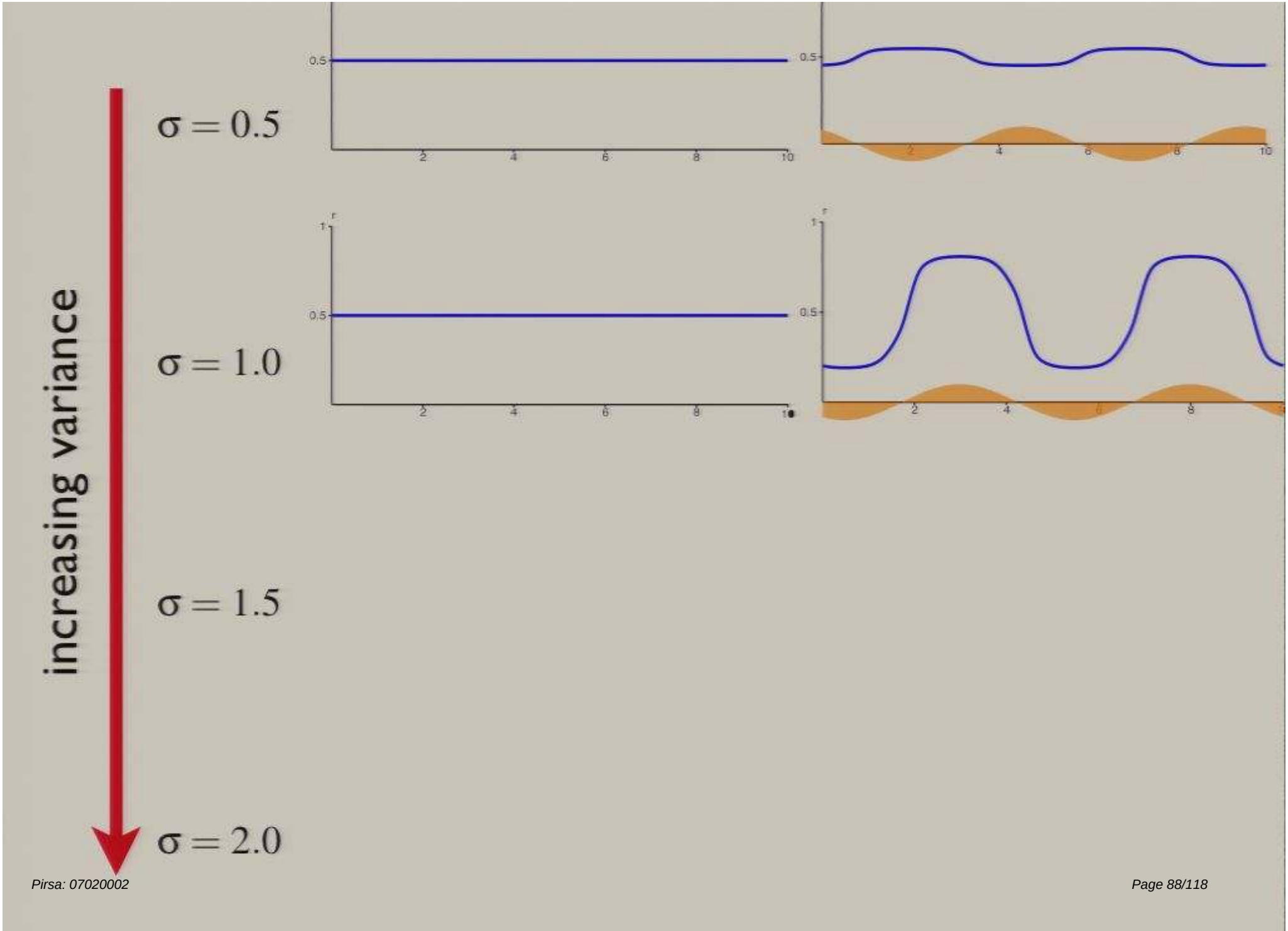
$\sigma = 1.5$

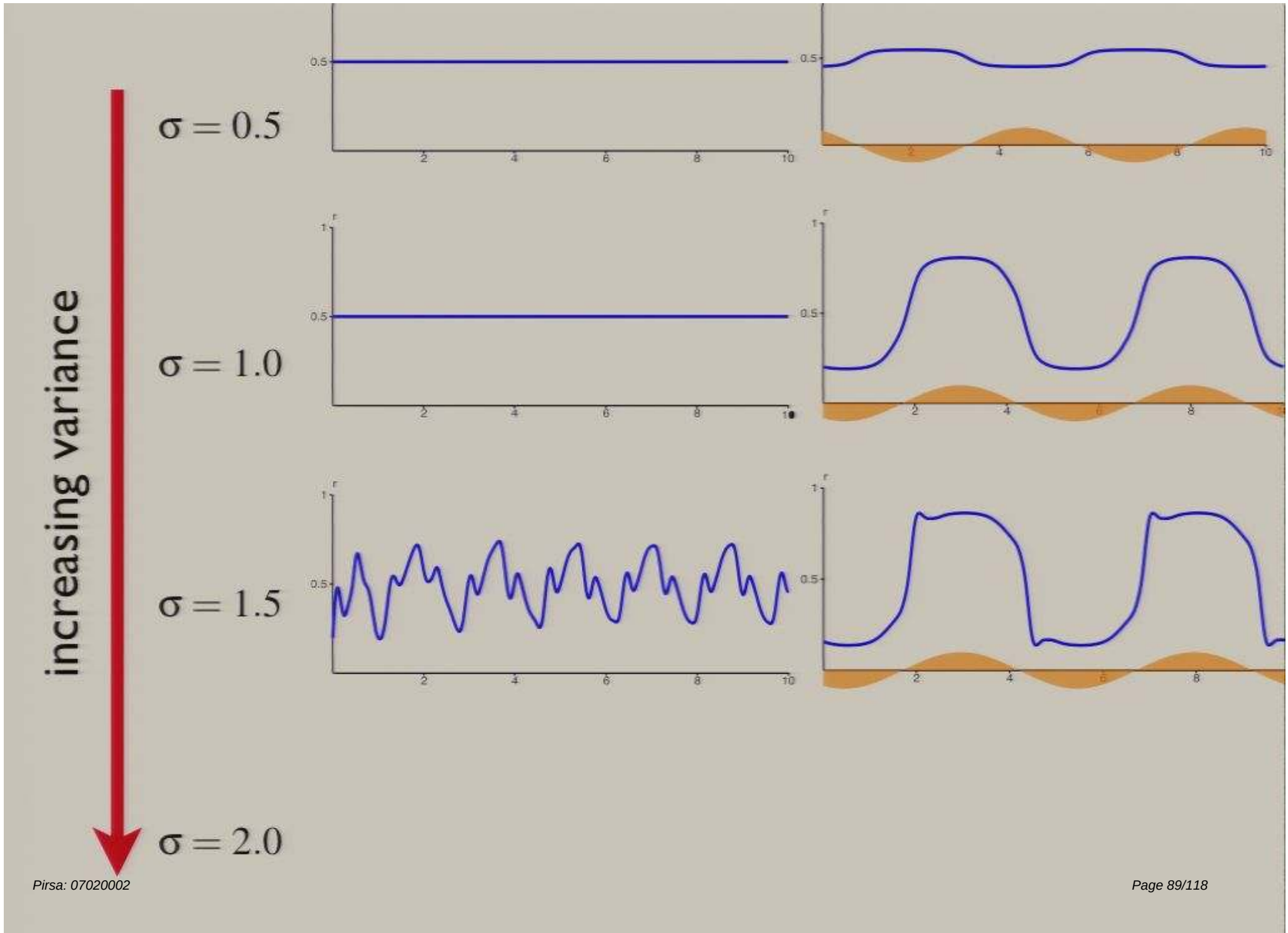
$\sigma = 2.0$

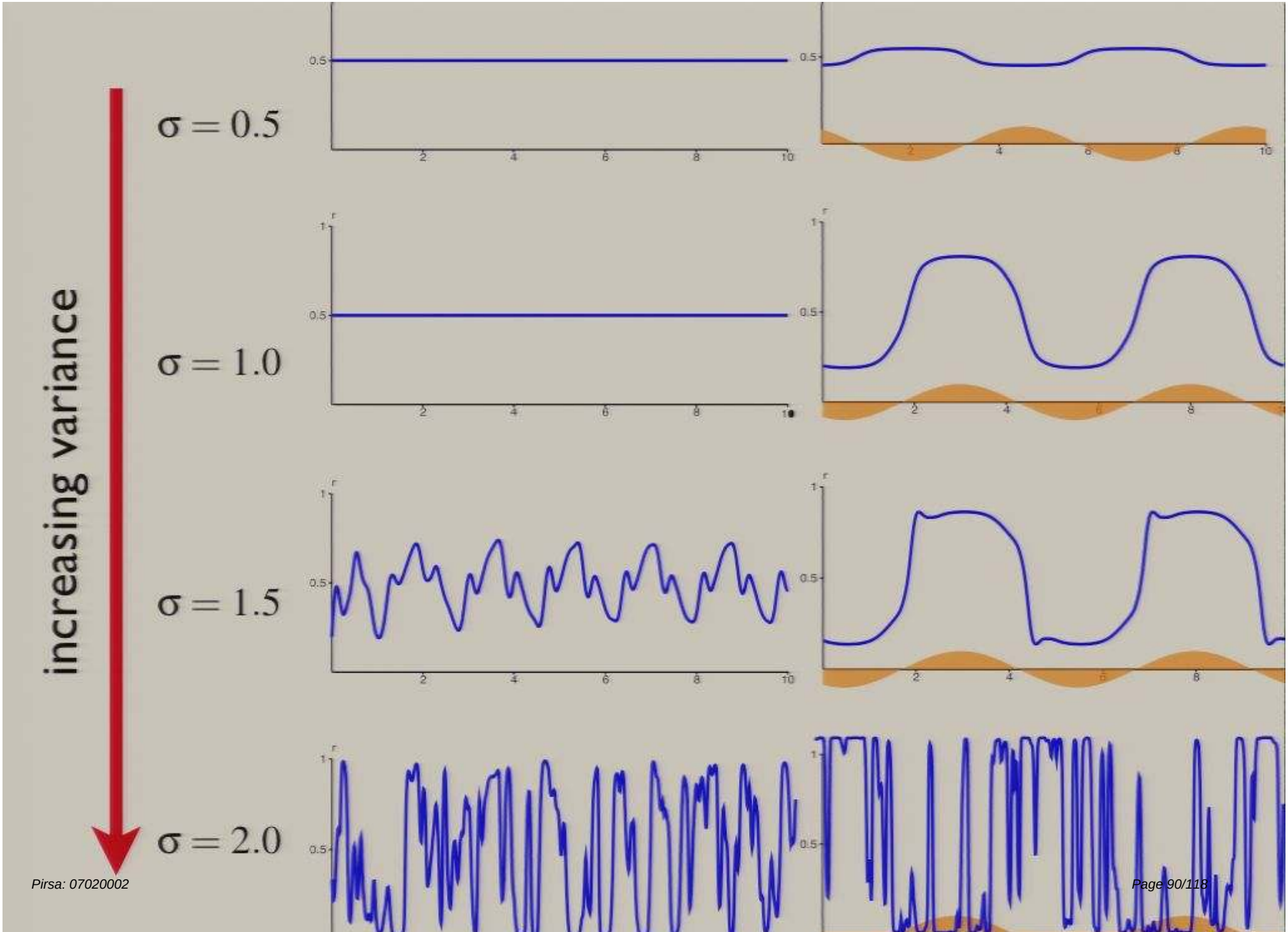
increasing variance

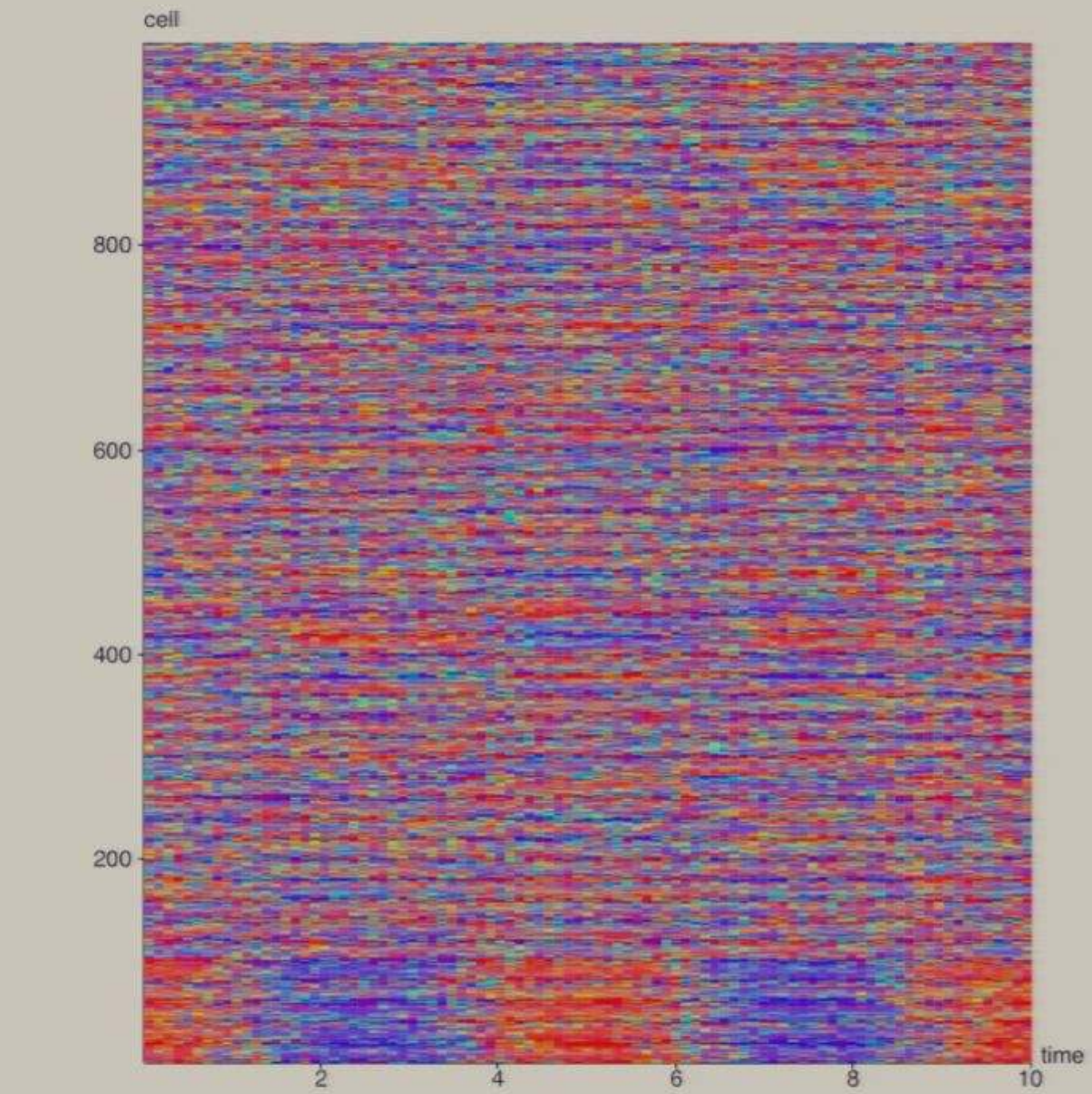

increasing variance
σ = 0.5
σ = 1.0
σ = 1.5
σ = 2.0

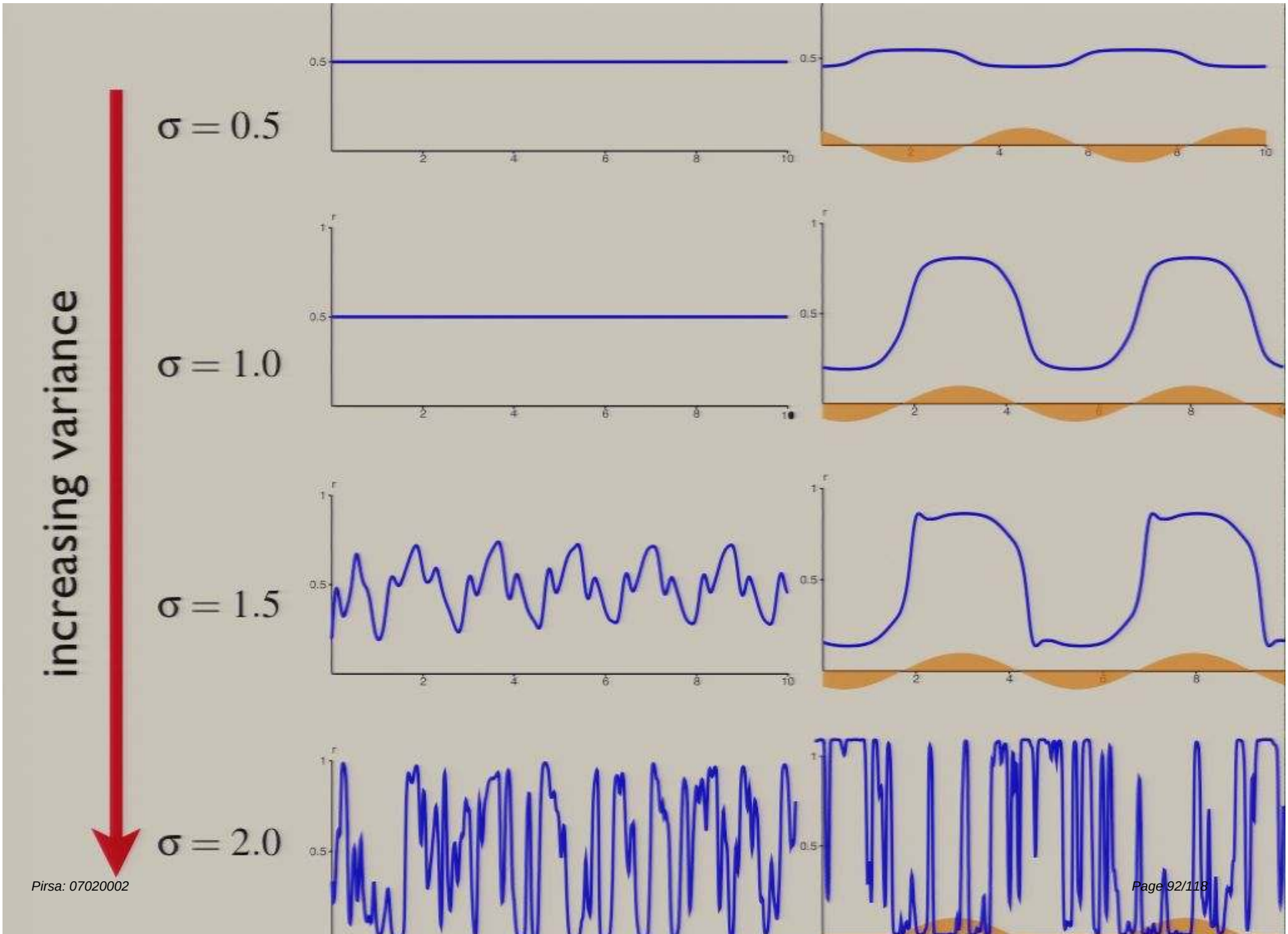

firing rate network
2
3
1
i
10% input cells
1% connectivity

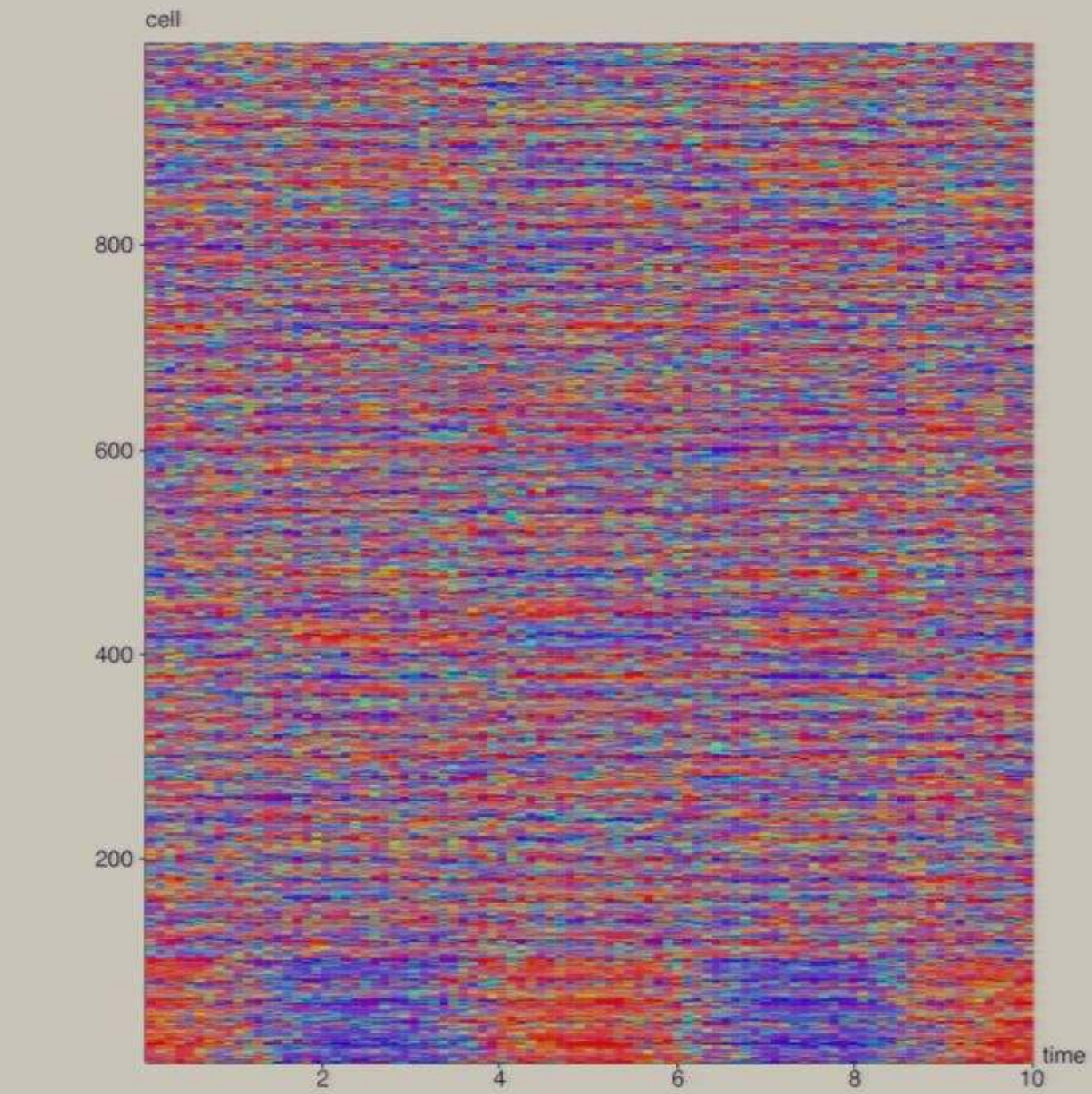

increasing variance
σ = 0.5
σ = 1.0
σ = 1.5
σ = 2.0


increasing variance
σ = 0.5
σ = 1.0
σ = 1.5
σ = 2.0


increasing variance
σ = 0.5
σ = 1.0
σ = 1.5
σ = 2.0

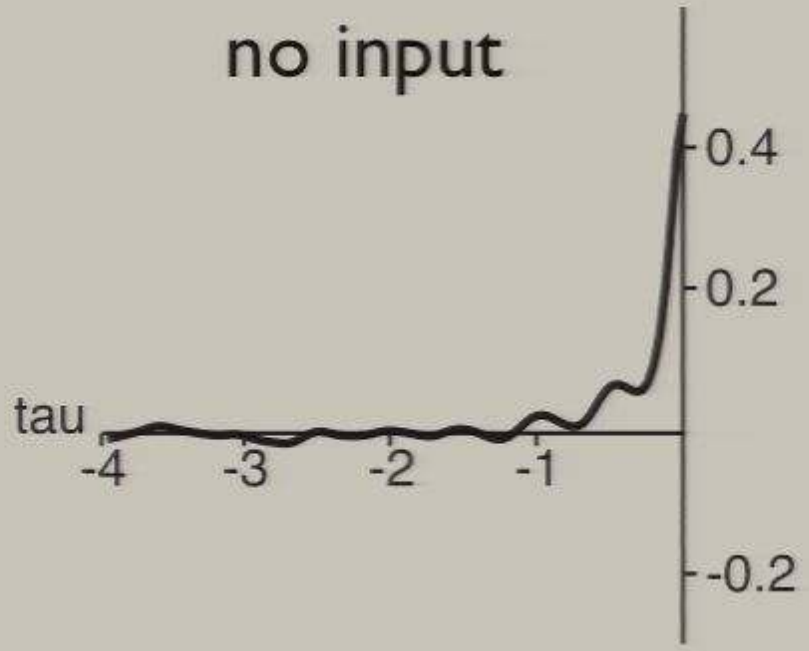

cell
800
600
400
200
2
4
6
8
10
time

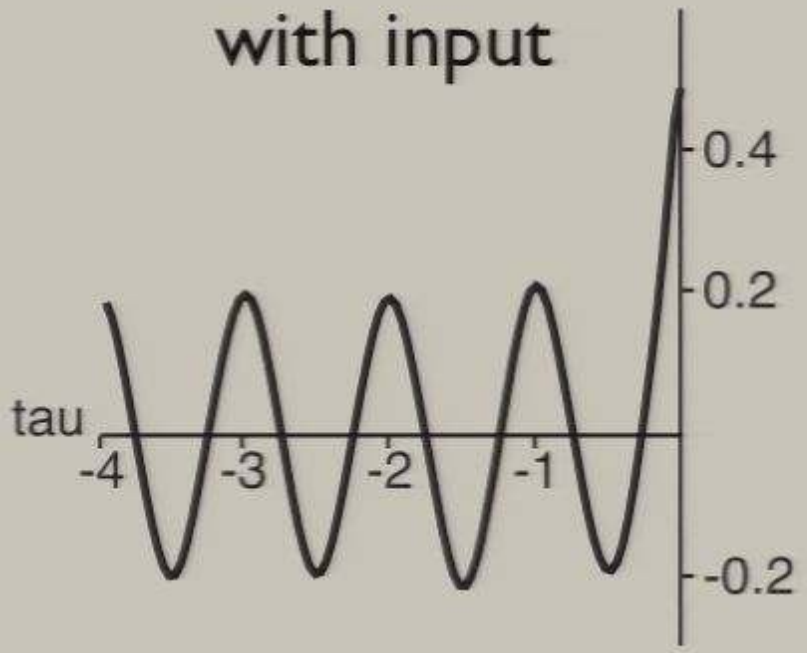

increasing variance
σ = 0.5
σ = 1.0
σ = 1.5
σ = 2.0

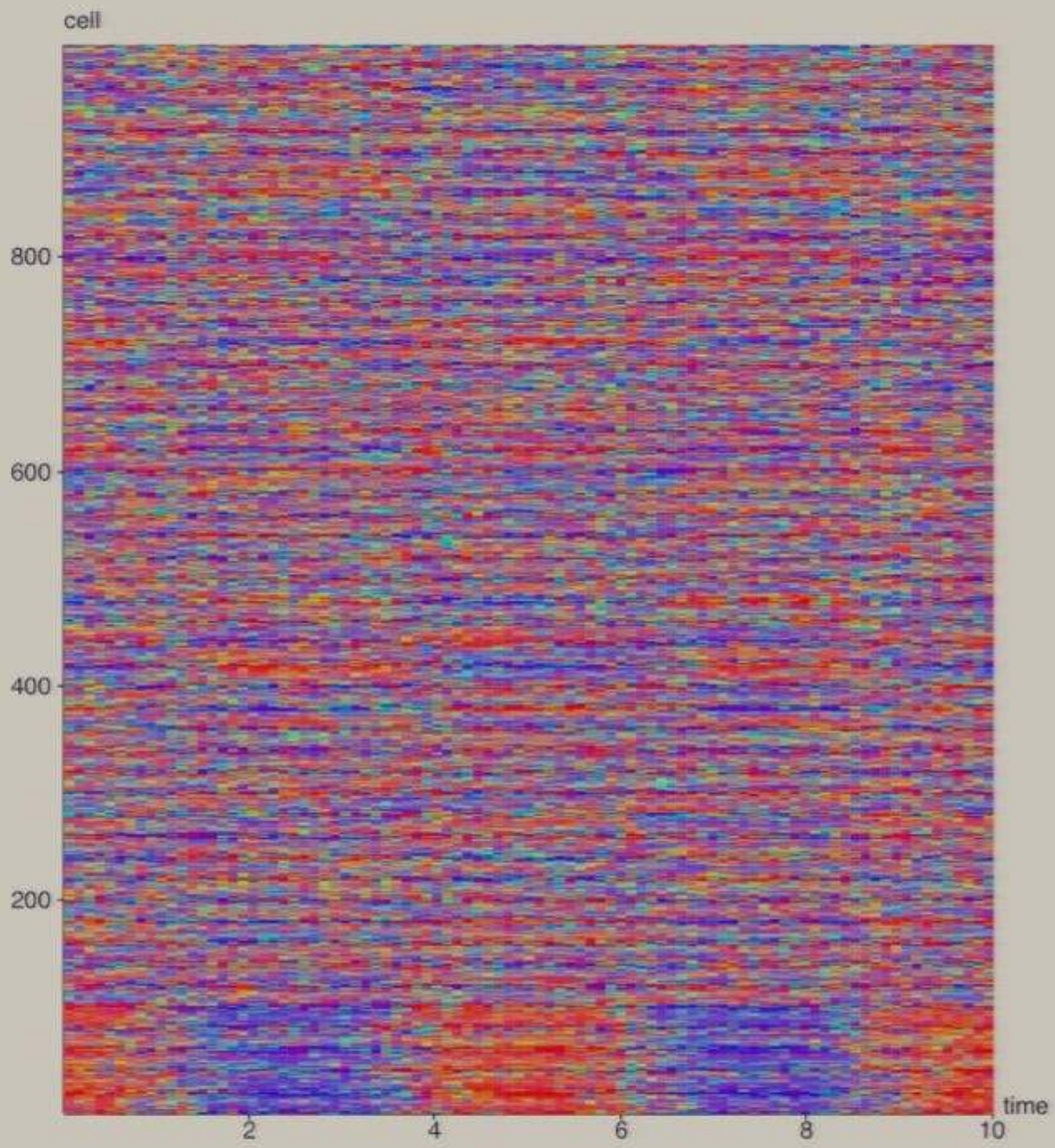

cell
800
600
400
200
2
4
6
8
10
time

correlation

$$D = \frac{1}{NT} \int_0^T dt \sum_i x_i(t) x_i(t+\tau)$$

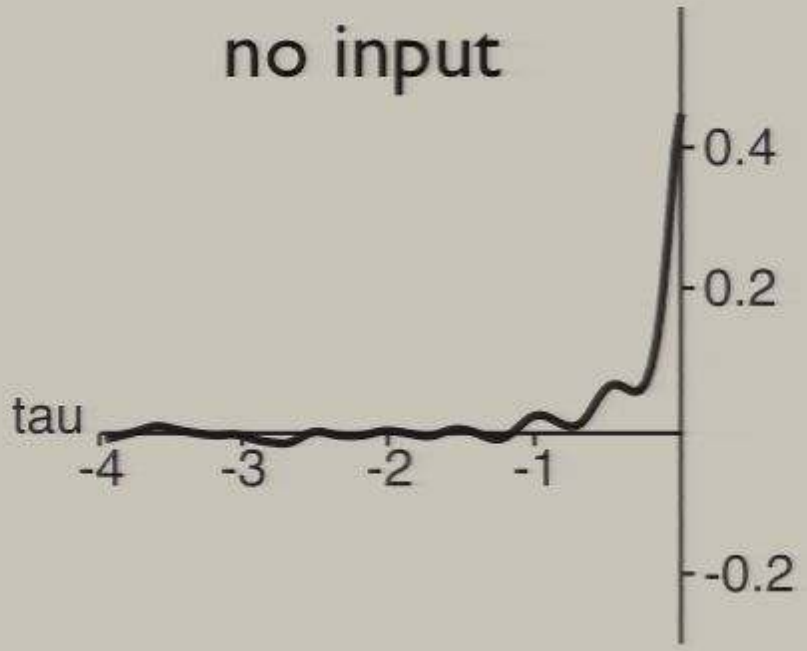


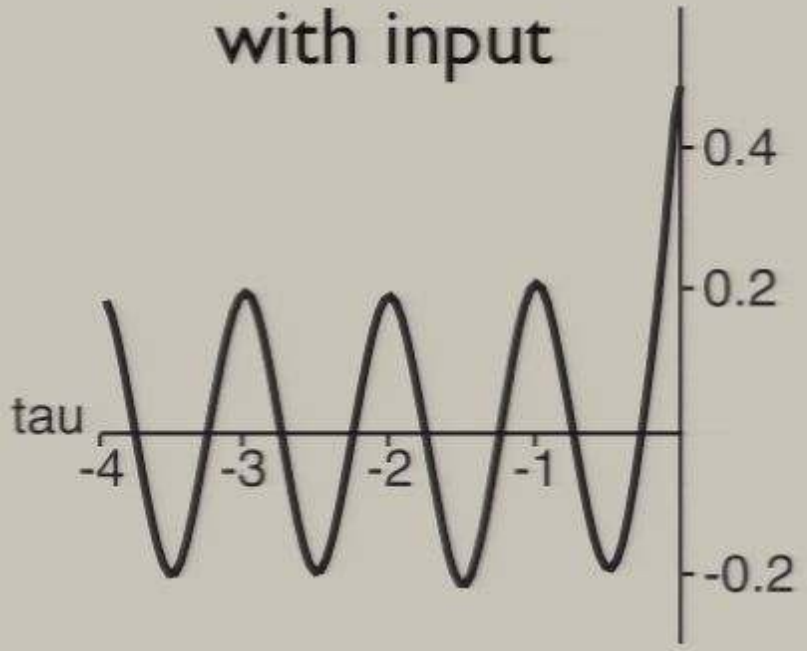


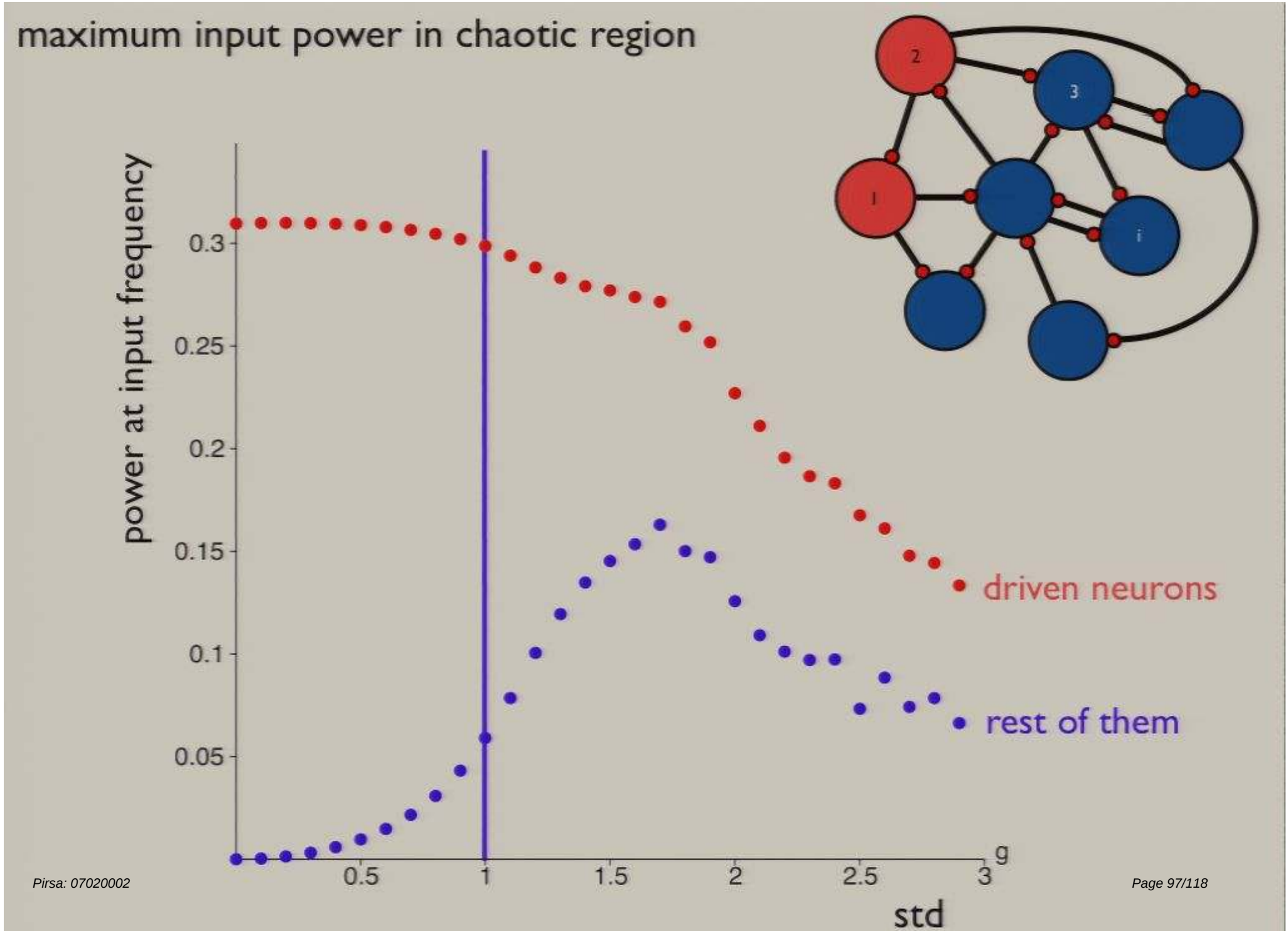

cell
800
600
400
200
2
4
6
8
10
time

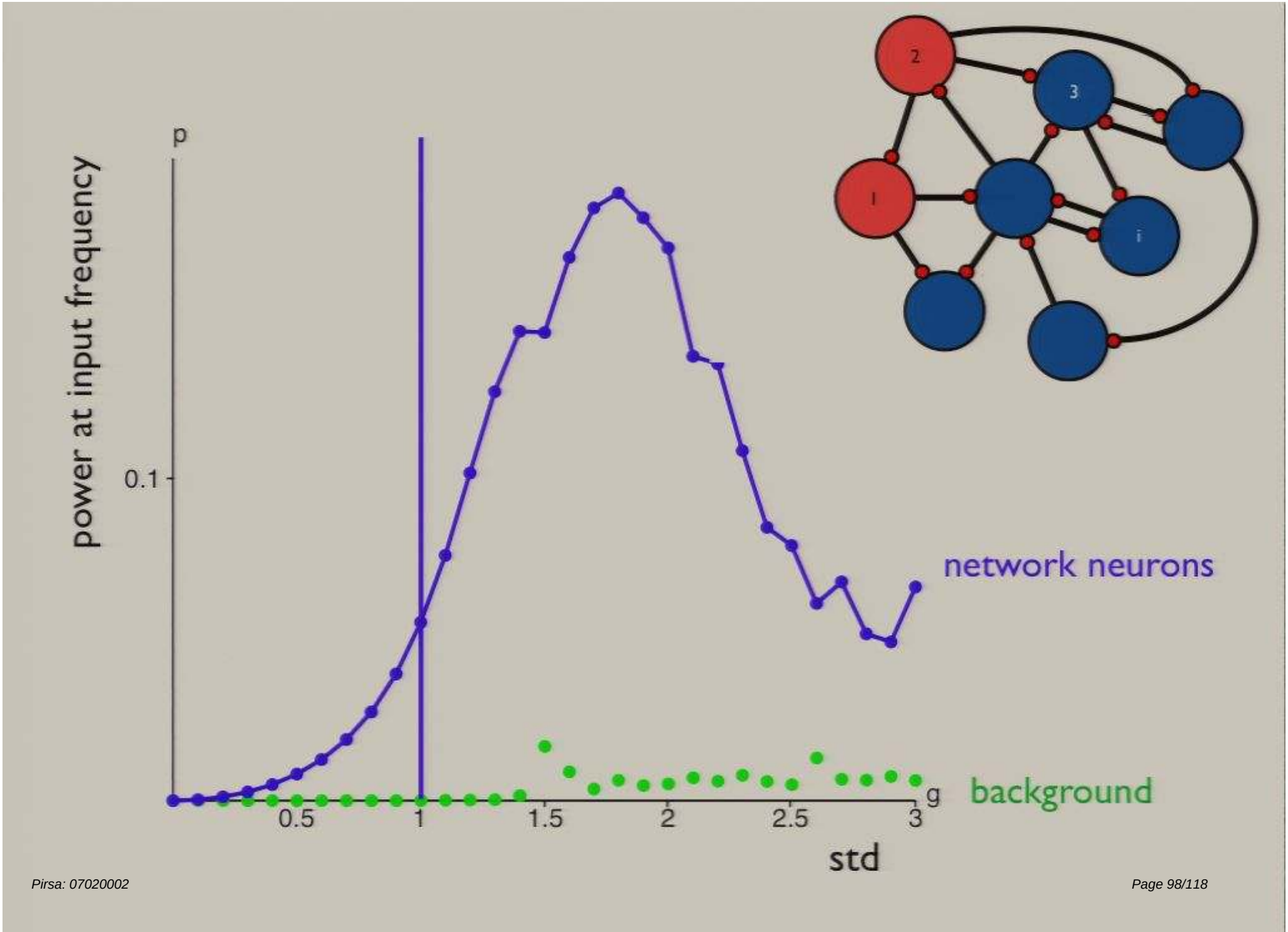
correlation

$$D = \frac{1}{NT} \int_0^T dt \sum_i x_i(t) x_i(t+\tau)$$

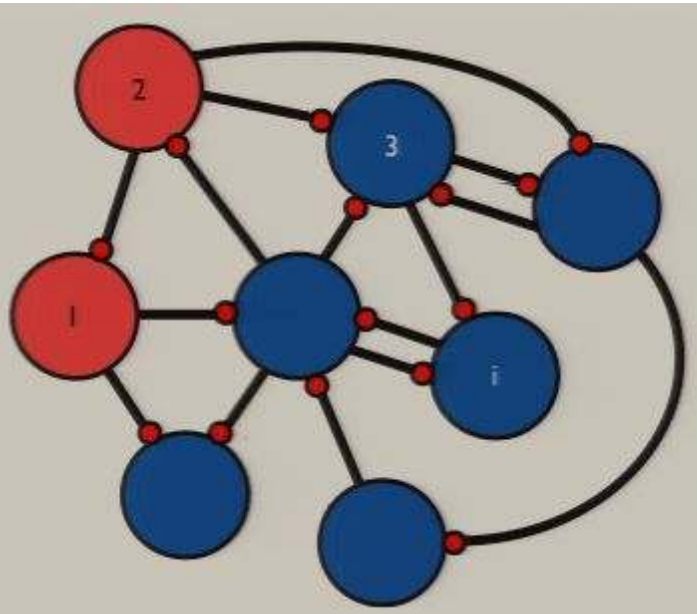





maximum input power in chaotic region
power at input frequency
0.3
0.25
0.2
0.15
0.1
0.05
0.5
1
1.5
2
2.5
3
g
std
driven neurons
rest of them
2
3
1
i


power at input frequency
p
0.1
0.5
1
1.5
2
2.5
3
g
std
network neurons
background
2
3
1
i

maximum input power in chaotic region



power at input frequency

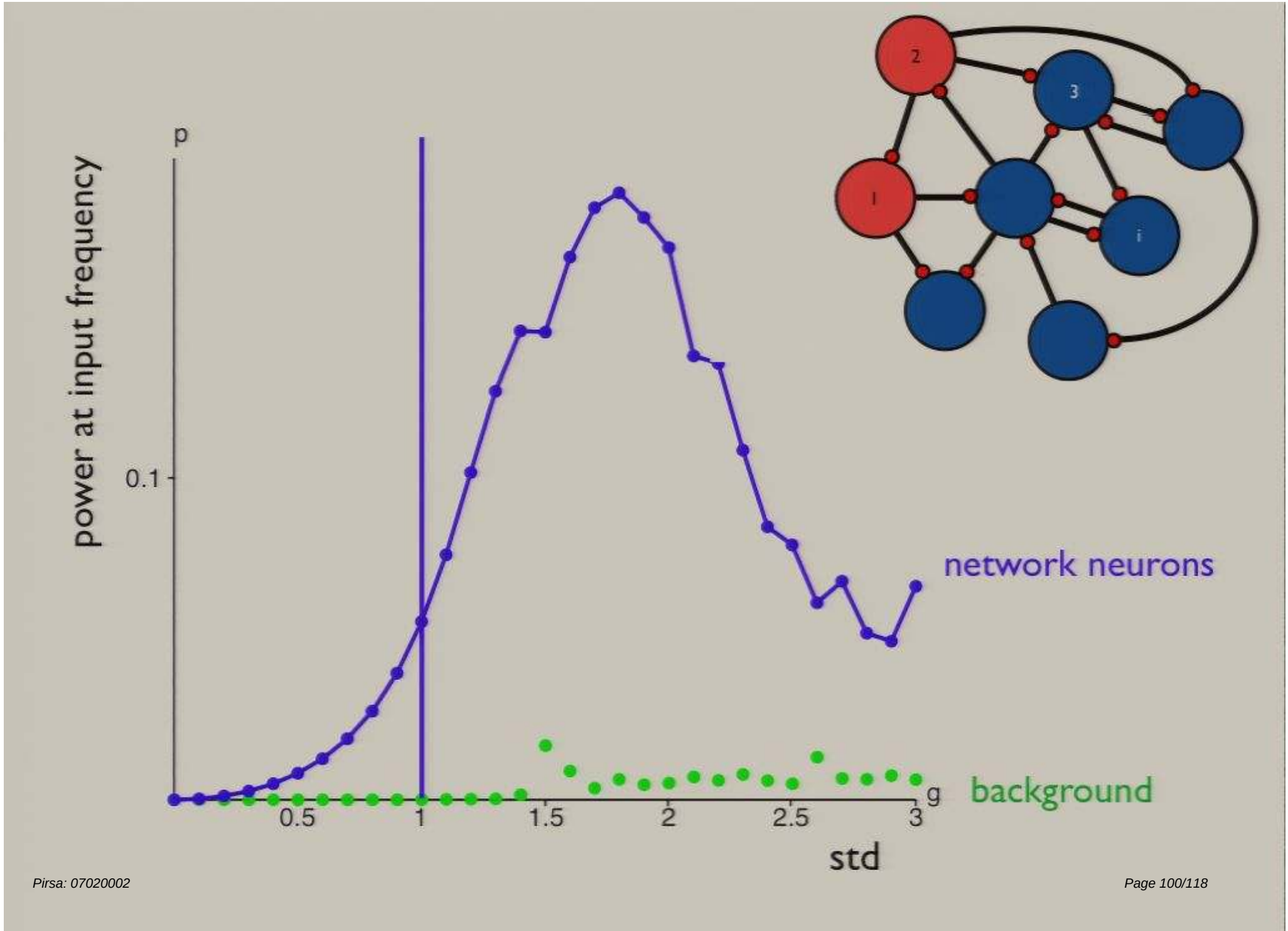
0.3
0.25
0.2
0.15
0.1
0.05

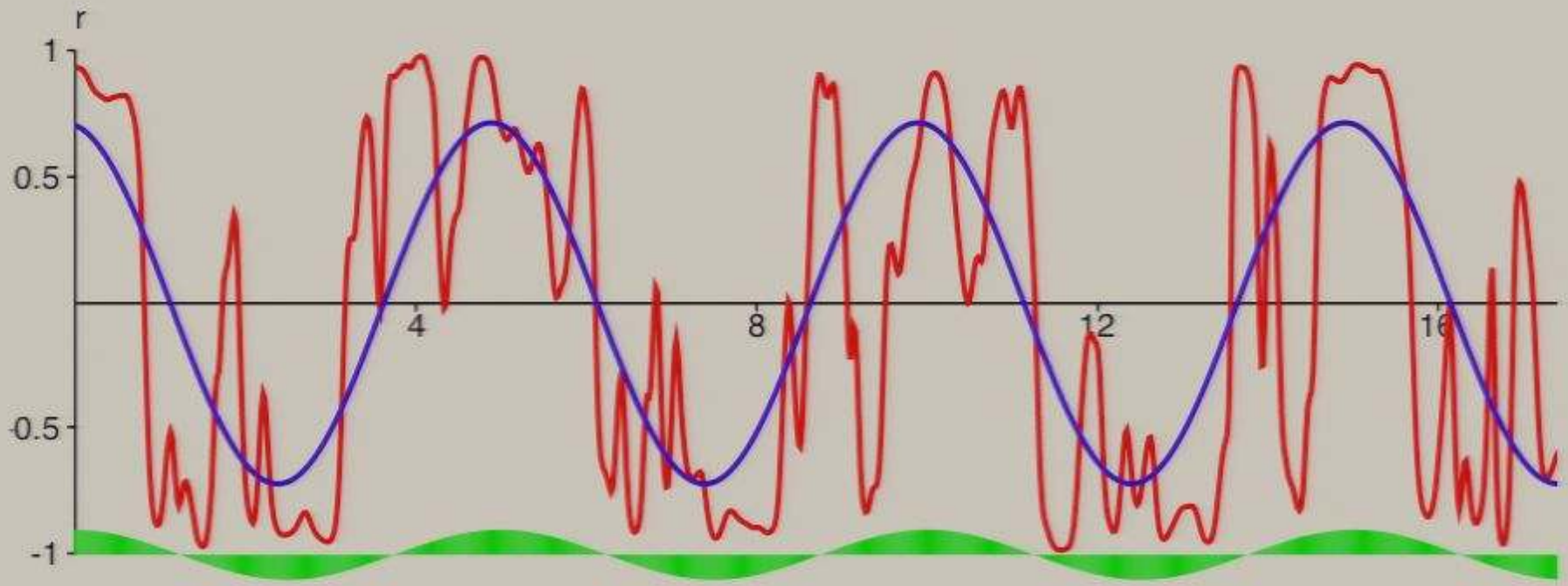
0.5
1
1.5
2
2.5
3
g

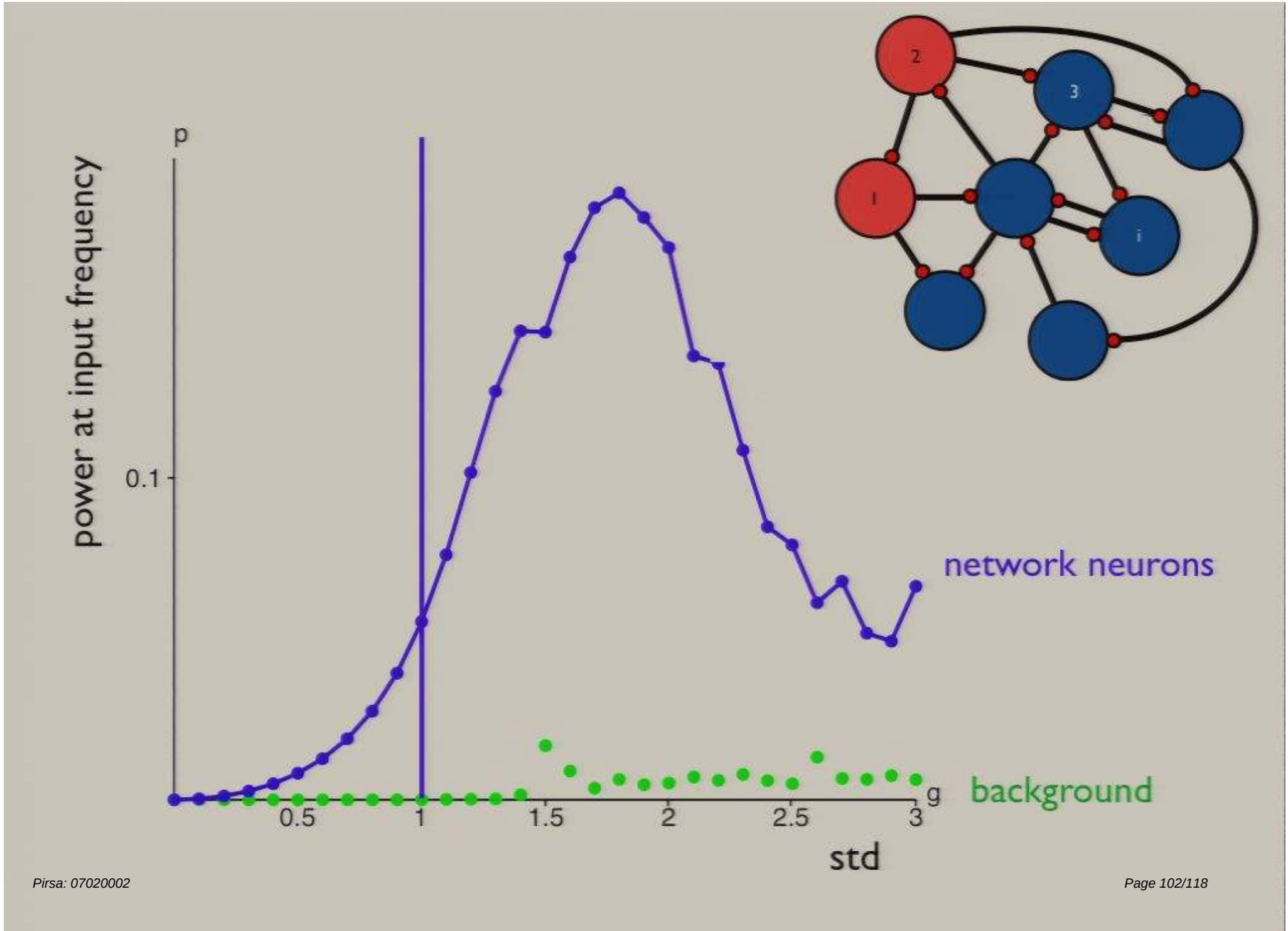
std

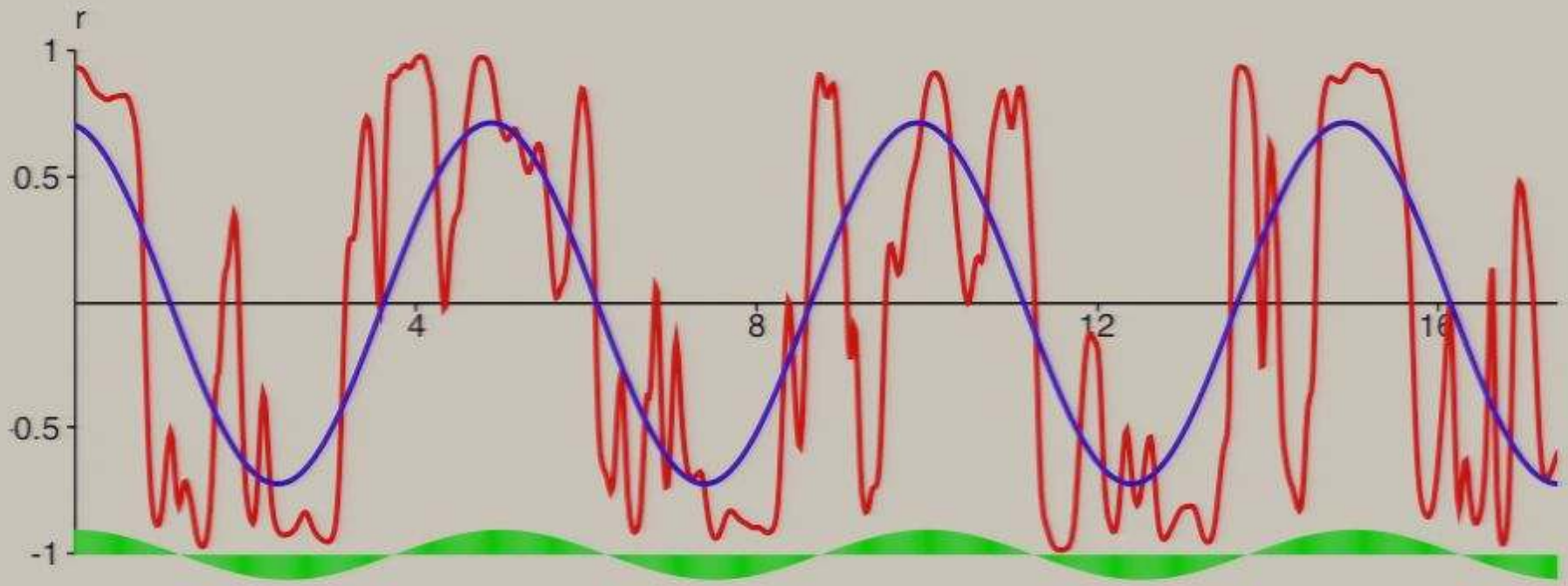
driven neurons

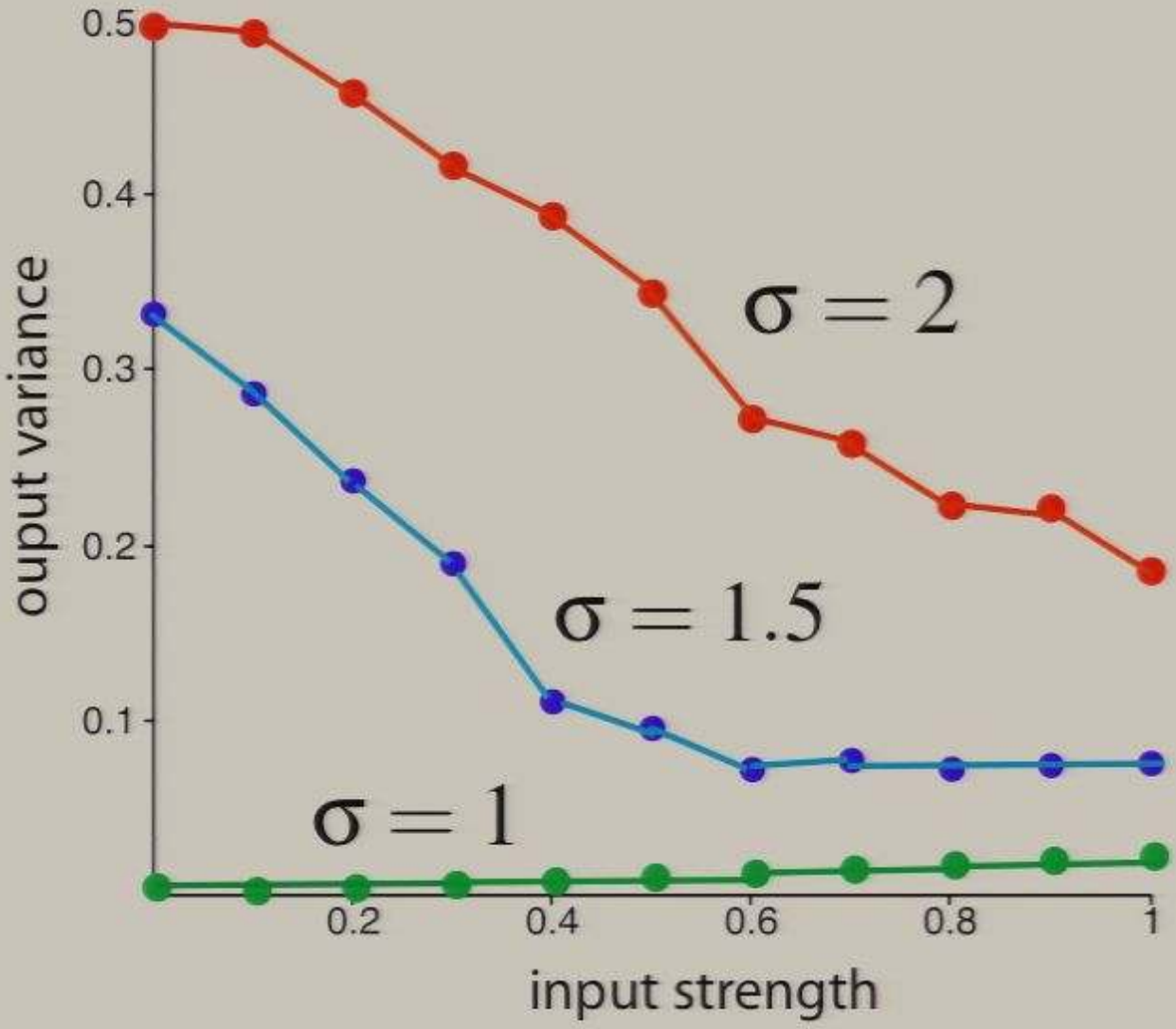
rest of them

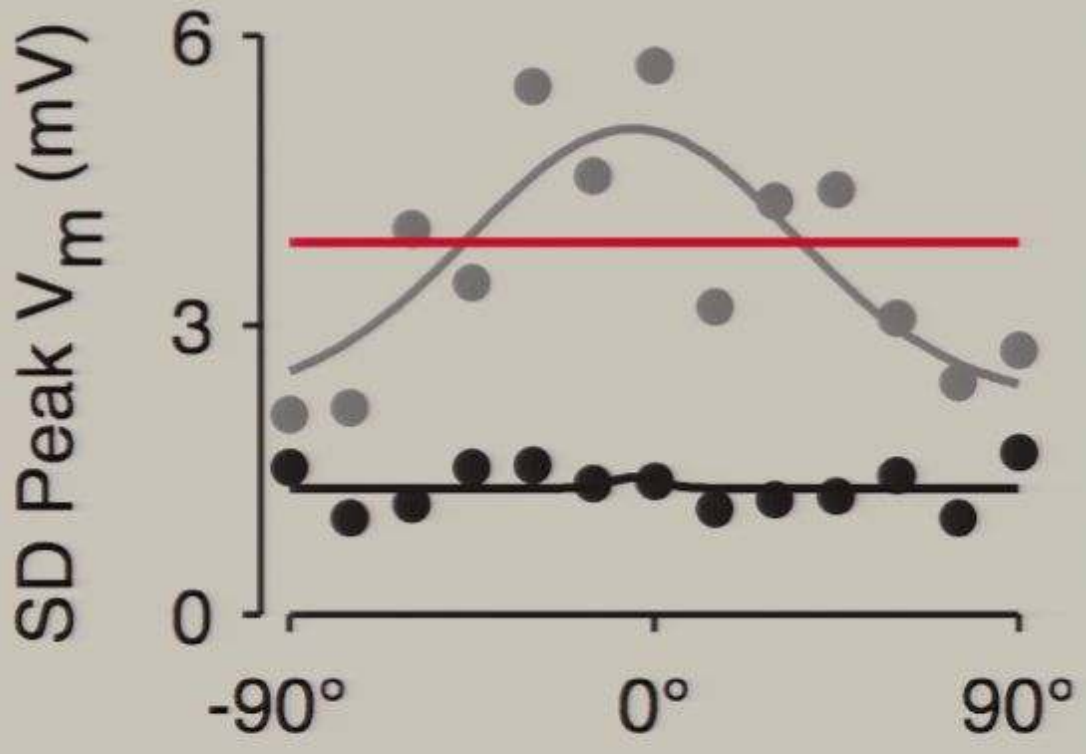

p
power at input frequency
0.1
0.5
1
1.5
2
2.5
3
g
std
network neurons
background
2
3
1
i


r
1
0.5
0.5
-1
4
8
12
16

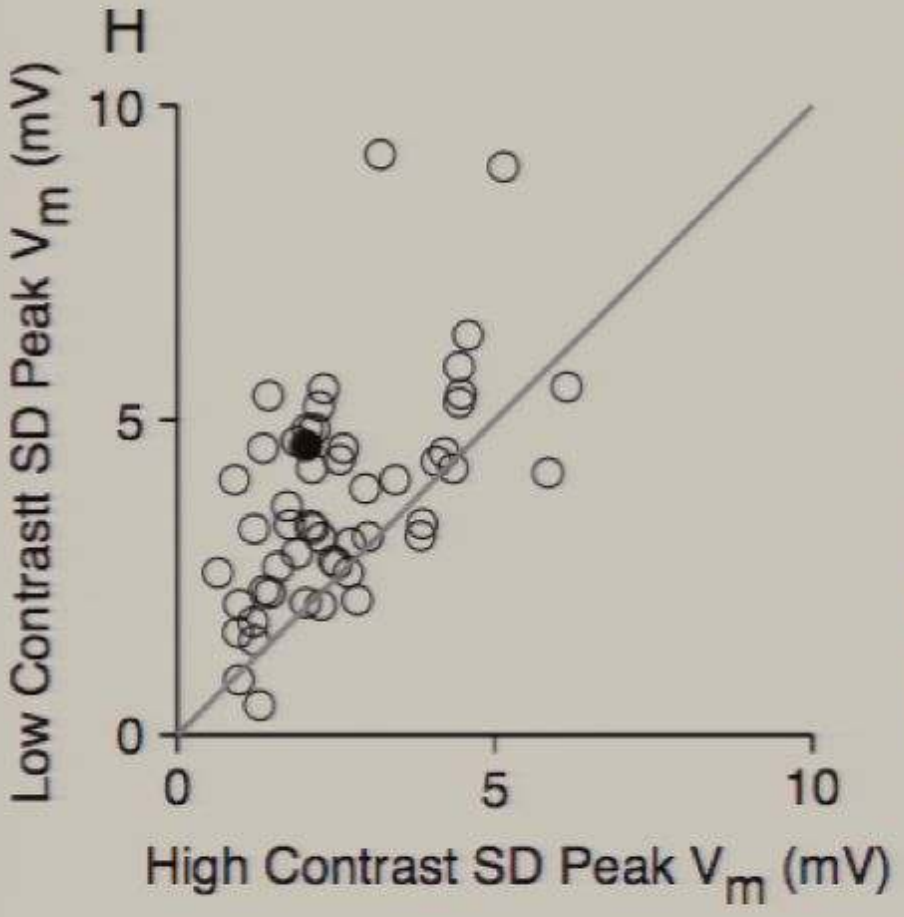

power at input frequency
p
0.1
0.5
1
1.5
2
2.5
3
g
std
network neurons
background
2
3
1
i

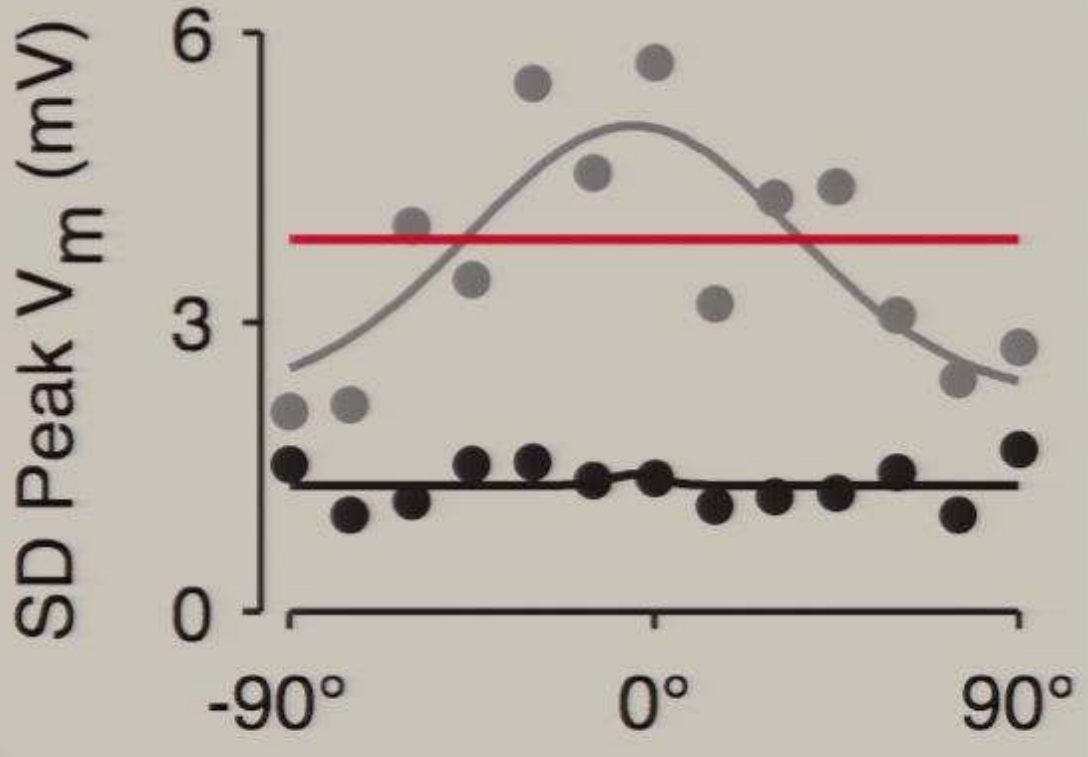

r
1
0.5
0.5
-1
4
8
12
16


0.5
0.4
0.3
0.2
0.1
ouput variance
σ = 2
σ = 1.5
σ = 1
0.2
0.4
0.6
0.8
1
input strength

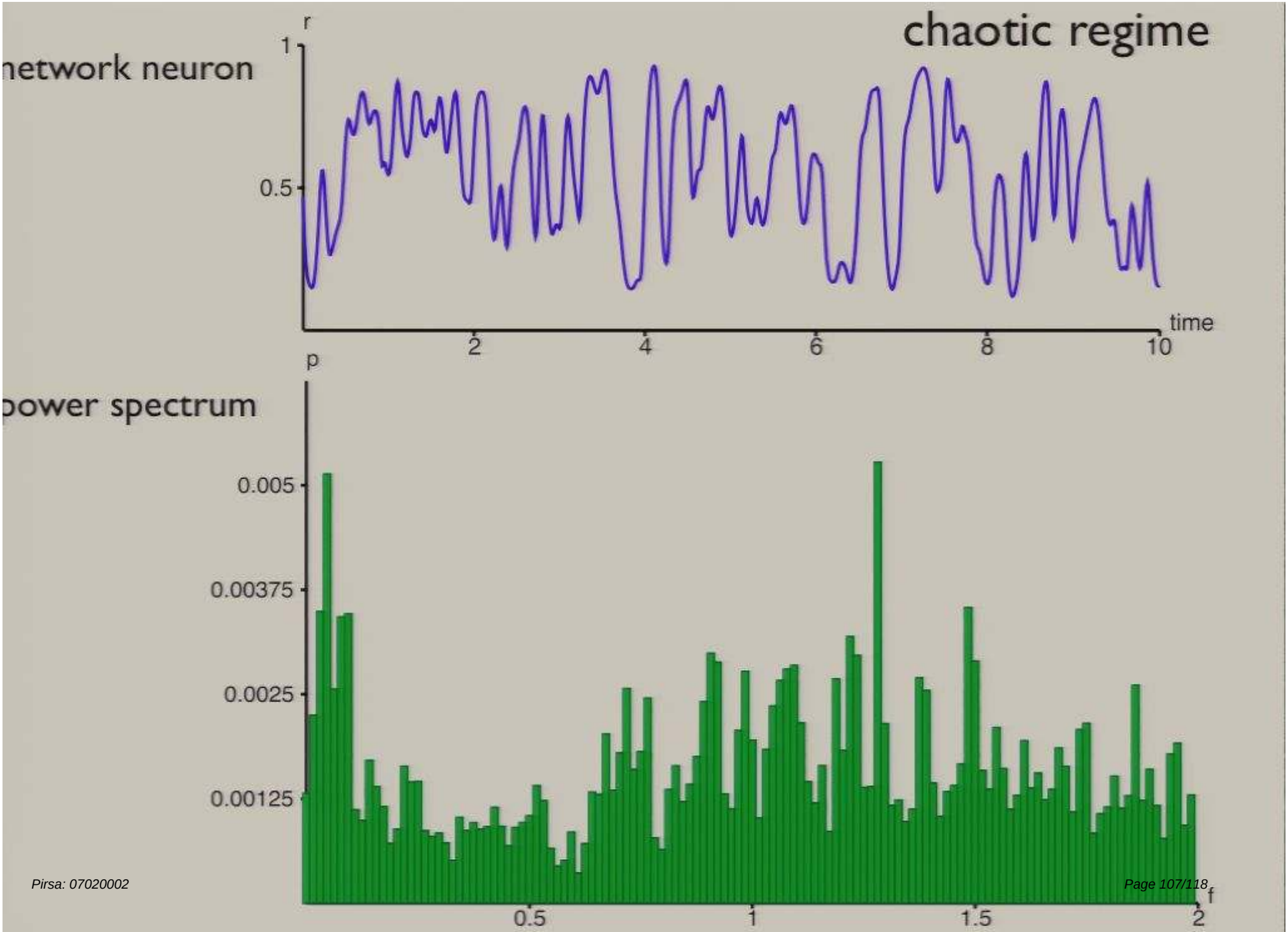


Finn, Priebe, Ferster, 2006





Finn, Priebe, Ferster, 2006


network neuron
chaotic regime
r
1
0.5
time
2
4
6
8
10
power spectrum
p
0.005
0.00375
0.0025
0.00125
0.5
1
1.5
2
f

Mean-Field Analysis

$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} r_j + I_i \longrightarrow \tau \frac{dx}{dt} = -x + \eta + I$$

H Sompolinsky and K Rajan

Mean-Field Analysis

$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} r_j + I_i \longrightarrow \tau \frac{dx}{dt} = -x + \eta + I$$

$$\langle \eta(t)\eta(t') \rangle_\eta = \left\langle \sum_j J_{ij} r_j(t) \sum_k J_{ik} r_k(t') \right\rangle_J$$

H Sompolinsky and K Rajan

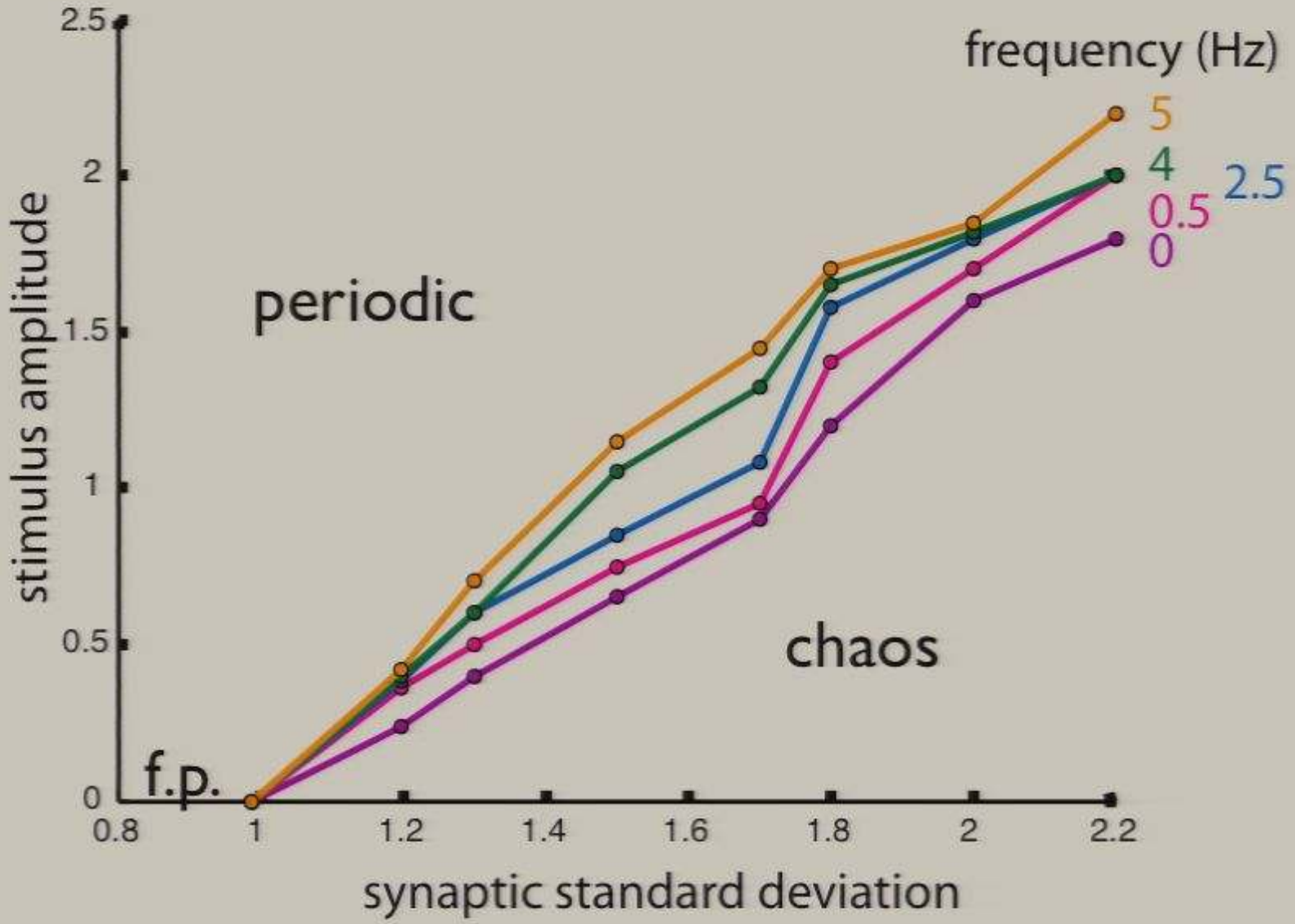
Mean-Field Analysis

$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} r_j + I_i \longrightarrow \tau \frac{dx}{dt} = -x + \eta + I$$

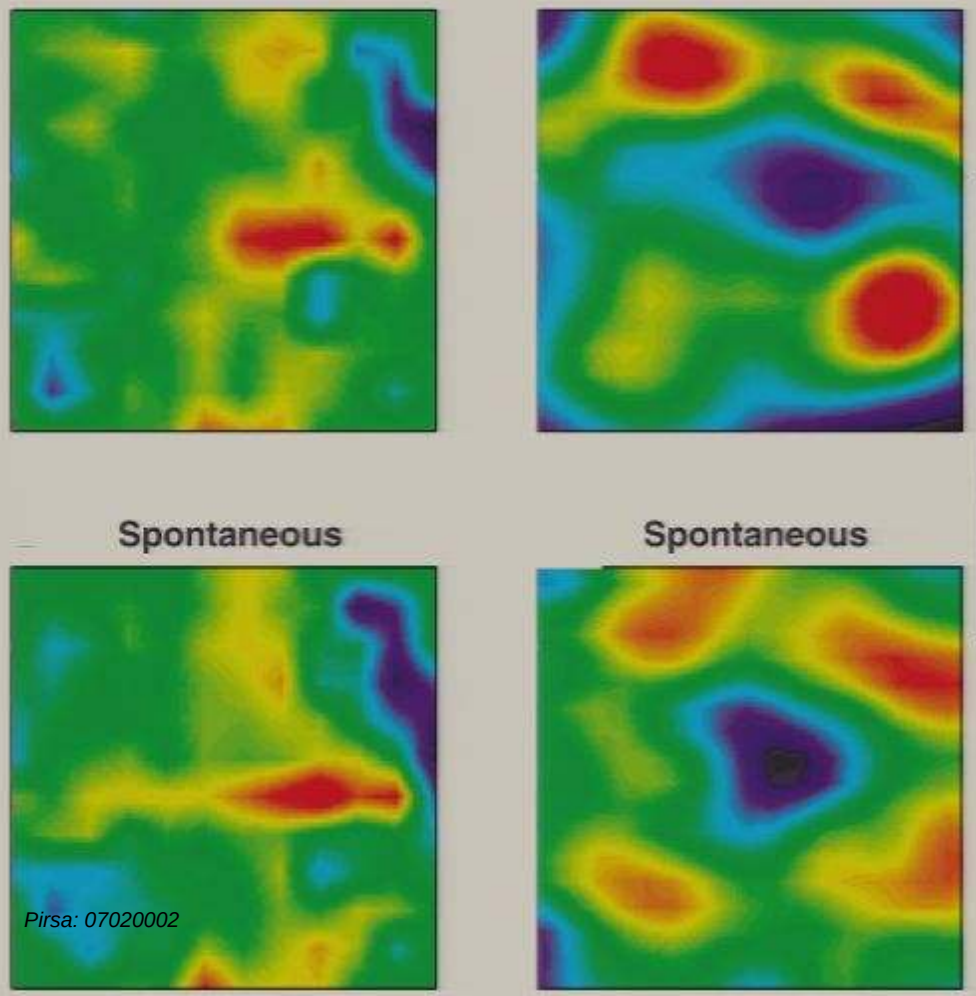
$$\langle \eta(t)\eta(t') \rangle_\eta = \left\langle \sum_j J_{ij} r_j(t) \sum_k J_{ik} r_k(t') \right\rangle_J$$

$$\frac{1}{N} \left\langle \sum_i x_i(t) x_i(t') \right\rangle_J \longrightarrow \frac{1}{N} \langle x(t) x(t') \rangle_\eta$$

H Sompolinsky and K Rajan


frequency (Hz)
5
4
2.5
0.5
0
periodic
chaos
f.p.
stimulus amplitude
synaptic standard deviation
2.5
2
1.5
1
0.5
0
0.8
1
1.2
1.4
1.6
1.8
2
2.2

spontaneous activity ⟷ evoked activity

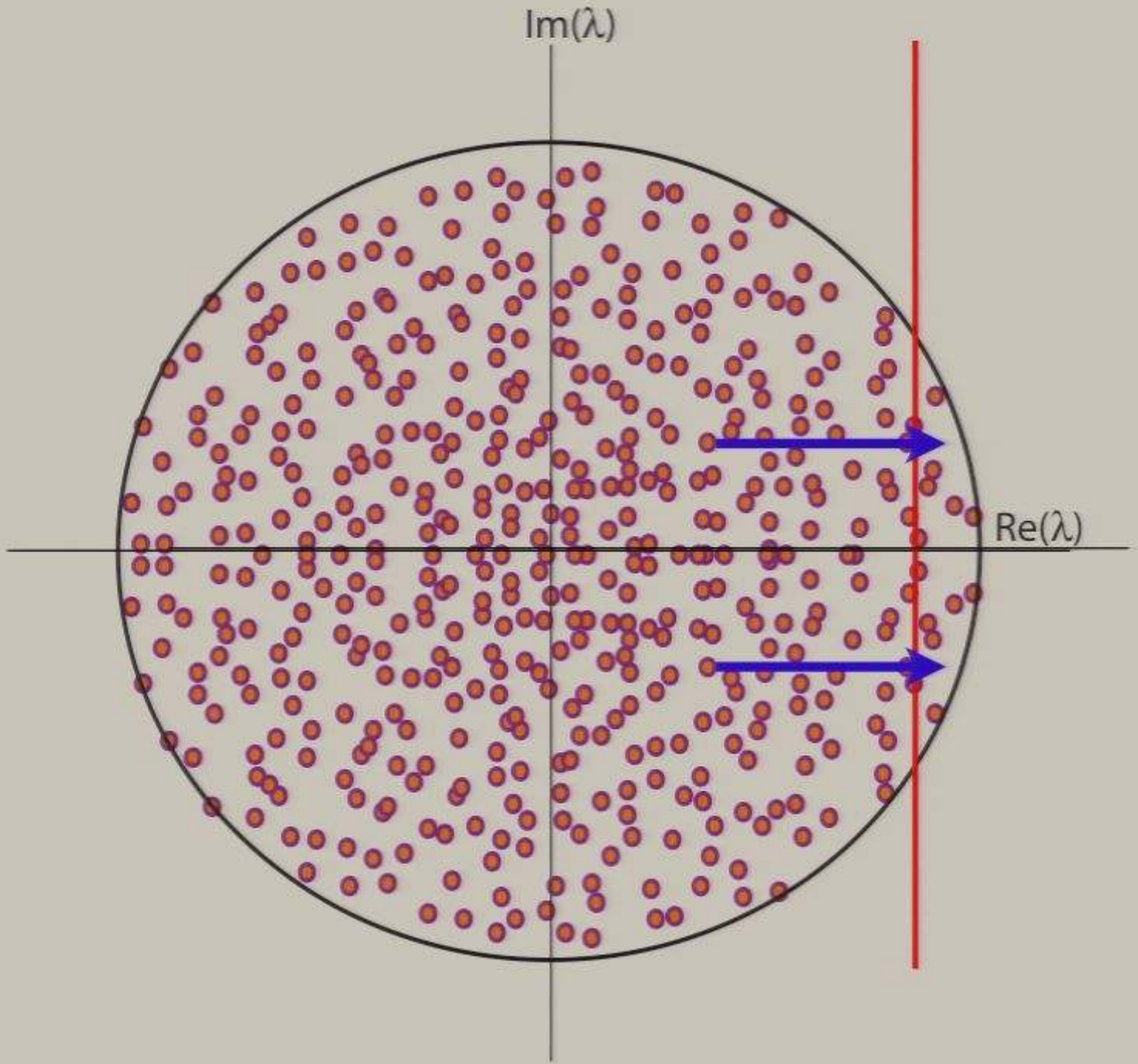


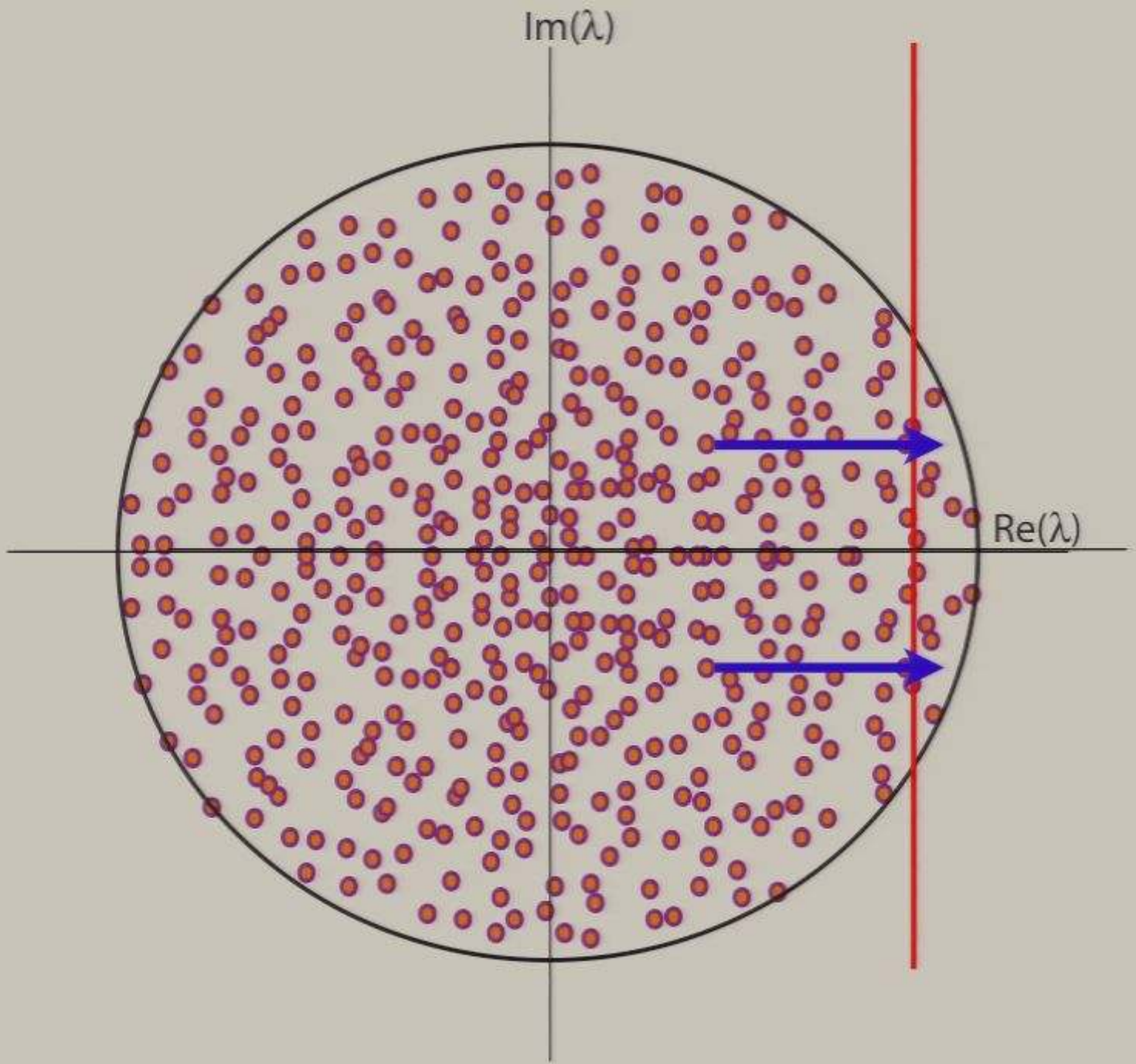
"Noisy" Spontaneous Activity

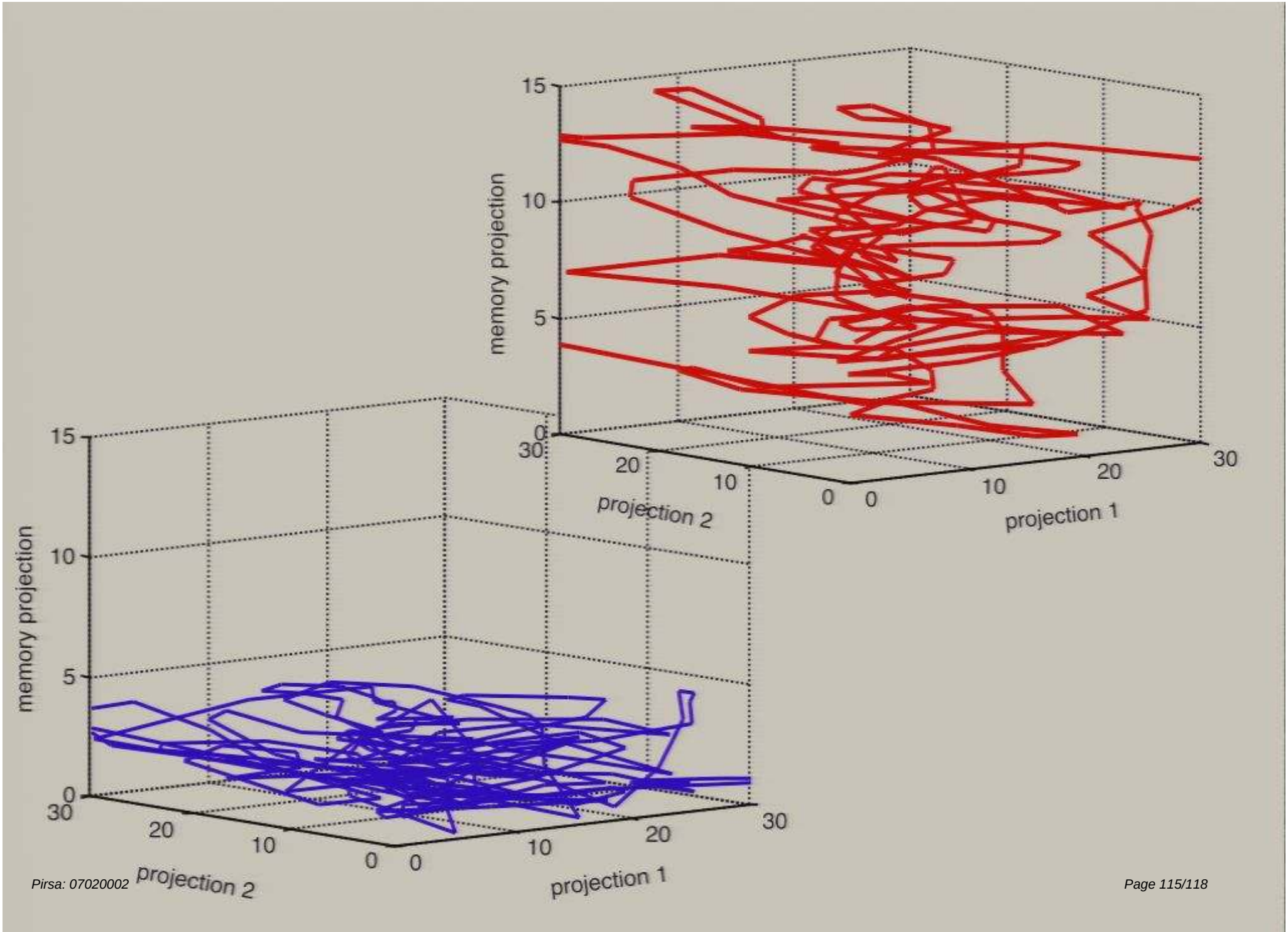
1) The price of strong connectivity

2) Variance decreases with signal

3) Function?


Im(λ)
Re(λ)


Im(λ)
Re(λ)


memory projection
15
10
5
0
30
20
10
0
projection 2
0
10
20
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projection 1
memory projection
15
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projection 2
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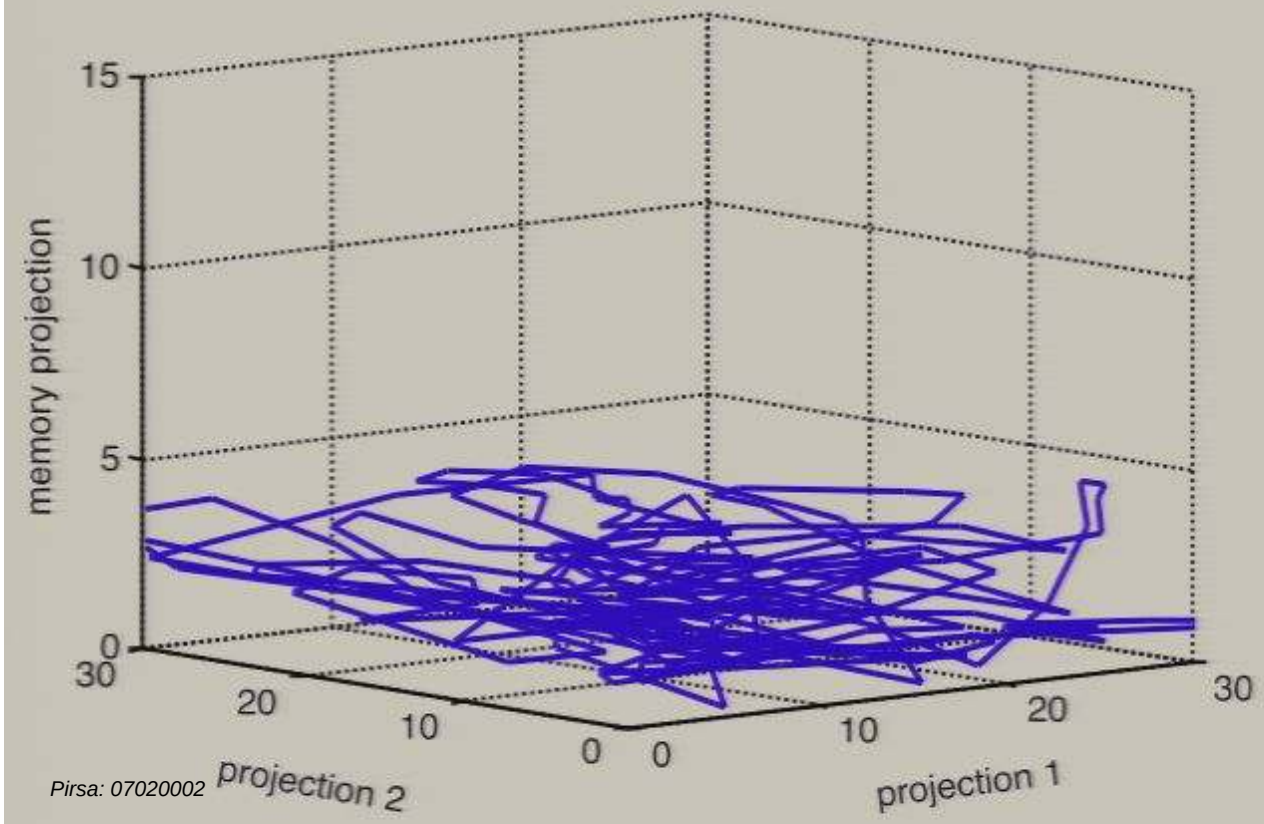
20
30
projection 1


15
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memory projection
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projection 2
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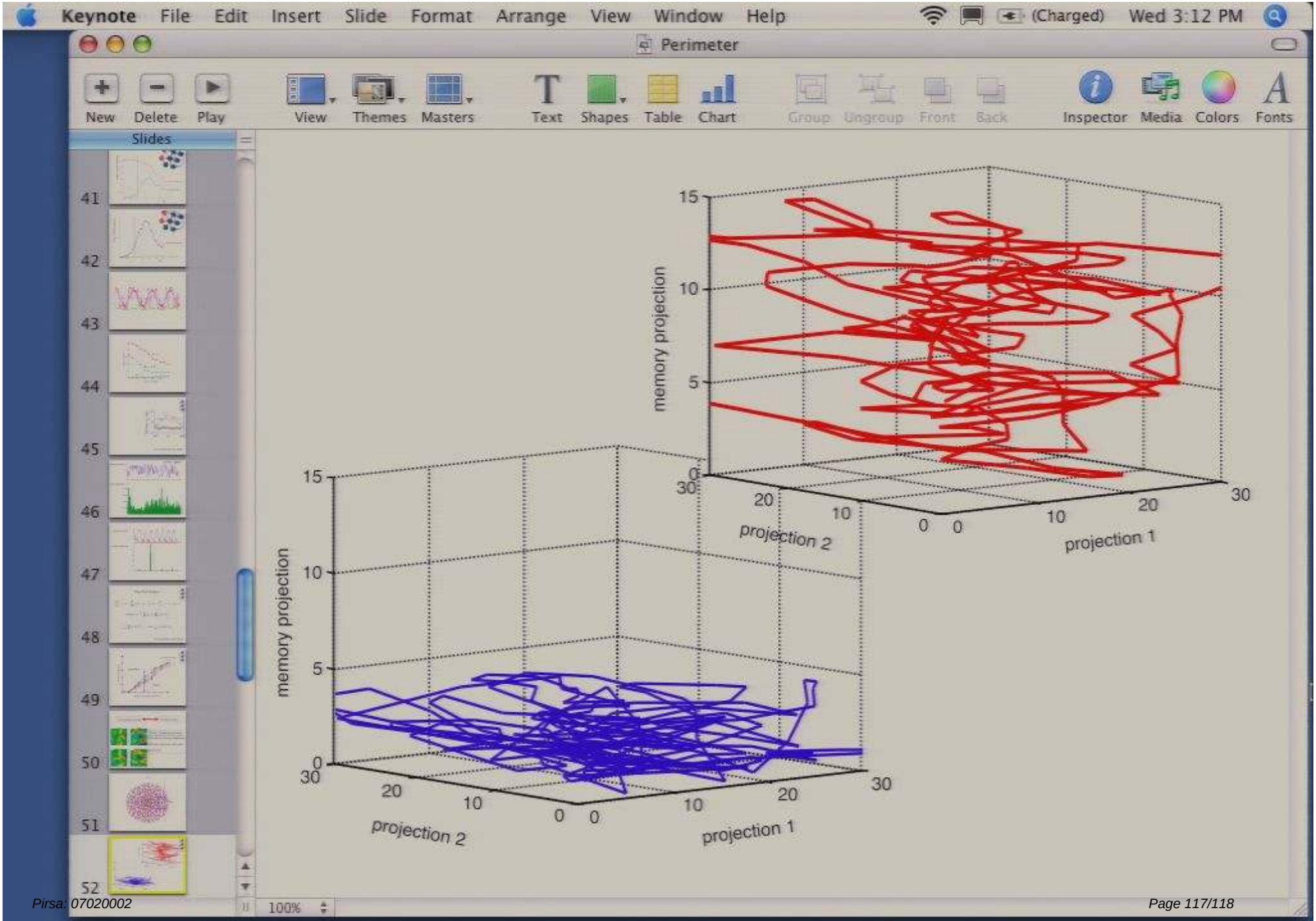
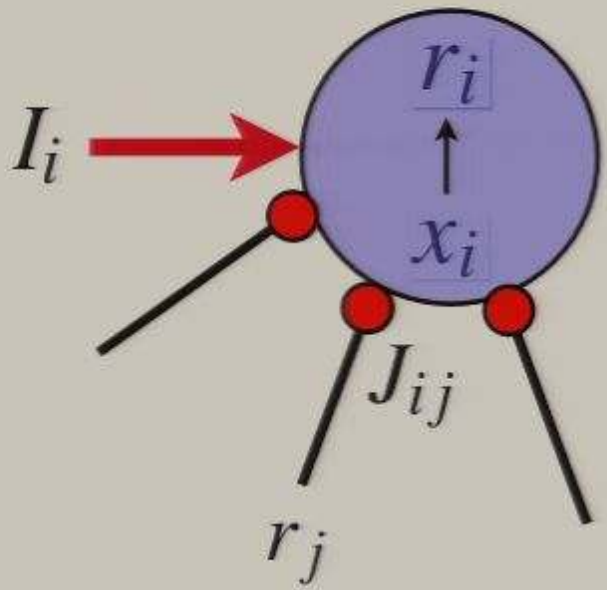
projection 1


Keynote File Edit Insert Slide Format Arrange View Window Help
(Charged) Wed 3:12 PM
Perimeter
New Delete Play View Themes Masters Text Shapes Table Chart Group Ungroup Front Back Inspector Media Colors Fonts
Slides
41 42 43 44 45 46 47 48 49 50 51 52
memory projection
projection 1
projection 2
100%

synaptic weights

$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} r_j + I_i$$

"stimulus" external input

"firing rate"

$$r_j = f(x_j)$$